CHAPTER 2

THE ENERGY APPROACH

Our interest in direct methods for transient stability analysis stems from the problem of real-time prediction of instability and control. As noticed in Chapter 1, the direct method has to be opposed to the conventional transient stability analysis, which numerically integrate the state equations. Assuming that the disturbance includes the initiation and isolation of a fault on a power system, conventional analyses proceed as follows. The initial system state is obtained from the pre-fault system. This is the starting point used for the integration of the fault-on dynamic equations. After the fault is cleared, the post-fault dynamic equations are numerically integrated. The machine angles may be plotted versus time and analyzed. If these angles are bounded, the system is stable, otherwise it is unstable. On the contrary, a direct method in transient stability analysis offers the opportunity of assessing the transient stability of power systems without explicitly solving differential equations. Therefore, they are computationally fast and very suitable for real-time transient stability analysis. It has become possible to achieve such tasks thanks to the newly developed technique of synchronized phasor measurements. Further details on the Phasor Measurement Units (PMUs) are given in appendix A. Assuming the availability of PMUs at each generator, the dynamic, state and transient energy equations are presented in the first part of this chapter. In the second part, the Energy Approach is described first on a one-machine-infinite-bus system and then extended to a multimachine system. Athay's 3-Machine system is then used to illustrate the Energy Approach.

2.1 General equations of a multimachine system

2.1.1 Power system representation

In order to simplify the system analysis, it is useful to make the following assumptions. The synchronous machines are represented as a constant voltage source behind the direct-axis transient reactance, X_d'. The loads are modeled as constant admittance's, and the input power from the prime mover, P_m , is assumed constant [1]. These parameters constitute the classical representation of a power system. In order to get an easy expression for the electrical power at the internal nodes of the machines, it is convenient to reduce the system to the internal nodes [1]. As it is shown bellow, it is useful to be able to monitor the power angle of the internal voltage of machines in the system to compute the dynamic, state, transient energy equations. These internal angles are the one that are monitored to determine the stability of the system. Hence the need to reduce the Y_{bus} matrix. This reduction is called the Kron reduction.

To obtain this reduction, we have to use the pre-fault load-flow to obtain the admittance values of the loads, yielding $Y_1=(P_{load}\ -jQ_{load})/V^2$. These admittances are included in the diagonal elements of the admittance matrix, Y_{bus} ($Y_{bus}^{new}(i,i)=Y_{bus}(i,i)+Y_l(i)$) . To get the final matrix of the reduced system, we have to include the transient reactance's. Therefore, we create the Y_{12} ,Y_{21}, and Y_{22} matrices. These three matrices are part of the extended matrix, Y_{ext}, which includes all of the original buses plus the internal nodes. The partition of Y_{ext} is given by :

$$Y_{ext}=\begin{bmatrix} Y_{bus}^{new} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \tag{2.1}$$

As no current injection is considered at the load buses, the extended system leads to the following system of equation:

$$\begin{bmatrix} 0 \\ I_{\text{int}} \end{bmatrix}=\begin{bmatrix} Y_{bus}^{new} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}*\begin{bmatrix} V \\ E \end{bmatrix} \tag{2.2}$$

By eliminating all the external nodes, we get the admittance matrix of the reduced system, which is written as:

$$Y_{red}=Y_{22}-Y_{21}\left[Y_{bus}^{new}\right]^{-1}Y_{12} \tag{2.3}$$

2.1.2 The swing equation

Once the system is reduced to the internal nodes, we can easily find the electrical power at the internal node i. It is given by:

$$P_{gi}+jQ_{gi}=E_iI_i^* \tag{2.4}$$

As for the current injection, I_i , it is expressed as:

$$I_i=\sum_{k=1}^{n}\left[\left(G_{ik}+jB_{ik}\right)E_k\right] \tag{2.5}$$

where $G_{ik}+jB_{ik}$ is the ik element of the reduced admittance matrix. After substituting the expression of the current given by (2.5) into (2.4) and ignoring the imaginary term, we get the expression of the real power yielding:

$$P_{gi}=G_{ii}E_i^2 + \sum_{k=1,\neq i}^{n}\left[B_{ik}E_iE_k \sin(\delta_i-\delta_k)+G_{ik}E_iE_k \cos(\delta_i-\delta_k)\right] \qquad (2.6)$$

