CHAPTER 3
THE INVERTED-L ANTENNA AND VARIATIONS

3.1 Introduction

As the demand for portable and convenient wireless devices becomes stronger, the need for device miniaturization increases. [1] The size of a wireless device is often limited by the dimensions of the battery and the antenna. [1] Generally, the quarter-wave monopole antenna is used on portable wireless devices due to its characteristically high radiation efficiency and wide bandwidth. [2] However, the monopole antenna is potentially obtrusive. It is desirable that the antenna in a wireless device be as small and inconspicuous as possible while still meeting performance demands. Therefore, a need exists for electrically small, low-profile antenna designs with broad impedance bandwidth, an input impedance that is easily matched to a feed line, omnidirectional radiation, and high radiation efficiency. [3]

One low-profile antenna design that is used in wireless applications is the Inverted-L antenna (ILA). The ILA is a short monopole with the addition of a horizontal segment of wire at the top. The ILA and a few of its variations are illustrated in Figure 3.1.
This chapter presents design issues for the ILA (#1 in Figure 3.1) and its variations. In Sections 3.2 and 3.3, closed form solutions for the radiation patterns and input impedance of the ILA are derived using an assumed current distribution on the antenna. Section 3.4 covers design issues for variations of the ILA, including the Inverted-F Antenna (IFA) (#2 in Figure 3.1), the Planar Inverted-F Antenna (PIFA) (#3 in Figure 3.1), and the Dual Inverted-F Antenna (DIFA) (#4 in Figure 3.1).

### 3.2 The Far-Field Radiation Pattern of the Inverted-L Antenna

The Inverted-L Antenna (ILA) in Figure 3.2 is an electrically small end-fed monopole with a section of horizontal wire added at the top as a capacitive load.
The ground plane image of the antenna, shown in Figure 3.2 with dotted lines, is treated as part of the antenna structure to simplify calculations. [4] The calculations are further simplified by rotating the Cartesian coordinate system as shown in Figure 3.3.

In Figure 3.3, the vertical element of the ILA is located on the x-axis, and the top horizontal segment and its image are z-directed with opposing currents. The resonant current distribution on each of the arms is assumed to be sinusoidal. The distributions are [4]
Using the current distributions in (3.1) through (3.3), the fields due to each arm are determined by calculating magnetic vector potentials, $A$, and integrating with respect to the radiation vector. [4] Arm 1 is $z$-directed with positive running current. The far-field magnetic vector potential on arm 1 is

$$A_z = \frac{e^{-jkr}}{4\pi r} \int_0^L I_1(z)e^{jkc\cos(\theta)} dz = \frac{e^{-jkr}}{4\pi r} \int_0^L I_z \sin(k(L-z')) \cos(\theta)e^{jkc\cos(\theta)} dz'$$

(3.4)

where $r = \sqrt{x^2 + y^2 + z^2}$. Using (3.4), the E-field for an offset $z$-directed line source is written as [4]

$$E = j\omega\mu \sin(\theta) \sin(kh \cdot \sin(\theta) \cdot \cos(\phi)) A_z a_\theta$$

(3.5)

Substituting (3.4) into (3.5) gives

$$E = j\omega\mu \frac{e^{-jkr}}{4\pi r} \sin(\theta) \sin(kh \cdot \sin(\theta) \cdot \cos(\phi)) I_z \cos(kh) \int_0^L \sin(k(L-z')) e^{jkc\cos(\theta)} dz'$$

(3.6)

The integration in (3.6) is solved using

$$\int \sin(a + bx) e^{cx} dx = \frac{e^{cx}}{b^2 + c^2} [c \sin(a + bx) - b \cos(a + bx)]$$

(3.7)

using (3.7) in (3.6) leads to [4]
\[
E_\theta = \frac{j\omega \mu}{4\pi \rho} e^{-jkr} \sin\theta (\text{SIN}) \cdot \cos(kh) \frac{e^{jkr\cos\theta}}{k^2 - k^2 \cos^2 \theta} \left( jk \cos \theta \sin(kL - k'z) + k \cos(kL - k'z) \right)
\]

where \( \text{SIN} = \sin(kh \cdot \sin(\theta) \cdot \cos(\phi)) \). When evaluated and simplified, this expression yields the \( \theta \)-component of the E-field produced by the currents on arm 1 of the ILA. The result is [4]

\[
E_\theta = -\frac{\omega \mu}{4\pi k} I_x \frac{e^{-jkr}}{r} \cos(kh) \frac{\text{SIN}}{\sin(\theta)} \left[ e^{jkl\cos\theta} - j\{\cos\theta \cdot \sin(kL) - \cos(kL)\} \right]
\]  (3.8)

Since the element is z-directed, \( E_\phi = 0 \).

By symmetry, the fields produced by the currents on arm 2 of the ILA are identical to those produced by arm 1. The fields produced by the currents on arm 3 of the structure are summarized without derivation as [4]

\[
E_\theta = -\frac{j\omega \mu}{4\pi k} I_x \frac{e^{-jkr}}{r} \sin(kL) \cos(\phi) \cdot f(\theta, \phi, h)
\]  (3.9)

\[
E_\phi = \frac{j\omega \mu}{4\pi k} I_x \frac{e^{-jkr}}{r} \sin(kL) \sin(\phi) \cdot f(\theta, \phi, h)
\]  (3.10)

where

\[
f(\theta, \phi, h) = \frac{\sin(kh) \cdot \cos(\phi) - \sin(\theta) \cdot \cos(\phi) \cdot \cos(kh) \cdot \text{SIN}}{1 - \sin^2 \theta \cdot \cos^2 \phi}
\]

in which \( \text{COS} = \cos(kh \cdot \sin(\theta) \cdot \cos(\phi)) \) and \( \text{SIN} \) was defined earlier.

