STUDIES OF RESONANCES IN

$^{23}$Na, $^{26}$Mg, $^{41}$K, $^{55}$Mn AND $^{59}$Co

by

Elias Peter Stergakos

Thesis submitted to the Graduate Faculty of the
Virginia Polytechnic Institute
in partial fulfillment of the requirements for the degree of
DOCTOR OF PHILOSOPHY
in
NUCLEAR SCIENCE AND ENGINEERING

APPROVED:

Dr. A. K. Furr, Chairman

Dr. James A. Jacobs, Head
Physics Department

Dr. A. G. Bullard

Dr. Andrew Robeson

Dr. Karl Horhyik

April, 1970

Blacksburg, Virginia
LD
5655
V856
1970
S73
0.2
ACKNOWLEDGEMENTS

The author is indebted to his faculty adviser, Dr. A. K. Furr, and wishes to express his appreciation for his guidance and cooperation in solving the problem. Also, the author would like to thank Messrs Gordon Lindsay and Larry Robinson for their help in operating the 100 kW reactor and setting up the detection apparatus.
# TABLE OF CONTENTS

ACKNOWLEDGEMENTS ........................................... ii

LIST OF FIGURES AND GRAPHS .............................. iv

LIST OF TABLES ............................................. v

INTRODUCTION ............................................... 1

EXPERIMENTAL PROCEDURES ................................. 3
  Preparation of Samples ..................................... 3
  Boron Filters, Sample Holders and Activation Box .... 4
  Activation .................................................. 6
  Counting System .......................................... 11

CORRECTIONS .................................................. 13
  Impurities in the Filters .................................. 13
  Activity of Samples due to Previous Irradiation .... 14
  Self Shielding ............................................. 17

THEORY AND METHOD OF ANALYSIS .......................... 22
  Attenuation of Neutron Beam by the $^{10}\text{B}$ Filters .. 22
  Determination of Resonance Energy, $E_r$ ................. 23
  Analysis of the Activity of the Samples ................. 24
  Radiation Partial Width, $\Gamma_\gamma$ .................... 33
  Normalization and Resonance Integral, $I_{R_1}$ .......... 35

RESULTS AND CONCLUSIONS .................................. 40

BIBLIOGRAPHY ............................................... 51

VITA ....................................................... 53
LIST OF FIGURES AND GRAPHS

ACTIVATION BOX .............................................. 8
NEUTRON BEAM MAPPING ..................................... 10
BLOCK DIAGRAM OF THE NaI(Tl) DETECTOR'S SYSTEM ......... 12
SELF SHIELDING .............................................. 19
ACTIVATION CURVES FOR $^{23}_{\text{Na}}$ .......................... 45
ACTIVATION CURVE FOR $^{26}_{\text{Mg}}$ ............................ 46
ACTIVATION CURVE FOR $^{41}_{\text{K}}$ ............................. 47
ACTIVATION CURVES FOR $^{55}_{\text{Mn}}$ .......................... 48
ACTIVATION CURVES FOR $^{59}_{\text{Co}}$ .......................... 49
<table>
<thead>
<tr>
<th>Table Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>INFORMATION ABOUT THE FILTERS</td>
<td>5</td>
</tr>
<tr>
<td>ELEMENTAL CONTENTS OF THE FILTERS</td>
<td>7</td>
</tr>
<tr>
<td>CALCULATED IMPURITIES CORRECTIONS</td>
<td>15</td>
</tr>
<tr>
<td>SELF SHIELDING CORRECTION</td>
<td>21</td>
</tr>
<tr>
<td>EXPERIMENTAL RESULTS</td>
<td>50</td>
</tr>
</tbody>
</table>
INTRODUCTION

Reactor engineering and the desire for pure theoretical nuclear knowledge have prompted much of the research in neutron capture cross sections and the determination of resonance parameters in the keV energy region. Much of the cross section data found in the literature were obtained through the use of monoenergetic neutrons or by time-of-flight techniques (12, 18, 19, 20, 24). The present work, however, was done through the use of a polyenergetic neutron beam obtained from the V.P.I. research reactor. The activation method employed here did not require that the isotopes activated be pure, but, on the other hand, it was restricted to isotopes that activate and decay with appropriate half-lives to allow observations. Positive identification of the isotopes studied was aided by the fact that their half-lives have been well established.

The activity of the samples studied, which were exposed to a cadmium filtered neutron beam, was due to their activation by epithermal and resonance energy neutrons. The ability to isolate the activity due to the different resonances was based on the $^{10}$B absorption cross section which varies inversely as the velocity of the incident neutrons over a broad range of neutron energy (8). Advantage of this property of $^{10}$B was taken through the use of $^{10}$B filters of different thicknesses inserted into the neutron beam prior to the samples. This is an old technique originally used to locate resonances (5, 24) but used in this work to isolate the individual resonances in order to study the resonance parameters.
The isotopes selected for investigation in this experiment were $^{23}$Na, $^{26}$Mg, $^{41}$K, $^{55}$Mn and $^{59}$Co with isolated resonances from 132 eV to 305 keV. The number of resonances studied in any one of the samples varied from one to three. It should be mentioned that the resonance energies employed in this work had already been determined (by using accelerator-produced neutrons) (18, 19, 20) and were employed here to verify the identification of the resonance energies being studied. The above mentioned technique was used successfully to measure the resonance integrals and radiative capture widths with the assumption that the reactor beam flux had a $1/E$ energy dependence. For normalization of the data, the above mentioned quantities were determined relative to the resonance integral and thermal cross section of $^{197}$Au (21).
EXPERIMENTAL PROCEDURES

Preparation of Samples

In order not to waste any reactor time while performing the experiment, two samples of the same element were prepared. Thus, while one was activated the other was counted. The samples of each element were made as identical as possible with regard to areal density and dimensions so that corrections between them were unnecessary. It was attempted also to make the samples as pure as possible in order to eliminate corrections for unwanted activities. The material of the samples was in a powder form and was formed into disks by compressing it in a die at an approximate pressure of twenty-two thousand pounds per square inch. The diameter of the disk was 5.71 cm and the area of the base of all the samples was 25.7 cm².

The magnesium samples were formed by compressing 40.31 gm of magnesium oxide, MgO. Only 11.17% of the magnesium was ²⁶Mg, i.e., the isotope of interest, and, thus, in terms of ²⁶Mg content, the areal density of the sample was 0.104 gm/cm².

The two manganese samples were formed by compressing a 17.88 gm homogenous mixture of manganese dioxide, MnO₂, and sulfur. The sulfur was added because manganese dioxide alone is not cohesive while the mixture would cohere to itself. The mixture ratio was three parts of sulfur and fifteen parts of manganese dioxide. Due to the low amount of sulfur in the mixture and also because of its relatively low absorption cross section, it was calculated that no sulfur activity would be noticed and none was observed. The
manganese was 100% $^{55}\text{Mn}$, i.e., the isotope of interest. Taking into consideration the above facts, the areal density of $^{55}\text{Mn}$ was calculated to be 0.367 gm/cm$^2$.

An amount of 14.98 gm of potassium nitrate, $\text{KNO}_3$, was used to make the potassium samples. Considering the fact that only 6.88% of the element was $^{41}\text{K}$, i.e., the isotope of interest, an areal density of 0.016 gm/cm$^2$ $^{41}\text{K}$ was in the sample.

The sodium samples were made by compressing a 14.94 gm homogenous mixture of sodium bicarbonate, $\text{NaHCO}_3$, and sulfur. The ratio of the mixture was two parts of sulfur to thirteen parts of sodium bicarbonate. Again, no activity due to sulfur, which served as a binding agent, was observed. Sodium is isotopically pure so the areal density was 0.138 gm/cm$^2$.

Finally, the cobalt samples were obtained by compressing a 19.87 gm homogenous mixture of cobalt and sulfur. Once more, sulfur served as a binding agent and no activity due to it was observed. The ratio of the mixture was one part of sulfur to five parts of cobalt. Using the previous information and the fact that the sample was 100% $^{59}\text{Co}$, i.e., the isotope of interest, the areal density of $^{59}\text{Co}$ was 0.643 gm/cm$^2$.

