

ESSAYS ON BIDS AND OFFER MATCHING IN THE LABOR MARKET

by

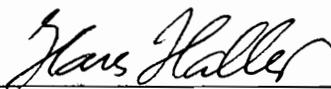
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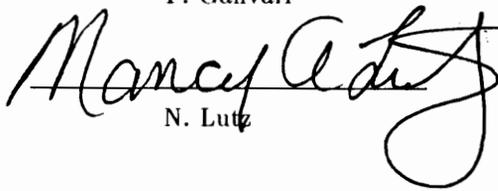
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(ABSTRACT)

This dissertation is a collection of essays on bids and offer matching in a labor market for new entrants to white-collar jobs. The papers compare some of the different institutions for determining wages and conducting the hiring process in the market for new entrants to white-collar jobs.

The first essay analyzes how does a firm announce and commit to a wage prior to deriving specific information about applicants' productivity and the consequences of following this hiring process. In the model there are two firms and at least as many applicants as the number of firms. All applicants apply simultaneously to both firms in response to the job advertisement which also mentions a wage. Each firm derives the firm-specific productivity of the applicants from their applications which is private information to each firm. None of the applicants have any information about the firms' evaluation. There are four pure strategy Nash Equilibria in wage announcements. Both firms announce a high wage, both firms announce a low wage, both firms announce a high or a low wage, and one firm announces a high wage and the other firm announces a low wage. In the latter case there also exists a unique mixed strategy equilibrium reflecting a firm's uncertainty about the choice of the other firm. In equilibrium one or both firms may not hire and the equilibrium may not exhibit wage dispersion.

The second essay analyzes the question; which is better, to announce and commit to a particular wage prior to deriving specific information about applicants' productivity or to offer wages privately after deriving the firm-specific productivity. The equilibrium policy, to be followed by the firms in the first place, is determined endogenously by comparing the ex ante expected profits associated with the equilibria under the different policies. Lack of prior information and the uncertainty about the possible match results in "*offer wages privately*" as always an equilibrium policy. However, if a low wage is the equilibrium strategy under all the policies, then "*any pair of policies*" is an equilibrium. This justifies one of the circumstances in which different policies might coexist. In equilibrium a firm's position is always filled and the equilibrium outcome may not exhibit wage dispersion.

The third essay analyses the question, if "*announcing a wage*" is the strategy rule to be followed by the firms, then what should be the equilibrium timing of wage announcement, before or after receiving specific information about applicants' productivity. Two policies are compared. Under the first policy a firm announces and commits to a particular wage prior to deriving the match-specific productivity. Under the second policy a firm solicits applications, derives the firm-specific productivity, and then announces and commits to a wage. The equilibrium timing of wage, to be followed by the firms in the first place, is determined endogenously by comparing the ex ante expected profits associated with the equilibrium strategy under the different timings. It turns out that announcing and committing to a particular wage after deriving specific information is always an equilibrium timing because of the informational advantage. However, if a low wage is the equilibrium strategy under all the policies then any pair of policies is an equilibrium. In equilibrium one of the firm's position may remain unfilled. The equilibrium outcome may not exhibit wage dispersion.

To
Ma
for everything!

This collection of essays is dedicated to Baba.

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Chapter 1

Theories of Wage Differentials

I. INTRODUCTION

The essential features of a perfectly competitive labor market is that workers who accept jobs should receive compensation equal to their opportunity cost. Firms pay a wage that is just sufficient to attract workers of the quality they need. Under the competitive framework firms can effortlessly find workers seeking employment and vice versa, since labor is fully mobile.

However, reality is quite different from this competitive paradigm. Observers of the labor market have long noted that large, systematic wage differences exist among industries. This is even true for workers who have similar observed characteristics and work in the same well-defined job classification.

Slichter's (1950) examination of the average hourly earnings of skilled and unskilled male workers in manufacturing between 1923 and 1946 revealed persistent industry wage differences over time. The pattern of these industry wage differences was similar for skilled and unskilled workers. Slichter also found that the average wage for unskilled workers was positively correlated with industry value added per worker and the profit margins.

Slichter along with many other researchers in this period - Reynolds (1951), Dunlop (1957) - concluded that differences in labor quality and liberal wage policies lead to interindustry wage differences. Several recent studies, discussed later, have also shown persistent industry wage differences even after controlling for a variety of worker quality and job characteristics.

From the search-theoretic point of view, it is often observed that there is a coordination problem in the labor market between firms and workers seeking employment. This coordination problem causes difficulty in forming matches. While some workers find jobs rapidly, others might have to spend more time before receiving an offer. Lang (1991) and Montgomery (1991) have given a search-theoretic explanation of industry wage differentials.

Theories on wage differentials can be broadly divided into three categories. Market clearing explanation includes unobserved ability differences and compensating wage differentials. Non-market clearing explanation postulates a variety of efficiency wage models. The most recent one is the search -theoretic explanation which focuses on the problem of coordination in the labor market and how it leads to industry wage differentials.

II. MARKET CLEARING EXPLANATIONS OF WAGE DIFFERENTIALS

The market clearing model offers two types of explanations for persistent interindustry wage differentials. These differentials may compensate for nonpecuniary differences in job characteristics or they may reflect differences in unmeasured labor quality. The theory of compensating wage differentials was originally conceived by Adam Smith.

Robert Smith (1979) summarized the concept of compensating wage differentials in the context of hedonic price theory. He exclusively focused on the tradeoff between wages and disagreeable job characteristics. A job can be characterized as consisting of numerous dimensions - pace of work, probability of injury, unpleasantness of tasks, and so on - which are elements of both the demand and supply functions relevant to the job. When a worker is hired, employer and employee agree to a single wage rate. This single wage rate embodies a series of implicit prices at which each of the job characteristics are traded. These prices are called the

compensating wage differentials.

Assume that all the job characteristics are traded at their market clearing value except risk of injury, whose value needs to be determined. To maintain a given level of profit, firms will reduce risk if they can simultaneously reduce the wage. So the firms' isoprofit curves are concave from below, reflecting the fact that greater safety is provided at increasing wage reductions. Firms differ in terms of technologies, products, and factor prices, but they compete in the same labor market. So the cost at the margin to achieve a given level of safety differs across firms. The envelope of the wage-risk offer curves of the different firms is the "market clearing implicit price" curve.

For higher risk of injury, workers need to be compensated at an increasing rate. So their indifference curves are convex from the bottom. Workers vary in their taste for job safety. Each worker maximizes his utility at the point where the indifference curve is tangent to the implicit price curve. Workers who value safety highly tend to accept jobs with firms that can offer it cheaply.

Numerous studies - Gordon (1973), Smith (1973), Thaler and Rosen (1975) - have shown that risk of injury or death or both have a significant positive effect on wage differentials. Results with respect to the risk of injury is not quite clear. Death rates by industry have a larger wage-risk tradeoff than death rates by occupation. Such differences arise because it might be possible that safe occupations in dangerous industries get paid more than equally safe occupations in safe industries. Studies attempting to test the theory of compensating wage differentials using job characteristics other than risk of injury have provided very little predictive power of the theory.

Katz and Summers (1989) in an attempt to test the effect of unmeasured labor quality on wage differentials found that most of the effect come from adding controls for occupation, region, and sex. Further proxies like education and experience reduce the wage differential. Katz

and Summers argued that high-wage-premium industries do not disproportionately employ highly educated or experienced workers sufficiently to significantly affect industry wage differentials.

Murphy and Topel (1987) concluded that unobserved ability accounts for the bulk of the industry wage differentials. But when sorting across industries is applied rather than sorting across both occupations and industries, the results are less favorable to the unobservable ability hypothesis. The same is true when education is taken as the sorting variable.

III. NON-MARKET CLEARING EXPLANATIONS OF WAGE DIFFERENTIALS

Competitive theory of labor market predicts that job attributes which do not directly affect the utility of workers should not affect the level of wages. Alternative theories that predict linkages between job attributes and wages has the property that at least some firms must be paying more than the market wage for the type they attract. This behavior can be rationalized only by assuming that some firms do not maximize profit or that some firms maximize profit by increasing wages above the market rate. The latter rationale is the defining characteristic of efficiency wage theories. There are four efficiency wage theories.

The labor turnover model - Salop (1979) - postulates that, if labor turnover is a decreasing function of wages, there may be an incentive to raise wages to minimize turnover costs of the firms. The shirking model - Shapiro and Stiglitz (1984) - predicts that increasing wages, increases workers' effort level. By raising wages the cost of losing a job increases and thereby encourage good performance. The fair wage model - Akerloff (1982), Akerloff and Yellen (1988) - says that workers' feeling of loyalty to their firm increases with the extent to which the firm shares its profit with them. This has a direct effect on productivity. The adverse selection

model - Weiss (1980, 1990) - is based on selection rather than incentive efforts. Firms which pay higher wages attract a higher quality pool of applicants. If quality is not directly observable, this will be desirable.

If firms differ in their ability to bear turnover cost, to monitor or to measure labor quality, either because of differences in management capacity or because of differences in the technology of production, then the optimal wage to pay will vary. Thus efficiency wage models can explain why firm attributes that do not directly affect workers' utility affect wage rates.

Dickens and Katz (1987a, b) and Krueger and Summers (1987, 1988) have provided a wide body of empirical evidences regarding the existence of wage differentials. They have shown that industry attributes like profitability and capital-labor ratio are positively correlated with these differentials.

In a recent interview survey, Blinder and Choi (1990), interviewed 19 managers with respect to different theories of wage stickiness. Four of the questions were framed in tune with the four models of efficiency wage hypothesis. The labor turnover, shirking, and fair wage wage postulates received a positive response but all the managers negated the adverse selection hypothesis.

IV. SEARCH THEORETIC EXPLANATION OF WAGE DIFFERENTIALS

Lang (1991) and Montgomery (1991) suggested that search theory may provide a more complete explanation of wage differentials. Though the generic "efficiency wage hypothesis" provides an explanation of interindustry wage differentials, different stylized facts seem to be better explained by different versions. The correlation between profitability and wage

differentials fits well with the fair wage model of efficiency wage hypothesis. The correlation between the average capital-labor ratio and wage differentials fits well with the monitoring model; shirking becomes a bigger problem in firms using expensive machinery. Also, the efficiency wage models ignore the large wage dispersion within narrowly defined industries. Groshen (1987, 1988) and Leonard (1988) provide evidence of significant intraindustry wage dispersion.

The basic argument in Lang's paper is: if it is difficult for firms and workers to form a match, then announcing a high wage in advance will increase the probability of filling a vacancy. Otherwise, the expensive machinery of highly profitable firms or firms with high capital-labor ratio would sit idle. Lang's model had the following structure. Firms differ in terms of their value of vacancy. Firms simultaneously announce a job opening and a wage in advance. Workers who are assumed to be of equal productivity can apply simultaneously to more than one firm. There will be wage dispersion in equilibrium, and some workers will randomly fail to receive any wage offers, provided that workers apply to at least two firms. The wage dispersion in equilibrium arises because of differences in the value of vacancy. Montgomery considered a similar situation but with stochastic applications. The results are the same. Lang and Montgomery argued that announcing a high wage in advance will reduce the probability of a firm's position remaining vacant because workers with better opportunities will not apply.

V. PREVIEW OF MY ESSAYS

During the course of the survey, I have outlined the development of the different theories of wage differentials. In my dissertation I have considered a labor market for new entrants to white-collar jobs. White-collar labor market is characterized by the fact that an

applicant can apply simultaneously to different firms and can also have more than one offer at the same time. New entrants to white-collar jobs do not have any information about their possible productivity in a firm at the time they are applying. Firms do not have any a priori information about the applicants' productivity but derive it from their applications. As aptly said by Mortensen (1986) - "Many relevant characteristics of a job-worker match cannot be observed without error but must be experienced. These experiences as they occur provide information about the expected future quality of a specific job-worker match relative to the set of alternatives. This process of learning about jobs and occupations as a means of finding a satisfactory place in the work world has long been thought an explanation for high turnover rates among young workers".

Given this lack of information, and taking into account match-specific productivity, in chapter 2 of my dissertation I have analyzed the consequences of announcing and committing to a wage in advance. The equilibrium wage to be announced depends on the constellation of parametric values. While the theories of wage differentials predict that workers with equal productivity receive different wages, two important equilibrium outcomes in this chapter are, workers with different productivities might receive the same wage, and workers with different productivities might receive different wages. Also, it is possible that workers with the same productivity receive different wages, and announcing a wage a priori might leave one or both firms' positions vacant.

In chapter 3, I have addressed the question whether it is better, to announce a wage a priori or to offer wages privately after deriving information about applicants' productivity. The latter scheme gives the firms the flexibility to alter their offers in the face of a rejection. The uncertainty associated with a firm's position getting filled when wages are announced a priori leads to offer wages privately as always an optimal policy.

In chapter 4, I have raised the question about the timing of wage announcement.

Whether firms should announce and commit to a wage prior to or after deriving specific information about applicants' productivity. The risk of a firm's position remaining vacant is reduced when a wage is announced after deriving information about applicants' productivity, and hence, turns out to be the optimal timing of wage announcement. In equilibrium a firm's position might remain vacant.

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Chapter 2

Bids in a Labor Market With Ex Ante Unobservable Productivity

I. INTRODUCTION

An advertisement published in one of the leading dailies in India read: Applications invited from young graduates, in the age group of 21 to 25 years, in any discipline for the post of Trainee Sales Executive. Lack of experience will not be a limiting factor. The pay package is Rs. 2500.00 per month.

There are a wide variety of institutions for determining wages and conducting the hiring process in the market for new entrants to white-collar jobs. The advertisement indicates that in some cases firms announce and commit themselves to a particular wage along with the announcement of a job opening. This paper analyzes how a firm selects a wage, and announces and commits to it before receiving specific information about its applicants' productivity, and the consequences of adopting such a hiring procedure. New entrants to white-collar jobs, say, trainee sales executives, may not know their suitability to a particular type of job, job satisfaction in a particular firm, and various other match-specific elements. In short, such an applicant may not know his possible productivity in a firm before he starts working. Firms also may not have prior information about an applicant's firm-specific productivity. I offer the following story.

The labor market consists of two firms - each having one job opening - and at least as many applicants. All applicants are new comers to white-collar jobs, like fresh college graduates, whose firm-specific productivities might differ across firms. Both firms simultaneously and costlessly

announce a wage along with the announcement of a job opening, in a newspaper. All applicants apply simultaneously to both the firms because they do not know their productivity in a firm and it is practically costless to apply - which might only include a letter of application and a resume. From the applications (and the interview), a firm derives specific information about the applicants' productivity which is private information. Each firm then makes at most one job-offer which depends on the wage it has announced. An applicant receiving one offer accepts it because he is not sure of getting any further offers. An applicant receiving offers from both firms accepts the highest one or randomly chooses one if both firms have announced the same wage. The rejected firm may or may not make any more offers.

As observed by Lang (1991), realistically, in white-collar jobs there is often a delay between the announcement of a job opening and the selection of an employee. This delay makes it possible for applicants to apply for a number of jobs simultaneously. Also, there is some time gap between the receipt of an offer and its acceptance by an applicant. During that time period, an applicant might receive a better offer and hence, reject the first one. This justifies the consideration of simultaneous job offers by firms. Both the firms and the applicants search nonsequentially (Wilde, 1977; Morgan and Manning, 1985), but in contrast to Wilde's model, where the firms play a passive role, here the applicants play a passive role.

There are four pure strategy combinations in wage announcements and depending on the parameter values two kinds of equilibria can result. In symmetric equilibria both firms announce a high wage or both firms announce a low wage. In asymmetric equilibria one firm announces a high wage and the other firm announces a low wage. In the case of symmetric equilibria each firm selects a low wage which is the payoff dominant strategy. However, in the case of asymmetric equilibria, there also exists a mixed strategy equilibrium. Since firms are advertising simultaneously, a firm may not know which other firm might advertise in the same newspaper. This lack of information does not matter if either a high or a low wage turns out to

be the dominant strategy. But the lack of information leads to the problem of coordination in the case of asymmetric equilibria in which case a firm needs to have some belief about the behavior of the other firm and this is reflected by the mixed strategy equilibrium (Harsanyi, 1973). The results depend on the relationship between the different parameters.

The uncertainty about hiring a high productivity applicant disappears if the number of applicants is large compared to the number of vacancies, in which case announcing a low wage is the dominant strategy. On the contrary, competition between the firms increases as the difference between the number of applicants and the vacancies diminishes and we can get the above mentioned results depending on the relationship between the different parameters.

Questions about firms' strategy to announce a wage in advance has been addressed by Lang (1991), Montgomery (1991), and Weitzman (1989). Lang and Montgomery analyzed this strategy, for white-collar jobs, to provide a search-theoretic explanation of interindustry and intraindustry wage differentials as an alternative to the efficiency wage hypothesis. Interindustry and intraindustry wage differentials are positively correlated with industry attributes like profitability and capital-labor ratio (Dickens and Katz, 1987; Krueger and Summers, 1987, 1988; Murphy and Topel, 1987).

The only equilibrium in a sequential search model (Stiglitz, 1979) involving search cost, in which workers receive one "take-it-or-leave-it" offer, is one of no search and no positions filled. Lang and Montgomery suggested that the strategy to announce a wage in advance, in a labor market where both firms and workers are searching nonsequentially, will reduce the probability of a firm's position remaining vacant because workers with better opportunities will not apply. They argued, that given coordination problems in the labor market, firms with higher opportunity cost of vacancy will pay higher wages to avoid expensive machineries remaining idle. In their models workers were assumed to have the same productivity but valuation of vacancies differed across firms and this resulted in persistent wage dispersion in equilibrium.

Weitzman showed that announcing and committing to a wage, which reflects wages being sticky in the short-run, leads to wage dispersion and job market segmentation. In Black and Loewenstein (1992) equilibrium variation in wages occurs due to variations in match-specific productivity.

In my paper, which describes a white-collar labor market for new entrants with sticky wages in the sense that firms a priori announce and commit to a wage, the equilibrium outcomes are more general. One or both the firms may not hire in equilibrium. Applicants with the same productivity might receive different wages, applicants with different productivities might receive the same wage, or applicants with different productivities might receive different wages. Thus the equilibrium outcome may but need not exhibit wage differential.

The paper is arranged as follows. In section II, I describe the model. In section III, I analyze the reduced game beginning after the post-application period. In section IV, I deduce the subgame perfect equilibria of the reduced game and discuss their implications in section V.

II. THE MODEL

I assume that there are n applicants, a_b ($b=1, 2, \dots, n$) and two firms F_1 and F_2 . The number of applicants is at least as large as the number of firms ($n \geq 2$). All applicants are newcomers to the white-collar labor force. For example, all applicants are college graduates with no prior work experience and are searching for jobs. Each firm has only one vacancy.

All applicants and firms know that an applicant can be of high productivity (H) or of low productivity (L) in a firm and these are firm-specific productivities. Productivity of an applicant in a firm is independent of his productivity in the other firm. So there are three

possibilities: an applicant may be of H-productivity in both firms, an applicant may be of H-productivity in one firm and of L-productivity in the other, or an applicant may be of L-productivity in both firms. There is heterogeneity in the model in terms of the firm-specific productivities of the applicants in the two firms.

All applicants and firms also know that there are two wages, a high wage (ω_h) and a low wage (ω_l), from which a firm chooses and announces. Let $W = \{\omega_h, \omega_l\}$. I assume $H > L$, $\omega_h \geq \omega_l$, $\omega_h \geq L$, $H - \omega_l > H - \omega_h \geq L - \omega_l \geq 0$. The last string of inequalities imply that a firm gets maximum profit if it hires an H-productivity applicant by announcing ω_l . Also, it is weakly more profitable to announce ω_h and hire an H-productivity applicant than to announce ω_l and hire an L-productivity applicant. The inequality $\omega_h \geq L$ implies that if a firm announces ω_h , it will hire, if any, only an H-productivity applicant.

Hiring by a firm depends on the wage announced by both the firms, productivity of the applicants in both the firms, and the recipients of the firms' job offer. The hiring process can be described as an extensive form game consisting of the following stages.

Stage 0: Let \hat{X}_{bi} be a family of independent random variables ($b=1,2,\dots,n; i=1,2$) where \hat{X}_{bi} assumes value H and L each with probability $\frac{1}{2}$. There are $\sum_{q=0}^n \binom{n}{q} = 2^n$ possible combinations of the firm-specific productivities of the n applicants in a firm, where $\binom{n}{q}$ means that q out of n applicants are of H-productivity in a firm. There are 2^{2n} possible combinations of the firm-specific productivities of the n applicants in the two firms. Each firm knows this distribution ex ante. Let $T^i = \{H, L\}^n$ be the set of all possible types of firm F_i ($i=1,2$), i.e., a type is a list of the firm-specific productivities, one for each applicant. T^i has 2^n elements which we label t_j^i ($j = 1, 2, \dots, 2^n$) such that (a) - (c) hold.

(a) $T_{2h}^i = \{t_j^i | j=1, \dots, 2^{n-1}\}$ be the set of types of F_i having at least two H-productivity applicants. This set has $2^n - n - 1$ elements.

(b) $T_{1h}^i = \{t_j^i \mid j=2^{n-n}, \dots, 2^n-1\}$ be the set of types of F_i having exactly one H-productivity applicant. This set has n elements.

(c) $T_1^i = \{t_{2^n}^i\}$ is a singleton with $t_{2^n}^i = (L, L, \dots, L)$.

Stage 1: In this stage the firms, simultaneously, announce and commit to a wage $\alpha_i \in A_i \equiv W$ ($i=1,2$), along with the announcement of a job opening.