The expressions for fields from the various arms of the ILA can be simplified if it is assumed that \( I_x = I_z = I_o \), \( L = \lambda/4 \), and \( h \ll \lambda \). Then the fields from arms 1 and 2 of the ILA are re-written as

\[
E_\theta = -\frac{\omega \mu}{4\pi k} I_o \frac{e^{-jkr}}{r} \cos(\phi) \left[ \cos\left(\frac{\pi}{2} \cos\theta\right) + j\{\sin\left(\frac{\pi}{2} \cos\theta\right) - \cos\theta\} \right]
\]  (3.11)

and \( E_\phi = 0 \). The fields due to the currents on arm 3 of the ILA are
The fields expressed in (3.11) through (3.13) define the radiation fields of the ILA in the three principal planes, x-y, y-z, and x-z. In the x-y plane, $\theta = \pi/2$, and the E-fields are

$$E_\theta = -60I_o \frac{e^{-jkr}}{r} kh\cos(\phi) \text{ from arms 1 and 2},$$

and

$$E_\phi = j60I_o \frac{e^{-jkr}}{r} kh\sin\phi \text{ from arm 3}.$$  

In the x-z plane, $\phi = 0$, and the E-fields are

$$E_\theta = -\frac{j\omega\mu}{4\pi k} I_o \frac{e^{-jkr}}{r} kh\left[\cos\left(\frac{\pi}{2}\cos\theta\right) + j\left\{\sin\left(\frac{\pi}{2}\cos\theta\right) - \cos\theta\right\}\right] \text{ from arms 1 and 2},$$

and

$$E_\phi = -\frac{j\omega\mu}{4\pi k} I_o \frac{e^{-jkr}}{R} kh\cos\theta \text{ from arm 3}.$$ 

In the y-z plane, there is no field produced by arms 1 and 2. The field due to arm 3 is

$$E_\phi = \frac{j\omega\mu}{4\pi k} I_o \frac{e^{-jkr}}{R} kh\sin\phi \text{ from arm 3}.$$ 

Thus, the normalized pattern factors in the x-y plane are, for $E_\theta$,

$$F(\phi) = \cos(\phi) \text{ from arms 1 and 2}$$

and, for $E_\phi$,

$$F(\phi) = \sin(\phi) \text{ from arm 3}$$

and in the x-z plane
\[ F(\theta) = \cos\left(\frac{\pi}{2}\cos\theta\right) + j\left\{\sin\left(\frac{\pi}{2}\cos\theta\right) - \cos\theta\right\} \text{ from arms 1 and 2} \quad (3.21) \]

and

\[ F(\theta) = \cos(\theta) \text{ from arm 3.} \quad (3.22) \]

and in the y-z plane

\[ F(\theta) = 1 \quad (3.23) \]

The radiation patterns of the ILA are plotted in Figure 3.4 using the normalized pattern factors given by (3.19) through (3.23).

**Figure 3.4.** The modeled radiation patterns of the Inverted-L Antenna, using the antenna system in Figure 3.3. [4]

The coordinate system used in Figure 3.4 was defined in Figure 3.3. Figure 3.4(c) shows that the radiation pattern of the ILA is omni-directional in azimuth. The patterns are identical to those of a monopole in the y-z and x-y planes. In the x-z plane, however, two \( E_\theta \) components are generated, one by arms 1 and 2 and one by arm 3. The components
vary in phase and have different points of maximum radiation. When the two components combine, the nulls in both patterns are filled to give nearly omnidirectional coverage. [4]

Figure 3.4 illustrates how the various current components on the ILA affect the far-field radiation. In the next section, the input impedance of the ILA is derived by assuming a sinusoidal current distribution on the antenna.

### 3.3 The Input Impedance of the Inverted-L Antenna

Before applying Pocklington’s equation to the Inverted-L Antenna (ILA), it is worthwhile to review the derivation for a z-directed current source. The electric field induced by a distributed current on an antenna structure is written as a combination of a vector potential $A$ and a scalar potential $\Phi$, \[ 5 \]

$$ E = - j \omega \mu_o A - \nabla \Phi $$ \hspace{1cm} (3.24)

For a z-directed current, (3.24) can be written in scalar form. The result is \[ 5 \]

$$ E_z = - j \omega \mu_o A_z - \frac{\partial \Phi}{\partial z} $$ \hspace{1cm} (3.25)

The Lorentz gauge condition for a z-directed current is \[ 5 \]

$$ \frac{\partial A_z}{\partial z} = - j \omega \mu_o \Phi $$ \hspace{1cm} (3.26)

using the derivative of (3.26) in (3.25), gives \[ 5 \]

$$ E_z = - \frac{1}{j \omega \mu_o} \left( \frac{\partial^2 A_z}{\partial z^2} + \beta^2 A_z \right) $$ \hspace{1cm} (3.27)

where $\beta = \omega \sqrt{\mu_o \varepsilon_o}$ is the phase constant for a plane wave. The free space Green’s function is given by

$$ \Psi = \frac{e^{-j \beta R}}{4\pi R} $$ \hspace{1cm} (3.28)
where \( R \) is defined as the distance between the source point, \((x', y', z')\), and the observation point, \((x, y, z)\), written as

\[
R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}
\]  

(3.29)

For a \( z \)-directed element of current, \( J \, dv' \) [5]

\[
dE_z = \frac{1}{j\omega \varepsilon_0} \left( \frac{\partial^2 \Psi(z,z')}{\partial z'^2} + \beta^2 \Psi(z,z') \right) J \, dv'
\]  

(3.30)

The total contribution to the electric field is the integral of (3.30) over the volume in which the current exists. If the current is running along a cylindrical wire of very high conductivity, such as that illustrated in Figure 3.5(a), nearly all of the current will exist on the surface of the wire. [5]

![Diagram](image)