**Boron Filters, Sample Holders and Activation Box**

The boron filters were made by Mr. Tucker (9) and consisted of finely crushed boron. The diameter of all the filters was 2.856 cm. However, their masses were slightly different and are given, with some more information, in Table I. The filters were covered with 0.0063 cm
<table>
<thead>
<tr>
<th>Filter No.</th>
<th>Mass in gm</th>
<th>Nominal Thickness in cm</th>
<th>Areal Density in gm/cm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/16</td>
<td>2.672 (B+S)</td>
<td>0.159</td>
<td>0.208</td>
</tr>
<tr>
<td>1/8</td>
<td>2.906</td>
<td>0.318</td>
<td>0.453</td>
</tr>
<tr>
<td>1</td>
<td>5.556</td>
<td>0.635</td>
<td>0.866</td>
</tr>
<tr>
<td>2</td>
<td>5.555</td>
<td>0.635</td>
<td>0.866</td>
</tr>
<tr>
<td>3</td>
<td>5.424</td>
<td>0.635</td>
<td>0.846</td>
</tr>
<tr>
<td>4</td>
<td>5.603</td>
<td>0.635</td>
<td>0.874</td>
</tr>
<tr>
<td>5</td>
<td>5.619</td>
<td>0.635</td>
<td>0.876</td>
</tr>
<tr>
<td>6</td>
<td>5.582</td>
<td>0.635</td>
<td>0.872</td>
</tr>
<tr>
<td>7</td>
<td>5.029</td>
<td>0.635</td>
<td>0.784</td>
</tr>
<tr>
<td>8</td>
<td>5.041</td>
<td>0.635</td>
<td>0.786</td>
</tr>
<tr>
<td>9</td>
<td>5.035</td>
<td>0.635</td>
<td>0.785</td>
</tr>
<tr>
<td>10</td>
<td>5.131</td>
<td>0.635</td>
<td>0.800</td>
</tr>
<tr>
<td>11</td>
<td>5.295</td>
<td>0.635</td>
<td>0.829</td>
</tr>
</tbody>
</table>
mylar to retard oxidation. The elemental contents of the filters had been determined by Dr. Furr and Mr. Tucker (9) and are listed in Table II. It had been found necessary to add sulfur to the 0.159 cm thick filter in order for it to hold together. The holders of the filters were made of aluminum sheet and were designed to minimize the amount of scattering into the sample.

The holders of the samples were also made of aluminum sheet. In their center was a hole of 5.715 cm in diameter where the samples were placed. The samples (which in turn means the holes in the center of the holder) were considerably larger than the beam at the activation point so that the aluminum holder would not be activated.

The activation box, Fig. 1, was constructed with slots which permitted the filters and samples to be accurately inserted in it. It had a full length door to facilitate the rapid insertion and removal of the samples to permit accurate irradiation times. The box and its door were covered with 7.62 x 10^{-2} cm cadmium to prevent thermal neutrons from the outside from entering the system. The activation box was affixed permanently to a collimator in such a way that the emergent beam from the core of the reactor coincided with its central axis.

Activation

The samples were activated through the use of the top central access port of the V.P.I. reactor. To reduce the probability of activation by neutrons scattered from the filters and/or filter holders, the samples were always placed 58.43 cm above the emergent point of the neutron beam. The diameter of the beam emerging from the
<table>
<thead>
<tr>
<th>Component</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>B&lt;sub&gt;10&lt;/sub&gt;</td>
<td>94.3</td>
</tr>
<tr>
<td>B&lt;sub&gt;11&lt;/sub&gt;</td>
<td>3.4</td>
</tr>
<tr>
<td>C</td>
<td>0.7</td>
</tr>
<tr>
<td>Fe</td>
<td>0.1</td>
</tr>
<tr>
<td>O</td>
<td>1.0</td>
</tr>
<tr>
<td>Si</td>
<td>0.5</td>
</tr>
<tr>
<td>S</td>
<td>50.047 (for #1/16 only)</td>
</tr>
</tbody>
</table>
Cadmium covered box and door

Target, mounted in sample holder

10B filters

Neutron beam

Activation box
collimator was 2.223 cm. The irradiation set up was such that the beam had to pass first through a removable cadmium sheet of 7.62 x \(10^{-2}\) cm thick and then through the boron filters before it reached the sample to be irradiated. The samples were activated starting with the cadmium and a maximum boron thickness between them and the neutron beam and then the boron was reduced, by the removal of the filters, down to zero thickness. The maximum number of filters was not always the same for the irradiation of the different elements but instead was chosen in each case to suit the expected experimental results. The above procedure of filter removal was used to minimize the induced activity in the sample caused by the preceding irradiation since only two samples were used alternatively in obtaining the data for each isotope. The activity of the samples due to the previous irradiation was accounted for when the data were analyzed. The activation and counting time of the samples depended upon their half-lives. Cadmium ratio measurements (activity of sample without the cadmium sheet in neutron beam to activity of sample with cadmium) were taken for all the samples and for gold also (17) since it was used as a standard in the normalization procedure to be discussed later.

Before any activation was done, a check was made and it was verified, by plotting the neutron flux across the beam, that the neutron beam did indeed remain fully within the filters throughout their length and within the area covered by the samples. This beam check was accomplished by the use of 0.318 cm diameter indium foils placed at recorded distances from the beam center and then measuring their activity as shown in Graph 1.
Counting System

The activity of most of the samples was determined through the use of two 3 x 3 NaI(Tl) scintillation counters. The samples, when counted, were placed between the counters in such a way that the subtended solid angle was as large as possible. Despite the effort for better detection efficiency, it was necessary to use a plastic beta scintillator in the case of potassium and sodium in order to improve the statistical error. The output signal of the photomultiplier tubes was fed to an Intertechnique Didac model 44 4000 channel analyzer. The detectors were housed in a large shielded enclosure to minimize background. The NaI(Tl) system was calibrated using the 1.17 MeV, 1.33 MeV and 2.50 MeV sum peak of $^{60}$Co and 0.662 MeV $^{137}$Cs gammas (23) while the plastic scintillator was roughly calibrated using the beta spectra of the two sources. The counting system is shown in Fig. 2 in block diagram. The multichannel analyzer was employed in the multiscalar mode so that the output of the analyzer consisted of a number of consecutive measurements of the total counts during a brief period involving the entire spectrum. The induced activities were low except in the case of cobalt and manganese and therefore several runs were necessary for each element.
BLOCK DIAGRAM OF THE NaI(Tl) DETECTOR'S SYSTEM
CORRECTIONS

Impurities in the Filters

As has been mentioned already, the results of this work depend on the \( \frac{1}{v} \) transmission cross section of \( ^{10}B \). It was necessary, therefore, to make the proper corrections to the experimental data since, as shown in Table II, the filters did have impurities. As a result of the impurities, the neutron beam was attenuated. Assuming no multiple scattering in the filters, the reduced beam as it emerged from the filters was (7):

\[
I = I_0 e^{-\sum_i N_i \sigma_i}
\]

where

\( I_0 \) = initial neutron beam intensity

\( N_i \) = number of atoms per cm\(^2\) of specific impurity \( i \)

\( \sigma_i \) = total microscopic cross section of specific impurity \( i \) at the energies of interest.

It should be noted that the scattering cross section of \( ^{10}B \) remains constant at the energies of interest of this work and thus was effectively considered as an impurity (6). As indicated above, the assumption was made here that once a neutron was removed from the neutron beam, it never re-entered it (the steps taken in this work to achieve this were mentioned in the chapter on Experimental Procedure).

Since the activity of the sample was directly related to the intensity of the neutron beam, to account for the impurities, the experimental data were multiplied by the factor
The total calculated impurities corrections are given in Table III. As can be noted, the corrections were quite appreciable. The different correction factors for magnesium were due to the fact that its resonance was at 305 keV; i.e., far above the resonance energies of the other elements and the total cross sections of the impurities at 305 keV were appreciably different from the cross sections (of the impurities) at the lower resonance energies of the other elements (18, 19).

