

A MARKOVIAN MANPOWER  
PLANNING, HUMAN RESOURCE VALUATION MODEL

by

JACKSON F. GILLESPIE

Dissertation submitted to the Graduate Faculty of the  
Virginia Polytechnic Institute and State University  
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

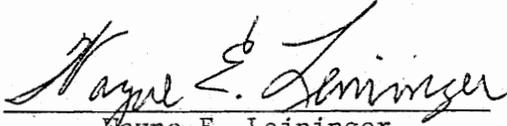
in

Business

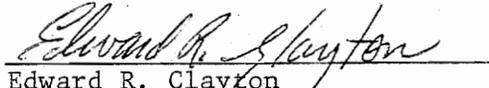
with a major in

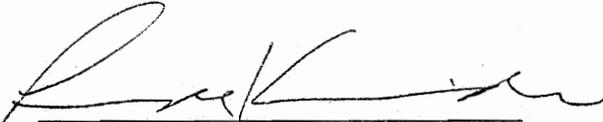
Accounting

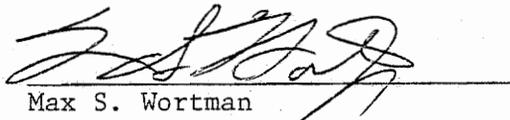
APPROVED:

  
Wayne E. Leininger  
Chairman

  
Floyd A. Beams

  
Edward R. Clayton

  
Larry N. Killough

  
Max S. Wortman

May, 1978

Blacksburg, Virginia

LD  
5655  
4856  
1978  
G54  
c.2

## ACKNOWLEDGMENTS

I would like to express my deepest gratitude to my family and friends for their understanding and moral support while I was working on this project. These people had confidence in me, and that was a great help, particularly during those times when the task seemed to be interminable.

I would also like to thank the members of my committee for their helpful comments and suggestions, particularly Dr. Floyd Beams and Dr. Larry Killough. I am grateful to Dr. Max Wortman and Dr. Edward Clayton for being on my final committee with such short notice.

There is one person in particular without whom this project would have never been completed. It was he who cajoled, coaxed, needed, berated, advised, and praised me when each was needed. He was both my committee chairman and my friend, and it is to Dr. Wayne E. Leininger that I would like to dedicate this dissertation.

TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS . . . . .	ii
LIST OF TABLES . . . . .	ix
LIST OF FIGURES . . . . .	x
Chapter	
I.    INTRODUCTION . . . . .	1
Manpower Planning--Human Resource Valuation . . . . .	1
The Model . . . . .	2
Outline of the Study . . . . .	3
II.   MANPOWER PLANNING . . . . .	5
Background . . . . .	6
Why Plan Manpower? . . . . .	7
Definition . . . . .	9
Stressing Integration . . . . .	12
Manpower Forecasting . . . . .	13
Judgmental Techniques . . . . .	15
Supervisor Estimates . . . . .	15
Rule-of-Thumb Techniques . . . . .	16
Replacement Charts . . . . .	16
Delphi Techniques . . . . .	17
Matrix Models . . . . .	18
Executive Planning Development Model . . . . .	18
Management Manpower Planning Matrix . . . . .	19

	Page
Computer Simulation . . . . .	19
MANPLAN . . . . .	20
Weber Model . . . . .	22
Quantitative Techniques . . . . .	23
Linear Programming . . . . .	24
Goal Programming . . . . .	24
Dynamic Programming . . . . .	25
Network Flows . . . . .	25
Markov Models . . . . .	25
Examples of Models . . . . .	26
Choosing a Model . . . . .	28
Accounting? . . . . .	30
Summary . . . . .	31
III. HUMAN RESOURCES ACCOUNTING . . . . .	33
Human Resources Accounting (HRA) . . . . .	34
Objections to HRA . . . . .	38
Cultural Objections . . . . .	38
Human Assets . . . . .	38
Measurement . . . . .	39
Objectivity . . . . .	39
Conservatism . . . . .	39
Visibility Bias . . . . .	40
Magnitude of Task . . . . .	40
Research Objective . . . . .	41
Human Resource Models . . . . .	42
Hermanson . . . . .	42
Lev and Schwartz . . . . .	44
Hekimian and Jones . . . . .	45
Flamholtz . . . . .	46
Jaggi and Lau . . . . .	47
Likert . . . . .	48
R. G. Barry Corporation . . . . .	51

	Page
Valuation Bases . . . . .	52
Acquisition (historical) Cost . . . . .	53
Replacement Cost . . . . .	53
Opportunity Cost . . . . .	54
Economic Value . . . . .	55
Behavioral Aspects of HRA . . . . .	55
Summary . . . . .	58
IV. THE CONCEPTUAL MODEL--I . . . . .	60
Foundation of the Model . . . . .	61
The Markov Property . . . . .	62
The Transition Matrix . . . . .	63
Classification of States . . . . .	64
Canonical Transition Matrix . . . . .	65
Testing the Markov Property . . . . .	66
The Basic Manpower Model . . . . .	69
States . . . . .	69
The Transition Matrix . . . . .	70
The Transition Period . . . . .	73
Extensions and Applications . . . . .	73
Rolling Over the P Matrix . . . . .	74
Initial Vectors . . . . .	74
The Fundamental Matrix . . . . .	77
Absorption Information . . . . .	79
Steady State Uses . . . . .	80
Time In State . . . . .	84
Chance of Particular Movements . . . . .	85
Number of State Changes . . . . .	86
Conditional Probabilities . . . . .	86
Summary . . . . .	87
V. THE CONCEPTUAL MODEL--II . . . . .	89
Economic Valuation . . . . .	89
Valuation Model . . . . .	91
Valuation Period . . . . .	93
Gains or Losses . . . . .	94

	Page
Historical Cost . . . . .	94
A Better Method? . . . . .	97
No Amortization Period . . . . .	98
Opportunity Cost . . . . .	100
Replacement Cost . . . . .	100
Conclusion . . . . .	101
VI. APPLICATION OF THE MODEL--I . . . . .	103
Background . . . . .	103
The Hierarchy . . . . .	103
Areas of Specialization . . . . .	104
Promotional Policy . . . . .	104
"Up-or-Out" . . . . .	104
The Model: Transition Matrix . . . . .	106
The Transition Period . . . . .	109
Testing the Transition Matrix . . . . .	110
Test for Markov Property . . . . .	110
Testing for Stationarity . . . . .	111
The Model: Manpower Planning . . . . .	112
The Transition Matrix . . . . .	112
The Fundamental Matrix . . . . .	113
Chances of Absorption . . . . .	114
Conditional Matrices . . . . .	114
Steady State . . . . .	115
The Model: Human Resource Valuation . . . . .	116
Economic Valuation . . . . .	116
Cost Bases . . . . .	118
Summary . . . . .	118
VII. APPLICATION OF THE MODEL--II . . . . .	120
Development and Tests . . . . .	120

	Page
The Data . . . . .	120
Developing the Transition Matrix . . . . .	125
Stability Test . . . . .	128
The Markov Property Test . . . . .	131
Predictive Ability . . . . .	137
Summary . . . . .	140
VIII. APPLICATION OF THE MODEL-III . . . . .	141
Computer Program . . . . .	141
Running the MARKOV Program--I . . . . .	143
The Fundamental Matrix . . . . .	144
The t Vector . . . . .	145
Absorption . . . . .	146
Consecutive Time in State . . . . .	147
The H Matrix . . . . .	148
Running the MARKOV Program--II . . . . .	149
Present Employees Absorbed . . . . .	150
Steady State . . . . .	150
Running the LEVELS Program . . . . .	151
Running the TESTLEVELS Program . . . . .	156
Running the CONDITIONAL Program . . . . .	156
Using the Model in the Firm . . . . .	160
Manpower Planning . . . . .	161
Hypothetical Questions . . . . .	166
Recruiting . . . . .	170
Human Resource Valuation . . . . .	171
Cost Basis . . . . .	171
The Flow Method . . . . .	176
Economic Valuation . . . . .	181
Summary . . . . .	185

	Page
IX. SUMMARY AND FURTHER RESEARCH . . . . .	187
Summary . . . . .	187
Further Research . . . . .	190
Markov Property Test . . . . .	190
Employee Levels . . . . .	193
Staff Entry . . . . .	193
Termination Reasons . . . . .	194
Specialists . . . . .	194
Minorities . . . . .	195
Geographic Regions . . . . .	195
Other Organizations . . . . .	196
Conclusions . . . . .	196
BIBLIOGRAPHY . . . . .	198
APPENDICES . . . . .	210
A Notation . . . . .	211
B Formulae . . . . .	216
C Data . . . . .	221
D Transition Matrices . . . . .	234
E Computer Programs . . . . .	245
F Computer Output . . . . .	259
VITA . . . . .	276
ABSTRACT	

LIST OF TABLES

Table		Page
1.	Professional Personnel (at January 31 of each year) . . . . .	121
2.	Hirings and Promotions (through January 31 of each year) . . . . .	122
3.	Personnel Flow--Year One . . . . .	124
4.	Personnel Flow--Ten Year Period . . . . .	126
5.	Stability Test . . . . .	130
6.	Simulated One-Period and Two-Period Movements . . . . .	135
7.	Markov Property Test . . . . .	136
8.	Actual vs. Program Results (1/31/76) . . . . .	138
9.	Actual vs. Program Results (1/31/77) . . . . .	138
10.	Actual vs. Program Results (1/31/77) . . . . .	139
11.	Steady State Levels . . . . .	152
12.	Yearly Employee Levels (50 years) . . . . .	154
13.	Yearly Employee Levels (2 years) . . . . .	158
14.	Six-Way Variance Analysis . . . . .	182

LIST OF FIGURES

Figure	Page
1. GERT Network . . . . .	133

## Chapter I

### INTRODUCTION

The intent of this dissertation is to formulate one comprehensive quantitative model which can be used in two areas: (1) manpower planning, and (2) human resource valuation. This introductory chapter provides a brief discussion on the combining of these topics into one model, as well as a general explanation of the model itself. A brief outline of the remainder of the study is also presented in this chapter.

#### Manpower Planning--Human Resource Valuation

"Manpower planning" has traditionally been a management topic. As the term implies, manpower planning is concerned with the future of the firm. For this dissertation, the most important aspect of manpower planning is the forecasting of the manpower supply that will be available to the firm at some point in time in the future. If there are different positions, or job levels, within the firm it would be important to know the availability of human resources at each of these different positions. Knowledge of the characteristics of both the firm and its internal job levels would help to determine availability of manpower for each position. Factors such as present manpower available at each level, hiring and promotion policies of the firm, and specific talents needed at each position could dictate future availability. After the supply of manpower has been forecast, comparisons can be made against

expected demand for manpower in order to detect problem areas. The management of the firm would then be able to make the appropriate decisions needed to avert the problem.

Human resource valuation has become of particular interest to accountants in the last 15 years. The main concern in human resource accounting is to somehow measure the value of the human resources employed by the firm with the intent of reporting this measurement on the financial statements. If the measurement is based on an economic valuation of the human resources, the future supply of manpower at each job level in the firm is important. If a cost base is used as a surrogate for economic value, the past and present personnel conditions of the firm are important.

Whether the topic is manpower planning or human resource valuation, it is to the advantage of the decision-maker to know as much as possible about the past, present, and future human resources of the firm. In the area of manpower planning, such knowledge is important for effective human resource management. In the area of human resource valuation, such knowledge is important for human resource measurement. It would appear that much of the same data are needed in both areas. If one comprehensive model could be formulated to supply a data bank for use in both manpower planning and human resource valuation, any duplication of effort could be circumvented.

### The Model

One quantitative model which has been applied to both manpower planning and human resource valuation is the Markovian model. Input

to the basic model includes historical data on all personnel movements, i.e., movements into the firm (hirings), movements within the firm (promotions, demotions, etc.), and movements out of the firm (termination for any reason). These historical data are summarized in the form of a transition matrix. This matrix is the foundation of the Markov model. The transition matrix portrays past personnel movements, and as such is used to predict future personnel movements. Thus, if present manpower levels are known, future manpower levels can be predicted using the model. These data would be important for manpower planning and for an economic valuation of human resources. Extensions of the basic model will provide other data which will be useful for both manpower planning and human resource valuation.

#### Outline of the Study

This dissertation consists of nine chapters. Chapters II and III consist of detailed reviews of the literature relating to manpower planning and human resource accounting, respectively. The nonfinancial aspects of the general model are developed in Chapter IV and the financial aspects (valuation) are presented in Chapter V. In Chapters VI through VIII, the model is specifically applied<sup>1</sup> to a large Certified Public Accounting (CPA) firm. The clarifications of the general model needed for application to a CPA firm are discussed in Chapter VI. In Chapter VII, certain validation tests are performed on an application of the

---

<sup>1</sup>Because not all of the data needed to apply the model were available, there are limitations to this "specific application" to a CPA firm.

model developed from actual data. The application of the model continues in Chapter VIII with the presentation of the uses of the applied model to manpower planning and human resource valuation for the actual firm. A summary of the research and suggested areas of further research are presented in Chapter IX.

## Chapter II

### MANPOWER PLANNING

"It would be an exaggeration to say that the manpower planner is the newest man in the executive suite. But it would not be an exaggeration to say that this is the newest and hottest area of interest right now."<sup>1</sup>

The above quotation, written in 1974, reflects the fact that manpower planning is not a new concept. The quotation also relates that the area of manpower planning is currently of considerable interest to corporate executives. When this statement was made, executives and consultants agreed that "sweeping and unprecedented" changes were taking place in the area of manpower planning, and that this area was "one of the top challenges in industry today."<sup>2</sup> One consultant even went so far as to say that companies were "running scared" because of possible manpower problems.<sup>3</sup> Even though the concept of manpower planning is not new, interest in the area has evolved slowly. However, this interest is now reaching a peak. This chapter will briefly discuss some of the background leading to this increased interest in manpower planning. Further discussion will include an explanation of manpower planning and the techniques of implementing this type of planning in a firm.

---

<sup>1</sup>"Big Push in Manpower Planning," Executive Ledger, Dun's Review, 104 (November 1974):103.

<sup>2</sup>Ibid.

<sup>3</sup>Ibid.

## Background

During the period before World War II, behavioral scientists who were studying organizations focused their attention on the rank and file worker.<sup>4</sup> During this period, the internal labor supply was the major source of managerial talent. This was true for several reasons. First, the educational level of most employees was lower than it is today. The emphasis on employees having a college education had not yet begun. The main employee education consisted of on-the-job training and experience. Second, by today's standards, technology was still at an unsophisticated level. Line workers were needed to perform many of the manual tasks that today have been automated. These workers were thus a large and important part of the work force. Third, change occurred slowly during this period. Even though employees were experience-based and also less educated, they were not as subject to obsolescence as these same employees are today.<sup>5</sup>

During the time period from after World War II to the early 1960's, the situation slowly changed. Technological advances decreased the need for less-skilled workers. At the same time, alternative external sources of managerial talent became abundantly available. More people were able to attend college, due to a rising level of affluence in the country and also to the passage of the G.I. bill. Another

---

<sup>4</sup>Victor H. Vroom and Kenneth R. MacCrimmon, "Toward a Stochastic Model of Managerial Careers," Administrative Science Quarterly, 13 (June 1968):26-7.

<sup>5</sup>F. J. Staszak and N. J. Mathys, "Organization Gap: Implications for Manpower Planning," California Management Review, 17 (Spring 1975):32.

reason for these external sources becoming available was that industry pay scales for these jobs were higher than the pay scales for education, government, and other non-industry positions.<sup>6</sup>

The availability of managerial material does not in itself mean that these employees are in demand, but in this case the business environment was such that college degrees were beginning to be stressed as a prerequisite for staff employees. In fact, by 1960, studies were already being done which estimated that as early as 1965 there would be a shortage of people qualified for management positions.<sup>7</sup> Emphasis was increasingly being placed upon the staff employee rather than the line worker.

Until the late 1950's, there were few systematic plans for managing human resources. However, as the situation became more pressing, both academicians and practitioners began to realize that manpower planning at the firm level was needed. Even so, solutions to the problem have been slow in coming. Implementation of formal manpower plans by firms has been even slower to evolve, even in firms with large investments in human assets.

#### Why Plan Manpower?

In order to better understand the need for a formal plan of human resource management, the term "manpower planning" must be

---

<sup>6</sup> Ibid.

<sup>7</sup> C. S. Myers, "Challenge of the 1960's: Manpower Management," Personnel, 36 (May 1959):15-21.

specifically explained. One of the weaknesses in the literature on manpower planning is that there is no one generally accepted conception of the term. Manpower planning means different things to different people. There are those that look at manpower planning on a "macro" basis. In this context, the manpower supply and demand for an industry or even an entire nation are studied. The economic effects of supply and demand are important for "macro" manpower planners.

More often, manpower planning is referred to in a "micro" sense. This means that human resources are studied on an individual firm basis. However, even researchers who study manpower on a micro basis do not necessarily agree on the important points of manpower planning. To some researchers, the main aspect of manpower planning is maximizing the productivity of positions and individuals. Other researchers are primarily concerned with the assignment of a set number of people to a set number of positions. To still others, manpower planning applies only to certain types of personnel skills, usually managerial skills. Finally, there are researchers who tend to look at the area as including only forecasting activities rather than the full range of planning functions.<sup>8</sup> All of the above facets of manpower planning are needed for a comprehensive plan for human resource management.

---

<sup>8</sup>B. P. Coleman, "Integrated Systems for Manpower Planning," Business Horizons, 13 (October 1970):89.

Definition. One definition simply states that manpower planning "is a strategy for the acquisition, utilization, improvement, and preservation of an enterprise's human resources."<sup>9</sup> While this definition implies that manpower planning deals with all facets of the human resource activities of the firm, it does not reflect the integration of all planning aspects of the firm (see the next section). A more comprehensive definition was set forth by Cassell. He defined manpower planning as:

a process designed to translate the corporate or institutional plans and objectives into future quantitative and qualitative manpower requirements, together with plans to fulfill those requirements over both the shorter and longer terms, through manpower utilization, human resource development, employment and recruiting, and manpower information systems. The former part of the process consists of forecasting whereas the latter involves management to meet requirements.<sup>10</sup>

With at least this general idea of the area of manpower planning, why plan human resources and what are the benefits of performing such functions?

There are two reasons why companies need to plan human resources at the micro level. The first reason has already been alluded to above. Factors such as an increasing rate of technological change have helped to bring about a change in the labor mix. The demand is for high-talent technical and managerial personnel, and the supply of

---

<sup>9</sup> John Lawrence, "Manpower and Personnel Models in Britain," Personnel Review, Great Britain (Summer 1973), p. 4.

<sup>10</sup> F. H. Cassell, "Manpower Planning: State of the Art at the Micro Level," MSU Business Topics, 21 (Autumn 1973):15.

these people is low, whether the source is internal or external.<sup>11</sup>  
With this increased demand, the firm also needs to realize the possible leverage effects of dealing with these personnel. The firm is making an increasingly large investment in these types of people, and should make some type of cost/benefit analysis on this investment just as it would any other. There should be concern over the future value of the individual, and thus the future value of the firm.<sup>12</sup>

The second big reason for having a formal manpower plan is the increasing cost of managing human resources. The management of this resource includes such actions as recruiting, hiring, training, and maintaining people. In fact, the importance of this cost may be surprising to some.

Cost is the key. With the inexorable rise in the cost of just about everything it takes to do business, companies are finally realizing that the care and feeding of personnel is one of the most expensive items of all.<sup>13</sup>

Another reason for emphasis on manpower planning has to do with the changing culture. The situation during the 1930's was that the country expected industry to provide jobs for the masses. During the 1940's and 1950's, industry was expected to provide production for the country. Today, industry is expected to provide careers for people.

---

<sup>11</sup>D. R. Bryant et al., "Manpower Planning Models and Techniques," Business Horizons, 16 (March 1973):69.

<sup>12</sup>Mason Haire, "A New Look at Human Resources," Industrial Management Review (Winter 1970), p. 20.

<sup>13</sup>Executive Ledger, p. 13.

This leads to the notion that the firm is not just hiring a pair of hands anymore. Instead, the firm is hiring characteristics such as initiative, innovation, and commitment.

Other reasons for utilizing human resource plans are reflected in the benefits of a successful manpower planning system:

(1) There is a reduction in personnel costs because of the ability of management to foresee possible shortages or surpluses of personnel. This allows management to correct the anticipated problem before it becomes unmanageable and expensive.

(2) The firm has a better basis for planning employee development with a system that is designed to make optimum use of workers' abilities within the organization.

(3) There is improvement in the overall business-planning process.

(4) The firm has more opportunity to include women and people from minority groups in future growth plans. This could prevent problems with interest groups and/or the government. The firm can also use the system to help identify the training and development programs that need to be implemented today in order to have specific skills available tomorrow.

(5) There is a greater awareness of the importance of sound manpower management throughout all levels of the organization.

(6) The system can be used as a tool for evaluating the effects of alternative manpower actions and policies.<sup>14</sup>

---

<sup>14</sup>R. J. Brown and J. D. Somerville, "What the Supervisor Should Know About Manpower Planning," *Supervisory Management*, 19 (September 1974):23-24.

In short, there needs to be emphasis on human resources planning because of the size of the problem. Manpower is just too big a resource to be managed casually.<sup>15</sup>

### Stressing Integration

One weakness in the literature has already been pointed out: the lack of a generally accepted definition of manpower planning. A second weakness is that authors rarely attempt the integration of manpower planning with other planning activities.<sup>16</sup>

The planning process needs to be integrated in two ways. First, manpower planning should be related to organizational planning on a company-wide basis. The organizational planning structure can and should be used to give direction and lend credence to the manpower planning procedures. For example, Hardt has set forth a list of procedures to follow when implementing a manpower plan. As with almost all authors, he specifies that the planner should analyze personnel needs for the projected period. It is interesting to note that Hardt puts this particular analysis fourth on his list of procedures. Before this analysis can be performed, however, he lists three other analyses that need to be developed: (1) study the marketing plan for the projected period; (2) analyze material and facility needs for the projected period; (3) analyze financial needs for the projected period.<sup>17</sup> Hardt obviously

---

<sup>15</sup>Haire, p. 19.

<sup>16</sup>Coleman, p. 90.

<sup>17</sup>E. Hardt, "Manpower Planning," Personnel Journal, 46 (March 1967):157-158.

believes that a thorough knowledge of other aspects of the firm are needed before a successful manpower plan can be implemented.

The second method of integration has to do with an overall comprehensive human resource plan, again on a company-wide basis. There needs to be a plan that determines the extent and depth of the firm's available human resources. This plan should then be used as a foundation (along with other plans) for recruitment, selection, and training procedures.<sup>18</sup>

### Manpower Forecasting

No formal manpower plan can be complete without some attempt at manpower forecasting. Regardless of the method used, there has to be some estimation of future human resource availability and needs.

Forecasting has been defined by Hopes as:

the rigorous building up of relationships between tasks and manpower, the expression of given policies in terms of a program of tasks to be undertaken, and then a bringing together of these analyses to derive future manpower requirements.<sup>19</sup>

Hopes also states that manpower forecasting can be related to economic and other indicators, which means that these indicators must also be forecast.<sup>20</sup> Otherwise, his definition implies a system where the only two factors are (1) projected tasks, and (2) required manpower

---

<sup>18</sup>R. E. Cassidy, "Manpower Planning: Coordinated Approach," Personnel, 40 (September 1963):36.

<sup>19</sup>R. F. A. Hopes, "Some Statistical Aspects of Manpower Planning in the Civil Service," Omega, The International Journal of Management Science, 1 (April 1973):169.

<sup>20</sup>Ibid.

to meet these tasks. The wording of the definition appears to overlook the several possible flows of human resources, although Hopes includes a projection of these flows in his manpower analysis. The possible flows are discussed by Haire and include movements into and out of the system as well as internal movements (promotion, demotion, and lateral movements). These possible movements will be discussed in a later section.

These internal and external manpower movements form the basis for the supply-demand relationship that needs to be forecast before a successful manpower planning system can be implemented. To further stress the importance of forecasting, Brown and Somerville state three major objectives of manpower planning that must always be included in a successful system.<sup>21</sup>

(1) To project demand for and potential supply of hourly and salaried manpower at every skill, salary, or job-classification level within the organization--at any given future time.

(2) To see how the projected supply of personnel stacks up against anticipated demand--so that specific personnel programs, policies, and procedures can be set up in time to ensure the availability of necessary manpower.

(3) To function as a necessary prelude to sound business planning.

Thus, two of the three ever-present objectives emphasize the supply-demand relationship, while the third emphasizes the integration of plans, which was discussed in the previous section.

---

<sup>21</sup>Brown and Somerville, p. 24.

As with any attempts to foretell the future, there have been a multitude of methods set forth in the literature for forecasting manpower. These methods range from unsophisticated subjective techniques to extremely sophisticated quantitative models, but there are four broad areas into which all can be categorized:<sup>22</sup>

- (1) Judgmental techniques
- (2) Matrix models
- (3) Computer simulation
- (4) Quantitative techniques

#### Judgmental Techniques

Judgmental techniques do not involve the use of any forecasting models. These techniques do involve the use of opinions or arbitrary rules, and can be broken down into several methods.

Supervisor estimates. This method simply utilizes the intuition and experience of the man closest to the situation for forecasting manpower. This is still probably one of the most-used techniques for several reasons. It is certainly simple and quick. Also, the decision-maker can usually make these estimates without much, if any, data. Another reason for its use is that in many cases this method can result in a fairly accurate short-run forecast. However, it lacks sophistication and should be viewed in such a manner.<sup>23</sup>

---

<sup>22</sup>Bryant et al., pp. 70-74.

<sup>23</sup>Ibid., p. 70.

Rule-of-thumb techniques. In this case the firm sets up decision models and then bases its manpower policies on past results in relation to these heuristics for certain environmental conditions. For instance, the decision to hire an extra production worker may be based on the amount of overtime that is needed to get the job done. When overtime costs exceed a specified amount for a particular time period, a new worker is added to the line. The technique may be useful, but it is definitely a short-run decision, and it is normally designed to maintain the status quo rather than adequately deal with future manpower requirements.<sup>24</sup>

Replacement charts. The use of a replacement chart is an attempt to graphically depict the internal labor situation so that suitable replacements will be available for vacancies as they occur. Data used to construct these charts may include such things as the present position-holder's age, level of performance, promotability, and actuarial statistics. For a backup person, data may include age and degree of readiness. Opinions on when and where vacancies should occur also may be used. These charts require a lot of time and labor to construct, and they usually depict a static rather than a dynamic picture of the situation.<sup>25</sup>

---

<sup>24</sup>Ibid.

<sup>25</sup>W. R. Dill et al., "Models and Modelling for Manpower Planning," Management Science, 13B (December 1966)B142-167.

Delphi Techniques. Undoubtedly the most sophisticated method of obtaining opinions or judgments is the use of the Delphi method. This technique was originally developed by the Rand Corporation in the late 1940's, and is designed to obtain the most reliable opinion from a group of experts. The Delphi method is an iterative procedure where the experts are asked certain questions and each expert has the capability of controlled feedback. However, there is no direct confrontation among the experts. This is supposedly more conducive to independent thinking, and allows more gradual formulation of an opinion. A "round" consists of a questionnaire being filled out by each expert. The expert is also exposed to information from previous rounds about all of the experts. The process ends when the opinions of the experts have converged to a level where differences are considered to be insignificant. This does not necessarily mean a general consensus of opinion.<sup>26</sup>

In one application of the technique, experts were asked the primary question about the number of employees required for a specified time period in the future. The results were compared with the results of an applied quantitative technique (regression analysis) after the forecast time period had elapsed. In other words, both forecasted results were compared against the actual needs for the period. In this case, the Delphi method came out as a better method of forecasting.<sup>27</sup> It should be noted that this was a short-run forecast.

---

<sup>26</sup>G. T. Milkovich, "Use of the Delphi Procedures in Manpower Forecasting," Management Science, 19 (December 1972:381-382).

<sup>27</sup>Ibid., p. 387.

### Matrix Models

These models do not provide the decision-maker with any added information. They do, however, provide him with a systematic method of viewing and evaluating the available data. Matrix models are just logic aids and cannot compensate for faulty information. Two such models will be presented as examples.

Executive Planning Development Model.<sup>28</sup> This matrix model utilizes a sophisticated data base and is composed of five matrices for both present and future purposes. These are as follows:

(1) Job analysis--talent task relationship matrix. A matrix of this type is needed for each executive position. It provides a needed overview for the job under consideration.

(2) Man-job relationship matrix. The formulation of this matrix is for the purpose of adjusting either the man (training, testing, etc.) or the job (to fit available talents). Neither is assumed to be rigid.

(3) Talent-task composition of the enterprise matrix. The talent requirements for the company as a whole are determined in this part of the model.

(4) Executive personnel requirements matrix. This matrix is an analysis of present and future requirements of executive personnel.

(5) Talent flows matrix. The training needed for the development of the executive is depicted by this matrix.

The study of all of these matrices brings together the available information in a logical manner for the planner. The model does not dictate policy; it only enhances the usefulness of the data.

---

<sup>28</sup> J. E. William Crissy, Robert M. Kaplan, and Louis H. Grossman, "Matrix Models for Planning Executive Development," MSU Business Topics (Summer 1969), p. 8.

Management Manpower Planning Matrix.<sup>29</sup> The possible movements of personnel as presented by Mason Haire in this model have been mentioned earlier. These flows include: (1) moving into the system from an external source, (2) moving out of the system, (3) moving upward (promotion), (4) moving over (taking another position at the same level within the firm), and (5) changing potential and/or behavior. The possibility of moving downward was intentionally omitted as being rare and unimportant in most organizations. It could be easily added to the model if desired.

Haire's model only uses one matrix. This matrix charts the possible personnel flows against the organizational factors (recruitment, pay, training, promotion, etc.). These organizational factors are those that the firm has control over in relation to an effect on employees. Once the matrix has been developed, it can be used to monitor the probabilities of movement in jobs. The model allows the firm to get some idea of how each of the organizational factors affects personnel flow. This knowledge would be helpful in setting policies in each of the controllable areas.

#### Computer Simulation

The following two sections on computer simulation and quantitative techniques list some of the more sophisticated attempts at manpower planning. The planner is actually trying to model the system as

---

<sup>29</sup>Mason Haire, "Managing Management Manpower," Business Horizons, 10 (Winter 1967):23-28

realistically as possible in order to test how it reacts to certain variables. Models have been applied in the areas of civil service systems, manpower defense systems, and in education, as well as in industrial settings. These models are more often intended to forecast supply of manpower at some future date instead of estimating the demand for human resources.

The advent of the computer has aided progress in sophisticated quantitative techniques by making possible the programming of tedious calculations that were previously untenable. These computer capabilities also improved known methods of simulating a system. There are many variables, both technical and behavioral, which affect manpower in a system. Behavioral science techniques are not very helpful for studying all of these variables at this point. Simulation is a way of dealing with many factors because it can be used to represent and manipulate multiple causes, multiple effects, and complex interdependencies. It is easier, cheaper, and less dangerous to use computer simulation to test the effects of each of these variables on the system rather than dealing with the organization itself.<sup>30</sup> Two applications of computer simulation in manpower planning are discussed below.

MANPLAN. This computer simulation program has been developed jointly by the General Electric Company, Massachusetts Institute of Technology, and Stanford University. At least four versions of MANPLAN

---

<sup>30</sup> James R. Miller, "Micro-Simulation as an Aid to Managing Human Resources," Industrial Management Review (Winter 1970), p. 25.

have been programmed and at least partially validated at General Electric.

MANPLAN deals mainly with a single-supervisor, multi-subordinate group relationship. It assumes that every individual in the organization has one supervisor to whom he reports. This allows analysis of strictly hierarchical organizations, and also permits generalization of programs.

There are four types of descriptive inputs needed for MANPLAN. The program needs structural inputs of the situation such as the size of the work group, communication channels, and authority relationships. Second, functional inputs are needed, such as production flows, activities performed, and the impact of resource inputs on production outputs. Organizational inputs such as institutional policies, reward policies, and promotional opportunities are also needed. Finally, inventory inputs are needed, such as information on the skills, history, and aspirations of all individuals involved.<sup>31</sup> (These input areas are needed for any simulation program, not just MANPLAN.)

Using the above inputs, MANPLAN simulates the behavioral processes within the group. In application, MANPLAN has generated "reasonably valid" results on financial summary performance measures, but its predictions of individual behavior have yet to be validated.<sup>32</sup>

---

<sup>31</sup>Ibid., p. 26.

<sup>32</sup>Ibid.

Weber Model.<sup>33</sup> This model simultaneously considers a large number of tasks, objectives, and interrelationships instead of looking at one variable while holding all else constant. Again the model is for use in a hierarchical organization, and represents the behavior of the individuals, management decisions, and aspects of the environment.

Factors included in the Weber model include characteristics of both the individual and the organization, and of external factors. The environment is represented as a job market, generating the probability that an individual will receive a job offer. Organizational policies such as recruiting, attribute evaluation, promotion, and salary increases are included in the model. Individual attributes constitute a major portion of the inputs needed. A list of 115 attributes are included for each person.

The model was designed for practical use, specifically for evaluating the effects of alternative personnel policies that the organization may wish to implement. Other uses may be in the areas of planning personal careers or the study of organizational behavior.

The above two computer simulation models are certainly not the only important simulation models that have been developed. Models

---

<sup>33</sup>W. L. Weber, "Manpower Planning in Hierarchical Organizations: A Computer Simulation Approach," Management Science, 18 (November 1971):B119-B144.

have been formulated for military systems<sup>34</sup> as well as for general industrial situations.<sup>35</sup>

### Quantitative Techniques

More space in the manpower literature has been devoted to the use of quantitative techniques than any other planning method. This voluminous amount of research does not mean that any generalizable model has been developed which stands above all others. Quite the contrary, many models have been formulated, each one an attempt to more realistically model the manpower system. A few of the techniques used will be presented below, along with a short discussion of at least one author's view of the technique. Because of its importance, both in the literature and to this dissertation, the use of Markov chains will be presented in a special section.

---

<sup>34</sup>For example, see:

Robert O. Groover, "PERSYM: A Generalized Entity-Simulation Model of a Military Personnel System," AD 696 986 (Oporto, Portugal: NATO Conference Paper, 1969).

Robert W. Moss and Stephen R. Wax, "A Computerized System for Projecting Reserve Manpower Strengths and Costs." AD 831 565 (McLean, Va.: Research Analysis Corporation, 1967).

<sup>35</sup>For example, see:

T. A. Mahoney and G. J. Milkovich, "Computer Simulation: A Training Tool for Manpower Managers," Personnel Journal, 54 (December 1975):609-612.

T. W. Bonham, Edward R. Clayton, and Lawrence J. Moore, "A GERT Model to Meet Future Organizational Manpower Needs," Personnel Journal, 54 (July 1975):402-406.

Linear programming. Vajda<sup>36</sup> used linear programming (LP) in a situation where transfers in a hierarchical organization were assumed to be known, as was "wastage" (employees leaving the organization). The LP model was used to study policies for recruitment. Vajda uses both the primal and the dual simplex solutions to analyze the movement by "steps" through the system. The model is not necessarily used as an optimization technique. The simplex algorithms are used because they help the modeler find desired regional vertices. The model has not been proven to be very practicable or applicable, nor has any other linear programming model received much practical attention.

Goal programming. The use of goal programming is more prevalent as a discussed programming method in manpower planning. Several authors<sup>37</sup> developed a model that incorporated goal programming along with other techniques (input-output, Markov) for the Office of Civilian Manpower Management (OCMM). The goal programming aspect of the model is an attempt to balance the input resources with the output goals. It also supplies flexibility to the model for examining the effects on final user demands of changes in resource inputs as well as manpower requirements. This model was a follow-up to other Naval models that

---

<sup>36</sup> S. Vajda, "Mathematical Aspects of Manpower Planning," Operational Research Quarterly, 26 (September 1975):527-542.

<sup>37</sup> A. Charnes, W. W. Cooper, R. J. Niehaus, and D. Scholtz, "Multi-Level Models for Career Management and Resource Planning," in Manpower Planning Models, edited by D. J. Clough, C. C. Lewis, and A. L. Oliver (New York: Crane, Russak, and Co., Inc., 1974), pp. 91-113.

used just input-output analysis. Both a static and a dynamic goal programming model were developed. The authors declare the model to be a powerful tool for management use.

Dynamic programming. A third programming technique was developed by Jewett. He used dynamic programming as part of his "Minimum Risk Manpower Scheduling Technique."<sup>38</sup> This model attempts to forecast optimum levels of employment for certain time periods when the required workload for the time period is unknown. Jewett's model is designed to minimize the expected transition costs for the set of possible workloads.

Network flows. Gorham<sup>39</sup> has developed a network flow model where the nodes in the network represent a position within the organization. The links between the nodes show the possible movements from one position to another. The network also specifies the training necessary for the movement to become possible. The network denotes a dynamic personnel situation in an uncertain setting, dealing with forecasting, training, and development activities.

#### Markov Models<sup>40</sup>

Markov models can best be applied in complex hierarchical organizations where manpower is developed internally. In such a situation,

---

<sup>38</sup>Roger F. Jewett, "A Minimum Risk Manpower Scheduling Technique," Management Science (June 1967), pp. B578-B592.

<sup>39</sup>William Gorham, "An Application of a Network Flow Model to Personnel Planning," IEEE Transactions of Engineering Management (September 1963), pp. 113-23.

<sup>40</sup>See Chapter 4 for an extensive discussion of the Markov model.

it is important for planning purposes to determine <sup>①</sup> what the available human resources will be at some point in the future, and also <sup>②</sup> what level in the organization the people can be expected to attain by that time. This gives the firm an idea of what the manpower supply will be; then personnel policies can be tested for comparison against the manpower demand.

Examples of models. Rowland and Sovereign<sup>41</sup> estimate manpower supply on the basis of probabilities of employee losses. These losses are based on the assumption that the proportion of losses in any particular classification is constant. The probabilities are then applied to the present supply of manpower for purposes of forecasting. This model specifically represents the internal labor flow for the organization from one position to another (assembler, machinist, manager), or the flow out of the firm altogether. The Markov-based supply forecast is compared with the prediction of manpower required. Any supply shortages must be made up by hiring or training (assuming, that is, that the management decides to cover the shortage).

Vroom and MacCrimmon developed the same type of model in an analytical attempt to "obtain some understanding of the career-development processes of managers within a single industrial

---

<sup>41</sup>Kenneth M. Rowland and Michael G. Sovereign, "Markov-Claim Analysis of Internal Manpower Supply," Industrial Relations (October 1969), pp. 88-99.

organization."<sup>42</sup> This model can be used to forecast management supply and also to represent an individual's development within the firm for study purposes. The basic input requirements are historic personnel data on movements within the organization. Demographic data can also be used. The transition possibilities for the managers include leaving the firm, promotion, demotion, interfunctional mobility, and no change. The authors stress that the model is a descriptive representation rather than being normative. It describes an existing system, not an ideal one. This is reflected in the fact that historic data from the system are used to formulate the model.

