ESSAYS ON CREDIT RATIONING AND BORROWING CONSTRAINTS

by

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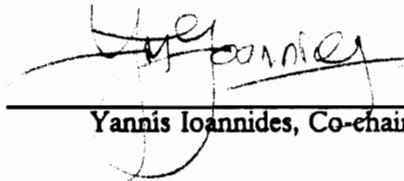
DOCTOR OF PHILOSOPHY

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(ABSTRACT)

The problem of credit rationing/borrowing constraint has recently received considerable attention. Individuals who are denied any credit by a financial institution, or who find it difficult to borrow against future incomes, are said to be credit rationed or borrowing constrained in the credit markets. This dissertation tries to identify the circumstances under which individuals may be rationed (or not), and analyses the actions undertaken to overcome future constraints.

Chapter 2 analyses the problem of credit rationing as it arises in equilibrium, when borrowers differ with respect to their demands for loans. It is shown that if the principal can costlessly observe the agent's type, then (i) the agents who meet the collateral requirements are not rationed in the sense of Stiglitz-Weiss (1981), (ii) the agents who do not meet the collateral requirements are rationed in the sense of Jaffee-Russell (1976). We further show that if the principal cannot distinguish between different agents, then the previous rationing results still hold in the second best contract which is pooling : agents of differnt types pick the same contract.

Chapter 3 analyses the problem of credit rationing as it emerges in a dynamic setting, when a re-negotiation of the original contract may be undertaken. It is conjectured that (i) the principal uses the information revealed about an agent's type at the time of first repayment, to design future contracts, (ii) the agents who show consistently honest behavior are never rationed, (iii) the agents who showed dishonest behavior impose a negative externality on the agents who were honest; they are rationed in later periods.

Finally, in chapter 3, we analyse the role of an exogenously imposed borrowing constraint prompting the individuals to change their life-cycle decisions. This chapter provides an explicit link

between human and non-human wealth by making income endogenous through investment in human capital. The chapter also discusses the econometric aspects of the problem: the possible empirical work that can be undertaken in the future using a micro data set.

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Chapter I

Introduction

Credit markets, or more generally, the financial markets have recently attracted considerable attention for several reasons. They perform crucial functions in any advanced, monetary economy, like the sharing of risk between different economic agents, the allocation of loanable funds or the transmission of monetary impulses, to name just a few. They also have led many observers to conclude that they operate quite differently from most other markets - with non-market clearing, or apparently inefficient equilibria being the rule rather than exception. Despite the complexity and sophistication of the financial markets, they are typically represented by two variables - the money stock and the interest rate. However, it is a recurrent theme in the existing literature, that neither the money stock nor the interest rate adequately reflect the links between financial markets and the rest of the economy.

Recently, there has been much emphasis on the role of credit in the business cycle, and particularly in the transmission of monetary policy to the economy. [1] If credit is rationed, then it is possible that the interest rate is not a reliable indicator of the impact of financial variables on the aggregate demand. It is quite likely that in that case quantity variables such as the amount of credit, have to be looked at in appraising monetary and financial policy. The phenomenon of credit rationing is therefore of continued interest for the following reasons:

(a) Whether rationing of credit implies inefficiency and if so what remedies could be applied.

(b) how exactly the rationing of credit affects the working of monetray policy.

(c) Whether there exist plausible conditions which make rationing compatible with individual rationality without resorting to exogenous factors like interest.rate regulations.

The recent literature on credit rationing builds on the theory of imperfect information. Particularly fruitful proved models of asymmetric quality information as initiated in the seminal paper by Akerlof (1970). The basic idea is that one party has to pay or to commit himself before knowing with certainty what it will get in exchange. This problem is particularly important in credit markets because of the time lag involved between delivery and repayment.

The credit markets are plagued by adverse selection and moral hazard effects - the bank faces the risk of default of the borrowers, and this risk may be increased as the interest rate of loan and/or the volume of credit are increased, implying that the gross return of a bank need not be a monotonically increasing function of the loan rate and the loan size. An increase in the rate of interest would cause more risk averse individuals to drop out of the borrowing pool - this adverse selection effect may more than offset the increased revenue due to the higher premium. Similarly,

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[1] See for example Bernanke and Gertler (1987), Blinder and Stiglitz (1983), Greenwald and Stiglitz (1988)

the borrowers may become careless as the premium is increased, and this moral hazard effect would again reduce the expected return of the lender.

The explanations of credit rationing based on adverse selection and moral hazard are quite appealing because they appear fairly robust to assumptions concerning the behavior of the economics agents, and also to institutional details of the credit markets. Also, they do not require any ad hoc assumptions about price or interest rate rigidities.

There are several definitions of credit rationing, all arising from the view and experience of capital market participants, that borrowers cannot borrow as much as they would like to even when the markets appear to be operating well. Keeton (1979) proposed a sharp distinction between different types of rationing :

Type I rationing : some or all loan applicants get a smaller loan than they desire at the quoted interest rate.

Type II rationing : Some loan applicants are denied a loan even though for the bank they are indistinguishable from the accepted pool of applicants.

Among all types of models considered in the rationing literature, the Jaffee-Russell (1976) model and the Stiglitz-Weiss (1981) model are certainly the path-breaking ones. The Jaffee-Russell (1976) paper produces type I rationing when borrowers differ with respect to their costs of default. Type I rationing arises in this model due to adverse selection effects : lenders are unable to distinguish between borrowers on an ex-ante basis. In contrast, the Stiglitz-Weiss (1981) paper produces type II rationing when the projects differ with respect to their riskiness, and the riskier projects are more profitable for the borroowers. Type II rationing is the outcome in this model due to the negative adverse selection effects - increase in the interest rate causes only the borrowers with riskier investments to apply for loans.

In this dissertation, we analyze how different types of credit rationing may result from the asymmetry of information between borrowers and lenders in a static as well as in a dynamic framework, and how a credit constrained individual behaves under such circumstances.

There are several caveats against the existing literature on credit rationing. Firstly, the literature consists mostly of models where the borrower is another firm, thereby largely ignores how a consumer may also be rationed in a credit market. Secondly, the literature emphasizes much the role of a collateral but never analyzes how the collateral itself may be acquired. Thirdly, most of the models in this area are static in the sense that they do not consider how the rationing results may change over time, when the lender learns more about the borrower's type through "learning by experience".

There is a close link between the literature on credit rationing and the literature on borrowing constraints. The literature on borrowing constraints provides an explanation about the apparent excess sensitivity of consumption to current income. A prominet explanation of this finding is that imperfections in capital markets impose constraints on borrowing that prohibit households from smoothing fluctuations in their incomes. While the literature emphasizes much the existence of binding constraints and the severity with which the constraint binds, it does not thoroughly analyse the actions undertaken by such an individual to loosen the constraints over time.

The purpose of this dissertation is to investigate the circumstances under which credit rationing may arise when *consumers* are the borrowers, and to analyze the behavior of a constrained individual who tries to overcome the constraints by undertaking human capital investment.

In chapter 2, we consider the problem faced by a monopolist lender who faces borrowers of different types, in a static framework. In that chapter we also try to give an explanation of the way the collateral itself may be acquired though loan financing. The agents are differentiated on the basis of the diversity of preferences. We assume that there are some agents who do not acquire the collateral because they receive no utility out of its consumption. On the other hand, there are some

agents who want to acquire the collateral because consumption of it increases their utilities substantially.

We show that when the principal can identify the agent's type perfectly, different contracts are offered to suit the agents' types, and both types are rationed in Jaffee-Russell (1976) manner if they are rationed at all. We further show that when the principal cannot identify the agent's type costlessly, only one contract may be offered depending on the parameters of the principal's payoff function. We find that if the agents are rationed, they are rationed again in the Jaffee-Russell manner.

In the third chapter of this dissertation, we consider the problem of a monopolist lender who tries to infer about the agent's type based on the agent's behavior in previous periods. We model the behavior of a credit market, where the original loan contracts may sometimes be renegotiated following a default, and where the lender uses that information to decide whether contracts will be offered in the future. In this dynamic setting we investigate whether we will have rationing equilibrium (or not) in each period. This paper is still in its nascent stage, but it is anticipated that along with the 'dishonest' agents, the 'honest' agents are likely to be rationed too, due to the negative externality effects exercised by the dishonest ones on the honest ones.

In chapter 4, we formulate a two-period dynamic programming problem to analyse the effects of human capital investment on the individual's holding of non-human wealth in the later period of his life. The model assumes that the agent is uncertain about the future realization of the exogenous variables and the second-period income is determined endogenously by the extent of human capital investment. We propose a switching-regression version of the model to incorporate the changes of regimes. The chapter also discusses the possible empirical work that can be undertaken using the 1983-1986 Survey of Consumer Finances data set. We anticipate that the empirical work would generally support the hypothesis that investment in human capital will eventually help the individual overcome the borrowing constraints in the future periods of his life. However further extensive econometric research is necessary in order to come to any definite conclusions. 1 spbf 0

Chapter II

Consumer Credit Rationing and Screening with

Endogenous Collateral

1. Introduction

The mechanism and rationale of credit rationing is now the focus of an extensive literature. There exist two concepts of credit rationing in the literature. The first one developed by Jaffee and Russell (1976) argues that credit rationing occurs when lenders quote a certain interest rate and then proceed to supply a smaller loan size than that demanded by the borrowers. This type of rationing occurs because the lenders have imperfect information about the type of the borrower (whether honest or dishonest). Therefore they are unable to distinguish between both types of borrowers on an ex ante basis and hence credit rationing arises due to market response to an adverse selection

problem. The second type of credit rationing, addressed by Stiglitz and Weiss (1981), occurs when some loan applicants receive a loan while others do not. This type of credit rationing occurs due to negative adverse selection and incentive effects : for a given collateral, an increase in the rate of interest causes adverse selection, since only borrowers with riskier investments will apply for a loan at a higher interest rate. On the other hand, for a fixed rate of interest, an increase in the collateral requirement may result in a decline in a bank's profits as well - this happens particularly if more risk averse borrowers, who choose relatively safe investment projects, drop out of the market. Wette (1983) has shown that increasing the collateral requirement may result in adverse selection even with risk-neutral investors. Bester (1985) shows that no credit rationing occurs in equilibrium, if banks compete simultaneously with respect to collateral requirements and the rate of interest, to screen investors' riskiness. This argument relies on the fact that when banks decide upon the rate of interest and the collateral simultaneously rather than separately, it becomes possible to use multiple contracts as a self-selection mechanism.

The main purpose of this paper is to show that if the bank can identify the agent's type perfectly well, based on the agent's stated loan-demand functions, then it will use that information as a self-selection device and offer different agents different contracts. Our model relies on the modification of the assumption made about the collateral - that borrowers can acquire collateral through loan-financing, in contrast to the prevailing approach where the collateral is assumed to be already in the possession of the borrower, to produce this result.

We assume some diversity in agents' preferences. For simplicity, only two types of consumption goods are considered in this paper : durable consumption goods and non-durable consumption goods. There are some agents who do not care about the consumption of durable goods and need loans to purchase only the non-durable goods (type 1 agents). There are some other agents who consume both types of goods but consider non-durable goods as imperfect substitutes for the durable good, so that their utilities are substantially reduced if they do not consume the durable good (type 2 agents).

In the simple version of the model we assume that the principal can costlessly observe the agent's type. Hence the agent who needs the loan primarily to purchase the durable good, is required to keep the good as collateral. On the other hand, those agents who use the entire loan to purchase non-durable consumption goods are not required to put up any collaterals. We show that the agents who need to keep the durable good as a collateral are not credit-rationed in equilibrium in the sense discussed by Stiglitz and Weiss (1981). Our non-rationing equilibrium result does not depend on the assumption about bank's choosing rate of interest and collateral simultaneously, as in Bester (1985). Instead it depends on the assumption that the bank has perfect information about the borrower's type. We further show that the agents who do not have to meet any collateral requirements may sometimes be credit rationed in the sense of Jaffee-Russell (1976). The reason for this result is that the agents can misreport their future incomes out of which the payment is to be made. Although the principal can monitor the agent's second-period income at a cost, he does not have the perfect monitoring capability. Our result is similar to the result obtained by Jaffee and Russell, because, like them, we deal with problems of 'honest and dishonest' borrowers. We show that the penalty for cheating is non-zero.

In the more general version of the model we relax the assumption that the principal can costlessly observe the agent's type on an ex-ante basis. Because of the imperfect information about the agent's type, the principal tries to design incentive-compatible contracts in equilibrium such that the agent reveals his true type and picks the contract designed for him. But we show that it is profitable for the principal to offer only one type of contract in equilibrium most of the time. Thus the equilibrium in this case is characterised by a pooling one where agents of differnt types pick the same contract. We show that in equilibrium, (i) type 1 agents may or may not participate in the contract, but if they do they will be rationed in the Jaffee-Russell (1976) manner, (ii) type 2 agents always participate in the contract and hence they are also rationed in Jaffee-Russell manner if they are rationed at all. This result is due to the fact that in equilibrium type 1 agents may sometimes be indifferent between receiving and not receiving loans, whereas type 2 agents strictly prefer loans to

no loans. The results of this section is sensitive to the assumptions made about cost of acquiring capital and specification of the monitoring cost function.

Our model is similar to the costly state verification setups of Townsend (1979), Gale and Hellwig (1984) and Williamson (1986, 1987), except that in our model the principal does not always monitor the agent's second-period income. Whether monitoring will occur or not depends on the agent's type.

The remainder of the paper is organized as follows. Section 2 presents the basic setup of the model. Section 3 analyses the type 1 agent's problem. Section 4 analyses the type 2 agent's problem. Section 5 analyses the problem of the principal when the principal cannot distinguish between the two types of agents on an ex-ante basis. Section 6 presents some concluding remarks.

2. The Model

The individual agents or borrowers in our economy live for two periods and can consume two types of goods : a non-durable consumption good and a durable consumption good. Let $c_i{}^j$ be the consumption of the jth good in period i, i = 1,2, j = n,d, where n represents the nondurable good and d represents the durable good. For simplicity, we assume that the income accruing to the agents in period one is zero. Hence all agents need loans to finance their consumption expenditures. Assume that there is a risk-neutral monopolist lender or principal (financial institution) who makes loans to different agents. Let B denote the amount of loan taken in period 1. Then in period 2, the agent is supposed to repay the amount RB, where R is the gross rate of interest. We distinguish between two types of agents who differ in their preferences towards consumption of the durable good. Specifically, those agents who do not consume the durable good and whose utility depends

only on the consumption of non-durable goods, are called type 1 agents, whereas those agents whose utility depends on the consumption of both goods, are called type 2 agents. In fact we assume that the type 2 agent's preference is such that if he cannot consume the durable good, his utility is substantially lowered - no matter how much of the non-durable good is given to him, i.e. the type 2 agent regards the non-durable good as imperfect substitute for the durable good. Hence we can write the utility function of the type 2 agent as $u = u(c^n, c^d)$ for each period. On the other hand, the utility function of the type 1 agent can be written only as a function of the non-durable good, i.e. for these agents, $u = u(c^n)$ for each period. The exact forms of the utility functions will be described in the later sections.