Activity of Samples due to Previous Irradiation

Due to the fact that two samples of each element were used in the experiment and they were irradiated over and over again, it was inevitable that, after the first counting of the samples, they would have some residual activity due to previous irradiations. It was necessary, therefore, to make a correction for this activity. Let:

\[ A = \text{number of atoms of the radioactive isotope} \]
\[ \phi = \text{neutron flux} \]
\[ \lambda = \text{decay constant of radioactive isotope} \]
\[ \Sigma = \text{absorption macroscopic cross section of element.} \]

Through the use of the above notation, it can be said that the change in the number of atoms of the radioactive isotope, while the sample is activated, is given by the following equation (11):
TABLE III. CALCULATED IMPURITIES CORRECTIONS

<table>
<thead>
<tr>
<th>Areal Density of Filters gm/cm²</th>
<th>( M = \exp\left(\sum_{i}N_{i}C_{i}\right) ) for 59Co, 23Na, 41K, 55Mn</th>
<th>( M = \exp\left(\sum_{i}N_{i}C_{i}\right) ) for Mg</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.284</td>
<td>3.337</td>
<td>3.284</td>
</tr>
<tr>
<td>8.458</td>
<td>3.001</td>
<td>2.956</td>
</tr>
<tr>
<td>7.676</td>
<td>2.710</td>
<td>2.678</td>
</tr>
<tr>
<td>6.922</td>
<td>2.460</td>
<td>2.433</td>
</tr>
<tr>
<td>6.182</td>
<td>2.235</td>
<td>2.214</td>
</tr>
<tr>
<td>5.440</td>
<td>2.099</td>
<td>2.015</td>
</tr>
<tr>
<td>4.701</td>
<td>1.846</td>
<td>1.824</td>
</tr>
<tr>
<td>3.879</td>
<td>1.659</td>
<td>1.652</td>
</tr>
<tr>
<td>3.255</td>
<td>1.524</td>
<td>1.513</td>
</tr>
<tr>
<td>3.055</td>
<td>1.492</td>
<td>1.487</td>
</tr>
<tr>
<td>2.431</td>
<td>1.369</td>
<td>1.362</td>
</tr>
<tr>
<td>2.257</td>
<td>1.346</td>
<td>1.345</td>
</tr>
<tr>
<td>1.634</td>
<td>1.235</td>
<td>1.231</td>
</tr>
<tr>
<td>1.441</td>
<td>1.211</td>
<td>1.212</td>
</tr>
<tr>
<td>1.244</td>
<td>1.174</td>
<td>1.171</td>
</tr>
<tr>
<td>1.013</td>
<td>1.146</td>
<td>1.149</td>
</tr>
<tr>
<td>0.817</td>
<td>1.112</td>
<td>1.110</td>
</tr>
<tr>
<td>0.624</td>
<td>1.089</td>
<td>1.092</td>
</tr>
<tr>
<td>0.427</td>
<td>1.057</td>
<td>1.056</td>
</tr>
<tr>
<td>0.196</td>
<td>1.031</td>
<td>1.035</td>
</tr>
</tbody>
</table>
\[
\frac{dA}{dt} = \Sigma \phi - \lambda A \quad (3)
\]

Multiplying the above equation by the factor \(e^{\lambda t}\) and integrating it, one obtains:

\[
Ae^{\lambda t} = \frac{\Sigma \phi}{\lambda} e^{\lambda t} + C \quad \text{where } C = \text{constant} \quad (4)
\]

However, at time \(t = 0\), \(A = 0\) and thus the number of radioactive atoms in the sample after the first activation period, \(T\), is

\[
A(T) = \frac{\Sigma \phi}{\lambda} (1 - e^{-\lambda T}) \quad (5)
\]

After the sample is out of the neutron beam, the radioactive isotope decays in the regular exponential manner \((7, 11)\). Thus, if \(t_1\) represents the total time that the sample is out of the neutron beam, then the number of radioactive atoms left in the element are:

\[
A(t_1 + T) = A(T)e^{-\lambda t_1} = \frac{\Sigma \phi}{\lambda} (1 - e^{-\lambda T}) e^{-\lambda t_1} \quad (6)
\]

After the second irradiation of the samples the number of radioactive atoms in it is given again by the Eq. (4), i.e.,

\[
A = \frac{\Sigma \phi}{\lambda} + C e^{-\lambda t} \quad (7)
\]

But, now, the number of radioactive atoms in the sample at time \(t = 0\) is not equal to zero but instead is equal to those given by Eq. (6). Thus, the number of radioactive atoms after the second irradiation period, \(T'\), is:
\[ A(t_1 + T' + T) = \sum_{\lambda} \frac{\phi_{\lambda}}{\lambda} (1 - e^{-\lambda T'}) + A(T) e^{-\lambda (t_1 + T')} \]  

(8)

In this work, however, the irradiation period for a particular sample was kept always constant (i.e., \( T = T' \)) and thus the above equation can be written as follows:

\[ A(t_1 + T + T) = A(t_1 + 2T) = \sum_{\lambda} \frac{\phi_{\lambda}}{\lambda} (1 - e^{-\lambda T}) + A(T)e^{-\lambda (t_1 + T)} \]  

(9)

Looking at the above equation, we see that it is Eq. (6) with the factor \( A(T)e^{-\lambda (t_1 + T)} \) added to it. This added factor is due to the previous irradiation and was, as stated before, to be eliminated.

Observe it, we recognize that \( A(T) \) is the activity of the sample due to the previous irradiation and the time factor in the exponential, \( t_1 + T \), is a complete cycle time, i.e., the time elapsed from when the sample was first started to be counted to the next time it was done. Therefore, to correct for the activity of the sample due to previous irradiation all that was necessary to be done was to multiply the previous activity of the sample by the factor \( e^{-\lambda (t_1 + T)} \) and subtract the product from the new activity.

**Self Shielding**

The activated samples were self shielding due to the fact that they were not infinitesimally thin. This means that the number of resonance energy neutrons entering the samples was higher than that exiting it; which, in turn, means that the activation at every thin layer of the sample, due to the resonance-energy neutrons, was different. It has been shown by Bethe that the activity for a beam
geometry, induced by the absorption of resonance energy neutrons, in a
layer at a given depth of a sample is given by (1, 2)

\[ A = A_0 e^{-\frac{N_o}{2}} I_0 \left( \frac{jN_o}{2} \right) \]  \hspace{1cm} (10)

where

- \( A_0 \) = the induced activity of the sample at the initial thin layer
- \( N \) = number of atoms/cm\(^2\) in the layer of the sample acting as an absorber
- \( \sigma_0 \) = the peak microscopic cross section at exact resonance energy
- \( I_0 \) = the Bessel function of zero order.

With the above expression, it was possible to correct the self shielding in our samples in the following manner. The total number of activated sample-atoms were equivalent to the area under the curve given by the above equation. If there were no attenuation of the resonance-energy neutrons, then, the total number of activated sample-atoms would have been equivalent to the area under the horizontal line whose starting and ending points were \( (\frac{A}{A_0}, 0) \) and \( (\frac{A}{A_0}, \frac{N_o}{2}) \), respectively. Thus the fraction, \( f \), (given in Graph 2) of the sample-atoms that were activated to those that would have been if there were no attenuation of the resonance energy neutrons is equivalent to the area under the curve, given by the above expression, to the area under the horizontal line. Thus, the correction for self shielding was to divide the activity, found experimentally to be due to the absorption of the resonance-energy neutrons, by \( f \). The
values of $\sigma_0$ were obtained from BNL 325 (18, 19, 20). Information on the self shielding of each isotope is given in Table IV. As can be seen, even though the samples were kept as thin as possible and yet still be thick enough to hold together, this correction was significant for most of the resonances.
<table>
<thead>
<tr>
<th>Isotope</th>
<th>Resonance Energy, $E_o$ (keV)</th>
<th>$\frac{N\sigma}{o}$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{11}\text{Na}^{23}$</td>
<td>2.85</td>
<td>$6.508 \times 10^{-1}$</td>
<td>0.750</td>
</tr>
<tr>
<td></td>
<td>54.0</td>
<td>$5.062 \times 10^{-2}$</td>
<td>0.969</td>
</tr>
<tr>
<td>$^{12}\text{Mg}^{26}$</td>
<td>305.0</td>
<td>$9.681 \times 10^{-3}$</td>
<td>1.0</td>
</tr>
<tr>
<td>$^{19}\text{K}^{41}$</td>
<td>5.4</td>
<td>$3.001 \times 10^{-4}$</td>
<td>1.0</td>
</tr>
<tr>
<td>$^{25}\text{Mn}^{55}$</td>
<td>0.337</td>
<td>6.841</td>
<td>0.299</td>
</tr>
<tr>
<td></td>
<td>1.098</td>
<td>1.911</td>
<td>0.537</td>
</tr>
<tr>
<td></td>
<td>2.375</td>
<td>1.207</td>
<td>0.634</td>
</tr>
<tr>
<td>$^{27}\text{Co}^{59}$</td>
<td>0.132</td>
<td>21.349</td>
<td>0.175</td>
</tr>
<tr>
<td></td>
<td>5.015</td>
<td>$7.784 \times 10^{-1}$</td>
<td>0.722</td>
</tr>
<tr>
<td>$^{79}\text{Au}^{197}$</td>
<td>0.0049</td>
<td>1.951</td>
<td>0.530</td>
</tr>
</tbody>
</table>
THEORY AND METHOD OF ANALYSIS

Attenuation of Neutron Beam by the $^{10}$B Filters

The use of the $^{10}$B filters, placed in the neutron beam, attenuates the beam in a similar manner to that described in the section about "Impurities in the Filters". It can be said, then, that the intensity of the neutron beam at any energy emerging from the filters is:

$$ I = I_0 e^{-N\sigma_a(E)x} $$

(11)

where

- $I_0$ = original beam intensity
- $N$ = number of $^{10}$B atoms in filters per cm$^3$
- $\sigma_a(E)$ = absorption cross section of $^{10}$B
- $x = ^{10}$B thickness through which the beam passes.