Putting

$$C_{ik}=B_{ik}E_iE_k ,$$
$$D_{ik}=G_{ik}E_iE_k , \qquad (2.7)$$
$$P_{ei}= \sum_{k=1,\neq i}^{n}\left[B_{ik}E_iE_k \sin(\delta_i-\delta_k)+G_{ik}E_iE_k \cos(\delta_i-\delta_k)\right] ,$$

The swing equation in pu for machine i results in the following equation:

$$M_i\frac{d^2\delta_i}{dt^2}= P_{mi}-P_{gi} \text{ , pu} \qquad (2.8)$$

$$M_i\frac{d^2\delta_i}{dt^2}= P_{mi}-G_{ii}E_i^2 -P_{ei} \text{ , pu} \qquad (2.9)$$

We can define : $P_i=P_{mi}-G_{ii}E_i^2$ (2.10)

For an n-machine system, we can write n swing equations namely:

$$M_i\frac{d^2\delta_i}{dt^2}=P_i-P_{ei} \text{ , } i=1,...,n \qquad (2.11)$$

2.1.3 The transient energy

With the swing equations, we only have n second-order differential equations. For the state equations, we need 2n first-order differential equations, which are :

$$\frac{d\delta_i}{dt}=\omega_i \text{ , rad/s ;}$$
$$\frac{d\omega_i}{dt}= \frac{P_i - P_{ei}}{M_i} \text{ , rad/s}^2 \text{ , for } i=1,...,n \qquad (2.12)$$

In the right-hand side of the swing equation, angle differences are used instead of the angles with respect to a synchronously rotating axis, i.e. we have $\delta_i-\delta_k$ instead of only δ_i. By considering relative angles instead of the actual angles in a synchronous rotating frame, the number of state variables is 2(n-1) instead of 2n. The Center of Angle formulation (COA) is a different way of characterizing the angles of the internal nodes of the machines that uses as its reference a weighted average of all the angles in the system. With this new representation, the physical meaning of the angles has not changed. The difference between this notation and the original notation is that COA notation measures angles relative to a rotating reference frame whereas the original notation measures angles in reference to a stationary frame. Moreover, the transformation of the former equations into the center of angle coordinates not only offers physical insight to the transient stability problem formulation in general, but in particular provides a concise framework for the analysis of systems with transfer conductance's [9]. Using this new notation, the dynamic and state equations can be re-writen. However, it is useful to first derive the dynamics of the center of angle. They are given by :

$$\delta_0 = 1/M_t \sum_{i=1}^{n} M_i \delta_i \ , \ M_t = \sum_{i=1}^{n} M_i$$

$$\frac{d^2\delta_0}{dt^2} = P_{coa} / M_t \text{ , rad/s}^2$$

$$P_{coa} = \sum_{i=1}^{n} P_i - 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} D_{ij} \cos\delta_{ij}$$

By defining new angles and speeds relative to this reference, namely $\tilde{\delta}_i = \delta_i - \delta_0$ and $\tilde{\omega}_i = \omega_i - \omega_0$, the state equations become

$$\frac{d\tilde{\delta}_i}{dt} = \tilde{\omega}_i \text{ , rad/s}$$

$$\frac{d\tilde{\omega}_i}{dt} = \frac{P_i - P_{ei}}{M_i} - \frac{P_{coa}}{M_t} \text{ , rad/s}^2 \text{ for i=1,2,...,n-1} \qquad (2.13)$$

Note: Only 2(n-1) equations are required. It stems from the fact that the summation over all the angles in COA notation will always be zero,that is, $\sum_{i=1}^{n} M_i\tilde{\delta}_i = 0$