**Figure 3.5.** (a) Surface current on a cylindrical wire with cross sectional curve \( c \). (b) Equivalent filamentary line source for (a). [5]
If infinite conductivity is assumed, all of the current exists on the surface of the wire, and the volume integral reduces to [5]

\[
E_z = \frac{1}{j\omega\epsilon_0} \oint \left( \frac{\partial^2 \Phi(z,z')}{\partial z^2} + \beta^2 \Phi(z,z') \right) J_z d\phi \tag{3.31}
\]

where \( c \) is the cross sectional curve of the wire in Figure 3.5(a), and \( J_s \) is the current on the surface of the wire. If the surface current is observed from a point along the wire axis in Figure 3.5(b),

\[
R = \sqrt{(z - z')^2 + a^2} \quad (3.32)
\]

where \( a \) is the radius of the wire. If the wire is sufficiently thin, so that \( a \ll \lambda \), circumferential current does not exist, and the longitudinal current is nearly uniform around the circumference of the wire. Using this thin wire approximation, (3.31) reduces to [5]

\[
E_z = \frac{1}{j\omega\epsilon_0} \int_{-L/2}^{L/2} \left( \frac{\partial^2 \Phi(z,z')}{\partial z^2} + \beta^2 \Phi(z,z') \right) I(z') dz \tag{3.33}
\]

that is the integration of current over the equivalent filamentary line source illustrated in Figure 3.5(b). Although the integration is over an infinitely thin filament of current, the cross sectional area of the wire is included since the wire radius, \( a \), is included in (3.32) and thus in (3.28) and (3.33). [5]

The expression in (3.33) gives the scattered field. [5] At the surface of the perfectly conducting wire, the scattered field is equal to the incident or impressed field. [5] Therefore, \( E_{z,s} = -E_{z,i} \), and

\[
-E_{z,i} = \frac{1}{j\omega\epsilon_0} \int_{-L/2}^{L/2} \left( \frac{\partial^2 \Phi(z,z')}{\partial z^2} + \beta^2 \Phi(z,z') \right) I(z') dz \tag{3.34}
\]

This is the general form of Pocklington’s integral equation, valid for \( z \)-directed currents with the thin wire approximation. [5]
Pocklington’s integral equation is formulated to express the fields radiated by an electrically small ILA by using the \( z \)-directed currents on the vertical segment as well as the \( x \)-directed currents on the horizontal segment. In Figure 3.2, the ILA is modeled with an infinite perfectly conducting ground plane. The ground plane image of the ILA is treated as part of the antenna structure. Currents flow on the ILA in both the \( x \) and \( z \) directions. Therefore, the vector magnetic potential, \( \mathbf{A} \), exists in the \( x \) and \( z \) directions and the scalar potential, \( \Phi \), includes the \( x \) and \( z \) dimensions. [6] The vector magnetic potential, and scalar potentials are related to one another by the Lorentz gauge equation: [6]

\[
\frac{\partial A_x}{\partial x} + \frac{\partial A_z}{\partial z} + j\omega\varepsilon \Phi = 0
\]  

(3.35)

Differentiating with respect to \( z \) yields

\[
-\frac{\partial \Phi}{\partial z} = \frac{1}{j\omega\varepsilon} \left( \frac{\partial^2 A_x}{\partial z^2} + \frac{\partial}{\partial z} \frac{\partial A_x}{\partial x} \right)
\]

(3.36)

Substituting this expression into (3.25) gives

\[
E_z = \frac{1}{j\omega\varepsilon} \left( \beta^2 A_z + \frac{\partial^2 A_z}{\partial z^2} + \frac{\partial}{\partial z} \frac{\partial A_z}{\partial x} \right)
\]

(3.37)

The derivation of Pocklington’s integral equation in two dimensions is the same as that for the general form of Pocklington’s equation, except that (3.25) is replaced by (3.37). The result is the electric field in the \( z \) direction due to both the \( x \) and \( z \)-directed currents on the ILA and its image. A two dimensional form of Pocklington’s integral equation that is valid for radiation problems involving the ILA is [6]

\[
-E_z^i = \frac{1}{j\omega 4\pi\varepsilon} \int_{-h}^{h} I(z') \left[ \frac{\partial^2 \psi_z(z, z')}{\partial z'^2} + \beta^2 \psi_z(z, z') \right] dz'
\]
\[ + \frac{1}{j\omega 4\pi\varepsilon} \left[ \int_{a}^{L+a} I_x(x') \left[ \frac{\partial}{\partial z} \frac{\partial \psi_\alpha(x,x',z)}{\partial x} \right] \, dx' \right] + \frac{1}{j\omega 4\pi\varepsilon} \int_{a}^{L+a} I_{ab}(x') \left[ \frac{\partial}{\partial z} \frac{\partial \psi_{ab}(x,x',z)}{\partial x} \right] \, dx' \]  

(3.38)

where the current distributions on each arm of the ILA, \( I_z(z') \), \( I_x(x') \), and \( I_{ab}(x') \) are unknown and the Green’s functions are

\[
\psi_z(z, z') = \frac{e^{-\beta R}}{R}, \quad \psi_x(x, x', z) = \frac{e^{-\beta R_x}}{R_x}, \quad \psi_{ab}(x, x', z) = \frac{e^{-\beta R_b}}{R_b}
\]

where

\[
R = \sqrt{(z'-z)^2 + a^2} \\
R_x = \sqrt{(h-z)^2 + (x'-x)^2} \\
R_b = \sqrt{(h+z)^2 + (x'-x)^2}
\]

If the ILA is electrically small (\( kh < 0.5 \) and \( kL < 0.5 \) for \( k = 2\pi / \lambda \)), the currents vary linearly with a maximum at the generator and zero at the free end. [6] Thus, the unknown current distributions in (3.38) are replaced by

\[
I_z(z') = I_a \left( 1 - \frac{|z'|}{h+L} \right) \quad \text{for } -h \leq z' \leq h
\]

(3.39)

\[
I_x(x') = I_a \left( 1 - \frac{x-a+h}{h+L} \right) \quad \text{for } a \leq x' \leq L + a \quad \text{and} \quad z' = h
\]

(3.40)

\[
I_{ab}(x') = -I_a \left( 1 - \frac{x-a+h}{h+L} \right) \quad \text{for } a \leq x' \leq L + a \quad \text{and} \quad z' = -h
\]

(3.41)

An accurate analysis of the current in the bend between the vertical and horizontal segments is not performed. However, the current entering the bend must equal the current leaving the bend. Therefore, the current at \( z' = h \) is equivalent to that at \( x' = a \). [6]

Once the current distribution on the ILA is assumed, the radiated electric fields
follow immediately from (3.38). Substituting (3.39) through (3.41) into (3.38) gives the impressed, or incident $z$-directed electric field, $-E_z^i$, in terms of an unknown current at the generator, $I_o = I_z(0)$.

