

Mathematical Discussion and Self-Determination Theory

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ABSTRACT

This dissertation focuses on the development and testing of a conceptual framework for student motivation in mathematical discussion. Specifically, this document integrates Yackel and Cobb's (1996) framework with aspects of Self-Determination Theory (SDT), as described by Ryan and Deci (2000). Yackel and Cobb articulated the development of students' mathematical dispositions through discussion by facilitating student autonomy, incorporating appropriate social norms and co-constructing sociomathematical norms. SDT mirrors these factors and describes a similar process of self-regulation through fulfillment of the individual needs of autonomy, social relatedness, and competence. Given the conceptual overlap, this dissertation examines the connection of SDT with mathematical discussion with two studies.

The first study examined the effect of student frequency of explaining mathematics on their perceived autonomy, competence and relatedness. Results of HLM analyses found that more frequent explanation of mathematics had a positive effect on students' perceived mathematics autonomy, mathematics competence, and relatedness. The second study used a triangulation mixed methods approach to examine high school geometry students' classroom discourse actions in combination with their perceived autonomy, competence, and relatedness. Results of the second study suggest a higher perceived sense of autonomy is indicative of more engagement in mathematical talk, but a measure of competence and relatedness are needed for such engagement to be fully indicative of mathematical discourse. Rather, students who lacked a measure of perceived competence or relatedness would cease participation in mathematical discussion when challenged by peers. While these results need further investigation, the results of the second study provide evidence that indicates the necessity of fulfilling all three SDT needs for engagement in mathematical discussion. Evidence from both the first and second studies presented in this dissertation provides support for the conceptual framework presented.

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In closing, I wish to thank a gentleman I met on an old dusty bookshelf. Even after all I have learned over the past few years, and all I shall learn, I believe Whittier's words put us all in our place:

I am: how little more I know!
Whence came I? Whither do I go?
A centered self, which feels and is;
A cry between the silences;
A shadow-birth of clouds at strife

With sunshine on the hills of life;
A shaft from Nature's quiver cast
 Into the Future from the Past;
Between the cradle and the shroud,
A meteor's flight from cloud to cloud.

John Greenleaf Whittier, 1852

Questions of Life

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Chapter One

Introduction

Overview

The National Council of Teachers of Mathematics (NCTM) states that discussion in mathematics classrooms “will develop and deepen students’ understanding of mathematics” (NCTM, 2000, p. 194). Discussion involves giving students opportunities to explain, justify, and defend their mathematical ideas and this allows for a deepened understanding of mathematics. Having students share these opportunities through discussion encourages them to reflect upon their ideas about mathematics and adjust them (Goos, 1995; Lee, 2006; Pimm, 1987). Therefore, discussion has positive effects on mathematics achievement (D’Ambrosio, Johnson, & Hobbs, 1995; Grouws, 2004; Hiebert & Wearne, 1993; Mercer & Sams, 2006). There is evidence that mathematics discussion is valued by mathematics educators, supervisors, inservice, and preservice teachers (Watanabe & Yarnevich, 1999) and there have been repeated attempts to emphasize the importance of mathematics discussion over the past hundred years. Yet even with these repeated calls to increase discussion in the mathematics classroom, it is still not widely implemented (Pimm, 1987). Implementing effective discussion in mathematics classrooms is difficult, even when the teachers involved have a thorough understanding of what effective discourse is (Lloyd, 2008; McGraw, 2002; Mendez, Sherin, & Lewis, 2007). Additionally, teachers may believe they are implementing effective discourse when in fact they are not (Kazemi & Stipek, 2001; Nelson, 1997).

The benefits of mathematical discussion advocated by various authors (D’Ambrosio et al., 2006; Lee, 2006; Pimm, 1987) would suggest that it would be implemented by teachers

across the country. However, as stated previously, effective discussion is not often implemented (Pimm, 1987). This is not to say that implementation of mathematical discussion is not attempted, simply that such implementation is not always effective (Kosko & Miyazaki, 2009). Part of the reason for such ineffective discussion is that many teachers may not realize they are not engaging students in mathematical discussion properly. Manouchehri and St. John (2006) compared two episodes of classroom talk where there was a large degree of student participation. On the surface the two episodes appeared to be similar in that both teachers actively engaged students in the topics they discussed. Yet one teacher appeared to control the course of discussion more than another teacher. Similar results were found by Kazemi and Stipek (2001) where some teachers emphasized student autonomy and others did not:

On the surface, all four teachers...observed implemented qualities of inquiry-oriented mathematics instruction. Students solved open-ended problems in groups, documented their work graphically and numerically, and shared their different strategies enthusiastically. The teachers and classmates accepted and supported students who made mistakes. All four teachers encouraged their students to describe how they solved the problems. Thus, a number of social norms...were in place... (p. 78)

The characteristics described in the above quote by Kazemi and Stipek (2001) illustrate how difficult it can be for a teacher or researcher to judge characteristics of effective mathematical discussion. Certain features in a classroom can mislead one to believe effective discussion is being implemented when it is not. While all teachers observed by Kazemi and Stipek had implemented appropriate social norms, it was only those students whose teachers

gave a higher press for learning who were engaged in more effective mathematical discussion. This high press for learning was characteristic of what Ryan and Deci (2000) would call autonomy supportive.

Autonomy supportive teachers, as described by Ryan and Deci (2000), enact practices that give students control over their own learning. However, Ryan and Deci discuss the importance of autonomy in context with internalizing actions for motivation. This process of internalization is described under the theory for motivation, Self-Determination Theory. Along with autonomy, competence and social relatedness are psychological needs described as facilitating the internalization of actions. When all three psychological needs are met, the process of internalizing actions and content can take place. This internalization in turn fosters motivation on the part of the student to engage in and intrinsically value the actions they are taking part (Ryan & Deci, 2000). In addition to the apparent connection with autonomy, there are numerous connections between mathematical discussion and Self-Determination literature. A full description of this connection will be provided in Chapter 2.

Current mathematics communication researchers do a good job of describing what effective mathematical discussion looks like. However, after a review of the literature, I found an incomplete description of how and why various characteristics of effective discussion interrelate and work. Self-Determination Theory can provide the necessary frame for explaining the interrelationship. The purpose of this dissertation is to use Self-Determination Theory to help explain the effectiveness of mathematical discussion.

Outline of the Dissertation

In Chapter 2, I describe a conceptual framework for explaining mathematical discussion in context with Self-Determination Theory of Motivation. This framework discusses connections

apparent in the mathematics communication literature with the three psychological needs described by Self-Determination Theory. Additionally, a detailed explanation of the process of internalization in mathematics discussion is provided within the context of Self-Determination Theory.

Chapter 3 includes two studies that are inherently related. In the first study, I use hierarchical linear modeling (HLM) to examine the effect of student frequency in explaining mathematics on their perceived level of autonomy, competence and relatedness. The HLM analysis allowed for the variability due to class assignment to be taken into account when examining the effect of explanation frequency. Additionally, the effect of classroom level factors (i.e., average student frequency in explaining mathematics & teacher use of small groups) on individual student frequency of explaining mathematics was examined. Due to low correlations among the student-level variables, a second study was included in Chapter 3 to evaluate the validity of variables used in the HLM analysis. Results and implications of both studies are included.

Results of a mixed methods study were reported in Chapter 4. The purpose of this study was to evaluate observable mathematics discourse actions in the classroom in context with measured individual SDT needs. Observed students completed a questionnaire that quantitatively measured their perceived needs of autonomy, competence, and relatedness. Student engagement in discourse-related actions was also assessed by the survey. Students' actions were observed as they participated in various discourse settings and these observed actions were examined in context with the quantitative data. The merging of this data allowed for a deeper interpretation of both the quantitative and qualitative data. Implications of this analysis are discussed.

Chapter 5 provides an overview of the studies included in Chapter 3 and 4. Additionally, in Chapter 5, I utilize the conceptual framework presented in Chapter 2 to illustrate the connections between Chapters 3 and 4, as well as providing a discussion of these connections. The implications of the studies included within this dissertation is provided in the context of potential impact on mathematics education pedagogy and research. Also, future directions in the research program of the author are described.

Summary

The development and successful evaluation of aspects of the conceptual framework presented in this dissertation can provide researchers and mathematics educators with a new understanding of mathematical discussion. In particular, by successfully connecting Self-Determination Theory with characteristics of effective mathematical discussion, we can better understand how students become engaged in mathematical discussion. Further, mathematics education researchers can better understand how engagement in discussion facilitates the internalization of mathematics through discussion, resulting in the formation of a positive, sophisticated mathematical disposition on the part of the student. Students with such mathematical dispositions should be considered intrinsically motivated to learn mathematics and such motivation should result in a deeper understanding of the content.

References

- D'Ambrosio, B., Johnson, H., & Hobbs, L. (1995). Strategies for increasing achievement in mathematics. In R.W. Cole (Ed.), *Educating everybody's children: Diverse teaching strategies for diverse learners* (121 – 138). Alexandria, VA: Association for Supervision and Curriculum Development.
- Goos, M. (1995). *How do you know when you understand? Using explanation to monitor and construct mathematical understanding*. Paper presented at the Postgraduate Research Conference Commemorating 50 Years of Educational Studies at the University of Queensland, Brisbane, Queensland, Australia.
- Grouws, D.A. (2004). Mathematics. In G. Cawelti (Ed.), *Handbook of research on improving student achievement* (3rd Ed.) (pp. 160 – 178). Arlington, VA: Education Research Service.
- Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal*, 30(2), 393 – 425.
- Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal*, 102(1), 59 – 80.
- Kosko, K. W., & Miyazaki, Y. (2009). *The impact of student discussion on fifth graders' mathematics achievement in the U.S.* Manuscript submitted for publication.
- Lee, C. (2006). *Language for learning mathematics: Assessment for learning in practice*. New York: Open University Press.
- Lloyd, G. M. (2008). Teaching mathematics with a new curriculum: Changes to classroom organization and interactions. *Mathematical Thinking and Learning*, 10(2), 163 – 197.

- Manouchehri, A., & St. John, D. (2006) From classroom discussions to group discourse. *Mathematics Teacher*, 99(8), 544 – 551.
- McGraw, R. H. (2002). *Facilitating whole-class discussion in secondary mathematics classrooms*. Unpublished doctoral dissertation, Indiana University, Bloomington, IN.
- Mendez, E. P., Sherin, M. G., & Louis, D. A. (2007). Multiple perspectives on the development of an eighth-grade mathematical discourse community. *The Elementary School Journal*, 108(1), 41 – 61.
- Mercer, N., & Sams, C. (2006). Teaching children how to use language to solve maths problems. *Language and Education*, 20(6), 507 – 528.
- National Council of Teachers of Mathematics. (2002). *Principles and standards for school mathematics*. Reston, VA: Author.
- Nelson, R. S. (1997). *Developing mathematical knowledge through class discussion: One teacher's struggles in implementing reform*. Unpublished doctoral dissertation, Virginia Polytechnic and State University, Blacksburg, VA.
- Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. New York: Routledge & Kegan Paul.
- Ryan, R. M., & Deci, E. L. (2000). Self-determination theory and facilitation of intrinsic motivation, social development, and well-being. *American Psychologist*, 55(1), 68-78.
- Watanabe, T., & Yarnevich, M. (1999). *What really should be taught in the elementary mathematics methods course?* Paper presented at the Annual Meeting of the Association of Mathematics Teacher Educators, Chicago, IL.

Chapter Two:
A Conceptual Framework for Mathematical Discussion using Self-Determination Theory
of Motivation.

Introduction

The content and actions occurring in mathematical discussions “serve to invoke one’s reflective consciousness” (Wood, 1999, p. 174). As a result, mathematical discussion has consistently been linked to student reflection of mathematics content (Cassel & Reynolds, 2007; Hoffman, 2004; Goos, 1995; Wood, 1999). This reflection upon mathematics content through discussion allows students to gain a deeper understanding of the content and, therefore, increased achievement in mathematics. Various researchers have linked effective mathematical discussion with increased academic achievement (D’Ambrosio, Johnson, & Hobbs, 1995; Grouws, 2004; Hiebert & Wearne, 1993; Mercer & Sams, 2006). Additionally, these studies have outlined specific characteristics of effective discourse in mathematics.

Providing students with a level of control, or autonomy, in how they engage in mathematics and their discussion about mathematics has been characterized as a critical component to effective mathematics discussion (Lee, 2006; Pimm, 1987; Turner, Meyer, Midgley, & Patrick, 2003). Certain classroom social norms and ground rules for discussion have also been identified as important (Kazemi & Stipek, 2001; Kotsopoulos, 2008; Sfard, 2007; Springer & Dick, 2006; Wood, 1999). Additionally, asking students to describe, explain and justify their mathematics is arguably what explicitly elicits discussion from students (Hiebert & Wearne, 1993; Stein, 2007; Truxaw & DeFranco, 2007; Wood, 1999). The researchers describing these characteristics of effective mathematical discussion does an effective job of telling us *what* mathematical discussion should encompass. However, there is a need for a

conceptual model describing *why* these and other characteristics of effective mathematics discussion increase understanding of the content and how each of these characteristics interact with each other to do so.

It is the primary purpose of this chapter to propose a theoretical framework that explains why effective mathematical discussion facilitates the internalization of the content for engaged students. This will be done by incorporating aspects of Self-Determination Theory (SDT) with aspects of effective mathematical discussion. By doing so, I seek to link psychological factors established by SDT with the social interactions involved in mathematical discussions. I believe that such a connection will facilitate a deeper understanding of how effective mathematical discussion works to deepen student understanding of the content and enable future avenues of research based on this conceptual framework to develop.

A description of different types of mathematical talk and discourse is necessary to establish an understanding of when discussion and talk in the mathematics classroom does and does not elicit understanding and reflection on the part of the student. Given the variability in the effectiveness of mathematical discussion (Kosko & Miyazaki, 2010), a review of the types of mathematical talk that have been described in the context of the classroom will help in establishing the characteristics of effective talk and discourse. It is also necessary to discuss the major concepts of Self-Determination Theory of Motivation in order to outline how the theory describes the internalization of content and actions of students. An understanding of this process of internalization, and the major components that drive it, are inherently useful in later describing the internalization of mathematics through the act of discussion.

Following the review of the relevant literature, two sections are presented that carefully outline the background and development of the conceptual framework. First, evidence of the

three psychological needs in mathematics discussion literature are presented and linked to the SDT literature. Second, a description of how different types of mathematical talk and discourse identify with different stages of internalizing mathematics, in accordance with SDT, is described. This second section makes extensive use of a framework previously developed by Erna Yackel and Paul Cobb (Yackel & Cobb, 1996). Although other literature exists to support the conceptual framework described here, the focus on Yackel and Cobb's framework is done specifically for the purpose of redefining their model and extending it with SDT. Finally, an overview of a theoretical framework that uses the process of internalization described by SDT to explain the development of internally reflective practices brought about through engagement in mathematical discussion is provided.

Mathematical Talk and Discourse

There are various types of mathematical talk and various names for each of them. In a synthesis of current mathematics education research, Schleppegrell (2007) summarized different types of talk. Of particular interest from this synthesis is procedural and conceptual talk. Procedural talk focuses on specific steps and strategies for solving problems while conceptual talk (e.g., evaluation and analysis) focuses on the reasons for doing certain procedures.

Both procedural talk and conceptual talk are used with specific purposes in mind. However, both of these types of talk describe *what* is being said. Certain settings for mathematical talk, such as univocal and dialogic discourse, focus on *who* is talking and *how* they are engaging in discourse. Gadanidas, Kotsopoulos, and Guembel (2006) described two such types of discourse, authoritative and internally persuasive. Authoritative discourse demands adherence to specific criteria while internally persuasive discourse is “reflective of autonomous thought and action” (p. 1). Kitchen (2004) also identified issues of power in mathematical

language when classifying mathematical communication as univocal and dialogic, rather than authoritative and internally persuasive. Univocal discourse was described as discussions where the teacher is the dominant voice and student participation in the direction of the discussion is limited. Dialogic discourse was described as discussion that includes teacher and students in describing and discussing mathematical concepts. In other words, students are involved in the directing of the discourse as well as the teacher. A teacher conducting dialogic discussions often must put a great deal of effort into involving students in the discussion and eliciting their views of the topic. Students engaged in dialogic discourse provide in depth descriptions in their discussions and engage in discussions with their fellow students as well as their teacher (Kitchen, 2004).

Mercer and Littleton (2007) also identified dialogic teaching as that which involves both teachers and students. Three tenets of dialogic discourse include the use of questions to guide instruction, promoting the linkage of content knowledge to that of student experiences, and the encouragement of students actively participating in the dialogue by giving richer descriptions and justifications of the content (Mercer & Littleton, 2007). There is some emphasis on students posing questions and making contributions to the dialogue versus the teacher being the dominant talker. Both Kitchen (2004) and Mercer and Littleton (2007) described dialogic discourse as a type of talk that characterizes how people engage in discussion and not specifically what is being said. Whereas procedural and conceptual talk characterize the content of discussion (Schleppegrell, 2007), both univocal and dialogic discourse can be described as how procedural and conceptual talk are brought about in the mathematics classroom. Just as there are differences between dialogic and univocal discourse, there are differences in what procedural and conceptual talk look like in dialogic and univocal discourse.

Descriptions by Hancewicz (2005) add to the list of possible types of talk by identifying three main scenarios for discourse in the mathematics classroom: traditional, probing, and discourse-rich. The first is the traditional setting where the teacher talks with several students. The discourse is teacher centered with the teacher asking most of the questions. The probing setting is still teacher centered but the questions are more open. In this case, the teacher's questioning strategies are guided by a desire to know what the students understanding of a concept is rather than trying to reach a lesson objective at a certain point in time. The discourse-rich setting has students asking the majority of questions of each other instead of the teacher leading what questions are being asked all the time. Students take a certain level of control in deciding what topics are discussed in more depth and what depth they are discussed.

Hancewicz's (2005) scenarios can be likened to a transition between univocal and dialogic discourse. Traditional settings, as described by Hancewicz (2005), are similar to univocal practices and discourse-rich settings appear similar to dialogic discourse. However, a setting between univocal and dialogic discourse, probing, has been introduced. Rather than add another possible type of classroom talk, it may be worthwhile to consider both univocal and dialogic discourse as being two ends of a spectrum where at some point discourse ceases to be univocal and becomes dialogic, or vice versa. The probing setting, which is teacher-centered, represents a phase of transition from univocal to dialogic. Since probing is teacher-centered discourse, it is univocal in nature. It should not be considered a separate or intermediate phase between univocal and dialogic discourse, but as univocal discourse on the verge of becoming dialogic.

Four key terms have been described from the literature above: procedural talk, conceptual talk, univocal discourse, and dialogic discourse. Procedural and conceptual talk convey specific

purposes while univocal and dialogic discourse deal more with how discussions take place. Schleppegrell (2007) described procedural and conceptual talk as facilitated by the manner in which the teacher engages students in mathematical talk. Although neither univocal nor dialogic discourse is described by Schleppegrell, the very definitions of these two types of discourse argue their place in such a description of facilitating mathematical talk. Therefore, univocal and dialogic discourse should be viewed as a means of incorporating both procedural and conceptual talk. Students engaged in univocal discourse may produce procedural and conceptual talk and students engaged in dialogic discourse may do the same. As stated before, terms used throughout the literature vary considerably even when the definitions themselves are almost identical. For the purposes of this manuscript, the terms procedural, conceptual, univocal, and dialogic will be used when referring to different types of mathematical talk or discourse.

Self-Determination Theory of Motivation

Theory concerning motivation focuses on why people act in certain ways. In other words, what causes behavior? What sustains, stops, or alters it? Why do people do what they do and why is there such variation in the intensity of their actions? These types of questions are the driving force behind understanding motivation. It is from questions such as these that motivation theory has developed (Reeve, 2005; Stipek, 2002).

Although theories on motivation can be traced back to Plato and other ancient Greeks, current motivational theory evolved from previous failed attempts at construction of grand theories in motivation (Reeve, 2005). Grand theories attempted to explain all aspects of motivation under a unifying umbrella of a particular causal agent (e.g. Descartes' motivational force of 'will,' see Reeve, 2005 for further explanation). One of the problems associated with these grand theories was that each tried to explain the full range of motivation within somewhat

confined parameters. Therefore, current motivation theory is comprised of numerous micro-theories which explain parts or aspects of motivation (Reeve, 2005). These include but are not limited to Intrinsic Motivation Theory (e.g. Deci, Betley, Kahle, Abrams, & Parac, 1981; Fairchild, Horst, Finney, & Barron, 2005), Attribution Theory (e.g. Weiner, 2000), Self-Efficacy Theory (e.g. Bandura, 1977), Goal Theory (e.g. Anderman, Austin, & Johnson, 2002; Wolters, 2004) and Self-Determination Theory (e.g. Reeve & Jang, 2006; Ryan & Deci, 2000).

It is in the context of one particular micro-theory that this manuscript takes interest. Self-Determination Theory (SDT) is concerned with the degree to which motivation is regulated by factors external or internal to the individual (Ryan & Deci, 2000). SDT explores not only motivation which is determined by the individual but also that which is controlled by others. Actions that are self determined are those that the individual has the most power over. Control is described in context with decisions or the availability of decisions being in the hands of those other than the individual (Deci, Vallerand, Pelletier, and Ryan, 1991; Reeve, 2006). Autonomy-supportive teachers “facilitate congruence by identifying and nurturing students’ needs, interests, and preferences” (Reeve, 2006, p. 228) into their instruction and lessons. Controlling teachers inhibit students’ connections to curricula content by forcing their own values and interests onto the students. Deci et al. (1991) describe a relationship between autonomy-supportive and controlling environments as corresponding with the degree to which student behaviors are internalized or externalized respectively. Deci et al. (1991) relate this aspect of Self-Determination Theory (SDT) to locus of control where actions under the individuals control are internal in nature and those in the control of others are external to the individual.

SDT is best described with three psychological needs: relatedness, competence, and autonomy. Relatedness describes the degree to which satisfying social connections are made.

Competence describes the degree to which the individual feels able to accomplish different external and internal tasks. Autonomy describes the degree to which the self is in control of initiating and maintaining different behaviors (Deci et al., 1991; Ryan & Deci, 2000). When these needs are met, “motivation, performance, and development will be maximized” (Deci et al., 1991a, p. 327). The impact of competence on increasing intrinsic motivation tends to depend on the degree of autonomy individuals have, while the development of autonomy does so best when there is a strong sense of relatedness. Any of the three needs can and do develop independently of each other but Deci et al. (1991), Reeve (2006), and Katz and Assor (2007) suggest that the interdependence of each on one another requires all three needs be met in order for motivation to be self determined. This claim can be supported by various studies including one conducted by Reeve and Sickenius (1994) where autonomy, relatedness, and competence were significantly correlated with each other and correlated with intrinsic motivation, therefore signifying a statistical relationship between these constructs. Asking adults to recall a satisfying event in the past six months, Hahn and Oishi (2006) found autonomy, relatedness, and competence were identified as three of the top four needs identified in importance of making the event satisfying. The results found by Hahn and Oishi (2006) also lend credit to the presence of a relationship between the three psychological needs central to SDT.

While the psychological needs of autonomy, competence, and relatedness demonstrate a degree of interrelatedness and interdependence (Deci et al., 1991; Reeve, 2006), it is arguable that self-determination is, in essence, autonomy. However, Ryan and Deci (2000) describe autonomy as facilitating internalization within the context of competence and relatedness, which in turn facilitate internalization. Therefore, while all three needs must be met, competence and relatedness are met through contexts and autonomy is facilitated in those contexts. So, while

autonomy and self-determination can arguably be used as synonymous terms, each psychological need is essential for actions to become self-determined.

Although much of the literature on SDT focuses on autonomy, competence, and relatedness, another aspect of SDT is the transition of motivation from external factors (i.e. external or extrinsic motivation) to motivation from internal factors (i.e. interest, intrinsic motivation) (Deci et al., 1991). Ryan and Deci (2000) describe a continuum of types of regulated actions from external to internal regulation. More specifically, external motivation is broken into four phases in which each subsequent phase is more internalized than the prior; external regulation, introjected regulation, identified regulation, and integrated regulation. External regulation describes the cause of a behavior as being completely external to the individual. A student completing an assignment for a reward or the threat of punishment can be described as being externally regulated. Introjected regulation describes an initial internalization of a regulation but not accepting it as one's own. Students who are motivated by the pride felt for achieving good grades or the guilt of doing poorly are appropriate examples of introjected regulation. Pride or guilt are internal thoughts but are not considered self determined because of significant external factors.

Identified regulation occurs when an individual identifies with the regulatory process (Deci et al., 1991; Ryan & Deci, 2000). Deci et al. (1991a) illustrate identified regulation with the example of a student who completes extra mathematics assignments because they know it will be useful in helping them succeed in the subject. External factors still motivate the student, but they are able to begin making their own choices and their actions are self determined. Integrated regulation occurs when externally motivating factors are fully internalized and integrated into the individual's identity. A student who studies trigonometry because he/she

values the content itself is characteristic of integrated regulation. Deci et al. (1991) note there is little distinction between integrated regulation and intrinsic motivation. Because of this degree of closeness, actions performed during integrated regulation are seen as fully self determined.

The transition from extrinsic to intrinsic motivation is a central concept in SDT. SDT posits that meeting the psychological needs of autonomy, competence and relatedness facilitates the transition from more externally regulated behaviors to more self-regulated behaviors (Deci et al., 1991; Ryan & Deci, 2000). Therefore, if there is a conceptual connection between SDT and mathematical discussion, then aspects of each psychological need and the transition from external to internal regulation should be evident in at least some of the research concerning mathematical discussion. In the following two sections, these connections will be described. First, a detailed description of how each psychological need manifests itself in the mathematical discussion literature is provided. Following this section is a description of the transition from external to internal regulation that relies heavily on the work of Yackel and Cobb (1996). It is in this second section that a clear connection between SDT and mathematical discussion is articulated and a description of the theoretical framework is provided.

Psychological Needs Facilitating Mathematical Discussion

Autonomy

Issues of student autonomy and teacher control of mathematical discussion are commonly discussed in mathematics communication literature (e.g. Goos, 1995; Sfard, 2007). However, autonomy is also a central theme in SDT literature (e.g. Deci et al., 1991; Ryan & Deci, 2000). In general, the teacher is the dominant talker in the mathematics classroom and has ultimate control over the mathematics discourse that takes place (Fey, 1970; Pimm, 1987). Yet varying mathematics communication literature argues against teacher dominance of talk. In their

observations of an inquiry-oriented class, Alro & Skovsmose (2002) observed a great deal of openness and a ceding of control by the teacher to the students. It is suggested that the benefit of such open lessons allow students to take ownership of their own learning. This in turn facilitates the internalization of mathematics content. Borenson (1986) described one teacher's class where students thought a rectangle had two right angles and two left angles. By asking the students to 'flip' the rectangle the class agreed that any 'left angle' was also a 'right angle' and therefore the rectangle had four right angles. Instead of telling students they were wrong, the teacher gave students a certain amount of power in deciding what types of angles the rectangle had. Borenson (1986) stated that this type of approach to discussion in a mathematics classroom makes students "more comfortable in handling new mathematical situations" (p. 70). This comfort comes in the degree to which students can identify with the topic. It is no longer foreign to them but relates to their previous ways of thinking. Brown and Hirst (2007) also suggested that giving students a certain amount of power in the dialogue increases their understanding. Analysis of a seventh grade class found that when the teacher sought to have students compare each other's understanding it seemed to help students connect prior knowledge to formal concepts. This type of approach is different than traditional classrooms where it is the teacher who is comparing, explaining, and justifying (Pimm, 1987).

In each of the examples above, student autonomy is stressed as enabling internalization of mathematics content knowledge. These examples from the mathematics communication literature are supported by Reeve and Jang (2006) who state that it is beneficial to students for teachers to provide opportunities and experiences for individuals to make decisions and take a measure of control. SDT literature also provides support for claims made by the above mathematics communication literature that autonomous support facilitates internalization of the

content. For example, Black and Deci (2000) and Miserandion (1996) found evidence that higher levels of perceived student autonomy predicted academic gains in the classroom.

Having a certain amount of control in the direction of class discussions motivates students to participate.

The pupils are involved when they take a full part in the classroom discourse.... Focusing on language in mathematics aims to allow the pupils to take control over their own mathematical thoughts and ideas.

Facilitating pupils' ability to express their mathematical ideas is a big step towards securing their involvement in the learning process

(Lee, 2006, pp. 69 – 70).

One part of the learning process that mathematics teachers often deal with is deciding when a topic or concept has been explained to the point where students should understand it. Observing whole class discussions, Goos (1995) noticed that “by ceding control of the debate the teacher provided [an] opportunity for students to ask for, and receive, explanations from each other until they were satisfied that they understood” (p. 16). Therefore, instead of the teacher, it was the students who decided the degree of explanation they needed to understand math concepts.

Acceptance of student contributions to the classroom dialogue requires the teacher to relinquish a certain degree of their control in the classroom. However, as teachers can exert too much control they can also give up too much as well. Investigating why a lesson engaging students in dialogue failed, Sfard (2007) found that the teacher relinquished too much control to the point that the students did not know the rules of discourse being employed. In other words, the students didn't know how they were expected to talk about the mathematics. The activity the students were given was so open that they did not know where to begin and there was an

understandable degree of confusion as a result. Students need a degree of autonomy, but as illustrated by Sfard (2007), too much can be as limiting to mathematical learning as complete dominance by the teacher. This situation is supported by SDT literature. Alderman (2008) stated that “autonomy is not just a matter of open-ended choices for students. It is a feeling of ownership by students...more choices than students can handle can be demotivating” (p. 219). Levinson (1999) also identified the need for a certain amount of structure in the form of “norms of critical inquiry and reflection...and mutual respect and toleration...” (p. 61). In other words, for students to truly be able to participate autonomously in mathematical discussions they must feel competent in their ability to discuss mathematics and they must feel a sense of relatedness as well.

Competence

Of the three psychological needs, competence is the least discussed in mathematics communication literature. However, there is evidence of its importance in two regards. The first relates to a previously cited example from Sfard (2007) where students failed to engage in mathematical discussion because they were at a loss on how to go about discussing mathematics. Students therefore lacked a sense of competence in how to engage in mathematics discussion.

Sfard’s (2007) observation makes the case for a degree of structure or set of ground rules for mathematical discourse. Coincidentally, social norms and ground rules are advocated by various researchers when discussing mathematics discussion (e.g. Mercer & Sams, 2006; Williams & Baxter, 1996; Wood, 1999). Although norms and ground rules are not discussed in context in SDT literature, it is arguable that by providing these rules, teachers are giving students the guidance and structure needed for them to engage in an activity they have little to no exposure to. Therefore, these initially external rules may quickly become more internally

regulated as to provide students with a sense of competence in understanding how to engage in mathematical discourse. The specifics on this process of internalizing norms for mathematics discussion will be described in a later section.

The other factor of competence that must be addressed is that of mathematical content. According to teachers interviewed by Kitchen (2004), students who know more about mathematics also talk more about mathematics. In other words, students who have a higher self perception of competence are more dominant talkers in mathematics discussion than students with lower self perceptions of competence. This appears to be supported, in theory, by SDT literature that states that having a higher sense of competence can positively impact students' achievement (Miserandion, 1996), which can be related to their actions in class, such as engagement in mathematics discussion. However, there is more to this issue of competence as outlined in mathematics communication literature. Specifically, there is evidence that transitions from less competent and less engagement in discussion to more competent and more engagement in discussion may not often occur.

