Chapter 3

Model Formulation

In this chapter, the pertinent vehicle representation and control schemes are modeled and formulated for use within the simulation program. The first section of the chapter will explain and formulate the roll-plane vehicle model used for this research. This is a planar model with four degrees of freedom that represent the heave and roll of the vehicle body, as well as the wheel hop of the left and right tires. In the second and final section of the chapter, the equations are developed for the semiactive damping control schemes as they pertain to the roll-plane model. The semiactive control schemes formulated for this research include on-off skyhook, continuous skyhook, on-off groundhook, and fuzzy logic damping control.

3.1 Roll-Plane Vehicle Model

The purpose of this research is to examine the effect of different control methods on the transient response of the heave and roll motion of a class 8 truck. The front end of the truck is represented by a four-degree-of-freedom, roll-plane model, as shown in Figure 3.1.

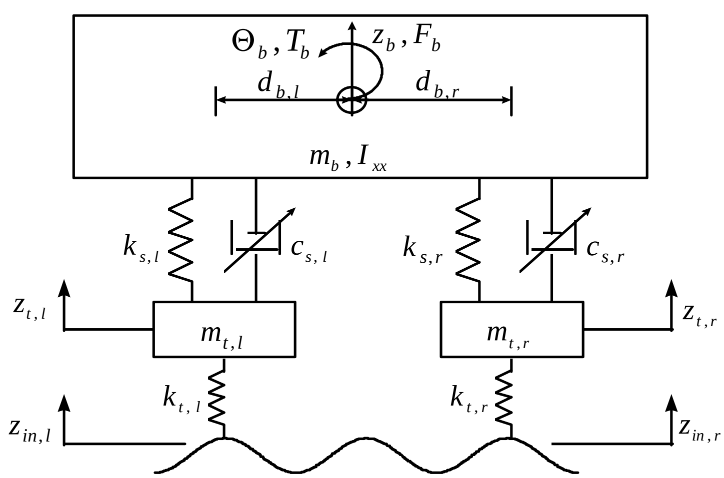


Figure 3.1. Roll-Plane Vehicle Model

The model consists of three masses. The top mass, m_b, represents the truck body, while the two lower masses, $m_{t,l}$ and $m_{t,r}$, represent the left and right tires of the front axle, respectively. The parallel spring and damper combinations located between the truck body and each tire ($k_{s,l}$, $c_{s,l}$ and $k_{s,r}$, $c_{s,r}$) represent the stiffness and damping of the vehicle suspension system. The respective stiffness' of the left and right tires are represented by the lower springs $k_{t,l}$ and $k_{t,r}$. On the truck body, a heave force, F_b, and a roll force (torque), T_b, can be exerted. The model parameters and their respective units are summarized in Table 3.1.

Table 3.1. Roll-Plane Vehicle Model Parameter Definition

Symbol	**Description**	**Units**
$c_{s,l}$	suspension damper, left side	lb-sec/in
$c_{s,r}$	suspension damper, right side	lb-sec/in
$d_{b,l}$	dimension from suspension to CG, left side	inch
$d_{b,r}$	dimension from suspension to CG, right side	inch
F_b	body heave force	lb
I_{xx}	body roll moment of inertia	slug-in^2
$k_{s,l}$	suspension stiffness, left side	lb/in
$k_{s,r}$	suspension stiffness, right side	lb/in
$k_{t,l}$	tire stiffness, left side	lb/in
$k_{t,r}$	tire stiffness, right side	lb/in
m_b	vehicle body mass	slug
$m_{t,l}$	tire mass, left side	slug
$m_{t,r}$	tire mass, right side	slug
T_b	body roll force (torque)	in-lb

The states of the model are shown by z_b, Θ_b, $z_{t,l}$, $z_{t,r}$, $z_{in,l}$, and $z_{in,r}$, where z_b and Θ_b represent the heave and roll motions of the truck body, $z_{t,l}$ and $z_{t,r}$ represent the heave motions of the left and right vehicle tires, and $z_{in,l}$ and $z_{in,r}$ represent the base (or road) inputs into the left and right tires of the model. The displacement, velocity, and acceleration components of the vehicle states are described in Table 3.2.