Let λ denote the fraction of the loan used to purchase the durable good and $(1 - \lambda)$ the fraction of loan used to purchase the non-durable good, $1 \geq \lambda \geq 0$. The type 1 agent's utility is independent of the consumption of the durable good, hence he uses the entire loan to buy the non-durable good. Thus for a type 1 agent, $\lambda = 0$. In contrast, a type 2 agent uses the loan first to purchase the durable good and then the residual budget is used for buying non-durable goods. Hence for a type 2 agent, $1 \geq \lambda > 0$.

For simplicity, we assume that the agent consumes a fixed amount of the durable good, if any. We take the non-durable good as the numeraire and assume exogenously fixed relative prices. Let W denote the expenditure on the durable good. By suitably choosing the unit of measurement of the durable good, we can assume, without loss of generality, that the unit price of the durable good is one and the agent consumes W units of the durable good, if he consumes it at all. Thus a further difference between the non-durable good and the durable good lies in the fact that consumption of the non-durable good is divisible whereas the durable good is treated as indivisible. Thus the agent either consumes W units of the durable good, in which case $c^d = W$ or he consumes zero units of the good, in which case $c^d = 0$. Assuming that the durable good does not depreciate in value, W also represents the consumption of the durable good in period 2.

With the above specifications about the nature of the durable good, the expenditure on the durable good by the type 2 agent can be written as $W = \lambda B$. Note that since the expenditure on the durable good is fixed at W, as the size of the loan obtained increases, the agent needs to spend a successively smaller fraction of the loan to buy the good, i.e. from the identity $\lambda \equiv \frac{W}{B}$, it follows that $\lambda = \lambda(B)$ with $\lambda'(B) < 0$. The period-one budget constraints for these agents can now be written as follows : for type 1 agents the budget constraint is $c_1^p = B$ and for type 2 agents the budget constraint is given by $c_1^p = (1 - \lambda)B$, where $(1 - \lambda)B \equiv B - W$. From the budget constraint of the type 2 agent, we observe that c_1^p would be zero for these agents if the size of the loan obtained is just enough to cover the cost of the durable good.

Let us introduce the following notation :

y_2^t - true value of the second-period income, private information for the agents.

y_2^s - reported value of the second-period income.

Thus the agents can lie about their second period income so as to avoid the repayment. Let us now make the following assumptions -

A1. The future (true) income y_2 is distributed i.i.d with density function f for $y_2 \in [0, \bar{y}]$. The distribution of y is common knowledge. The realization of y_2 is not known to either party in period 1, but is observed by the agent in period 2.

A2. If $\lambda = 0$ (and thus the agent is considered to be of type 1) then there is no collateral requirement, but the principal monitors the agent's second-period income incurring a cost. If $\lambda > 0$ (thus the agent is considered to be of type 2), then monitoring does not occur, but there is a collateral requirement.

A3. For type 2 agents (or more precisely, those with $\lambda > 0$) the durable good is used as collateral.

The following assumption 4 is a simplifying assumption, which we relax later.

A4. The principal can costlessly observe the type of the agent, i.e. the agents truthfully report their λ values to the principal.

The rationale behind assumption 2 is that, when there is a collateral requirement the principal is guaranteed to receive some repayment (equal to the value of the durable good) in the event that the individual defaults. Hence, since monitoring is costly, the principal does not monitor income of collateralizing agents. However, when there is no collateral requirement, then the lender may not get any money back, if he does not monitor, and therefore it pays to monitor. For the next two sections, we make assumption 4, which justifies separate treatment of the problems of type 1 and type 2 agents. We shall revoke assumption 4 when we deal with the adverse selection problem in section V.

3. The problem of the principal and the type 1 agent

Type 1 refers to the agents for whom $\lambda = 0$, i.e., those who consume the non-durable good only and whose second period income is monitored by the principal. Let c denote the intensity of monitoring and let $\gamma(c)$ be the cost of monitoring associated with intensity level c. Let $\pi(c)$ be the probability that the principal observes the true income y_2^i when monitoring occurs with intensity c, and let $1 - \pi(c)$ be the probability that the principal does not observe the true income, when monitoring occurs with intensity c. We assume that c, $\gamma(c)$, and $\pi(c)$ satisfy the following properties :-

i) $c = c(\lambda, \ RB - y_2^i)$

ii) $c(\lambda > 0,\ RB - y_2^i) = 0$

iii) $c(\lambda = 0,\ y_2^i \geq RB) = 0$

iv) $c(\lambda = 0,\ RB - y_2^i) > 0$ for $RB > y_2^i$, $c'(RB - y_2^i) > 0$,

v) $c(\lambda = 0,\ RB - y_2^i) \leq c_{max}$, for any $(RB - y_2^i) > 0$

vi) $c(\lambda = 0,\ RB - y_2^i) > c_{min} \simeq 0$, for any $(RB - y_2^i) > 0$

vii) $\gamma'(c) > 0$

viii) $\pi(c) \simeq 0$ for $c = c_{min}$

ix) $\pi(c) = 1$ for $c = c_{max}$

x) $\pi'(c) > 0,\ \pi(0) = 0$

If the agent takes a loan of size B then he is supposed to repay the amount RB next period. The rule of the game is that the repayment should be min $\{y_2^i,\ RB\ \}$. The term $(RB - y_2^i)$ represents the extent of potential cheating or evasion. Thus the <u>intensity of monitoring</u> is an exogenously given function of the λ value and the extent of cheating. Property (ii) implies that monitoring does not occur if the agent is of type 2. Property (iii) implies that the principal does not care whether the agent is telling the truth or not as long as $y_2^i \geq RB$ and hence monitoring does not occur in that case. Property (iv) implies that monitoring occurs whenever $y_2^i < RB$, and the intensity of monitoring increases as the extent of potential cheating increases. For sufficiently large value of c_{max}, property (v) and property (ix) together imply that the true income may not be observed with some small positive probability even though a very high extent of evasion induces a very high intensity level of monitoring. These two assumptions are needed to guarantee that the principal does not have perfect monitoring capabilitity.

If the agent is caught cheating when $y_2^i \geq RB$, then the the principal can impose a penalty according to the penalty function $\delta = \delta(y_2^i - RB)$ having the properties that it is continuous in its argument with $\delta'(.) > 0$, $\delta(0) = 0$. Now the principal (the financial institution), although able to penalise the agent depending on the extent of cheating, may not be in a position to punish the individual too severely. In fact, the penalty imposed by the financial institution can only be of two kinds - (i) denial of credits to the individual in the future, and/or (ii) an increase in the interest rate in case of default. In this paper, we assume the penalty to be of the second kind. However, in that case the lender faces a constraint of the form $(RB + \delta(.)) \leq y_2^i$ (i.e. the penalty factor can reduce the second period consumption to zero). In particular, if the true income is less than or equal to the repayment obligation, the bank cannot do anything beyond claiming the entire second-period income, and the penalty is zero in that case. From now on, we will work with the <u>following functional form of the penalty function</u> : $\delta = \bar{\delta}.(y_2^i - RB)$, $1 \geq \bar{\delta} > 0$, for $y_2^i > RB$. We will also assume that (i) the utility function of the agent can be represented by $u(c) = c$. For simplicity, we will work with the linear functional forms for π, c, and y. In particular, we assume that $c = a_0(RB - y_2^i)$, $\pi(c) = a_1 c$, and $y(c) = \bar{y}c$ where $a_0, a_1, \bar{y}$ are constant coefficients. Denote $a_0 a_1$ by α and $\bar{y}a_0$ by ϕ. We normalise α as $\alpha = \frac{1}{\bar{y}}$, so that properties (v) and (ix) are satisfied. Therefore, $\pi(c) = \alpha(RB - y_2^i)$, and $y(c) = \phi(RB - y_2^i)$.

The game between the principal and the agent is as follows:

<u>First stage</u> : At the beginning of the first period the agent goes to the bank and reveals his λ value. If the principal is not willing to make a loan, the game stops there. Otherwise we move on to the second stage.

<u>Second Stage</u> : Based on the stated value of λ the principal offers a contract $\beta = (R, B, \bar{\delta})$. If the agent accepts the contract, we move on to the third stage.

Third stage : Now we are actually in the second period. Based on the realization of true second-period income, the agent chooses the value of y_2^r (whether or not to report the true income) so as to maximise his utility and reports that income to the principal.

Fourth stage : Based on the reported value of second-period income, the principal monitors the agent's income.

Fifth stage : The agent is penalised if caught cheating according to the penalty schedule specified in the contract. If not caught cheating, the agent pays $\min\{y_2^r, RB\}$ to the principal.

The game needs to be solved backwards, starting with the agent's optimum choice of y_2^r conditional on R, B, δ etc. Let p be the probability that the principal observes y_2^r to be greater than or equal to RB and 1 - p be the probability that y_2^r is not greater than RB. The probabilities p and 1 - p will therefore be a function of y_2^r which on the other hand is a function of R, B, δ etc. Using these probabilities the principal will pick the contract that maximizes his expected payoff.

The agent's optimal choice of y_2^r is obtained from the following claims.

Claim 1 : If the individual lies about his true income then he will always underreport his true income, i.e. if $y_2^r \neq y_2^t$ is true then $y_2^r < y_2^t$ will hold, with $y_2^r = y_2^t$ only when $y_2^t = 0$

Proof : $y_2^r > y_2^t$ is not possible because, in that case the agent must pay y_2^r which he does not have. Hence if reported income is not same as the true income then it must be strictly less than the true income.

Claim 2 : If $y_2^r \geq$ RB holds then $y_2^r = y_2^t$. The proof is obvious.

Claim 3 : If $y_2^r < y_2^t$, then $y_2^r <$ RB holds. The proof is obvious.

The agent decides how much of second period income to report after the realisation of the second period income. We have the following reporting rule for the agent.

Proposition 1 : For given values of $\overline{\delta}$, RB and $\pi \neq 1$, cheating is the optimal strategy for the type 1 agents.

Proof : When $y_2^t < RB$ holds, then expected utility from truth telling is $EU_t = u(0)$. And the expected utility from cheating for all $y_2^r < y_2^t$ is $EU_c = \pi(.)u(0) + (1 - \pi(.))u(y_2^t - y_2^r)$. Since $EU_c > EU_t$ cheating is the optimal strategy in this case. For $y_2^t \geq RB$, if $y_2^r < y_2^t$ then let $y_2^r = RB - \varepsilon$, where ε is any positive number. In this case, the expected utility from truth telling is $EU_t = u(y_2^t - RB)$, and the expected utility from cheating is

$EU_c = \pi(.)u((1 - \overline{\delta})(y_2^t - RB)) + (1 - \pi(.))u(y_2^t - RB + \varepsilon)$

. Cheating is preferable if $EU_c > EU_t$. With the help of assumptions made about the utility function and $\pi(.)$ we can write the expected utilities in the following forms:

$$EU_t = y_2^t - RB$$

and

$$EU_c = \alpha\varepsilon(1 - \overline{\delta})(y_2^t - RB) + (1 - \alpha\varepsilon)(y_2^t - RB) + (1 - \alpha\varepsilon)\varepsilon$$

or,

$$EU_c = (1 - \alpha\varepsilon\overline{\delta})(y_2^t - RB) + \varepsilon - \alpha\varepsilon^2$$

Cheating is preferable if $EU_c > EU_t$ holds, or written alternatively, if the following condition holds :

$$1 > \alpha\varepsilon + \alpha\overline{\delta}(y_2^t - RB)$$

Note that given the specifications about π, c and γ, y_2^r cannot deviate from RB too much, because that induces monitoring at a greater intensity level, and hence higher would be the probability of

detection. But that implies that ε cannot be too large. Without any loss of generality, we can also assume that RB is positive. Hence, for $RB > 0$, and ε small enough, the above condition can be satisfied. Thus cheating is the optimal strategy in this case too. Hence cheating is the optimal strategy in both cases. Q.E.D.

