AN INVESTIGATION OF THE VALIDITY OF AUDITING PROCEDURES

USED IN MEAN-PER-UNIT SAMPLING PLANS

by

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Chapter I

INTRODUCTION

This study is concerned with the effect of different distribution assumptions of infinite auditing populations on the validity of existing auditing procedures used in mean-per-unit sampling plans. The introductory chapter has a dual role. First, the chapter provides information necessary to form a foundation from which a statement of the problem can be developed. Secondly, the chapter provides a general overview of the research methodology and an outline of the dissertation.

Acceptability of Statistical Sampling within the Auditing Environment

Sampling is used by auditors to gather information about auditing populations. The information is then used in conjunction with other audit evidence to form sufficient competent evidential matter upon which an opinion is expressed as to the fairness of financial statements. Sampling a population, rather than investigating each unit in a population, is considered "a necessary and accepted practice."[1]

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[1] R. K. Mautz and Hussein A. Sharaf, The Philosophy of Auditing (Sarasota, Florida: American Accounting Association, 1961), p. 33.

Justification for sampling "arises from the relationship between such factors as the cost and time required to examine the data and the adverse consequences of possible erroneous decisions based on the resulting conclusions."[2] However, in using statistical sampling to gather audit evidence some auditors may have done so without recognizing potential deficiencies in their sampling plans.

A recent study supports the idea that there may be deficiencies in some of the statistical techniques employed by auditors.[3] As pointed out by Anderson and Leslie,

> There has sometimes been a tendency for auditors to assume that because various sampling plans were well documented in accepted statistical literature they were automatically appropriate for whatever (possibly quite different) circumstances are involved in an audit. We hope that the Loebbecke-Neter paper, and the underlying study, will end such cases of blind acceptance and promote the view that the real sampling risks arising in various audit situations are worthy of careful consideration.[4]

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[2]American Institute of Certified Public Accountants, Statement on Auditing Standards 1 (New York: American Institute of Certified Public Accountants, 1973), p. 44. (Hereafter referred to as SAS 1.)

[3]John Neter and James K. Loebbecke, Auditing Research Monograph 2, Behavior of Major Statistical Estimators in Sampling Accounting Populations (New York: American Institute of Certified Public Accountants, Inc., 1975).

[4]R. J. Anderson and Donald A. Leslie, "Discussion of Considerations in Choosing Statistical Sampling Procedures in Auditing," Journal of Accounting Research: Supplement 1975, p. 54.

The Neter-Loebbecke study indicates that various sampling methods are unreliable because the specified reliability level[5] is not attained under conditions of highly skewed populations. In other words, the confidence level actually attained was lower than the specified confidence level. One of the methods pointed out by the Neter study as being unreliable in certain circumstances was the mean-per-unit estimator. The current research project centers around mean-per-unit sampling techniques. Reasons for selecting this sampling technique are indicated in subsequent sections.

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[5]Reliability is the confidence with which we can state that the proportion of sample means, from all possible samples of the same size from a given population, will lie within a confidence (precision) interval. A confidence interval is computed using the expected standard error of the mean which is symbolized by $s_{\bar{y}}$.

Computationally:

$$s_{\bar{y}} = \frac{s}{\sqrt{n}}$$

where "s" is an estimate of the population standard deviation and "n" represents the sample size. The formula for a 95% confidence interval for a sample mean is:

$$\bar{y} \pm 1.96\, s_{\bar{y}}$$

where "$\bar{y}$" represents the sample mean and "1.96" is the factor from the normal table related to the 95% confidence level.

The Mean-Per-Unit Estimator

General Uses

The mean-per-unit (MPU) estimator is used in a variables sampling plan in which the objective is generally to measure some dollar amount in the population as opposed to attributes sampling which attempts to measure the frequency of errors in a population. For instance, the MPU estimator can be used (1) to test the reasonableness of an account balance where an account balance exists and is known; (2) to estimate an account balance where one exists but the amount is not known; and (3) where the individual book value or other documentary value is not available at the time of audit [to be used in conjunction with (1) or (2) above].

When no account balance exists or the individual book values are not available at the time of the audit, the only statistical alternative available within the variables estimation framework is a MPU technique.[6] Differences, advantages and disadvantages of different variables estimators are discussed in Chapter III when literature documentation in support of and limitations of the MPU estimator are presented. Here, the differences are not discussed but they are pointed out to indicate situations advantageous to the MPU estimator. The MPU technique does not require particular error patterns or rates of error like the ratio and difference estimators. Another advantage of the MPU estimator over the other two estimators

[6]Ernst & Ernst, Audit Sampling (Ernst & Ernst, 1976), p. 21.

is that compilation errors are disclosed with this technique. Consequently, the MPU estimator appears to be a potentially valuable auditing tool and for this reason it was selected for this research project. Practical applications of the MPU estimator will be discussed in the next section followed by an explanation of MPU procedures.

