ESSAYS ON INFORMATION GATHERING IN PRINCIPAL - AGENT CONTRACTS

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(ABSTRACT)

This dissertation is a collection of essays on principal-agent contracts under asymmetry of information. The papers investigate how the possibility of acquiring information influences contracts.

The first essay analyzes the contract between a principal and an agent, when the principal can conduct an audit of the agent's cost of production. The principal can choose an audit policy after output is produced – but he cannot commit to an audit policy at the begining. The probability of audit is a best reply to the agent's probability of misreporting given the contract. The interaction between the contract, the audit strategy and the reporting strategy is analyzed. The main result obtained is that, when the cost of production is high the optimal contract requires the agent to produce an amount greater than the output under full information. The principal audits randomly and truthful cost announcements cannot be induced with certainty. It is also shown that the principal audits with a higher probability when he cannot commit as compared to when he can.

The second essay considers an effort monitoring problem. It analyzes the contract the principal will offer an agent when the monitoring strategy cannot be committed to. Given the contract, the monitoring strategy is a best reply to the agent's effort strategy. The interaction between the contract, the monitoring strategy and the effort strategy is analyzed. The source of

the principal's gain from monitoring is explained. It is shown that the wage payments to the agent may be decreasing in the outcome of the agent's effort.

The third essay endogenizes the amount of information the agent will rely on when deciding whether or not to accept the contract. By incurring an observation cost, the agent can observe the state of nature after the contract is offered. If he does so he will be able to turn it down whenever his payoff is negative. It is shown that the principal will always find it in his best interest to offer a contract such that the agent has no incentive to use his ability to observe the state of nature. Furthermore, an increase in the cost of observation is very valuable to the principal. The paper also looks at the case in which the principal is allowed to put several agents in competition for the contract. It is shown that, though the principal has monopoly power and can force the single agent to his reservation utility, having several agents compete for the contract increases the principal's payoff.

To

Amma and Abba

and

Dada who started it all !

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Chapter 1

Principal-Agent Contracts with Monitoring: A Survey

1. INTRODUCTION AND SCOPE

There are many interesting problems where incentive schemes are used and monitoring of economic agents occurs. For example in the case of fire insurance the insurance company faces the following dilemma: If it provides insurance to the homeowner, then the homeowner may not take appropriate precautions any more. This problem is resolved by making the insurance contract an incentive scheme, which motivates the homeowner to be cautious — this is one explanation for deductibles in insurance contracts. The problem of a life insurance company is somewhat different: It does not know how risky it is to insure a particular person – the person could be critically ill and appear normal. What should be the appropriate benefit-premium package (incentive scheme) that will induce insurees to reveal their current state of health ? In both cases monitoring is often used: fire insurance claims are investigated to check if 'due care' was taken and companies follow up on medical records to find out more about insurees.

Incentive schemes and monitoring have interested economists for a long time, but the literature on contracts with monitoring is fairly recent. It was not until the early seventies that formal insights were gained, about how asymmetry of information alters economic behaviour. Asymmetry of information lies at the heart of the monitoring problem — in the absence of asymmetric information there would be no need for monitoring. Under symmetric information, contracts can be made contingent upon uncertain events without loss of welfare. However, since contracts have to be conditioned on information observed by all parties to the contract, information asymmetry can limit the scope of contingent contracts. Verification or monitoring of information that was previously unobserved by one of the parties to a contract, can improve the contract.

Situations involving two basic sources of asymmetry of information have been analyzed in the literature on contracts with monitoring. It is always the agent who has superior information. In one case, the agent knows some parameter important to the environment and the principal does not: the insuree's health for example in the case of life insurance. In the other case, the agent can take an 'unobservable' action which influences the outcome, but cannot be inferred from the outcome: 'due care' in the example of fire insurance. The first is the case of hidden information or adverse selection, and the second is the case of hidden action or moral hazard. It was typical in the early models to focus only on one kind of asymmetry. Models motivated by managerial accounting issues of motivation would focus on hidden action, whereas hidden information was central to the models that focused on designing a regulatory mechanism for a monopolist with unknown costs. Later on, authors have combined both adverse selection and moral hazard in the same framework and have studied the interaction between the two. I do not want to press the issues of differences between adverse selection and hidden information or the differences between hidden action and moral hazard. Quoting Kreps(1990) (footnote in page 578) "………you should pay less attention to labels and more attention to the "rules of the game", the author specifies—who knows what when, who does what when."

The literature has used both costless and costly monitoring devices. Costless monitoring leads to a trivial monitoring strategy and we learn very little about monitoring schemes. But this allows us to focus on other issues and still have the use of a verification technology. Models which emphasize the monitoring strategy itself must therefore, have costly verification.

I have restricted this survey to contracts which model situations with production and monitoring. Surveys of the literature on principal-agent contracts can also be found in Baiman (1982), Besanko and Sappington (1987) and Caillaud et al (1988). I have divided papers into three main categories: Section 2 contains papers mainly concerned with hidden action problems; Section 3 discusses papers where adverse selection is the only issue; and Section 4 has papers which treat hidden action and hidden information problems simultaneously. A classification of the papers is provided below. The classification also indicates whether the models use costly monitoring and which variable is monitored in each model. I have cross referenced some papers, in which case I provide the section where the paper is discussed.

	Monitoring	Variable Monitored
Hidden Action		
1. Baiman and Demski (1980)	costly	action
2. Dye (1986)	costly	action
3. Evans (1980)	costly	action and state
4. Harris and Raviv (1979)	costless	action
5. Holmstrom (1979)	costless	action
6. Jewitt (1988)	costly	action
7. Kanodia (1985)	costly	action and state
8. Mukherjee and Png (1989)	costly	outcome report
9. Shavell (1979 a, b)	costless	action
Hidden Information		
1. Baron (1984)	costly	state
2. Baron and Besanko (1984)	costly	state
3. Demski, Sappington and Spiller (1986)	costly	state
4. Townsend (1979)*	costly	state
Hidden Action and Hidden Information		
1. Baron and Besanko (1987, 1988)	costless	joint signal
2. Harris and Raviv (1979)*	costless	action
3. Kanodia (1985)*	costly	action and state
4. Kumar (1989)	costly	action and state
5. Laffont and Tirole (1986)	costless	joint signal
6. West (1991)	costless	joint signal

* These papers are discussed under hidden action in section 2.

2. HIDDEN ACTION

This section discusses papers which are mainly concerned with moral hazard issues. For the models in this section, the typical story is the following: a risk neutral principal (entrepreneur) must hire a risk averse agent (manager) to run his firm; the outcome (e.g., profit or just output) depends on the agent's effort, and also on other factors modelled as a random component. Higher effort results in better outcomes for the principal, but the agent dislikes effort.

The first set of papers have costless and imperfect monitoring and the second set of papers deal with costly, perfect or imperfect monitoring. If the variable monitored is revealed with certainty or without error, then the monitor is perfect; if the monitoring information contains error, then it is imperfect. Costless monitoring has to be imperfect, because it can be carried out all the time; if it were perfect, the contingent contracts could be written directly on the "unobservable" variable. On the other hand, costly monitoring can be either perfect or imperfect. There are three potential unobservables in this story, the effort of the agent, the random component in the outcome and the outcome itself. Typically, effort or the random component is not observable unless there is ex-post monitoring after the outcome occurs. If the outcome is observable, then the contract is made contingent on outcome. If the outcome is not observable, then the contract depends either on a report about the outcome by the agent or on the transfer, from the agent to the principal, after the outcome occurs.

A contract, in this framework, will contain all the information the agent needs, to compute his expected payoff from different actions. The contract contains a payment schedule to the agent as a function of the observable variable(s). In models with costless monitoring, the monitoring strategy is trivial — always monitor. But, when monitoring is costly, the principal can save resources by investigating with a higher or lower probability contingent on the value of the observable(s). Which means there is a non-trivial monitoring strategy when investigation is costly, and this strategy might be part of the contingent contract. In this case, the contract will include a payment schedule as well as a monitoring schedule.

There are basically two approaches that have been used to analyze the monitoring problem. One is the so called "first-order approach", and the other is based on the work of Grossman and Hart (1983). The early literature on hidden action models has exclusively used the former approach. Given that the principal can commit to what will happen in each contingency, the contract can be designed to induce the agent to take a particular action. The payment schedule and the monitoring schedule can be altered so that a particular action is preferred by the agent. The above observation is fundamental to both approaches. However, the approaches differ in incorporating this observation into the principal's problem of choosing a contract. The first-order approach uses the first-order condition from the agent's maximization problem. Given the contract, the agent computes his best response (chooses an action) by solving a maximization problem. The first order necessary condition from the agent's problem is then inserted as a constraint in the principal's problem. The claim being that the principal can "choose" the agent's action, as long as the contract satisfies the first order condition of the agent's problem. The criticism of course, is that this constraint is not a sufficient condition for the agent's choice of action. If the contract were such that the agent's payoff as a function of effort was concave, then the first order condition would be necessary and sufficient. But, examples can be constructed where the agent's problem is not concave (Grossman and Hart (1983)). Mirlees (1975) had pointed out this problem, but it was still used by many authors, since it made models analytically tractable and provided a good starting point. Grossman and Hart (1983) circumvent the problem associated with the first order approach, by dividing the principal's problem into two parts. First, contracts are chosen which minimize the cost of implementing each action. Second, the benefit from each action is compared with the cost of implementing it, to choose the particular action. The trouble with this approach is that results are very hard to find. Additional conditions have to be imposed to get even the most basic results. Ironically, Jewitt (1988) has shown that the usual conditions that accompany the Grossman-Hart approach, would validate the first-order approach in the first place. Before Jewitt, Rogerson (1985) had provided sufficient conditions under which the first-order approach could be used. But Rogerson had not looked at the monitoring problem using the first-order approach. Jewitt provided sufficient conditions when imperfect monitoring was possible. The results on the sufficient conditions are partially credited to the work of Mirrlees (1975).

There was an outburst of papers (all published in 1979) dealing with the same problem: How to motivate an agent into taking "proper" actions when only the outcome from the action is observable (and the action cannot be inferred by the resulting outcome). In this context, would additional information (monitoring) on action be useful in designing the contract ? All of these models deal with imperfect observability of action, i.e., in addition to the outcome, another variable is observed which is also related to the action. The variable other than the outcome is also called the signal or monitor (for the action taken). In each model, the contract is offered and agreed upon at the outset under symmetric information. In Holmstrom(1979), Shavell (1979a, b) and Harris and Raviv (1979-model 1) the agent takes the action knowing the contract, but not knowing the random component determining output. Then the output and (when available) additional information on effort (action) occur. In Harris and Raviv (1979-model 2), the agent learns the random component before taking the action but after receiving the contract.

The trade-off between risk sharing and providing motivation to the agent is the main issue in these models. If the action could be observed with complete accuracy, then the first best level of effort could be implemented --- provide the agent with full insurance and penalize every effort level other than the first best level of effort. However, when action is not directly observable, providing full insurance is not going to give the agent any motive to put in effort. Therefore, the contract must impose some risk on the agent to induce the agent to put in effort. If a signal on effort is available (other than the outcome), then the contract could be based on both signal and outcome. But, if the signal on effort is imperfect, then basing contracts on that information poses additional risk on the agent. It is not obvious that this additional information is going to be useful. In this context 'useful' means that use of this information results in a Pareto improvement. This problem was first posed by Harris and Raviv (1976, 1978), and they concluded that the signal would be useful only under limited conditions. But later, Harris and Raviv (1979) revised their previous claim to show that the signal would always be useful. Holmstrom (1979) and Shavell (1979a, b) also characterize the optimal contract when effort is unobservable. They also conclude that imperfect observability of effort is useful. Holmstrom provides a necessary and sufficient condition for this additional information to be useful, which is referred to as the Holmstrom informativeness condition.

The main conclusions from the first set of models are: (i) effort is less than the first best; (ii) the optimal contract is contingent on both outcome and the monitor or signal; (iii) if the agent were risk neutral, then the additional information would be useless and effort can be induced to be first best.

The early models of Holmstrom (1979), Shavell (1979a, b) and Harris and Raviv (1979) proved that there is a demand for imperfect monitoring of action. In their analysis, the additional information was costlessly available. Therefore, the question of a monitoring strategy was never investigated. Baiman and Demski (1980), Evans (1980), Kanodia (1985), Dye (1986) and Mukerjee and Png (1989) have investigated monitoring strategies by introducing costly monitoring. In all of these models, monitoring is perfect. It reveals the variables under investigation with certainty. In each of the papers, the principal can commit to the monitoring strategy, i.e., the probability of monitoring contingent on outcomes can be made part of the contract. If the principal can include the monitoring strategy in the contract, then the agent's best response (choice of action) can be computed. Therefore, when the principal's turn to investigate comes he already knows the level of effort that has been put in, and the investigation is totally uninformative. Since monitoring is costly, the principal can save resources by not conducting an uninformative investigation. If the principal can not be relied upon to execute his promised contract even when he would like to renege, then the contract becomes non-credible and will fail to explain equilibrium behaviour. However, reputational considerations are important; if the 'one shot' model is trying to capture a long-term relationship the pre-commitment assumption might be credible.

Baiman and Demski (1980) extended the Holmstrom (1979) model to the case of costly monitoring. They restricted their attention to deterministic investigation strategies; i.e. when investigation is called for, it occurs with certainty. Using a particular utility function, they show that the monitoring strategy is either lower tailed or higher tailed depending on the properties of the agent's utility function. A lower tailed monitoring policy means that all outcomes below a certain outcome (assuming outcomes can be ranked) will trigger an investigation of action with probability one. A higher tailed monitoring policy means that all outcomes above a certain outcome will trigger an investigation of action with certainty. Baiman and Demski (1980) assume the first-order approach to be valid. Jewitt (1988) provided sufficient conditions for the first-order approach to be valid for the monitoring problem. One of the conditions imposes restrictions on the agent's utility function. It turns out that this restriction implies, for the

Baiman and Demski model, that investigation will be lower tailed.

Dye (1986) analyzes the same problem as Baiman and Demski (1980), using the Grossman-Hart (1983) approach. He presents sufficient conditions for which the monitoring policy will be lower tailed. Interestingly, the conditions he finds are the same as those found by Jewitt, to justify the first-order approach. Jewitt (1988, page 1178) points out the difficulty of using the Grossman-Hart approach: "Indeed, to get results one often has to impose strange conditions, and these may sometimes be sufficient for the first-order approach to have been valid in the first place."

Evans (1980) considers a slightly different problem. The agent's effort together with the state of nature determines a monetary outcome. The outcome is not observable by the principal, unless there is an investigation. The agent makes a transfer to the principal and the monitoring strategy is made contingent on this transfer. If there is an investigation, where investigation is deterministic, then both the state of nature and the effort choice of the agent is revealed. This model brings into focus the financial reporting part of many principal-agent relationships. Based on the financial report of the firm, the entrepreneurs may audit (investigate) the firm. And this audit will reveal the effort put in, as well as the true financial situation of the firm (without error). Evans shows that the optimal monitoring strategy will require an audit only when low outcomes occur.

In a paper related to the work of Evans, Townsend (1979) characterized debt contracts as the outcome of a principal-agent relationship with costly monitoring. In Townsend's model there is no productive action, therefore, strictly speaking it falls outside the scope of the current survey. However, it sheds a better light on the problem Evans (1980) looks at. It is also one of the seminal papers with costly monitoring and is the motivation for the important paper written by

Mukerjee and Png(1989). Townsend's analysis is motivated by the empirical observation of non-state contingent contracts between economic agents. He cites examples of deductibles in insurance policies and debt contracts. A debt contract has the following characteristics : (i) above a cut-off level of income (of the agent), the principal is paid a constant amount; (ii) below the cut-off level the agent declares bankruptcy and the principal tries to recover his debt as much as possible — the agent is protected by limited liability so he gets a constant consumption below the cut-off level of income. In Townsend's model a risk neutral principal sets up an insurance contract with a privately informed risk averse agent. The agent privately observes his income realisation, and reports it to the principal. The principal can investigate the agent's report and learn the true income. Investigation is costly and deterministic. Townsend shows that the optimal contract involves investigating all income reports below a cut-off level of income, and no investigation above the cut-off level. Thus, the agent must pay a constant sum to the principal above the cut-off, and receive a constant sum below the cut-off. These contracts resemble debt contracts, if investigation is identified with bankruptcy.

Mukherjee and Png (1989) have extended Townsend's (1979) model in two directions. They introduce moral hazard by making the probability of income realizations depend on effort put in by the agent, and they allow for random monitoring. The agent is privately informed of the income realisation and makes an income report to the principal. The principal can investigate this income of the agent, but never the effort applied. Mukherjee and Png use the Grossman-Hart (1983) approach and show that (i) investigation must be random; and (ii) debt contracts are not optimal, i.e. consumption depends on the income state. Therefore, they point out that Townsend's explanation of debt contracts is not valid in the presence of moral hazard and random state verification. They also show that between any two transfers, the higher one will be associated with a lower probability of investigation. But, the general structure of transfers or the monitoring strategies are not characterized.

Finally, Kanodia (1985) discusses a situation when the production technology is deterministic. After the contract is agreed upon, the agent observes the state of nature (a productivity parameter). With the knowledge of the state of nature, he chooses an effort level which results in an observable outcome (in a deterministic manner). The principal can investigate (investigation can be random) any outcome and find out the actual effort level and therefore, the state of nature too. The only randomness faced by the agent when choosing the action is that imposed by the monitoring strategy. He uses the Grossman-Hart approach to show that (i) the optimal output schedule is flatter than the one under full information (i.e., effort is greater than the full information effort for less favourable states and lower for more favourable states); and (ii) higher output is investigated with lower probability. These are very interesting results, but are obtained under a rather troubling condition: the agent is paid a constant wage regardless of output. In this model, there is a one to one correspondence between effort and output. Thus, he emphasises the relationship between monitoring and the effort schedule (or equivalently the output schedule), with wages held constant.

To conclude this section on costly monitoring under moral hazard, let me collect the main contributions and point out something that is obviously missing. Sufficient conditions have been identified under which deterministic monitoring is optimal in the basic model. But, in other models, it has been shown that random investigation is optimal. The contracts consist of two parts — the monitoring schedule and the wage or transfer schedule. While the monitoring schedule has received a lot of attention, little is known about the other part of the contract. Even basic monotonicity properties have not been established (other than in a special case by Mukherjee and Png (1989)). Our understanding of how the rest of the contract influences the monitoring scheme is also quite shallow.

3. HIDDEN INFORMATION

The main difference between sections 2 and 3 is that models in this section do not explicitly contain any action taken by the agent. Moreover, contracts are signed between asymmetrically informed economic agents. The typical story for the asymmetry of information is: the principal hires an agent who is experienced in a particular task. Therefore, the agent knows more about the environment then the principal. In particular, the agent may know some parameter which determines the cost of production, and the principal does not. The asymmetry is captured in a single parameter whose realized value the agent knows, whereas the principal only knows its distribution. In the simplest case the parameter is the marginal cost itself.

All the papers in this section, though generalizable to other settings, are presented in a regulatory environment. A risk neutral regulator (principal) has to design a regulatory policy (contract) for a risk neutral monopolist (agent) who has superior information about the cost of production. With the knowledge of the cost parameter the monopolist decides whether to accept the policy. If he accepts the policy, he makes an announcement of the cost parameter. Contingent on the announcement, the policy assigns a particular price-tax combination for the monopolist. The demand curve is publicly known, therefore assigning a price is equivalent to assigning an output.

In this framework, an investigation takes the form of an audit of the true cost of production. The principal can commit to the investigation strategy in the contract. Given the ability to commit, the analysis can be restricted to incentive compatible contracts which leave the agent no incentive to misrepresent cost. The criticism made earlier about pre-commitment to monitoring strategy applies here also (as in section 2). If the agent is induced into announcing the true cost then the audit will not reveal anything new — thus investigation at this stage is not optimal.