The number of $^{10}$B atoms can be expressed in terms of its density, $\rho$, atomic mass, $A$, and Avogadro's number, $N_a$, in the following manner.

$$ N = \frac{\rho N_a}{A} $$

(12)

$$ = \frac{g}{x A} N_a $$

Where in the second expression of the above equation, $g$ stands for the number of $^{10}$B grams, $x$ and "a" for the thickness and area of the filters, respectively. Thus

$$ N x = \frac{g}{A} a N_a $$

(13)

Letting, in the above equation, $g/a = t$ (gm/cm$^2$) (from now on $t$ will be called areal density or thickness) and substituting the resultant
expression into equation (11), the following expression is obtained:

$$ I = I_0 e^{-\frac{tN_a}{A} \sigma_a(E)} $$

(14)

It is given by Gibbons (10) that $\sigma_a(E) = \frac{611}{\sqrt{E}} \times 10^{-24} \text{ cm}^2$ with $E$ expressed in eV up to about 200 keV. In the 300 keV region (where the $^{26}\text{Mg}$ resonance was observed) the absorption cross section found by Bichsel and Bonner is different from that found by Mooring or Gibbons. However, the average of the results of the above mentioned experimenters comes out to be approximately equal to the expression given above (20b). (It should be stated that, according to Dimnt (6), $^{10}\text{B}$ has a scattering cross section of $1.95 \times 10^{-24} \text{ cm}^2$ in the energy region being studied. It is not used in equation (14), for scattering was already taken into consideration when the impurities of the filters were discussed). In addition, using the fact that $A = 10$, and $N_a = 6.023 \times 10^{23}$ (13), then equation (14) becomes

$$ I = I_0 e^{-\frac{36.8}{\sqrt{E}} t} $$

(15)

Thus, the original neutron beam, $I_0$, reduces by a factor of $\exp \left(-\frac{36.8}{\sqrt{E}} t\right)$.

**Determination of Resonance Energy, $E_R$**

As shown in equation (15), the neutron beam reduces exponentially with $^{10}\text{B}$ thickness. It follows, therefore, that the activation of the sample should behave in the same manner, for activation is directly related to flux intensity. Letting $A_r(0)$ represent the
activation of the sample due to resonance-energy neutrons with no boron filters, it can then be said that the activation of the sample with $^{10}\text{B}$ thickness $t$ (inserted into the neutron beam prior to the samples) is given by the following equation.

$$A_r(t) = A_r(0) e^{-\frac{36.8}{\sqrt{E_r}} t}$$  \hspace{1cm} (16)

Where $E_r$ represents the energy of the resonance. Letting $t_{1/2}$ denote the $^{10}\text{B}$ thickness which will reduce the activation down to one half that when there are no boron filters (i.e., $A_r(t_{1/2}) = \frac{A_r(0)}{2}$), then the above equation becomes

$$A_r(t_{1/2}) = \frac{A_r(0)}{2} = A_r(0) e^{-\frac{36.8}{\sqrt{E_r}} t_{1/2}}$$  \hspace{1cm} (17)

Solving the above expression for the resonance energy, $E_r$, the following result is easily obtained which will prove convenient in ascertaining the contributions due to the resonance $E_r$.

$$E_r = 2818 \cdot t_{1/2}^2$$  \hspace{1cm} (18)

The value of $t_{1/2}$ can be obtained experimentally from the graphs where the activity is plotted against $^{10}\text{B}$ thickness, $t$. (This is fully explained in the section entitled, "Analysis of the Activity of the Samples"). Since the resonance energies are known, equation (18) was employed primarily to draw resonance attenuation curves of the proper slope.

**Analysis of the Activity of the Samples**

The neutron beam used to activate the samples was polyenergetic,
i.e., it consisted of thermal, epithermal and resonance-energy neutrons; where the term "epithermal" is used here to denote specifically the neutrons whose energy lies above the cadmium cut-off energy, i.e. about 0.4 eV, and below the resonances. Thus, when there was nothing between the beam and the sample, the activation of the sample was due to the absorption of neutrons of all energies present in the beam. Let $A_{th}$, $A_{1/v}$ and $A_r$ represent the activation of the sample due to its absorption of thermal, epithermal and resonance-energy neutrons, respectively. The activity due to the absorption of the epithermal neutrons is denoted by $A_{1/v}$ because the absorption cross section in that energy region is an inverse function of velocity, $v$. It was necessary, in analyzing the data, to determine $A_{th}$, $A_{1/v}$ and $A_r$. The $A_{th}$ component of the activation could easily be separated from $A_{1/v}$ and $A_r$ through the use of a cadmium filter which has a high thermal cross section. In determining the $A_{1/v}$ component of the activation of the sample, it was necessary to proceed mathematically as follows. The activation rate of the sample is given by the equation (3, 11)

$$A(E) = N\sigma(E)\phi(E)$$

where

$N$ = number of atoms/cm$^3$ of isotope (sample) being activated

$\sigma(E)$ = absorption cross section

$\phi(E)$ = neutron flux.

However, as explained in the section, "Attenuation of Neutron Beam by the $^{10}$B Filters" of this chapter, the $^{10}$B filters attenuate the
neutron beam (i.e. flux) by the factor of \( \exp\left(-\frac{36.8}{\sqrt{E}} t\right) \). (Where, once again, \( E \) represents the neutron energy in eV and \( t \) the 10\(^{B}\) thickness in gm/cm\(^2\)). It follows, therefore, that the activation of the sample is reduced by the same factor and thus the above equation should be written as follows.

\[
A(E,t) = N_0 \sigma(E) \phi(E) e^{-\frac{36.8}{\sqrt{E}} t}
\]  

(20)

It will be assumed for the time being that

\[
\phi(E) = \frac{C}{E}
\]

(21)

where \( C \) is a constant and can be solved for in terms of a specific energy and the neutron flux at that specific energy. For example, according to Eq. (21), the flux at a given energy \( E_o \) is

\[
\phi(E_o) = \frac{C}{E_o}
\]

(22)

It follows

\[
C = E_o \phi(E_o)
\]

(23)

Substituting equation (23) into equation (21) and the resultant expression into equation (20), the following equation is obtained

\[
A(E,t) = E_o \phi(E_o) \frac{N_0(E)}{E} e^{-\frac{36.8}{\sqrt{E}} t}
\]

(24)

For isolated resonances, about which this problem deals, the capture cross section is given by the general Breit-Wigner formula, i.e. (16)
\[ \sigma_{n,\gamma}(E) = \frac{\Gamma_n \Gamma_{\gamma}}{(E - E_r)^2 + \frac{\Gamma_r^2}{4}} \left( \frac{E_r}{E} \right)^{1/2} \pi \lambda_r^2 g . \]  

(25)

where

- \( E \) = energy of incident neutron
- \( E_r \) = energy of resonance
- \( \Gamma \) = level width
- \( \Gamma_n \) = neutron partial width
- \( \Gamma_{\gamma} \) = radiation partial width
- \( \lambda_r \) = reduced wave length of incident neutron with resonance-energy

\[ g = \begin{cases} 1 & \text{I = 0} \\ \frac{2J + 1}{2(2I + 1)} & \text{I \neq 0} \end{cases} \]

- \( J \) = angular momentum of compound nucleus
- \( I \) = angular momentum of target nucleus

It should be observed from Eq. (25) that when \( E \) is very much less than \( E_r \), \( \sigma_{n,\gamma} \) behaves as \( \frac{1}{\sqrt{E}} \), that is, as \( \frac{1}{v} \), where \( v \) is the speed of the neutron. With the above observation, substituting Eq. (25) into (24) and integrating over energy, the activation of the sample due to the epithermal neutrons is given by

\[ \lambda_{1/v}(t) = E_0 \phi(E_0) N \pi \lambda_r^2 \Gamma \Gamma_{\gamma} \int_{E_r - 2\Gamma}^{E_r} \left( \frac{E_r}{E} \right)^{1/2} \frac{-36.8}{v^2} \frac{1}{(E - E_r)^2 + \frac{\Gamma_r^2}{4}} dE . \]

(26)
The upper limit on the integral is chosen rather arbitrarily as a point where the absorption cross section no longer behaves as $1/v$.