Stage 2: In response to the advertisement in stage 1, all applicants apply simultaneously to both firms. I assume that there is no transaction cost to applying, hence, applying to both firms is the dominant strategy. From the applications each firm derives the firm-specific productivities of the applicants, i.e., each firm knows its own type which is private information. In this stage \hat{X}_{ji} is realized. Let X_{ji} be the realized value of \hat{X}_{ji} . An applicant has no information about the firms' evaluation.

Stage 3: In this stage both firms make their job offers simultaneously. Each firm can make at most one job offer. A firm having q H-productivity applicants ($q=1,2,\dots,n$) makes an offer to each with probability $\frac{1}{q}$. A firm has no information about the recipient of the job offer of the other firm. An applicant receiving offers from both firms accepts the highest offer. If both firms have announced the same wage then the applicant accepts each offer with probability $\frac{1}{2}$. An applicant receiving only one offer accepts it because he has no information about the firms' evaluation and hence is not sure of getting any further offers.

Stage 4: This stage regards the behavior of a rejected firm, if any, in stage 3. A rejected firm may or may not make an offer in this stage. This depends on the wage announced by that firm and the productivity of the remaining applicants.

I define the following variables; $\pi^l := L - \omega_l$, $\pi^h := H - \omega_h$, and $\omega := \omega_h - \omega_l$. Without loss of generality this model also holds if the firms announce a wage range, as long as the wage range either belongs to the high or low wage intervals which are nonoverlapping. For simplicity I have assumed that the firms announce a specific wage. Let us determine the equilibrium wage announced by the firms and its consequences on the firms' recruitment. I only consider the **subgame perfect equilibria** of the game in extensive form.

III. THE REDUCED GAME

At the pre-application stage, the firms F_1 and F_2 make simultaneous wage announcements, (α_1, α_2) , where $\alpha_i \in A_i \equiv W$ ($i=1,2$). For each $(\alpha_1, \alpha_2) \in A_1 \times A_2$, let $G(\alpha_1, \alpha_2)$ denote the subgame beginning immediately after the announcements α_1 and α_2 have been made. It turns out that each of the subgames $G(\alpha_1, \alpha_2)$ has unique subgame perfect equilibrium payoffs $\pi_i(\alpha_1, \alpha_2)$ for each firm F_i ($i=1,2$). Given the knowledge of these payoffs and the underlying equilibria, the analysis of the entire game amounts to the analysis of the **reduced game in normal form**:

$$\Gamma = (\{F_1, F_2\}, A_1, A_2, \pi_1, \pi_2).$$

The ex ante expected profits $\pi_1(\alpha_1, \alpha_2)$ and $\pi_2(\alpha_1, \alpha_2)$ are calculated by the process of backward induction. Let $P_h^i(\alpha_i, \alpha_m | t_j^i, t_k^m)$ be the conditional probability that a firm F_i hires an H-productivity applicant, if (α_i, α_m) are the wages announced by the firms given their types ($i, m = 1, 2; i \neq m$). Now F_m can be any one of the types with equal probability $\frac{1}{2^n}$. So the conditional probability that F_i hires an H-productivity for each $t_j^i \in \{T_{2h}^i \cup T_{1h}^i\}$ is

$$P_h^i(\alpha_i, \alpha_m | t_j^i) = \frac{1}{2^n} \sum_{k=1}^{2^n} P_h^i(\alpha_i, \alpha_m | t_j^i, t_k^m).$$

A firm F_i can have at least two H-productivity applicants in $\sum_{q=2}^n \binom{n}{q} = 2^n - n - 1$ possible ways and it can have exactly one H-productivity applicant in n possible ways.

So the probability that a firm F_i hires an H-productivity applicant, if (α_i, α_m) are the wages announced by the firms is

$$P_h^i(\alpha_i, \alpha_m) = \frac{2^n - n - 1}{2^n} \sum_{t_j^i \in T_{2h}^i} \left(\frac{1}{2^n} \sum_{k=1}^{2^n} P_h^i(\alpha_i, \alpha_m | t_j^i, t_k^m) \right) + \frac{n}{2^n} \sum_{t_j^i \in T_{1h}^i} \left(\frac{1}{2^n} \sum_{k=1}^{2^n} P_h^i(\alpha_i, \alpha_m | t_j^i, t_k^m) \right)$$

The probability that F_i hires an L-productivity applicant, if at all, is

$$P_l^i(\alpha_i, \alpha_m) = P_h^i(\alpha_i, \alpha_m | t_8^i) + (1 - P_{1h}^i(\alpha_i, \alpha_m | T_{1h}^i)).$$

So the ex ante expected profit of a firm F_i ($i=1, 2$) is:

$$\pi_i(\alpha_i, \alpha_m) = (H - \alpha_i) \times P_h^i(\alpha_i, \alpha_m) + (L - \alpha_i) \times P_l^i(\alpha_i, \alpha_m).$$

Main Lemma.

The payoff functions π_1 and π_2 of the reduced game Γ are represented in the following matrix:

		A_2	
		ω_h	ω_l
A_1	ω_h	$\frac{2^n(2^{n+1} - 3) + 1}{2^{2n+1}} \pi^h$	$\frac{2^{n+1}(2^n - 1) \pi^h}{2^{2n+1}}$
	ω_l	$\frac{2^n(2^{n+1} - 3) + 1}{2^{2n+1}} \pi^h$	$\frac{(2^{2n+1} - 2^{n+2} + 2)(\pi^h + \omega) + (2^{n+2} - 2) \pi^l}{2^{2n+1}}$
ω_1	ω_h	$\frac{(2^{2n+1} - 2^{n+2} + 2)(\pi^h + \omega) + (2^{n+2} - 2) \pi^l}{2^{2n+1}}$	$\frac{2^n(2^{n+1} - 3)(\pi^h + \omega) + 3(2^n) \pi^l}{2^{2n+1}}$
	ω_l	$\frac{2^{n+1}(2^n - 1) \pi^h}{2^{2n+1}}$	$\frac{2^n(2^{n+1} - 3)(\pi^h + \omega) + 3(2^n) \pi^l}{2^{2n+1}}$

The derivation of the payoffs is shown in appendix A.

IV. COMPLETE EQUILIBRIUM ANALYSIS

We now exploit the information condensed in the main lemma and the insights gained in its derivation, to determine the equilibrium set of the entire game. This is achieved by determining the equilibria of the reduced game Γ . First I shall state and prove the Nash Equilibria in pure strategies and then I will show the existence of Nash Equilibria in mixed strategies. I shall use a few general terms.

$$\begin{aligned} A &:= \pi_1(\omega_h, \omega_h) - \pi_1(\omega_l, \omega_h) = \pi_2(\omega_h, \omega_h) - \pi_2(\omega_h, \omega_l) \\ &= \frac{1}{2^{2n+1}} \left((2^n - 1)\pi^h - (2^{n+2} - 2)\pi^l - (2^{2n+1} - 2^{n+2} + 2)\omega \right) \end{aligned} \quad [1]$$

$$\begin{aligned} B &:= \pi_1(\omega_h, \omega_l) - \pi_1(\omega_l, \omega_l) = \pi_2(\omega_l, \omega_h) - \pi_2(\omega_l, \omega_l) \\ &= \frac{1}{2^{2n+1}} \left(2^n \pi^h - 3(2^n)\pi^l - 2^n(2^{n+1} - 3)\omega \right) \end{aligned} \quad [2]$$

$$\text{Then, } B - A = \frac{1}{2^{2n+1}} \left(\pi^h + (2^n - 2)(\pi^l - \omega) \right) \quad [3]$$

which a priori can be of any sign.

IV.1. Pure Strategy Equilibria

If $(B - A) \geq 0$ we get the following results.

Proposition 1

If $A \geq 0$, then (ω_h, ω_h) is the *unique Nash Equilibrium* wage announcement strategy profile.

Proof: Since $B - A \geq 0$, $A \geq 0 \Rightarrow B \geq 0$. Hence, (ω_h, ω_h) is the unique Nash Equilibrium. \square

Proposition 2

If $B \leq 0$, then (ω_l, ω_l) is the *unique Nash Equilibrium* wage announcement strategy profile.

Proof: Since $B - A \geq 0$, $B \leq 0 \Rightarrow A \leq 0$. Hence, (ω_l, ω_l) is the unique Nash Equilibrium. \square

Proposition 3

If $B \geq 0 \geq A$, then (ω_h, ω_l) and (ω_l, ω_h) are the two *asymmetric Nash Equilibrium* wage announcement strategy profiles.

Proof: If $B \geq 0 \geq A$, then announcing ω_l (or ω_h) is a firm's best response to the announcement of ω_h (or ω_l) by the other firm. Hence, (ω_h, ω_l) and (ω_l, ω_h) are the two asymmetric equilibria. \square

If $B - A \leq 0$ we get the following results. This inequality does not hold when $H = 2L$.

Proposition 4

If $B \geq 0$, then (ω_h, ω_h) is the *unique Nash Equilibrium* wage announcement strategy profile.

(The proof is similar to the proof of proposition 1)

Proposition 5

If $A \leq 0$, then (ω_l, ω_l) is the *unique Nash Equilibrium* wage announcement strategy profile.

(The proof is similar to the proof of proposition 2.)

Proposition 6

If $A \geq 0 \geq B$, then (ω_h, ω_h) and (ω_l, ω_l) are the two *symmetric Nash Equilibrium* wage

announcement strategy profiles. In this case, the firms will choose the *strictly payoff-dominant* strategy profile (ω_1, ω_1) .

Proof: If $A \geq 0 \geq B$, then announcing ω_1 (or ω_h) is a firm's best response to the announcement of ω_1 (or ω_h) by the other firm. $\pi_i(\omega_h, \omega_h) - \pi_i(\omega_1, \omega_1) = \frac{1}{2^{2n+1}}(\pi^h - 3(2^n)\pi^1 - 2^n(2^{n+1} - 3)\omega) < B \leq 0$. Hence, (ω_1, ω_1) is the strictly payoff-dominant strategy. \square

IV.2. Mixed Strategy Equilibria

Let $(p, (1 - p))$ be the probability distribution over the pure strategy space $A_2 \equiv W = \{\omega_h, \omega_1\}$ of firm F_2 and $(r, (1 - r))$ be the same over the pure strategy space $A_1 \equiv W = \{\omega_h, \omega_1\}$ of firm F_1 , $(p, r \in [0, 1])$. This means that if F_1 is uncertain about the choice of F_2 , F_1 believes that F_2 chooses ω_h and ω_1 with probabilities p and $(1 - p)$, respectively, (Harsanyi 1973). The probabilities r and $1 - r$ can be analogously interpreted. Given F_1 's belief, let us calculate its best response, $r^*(p)$. Since this is a symmetric situation F_2 's best response to r , denoted by $p^*(r)$, is the same.

Proposition 7

There exists a unique mixed strategy equilibrium $(r^*(p) = \frac{B}{B - A}, p^*(r) = \frac{B}{B - A})$ if and only if there are two asymmetric Nash Equilibria, (ω_h, ω_1) and (ω_1, ω_h) , in pure strategies.

Proof: Let $E\pi_1(r; p)$ denote the expected payoff of F_1 when that firm chooses ω_h and ω_1 with probability r and $(1 - r)$, respectively, given its belief p about the behavior of F_2 .

$$E\pi_1(r; p) = \frac{1}{2^{2n+1}} \left\{ rp(-\pi^h - (2^n - 2)(\pi^1 - \omega)) + r(2^n\pi^h - 3 \cdot 2^n\pi^1 - 2^n(2^{n+1} - 3)\omega) + p(-(2^n - 2)\pi^h + (2^n - 2)(\pi^1 - \omega)) + 2^n(2^{n+1} - 3)(\pi^h - \omega) + 3 \cdot 2^n\pi^1 \right\}$$

$$= -rp(B - A) + rB - p(2^n - 1)\pi^h + pB + 2^n((2^{n+1} - 3)(\pi^h - \omega) + 3\pi^l)$$

Firm F_1 chooses r so as to maximize its expected payoff, given its belief p and $(1 - p)$ about F_2 's choice of ω_h and ω_l , respectively. We get:

$$\frac{dE\pi_1(r; p)}{dr} = -p(B - A) + B = 0 \Rightarrow p = \frac{B}{B - A} \in [0, 1].$$

The condition $1 \geq \frac{B}{B - A} \geq 0$ is fulfilled only in the following two cases:

(i) $B \geq 0$, $B - A > 0$, and $A \leq 0$. This is the same as the condition for the two asymmetric Nash Equilibria in pure strategies, (ω_h, ω_l) and (ω_l, ω_h) , stated in proposition 3.

(ii) $B \leq 0$, $B - A < 0$, and $A \geq 0$. This is the same as the condition for the two symmetric Nash Equilibria in pure strategies, (ω_h, ω_h) and (ω_l, ω_l) , stated in proposition 6. In this case I have shown that (ω_l, ω_l) is the strictly payoff-dominant strategy. So if the above inequalities hold then each firm will choose ω_l with probability 1. Since ω_h is a strictly dominated strategy for each firm, there is no belief that a firm F_i could hold about the strategies of the other firm such that it would be optimal to choose ω_h (Pearce 1984).

So there can be a mixed strategy equilibrium only when there are two asymmetric equilibria in pure strategy (case i). The best-response correspondence of F_1 is as follows. $\frac{dE\pi_1}{dr} > 0$ if $p < \frac{B}{B - A}$ in which case $r^*(p) = 1$, i.e., F_1 will choose ω_h . $\frac{dE\pi_1}{dr} < 0$ if $p > \frac{B}{B - A}$ in which case $r^*(p) = 0$, i.e., F_1 will choose ω_l . If $p = \frac{B}{B - A}$ then $r^*(p) = [0, 1]$. The analysis is symmetric for F_2 . The intersection of the best-response correspondences $r^*(p)$ and $p^*(r)$ yields the unique mixed strategy Nash Equilibrium $r^*(p) = p^*(r) = \frac{B}{B - A}$. \square

V. IMPLICATIONS OF THE EQUILIBRIA

Let us analyze the implications of the various equilibria asserted in propositions 1–7. If (ω_l, ω_l) is the unique Nash Equilibrium wage announcement strategy profile then both firms hire

an applicant and neither positions remain vacant. There can be three possible cases. Both firms hire an H-productivity applicant, both firms hire an L-productivity applicant, or one firm hires an H-productivity applicant and the other firm hires an L-productivity applicant. The third situation implies applicants having productivity differences receive the same wage.

On the contrary if (ω_h, ω_h) is the unique Nash Equilibrium strategy, then either one or both firms' positions might remain vacant. Suppose only the same applicant is of H-productivity in both firms. Then both firms will make the job offer to the same applicant who will accept the offer of one of the firms randomly. The rejected firm's position will remain vacant because the other two applicants are of L-productivity in that firm. Also it might be the case that after announcing ω_h , both firms find that all the applicants are of L-productivity. Then neither firm will hire. If one firm had only L-productivity applicants and the other firm had at least one H-productivity applicant then the former will not hire.

The two asymmetric equilibria in pure strategies, (ω_h, ω_l) and (ω_l, ω_h) , result because of the particular relationship between high and low productivity and that between high and low wage. In this case the firm announcing a high wage either hires an H-productivity applicant or do not hire. The firm announcing a low wage either hires an H or an L-productivity applicant. Thus both the firms might hire an H-productivity applicant but offer them different wages. In Lang (1991) a wage differential occurs because of the difference in the value of the vacancies across firms and in Black - Loewenstein 1992 the same results because of the difference in the firm-specific productivities of the applicants across firms.

Realistically, when a firm advertises a job opening, it has no prior information about what other firms might advertise in the same newspaper. This lack of prior information does not matter if the relationship between the parameters are such that announcing a high wage or a low wage is the dominant strategy. However, lack of information leads to the problem of coordination if the relationship between the parameters are such that asymmetric equilibria

result. The mixed strategy equilibrium reflects a firm's belief about the other firm's wage selection, in the face of uncertainty which arises when there is asymmetric equilibria.

In this paper I have assumed that the number of applicants is at least as much as the number of firms which is two ($n \geq 2$). If the number of applicants is very large, then a firm surely hires an H-productivity applicant, irrespective of the wage announced by the firms. That is, $\lim_{n \rightarrow \infty} P_h^i(\alpha_1, \alpha_2) \rightarrow 1 \forall (\alpha_1, \alpha_2) \in A_1 \times A_2 \equiv W \times W$. In such a situation announcing a low wage is the strictly dominant strategy and it does not matter whether firms announce and commit to a wage a priori or offer wages privately, after deriving the applicants firm-specific productivity. In general, as the difference between the number of jobs and the number of applicants decreases, the increased competition between the firms reduces the probability of hiring an H-productivity applicant. Hence, different equilibria in wage announcements are possible depending on the relationships between the parameters.

Fact: If there is one possible productivity and each firm and each applicant knows that, then (ω_1, ω_1) is the *unique Nash Equilibrium strategy* of both firms.

Proof: Suppose $H = L$ and every applicant and every firm knows it. Then both firms would announce ω_1 and still be able to hire an applicant. At worst, both firms might make the job offer to the same applicant who will select one firm randomly. The rejected firm will then make an offer to one of the remaining two applicants and will be able to hire him because of the assumption that an applicant receiving only one offer accepts it. Thus both firms will have their positions filled. □

This result depends on the assumption that the number of applicants is at least as large as the number of jobs. If the number of jobs exceeded the number of applicants whose

productivity are the same across all firms then all firms would have announced a high wage. Commonly, for unskilled jobs in manufacturing or jobs in restaurants workers do not have any difference in their productivity. The above fact describes this situation and it does not matter whether the wage is announced or offers are made privately to individual workers. There is usually no delay between the announcement of an opening and the selection of an employee. Frequently, firms hire the first person to show up after a job opening in such occupations.

If $\omega_h = H$ and $\omega_l = L$, then it is optimal for the firms to announce a low wage. A firm can either make a profit of $(H - \omega_l)$ if it hires an H-productivity applicant, or make zero profit if it hires an L-productivity applicant. On the contrary, if a firm announces ω_h then it will be restricted to hiring an H-productivity applicant only, and whether the firm hires or not it will always make zero profit. Even if the wages are set as $\omega_h = H$ and $\omega_l = L$, the inequalities $\omega_h \geq L$ and $H - \omega_l > H - \omega_h \geq L - \omega_l \geq 0$ hold because $H - \omega_h = L - \omega_l = 0$ and $\omega_h = H > L$.

VI. CONCLUSION

In this chapter firms' mechanism of hiring employees who are new-comers to the labor force by announcing a wage a priori was analyzed. Firms announced and committed to a particular wage without having and receiving any specific information about applicants' productivity. Firm-specific productivity was taken into consideration and it was assumed that an applicant's productivity in a firm was independent of his productivity in another firm.

Competition between the firms and the composition of the productivity of the applicants in the firms endogenously determined the probability of hiring a high and a low productivity applicant which in turn determined the ex ante expected profits of the firms. The probability of hiring a high and a low productivity applicant also depended upon the number of

applicants. There were three (pure strategy) Nash Equilibria in wage announcement: both firms announced a high wage, both firms announced a low wage, or one firm announced a high wage and the other firm announced a low wage. The problem of coordination and the uncertainty about the wage announced by each firm, in the case of asymmetric equilibria, led to a mixed strategy equilibrium which reflected a firm's belief about the other firm's choice of strategy.

One important feature of the equilibrium outcome was that either one or both firms' position remaining vacant though the number of applicants was at least as much as the number of jobs. Also, the equilibrium outcomes may not exhibit wage differential. Applicants with different productivities might receive the same wage or applicants with the same productivity might receive different wages. The other possibilities were, applicants with different productivities might receive different wages or applicants with the same productivity might receive the same wage.

It was also seen that as the difference between the number of applicants and the number of job openings diminished, the uncertainty in hiring a high productivity applicant increased. However, if the number of applicants is very large compared to the number of vacancies, then the optimal strategy for each firm was to announce a low wage because each firm could surely hire a high productivity applicant.

APPENDIX A

In this section I will analyze each of the subgames $G(\omega_h, \omega_h)$, $G(\omega_l, \omega_l)$, $G(\omega_h, \omega_l)$, and $G(\omega_l, \omega_h)$. Since there are two firms each firm can hire in at most two rounds. If a firm has at least two H-productivity applicants that firm hires an H-productivity applicant for sure

irrespective of the wage announced and the type of the other firm.

A firm can have at least two H-productivity applicants in $\sum_{q=2}^n \binom{n}{q} = (2^n - n - 1)$

number of situations. T_{2h}^i is the set of types of a firm F_i having at least two H-productivity applicants.

So $P_h^i(\alpha_i, \alpha_m | t_j^i) = \frac{1}{2^n}$ for each $t_j^i \in T_{2h}^i$. Therefore, $P_h^i(\alpha_i, \alpha_m | T_{2h}^i) = \frac{2^n - n - 1}{2^n}$.

A firm can have exactly one H-productivity applicant in $\binom{n}{1} = n$ number of situations. T_{1h}^i is the set of types of a firm having one H-productivity applicant. So the conditional probability that a firm hires an H-productivity applicant if there is exactly one of them, is

$$P_h^i(\alpha_i, \alpha_m | T_{1h}^i) = \sum_{t_j^i \in T_{1h}^i} \sum_{t_k^m \in T^m} P_h^i(\alpha_i, \alpha_m | t_j^i, t_k^m).$$

A firm having only L-productivity applicants is represented by t_{2n}^i . So the overall probability of hiring an H-productivity applicant is $P_h^i(\alpha_i, \alpha_m) = P_h^i(\alpha_i, \alpha_m | T_{2h}^i) + P_h^i(\alpha_i, \alpha_m | T_{1h}^i)$. In each of the subgames I need to derive a firm's probability of hiring an H-productivity applicant if there is exactly one of them, and hence, calculate the total probability of hiring an H-productivity applicant and the ex ante expected profits.

