

## Chapter 3

# STATIC ANALYSIS

This chapter discusses the procedures involved in computing the equilibrium configurations of the inflatable dams.

The dam is assumed to be elastic, with negligible weight. The assumption of negligible weight is reasonable since it was found that the effect of the weight of the dam on its vibrations was insignificant (Fagan, 1987). For the dam with no external water, the equilibrium shapes can be computed once the cross-sectional perimeter, the material thickness, the modulus of elasticity, Poisson's ratio, and the internal pressure are known. For the dam with external water, the external water height is also needed.

### 3.1 THE MODEL

The dam is modeled as a thin, isotropic, elastic shell and is analyzed using the finite element method (Figure 3.1). The finite element package ABAQUS is used for performing the static and dynamic analysis. A four-node shell element (the S4R element in ABAQUS (Hibbitt et al., 1994) is adopted, which has been developed under the assumptions of large deflections, large rotations, and small strains. Each node of the element has six generalized displacements,  $u$ ,  $v$ ,  $w$ ,  $\theta_1$ ,  $\theta_2$ ,  $\theta_3$ , with a total of 24 degrees of freedom per element (Figure 3.2). A total of 1500 such elements are used to model the dam.

The dam is modeled as two sheets of rubber, one lying on top of the other. Each of the sheets is modeled using 750 S4R elements, with 25 elements in the circumferential direction and 30 in the longitudinal direction. The boundary conditions are such that one end of each sheet in the longitudinal direction is clamped to the ground while the other ends of the sheet are tied to each other such that they are together at all times. The ends which are tied together form the fin of the dam with a length which is about 5 percent of the cross-sectional perimeter of the dam.

The surface on which the dam rests is modeled as a rigid surface. A four-node rigid element (the R3D4 element in ABAQUS) is used to model the surface. A total of 50 such elements is used. The boundary conditions are such that there are no deflections or rotations for nodes constituting the surface.

The following parameters were used for the dam :

Modulus of elasticity - 0.1038 GPa

Density of rubber - 1,005 kg/m<sup>3</sup>

Poisson's ratio - 0.3

Material thickness - 12.7 mm

Cross-sectional perimeter - 9.14 m

Length of the dam - 30 m

The internal pressure, according to the manufacturers, should be equal to the maximum hydrostatic pressure for a water level equal to the height of the dam. In this study, internal pressures ranging from 1kPa to 30 kPa are used. The internal pressure of 30 kPa is approximately equal to the ideal internal pressure as suggested by the manufacturers.

## **3.2 ANALYSIS PROCEDURE**

The equilibrium shapes are obtained numerically. ABAQUS uses Newton's method as a numerical technique for solving the nonlinear equilibrium equations (Hibbitt et al., 1995). The basic idea is to reduce the set of nonlinear equations into a set of linear equations by choosing to solve the equilibrium equations at "small" increments, the size of which depends upon the nonlinearity of the problem.

### **3.2.1 WITHOUT EXTERNAL WATER**

The dam is assumed to lie flat initially. The internal pressure is increased gradually until it reaches the desired value. No boundary conditions are specified on the ends of the dam for this case. In ABAQUS, the load is applied in a set of small increments and the matrix equations are solved for each of those incremental pressures. The procedure is continued until the desired value of pressure is reached and then the final equilibrium shape is computed.

In the present work, internal pressures ranging from 1 kPa to 30 kPa are considered. The corresponding cross-sectional equilibrium shapes of the dam are depicted in Figure 3.3. The material shows a nonlinear deformation behavior as can be seen in Figure 3.3. The change in the equilibrium shapes is more pronounced for the lower pressures (0.5 kPa to 5 kPa) while the shape tends to become almost circular as the internal pressure is increased further. The rigid support provides an additional reaction force which forces the dam to rise higher. Figures 3.4 (a) and 3.5 (a) illustrate the three-dimensional equilibrium shapes of the dam without external water. The height of the center of the dam is 2.4 m and 3.0 m in Figures 3.4 (a) and 3.5 (a), respectively.

### **3.2.2 WITH EXTERNAL WATER**

For the dam with external hydrostatic water, the analysis is completed in two steps. First, for a given internal pressure, the equilibrium shape of the dam is computed for the dam without external water as

described in the above section. Then the ends of the dam are fixed and the external fluid is added to the anchored side of the dam, with its height less than the dry equilibrium height of the dam. The density of the fluid is gradually increased from zero to the density of the water ( $1,000 \text{ kg/m}^3$ ), and the equilibrium shape is obtained.