Table 3.2. Roll-Plane Vehicle Model States

Symbol	Description	Units
z_b	body heave displacement	inch
$\dot{z}_b$	body heave velocity	in/sec
$\ddot{z}_b$	body heave acceleration	in/sec^2
$z_{in,l}$	road input displacement, left side	inch
$z_{in,r}$	road input displacement, right side	inch
$z_{t,l}$	tire heave displacement, left side	inch
$\dot{z}_{t,l}$	tire heave velocity, left side	in/sec
$\ddot{z}_{t,l}$	tire heave acceleration, left side	in/sec^2
$z_{t,r}$	tire heave displacement, right side	inch
$\dot{z}_{t,r}$	tire heave velocity, right side	in/sec
$\ddot{z}_{t,r}$	tire heave acceleration, right side	in/sec^2
Θ_b	body roll displacement, left side	rad
$\dot{\Theta}_b$	body roll velocity, left side	rad/sec
$\ddot{\Theta}_b$	body roll acceleration, left side	rad/sec^2

The dynamics of the roll-plane model in Figure 3.1 are described by:

$$\begin{aligned} m_b\ddot{z}_b + c_{s,l}(\dot{z}_b - \dot{z}_{t,l} - d_{b,l}\dot{\Theta}_b) + c_{s,r}(\dot{z}_b - \dot{z}_{t,r} + d_{b,r}\dot{\Theta}_b) + k_{s,l}(z_b - z_{t,l} - d_{b,l}\Theta_b) \\ + k_{s,r}(z_b - z_{t,r} + d_{b,r}\Theta_b) = F_b \end{aligned} \tag{3.1}$$

$$m_{t,l}\ddot{z}_{t,l} - c_{s,l}(\dot{z}_b - \dot{z}_{t,l} - d_{b,l}\dot{\Theta}_b) - k_{s,l}(z_b - z_{t,l} - d_{b,l}\Theta_b) + k_{t,l}z_{t,l} = k_{t,l}z_{in,l} \tag{3.2}$$

$$m_{t,r}\ddot{z}_{t,r} - c_{s,r}(\dot{z}_b - \dot{z}_{t,r} - d_{b,r}\dot{\Theta}_b) - k_{s,r}(z_b - z_{t,r} + d_{b,r}\Theta_b) + k_{t,r}z_{t,r} = k_{t,r}z_{in,r} \tag{3.3}$$

$$\begin{aligned} I_{xx}\ddot{\Theta}_b + c_{s,l}d_{b,l}(\dot{z}_b - \dot{z}_{t,l} - d_{b,l}\dot{\Theta}_b) + c_{s,r}d_{b,r}(\dot{z}_b - \dot{z}_{t,r} + d_{b,r}\dot{\Theta}_b) - k_{s,l}d_{b,l}\,(z_b \\ - z_{t,l} - d_{b,l}\Theta_b) + k_{s,r}d_{b,r}(z_b - z_{t,r} + d_{b,r}\Theta_b) = T_b \end{aligned} \tag{3.4}$$

where all parameters and variable states are defined in Tables 3.1 and 3.2. Equations (3.1) through (3.4) can be represented in matrix form as:

$$M\ddot{\bar{z}} + C\dot{\bar{z}} + K\bar{z} = f \tag{3.5}$$

where M, C, and K represent the mass, damping, and stiffness matrices described by:

$$M = diagonal\left[m_b,\ m_{t,l},\ m_{t,r},\ I_{xx}\right],$$

$$C = \begin{bmatrix} (c_{s,l} + c_{s,r}), & -c_{s,l}, & -c_{s,r}, & (-c_{s,l}d_{b,l} + c_{s,r}d_{b,r}) \\ -c_{s,l}, & c_{s,l}, & 0, & c_{s,l}d_{b,l} \\ -c_{s,r}, & 0, & c_{s,r}, & -c_{s,r}d_{b,r} \\ (-c_{s,l}d_{b,l} + c_{s,r}d_{b,r}), & c_{s,l}d_{b,l}, & -c_{s,r}d_{b,r}, & (c_{s,l}d_{b,l}^2 + c_{s,r}d_{b,r}^2) \end{bmatrix},$$