In observations of a fifth grade class, Black (2004) found that the observed teacher gave a more competent student more autonomy in mathematics discussions and less competent students were given more closed questions. Similarly, Van der Aalsvoort, Harinck, and Gosse (2006) found that teachers working with students of varying math competencies in small groups gave more competent students more engaging questions and opportunities to engage in the mathematical activity. On the contrary, less competent students received less support from the teacher and on some occasions were not given the opportunity to solve the problem on their own. Alfi, Assor, and Katz (2004) would describe these types of actions as not being competence supportive. Rather, students perceived as less competent should be given tasks that are optimally

challenging for them so that they can increase their sense of competence while improving their ability to solve mathematical problems. When optimizing the challenge of a task that is to be discussed in the mathematics classroom, it may be that the task is not optimally challenging for the entire class. However, if students believe that they are capable of solving the problem, then the nature of the mathematics discourse can act as an initially external support that guides students into becoming more competent in the mathematics that is discussed.

Relatedness

Interviewing middle school students who were members of discussion-oriented math classrooms, Hoffman (2004) found that many students voiced fears of social penalties for speaking in their mathematics class. These students were more likely to describe the need to obtain knowledge from their teacher and to reproduce it accordingly than students who did not fear social penalties. Students who did not express this fear described discussing mathematics with others as a way to reflect on their own knowledge and identify misconceptions. Similar to these findings, Jansen (2008) identified students who believed participation in whole class discussion to pose some sort of social threat as less likely to participate in discussion. When these students did participate, their answers were brief and their participation during discussion was often of procedural knowledge and not conceptual knowledge. Both of these studies describe the necessity to meet students' need of relatedness in order for them to fully engage in mathematical discussion.

In SDT literature, Alfi et al. (2004) stated that for teachers to foster a sense of relatedness, they should enact rules and procedures that protect students from social penalties when they fail or make a mistake in class. Additionally, social interaction between students is also seen as facilitating relatedness. This is characteristic of social norms described by

mathematics communication researchers (e.g. Kazemi & Stipek, 2001; Martin, McCrone, Bower, & Dindyal, 2005; Wood, 1999) in which teachers create a social environment where students feel comfortable in discussing their mathematical beliefs. These norms, when implemented, protect students who may be fearful of giving a wrong answer, focus more on the reasoning of process rather than the final answer, and modeling how students respond to statements they feel are incorrect. Student mistakes are supported and accepted rather than rejected. In this regard, such norms are not only supportive of relatedness but also competence. Minnaert, Boekaerts, and De Brabander (2007) found that social relatedness played a part in raising correlations with a program's implementation and the degree to which competence and autonomy were met. Therefore, by implementing the social norms described by mathematics communication literature, a sense of relatedness is met but the needs of autonomy and competence are also facilitated.

The Process of Internalizing Mathematics through Discourse

The previous section discussed how each psychological need identified in SDT is discussed in the mathematics discussion literature. In this section, the connection between SDT and mathematics discussion will be clarified by defining mathematical talk through the transition from external to internal regulation. As described in the previous section, SDT claims that by meeting the psychological needs of autonomy, competence, and relatedness, students move from more external to internally motivated behaviors. Also described in a previous section are the four types of mathematical talk this manuscript discusses: procedural, conceptual, univocal, and dialogic.

Interaction of Types of Mathematical Talk

Mathematical talk in the classroom can typically be described as procedural or conceptual. Although there are other types of talk that occur in the mathematics classroom (e.g. regulatory talk as described by Schleppegrell, 2007; simple facts as described by Fey, 1970), they are not in and of themselves mathematical. Yackel and Cobb (1996) described mathematical discourse as being inherently linked to the process of mathematics. As simple facts might be described simply as a kernel type statement, they are not in and of themselves a type of mathematical talk. A fifth grade student stating that two plus two is four is no more evident of mathematical thought than a man stating he bought gasoline for two dollars per gallon. Although the first statement is one relevant to mathematics and likely to be spoken within a mathematics classroom, because it is not evident of the process of mathematics, it is not *mathematical* talk. Also not mathematical, regulatory talk deals with managerial tasks that can be found in classrooms of any subject. Therefore, this manuscript takes the view that there are two main types of mathematical talk: procedural and conceptual.

Also within the classroom are two types of discourse: univocal and dialogic. As stated in a previous section, the characteristic difference between a type of talk and a type of discourse is that talk describes what is said and discourse describes how it is said. When viewing the types of talk in context with the types of discourse, Schleppegrell (2007) discussed mathematical talk such that both univocal and dialogic discourse can be described to incorporate both procedural and conceptual talk. It is the way that procedural and conceptual talk are used that marks the difference between the two. Specifically, during univocal discourse it is the teacher who is the dominant talker. As described by Pimm (1987), the teacher is the dominant talker and the one who typically gives explanations and justifications. Therefore, in univocal discourse, it is the

teacher who generally produces conceptual talk. The student is typically given the opportunity to produce simple answers or a description of their procedures. As identified by Mercer and Littleton (2007), dialogic discourse is characterized as having students provide descriptions and justifications of the content along with the teacher. Therefore, in dialogic discourse, students are more likely to engage in conceptual talk than those in a univocal setting. Additionally, both dialogic and univocal settings provide opportunities for procedural talk.

These descriptions of the interaction between mathematical talk and discourse are supported by Cobb, Wood, Yackel, and McNeal's (1992) observations of two elementary school classrooms. Through their observations, Cobb et al. described two different types of classrooms. In one, the teacher guided students through a process by providing them with procedures that they were to follow. In the other classroom, the teacher and students both validated strategies presented to solve mathematical problems. The first teacher was described as engaging in more univocal discourse that required students to follow procedures and, at times, describe the procedures. This was the extent of their explanation. In the second classroom, the teacher had students explain and justify what they said. Since both the teacher and students were involved in the explanation and justification of mathematical content, the discourse was more dialogic in nature. Therefore, univocal discourse appears to exhibit more procedural talk than conceptual talk from students and dialogic discourse appears to exhibit more conceptual talk from students than a univocal setting. This relationship can be observed in Figure 2.1, created here for purpose of illustrating the relationship. As the figure illustrates, the different types of talk overlap within the different discourse settings. Additionally, the figure describes univocal and dialogic discourse as a continuum where, at a certain point, discourse ceases to be univocal and becomes dialogic, or vice versa.

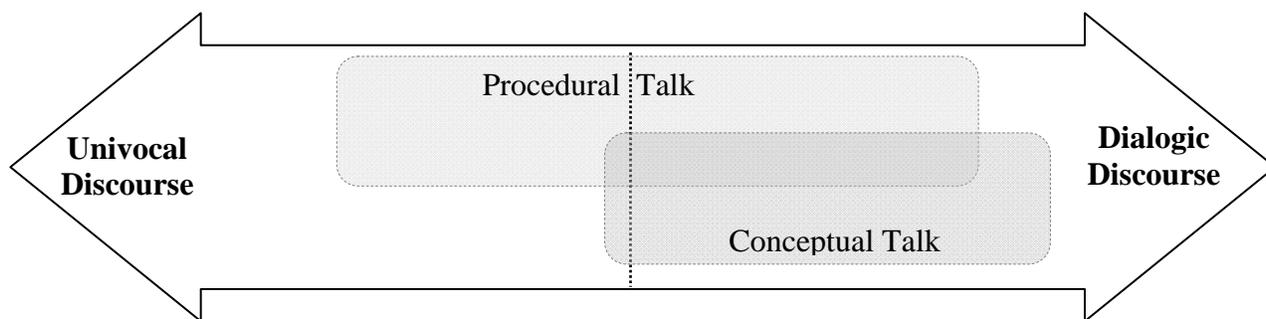


Figure 2.1. The Interaction Between Mathematical Talk and Discourse

A student who participates in univocal discourse may not produce procedural or conceptual talk. In fact, such a student may provide simple facts or other memorized content when prompted by their teacher. Such simple facts may also be used in context with justifying procedures when solving a problem. For example, a tenth grade student who states that you solve operations within grouping symbols or parenthesis is most likely stating a simple mathematical fact. Such a situation is not represented explicitly in Figure 2.1. However, the open space within the figure is meant to allow for situations that are not accounted for by procedural or conceptual talk.

Although the “order of operations” may be viewed by the tenth grade student in the example above as a simple fact, the same may not be true if instead it were a fourth grade student who is learning about the order of operations. What constitutes a simple mathematical fact is dependent on the mathematical development of the individual student. Therefore, what may be viewed as conceptual talk for a fourth grade student may be procedural talk or a simple mathematical fact for a tenth grade student. This example stresses that the model represented in Figure 2.1 is content independent and context specific. As a student is introduced to new concepts within mathematics, they may fall into a different position along the continuum. Therefore, the model is independent of specific mathematics content, but it is dependent on

specific contexts for each individual student. With that said, it is also logical to state that a student's participation in previous contexts can influence their initial placement in a new or different context.

As previously stated, students engaging in univocal discourse are more likely to engage in procedural talk than conceptual talk (Cobb et al., 1992; Pimm, 1987). However, there are potential situations where students engage in univocal discourse with the teacher but produce conceptual talk. For example, a teacher in a univocal setting may ask students to explain why the missing angle in a triangle is a certain measurement. It is important to note that while students may answer using conceptual talk, it is a situation where the teacher is directing the discourse. Additionally, the conceptual talk may in some ways be biased towards procedural understanding. This situation, while univocal, is representative of what Hancewicz (2005) described as the probing setting. As previously mentioned, the probing setting can be likened to univocal discourse that is on the verge of becoming dialogic. Figure 2.1 represents this, because conceptual talk occurring in univocal settings is near to becoming dialogic.

While it is possible to provide any number of situations or examples that may be exemplified within the model of mathematical talk outlined here, such descriptions would be tedious. Therefore, it is presumed that situations occur within univocal and dialogic settings in which procedural and conceptual talk are used together, separately, or not at all. The main intent of Figure 2.1 is to provide a basic understanding of when conceptual and procedural talk occur in univocal and dialogic settings. It is not meant to represent all possible situations of discourse actions in the mathematics classroom, but at the same time allow for the possibility of such situations.

In addition to the characteristics outlined above, it is important to remember that the model of mathematical talk as outlined in Figure 2.1 represents the student as participant in mathematical discourse. For mathematical discussion to take place and be effective, all participants in the discourse setting must *willingly* participate (Cobb et al., 1992). In other words, a teacher may establish a classroom atmosphere characteristic of dialogic discourse, but it is still possible for some or all of the students to engage in such a way characteristic of student roles in a univocal setting. Yackel and Cobb (1996) found evidence of such a situation when examining student engagement in mathematical discourse in an elementary classroom. Many students observed were used to the teacher being the sole provider of explanation and were therefore unsure of how to engage in dialogic discourse. Even though the classroom teacher facilitated student autonomy, and established social and sociomathematical norms, the students engaged in dialogue as if in a univocal setting. The teacher had provided a dialogic setting, but the students initially participated in a univocal manner. Over time, students slowly transitioned into engagement in more dialogic discourse.

From Externalized to Internalized Mathematical Discourse

The previous section described the interaction between mathematical talk and mathematical discourse. Understanding this interaction is critical when looking at how SDT contributes to describing mathematical discourse. In the section of this paper that described how autonomy, competence, and relatedness are met through mathematical discussion, one may begin to see how meeting these three psychological needs can coincide with dialogic discourse. In the following section, I elaborate on how aspects of SDT not only coincide with types of mathematical discourse but actually facilitate it. To do this, I provide examples from research conducted by Paul Cobb, Erna Yackel, and others that describes aspects of the internalization of

content through dialogue and describe how these examples provide support for the framework that will be presented later in this section.

In an example used in the previous section, Cobb et al. (1992) described two types of classrooms in which one teacher was observed to use more univocal discourse while the other used more dialogic discourse. The teacher that incorporated dialogic discourse engaged their students in much more conceptual talk than the teacher who incorporated univocal discourse. However, what is interesting is the nature of how the conceptual talk was observed to develop. Cobb et al. described the difference in how students in both types of classrooms came about making meaning of mathematics. In the univocal classroom, students were described as understanding mathematics if they could successfully follow procedures. Students in the dialogic classroom understood mathematics when they could “create and manipulate mathematical objects in ways that they can explain and...justify” (p. 598). When focusing on this aspect of Cobb et al.’s findings, it appears that students in the more univocal classroom had understandings based on more external factors. Mainly, these factors are procedures that are typically provided them by external sources such as a textbook or the teacher. Even when these students happen to form procedures of their own, it could still be considered external in nature since these self-created procedures mimic the teacher’s or textbook’s procedures.

When focusing on the understandings of mathematics formed by students in the more dialogic classroom, we see that these understandings are based upon more internal factors. These factors come from mathematical objects that the students themselves create. These autonomous acts are different from the self-created procedures that students in a univocal classroom might develop. In the univocal classroom, procedures need not be explained or justified; unless the teacher directs the discourse in such a way. The procedure itself is the explanation and the fact

that it provides an answer is the justification. Yet in the dialogic classroom it is the process that is explained and justified. Cobb et al. (1992) are careful to note that the teacher and students in the dialogic classroom appeared less concerned about an answer than the process of obtaining a solution. An answer can be seen as a more externally valued aspect of a mathematical problem since it is usually developed to satisfy the teacher. The process, although it can be externally motivated initially, is more internally motivated and valued since it is developed and fostered by the individual.

In a later piece, Yackel and Cobb (1996) elaborated on what constitutes true mathematical discourse. This later study sought to develop a theoretical framework for how student autonomy and effective mathematics discourse related. True mathematical discourse was described as being fundamentally concerned with the process of mathematics as opposed to an answer. Rather than describing this process-oriented talk as being more internal, Yackel and Cobb describe it as providing the student with a certain degree of autonomy. Both descriptions, however, are characteristic of Self-Determination Theory since more autonomous acts are more internalized. Supporting this relationship, Shi (2008) found that eighth graders in Taiwan who reported higher levels of autonomy support were more likely to have higher levels of introjected regulation and intrinsic motivation than students with less autonomy support. Additionally, Liu et al. (2007) also found that autonomy predicted more internally regulated behaviors, and relatedness and competence did so as well. Therefore, it would seem that conceptual talk supports students' autonomy, and conceptual talk's autonomy-supportive nature facilitates more internally regulated behaviors.

In addition to describing the autonomy of students in discussing mathematics, Yackel and Cobb (1996) discussed what they termed sociomathematical norms. These norms are discussed

in context with student autonomy through an attempt to incorporate two different paradigms. Specifically, Yackel and Cobb described mathematical learning as being a process of individual construction and social construction.

Sociomathematical norms, as described by Yackel and Cobb (1996), are specific norms developed by both teacher and students in mathematics classrooms:

Normative understandings of what counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant in a classroom are sociomathematical norms. Similarly, what counts as an acceptable explanation and justification is a sociomathematical norm (p. 461).

Mathematical difference involves teacher and students' understandings as to what constitutes a different type of answer or process in mathematics discussion. Mathematical sophistication, efficiency and elegance are described as aspects that are typically validated by the teacher in some way. Explanations and justifications are also validated by the teacher. Although many of these norms are validated by the teacher, and at times explicitly identified, it is not necessarily a controlling action on the part of the teacher. It is, however, initially an external act on the part of the student, whether in a univocal or dialogic classroom setting. Additionally, these sociomathematical norms can be described as facilitating the need for competence in mathematical discussion, as it provides a set of expectations for students when they are engaging in discourse. As described by Yackel and Cobb, the interaction between student autonomy and sociomathematical norms, as well as social norms (i.e. mutual respect between students, etc.), facilitate the success of the other. This interactive relationship can be likened to the interaction between autonomy, competence, and relatedness. Sociomathematical norms are supportive of

competence and social norms are supportive of relatedness. Therefore, the interaction described by Yackel and Cobb appears to support a relationship between their framework and SDT. Yet, Yackel and Cobb also described a process in which both autonomy and sociomathematical norms interact to produce more and more effective mathematical discourse:

Initially, students' explanations may have a social rather than a mathematical basis. As their participation in inquiry mathematics instruction increases, they differentiate between various mathematical reasons. For example, they distinguish between explanations that describe procedures and those that describe actions on experientially real mathematical objects. Finally, some students progress to being able to take explanations as objects of reflection (p. 467).

These stages are illustrated in Figure 2.2. Although, Yackel and Cobb (1996) did not use a figure in describing these stages, such a figure illustrating the stages they posited was included as a useful visualization of their ideas. The first stage described by Yackel and Cobb involves students who initially came into the observed classroom as having a perception of the teacher and textbook as the authority figures in mathematical reasoning. Only the authority figures could provide effective mathematical reasoning. Therefore, Yackel and Cobb argued that student statements in whole class discussion were initially more socially geared, meaning that they were done to satisfy the teacher or present a certain image to fellow students. Yet the teacher can use such situations to orient these students to statements having a mathematical reasoned basis rather than a socially reasoned basis.

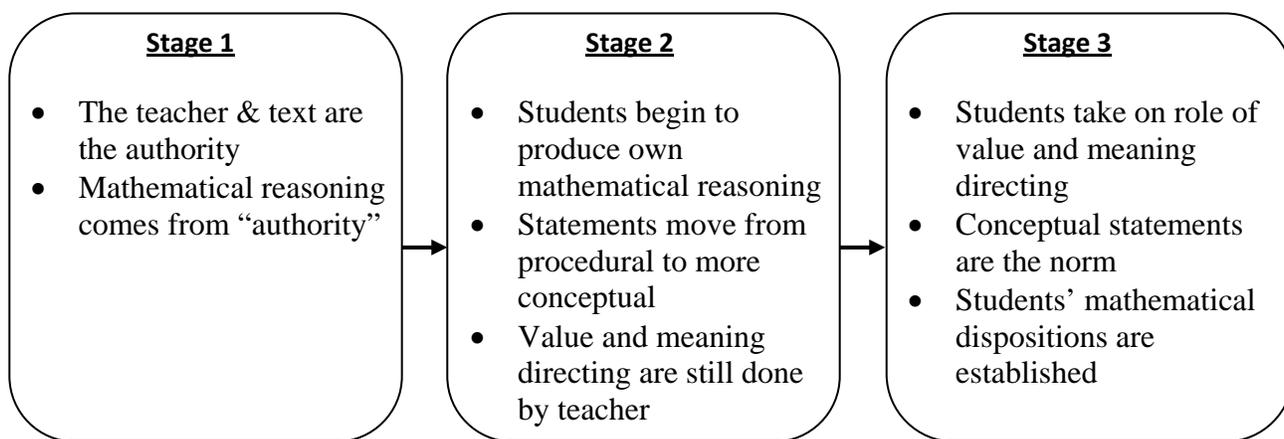


Figure 2.2. Stages of Mathematical Discourse as Advocated by Yackel and Cobb (1996)

The next progression described by Yackel and Cobb (1996) is from procedural to conceptual mathematical reasons. Yackel and Cobb argued that procedural descriptions were preprescribed and were simply carried out by students. They were directions instead of descriptions. However, when students generated “their own personally meaningful ways of solving problems instead of following procedural instructions” (p. 470) they were able to look at such mathematical ideas conceptually and their “explanations were conceptual rather than calculational” (p. 470). At the same time, such explanations and types of explanations were negotiated between other students and the teacher as to their acceptability as a mathematical reason. This process was facilitated by the teacher acknowledging challenges to student procedural statements.

The final stage in the evolution of autonomous discussion described by Yackel and Cobb (1996) occurs when the student’s statements become objects of reflection by the student. Up until this point, such valuing and meaning-directing is done by the teacher. However, Yackel and Cobb stated that students can take this role after mastering previous stages as well as understanding what constitutes a true mathematical reasoned statement. This stage is evidenced by the student trying to make a statement understandable to other students, not just themselves.

These actions are identified as being evident of a mathematical disposition of the student since they are not only advocating a stance on a mathematical situation but attempting to have others understand it as well. By developing their own mathematical disposition, Yackel and Cobb argued that these students have reached a higher level of mathematical autonomy. Yet it is through the process of discussion that such dispositions can be formed.

The specific nature of the described stages in developing mathematical dispositions appears identical to self-regulated behaviors as described by Ryan and Deci (2000). A mathematical disposition, as described by Yackel and Cobb, is equivalent to “intellectual autonomy” in mathematics. Since SDT is posited to facilitate the internalization of content (Deci et al., 1991), a mathematical disposition can be described as a structure of beliefs that is self-determined. Moreover, each stage in Yackel and Cobb’s framework can be described using the transition from external to internal regulation as advocated by SDT.

The first stage in Yackel and Cobb’s (1996) framework describes mathematical reasoning as coming from external sources such as the teacher or text. Students in this stage are characterized as engaging in mathematics discussion for “social” reasons. In SDT, this stage would actually be separated into two phases of regulation. The first is external regulation in which engaging in mathematical discussion would be caused by a purely external source (Ryan & Deci, 2000). The student might engage in discussion because they fear the teacher will punish them if they don’t or because they believe they will get some sort of reward for doing so. The second phase is introjected regulation where students begin to internalize the process of discussion but have yet to accept it as their own process. These students might engage in mathematical discussion because of the pride they feel from giving good statements or the guilt they may feel from not responding.

The second stage described by Yackel and Cobb (1996) describes the student as beginning to produce their own mathematical reasoning but at the same time, any meaning making is still done by the teacher. This is characteristic of identified regulation as described by Ryan and Deci (2000). In this phase, the student has identified with the process by producing their own mathematical reason, but there are still external factors that regulate their behavior; particularly the meaning directing that the student allows the teacher to do for them.

The third stage, which characterizes the establishment of mathematical dispositions, is characteristic of the integrated and internal regulation phases described by Ryan and Deci (2000). It is during these two phases that all external factors are internalized as part of the student's identity. As described by Yackel and Cobb (1996), it is during this phase that students take on the role of value and meaning directing and become intellectually autonomous. Therefore, since they have formed an identity based upon the content, they have fully internalized it. Figure 2.3 illustrates the relationship between Yackel and Cobb's stages and the transition from external to internal regulation described by SDT.

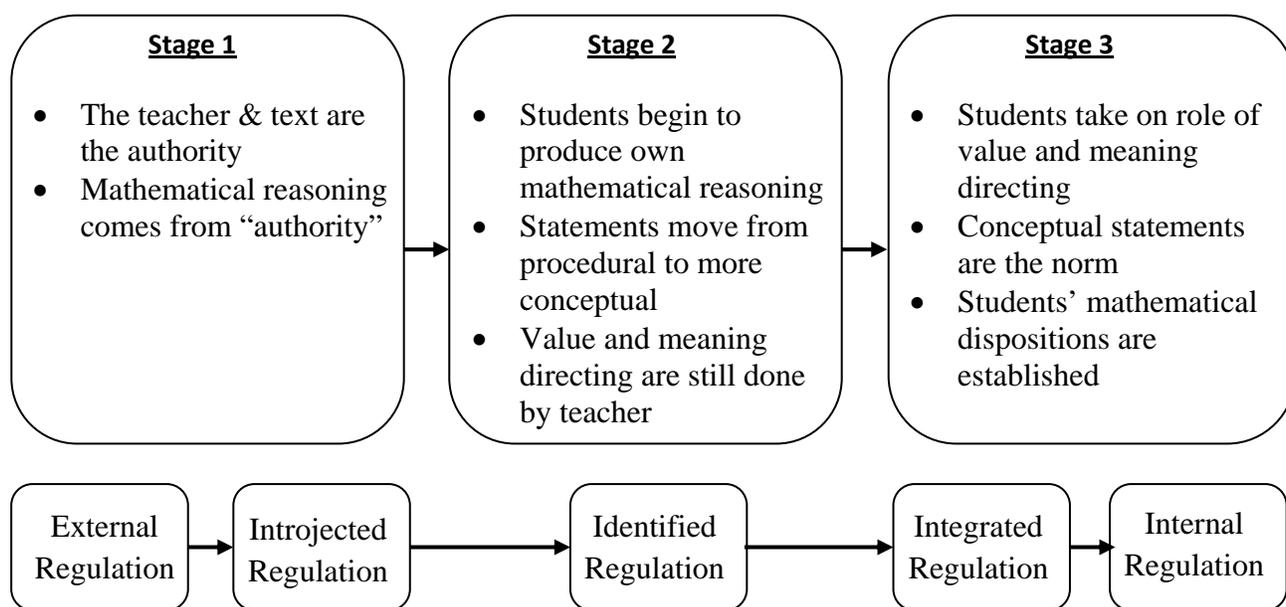


Figure 2.3. Relationship between Yackel and Cobb's (1996) Development of Mathematical Dispositions and Internalization of Regulations.

A Theoretical Framework for Mathematical Discussion

Self-Determination Theory posits that the more a student's needs of autonomy, competence, and relatedness are met, the more internalized their motives for performing certain actions are (Ryan & Deci, 2000). The model of mathematical discussion presented by Yackel and Cobb (1996) suggests that by incorporating social norms, sociomathematical norms, and providing the student with a degree of autonomy, the act of mathematical discussion aids in the development of mathematical dispositions. Termed "intellectual autonomy" by Yackel and Cobb, the mathematical disposition is characteristic of what Ryan and Deci would term a self-determined set of mathematical beliefs. Additionally, students described by Yackel and Cobb as engaging in a more univocal setting, where the teacher and textbook are seen as the source of mathematical reasoning, would be identified as being externally regulated by Ryan and Deci. The transition from univocal to dialogic mathematical discussion described in the previous

section is therefore characteristic of the transition from externally to internally regulated behaviors.

Yackel and Cobb (1996) stated that “the development of individuals' reasoning and sense-making processes cannot be separated from their participation in the interactive constitution of taken-as-shared mathematical meanings” (p.460). This reflexive nature of individual sense-making and their role in mathematical discussion is a critical concept in integrating SDT with Yackel and Cobb’s framework. So, the psychological needs of SDT interact directly with students’ “reasoning and sense-making processes” and are thus influenced by student engagement in mathematical discourse, and vice versa. Therefore, the integration of SDT with Yackel and Cobb’s framework does not focus explicitly on the context of discourse in the classroom, but the student’s *engagement* in discourse.

A teacher who attempts to develop a classroom that facilitates dialogic discourse may have many students who are well adapted to univocal mathematical discourse. Such a student may engage their teacher and classmates in mathematical discourse in a univocal manner, no matter what atmosphere the teacher attempts to develop (Yackel and Cobb, 1996). This student is typically externally regulated. As they become accustomed to the dialogic atmosphere the teacher facilitates, they begin engaging in procedural talk. This is done not because they see any value in it, but because the teacher asks them to engage in it. It is still an externally regulated act. Such a student is incapable of engaging successfully in conceptual talk because all reasoning is provided by the external source of teacher or textbook (Yackel and Cobb, 1996)). For them, the procedural talk they produce is equivalent to justification (Cassel & Reynolds, 2007).

As the student reaches the stage of introjected regulation, they continue to engage in procedural talk. However, the student does not engage in it solely because the teacher tells them

to, but because of the positive social benefits and feedback they receive when they produce it. Transition to introjected regulation is therefore facilitated by the social and sociomathematical norms the teacher constructs with their students. Social and sociomathematical norms, as outlined by Yackel and Cobb (1996), create a positive social environment for students as they engage in discussion and enables them to understand what an acceptable mathematical statement looks like.

The transition from univocal to dialogic mathematical discourse occurs when students' discourse actions are in the phase of identified regulation. Similar to the probing setting described by Hancewicz (2005), the identified regulation stage in mathematical discourse characterizes the student as valuing the process of discussion while not necessarily valuing what is discussed. They may see the teacher as the provider of the questions that lead to mathematical reasoning, which is characteristic of the probing setting identified by Hancewicz. At the same time, the student's actions may become dialogic as they engage in the process of describing and/or justifying mathematics, without necessarily valuing the content discussed. In other words, it is entirely possible for one student who is in identified regulation to participate in discussion in a univocal manner while another student in identified regulation participates in a dialogic manner.

In a previous section, I described how procedural and conceptual talk manifest themselves in univocal and dialogic discourse. As a student becomes more engaged in dialogic rather than univocal discourse, they elicit more conceptual rather than purely procedural talk (Cobb et al., 1992; Yackel & Cobb, 1996). The probing setting outlined by Hancewicz (2005) was described as univocal discourse on the verge of becoming dialogic. Additionally, it is during the probing setting that conceptual talk, as well as procedural talk, may appear in univocal

discourse. Recalling the previous paragraph where the probing setting was described as part of the identified regulation stage, it is apparent that as a student transitions from univocal to dialogic discourse, they initially engage in conceptual talk whether it is univocal or dialogic. In either case, such initial engagement in conceptual talk occurs during the identified regulation stage.

Transition into integrated regulation is represented by the student beginning to develop their own mathematical conjectures and perspectives. The teacher and textbook are no longer the sole source of mathematical reasoning (Yackel and Cobb, 1996). Such reasoning can be produced by the student and their mathematical reasoning is therefore self-determined. Additionally, conceptual talk is commonplace. Procedural talk no doubt still occurs, but the content of dialogue is becoming more conceptual. This process continues when transitioning to internal regulation. However, the distinction between integrated and internal regulation is small (Deci et al., 1991).

Figure 2.4 provides an illustration of the conceptual framework and the processes described in the previous paragraphs. As the student moves from externally to more internally regulated discussion, they participate less univocally and more dialogically. During the phase of identified regulation, the student's engagement in discourse ceases to be univocal and becomes dialogic. As the student moves from left to right in the model, they will begin to produce procedural and then conceptual talk. At times, both types of talk may be used in conjunction or isolation from the other. It is important to note that Figure 2.4 is not meant to discount other forms of talk from occurring nor is it meant to imply that such forms of talk (e.g. regulatory talk as described by Schleppegrell, 2007) are not helpful in facilitating the transition from univocal to dialogic student engagement. Additionally, this model is not meant to represent the context of

discourse in the classroom, but the student's *engagement* in discourse. As previously noted, a teacher may construct a dialogic setting in their classroom and have students who engage in discourse univocally (Yackel & Cobb, 1996).

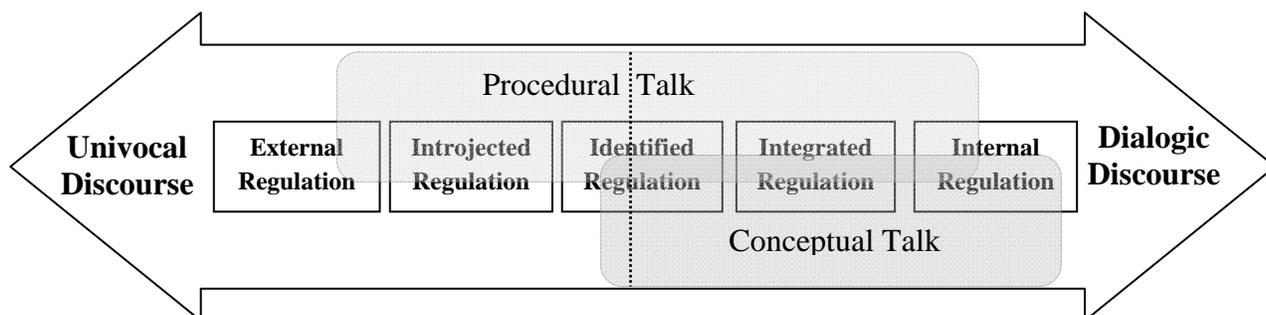


Figure 2.4. Theoretical Framework Relating Mathematical Discussion to Self-Determination Theory

Figure 2.4 represents an adaptation of Yackel and Cobb's (1996) model in context with SDT. In adapting this model, it is important to remember that Yackel and Cobb described the transition to autonomous action as something done by the student in context with certain classroom factors (i.e. social norms and sociomathematical norms). This is also the case with the model represented in Figure 2.4, but specifically representative of the individual within such contexts. As Yackel and Cobb suggested, social and sociomathematical norms are negotiated with the involvement of the individual student as a member of the class. Yet the student chooses to engage in or participate in the social and sociomathematical norms (Yackel and Cobb, 1996). As the student chooses to utilize the social and sociomathematical norms, they internalize mathematics through discussion and become more and more autonomous, or self-regulated.