Making the assumptions that $\Gamma << E_r$ and $E << E_r$, performing the change of variables $E^{1/2} = E$ and then carrying out the integration, we obtain

$$A_{1/v}(t) = \frac{2E_0^2\Phi(E_0)\pi N_a^2 r^2}{36.8 \sqrt{E_r}} \left( e^{\frac{36.8}{\sqrt{E_r-2\Gamma}t}} - e^{-58.2t} \right)$$  (27)

The above equation gives the activation of the sample due to its absorption of neutrons of energy above the cadmium cut-off energy but less than the resonance energy and with any $^{10}$B thickness, $t$, inserted in the beam prior to the sample. To obtain the activation of the sample with no filters (i.e., $t = 0$) l'Hopital's rule is used to give

$$A_{1/v}(0) = \frac{2E_0^2\Phi(E_0)\pi N_a^2 r^2}{36.8 \sqrt{E_r}} (58.2 - \frac{36.8}{\sqrt{E_r-2\Gamma}})$$  (28)

or

$$\frac{2E_0^2\Phi(E_0)\pi N_a^2 r^2}{36.8 \sqrt{E_r}} = \frac{A_{1/v}(0)}{(58.2 - \frac{36.8}{\sqrt{E_r-2\Gamma}})}$$  (29)

Substituting Eq. (29) into Eq. (27), the following result is obtained.

$$A_{1/v}(t) = \frac{A_{1/v}(0)}{(58.2 - \frac{36.8}{\sqrt{E_r-2\Gamma}})} \left( e^{\frac{36.8}{\sqrt{E_r-2\Gamma}t}} - e^{-58.2t} \right)$$  (30)
Having obtained the equation for the activation of the sample
due to its absorption of epithermal neutrons, we proceed now to find
a similar equation for the activation of the sample due to the absorption
of resonance-energy neutrons. Proceeding in the exact same way as
was done in determining $A_{1/V}(t)$, the following equation is derived.

$$A_r(t) = E_0 \phi(E_0) N \pi \lambda^2 \Gamma \frac{\Delta E}{E} \left( \frac{E - E_0}{E} \right)^{1/2} \frac{e^{-\frac{36.8}{\sqrt{E} t}}}{(E - E_r)^2 + \gamma^2} \frac{dE}{E}$$

Since the resonances are assumed narrow compared to $E_r$, then the
quantities depending on $E$ alone may be assumed constant and thus
taken out of the integral. Moreover, nearly all the contribution to
$A_r(t)$ comes from the absorption of neutrons from the immediate
neighborhood of the resonance and thus no significant error is
introduced by allowing the limits of integration to approach infinity.

These assumptions lead to

$$A_r(t) = \int_{-\infty}^{\infty} E_0 \phi(E_0) N \pi \lambda^2 \Gamma \frac{\Delta E}{E} \frac{e^{-\frac{36.8}{\sqrt{E} t}}}{(E - E_r)^2 + \gamma^2} \frac{dE}{E}$$

Making a change of variables, $E - E_r = X$, and then performing the
integration, the following result is obtained.

$$A_r(t) = \frac{2E_0 \phi(E_0) N \pi \lambda^2 \Gamma \frac{\Delta E}{E} \frac{e^{-\frac{36.8}{\sqrt{E} t}}}{(E - E_r)^2 + \gamma^2}}{E_r \Gamma}$$
Letting the $^{10}\text{B}$ thickness, $t$, approach zero, the above equation becomes

$$A_r(0) = \frac{2E_o \phi(E_o) N \frac{2}{3} \Gamma \gamma^2}{E_r \Gamma}$$

(34)

It follows from Eq. (34) that $A_r(t)$, i.e., Eq. (33), can be written in the following manner

$$A_r(t) = A_r(0) e^{-\frac{36.8 t}{\sqrt{E_r}}}$$

(35)

The $A_{1/\nu}(t)$ and $A_r(t)$ components of activation given by Eqs. (30) and (35), respectively, cannot be determined explicitly. This is true because the quantities $A_{1/\nu}(0)$ and $A_r(0)$ (found in the above mentioned equations, respectively) given by Eqs. (28) and (34), respectively, contain the radiative capture widths, $\Gamma_r$, which are not generally known for light elements. The seemingly unresolvable problem can be overcome by using information obtained experimentally.

Plots of activity of the sample versus $^{10}\text{B}$ thickness were made. The activity of the sample, was, of course, a summation of the components $A_{1/\nu}(t)$ and $A_r(t)$ of all the resonances. It was necessary to make an assumption of how many resonances could be resolved using the experimental data. Since, as a function of energy, terms of $A_r$ fall off as $\frac{1}{E_r^2}$ and for $A_{1/\nu}$ as $\frac{1}{E_r^{5/2}}$ only the lowest few resonances would be expected to have individual contribution big enough to be seen.

Having done that, the following set of linear equations were used to
determine the individual components of the activity.

\[ \Sigma_i A_i^{1/\nu}(t_j) + \sum_i A_i(t_j) = A(t_j) \]  
(36)

where

\[ i = \text{the number of resolvable resonances} \]

\[ A(t_j) = \text{total activity of the sample with } ^{10}{\text{B}} \text{ thickness } t_j \]

(read off from graph)

Let us, now, define another quantity, \( K \)

\[ K_i(t_j) = \frac{A_i(t_j)}{A_i^{1/\nu}(t_j)} \]

\[ = \frac{36.8}{\pi t_j E_{r_i}^{1/2}} \cdot \frac{36.8}{\sqrt{V_{r_i}}} t_j \]

\[ \left( \frac{\sqrt{E_{r_i}} - 2\Gamma_{r_i} - t_j}{\Gamma_{r_i}} \right) \]

\[ \left( e^{58.2t_j} - e \right) \]  
(37)

Where, in deriving the second expression above, Eqs. (27) and (33) were used. Note that in \( K \), \( \Gamma \) is not present but instead only \( \Gamma \) which is available from the literature. In order to evaluate \( K_i \) at \( t_i = 0 \), all one has to do is to apply l'Hopital's rule at the above equation to obtain,

\[ K_i(0) = \frac{36.8 \pi E_{r_i}^{1/2}}{\Gamma_{r_i} \left( 58.2 - \frac{36.8}{\sqrt{E_{r_i}} - 2\Gamma_{r_i}} \right)} \]  
(37b)
Solving Eq. (37) for $A_{r_i}^{1/v}(t_j)$ and putting the resultant into Eq. (36),
the following equation is obtained

$$
\Sigma_1 A_{r_i}^{1/v}(t_j)[1 + K_i(t_j)] = A(t_j)
$$

(38)

Through the use of Eq. (30), it can be said that:

$$
A_{r_i}^{1/v}(t_j) = \frac{A_{r_i}^{1/v}(0) (e^{\frac{36.8}{\sqrt{E_{r_i}} - 2\Gamma_{r_i}} t_j} - e^{-58.2 t_j})}{(58.2 - \frac{36.8}{\sqrt{E_{r_i}} - 2\Gamma_{r_i}}) t_j}
$$

(39)

Substituting Eq. (39) into (38), the following result is obtained.

$$
\Sigma_1 \left[ A_{r_i}^{1/v}(t_j) \left( e^{\frac{36.8}{\sqrt{E_{r_i}} - 2\Gamma_{r_i}} t_j} - e^{-58.2 t_j} \right) \right]
$$

$$
1 + K_i(t_j)
$$

(40)

All quantities involved in the above set of linear equations are known and thus explicit solutions for $A_{r_i}^{1/v}(0)$ can be gotten. The solutions of the above equations give as many $A_{r_i}^{1/v}(0)$ as the number of resonances that can be resolved from the data. To find the $A_{r_i}^{1/v}(t_j)$ and $A_{r_i}(t_j)$ components of the activity Eqs. (39) and/or (35) should be used, respectively.

