CHAPTER 5:

CONTROLLING UNSTABLE EQUILIBRIUM POINT

The main purpose in the Curve-Fitting method is to calculate the exit point in order to locate the relevant part of the stability boundary to which the post-fault trajectory is heading. Starting at the exit point, the key step of the proposed approach is the calculation of the controlling u.e.p. Indeed, through the application of the energy approach, transient stability assessment is performed by evaluating the energy margin function. This energy margin is dependent on the controlling unstable equilibrium point (c.u.e.p.). Without finding the c.u.e.p. accurately, the method can either over-estimate or under-estimate the stability margins. Generically, there is no two u.e.p's. that have the same values of the energy function.

Due to the importance of finding the correct c.u.e.p., methods such as the BCU method have been developed to derive theoretic-based algorithm for locating the c.u.e.p. The first method presented in this chapter exploits the relationship between the stability boundary of the post-fault classical power system model and the stability boundary of the post-fault reduced system. The essence of the method is that it finds the c.u.e.p. via that of the reduced, system which is defined in the angle space only and is easier to compute. The method consists of two steps. The first step focuses on the detection of the so-called minimum gradient point. In the second step, a set of nonlinear algebraic equations is solved with the minimum gradient point as an initial guess yielding a c.u.e.p. However, it has been observed that this exit point method lacks robustness. Those drawbacks are underlined in this chapter and an other method called the Shadowing method is proposed and explained.

5.1 C.U.E.P. detection using the reduced system

In this section, we describe the two steps taken from the BCU method to find the c.u.e.p. for the classical power system model. As defined before, the method exploits the relationship between the stability boundary of the post-fault system and the stability boundary of the following post-fault reduced system:

$$\frac{d\tilde{\delta}_i}{dt} = P_i - P_{ei} - \frac{M_i}{M_t} P_{coa} \quad \text{for i=1,......,n-1} \qquad (5.1)$$

In (5.1), the state variables of the reduced system are the machine angles, whose number is only of (n-1) while the dimension of the original system (2.13) is of 2(n-1). It can be seen that an equilibrium point of the reduced system is also an equilibrium point of the original system. Furthermore, it can be shown that [14]

1) δs is a stable equilibrium point of the reduced system defined by (5.1) if and only if (δs,0) is a stable equilibrium point of the original system.

2) Under certain circonstancies introduced in chapter 3, δ is on the stability boundary of the reduced system if and only if (δ, 0) is on the stability boundary of the original system given by (2.13).

Based on those properties, a method has been set up to compute the c.u.e.p. as shown on figure 5.1.

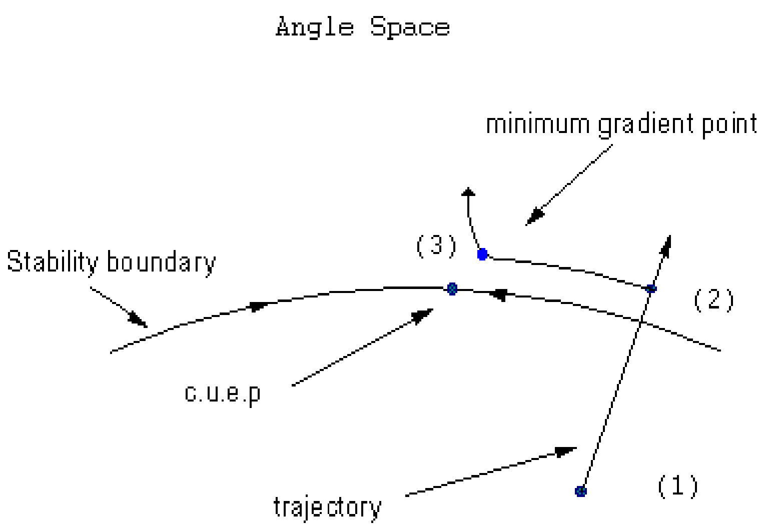

Figure 5.1: c.u.e.p. detection using the reduced system

From (5.1), it is seen that the velocities of each machine in the reduced system are functions of the acceleration of the original system. In other words, the reduced system is designed such that the velocity of any state tends to move in the direction of the maximum slope of the potential energy surface of the original system. Using this property, the minimum gradient point is computed using the exit point as initial condition. This represents steps 1 and 2 in Figure 5.1. It requires the integration of the associated differential equations given by (5.1), which can be written as

$$\frac{d\tilde{\delta}_i}{dt} = f_i^{red}\left(\underline{\tilde{\delta}}\right) \qquad (5.1\ \text{bis})$$