Now consider the problem faced by the principal. Observe that the probability p (i.e. the probability of finding reported income to be greater than the repayment) is zero in both cases, since agent's optimal strategy is to underreport his income. For simplicity we assume the discount factor of the future utility to be equal to one. To keep the analysis simple we will assume that the <u>principal can acquire an unlimited amount of funds at zero cost.</u> Then the problem of the principal is :

$$\max_{R,\,B,\,\bar{\delta}} \int_0^{RB} \{\, \pi(c)y_2^t + (1-\pi)y_2^{\cdot\cdot} - \gamma(c) \,\} f(y_2^t) dy_2^t \;+\; \int_{RB}^{\bar{y}} \{\, \pi(c)(RB+\delta) \;+\;$$

$$(1-\pi(c))y_2^{\cdot\cdot} - \gamma(c) \,\} f(y_2^t) dy_2^t$$

subject to

$$u(B) \;+\; \int_0^{RB} \{\, \pi(c)u(0) + (1-\pi(c))u(y_2^t - y_2^{\cdot\cdot}) \,\} f(y_2^t) dy_2^t \;+\;$$

$$\int_{RB}^{\bar{y}} \pi(c)u(y_2^t - (RB+\delta) + (1-\pi(c))u(y_2^t - y_2^{\cdot\cdot}) \; f(y_2^t) dy_2^t \;\geq\; u(0) + \int_0^{\bar{y}} u(y_2^t)f(y_2^t) dy_2^t \;, \tag{3.1}$$

$$(RB \;+\; \bar{\delta}(y_2^t - RB) \leq y_2^t \text{ for } RB < y_2^t \tag{3.2}$$

$$0 \leq \bar{\delta} \leq 1 \tag{3.3}$$

where $\pi(c) = \pi(c(RB - y_2^{\cdot}))$, $\gamma(c) = \gamma(c(RB - y_2^{\cdot}))$. Equation (3.1) gives the individual rationality (IR) constraint of the borrower. Equation (3.2) gives the penalty constraint faced by the lender. Once the loan size B and the interest rate R are determined, the repayment RB is deter-

mined, and the amount of penalty to be paid is determined too, for a given value of $\bar{\delta}$. Note that in the above stated problem, while the agent takes $\bar{\delta}$ as given, the principal actually **chooses** the value of $\bar{\delta}$. Moreover, the choice of $\bar{\delta}$ must be made in such a way so as to satisfy the constraint (3.3) and consequently, (3.2). Therefore we shall drop the redundant constraint (3.2) in the sequel. It can be easily verified that the optimal reporting rule of the consumer is given by

$$y_2^{\cdot} = \frac{y_2^t + RB}{2} - \frac{1}{2\alpha} \quad for \ \ 0 < y_2^t \le RB,$$

$$= RB + \frac{\bar{\delta}}{2}(y_2^t - RB) - \frac{1}{2\alpha} \quad for \ \ y_2^t > RB,$$

Let π_1, γ_1 denote the probability that the true income is observed, and the monitoring cost, respectively, when $0 < y_2^t \le RB$ holds. Let π_2, γ_2 denote the same as above, when $y_2^t > RB$ holds. Then we have the following values for π_i and γ_i for i = 1,2 :

$$\pi_1 = 1/2[\ 1 - \alpha(y_2^t - RB)\ ], \quad \pi_2 = 1/2[\ 1 - \bar{\delta}\alpha(y_2^t - RB)\ ],$$

$$\gamma_1 = \frac{\phi}{2}[\ \frac{1}{\alpha} + (RB - y_2)], \quad \gamma_2 = \frac{\phi}{2}[\ \frac{1}{\alpha} - \bar{\delta}(y_2^t - RB)\ ].$$

The IR constraint can now be simplified as :

$$B + \int_0^{RB} (1 - \pi_1)(\frac{y - RB}{2} + \frac{1}{2\alpha})f\,dy + \int_{RB}^{\bar{y}} [\ \pi_2(1 - \bar{\delta})(y - RB) + (1 - \pi_2)\{(1 - \frac{\bar{\delta}}{2})(y - RB)$$

$$+ \frac{1}{2\alpha}\}]f\,dy \ge \int_0^{\bar{y}} yf\,dy \tag{3.4}$$

where y denotes y_2^t, f denotes $f(y_2^t)$, dy denotes $d(y_2^t)$. Also, from now on we will denote $y_2^{\cdot}$ by simply y_r. Let us now make a few observations about the principal's problem.

First, we note that we do not need to have the loan size B and the interest rate R as separate arguments. That is because the principal, being able to acquire unlimited amount of funds at zero cost, will in fact supply it if the agents demand it. The source of the principal's profit therefore lies in how much repayment he can extract from the agents in the second period. Therefore, it is the size of the repayment, and not the loan size, which appears as an argument in the above stated problem.

Second, we observe that if there exists an equilibrium, i.e. a solution to the principal's problem, then the IR constraint can be assumed not to be binding in the equilibrium. The argument for that goes as follows : the agent being endowed with zero income in the first period, will always prefer to have more loans. Since the principal has costless access to unlimited amount of capital, he will supply enough loans to the agents in the first period, because by manipulating the interest rate R, he can always increase the size of the repayment and consequently the amount of profit. Therefore it is always possible to design a contract consisting of RB, such that B is large enough to make the agent's first period utility high enough to offset the decrease in utility due to repayment in the second period. Therefore, we do not need to deal with the constraint (3.1) any more since it can be assumed not to be binding.

Finally, we observe that the solution to the principal's problem exists. Note that RB lies in the compact interval, $0 \leq RB \leq \bar{y}$, and $\bar{\delta}$ lies in the compact interval, $0 \leq \bar{\delta} \leq 1$. The profit function of the principal is continuous in RB. It is also continuous in $\bar{\delta}$. Namely the continuity of the profit function really depends on the continuity of the optimal reporting rule y_r. y_r is continuous in RB, for $y \in [0, RB]$; it is continuous in RB and $\bar{\delta}$ for $y \in (RB, \bar{y}]$. Taking the limit for $y \rightarrow RB$ establishes the continuity at $y = RB$. Thus y_r is continuous everywhere in RB and $\bar{\delta}$ and so is the profit function. Since the profit function is continuous in the compact intervals of RB and $\bar{\delta}$, therefore it will have a maximum. Hence there exists a solution to the principal's maximization problem. We can now characterize the solution.

Define $x = RB$, and $\Pi =$ principal's profit. At the equilibrium, the value of $\bar{\delta}$ will be nonzero. Because,

$$\frac{\partial \Pi}{\partial \bar{\delta}} \Big|_{\bar{\delta}=0} = \int_{RB}^{\bar{y}} \left[\frac{1}{2}(y - RB) + \frac{\phi}{2}(y - RB) \right] f dy > 0.$$

Therefore, we can now restate the problem of the principal in its simplified form as follows :

$$\max_{x, \delta} \int_0^x \pi_1 y + (1 - \pi_1) y_r - \gamma_1 f dy + \int_x^{\bar{y}} \pi_2(x + \delta) + (1 - \pi_2) y_r - \gamma_2 f dy$$

subject to

$$\bar{\delta} \leq 1.$$

The optimal solution will have $\bar{\delta} = 1$. Let θ be the nonnegative Lagrange multiplier associated with the $\bar{\delta}$ - constraint. Since, we have already argued that there exists a maximum, hence the value of θ is obtained from the first order condition with respect to $\bar{\delta}$, as

$$\theta = \int_{RB}^{\bar{y}} \pi_2(y - RB) + \frac{\phi}{2}(y - RB) f dy$$

which is positive implying that the constraint holds with equality. The value of $\bar{\delta}$ equal to 1 makes sense, because that is the only way the redundant penalty constraint (3.2) can be satisfied. Also the first order condition with respect to x using Leibniz' rule and after substituting for $\bar{\delta} = 1$, is given by

$$\int_0^x \frac{1}{2} \{ 1 + \alpha(y - x) - \phi \} f dy + \int_x^{\bar{y}} \frac{1}{2} \{ 1 + \alpha(y - x) - \phi \} f dy \leq 0 \text{ for } x \geq 0$$

Note that since the principal is interested in extracting the maximum amount of money possible from the agents in the second period, the interest rate cannot be zero (and also not negative of

course). Since the principal has already offered enough loans to the agents in the first period, so as to make them accept the contract, B is positive. Hence, if a contract is offered, the associated repayment size must be non-zero. Hence the above equation holds with equality. Thus, the optimal repayment size solves

$$x^* = \frac{1 - \phi}{\alpha} + E(y)$$

where $E(y)$ is the agent's expected income. The value of x^* therefore depends on the monitoring parameter ϕ. If the value of ϕ is extremely high, then no contract will be offered. The following proposition gives the credit-rationing result.

Proposition 2 The type 1 agents are credit rationed whenever $R < (2\bar{y})/(\bar{y} + \phi - 1)$ holds.

Proof. Let $\underline{B}$ be the loan size such that the IR constraint holds with equality. Let $\underline{R}$ be the corresponding interest rate such that $\underline{RB} = x^*$, i.e. $\underline{R} = (\frac{1 - \phi}{\alpha} + E(y))/\underline{B}$. If the agent is rationed at $\underline{B}$, then the agent will also be rationed for any loan size $B > \underline{B}$. To find out whether the agent is rationed at $\underline{B}$ we evaluate the following:

$$\frac{\partial u}{\partial B} \mid R = \underline{R} = 1 - \frac{R}{2} [\alpha E(y) - \alpha x^* + 1]$$

Sustituting the value of x^* in the above equation we get

$$\frac{\partial u}{\partial B} \mid R = \underline{R} = 1 - \frac{R}{2\bar{y}} \{\bar{y} + \phi - 1\}$$

The above derivative is positive if and only if R is less than $(2\bar{y})/(\bar{y} + \phi - 1)$, but that would imply that the agent will be rationed. Q.E.D.

Whether the agent will be rationed or not therefore depends on the specifications of the parameters. Since the implicit interest rate depends on the parameter specifications too, therefore we may have a non-rationing equilibrium for certain values of ϕ. But if the agent is rationed, he will be rationed

in the Jaffee-Rusell sense, because at a certain interest rate, he ends up getting a loan smaller in size than the what he would like to have. The analysis of this section is summarised in the form of the following result.

Result 1 If the type 1 agents are credit rationed, they will be rationed in the Jaffee-Russell (1976) manner.

The type 1 agents are not rationed in the sense of Stiglitz-Weiss (1981) because they all receive loans in equilibrium if a contract is offered. However they may be rationed in Jaffee-Russell (1976) manner because they prefer to obtain more loans than what is provided to them. The result of this section depends heavily on (i) the divisible nature of the non-durable good, and (ii) that the principal must supply a non-zero amount of loan in order to make profits. The result of this section can be contrasted with the result obtained by Stiglitz and Weiss in their 1981 paper - where in equilibrium some agents received loans and others did not. Instead in this paper, (i) we cannot rule out the possibility of a non-rationing equilibrium, (ii) all agents are rationed to the same extent if they are rationed at all. Moreover, the Stiglitz-Weiss result was due to the variability of the degree of riskiness associated with the return of the project, whereas in our model the rationing equilibrium might occur due to the principal's inability to identify the agent's truthfulness along with the fact that he does not have perfect monitoring capability. Thus our model is more like Jaffee-Russell, because we also deal with 'honest and dishonest' borrowers but unlike Jaffee-Russell here we allow for the possibility of a costly monitoring.

4. The problem of the principal and type 2 agents

Type 2 refers to agents for whom $\lambda > 0$. Type 2 agents consume both non-durable goods and the durable good but consider non-durable goods as imperfect substitutes for the durable good. In fact we assume that the two goods are almost non-substitutable in consumption which can be alternatively represented by the condition : $\lim\limits_{c^n \to \infty} u(c^n, 0) \leq u(0, W)$. The type 2 agent therefore always wants the loan to at least cover the cost of the durable good. If he obtains a loan exceeding the cost of the durable good, he spends the difference buying the non-durable consumption good.

Note, in this section we deal with the problem of the principal and the agents when <u>only the collateralized contracts are offered.</u> Therefore, if a contract is offered, it must consist of a loan of size at least as big as the value of the collateral (the durable good in this case), otherwise the agents cannot buy the collateral. The type 2 agent's problem is very similar to the type 1 agent's problem except that his second period income is not monitored by the principal and that he keeps the durable good as collateral. If the agent takes a loan of size B, he is supposed to repay the amount RB next period. If his reported income is less than RB, he loses the durable good. The difference RB - W is paid out of his reported income. Here, the rule of the game is that the repayment should be $W + \min \{ RB - W, y_2^s \}$ whenever $y_2^s \leq RB$. Let the contract offered to type 2 agents be denoted by α. Then $\alpha = (R, B, W)$, where W is the collateral. The game between principal and agent is as follows :

<u>First stage</u> : At the beginning of the first period, the agent goes to the bank and states his $\lambda(B)$ function (where λ is greater than 0). If the principal agrees to give a loan, we move to second stage.

<u>Second stage</u> : The principal offers the contract α to the agent. If the agent accepts the contract, we move to third stage.

<u>Third stage</u> : Now we are in second period. Based on the realized value of income, the agent makes his optimal choice of reported income. If the reported income is less than the repayment, the agent loses the durable good.

The problem looks similar to type 1 agent's problem, but somewhat simpler than that. Once again, we solve the game backward starting with agent's optimal choice of reported income. We have the following reporting rule for the type 2 agents :

Proposition 3 : (i) For $y_2^r < RB$, cheating is the optimal strategy of the agent. In particular, $y_2^{r*} = 0 \leq y_2^r < RB$ holds. (ii) For $y_2^r \geq RB$ truth telling is the optimal strategy.

Proof. (i) When $y_2^r < RB$ holds, then $c_2^d = 0$ and if the agent tells the truth then $c_2^q = \max\{0, y_2^r - (RB - W)\}$. Hence, $EU_t = u[\max\{0, y_2^r - (RB - W)\}, 0]$. Now, we know that if the agent lies about his income then he will underreport his income. In that case, $c_2^d = 0$ and $c_2^q = y_2^r - (RB - W)$ if $y_2^r \geq y_2^s \geq (RB - W)$, and $c_2^d = 0$, $c_2^q = y_2^r - y_2^s$ if $(RB - W) > y_2^s$ is true. Because in the former case, the expected utility from truth telling is same as the expected utility from cheating, there is no gain from underreporting the income. Hence, if the agent cheats, he will report income to be less than (RB - W) and in that case, $EU_c = u(y_2^r - y_2^s, 0)$. Now since y_2^s is the choice variable of the agent, by reporting zero income he can always make $EU_c = u(y_2^r, 0)$ to be greater than EU_t. Because the principal does not monitor income of the type 2 agents, the type 2 agent will in fact do so. Thus (i) is proved. Part (ii) : For $y_2^r \geq RB$, $EU_t = u(y_2^r - RB, W)$. Once again if the agent lies seriously about his true income he will underreport it and $y_2^r \geq RB > y_2^s$, $EU_c = u(y_2^r - y_2^s, 0)$ will hold. In this case EU_c is highest when $y_2^s = 0$. However because the durable good good is almost never replaceable by the non-durable good under our assumption on preferences, EU_t is always greater than EU_c. Therefore truth telling is the optimal strategy in this case.

We proceed with the following utility function for the agent - consistent with the assumption that at best an infinite amount of non-durable good can replace the durable good :

$$u(c^n, c^d) = c^n + a \ for \ W > 0 \,,$$

$$= \min\{ c^n, \bar{y} \} \ for \ W = 0 \,,$$

where $a \geq \bar{y}$.

In this section we assume that the <u>cost of obtaining capital is non-zero.</u> Let r denote the unit cost of obtaining capital. For simplicity we would again assume the discount factor to be equal to unity. The analysis of this section depends heavily on the value of the durable good W. Based on different values of W, we have the following cases to consider.

Case (i) $W \geq \bar{y}$ or W is slightly less than $\bar{y}$.

In this case, the agent always loses the collateral, and since there is no monitoring, $y_r = 0$ by proposition 3. Therefore the principal's payoff Π in this case is always equal to $W - rB$. Obviously, if a contract is offered then the optimal loan size B^* will be no greater than W. However no contract may be offered if $r > 1$. If a contract is offered, then the agent is rationed in Jaffee-Russell (1976) sense because of the specification of the agent's utility function. If no contract is offered then also the agent is rationed because the agent's utility with loan is always greater than the agent's utility with no loan (recall that $a \geq \bar{y}$).

Case (ii) When W is sufficiently less than $\bar{y}$.