A.1. Analysis of $G(\omega_h, \omega_h)$: Both firms announce ω_h .

In this case each firm will hire, if any, an H-productivity applicant only. I calculate the probability that F_1 hires an H-productivity applicant which would be the same for F_2 .

Suppose F_1 has one H-productivity applicant and it is of type t_{2n-n}^1 , i.e., applicant a_1 is the only H-productivity applicant in that firm. There can be two possible situations:

(i) Applicant a_1 is of L-productivity in F_2 . Then F_1 hires the H-productivity applicant. The

conditional probability that this situation occurs is $\frac{2^{n-1}}{2^n}$, because there are 2^{n-1} out of 2^n arrangements where a_1 is of L-productivity in F_2 .

(ii) Applicant a_1 is of H-productivity in F_2 .

(a) All applicants are H-productivity in F_2 . The conditional probability that F_2 makes the offer to a_1 and F_1 hires him is $= \frac{1}{2^n}$. If F_2 makes the offer to any one of the other $n-1$ applicants then F_1 hires a_1 , and this occurs with the conditional probability $\frac{n-1}{n}$. So the conditional probability that F_1 hires a_1 given that all applicants are of H-productivity in F_2 is $\left(\frac{1}{2^n} + \frac{n-1}{n}\right)$.

(b) F_2 has $n-1$ H-productivity applicants including a_1 . There are $\binom{n-1}{n-2}$ such types out of 2^n types. The conditional probability that F_2 makes the offer to a_1 and F_1 hires him is $\frac{1}{2}$ $\left(\frac{1}{n-1}\right) = \frac{1}{2n-2}$. If F_2 makes the offer to any one of the other H-productivity applicants then F_1 hires a_1 for sure and this occurs with conditional probability $\frac{n-2}{n-1}$. So the conditional probability

that F_1 hires a_1 given that F_2 has $n-1$ H-productivity applicants including a_1 is $\frac{\binom{n-1}{n-2}}{2^n} \left(\frac{n-2}{n-1} + \frac{1}{2n-2}\right)$.

In general if F_2 has $n-q$ H-productivity applicants including a_1 ($q=0,1,2,\dots,n-1$) then the

conditional probability that F_1 hires a_1 is $\frac{\binom{n-1}{n-q-1}}{2^n} \left(\frac{n-q-1}{n-q} + \frac{1}{2(n-q)}\right)$

The conditional probability that F_1 hires a_1 given that F_2 has $n-q$ H-productivity applicants

($q=0,1,\dots,n-1$) including a_1 is $c = \sum_{q=0}^{n-1} \frac{\binom{n-1}{n-q-1}}{2^n} \left(\frac{n-q-1}{n-q} + \frac{1}{2(n-q)}\right)$.

So, $P_h^1(\omega_h, \omega_h | t_j^1) = \frac{2^{n-1}}{2^n} + c$ for each $t_j^1 \in T_{jh}^1$. Hence, $P_h^1(\omega_h, \omega_h) = \frac{2^n - n - 1}{2^n} + \frac{n}{2^n} \frac{2^{n-1}}{2^n} + \frac{n}{2^n} c$

Lemma 2 :
$$\sum_{q=0}^{n-1} n \frac{\binom{n-1}{n-q-1}}{2^{2n}} \left(\frac{n-q-1}{n-q} + \frac{1}{2(n-q)} \right) = \frac{n2^n - 2^n + 1}{2^{2n+1}}$$

Proof : L.H.S =
$$\frac{1}{2^{2n}} \left(\sum_{q=0}^{n-1} n \frac{(n-1)!}{(n-q-1)! q!} \left(1 - \frac{1}{2(n-q)} \right) \right) = \frac{n2^{n-1}}{2^{2n}} - \frac{1}{2^{2n+1}} \sum_{q=0}^{n-1} \frac{n(n-1)!}{(n-q)(n-q-1)! q!}$$

$$= \frac{n2^n}{2^{2n+1}} - \frac{1}{2^{2n+1}} \left(\sum_{q=0}^n \binom{n}{q} - \binom{n}{n} \right) = \frac{n2^n - 2^n + 1}{2^{2n+1}} \quad \square$$

The overall probability that a firm F_i ($i=1, 2$) hires an H-productivity applicant and make a profit of π^h when both the firms have announced ω_h is $\frac{2^n(2^n-n-1)}{2^{2n}} + \frac{n2^{n-1}}{2^{2n}} + \frac{n2^n-2^n+1}{2^{2n+1}} = \frac{2^{2n+1}-2^{n+1}-2^n+1}{2^{2n+1}}$. Therefore the ex ante expected profit of each firm is :

$$\pi_i(\omega_h, \omega_h) = \frac{2^n(2^{n+1} - 3) + 1}{2^{2n+1}} \pi^h \quad (i=1, 2) \quad [\text{A.1}]$$

A.2. Analysis of $G(\omega_1, \omega_1)$: Both firms announce ω_1 .

In this case both firms will hire an applicant. If one or both the firms fail to hire an H-productivity applicant then they will hire an L-productivity applicant.

Suppose F_1 is of type $t_{2^n-n}^1$, i.e., a_1 is the only H-productivity applicant. There can be two situations.

(i) a_1 is of L-productivity in F_2 .

(a) If F_2 has at least one H-productivity applicant then both firms will hire an H-productivity applicant because different applicants are of H-productivity in the two firms and an applicant receiving only one offer accepts it. There are $\sum_{q=1}^{n-1} \binom{n-1}{q} = n(2^{n-1}-1)$ such situations. So the

conditional probability that firm F_1 hires a_1 is $\frac{n(2^{n-1}-1)}{2^{2n}}$.

(b) F_2 has only L-productivity applicants. The conditional probability that both firms make their first offer to a_1 and F_1 hires him is $\frac{1}{2n}$. If F_2 makes the first offer to one of the other applicants then F_1 hires a_1 and this occurs with conditional probability $\frac{n-1}{n}$. So the conditional probability that F_1 hires a_1 is $\left(\frac{n-1}{n} + \frac{1}{2n}\right)$.

(ii) If a_1 is of H-productivity in F_2 , the conditional probability that F_1 hires a_1 is the same as in A.1.

Therefore, $P_h^1(\omega_1, \omega_1 | t_j^1) = \frac{2^{n-1}-1}{2^n} + \left(\frac{n-1}{n} + \frac{1}{2n}\right) + c$ for each $t_j^1 \in T_{lh}^1$. The overall probability that a firm F_i ($i=1, 2$) hires an H-productivity applicant and make a profit of $(H-\omega_1)$ when both firms have announced ω_1 is $\frac{2^n(2^n-1)}{2^{2n}} + \frac{n(2^{n-1}-1)}{2^{2n}} + \frac{n}{2^{2n}}\left(\frac{n-1}{n} + \frac{1}{2n}\right) + \frac{n2^n-2^n+1}{2^{2n+1}} = \frac{2^{2n+1}-2^{n+1}-2^n}{2^{2n+1}}$.

So the probability that F_i hires an L-productivity applicant and make a profit of π^l is $\frac{2^{n+1}+2^n}{2^{2n+1}}$.

Therefore, the expected profit of each firm is:

$$\pi_i(\omega_1, \omega_1) = \frac{2^n(2^{n+1}-3)}{2^{2n+1}} \pi^h + \frac{3(2^n)}{2^{2n+1}} \pi^l + \frac{2^n(2^{n+1}-3)}{2^{2n+1}} \omega \quad (i=1, 2) \quad [A.2]$$

A.3. Analysis of $G(\omega_h, \omega_l)$: F_1 announces ω_h and F_2 announces ω_l .

In this case F_1 will hire an H-productivity applicant only and will be able to do so if that firm has at least one H-productivity applicant. Even if F_1 has one H-productivity applicant and both firms make their offer to the same applicant, F_1 will hire him because that firm has announced a higher wage. The probability that there is at least one H-productivity applicant in F_1 is $\frac{2^{2n}-2^n}{2^{2n}}$. Therefore, the ex ante expected profit of F_1 is: $\pi_1(\omega_h, \omega_l) = \frac{2^{2n+1}-2^{n+1}}{2^{2n+1}} \pi^h$.

Let us consider the situation for F_2 . It can hire either an H or an L-productivity applicant. Suppose F_2 is of type t_{2n-n}^2 , i.e., a_1 is the only H-productivity applicant in F_2 . There

can be two situations.

(i) a_1 is of L-productivity in F_1 . Then F_2 hires a_1 . The conditional probability that this occurs is $\frac{2^{n-1}}{2^n}$ because there are $\sum_{q=0}^{n-1} \binom{n-1}{q} = 2^{n-1}$ such types out of 2^n .

(ii) a_1 is of H-productivity in F_1 .

(a) F_1 has only H-productivity applicants. F_2 hires a_1 only if F_1 makes the offer to any one of the other $n-1$ H-productivity applicants which occurs with conditional probability $\frac{n-1}{n2^n}$.

(b) F_1 has $n-1$ H-productivity applicant including a_1 . F_2 hires a_1 only if F_1 does not make the

offer to the same applicant which occurs with conditional probability $\frac{\binom{n-1}{n-2}}{2^n} \frac{n-2}{n-1}$.

Therefore, in general, if F_1 has q H-productivity applicants including a_1 ($q=2,3,..,n$) then the

conditional probability that F_2 hires a_1 is $\frac{\binom{n-1}{q-1}}{2^n} \frac{q-1}{q}$. If only a_1 is of H-productivity in both the firms then F_1 will hire him since that firm has announced a higher wage. Therefore, the conditional probability that F_1 has at least two H-productivity applicants including a_1 and F_2

hires him is $d = \sum_{q=2}^n \frac{\binom{n-1}{q-1}}{2^n} \frac{q-1}{q}$.

Hence, $P_h^2(\omega_h, \omega_1 | t_k^2) = \frac{2^{n-1}}{2^n} + d$ for each $t_k^2 \in T_{/h}^2$. So, $P_h^2(\omega_h, \omega_1) = \frac{2^n - n - 1}{2^n} + \frac{n}{2^n} \frac{2^{n-1}}{2^n} + \frac{n}{2^n} d$.

Lemma 3 : $\sum_{q=2}^n n \frac{\binom{n-1}{q-1}}{2^{2n}} \frac{q-1}{q} = \frac{1}{2^{2n+1}} (n2^n - 2^{n+1} + 2)$

Proof: L.H.S = $\frac{1}{2^{2n}} \sum_{q=2}^n n \frac{(n-1)!}{(q-1)! (n-q)!} (1 - \frac{1}{q}) = \frac{n}{2^{2n}} \sum_{k=1}^{n-1} \binom{n-1}{k} - \frac{1}{2^{2n}} \sum_{q=2}^n \frac{n(n-1)!}{q(q-1)! (n-q)!} (k = q-1)$

$$= \frac{n2^{n-1} - n}{2^{2n}} - \frac{2^n - n - 1}{2^{2n}} = \frac{n2^n - 2^{n+1} + 2}{2^{2n+1}} \quad \square$$

So, the overall probability that F_2 hires an H-productivity applicant and make a profit of $(H-\omega_1)$

is $\frac{2^n(2^n - n - 1)}{2^{2n}} + \frac{n2^{n-1}}{2^{2n}} + \frac{n2^n - 2^{n+1} + 2}{2^{2n+1}} = \frac{2^{2n} - 2^{n+1} + 1}{2^{2n}}$. The probability that F_2 hires an L-

productivity applicant and make a profit of π^l is $\frac{2^{n+1} - 1}{2^{2n}}$. Therefore, $\pi_2(\omega_h, \omega_1) = \frac{2^{2n} - 2^{n+1} + 1}{2^{2n}} H + \frac{2^{n+1} - 1}{2^{2n}} L - \omega_1$. We have:

$$\pi_1(\omega_h, \omega_1) = \frac{2^{n+1}(2^n - 1)}{2^{2n+1}} \pi^h$$

[A.3]

$$\pi_2(\omega_h, \omega_1) = \frac{2^{2n+1} - 2^{n+2} + 2}{2^{2n+1}} \pi^h + \frac{2^{n+2} - 2}{2^{2n+1}} \pi^l + \frac{2^{2n+1} - 2^{n+2} + 2}{2^{2n+1}} \omega$$

A.4. Analysis of $G(\omega_1, \omega_h)$: F_1 announces ω_1 and F_2 announces ω_h .

This is symmetric to A.3. and the expected profits of the firms are:

$$\pi_1(\omega_1, \omega_h) = \frac{2^{2n+1} - 2^{n+2} + 2}{2^{2n+1}} \pi^h + \frac{2^{n+2} - 2}{2^{2n+1}} \pi^l + \frac{2^{2n+1} - 2^{n+2} + 2}{2^{2n+1}} \omega$$

[A.4]

$$\pi_2(\omega_1, \omega_h) = \frac{2^{2n+1} - 2^{n+1}}{2^{2n+1}} \pi^h$$

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Chapter 3

What Is In The Advertisement?

I. INTRODUCTION

In the market for new entrants to the white-collar labor force there exists a variety of institutions or policies for determining wages and conducting the hiring process. In many cases firms advertise an opening for an entry level white-collar position and then make a one time take-it-or-leave-it wage offer, privately, after deriving the specific information about applicants' productivities from their application. This leaves the firms with the option to alter their wage offers. In other cases firms publicly announce and commit themselves to a particular wage along with the announcement of the position prior to receiving specific information about applicants' productivities. This paper provides an explanation of which policy is likely to emerge as an equilibrium policy in a market for newcomers to white-collar jobs.

Firms' strategy to announce a wage in advance has been considered by Lang (1991), Montgomery (1991), and Banerjee (Chapter 2). Lang and Montgomery addressed this issue to provide a search theoretic explanation of interindustry and intraindustry wage differentials which are positively correlated with industry attributes like profitability and capital-labor ratio (Dickens and Katz 1987; Krueger and Summers 1987, 1988; Murphy and Topel 1987). They argued that given coordination problems in the labor market, firms with higher "value of vacancy" will pay higher wages to avoid their expensive machineries remaining unused. Since workers with better opportunities will not apply firms' announcement of a high wage in advance will reduce the probability of a firm's position remaining vacant. In their models equilibrium

variation in wages occurred due to variations in the value of vacancy across firms. Match specific considerations were not taken into account.

In chapter 2 the consequences of announcing and committing to a wage in advance was studied in the context of recruiting newcomers to the white-collar labor force. New entrants to such jobs do not have any information about their possible productivity in a firm. By taking into consideration match-specific productivity, it was shown in chapter 2 that announcement of a wage in advance may lead to firms' position remaining vacant and the equilibrium outcome may not exhibit wage dispersion.

Questions about commitment to a specific strategy has been addressed before. Gal-Or (1985) and Dowrick (1986) have shown, that in a Stackelberg type of game with time of commitment as the strategic choice, firms are unwilling to commit first when the reaction curves are positively sloping, but with negatively sloping reaction curves, it is advantageous for the firms to commit first. Singh and Vives (1984) have shown that for substitutes (complements) reaction curves in terms of output are negatively (positively) sloping. The opposite is true for reaction curves in terms of prices. If goods are substitutes it is optimal for the follower to reduce output (increase price) if the leader increases its output (increases its price).

Bester (1993) has compared the performance of negotiated pricing with posted-offer pricing and showed that posted pricing induce sellers to select a suboptimal quality while negotiated pricing always leads to an efficient selection of quality. The trade-off between the lack of competition and customers being exploited by sellers uniquely determines the pricing mechanism which can be either posted pricing or negotiated pricing. Gale (1988) explored the relationship between ex ante and ex post pricing. He showed that different parameter values induced competitive or monopoly outcome in a setting where sellers are uncertain whether the buyer is receiving offers from both or one of the firms.

In this chapter firms first simultaneously choose a policy, whether to announce a wage

in advance or to offer wages privately, and then compete contingent on the chosen policy. The equilibrium policy is determined by comparing the ex ante expected profit associated with the equilibrium strategy under the different policies. For different constellations of parametric values we get different equilibrium strategies for the two policies. Our analysis shows that *offer wages privately* is always an equilibrium policy. However, if a low wage is the equilibrium strategy under all the policies then announcing a wage in advance and offer wages privately are both equilibrium policies. This result justifies one of the circumstances under which different policies may coexist. This is because the ex ante expected profit of the firms associated with a low wage equilibrium is the same under all the policies.

The tradeoff between the restriction under the wage announcement policy, which may cause a firms' position remain vacant, and the possibility to alter offers across applicants under the private wage offer policy drives the above mentioned results. Also, lack of prior information about applicants' productivities and the uncertainty about the behavior of the competing firms (the evaluation of the applicants' specific productivities and the recipient of other firms' offers) explains the result. When a low wage is the equilibrium strategy under all the policies, the firms have the same chance to exploit the applicants under all the policies, hence, both the policies are equilibrium policy.

This paper is organized as follows. In section II, I discuss the model and in section III, I provide results of the different subgames. The analysis of the different subgames are given in the appendix. In section IV, I endogenize the equilibrium policy to be followed by the firms, at the beginning of the hiring process.

II. THE MODEL

There are two firms F_m ($m=1, 2$) and three applicants a_x ($x=1, 2, 3$). All firms and applicants know that an applicant can be of high (H) or low (L) productivity in a firm and these are firm-specific productivities. Let $W=\{\omega_h, \omega_l\}$ be the set of high and low wage, respectively, from which each firm selects a wage. All applicants are newcomers to white-collar jobs. Each firm has one vacancy.

It is also assumed that $H > L$, $\omega_h \geq \omega_l$, $\omega_h \geq L$, $H - \omega_l > H - \omega_h \geq L - \omega_l \geq 0$. The last string of inequalities imply that a firm gets maximum surplus by hiring an H-productivity applicant and paying him ω_l . Also, it is weakly more profitable to hire an H-productivity applicant by paying him ω_h than by hiring an L-productivity applicant and paying him ω_l . The inequality $\omega_h \geq L$ implies that a firm will pay a high wage, if at all, only to an H-productivity applicant. Throughout the paper I will use the following notations: $\pi^h = H - \omega_h$, $\pi^l = L - \omega_l$, and $\omega = \omega_h - \omega_l$

The hiring process of the firms can be described as consisting of the following stages.

Stage 0: Let \hat{X}_{jm} be a family of independent random variables, $j=1,2,3$; and $m=1,2$. \hat{X}_{jm} assumes value H and L each with probability $\frac{1}{2}$. There are $\sum_{q=0}^3 \binom{3}{q} = 2^3 = 8$ possible combinations of the firm-specific productivities of the 3 applicants in a firm F_m ($m=1,2$). $\binom{3}{q}$ denotes that q out of 3 applicants can be of H-productivity in a firm, ($q=0,1,2,3$). There are 2^6 possible combinations of the productivity of the 3 applicants in both the firms. Each firm knows this distribution ex ante. Let $T^m = \{H, L\}^3$ be the set of all possible types of a firm F_m , i.e., a type is a list of the firm-specific productivities, one for each applicant. T^m has 2^3 elements which we label t_j^m ($j=1,2,\dots,8$). We can Assume (a) to (c).

(a) $T_{2h}^m = \{t_j^m \mid j=1,2,3,4\}$ is the set of types of a firm F_m having at least two H-productivity

applicant.

(b) $T_{jh}^m = \{t_j^m \mid j=5,6,7\}$ be the set of types of a firm F_m having exactly one H-productivity applicant.

(c) $T_l^m = \{t_l^m\}$ is the singleton containing the type of F_m that has only L-productivity applicants: $t_l^m = (L, L, \dots, L)$.

Stage 1: In this stage each firm chooses a policy $p_m \in P_m = \{b, o\}$, ($m=1,2$), where ‘b’ denotes the policy to **announce a wage before** receiving specific information about applicants’ productivities and ‘o’ denotes the policy to **offer wages privately** after receiving specific information. Let (p_1^*, p_2^*) denote the equilibrium policies of the two firms.

Stage 2: After adopting a policy both firms announce a job opening simultaneously. If a firm’s policy is to announce a wage, then that firm announces and commits itself to a particular wage in this stage.

Stage 3: In this stage all applicants apply to both the firms simultaneously. There is no transaction cost in applying. From the applications each firm derives the firm-specific productivities of the applicants. A firm knows its own evaluation but has no information about the evaluation of the other firm. Also, an applicant has no information about the firm’s evaluation at any stage. Hence, applying to both the firms is the dominant strategy.

Stage 4: In this stage a firm may or may not make an offer which depends on the policy that the firm has adopted. If both firms are making an offer they do it simultaneously. A firm having q ($q=1,2,3$) H-productivity applicants will make an offer to each with probability $\frac{1}{q}$. An applicant receiving offers from both the firms accepts the highest offer. If both offers are the same then

that applicant chooses each with probability $\frac{1}{2}$. An applicant receiving only one offer accepts it because he has no information about the firms' evaluation and hence is not sure of getting any further offers.

Stage 5: This stage regards the decision of a rejected firm, if any, from stage 4. The rejected firm's decision depends on the policy that firm has adopted, the composition of the productivity of the applicants in that firm, the amount and the recipient of the other firm's offer. In this stage the rejected firm may or may not make any more offers.

Let $G^{P_1P_2}$ be the subgame beginning at stage 2 after each firm has chosen a policy $p_m \in P_m$ ($m=1,2$) in stage 1. Note that this subgame begins at the **pre-application stage**. Let $W_m^{P_1P_2}$ denote the strategy space of a firm F_m ($m=1,2$) for each of the subgames $G^{P_1P_2}$. The equilibrium policy is endogenously determined by comparing the *ex ante expected profit* $\Pi_m^{P_1P_2}$ ($m=1,2$) associated with the set of equilibrium strategies $\{(\omega_1^{P_1P_2}, \omega_2^{P_1P_2})\}$, $\omega_m^{P_1P_2} \in W_m^{P_1P_2}$, of each of the subgames $G^{P_1P_2}$.