By expanding Yackel and Cobb's (1996) description of stages for developing mathematical autonomy of students through discussion with the incorporation of concepts from SDT, a more complete explanation of how mathematical discussion facilitates internalization has

been provided. This expanded model describes the internalization of mathematics content through mathematical discussion. However, it also directly links this process of internalization to the student engaging in univocal or dialogic discourse as mathematical content became more internalized. Therefore, the internalization process as identified by Ryan and Deci (2000) was incorporated in the conceptual framework as well. By linking the description of the interaction of types of mathematical talk with the description of internalization of mathematics through discourse, we are able to do two things. First, we are able to describe how students internalize mathematics as they proceed from univocal to dialogic discourse. Second, we are able to see what stages of internalization typically exhibit procedural or conceptual talk.

The last piece of the conceptual framework to be described is not something that can be adequately placed in Figure 2.4. The process of internalization as advocated by SDT proceeds when a student's autonomy, competence and relatedness are met. Furthermore, all three needs must be met for actions to become fully self-determined (Deci et al., 1991; Liu et al., 2007; Ryan & Deci, 2000). By giving students a certain degree of autonomy in how discussions in the classroom proceed, the students are able to take ownership of the dialogue and increase their understanding of the content (Alro & Skovsmose, 2002; Borenson, 1986; Brown & Hirst, 2007; Lee, 2006). Students also need to know how to talk about mathematics (Sfard, 2007) and this need for competence can be facilitated through incorporation of sociomathematical norms as described by Yackel and Cobb (1996). Additionally, certain social norms that foster a sense of relatedness by promoting mutual respect between students and decreasing student fear of failure are also advocated for incorporating effective mathematical discussion (Kazemi & Stipek, 2001; Martin et al., 2005; Wood, 1999). Therefore, Figure 2.4 illustrates the process of an individual student within a classroom where dialogic discourse is encouraged and facilitated. The model

represented in Figure 2.4 suggests that by giving students a degree of autonomy, guidance in how to talk about mathematics (competence), and fostering certain social norms in the classroom (relatedness), internalization of mathematics content through discussion can take place.

Overview and Conclusion

Self-Determination Theory posits that student actions are more internally regulated with the fulfillment of their needs for autonomy, competence, and relatedness (Ryan & Deci, 2000). The characteristics of dialogic discourse, sociomathematical norms, and social norms fulfill these needs and, over time, *facilitate* the individual student's transition to more internalized conceptions of mathematics. As students enter the mathematics classroom they may be more responsive to externally motivating factors when engaging in mathematics discussion. As the teacher, along with the students, develops sociomathematical and social norms that facilitate dialogic discourse, the student makes more conceptual statements about mathematics and internalizes these statements as their own. The final result in such a transition is a student with a distinctly unique mathematical disposition that is self-regulated in their engagement in mathematics and mathematical discussion.

The reflexive nature of the individual with the discourse they engage in necessitates a certain commitment and fulfillment of that individual to such discourse. The model presented here posits that such commitment comes in the form of procedural and conceptual talk produced in specific discourse roles (i.e. univocal and dialogic). The fulfillment of the individual in mathematical discourse is actualized by the manifestation of a mathematical disposition, which is achieved through the fulfillment of their autonomy, competence, and relatedness. Therefore, an individual is committed, or engaged, in mathematical discourse by the statements they make and the roles they take, but simultaneously and reflexively fulfilled by such engagement through the

SDT supportive features of autonomy-supportive practices, sociomathematical norms, and social norms.

A new way of looking at mathematical discussion has been presented. This model provides explanations as to why factors of effective discussion work together to internalize mathematical content knowledge for students. Yet to truly capitalize on the usefulness of this model, it must be tested. One way to examine this model is to study the relationship of the SDT psychological needs within the context of mathematical discourse. Another investigation might examine mathematical statements within discourse settings and compare the results of such analysis to the model presented in this manuscript. Students' self-determination should also be compared with their roles in univocal and dialogic discourse settings. Teachers' incorporation of autonomy-supportive practices, social norms, and sociomathematical norms should also be investigated to determine what specific practices are more supportive of students' psychological needs, and therefore the development of positive, sophisticated mathematical dispositions.

The topics described in the previous paragraph represent a small sample of potential studies to examine the relationships presented in this manuscript. Such studies may not only support the model proposed here, but provide answers that revolutionize the way we view student interactions when they discuss mathematics. Therefore, while the current chapter provides theory on why mathematical discussion facilitates understanding of the content, the actualization of such theory must take place.

References

- Alderman, M. K. (2008). *Motivation for achievement: Possibilities for teaching and learning* (3rd ed.). New York: Routledge.
- Alfi, O., Assor, A., & Katz, I. (2004). Learning to allow temporary failure: Potential benefits, supportive practices and teacher concerns. *Journal of Education for Teaching*, 30(1), 27 – 41.
- Alro, H., & Skovsmose, O. (2002). *Dialogue and learning in mathematics education: Intention, reflection, critique*. Boston, MA: Kluwer Academic Publishers.
- Anderman, E. M., Austin, C. C., & Johnson, D. M. (2002). The development of goal orientation. In A. Wigfield, & J. S. Eccles (Eds.), *Development of achievement motivation* (pp. 197-220). San Diego: Academic Press.
- Bandura, A. (1977). Self-efficacy: Toward a unifying theory of behavioral change. *Psychological Review*, 84(2), 191 – 215.
- Brown, R., & Hirst, E. (2007). Developing an understanding of the mediating role of talk in the elementary mathematics classroom. *Journal of Classroom Interaction*, 41/42(2/1), 18 – 28.
- Black, L. (2004). Teacher-pupil talk in whole-class discussions and processes of social positioning within the primary school classroom. *Language & Education*, 18(5), 347 – 360.
- Black, A. E., & Deci, E. L. (2000). The effects of instructors' autonomy support and students' autonomous motivation on learning organic chemistry: A self-determination theory perspective. *Science Education*, 84(6), 740 – 756.

- Borenson, H. (1986). Teaching students to think in mathematics and to make conjectures. In M. Driscoll & J. Confrey (Eds.), *Teaching mathematics: Strategies that work K-12* (pp. 63 – 70). Portsmouth, NH: Heinemann.
- Cassel, D., & Reynolds, A. (2007). Listening as a vital element of synergistic argumentation during mathematics problem solving. *Focus On Learning Problems in Mathematics*, 29(4), 30 – 47.
- Cobb, P., Wood, T., Yackel, E., & McNeal, B. (1992). Characteristics of classroom mathematics traditions: An interactional analysis. *American Educational Research Journal*, 29(3), 573 – 604.
- D'Ambrosio, B., Johnson, H., & Hobbs, L. (1995). Strategies for increasing achievement in mathematics. In R.W. Cole (Ed.), *Educating everybody's children: Diverse teaching strategies for diverse learners* (121 – 138). Alexandria, VA: Association for Supervision and Curriculum Development.
- Deci, E. L., Betley, G., Kahle, J., Abrams, L., & Parac, J. (1981). When trying to win: Competition and intrinsic motivation. *Personality and Social Psychology Bulletin*, 7(1), 79-83.
- Deci, E. L., Vallerand, R. J., Pelletier, L. G., & Ryan, R. M. (1991). Motivation and education: The self-determination perspective. *Educational Psychologist*, 26(3 & 4), 325 – 346.
- Fairchild, A. J., Horst, S. J., Finney, S. J., & Barron, K. E. (2005). Evaluating existing and new validity evidence for the Academic Motivation Scale. *Contemporary Educational Psychology*, 30(3), 331-358.
- Fey, J.T. (1970). *Patterns of verbal communication in mathematics classes*. New York: Teachers College Press.

- Gadanidas, G., Kotsopoulos, D., & Guembel, K. W. (2006). *Factors effecting middle school teachers' choice of authoritative discourse in teaching mathematics*. In S. Alatorre, J. L. Cortina, M. Saiz, & A. Mendez (Eds.). *Proceedings of the 28th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics & Education*. Merida, Yucatan, Mexico.
- Goos, M. (1995). *How do you know when you understand? Using explanation to monitor and construct mathematical understanding*. Paper presented at the Postgraduate Research Conference Commemorating 50 Years of Educational Studies at the University of Queensland, Brisbane, Queensland, Australia.
- Grouws, D. A. (2004). Mathematics. In G. Cawelti (Ed.), *Handbook of research on improving student achievement* (3rd Ed.) (pp. 160 – 178). Arlington, VA: Education Research Service.
- Hahn, J., & Oishi, S. (2006). Psychological needs and emotional well-being in older and younger Koreans and Americans. *Personality and Individual Differences*, 40(4), 689 – 698.
- Hancewicz, E. (2005). Discourse in the mathematics classroom. In J.M. Kenney (Ed.), *Literacy strategies for improving mathematics instruction* (pp. 72 – 86). Alexandria, VA: Association for Supervision and Curriculum Development.
- Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal*, 30(2), 393 – 425.
- Hoffmann, A. J. (2004). *Motivational beliefs and goals of middle school students in discussion-oriented mathematics classrooms*. Paper presented at the annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Toronto, Ontario, Canada.

- Jansen, A. (2008). An investigation of relationships between seventh-grade students' beliefs and their participation during mathematics discussions in two classrooms. *Mathematical Thinking & Learning, 10*(1), 68 – 100.
- Katz, I., & Assor, A. (2007). When choice motivates and when it does not. *Educational Psychology Review, 19*(4), 429 – 442.
- Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal, 102*(1), 59 – 80.
- Kitchen, R. S. (2004). Challenges associated with developing discursive classrooms in high-poverty, rural schools. *Mathematics Teacher, 97*(1), 28 – 31.
- Kosko, K. W., & Miyazaki, Y. (2010). *The impact of student discussion on fifth graders' mathematics achievement in the U.S.* Manuscript submitted for publication.
- Kotsopoulos, D. (2008). Beyond teachers' sight lines: Using video modeling to examine peer discourse. *Mathematics Teacher, 101*(6), 468 – 472.
- Lee, C. (2006). *Language for learning mathematics: Assessment for learning in practice.* New York: Open University Press.
- Levinson, M. (1999). *The demands of liberal education.* Oxford, UK: Oxford University Press.
- Liu, W. C., Tan, O. S., Wang, C. K. J., Koh, C., & Ee, J. (2007). Motivation in the context of project work: The self-determination perspective. In D. M. McInerney, S. Van Etten, & M. Dowson (Eds.), *Standards in education.* Charlotte, NC: Information Age Publishing, INC.
- Martin, T., McCrone, S., Bower, M., & Dindyal, J. (2005). The interplay of teacher and student actions in the teaching and learning of geometric proof. *Educational Studies in Mathematics, 60*(1), 95 – 124.

- Mendez, E. P., Sherin, M. G., & Louis, D. A. (2007). Multiple perspectives on the development of an eighth-grade mathematical discourse community. *The Elementary School Journal*, *108*(1), 41 – 61.
- Mercer, N., & Littleton, K. (2007). *Dialogue and the development of children's thinking: A sociocultural approach*. London: Routledge.
- Mercer, N., & Sams, C. (2006). Teaching children how to use language to solve maths problems. *Language and Education*, *20*(6), 507 – 528.
- Minnaert, A., Boekaerts, M., & De Brabander, C. (2007). Autonomy, competence, and social relatedness in task interest within project-based education. *Psychological Reports*, *101*(2), 574 – 586.
- Miserandion, M. (1996). Children who do well in school: Individual differences in perceived competence and autonomy in above-average children. *Journal of Educational Psychology*, *88*(2), 203 – 214.
- Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. New York: Routledge & Kegan Paul.
- Reeve, J. (2005). *Understanding motivation and emotion*. Hoboken, NJ: Wiley.
- Reeve, J. (2006). Teachers as facilitators: What autonomy-supportive teachers do and why their students benefit. *The Elementary School Journal*, *106*(3), 225 – 236.
- Reeve, J., & Jang, H. (2006). What teachers say and do to support students' autonomy during a learning activity. *Journal of Educational Psychology*, *98*(1), 209-218.
- Reeve, J., & Sickenius, B. (1994). Development and validation of a brief measure of the three psychological needs underlying intrinsic motivation: The AFS scales. *Educational and Psychological Measurement*, *54*(2), 506 – 515.

- Ryan, R. M., & Deci, E. L. (2000). Self-determination theory and facilitation of intrinsic motivation, social development, and well-being. *American Psychologist*, 55(1), 68-78.
- Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly*, 23(2), 139 – 159.
- Sfard, A. (2007). When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from a commognitive standpoint. *Journal of the Learning Sciences*, 16(4), 565 – 613.
- Shi, S. S. (2008). The relation of self-determination and achievement goals to Taiwanese eighth graders' behavioral and emotional engagement in school. *Elementary School Journal*, 108(4), 313 – 334.
- Springer, G. T., & Dick, T. (2006). Making the right (discourse) moves: Facilitating discussions in the mathematics classroom. *Mathematics Teacher*, 100(2), 105 – 109.
- Stein, C. C. (2007). Let's talk: Promoting mathematical discourse in the classroom. *Mathematics Teacher*, 101(4), 285 – 289.
- Stipek, D. (2002). *Motivation to learn: Integrating theory and practice*. Boston: Allyn and Bacon.
- Truxaw, M. P., & DeFranco, T. C. (2007). Lessons from Mr. Larson: An inductive model of teaching for orchestrating discourse. *Mathematics Teacher*, 101(4), 268 – 272.
- Turner, J. C., Meyer, D. K., Midgley, C., & Patrick, H. (2003). Teacher discourse and sixth graders' reported affect and achievement behaviors in two high-mastery/high-performance mathematics classrooms. *The Elementary School Journal*, 103(4), 357 – 382.

- Van der Aalsvoort, G. M., Harinck, F. J. H., & Gosse, G. (2006). Script identification: A way to understand how students choose strategy during mathematization. *European Journal of Special Needs Education, 21*(3), 301 – 319.
- Weiner, B. (2000). Intrapersonal and interpersonal theories of motivation from an attributional perspective. *Educational Psychology Review, 12*(1), 1-14.
- Williams, S. R., & Baxter, J. A. (1996). Dilemmas of discourse-oriented teaching in one middle school mathematics classroom. *The Elementary School Journal, 97*(1), 21 – 38.
- Wolters, C. A. (2004). Advancing achievement goal theory: Using goal structures and goal orientations to predict students' motivation, cognition, and achievement. *Journal of Educational Psychology, 96*(2), 236-250.
- Wood, T. (1999). Creating a context for argument in mathematics class. *Journal for Research in Mathematics Education, 30*(2), 171 – 191.
- Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458 – 477.

Chapter Three

Self-Determination Theory and the Frequency of Students' Explanation in Mathematics

Classrooms

Introduction

Empirical evidence suggests benefits in having students explain and justify their mathematical ideas (Hiebert et al., 2005; Schleppenbach et al., 2007a; Schleppenbach et al., 2007b). Hiebert and Wearne (1993) found that when second grade students were asked to describe, explain, and analyze mathematical statements and concepts they had greater gains in mathematical achievement. In another study of an effective mathematics teacher, Leinhardt and Steele (2005) found that the teacher and students often shared the construction of definitions and explanations. Students were asked to justify their positions throughout the process of these shared constructions. Wood (1999) also identified the characteristics of explanation and justification as integral in effective dialogue in her observations of a second grade teacher. Therefore, by focusing on student descriptions and explanations, teachers can encourage deeper understanding of mathematical concepts and thereby increase mathematics achievement (Hiebert & Wearne, 1993).

The empirical evidence presented above would suggest that requiring explanation and justification of mathematics students fosters effective discussion. Yet a common misconception about effective discussion in mathematics is that it is equivalent to simply not giving students the answer (Stein, 2007). It is more accurately described as a complex interaction of the teacher with the students where the teacher posits questions that lead students to understanding. “questioning of students allows their responses to enter the classrooms’ discourse space to be assessed and

built on by others” (Hufford-Ackles, Fuson, & Sherin, 2004, p. 92). It encourages students to think more deeply about topics by getting them to defend, explain, and justify their answers.

The main types of question use primarily addressed in the mathematics discourse literature include descriptions, justifications, and explanations (Dalton-Puffer, 2006; Goos, 1995; Hufford-Ackels et al., 2004; Lee, 2006; Pimm, 1987; Truxaw & DeFranco, 2007). Typically, investigations in mathematical discourse focus solely on justification or a combination of justification and explanation. The focus of the current chapter is on mathematical explanations only. However, rather than focusing on what student engagement in explaining mathematics can do, this chapter seeks to investigate whether certain psychological factors predict engagement in explaining mathematics.

Much of the literature about mathematical discussion cites social norms and student autonomy as components of effective discourse (e.g. Alro & Skovsmose, 2002; Turner, Meyer, Midgley, & Patrick, 2003; Sfard, 2007; Williams & Baxter, 1996; Wood, 1999). These characteristics can also be described as effective pedagogy through the lens of Self-Determination Theory of Motivation (SDT) as posited by Ryan and Deci (2000). This theory and its connections to mathematical explanation will be outlined in the following review of the literature. Additionally, it is this connection that will be outlined in the following section that will be tested in this chapter.

As stated above, the focus of the current chapter is on student explanation of mathematics. Rather than examining the effects of explanation on student learning or achievement, the first study of this chapter seeks to examine explanation as an outcome with aspects of Self-Determination Theory as predictors. These aspects are the psychological needs of autonomy, competence, and relatedness which Ryan and Deci (2000) state facilitate

internalization of content and actions. Therefore, the primary purpose of this chapter is to determine whether or not the combined impact of autonomy, competence and relatedness positively predicts the frequency students engage in explaining mathematics in their classroom. A secondary purpose of this chapter is to evaluate the validity of variables used in the first study. Therefore, a second study is included in the current chapter for this validation.

Overview of SDT

Theory concerning motivation focuses on why people act in certain ways. In other words, what causes behavior? What sustains, stops or alters it? Why do people do what they do and why is there such variation in the intensity of their actions? These types of questions are the driving force behind understanding motivation. It is from questions such as these that motivation theory has developed (Reeve, 2005; Stipek, 2002).

Although theories on motivation can be traced back to Plato and other ancient Greeks, current motivational theory evolved from previous failed attempts at construction of grand theories in motivation. Grand theories attempted to explain all aspects of motivation under a unifying umbrella of a particular causal agent (Reeve, 2005). Partly because of the inability of these grand theories to explain many of the behaviors they were supposed to predict, there was a shift over the course of the 20th Century in motivation theory from behavioral models to more cognitive models (Pintrich & Schunk, 1996). Therefore current motivation theory is comprised of numerous mini-theories which explain parts or aspects of motivation (see p. 3, Reeve, 2005 for further information). It is in the context of one particular micro-theory, Self-Determination Theory, that this chapter examines mathematical explanations of students.

SDT was originally posited by Edward L. Deci and Richard M. Ryan (Deci, Vallerand, Pelletier, & Ryan, 1991). Actions that are self-determined are those that the individual has the

most power over. Therefore, self-determination can be described as “the process of using one’s capacity to choose how to satisfy one’s needs” (Winne & Alexander, 2006, p. 359).

SDT is best described with three psychological needs: relatedness, competence, and autonomy. Relatedness describes the degree to which satisfying social connections are made. Competence describes the degree to which the individual feels able to accomplish different external and internal tasks. Autonomy describes the degree to which the self is in control of initiating and maintaining different behaviors (Deci et al., 1991; Ryan & Deci, 2000). Although any of these three needs can and does develop independently of the others, it is the combination of all three needs that facilitates internalization of actions, and thus intrinsic motivation. More specifically, it is the interdependence of each need on the other that requires all three be met in order for motivation to be self-determined (Deci et al., 1991; Katz & Assor, 2007; Reeve, 2006).

Mathematical Explanation in Context with SDT

The description of Self-Determination Theory above is by no means comprehensive (see Ryan & Deci, 2000 for a full description of SDT). It was not meant to be. Rather, it is meant to provide an overview of a motivation theory that appears to explain why students engage more in explanation. In the following section, I outline the relationship between explanation in mathematics classrooms and the three psychological needs posited by SDT.

In much of the mathematics communication literature, there appears to be two types of teachers. There are those who provide students opportunities to explain their mathematics and then there are those who choose to do the explaining themselves. This is characterized in interviews and observations of teachers by Casa and DeFranco (2005). While interviewing two veteran teachers, Casa and DeFranco found that one teacher believed in using discussion in the classroom to get students involved in explaining mathematical concepts as well as assessing their

current level of understanding. The other veteran teacher believed that students needed to have the content explained to them. More accurately, the second teacher believed it was their job to provide the correct explanation for students. SDT research would refer to the first teacher as supportive of student autonomy and the second veteran teacher as controlling (Reeve & Jang, 2006; Ryan & Deci, 2000). Autonomy support implies providing opportunities and experiences for individuals to make decisions and take some measure of control (Reeve & Jang, 2006; Katz, 2007). Since the second veteran teacher interviewed by Casa and DeFranco chose to give explanations to students, she denied the students the opportunity to take control or ownership of their own mathematics.

Similar to Casa and DeFranco (2005), Turner et al. (2003) found differences in the amount of explanation expected of students and the degree to which teachers were autonomy supportive. Turner et al., however, specifically looked for autonomy supportive characteristics. One teacher used autonomy supportive language significantly less than the other. Turner et al. described the difference between the two teachers such that one would focus on explaining the content to the students while the other, autonomy-supportive teacher, required more explanation from their mathematics students. The findings based on these observations imply that students are more likely to be the ones to explain and describe mathematics rather than the teacher when they are in a mathematics classroom that supports their need for autonomy.

Pimm (1987) stated that “it is uncommon for pupils to be asked how they attempted to solve a problem, rather than for the answer they obtained. They are provided with few opportunities to practice giving explanations and justifications...” (p. 43). Simple answers require little detail. However, when asking students to “explain,” it prompts them to give descriptions and details rather than shortened, circular responses. It therefore fosters more

complex thinking (Dalton-Puffer, 2006). Because the simpler answers require such little detail from the student, any explanation comes from the teacher or textbook (Yackel & Cobb, 1996). It is therefore external to the individual as well as less autonomous. When students are expected to provide explanations or descriptions, students produce their own mathematical reasoning.

Mathematical statements become more sophisticated and autonomous. In other words, having students provide explanations facilitates the development of mathematical dispositions, which Yackel and Cobb describe as characteristic of mathematical autonomy.

In addition to the examples given above, the idea of autonomy and student explanation of mathematics as being inherently linked can be found in various literature (e.g. Brown & Hirst, 2007; Cassel & Reynolds, 2007; Choppin, 2007; Hiebert & Wearne, 1993). However, it should be noted that supporting the need for autonomy will not, in and of itself, exhibit student explanation in mathematics. In a study of mathematical discourse in a middle school classroom conducted by Sfard (2007), students actually provided little effective explanation in an autonomy supportive classroom. Investigating why the lesson engaging students in dialogue failed, Sfard stated that the teacher relinquished too much control to the point that the students did not know the rules of discourse that were to be employed. In other words, they didn't know how they were expected to talk about the mathematics.

The lack of rules for discourse, as described by the example from Sfard's (2007) study, is characteristic of a lack in competence. Remembering that the psychological need for competence reflects a student's belief that he/she can complete an activity or action, the action that students in Sfard's observed classroom could not complete was discussion of mathematics. So called rules for discourse go by many names. Yackel and Cobb (1996) term such rules as sociomathematical norms. Yackel and Cobb identify a sociomathematical norm for explanation as what counts as an

acceptable explanation of the mathematical concept. Development of an understanding of what an acceptable mathematical explanation looks like is done gradually with the use of feedback from the teacher and students. Ryan, Connell, and Deci (1984) describe feedback as supportive of competence when it is positive, effective, and relevant concerning one's performance at an activity. "However, negative feedback in the form of critical messages, statements of failure, and communication conveying incompetence will tend to undermine..." (p. 17) any benefits of feedback.

In addition to fulfilling the psychological needs of autonomy and competence, relatedness must also be maintained if students are to provide their own mathematical explanations. The psychological need of relatedness, which focuses on social relationships, can be fulfilled with proper implementation of social norms in the mathematics classroom. Many students who do not participate in mathematical discussion fear social penalties (Hoffman, 2004; Jansen, 2008). Yet it is possible for a teacher to create an atmosphere where students do not need to have these fears. Martin, McCrone, Bower, and Dindyal (2005) observed that students were willing to provide statements and arguments even if what they said was rejected. The students in the classroom were comfortable in participating and offering their ideas readily. Kazemi and Stipek (2001) observed four classrooms with similar positive social environments. Wood (1999) also studied implementation of discussion and outlined how the observed teacher established social norms to protect students who may be fearful of giving a wrong answer. The focus of discussing mathematics was placed on justifying and explaining the answer itself and not its correctness. Therefore, by focusing on the content of a student's mathematical explanation a teacher can facilitate a student's need for relatedness.

As is apparent from the descriptions above, much of what exists in the mathematics communication literature that discusses explanation explicitly describes it in context with autonomy. However, it is also apparent from the above literature that fulfilling the psychological need of autonomy does not necessarily guarantee that students will participate in effective discourse. The implication from the literature above is that, as is suggested by Ryan and Deci (2000), all three psychological needs must be met for students to actively engage in meaningful mathematical discussion, and therefore, provide meaningful mathematical explanations.

Summary

Mathematical explanations are an essential component of effective mathematical discussion (Lee, 2006; Pimm, 1987; Wood, 1999). Self-Determination Theory provides a possible explanation as to what types of individual processes foster mathematical explanation, and vice versa. Because most teachers still do not implement mathematical discussion effectively, despite its benefits (Pimm, 1987), a better understanding of the primary factors that make implementation effective could potentially change implementation of mathematical discussion in the classroom. The primary purpose of this chapter as outlined by the literature is to determine whether or not the frequency students engage in explaining mathematics in their classroom positively related to their perceived needs of autonomy, competence and relatedness. This primary purpose is examined within the first study presented using data from the Trends in International Mathematics and Science Study (TIMSS). Yet, a secondary purpose of this chapter focuses on examining the validity of items used from TIMSS. Therefore, the second study used a separate sample and additional items to examine the validity of the TIMSS items used in the first study. The questions for the first and second study are outlined below:

Research Questions, First Study

1. Is there a relationship between the frequency eighth grade students report they explain their mathematical answers and their reported autonomy, competence, and relatedness?
2. Can the relationship between SDT psychological needs and student frequency of explanation be modeled by regressing frequency of explanation on each of the SDT psychological needs (i.e. autonomy, competence, relatedness)?
3. Do the relationships between the frequency eighth grade students explain their mathematical answers and their perceived SDT needs vary significantly between classrooms?
4. Do the classroom level factors of small group use and average frequency of students explaining mathematics in the classroom have an effect on the relationship between individual students' frequency of explanation and their SDT needs?

Research Questions, Second Study

1. Is there a relationship between SDT needs measured by the TIMSS items and SDT needs measured by items adapted from Deci and Ryan (2008)?
2. Is there a relationship between the TIMSS item measuring frequency of mathematical explanation and non-TIMSS items measuring frequency and quality of mathematical discussion?
3. Are the relationships found between TIMSS items and constructs statistically and meaningfully different from the relationships of non-TIMSS items and constructs?

Study 1: Methods

Sample

This study used data from the Trends in International Mathematics and Science Study 2003 (TIMSS 2003). TIMSS is a study conducted by the International Association for the Evaluation of Educational Achievement (IEA) and is conducted approximately every four years. Data were collected from approximately 50 different countries at the fourth grade level, eighth grade level, or both (IEA, 2008). Mathematics and science achievement scores are collected along with information from students, teachers, and school officials in an effort to understand how different factors influence the “quantity, quality, and content of instruction” (p. 1, IEA, 2008).

While TIMSS 2003 collected data from almost 50 countries and from both fourth and eighth grade, the current study used only data from eighth grade students in the United States. The eighth grade sample was collected using a stratified sampling technique. First, all schools in the U.S. that had an eighth grade class were included in the sampling population. In the next step 301 schools were selected, of which 232 agreed to participate in the study. Following selection of the schools, two classrooms per school were randomly selected for inclusion in the study. If a classroom had less than 15 students in it, it was included as a pseudo-classroom and therefore combined with another classroom to make sure that no class had too few students. All students within a sampled classroom were included within the study. Approximately 1090 teachers were sampled in the study with 957 completing questionnaire forms. Of the 9891 students sampled, 8912 were assessed. Student attrition was typically due to absence or the student no longer being in the class (Kastberg, Roey, Williams, & Gonzales, 2006).

Measures

Dependent Variables

Three different dependent variables were used to address the psychological needs posited by SDT. Aspects of SDT were not purposely measured by TIMSS. However, several variables were selected from the student questionnaire representing student reported competence in mathematics, mathematical autonomy, and general relatedness.

Five items were found that appear to relate directly to student competence in mathematics. Students were asked how much they agreed with certain statements about learning mathematics (*1=agree a lot, 2=agree a little, 3=disagree a little, 4=disagree a lot*). These items were:

- I usually do well in mathematics;
- Mathematics is more difficult for me than many of my classmates;
- Sometimes, when I do not initially understand a new topic in mathematics, I know that I will never really understand it;
- Mathematics is not one of my strengths;
- I learn things quickly in mathematics

(Kastberg et al., 2006, p. A-62).

Each of these variables address the degree to which the students feel able to accomplish mathematical tasks, which is the very nature of competence as described by Deci, et al. (1991). This suggests that the selected items have sufficient face validity. The scores of all five items were averaged to create a composite score representing student mathematical competence (*competence*). As a higher number on the second, third, and fourth question would represent a higher sense of competence, the first and fifth questions were reverse coded before being

averaged with the other three items. The score of *competence* was tested for reliability using Cronbach's alpha ($\alpha = .824$). Therefore, at least 82.4% of the total score variance is due to the true score variance for *competence*. An alpha coefficient of .70 or higher is generally accepted as reliable in the social sciences (e.g. de Vaus, 2002; Nunnally, 1978). So the composite score for *competence* was accepted as a reliable measure of the psychological need of competence for the current study.

Two questions assessing relatedness and one assessing autonomy were selected. The questions assessing relatedness asked students how much they agreed with the statements:

- I think that most teachers in my school care about the students;
- I think that most teachers in my school want students to do their best

(Kastberg et al., 2006, p. A-69).

Relatedness describes the degree to which satisfying social connections are made with students, teachers, parents, etc. (Deci et al., 1991; Ryan & Deci, 2000). These variables specifically address a student's relatedness with their teacher(s). Although relatedness pertains not just to teachers but also an individual's relationships with other students and their parents, only teacher relatedness was assessed in TIMSS.

The coding on the items addressing relatedness is identical with that of the competence items. In order to have higher numbers representing higher degrees of relatedness, both variables were reverse coded before being averaged together for a composite score (*relatedness*). The score for the composite variable *relatedness* was also tested for reliability using Cronbach's alpha. A score of $\alpha = .815$ was obtained, which represents an acceptable level of reliability for *relatedness* scores.