Given the impressed electric field, the input impedance of the ILA is derived using the equivalent circuit in Figure 3.6.

![Figure 3.6. Equivalent circuit of the ILA [6]](image)

In Figure 3.6, $Z_A$ is the input impedance of the antenna, and the current source at the feed point of the ILA is replaced by a complex impedance, $Z_L$. The Thevenin voltage, $V_T$, is the phasor potential across $Z_L$ when $Z_L$ is infinite. [6] To determine the phasor potential, $V_T$, the ILA radiation problem is related to a scattering problem using superposition. That is, $E_z^i$ is treated as the $z$-directed electric field induced by a plane wave polarized opposite the spherical unit vector $a_\theta$. [6] The situation is illustrated in Figure 3.7.

![Figure 3.7. A center fed ILA, and the spherical unit vector $a_\theta$. [6]](image)
The Thevenin voltage induced by the plane wave in Figure 3.7 is \( V_T = E_\theta L_\theta \) where \( L_\theta \) is the component of the vector equivalent length of the ILA in the direction of \( E_\theta \). [6] If \( E_\theta \) approaches perpendicular to the \( x \)-axis (\( \theta = \pi/2 \)), the vector equivalent length of the ILA is [6]

\[
L_\theta \left( \frac{\pi}{2} , 0 \right) = h \left( 2 - \frac{h}{h + L} \right)
\]  

(3.42)

Assuming a lossless current source (\( Z_L = 0 \)), the current \( I_o \) in Figure 3.6 is \( V_T / Z_A = E_\theta L_\theta / Z_A = -E_z^i L_\theta / Z_A \) and an expression for input impedance follows directly as [6]

\[
Z_A = \left( \frac{E_z^i}{I_o} \right) L_\theta
\]

(3.43)

which is in terms of the unknown current \( I_o = I_z(0) \). The expression in (3.38) for impressed electric field, \( -E_z^i \), is also in terms of \( I_o \). Thus, a solution for \( I_o \) is necessary to determine the \( z \)-directed electric field and input impedance of the ILA.

A point matching technique is used to solve for \( I_o \). It is known that the magnitude of the tangential electric field at the boundary of a perfect conductor is zero. Thus, at any point on the vertical segment of the ILA, the \( z \)-directed electric field must go to zero. This boundary condition is enforced by setting \( E_z^i \) in (3.38) to zero at \( z = h / 2 \). By symmetry, this enforces the same condition at \( z = -h / 2 \). [6] Solving (3.38) using the boundary condition yields an approximation for \( I_o \).

Using the point matching approximation for \( I_o \), the expression in (3.38) is integrated to derive \( E_z^i \). The exponentials in the free space Green’s functions are estimated using third order Maclaurin series representations such as [6]

\[
e^{-jkR} \approx \left[ 1 - jkR - \frac{(kR)^2}{2!} + \frac{j(kR)^3}{3!} \right]
\]  

(3.44)
The results of the integration are expressions for the input resistance and reactance of an electrically small ILA. The expression for input resistance is [6]

\[ R_{\text{IL}} = 15(kh)^2 \left[ 2 - \frac{h}{h+L} \left( 2 - \frac{10}{9} \left( \frac{h^2 + \frac{2}{3} L^2 + \frac{2}{9} h(L+a)}{(h+L)^2} \right) \right) \right] \]  

(3.45)

and the reactance is [6]

\[
X_{\lambda} = \frac{-60h}{k(h+L)^2} \left[ \ln(\frac{\sqrt{3}h}{a}) - 3h - 20a \frac{L_T - \frac{h}{4}}{9h} + \frac{L_T - 3h}{4} \frac{L_T + \frac{h^2}{4}}{9h^2} - \frac{3(kh)^2}{8} \right. \\
+ \left. \frac{1}{2} \frac{k^2h^2}{4} \left[ 1 - \left( 1 + \frac{L}{h} \right)^2 \right] \right] \ln(\frac{\sqrt{3}h}{a}) + \frac{k^2h^2}{8} \frac{1 + L_T}{h} + \frac{L_T}{8} \frac{L_T + \frac{9h^2}{4}}{8} \frac{9 + L_T}{h^2} \\
+ \frac{k^2h^2}{8} L_T \ln \left( \frac{L_T + \frac{L_T^2 + \frac{h^2}{4}}{h^2 + a} \right) + \frac{3k^2h^2}{8} T \ln \left( \frac{L_T + \frac{L_T^2 + \frac{9h^2}{4}}{\frac{5h}{2} + a} \right) \right] 
\]

(3.46)

where \( L_{\lambda} = L + a \) and \( T = 1 - a / h \).

