# Growth Models and Mortality Functions for Unthinned and Thinned Loblolly Pine Plantations

by

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(ABSTRACT)

Effects of thinning, such as increased diameter growth and decreased mortality in the residual stand, have been recognized by foresters for many years. These effects are largely the result of increased tree vigor which is induced by a decreased level of competition. These relationships are reflected in many of the models that are central to PTAEDA2, a growth and yield simulator which was developed for use with loblolly pine (*Pinus taeda*) plantations established on cut-over, site prepared lands.

Data from a long-term thinning study served as a basis for attempting to improve the predictive output of PTAEDA2. Assessment of differences in model parameter estimates between three levels of thinning intensity led to various approaches to reach this goal. Height increment and mortality models were found to need no additional refinement and were re-fit using all available data. Diameter increment and crown ratio model forms could not account for thinning effects in their present form and thinning response functions that could provide the proper behavioral response were added to these models.

Models were evaluated individually and in combinations in a reduced growth simulator. This reduced simulator is a modified form of the growth subroutines in PTAEDA2 and is designed to utilize external data. Results of growth simulation runs show improvements in predictive ability for the crown ratio model fit to all data and for the re-fit height increment model/crown ratio model with thinning response variable combination. The diameter increment model with a thinning response variable significantly improved diameter prediction within the simulator, but predicted stand volumes were poor. The re-fit mortality function resulted in greater prediction error for mortality than the original PTAEDA2 mortality function.

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## Introduction

In today's increasingly competitive timber industry, the ability of forest managers to accurately project the effects of treatments on timber quality and quantity is critical. A method for assessing different types of thinning regimes is necessary for effective forest management. Foresters have long recognized thinning effects such as increases in volume growth of residual trees, improvements in wood quality, and reduction of mortality in the residual stand. However, quantifying these attributes and making reasonably accurate projections of future stand development can be a difficult task. There have been numerous approaches taken by forest growth and yield researchers to use data from thinned stands to create models that will give reliable estimates of responses to thinning.

Unfortunately, thinning studies require many years to obtain useful data. Also, tree data is usually not collected on an annual basis, but on intervals of approximately 5 years. These data collection protocols result in analyses that typically assume linear relationships between the measurement periods and the possibility that some important nuances of tree growth and stand development may remain unnoticed or unaccounted for. The data collected as part of the Loblolly Pine Growth and Yield Research Cooperative thinning study that were used for this research is quite extensive in both tree attributes and time length. Since establishment, there have been five remeasurement periods at three year intervals. This vast amount of data should provide answers to many questions regarding the response of the residual stand to various levels of thinning.

Primary growth aspects to be targeted are height and diameter growth, crown ratio, and mortality. The models from the loblolly pine growth and yield simulator PTAEDA2 will serve as the framework for the analysis. The goal is to improve predictive ability while still utilizing a single model. Previous work has shown that adding a function which modifies response based on various thinning factors can provide improved prediction for thinned stands. Greater predictive ability for any given model(s) does not ensure improvement in growth and yield estimation. The resultant models were assessed both singularly and in combination in a reduced form of the PTAEDA2 growth simulator to evaluate the effect the model(s) have on the system as a whole.

## The Data

The data used in this research is from a long-term thinning study maintained by the Loblolly Pine Growth and Yield Research Cooperative. During 1981-82, 186 locations were chosen in cutover, site-prepared loblolly pine plantations. These locations cover most of the natural range of loblolly pine. Three plots were established at each location which represent control (no thin), light thinning (approximately one-third of the basal area removed), and heavy thinning (approximately one-half of the basal area removed). All trees (including hardwoods in the main canopy) within the plot were measured and stem mapped before the thinning was performed. The thinning was carried out in a manner consistent with operational methods.

Since establishment, these plots have been remeasured five times at three year intervals. Numerous measurements, including but not limited to, age, dbh, height, height-to-crown, stem quality, tree vigor, and crown class have been made on each tree. For the purposes of this study, only interior trees whose competitors are within the plot (BAF 10 sweep) were used. This restriction ensured valid competition index values.

Many of these plots have reached or are approaching rotation age. Thus, this data set essentially covers the range of response to thinning in the residual stand. A summary of the data can be found in Table 1.

Table 1. Summary of stand and tree level data at plot establishment and at the fifth remeasurement. Planted loblolly pines only.

| | **Plot Establishment** | | | **Fifth Remeasurement** | | |
|---|---|---|---|---|---|---|
| **STAND** | Unthinned | Light Thinned | Heavy Thinned | Unthinned | Light Thinned | Heavy Thinned |
| Age | 15 | 15 | 15 | 30 | 30 | 30 |
| Vol/Acre | 2156 | 1592 | 1292 | 4861 | 3772 | 3359 |
| MTH | 40.6 | 40.9 | 41.0 | 65.3 | 66.2 | 66.7 |
| I | 1.00 | 0.73 | 0.59 | NA | NA | NA |
| **TREE** | | | | | | |
| HT | 29.5 | 34.7 | 35.5 | 54.0 | 61.5 | 61.0 |
| DBH | 4.7 | 5.6 | 5.8 | 7.0 | 8.7 | 9.0 |
| CR | 0.52 | 0.51 | 0.51 | 0.33 | 0.36 | 0.36 |
| CI | 0.93 | 0.60 | 0.43 | 1.36 | 0.95 | 0.80 |

Stand

Age = Age in years since planting.

Vol/Acre = Total stem-to-tip volume in cubic feet.

MTH = Mean total height of domininant/co-dominant trees in feet.

I = Thinning intensity (basal area after thinning divided by basal area before thinning).

Tree

HT = Total tree height in feet.

DBH = Diameter at breast-height in inches.

CR = Crown ratio

CI = Competition index

## Literature Review

<u>Height Increment Model</u>

Individual tree models are often based on a potential growth multiplied by a modifying function. In the loblolly pine growth simulator PTAEDA, developed by Daniels and Burkhart (1975), height and diameter increment models are expressed as potential times modifier. These models are also used in the updated version, called PTAEDA2 (Burkhart et al. 1987).

Many studies have provided evidence that height growth of dominant trees is, in most cases, independent of stand density. However, when considering the height growth of individual trees, competition from adjacent trees causes height growth to be less than optimal. In distance-dependent models, the level of competition can be accounted for by evaluating the number and size of competing trees. The competition index equation by Hegyi (1974), with a modified definition of competitor identification proposed by Daniels (1976), is employed:

$$CI_i = \Sigma\,[(D_j/D_i)/Dist_{ij}]$$

where: $CI_i$ = competition index of the $i^{th}$ subject tree

$D_j$ = diameter breast-height of $j^{th}$ competitor

$D_i$ = diameter breast-height of $i^{th}$ subject tree

$Dist_{ij}$ = distance between subject tree i and $j^{th}$ competitor

The rationale behind the diameter ratio portion of this measure of competition is that a competitor that is smaller than the subject tree cannot obtain resources as well as the subject tree. Similarly, a competitor of equal size as the subject tree has approximately the same capability to gather resources and competitors of greater size have greater access to resources than a smaller subject tree. Diameter is used as a measure of overall size because many other tree attributes, such as height and various crown measures, are highly correlated with diameter. The other factor in the competition index is distance. The further a competitor is from the subject tree, the less influence it will have. The form of the equation ensures that the level of competition decreases as distance increases. The total amount of competition a subject tree receives is an accumulation of values from each competing tree.

Crown ratio can be considered a reflection of a tree's vigor and an indicator of potential growth. Thus, both competition index and crown ratio are often employed in the development of height increment models. The height increment model used for tree growth simulation in the PTAEDA2 (Burkhart et al. 1987) growth and yield simulator is shown below:

$$HIN = PHIN * \{ß_1 + ß_2 CR^{ß_3} * \exp(-ß_4 CI - ß_5 CR)\}$$

where: HIN = predicted height increment

PHIN = potential height increment

CR = tree crown ratio

CI = competition index

An alternate height increment model is specified if competition from hardwoods is to be considered as well:

$$HIN = PHIN * \{ß_1 + ß_2 CR^{ß_3} * \exp(-ß_4 CI - ß_5 MCI_{hw} - ß_6 CR)\}$$

where: $MCI_{hw}$ = mean competition index of hardwoods all other variables as previously defined

The potential height increment is determined by taking the first difference with respect to age of the change in the average height of the dominant and co-dominant stand. These average heights are found by using a model developed by Amateis and Burkhart (1985) fitted to Coop thinning study plot data.

$$\ln(HD) = \ln(SI)(25/A)^{-02205}\ e^{-2.83285(1/(A-25))}$$

where: HD = mean height of dominant and co-dominant trees

SI = site index (base age 25)

A = stand age

These height models provide decreased growth as the level of competition increases. Also, height increment increases as crown ratio increases, up to a maximum point. The maximum height growth is attained when crown ratio is near 60%. To account for variability in height increment, a random component is assigned to the growth calculated by the model. Both height increment models were fitted using height data from unthinned plots only (Burkhart et al. 1987).

Smith (1994) attempted to improve height prediction by fitting the existing PTAEDA2 height increment model to data from thinned plots. At the time, there were data available from the first three remeasurement periods. Fitting the equation to data from thinned plots proved troublesome. Each attempt resulted in most or all parameter estimates containing zero in their 95% confidence intervals and/or the standard errors of the estimates being zero (singular estimates). All efforts to fit the model to data from thinned plots failed to yield any significant results.

A possible explanation for the difficulty in fitting the model is the high correlation between parameters. In an attempt to overcome this problem, certain parameters can be set at a fixed value instead of being estimated. This approach was taken and setting the parameter value multiplied by crown ratio in the exponent resulted in the greatest improvements. This parameter was fixed at 0.5 for unthinned, 0.6 for light thinned, and 0.3 for heavy thinned model fits. In no case did all of the other four parameters that were estimated become significant, but some parameters that were previously insignificant were now significant.

A final attempt to obtain significant parameter estimates involved fitting the model with a thinning response variable (TRV) included. A TRV developed by Liu et al. (1995) was selected. This TRV provided an initial response of zero, an increased response to a culmination point over a period of time, then a decrease in response until zero is reached again:

$$TRV = (BA_a/BA_b)^{**}r\{-(A_s - A_t)^2 + K(A_s - A_t)\}/A_s^2$$

where: $BA_a$ = stand basal area before thinning

$BA_b$ = stand basal area after thinning

r = rate parameter

K = duration parameter

$A_s$ = current stand age

$A_t$ = stand age at thinning

The TRV was multiplied against CI in the height increment model. The parameter estimates for r and K were both insignificant. Also, the estimate for K was 725. On the basis of these results, the idea of incorporating the TRV into the model was dropped. In order to obtain significant estimates, the height increment model was fit to data from unthinned plots only.

## Diameter Increment Model

The diameter increment model for PTAEDA2 is also based on a modification of potential increment. It is the same model that was used in the original growth simulator PTAEDA (Daniels and Burkhart 1975). An equation for the diameter of an open grown tree was used to obtain a model for potential diameter increment. The open-grown tree diameter model is:

$$D_0 = -2.422297 + 0.286583H + 0.209472A$$

where: $D_0$ = open-grown tree DBH (in.)
H = tree height (ft.)
A = age (yrs.)

The first difference of the open grown equation gives the potential diameter increment model:

$$PDIN = 0.286583HIN + 0.209472$$

where: PDIN = potential diameter increment (in.)
HIN = height increment (ft.)

As with the height increment models, the diameter increment models in PTAEDA2 also allow for the option of including competition from hardwoods. These models are:

$$DIN = PDIN^*(\beta_1 CR^{\beta 2}\, e^{-\beta 3 CIp})$$

and

$$DIN = PDIN^*(\beta 1 CR_{hw}^{\beta 2}\, e^{-\beta 3(CIp\beta 4 + MCIhw\beta 5)})$$

where: $DIN$ = predicted diameter increment

$PDIN$ = potential diameter increment

$CR$ = tree crown ratio

$CR_{hw}$ = crown ratio when hardwood competition specified

$CI_p$ = pine competition index

$MCI_{hw}$ = mean competition index of hardwoods

Again, these models were fitted to data from unthinned plots only. Diameter increment becomes smaller as competition increases and grows larger as crown ratio increases. A random component is added to the final diameter increment calculation to account for variability among trees.

Smith (1994) fitted the PTAEDA2 diameter increment model separately for each of the three thinning treatments and for all the data combined. Analysis of the residuals from these fits showed that a model based on all the data would underpredict diameter increment in heavily thinned stands and overpredict in unthinned stands.

As with the height increment model, the TRV developed by Liu et al. (1995) was added to the DIN model. Here, the TRV was multiplied against the potential increment (PDIN). The fits to light and heavy thin data, respectively, resulted in improved prediction for first remeasurement data, but were worse for the third remeasurement. An alternative model form was chosen where the TRV was multiplied by CI. These fits gave poorer results than those having the TRV times PDIN, but were still better than those without the TRV in the model.

Smith (1994) also created an alternative form of the TRV in which the before and after thin basal area ratio term was replaced with a before and after competition index ratio. This approach allows tree-level variation where the basal area method maintains the same value for all trees in the stand. This new TRV was applied in the same two model forms as the previous TRV. This approach did not yield any improvements over the Liu TRV models.

Smith (1994) also experimented with a TRV created by Short and Burkhart (1992). This TRV is slightly different than that of Liu et al. (1995).

$$TRV = (BA_a/Ba_b)^{TA/A}$$

where: $BA_a$ = basal area after thinning

$BA_b$ = basal area before thinning

TA = stand age at thinning

A = current stand age

This TRV was also utilized in both basal area ratio and competition index ratio forms. Again, these fits offered no improvement over the Liu basal area ratio TRV models.

The TRV was again altered to change the elapsed time since thinning. Because the model predicts on an annual basis, the elapsed time was changed to be years since thinning plus one. This methodology was employed and the six candidate models were fit again. The best overall model was the one with the Liu TRV with modified elapsed time multiplied against CI.

Zhang et al. (1995) also used thinning study data to develop diameter increment models. These models are based on the original increment models found in TRULOB (Amateis and Burkhart, 1989). These models are distance-independent and, therefore, do not utilize a distance-based competition index such as that used in many of the PTAEDA2 components. Two models are offered. The second model allows for specification of hardwood competition while the first does not. As with many increment prediction models, these models are based on modified potential growth:

$$DIN = PDIN\{ß_1 CR^{ß_2} \exp(ß_3(1-(D_q/D) + ß_4(1 - (BA_b/Ba_a))\}$$

where: $DIN$ = diameter increment

$PDIN$ = potential diameter increment

$CR$ = crown ratio

$D_q$ = quadratic mean diameter

$D$ = diameter at breast height

$BA_a$ = stand basal area after thinning

$BA_b$ = stand basal area before thinning

and:

$$DIN_{hw} = PDIN\{ß_1 CR^{ß_2} \exp(ß_3(1-(D_{hw}/D) + ß_4(1 - (BA_a/Ba_b))\}$$

where: $D_{hw} = \{(BA + BA_{hw})/N/0.005454\}^{0.5}$

$BA$ = basal area (loblolly pine)

$BA_{hw}$ = hardwood basal area

all other variables as previously defined.

TRULOB uses two diameter increment models. One is specified for unthinned stands and the other for thinned stands. The above models were fitted to combined thinned and unthinned data through the first three remeasurement periods. Fit statistics indicate that these models will give reliable diameter increment predictions for both thinned and unthinned stands.

## Mortality Function

The PTAEDA2 growth simulator determines individual tree mortality by calculating a probability that a tree will remain alive. This probability is a function of competition and tree vigor. These characteristics are assessed by competition index and crown ratio values, respectively. The mortality function is fit to (0,1) data where dead trees are assigned a 0 and living trees have a value of 1:

$$PLIVE = \beta_1 CR^{\beta_2} \exp(-\beta_3 CI^{\beta_4})$$

As with other growth relationships in PTAEDA2, an alternate model is used when the level of hardwood competition is specified:

$$PLIVE = \beta_1 CR^{\beta_2} \exp(-\beta_3 CI^{\beta_4} + MCI_{hw}^{\beta_4})$$

where: PLIVE = probability that a tree remains alive
all other variables as previously defined

Both models were fitted to Coop thinning study data. The forms of these models provide increased chance of survival with increasing crown ratio and decreasing competition index. The probability of survival for each tree is determined by the above model and this value is compared to a computer-generated uniformly distributed random number whose value is between zero and one. If the PLIVE value is greater than the generated value, the tree remains alive. If PLIVE is smaller then the tree is assumed to have died (Burkhart et al. 1987).

Amateis et al. (1989) developed survival equations based on the same fundamental principles adopted by Burkhart et al. (1987). Competition and tree vitality are used to generate a probability that a tree will remain alive for another growing season. This survival model is distance independent and thus uses the ratio of stand quadratic mean diameter to subject tree diameter to determine the level of competition:

$$P = \beta_1 CR^{\beta_2} \exp(\beta_3 (D_q/D)^{\beta_4}$$

where: P = probability a tree remains alive

CR = crown ratio

$D_q$ = stand quadratic mean diameter

D = subject tree diameter

This model was fitted to the Coop thinning study data which is comprised of approximately 5% (by basal area) overstory hardwoods. This model can be applied when the level of hardwood competition is unknown or is near 5% of the total overstory basal area. The effects of additional competition from hardwoods on mortality can be evaluated by a similar model, also developed by Amateis et al. (1989):

$$P = \beta_1 CR^{\beta_2} \exp(\beta_3 (D_h/D)^{\beta_4}$$

where: $D_h = (B_p/(1 - (P_h/100))/N_p/0.005454)^{1/2}$

The $D_h$ variable is a function of $B_p$ = basal area of planted loblolly pines, $P_h$ = percent of total overstory basal area comprised by hardwoods, and $N_p$ = number of planted loblolly pines (trees/ac). $D_h$ increases as the percentage of overstory hardwoods increases, which results in an increased level of competition. This equation is most useful when the hardwood component is known to be different than the 5% mean level found in the data.

Avila and Burkhart (1992) developed survival functions for both thinned and unthinned loblolly pine plantations. Separate models were presented for both unthinned and thinned conditions. Additionally, distance-dependent and distance independent model forms were given for both thinned and unthinned treatments. For unthinned stands, the distance-dependent mortality function is:

$$P = 1/[1 + \exp(-(\beta_0 + \beta_1 CR + \beta_3 HH + \beta_4 CI))]$$

where: P = probability that a tree remains alive

CR = crown ratio

HH = total height/mean height of dominant and co-dominant trees
CI = competition index

The distance-independent mortality function for unthinned stands is:

$$P = 1/[1 + \exp(-(\beta_0 + \beta_1 CR + \beta_3 HH + \beta_4 DD))]$$

where: DD = quadratic mean diameter/tree dbh
all other variables as previously defined

The distance-dependent and distance-independent survival models for thinned stands are identical to those shown above except that the CR variable is raised to the 1.5 power. This accounts for the increased survival of trees with small crown ratios in thinned stands. The distance-dependent and distance-independent equations for thinned stands are:

$$P = 1/[1 + \exp(-(\beta_0 + \beta_1 CR^{1.5} + \beta_3 HH + \beta_4 CI))]$$

and

$$P = 1/[1 + \exp(-(\beta_0 + \beta_1 CR^{1.5} + \beta_3 HH + \beta_4 DD))]$$

where all variables as previously defined.

The models developed by Avila and Burkhart (1992) were compared to those of Amateis et al. (1989) and Burkhart et al. (1987). The comparison was done for unthinned stands only. The Avila and Burkhart (1992) models showed only a minor improvement in prediction over the Amateis et al. (1989) and Burkhart et al. (1987) mortality equations.

Crown Ratio Model

Crown ratio is a very important aspect of growth and yield modeling. The crown ratio of a given tree reflects the tree's vigor and, therefore, is an indicator of the amount of diameter and height growth that may be attained. Determination of crown ratio in PTAEDA2 is accomplished through models that are dependent upon whether or not the level of hardwood competition is specified. The first model, that of Dyer and Burkhart (1987), is used when hardwood competition is not specified:

$$CR = 1 - \exp[(\beta_1 - \beta_2 A^{-1})D/H]$$

where: CR = crown ratio

D = diameter at breast height (in.)

H = tree height (ft.)

A = age (yrs.)

The model for crown ratio when hardwood competition is specified assumes an equal effect on all pines in the stand. Also, for any given size and distance from a subject tree, the competition from a hardwood is considered to be the same as that of a pine. This model was specified by Burkhart et al. (1987) and is similar to the model shown above:

$$CR_{hw} = 1 - \exp[(\beta_1 - \beta_2 A^{-1})(D/H)(1-PHDWD)]$$

where: $CR_{hw}$ = crown ratio when hardwood competition is specified

PHDWD = percent overstory basal area in hardwoods

all other variables as previously defined

Model parameters for both of the above crown ratio models were estimated using data from unthinned plots only.

Short and Burkhart (1992) developed a continuous thinning response variable (TRV) to improve crown height increment prediction for thinned stands. This TRV was used by Smith (1994) in analysis of diameter increment and is restated below:

$$TRV = BA_a/BA_b{}^{TA/A}$$

This TRV provides decreasing response over time and produces greater response as age at thinning increases. This TRV was incorporated into several crown height increment models to determine it's effectiveness. The greatest improvement was attained when the TRV was raised to an exponent whose value was estimated from the data.

Lui et al. (1995) developed a prediction equation that would provide better crown ratio estimates for thinned loblolly pine plantations. This was accomplished by adding a thinning response variable (TRV) to the Dyer and Burkhart (1987) crown ratio model. This is the TRV used by Smith (1994) in his analysis of height and diameter increment. The proposed crown ratio model form is:

$$CR = 1 - TRV*\exp[(\beta_1 - \beta_2 A^{-1})(D/H)]$$

where all variables as previously defined.

For comparative purposes, this same model form was fitted using the thinning response variable presented by Short and Burkhart (1992) in place of the Liu et al. (1995) TRV. The Liu model showed less overall bias across the three levels of thinning and over elapsed time since thinning.

Zhang et al. (1995) updated the crown ratio model for TRULOB. The model developed by Zhang et al. (1995) is based on the crown ratio model that incorporates the thinning response variable presented by Liu et al. (1995). Data analysis revealed that including the ratio of tree height to dominant height in the model improved the predictive ability. This ratio represents the amount of sunlight a tree would receive in comparison to neighboring trees. The model presented by Zhang et al. (1995) is given below:

$$CR = (1 - T^*\exp((ß_1 + ß_2/A)D/H + ß_3 H/HD))^{ß_4}$$

where: HD = height of dominant stand (ft.)
all other variables as previously defined.

This model had a lower mean square error than either the original TRULOB or Liu crown ratio models.

A crown ratio prediction model for thinned and unthinned Scots pine (*Pinus sylvestris* L.) was constructed by Hynynen (1995). The Scots pine data used by Hynynen (1995) is similar to that of the Loblolly Pine Research Co-op thinning study. Research plots were established in even-age Scots pine stands and plots were either left unthinned, lightly thinned (~30% stems removed), or heavily thinned (~60% stems removed). These plots were measured at 5-year intervals over a period of 15 years.

Initial analysis was performed using the Dyer and Burkhart (1987) crown ratio model. Analysis of the data resulted in the inclusion of basal area and height of dominants as additional model inputs and the deletion of the age variable. By replacing age with dominant height, both age and site index can be accounted for. The form of the model for unthinned stands is:

$$CR = 1 - \exp[-(ß_1(\exp(-ß_2 G)) + ß_3 HD^{-1})(d/h)^{ß_4}]$$

where: $G$ = basal area ($m^2$/ha)

$HD$ = dominant height (m)

$d$ = diameter breast-height (cm)

$h$ = tree height (m)

In order to account for the effects of thinning on crown ratio, a thinning response function was developed. This model includes thinning intensity as a function of basal area differences and elapsed time since thinning is represented by changes in dominant height. The underlying assumption of the thinning variable is that the crown ratio of trees in a thinned stand will converge to the crown ratio of trees in an unthinned stand of equal basal area. This function takes the form of:

$$THIN = (G_b - G_a)\exp[-((H_{dom} - H_{domt})/ß_5)^{ß_6}]$$

where: THIN = thinning response function

$G_b$ = basal area before thinning ($m^2$/ha)

$G_a$ = basal area after thinning ($m^2$/ha)

$H_{dom}$ = height dominants (m)

$H_{domt}$ = height dominants at time of thinning (m)

This thinning function was combined with the unthinned crown ratio model to form a crown ratio model for thinned stands. The model for thinned stands is:

$$CR = 1 - \exp[-(ß_1(\exp(-ß_2 G + THIN)) + ß_3 H_{dom}^{-1})(d/h)^{ß_4}]$$

where all variables as previously defined.