Nielsen and Young<sup>43</sup> view Markov chains as one way to analyze complex needs in a progression of position within a firm. They feel that the model deals with these complex needs in a fairly simplistic manner that is easy to comprehend by management. The model can be used to project manpower supply like the other models, but it can also be used in another way. An alternative method of finding manpower recruitment needs would be to specify through the model what the supply should be, and then working backwards to find the needed additions. This incorporates manpower demand into the model rather than just comparing supply and demand after the Markov model has been applied.

---

<sup>42</sup>Victor H. Vroom and Kenneth R. MacCrimmon, "Toward a Stochastic Model of Managerial Careers," Administrative Science Quarterly, 13 (June 1968):26-46.

<sup>43</sup>Gordon L. Nielsen and Allan R. Young, "Manpower Planning: A Markov Claim Application," Public Personnel Management (March-April, 1977), pp. 133-143.

None of the above discussions on manpower models are meant to be an exhaustive list of the techniques that have been applied in the area. Nevertheless, one can begin to recognize the multitude of options available to the manpower planner. It is not an easy task for the firm to decide which particular model should be implemented.

### Choosing A Model

The firm should certainly choose the model that best fulfills its objective for manpower planning. The first decision is whether to implement a formal model or to just use an informal plan. The advantages of informal plans are that they are quick, easy, and inexpensive. There are many advantages to using a formal manpower model. These are presented by Haire:<sup>44</sup>

- (1) The process of modeling forces one to make the variables and their purposes explicit and communicable.
- (2) Having made the variables explicit leads us directly to an empirical operational identification of what we are discussing.
- (3) Modeling the process leads us to treat the organization as a system rather than dealing piecemeal with the parts.
- (4) Viewing the system as a whole highlights the interdependencies among the variables and helps us reassess the consequences of optional interventions in the process.
- (5) The system view leads us to a statement of a continuous flow function rather than intermittent models.
- (6) The statement of the system and the explicit formulation of the variables allow us to conceive of alternative strategies and to ask questions about the differential cost/benefit ratios for them.

---

<sup>44</sup> Mason Haire, "A New Look at Human Resources," Industrial Management Review (Winter 1970), p. 22.

(7) Most models have analytic spin-offs which generate insights that were previously overlooked.

(8) Most models have simulation possibilities which allow us to explore, on an "as if" basis, a whole family of values for alternative discretionary inputs into the system.

(9) Most models do not usurp the managerial decision-making process, but instead sharpen and support it.

If the decision by the firm is to go with a formal model, then a choice must be made among the available alternative techniques.

Milton Lavin explains that implementation of a model can involve a major commitment of the firm's resources. Therefore, care should be exercised in the choice of a model. Lavin considers three areas to be important.<sup>45</sup>

(1) Goals. Attention should be directed towards the specific output of the organization and to the effect of firm policies on these outputs. The problem is to make these variables (outputs, policies) operational in terms of data.

(2) Formulation. Formulation of a model involves two questions. First, should the model be static or dynamic? Lavin implies that in most cases a dynamic model is needed, although he does state that in a large organization with a fairly stable set of roles, it would be appropriate to treat the promotion policy as static.

The second question is whether to use an analytic or a non-analytic model. Again, Lavin gives the impression that the situation dictates which is better, but that both types of models are needed.

---

<sup>45</sup>Milton Lavin, "Strategy in Manpower Planning Problems," Industrial Management Review (Winter 1970), pp. 47-50.

(3) Evaluating model performance. This discussion by Lavin is in terms of the validity of the model. "Face" validity relates to whether the output of the model seems reasonable. "Hypothesis" validity seeks to verify that each part of the model is empirically justifiable. This standard is automatically applied to analytic models. Finally, "event" validity deals with whether the model accurately portrays actual sequences of behavior. This is the most crucial test. All three validity tests need to be a part of the decision as to which model best fits the needs of the organization.

#### Accounting?

A survey of accounting literature would not supply the reader with much information on manpower planning. This may lead to a question as to whether the topic is appropriate as part of an accounting dissertation.

The 1958 AAA Committee on Management Accounting defined

"management accounting" as:

the application of appropriate techniques and concepts in processing the historical and projected economic data of an entity to assist management in establishing plans for reasonable economic objectives and in the making of rational decisions with a view towards achieving these objectives.

This definition implies that manpower planning is indeed an accounting topic. As was mentioned earlier in this chapter, the size and cost of the manpower problem have grown significantly in the last two decades. During this same period, the boundaries of management accounting have been broadened into new areas (such as the behavioral sciences) and

techniques (from management science). Both areas have had an effect on the emphasis on manpower planning. The size of the problem has brought it to the attention of management, which now realizes that there have to be "rational decisions" in relation to human resource planning and development. Once it is realized that there are important decisions that need to be made, it seems logical that the management accountant should, if possible, supply any information to the decision-maker that would be relevant to the decision. This is precisely the aim of manpower planning. On this basis, there is no disharmony in discussing manpower planning in an accounting dissertation.

#### Summary

This chapter presented some of the factors leading to the current interest in manpower planning, particularly at the managerial level. These factors center on the size and cost of manpower employed in most firms. Reducing these costs is one benefit of a formal manpower planning system. Other benefits are presented and include such aspects as filling future skill needs and making optimum use of workers' abilities. The manpower plan should be integrated into overall organizational planning.

Manpower forecasting is an important part of a formal manpower planning system. Four techniques for implementing a forecast are discussed. These areas include: (1) judgmental techniques; (2) matrix models; (3) computer simulation; and (4) quantitative techniques. The problem of choosing a model is briefly discussed.

Finally, it is argued that manpower planning should be a concern of the managerial accountant, regardless of the paucity of accounting literature on the subject.

## Chapter III

### HUMAN RESOURCE ACCOUNTING

The previous chapter showed that executives have slowly come to realize the importance of a formal plan for human resource management. The high and rising cost of acquiring, developing, and maintaining a capable work force is the main reason that manpower planning models have been implemented by some companies. However, manpower planning is only one step in human resource management; it is only part of an overall system for dealing with people as a manageable resource. This chapter will deal with an accounting topic which could also be a component of this overall system, i.e., human resource accounting.

As the importance of human resources increased, and as the interest by management in information on these resources did the same, it was inevitable that accounting would become involved.

A favorite cliché for the president's letter in corporate reports is "our employees are our most important--our most valuable asset" . . . looking to the remainder of the report, one might ask, "Where is this human asset? What is the value of the most important . . . asset? Is it increasing, decreasing, or remaining unchanged?"<sup>1</sup>

---

<sup>1</sup>R. Lee Brummet, Eric G. Flamholtz, and William C. Pyle, "Human Resource Measurement--A Challenge to Accountants," The Accounting Review 43 (April 1968):215.

Historically accountants have not attempted to answer questions such as these in their reports. William Paton recognized this situation as far back as 1922 when he stated that ". . . a well-organized and loyal personnel may be a more important 'asset' than a stock of merchandise."<sup>2</sup> However, it was not until the mid 1960's that serious attempts began to be made to answer these human resource questions. It was about this same time that the phrase "human resource accounting" became popular.

#### Human Resource Accounting (HRA)

The 1973 AAA Committee on Human Resource Accounting defined human resource accounting as "the process of identifying and measuring data about human resources and communicating this information to interested parties."<sup>3</sup> As follows from their definition, the Committee stated the purpose of HRA as being ". . . to improve the quality of financial decisions made both internally and externally concerning an organization."<sup>4</sup> This assumes that decision-makers will actually use the data in order to make more informed decisions. Caplan and

---

<sup>2</sup>William A. Paton, Accounting Theory (New York: The Ronald Press Company, 1922).

<sup>3</sup>American Accounting Association, "Report of the Committee on Accounting for Human Resources," Supplement to the Accounting Review 48 (1973):169.

<sup>4</sup>Ibid.

Landekich state, "The rationale for human resource accounting rests on the premise that accounting data have a significant behavioral impact on the people who rely on accounting information in the conduct of their business affairs . . ." <sup>5</sup> Thus, human resource accounting is simply an attempt to provide accounting data in an area which has been lacking in such data in the past. Proponents of HRA would say that it is up to the accountant to take the initiative to provide this information, and that if it were available, at least managers, if not others, would make good use of human resource information. <sup>6</sup>

Caplan and Landekich consider some theoretically specific assumptions which they say underlie HRA. <sup>7</sup> These assumptions are as follows:

- "1. Human resources provide benefits to an organization in a fashion similar to the manner in which financial and physical resources provide benefits.
- "2. The benefits associated with both conventional assets and human resources have value to the organization because these benefits contribute in some way to the accomplishment of the organization's goals.
- "3. The acquisition of human resources typically involves an economic cost, and the benefits associated with such resources can reasonably be expected to contribute to economic effectiveness. It follows, therefore, that these benefits are essentially economic in nature and are subject to measurement in financial terms.

---

<sup>5</sup> Edwin H. Caplan and Stephen Landekich, Human Resource Accounting: Past, Present, and Future (New York: National Association of Accounting, 1974), p. 56.

<sup>6</sup> Ibid., p. 35.

<sup>7</sup> Ibid., p. 2.

- "4. Since the usual accounting definition of an asset involves the right to receive economic benefits in the future, human resources are appropriately classified as accounting assets.
- "5. It is theoretically possible to identify and measure human resource costs and benefits within an organization.
- "6. Information with respect to human resource costs and benefits should be useful in the processes of planning, controlling, evaluating and predicting organizational performance."

Some of these assumptions are open to discussion, as will be seen in the next section of this chapter. However, before studying the assumptions further, it seems desirable to consider the benefits of a useable human resource accounting system. An ultimate objective of HRA is to measure human resource cost and value to the organization in quantitative terms. Such quantification would help to provide a basis for cost/benefit analyses by managers when making certain personnel decisions related to acquisition, development, and replacement of human resources. After such decisions have been made and carried out, a quantitative HRA system could be used as a control factor. Stated differently, HRA could aid in monitoring the effectiveness of the company's utilization of human assets. This could be done by measuring the gain or loss in human resource value.<sup>8</sup> Such measurements could lead to a smaller number of suboptimal decisions made by managers who use short-run, high-pressure techniques to

---

<sup>8</sup>Eric Flamholtz, "Human Resource Accounting: A Review of Theory and Research," Journal of Management Studies, February 1974, p. 45.

increase conventional accounting profit. These suboptimal decisions may result in immense deterioration of human resource value, hurting the organization in the long run.<sup>9</sup>

The above benefits relate to managerial decision-making, and as such are internal benefits. They are also indirect benefits to the stockholders because the attention of management is focused on long-range profitability. There are other benefits to outsiders that are more direct. First, creditors and investors may be better able to appraise the abilities of management for dealing with all available resources. Second, reporting on the performance of human resources as well as the company's normal physical resources would allow for more accurate predictions of future earnings by external users of financial statements.<sup>10</sup>

Social benefits may also result from the use of HRA. If HRA became an established practice, comparability among financial statements of different companies could be enhanced. This would permit a better allocation of resources in the economy. In addition, utilization of resources could be more efficient because of a higher quality of both internal and external decisions.<sup>11</sup>

---

<sup>9</sup>Michael H. Gilbert, "The Asset Value of the Human Organization," Management Accounting, July 1970, p. 26.

<sup>10</sup>Nabil S. Elias, "Some Aspects of Human Resource Accounting," Cost and Management, November-December 1971, p. 40.

<sup>11</sup>Ibid.

### Objections to HRA

A human resource accounting system can be a valuable tool in an overall plan of human resource management. If this is true, why do not all firms with significant investments in people have some type of human resource accounting system? The obvious answer is that there are problems, biases, and objections associated with the development and implementation of such a system.

Cultural objections. This first complaint has nothing to do with accounting limitations or restrictions. There seems to be a reluctance on the part of some people to seeing a dollar value associated with a person. This is probably due to some fear of behavioral repercussions.<sup>12</sup>

Human assets. If human resources were to be recorded and reported on the financial statements, the account would presumably be shown under the asset section of the balance sheet. Many people equate asset status with a proprietary condition of "ownership."<sup>13</sup> These people argue that employers only own their employees in a society where slavery is accepted. Thus, the argument is that since employees have freedom of movement, they are not owned by the organization, and therefore should not be regarded as assets.

---

<sup>12</sup>R. Lee Brummet, "Accounting for Human Resources," The New York CPA 40 (July 1970):549.

<sup>13</sup>David E. Robinson, "Human Asset Accounting," Long Range Planning 7 (February 1974):58.

Measurement. In order to put human resources on the financial statements, some method of quantification in dollars and cents must be made. This is by far the biggest problem that must be overcome before HRA will become widespread. A number of models have been formulated, as will be seen later in this chapter. Nevertheless, no one generalizable model has been developed and implemented. Measurements using acquisition cost, replacement cost, or economic valuation have been advocated, but none of these bases displays a clear-cut advantage over the others. Measurement bases will be discussed later. For now, it will suffice to say that, regardless of the mathematical models that have been proposed, measurement of human resources is still a problem.

Objectivity. Closely associated with the measurement problem is the accountant's desire for objectivity. Many accountants feel that the subjectivity involved in valuing human resources is just too great. These accountants would rather record no value at all than to record a value that is highly subjective.<sup>14</sup> A better method of measurement could possibly minimize this problem.

Conservatism. Another concept which is firmly entrenched in the minds of many accountants is the idea of conservatism. This concept leads to the practice of reporting assets in a "conservative" manner. This could impede the adoption of HRA by accountants. On the

---

<sup>14</sup>Elias, "Some Aspects of Human Resource Accounting," p. 40.

other hand, the change in emphasis from the balance sheet to the income statement has shown that conservative practices towards the balance sheet can cause a non-conservative income statement. Thus, the conservative argument of leaving human resources off the balance sheet may inflate the profit figure on the income statement.<sup>15</sup>

Visibility bias. Another belief which may hinder the adoption of HRA is the "visibility bias" of investments in human resources. If a machine is overhauled or improved in some other manner, positive results can normally be seen immediately in a physical sense. However, if a manager is sent to a development program, results may not be immediate, and also may not ever be easily "seen." This leads to a write-off of these development costs rather than dealing with the outlay as an investment (which implies capitalization).<sup>16</sup>

Magnitude of task. Finally, a condition which has been detrimental to HRA is the magnitude of the problem.<sup>17</sup> No one person is likely to come up with a solution. In fact, interdisciplinary approaches may be needed. The area of human resource management should be of interest to accountants, economists, financial managers,

---

<sup>15</sup> Ibid.

<sup>16</sup> Brummet, "Accounting for Human Resources," p. 549.

<sup>17</sup> H. C. Eggers, "Evaluation of Human Assets," Management Accounting 53 (November 1971):28.

sociologists, psychologists, and information technocrats.<sup>18</sup> Cooperation among disciplines is certainly desirable, but not always easily brought about.

The above objections and conditions give a general idea as to why common acceptance of HRA has been deterred. Most of these conditions could be overcome if it were not for the awesome problem of measurement. As long as quantifying the value of human resources in an acceptable manner to a general majority is a fantasy rather than a reality, then HRA will be an idea rather than a generally accepted principle of accounting.

#### Research Objective

While the above conditions may have hindered the adoption of HRA, the amount of research done in the area does not seem to have been impeded. Admittedly there has been much duplication of effort, and some of the literature on HRA has been merely a rehash of work that has already been published. Nevertheless, much positive research has been done. While reviewing some of this research, the objectives of HRA research should be kept in mind. Flamholtz<sup>19</sup> lists the objectives towards which much of the research has been directed:

1. to develop a theory that explains the nature and determinants of the value of people to formal organization;

---

<sup>18</sup>Brummet, "Accounting for Human Resources," p. 549.

<sup>19</sup>Flamholtz, "Human Resource Accounting," pp. 45-6.

2. to develop valid and reliable methods for measuring the cost and value of people to organizations (both monetary and non-monetary measurement);

3. to design operational systems to apply (implement) these measurement methods in actual organizations, and

4. to determine cognitive and behavioral impact of human resource accounting measurements and frameworks.

Research in HRA does not necessarily apply to any one of these objectives, but most of the positive research that has been done does fall into at least one of the areas. This can be seen by looking at some of the models that have been set forth by researchers.

#### Human Resource Models

There are six popular models for measuring human resources. A seventh model is the one that was actually implemented at the R. G. Barry Corporation. Most of these models are included in survey articles on research in HRA. All are discussed below, in no particular order. The models will be specified by the researcher(s) who developed them, except for the Barry model.

Hermanson.<sup>20</sup> Roger Hermanson presented two models in his 1964 book. His first technique, the Unpurchased Goodwill Method, was an attempt to use objectively verifiable figures rather than prophesying future amounts, such as income. Therefore, he only used

---

<sup>20</sup>Roger H. Hermanson, Accounting for Human Assets, Occasional Paper No. 14 (East Lansing, Michigan: Michigan State University, 1964).

the income figure for the current year in his computations. This income figure is compared to an average income figure based on a normal firm in the industry having the same amount of owned assets. For example, if the industry average return is 10%, then the expected return for an individual company is 10% of owned assets.<sup>21</sup> The dollar income figures are compared. Any deviation from expectations is capitalized at the normal return rate. An income higher than normal would denote that there were more than just the owned assets at work during the period. Hermanson calls these "operational assets" and equates them with human resource value. His rationale is that "the human asset is the most neglected of all the resources operating within a firm." Any difference between total operational assets and just the human assets would be immaterial, according to Hermanson. This is an indirect valuation method and is called the residual method by real estate appraisers.

Hermanson's second method, the Adjusted Present Value Method, is more popular. Here he uses a weighted average of the last five years' performance. This average is used to modify the present value of the expected wage rates over the next five years. These wages are discounted at the economy rate of return on owned assets for the latest year.

---

<sup>21</sup>Hermanson defines "owned assets" as ". . . all scarce resources, legally or constructively owned by the entity, that have a separate determinable market value and therefore could conceivably be directly used or converted for the payment of its debt." (See Hermanson, p. 5).

Once the present value of the wages has been computed, Hermanson presents an Efficiency Ratio. This ratio is designed to measure the efficiency of the human resources utilized by the entity over the five-year period. This ratio is based on yearly differences between the rate of accounting income for the company and the same rate for all firms in the economy. The discounted wages are multiplied by the Efficiency Ratio. Any of the product above the discounted wages figure is assumed to be "excess worth created by relatively efficient human resources." An Efficiency Ratio of below one would denote inefficiencies in human resource utilization.

Lev and Schwartz.<sup>22</sup> Baruch Lev and Aba Schwartz also use salaries in their determination of human resource value. They specify that the value of human capital embodied in an employee is equal to the present value of all future salary that will be earned from the valuation date to retirement. Since these future earnings cannot be known, an estimate must be used. Lev and Schwartz use census and other data as a basis for their estimations. These data can be used to find the average income of a particular "position-holder" (such as an industrial engineer) at a particular age. The incomes at each age level between the position-holder's present age and his retirement age becomes the estimated stream of earnings to retirement. This stream is discounted at the firm's cost of capital.

---

<sup>22</sup>Baruch Lev and Aba Schwartz, "On the Use of the Economic Concept of Human Capital in Financial Statements," The Accounting Review, January 1971, pp. 103-112.

Because of the possibility of death before retirement age, Lev and Schwartz go one step further. The use of mortality tables is incorporated into the model. These tables show the probabilities of a person in a group with like characteristics (race, sex, education, etc.) dying at a particular age. These probabilities are then used with the present value of the earnings stream to find the expected value of the person's human capital. The firm's value for human capital is therefore based on average data (earnings and death rates) of homogeneous groups in the United States.

Hekimian and Jones.<sup>23</sup> James Hekimian and Curtis Jones attempt to develop a method for valuing human resources that would be consistent with valuation techniques used on physical resources. With this in mind, they propose a competitive bidding system for scarce employees. The manager of the investment center which succeeds in acquiring the services of a particular employee in a competitive situation would have to include the "price" of the employee in his investment base. Benefits to the acquiring center would be in the form of increased profits because of the employee acquired. Specifying an employee as "scarce" denotes that there is competition for the employee's services and that he could not be hired readily from outside. Hekimian and Jones feel that this method of valuing human

---

<sup>23</sup>James S. Hekimian and Curtis H. Jones, "Put People on Your Balance Sheet," Harvard Business Review, January-February 1967, pp. 105-113.

resources would lead to a more optimal allocation of people in relation to the company's goals. They also feel that including these employees in the investment base will motivate managers to strive towards efficient use of these employees.

Flamholtz.<sup>24</sup> Eric Flamholtz has done much research in HRA. His model focuses upon an individual's value to an organization. He devises a model for measuring the economic value of an employee. This valuation is based on the premise that individuals can move through a set of mutually exclusive roles in the organization, specified as "service states." Each service state has certain service expectations associated with it. These service states can be broken down into "service levels" (position and salary grade levels) and "service groups" (performance levels). For each individual employee, it is possible to estimate his total expected service time to the organization. It is also possible to estimate which service states the individual will fill during that period, and for how long. This gives a picture of the total expected services of the employee to the organization during his tenure. These estimations are based on a stochastic model which gives the probabilities of moving from one service state to each other state during a particular transition period, and is called a Markov model. The model would be the basis for putting a dollar value on the individual using either of two

---

<sup>24</sup>Eric Flamholtz, "A Model for Human Resource Valuation: A Stochastic Process with Service Rewards," The Accounting Review, April 1971, pp. 253-267.

methods. The price-quantity method multiplies the amount of future expected services by the price of these services. The income method calculates expected income derived from the use of the future services. Either method would give the economic valuation desired.

Flamholtz concludes that because of the problems associated with an economic valuation, some kind of surrogate measure is needed. He cites as possibilities historical cost, replacement cost, current cost, performance measures, and compensation measures.

Jaggi and Lau.<sup>25</sup> Bikki Jaggi and Hon-Shiang Lau state that a human resource model can be evaluated on the basis of three standards. The first standard of evaluation is the extent to which the model includes important variables. The important variables in human resource models are specified as employee service lengths and career movements. The second standard is whether the model is operational, which means the required data is available and usable. Finally, the model must generate data which are useful.

Jaggi and Lau feel that the Flamholtz model falls short on the latter two standards. Dealing with each individual causes highly subjective probabilities in the model. The data for his model are felt to be suspect because of the high statistical variance inherent in its use by only looking at one person at a time.

---

<sup>25</sup>Bikki Jaggi and Hon-Shiang Lau, "Toward a Model for Human Resource Valuation," The Accounting Review, April 1974, pp. 321-329.

The model presented by Jaggi and Lau is a modification of the Flamholtz model, but without its limitations. This is accomplished by using a group valuation instead of valuing each individual. This group is a homogeneous set of employees, and not necessarily a department or a division. A Markovian model is used to estimate the probabilities of the group's career movements. These probabilities are based on historical data on the pattern of movements of employees.

The assumption is that the economic value of the employees is dependent upon the positions they hold at the end of the period. Each position has an economic value associated with it. The model is based upon knowledge of where the employees will be at the end of each period during their service life. The probability of holding each position is multiplied by the economic value associated with that position. This product is multiplied by the number of employees at each level at the beginning of the period before movement occurs. The result is an economic valuation total for each position or rank in the organization. These can be totaled over the ranks to arrive at the total human resource value of the firm.

Likert.<sup>26</sup> None of the aforementioned models has explicitly outlined any behavioral dimensions. This is not to say that they had no behavioral aspects.

---

<sup>26</sup>Rensis M. Likert, The Human Organization, Its Measurement and Value (New York: McGraw-Hill Publishing Company, 1966); see also Likert and David G. Bowers, "Improving the Accuracy of Profit/Loss Reports by Estimating the Change in Dollar Value of the Human Organization," Michigan Business Review 25 (March 1973):15-24; and "Organizational Theory and Human Resource Accounting," American Psychologist 23 (September 1969):585-592.

It may be appropriate to observe here that the human behavior dimension underlies all the approaches to human resource accounting. In a sense, the human behavior dimension underlies all the value systems, since the perception and application of any value system is a behavioral phenomena.<sup>27</sup>

Rensis Likert and his associates attempt to deal explicitly with behavioral variables in the firm. Likert has directed his research efforts towards the basic elements in organizational behavior. These efforts lead him to conclude that the human organization of any firm can be measured by a small number of variables, as follows:

- I. Causal variables
  - A. Managerial leadership behavior
  - B. Organizational climate
- II. Intervening variables
  - A. Subordinate (peer) leadership behavior
  - B. Group processes
  - C. Satisfaction
- III. End-result variable
  - A. Performance of effectiveness

Likert cites a follow-through effect for the three sets of variables. Management can control and alter the causal variables, thus producing changes in the intervening variables. These changes follow through to the end-result variable, affecting productivity or earnings. Both favorable and unfavorable effects follow through to the end.

---

<sup>27</sup>Caplan and Landekich, Human Resource Accounting, p. 39.

Likert also sets forth a four part spectrum of prevailing management styles. At one extreme is the exploitive-authoritative style. Then comes the benevolent-authoritative style, the consultative style, and finally the participative style. Likert's basic premise is that the closer an organization is to a participative style of management, the more favorable the end result variables.

Accounting practice, Likert contends, leans towards authoritative systems when evaluating performance. For example, participative systems emphasize investments in human resources. Accounting practice expenses these outlays. This leads to short-run, suboptimal decision-making. Explicitly measuring human resources would reward a manager for moving towards a participative style.

Likert uses a set of questionnaires to measure both the key variables as specified, and also the managerial style present in the organization. Using these measurements, he returns to his premise that changes in causal variables lead to changes in the intervening variables, and finally, over time, affect end results. Thus, changes in the value of a firm's human resources can be estimated in dollars on the basis of expected changes in the causal and intervening variables. In later work, Likert and others extended the above to compute asset values by using sophisticated statistical correlation techniques.

R. G. Barry Corporation.<sup>28</sup> A human resource accounting system has actually been implemented at the R. G. Barry Corporation in Ohio. Much of the work done on the system was performed by researchers from the University of Michigan, including Flamholtz and Likert. Others included R. Lee Brummet and William Pyle. The initial research was limited to dealing with human resource costs. After considering other possibilities, outlay cost was decided upon for three reasons. (1) There is no "return" involved, which would be behaviorally threatening. (2) Outlay cost is more objective than replacement cost or economic value. (3) Outlay cost is also more consistent with conventional practice than other bases.

The outlay costs were classified into seven categories, as follows:

1. Recruiting costs
2. Acquisition costs
3. Orientation costs
4. Training costs
5. Familiarization costs
6. Informal development costs
7. Formal development costs

The amortization of these costs was meant to show expiration of expected benefits. Two different amortization periods were used. For

---

<sup>28</sup>This explanation is taken from Caplan and Landekich, Chapter 7, pp. 87-108.

some investment cost categories such as recruiting and acquisition, the amortization period was equal to the expected working period of the employee. This period is equal to the maximum working life multiplied by the probability of continued tenure. These probabilities were based on age, existing tenure, and the level of position held in the company. For costs such as orientation that decline in usefulness, a predetermined time period was used for amortization purposes.

At first, only managers used the data. An account was kept on each employee. If the employee left, the account was written off as a loss. At any given time, the company's "net investment in human resources" is the summation of unamortized employee account balances.

The long-run goal of the system is to integrate cost measurements with behavioral measurements. Because of this, behavioral measurements have been taken every year since the inception of the system in 1968.

#### Valuation Bases

There are two issues of HRA that have been alluded to earlier in the chapter; these issues will be discussed in more detail in the next two sections. This section looks at the different valuation bases that can be used for putting a dollar figure on human resources. The next section is on the behavioral aspects of HRA.

If valued using conventional accounting practices, human resources would be put on the financial statements at acquisition

cost. Arguments have been offered that a different base, such as current or replacement cost, would be an improvement over acquisition cost. These arguments do not differ significantly from the well-known discussions by different authors on historical cost versus replacement cost, etc. Some of the different valuation bases will be covered briefly below.

Acquisition (historical) cost. For the most part the R. G. Barry Corporation HRA system capitalized outlay costs. These outlay costs were historical costs incurred by the company. Brummet states that "the development of an outlay cost system for human resource accounting should be viewed as a first step or the providing of an important first installment of a useful set of human resource information."<sup>29</sup> Such a valuation could be used to provide a base for measurement of the company's return on investment in its human resources. This would at least draw attention to the importance of these assets. A historical cost base is recognized and understood by management, and can be reasonably objective.<sup>30</sup>

Replacement cost. Although Brummet felt that an outlay (historical) cost system was a needed first step, he goes on to say that replacement cost data should be developed for human resources,

---

<sup>29</sup> Brummet, "Accounting for Human Resources," p. 552.

<sup>30</sup> American Accounting Association, "Report of the Committee on Accounting," p. 171.

and that these data would be particularly useful for planning and capital budgeting.<sup>31</sup> Flamholtz attempts to measure positional replacement costs, stating that the manager is the primary intended user of such information.<sup>32</sup> It seems to be widely believed that replacement cost of any asset is a more relevant and useful tool for managerial purposes than historical cost. However, the problem of increased subjectivity is the primary objection to the use of replacement cost. As stated by Banner and Baker, "substituting replacement cost for actual cost appears to do little more than update the valuation, at the expense of importing considerably more subjectivity into the measure."<sup>33</sup>

Opportunity cost. The model of Hekimian and Jones is an attempt at a valuation base that is closely related to the concept of opportunity cost. The authors see the model as an improvement over replacement cost, particularly in human resource allocation.<sup>34</sup>

---

<sup>31</sup>Brummet, "Accounting for Human Resources," p. 552.

<sup>32</sup>Eric Flamholtz, "Human Resource Accounting: Measuring Positional Replacement Cost," Human Resource Management, Graduate School of Business Administration, University of Michigan (Spring 1973), pp. 8-16.

<sup>33</sup>D. Banner and G. Baker, "Human Resource Accounting: A Critical View," MSU Business Topics 21 (Autumn 1973):45-52.

<sup>34</sup>Hekimian and Jones, "Put People on Your Balance Sheet," p. 108.

Elovitz states that the approach is ". . . so artificial as to impair its effectiveness."<sup>35</sup>

Economic value. The economic valuation of an asset implies that the value of the asset is equal to the present value of its estimated future benefits. Hence, the timing and amount of the future services provided by the asset have to be estimated, and then the present value of all of these services computed. Several of the techniques in the models discussed earlier are attempts at economic valuations. Lev and Schwartz use future salaries as the price for future services and discount the salaries as a type of surrogate measure. Flamholtz and Jaggi and Lau use Markovian models to predict future movements as a basis for measuring services. Even the Unpurchased Goodwill Method of Hermanson is an indirect method of determining economic value. Still another valuation technique is the use of social-psychological measurements by Likert.

#### Behavioral Aspects of HRA

The implementation of a human resource accounting system definitely has behavioral connotations in at least two areas. The first area is the effect of HRA on those who use the information for decision-making. The assumption, as stated earlier, is that managers will actually use these data in decision-making situations. Thus, HRA

---

<sup>35</sup>David Elovitz, "From the Thoughtful Businessman," Harvard Business Review (May-June 1967), p. 59.

influences managerial behavior by influencing decisions. These decisions are of two basic types. Decisions will be made that will deal more explicitly with people as resources of the organization, and managers may better realize the importance of decisions relating to these resources. The second type of decision that may be affected would be those decisions that deal with people as people. Wright feels that just by capitalizing investments in human resources, managerial behavior will be altered significantly. He states, "It is apparent that this reconceptualization of the only vital factor of production will have a profound impact on the way managers manage."<sup>36</sup> Likert emphasizes the positive effect of this behavior change:

. . . but when current financial reports are accompanied by dollar estimates of the change in the value of the human organization for the same reporting period, the kind of management that builds more productive human organizations will be fostered, because such management creates the will to work and at the same time contributes to employee health and satisfaction.<sup>37</sup>

Unfortunately, little work has been done to show just how significant the effects on decision-making would be if HRA were implemented. The reporting of human resource information could affect decisions other than those by managers. Elias did a study of HRA data and its effects

---

<sup>36</sup>Robert Wright, "Managing Man as a Capital Asset," Personnel Journal, April 1970, p. 290.

<sup>37</sup>Rensis Likert, "Human Resource Accounting: Building and Assessing Productive Organizations," Personnel, May 1973, p. 8.

on the investment decision. He found that there was an effect on some people, but his study was not conclusive.<sup>38</sup>

The second area of behavioral considerations is how those who are measured by the HRA system might react. How will the employees feel about having a dollar value put on them (or their future services)? Research done in the area of budgeting has shown that accounting information can have an effect on the performance of workers. Modern organizational theory stresses that money is not the sole motivator of people. According to Landekich and Caplan, "high levels of motivation and goal congruence can be achieved only if an organization is able to meet the social and psychological needs of its people. These needs include self-esteem, recognition, and a sense of achievement."<sup>39</sup> If having a dollar value put on them, or if decisions made by management on the basis of these dollar values has an effect on employee productivity, behavior has certainly been altered and this aspect of HRA must be studied. Finally, the presence of such a possibility was expressed in the following statement made by an employee to Brummet, "I would not mind being depreciated if I really thought I was appreciated."<sup>40</sup>

---

<sup>38</sup>Nabil Elias, "The Effects of Human Asset Statements on the Investment Decision: An Experiment," Empirical Research in Accounting, Selected Studies, 1972, Supplement to Journal of Accounting Research, Vol. 10, pp. 215-233.

<sup>39</sup>Caplan and Landekich, Human Resource Accounting, pp. 6-7.

<sup>40</sup>Brummet, "Accounting for Human Resources," p. 549.

### Summary

This chapter has been a survey of the literature on human resource accounting. Benefits of HRA include enhanced decision-making in both planning and control in a company. Appraisals and predictions by outsiders (creditors, investors) could also be improved. A system of HRA could also have social benefits because of increased comparability among financial statements of different companies. This permits better allocation and more efficient utilization of resources in the economy. Objections to HRA range from cultural objections (placing a dollar value on people) to accounting objections (measurement problems, objectivity, and conservatism) to practical objections (the magnitude of the task).

Seven human resource measurement models are mentioned in the chapter. These models used such valuation bases as historical cost, replacement cost, opportunity cost, and economic value. No conclusion as to the "best" valuation base is presented. The behavioral aspects of human resource accounting are stated as being in two areas: (1) the behavioral effect on decision-makers, and (2) the behavioral effect on those people "valued" by HRA.

Chapters II and III survey two possible components of a comprehensive program of human resource management, manpower planning and human resource accounting. Chapter II presented models for manpower planning, and Chapter III presented models for measuring human resources. One technique that was common to both sets of models

was the application of Markov chains to the problem. One Markov model that can be implemented for both manpower planning and human resource valuation is developed in the next two chapters.

## Chapter IV

### THE CONCEPTUAL MODEL--I

This chapter will set forth a comprehensive conceptual model for the management of human resources in an organization. The model is based upon the premise that the mobility of manpower within the organization can be envisaged as a Markov Process. Many situations have been viewed in such a manner. Markov models have been used to simulate blood-bank inventories,<sup>1</sup> product brand-switching,<sup>2</sup> newspaper subscription lives,<sup>3</sup> and even the Peter Principle.<sup>4</sup> These are but a few of the applications. Another popular use has been to model the flow of manpower through the different jobs, levels, or positions within an organization. It is this application which is the basis for the model in this chapter.

The model is a special type of formulation; it is an absorbing Markov process. This is important because Markov chain theory differs in certain areas depending on whether the process is absorbing or non-absorbing.

---

<sup>1</sup>C. Carl Pegels and Andrew E. Jelmer, "An Evaluation of Blood-Inventory Policies: A Markov Chain Approach," Operations Research 18 (November 1970):1087-98.

<sup>2</sup>For example, see R. B. Maffei, "Brand Preferences and Simple Markov Processes," Operations Research 8 (March 1960):210-18.

<sup>3</sup>W. Edwards Deming and Gerald J. Glasser, "A Markovian Analysis of the Life of Newspaper Subscriptions," Management Science 14 (February 1968):B283-B293.

<sup>4</sup>Julius Kane, "Dynamics of the Peter Principle," Management Science 16 (August 1970):800-11.

### Foundation of the Model<sup>5</sup>

In using a Markovian model to simulate a real-world situation, there must be a set of states that can be distinguished from one another in some way. There does not need to be any special relationship among these states (i.e., they need not have a hierarchical, chronological, equivalent, etc. sort of relationship). The formulation of any Markov model is based on finding the probabilities of movement from one state to another during a specified transition period. These probabilities are based on historical data in some form and are found for all possible movements between states within the system. Each probability figure is actually a percentage, and as such reflects the proportion of actual occurrences (movement from one state to another) to the total possible number of occurrences for that particular movement. In other words, very simply, if there were 100 units in State II at the beginning of the period, and if 40 of these units moved to State III in the system during the period, then the probability of moving from State II to State III during the period would be 40% (40 out of a possible 100 chances). This figure could then be used for estimation purposes in future time periods. This necessarily assumes that what has happened in the past will happen in the future, but anyone using quantitative modeling techniques should be aware of this inherent limitation.

---

<sup>5</sup>See John G. Kemeny and J. Laurie Snell, Finite Markov Chains (Princeton, New Jersey: D. Van Nostrand, 1960), see especially Chapters 2 and 3.

<sup>6</sup>The "states" of a Markov analysis are the mutually exclusive positions within the system as defined by the user modeling the system.

The transition probabilities can be formed in one of two ways. The 40% example above shows how the probability can be found by using aggregate figures. The second method is to follow the flow of individual experiments through the model. With a large amount of historical data, a sample of the total number of occurrences could be taken, and the probabilities for the whole system could be based upon this sample.

The Markov property. The definition of a Markov process specifies another assumption about these probabilities: that they possess the Markov property. As Kemeny and Snell state, "knowing the outcome of the last experiment we can neglect any other information we have about the past in predicting the future."<sup>7</sup> Popularly stated, the presence of the Markov property means that the position of the system at time  $(t + 1)$  is only dependent on the position of the system at time  $(t)$ . In the example above, the probability of a particular unit moving from State II at time  $(t)$  to State III at time  $(t + 1)$  depends only on the fact that the particular unit was in State II at time  $(t)$ . How the unit arrived at State II and the length of time that it has been there should have no effect on the probability of movement. ?

---

<sup>7</sup>Kemeny and Snell, p. 24.

The transition matrix. The set of states within the system must be exhaustive and mutually exclusive.<sup>8</sup> The stipulation that the set of states must be exhaustive simply means that each possible occurrence must be able to be classified into a particular state. The set is mutually exclusive in that each occurrence can fall into one and only one state at any given point in time.