The exit point is used as the starting point for the integration of the reduced system trajectory. Along this trajectory, the Euclidean norm of each state is monitored by means of (5.2).

$$F = \sum_{i=1}^{n-1} \left\| f_i^{rd}\left(\underline{\tilde{d}}\right) \right\| \qquad (5.2)$$

As shown in Figure 5.1, this norm tend to decrease as the trajectory is integrated, until it reaches some minimum value before starting to increase. The calculation of equation (5.2) is stopped when it has reached a minimum. Then, the associated state is used as initial guess for a Newton-Raphson load flow analysis to find the c.u.e.p. Further, using the minimum gradient point as initial guess, the controlling unstable equilibrium point is obtained by solving a system of nonlinear equations given by

$$f_i^{red}\left(\underline{\tilde{d}}\right) = 0 \quad \text{for i=1,...,n-1} \qquad (5.3)$$

It has been observed that this method suffers from the following two problems [3], [4]. The first problem is that there may be no minimum gradient point encountered. In that case, the system will either go off to infinity or converge to a stable equilibrium point; the method will fail and in general the resulting transient stability assessment will be flawed. The stable equilibrium point may not be the s.e.p of the post-fault trajectory but a neighboring s.e.p.

The second problem is that when the minimum gradient point is encountered, it is expected that the c.u.e.p will result from a nonlinear solution of $f_i^{red}\left(\underline{\tilde{d}}\right) = 0$ with the minimum gradient point as an initial guess. Unfortunately, this may not be true since the minimum gradient point may not be in the domain of convergence of the c.u.e.p for the particular solving algorithm used. To develop this idea, it is useful to separate the u.e.p.'s in different categories. For example, let us consider the potential energy boundary surface and the analogy with a saddle as shown in figure 5.2, the bottom of the saddle is considered as a type-1 u.e.p. and the top of the saddle is considered to be a type-2 u.e.p. In a more generic way, the p.e.b.s. is composed of the stable manifolds of type-k u.e.p.'s. The type of a u.e.p. is defined by its degree of instability or in a more mathematical way, is defined by the number of Jacobian matrix unstable eigenvalues. Note that the Jacobian matrix is the derivative of the left part of the state equations given by (2.13) with respect to the state variables.