Let $Q_m^{P_1P_2}(w_1^{P_1P_2}, w_2^{P_1P_2} | t_j^1, t_k^2)$ be the conditional probability that a firm F_m hires an H-productivity applicant when $(w_1^{P_1P_2}, w_2^{P_1P_2})$ is the wage profile of the two firms in the subgame $G^{P_1P_2}$ given the types of the firms. The corresponding conditional probability of hiring an L-productivity applicant, if at all, is denoted by $R_m^{P_1P_2}(w_1^{P_1P_2}, w_2^{P_1P_2} | t_j^1, t_k^2)$. The overall probability of hiring an H-productivity applicant when $(w_1^{P_1P_2}, w_2^{P_1P_2})$ is the strategy profile is

$$Q_m^{P_1P_2}(w_1^{P_1P_2}, w_2^{P_1P_2}) = \frac{4}{8} \sum_{t_j^m \in T_{2k}^m} \left\{ \frac{1}{8} \sum_{k=1}^8 Q_m^{P_1P_2}(w_1^{P_1P_2}, w_2^{P_1P_2} | t_j^m, t_k^i) \right\} +$$

$$\frac{3}{8} \sum_{t_j^m \in T_{1k}^m} \left\{ \frac{1}{8} \sum_{k=1}^8 Q_m^{P_1P_2}(w_1^{P_1P_2}, w_2^{P_1P_2} | t_j^m, t_k^i) \right\}$$

The overall probability of hiring an L-productivity applicant is

$$R_m^{P_1P_2}(w_1^{P_1P_2}, w_2^{P_1P_2}) = \frac{3}{8} \left(1 - \sum_{t_j^m \in T_{1h}^m} \left\{ \frac{1}{8} \sum_{k=1}^8 Q_m^{P_1P_2}(w_1^{P_1P_2}, w_2^{P_1P_2} \mid t_j^m, t_k^i) \right\} \right) + \frac{1}{8} \left(\frac{1}{8} \sum_{k=1}^8 R_m^{P_1P_2}(w_1^{P_1P_2}, w_2^{P_1P_2} \mid t_8^m, t_k^i) \right)$$

Note that if a firm has at least two H-productivity applicants that firm hires an H-productivity applicant for sure because a firm can hire an applicant in at most two rounds. In the next section I shall provide the results for each of the subgames G^{aa} , G^{oo} , G^{ao} and G^{oa} . A detailed analysis of each of the subgames is given in the appendix.

III.1. RESULTS OF THE SUBGAME G^{bb}

If announcing a wage is the policy chosen by both the firms, then each firm F_m will announce and commit themselves to a wage $\alpha_m \in W_m^{bb} \equiv W$, ($m=1,2$), a priori, along with the announcement of a job opening. The consequences of this decision along with the equilibrium wage announced by each firm was analyzed in chapter 2. Here I will briefly mention the results.

At the pre-application stage, the two firms make simultaneous wage announcements. For each $(\alpha_1, \alpha_2) \in W_1^{bb} \times W_2^{bb}$, let $G^{bb}(\alpha_1, \alpha_2)$ denote the subgame beginning immediately after the announcements α_1 and α_2 have been made. In this case $G^{bb}(\alpha_1, \alpha_2) \equiv G^{bb}$. It turns out that each of the subgames $G^{bb}(\alpha_1, \alpha_2)$ has unique subgame perfect expected ex ante equilibrium payoffs $\Pi_m^{bb}(\alpha_1, \alpha_2)$ for each firm F_m ($m=1,2$). Given the knowledge of these payoffs, the analysis of the subgame reduces to the analysis of the *reduced game in normal form*: $\Gamma^{bb} = (\{F_1, F_2\}, W_m^{bb}, \Pi_m^{bb}; (m=1, 2))$.

Lemma 1

The payoff functions Π_1^{aa} and Π_2^{aa} of the reduced game Γ^{aa} are represented in the following matrix:

$$W_2^{bb}$$

		ω_h	ω_l
W_1^{bb}	ω_h	$\frac{105}{128}\pi^h$	$\frac{112}{128}\pi^h$
	ω_l	$\frac{105}{128}\pi^h$	$\frac{98}{128}\pi^h + \frac{30}{128}\pi^l - \frac{98}{128}\omega$
	ω_h	$\frac{98}{128}\pi^h + \frac{30}{128}\pi^l - \frac{98}{128}\omega$	$\frac{104}{128}H + \frac{24}{128}L - \omega_1$
	ω_l	$\frac{112}{128}\pi^h$	$\frac{104}{128}H + \frac{24}{128}L - \omega_1$

The derivation of this matrix is discussed in Banerjee (chapter 2).

III.2. RESULTS OF THE SUBGAME G^∞

In this case both firms simultaneously make one time take-it-or-leave it wage offers, privately, after receiving specific information about applicants' productivities, i.e., after learning their own type. So each firm has the option of altering its wage offer, if rejected.

A firm has no information about the type, the amount and the recipient of the wage offer of the other firm. Each firm believes that the other firm could be any one of eight types with probability $\frac{1}{8}$. Since there are two firms and three applicants, a firm will hire an applicant in at most two rounds.

The equilibrium wage offer of a firm at the post-application stage is defined as follows.

Definition 1

The equilibrium wage offer of a firm F_m is defined as a mapping $S^m: T^m \rightarrow W_m^{\text{oo}} \equiv W$ ($m=1, 2$)

such that, for each $t_j^m \in T^m$, $s^m(t_j^m) \in \operatorname{argmax}_{\omega_1^m, \omega_2^m \in W} \frac{1}{8} \sum_{k=1}^8 \pi_m^{\text{oo}}(\omega_1^m, \omega_2^m, s^n(t_k^n), t_j^m, t_k^i);$

($m, i=1, 2; m \neq i$), where ω_r^m denotes the wage offered in round r ($r=1, 2$) by F_m .

For each $(s^1, s^2) \in S^1 \times S^2$, let $G^{\text{oo}}(s^1, s^2)$ denote the subgame beginning at the **post-application stage**. The ex ante expected payoffs of a firm F_m ($m=1,2$) of the subgame G^{oo} beginning at the **pre-application stage** for each (s^1, s^2) is: $\Pi_m^{\text{oo}}(s^1, s^2) = \frac{1}{8} \sum_{j=1}^8 \left\{ \frac{1}{8} \sum_{k=1}^8 \pi_m^{\text{oo}}(s^m(t_j^m), s^i(t_k^i), t_j^m, t_k^i) \right\}$, $m, i=1,2; m \neq i$.

Given the knowledge of these ex ante expected payoffs, the analysis of the entire subgame G^{oo} reduces to the analysis of the **reduced game in the normal form**: $\Gamma^{\text{oo}} = (\{F_1, F_2\}, S^1, S^2, \Pi_1^{\text{oo}}, \Pi_2^{\text{oo}})$. A detailed analysis of each of the subgames $G^{\text{oo}}(s^1, s^2)$ and the derivation of the ex ante expected payoffs of the subgame G^{oo} for each (s^1, s^2) is given in appendix A. In the current section I will mention the results only.

Proposition 1

If a firm learns that there are at least two H-productivity applicants or only L-productivity applicants then that firm will offer ω_1 in all possible rounds irrespective of the type and the wage offer of the other firm. (*For a proof see appendix A.*)

A firm having one H-productivity applicant (i.e., t_j^m ($j=5,6,7$)) may offer ω_h or ω_1 to him. Clearly, if the firm is rejected by the H-productivity applicant, that firm will offer ω_1 to an L-productivity applicant and hire him. Here I assume that $s^m(t_5^m) = s^m(t_6^m) = s^m(t_7^m)$, ($m=1,2$).

This means that if a firm has only one H-productivity applicant, that firm will offer the same equilibrium wage irrespective of which applicant is of H-productivity in that firm.

There are two possibilities. A firm might offer ω_1 in all possible rounds of offers irrespective of its type, i.e., $s^m(t_j^m) = \omega_1 \forall j$. The second possibility is, if a firm has one H-productivity applicant then it offers ω_h to him and otherwise offers ω_1 , i.e., $s^m(t_j^m) = \begin{cases} \omega_h & j=5,6,7 \\ \omega_1 & \text{otherwise} \end{cases}$. Let the latter strategy be denoted by s_d^m with 'd' standing for discriminating.

Lemma 2

The payoff functions Π_1^{oo} and Π_2^{oo} of the reduced game Γ^{oo} are represented in the following matrix:

$$S^2$$

	$s^2 = s_d^2$	$s^2 \equiv \omega_l$
$s^1 = s_d^1$	$\frac{109}{128}\pi^h + \frac{19}{128}\pi^l + \frac{64}{128}w$	$\frac{112}{128}\pi^h + \frac{16}{128}\pi^l + \frac{64}{128}w$
s^1	$\frac{109}{128}\pi^h + \frac{19}{128}\pi^l + \frac{64}{128}w$	$\frac{101}{128}\pi^h + \frac{27}{128}\pi^l + \frac{101}{128}w$
$s^1 \equiv \omega_1$	$\frac{101}{128}\pi^h + \frac{27}{128}\pi^l + \frac{101}{128}w$	$\frac{104}{128}\pi^h + \frac{24}{128}\pi^l + \frac{104}{128}w$

The derivation of this matrix is provided in appendix A.

III.3. RESULTS OF THE SUBGAME G^{bo}

In this subgame firm F_1 announces a wage and firm F_2 offer wages privately. F_1 has no information about the amount and recipient of F_2 's offer. On the contrary, F_2 knows the wage announced by firm F_1 but has no information about the recipient of F_1 's offer.

This subgame has a leader-follower structure. F_1 is announcing a wage which F_2 observes, incorporates that wage into its expected profit function and chooses a wage that maximizes its expected profit. F_1 announces a wage that maximizes its expected profit by taking into consideration F_2 's possible strategies. Thus F_1 behaves like a leader while F_2 behaves like a follower.

Definition 2

The equilibrium in this subgame is defined as a pair (ω^*, s^2) which satisfies the following conditions.

(i) At the pre-application stage firm F_1 announces a wage $\omega^* \in A_1 \equiv W$ such that,

$$\omega^* \in \operatorname{argmax}_{\omega \in W} \frac{1}{8} \sum_{j=1}^8 \left\{ \frac{1}{8} \sum_{k=1}^8 \pi_1^{\text{bo}}(\omega, s^2(t_k^2), t_j^1, t_k^2) \right\}$$

(ii) Firm F_2 offers a wage $s^2(t_k^2) \in S^2$ at the post-application stage, such that for each $t_k^2 \in T^2$,

$$s^2(t_k^2) \in \operatorname{argmax}_{\omega \in S^2} \frac{1}{8} \sum_{j=1}^8 \pi_2^{\text{bo}}(\omega^*, \omega, t_j^1, t_k^2).$$

Let $G^{\text{bo}}(\omega^*, s^2)$ be the subgame beginning at the post-application stage. It turns out that each of the subgames $G^{\text{bo}}(\omega^*, s^2)$ has unique equilibrium payoffs as mentioned above. So the ex ante expected payoffs of the firms in the subgame G^{ao} beginning at the pre-application stage for each (ω^*, s^2) are:

$$\Pi_1^{\text{bo}}(\omega^*, s^2) = \frac{1}{8} \sum_{j=1}^8 \left\{ \frac{1}{8} \sum_{k=1}^8 \pi_1^{\text{bo}}(\omega^*, s^2(t_k^2), t_j^1, t_k^2) \right\}$$

$$\Pi_2^{\text{bo}}(\omega^*, s^2) = \frac{1}{8} \sum_{k=1}^8 \left\{ \frac{1}{8} \sum_{j=1}^8 \pi_2^{\text{bo}}(\omega^*, s^2(t_k^2), t_j^1, t_k^2) \right\}$$

Given the knowledge of these payoffs the analysis of the subgame G^{ao} reduces to the

analysis of the *reduced game in normal form*: $\Gamma^{\text{bo}} = (\{F_1, F_2\}, W, S^2, \Pi_1^{\text{bo}}, \Pi_2^{\text{bo}})$

The derivation of the payoffs Π_m^{ao} ($m=1,2$) is given in appendix B.

Lemma 3

The payoff functions Π_1^{ao} and Π_2^{ao} of the reduced game Γ^{ao} are represented in the following matrix:

$$S^2$$

		$s^2 = s_d^2$	$s^2 \equiv \omega_1$
W	ω_h	$\frac{109}{128}\pi^h$ $\frac{105}{128}\pi^h + \frac{23}{128}\pi^l + \frac{64}{128}w$	$\frac{112}{128}\pi^h$ $\frac{98}{128}\pi^h + \frac{30}{128}\pi^l + \frac{98}{128}w$
	ω_l	$\frac{101}{128}\pi^h + \frac{27}{128}\pi^l + \frac{101}{128}\omega$ $\frac{112}{128}\pi^h + \frac{16}{128}\pi^l + \frac{64}{128}\omega$	$\frac{104}{128}\pi^h + \frac{24}{128}\pi^l + \frac{104}{128}\omega$ $\frac{104}{128}\pi^h + \frac{24}{128}\pi^l + \frac{104}{128}\omega$

The ex ante expected profits of the firms for each (s^1, ω^*) of the subgame G^{oa} are symmetric to those of the subgame G^{ao} .

IV. ENDOGENIZING THE EQUILIBRIUM POLICY

We will use the information condensed in the derivation of the ex ante expected profit of the firms in each of the subgames for endogenizing the equilibrium policy. In a first step firms decide upon which policy to follow. Let G be the game beginning in stage 1.

Given the knowledge of the ex ante expected payoffs Π_m^{P1P2} ($m=1,2$) and the underlying equilibria of each of the subgames G^{P1P2} , the analysis of the entire game G reduces

to the analysis of the *reduced game in normal form*

$$\Gamma = (\{F_1, F_2\}, P_1, P_2, \Pi_1^{P_1 P_2}, \Pi_2^{P_1 P_2}).$$

The equilibrium policy is determined by comparing the ex ante expected profit of the firms associated with the equilibria under the three policies. Let the set $\{(\alpha_1, \alpha_2), (s^1, s^2), (\omega^*, s^2)\}$ denote the equilibria of the subgames G^{bb} , G^{oo} , and G^{bo} , (the equilibria of the subgame G^{oa} is symmetric to that of G^{ao}), respectively. I shall state the results in this section. The derivation of the equilibrium sets of strategies and the proofs are provided in appendix C.

Theorem 1

If $\{(\omega_h, \omega_h), (s^1 = s_d^1, s^2 = s_d^2), (\omega_h, s^2 = s_d^2), (s^1 = s_d^1, \omega_h)\}$ is the set of equilibria, then *“offer wages privately”* is the equilibrium policy.

If a high wage is the equilibrium for a firm announcing a wage, it means that the firm would hire, if at all, an H-productivity applicant only. It restricts the firms’ selection of quality and increases the competition. This coupled with the uncertainty about forming a possible match might result in one or both the firm’s positions remaining vacant which lowers the expected payoff.

By offering a high wage to a high productivity applicant, when a firm has one of them, and offering a low wage otherwise, firms do not face the risk of having their position unfilled. It increases the probability of hiring an H-productivity applicant when there is only one of them. If a firm cannot hire the H-productivity applicant, that firm will hire an L-productivity applicant. Under this policy the quality selection is not restricted. The structure of the policy allows a firm to exploit the applicants in the sense that a firm might hire an H- productivity applicant by paying him a low wage.

Theorem 2

If $\{(\omega_l, \omega_l), (s^1 = s_d^1, s^2 = s_d^2), (\omega_h, s^2 = s_d^2), (s^1 = s_d^1, \omega_h)\}$ is the set of equilibria, then there exists two *symmetric equilibria* (b, b) and (o, o) . Each firm will choose the payoff dominant policy, *“offer wages privately”*.

Though announcing a low wage is the equilibrium strategy profile in the subgame G^{bb} which means that none of the firms' positions remain vacant, the ex ante expected profit associated with the equilibria of the subgame G^{oo} exceeds that of the subgame G^{bb} . This is because in the subgame G^{oo} none of the firms' positions remain vacant and by offering a high wage, if a firm has one H-productivity applicant, increases the probability of hiring an H-productivity applicant.

Theorem 3

If $\{(\omega_l, \omega_l), (s^1 = s_d^1, s^2 = s_d^2), (\omega_l, s^2 = s_d^2), (s^1 = s_d^1, \omega_l)\}$ is the set of equilibria, then *“offer wages privately”* is the equilibrium policy.

In this case a firm which announces a wage, (hence, moves first), and a firm which offer wages privately, (hence, moves second) have partially asymmetric equilibria. A firm offering a wage privately, offers a high wage if there is only one high productivity applicant, otherwise, offers a low wage. A firm announcing a low wage in equilibrium, commits to it. According to Gal-Or (1985) it is advantageous for the players to commit first if they have asymmetric reaction functions. From theorem 3 we see, though there is partially asymmetric equilibria, *“offer wages privately”* is the equilibrium policy.

Theorem 4

If $\{(\omega_l, \omega_l), (s^1 \equiv \omega_l, s^2 \equiv \omega_l), (\omega_l, s^2 \equiv \omega_l), (s^1 \equiv \omega_l, \omega_l)\}$ is the set of equilibria, then announcing a wage in advance and offering wages privately are both equilibrium policies.

It is only in this case a possible outcome may be that one firm announces and the other firm offers. In this case the assignment of the role of the firms, which firm should announce and which firm should offer, do not matter as the ex ante profits are the same in all the subgames. This justifies the coexistence of different labor market policies.

Theorem 5

If $\{((\omega_h, \omega_l), (\omega_l, \omega_h)), (s^1 = s_d^1, s^2 = s_d^2), (\omega_h, s^2 \equiv \omega_l), (s^1 \equiv \omega_l, \omega_h)\}$ is the set of equilibria, then *“offer wages privately”* is the equilibrium policy.

In this case we see that the subgame G^{bb} has two asymmetric equilibria. This causes a potential problem of coordination.

V. CONCLUSION

In this paper we endogenously determined the equilibrium policy which is likely to emerge in a market for new entrants to the white-collar labor force. Two policies followed by the firms at the time of hiring newcomers to white-collar jobs were examined: announce and commit to a particular wage prior to receiving specific information about applicants' productivities and offer wages privately after deriving specific information about applicants' productivities. In a first step the firms had to decide upon which policy to follow.

It turned out that “offer wages privately” is always an equilibrium policy. However, if a low wage is the equilibrium strategy for all the policies, then any policy is an equilibrium policy. One implication of this result is that it justifies one of the circumstances in which different policies might coexist. This equilibrium outcome do not have the problem of assignment of the role of the firms: which firm should announce and which firm should offer. Any pair of policies is an equilibrium.

It is important to observe that in all the situations offer wages privately is an equilibrium policy. The lack of prior information about the applicants’ productivities, a firm’s lack of information about the productivity of the applicants in the other firm, and at least, about the recipient of the other firm’s offer, resulted in the uncertainty about the possible match between a firm and an applicant. The policy to announce a wage restricts the firms to increase their expected profits. On the contrary, the policy to “offer wages privately” provided firms with the option to alter their offer and hence to increase their expected profit.

If the wages are set at the competitive level, that is, $H = L = \omega_1$, then any policy is an equilibrium. This is because under all the policies low wage is the equilibrium strategy. Intuitively, by announcing a high wage each firm can make zero profit. Same is the case if a firm having only one H-productivity applicant offers a high wage to him and offers a low wage otherwise. So it turns out that low wage is the dominant strategy under all the policies because a firm might make a positive profit by hiring an H-productivity applicant and paying him a low wage.

We have seen that in all the cases the equilibrium policies ensured that a firm’s position do not remain vacant. There can be four possible equilibrium outcomes. Applicants with productivity differences might receive the same wage, applicants with the same productivity might receive different wages, applicants with different productivities receive different wages, and applicants with the same productivity receive the same wage. So the equilibrium outcome

may not exhibit a wage differential.

There is a considerable literature studying the endogenous determination of different market institutions. Singh and Vives (1984) had shown that in a differentiated duopoly with a choice between a price and a quantity contract, quantity contract is the dominant strategy for each firm if the goods are substitutes and price contract is the dominant strategy for each firm if the goods are complements.

Klemperer and Meyer (1986) have analyzed the effect of an exogenous demand shock on the choice between price or quantity as strategic variables. If the marginal cost is steep then quantity is the equilibrium choice. If it is flat then price is the equilibrium choice. Kats and Thisse have shown that in a spatial oligopoly the choice of uniform delivered pricing is always an equilibrium. The choice of mill pricing is an equilibrium if the reservation price is high enough.

In chapter 4, I have addressed the question, if “announcing a wage” is the strategy rule to be followed by the firms, then what should be the equilibrium timing of the wage announcement: before or after receiving information about applicants’ productivities.

APPENDIX A: ANALYSIS OF THE SUBGAME G^{∞}

I shall first prove proposition 1 and then analyze each of the subgames $G^{\infty}(s^1, s^2)$ and consequently derive the ex ante expected profit of each firm.