It is also possible to estimate the input resistance of the ILA using Poynting’s theorem and the current distributions in (3.39) through (3.41). The complex Poynting vector is integrated over all points in the far field of the antenna, and the result is the total radiated power from the antenna, \( P_R \). The result of the integration is used in

\[ P_R = \frac{I_0^2 R_{LP}}{2} \]  

(3.47)

where \( R_{LP} \) is the input resistance of the ILA. If terms of the order \([kh]^4\), \([kL]^4\), and \([(kh)^2(kL)^2]\), and smaller are dropped from the result of the integration of the Poynting vector, the input resistance of the ILA is

\[ R_{LP} = 40(kh)^2 \left( 1 - \frac{h}{2(h+L)} \right)^2 \]  

(3.48)

The input impedance of the ILA is calculated using (3.45) and (3.48). The results are given in Figure 3.8 which shows that when Pocklington’s integral equation is used to
model the input resistance of the ILA, the results depend on wire diameter. Poynting’s theorem, however, is independent of wire diameter. There is a 20% discrepancy between the results predicted by Pocklington’s equation (see (3.45)) and by Poynting’s theorem (see (3.48)). The input resistance of the ILA is small, below $7 \, \Omega$, and there is a sharp decrease in input resistance as the size of the ILA is reduced.

![Figure 3.8](image)

**Figure 3.8.** (a) Input resistance of the ILA illustrated in Figure 3.2 calculated using (3.45) and (3.48) with $kh = 0.3$, with $a/h = 0.01$ and $a/h = 0.1$. (b) Input resistance of the ILA calculated using (3.45) and (3.48) with $kh = 0.5$, with the same variations in $a/h$ used in (a). [6]

The accuracy of equations (3.45) and (3.46) is limited by the point matching technique used to estimate $I_o$. If the boundary condition is enforced on the vertical segment of the antenna at a point $z > 0$, the symmetry of the antenna enforces another sample point below the ground plane. It is possible to simplify (3.46) by enforcing the boundary condition at $z = 0$. However, the increased precision due to antenna symmetry is lost. The modeled input reactance of the ILA with the boundary condition enforced at $z = 0$ is
The input reactances of two electrically small ILAs with different wire diameters are modeled using (3.46) and (3.49) and the results are compared in Figure 3.9.

\[
X_L = \left[ -60h \left( 2 - \frac{h}{h + L} \right) \right] \ln \left( \frac{2h}{a} \right) - \frac{a}{h} + \frac{L_0 T - h}{\sqrt{L_a^2 + h^2}} + \frac{5(kh)^2}{8} \\
+ \left\{ \frac{k^2 h^2}{2} \left( 1 + \frac{L}{h} \right)^2 \right\} \ln \left( \frac{2h}{a} \right) + \frac{k^2 h^2}{2} \sqrt{1 + \frac{L_0^2}{h^2}} \\
+ \frac{k^2 h^2}{2} T \ln \left( \frac{L_0 + \sqrt{L_a^2 + h^2}}{h + a} \right) 
\]  

(3.49)

Figure 3.9. Input reactance of the ILA illustrated in Figure 3.2 calculated using (3.46) and (3.49) with \( kh = 0.3 \) and with \( a/h = 0.01 \) and \( a/h = 0.1 \). [6]
Figure 3.9 shows that agreement between (3.46) and (3.49) improves as $L$ increases. The estimations are closest for large values of $L$ and small values of $a$. In the absence of experimental data, (3.46) is more accurate due to antenna symmetry.

The low input resistance and high input reactance of the ILA make it difficult to match to a standard coaxial feedline. One way to tune the input impedance of the ILA is to change the structure of the antenna. In the next section, a number of variations on the ILA are treated that optimize input impedance and frequency bandwidth.

### 3.4 Variations on the Inverted-L Antenna

#### 3.4.1. The Inverted-F Antenna

The ILA consists of a short vertical monopole with the addition of a long horizontal arm at the top. Its input impedance is nearly equivalent to that of the short monopole with the addition of the reactance caused by the horizontal wire above the ground plane. [4] The ILA is generally difficult to impedance match to a feedline since its input impedance consists of a low resistance and high reactance. Since loss due to mismatch decreases radiation efficiency, it is desirable to modify the structure of the ILA to achieve a nearly resistive input impedance that is easily matched to a standard coaxial line.

The ILA structure is commonly modified by adding another inverted-L element to the end of the vertical segment to form the Inverted-F Antenna (IFA) shown in Figure 3.10.
The IFA in Figure 3.10 is identical to a transmission line antenna of length \((h + L)\) fed at the tap point, \(t\). Alternately, the configuration is treated as a small loop inductor, consisting of the feed probe and the inverted-L element behind the feed, resonated with the capacitance of a horizontal wire above a ground plane.

The addition of the extra inverted-L element behind the feed tunes the input impedance of the antenna. [4] The impedance is adjusted by changing the tap point. [4] Figure 3.11 illustrates how the input impedance of an IFA changes when the tap point is altered.

**Figure 3.11.** a) The dimensions of the IFA (in mm) used to determine the effect of tap point placement. b) Impedance of an Inverted-F Antenna for various tap points. Points on the graph are frequency, denoted in MHz. [4]
The antenna in Figure 3.11(a) was designed to resonate at 730 MHz with $l_s = 7$ mm. Figure 3.11(b) shows that moving the tap point away from the short circuit stub increases the resonant frequency of the ILA and decreases the input resistance at resonance. [4] The impedance tuning feature has made the IFA more popular than the ILA in practical low-profile applications. [4]

One disadvantage of an IFA constructed using thin wires is low impedance bandwidth. Typically, a single IFA element experiences an impedance bandwidth of less than 2% of the center frequency. [4] One way to increase the bandwidth of the IFA is to replace the top horizontal arm with a plate oriented parallel to the ground plane to form the Planar Inverted-F Antenna (PIFA). The PIFA is the subject of the next section.

