

6.0 Extension of Four and Five-Unit Analyses to Six-Unit Configurations

The analyses of the four-unit and five-unit configurations can be extended to configurations of six units. The results for some of the six-unit configurations are identical to those obtained for configurations of four and five units. These configurations are summarized in Section 6.1. However, there are two groups of six-unit configurations which require individual analysis.

The first group of six-unit configurations which require individual analysis is the 1-2-2-1 configurations. In the 1-2-2-1 configurations, one unit transmits an alarm message to two units, which in turn transmit the message to two more units. These two units then relay the message to a sixth unit. The 1-2-2-1 configurations along with their analytical results are summarized in Table 6.1. Derivation of the analytical results follows Table 6.1 in Sections 6.2.1 - 6.2.4.

The second set of six-unit configurations requiring individual analysis are the 1-4-1 configurations. In these configurations, one unit transmits to four *middle* units. The middle units then relay the message to a sixth unit. The 1-4-1 configurations are summarized in Table 6.2 with their derivations following in Sections 6.3.1-6.3.10.

6.1 Six-Unit Configurations Having Results Identical to the Four and Five-Unit Configurations

The following table summarizes those six-unit configurations having analytical results which are identical to those for the four-unit and five-unit configurations. In the configuration diagrams, a solid arrow between two units indicates that the units are within range of one another, while a dotted line indicates that units may or may not be within range of each other depending upon the distance and the amount of attenuation between them. The results shown in the table assume that the alarm originates from unit one and then propagates throughout the system.

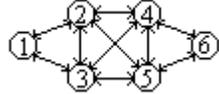
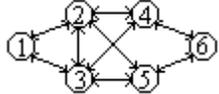
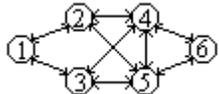
Table 6.1: Six-Unit Configurations Having Analytical Results Identical to Those of the Four-Unit and Five-Unit Configurations

Six-Unit Configuration	Description	Pr{Link Failure(s)}
	Identical to four-unit intermediate or worst-case configuration with one, two or three failing links depending upon the topology and which units are in range of one another	0.0622 intermediate, 0.6043 worst
	Identical to four-unit intermediate or worst-case configuration with two or three possible failing links depending upon the topology and which units are in range of one another	0.0622 intermediate, 0.6043 worst
	Identical to four-unit worst-case or intermediate configurations depending upon which units are in range of one another.	0.0622 intermediate, 0.6043 worst
	Identical to five-unit configurations in Table 5.2	See Table 5.2 for possible values.
	Identical to five-unit configurations in Table 5.2. Now there are two possible failing links.	See Table 5.2 for possible values.

6.2 Analysis of 1-2-2-1 Six-Unit Configurations

The 1-2-2-1 six-unit configurations are summarized in Table 6.2. Note that the analysis for these configurations assumes that the alarm message originates from unit one and then propagates throughout the system.

Table 6.2: 1-2-2-1 Six-Unit Configurations

Six-Unit Configuration	Description	Pr{Link Failure to Unit 6}
	All four middle units can hear one another	0.1211
	First pair of middle units can hear one another	0.6290
	Second pair of middle units can hear one another	0.4049
	Neither pair of middle units can hear one another	0.7488

6.2.1 Analysis of 1-2-2-1 Six-Unit Configuration, All Middle Units in Range of Each Other

In the six-unit configuration shown in row one of Table 6.2, the two pairs of middle units are in range of one another. Unit six will not receive an alarm message if either of the following two conditions occurs:

- (1) The holdoff timers in units two and three are synchronized with one another.
- (2) The holdoff timers in units four and five are synchronized, but the holdoff timers in units two and three are not.

The probability that unit six does not receive an alarm message in this configuration is given by equation (6.1).

$$\begin{aligned}
 &Pr\{\text{Unit six does not receive message}\} \\
 &= Pr\{\text{Holdoff timers in units two and three are synchronized}\} + \\
 &\quad Pr\{\text{Holdoff timers in units four and five are synchronized}\} \times \\
 &\quad Pr\{\text{Holdoff timers in units two and three are not synchronized}\} \\
 &= (0.0625) + (0.0625)(1-0.0625) \\
 &= 0.1211 \tag{6.1}
 \end{aligned}$$

6.2.2 Analysis of 1-2-2-1 Six-Unit Configuration, First Pair of Middle Units in Range of Each Other

In the six-unit configuration shown in row two of Table 6.2, the second pair of middle units is no longer within range of one another. Unit six will not receive an alarm message if either of the following two conditions occurs:

- (1) The holdoff timers in units two and three are synchronized with one another.
- (2) The holdoff timers in units two and three are *not* synchronized and none of the messages transmitted by units four and five are successfully received by unit six because units four and five cannot hear one another's transmissions.

The probability that unit six does not receive an alarm message in this configuration is represented by equation (6.2).

$$\begin{aligned}
 &Pr\{\text{Unit six does not receive message}\} \\
 &= Pr\{\text{Holdoff timers in units two and three are synchronized}\} + \\
 &\quad Pr\{\text{Holdoff timers in units two and three are not synchronized}\} \times \\
 &\quad Pr\{\text{None of the messages generated by units four and five are received by unit six}\} \\
 &= (0.0625) + (1-0.0625)(0.6043) \\
 &= 0.6290 \tag{6.2}
 \end{aligned}$$

6.2.3 Analysis of 1-2-2-1 Six-Unit Configuration, Second Pair of Middle Units in Range of Each Other

In the six-unit configuration shown in row three of Table 6.2, the first pair of middle units is no longer within range of one another. Unit six will not receive an alarm message if either of the following two conditions occurs:

- (1) Messages generated by units two and three collide with respect to the receive intervals of both units four and five since units two and three cannot hear one another's transmissions.
- (2) The transmissions generated by either units two or three are successfully received by both units four and five, but the holdoff timers in units four and five are synchronized.