Proof of Proposition 1

A firm F_m having at least two H-productivity applicants means that it is one of the

following types t_j^m , $j=1,2,3,4$; $m=1, 2$. In this case a firm is sure to hire an H-productivity applicant, because a firm can hire an applicant in at most two rounds of offers. Clearly, the equilibrium wage offer of a firm having at least two H-productivity applicants is $s^m(t_j^m) = \omega_1$, ($m=1,2$; $j=1,2,3,4$), irrespective of the type and the wage offer of the other firm, because $(H - \omega_1) > (H - \omega_h)$. At worst the other firm might offer ω_h to the same applicant, in which case the firm in question will get rejected. Consequently, the firm will offer ω_1 to another H-productivity applicant and hire him. So the equilibrium expected payoff of a firm is: $\frac{1}{8} \sum_{k=1}^8 \pi_m^{oo}(s^m(t_j^m)=\omega_1, s^i(t_k^i), t_j^m, t_k^i) = H - \omega_1$ ($m,i=1,2$; $m \neq i$; $j=1,2,3,4$; $s^i \in S^i$; $t_k^i \in T^i$)

A firm having only L-productivity applicants means it is of type t_8^m . In this case that firm can only hire an L-productivity applicant. By assumption a firm will offer ω_1 , if at all, to an H-productivity applicant only. So the equilibrium wage offer of firm having only L-productivity applicant is ω_1 , irrespective of the type and the wage offer of the other firm. So the expected profit of the firm is: $\frac{1}{8} \sum_{k=1}^8 \pi_m^{oo}(s^m(t_8^m)=\omega_1, s^i(t_k^i), t_8^m, t_k^i) = L - \omega_1$ ($m,i=1,2$; $m \neq i$; $t_k^i \in T^i, s^i \in S^i$) □

Let us derive the ex ante expected payoffs of a firm F_m ($m=1,2$) for $s^m \equiv \omega_1$, and for $s^m = s_d^m$. I will derive the ex ante expected profits of firm F_1 which would be the same for F_2 . We know that if a firm has at least two H-productivity, that firm will surely hire an H-productivity applicant. So $Q_1^{oo}(s^1, s^2 | T_{2h}^1) = \frac{4}{8}$. So F_1 's expected profit when it has at least two

H-productivity applicant is $\frac{1}{8} \sum_{k=1}^8 \pi_1^{oo}(\omega_1, s^2(t_k^2), t_j^1, t_k^2) = H - \omega_1$ ($j=1,2,3,4$) and with probability

$\frac{1}{8}$, F_1 's expected profit is $\frac{1}{8} \sum_{k=1}^8 \pi_1^{oo}(\omega_1, s^2(t_k^2), t_8^1, t_k^2) = L - \omega_1$, irrespective of the type and strategy of firm F_2 .

A.1. Analysis of $G^{\infty}(s^1 \equiv \omega_1, s^2 \equiv \omega_1)$

This means that the strategy of firm F_1 is to offer ω_1 in all possible rounds irrespective of its type, given that firm F_2 follows the same strategy. Let us derive the expected profit of F_1 if that firm learns that it has one H-productivity applicant. Assume that F_1 learns it is of type t_7^1 , i.e., a_1 is the only H-productivity applicant.

(i) If firm F_2 is of type t_k^2 ($k=1,8$), then with probability $\frac{2}{3}$, that firm will make an offer to any one of the other two applicants in which case firm F_1 will hire the H-productivity applicant. With probability $\frac{1}{3}$ firm F_2 can make the offer to applicant a_1 , in which case firm F_1 will succeed in hiring him with probability $\frac{1}{2}$. So $Q_1^{\infty}(s^1 \equiv \omega_1, s^2 \equiv \omega_1 | t_7^1, t_k^2) = \frac{5}{6}$ ($k=1,8$) and the corresponding $R_1^{\infty} = \frac{1}{6}$.

(ii) If firm F_2 is of type t_k^2 ($k=2, 3$), then with probability $\frac{1}{2}$ firm F_2 will make an offer to applicant a_1 . In that case firm F_1 succeeds in hiring applicant a_1 with probability $\frac{1}{2}$. With probability $\frac{1}{2}$ firm F_2 will make an offer to the other H-productivity applicant in which case F_1 will surely hire applicant a_1 . So $Q_1^{\infty}(s^1 \equiv \omega_1, s^2 \equiv \omega_1 | t_7^1, t_k^2) = \frac{3}{4}$ ($k=2,3$) and the corresponding $R_1^{\infty} = \frac{1}{4}$.

(iii) If firm F_2 is of type t_k^2 ($k=4,5,6$), then applicant a_1 who is of H-productivity in F_1 is of L-productivity in F_2 . This means that firm F_2 will not make its first offer to applicant a_1 and hence F_1 will hire the H-productivity applicant for sure. So $Q_1^{\infty}(s^1 \equiv \omega_1, s^2 \equiv \omega_1 | t_7^1, t_k^2) = 1$ ($k=4,5,6$).

(iv) If firm F_2 is also of type t_7^2 then both the firms will make their first offer to the same applicant a_1 . Firm F_1 hires him probability $\frac{1}{2}$. With probability $\frac{1}{2}$ firm F_1 's offer will be rejected in which case that firm will hire an L-productivity applicant.

So $Q_1^{oo}(s^1 \equiv \omega_l, s^2 \equiv \omega_l | T_{lh}^1) = \frac{40}{48}$ and $R_1^{oo}(s^1 \equiv \omega_l, s^2 \equiv \omega_l | T_{lh}^1) = \frac{8}{48}$. So the ex ante expected profit Π_m^{oo} of a firm F_m when $s^m(t_j^m) = \omega_l \forall j$ ($m=1,2$) is:

$$\Pi_m^{oo}(s^1 \equiv \omega_l, s^2 \equiv \omega_l) = \frac{4}{8}(H-\omega_l) + \frac{3}{8}\left(\frac{40}{48}H + \frac{8}{48}L - \omega_l\right) + \frac{1}{8}(L-\omega_l) = \frac{104}{128}\pi^h + \frac{24}{128}\pi^l - \frac{104}{128}\omega \quad [A.1]$$

A.2. Analysis of $G^{oo}(s^1 = s_d^1, s^2 \equiv \omega_l)$

In this case the strategy of firm F_1 having one H-productivity applicant is to offer ω_h to him and to offer ω_l otherwise, given that the strategy of firm F_2 is to offer ω_l in all rounds irrespective of its type. If F_1 has one H-productivity applicant, that firm will surely hire the H-productivity applicant even if both firms make the offer to the same applicant, because firm F_1 offered a higher wage. Let us now calculate the probability that F_2 hires an H-productivity applicant.

Assuming that F_2 has exactly one H-productivity applicant (type t_7^2) we get the following conditional probabilities.

(i) $Q_2^{oo}(s^1 = s_d^1, s^2 \equiv \omega_l | t_7^2, t_k^1) = \frac{5}{6}$ and $R_2^{oo} = \frac{1}{6}$ ($k=1,8$). The reason is already explained.

(ii) $Q_2^{oo}(s^1 = s_d^1, s^2 \equiv \omega_l | t_7^2, t_k^1) = \frac{3}{4}$ and $R_2^{oo} = \frac{1}{4}$ ($k=2, 3$).

(iii) $Q_2^{oo}(s^1 = s_d^1, s^2 \equiv \omega_l | t_7^2, t_k^1) = 1$ ($k=4,5,6$).

(iv) If F_1 is of type t_7^1 , it means that applicant a_1 is the only H-productivity in both the firms. So both the firms will make their first offer to a_1 . F_2 's offer will be rejected since F_1 has offered a higher wage in which case F_2 will hire an L-productivity applicant. So $R_2^{oo}(s^1 = s_d^1, s^2 \equiv \omega_l | t_7^1, t_7^2) = 1$. So $Q_2^{oo}(s^1 = s_d^1, s^2 \equiv \omega_l | T_{1h}^2) = \frac{37}{48}$ and the corresponding $R_2^{oo} = \frac{11}{48}$.

Therefore the ex ante expected profits of the two firms when $s^1 = s_d^1$ and $s^2 \equiv \omega_1$ are

$$\Pi_1^{oo}(s^1 = s_d^1, s^2 \equiv \omega_1) = \frac{112}{128}\pi^h + \frac{16}{128}\pi^l + \frac{64}{128}\omega \quad [\text{A.2}]$$

$$\Pi_2^{oo}(s^1 = s_d^1, s^2 \equiv \omega_1) = \frac{4}{8}(H - \omega_l) + \frac{3}{8}\left(\frac{37}{48}H + \frac{11}{48}L - \omega_l\right) + \frac{1}{8}(L - \omega_l) = \frac{101}{128}\pi^h + \frac{27}{128}\pi^l + \frac{101}{128}\omega$$

The analysis of $G^{oo}(s^1 \equiv \omega_1, s^2 = s_d^2)$ is symmetric.

A.3. Analysis of $G^{oo}(s^1 = s_d^1, s^2 = s_d^2)$

In this case the strategy of a firm F_m ($m=1,2$) having one H-productivity applicant is to offer ω_h to him and to offer ω_l otherwise. The conditional probability that F_1 has one H-productivity applicant (say, it is of type t_7^1) and hires him for each type of F_2 are as follows:

(i) $Q_1^{oo}(s^1=s_d^1, s^2=s_d^2 | t_7^1, t_k^2) = k=1,2,3,4,5,6,8$

(ii) If F_2 is of type t_7^2 , it means that applicant a_1 is the only H-productivity applicant in both the firms. So both firms will make their first offer to applicant a_1 . F_1 hires the H-productivity applicant or an L-productivity applicant each with probability $\frac{1}{2}$.

Therefore, $Q_m^{oo}(s^1=s_d^1, s^2=s_d^2 | T_{1h}^m) = \frac{15}{16}$ and the corresponding $R_m^{oo} = \frac{1}{16}$. So the ex ante expected profit Π_m^{oo} of a firm F_m ($m=1, 2$) when $s^m = s_d^m$ is:

$$\Pi_m^{oo}(s^1 = s_d^1, s^2 = s_d^2) = \frac{109}{128}\pi^h + \frac{19}{128}\pi^l + \frac{64}{128}\omega \quad [\text{A.3}]$$

APPENDIX B: ANALYSIS OF THE SUBGAME G^{bo}

In this subgame F_1 announces and commits to a particular wage prior to receiving specific information about applicants' productivities. F_2 solicits applications from the applicants and after deriving their firm-specific productivities offer wages privately to them. Thus F_2 has the option of altering its wage offers. I will analyze each of the subgames $G^{\text{bo}}(\omega^*, s^2)$ and consequently derive the ex ante profit Π_m^{bo} of each firm ($m=1,2$). The analysis of the subgame G^{ob} is symmetric.

B.1. Analysis of the Subgame $G^{\text{bo}}(\omega_1, s^2 \equiv \omega_1)$

Firm F_1 announces ω_1 and the strategy of firm F_2 is to offer ω_1 in all possible rounds irrespective of its type. The ex ante expected profits F_1 and F_2 are:

$$\Pi_1^{\text{bo}}(\omega_1, s^2 \equiv \omega_1) = \frac{104}{128}\pi^h + \frac{24}{128}\pi^l + \frac{104}{128}\omega \quad [\text{B.1a}]$$

$$\Pi_2^{\text{bo}}(\omega_1, s^2 \equiv \omega_1) = \frac{104}{128}\pi^h + \frac{24}{128}\pi^l + \frac{104}{128}\omega \quad [\text{B.1b}]$$

B.2. Analysis of the Subgame $G^{\text{bo}}(\omega_h, s^2 \equiv \omega_1)$

Firm F_1 announces ω_h , and the strategy of firm F_2 is to offer ω_1 in all possible rounds of offer irrespective of its type. Firm F_1 will hire, if any, an H-productivity applicant only. The ex ante expected profit of F_1 and F_2 are:

$$\Pi_1^{\text{bo}}(\omega_h, s^2 \equiv \omega_1) = \frac{112}{128}\pi^h \quad [\text{B.2a}]$$

$$\Pi_2^{\text{bo}}(\omega_h, s^2 \equiv \omega_1) = \frac{98}{128}\pi^h + \frac{30}{128}\pi^l + \frac{98}{128}\omega \quad [\text{B.2b}]$$

B.3. Analysis of the Subgame $G^{bo}(\omega_1, s^2 = s_d^2)$

Firm F_1 announces ω_1 and the strategy of firm F_2 having one H-productivity applicant is to offer ω_h to him and to offer ω_l otherwise. So the ex ante expected profits F_1 and F_2 are:

$$\Pi_1^{bo}(\omega_1, s^2 = s_d^2) = \frac{101}{128}\pi^h + \frac{27}{128}\pi^l + \frac{101}{128}\omega \quad [\text{B.3a}]$$

$$\Pi_2^{bo}(\omega_1, s^2 = s_d^2) = \frac{112}{128}\pi^h + \frac{16}{128}\pi^l + \frac{64}{128}\omega \quad [\text{B.3b}]$$

B.4. Analysis of the Subgame $G^{bo}(\omega_h, s^2 = s_d^2)$

The strategy of firm F_1 is to announce ω_h and the strategy of F_2 having one H-productivity is to offer ω_h to him and to offer ω_l otherwise. So the ex ante expected profits of F_1 and F_2 are:

$$\Pi_1^{bo}(\omega_h, s^2 = s_d^2) = \frac{109}{128}\pi^h \quad [\text{B.4a}]$$

$$\Pi_2^{bo}(\omega_h, s^2 = s_d^2) = \frac{105}{128}\pi^h + \frac{23}{128}\pi^l + \frac{64}{128}\omega \quad [\text{B.4b}]$$

APPENDIX C: DERIVATION OF THE EQUILIBRIUM POLICY

I define the following values used for deriving the equilibrium policy.

$$X1 := \Pi_1^{bb}(\omega_h, \omega_h) - \Pi_1^{bb}(\omega_l, \omega_h) = \Pi_2^{bb}(\omega_h, \omega_h) - \Pi_2^{bb}(\omega_h, \omega_l) = \frac{7}{128}\pi^h - \frac{30}{128}\pi^l - \frac{98}{128}\omega \quad [\text{C.1}]$$

$$X2 := \Pi_1^{bb}(\omega_h, \omega_l) - \Pi_1^{bb}(\omega_l, \omega_l) = \Pi_2^{bb}(\omega_l, \omega_h) - \Pi_2^{bb}(\omega_l, \omega_l) = \frac{8}{128}\pi^h - \frac{24}{128}\pi^l - \frac{104}{128}\omega \quad [\text{C.2}]$$

$$X3 := \Pi_m^{oo}(s^m = s_d^m, s^i = s_d^i) - \Pi_m^{oo}(s^m \equiv \omega_1, s^i = s_d^i) = \frac{8}{128}\pi^h - \frac{8}{128}\pi^l - \frac{37}{128}\omega \quad [C.3]$$

$$X4 := \Pi_m^{oo}(s^m = s_d^m, s^i \equiv \omega_1) - \Pi_m^{oo}(s^m \equiv \omega_1, s^i \equiv \omega_1) = \frac{8}{128}\pi^h - \frac{8}{128}\pi^l - \frac{40}{128}\omega \quad [C.4]$$

$$X5 := \Pi_1^{bo}(\omega_h, s^2 = s_d^2) - \Pi_1^{bo}(\omega_1, s^2 = s_d^2) = \frac{8}{128}\pi^h - \frac{27}{128}\pi^l - \frac{101}{128}\omega \quad [C.5]$$

$$X6 := \Pi_1^{bo}(\omega_h, s^2 \equiv \omega_1) - \Pi_1^{bo}(\omega_1, s^2 \equiv \omega_1) = \frac{8}{128}\pi^h - \frac{24}{128}\pi^l - \frac{104}{128}\omega \quad [C.6]$$

$$X7 := \Pi_2^{bo}(\omega_h, s^2 = s_d^2) - \Pi_2^{bo}(\omega_h, s^2 \equiv \omega_1) = \frac{7}{128}\pi^h - \frac{7}{128}\pi^l - \frac{34}{128}\omega \quad [C.7]$$

$$X8 := \Pi_2^{bo}(\omega_1, s^2 = s_d^2) - \Pi_2^{bo}(\omega_1, s^2 \equiv \omega_1) = \frac{8}{128}\pi^h - \frac{8}{128}\pi^l - \frac{40}{128}\omega \quad [C.8]$$

Comparing the values X1 to X8 we get:

$$X3 \geq X4 = X8 \text{ and } X5 \geq X6 = X2$$

$$X3 - X1 = \frac{1}{128}\pi^h + \frac{22}{128}\pi^l + \frac{61}{128}\omega \geq 0$$

$$X4 - X1 = \frac{1}{128}\pi^h + \frac{22}{128}\pi^l + \frac{58}{128}\omega \geq 0 \Rightarrow X3 \geq X4 = X8 \geq X1$$

$$X4 - X2 = \frac{16}{128}\pi^l + \frac{64}{128}\omega \geq 0 \Rightarrow X3 \geq X4 = X8 \geq X2 = X6$$

$$X7 - X1 = \frac{23}{128}\pi^l + \frac{64}{128}\omega \geq 0 \Rightarrow X7 \geq X1$$

$$X4 - X5 = \frac{19}{128}\pi^l + \frac{61}{128}\omega \geq 0$$

So we get the following string of inequalities.

$$(a) X3 \geq X8 = X4 \geq X5 \geq X2 = X6; (b) X3 \geq X8 = X4 \geq X1; (c) X7 \geq X1$$

Proof of Theorem 1

If $X1 \geq 0 \Rightarrow X7 \geq 0$ and $X3 \geq X8 = X4 \geq X5 \geq X2 = X6 \geq 0$, then the set of equilibria is

$\{(\omega_h, \omega_h), (s^1 = s_d^1, s^2 = s_d^2), (\omega_h, s^2 = s_d^2), (s^1 = s_d^1, \omega_h)\}$. We get the following matrix:

		b	P ₂	o
P ₁	b	$\frac{105\pi^h}{128}, \frac{105\pi^h}{128}$		$\frac{109\pi^h}{128}, \frac{105\pi^h}{128} + \frac{23\pi^l}{128} + \frac{64w}{128}$
	o	$\frac{105\pi^h}{128} + \frac{23\pi^l}{128} + \frac{64w}{128}, \frac{109\pi^h}{128}$		$\frac{109\pi^h}{128} + \frac{19\pi^l}{128} + \frac{64w}{128},$ (same)

From this matrix it is seen that (o, o) is the dominant policy profile. \square

Proof of Theorem 2

(i) If $X1 \geq 0 \Rightarrow X7 \geq 0$ and $X3 \geq X8=X4 \geq X5 \geq 0 \geq X2=X6$, then the set of equilibria is

$\{(\omega_h, \omega_h), (\omega_l, \omega_l)\}, (s^1 = s_d^1, s^2 = s_d^2), (\omega_h, s^2 = s_d^2), (s^1 = s_d^1, \omega_h)\}$. There are two symmetric equilibria in the subgame G^{bb} . $\Pi_m^{bb}(\omega_h, \omega_h) - \Pi_m^{bb}(\omega_l, \omega_l) = \frac{1}{128}\pi^h - \frac{24}{128}\pi^l - \frac{104}{128}\omega < 0$ because by the condition $X2 = \frac{8}{128}\pi^h - \frac{24}{128}\pi^l - \frac{104}{128}\omega \leq 0$. Each firm will choose the payoff dominant strategy (ω_l, ω_l) . We get the following matrix:

		b	P ₂	o
P ₁	b	$\frac{104\pi^h}{128} + \frac{24\pi^l}{128} + \frac{104w}{128}, \frac{104\pi^h}{128} + \frac{24\pi^l}{128} + \frac{104w}{128}$		$\frac{109\pi^h}{128}, \frac{105\pi^h}{128} + \frac{23\pi^l}{128} + \frac{64w}{128}$
	o	$\frac{105\pi^h}{128} + \frac{23\pi^l}{128} + \frac{64w}{128}, \frac{109\pi^h}{128}$		$\frac{109\pi^h}{128} + \frac{19\pi^l}{128} + \frac{64w}{128}, \frac{109\pi^h}{128} + \frac{19\pi^l}{128} + \frac{64w}{128}$

$$\Pi_1^{ob} - \Pi_1^{bb} = \Pi_2^{bo} - \Pi_2^{bb} = \frac{1}{128}\pi^h - \frac{1}{128}\pi^l - \frac{40}{128}\omega \leq 0 \text{ (because, } X2 \leq 0)$$

$$\Pi_1^{oo} - \Pi_1^{bo} = \Pi_2^{oo} - \Pi_2^{ob} = \frac{19}{128}\pi^l + \frac{64}{128}\omega \geq 0.$$

So $\{(b, b), (o, o)\}$ are the two symmetric equilibria.

Now, $\Pi_m^{oo} - \Pi_m^{bb} = \frac{5}{128}(\pi^h - \pi^l - 9\omega) \geq 0$ because $X1 \geq 0 \Rightarrow \pi^h - \frac{30}{7}\pi^l \geq 14\omega \Rightarrow \pi^h - \pi^l \geq \pi^h - \frac{30}{7}\pi^l \geq 14\omega \geq 9\omega$. Hence, (o, o) is the payoff dominant policy profile.

(ii) We get the same set of equilibria and hence the same result if the inequality: $X7 \geq 0$, $X1 \leq 0$ and $X3 \geq X4=X8 \geq X5 \geq 0 \geq X6=X2$ holds. \square

Proof of Theorem 3

(i) If $X1 \geq 0 \Rightarrow X7 \geq 0$ and if $X3 \geq X4=X8 \geq 0 \geq X5 \geq X2=X6$, then the set of equilibria is:

$\{(\omega_h, \omega_h), (\omega_l, \omega_l), (s^1 = s_d^1, s^2 = s_d^2), (\omega_l, s^2 = s_d^2), (s^1 = s_d^1, \omega_l)\}$. There are two symmetric equilibria in the subgame G^{aa} . As shown above, each firm will choose the payoff dominant strategy (ω_l, ω_l) .

(ii) If $X7 \leq 0 \Rightarrow X1 \leq 0$ and $X3 \geq X4=X8 \geq 0 \geq X5 \geq X2=X6$ then the set of equilibria is

$\{(\omega_l, \omega_l), (s^1 = s_d^1, s^2 = s_d^2), (\omega_l, s^2 = s_d^2), (s^1 = s_d^1, \omega_l)\}$.