Model validation results indicated acceptable model behavior and no significant bias across a range of thinning intensities and time periods.

## Analysis

Statistical analyses for this research was accomplished through the use of statistical software from SAS Institute Incorporated, Cary, NC.

The nature of this thinning study data requires that observations be taken over time on the same subject. There is no question that there is a high degree of temporal correlation present. Previous investigation into the effects of temporal correlation related to this data have shown that this correlation has no practical effect on statistical analyses (Liu et al. 1995, Smith 1994). The analyses for this research will proceed under the assumption that removal of temporal correlation would provide no benefit.

Initial analyses consisted of determining if there were statistical differences in height increment, diameter increment, crown ratio, and mortality model parameters across the different thinning treatments. These tests follow the methodology proposed by Ratkowsky (1983). This required that each model be fit to unthinned, light thinned, and heavy thinned data as well as all data combined. The residual sum-of-squares and residual degrees of freedom from each fit were used to calculate an F-statistic for hypothesis testing. The full model is the separate parameter estimates for each thinning treatment. The restricted model is where the parameters were estimated using all data. The F-statistic is given by:

$$F = \frac{(SSR_r - SSR_f) \;/\; (df_r - df_f)}{SSR_f \;/\; df_f}$$

where $SSR_r$ and $SSR_f$ are the residual sum-of-squares and $df_r$ and $df_f$ are the degrees of freedom for the restricted and full models, respectively.

The null hypothesis in each case was that there were no differences in parameter estimates for a given model across the thinning regimes. The alternative hypothesis was that at least one parameter had significantly different estimates across fits to each level of thinning. For a model with two parameters, these hypotheses would be:

$H_o : \beta_{11} = \beta_{21} = \beta_{31}$ and $\beta_{12} = \beta_{22} = \beta_{32}$

$H_a : \beta_{11} \neq \beta_{21} \neq \beta_{31}$ or $\beta_{12} \neq \beta_{22} \neq \beta_{32}$

where $\beta_{11}$ , $\beta_{21}$ , and $\beta_{31}$ refer to the first model parameter in fits to unthinned, lightly thinned, and heavily thinned data, respectively. Similarly, $\beta_{12}$ - $\beta_{32}$ represent the second model parameter. This scheme can be extended to accommodate any number of model parameters.

Prior to fitting the models, the data was split into fit and validation sets. Approximately 60% of the available data was used to fit models and the remaining 40% served as model validation data.

Height Increment

In PTAEDA2, height increment is a function of potential height increment, crown ratio, and competition index. Potential height increment is calculated by finding the difference between the average height of the dominant and co-dominant trees on each plot between successive remeasurement periods. This difference is divided by three in order to obtain potential height growth on an annual basis. The height increment model is shown again:

$$HIN = PHIN*\{ß_1 + ß_2CR^{ß_3} * \exp(-ß_4CI - ß_5CR)\}$$

Fitting the HIN model to data from each thinning regime and to all data was quite troublesome. Obtaining reasonable parameter estimates required very precise starting values for some parameters. After much effort, acceptable parameter values were obtained for each of the separate fits. However, all fits resulted in insignificant parameter estimates at the 95% confidence level. Similar difficulties were encountered by Smith (1994) when fitting this height increment model. Probable causes for this were proposed by Smith (1994) and included lack of variation in height growth across thinning treatments, high correlation between parameters, and that the response may already be present in the potential height increment. It is likely that all of these factors contributed to the fitting problems.

Figure 1 shows the average observed height increment for each thinning treatment by remeasurement period. These mean values have a maximum difference of 0.94 ft across the range of the data. For any given remeasurement period, the largest difference (0.35 ft.) across thinning regimes occurred in the first three years after plot establishment. This result was anticipated for thinned plots because height growth typically slows for a period immediately after thinning. Apart from this initial response in thinned plots, height increment values were nearly identical across thinning treatments through the fourth remeasurement period.

There was a notable decrease in height increment in unthinned plots at the fifth remeasurement. This suggests that trees in thinned plots have the ability to maintain consistent height increment values for a longer period than trees in unthinned plots. This may be occurring because of greater competition in unthinned stands. As intermediate and suppressed trees become weaker, height increment decreases. The decreased increment found in these trees causes the mean increment to be lower. Not only are there fewer intermediate/suppressed trees in thinned stands but competition is much lower, which allows these trees to maintain consistent growth for a longer time. Figure 2 provides insight into how the degree of competition has progressed throughout the course of the Coop thinning study. Note that for unthinned plots there is a much greater increase in competition at the fifth remeasurement than there was for earlier remeasurement periods. At this level of competition, a certain number of trees are likely to exhibit reduced growth as resources are lost to larger competitors.

Another possible explanation for the problems encountered while fitting the height increment model was multicollinearity. A high correlation between parameters can often result in highly unstable, insignificant estimates. There was a very high correlation between parameters $\beta_3$ and $\beta_5$ in the HIN model. Correlation values for these two parameters were greater than 0.99 in all height increment fits. Each fit

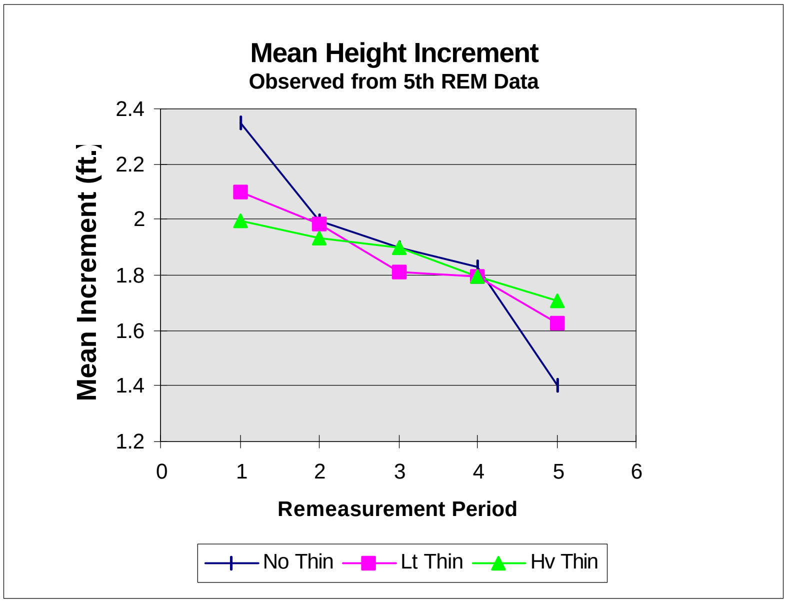



Figure 1. Mean height increments from observed data.

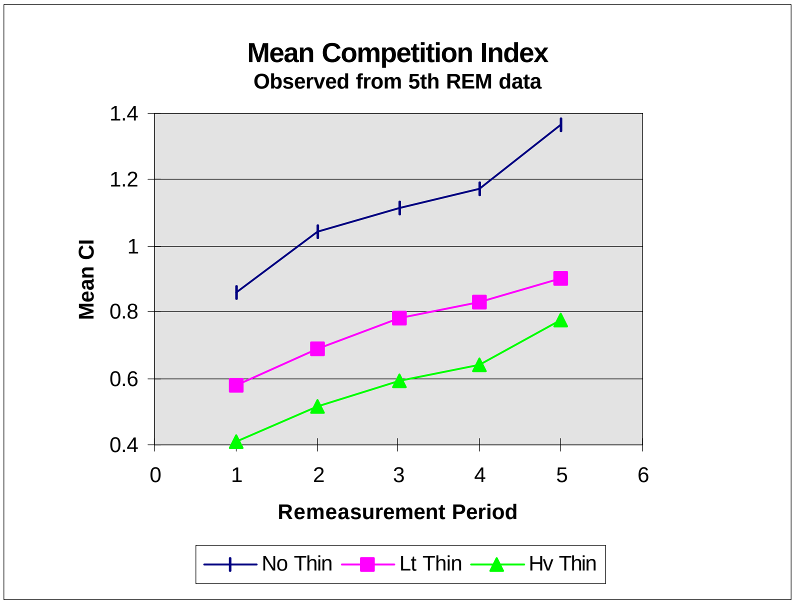



Figure 2. Mean competition index values from observed data.

also had at least one other pair of parameters that exhibited a correlation greater than 0.90. These parameters varied depending on which data was used to fit the model. The SAS regression analysis output only gives pairwise correlation values (i.e. correlation between two parameters only). It is possible that a high correlation exists between three or more of the model parameters.

Fitting difficulties may also be attributed to the possibility that the potential height increment contains most of the response. This would mean that the modifying portion of the model containing independent variables CI and CR has very little influence on HIN. Figure 3 shows the mean potential height increments for each thinning treatment across the five remeasurement periods. Except for the first remeasurement, where potential is slightly lower in thinned plots due to effects of thinning, there is very little difference in potential height growth across thinning regimes. This pattern is nearly identical to that of the graph displaying mean predicted height increments (Figure 4) from the PTAEDA2 height increment model. This comparison indicates that the potential height increment is a highly influential variable in the HIN model.

The hypothesis test for model parameter differences across the three thinning treatments was nonsignificant at the .05 level ($p = 0.2061$). This conclusion leads to the question of whether the data could be classified into unthinned and thinned categories where observations from the light thin and heavy thin plots would be combined. The test for parameter differences between the models fit to unthinned and thinned data also returned a conclusion of nonsignificance ($p = 0.7458$).

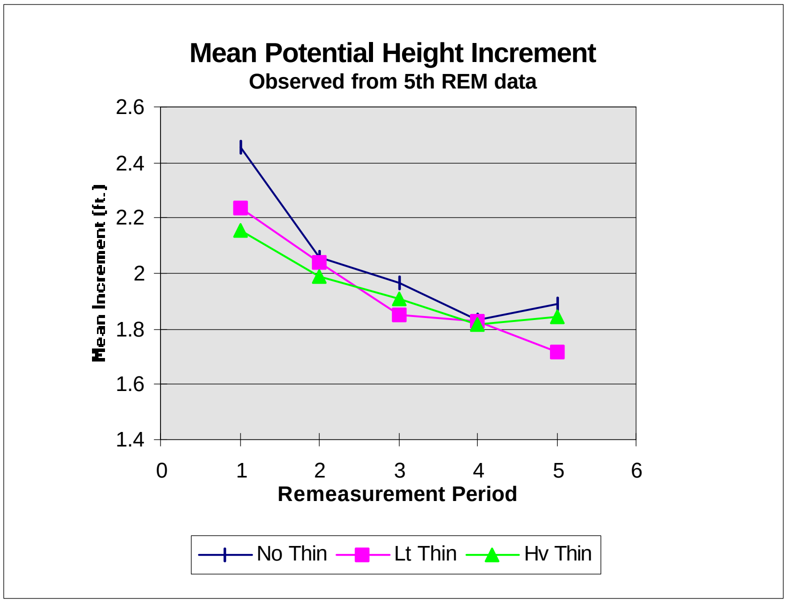



Figure 3. Mean potential height increment from observed data.

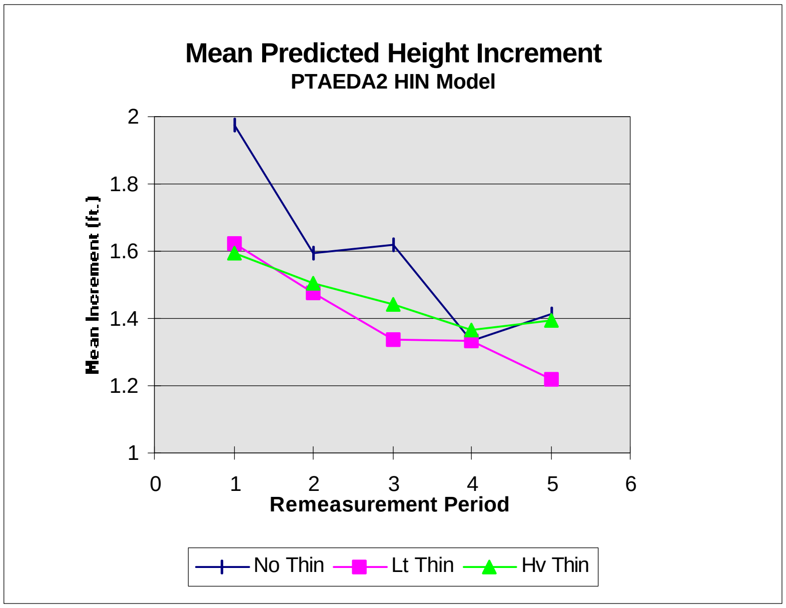



Figure 4. Mean predicted height increment from PTAEDA2 height increment model.

This was expected as the independence of height growth and tree density (except at extremely low or high densities) is a widely-accepted forestry phenomenon. The lack of differences in height increment model parameters across the thinning regimes means that the data from unthinned, light-thinned, and heavy-thinned plots can be combined and the height increment model can be fit to all data.

The height increment model was fit to the combined data from all three thinning treatments. Again, all parameter estimates were nonsignificant.

The results of this fit indicate that the value of $\beta_3$ is near 0.5 and the value of $\beta_5$ is approximately 1.0. In an attempt to overcome multicollinearity problems, these parameters were fixed at these values. The HIN model was re-fit and the remaining three parameter estimates were now all significant and retained similar estimates to those of the previous fit. This result provides evidence that the difficulties encountered while fitting the HIN model were largely due to the high correlation between parameters $\beta_3$ and $\beta_5$. However, differing results were obtained when this approach was taken for the separate fits to unthinned, light-thinned, heavy-thinned, and light/heavy thinned data. In all cases except the fit to heavy-thinned data, parameters $\beta_1$ and $\beta_2$ became significant, but $\beta_3$ remained insignificant. The fit to heavy-thin data resulted in only $\beta_1$ being significant. It appears that lack of variation in height growth and the dominance of the potential height increment in the model are more problematic in the fits to the various subsets of the data than in the fit to all data.

Despite these fitting difficulties, the resultant models provide good height increment predictions. Height increment model statistics can be found in Table 2. Standard error calculations and residual analysis based on validation data indicate reliable predictive ability across remeasurement periods. The re-fit HIN model gives improved prediction over the PTAEDA2 HIN model for the first and fifth remeasurement periods. The PTAEDA2 model is superior at the second remeasurement and the models perform very similarly at the third and fourth remeasurements (See Fig. 5). The slope of the line connecting the mean residuals for the newly fit HIN model is much smaller than that of the PTAEDA2 model. This indicates that the predictive ability of the new model is more consistent at later remeasurement periods.

In unthinned plots, the re-fit HIN model shows improvements at each remeasurement period except the fifth. At the fifth remeasurement, height prediction worsens dramatically when compared to previous remeasurement periods (See Figure 6). Not only is the magnitude of error notably greater, but height is also now being overpredicted. A similar situation exists for light-thinned plots. In this case, the new HIN model is superior at each remeasurement except the second. The mean residuals for light thinned plots closely resemble those for the data set as a whole. This is likely due to the greater number of observations from light-thinned plots in the data set. The range of mean residual values is greatest for heavy-thinned plots. The PTAEDA2 HIN model provides better increment prediction than the newly parameterized model at both the third and fourth remeasurement periods. The new model appears to have the ability to mitigate some of the exceptional prediction error that occurs with the PTAEDA2 model at the first and fifth remeasurements.

The final height increment model as fitted to combined fit/validation data is:

$$HIN = PHIN*\{0.58308 + 1.07573CR^{0.58205} \exp(-0.14611CI - 1.09172CR)\}$$

$$S_{y.x} = 0.59843 \qquad R\text{-square} = 0.51$$

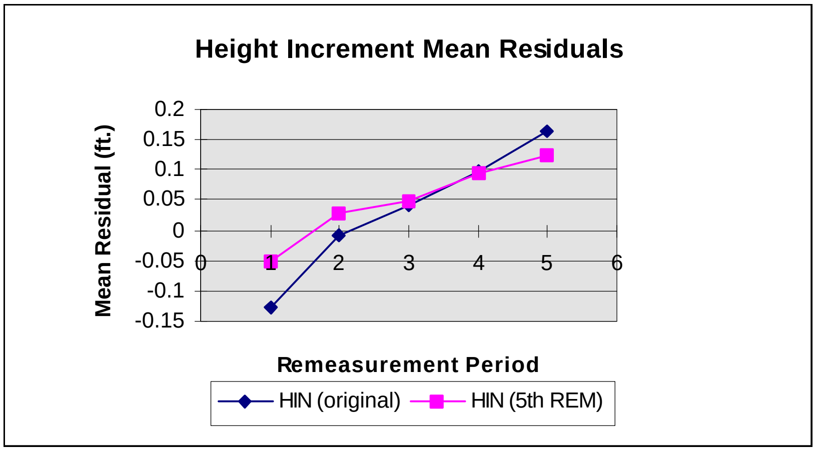



Figure 5. Mean residual comparison between HIN models.

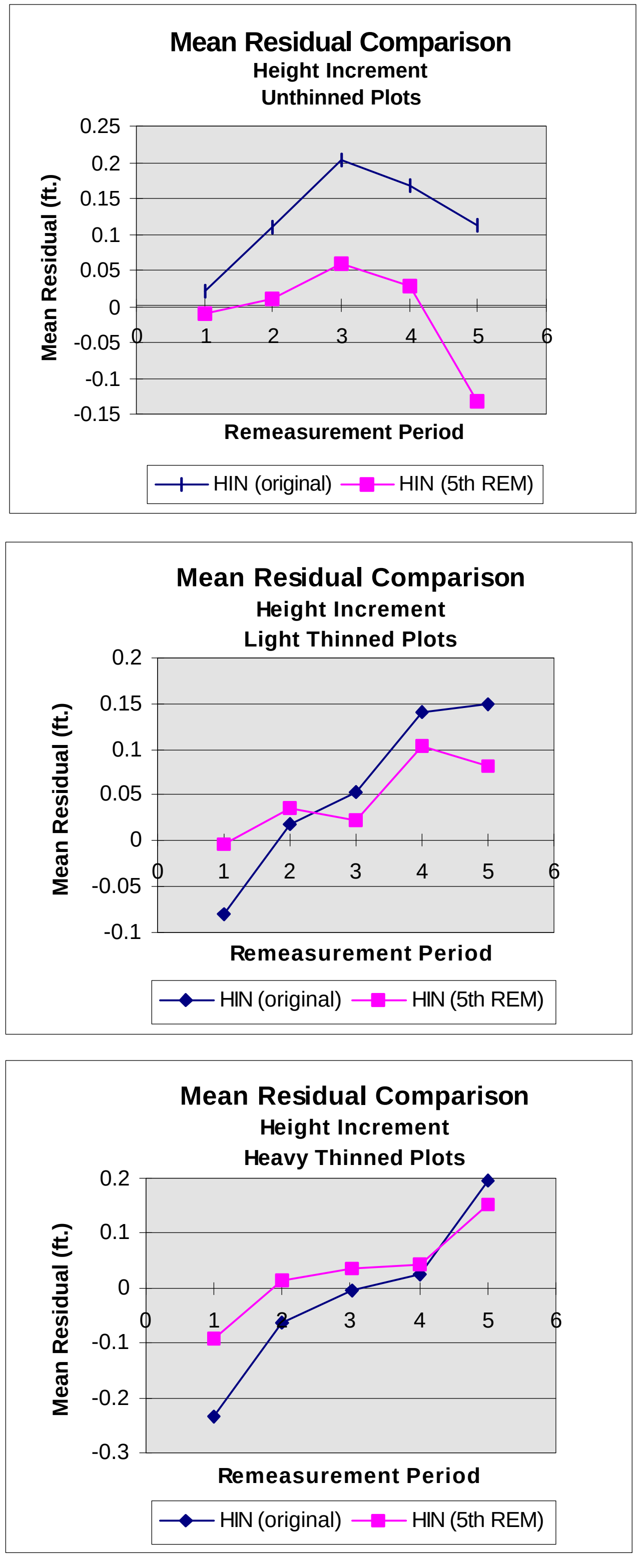



Figure 6. Mean residual comparison between HIN models for each thinning treatment.

Table 2. Statistics for various fits of height increment model.

| Model | df | MSE | Sy.x | R-square |
|---|---|---|---|---|
| A | 466 | 0.31835 | 0.56423 | 0.57 |
| B | 468 | 0.30541 | 0.55264 | 0.59 |
| C | 2521 | 0.35844 | 0.59870 | 0.49 |
| D | 2523 | 0.35793 | 0.59827 | 0.49 |
| E | 1922 | 0.36300 | 0.60249 | 0.47 |
| F | 1924 | 0.36204 | 0.60170 | 0.47 |
| G | 4448 | 0.36087 | 0.60072 | 0.48 |
| H | 4450 | 0.36016 | 0.60013 | 0.49 |
| I | 4919 | 0.35667 | 0.59722 | 0.50 |
| J | 4921 | 0.35563 | 0.59635 | 0.50 |
| K | 8208 | 0.35812 | 0.59843 | 0.51 |
| L | 8210 | 0.35804 | 0.59836 | 0.51 |

Model Description:

A HIN model fit to unthinned data
B HIN model fit to unthinned data with fixed $\beta_3 = 0.5$, $\beta_5 = 1.0$
C HIN model fit to light thinned data
D HIN model fit to light thinned data with fixed $\beta_3 = 0.5$, $\beta_5 = 1.0$
E HIN model fit to heavy thinned data
F HIN model fit to heavy thinned data with fixed $\beta_3 = 0.5$, $\beta_5 = 1.0$
G HIN model fit to thinned data
H HIN model fit to thinned data with fixed $\beta_3 = 0.5$, $\beta_5 = 1.0$
I HIN model fit to all data
J HIN model fit to all data with fixed $\beta_3 = 0.5$, $\beta_5 = 1.0$
K HIN model fit to combined fit/validation data
L HIN model fit to combined fit/validation data with fixed $\beta_3 = 0.5$, $\beta_5 = 1.0$

Diameter Increment

The PTAEDA2 diameter increment model is also based upon a modified potential. The potential diameter increment is assumed to be that of an open-grown tree at any given height. This potential is modified by a function of crown ratio and competition index:

$$DIN = PDIN*(ß_1CR^{ß2} *exp(-ß_3CI^{ß4}))$$

Fits of the diameter increment model to unthinned, light thinned, heavy thinned, and combined data were not as problematic as the height increment fits. Diameter measurements differ from height measurements in that there is very little error and there is a wide variation of values (especially across thinning treatments). Additionally, correlation between parameters was not very high. Each fit produced parameter estimates that were significant. Fit statistics from DIN model fits can be found in Table 3.

Results of the fits were used to test the hypothesis of differences in model parameter estimates across the three levels of thinning. This test showed that there is a very significant difference ($p < .00001$) between unthinned, light thinned, and heavy thinned plots. This outcome was somewhat anticipated as increased diameter growth is one of the primary effects of thinning (See Figure 7). However, it was not clear how much the differences in crown ratio and competition index across thinning regimes would account for the changes in diameter growth.

In order to effectively predict diameter growth, effects of thinning must be taken into account. Some easily discerned factors that influence

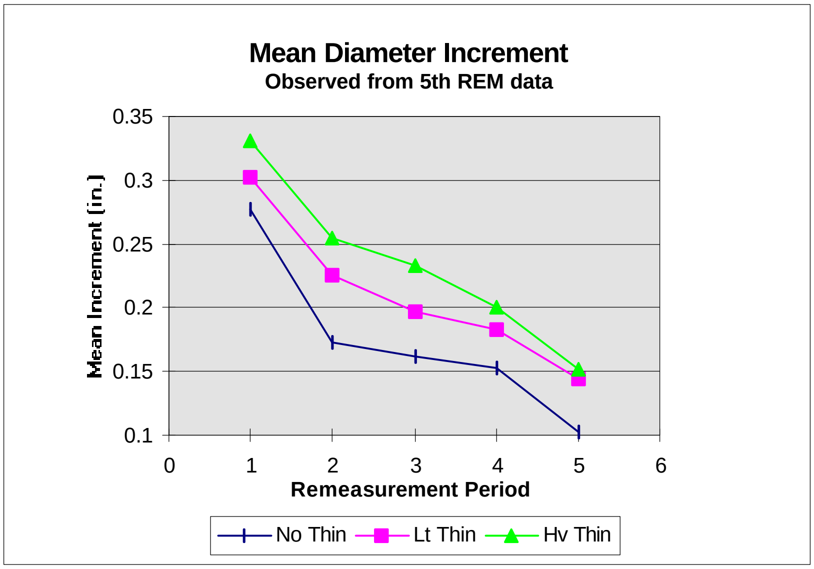



Figure 7. Mean diameter increments from observed data.

thinning effects are intensity, elapsed time since thinning, and stand age at time of thinning. Undoubtedly, there are many other factors which are not as clearly evident. A thinning response function that incorporates such factors would allow diameter increment to still be predicted with a single model for both unthinned and thinned stands.