When all the states have been defined and all of the probabilities have been computed, the information can be set forth in the form of a square matrix, called the transition matrix. A transition matrix with a set of  $m$  states would appear as follows:

$$P = \begin{array}{c} \begin{array}{cccccc} P_{11} & P_{12} & \dots & P_{1j} & \dots & P_{1m} \\ P_{21} & P_{22} & \dots & P_{2j} & \dots & P_{2m} \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ P_{i1} & P_{i2} & \dots & P_{ij} & \dots & P_{im} \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ P_{m1} & P_{m2} & \dots & P_{mj} & \dots & P_{mm} \end{array} \end{array}$$

The  $m$  states in the above matrix would be defined and listed down the left side of the matrix (defining the rows of the matrix)

---

<sup>8</sup>William W. Thompson, Jr. and James U. McNeal, "Sales Planning and Control Using Absorbing Markov Chains," Journal of Marketing Research 4 (Fall 1967):62-66.

and across the top (defining the columns of the matrix). The states would be listed exactly the same in both places. This explains why the transition matrix is square. The row state denotes the position of the system at time  $(t)$ , while the column state shows the position of the system at time  $(t + 1)$ . Thus,  $p_{ij}$  gives the probability of moving from state  $i$  at time  $(t)$  to state  $j$  at time  $(t + 1)$ . All of the other elements in the  $P$  matrix can be explained in the same manner.

Since the set of states is exhaustive as defined above, then each row covers all possible movements from a particular state. Since the  $p$  elements are probabilities, each row should thus sum to 1, or 100%. This means that  $P$  is a stochastic matrix, such that

$$\sum_{j=1}^m p_{ij} = 1. \text{ (for } i = 1, 2, \dots, m)$$

Classification of states. The set of states in the transition matrix can be classified into subsets according to the possible transitions within the system. These subsets can be either transient or ergodic. If the transition probabilities are such that, once a subset of states has been exited it can never be entered again, then these states are called transient sets. Every Markov chain must have a subset of states where once the state or states is entered, it can never be exited. This is an ergodic set.

A special ergodic set occurs when there is only one state in the set. This is an absorbing state. From the definition, the

state cannot be exited, so  $p_{ii} = 1$ , and all other elements in the row are equal to zero. The Markov process that has an absorbing state is called an absorbing Markov chain.

Canonical transition matrix. Grouping the  $m$  states in the transition matrix as to their classification,  $P$  can be ordered and partitioned into its canonical form. In this form, the absorbing states are listed first, then the transient states. If there are  $c$  absorbing states and  $d$  transient states, the canonical form of  $P$  is as follows:

$$P = \begin{array}{c} \begin{array}{c} c \\ \hline d \end{array} \begin{array}{c} \begin{array}{c} c \\ \hline d \end{array} \begin{array}{c} \begin{array}{c} I \\ R \end{array} \begin{array}{c} \begin{array}{c} d \\ \hline d \end{array} \begin{array}{c} \begin{array}{c} O \\ Q \end{array} \end{array} \end{array}$$

where

$I$  is an identity matrix with dimensions  $(c \times c)$ .

$O$  is a matrix of zeros with dimensions  $(c \times d)$ .

$R$  is a  $(d \times c)$  matrix showing the relationships between the transient states and the absorbing states.

$Q$  is a  $(d \times d)$  matrix which shows the relationships among the transient states.

If the transition matrix has a total of  $m$  states, then  $c + d = m$  for an absorbing Markov chain.

The canonical form of  $P$  segregates the matrix into submatrices which can be analyzed both individually and together, as will be shown later in this chapter.

Testing the Markov property.<sup>9</sup> The Markov property, as previously stated, means that the process is a first-order Markov chain. It is possible for a chain to be of higher order. Consider a second-order chain. The probabilities in the transition matrix would not only depend on the position of the system one step previously, but also upon the position of the system two steps prior to the present position. The probability  $p_{ijk}(t)$  would denote that the system is in state  $k$  at time  $(t)$ . The probability depends on the knowledge that the system was in state  $i$  at time  $(t - 2)$  and also in state  $j$  at time  $(t - 1)$ . (3-D)

Testing for the presence of the Markov property is the same as testing to see whether the process is a first-order chain. The null hypothesis would be that the chain is of first-order. The alternative hypothesis would be that the chain is of second-order. The null hypothesis can be stated symbolically as:

$$H_0: p_{1jk} = p_{2jk} = \dots = p_{mjk} = p_{jk} \quad (\text{for } j, k = 1, 2, \dots, m)$$

The chi square formula for testing this hypothesis is

---

<sup>9</sup>T. W. Anderson and L. A. Goodman, "Statistical Inference About Markov Chains," Annals of Mathematical Statistics 28 (March 1957): 89-110.

$$\chi_j^2 = \sum_{i,k} n_{ij}^* (\hat{p}_{ijk} - \hat{p}_{jk})^2 / \hat{p}_{jk} \quad (4-1)^{10}$$

where  $j$  is the row of the transition matrix being tested. A test statistic is computed for each row. Each test statistic would then be compared with the critical statistic in a Chi square table. The degrees of freedom for the test would equal  $(m-1)^2$ , where  $m$  is again the total number of states in the matrix. In the formula,  $p_{jk}$  and  $\hat{p}_{ijk}$  are the one-period and two-period probabilities, respectively, and  $n_{ij}^*$  is an actual number of observations. The formulae for these variables are

$$\hat{p}_{jk} = \frac{\sum_{t=2}^T n_{jk}(t)}{\sum_{t=1}^{T-1} n_j(t)} \quad (4-1A)$$

$$\hat{p}_{ijk} = \frac{\sum_{t=2}^T n_{ijk}(t)}{\sum_{t=2}^T n_{ij}(t-1)} \quad (4-1B)$$

$$n_{ij}^* = \sum_{t=1}^{T-1} n_{ij}(t) \quad (4-1C)$$

In the above formulae, the  $n_{ij}(t)$  denote the actual observations of individuals in state  $i$  at time  $(t - 1)$  and also in state  $j$  at time  $(t)$ . The explanation is simply expanded for any  $n_{ijk}$ . The overall time period of these observations is  $T$ , where  $t = 0, 1, 2, \dots, T$ .

If the test statistic for each row is less than the critical table statistic, the null hypothesis would not be rejected, i.e., the test does not disprove the existence of the Markov property.

Testing for stationarity. The assumption that the Markov chain is stationary means that the transition probabilities remain constant

---

<sup>10</sup>In this and the next two chapters, formulae will be numbered according to chapter (first number) and order within the chapter (second number).

over time. Fluctuations in the  $p_{ij}$  during different time periods decrease the predictive ability of the model and thus invalidate its use. Therefore, some test must be performed to ascertain whether the probabilities are stationary. Again, a Chi-square test can be used for this purpose.<sup>11</sup>

The null hypothesis would be that the transition probabilities for each time period are the same as the overall probability that an individual will move from state  $i$  at time  $(t - 1)$  to state  $j$  at time  $(t)$ . Symbolically, this can be expressed as

$$H_0: p_{ij}(t) = p_{ij}$$

The alternative hypothesis would be that the transition probabilities depend on the time period in which they are observed. The chi square formula for testing the null hypothesis is

$$\chi^2 = \sum_{t,j} n_i(t-1) \hat{p}_{ij}(t) - \hat{p}_{ij}^2 / \hat{p}_{ij} \quad (4-2)$$

where

$$\hat{p}_{ij}(t) = n_{ij}(t) / n_i(t - 1) \quad (4-2A)$$

$$\hat{p}_{ij} = \frac{\sum_{t=1}^T n_{ij}(t)}{\sum_{t=0}^{T-1} n_i(t)} \quad (4-2B)$$

The degrees of freedom for the test would be equal to  $(m - 1)(T - 1)$ . As with the Markov property test, this is a row test. The test statistic for each row would have to be compared to the critical table statistic. Failure to reject the null hypothesis implies that the transition probabilities are indeed stationary.

---

<sup>11</sup> Anderson and Goodman, pp. 89-110.

The above foundations of the Markov model are by no means the extent of the model. The discussion was meant to give a general description of the transition matrix and the associated tests and assumptions. This is the basis for any Markov model. The rest of the chapter will describe the basic model and extensions in terms of its application for managing human resources.

#### The Basic Manpower Model

The model will be discussed here in a very simplistic manner. For any particular application, such as in an insurance company or a legal firm, the model would be modified to fit the situation and needs of the company. An example of how the general model can be used will be set forth in later chapters. For now, the discussion will be very general.

States. The transient states would be the different jobs, levels, or positions within the organization that are to be covered by the model. These positions do not have to be hierarchical in nature, such as the situation where the employees are trying to "climb the ladder" to success in the firm. However, the positions do have to be distinguishable from one another to ensure that the states are mutually exclusive. The number of transient states would equal the number of positions or jobs in the firm.

There is at least one absorbing state in the model. If only one, this state would be for the individuals that leave the system for any reason, i.e., death, resignation, firing. Depending on the desired information, there could easily be more absorbing states. For example, there could be an absorbing state for each reason for leaving the system given above.

The transition matrix. Once the states have been defined, the transition matrix can be formulated. Assume a situation where a company has three job levels, specified as levels A, B, and C. Using only one absorbing state, the transition matrix in its canonical form would be

	Leave		A	B	C	
Leave	1		0	0	0	
-----						
A	$P_{AL}$		$P_{AA}$	$P_{AB}$	$P_{AC}$	
B	$P_{BL}$		$P_{BA}$	$P_{BB}$	$P_{BC}$	
C	$P_{CL}$		$P_{CA}$	$P_{CB}$	$P_{CC}$	

The "L" in the subscripts corresponds with the "leave" absorbing state.

The I matrix in this case is just a scalar with a value of 1. The 0 partition is a (1 X 3) vector of zeros. This vector specifies that once the employee leaves the firm, there is no probability of re-entering the system into one of the three job

levels. It is easily conceivable that someone could leave the firm temporarily and then come back to the firm in the same or even a different position. This would imply that "L" is not truly an absorbing state, which is essential to the applicable theory. This can be dealt with in either of two ways. First, another state could be built into the model which would take care of the employees who left the firm but were expected to return. This would not be an absorbing state. Unfortunately, this would not completely solve the problem since some of the employees who returned to the firm would almost certainly do so unexpectedly. The second possibility would be to treat these people as entering the firm for the first time. Again, however, this may not solve the problem. These people who re-enter the firm may have different job mobility probabilities than one who is entering a position for the first time. An example of this may be someone who takes a leave of absence to perform research or continue their education in an area relevant to their job. If there are enough of these people to affect the stationarity of the transition matrix, it is possible to deal with the problem by developing a probability matrix for just this type of situation, along with the matrix for "normal" employees.

The R vector in the above example is a  $(3 \times 1)$  column vector which gives the probabilities of entering the single absorbing state from each job, during a particular time period. The Q partition is a  $(3 \times 3)$  matrix which shows the probabilities of moving from one

job to another during one time period. These two partitions (R and Q) are very important for the extensions of the basic model.

The probabilities for the stochastic transition matrix are derived from actual historical data, and can be found in one of two ways. First, a sample of employees can be traced from their original employment by the firm to their present status. The flow of these sample employees would be used to find the probabilities of movement between jobs for the entire population of employees for the particular firm.

The second method would be to deal with aggregates. The needed data would include the number of employees in each job at the beginning of the period, the movements between jobs during the period, and the number of employees who entered and left the firm during the period. It would also be essential to know what job level the entering employee filled, and what level the exiting employee was in just before he left. This information would be used to develop a transition matrix for each time period. These individual time period matrices would be averaged, as a general rule, to form the model P for the firm.<sup>12</sup> The importance of stationarity in the transition matrix becomes obvious here. Periodic transition matrices that fluctuate widely detract from the dependability and validity of the average  $p_{ij}$ . Hence, there is a need to test the matrix for

---

<sup>12</sup> If later periods are more relevant for modeling mobility practices of the firm, exponential smoothing may be used rather than a simple averaging technique.

stationarity, using a chi-square test. (See formulae (4-2), (4-2A), and (4-2B) above.) Testing for constant  $p_{ij}$  when modeling human resource movements refers to testing whether the particular time period affects the transition probabilities of job mobility.

The test for the presence of the Markov property would also need to be performed. (See formulae (4-1), (4-1A), and (4-1B) above.) This test would ensure that the future mobility of an employee is dependent only upon his present position with the firm. The length of time in the present position should not have an effect on the  $p_{ij}$ 's, according to the assumptions of the model.

The transition period. Much of the discussion of the transition matrix has referred to "a particular time period," during which certain occurrences are relevant. This time period is commonly called the transition period. The primary stipulation is that the time period be short enough that only one movement can be made during each period. For example, an employee would not move from level A to level B to level C during only one transition period. Most manpower models use a transition period of one year.

#### Extensions and Applications

Once the transition matrix has been formulated and the proper tests performed, the model is ready to be used to provide information to the researcher about the firm. The extensions and applications as discussed below are not in any special order of importance. The information that is desired most by the firm would be the most

important application in that particular situation. The purpose of the rest of the chapter is to show what information can be gleaned by the use of the model.

Rolling over the P matrix. Once the P matrix has been formulated from past data, it becomes the mobility model for one transition period. This can be extended to a transition matrix for longer time periods simply by "rolling-over" the P matrix or multiplying it by the original P. For a two-period transition matrix,

$$P_2 = P \cdot P \text{ or } P_2 = P_1 \cdot P_1$$

Generalizing, the n-period transition is given by the following: <sup>13</sup>

$$P_N = P_{N-1} \cdot P \quad (4-3)$$

This development is important for predicting manpower movements farther than just one period into the future. This can be understood better after initial state vectors and steady state developments have been explained.

Initial vectors. The initial vector can be stated in terms of probabilities or individuals. When stated in probabilities, the vector gives the chance that, in its initial position, an individual employee is in a particular state or job. This initial probability vector will be called  $y_0$ , where the subscript denotes the time period. In a first-order Markov chain, the initial probability

---

<sup>13</sup> Notation in the rest of the chapter abides by the rule that matrices are designated by capital letters, while vectors are designated by small letters.

vector can be used in conjunction with the transition matrix to find the probabilities of being in each job after one period. The formula is simply

$$y_1 = y_0 \cdot P \quad (4-4)$$

Generalizing, these probabilities can be found in the form of a state vector for any period, as follows

$$y_N = y_0 \cdot P_N \quad (4-5)$$

The state vector can be used to predict the probability of being in a certain job at a specified point in the future. Another way of interpreting this vector is that it would show the percentage make-up of the firm's employees in each job at some point in the future.

The initial state vector can also give the number of employees in each job at the commencement of the model. The information can be used in much the same manner as the probability vector above. Using  $w_0$  to denote the initial vector of employees, the general formula would be

$$w_N = w_0 \cdot P_N \quad (4-6)$$

This formula would give the predicted number of people in each job at some future period. A percentage breakdown of this vector should yield the  $y_N$  vector.

It should be noted that neither formula (4) nor formula (5) recognizes the influx of new employees during future time periods. These formulae only give the disposition of present employees. Since the model includes at least one absorbing state, it is evident that as  $n$  gets larger, the probability of being in the absorbing state tends to one. In other words, sooner or later all of the employees will leave the firm for some reason. To be more relevant for manpower planning purposes, some provision needs to be made for future hiring practices.

If the absorbing states are disregarded, the number of states would just be equal to the number of transient states. Let  $x_N$  be a vector that gives the number of employees at each level within the firm at the end of period  $N$ . It is easily seen that the number of elements in this vector would equal the number of transient states in the model, which also equals the rows (and columns) in the  $Q$  matrix. Let  $k_N$  be the vector which gives the number of people entering the firm at each level during transition period  $N$ . The vector  $x_N$  can now be computed as follows:

$$x_N = x_{N-1} Q + k_N \quad (4-7)$$

This formula simply means that the number of employees at each level at the end of a period is based on internal movements of employees who were present at the beginning of the period plus new employees entering during the period. This is consistent with the definition of the

transition period which says that it is short enough to allow for only one movement. Thus, employees who enter the system are not expected to change positions during their first transition period with the firm. The implications of this formula for manpower planning should be rather obvious. Before extending  $x$  into its steady state, the fundamental matrix must be explained.

The fundamental matrix. The formula for the fundamental matrix is

$$F = (I - Q)^{-1} \quad (4-8)$$

The elements of the fundamental matrix give the number of times the process is in state  $j$ , given that the process began in state  $i$ . Since  $Q$  is used in the formula, the fundamental matrix only relates to the transient states. This makes sense, since once an absorbing state is entered, it is never left, by definition. Thus,  $f_{ij}$  for any absorbing state should be infinity and tells nothing. Each  $f_{ij}$  for the transient states would tell how many transition periods, on the average, an employee would be in each job before he is absorbed, or leaves the firm, depending on the position he filled when he entered the firm. This could especially be of interest to someone who is trying to make a decision on whether to go to work for the firm. As with any figure that is an average, using the value as an absolute for predicting could cause problems. Hence, the variances of the elements in the fundamental matrix may be relevant. These are given by

---

<sup>14</sup>For proof, see Kemeny and Snell, pp. 46-47.

$$F_{\text{VAR}} = F (2 F_{\text{dg}} - I) - F_{\text{sq}} \quad (4-9)$$

where the subscript (dg) means that all elements that are not on the primary diagonal equal zero, but the primary diagonal elements are left intact.<sup>15</sup> Also, the subscript (sq) means that each element in the matrix should be individually squared.

This matrix of variances can be used as any variance is used. The square root of the variance is the standard deviation. This can be used to find confidence intervals around each  $f_{ij}$  in the normal statistical fashion.

Based on the definition of  $F$ , summing up each row would give the total average amount of time an employee spends with the firm, based upon his entering position. Symbolically, this can be written:

$$t = Fe \quad (4-10)$$

where  $e$  is a column vector of all ones. In some situations, this may yield very misleading results. It must be remembered that, since this is an average, it is affected by those who leave the firm after only a short tenure, as well as those who build a long-term career with the company. Again, since this is an average, a variance can be computed, as follows:<sup>16</sup>

$$t_{\text{VAR}} = (2 F - I) t - t_{\text{sq}} \quad (4-11)$$

As with the  $f_{ij}$  and their variances, both  $t$  and  $t_{\text{VAR}}$  must be studied when using the figures as bases for prediction.

<sup>15</sup>For proof, see Kemeny and Snell, pp. 49-50.

<sup>16</sup>For proof, see Kemeny and Snell, p. 51.

Absorption information. If the model has only one absorbing state (leaving the firm), then no matter what job the person enters when he is hired, he will eventually be absorbed into this one state. Thus, the probability of being absorbed in state  $j$ , given that entrance into the system occurred at state  $i$ , would always be one. However, if there were more than one absorbing state in the model, common sense will not normally tell in which state the employee will be absorbed. Nevertheless, the probabilities of absorption into each particular state in the ergodic set, based on the entering state, are given as<sup>17</sup>

(absorbing state)

$$B = FR \quad (4-12)$$

where  $R$  is one of the original partitions of  $P$ . The importance of this information depends on the particular situation of the firm, and also the ingenuity of the person who is using the model. For instance, Churchill and Shank built into their model an absorbing state that designated that the employee had reached "top management." The  $b_{ij}$  in that case would give the chances of reaching "the top," again depending on where the employee began with the firm.<sup>18</sup>

Once the chances of being absorbed into each state have been computed, then the initial number of employees (the  $w$  vector) that will ultimately be absorbed into each state can also be found, as follows:

$$g = wB \quad (4-13)$$

<sup>17</sup>For proof, see Kemeny and Snell, pp. 52-53.

<sup>18</sup>Neil C. Churchill and John K. Shank, "Accounting for Affirmative Action Programs: A Stochastic Flow Approach," The Accounting Review 50 (October 1970):653-654.

The variance of each element of the  $g$  vector can also be found by<sup>19</sup>

$$g_{VAR} = w_t [yB - (yB)_{sq}] \quad (4-14)$$

The term  $w_t$  in the above formula is the number of times the Markov process is started. This corresponds to the total number of employees in the system and is equal to the sum of the individual elements of the  $w$  vector. The  $y$  vector in the above formula is the initial probability vector as previously defined. All elements of this  $g_{VAR}$  vector will be equal.

Steady state uses. With the fundamental matrix, steady state vectors can be computed which give the "plateau" of manpower which will be reached if the firm continues on a certain course. Remember that  $k_N$  is the hiring policy vector (number of incoming employees during a transition period) for the firm. Assume that this vector can be taken as a constant for all transition periods. This may not be as outlandish an assumption as it appears on the surface. A company projecting a fairly steady growth over the foreseeable future may have a hiring policy which is about the same each year. If the  $k$  vector remains the same, and the transient states of  $P$  remain constant (as defined), then steady-state levels of employees in each job can be computed as follows:

---

<sup>19</sup>For detailed discussion and proofs, see R. M. Cyert, H. J. Davidson, and G. L. Thompson, "Estimation of the Allowance for Doubtful Accounts by Markov Chains," Management Science 8 (April 1962):287-303.

$$x_{SS} = k F \quad (4-15)$$

where the subscript (SS) designates that the vector is a steady-state vector. Once the steady state is reached, the number of employees in each position would stay the same from one period to the next, as given by  $x_{SS}$ .

The variance of each component of  $x_{SS}$  can be computed as follows:<sup>20</sup>

$$x_{SS} \text{ (VAR)} = K[\mu F - \sum_{i=0}^{\infty} (\mu Q^i)_{sq}] \quad (4-16)$$

where  $K$  is a constant equal to the total number of employees hired each period, and  $\mu$  is the vector of hirings ( $k$ ) stated as percentages of total hirings rather than number of employees hired. The upper bound of each variance can be estimated by the following formula:<sup>21</sup>

$$x_{SS} \text{ (VAR)} \leq K[\mu F - \mu_{sq} (I - Q_{sq})^{-1}] \quad (4-17)$$

This upper bound estimate can be used for computations in place of the variance, although the user should remember that this is an estimate and that the actual variance could be a lesser amount.

Since the number of employees at each level will remain constant at steady state, it follows that the total number of employees would also remain constant. This figure can be found by summing the employees in each state, or symbolically as:

$$X = (x_{SS}) e = k F e \quad (4-18)$$

<sup>20</sup>For proof, see Cyert, Davidson, and Thompson, p. 296.

<sup>21</sup>For proof, see Cyert, Davidson, and Thompson, p. 297.

The variance and upper bound estimate of the variance can be computed<sup>22</sup>

$$X_{\text{VAR}} = K[\mu F e - \sum_{i=0}^{\infty} (\mu Q^i e)_{\text{sq}}] \quad (4-19)$$

$$X_{\text{VAR}} \leq K[\mu f - \mu_{\text{sq}} (I - Q_{\text{sq}})^{-1}]e \quad (4-20)$$

Notice that the upper bound estimate as given by formula (4-20) is simply a sum of the elements for the upper bound estimates of the variances of  $x_{\text{SS}}$  as given by formula (4-17).

The steady state also means that the number of employees leaving the firm would equal the number of people entering the firm. The assumed number of entrants would be attained by summing the elements in the  $k$  vector, or symbolically by  $(k \cdot e)$ . If there is more than one absorbing state in the model, the steady state number of employees entering each of these states (and thus leaving the system) can be found by

$$l_{\text{SS}} = k B \quad (4-21)$$

Thus, by definition of steady state,  $(k \cdot e) = (l_{\text{SS}} \cdot e)$ . The  $e$  column vector of all ones is adjusted dimensionally to fit multiplicative purposes in each case.)

The variances and upper bound estimates of the variances for  $l_{\text{SS}}$  can be computed as follows:<sup>23</sup>

$$l_{\text{SS}} (\text{VAR}) = K[\mu B - \sum_{i=0}^{\infty} (\mu Q^i R)_{\text{sq}}] \quad (4-22)$$

$$l_{\text{SS}} (\text{VAR}) \leq K[\mu B - \mu_{\text{sq}} (I - Q_{\text{sq}})^{-1} R_{\text{sq}}] \quad (4-23)$$

where all notation has been previously defined.

<sup>22</sup>Cyert, Davidson, and Thompson, pp. 296-7. The proofs of these two formula are discussed very briefly, but not formally presented.

<sup>23</sup>Ibid.

Each element of  $\lambda_{SS}$  gives the number of employees entering an absorbing state during each particular time period at steady state. The ultimate disposition of the total employees at steady state at any particular time can be formed by:<sup>24</sup>

$$m = k F^2 R \quad (4-24)$$

In this formula the fundamental matrix was squared rather than squaring each element. The sum of the elements of  $m$  should equal  $X$ , the total employees at steady state.

The variances of the elements of  $m$  are given by:<sup>25</sup>

$$m_{VAR} = X [(1/X)kF^2R - ((1/X)kF^2R)_{sq}] \quad (4-25)$$

In all cases with the variance vectors as given above, standard deviations are found by computing the square root of each element in the variance vector (or upper bound estimate vector).

Steady state information can have implications for the promotion and hiring practices of the firm. The model tells what mix of employees will occur once the steady state is reached, and also what happens to the people that leave the firm (depending on the set-up of the model). These figures can be compared to the type of steady state situation the management of the company desires. A significant variance would mean that some change needs to be made. These changes can be made in one of two areas, coinciding with the input that was held constant to compute the steady state figures. First, the hiring policy could be changed;

---

<sup>24</sup>For discussion, see Cyert, Davidson, and Thompson, pp. 295-6. No proof is presented.

<sup>25</sup>Ibid.

i.e., change the  $k$  vector. In fact, the flexibility of the model allows management to test any new hiring policy by simply plugging the new  $k$  vector into formulae (12) and (13), and again comparing the result against the desired situation.

The alternative is to change promotion policies; i.e., change  $Q$  (and thus implicitly  $R$ , since the transition matrix must remain stochastic). Again, different policies can be tested by changing  $Q$  and  $R$  and following through to the resulting  $x_{SS}$  and  $l_{SS}$ .

Time in state. The elements of the fundamental matrix give the number of transition periods an average individual is in each state, given the starting point. This does not mean that the time periods are consecutive. It is possible for the individual to leave the state and return during a future transition period. The  $f_{ii}$  would include all periods spent in the state.

It could be useful to know how long an individual stays in one job once he has entered it. This does refer to consecutive time periods, and can be formed by the following:<sup>26</sup>

$$a_i = 1/(1 - p_{ii}) \quad (4-26)$$

As with the  $f_{ij}$ , this is an average and the variance can be computed<sup>27</sup>

$$a_{VAR} = p_{ii}/(1 - p_{ii})^2 \quad (4-27)$$

There is one situation where the  $f_{ii}$  should equal the  $a_i$  in the above vector. This would occur when there is no chance of the process

---

<sup>26</sup>For proof, see Kemeny and Snell, p. 61

<sup>27</sup>Ibid.

returning to a transient state once it has been exited. This is possible when the process models a hierarchy of jobs in the firm, and there are no demotions. The employee stays where he is, moves up, or moves out. Thus, no return to a state is possible. It should be noted the  $a_i$  includes the entering step.

Chance of particular movements. If it is known that an individual is going to leave his present job, new  $p_{ij}$  can be computed<sup>28</sup>

$$p'_{ij} = P_{ij}/(1 - P_{ii}) \quad (4-28)$$

Many universities have a policy that a faculty member receives tenure after the sixth year, or his contract is not renewed. Given that there is going to be a move after the sixth year,  $p'_{ij}$  can be computed to show the chances of moving to a tenured state or of leaving the university.

It may also be desirable to know the probability, if a process starts in one transient state, of the individual ever being in the other transient states. These probabilities for each state are found by the following:<sup>29</sup>

$$H = (F - I)^{-1} F_{dg} \quad (4-29)$$

The  $h_{ij}$  would give the probability of an employee occupying a particular job at any time in the future, based upon his entry point into the firm.

---

<sup>28</sup>Ibid.

<sup>29</sup>For proof, see Kemeny and Snel, p. 62.

Number of state changes. The number of job changes before leaving the firm (being absorbed) may be useful information, particularly in a non-hierarchical situation. The first step for arriving at this information is to develop a new transition matrix. Each  $p_{ii}$  is set equal to zero. Each  $p_{ij}$  is then found by summing the elements in the row, and dividing each element by this row sum. Using formulae (4-8), (4-10), and (4-11), a new fundamental matrix,  $t$  vector, and variances of the  $t$  vector can be computed. The new  $t$  vector, designated as  $t_c$ , gives the mean number of changes of states for the original process, based on entry point. The new  $t_{VAR}$ , designated  $t_{c(VAR)}$ , gives the variance of the same function. Thus, the average number of times an employee changes jobs, but still remains with the firm, can be found, along with its variance.

Conditional probabilities. If it is somehow known that a process is going to be absorbed in a particular state, or even if this is just assumed, a conditional  $Q$  matrix can be formulated

$$\hat{Q} = v^{-1} Q v \quad (4-30)$$

The  $V$  matrix above has zeros for all elements that are not on the primary diagonal. The primary diagonal entries are the appropriate entries from the computed  $B$  matrix (formula (4-12)), where  $V_{ii} = B_{ji}$  for  $i = 1$  to  $d$  (the number of transient states) and  $j$  is the absorbing state conditionally assumed.

The elements of the conditional matrix would give the mobility probabilities for the transient states, assuming that the end result is

---

<sup>30</sup>For proof, see Kemeny and Snell, p. 63.

<sup>31</sup>For discussion, see Kemeny and Snell, p. 64. No formal proof is presented.

known. This assumption of certainty as to the final disposition of the process could significantly change the transition probabilities. The assumption that the president's son will one day automatically become the president changes the probabilities of his movements before his transition into the presidency.

A conditional fundamental matrix can be found in the same manner as in formula (4-8),

$$\hat{F} = (I - \hat{Q})^{-1} \quad (4-31)$$

Each element  $\hat{f}_{ij}$  gives the average number of transition periods the process is in state  $j$ , given that the process began in state  $i$ , and also given that the absorbing state is known. Not only would the son of the president's transition probabilities be different, but also the amount of time he spends in each job. Other conditional information may be attained by using  $\hat{Q}$  and  $\hat{F}$  in place of the original  $Q$  and  $F$  in the model. In each case, the meaning of the elements is the same as before, except that the absorbing state is given.

### Summary

This chapter has set forth the formulae for the conceptual Markov model for use by a firm. Non-financial aspects of the model were all that were covered in this chapter. While the manpower planning facets of the model were stressed, in total the conceptual model relates to human resource management. Some of the information derived from the model may not be directly useful to manpower planning, but may be

relevant for decision-making purposes in a comprehensive program for development and management of human resources.

The development of the model has been very general. Certain parts may become extraneous for some firms. The purpose of the chapter was to show the information that could be gleaned from the model, and not all of this is necessary in every case.

Furthermore, there is more information that can possibly be inferred from the model. This information is of the common-sense type that may be extracted when studying both the input and results. This was not discussed in the chapter since the actual numbers must be examined to make this type of inference, and also since the particular situation of the firm being modeled may be important.

## Chapter V

### THE CONCEPTUAL MODEL--II

The previous chapter discussed the nonfinancial aspects of the Markov model as it relates to human resource management. In Chapter III, the well documented models of human resource valuation were briefly outlined. The financial phase of the conceptual model, as presented in this chapter, will show the applicability of the Markov model to the different valuation bases, i.e., economic valuation, historical cost, opportunity cost, and replacement cost.

Possible model applicability can be in two areas. First, the model can be used to determine the value of human assets to be recorded. Second, the model may aid in the determination of an amortization period over which the asset value is to be allocated. Each valuation base will be viewed in these two areas of applicability.

#### Economic Valuation

The economic valuation of an asset is defined as the present value of all future expected benefits.<sup>1</sup> The problem of uncertainty of these future benefits has led accountants towards the more

---

<sup>1</sup>AAA Committee on Accounting Concepts and Standards, "Accounting and Reporting Standards for Corporate Financial Statements and Preceding Statements and Supplements," (Columbus, Ohio: American Accounting Association, 1957), p. 4.

objective practice of using cost as a basis for recording assets, even though an economic valuation is more theoretically correct. In the case of human resources, the Markov model allows a simulation of future events relating to personnel policies and movements. This means that an economic valuation can be found, if (1) the Markov model is accepted as being a valid representation of manpower mobility, and (2) some method of actually placing a financial valuation on each employee based on expected benefits can be established.

The validation of the Markov model would be based upon the tests of the model set forth in the previous chapter. Acceptance of the model would also be based upon the predictive ability of the model.

The method of putting a "dollars-and-cents" valuation figure on employees will vary according to the situation of the firm. Some possibilities were discussed in Chapter III. For the model to be helpful, it is likely that a valuation can be put on each transient state of the model. If these states correspond to possible jobs or positions within the company, then the valuation is not a personal one. The set of people being valued are viewed as a set of jobholders rather than allowing personal characteristics to affect the valuation. It is important to note that it is a homogeneous group of people being valued instead of each individual. If each person were being valued, personal characteristics and other exogeneous factors may have a profound effect on actual results as compared to the model. However, applying the model to groups of people in each state

minimizes the effect of the experience of any one person. The model thus portrays an "average" person out of the group. The probabilities in the transition matrix indicate the percentage of the number of employees in the applicable group which, on the average, are expected to move from one state to another. This is opposed to the view that the probabilities specify the chance that a particular employee has of making the transition. This may sound like just a question of semantics, but it may become particularly important when applying the model, especially when explaining the test for the Markov property in the transition matrix.

In the conceptual model as continued below, it is assumed that a value,  $z$ , can be put on each member of a group situated in a certain state within the transition matrix. The particular method can differ by firms and will not be discussed. However, one method will be used when the model is applied in the next chapter.

Valuation model. This human resource valuation will be based upon the use of the Markov model to predict the number of employees in each state (professional level) at the end of each transition period in the future. The number of employees will include only those who are with the firm on the date of valuation. Any employees entering the firm after that date should not be included in the human asset valuation. Once the prediction of employee distribution has been ascertained, it can be multiplied by the value of each member in a particular state, giving a total value for each state.

Summing the states' values gives the total value of the firm for a particular period, as will be shown.

Formula (4-3) can be used to "roll-over" the transition matrix (P). This gives transition matrices for longer than just one period. Formula (4-5) can be used to predict the number of employees (who are presently part of the firm) in each state at the end of any period in the future. Thus, multiplying this vector of employees in each state by a vector of the values associated with the member of each state will provide the total value of benefits derived from human assets for a period. This is shown symbolically as:

$$Z = w_n \cdot z \quad (5-1)$$

where  $w$  is a row vector and  $z$  is a column vector. The multiplication of the two yields a scalar. However, this figure is not the value of all human assets as it should be recorded on a statement of financial position for the firm. This denotes only the value of benefits derived from human assets for one particular period. The theoretical definition of an asset says that these values should be summed over all years. Thus, the formula should be

$$Z = \sum_{n=0}^{\infty} (w_n \cdot z) \quad (5-2)$$

This assumes that the  $z$  vector of values for a member of each state is the same for each year. The summation is from zero to infinity because the initial (or present) vector of employees is designated as  $w_0$ .

The asset definition also says that it is the present value of the future benefits which is relevant for the valuation. As it stands, formula (4-2) does not specify present value. The formula needs to be modified to incorporate this into the model, as follows:

$$Z = \sum_{n=0}^{\infty} \frac{(w_n \cdot z)}{(1 + r)^n} \quad (5-3)$$

where  $r$  is the discount rate to be used. Since the comprehensive model is an attempt to better manage human assets and the investments in them, the best  $r$  would be the cost of capital for the company.

Valuation period. Notice that both (5-2) and (5-3) sum over infinity as implied by the definition of an asset as set forth above. This is theoretically correct, but not practical. The  $p_{ij}$ 's in the original transition matrix are probabilities and as such are between zero and one, inclusive. In fact, only  $p_{ii}$  for the absorbing state(s) of the model would be equal to one. All other elements of  $P$  would be either zero or  $0 < p_{ij} < 1$ . When the transition matrix is multiplied by itself, or rolled over, these elements of  $P$  decrease as  $n$  increases. These elements which are between zero and one tend to become close to zero quicker when the effect of finding the present value in (5-3) is taken into account. Hence, Jaggi and Lau state that it is unlikely that the value of  $p_{ij}^{(n)}$  would be greater than zero for a valuation period of 40 years.<sup>2</sup>

---

<sup>2</sup>Bikki Jaggi and Hon-Shiang Lau, "Toward a Model for Human Resource Valuation," The Accounting Review, 49 (April 1974):326.

Using this rationale for a valuation period, formula (5-3) would be modified as follows:

$$Z = \sum_{n=0}^{40} \frac{w_n \cdot z}{(1+r)^n} \quad (5-4)$$

This expression gives the economic valuation of the human assets in the firm.

Gains or losses. Since this same type of economic valuation can be performed each period, an arbitrary amortization period does not need to be chosen as a method of allocation. Simply the difference between Z for year one and year two is the gain or loss on human assets for the period. If Z increases from one year to the next, there would be a gain, and if Z decreases, a loss would have occurred.

#### Historical Cost

Using historical cost as a basis for recording human assets would be consistent with current accounting practice.<sup>3</sup> The "valuation" method records and capitalizes actual costs associated with acquisition and development of personnel. These costs can be attributed to many areas (see the R. G. Barry model in Chapter III). However, it is not the problem of cost capitalization which can be alleviated by the Markov model. The historical cost principle

---

<sup>3</sup>American Institute of Certified Public Accountants, APB Accounting Principles: Original Pronouncements, "Statement No. 4," Chapter 6, paragraphs 145 (P-1) and 182 (M-1), (Chicago: Commerce Clearing House, Inc., 1972).

records assets "at cost," and then amortizes or depreciates these assets over a "useful life." It is the problem of choosing an appropriate amortization period that is more relevant here. For human assets, the "useful life" should be the employee's tenure with the firm. Unfortunately, there is no way of knowing how long any person is going to be with the company. Nevertheless, employee tenure and also career movements inside the organization can be predicted using the conceptual Markov model.

The Markov prediction of the length of time a person would remain with the firm is based on an "average employee" and is given by the fundamental matrix and the  $t$  vector (see formulae (4-8) and (4-10)). The fundamental matrix gives the number of transition periods the person would be in each transient state before absorption. Again it is stressed that this is an average figure and is affected by employees who stay only a short time as well as by those who make a career with the firm. The elements in the fundamental matrix are also based upon the position the employee filled upon entrance into the company. The  $t$  vector simply sums the rows of the fundamental matrix to give the total predicted time with the firm, dependent upon entering position.

In the example in the previous chapter, there were three transient states corresponding to three job positions within the company. It is conceivable that an employee would be expected, on the average, to spend more or less time with the company depending on

his initial job. The  $t$  vector would thus show expected tenure for each of the three positions being the initial state. These elements of the  $t$  vector could be used as the amortization period over which costs could be allocated. In this example there could be as many as three different amortization periods. Capitalization costs would not have to be associated with a particular employee, but these costs would at least have to be associated with an initial state before the appropriate amortization period can be chosen. One method of computing an amortization expense for a period would be

$$\text{Amortization Expense} = \sum_{i=1}^d \frac{C_i}{t_i} \quad (5-5)$$

where  $C_i$  is the capitalized cost of the  $i$ th transient state,  $t_i$  is the  $i$ th element of the  $t$  vector, and  $d$  equals the number of transient states. Since the model can change from period to period, it may be better for the company to use an amortization percentage for each group for each year. These percentages would be given by

$$\text{Amortization \%} = \frac{1}{t_i} \quad \text{for } i = 1 \text{ to } d \quad (5-6)$$

The simplest amortization method based on the above would be straight-line amortization computed for each group for each transition period. The key word here is that the "group" is being amortized rather than the single employee. Certain members of this group will stay with the firm much longer than the average period specified by  $t_i$ , while others will be with the firm a much shorter time. These

occurrences would be disregarded. Using this technique, an average amortization is being recorded. Thus, amortization for the first  $t_i$  periods would most likely be higher than a theoretical charge against income based on amortization and losses from termination of employment. After  $t_i$  years, there would be no amortization under this technique, but theoretically amortization should still be taken if (1) any of the personnel hired the period in question are still with the firm, and (2) assuming the company is still deriving benefits because of these employees.