In this case the problem of the principal is stated as follows :

$$\max_{B, R} \int_0^{RB} W f(y_2^t) dy_2^t \ + \ \int_{RB}^{\bar{y}} RB(y_2^t) dy_2^t \ - \ rB$$

subject to

$$B \geq W \ \ \text{if} \ B > 0 \tag{4.1}$$

$$u((1-\lambda)B,\ W) + \int_0^{RB} u(y_2^t,\ 0)f(y_2^t)dy_2^t + \int_{RB}^{\bar{y}} u(y_2^t - RB,\ W)f(y_2^t)dy_2^t \geq$$

$$u(0,0) + \int_0^{\bar{y}} u(y_2^t,\ 0)f(y_2^t)dy_2^t \tag{4.2}$$

Constraint (4.1) appears because only the collateralised contract is offered. Constraint (4.2) represents the individual rationality (IR) constraint, which says that the utility with loan should at least be as big as the utility without loan if the agent has to accept a loan. The principal's expected payoff can be simplified to

$$\Pi = WF(RB) + RB(1 - F(RB)) - rB$$

With the particular specification of the utility function, the IR constraint can be simplified as follows :

$$(1-\lambda)B + a + \int_0^{RB} y_2^t f(y_2^t)dy_2^t + \int_{RB}^{\bar{y}} \{y_2^t - RB + a\} f(y_2^t)dy_2^t \geq$$

$$\int_0^{RB} y_2^t f(y_2^t)dy_2^t + \int_{RB}^{\bar{y}} y_2^t f(y_2^t)dy_2^t$$

or,

$$(B - W) + a + (a - RB)(1 - F(RB)) \geq 0$$

Because $a \geq \bar{y}$ and $\bar{y} \geq RB$, the above constraint always holds with strict inequality, and therefore we can get rid of that constraint. Note that in the presence of cost of obtaining capital, we must treat R and B as separate arguments (as opposed to section 3 where the problem was simply an

optimal extraction problem). Let θ denote the Lagrange multiplier associated with the constraint (4.1). The first order conditions are as follows :

$$\{(W - RB)f(RB) + (1 - F(RB))\}R - r + \theta \le 0 \text{ for } B \ge 0$$

$$\{(W - RB)f(RB) + (1 - F(RB))\}B \le 0 \text{ for } R \ge 0$$

The implications of the above first order conditions give rise to the following proposition.

Proposition 4 Assume $f(y) \ne 0$ for $y \ne 0$. Then if a contract is offered, the optimal loan size B^* is equal to W and the optimal interest rate R^* is greater than 1. At the optimal B^* and R^* the type 2 agents are credit rationed.

Proof. R cannot be zero if B is positive - because in that case the repayment becomes zero and principal's profit cannot have a maximum. Also B must be non-zero (and hence R positive too), because in that case by choosing B and R simultaneously the principal can always increase his profit and if so then a maximum cannot be achieved at B = 0. Therefore, in equilibrium the amount of loan supplied must be positive. Hence we have the following versions of the above first order conditions :

$$\{\{(W - RB)f(RB) + (1 - F(RB))\}\}R - r + \theta = 0 \tag{4.3}$$

$$\{(W - RB)f(RB) + (1 - F(RB))\}B = 0 \tag{4.4}$$

Substitution of equation (4.4) into the equation (4.4), for positive B implies $\theta = r > 0$. Hence $B^* = W$, where B^* is the equilibrium loan size. Now $(W - RB)f(RB) + (1 - F(RB)) = 0 \Rightarrow (RB)^* > W$ because $F(RB) \ne 0$ and $f(RB) \ne 0$. Hence $R^* > 1$. The agents are rationed because for given $(RB)^*$, u_1 is monotonically increasing in B, where u_1 is the first period utility of the agent. Q.E.D.

Let z^* denote the optimal expected repayment size. If a contract is offered, then $(RB)^*$ must be less than $\bar{y}$. That is because if the principal sets R^* high enough to make $(RB)^*$ equal to $\bar{y}$, then $z^* = W$, because in absence of monitoring, the agent always reports zero income by proposition 3. However, if he sets R^* in such a manner so that $(RB)^* < \bar{y}$ then $z^* = WF + RB\,(\,1 - F\,)$ which is greater than W. Thus the principal would not charge an abnormally high interest rate, because the increasing probability of default will eventually reduce his expected profits even though collateral covers some part of the repayment. However, whether a contract will be offered at all, depends on the value of r - the cost of obtaining capital. The above contract will be offered if for a given value of r, the principal's payoff at the optimum contract $\Pi^* \geq 0$ which is equivalent to stating as long as the following holds : $(R^* - r) \geq (R^* - 1)F$. This is easily satisfied when r equals one or when r is slightly greater than one. If the cost of acquiring capital is prohibitively high then even by choosing R^* greater than one and $(RB)^* < \bar{y}$, the principal incurs a loss and no contract will be offered. Once again, the agent is rationed if no contract is offered because the agent's utility with the collateralised loan is always greater than the utility with no loan.

The analysis of this section is summarised in the form of the following result.

Result 2 If a contract is offered then the type 2 agents are credit rationed in the sense of Jaffee-Russel (1976).

The type 2 agents are not rationed in the Stiglitz-Weiss (1981) sense because they all obtain a loan of size as big as the value of the durable good, if they obtain a loan at all. Since all type 2 agents are alike therefore they all obtain the loan of same size and in that sense nobody is denied credit. However the agents end up getting a loan smaller in size than what they would like to have and hence they are rationed in the Jaffee-Russell (1976) manner. The result in this section depends on the fact that only the collateralised contract is offered, if a contract is offered at all, and that the collateral itself is acquired through loan financing. Because there is a cost of obtaining capital the principal does not supply unlimited amount of loans. Instead he chooses to supply a loan of size equalling W and manipulates the interest rate so as to maximize his profit. However he does not

charge R abnormally high because in that case the high default rate actually reduces his expected profits.

5. The problem of the principal when agents are indistinguishable ex-ante

In this section we deal with the problem when the principal cannot distinguish between the two types of agents on an a-priori basis. Therefore the assumptions A2, A3, and A4, made in section II are no longer valid. Let λ^t represent the true value of λ which is private information of the agent, and let λ^r represent the reported value of λ which the principal actually observes (recall that λ represents the fraction of loans used to purchase the durable good). As before, the type 1 agent's utility depends on the consumption of the non-durable good only, and the type 2 agent's utility depends on the consumption of both types of goods, and specifications of the agents' utility functions are the same as in sections 3 and 4.

Let W denote the expenditure for or the value of the durable good. Now, if the principal decides to offer contracts smaller in size than W, then he offers only the non-collateralised contract. In that case, all agents are treated as if of type 1 only, since the type 2 cannot buy the collateral. However, if the principal decides to offer contracts consisting of loans bigger in size than the value of the durable good, then he can potentially offer both the collateralised and the non-collateralised contracts. In that case, there exist borrowers of two types - those who are interested in consuming only the non-durable goods and the borrowers who would like to consume the durable good first and then use the rest of the loans to purchase the non-durable good. Thus the distinction between different types of agents matters only when different contracts are offered. Whether different contracts will be offered or not, needs to be determined. Since now the agents can lie about their types, therefore the rationale behind offering two types of contracts is that, by making the contracts

incentive-compatible the principal could actually design a self-selection mechanism which would help the pincipal identify the agen's true type, thereby minimizing the loss associated with default. It is important to note however, that the decision to offer a loan of a particular size and the decision to offer different contracts are interrelated. If the optimal loan size is less than W, then the principal can offer only the non-collateralised contract. In contrast if the optimal loan size turns out to be greater than W, then he can decide whether to offer different contracts or not.

We would assume as before that if a non-collateralised contract is offered then the principal monitors the agent's second-period income incurring a cost, and penalises the agent if he is found cheating. On the other hand, if a collateralised contract is offered, the principal does not monitor the agent's second period income but repossesses the durable good (the collateral) if the agent cannot make the loan payment. We will assume that monitoring cost function $\gamma = \gamma(c)$ and the penalty function $\delta = \bar{\delta}(y_2^t - RB)$ satisfy the same properties as described in section 3. Then once again the principal chooses $\bar{\delta}$ in such a manner so as to satisfy in each period the constraint $[(RB)_1 + \delta(.)] \leq y_2^t$ for $(RB)_1 \leq y_2^t$, where $(RB)_1$ represents the repayment size if the agent picks the non-collateralised contract. In fact the principal would choose $\bar{\delta}$ equal to 1, because by doing so he can extract most money from the agent. Let $\alpha_1 = (R_1, B_1, \bar{\delta})$ represent the non-collateralised contract and $\alpha_2 = (R_2, B_2, C)$ represent the collateralised contract, where R_i and B_i represent the interest rate and the loan size for the contract α_i, $i = 1, 2$ and C represents the collateral, $C = W$.

λ^r can differ from λ^t in two ways - (i) for type 1 agent $\lambda^t = 0$ but $\lambda^r > 0$ (i.e. when type 1 reveals himself to be of type 2), and (ii) for type 2 agent $\lambda^t > 0$ but $\lambda^r = 0$ (i.e. when type 2 reveals himself to be of type 1). Now the nature of the contract is such that if the agent reveals himself to be of type 2, then he must meet the collateral requirement i.e. he must pick the second contract, otherwise he can pick the first contract. Possibly each type reveals himself to be of the other type. The type 1 can reveal himself to be of type 2 because by doing so he can avoid being monitored. On the other hand, the type 2 can reveal himself to be of type 1, because by doing that, the type 2 can get rid of the collateral requirement and can always keep the durable good even in the case when the true income is less than the promised repayment. Of course we could assume that the principal could

monitor the agent's type as well incurring a cost, but to keep the problem simple and tractable we assume that the lender does not try to discover the agent's true type - because the lender is only interested in receiving the repayment. Therefore, it does not matter to the principal whether the agent is honest or dishonest as long as he receives the repayment.

Thus the principal knows that for a loan offered of size less than W, λ^t equals zero for both types, and that of a size greater than W λ^r may not equal λ^t and that for some agents $\lambda^r \neq 0$. The question is if the principal would want to design contracts in such a manner so as to make the agents reveal their true types. Let n_1 represent the total number of type 1 agents, and n_2 represent the total number of type 2 agents. It can be easily verified that the claims 1,2,3, and proposition 1 of section 3 are valid when the agents are of type 1 and proposition 3 of section 4 is valid when the agents are of type 2. Let us denote y_s^t by y, y_r^t by y_r, $f(y_s^t)$ by f and dy_s^t by dy.

Denote as in section III, the probability of observing true income and the monitoring cost for $0 < y \leq RB$ by π_1 and γ_1 and the same for $y > RB$ by π_2 and γ_2.

It can be easily verified that the optimal reporting rule for **both** type of agents when both types pretend to be of type 1 (i.e. when the agents accept the non-collateralised contract) are going to be the following (same as obtained in section III)

$$
\begin{aligned}
y_r^* &= \frac{y + RB}{2} - \frac{1}{2\alpha} \quad for \ 0 < y \leq RB \\
&= RB + \frac{\bar{\delta}}{2}(y - RB) - \frac{1}{2\alpha} \quad for \ y > RB
\end{aligned}
$$

Hence the principal cannot really distinguish between the agents' types based on the reported value of the income (the fact that the reporting rules are the same for both type of agents, follows from the specification of the utility functions. Since the agents are alike incomewise (the income distribution function being the same for both types), hence there exists no difference in the reported value of the second period income, and the principal cannot distinguish between the agents). The corresponding $\pi_i, \gamma_i, \ i = 1, 2$ are also the same as obtained in section III, namely

$$\pi_1 = 1/2[1 + \alpha(RB + y)], \quad (1 - \pi_1) = 1/2[1 - \alpha(RB + y)],$$

$$\pi_2 = 1/2[1 - \overline{\delta}\alpha(y - RB)], \quad (1 - \pi_2) = 1/2[1 + \overline{\delta}\alpha(y - RB)],$$

$$\gamma_1 = \frac{\phi}{2}[\frac{1}{\alpha} + (RB - y)], \quad \gamma_2 = \frac{\phi}{2}[\frac{1}{\alpha} - \overline{\delta}(y - RB)].$$

It is obvious that when the type 1 agents claim to be of type 2, and accept the collateralised contract then for them $y_2^s = 0$ for any y because in absence of monitoring the type 1 agents will always report zero second period income to maximize utility. The analysis of this section depends once again on the value of W and r. Hence we have the following cases to consider.

Case (i) The cost of obtaining capital $r = 0$, $W \geq \overline{y}$.

Proposition 5 Only the collateralised contract will be offered in equilibrium. Moreover, the optimal loan size in the collateralised contract is $B^* = W + \varepsilon$, where ε is any arbitrary positive number.

Proof. Suppose not. Suppose both α_1 and α_2 are offered. Let x_1 and x_2 denote the *proposed* repayment sizes and let z_1^* and z_2^* denote the *expected* repayments of the principal, when the contracts α_1 and α_2 are offered respectively. In absence of refinancing costs, the principal can always choose to make x_1 and x_2 greater than $\overline{y}$ assuming that the corresponding interest rates are non-zero. In that case we have $z_1^* \leq \overline{y}$ and $z_2^* = W \geq \overline{y}$. Since $z_2^* > z_1^*$, the principal's profit is maximized by offering the collateralised contract only. If the collateralised contract is to be offered then the loan size must at least be as big as W. Offering a loan of size W will only make the type 2 agents participate in the contract. The type 1 agents accept the contract if

$$B - W + E(y) \geq E(y)$$

For $B^* = W$ the type 1 agents are indifferent between participating or not participating. However a loan size of $B^* = W + \varepsilon$ will make all type 1 agents accept the contract. Since the principal increases his payoff by offering $B^* = W + \varepsilon$, he will indeed do so. Q.E.D.

The above solution characterizes a pooling equilibrium where only one type of contract will be offered and where agents of both types accept the same contract. In this case we may have a non-rationing equilibrium. Because with $r = 0$, the principal can supply as much loans as the agents demand. The agents will be rationed (in Jaffee-Russell manner) only if the principal decides to stop supplying loans at some point. This particular result of case (i) is due to the fact that the principal is interested only in maximizing his second-period payoff and that there is no cost of obtaining capital. The result is going to be changed when we introduce $r > 0$ as in the following case.

Case (ii) $r > 0$, $W \geq \bar{y}$.