In order to obtain a baseline for comparative purposes, two existing thinning response variables (TRV) were evaluated. Both Short and Burkhart (1992) and Liu et al. (1995) developed TRV's for predicting crown measures in unthinned and thinned stands. The TRV from Short and Burkhart (1992) is restated:

$$TRV = BA_a/BA_b{}^{TA/A}$$

The Liu et al. (1995) is also repeated:

$$TRV = (BA_a/BA_b)^{**}r\{-(A_s - A_t)^2 + K(A_s - A_t)\}/A_s{}^2$$

Incorporation of these existing TRV's into the DIN model included assessment of where the TRV should be placed in the model. There are three potential locations:

1) Keep the TRV separate from the model (i.e. multiply the TRV times the model);
2) Place the TRV in the crown ratio exponent (multiply TRV times $\beta_2$);
3) Put the TRV in the exponential portion of the model (multiply TRV times competition index);

The TRV of Short and Burkhart (1992) was evaluated first. The diameter increment model with TRV was fit to all data in each of the above placement possibilities. These fits converged quickly and all parameter estimates were significant. The fit statistics were virtually identical when the TRV was actually within the model. Multiplying the TRV by the model resulted in the worst fit statistics. Comparison between the fits of the diameter model with the Short and Burkhart (1992) TRV and the fit to all data without the TRV generally show that the DIN model performs equally well or better when the TRV is not present.

The performance of the Liu et al. (1995) TRV was evaluated in the same manner. The TRV was added to the DIN model in each potential location. Fit statistics from each of these models show that the Liu et al. (1995) TRV performs equally well at any location in the model. The Liu et al. (1995) TRV also performed better than the TRV of Short and Burkhart (1992) in all fits. The improvements offered by the Liu TRV were initially attributed to biological soundness. This TRV provides an initial response of zero, an increase to a culmination point, and a decline thereafter. The Short and Burkhart TRV gives maximum response at time of thinning with decreasing effect thereafter.

Examination of the parameter estimates within the Liu TRV revealed that the TRV exerted an inverted response pattern in each of the three locations within the model. Diameter increment prediction was decreased initially after thinning until a minimum was reached and then began to increase slowly. This behavior was caused by the k parameter being negative and is contrary to the intended effect of the TRV. In an attempt to overcome this problem, the DIN model was divided by the TRV. This would provide the appropriate response if the k parameter did not change sign when fitted.

This fit gave a positive estimate for k, which again caused undesirable bevavior in the response. Surprisingly, these fits provided a slight improvement over the DIN model without a TRV.

Smith (1994) attempted to improve TRV performance by addressing thinning intensity at the individual tree level. This was accomplished by calculating before thinning and after thinning competition index values for each tree. These values were used in place of the before and after thinning basal area measures in the TRV. It was found that evaluation of thinning at the tree level performed worse than the stand level approach in all cases. This result indicates that a stand level measure of thinning intensity is desirable.

For this research, an alternate measure of thinning intensity besides basal area was sought to see if existing TRV performance could be improved. Before and after thinning mean competition index values were calculated for each plot. This would apply a tree level variable at the stand level. The ratio of competition indices were used in place of the basal area ratio and the models were refit. The number of observations for these fits was reduced by 202 because there were four thinned stands that did not have any interior trees at establishment. This was not a concern for unthinned stands as the TRV reduces to 1 in all cases.

This new measure of thinning performed nearly as well as the basal area form for all fits except for the Short and Burkhart (1992) TRV multiplied by the entire model. These results indicate that there is no advantage in using competition index at the tree or stand level. Given the relative ease of computation of basal area compared to competition index, it will be assumed that basal area differences provide the best measure of thinning intensity for diameter increment prediction.

In order to obtain meaningful improvements in diameter increment prediction, a TRV that provides the correct response pattern is needed. Initially, a TRV having the same general form (base raised to exponent) as that of Liu was developed. It was thought that a slightly altered exponent and differing variable usage would provide the necessary TRV response. This TRV is shown below:

$$\text{TRV1} = \left(\frac{A_s}{A_t}\right)^{\frac{B_1(BA_b - BA_a)/BA_b}{A_s{}^{B_2}}}$$

where all variables as previously defined

This TRV will give the desired response when $\beta_1$ and $\beta_2$ are positive. However, this TRV was plagued with the same problems as the Liu TRV. Although the TRV provided improvement over the DIN model alone, estimated parameters within the function resulted in inverted behavior. Also, the estimated parameter $\beta_1$ was insignificant in all fits. Removal of the $\beta_1$ parameter provided much poorer results. Apparently, TRV's in base to exponent form will not behave correctly when fitted to diameter increment data. Fit statistics for diameter increment models containing newly developed TRV's are located in Table 4.

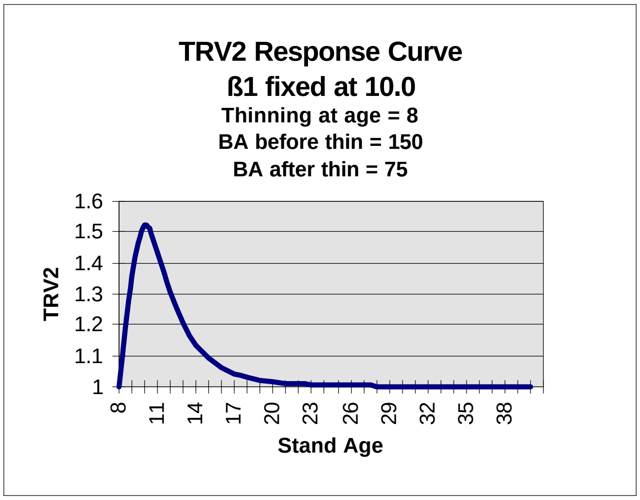
A totally different form was sought for the TRV that would still maintain the necessary behavioral properties when fit to the data. A function that utilizes an exponential term was chosen and the following TRV was constructed:

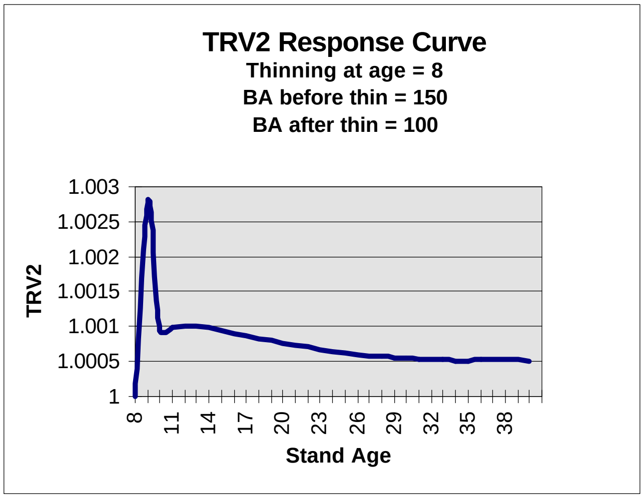
$$\mathrm{TRV2} = \left(\frac{\mathrm{BA_b}}{\mathrm{BA_a}}\right)^{\frac{(A_s - A_t)}{HD^2}} \exp\left[\frac{(A_s - A_t)^2}{(As / At)^{B_1}}\right]$$

where all variables as previously defined

This TRV is more complex than the base to exponent forms, but only has one parameter to estimate. During development, it was thought that the estimated parameter would be a positive number with a value approximately in the range of 0-20. The behavior of TRV2 when the estimated parameter, $ß_1$, is fixed at a value of 10.0 is shown in Figure 8. When incorporated into the DIN model, the above TRV behaved correctly only when multiplied times the model. The value of $ß_1$ was estimated to be 49.05654704 and was not significant at the $\alpha = 0.05$ level. This value of $ß_1$ caused the behavior of TRV2 to alter dramatically. The maximum response was slightly over 1.003 for heavy thinned stands and slightly less than 1.003 for light thinned stands. Also, the shape of the response curve became much more erratic and developed an inverted spike at two years after thinning. Despite the presence of a TRV which is designed to increase predicted increment values for thinned stands, there is very little realized effect due to the high value of the estimated parameter. There was no improvement in model statistics over the DIN model alone.

Analysis of mean residuals shows that, on an overall level, the re-fit DIN model and the DIN model with TRV2 perform very similarly (Figure 9).



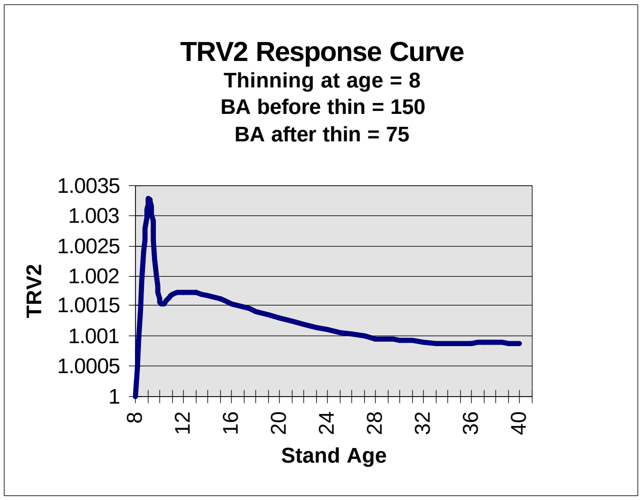




Figure 8. Intended and actual behavior of TRV2 when multiplied against DIN model.

Both of these equations provide better prediction through the fourth remeasurement. The original PTAEDA2 DIN model has better predictive ability for the fifth remeasurement. This is caused by a shift in bias from under-prediction to over-prediction.

Comparison of residuals between the three models for unthinned plots indicates no consistent pattern of superiority for any given model. Model performance is approximately equivalent at the first, third, and fifth remeasurements. The predictive ability of the newer models is obvious for light thin plots as predictive improvements are evident across the range of the data. The mean residual for the original PTAEDA2 DIN model approaches those of the newer models at the fifth remeasurement. For heavy thin plots, the newer models work better through the third remeasurement. All models are approximately equal at the fourth remeasurement and the original DIN model again is superior at the fifth remeasurement. It is notable that the re-fit DIN model and the DIN model with TRV2 perform almost identically in each analysis (Figure 10). The improper behavior and overall lack of effect of the TRV's on diameter increment prediction leads to the conclusion that a TRV may not be appropriate. Although the hypothesis test indicated a significant difference in model parameters across thinning regimes, there appears to be no meaningful difference. It is possible that the null hypothesis was rejected due to the large number of degrees of freedom. This statistical difference is inconsequential for all practical purposes of assessing differences in diameter increment across thinning levels.

Analysis of mean residuals for each of the three thinning levels indicate that there is a certain level of bias inherent in the DIN model that

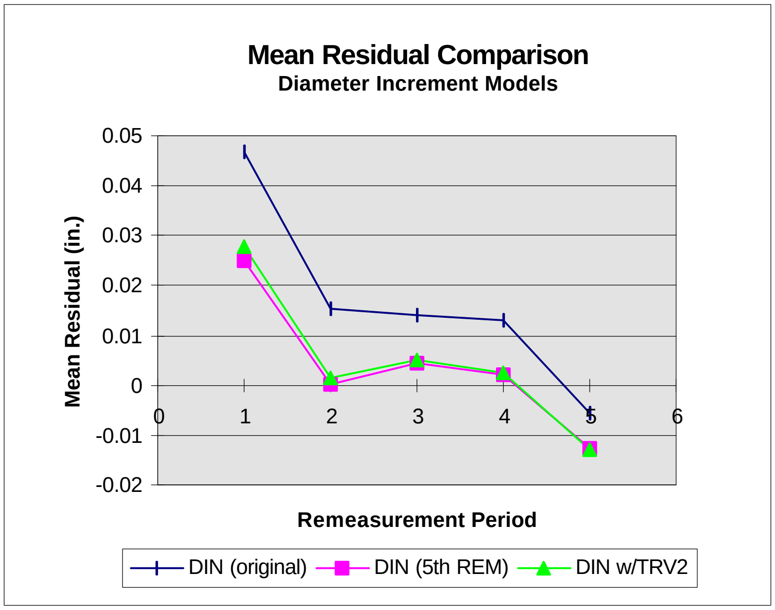



Figure 9. Mean residual comparison between DIN models.

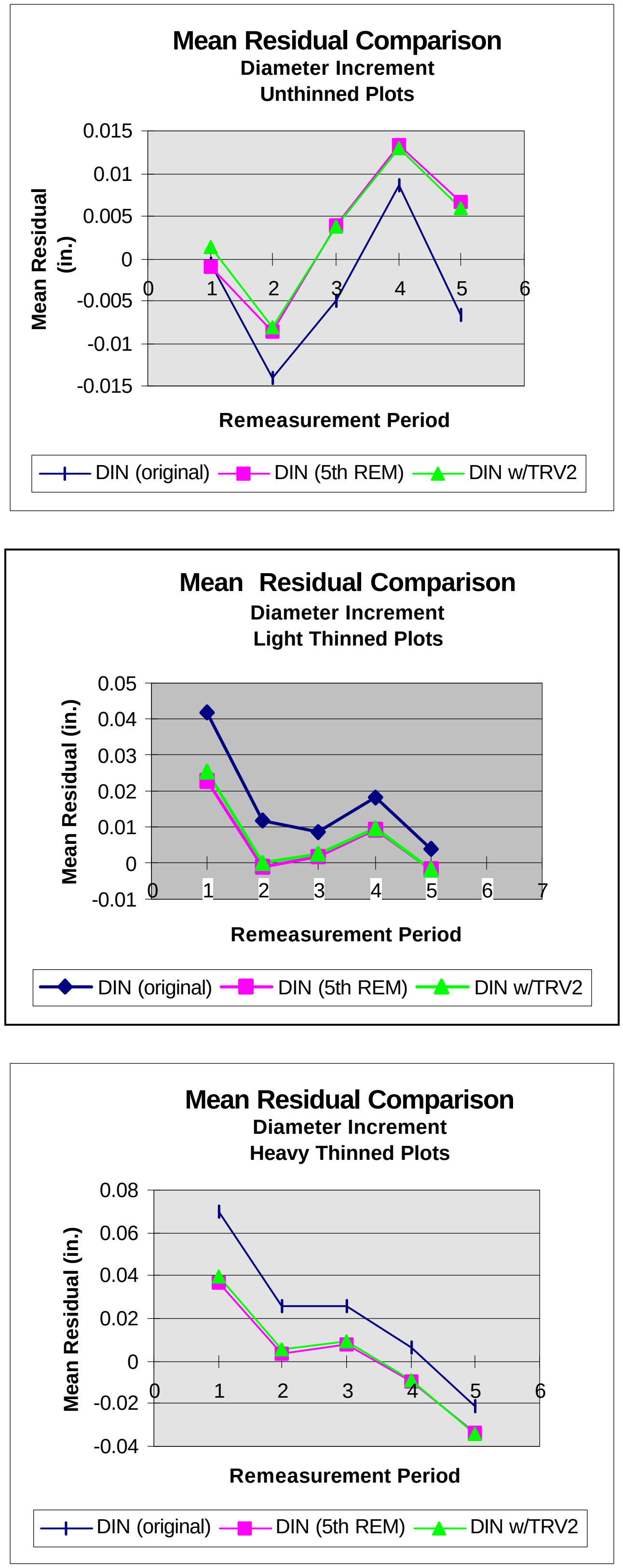



Figure 10. Mean residual comparison between DIN models by thinning treatment.

cannot be alleviated through implementation of a TRV. The trend of mean residual values over time is the same for each thinning treatment. This would indicate that the bias is consistent in behavior. The magnitude of bias for thinning treatments fluctuates erratically.

The values for the light thin and heavy thin plots do not exhibit a pattern that would indicate that a TRV would improve prediction. The overall mean residual for unthinned stands (0.0070) was slightly lower than those of light thin (0.0090) and heavy thin (0.0087) stands. The mean residuals reveal that diameter increment is nearly equally under-predicted for all three thinning intensities.

The DIN model incorporating TRV2 appears to have no significant effect on diameter increment prediction when compared to the DIN model alone. Despite this lack of improved behavior, the DIN model with TRV2 was fit to combined fit/validation data and the results are offered below:

$$DIN = TRV2*PDIN*(0.79570000CR^{0.74920719} \exp(-0.68966795CI)$$

where: $$TRV2 = \left(\frac{BA_b}{BA_a}\right)^{\frac{(A_s - A_t)}{HD^2}} \exp\left[\frac{(A_s - A_t)^2}{(As / At)^{52.10901872}}\right]$$

$S_{y.x} = 0.09050$ R-square = 0.58

The lack of differences across thinning regimes, for practical purposes, is sufficiently large and the lack of any trend to show the need for a TRV resulted in the decision to fit the DIN model to all data. The fit and validation data was combined to get final parameter estimates. The re-fit

DIN model is:

$$DIN = PDIN*(0.803751CR^{0.761097} \exp(-0.694711CI)$$

$$S_{y.x} = 0.09044 \qquad \text{R-square} = 0.58$$

Table 3. Statistics for fits of DIN model and DIN model with pre-existing thinning response variables.

| Model | df | MSE | Sy.x | R-square |
|---|---|---|---|---|
| A | 468 | 0.00595 | 0.07714 | 0.69 |
| B | 2523 | 0.00780 | 0.08832 | 0.57 |
| C | 1924 | 0.00875 | 0.09354 | 0.51 |
| D | 4921 | 0.00805 | 0.08972 | 0.56 |
| E | 4921 | 0.01020 | 0.10100 | 0.44 |
| F | 4921 | 0.00809 | 0.08994 | 0.56 |
| G | 4921 | 0.00810 | 0.09000 | 0.56 |
| H | 4921 | 0.01205 | 0.10977 | 0.34 |
| I | 4921 | 0.00821 | 0.09061 | 0.56 |
| J | 4921 | 0.00811 | 0.09006 | 0.55 |
| K | 4919 | 0.00785 | 0.08860 | 0.57 |
| L | 4919 | 0.00790 | 0.08888 | 0.57 |
| M | 4919 | 0.00792 | 0.08899 | 0.57 |
| N | 4919 | 0.00794 | 0.08911 | 0.57 |
| O | 4919 | 0.00796 | 0.08922 | 0.57 |
| P | 4919 | 0.00797 | 0.08927 | 0.56 |

Model Description:

A DIN model fit to unthinned data
B DIN model fit to light thinned data
C DIN model fit to heavy thinned data
D DIN model fit to all data
E DIN model with Short TRV (BA ratio) times model fit to all data
F DIN model with Short TRV (BA ratio) in crown ratio exponent fit to all data
G DIN model with Short TRV (BA ratio) in exponential term fit to all data
H DIN model with Short TRV (CI ratio) times model fit to all data
I DIN model with Short TRV (CI ratio) in crown ratio exponent fit to all data
J DIN model with Short TRV (CI ratio) in exponential term fit to all data
K DIN model with Liu TRV (BA ratio) times model fit to all data
L DIN model with Liu TRV (BA ratio) in crown ratio exponent fit to all data
M DIN model with Liu TRV (BA ratio) in exponential term fit to all data
N DIN model with Liu TRV (CI ratio) times model fit to all data
O DIN model with Liu TRV (CI ratio) in crown ratio exponent fit to all data
P DIN model with Liu TRV (CI ratio) in exponential term fit to all data

Table 4. Statistics for fits of DIN model incorporating newly developed thinning response variables.

| Model | df | MSE | Sy.x | R-square |
|---|---|---|---|---|
| A | 4919 | 0.00786 | 0.08866 | 0.57 |
| B | 4919 | 0.00805 | 0.08972 | 0.56 |
| C | 4919 | 0.00793 | 0.08905 | 0.57 |
| D | 4920 | 0.00806 | 0.08978 | 0.56 |
| E | 4920 | 0.00805 | 0.08972 | 0.56 |
| F | 4920 | 0.00805 | 0.08972 | 0.56 |
| G | 8209 | 0.00819 | 0.09050 | 0.58 |

Model description

A DIN model with TRV1 times model fit to all data
B DIN model with TRV1 in crown ratio exponent fit to all data
C DIN model with TRV1 in exponential term fit to all data
D DIN model with TRV2 times model fit to all data
E DIN model with TRV2 in crown ratio exponent fit to all data
F DIN model with TRV2 in exponential term fit to all data
G DIN model with TRV2 times model fit to combined fit/validation data

Crown Ratio

The crown ratio model differs from the height and diameter increment equations as it does not attempt to predict yearly growth. As such, it does not assume the potential times modifier form found in the incremental models. PTAEDA2 utilizes the model developed by Dyer and Burkhart (1987), which is reiterated:

$$CR = 1 - \exp[(\beta_1 - \beta_2 A^{-1})D/H]$$

The crown ratio model was separately fit to unthinned, light thinned, and heavy thinned data (Table 5). Fit statistics from the regression analyses were used to test for model parameter differences across thinning regimes. The F-statistic for the test was 97.19 ($p < .00001$). As with diameter increment, it is generally known that crown ratio becomes larger as distance between trees increases (Figure 11). Changes in model variables diameter and height do not fully account for the differences in crown ratio across the three thinning levels found in this study. Both the Short and Burkhart (1992) and Liu et al. (1995) thinning response variables were developed for crown ratio prediction in unthinned and thinned stands. However, Short and Burkhart (1992) did not apply their TRV to the model of Dyer and Burkhart (1987) that is used in PTAEDA2. Thus, it will be necessary to examine the performance of this TRV in various locations in the crown ratio model. The Liu et al. (1995) TRV was evaluated using the PTAEDA2 crown ratio model. Initial assessment of the TRV will consist of fitting the model form specified in the previous work to this data set.

Placement of the Short and Burkhart (1992) TRV in the crown ratio model offered several locations:

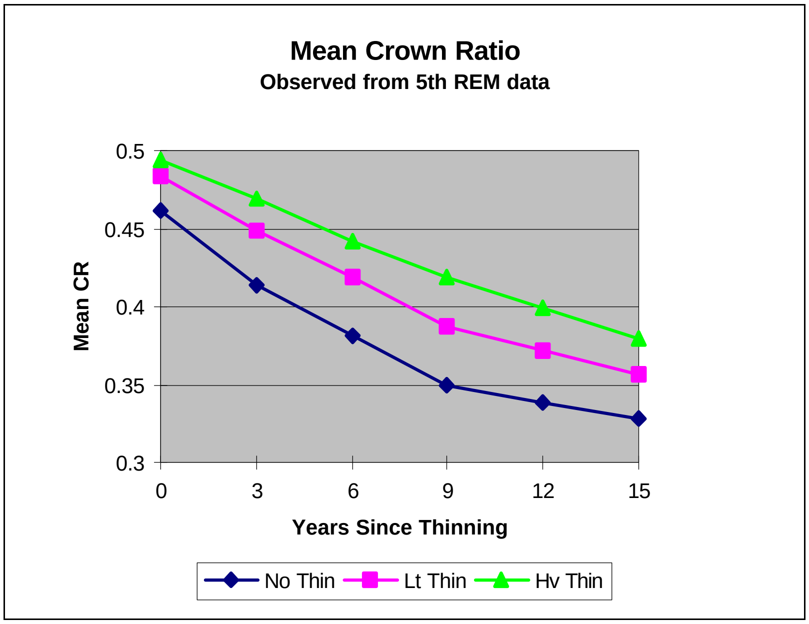



Figure 11. Mean crown ratio values from observed data.

1) Multiply the TRV times the entire model
2) Multiply the TRV times the exponential portion of the model
3) Include the TRV within the exponential term (multiply by D/H)

Fits to the crown ratio model with the Short TRV in each of the listed locations provided no practical improvement over the CR model alone. This was true for both the basal area ratio and competition index ratio forms. This was not surprising due to the inherent behavior of this TRV.