$$K = \begin{bmatrix} (k_{s,l} + k_{s,r}), & -k_{s,l}, & -k_{s,r}, & (-k_{s,l}d_{b,l} + k_{s,r}d_{b,r}) \\ -k_{s,l}, & (k_{s,l} + k_{t,l}), & 0, & k_{s,l}d_{b,l} \\ -k_{s,r}, & 0, & (k_{s,r} + k_{t,r}), & -k_{s,r}d_{b,r} \\ (-k_{s,l}d_{b,l} + k_{s,r}d_{b,r}), & k_{s,l}d_{b,l}, & -k_{s,r}d_{b,r}, & (k_{s,l}d_{b,l}^2 + k_{s,r}d_{b,r}^2) \end{bmatrix},$$

and the force vector, f, is defined as:

$$f = \begin{bmatrix} F_b \\ k_{t,l} \times z_{in,l} \\ k_{t,r} \times z_{in,r} \\ T_b \end{bmatrix}.$$

The displacement vector $\bar{z}$ is described by:

$$\bar{z} = \begin{bmatrix} z_b \\ z_{t,l} \\ z_{t,r} \\ \Theta_b \end{bmatrix}.$$

The velocity and acceleration vectors, $\dot{\bar{z}}$ and $\ddot{\bar{z}}$, are similar to $\bar{z}$, and are defined as:

$$\dot{\bar{z}} = \begin{bmatrix} \dot{z}_b \\ \dot{z}_{t,l} \\ \dot{z}_{t,r} \\ \dot{\Theta}_b \end{bmatrix},$$

and

$$\ddot{\bar{z}} = \begin{bmatrix} \ddot{z}_b \\ \ddot{z}_{t,l} \\ \ddot{z}_{t,r} \\ \ddot{\Theta}_b \end{bmatrix}.$$

3.2 Semiactive Damping Control Models

In Chapter 2, a quarter-car model was used to explain the semiactive damping schemes of on-off skyhook, continuous skyhook, on-off groundhook, and fuzzy logic control. In the following sections, each of these control schemes will be formulated for the roll-plane model of Figure 3.1.

3.2.1 On-Off Skyhook Control

In Section 2.3.1, the on-off skyhook control policy was explained for a quarter-car model. In Equations (2.2a) and (2.2b), the basis for on-off skyhook control was presented. When the product of the absolute velocity of the vehicle body and the relative velocity across the damper is greater than or equal to zero, the damper is adjusted to its high state. If this product is negative, then the low state of the damper is applied.

For the roll-plane model of Figure 3.1, the relative velocity across each of the suspension dampers is computed as:

$$v_{rel,l} = \dot{z}_b - \dot{z}_{t,l} - d_{b,l}\dot{\Theta}_b \tag{3.6a}$$

$$v_{rel,r} = \dot{z}_b - \dot{z}_{t,r} + d_{b,r}\dot{\Theta}_b \tag{3.6b}$$

where $v_{rel,l}$ and $v_{rel,r}$, respectively represent the relative velocity across the left and right dampers of the roll-plane model. The absolute velocity of the vehicle body mass attached to the left and right side of each damper is thus calculated as:

$$v_{abs,l} = \dot{z}_b - d_{b,l}\dot{\Theta}_b \tag{3.7a}$$

$$v_{abs,r} = \dot{z}_b + d_{b,r}\dot{\Theta}_b \tag{3.7b}$$

Therefore, the on-off skyhook control policy as it applies to the roll-plane model of Figure 3.1 can be formulated by:

<u>Left Damper</u>

$$v_{abs,l} \times v_{rel,l} \geq 0; \qquad c_{s,l} = high\ state \tag{3.8a}$$

$$v_{abs,l} \times v_{rel,l} < 0; \qquad c_{s,l} = low\ state \tag{3.8b}$$

<u>Right Damper</u>

$$v_{abs,r} \times v_{rel,r} \geq 0; \qquad c_{s,r} = high\ state \tag{3.9a}$$

$$v_{abs,r} \times v_{rel,r} < 0; \qquad c_{s,r} = low\ state \tag{3.9b}$$

3.2.2 Continuous Skyhook Control

An extension of the on-off skyhook control policy was used as one method of continuous control. As explained in Section 2.3.2, when the product of the absolute velocity of the vehicle body and the relative velocity across the damper is less than zero, low state damping is applied. When this product is positive or equal to zero, the damping value applied by the damper is equal to a gain times the absolute velocity of the vehicle body, maintained within the high and low state limits of the damper. This concept, as it pertains to the left and right dampers of the roll-plane model, is shown by:

Left Damper

$$v_{abs,l} \times v_{rel,l} \geq 0; \qquad c_{s,l} = \max\{\textit{low state}, \min[(\Omega \times v_{abs,l}), \textit{high state}]\} \tag{3.10a}$$

$$v_{abs,l} \times v_{rel,l} < 0; \qquad c_{s,l} = \textit{low state} \tag{3.10b}$$

Right Damper

$$v_{abs,r} \times v_{rel,r} \geq 0; \qquad c_{s,r} = \max\{\textit{low state}, \min[(\Omega \times v_{abs,r}), \textit{high state}]\} \tag{3.11a}$$

$$v_{abs,r} \times v_{rel,r} < 0; \qquad c_{s,r} = \textit{low state} \tag{3.11b}$$

The symbol Ω represents the gain of the damper. Due to the symmetry of the model, the same gain is used for each damper.

3.2.3 On-Off Groundhook Control

For the roll-plane model of Figure 3.1, the relative velocity across each damper was previously determined in Equations (3.6a) and (3.6b), and the absolute velocities of the vehicle tire masses attached to the left and right side suspension dampers are $\dot{z}_{t,l}$ and $\dot{z}_{t,r}$, respectively. Thus, the on-off groundhook control policy as it applies to the roll-plane model can be summarized by:

Left Damper

$$\dot{z}_{t,l} \times v_{rel,l} \leq 0; \qquad c_{s,l} = \textit{high state} \tag{3.12a}$$

$$\dot{z}_{t,l} \times v_{rel,l} > 0; \qquad c_{s,l} = \textit{low state} \tag{3.12b}$$

Right Damper

$$\dot{z}_{t,r} \times v_{rel,r} \leq 0; \qquad c_{s,r} = \textit{high state} \tag{3.13a}$$

$$\dot{z}_{t,r} \times v_{rel,r} > 0; \qquad c_{s,r} = \textit{low state} \tag{3.13b}$$

As explained in Section 2.3.3, in on-off groundhook control, the product of the tire heave velocity and the relative velocity across the damper is used to determine the damper state. If this product is negative or zero, the high state of the damper is applied. Otherwise, the damper is adjusted to its low state.

3.3 Fuzzy Logic Control

In Chapter 2, Section 2.4, the three basic steps of a fuzzy logic controller were presented and explained. These steps were referred to as fuzzification, execution of rules, and defuzzification. In the following paragraphs, these three steps are presented as they pertain to the fuzzy logic controller design used for this research.

3.3.1 Step One: Fuzzification

As explained previously, the first step of a fuzzy logic controller is the fuzzification of the controller inputs, based on the membership function design for each input. For the roll-plane model of Figure 3.1, there exist two controllable dampers and, therefore, two controllers. Each fuzzy logic controller was designed with four inputs. For each damper controller, these inputs include the heave velocity of each suspension tire, $\dot{z}_{t,l}$ and $\dot{z}_{t,r}$, the relative velocity across the damper and the absolute velocity of the vehicle body, defined by Equations (3.6) and (3.7), and the absolute acceleration of the vehicle body, defined as:

$$a_{abs,l} = \ddot{z}_b - d_{b,l}\ddot{\Theta}_b \tag{3.14a}$$

$$a_{abs,r} = \ddot{z}_b + d_{b,r}\ddot{\Theta}_b \tag{3.14b}$$

where $a_{abs,l}$ and $a_{abs,r}$ respectively represent the absolute acceleration of the left and right sides of the vehicle body.