In this case also only the collateralised contract will be offered by the same argument as in propositon 5, if a contract is offered at all. The principal's payoff from offering the collateralised contract will equal $(n_1 + n_2)(1 - r)W$ if all types participate in the contract, and equals $n_2(1 - r)W$ if only the type 2 agents participate. However no contract will be offered if $r \geq 1$ or even when r is quite close to 1. Moreover depending on the value of r, the principal decides whether or not to offer B^* greater than or equal to W. The type 2 agents will always participate in the contract, the type 1 agents may be indifferent if the optimal loan size equals W. In either case, in presence of positive r, the principal will not make arbitrarily large amount of loans and the agents will be rationed in the sense of Jaffee-Russell (1976).

Case (iii) $r = 0$ and $W < \bar{y}$.

In this case, the principal's expected payoff from offering the collateralised contract, Π_c is given by

$$\Pi_c = n_1 W + n_2\{WF(RB) + RB(1 - F(RB))\} < (n_1 + n_2)\bar{y}$$

where $RB < \bar{y}$ (by the argument put in section 4). If the principal decides to offer only the noncollateralised contract, then again he is interested in extracting the maximum amount possible from the second- period income of the type 1 agents. In that case a possible solution is

$B^{\cdot} \geq \underline{B}$, $(RB)^{\cdot} \geq \bar{y}$, where $\underline{B}$ is the minimum loan size at which the agents accept the contract. The principal's expected payoff from offering the non-collateralised contract, Π_{nc} is given by

$$(n_1 + n_2)[\int_0^{\bar{y}} \{(1 - \pi_1)y_r + \pi_1 y - \gamma_1\} f \, dy \; < \bar{y}$$

Now we have the following proposition.

Proposition 6 As a rule, only one type of contract will be offered.

Proof. The contract α_1 will be offered if $\Pi_{nc} > \Pi_c$, and the contract α_2 will be offered if $\Pi_c > \Pi_{nc}$. Unless $\Pi_c = \Pi_{nc}$, only one contract will be offered. Because if the expected payoff from one of the contracts is higher than the other, then the principal can always increase his payoff by offering that contract only. Q.E.D.

The collateralised contract will be offered if the value of W is sufficiently large yet less than $\bar{y}$, and/or the monitoring cost is high enough. Thus in the case when the cost of obtaining capital is zero, the principal will decide which contract to offer based on the value of W and the parameters of the monitoring cost function. If the contract α_2 is offered, then $B^{\cdot}$ equals $W + \varepsilon$ where ε is any positive number, so as to make the type 1 agents participate in the contract. On the other hand, if the contract α_1 is offered, then $B^{\cdot}$ need only be greater than $\underline{B}$. In either case, we will have a pooling equilibrium where all agents accept the same contract, and the agents will be rationed in the Jaffee-Russell 1976 manner. The principal will offer different contracts only if he is legally obligated to do so.

Case (iv) $r > 0$, $W < \bar{y}$

The analysis in this case depends heavily on the value of r and the monitoring cost parameters. Of course if the value of r is too high then no contracts will be offered at all, and in that case all

agents will be rationed. However, depending on the value of W, r, and the monitoring cost parameters, we are likely to have a separating equilibrium.

The result of this section is summarised below in two parts.

Result 3 (i) When the value of the durable good is high enough, then only the collateralised contract will be offered. If the agents are rationed, they will be in the Jaffee-Russell (1976) manner. (ii) When the value of the durable good is not sufficiently large, then depending on the refinancing costs of capital and the monitoring cost parameters, more than one contract may be offered. In this case too if the agents are rationed, they will be in the Jaffee-Russell manner.

The basic credit rationing result of this section is therefore the same as in the previous sections. Thus if the agents are rationed they are always rationed in the same Jaffee-Russell manner, no matter whether the principal can distinguish between the agents perfectly well or not. Although we cannot rule out the possibility that the principal is successful in designing optimal contracts which screen for the agents' types, under most circumstances only one type of contract will be offered. Therefore, in most cases we have pooling equilibria where agents of different types buy the same contract. It is important to note that whether we have a separating equlibrium or a pooling equilibrium, depends very much on the parameters of the principal's payoff function.

It is interesting to note that even when only the collateralised contract is offered we do not have rationing in the Stiglitz-Weiss manner. That is because some of the agents consume the durable good which also serves the purpose of a collateral. Hence if the principal has to offer a contract, he must supply a loan of a size at least as big as the value of the durable good itself.

Whether the extent of rationing in this case increases or not also depends on the value of the durable good. If the objective of the principal is only to make the type 1 agents participate in the contract so that he can repossess the collateral, then offering a loan size slightly greater than W will serve the purpose, and in that case the extent of rationing is likely to increase among the type 1

agents. However, the situation with type two agents remains the same as before - they get at least W. In fact the situation with the type 2 agent may be better in this case, if the loan offered equals $W + \varepsilon$, as compared to the previous case where they were always getting a loan equalling the cost of the durable good.

6. Conclusion

In this paper we have constructed a credit market model where different agents have different attitudes towards the consumption of durable goods. This induces the principal to offer different contracts to different agents. The models presented in sections 3 and 4 of the paper assume that the principal can costlessly identify the type of the borrowers ex-ante, based on which he offers different contracts to the different borrowers. It is shown that the type 2 agent i.e. the one who borrows to finance a durable good, which then is used for collateral, is not rationed in the sense of Stiglitz and Weiss (1981), even though he wants to have a loan at least as big as the value of the durable good. However the agent is credit rationed in the Jaffee-Russell manner, since he gets a loan of size only equalling the value of the durable good. On the other hand, type 1 agents i.e. those who do not consume the durable good and only need loans to finance the consumption of non-durable goods, may not be rationed, if the principal keeps on supplying loans as long as there is demand for them. This is possible because the principal can acquire unlimited amount of funds at zero cost. But the agents will be rationed the moment the principal stops providing loans to them. If the type 1 agents are rationed, they are rationed in Jaffee-Russell manner i.e. they receive loans smaller in size than what they would demand.

Section 5 relaxes the assumption that that the principal can identify the agent's type from costlessly observable information. It is shown that most of the time the resulting equilibrium will be a pooling

one, where only one type of contract will be offered. The results of this section however depends heavily on the assumptions made about the capital acquisition costs and other parameters of the payoff function of the principal. The type 1 agents are still rationed in Jaffee-Russell (1976) manner as they were in section 4. But the type 2 agents may be better-off in this case compared to the previous case, if the loan-size of the collateralised contract is bigger than the value of the durable good.

The model presented in this paper is attractive because here the collateral itself is acquired through loan financing - this possibility has not been dealt with in the existing literature. Also, most of the existing literature on credit rationing discusses the problem where the borrower is another firm. How consumers can be rationed in the credit market, has largely been ignored by the literature. This paper therefore tries to fill these gaps.

Chapter III

Consumer Credit Rationing and Screening with

Renegotiation of Contracts

1. Introduction

This paper investigates the phenomenon of credit rationing, as it emerges in a dynamic setting, when a renegotiation of the original contract is allowed. The need for renegotiation of a contract arises mainly because of the limited computational power on the part of the agent on one hand (the agent can at best choose to specify the most likely actions in the most likely contingencies), and due to limited enforcement power of the agent if the original contract is not obeyed, on the other hand. Of course, the original contract can be enforced if appropriate actions can be undertaken. However, sometimes undertaking such actions can be too costly (for example, if a loan recipient

simply disappears without making a repayment, the cost of search for that person may be so high that the lending institution would rather forego the repayment). In such cases, it is mutually beneficial for the contracting parties to revise the original contract.

The revision of a contract is most likely to occur in a credit market. In most loan defaults, the bank does not foreclose, yet foreclosure is the action specified in the original loan contract. In this paper, we try to model a credit market, where the original loan contracts can sometimes be renegotiated if a default occurs, and where the lender uses the knowledge about such an event when deciding whether to offer contracts in the future. Therefore, depending on the type of the default - unintentional or intentional, the original contract is revised (or not revised) so as to give the agents another chance to repay the loan. This paper is similar to Stiglitz-Weiss (1983), but allows for intentional default. Also, in their paper, the terms of new contracts offered in subsequent periods depends on whether or not default occurred previously, but they do not allow for a revision of the original contract. In contrast, in the present paper we allow for the possibility of an intentional default (cheating). However, whether the default would be intentional (cheating) or not (pure bad luck) depends on the agent's type - i.e whether the agent is honest or dishonest. We assume that the lender (the principal) does not know the agent's type ex ante, but can find out by incurring a cost after the default has occurred. We assume that the principal does not have perfect monitoring capability. Therefore there is always a possibility that the agent's true type will never be revealed. Also, it is possible for an agent who acted honestly in the first period to become dishonest in the subsequent period. Hence the contracts must be designed in each period in such a way so as to make the agents act honestly in all periods. We further assume that the principal monitors agent's type only if a default has occurred because monitoring is costly. If the agent is found to have defaulted because of pure bad luck, the principal gives him another chance to pay back the loan in the future at the different terms. It is at this point that the original contract is revised. The revision of contract here is justified on the ground that there is a chance of an outcome with renegotiation. The source of the principal's profit lies in the increased expected repayments from the agents. Therefore, offering a revised contract and plus possibly a new contract in place of complete denial

of credits in the future, improves the principal's profit situation. Agents also are better off with the renegotiated contract than with none at all. Hence renegotiation is preferred by both the contracting parties.

However, if the agent is found to have defaulted dishonestly, the principal merely punishes him by denying loans in the future. The information revealed at the time of repayment of the first period is then used by the principal to form probabilistic beliefs about the type of the agent in the future, and in deciding whether or not to offer further contracts in the future. Therefore the agent's actions today determine to some extent the agent's fate in the future. We are interested in finding out whether or not credit rationing would be a feature of the equilibrium contract in each period, under the above set up.

The rest of the paper is organized as follows. Section II presents the basic set up of the model. Section III poses the problem of the principal. Section IV presents some concluding remarks and plans for possible future extensions.

2. The model

We consider the relationship between a risk-neutral individual borrower (the agent) and a risk-neutral monopolist lender. The principal and the agent write a contract at some initial date '0' which specifies the terms of the contracts between them in the future. The agent (the consumer in the economy), endowed with zero first-period income at the time when the contract is first signed, needs loans to purchase the consumption good. Let c_i denote the agent's consumption in period i. The agent's utility in period i is represented by $u(c_i)$. Let B_i denote the amount of loan taken at the beginning of period i, which is then used to purchase the consumption good. Let R_i denote the

corresponding gross interest rate. Hence in period i, the repayment size is $R_i B_i$ which we simply denote by x_i. To simplify, we assume that the repayment should be made at a single date, date 1. After the initial contract is signed, but before the date at which the repayment is to be made, the agent realizes his income, which is unobserved by the principal. Let y_i denote the (true) income realized by the agent in period i. After the realization of y_i the agent decides how much of the true income to report to the principal. Let y_i^r denote the reported value of the income in period i. We say that the agent has defaulted if $y_i^r < x_i$. The term of the contract is such that if a default occurs then the agent must pay $\min\{y_i^r, x_i\}$. Let us now make the following assumptions.

<u>A1</u> The true income y_i is distributed i.i.d across individuals and over time with p.d.f $f(y_i)$, $f(.) > 0$ and c.d.f F for $y_i \in [0, \bar{y}]$. The distribution is common knowledge but only the agent observes costlessly the true value of y_i after it is realized.

<u>A2</u> The principal monitors the agent's type by incurring a cost only if a default occurs so as to find out whether the default has been intentional or not.

We say that a default has been unintentional (the agent defaulted due to a bad realization of income) if $x_i > y_i^r = y_i$ occurs. We say that the default is intentional if $y_i \geq x_i > y_i^r$ holds. Let γ denote an exogenously specified monitoring cost function which should depend on the extent of evasion. But just for the sake of of simplification, let us work with a cost function having only a fixed value. Thus we assume, following A2, that

$$
\gamma = \begin{cases} \bar{\gamma} > 0 & \text{for } y_i^r < x_i \\ 0 & \text{for } y_i^r \geq x_i \end{cases}
$$

We further assume that the principal does not have perfect monitoring capability. Let p denote the probability that the true income will be discovered following monitoring, and let $1 - p$ denote the probability that true income will not be observed even though monitoring occurs, where $1 > p \geq 0$. This assumption merely says that the true income may never be revealed even if a monitoring takes place following a default.

Following a default, if it is found that the default has been intentional, then the agent is denied credit in the future. Therefore we can say that the punishment in this model takes the form of complete denial of future credit. However if it is found that the default has been unintentional, then the principal gives the agent another chance to repay the loan and revises the original contract, because as already mentioned in the introduction, that renegotiation is mutually beneficial.

However when the initial contract was signed the agent did not know that the principal might revise the original contract. Because if they knew, then the new terms of contracts would already have been specified in the original contract and then we no longer have 'renegotiation of contracts'. Therefore we assume that the agent knew that if he defaults then the principal will monitor to find out whether the default has been intentional or not, and that if it is revealed that the default has been on purpose, then he will be denied of future credit. The contract is said to be revised, because now the terms of contracts differ from that specified in the original contract - a different rate of interest is now applied on the **old** loan. The sequence of events is illustrated in figure 1.

DATE 0   DATE 1   DATE 2

                  Period 1   Period 2

Initial   Revision ?   Repayment

Contract   Income y_1 Denial ?   Income y_2

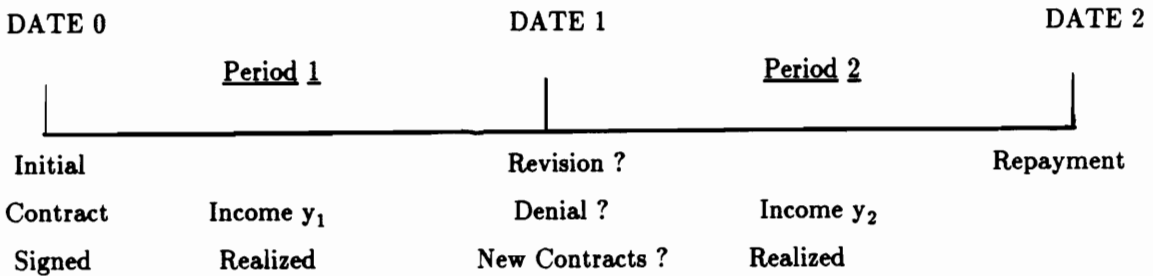Signed   Realized   New Contracts ? Realized

Figure 1

Let us now describe the game between the principal and the agents in full details. In the first stage of the game, at date '0', an initial contract $\alpha_1 = \{R_1, B_1, \gamma(y_f)\}$ is offered to all agents. Since the

principal does not know at this stage about the agent's type, the same contract α_1 is offered to all agents. If the contract is accepted, we move on to the second stage, or else the game stops here. Now we have passed date '0', and we are in period 1. Based on the realization of period one income, the agent chooses y_1' so as to maximize his utility, knowing that the choice of y_1' today will influence future availability of credits, and reports that value of income to the principal at the time of re-payment. The rule of the game is such that the agent pays $\min\{y_1', x_1\}$.