The Liu TRV was also assessed in both basal area ratio and competition index ratio forms. However, it was only placed into the crown ratio model in the same location that was reported by Liu et al. (1995), as this was deemed likely to provide the best result. As with the diameter increment research, the fitted parameter estimates within the TRV forced improper behavior. The Liu TRV resulted in no practical improvement over the CR model alone.

The base to exponent and exponential form TRV's developed for this project that were applied to the DIN model were evaluated as well. The same problems that plagued the DIN fits were encountered. TRV1 exhibited inverted behavior in all locations due to the sign of the estimated parameters. TRV2 also behaved improperly and, when multiplied times the exponential portion of the model, had an illogical parameter estimate. The feasibility of constructing a TRV that provides the proper response for crown ratio in thinned stands using the variables available in the Coop data set came into question.

The lack of prediction improvement using the Short, Liu, and newly developed TRV's led to the conclusion that a multiplicative TRV may not be appropriate. A new TRV was created that was additive:

$$\text{TRV3} = T\,\frac{(BA_b - BA_a)}{BA_b}\left[b_1 D^{b_2}\right]\exp\left[\frac{-(A_s - A_t)}{\sqrt{A_s}}\right]$$

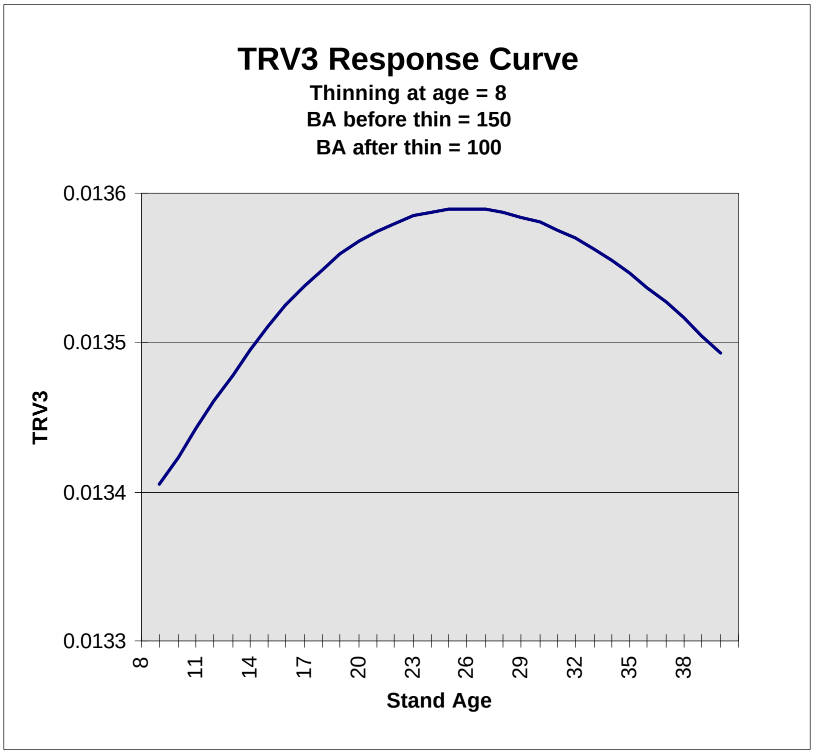
where: T = indicator variable
(= 0 for unthinned stands)
(= 0 for thinned stands at time of thinning)
(= 1 for thinned stands after first growth season)

This TRV allows the response curve of the original model to be maintained while providing an increase in response for thinned stands. The inclusion of an indicator variable reduces many of the constraints encountered in the development of the other TRV's. The additive property of the TRV allows the TRV to assume a value of zero when no response to thinning is appropriate. The multiplicative TRV's had to take on a value of one for no response. Variable selection and usage for an additive TRV is not as greatly restrained as in the multiplicative TRV's.

The fit statistics of TRV3 combined with the crown ratio model show slight improvements in MSE and r-square values when compared to any of the fits of other TRV's or the CR model alone (Table 6). While the additive form of the TRV does provide better predictive ability, the level of improvement was less than expected. The behavior of TRV3 is illustrated in Figure 12 using the parameter estimates from the fitted regresssion.

Comparisons between the PTAEDA2 CR model, the re-fit CR model, and the CR model with TRV3 over all data show that the PTAEDA2 model outperforms the other two models only at plot establishment (Figure 13). Thereafter, the newer models are conspicuously superior in predictive ability. Analysis of model performance by thinning treatment is similar to that of all data in that the original CR model has acceptable predictive ability only at the time of plot establishment (Figure 14). Comparisons between the re-fit CR model and the CR model with TRV3 show equivalent performance for unthinned stands. The effects of TRV3 begin to show in light thinned plots. The CR model incorporating TRV3 provides slightly smaller mean residual values at each remeasurement except the first. In heavy thinned plots, where the effect of a TRV should be most obvious, the TRV3 CR model shows markedly better predictive ability when compared to the re-fit CR model at each remeasurement except the first.

This lack of predictive success at the first remeasurement led to an investigation of the data. It was surmised that the ineffectiveness of the TRV may be due to a lack of thinning response in the data. This phenomenon could occur in two ways. First, the actual data indicates little or no difference in tree growth across thinning treatments. Second, the number of observations in which a thinning response is present is greatly outweighed by the amount of data containing no thinning response. Analyses have shown that, except for height increment, there is indeed a response to thinning for diameter increment, crown ratio, and tree mortality. Effects of thinning on crown ratio and competition account for the response in mortality levels, and to a lesser extent, diameter growth. However, changes in tree dbh due to



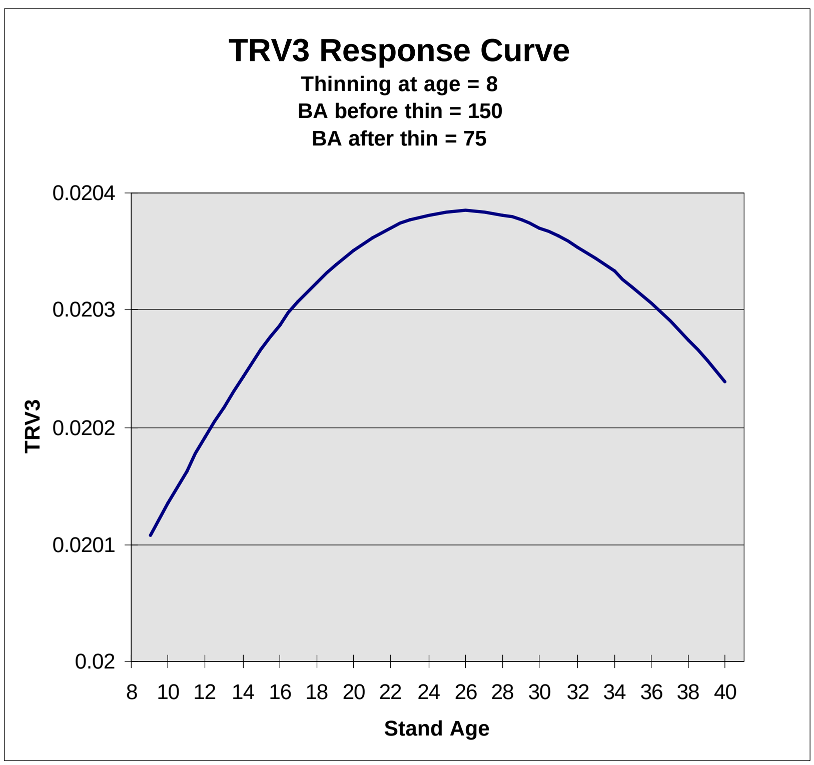




Figure 12. Behavior of TRV3 for light thinned and heavy thinned stands.

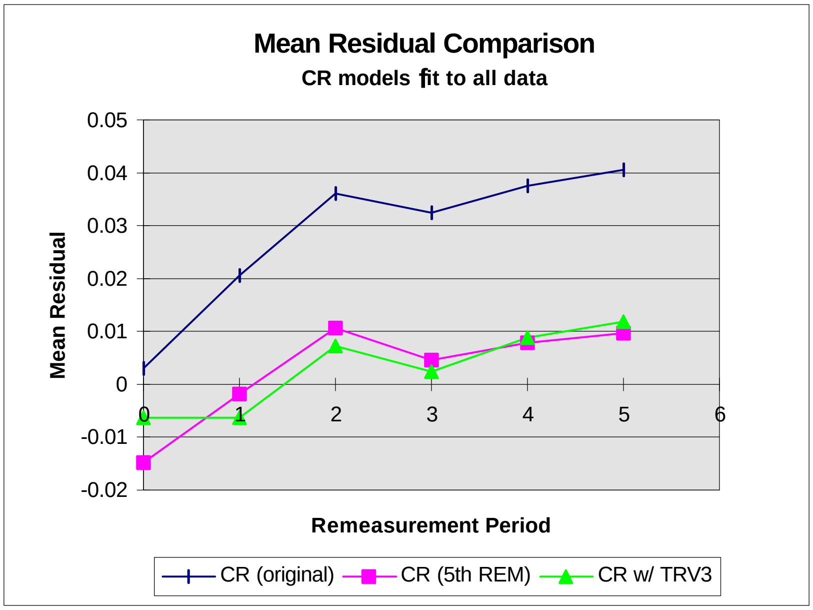



Figure 13. Mean residual comparison between PTAEDA2 CR model, re-fit CR model, and CR model with TRV3.

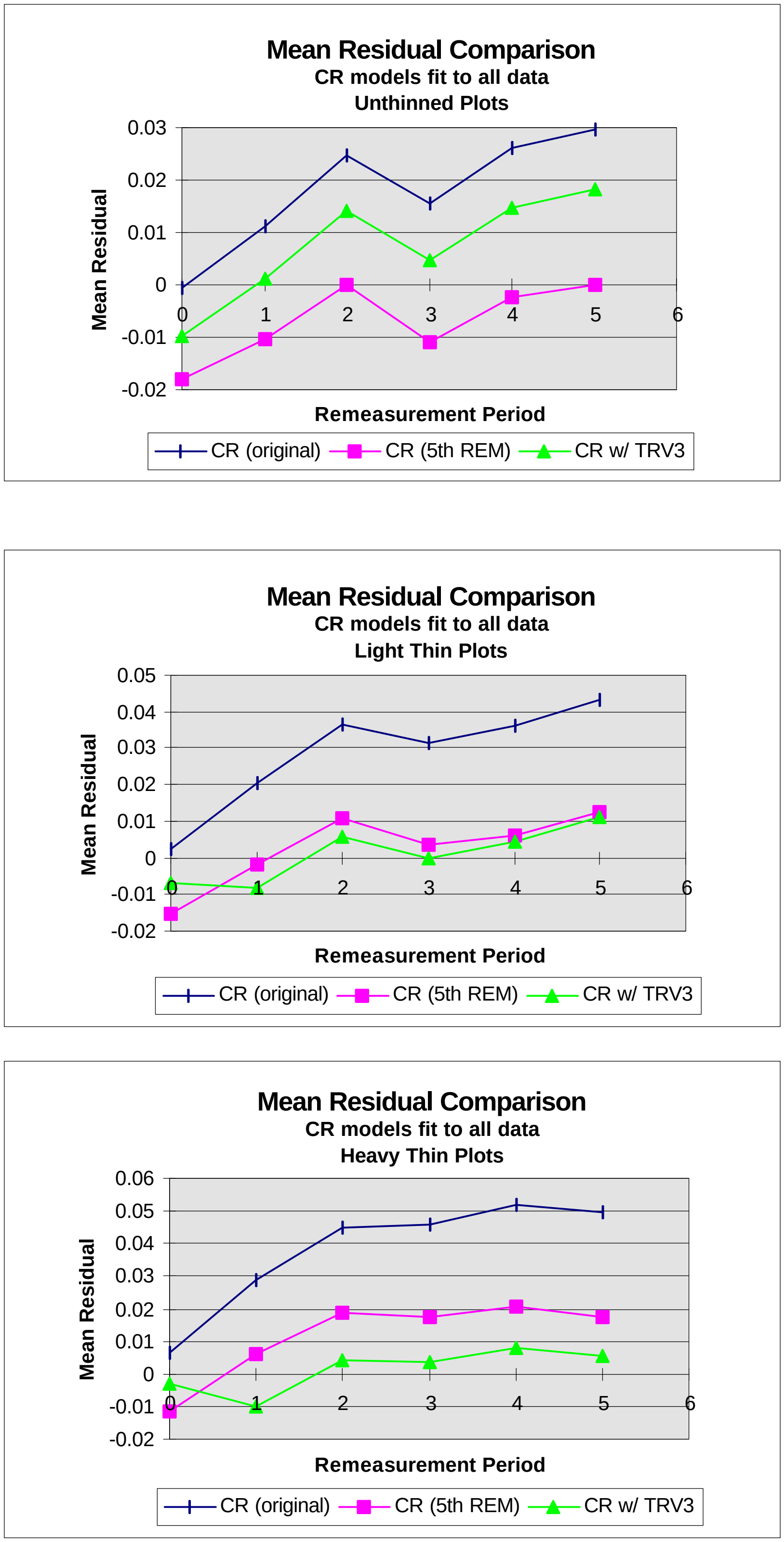



Figure 14. Mean residual comparison between CR models.

thinning do not account for the response of crown ratio (assuming height growth is largely unaffected by thinning). Crown ratio prediction should benefit from some type of thinning response adjustment.

The presence of thinning response in the observed data leads to the conclusion that the latter explanation should be investigated. Observations in which no thinning response is needed include all data from unthinned stands and data from thinned stands at time of thinning. If these observations make up a disproportionate amount of the data, the effects of thinning may be overshadowed. Analysis of the data reveals that less than half (42.5%) of the data fit this description. Furthermore, the bulk of the data (65.5%) comes from plot establishment and the first remeasurement period. Obviously, there is no thinning response at the time of plot establishment and it is likely that the first remeasurement data exhibits minimal thinning response due to the necessary assumption of linearity in growth over the three year period.

This phenomenon may prevent a TRV from providing improved prediction in the first three years of the study because model parameters are estimated from data that is dominated by observations that contain little or no thinning response. In an attempt to overcome this, a new data set was created which contained an equal number of observations from each remeasurement period. Randomly selected observations were taken from each remeasurement period.

The CR model with each of the TRV's developed for this project were re-fit to this balanced data set to see if an appropriate response would occur. In each case, MSE dropped approximately 6%. However, TRV1 and TRV2 still exhibited improper behavior. TRV3 maintained appropriate behavior. The results of these fits indicate that a certain level of bias towards data from earlier remeasurement periods is present. While the balancing of the data did provide improved fit statistics, the actual numerical difference in MSE (-.0003) suggests that meaningful improvements in crown ratio prediction may be minimal.

Comparison of residuals between the CR models with TRV3 fit to all data and balanced data show that the fit to all data provides better prediction at the first and second remeasurement periods. The fit to balanced data has smaller mean residuals for the remainder of the remeasurement periods (Figure 15). This provides further evidence that the data structure is detrimental to predictive success at later remeasurement periods. The use of parameter estimates from the fit to balanced data may be of some value as elapsed time since thinning increases.

The CR model with TRV3 as fit to balanced combined fit/validation data is shown below:

$$CR = 1 - \exp[(-1.55225903 - 36.27559592A^{-1})D/H] + TRV3$$

where: $$TRV3 = T\frac{(BA_b - BA_a)}{BA_b}\left[0.02788582D^{0.47736085}\right]\exp\left[\frac{-(A_s - A_t)}{\sqrt{A_s}}\right]$$

The CR model with TRV3 as fit to all combined fit/validation data is also given:

$$CR = 1 - \exp[(-1.48380653 - 36.84780194A^{-1})D/H] + TRV3$$

where: $$TRV3 = T\frac{(BA_b - BA_a)}{BA_b}\left[0.03206053D^{0.43664658}\right]\exp\left[\frac{-(A_s - A_t)}{\sqrt{A_s}}\right]$$

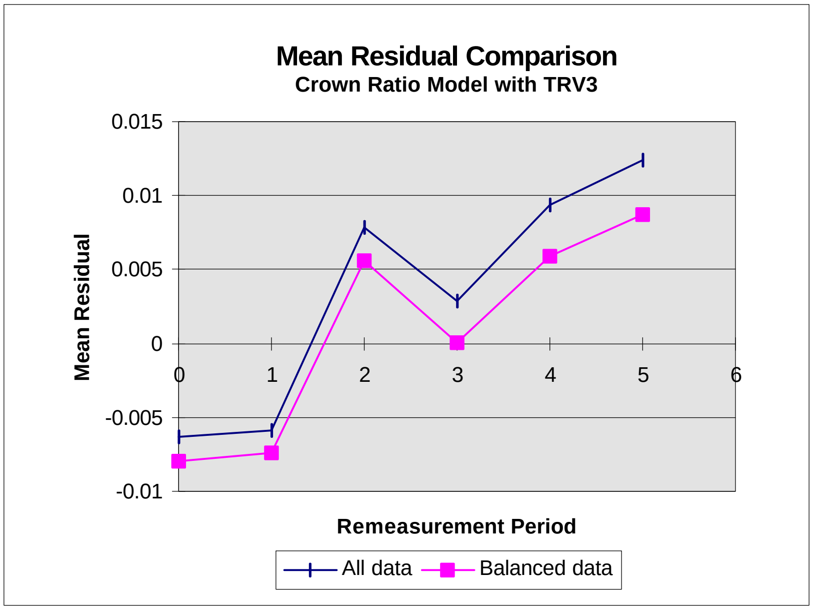



Figure 15. Mean residual comparison between fits of CR model with TRV3 to all data and to balanced data.

Table 5. Statistics for fits of CR model and CR model with pre-existing thinning response variables.

| Model | df | MSE | Sy.x | R-square |
|---|---|---|---|---|
| A | 13550 | 0.00527 | 0.07259 | 0.62 |
| B | 19336 | 0.00530 | 0.07280 | 0.61 |
| C | 15767 | 0.00506 | 0.07113 | 0.60 |
| D | 48657 | 0.00526 | 0.07253 | 0.62 |
| E | 81095 | 0.00524 | 0.07239 | 0.62 |
| F | 20200 | 0.00494 | 0.07029 | 0.62 |
| G | 33730 | 0.00507 | 0.07120 | 0.61 |
| H | 48656 | 0.00524 | 0.07239 | 0.62 |
| I | 48656 | 0.00522 | 0.07225 | 0.62 |
| J | 48656 | 0.00523 | 0.07232 | 0.62 |
| K | 48454 | 0.00524 | 0.07239 | 0.62 |
| L | 48454 | 0.00521 | 0.07218 | 0.62 |
| M | 48454 | 0.00523 | 0.07232 | 0.62 |
| N | 48453 | 0.00522 | 0.07225 | 0.62 |
| O | 48655 | 0.00521 | 0.07218 | 0.62 |
| P | 20198 | 0.00489 | 0.06993 | 0.62 |

Model Description:

| | |
|---|---|
| A | CR model fit to unthinned data |
| B | CR model fit to light thinned data |
| C | CR model fit to heavy thinned data |
| D | CR model fit to all data |
| E | CR model fit to combined fit/validation data |
| F | CR model fit to balanced data |
| G | CR model fit to balanced combined fit/validation data |
| H | CR model with Short TRV (BA ratio) multiplied times model fit to all data |
| I | CR model with Short TRV (BA ratio) multiplied times exponential term fit to all data |
| J | CR model with Short TRV (BA ratio) in exponential term fit to all data |
| K | CR model with Short TRV (CI ratio) multiplied times model fit to all data |
| L | CR model with Short TRV (CI ratio) multiplied times exponential term fit to all data |
| M | CR model with Short TRV (CI ratio) in exponential term fit to all data |
| N | CR model with Liu TRV (BA ratio) multiplied times exponential term fit to all data |
| O | CR model with Liu TRV (CI ratio) multiplied times exponential term fit to all data |
| P | CR model with Liu TRV (BA ratio) multiplied times exponential term fit to balanced data |

Table 6. Statistics for fits of CR model incorporating newly developed thinning response variables.

| Model | df | MSE | Sy.x | R-square |
|---|---|---|---|---|
| A | 48656 | 0.00525 | 0.07246 | 0.62 |
| B | 48656 | 0.00524 | 0.07239 | 0.62 |
| C | 48656 | 0.00525 | 0.07246 | 0.62 |
| D | 20199 | 0.00492 | 0.07014 | 0.62 |
| E | 48656 | 0.00532 | 0.07294 | 0.62 |
| F | 48656 | 0.00523 | 0.07232 | 0.62 |
| G | 48656 | 0.00529 | 0.07273 | 0.62 |
| H | 20199 | 0.00490 | 0.07000 | 0.62 |
| I | 48655 | 0.00511 | 0.07148 | 0.63 |
| J | 81093 | 0.00510 | 0.07141 | 0.63 |
| K | 20198 | 0.00480 | 0.06928 | 0.63 |
| L | 33728 | 0.00494 | 0.07029 | 0.62 |

Model description

| | |
|---|---|
| A | CR model with TRV1 multiplied times model fit to all data |
| B | CR model with TRV1 multiplied times exponential term fit to all data |
| C | CR model with TRV1 in exponential term fit to all data |
| D | CR model with TRV1 multiplied times exponential term fit to balanced data |
| E | CR model with TRV2 multiplied times model fit to all data |
| F | CR model with TRV2 multiplied times exponential term fit to all data |
| G | CR model with TRV2 in exponential term fit to all data |
| H | CR model with TRV2 multiplied times exponential term fit to balanced data |
| I | CR model with TRV3 added to model fit to all data |
| J | CR model with TRV3 added to model fit to combined fit/validation data |
| K | CR model with TRV3 added to model fit to balanced data |
| L | CR model with TRV3 added to model fit to balanced combined fit/validation data |

Mortality Function

The mortality function estimates a probability that a tree will remain alive. Thus, a number between 0 and 1 is calculated based on tree crown ratio and competition index. Trees having large crown ratios and low competition index values have a greater probability of staying alive than trees with small crown ratios and a high number of competitors. The mortality function is shown again:

$$PLIVE = \beta_1 CR^{\beta_2} * \exp(-\beta_3 CI^{\beta_4})$$

This function is fit to binary (0, 1) data. Trees that were alive at each remeasurement period were assigned a 1. Trees that have died are only assigned a 0 value if they were assumed to have died in the first year after the previous remeasurement. This was necessary to assure valid crown ratio and competition index values.

Analysis of the data revealed the existence of what can best be described as 'measurement bias.' Given the wide range of ages, site qualities, densities, etc. it seems to be a valid assumption that approximately one-third of the mortality between remeasurements would occur in the first year, one-third in the second year, etc. However, this data set shows that only slightly over twenty percent of all mortality occurred in the first year after remeasurement. Additionally, mortality for the second year after remeasurement is over forty seven percent. Mortality that occurred in the third year following remeasurement is just under thirty three percent.

These results prompted a similar investigation into the data set consisting of interior trees only. This was done because competition index is an independent variable in the mortality function, and thus, the fitting data will be comprised of interior trees exclusively. The interior trees exhibited the same bias as the whole data set. Interior trees having a year of death immediately following remeasurement make up only 18.6% of the total mortality. Death occurring in the second year was 49.2% and death in the third year was 32.3%.

It is proposed that this phenomenon is a function of the cruiser's ability to determine how long a given tree has been dead. If a tree has died recently, it is easily noticeable and the year of death is assigned as the third year past the previous remeasurement. However, it is more difficult to determine a year of death for trees that have been dead for some time. Analysis of the Coop thinning study data indicates that cruisers are reluctant to say that trees that have died since the previous remeasurement have been dead for more than two years. Thus, very few trees are assigned a year of death that immediately succeeds remeasurement. This would account for the large discrepancy in mortality levels between the first and second years following remeasurement. The third year following remeasurement is very close to the expected value (33.3%).

In order to compensate for this bias, the data for fitting the mortality function was adjusted to obtain approximately equal levels of mortality for each of the three years between remeasurements. This was accomplished by taking a random portion of second year trees and placing them in the first year category. Crown ratio and competition index values were calculated as though these trees had died in the first year following remeasurement. This alteration of interior tree data resulted in mortality levels of 33.6%, 34.2%, and 32.2% for the first, second, and third years following remeasurement, respectively.