The probability that unit six does not receive an alarm message in this configuration is given by equation (6.3).

$$\begin{aligned}
 &Pr\{\text{Unit six does not receive message}\} \\
 &= Pr\{\text{None of the messages generated by units two and three are received by either units four or five}\} + \\
 &\quad Pr\{\text{Messages generated by either units two or three are successfully received}
 \end{aligned}$$

$$\begin{aligned}
& \text{by both units four and five} \} \times \\
& Pr\{\text{Holdoff timers in units four and five are synchronized}\} \\
& = (0.6043)(0.6043) + (0.0625)[1 - (0.6043)(0.6043)] \\
& = 0.4049 \tag{6.3}
\end{aligned}$$

6.2.4 Analysis of 1-2-2-1 Six-Unit Configuration, Neither Pair of Middle Units in Range of Each Other

In the six-unit configuration shown in row four of Table 6.2, the first pair of middle units are not within range of one another, and the second pair of middle units are not in range of one another. Unit six will not receive an alarm message if either of the following two conditions occurs:

- (1) Messages generated by units two and three collide with respect to the receive intervals of both units four and five since units two and three cannot hear one another's transmissions.
- (2) Transmissions generated by either units two or three are successfully received by both units four and five. However, messages generated by units four and five are not successfully received by unit six because units four and five cannot hear one another's transmissions.

The probability that unit six does not receive an alarm message in this configuration is shown in equation (6.4).

$$\begin{aligned}
& Pr\{\text{Unit six does not receive message}\} \\
& = Pr\{\text{None of the messages generated by units two and three are received by} \\
& \quad \text{either units four or five}\} + \\
& \quad Pr\{\text{Messages generated by units two and three are successfully received by} \\
& \quad \text{both units four and five}\} \times \\
& \quad Pr\{\text{None of the messages generated by units four and five are successfully} \\
& \quad \text{received by unit six}\} \\
& = (0.6043)(0.6043) + (0.6043)[1 - (0.6043)(0.6043)] \\
& = 0.7488 \tag{6.4}
\end{aligned}$$

6.3 Analysis of 1-4-1 Six-Unit Configurations

The analysis for the 1-4-1 six-unit configurations is similar to that performed for the 1-3-1 five-unit configurations in Section 5.2. The results of the 1-4-1 six-unit analysis are summarized in Table 6.3. The interested reader can find details of the analysis of 1-4-1 configurations in Appendix A.

Table 6.3: 1-4-1 Six-Unit Configurations

Six-Unit Configuration	Description	Pr{Link Failure to Unit Six}
	None of the middle units can hear one another	0.2207
	One pair of middle units can hear one another	0.0231
	Two non-adjacent pairs of middle units can hear one another	0.0026
	Two adjacent pairs of middle units can hear one another	0.1497
	Three adjacent pairs of middle units can hear one another	0.0026
	Two adjacent pairs of middle units can hear one another. The outer two units of the adjacent pairs can also hear one another.	0.0368
	Three adjacent pairs of middle units can hear one another. An additional pair of middle units that are separated by one middle unit can hear one another.	0.0444

	<p>Three adjacent pairs of middle units can hear one another. An additional pair of middle units that are separated by two middle units can hear one another.</p>	<p>0.0202</p>
	<p>Three adjacent pairs of middle units can hear one another. One of the middle units can hear all of the other middle units.</p>	<p>0.0570</p>
	<p>All middle units can hear one another.</p>	<p>0.0506</p>

6.4 Six-Unit Installation Recommendations

Recall from the five-unit analysis that an effective installation plan requires not only the analytical results in Tables 6.1-6.3, but also a topological analysis for each configuration. Tables 6.1-6.3 provide a prediction of the system reliability when the alarm originates from unit one. We must also consider the reliability of the system when the alarm originates from any other unit in the system.

The topologies shown in Table 6.1 have results similar to those obtained for configurations of four and five units. In fact, all of the configurations shown in Table 6.1 are acceptable installations provided that units connected by dotted lines are within range of one another.

Table 6.2 depicts the 1-2-2-1 configurations. The probability of link failure is greater than 0.1 for each of these topologies when the alarm originates from unit one. It is not even necessary to consider the effect on system performance when the alarm originates from other units because this high rate of failure is not acceptable. Thus, it is recommended that the 1-2-2-1 configurations be avoided.

Of the 1-4-1 configurations shown in Table 6.3, there are eight in which the probability of link failure is less than 0.1 when the alarm originates from unit one. However, in most of these configurations, it can be shown that the reliability worsens significantly when the alarm originates from other units. In fact, the only acceptable configuration in Table 6.3 is the last one in which all of the middle units are within range of each other. When the alarm originates from any of the four middle units, there is no chance of link failure, as all of the middle units are within range of every other unit in the system. When the alarm originates from unit six, the topology is identical to the case in which the alarm originates from unit one, making the configuration acceptable.

In summary, there are eleven effective installation configurations for a six-unit system. Ten of these are those shown in Table 6.1, provided that units connected by a dotted line are within range of each other. The eleventh configuration is displayed in the last row of Table 6.3. It is a 1-4-1 configuration in which all four middle units are within range of each other. Additionally, the six-unit configurations in which message collisions are not a threat are also acceptable. These include a configuration in which all six units are within range of one another and a serial configuration of six units.