Now we are in the third stage of the game and at date 1. Based on the reported value of income the principal decides whether or not to monitor. If the monitoring takes place, and if it reveals the agent's type, then the principal has two options : (i) denial of future credits to those who are found cheating, and (ii) revision/renegotiation of contracts to those who displayed honest behavior. We say that (i) the agent defaulted, but acted honestly if $y_1' = y_1 < x$ holds, (ii) the agent dishonestly defaulted if $y_1' < x_1 < y_1$ holds, and (iii) the agent acted dishonestly upon default if $y_1' < y_1 < x_1$ holds. The principal punishes dishonest agents of both the cases of (ii) and (iii) by completely denying credits to them in future and not allowing any further revision of contracts. The rule of the game specifies that the agent must then pay y_1 irrespective of the particular cases.

If monitoring takes place but it does not reveal agent's type (honest or dishonest), a new contract may be offered and is either accepted or rejected. If monitoring does not take place (if the agent makes full payment at date 1), then a new contract is offered and is either accepted or refused. Therefore, the principal offers four different contracts at date 1 as follows :

α_{dd} = contract offered to the dishonest agents of of period 1. Since no contract is offered to them, α_{dd} is an empty set.

$\alpha_{dh} = \{R_{dh}, B_{dh}, R'_{dh}\}$ is the contract offered to the honest agents who defaulted in period 1, where R_{dh} is the new interest rate to be applied to the new loan size B_{dh} offered to them, and R'_{dh} is the interest rate applied on the unpaid previous loan $(y_1 - x_1)$. We need to find out whether R'_{dh} will be any different from R_{dh}.

$\alpha_{du} = \{R_{du}, B_{du}\}$ is the contract offered to the agents who defaulted in period 1, but whose true income was not verified.

$\alpha_{nd} = \{R_{nd}, B_{nd}\}$ is the contract offered to those agents who did not default in perion 1.

If a contract is offered and accepted, the agent is supposed to make the repayment at the end of the period 2, i.e. at date 2. In this model we have assumed that if monitoring takes place at all, it will occur only once - only if a default is observed. Of course we could model that monitoring would take place again if the agent defaults in period 2. However, it is not necessary, because, the information revealed about the type of the agent at the time the first repayment is made is used by the principal to form probabilistic beliefs about the behavior of the agents in future, which is then utilized to design the contracts for period 2. Now we present the problem of the principal in the following section.

3. The problem of the principal

We assume that the principal can acquire capital at no cost. He chooses contracts in each period to maximize his expected payoff. He must make sure that the agent accepts the contract in each period, in each state. The principal calculates his expected payoff by incorporating the agent's best response about y_i, $i = 1, 2$. The agents on the other hand decides on y_i, knowing that the choice of y_1 is to some extent responsible in deciding whether further contracts will be offered to agents. Therefore, the optimal choice of y_1 depends on the agent's beliefs about principal's behavior in future. Hence each contract induces probabilities of cheating and denial/offering of credits which are best responses to each other.

Let us first calculate the expected payoff of the principal, assuming that the symbol y_i' denotes the agent's **optimal** choice of the reported income (which is actually a function of the agent's (true) income and the terms of the contract. Now since y_i and y_i^r are not independent, (in fact we can express y_i as a an inverse function of y_i^r) let $f(y_i | y_i^r < x_i)$ represent the conditional p.d.f of y_i given that the event $y_i^r < x_i$ has occurred, and let $f(y_i | y_i^r \geq x_i)$ represent the conditional probability that the event $y_i^r \geq x_i$ has occurred. Denoting the principal's profit by π, the expected profit of the principal in period 1 is given by

$$\pi_1 = \int_0^{\bar{y}} [py_1 + (1-p)y_1^r - \bar{\gamma}] f(y_1 | y_1^r < x_1) dy_1 + \int_{x_1}^{\bar{y}} x_1 f(y_1 | y_1^r \geq x_1) dy_1$$

In period 2, the principal effectively offers three different contracts, α_{dh}, α_{du}, α_{nd}, because the contract α_{dd} is empty. Let us calculate the expected profits in each of these cases by defining the following probabilities :

τ_i = unconditional probability that the agent is honest in period 1. Therefore

$$\tau_i = Prob(y_i = y_i^r); \quad i = 1, 2$$

τ_{12} = probability that the agent who displayed honest behavior in period 1 will also display honest behavior in period 2. Therefore,

$$\tau_{12} = Prob(y_2^r = y_2 | y_1^r = y_1)$$

Denote $R_{dj}B_{dj} = x_{dj}$, $j = h, u$ and $R_{nd}B_{nd} = x_{nd}$. Therefore, the principal's expected profit from offering the contract α_{dh} is given by

$$\pi(\alpha_{dh}) = \tau_{12} \left\{ \int_0^{R'_{dh}(y_1 - x_1) + x_{dh}} y_2 f(y_2) dy_2 + \int_{R'_{dh}(y_1 - x_1) + x_{dh}}^{\bar{y}} \{ R'_{dh}(y_1 - x_1) + x_{dh} \} f(y_2) dy_2 \right.$$

$$+ \quad (1 - \tau_{12}) \int_0^{\bar{y}} y_2^r f(y_2) dy_2$$

The principal's expected profit from offering the contract α_{du} is given by

$$\pi(\alpha_{du}) = \tau_1 \left[\tau_{12} \left\{ \int_0^{x_{du}} y_2 f(y_2) dy_2 + (1 - \tau_{12}) \int_0^{\bar{y}} y_2^r f(y_2) dy_2 + (1 - \tau_1) [\tau_{21} \int_0^{\bar{y}} y_2^r f(y_2) dy_2 + \right. \right.$$

$$(1 - \tau_{21}) \{ \int_0^{x_{du}} y_2 f(y_2) dy_2 + \int_{x_{du}}^{\bar{y}} x_{du} f(y_2) dy_2 \}$$

where $\tau_{21} = Prob(y_2^s \neq y_2 | y_1^s \neq y_1)$. If the agent has repaid the first period loan, the principal as-sumes him to an honest agent, and computes the expected profit from offering contract α_{nd} as

$$\pi(\alpha_{nd}) = \tau_{12} \left\{ \int_0^{x_{nd}} y_2 f(y_2) dy_2 + \int_{x_{nd}}^{\bar{y}} x_{nd} f(y_2) dy_2 \right\} + (1 - \tau_{12}) \int_0^{\bar{y}} y_2^r f(y_2) dy_2$$

Let δ be the rate at which the future profit is discounted. Hence the total expected profit of the principal is given by

$$\pi = \pi_1 + \delta [\pi(\alpha_{dh}) + \pi(\alpha_{du}) + \pi(\alpha_{nd}]$$

The principal therefore maximizes the above expected profit subject to individual rationality con-ditions for the agent.

4 Conclusion and plans for possible future extensions

In order to solve the above problem, we need to first find out the optimal reporting rule of the agent, knowing that the agent does not know for sure whether the contract would be renegotiated or not. The reporting rule thus obtained can be used in the expression for the expected profit to find out explicitly whether different contracts would be offered or not. The anticipated results are as follows : (i) honest agents who defaulted will not be denied credits, although it is difficult to say a-priori whether R_{dh} would be different from R'_{dh}, we need to solve the problem explicitly for that. (ii) agents who defaulted but whose first-period income was not verified, may be denied credit. In this case, there would be a negative externality exerted by dishonest agents on honest agents, incomes for both of whom were not revealed. (iii) the agents who made full payment of loans in period one are most likely to get future loans; moreover they are also very likely to get the most favorable interest rate. Idea (iii) merely expresses the observed fact that agents who have good credit history are not usually credit-rationed. Lastly we are interested in finding out what kind of rationing, if any, would characterize the equilibrium outcome. To make a definite conclusion, however, furter research is necessary.

Chapter IV

Borrowing Constraints, Human Capital

Accumulation, and the Life-Cycle

1. Introduction

Most of the theoretical and empirical literature on the life-time utility maximization problem of an individual deals with life-cycle decisions about consumption, labor supply, and portfolio allocation, separately. These papers emphasize the role of non-human wealth as a major determinant of life-cycle decisions but largely ignore the importance of human wealth in such a decision process. But it is the total wealth - the sum of human and non-human wealth - which should appear as a major factor in life-cycle decision making processes. This issue, although long recognized in the existing literature, has not been addressed explicitly, particularly in the empirical literature. Perhaps the

reason for this omission is the dearth of data on human wealth (also referred to as human capital) - human wealth after all is a theoretical concept and therefore is hard to measure [2]. Most of the research undertaken in the empirical literature is engaged in estimating the preference parameters of the individual without incorporating the role played by the human wealth. However, because the stock of human wealth largely determines an individual's stock of non-human wealth, estimates of the preference parameters which omit wealth may be biased.

The literature on liquidity constraints (also referred to as the borrowing constraint and often cited as a reason for observed excess sensitivity of consumption to current income) expresses another view point. These papers consider restrictions that the individual faces with respect to his non-human wealth holdings and analyse individual's life-cycle consumption and other decisions conditional on these restrictions. Once we recognize the dependence of one form of wealth on the other, failure to incorporate the role of human wealth while analysing the impact of liquidity constraint on person's life-cycle decisions, may leave the analysis incomplete.

In this paper we extend these literatures by explicitly incorporating the individual's decision to invest in human capital into his life-time optimization problem. We refer to the literature on human capital accumulation in order to provide an explicit link between human capital investment and non-human wealth holdings. Research in the area of human capital accumulation accounts for the endogeneity of the wage rate, by noting that current hours of work indirectly increase future wage rates by entering the production function as human capital investment. Thus many of these papers investigate the interaction between labor supply and human capital investment decisions (see, for example, Heckman (1976), Haley (1976), Rosen (1976), Brown (1976), and Shaw (1989)).

In this paper we do not explicitly model the individual's labor supply decision. Instead we assume that the (labor) income earned by an individual is a function of his stock of human wealth - referred to as the individual's ability in this paper. Investment in human capital is likely to increase the

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[2]   See Ioannides (1989) for an elaboration of this argument

person's ability, thereby increasing the probability of earning higher income in future periods. Thus the decision to invest in human capital is, to a large extent, motivated by a desire to earn higher income in the future - this way the future income becomes endogeneous in our model. Because higher income also increases the stock of non-human wealth, a model which incorporates human capital investment decisions into the individual's life-time optimization problem provides a superior setting for testing the impact of liquidity constraints.

The model we formulate is a two-period version of the life-cycle model. In the first period the individual consumes, may invest in human capital, and allocates savings into risky and safe assets. In the second period, he consumes, allocates his portfolio and leaves a bequest for the next generation. The model assumes that the agent is uncertain about the future realization of the exogeneous variables and that the second-period wage income is determined endogeneously by the amount of human capital investment. Since households may be constrained at some point in their lifetimes but unconstrained at others, we propose a switching-regression version of the theoretical model presented in section II. We show that the probability that the household moves to a different regime depends heavily upon the decision made about human capital investment - in this way switching is endogenized in this paper. Thus our empirical specification differ from most papers on liquidity constraints in that, like Ioannides and Hajivassiliou (1990), we deal with the possibilty of regime switching. Because, inclusion of human capital investment immediately changes a household's asset holdings, the assumption of unchanged regimes is not justified. We closely follow Ioannides and Hajivassiliou (1990) in developing the econometric specification of the switching model.

We utilize 1983 and 1986 data from the Survey of Consumer Finances which contains detailed descriptions about households' socio-economic characteristics, financial services and attitudes as well as extensive details on the households' asset holdings and liabilities.

The rest of the paper is organized as follows. Section II describes the theoretical model, section III gives the econometric specification. Section IV describes the data and contains some empirical results. Section V presents some concluding remarks.

2. The Model

Consider an individual who lives in an economy for three periods and makes consumption and investment decisions in the following way - after the wage income is accrued at the beginning of each period, the individual decides how much to consume and save. The wage income earned is a function of the individual's ability which can also be called the individual's marketable skills. The utility each period depends only on consumption and is represented by u(c) ; we do not consider the individual's labor supply decision here. In order to justify the role of inheritance or the endowments that an individual might have at the beginning of his working life, we consider period one as the period when the individual does not make any life-cycle decisions but builds up marketable skills and/or wealth for period two. Period one is therefore captures that portion of a person's life when the person acquires education rather than working full time.

The individual starts working from period 2 and receives wage income at the beginning of the period. After receiving wage income, he makes consumption and human capital investment decisions (so as to increase his marketable skills) out of his total income, which is the sum of wage income and financial wealth. After the decisions about consumption and human capital investments have been made, the individual decides how to allocate his savings (the residual income) among safe and risky asset holdings.

Period three is the last period in a person's life, hence the individual no longer invests in human capital. However, he still decides how to allocate his savings (after the consumption decision has been made) and leaves the financial wealth accumulated at the end of period three, as a bequest. Let us introduce the following notation -

c_t - consumption in period t, $t = 2,3$.

k_t - human capital investment in period t, $t = 2$ only since k_3 is zero.

y_t - wage income accruing at the beginning of period t, $t = 2,3$.

w_{t+1} - amount of financial wealth accumulated at the end of period t, $t = 1,2,3$. For simplicity we assume that the end-of-period wealth is same as the beginning-of-next-period wealth. Hence, w_{t+1} is also the wealth available at the beginning of period $t+1$.

I_t - period t's full income, that is $I_t = y_t + w_t$, $t = 2,3$.

α_t - fraction of residual income (after deciding on c_t and k_t) allocated to the risky asset. Therefore, $(1 - \alpha_t)$ is the fraction allocated to the safe asset, $t = 2,3$.