For each input, a trapezoidal membership function was used. Figure 3.2 shows the shape of the trapezoidal membership function as it was used for the fuzzification of the controller inputs.

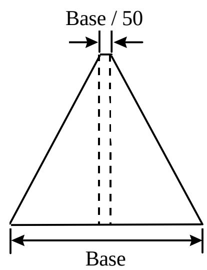


Figure 3.2. Trapezoidal-Shaped Membership Function Definition

For the trapezoidal-shaped membership functions used for this research, the width of the top of the trapezoid was defined as one-fiftieth of the width of the trapezoid base. The reason for using this flattened top triangle instead of a completely triangular-shaped function is that the trapezoid has been shown to smooth the response of the controller [21].

In Figure 3.3, each input and its membership function are defined for the left damper controller.

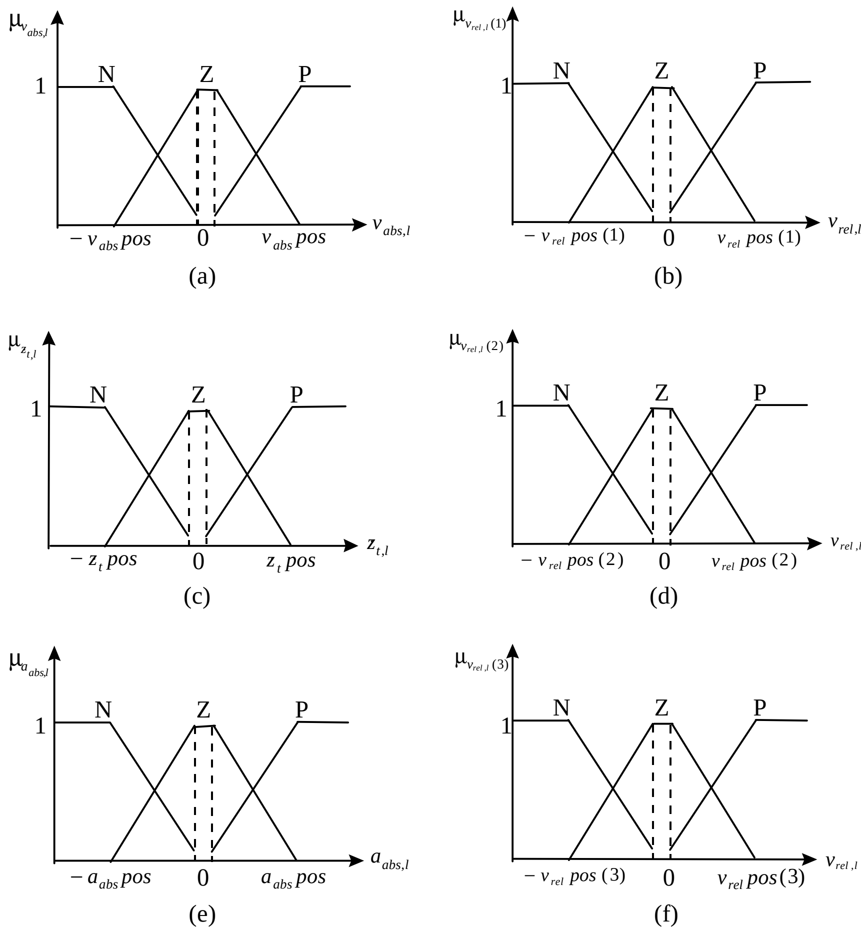


Figure 3.3. Left Damper - Controller Input Membership Functions

Each membership function is defined by three linguistic variables, Negative(N), Zero(Z), and Positive(P), and is symmetric about zero. There is a total of six membership functions used for four inputs because the relative velocity across the damper is fuzzified three times, using three separate membership functions.

In Figure 3.4, each input and its membership function are defined for the right damper controller.