(iii) If $X7 \geq 0$ and $X1 \leq 0$, $X3 \geq X4=X8 \geq 0 \geq X5 \geq X2=X6$ then also we get the same set of equilibria. We get the following matrix:

		b	P ₂	o
b		$\frac{104}{128}\pi^h + \frac{24}{128}\pi^l + \frac{104}{128}\omega$	$\frac{104}{128}\pi^h + \frac{24}{128}\pi^l + \frac{104}{128}\omega$	$\frac{101}{128}\pi^h + \frac{27}{128}\pi^l + \frac{101}{128}\omega, \frac{112}{128}\pi^h + \frac{16}{128}\pi^l + \frac{64}{128}\omega$
P ₁		$\frac{112}{128}\pi^h + \frac{16}{128}\pi^l + \frac{64}{128}\omega$	$\frac{101}{128}\pi^h + \frac{27}{128}\pi^l + \frac{101}{128}\omega$	$\frac{109}{128}\pi^h + \frac{19}{128}\pi^l + \frac{64}{128}\omega, \frac{109}{128}\pi^h + \frac{16}{128}\pi^l + \frac{64}{128}\omega$
o				

$$\Pi_1^{ob} - \Pi_1^{bb} = \Pi_2^{bo} - \Pi_2^{bb} = \frac{8}{128}\pi^h - \frac{8}{128}\pi^l - \frac{40}{128}\omega = X4 \geq 0$$

$$\Pi_1^{oo} - \Pi_1^{bo} = \Pi_2^{oo} - \Pi_2^{ob} = \frac{8}{128}\pi^h - \frac{8}{128}\pi^l - \frac{37}{128}\omega = X3 \geq 0$$

So, (o, o) is the equilibrium policy profile. \square

Proof of Theorem 4

We have the following cases.

(i) If $0 \geq X7 \Rightarrow 0 \geq X1$ and $X3 \geq 0 \geq X4=X8 \geq X5 \geq X2=X6$ holds, then the set of equilibria is:

$\{(\omega_1, \omega_1), \{(s^1 \equiv \omega_1, s^2 \equiv \omega_l), (s^1 = s_d^1, s^2 = s_d^2)\}, (\omega_1, s^2 \equiv \omega_1), (s^1 \equiv \omega_l, \omega_l)\}$. There are two symmetric equilibria in the subgame G^{oo} . Since $X4 \leq 0 \Rightarrow \frac{8}{128}(H-L) - \frac{40}{128}(\omega_h - \omega_l) \leq 0$.

$$\Pi_m^{oo}(s^1=s_d^1, s^2=s_d^2) - \Pi_m^{oo}(s^1 \equiv \omega_1, s^2 \equiv \omega_l) = \frac{5}{128}(H-L) - \frac{40}{128}(\omega_h - \omega_l) \leq 0$$

So each firm will choose the payoff dominant strategy strategy $(s^m(t_j^m) = \omega_1 \forall j; (m=1,2))$.

Therefore, the set of equilibria is $\{(\omega_1, \omega_1), (s^1 \equiv \omega_1, s^2 \equiv \omega_l), (\omega_1, s^2 \equiv \omega_1), (s^1 \equiv \omega_l, \omega_l)\}$.

We get the same set of equilibria if the following inequalities hold.

(ii) $0 \geq X7 \Rightarrow 0 \geq X1$ and $0 \geq X3 \geq X8=X4 \geq X5 \geq X2=X6$;

(iii) $X7 \geq 0$ and $X1 \leq 0, X3 \geq 0 \geq X4=X8 \geq X5 \geq X2=X6$;

(iv) $X7 \geq 0$ and $X1 \leq 0, 0 \geq X3 \geq X4=X8 \geq X5 \geq X2=X6$.

The ex ante expected profit of a firm F_m ($m=1,2$) for any pair of policies (p_1, p_2) is:

$$\Pi_m^{p_1 p_2} = \frac{104}{128}H + \frac{24}{128}L - \omega_l. \text{ Therefore, any policy is an equilibrium policy. } \square$$

Proof of Theorem 5

(i) If $X7 \leq 0 \Rightarrow X1 \leq 0$ and $X3 \geq X4=X8 \geq X5 \geq X2=X6 \geq 0$ hold, then the set of equilibria is

$\{(\omega_h, \omega_1), (\omega_1, \omega_h)\}, (s^1 = s_d^1, s^2 = s_d^2), (\omega_1, s^2 \equiv \omega_1), (s^1 \equiv \omega_l, \omega_h)\}$. Suppose (ω_h, ω_1) is the

resultant equilibrium in the subgame G^{aa} , we get the following matrix:

		b	P ₂	o
b		$\frac{112}{128}\pi^h, \frac{98}{128}\pi^h + \frac{30}{128}\pi^l + \frac{98}{128}w$		$\frac{112}{128}\pi^h, \frac{98}{128}\pi^h + \frac{30}{128}\pi^l + \frac{98}{128}w$
P ₁		$\frac{98}{128}\pi^h + \frac{30}{128}\pi^l + \frac{98}{128}w, \frac{112}{128}\pi^h$		$\frac{109}{128}\pi^h + \frac{19}{128}\pi^l + \frac{64}{128}w, \text{ (same)}$
o				

$X7 \leq 0 \Rightarrow \pi^h - \pi^l \leq \frac{34}{7}\omega \Rightarrow \pi^h - \frac{30}{14}\pi^l < \frac{98}{14}\omega$. We get: $\Pi_1^{bb} - \Pi_1^{ob} = \frac{14}{128}\pi^h - \frac{30}{128}\pi^l - \frac{98}{128}\omega < 0$.

$$\Pi_1^{bo} - \Pi_1^{oo} = \Pi_2^{ob} - \Pi_2^{oo} = \frac{3}{128}\pi^h - \frac{19}{128}\pi^l - \frac{64}{128}\omega < 0$$

$$\Pi_2^{bb} - \Pi_2^{bo} = 0$$

So (o, o) is the dominant policy profile. The same is true if (ω_1, ω_1) is the resultant equilibrium in the subgame G^{bb} .

(ii) If $X7 \geq 0$ and $X1 \leq 0$ and $X3 \geq X4=X8 \geq X5 \geq X6=X2$ holds, then the set of equilibria is the same as (i). In this case also (o, o) is the dominant policy profile. \square

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Chapter 4

Timing of Wage Tenders

I. INTRODUCTION

In the market for newcomers to the white-collar labor force it is sometimes observed that firms announce and commit to a particular wage after receiving specific information about applicants' productivity. A typical example is the British academic market. Some British universities and colleges announce a job opening for lectureships and after soliciting applications send the applicants particulars about the post which also include nonoverlapping wage ranges that are fixed and nonnegotiable.

The motivation for this paper stems from the above type of labor market institution. The question analyzed is that if "announcing a wage" is the strategy rule to be followed by the firms, then what should be the equilibrium timing of the wage announcement: before receiving specific information about applicants productivity or after receiving specific information about applicants' productivity. Firms first simultaneously commit themselves to the timing of a wage tender and afterwards compete contingent on the chosen types of timing.

In chapter 2, the consequences of announcing and committing to a wage prior to receiving information about applicants' productivity was discussed. In chapter 3, two recruitment policies, whether to announce and commit to a wage a priori or to offer wages privately after deriving specific information about applicants' productivity, were compared. The equilibrium policy to be followed by the firms at the start of the hiring process was endogenously determined.

Questions about commitment to a particular strategy have been addressed before by Gal-Or (1985) and Dowrick (1986). They have shown that in a Stackelberg type of game with time of commitment as the strategic choice, firms are unwilling to commit first when the reaction curves are downward sloping. With upward sloping reaction curves, it is advantageous to commit first. The outcome is a conflict over the choice of roles: which firm should be the leader and which firm should be the follower. When goods are substitutes (complements) the reaction curves in terms of quantities are downward (upward) sloping and the reaction curves in terms of prices are upward (downward) sloping (Singh and Vives, 1984).

When goods are substitutes, an increase in the output by the leader requires the follower to reduce its output to at least partially restore its profit margin. Correspondingly, it is optimal for the follower to increase its price when the leader increases its price if the marginal cost of the follower is constant. This intuitively explains the nature of the reaction curves when the goods are substitutes. The nature of the reaction curves if the goods are complements can be similarly explained.

In this chapter the problem of time of commitment to a wage is studied in the context of a labor market where the productivity of applicants is firm-specific and hence, is independent across firms. Each firm has no prior information about individual productivity and only knows the distribution of the applicants' productivity. Gal-Or and Dowrick considered substitutes and complements but did not take into account market uncertainty. They analyzed the question of time of commitment to a strategy from the seller's point of view where the firms were competing with each other in prices or output. In contrast I investigate the timing of announcement and commitment to a wage when the firms are competing with each other to buy labor services in the presence of uncertainty about a possible match.

The equilibrium timing of wage tender is determined endogenously by comparing the ex ante expected profits of the firms associated with the equilibrium wage announced under the two

policies. For certain constellations of parametric values, we get different equilibrium wages to be announced under the different policies. It turns out that the lack of prior information and, hence the uncertainty about the possible match, always yields announcing a wage after receiving specific information about applicants' productivity as an equilibrium policy. However, if a low wage is the equilibrium outcome under both timings, then announcing a wage ex ante and announcing a wage ex post are consistent with equilibrium. This explains the coexistence of different timings of wage tender.

The paper is organized as follows. The model is presented in section II. In Section III, I discuss the results of the different subgames, with proofs in appendix 1. In Section IV, I endogenously determine the equilibrium of wage tender, the derivations are provided in appendix 2.

II. THE MODEL

There are three applicants a_1, a_2, a_3 , and two firms F_1 and F_2 . All applicants and firms know that an applicant can be of high (H) or low (L) productivity and these are firm-specific productivities. Let $W = \{\omega_h, \omega_l\}$ be the set of high and low wage, respectively, from which each firm selects a wage. All applicants are newcomers to white-collar jobs. It is assumed that each firm has one vacancy.

It is also assumed that $H > L, \omega_h \geq \omega_l, \omega_h \geq L, H - \omega_l > H - \omega_h \geq L - \omega_l \geq 0$. The last string of inequalities imply that a firm gets maximum surplus by hiring an H-productivity applicant by paying him ω_l . Also it is weakly more profitable to hire an H-productivity applicant and pay him ω_h than to hire an L-productivity applicant and pay him ω_l . The inequality $\omega_h \geq L$ implies that a firm will pay a high wage, if at all, to an H-productivity

applicant only.

Hiring by a firm depends on the wage announced by both the firms, productivity of the applicants in both the firms, and the recipients of the firms' job offer. The hiring process can be described as an extensive form game consisting of the following stages.

Stage 0: Let \hat{X}_{bi} be a family of independent random variables ($b=1,2,3; i=1,2$) where \hat{X}_{bi} assumes value H and L each with probability $\frac{1}{2}$. There are $\sum_{q=0}^3 \binom{3}{q} = 2^3 = 8$ possible combinations of the firm-specific productivities of the three applicants in a firm which is shown in Table 1 below. $\binom{3}{q}$ means that q out of three applicants are of H-productivity in a firm. There are 2^6 possible combinations of the firm-specific productivities of the three applicants in the two firms. Each firm knows this distribution ex ante. Let $T^i = \{H, L\}^3$ be the set of all possible types of a firm F_i ($i=1,2$). A type is a list of the firm-specific productivities, one for each applicant. T^i has 2^3 elements which we label t_j^i ($j=1,2,..8$). We can assume (a) to (c).

- (a) $T_{2h}^i = \{t_j^i | j=1,2,3,4\}$, is the set of types of firm F_i having at least two H-productivity applicants.
- (b) $T_{1h}^i = \{t_j^i | j=5,6,7\}$, is the set of types of firm F_i having exactly one H-productivity applicant.
- (c) $T_1^i = \{t_8^i\}$ is the singleton containing the types of firm F_i that has only L-productivity applicants: $t_8^i = (L, L, L)$.

Stage 1: In this stage the firms simultaneously, choose a policy $p_i \in \{b, a\}$. '**b**' denotes the policy to announce and commit to a wage before receiving specific information about applicants' productivity. '**a**' denotes the policy to announce and commit to a wage after receiving specific information about applicants' productivity.

Stage 2: After choosing a policy, both firms announce a job opening, simultaneously. If a firm chooses the policy 'b', then that firm announces and commits itself to a particular wage in this stage.

Stage 3: In response to the advertisement in stage 2, all applicants apply simultaneously to both the firms. I assume that there is no transaction cost to applying. From the applications each firm derives the firm-specific productivities of the applicants, i.e. each firm knows its own type which is private information. An applicant has no information about the firm's evaluation. So applying to both the firms is the dominant strategy.

Stage 4: In this stage each firm makes at most one offer. A firm having q number of H -productivity applicants makes an offer to each with probability $\frac{1}{q}$ ($q=1,2,3$). An applicant receiving offers from both firms then he accepts the highest offer. If both firms make the same offer to the same applicant then he chooses each firm's offer with probability $\frac{1}{2}$. An applicant receiving one offer accepts it because he has no information about the firms' evaluation and, hence is not sure of getting any further offers.

Stage 5: This stage regards the behavior of a rejected firm, if any, in stage 3. A rejected firm may or may not make an offer in this stage. This depends on the policy chosen by the firm, the wage announced by that firm, and the productivity of the remaining two applicants in that firm.

Let $G^{P_1 P_2}$ be the subgame beginning at stage 2 after each firm has chosen a policy p_i ($i=1,2$) in stage 1. Note that each of these subgames begins at the **pre-application stage**. Let $W_i^{P_1 P_2}$ be the pure strategy space of a firm F_i ($i=1,2$) for each of the subgames $G^{P_1 P_2}$. The

equilibrium policy is endogeneously determined by comparing the *ex ante expected profit* $\Pi_i^{P_1 P_2}$ ($i=1,2$) associated with the equilibrium set of strategies of each of the subgames.

The *ex ante expected profits* for each subgame are calculated by the process of backward induction. Let $Q_i^{P_1 P_2}(w_1^{P_1 P_2}, w_2^{P_1 P_2} | t_j^1, t_k^2)$ be the conditional probability that a firm F_i ($i=1,2$) hires an H-productivity applicant when $(w_1^{P_1 P_2}, w_2^{P_1 P_2})$ is the strategy profile of the two firms given their types. So the overall probability that a firm F_i hires an H-productivity applicant if (p_1, p_2) are the policies chosen by the firms and the strategy profile is $(w_1^{P_1 P_2}, w_2^{P_1 P_2})$ is given by

$$Q_i^{P_1 P_2}(w_1^{P_1 P_2}, w_2^{P_1 P_2}) = \frac{4}{8} \sum_{t_j^i \in T_{2h}^i} \left(\frac{1}{8} \sum_{k=1}^8 Q_i^{P_1 P_2}(w_1^{P_1 P_2}, w_2^{P_1 P_2} | t_j^i, t_k^2) \right) + \frac{3}{8} \sum_{t_j^i \in T_{1h}^i} \left(\frac{1}{8} \sum_{k=1}^8 Q_i^{P_1 P_2}(w_1^{P_1 P_2}, w_2^{P_1 P_2} | t_j^i, t_k^2) \right)$$

The probability that a firm F_i hires an L-productivity applicant, if at all, is

$$R_i^{P_1 P_2}(w_1^{P_1 P_2}, w_2^{P_1 P_2}) = \frac{3}{8} (1 - Q_i^{P_1 P_2}(w_1^{P_1 P_2}, w_2^{P_1 P_2} | T_{1h}^i)) + \frac{1}{8}.$$

In the next section I shall provide the results for each of the subgames G^{bb} , G^{aa} , G^{ab} , and G^{ba} . The derivation of the results are given in the appendix. Throughout the paper I will use the following notations: $\pi^h = H - \omega_h$; $\pi^l = L - \omega_l$; and $\omega = \omega_h - \omega_l$.

III.1. RESULTS OF THE SUBGAME G^{bb}

If announcing a wage without receiving specific information about applicants' productivity is the policy chosen by both the firms, then each firm each firm F_i will announce and commit to a wage $w_i^{bb} \in W_i^{bb} \equiv W$, a priori, along with the announcement of a job

opening. For each $(w_1^{bb}, w_2^{bb}) \in W_1^{bb} \times W_2^{bb}$, let $G^{bb}(w_1^{bb}, w_2^{bb})$ denote the subgame beginning immediately after the wage announcements have been made. In this case $G^{bb}(w_1^{bb}, w_2^{bb}) \equiv G^{bb}$. It turns out that each of the subgames $G^{bb}(w_1^{bb}, w_2^{bb})$ has unique subgame perfect expected ex ante equilibrium payoffs $\Pi_i^{bb}(w_1^{bb}, w_2^{bb})$. Given the knowledge of these payoffs, the analysis of the payoffs reduces to the analysis of the *reduced game in normal form*:

$$\Gamma^{bb} = (\{F_1, F_2\}, W_i^{bb}, \Pi_i^{bb}; (i = 1, 2)).$$

Lemma 1

The payoff functions $\Pi_1^{bb}(w_1^{bb}, w_2^{bb})$ and $\Pi_2^{bb}(w_1^{bb}, w_2^{bb})$ are represented in the following matrix:

	W_2^{bb}	
	ω_h	ω_l
ω_h	$\frac{105}{128} \pi^h$	$\frac{112}{128} \pi^h$
	$\frac{105}{128} \pi^h$	$\frac{98}{128} \pi^h + \frac{30}{128} \pi^l + \frac{98}{128} \omega$
W_1^{bb}	$\frac{98}{128} \pi^h + \frac{30}{128} \pi^l + \frac{98}{128} \omega$	$\frac{104}{128} \pi^h + \frac{24}{128} \pi^l + \frac{104}{128} \omega$
ω_l	$\frac{112}{128} \pi^h$	$\frac{104}{128} \pi^h + \frac{24}{128} \pi^l + \frac{104}{128} \omega$

The derivation of this matrix is discussed in chapter 2.

III.2. RESULTS OF THE SUBGAME G^{aa}

In this case both firms announce and commit to a wage, simultaneously, after deriving specific information about applicants' productivity, i.e. each firm announces and commits to a wage after learning its own type. Each firm believes that the other firm could be any one of the

types with equal probability. The equilibrium wage announced by a firm F_i ($i=1,2$) at the post-application stage is defined as follows.

Definition 1

The Bayesian Equilibrium wage announced by a firm is defined as a mapping (pure strategy rule) $S_i^{aa}: T^i \rightarrow W_i^{aa}$ such that, for each $t_j^i \in T^i$, $s_i^{aa}(t_j^i) \in \operatorname{argmax}_{\omega_i \in W_i^{aa}} \frac{1}{8} \sum_{k=1}^8 \pi_i^{aa}(\omega_i, s_i^{aa}(t_k^m), t_j^i, t_k^m)$, ($i, m = 1, 2; i \neq m$).

For each $(s_1^{aa}, s_2^{aa}) \in S_1^{aa} \times S_2^{aa}$, let $G^{aa}(s_1^{aa}, s_2^{aa})$ denote the subgame beginning immediately after the announcements s_1^{aa} and s_2^{aa} have been made. It turns out that each of the subgames $G^{aa}(s_1^{aa}, s_2^{aa})$ has unique subgame perfect payoffs $\frac{1}{8} \sum_{k=1}^8 \pi_i^{aa}(s_i^{aa}(t_j^i), s_m^{aa}(t_k^m), t_j^i, t_k^m)$, ($i, m = 1, 2; i \neq m$).

Consequently, the ex ante expected payoff of a firm F_i ($i = 1, 2$) of the subgame G^{aa} beginning at the pre-application stage (i.e. at stage 2) for each (s_1^{aa}, s_2^{aa}) is:

$$\Pi_i^{aa}(s_i^{aa}, s_m^{aa}) = \frac{1}{8} \sum_{j=1}^8 \left\{ \frac{1}{8} \sum_{k=1}^8 \pi_i^{aa}(s_i^{aa}(t_j^i), s_m^{aa}(t_k^m), t_j^i, t_k^m) \right\}, (i, m = 1, 2; i \neq m).$$

Given the knowledge of these payoffs and the underlying equilibria, the analysis of the entire subgame G^{aa} reduces to the analysis of the *reduced game in normal form*:

$$\Gamma^{aa} = (\{F_1, F_2\}, S_1^{aa}, S_2^{aa}, \Pi_1^{aa}, \Pi_2^{aa}).$$

An analysis of the subgame G^{aa} for each (s_1^{aa}, s_2^{aa}) and the derivation of the ex ante expected payoffs is provided in appendix 1.

Let us consider the subgame beginning at the post-application stage. If a firm learns that there are at least two H-productivity applicants or only L-productivity applicants, that firm

will announce ω_l . Since there are two firms each firm can hire in at most two rounds. So if a firm has at least two H-productivity applicants it will surely hire one of them, hence, will get maximum surplus by announcing ω_l . If a firm has only L-productivity applicants it will announce ω_l , by assumption. If a firm has exactly one H-productivity applicant it can either announce ω_h or ω_l . So the strategy space S_i^{aa} of a firm F_i ($i=1,2$) in this subgame G^{aa} is:

$$S_i^{aa} = \{s_{id}^{aa}, s_{ic}^{aa}\} \text{ with } s_{id}^{aa}(t_j^i) = \begin{cases} \omega_h & j=5,6,7 \\ \omega_l & \text{otherwise} \end{cases} \text{ and } s_{ic}^{aa}(t_j^i) \equiv \omega_l.$$

Lemma 2

The payoff functions Π_i^{aa} ($i = 1, 2$) of the reduced game Γ^{aa} are represented in the following matrix:

	S_2^{aa}	
	s_{2d}^{aa}	s_{2c}^{aa}
s_{1d}^{aa}	$\frac{109}{128} \pi^h + \frac{16}{128} \pi^l + \frac{64}{128} \omega$	$\frac{112}{128} \pi^h + \frac{16}{128} \pi^l + \frac{64}{128} \omega$
s_{1c}^{aa}	$\frac{109}{128} \pi^h + \frac{16}{128} \pi^l + \frac{64}{128} \omega$	$\frac{101}{128} \pi^h + \frac{27}{128} \pi^l + \frac{101}{128} \omega$
S_1^{aa}	$\frac{101}{128} \pi^h + \frac{27}{128} \pi^l + \frac{101}{128} \omega$	$\frac{104}{128} \pi^h + \frac{24}{128} \pi^l + \frac{104}{128} \omega$
s_{1c}^{aa}	$\frac{112}{128} \pi^h + \frac{16}{128} \pi^l + \frac{64}{128} \omega$	$\frac{104}{128} \pi^h + \frac{24}{128} \pi^l + \frac{104}{128} \omega$

III.3. RESULTS OF THE SUBGAME G^{ba}

In this subgame firm F_1 announces and commits to a wage before receiving specific information about applicants' productivity. Firm F_2 announces a job opening only, solicits

applications, and **after** deriving specific information about applicants' productivity, announces and commits to a wage.