Baron and Myerson (1982) look at a model without any monitoring. But the analysis is so central to understanding contracts under adverse selection, that a discussion on adverse selection contracts is incomplete without it. Moreover, it provides a good benchmark for the contracts which do have monitoring. Baron and Myerson find the optimal regulatory policy for a monopolist who has private information on the cost of production. The policy assigns price and tax schedules to each announcement of the cost parameter. After the policy is agreed upon, the agent makes a cost announcement which is equivalent to choosing a price-tax combination. If there was full information the optimal policy would set marginal benifit of output to the principal equal to marginal cost, and leave the agent with zero rent (his profit is equal to his opportunity profit). The main results obtained from the Baron-Myerson model are: (i) the agent receives rents from private information (in all but the highest cost realization) ; and (ii) marginal benefit of output to the principal is raised above marginal cost to limit rents to the agent (in all but the lowest cost realization). The regulatory policy is designed to discourage the firm from exaggerating cost. It achieves its goal by rewarding reports of low costs and punishing reports of high costs by raising the price (marginal benefit) above marginal cost. These results are quite robust and appear in almost every contract with adverse selection. In particular, they survive the introduction of monitoring into the model as long as the monitoring strategy is made part of the contract.

There are two ways in which monitoring has been introduced into the regulatory framework. The first, used by Baron (1984) and Demski, Sappington and Spiller (1987), has the regulator investigating the cost report of the firm before price is set. Thus, the price of output can be contingent on the outcome of the investigation. The contract enumerates what will happen with or without investigation. Both papers are restricted models with two states of nature. The main results obtained are, (i) only high cost reports are subjected to (random) verification; (ii) price is raised above marginal cost for high cost reports and price is raised even further as a penalty, if investigation reveals cost exaggeration; and (iii) the agent receives rents when cost is low. The intuition carries over from the Baron-Myerson model. High cost reports are verified (randomly) to discourage exaggeration of costs. Price is also raised to penalize high cost reports. A low cost report is encouraged by not distorting price when cost is low and by directly giving rewards in the form of rents.

The second way of introducing monitoring in the regulatory environment is found in Baron and Besanko (1984). This is the classic ex-post monitoring or auditing which occurs after output is produced. In this case, output or prices cannot be based on the outcome of the audit. With the knowledge of the contract, the firm announces costs, which is equivalent to choosing a price-tax combination. When output is realized, the regulator can audit the cost of production. If the audit signals a misreport, then the agent is penalized. The presence of monitoring improves upon the Baron-Myerson contract by reducing the agent's rent from private information and by increasing the efficiency of the contract by allowing the regulatory policy to have marginal benefits (price) closer to marginal costs.

These papers characterize the complete contract — the monitoring strategy as well as the rest of the incentive scheme. They explain the interaction between the monitoring scheme and the rest of the contract. The agent receives rents from private information and output distortion (marginal benefit greater than marginal cost) is created to limit these rents. Monitoring can be used to alleviate, but not to eliminate these distortions.

4. HIDDEN ACTION AND HIDDEN INFORMATION

In most situations of interest, hidden action and hidden information are present simultaneously. The papers discussed in this section recognize this fact in different frameworks. Kumar (1989) has a costly monitoring technology which can reveal both effort and state. The rest of the papers, Laffont and Tirole (1986), Baron and Besanko (1987, 1988) and West (1990) have a costless monitoring technology which reveals a joint signal of action and state. Therefore, these authors focus more on the tradeoff between the moral hazard and the adverse selection problems. The papers other than Kumar (1989) are mostly built on the Baron-Myerson (1982) model, while Kumar (1989) extends the Holmstrom (1979) model.

Laffont and Tirole (1986) motivate their analysis by making the following observation about the Baron-Myerson (1982) model : since the cost of production is not observable, it is borne entirely by the firm; therefore, if the firm can reduce cost by putting in effort, the benefits would also belong to the firm. Thus, effort would be efficient conditional on output. In this framework, if the regulator (principal) had information on total cost, should he use that information to formulate the regulatory policy ? Making the contract contingent on cost observation may distort the effort incentives. In particular, if cost is no longer borne entirely by the firm then effort may no longer be optimal given output. Laffont and Tirole (1986) analyze this problem using a linear cost function where effort reduces the marginal cost. The marginal cost and effort are both private information to the agent. The principal only observes total cost plus an additive error term with zero mean. Since the principal and the agent are both risk-neutral, unlike the hidden action models of section 2, risk sharing is not the issue here. The focus here is the trade-off between inducing effort and inducing truthful reports of cost. Whenever the contract is based on ex-post observables (always associated with monitoring), this tradeoff is likely to be important.

Therefore, Laffont and Tirole's (1986) analysis contribute significantly to our understanding of contracts with monitoring even though the monitoring strategy is trivial in their model. They show that linear incentive schemes are optimal; i.e., the optimal contract has a fixed part and a part based on observed costs. Also, there is partial reimbursement of cost by the principal. The partial reimbursement alleviates the adverse selection problem and allows improvements in the efficiency of the output. Partial reimbursement also induces effort by leaving the agent some of the benefits of effort. Since the agent does not get the entire benefits of effort, effort is less than what the principal would like the agent to put in.

Baron and Besanko (1987, 1988) build on the work of Laffont and Tirole (1986) by introducing risk aversion and a more general cost function. In Laffont and Tirole, it does not make a difference if the cost has a random component or if the monitor is imperfect. But if the agent is risk-averse, then this distinction is important. A higher output results in greater benefits to the agent from cost reducing effort. But, a high output also implies greater rents from private information. Therefore, to tackle the moral hazard problem output should be raised (to induce effort), but that worsens the adverse selection problem. Under a linear cost function, the two effects cancel out and we get marginal cost pricing. If the cost function is linear, effort itself does not influence the adverse selection problem. For more general cost functions effort could increase or decrease rents from private information. In which case the agent may put in more or less effort than the principal would like.

West (1990) analyzes a setting where an initial investment (effort) must be made by a risk-averse agent, before private information about cost (productivity) is generated. Contracting occurs between symmetrically informed parties and the agent chooses effort not knowing the state of nature. Effort reduces the marginal cost of production, and together with the state of nature, determines marginal cost. Therefore, risk-aversion is more emphasized than in Baron and

Besanko (1987, 1988) . Using the same cost function as Baron and Besanko, West (1990) shows that effort will always be less than the level the principal desires. Effort is like a sunk investment, the return to which is higher under low cost realizations. This discourages the agent to overstate cost and therefore, the principal can get away by paying lower rents. However, the agent will not take into account this positive externality to the principal when he chooses effort and thus, effort will be less than what the principal would like.

In Kumar (1989), a risk-neutral principal designs a contract for a risk-averse agent who is informed of a productivity parameter before signing the contract. After the contract is accepted, the agent takes a productive action, which together with the productivity parameter determines output. Kumar uses the first order approach and provides sufficient conditions for this approach to be valid in the presence of asymmetric information. Kumar uses a costly monitoring technology which can investigate either effort or state or both. Monitoring is contingent on outcome realization. He shows that monitoring is lower tailed with respect to outcome. He also shows that the action will be monitored for a report of high productivity and the state will be monitored for a low productivity report.

It is easily perceived that the problem becomes quite complicated when hidden action and information is jointly present. The authors have used various methods to simplify the structure of the problem. Unfortunately, these authors have chosen not to emphasize the monitoring strategy. Only Kumar (1989) has some results on the monitoring scheme. The interaction between monitoring and the rest of the scheme is almost absent. However, important insights about the tradeoff between the hidden information and hidden action problems have been gained. A fixed price contract independent of the realized cost is good for the moral hazard problem but does not address the adverse selection problem. Cost reimbursement takes care of the adverse selection problem but does a poor job with the moral hazard problem. Therefore, the optimal policy must be based on both sources of information, the cost parameter report (fixed price part) and the realized cost (cost reimbursement part).

5. CONCLUSION
(preview of my essays)

During the course of the survey I indicated in several places the inconsistency associated with pre-commitment to monitoring strategies. I have addressed this issue in two separate models. I analyze the designing of an optimal contract, when the principal cannot commit to monitoring strategies. In chapter 2 I look at the optimal contract in an adverse selection framework and I investigate the same issue in a moral hazard context in chapter 3. The contract only contains a wage schedule in the moral hazard problem and a quantity-tax schedule in the adverse selection problem. In response to the contract the agent must choose his announcement or effort strategy while the principal chooses the investigating strategy.

In the adverse selection problem the contracted output is such that the marginal benefit of output is less than the marginal cost, in order to lower the agent's incentive to cheat (overstate cost). Under moral hazard the wage scheme may be non-monotonic in the outcome to motivate the agent to put in high effort. In both setups investigation has to be random and the principal cannot induce the agent either to make truthful cost reports or to put in high effort with certainty.

Taking away the power to commit to monitoring strategy gives the principal-agent relationship an interesting twist. The principal now has two functions, designing the contract and choosing the investigation strategy. The two functions could be thought of as being performed by two different people — a 'three-person' scenario is thus introduced. Since the investigation strategy cannot be committed to, the contract must be designed to induce the (second) principal to investigate optimally. If it is not optimal for the principal to investigate, the agent will not believe that the principal will investigate. The principal can increase the agent's incentive to report truthfully or to put in high effort, only by increasing his own incentive to investigate (when he designs the contract). The three-person story is probably the most useful way to understand the forces in the model. It brings out clearly the interaction between the contract, the monitoring strategy and the agent's response.

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Chapter 2

Commitment and Auditing

1. INTRODUCTION

The literature on mechanism design has studied contracts that a principal will offer an agent who has private information on production costs — the most influential work being that of Baron and Myerson. Baron and Besanko have extended the Baron–Myerson model to the case where the principal can gather information after production, by auditing at a cost. A criticism of this model noted by Baron and Besanko themselves is that the principal is given the power to commit to an audit policy before the output is produced[1]. The question that I ask in this paper is what type of a contract will the principal offer if commitment to audit policy is not credible?

There are many instances when an employer will monitor the work of an employee. For instance, when a sales representative is hired for a region, the employer will in some cases call the serviced customers directly to check on the employee. The contract between the employer and the employee will typically not say anything about the likelihood of monitoring. Some representatives will be found guilty of not working properly and some guilty representatives will go undetected. However, it is standard in models of principal-agent contracts with auditing, that the likelihood of monitoring is contracted upon and contractual terms are never violated in equilibrium. Important departures from the standard results occur if the likelihood of monitoring cannot be contracted upon. The most immediate being, violation of contractual terms in equilibrium.

A standard auditing contract with adverse selection will go as follows. A principal hires an agent with private information on production costs. The agent produces output and bears the cost of production. The contract specifies the output to be produced for each state of nature and the corresponding compensation; the principal can audit after production has occurred. *Commitment to an audit policy means the principal can announce before production occurs, the probability of audit for each level of output.* The principal can therefore include these probabilities in the contract. The optimal contract then induces "truth telling", i.e in equilibrium the agent always produces the output specified for the state of nature that has indeed occurred. This obviates the need to audit since actually performing an audit at this stage will only result in the loss of audit cost. The principal will want to shirk. Even if he is bound by the contract to audit, at least two problems of implementation could appear. One, if the announced probability is positive but less than one, it will be hard to monitor whether the principal is sticking to the contract. Two, if the principal can save on audit cost by lowering audit effort, he will try to do so. Important exceptions to the criticism above could occur if the principal-agent relationship is long term in nature and if there were important reputational considerations at stake. Both of these considerations would be present in the case of a regulator for instance.

In this paper I analyze the contract a principal will offer an agent, who has private information on production cost, when the principal can audit but cannot commit to audit policy. Some recent literature on auditing has also addressed the issue of commitment in other frameworks. Graetz, Reinganum and Wilde(1986) have modeled tax compliance and Banks(1988) has modeled regulatory auditing without commitment. In the tax–compliance game an individual files his income tax report with the I.R.S. which does not know ex–ante what the individual's true income is. The I.R.S. can audit once it receives a report and penalize non—compliers. Individual income is exogenous and the equilibrium gives the reporting and auditing strategies of individuals and the I.R.S. In Banks' model of regulatory auditing, a monopolist announces a price — the regulator does not know the cost of production but can audit and find the cost once price is announced. If an audit occurs the price is set equal to the marginal cost, otherwise the monopolist sells at whatever price he announced. The equilibrium pricing and auditing strategies of the monopolist and the regulator are determined. However, these authors do not use a contractual framework and therefore have not investigated how the terms of the contract change when the principal cannot commit to an audit policy.

I assume that there are only two states of nature, production cost is high in one state and low in the other. The audit reveals the state of nature with certainty. The main result I obtain is: when cost is high the optimal contract requires the agent to produce an amount greater than the full information output for high production cost. In standard models of adverse selection, including the case when commitment to auditing is allowed, production is lower than the full information output when cost is high [2]. Further results that I obtain are: first, the optimal contract does not reduce the probability of cheating or auditing to zero, i.e when costs are low the agent produces the output specified for the high cost state with a positive probability and when the output specified for the high cost state is produced the principal audits with a positive probability; second, the probability of audit is higher when the principal cannot commit compared to when he can; third, as the penalty for cheating becomes very large the contract approaches the full information contract while the probability of cheating and auditing approach zero.

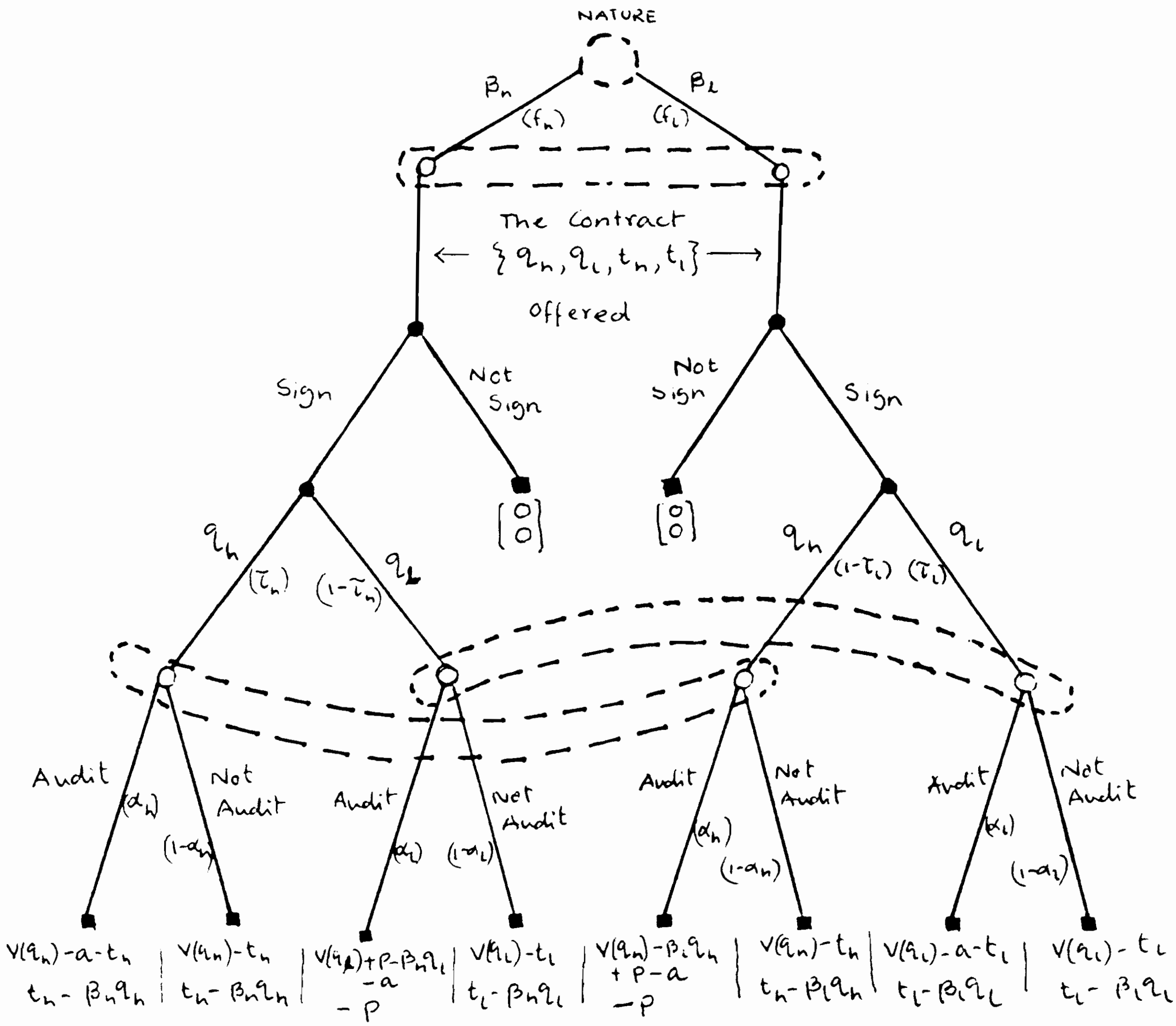
The model is presented in Section 2. The principal's problem is developed in Section 3 and Section 4 contains the solution with the main result. Section 4 also contains comparative statics results when the penalty changes and Section 5 compares the case of auditing with commitment.

2. A MODEL OF AUDITING WITHOUT COMMITMENT

I will start with a brief description of the physical setup, and then present the timing of moves and the information structure with the help of a game tree. Consider a principal who must hire an agent to carry out the production of some good. The value of the output to the principal is measured by a strictly concave function V, where V(q) is the value, measured in dollars, of q units of output with $V(0) = 0$. To ensure some strictly positive but bounded output level I assume that $\lim_{q \to 0} V'(q) = \infty$ and $\lim_{q \to \infty} V'(q) = 0$.

The state of nature determines the marginal cost parameter β and the agent bears the cost βq for producing q units of output. The cost parameter can take only two values, β_h in state 'h' and β_l in state 'l', with $\beta_h > \beta_l > 0$. The state of nature is private information to the agent at the time the contract is signed. The principal can determine the state of nature after the production has occured only by employing a costly audit technology. The cost of an audit is 'a' and the audit reveals the state of nature with certainty [3].

A contract is a vector in $\Re^4$, $\{q_h, q_l, t_h, t_l\}$. The agent is required to produce q_i in the state of nature 'i' and is paid t_i in exchange. If the agent produces q_i in state 'i' then I say he is telling the 'truth'. If he produces $q_{j \neq i}$ in state 'i' then he is 'cheating'. When q_i is produced in state 'i', the agent gets a transfer t_i whether or not the audit takes place. The agent can also



- ○ Principal's Move
- ● Agent's Move
- ◌ Chance Move
- ■ Terminal Nodes with the Principal's Payoff above the Agent's Payoff

GAME TREE INDUCED BY A CONTRACT

Fig 1

produce $q_{j \neq i}$ in the state 'i' and get $t_{j \neq i}$ if the principal does not audit. If there is an audit and it reveals the true state to be 'i' when $q_{j \neq i}$ is produced the agent only receives the true cost $\beta_i q_j$ but pays a penalty 'p' to the principal. I will adhere to the tradition in the auditing literature and keep the penalty exogenous. Baron and Besanko have defended this assumption: according to them, in many cases there are statutory limits on the maximum penalty that can be imposed. They cite as examples the Federal Water Pollution Control Act and the Price-Anderson Act for electric utilities which set liability limits in case of accidents. In my model as in Baron—Besanko, the penalty when applicable, will always equal the maximum allowed.

The game tree in figure 1 summarises the structure of the game. In the first stage the contract is offered and accepted or refused. If the contract is accepted the second stage occurs otherwise the game ends. Due to the asymmetry of information the second stage is equivalent to a simultaneous move game with incomplete information. The two stages are presented below.

Nature has the first move; it reveals the state of nature to the agent. The principal does not know the state but it is common knowledge that he assigns the probabilities f_h and f_l to states 'h' and 'l' occuring. The principal's move is the offer of a contract, $\{q_h, q_l, t_h, t_l\}$. With full information on the state of nature the agent decides whether to sign or refuse the contract. If the contract is signed the game continues to the next stage otherwise it ends at this point and both players get a zero payoff. The agent will sign the contract if his expected payoff from participation is at least as great as his opportunity profit level.

The second stage begins after the contract is signed. The agent has a mixed strategy; in state 'i' he produces q_i with a probability 'τ_i', i.e he tells the 'τ'ruth with a probability 'τ_i'. When q_i is produced the principal does not know if the state is 'h' or 'l' — he does not know if the agent is cheating or telling the truth. From the principal's viewpoint, q_i will be produced with probability $\pi_i = f_i\tau_i + f_j(1-\tau_j)$. The principal also has a mixed strategy; he 'α'udits with a probability 'α_i' when he observes q_i and expects to find the true state to be 'i' with probability $\delta_i = \frac{f_i\tau_i}{\pi_i}$.

