

### 4.2.3 A second-order auto-regressive model

An auto-regressive model of order p can be defined as follows

$$\begin{aligned} \tilde{d}_{i,p+1} &= a_{i,0} + a_{i,1}\tilde{d}_{i,p-1} + \dots + a_{i,p}\tilde{d}_{i,1} + e_{i,p} \\ \cdot & \\ \tilde{d}_{i,p+k} &= a_{i,0} + a_{i,1}\tilde{d}_{i,p+k-1} + \dots + a_{i,p}\tilde{d}_{i,k} + e_{i,p+k} \end{aligned} \quad \text{for } i=1, \dots, n-1$$

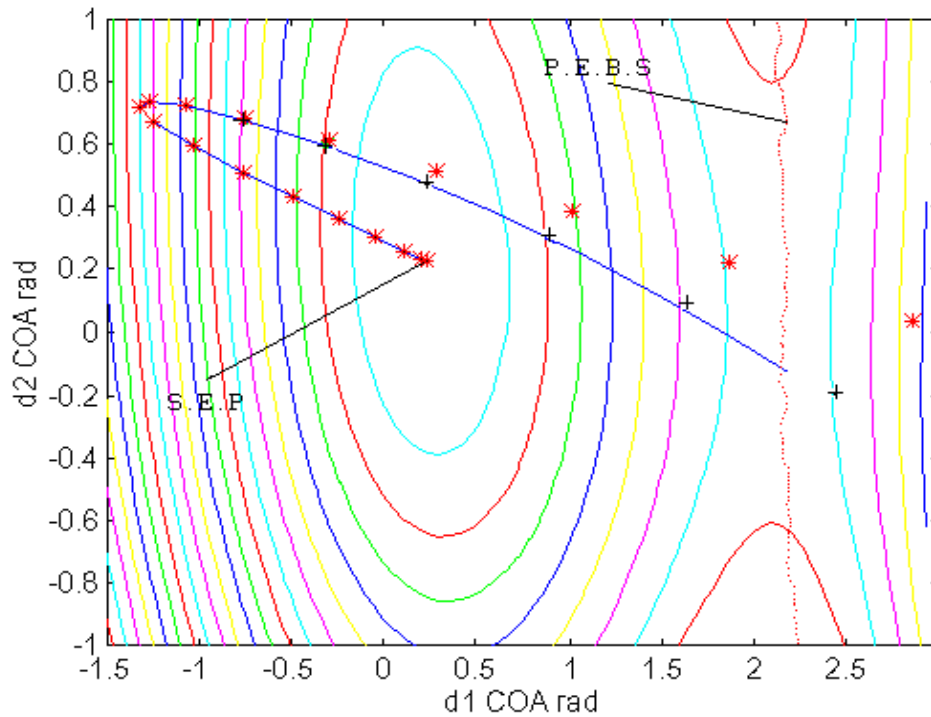
where

- k : number of point taken for the estimation
- a : parameters to estimate
- e : error in the Phasor measurements(neglected in our case)

Let us write the equations in a matrix form as follows:

$$\Rightarrow \begin{bmatrix} \tilde{d}_{i,p+1} \\ \cdot \\ \cdot \\ \cdot \\ \tilde{d}_{i,p+k} \end{bmatrix} = \begin{bmatrix} 1 & \tilde{d}_{i,p} & \cdot & \tilde{d}_{i,1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 1 & \tilde{d}_{i,p+k-1} & \cdot & \tilde{d}_{i,k} \end{bmatrix} * \begin{bmatrix} a_{i,0} \\ a_{i,1} \\ \cdot \\ \cdot \\ a_{i,p} \end{bmatrix} + e$$