This subgame has a leader-follower structure. F_1 announces a wage which F_2 observes, incorporates that wage into its expected profit function, and then announces a wage that maximizes its expected profit. F_1 announces a wage that maximizes its expected profit by taking into consideration F_2 's possible strategies. Thus F_1 behaves like a leader while F_2 behaves like a follower.

Definition 2

The equilibrium in this subgame is defined as a pair $(\omega_1^{ba}, s_2^{ba} \equiv s_2^{aa})$ which satisfies the following conditions.

(i) At the pre-application stage firm F_1 announces a wage $\omega_1^{ba} \in W$ such that

$$\omega_1^{ba} \in \operatorname{argmax}_{\omega \in W} \frac{1}{8} \sum_{j=1}^8 \left\{ \frac{1}{8} \sum_{k=1}^8 \pi_1^{ba}(\omega, s_2^{ba}(t_k^2), t_j^1, t_k^2) \right\}.$$

(ii) Firm F_2 announces a wage $s_2^{ba}(t_k^2) \in S_2^{ba} \equiv S_2^{aa}$ at the post-application stage such that

$$\text{for each } t_k^2 \in T^2, s_2^{ba}(t_k^2) \in \operatorname{argmax}_{\omega \in S_2^{ba}} \frac{1}{8} \sum_{j=1}^8 \pi_2^{ba}(\omega^*, \omega, t_j^1, t_k^2).$$

The ex ante expected payoffs of the firms in the subgame G^{ba} beginning at the pre-application period for each $(\omega_1^{ba}, s_2^{ba})$ are:

$$\Pi_1^{ba}(\omega_1^{ba}, s_2^{ba}) = \frac{1}{8} \sum_{j=1}^8 \left\{ \frac{1}{8} \sum_{k=1}^8 \pi_1^{ba}(\omega_1^{ba}, s_2^{ba}(t_k^2), t_j^1, t_k^2) \right\};$$

$$\Pi_2^{ba}(\omega_1^{ba}, s_2^{ba}) = \frac{1}{8} \sum_{k=1}^8 \left\{ \frac{1}{8} \sum_{j=1}^8 \pi_2^{ba}(\omega_1^{ba}, s_2^{ba}(t_k^2), t_j^1, t_k^2) \right\}.$$

Given the knowledge of these payoffs the analysis of the subgame G^{ba} reduces to the

analysis of the *reduced game in normal form*: $\Gamma^{ba} = (\{F_1, F_2\}, W_1^{ba}, S_2^{ba}, \Pi_1^{ba}, \Pi_2^{ba})$. The derivation of the ex ante expected payoffs is shown in appendix 1.

Lemma 3

The payoff functions Π_1^{ba} and Π_2^{ba} are represented in the following matrix:

		S_2^{ba}	
		s_{2d}^{ba}	s_{2c}^{ba}
ω_h	$\frac{109}{128}\pi^h$ $\frac{103}{128}\pi^h + \frac{16}{128}\pi^l + \frac{64}{128}\omega$	$\frac{112}{128}\pi^h$ $\frac{98}{128}\pi^h + \frac{30}{128}\pi^l + \frac{98}{128}\omega$	
W_1^{ba}	$\frac{92}{128}\pi^h + \frac{36}{128}\pi^l + \frac{92}{128}\omega$	$\frac{104}{128}\pi^h + \frac{24}{128}\pi^l + \frac{104}{128}\omega$	
ω_l	$\frac{112}{128}\pi^h + \frac{16}{128}\pi^l + \frac{64}{128}\omega$	$\frac{104}{128}\pi^h + \frac{24}{128}\pi^l + \frac{104}{128}\omega$	

The payoffs of the subgame G^{ab} are symmetric to those of the subgame G^{ba} .

IV. ENDOGENIZING THE TIMING OF WAGE TENDER

We will use the information condensed in the derivation of the ex ante expected profit of the firms in each of the subgames for endogenizing the equilibrium timing of wage tender. Let G be the game beginning in stage 1.

Given the knowledge of the expected ex ante payoffs $\Pi_i^{P_1P_2}$ ($i = 1, 2$) and the underlying equilibria of each of the subgames $G^{P_1P_2}$, the analysis of the entire game G reduces to the analysis of the *reduced game in normal form*:

$$\Gamma = (\{F_1, F_2\}, P_1, P_2, \Pi_1^{P_1P_2}(w_1^{P_1P_2}, w_2^{P_1P_2}), \Pi_2^{P_1P_2}(w_1^{P_1P_2}, w_2^{P_1P_2})).$$

I shall state the results in this section. The proofs are provided in the appendix 2.

The equilibria for each of the subgames $G^{P_1P_2}$ is determined by the method of backward induction. For different conditions (shown in appendix 2) we get different sets of equilibria. The set of equilibria is defined as:

$E = \{(w_1^{bb}, w_2^{bb}), (s_1^{aa}, s_2^{aa}), (\omega_1^{ba}, s_2^{ba}), (s_1^{ab}, \omega_2^{ab})\} \in \{(W_1^{bb} \times W_2^{bb}) \times (S_1^{aa} \times S_2^{aa}) \times (W_1^{ba} \times S_2^{ba}) \times (S_1^{ab} \times W_2^{ab})\}$. The equilibrium timing of wage tender is determined by comparing the ex ante expected profits associated with the equilibria in the set E. I will only consider the equilibrium timing for pure strategy equilibria of each of the subgames $G^{P_1P_2}$.

Theorem 1

If $\{(\omega_h, \omega_h), (s_{1d}^{aa}, s_{2d}^{aa}), (\omega_h, s_{2d}^{ba}), (s_{1d}^{ab}, \omega_h)\}$ is the set of equilibria then both firms will announce and commit to a wage *after* receiving specific information about applicants' productivity.

In this case announcing a high wage is the equilibrium strategy for a firm announcing a wage a priori. The equilibrium strategy for a firm announcing a wage after receiving specific information is to announce a high wage, if there is exactly one H-productivity applicant; otherwise announce a low wage. The ex ante expected profit associated with the equilibria when both firms announce a wage after receiving specific information exceeds those associated with the equilibria in the other subgames. So announcing a wage after receiving specific information is the unique equilibrium timing.

There is no first-mover advantage for firms announcing a wage a priori because the job offers are made simultaneously. On the contrary, it is costly for firms to announce a wage without receiving specific information in the sense that it increases the probability of a firm's position remaining vacant. This is evident from the payoffs associated with the above mentioned strategies under the four subgames. So in the Gal-Or sense, both firms act as followers in

equilibrium. The informational advantage of a firm announcing a wage after receiving specific information, is clearly reflected by its strategy choice.

Even though firms commit to a wage after soliciting applications, which is the equilibrium timing in this case, it is possible that a firm may not hire in equilibrium. It might be the case that both firms have exactly one and the same H-productivity applicant. Then both firms will announce a high wage and make the offer to the same applicant who then will accept a firm's offer randomly. Since the rejected firm has also committed itself to a high wage it will not hire. The other possible equilibrium outcomes are: applicants with different productivities receiving the same wage, applicants with different productivities receiving the same wage, applicants with the same productivity receiving the same wage, and applicants with different productivities receiving different wages. So the equilibrium outcome may not result in a wage differential.

Theorem 2

If $\{((\omega_h, \omega_l), (\omega_l, \omega_h)), (s_{1d}^{aa}, s_{2d}^{aa}), (\omega_h, s_{2d}^{ba}), (s_{1d}^{ab}, \omega_h)\}$ is the set of equilibria then both firms will announce and commit to a wage *after* receiving specific information about applicants' productivity.

Under the hypothesis of theorem 2, we see that there are two asymmetric equilibria in the subgame where both firms announce a wage a priori. A comparison of the expected ex ante profits associated with the equilibria inumerated in theorem 2 yields announcement of a wage after receiving specific information about applicants' productivity as the equilibrium timing. The other implications are the same as before.

Theorem 3

If $\{((\omega_h, \omega_l), (\omega_l, \omega_h)), (s_{1d}^{aa}, s_{2d}^{aa}), (\omega_h, s_{2c}^{ba}), (s_{1c}^{ab}, \omega_h)\}$ is the set of equilibria then both firms will announce and commit to a wage **after** receiving specific information about applicants' productivity.

In this case we hypothesize two asymmetric equilibria (ω_h, ω_l) and (ω_l, ω_h) in the subgame where both firms announce a wage in advance. There are also asymmetric equilibria in the subgame where one firm announces a wage in advance while the other firm announces a wage after receiving specific information about applicants' productivity. Namely, the firm announcing a wage a priori announces high wage while the other firm announces a low wage. Here, too, both firms would commit to a strategy after receiving specific information and hence behave as followers.

Theorem 4

If $\{((\omega_l, \omega_l), (s_{1d}^{aa}, s_{2d}^{aa}), (\omega_h, s_{2d}^{ba}), (s_{1d}^{ab}, \omega_h)\}$ is the set of equilibria then both firms will announce a wage **after** receiving specific information about applicants' productivity.

Here, announcing a low wage is the equilibrium strategy when both firms announce a wage a priori. So neither firm's position remains vacant. Announcing a high wage if there is exactly one H-productivity applicant and otherwise announcing a low wage is the equilibrium strategy, when both firms announce a wage after receiving specific information. It is possible that a firm's position might remain vacant. In the leader-follower subgame, announcing a high wage is the equilibrium strategy for a firm announcing a wage in advance, whereas announcing a high wage if there is exactly one H-productivity applicant and otherwise announcing a low wage is the equilibrium strategy for a firm announcing a wage after receiving information about

applicants' productivity. In this case, though a firm's position might remain vacant when both firms announce a wage ex post, it turns out to be the equilibrium timing. The cost of a firm's position remaining vacant is less than the benefit of the firm's position being filled - the latter occurring when both firms announce a low wage a priori.

Theorem 5

If $\{(\omega_l, \omega_l), (s_{1c}^{aa}, s_{2c}^{aa}), (\omega_l, s_{2c}^{ba}), (s_{1c}^{ab}, \omega_l)\}$ is the set of equilibria then announce a wage *before* receiving specific information about applicants' productivity and announce a wage *after* receiving specific information about applicants' productivity are both equilibrium policies.

In this case we see that announcing a low wage is the equilibrium in all the subgames. The ex ante expected profit associated with a low wage strategy profile is the same for all the policies. Hence, to announce a wage a priori and to announce a wage ex post are both equilibrium timings. This supports the coexistence of different timing of wage tender. The British colleges and universities always announce the same wage range for a lecturer whether they announce it along with the job advertisement or after soliciting applications. In this case we see that they are no conflict about which firm should announce it a priori and which firm should announce ex post.

On the contrary, in the U. S. academic market it is usually the choice between announcing a wage in advance and offering a wage privately. This issue has been addressed in chapter 2.

V. CONCLUSION

In this paper we endogenized the timing of the wage tender. When a firm announced a wage in advance, its choice was either to announce a high wage or a low wage. On the contrary, when a firm announced a wage after soliciting applications, its choice was either to announce a high wage if there was exactly one H-productivity applicant and announce a low wage otherwise, or to announce a low wage irrespective of its type. The informational advantage in the second case was that a firm knew the productivity of the applicants before announcing a wage.

The equilibrium timing of wage announcement was determined by the method of backward induction. For different sets of inequalities we got different sets of equilibria for the subgames. Comparison of the expected ex ante profit associated with these equilibria determined the equilibrium timing of wage tender endogenously.

The uncertainty of a possible match and the risk of a firm's position remaining vacant lead announcing and committing to a wage after receiving specific information about applicants' productivity to be always an equilibrium timing of wage tender. Both firms gained by being the "second-mover" and hence there was no conflict regarding the role of the firms in terms of the timing of wage tender. However, if a low wage was the equilibrium strategy in all the subgames then announcing a wage in advance and announcing a wage ex post were both equilibria. There were no conflict regarding which firm should follow which policy. In equilibrium one firm's position may remain vacant and the equilibrium outcomes may not exhibit wage differential. Applicants with different productivities might receive the same wage, applicants with the same productivity might receive different wages, and applicants with different productivities might receive different wages.

APPENDIX 1

In this section I will analyze the subgames G^{aa} (both firms announce a wage after soliciting applications), G^{ba} (F_1 announces a wage before receiving applications and F_2 announces a wage after receiving applications), and G^{ab} which is symmetric to the subgame G^{ba} .

Since there are two firms each firm can hire in at most two rounds. If a firm has at least two H-productivity applicants that firm will surely hire an H-productivity applicant irrespective of the type and the wage announced by the other firm. This holds true for all the subgames.

$$\text{So } Q_i^{P_1P_2}(w_1^{P_1P_2}, w_2^{P_1P_2} | T_{2h}^i) = \frac{4}{8} \sum_{t_j^i \in T_{2h}^i} \left\{ \frac{1}{8} \sum_{k=1}^8 \pi_i^{P_1P_2}(w_1^{P_1P_2}, w_2^{P_1P_2} | t_j^i, t_k^m) \right\} = \frac{4}{8}. \text{ I will}$$

determine the probability of hiring an H-productivity applicant if there is exactly one of them in a firm under the different subgames. I assume that $s_i^{P_1P_2}(t_5^i) = s_i^{P_1P_2}(t_6^i) = s_i^{P_1P_2}(t_7^i)$, i.e., a firm having one H-productivity applicant have symmetric strategies. I will determine the probability of hiring an H-productivity assuming that F_1 is of type t_5^1 i.e., a_1 is the only H-productivity applicant in F_1 . This would be the same for t_6^1 and t_7^1 because the strategies are symmetric.

I. ANALYSIS OF THE SUBGAME G^{aa}

In this subgame both firms announce a wage after receiving specific information about applicants' productivity. The strategy space of a firm F_i is $S_i^{aa} = \{s_{id}^{aa}, s_{ic}^{aa}\}$, ($i=1,2$). For each pair of strategy profile $(s_1^{aa}, s_2^{aa}) \in S_1^{aa} \times S_2^{aa}$ I will determine the probability of hiring an H-

productivity applicant if there is exactly one of them. Consequently, I will determine the overall probability of hiring an H-productivity applicant, the probability of hiring an L-productivity applicant, and the ex ante expected profit of a firm for each pair of strategy profile.

I.1. Analysis of $G^{aa}(s_{1d}^{aa}, s_{2d}^{aa})$

In this case each firm announces a high wage after it learns that there is exactly one H-productivity applicant in that firm, otherwise announces a low wage. The probability of hiring an H-productivity applicant is: $Q_1^{aa}(s_{1d}^{aa}, s_{2d}^{aa} | t_5^1, t_k^2) = \begin{cases} 1 & (k=1,2,3,4,6,7,8) \\ \frac{1}{2} & (k=5) \end{cases}$. If firm F_2 has at least two H-productivity applicant then that firm will announce ω_l . In this case F_1 will surely hire the H-productivity applicant because even if both the firms make the offer to the same applicant F_1 will hire him because it has announced a high wage. The same holds true if F_2 has only L-productivity applicants (t_8^2). If F_2 is of type t_6^2 or t_7^2 , it means that different applicants are of H-productivity in the two firms, hence F_1 will surely hire its only H-productivity applicant. If both firms have exactly one and the same H-productivity applicant, then both firms will make the offer to him and he will accept each offer with probability $\frac{1}{2}$. Therefore, the probability that F_1 has exactly one H-productivity applicant and that firm hires him is $Q_1^{aa}(s_{1d}^{aa}, s_{2d}^{aa} | T_{1h}^1) = (\frac{7}{8} + \frac{1}{2} \times \frac{1}{8}) = \frac{15}{16}$. The probability of not hiring is $\frac{1}{16}$.

So the overall probability that a firm F_i ($i=1,2$) hires an H-productivity applicant is:

$Q_i^{aa}(s_{1d}^{aa}, s_{2d}^{aa}) = \frac{4}{8} + \frac{45}{128} = \frac{109}{128}$. The overall probability of hiring an L-productivity applicant is:

$R_i^{aa}(s_{1d}^{aa}, s_{2d}^{aa}) = \frac{16}{128}$. In this subgame the probability of a firm's position remaining vacant is $\frac{3}{128}$. Therefore, the ex ante expected profit of each firm F_i ($i=1,2$) is:

$$\Pi_i^{aa}(s_{1d}^{aa}, s_{2d}^{aa}) = \frac{109}{128} H + \frac{16}{128} L - \frac{45}{128} \omega_h - \frac{80}{128} \omega_l = \frac{109}{128} \pi^h + \frac{16}{128} \pi^l + \frac{64}{128} \omega \quad [1.1]$$

I.2. Analysis of $G^{aa}(s_{1c}^{aa}, s_{2c}^{aa})$

In this subgame each firm announces ω_1 irrespective of its type. Assuming F_1 is of type t_5^1 we get the following conditional probabilities.

(i) $Q_1^{aa}(s_{1c}^{aa}, s_{2c}^{aa} | t_5^1, t_k^2) = \frac{5}{6}$ and the corresponding $R_1^{aa} = \frac{1}{6}$ ($k=1, 8$). If F_2 has only H or L-productivity applicants then that firm can make the offer to applicant a_1 with probability $\frac{1}{3}$ in which case he will accept each firm's offer with probability $\frac{1}{2}$. If F_2 makes an offer to one of the remaining two applicants F_1 hires a_1 for sure.

(ii) $Q_1^{aa}(s_{1c}^{aa}, s_{2c}^{aa} | t_5^1, t_k^2) = \frac{3}{4}$ and the corresponding $R_1^{aa} = \frac{1}{4}$ ($k=2, 3$). The reasoning is similar as (i).

(iii) $Q_1^{aa}(s_{1c}^{aa}, s_{2c}^{aa} | t_5^1, t_k^2) = 1$ ($k=4, 7, 6$). In this case applicant a_1 who is of H-productivity in firm F_1 is of L-productivity in firm F_2 hence, firm F_1 will hire him for sure.

(iv) $Q_1^{aa}(s_{1c}^{aa}, s_{2c}^{aa} | t_5^1, t_5^2) = \frac{1}{2}$ and the corresponding $R_1^{aa} = \frac{1}{2}$. If firm F_2 is also of type t_5^2 then both firms will make the offer to a_1 who will accept each firm's offer with probability $\frac{1}{2}$.

So the conditional probability that a firm having exactly one H-productivity applicant hires him is: $Q_i^{aa}(s_{1c}^{aa}, s_{2c}^{aa} | T_{1h}^i) = \frac{1}{8} \times (\frac{10}{6} + \frac{6}{4} + 3 + \frac{1}{2}) = \frac{20}{24}$. The conditional probability that a firm having one H-productivity applicant hires an L-productivity applicant is $= \frac{4}{24}$. So the overall probability of hiring an H-productivity applicant is $Q_i^{aa}(s_{1c}^{aa}, s_{2c}^{aa}) = \frac{104}{128}$. The probability of hiring an L-productivity applicant and realizing a profit of is $R_i^{aa}(s_{1c}^{aa}, s_{2c}^{aa}) = \frac{24}{128}$. In this subgame none of the firm's position remains vacant. Therefore, the ex ante expected profit of each firm F_i ($i= 1, 2$) is:

$$\Pi_i^{aa}(s_{1c}^{aa}, s_{2c}^{aa}) = \frac{104}{128} H + \frac{24}{128} L - \omega_l = \frac{104}{128} \pi^h + \frac{24}{128} \pi^l + \frac{104}{128} \omega \quad [1.2]$$

I.3. Analysis of $G^{aa}(s_{1d}^{aa}, s_{2c}^{aa})$

In this case firm F_1 announces ω_h if it learns that there is exactly one H-productivity applicant, otherwise announces ω_l . Firm F_2 announces ω_l irrespective of its type. In this subgame F_1 will surely hire an H-productivity applicant even if both firms make the job offer to the same applicant. So the overall probability that F_1 hires an H-productivity applicant is $Q_1^{aa}(s_{1d}^{aa}, s_{2c}^{aa}) = \frac{112}{128}$ and the probability of hiring an L-productivity applicant is $R_1^{aa}(s_{1d}^{aa}, s_{2c}^{aa}) = \frac{16}{128}$.

Let us now determine the probability that F_2 hires an H-productivity applicant assuming it is of type t_5^2 .

(i) $Q_2^{aa}(s_{1d}^{aa}, s_{2c}^{aa} | t_5^2, t_j^1)$ and the corresponding R_2^{aa} for $j=1,2,3,4,6,7,8$, are the same as in the subgame $G^{aa}(s_{1c}^{aa}, s_{2c}^{aa})$.

(ii) $Q_2^{aa}(s_{1d}^{aa}, s_{2c}^{aa} | t_5^2, t_5^1) = 0$ because both firms have a_1 as the only H-productivity applicant and so both firms will make an offer to him in which case F_1 will hire him for sure because that firm has announced ω_h and F_2 will hire an L-productivity applicant.