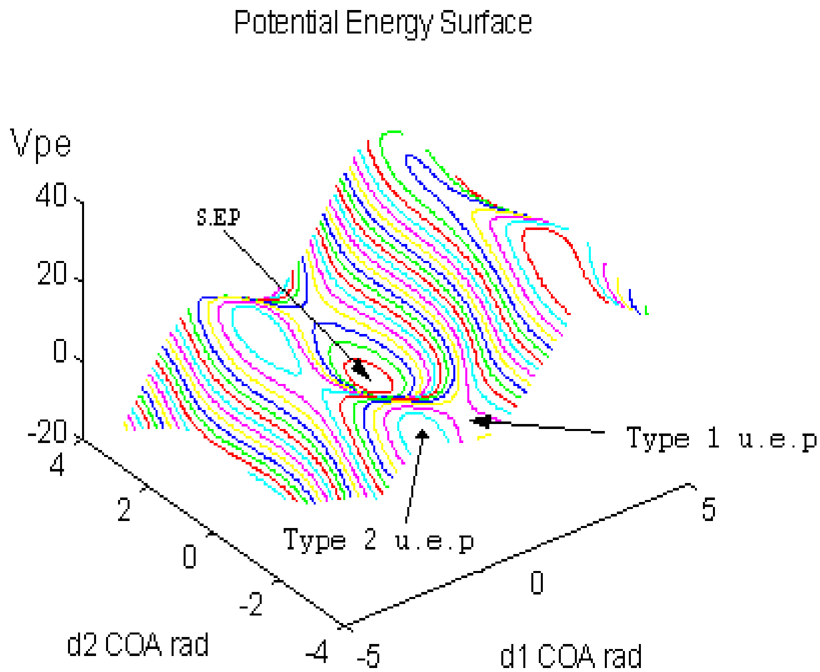

Figure 5.2: Potential Energy Boundary Surface

Due to the fact that the exit point (located on the p.e.b.s.) may be in the vicinity of a type-k u.e.p., this method of detection may lead to a type-k u.e.p. instead of the c.u.e.p., which is assumed to be a type-1 u.e.p. As illustrated in Figure 5.3, the type-2 u.e.p.'s around a type-1 u.e.p will, in general, have a higher value of potential energy than the type-1 u.e.p., since the energy decreases along the trajectories of the gradient system. Therefore, assessing the stability margin based on a different u.e.p. around the controlling u.e.p. can lead to a nonconservative stability assessment. This can be seen in Figure 5.3. For this simulation, a self-cleared fault on bus 3 with Zf =(1+j)10^(-5) is considered. The fault is cleared at 0.44 second after it occurred on the system. Figure 5.3 shows a simulation of the real trajectory using a time domain simulation method that gives us the real point where the trajectory crosses the p.e.b.s. It also shows prediction given by the Curve-Fitting method (step 1) and the detection of the c.u.e.p. using the reduced system (step2). As it can be seen, the method has failed to detect the c.u.e.p. by leading toward the surrounding type-2 u.e.p.

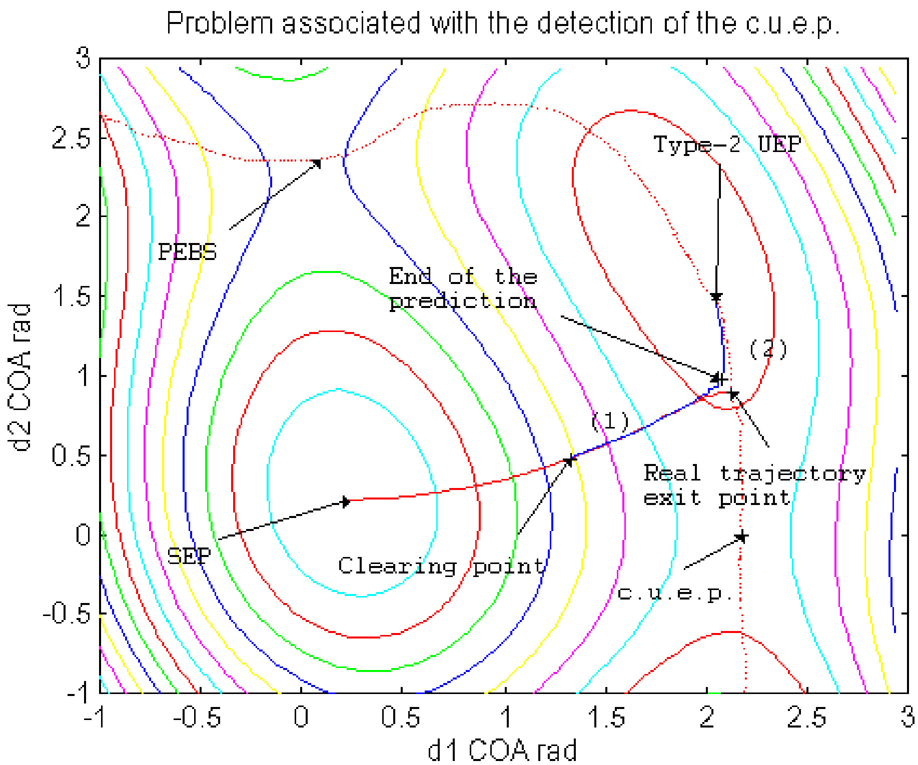

Figure 5.3: Problem associated with the detection of the c.u.e.p. using the reduced system