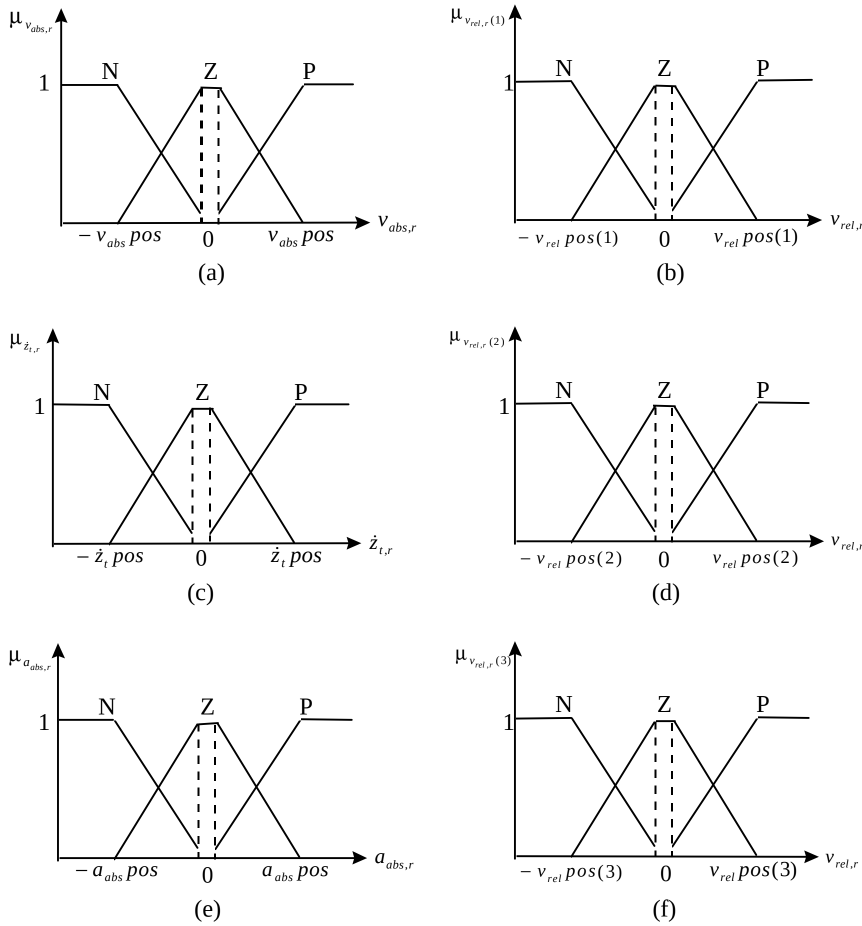
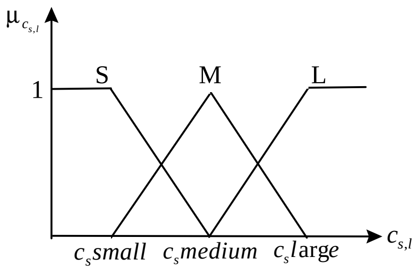


Figure 3.4. Right Damper - Controller Input Membership Functions

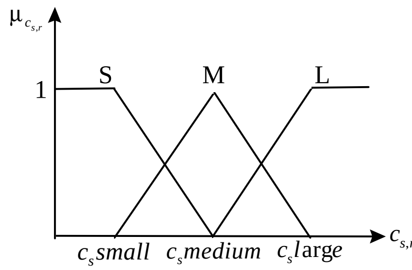
Because of the symmetry of the roll-plane model, the shapes of the membership functions for the left and right dampers of the model are the same, and the values used to describe the maximum values of each linguistic variable will be identical.

3.3.2 Step Two: Execution of Rules

As detailed in Section 2.4.2, prior to establishing the rules of the rule-base, the output membership function must be defined. For this research, the output of the controller is the value of damping to be exerted by the left and right dampers of the roll-plane model, $c_{s,l}$ and $c_{s,r}$. For the outputs, a triangular-shaped membership function is used. These functions are shown in Figure 3.5.