The swing equation then becomes:

$$M_i \frac{d^2\tilde{\delta}_i}{dt^2} = P_i - P_{ei} - \frac{M_i}{M_t} P_{coa} \ , \quad \text{for i=1,...,n} \qquad (2.14)$$

Once the swing equations are expressed in the COA notation, the transient energy function V can be derived in several ways. One way is to multiply the equation above by $\frac{d\tilde{\delta}_i}{dt}$ and add all n equations so obtained to get:

$$V = \sum_{i=1}^{n} \frac{M_i}{2} \tilde{\omega}_i^2 - \sum_{i=1}^{n} P_i \left(\tilde{\delta}_i - \tilde{\delta}_i^s \right) - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} C_{ij} \left(\cos\tilde{\delta}_{ij} - \cos\tilde{\delta}_{ij}^s \right) + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left[\int_{\tilde{\delta}_i^s + \tilde{\delta}_j^s}^{\tilde{\delta}_i + \tilde{\delta}_j} D_{ij} \cos\left(\tilde{\delta}_i - \tilde{\delta}_j \right) d\left(\tilde{\delta}_i + \tilde{\delta}_j \right) \right] \quad (2.15)$$

The transient energy function consists of 3 terms, which can be physically interpreted as follows:

* The first term is the transient kinetic energy : $V_{ke} = \sum_{i=1}^{n} \frac{M_i}{2} \tilde{\omega}_i^2$

* The remainder is the transient potential energy, which, may be decomposed in the following two terms:

$$V_p = -\sum_{i=1}^{n} P_i \left(\tilde{\delta}_i - \tilde{\delta}_i^s \right) - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} C_{ij} \left(\cos\tilde{\delta}_{ij} - \cos\tilde{\delta}_{ij}^s \right)$$

$$V_d = +\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left[\int_{\tilde{\delta}_i^s + \tilde{\delta}_j^s}^{\tilde{\delta}_i + \tilde{\delta}_j} D_{ij} \cos\left(\tilde{\delta}_i - \tilde{\delta}_j \right) d\left(\tilde{\delta}_i + \tilde{\delta}_j \right) \right]$$

Note that the last term is path dependent for n>2. Hence, the integral needs to be performed along a given path [1]. This path dependent term is the most difficult term to evaluate. However, as it is a well-known and well-described subject in numerous publication (for example [9]), several methods have been set up to come up with an immediate way of assessing it. The most widely used method in the literature is the ray approximation. For further details on the calculation, see the reference [1]. The ray approximation provides an estimated value for Vd so that an instantaneous potential energy can be found without paying any consideration to the actual trajectory.

2.2 One-machine-infinite-bus system

2.2.1 Theory

As noticed before, assuming the availability of synchronized phase angle measurements at each generator and using the energy equation, the energy level of a multimachine power system can be quickly computed so that corrective actions can be initiated as soon as possible. This represents the main concept of the Energy Approach and it is explained in details bellow. Let us first consider a machine connected to an infinite bus through two parallel lines (Figure2.1). Assume that a fault occurs at the middle of one of these lines. The swing equation will be given by using n=2 in the equations (2.8),yielding:

$$\mathrm{M}\frac{d^2\delta}{dt^2} = P_m - P_{\max}\sin(\delta)$$

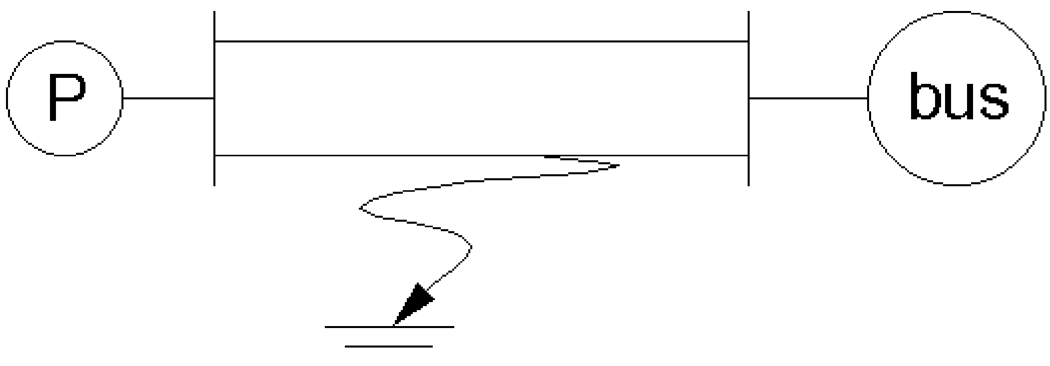

Figure 2.1: Machine connected to an infinite bus

The post-fault stable equilibrium point (s.e.p) is given by $\delta_s = \sin^{-1}\left(\frac{P_m}{P_{\max}}\right)$ and the nearest unstable equilibrium point (u.e.p) is $\delta_u = \Pi - \delta_s$. The right-hand-side of the post-fault swing equation can be expressed as the negative gradient of a potential energy function V_{pe}, that is $M\frac{d^2\delta}{dt^2} = -\frac{dV_{pe}(\delta)}{d\delta}$. We can either integrate the potential energy and add the kinetic energy to get the total energy or use the expression (2.15) to get:

$$V(\delta,\ \omega) = V_{ke}(\omega) + V_{pe}(\delta)$$

$$V(\delta,\omega) = \frac{1}{2}M\omega^2 - P_m(\delta - \delta_s) - P_{\max}(\cos\delta - \cos\delta_s) \qquad (2.16)$$

Since the provided model is ideal and the damping is neglected, the total energy is constant along the post-fault trajectory and equal to the total energy level at the clearing time. In other words, the only form of energy in the system are either kinetic energy or potential energy or the sum of these two energies, called the total energy. The latter is always constant at any time in the post-fault period. From the expression of the total energy, the transient energy method can easily be explained. Most of the stability concepts can be interpreted by considering a ball sliding without friction in a bowl having a shape similar to that of the potential energy surface $V_{pe}(\delta)$ as depicted in Figure 2.2. During the fault-on period, an additional energy is injected into the system in the same way as the ball in the bowl is given energy when it is initially pushed. During the post-fault period, the total energy remains constant.

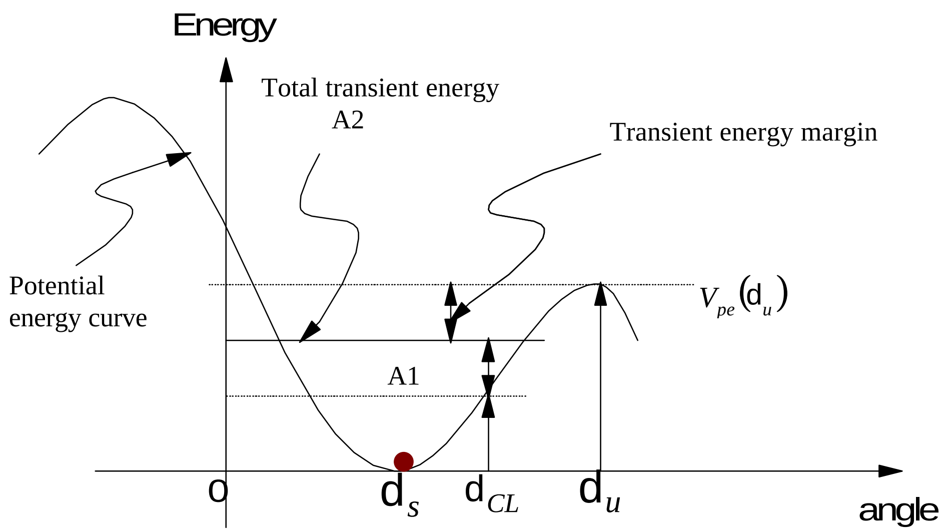

Figure 2.2: TEF approach

In Figure 2.2, A_1 represents the excess of kinetic energy injected into the system during the time period where the fault is on and A_2 represents the total energy present in the system at the clearing time t_c. The stability of the system is determined by the ability of the post-fault system to convert the excess of kinetic energy $V_{ke}(\omega_c)$ [2]. If the kinetic energy at clearing time exceeds the difference between the potential energy at clearing time and that at the u.e.p, the system will be unstable. Formally, we have