It should be mentioned that $^{59}$Co and $^{55}$Mn had numerous higher energy resonances besides those which could be resolved so that they contributed "extra" activity. This "extra" activity, however, was
not enough to give us any information about the resonances and had to be subtracted before the $A_{i}^{1/v}(t_{j})$ and $A_{rj}^{i}(t_{j})$ components of the resolved resonances could be determined. This was done by drawing a line representing the average $A_{r}$ for the higher energy resonances through the last pair of points on the $^{10}$B curve, or an estimate of the asymptote which the $^{10}$B curve was approaching, and subtracting it from the original data. If this did not fully account for the higher energy resonances, it, at least, reduced their $A_{r}$ contribution to a negligible level.

It was observed, that at $t = 0$, the $A_{r}$ contribution for these higher energy resonances was very small compared to the other resonances and since the $A_{1/v}$ term falls off even more rapidly than $A_{r}$ no average $A_{1/v}$ was subtracted from the original data. Having gotten the $A_{r}$ components they were plotted on a semilog paper. The plot gave a straight line with negative slope; as is indicated to be the case by Eq. (35). From this straight line the $^{10}$B thickness, $t$, that would reduce $A_{rj}^{i}(0)$ by half, was found. Using then Eq. (18), the energies of the resonances were calculated and compared to the expected resonance energies. They agreed well.

Radiation Partial Width, $\Gamma_{\gamma}$

We define a quantity called the resonance integral for a single resonance in the following way (22).

$$I_{rj}^{i}(t) = \int_{-\Delta E}^{\Delta E} \phi(E)\sigma_{C}(E)e^{-36.8E/t} \, dE$$

(41)
Using the fact that the capture cross section is given by the Breit-Wigner formula, the assumption that the flux at the V.P.I. reactor is an inverse function of energy, and the same assumptions made for Eq. (31), after integrating the previous equation the result, at \( t = 0 \), is:

\[
I_{r_i} = \frac{2\pi^2\lambda^2 r_i}{\pi^2 r_i^2} \frac{r_i}{\Gamma_i} \frac{r_i}{\Gamma_i} \frac{r_i}{\Gamma_i} \frac{e_{r_i}}{n_{\gamma} \Gamma_i} \tag{42}
\]

However, the reduced wave length of the incident neutron with resonance energy can be expressed as follows (4):

\[
\lambda_{r_i} = \frac{\hbar}{2M_{r_i} E_{r_i}}
\]

Thus Eq. (42) can be written in the following manner:

\[
I_{r_i} = \frac{2\pi^2\lambda^2 r_i}{\pi^2 r_i^2} \frac{r_i}{\Gamma_i} \frac{r_i}{\Gamma_i} \frac{r_i}{\Gamma_i} \frac{e_{r_i}}{n_{\gamma} \Gamma_i} \tag{43}
\]

Substituting the proper constants, i.e., \( \pi^2 = 9.870 \), \( M = 1.675 \times 10^{-24} \) gm, and \( \hbar = 1.112 \times 10^{-54} \) (erg-sec)\(^2\) (13), the above expression becomes

\[
I_{r_i} = 6.553 \times 10^{-30} \frac{r_i}{\Gamma_i} \frac{r_i}{\Gamma_i} \frac{e_{r_i}}{n_{\gamma} \Gamma_i} \tag{44}
\]

Remembering that \( \Gamma_i = \Gamma_{r_i} + \Gamma_{\gamma} \), the previous equation can be solved for the radiation partial width, \( \Gamma_{\gamma} \), to give
\[ r_i \gamma = \frac{I \gamma_1 E_{\gamma 1} r_i }{6.553 \times 10^{-30} g_{\gamma 1} r_i n - I \gamma_1 E_{\gamma 1}^2 r_i } \text{ ergs} \quad (45) \]

All quantities can be evaluated in the above equation except \( I_{\gamma_1} \), for which an expression, in terms of known quantities, will be derived in the next section. It should be mentioned that, for low \( z \), it is safe to assume that \( \Gamma_n \gg \Gamma_\gamma \) and thus from Eq. (44) it is derived that

\[ r_i \gamma = 1.527 \times 10^{29} \frac{I \gamma_1 E_{\gamma 1}^2 r_i }{g_{\gamma 1}} \text{ ergs.} \]

or in eV

\[ r_i \gamma = 2.435 \times 10^{17} \frac{I \gamma_1 E_{\gamma 1}^2 r_i }{g_{\gamma 1}} \text{ eV.} \]

Normalization and Resonance Integral, \( I_{\gamma_1} \)

In normalizing the data, the following equation was used (22)

\[ r^x = \left( \frac{A_\gamma}{A_{th}} \right)^x \left( \frac{A_{th}}{A_\gamma} \right)^s \quad (46) \]

where \( A_\gamma \) represents the activity of the sample due to its activation by resonance energy neutrons and \( A_{th} \) is its activity due to its activation by thermal neutrons; \( x \) stands for the sample to be analyzed and \( s \) for the standard one, which in this case, is gold. Looking at Eqs. (33) and (42), we can say that, at \( t = 0 \), the following equation
holds true,

$$A_x^r = E_o \phi(E_o) N_I^r$$  \hspace{1cm} (47)

It should be recalled that the activation (and thus the activity) of the sample due to its absorption of thermal neutrons can be expressed as follows:

$$A_{th}^s = N_{th}^s \phi_{th}^s$$  \hspace{1cm} (48)

Substituting Eqs. (48) and (47) into Eq. (46), the following result is obtained

$$I_x^r = \frac{E_o \phi(E_o) N_I^r}{N_{th}^s \phi_{th}^s} \cdot \frac{N_{th}^s \phi_{th}^s}{E_o \phi(E_o) N_I^r} \cdot s = Au$$

$$= \frac{I_r^s}{\sigma_{th}} \cdot \frac{\sigma_{th}}{I_r} \cdot s = Au$$

or

$$I_x^r = r_{th}^x \left( \frac{I_r}{\sigma_{th}} \right)_{Au}$$  \hspace{1cm} (49)

It should be noted that $\phi_{th}^x = \phi_{th}^Au$; for all samples were activated with the same reactor and at the same power. In using Eq. (49), $\sigma_{th}^x$ and $\sigma_{th}^Au$ can be found in tables but $I_{th}^Au$ and $r^x$ (which in turn requires the knowledge of $A_{th}^Au$ to be solved in terms of known $A_{th}^r$ quantities. First, the quantity $I_{th}^Au$ will be determined. In doing so, Eq. (44) will be used, i.e.,
\[
I_{x}^{Au} = 6.553 \times 10^{-30} \left( \frac{g_{n}^{r} r_{n}^{r}}{E_{r}^{r}} \right)_{Au}
\]  

(50)

Using the values \(E_{r}^{r} = (6.178 \pm 0.023) \times 10^{-23}\) erg, \(\Gamma_{y} = (1.987 \pm 0.032) \times 10^{-13}\) erg, \(\Gamma_{n} = (2.499 \pm 0.065) \times 10^{-14}\) erg and \(g = 6.25 \times 10^{-1}\) (21) into the above equation, then the resonance integral of gold is found to be:

\[
I_{r}^{Au} = 1,472 \pm 46 \text{ barns}
\]  

(51)

Compared with \(1533 \pm 40\) b as given by Jirlov et al., (15). Applying 98.8 barns for the thermal cross section of gold and 1,472 b for its resonance integral (as found in Eq. (51)) into Eq. (49), the following result is obtained.

\[
I_{r}^{x} = 14.9 \, r_{x}^{x} \sigma_{th}^{x}
\]  

(52)

In determining \(r_{x}^{x}\) in the above expression, Eq. (46) is recalled where

\[
r_{x}^{x} = \left( \frac{A_{x}^{r}}{A_{th}^{r}} \right)_{x} \left( \frac{A_{th}^{r}}{A_{x}^{r}} \right)_{Au}
\]  