A better method? A better method of amortization would allocate costs over the useful service life of each employee rather than using an average. Although the method presented below does not completely alleviate this problem, it does take into account deviation from an average. In fact, it is standard deviations which are used.

Formula (4-11) gives the computation for the variances of the elements in the  $t$  vector. The standard deviation of  $t_i$  is the square root of the variance. Multiply this standard deviation by three and add the product to  $t_i$ . This would give a time period equaling the mean plus three standard deviations. Of the employees hired this period, over 99% should leave the firm before  $(t_i + 3\sigma)$  periods have passed. This time period now becomes the basis for allocation. Amortization would be on a straight-line basis, but the percentage would now be

$$\text{Amortization \%} = \frac{1}{t_i + 3\sigma} \text{ for } i = 1 \text{ to } d \quad (5-7)$$

Losses from termination of employees can now be reflected in the accounts. To do this, some value has to be allocated to each member of the group. Since up to this point the group has been assumed to be homogeneous for purposes of the model, the "value" of each member of the group would be the same. Thus, at any point in time, the basis of an employee would be the total unamortized capitalized cost associated with the group divided by the number of employees still left in the group. When an employee leaves the firm, the amount of his basis would be a charge against income.

At the end of the  $(t_i + 3\sigma)$  time period, less than 1% of the members of the original group would still be with the firm. Theoretically, there should be amortization on these human assets until their termination from the firm, again assuming that the firm is deriving benefits by keeping these employees. The second amortization method presented above disregards this final amortization, inherently assuming it to be insignificant.

No amortization period. A final method for charging costs against the income of the period is not based on an amortization period. Any charge would be from a loss caused by personnel leaving the firm. The human resource costs would follow the flow of the human resources. This flow is predicted by the transition matrix. A vector of the balances in the asset account at each level as of the beginning of the year is multiplied by the transition matrix to find where these costs should be at the end of the period. Any costs incurred during the year

would be added to these ending totals to find the ending balance in the human asset account, broken down into balances for each level. In order to use this method, costs must be categorized as to which homogeneous group of employees (staff, managers, or partners) the costs apply.

The formula to use this method is as follows:

$$(C_{n-1} \cdot P) + C_c = C_n \quad (5-8)$$

where  $C_{n-1}$  is the capitalized cost at the beginning of the period,  $C_c$  is the current cost to be capitalized, and  $C_n$  is the ending distribution of costs. This formula is analogous to formula (4-7) which computes the flow of personnel during the period. The amount which would be charged against income would be the sum of the elements in the  $C_n$  vector which signify the employee has left the firm. In the model from Chapter IV, this would simply be the "Leave" state.

This method of applying costs would be useful if the firm desired to set up a budget for human resource costs. Using the model transition matrix for the period and budgeted current costs, budgeted end-of-year balances for each level could be computed using formula (5-7). At the end of the period, an actual transition matrix for the year and actual current costs would be used to give the actual end-of-the year balances. Any variance could be caused by one of two factors. Staff flows could have been different than expected, i.e., the transition matrices were different (a flow variance). The other cause would be that actual current costs were different than expected current costs (a cost variance).

### Opportunity Cost

Hekimian and Jones (see Chapter III) presented a model using opportunity costs (competitive bidding) for human resource valuation. The human assets to be recorded are only those that are considered to be "scarce resources." This does not include personnel that can be readily hired from outside the company. The assets would be recorded at the bid price which was successful in acquiring the employees. Their scheme does not take into account any periodic allocation of the recorded asset value. They do recognize that humans can lose their value and that some procedure may be needed to recognize this, but that is as far as they go. Under these circumstances, the Markov model does not appear applicable for valuing human assets. It is doubtful that enough successful competitive bidding occurs that even allows a researcher to develop the conceptual model for a firm.

### Replacement Cost

Flamholtz defines human resource replacement cost as "the sacrifice that would have to be incurred today to acquire a substitute capable of rendering a set of services equivalent to that of the person presently employed."<sup>4</sup> He then modifies this to refer to what he calls "positional replacement cost," which is defined as "the sacrifice that would have to be incurred today to replace an individual in a specified position with a substitute capable of providing

---

<sup>4</sup>Eric G. Flamholtz, "Human Resource Accounting: Measuring Positional Replacement Costs," Human Resource Management, 12 (Spring 1973):10.

an equivalent set of services in the given position."<sup>5</sup> Since the conceptual model presented deals with homogeneous groups of people in a job or position, this positional replacement cost would be more relevant.

Flamholtz classified the elements of positional replacement cost into three main categories: acquisition cost, learning cost, and operation cost. These categories respectively relate to costs of acquiring a new position-holder, costs to train and educate the new position-holder, and costs of losing the position-holder. As with historical costing, the conceptual model does not appear applicable to the problem of which costs should be aggregated and capitalized. The applicability of the model is in the same aspect as historical costing, the amortization period. There is no reason why the same amortization methods as presented for historical costs should not be pertinent to replacement costing also. The only change in formula (5-5) is that the capitalized replacement cost would replace capitalized historical cost ( $C_i$ ).

### Conclusion

The conceptual Markov model for manpower planning can be an integral part in the development of a human resource valuation system. This chapter showed how the model could be used with different valuation bases, such as economic valuation, historical costing, and replacement costing. The model was not found to be applicable with an

---

<sup>5</sup>Ibid.

opportunity cost valuation base as set forth by Hekimian and Jones. The methods presented in this chapter are by no means the only utilizable techniques for applying the Markov model to human resource accounting.

Like Chapter IV, this chapter is meant to show generalizable models that may be used. The next chapter will relate the conceptual model, both financial and nonfinancial, to a Certified Public Accounting firm as an example of how the model can be used in a special situation.

## Chapter VI

### APPLICATION OF THE MODEL--I

The model will be presented in this chapter as it could apply to a public accounting firm.

#### Background

There can be little doubt that the professional staff of a Certified Public Accounting (CPA) firm is its most important resource. The importance of the resource should be reflected by the importance attributed to the management of these personnel. Manpower planning should be an essential element in the comprehensive planning program of the firm. Very significant amounts are invested by these firms in recruiting, training, and developing their professional staff. Finally, human resource management in public accounting firms presents multiple problems. All of the above are justifications for a CPA firm to utilize available sophisticated techniques to aid in the management of personnel.

The hierarchy. The professional staff<sup>1</sup> of a CPA firm occupy a hierarchy of positions within the organization. Although terminology and the organization chart will differ from firm to firm, there are

---

<sup>1</sup>Although a CPA firm hires other employees, such as secretaries, the model will only be developed for professional accounting staff.

basically four levels in the hierarchy. For use in the model, these levels will be specified as junior, senior, manager, and partner (listed in order from the bottom to the top of the organizational ladder).

Areas of specialization. There are three broad areas of specialization within most public accounting firms.<sup>2</sup> These areas are auditing, taxation, and management advisory services (MAS). For employees in the auditing or taxation areas, normal entry into the firm is at the lowest or junior level. Employees who are recruited specifically for the MAS area may enter the firm at any level, depending on the education and experience of the particular recruit.

Promotional policy. Regardless of the area of specialization, promotions only allow the employee to move up one level at a time. A junior will not be promoted directly to the manager or partner level, and a senior will not be promoted directly to partner. There are no demotions in a CPA firm. This is a result of the "up-or-out" policy as explained below.

"Up-or-out." Personnel policies in CPA firms follow the pattern of the employee moving upward in the firm or ultimately being dismissed. An employee at the junior, senior, or manager level

---

<sup>2</sup>The size of the firm dictates whether a single employee will be classified as having only one area of specialization. Also, not every firm necessarily performs services in all three areas.

will not remain at that level indefinitely. This does not mean that promotional policies are a constant, e.g., at the end of two years as a junior, the employee is either promoted or dismissed. Time in grade before promotion will fluctuate according to employee performance and manpower needs of the firm.

The "up-or-out" policy ensures that employees who remain with the firm will eventually man upper level staffing positions with the firm. Another important aspect of personnel management in public accounting firms is that the primary source of manpower for these upper level positions is through promotion from lower levels. Firms almost exclusively look internally for staff to promote to middle and upper management positions.<sup>3</sup> This accentuates the need for a formal manpower planning program. There must be an adequate supply of personnel in the internal labor market to adequately man upper level positions as needed by the firm.

These policies may have certain motivational effects which would need to be taken into account when managing human resources. An employee who stays with the firm has expectations with relation to promotions, and non-fulfillment of these expectations could have an adverse effect on behavior. These expectations are probably based, at least to some degree, on the employee's perception of past

---

<sup>3</sup> While this is true in the areas of auditing and taxation, it may not necessarily be the case with employees performing management advisory services. This is consistent with the fact that MAS people may be recruited at any employment level.

promotional policies of the firm. Employees view these promotional policies as part of the reward structure of the firm. Any drastic altering of these policies will influence morale and/or productivity and may even result in undesired turnover.

As a direct result of the factors discussed above, reaching the partnership or top level in the firm is stressed in public accounting firms. Any employee who has aspirations of a career with a CPA firm is synonymously aspiring to become a partner with the firm. The development of the model will proceed in such a way as to generate data about the achievement of the partnership level. These data are be used in the area of recruiting, as well as in manpower planning.

#### The Model: Transition Matrix

The development of the model begins with the formulation of the transition matrix. The first step is to define the states of the matrix. In this case, the transient states would be the different attainment levels within the firm: staff, manager, and partner.<sup>4</sup> The "staff" level is a combination of the two lowest levels of the firm, the junior and senior levels.<sup>5</sup> The absorbing states would be for the personnel who leave the firm. Because of the importance attributed

---

<sup>4</sup>State definition is left to the creative ability of the researcher. States may be modified or drastically altered according to the results of tests on the model and also according to the informational needs of the firm.

<sup>5</sup>The model is being developed in a consistent manner as applied to a specific CPA firm in Chapters VII and VIII. Available data segregated employees into only three levels rather than the normal four levels.

to attaining the partnership level, there will be an absorbing state for partners who leave the firm for any reason. The other absorbing state is for employees who leave the firm from any level other than partner. Presented in the canonical form,<sup>6</sup> the transition matrix would be as follows:

TO: FROM:	(L) Leave	(LP) Leave as Partner	(S) Staff	(M) Manager	(P) Partner
Leave	$P_{LL}$	$P_{L(LP)}$	$P_{LS}$	$P_{LM}$	$P_{LP}$
Leave as Partner	$P_{(LP)L}$	$P_{(LP)(LP)}$	$P_{(LP)S}$	$P_{(LP)M}$	$P_{(LP)P}$
P = Staff	$P_{SL}$	$P_{S(LP)}$	$P_{SS}$	$P_{SM}$	$P_{SP}$
Manager	$P_{ML}$	$P_{M(LP)}$	$P_{MS}$	$P_{MM}$	$P_{MP}$
Partner	$P_{PL}$	$P_{P(LP)}$	$P_{PS}$	$P_{PM}$	$P_{PP}$

The partitioning of the P matrix specifies the I, O, R, and Q matrices as shown by the canonical matrix in Chapter IV. The I matrix has dimensions of (2 x 2) and is as follows:

$$I = \begin{matrix} & \begin{matrix} \text{Leave} \\ \text{Leave as Partner} \end{matrix} \\ \begin{matrix} \text{Leave} \\ \text{Leave as Partner} \end{matrix} & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{matrix}$$

<sup>6</sup>The canonical transition matrix and its partitions are presented in Chapter IV, as follows:

$$P = \begin{bmatrix} I & O \\ R & Q \end{bmatrix}$$

The ones on the primary diagonal of the 2-state matrix denote that once an employee has left the firm, he will not return.<sup>7</sup> The first state shows this for employees who leave before becoming partner, while the second state is for those who leave after becoming partner.

The O Matrix is a (2 x 4) matrix of all zeroes, as shown:

$$O = \begin{array}{cc} & \begin{array}{ccc} \text{Staff} & \text{Manager} & \text{Partner} \end{array} \\ \begin{array}{c} \text{Leave} \\ \text{Leave as Partner} \end{array} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

This formally denotes that there is no probability of movement back into one of the employment levels of the firm by someone who has left the firm for any reason.

The R Matrix gives the probabilities of moving from a transient state to an absorbing state.

$$R = \begin{array}{cc} & \begin{array}{cc} \text{Leave} & \text{Leave as Partner} \end{array} \\ \begin{array}{c} \text{Staff} \\ \text{Manager} \\ \text{Partner} \end{array} & \begin{bmatrix} P_{SL} & 0 \\ P_{ML} & 0 \\ 0 & P_P(PL) \end{bmatrix} \end{array}$$

The zero in the (1, 2) position designates that a staff employee cannot leave the firm as a partner. The remaining zeroes

---

<sup>7</sup>How to deal with the situation in which employees actually do return to the firm is discussed in Chapter IV.

in the matrix can be interpreted in the same manner.

The Q Matrix gives the relationships among the transient states.

Q =	Senior	P <sub>SS</sub>	P <sub>SM</sub>	0
	Manager	0	P <sub>MM</sub>	P <sub>MP</sub>
	Partner	0	0	P <sub>PP</sub>

The zero in the (2, 1) position specifies that there is no probability of movement from manager back to staff, i.e., there is no demotion. The other zeroes below the primary diagonal can be similarly interpreted.

The zero in the (1, 3) position of the matrix signifies that an employee will not be promoted directly from the staff position to the partner position. The other zeroes above the primary diagonal may be interpreted in this same manner. The  $p_{ij}$ 's in both the R and Q matrices are the probabilities that must be computed by applying the model to actual historical data (see the next chapter).

The transition period. As stated in Chapter IV, the transition period should be short enough so as to allow only one movement per period. For CPA firms, a transition period of one year should suffice. However, if the firm hires extra professional help during the tax season, a problem arises. If these employees are included in the model, the transition period would need to be short enough so that

they are not hired and fired in the same period. An alternative would be to disregard these employees since they are not permanent personnel. Unless otherwise stated, the transition period will be assumed to be one year.

### Testing the Transition Matrix

Once the transition matrix has been formulated, tests must be performed to ensure the validity of the model. These are the tests for the Markov property and for stationarity.

Test for Markov property. The formulae for testing the Markov property are given in Chapter IV (equations (4-1), (4-1A), (4-1B), and (4-1C)). Each of the  $p_{ij}$ 's which were not specified as being either zero or one in the preceding section must be tested. The test tells whether the model is a first-order Markov chain. Thus, the formulae are comparing employee movements over one period ( $n_{jk}$ ) with employee movements over two periods ( $n_{ijk}$ ). The results of the tests tell whether the two-period information is needed to validly build the model, i.e., the chain is at least a second-order Markov chain. The one-period employee movements refer to any possible movement that could occur during one year (staff to staff, staff to manager, staff leaves, etc.). The two-period movements refer to the employees who make the same consecutive movements (staff to staff to manager, etc.).

The historical data used to formulate the transition matrix will cover a certain number of years of actual data for the firm. The

one- and two-period employee movements are those that occur during this historical time period. The (t)'s in the test formulae refer to the most recent year in the subset of movements being studied. The (T)'s in the formula refer to the complete time period being studied for development of the model. For example, assume the data covers the years 1962-72. The time period T would cover all these years. If just the one-period movements during 1968-69 are looked at, then 1969 is the period t.

If using Chi-square tests on the model as developed so far, the degrees of freedom for the test would equal  $(m - 1)^2 = (5 - 1)^2 = 16$ . The m in the above formula denotes the number of states in the P matrix.

Testing for stationarity. When testing for stationarity in the transition matrix, the test compares each year of employee movements to all employee movements over the historical period. Again, each  $p_{ij}$  which is not 0 or 1 would have to be tested. The formulae for this test can also be found in Chapter IV (equations (4-2), (4-2A), and (4-2B)). The notation in these formulae has the same meaning as in the Markov property formulae.

If Chi square tests are used, the appropriate degrees of freedom for the model as developed so far would be  $(m - 1)(T - 1) = (5 - 1)(T - 1)$ . The T would be the total number of years included in the historical period.

If the above tests fail to reject the Markov property and stationarity in the transition matrix, the model may be implemented. If

either of the tests cause rejection of the null hypothesis, the model must be modified to be valid.

#### The Model: Manpower Planning

All of the data which can be generated by the conceptual manpower planning model as presented in Chapter IV can be related to the accounting firm model. Discussion of all aspects here would be repetitious. However, the more relevant aspects from the standpoint of a CPA firm will be presented below.

The transition matrix. Those elements of the transition matrix not specified as being 0 or 1 above are found by computing percentages of employee groups which moved from staff to manager, etc. during the historical period. These percentages are then used as predictions of such movements occurring during a transition period in the future. Thus, if  $p_{SM} = .20$ , this would lead to a prediction that, of the total number of employees now employed at the staff level in the firm, 20% will be promoted to managers during the next year.

To transform the percentages into numbers of employees, the initial state vector ( $w_0$ ) is multiplied by the transition matrix (formula (4-6)). However, to be more relevant as a manpower planning tool, the new employees hired during the year must be included. This is achieved in formula (4-7). If the initial state vector of employees within the firm ( $x_0$ ) is multiplied by the Q partition of the transition matrix, the result is a prediction of staff levels one year in the future, but only after the vector of expected employees ( $k$ ) has

been added. This process can be repeated as many times as needed to get the prediction for the desired year. The relevance of this prediction to manpower planning should be obvious.

The original transition matrix gives one-period percentages for group movement. Two-period percentages can be easily found by multiplying the matrix by itself. The elements of the resulting matrix would give the percentage of the initial group that would make the transition from staff to manager, etc., during a two-year period. The process can be extended to  $n$  years.

The fundamental matrix. The elements of the fundamental matrix (formula (4-8)) are associated with predictions for the length of time an employee might stay at each level of the firm. Actually, this prediction would be based on an average person of the group included in the model during the data-gathering period. This minimizes the importance of the  $f_{ij}$  elements. The  $t$  vector can be computed from the fundamental matrix (formula (4-10)) and gives predictions of an employee's total length of time with the firm, depending on the present level of attainment. Again this is an average, but the prediction may be tempered somewhat by the use of the standard deviations of the  $t$  vector.

The  $f_{ij}$  elements give total expected time in each state before being absorbed. Formula (4-26) could be used to compute the expected consecutive periods in a state before leaving (not necessarily into absorption). In the case of a CPA firm, the results should be the same. Because of the "up-or-out" policy, an employee would be expected

to be in each state either 0 or 1 time. Thus, if an employee does enter a job level, his total number of transition periods in that job should equal his consecutive number of periods in that job.

Chances of absorption. Every employee will enter an absorbing state eventually, so the chances of being absorbed are 100%. In the model as presented above, there are two absorbing states. Formula (4-12) can be used to find out the percentages of employees that will be absorbed into each of these states. Thus, the percentages should sum to 100%.

The importance of this information is based upon the importance put upon the attainment of the partnership level. By segregating the people leaving the firm into the two absorbing states, those who leave before becoming a partner and those who leave as partners, the model generates a prediction of the percentage of employees who will reach the partnership level. Even if not used in manpower planning, this information may be used in recruiting.

Conditional matrices. As explained above, much of the model is based upon an average employee. Actual employees may vary drastically from this average, which is affected by those who leave the firm after only a short time as well as those who make a career with the firm. If the assumption were made that the employee is going to reach the partnership level, he is no longer part of the group of average employees. This assumption allows a conditional model to be developed. Formula (4-30) gives the computation for the  $\hat{Q}$  matrix, the

conditional matrix of transition relationships among the transient states. Only the  $q_{ij}$  elements which were not zero or one would change. The  $\hat{Q}$  matrix would thus give the percentages of each group that make a particular transition, assuming that the whole group will eventually make partner.

A conditional fundamental matrix ( $\hat{F}$ ) can also be computed (formula (4-31)). The  $\hat{f}_{ij}$  elements may be more significant than the unconditional  $f_{ij}$  elements. These conditional elements would tell how long the employee is at each job level on his way to becoming a partner. This gives an idea of the path of an employee who will make a successful career with the firm. A conditional  $t$  vector would tell the total expected length of time these employees would remain with the firm. These results are based on an average partner rather than on an average employee.

Steady state. The firm may be interested in the amounts and distributions of manpower if it follows a steady continuation of present promotional and hiring policies. Such information can be obtained by computing the steady-state vector of employment levels ( $X_{SS}$ ) as in formula (4-15). In effect, this vector tells the plateau of manpower towards which the firm is moving.<sup>8</sup> The firm can then decide whether these are the manpower levels for which it is striving. If not, other promotional and/or hiring policies can be substituted into the model in order to test whether this different policy is acceptable. This

---

<sup>8</sup>Variances for these steady-state levels can also be computed. This was presented in Chapter IV.

also can be done for years before the steady-state levels are reached by using different policy changes in conjunction with formula (4-8). Promotional policy changes will affect the Q matrix, while hiring policy changes will affect the k vector.

The Model: Human Resource Valuation

The human resource situation with a CPA firm is conducive to valuation. Any of the valuation bases referred to in Chapter V may be used to arrive at this valuation. The discussion below will be an economic valuation of these human resources.

Economic valuation. The economic valuation of a CPA firm is based on the "excess earnings" of the firm, as computed in the income statement presentation below.

Income Statement

Revenue		XX
Less:		
Employee compensation (managers & staff)	XX	
All other expenses	<u>XX</u>	
Total expenses		<u>(XX)</u>
Net income		XX
Less: salary allowance for partners		<u>(XX)</u>
Excess earnings		<u><u>XX</u></u>

The revenue figure in the above statement includes the total billings to clients. Most CPA firms have an hourly billing rate for each level within the firm. In the model as developed in this chapter,

the firm would have a separate billing rate for staff, managers, and partners. These rates would be multiplied by the corresponding total billable hours for all employees at each level, and then summed over the three levels to get total revenue for the firm.

The expenses that were deducted on the above statement include only the salaries of staff and managers. The partners are owners in the firm and, as such, receive a share of the profits rather than a true salary. These partners do receive a salary allowance. The allowance in the above statement is based on an average allowance to all partners in large CPA firms. This is an opportunity cost for the time and effort of each partner. The partner should be able to receive this amount from any "average" CPA firm. Any net income above these salary allowances is the "excess earnings" of the firm. This is the amount that should be capitalized each year to compute the value of the unrecorded intangible assets of the firm. Since professional personnel are the predominant resource of the firm, then this valuation figure can be viewed as predominantly a human resource valuation. Thus, the capitalization of the excess earnings of the firm will be used as a maximum valuation of the firm's human resources. This capitalized figure is not simply the excess earnings for the current year, it is the excess earnings for all years that any of the current employees remain with the firm. The future flows of the current employees within the firm and out of the firm can be predicted by using the transition matrix to show probabilities of movements. Based on predicted flows, an excess earnings figure can be computed for each year in the future. This will be done over a time period of 40

years (see Chapter V). Each excess earnings figure should be discounted at the cost of capital for the firm.

Based on the computations above, valuations can be derived for (1) staff and managers together, and (2) partners. If only revenues and expenses related to the staff and manager levels are used, a net income for these two levels can be computed. This net income can be capitalized over the next 40 years using the same techniques as the total valuation above. The result would be a valuation for only the lower two levels of the firm. The difference between the total valuation and this human resource valuation related to only staff and managers would be viewed as a valuation figure for the partners of the firm.

Cost bases. For either historical or replacement cost, the model could be used for amortization of these costs over some allocation period. Either of the methods presented in the preceding chapter could be used for this purpose. Applying these methods to a CPA firm causes no special problems. Hence, it would be repetitious to present these methods here. The same is true for the method based on employee flow according to the transition matrix. All three methods are discussed in Chapter VIII.

### Summary

This chapter developed the conceptual model into an application for a Certified Public Accounting Firm. Generalities relating to all CPA firms were presented so the reader could better understand the model as applied. The transition matrix and extensions were discussed

as they would relate to any accounting firm. Manpower planning and human resource valuation applications were presented and will be further developed in the next two chapters.

## Chapter VII

### APPLICATION OF THE MODEL--II

#### Development and Tests

In Chapter VI, the conceptual model was applied to a public accounting firm, although still on a conceptual basis. In Chapter VII, the model will be applied to a specific large Certified Public Accounting firm, using data supplied by the firm when this is possible. The identity of this firm will be kept confidential.<sup>1</sup>

#### The Data

The data for the foundation of the model were supplied by a large CPA firm. These data consist of yearly hirings and internal staff flows for the time period February 1, 1967 through January 31, 1977. Tables 1 and 2 show the raw data as they were supplied by the firm.

In Table 1, the number of employees at certain professional levels within the firm is given as of January 31 of each year. The "staff" designation is a combination of at least two levels within the firm. In many firms, this would be a combination of the job levels designated as "juniors" and "seniors."

In Table 2, the number of employees hired and the number of employees promoted is given for yearly time periods corresponding with the time periods for employee levels in Table 1. Although it is

---

<sup>1</sup>This condition (confidentiality of the firm) was expressly designated by the firm as a prerequisite to the availability of data.

TABLE 1

PROFESSIONAL PERSONNEL (AT JANUARY 31 OF EACH YEAR)

Year	Staff	Managers	Partners	Total
1967	3,451	926	387	4,764
1968	4,192	1,056	451	5,699
1969	4,599	1,174	515	6,288
1970	5,683	1,324	592	7,599
1971	6,829	1,560	666	9,055
1972	6,612	1,567	716	8,895
1973	6,708	1,666	751	9,125
1974	7,323	1,787	814	9,924
1975	7,963	1,942	879	10,784
1976	8,027	2,049	937	11,013
1977	8,316	2,134	1,015	11,465

TABLE 2

HIRINGS AND PROMOTIONS (THROUGH JANUARY 31 OF EACH YEAR)

Year	New Hires	Promotions	
		From Staff to Manager	From Manager to Partner
1967	1,465	222	60
1968	1,842	261	78
1969	1,725	295	69
1970	2,590	378	91
1971	2,647	414	97
1972	1,633	298	71
1973	2,074	409	66
1974	2,602	458	84
1975	2,747	506	93
1976	2,448	505	101
1977	2,752	510	104

true that the firm occasionally does hire an employee at some level above the "staff" designation, this occurs very infrequently, and this data was not available. It is thus assumed that all new hires enter the firm at the "staff" level.

There is one possible manpower flow which is not included in the above data. An employee can obviously leave the firm during any time period. However, the data can be constructed in such a way as to derive the figures for the number of employees leaving the firm each year. For example, the number of managers employed by the firm at January 31, 1967, and thus carried over to February 1, 1967 is shown in Table 1 to be 926. The same table gives the number of managers one year later as being 1,056. Increases in the number of managers can only occur through promotion from staff, since hirings are assumed to only occur at the staff level and also since demotions do not occur. From February 1, 1967 to January 31, 1968, the number of promotions to manager (261) is given in Table 2. This table also gives the promotions from manager to partner for the same time period (78). These promotions would obviously decrease the number of managers. The only other possible flow for a manager would be out of the firm completely (termination of employment for any reason). This is also the only figure needed to reconcile the beginning and end-of-year number of managers in the firm. Table 3 gives the personnel flow for the first year of data (February 1, 1967 through January 31, 1968). This table shows that 53 managers left the firm during the year. This is the "plug" figure needed to complete the table. Personnel flow tables

TABLE 3  
 PERSONNEL FLOW--YEAR ONE  
 (February 1, 1967 to January 31, 1968)

	Staff	Manager	Partner
Beginning of year	3,451	926	378
Add: new hires	1,842	---	---
promotions in	---	261	78
Maximum	5,293	1,187	465
Less: promotions out	(261)	(78)	--
terminations	(840)	(53)	(14)
End of Year	<u>4,192</u>	<u>1,056</u>	<u>451</u>

for each year of data are given in Appendix C.

After the ten annual personnel flow tables have been formulated, one table for all ten years can be developed. This can be done by simply adding the corresponding elements in the ten yearly tables. The result is given in Table 4.

Tables for any number of years can be formulated by summing the yearly personnel flow tables. This has been done for 5, 6, 7, 8, 9, and 10 years in Appendix C. These tables will be needed later for a test of stability in the model. The beginning and end-of-period levels in these tables do not represent the total actual number of employees over the time period. These figures are inflated because the yearly figures were simply summed. An employee who was at the staff level two years would have been included at least twice in the beginning-of-period total for staff.

#### Developing the Transition Matrix

The personnel flow tables can be used to develop a transition matrix for each year. Table 3 above provides the needed figures for developing a transition matrix for the first year. Computations for the first row of the matrix are explained below.

According to Table 3, there are 3,451 staff in the firm at the beginning of the year. Any "promotions out" or "terminations" come from these 3,451 employees. This is because of the assumption that the transition period is short enough to allow only one movement during

TABLE 4  
 PERSONNEL FLOW--TEN YEAR PERIOD  
 February 1, 1967 to January 31, 1977

	Staff	Manager	Partner
Beginning of period	61,387	15,051	6,708
Add: new hires	23,060	--	--
promotions in	--	4,034	854
	<hr/>	<hr/>	<hr/>
Maximum	84,447	19,085	7,562
Less: promotions out	(4,034)	(854)	--
terminations	(14,161)	(1,972)	(226)
	<hr/>	<hr/>	<hr/>
End of period	<u>66,252</u>	<u>16,259</u>	<u>7,336</u>

the period.<sup>2</sup> Hence, 261 of these staff were promoted to manager. This accounts for 7.56% of the 3,451 staff. Also, 840 staff, or 24.3%, left the firm. Finally, the net of these employees ( $3,451 - 261 - 840 = 2,350$ ), or 68.1%, remained at the staff level throughout the year. Since the other two elements in the row are equal to zero (an impossible move), the row is complete and the elements sum to unity. The other rows in the matrix can be found in the same manner as the first row. The transition matrix for the first year is given below:

TRANSITION MATRIX--YEAR ONE

February 1, 1967 to January 31, 1968

From	-To:	Staff	Manager	Partner	Leave	Leave (Partner)
Staff		.6810	.0756	0	.2434	0
Manager		0	.8586	.0842	.0572	0
Partner		0	0	.9638	0	.0362
Leave		0	0	0	1	0
Leave (Partner)		0	0	0	0	1

Using the corresponding personnel flow table, a transition matrix can be developed for each year, and also for multiple years. The transition matrix for all ten years, based on Table 4, is given below. The transition matrices corresponding to the personnel flow table are set forth in Appendix D.

<sup>2</sup>This assumption (one movement per transition period) is basic to the Markov model (see Chapter IV). The use of a one year transition period is discussed in Chapter VI.

## MODEL TRANSITION MATRIX BASED ON TEN-YEAR PERIOD

February 1, 1967 to January 31, 1977

From	-To:	Staff	Manager	Partner	Leave	Leave (Partner)
Staff		.7036	.0657	0	.2307	0
Manager		0	.8123	.0567	.1310	0
Partner		0	0	.9663	0	.0337
Leave		0	0	0	1	0
Leave (Partner)		0	0	0	0	1

Stability Test

After a transition matrix had been developed, a stability test was to be performed on this matrix as a condition for its acceptance as the basis of the model. The administration of this Chi-square test was explained in Chapter IV. The null hypothesis for the test was a statement that the transition probabilities for each year of the time period covered by the model are equal to the transition probabilities for the complete time period.

A computer program was written to compute the test statistic for each row of the transition matrix. The program simply made the computations required by formula (4-2). This program is shown in Appendix E. The test was only performed on the first three rows of each transition matrix tested. No stability question occurred in either of the

last two rows. These rows corresponded to the absorbing states of the model, and were identical in all transition matrices.

In each row, only the non-zero elements were needed to be tested in order to test the entire row. In all matrices developed from the data, all elements equal to zero were in the same location in the matrix. Since only elements which fluctuate can have an effect on the stability of the matrix, the other elements were disregarded.

The first transition matrix tested covered the entire ten years of data. This matrix is shown in the preceding section. The testing of the first row, corresponding to the staff movements, was the starting point. The test compares the non-zero elements in the row of the ten-year matrix with the corresponding elements in each of the ten yearly transition matrices. The test statistic for this row, from the computer program, was 174.2. This statistic was compared with the values in a Chi-square table<sup>1</sup> with  $(m - 1) (T - 1)$  degrees of freedom, where  $m$  = the total number of states in the model and  $T$  = the number of transition periods. In this initial test, the degrees of freedom equal  $(5 - 1) (10 - 1) = 36$ . At the .05 level, the critical statistic from the table was only 51.0. The null hypothesis for the first row was rejected. This row was not stable over a ten-year period and cannot be validly used in the model.

The test statistic for the second and third rows of the matrix were found in the same manner. These values were computed to be 182.7 and

---

<sup>1</sup>The chi-square table used in this chapter came from Statistical Tables, by F. James Rohlf and Robert R. Sokal (San Francisco: W. H. Freeman and Company, 1969), Table R, pp. 163-167.

18.5, respectively. Since the number of degrees of freedom has not changed, the critical statistic from the table is still 51.0 at the .05 level. Thus, the null hypothesis for the second row, the movement of managers, was also rejected. However, the null hypothesis for the third row was not rejected. The movements of the partners over the entire ten-year period were found to be stable.

Obviously, the ten-year transition matrix cannot be used in the model, since the first two rows are not stable. The next matrix tested was for the most recent nine years of the data (February 1, 1968 through January 31, 1977). Tests were performed on decreasing time periods until a period was found for which none of the rows were found to be unstable. This did not occur until the most recent five years of data were tested (February 1, 1972 through January 31, 1977). Results of each test can be found in Table 5 below.

TABLE 5  
STABILITY TEST

Time period (T)	df	Computed test statistic			Critical Statistic (.05)
		Row 1 Staff	Row 2 Managers	Row 3 Partners	
10	36	174.2*	182.7*	18.5	51.0
9	32	164.4*	123.7*	18.5	46.2
8	28	156.9*	110.1*	10.4	41.3
7	24	120.6*	92.2*	9.2	36.4
6	20	68.8*	5.7	9.1	31.4
5	16	18.4	3.6	8.5	26.3

\* This test statistic is significant at the .05 level.

Each asterisk in the above table represents a situation where the test statistic is greater than the critical statistic, and the null hypothesis is rejected. The null hypothesis is rejected for at least one row in each matrix until the five-year period. Thus, the transition matrix for February 1, 1972 through January 31, 1977, was chosen as the transition matrix to be used as the basis for the model.<sup>3</sup> Hereafter, this matrix will be referred to as the "model transition matrix." This matrix is shown below.

MODEL TRANSITION MATRIX BASED ON FIVE-YEAR PERIOD  
February 1, 1972 to January 31, 1977

From	-To:	Staff	Manager	Partner	Leave	Leave (Partner)
Staff		.7019	.0652	0	.2329	0
Manager		0	.7979	.0497	.1524	0
Partner		0	0	.9636	0	.0364
Leave		0	0	0	1	0
Leave (Partner)		0	0	0	0	1

#### The Markov Property Test

While the procedures discussed below are an example of the testing of data for the Markov Property, the test itself is not valid for the CPA firm in this model. This is because the data needed for the test was not available from the firm. The personnel flow data as released by the firm was aggregate data on a one-period basis. In order to test

<sup>3</sup>As expressed by one employee of the firm, one reason for relative stability after 1972 could be because of the inception of the volunteer army. The employee stated that this event had an initial impact on personnel policies of the firm, although the effects soon faded.

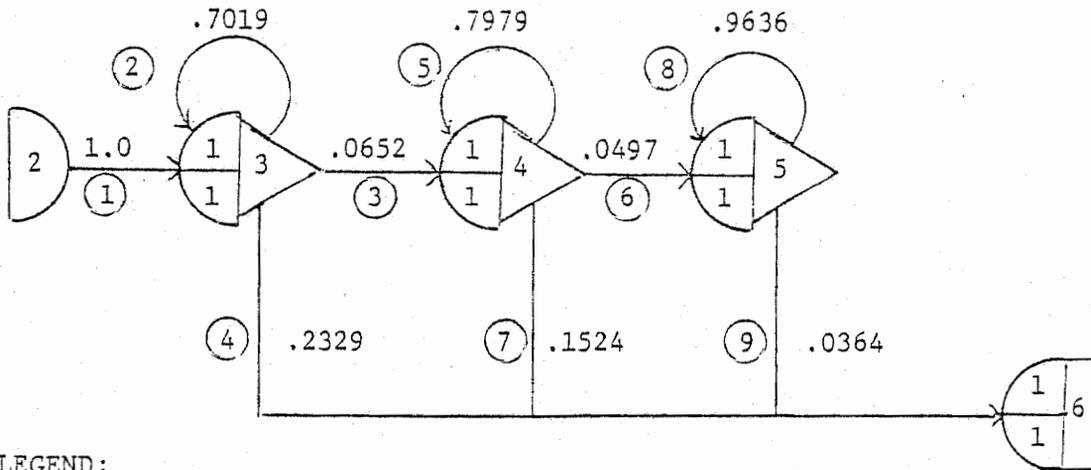
for the Markov Property, the data must be available in one of two ways. Aggregate data could be used, if the data were in the form of two-period personnel movements. For example, of the managers who were promoted to partner in a particular year, how many of these employees had been managers the previous year? This would mean that the two-period movement would be from manager to manager to partner. All two-period aggregates would need to be known. Rather than using aggregates, data collected on the career flow of individuals through the firm could be used, although this form of data would then have to be aggregated. Data on only a random sample of the individual employees of the firm would be needed.

Since the data in either of the usable forms were not available,<sup>4</sup> data on individual careers were simulated. The simulation method used was a computer program which applied the Graphic Evaluation and Review Technique (GERT). This is a network technique similar to the more common Program Evaluation and Review Technique (PERT) and the Critical Path Method (CPM). The main difference is that GERT utilizes probabilistic branching through the network while PERT and CPM only allow deterministic branching (all events in the network must occur). It is this probabilistic capability which makes GERT a powerful simulation tool when appropriate computer facilities are available.

The GERT network for the CPA firm is shown in Figure 1. Explanation of the activities are included in the figure. The probabilities for the network are the same probabilities that can be found in the model transition matrix discussed in the previous section. These are stable

---

<sup>4</sup>The firm that supplied the data would give out no information whatsoever on individuals, citing the privacy of the individual as the reason for this.



LEGEND:

Activity	Start Node	End Node	Description
(1)	2	3	Employee enters the firm as staff.
(2)	3	3	Employee stays at the staff level.
(3)	3	4	Employee is promoted from staff to manager.
(4)	3	6	Employee leaves the firm as staff.
(5)	4	4	Employee stays at the manager level.
(6)	4	5	Employee is promoted from manager to partner.
(7)	4	6	Employee leaves the firm as a manager.
(8)	5	5	Employee stays at the partner level.
(9)	5	6	Employee leaves the firm as a partner.