(a) Left Damper Controller Output



(b) Right Damper Controller Output

Figure 3.5. Controller Output Membership Functions

The same membership function is used for the controller output to the left and right damper. Each is described by three linguistic variables, Small(S), Medium(M), and Large(L), and the maximum value of each linguistic variable is the same for each damper. The output membership function will always be symmetric about $c_s medium$, which is defined as:

$$c_s medium = c_s small + \left[\frac{\left(c_s large - c_s small\right)}{2} \right] \tag{3.15}$$

The rules of the system can now be developed. The fuzzy logic controller rule-base for the left damper of the half-car model is detailed in Figure 3.6.

$v_{abs,l}$ \ $v_{rel,l}(1)$	N	Z	P
N	L	M	S
Z	M	M	M
P	S	M	L

(a) Rule Table 1L

$\dot{z}_{t,l}$ \ $v_{rel,l}(2)$	N	Z	P
N	S	M	L
Z	M	M	M
P	L	M	S

(b) Rule Table 2L

$a_{abs,l}$ \ $v_{rel,l}(3)$	N	Z	P
N	L	M	S
Z	M	M	M
P	S	M	L

(c) Rule Table 3L

Figure 3.6. Left Damper - Controller Rule-Base

The controller for this damper consists of three rule tables in the rule-base. For future reference, the rule tables of Figure 3.6 (a), (b), and (c), will be referred to as 1L, 2L, and 3L, respectively, as indicated on the figure. Each table is a function of either the absolute body velocity, body acceleration, or tire heave velocity and the relative velocity across the damper. As discussed previously, an individual membership function is used to describe the relative velocity of each rule table.

The logic of rule table 1L follows what may be called a *fuzzy skyhook* policy. By examining this rule table, it can be seen that when the $v_{abs,l}$ and $v_{rel,l}$ are both positive or both negative, corresponding to a positive product, the output is large, and when their signs are opposite, corresponding to a negative product, the output is small. When $c_s large$ is defined as the high-state damping and $c_s small$ is defined as the low-state damping and when each input is fully Negative or fully Positive, a skyhook policy is observed. However, when each input is not fully Negative or fully Positive, it is fuzzified according to the membership function, and a fuzzy skyhook algorithm is used.

In a similar manner, rule table 2L is deemed a *fuzzy groundhook* policy. When $\dot{z}_{t,l}$ and

$v_{abs,l}$ are the same sign, the output is small, and when they have opposite signs, the output is large. When each input is either fully Negative or fully Positive, a groundhook policy is observed; otherwise, the input is fuzzified according to its membership function and a fuzzy groundhook result is obtained.

Lastly, rule table 3L is identical to table 1L, with the exception that the control is based on the absolute body acceleration, not its velocity. During tuning of the controller, it was found that rule tables 1L and 2L were successful at reducing the body and tire displacements, but not the body accelerations. Thus, rule table 3L was added to assist in reducing the body accelerations to the desired level. This table may be considered a *fuzzy, acceleration-based skyhook* policy.

In Figure 3.7, the rule-base for the right damper of the roll-plane model is shown.

$v_{abs,r}$ \ $v_{rel,r}$ (1)	N	Z	P
N	L	M	S
Z	M	M	M
P	S	M	L

(a) Rule Table 1R

$\dot{z}_{t,r}$ \ $v_{rel,r}$ (2)	N	Z	P
N	S	M	L
Z	M	M	M
P	L	M	S

(b) Rule Table 2R

$a_{abs,r}$ \ $v_{rel,r}$ (3)	N	Z	P
N	L	M	S
Z	M	M	M
P	S	M	L

(c) Rule Table 3R

Figure 3.7. Right Damper - Controller Rule-Base

As with the left damper, the controller for the right damper consists of three rule tables in the rule base. Because of the symmetry of the model, the rule-base for the right tire is identical to the left tire rule-base of Figure 3.6. For future reference, the rule tables of

Figures 3.7 (a), (b), and (c) will be referred to as 1R, 2R, and 3R, as indicated on the figure.