The question that assessed autonomy on the student questionnaire asked students how often “We decide on our own procedures for solving complex problems” (Kastberg et al., 2006, p. A-70). Students selected from the following choices in answering the item: *1=ever or almost every lesson, 2=about half the lessons, 3=some lessons, 4=never*. Since Deci et al. (1991) and Ryan and Deci (2000) identify autonomy as the degree to which the self is in control of initiating and maintaining different behaviors, a student’s ability to decide on their own mathematical procedures would meet such a description. The variable’s score was reverse coded to ensure that a higher score represented a higher sense of autonomy. The new variable is identified as *autonomy*.

Independent Variables

The independent variables used in Study 1 were created from a single item. Students were asked how often, “we explain our answers” (Kastberg et al., 2006, p. A-70) in their math class (*1=ever or almost every lesson, 2=about half the lessons, 3=some lessons, 4=never*). Since explanation of mathematics is considered an important part of effective mathematical discourse (Hiebert & Wearne, 1993; Leinhardt & Steele, 2005; Wood, 1999), this variable was selected to represent an aspect of mathematical discussion. Furthermore, since student responses were self-reported frequencies, the mathematics explanation variable was dummy coded to compare each new variable (*exp_Often, exp_Half, exp_Some*) with students who never explain mathematics (*exp_Never*)¹.

¹ More frequent student explanation of mathematics does not necessarily represent better or more effective explanation (Kosko & Wilkins, 2008). However, students who explain their mathematics generally have higher achievement than students who never explain their mathematics.

Covariates

Student Level

Covariates included at the student level included gender and race/ethnicity. The variable assessing gender was dummy coded with males as the reference group (*d_Female*). The variable assessing race/ethnicity was dummy coded with whites as the reference group for blacks, Hispanics, Asians, and Native Americans (*d_Black*, *d_Hispanic*, *d_Asian*, *d_NatAm*).

Another covariate included at the student level asked students how frequently “we work together in small groups” (Kastberg et al., 2006, p. A-70). Student responses (*1=ever or almost every lesson*, *2=about half the lessons*, *3=some lessons*, *4=never*) were dummy coded so that students who stated they never worked in small groups were the reference group (*Grps_Often*, *Grps_Half*, and *Grps_Some*). Although not in and of itself a type of mathematical discussion, small groups are a situation where such talk can take place. Additionally, it is an atmosphere where a sense of relatedness can potentially be fostered. It is for these reasons that this item was included as a student level covariate.

Classroom Level

Covariates at the classroom level included the classroom mean of students’ frequency explaining their mathematics (*class_explain*) and the teacher reported frequency that students worked in small groups. The mean level of frequency for explanation was constructed from reverse coding the student level item for explaining mathematics and aggregating it for the classroom. Therefore, a higher mean represents higher student reported engagement in explanation for the classroom. The mean student reported frequency of engaging in explanation was used instead of a teacher level variable that assessed how often the teacher asked students to explain their mathematics. This was done because there was no guarantee that the teacher asking

students to explain mathematics would result in individual students actually explaining mathematics.

Contrary to the decision regarding the level-2 variable for explanation, the teacher reported frequency of small groups was used since a teacher asking students to work in small groups will typically result in students being physically placed in small groups. It is important to note that this classroom level variable does not report student activity in the small group setting, but merely the frequency they are physically in a small group setting.

The variable for small group settings in the classroom was dummy coded so as to compare classroom contexts where students were assigned to small groups (*TchrGrps_Often*, *TchrGrps_Half*, *TchrGrps_Some*) to contexts in which they were never placed in small groups (*TchrGrps_Never*). The frequency other students engage in explanation and how often students work in small groups may demonstrate contextual factors that interact with how frequently the individual student explains mathematics and therefore may impact one or more of the psychological needs being investigated in this study.

Study 1: Analysis and Results

Analysis

Correlation Analyses

Pearson correlations were calculated as an initial step in examining the relationship between eighth grade students' frequency in explaining their mathematics with their SDT needs (i.e. autonomy, competence, relatedness). Therefore, the variables, *autonomy*, *competence*, and *relatedness* were correlated with the small groups and explanation variables to see if a statistically significant relationship existed between the frequency students explain mathematics, the frequency they state they work in small groups, and the degree to which their needs of

autonomy, competence, and relatedness. A second correlation analysis was performed for the classroom level variables to determine if there was a statistically significant relationship between the mean frequency students within a class explain their mathematics and how often they are placed into small groups by their teacher.

Hierarchical Linear Modeling

Hierarchical Linear Modeling techniques were used to evaluate the degree to which the frequency of explanation affected the degree to which each of the psychological needs was met. The between-classroom variance in the frequency of explanation variables was also examined. Finally, the classroom level variables were examined to evaluate whether they had an effect on the student level variables for explanation and the class average for each psychological need that was modeled. By conducting this last step in modeling, the contexts of the average student frequency in explaining mathematics as well as the frequency students worked in small groups could be examined with respect to how such contextual factors affected individual students' frequency of explaining mathematics. More specifically, the effect of how such individual engagement in explaining predicted specific SDT needs could be compared in regards to different classroom contexts (i.e. mean explanation and frequency of small groups).

As mentioned above, the current study uses a hierarchical linear model, or HLM (Raudenbush & Bryk, 2002). Hierarchical linear models have been used in evaluating data nested in many groups. Since much of what is studied in education exists in hierarchical structures (e.g. students within classes, teachers within schools) HLM is particularly useful in studying differences within and between nested units (Burstein, 1980). The current analysis uses a two level hierarchical linear model (HLM-2), which incorporates two levels of analysis simultaneously. The first level, or level-1, focuses on the micro-units in the data such as students.

These micro-units are nested or grouped within macro units. The second level, or level-2, represents the macro units, such as classrooms, which the micro units are nested within. HLM-2 allows factors examined at the first level to be compared at the second level unit of analysis which they are nested in. Additionally, HLM-2 allows the interaction between factors at level-2 and level-1 to be examined (see Raudenbush & Bryk, 2002 for further information).

Hierarchical linear modeling uses a regression equation for each classroom to model student data. Only variables associated with the student are used in the level-1 regression equation. Therefore, variables associated with the classroom are not included at this level. Level-2 uses the coefficients from level-1 as outcomes for the regression equations at level-2. Since level-2 is associated with the classroom, it is here that variables associated with the classroom are placed. Therefore, the impact of classroom-level variables on the slopes or coefficients of individual level variables within a group can be examined (Bryk & Raudenbush, 1986; Raudenbush & Bryk, 2002).

Procedures of HLM Analysis

Unconditional Model

HLM analysis of the data occurred in stages. First, an unconditional model was estimated to examine the intraclass correlation (ICC), or variance accounted for by macro-units, by excluding the independent variables and covariates. An overview of this model is presented below:

Level-1:

$$(SDT_{A,C,R})_{ij} = \beta_{0j} + r_{ij}$$

Level-2:

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

Within the model shown above, β_{0j} represents the average level of autonomy, competence, or relatedness for students within classroom j . β_{0j} is modeled at level to by the sum of the control group mean, γ_{00} , and the classroom error, u_{0j} . Since, in the above model, no independent variables are included, γ_{00} is best interpreted as the mean of the whole sample.

Baseline Model

After examination of the unconditional model, a baseline model was constructed to examine the effects of the independent variables before including covariates at either level-1 or level-2. The slopes of the mathematical explanation variables were set to vary randomly between classrooms at level-2 since the contextual nature of the classroom may have an impact on the frequency students explain mathematics. Because each classroom may have any number of factors that affect the classroom environment, such effects are considered random and are best measured by setting the slope of the explanation variables (*exp_Often*, *exp_Half*, *exp_Some*) as random. Additionally, the development of a baseline model allows for examination of the variables of interest before covariates are included in the model, which is a standard practice in other forms of statistical analysis. The level-1 and level-2 equations of the baseline model are presented below:

Level-1:

$$(SDT_{A,C,R})_{ij} = \beta_{0j} + \beta_{1j}(exp_{Often})_{ij} + \beta_{2j}(exp_{Half})_{ij} + \beta_{3j}(exp_{Some})_{ij} + r_{ij}$$

Level-2:

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + u_{1j}$$

$$\beta_{2j} = \gamma_{20} + u_{2j}$$

$$\beta_{3j} = \gamma_{30} + u_{3j}$$

Within the model above, β_{0j} represents the average level of autonomy, competence, or relatedness for students who never explain their mathematics. β_{1j} represent the association between students who explain their mathematics often with their psychological need of autonomy, competence, or relatedness. Similarly, β_{2j} , and β_{3j} represent this association for students who explain their mathematics ‘half of the lessons’ and ‘some of the lessons,’ respectively. At level-2, γ_{00} represents the control group mean. γ_{10} , γ_{20} , and γ_{30} represent the mean effect of each frequency of explanation for classroom j . u_{0j} , u_{1j} , u_{2j} , and u_{3j} each represent the classroom error for each effect modeled in level-1.

Final Model

After the baseline model was developed, covariates were added at level-1. These covariates included dummy coded variables for frequency of small group involvement, race/ethnicity, and gender. Incorporation of the covariates in this manner allowed for the intercept at level-1 to be interpreted as the amount of discussion for a particular white, male student who never is asked to work in small groups in math class. In order to simplify the model, all covariates were fixed at level-2.

Once covariates were added at level-1, a fourth model, the final model, was constructed by including variables at level-2. As stated before, these variables included the mean classroom frequency students explained mathematics (*class_explain*) and the use of small groups within the classroom as reported by the teacher (*TchrGrps_Often*, *TchrGrps_Half*, *TchrGrps_Some*). These level-2 variables were used to explain any statistically significant variance in *exp_Often*, *exp_Half*, and *exp_Some* between classrooms. Therefore, they were added to the equation for the coefficient of the intercept and the explanation variables. For the simplification of the model, explanation of variability in other level-1 variables was not attempted. Only the variables of

interest for the current study were examined at level-2 (i.e. explanation variables). Additionally, incorporation of level-2 covariates was done to observe any interactions between the contextual classroom effects of average classroom engagement in explaining mathematics and the frequency the teacher said students worked in small groups on how frequently the individual student explained mathematics. This interaction was also observed on the intercept of each psychological need and therefore helped describe any contextual affects these two classroom factors had on the facilitation of the individual student's psychological needs. The level-1 and level-2 equations for the final model are presented and described in detail in the following sections.

Level 1 – Student Level

Three HLM models were constructed; one for each SDT need (autonomy, competence, relatedness). Although each variable representing a psychological need was modeled separately, current Self-Determination Theory posits that all three needs must be met for actions to be fully self-determined (Deci et al., 1991; Katz & Assor, 2007; Reeve, 2006). Specifically, the effectiveness and degree to which each of the needs is fulfilled is interdependent on the other two. However, no SDT literature currently describes the specific degree to which each psychological need interacts with the other. Therefore, each psychological need was examined independently and a composite score for self-determination was not used.

Each of the three models was identical except for the outcome measure. The first level of each HLM model used the student as the unit of analysis. Level-1 modeled student autonomy, competence, or relatedness as a function of student frequency in explaining mathematics (*exp_Often*, *exp_Half*, *exp_Some*) while controlling for the frequency they worked in small groups (*Grps_Often*, *Grps_Half*, *Grps_Some*), their gender (*d_Female*), and ethnicity (*d_Black*,

$d_{Hispanic}$, d_{Asian}). As is typical with dummy coding, the variables exp_Never and $Grps_Never$ were used as comparison groups and therefore do not appear in the level-1 equation.

The level-1 equation used for the final model is presented in the equation below:

$$\begin{aligned} (SDT_{A,C,R})_{ij} = & \beta_{0j} + \beta_{1j}(exp_{Often})_{ij} + \beta_{2j}(exp_{Half})_{ij} + \beta_{3j}(exp_{Some})_{ij} + \\ & \beta_{4j}(Grps_{Often})_{ij} + \beta_{5j}(Grps_{Half})_{ij} + \beta_{6j}(Grps_{Some})_{ij} + \\ & \beta_{7j}(d_{Female})_{ij} + \beta_{8j}(d_{Black})_{ij} + \beta_{9j}(d_{Hispanic})_{ij} + \\ & \beta_{10j}(d_{Asian})_{ij} + \beta_{11j}(d_{NatAm})_{ij} + r_{ij} \end{aligned}$$

In each of the level-1 equations, one of the three psychological needs is the outcome measure for student i in classroom j . This is represented above by $(SDT_{A,C,R})_{ij}$ which signifies one of the psychological needs of SDT with the subscripts A, C, and R for *autonomy*, *competence*, and *relatedness*, respectively. The level-1 equation for each psychological need is identical for purposes of comparing each measure under the same premises. As each need is examined, it is done so in the context of the same level-1 and level-2 setup.

The intercept for each level-1 equation, β_{0j} , represents the adjusted average level of autonomy, competence, or relatedness for students within classroom j , who never explain their mathematics and are white males. β_{1j} represents the association between students who explain their mathematics every or almost every lesson with their psychological need of autonomy, competence, or relatedness. β_{2j} represents the association between students who explain their mathematics about half of the lessons with their psychological need of autonomy, competence, or relatedness. β_{3j} represents the association between students who explain their mathematics some lessons with their psychological need of autonomy, competence, or relatedness. β_{4j}

represents the association between students who are asked to be in small groups at least half of their lessons with their own psychological need of autonomy, competence, or relatedness. Similarly, β_{5j} , β_{6j} , and β_{7j} represent the effects of being female, black, Hispanic, or Asian, respectively, on the level of a student's autonomy, competence, or relatedness. The random effect for student i in classroom j is represented as r_{ij} and is assumed to be independent and identically distributed (*i.i.d.*) as normal with a mean of zero and variance σ^2 .

Level 2 – Classroom Level

The second level unit of analysis was the classroom. It is at level-2 where variance in the effect of student explanation between classrooms was examined. Incorporation of variables at level-2 was done in part to explain variability between classrooms with contextual factors, such as mean frequency of explanation in the classroom and teacher use of small groups. Additionally, these classroom variables were included to examine interactions between classroom contexts and individual factors (i.e. student frequency of explaining mathematics).

$$\begin{aligned} \beta_{0j} = & \gamma_{00} + \gamma_{01}(\text{class}_{\text{explain}})_j + \gamma_{02}(\text{TchrGrps}_{\text{Often}})_j + \gamma_{03}(\text{TchrGrps}_{\text{Half}})_j \\ & + \gamma_{04}(\text{TchrGrps}_{\text{Some}})_j + u_{0j} \end{aligned}$$

$$\begin{aligned} \beta_{1j} = & \gamma_{10} + \gamma_{11}(\text{class}_{\text{explain}})_j + \gamma_{12}(\text{TchrGrps}_{\text{Often}})_j + \gamma_{13}(\text{TchrGrps}_{\text{Half}})_j \\ & + \gamma_{14}(\text{TchrGrps}_{\text{Some}})_j + u_{1j} \end{aligned}$$

$$\begin{aligned} \beta_{2j} = & \gamma_{20} + \gamma_{21}(\text{class}_{\text{explain}})_j + \gamma_{22}(\text{TchrGrps}_{\text{Often}})_j + \gamma_{23}(\text{TchrGrps}_{\text{Half}})_j \\ & + \gamma_{24}(\text{TchrGrps}_{\text{Some}})_j + u_{2j} \end{aligned}$$

$$\begin{aligned} \beta_{3j} = & \gamma_{30} + \gamma_{31}(\text{class}_{\text{explain}})_j + \gamma_{32}(\text{TchrGrps}_{\text{Often}})_j + \gamma_{33}(\text{TchrGrps}_{\text{Half}})_j \\ & + \gamma_{34}(\text{TchrGrps}_{\text{Some}})_j + u_{3j} \end{aligned}$$

At level-2, the intercept and slopes of level-1 become the outcome variables. For example, β_{0j} was expressed as the sum of the control group mean, γ_{00} , the coefficients for the level-2 variables γ_{01} , γ_{02} , γ_{03} , and γ_{04} , and the random group error for classroom j , u_{0j} . The slope of *exp_Often*, β_{1j} , was also expressed as the sum of the average impact of *exp_Often* on each psychological need, γ_{10} , the coefficients for the level-2 variables γ_{11} , γ_{12} , γ_{13} , and γ_{14} , and the random unique effect for school j , u_{1j} . The level-2 equations for *exp_Half* and *exp_Some* can be described likewise.

Within each level-2 equation, the level-2 parameter estimates represent the interaction effects of *class_explain*, *TchrGrps_Often*, *TchrGrps_Half*, and *TchrGrps_Some*. For example, in the level-2 equation with outcome measure β_{0j} , γ_{01} represents the interaction between the mean level of students' engagement in explaining their mathematics on the intercept for the psychological need (autonomy, competence, or relatedness) of a student who never explains their mathematics. Similarly, γ_{12} in the level-2 equation for *exp_Often* represents the interaction between the teacher asking students to work in small groups in every or almost every lesson and their engagement in explaining mathematics in every or almost every lesson. All other level-2 coefficients can be described similarly.

Study 1: Results

Correlational Analysis

Student Level Analysis

The variables *autonomy*, *competence*, and *relatedness* were correlated with *exp_Often*, *exp_Half*, *exp_Some*, and the small group variables. The results are presented in Table 3.1. The variables *autonomy* and *exp_Often* were found to be positively correlated and statistically significant ($r = .26, p < .01$). However, the correlation between *autonomy* and *exp_Half* was

virtually zero ($r = -.04$) and the correlation with *exp_Some* was statistically significant in the negative direction ($r = -.23, p < .01$). These relationships suggest that more frequent engagement in explanation happens in conjunction with higher levels of perceived autonomy. The correlation between *autonomy* and engagement in small groups was positive and statistically significant ($r = .22, p < .01$).

Correlations between the mathematical explanation variables and *competence* and *relatedness* were quite low. In fact, all of these correlations, with the exception of *exp_Often* and *relatedness*, were virtually zero. However, when variables are dummy-coded, correlations of the dummy-coded variables can often yield smaller r-values. The correlations between the non-dummy-coded variables with autonomy ($r = .31, p < .01$), competence ($r = .09, p < .01$), and relatedness ($r = .19, p < .01$) provide additional but similar information as those presented in Table 3.1.

Table 3.1. *Correlation of Level-1 Variables.*

	<i>exp_Often</i>	<i>exp_Half</i>	<i>exp_Some</i>	<i>aut.</i>	<i>comp.</i>	<i>relat.</i>
<i>exp_Often</i>	1 <i>n</i> = 8686	-	-	-	-	-
<i>exp_Half</i>	-.62* <i>n</i> = 8686	1 <i>n</i> = 8686	-	-	-	-
<i>exp_Some</i>	-.50* <i>n</i> = 8686	-.25* <i>n</i> = 8686	1 <i>n</i> = 8686	-	-	-
<i>autonomy</i>	.26* <i>n</i> = 8633	-.04* <i>n</i> = 8633	-.23* <i>n</i> = 8633	1 <i>n</i> = 8714	-	-
<i>competence</i>	.09* <i>n</i> = 8512	-.04* <i>n</i> = 8512	-.04* <i>n</i> = 8612	.06* <i>n</i> = 8546	1 <i>n</i> = 8650	-
<i>relatedness</i>	.16* <i>n</i> = 8517	-.06* <i>n</i> = 8517	-.09* <i>n</i> = 8517	.11* <i>n</i> = 8546	.15* <i>n</i> = 8474	1 <i>n</i> = 8649
<i>Grps_Often</i>	.09* <i>n</i> = 8625	-.05* <i>n</i> = 8625	-.09* <i>n</i> = 8625	.20* <i>n</i> = 8653	.02 <i>n</i> = 8533	.04* <i>n</i> = 8539
<i>Grps_Half</i>	-.01 <i>n</i> = 8625	.09* <i>n</i> = 8625	-.06* <i>n</i> = 8625	.07* <i>n</i> = 8653	.00 <i>n</i> = 8533	.01 <i>n</i> = 8539
<i>Grps_Some</i>	-.01 <i>n</i> = 8625	.00 <i>n</i> = 8625	.04* <i>n</i> = 8625	-.06* <i>n</i> = 8653	.02 <i>n</i> = 8533	.07* <i>n</i> = 8539

*represents significance at the .01 level

Correlations between *autonomy* and *competence* ($r = .06, p < .01$) as well as *autonomy* and *relatedness* ($r = .11, p < .01$) were statistically significant but small. Additionally, the correlation between *competence* and *relatedness* was small but statistically significant ($r = .15, p < .01$). At first glance, this suggests little meaningful relationship between the three psychological needs advocated by SDT. Yet, the variables assessing relatedness were not specific to the mathematics teacher. Also, student relatedness was not assessed; only teacher relatedness was addressed and only for teachers in general. Therefore, the measurement of relatedness may have been limited in its breadth and could account for the low correlations.

The variable *competence* measured students' perceived competence in mathematics. A separate correlation found that *competence* was significantly related to mathematics achievement ($r = .42, p < .01$). Therefore, *competence* appears to reliably predict the psychological need of competence.

The correlation between the small group variables (*Grps_Often*, *Grps_Half*, *Grps_Some*) and the mathematical explanation variables (*exp_Often*, *exp_Half*, *exp_Some*) yielded small r-values (see Table 3.1). As with *competence* and *relatedness*, these values did show a positive trend, but the small size of the r-values causes such a trend to have little meaningful value. The correlation between *Grps_Often* and *Autonomy* was moderate ($r = .20, p < .01$). Yet similar trends between the three psychological needs and the small group variables emerged as they did with the explanation variables. The correlations between the small group variable prior to dummy-coding with autonomy ($r = .25, p < .01$), competence ($r = .04, p < .01$), and relatedness ($r = .04, p < .01$) provided similar results as those presented in Table 3.2.

Classroom Level Analysis

Pearson correlation analysis of the classroom level variable *class_explain* with *TchrGrps_Often*, *TchrGrps_Half*, and *TchrGrps_Some* yielded small r values. These correlations can be viewed in Table 3.2. These results suggest that average student engagement in explaining mathematics has no relationship with the teacher asking students to work in groups. However, this is not to say that placing students in small groups does not have a relationship with individual students engaging in explanation of mathematics. It does mean that asking students to work in groups does not appear to have a relationship with the average amount of student explanation within the classroom.

Table 3.2. *Correlations of Level-2 Variables.*

	<i>class_explain</i>	<i>TchrGrps_Often</i>	<i>TchrGrps_Half</i>	<i>TchrGrps_Some</i>
<i>class_explain</i>	1 <i>n</i> = 456	-	-	-
<i>TchrGrps_Often</i>	.05 <i>n</i> = 331	1 <i>n</i> = 396	-	-
<i>TchrGrps_Half</i>	-.03 <i>n</i> = 331	-.30* <i>n</i> = 396	1 <i>n</i> = 396	
<i>TchrGrps_Some</i>	.02 <i>n</i> = 331	-.44* <i>n</i> = 396	-.63* <i>n</i> = 396	1 <i>n</i> = 396

*represents significance at the .01 level

Hierarchical Linear Modeling

Unconditional Model

Analysis of the unconditional model found an ICC of approximately 5.5% for the autonomy model, 10.7% for the competence model, and 10.7% for the relatedness model.

Hedges and Hedberg (2007) suggest an ICC of at least 15-25% should be used with nationally representative datasets. This would seem to indicate that there is not enough variance between

classrooms in students' level of autonomy, competence, or relatedness to warrant a between-class analysis. However, part of the model being developed looks at how classroom factors affect individual factors and HLM analysis is the most reliable way to examine such relationships. Therefore, while the ICC for each model was low, it was decided that HLM analysis would still be conducted.

Baseline Model

Autonomy-Model

Baseline model results for the autonomy-model can be found in the Model 2 column of Table 3.5. Baseline results of the autonomy-model indicated that *exp_Often* ($\beta_{1j} = .88, p < .01$), *exp_Half* ($\beta_{2j} = .57, p < .01$), and *exp_Some* ($\beta_{3j} = .19, p < .01$) all positively predicted *autonomy* significantly more than never explaining mathematics and the size of coefficients increased with the frequency of explanation. However, the level-2 variance of *exp_Half* was found to be statistically significant ($SD = .39, p < .01$). Comparison of standard deviation measures to Beta coefficients indicated that in some classrooms the effect of *exp_Half* was similar to that of *exp_Some* and in others the impact was larger than that of *exp_Often*. This finding appeared to hold even after the addition of level-1 covariates and level-2 variables.

Competence-Model

Baseline model results for the competence-model can be found in the Model 2 column of Table 3.7. Baseline results of the competence-model indicated that *exp_Often* ($\beta_{1j} = .26, p < .01$), *exp_Half* ($\beta_{2j} = .18, p < .01$), and *exp_Some* ($\beta_{3j} = .15, p < .01$) all positively predicted *competence* significantly more than never explaining mathematics. These coefficients were smaller than in the autonomy-model, but still increased with the frequency of explanation. Similar to the autonomy-model, the variance for *exp_Half* was found to be statistically

significant ($SD = .17, p < .05$), indicating that in classrooms within one standard deviation of the mean, the effect of *exp_Half* was virtually zero and in others it was greater than that of *exp_Often*. The variance for *exp_Often* ($SD = .18, p = .09$) and *exp_Some* ($SD = .19, p = .07$) were found not to be statistical significance.

Relatedness-Model

Baseline model results for the relatedness-model can be found in the Model 2 column of Table 3.9. Baseline results of the relatedness-model showed that *exp_Often* ($\beta_{1j} = .71, p < .01$), *exp_Half* ($\beta_{2j} = .55, p < .01$), and *exp_Some* ($\beta_{3j} = .46, p < .01$) all positively predicted students' levels of relatedness. Additionally the variance of the relationship across classrooms was found to be statistically significant for each effect: *exp_Often* ($SD = .67, p < .01$); *exp_Half* ($SD = .56, p < .01$); and *exp_Some* ($SD = .57, p < .01$). Therefore, the effect of each frequency of explanation varied from near zero in some classrooms to over a 1.0.

Final Model

Results of the final model for the autonomy-model, competence-model, and relatedness-model are presented in Table 3.3 and Table 3.4. For each model, the intercept represents the average level for a specific psychological need of a typical white male student who 'never explains their mathematics.' The coefficients for each variable represent the slope for those variables while adjusting for covariates in the model. For example, γ_{10} can be interpreted as the effect that 'explaining mathematics often' has on the respective psychological need (i.e. autonomy, competence, or relatedness), while adjusting for gender and race/ethnicity.

Table 3.3. *Results of the Final Model.*

Fixed Effect	Coefficient			S.E.		
	A	C	R	A	C	R
Intercept, γ_{00}	1.66**	2.72**	2.49**	.09	.07	.10
class_explain, γ_{01}	-.32*	.01	-.28	.17	.14	.19
TchrGrps_Often, γ_{02}	-.18	.05	-.35	.24	.25	.31
TchrGrps_Half, γ_{03}	-.06	.25	.03	.24	.22	.30
TchrGrps_Some, γ_{04}	-.18	.07	-.13	.23	.22	.29
exp_Often, γ_{10}	.87**	.26**	.74**	.09	.08	.09
class_explain, γ_{11}	.44**	.09	.35	.17	.13	.19
TchrGrps_Often, γ_{12}	.20	-.14	.39	.26	.26	.33
TchrGrps_Half, γ_{13}	.06	-.36	.01	.25	.24	.33
TchrGrps_Some, γ_{14}	.20	-.18	.18	.24	.24	.32
exp_Half, γ_{20}	.60**	.19*	.59**	.09	.08	.09
class_explain, γ_{21}	.39*	.05	.29	.18	.16	.19
TchrGrps_Often, γ_{22}	-.00	.05	.54	.27	.23	.29
TchrGrps_Half, γ_{23}	-.01	-.13	.11	.25	.21	.28
TchrGrps_Some, γ_{24}	.18	.03	.24	.24	.21	.28
exp_Some, γ_{30}	.25**	.17*	.52**	.09	.08	.09
class_explain, γ_{31}	.31	.06	.36*	.18	.14	.18
TchrGrps_Often, γ_{32}	.06	-.11	.42	.26	.26	.34
TchrGrps_Half, γ_{33}	.05	-.33	-.01	.26	.24	.33
TchrGrps_Some, γ_{34}	.13	-.10	.17	.24	.24	.32
Grps_Often, γ_{40}	.62**	.07*	.15**	.05	.04	.04
Grps_Half, γ_{50}	.39**	.06	.15**	.04	.03	.04
Grps_Some, γ_{60}	.18**	.07**	.18**	.03	.03	.03
dFemale, γ_{70}	-.05*	-.15**	.15**	.02	.02	.02
dBlack, γ_{80}	.12**	.01	-.16**	.04	.04	.04
dHispanic, γ_{90}	.09**	-.09**	-.04	.03	.03	.03
dAsian, $\gamma_{10,0}$.15*	.11*	.06	.06	.05	.05

* $p < .05$, ** $p < .01$

Table 3.4. *Level-2 Variance for Outcome Measures Autonomy, Competence, and Relatedness.*

	Autonomy		Competence		Relatedness	
	S.D.	Variance	S.D.	Variance	S.D.	Variance
Intercept, u_{0j}	.31*	.10	.31**	.10	.67**	.45
exp_Often, u_{1j}	.28	.08	.23*	.05	.61**	.37
exp_Half, u_{2j}	.39**	.15	.22	.05	.50**	.25
exp_Some, u_{3j}	.26	.04	.23	.05	.48**	.23
random error, r_{ij}	.90	.80	.70	.49	.72	.51

* $p < .05$, ** $p < .01$

Autonomy-Model

Results of the autonomy-model at level-1 show that the independent variables were all statistically significant (exp_Often , $\gamma_{10} = .87$, $p < .01$; exp_Half , $\gamma_{20} = .60$, $p < .01$; exp_Some , $\gamma_{30} = .25$, $p < .01$). These results are illustrated in Table 3.5. As can be observed, the value of the coefficients is sequentially higher with variables associated with more frequent explanation about mathematics. This suggests that students who explain mathematics have higher perceived autonomy than students who do not explain mathematics, and as the frequency increases, so does the difference between the two types of students. What should be noted from the results from the independent variables is that they adjust not only for race/ethnicity and gender, but also for participation in small groups. Additionally, the size of the coefficients should be noted. Recalling that the range for the outcome measure, *autonomy*, is 1.00 to 4.00, a coefficient of .87 or .60 as given for the variables exp_Often and exp_Half is relatively large.

The results of the small group variables were similar to those of the explanation variables. $Grps_Often$ ($\gamma_{40} = .62$, $p < .01$), $Grps_Half$ ($\gamma_{50} = .39$, $p < .01$), and $Grps_Some$ ($\gamma_{60} =$

.18, $p < .01$) were each found to be statistically significant from *Grps_Never*. This suggests that, controlling for mathematical explanation, student participation in small groups is positively associated with higher perceived autonomy.

Results for the race/ethnicity covariates showed that, on average, students who were black ($\gamma_{80} = .12, p < .01$), Hispanic ($\gamma_{90} = .09, p < .01$), or Asian ($\gamma_{10,0} = .15, p < .05$) perceived themselves as having a higher degree of autonomy than white students. Female students appeared to have slightly lower perceived autonomy than male students ($\gamma_{70} = -0.05, p < .05$).

Examination of between-class variance for the autonomy-model showed that only *exp_Half* had a statistically significant amount of variability ($S.D. = .39, p < .01$; see Table 3.4). This suggests that in some classes, the effect of explaining mathematics half the time was lower than explaining some of the time and in some classes it was higher than explaining often.

Looking further at the level-2 results (see Table 3.3), a positive interaction effect of the mean number of students who engage in explaining their mathematics was found. This interaction effect was statistically significant for the intercept ($\gamma_{01} = -.32, p < .05$), *exp_Often* ($\gamma_{11} = .44, p < .01$), and *exp_Half* ($\gamma_{21} = .39, p < .05$). However, it was not statistically significant for *exp_Some* ($\gamma_{31} = .31, p = .09$). Therefore, students who never explain their mathematics have a lower sense of autonomy in classrooms where more students *do* explain mathematics. However, students who *do* explain their mathematics have an increased sense of autonomy as more students within their class do the same.