$$V_{ke}(\omega_{cl}) + V_{pe}(\delta_{cl}) < V_{pe}(\delta_u) \qquad (2.17)$$

This inequality is the mathematical way of stating the Energy Approach. The above inequality must hold not only at clearing time, but also during the post-fault time[1], that is,

$$V_{ke}(\omega) + V_{pe}(\delta) < V_{pe}(\delta_u) \qquad (2.18)$$

According to the inequality (2.18), a transient stability criterion can be defined and extended to a multimachine systemas follows:

Following a disturbance, the system is transiently stable if the total transient energy is less than the potential energy evaluated at the closest Unstable Equilibrium Point (u.e.p.).

2.2.2 Transient stability analysis in a 3-machine system

In this section, Athay's three-machine system is used to illustrate the Energy Approach to a multimachine system. Please refer to chapter 3 for more details about the simulations. The system shown in Figure 2.3 has three buses, each of them having a generator attached to it. The active power supplied by each machine is indicated above the arrow pointing toward each machine.

For this system, the C_{ij} and D_{ij} are given by:

i	j	Cij	Dij
1	2	1.3023	0.2180
1	3	3.0004	0.8052
2	3	7.2806	1.4865

The angle at the s.e.p, the powers Pi, and the voltages, Ei, are as follows:

i	s.e.p	Pi	Ei
1	12.7827	1.6510	1.0736
2	13.4473	3.6167	1.0573
3	-5.4947	-2.6751	1.0530

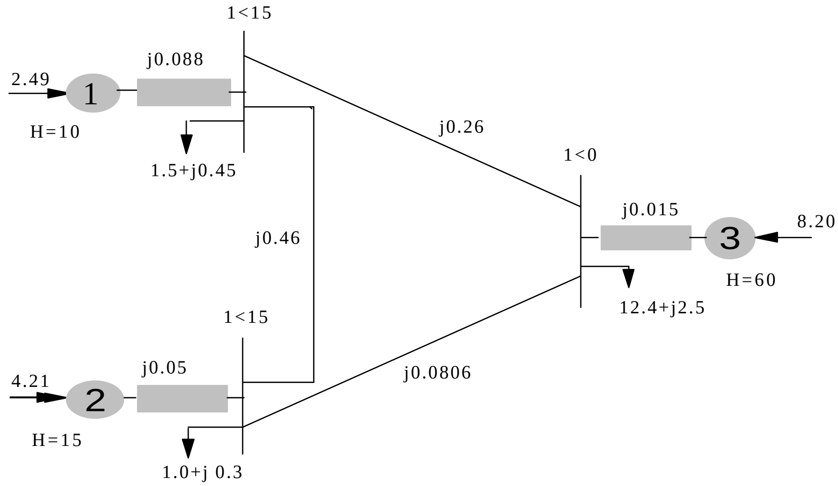

Figure2.3: Athay's Three-machine System

Let us consider a case where a fault of impedance Zf=(1+j)*10^(-5) pu occurs on bus 2. The fault, self-cleared at time tcl=0.21 sec, drives the system into instability. Figure 2.4 shows the post-fault path within the transient energy surface.