5.2 Description of the Shadowing Method

As already described, the previous method utilizes the stable manifold of the u.e.p. and the flow surrounding it to find the c.u.e.p. However, this method may fail as seen. The shadowing method [4] as described in this section is a new technique to determine the c.u.e.p. This method not only utilizes the stable manifold and its neighboring flow, but also the shape of the equipotential energy surfaces around the stable manifold of the c.u.e.p. The idea behind the shadowing method [4] is to use the flow of the gradient system in the vicinity of the stable manifold for a relatively short time interval. The resulting point is then corrected to a new point closer to the stable manifold, and the procedure is repeated. In this way, we obtain a sequence of points, which, under certain assumptions [4] such as the c.u.e.p. is of a type-1, converges to the c.u.e.p. The endpoint of the sequence should be close enough to the c.u.e.p. so that an algebraic equation solver can determine the c.u.e.p. correctly. Hence, the first problem does not exist for the Shadowing method because no minimum gradient point is computed and the second problem is alleviated. The application of two cycles of the shadowing method is shown in figure 5.4, which depicts the equipotential energy surfaces around the stable manifold of the c.u.e.p $W^s(\delta^u)$.

The initial point is d^{mo}. This point is the first one along the trajectory at which V_{pe} attains a local maximum.

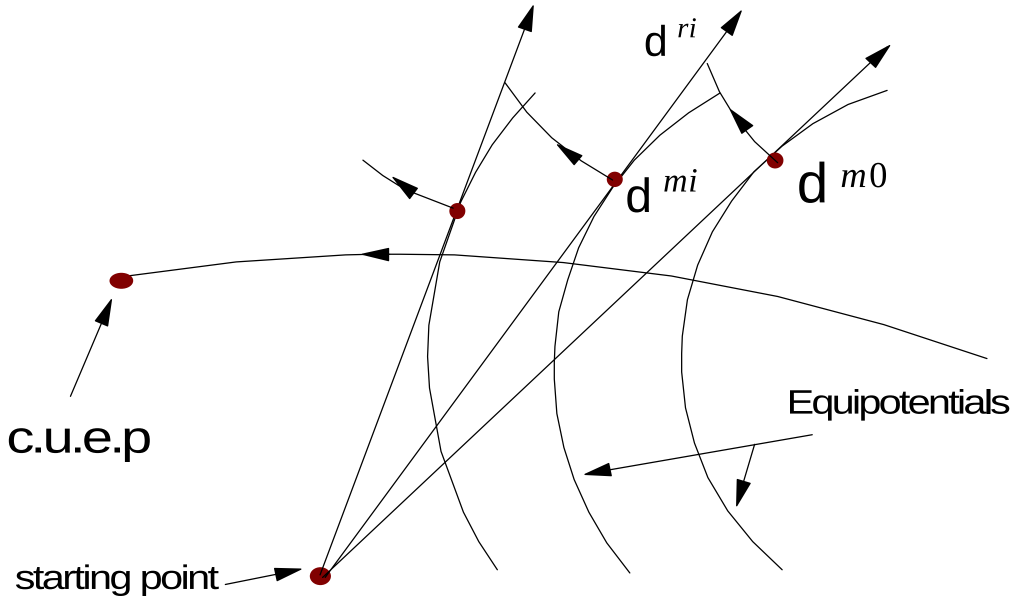

Figure 5.4: Illustration of the shadowing method

The ith cycle of the method consists of the following steps:

> <u>Step 1:</u> d^{ri} being the solution of the integration of the gradient system during a relatively small time.
>
> <u>Step 2:</u> Form an affine ray set, $\left\{\mathrm{d} : \mathrm{d} = \left(\mathrm{d}^{ri} - \mathrm{d}^{s}\right)a + \mathrm{d}^{s}, a \geq 0\right\}$.
>
> <u>Step 3:</u> Determine the point at which V_{pe} attains a local maximum on the ray established in the former part. This can be done by applying the Newton's method with d^{ri} as an initial guess.

The criterion to determine the number of cycles is similar to the one that is checked when the minimum gradient point is to be found. At the end of the ith cycle the following sequence of points will have been produced, d^{m0},.......,d^{mi} . If the norm of the gradient vector is less than a threshold value, the resulting point is close enough to the c.u.e.p so that an algebraic equation solver can determine the c.u.e.p correctly. This technique brings a substantial improvement because it does not attempt to detect the minimum gradient point. It can also identify a point that is close to the controlling unstable equilibrium point, thus avoiding the domain of convergence problem.