Table 3.5. *Autonomy-Model Results.*

Fixed Effect	Model 1	Model 2	Model 3	Model 4
Intercept, γ_{00}	2.61**	1.95**	1.79**	1.66**
class_explain, γ_{01}				-0.32
TchrGrps_Often, γ_{02}				-0.18
TchrGrps_Half, γ_{03}				-0.06
TchrGrps_Some, γ_{04}				-0.18
exp_Often, γ_{10}		.88**	.76**	.87**
class_explain, γ_{11}				.44*
TchrGrps_Often, γ_{12}				.20
TchrGrps_Half, γ_{13}				.06
TchrGrps_Some, γ_{14}				.20
exp_Half, γ_{20}		.57**	.47**	.60**
class_explain, γ_{21}				.39*
TchrGrps_Often, γ_{22}				.00
TchrGrps_Half, γ_{23}				-0.01
TchrGrps_Some, γ_{24}				.18
exp_Some, γ_{30}		.19**	.15*	.25**
class_explain, γ_{31}				.31
TchrGrps_Often, γ_{32}				.06
TchrGrps_Half, γ_{33}				.05
TchrGrps_Some, γ_{34}				.13
Grps_Often, γ_{40}			.60**	.62**
Grps_Half, γ_{50}			.38**	.39**
Grps_Some, γ_{60}			.18**	.18**
dFemale, γ_{70}			-0.05*	-0.05*
dBlack, γ_{80}			.13**	.12**
dHispanic, γ_{90}			.09*	.09**
dAsian, $\gamma_{10,0}$.15*	.15*
dNatAm, $\gamma_{11,0}$.22*	.23*

* $p < .05$, ** $p < .01$

Table 3.6 illustrates the variance of the model for within and between classrooms. The intercept illustrates that through Model 3, the between-class variance increased with the addition of level-1 variables. The addition of the explanation variables substantially reduced the within-class variance shown in Model 1.

Table 3.6. *Autonomy-Model Variance Explained.*

	Model 1	Model 2	Model 3	Model 4
Intercept, u_{0j}	5.5%	9.7%	14.6%	8.2%
exp_Often, u_{1j}	-	7.7%	13.7%	6.3%
exp_Half, u_{2j}	-	11.9%	18.4%	12.8%
exp_Some, u_{3j}	-	6.0%	12.2%	5.7%
random error, r_{ij}	94.5%	64.7%	41.1%	67.1%

* $p < .05$, ** $p < .01$

Competence-Model

Level-1 results of the competence-model, presented in Table 3.7, show that the independent variables were all statistically significant (exp_Often , $\gamma_{10} = .26$, $p < .01$; exp_Half , $\gamma_{20} = .19$, $p < .05$; exp_Some , $\gamma_{30} = .17$, $p < .05$). However, the relative size of each coefficient is much smaller than was shown in the autonomy-model. Yet, each coefficient is still larger than those of the race/ethnicity and gender covariates.

The results of the small group variables were similar to those of the explanation variables. $Grps_Often$ ($\gamma_{40} = .07$, $p < .05$) and $Grps_Some$ ($\gamma_{60} = .07$, $p < .01$) were both found to be statistically significant from $Grps_Never$, but $Grps_Half$ was not statistically significant from $Grps_Never$. These results suggest that some frequencies of small group participation are more effective than others in their association with student competence in mathematics.

Results for the race/ethnicity covariates showed that, on average, students who were Hispanic ($\gamma_{90} = -0.09$, $p < .01$) had lower perceptions of competence than white students, and Asian ($\gamma_{10,0} = .11$, $p < .05$) perceived themselves as having a higher degree of competence than

white students. There was no statistical difference between black students and white students in terms of perceived competence. Female students appeared to have lower perceived competence in mathematics than male students ($\gamma_{70} = -0.15, p < .01$).

Table 3.7. *Competence-Model Results.*

Fixed Effect	Model 1	Model 2	Model 3	Model 4
Intercept, γ_{00}	2.89**	2.68**	2.74**	2.72**
class_explain, γ_{01}				.01
TchrGrps_Often, γ_{02}				.05
TchrGrps_Half, γ_{03}				.25
TchrGrps_Some, γ_{04}				.07
exp_Often, γ_{10}		.26**	.25**	.26**
class_explain, γ_{11}				.09
TchrGrps_Often, γ_{12}				-0.14
TchrGrps_Half, γ_{13}				-0.35
TchrGrps_Some, γ_{14}				-0.17
exp_Half, γ_{20}		.18**	.17**	.19*
class_explain, γ_{21}				.05
TchrGrps_Often, γ_{22}				.05
TchrGrps_Half, γ_{23}				-0.13
TchrGrps_Some, γ_{24}				.03
exp_Some, γ_{30}		.15**	.15*	.17*
class_explain, γ_{31}				.06
TchrGrps_Often, γ_{32}				-0.11
TchrGrps_Half, γ_{33}				-0.33
TchrGrps_Some, γ_{34}				-0.10
Grps_Often, γ_{40}			.08*	.07*
Grps_Half, γ_{50}			.07*	.06
Grps_Some, γ_{60}			.07**	.07*
dFemale, γ_{70}			-0.15**	-0.15**
dBlack, γ_{80}			.01	.01
dHispanic, γ_{90}			-0.09**	-0.09**
dAsian, $\gamma_{10,0}$.11*	.11*
dNatAm, $\gamma_{11,0}$			-0.18*	-0.18*

* $p < .05$, ** $p < .01$

Examination of the between-class variance showed statistically significant findings for *exp_Often* (See Table 3.4). These results suggest that in some classrooms, the effect of explaining mathematics often was near zero and in other classrooms it was almost twice the size of the general effect. The level-2 results for the competence-model showed no statistically significant interactions between level-2 and level-1 variables.

Table 3.8 illustrates the variance of the competence-model for within and between classrooms as different variables were included. The intercept illustrates that through Model 4, the between-class variance increased with the addition of level-1 variables. The addition of the explanation variables substantially reduced the within-class, r_{ij} , variance shown in Model 1.

Table 3.8. *Competence-Model Variance Explained.*

	Model 1	Model 2	Model 3	Model 4
Intercept, u_{0j}	10.7%	12.0%	12.6%	13.2%
<i>exp_Often</i> , u_{1j}	-	4.9%	6.1%	7.1%
<i>exp_Half</i> , u_{2j}	-	4.5%	4.8%	6.4%
<i>exp_Some</i> , u_{3j}	-	5.3%	6.2%	7.1%
random error, r_{ij}	89.3%	73.3%	70.3%	66.2%

* $p < .05$, ** $p < .01$

Relatedness

Results of the relatedness-model at level-1 show that the independent variables were all statistically significant (*exp_Often*, $\gamma_{10} = .74$, $p < .01$; *exp_Half*, $\gamma_{20} = .59$, $p < .01$; *exp_Some*, $\gamma_{30} = .52$, $p < .01$). These results are presented in Table 3.9 and suggest that students who

explain their mathematics have a higher sense of relatedness than students who never explain their mathematics.

Table 3.9. *Relatedness-Model Results.*

Fixed Effect	Model 1	Model 2	Model 3	Model 4
Intercept, γ_{00}	3.31**	2.71**	2.58**	2.49**
class_explain, γ_{01}				-0.28
TchrGrps_Often, γ_{02}				-0.35
TchrGrps_Half, γ_{03}				.03
TchrGrps_Some, γ_{04}				-0.13
exp_Often, γ_{10}		.71**	.65**	.74**
class_explain, γ_{11}				.35
TchrGrps_Often, γ_{12}				.39
TchrGrps_Half, γ_{13}				.01
TchrGrps_Some, γ_{14}				.18
exp_Half, γ_{20}		.55**	.49**	.59**
class_explain, γ_{21}				.29
TchrGrps_Often, γ_{22}				.54
TchrGrps_Half, γ_{23}				.11
TchrGrps_Some, γ_{24}				.24
exp_Some, γ_{30}		.46**	.41**	.52**
class_explain, γ_{31}				.36*
TchrGrps_Often, γ_{32}				.42
TchrGrps_Half, γ_{33}				-0.01
TchrGrps_Some, γ_{34}				.17
Grps_Often, γ_{40}			.16**	.15**
Grps_Half, γ_{50}			.16**	.15**
Grps_Some, γ_{60}			.19**	.18**
dFemale, γ_{70}			.15**	.15**
dBlack, γ_{80}			-0.16**	-0.16**
dHispanic, γ_{90}			-0.04	-0.04
dAsian, $\gamma_{10,0}$.05	.06
dNatAm, $\gamma_{11,0}$			-0.14	-0.15

* $p < .05$, ** $p < .01$

The results of the small group variables did not yield results similar to those presented in the autonomy model. *Grps_Often* ($\gamma_{40} = .15, p < .01$), *Grps_Half* ($\gamma_{50} = .15, p < .01$), and *Grps_Some* ($\gamma_{60} = .18, p < .01$) were each found to be statistically significant from *Grps_Never*. This suggests that, controlling for mathematical explanation, student participation in small groups is positively associated with higher perceived relatedness. However, there appears to be little difference in the size of coefficients for the different frequencies. Therefore, it appears that any frequency of small group participation may have as likely an effect on students' perceived relatedness.

Results for the race/ethnicity covariates showed that, on average, students who were black ($\gamma_{80} = -0.16, p < .01$) had lower perceptions of relatedness than white students. There was no statistical difference between either Hispanic or Asian students as compared to white students. Female students appeared to have higher perceived relatedness than male students ($\gamma_{70} = .15, p < .01$).

Examination of between-class variance for the relatedness-model showed statistically significant levels of variance for *exp_Often* ($S.D. = .61, p < .01$), *exp_Half* ($S.D. = .50, p < .01$), and *exp_Some* ($S.D. = .48, p < .01$). These results suggest that in some classes (those within one standard deviation of the mean effect), the effect of explaining mathematics on relatedness was near zero and in others it was near double its size for each frequency.

Level-2 results showed that mean explanation for a class, *class_explain*, positively predicted the slope of *exp_Some* ($\gamma_{31} = .36, p < .05$). However, this level-2 variable was not found to be a statistically significant predictor of any other level-1 slope. Interestingly, a teacher's assignment of students to small groups was not found to have a significant interaction

with any of the level-1 variables for the relatedness-model. While individual student participation had a nominal effect, it is notable that teacher assignment was not significant.

Table 3.10 illustrates the variance of the relatedness-model for within and between classrooms as different variables were included. The intercept illustrates that through Model 4, the between-class variance increased with the addition of level-1 variables. The addition of the explanation variables in Model 2 reduced the within-class variance shown by 72.7% (from 10.7% to 24.6%), and increased the between-class variance by 29.9% (from 89.3% to 24.4%).

Table 3.10. *Relatedness-Model Variance Explained.*

	Model 1	Model 2	Model 3	Model 4
Intercept, u_{0j}	10.7%	24.6%	24.0%	24.7%
exp _Often, u_{1j}	-	21.2%	20.3%	20.4%
exp _Half, u_{2j}	-	14.8%	14.2%	14.0%
exp _Some, u_{3j}	-	15.1%	13.8%	12.7%
random error, r_{ij}	89.3%	24.4%	27.6%	28.3%

* $p < .05$, ** $p < .01$

Overview of Study 1 Results

Overall results of each model suggest that more frequent explanation of mathematics has a positive effect on student perceived autonomy, competence, and relatedness. Mathematical explanation appears to have a larger effect on autonomy and relatedness than mathematical competence.

While the results of the HLM analysis were generally supportive of the framework presented in the literature review, the results of the correlation analysis were not. Specifically,

SDT research generally suggests that the different psychological needs (autonomy, competence, and relatedness) are interdependent and often found to be statistically correlated. However, the Pearson correlations obtained in the current study were rather small. Examination of the face validity of the SDT variables themselves suggested a possibility that the variables for autonomy and relatedness may not have been completely adequate. Specifically, there was only one autonomy variable and the two relatedness variables were not math-specific and referred only to teachers. It was this concern over the validity of the TIMSS items that resulted in the second study presented here.

Study 2 was conducted to evaluate the validity of the items used in study 1. This was accomplished by asking a sample of high school students the same questions asked in TIMSS as well as other items designed to illustrate the relationship between SDT and mathematical discussion. This second sample of students was originally a part of a separate study within the same program of research. The specific methods and measures used are described in the following pages.

Study 2: Methods

Sample

Data from 92 high school students was collected in spring 2009. Students were enrolled in either geometry or algebra II in a rural high school in Southwest Virginia. All students were taught by the same teacher. The teacher taught three sections of geometry and two sections of algebra II. Students enrolled in the geometry course were a part of a sample that had completed a survey in fall 2008 and the spring collection of data had been done at the request of the teacher.

Measures Specific to Study 2.

A questionnaire adapted from Deci and Ryan (2008) was used to assess student psychological needs: autonomy, competence, and relatedness (see Appendix). This survey has been used in various analyses and has been found to be supportive of general SDT theory (Gagne, 2003; La Guardia, Ryan, Couchman, & Deci, 2000; Vlachopoulos & Michailidou, 2006). In addition to questions assessing SDT needs, three other items were included to assess the quality of student engagement in mathematical discussion. These items asked students how true the following statements were: when I talk about math with others, I back up any claims I make; when I talk about math with others, I explain what I mean in detail; when others talk about math, I listen to what they are saying. Student responses ranged from 1 to 7 with 1 representing not at all true, 4 representing somewhat true, and 7 representing very true. Explanation and justification have been advocated as essential components of mathematics communication. Additionally, listening has been characterized as signaling student participation in mathematical discussion, even when they may not be the one speaking (Yackel and Cobb, 1996). All items from the survey included in the analysis are presented in Table 3.11, along with the reliability of their composite variables. Alpha coefficients for each composite were at or near 0.70, indicating an acceptable level of reliability (de Vaus, 2000; Nunnally, 1978). Student responses to the autonomy, competence, and relatedness items were identical to those of the discussion items.

Table 3.11. *Questions from Adapted Deci & Ryan Survey*

	Question	Alpha
<i>Autonomy</i>	I feel like I can make a lot of decisions in how I do my math; I am free to express my ideas and opinions about math in my math class; When I am in math class, I have to do things the way the teacher tells me to; My feelings are taken into consideration in math class; I feel like I can pretty much be myself in math class; There is not much opportunity for me to decide how to do my work in math class.	.67
<i>Competence</i>	People in math class tell me I am good at math; I have been able to learn interesting things about math in this math class; Most days I feel a sense of accomplishment after being in math class; When I am doing math, I often do not feel very capable.	.67
<i>Relatedness</i>	I really like the people in my math class; I get along with the students in my math class; I pretty much keep to myself when I am in math class; I consider the people in my math class to be my friends; People in math class care about me; There are not many students in my math class that I am close to; The students in my math class do not seem to like me much; People in my math class are pretty friendly towards me.	.87
<i>Mathtalk</i>	When I talk about math with others, I back up any claims I make; When I talk about math with others, I explain what I mean in detail; When others talk about math, I listen to what they are saying.	.72

In addition to the questions from the second survey addressing SDT and mathematical discussion (Alternate Survey), questions from the TIMSS survey were included to compare measures of autonomy, competence, and relatedness. The single question from TIMSS assessing frequency of student explanation was also included. Yet, the original TIMSS question assessing student explanation appeared to be somewhat ambiguous. Specifically, the question asked students how often “we explain our answers.” This question does not provide a context such as

classroom, student-to-student, or small group discussions. By neglecting the context of explaining, the question also neglects the audience to whom the student is explaining; to the teacher, other students, or themselves. Therefore, an additional question was included for comparison to the TIMSS explanation item. This question asked students to select the frequency they participated, “when we have class discussions about mathematics.” The options were modeled after those for the original TIMSS explanation question. However, instead of offering the choice “every or almost every lesson,” the option was written as “every or almost every discussion.” The other choices for the item were altered accordingly.

Study 2: Analysis and Results

Alternate Survey (AS) items and TIMSS survey items were correlated to determine the degree to which they were related. This was done to assess the validity of the TIMSS items selected for analysis in the initial study. Items from the AS were adapted from a survey used regularly in SDT research (e.g. Gagne, 2003; La Guardia et al., 2000; Vlachopoulos & Michailidou, 2006).

Prior to comparing the TIMSS and AS items, student responses to the original TIMSS items were compared to those of the original TIMSS dataset. Specifically, the correlations found in the first study between TIMSS assessed autonomy, competence, and relatedness with the frequency that students explain their answers was compared between both groups. These results are presented in Table 3.12. Using z-scores, an examination of the differences in the correlation coefficients from the two samples found that there were no statistically significant differences in the correlation coefficients. Therefore, in terms of responses, the sample from the newer or second set of students appears similar to that of the original TIMSS sample of students.

Table 3.12. *Comparison of Correlation between SDT Needs with the TIMSS Explanation**Variable from both Data Sets.*

	TIMSS Data	Second Set of Data	Test of Difference
TIMSS Autonomy	.31 <i>n</i> = 8633	.37 <i>n</i> = 90	<i>z</i> = .63 <i>p</i> = .53
TIMSS Competence	.09 <i>n</i> = 8512	.19 <i>n</i> = 90	<i>z</i> = .95 <i>p</i> = .34
TIMSS Relatedness	.19 <i>n</i> = 8517	.15 <i>n</i> = 90	<i>z</i> = .38 <i>p</i> = .70

A correlation analysis between SDT needs from TIMSS survey and the AS was conducted to examine how well the TIMSS SDT constructs related to those from the AS. These correlations are presented in Table 3.13. The results suggest there is a weak relationship between the different assessments of autonomy ($r = .18, p = .10$) and relatedness ($r = .24, p < .05$). Keeping in mind that studies evaluating the relationships between all three needs have shown strong correlations (e.g. Black & Deci, 2000; Connell & Wellborn, 1990; Minneart, Boekarts, & De Brabander, 2007), low correlations presented between any of the needs in Table 3.13 would cause some concern in terms of the validity of the TIMSS items. While TIMSS competence was found to have a positive relationship with AS competence ($r = .62, p < .01$), the other TIMSS assessed SDT needs did not have particularly large correlation coefficients. These results suggest that TIMSS competence may have been a valid measure of the psychological need of competence, but TIMSS autonomy and TIMSS relatedness were not as valid of constructs.

Table 3.13. *Correlation between SDT Needs from Different Surveys*

	TIMSS Autonomy	TIMSS Competence	TIMSS Relatedness
Autonomy	.18 <i>n</i> = 90	.41** <i>n</i> = 91	.32** <i>n</i> = 91
Competence	.20 <i>p</i> = .07 <i>n</i> = 90	.62** <i>n</i> = 91	.14 <i>p</i> = .18 <i>n</i> = 91
Relatedness	-.06 <i>n</i> = 90	.05 <i>n</i> = 91	.24* <i>n</i> = 91

p* < .05, *p* < .01

The next correlation analysis was between the math explanation item from the TIMSS survey, “we explain our answers,” the composite variable for discussion from the AS, and the more specific frequency question written as a possible alternative for the TIMSS item. These results are displayed in Table 3.14.

Table 3.14. *Correlation between Different Assessments of Discussion*

	We explain our answers.	When we have class discussions about mathematics, I participate (frequency).
When we have class discussions about mathematics, I participate (frequency).	.24* <i>n</i> = 90	-
Math Talk Composite Variable	.16 <i>n</i> = 89	.49** <i>n</i> = 90
Math Talk: Justify	.12 <i>n</i> = 89	.32** <i>n</i> = 90
Math Talk: Explain	.19 <i>n</i> = 88	.43** <i>n</i> = 88
Math Talk: Listen	.06 <i>n</i> = 88	.42** <i>n</i> = 88

p* < .05, *p* < .01

There were dramatic differences in the correlations with “Math Talk” discussion items between the original TIMSS question assessing frequency of explanation and the rewritten question included. Specifically, the differences in the correlation of both frequency of discussion variables with the composite math talk variable was statistically significant ($z = 2.46, p = .01$). Focusing specifically on the components of this composite variable, there was a statistically significant difference in the “listen” variable ($z = 2.53, p = .01$) and a near significant difference in the “explain” variable ($z = 1.4, p = .08$). While not statistically significant, the difference in the “justify” variable should also be noted. As a whole, these differences indicate that the original TIMSS explanation item is less related to students’ perceived engagement in discussion than is the rephrased item. Additionally, the TIMSS item was not strongly related to any of the other discussion variables assessed.

It is particularly interesting that an item asking students “when I talk about math with others, I explain what I mean in detail,” was weakly correlated with a question specifically asking students how frequently they explained mathematics in their class ($r = .19, p = .07$). By itself, such a finding would indicate cause for concern in the validity of this particular TIMSS item. Anecdotal evidence collected via feedback from the teacher and students suggested that students felt the TIMSS explain item was vague. Some students stated that they were not sure whether the item was referring to explaining math to the class, to a small group, or to a fellow student. However, such concerns did not appear to be prevalent in regards to the other discussion items.

After comparing both sets of SDT items and both sets of discussion items, it seemed prudent to examine differences between the SDT correlations with both items assessing frequency of discussion. Table 3.15 shows the differences between correlations for the original

TIMSS explain item and the reworded frequency of classroom discussion item. For both assessments of autonomy, the correlations appear to be similar between both discussion questions. However, the correlations for competence and relatedness are considerably higher for the second discussion item. These findings provide further evidence of the ambiguity of the TIMSS explain item.

Table 3.15. *Correlation of Different SDT Needs with Discussion Variables*

	We explain our answers.	When we have class discussions about mathematics, I participate (frequency).
TIMSS Autonomy	.37** <i>n</i> = 89	.37** <i>n</i> = 90
TIMSS Competence	.19 <i>n</i> = 90	.30** <i>n</i> = 91
TIMSS Relatedness	.15 <i>n</i> = 90	.22* <i>n</i> = 91
Alternative Autonomy	.32** <i>n</i> = 90	.26* <i>n</i> = 91
Alternative Competence	.32** <i>n</i> = 90	.53** <i>n</i> = 91
Alternative Relatedness	-.01 <i>n</i> = 90	.07 <i>n</i> = 91

* $p < .05$, ** $p < .01$

Overview of Study 2

Overall results of Study 2 indicate that items used to examine autonomy and relatedness from the TIMSS survey did not have sufficient validity. Low correlations, displayed in Table 3.13, indicated a weak relationship between the autonomy and relatedness constructs as measured by TIMSS to the constructs as measured by AS. The competence construct based on TIMSS items, however, did appear to have sufficient validity.

Examination of the explain variable from TIMSS suggests that it also did not have sufficient validity. Comparisons to other discussion-related items from the AS, as displayed in Table 3.14, show that the TIMSS explain item had a weak relationship with every discussion-related item asked of students in Study 2. Anecdotal evidence suggested that the TIMSS explain item may have been too vague in its wording, adding evidence of the item's lack of validity. Further comparisons between the TIMSS explain item and the alternatively worded explain item (see Table 3.15) showed that the reworded item had a stronger relationship with the SDT psychological needs than the original TIMSS explain item. This also provided evidence that the TIMSS explain item lacked validity.

Discussion

The results of Study 1 suggest that more frequent explanation of mathematics has a positive relationship with students' mathematical autonomy, mathematical competence, and relatedness. However, the results from the correlation analysis in Study 1 (see Table 3.1) suggest that the measures used for the psychological needs may not have had sufficient validity. Such results might tempt one to suggest that HLM results of Study 1 are invalid. Additionally, results from Study 2 illustrated that not all of the variables used in Study 1 may have had sufficient validity. While TIMSS assessment of competence appeared relatively valid, neither TIMSS assessed autonomy or relatedness appeared to correlate well with those from the AS. Undoubtedly, part of this is likely due to the fact that TIMSS was not designed with evaluating SDT needs in mind. Yet, the aspects specifically identified by SDT have traditionally been of interest to national surveys, if not in perfect unison with the specific way these needs are examined by SDT. Nevertheless, it appears that the use of the autonomy and relatedness constructs from the TIMSS study was an error on the part of the researcher.

When selecting the TIMSS explain variable in Study 1, I noticed the item was not context specific and was somewhat concerned that this would affect the generalizability of the study. After examining the results and deciding to conduct Study 2, it was decided that a more context specific version of the item may be useful. The results of Study 2 supported including the new item, as it appeared to correlate better with the discussion items from the AS. With the exception of autonomy, the correlations between the TIMSS assessed needs were stronger with the discussion frequency variable than they were with the TIMSS explain variable. Therefore, had the TIMSS dataset used a more context specific item to assess student explanation and/or discussion, and such results were similar to those found in Study 2, Study 1 may have yielded more encouraging results than what were found in HLM.

The initial purpose of this chapter was to examine whether students' SDT needs helped predict their engagement in explaining mathematics. Although the course of this investigation took a decidedly different route than initially planned, the initial purpose of the chapter was not completely abandoned. Results from Study 2 show support for a connection between SDT and the frequency students engage in discussion. Additionally, Study 2 helped emphasize the importance of context in examining this relationship. While Study 1 was designed in part to examine the effects of the classroom context, the TIMSS explain item failed to take context into account. Also, while some variables used in Study 1 lacked validity, the results from Study 1 still support the hypothesized relationship between mathematical explanation and SDT needs.

As stated in the previous paragraph, the most significant implication of this chapter is the need for large-scale studies, such as TIMSS and others, to be mindful of the wording of items included on surveys. While it is necessary to limit the number of items included on a survey for a large dataset, it is equally necessary that such items not be too general or ambiguous. Otherwise,

such items, as has been demonstrated in this chapter, lack validity and thus any meaningful use in research.

While specific wording of items is the most significant implication and finding of this chapter, several other implications emerged. One additional implication is the need for more statistical examinations of students' perceived autonomy in mathematics. Although cited frequently in qualitative research (e.g. Casa & DeFranco, 2005; Turner et al., 2003; Yackel & Cobb, 1996), only a single item assessing autonomy was found in the TIMSS 2003 dataset. Additionally, questions assessing either relatedness or social norms as perceived by the student were rare; even though this is also a frequently cited characteristic of effective mathematics classrooms (e.g. Hoffman, 2004; Jansen, 2008; Martin et al., 2005). Lastly, additional research should be conducted investigating the interactive relationship between students' self-determination in mathematics and their engagement in mathematical discussion. Although the current chapter provided some support for this relationship, more information is needed and should be collected using a variety of methods.

Conclusion

Students' perceived autonomy, competence, and relatedness appear to be related to the frequency they engage in mathematical classroom discussion. Yet, investigations of such a relationship should be mindful of context and the perceptions of individuals studied. The wording of survey items should be specific so that it is clearly known what the students are being asked, and therefore what is being assessed of students. Additionally, when data from large-scale studies is used, wording of items should be examined critically before analysis.

The reoccurring issue in lack of validity for the items used in Study 1 was a lack of meaningful contextual wording in the items. The items assessing relatedness were not specific to

the context of mathematics classrooms, and the explanation item from TIMSS did not specify the specific context or situation in which students were meant to explain their mathematics. A simple rewording of the item, as done in Study 2, showed clear results and a specific interpretation of such results.

Neglecting aspects of context and individual perception in research can provide misleading results, as has been demonstrated here. It is therefore ever important to be mindful of such matters when conducting research. Otherwise, misleading results may lead to misinterpretations of data meant to provide support for reform-oriented pedagogy. In a world filled with advocates of traditional mathematics teaching practices, we cannot afford to make such careless and costly mistakes.

References

- Alro, H. & Skovsmose, O. (2002). *Dialogue and learning in mathematics education: Intention, reflection, critique*. Boston, MA: Kluwer Academic Publishers.
- Black, A. E. & Deci, E. L. (2000). The effects of instructors' autonomy support and students' autonomous motivation on learning organic chemistry: A self-determination theory perspective. *Science Education*, 84(6), 740 – 756.
- Brown, R. & Hirst, E. (2007). Developing an understanding of the mediating role of talk in the elementary mathematics classroom. *Journal of Classroom Interaction*, 41/42(2/1), 18 – 28.
- Bryk, A. S., & Raudenbush, S. W. (1986). A hierarchical model for studying school effects. *Sociology of education*, 59, 1 – 17.
- Burstein, L. (1980b). The rule of levels of analysis in the specification of education effects. In R. Dreeben & J. A. Thomas (Eds.), *The analysis of educational productivity: Issues in microanalysis* (v. I). Cambridge, MA: Ballinger Publishing Company.
- Casa, T. M. & DeFranco, T. C. (2005). Making decisions about discourse: Case studies of three elementary-level teachers with various backgrounds and years of teaching experience. In G. M. Lloyd, M. Wilson, J. L. M. Wilkins, & S. L. Behm (Eds.). *Proceedings of the 27th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics & Education*. Blacksburg, VA, USA.
- Cassel, D. & Reynolds, A. (2007). Listening as a vital element of synergistic argumentation during mathematics problem solving. *Focus On Learning Problems in Mathematics*, 29(4), 30 – 47.

- Choppin, J. (2007). Engaging students in collaborative discussions: Developing teachers' expertise. In W. G. Martin, M. E. Strutchens, & P. C. Elliot (Eds.), *The learning of mathematics: Sixty-ninth yearbook* (pp. 129 – 39). Reston, VA: The National Council of Teachers of Mathematics, Inc.
- Connell, J. P. & Wellborn, J. G. (1990). Competence, autonomy, and relatedness: A motivational analysis of self-system processes. In M. R. Gunnar & L. A. Sroufe (Eds.), *Self processes and development: The Minnesota symposia on child development*. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.
- Dalton-Puffer, C. (2006). Questions as strategies to encourage speaking in content-and-language-integrated classrooms. In E. Uso-Juan & A. Martinez-Flor (Eds.), *Current trends in the development and teaching of the four language skills* (pp. 187 – 214). Berlin: Mouton de Gruyter.
- de Vaus, D. (2002). *Surveys in social research* (5th Ed.). New York: Routledge.
- Deci, E. L., Vallerand, R. J., Pelletier, L. G., & Ryan, R. M. (1991). Motivation and education: The self-determination perspective. *Educational Psychologist*, 26(3 & 4), 325 – 346.
- Deci, E. L., & Ryan, R. M. (2008). *Basic psychological needs scales*. Retrieved September 4, 2008, from http://www.psych.rochester.edu/SDT/measures/needs_scl.html
- Gagné, M. (2003). The role of autonomy support and autonomy orientation in prosocial behavior engagement. *Motivation and Emotion*, 27, 199-223.
- Goos, M. (1995). *How do you know when you understand? Using explanation to monitor and construct mathematical understanding*. Paper presented at the Postgraduate Research Conference Commemorating 50 Years of Educational Studies at the University of Queensland, Brisbane, Queensland, Australia.

- Hedges, L. V., & Hedberg, E. C. (2007). Intraclass correlation values for planning group-randomized trials in education. *Educational Evaluation and Policy Analysis*, 29(1), 60-87.
- Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal*, 30(2), 393 – 425.
- Hiebert, J., Stigler, J. W., Jacobs, J. K., Givven, K. B., Garnier, H., Smith, M., Hollingsworth, H., Manaster, A., Wearne, D., & Gallimore, R. (2005). Mathematics teaching in the United States today (and tomorrow): Results from the TIMSS 1999 video study. *Education Evaluation and Policy Analysis*, 27(2), 111 – 132.
- Hoffmann, A. J. (2004). *Motivational beliefs and goals of middle school students in discussion-oriented mathematics classrooms*. Paper presented at the annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Toronto, Ontario, Canada.
- Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education*, 35(2), 81 – 116.
- IEA (2008). TIMSS 2003: Trends in international mathematics and science study. Retrieved January 6, 2009 from <http://timss.bc.edu/timss2003.html>
- Jansen, A. (2008). An investigation of relationships between seventh-grade students' beliefs and their participation during mathematics discussions in two classrooms. *Mathematical Thinking & Learning*, 10(1), 68 – 100.