Figure 1. GERT Network

probabilities as tested, and are the best probabilities to use in the network.

The large number in the right-hand side of the node is the node number. Node 2 is the start node of the network. Nodes 3, 4, and 5 are the staff, manager, and partner levels of the firm, respectively. Node 6 is the sole end node of the network and corresponds to the state of the employee leaving the firm.

The decimal fractions are the probabilities that the particular branch will be taken from the node. The top number in the left-hand side of the node tells the number of activities which must reach the node before it can be realized the first time. The bottom number tells the number of activities which must reach the node before it is realized all subsequent times.

The computer program was run to simulate the careers of 200 employees. A "tracer" was put on each of the simulated careers to find the exact personnel movements for each one. Then the number of each possible one-period and two-period movements was manually counted. These movements, as needed for the Chi-square test, are given in Table 6. The  $n^*$ 's needed for the Chi-square calculations (see formula 4-1C) are also given in this table.

TABLE 6  
SIMULATED ONE-PERIOD AND TWO-PERIOD MOVEMENTS

One-Period Movements	Two-Period Movements	$n^*$
SS = 327	SSS = 327	
SM = 30	SSM = 30	$n_{SS}^* = 457$
SL = 100	SSL = 100	
MM = 133	SMM = 38	$n_{SM}^* = 43$
MP = 9	SMP = 0	
ML = 33	SML = 5	$n_{MM}^* = 132$
PP = 151	MMM = 95	
PL = 6	MMP = 9	$n_{MP}^* = 9$
	MML = 28	
	MPP = 9	$n_{PP}^* = 148$
	MPL = 0	
	PPP = 142	
	PPL = 6	

The abbreviations used in the table relate to the possible states in the model (S = staff, M = managers, P = partner, and L = leave). Once the manual count had been accomplished, the remaining calculations for the test were performed. These calculations included computing percentages (formulae (4-1A) and (4-1B)) and inserting these percentages into formula (4-1), along with the appropriate  $n^*$  above. A computer program was not deemed necessary for these calculations.

As with the stability test, the Markov Property test is a row-by-row test. Only the non-zero elements of the row are important for the test, and only the first three rows of the transition matrix need to be tested. The computed test statistics for each row are as follows:

TABLE 7

## MARKOV PROPERTY TEST

<u>Row</u>	<u>Test Statistics</u>
(1) Staff	0
(2) Managers	5.6609
(3) Partners	.3784

The degrees of freedom for the test would be equal to  $(m-1)^2$ , where  $m$  is again the total number of states in the model (which corresponds to the number of elements in each row). Thus, the degrees of freedom would equal  $(5-1)^2$  or 16 for each row. Each test statistic above was compared to a critical statistic of 26.296. This is the figure in the Chi-square table for 16 degrees of freedom at the .05 level. In each case, the test statistic was found to be less than the critical statistic, and the null hypothesis was not rejected.

### Predictive Ability

The stability test performed earlier in this chapter is one test of the predictive ability of the model. The stable matrix implies that hiring and promotion policies in the firm have been fairly constant over the last five years. The assumption is that this relative stability will continue into the future, thus allowing the user of the model to predict the future based on a stable past. If no stable matrix had been found, this would have implied that fluctuations in prior results were so pronounced that any predictions based on these occurrences would have little credibility.

A computer program was written to predict employee levels. The program uses the model transition matrix as input, along with the number of employees at each level at the beginning of the period and the hirings during the period. The output of the program is simply the number of employees at each level at the end of the transition period. After the first period, the predictions can be extended to any number of future periods, with the only input needed being the number of employees hired each year.

According to the actual data, on February 1, 1975 the firm had 7,963 staff, 1,942 managers, and 879 partners. During the next year there were 2,448 employees hired. As stated before, it is assumed that all new employees enter the firm at the staff level. These data were input into the program with the model (5-year) transition matrix. The results and comparisons with actual data were as follows:

TABLE 8

## ACTUAL vs. PROGRAM RESULTS (1/31/76)

	<u>Staff</u>	<u>Managers</u>	<u>Partners</u>
Actual (1/31/76)	8,027	2,049	937
Program Results (1/31/76)	<u>8,037</u>	<u>2,069</u>	<u>944</u>
Difference	<u>(10)</u>	<u>(20)</u>	<u>(7)</u>

In each case, there is less than 1% difference between the actual and predicted levels. The same procedures were followed for the 1976-77 transition period, using the actual data for employee's levels and hirings. The results are as follows:

TABLE 9

## ACTUAL vs. PROGRAM RESULTS (1/31/77)

	<u>Staff</u>	<u>Managers</u>	<u>Partners</u>
Actual (1/31/77)	8,316	2,134	1,015
Program Results (1/31/77)	<u>8,386</u>	<u>2,158</u>	<u>1,004</u>
Difference	<u>(70)</u>	<u>(24)</u>	<u>11</u>

The results are somewhat less accurate, with the difference between actual and predicted employees at the manager and partner level being slightly more than 1% of the actual levels. The program was again executed by inputting the 1975-76 actual data. These data were used to predict January 31, 1976 employee levels and also January 31, 1977 employee levels.

TABLE 10

## ACTUAL vs. PROGRAM RESULTS (1/31/77)

	<u>Staff</u>	<u>Managers</u>	<u>Partners</u>
Actual (1/31/76)	8,027	2,049	937
Program Results (1/31/76)	<u>8,037</u>	<u>2,069</u>	<u>944</u>
Difference	<u>(10)</u>	<u>(20)</u>	<u>(7)</u>
Actual (1/31/77)	8,316	2,134	1,015
Program Results (1/31/77)	<u>8,393</u>	<u>2,175</u>	<u>1,012</u>
Difference	<u>(77)</u>	<u>(41)</u>	<u>3</u>

The results for January 31, 1976 are the same as before, as would be expected. However, the results for January 31, 1977 do change since the beginning-of-year employee levels are those computed by the program rather than the actual figures. This does not imply that any deviation errors from actual levels in the first period will be compounded in the second period. In the case of the partner level, the prediction is better when the 1975 data is projected two periods into the future.

No formal test of predictive ability has been performed due to a lack of observations.<sup>5</sup> However, predicted results were manually compared to actual results for 1976 and 1977; every comparison yielded less than 2% deviation. This is hardly unexpected since the time periods for which the predictions were made had been used to formulate the model.

---

<sup>5</sup>A Chi-square goodness-of-fit test needs at least five observations to be a statistically valid test.

The real test of the model's predictive ability would be to project employee levels in the future and then compare these predictions with actual results as they become known.

### Summary

Actual data were used in the chapter to develop a transition matrix for a CPA firm. When these data were tested for stability, it was found that all employee levels (staff, manager, partner) are statistically stable for the 5-year time period from February 1, 1972 through January 31, 1977. Therefore, the model transition matrix hereafter will be the matrix based on these five years of data.

An example of the Markov Property test was also performed in the chapter. However, this was not a valid test for the CPA firm in the model. Since needed data were unavailable, simulated data were used in the test. The text does show how the Markov Property test could be undertaken. The results of the simulated test did not lead to the rejection of the null hypothesis; i.e., the existence of the Markov property could not be disproven.

## Chapter VIII

### APPLICATION OF THE MODEL--III

In Chapter VII, the foundation of the Markov model, the transition matrix, was developed for a large CPA firm. In this chapter, the model will be extended into the areas of manpower planning and human resource valuation.

As the results of the application of the model are presented, the formulae used to derive these results will be referred to by number, e.g., formula (4-1). The actual formula may be found in either of two places. If an explanation of the formula is desired, the reader should return to the chapter specified by the number of the formula. For example, formula (4-1) is the first formula in Chapter IV. If only a presentation of the formula is desired, this can be found in Appendix B. A brief explanation of the notations used in these formulae can be found in Appendix A.

#### Computer Programs

There were a total of five computer programs written to aid in compiling the results of the model. These programs were written in a computer language called BASIC. Listings of these programs can be found in Appendix E. Validation procedures for each program are also briefly discussed in the same appendix. The names of the five programs and a brief discussion of each are given below.

(1) MARKOV. This program embodies much of the nonfinancial aspects of the model as presented in Chapter IV. The main data inputs for the program are the canonical Q and R partitions of the transition matrix (see Chapter IV). For steady state results, other inputs include the present number of employees at each level in the firm and hirings at each level during a transition period. In this program, the hiring figures are held constant for each period in the future.

(2) TESTLEVELS. The TESTLEVELS program has already been used in Chapter VII to check the predictive ability of the model. The program takes as inputs the transition matrix, present employees at each level, and hirings. However, this program is more flexible than the MARKOV program in that the yearly hirings figures are not held constant. The output of this program is simply the predicted number of employees at each level at the end of a year. The program can be executed over any number of years, but the hirings figure for each year must be entered into the program.

(3) LEVELS. The MARKOV program will calculate the number of employees at each level when the firm reaches a steady-state situation. The purpose of the LEVELS program is to trace the flow of employees until the program reaches the steady state. Inputs needed for the program are the same as for the MARKOV program. Output of the program is simply the predicted number of employees at each level at the end of each year until the firm reaches steady state.

(4) CONDITIONAL. This program prints the conditional Q matrix ( $\hat{Q}$ ) and the conditional fundamental matrix ( $\hat{F}$ ). If model results

based on conditional probabilities were desired, the  $\hat{Q}$  matrix and the appropriate  $\hat{R}$  matrix could be entered as inputs into the MARKOV program.

(5) VALUATION. This program computes the economic value of the human resources of the firm. Inputs include the transition matrix, the present number of employees at each level in the firm, the cost of capital of the firm, and the average salary, billing rate, and chargeable hours of one employee at each level in the firm.

These programs will be referred to by name throughout this chapter. Unless specified otherwise, any results presented below have been found by the execution of one of the five programs. More complete output of each program is given in Appendix F.

#### Running the MARKOV Program--I

The only data entered into the initial phase of the MARKOV program are the Q and R partitions of the canonical transition matrix, as shown below.<sup>1</sup>

$$Q = \begin{array}{c} S \\ M \\ P \end{array} \begin{bmatrix} .7019 & .0652 & 0 \\ 0 & .7979 & .0497 \\ 0 & 0 & .9636 \end{bmatrix} \quad R = \begin{array}{c} S \\ M \\ P \end{array} \begin{bmatrix} .2329 & 0 \\ .1524 & 0 \\ 0 & .0364 \end{bmatrix}$$

Output from the first phase of the program will be discussed in detail below. The results have been rounded to two decimal places.

---

<sup>1</sup>The reader should remember the abbreviations used to denote states of the model. These abbreviations are as follows: S - staff, M - manager, P - partner, L - leave, L(P) - leave as partner.

The fundamental matrix. The matrix below is the fundamental matrix (formula 4-8) for the firm.

$$F = \begin{matrix} & \begin{matrix} S & M & P \end{matrix} \\ \begin{matrix} S \\ M \\ P \end{matrix} & \begin{bmatrix} 3.35 & 1.08 & 1.48 \\ 0 & 4.95 & 6.76 \\ 0 & 0 & 27.47 \end{bmatrix} \end{matrix}$$

This matrix gives the average amount of time all employees have spent in each state, depending upon the entry level into the firm. Thus, the first row of the matrix gives the average number of years that employees that begin as staff accountants have spent at the staff level (3.35 years), the manager level (1.08 years), and the partner level (1.48 years). The second and third rows of the matrix relate the same information for employees beginning at the manager level and the partner level, respectively. The elements in the matrix below the primary diagonal are all zeroes because there are no demotions in the firm.

Since these results are all averages, the figures in the first row for managers (element (1,2)) and partners (element (1,3)) would be affected by employees who never reached these levels during their tenure with the firm. Adding in zeroes for the time spent by these employees at the manager and partner levels decreases the average. The figures in the second row are not affected by employees who began at the staff level. Thus, these results are larger than the corresponding results for managers and partners in the first row. However, element (2,3) is affected by employees who never reach the partner

level. Since the elements in the third row are not affected by such employees, element (3,3) is significantly larger than element (2,3).

The following matrix gives the standard deviation for the corresponding element in the fundamental matrix.

$${}^F \text{ST.DEV.} = \begin{matrix} & \begin{matrix} S & M & P \end{matrix} \\ \begin{matrix} S \\ M \\ P \end{matrix} & \begin{bmatrix} 2.81 & 2.91 & 8.81 \\ 0 & 4.42 & 17.86 \\ 0 & 0 & 26.97 \end{bmatrix} \end{matrix}$$

For example, the standard deviation of element (1,1) in the fundamental matrix is element (1,1) of this matrix. This means that an employee who enters the firm at the staff level will remain at that level an average of 3.35 years (element (1,1) of the F matrix) with a standard deviation of 2.81 years. The above matrix was found by taking the square root of the variances of the elements of the F matrix, as found by formula (4-9).

The t vector. The vector below is the t vector for the firm (formula 4-10).

$$t = \begin{matrix} S \\ M \\ P \end{matrix} \begin{bmatrix} 5.91 \\ 11.71 \\ 27.47 \end{bmatrix}$$

The elements of the t vector are calculated by summing the elements in each row of the F matrix. For example, the elements in the first row of the F matrix are 3.35, 1.08, and 1.48. The sum of

these three figures is 5.91, the first element in the above column vector. Since the elements of the F matrix give the average amount of time spent in each state, the elements of the t vector give the employee's total average tenure with the firm. As with the F matrix, these results are based on the entering position with the firm. Hence, an employee who begins as a staff member spends an average of 5.91 years with the firm. An employee who begins as a manager stays with the firm an average of 11.71 years, and an employee who begins as a partner remains with the firm an average of 27.47 years.

The standard deviations of the elements of the t vector are shown below.

$${}^t\text{ST.DEV.} = \begin{matrix} \text{S} \\ \text{M} \\ \text{P} \end{matrix} \begin{bmatrix} 10.26 \\ 18.39 \\ 26.97 \end{bmatrix}$$

These figures are found by taking the square root of the variance of each element in the t vector. The variances are found in formula (4-11). The results in the above vector cannot be found by summing the rows of the matrix of standard deviations of the F matrix.

Absorption. The B matrix (formula 4-12) gives the probability of being absorbed into each state, depending on the entry level of the employee. This matrix is shown below.

$$B = \begin{matrix} & & L & L(P) \\ S & \left[ \begin{array}{cc} 0.95 & 0.05 \\ 0.75 & 0.25 \\ 0 & 1.00 \end{array} \right. \\ M \\ P \end{matrix}$$

Remember that the two absorbing states of the firm are:

(1) leave the firm before becoming partner (L), and (2) leave the firm after becoming partner (L(P)). From the B matrix above, an employee who enters the firm at the staff level will leave the firm before becoming a partner 95% of the time. Only 5% of entering staff members will reach the partner level. For employees who enter the firm as a manager, there is only a 75% chance that the employee will leave the firm before reaching the partner level. The chances of making partner have increased to 25%. Obviously an employee who is already a partner cannot leave the firm before becoming a partner. Hence, his chances of entering the two absorbing states are 0% and 100%, respectively.

Consecutive time in state. The consecutive number of transition periods that an average employee would spend in each state, once that state has been entered, is given by the following vector.

$$a = \begin{matrix} S \\ M \\ P \end{matrix} \left[ \begin{array}{c} 3.35 \\ 4.95 \\ 27.47 \end{array} \right]$$

These results are found by using formula (4-26). This vector supplies no new results for a CPA firm. This is because of a combination of two inherent characteristics of CPA firms; the firm is a

hierarchical organization and there are no demotions. Thus, once an employee leaves a job level, he will never return to that same level. This means that an employee's total time at a job level will equal the consecutive number of years at that level. As would be expected under these circumstances, the elements of the vector above are identical to the elements on the primary diagonal of the F matrix. In a situation where an employee can enter a job level more than once, the a vector would yield different results from the F matrix.

The above analysis is also true for the vector of standard deviations of the a vector, as shown below.

$${}^a\text{ST.DEV.} = \begin{matrix} S \\ M \\ P \end{matrix} \begin{bmatrix} 2.81 \\ 4.42 \\ 26.97 \end{bmatrix}$$

These figures were found by taking the square root of each variance of the elements of the a vector. These variances were found by using formula (4-27). The standard deviations in the above vector are identical to the elements on the primary diagonal of the  $F_{\text{ST.DEV.}}$  matrix (presented earlier in the chapter).

The H matrix. The H matrix gives the probability that an employee will ever be in each particular state any time in the future. The results are based on the present position of the employee. The H matrix is shown below.

$$H = \begin{matrix} & \begin{matrix} S & M & P \end{matrix} \\ \begin{matrix} S \\ M \\ P \end{matrix} & \begin{bmatrix} .7019 & .2187 & .0538 \\ 0 & .7979 & .2459 \\ 0 & 0 & .9636 \end{bmatrix} \end{matrix}$$

The primary diagonal of the original Q matrix shows the probability of an employee staying at his present level in the firm. The primary diagonal of the H matrix gives the probability of an employee ever being in his present state again. For a CPA firm that has a hierarchical organization and no demotions, the primary diagonals of the Q matrix and the H matrix should be identical. For comparison purposes, the H matrix is given to four decimal places. The primary diagonals are indeed identical (the Q matrix is shown near the beginning of this chapter).

The probability that a staff member would become a partner was given by the B matrix as being 5%. The corresponding element of the H matrix (element (1,3)) is consistent with this figure, although it has been carried to 4 decimal places (.0538) rather than two. In the same manner, the probability that a manager would become partner according to the B matrix (25%) is consistent with the corresponding element in the H matrix (.2459). The H matrix also shows that a staff member has a 21.87% chance of ever being a manager.

#### Running the MARKOV Program--II

Before the second phase of the MARKOV program can be executed, more inputs must be entered. This input consists of: (1) the present

number of employees at each level, and (2) hirings. It is assumed that the number of employees hired will remain constant for each transition period in the future. For this run of the program, the figures chosen for the employee levels were the figures as of January 31, 1977 (the last known figures given in the data). These figures were: 8,316 staff, 2,134 managers, and 1,015 partners. Summing these figures, the firm had a total of 11,465 employees. All hirings occur at the staff level, and this figure was assumed to be 2,750 per year. Since the remaining output of the program is in terms of numbers of employees, the results have been rounded to the nearest whole number.

Present employees absorbed. The 11,465 employees of the firm as of January 31, 1977 will ultimately be absorbed into one of the two absorbing states (formula (4-13)). According to the output of the program, 9,478 of the employees will leave the firm before becoming partner. The other 1,987 employees will become partners before leaving the firm. The standard deviation of each of these figures is 41. The standard deviation is the square root of the variances which were found by formula (4-14).

Steady State. Because a constant number of employees are assumed to be hired each year, the firm will eventually reach a steady state condition. The characteristics of steady state are: (1) the same number of employees will leave the firm each year as are hired (2,750), and (2) the number of employees at each level will remain constant from year to year. The steady state situation is the long-run manpower plateau of the firm. These results are dependent on the

hirings figure as well as the other inputs. Part of the steady state results are summarized in Table 11. Table 11 shows that the total number of employees at steady state would be 16,266. The number of employees at each level is also given by the table, as well as upper bound estimates of the standard deviation for each figure. The standard deviation is not a summation of the standard deviations for the individual job levels. These standard deviation estimates are small compared to the computed employees levels, and should have little effect on the distribution of employees at each level.

It has already been stated that the number of employees leaving the firm should be equal to 2,750. A breakdown of this figure shows that 2,602 employees will leave the firm from some level other than the partner level, and 148 partners will leave the firm. These figures (the 2,602 and 148) are yearly figures that will occur at steady state. The program also gives how many of the total employees at steady state (the 16,266) will ultimately be absorbed into each state. Of these employees, 10,974 will leave the firm before becoming a partner, while 5,292 will leave the firm as partners. The standard deviation for each of these latter two figures is 60.

The complete run of the MARKOV program, showing both the inputs and the outputs discussed in this chapter, can be found in Appendix F.

#### Running the LEVELS Program

The LEVELS program can also be used to find steady state information for the firm. Although the MARKOV program tells the steady

TABLE 11  
STEADY STATE LEVELS

	Staff	Managers	Partners	Total
Employees	9,225	2,976	4,065	16,266
Upper bound estimate of standard deviation	62	54	64	104

state number of employees at each level, the program does not give the pattern of employee levels as the firm approaches steady state. The program also does not give the number of years it takes the firm to get to steady state. These data can be determined by executing the LEVELS program.

As with the MARKOV program, the level of new hires is assumed to be constant at 2,750. The same levels for the transient states (8,316, 2,134, and 1,015 for staff, managers, and partners, respectively) as were input to the MARKOV program are used here. The user of the LEVELS program also must input the number of years into the future that should be projected by the program. The output for the first 50 years, stated in terms of whole numbers, is given in Table 12.

From Table 11, it can be seen that there would be 9,225 staff accountants after 21 years. This is the same figure as given in the MARKOV program as being the steady state level for staff. It can be seen that the number of employees at the staff level would be 9,199 after only 10 years and 9,221 after 15 years. The staff level stabilizes much earlier than the 21-year steady state figure. This same pattern can be seen at the manager level. The steady state level (2,976) is reached after 35 years. However, after only 25 years, the level has already reached 2,971.

The partner level is the only level which has not stabilized by the end of year 50. In fact, this level will not reach a true steady state figure (4,063) until the 235th year.

TABLE 12  
 YEARLY EMPLOYEE LEVELS (50 YEARS)

Year	Staff	Managers	Partners	Total
1	8587	2245	1084	11916
2	8777	2351	1156	12284
3	8911	2448	1231	12590
4	9004	2534	1308	12846
5	9070	2609	1386	13065
6	9116	2673	1465	13254
7	9149	2727	1545	13421
8	9172	2773	1624	13569
9	9188	2810	1703	13701
10	9199	2841	1781	13821
11	9207	2867	1857	13931
12	9212	2888	1932	14032
13	9216	2905	2005	14126
14	9219	2919	2077	14215
15	9221	2930	2146	14297
16	9222	2939	2213	14374
17	9223	2946	2279	14448
18	9224	2952	2342	14518
19	9224	2957	2404	14585
20	9224	2961	2463	14648
21	9225	2963	2521	14709
22	9225	2966	2576	14767
23	9225	2968	2630	14823
24	9225	2970	2682	14877
25	9225	2971	2732	14928
26	9225	2972	2780	14977
27	9225	2973	2827	15025
28	9225	2974	2871	15070
29	9225	2974	2915	15114
30	9225	2974	2956	15155
31	9225	2975	2997	15197
32	9225	2975	3035	15235
33	9225	2975	3072	15272
34	9225	2975	3109	15309
35	9225	2976	3143	15344
36	9225	2976	3177	15378
37	9225	2976	3209	15410
38	9225	2976	3240	15441
39	9225	2976	3270	15471
40	9225	2976	3299	15500

TABLE 12 (Cont'd.)

Year	Staff	Managers	Partners	Total
41	9225	2976	3327	15528
42	9225	2976	3354	15555
43	9225	2976	3380	15581
44	9225	2976	3404	15605
45	9225	2976	3428	15629
46	9225	2976	3452	15653
47	9225	2976	3474	15675
48	9225	2976	3495	15696
49	9225	2976	3516	15717
50	9225	2976	3536	15737

### Running the TESTLEVELS Program

The TESTLEVELS program was used in Chapter VII to predict employee levels within the data period so as to compare predicted and actual results. This program can also be used to predict future levels. The advantage of this program over LEVELS is that a new figure for hirings can be input each year. For this reason, the TESTLEVELS program would be the better program to use when predicting only a few years into the future. The program was executed by inputting the same levels (8,316, 2,134, and 1,015) as before and also the same hirings for the first year (2,750). The predicted levels for this program for the end of year 1 would be the same results as for the LEVELS program (see Appendix F). However, a different level of hirings was assumed for year 2, and thus the results for the two programs differ for that year. For the TESTLEVELS program, a hirings figure of 2,800 was used. The levels for each state as given by the program, projecting two years into the future, are given below.

### Running the CONDITIONAL Program

The conditional Q matrix ( $\hat{Q}$ ) and the conditional fundamental matrix ( $\hat{F}$ ) are based on the assumption that it is known into which state the system will be absorbed. In the case of the CPA firm, it is assumed that the employee will make partner before leaving the firm. The data needed to compute these matrices are the original Q matrix and the column of the B matrix corresponding with the assumed absorbing state. This would be the second column of the B matrix in the

output from the MARKOV program. Earlier in the chapter, the elements of the B matrix were rounded to two decimal places for expediency. As input to this program, these figures should not be rounded, and thus would be .05378679849, .24591786244, and 1.0 (see execution of the MARKOV program in Appendix F).

The conditional matrices are given below.

$$\hat{Q} = \begin{array}{c} \text{S} \\ \text{M} \\ \text{P} \end{array} \begin{array}{ccc} \text{S} & \text{M} & \text{P} \\ \left[ \begin{array}{ccc} .7019 & .2981 & 0 \\ 0 & .7979 & .2021 \\ 0 & 0 & .9636 \end{array} \right] \end{array}$$

$$\hat{F} = \begin{array}{c} \text{S} \\ \text{M} \\ \text{P} \end{array} \begin{array}{ccc} \text{S} & \text{M} & \text{P} \\ \left[ \begin{array}{ccc} 3.35 & 4.95 & 27.47 \\ 0 & 4.95 & 27.47 \\ 0 & 0 & 27.47 \end{array} \right] \end{array}$$

The  $\hat{Q}$  matrix has the same primary diagonal as the original Q matrix (see the third page of this chapter). This means that the percentage of people remaining in a particular state has not changed. The situation above the primary diagonal has changed, however, and this is specifically because it is assumed that all personnel will eventually become a partner with the firm. This means that no one leaves the firm from the staff or manager levels. These employees each have only one place which they can go: the staff member can only go to manager, and the manager can only go to partner. Since all

TABLE 13  
YEARLY EMPLOYEE LEVELS (2 YEARS)

Year	Staff	Managers	Partners	Total
1	8,587	2,245	1,084	11,916
2	8,827	2,351	1,157	12,335

staff accountants either stay where they are or move up to manager, the sum of elements (1,1) and (1,2) in the  $\hat{Q}$  sum to one. A similar analysis for managers would explain why the elements (2,2) and (2,3) sum to one. However, the third row of the  $\hat{Q}$  matrix will not sum to unity because these employees can leave the firm. This row is the only outlet from  $\hat{Q}$  into a  $\hat{R}$  partition of a conditional transition matrix for the firm. The  $\hat{R}$  matrix for the above situation would be as follows:

$$\hat{R} = \begin{matrix} & & L & L(P) \\ \begin{matrix} S \\ M \\ P \end{matrix} & \left[ \begin{array}{cc} 0 & 0 \\ 0 & 0 \\ 0 & .0364 \end{array} \right] \end{matrix}$$

The primary diagonal of the  $\hat{F}$  matrix is identical to the primary diagonal of the original F matrix (see the initial phase of the MARKOV program earlier in this chapter). The elements above the primary diagonal in  $\hat{F}$  are equal to the elements on the primary diagonal in that column. This shows that the person's expected tenure with the firm is governed by the knowledge that he will eventually become a partner, and that the path to this end, as given by  $\hat{F}$ , is still an average. The  $\hat{F}$  matrix gives the average time in each state of only those employees who reach partner rather than the average of all employees in the firm. Thus, it should take an employee an average of 8.3 years to become a partner with the firm if he began at the staff level. The 8.3 years is derived by adding the first two elements of the first row of the F matrix (3.35 + 4.95).

Using the Model in the Firm

The computer programs and the corresponding output from these programs discussed above were presented for two reasons. First, the capabilities of the programs in relation to the model can be seen. Second, the information from the program output can be referred to in the following discussion on the uses of the model in the CPA firm.

The model can be used in the areas of manpower planning and human resource valuation. Before discussing these uses, a review of the general areas of information derived from the available data is needed. A summary of these general areas is given below.

- (1) Promotion policies and terminations, both based on percentages (the transition matrix).
- (2) The average number of years spent with the firm.
- (3) The probabilities of making manager and partner.
- (4) The ultimate distribution of employees into the absorbing states.
- (5) Long-run projections of employee levels in each state, assuming a constant number of hirings each year.
- (6) Short-run projections of employee levels in each state, assuming a different number of hirings each year.
- (7) Steady-state information, including levels of employees, ultimate disposition of employees

into absorbing states, and length of time to reach steady state.

- (8) Conditional Q and F matrices assuming that the employee will make partner.
- (9) Standard deviations of all averages.

### Manpower Planning

There are two areas in which the model will provide information for the firm in relation to manpower planning: (1) projecting the known information into the future as an attempt to predict the direction of the firm, and (2) using the model to answer hypothetical questions for the firm. The first area involves informing management of the predicted results if the firm continues with its present policies. The latter area would allow the management to use the model to test changes in policy. This would be useful if it were found that present policies would not fulfill the objectives of the top management of the firm.

The model could also be used in the area of recruiting. The utility of the model would be the information that could be presented to the recruit pertaining to his possible future with the firm. This area of interest will also be discussed below.

Projected known data. The aggregate data supplied by the firm were the basis for the model transition matrix developed in Chapter VII. This matrix, as shown below, became the foundation of the model.

	S	M	P	L	L(P)
S	.7019	.0652	0	.2329	0
M	0	.7979	.0497	.1524	0
P = P	0	0	.9636	0	.0364
L	0	0	0	1	0
L(P)	0	0	0	0	1

This matrix was formulated by using historical data. Thus, any extension of the model using this transition matrix is also based on historical data. If this model is used to project the future direction of the firm, the inherent assumption is that what has happened in the past will continue to occur in the future. If the user of the model has no knowledge to the contrary, this assumption can be accepted. However, if the user does have knowledge of circumstances which would alter future occurrences, the model should be modified to reflect this knowledge. An example of such knowledge would be that the management has decided to change the personnel policies of the firm so that demotions are possible.

Based upon the transition matrix, the firm can expect 70.19% of its staff accountants in 19X1 to also be staff members in 19X2, while 6.52% will have been promoted to manager and 23.29% will have left the firm. In this manner, percentages according to the past become probabilities for the future. The transition matrix projects the promotion policy of the firm as well as the projections for personnel who leave the firm for any reason.

When the transition matrix is used in connection with employee levels and hirings (as done in LEVELS and TESTLEVELS), projections of future manpower can be developed. These projections not only predict the number of employees, but also the state in which these personnel will be located (staff, manager, partner). The importance of this information for manpower planning cannot be over-emphasized. For example, using the TESTLEVELS program, it was found that the firm should have 8,587 staff, 2,245 managers, and 1,084 partners on January 31, 1978. These predictions are based on the transition matrix, the known levels as of February 1, 1977, and an assumed hirings figure (2,750). Once the January 31, 1978 levels have been predicted, the same program can be used to predict employee levels for January 31, 1979. For this second transition period, a hirings figure of 2,800 was input into the program. The assumption was that there would be a slight increase in hirings over the first transition period. The result was a prediction of 8,827 staff, 2,351 managers, and 1,156 partners at the end of the period. The main limitation in these predictions is the hirings figure. Management must be consulted before arriving at a figure to be used in the program. Two factors can affect the future hirings: (1) management may have a goal with respect to short-run growth of the firm, and (2) expected service demand for the firm.

While the projected employee levels may be important information for manpower planning, it is certainly not the only useful data that can be developed from the model. Projecting the employee levels a number of years into the future would bring the firm to a steady state

existence. The steady state levels for the firm, using the MARKOV program, were found to be 9,225 staff, 2,976 managers, and 4,063 partners based on constant hirings of 2,750 per year. This distribution of employees at steady state is very different from the present distribution. As of January 31, 1977 the breakdown of employees was approximately 72% staff, 19% managers, and 9% partners. The steady-state distribution is approximately 57%, 18%, and 25% for staff, managers, and partners, respectively. In this steady-state situation, there may easily be "too many chiefs and not enough Indians" for the firm to run efficiently. Although it takes the firm 235 years to reach a technical steady state in all three job levels, the staff and manager levels stabilize long before this. True steady state is reached for these levels at the end of 21 and 35 years. These figures were provided by the LEVELS program. The program also shows that the number of partners in the firm becomes greater than the number of managers in the firm after only 31 years.

Once the steady state level has been reached, the MARKOV program shows that there are 16,266 total employees. This number compares with only 11,465 employees as of January 31, 1977. The program also provides the information that at steady state, there will be 148 partners leaving the firm each year and 2,602 employees leaving the firm each year who are not partners. The steady state information portrays the long-run impact of present policies. In this case, the results point towards a needed change in policy at some point in time.

This policy change can be in the area of promotional policy, in hiring policy, or in both areas.

The original fundamental matrix ( $F$ ) and the conditional fundamental matrix ( $\hat{F}$ ) yield information on the expected average tenure of an employee. The average employee will spend 5.91 years with the firm (from the  $t$  vector found earlier in this chapter). This employee will spend 3.35 years at the staff level, 1.08 years as a manager, and 1.48 years as a partner. Only the 3.35 years at the staff level is a meaningful figure here, since all employees are assumed to enter the firm at this level. The averages for the manager and partner levels are distorted because many employees never reach these levels, even though the averages are affected by a zero amount of time in each state. Although the average path may not be helpful, the total (5.91) is relevant.

The conditional  $F$  matrix gives the average path and tenure for just employees who make partner in the firm. The first row of this matrix shows that these employees spend an average of 3.35 years at the staff level, 4.95 years as a manager, and 27.47 years as a partner. Thus, an employee who is hired by the firm at the staff level and who leaves the firm as a partner will be with the firm an average of 35.77 years.

As seen by the  $F_{ST.DEV.}$  matrix, the elements of the  $F$  matrix have relatively high standard deviations when compared to the mean (both of these matrices ( $F$  and  $F_{ST.DEV.}$ ) can be found in the earlier section on running the MARKOV program). These high standard deviations

reflect a short tenure with the firm for many employees. For example, the element in the (1,1) position of the fundamental matrix (3.35) has a standard deviation of 2.81 (element (1,1) in the  $F_{ST.DEV.}$  matrix). The mean minus one standard deviation ( $3.35 - 2.81 = .54$ ) would cover approximately 84% of the area under the normal curve. Thus, 16% of the employees hired would be expected to spend less than .54 years with the firm.

The final two pieces of information in the MARKOV program which may be relevant to management both have to do with probabilities of reaching a level some time in the future. Accordingly, the B matrix shows that a staff member has a 5% chance of making partner, while a manager has a 25% chance of reaching that level. The H matrix corroborates these figures, plus it shows that a staff member has almost a 22% chance of becoming a manager. These figures would show management the results of the promotion policies of the firm (over which management has control) and results of terminations (over which management has much less control).

Hypothetical questions. After the projected direction of the firm has been evaluated using the above information, management must decide whether this is the desired course for the firm. In other words, will these projections fulfill the goals and objectives of the firm? If not, then changes must be made. The model also has applications in the area of deciding what specific changes are needed.

Short-run predictions of employee levels, using the TESTLEVELS program, projected a total of 8,827 employees at the staff level on

January 31, 1979. This was based on an assumption of 2,750 new hires in 1977-78 and 2,800 new hires in 1978-79. Suppose that the management of the firm feels that in order to achieve the desired growth, the firm will need 9,000 staff on January 31, 1979. To achieve this goal, three courses of action may be considered. The company could hire more employees during the ensuing two years. The firm could also alter its promotional policies, leaving more employees at the staff level. Finally, the firm could initiate an effort to reduce terminations at the staff level.

Management must keep in mind the qualitative aspects of the alternatives as well as the quantitative aspects which can be tested by the model. For instance, slowing the promotion process from staff to manager may have a behavioral impact on certain employees. Based on previous policies of the firm, the employees may have expectations as to when they should be promoted. If this promotion did not occur near the expected time period, adverse behavior may result, even to the extent that employees may leave the firm. In this way, holding back employees to increase the number of staff employees could cause a reaction which would mean that the goal of 9,000 staff could not be attained because a higher than usual number of employees leave the firm. Whenever altering promotion policies, the management should be aware of behavioral implications.

When one of the alternatives has been chosen, the next step is to decide specific actions needed to reach the goal. For instance, if the company decides to hire more people in order to achieve the

desired 9,000 level for staff, how many employees should be hired during the two years? It is this type of question which can be answered by using the model. In this case, different hiring levels can be input into the TESTLEVELS program, comparing the output to the desired goal. If 2,800 and 2,900 hirings for the two years, respectively, were used in the program, there would be only 8,962 staff projected for the end of the second year. If the hirings in the second year were increased to 2,950, then the projected level would be 9,012. One strategy that could be used to achieve the goal would be to hire 2,800 people the first year and 2,950 the second year. This is certainly not the only strategy that would attain the desired result. The ultimate choice would be the responsibility of the management of the firm.

If one of the other alternatives were chosen (change promotion policies or try to stop people from leaving), the probabilities of the transition matrix would be affected. Different probabilities could be used in the matrix, remembering that each row must sum to unity to keep the matrix stochastic. If the probability of staying at the staff level were increased to 72% and the probability of promotion decreased to 4.71%, this change could be used in the program with hiring levels to find a strategy. Using the originally assumed hiring levels of 2,750 and 2,800, the number of staff would be 9,091 at the end of the second year. Another strategy has been found to achieve the goal.

The long-run course of the firm can also be a problem. As was stated above, as the firm moves towards steady state, the number of partners will become greater than the number of managers in the firm. It is very unlikely that this situation is desired by the top management of the firm. Again something needs to be done. The results were found by assuming a hiring level of 2,750 each year. If this figure were increased to 3,000, the result would be steady state employees of 10,063 staff, 3,246 managers, and 4,432 partners, a distribution of approximately 57%, 18%, and 25%, respectively. This is the same distribution as when the 2,750 figure was used. Using the present policies, the steady state situation will always have more partners than managers because of the fact that partners spend a much longer period of time with the firm (27.47 years), on the average, than managers (4.95 years). The answer to this long-run problem would appear to be in promotion policy changes. Once these policy changes have been put into effect, then it may be necessary to alter the hiring levels to avoid having bottlenecks at the lower levels because of too many employees. The MARKOV program can be used to test possible changes.

By trial-and-error, a strategy was found that would keep the company near the same distribution of employees as at present, with the hiring level the same as before (2,750). The transition matrix would be as follows:

$$P = \begin{bmatrix} .74 & .03 & 0 & .23 & 0 \\ 0 & .83 & .02 & .15 & 0 \\ 0 & 0 & .9636 & 0 & .0364 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

This matrix reflects changes in the promotional policies at both the staff and manager levels. No change was made in the partnership probability for leaving the firm. The steady state levels for the transient states would be 10,576 staff, 1,866 managers, and 1,025 partners. These figures may or may not be the desired plateau of manpower for the firm. If not, then more trial-and-error tactics can be used, testing both promotional policy changes and hiring level changes. Again the management must be aware of the fact that altering promotion policies can have adverse behavioral effects.

The long-run problem discussed above was that the firm would have too many partners. Instead of using promotion policies to meet the problem, the firm could change its retirement policies. For instance, a mandatory retirement age of 65 could be incorporated into the personnel policies of the firm. The effect of such a policy change would depend on the number of partners which would remain with the firm past this age. It is unlikely that this mandatory retirement requirement would alleviate the problem entirely. However, instituting such a policy could minimize the change in promotion policies needed. This would also minimize the behavioral effects of the change in promotion policies.