In a model with commitment (e.g. Baron and Besanko) the principal can commit to the probabilities of audit in the first stage. These probabilities can therefore be incorporated in the contract, truth telling can be induced and cheating eliminated. The principal would be bound by the contract to carry out an audit according to the announced probability even though the agent never cheats. Of course in the absence of cheating, the principal is not going to want to perform a costly audit. The main modification in my model is that the principal cannot commit to the probabilities of auditing before the output is produced. Each contract can only induce auditing and cheating which are best responses to each other.

The principal collects the output once production has taken place. If there is no audit his payoff is the value of output minus the payment to the agent. The agent bears the cost of production and is paid the transfer in return. If an audit reveals 'truth telling', then the principal receives the output, pays the transfer and occurs audit cost. But, if the audit reveals cheating, the principal only has to pay the 'true' cost of production instead of the transfer, incurs the cost of auditing and collects the penalty. When caught cheating, the agent only bears the penalty. When he is telling the truth, the agent receives the transfer and bears the cost of production.

3. THE PRINCIPAL'S PROBLEM:

How To Choose The Optimal Audit Contract

The principal and the agent are both risk neutral. The principal chooses the contract $\{q_h, q_l, t_h, t_l\}$ to maximize his expected payoff. He must however make sure the agent accepts the contract in each state of nature. Therefore the contract must ensure the agent his opportunity profit level, normalized to zero, in each state of nature.

The principal can calculate his expected payoff for each contract by calculating the induced probabilities. Each contract induces, in the second stage, probabilities of cheating and auditing which are best responses to each other. These in turn determine the probabilities π_i and δ_i. Therefore the problem has to be solved backwards starting with the computation of the probabilities 'α_i' and 'τ_i' as functions of the terms in the contract. The principal can then choose the contract which maximizes his expected payoff. Keeping in mind that the probabilities α_i, τ_i, π_i and δ_i are really functioins of $\{q_h,\ q_l,\ t_h,\ t_l\}$, the principal's problem [PP] is:

$$\max_{q_l, q_h, t_l, t_h} \quad \sum_{i=h,l} \pi_i \Bigg([1-\alpha_i] \Big\{ V(q_i) - t_i \Big\} +$$

$$\alpha_i \Big\{ \delta_i \Big(V(q_i) - a - t_i \Big) + [1-\delta_i] \Big(V(q_i) - \beta_{j \neq i} q_i - a + p \Big) \Big\} \Bigg), \tag{1}$$

subject to

$$\text{IR(h)} \quad \tau_h \Big\{ t_h - \beta_h q_h \Big\} + [1-\tau_h] \Big\{ [1-\alpha_l] \Big(t_l - \beta_h q_l \Big) - \alpha_l \, p \Big\} \geq 0 \tag{2}$$

$$\text{IR(l)} \quad \tau_l \Big\{ t_l - \beta_l q_l \Big\} + [1-\tau_l] \Big\{ [1-\alpha_h] \Big(t_h - \beta_l q_h \Big) - \alpha_h \, p \Big\} \geq 0 \ . \tag{3}$$

The IR(h) and IR(l) are the individual rationality constraints for the two states of nature. These constraints must take into account the possibility of cheating and the possibility of being penalised if caught cheating. In Section V, I briefly present the model above, but allow the principal to commit to audit probabilities. Then the principal's problem includes incentive compatibility constraints which eliminate cheating in equilibrium. But without commitment, cheating and auditing occur as best responses to each other, and then the principal will audit with a positive probability only if there is a positive probability of cheating.

As a benchmark it is useful to look at the contract that will be offered if the principal knew the state of nature. This contract is called the **full information contract** $\{q_h^*, q_l^*, t_h^*, t_l^*\}$ and this contract satisfies the following conditions

$$V'(q_i^*) = \beta_i \qquad i = h, l$$

$$t_i^* - \beta_i q_i^* = 0 \qquad i = h, l$$

The marginal value of output equals the marginal cost and the agent gets a zero payoff. It is worth mentioning that, since there cannot be any cheating there will be no auditing,

$$\alpha_i^* = [1 - \tau_i^*] = 0$$

$$\text{or} \qquad \pi_i^* = f_i .$$

On the other hand when the state of nature is known only to the agent, the principal can still offer the contract $\{q_h^*, q_l^*, t_h^*, t_l^*\}$ but this contract will induce cheating which will create a wedge between π_i^* and f_i. Below, I show that when information is asymmetric the full information contract is not optimal because it induces 'too much' cheating. The task of the principal is to find a contract that lowers cheating without inducing too much auditing.

Depending on the magnitudes of the penalty and the audit costs there can be two types of contracts: contracts which induce a zero probability of audit and **audit contracts** which induce a positive probability of audit. For example, if the penalty is too low compared to the audit cost the principal will not want to audit. He will then offer a contract which induces truth telling and therefore no auditing. Among the contracts that do not induce audit the best contract is the Baron–Myerson contract, $\{q_h{}^b, q_l{}^b, t_h{}^b, t_l{}^b\}$. The Baron–Myerson contract tailored to fit my model is presented in Appendix A. In this contract $\pi_i{}^b = f_i$ for i = h, l; $q_l{}^b = q_l{}^*$; $q_h{}^b < q_h{}^*$; $t_h{}^b = \beta_h q_h{}^b$ and $t_l{}^b > t_l{}^*$. The principal's payoff in the Baron-Myerson contract is strictly less than his payoff when he has full information. Losses come from two sources: 'under' production if the cost is high and 'over' payment if the cost is low.

The focus of my paper is on contracts which induce some audit at the optimum. Therefore in the rest of the paper I assume that the penalty and the audit cost are such that the principal prefers an audit contract over the Baron-Myerson contract. In particular I assume that $p > a$. I show later in Proposition 2.1, that for every audit cost if the penalty is high enough an audit contract will be offered.

It can be proved that in the optimal audit contract the agent will never have an incentive to produce q_l in state 'h' i.e. $\tau_h = 1$ and this implies $\alpha_l = 0$. For expositional purposes I take these conditions as given and derive the optimal audit contract. Then I show that the optimal payoffs indeed induce truth telling in state 'h'. Lemma 1 describes the equilibrium strategies in the second stage game for the optimal contract when there is auditing.

Lemma 1. **An optimal audit contract will induce a unique equilibrium in the second stage game with the following properties: (i) $\alpha_h < 1$ and (ii) $0 < \tau_l < 1$.**

Proof:

(i) If the principal audits everytime q_h is produced then there will be no cheating, but then the principal could reduce audit costs by not auditing.

(ii) I show that $\tau_l > 0$ in appendix B, but the main idea of the proof is provided here. If $\tau_l = 0$ then q_l would never be produced, but then the principal would be better off offering the Baron-Myerson contract. The other side of the inequality $\tau_l < 1$ is proved using $0 < \alpha_h < 1$. The probability of audit will be strictly between zero and one only if the principal's return from not auditing is equal to the expected return from auditing when q_h is produced. Therefore,

$$V(q_h) - t_h = \frac{f_h}{f_h + f_l(1-\tau_l)}\left\{V(q_h) - t_h - a\right\} + \frac{f_l(1-\tau_l)}{f_h + f_l(1-\tau_l)}\left\{V(q_h) - \beta_l q_h - a + p\right\} \tag{4}$$

$$\left.\begin{aligned} &\Rightarrow \quad (1 - \tau_l) = \frac{af_h}{(p + t_h - \beta_l q_h - a)\, f_l} \\ &\Rightarrow \quad \tau_l = \frac{(p + t_h - \beta_l q_h) f_l - a}{(p + t_h - \beta_l q_h - a)\, f_l} < 1 \end{aligned}\right\} \tag{5}$$

Finally, I complete the proof by computing the equilibrium probability α_h. Since $0 < \tau_l < 1$ the agent must be indifferent between producing q_l and q_h in state 'l',

$$t_l - \beta_l q_l = (1 - \alpha_h)(t_h - \beta_l q_h) - \alpha_h p \tag{6}$$

$$\Rightarrow \quad \alpha_h = \frac{(t_h - \beta_l q_h) - (t_l - \beta_l q_l)}{t_h + p - \beta_l q_h} \tag{7}$$

□

By rearranging the terms equation (4) can be written as

$$f_l(1-\tau_l)\left\{\left(V(q_h)-\beta_l q_h - a + p\right)-\left(V(q_h)-t_h\right)\right\}$$

$$= f_h\left\{\left(V(q_h)-t_h\right)-\left(V(q)-t_h-a\right)\right\} \tag{8}$$

The left hand side in equation (8) is the expected gain from detecting cheating when the principal has observed q_h; instead of paying t_h the principal pays the true cost $\beta_l q_h$ and receives p by incurring the audit cost a — therefore the ex-post gain to auditing is $(p + t_h - \beta_l q_h - a)$. The right hand side in equation (8) is the expected cost of finding that the agent has told the truth; the principal incurs the cost of audit but gains nothing and the ex-post cost of auditing is simply a. In equilibrium $(1-\tau_l)$ will equate the expected gain and the expected cost of auditing when q_h is observed. The probability of cheating $(1-\tau_l)$ decreases as the ex-post gain from auditing increases and when the ex-post gain decreases cheating increases. Rearranging the terms equation (6) can be written as

$$\alpha_h\left\{\left(t_l-\beta_l q_l\right)-(-p)\right\}=(1-\alpha_h)\left\{\left(t_h-\beta_l q_h\right)-\left(t_l-\beta_l q_l\right)\right\} \tag{9}$$

The left hand side in equation (9) is the expected cost of getting caught when cheating; instead of obtaining $(t_l-\beta_l q_l)$ the agent must pay p, therefore the ex-post cost of cheating is $\left\{(t_l-\beta_l q_l) + p\right\}$. The right hand side in (9) is the expected gain from going undetected when cheating; instead of receiving $(t_l-\beta_l q_l)$ the agent gets $(t_h-\beta_l q_h)$ — the ex-post gain is $\left\{(t_h-\beta_l q_h) - (t_l-\beta_l q_l)\right\}$. In equilibrium α_h equates the expected gain and the expected cost of cheating when the state 'l' occurs. The probability of auditing increases as the ex-post gain from cheating increases and auditing decreases as the ex-post gain decreases.

I can use lemma 1 to make the principal's problem [PP] much simpler. The objective function of the principal is,

$$\pi_l \left\{ V(q_l) - t_l \right\} + \pi_h \left\{ (1-\alpha_h)\Big(V(q_h)-t_h\Big) + \alpha_h\Big(\frac{f_l(1-\tau_l)}{f_h+f_l(1-\tau_l)} \Big(V(q_h)-\beta_l q_h - a + p\Big) \right.$$

$$\left. + \frac{f_h}{f_h + f_l(1-\tau_l)} \Big(V(q_h) - a - t_h\Big) \Big) \right\}.$$

Using equation (4) this expression can be rewritten as

$$\pi_l \Big(V(q_l) - t_l \Big) + \pi_h \Big(V(q_h) - t_h \Big) \tag{10}$$

The constraints in the problem [PP] are the two individual rationality constraints for the two states of nature. Using the condition $\tau_h = 1$ and equation (6) the constraints in (2) and (3) can be rewritten as

$$\left.\begin{array}{ll} IR(h)' & t_h - \beta_h q_h \geq 0 \\ & \\ IR(l)' & t_l - \beta_l q_l \geq 0 \end{array}\right\} \tag{11}$$

Equation (5) also provides the following probabilities as functions of the terms in the contract,

$$\left.\begin{array}{l} \pi_l = f_l \tau_l = \dfrac{(p + t_h - \beta_l q_h) f_l - a}{p + t_h - \beta_l q_h - a} \\ \\ \pi_h = f_h + f_l\,(1 - \tau_l) = \dfrac{(p + t_h - \beta_l q_h) f_h}{p + t_h - \beta_l q_h - a} \ . \end{array}\right\} \tag{12}$$

Using equations (10) — (12) the the principal's problem can be simplified into [SP],

$$\underset{q_h, q_l, t_h, t_l}{\text{Max}} \quad \pi_h \Big\{V(q_h) - t_h\Big\} + \pi_l \Big\{V(q_l) - t_l\Big\}$$

subject to $\quad$ IR(h)' $\quad t_h - \beta_h q_h \geq 0$

$\qquad\qquad\quad$ IR(l)' $\quad t_l - \beta_l q_l \geq 0$.

Note that the only effect of auditing and penalties is in improving the incentives of the agent. Auditing and penalties affect the principal's objective function only by their effect on the probability of cheating $(1-\tau_l)$. The principal cannot use them directly to extract surplus.

4. THE SOLUTION: Properties of the Optimal Audit Contract

The Lagrangian of the problem [SP] is,

$$\mathcal{L} = \frac{(p + t_h - \beta_l q_h)\, f_h}{p + t_h - \beta_l q_h - a} \left\{ V(q_h) - t_h \right\} + \frac{(p + t_h - \beta_l q_h)\, f_l - a}{p + t_h - \beta_l q_h - a} \left\{ V(q_l) - t_l \right\}$$

$$+ \ \theta_h \left\{ t_h - \beta_h q_h \right\} + \theta_l \left\{ t_l - \beta_l q_l \right\} \qquad (13)$$

where the multipliers θ_h and θ_l are non-negative. It will be shown later that this problem has a unique solution.The 'optimal audit contract' $\{\hat{q}_h, \hat{q}_l, \hat{t}_h, \hat{t}_l\}$ will satisfy the following first order conditions

$$\frac{\partial \mathcal{L}}{\partial q_h} = \frac{(p + t_h - \beta_l q_h)\, f_h}{p + t_h - \beta_l q_h - a} V'(q_h)$$

$$+ \left\{ V(q_h) - t_h - V(q_l) + t_l \right\} \frac{a \beta_l f_h}{(p + t_h - \beta_l q_h - a)^2} - \theta_h \beta_h = 0 , \qquad (14)$$

$$\frac{\partial \mathcal{L}}{\partial q_l} = \frac{(p + t_h - \beta_l q_h)\, f_l - a}{p + t_h - \beta_l q_h - a} V'(q_l) - \theta_l \beta_l = 0 , \qquad (15)$$

$$\frac{\partial \mathcal{L}}{\partial t_l} = - \frac{(p + t_h - \beta_l q_h)\, f_l - a}{p + t_h - \beta_l q_h - a} + \theta_l = 0 , \qquad (16)$$

$$\frac{\partial \mathcal{L}}{\partial t_h} = -\frac{af_h}{(p + t_h - \beta_l q_h - a)^2}\left\{V(q_h) - t_h - V(q_l) + t_l\right\} - \frac{(p + t_h - \beta_l q_h)f_h}{p + t_h - \beta_l q_h - a} + \theta_h = 0 \quad (17)$$

Equation (16) implies that $\hat{\theta}_l = \hat{\pi}_l = f_l\hat{\tau}_l > 0$. Then, it follows from (15) that $\hat{q}_l = q_l^*$. Next, I will show that $\hat{\theta}_h > 0$ by deriving a contradiction when $\hat{\theta}_h = 0$. If $\hat{\theta}_h = 0$, then (14) and (17) together imply $V'(\hat{q}_h) = \beta_l$ or $\hat{q}_h = q_l^*$. But then from (17) it must be true that,

$$p(p-a) + (t_h - \beta_l q_l^*)(p-a)2 + (t_h - \beta_l q_l^*)^2 = 0.$$

This provides the contradiction, since $t_h \geq \beta_h q_l^* > \beta_l q_h^*$ and $p > a$. Therefore, the IR(h)′ and IR(l)′ are both binding and the agent receives no rents regardless of the state of nature. Rents are not the source of divergence of the principal's payoff from his full-information payoff. The sources of divergence are the presence of cheating and the inefficiency in $\hat{q}_h$.

For ease of exposition it was assumed in section 3 that when the cost is high the agent will always tell the truth, that is $\hat{\tau}_h = 1$. The binding individual rationality constraints will be used to show this result. The payoff from telling the truth, producing q_h, is $\hat{t}_h - \beta_h\hat{q}_h = 0$. The expected payoff from cheating is the convex combination of two negative numbers, $\hat{t}_l - \beta_h\hat{q}_l$ and $(-p)$. I will show next the nature of the inefficiency in $\hat{q}_h$.

Theorem: The optimal audit contract sets $\hat{q}_h$ greater than $q_h{}^*$.

Proof I use (17) to replace $\hat{\theta}_h$ in (14) to get,

$$\frac{af_h\,(\beta_h-\beta_l)}{(p+(\beta_h-\beta_l)\hat{q}_h-a)^2}\left\{\left(V(\hat{q}_l)-\beta_l\hat{q}_l\right)-\left(V(\hat{q}_h)-\beta_h\hat{q}_h\right)\right\}$$
$$+\ \frac{(p+(\beta_h-\beta_l)\hat{q}_h)\,f_h}{p+(\beta_h-\beta_l)\hat{q}_h-a}\left\{V'(\hat{q}_h)-\beta_h\right\}=0\,.$$

The expression $\left\{\left(V(\hat{q}_l)-\beta_l\hat{q}_l\right)-\left(V(\hat{q}_h)-\beta_h\hat{q}_h\right)\right\}$ is strictly positive since $\hat{q}_l = q_l{}^*$, $\beta_h > \beta_l$ and V is strictly concave. Therefore, $V'(\hat{q}_h) < \beta_h$. □

This is the main result of the paper. In models of adverse selection, even when commitment to audit probabilities is possible, it is typical to have under production when the cost is high. By taking away commitment this result has been reversed. This somewhat surprising result can be explained through quite intuitive arguments. Three things should be noted about the objective function in [SP]. First, $V(q_l{}^*) - \beta_l q_l{}^* > V(q_h) - \beta_h q_h$ for all q_h. Second, since the constraint $IR(h)'$ is binding, π_h falls and π_l rises when q_h increases; therefore when $\hat{q}_h$ is increased the probability of getting the higher payoff $V(\hat{q}_l) - \beta_l\hat{q}_l$ is increased. Third, since the derivative of $V(q_h)-\beta_h q_h$ w.r.t. q_h is zero at $q_h{}^*$, changes in q_h near $q_h{}^*$ do not lower the value $V(q_h{}^*) - \beta_h q_h{}^*$ by much. Now consider the principal's decision at $q_h{}^*$ when all the other variables are at their optimal levels: if he raises q_h he increases the probability of getting the higher payoff but lowers $V(\hat{q}_h)-\beta_h\hat{q}_h$ —— will the expected payoff increase? Yes, because there is a first-order gain from reducing $\hat{\pi}_h$ at $q_h{}^*$ but at most a second-order loss in $V(\hat{q}_h)-\beta_h\hat{q}_h$. In fact $\hat{q}_h$ will be increased beyond $q_h{}^*$ to equate the gain from lowering $\hat{\pi}_h$ to the loss in $V(\hat{q}_h)-\beta_h\hat{q}_h$.

The principal cannot directly choose the probability of audit. He chooses a contract which induces probabilities of auditing and cheating according to equations (8) and (10). From equation (8) it is clear that cheating will go down only if the ex-post gains from auditing goes up. That is, the agent will believe that the principal will audit, only if the principal creates incentives for himself to audit! Keeping in mind that the $IR(h)'$ is binding, the principal can raise his gains from auditing only by raising $\hat{q}_h$. To summarize, the principal lowers cheating by increasing $\hat{q}_h$, and since the first order gains from increasing $\hat{q}_h$ beyond q_h^* outweighs the second order losses, overproduction occurs in equilibrium.[4]

In other models of adverse selection underproduction typically obtains because incentive compatibility constraints are present. Contracts in these models induce truth telling and the amount the principal has to pay the agent to induce truth telling increases with q_h. Thus by lowering q_h below q_h^* the principal saves on the rents he pays to the agent. See Section V for a brief exposition.