- Kastberg, D., Roey, S., Williams, T., & Gonzales, P. (2006). *Trends in international mathematics and science study (TIMSS) 2003: Data analysis user's guide*. Washington D.C.: Institute of Education Sciences.
- Katz, I., & Assor, A. (2007). When choice motivates and when it does not. *Educational Psychology Review*, 19(4), 429 – 442.
- Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal*, 102(1), 59 – 80.
- Kosko, K. W., & Wilkins, J. L. M. (2008). *Is mathematics discussion effective: Interesting results from a national data set and what they might mean*. Presented at the 2008 Annual Conference of the School Science and Mathematics Association, Raleigh Durham, NC.
- La Guardia, J. G., Ryan, R. M., Couchman, C. E., & Deci, E. L. (2000). Within-person variation in security of attachment: A self-determination theory perspective on attachment, need fulfillment, and well-being. *Journal of Personality and Social Psychology*, 79, 367-384.
- Lee, C. (2006). *Language for learning mathematics: Assessment for learning in practice*. New York: Open University Press.
- Leinhardt, G. & Steele, M. D. (2005). Seeing the complexity of standing to the side: Instructional dialogues. *Cognition & Instruction*, 23(1), 87 – 163.
- Martin, T., McCrone, S., Bower, M., & Dindyal, J. (2005). The interplay of teacher and student actions in the teaching and learning of geometric proof. *Educational Studies in Mathematics*, 60(1), 95 – 124.
- Minnaert, A., Boekaerts, M., & De Brabander, C. (2007). Autonomy, competence, and social relatedness in task interest within project-based education. *Psychological Reports*, 101(2), 574 – 586.

- Nunnally, J. (1978). *Psychometric Theory*. New York: McGrall-Hill.
- Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. New York: Routledge & Kegan Paul.
- Pintrich, P. R. & Schunk, D. H. (1996). *Motivation in education: Theory, research, and applications*. Columbus, OH: Prentice Hall.
- Raudenbush, S. W., & Bryk, A. S. (2002). *Hierarchical linear models: Applications and data analysis methods* (2nd Ed). Thousand Oaks, CA: Sage Publications.
- Reeve, J. (2005). *Understanding motivation and emotion*. Hoboken, NJ: Wiley.
- Reeve, J. (2006). Teachers as facilitators: What autonomy-supportive teachers do and why their students benefit. *The Elementary School Journal*, 106(3), 225 – 236.
- Reeve, J., & Sickenius, B. (1994). Development and validation of a brief measure of the three psychological needs underlying intrinsic motivation: The AFS scales. *Educational and Psychological Measurement*, 54(2), 506 – 515.
- Ryan, R. M., Connell, J. P., & Deci, E. L. (1984). A motivational analysis of self-determination and self-regulation in education. In C. Ames & R. Ames (Eds.), *Research on motivation in education: The classroom milieu*. Orlando, FL: Academic Press, INC.
- Ryan, R. M., & Deci, E. L. (2000). Self-determination theory and facilitation of intrinsic motivation, social development, and well-being. *American Psychologist*, 55(1), 68-78.
- Schleppenbach, M., Flevares, L. M., Sims, L. M., & Perry, M. (2007a). Teachers' responses to student mistakes in Chinese and U.S. mathematics classrooms. *The Elementary School Journal*, 108(2), 131 – 147.

- Schleppenback, M., Perry, M., Miller, K. L., Sims, L., & Ge, F. (2007b). The answer is only the beginning: Extended discourse in Chinese and U.S. mathematics classrooms. *Journal of Educational Psychology, 99*(2), 380 – 396.
- Sfard, A. (2007). When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from a commognitive standpoint. *Journal of the Learning Sciences, 16*(4), 565 – 613.
- Stein, C. C. (2007). Let's talk: Promoting mathematical discourse in the classroom. *Mathematics Teacher, 101*(4), 285 – 289.
- Stipek, D. (2002). *Motivation to learn: Integrating theory and practice*. Boston: Allyn and Bacon.
- Truxaw, M. P. & DeFranco, T. C. (2007). Lessons from Mr. Larson: An inductive model of teaching for orchestrating discourse. *Mathematics Teacher, 101*(4), 268 – 272.
- Turner, J. C., Meyer, D. K., Midgley, C., & Patrick, H. (2003). Teacher discourse and sixth graders' reported affect and achievement behaviors in two high-mastery/high-performance mathematics classrooms. *The Elementary School Journal, 103*(4), 357 – 382.
- Vlachopoulos, S. P., & Michailidou, S. (2006). Development and initial validation of a measure of autonomy, competence, and relatedness in exercise: The Basic psychological needs in exercise scale. *Measurement in Physical Education and Exercise Science, 103*, 179-201.
- Williams, S. R. & Baxter, J. A. (1996). Dilemmas of discourse-oriented teaching in one middle school mathematics classroom. *The Elementary School Journal, 97*(1), 21 – 38.
- Winne, P. H. & Alexander, P. A. (Eds.). (2006). *Handbook of educational psychology* (2nd ed.). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Wood, T. (1999). Creating a context for argument in mathematics class. *Journal for Research in Mathematics Education*, 30(2), 171 – 191.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, 27(4), 458 – 477.

Appendix

The following questions concern your feelings about your math class during the current school year. Please indicate how true each of the following statements is for you given your experiences in this math class. Remember that your teacher will never know how you responded to the questions. Please mark the number that best represents your answer.

1	2	3	4	5	6	7
not at all true			somewhat true			very true

1. I feel like I can make a lot of decisions in how I do my math.
2. I really like the people in my math class.
3. I do not feel very competent when I am in math class.
4. People in math class tell me I am good at math.
5. I feel pressured in math class.
6. I get along with the students in my math class.
7. I pretty much keep to myself when I am in math class.
8. I am free to express my ideas and opinions about math in my math class.
9. I consider the people in my math class to be my friends.
10. I have been able to learn interesting things about math in this math class.
11. When I am in math class, I have to do things the way the teacher tells me to.
12. When I am in math class, I have to do things the way my classmates tell me to.
13. Most days I feel a sense of accomplishment after being in math class.
14. My feelings are taken into consideration in math class.
15. In math class, I do not get much of a chance to show how capable I am.
16. People in math class care about me.
17. There are not many students in my math class that I am close to.
18. I feel like I can pretty much be myself in math class.
19. The students in my math class do not seem to like me much.
20. When I am doing math, I often do not feel very capable.
21. There is not much opportunity for me to decide how to do my work in math class.
22. People in my math class are pretty friendly towards me.
23. My math teacher has described how we should talk about math with each other.
24. When I talk about math with others, I back up any claims I make.
25. When I talk about math with others, I explain what I mean in detail.
26. When others talk about math, I listen to what they are saying.
27. What gender are you: (1) Male (2) Female
28. Are you: (1) Caucasian (2) African American (3) Hispanic (4) Asian (5) Other
29. What grade are you in: (1) 9th Grade (2) 10th Grade (3) 11th Grade (4) 12th Grade
30. How old are you: (1) 13 (2) 14 (3) 15 (4) 16 (5) 17 (6) 18 (7) 19+

Thank you for completing the survey!

Please place it in the sealed envelope provided your teacher.

How often do you do these things in mathematics lessons?		<i>Please fill in the number of your choice</i>			
We explain our answers:	Every or almost every lesson ①	About half the lessons ②	Some lessons ③	Never ④	
We decide on our own procedures for solving complex problems:	Every or almost every lesson ①	About half the lessons ②	Some lessons ③	Never ④	
When we have class discussions about mathematics, I participate:	Every or almost every discussion ①	About half the discussions ②	Some discussions ③	Never ④	
How much do you agree with these statements about learning mathematics?		<i>Please fill in the number of your choice</i>			
I usually do well in mathematics.	Agree a lot ①	Agree a little ②	Disagree a little ③	Disagree a lot ④	
Sometimes, when I do not initially understand a topic in mathematics, I know that I will never understand it.	Agree a lot ①	Agree a little ②	Disagree a little ③	Disagree a lot ④	
I learn things quickly in mathematics.	Agree a lot ①	Agree a little ②	Disagree a little ③	Disagree a lot ④	
Mathematics is not one of my strengths.	Agree a lot ①	Agree a little ②	Disagree a little ③	Disagree a lot ④	
Mathematics is more difficult for me than many of my classmates.	Agree a lot ①	Agree a little ②	Disagree a little ③	Disagree a lot ④	
How much do you agree with these statements about your school?		<i>Please fill in the number of your choice</i>			
I think that most teachers in my school care about the students.	Agree a lot ①	Agree a little ②	Disagree a little ③	Disagree a lot ④	
I think that most teachers in my school want students to do their best.	Agree a lot ①	Agree a little ②	Disagree a little ③	Disagree a lot ④	

Chapter Four:
Self-Determination Theory in Mathematical Discussion:
A Triangulation Mixed Methods Study

Introduction

In their widely influential document identifying mathematics principals and standards, the National Council of Teachers of Mathematics (NCTM) identified communication as “an essential part of mathematics and mathematics education” (NCTM, 2000, p. 60). Furthermore, mathematical discussion engages students in reflection about mathematics, having them organize their reasoning, and encourages them to justify their solution strategies (D’Ambrosio, Johnson, & Hobbs 1995; NCTM, 2000). Student reflection and engagement in the mathematics classroom as a result of discussion has been documented by several qualitative studies (i.e. Jansen, 2008; Kazemi & Stipek, 2001), as well as some quantitative studies (i.e. Gadanidas, Kotsopoulos, & Guembel, 2006; Hiebert et al., 2005). Yet studies describing the impact of mathematics discussion on mathematics achievement have been contradictory.

Some studies document a positive impact of mathematical discussion on mathematics achievement (e.g. Koichu, Berman, & Moore, 2007; Mercer & Sams, 2006; Stigler & Hiebert, 1997), while Shouse (2001) found a negative effect of discussion on achievement. Investigating such contrary findings, Kosko and Miyazaki (under review) found that, in general, mathematical discussion had no statistically significant impact on fifth grade students’ mathematics achievement. However, when taking into account that discussion in some classrooms and schools may be more or less effective than in others, different results emerged. Specifically, discussion in some schools and classrooms was found to have a largely negative impact on mathematics

achievement, while in other settings the impact was overwhelmingly positive (Kosko & Miyazaki, under review).

The varying effectiveness of discussion found by Kosko and Miyazaki (under review) is arguably characteristic of different contextual factors that exist in some classrooms and schools but not in others. Such factors have been identified by mathematics education literature. One such factor is a feeling of student autonomy in the discussion process (Kazemi & Stipek, 2001; Manke, 1997; Sfard, 2007). Another often discussed characteristic of effective discussion is a positive social atmosphere between all students and between the students and teacher (Kazemi & Stipek, 2001; Martin et al., 2005; Wood, 1999). To a slightly smaller degree there is agreement on establishing social norms where teachers do not focus specifically on the correctness of an answer but rather on the solution strategies students used (Kazemi & Stipek, 2001; Martin, McCrone, Bower, & Dindyal, 2005; Wood, 1999). This last aspect of establishing effective discussion could be seen as reinforcing students' perception of their competence. Taken altogether, these three aspects of effective discussion comprise elements of Self-Determination Theory (SDT).

SDT is a theory of motivation that claims the fulfillment of three innate individual needs allows for the internalization of content and actions (Ryan & Deci, 2000). These needs, autonomy, social relatedness and competence, seem to align theoretically with characteristics of an effective mathematical discourse setting as outlined by mathematics education literature. However, empirical evidence of such a connection has not yet been documented. Therefore, it is the primary purpose of this study to establish an empirical basis for such a connection. To do this, I used mixed methods research, which allows for the comparison of individual, quantitative

measures of autonomy, competence and relatedness with qualitative data gathered from observations of the social context of mathematics classroom discussion.

Discourse Settings

Kitchen (2004) identified two main types of discourse in mathematics classrooms: univocal and dialogic. Univocal discourse is teacher centered discussion in which the student has little influence on the direction of discussion. Dialogic discourse is described as discussion that includes teacher and students in describing and discussing mathematical concepts (Kitchen, 2004). While univocal discourse involves the teacher having sole control over the course of the discussion and dialogic discourse involves shared control with students, there are other differences in each setting. Mercer and Littleton (2007) characterize dialogic discourse as involving students in providing richer descriptions and justifications of content discussed.

Similar to the univocal and dialogic discourse settings identified by Kitchen (2004) and Mercer and Littleton (2007), Hancewicz (2005) characterized two different discourse settings in the mathematics classroom; the traditional setting and the discourse-rich setting. Hancewicz characterized the traditional setting as teacher centered with the teacher asking the majority of the questions during the discussion. Therefore, the traditional setting described by Hancewicz is identical to the univocal setting. In a discourse-rich setting, the students share in the role of asking questions and guiding instruction. Thus, the discourse-rich setting is identical to a setting with dialogic discourse. In addition to the traditional and discourse-rich settings, Hancewicz described a transitional stage called the probing setting. The probing setting is still teacher centered, but questions are more oriented in engaging students in answering more conceptual topics. Therefore, such a setting is still characteristic of univocal discourse, but on the verge of becoming dialogic discourse.

Types of Mathematical Talk

Within each discourse setting, students and teachers produce varying types of mathematical talk. In a review of mathematics communication literature, Schleppegrell (2007) summarized different types of mathematical talk, including procedural and conceptual talk. Procedural talk focuses on specific steps and strategies for solving problems while conceptual talk (e.g. evaluation and analysis) focuses on the reasons for doing certain procedures.

While there are varying types of talk produced within a mathematics classroom, not all of these types of talk are mathematical. Yackel and Cobb (1996) describe mathematical discourse as being inherently linked to the process of mathematics. Therefore, since procedural talk involves aspects of mathematics procedures, it is mathematical in nature. Additionally, since conceptual talk focuses on the deeper meanings of mathematics, it is also inherently mathematical. Using Yackel and Cobb's definition of what consists of mathematical discourse, a student who states that the answer to a problem is 42 may not be producing mathematical talk. However, if such a statement is related in context to the process of mathematics, it is then mathematical and likely will be either procedural or conceptual. With such scenarios in mind, the current study takes the view that there are two main types of mathematical talk: procedural and conceptual.

Procedural and conceptual talk occur in both univocal and dialogic discourse settings (Schleppegrell, 2007). However, in univocal settings, it is the teacher who predominately produces conceptual talk (Pimm, 1987) and in dialogic settings, the focus is on having students produce conceptual talk (Mercer & Littleton, 2007). Therefore, it is logical to perceive that students produce more conceptual talk as a discourse setting becomes more dialogic. The

probing setting described by Hancewicz (2005) can be likened to a transitional phase of univocal discourse in which a teacher scaffolds students' engagement in producing conceptual talk.

Self-Determination Theory in Mathematical Discussion

Overview of Self-Determination Theory

According to Deci, Vallerand, Pelletier, and Ryan (1991) the majority of current motivational research advocates the concept of intent as a fundamental concept of motivation. Self-Determination Theory (SDT) explores motivation which is not only determined by the individual, but also that which is controlled by others. Actions that are self determined are those that the individual has the most power over (Deci et al., 1991; Reeve, 2006). Autonomy-supportive teachers “facilitate congruence by identifying and nurturing students’ needs, interests, and preferences...” (Reeve, 2006, p. 228) into their instruction and lessons. Controlling teachers inhibit students’ connection to curricula content by forcing their own values and interests onto the students. Deci et al. (1991) describe a relationship between autonomy-supportive and controlling environments as corresponding with the degree to which student behaviors are internalized or externalized respectively.

SDT is best described with three psychological needs: relatedness, competence, and autonomy. Relatedness describes the degree to which satisfying social connections are made. Competence describes the degree to which the individual feels able to accomplish different external and internal tasks. Autonomy describes the degree to which the self is in control of initiating and maintaining different behaviors (Deci et al., 1991; Ryan & Deci, 2000). When these needs are met, “motivation, performance, and development will be maximized” (Deci et al., 1991, p. 327). The impact of competence on increasing intrinsic motivation tends to depend on the degree of autonomy individuals have, while the development of autonomy does so best

when there is a strong sense of relatedness. Any of the three needs can and do develop independently of each other but Deci et al. (1991), Reeve (2006), and Katz and Assor (2007) suggest that the interdependence of each on the other(s) requires all three needs be met in order for motivation to be self determined.

Connections to Mathematics Communication Literature

Varying sources within mathematics communication literature provide support for SDT as a basis for explaining effective mathematical discussion. One example is Yackel and Cobb's (1996) description of a transition in the way students engage in discourse. Yackel and Cobb described students in an elementary classroom as initially lacking autonomy in the way they engaged in discussion. Their statements were initially simple or procedural in nature and the teacher was viewed as the source of mathematical reasoning. However, through the incorporation of sociomathematical norms, or socially created knowledge of what consists of proper mathematical explanations and justification, and the creation of a positive social environment, students became more autonomous in the way they engaged in mathematical discussion.

Yackel and Cobb's (1996) framework for creating a dialogic setting for mathematical discussion relates well with Ryan and Deci's (2000) description of SDT and the three needs of autonomy, competence, and relatedness. Autonomy was clearly outlined by Yackel and Cobb as an essential component of facilitating discourse while sociomathematical norms could be characterized as supportive of student competence. Since sociomathematical norms as defined by Yackel and Cobb consist of knowledge in how to effectively communicate about mathematics, it is logical to deduce that such norms facilitate students' perceived competence in engaging in mathematical discussion. The establishment of a positive and safe social environment is part of what facilitates students' relatedness (Deci et al., 1991; Ryan & Deci, 2000). Additionally,

students who appear to have a lower degree of relatedness produce more procedural talk than conceptual (Jansen, 2008). Taken together, Yackel and Cobb's framework for facilitating mathematical dispositions of students through discussion incorporates much of what Deci and Ryan suggest consists of self-determined behavior (Deci et al., 1991; Ryan & Deci, 2000).

Research Approach and Questions

Yackel and Cobb's work on mathematical discourse is predominately qualitative in nature. Further, their investigations have led to philosophical discussions of individual versus socially constructed knowledge and the location of the mind (e.g. Cobb, 1994; Cobb & Yackel, 1996). Deci and Ryan's approach to investigating self-determination has been predominately quantitative, yet also lends itself to a more interactive view of individual and society (Deci et al., 1991; Ryan & Deci, 2000). A discussion of the interaction between the individual and the social context is beyond the scope of this study. However, it is key to acknowledge such an interaction so that the intent of this study is clear. The individual psychological needs of autonomy, competence, and relatedness as advocated by SDT are characterized here as interacting with contextual factors that can facilitate or hinder these needs. If these needs are facilitated, it is hypothesized that a student will engage in more dialogic discourse. If these needs are not facilitated or an individual student does not perceive them as being facilitated, then that student will participate in a more univocal manner.

In order to examine the interaction of individual needs with social context, a mixed methods design was chosen. This design, as outlined in the methods section, provides a basis for combining observational data of the social context with measured quantitative data of individual psychological needs. It is with this interactive mindset in which the following research questions are examined:

1. How do students' psychological needs of autonomy, competence, and relatedness relate to their actions in mathematics classroom discussion?
2. Do students' perceptions of their mathematical talk align with their observed discourse actions in mathematics classroom discussion?

Methods

Overview of Mixed Methods Design

Mixed methods research uses both quantitative and qualitative analyses in such a way that interpretations of the data are combined so they inform and complement each other (Creswell & Plano Clark, 2007). The criteria for a mixed methods design is not the inclusion of both quantitative and qualitative data, but the mixing of the analysis of both so that a more complete picture of the topic being investigated is formed. There are several different types of mixed methods designs. The current study uses a triangulation design. A triangulation mixed methods design collects both quantitative and qualitative data concurrently (Creswell & Plano Clark, 2007). That is to say, both types of data are collected during the same phase of the study. The data is then merged during analysis to form a more complete picture of the issue at hand.

To assess the degree to which these needs are met, an adaptation of the Basic Psychological Needs Scale available from Deci and Ryan (2008) was used. Items relating to mathematics discussion were also included on the survey. In conjunction with this quantitative data, qualitative data in the form of classroom observations were collected to gain a deeper understanding of how these needs manifest themselves in classroom discussion. After the analysis of both quantitative and qualitative components, both types of data were merged in an effort to link specific actions and themes with the psychological needs so that self-determined actions in mathematics discussion could be identified.

Since this study seeks to understand how individual actions observed in mathematics classroom discussion interact with internal psychological needs, a mixed methods approach was the most suitable. A quantitative score for self-determined motivation and the needs of autonomy, competence, and relatedness allowed for the examination of these traits as they related to discussion. A qualitative assessment of student actions in discussion allowed for the detection of actions and interactions that cannot be measured with a survey. Both quantitative and qualitative data were then merged; a process of mixed methods analysis that allows both types of data to simultaneously inform the other. By merging the data in a triangulation mixed methods approach, it is believed that a better perception of the internal processes represented by the quantitative assessment, the external processes assessed through classroom observations, and the interaction between the two was gained. See Figure 4.1 below for an outline of the proposed study.

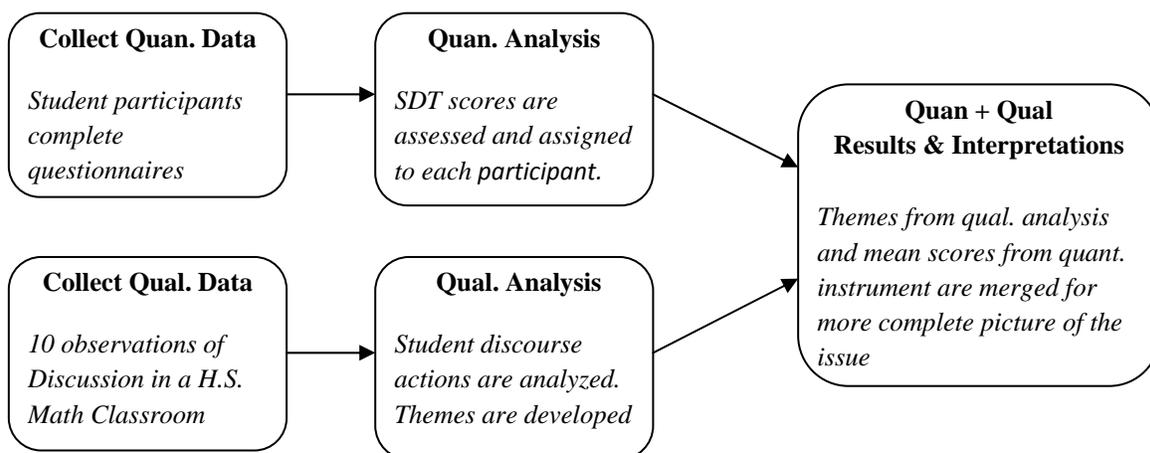


Figure 4.1. *Triangulation Mixed Methods Design*

In the current study, data were collected during the same phase of investigation. Specifically, quantitative data and qualitative data were collected within the same academic

semester. Quantitative and qualitative analyses were also conducted during the same phase of investigation. As Figure 4.1 illustrates, neither analysis influenced the other prior to merging the data. Therefore, quantitative scores were not used to analyze the qualitative data, or vice versa, before the final phase where the results of both analyses were merged.

Quantitative data was analyzed using Pearson correlation analysis. The results of the correlation analysis were used as a measure of the validity when mixing data, but were not directly merged with the qualitative data. The only quantitative data used during the actual merging phase of the study were the composite scores of the survey.

Qualitative data were analyzed using a microethnographic approach (Bloome, Carter, Christian, Otto, & Shuart-Faris, 2005; Streeck & mehus, 2005). Microethnography investigates how “people use language and other systems of communication...” (Bloome et al., 2005, p. xv). Discourse events are analyzed as groups of actions, with actions being the unit of analysis. For the current study, students’ discourse actions were of primary interest, and therefore the microethnographic approach seemed most suitable. Student discourse actions were analyzed within the context of different discourse settings, as well as the general context of the classroom they were placed. The observed discourse actions, and the results from this analysis were used in the merging of data.

The merging of data was conducted in the final stage of the study. This merging of data did not include any further analysis other than comparison of both sets of data in relation to the other. Specifically, a correlation table merging qualitative discourse actions and themes with quantitative measures of psychological needs was used to examine selected episodes of mathematics discussion. The merging of data allowed for the quantitative scores of psychological needs to further inform discourse actions observed in the qualitative analysis.

Simultaneously, merging of data allowed for discourse actions analyzed and observed in the qualitative analysis to illustrate what the quantitative scores for psychological needs may represent in the classroom.

Sample Participants and Classroom

Data were collected from high school students enrolled in Geometry in a rural high school in Southwest Virginia. A former mathematics curriculum administrator of the high school and school district referred the participating teacher to the author. The teacher was described as one who regularly implements mathematics discussion in their instruction. It was believed that this characterization increased the likelihood that student engagement in mathematical discussion would be observed. The particular class where observations took place included 20 students. The majority of these students were Caucasian ($n=16$), which is representative of the school and community population. There was also one African American student, one Asian, one Hispanic, and one of multicultural ethnicity. The class was evenly divided in terms of gender (*10 male, 10 female*).

The participating teacher was a white, female, veteran teacher with 25 years experience teaching both high school and middle school mathematics. She frequently vocalized her support for discussion and writing in the mathematics classroom and her use of both in her teaching. Additionally, the participating teacher regularly pursued positive relationships with her current and former students as a means of developing a positive social atmosphere.

Survey Instrument

A questionnaire adapted from Deci and Ryan (2008) was used to assess student psychological needs: autonomy, competence, and relatedness (see Appendix A). Three additional items were also included to assess the quality of student engagement in mathematical discussion.

These items asked students how true the following statements were ($1 = \text{not at all true}$; $4 = \text{somewhat true}$; $7 = \text{very true}$): (a) When I talk about math with others, I back up any claims I make; (b) When I talk about math with others, I explain what I mean in detail; (c) When others talk about math, I listen to what they are saying. These three items were written to address three areas of effective mathematical talk respectively: justification; explanation; and opportunity for reflection. In her description of mathematical linguistics, Schleppegrell (2007) characterized student discourse actions of explaining and justifying as those that produce procedural and conceptual talk. Additional literature characterizes discussion as encouraging students to reflect on their mathematical understanding (Goos, 1995; Lee, 2006; Pimm, 1987). To take such an opportunity to reflect, it was decided that a student participant in the discussion must listen. Therefore, while the last of the three questions did not ask students about reflecting on what other students say about math, it did ask them if they listen to what other students say about math. Listening may signal that the student engages as a participant in discussion even when they may not be the one speaking (Yackel and Cobb, 1996). A composite score incorporating all three mathematical talk variables (*mathtalk*, $\bar{X} = 3.91$, $SD = 1.57$) was created and examined for reliability using a larger sample from the entire set of Geometry classes within the school that students were observed ($n = 180$). A Cronbachs alpha coefficient of .75 was found, suggesting that the composite score is sufficiently reliable. By incorporating students' scores on the three items focusing on mathematics talk and their composite, student perceptions on how they engage in mathematics talk were compared with their actions in classroom discussion. Additionally, relationships between their psychological needs and their observed discourse actions were evaluated within context of these scores.

In total, there were six items on the questionnaire focusing on competence, eight on autonomy, eight on relatedness, and three on mathematical talk. Students were asked how true each of the items were on a scale of 1 to 7 (1 = not at all true; 4 = somewhat true; 7 = very true). As with the composite for *mathtalk*, item analysis was conducted to examine the reliability and validity of items and the composite variables they made up. The outcome of this preliminary analysis is described for each psychological need.

Autonomy describes the degree to which the self is in control of initiating and maintaining different behaviors (Deci et al., 1991; Ryan & Deci, 2000). Therefore the statements displayed in Table 4.1 were used to address autonomy in the mathematics classroom.

Table 4.1

Autonomy Construct Items.

-
- | | |
|-----|---|
| 1. | I feel like I can make a lot of decisions in how I do my math; |
| 5. | I feel pressured in math class; |
| 8. | I am free to express my ideas and opinions about math in my math class; |
| 11. | When I am in math class, I have to do things the way the teacher tells me to; |
| 12. | When I am in math class, I have to do things the way other students tell me to; |
| 14. | My feelings are taken into consideration in math class; |
| 18. | I feel like I can pretty much be myself in math class; |
| 21. | There is not much opportunity for me to decide how to do my work in math class. |
-

Preliminary item analysis found that item 5 ($r = .20$) and item 12 ($r = .05$) had low item-total correlations, signifying that when the item was compared to the construct with all other items

included, there was a low correlation. Both items were removed due to these low item-total correlations. The remaining items were summed to create the composite variable *autonomy* ($\bar{X} = 4.41$, $SD = 1.04$). As was the case with the variable *mathtalk*, *autonomy* was tested for reliability using a larger sample selected from all Geometry classrooms within the school. A Cronbach's alpha coefficient of .65 was computed. Although this score is not at the .70 threshold typically accepted for reliability (Nunnally, 1978), it was determined to be near enough to accept it as reliable for the purpose of the current analysis.

Competence describes the degree to which the individual feels able to accomplish different external and internal tasks (Deci et al., 1991; Ryan & Deci, 2000). For the mathematics classroom, the statements listed in Table 4.2 were used to evaluate competence.

Table 4.2

Competence Construct Items.

3.	I do not feel very competent when I am in math class;
4.	People in math class tell me I am good at math;
10.	I have been able to learn interesting things about math in this math class;
13.	Most days I feel a sense of accomplishment after being in math class;
15.	In math class, I do not get much of a chance to show how capable I am;
20.	When I am doing math, I often do not feel very capable.

During the item analysis, item 3 ($r = .23$) and item 15 ($r = .26$) were found to have lower item-total correlations. In examining the face validity of the questions, item 15 appeared to ask students about their 'opportunity' to demonstrate competence and not their self perception of

competence. The face validity of item 3 might be lower because it was determined that many students may not have understood the term “competent.” This assessment was made via anecdotal evidence provided by the teacher when surveys were administered. The remaining items were summed to create the composite variable *competence* ($\bar{X} = 4.20, SD = 1.14$) with a Cronbach’s alpha coefficient of .69.

Relatedness describes the degree to which satisfying social connections are made (Deci et al., 1991; Ryan & Deci, 2000). Therefore, the statements presented in Table 4.3 were used to evaluate student relatedness within the mathematics classroom.

Table 4.3

Relatedness Construct Items.

-
- | | |
|-----|--|
| 2. | I really like the people in my math class; |
| 6. | I get along with the students in my math class; |
| 7. | I pretty much keep to myself when I am in math class; |
| 9. | I consider the people in my math class to be my friends; |
| 16. | People in math class care about me; |
| 17. | There are not many students in my math class that I am close to; |
| 19. | The students in my math class do not seem to like me much; |
| 22. | People in my math class are pretty friendly towards me. |
-

Preliminary item analysis found that all items were sufficiently related. Therefore, these items were summed to create the composite variable *relatedness* with a Cronbach’s alpha coefficient of .83 ($\bar{X} = 4.59, SD = 1.20$).