### 3.4.2. The Planar Inverted-F Antenna

To improve the narrow impedance bandwidth of the IFA, the thin wire horizontal segment is replaced by a flat conducting plate oriented parallel to the groundplane. The result is the Planar Inverted-F Antenna (PIFA) shown in Figure 3.12. The PIFA is widely applied as a low profile antenna design. [4] Specifically, it has found wide spread use in hand-held radio phone units in Japan. [8]

*Figure 3.12. Geometry of the Planar Inverted-F Antenna (PIFA)*
The PIFA in Figure 3.12 consists of a planar element with an off-center probe feed. The feed line is coaxial with the outer conductor connected to ground and the center conductor emerging from beneath the ground plane to contact the planar element. One edge of the PIFA is shorted to ground using a plate of width $W < L_1$. The physical action of the PIFA is a combination of the IFA and the short-circuited air substrate rectangular microstrip patch (see Chapter 5). [7]

In [7], the currents on the PIFA are modeled using a three dimensional electromagnetic field time-domain numerical method called the spatial network method (SNM). In the SNM, The PIFA is broken up into a three dimensional grid of cubes. The dimension of each cube is $\Delta d$. The feed is modeled as a coaxial line with a center and outer conductor. The width of the outer conductor is $2\Delta d$. The extent of the gridding affects the results of the analysis, and is selected so that the numerical results converge. The PIFA is excited at resonance through the coaxial feed. The resulting electric and magnetic node components correspond to the amplitude of the electric field and current distribution, respectively.

The magnitude of the electric fields in the $x$, $y$, and $z$-planes are modeled using the SNM and illustrated in Figure 3.13. Figure 3.13 shows that the electric field under the planar element of the PIFA is $z$-directed. The $z$-directed electric field, $E_z$, is zero at the short-circuit and maximum at the free end of the planar element. [7] The distribution of $E_z$ is similar to that under the rectangular short-circuited microstrip patch antenna. [7] The electric fields, $E_x$ and $E_y$ are generated at all open edges of the planar element. These fringing fields are a radiating mechanism of the PIFA. [7]

The surface current distributions on a resonant PIFA, modeled using the SNM, are illustrated in Figure 3.14 for different widths, $W$, of the short-circuited plate, and feed point locations, $F$. In Figure 3.14 the distributions in the top row correspond to the current on the upper side of the planar element. The distributions in the middle row correspond to the currents on the lower face of the planar element. The distributions in the bottom row are the currents that flow back along the ground plane under the planar element. [7] The currents on the short-circuited plate are shown in all cases, and the feed point is denoted
by a filled black circle. The direction of the current is shown using an arrow and the magnitude of the current is given by the area within the arrow. [7] Figure 3.14 shows that currents flow out along the lower side of the planar element, and return on the surface of the ground plane. These currents set up the inner electric and magnetic fields between the planar element and the ground plane. The currents on the ground plane produce the image of the PIFA element. [7] The currents on the top face of the planar element are much smaller than those on the lower face except at the edges of the planar element. The edge currents on the plate contribute to the fringing fields which are a radiating mechanism of the PIFA. [7]
Figure 3.13. Distribution of the electric fields, $E_x$, $E_y$, and $E_z$ in the x-y plane calculated using the SNM where the observed plane is $2.5\Delta d$ above the ground plane for $E_z$, and $2\Delta d$ above the ground plane for $E_x$ and $E_y$. [7]
Figure 3.14. Surface current distributions on the PIFA calculated using the SNM for different widths, \( W \), of the short circuit stub, and different feed point locations, \( F \). Part c) represents the currents under a quarter-wave rectangular air substrate microstrip patch antenna for reference. [7]

Comparison of Figures 3.14(a&b) to Figure 3.14(c) shows that the currents on the PIFA are similar to the currents on the short-circuited rectangular microstrip patch antenna. However, the PIFA has the advantage of a partially shorted edge. As the width of the short circuited plate, \( W \), is decreased, the effective path of the current flow on the planar element increases. The result is a lower resonant frequency, and a corresponding decrease of \( L_2 \). [7] For example, if the ratio of the width of the short circuit element to the resonant length of the PIFA is \( W/L_1 = 0.125 \), then the resonant length of a PIFA with \( L_1/L_2 = 2.0 \) is 60% shorter than the resonant length of the same PIFA with a fully shorted edge. If the planar element is square, so that \( L_1/L_2 = 1.0 \), the reduction factor is 40%. [7]
The resonant frequency of the PIFA depends upon the width of the short circuited stub, \( W \), the height of the element, \( h \), and the dimensions of the planar element, \( L_1 \) and \( L_2 \). To derive the resonant frequency of the PIFA, a factor, \( \gamma_1 \), is calculated that involves the height, \( h \), and the length of the planar element, \( L_2 \). This factor is

\[
\gamma_1 = 4(L_2 + h) \tag{3.50}
\]

Another factor, \( \gamma_2 \), is calculated that treats the effectively lengthened current path due to the partial short-circuit. The second factor is

\[
\gamma_2 = 4(L_1 + L_2 + h - W) \tag{3.51}
\]

The factors \( \gamma_1 \) and \( \gamma_2 \) are substituted into one of the following equations to determine the resonant frequency of the PIFA:

\[
f_r = \frac{r c}{\gamma_1} + \frac{(1-r)c}{\gamma_2} \quad \frac{L_1}{L_2} \leq 1 \tag{3.52}
\]

and

\[
f_r = r^k c \frac{1-r^k c}{\gamma_1} \quad \frac{L_1}{L_2} > 1 \tag{3.53}
\]

where \( r = W / L \), and \( k = L_1 / L_2 \). [7]

The radiation pattern of the IFA and PIFA are the same as that of the ILA if \( t \) is small compared to the length of the horizontal or planar element. The ILA radiation patterns are in Figure 3.4.