The results are presented in Figures 3.4-3.6. Figures 3.4 (b) and 3.5 (b) illustrate the three-dimensional equilibrium shapes of the dam with external water for internal pressures of 1 kPa and 30 kPa, respectively. In Figures 3.4 (b) and 3.5 (b) the height of the external water is 0.5 m and 1.5 m, respectively. The external water tends to push the dam towards the right. Figure 3.6 depicts the change in the cross-sectional equilibrium shape at the center of the dam with increasing water level, for an internal pressure of 30 kPa.

## Model Information

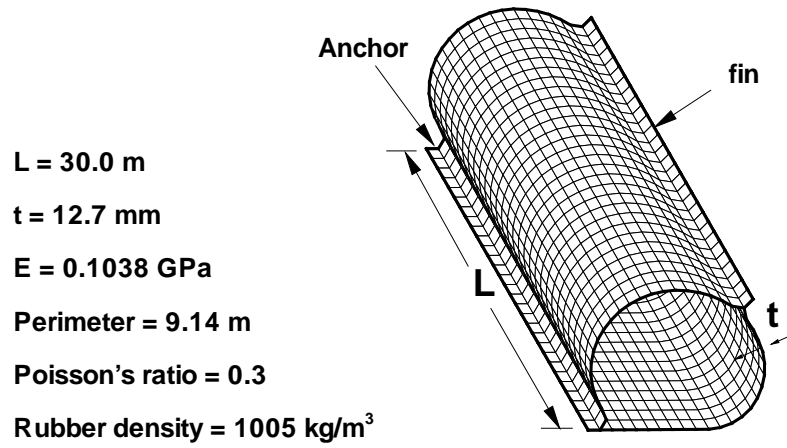


Figure 3.1 Figure showing the model information.

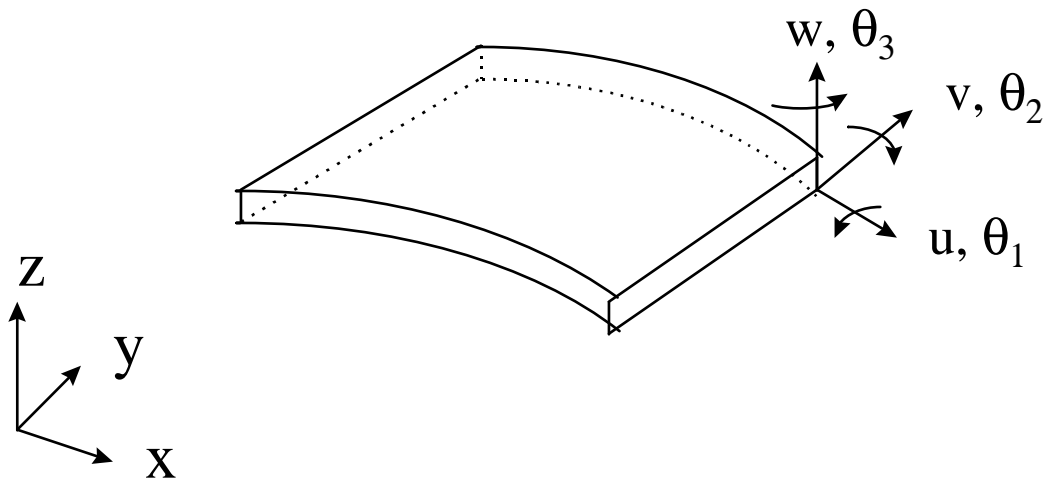


Figure 3.2 A shell finite element.