(53)

The first thing that will be evaluated for the above equation is \(\frac{A_{th}}{A_{x}^{r}}\) . In doing so, the term cadmium ratio, Cd R, is defined as the total activity of the sample, \(A_{T} = A_{th} + A_{\nu/V} + A_{r}^{e}\), to its activity due to the activation by epithermal and resonance energy neutrons, \(A_{e} = A_{\nu/V} + A_{r}^{e}\). That is
\[
Cd R = \frac{A_T}{A_e} = \frac{A_{th} + A_{1/v} + A_r^e}{A_{1/v} + A_r^e} = 1 + \frac{A_{th}}{A_{1/v} + A_r^e}
\] (54)

The superscript \(e\) on the \(A_r\) denotes the experimental value of the activity of the sample due to its absorption of resonance energy neutrons. As explained in the section "Self Shielding" the experimental value of the activity, due to absorption of resonance energy neutrons, \(A_r^e\), is a fraction, \(f\), of the theoretical, \(A_r^0\). That is

\[
A_r^e = f A_r^0
\] (55)

Similarly, for the quantity \(K\) (defined already in the section "Analysis of the Activity of the Samples") we have

\[
K^e = iK^0 = \frac{f A_r^0}{A_{1/v}}
\]

or

\[
A_{1/v} = \frac{A_r^0}{K^0}
\] (56)

Substituting Eqs. (55) and (56) into Eq. (54), the following expression is obtained

\[
Cd R = 1 + \frac{A_{th}}{A_r^0 \left( f + \frac{1}{K^0} \right)}
\]

or
\[
\frac{A_{\text{th}}}{\frac{A_0^\theta}{r}} = (C_0 \, R - 1) \left( f + \frac{1}{K^\theta} \right) \tag{57}
\]

For gold, we have the following experimental values.

\[
C_0 \, R = 3.83
\]

\[
f = 0.530
\]

By the use of Eq. (37b), it is obtained that \( K^\theta_{\text{Au}} = 44.7 \). Thus

\[
\frac{A_{\text{th}}}{\frac{A_0^\theta}{A_r^\theta}}_{\text{Au}} = 1.56 \tag{58}
\]

Substituting Eqs. (55) and (58) into Eq. (53), the following result is obtained

\[
r^x = 1.56 \, \frac{\frac{A_r^\theta}{fA_{\text{th}}^x}}{A_{\text{th}}}
\tag{59}
\]

Putting the previous expression for \( r^x \) into Eq. (52), the result is

\[
I^x_r = 23.2 \, \frac{\frac{A_r^\theta}{fA_{\text{th}}^x}}{A_{\text{th}}^{\text{th}}}
\tag{60}
\]

In this last equation, \( A_{\text{th}} \) is the subcadmium activity and \( A_r^\theta \) is the zero intercept of the resonance components of the \( ^{10}B \) activation curve. Thus, the resonance integral of the element \( x \), expressed in the above equation, is in terms that can be calculated, determined experimentally or can be found in tables.
RESULTS AND CONCLUSIONS

The results of this work are shown in graphs three through seven and on Table V. Looking at the graphs of $^{59}\text{Co}$ and $^{55}\text{Mn}$, it can be seen that there is extra activity (indicated as Residual Data) which is attributed to higher energy resonances which have attenuation curves with nearly the same slope and hence can not be resolved. This activity of the higher or residual resonances was subtracted before the analysis of the data was performed. The resonance integral of the residual resonances of $^{55}\text{Mn}$ and $^{59}\text{Co}$ were calculated to be $0.053 \pm 0.003\text{ b}$ and $0.074 \pm 0.007\text{ b}$, respectively. It should be observed, in graphs six and seven that the sum of the activities of all the resolvable resonances is indicated by the solid-line-curve and to obtain the original data (indicated by circles) the residual data had to be added. In analyzing the collected data, it was necessary to assume the existence of a certain number of resolvable resonances and then proceed to verify this assumption through the use of theory and experimental information. As can be seen from the graphs, the experimental data can be analyzed to give two resolvable resonances for sodium and cobalt, one for magnesium and potassium and three for manganese.

Columns three and four in Table V give the single resonance integrals, given by Eq. (60), and the radiation partial widths, obtained by Eq. (45), respectively. Comparing the results of this work with those of other researchers, a close agreement, for some of them, is observed. For example, it was found in this work that the $\Gamma_Y$
for the 0.132 keV resonance of $^{59}$Co to be 0.468 $\pm$ 0.047 eV compared to 0.45 $\pm$ 0.05 eV recommended in BNL 325 (20). For a further check, the $\Gamma_\gamma$'s evaluated in this work were used in the Breit-Wigner one level formula (4) and the thermal absorption cross section for $^{59}$Co was evaluated. The value obtained (added to it was an estimate of the contribution of the higher energy resonances which could not be resolved in this work) was 34 $\pm$ 3 b compared to 37.2 $\pm$ 0.6 b found by other experimental methods and given by BNL 325 (20). To derive the above mentioned values of $\Gamma_\gamma$'s, the single resonance integrals had to be used. The single resonance integrals found in this work add up to 57.0 $\pm$ 5 b (for the resolvable resonances) compared to the total resonance integral of 72.3 $\pm$ 5 b given by Dahlenberg (21).

The values of the $\Gamma_\gamma$'s for the 0.337 keV and 1.098 keV resonances of $^{55}$Mn given in BNL 325 (20) are - 0.5 eV and - 0.6 eV respectively; compared to 0.293 $\pm$ 0.015 eV and 0.780 $\pm$ 0.031 eV found in this work. Again, by using the $\Gamma_\gamma$'s found in this work into the Breit-Wigner formula a check was made to see whether the thermal absorption cross section found in this manner was compatible with that derived through other experimental methods. The results of this work gave us 9.80 b for the absorption cross section for the three resolvable resonances compared to 13.3 b for the total absorption cross section given in BNL 325 (20). The single resonance integrals for the resolvable resonances add up to 6.33 $\pm$ 0.24 b; which is quite low compared to 15.6 $\pm$ 0.6 b given by Feiner (21) in an unpublished work. It should be mentioned that if one takes Feiner's answer and proceeds to evaluate the thermal absorption cross section, the result will be close to 28b;
far above the \(13.3\) b found by other experimental methods and given in BNL 325 (20).

The author of this work could not find any published values for comparison for the single resonance integrals and radiation partial widths for the potassium and magnesium resonances. By using the \(\Gamma_Y\) evaluated here for \(^{41}\text{K}\) into the Breit-Wigner formula, it was found that the thermal absorption cross-section for the single resolvable resonance was 0.164 b; which is far below the value of 1.1 b given as the result of other experimental methods for the total absorption cross section of \(^{41}\text{K}\). It should be mentioned that the \(\Gamma_n\) of \(^{26}\text{Mg}\) given in the literature (18, 19) as being \(= 50\) keV gave the activity due to the 305 keV resonance to be far too low from what was obtained experimentally in the present work. If it were assumed that the total activity was due to more than one resonance, then the "1/\(\nu\)" contribution of these resonances would have been even higher than that due to one resonance and relative to the one resonance contribution the "1/\(\nu\)" contribution is already too small. It was thus decided to take the experimental value of the resonance activity and proceed to evaluate the \(\Gamma_n\) of \(^{26}\text{Mg}\). To accomplish this, Eq. (37b) was used; i.e.,

\[
K(0) = \frac{\pi E_r^{1/2}}{\Gamma_r \left(1.58 - \frac{1}{\sqrt{E_r}} - \frac{2\Gamma}{\Gamma_r}\right)}
\]

Making the assumption that for light elements \(\Gamma \approx \Gamma_n\) (4) and also \(1.58 \frac{\Gamma}{\Gamma_n} \sqrt{E_r} - 2\Gamma_n >> 1\), it follows from the above equation that
\[ \Gamma_n = \frac{\pi E \Gamma_i^{1/2}}{1.58 K(0)} \]

Making use of the previous equation, the value of \( \Gamma_n \) for \(^{26}\text{Mg}\) was found to be \( 6.95 \pm 0.18 \text{ keV} \). Using this value of \( \Gamma_n \), the value of \( \Gamma_\gamma \) was evaluated to be \( 53.1 \pm 1.3 \text{ eV} \). To check these values of \( \Gamma_n \) and \( \Gamma_\gamma \), they were used with the Breit-Wigner formula to evaluate the absorption cross section. The result obtained for the absorption cross section was \( 0.029 \text{ b} \); remarkably close to \( 0.03 \text{ b} \) given in BNL 325.