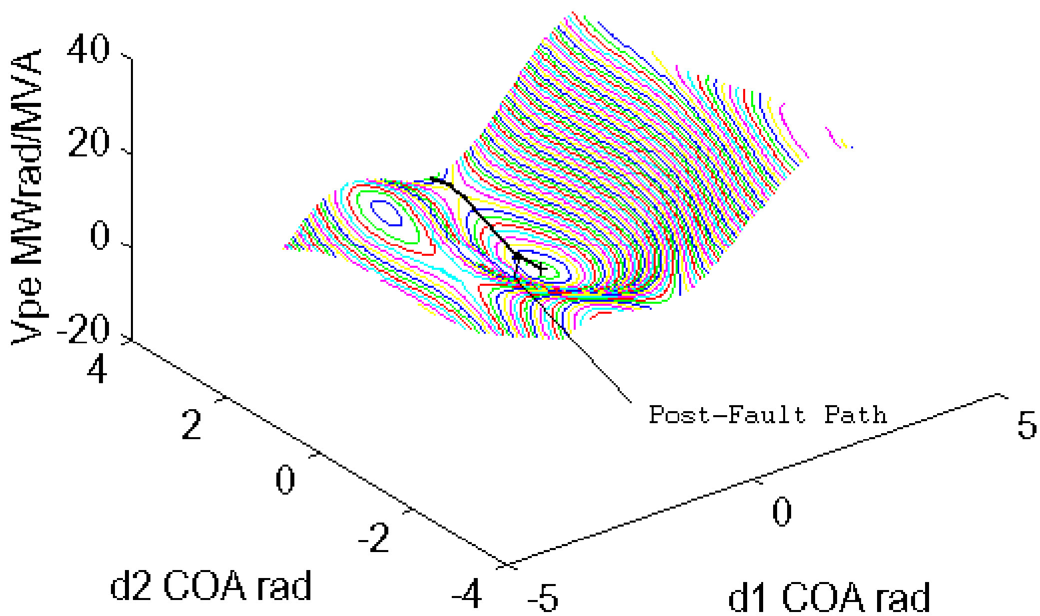

Figure 2.4: Potential Energy Surface and Unstable post-fault Path

The simulation program gives a total energy at the clearing time of 8.51 MWrad/MVA. The controlling u.e.p. is found to be the following point: **delta1**=0.464 rad; **delta2**=2.183 rad;

delta3=-0.546 rad. The potential energy at the c.u.e.p is equal to 7.87 MWrad/MVA. At any time during the post-fault period, the total transient energy of the system is bigger than the transient potential energy of the dominant c.u.e.p. The system is then assessed to be unstable. The energy level of the system can be checked in Figure 2.5. Notice that the total energy during the post-fault time is constant.

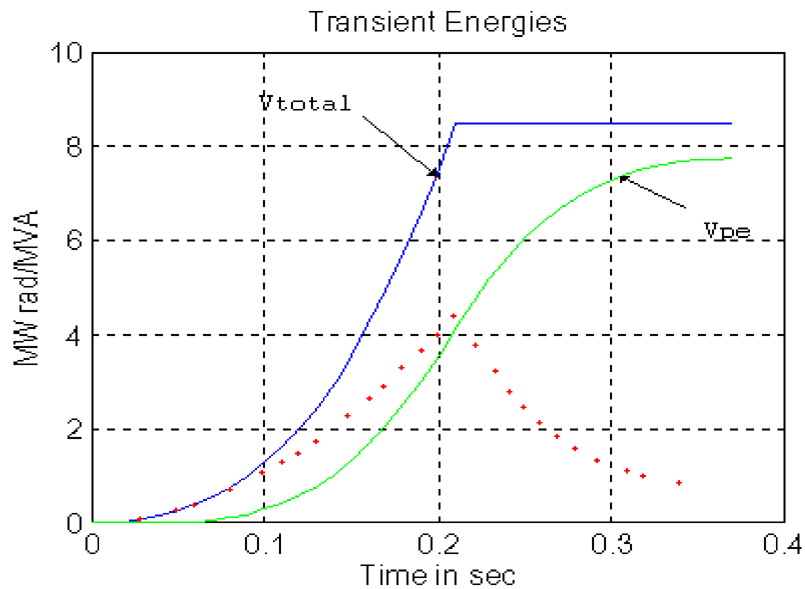

Figure 2.5: Transient energies
(Dash: kinetic Energy, Green: Potential Energy, Blue: Total Energy)