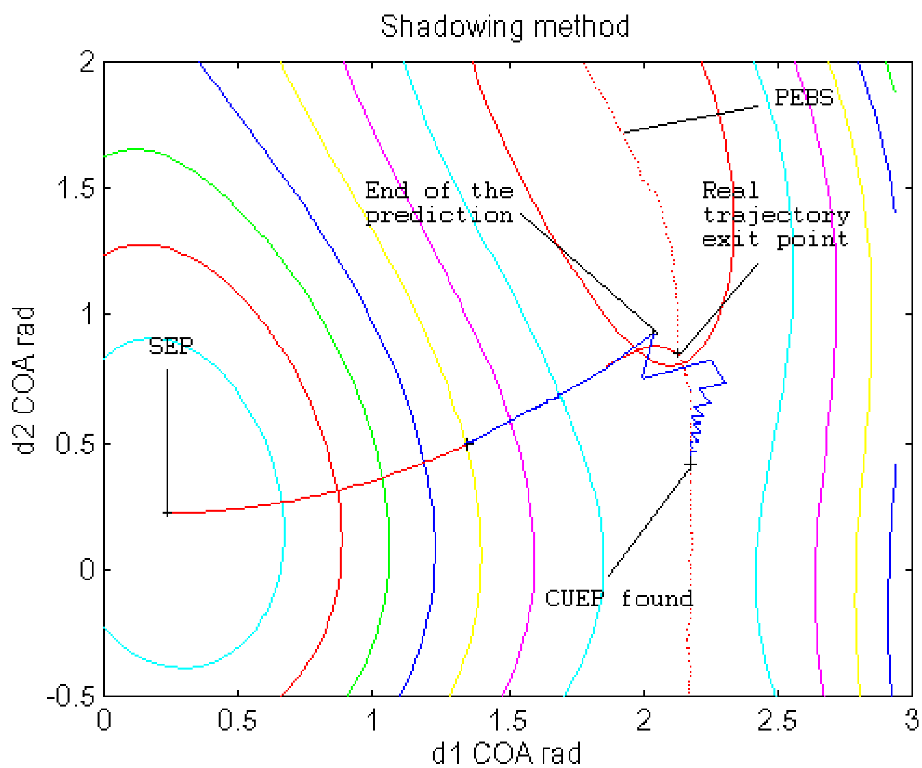

Figure 5.5: Shadowing method

Figure 5.5 shows the efficiency of the Shadowing method [4] for the case of Figure 5.3. This method is based on several assumptions. The two first assumptions are standard assumptions for the mathematical analysis of stability boundary of power systems. The first assumption guarantees that the exit point in the gradient system lies into a stable manifold of an u.e.p. and that this u.e.p. is the c.u.e.p. of the swing system. The second assumptions supposes that the c.u.e.p. is of type-1. The third assumption is one of the crucial assumptions for the Shadowing method. It supposes that along the ray created in the step 2 of the procedure, the function V_{pe} has a unique maximum. The validity of this assumption depends on the geometry of the level surfaces of V_{pe}. Unfortunately, this geometry is very complex for a multi-machine system. Although it has not been mathematically proven [4], its validity has been observed in most of the simulations run on Athay's three-machine system as well as on the New-England power system. However, in chapter 2, it has been shown that the p.e.b.s. shape was different for the ray method of p.e.b.s. detection than it was for the potential energy gradient method. Using the ray method, at some positions on the p.e.b.s. (see top and bottom left of figure 3.2), no local maximums were found. One solution would be to use the Ball-Drop method as back up. But, it may slow down the method considerably. Finally, it can be underlined that even if the third assumption is violated in a neighborhood of the c.u.e.p., the Shadowing method will still produce a finite sequence of points that approaches this neighborhood. The endpoint of this sequence may be a better starting point for an algebraic equation solver to locate the c.u.e.p. that the minimum gradient point employed by the previous method. The fourth assumption of the Shadowing method guarantees that the exit point of the gradient system and the first maximum of the potential energy along rays through the s.e.p. lie in the vicinity of the same u.e.p. This assumption holds true in all systems with type-1 controlling u.e.p.'s that have been analyzed. To conclude, the availability and the accuracy of the Shadowing method hold with respect to those assumptions. Indeed, they ensure us of the convergence of the Shadowing method towards the c.u.e.p.