The bandwidth of the PIFA can be improved using a proximity coupled feed. The proximity coupled feed does not touch the planar element of the PIFA. Instead, the feedline is coupled capacitively to the planar element of the PIFA using a plate at the end of the feedline that is oriented parallel to the planar element. The proximity coupled PIFA is the subject of the next section.
3.4.3. Proximity Coupled Planar Inverted-F Antenna

Normally the Planar Inverted-F Antenna (PIFA) is fed coaxially with the shielding of the feedline connected to the ground plane and the center conductor attached to the planar element. [2] An alternative to the contacting feed is the proximity coupled feed. In proximity coupling, the center conductor of the coaxial line is terminated with a plate that is oriented parallel to the planar element of the PIFA. Figure 3.15 shows a PIFA with a proximity coupled feed.

![Diagram of Planar Element, Proximity Coupled Feed, and Ground Plane](image)

Figure 3.15. Planar inverted-F antenna fed using proximity coupling

In Figure 3.15 the plate at the end of the feedline acts as an additional load that stores charge and makes the current distribution on the feedline more uniform. [1] The proximity coupled feed adds three degrees of freedom to the impedance matching of the antenna: the length and width of the feed plate, and the distance between the feed plate and the planar element. [8] By adjusting these parameters along with the location of the feed, the proximity coupled PIFA can be impedance matched to a transmission line.

A proximity coupled feed offers a number of advantages. First it is easier to change the location of the proximity coupled feed with respect to the planar element since there is not a direct connection. [8] Second, the loading effect of the capacitive feed causes the proximity fed PIFA to have a lower resonant frequency than a typical PIFA. At resonance, the planar element of the PIFA has a length, \( L_2 \), slightly less than \( \lambda/4 \). Using proximity coupling, \( L_2 \) can be reduced to \( \lambda/8 \). [8]

Proximity coupling also has disadvantages. Since the feed plate must be oriented parallel to the planar element and the ground plane, complexity and manufacturing
tolerances are added to the design. In addition, currents on both sides of the feed plate must be modeled to achieve accurate results. Moment Method codes often neglect the currents on one side of a planar element. Therefore, brute force numerical methods such as the Finite Difference Time Domain (FDTD) theory are sometimes necessary. [2]

The primary purpose a proximity coupled feed is manipulation of the input impedance of the PIFA. Increasing the distance between the feed plate and the planar element reduces the input impedance. [1] A single PIFA element fed using proximity coupling and optimized for frequency bandwidth experiences a frequency bandwidth of about 5% of the center frequency. [1]

In [1], further size reduction of the PIFA is achieved by placing a capacitive load at the end of the planar element opposite the feed, as illustrated in Figure 3.16. [1]

![Figure 3.16. A planar inverted-F antenna with a capacitive end load](image)

A proximity coupled PIFA like that in Figure 3.16 was built using the dimensions specified in Table 3.1.

![Table 3.1. Dimensions of a proximity fed PIFA with a capacitive end load](table)
<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resonant Frequency</td>
<td>$f_r$</td>
<td>1.8</td>
<td>GHz</td>
</tr>
<tr>
<td>Width of Planar Element</td>
<td>$W$</td>
<td>10</td>
<td>mm</td>
</tr>
<tr>
<td>Length of Planar Element</td>
<td>$L$</td>
<td>10</td>
<td>mm</td>
</tr>
<tr>
<td>Element Height</td>
<td>$h$</td>
<td>5</td>
<td>mm</td>
</tr>
<tr>
<td>Feed Point</td>
<td>$t$</td>
<td>5</td>
<td>mm</td>
</tr>
<tr>
<td>Width of Feed Plate</td>
<td>$W_f$</td>
<td>8</td>
<td>mm</td>
</tr>
<tr>
<td>Length of Feed Plate</td>
<td>$L_f$</td>
<td>8</td>
<td>mm</td>
</tr>
<tr>
<td>Height of Feed Plate</td>
<td>$h_f$</td>
<td>3.5</td>
<td>mm</td>
</tr>
<tr>
<td>Load Capacitor Separation</td>
<td>$s$</td>
<td>0.5</td>
<td>mm</td>
</tr>
<tr>
<td>Load Capacitor Length</td>
<td>$L_L$</td>
<td>4</td>
<td>mm</td>
</tr>
</tbody>
</table>

The antenna specified in Table 3.1 was built and measured. [1] The measured impedance bandwidth (VSWR < 2 on a 50 Ω feedline) was 4.7% of the resonant frequency. The current distribution on the PIFA was not affected by the proximity coupled feed or the capacitive end load. [1] Therefore, the proximity coupled feed had no effect on radiation pattern. [1] As the reactance of the load increased, the resonant frequency of the antenna decreased. Maximum tuning of the resonant frequency was achieved by changing the plate separation, $s$, of the load capacitor. The resonant frequency is also affected by the load capacitor length, $L_L$. [1] A disadvantage of the capacitive end load is increased input reactance and narrower impedance bandwidth. [1] The proximity coupled feed is used to tune out the increased reactance, and preserve the impedance bandwidth of the end loaded PIFA. [1]

Another way to improve the bandwidth of the inverted-F antenna is to add a parasitic element to the design. The coupled element resonates at a different frequency than the fed element. Impedance bandwidth is increased if the resonant frequencies of the two elements are closely spaced. The Dual Inverted-F Antenna (DIFA) is the subject of the next section. [2]
3.4.4. The Dual Inverted-F Antenna

The Dual Inverted-F Antenna (DIFA) in Figure 3.17 is formed by adding a second inverted-L element to the Inverted-F Antenna (IFA). The second element is parasitic and is not directly fed. This antenna configuration is frequently referred to as a dual-L antenna. [2]

In Figure 3.17, the fed element, of length  \( L_f \), and the coupled element, of length  \( L_c \), are separated by an offset,  \( S \). The coupled element has a different resonant frequency than the fed element. If the resonant frequencies of the fed and coupled elements are nearly the same, the impedance bandwidths of the elements blend together. [2] This serves to widen the overall impedance bandwidth of the PIFA without an increase in element height,  \( h \), or an alternate feeding technique. [2] An experimental DIFA with planar elements as reported in [2] is shown in Figure 3.18. The dimensions of the antenna are noted in the figure.
Figure 3.18. An experimental Dual Inverted-F Antenna and its dimensions in mm. [2]

The antenna in Figure 3.18 was designed for a center frequency of 900 MHz. The individual elements were designed at resonant frequencies of 850 and 950 MHz. [2] The measured impedance of the DIFA in Figure 3.18 is shown in Figure 3.19. The impedance is expressed in terms of return loss. A return loss of 10 dB corresponds to a VSWR of 2 and a 10% reflection of input power.