Separate fits to the mortality function were made for unthinned, light thinned, heavy thinned, light/heavy thinned, and all data. The hypothesis test for differences in model parameters across all three thinning treatments was nonsignificant (p = 0.1878). Fits to data divided into unthinned and thinned categories also resulted in models having nonsignificant differences in parameter estimates (p = 0.3954). This outcome indicates that changes in crown ratio and competition index account for the different rates of mortality across the thinning regimes and that it is appropriate to fit the mortality function to all data.

Validation data was used to compare the PLIVE values generated with parameter estimates from PTAEDA2 to the PLIVE values obtained using the parameter estimates from this research (Table 7.). It was found that the mean PLIVE values were virtually identical but that the minimum calculated value was much smaller using the newly estimated parameters. Also, the new parameter estimates generated a smaller number of PLIVE values greater than one. This translates into a smaller probability of remaining alive for all trees. This should improve mortality prediction as the current mortality function being used in PTAEDA2 underestimates mortality (Avila and Burkhart, 1992).

In order to assess the behavior of the mortality function with new parameter estimates, a number of simulation runs were made. PLIVE values were calculated for each tree using the parameter estimates currently found in PTAEDA2. PLIVE values were also generated from the re-fit mortality function. These values were compared to uniformly-distributed random values between 0 and 1. If PLIVE is greater than the random value, the tree remains alive. A tree is presumed to have died if PLIVE is less than the random number. This process was repeated 10 times and the results are summarized in Table 8.

The re-fit mortality function performed better than the PTAEDA2 equation in prediction of number of dead trees and in prediction of death for individual trees in the validation data. It was surprising to find that the PTAEDA2 mortality function provided a higher number of dead trees than the re-fit model. Given the wider range of PLIVE values and the number of PLIVE less than one that are associated with the re-fit function, it was expected that a greater amount of mortality would be realized.

Although the new parameter estimates provide better identification of trees that were actually dead in the validation data, the number of dead trees that were correctly predicted to have died was very low. On average, only 1 in 43 dead trees were correctly assigned to the dead category. This prompted an investigation into the data to see if there was some type of explanation for this discrepancy.

The cause of death for 12 of the 43 (28%) dead trees was insect damage or disease. The failure of the mortality function to accurately predict death for these trees is acceptable because the mortality model is specifically designed to capture death due to suppression. However, this still leaves a large percentage of dead trees whose death was not predicted. Analysis of mean crown ratio and mean competition index for trees whose death is attributed to suppression and for live trees show that the dead trees do

Table 7. Statistics for calculated PLIVE values comparing the PTAEDA2 mortality function and the re-fit mortality function.

| Model | Mean | Std. Dev. | High | Low | Percent > 1 |
|---|---|---|---|---|---|
| A | 0.9963 | 0.0201 | 1.0239 | 0.8871 | 42.33 |
| B | 0.9942 | 0.0187 | 1.0099 | 0.7810 | 22.62 |

Model Description

A Original PTAEDA2 mortality function
B PTAEDA2 mortality function fit to 5th REM data

Table 8. Mean values for 10 simulation runs comparing mortality prediction between the PTAEDA2 mortality function and re-fit mortality function.

| | Dead Trees | | | | Live Trees | | | |
|---|---|---|---|---|---|---|---|---|
| Model | Actual | Predicted | Correctly Predicted | Incorrectly Predicted | Actual | Predicted | Correctly Predicted | Incorrectly Predicted |
| A | 43 | 45.9 | 0.9 | 45.0 | 7171 | 7168.1 | 7126.0 | 42.1 |
| B | 43 | 44.0 | 1.5 | 42.5 | 7171 | 7170.0 | 7128.5 | 41.5 |

Model Description

A Original PTAEDA2 mortality function
B PTAEDA2 mortality function fit to 5th REM data

exhibit smaller crown ratios and greater competition indices when compared to live trees (See Table 9). Yet, when these values are examined in the context of PLIVE values, there is very little difference. The calculated PLIVE using mean crown ratio and competition index values for dead trees is 0.98132. Similarly, substitution of mean values for live trees into the mortality function produces a PLIVE of 0.99648. The difference in average PLIVE values between trees suffering mortality through suppression and live trees is approximately 0.015. This would seem to be the explanation for the inability of the mortality function to accurately identify dead trees on a specific tree basis.

The mortality function does predict overall mortality levels quite well. The form of the model does not directly account for death due to insect damage, lightning, windthrow, etc. However, these causes of death appear to be accounted for by the model prediction as far as numbers of trees dying is concerned. Inclusion of all causes of death in the fitting data results in model parameter estimates that can predict overall mortality at the stand level with good accuracy.

A single fit to the combined fit/validation data was done to get final parameter estimates for the mortality function:

$$PLIVE = 1.00893CR^{0.01486} * \exp(-0.00463CI^{3.16075})$$

Table 9. Mean crown ratio, competition index, and PLIVE values for live trees and for trees having died from natural suppression.

| Status | N | Mean CR | Mean CI | Mean PLIVE |
|---|---|---|---|---|
| Live Trees | 7171 | 0.4734 | 0.6696 | 0.99648 |
| Dead Trees* | 31 | 0.3499 | 1.3565 | 0.98132 |

* Trees dying from natural suppression

Reduced Growth Simulator

The primary function of the models addressed in this research is to project stand development over time by growing trees on an annual basis. In order to accomplish this, these models must work together as elements of an entire system. The predictive ability of a given model may heavily rely on the performance of one or more of the other models. In PTAEDA2, these models are included in the subroutine GROW2. GROW2 is the portion of the computer program that performs yearly tree growth functions. The main program, the GROW2 subroutine, and several other supporting subroutines were extracted from PTAEDA2 by Smith (1994). These portions of PTADEA2 were modified to create a reduced growth simulator. This reduced simulator is designed to read external data to provide the basis for an existing stand. In this case, the external data is the data from the Coop thinning study at plot establishment.

The reduced simulator takes the initial data and grows the trees on an annual basis. Outputs are generated at three year intervals for comparison with observed data from each remeasurement period. For this research, the reduced simulator by Smith (1994) had to be modified to output data for the 12$^{th}$ and 15$^{th}$ growing season (4$^{th}$ and 5$^{th}$ remeasurement period). The actual models in the GROW2 subroutine were changed to those developed earlier in this project.

The FORTRAN source code for the reduced growth simulator is listed in Appendix A.

## Results and Conclusions

Results

An initial simulation run was performed using the PTAEDA2 height increment, diameter increment, crown ratio, and mortality models. This was done to establish a baseline for comparative purposes.

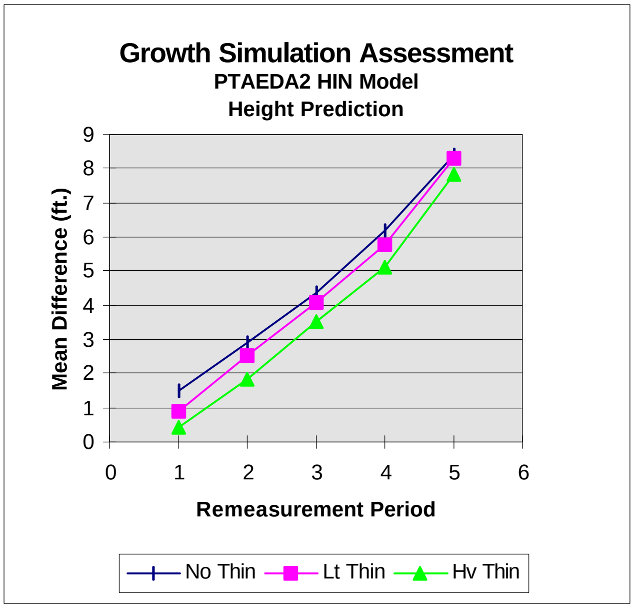
The first model in the subroutine GROW2 is height increment. The height increment model performance is critical to the performance of the other models. The improved predictive ability of the re-fit height model may provide better diameter growth, crown ratio, and mortality prediction without changing any of these other models.

As expected, the simulation produced a notable improvement in height prediction across the range of the data when the results from the new model were compared to the PTAEDA2 HIN model. This improvement is especially notable at later remeasurement periods where mean residuals improved by 1-2 feet across all thinning treatments (Figure 16).

The effects of the new HIN model on prediction of diameter growth, crown ratio and mortality were either non-existent or slightly negative. The mean differences between predicted and observed tree diameters were virtually identical for both the PTAEDA2 and re-fit HIN models across the range of the data. Crown ratio prediction was worsened across all data. Mean residuals were increased by approximately 0.005 to 0.01. Predicted mortality levels were mostly equivalent for each HIN model simulation. There were several points where there was a slight improvement or decrease in the mean number of trees predicted to have died. These differences were generally not greater than 1. The anticipated improvement in predictive ability of the other growth functions failed to materialize.

The simulation runs for the re-fit DIN model and the DIN model with TRV2 produced unexpected results. The re-fit DIN model could not match the predictive ability of the PTAEDA2 DIN model at any point (Figure 17). It was thought that the DIN model fit to the 5th remeasurement data would have superior predictive ability, especially for the first through fourth remeasurement periods. Comparisons made between the PTAEDA2 DIN model and the re-fit model earlier in this research showed that, on a mean level, the re-fit model performed better at all remeasurements except the fifth.

Previous analysis of the DIN models also showed that there was no significant difference in predictive ability between the re-fit DIN model and the DIN model with TRV2. These models performed very similarly by remeasurement period and by thinning treatment. The simulation run in which the DIN model with TRV2 was isolated resulted in dramatic improvements in diameter prediction. Apparently, the desired effect of the TRV was achieved when the model had to perform within the growth simulator. The most noteworthy gains were for the light thinned and heavy thinned plots. For heavily thinned stands, the maximum mean residual the Previous analysis of the DIN models also showed that there was no significant difference in predictive ability between the re-fit DIN model and DIN model with TRV2. These models performed very similarly by remeasurement period and by thinning treatment. The simulation run in which the DIN model with TRV2 was isolated resulted in dramatic dropped from approximately 1.6 to 0.03. Similar gains were found in light thin plots



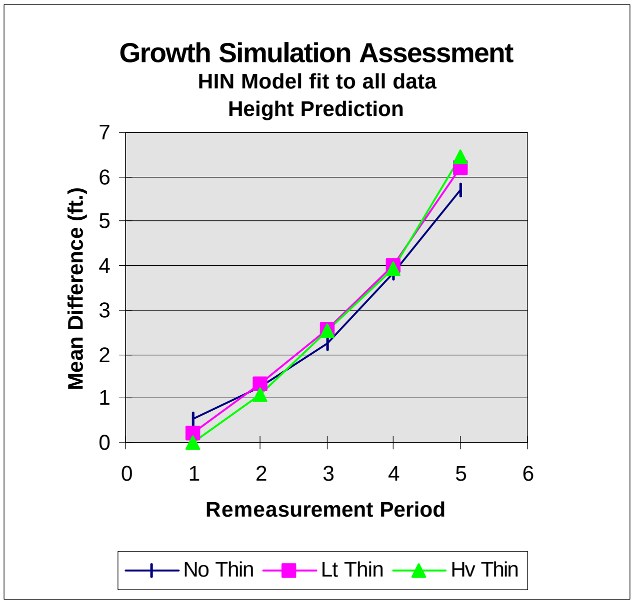




Figure 16. Growth simulation comparison between HIN models

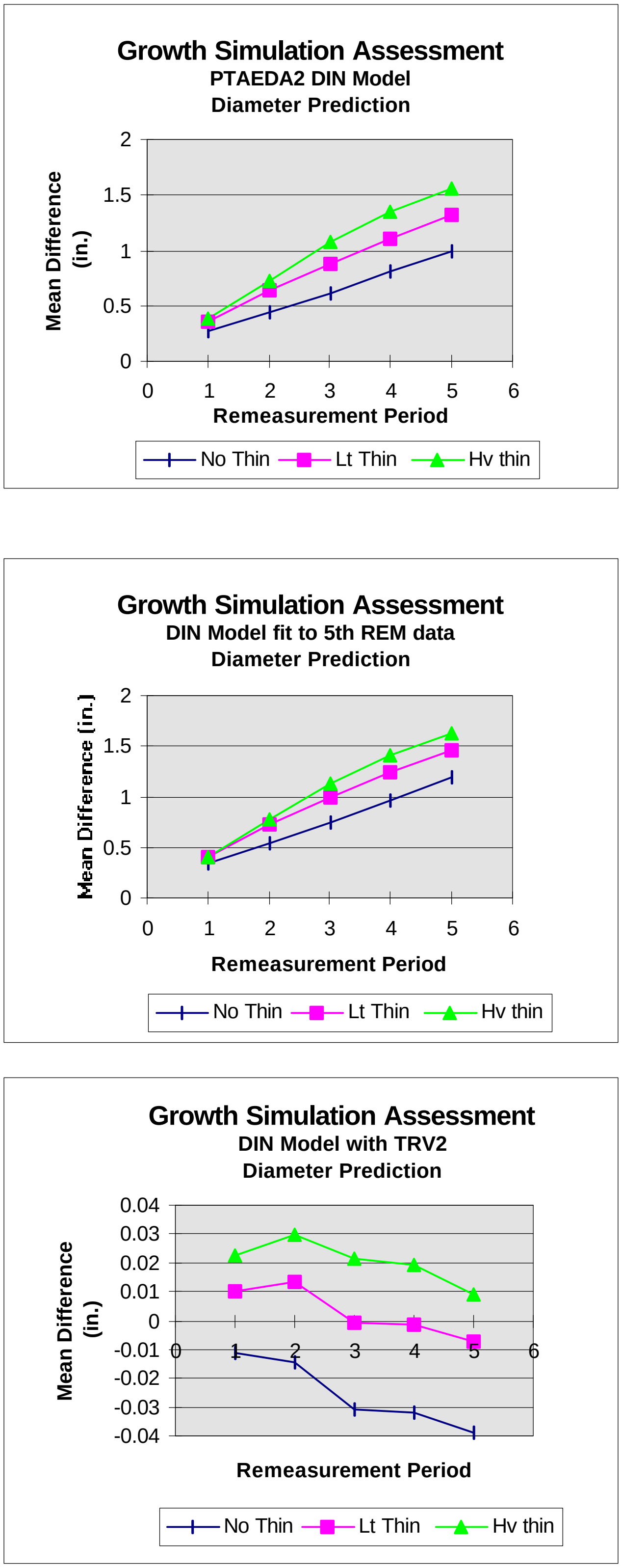



Figure 17. Growth simulation comparison between DIN models

where the maximum mean residual dropped from 1.5 to 0.015. Tangible improvements in diameter prediction for unthinned stands were also obtained. The notable change in prediction for unthinned plots is that there was a shift in predictive bias from under-prediction to over-prediction. However, given the relatively small magnitude of the residuals, this is preferable to the much larger predictive error found in the other DIN models.

The performance of crown ratio models was also evaluated within the growth simulator. In addition to the PTAEDA2 CR model that was a component of the baseline runs, the CR model fit to 5$^{th}$ remeasurement data and the CR model with TRV3 were evaluated. First, the re-fit CR model performance was compared to that of the PTAEDA2 model. The newly fit CR model provided much better prediction for thinned stands (Figure18). Mean residuals for heavy thinned stands dropped roughly 50% across all remeasurement periods. Even greater improvements exist for light thinned plots, where mean residuals dropped in the range of 60-80%. These results were obtained while still maintaining an under-prediction bias. Unthinned stands showed no meaningful difference in mean residual magnitude except at the second remeasurement. This simulation provided a similar result to that of the DIN model with TRV2 in that the predictive bias for unthinned stands shifted from under-prediction to over prediction. Unfortunately, the re-fit CR model did not provide notable predictive improvements to offset this shift.

The simulation results for the CR model with TRV3 produced an entirely different pattern of residual values for thinned stands. Both light thin and heavy thin stands exhibit a marked shift from under-prediction at the first remeasurement to over-prediction throughout the remainder of the data. This type of behavior was not found in simulation runs of the other two CR models. Although over-prediction is evident, mean residuals dropped roughly 50% for each remeasurement period for heavy thinned stands. Modest gains were obtained for light thinned stands, most notably at the second and fourth remeasurement. The CR model with TRV3 is inferior to the re-fit CR model for thinned stands at all remeasurement periods. An unexpected effect of the CR model with TRV3 is the superior predictive ability for unthinned stands. The mean residual values range from nearly 0 to 0.0081 across all remeasurement periods. Neither the PTAEDA2 CR model nor re-fit CR model provide this level of predictive success for unthinned stands. This result is contrary to the intended effect of TRV3 in the CR model. One would expect to see improvements in prediction for thinned stands while possibly a slight decrease in performance for unthinned stands.

The performance of mortality functions within the growth simulator were assessed as well. Comparisons were made between the PTAEDA2 mortality function and the same mortality function fit to 5$^{th}$ remeasurement data. Differences in observed vs. predicted numbers of trees suffering mortality were calculated for each remeasurement period by thinning treatment (Figure 19). The behavior of both mortality functions is very similar. The re-fit model exhibits a much higher level of predictive error than the PTAEDA2 mortality function at the first remeasurement period where mortality is over-predicted. This over-prediction is most evident in unthinned stands. After this initial over-prediction, mortality estimates are consistent and reliable throughout remeasurement periods and across thinning treatments.

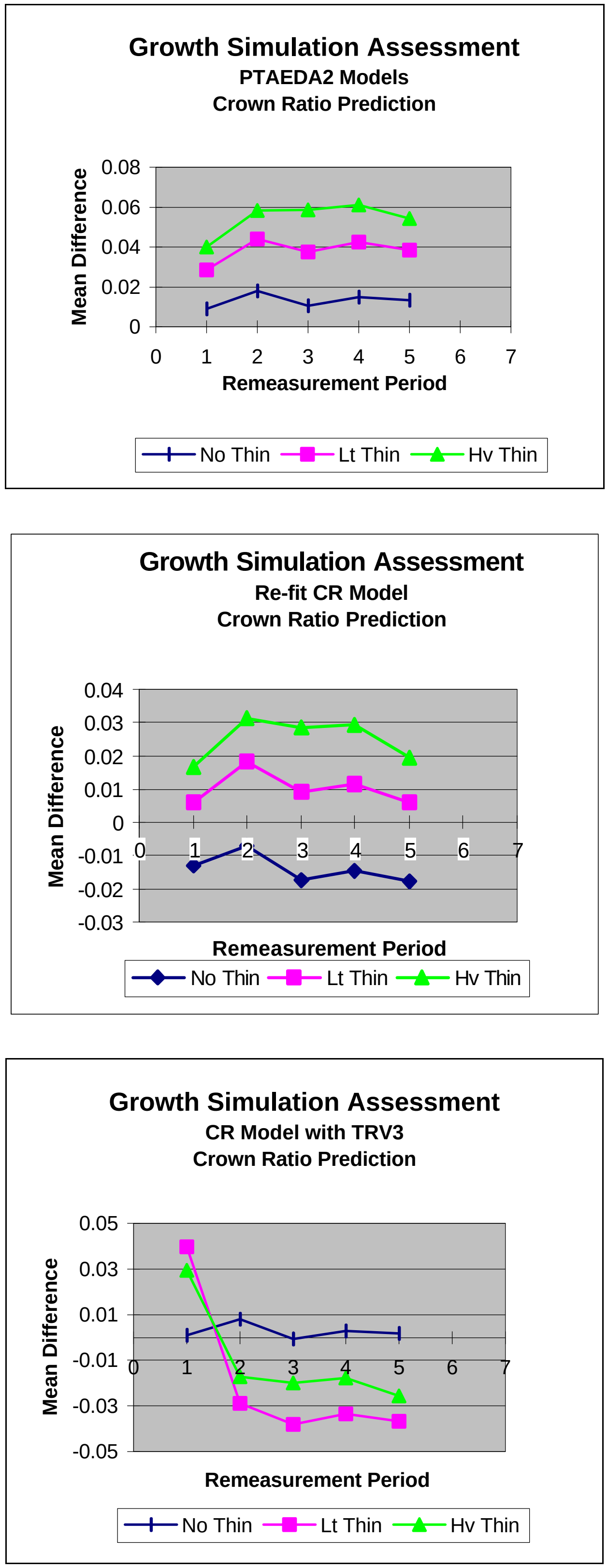
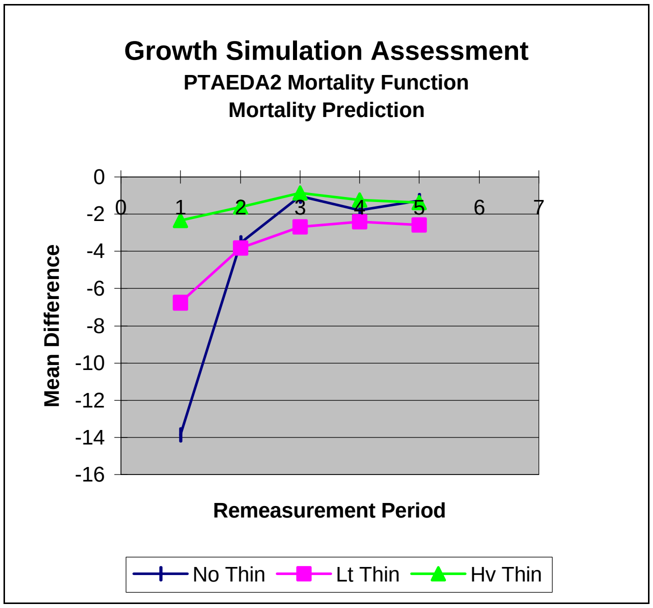



Figure 18. Growth simulation comparison between CR models.

It is thought that the cause of the greater predictive error at the first remeasurement for the re-fit mortality function is two-fold. Analyses of the data reveals that the number of observations from unthinned stands is much lower than those of thinned stands. Unthinned plot data comprised only 10.7% of the fitting data, while 51.8% of the data came from light thin plots and 37.5% came from heavy thin plots. This was largely due to the fact that most of the unthinned plots are small compared to the thinned plots and thus the unthinned plots had much fewer interior trees. This data structure results in parameter estimates that provide appropriate levels of mortality for crown ratio and competition index values that occur in thinned stands. This results in the excessive mortality prediction at the first remeasurement for unthinned stands.

The over-prediction that is found in thinned stands can be attributed to growth patterns that exist immediately after thinning. Height growth for trees in thinned stands slows immediately after thinning as nutrients are re-distributed to increase foliage growth. This results in lower crown ratio values. Additionally, if the lowest whorl of live branches was on the verge of death before thinning, these branches are still likely to die even though thinning has increased available light. This combination of reduced height growth and equal or greater height-to-crown measurements produces crown ratio values that would usually be found in denser stands. In terms of mortality prediction, competition index values are small because of decreased density due to thinning, but crown ratio values are also small. Thus, mortality is also over-predicted for thinned stands at the first remeasurement. This overprediction is not as great as is found in unthinned stands because the lower competition index values in thinned stands help increase the calculated PLIVE somewhat. Overall, the PTAEDA2 mortality



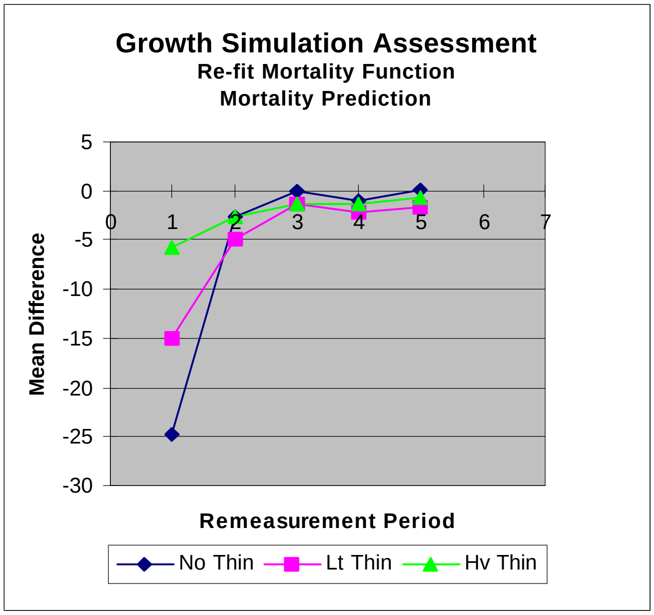




Figure 19. Growth simulation comparison between mortality functions.

function provided better prediction for thinned stands, except at the fifth remeasurement where the re-fit model was slightly better. The re-fit function provided better mortality prediction for unthinned stands for the second through fifth remeasurement periods.