Uniqueness of Equilibrium and Comparative statics:

It is not clear a priori that the objective function in [SP] is concave, therefore I must check if the solution to [SP] is unique. In the first part of this section I show that the solution to [SP] is unique. Then I present some comparative results showing how the optimal audit contract changes as the penalty or the audit cost changes. As long as there is auditing, $\hat{q}_l$ and $\hat{t}_l$ are going to be fixed regardless of changes in the parameters p and a. Given the binding $IR(h)'$, $\hat{t}_h$ is a linear function of $\hat{q}_h$. Therefore the discussion on comparitive statics will mostly be driven by changes in $\hat{q}_h$.

For uniqueness of the solution all I need to show is that at the optimum, q_h can only take one value. This is done in proposition 1. In models where incentive compatibility constraints are present, $\hat{q}_h$ cannot be greater than $\hat{q}_l$. In the present model incentive compatibility constraints are absent and $\hat{q}_h$ is greater than q_h^*, but is it greater than $\hat{q}_l$? It would certainly not violate any feasibility conditions like it would in the case with incentive compatibility. However, the classical result is preserved even without the incentive compatibility constraints present; $\hat{q}_h$ is indeed strictly less than $\hat{q}_l$. This will be proved along with the uniqueness of the optimal q_h. Proposition 1 summarises these results,

Proposition 1: The optimal audit contract is unique and it satisfies $q_h^* < \hat{q}_h < \hat{q}_l$.

Proof: I use (17) to replace θ_h in (14) to get the following

$$\frac{\partial \mathcal{L}}{\partial q_h} = \frac{f_h(p + (\beta_h - \beta_l)q_h)}{p + (\beta_h - \beta_l)q_h - a}\left\{ V'(q_h) - \beta_h \right\}$$

$$+ \quad \frac{a(\beta_h - \beta_l)f_h}{(p + (\beta_h - \beta_l)q_h - a)^2}\left\{V(\hat{q}_l) - \beta_l\hat{q}_l - V(q_h) + \beta_h q_h\right\}$$

$$= \frac{f_h}{(p + (\beta_h - \beta_l)q_h - a)^2}\ \phi\left(q_h; p, a \right) \tag{18}$$

where

$$\phi\left(q_h; p, a, \right) \equiv \left(p + (\beta_h - \beta_l)q_h\right)\left(p + (\beta_h - \beta_l)q_h - a\right)\left(V'(q_h) - \beta_h\right)$$

$$+ \quad a(\beta_h - \beta_l)\Big(V(\hat{q}_l) - \beta_l\hat{q}_l - V(q_h) + \beta_h q_h\Big) \tag{19}$$

I need to show that there exists a unique q_h such that $\phi\left(\hat{q}_h; p, a \right) = 0$.

$$\frac{\partial \phi}{\partial q_h} = \left(p + (\beta_h - \beta_l)q_h\right)\left(p + (\beta_h - \beta_l)q_h - a\right)V''(q_h)$$

$$+ \quad 2(\beta_h - \beta_l)\Big(p + (\beta_h - \beta_l)q_h - a\Big)\left(V'(q_h) - \beta_h\right) \tag{20}$$

From the theorem I can restrict attention to $q_h > q_h^*$. Since $V'(q_h) < \beta_h$ for $q_h > q_h^*$ the derivative in (20) is strictly less than zero. Substituting q_h by q_h^* and $\hat{q}_l$ I get

$\phi(q_h^*; p, a)$ and $\phi(\hat{q}_l; p, a)$,

$$\phi\,(q_h^*; p, a) = a(\beta_h - \beta_l)\Big(V(\hat{q}_l) - \beta_l \hat{q}_l - V(q_h^*) + \beta_h q_h^* \Big) > 0 \quad \text{since } \hat{q}_l = q_l^* \tag{21}$$

and

$$\phi\,(\hat{q}_l; p, a) = -\Big(2(\beta_h - \beta_l)(p - a)\hat{q}_h + p(p-a) + (\beta_h - \beta_l)^2 \hat{q}_l^{\,2}\Big)\Big(\beta_h - \beta_l\Big) < 0 \tag{22}$$

he expressions (20), (21) and (22) imply that there is **a unique q_h such that**

$\phi(\hat{q}_h; p, a) = 0$ and $q_h^* < \hat{q}_h < \hat{q}_l$. □

Comparative statics exercises can now be presented to show the changes induced in the optimal contract because of changes in the penalty and the audit cost. To see how $\hat{q}_h$ changes with the penalty note that:

$$\frac{\partial \hat{q}_h}{\partial p} = - \frac{\dfrac{\partial^2 \mathcal{L}}{\partial q_h \partial p}(\hat{q}_h; p, a)}{\dfrac{\partial^2 \mathcal{L}}{\partial q_h^{\,2}}(\hat{q}_h; p, a)} \tag{23}$$

By equation (18), and because $\phi\Big(\hat{q}_h; p, a\Big) = 0$ I get

$$\frac{\partial^2 \mathcal{L}}{\partial q_h^{\,2}}(\hat{q}_h; p, a) = \frac{f_h}{\Big(p + (\beta_h - \beta_l)\hat{q}_h - a\Big)^2} \times \frac{\partial \phi}{\partial q_h}(\hat{q}_h; p, a) < 0 \tag{24}$$

$$\frac{\partial^2 \mathcal{L}}{\partial q_h\, \partial p}(\hat{q}_h;\, p,\, a) = \frac{f_h}{\left(p + (\beta_h - \beta_l)\hat{q}_h - a\right)^2} \times \frac{\partial \phi}{\partial p}\,(\hat{q}_h;\, p,\, a) \;<\; 0 \tag{25}$$

The expressions (23), (24) and (25) together imply that $\hat{q}_h$ decreases with the penalty. A similar reasoning shows that $\hat{q}_h$ increases with the audit cost. In the audit contract, increases in the penalty or decreases in the audit cost can be used to improve efficiency by decreasing $\hat{q}_h$. For given 'p' and 'a' the optimal q_h had to be increased from q_h^* to reduce cheating. But with a higher penalty efficiency gains can be obtained by lowering $\hat{q}_h$ without inducing an increase in cheating.

Earlier in the paper, in section 3, I assumed that penalties and audit costs existed such that the principal would want to offer an audit contract. I will now show that this is true: for any 'a' there exists p(a) such that the principal will prefer an audit contract to a Baron-Myerson contract. In fact for every audit cost, a certain level of penalty will make the principal indifferent between a Baron-Myerson and an audit contract and for higher penalties the audit contract will be strictly preferred.[5] For very large penalties the principal's payoff from the audit contract will approximate his payoff when he has full information. These results are summarised in Proposition 2.

Proposition 2: Observing that $\hat{q}_i$, $\hat{\pi}_i$ and $\hat{t}_i$ are just condensed forms of $q_i(p, a)$, $\pi_i(\, q_h(p, a);\, p, a)$ and $t_i(\, q_h(p, a))$ the following are true :

(2.1) $\forall$ a $\exists$ p(a) such that p > p(a) $\Rightarrow \sum_{i=h,\, l} \hat{\pi}_i \left(V(\hat{q}_i) - \hat{t}_i \right) > \sum_{i=h,\, l} f_i \left(V(q_i^r) - t_i^r \right)$

i.e for penalties higher than p(a) an audit contract will be offered.

(2.2) $\lim_{p \to \infty} \sum_{i=h,\, l} \hat{\pi}_i \left(V(\hat{q}_i) - \hat{t}_i \right) = \sum_{i=h,\, l} f_i \left(V(q_i^*) - t_i^* \right)$

Proof

(2.1) I show first that the contract $\{q_h^*, q_l^*, t_h^*, t_l^*\}$ is an audit contract. Then I show that for very large p, the principal's payoff from $\{q_h^*, q_l^*, t_h^*, t_l^*\}$ is greater than his payoffinthe Baron-Myerson contract. But if this is true the principal can do even better by offering the optimal audit contract. I then use the envelope theorem to show the principal's payoff is positively related to p and define p(a) as the penalty which makes his payoff from the optimal audit contract equal his payoff in the Baron-Myerson contract.

(a) For a sufficiently high penalty the contract $\{q_h^*, q_l^*, t_h^*, t_l^*\}$ will induce a unique equilibrium with $\tau_h^* = (1 - \alpha_l^*) = 0$; $0 < \tau_l^* < 1$ and $0 < \alpha_h^* < 1$. The proof is easy and thus omitted.

(b) From (9) I have $\lim_{p\to\infty} \pi_h^* = f_h$ and $\lim_{p\to\infty} \pi_l^* = f_l$. Therefore,

$$\lim_{p\to\infty} \left\{\sum_{i=h,\ l} \pi_i^* \left(V(q_i^*) - t_i^*\right)\right\} > \sum_{i=h,\ l} f_i \left(V(q_i^b) - t_i^b\right)$$

(c) Since $\frac{\partial \pi_h}{\partial p} < 0$; $\frac{\partial \pi_l}{\partial p} > 0$; $\pi_h + \pi_l = 1$ and $V(\hat{q}_l) - \beta_l\hat{q}_l > V(\hat{q}_h) - \beta_h\hat{q}_h$,

I have $$\frac{\partial \mathcal{L}}{\partial p}(\hat{q}_h;\ p,\ a) = \frac{\partial \hat{\pi}_h}{\partial p}\left(V(\hat{q}_h) - \beta_h\hat{q}_h\right) + \frac{\partial \hat{\pi}_l}{\partial p}\left(V(\hat{q}_l) - \beta_l\hat{q}_l\right) > 0 .$$

(2.2) $\lim_{p\to\infty} \hat{q}_h = q_h^*$:

Note from equation (17) $\lim_{p\to\infty} \hat{\theta}_h = \lim_{p\to\infty} \hat{\pi}_h = f_h$ and then (14) implies $\lim_{p\to\infty} V'(\hat{q}_h) = \beta_h$. This result together with the limits in proof (2.1) (b) prove the proposition.

□

I can now give a summary of the comparative statics results. Take the audit cost as given. When the penalty is very high the contract looks very similar to the full information contract. There is very little cheating and very little auditing, $\hat{q}_l$ equals q_l^* and $\hat{q}_h$ is very close to q_h^*. As the penalty goes down, $\hat{q}_h$ goes up and the probabilities of cheating and auditing increase. The principal's expected payoff falls but the agent's expected payoff does not change. These properties hold until the penalty is such that the principal's expected payoff equals his expected payoff for the Baron-Myerson contract. At this point $\hat{q}_h$ reaches the maximum; for lower penalties the Baron-Myerson contract is offered and $\hat{q}_h$ drops to q_h^b below q_h^*.

5. THE PROBABILITY OF AUDIT WHEN THERE IS COMMITMENT

Is there more audit with or without commitment? To answer this question I present in this section a problem similar to the principal's problem above but in which commitment to audit probabilities is allowed. If the principal can commit to audit probabilities, then the contract will be a vector in $\Re^6$, $\{ q_h, q_l, t_h, t_l, \alpha_h, \alpha_l \}$. The principal's problem will be the following:

$$\max_{q_h,\, q_l,\, t_h,\, t_l,\, \alpha_h,\, \alpha_l} \quad f_h \left\{ V(q_h) - t_h - \alpha_h a \right\} + f_l \left\{ V(q_l) - t_l - \alpha_l a \right\} \tag{26}$$

subject to

$$\text{ICC(l)} \qquad t_l - \beta_l q_l \geq (1 - \alpha_h)(t_h - \beta_l q_h) - \alpha_h p \tag{27}$$

$$\text{ICC(h)} \qquad t_h - \beta_h q_h \geq (1 - \alpha_l)(t_l - \beta_h q_l) - \alpha_l p \tag{28}$$

$$\text{IR(l)} \qquad t_l - \beta_l q_l \geq 0 \tag{29}$$

$$\text{IR(h)} \qquad t_h - \beta_h q_h \geq 0 \tag{30}$$

Constraints (27) and (28) are the incentive compatability constraints. Since truth telling is induced and auditing is perfect, the individual rationalities (29) and (30) do not have to take into account the expected penalities. Also, since there is no cheating in equilibrium, the probabilities π_h and π_l reduce to f_h and f_l, and the principal does not receive any penalties.

Denoting $\{\tilde{q}_h, \tilde{q}_l, \tilde{t}_h, \tilde{t}_l, \tilde{\alpha}_h, \tilde{\alpha}_l\}$ as the solution to the above problem, it is easy to show that $\tilde{q}_h < q_h^*$, (27) and (30) are binding but (28) is not. Therefore, $\tilde{\alpha}_l = 0$ and

$$\tilde{\alpha}_h = \frac{(\beta_h - \beta_l)\tilde{q}_h - (\tilde{t}_l - \beta_l\tilde{q}_l)}{p + (\beta_h - \beta_l)\tilde{q}_h} \quad .$$

Keeping in mind that $\tilde{q}_h < q_h^* < \hat{q}_h$, I can subtract the probability of audit with commitment $\tilde{\alpha}_h$, from the probability of audit without commitment $\hat{\alpha}_h$, to get the following:

$$\hat{\alpha}_h - \tilde{\alpha}_h = \frac{(\beta_h - \beta_l)\,\hat{q}_h}{p + (\beta_h - \beta_l)\hat{q}_h} - \frac{(\beta_h - \beta_l)\,\tilde{q}_h}{p + (\beta_h - \beta_l)\tilde{q}_h} + \frac{\tilde{t}_l - \beta_l\tilde{q}_l}{p + (\beta_h - \beta_l)\tilde{q}_h} > 0.$$

Therefore, I have the following proposition,

Proposition 3. Production of q_h triggers off a higher probability of audit, when there is no commitment to audit probabilities, compared to when commitment is allowed.

The objective of the principal is to lower cheating with as little cost as possible. When the principal can commit to audit probabilities, he has two tools at his discretion — $\tilde{q}_h$ and $\tilde{\alpha}_h$. He can lower $\tilde{q}_h$ to lower the gains from cheating and thereby lower $\tilde{\alpha}_h$ too. On the other hand, when he cannot commit, he only has $\hat{q}_h$ to work with. He has to raise $\hat{q}_h$ to credibly convey to the agent that he is going to audit more. But this also increases the gains to cheating pushing up $\hat{\alpha}_h$ even further.

6. CONCLUSION

The most obvious extension of this model is to look at the case with many states of nature. This exercise can be problematic as it is not clear what will happen in the second stage. It is not even clear if a unique equilibrium exists. In this analysis I have abstracted from moral hazard considerations. However it is well known that basing payments on ex-post observables can introduce moral hazard considerations. Laffont and Tirole have looked at this problem, but in their model auditing is costless. Without investigating the moral hazard considerations one must be careful in thinking about applications of this model.

FOOTNOTES

1. Other prominent models in this tradition are Baron(1984) and Demski, Sappington and Spiller(1987).

2. Writing about the Baron—Besanko model (Handbook of Industrial Organization, p. 1386) Baron has said that the price or the marginal value of output to the principal can be below the marginal production cost. In my terminology this would be overproduction— but some clarification is required. In their model, at the time of signing the contract, the agent only knows a 'cost' parameter which determines the distribution of the marginal cost; the marginal cost is revealed to the agent only after production has ocurred. The price or marginal value of output to the principal is always greater than the expected marginal cost given the cost parameter but could of course be less than the realized marginal cost. Therefore ex-ante, there is

underproduction — overproduction can occur only ex-post.

Lewis and Sappington's work (1989 a & b) on countervailing incentives also provide examples of overproduction. But, in their models, a regulator expands output output beyond the efficient level to mitigate the firm's incentive to understate cost. In my model there are no incentives to understate cost.

3. The assumption of a perfect audit technology is shared by Townsend(1979) in his seminal paper on costly state verification, Graetz, Reinganum and Wilde(1986) in their tax compliance game and Banks(1988) in his paper on regulatory auditing. This assumption allows me to present the essential results wihout increasing notation.

4. The equilibrium probabilities of cheating and auditing will be independent of the terms in the contract if the principal had to pay t_h before auditing, i.e. as soon as q_h is produced. This might be true in the case where the principal is a regulator controlling a monopolist selling in the market. In this case t_h is interpreted as the revenue of the monopolist net of taxes. The contract will have full information outputs in both states and the probabilities of auditing and cheating will be determined by the parameters f_l, f_h, p and a.

5. This is not quite correct. It may be the case that an audit contract is offered for all p and there is no p > a for which the principal is indifferent between the Baron-Myerson and the audit contract. I will ignore these cases.

APPENDIX A : The 'Baron-Myerson' Contract

When there is no audit the best contract for the principal is the Baron-Myerson 'Baron-Myerson' contract. The principal's problem is

$$\max_{\{q_h,\, q_l,\, t_h,\, t_l\}} \sum_{i=h,\, l} f_i \left\{ V(q_i) - t_i \right\}$$

$$\text{subject to} \qquad t_i - \beta_i q_i \;\geq\; t_j - \beta_i q_j \qquad i = h, l; \;\; j = h, l; \;\; i \neq j$$

$$t_i - \beta_i q_i \;\geq\; 0 \qquad\qquad i = h, l$$

The solution $\{q_h{}^b, q_l{}^b, t_h{}^b, t_l{}^b\}$ will satisfy the following

$$t_h{}^b - \beta_h q_h{}^b \;=\; 0$$

$$t_l{}^b - \beta_l q_l{}^b \;=\; t_h{}^b - \beta_l q_h{}^b = (\,\beta_h - \beta_l\,)\, q_h{}^b > 0$$

$$V'(q_h{}^b) \;=\; \beta_l + \frac{1}{f_h}(\,\beta_h - \beta_l\,) \;>\; \beta_h$$

$$V'(q_l{}^b) \;=\; \beta_l$$

The agent gets a zero payoff in the high-cost state. There is no cheating. The agent gets a strictly positive payoff in the low cost state. There is under production in the high cost state and production in the low cost state is equal to the output in the full information contract. The principal's payoff is strictly less than his payoff when there is no asymmetry of information. The losses come from two sources, the 'low' $q_h{}^b$ and the payment that has to be made to induce truth telling.

APPENDIX B

Claim : In the optimal contract $0 < \alpha_h < 1 \Rightarrow \tau_l > 0$.

(i) If $\tau_l = 0$ in the optimal contract then q_l is only produced in the high cost state. Knowing this the principal will always audit when q_l is produced and the agent will never cheat. Therefore $\tau_l = 0$ implies $\tau_h = 1$. But these together imply $\pi_h = 1$.

(ii) $0 < \alpha_h < 1$ and $\tau_l = 0 \Rightarrow t_h = \frac{a}{f_l} - p$. This follows from equation(4).

(iii) The above are equivalent to having $q_l = q_h$ and $t_l = t_h$ which is clearly feasible under the Baron-Myerson contract but never chosen. The principal's payoff in the Baron-Myerson contract is therefore higher. Thus if the principal cannot induce $\tau_l > 0$ he is better off with the Baron-Myerson contract.

$\square$

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Chapter 3

Effort Monitoring without Commitment

1. INTRODUCTION

The early literature on contracts under moral hazard had used costless monitoring of the agent's effort choice and the focus was not on the monitoring scheme.[1] Later, costly monitoring was introduced and monitoring schemes were analyzed. In each model, however the principal is allowed to pre-commit to a monitoring strategy, even though the principal would like to renege on his commitment.[2] Also the literature has not emphasized the interaction between monitoring and the rest of the incentive schemes.[3] I look at the contract that will be offered if the principal cannot pre-commit to a monitoring strategy. In this case the contract itself induces the monitoring strategy and the interaction between monitoring and the rest of the incentive scheme becomes clear.

The model is based on a very simple moral hazard framework. A principal hires a risk-averse agent to carry out some task. With the knowledge of the contract, the agent puts in some effort. The effort together with a random component determines the outcome. The output is publicly observable, but the effort level is private information to the agent. The principal can investigate effort at a cost after the outcome has occured.

The contract is an outcome contingent wage scheme. The principal and the agent can both *anticipate* the probability with which investigation will occur, but these probabilities cannot themselves be included in the contract. Monitoring effort (as opposed to state information) has to be somewhat subjective; therefore, I assume the following: the principal can 'hide' the result of an investigation. Thus the agent believes that the principal, whenever he investigates, is only going to reveal the results of his investigation when he has caught the agent shirking. This rules out the possibility of the principal promising to reward or insure the agent when he is not shirking but the outcome is bad.[4] The agent believes that when such a situation occurs the principal is just going to ignore the results of the investigation. The upshot of this belief is, payments to the agent will depend on outcome alone unless the principal can prove that the agent had shirked. If the agent is caught shirking, he is penalized. Consider a production process where the agent's action is recorded on a TV monitor. Based on outcome the principal can review the tape. The agent believes that the tape will be used as evidence only to prove shirking.