Figure 3.19. Measured impedance versus frequency characteristic of DIFA in Figure 3.18. [2]

Figure 3.19 shows a clear dual resonance. The frequency bandwidth of the DIFA below 10 dB return loss is 14.1% of the center frequency, from 0.872 to 1.004 GHz. The factors that have the greatest effect on the frequency bandwidth are the height of the element, the
width and shape of the gap between the planar elements, and the length of the coupled arm. [2] The radiation patterns of the DIFA are shown in Figure 3.20.

![Figure 3.20](image)

**Figure 3.20.** Measured radiation patterns of the Dual Inverted-F Antenna in Figure 3.18. [2]

Figure 3.20 shows that the radiation pattern of the DIFA is relatively omnidirectional in the x-y and y-z planes. [2] The high cross-polarization level is a desirable characteristic in hand-held multi-path environments. [2]

A variation of the DIFA is presented by Liu and Hall in [9]. A rectangular notch is cut into the corner of the planar element of a PIFA, and another PIFA with a different resonant frequency is inserted into the empty space. The resulting geometry is illustrated in Figure 3.21, along with a side view of the antenna mounted on a rectangular conducting case.

The DIFA in Figure 3.21 is fed in two locations. The result is a configuration that is switchable between the 900 MHz cellular band, and the 1800 MHz PCS band. [9] The volume occupied by the antenna is almost identical to that of a PIFA designed at 900 MHz. [9] Spurious radiation and a loss of radiation efficiency occurs if there is mutual coupling occurs between the elements of a dual band antenna. Mutual coupling between the elements of the DIFA in Figure 3.21 is below -17 dB at both resonant frequencies. [9]
The input impedance of the antenna is illustrated in Figure 3.22 for both resonant frequencies in terms of return loss.

![Figure 3.21. Geometry and dimensions of a dual-band DIFA. [9]](image1)

![Figure 3.22. Modeled and empirical impedance data for the dual-band DIFA in Figure 3.21. [9]](image2)
Figure 3.22 shows that the antenna is well matched to a 50 Ω coaxial line at both resonant frequencies. The frequency bandwidth of the antenna at 0.9 GHz is 63 MHz, or 7% of the resonant frequency. At 1.76 GHz, the frequency bandwidth of the antenna is 110 MHz, or 6.25% of the center frequency. [9] The radiation patterns of the dual-band DIFA are illustrated in Figure 3.23.

Figure 3.23. Radiation patterns of the dual-band DIFA in Figure 3.21, modeled using the FDTD technique in the x-y plane at (a) 1.76 GHz and (b) 1.9 GHz. [9]

The radiation patterns illustrated in Figure 3.23 show that the $E_\phi$ field of the dual-band DIFA is omni-directional in the x-y plane at both resonant frequencies. This is a desirable feature for hand-held applications. [9]

The DIFA has a number of features that are attractive for hand-held applications. It is low-profile, relatively compact, has a wide frequency bandwidth and omni-directional coverage in azimuth. It exhibits good performance on a conducting box of finite dimensions and lends itself to dual-banding. Thus, the DIFA is an excellent candidate for use in the hand-held environment.
3.5. Conclusion

In this chapter, the Inverted-L Antenna (ILA) and a number of its variations were presented. The variations on the ILA included the Inverted-F Antenna (IFA), the Planar Inverted-F Antenna (PIFA) and the Dual Inverted-F Antenna (DIFA). The variations on the ILA are summarized in Table 3.2.

Table 3.2. Summary of variations on the Inverted-L Antenna

<table>
<thead>
<tr>
<th>Name</th>
<th>Figure Containing Geometry</th>
<th>Critical Dimensions</th>
<th>Typical Bandwidth (% (f_{\text{resonant}}))</th>
<th>Azimuthal Coverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverted-F Antenna (IFA)</td>
<td>3.10</td>
<td>(h \approx \lambda / 16) (L \approx \lambda / 4)</td>
<td>&lt; 2 % omni</td>
<td></td>
</tr>
<tr>
<td>Planar Inverted-F Antenna (PIFA)</td>
<td>3.12</td>
<td>(h \approx \lambda / 16) (L &lt; \lambda / 4) (W \text{ varies})</td>
<td>5 % omni</td>
<td></td>
</tr>
<tr>
<td>Dual Inverted-F Antenna (DIFA)</td>
<td>3.17</td>
<td>(h \approx \lambda / 16) (L_{\text{fed}} \approx \lambda / 4) (L_{\text{coupled}} \neq L_{\text{fed}})</td>
<td>14 % possible (5-7 % \text{ per dual banded frequency}) omni</td>
<td></td>
</tr>
</tbody>
</table>

The ILA variations experience higher input resistance at resonance, and wider impedance bandwidth than the ILA. The antenna with the widest impedance bandwidth is the DIFA. In addition, the DIFA is as compact as the PIFA and matches the ILA, IFA, and PIFA in height reduction. The DIFA provides omni-directional coverage, good performance on size-constrained ground planes, and lends itself to dual-banding. This makes the DIFA ideal for hand-held applications.

In the next chapter the DIFA is studied in further detail. A numerical model of the DIFA is developed using the Method of Moments (MoM). This model is implemented and the results are compared to empirical measurements. The performance of the DIFA is
compared to that of the IFA over an infinite ground plane and over conducting boxes of various finite dimensions.

REFERENCES FOR CHAPTER 3