We assume that the safe asset earns a gross interest rate r, which is constant over time and the risky asset earns an interest rate (gross) i_t each period, which is distributed with mean μ and variance σ_i^2 over time, but which is same across individuals. The interest income accruing to the assets is obtained at the end of each period, after the portfolio allocation decision has been made. Under this set up, the following equations describe the the dynamic budget constraints of the individual -

For period two, the budget constraint is

$$w_3 = \{\alpha_2(i_2 - r) + r\}[I_2 - c_2 - k_2]$$

For period three, the budget constraint is

$$w_4 = \{\alpha_3(i_3 - r) + r\}[I_3 - c_3]$$

The return to human-capital investment is reflected in the income earned in the following period. We assume that wage income earned by a person is a function of his ability and that investment in human capital increases a person's ability. Therefore a person making a human capital investment

this period should be able to earn higher income in the next period. Let A_t denote a person's ability in period t. Then the relationship between a person's ability and income earned in a period t is represented by the following equation :-

$$y_t = m_t A_t \quad for \ t = 2, \ 3$$

where for simplicity, we have assumed that income earned and ability are linearly related. We can also think of m_t as the market rental rate for ability. The individual knows m_t in period t, since income is accrued at the beginning of period t, but he does not know the future values of m. Therefore, the expectations about future values of m are formed based on the past realizations of 'm's. Since different individuals will realize different incomes depending on their abilities, we assume that m_t is distributed with mean $\overline{m}$ and variance σ_m^2 which are constant over time but differ among individuals (omitting the index for the individual for the time being in the mean and variance notation).

The relationship between ability and human capital investment can be represented as follows :-

$$A_3 = f(k_2, A_2), \quad f_k > 0, \quad f_A > 0, \quad f_{kk} < 0, \quad f_{AA} = 0$$

The above formulation means that ability next period is a function of the extent of investment (in human capital) undertaken as well as the ability that the person already has. The function is concave in k_2 but is linear in ability, implying that the person always retains at least some part of his ability. For econometric purposes, we will actually work with the following functional form -

$$f(k_2, \ A_2) = \beta_1 k_2 + \beta_2 k_2^2 + \beta_3 A_2$$

In addition to the budget constraints, the individual also faces an exogeneously imposed borrowing constraint - the end-of-period wealth cannot be negative. The borrowing constraint can be represented in the following form-

$$w_{t+1} \geq 0, \quad t = 3, 4$$

Because the end-of-period wealth is the same as beginning- of-next-period wealth, the above constraint can be equivalently expressed as

$$[I_t - c_t - k_t] \geq 0, \quad t = 2, 3$$

The agent's decision process can now be formulated as the maximization of the value function in each period. Let V_3 denote the value function (maximum value of remaining life-time utility) for period 3 and let V_2 denote the value function for period 2. Assume for simplicity that the discount factor for future utility is equal to 1. Then the final period's maximization problem of the agent is given by

$$V_3(I_3) = \max_{c_3, \alpha_3} \{ u(c_3) + E_3(B(w_4)) \}$$

subject to

$$w_4 = [\alpha_3(i_3 - r) + r][I_3 - c_3] \tag{1}$$

and

$$I_3 \geq c_3 \tag{2}$$

where $B(w_4)$ is the concave bequest function and the utility is assumed to be additively separable between consumption and bequest. The expectation is taken with respect to i_3 since it becomes known only at the end of the period. Similarly, the two-period maximization problem is formulated as

$$V_2(I_2) = \max_{c_2, \alpha_2, k_2} \{ u(c_2) + E_2[V_3(I_3)] \}$$

subject to

$$w_3 = [\alpha_2(i_2 - r) + r][I_2 - c_2 - k_2] \tag{3}$$

$$y_3 = m_3 f(k_2, A_2) \tag{4}$$

$$I_2 \geq c_2 + k_2 \tag{5}$$

$$k_2 \geq 0 \tag{6}$$

Let $\lambda_w(3)$, $\lambda_w(2)$, and λ_k be the lagrange multipliers associated with the constraints (2), (5) and (6) respectively. Observe that when $\lambda_w(3) > 0$, then the borrowing constraint is binding in period 3, and $I_3 = c_3$ holds, implying that $w_4 = 0$.

When $\lambda_w(2) > 0$, the borrowing constraint is binding in period 2, implying that $I_2 = c_2 + k_2$ holds. In that case $I_3 = y_3$ since w_3 is zero. The first order conditions for the last period problem (assuming an interior solution) are as follows -

when $\lambda_w(3) = 0$

c_3: $u'(c_3) = E_3\{B'(w_4)[\alpha_3(i_3 - r) + r]\}$

α_3: $E_3\{B'(w_4)(i_3 - r)(I_3 - c_3)\} = 0$

For analytical convenience, we will work with following quadratic form of the utility and bequest functions :- $u(c) = c - c^2$ and $B(w_4) = w_4 - w_4^2$ where some restrictions on the parameters of the utility and bequest functions apply. From the f.o.c. s we obtain the following optimal values.

$$c_3^* = \frac{(\mu - r)^2 + \sigma_i^2(1 - r - 2I_3 r^2)}{2[(\mu - r)^2 + \sigma_i^2(1 + r^2)]}$$

$$\alpha_3^* = \frac{(r - \mu)(1 + r - 2rI_3)}{(1 - 2I_3)(\mu - r)^2 + \sigma_i^2(1 - r - 2I_3)}$$

The value function V_3, when $\lambda_w(3) = 0$ is given by $V_3 = u(c_3^*) + E_3(B(w_4^*))$.

When $\lambda_w(3) > 0$, the f.o.c. is given by

$$u'(c_3) = \lambda_w(3)$$

In this case, the optimal values are -

$$c_3^* = I_3, \quad \alpha_3^* = 0, \quad V_3 = u(I_3).$$

The value of lagrange multiplier here is $\lambda_w(3) = 1 - 2I_3$, obtained directly from the first-order conditions. Note that the value of the lagrange multiplier is directly affected by the decision variables of period 2. Variables k_2, c_2, α_2 of period 2 affect y_3 and w_3 and therefore I_3 as well. Therefore the higher the value of I_3, the lower would be the value of the multiplier and those borrowers who initially were initially borrowing constrained may eventually become unconstrained. In order to find out how the individual will make decisions, given that he has all the relevant informations, we need to look at the f.o.c. for the two-period problem. The equations are the following (assuming interior solutions for c_2 and α_2) -

For $\lambda_w(2) = 0$,

$$c_2: \qquad u'(c_2) = E_2 \left\{ \frac{\partial V_3}{\partial I_3} \frac{\partial I_3}{\partial w_3} [\alpha_2(i_2 - r) + r] \right\} \qquad (7)$$

$$\alpha_2: \qquad E_2 \left\{ \frac{\partial V_3}{\partial I_3} \frac{\partial I_3}{\partial w_3} (i_2 - r) [I_2 - c_2 - k_2] \right\} = 0 \qquad (8)$$

$$k_2: \qquad E_2 \left\{ \frac{\partial V_3}{\partial I_3} \frac{\partial I_3}{\partial w_3} [\alpha_2(i_2 - r) + r] \right\} = E_2 \left\{ \frac{\partial V_3}{\partial I_3} \frac{\partial I_3}{\partial y_3} m_3 f_k \right\} + \lambda_k \qquad (9)$$

In the above equations the value function V_3 will take different forms depending on whether the borrowing constraint is binding in period 3 or not. The f.o.c. s can be combined to yield the following equation -

$$E_2 \left\{ \frac{\partial V_3}{\partial I_3} \frac{\partial I_3}{\partial y_3} m_3 f_k \right\} + \lambda_k = E_2 \left\{ \frac{\partial V_3}{\partial I_3} \frac{\partial I_3}{\partial w_3} i_2 \right\} \qquad (10)$$

When the borrowing constraint is binding in period 2, then $I_2 = c_2 + k_2$ and $\alpha_2^{\cdot} = 0$ hold and the f.o.c. s have the following forms -

$$u'(c_2) = \lambda_w(2)$$

$$E_2\{\frac{\partial V_3}{\partial I_3} \frac{\partial I_3}{\partial y_3} m_3 f_k\} + \lambda_k = \lambda_w(2)$$

The value of lagrange multiplier for period 2 is therefore given by $\lambda_w(2) = 1 - 2(I_2 - k_2)$. We thus have the following four cases to analyse :

Case (i) - when $\lambda_w(2) > 0$, and $\lambda_w(3) > 0$. In this case, the f.o.c. conditions for the maximization problem take the form -

$$\lambda_w(2) = E_2\{\frac{\partial V_3}{\partial I_3} \frac{\partial I_3}{\partial y_3} m_3 f_k\} + \lambda_k \qquad (11)$$

or,

$$\lambda_w(2) = E_2\{(1 - 2y_3)m_3 f_k\} + \lambda_k$$

which can be simplified as

$$\lambda_w(2) = \overline{m} f_k - 2(\sigma_m^2 + \overline{m}^2) f f_k + \lambda_k$$

or

$$1 - 2(I_2 - k_2) = \overline{m} f_k - 2(\sigma_m^2 + \overline{m}^2) f f_k + \lambda_k \qquad (12)$$

Case (ii) - when $\lambda_w(2) > 0$ but $\lambda_w(3) = 0$. In this case the f.o.c. is same as in equation (11), but the value function for period three has the form given by $V_3 = u(c_3^{\cdot}) + E_3\{B(w_4^{\cdot})\}$. Now,

$$\frac{\partial V_3}{\partial I_3} = u'(c^{\cdot}) \frac{\partial c_3^{\cdot}}{\partial I_3} + E_3\{B'(w_4^{\cdot})[(I_3 - c_3^{\cdot})(i_3 - r)\frac{\partial \alpha_3^{\cdot}}{\partial I_3} + \{\alpha_3^{\cdot}(i^3 - r) + r\}(1 - \frac{\partial c_3^{\cdot}}{\partial I_3})]\}$$

or,

$$\frac{\partial V_3}{\partial I_3} = [1 - 2c_3^* - r + 2rE_3(w_4^*)]\frac{\partial c_3^*}{\partial I_3} + E_3\{(1 - 2w_4^*)(i_3 - r)[(I_3 - c_3^*)\frac{\partial \alpha_3^*}{\partial I_3} - \alpha_3^*\frac{\partial c_3^*}{\partial I_3}]\}$$

$$+ E_3\{(1 - 2w_4^*)[\alpha_3^*(i_3 - r) + r]$$

After taking the expectation over i_3 and substituting the optimal values of c_3^* and α_3^* we have the following [3] -

$$\frac{\partial V_3}{\partial I_3} = r - \frac{(\mu - r)^2(1 + r - 2rI_3)[(\mu - r)^2 - \sigma_i^2(1 - r - 2I_3)r]}{[(1 - - 2I_3)(\mu - r)^2 + \sigma_i^2(1 - r - 2I_3)][(\mu - r)^2 + \sigma_i^2(1 + r^2)]}$$

$$\simeq r - \frac{\sigma_i^2(1 + r - 2rI_3)[1 - (1 - r - 2I_3)r]\sigma_i^2}{\sigma_i^2(2 - r - 4I_3)(2 + r^2)\sigma_i^2}$$

$$\therefore \quad \frac{\partial V_3}{\partial I_3} \simeq \frac{2r}{2 + r^2} + \frac{r^3(1 - r - 2I_3)}{(2 - r - 4I_3)(2 + r^2)} \tag{13}$$

Therefore, the f.o.c. in this case can be written as

$$1 - 2(I_2 - k_2) = E_2\{[\frac{2r}{(2 + r^2)} + r^3\frac{(1 - r - 2I_3)}{(2 - r - 4I_3)(2 + r^2)}\; m_3 f_k\} \tag{14}$$

Case (iii) - when $\lambda_w(2) = 0$ but $\lambda_w(3) > 0$. In this case, the first-rder condition takes the following form :-

$$E_2\{\frac{\partial V_3}{\partial I_3}\frac{\partial I_3}{\partial w_3}(i_2 - m_3 f_k)\} = \lambda_k$$

where $V_3 = I_3 - I_3^2$. After some algebraic manipulation we obtain the following form of the equation -

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[3]   see appendix for derivation.

$$(\mu - \overline{m}f_k) - 2\{(\mu - \overline{m}f_k)f\overline{m} - ff_k \sigma_m^2\} \tag{15}$$

$$+ \text{NW} \, \alpha_2 \, \sigma_i^2 + 2\text{NW}\{\alpha_2(\mu - r) + r\}(\mu - \overline{m}f_k) = \lambda_k \tag{15}$$

where 'NW' refers to the net wealth the individual has after deciding on consumption and human capital investment. Note that $\text{NW}\{\alpha_2(\mu - r) + r\}$ denotes the average return on the individual's savings, which we will henceforth refer to as R.

case (iv) - this is the case when both $\lambda_w(2)$ and $\lambda_w(3)$ are zero. In this case, the relevant equation is equation (10) and the form of $\dfrac{\partial V_3}{\partial I_3}$ is given by equation (13). Therefore the equation can be written as

$$E_2 \left\{ \left[\frac{2r}{(2+r^2)} + \frac{r^3(1 - r - 2I_3)}{(2 - r - 4I_3)(2 + r^2)} \right] (i_2 - m_3 f_k) \right\} = \lambda_k \tag{16}$$

For estimation purposes some simplifications of equations (14) and (16) are necessary. Observe, that in equation (14), the period 3 full income (I_3) consists only of wage income obtained in period 3. That is because the binding borrowing constraint in period 2 makes end-of-period-2 financial wealth (w_3) equal zero. But in equation (16) we still have $I_3 = y_3 + w_3$. In order to simplify the equations, we assume that the interest rate on risky assets i and market rental rate of ability 'm' are uncorrelated. After expanding the series $(4I_3 + r - 2)^{-1}$ up to the second term and then using the method of statistical differentials, we obtain the following two versions of equation (14) and equation (16) [4]

$$1 - 2(I_2 - k_2) = \frac{f_k}{2 + r^2} \left[r^2(1 + r)\overline{m} + 2rf(4 - r)(\sigma_m^2 + \overline{m}^2) - 8rfE_2(m_3^3) \right] + \lambda_k \tag{14'}$$

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[4] Using the method of statistical differentials, we can approximate the function $Y = f(X)$ by Taylor's series expansion as follows

$$Y \simeq f(E[X]) + (X - E[X])f'(E[X]) + \frac{1}{2!}(X - E[X])^2 f''(E[X]) + \ldots\ldots\ldots$$

and

$$\frac{1}{2+r^2} \left[r^2(1+r)(\mu - \overline{m} f_k) + 8r\{NW\alpha_2 \sigma_i^2 - \sigma_m^2 f f_k\} (1 - \overline{m}f - R) \right.$$

$$\left. - 2r^2\{(\overline{m}f + R)(\mu - \overline{m} f_k) - \sigma_m^2 f f_k + NW\alpha_2 \sigma^2\} \right] = \lambda_k \qquad (16)'$$

where 'NW' represents the term $[I_2 - c_2 - k_2]$ and R denotes the term $NW[\alpha_2(\mu - r) + r]$. Equations (12), (14)′, (15) and (16)′ will form the basis of estimation.