Recruiting. The model yields data which can be used to inform recruits of the policies of the firm, and how these policies may affect that person. Most recruits would probably be interested in their chances of reaching the partner level within the firm. From the B matrix, the hiree entering at the staff level has a 5% chance of

making partner. If he reaches the manager level (a 22% chance), his chances have increased to 25%. The recruit could be made aware of the fact that these probabilities are not only affected by promotion policies, but also by the large number of people who leave the firm of their own accord. The person could also be told his probable route through the firm if he were to become a partner. He could expect to spend an average of 3.35 years as a staff accountant and 4.95 years as a manager, or a total of 8.3 years with the firm before becoming a partner. Such information could be instrumental in the person's decision to accept or reject a position with the firm. However, once the recruit decides to enter the firm, it is likely that his expectations of promotion will be based on the average figures given to him by the recruiter. If the employee is below average, or if promotion policies change, it may take longer than 3.35 years (the average) for him to become a manager. This is another situation which can lead to discouragement and adverse behavior.

#### Human Resource Valuation

As presented in Chapter VI, the MARKOV model can be useful in the area of human resource valuation on a homogeneous group basis. This is true whether a cost basis (historical or replacement) model or an economic valuation model is used. Both methods will be discussed here.

Cost basis. Flamholtz states that there are certain problems

associated with computing an economic valuation for human resources.<sup>2</sup> Because of these problems (see Chapter III), a surrogate measure is needed. Three of the examples he cites as possible surrogates are (1) historical cost, (2) replacement cost, and (3) current cost. For each cost base, decisions must be made as to which costs should be capitalized. Examples of costs that could be capitalized for a CPA firm would include recruiting costs, costs of training schools for employees, costs of on-the-job training, and costs for continuing education paid for by the firm. However, the MARKOV model has no application in the area of choosing which costs are to be capitalized.

After human resource costs have been capitalized, an amortization period must be chosen over which these costs should be allocated to separate time periods. If the useful life were known for a particular cost, then this life would be the amortization period for that cost. Unfortunately, it is very unlikely that the useful life of a human resource cost can be known. For this reason, some method is needed for finding a suitable amortization period for capitalized human resource costs. Two possible methods of amortization were discussed in Chapter VI.

The first method of amortization is based solely on the total average time employees spend with the firm. This average time is dependent upon the level at which the employee enters the firm. If personnel entered the firm at all three levels (staff, manager, partner), then there would need to be three amortization periods, one for

---

<sup>2</sup>Flamholtz, "A Model for Human Resource Valuation . . .," p. 266.

each level. Incurred costs would have to be categorized as to which entry level or levels these costs should be apportioned. However, in the situation where all employees enter the firm at the staff level, these complexities are no longer part of the model. When the decision is made to capitalize a human resource cost, this cost can be capitalized at the staff level. The amortization period for this cost would be the average time that employees entering at the staff level remain with the firm.

The amortization period in the case of the CPA firm used in the model would be 5.91 years, as found in the  $t$  vector earlier in the chapter. The percentage of amortization which would be taken each period is computed using formula (5-6), as shown below.

$$\text{Amortization \%} = \frac{1}{t_1} = \frac{1}{5.91} = 16.9\%$$

Each period the firm could amortize 16.9% of all capitalized human resource costs. Since the average of all employees is used, any new costs incurred would be added to the capitalized figure, even if these new costs were to develop managers or partners. Regardless of how long an employee stayed with the firm, any human resource costs would be amortized on a straight-line basis over 5.91 years. Since human resource costs could be incurred at any time during the period, the amortization percentage could be taken against the average balance in the human resource cost account during the period.

The above method does not recognize any losses due to personnel leaving the firm. A more theoretically correct method would amortize

capitalized human resource costs over the entire career of each employee. The inherent assumption in the preceding statement is that the firm will derive benefits from an employee as long as that employee remains with the firm. No one can know how long each employee will stay with the firm. However, a time period can be computed during which 99% of the present employees will leave the firm. This time period would be equal to the average length of time an employee remains with the firm plus three standard deviations. The average tenure was shown above to be 5.91 years. The standard deviation of this figure, as given in the  $t_{ST.DEV.}$  vector, would be 10.26 years. Thus, the amortization period would be  $5.91 + 3(10.26) = 36.69$  years. This period would be long enough to ensure that for 99% of the employees, an amount for amortization would be charged off each year they remain with the firm. Whenever an employee leaves before they have been with the firm 36.69 years, a loss would occur. Recognition of this loss means that a valuation figure must be placed on each employee. The easiest way to achieve this is simply to divide the total unamortized cost by the total number of employees with the firm (since each started at the staff level). The charge against income each year would include a loss from personnel terminations plus an amortization of human resource costs.

Assume that the firm had 5,000 total employees at the beginning of the year, and that the average unamortized human resource cost for the year is \$1,000,000. The average valuation for an employee would be  $(\$1,000,000/5,000) = \$200$ . For each employee who left the firm

during the year, a human resource loss of \$200 would be recognized. Amortization expense would also be recognized for the period. This expense would be equal to 2.7% of the average unamortized human resource cost for the period. This amortization rate is also found using formula (5-6), and is equal to  $(1/36.69) = 2.7\%$ . Hence, if 500 employees left the firm during the year, the amounts charged against income would be as follows:

Loss (500)(\$200)	=	\$100,000
Expense (.027)(\$1,000,000)	=	<u>27,000</u>
Total		<u>\$127,000</u>

The number of employees hired during the period does not affect the above calculations.

Recognizing the loss on the basis of the number of employees at the beginning of the year is consistent with the model assumption that the transition period is short enough to permit only one movement during each period. Thus, the employees who leave during each period are among the employees who were already with the firm as of the beginning of the period.

Both the loss and the amortization expense were based on the average human resource cost for the year. The loss is based on average human resource cost because it is assumed that employees can leave the firm any time during the period. If an employee left the firm on the first day of the year, the loss should be based on the beginning balance in the human resource cost account. If the employee leaves the firm at the end of the year, the ending balance in the cost

account should be the basis for the loss. Thus, with employees leaving continuously during the year, a loss based on the average figure in the cost account is best. The amortization expense is based on the average human resource cost balance because new costs occur continuously throughout the year.

The flow method. The final method which was presented in Chapter V for charging human resource costs against income applies the flow of the costs to the flow of the personnel. The total balance in the human resource cost account would be the sum of the balances of three subsidiary accounts. One subsidiary account is needed for each job level with the firm (staff, manager, partner). The flow method carries the beginning-of-the-period balance in each subsidiary account through the transition matrix to determine the status of the human resource costs at the end of the year. This is analogous to carrying the vector of employees at each level at the beginning of the year through the transition matrix to determine the status of these employees at the end of the year. Thus, the human resource costs flow through the firm in the same manner as the employees. Human resource costs incurred during the period must be added to the appropriate subsidiary account. An example is shown below.

Assume that the beginning balance in each of the subsidiary accounts for job levels is \$100,000. Balances are not carried forward from year to year for the absorbing states (any amounts flowing into these states are charged against income). Thus, the vector of human resource costs at the beginning of the year would be

[\$100,000    \$100,000    \$100,000    \$0    \$0]

The budgeted ending distribution of the above costs would be found by multiplying the vector of beginning balances by the model transition matrix for the firm.

$$\begin{array}{r}
 \\
 \\
 \\
 \\
 \\
 \end{array}
 \begin{array}{l}
 [\$100,000 \ \$100,000 \ \$100,000 \ 0 \ 0]
 \end{array}
 \begin{array}{|c|c|c|c|c|}
 \hline
 .7019 & .0652 & 0 & .2329 & 0 \\
 \hline
 0 & .7979 & .0497 & .1524 & 0 \\
 \hline
 0 & 0 & .9636 & 0 & .0364 \\
 \hline
 0 & 0 & 0 & 1 & 0 \\
 \hline
 0 & 0 & 0 & 0 & 1 \\
 \hline
 \end{array}$$

If capitalized human resource costs budgeted for the year were \$20,000 at each level, the budgeted ending balances in the subsidiary accounts and the ending balance in total would be as follows:

Staff	(\$70,190 + \$20,000)	=	\$ 90,190
Managers	(\$86,310 + \$20,000)	=	\$106,310
Partners	(\$101,330 + \$20,000)	=	<u>\$121,330</u>
TOTAL	(\$257,830 + \$60,000)	=	<u>\$317,830</u>

The budgeted amount of costs which would be charged against income during the period can be found from the computed vector of ending distribution of costs shown above. This figure is shown by the vector elements corresponding to the absorbing states (elements (1,4) and (1,5)). Thus, the amount charged against income in the example above would be \$38,530 + \$3,640 = \$42,170. This figure does not include

any human resource costs which were incurred during the period, but were not capitalized.

The budgeted figures above can be compared against actual human resource figures for the year, and variances can be computed. The beginning balances would not differ, but the transition matrix and actual costs incurred could differ from the budgeted amounts. Assume that the transition matrix for actual personnel movements during the year (promotions, terminations, etc.) is as shown below.

$$\text{Actual P} = \begin{matrix} & \begin{matrix} \text{S} & \text{M} & \text{P} & \text{L} & \text{L(P)} \end{matrix} \\ \begin{matrix} \text{S} \\ \text{M} \\ \text{P} \\ \text{L} \\ \text{L(P)} \end{matrix} & \left[ \begin{array}{ccccc} .70 & .06 & 0 & .24 & 0 \\ 0 & .79 & .05 & .16 & 0 \\ 0 & 0 & .96 & 0 & .04 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right] \end{matrix}$$

Using the above matrix for actual movements, the actual end-of-year distribution of the beginning balances in human resource costs would be found as follows:

$$[\$100,000 \ \$100,000 \ \$100,000 \ 0 \ 0] \left[ \begin{array}{ccccc} .70 & .06 & 0 & .24 & 0 \\ 0 & .79 & .05 & .16 & 0 \\ 0 & 0 & .96 & 0 & .04 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right]$$

Assume also that actual human resource costs capitalized during the period at the staff, manager, and partner levels were \$21,000, \$18,000, and \$23,000, respectively. The ending subsidiary balances (actual) would be as follows:

Staff	$(\$70,000 + \$21,000)$	= \$ 91,000
Managers	$(\$85,000 + \$18,000)$	= \$103,000
Partners	$(\$101,000 + \$23,000)$	= <u>\$124,000</u>
TOTAL	$(\$256,000 + \$62,000)$	= <u><u>\$318,000</u></u>

The actual amount that would be charged against income for the period would be  $(\$40,000 + \$4,000) = \$44,000$ . These are the (1,4) and (1,5) elements of the computed vector of ending distributions of human resource costs, shown above.

The total variance would be the difference between the budgeted account balance at the end of the year (\$317,830) and the actual balance (\$318,000). This total variance (\$170) can be broken down into a flow variance and a cost variance. The flow variance would be the difference in the asset balances (budgeted vs. actual) before the actual costs for the year were added. The same result is found by comparing the actual amount charged against income to the budgeted figure. This is shown by the following computations.

Budgeted asset (before current costs)	\$257,830
Actual asset (before current costs)	<u>\$256,000</u>
Flow variance	<u>\$ 1,830</u>
Budgeted charge against income	\$ 42,170
Actual charge against income	<u>\$ 44,000</u>
Flow variance	<u>(\$ 1,830)</u>

The cost variance in the above example would be the difference in budgeted current costs and actual current costs, as shown below.

Budgeted current costs		
(\$20,000 + \$20,000 + \$20,000)	=	\$ 60,000
Actual current costs		
(\$21,000 + \$18,000 + \$23,000)	=	<u>\$ 62,000</u>
Cost variance		<u>\$ 2,000</u>

Based on the above example, the flow variance would be unfavorable. The actual charge against income (\$44,000) is greater than the budgeted charge against income (\$42,170). The actual flow of costs results in a lower net income than budgeted costs, as well as a lower balance in the human resource account. The cost variance in the above example would be favorable, even though actual current costs (\$62,000) are greater than the budgeted current costs (\$60,000). The reason that this variance is favorable is that current costs have no effect on the current period net income figure, but these costs do increase the balance in the human resource account.

The unfavorable flow variance (\$1,830) plus the favorable cost variance (\$2,000) will give the total variance (\$170 favorable). This total variance reflects a difference between the actual ending balance in the human resource account and the budgeted ending balance in this account. The causes of this difference in the ending balances can be seen in the following reconciliation.

Budgeted ending balance	\$317,830
Add: cost variance (favorable)	<u>2,000</u>
	\$319,830
Less: flow variance (unfavorable)	<u>(1,830)</u>
Actual ending balance	<u>\$318,000</u>

Both the flow variance and the cost variance could be segregated into three separate variances. Such an analysis would break the total cost variance into a staff cost variance, a manager cost variance, and a partner cost variance. The flow variance could also be broken down into variances for each job level. All figures needed to compute these six variances have already been given in the above example. These variances would be as follows in Table 14. As would be expected, the three individual flow variances sum to the total flow variance. This is also true for the cost variance.

Economic valuation. The economic valuation of the human resources of the firm is based on the excess earnings of the firm as discussed in Chapter VI. The following data are assumed data for the firm and will be used in the valuation computation:

TABLE 14  
SIX-WAY VARIANCE ANALYSIS

	Budgeted	Actual	Variance
<u>Flow variance</u>			
Staff	\$ 70,190	\$ 70,000	\$ 190 (UNFAV)
Manager	86,310	85,000	1,310 (UNFAV)
Partner	101,330	101,000	<u>330</u> (UNFAV)
			<u>\$1,830</u> (UNFAV)
<u>Cost variance</u>			
Staff	\$ 20,000	\$ 21,000	\$1,000 (FAV)
Manager	20,000	18,000	2,000 (UNFAV)
Partner	20,000	23,000	<u>3,000</u> (FAV)
			<u>\$2,000</u> (FAV)

	<u>Staff</u>	<u>Manager</u>	<u>Partner</u>
Billing rate	\$ 25	\$ 40	\$ 65
Annual chargeable hours	1,700	1,500	1,300
Average annual salary	\$18,000	\$34,000	\$74,000 <sup>3</sup>
Other expenses per chargeable hour	\$ 7.75	\$ 7.75	\$ 7.75

Expenses other than personal salaries are assumed to be \$7.75 per chargeable hour, or about 25% of total revenue. This figure is assumed to be constant at all levels. The partners will have lower travel expenses than the staff and managers, but it is assumed that this difference will be offset by higher facility costs incurred by the partners.

The above data can be used to compute the excess earnings for the firm for the period ending January 31, 1976. The number of employees at that date were 8,027 staff, 2,049 managers, and 937 partners (based on the actual data from the firm).

Income Statement  
(in thousands)

Revenue:

Staff (1700 x 8027 x \$25)	\$341,147.5
Managers (1500 x 2049 x \$40)	122,940.0
Partners (1300 x 937 x \$65)	<u>79,176.5</u>
Total Revenue	\$543,264.

Expenses:

Salaries:

Staff (8027 x \$18000)	(144,486)
Managers (2049 x \$34000)	( 69,666)
Other expenses (\$7.75 per chargeable hour)	<u>(139,014.9)</u>
Net Income	\$190,097.1
Less: partners salary allowances (937 x \$74,000)	<u>(69,338)</u>
Excess Earnings	<u><u>\$120,759.1</u></u>

<sup>3</sup>The basis for these salary figures was found in "News Report," Journal of Accountancy (July 1977), p. 22. The amount for partners is assumed to be a salary allowance rather than a profit distribution.

To compute the total economic valuation figure, the excess earnings each year for the next 40 years would be discounted to the present (January 31, 1976 in the above example). The discount rate would be the cost of capital for the firm, and is assumed to be 10%. The excess earnings for each year would be computed in the same manner as above. However, the number of employees at each level will change. These predicted levels are computed using the transition matrix and the number of current employees at the date of valuation.

The above methodology also can be used to compute the valuation of only the staff and manager levels. The net income figures which should be capitalized are derived from the revenue and expenses that relate to these two levels.

A computer program was written to make all of the above computations. The program is called "VALUATION" and a listing can be found in Appendix E. The program was executed using the above data as of January 31, 1976. According to the output of the program, the total human resource valuation as of that date was \$432,314,000. The program was executed a second time, but all data for partners was deleted. The valuation for staff and managers was \$427,239,000. The valuation for partners can be found as follows:

Total human resource valuation	\$432,314,000.
Staff and manager valuation	<u>427,239,000.</u>
Difference (partner valuation)	<u>\$ 5,075,000.</u>

This program was also executed twice using the employee levels as of January 31, 1977. The results can be summarized as follows:

Total human resource valuation	\$448,624,000
Staff and manager valuation	<u>443,209,000</u>
Difference (partner valuation)	<u>\$ 5,415,000</u>

The above information can be used for internal decision-making purposes. If the firm desired to capitalize human resources on the balance sheet, the amount of \$432,314,000 would be capitalized as of January 31, 1976 (assuming that was the first year of capitalization). As of January 31, 1977 the valuation had increased to \$448,624,000. The increase of \$16,310,000 would be the amount capitalized during the period (January 31, 1976 to January 31, 1977).

#### Summary

Even though a valid MARKOV property test was not performed on the data in Chapter VII, it was assumed that the model could be used for planning and control in a CPA firm. As much as possible data from a large CPA firm was used to develop the model for use in manpower planning and human resource valuation.

The model is used to find nonfinancial information such as average tenure with the firm, average time at each level, probability of reaching partner, projected steady state information, and projected short-run employee levels. These data have specific use in the area of manpower planning.

Two methods were used to amortize a cost-based human resource account. The first method used an average time period with the firm (5.91 years) and amortized all costs over this period. The second

method used a time period equal to the average tenure plus three standard deviations ( $5.91 + 3(10.26) = 36.69$  years) for purposes of amortization. Whenever an employee leaves the firm, a loss is recognized, an aspect that was not characteristic of the first method. A third method of dealing with a cost-based asset account assumed that the costs would flow with the personnel. Budgeting and variances were discussed in relation to this method. Finally, an economic valuation of the human assets was used at the beginning and end of a time period to compute the increase or decrease (gain or loss) in the account.

## Chapter IX

### SUMMARY AND FURTHER RESEARCH

This concluding chapter will present a brief summary of Chapters II-VIII, although no numerical results will be given here. A number of areas in which further research could extend the preceding results will also be presented, along with the data needed for this research.

#### Summary

Chapters II and III were surveys of the literature in the areas of manpower planning and human resource valuation, respectively. The nonfinancial aspects of the model are presented in Chapter IV. Assumptions of the Markov model are discussed and the formulae for testing these assumptions are given. Steps for the development of the foundation of the model, the transition matrix, are discussed. Finally, it is shown how the transition matrix can be manipulated, both alone and with other data, to compute nonfinancial data which can be used in manpower planning.

The financial facets of the model and their relation to human resource valuation are presented in Chapter V. Cost bases and an economic valuation base for valuing human assets are discussed. The cost bases include historical cost and replacement cost. The applicability of the model is not in the distinction of which personnel

costs should be capitalized. Once certain costs have been capitalized, the model can be used to determine which costs should be allocated to the income statement in successive periods. Three methods for making this allocation were discussed. The first method amortizes capitalized costs over the average time all employees spend with the firm. Losses from personnel leaving the firm would not be shown if this method were used. The second method uses an amortization period equal to the average tenure with the firm plus 3 standard deviations. The use of this method does allow recognition of losses from employees leaving the firm. The third method of dealing with the capitalized costs assumes that these costs flow through the firm in the same manner as do the employees. This flow is modeled by the transition matrix. This method can easily be used to establish a human resource budget and to compute variances from this budget ("flow" and "cost" variances).

The economic valuation of the human assets of the firm is found by computing the present value of expected future benefits accruing to the firm from the employment of these human assets. In order to make these computations, some method is needed for placing a monetary value on the benefits of each employee to the firm. If this can be done, the model can be used to project how many employees would be with the firm in the future, based solely on present employees (no hirings). The economic valuation is the number of employees at the end of each period multiplied by the "value" of each employee, discounted to the present. These computations are projected over a 40-year time period.

The application of the conceptual model to a large Certified Public Accounting firm is presented in Chapters VI through VIII. Chapter VI sets up the model for any CPA firm without dealing with quantitative data. These data are used in the next two chapters to relate the model to one specific firm as much as possible. The true applicability of the figures in these chapters to the particular firm was limited because not all needed data were available. However, if these data had been available, the techniques presented in Chapters VII and VIII would not change.

Tests on the validity of the model were performed in Chapter VII. Based on the stability test, a transition matrix was developed covering the time period from February 1, 1972 to January 31, 1977. The Markov property test was an example of how the test would be performed, although in this case the test was only simulated because it was not based on actual data from the firm. The data were simulated by the use of a GERT computer program.

The manpower planning data which can be gleaned from the model are presented in Chapter VIII. Examples of the amortization techniques and other methods related to human resource valuation are also presented in this chapter. A method of budgeting human resources on a monetary basis is discussed and a two-way analysis of variance is shown. An economic valuation of the firm's human resources is also based upon the Markovian transition matrix in that this matrix is used to project the number of present employees at each level over the next

forty years. The present value of the net expected benefits from these employees over the forty-year period is the economic valuation of the firm's human resources.

#### Further Research

If a researcher were to have complete access to the personnel records of a company, there are many areas in which further research would yield useful data. These areas are discussed below in terms of the data needs (inputs) and the information yield (outputs).

Markov property test. If the data are available, a Markov property test should be performed on the model. The movements of individuals, rather than aggregate movements would be needed in order to perform this test. The career movements of only a sample of the total number of employees would be sufficient.

A knowledge of the promotional policies of a CPA firm would make one believe that the model is not a first-order Markov chain. Promotions are affected by the amount of time an individual has been in each state, i.e., the probability of a manager being promoted to partner would be different if the person has been a manager three years than if the person has been a manager only two years. Probabilities thus become time-dependent and the chain is not first-order. However, in looking at the model as if the employees were parts of homogeneous groups (aggregates), then it is possible that

the model will statistically react as if it were a first-order Markov chain. This would justify the use of the model, as stated by Churchill and Shank:

In summary, the results of our tests indicate that we cannot reject the Markov assumptions for our sample data. This does not say that the management succession process for the test firm is a first-order Markov Chain because it clearly is not. No complex system such as management succession for an actual business firm can ever be accurately described by such a simple mathematical model. Since, however, the management succession data behave as they would in a first-order Markov Chain it does seem justifiable to consider such a model to be a useful planning and control tool--as long as the results are tempered with a healthy dose of management judgment.<sup>1</sup>

It is possible that a valid test of the data would reject the hypothesis that the chain is of first-order. The model would then have to be modified in order to be mathematically valid. One way to ensure that the chain would be first-order is to define each state as a job level (as done before), but go further and denote different states for each year the employee is at each job level. The person who enters the firm at the staff level would be in the  $S_1$  state (staff level, first year). During the next transition period, the employee could make one of three movements; (1) from  $S_1$  to  $S_2$  (staff level, second year), (2) from  $S_1$  to  $M_1$  (manager level, first year), or (3) from  $S_1$  to L (leave the firm). The person could not stay at the  $S_1$  level, so  $p_{i1} = 0$ . A possible transition matrix based on this definition of states is shown below.

---

<sup>1</sup>Neil C. Churchill and John K. Shank, "Accounting for Affirmative Action Programs: A Stochastic Flow Approach," The Accounting Review, Vol. 50, No. 4 (October 1975), p. 652.

	$S_1$	.	.	.	$S_5$	$S_A$	$M_1$	.	.	$M_6$	$M_A$	P	L	$L(P)$
$S_1$	$P_{s_1 s_1}$	.	.	.	.	.	.	.	.	.	.	.	.	$P_{s_1 L(P)}$
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$S_5$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$S_A$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$M_1$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
P =	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$M_6$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$M_A$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
P	.	.	.	.	.	.	.	.	.	.	.	.	.	.
L	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$L(P)$	$P_{L(P) s_1}$	.	.	.	.	.	.	.	.	.	.	.	.	$P_{L(P) L(P)}$

The subscript (<sub>A</sub>) means that the state includes all employees who have been at the same job level longer than the time specified in the immediately preceding state. For example, the state  $S_A$  in the above matrix would include all employees at the staff level who have been there longer than 5 years (the preceding state is  $S_5$ ). This state would only be valid if the probabilities of being promoted to manager or of leaving the firm are the same for all employees included in the state.

Although the model has been modified, much of the informational output should be the same as before. This new model should show a more precise flow of each employee through the firm, since it is on a year-by-year basis.

Employee levels. Returning to the original model, there is another way in which a more precise flow of the employee through the firm could be found. In the given data, there were only three levels of employment. The staff level in this firm includes what would be equal to at least two levels in many firms. If the data had been on a basis of four levels instead of three, the policies at these lower levels could have been included in the model. Many firms even have sub-levels within these four levels. It would be possible to incorporate all levels and sub-levels into the model. This would analyze the employee's internal movements in the most complete manner.

Staff entry. It was assumed in the application of the model that all new personnel entered the firm at the staff level. This is not necessarily the case, even in the particular firm used in the application of the model. According to one employee with the firm, hirings do occur at other levels, but that the number of these hirings is "insignificant" and "not readily available." If known, these upper-level hirings could be brought into the model easily, simply by including the employees in the hirings vector (k).

One problem associated with upper-level hirings is that these entering employees may not have the same probabilities of movement as a person who has moved up through the firm. This could create a need for a second transition matrix for just these employees. This would depend on the number of employees falling into this category, and also whether the probabilities for these employees is

significantly different than for the "normal" employees. If the transition matrix were in the form as discussed in the earlier section of this chapter on the Markov property test, it would be easier to find a level analogous to the entry of the employee.

Termination reasons. Reasons for leaving the firm were not dealt with by the model at all. If these reasons were known by the researcher, they could be built into the model by re-defining and adding absorbing states. Presently, the two absorbing states are defined as (1) leaving the firm before becoming partner (for any reason), and (2) leaving the firm after becoming partner (for any reason). The first absorbing could be expanded into at least three absorbing states denoting why the employee left the firm before becoming partner, i.e., death, resignation, or firing. The second absorbing state could be realistically expanded to four states denoting why the partner left the firm. These reasons would include the three possibilities above plus retirement. This termination information could be relevant to planning future policies, and would be the type of information needed for manpower planning in a comprehensive human resource management system.

Specialists. In most CPA firms, there are three main areas in which an employee may specialize. These areas are auditing, taxation, and management advisory services (MAS). It is possible that the transition probabilities for these three areas may differ. If the aggregate data were segregated into areas of specialization, three

transition matrices could be developed. Both manpower planning and human resource valuation could be carried out separately for each area.

Minorities. It is also very possible that the transition probabilities would be different for white males as opposed to blacks or women. No distinction of this type was made in the original data. It may be that there have not been enough blacks and women hired by the firm to allow adequate modeling of their experiences. Any segregation of data into these groups could at least give the researcher a feel for whether the transition probabilities would be significantly different from the same probabilities for the male Caucasians employed by the firm.

Geographical regions. The model was based on the total number of domestic employees of the firm. Transition probabilities could differ from one geographical area to another. In making comparisons, there must be enough data to ensure the model is a valid representation of the region. If the model could be developed for individual regions, then manpower planning could be carried out on this same regional basis. This would be more relevant for some of the decentralized decision-making that must be done, particularly if the firm allows a large amount of autonomy. The best transition matrix for an autonomous office would be the matrix for that office alone. The main problem here is whether there are enough employees in the office to develop a valid model (as with regions). This would

probably only be true of very large offices, such as those in New York City. This is an important area for future research. Manpower planning on a one-office basis would be invaluable to that office.

Other organizations. The above areas of further research have been discussed in the context of a large CPA firm. This does not mean that this is the only type of organization for which the conceptual model will apply. If an organization has different job positions, and if personnel movements from one position to another normally occur, then the model should be applicable. The jobs do not have to be hierarchical as is the case in the CPA firm. Further research could reveal other types of organizations which could use the model as an aid in manpower planning and human resource valuation. Examples of organizations which might use the model would be civil service systems, branches of the armed services, insurance companies, and universities.

### Conclusions

A Markov model can be a valuable addition to a comprehensive human resource management system. The model has applications in the areas of manpower planning and human resource valuation. Because of the lack of data, not all of these applications were presented in the example of modeling a CPA firm. However, this does not mean that these applications are not important. Some relevant applications are presented in the "Further Research" section of this chapter.

Manpower planning and human resource valuation are not mutually exclusive areas. The areas are complementary and both can

be an integral facet in personnel management. Manpower planning has not traditionally been an accounting topic. The accountant should be concerned with the planning in relation to a resource as important as the personnel of the company. Concern over human resources, from an accounting point of view, has manifested itself in the form of human resource valuation. The problem of manpower planning is the next logical step that the accountant can and should take.

BIBLIOGRAPHY

- American Accounting Association. "Report of the Committee on Accounting for Human Resources." Supplement to The Accounting Review 48 (1973):169-185.
- American Accounting Association. "Report of the Committee on Accounting for Human Resources." Supplement to The Accounting Review 49 (1974):115-124.
- American Accounting Association, Committee on Accounting Concepts and Standards. "Accounting and Reporting Standards for Corporate Financial Statements and Preceding Statements and Supplements." Columbus, Ohio: American Accounting Association, 1957.
- American Institute of Certified Public Accountants. APB Accounting Principles: Original Pronouncement. "Statement No. 4," Chapter 6. Chicago: Commerce Clearing House, Inc., 1972.
- Anderson, T. W., and Goodman, L. A. "Statistical Inference about Markov Chains." Annals of Mathematical Statistics 28 (March 1957): 89-110.
- Baker, Geoffrey M. N. "Feasibility and Utility of Human Resource Accounting." California Management Review 16 (Summer 1974):17-23.
- Banner, D., and Baker, G. "Human Resource Accounting: A Critical View." MSU Business Topics 21 (Autumn 1973):45-52.
- Bartlett, J. B. "Problems in Manpower Planning." Personnel Management 5 (Fall 1973):30-31.
- Bassett, Glenn A. "Manpower Forecasting and Planning: Problems and Solution." Personnel 47 (September 1970):8-16.
- Bassett, Glenn A. "Employee Turnover Measurement and Human Resource Accounting." Human Resource Management (Fall 1972), pp. 21-30.
- Bierman, Harold. "Bond Refunding Decision as a Markov Process." Management Science 12 (August 1966):B545-B551.
- Blaine, E., and Stanbury, W. J. "Accounting for Human Capital." Canadian Chartered Accountant (January 1971), pp. 68-72.
- Blakely, R. J. "Markov Models and Manpower Planning." Industrial Management Review 11 (Winter 1970):39-46.
- Bonham, J. W.; Clayton, Edward R.; and Moore, Lawrence J. "A GERT Model to Meet Future Organizational Manpower Needs." Personnel Journal 54 (July 1975):402-406.

- Brown, R. J., and Somerville, J. D. "What the Supervisor Should Know About Manpower Planning." Supervisory Management 19 (September 1974):22-29.
- Brummet, R. Lee. "Accounting for Human Resources." The New York CPA 40 (July 1970):547-555.
- Brummet, R. Lee. "Human Resource Accounting." Massachusetts CPA Review 45 (March/April 1971):16.
- Brummet, R. Lee; Flamholtz, Eric G.; and Pyle, William C. "Accounting for Human Resources." Michigan Business Review (March 1968), pp. 20-25.
- Brummet, R. Lee; Flamholtz, Eric G.; and Pyle, William C. "Human Resource Management--A Challenge to Accountants." The Accounting Review 43 (April 1968):217-224.
- Brummet, R. Lee; Flamholtz, Eric G.; and Pyle, William C. "Human Resource Accounting in Industry." Personnel Administration (July/August 1969), pp. 34-46.
- Brummet, R. Lee; Flamholtz, Eric G.; and Pyle, William C. "Human Resource Accounting: A Tool to Increase Managerial Effectiveness." Management Accounting (August 1969), pp. 12-15.
- Bryant, Don R.; Maggard, Michael J.; and Taylor, Robert P. "Manpower Planning Models and Techniques." Business Horizons 16 (April 1973):69-78.
- Caddy, G. K. "Human Value Model." Management Accounting 56 (April 1975):27-30.
- Caplan, Edwin H., and Landekich, Stephen. Human Resource Accounting: Past, Present and Future. New York: National Association of Accountants, 1974.
- Cassell, F. H. "Manpower Planning: State of the Art at the Micro Level." MSU Business Topics 21 (Autumn 1973):13-21.
- Cassidy, R. E. "Manpower Planning--Coordinated Approach." Personnel 40 (September 1963):35-41.
- Churchill, Neil C., and Shank, John K. "Accounting for Affirmative Action Programs: A Stochastic Flow Approach." The Accounting Review 50 (October 1975):643-656.
- Coleman, B. P. "Integrated Systems for Manpower Planning." Business Horizons 8 (October 1970):89-95.

- Comerford, Robert. "What's Wrong with Today's Human Resource Accounting?" Massachusetts CPA Review 47 (March/April 1973):24-27.
- Cooper, Allen A., and Parker, James E. "Human Resource Accounting: An Examination." Cost and Management (Canada) 47 (January/February 1973):21-26.
- Corcoran, A. Wayne, and Leininger, Wayne E. "Stochastic Process Costing Models." Accounting Review 48 (January 1973):105-114.
- Craft, J. A., and Birnberg, J. G. "Human Resource Accounting: Perspective and Prospects." Industrial Relations 15 (February 1976):2-12.
- Croasdale, Robert. "Markov Chain Models for Motor Insurance." Accounting and Business Research 2 (Summer 1972):177-181.
- Crumbly, D. Larry, and Savich, S. Richard. "Human Resource Accounting in Taxation." The Accounting Review 50 (January 1975):112-117.
- Cyert, R. M.; Davidson, H. J.; and Thompson, G. L. "Estimation of the Allowance for Doubtful Accounts by Markov Chains." Management Science 8 (April 1962):287-303.
- Davies, G. S. "Structural Control in a Graded Manpower System." Management Science 20 (September 1973):76-84.
- Deming, W. Edwards, and Glasser, Gerald J. "A Markovian Analysis of the Life of Newspaper Subscriptions." Management Science 14 (February 1968):B283-B293.
- Dermer, Jerry, and Siegel, Jacob. "The Role of Behavioral Measures in Accounting for Human Resources." The Accounting Review 49 (January 1974):88-97.
- Dill, W. R.; Gaver, D. P.; and Weber, W. L. "Models and Modelling in Manpower Planning." Management Science 13(B) (December 1966): B142-B167.
- Dittman, David A.; Juris, Hervey A.; and Revsine, Lawrence. "On the Existence of Unrecorded Human Assets, An Economic Perspective." Journal of Accounting Research (April 1976), pp. 49-65.
- Drandell, Milton. "Composite Forecasting Methodology for Manpower Planning." Academy of Management Journal 18 (September 1975): 510-519.
- Dryden, M. M. "Share Price Movements: A Markovian Approach." Journal of Finance 24 (March 1969):49-60.

- Eggers, H. C. "Evaluation of Human Assets." Management Accounting 53 (November 1971):28-30.
- Ehrenburg, A. S. C. "An Appraisal of Markov Brand Switching Models." Journal of Marketing Research 2 (November 1965):347-362.
- Elbert, N. F., and Kehoe, W. J. "How to Bridge Fact and Theory in Manpower Planning." Personnel 53 (November 1976):31-39.
- Elias, Nabil S. "Some Aspects of Human Resource Accounting." Cost and Management 45 (November-December 1971):38-43.
- Elias, Nabil. "The Effects of Human Asset Statements on the Investment Decision: An Experiment." Empirical Research in Accounting, Selected Studies, 1972, Supplement to Journal of Accounting Research, V. 10, pp. 215-233.
- Elias, Nabil S. "Behavioral Impact of Human Resource Accounting." Working Paper No. 91, Faculty of Administrative Studies, The University of Manitoba, 1973.
- Elias, Nabil. "Behavioral Impact of Human Resource Accounting." Management Accounting 47 (February 1976):43-45.
- Elovitz, David. "From the Thoughtful Businessman." Harvard Business Review 45 (May-June 1967):56, 59.
- Executive Ledger. "Big Push in Manpower Planning." Dun's Review 104 (November 1974):103-105.
- Figler, H. R. "Accounting for Human Assets." Management Accounting 47 (November 1975):23-26.
- Flamholtz, Eric. "A Model for Human Resource Valuation: A Stochastic Process with Service Rewards." The Accounting Review 46 (April 1971):253-267.
- Flamholtz, Eric G. "Should Your Organization Attempt to Value Its Human Resources?" California Management Review 14 (Winter 1971):40-45.
- Flamholtz, Eric G. "On the Use of the Economic Concept of Human Capital in Financial Statements: A Comment." (Correspondence), The Accounting Review 47 (January 1972).
- Flamholtz, Eric G. "Assessing the Validity of a Theory of Human Resource Value: A Field Study." Empirical Research in Accounting: Selected Studies, 1972 (Supplement to the Journal of Accounting Research), pp. 241-266.

- Flamholtz, Eric. "Human Resource Accounting: Measuring Positional Replacement Cost." Human Resource Management, Graduate School of Business Administration, University of Michigan (Spring 1973).
- Flamholtz, Eric. "Human Resource Accounting: A Review of Theory and Research." Journal of Management Studies 11 (February 1974):44-61.
- Flamholtz, Eric G. Human Resource Accounting. Encino, California: Dickenson Publishing Co., 1974.
- Flowers, V. S., and Coda, B. A. "Human Resource Planning: Foundation for a Model." Personnel 51 (January 1974):20-32.
- Frantzred, Richard G.; Landau, Linda J.; and Lundberg, Donald P. "Valuation of Human Resources." Business Horizons 17 (June 1974): 73-80.
- Friedland, M. "Putting a Value on Human Resources." International Management (October 1970), pp. 42-44.
- Friedman, Abraham, and Lev, Baruch. "A Surrogate Measure for the Firm's Investment in Human Resources." Journal of Accounting Research 12 (Autumn 1974):235-250.
- Gambling, Trevor E. "A System Dynamics Approach to Human Resource Accounting." The Accounting Review 49 (July 1974):538-546.
- Gilbert, Michael H. "The Asset Value of the Human Organization." Management Accounting, (July 1970), pp. 25-28.
- Glautier, M. W. E., and Underdown, B. "Problems and Prospects of Accounting for Human Assets." Management Accounting (England) 51 (March 1973):98-102.
- Groover, Robert O. "PERSYM: A Generalized Simulation Model of a Military Personnel System." AD 696 986. Oporto, Portugal: NATO Conference Paper, 1969.
- Guiles, Roger. "Human Resource Accounting: A Price Tag on People." Modern Healthcare 2 (July 1974):62-65.
- Haire, Mason. "Managing Management Manpower." Business Horizons 10 (Winter 1967):23-28.
- Haire, Mason. "A New Look at Human Resources." Industrial Management Review 11 (Winter 1970):17-23.
- Hardt, E. "Manpower Planning." Personnel Journal 46 (March 1967):157-161.