The literature on contracts with monitoring under moral hazard has allowed the principal to commit to monitoring schemes. A monitoring scheme contains the probabilities of investigation occuring contingent on output. The principal is able to commit, if he can announce the monitoring scheme before the agent applies effort and before the outcome occurs. In this case the principal can include the monitoring strategy in the contract along with the wage scheme. And this is what leads to a consistency problem. Because of the commitment, the principal can always predict the effort put in by the agent— the investigation itself is uninformative. Therefore, carrying out the investigation is not optimal. Reputational considerations are usually cited to solve this problem of consistency. In the case of a government regulator, reputational arguments might make such a contract credible. But, surely in many situations there will be serious doubts about the credibility of such a contract.

There has been surprisingly little work done on the issue of commitment to monitoring strategies. I know of no paper which deals with it in a framework with moral hazard . Some recent literature on auditing has addressed the issue of commitment in other frameworks. Graetz, Reinganum and Wilde (1986) have modelled tax compliance and Banks (1988) has modelled regulatory auditing without commitment. In the tax-compliance game an individual files his income tax report with the I.R.S. which does not know ex-ante what the individual's true income is. The I.R.S can audit once it receives a report and penalize non-compliers. Individual income is exogenous and the equilibrium gives the reporting and auditing strategies of individuals and the I.R.S. In Bank's model of regulatory auditing, a monopolist announces a price — the regulator does not know the cost of production but can audit and find the cost once price is announced. If an audit occurs the price is set equal to the marginal cost, otherwise the monopolist sells at whatever price he announced. The equilibrium pricing and auditing strategies of the monopolist and the regulator are determined. However, these authors do not use a contractual framework and therefore have not investigated how the terms of the contract change when the principal cannot commit to an audit policy.

Taking away the principal's power to commit to a monitoring strategy gives the framework an interesting twist. It introduces a 'three-person' hierarchical structure into the 'principal-agent' relationship. There is the principal who offers a wage contract to the agent who *reacts* to the contract by choosing his effort strategy. But, there is the principal again (a step removed) who also *reacts* like an agent, to the contract he himself designed in the first step, when he chooses his investigation strategy. The agent will put in effort if he believes that the principal will investigate. The agent will believe that there will be an investigation, only if the contract provides an incentive for the principal to investigate.

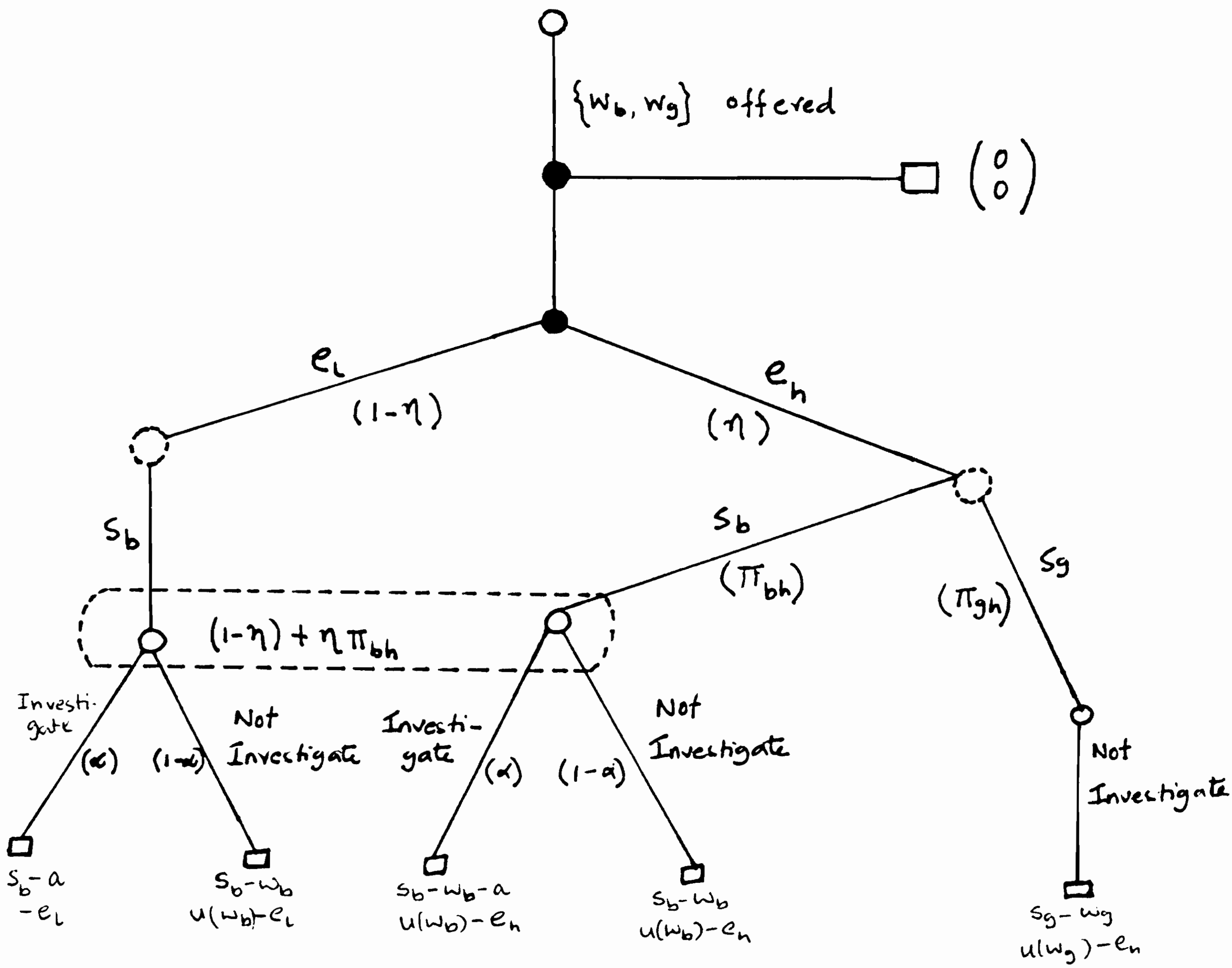
I analyze the optimal contract to show where the gains from monitoring occur. I show that the optimal monitoring contract cannot eliminate shirking and that investigation must be random. The optimal contract depends on the trade off between how much the principal values effort and how costly it is to induce effort. I show that if effort is valued very highly then the optimal contract might have wages decreasing in outcome. Section 2 presents the model. Section

3 sets up the principal's problem. Section 4 analyzes and explains the optimal contract. Section 5 is the conclusion containing directions for future research.

2. THE MODEL

I will first present the environment under which the principal and agent interact, and then present the timing of the moves and the information structure with the help of a game tree. A risk-neutral principal hires a risk-averse agent to perform a task. The outcome depends on the agent's effort, and on other factors modelled as a random component. The outcome, which can be good or bad, is publicly observable, while the agent's effort is private information. In particular, when the outcome is bad the principal does not know if the agent had shirked or if he was simply unlucky. The principal has to investigate at a cost to find out whether the agent had indeed shirked.

The contract is an outcome contingent wage scheme $\{ w_b, w_g \}$, where the subscripts refer to the quality of the outcome. The principal and the agent agree upon the contract before the agent puts in any effort. The parameters of the problem are such that the principal would like the agent to put in high effort e_h but the agent can shirk and put in e_l ($e_h > e_l \geq 0$). There are two possible outcomes, good and bad where s_g is the good and s_b is the bad outcome ($s_g > s_b > 0$). If the agent shirks the outcome is certain to be bad. However, when the agent puts in a high effort s_g occurs with a probability $\pi_{gh} < 1$, thus s_b can also occur (due to factors beyond the agent's control) with a probability $\pi_{bh} = 1 - \pi_{gh} > 0$. The agent's payment must be non-negative, because the agents have limited resources, for instance. The agent is paid w_i when outcome s_i occurs unless there is an investigation. The principal can investigate by incurring a



○ Principal's Move

● Agent's Move

◌ Chance Move

□ Terminal Nodes with the Principal's Payoff above the Agent's Payoff

<u>GAME TREE INDUCED BY A CONTRACT</u>

<u>Fig 1</u>

monitoring cost, $a > 0$. If investigation occurs and it reveals shirking, then the agent is not paid anything.[5] If the investigation reveals high effort then the agent is paid according to the outcome. The principal is risk-neutral and the agent, risk-averse with a separable utility function $V(.\,,.) = u(\,w\,) - e$; where $u(0) = 0 \quad u' > 0\,, \quad u'' < 0$ and $\lim_{w \to 0} u'(w) = \infty$.

The game tree in fig. 1 helps to show the sequence of moves and the information available at each move. The contract $\{w_g, w_b\}$ must guarantee the agent his opportunity profit level (normalised to zero), otherwise the agent will not accept the contract. The second stage begins if the contract is accepted. Then the agent must choose his effort strategy. He puts in a high effort with probability η. When the principal observes s_b he does not know whether the agent shirked or if he was simply unlucky. If s_g occurs, then the agent must have put in a high effort. In each outcome the principal must decide whether or not to investigate. The principal investigates the outcome s_b with probability α and expects to find low effort or shirking with a probability $(1-\eta)/(1-\eta+\eta\pi_{bh})$. Investigation of a good outcome will only result in the loss of monitoring cost. Therefore, the principal will never investigate a good outcome and I have ignored that possibility in the game tree. The payoffs are presented at the terminal nodes with the agent's payoff written under the principal's payoff.

3. CHOOSING THE OPTIMAL MONITORING CONTRACT:

A monitoring contract is one which induces a positive probability of investigation or monitoring. In this section I will set up the problem the principal has to solve to derive the optimal monitoring contract. In section 3.1 I discuss two benchmark contracts, the optimal no-investigation and the full-information contracts. In section 3.2 the principal's problem is presented in its complete form and lemma 1 is established. In section 3.3 lemma 1 is used to present a simpified principal's problem, which is equivalent to the complete problem in section 3.2.

3.1 Two Benchmark Contracts :

If the monitoring technology is not available to the principal, the principal must solve the program NIP,

$$\max_{w_b,\, w_g} \quad \pi_{bh} [\, s_b - w_b \,] + \pi_{gh} [\, s_g - w_g \,] \tag{1}$$

$$\text{s.t.} \qquad \pi_{bh}\, u(w_b) + \pi_{gh}\, u(w_g) - e_h \geq u(w_b) - e_l \quad , \tag{2}$$

$$\text{and} \qquad \pi_{bh}\, u(w_b) + \pi_{gh}\, u(w_g) - e_h \geq 0 \quad . \tag{3}$$

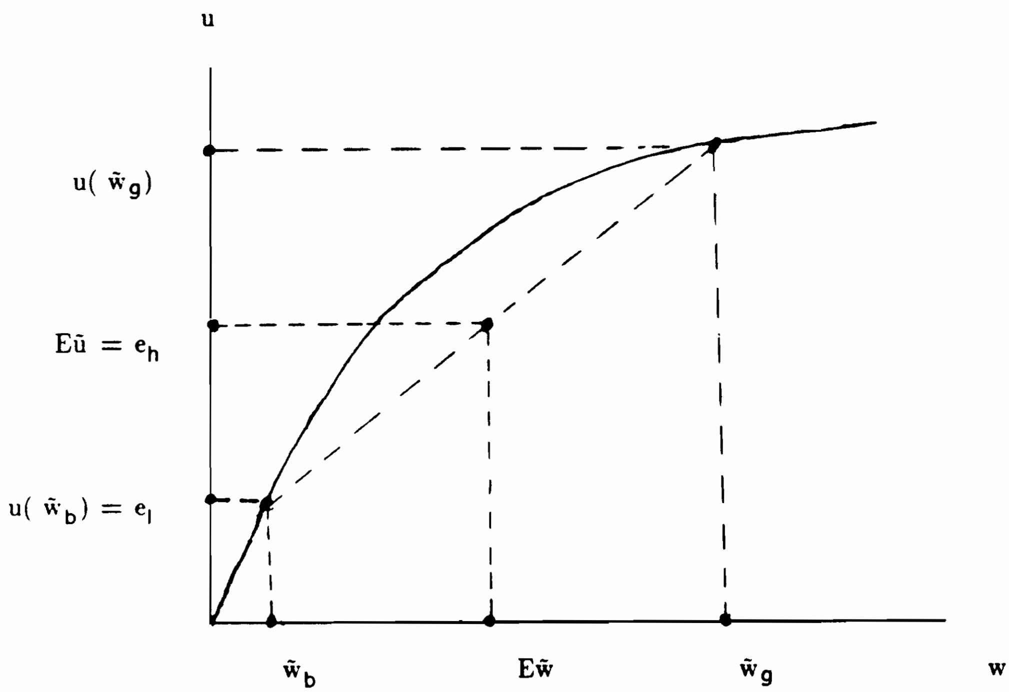
The incentive constraint (2) ensures that the contract induces a high effort. The participation constraint (3) ensures that the agent accepts the contract. The objective function of the principal (1) reflects the fact that the agent is induced to put in e_h. The optimal no investigation contract is $\{\tilde{w}_b, \tilde{w}_g\}$ and it has the following properties ;

(i) constraints (2) and (3) are binding.

(ii) $u(\tilde{w}_g) > e_h > u(\tilde{w}_b) = e_l$ i.e., the agent must be exposed to some risk.

Note also that the incentive constraint (2) implies

$$u(\tilde{w}_g) - u(\tilde{w}_b) \;\geq\; \frac{e_h - e_l}{\pi_{gh}} \quad . \tag{4}$$



THE OPTIMAL NO - INVESTIGATION CONTRACT

Fig 2

The solution is presented in fig. (2). The expected utility of the agent net of effort cost is $E\tilde{u}$ and $E\tilde{w}$ is the expected wage payments by the principal. Incentive compatibility considerations (4), prevents the principal from bringing $\tilde{w}_b$ and $\tilde{w}_g$ closer and reducing $E\tilde{w}$. If there was full information i.e., effort was observable then there would be no need for constraint (2) and the full information contract would satisfy $w_g = w_b = w$ with $u(w) = e_h$.

3.2 The Principal's Problem :

I present first the problem the principal must solve when he chooses the optimal monitoring contract. Then Lemma 1 is established which contains properties of the optimal monitoring contract.

The principal must solve the problem PP to derive the optimal monitoring contract,

$$\max_{w_g,\, w_b} \; (1 - \eta + \eta\pi_{bh}) \left\{ (1-\alpha)(s_b - w_b) + \alpha \left(\frac{1}{1 - \eta + \eta\pi_{bh}} \Big((1-\eta)\, s_b + \eta\pi_{bh}(s_b - w_b) \Big) - a \right) \right\} + \eta\pi_{gh}\,(s_g - w_g) \quad , \tag{5}$$

$$\text{s.t.} \quad \eta \left\{ \pi_{bh}\, u(w_b) + \pi_{gh}\, u(w_g) - e_h \right\} + (1-\eta) \left\{ (1-\alpha)\, u(w_b) - e_l \right\} \geq 0 \tag{6}$$

$$w_b \geq 0 \; , \quad w_g \geq 0 \; ; \tag{7}$$

$$\text{where} \quad \eta = \eta(w_b, w_g)\; ; \text{ and } \alpha = \alpha(w_b, w_g)\; . \tag{8}$$

The objective function (5) is the principal's expected pay-off from the contract $\{ w_b, w_g \}$. The principal expects the agent to put in high effort with probability η. Therefore, the bad outcome occurs with probability $\{(1-n) + \eta\pi_{bh}\}$ and the good outcome with probability $\eta\pi_{gh}$. When s_g occurs, there is no investigation and the principal's payoff is $(s_g - w_g)$. When s_b occurs the principal investigates with a probability α. If he does not investigate his pay-off is $(s_b - w_b)$. When investigation does occur he expects to find shirking with a probability $(1-\eta)/(1-\eta + \eta\pi_{bh})$, in which case he does not have to pay w_b; otherwise he has to pay w_b. The cost of the investigation is a. The constraint (6) is the participation constraint. It ensures that the contract $\{w_b, w_g\}$ leaves the agent with an expected utility greater than his opportunity profit level (normalized to zero). When the agent puts in high effort (with probability η), the investigation does not affect him. If he shirks (with probaility $(1-\eta)$), an investigation results in a zero wage; but, when he is not investigated (with probability $(1-\alpha)$), he gets $u(w_b)$. The set of functions (8) is just a reminder that the probabilities of monitoring and putting in high effort are best replies to each other, and are induced by the contract $\{w_b, w_g\}$. In order to solve the program PP, the equilibrium investigation and effort probabilities must first be computed for any given contract. Then knowing how these probabilities change in response to differrent contracts offerred the principal can choose the contract which maximizes his expected pay-off (5) subject to the participation constraint (6) and the non-negativity constraints (7). At this point I assume, for the sake of continuity, that monitoring costs are low enough so that a monitoring contract is offered i.e., $\alpha > 0$. Later in proposition 3, I prove that if a is low enough a monitoring contract will dominate the no investigation contract. Lemma 1 describes the equilibrium probabilities of effort and investigation induced by the optimal monitoring contract.

Lemma 1. An optimal monitoring contract will induce unique equilibrium probabilities of investigation and effort with the following properties: (i) $\alpha < 1$ and (ii) $0 < \eta < 1$.

Proof :

(i) If $\alpha=1$ then the agent loses e_l if he shirks, but receives $\pi_{bh}\, u(w_b) + \pi_{gh} u(w_g) - e_h \geq 0$ from putting in a high effort. Therefore, the agent will always work hard, $\eta=1$. But, if $\eta=1$ then an investigation only results in the loss of monitoring cost, which implies $\alpha=0$.

(ii a) $\eta > 0$: Suppose not, then only s_b is produced and the principal gets $s_b - \overline{w}_b - \alpha a$. But in that case the principal can do better by offering the optimal no investigation contract $\{ \tilde{w}_b ,\ \tilde{w}_g \}$, because

$$\pi_{bh} [s_b - \tilde{w}_b] + \pi_{gh} [s_g - \tilde{w}_g] \geq s_b - \tilde{w}_b > s_b - \overline{w}_b - \alpha a.$$

The left inequality follows from the assumption made about the parameters in program NIP. In NIP, the principal could always ensure $s_b - \tilde{w}_b$ by setting $\tilde{w}_b = \tilde{w}_g = \tilde{w}$, satisfying $u(\tilde{w}) = e_l$, but he does not. The second inequality holds since the participation constraint (6) is reduced to $(1-\alpha)\, u(\overline{w}_b) \geq e_l$, which implies $\overline{w}_b > \tilde{w}_b$.

(ii b) $\eta < 1$: $0 < \alpha < 1$ implies, the principal is indifferent between not investigating (paying w_b) or investigating (paying $w_b(\eta\pi_{bh})/((1 - \eta) + \eta\pi_{bh}) + a$). Therefore,

$$w_b = \frac{\eta\pi_{bh}}{(1 - \eta) + \eta\pi_{bh}}\, w_b + a$$

$$\text{or} \qquad \eta = \frac{w_b - a}{w_b - \pi_{gh} a} < 1 \quad . \tag{9}$$

The uniqueness of α is demonstrated by computing α, using $0 < \eta < 1$. The agent is indifferent between putting in high and low effort,

$$\pi_{bh}\, u(w_b) + \pi_{gh}\, u(w_g) - e_h = (1 - \alpha)\, u(w_b) - e_l$$

$$\text{or} \quad \alpha = \frac{\pi_{gh} (u (w_b) - u(w_g)) + e_h - e_l}{u(w_b)} \quad . \tag{10}$$

□

Lemma 1 shows that an optimal monitoring contract can only induce random investigations and that shirking can not be eliminated. The expression for η in (9) shows, an increase in the wage for the bad outcome (w_b) increases effort (η). On first sight this is somewhat puzzling. It occurs due to the mixed strategies. In a mixed strategy equilibrium η will be chosen to make the principal indifferent between investigating and not investigating. In (11) I have rearranged (9) to depict better what is happening;

$$\frac{(1 - \eta)}{(1 - \eta) + \eta\pi_{bh}} \left\{(w_b - 0) - a \right\} = \frac{\eta\pi_{bh}}{(1 - \eta) + \eta\pi_{bh}} \left\{ w_b - w_b + a \right\} \quad . \tag{11}$$

The left hand side of (11) is the expected gain from detecting shirking when investigating outcome s_b. Instead of paying w_b, the principal pays nothing but incurs an investigation cost a. Thus, the expost gain from investigating is ($w_b - 0 - a$). The right hand side is the expected cost of finding that the agent was simply unlucky. He has to pay the agent w_b anyway, moreover he incurs the cost a. Thus, the expost cost of finding high-effort leading (unluckily) to bad outcome is simply the investigation cost a. An increase in w_b increases the ex-post gains from investigating, since the principal can save more when he detects shirking. In order to counter this gain (the principal must be kept indifferent between investigating and not-investigating), the agent must raise η and reduce the expected gains.