These results are contrary to the mortality simulation comparison between these models that was performed earlier in this project. Recall that although the re-fit model generated a lower minimum PLIVE value and had fewer PLIVE values less than 1.0, the simulation results showed that the PTAEDA2 mortality function predicted a greater mortality level. It has been noted that mortality is generally under-predicted in PTAEDA2 (Avila and Burkhart, 1992). The re-fit mortality function tends to initially increase mortality levels for thinned stands when compared to the PTAEDA2 model, but then gives better mortality estimates at the later remeasurment periods.

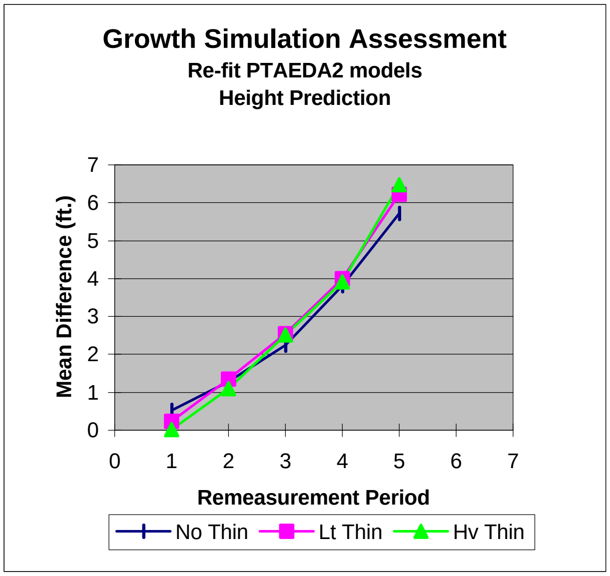
The growth simulations performed up to this point have been useful to evaluate the behavior of various individual models. It would seem appropriate to evaluate the effects of certain combinations of these models. The results from the runs in which each model was isolated can be used to judge which combinations of models may be appropriate.

One of the obvious selections would be to run the height increment, diameter increment, crown ratio, and mortality functions using parameter estimates from fits to 5$^{th}$ remeasurement data. This would seem to provide good prediction as each model is utilizing all of the available growth data. The re-fit models were run simultaneously in the reduced growth simulator with positive results.

Height prediction was greatly improved when compared to the PTAEDA2 simulation. Also, mean residual values decreased across the range of the data, except for heavy thin plots at the first remeasurement period, when compared to the simulation run in which the re-fit HIN model was isolated (Figure 20). The greatest improvements were found at the later remeasurement periods, were mean residuals were decreased approximately 0.5 feet for each thinning treatment. Predictive bias was very consistent across the thinning treatments. This provides further evidence of no difference in model parameter estimates across thinning treatments.

Re-fitting of all models had mixed effects on diameter prediction. At the first remeasurement period, the PTAEDA2 configuration resulted in slightly better predictive ability. The PTAEDA2 DIN model and the re-fit DIN model provided virtually identical levels of predictive error at the second remeasurement. The third through fifth remeasurement periods show the re-slightly better predictive ability. The PTAEDA2 DIN model and the re-fit fit model emerging as the better model with the greatest improvements coming at the fifth remeasurement, where mean predictive bias is reduced by roughly 0.1 inches for each thinning intensity.

The ability of the re-fit models to provide accurate crown ratio prediction increased significantly over the baseline PTAEDA2 simulations (Figure 21). Prediction for unthinned stands improved dramatically, where the highest mean residual across remeasurement periods is 0.0087 (2nd remeasurement). This is compared to the maximum residual of 0.0180 from the PTAEDA2 simulation run. Similar reductions in predictive error are also noted for thinned stands.



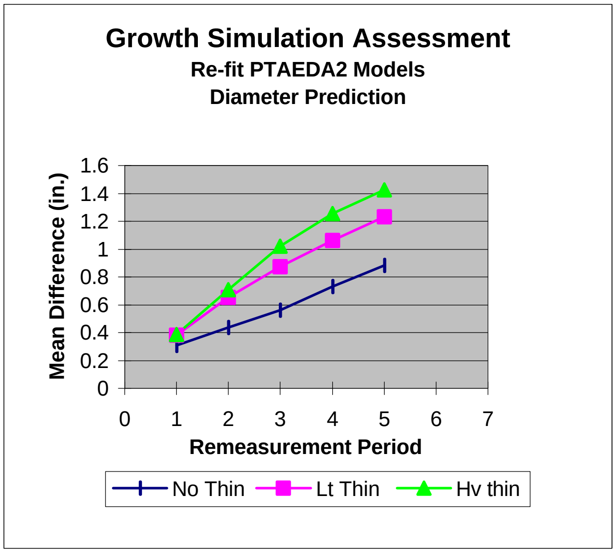
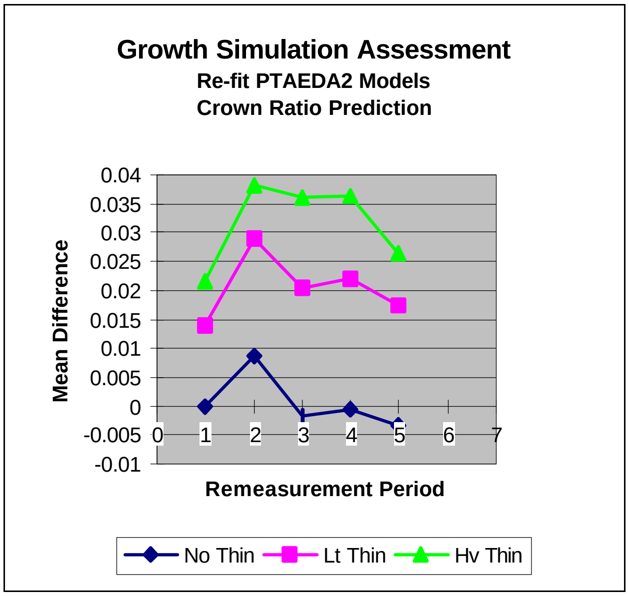




Figure 20. Growth simulation results for height and diameter increment when all models re-fit to 5th REM data.



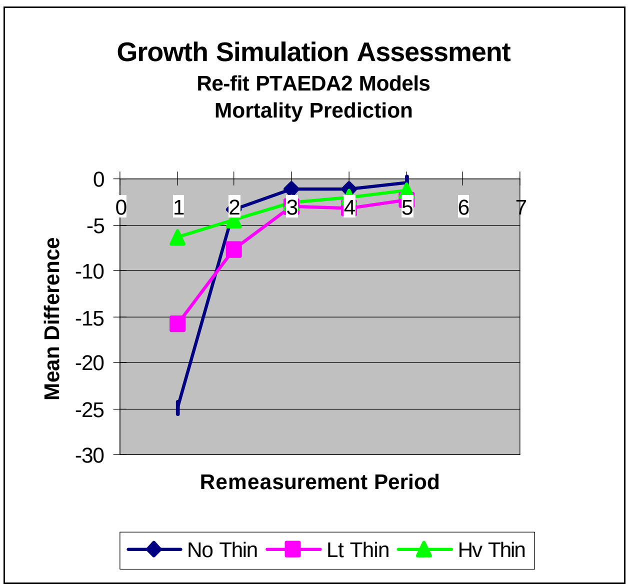
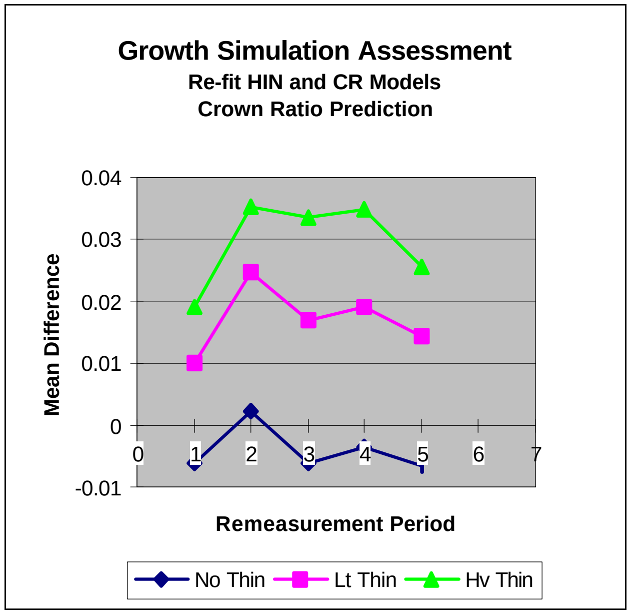




Figure 21. Growth simulation results for CR and mortality models when all models re-fit to 5th REM data.

The overall effect of re-fit models on mortality prediction is to increase the level of over-prediction that was found in the baseline simulation. This error is most notable at the first remeasurement period and decreases with each remeasurement period thereafter. Mortality estimation is slightly better for the re-fit model simulation at the fifth remeasurement. The greatest gains in predictive ability when all models were re-fit were obtained for height and crown ratio prediction. The apparent relationship between the height increment and crown ratio models leads to the conclusion that these models should be evaluated as a combination within the growth simulator. In this case, the re-fit HIN and CR models can form one combination and the re-fit HIN model and CR model with TRV3 can also be assessed. This would provide some insight into the effect of the HIN model on the performance of TRV3 within the CR model.

The combination of re-fit HIN and CR models in the growth simulator provides better crown ratio prediction than the re-fit CR model alone for unthinned stands only (Figure 22). The re-fit CR model alone is superior for both light and heavy thinned plots. The patterns of predictive predictive bias are the same between the two comparisons. The main effect of the HIN model on CR prediction is to slightly shift the bias in a positive direction.

The re-fit HIN model and CR model with TRV3 combination had the opposite effect on crown ratio prediction when compared to the results of the previous combination of HIN and CR models. The re-fit HIN model helped improve crown ratio prediction for thinned stands when compared to the predictive ability of the CR model with TRV3 alone. The effect of the re-fit HIN model was to lessen the amount of overprediction. Unthinned stand



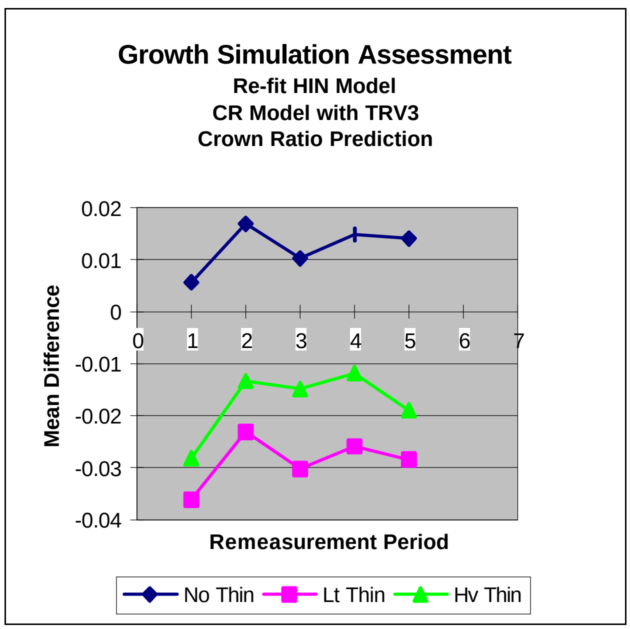




Figure 22. Growth simulation comparison between the re-fit CR model and the CR model with TRV3 when both are combined with the re-fit HIN model.

crown ratio prediction suffered worse prediction in roughly the same amount that the thinned stands improved.

Analyses of mortality prediction for simulations of isolated models, the PTAEDA2 models, and re-fit models shows that the re-fit model provides greater levels of mortality, especially at early remeasurement periods. The effects of this increased mortality on growth attributes would be similar to increases in thinning intensity (i.e. increased diameter growth and larger crown ratio values).

It seems unlikely that artificial density decreases resulting from over-prediction of mortality would provide better levels of prediction for tree diameter growth than is found in the simulation having the isolated DIN model with TRV2. Thus, application of the re-fit mortality function in combination with other models will be aimed toward improvements in crown ratio prediction. The combination of the re-fit mortality model and re-fit CR model would appear to benefit thinned stands, as crown ratio is underpredicted when the PTAEDA2 mortality function is utilized. Crown ratio predictions for unthinned stands may suffer, however, as crown ratio is already overpredicted.

Crown ratio prediction error in the re-fit HIN model/CR model with TRV3 combination is opposite that of the re-fit CR model alone. The results of the combined model simulation indicate that over-prediction is present in thinned plots, while unthinned plot crown ratios are under-predicted. The re-fit mortality function will be used in place of the PTAEDA2 mortality model for these simulations.

The re-fit mortality function had no practical effect on crown ratio prediction when combined with the re-fit CR model or the re-fit HIN model/CR model with TRV3 in growth simulations. In each case, trends in predictive error were the same as those found when the PTAEDA2 mortality function was used. The increased over-prediction of mortality at early remeasurements exhibited by the re-fit model has no effect on crown ratio prediction.

While accurate prediction of individual tree growth is important, most forest managers are interested in reliable stand-level volume prediction. One of the primary outputs of PTAEDA2 is predicted stand volume. Improved prediction of any given tree-level variable(s) does not ensure that stand volume prediction will improve as well. With this in mind, plot volumes based on growth simulation data were calculated and compared to observed plot volumes from the Coop thinning study data. The volume equation in PTAEDA2 was used to calculate tree volumes:

$$V_{tob} = 0.18658 + 0.00250D^2H$$

where: $V_{tob}$ = cubic-foot volume, outside bark, stump to tip
all other variables as previously defined

Mean plot volume differences were calculated for each thinning intensity across the five remeasurement periods. This volume comparison was made for the following simulation runs:

1) All models from PTAEDA2 with original parameter estimates
2) All models from PTAEDA2 re-fit to 5$^{th}$ remeasurement data

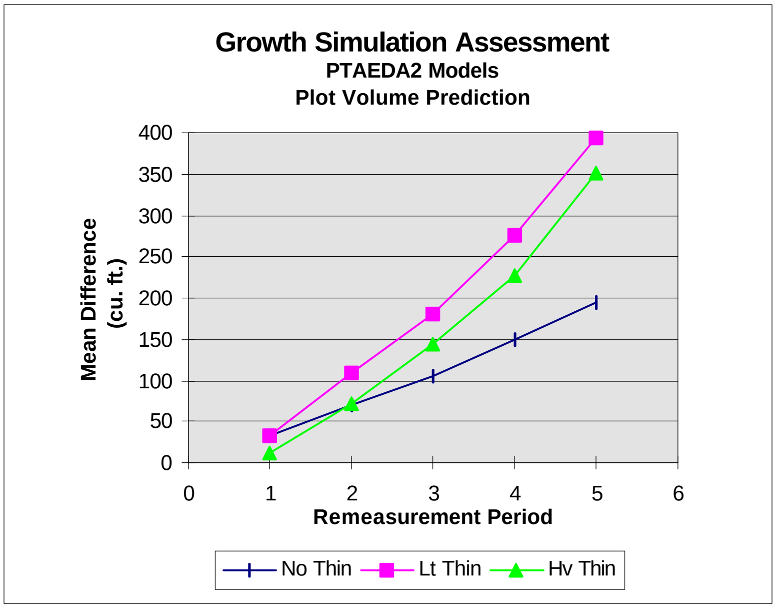
3) Isolated re-fit HIN model
4) Isolated DIN model with TRV2
5) Isolated re-fit CR model
6) Combined re-fit HIN and CR models
7) Combined re-fit HIN model and CR model with TRV3

Only two of the above comparisons exhibited improved plot volume prediction when compared to the volume estimates of the original PTAEDA2 models (Figure 23). The isolated re-fit CR model (4) and combined re-fit HIN model and CR model with TRV3 (6) produced better plot volume estimates than the PTAEDA2 configuration (Figure 24). The re-fit CR model predictive bias has a trend similar to that of PTAEDA2 and all volumes are under-predicted. This simulation produced slightly worse estimates for unthinned stands until the fourth and fifth remeasurements, where volume prediction is equivalent or slightly better. Thinned stand volume prediction was improved across the range of the data. The most notable improvements occur at later remeasurement periods where there is a 6-8% reduction in predictive bias.

The re-fit HIN model and CR model with TRV3 combination resulted in much different plot volume estimates for thinned stands. Plot volume predictive bias is very consistent across the five remeasurement periods for both light thinned and heavy thinned stands. Unfortunately, volume prediction is very poor for the first and second remeasurement periods. However, volume prediction for the third through fifth remeasurements is greatly improved. At the fifth remeasurement period, volume prediction is improved by approximately 70% for both light and heavy thinning treatments. Improvements are also noted for unthinned stands, except at the first remeasurement where volume estimates are virtually identical. The gains in volume prediction are again most evident at later remeasurement periods. It should be noted that unthinned stands maintained an under-prediction bias while the thinned stand error shifted to over-prediction.

The simulations for the DIN model with TRV2, the isolated re-fit HIN model, and the re-fit HIN and CR models combined all resulted in inferior volume estimates when compared to the PTAEDA2 simulation (Figures 25, 26). The DIN model with TRV2 produced the worst volume estimates. Volume was grossly underpredicted across all remeasurement periods and all thinning treatments. It was thought that the improvements in diameter prediction that occured during the simulation runs would translate into better volume predictions, as volume is predicted from tree diameter and height. The lack of improvement for height and mortality prediction in this simulation may have contributed to this result.

The re-fit HIN model and the combined re-fit HIN and CR models were very similar in volume prediction error. The re-fit HIN model was slightly better at earlier remeasurement periods, but produced poorer volume estimates than the re-fit model combination at later remeasurements. These differences are quite small and neither simulation could be judged as being superior to the other. The volume prediction of the PTAEDA2 configuration provided better volume estimates than either of these simulations.



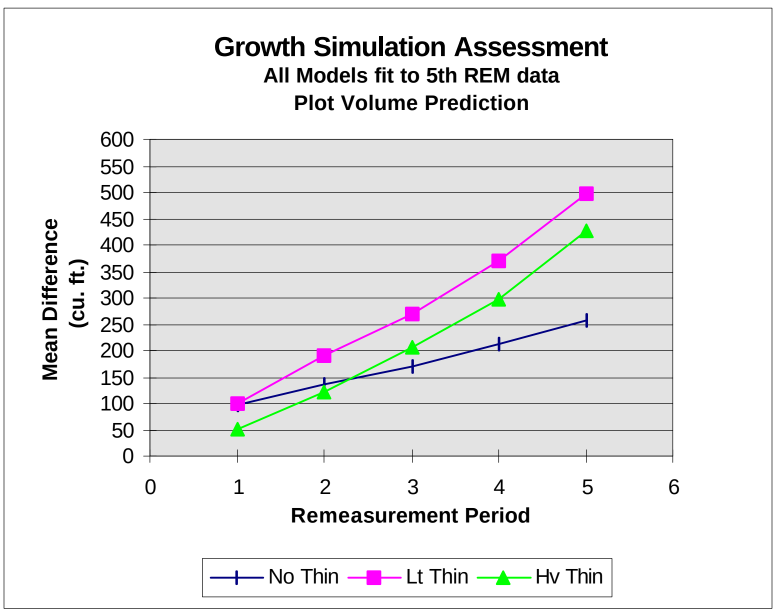
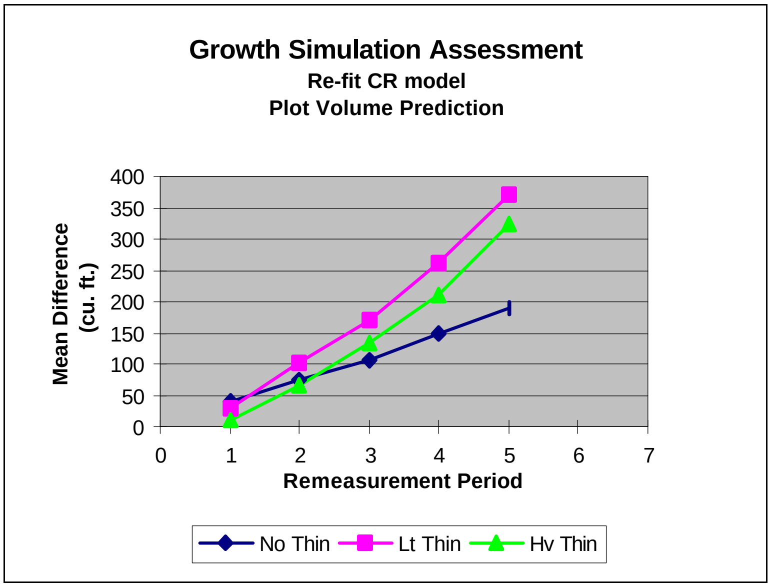




Figure 23. Growth simulation results for predicted plot volumes for the original PTAEDA2 models and the re-fit PTAEDA2 models.



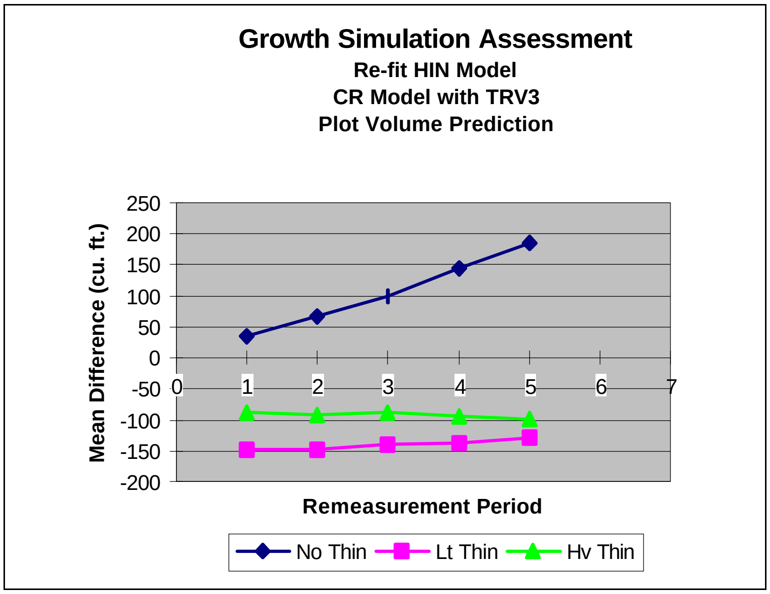
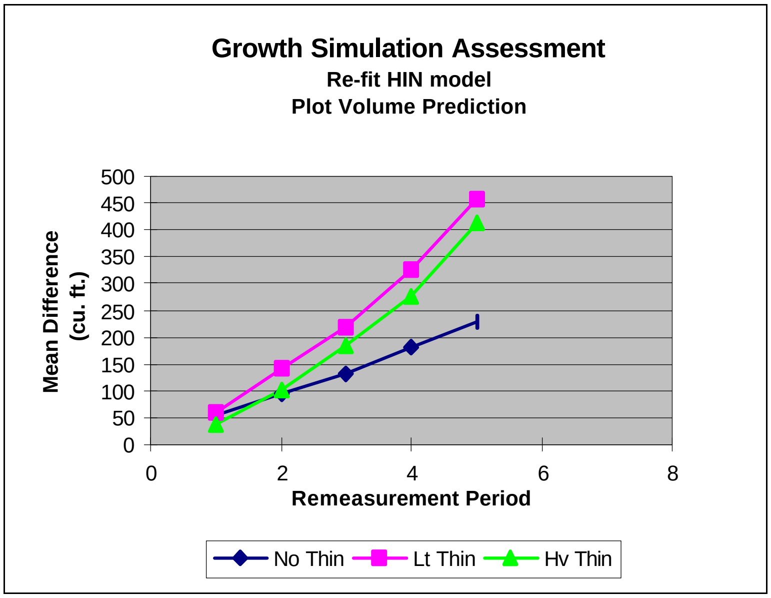




Figure 24. Growth simulation results for predicted plot volumes for the isolated re-fit CR model and re-fit HIN model and CR model with TRV3 combined.