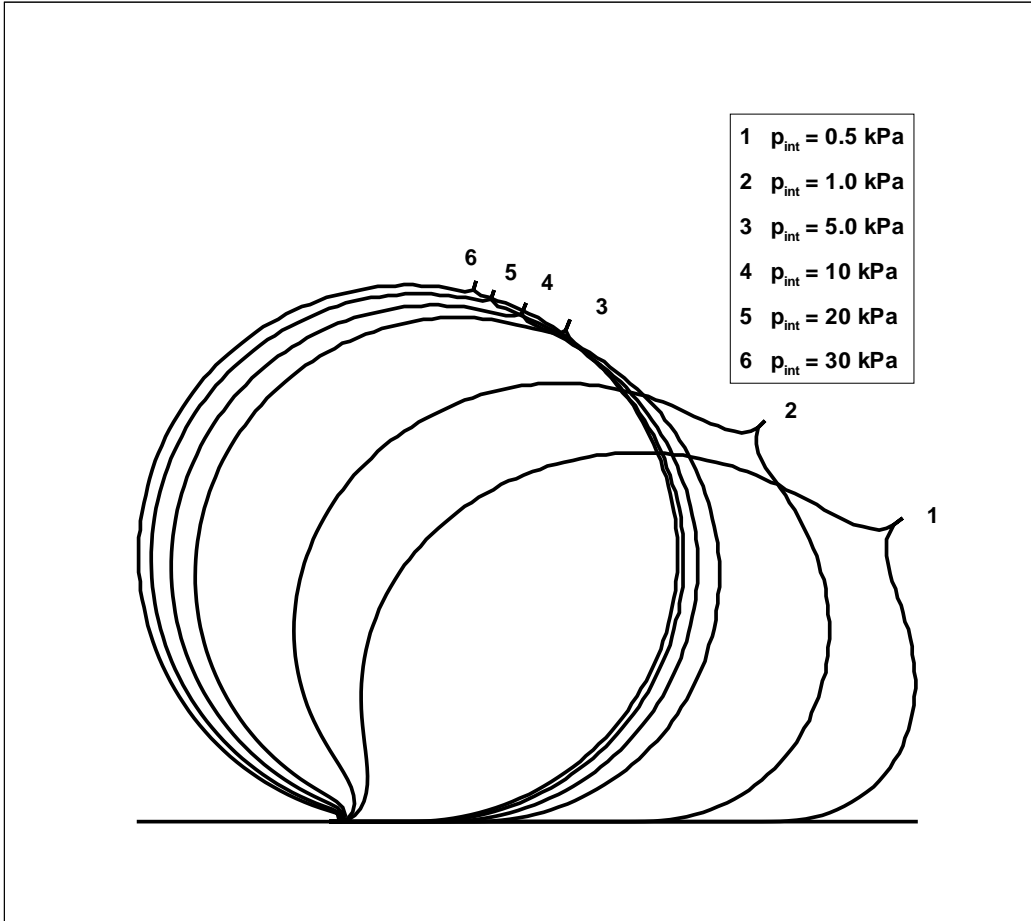


Figure 3.3 Cross-sectional equilibrium shapes of the dam (at the center) without external water at different internal pressures.

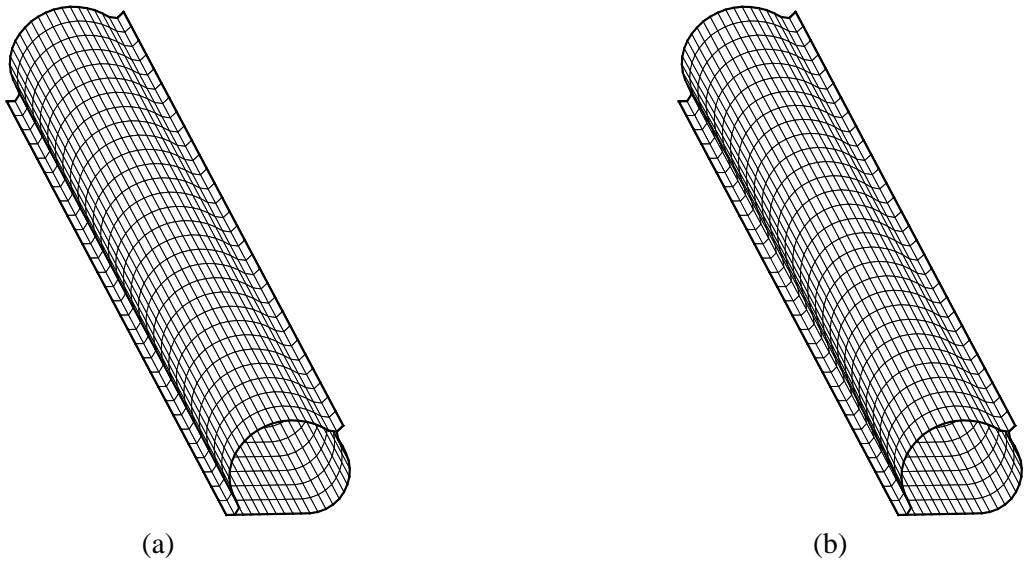


Figure 3.4 Equilibrium shapes of the dam (a) without water (b) with water,  $p_{int} = 1 \text{ kPa}$ .