Another way of motivating this analysis would be to think of this as a three person game. The fact that the principal cannot commit to a monitoring strategy induces a hierarchical structure on the relationship. There is the principal who offers the contract to the agent, but there is the principal again who reacts like an agent to his own contract, while he decides on a monitoring strategy. The agent will only put in high effort (raise η) if he sees that the contract provides incentives for the principal to investigate in the second stage. Therefore, the only way the principal can raise η is by increasing his own ex-post gains from investigating.

Rewriting expression (10) as (12a), allows me to make an interesting comparison of the monitoring problem with the program NIP in section 3.1.

$$\alpha = \frac{[\, u(w_b) - e_l \,] - [\, \pi_{bh} u(w_b) + \pi_{gh} u(w_g) - e_h \,]}{u(w_b)} , \tag{12a}$$

This implies that whenever $\alpha > 0$ it must be that

$$u(w_g) - u(w_b) < \frac{e_h - e_l}{\pi_{gh}} . \tag{12b}$$

Note that the numerator in (12a) is the difference between the agent's expected utilities for tthe two effort levels in program NIP. The incentive constraint (2) imposes a restriction on how close the principal can bring w_a and w_b. The principal is allowed to choose the optimal investigation contract $\{\tilde{w}_b, \tilde{w}_g\}$ in the monitoring program PP. But expression (12b) says that if the solution to PP is a monitoring contract ($\alpha > 0$), then it must be different from $\{\tilde{w}_b, \tilde{w}_g\}$. In fact if $w_g > w_b$, the principal benefits from being able to bring the wages closer in the monitoring contract.

3.3 The Principal's Problem Simplified :

Using Lemma 1, I can simplify the program PP. The principal is indifferent between investigating and not investigating, therefore in (5) I can use $s_b - w_b$ as the principal's expected payoff when s_b occurs. Therefore, the objective function (5) can be simplified into,

$$(1 - \eta\pi_{gh}) (s_b - w_b) + \eta\pi_{gh} (s_g - w_g) \quad . \tag{13}$$

The agent is indifferent between putting in a high or a low effort. Therefore, the participaton constraint (6) can be simplified into any of the following two equivalent forms,

$$\pi_{bh}\, u (w_b) + \pi_{gh}\, u (w_g) - e_h \geq 0 \tag{14a}$$

or

$$(1 - \alpha)\, u (w_b) - e \geq 0 \tag{14b}$$

Collecting (13) and (14), the solution to the following simplified program [SP] is going to give the optimal monitoring contract. Lemma 1 shows that solving the program PP is equivalent to solving the program SP,

$$\underset{w_g,\, w_b}{\text{Max}} \quad (1 - \eta\pi_{gh})(s_b - w_b) + \eta\pi_{gh} (s_g - w_g)$$

s.t.

$$\pi_{bh}\, u (w_b) + \pi_{gh}\, u (w_g) - e_h \geq 0$$

$$w_b \geq 0 , \quad w_g \geq 0; \qquad \text{with} \qquad \eta = \frac{w_b - a}{w_b - \pi_{gh}\, a} \quad .$$

Unlike the program NIP, this program does not have an incentive constraint. Also Lemma 1 shows that η depends on the contract and that $0 < \eta < 1$. Therefore, the probabilities of s_g and s_b occuring are different from π_{gh} and π_{bh}.

The nature of the problem is more complicated than what has been standard in the moral hazard literature. If an incentive constraint were present, then $\eta=1$ and the objective function would be separable into two parts. (This can also be seen in (5)). One part is the expected benefits $\pi_{gh}s_g+\pi_{bh}s_b$ and it does not depend on the contract. The other part is the expected cost $\pi_{gh}w_g+\pi_{bh}w_b$, which depends on the contract. In this case, the values s_g and s_b determine whether the principal should induce e_h or e_l, but they do not show up in the first order conditions. Therefore, they do not determine the shape of the contract. The maximization program is then equivalent to a cost minimization program. This is the observation Grossman and Hart (1983) have utilized to analyze a model without monitoring. In their approach, the problem is broken up into two steps. First, the minimum cost of implementing each effort is determined and that provides the shape of the wage scheme. Then, the best effort level is chosen by comparing the expected values of output for each effort level to the cost of implementing that effort level. The same separation occurs when a principal can commit to a monitoring scheme. In this case also, incentive constraints are part of the principal's problem and expected benefits can be separated from expected costs. Dye (1986) uses this observation to investigate the minimum cost of inducing a particular effort level, when the principal can commit to a monitoring strategy. He derives properties of the optimal contract. If the shape of the optimal contract depended on the value of output, then Dye's exercise would not be of much use. In the program SP, expected benefit and expected cost both depend upon the contract. Therefore, the separation which helped the above authors is not available here.

The principal's problem therefore, hinges on the trade-off between the value of effort (given by the difference between s_g and s_b) and the cost of inducing effort (in the form of wage payments and monitoring costs) .

4. THE OPTIMAL MONITORING CONRACT

The Lagrangian of the program SP is,

$$\mathcal{L} = (1-\eta\pi_{gh})\,(\,s_b - w_b\,) + \eta\pi_{gh}\,(\,s_g - w_g\,)$$

$$+\,\theta\left(\,\pi_{bh}\,u(w_b) + \eta_{gh}\,u(w_g) - e_h\,\right)\;; \tag{15}$$

where, $\eta = \dfrac{w_b - a}{w_b - \pi_{gh}a}$ and the multiplier $\theta \geq 0$.

I have left out the non-negativity constraints, and I will talk about them shortly. The derivative of the Lagrangian with respect to w_g and w_b are,

$$\frac{\partial \mathcal{L}}{\partial w_g} = -\,\eta\pi_{gh} + \theta\pi_{gh}\,u'(w_g) \tag{16}$$

and

$$\frac{\partial \mathcal{L}}{\partial w_b} = -\,(\,1-\eta\pi_{gh}\,) - \pi_{gh}\left(\,s_g - w_g - s_b + w_b\,\right)\frac{\partial \eta}{\partial w_b} + \theta\pi_{gh}\,u'(w_b)\;. \tag{17}$$

Since $\eta > 0$, it must be that $w_b > a > 0$. But it is not obvious that w_g will also be greater than zero. There can be two cases (i) $\theta > 0$, and (ii) $\theta = 0$. If $\theta > 0$ then from (16) $w_g > 0$, because $\lim_{w\to 0} u'(w) = \infty$. If $\theta = 0$ then the constraint is not binding. Suppose $\theta = 0$ and by (16) $w_g = 0$, I can show that if the monitoring cost is small enough, the principal can improve his payoff by relaxing the constraint and therefore $\theta > 0$. These results are included in proposition 1.

Proposition 1 : In an optimal monitoring contract $\{\, w_b^*,\, w_g^* \,\}$ the following will hold,

(1.1) $u\,(w_b^*) > e_l$.

(1.2) For a low enough monitoring cost the participation constraint is binding and $w_g^* > 0$

(1.3) $\lim_{a\to 0} u\,(w_b^*) = \lim_{a\to 0} u(w_g^*) = e_h$.

Proof:

(1.1) Follows from (14b).

(1.2) Suppose $\theta = 0$ and therefore $w_g{}^* = 0$. Then, after replacing η, (17) must satisfy,

$$\frac{\pi_{bh}}{(w_b - \pi_{gh}\, a)^2}\left\{(s_g - s_b)\pi_{gh}\, a + 2\pi_{gh}\, a\, w_b{}^* - (w_b{}^*)^2\right\} = 0 \ . \tag{18}$$

If $w_g = 0$, then $w_b \geq u^{-1}\left(\frac{e_h}{\pi_{bh}}\right)$, by (14).

The expression $\left\{(s_g - s_b)\pi_{gh}\, a + 2\pi_{gh}\, a\, w_b{}^* - (w_b{}^*)^2\right\}$ attains the maximum at,

$$w_b{}^+ = a\,\pi_{gh} + \left(a^2\pi_{gh}{}^2 + a\,\pi_{gh}\,(s_g - s_b)\right)^{1/2}.$$

For low monitoring costs $w_b{}^+ < u^{-1}\left(\frac{e_h}{\pi_{bh}}\right)$. Then the principal can clearly gain from relaxing the constraint; therefore $\theta > 0$, and from (16) $w_g > 0$.

(1.3) If $\theta > 0$ and $w_g > 0$, which will be the case as $a \rightarrow 0$, then (16) and (17) will both equal zero. Using (16) to replace θ, it is straightforward to show that (1.3) holds. □

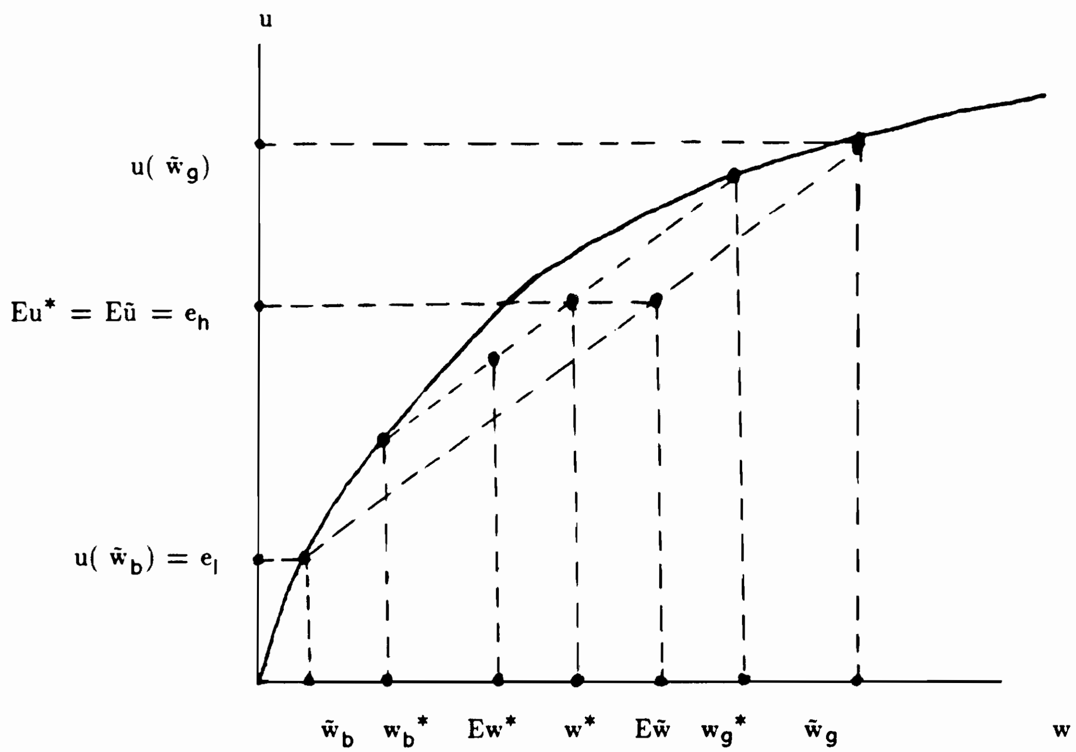
In this paper I assume that, since monitoring cost is low enough, the optimal monitoring contract dominates the optimal no-investigation contract $\{\tilde{w}_b, \tilde{w}_g\}$. I wiil explore where the improvements in the principal's payoff is coming from. For the moment I will assume that in the optimal monitoring contract, $w_g{}^* > w_b{}^*$ and proposition 2 demonstrates that this assumption does hold under certain conditions. When $w_g{}^* > w_b{}^*$ the binding constraints (14a, b) imply that $u(w_g{}^*) > e_h > u(w_b{}^*) > e_l$. More importantly, $u(w_b{}^*) > e_l$ implies $u(w_g{}^*) < u(\tilde{w}_g)$. Refer to fig. 3 for the discussion in this paragraph. In the optimal no investigation contract the expected wage payment is $E\tilde{w} = \pi_{bh}\,\tilde{w}_b + \pi_{gh}\,\tilde{w}_g$. However, in the optimal monitoring contract, because of shirking, the expected wage payment becomes $Ew^* = (1 - \eta\pi_{gh})\, w_b{}^* + \eta\pi_{gh}\, w_g{}^*$, which is lower than $w^* = \pi_{bh}\, w_b{}^* + \pi_{gh}\, w_g{}^*$. Thus, expected payments are lower, not only for the lower variance but also because $\eta < 1$. Since there is shirking, w_g is paid less often than π_{gh}. However, there is a flip side to $\eta < 1$—— s_g also occurs less often! Because of shirking, expected benefits to the principal is lower in the optimal monitoring contract compared to the optimal no investigation contract. Therefore, when $w_g^* > w_b^*$, the reduction in expected wages must more than compensate the loss in expected benefits due to shirking.

When there is a big difference in value between the good and the bad outcome, the presence of shirking can lead to big losses in expected benefits. Therefore, limiting shirking becomes crucial. The principal can lower shirking (raise η) only by increasing w_b. Using a particular utility function $u(w)=w^{1/2}$, I show in proposition 2 that w_b can actually exceed w_g.

Proposition 2: If the following three conditions hold, (i) $u(w) = w^{1/2}$; (ii) the monitoring cost 'a' very small; and (iii) s_g is sufficiently greater than s_b ; then in the optimal monitoring contract w_b^* is greater than w_g^*.

Proof: In appendix A.

I will discuss an artificial situation to show that it is indeed the difference in the values of s_g and s_b that is driving the result in proposition 2. Imagine that $s_g = s_b$. It is important to keep in mind that if this condition were true, the principal would never want to induce a positive η and it is only to make a point that I emphasize this condition. If there is no difference in value between the good and the bad outcome, then w_b^* is less than w_g^*. This can be easily demonstrated from the last step of the proof of proposition 2. Finally, proposition 3 shows that there are indeed monitoring costs for which monitoring contracts will be offered.



THE OPTIMAL MONITORING CONTRACT WHEN $w_g^* > w_b^*$

<u>Fig 3.</u>

Proposition 3: For small enough a, the optimal monitoring contract will dominate the optimal no-investigation contract.

Proof: The proof consists of three steps. In the first step I construct a contract. In the second step I show that my constructed contract will be a monitoring contract if monitoring costs are low. I demonstrate in the third step that this contract will dominate the optimal no-investigation contract if monitoring costs are low enough.

(i). Suppose $w_g = w_b = w_c$, with w_c satisfying $u(w_c) = e_h$. The constructed contract is $\{w_c, w_c\}$.

(ii). Claim: For $a < w_c$, the contract $\{w_c, w_c\}$ will be a monitoring contract with $0 < \alpha < 1$.

Proof: Arguments similar to those in Lemma 1 can be used to show that $\alpha < 1$. If $\alpha = 0$ then the agent would get a strictly higher utility from putting in e_l, which implies $\eta = 0$. But then the principal would always monitor if $a < w_c$.

(iii). Note that $0 < \alpha < 1$ implies (as in Lemma 1), $\eta = (w_c - a)/(w_c - a\,\pi_{gh})$ with $\lim_{a \to 0} \eta = 1$. Therefore,

$$\lim_{a \to 0} \left\{ (1 - \eta\pi_{gh})\,(s_b - w_c) + \eta\pi_{gh}(s_g - w_c) \right\} > \pi_{bh}\,(s_b - \tilde{w}_b) + \pi_{gh}(s_g - \tilde{w}_b)\,.$$

□

5. CONCLUSION

I have analyzed the optimal monitoring contract when the principal cannot commit to a monitoring startegy in the contract. I have obtained most of my results for very low monitoring costs. However I think I can use the expression for w_b^+ in the proof of proposition 1, to separate high and low monitoring costs. My conjecture is that, my results will hold when $w_b^{+} < u^{-1}\left(\frac{e_h}{\pi_{bh}}\right)$ (this is the condition which ensures that $w_g^{*} > 0$). Demonstrating this is left for future work. I would also like to show that the results in this paper carry over to the case where low effort can also result in the good outcome. Finally, the most interesting extension would be to incorporate private information together with the problem of moral hazard.

APPENDIX

Proof of Proposition 2:

It is easier to use the transformed problem where $u^{-1}(\,.\,) = v\,(\,.\,)$, when using the function $u\,(w) = w^{1/2}$. Substitute x for $u\,(w)$ and $v(x)$ for w in the lagrangian and use $v(x) = x^2$. Take derivatives w. r. t. x_g and x_b to get the following,

$$\theta = \eta\, v'(\,x_g\,) \tag{A1}$$

and
$$\theta = \frac{v(x_b)\, v'(x_b)}{v(x_b) - a\,\pi_{gh}} + \pi_{gh}\left\{s_g - v(x_g) - s_b + v(x_b)\right\}\frac{a\, v'(x_b)}{\left(v(x_b) - a\,\pi_{gh}\right)^2}\ . \tag{A2}$$

Using (19) and (20) to replace θ and simplifying I get,

$$\left\{v(x_b) - a\,\pi_{gh}\right\}\left\{\left(v(x_b) - a\right) v'(x_g) - v(x_b)\, v'(x_b)\right\} + a\,\pi_{gh}\left\{s_g - v(x_g) - s_b + v(x_b)\right\} v'(x_b) = 0\ . \tag{A3}$$

It is easy to see that $\lim_{a\to 0} v'(x_b) = \lim_{a\to 0} v'(x_g)$, which implies through (14a),

$$\lim_{a\to 0} x_b = \lim_{a\to 0} x_g = e_h. \tag{A4}$$

First I replace x_g by x_b using the binding constraint (14a). Then employing $v(x) = x^2$ and simplify I get,

$$F(x_b, a) = -x_b^5 + e_h x_b^4 + 2a\pi_{gh} x_b^3 + (1 - 3\pi_{gh}) a e_h x_b^2$$

$$+ \left\{ (s_g - s_b)\pi_{gh}^2 - a\pi_{gh}\pi_{bh} - e_h^2 \right\} a x_b + a^2 \pi_{gh} e_h = 0 . \tag{A5}$$

Keeping in mind that $F(x_b, a) = F(x_b(a), a)$, I take the derivative with respect to a,

$$\frac{dF(x_b(a), a)}{d\,a} = F_{x_b} \cdot x_b'(a) + F_a \equiv 0$$

or

$$x_b'(a) = -\frac{F_a}{F_{x_b}} . \tag{A6}$$

Taking partial derivatives of expression (A5) w. r. t. x_b and a, taking limits and using (A4) I get

$$\lim_{a\to 0} F_a = \pi_{gh}\left\{ (s_g - s_b)\pi_{gh} - e_h^2 \right\} e_h ,$$

and

$$\lim_{a\to 0} F_{x_b} = -e_h^4 .$$

Therefore,

$$\lim_{a\to 0} x_b'(a) = \frac{\pi_{gh}\left\{ (s_g - s_b)\pi_{gh} - e_h^2 \right\}}{e_h^3} . \tag{A7}$$

Depending upon the difference $(s_g - s_b)$, x_b is going to approach e_h from the right or left. This implies,

$$s_g - s_b \gtrless \frac{e_h^2}{\pi_{gh}} \quad \Leftrightarrow \quad x_b \gtrless x_g . \qquad \square$$

FOOTNOTES

1. e. g. Holmstrom (1979), Shavell (1979) and Harris and Raviv (1979).

2. e. g. Baiman and Demski (1980) and Dye (1986).