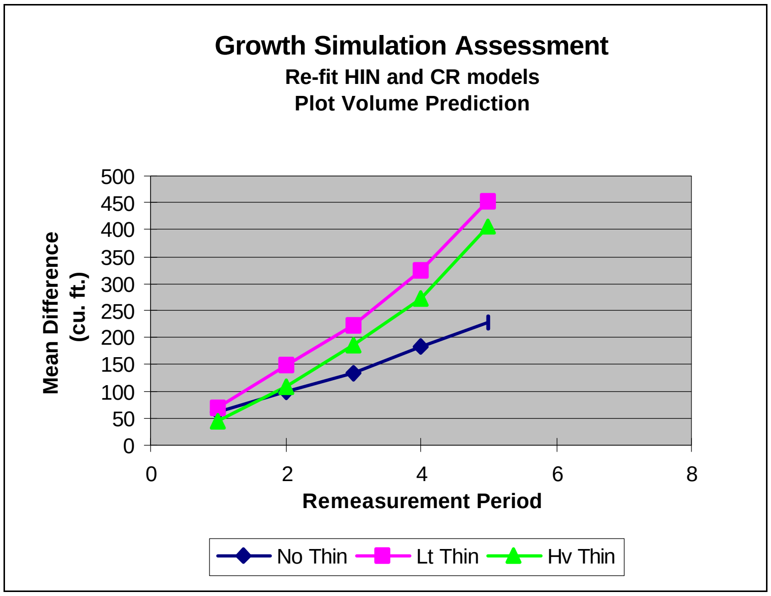




Figure 25. Growth simulation results for predicted plot volumes for the re-fit HIN model and re-fit HIN and CR models combined.

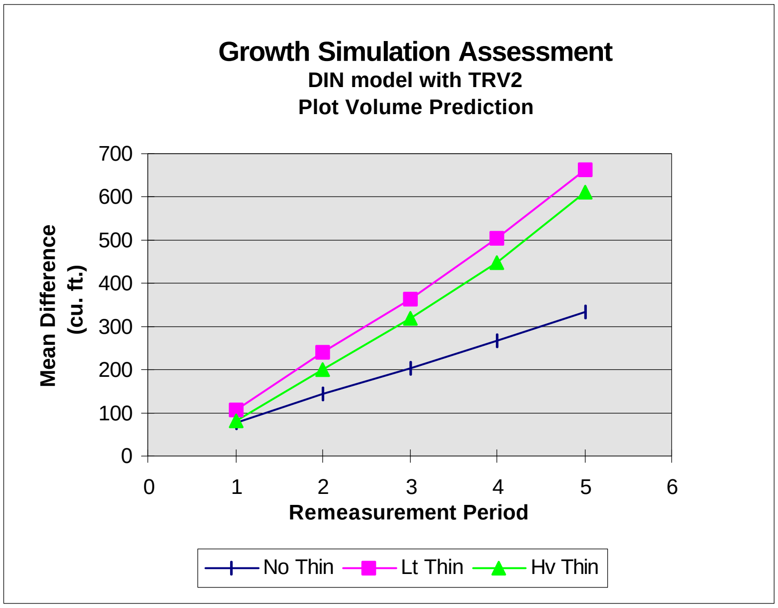



Figure 26. Growth simulation results for predicted plot volumes for the re-fit DIN model with TRV2.

Conclusions

The height increment model benefited greatly from re-fitting to all available data. There were notable improvements in predictive ability when compared to the HIN model in PTAEDA2. However, this improvement failed to yield any positive results when the model was evaluated within the reduced growth simulator. The anticipated improvement in the predictive ability of other components due to better height increment prediction was not realized.

The statistical indication that a thinning response function would be appropriate for diameter increment prediction resulted in the application of TRV2. TRV2 was designed to increase diameter increment for thinned stands. Initial analyses comparing the DIN model with TRV2 to the DIN model re-fit to all data showed no meaningful difference in predictive ability between the two models. However, the DIN model with TRV2 produced significantly better diameter increment prediction when incorporated into the reduced simulator. These results were encouraging as improvement in diameter increment prediction, especially for thinned stands, was the purpose for the development and application of a thinning response function. The effect of the DIN model with TRV2 on the predictive capacity of the other models was negative and resultant estimated plot volume prediction was poor.

Based on the initial hypothesis testing, the CR model appeared to be the best candidate for prediction improvement by the incorporation of a thinning response variable. Also, previous work on crown ratio had shown increases in performance when a TRV was utilized. Due to behavioral problems with existing TRV's and TRV's developed for this project, TRV3 was developed that was incorporated as an added, not multiplicative term. The CR model with TRV3 provided better crown ratio prediction than either the PTAEDA2 CR model or the CR model re-fit to all data for thinned plots. The CR model with TRV3 combined with the re-fit HIN model produced very consistent plot volume predictive error when evaluated within the reduced simulator. Further investigation is needed to determine if this consistency can be maintained while bringing the magnitude of error down for early remeasurement periods.

The re-fit CR model also provided improved crown ratio prediction over the PTAEDA2 model. This re-fit model lacked some of the predictive ability of the CR model with TRV3 for thinned stands, but worked equally well for unthinned stands. Evaluation of the re-fit model in the reduced simulator showed slight improvements in volume prediction over the PTAEDA2 configuration across the range of the data.

Initial statistical analyses of the re-fit mortality function indicated that higher levels of mortality would be produced than were presently found using the PTAEDA2 mortality function. However, mortality prediction simulations performed as part of the analyses indicated that mortality levels would be reduced. Comparisons between the PTAEADA2 mortality function and the mortality function re-fit to all data in the growth simulator resulted in higher levels of mortality prediction for the re-fit model. This was the desired effect of re-fitting because PTAEDA2 mortality levels are considered to be under-predicted. However, the change in mortality prediction provided by the re-fit model had no obvious effect on growth and yield prediction.

The results of the reduced simulator runs reveal that the predictive ability of any given model outside the simulator does not guarantee predictive success within the simulator. The interactions among the various growth models are far more complex than what they appear to be. The simulations that provided the best volume predictions and greatest shifts in volume prediction were both based on new CR models. The presence of the crown ratio variable in the height increment, diameter increment, and mortality models makes crown ratio prediction very important to the success of the simulator as a whole. Future work on PTAEDA2 should be focused on crown ratio prediction, as this is where the greatest advances have been made. Improvements in other areas, such as incremental growth, cannot provide improved volume predictions unless combined with a CR model that exhibits superior predictive ability.

This research indicates that the predictive output of the PTAEDA2 growth and yield simulator can be improved by utilizing long-term thinning study data. These improvements can largely be attributed to advances in predictive ability for crown ratio and, to a lesser extent, height increment models. Re-fitting of the crown ratio model provided modest gains in plot volume prediction, especially in thinned stands as elapsed time since thinning increases. Incorporation of a thinning response function into the crown ratio equation combined with the effects of the re-fit height increment model greatly affected stand volume estimates for thinned stands. While volume estimates initially suffer from notable over-prediction, there is significant improvement found in the nine to fifteen years since thinning range. The initial volume error should not be problematic, as thinned stands are usually not harvested within 6 years of thinning. Incorporation of these models into PTAEDA2 will provide forest managers with more reliable volume estimates for most stand conditions in both thinned and unthinned loblolly pine plantations.