Observation Data

Ten class sessions over the course of approximately five weeks were videotaped, audio recorded, and observed. A video camera was placed to capture students' actions during whole class discussion and two digital audio recorders were placed on opposite ends of the room to help ensure the capture of all audio occurring during class discussions. Field notes were compiled during observations to collect information that may not be picked up via the video or audio recording devices. Worksheets used during class also were collected. I used all of these data sources when transcribing the data and referred back to them during analysis. This included review of video before, during, and after transcription as well as repeated reference to both field notes and video during analysis of the transcripts.

Before episodes of discussion were selected, all classroom observations were evaluated for their overall discourse features. This included categorization of different discourse settings within each lesson, evaluating which students were more engaged in discussion and what types of mathematical talk were produced. It is important to note that this initial analysis of observations was meant to provide an overall view of the nature of discourse in the classroom. After evaluating all observed class sessions, five episodes of classroom discourse were selected to represent three specific discourse settings; two episodes for each main discourse setting of focus and one for the transitional phase of univocal discourse. Episodes were selected based on how well they were observed to fit with the definitions of the different discourse settings as presented in the literature. The two main types of discourse that this study is concerned with are univocal and dialogic. Univocal discourse is primarily teacher centered while dialogic discourse involves both the teacher and students as leading the discussion (Kitchen, 2004). In addition to selecting episodes that represented univocal and dialogic settings, a third category was included

for analysis. The probing setting identified by Hancewicz (2005) is a teacher centered discourse setting where the teacher prompts students to engage in conceptual talk. Since the teacher is still the dominant voice in the discussion, the probing setting is a type of univocal discourse, but such that it is on the verge of becoming dialogic.

After the selection of episodes that characterized the probing, univocal, and dialogic settings, deeper analysis of these episodes began. During analysis, special interest was paid to the type of talk the students and teacher produced as well as the roles the students and teacher played in the discourse. The two main types of mathematical talk of interest were procedural and conceptual. Procedural talk focuses on specific steps and strategies for solving problems while conceptual talk focuses on the reasons for doing certain procedures (Schleppegrell, 2007). Therefore, spoken statements and physical gestures were the unit of analysis when analyzing episodes of discourse (Bloome, Carter, Christian, Otto, & Sshuart-Faris, 2005; Streeck & Mehus, 2005).

During initial analysis of the transcripts, the roles that students and teachers portrayed in the episodes of whole class discussion analyzed helped to categorize the type of discourse setting that individuals were participating. However, these roles also characterized how different students acted within particular discourse settings. For example, in a dialogic setting, students help direct the course of the conversation. Yet simply because a classroom discussion is dialogic does not mean that all students are participating dialogically (Yackel & Cobb, 1996). Therefore, while the roles students portrayed helped to categorize the type of setting, not *all* students were required to take on such roles. Rather, students who did not take on the roles typical of a discourse setting were of interest in the analysis as well as students who did take on typical roles of discourse settings.

Results

Quantitative Results

In Table 4.4, the mean scores for the observed class ($n = 20$) are presented along with the range of those scores. Recalling that the possible range for any of these scores was from 1.00 to 7.00, we can see that many students had a more positive degree of autonomy and relatedness, but medium scores for *competence* and *mathtalk*. Notably, no students in the observed class considered themselves as having a low sense of autonomy. The range for all other scores was larger.

Table 4.4. *Descriptive Statistics for SDT Needs and Mathtalk*

	Range	Score
<i>Autonomy</i>	4.00 – 7.00	$\bar{X} = 5.11$ $SD = 0.85$
<i>Competence</i>	2.33 – 6.17	$\bar{X} = 4.54$ $SD = 1.05$
<i>Relatedness</i>	2.75 – 7.00	$\bar{X} = 5.52$ $SD = 1.18$
<i>Mathtalk</i>	1.00 – 6.67	$\bar{X} = 4.06$ $SD = 1.49$

Note: $n = 20$

While, in general, students within the observed classroom appeared to have higher perceptions of autonomy and relatedness, there were many students who did not appear to have a high degree of either, as is reported by the range. Additionally, the relationship between the SDT measures appeared strong for the observed class, as well as in comparison to the remainder of the sample of the study. The results of the correlation analysis are presented in Table 4.5.

Table 4.5. *Correlations for Observed Class and Larger Sample.*

		Overall Sample			
		Autonomy	Competence	Relatedness	Mathtalk
Observed Class	Autonomy	-	.51** <i>n</i> = 156	.41** <i>n</i> = 156	.42** <i>n</i> = 156
	Competence	.62** <i>n</i> = 20	-	.30** <i>n</i> = 156	.47** <i>n</i> = 156
	Relatedness	.54* <i>n</i> = 20	.35 <i>n</i> = 20	-	.31** <i>n</i> = 156
	Mathtalk	.48* <i>n</i> = 20	.40 <i>n</i> = 20	.48* <i>n</i> = 20	-

* $p < .05$, ** $p < .01$

Note: The “overall sample” represented in this table does not include the observed class.

While two r -coefficients were not found to be statistically significant for the observed class, the lower sample size for the observed class should be noted as this reduced the statistical power. Rather, the purpose of reporting these coefficients along with those of the larger sample is to show the consistency of the relationship overall and within the observed class. Using the Fisher r -to- z transformation, differences between the correlations of the overall sample and the class sample were compared. The correlations for *competence* and *autonomy* ($Z = .63$, $p = .26$), *competence* and *relatedness* ($Z = .22$, $p = .41$), *autonomy* and *relatedness* ($Z = .66$, $p = .25$), *autonomy* and *mathtalk* ($Z = .29$, $p = .39$), *competence* and *mathtalk* ($Z = -.34$, $p = .37$), and *relatedness* and *mathtalk* ($Z = .79$, $p = .21$) were all found to be statistically similar. Even with the lower sample size, the correlations appeared strong within the class and were not statistically different than those of the larger sample.

Taken together, the correlation analysis of both the class and the rest of the sample suggest that there is a relationship between students' perceived needs of autonomy, competence and relatedness with their self-reported discursive actions (i.e. justification, explanation, listening). Therefore, the quantitative findings presented here provide some support for the proposed relationship outlined in the literature review as well as a partial answer to the first research question. The more self-regulated or self-determined these students perceived themselves to be, the higher their *mathtalk* scores appeared to be.

Qualitative Findings

Univocal Discourse.

The episodes selected that characterizes mathematical discourse in a univocal setting were taken from the third and fifth observations. The topic of the lesson in the third observation was geometric constructions, and the topic in the fifth observation was angle relations. During the third observation, the teacher was directing students in making constructions within class. The teacher was the dominant voice and provided mostly procedural instructions. As is typical in univocal discourse, the discussion followed the familiar IRE (initiation-response-evaluation) pattern (Schleppegrell, 2007). In the excerpt below, the teacher was prompting students to “do some thinking” on how they were going to construct angles measuring 22.5 degrees, given their current construction. This prompting came directly after the class had constructed a perpendicular line.

Teacher: Okay, so I'm thinking that we need to construct a $22\frac{1}{2}$ and a $22\frac{1}{2}$ that share a common side, \overline{TA} . Now, let's do some thinking. We're not going to use any protractors, so how am I going to get a 22 and –

Liz: Do one fourth.

Teacher: Hmm?

Liz: A fourth. One fourth of 90.

Teacher: One fourth of 90 works, beautifully. Cause half of 90 is what?

Liz: 30.

Robert: 45.

Teacher: 45, and half of 45 is?

Greg: Twenty two point five.

Teacher: 22.5, okay. But I need to be up here. Cause if I use these two sides, my angles will go this way. So, guess what I need to do.

Liz: Gonna have to draw it there.

As one can tell, the focus of this conversation is on the procedures of constructing angles and not on the concepts underlying their properties. Therefore the participants are using predominately procedural talk in this episode. The first procedural response from a student, Liz, is given after the teacher asks ‘how’ she was going to construct a $22\frac{1}{2}$ degree angle. Liz’s procedural response prompts the teacher to begin the procedure “do one fourth” and as the teacher engages in this, she asks the class for the calculated answers to her steps: “Cause half of 90 is what?...and half of 45 is?” This line of simplistic questioning is punctuated by another prompt for a procedural response by Liz. Such a sequence was typical of the class session in which the majority of dialogue included the teacher providing procedures and prompting students for simplistic replies to keep them vocally engaged. There was occasional procedural talk on the part of the students, but little, if any, conceptual talk.

What is interesting in this excerpt of univocal discourse is Liz’s interruption of the teacher to provide the procedural response the teacher was seeking. Directly after the teacher

outlines what the class needs to do, Liz's hand goes to the side of her head and her brow scrunches as she visually appears to think, preempting the teacher's comment "now, let's do some thinking," by a fraction of a second. Since this concentration appeared to have been prompted by the teacher's stating of the task, we can reasonably assume that Liz had taken it upon herself to solve this task. She most likely felt competent enough and did not seem to feel that she needed to wait for the teacher's explanation of how to go about solving the problem. As soon as Liz heard the teacher's prompting question "how," Liz voiced the procedure which she had appeared to have come up with when presented with the mathematical task, without fear of social penalty.

In examining other students' physical reactions, the majority of the students appeared to be working on their constructions. Robert did not provide a procedural response here, but in many observations he was observed to state answers, procedures, or conceptual reasons but not loud enough for the entire class or teacher to hear them. In this particular episode, Robert was observed exhaling, a gesture that appeared to represent boredom in this scenario. Greg scrunched his brow around the same time as Liz, but after looking briefly at his paper and twirling his pencil, he looked back up at the teacher. Greg's gesturing seemed to indicate his mental examination of the problem, but he appeared content to wait for the teacher's explanation, rather than initiate the development of his own.

Robert, Greg, and Liz appeared to have slightly different physical reactions to the posed mathematical task by the teacher. Yet it was Liz who initially responds, and does so before the teacher finished asking for the procedure. As the teacher approved of, and then carried out Liz's procedure, Robert and Greg contributed to the discourse, although with very simplistic responses. The other students in the classroom appeared focused on what the teacher was doing

on the overhead during this exchange. Yet most students remain silent and do not contribute to this brief exchange. When the teacher prompted for another procedure, the only one who voiced a possible procedure was, again, Liz.

All three students who engaged in this short episode of discourse were, by all accounts, bright students. Throughout all observations and visits to the class, each student appeared to be very competent in mathematics and were regarded as such by their teacher. However, Liz typically favored the quickest solution strategy for the sake of its quickness. Therefore, while she appeared self motivated, competent, and related well with her peers, she did not appear to value the mathematics but rather the successful completion of her mathematics course. Both Greg and Robert did appear to value the mathematics but Robert appeared to perceive his fellow classmates did not like him, often making comments in this regard, and this appeared to influence the frequency he spoke in class, as was evident throughout various observations of the class. Greg seemed to relate well with his peers but appeared to value the teacher's explanations over his own. For example, on one separate occasion, Greg requested that the teacher tell him all possible explanations for how a certain problem could be solved so that he could memorize them.

During the fifth observation, the teacher had passed out a sheet with different figures that students were to work on together to find the values of missing angles. After students had worked on the problems, the teacher began going over the problems in class. It is in this review that a univocal discourse setting manifested. The teacher would primarily ask students for the answers and how they got them. The discourse between students and teacher was primarily teacher directed, as was evident by students referring to the teacher, rather than the class or other students. The excerpt that follows was taken from a very interactive session of the discussion

during this lesson. The figure that students had worked on with partners and small groups was displayed on the overhead and the teacher was asking students to fill in the missing values they had gotten as well as how they knew their answers were correct.

Teacher: I know that—what’s this one right here? Cause that’s a linear pair.

Mark: 68?

Teacher: 68 and 68. Okay.

Mark: 62 and 62.

Teacher: 62 and 62 here and what? 112?

Beth: Yeah 112, up (pointing at a different angle on the projection).

Mark: No that’s 118 (talking to the teacher).

Mark: 112 is where it equals, the bold one (both Mark and Beth are pointing and directing the teacher) and then you can figure the vertical.

Greg: You can figure out B and S.

As can be seen from the excerpt, many of the responses were very simplistic in that students only provided an answer. This was characteristic of much of the discussion. However, Beth and Mark physically directed the teacher on where to go next in the figure and, by de facto, what steps should be taken next. While Beth, Mark, and Greg did not describe the procedures explicitly, they did direct the teacher through gesturing, head nods, and partial sentences to specify which angle should be looked at next. Given the nature of finding missing angles in figures, this was considered a form of procedural description, albeit without verbal or written communication.

Throughout classroom observations, Beth and Mark appeared to be average students in respect to their mathematical ability. Both were well liked by other students and the teacher. They were, in a sense, very typical students. Therefore, the way they interacted in this univocal setting is of particular interest. In the prior excerpt that included Liz, Greg, and Robert, each student could be characterized as demonstrating higher math ability in class. This appeared to play a role in the prior excerpt since it was these individuals who spoke procedurally in a more “toned down” discussion. This latter excerpt contrasts the prior in that the vast majority of students were interactively involved in the discussion over the course of the lesson, while the teacher directed the course of the discussion. Various students called out answers and directed the teacher. Beth and Mark’s actions typified those of the majority of students in the lesson.

What seemed to characterize individuals who provided procedural responses in the first excerpt was that such students appeared to be more competent math students. Given the way the class generally acted towards them, students in the second excerpt appeared to be of average math ability. Yet, in both excerpts there were students who did not engage in class dialogue. Two of these students, Carol and Cindy, did not provide any statements in either lesson portrayed here. The rare participation of these two students will be discussed in a later section.

Probing Setting.

While the probing setting is still a univocal setting, it can be described as univocal discourse on the verge of being dialogic. The episode selected for deeper analysis came from the fifth observation. The class was reviewing a set of homework problems dealing with properties of isosceles, equilateral, and scalene triangles. Much of the conversation focused on an end result of finding ‘the answer,’ but the teacher used the topic to encourage students to describe their procedures by simultaneously discussing the underlying concepts. In the excerpt below, the class

discussed first a problem dealing with an isosceles triangle and then one with an equilateral triangle. The excerpt begins after the class has found the measures of two congruent sides of an isosceles triangle.

Teacher: Does that help us find the length of the base?

Greg: Yeah.

Teacher: How?

Greg: Well, uh. No it couldn't (motions hands to show moving sides with varying angles then puts his hand up to his mouth).

Beth: No...no consistent angle.

Teacher: Yeah, your angle could...is certainly not. You've gotta find out about the angle right? (Greg nods). An equilateral triangle has a perimeter of 54. Now, Chase would you like to help us out with this one because you were doing a good job before.

Chase: Really?

Teacher: Mmm hmm.

Chase: Uh, okay. Are, are you really sure?

Laura: Chase!

Chase: Okay. So you've got yourself an equilateral triangle and since all three sides are the same length and one side is $5x - 2$, you apply $5x - 2$ to all of it. Instead of writing $5x - 2$, yeah, you just multiply it by three to apply it to all three sides. And that equals the perimeter. Cause that [gets] you the sides (*makes triangle with his arms*). So, that's—

Larry: What?

Chase: See I can't explain things.

Teacher: No, that's a—I wouldn't have done it that way but that's fine. What you're saying is to multiply this by three, right? Because you've got three of them.

Chase: Yeah.

Characteristic of the probing setting described by Hancewicz (2005), the teacher is prompting students to provide conceptual reasons. The teacher was still the dominant voice in the discussion, since the students were not controlling the direction of the discourse. In the first part of this scene, Greg provided a simple yes answer to the teacher's first attempt to get students to provide a conceptual response. In response, the teacher asked him to justify "how" knowing two sides of an isosceles triangle helps you find the base. Greg, realizing his error, made an attempt to describe why it couldn't, but when he was unable to verbalize his thoughts, he began trying to illustrate the reason with hand gestures. Beth used this verbal pause in talk to provide a conceptual reason "no consistent angle." Both Greg and Beth's attempts at producing conceptual talk appeared inherently tied to the teacher's prompting. By presenting the students with a procedural task that would not produce an answer, the teacher forced the students to provide a conceptual reason why a procedure was not possible. Typically the teacher asking students "how" would produce procedural talk as a student may explain the steps one needs to take in order to find a solution. However, since a reliable solution did not seem possible to the students, they produced conceptual talk instead.

Another characteristic of both Greg and Beth's responses was the brevity of their statements. Even though Beth provided a reason for why having the measurement of the two congruent sides could not tell you the measurement of the base, her description was short. It included a description of a property of triangles with two congruent sides that would allow for

someone to reach the reasoned conclusion. Therefore, in some ways, Beth's conceptual description was incomplete. It did not provide enough detail to properly support her claim. In this sense, Beth provided a conjecture as to why one could not determine the measurement of the base, but did nothing to mathematically prove this statement. Rather, the teacher's verification of the conjecture seemed to be proof for the class.

The teacher made a quick transition to the next problem after confirming the conceptual reason provided verbally by Beth and physically by Greg. Such a quick transition between problems is typical of many teachers when reviewing homework. After the teacher asked Chase to go over the problem with an equilateral triangle, Chase produced both procedural and conceptual talk. The equilateral triangle problem provided one side of the triangle, $5x - 2$, and the perimeter, 54. The students were required to find the value of x .

As Chase explained his procedures, thereby producing procedural talk, he provided justification based on underlying concepts for these procedures. For example, since all three sides were congruent and the perimeter equaled 54, Chase stated that if one were to multiply one side by three that it would be the same measure as the perimeter. The justification provided by Chase, however brief, is characteristic of conceptual talk. Further, it was integrated with procedural talk rather than isolated from it. Notably, the teacher was still needed for verification of both the procedural and conceptual talk produced by Chase.

Chase's reluctance to explain the problem was not characteristic of any fear of social penalty. Throughout observations of the class, Chase showed little fear of speaking or presenting to the class and appeared to be well liked by his classmates. He did, however, seem unsure of his ability to do the mathematics correctly. Often in class, he would voice concerns over the correctness of his methods or answers. Therefore, the reluctance expressed in the excerpt above

can be described as fairly typical of Chase. When another individual, Larry, challenged Chase's strategy with a simple question of "what," it was enough to recall Chase's attempts to explain. Again, this did not seem to be due to Chase's connection to the social environment, except for the response from Larry acting as a catalyst, but rather the ineptness of Chase's perceived competence in mathematics.

In response to Chase's cessation of explaining and justifying his solution strategy, the teacher provided supportive feedback. The teacher confirmed that Chase's strategy was acceptable and that it would get the answer to the problem. However, a key statement at the beginning of the teacher's response was "I wouldn't have done it that way but that's fine." This short statement validated Chase's strategy as well as validating other strategies that had not been presented. We can assume this was the teacher's reasoning since shortly afterwards she asked for solution strategies that may have been somewhat different than Chase's. Additionally, the teacher would frequently ask students for other ways of obtaining solving problems. By identifying Chase's strategy as one of many valid ways to obtain the answer, it may have improved Chase's perception of competence while maintaining that other students' methods may also have been valid.

In reviewing both problems, the teacher was a critical part of the discourse that took place. The teacher validated both the procedural and conceptual talk produced as well as prompting the students to produce it. Such actions are typical of the probing setting described by Hancewicz (2005) and also of univocal discourse described by Kitchen (2004). What is interesting in examining this episode of discourse in a probing setting is the way that conceptual talk was used. Greg and Beth made conjectures without an attempt to prove them. No one challenged their statements or the teacher's validation of them. The same holds true for Chase's

conceptual statements. Larry expressed some confusion as to the procedure Chase used, but not his reasoning behind it. In searching for disconfirmatory evidence, such actions seem characteristic of the univocal discourse in this class. Students may have challenged the teacher's or another student's procedural method, but did not appear to challenge their reasoning. However, such distinctions break down when one or more individuals attempt to guide the conversation in a different direction. Such situations appear characteristic of dialogic discourse.

Dialogic Discourse.

In the class sessions observed, dialogic discourse seemed most prevalent when the teacher had students come to the front of the classroom. The typical scenario would include the teacher assigning each pair of students with a problem and having each pair come up to the front of the class and explain their strategy. If the problem was not pre-assigned, such as a homework problem, then the class would have a certain amount of time to prepare themselves before presenting their problems. The teacher would generally stand over to the side or find a place amongst the students to position herself. In either case, her repositioning of herself yielded the physical position that the teacher would typically be found in most math classrooms to the students.

During the ninth observation, the class was studying how to construct proofs. Students were provided a worksheet with mathematical situations they were asked to construct proofs for. After the teacher went over a few problems with the class, she asked for student pairs to volunteer and present different problems. At one point the teacher asked Greg and Robert to explain their problem, in which they were to prove two triangles were congruent. The teacher moves to the adjoining room to answer the phone but returns briefly after Robert and Greg begin their description.

Robert: Okay. So obviously we start out with sta-oh yeah, with statement (begins writing slowly).

Rebecca (pointing to the overhead): Can you guys say, can you say if they're parallel?

Robert: Yeah.

Rebecca: Thank you.

Teacher: And could you please mark the givens.

Rebecca: Or just put them at the bottom of that sheet.

Robert: Yeah sure. Well this, there's plenty of room here to...

Xavier: No wait, you're supposed to mark it on the picture.

Teacher: Okay so we've got that big bad angle.

Robert: Okay, so then we have our givens. So we have that. Okay.

Greg: Hold up. You've got to mark up the givens, B A D.

Robert: Yeah I was going to do that after I wrote the givens statement.

As Robert began the proof, Rebecca, Xavier, and Greg made comments about the procedures he was taking in writing it up. Although none of the students were discussing concepts at this point, it is important to note that the direction of the discourse was guided by the students and their procedural talk. We call this procedural talk here because it is distinctly mathematical in that the students were concerned with how to go about setting up a proper proof.

Once the issue of setting up the proof was resolved, Robert proceeded to explain what he and Greg did to prove the triangles were congruent, while Greg made the occasional comment. At one point, Robert provided his reasoning as to what needed to be done to prove the triangles were congruent:

Robert: Okay. AB equals BC. Now from there, we're trying to figure—we want to have three things. Right now we have side side angle but we know that can't work because sides could just go in any direction. So we have to find the angle here or a side here (pointing at parts of the triangles) to prove the triangles are the same.

Teacher: Or?

Robert: Or, an ang-

Greg: Interior angle?

Liz: Hypotenuse leg.

Rebecca: The definition of a right triangle, hypotenuse leg.

Teacher (speaking at same time as Liz & Rebecca): What angle do you use if you've got it? (acknowledging Liz & Rebecca) Hypotenuse leg.

Robert: Hypotenuse leg. You are absolutely right.

Chase: I was going to say that!

Greg: I don't understand how that works.

Robert: Of course—okay, well I guess you could do that but I also don't understand how it works so I think there's another way to do it.

The conversation concerning this proof continued for the majority of the class period. However, the beginnings of the conversation illustrate how students slowly moved from producing procedural talk to more conceptual talk. In the excerpt above, Robert was the one primarily directing the course of the conversation and producing the conceptual talk. The teacher then asked for an alternate reason or justification for proving the triangles congruent. At this point, Liz provided an alternate method for proving the triangles congruent, “hypotenuse leg,” while

Rebecca immediately provided the reasoning and the method, “definition of a right triangle, hypotenuse leg.” The teacher’s act of facilitating here is characteristic of dialogic discourse. Rather than having one individual act as the sole describer, she allowed other students the chance to provide their own reasoning. In turn, Robert acknowledged their reasoning and method but both he and Jarrett proposed an alternate method that the rest of the class was not aware of.

The class continued to discuss the congruency of the two triangles. As Robert and Greg described their method, several students voiced agreement through gestures such as head nodding and gasps or through comments such as Rebecca’s “I get this now, that’s exciting.” However, Liz was notably irritated that the class took the amount of time they did discussing the alternate method proposed by Robert and Greg. While Laura, Rebecca, and Chase were very supportive and encouraging of Robert’s description, Liz voiced her irritation:

Liz: Instead of spending like 2 seconds on homework we spend like three hours.

Robert: Well I didn’t realize that the hypotenuse leg was there. So I’m sorry.

Teacher: That’s alright.

Liz: Apology accepted.

Even though there seemed to be plenty of praise and positive feedback from several students, Liz’s words seemed to stick with Robert. As Robert and Greg finished and were making their way to their seats Robert commented, “I am very sorry for wasting our time and I know you hate me...” This was followed by encouraging words from Laura and Rebecca. Additionally, the teacher made sure to spend time addressing the issue.

Teacher: You know what? I want to nip that in the bud. What did I tell you all in the beginning of the year?

Robert: Think.

Teacher: And, in what did I say that I would celebrate? If you could find another way of going about it, and I wasn't meaning just proofs. I mean in finding out the answers to problems or whatever. And I still mean that. And I love people who think outside the box.

Even though the teacher had made specific attempts to address the negative comment made by Liz, the damage to the discourse was done. After Liz made her comment, Robert and Greg made briefer comments than before and the teacher took a more dominant role in the conversation as the setting began to resemble the probing setting rather than the dialogic setting that had been present.

Another example of dialogic discourse was selected from the sixth observation. During this lesson the students and teacher were talking about angle relationships. Prior to the excerpt presented here, the discussion had become too unstructured to the point that the teacher needed to intervene so that all students would have a chance to participate.

Teacher: Okay you guys. Shhh. We're going to go table to table. When other people are talking—and for now it's going to be ya'll (pointing to Noah and Xavier) that's going to give us some words of wisdom. The rest of us will need to listen...cause they may not be right.

Xavier: No, we're right.

Chase: Xavier is going to be right.

Teacher: Okay. So give us a pearl.

Xavier: Does it matter where? Okay. I. Okay, I is 40.

Teacher: This one?

Xavier: Yeah.

Jasmine: That's what I said.

Teacher: Please tell me how.

Xavier: 70...

Teacher: Yeah?

Xavier: The, is a linear pair. Or no, you could also do that (pointing at the overhead display). The linear pair is one and vertical is 70. And then you subtract 40 from 70 which is...no no no wait. We're trying to get L. Nevermind. So then (looking at Noah)—
How did we get that part?

(class laughs)

Noah: No. Okay. We did. We have 70 up in the left corner and we did 70 times, I mean 180 minus 70 (gestures to teacher as she writes on the overhead). No like, left...there you go. And we got 110 and then we do—

Teacher: 110 here?

Noah: Mmm hmm. Then opposite interior we get 110 on the other side right there (pointing). Then when you do 180 minus 110 you get 70 and then...

This exchange continued between Noah and the Teacher until Chase interrupts:

Chase: Um, can't you not go all through that and hit the little, the one that has two dashes (pointing at the figure). No the other one that automatically equals 70. Consecutive, um, interior pair ones, whatever that's called for K. Then use the whole triangle to get 40 instead of getting all those little set pieces 110 and everything.

The above excerpt contrasts the prior dialogic excerpt in that Chase's interruption was seen as a justified and welcomed redirection of the discourse. In the prior excerpt, Liz's interruptions and attempts to redirect the discourse were not welcomed by the teacher or several of the students. Additionally, Chase's redirection was mathematical in nature while Liz's redirection was not.

Similar to the prior excerpt, this second excerpt shows students using both procedural and conceptual talk. However, many of the conceptual components of their talk was often used in a procedural manner. Rather, it was inherently linked to their procedural talk. For example, when Noah stated "then opposite interior we get 110 on the other side right there," he casually described why a different angle was 110 degrees by using the justification as a procedure. Instead of saying, "this angle is 110 because it is an opposite interior angle from the one above," he reversed this order so that the justification was used to imply an action rather than simply supporting one.

One interesting aspect of the second excerpt is Chase's willingness to posit a different approach. Throughout observations of the class it was apparent that Noah and Xavier were considered by their peers to be the smartest students in the class. Yet Chase seemed to be good friends with both Noah and Xavier, thereby making such an interruption more common than it

might otherwise have been. As a contrast, Robert, who seemed to feel he was not well liked by his classmates, rarely interrupted any of his fellow students. The exception to this was his partner, Greg, whom Robert worked with regularly. Robert's seemingly low self perception of his social status was somewhat strange in that almost all of his fellow students treated him well and seemed to value his insights when he did give them.

Rare-participants in Discourse

The previous examples concerning univocal and dialogic discourse described those students that actually participated. However, there were some students that rarely participated in class discussions. Of interest are Bruce, Carol and Cindy. None of these students typified the other, but, rather, each was unique in how they interacted with the class. Bruce sat behind Chase and was a friend of his. This was apparent in their conversations before, during, and after class. Bruce would often not talk voluntarily in class but when the teacher would ask him to explain or describe something, he would do so readily. Carol similarly would explain or describe her mathematics when asked, but was less willing to talk in front of her fellow students. During one class with an intense dialogic discussion, Carol sat quietly until the end of class when she walked up to the teacher to ask questions that she had not wanted to ask during class. Cindy could be characterized as a student who had math anxiety. She would engage socially with her fellow students but actively avoided any discussion about mathematics and acted as if she knew she could not do mathematics. For example, in one class Cindy expressed concern because she actually *understood* the mathematics.

Cindy: I was confused with this homework because it was too easy and that concerned me.

Teacher: (laughs) I told you it was easy.

Cindy: No but it concerned me because everytime I do that, I totally do it wrong. Cause I thought it was something...

Chase: Take a chill pill Cindy.

(class laughs)

During a separate class, the teacher was asking students to explain different problems through proof to the rest of the class. Chase had finished writing a proof and handed the overhead pen to Cindy, indicating she was next to write a proof.

Cindy: No! No don't! (tries to hand pen back to Chase) No, get that thing away from me—here!

Xavier: We'll take it Cindy. We'll take it again Chase.

Teacher: Cindy. Cindy. This is easy, we'll help you.

Cindy: (squealing voice) I don't know how to do it!

Teacher: Yeah you do.

Liz: You can do it.

After a few more attempts to escape from explaining a problem to the class, the Teacher tells Cindy that Carol will go up with her to help.

Cindy: Yeah she can do it (laughs and hands pen to Carol, then looking at Carol) Sorry, I don't know what to do.

Carol: I don't know what to do either.

Cindy: Oh that's not good.

Teacher: So, folks. Let's help them out. (3 seconds). We're going to use the collective brain and help them out. Say it loudly please.

Several students provide suggestions on how Cindy and Carol can go about starting the proof. After Cindy finishes writing the proof, the teacher makes a reassuring statement.

Teacher: It wasn't so bad. Got all you know. You made it, right?

Cindy: Well I didn't do anything.

This active avoidance of mathematics was characteristic of Cindy and she rarely, if ever engaged in talking about mathematics. Carol, also present in this episode, avoided explaining how to write the proof, indicating that she was afraid of making a mistake in front of the class. Since Carol was willing, when asked to explain problems she seemed to understand, this situation appeared to exemplify a situation where Carol did not fully understand how she was to do a mathematical task.

Cindy's last statement is of significance in establishing her math anxiety. The other students helped her to begin the proof, but Cindy did a large portion of the work. Yet, Cindy did not seem to see herself as having a significant role in writing the proof. "I didn't do anything," appeared to imply that she did not feel that she contributed anything of value.

*Mixing of Data**Univocal Discourse.*

Qualitative analysis of the univocal setting illustrated how different students engaged in producing procedural talk. What was apparent in the qualitative analysis was that students who perceived themselves as more competent were the ones involved in the discourse. In Table 4.6, scores from the SDT survey support this observation. All but one student in this exchange had relatively high perceptions of mathematical competence. While all students seemed to have a positive perception of autonomy, Liz's perceived autonomy and relatedness were extremely high. This could help explain why she felt able to voice her procedures as soon as she saw the teacher provide the opportunity to do so.