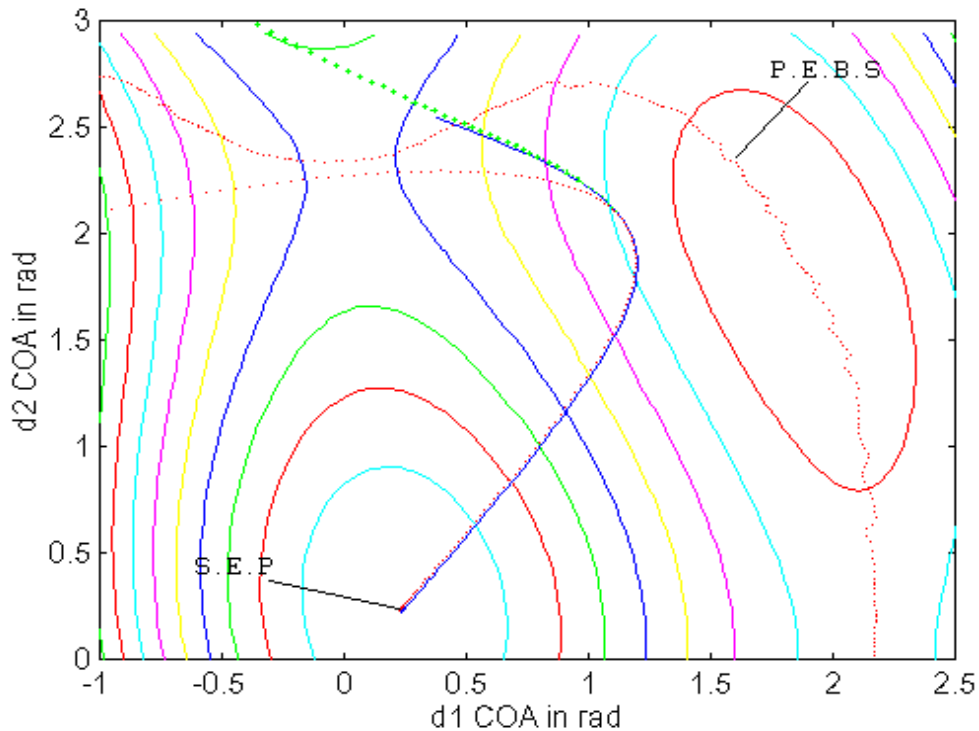
In a closed form, we have the formulation  $Z = H^* \underline{x} + \underline{e}$  where  $\underline{x}$  is the vector to be estimated in the least-squares sense. The estimate vector is given by  $\hat{\underline{x}} = (H^* H)^{-1} H^* Z$ . An example of the second-order auto-regressive model is displayed in Figure 4.6. It is compared to the second-order polynomial function. A self-cleared fault on bus 2, where  $Z_f = 0.032$  pu is considered. The clearing time is equal to 0.2 second. The generator angle data are recorded over 150 ms after the clearing time to estimate the different parameters. The sampling rate of the generator angle measurements is 30 milliseconds. Thus, in both cases, 5 points are used for the estimation of the different parameters.



(\* : polynomial model & + : auto-regressive model)

**Figure 4.6 :** Comparison between a 2<sup>nd</sup>-order polynomial and auto-regressive models

As shown in Figure 4.6, the second-order auto-regressive model is a closer approximation to the real path than the polynomial function. As shown in Figure 4.7, the second-order polynomial function may lead to an exit point that is not in the vicinity of the c.u.e.p. The polynomial approximation slides closely next to the p.e.b.s. without crossing it. As a result, the predicted exit point may be assessed on the vicinity of a u.e.p. that presents a potential energy higher than the c.u.e.p. Using in this case the energy criterion defined in chapter 2 and the polynomial model, the approach proposed in this paper might lead to a stability evaluation whereas it is unstable. However, several tools have been set up to overcome these drawbacks. Tools such as the directional derivative and the Ball-Drop [22] method are explained in the last part of this chapter.



(+ : auto-regressive model & Dash: polynomial model)  
**Figure 4.7:** Drawbacks of the 2<sup>nd</sup>-order polynomial model

For the previous simulation, a self-cleared fault on bus 3 has been simulated. The fault impedance is equal to  $Z_{f3}=0.032$  pu and the clearing time is equal to 0.26 second. The sampling rate is here equal to 5 milliseconds and a collection of generator angles over 150 ms are saved after the clearing time to estimate the different parameters.

#### 4.2.4 Y. Ohura's model [6]

As explained in chapter 3, the generator angles are approximated by a sine wave that diverges or converges. Using the angle values for the present time and the previous time, the future generator angle can be predicted. The method is described using Figure 4.8.