3. Plans for future econometric research

In order to analyse the impact of liquidity constraints in each period of time, we need to introduce a switching regression version of the model specified above. Let S(t) denote a binary regime indicator which takes on values 0 or 1, depending on whether the liquidity constraint is binding in period t or not, t = 2, 3. Specifically,

$S(2) = 0 \quad if \ \lambda_w(2) > 0$
$\quad\;\; = 1 \quad if \ \lambda_w(2) = 0$

and

$S(3) = 0 \quad if \ \lambda_w(3) > 0$
$\quad\;\; = 1 \quad if \ \lambda_w(3) = 0$

Our interest is best served if we assume that the switching follows a first-order Markov process with the following transition probability matrix -

$$P = \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix}$$

where P_{jk} = probability that the individual i is in regime k, k = 0,1, in period 3, given that he was in regime j, j = 0,1, in period 2 and the matrix has the usual properties of a Markov matrix namely (i) every element of P is nonnegative , (ii) the sum of each row is unity. Note that in our model P_{jk} is not necessarily the same for all i and therefore the Markov model we have here is not homogeneous.

Given the above setup, we now discuss the maximum-likelihood estimation framework of the model developed in section II. We assume that the investigator has a priori information on how the sample is partitioned into the underlying regimes, i.e. we assume that it is a switching model with known sample separation. In particular, we assume that the econometrician can perfectly infer whether the constraint is binding or not based on whether his asset income is positive or not. Hence suppressing the index for households, we can write the likelihood function for each individual i as,

$$L = P_{11}^{S(2)S(3)} \, P_{10}^{S(2)(1-S(3))} \times P_{01}^{(1-S(2))S(3)} \, P_{00}^{(1-S(2))(1-S(3))}$$

In general, when we have more than 2 periods the above likelihood function would be conditional on the initial state S(1) where S(1) denotes the regime in period 1. Here we have only two periods and we know that either S(2) = 0 or S(2) = 1 (since period 2 is the first period in this model). Hence calculation of the likelihood function does not pose any computational problems at all since we have only 4 cases to deal with. Next we discuss the parameterization of the the transition probabilities P_{jk} s. Referring to the model developed in section II we find that - (i) with probability P_{00} the relevant equation is the equation (12) , (ii) with probability P_{01} the relevant equation is equation (14)', (iii) with probability P_{10} the relevant equation is (15) and (iv) with probability P_{11} the equation is (16)'. The above set of equations will have different forms depending on whether k_2 is positive ($\lambda_k > 0$) or not ($\lambda_k = 0$).

In this model the (labor) income y is the dependent variable (this follows directly from the endogeneity of the labor income) and expenditures on human capital, individual's holdings of safe and risky assets and all other household characteristics are the independent variables. In general, if the dependent variable is unobservable, we can formulate the model as a limited dependent variable model and estimate it by defining P_{jk} as $P^i_{jk} = F(\beta' X_i)$ where F is some distribution function and X is the vector of exogeneous variables. But in this model we do observe y_t, t = 2,3, for different cases when the borrowing constraint binds or not. Not only that, we also observe different values of y for the cases when k_2 is positive or zero. Because the variable k_2 is actually a decision variable of the individual, we need to treat our model essentially as a self-selection model. If the variable k_2 is important for the endogeneity of income and for altering the holdings of non-human wealth, then exclusion of k_2 from the model, would reduce the fit of the model and or give implausible values for parameter estimates.

In order to carry out estimation, we first need to obtain the equations (12), (14)', (15) and (16)' in their reduced forms. To derive the reduced forms, we first rewrite the equations, substituting the human capital production function f with $\frac{y_3}{m_3}$ using the relation $y_3 = m_3 f(2, A_2)$ and treating it as a dependent variable for the estimation. There would actually be two groups of people - those who invest in human capital and those who do not. The set of equations relevant for the people who invest in human capital (i.e. people with $k_2 > 0$) is given by (for each individual i) equations (12), (14)', (15) and (16)'.

$$\frac{y_3}{m_3} = \frac{\overline{m}\beta_1 - 1}{2(\sigma_m^2 + \overline{m}^2)} + \frac{1}{(\sigma_m^2 + \overline{m}^2)} I_2 + \frac{(\overline{m}\beta_2 - 1)}{(\sigma_m^2 + \overline{m}^2)}\beta_1 k_2 - 2\frac{\beta_2}{\beta_1}k_2\frac{y_3}{m_3} \qquad (12)'$$

$$\frac{y_3}{m_3} = \frac{(2 + r^2 - \beta_1 r^2(1 + r)\overline{m}}{2\beta_1 r a} - \frac{2 + r^2}{\beta_1 r a}I_2 + \frac{2 + r^2 - \beta_2 r^2(1 + r)\overline{m}}{\beta_1 r a}k_2$$

$$- 2\frac{\beta_2}{\beta_1}k_2\frac{y_3}{m_3} \qquad (14)''$$

where $a = [(4 - r)(\sigma_m^2 + \overline{m}^2) - 4E_2(m_2^2)]$

$$\frac{y_3}{m_3} = \frac{(\mu - \overline{m}\beta_1)}{[\,\cdot\,]} - \frac{\overline{m}\beta_2}{[\,\cdot\,]} k_2 + \frac{[\sigma_i^2 + 2\mu(\mu - \overline{m}\beta_1)]}{2[\,\cdot\,]} RNW + \frac{r(\mu - \overline{m}\beta_1)}{[\,\cdot\,]} SNW -$$

$$\frac{2\mu\overline{m}\beta_2}{[\,\cdot\,]} RNW \, k_2 - \frac{2r\overline{m}\beta_2}{[\,\cdot\,]} SNW \, k_2 \tag{15$'$}$$

where $RNW = \alpha_2 NW$ is the amount of wealth put in the risky asset and $SNW = (1 - \alpha_2)NW$ is the amount of wealth put in the safe asset and $[\,\cdot\,] = [\mu\overline{m} - (\overline{m}^2 + \sigma_m^2)\beta_1]$.

The form of equation (16)$'$ is really complicated, hence we write it in the following form -

$$[\gamma_1 + \gamma_2 SNW + \gamma_3 k_2 + \gamma_4 (SNW)k_2 + \gamma_5 RNW] \frac{y_3}{m_3} = \gamma_0 + \gamma_6 k_2 + \gamma_7 (RNW)^2 +$$

$$\gamma_8 (RNW)(SNW) + \gamma_9 RNW + \gamma_{10} SNW + \gamma_{11}(RNW) k_2 + \gamma_{12}(SNW) k_2 \tag{16$'$}$$

where the coefficients will be the functions of the parameters defined earlier.

We assume that the individuals have rational expectations and that for individual i

$$E_t(m_{t+1}) = m_t, \quad \text{i.e.} \quad m_{i\,t+1} = m_{it} + \eta_i$$

where η_i is an error term with mean zero and variance σ_η^2. Hence the system of equations used for the purpose of estimation consists of equations (17) through (20) and can be written as follows :-

$$\frac{y_3}{m_3} = \Pi_{11} + \Pi_{12} k_2 + \Pi_{13} l_2 + \Pi_{14} l_2 k_2 + \Pi_{15} k_2^2 + \varepsilon_1 \tag{17}$$

appearing with probability P_{00}.

$$\frac{y_3}{m_3} = \Pi_{21} + \Pi_{22} k_2 + \Pi_{23} l_2 + \Pi_{24} l_2 k_2 + \Pi_{25} k_2^2 + \varepsilon_2 \tag{18}$$

appearing with probability P_{01}

$$\frac{y_3}{m_3} = \Pi_{31} + \Pi_{32}k_2 + \Pi_{33}\,RNW + \Pi_{34}\,SNW + \Pi_{35}\,(RNW)k_2 + \Pi_{36}\,(SNW)k_2 + \varepsilon_3 \quad (19)$$

appearing with probability P_{10}.

$$Y = B\,X' + \varepsilon_4 \qquad (20)$$

where B and X are the vectors defined as follows -

$$B = (\Pi_{4,j})_{j=1,\ldots,17}$$

with $\Pi_{4,j}$ being the coefficients of the right hand side variables in equation (20)

$$[\,X\,]'_{17 \times 1} = [1,\ X_j,\ X_j^2,\ X_j^3,\ X_i X_j,\ X_i X_j^2,\ X_i X_j^3,\ ]\ j = 1,2,3,\ i = 1,2,3,\ \ i \neq j$$

where $X_1 = RNW,\ X_2 = SNW,\ X_3 = k_2,\ Y = \frac{y_3}{m_3}$. 2 . The $\Pi_{ij}\ i = 1,2,3,$ and $j = 1,\ldots\ldots,6$ are the coefficients appearing in equations (17) through (19) and ε_i s are the error terms. The coefficients in the reduced form equations are derived from equations (12)′ through (16)′ and are not exactly the same as the coefficients appearing in those equations.

The set of equations relevant for the people who do not invest in human capital is given by -

$$\frac{y_3}{m_3} = \tilde{\Pi}_{11} + \tilde{\Pi}_{12}\,I_2 + \tilde{\varepsilon}_1 + \lambda_k \qquad (17)'$$

appearing with probability P_{00}.

$$\frac{y_3}{m_3} = \tilde{\Pi}_{21} + \tilde{\Pi}_{22}\,I_2 + \tilde{\varepsilon}_2 + \lambda_k \qquad (18)'$$

appearing with probability P_{01}.

$$\frac{y_3}{m_3} = \tilde{\Pi}_{31} + \tilde{\Pi}_{32}RNW + \tilde{\Pi}_{33}SNW + \tilde{\varepsilon}_3 + \lambda_k \qquad (19)'$$

appearing with probability P_{10}.

$$Y = \tilde{B}\, \tilde{X}' + \lambda_k + \tilde{\varepsilon}_4 \qquad\qquad\qquad (20)'$$

appearing with probability P_{11}, where the vectors $\tilde{B}$ and $\tilde{X}$ are same as in equation (20) except they do not contain k_2.

Note that although we may have data on y_3, the dependent variable is actually $\frac{y_3}{m_3}$ and m_3 is not in general observable. Therefore we must have an estimate of m_3 in order to compute the dependent variable. Now m_3 is actually the coefficient of the ability vector in period three. But because m_3 may itself be a function of ability A_3 in period three, estimating m_3 from the equation $y_3 = m_3 A_3 + \varepsilon_{3m}$ and using that estimated value to compute the dependent variable to be used in equations (17) through (20), would produce biased results. Therefore the best instrument for m_3 would be $\hat{m}_2$. Hence we should first estimate the equation $y_2 = m_2 A_2 + \varepsilon_{2m}$ and use those parameter values in the second stage of estimation. Because different individuals may have different realizations of income depending on individual characteristics therefore we should use the fixed-effect technique to estimate m_2. First we can do bivariate probit regressions corresponding to different regimes, second we can estimate the four equations stated out before, using the selection method twice - including and excluding k_2 respectively from regression equations.

Note that λ_k appears as an additive effect in the equations (17)' through (20)'. One implication of this would be that effect of λ_k may not be separable from that of error term. In that case the variance of the error term is likely to be high.

To carry out these estimations we need data on labor income, financial wealth and data on the expenditure on human capital. The 1983 and 1986 survey of consumer finances are micro data sets which contain detailed data on labor income and on financial wealth. The expenditure on human capital would normally include educational expenses, income foregone while being trained for the job, and income foregone while being in school. The data set does not have information about

on-the-job-training - it contains data only on the educational expenses. But we can compute the wage foregone while being a student and not working full time. We could use a dummy to indicate whether the individual is investing in human capital. In particular, if the individual has taken a loan for educational purposes (irrespective of whether he makes the loan payment on a regular basis or not) or if the individual is a student (irrespective of whether she works part-time or not at all), we assign the value 1 to the dummy for k_2, else the dummy takes the value 0. We can compute the wage foregone while being a student as follows - if the individual is a student in 1983 then we calculate the wage foregone in 1983 as the difference between income the he could have earned if he had worked full-time and the income he is currently earning (zero for full-time students). The 'income he could have earned' can easily be calculated given the individual's characteristics by using the equation $y_t = m_t A_t + \varepsilon_{tm}$.

We try two different measures of net worth defined to be the difference between total assets and total debt. One measure of total assets includes real assets plus the financial assets and the other includes only financial assets, because of the possibility that real assets like the household's principal residence, are not liquid and could not easily be borrowed against. Real assets include the household's principal residence, the gross value of all real estate holdings other than the principal residence, business assets and the gross value of vehicles owned by the household. Business assets include net household holdings of privately-held businesses, farms, and professional practices or partnerships. Financial assets include all kinds of paper assets. We include stocks, bonds and mutual funds in the category of risky assets and checking, saving, CDs etc. in the category of safe assets.

Next we can estimate $\hat{m}_2$, using the above equation, by regressing total household (labor) income against the household's demographic characteristics (age, sex, marital status, race), health, education level (high-school and and college education), type of the household's current employer, past employer and experience (in number of years) by defining several dummies for each of the above variables in order to indicate whether the individual is the head of the household (same as the respondent in the survey) or the spouse. In the preliminary regression, we define household's total labor income to be the sum of income of the head and the income of the spouse, where by labor

income we mean the income earned only from working (i.e. it does not include income earned from any other sources like interest income, alimony, etc.). If the household is a single unit, then total income is the same as the respondent's income. If the household is a family unit, then the total income earned is the sum of the husband and spouse's incomes if both are working, and it is same as the income of the working member if only one of them is working. We should use the dummy-variable method because although we have data on the total net worth (difference between total asset and total debt) of the household **unit**, we do not have data on total labor income of the household unit. Hence in order to make the two regressions compatible with each other, we need to use dummy-variable method for the estimation of m_2.

4. Concluding Comments

This paper investigates the effect of investment in human capital on the individual's holding of non-human wealth. We have constructed a dynamic programming model, where the effect of investment in human capital is reflected in the person's ability to earn higher income in the future. It is conjectured in the paper that if the individual cannot hold a negative amount of non-human wealth in each period of his life, then he might be tempted to make investment in human wealth, since the increased future income due to increased ability implies an increase in the holding of non-human wealth as well. It is shown in the paper that the probability with which the constraint binds depends heavily on the extent of investment undertaken. A switching-regression version of the theoretical model can be developed and can be estimated using the 1983 and 1986 surveys of consumer finances. We anticipate that this model would generally support the hypothesis that investment in human capital would eventually help the individual overcome the borrowing constraint in future periods of his life.

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