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# Appendix A. Reduced Growth Simulator Source Code

```
C
C
C     THIS PROGRAM DOES FIFTEEN YEARS OF PREDICTION, USING EXTERNAL DATA
C  OBSERVED ON 186 SITES IN THE COASTAL PLAINS AND PIEDMONT AREAS OF THE
C  SOUTHEAST.  IT GENERATES AN OUTPUT TO AN ASCII DATASET EVERY THREE YEARS
C  FOR EACH PLOT.  THE PROGRAM USES EXCERPTS FROM THE GROWTH SIMULATION
C  MODEL PTAEDA2.  THE PURPOSE OF THIS PROGRAM IS TO DETERMINE THE VALIDITY
C  OF THE MORTALITY FUNCTION AND THE HEIGHT INCREMENT, DIAMETER INCREMENT,
C  AND CROWN RATIO  MODELS IN PTAEDA2 BY COMPARING THE PREDICTED OUTPUT TO
C  THE OBSERVED DATA OVER THE FIFTEEEN YEAR PERIOD.
C
C  VARIABLES:
C
C  ODEAD = 1    CODE FOR TREE STATUS OF DEAD
C  OALIVE = 2   CODE FOR TREE STATUS OF ALIVE
C
C  COMMON BLOCK 1
C
C  X, Y, D, H, CL, CIP, LMORT, KMORT, TAG, LVIGR(MTREES) =
C     ARRAYS OF THESE VARIABLES FROM 1 TO MTREES.
C  ACRES = PLOT SIZE (from PLOT DATA)
C  ID = COMPANY/LOCATION CODE
C  PLOT = 1, 2, OR 3.  INDICATES NO, LIGHT, OR HEAVY THINNED
C  PERCENT = % OF TOTAL BASAL AREA IF QFERT TRUE (from INIT1)
C  HD = AVERAGE HEIGHT OF DOMINANT AND CODOMINANT TREES (from PLOT DATA)
C  IX = A RANDOM NUMBER (from INIT1)
C  N = NROWS*NROWS (from STANDARD)
C  K = PRESENT STAND AGE & CURRENT GROWING SEASON (from PLOT DATA)
C  M = TREE COUNT FOR EACH PLOT (from PLOT DATA)
C  V = 3, 6, 9, 12, OR 15 YEARS OF PREDICTED GROWTH, OR REMEASUREMENT PERIOD
C  KTHIN = PLOT AGE AT THINNING (from MAIN PROGRAM)
C  LTHIN = INDICATES THINNED (= 1) OR UNTHINNED (= 0)
C  NROWS = # OF PLANTED ROWS AND TREES/ROW (from INIT1 or STANDARD)
C  RESBA = RESIDUAL BASAL AREA (from PLOT DATA)
C INITBA = BASAL AREA BEFORE THINNING (from PLOT DATA)
C  K3, K6, K9, K12, K15 = AGES AT 1ST,  2ND,  3RD, 4TH, AND 5TH REMEASUREMENTS (from PLOT
C  DATA)
C  HD3, HD6, HD9, HD12, HD15 = HD'S AT 1ST,  2ND,  3RD, 4TH, AND 5TH REMEASUREMENTS (from
C  PLOT DATA)
C  DUM=INDICATOR VARIABLE (=0 FOR UNTHINNED AND ESTABLISHMENT DATA,
C  =1 FOR THINNED AFTER ESTABLISHMENT)
C
C  COMMON BLOCK 2
C
C  SITE = SITE INDEX (from PLOT DATA)
C  PX = DISTANCE RATIO BETWEEN TREES (from INIT1)
C  PY = DISTANCE RATIO BETWEEN ROWS (from INIT1)
C  PLOTX = NROWS*PX (from INIT2)
C  PLOTY = NROWS*PY (from INIT2)
C  VARX = % VARIANCE BETWEEN TREES (from INIT1)
C  VARY = % VARIANCE BETWEEN TREES (from INIT1)
C  NYEARS = # OF GROWING SEASONS TO SIMULATE (from MAIN PROGRAM)
C
C     MAIN PROGRAM
C
      IMPLICIT       INTEGER*4 (I-N)
      INTEGER*4      IX,ID,PLOT,TAG
      INTEGER*4      ODEAD,OALIVE
      INTEGER*4      K3,K6,K9,K12,K15,Z,Q,V,DUM,M
      REAL           HD3,HD6,HD9,HD12,HD15,INITBA,RESBA
      CHARACTER*80   FILEN
      PARAMETER      (MROWS=20,MTREES=MROWS*MROWS)
      PARAMETER      (ODEAD=1,OALIVE=2)
      PARAMETER      (ZERO=0.0)
      COMMON /BLOK1/ X(MTREES),Y(MTREES),D(MTREES),H(MTREES),CL(MTREES),
     1 CIP(MTREES),LMORT(MTREES),KMORT(MTREES),TAG(MTREES),ACRES,ID,
     2 PLOT,PERCNT,HD,IX,N,K,M,V,NROWS,INITBA,RESBA,K3,K6,K9,K12,K15,HD3,HD6,HD9,
```

```
     3 HD12,HD15,LVIGR(MTREES),LTHIN,KTHIN ,DUM
      COMMON /BLOK2/ SITE,PX,PY,PLOTX,PLOTY,VARX,VARY,NYEARS
C
      CALL HEADER
C
C  GET THE NAME FOR THE OUTPUT FILE.
C
C
   10  WRITE(*,20)
   20  FORMAT(5(/),31X,'ASCII Output File',//,
     1 ' Please enter the file name (or CON or PRN): ')
       READ(*,30) FILEN
   30  FORMAT(A80)
      IF (FILEN.EQ.' ') GO TO 10
C
C  RUN ENTIRE PROGRAM ON FIRST ID, THEN LOOP TO START, RESET, AND RUN
C  PROGRAM ON EACH SUCCESSIVE ID UNTIL LAST ID IS REACHED.
C
      OPEN(1,FILE=FILEN,STATUS='NEW')
      OPEN(2,FILE='P1.DAT',STATUS='OLD')
      OPEN(3,FILE='T1.DAT',STATUS='OLD')
      Q = 1
      DO 180 Z=1,186
      CALL INIT1
      CALL INIT2
C
C  READ IN PLOT DATA FOR THIS ID.
C
      READ (2,110) ID,PLOT,K,HD,ACRES,RESBA,INITBA,K3,K6,K9,
     1 K12,K15,HD3,HD6,HD9,HD12,HD15,M
  110 FORMAT(3I4,F6.1,F8.4,2F9.5,5I4,5F6.1,I4)
      KTHIN = K
      CALL SI
C
C  READ IN TREE DATA FOR THIS ID.
C
      DO 130 I=1,M
      READ (3,120,END = 140) ID,PLOT,TAG(I),X(I),Y(I),CL(I),D(I),H(I),
     1 LMORT(I)
  120 FORMAT(3I4,3F4.0,F5.1,F4.0,I4)
C
C  ADD CIB(I) AND F9.5 IF RUNNING PLOTS 101 TO 1109 TO TREE INPUT.
C
C  CONVERT FROM VIGR CODE TO LMORT CODE.  VIGR CODES ARE: 1 = ALIVE,
C  2 = DEAD, AND 3 = THINNED.  VIGR VALUES ARE READ IN AS LMORT.
C
        IF (LMORT(I).EQ.1) THEN
          LMORT(I) = OALIVE
          ELSE
          LMORT(I) = ODEAD
        ENDIF
      N = N+1
  130 CONTINUE
  140 CALL STANDARD
C
C  ENABLE CALL DATALIST TO DISPLAY A LIST OF THE INITIAL DATA.
C
C     CALL DATALIST
      KBEGIN = K+1
C
C  CHANGE LENGTH OF GROWTH SIMULATION TO MATCH ACTUAL PLOT SURVIVAL.
C
      IF (HD3.EQ.0.0.AND.HD6.EQ.0.0.AND.HD9.EQ.0.0) THEN
          WRITE (*,145)
  145 FORMAT(/,1X,'PLOT DESTROYED PRIOR TO FIRST REMEASUREMENT.')
          Q = Q+1
          GO TO 165
        ELSEIF (HD6.EQ.0.0.AND.HD9.EQ.0.0.AND.HD12.EQ.0.0) THEN
          NYEARS = K+3
        ELSEIF (HD9.EQ.0.0.AND.HD12.EQ.0.0.AND.HD15.EQ.0.0) THEN
          NYEARS = K+6
       ELSEIF (HD12.EQ.0.0.AND.HD15.EQ.0.0) THEN
          NYEARS = K+9
       ELSEIF (HD15.EQ.0.0) THEN
```

```
          NYEARS = K+12
        ELSE
          NYEARS = K+15
      ENDIF
C
C  COMPUTE COMPETITION INDEX AND GROW TREES.
C
      DO 160 K=KBEGIN,NYEARS
      WRITE(*,150) K
  150 FORMAT(/,20X,'Computing Growth for Growing Season',I4)
      CALL COMP1
      CALL GROW2
C
C  GET OUTPUTS FOR 3,6,9,12 AND 15 YEARS OF GROWTH. OUTPUTS ARE ONLY GENERATED
C  IF GROWTH WAS ACTUALLY DONE FOR THAT REMEASUREMENT PERIOD.
C
      IF (K.EQ.KBEGIN+2) THEN
          V = 3
          CALL TREES
        ELSEIF (K.EQ.KBEGIN+5) THEN
          V = 6
          CALL TREES
        ELSEIF (K.EQ.KBEGIN+8) THEN
          V = 9
          CALL TREES
        ELSEIF (K.EQ.KBEGIN+11) THEN
          V = 12
          CALL TREES
        ELSEIF (K.EQ.KBEGIN+14) THEN
          V = 15
          CALL TREES
        ELSE
          GO TO 160
      ENDIF
  160 CONTINUE
      K = NYEARS
      Q = Q+1
  165 IF (Q.EQ.187) GO TO 180
      WRITE (*,170) Q
  170 FORMAT(//,24X,'Growing Plot',I4,' of 186 Plots')
  180 CONTINUE
      CLOSE(1)
      CLOSE(2)
      CLOSE(3)
      STOP 'END OF PROGRAM'
      END
C
      SUBROUTINE HEADER
C
C  Write program heading
C
      WRITE(*,20)
   20 FORMAT(10(/),1X,78('-'),/,' |',T79,'|',/,' |',5X,4(8('*'),2X),
     1 7('*'),3X,8('*'),2X,6('*'),T79,'|',/,' |',5X,'**    ',
     2 2('**',5X),'**    **  **',8X,'**    **  **    **',8X,'**',
     3 T79,'|',/,' |',5X,8('*'),2(5X,'**'),6('*'),2X,6('*'),
     4 2(4X,'**'),2X,8('*'),3X,6('*'),T79,'|',/,' |',5X,'**',11X,
     5 '**     **    **  **',8X,2('**    **  '),'**',T79,'|',/,' |',
     6 5X,'**',11X,'**     **    **  ',8('*'),2X,7('*'),
     7 '   **    **  ',8('*'),T79,'|',/,' |',T79,'|')
      WRITE(*,30)
   30 FORMAT(' |',23X,'COPYRIGHT 1987     VERSION 1.0',T79,'|',/,' |',
     1 7X,'Simulation of Individual Tree Growth and Stand ',
     2 'Development in',T79,'|',/,' |',9X,'Loblolly Pine Plantations ',
     3 'on Cutover, Site-Prepared Areas',T79,'|',/,' |',T79,'|',/,
     4 ' |',5X,'By  K. D. Farrar, R. L. Amateis, H. E. Burkhart, ',
     5 'and R. F. Daniels',T79,'|',/,' |',18X,
     6 'School of Forestry and Wildlife Resources',T79,'|',/,' |',13X,
     7 'Virginia Polytechnic Institute and State University',T79,'|',/,
     8 ' |',25X,'Blacksburg, Virginia 24061',T79,'|',/,' |',T79,'|',/,
     9 ' |',21X,'Modified by Michael C. Smith, 1993', T79,'|',/,' |',T79,'|',/,
     * ' |',21X,'Modified by James A. Westfall, 1998',T79,'|',/,'|',
     * T79,'|',/,1X,78('-'))
      CALL CONT
```

```
      RETURN
      END
C
      SUBROUTINE INIT1
C
C  This routine initializes common variables to initial default values
C
      IMPLICIT       INTEGER*4 (I-N)
      INTEGER*4      IX,ID,PLOT,TAG
      INTEGER*4      K3,K6,K9,K12,K15
      REAL           HD3,HD6,HD9,HD12,HD15
      PARAMETER      (MROWS=20,MTREES=MROWS*MROWS)
      COMMON /BLOK1/ X(MTREES),Y(MTREES),D(MTREES),H(MTREES),CL(MTREES),
     1 CIP(MTREES),LMORT(MTREES),KMORT(MTREES),TAG(MTREES),ACRES,ID,
     2 PLOT,PERCNT,HD,IX,N,K,M,V,NROWS,INITBA,RESBA,K3,K6,K9,K12,K15,HD3,HD6,HD9,
     3 HD12,HD15,LVIGR(MTREES),LTHIN,KTHIN,DUM
      COMMON /BLOK2/ SITE,PX,PY,PLOTX,PLOTY,VARX,VARY,NYEARS
C
      PERCNT = 0.048
      IX = 68767
      NYEARS = 35
      NROWS = 15
      SITE = 60.0
      PX = 1.0
      PY = 1.0
      VARX = 10.0
      VARY = 10.0
      END
C
      SUBROUTINE INIT2
C
C    Subroutine INIT2 initializes the individual tree data to zero
C  before each simulation begins.
C
      IMPLICIT       INTEGER*4 (I-N)
      INTEGER*4      IX,ID,PLOT,TAG
      INTEGER*4      ODEAD,OALIVE
      INTEGER*4      K3,K6,K9,K12,K15,DUM
      REAL           HD3,HD6,HD9,HD12,HD15
      PARAMETER      (MROWS=20,MTREES=MROWS*MROWS)
      PARAMETER      (ODEAD=1,OALIVE=2)
      PARAMETER      (ZERO=0.0)
      COMMON /BLOK1/ X(MTREES),Y(MTREES),D(MTREES),H(MTREES),CL(MTREES),
     1 CIP(MTREES),LMORT(MTREES),KMORT(MTREES),TAG(MTREES),ACRES,ID,
     2 PLOT,PERCNT,HD,IX,N,K,M,V,NROWS,INITBA,RESBA,K3,K6,K9,K12,K15,HD3,HD6,HD9,
     3 HD12,HD15,LVIGR(MTREES),LTHIN,KTHIN,DUM
      COMMON /BLOK2/ SITE,PX,PY,PLOTX,PLOTY,VARX,VARY,NYEARS
C
      DO 10 I=1,MTREES
        X(I) = ZERO
        Y(I) = ZERO
        D(I) = ZERO
        H(I) = ZERO
        CL(I) = ZERO
        CIP(I) = ZERO
        TAG(I) = ZERO
        LVIGR(I) = 1
        LMORT(I) = OALIVE
   10   KMORT(I) = -1
      N = 0
      M = 0
      K = 0
      K3 = 0
      K6 = 0
      K9 = 0
      K12 = 0
      K15 = 0
      HD = ZERO
      HD3 = ZERO
      HD6 = ZERO
      HD9 = ZERO
      HD12 = 0
      HD15 = 0
      RESBA = ZERO
```

```
      INITBA= ZERO
      ACRES = ZERO
      PLOTX = ZERO
      PLOTY = ZERO
      NROX = 0
      NROY = 0
      KTHIN = 0
      LTHIN = 0
      DUM=0
      RETURN
      END
C
C
C
      SUBROUTINE SI
C
C     THIS ROUTINE DETERMINES WHICH REMEASUREMENT PERIOD IS CLOSEST TO THE
C   SI BASE AGE OF 25, THEN USES THE DOMINANT HEIGHT FROM THAT REMEASUREMENT
C   PERIOD TO DETERMINE THE SITE INDEX.  PLOT ESTABLISHMENT K AND HD ARE
C   RETAINED IN THE MAIN PROGRAM.
C
      IMPLICIT        INTEGER*4 (I-N)
      INTEGER*4       IX,ID,PLOT,TAG
      INTEGER*4       ODEAD,OALIVE
      INTEGER*4       K3,K6,K9,K12,K15,DUM
      REAL            HD3,HD6,HD9,HD12,HD15
      PARAMETER       (MROWS=20,MTREES=MROWS*MROWS)
      PARAMETER       (ODEAD=1,OALIVE=2)
      PARAMETER       (ZERO=0.0)
      COMMON /BLOK1/ X(MTREES),Y(MTREES),D(MTREES),H(MTREES),CL(MTREES),
     1 CIP(MTREES),LMORT(MTREES),KMORT(MTREES),TAG(MTREES),ACRES,ID,
     2 PLOT,PERCNT,HD,IX,N,K,M,V,NROWS,INITBA,RESBA,K3,K6,K9,K12,K15,HD3,HD6,HD9,
     3 HD12,HD15,LVIGR(MTREES),LTHIN,KTHIN,DUM
      COMMON /BLOK2/ SITE,PX,PY,PLOTX,PLOTY,VARX,VARY,NYEARS
C
      A = ABS(25 - K)
      B = ABS(25 - K3)
      C = ABS(25 - K6)
      E = ABS(25 - K9)
      F = ABS(25-K12)
      G = ABS(25-K15)
      IF (HD3.EQ.0.0.AND.HD6.EQ.0.0.AND.HD9.EQ.0.0) THEN
          TEMP = A
        ELSEIF (HD6.EQ.0.0.AND.HD9.EQ.0.0.AND.HD12.EQ.0.0) THEN
          TEMP = MIN(A,B)
        ELSEIF (HD9.EQ.0.0.AND.HD12.EQ.0.0.AND.HD15.EQ.0.0) THEN
          TEMP = MIN(A,B,C)
        ELSEIF (HD12.EQ.0.0.AND.HD15.EQ.0.0) THEN
          TEMP = MIN(A,B,C,E)
        ELSEIF (HD15.EQ.0.0) THEN
          TEMP = MIN(A,B,C,E,F)
        ELSE
          TEMP = MIN(A,B,C,E,F,G)
      ENDIF
      IF (TEMP.EQ.A) THEN
        L = K
        DH = HD
      ELSEIF (TEMP.EQ.B) THEN
        L = K3
        DH = HD3
      ELSEIF (TEMP.EQ.C) THEN
        L = K6
        DH = HD6
      ELSEIF (TEMP.EQ.E) THEN
        L = K9
        DH = HD9
      ELSEIF (TEMP.EQ.F) THEN
        L = K12
        DH = HD12
      ELSE
      L=K15
      DH=HD15
      ENDIF
C
```

```
C
C
C  Equation (Site Index) from Amateis and Burkhart (unpublished)
C
      SITE = (1.0/ALOG(DH))*((1.0/REAL(L))/(1.0/25.0))**(-0.02205)*
     1 EXP(-2.83285*(1.0/REAL(L)-1.0/25.0))
      SITE = 1.0/SITE
      SITE = EXP(SITE)
C     WRITE (*,100) SITE,DH,L
C 100 FORMAT(/,1X,'SITE INDEX = ',F6.1,', FROM HD = ',F6.1,
C    1 ', AND K = ',I4)
      RETURN
      END

      SUBROUTINE STANDARD
C
C  This routine standardizes a plot to rectangular for input by adding
C  dead trees to make NROWS a whole number.
C
      IMPLICIT       INTEGER*4 (I-N)
      INTEGER*4      IX,ID,PLOT,TAG,DUMMIES
      INTEGER*4      ODEAD,OALIVE
      INTEGER*4      K3,K6,K9,K12,K15
      REAL           HD3,HD6,HD9,HD12,HD15,DUM
      PARAMETER      (MROWS=20,MTREES=MROWS*MROWS)
      PARAMETER      (ODEAD=1,OALIVE=2)
      PARAMETER      (ZERO=0.0)
      COMMON /BLOK1/ X(MTREES),Y(MTREES),D(MTREES),H(MTREES),CL(MTREES),
     1 CIP(MTREES),LMORT(MTREES),KMORT(MTREES),TAG(MTREES),ACRES,ID,
     2 PLOT,PERCNT,HD,IX,N,K,M,V,NROWS,INITBA,RESBA,K3,K6,K9,K12,K15,HD3,HD6,HD9,
     3 HD12,HD15,LVIGR(MTREES),LTHIN,KTHIN,DUM
C
      N2 = N
      ROWS = ANINT(SQRT(REAL(N))+0.499999)
      NROWS = INT(ROWS)
      N = INT(ROWS)*INT(ROWS)
      DUMMIES = N - N2
      IF (DUMMIES.EQ.0) RETURN
      DO 20 I = N2+1,N
        X(I) = 0.0
        Y(I) = 0.0
        D(I) = 0.0
        H(I) = 0.0
        CL(I) = 0.0
        LMORT(I) = ODEAD
        KMORT(I) = K
   20 CONTINUE
      RETURN
      END
C
      SUBROUTINE DATALIST
C
      IMPLICIT       INTEGER*4 (I-N)
      INTEGER*4      IX,ID,PLOT,TAG
      INTEGER*4      ODEAD,OALIVE
      INTEGER*4      K3,K6,K9,K12,K15,Z,Q
      REAL           HD3,HD6,HD9,HD12,HD15
      CHARACTER*80   FILEN
      PARAMETER      (MROWS=20,MTREES=MROWS*MROWS)
      PARAMETER      (ODEAD=1,OALIVE=2)
      PARAMETER      (ZERO=0.0)
      COMMON /BLOK1/ X(MTREES),Y(MTREES),D(MTREES),H(MTREES),CL(MTREES),
     1 CIP(MTREES),LMORT(MTREES),KMORT(MTREES),TAG(MTREES),ACRES,ID,
     2 PLOT,PERCNT,HD,IX,N,K,M,V,NROWS,INITBA,RESBA,K3,K6,K9,K12,K15,HD3,HD6,HD9,
     3 HD12,HD15,LVIGR(MTREES),LTHIN,KTHIN,DUM
C
C  THIS SUBROUTINE HAS NOT BEEN UPDATED FOR CHANGES IN THE INPUT DATASET
C
      CALL COMP1
      WRITE(*,10)
   10 FORMAT(//1X,' ID  PLOT   X     Y    K   CL    D    H     HD  LMORT
     1 ACRES  SITE    CIP',/,1X,73('-'))
      DO 30 I = 1,N
      WRITE (*,20) ID,PLOT,X(I),Y(I),K,CL(I),D(I),
```

```
     1 H(I),HD,LMORT(I),ACRES,SITE,CIP(I)
   20 FORMAT(1X,2I4,2F6.1,I4,F6.1,F5.1,2F6.1,I4,F8.4,F6.1,F8.4)
   30 CONTINUE
      RETURN
      END
C
      SUBROUTINE COMP1
C
C    Subroutine COMP1 calculates a modified Hegyi competition index
C  on all live trees in a stand.  Competitors are found by sampling
C  neighbors based on their size and distance away by essentially
C  taking a point sample at each subject tree with a BAF-10 prism.
C
      IMPLICIT       INTEGER*4 (I-N)
      INTEGER*4      IX,ID,PLOT,TAG
      INTEGER*4      ODEAD,OALIVE
      INTEGER*4      K3,K6,K9,K12,K15,DUM
      REAL           HD3,HD6,HD9,HD12,HD15
      PARAMETER      (MROWS=20,MTREES=MROWS*MROWS)
      PARAMETER      (ODEAD=1,OALIVE=2)
      PARAMETER      (PLOTR=2.75)
      DIMENSION      JDIS(9),MID(MTREES),IDIS(4),DIST(9)
      COMMON /BLOK1/ X(MTREES),Y(MTREES),D(MTREES),H(MTREES),CL(MTREES),
     1 CIP(MTREES),LMORT(MTREES),KMORT(MTREES),TAG(MTREES),ACRES,ID,
     2 PLOT,PERCNT,HD,IX,N,K,M,V,NROWS,INITBA,RESBA,K3,K6,K9,K12,K15,HD3,HD6,HD9,
     3 HD12,HD15,LVIGR(MTREES),LTHIN,KTHIN,DUM
      COMMON /BLOK2/ SITE,PX,PY,PLOTX,PLOTY,VARX,VARY,NYEARS
      DATA           JDIS/1,9,8,7,6,5,4,3,2/
C
C  Initialize
C
      DO 5 I=1,MTREES
    5   CIP(I) = 0.0
      IDIS(1) = 1
C
C  Find internal trees
C
      DMAX = 0.0
      DO 10 I=1,N
   10   DMAX = AMAX1(DMAX,D(I))
      D2 = PLOTR*DMAX
      DISMAX = D2-PX/2.0
      DISMAY = D2-PY/2.0
      DMX = PLOTX-DISMAX
      DMY = PLOTY-DISMAY
      DO 20 I=1,N
        MID(I) = 2
   20   IF (X(I).GT.DISMAX.AND.X(I).LT.DMX.AND.
     1       Y(I).GT.DISMAY.AND.Y(I).LT.DMY) MID(I) = 1
C
C  Calculate competition index
C

      DO 130 I=1,N-1
        IF (LMORT(I).NE.OALIVE) GO TO 130
        DO 120 J=I+1,N
          IF (LMORT(J).NE.OALIVE) GO TO 120
          INTIOR = MID(I)+MID(J)
          XDIST = X(J)-X(I)
          YDIST = Y(J)-Y(I)
          DIST(1) = SQRT(XDIST*XDIST+YDIST*YDIST)
          IF (INTIOR.LT.3) GO TO 100
          IF (XDIST.LT.0.0) GO TO 30
          DIST(5) = SQRT(((XDIST-PLOTX)**2)+(YDIST*YDIST))
          IDIS(2) = 5
          GO TO 40
   30     DIST(6) = SQRT(((XDIST+PLOTX)**2)+(YDIST*YDIST))
          IDIS(2) = 6
   40     IF (YDIST.GE.0.0) GO TO 50
          DIST(3) = SQRT((XDIST*XDIST)+((YDIST+PLOTY)**2))
          IDIS(3) = 3
          ICODE = IDIS(2)+IDIS(3)-7
          GO TO (60,70,100,100,100,80,90),ICODE
   50     DIST(8) = SQRT((XDIST*XDIST)+((YDIST-PLOTY)**2))
```

```
          IDIS(3) = 8
          ICODE = IDIS(2)+IDIS(3)-7
          GO TO (60,70,100,100,100,80,90),ICODE
   60     DIST(2) = SQRT(((XDIST-PLOTX)**2)+((YDIST+PLOTY)**2))
          IDIS(4) = 2
          GO TO 100
   70     DIST(4) = SQRT(((XDIST+PLOTX)**2)+((YDIST+PLOTY)**2))
          IDIS(4) = 4
          GO TO 100
   80     DIST(7) = SQRT(((XDIST-PLOTX)**2)+((YDIST-PLOTY)**2))
          IDIS(4) = 7
          GO TO 100
   90     DIST(9) = SQRT(((XDIST+PLOTX)**2)+((YDIST-PLOTY)**2))
          IDIS(4) = 9
  100     RJI = D(J)/D(I)
          RIJ = 1.0/RJI
          DO 110 L=1,4
            LC = IDIS(L)
            LCC = JDIS(LC)
            IF (DIST(LC).EQ.0.0) GO TO 105
            IF (DIST(LC).LT.D(J)*PLOTR) CIP(I) = CIP(I)+RJI/DIST(LC)
            IF (DIST(LC).LT.D(I)*PLOTR) CIP(J) = CIP(J)+RIJ/DIST(LC)
  105       IF (INTIOR.LE.3) GO TO 120
  110       CONTINUE
  120     CONTINUE
  130   CONTINUE
      RETURN
      END
C
      SUBROUTINE GROW2
C
C    Subroutine GROW2 does the annual growth of the individual trees
C  in the stand.  This routine does NOT take into consideration the
C  amount of hardwood competition in terms of percent of total basal
C  area - it assumes a 4.8% basal area.
C
      IMPLICIT       INTEGER*4 (I-N)
      INTEGER*4      IX,ID,PLOT,TAG
      INTEGER*4      ODEAD,OALIVE
      INTEGER*4      K3,K6,K9,K12,K15,DUM
      INTEGER*4      KT,KT1,K1
      REAL           HD3,HD6,HD9,HD12,HD15,TRV1,TRV2,TRVD
      PARAMETER      (MROWS=20,MTREES=MROWS*MROWS)
      PARAMETER      (ODEAD=1,OALIVE=2)
      COMMON /BLOK1/ X(MTREES),Y(MTREES),D(MTREES),H(MTREES),CL(MTREES),
     1 CIP(MTREES),LMORT(MTREES),KMORT(MTREES),TAG(MTREES),ACRES,ID,
     2 PLOT,PERCNT,HD,IX,N,K,M,V,NROWS,INITBA,RESBA,K3,K6,K9,K12,K15,HD3,HD6,HD9,
     3 HD12,HD15,LVIGR(MTREES),LTHIN,KTHIN,DUM
      COMMON /BLOK2/ SITE,PX,PY,PLOTX,PLOTY,VARX,VARY,NYEARS
C
C  Initialize tree counter
C
        NTSIM = 0
C
C  Compute potential height increment for all trees and begin
C  individual tree growth
C
C
C  Equation (Site Index) from Amateis and Burkhart (unpublished)
C
      POTH = ALOG(SITE)*((1.0/K)/(1.0/25.0))**(-0.02205)*
     1 EXP(-2.83285*(1.0/K-1.0/25.0))
      POTH = EXP(POTH)
      PHIN = POTH-HD
C
C
      DO 10 I=1,N
        IF (LMORT(I).NE.OALIVE) GO TO 10
      CR = CL(I)/H(I)
C
C
C  Determine tree mortality
C
C  Originally fit PLIVE model from PTAEDA2.
```

```
C
      PLIVE = 1.02797295*CR**0.03789773*EXP(-0.00230209*CIP(I)**
     1 2.65206263)
C
C   New fit of PLIVE to all data
C
C       PLIVE=1.00892752*CR**0.01485757
C     1 *EXP(-0.00463362*CIP(I)**3.16075490)
        IF (U(IX).GE.PLIVE) THEN
           NLIVE = NLIVE-1
           LMORT(I) = ODEAD
           KMORT(I) = K
           GO TO 10
         ENDIF
C
C  Compute Height and Diameter increment on all trees
C
         R = STNORM(IX)
C
C  New fit of HIN model to REM5 data:
C
      HRED = 0.58307963+1.075729902*CR**0.582052709*EXP(-0.146110716*
     1 CIP(I)-1.091722629*CR)
C
C  Originally fit HIN model from PTAEDA2:
C
C       HRED = 0.26324665+2.11118696*CR**0.56188187*EXP(-0.26375086*
C     1   CIP(I)-1.03076126*CR)
         HIN =AMAX1(PHIN*HRED,0.0)
C
C  HINMAX is not actually used
C        HINMAX = 0.72785206*PHIN+0.88373520
C        HIN =AMIN1(HIN,HINMAX)
C
      NTSIM = NTSIM+1
      HIN = AMAX1((HIN+R*0.6723),0.0)
C
C  New fit of DIN model to REM5 data
C
C
C     DRED = 0.80375055*CR**0.76109658*
C     1 EXP(-0.69471138*CIP(I))
C
C
C
C   DIN model with TRV2 fit to all data
C
C
C       IF (KTHIN.EQ.0.0.OR.DH.EQ.0.0.OR.RESBA.EQ.0.0)
C     1 GO TO 10
C       DRED=(0.7957*CR**0.74920719*EXP(-0.68966795*CIP(I)))
C       TRV=((INITBA/RESBA)**((K-KTHIN)/DH**2))
C     1 *EXP(((K-KTHIN)**2)/((K/KTHIN)**52.109019))
C       DRED=DRED*TRV
C
C
C  Originally fit DIN model from PTAEDA2:
C
        DRED = 0.72511188*CR**0.98014576*
     1 EXP(-0.37397613*CIP(I))
         PDIN = 0.28658336*HIN+0.2094718
         DIN = AMAX1((PDIN*DRED+R*0.085),0.0)
         D(I) = D(I)+DIN
         H(I) = H(I)+HIN
   10    CONTINUE
      HD = POTH
C
C  Determine crown length:
C  Crown ratio equation by Dyer and Burkhart, 1986
C
      DO 20 I=1,N
C        IF (LMORT(I).NE.OALIVE) GO TO 20
C        CR = 1.0-EXP((-1.35243-37.02600/K)*D(I)/H(I))
C
```

```
C
C  Crown ratio equation fit to all data
C
        IF (K.EQ.0.0.OR.H(I).EQ.0.0.OR.
     1 INITBA.EQ.0.0.OR.KTHIN.EQ.0.0) GO TO 20
C         CR = 1.0-EXP((-1.7824575-34.1967186/K)*D(I)/H(I))
C
C
C  Crown ratio equation fit to balanced all data
C
C       CR = 1.0-EXP((-1.82202000-33.99552921/K)*D(I)/H(I))
C
C
C   Set DUM=1 for thinned after first growing season for thinned plots
C
      IF (PLOT.NE.1.AND.K.NE.KTHIN) THEN
         DUM=1
      ENDIF
C
C  Crown ratio equation w/TRV3
C
        CR = 1.0-EXP((-1.48380653-36.84780194/K)*D(I)/H(I))+
     1  (DUM*((INITBA-RESBA)/INITBA)*( 0.03206053*D(I)** 0.43664658)
     2  *EXP(-(K/KTHIN)/K**.5))
C
C
C  Crown ratio equation w/TRV3 fit to balanced data
C
C     CR = 1.0-EXP((-1.55225903-36.27559592/K)*D(I)/H(I))+
C    1  (DUM*((INITBA-RESBA)/INITBA)*( 0.02788582*D(I)** 0.47736085)
C    2  *EXP(-(K/KTHIN)/K**.5))
C
C
      CL(I) = AMIN1(AMAX1(0.0,(H(I)*CR)),H(I))
   20   CONTINUE
      WRITE(*,30) NTSIM
   30 FORMAT(I33,' Trees Simulated')
      RETURN
      END
C
      SUBROUTINE TREES
C
C    This subroutine TREES outputs a file containing the individual
C  tree data in ASCII format.
C
      IMPLICIT       INTEGER*4 (I-N)
      INTEGER*4      IX,I4,ID,PLOT,TAG
      INTEGER*4      ODEAD,OALIVE
      INTEGER*4      K3,K6,K9,K12,K15,V
      REAL           HD3,HD6,HD9,HD12,HD15
      PARAMETER      (MROWS=20,MTREES=MROWS*MROWS)
      PARAMETER      (ODEAD=1,OALIVE=2)
      COMMON /BLOK1/ X(MTREES),Y(MTREES),D(MTREES),H(MTREES),CL(MTREES),
     1 CIP(MTREES),LMORT(MTREES),KMORT(MTREES),TAG(MTREES),ACRES,ID,
     2 PLOT,PERCNT,HD,IX,N,K,M,V,NROWS,INITBA,RESBA,K3,K6,K9,K12,K15,HD3,HD6,HD9,
     3 HD12,HD15,LVIGR(MTREES),LTHIN,KTHIN,DUM
      COMMON /BLOK2/ SITE,PX,PY,PLOTX,PLOTY,VARX,VARY,NYEARS
C
      WRITE (*,10)
   10 FORMAT(/)
C     WRITE(1,20)
C  20 FORMAT(1X,'ID',2X,'PLOT',1X,'TAG',3X,'X',5X,'Y',4X,'DBH',2X,
C    1 'Height',3X,'CL',5X,'CIP',2X,'DETHVIGR',2X,'HD',4X,'V',1X,'AGE')
      DO 30 I=1,N
C
C  CONVERT FROM LMORT CODE TO VIGR CODE FOR USE WITH COOP DATASET.
C
      IF (TAG(I).EQ.0) RETURN
        IF (LMORT(I).EQ.OALIVE) THEN
          LVIGR(I) = 1
          ELSE
          LVIGR(I) = 2
        ENDIF
      WRITE (1,40) ID,PLOT,TAG(I),X(I),Y(I),D(I),H(I),CL(I),SITE,CIP(I),
```

```
     1 KMORT(I),LVIGR(I),HD,V,K
   30   CONTINUE
   40 FORMAT(3I4,2F6.1,F6.2,2F7.2,F7.1,F8.4,2I4,F6.1,2I4)
      RETURN
      END
C
      SUBROUTINE CONT
C
C    This subroutine asks the user if he/she wishes to continue
C  running PTAEDA2.  If not, the program is halted.
C

      IMPLICIT       INTEGER*4(I-N)
      CHARACTER*1    ANS
C
      WRITE(*,10)
   10 FORMAT(/,' Do you wish to continue? (YES or NO): ')
      CALL GETYN(ANS)
      IF (ANS.EQ.'N') THEN
        WRITE(*,20)
   20   FORMAT(25(/))
        STOP 'Program Terminated By User'
      ENDIF
      RETURN
      END
C
      SUBROUTINE GETYN(ANS)
C
C    This routine reads in character input for YES/NO questions.
C
      CHARACTER*1    ANS,TEMP
C
C  Read in string
C
   10 READ(*,20) TEMP
   20 FORMAT(A1)
C
C  Convert to upper case
C
      LETTER = ICHAR(TEMP)
      IF ((LETTER.GE.97).AND.(LETTER.LE.122)) LETTER = LETTER-32
      TEMP = CHAR(LETTER)
C
C  Check to be sure value is "Y" or "N"
C
      IF ((TEMP.EQ.'Y').OR.(TEMP.EQ.'N')) GO TO 40
      WRITE(*,30) CHAR(7)
   30 FORMAT(1X,A1,'Please enter YES or NO: ')
      GO TO 10
C
C  End: wrap-up
C
   40 ANS = TEMP
      RETURN
      END
C
      FUNCTION GAMMA(XX)
C
C    Based on program from Scientific Subroutine Package, IBM
C
      IMPLICIT       INTEGER*4 (I-N)
      IF (XX.LE.10.0) GO TO 20
   10 GX = 0.0
      GO TO 100
   20 X = XX
      ERR = 1.0E-6
      GX = 1.0
      IF (X-2.0) 50,50,40
   30 IF (X.LE.2.0) GO TO 90
   40 X = X-1.0
      GX = GX*X
      GO TO 30
   50 IF (X-1.0) 60,100,90
C
```

```
C  See if X is near negative integer or zero
C
   60 IF (X.GT.ERR) GO TO 80
      Y = FLOAT(INT(X))-X
      IF (ABS(Y).LE.ERR) GO TO 10
C
C  X not near a negative integer or zero
C
   70 IF (X.GT.1.0) GO TO 90
   80 GX = GX/X
      X = X+1.0
      GO TO 70
   90 Y = X-1.0
      GY = 1.0+Y*(-0.5771017+Y*(0.9858540+Y*(-0.8764218+Y*(0.8328212+
     1 Y*(-0.5684729+Y*(0.2548205+Y*(-0.05149930)))))))
      GX = GX*GY
  100 GAMMA = GX
      RETURN
      END
C
      FUNCTION STNORM(IX)
C
C    Generates a standard Normal random variate
C  Based on the routine "GAUSS" in "Scientific Subroutine Package";
C  IBM; 1968; page 77.  The algorithm is based on "Numerical Methods
C  for Scientists and Engineers"; R. W. Hamming; McGraw-Hill Pub.;
C  1962; pages 34 and 389.
C    Assumes a standard deviation (S) of 1.0 and a mean (AM) of 0.0
C
      IMPLICIT       INTEGER*4 (I-N)
      INTEGER*4      IX
      PARAMETER      (S=1.0,AM=0.0)
C
      A = 0.0
      DO 10 I=1,12
   10   A = A+U(IX)
      STNORM = (A-6.0)*S+AM
      RETURN
      END
C
      FUNCTION TRIANG(IX)
C
C    This routine TRIANG returns a triangular distributed random
C  number from -1.0 to 1.0.
C
      INTEGER*4      IX
      REAL           X,Y
C
      X = U(IX)
      IF (X.GT.0.5) THEN
        Y = 1.0-SQRT(0.5*(1.0-X))
      ELSE
        Y = SQRT(0.5*X)
      ENDIF
C  This converts the range from (0,1) to (-1,1)
      TRIANG = (2.0*Y)-1.0
      RETURN
      END
C
      FUNCTION U(ISEED)
C
C  From: 'Simulation, Statistical Foundations and Methodology'
C        'Mathematics In Science and Engineering', Vol. 92
C        by G. Arthur Mihram, 1972, Academic Press, NY.
C        pages 44-57.
C
C  M = 2**b where b = number of bits in integer
C  A = M-3  (actually A = (M-((4*I)-1)), I = 1,2,...)
C  C = (M/2)-1
C
      INTEGER*4 ISEED
      REAL*8    A,C,M
      PARAMETER (M=2147483648.D0,A=2147483645.D0,C=1073741823.D0)
C
```

```
ISEED = IDINT(DMOD(((A*DBLE(ISEED))+C),M))
U = DBLE(ISEED)/M
RETURN
END
```

## Vita

James A. Westfall was born in Ithaca, New York on December 30, 1964. Upon completion of high school, he remained in the Ithaca area and worked in the local community. He married Lisa M. White on July 25, 1987. In 1992, he attended the SUNY Environmental Science and Forestry Ranger School and earned an A. A. S. in Forestry. He continued his education at Cornell University to get a B.S. in Natural Resources in 1995. He is currently pursuing a M.S. in the Forest Biometrics Department at Virginia Polytechnic Institute and State University. Upon completion of the M.S. in Forestry, James will remain in the Forest Biometrics Dept. at VPI & SU to begin work towards a Ph.D in Forestry.