3. See Mukherjee and Png (1989) and Kanodia (1985) for some results on the interaction between the monitoring scheme and the rest of the contract.

4. It is not obvious and I have not checked that even if rewards were allowed, the principal would indeed reward the agent.

5. There is another way of explaining the agent's payoffs. He is paid w_i when s_i occurs. Investigation occurs after the payment is made. When shirking is revealed the principal imposes the maximum penalty, which can only be w_i, and the agent's payment net of penalties is zero.

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Chapter 4

Gathering information before signing a contract[1]

Asymmetry of information has been the cornerstone of much recent economic theory, but in reality, this asymmetry exists because agents acquire different pieces of information: The fundamental asymmetry lies in the ability to acquire information. This paper shows that there can be a substantial difference between assuming that there is asymmetry of information and assuming that there is asymmetry in the ability to gather information. More precisely, under this last assumption, there can be no asymmetry of information at equilibrium, although the contracts signed by a principal and an agent are substantially influenced by the fact that the agent *could* acquire information at very low cost.

We use a very simple adverse selection model. A principal offers a contract for the production of a good. Before deciding whether to accept it, the agent can, at some cost, acquire information on the disutility[2] of production. We have in mind the following type of situations. Firm A produces spare parts under contract for firm P, who suddenly asks whether some of them could be specially packaged. At the same time, it offers a schedule

[1]This paper, forthcoming in the *American Economic Review* was written jointly with Professor Jacques Crémer.

[2]In this paper, we have two costs: the cost of acquiring information and the cost of production. To avoid confusion, we will refer to this second cost as the "disutility of production". We are thankful to a referee for insisting that we clarify our terminology.

of payment dependent on the number of parts packaged. Firm A does not know precisely how much resources it will have to commit to this modification, but it can find out, at some cost. Our first problem is to determine whether firm P will offer a contract that will induce firm A to conduct this cost analysis, before accepting it, or whether it should make the contract attractive enough to induce it to sign without this analysis. It turns out that the second branch of this alternative is always preferable, and we identify the way in which firm P alters the contract to obtain this result.

Examples of actual economic situations in which our setup applies abound. Before trying to increase enrollment in principles, in exchange for special favors if he is successful, a professor may or may not conduct an enquiry on the difficulty of obtaining the desired result. Before accepting to clear land for a developer, a contractor can choose to gather more or less information on the specific piece of property. Finally, before accepting a new twist on an old telephone tariff, a firm can eyeball its acceptability or go through a detailed analysis of the changes.

The examples that we have chosen share important common characteristics. First, even though the agents are endowed with the same information, there is an asymmetry in the cost of accumulating information. This asymmetry is two fold:

- After the contract is signed, the agent will acquire information about the disutility of production at no cost.
- Before the contract is signed, the agent can discover at relatively low cost the disutility of production, and the principal cannot.

Second, the acquisition of information by the agent before the contract is signed is socially wasteful: he will in any case discover the disutility of production at lower cost later. Third, the threat of this acquisition of information has important strategic consequences, and yields informational rents to the agent. We have also chosen examples where lengthy negotiations and renegotiations are not very likely, and where it seems reasonable to assume that the decision to seek further information before signing the contract is a straightforward yes or no decision.[3]

Our model goes as follows: a principal must delegate the execution of a task to an agent. Once the contract is signed but before production begins, the agent will in any case discover the disutility of production, but he can also acquire this information before signing the contract, at some cost. We will assume that at the outset the principal and the agent are symmetrically informed, so that the terms of the contract that is proposed do not provide any information about the principal's information. We must first determine whether the principal will ever choose a contract that induces the agent to acquire information before signing. The answer is negative. The optimal contract never induces pre-contractual observation or pre-contractual investigation, to use the terminology of Craswell (1988).

As soon as the formal model is written down, it is obvious that the principal prefers agents whose cost of acquisition of information is high. We compute the increases in the expected welfare of the principal when this cost increases, and we show that they can be important. Although we do not model this explicitly, this indicates that the principal would be willing to spend relatively large sums to increase the cost of observation of the agent. Most of the action may take place before the drafting and the signing of the contract.

[3] It would be of great interest to integrate acquisition of information in a model where renegotiation is possible, but this has to be left to future work.

The model that we build has the added benefit of allowing us to treat in a unified framework two different types of adverse selection problems, those in which the agent is informed and those where he is not informed when he signs the contract.

In section II we examine the benefits for the principal of competition between agents. For instance our developer offers the contract to several contractors, and randomly chooses one among those who accept the contract. The value to the agents of precontractual investigation decreases, and the welfare of the principal increases. Although the result is technically straightforward, it points out a fact that is not true in informed or uninformed agent problems. Competition between agents limits the incentives for opportunistic behavior during the precontractual phase, it can therefore be beneficial even when agents are identical.

There has been remarkably little literature on the relationship between contract theory and the accumulation of information. Craswell studies a related question. His typical problem involves a contractor who must decide whether to accept to build a house. Before accepting the contract he spends resources investigating the disutility of production. Ex-post, after he begins work he obtains better information and breaches the contract if this disutility is too high. When the contract is signed the client sinks resources of his own in the project, and looses this investment in case of breach. Therefore, Craswell builds a model where precontractual investigation is productive, because it reduces the probability of breach. His problem is to identify the optimal legal environment under these circumstances. In a similar spirit Lee (1982) and Matthews (1984) study the incentives of agents to acquire information about the value of an object before participating in an auction where it is sold.

See also Milgrom (1981) and Milgrom and Weber (1982).

Sobel (undated) compares the payoff of the principal depending on the time of information acquisition by the agents. However, there is no cost to the agent to acquire the information earlier or later. See also Sappington (1983).

Finally Demski and Sappington (1986) study the acquisition of information by an agent. Their problem is different from ours because their principal is trying to encourage a reluctant agent to accumulate information.

I One agent

In this section, we consider the relationship between a principal and a single agent. The extension to the case of several agents will be presented in the next section.

A Formal description of the game

The task that the principal wants performed consists in the production of some quantity $q \geq 0$ of output, the precise amount to be determined as a function of the state of nature. The value of production to the principal is measured by a strictly concave, increasing, utility function V, where $V(q)$ represents the value, measured in dollars, that he attaches to q units of output. Without loss of generality we assume that $V(0)$ is equal to 0. To ensure that some production will take place in every state of nature, we assume that $V'(0)$ is infinite, and to ensure that production is always finite, we assume that $\lim_{q \to +\infty} V'(q) = 0$.

The disutility of producing q units is βq, $\beta > 0$, and the agent bears this disutility. Utility is transferable. If q units of output are produced and the principal pays the agent t

dollars, the payoff to the principal is $V(q) - t$ and the net benefit of the agent is $t - \beta q$.

All the parameters of the problem are known to both parties, except for the parameter β, which can take any of a finite number of strictly positive values, β_i, $i = 1, \ldots, n$, with

$$0 < \beta_1 < \beta_2 < \ldots < \beta_n.$$

(When we speak of state of nature i, we mean that β is equal to β_i.) The efficient production q_i^* in state of nature i is unambiguously determined by

$$V'(q_i^*) = \beta_i.$$

As soon as he has accepted the terms of the contract the agent learns β, and can use this knowledge in the determination of the amount to be produced. He can also observe β immediately after learning the terms of the contract proposed by the principal and before accepting them, but at a cost e.

This cost should be interpreted as the difference of cost between acquiring information in the precontractual and in the postcontractual phases. In most situations of interest, before beginning production the agent will have to spend resources to analyze the problem, conduct the necessary engineering studies, and organize workers. The cost e should be understood as the extra cost of acquiring, before signing the contract, enough information to evaluate the disutility of production. This extra cost should be strictly positive for the following reasons:

- There is less time to conduct the studies between the time at which the contract is offered and the time at which it must be accepted or turned down than would be optimal.

- Some of the studies could benefit from the cooperation of the principal, which is not provided before the contract is signed.

- The studies have to be done earlier and e refers to the interest lost by advancing the date of expenditures.

To summarize, the game between the two parties has 4 stages.

- At stage $t = 1$, the principal proposes a contract. This is his only move.

- At stage $t = 2$, with full information about the terms of the contract, the agent decides whether or not to observe the state of nature.

- At stage $t = 3$, the agent decides whether or not to accept the contract. If he does not the game ends at this point. If he does, we go to stage 4. (For definitiveness, we assume that when the agent is indifferent between accepting and not accepting the contract, he chooses to accept it.)

- At stage $t = 4$, the agent observes the state of nature (if at stage 2, the agent has decided to incur cost e and observe the state of nature, he does not learn anything new at this stage.) Then, he chooses the actions that he prefers according to the terms of the contract. When he is indifferent between several actions, he chooses the action that the principal prefers.

An observation of the state of nature at stage 2 has no productive function because we want to focus attention on its strategic consequences. As a consequence the expenditure of e by the agent at stage 2 is socially wasteful.

The contract, or mechanism, offered by the principal in the first stage is a triplet $\{\mathcal{M}, q, t\}$. In the fourth stage, when he knows the state of nature, the agent chooses a message $\mathbf{m}$ in the set $\mathcal{M}$ and sends it to the principal. He must then produce the amount $q(\mathbf{m})$ and receives a payment $t(\mathbf{m})$.

Let $\mathbf{m}_i$ be the message[4] chosen by the agent in state of nature i. The incentive compatibility constraint of the agent in state of nature i is written:

$$(1) \qquad t(\mathbf{m}_i) - \beta_i q(\mathbf{m}_i) \geq t(\mathbf{m}) - \beta_i q(\mathbf{m}) \quad \text{for all } \mathbf{m} \in \mathcal{M}.$$

Furthermore, for any $\mathbf{m} \neq \mathbf{m}_i$ for which (1) is an equality, the following inequality must hold:

$$(2) \qquad V(q(\mathbf{m}_i)) - t(\mathbf{m}_i) \geq V(q(\mathbf{m})) - t(\mathbf{m}).$$

Let $\pi_i > 0$ be the probability of state of nature i (ex ante the principal and the agent agree on these probabilities). If he has chosen not to observe the state of nature the agent will accept the contract if

$$(3) \qquad \sum_{i=1}^{n} \pi_i [t(\mathbf{m}_i) - \beta_i q(\mathbf{m}_i)] \geq 0.$$

The value of the contract to an agent who does not observe the state of nature is therefore

$$(4) \qquad \max\left[0, \sum_{i=1}^{n} \pi_i (t(\mathbf{m}_i) - \beta_i q(\mathbf{m}_i))\right].$$

[4] For completeness, we should take into account two possible complications:

1. The agent could choose different messages depending on the decision he has made to observe or not to observe the state of nature before accepting the contract,

2. If the agent is indifferent between several messages, we might have a mixed strategy equilibrium.

The reader will easily convince himself that we are not changing the results by assuming these complications away.

If he has observed the state of nature i, the agent will accept the contract if:

$$t(\mathbf{m}_i) - \beta_i q(\mathbf{m}_i) \geq 0. \tag{5}$$

If $i < j$, we have:

$$t(\mathbf{m}_i) - \beta_i q(\mathbf{m}_i) \geq t(\mathbf{m}_j) - \beta_i q(\mathbf{m}_j) > t(\mathbf{m}_j) - \beta_j q(\mathbf{m}_j). \tag{6}$$

Hence there exists a state of nature $\bar{\imath}$ such that the following inequalities hold:

$$t(\mathbf{m}_i) - \beta_i q(\mathbf{m}_i) \geq 0 \quad \text{for all } i \leq \bar{\imath},$$

$$t(\mathbf{m}_i) - \beta_i q(\mathbf{m}_i) < 0 \quad \text{for all } i > \bar{\imath}.$$

The agent will accept the contract if $i \leq \bar{\imath}$ and refuse it if $i > \bar{\imath}$ (if he always refuses it we let $\bar{\imath} = 0$). The value of the contract once he has decided to observe the state of nature, but before he actually does, is therefore

$$\sum_{i=1}^{\bar{\imath}} \pi_i[t(\mathbf{m}_i) - \beta_i q(\mathbf{m}_i)] - e, \tag{7}$$

which is equal to

$$\max_{j=1,\ldots,n} \left[\sum_{i=1}^{j} \pi_i[t(\mathbf{m}_i) - \beta_i q(\mathbf{m}_i)]\right] - e.$$

The agent will observe the state of nature when the value of the contract with observation is greater than the value without observation. From equations (4) and (7), he will therefore choose to observe if

$$\sum_{i=1}^{\bar{\imath}} \pi_i[t(\mathbf{m}_i) - \beta_i q(\mathbf{m}_i)] - \max\left[0, \sum_{i=1}^{n} \pi_i(t(\mathbf{m}_i) - \beta_i q(\mathbf{m}_i))\right] > e. \tag{8}$$

(If the left hand side is equal to e, the agent is indifferent between observing and not observing. We assume that he chooses not to observe.) A contract *induces pre-contractual* *observation* if inequality (8) holds and it does not induce pre-contractual observation in the opposite case. If a contract induces observation, there must be at least one state of nature in which the agent will accept it and in this case we must therefore have $\bar{\imath} \geq 1$.

If the contract does not induce observation, the payoff of the principal is

$$\sum_{i=1}^{n} \pi_i [V(q(\mathbf{m}_i)) - t(\mathbf{m}_i)]$$

if the agent accepts the contract and 0 otherwise. If it does induce observation, the payoff is

$$\sum_{i=1}^{\bar{\imath}} \pi_i [V(q(\mathbf{m}_i)) - t(\mathbf{m}_i)].$$

The principal chooses among all contracts a contract that maximizes his payoff.

B The optimal contract does not induce observation

We first prove that without loss of generality we can limit ourselves to contracts that do not induce pre-contractual observation.

Lemma 1 *To every mechanism that induces pre-contractual observation corresponds a mechanism that does not induce observation, provides the agent with the same utility and makes the principal better off (strictly better off if $e > 0$).*

Proof: The basic idea of the proof is very simple. Take a contract $\{\mathcal{M}, q, t\}$ that induces observation. The net outcome of this contract is production $q(\mathbf{m}_i)$ and transfer $t(\mathbf{m}_i)$ for $i \leq \bar{\imath}$, and production 0 and transfer 0 for $i > \bar{\imath}$. We first construct a new contract that does not induce precontractual observation and yields the same outcome. The principal will be indifferent between these two contracts, and the agent will prefer the second as he saves e.

We then improve on this second contract from the viewpoint of the principal, to make him strictly better off.

The new contract is $\{\mathcal{M}', q', t'\}$ where $\mathcal{M}' = \mathcal{M} \cup \{\text{no!}\}$ (we assume that the message no! does not belong to $\mathcal{M}$), and q' and t' satisfy:

$$q'(\mathbf{m}') = \begin{cases} q(\mathbf{m}') & \text{if } \mathbf{m}' \neq \text{no!} \\ 0 & \text{if } \mathbf{m}' = \text{no!}, \end{cases} \tag{9}$$

$$t'(\mathbf{m}') = \begin{cases} t(\mathbf{m}') & \text{if } \mathbf{m}' \neq \text{no!} \\ 0 & \text{if } \mathbf{m}' = \text{no!}. \end{cases} \tag{10}$$

It is clear that we have $\mathbf{m}'_i = \mathbf{m}_i$ for $i \leq \bar{\imath}$ and $\mathbf{m}'_i = \text{no!}$ for $i > \bar{\imath}$, and this proves the contract is equivalent to the original from the viewpoint of the principal, and at least as good from the viewpoint of the agent.

Assume now $e > 0$. We have

$$\sum_{i=1}^{n} \pi_i[t'(\mathbf{m}'_i) - \beta_i q'(\mathbf{m}'_i)] = \sum_{i=1}^{\bar{\imath}} \pi_i[t(\mathbf{m}_i) - \beta_i q(\mathbf{m}_i)] > e,$$

otherwise, it is not even worthwhile for the agent to observe the state of nature when offered $\{\mathcal{M}, q, t\}$. This implies that the agent would also accept without observation the contract $\{\mathcal{M}', q', t''\}$, where $t''(\mathbf{m}') = tf'(\mathbf{m}') - e$ for all $\mathbf{m}' \in \mathcal{M}'$, and this contract clearly makes the principal strictly better off. ∎

C The problem of the principal

For the remainder of this paper, to make the exposition simpler and more intuitive, we assume that there are only two states of nature. In the appendix we show that all our more important results hold if we relax this assumption. State of nature 1, where the disutility of production is low will be denoted l, whereas state of nature 2, where the disutility of production is high, will be denoted h. We have $\beta_h > \beta_l > 0$.

On the basis of Lemma 1, we can apply the revelation principle to restrict ourselves to contracts where $\mathcal{M} = \{h, l\}$, and where the agent's best strategy is to truthfully reveal the state of nature that he has observed: $\mathbf{m}_i = i$ for $i = h, l$. This enables us to simplify the notation and write q_i for $q(i)$ and t_i for $t(i)$. We could not easily apply the revelation principle before proving Lemma 1: it states that any contract is equivalent to a direct mechanism where the agent reveals his information. But the principal knows that the agent has no information at the outset!

On the basis of these remarks, we can write the principal's problem as follows:

$$\max_{q(\cdot),t(\cdot)} \sum_{i=h,l} \pi_i(V(q_i) - t_i)$$

(11)
$$\text{subject to} \begin{cases} t_l - \beta_l q_l \geq t_h - \beta_l q_h \\ -\pi_h(t_h - \beta_h q_h) \leq e \\ \pi_h(t_h - \beta_h q_h) + \pi_l(t_l - \beta_l q_l) \geq 0 \end{cases}$$

The first constraint is an incentive compatibility constraint which states that the agent will not choose to announce a high disutility of production when it is in reality low. The complete problem should also include another incentive compatibility constraint that states that he will not announce a low β when it is high:

$$t_h - \beta_h q_h \geq t_l - \beta_h q_l,$$

but, as usual in adverse selection problems, this constraint is not binding and we omit it.

The third constraint is an individual rationality constraint: The expected payoff of the agent is non-negative, and he will therefore accept the contract.

The second constraint expresses the fact that the optimal contract does not induce observation. Its totally general form would be:

$$(12) \qquad - \sum_{i=h,l} \pi_i \min[0, t_i - \beta_i q_i] \leq e.$$

The left hand side represents the value to the agent of refusing the contract in the states of nature where he would obtain a negative payoff. The inequality states that this quantity is less than the cost of observation. By the individual rationality constraint, we know that the agent would refuse the contract in at most one state, and equation (6) shows that this must be the state h. Hence the second constraint is equivalent to equation (12).

It is of some interest to note that in the two agents case, our problem is formally equivalent to a adverse selection problem with limited liability as in Sappington (1983). This is not true if there are more than two states of nature.[5]

In writing problem (11), we have omitted the non-negativity constraints on q_h and q_l. Because $V'(0) = +\infty$, these constraints will not be binding.

If $e = \infty$, problem (11) becomes a standard uninformed agent problem. The agent will never want to observe the state of nature. If $e = 0$, the problem is mathematically equivalent to a standard informed agent problem (see Baron and Myerson (1982)), but a small problem of interpretation arises. In our set-up, the agent is actually uninformed. There is no contradiction. The set of feasible contracts is the same when the agent is informed and when he can observe the state of nature at no cost. In both cases, his payoff must be non-negative in all states.

[5] A short note explaining the difference in detail for $n = 3$ is available from the authors.

D Solution: value to the principal of an increase in the cost of observation

The Lagrangian of problem (11) is:

$$\mathcal{L} = \pi_h(V(q_h) - t_h) + \pi_l(V(q_l) - t_l) + \lambda[(t_l - \beta_l q_l) - (t_h - \beta_l q_h)]$$

$$+ \mu[\pi_h(t_h - \beta_h q_h) + e] + \theta[\pi_h(t_h - \beta_h q_h) + \pi_l(t_l - \beta_l q_l)],$$

where the multipliers λ, θ, and μ are non-negative.