3.3.3 Step Three: Defuzzification

As discussed previously, defuzzification converts the fuzzy values obtained from execution of the rule tables into a single, crisp value. The method of defuzzification used for this research is the weighted average method of Equation (2.5). However, the method has been expanded and revised to account for the existence of the three rule tables in the rule-base.

The weighted average method (explained in Section 2.4.3), as it pertains to the left damper of the vehicle model, is given by:

$$c_{s,l}^{*} = \frac{\Gamma_{1L}\Sigma\left[\mu_{c_{s,l}} \times \bar{c}_{s,l}\right]_{1L} + \Gamma_{2L}\Sigma\left[\mu_{c_{s,l}} \times \bar{c}_{s,l}\right]_{2L} + \Gamma_{3L}\Sigma\left[\mu_{c_{s,l}} \times \bar{c}_{s,l}\right]_{3L}}{\Gamma_{1L}\Sigma\left[\mu_{c_{s,l}}\right]_{1L} + \Gamma_{2L}\Sigma\left[\mu_{c_{s,l}}\right]_{2L} + \Gamma_{3L}\Sigma\left[\mu_{c_{s,l}}\right]_{3L}} \qquad (3.16)$$

where $\mu_{c_{s,l}}$ represents the weighting function of the output, $\bar{c}_{s,l}$ represents the maximum (median) linguistic variable value based on the output membership function, and Γ_{1L}, Γ_{2L}, and Γ_{3L} represent the ranking functions of rule tables 1L, 2L, and 3L, respectively, from Figure 3.6. In the defuzzification method of Equation (3.16), in the numerator, the products of the weighting function and corresponding maximum linguistic variable value are summed for each rule. In turn, these summations are each multiplied by the ranking function of their corresponding rule table before they are added. In the denominator, the summation of the weighting functions from each rule are also multiplied by their corresponding ranking function before their products are added. Thus, the ranking functions are used to rank the importance of each rule table during defuzzification.

In Equation (3.17), the weighted average defuzzification method is shown as it applies to the right damper of the roll-plane model.

$$c_{s,r}^{*} = \frac{\Gamma_{1R}\sum\left[\mu_{c,r}\times\bar{c}_{s,r}\right]_{1R} + \Gamma_{2R}\sum\left[\mu_{c_{s,r}}\times\bar{c}_{s,r}\right]_{2R} + \Gamma_{3R}\sum\left[\mu_{c_{s,r}}\times\bar{c}_{s,r}\right]_{3R}}{\Gamma_{1R}\sum\left[\mu_{c_{s,r}}\right]_{1R} + \Gamma_{2R}\sum\left[\mu_{c_{s,r}}\right]_{2R} + \Gamma_{3R}\sum\left[\mu_{c_{s,r}}\right]_{3R}} \qquad (3.17)$$

The defuzzification method for the right damper is identical to the defuzzification method used for the left damper. The symbols $\mu_{c_{s,r}}$ and $\bar{c}_{s,r}$ respectively represent the weighting function of the output and the maximum (median) linguistic variable value based on the output membership function, while Γ_{1R}, Γ_{2R}, and Γ_{3R} represent the ranking functions of rule tables 1R, 2R, and 3R, respectively, from Figure 3.7. Due to the symmetry of the model, the ranking values for the right damper will be identical to those used for the left damper.

Because the semiactive damper is limited by its high and low state, the crisp output values from Equations (3.16) and (3.17) must be maintained within the allowable damping range. This is accomplished for the left controller output by:

$$c_{s,l} = \max\{\textit{low state}, \min[c_{s,l}^{*}, \textit{high state}]\} \qquad (3.18)$$

Equation (3.18) implies that the crisp output value of the left controller, $c_{s,l}^{*}$, is limited to within the high-state and low-state damping, and that the actual value applied by the damper is determined by the computation result, $c_{s,l}$.

The crisp output of the right controller, $c_{s,r}^{*}$, is limited to within its high-state and low-state damping through:

$$c_{s,r} = \max\{\textit{low state}, \min[c_{s,r}^{*}, \textit{high state}]\} \qquad (3.19)$$