So the conditional probability that F_2 has exactly one H-productivity applicant and it hires him is $Q_2^{aa}(s_{1d}^{aa}, s_{2c}^{aa} | T_{1h}^2) = \frac{1}{8} \times (\frac{10}{6} + \frac{6}{4} + 3) = \frac{37}{48}$. The conditional probability that F_2 hires an L-productivity applicant is $R_2^{aa}(s_{1d}^{aa}, s_{2c}^{aa} | T_{1h}^2) = \frac{11}{48}$. So the overall probability that F_2 hires an H-productivity applicant is $Q_2^{aa}(s_{1d}^{aa}, s_{2c}^{aa}) = \frac{101}{128}$. The overall probability that a firm hires an L-productivity applicant is $R_2^{aa}(s_{1d}^{aa}, s_{2c}^{aa}) = \frac{27}{128}$. So the ex ante expected profits of the two firms are:

$$\Pi_1^{aa}(s_{1d}^{aa}, s_{2c}^{aa}) = \frac{112}{128} \pi^h + \frac{16}{128} \pi^l + \frac{64}{128} \omega$$

[1.3]

$$\Pi_2^{aa}(s_{1d}^{aa}, s_{2c}^{aa}) = \frac{101}{128} \pi^h + \frac{27}{128} \pi^l + \frac{101}{128} \omega$$

The analysis of the subgame $G^{aa}(s_{1c}^{aa}, s_{2d}^{aa})$ is symmetric.

II. ANALYSIS OF THE SUBGAME G^{ba}

In this subgame F_1 announces a wage before receiving specific information about applicants while F_2 announces a wage after receiving specific information about applicants' productivity. The strategy spaces of F_1 and F_2 are, $W_1^{ba} = \{\omega_h, \omega_l\}$ and $S_2^{ba} \equiv S_2^{aa} = \{s_{2d}^{ba}, s_{2c}^{ba}\}$, respectively. I will determine the probability of hiring an H-productivity applicant if there is exactly one H-productivity applicant in each firm, for each pair of strategies (w_1^{ba}, s_2^{ba}) . I will assume that F_1 and F_2 are of types t_5^1 and t_5^2 , respectively.

II.1. Analysis of $G^{ba}(\omega_h, s_{2d}^{ba})$

In this subgame F_1 announces ω_h while F_2 announces ω_l if there are at least two H-productivity applicants or only L-productivity applicants and announces ω_h if there is exactly one H-productivity applicant. I will first determine the probability that F_1 hires an H-productivity applicant and then determine the probability that F_2 hires an H-productivity applicant.

$$Q_1^{ba}(\omega_h, s_{2d}^{ba} \mid t_5^1, t_k^2) = \begin{cases} 1 & \text{for } k=1,2,3,4,6,7,8 \\ \frac{1}{2} & \text{for } k=5 \end{cases}. \text{ The reason is the same as in I.1.}$$

So the conditional probability probability that F_1 has exactly one H-productivity applicant and hires him is $Q_1^{ba}(\omega_h, s_{2d}^{ba} \mid T_{1h}^1) = \frac{15}{16}$. So the overall probability that F_1 hires an H-productivity applicant is $Q_1^{ba}(\omega_h, s_{2d}^{ba}) = \frac{109}{128}$.

The conditional probability that F_2 is of type t_5^2 and hires the H-productivity applicant for each type of F_1 are as follows.

$$Q_2^{ba}(\omega_h, s_{2d}^{ba} \mid t_1^1, t_5^2) = \frac{5}{6}, \quad Q_2^{ba}(\omega_h, s_{2d}^{ba} \mid t_j^1, t_5^2) = \frac{3}{4} \quad (j=2,3), \quad Q_2^{ba}(\omega_h, s_{2d}^{ba} \mid t_j^1, t_5^2) = 1 \quad (j=4,6,7,8), \quad Q_2^{ba}(\omega_h, s_{2d}^{ba} \mid t_5^1, t_5^2) = \frac{1}{2}, \text{ for similar reasons as discussed in I. So the conditional}$$

probability that F_2 has exactly one H-productivity applicant and hires him is $Q_2^{\text{ba}}(\omega_h, s_{2d}^{\text{ba}} | T_{1h}^2) = \frac{41}{48}$ and the probability of not hiring is $\frac{7}{48}$.

So the overall probability that F_2 hires an H-productivity or an L-productivity applicant are

$Q_2^{\text{ba}}(\omega_h, s_{2d}^{\text{ba}}) = \frac{105}{128}$ and $R_2^{\text{ba}}(\omega_h, s_{2d}^{\text{ba}}) = \frac{16}{128}$. So the ex ante expected profits are:

$$\Pi_1^{\text{ba}}(\omega_h, s_{2d}^{\text{ba}}) = \frac{109}{128} \pi^h \tag{2.1}$$

$$\Pi_2^{\text{ba}}(\omega_h, s_{2d}^{\text{ba}}) = \frac{105}{128} \pi^h + \frac{16}{128} \pi^l + \frac{64}{128} \omega$$

II.2. Analysis of $G^{\text{ba}}(\omega_l, s_{2c}^{\text{ba}})$

In this subgame F_1 announces ω_l before receiving specific information and F_2 announces ω_l after receiving specific information, irrespective of its type. The ex ante expected profits of the firms are the same as in I.2.

$$\Pi_1^{\text{ba}}(\omega_l, s_{2c}^{\text{ba}}) = \frac{104}{128} \pi^h + \frac{24}{128} \pi^l + \frac{104}{128} \omega \tag{2.2}$$

$$\Pi_2^{\text{ba}}(\omega_l, s_{2c}^{\text{ba}}) = \frac{104}{128} \pi^h + \frac{24}{128} \pi^l + \frac{104}{128} \omega$$

II.3. Analysis of $G^{\text{ba}}(\omega_h, s_{2c}^{\text{ba}})$

In this subgame F_1 announces ω_h before receiving specific information while F_2 announces ω_l irrespective of its type, after receiving specific information. In this case F_1 will surely hire an H-productivity applicant if there is at least one of them. Even if both firms make the offer to the same applicant F_1 will hire him because it has announced a higher wage. So

$$Q_1^{\text{ba}}(\omega_h, s_{2c}^{\text{ba}}) = \frac{112}{128}.$$

Assuming F_2 is of type t_5^2 , the conditional probabilities that F_2 hires the H-productivity applicant or an L-productivity applicant for each type of F_1 are: $Q_2^{ba}(\omega_h, s_{2c}^{ba} | t_1^1, t_5^2) = \frac{2}{3}$, $Q_2^{ba}(\omega_h, s_{2c}^{ba} | t_j^1, t_5^2) = \frac{1}{2}$ ($j=2,3$), $Q_2^{ba}(\omega_h, s_{2c}^{ba} | t_j^1, t_5^2) = 1$ ($j=4,6,7,8$), $R_2^{ba}(\omega_h, s_{2c}^{ba} | t_1^1, t_5^2) = \frac{1}{3}$, $R_2^{ba}(\omega_h, s_{2c}^{ba} | t_j^1, t_5^2) = \frac{1}{2}$ ($j=2,3$), $R_2^{ba}(\omega_h, s_{2c}^{ba} | t_5^1, t_5^2) = 1$. The reasoning is the same as before. So the conditional probabilities that F_2 has exactly one H-productivity applicant and hires him or an L-productivity applicant are $Q_2^{ba}(\omega_h, s_{2c}^{ba} | T_{1h}^2) = \frac{34}{48}$ and $R_2^{ba}(\omega_h, s_{2c}^{ba} | T_{1h}^2) = \frac{14}{128}$.

So the overall probability that F_2 hires an H-productivity or an L-productivity applicant are:

$Q_2^{ba}(\omega_h, s_{2c}^{ba}) = \frac{98}{128}$ and $R_2^{ba}(\omega_h, s_{2c}^{ba}) = \frac{30}{128}$. So the ex ante expected profits of the firms are:

$$\Pi_1^{ba}(\omega_h, s_{2c}^{ba}) = \frac{112}{128} \pi^h$$

[2.3]

$$\Pi_2^{ba}(\omega_h, s_{2c}^{ba}) = \frac{98}{128} \pi^h + \frac{30}{128} \pi^l + \frac{98}{128} \omega$$

II.4. Analysis of $G^{ba}(\omega_l, s_{2d}^{ba})$

In this subgame F_1 announces ω_l before receiving specific information about applicants' productivity. F_2 announces ω_h if there is exactly one H-productivity applicant and announces ω_l if there are at least two H-productivity or only L-productivity applicants, after receiving specific information about applicants' productivity.

Assuming that F_1 is of type t_5^1 , the probabilities that F_1 hires the H-productivity applicant or an L-productivity applicant are

$$Q_1^{ba}(\omega_l, s_{2d}^{ba} | t_5^1, t_k^2) = \begin{cases} \frac{2}{3} & (k=1) \\ \frac{1}{2} & (k=2,3) \\ 1 & (k=4,6,7) \end{cases} \text{ and } R_1^{ba}(\omega_l, s_{2d}^{ba} | t_5^1, t_k^2) = \begin{cases} \frac{1}{3} & (k=1) \\ \frac{1}{2} & (k=2,3) \\ 1 & (k=5,8) \end{cases}$$

So $Q_1^{ba}(\omega_l, s_{2d}^{ba} | T_{1h}^1) = \frac{14}{24}$ and $R_1^{ba}(\omega_l, s_{2d}^{ba} | T_{1h}^1) = \frac{10}{24}$. So the overall probabilities of F_1 hiring an H or an L-productivity applicant are $Q_1^{ba}(\omega_l, s_{2d}^{ba}) = \frac{92}{128}$ and $R_1^{ba}(\omega_l, s_{2d}^{ba}) = \frac{36}{128}$.

For F_2 the probability of hiring an H or an L-productivity applicant are the same as discussed in I.3. So the ex ante expected profits of the firms are:

$$\Pi_1^{ba}(\omega_l, s_{2d}^{ba}) = \frac{92}{128} \pi^h + \frac{36}{128} \pi^l + \frac{92}{128} \omega \quad [2.4]$$

$$\Pi_2^{ba}(\omega_l, s_{2d}^{ba}) = \frac{112}{128} \pi^h + \frac{16}{128} \pi^l + \frac{64}{128} \omega$$

The analysis of the subgame G^{ab} is symmetric.

APPENDIX 2

In this section I will derive the equilibrium timing of wage tender. I define the following variables:

$$C := \Pi_1^{bb}(\omega_h, \omega_h) - \Pi_1^{bb}(\omega_l, \omega_h) = \Pi_2^{bb}(\omega_h, \omega_h) - \Pi_2^{bb}(\omega_h, \omega_l) = \frac{7}{128}(\pi^h - 4.29\pi^l - 14\omega) = \frac{7}{128}c.$$

$$D := \Pi_1^{bb}(\omega_h, \omega_l) - \Pi_1^{bb}(\omega_l, \omega_l) = \Pi_2^{bb}(\omega_l, \omega_h) - \Pi_2^{bb}(\omega_l, \omega_l) = \frac{8}{128}(\pi^h - 3\pi^l - 13\omega) = \frac{8}{128}d.$$

Now $d \geq c$. So we get the following pairs of relationships between C and D: **(C ≥ 0, D ≥ 0),**

(C ≤ 0, D ≥ 0, C ≤ 0, D ≤ 0).

$$M := \Pi_1^{aa}(s_{1d}^{aa}, s_{2d}^{aa}) - \Pi_1^{aa}(s_{1c}^{aa}, s_{2d}^{aa}) = \Pi_2^{aa}(s_{1d}^{aa}, s_{2d}^{aa}) - \Pi_2^{aa}(s_{1d}^{aa}, s_{2c}^{aa}) = \frac{8}{128}(\pi^h - 1.375\pi^l - 4.625\omega) = \frac{8}{128}m.$$

$$N := \Pi_1^{aa}(s_{1d}^{aa}, s_{2c}^{aa}) - \Pi_1^{aa}(s_{1c}^{aa}, s_{2c}^{aa}) = \Pi_2^{aa}(s_{1c}^{aa}, s_{2d}^{aa}) - \Pi_2^{aa}(s_{1c}^{aa}, s_{2c}^{aa}) = \frac{8}{128}(\pi^h - \pi^l - 5\omega) = \frac{8}{128}n.$$

We get the following relationships: $M \geq N \geq C, M \geq N \geq D$

$$U_1 := \Pi_1^{ba}(\omega_h, s_{2d}^{ba}) - \Pi_1^{ba}(\omega_l, s_{2d}^{ba}) = \frac{17}{128}(\pi^h - 2.12\pi^l - 5.41\omega) = \frac{17}{128}u_1.$$

$$V_1 := \Pi_1^{ba}(\omega_h, s_{2c}^{ba}) - \Pi_1^{ba}(\omega_l, s_{2c}^{ba}) = \frac{8}{128}(\pi^h - 3\pi^l - 13\omega) = \frac{8}{128}v_1 \equiv D$$

We get the following relationships: $U_1 \geq V_1 \equiv D, U_1 \geq C.$

$$U_2 := \Pi_2^{ba}(\omega_h, s_{2d}^{ba}) - \Pi_2^{ba}(\omega_h, s_{2c}^{ba}) = \frac{7}{128}(\pi^h - 2\pi^l - 4.86\omega) = \frac{7}{128}u_2$$

$$V_2 := \Pi_2^{ba}(\omega_l, s_{1d}^{ba}) - \Pi_2^{ba}(\omega_l, s_{2c}^{ba}) = \frac{8}{128}(\pi^h - \pi^l - 5\omega) = \frac{8}{128}v_2 \equiv N$$

We get the following relationships: $U_2 \geq C, (U_2 \geq 0, M \geq 0), (U_2 \leq 0, M \geq 0), (U_2 \leq 0, M \leq 0).$

I will use these relationships between the different variables mentioned above to prove theorems 1 - 6.

Proof of Theorem 1

If $C \geq 0$, it implies that all other variables defined above are nonnegative. Then the set of equilibria is $\{(\omega_h, \omega_h), (s_{1d}^{aa}, s_{2d}^{aa}), (\omega_h, s_{2d}^{ba}), (s_{1d}^{ab}, \omega_h)\}$. We get, $\Pi_i^{bb}(\omega_h, \omega_h) - \Pi_i^{ab}(s_{1d}^{ab}, \omega_h) = \Pi_i^{ba}(\omega_h, s_{2d}^{ba}) - \Pi_i^{aa}(s_{1d}^{aa}, s_{2d}^{aa}) = -\frac{16}{128}\pi^l - \frac{64}{128}\omega \leq 0; (i=1,2)$. Therefore, announcing a wage after receiving specific information about applicants' productivity is the weakly dominant strategy.

Proof of Theorem 2

If $C \leq 0, U_2 \geq 0$, and $D \geq 0$, then $U_1 \geq 0$ and $M \geq N \geq 0$. Therefore, the set of equilibria is

$\{(\omega_h, \omega_l), (\omega_l, \omega_h)\}, (s_{1d}^{aa}, s_{2d}^{aa}), (\omega_h, s_{2d}^{ba}), (s_{1d}^{ab}, \omega_h)\}$. There are two asymmetric equilibria in

the subgame G^{bb} . Assuming (ω_h, ω_l) is the equilibrium in the subgame G^{bb} we get the following inequalities.

$$\Pi_1^{bb}(\omega_h, \omega_l) - \Pi_1^{ab}(s_{1d}^{ab}, \omega_h) = \frac{1}{128}(7\pi^h - 16\pi^l - 64\omega); \quad \Pi_2^{bb}(\omega_h, \omega_l) - \Pi_2^{ba}(\omega_h, s_2^{ba}) = \frac{-1}{128}(7\pi^h - 14\pi^l - 34\omega) \leq 0$$

$$\Pi_1^{ba}(\omega_h, s_2^{ba}) - \Pi_1^{aa}(s_{1d}^{aa}, s_{2d}^{aa}) = -\frac{1}{128}(16\pi^l + 64\omega) = \Pi_2^{ab}(s_{1d}^{ab}, \omega_h) - \Pi_2^{aa}(s_{1d}^{aa}, s_{2d}^{aa}) \leq 0$$

So F_2 announces a wage after receiving specific information because it is the weakly dominant strategy. So the optimal strategy for F_1 is to announce a wage after receiving specific information about applicants' productivity because the second inequality (above) is nonpositive. The result remains the same if (ω_l, ω_h) is the equilibrium in the subgame G^{bb} . \square

Proof of Theorem 3

If $C \leq 0$, $U_2 \leq 0$, and $D \equiv V_1 \geq 0$, then $U_1 \geq 0$ and $M \geq N \equiv V_2 \geq 0$. In this case the set of equilibria is $\{(\omega_h, \omega_l), (\omega_l, \omega_h)\}, (s_{1d}^{aa}, s_{2d}^{aa}), (\omega_h, s_{2c}^{ba}), (s_{1c}^{ab}, \omega_h)\}$. Assume (ω_h, ω_l) is the equilibrium in the subgame G^{bb} .

$$\Pi_2^{ba}(\omega_h, s_{2c}^{ba}) - \Pi_2^{bb}(\omega_h, \omega_l) = 0.$$

$$\Pi_2^{ab}(s_{1c}^{ab}, \omega_h) - \Pi_2^{aa}(s_{1d}^{aa}, s_{2d}^{aa}) = \Pi_1^{ba}(\omega_h, s_{2c}^{ba}) - \Pi_1^{aa}(s_{1d}^{aa}, s_{2d}^{aa}) = \frac{3}{128}(\pi^h - 5.13\pi^l - 21.33\omega) \leq 0.$$

This is because, the term in the parenthesis is weakly less than c . By assumption $C \leq 0$ which implies that $c \leq 0$. So F_2 announces a wage after receiving specific information about applicants' productivity because it is the weakly dominant strategy. This means that F_1 will also announce a wage after receiving specific information about applicants' productivity. \square

The proof is the same if we assume that (ω_l, ω_h) is the equilibrium in the subgame G^{bb} .

Proof of Theorem 4

If $C \leq 0$, $D = V_1 \leq 0$, $M \geq N = V_2 \geq 0$, $U_1 \geq 0 \geq V_1 = D$, $U_2 \geq 0$. then the set of equilibria is $\{(\omega_l, \omega_l), (s_{1d}^{aa}, s_{2d}^{aa}), (\omega_h, s_{2d}^{ba}), (s_{1d}^{ab}, \omega_h)\}$. So we get

$$\Pi_1^{bb}(\omega_l, \omega_l) - \Pi_1^{ab}(s_{1d}^{ab}, \omega_h) = \Pi_2^{bb}(\omega_h, \omega_h) - \Pi_2^{ba}(\omega_h, s_{2d}^{ba}) = -(\pi^h - 0.0625\omega_l - 0.325\omega) \leq 0$$

because the term in the parenthesis is weakly greater than m and $M \geq 0$ implies that $m \geq 0$.

$$\Pi_1^{ba}(\omega_h, s_{2d}^{ba}) - \Pi_1^{aa}(s_{1d}^{aa}, s_{2d}^{aa}) = \Pi_2^{ab}(s_{1d}^{ab}, \omega_h) - \Pi_2^{aa}(s_{1d}^{aa}, s_{2d}^{aa}) = -\frac{1}{128}(16\pi^l + 64\omega) \leq 0.$$

Hence, announcing a wage after receiving specific information about applicants' productivity is the weakly dominant strategy for both the firms. \square

Proof of Theorem 5

(1) $\{(\omega_l, \omega_l), \{(s_{1d}^{aa}, s_{2d}^{aa}), (s_{1c}^{aa}, s_{2c}^{aa})\}, (\omega_l, s_{2c}^{ba}), (s_{1c}^{ab}, \omega_l)\}$ is the set of equilibria if the following conditions hold.

- (i) $C \leq 0, D = V_1 \leq 0, M \geq 0 \geq N = V_2, U_1 \geq 0, U_2 \geq 0,$
- (ii) $C \leq 0, D = V_1 \leq 0, M \geq 0 \geq N = V_2, U_1 \geq 0, U_2 \leq 0,$
- (iii) $C \leq 0, D = V_1 \leq 0, M \geq 0 \geq N = V_2, U_1 \leq 0, U_2 \geq 0,$
- (iv) $C \leq 0, D = V_1 \leq 0, M \geq 0 \geq N = V_2, U_1 \leq 0, U_2 \leq 0.$

In this case we see that there are two symmetric equilibria in the subgame G^{aa} . Now,

$$\Pi_i^{aa}(s_{1d}^{aa}, s_{2d}^{aa}) - \Pi_i^{aa}(s_{1c}^{aa}, s_{2c}^{aa}) = \frac{1}{128}(5\pi^h - 8\pi^l - 40\omega) \leq V_2 = N \leq 0. \text{ So in this subgame the two firms will choose the payoff dominant strategy } (s_{1d}^{aa}, s_{2d}^{aa}). \text{ So announcing a low wage is the equilibrium strategy in all the subgames. Since the ex ante expected profit of the firms is the same when announcing a low wage is the equilibrium strategy in all the subgames, announcing a low wage before and after receiving specific information about applicants' productivity are both equilibrium policies.}$$

(2) $\{(\omega_l, \omega_l), (s_{1c}^{aa}, s_{2c}^{aa}), (\omega_l, s_{2c}^{ba}), (s_{1c}^{ab}, \omega_l)\}$ is the set of equilibria if the following conditions hold.

- (i) $C \leq 0, D = V_1 \leq 0, V_2 = N \leq M \leq 0, U_1 \geq 0, U_2 \leq 0,$
- (ii) $C \leq 0, D = V_1 \leq 0, V_2 = N \leq M \leq 0, U_1 \leq 0, U_2 \leq 0.$

The analysis is the same as in (1). \square

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Vita

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