The problem (11) maximizes a concave function over a convex set and the first order conditions are necessary and sufficient for an optimum. We must have

$$\frac{\partial \mathcal{L}}{\partial q_h} = \pi_h V'(q_h) + \lambda\beta_l - (\theta + \mu)\pi_h\beta_h = 0; \tag{13}$$

$$\frac{\partial \mathcal{L}}{\partial q_l} = \pi_l V'(q_l) - \lambda\beta_l - \theta\pi_l\beta_l = 0; \tag{14}$$

$$\frac{\partial \mathcal{L}}{\partial t_h} = -\pi_h - \lambda + (\theta + \mu)\pi_h = 0; \tag{15}$$

$$\frac{\partial \mathcal{L}}{\partial t_l} = -\pi_l + \lambda + \theta\pi_l = 0. \tag{16}$$

Furthermore, the multipliers must be equal to zero for any non binding constraint.

Adding equations (15) and (16), and remembering that $\pi_h + \pi_l = 1$, we obtain

$$\theta + \mu\pi_h = 1, \tag{17}$$

which implies

$$1 \leq \theta + \mu \leq \frac{\theta}{\pi_h} + \mu = \frac{1}{\pi_h}. \tag{18}$$

Because the multipliers are positive, we must have $\theta \leq 1$ and $\mu \leq 1/\pi_h$. These multipliers represent the marginal values of relaxing the incentive compatibility and the no-observation constraint. Equality (17) shows that each of these marginal values can be computed from the other.

There are other noteworthy equalities. Multiplying (16) by β_l and adding to (14), we obtain

$$V'(q_l) = \beta_l, \tag{19}$$

and from (13), (15), and (18)

$$V'(q_h) = \beta_l + (\theta + \mu)(\beta_h - \beta_l) \geq \beta_h. \tag{20}$$

Equations (19) and (20) are standard features of adverse selection problems. The first, (19), states that production is efficient for low disutilities of production ($q_l = q_l^*$), while the second states that for high disutilities there is underproduction ($q_h < q_h^*$) as long as the no-observation constraint is binding ($\mu > 0$).

Equations (18) and (20) imply

$$V'(q_h) \leq \beta_l + \frac{\beta_h - \beta_l}{\pi_h}.$$

Therefore, there exists a lower bound $\underline{q}$ on production, independent of e.

We now compute the value of μ when e is small. In these cases, the no-observation constraint is binding whereas the individual rationality is not. Hence, by (17), μ must be equal to $1/\pi_h$. To see this formally, we first prove that θ cannot be strictly positive for small values of e.

Assume that $\theta > 0$. Then, the individual rationality constraint is strictly binding, and we have:

$$\pi_h(t_h - \beta_h q_h) + \pi_l(t_l - \beta_l q_l) = 0 \tag{21}$$

This implies

$$
\begin{aligned}
t_h - \beta_h q_h &= \pi_h(t_h - \beta_h q_h) + \pi_l(t_h - \beta_h q_h) \\
&= -\pi_l(t_l - \beta_l q_l) + \pi_l(t_h - \beta_h q_h) \qquad \text{(by equation (21))} \\
&\leq -\pi_l(t_h - \beta_l q_h) + \pi_l(t_h - \beta_h q_h) \qquad \text{(incentive compatibility)} \\
&= \pi_l q_h(\beta_l - \beta_h) \\
&\leq \pi_l \underline{q}(\beta_l - \beta_h) < 0.
\end{aligned}
$$

For e small enough, this contradicts the no-observation constraint. Therefore θ must be equal to 0 for small e, and μ is equal to $1/\pi_h$. We have written the Lagrangian under a form where μ represents the value to the principal of a *decrease* in the left hand side of the constraint

$$\pi_h(t_h - \beta_h q_h) \geq -e.$$

Therefore μ is the benefit of an increase in e, and, because it is equal to $1/\pi_h$, we have proved the following theorem:

Theorem 2 . *There exists a $\delta > 0$ such that for $e < \delta$, the marginal value for the principal of an increase in the cost of observation is $1/\pi_h > 1$.*

There is a high payoff to the principal of an increase in the cost of observation: an increase in a small e by one dollar will increase the expected profit by more than one dollar. As we prove in the appendix, the result is even stronger with an arbitrary number of states of nature: The theorem remains valid with π_h replaced by π_n, the probability of the highest disutility of production state. As we approach a continuum of states, we find $V'(q_n)$ going to infinity.

Let us try to give a more precise intuition for theorem 2. Start from a situation where the no-observation constraint is tight and the individual rationality constraint is not binding. If the cost of observation increases by one dollar, the principal can reduce the payoff to the agent in the high disutility state of nature by $1/\pi_h$ dollars and keep the no-observation constraint satisfied. But in this case, he can also reduce the payment in the low disutility state of nature by the same amount, because that payment is only constrained by the incentive compatibility constraint whose right hand side has decreased by $1/\pi_h$. Hence, the total profit to the principal increases by this amount.

The reasoning of the last paragraph suggests that the gain to the principal due to a small increase in e is entirely a transfer from the agent, there is no change of social welfare. This is true, for small increases of e from 0. Indeed, by equations (19) and (20), production is constant, as θ and μ are locally constant, and hence the sum of the utilities of the principal and the agent are constant. For larger increases in e, on the other hand, $\theta + \mu$ decreases, and efficiency improves.

We can be more precise. Define $\underline{q_h}$, q_h^*, $\underline{e}$, and $\overline{e}$ by the following equalities:

$$V'(q_h^*) = \beta_h,$$

$$\overline{e} = q_h^*(\beta_h - \beta_l)\pi_h\pi_l,$$

$$V'(\underline{q_h}) = \frac{\beta_h - \beta_l}{\pi_h} + \beta_l,$$

$$\underline{e} = \underline{q_h}(\beta_h - \beta_l)\pi_h\pi_l.$$

(In a standard informed agent problem, the optimal production in state h would be $\underline{q}_h$.)

These quantities satisfy

$$\underline{e} < \overline{e},$$

$$\underline{q_h} < q_h^*.$$

Let us also call $q_h(e)$ the optimal production when β is high, obtained from equation (20). We can then prove the following theorem:

Theorem 3 *The optimal production when the disutility of production is high varies as follows with e:*

$$e \leq \underline{e} \Longrightarrow q_h(e) = \underline{q_h},$$

$$\underline{e} \leq e \leq \overline{e} \Longrightarrow q_h(e) = \frac{e}{(\beta_h - \beta_l)\pi_h \pi_l},$$

$$\overline{e} \leq e \Longrightarrow q_h(e) = q_h^*.$$

The graph of $q_h(e)$ is represented on figure 1.

We prove theorem 3 through a sequence of three claims:

Claim 1 *The Lagrange multiplier θ can only be equal to zero when $e \leq \underline{e}$. In this case, $q_h(e) = \underline{q_h}$.*

By equations (16), (20), and (17), $\theta = 0$ implies $\lambda > 0$, $\mu = 1/\pi_h$ and $q_h = \underline{q_h}$. The no-observation constraint is binding, and we have

$$t_h = -\frac{e}{\pi_h} + \beta_h \underline{q_h}.$$

Substituting in the incentive compatibility constraint, we obtain the bound on e.

Claim 2 *The Lagrange multiplier θ can only be equal to one when $e \geq \overline{e}$. In this case, $q_h(e) = q_h^*$.*

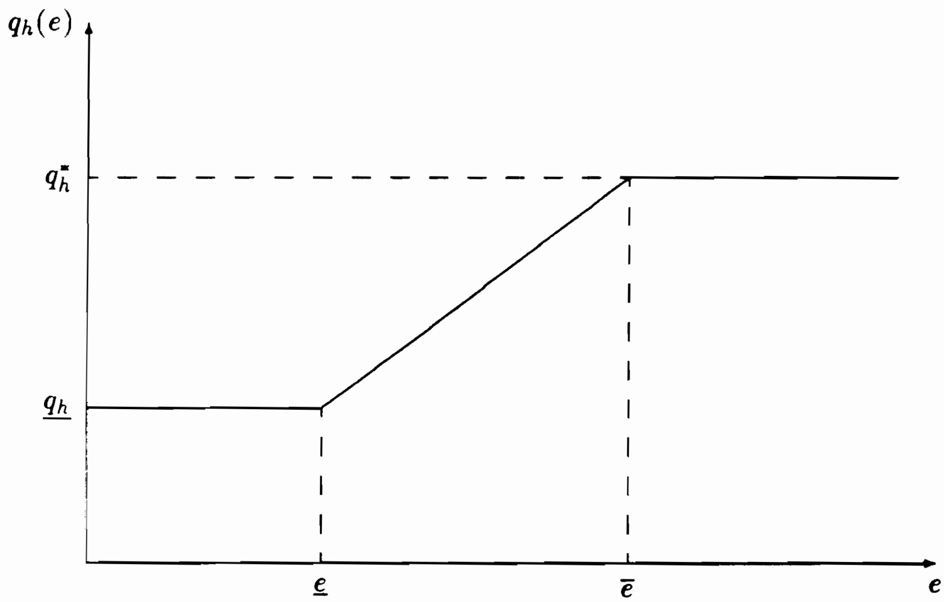

Figure 1: Optimal production in the high cost state of nature.

By equations (16), (17), and (20), $\theta = 1$ implies $\lambda = \mu = 0$ and $q_h = q_h^*$. The individual rationality constraint is binding, and we have

$$
\begin{aligned}
0 &= \pi_h(t_h - \beta_h q_h^*) + \pi_l(t_l - \beta_l q_l) && \text{(binding individual rationality)} \\
&\geq t_h - \pi_h \beta_h q_h^* - \pi_l \beta_l q_h^* && \text{(incentive compatibility)} \\
&\geq -e/\pi_h + q_h^*(\beta_h - \beta_l)\pi_l, && \text{(no-observation)}
\end{aligned}
$$

which proves the claim.

Claim 3 *The Lagrange multiplier θ can only belong to the interval* $(0,1)$ *if $\underline{e} < e < \overline{e}$. In this case,*

$$q_h(e) = \frac{e}{(\beta_h - \beta_l)\pi_h \pi_l}.$$

The first sentence follows immediately from claims 1 and 2. Equations (16) and (17) show that both λ and μ are strictly positive. All the constraints of problem (11) are binding, and the result follows from simple algebra.

Theorem 3 summarizes claims 1 to 3. When $e < \underline{e}$ or $e > \overline{e}$, small variations in e do not change production. On the other hand, with $\underline{e} < e < \overline{e}$ increases in e lead to more efficient production.

Very strong risk aversion on the part of the agents would substantially modify the results. To see this, take the extreme case where the agent's von Neuman-Morgenstern utility function is of the form

$$v(x) = \begin{cases} 0 & \text{if } x \geq 0, \\ -\infty & \text{if } x < 0. \end{cases}$$

Then, the individual rationality constraint

$$\pi_h v(t_h - \beta_h q_h) + \pi_l v(t_l - \beta_l q_l) \geq 0$$

is equivalent to

$$t_h - \beta_h q_h \geq 0,$$

$$t_l - \beta_l q_l \geq 0.$$

Whatever the state of nature the agent does not loose by accepting the contract, and therefore the no-observation constraint is not binding. The principal does not gain from an increase in ϵ, and hence μ is equal to 0. On the other hand, when the degree of risk aversion is low (when v' does not vary too much), μ is greater than 1. The proof is not very instructive as it follows the same general steps as the proof of Theorem 2, and we will not present it here.

II Several agents

The folk wisdom of transaction costs economics states that competition will mitigate opportunism. By putting potential contracting partners in competition with each other, a principal can limit the impact of asymmetry of information (see Williamson (1975), chapter 2). We know of little formal analysis of this statement (see however Hart (1983) and Scharfstein (1988)). In our setup, opportunism leads the agents to use resources to observe the state of nature, and we show that the principal is indeed better off when he can put several of them in competition. The results are technically straightforward, but, we believe, of some economic interest.

A Formal statement of the game

There are several potential agents, all identical. Although he needs only one to conduct the work, the principal offers a contract to all of them. We show that competition decreases the benefit to any one agent of observing β before signing the contract and that it improves the welfare of the principal.

We use the same notation as in section I but there are m agents. The value of production to the principal is still measured by the function $V(q)$, which has the same properties as above. The cost of production, β, is borne by the agents, is uncertain *ex ante*, never directly observable by the principal, and is independent of the agent who will conduct the work. Once an agent has signed the contract he observes the cost parameter. By incurring a cost e, any agent can observe the cost before accepting the terms of the contract.

The game between the $m + 1$ parties has therefore 4 stages.

- At stage $t = 1$, the principal proposes a contract, this is his only move.
- At stage $t = 2$, with full information about the terms of the contract, each agent decides whether or not to observe the state of nature. Let $\overline{m}^o$ be the number of agents who decide to observe.
- At stage $t = 3$, the agents decide whether or not to accept the contract. We call $\overline{m}^y$ the number of agents who accept it. If $\overline{m}^y = 0$, the game ends at this point. If $\overline{m}^y = 1$, the only agent that has accepted the terms of the contract conducts the work. If $\overline{m}^y > 1$, each of these agents is chosen with probability $1/\overline{m}^y$.
- At stage $t = 4$, the agent who has been chosen observes the state of nature and play

ends as in section I.

Note that the principal might wish to offer different contracts to the agents who have acquired information, but the acquisition of information is not observable by a third party, and this is impossible.

B Are contracts that do not induce observation optimal?

As in section I a contract, or mechanism, is a triplet $\{\mathcal{M}, q, t\}$, and we assume that a contract of this form is offered to all the agents. Is it optimal for the principal to offer a contract that will not induce the agents to observe the state of nature? The answer is more complicated than in the case of one agent, and we will not examine this question formally. First, because this is not necessary for our purposes. We only want to show that competition is useful to the principal. It suffices to show that one type of multi agents mechanism yields higher payoff than the one agent mechanism. Second, it requires the development of new notation which would distract from the main message of this paper. In an earlier version of this paper, available from the authors, we provide arguments to show that it is reasonable to restrict ourselves to contracts that induce all agents to sign without pre-contractual observation.

C Competition improves the welfare of the principal

The same incentive compatibility (1) holds in the framework of this section as in the case of one agent. The value of observing the state of nature before accepting the contract, conditional on the fact that the agent is chosen if he accepts, is the left hand side of equation (8). But, the agent will be chosen only with probability $1/\overline{m}^y$ and the expected value of observation is the unconditional value multiplied by this probability. It follows that the no-observation constraint is written:

$$\sum_{i=1}^{\bar{i}} \pi_i[t(\mathbf{m}_i) - \beta_i q(\mathbf{m}_i)] - \max[0, \sum_{i=1}^{n} \pi_i(t(\mathbf{m}_i) - \beta_i q(\mathbf{m}_i))] > e\overline{m}^y. \tag{22}$$

As discussed in the preceding subsection, we will only consider contracts that induce no agent to observe the state of nature, and all of them to accept the terms of the contract.

Turning back to the case where there are two states of nature, the problem of the principal becomes:

$$\max_{q(.),t(.)} \sum_{i=h,l} \pi_i(V(q_i) - t_i)$$

$$\text{subject to} \begin{cases} t_l - \beta_l q_l \geq t_h - \beta_l q_h \\ -\pi_h(t_h - \beta_h q_h) \leq me \\ \pi_h(t_h - \beta_h q_h) + \pi_l(t_l - \beta_l q_l) \geq 0 \end{cases} \tag{23}$$

This problem is exactly similar to problem (11), except for the no-observation constraint, whose right hand side has been modified. Increasing the number of agents is similar to increasing the cost of observation, e, in the one agent model. The following theorem follows immediately:

Theorem 4 *For e small enough, the expected payoff of the principal increases as the number of agents increases, and the marginal value of an increase in the cost of observation is proportional to the number of agents.*

The first part of this theorem states that increasing the number of agents increases the payoff of the principal. It points out that putting different agents in competition can be beneficial, even if the principal can bring a single agent to his reservation value, that is when he holds all the bargaining power. This property holds because the incentives of the agents to engage in pre-contractual gaming decrease when they have more competitors.

III Conclusion

In this paper, we stressed the role of asymmetry in the cost of acquisition of information, rather than in information itself. At equilibrium information can be symmetric, even if the agent finds observation to be very cheap, while the principal cannot accumulate it at any price. However, even though the information is symmetric, the terms of the contract can be substantially altered by the comparative advantage of the agent. This has very important consequences for applied work: principal-agent methods can be useful to study situations where *a priori* no party is better informed than any other. For instance, even though a supplier does not conduct engineering studies before signing a contract, the terms of the contract may be affected by his ability to do so.

Of course, much more work is needed before strong conclusions can be drawn: The timing of information acquisition could be crucial. For instance, the agent could accumulate information before being offered the contract: a public utility does research on the costs of investments before the hearings. Furthermore, in trading situations, both parties might find information about the other useful: the consulting firm needs data on the financial health of its client while the client needs data on the technical competence of the consultants. Much exciting work remains to be done.

Appendix

In this appendix we prove that theorem 2 holds when there are more than two states of nature. In this case problem (11) becomes

$$\max_{q(\cdot),t(\cdot)} \sum_{i=1}^{i=n} \pi_i(V(q_i) - t_i)$$

$$\text{(24)} \qquad \text{subject to} \begin{cases} t_i - \beta_i q_i \geq t_{i+1} - \beta_i q_{i+1} & \text{for } 1 \leq i < n, \\ -\sum_{i=j}^{i=n} \pi_i[t_i - \beta_i q_i] \leq e & \text{for all } j, \\ \sum_{i=1}^{i=n} \pi_i(t_i - \beta_i q_i) \geq 0. \end{cases}$$

The first constraints are the incentive compatibility constraints. We have, here again, eliminated all those that are irrelevant. The second set of constraints represent the no-observation constraints (see equation (7)). Finally, the last constraint is a standard individual rationality constraint.

Claim 1 *There exists a* $\underline{q}$ *such that for any* e *the optimal solution to problem (24) satisfies* $q_i > \underline{q}$ *for all* i.

Proof: Assume that the claim were not true. There would exist a state of nature k, and sequences of production $\{q_k^s\}_{s=1,\ldots,+\infty}$ and of observation costs $\{e^s\}_{s=1,\ldots,+\infty}$ such that q_k^s is an optimal q_k in the problem associated to e^s and such that

$$\lim_{s\to\infty} e^s = \lim_{s\to\infty} q_k^s = 0.$$

It is immediate to prove that if for s large enough we increase all q_i^s by a small enough $\epsilon > 0$ and t_i^s by $\beta_i\epsilon$, we obtain another feasible solution to problem (24), and that this solution is strictly better than the original because $V'(0) = +\infty$. The contradiction is established. ∎

Claim 2 *There exists an $\eta > 0$ such that for all e, $0 < e < \eta$, the payoff of the agent in state i, $t_i - \beta_i q_i$, is strictly negative if and only if $i = n$.*

Proof: When $e > 0$ the principal will never choose a contract with $t_i - \beta_i q_i \geq 0$ for all i and one of these numbers strictly positive. He would decrease all the t_i's by a small amount. This would not change the incentives of the agent to reveal his true cost and not to observe, and would increase the payoff of the principal.

Equation (6) shows that the payoff of the agent decreases when the cost of production increases. It is therefore sufficient to show that we cannot have $t_{n-1} - \beta_{n-1}q_{n-1} < 0$ when e is very small. Assume that this were the case. We would have:

$$\begin{aligned} t_n - \beta_n q_n &= (t_n - \beta_{n-1}q_n) - (\beta_n - \beta_{n-1})q_n \\ &\leq (t_{n-1} - \beta_{n-1}q_{n-1}) - (\beta_n - \beta_{n-1})q_n \quad \text{(incentive compatibility)} \\ &< -(\beta_n - \beta_{n-1})\underline{q}. \end{aligned}$$

Then $\pi_n(t_n - \beta_n q_n)$ would be bounded from above away from 0 and the no-observation constraint could not hold for very small e. ∎

Claim 2 implies that for $e < \eta$, we can rewrite problem (24) under the form

$$\max_{q(\cdot),t(\cdot)} \sum_{i=1}^{i=n} \pi_i(V(q_i) - t_i)$$

$$\text{subject to} \quad \begin{cases} t_i - \beta_i q_i \geq t_{i+1} - \beta_i q_{i+1} & \text{for } 1 \leq i < n, \\ -\pi_n[t_n - \beta_n q_n] \leq e & \\ \sum_{i=1}^{i=n} \pi_i(t_i - \beta_i q_i) \geq 0. & \end{cases}$$

The rest of the proof is exactly similar to that of theorem 2.

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Vita

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