Table 4.6. *Procedural Talk in Univocal Setting.*

	<i>Autonomy</i>	<i>Competence</i>	<i>Relatedness</i>	<i>Mathtalk</i>
Liz	7.00	5.67	7.00	6.67
Greg	4.86	5.33	4.38	4.00
Robert	4.71	5.17	3.63	3.00
Mark	4.71	5.00	5.38	5.33
Beth	4.43	3.33	6.75	3.00
Group Mean	5.14	4.90	5.43	4.40
Class Mean	5.07	4.54	5.48	4.06
School Mean	4.41	4.20	4.59	3.91

Mark and Beth were partners during math class and during observations appeared to typify average students in terms of ability. The two students demonstrated relatively similar ability as most of the students within the class. When examining their perceived competence, however, it appears that Mark considered himself slightly more competent and Beth considered

herself slightly less competent. Even though Beth had a lower perceived competence in mathematics, the overall trend for the students in these excerpts was that they perceived themselves as relatively competent in mathematics ($\bar{X} = 4.90$), autonomous ($\bar{X} = 5.14$), and related with their peers ($\bar{X} = 5.43$).

Probing Setting.

Results from the qualitative analysis of the probing setting illustrated two different ways of producing conceptual talk within that setting. Greg and Beth both gave brief statements when producing conceptual talk, while Chase gave a deeper explanation that was thoroughly connected to his procedures. Table 4.7 illustrates the differences in perceived autonomy, competence, and relatedness in these individuals. What is important to note is that Greg and Beth had similar perceptions of autonomy, but they had slightly different scores for *competence* and *relatedness*. Taking this into account, the difference between perceived needs for these individuals appears most dramatic in their perceptions of competence. All students appeared to have positive perceptions of autonomy, but Chase's was much higher than either Greg or Beth's.

Table 4.7. *Probing Setting.*

		<i>Autonomy</i>	<i>Competence</i>	<i>Relatedness</i>	<i>Mathtalk</i>
Brief Conceptual Talk	Greg	4.86	5.33	4.38	4.00
	Beth	4.43	3.33	6.75	3.00
Deeper Conceptual Talk	Chase*	5.40	3.80	6.00	-
Group Mean		4.90	4.15	5.71	3.50
Class Mean		5.07	4.54	5.48	4.06
School Mean		4.41	4.20	4.59	3.91

*Chase did not complete the items for the *mathtalk* composite.

Even though Chase did not appear to feel very competent from either the qualitative or quantitative data, his perceived autonomy allowed him to come up with and present a distinctly different method for finding the value of x on the equilateral triangle. Additionally, he was able to provide reasoning for his method without direct prompting from the teacher. Only his lower level of competence seemed to keep Chase from completing his explanation with a full justification of his solution strategy.

Beth's level of competence appeared to be at a similar level as Chase. Hypothetically, it would seem that if challenged in a similar manner as Chase was, Beth would demonstrate similar actions. Unfortunately, Beth was never observed being challenged by another student while giving an explanation. It may be that Beth never positioned herself in a situation where a challenge would be made. Greg did receive challenges to some of his explanations and was able to continue participating in discussions about mathematics. As shown in Table 4.7, Greg had a higher perception of mathematical competence and a relatively higher perception of autonomy.

Dialogic Discourse.

The beginning of the first excerpt analyzed for the dialogic setting displayed students discussing the proper procedures for beginning a proof. Similar to the univocal setting, a perceived level of competence appeared necessary for students to participate in this student-led discussion (see Table 4.8). The students who participated in this episode also appeared to have a higher level of autonomy, relative to the scale of the survey. Some students had a higher perception of relatedness while others, such as Robert, appeared able to engage in the discussion because of their higher sense of autonomy and competence.

Table 4.8. *Procedural Talk in Dialogic Episode.*

	<i>Autonomy</i>	<i>Competence</i>	<i>Relatedness</i>	<i>Mathtalk</i>
Rebecca	5.86	4.83	4.63	3.33
Xavier	5.43	5.33	5.63	-
Robert	4.71	5.17	3.63	3.00
Gregg	4.86	5.33	4.38	4.00
Group Average	5.22	5.17	4.57	3.44
Class Mean	5.07	4.54	5.48	4.06
School Mean	4.41	4.20	4.59	3.91

What appeared to be the most interesting part of the first episode presented in the qualitative analysis was Liz's irritation of how long the discussion over the proof took. In Table 4.9, mean scores for each psychological need and student scores for *mathtalk* are presented. While Liz's scores are all quite high for each SDT need and the quality of her math talk, she appears to value a quick answer rather than the mathematics itself. On the contrary, Chase, Laura, and Rebecca appear to have an unimpressive perception of their own competence in mathematics and do not perceive themselves as producing higher quality mathematical talk. Yet, these students appeared to be supportive of Robert's explanations and can therefore be considered supportive of other students' relatedness.

Table 4.9. *Student Support of Dialogic Discourse.*

		<i>Autonomy</i>	<i>Competence</i>	<i>Relatedness</i>	<i>Mathtalk</i>
	Chase	5.40	3.80	6.00	3.33
Supportive	Laura	5.57	3.67	6.00	-
	Rebecca	5.86	4.83	4.63	3.33
Unsupportive	Liz	7.00	5.67	7.00	6.67
Group Average		5.96	4.49	5.91	4.44
Class Mean		5.07	4.54	5.48	4.06
School Mean		4.41	4.20	4.59	3.91

What is particularly notable from the mixing of data in Table 4.6 is that Liz, a student who was vocally unsupportive of dialogic discourse, appeared to have such a high perception of all three SDT needs *and* the quality of her own mathematical talk. Students who were supportive of the dialogic discourse that was taking place also had acceptable levels of autonomy, competence, and relatedness, therefore implying that internalization of mathematics may have been supported by the fulfillment of the SDT needs. Yet, fulfillment of the needs and production of more sophisticated mathematical talk did not necessarily translate into the support of others' needs, as is the case with Liz.

Rare-participants.

Qualitative findings suggested that there were those students who rarely participated in classroom discussions about mathematics. Some of these students, such as Bruce, were willing and able to participate, but simply were not observed participating as frequently as other students. Additionally, when he did participate, Bruce would produce both procedural and conceptual talk. Students such as Cindy were observed to actively avoid discussing mathematics in any form, and seldom made mathematical statements more sophisticated than a simple answer. Students such as Carol appeared not to want to speak in front of their fellow students, but would

describe their procedures when asked. Carol, specifically, would talk about her mathematics in some classes, but would remain silent in other class sessions. In those class sessions where she remained silent, she would come up to the teacher at the end of the class and proceed to ask her about the topic of that day. This appeared to signify that Carol was willing to talk about her mathematics when she was relatively comfortable with the material, but would remain silent if she did not feel comfortable enough with the topic.

Table 4.10. *Rare-Participants in Mathematical Discussions.*

	Autonomy	Competence	Relatedness	Mathtalk
Cindy	4.00	4.33	6.13	2.00
Carol	4.86	2.33	5.43	5.33
Bruce	4.86	5.00	6.50	4.67
Class Mean	5.07	4.54	5.48	4.06
School Mean	4.41	4.20	4.59	3.91

Table 4.10 shows that Carol and Bruce had relatively positive autonomy as compared to the school and their autonomy scores were near that of the class mean. However, Cindy's autonomy was the lowest score in the class (range = 4.00 – 7.00). Only one other student, Lucy, had as low of an autonomy score (see Appendix B for further information). Carol's low competence score signifies her apparent actions to avoid talking in front of her classmates when she was unsure of her mathematics. What the quantitative and qualitative data indicate is that many students may have different reasons for not participating in mathematical discussions. For Bruce, he simply may not have felt the need to make statements at certain points. He did engage in discussion, but less so than many other students. For Carol, she only participated when she felt she was able to do the mathematics being discussed. For Cindy, there was an active avoidance of mathematics, as was apparent in the qualitative analysis.

Differences between Frequency of Participation and Quality of Mathematical Talk.

While the prior analysis characterizes specific aspects of mathematical discourse as related to participants, a larger view of the classroom's discourse environment should be provided. Prior to selecting the episodes of univocal and dialogic discourse, a surface analysis was conducted of each observed lesson. Certain students were categorized as more generally engaged in discussion and others as less generally engaged. This classification of engagement was based on their observed participation in different discussions. Additionally, certain students were observed to produce only answers, while others would make procedural and/or conceptual statements. For the purposes of simplicity, only the results from the second observation are presented here. Results of merging this qualitative analysis with available quantitative data are presented in Table 4.11. Students who produced at least one procedural statement were categorized as producing procedural talk. A similar criterion was used for conceptual talk and simplistic talk. Students were assigned to only one talk category; the category they produced their most sophisticated statement in the class session.

Table 4.11 *Trends in Frequency and Quality of Math Talk.*

	More Engaged	Less Engaged
Simplistic Talk	$n=0$	$n=8$ <i>Autonomy =4.69</i> <i>Competence =4.17</i> <i>Relatedness =5.99</i>
Procedural Talk	$n=4$ <i>Autonomy=5.14</i> <i>Competence=4.33</i> <i>Relatedness =5.44</i>	$n=2$ <i>Autonomy =5.43</i> <i>Competence =5.57</i> <i>Relatedness =6.19</i>
Conceptual Talk	$n=5$ <i>Autonomy =5.51</i> <i>Competence =5.03</i> <i>Relatedness=5.33</i>	$n=0$

Note: 1 student absent during observation.

As can be seen in Table 4.11, students who produced higher forms of mathematical talk had higher perceived autonomy. Students who produced simplistic talk had lower competence than all other students, but higher relatedness than students who produced conceptual talk. While the results presented for competence and relatedness may seem somewhat mixed, one should be reminded that it was autonomy that appeared to be the key psychological need that related to students' production of mathematical statements. To effectively sustain such statements, however, relatedness and competence needed to be higher, as is evidenced in the previous sections. Table 4.11 reflects this relationship since students were categorized for making at least one such statement type (simplistic, procedural, conceptual).

Table 4.11 provides a detailed example of one observed class session. A similar analysis of two randomly selected sessions found similar trends. Notably, individual students would sometimes appear in different cells of Table 4.11. For example, a student who was classified in Table 4.11 as less engaged, procedural talk would appear in the more engaged, conceptual talk category in another class session. Some students were categorized in the two empty cells in other class sessions as well (more engaged, simplistic talk; less engaged, conceptual talk), but this happened with only one or two students. What is important to note about Table 4.11 is that while the same students may participate in mathematical discussions differently in other class sessions, the general pattern observed here appeared to hold when other class sessions were examined. Namely, more engagement and more sophisticated talk was typically exemplified by students with higher autonomy and competence scores.

Validity, Trustworthiness, and Inference Quality

Internal validity of the scores from the quantitative analysis came in the form of alpha coefficients. These scores allowed for reasonable confidence in the reliability of the composites

used in the quantitative and mixed methods analysis. Additionally, the correlation analysis was conducted to ensure that the observed class was sufficiently illustrating the relationships posited by the theoretical framework of this study. Specifically, the lack of statistical difference in the r -coefficients, and not the statistical significance of the observed class's relationship, was used to determine similar trends within the class to that of the larger sample.

The validity or trustworthiness of the qualitative analysis came from the use of varying sources of data which included field notes, video and audio recordings, and artifacts collected from the class. Reflective and continuous use of these varying sources of data was used to ensure a more accurate illustration of the excerpts of discourse analyzed. Additionally, the use of the literature as criteria for selecting excerpts helped to limit the influence of researcher bias.

Inference quality is the term generally used for validity in mixed methods research (Teddlie & Tashakkori, 2003). It describes the validity of making certain inferences from the mixing of both quantitative and qualitative data. The inference quality of the current study was supported by the validity of both quantitative and qualitative analysis. Additionally, the inference quality of the mixing of data was supported by its match with the theoretical construct developed and the match of the types of data merged; discourse actions with psychological factors that may influence such actions. The correlation analysis conducted in the quantitative portion of the study supports such a construct, as a strong relationship appears to exist between the psychological needs and discourse actions.

Conclusion

In general, the results of the current investigation support a connection between Self-Determination Theory and students' mathematical discourse actions. Students who have higher degrees of autonomy, competence, and relatedness are more engaged in dialogic discourse. As

such, they produce more conceptual talk and at a deeper level than students with lower perceived needs. Specifically, autonomy appeared to be a defining characteristic of what prompted students to give deeper and lengthier descriptions. This was apparent in Liz's production of procedural talk in the univocal setting and Chase's lengthy description and justification of his solution strategy during the probing setting. However, these results concerning the role of autonomy are preliminary and need to be confirmed by further research.

While perceived competence and relatedness appeared to promote discursive actions of some students, it is important to note that students observed to produce conceptual talk, whether in a univocal or dialogic setting, had a higher perception of autonomy. Rather, it appeared that deficiencies in a student's perceived competence or relatedness could hinder or shorten their descriptions. Chase's cessation of his explanation after his method was questioned by a student is an example of how a lower perception of competence can hinder discursive action. Robert's actions after Liz voiced her irritation characterized how a lower perception of relatedness may hinder a student's participation in discourse. Taken altogether, it appears that students' autonomy must be fulfilled if they are to engage in more sophisticated discourse and mathematical talk. Yet, to maintain such talk so that students can discuss and answer challenges to their descriptions, their needs for competence and relatedness must be fulfilled. Otherwise, it appears that once challenged, an autonomous student will cease or limit their engagement in discussion. While the results of the current study support these conclusions, further research needs to take place to establish the generalizability of these findings.

Jansen's (2008) investigation of seventh grader's engagement in classroom discourse provides some support for the current findings. Jansen found that students who feared social penalty produced more procedural and less conceptual talk than students who did not fear social

penalty. The current study supports these findings while adding to them. Robert had a low perception of relatedness but still produced conceptual talk. However, it is believed that only his higher perceptions of autonomy and competence allowed him to do this. Another student in the class with low relatedness, Connie, also had low perceived competence and autonomy. While Robert's higher autonomy and competence are believed to have facilitated his discourse actions, Connie's lower perceived autonomy and competence appear to have done the opposite.

The determining factor in what encouraged initiation of participation in discourse in the current study appeared to be autonomy. This finding therefore supports Yackel and Cobb (1996) who characterized social norms (supportive of relatedness) and sociomathematical norms (supportive of competence) as facilitating a student's sense of autonomy. Further, the development of mathematical dispositions described by Yackel and Cobb are extended by the descriptions provided in the current study. This is evidenced by the finding that students with higher perceptions of autonomy produced more procedural and conceptual talk. Yackel and Cobb argued that students with more mathematical autonomy had formed certain mathematical dispositions that allowed them to discuss mathematics at a deeper level. The results presented in the current study support such a conclusion.

The current findings imply that mathematics education researchers can use the measures of autonomy, competence, and relatedness as advocated by SDT to further examine mathematical discourse actions and engagement. This adds a useful quantitative measure that can be used to examine mathematical discussion at a larger scale and, therefore, contribute to the current body of literature.

Since this study is, to the knowledge of the author, the first to specifically examine how students' autonomy, competence, and relatedness related to their discourse actions, future

research is needed. The role of each psychological need in mathematical discourse should be examined by other studies and compared to the findings of the current study. Further statistical analysis is needed to investigate the degree to which each measure of autonomy, competence, and relatedness relate to students' discourse actions and engagement. Also, additional quantitative and qualitative analysis of students' engagement in mathematical discussion in relation to their perceived SDT needs should take place in order to confirm and expand upon the findings of the current investigation.

The current findings support mathematics education literature that suggests the importance of encouraging students' autonomy in mathematics (e.g. Lee, 2006; Wood, 1999; Yackel & Cobb, 1996). Additionally, since students' with higher perceptions of competence and relatedness appear to maintain dialogic discourse when challenged, facilitating both of these needs may encourage deeper and more meaningful discussions about mathematics to take place.

References

- Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher*, 23(7), 13 – 20.
- Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist*, 31(3/4), 175 – 190.
- Creswell, J. W., & Plano Clark, V. L. (2007). *Designing and conducting mixed methods research*. Thousand Oaks, CA: Sage Publications, Inc.
- D'Ambrosio, B., Johnson, H., & Hobbs, L. (1995). Strategies for increasing achievement in mathematics. In R.W. Cole (Ed.), *Educating everybody's children: Diverse teaching strategies for diverse learners* (121 – 138). Alexandria, VA: Association for Supervision and Curriculum Development.
- Deci, E. L., & Ryan, R. M. (2008). *Basic psychological needs scales*. Retrieved September 4, 2008, from http://www.psych.rochester.edu/SDT/measures/needs_scl.html
- Deci, E. L., Vallerand, R. J., Pelletier, L. G., & Ryan, R. M. (1991). Motivation and education: The self-determination perspective. *Educational Psychologist*, 26(3 & 4), 325 – 346.
- Gadanidas, G., Kotsopoulos, D., & Guembel, K. W. (2006). *Factors effecting middle school teachers' choice of authoritative discourse in teaching mathematics*. In S. Alatorre, J. L. Cortina, M. Saiz, & A. Mendez (Eds.). *Proceedings of the 28th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics & Education*. Merida, Yucatan, Mexico.
- Hancewicz, E. (2005). Discourse in the mathematics classroom. In J.M. Kenney (Ed.), *Literacy strategies for improving mathematics instruction* (pp. 72 – 86). Alexandria, VA: Association for Supervision and Curriculum Development.

- Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal*, 30(2), 393 – 425.
- Hiebert, J., Stigler, J. W., Jacobs, J. K., Givven, K. B., Garnier, H., Smith, M., Hollingsworth, H., Manaster, A., Wearne, D., & Gallimore, R. (2005). Mathematics teaching in the United States today (and tomorrow): Results from the TIMSS 1999 video study. *Educational Evaluation and Policy Analysis*, 27(2), 111 – 132.
- Jansen, A. (2008). An investigation of relationships between seventh-grade students' beliefs and their participation during mathematics discussions in two classrooms. *Mathematical Thinking & Learning*, 10(1), 68 – 100.
- Katz, I., & Assor, A. (2007). When choice motivates and when it does not. *Educational Psychology Review*, 19(4), 429 – 442.
- Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal*, 102(1), 59 – 80.
- Kitchen, R. S. (2004). Challenges associated with developing discursive classrooms in high-poverty, rural schools. *Mathematics Teacher*, 97(1), 28 – 31.
- Koichu, B., Berman, A., & Moore, M. (2007). The effect of promoting heuristic literacy on the mathematical aptitude of middle-school students. *International Journal of Mathematical Education in Science and Technology*, 38(1), 1 – 17.
- Kosko, K. W., & Miyazaki, Y. (under review). *The impact of student discussion on fifth graders' mathematics achievement in the U.S.* Manuscript submitted for publication.
- Lee, C. (2006). *Language for learning mathematics: Assessment for learning in practice*. New York: Open University Press.

- Manke, M. P. (1997). *Classroom power relations. Understanding student-teacher interaction*. Mahwah, New Jersey: Lawrence Erlbaum Associates, Publishers.
- Martin, T., McCrone, S., Bower, M., & Dindyal, J. (2005). The interplay of teacher and student actions in the teaching and learning of geometric proof. *Educational Studies in Mathematics*, 60(1), 95 – 124.
- Mercer, N., & Littleton, K. (2007). *Dialogue and the development of children's thinking: A sociocultural approach*. London: Routledge.
- Mercer, N., & Sams, C. (2006). Teaching children how to use language to solve maths problems. *Language and Education*, 20(6), 507 – 528.
- National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics, Inc.
- Nunnally, J. (1978). *Psychometric theory*. New York: McGrall-Hill.
- Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. New York: Routledge & Kegan Paul.
- Reeve, J. (2006). Teachers as facilitators: What autonomy-supportive teachers do and why their students benefit. *The Elementary School Journal*, 106(3), 225 – 236.
- Ryan, R. M., & Deci, E. L. (2000). Self-determination theory and facilitation of intrinsic motivation, social development, and well-being. *American Psychologist*, 55(1), 68-78.
- Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly*, 23(2), 139 – 159.
- Sfard, A. (2008). *Thinking as communicating*. Cambridge, NY: Cambridge University Press.

- Sfard, A. (2007). When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from a commognitive standpoint. *Journal of the Learning Sciences*, 16(4), 565 – 613.
- Shouse, R. (2001). The impact of traditional and reform-style practices on students mathematical achievement. In T. Loveless (Ed.), *The Great Curriculum Debate: How Should We Teach Reading and Math?* Washington, D.C.: Brookings Institution Press.
- Stigler, J. W., & Hiebert, J. (1997). Understanding and improving classroom mathematics instruction. *Phi Delta Kappa*, 79(1), 14 – 21.
- Teddlie, C., & Tashakkori, A. (2003). Major issues and controvieries in the use of mixed methods in the social and behavioral sciences. In A. Tashakori & C. Teddlie (Eds.), *Handbook of mixed methods in social & behavioral research* (pp. 3 – 50). Thousand Oaks, CA: Sage Publications, Inc.
- Wood, T. (1999). Creating a context for argument in mathematics class. *Journal for Research in Mathematics Education*, 30(2), 171 – 191.
- Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, 27(4), 458 – 477.

Appendix A

The following questions concern your feelings about your math class during the current school year. Please indicate how true each of the following statements is for you given your experiences in this math class. Remember that your teacher will never know how you responded to the questions. Please mark the number that best represents your answer.

1	2	3	4	5	6	7
not at all true			somewhat true			very true

1. I feel like I can make a lot of decisions in how I do my math.
2. I really like the people in my math class.
3. I do not feel very competent when I am in math class.
4. People in math class tell me I am good at math.
5. I feel pressured in math class.
6. I get along with the students in my math class.
7. I pretty much keep to myself when I am in math class.
8. I am free to express my ideas and opinions about math in my math class.
9. I consider the people in my math class to be my friends.
10. I have been able to learn interesting things about math in this math class.
11. When I am in math class, I have to do things the way the teacher tells me to.
12. When I am in math class, I have to do things the way my classmates tell me to.
13. Most days I feel a sense of accomplishment after being in math class.
14. My feelings are taken into consideration in math class.
15. In math class, I do not get much of a chance to show how capable I am.
16. People in math class care about me.
17. There are not many students in my math class that I am close to.
18. I feel like I can pretty much be myself in math class.
19. The students in my math class do not seem to like me much.
20. When I am doing math, I often do not feel very capable.
21. There is not much opportunity for me to decide how to do my work in math class.
22. People in my math class are pretty friendly towards me.
23. My math teacher has described how we should talk about math with each other.
24. When I talk about math with others, I back up any claims I make.
25. When I talk about math with others, I explain what I mean in detail.
26. When others talk about math, I listen to what they are saying.
27. What gender are you: (1) Male (2) Female
28. Are you: (1) Caucasian (2) African American (3) Hispanic (4) Asian (5) Other
29. What grade are you in: (1) 9th Grade (2) 10th Grade (3) 11th Grade (4) 12th Grade
30. How old are you: (1) 13 (2) 14 (3) 15 (4) 16 (5) 17 (6) 18 (7) 19+

Thank you for completing the survey!

Please place it in the sealed envelope provided your teacher.

Appendix B

Student Scores for Quantitative Measures.

Student	Autonomy	Competence	Relatedness	Mathtalk
Liz	7.00	5.67	7.00	6.67
Carol	4.86	2.33	5.43	5.33
Stacey	5.57	5.80	6.38	6.00
Beth	4.43	3.33	6.75	3.00
Rebecca	5.86	3.67	6.00	3.33
Laura	5.57	3.67	6.00	3.33
Jeremy	5.00	4.50	6.75	4.33
Candy	7.00	6.17	6.75	3.67
Chase*	5.40	3.80	6.00	-
Connie	4.00	3.00	2.75	1.00
Xavier*	5.43	5.33	5.63	-
Greg	4.86	5.33	4.38	4.00
Bruce	4.86	5.00	6.50	4.67
Larry	5.29	5.00	5.88	5.67
Lynn	4.29	4.17	4.38	5.33
Robert	4.71	5.17	3.63	3.00
Lucy	4.00	3.00	4.00	2.67
Mark	4.71	5.00	5.38	5.33
Noah	5.29	5.33	6.00	3.67
Cindy	4.00	4.33	6.13	2.00

**These students did not complete the mathtalk items on the questionnaire.*

Chapter Five

Overview and Conclusion

The purpose of this dissertation was to use Self-Determination Theory to help explain the effectiveness of mathematical discussion. The conceptual model presented in Figure 5.1 and described in Chapter 2 suggests that as students engage in more dialogic discourse, they are more likely to produce conceptual talk and have a higher degree of autonomy. Students participating more univocally were predicted to have lower degrees of autonomy, competence, and relatedness and were predicted to be less likely to produce conceptual and, to some degree, procedural talk.

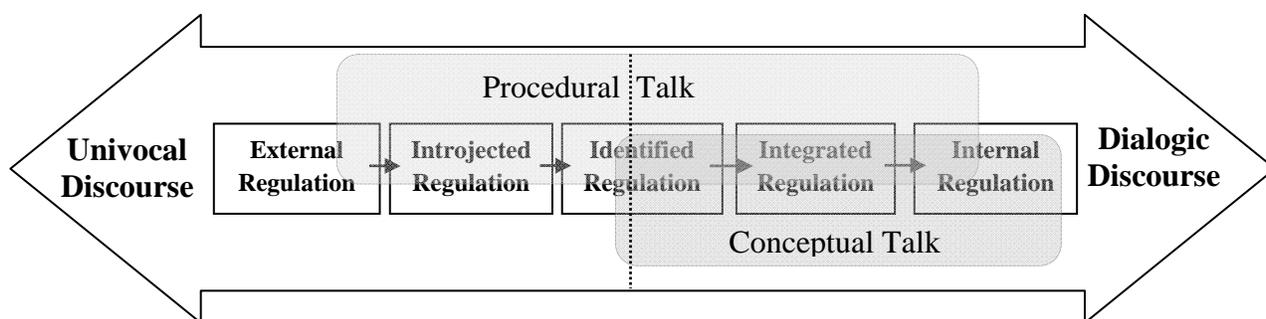


Figure 5.1. Theoretical Framework Presented in Chapter 2.

The results from both studies in Chapter 3 provide some support for the conceptual model described in Chapter 2. Even though correlations between frequency of explanation and SDT needs were somewhat low in the first study, HLM analysis showed that the more students stated they explained their mathematics, the higher their perceived autonomy, competence and relatedness. Further, the percent of students engaged in explaining mathematics had a positive interaction with individual students' own engagement, suggesting that the discourse environment students are in impacts both their engagement in discussion and their perceived SDT needs. Such

was the implication in Yackel and Cobb's (1996) description of student mathematical discourse. However, the evidence presented here extends this description to include aspects of Self-Determination Theory.

The second study conducted in Chapter 3 suggested that the main variable assessing student frequency of explanation used in Study 1 was too vague for students. Anecdotal evidence suggested the problem lay in the variable's lack of context specific language. While these findings provided a possible explanation for inconsistent results and significant variance found in Study 1, Study 2 also provided support for the model described in Chapter 2. Specifically, students who justified their math, explained their math, and listened to other students descriptions of mathematics typically had higher levels of autonomy, competence, and relatedness. Further, students engaging more frequently in class discussion about mathematics were also likely to be more self-determined. Therefore, the combined findings of Study 1 and 2 in Chapter 3 supported a relationship between individuals' perceived SDT needs and their mathematical discourse actions.

Findings from Chapter 4 indicated that students' autonomy was a main factor in their engagement in univocal or dialogic discourse. However, their perceived competence and relatedness appeared to affect their engagement as well. While Robert and others engaged in dialogic discussion and both conceptual and procedural talk, it was these students perceived autonomy that appeared to make them more likely to engage in the discourse. However, when Liz challenged the need for discussion, she violated the social norms set up by the teacher. For Robert, who had lower perceived relatedness, this challenge effectively ended his engagement in dialogic discourse and he relinquished his leading role to the teacher. In the probing setting, Chase ceased to provide his description of his solution process after the validity of his methods

were challenged by another student. Results from mixing of the data indicate that this appeared to be caused by his low sense of competence. Therefore, while autonomy appeared to be the deciding factor in what engaged some students in mathematical discussion, the lack of either competence or relatedness could cause them to cease participating. This is supportive of SDT literature that suggests all three needs must be met for actions to be *fully* self-determined (Deci et al., 1991; Reeve, 2006; Ryan & Deci, 2000). For both Chase and Robert, their participation in discourse seemed self-determined only to a point. Rather, Chase and Robert may have identified with the regulation of mathematical discussion, but this regulation may not yet have become integrated, allowing for them to slip back easily to allowing the teacher as the authority figure. Liz appeared to have been more internally regulated than many other students, but lacked the willingness to support other students.

Results from Chapter 4 also supported the premise of this dissertation in that the type of talk produced by students appeared to match the description provided in Chapter 2 and illustrated in Figure 5.1. Students appeared only to produce conceptual talk when they were in a probing setting, as described by Hancewicz (2005), or a dialogic setting, as characterized by Kitchen (2004). Additionally, students who were not observed to participate in discussions also had lower perceived SDT needs than their participating peers. While these specific results were not explicitly discussed in Chapter 4, the lack of such students' voices and actions in the transcripts should be noted.

Both Chapters 3 and 4 provided supporting evidence for the purpose of this dissertation and its conceptual framework. However, both chapters also provided evidence of a need for further investigation. Implications from Chapter 3 suggest a need for HLM analysis with more reliable variables than those provided by TIMSS. Additionally, context of discussion may play

an important role in its ability to predict SDT needs, as suggested by anecdotal evidence. Therefore, an examination of different types of mathematical discussion is warranted (i.e. class discussion, peer discussion, small group discussion). Chapter 4 provided evidence that even though an individual student may be internally regulated in regards to mathematical discussion, they may not be supportive of other students' process of internalization or of the value in other mathematical perspectives. Such a perspective warrants further investigation.

In addition to implications provided by the results within this dissertation, other research should be considered for the future development of the conceptual model presented here. First, competence in mathematical discussion, in addition to competence in mathematical content, should be investigated to determine what impact or relationship it has in the SDT relationship with discussion. Second, longitudinal research should be conducted throughout the school year to see how students' engagement in discussion and their SDT needs change within a discourse-supportive classroom. Next, examination of other contextual factors affecting students SDT and discursive actions should be conducted. Finally, this program of research should be extended into examining mathematical writing to determine the extent of SDT's relationship with mathematics communication in general.

The overall results of the studies presented in this dissertation support the bi-directional influence of mathematical discussion on SDT and vice versa. Further, by supporting the relationship between SDT and mathematical discussion, the results presented in this document also provide evidence of an interactive relationship between the individual student and their classroom environment. The findings presented and discussed here are the first of their sort, while supporting existing mathematics education literature. This dissertation has provided empirical evidence that successfully extends Yackel and Cobb's (1996) theoretical framework

with the incorporation of Self-Determination Theory. Such an extension provides a more coherent framework for investigating mathematical discussion, and can potentially provide more practical treatments within the classroom.

References

- Deci, E. L., Vallerand, R. J., Pelletier, L. G., & Ryan, R. M. (1991). Motivation and education: The self-determination perspective. *Educational Psychologist, 26*(3 & 4), 325 – 346.
- Hancewicz, E. (2005). Discourse in the mathematics classroom. In J.M. Kenney (Ed.), *Literacy strategies for improving mathematics instruction* (pp. 72 – 86). Alexandria, VA: Association for Supervision and Curriculum Development.
- Kitchen, R. S. (2004). Challenges associated with developing discursive classrooms in high-poverty, rural schools. *Mathematics Teacher, 97*(1), 28 – 31.
- Reeve, J. (2006). Teachers as facilitators: What autonomy-supportive teachers do and why their students benefit. *The Elementary School Journal, 106*(3), 225 – 236.
- Ryan, R. M., & Deci, E. L. (2000). Self-determination theory and facilitation of intrinsic motivation, social development, and well-being. *American Psychologist, 55*(1), 68-78.
- Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458 – 477.