The values of resonance integrals obtained in this work for the 2.85 keV and 54.0 keV resonances of sodium were \( 73.9 \pm 3.7 \text{ mb} \) and \( 1.64 \pm 0.08 \text{ mb} \), respectively; compared to \( 78.1 \text{ mb} \) and \( 10.0 \text{ mb} \) found by Salaita (22). Stephenson (20b) has evaluated the resonance integral for the 2.85 keV resonance to be \( 72 \text{ mb} \) while Bauman (16) has found it to be \( 75 \pm 10 \text{ mb} \). The radiation partial widths reported by Salaita (22) are \( 0.42 \text{ eV} \) and \( 3.92 \text{ eV} \) compared to \( 0.392 \pm 0.031 \text{ eV} \) and \( 1.34 \pm 0.11 \text{ eV} \) obtained in this work for the 2.85 keV and 54.0 keV resonances, respectively. However, Salaita's work was done at a reactor power of 10 KW instead of 100 KW which this work was performed and his data were much more scattered. Lynn, et al. (17b) estimated the radiation width, \( \Gamma_\gamma \), to be \( 0.34 \text{ eV} \), assuming that the thermal cross section is due entirely to the 2.85 keV resonance. Hockenbury, et al., (14) gave \( 0.61 \text{ eV} \) as the value of \( \Gamma_\gamma \) for the first resonance of \(^{23}\text{Na}\). For the second resonance of \(^{23}\text{Na}\), Hockenbury, et al., gave the value for \( \Gamma_\gamma \) to be \( 1.12 \text{ eV} \). It should be noted, however, that, in the above
mentioned source, the values of the resonance energy, $E_r$, and neutron partial width, $\Gamma_n$, used were 52.2 keV and 700 eV, respectively; compared to 54.0 keV and 800 eV, respectively, used in this work.

To check Hockenbury's results, his values were substituted into the Breit-Wigner equation to determine the thermal absorption cross section. The value of the cross section obtained was ~0.87 b compared to 0.592 b using the values of the $\Gamma_\gamma$'s found in this work. The value of the total cross section of $^{23}$Na given in BNL 325 is 0.534 b. Thus, in comparison, Hockenbury's values for the $\Gamma_\gamma$'s of $^{23}$Na would yield a significantly higher value of $\sigma_{th}$ than the experimental value where those obtained in this work are within experimental error.
GRAPH 4 ACTIVATION CURVES FOR $^{12}$Mg$^{2+}$

![Graph showing activation curves for $^{12}$Mg$^{2+}$]
Graph 5: Activation Curve for $^5K^{2+}$

Counts

$5B^{10} (gm/cm^2)$

5.4 keV

Original Data

$10^4$

$10^3$

0 1.0 2.0 3.0 4.0 5.0
GRAPH 6 ACTIVATION CURVES FOR $^{23}$Mn$^{55}$

- SUM OF 0.337 KEV, 1.08 KEV, AND 2.38 KEV RESONANCES
- ORIGINAL DATA
- RESIDUAL DATA

COUNTS vs. $^{5}$B$^{10}$ (gm/cm$^2$)
GRAPH 7 ACTIVATION CURVES FOR $^{56}_{27}$CO

COUNTS

$10^5$

$10^4$

$10^3$

$10^2$

$10^1$

0.132 KEV

SUM OF 0.132 KEV AND 5.01 KEV RESONANCES

ORIGIANL DATA

5.01 KEV RESIDUAL DATA

$\sigma_{10}^{56}_{5B} (gm/cm^2)$

0 1.0 2.0 3.0 4.0 5.0 6.0 7.0
<table>
<thead>
<tr>
<th>Isotope</th>
<th>Resonance Energy $E_r$</th>
<th>Resonance Integral $I_r$</th>
<th>Radiation Partial Width $\Gamma_y$</th>
<th>Cadmium Ratio Cd R</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{11}\text{Na}^{23}$</td>
<td>2.85 ± 0.05 keV</td>
<td>73.9 ± 3.7 mb</td>
<td>0.392 ± 0.031 ev</td>
<td>37.8 ± 0.1</td>
</tr>
<tr>
<td></td>
<td>54.0 ± 0.3 keV</td>
<td>1.64 ± 0.08 mb</td>
<td>1.34 ± 0.11 ev</td>
<td></td>
</tr>
<tr>
<td>$^{12}\text{Mg}^{26}$</td>
<td>305 ± 3 keV</td>
<td>2.32 ± 0.06 mb</td>
<td>53.1 ± 1.3 ev; $\Gamma_n = 6.95 ± 0.18$ keV</td>
<td>42.0 ± 0.6</td>
</tr>
<tr>
<td>$^{19}\text{K}^{41}$</td>
<td>5.4 ± 0.1 keV</td>
<td>0.301 ± 0.023 b</td>
<td>4.89 ± 0.43 ev</td>
<td>25.0 ± 1</td>
</tr>
<tr>
<td>$^{25}\text{Mn}^{55}$</td>
<td>0.337 ± 0.001 keV</td>
<td>4.34 ± 0.17 b</td>
<td>0.293 ± 0.011 ev</td>
<td>24.4 ± 0.05</td>
</tr>
<tr>
<td></td>
<td>1.098 ± 0.02 keV</td>
<td>1.47 ± 0.05 b</td>
<td>0.780 ± 0.031 ev</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.38 ± 0.01 keV</td>
<td>0.467 ± 0.018 b</td>
<td>1.11 ± 0.04 ev</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Residual Resonances</td>
<td>0.053 ± 0.003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{27}\text{Co}^{59}$</td>
<td>0.132 ± 0.001 keV</td>
<td>56.7 ± 5.3 b</td>
<td>0.468 ± 0.047 ev</td>
<td>22.2 ± 0.04</td>
</tr>
<tr>
<td></td>
<td>5.01 ± 0.01* keV</td>
<td>0.244 ± 0.022 b</td>
<td>3.45 ± 0.31 ev</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Residual Resonances</td>
<td>0.074 ± 0.007 b</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Contains effects of the 4.32 keV resonance.
BIBLIOGRAPHY


17. Lindsay, Gordon, private communication.


VITA

Elias P. Stergakos was born in Potamia, Lakonia, Greece on February 20, 1942. He attended public school in Potamia and had approximately two years of gymnasium in Sparta and Athens. On May 29, 1956 he entered the United States as an immigrant. On the same year he entered public school in Hempstead, New York and then continued into Hempstead High School from where he graduated in June, 1962. He entered Adelphi University in 1962 and received his B.A. on January of 1966. On the same month of that year he entered Virginia Polytechnic Institute for his Master of Science in Physics and received it on June of 1968. He, then, continued for his Ph.D. in Nuclear Science and Engineering. During his graduate study, he worked under teaching and research assistantships granted to him by the Physics Department.

Elias P. Stergakos

53
STUDIES OF RESONANCES IN 
$^{23}\text{Na}$, $^{26}\text{Mg}$, $^{41}\text{K}$, $^{55}\text{Mn}$ and $^{59}\text{Co}$

by

Elias Panagiotis Stergakos

ABSTRACT

The isotopes $^{23}\text{Na}$, $^{26}\text{Mg}$, $^{41}\text{K}$, $^{55}\text{Mn}$ and $^{59}\text{Co}$ were activated by the use of a well collimated, $^{10}\text{B}$ filtered neutron beam from a reactor. An analysis of the induced activity versus $^{10}\text{B}$ filter thickness permitted studies of individual resonances of the target elements. With the assumption of a "1/E" energy dependence for the neutron flux in the beam, it was possible to obtain resonance integrals and partial widths for radiative capture for the individual resonances. The results were normalized to the resonance integral and thermal cross section of gold.

Little work of this character has been done previously for these nuclides so that comparative results are not generally available. However, those values that were in the literature, did agree reasonably well with the results gotten by this author. A consistency check obtained by using the values calculated in this work to compute the thermal absorption cross sections showed that the computed cross sections were smaller than or equal to the tabulated absorption cross sections of the elements. Since additional contributions from other resonances would add on to the calculated values, the fact that the tabulated thermal cross sections were not exceeded is in the correct direction.