- Haywood, J. J. "Man-Management Myopia: Accounting for Human Resources." Accountant (England) 167 (September 28, 1972):384-387.
- Hekimian, James S., and Jones, Curtis H. "Put People on Your Balance Sheet." Harvard Business Review 45 (January-February 1967): 105-113.
- Hermanson, Roger H. Accounting for Human Assets, Occasional Paper No. 14. East Lansing, Michigan: Michigan State University, 1964.
- Hinomoto, Hirohide. "Optimum Strategies for Management Information Processing and Control." Journal of Accounting Research 9 (Autumn 1971):253-267.
- Hopes, R. F. A. "Some Statistical Aspects of Manpower Planning in the Civil Service." Omega, The International Journal of Management Science 1 (April 1973):165-180.
- Howard, Ronald A., and Matheson, James E. "Risk Sensitive Markov Decision Processes." Management Science 18 (March 1972):356-369.
- Jaggi, Bikki L. "Valuation of Human Resources: A Practical Model." Cost and Management (July/August 1975), pp. 29-34.
- Jaggi, Bikki. "Human Resources are Assets." Management Accounting 57 (February 1976):41-42.
- Jaggi, Bikki, and Lau, Hon-Shiang. "Toward a Model for Human Resource Valuation." The Accounting Review 49 (April 1974):321-329.
- Jarrett, F. G., and Dent, W. J. "Markov Chains and Wool Promotion Strategy." Economics Review 43 (March 1967):97-107.
- Jaunch, Roger, and Skigen, Michael. "Human Resource Accounting: A Critical Evaluation." Management Accounting 55 (May 1974):33-36.
- Jones, D. M. C. "Accounting for Human Assets." The Accountant's Review (June 1975), pp. 86-99.
- Kahalas, Harvey, and Leininger, Wayne E. "Manpower Planning Model for Organizational Effectiveness." Long Range Planning 8 (August 1975):85-90.
- Kahalas, Harvey, and Leininger, Wayne E. "A Stochastic Decision Model for Returnable Containers." (Unpublished paper.)
- Kane, Julius. "Dynamics of the Peter Principle." Management Science 16 (August 1970):800-811.

- Kemeny, John G., and Snell, J. Laurie. Finite Markov Chains. Princeton, New Jersey: D. Van Nostrand, 1960.
- Kollaritsch, Felix P. "Future Service Potential Value." Journal of Accountancy 119 (February 1965):57-62.
- Lavin, Milton. "Strategy in Modeling Manpower Problems." Industrial Management Review (Winter 1970), pp. 47-51.
- Lawrence, John. "Manpower and Personnel Models in Britain." Personnel Review (Great Britain) (Summer 1973), pp. 4-26.
- Lev, Baruch, and Schwartz, Aba. "On the Use of the Economic Concept of Human Capital in Financial Statements." The Accounting Review 46 (January 1971):103-112.
- Lev, Baruch, and Schwartz, Aba. "On the Use of the Economic Concept of Human Capital in Financial Statements: A Reply." (Correspondence), The Accounting Review 47 (January 1972):153-4.
- Lewin, C. G. "A Manpower Planning Study." Operational Research Quarterly (June 1971), pp. 99-116.
- Liao, Shu S. "Human Assets, Human Resources, Managerial Decisions." Management Accounting 56 (November 1974):19-22.
- Liebman, Leon H. "Markov Decision Model for Selecting Optimum Credit Control Policies." Management Science 18 (June 1972):519-525.
- Likert, Rensis M. The Human Organization, Its Measurement and Value. New York: McGraw-Hill Publishing Company, 1966.
- Likert, Rensis. "Human Resource Accounting: Building and Assessing Productive Organizations." Personnel (May 1973), pp. 8-24.
- Likert, Rensis M., and Bowers, David G. "Organizational Theory and Human Resource Accounting." American Psychologist 23 (September 1969):585-592.
- Likert, Rensis M., and Bowers David G. "Improving the Accuracy of Profit/Loss Reports by Estimating the Change in Dollar Value of the Human Organization." Michigan Business Review 25 (March 1973):15-24.
- Likert, Rensis, and Pyle, William C. "A Human Organizational Measurement Approach." Financial Analysts Journal (January/February 1971), pp. 75-84.

- Likert, Rensis, and Seashore, Stanley E. "Making Cost Control Work." Harvard Business Review (November/December 1963), pp. 96-108.
- Maffei, R. B. "Brand Preferences and Simple Markov Processes." Operations Research 8 (March 1960):210-218.
- Mahoney, T. A., and Milkovich, G. J. "Computer Simulation: A Training Tool for Manpower Managers." Personnel Journal 54 (December 1975): 609-612.
- Marshall, Kneale J. "A Comparison of Two Personnel Prediction Models." Operations Research (May-June 1973), pp. 810-822.
- McRae, Thomas W. "Human Resource Accounting as a Management Tool." Journal of Accountancy (August 1974 , pp. 32-38.
- Mehta, Dileep. "Markov Process and Credit Collection Policy." Decision Sciences 3 (April 1972):27-43.
- Meredith, Jack. "Markovian Analysis of a Geriatric Ward." Management Science 19 (February 1973):604-612.
- Mikovich, G. J.; Annoni, Anthony J.; and Mahoney, Thomas J. "Use of the Delphi Procedures in Manpower Forecasting." Management Science 19 (December 1972):381-388.
- Miller, James R. "Micro-Simulation as an Aid to Managing Human Resources." Industrial Management Review 11 (Winter 1970):24-30.
- Miller, James R., and Haire, Mason. "MANPLAN: A Micro-Simulator for Manpower Planning." Behavioral Science (November 1970), pp. 524-531.
- Morse, Wayne J. "Toward a Model for Human Resource Valuation: A Comment." The Accounting Review (April 1975), pp. 345-347.
- Moss, Robert W., and Was, Stephen R. " A Computerized System for Projecting Reserve Manpower Strengths and Costs." AD 831 565. McLean, Virginia: Research Analysis Corporation, 1967.
- Myers, C. S. "Challenge of the 1960's: Manpower Management." Personnel 36 (May 1959):15-21.
- Myers, S., and Flowers, V. S. "Framework for Measuring Human Assets." California Management Review (Summer 1974), pp. 5-16.
- Nielsen, G. L., and Young, A. R. "Manpower Planning: A Markov Chain Application." Public Personnel Management 2 (March 1973):133-143.

- Ogan, Pekin. "Human Resource Value Model for Professional Service Organizations." The Accounting Review 51 (April 1976):306-320.
- Paton, William A. Accounting Theory. New York: The Ronald Press Company, 1922.
- Pegels, C. Carl, and Jelmer, Andrew E. "An Evaluation of Blood Inventory Policies: A Markov Chain Approach." Operations Research 18 (November 1970):1087-1098.
- Peterson, R. B. "Growing Role of Manpower Forecasting in Organizations." MSU Business Topics 17 (Summer 1969):7-14.
- Peterson, Sandra E. "Accounting for Human Resources." Management Accounting 53 (June 1972):19-22.
- Powell, Reed M., and Wilkens, Paul L. "Design and Implementation of a Human Resource Information System." MSU Business Topics 21 (Winter 1973):21-27.
- Pyle, William C. "Human Resource Accounting." Financial Analysts Journal (September/October 1970), pp. 69-78.
- Pyle, William C., and Likert, Rensis. "Human Resource Accounting." Financial Analysts Journal (January 1971), pp. 75-87.
- Rhode, John G.; Lawler, Edward E., III; and Sundem, Gary L. "Human Resource Accounting: A Critical Assessment." Industrial Relations 15 (February 1976):13-25.
- Robinson, Bruce A. "Approach to Human Resource Accounting." Cost and Management 48 (May/June 1974):26-32.
- Robinson, David. "Progress in Human Resource Accounting." Personnel Management (March 1973), pp. 31-33.
- Robinson, David E. "Human Asset Accounting." Long Range Planning 7 (February 1974):58-60.
- Robinson, David. "Two Approaches to Human Resource Accounting." Accounting 86 (February 1975):46-48.
- Rowland, Kenneth M., and Sovereign, Michael G. "Markov Chain Analysis of Internal Manpower Supply." Industrial Relations (October 1969), pp. 88-99.
- Ryan, Terence M. "Security Prices as Markov Processes." Journal of Finance and Quantitative Analysis 8 (January 1973):17-36.
- Sadan, S., and Auerbach, L. B. "Stochastic Model for Human Resource Valuation." California Management Review 16 (Summer 197 ):24-31.

- Shank, John K. "Income Determination Under Uncertainty: An Application of Markov Chains." Accounting Review 46 (January 1971):57-74.
- Simpson, Robert R., Jr. "Measurement of Human Resources." Journal of Accountancy 132 (September 1971):42.
- Staszak, F. J., and Mathys, N. J. "Organization Gap: Implications for Manpower Planning." California Management Review 17 (Spring 1975): 32-38.
- Thompson, William W., and McNeal, James U. "Sales Planning and Control Using Absorbing Markov Chains." Journal of Marketing Research 4 (Fall 1967):62-66.
- Toikka, R. S. "Markovian Model of Labor Market Decisions by Workers." American Economic Review 66 (December 1976):821-834.
- Trinkl, Frank H. "Stochastic Analysis of Programs for the Mentally Retarded." Operations Research 22 (November-December 1974): 1175-1191.
- Uyar, Kivilcim M. "Markov Chain Forecasts of Employee Replacement Needs." Industrial Relations (February 1972), pp. 96-106.
- Vajda, S. "Mathematical Aspects of Manpower Planning." Operational Research Quarterly 26 (September 1975):527-542.
- Vassiliou, P. C. G. "Markov Chain Model for Wastage in Manpower Systems." Operations Research Quarterly 27 (No. 1, 1976):57-70.
- Vroom, Victor H., and MacCrimmon, Kenneth R. "Toward a Stochastic Model of Managerial Careers." Administrative Science Quarterly 13 (June 1968):26-46.
- Weber, W. L. "Manpower Planning in Hierarchical Organizations: A Computer Simulation Approach." Management Science 18 (November 1971):B119-B144.
- Weiss, Marvin. "Human Resource Accounting--A Neglected Area." CPA Journal (September 1972), pp. 735-743.
- Weiss, Marvin. "Accounting for Human Resources." Management Review (March 1973), pp. 58-66.
- Weiss, Marvin. "Human Resources Accounting." Management Accounting 57 (March 1976):45-6.

Woodruff, Robert L., Jr. "Human Resource Accounting." Canadian Chartered Accountant (September 1970), pp. 156-161.

Wright, Robert. "Managing Man as a Capital Asset." Personnel Journal 49 (April 1970):290-298.

## APPENDICES

APPENDIX A

NOTATION

## APPENDIX A

## NOTATION

This appendix will briefly define the notations as used in the formulae in Chapters 4 and 5. These formulae are also set forth in Appendix B. The notation will be defined in alphabetical order so as to facilitate the finding of each particular term needed. The first listing is for just subscripts found in the formulae.

<u>Subscript</u>	<u>Explanation</u>
------------------	--------------------

dg	The subscript (dg) denotes a matrix with zeros replacing the original elements that are <u>not</u> on the primary diagonal. The primary diagonal elements remain intact.
i,j,k	These are the classic symbols used to specify position in a matrix (i and j). All three are referenced by summation ( $\sum$ ) and multiplicative ( $\prod$ ) signs and are defined in the formula where used.
m	The total number of states in the transition matrix.
n,N	Both are used to specify a time period.
sq	The subscript (sq) is used to denote that all elements of the matrix should be individually squared.
SS	Steady-state.
VAR	Variance.

---

<u>Term</u>	<u>Explanation</u>
-------------	--------------------

a	The number of <u>consecutive</u> transition periods in a state, once that state has been entered. Computed in (4-26).
B	The matrix of the probability of being absorbed into each state, based on entering position. Computed in (4-12).

<u>Term</u>	<u>Explanation</u>
BR	Billing Rate. The rate at which the client of a CPA firm is charged per hour of services provided by an employee of the CPA firm. Used in (6-1).
C	Capitalized human resource costs associated with a particular state. Used in (5-5).
CH	Chargeable Hours. The number of hours worked by an employee of a CPA firm which can be billed to clients. Used in (6-1).
e	A vector with all elements being equal to one.
F	The fundamental matrix. Each element gives the number of transition periods the process is in each state before it is absorbed, depending upon the initial state entered. Computed by (4-8).
$\hat{F}$	The conditional fundamental matrix. Each element means the same as in F, but with the knowledge that the process will be absorbed into a particular state. Computed in (4-31).
g	The vector of the employees that are with the firm at the beginning of the process that will ultimately be absorbed into each state. Computed in (4-13).
H	The probability that the process will ever be in a particular transient state in the future, based on the present state of the process. Computed in (4-29).
I	The identity matrix, even when used as one of the partitions of the transition matrix.
k	The vector which gives the number of people entering the firm at each level during a transition period.
K	The total number of employees hired each period.
l	The vector of employees entering each absorbing state during a period. Used in (4-21).
m	The vector of steady state employees that will ultimately be absorbed into each state. Computed in (4-24).
n	Actual observations as found in the statistical tests.

<u>Term</u>	<u>Explanation</u>
O	A partition of the transition matrix where all elements are equal to zero.
p	A specific probability. Also, an element of the transition matrix.
P	The transition matrix.
Q	A partition of the transition matrix which shows the probabilities of moving from each transient state to each other transient state.
$\hat{Q}$	The conditional Q matrix. The elements of $\hat{Q}$ mean the same as the elements of the Q matrix, except that the single state into which the process will ultimately be absorbed is known. Computed in (4-30).
r	The discount rate for percent value calculations.
R	A partition of the transition matrix. Each element gives the probability of moving from a particular transient state to a particular absorbing state during the transition period.
S	The salary of an employee. Used in (6-1).
t	The vector of total average time spent in the model, based on entering position. Computed in (4-10).
(t)	Relates to actual observations for a particular transition period (n(t)).
T	The total time period over which data for the model is accumulated.
$\mu$	The vector of hirings each period (k) stated as percentages of total hirings for the period.
V	A diagonal matrix where $v_{ii} = b_{ji}$ for $i = 1$ to $d$ (the number of transient states) and $j$ is the absorbing state conditionally assumed.
w	The vector of employees in each state at the initiation of the model.

<u>Term</u>	<u>Explanation</u>
x	The vector of employees in each transient state after transitions and hirings have been taken into account each period.
X	The total number of employees in the firm under steady state conditions.
y	The vector of employees in each state (w) stated as a percentage of the total employees.
z	The column vector of values associated with one member in each state.
Z	The total value of all human assets.

APPENDIX B

FORMULAE

## APPENDIX B

## FORMULAE

This appendix has all formulae in order as numbered in Chapters IV and V. An explanation of the notation for these formulae is given in Appendix A.

Formula Number	Formula
(4-1)	$\chi_j^2 = \sum_{i,k} n_{ij}^* (\hat{p}_{ijk} - \hat{p}_{jk})^2 / \hat{p}_{jk}$
(4-1A)	$\hat{p}_{jk} = \sum_{t=2}^T n_{jk}(t) / \sum_{t=1}^{T-1} n_j(t)$
(4-1B)	$\hat{p}_{ijk} = \sum_{t=2}^T n_{ijk}(t) / \sum_{t=2}^T n_{ij}(t-1)$
(4-1C)	$n_{ij}^* = \sum_{t=1}^{T-1} n_{ij}(t)$
(4-2)	$\chi_i^2 = \sum_{t,j} n_i(t-1) [\hat{p}_{ij}(t) - \hat{p}_{ij}]^2 / \hat{p}_{ij}$
(4-2A)	$\hat{p}_{ij}(t) = n_{ij}(t) / n_i(t-1)$
(4-2B)	$\hat{p}_{ij} = \sum_{t=1}^T n_{ij}(t) / \sum_{t=0}^{T-1} n_i(t)$
(4-3)	$P_N = P_{N-1} \cdot P$
(4-4)	$y_1 = y_0 \cdot P$
(4-5)	$y_N = y_0 \cdot P_N$

<u>Formula Number</u>	<u>Formula</u>
(4-6)	$w_N = w_0 \cdot P_N$
(4-7)	$x_N = x_{N-1}Q + k_N$
(4-8)	$F = (I - Q)^{-1}$
(4-9)	$F_{VAR} = F (2 F_{dg} - I) - F_{sq}$
(4-10)	$t = F e$
(4-11)	$t_{VAR} = (2 F - I) t - t_{sq}$
(4-12)	$B = F R$
(4-13)	$g = w B$
(4-14)	$g_{VAR} = w_t [y^B - (y^B)_{sq}]$
(4-15)	$x_{SS} = k F$
(4-16)	$x_{SS} (VAR) = K[\mu F - \sum_{i=0}^{\infty} (\mu Q^i)_{sq}]$
(4-17)	$x_{SS} (VAR) \leq K[\mu F - \mu_{sq} (I - Q_{sq})^{-1}]$
(4-18)	$X = (x_{SS}) e = k F e$
(4-19)	$X_{VAR} = K[\mu F e - \sum_{i=0}^{\infty} (\mu Q^i e)_{sq}]$
(4-20)	$X_{VAR} \leq K[\mu F - \mu_{sq} (I - Q_{sq})^{-1}] e$

Formula  
Number

Formula

$$(4-21) \quad l_{SS} = k B$$

$$(4-22) \quad l_{SS} (\text{VAR}) = K[\mu B - \sum_{i=0}^{\infty} (\mu Q^i R)_{sq}]$$

$$(4-23) \quad l_{SS} (\text{VAR}) \leq K[\mu B - \mu_{sq} (I - Q_{sq})^{-1} R_{sq}]$$

$$(4-24) \quad m = k F^2 R$$

$$(4-25) \quad m_{\text{VAR}} = X[(1/X)k F^2 R - ((1/X) k F^2 R)_{sq}]$$

$$(4-26) \quad a_i = 1/(1 - P_{ii})$$

$$(4-27) \quad a_{\text{VAR}} = P_{ii}/(1 - P_{ii})^2$$

$$(4-28) \quad P'_{ij} = P_{ij}/(1 - P_{ii})$$

$$(4-29) \quad H = (F - I) F_{dg}^{-1}$$

$$(4-30) \quad \hat{Q} = V^{-1} Q V$$

$$(4-31) \quad \hat{F} = (I - \hat{Q})^{-1}$$

$$(5-1) \quad Z = w_n \cdot z$$

$$(5-2) \quad Z = \sum_{n=1}^{\infty} (w_n \cdot z)$$

$$(5-3) \quad Z = \sum_{n=1}^{\infty} \frac{(w_n \cdot z)}{(1 + r)^n}$$

Formula  
Number

Formula

$$(5-4) \quad Z = \sum_{n=1}^{40} \frac{(w_n \cdot z)}{(1+r)^n}$$

$$(5-5) \quad \text{Amortization Expense} = \sum_{i=1}^d \frac{C_i}{t_i}$$

$$(5-6) \quad \text{Amortization \%} = \frac{1}{t_i} \text{ for } i = 1 \text{ to } d$$

$$(5-7) \quad \text{Amortization \%} = \frac{1}{t_i + 3\sigma}$$

APPENDIX C

DATA

## APPENDIX C

## DATA

This appendix consists of two parts. The first part gives the data as received from a CPA firm. The second part constructs the data so as to be useable for the Markov model.

TABLE 1

PROFESSIONAL PERSONNEL (AT JANUARY 31 OF EACH YEAR)

Year	Staff	Managers	Partners	Total
1967	3,451	926	387	4,764
1968	4,192	1,056	451	5,699
1969	4,599	1,174	515	6,288
1970	5,683	1,324	592	7,599
1971	6,829	1,560	666	9,055
1972	6,612	1,567	716	8,895
1973	6,708	1,666	751	9,125
1974	7,323	1,787	814	9,924
1975	7,963	1,942	879	10,784
1976	8,027	2,049	937	11,013
1977	8,316	2,134	1,015	11,465

TABLE 2  
HIRINGS AND PROMOTIONS (THROUGH JANUARY 31 OF EACH YEAR)

Year	New Hires	Promotions	
		From Staff To Manager	From Manager To Partner
1967	1,465	222	60
1968	1,842	261	78
1969	1,725	295	69
1970	2,590	378	91
1971	2,647	414	97
1972	1,633	298	71
1973	2,074	409	66
1974	2,602	458	84
1975	2,747	506	93
1976	2,448	505	101
1977	2,752	510	104

The "staff" designation in the above tables is a combination of at least two levels within the firm. In many firms this would be a combination of the job levels designated as "juniors" and "seniors."

Although it is true that the firm occasionally does hire an employee at some level above the "staff" designation, it is very infrequent, and that data was not available. It is thus assumed that all new hires enter the firm at the "staff" level.

The above data is exactly as given by the CPA firms. It can be reconstructed so that one other piece of information can be derived: the number of people leaving the firm at each level during the year. For example, the maximum possible number of partners at the end of a

year would be the number of partners at the end of the previous year plus any managers promoted to partner during the year. Since the partnership level is the top level, if the actual number of partners is less than the maximum possible number, the difference would be the number of partners who have left the firm during the year.

## PERSONNEL FLOW--YEAR ONE

February 1, 1967 to January 31, 1968

	<u>Staff</u>	<u>Manager</u>	<u>Partner</u>
Beginning of year	3,451	926	387
Add: new hires	1,842	--	--
promotions in	--	261	78
	<u>          </u>	<u>          </u>	<u>          </u>
Maximum	5,293	1,187	465
Less: promotions out	(261)	(78)	--
terminations	(840)	(53)	(14)
	<u>          </u>	<u>          </u>	<u>          </u>
End of Year	4,192	1,056	451
	<u>          </u>	<u>          </u>	<u>          </u>

---

## PERSONNEL FLOW--YEAR TWO

February 1, 1968 to January 31, 1969

	<u>Staff</u>	<u>Manager</u>	<u>Partner</u>
Beginning of year	4,192	1,056	451
Add: new hires	1,725	--	--
promotions in	--	295	69
Maximum	5,917	1,351	520
Less: promotions out	(295)	(69)	--
terminations	(1,023)	(108)	(5)
End of Year	4,599	1,174	515

## PERSONNEL FLOW--YEAR THREE

February 1, 1969 to January 31, 1970

	<u>Staff</u>	<u>Manager</u>	<u>Partner</u>
Beginning of year	4,599	1,174	515
Add: new hires	2,590	--	--
promotions in	--	378	91
Maximum	7,189	1,552	606
Less: promotions out	(378)	(91)	--
terminations	(1,128)	(137)	(14)
End of Year	5,683	1,324	592

## PERSONNEL FLOW--YEAR FOUR

February 1, 1970 to January 31, 1971

	<u>Staff</u>	<u>Manager</u>	<u>Partner</u>
Beginning of year	5,683	1,324	592
Add: new hires	2,647	--	--
promotions in	--	414	97
Maximum	8,330	1,738	689
Less: promotions out	(414)	(97)	--
terminations	(1,087)	(81)	(23)
End of year	6,829	1,560	666

## PERSONNEL FLOW--YEAR FIVE

February 1, 1971 to January 31, 1972

	<u>Staff</u>	<u>Manager</u>	<u>Partner</u>
Beginning of year	6,829	1,560	666
Add: new hires	1,633	--	--
promotions in	--	298	71
Maximum	8,462	1,858	737
Less: promotions out	(298)	(71)	--
terminations	(1,552)	(220)	(21)
End of year	6,612	1,567	716

## PERSONNEL FLOW--YEAR SIX

February 1, 1972 to January 31, 1973

	<u>Staff</u>	<u>Manager</u>	<u>Partner</u>
Beginning of year	6,612	1,567	716
Add: new hires	2,074	--	--
promotions in	--	409	66
Maximum	8,686	1,976	782
Less: promotions out	(409)	(66)	--
terminations	(1,569)	(244)	(31)
End of Year	6,708	1,666	751

## PERSONNEL FLOW--YEAR SEVEN

February 1, 1973 to January 31, 1974

	<u>Staff</u>	<u>Manager</u>	<u>Partner</u>
Beginning of year	6,708	1,666	751
Add: new hires	2,602	--	--
promotions in	--	458	84
Maximum	9,310	2,124	835
Less: promotions out	(458)	(84)	--
terminations	(1,529)	(253)	(21)
End of Year	7,323	1,787	814

## PERSONNEL FLOW--YEAR EIGHT

February 1, 1974 to January 31, 1975

	<u>Staff</u>	<u>Manager</u>	<u>Partner</u>
Beginning of year	7,323	1,787	814
Add: new hires	2,747	--	--
promotions in	--	506	93
	<u>          </u>	<u>          </u>	<u>          </u>
Maximum	10,070	2,293	907
Less: promotions out	(506)	(93)	--
terminations	(1,601)	(258)	(28)
	<u>          </u>	<u>          </u>	<u>          </u>
End of Year	7,963	1,942	879
	<u>          </u>	<u>          </u>	<u>          </u>

## PERSONNEL FLOW--YEAR NINE

February 1, 1975 to January 31, 1976

	<u>Staff</u>	<u>Manager</u>	<u>Partner</u>
Beginning of year	7,963	1,942	879
Add: new hires	2,448	--	--
promotions in	--	505	101
	<u>          </u>	<u>          </u>	<u>          </u>
Maximum	10,411	2,447	980
Less: promotions out	(505)	(101)	--
terminations	(1,879)	(297)	(43)
	<u>          </u>	<u>          </u>	<u>          </u>
End of Year	8,027	2,049	937
	<u>          </u>	<u>          </u>	<u>          </u>

## PERSONNEL FLOW--YEAR TEN

February 1, 1976 to January 31, 1977

	<u>Staff</u>	<u>Manager</u>	<u>Partner</u>
Beginning of year	8,027	2,049	937
Add: new hires	2,752	--	--
promotions in	--	510	104
	<u>          </u>	<u>          </u>	<u>          </u>
Maximum	10,779	2,559	1,041
Less: promotions out	(510)	(104)	--
terminations	(1,953)	(321)	(26)
	<u>          </u>	<u>          </u>	<u>          </u>
End of Year	8,316	2,134	1,015
	<u>          </u>	<u>          </u>	<u>          </u>

In all ten yearly tables above, the "terminations" figures are plug figures to reconcile beginning and ending numbers of employees when all else is known. These tables are the bases for the yearly transition matrices as given in Appendix D.

The personnel flow for the firm for the ten-year period can be found by summing the corresponding elements in the ten tables above.

## PERSONNEL FLOW--TEN-YEAR PERIOD

February 1, 1967 to January 31, 1977

	Staff	Manager	Partner
Beginning of Period	61,387	15,051	6,708
Add: new hires	23,060	--	--
promotions in	--	4,034	854
Maximum	84,447	19,085	7,562
Less: promotions out	(4,034)	(854)	--
terminations	(14,161)	(1,972)	(226)
End of Period	<u>66,252</u>	<u>16,259</u>	<u>7,336</u>

The beginning and end-of-period levels in the above table do not represent the total actual number of employees over the ten years. These figures are inflated because yearly figures were summed. An employee who was at the staff level two years would be counted twice in the above table. This table is the basis for the overall model transition matrix in Appendix D. The rest of the tables in this appendix were found by adding the nearly personnel flow tables for the appropriate time period.

PERSONNEL FLOW--NINE-YEAR PERIOD  
February 1, 1968 to January 31, 1977

	Staff	Manager	Partner
Beginning of Period	57,936	14,125	6,321
Add: new hires	21,218	--	--
promotions in	<u>    --</u>	<u>3,773</u>	<u>776</u>
Maximum	79,154	17,898	7,097
Less: promotions out	(3,773)	(776)	--
terminations	<u>(13,321)</u>	<u>(1,919)</u>	<u>(212)</u>
End of Period	<u>62,060</u>	<u>15,203</u>	<u>6,885</u>

PERSONNEL FLOW--EIGHT-YEAR PERIOD  
(February 1, 1969 to January 31, 1977)

	Stadd	Manager	Partners
Beginning of Period	53,744	13,069	5,870
Add: new hires	19,493	--	--
promotions in	<u>    --</u>	<u>3,478</u>	<u>707</u>
Maximum	73,237	16,547	6,577
Less: promotions out	(3,478)	(707)	--
terminations	<u>(12,298)</u>	<u>(1,811)</u>	<u>(207)</u>
End of Period	<u>57,461</u>	<u>14,029</u>	<u>6,370</u>

PERSONNEL FLOW--SEVEN-YEAR PERIOD  
February 1, 1970 to January 31, 1977

	Staff	Managers	Partners
Beginning of Period	49,145	11,895	5,355
Add: new hires	16,903	--	--
promotions in	<u>    --</u>	<u>3,100</u>	<u>616</u>
Maximum	66,048	14,995	5,971
Less: promotions out	(3,100)	(616)	--
terminations	<u>(11,170)</u>	<u>(1,674)</u>	<u>(193)</u>
End of Period	<u>51,778</u>	<u>12,705</u>	<u>5,778</u>

PERSONNEL FLOW--SIX-YEAR PERIOD  
February 1, 1971 to January 31, 1977

	Staff	Managers	Partners
Beginning of Period	43,462	10,571	4,763
Add: new hires	14,256	--	--
promotions in	<u>    --</u>	<u>2,686</u>	<u>519</u>
Maximum	57,718	13,257	5,282
Less: promotions out	(2,686)	(519)	--
terminations	<u>(10,083)</u>	<u>(1,593)</u>	<u>(170)</u>
End of Period	<u>44,949</u>	<u>11,145</u>	<u>5,112</u>

PERSONNEL FLOW--FIVE-YEAR PERIOD  
February 1, 1972 to January 31, 1977

	Staff	Managers	Partners
Beginning of Period	36,633	9,011	4,097
Add: new hires	12,623	--	--
promotions in	<u>--</u>	<u>2,388</u>	<u>448</u>
Maximum	49,256	11,399	4,545
Less: promotions out	(2,388)	(448)	--
terminations	<u>(8,531)</u>	<u>(1,373)</u>	<u>(149)</u>
End of Period	<u>38,337</u>	<u>9,578</u>	<u>4,396</u>

APPENDIX D

TRANSITION MATRICES

APPENDIX D  
TRANSITION MATRICES

Using the data from Appendix C, transition matrices can be formulated. While the new hirings during the year were needed to complete the personnel flow tables, this data is not necessary for the computations to form the transition matrices. This is because of the assumption that a transition period is short enough that only one movement can occur during each period. Thus, any promotions or terminations would be assumed to be employees who were with the firm at the beginning of the period.

Each matrix below corresponds respectively with the tables in Appendix C. The first matrix is thus based on the first table. The number of employees at the staff level at the beginning of Year One is 3,451. Of these, 261 employees, or 7.56% of the total, were promoted to manager. Also, 840 of these employees, or 24.34% of the total, left the firm. This means that 2,350 of these employees stayed with the firm and remained at the staff level. This constitutes 68.1% of the total employees at the staff level at the beginning of the year. All possible movements for an employee at the staff level have been taken into account, and the first row of the matrix can be completed. The other rows in the matrix can be computed in the same manner.

## TRANSITION MATRIX--YEAR ONE

February 1, 1967 to January 31, 1968

From	To	Staff	Manager	Partner	Leave	Leave (Partner)
Staff		.6810	.0756	0	.2434	0
Manager		0	.8586	.0842	.0572	0
Partner		0	0	.9638	0	.0362
Leave		0	0	0	1	0
Leave (Partner)		0	0	0	0	1

The last two states ("Leave" and "Leave (Partner)") are the absorbing states and are never exited. The rest of the transition matrices below are found in the same manner as the above matrix.

## TRANSITION MATRIX--YEAR TWO

February 1, 1968 to January 31, 1969

From	To	Staff	Manager	Partner	Leave	Leave (Partner)
Staff		.6856	.0704	0	.2440	0
Manager		0	.8324	.0653	.1023	0
Partner		0	0	.9889	0	.0111
Leave		0	0	0	1	0
Leave (Partner)		0	0	0	0	1

## TRANSITION MATRIX--YEAR THREE

February 1, 1969 to January 31, 1970

From	To	Staff	Manager	Partner	Leave	Leave (Partner)
Staff		.6725	.0822	0	.2453	0
Manager		0	.8058	.0775	.1167	0
Partner		0	0	.9728	0	.0272
Leave		0	0	0	1	0
Leave (Partner)		0	0	0	0	1

## TRANSITION MATRIX--YEAR FOUR

February 1, 1970 to January 31, 1971

From	To	Staff	Manager	Partner	Leave	Leave (Partner)
Staff		.7359	.0728	0	.1913	0
Manager		0	.8656	.0733	.0612	0
Partner		0	0	.9611	0	.0389
Leave		0	0	0	1	0
Leave (Partner)		0	0	0	0	1

## TRANSITION MATRIX--YEAR FIVE

February 1, 1971 to January 31, 1972

From	To	Staff	Manager	Partner	Leave	Leave (Partner)
Staff		.7291	.0436	0	.2273	0
Manager		0	.8135	.0455	.1410	0
Partner		0	0	.9685	0	.0315
Leave		0	0	0	1	0
Leave (Partner)		0	0	0	0	1

## TRANSITION MATRIX--YEAR SIX

February 1, 1972 to January 31, 1973

From	To	Staff	Manager	Partner	Leave	Leave (Partner)
Staff		.7008	.0619	0	.2373	0
Manager		0	.8022	.0421	.1557	0
Partner		0	0	.9567	0	.0433
Leave		0	0	0	1	0
Leave (Partner)		0	0	0	0	1

## TRANSITION MATRIX--YEAR SEVEN

February 1, 1973 to January 31, 1974

From	To	Staff	Manager	Partner	Leave	Leave (Partner)
Staff		.7038	.0683	0	.2279	0
Manager		0	.7977	.0504	.1519	0
Partner		0	0	.9720	0	.0280
Leave		0	0	0	1	0
Leave (Partner)		0	0	0	0	1

## TRANSITION MATRIX--YEAR EIGHT

February 1, 1974 to January 31, 1975

From	To	Staff	Manager	Partner	Leave	Leave (Partner)
Staff		.7123	.0691	0	.2186	0
Manager		0	.8036	.0520	.1444	0
Partner		0	0	.9656	0	.0344
Leave		0	0	0	1	0
Leave (Partner)		0	0	0	0	1

## TRANSITION MATRIX--YEAR NINE

February 1, 1975 to January 31, 1976

From	To	Staff	Manager	Partner	Leave	Leave (Partner)
Staff		.7006	.0634	0	.2360	0
Manager		0	.7951	.0520	.1529	0
Partner		0	0	.9511	0	.0489
Leave		0	0	0	1	0
Leave (Partner)		0	0	0	0	1

## TRANSITION MATRIX--YEAR TEN

February 1, 1976 to January 31, 1977

From	To	Staff	Manager	Partner	Leave	Leave (Partner)
Staff		.6932	.0635	0	.2433	0
Manager		0	.7926	.0507	.1567	0
Partner		0	0	.9723	0	.0277
Leave		0	0	0	1	0
Leave (Partner)		0	0	0	0	1

A transition matrix for all ten years can also be developed. This matrix is derived from the personnel flow table for all ten years as depicted in Appendix C.

MODEL TRANSITION MATRIX BASED ON TEN-YEAR PERIOD

February 1, 1967 to January 31, 1977

FROM \ TO:	Staff	Manager	Partner	Leave	Leave (Partner)
Staff	.7036	.0657	0	.2307	0
Manager	0	.8123	.0567	.1310	0
Partner	0	0	.9663	0	.0337
Leave	0	0	0	1	0
Leave (Partner)	0	0	0	0	1

Matrices for any combination of years can be developed. The transition matrices below are for the most recent nine-year period in the data down to the most recent five-year period. Each matrix is derived from the appropriate personnel flow table found in Appendix C.

## MODEL TRANSITION MATRIX BASED ON NINE-YEAR PERIOD

February 1, 1968 to January 31, 1977

FROM: \ TO:	Staff	Manager	Partner	Leave	Leave (Partner)
Staff	.7050	.0651	0	.2299	0
Manager	0	.8092	.0549	.1359	0
Partner	0	0	.9665	0	.0335
Leave	0	0	0	1	0
Leave (Partner)	0	0	0	0	1

## MODEL TRANSITION MATRIX BASED ON EIGHT-YEAR PERIOD

February 1, 1969 to January 31, 1977

FROM: \ TO:	Staff	Manager	Partner	Leave	Leave (Partner)
Staff	.7065	.0647	0	.2288	0
Manager	0	.8073	.0541	.1386	0
Partner	0	0	.9647	0	.0353
Leave	0	0	0	1	0
Leave (Partner)	0	0	0	0	1

## MODEL TRANSITION MATRIX BASED ON SEVEN-YEAR PERIOD

February 1, 1970 to January 31, 1977

FROM: \ TO:	Staff	Manager	Partner	Leave	Leave (Partner)
Staff	.7096	.0631	0	.2273	0
Manager	0	.8075	.0518	.1407	0
Partner	0	0	.9640	0	.0360
Leave	0	0	0	1	0
Leave (Partner)	0	0	0	0	1

## MODEL TRANSITION MATRIX BASED ON SIX-YEAR PERIOD

February 1, 1971 to January 31, 1977

FROM: \ TO:	Staff	Manager	Partner	Leave	Leave (Partner)
Staff	.7062	.0618	0	.2320	0
Manager	0	.8002	.0491	.1507	0
Partner	0	0	.9643	0	.0357
Leave	0	0	0	1	0
Leave (Partner)	0	0	0	0	1

## MODEL TRANSITION MATRIX BASED ON FIVE-YEAR PERIOD

February 1, 1972 to January 31, 1977

FROM: \ TO:	Staff	Manager	Partner	Leave	Leave (Partner)
Staff	.7019	.0652	0	.2329	0
Manager	0	.7979	.0497	.1524	0
Partner	0	0	.9636	0	.0364
Leave	0	0	0	1	0
Leave (Partner)	0	0	0	0	1

APPENDIX E

COMPUTER PROGRAMS

## APPENDIX E

## COMPUTER PROGRAMS

The use of computer programs greatly facilitates the application of the conceptual model. There were a total of five programs written for this purpose. These programs were written in a language called BASIC. A listing of each program will be presented below, as well as a brief explanation.

MARKOV

This is the main application program for the model. Inputs include Q and R partitions of the model transition matrix, present employees at each level, and a constant annual hirings figure. The outputs of this program are discussed in Chapter VIII.

To validate this program, MARKOV was executed with inputs from three published articles in order to compare the outputs of the program with cited output in the articles. These articles were:

- (1) "A Model of Professional Staff Flows in a CPA Firm," by Jackson F. Gillespie and Wayne E. Leininger, Proceedings of the 1975 A.I.D.S. Conference, pp. 199-201.
- (2) "Estimation of the Allowance for Doubtful Accounts by Markov Chains," by R. M. Cyert, H. J. Davidson, and G. L. Thompson, Management Science, V. 8 (April 1962), pp. 287-303.
- (3) "Stochastic Process Costing Models," by A. Wayne Corcoran and Wayne E. Leininger, Accounting Review, Vol. 48 (January 1973), pp. 105-113.