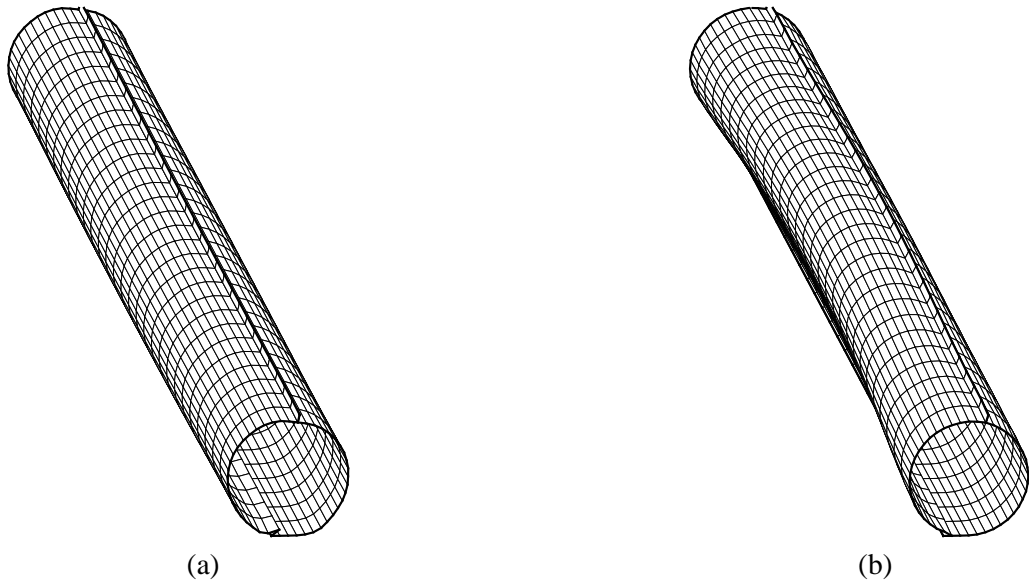


Figure 3.5 Equilibrium shapes of the dam (a) without external water (b) with water, for  $p_{int} = 30$  kPa.

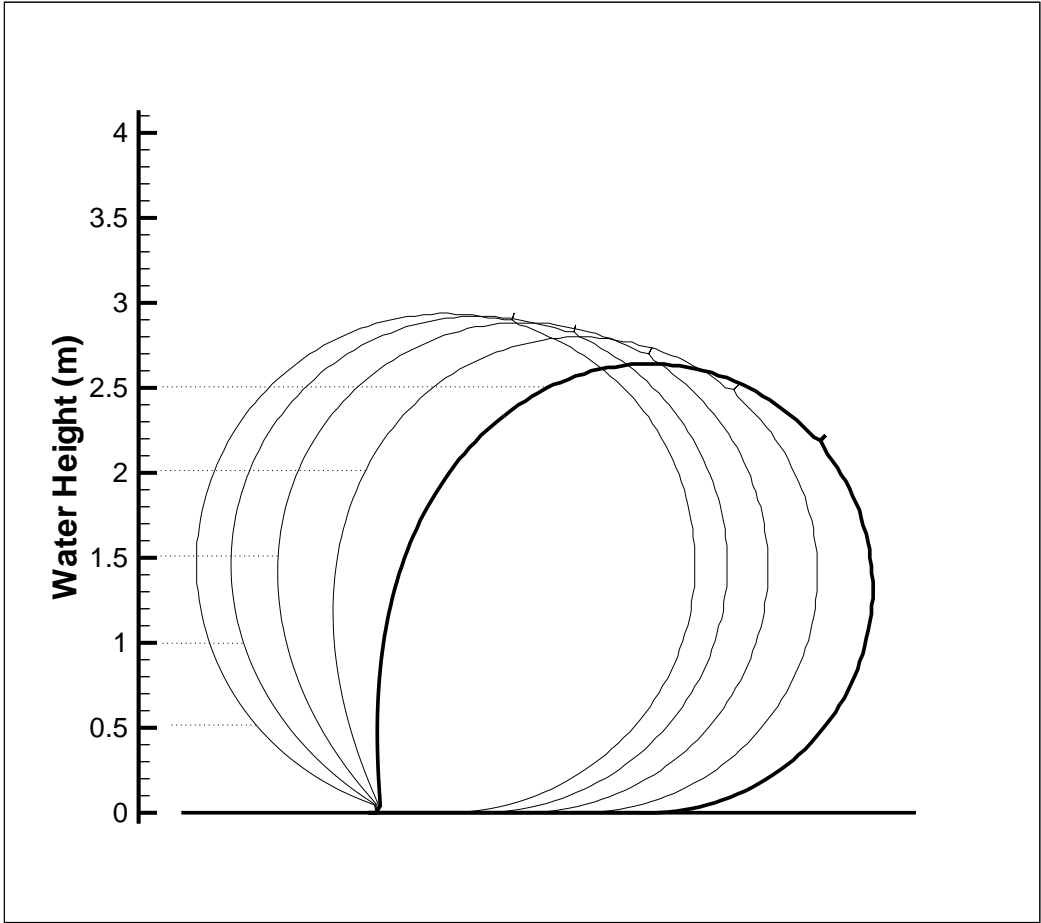


Figure 3.6 Cross-sectional equilibrium shapes of the dam (at the center) with different water levels for  $p_{\text{int}} = 30 \text{ kPa}$ .