In each case, the results of the MARKOV program were comparable to the results in the articles. The results of the program also passed the "reasonableness" test. A listing of the program follows:

```

PRINT
100 PRINT
110 PRINT "HOW MANY TOTAL STATES DOES"
120 PRINT "THE TRANSITION MATRIX HAVE?"
130 INPUT S3
140 PRINT
150 PRINT "HOW MANY ABSORBING STATES DOES"
160 PRINT "THE TRANSITION MATRIX HAVE?"
170 INPUT S2
180 LET S1=S3-S2
190 PRINT
200 PRINT "INPUT THE Q MATRIX (CANONICAL FORM) ROW BY ROW"
210 MAT INPUT Q(S1,S1)
220 PRINT
230 PRINT "INPUT THE R MATRIX (CANONICAL FORM) ROW BY ROW"
240 MAT INPUT R(S1,S2)
250 MAT I=IDN(S1,S1)
260 MAT J=ZER(S1,S1)
270 MAT F=ZER(S1,S1)
280 MAT J=I-Q
290 MAT F=INV(J)
300 PRINT
310 PRINT
320 PRINT "THE FUNDAMENTAL MATRIX IS"
330 MAT PRINT F
340 MAT D=ZER(S1,S1)
350 FOR N=1 TO S1
360 LET D(N,N)=F(N,N)
370 NEXT N
380 MAT J=ZER(S1,S1)
390 FOR N=1 TO S1
400 FOR M=1 TO S1
410 LET J(N,M)=F(N,M)^2
420 NEXT M
430 NEXT N
440 MAT V=ZER(S1,S1)
450 MAT C=ZER(S1,S1)
460 MAT C=(2)*D
470 MAT G=ZER(S1,S1)
480 MAT G=C-I
490 MAT H=ZER(S1,S1)

```

```

500 MAT H=F*G
510 MAT V=H-J
520 PRINT "AND THE CORRESPONDING VARIANCE FOR EACH ELEMENT IS"
530 MAT PRINT V
540 MAT S=ZER(S1,S1)
550 FOR N=1 TO S1
560 FOR M=1 TO S1
570 LET S(N,M)=SQR(V(N,M))
580 NEXT M
590 NEXT N
600 PRINT "THE CORRESPONDING ST. DEV. FOR EACH ELEMENT IS"
610 MAT PRINT S
620 MAT T=ZER(S1,1)
630 MAT E=CON(S1,1)
640 MAT T=F*E
650 PRINT "THE TOTAL AVERAGE TIME SPENT WITH"
660 PRINT "THE FIRM, BASED ON ENTERING POSITION, IS"
670 MAT PRINT T
680 MAT V=ZER(S1,1)
690 MAT C=ZER(S1,S1)
700 MAT C=(2)*F
710 MAT G=ZER(S1,S1)
720 MAT G=C-I
730 MAT H=ZER(S1,1)
740 MAT H=G*T
750 MAT L=ZER(S1,1)
760 FOR N=1 TO S1
770 LET L(N,1)=(T(N,1))^2
780 NEXT N
790 MAT V=H-L
800 PRINT "THE CORRESPONDING VARIANCE FOR EACH ELEMENT IS"
810 MAT PRINT V
820 MAT S=ZER(S1,1)
830 FOR N=1 TO S1
840 LET S(N,1)=SQR(V(N,1))
850 NEXT N
860 PRINT "THE CORRESPONDING STAND. DEV. FOR EACH ELEMENT IS"
870 MAT PRINT S
880 MAT B=ZER(S1,S2)
890 MAT B=F*R
900 PRINT "DEPENDING ON INITIAL POSITION, THE PROBABILITY"
910 PRINT "OF ENTERING EACH ABSORBING STATE IS"
920 MAT PRINT B
930 MAT A=ZER(1,S1)
940 MAT V=ZER(1,S1)
950 FOR N=1 TO S1
960 LET Q1=1-Q(N,N)
970 LET Q2=Q1^2
980 LET A(1,N)=1/Q1
990 LET V(1,N)=Q(N,N)/Q2
1000 NEXT N
1010 PRINT "THE NUMBER OF CONSECUTIVE PERIODS IN A STATE,"

```

```

1020 PRINT "ONCE THAT STATE HAS BEEN ENTERED, IS"
1030 MAT PRINT A
1040 PRINT "WITH THE VARIANCE OF EACH ELEMENT BEING"
1050 MAT PRINT V
1060 MAT S=ZER(1,S1)
1070 FOR N=1 TO S1
1080 LET S(1,N)=SQR(V(1,N))
1090 NEXT N
1100 PRINT "AND THE STANDARD DEVIATIONS ARE"
1110 MAT PRINT S
1120 MAT H=ZER(S1,S1)
1130 MAT C=ZER(S1,S1)
1140 MAT C=INV(D)
1150 MAT J=ZER(S1,S1)
1160 MAT J=F-I
1170 MAT H=J*C
1180 PRINT "BASED ON EACH TRANSIENT STATE, THIS"
1190 PRINT "GIVES THE PROBABILITY OF"
1200 PRINT "OCCUPYING EACH TRANSIENT STATE"
1210 PRINT "ANYTIME IN THE FUTURE"
1220 MAT PRINT H
1230 MAT K=ZER(1,S1)
1240 PRINT "INPUT THE VECTOR OF EMPLOYEES HIRED"
1250 PRINT "AT EACH LEVEL. SEPARATE WITH COMMAS."
1260 MAT INPUT K
1270 PRINT
1280 MAT J=ZER(1,S1)
1290 PRINT "WHAT IS THE PRESENT NUMBER"
1300 PRINT "OF EMPLOYEES AT EACH LEVEL?"
1310 MAT INPUT J
1320 PRINT
1330 PRINT
1340 MAT G=ZER(1,S2)
1350 MAT G=J*B
1360 PRINT "THE PRESENT EMPLOYEES ULTIMATELY ABSORBED INTO
      EACH STATE"
1370 MAT PRINT G
1380 LET B1=0
1390 FOR N=1 TO S1
1400 B1=B1+J(1,N)
1410 NEXT N
1420 MAT G=ZER(1,S1)
1430 FOR N=1 TO S1
1440 LET G(1,N)=J(1,N)/B1
1450 NEXT N
1460 MAT C=ZER(1,S2)
1470 MAT C=G*B
1480 MAT L=ZER(1,S2)
1490 FOR N=1 TO S2
1500 LET L(1,N)=(C(1,N))^2
1510 NEXT N
1520 MAT H=ZER(1,S2)

```

```

1530 MAT H=C-L
1540 MAT V=ZER(1,S2)
1550 MAT V=(B1)*H
1560 PRINT "THE VARIANCE OF EACH ELEMENT IS"
1570 MAT PRINT V
1580 MAT S=ZER(1,S2)
1590 FOR N=1 TO S2
1600 LET S(1,N)=SQR(V(1,N))
1610 NEXT N
1620 PRINT "THE STANDARD DEV. OF EACH ELEMENT IS"
1630 MAT PRINT S
1640 FOR N=1 TO S1
1650 LET K1=K1+K(1,N)
1660 NEXT N
1670 MAT F=ZER(1,S1)
1680 FOR N=1 TO S1
1690 LET F(1,N)=K(1,N)/K1
1700 NEXT N
1710 MAT Y=ZER(1,S2)
1720 MAT C=ZER(S1,S1)
1730 MAT C=F*F
1740 MAT G=ZER(1,S1)
1750 MAT G=K*C
1760 MAT Y=G*R
1770 PRINT
1780 PRINT
1790 PRINT "THE NEXT SEGMENT OF THE PROGRAM GIVES"
1800 PRINT "STEADY STATE INFORMATION FOR THE FIRM"
1810 PRINT "AND IS BASED ON THE VECTOR OF HIRINGS"
1820 PRINT "REMAINING CONSTANT IN THE FUTURE"
1830 PRINT
1840 PRINT
1850 PRINT "OF THE TOTAL EMPLOYEES AT STEADY STATE"
1860 PRINT "THE PREDICTED EMPLOYEES THAT WILL ULTIMATELY"
1870 PRINT "BE ABSORBED INTO EACH STATE EQUALS"
1880 MAT PRINT Y
1890 MAT X=ZER(1,S1)
1900 MAT X=K*F
1910 MAT J=ZER(1,1)
1920 MAT J=X*E
1930 MAT V=ZER(1,S2)
1940 LET K2=1/J(1,1)
1950 MAT C=ZER(1,S1)
1960 MAT C=(K2)*X
1970 MAT G=ZER(1,S1)
1980 MAT G=C*F
1990 MAT L=ZER(1,S2)
2000 MAT L=G*R
2010 MAT O=ZER(1,S2)
2020 FOR N=1 TO S2

```

```
2030 LET O(1,N)=L(1,N)^2
2040 NEXT N
2050 MAT W=ZER(1,S2)
2060 MAT W=L-O
2070 MAT V=(J(1,1))*W
2080 PRINT "THE VARIANCE OF EACH MEAN IS"
2090 MAT PRINT V
2100 MAT S=ZER(1,S2)
2110 FOR N=1 TO S2
2120 LET S(1,N)=SQR(V(1,N))
2130 NEXT N
2140 PRINT "THE STANDARD DEVIATION OF EACH MEAN IS"
2150 MAT PRINT S
2160 PRINT "THE STEADY STATE LEVELS OF EACH TRANSIENT STATE ARE"
2170 MAT PRINT X
2180 MAT C=ZER(1,S1)
2190 MAT G=ZER(S1,S1)
2200 FOR N=1 TO S1
2210 LET C(1,N)=(P(1,N))^2
2220 FOR M=1 TO S1
2230 LET G(N,M)=(Q(N,M))^2
2240 NEXT M
2250 NEXT N
2260 MAT H=ZER(S1,S1)
2270 MAT H=I-G
2280 MAT D=ZER(S1,S1)
2290 MAT D=INV(H)
2300 MAT W=ZER(1,S1)
2310 MAT W=C*D
2320 MAT L=ZER(1,S1)
2330 MAT L=P*F
2340 MAT T=ZER(1,S1)
2350 MAT T=L-W
2360 MAT U=ZER(1,S1)
2370 MAT U=(K1)*T
2380 PRINT "THE UPPER BOUND ESTIMATE OF EACH VARIANCE IS"
2390 MAT PRINT U
2400 MAT S=ZER(1,S1)
2410 FOR N=1 TO S1
2420 LET S(1,N)=SQR(U(1,N))
2430 NEXT N
2440 PRINT "THE UPPER BOUND ESTIMATE OF EACH ST. DEV. IS"
2450 MAT PRINT S
2460 PRINT "THE TOTAL EMPLOYEES AT STEADY STATE EQUAL"
2470 MAT PRINT J
2480 MAT W=ZER(1,1)
2490 MAT W=U*E
2500 PRINT "THE UPPER BOUND ESTIMATE OF THE VARIANCE IS"
2510 MAT PRINT W
2520 MAT S=ZER(1,1)
2530 LET S(1,1)=SQR(W(1,1))
```

```

2540 PRINT "THE UPPER BOUND ESTIMATE"
2550 PRINT "OF THE STANDARD DEVIATION IS"
2560 MAT PRINT S
2570 MAT Y=ZER(1,S2)
2580 MAT C=ZER(1,S1)
2590 MAT C=K*F
2600 MAT Y=C*R
2610 PRINT "THE STEADY STATE BALANCES FOR EMPLOYEES"
2620 PRINT "LEAVING THE FIRM (BEING ABSORBED) ARE"
2630 MAT PRINT Y
2640 MAT C=ZER(1,S1)
2650 MAT O=ZER(S1,S2)
2660 MAT J=ZER(S1,S1)
2670 FOR N=1 TO S1
2680 LET C(1,N)=(F(1,N))^2
2690 FOR M=1 TO S1
2700 LET J(N,M)=(Q(N,M))^2
2710 NEXT M
2720 NEXT N
2730 FOR N=1 TO S1
2740 FOR M=1 TO S2
2750 LET O(N,M)=(R(N,M))^2
2760 NEXT M
2770 NEXT N
2780 MAT V=ZER(S1,S1)
2790 MAT V=I-J
2800 MAT A=ZER(S1,S1)
2810 MAT A=INV(V)
2820 MAT Q=ZER(1,S1)
2830 MAT Q=C*A
2840 MAT H=ZER(1,S2)
2850 MAT H=Q*O
2860 MAT S=ZER(1,S2)
2870 MAT W=ZER(1,S2)
2880 MAT W=L*R
2890 MAT S=W-H
2900 MAT U=ZER(1,S2)
2910 MAT U=(K1)*S
2920 PRINT "THE UPPER BOUND ESTIMATE OF EACH VARIANCE IS"
2930 MAT PRINT U
2940 MAT S=ZER(1,S2)
2950 FOR N=1 TO S2
2960 LET S(1,N)=SQR(U(1,N))
2970 NEXT N
2980 PRINT "THE UPPER BOUND ESTIMATES OF"
2990 PRINT "THE STANDARD DEVIATIONS ARE"
3000 MAT PRINT S
#

```

TESTLEVELS

This program is for projecting the number of employees at each level at some future point. Different hiring levels for each year are allowable as input to the program. Other inputs include the model transition matrix and the number of employees at each level at some known point. Validation of this program consisted of comparing manual computations against program results.

```

PRINT
100 PRINT
200 PRINT "THIS PROGRAM WILL TAKE THE KNOWN HIRINGS FOR EACH
        YEAR."
300 PRINT "AND PREDICT THE LEVELS OF EMPLOYEES FOR THE NEXT
        YEAR."
400 PRINT
500 PRINT "HOW MANY TOTAL STATES IN THE TRANSITION MATRIX?"
600 INPUT S1
700 MAT K=ZER(1,S1)
800 MAT B=ZER(1,S1)
900 MAT P=ZER(S1,S1)
1000 PRINT
1100 PRINT "INPUT THE TRANSITION MATRIX ROW BY ROW."
1200 MAT INPUT P
1300 PRINT
1400 PRINT "INPUT THE NUMBER OF EMPLOYEES AT EACH"
1500 PRINT "LEVEL AT THE BEGINNING OF PERIOD ONE?"
1600 MAT INPUT K
1700 PRINT
1800 PRINT "HOW MANY YEARS WILL BE TESTED?"
1900 INPUT Y
2000 FOR N=1 TO Y
2100 MAT B=ZER(1,S1)
2200 PRINT
2300 PRINT "WHAT ARE THE HIRING LEVELS IN PERIOD";N
2400 MAT INPUT B
2500 MAT C=ZER(1,S1)
2600 MAT C=K*P
2700 MAT K=C+B
2800 PRINT
2900 PRINT "THE NUMBER OF EMPLOYEES AT EACH"
3000 PRINT "LEVEL AT THE END OF PERIOD";N
3050 FOR Z=1 TO S1
3100 LET R$="#####"
3150 PRINT USING R$,K(1,Z)
3151 NEXT Z
3200 NEXT N
#

```

LEVELS

The only difference between this program and the TESTLEVELS program is that hirings each year are assumed to be constant. This allows the user to see yearly levels of employees as the firm approaches steady state. Like the TESTLEVELS program, validation consisted of comparing manual computations with program results.

```

100 PRINT
200 PRINT "THIS PROGRAM ROLLS OVER THE ENTIRE TRANSITION"
300 PRINT "MATRIX. INPUT FOR NEW HIRINGS IS INCLUDED."
400 PRINT "FUTURE LEVELS CAN BE PREDICTED, AND"
500 PRINT "STEADY STATE LEVELS CAN BE FOUND."
600 PRINT
700 PRINT "HOW MANY TOTAL STATES IN THE TRANSITION MATRIX?"
800 INPUT S1
900 MAT A=ZER(1,S1)
1000 MAT B=ZER(1,S1)
1100 MAT C=ZER(1,S1)
1200 MAT P=ZER(S1,S1)
1300 PRINT
1400 PRINT "INPUT THE TRANSITION MATRIX ROW BY ROW."
1500 MAT INPUT P
1600 PRINT
1700 PRINT "INPUT THE NUMBER OF EMPLOYEES AT"
1800 PRINT "EACH LEVEL IN THE FIRST YEAR STUDIED."
1900 MAT INPUT A
2000 PRINT
2100 PRINT "WHAT ARE THE YEARLY HIRING LEVELS?"
2200 MAT INPUT B
2300 PRINT
2400 PRINT "HOW MANY PERIODS IN THE FUTURE"
2500 PRINT "ARE WE TRYING TO PREDICT?"
2600 INPUT S2
2700 FOR N=1 TO S2
2800 MAT C=A*P
2900 MAT A=C+B
3000 PRINT
3100 PRINT
3200 PRINT "THE NUMBER OF EMPLOYEES AT EACH"
3300 PRINT "LEVEL AT THE END OF PERIOD" ; N
3400 MAT PRINT A
3500 NEXT N
#

```

CONDITIONAL

This program computes the conditional Q matrix ( $\hat{Q}$ ) and the conditional fundamental matrix ( $\hat{F}$ ). Inputs include the canonical Q matrix and a known column of the B matrix. Validation of the program consisted of (1) comparing manual computations against program results, and (2) the reasonableness of the results as compared with the results of the MARKOV program.

```

PRINT
100 PRINT
200 PRINT "THIS PROGRAM COMPUTES THE CONDITIONAL Q"
300 PRINT "MATRIX AND THE CONDITIONAL F MATRIX."
400 PRINT "IT IS BASED ON THE KNOWLEDGE THAT THE"
500 PRINT "EMPLOYEE WILL BE ABSORBED INTO A CERTAIN STATE."
600 PRINT
700 PRINT "HOW MANY TRANSIENT STATES IN THE TRANSITION
      MATRIX?"
800 INPUT S1
900 MAT B=ZER(S1,1)
1000 MAT H=ZER(S1,S1)
1100 MAT G=ZER(S1,S1)
1200 MAT Q=ZER(S1,S1)
1300 PRINT
1400 PRINT "INPUT THE Q MATRIX (CANONICAL FORM) ROW BY ROW."
1500 MAT INPUT Q
1600 PRINT
1700 PRINT "INPUT THE COLUMN OF THE B MATRIX"
1800 PRINT "CORRESPONDING TO THE ASSUMED ABSORBING STATE."
1900 MAT INPUT B
2000 FOR N=1 TO S1
2100 LET G(N,N)=B(N,1)
2200 NEXT N
2300 MAT H=INV(G)
2400 MAT E=ZER(S1,S1)
2500 MAT E=H*Q
2600 MAT J=ZER(S1,S1)
2700 MAT J=E*G
2800 PRINT
2900 PRINT
3000 PRINT "THE CONDITIONAL Q MATRIX IS"
3100 MAT PRINT J
3200 MAT I=IDN(S1,S1)
3300 MAT C=ZER(S1,S1)

```

```
3400 MAT C=I-J
3500 MAT F=ZER(S1,S1)
3600 MAT F=INV(C)
3700 PRINT
3800 PRINT
3900 PRINT "THE CONDITIONAL F MATRIX IS"
4000 MAT PRINT F
#
```

VALUATION

The last program will compute an economic valuation of the human assets of the firm. Inputs needed include the hourly billing rates for each employee level (staff, manager, partner), annual chargeable hours for each level, and the number of employees at each level at the point of valuation. Other inputs to the program are the model transition matrix and the cost of capital for the firm. Validation of the program was undertaken by comparing simple manual computations with program output.

```

PRINT
100 PRINT
200 PRINT "THIS PROGRAM WILL COMPUTE THE ECONOMIC"
300 PRINT "VALUATION FOR THE HUMAN ASSETS OF A CPA FIRM."
400 PRINT
500 MAT A=ZER(5,1)
600 PRINT "WHAT ARE THE HOURLY BILLING RATES FOR EACH LEVEL?"
700 MAT INPUT A
800 MAT B=ZER(5,1)
900 MAT C=ZER(5,1)
1000 PRINT
1100 PRINT "WHAT ARE THE YEARLY CHARGEABLE HOURS FOR EACH
        LEVEL?"
1200 MAT INPUT B
1300 FOR X=1 TO 5
1400 LET C(X,1)=A(X,1)*B(X,1)
1500 NEXT X
1600 MAT S=ZER(5,1)
1700 PRINT
1800 PRINT "WHAT ARE THE AVERAGE SALARIES AT EACH LEVEL?"
1900 MAT INPUT S
2000 MAT Z=ZER(5,1)
2100 MAT Z=C-S
2200 MAT K=ZER(1,5)
2300 PRINT
2400 PRINT "HOW MANY EMPLOYEES AT EACH LEVEL?"
2500 MAT INPUT K
2600 MAT E=ZER(1,1)
2700 MAT E=K*Z
2800 LET Z2=E(1,1)
2900 PRINT

```

```
3000 MAT P=ZER(5,5)
3100 PRINT "INPUT THE TRANSITION MATRIX BY ROWS."
3200 MAT INPUT P
3210 PRINT
3220 PRINT "WHAT IS THE COST OF CAPITAL FOR THE FIRM?"
3300 MAT Q=IDN(5,5)
3330 INPUT R
3400 MAT W=ZER(1,5)
3500 MAT V=ZER(1,1)
3550 MAT L=ZER(5,5)
3600 FOR I=1 TO 2
3700 MAT L=Q*P
3800 MAT W=K*L
3900 MAT V=W*Z
4000 LET V1=V(1,1)
4100 LET R1=(1+R)
4150 LET R2=(R1)^I
4200 LET Z1=V1/R2
4300 LET Z2=Z2+Z1
4350 MAT Q=L
4400 NEXT I
4500 LET Y$="#####"
4600 PRINT
4700 PRINT "THE ECONOMIC VALUATION OF THE HUMAN"
4800 PRINT "ASSETS OF THE FIRM IS EQUAL TO"
4900 PRINT USING Y$, Z2
#
```

APPENDIX F

COMPUTER OUTPUT

## APPENDIX F

## COMPUTER OUTPUT

An example of each of the computer programs is presented below, showing both the input and the output of the program.

MARKOV

This is the main program. The example here is the same as shown in Chapter VIII, although here it is complete.

RUN

#RUNNING 4198

HOW MANY TOTAL STATES DOES  
THE TRANSITION MATRIX HAVE?

#?

?5

HOW MANY ABSORBING STATES DOES  
THE TRANSITION MATRIX HAVE?

?2

INPUT THE Q MATRIX (CANONICAL FORM) ROW BY ROW

? .7019, .0652, 0, &

? 0, .7979, .0497, &

? 0, 0, .9636

INPUT THE R MATRIX (CANONICAL FORM) ROW BY ROW

? .2329, 0, &

? .1524, 0, &

? 0, .0364

THE FUNDAMENTAL MATRIX IS

3.3545790003      1.0822293460      1.4776592993

0                    4.948045522      6.7559852317

0                    0                    27.472527472

AND THE CORRESPONDING VARIANCE FOR EACH ELEMENT IS

7.8986212691      8.4563904349      77.528935081

0                    19.535108966      318.80865808

0                    0                    727.26723822

THE CORRESPONDING ST. DEV. FOR EACH ELEMENT IS  
 2.8104485886    2.9079873513    8.8050516797  
 0                4.4198539530    17.855213751  
 0                0                 26.967892729

THE TOTAL AVERAGE TIME SPENT WITH  
 THE FIRM, BASED ON ENTERING POSITION, IS  
 5.9144676456  
 11.704030754  
 27.472527472

THE CORRESPONDING VARIANCE FOR EACH ELEMENT IS  
 105.30866523  
 338.34376704  
 727.26723822

THE CORRESPONDING STAND. DEV. FOR EACH ELEMENT IS  
 10.262001034  
 18.394123166  
 26.967892729

DEPENDING ON INITIAL POSITION, THE PROBABILITY  
 OF ENTERING EACH ABSORBING STATE IS  
 .94621320151    .05378679849  
 .75408213756    .24591786244  
 0                .99999999997

THE NUMBER OF CONSECUTIVE PERIODS IN A STATE,  
 ONCE THAT STATE HAS BEEN ENTERED, IS  
 3.3545790003    4.948045522    27.472527472

WITH THE VARIANCE OF EACH ELEMENT BEING  
 7.8986212691    19.535108966    727.26723822

AND THE STANDARD DEVIATIONS ARE  
 2.8104485886    4.4198539530    26.967892729

BASED ON EACH TRANSIENT STATE, THIS  
 GIVES THE PROBABILITY OF  
 OCCUPYING EACH TRANSIENT STATE  
 ANYTIME IN THE FUTURE  
 0.7019            .21871855082    .05378679849  
 0                 0.7979            .24591786244  
 0                 0                 0.9636

INPUT THE VECTOR OF EMPLOYEES HIRED  
 AT EACH LEVEL. SEPARATE WITH COMMAS.  
 ?2750,0,0

WHAT IS THE PRESENT NUMBER  
 OF EMPLOYEES AT EACH LEVEL?  
 ?8316,2134,1015

THE PRESENT EMPLOYEES ULTIMATELY ABSORBED INTO EACH STATE  
 9477.9202652 1987.0797347

THE VARIANCE OF EACH ELEMENT IS  
 1642.6849792 1642.6849792

THE STANDARD DEV. OF EACH ELEMENT IS  
 40.530050323 40.530050323

THE NEXT SEGMENT OF THE PROGRAM GIVES  
 STEADY STATE INFORMATION FOR THE FIRM  
 AND IS BASED ON THE VECTOR OF HIRINGS  
 REMAINING CONSTANT IN THE FUTURE

OF THE TOTAL EMPLOYEES AT STEADY STATE  
 THE PREDICTED EMPLOYEES THAT WILL ULTIMATELY  
 BE ABSORBED INTO EACH STATE EQUALS  
 10973.151074 5291.6349512

THE VARIANCE OF EACH MEAN IS  
 3570.0383429 3570.0383429

THE STANDARD DEVIATION OF EACH MEAN IS  
 59.749797848 59.749797848

THE STEADY STATE LEVELS OF EACH TRANSIENT STATE ARE  
 9225.0922509 2976.1307014 4063.5630730

THE UPPER BOUND ESTIMATE OF EACH VARIANCE IS  
 3804.6255661 2912.7145291 4061.3714876

THE UPPER BOUND ESTIMATE OF EACH ST. DEV. IS  
 61.681646914 53.969570400 63.728890525

THE TOTAL EMPLOYEES AT STEADY STATE EQUAL  
 16264.786025

THE UPPER BOUND ESTIMATE OF THE VARIANCE IS  
 10778.711583

THE UPPER BOUND ESTIMATE  
 OF THE STANDARD DEVIATION IS  
 103.82057399

THE STEADY STATE BALANCES FOR EMPLOYEES  
 LEAVING THE FIRM (BEING ABSORBED) ARE  
 2602.0863042 147.91369586

THE UPPER BOUND ESTIMATE OF EACH VARIANCE IS  
2306.5942390 147.91079209

THE UPPER BOUND ESTIMATES OF  
THE STANDARD DEVIATIONS ARE  
48.027015721 12.161858086

#ET=4:35.7 PT=0.7 IO=0.2

LEVELS

This program takes present employees and a constant hiring figure and projects future numbers of employees. Here it has been run through 50 years.

RUN  
#RUNNING 0709

THIS PROGRAM ROLLS OVER THE ENTIRE TRANSITION MATRIX. INPUT FOR NEW HIRINGS IS INCLUDED. FUTURE LEVELS CAN BE PREDICTED, AND STEADY STATE LEVELS CAN BE FOUND.

HOW MANY TOTAL STATES IN THE TRANSITION MATRIX?

?#?

3

INPUT THE TRANSITION MATRIX ROW BY ROW.

? .7019, .0652, .0, &

?0, .7979, .0497, &

?0, 0, .9636

INPUT THE NUMBER OF EMPLOYEES AT EACH LEVEL IN THE FIRST YEAR STUDIED.

?8316, 2134, 1015

WHAT ARE THE YEARLY HIRING LEVELS?

?2750

HOW MANY PERIODS IN THE FUTURE ARE WE TRYING TO PREDICT?

?50

THE NUMBER OF EMPLOYEES AT EACH LEVEL AT THE END OF PERIOD 1

8587.0004	2244.9218	1084.1138
-----------	-----------	-----------

THE NUMBER OF EMPLOYEES AT EACH LEVEL AT THE END OF PERIOD 2

8777.2155808	2351.0955303	1156.2246711
--------------	--------------	--------------

THE NUMBER OF EMPLOYEES AT EACH LEVEL AT THE END OF PERIOD 3

8910.7276161	2448.2135795	1230.9875410
--------------	--------------	--------------

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 4

9004.4397137 2534.4090556 1307.8558094

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 5

9070.2162351 2609.2944548 1386.2099880

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 6

9116.3847754 2673.3341440 1465.4338788

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 7

9148.7904738 2727.4416009 1544.9567926

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 8

9171.5360336 2772.7267922 1624.2742129

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 9

9187.501142 2810.3428569 1702.9551531

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 10

9198.7070516 2841.39764 1780.6416255

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 11

9206.5724795 2866.9068767 1857.0437331

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 12

9212.0932233 2887.7735226 1931.9326130

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 13

9215.9682334 2904.7829718 2005.1326099

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 14

9218.688103 2918.6074621 2076.5134966

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 15

9220.5971795 2929.8153583 2145.9831962

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 16

9221.9371603 2938.8826105 2213.4812312

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 17

9222.8776928 2946.2047378 2278.9729801

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 18

9223.5378525 2952.1083858 2342.4447391

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 19

9224.0012187 2956.8619490 2403.8995374

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 20

9224.3264554 2960.6850286 2463.3536331

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 21

9224.554739 2963.7566692 2520.8336067

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 22

9224.7149713 2966.2224153 2576.3739699

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 23

9224.8274384 2968.2002813 2630.0152115

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 24

9224.906379 2969.7857535 2681.8022117

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 25

9224.9617874 2971.0559486 2731.7829632

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 26

9225.0006785 2972.0730499 2780.0075440

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 27

9225.0279763 2972.8871308 2826.5272999

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 28

9225.0471366 2973.5384657 2871.3941966

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 29

9225.0605852 2974.0594151 2914.6603096

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 30  
9225.0700247 2974.4759575 2956.3774273

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 31  
9225.0766504 2974.8089321 2996.5967440

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 32  
9225.0813008 2975.0750445 3035.3686264

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 33  
9225.0845651 2975.2876788 3072.7424381

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 34  
9225.0868562 2975.4575526 3108.7664110

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 35  
9225.0884644 2975.5932442 3143.4875540

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 36  
9225.0895932 2975.7016175 3176.9515913

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 37  
9225.0903854 2975.7881620 3209.2029238

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 38

9225.0909415 2975.8572676 3240.2846090

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 39

9225.0913319 2975.9124432 3270.2383554

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 40

9225.0916058 2975.9564933 3299.1045277

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 41

9225.0917981 2975.9916587 3326.9221606

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 42

9225.0919331 2976.0197297 3353.7289794

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 43

9225.0920278 2976.0421364 3379.5614251

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 44

9225.0920944 2976.0600208 3404.4546834

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 45

9225.092141 2976.0742952 3428.4427160

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 46  
9225.0921738 2976.0856877 3451.5582936

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 47  
9225.0921968 2976.0947800 3473.8330304

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 48  
9225.0922129 2976.1020362 3495.2974186

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 49  
9225.0922243 2976.1078269 3515.9808638

THE NUMBER OF EMPLOYEES AT EACH  
LEVEL AT THE END OF PERIOD 50  
9225.0922322 2976.1124481 3535.9117193

TESTLEVELS

This program differs from the LEVELS program only in that it allows a different hirings figure to be input for each year. The example here begins with January 31, 1977 employees and projects to January 31, 1979.

RUN

#RUNNING 4238

THIS PROGRAM WILL TAKE THE KNOWN HIRINGS FOR EACH YEAR AND PREDICT THE LEVELS OF EMPLOYEES FOR THE NEXT YEAR.

HOW MANY TOTAL STATES IN THE TRANSITION MATRIX?

#?

?3

INPUT THE TRANSITION MATRIX ROW BY ROW.

? .7019, .0652, 0, &

? 0, .7979, .0497, &

? 0, 0, .9636

INPUT THE NUMBER OF EMPLOYEES AT EACH LEVEL AT THE BEGINNING OF PERIOD ONE?

? 8316, 2134, 1015

HOW MANY YEARS WILL BE TESTED?

? 2

WHAT ARE THE HIRING LEVELS IN PERIOD 1

? 2750

THE NUMBER OF EMPLOYEES AT EACH LEVEL AT THE END OF PERIOD 1

8587

2244

1084

WHAT ARE THE HIRING LEVELS IN PERIOD 2

? 2800

THE NUMBER OF EMPLOYEES AT EACH LEVEL AT THE END OF PERIOD 2

8827

2351

1156

#ET=1:39.3 FT=0.2 IO=0.2

CONDITIONAL

This program takes the Q and R partitions of the model transition matrix and computes the conditional Q and F matrices for the model.

RUN

#RUNNING 4314

THIS PROGRAM COMPUTES THE CONDITIONAL Q MATRIX AND THE CONDITIONAL F MATRIX. IT IS BASED ON THE KNOWLEDGE THAT THE EMPLOYEE WILL BE ABSORBED INTO A CERTAIN STATE.

HOW MANY TRANSIENT STATES IN THE TRANSITION MATRIX?

#?

?3

INPUT THE Q MATRIX (CANONICAL FORM) ROW BY ROW.

? .7019, .0652, 0, &

? 0, .7979, .0497, &

? 0, 0, .9636

INPUT THE COLUMN OF THE B MATRIX

CORRESPONDING TO THE ASSUMED ABSORBING STATE.

? .05378679849, .24591786244, 1

THE CONDITIONAL Q MATRIX IS

0.7019	.29810000002	0
0	0.7979	0.2021
0	0	0.9636

THE CONDITIONAL F MATRIX IS

3.3545790004	4.9480455225	27.472527475
0	4.9480455221	27.472527473
0	0	27.472527472

#ET=1:38.2 PT=0.2 IO=0.2

VALUATION

This program computes an economic valuation of the firm's human assets. The two examples below are for 1977 and 1976.

RUN

#RUNNING 4253

THIS PROGRAM WILL COMPUTE THE ECONOMIC VALUATION FOR THE HUMAN ASSETS OF A CPA FIRM.

WHAT ARE THE HOURLY BILLING RATES FOR EACH LEVEL?

#?

?25,40,65,0,0

WHAT ARE THE YEARLY CHARGEABLE HOURS FOR EACH LEVEL?

?1700,1500,1300,0,0

WHAT ARE THE AVERAGE SALARIES AT EACH LEVEL?

?18000,34000,74000

HOW MANY EMPLOYEES AT EACH LEVEL?

?8316,2134,1015

INPUT THE TRANSITION MATRIX BY ROWS.

? .7019, .0652, 0, .2329, 0, &

? 0, .7979, .0497, .1524, 0, &

? 0, 0, .9636, 0, .0364, &

? 0, 0, 0, 1, 0, &

? 0, 0, 0, 0, 1

WHAT IS THE COST OF CAPITAL FOR THE FIRM?

? .1

THE ECONOMIC VALUATION OF THE HUMAN ASSETS OF THE FIRM IS EQUAL TO

\*\$602955194

#ET=3:23.9 FT=0.3 IO=0.2

RUN  
#RUNNING 4284

THIS PROGRAM WILL COMPUTE THE ECONOMIC  
VALUATION FOR THE HUMAN ASSETS OF A CPA FIRM.

WHAT ARE THE HOURLY BILLING RATES FOR EACH LEVEL?  
#?  
?25,40,65

WHAT ARE THE YEARLY CHARGEABLE HOURS FOR EACH LEVEL?  
?1700,1500,1300

WHAT ARE THE AVERAGE SALARIES AT EACH LEVEL?  
?18000,34000,74000

HOW MANY EMPLOYEES AT EACH LEVEL?  
?8027,2049,937

INPUT THE TRANSITION MATRIX BY ROWS.

? .7019, .0652, 0, .2329, 0, &

? 0, .7979, .0497, .1524, 0, &

? 0, 0, .9636, 0, .0364, &

? 0, 0, 0, 1, 0, &

? 0, 0, 0, 0, 1

WHAT IS THE COST OF CAPITAL FOR THE FIRM?  
?.1

THE ECONOMIC VALUATION OF THE HUMAN  
ASSETS OF THE FIRM IS EQUAL TO

\*\$580167370

#ET=3;22.2 PT=0.3 IO=0.3



## VITA

Jackson F. Gillespie was born on August 17, 1951. He grew up in Narrows, Virginia. In 1969, he entered the undergraduate school at Virginia Polytechnic Institute and State University. In 1972 Mr. Gillespie completed the requirements for a Bachelor of Science degree in Business with a major in accounting. The requirements for a Master of Accountancy degree were completed in 1974, also at Virginia Polytechnic Institute and State University.

From 1974 to 1977, Mr. Gillespie taught undergraduate courses at V.P.I. & S.U. During the same period, he was working towards a Doctor of Philosophy degree at the same institution. In 1977, while still working towards this degree, he began teaching at the University of Delaware.

In 1978, requirements for the Ph.D. in Business with a major in accounting were completed, after which Dr. Gillespie returned to his duties at the University of Delaware

Jackson F. Gillespie

A MANPOWER PLANNING, HUMAN  
RESOURCE VALUATION MODEL

by

Jackson F. Gillespie

(ABSTRACT)

The purpose of this dissertation is to formulate a quantitative model which can be used in two areas: (1) manpower planning, and (2) human resource valuation. The techniques used are based on an absorbing Markov model. A conceptual model is developed for any firm that has employees who normally move from one position to another within the firm.

The conceptual model is applied to a large Certified Public Accounting (CPA) firm. Some of the data used in the application were supplied by an actual firm. These data were used to develop a transition matrix for the firm. The transient states of the matrix correspond to the employment positions within the firm, i.e., staff, manager, and partner. The absorbing states of the matrix were for employees leaving the firm (1) before becoming a partner, or (2) after becoming a partner. The elements of the transition matrix are the probabilities of employees making certain movements (staff to staff, staff to manager, etc.) during each one-year transition period. These probabilities are based on historic data and must pass a stability test (Chi square) in order to

be statistically valid. The original transition matrix covered a ten-year time period (February 1, 1967 to January 31, 1977). However, the transition probabilities for staff and managers were found to be unstable according to the Chi square test. A matrix covering the nine-year period from February 1, 1968 to January 31, 1977 was tested with the same results. After further tests, a transition matrix for the five-year period from February 1, 1972 to January 31, 1977, was found to be stable. This became the model transition matrix.

The transition matrix should also be tested for the Markov property, i.e., are the transition probabilities affected by the employee's time in grade at his present position? This test was only simulated for the CPA firm because the data needed for the test were not available.

After the tests, the model was extended into manpower planning and human resource valuation. Examples of model output that can be helpful for manpower planning include predicted numbers of employees at each level during future transition periods, probabilities of employees rising to each level within the firm, steady-state information on manpower supply, and average employee tenure with the firm. For human resource valuation, the model is the basis for three methods for charging cost-based valuations against income. The first method is an amortization technique based on an employee's average tenure with the firm. The second method amortizes human resource costs over a period equal to average tenure plus three standard deviations. This method recognizes