Practical Applications

Some examples of practical applications of MPU techniques involve inventory problems. Dollar estimates of (1) the total inventory or (2) total pricing errors in the inventory may be calculated. Alternatively, the MPU estimator can be used to test the reasonableness of a booked inventory value. Arthur Young & Company's Auditing Manual points out that:

> If a sample is drawn prior to the completion of the listing [of inventory] (from the original inventory count tickets for example) the direct extension estimate is a test of the compilation of the data from that point forward. It will help to verify that the count tickets have been properly recorded, that no significant number of tickets have been deleted or added and that the final inventory listing has been correctly compiled and totaled.[7]

Other practical applications include testing for the reasonableness of accounts receivable and notes receivable balances. These

[7] Arthur Young & Company, Auditing Manual (Arthur Young & Company, 1973), p. 7.424.

potential applications are illustrated in Robertson's and in Hermanson's auditing texts, respectively.[8]

The accounting literature also gives different types of applications of variables estimation but does not always specify a particular technique. For instance, Anderson states that:

> Variables sampling can be used for estimating the average age of accounting receivable from a sample, or estimating the average gross profit from a sample of sales invoices, or estimating inventory cost from inventory retail value using the average markups from a sample of inventory items.[9]

MPU Procedures

MPU procedures can be illustrated with a hypothetical inventory problem. In this example it is assumed that the sample size is given at 474 units and that the auditor's objective is to test the reasonableness of the inventory account balance. The auditor would like to be 95% confident that the true inventory balance falls within $5,000 (plus and minus) of the $100,000 book value, or within $.25 of the $5 average book value per unit. The following additional data are assumed:

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[8]Jack C. Robertson, Auditing (Dallas, Texas: Business Publications, Inc., 1976), p. 375; and Roger H. Hermanson, Stephen E. Loeb, John M. Saada, and Robert H. Strawser, Auditing Theory and Practice (Homewood, Illinois: Richard D. Irwin, Inc., 1976), p. 230.

[9]Rodney J. Anderson, "Audit Uses of Statistical Sampling," Internal Auditor (May/June, 1973), p. 34.

$BV = \$100{,}000$	Inventory account balance;
$N = 20{,}000$	Number of inventory items;
$BV/N = \$5$	Average book value;
$n = 474$	Sample size;
$\overline{y} = \$5.10$	Sample mean;
$s = \$1.86$	Sample standard deviation;
$Z_{\alpha/2} = 1.96$	Factor from the normal table for a 95% confidence level.

A two-sided test is considered necessary so that a materially understated or overstated book value will be detected at the 95% confidence level.

After the 474 sample items have been randomly selected and the book value for each item verified, the auditor will compute a confidence interval for the sample mean. The first step is to compute the mean and standard deviation for the sample of verified book values. The assumed values of these two statistics are given above. The 95% confidence limits (see footnote 5 of this chapter) can now be computed as follows:

$$\$5.10 \pm 1.96\ (\$1.86/\sqrt{474})$$

$$\$5.10 \pm \$.17$$

Lower confidence limit = \$4.93

Upper confidence limit = \$5.27.

Since the $5 average book value of the inventory falls within this confidence interval, the statistical procedure indicates that the inventory account balance is reasonably stated with 95% confidence.

Alternatively, the auditor could extent the mean and compute a confidence interval around the extended mean. This procedure is demonstrated below:

$$\$5.10 \times 20{,}000 = \$102{,}000$$

The confidence interval is then determined as follows:

$$\$102{,}000 \pm 1.96\ (1.86/\sqrt{474}\ )\ 20{,}000$$

$$102{,}000 \pm 3{,}349$$

Lower confidence limit = $ 98,651

Upper confidence limit = $105,349

This procedure also implies that the inventory account balance is reasonably stated since the book value of $100,000 falls within the interval. On the other hand, if the account balance, or the average value of the account, falls outside the respective confidence intervals, the statistical procedure indicates rejection of the account balance as being reasonably stated.

If the auditor sets the confidence (reliability) level at 95% when he is computing confidence intervals, he assumes the risk that 5% of the time his estimating procedures will not include the true population mean. Stating the proposition in auditing terminology, 5%

of the time the auditor will reject the account as being reasonably stated when in fact it is appropriately stated. This sampling error is called the alpha risk. If a reasonably stated account balance is rejected, a Type I error occurs. If the account balance is materially misstated, however, a Type I error cannot be committed, but a Type II error could occur. A Type II error occurs when a materially misstated account is accepted as reasonably stated. The potential commission of a Type II error involves the beta risk. Type I and Type II errors are discussed in the next section.

Classical Statistical Considerations

The statistical concepts of Type I and Type II errors are based on the distribution of a sample statistic. A sampling distribution of a sample statistic is a probability distribution computed from samples of the same size taken one at a time from a specified population. As the sample size $\underline{n}$ increases, the frequency distribution of the sample statistic tends to become normal. The normal approximation enables one to make inferences about a population based on the statistics of a sample and it is the basis for the central limit theorem.

Central Limit Theorem

The central limit theorem states that the distribution of sample means, of samples selected from an infinite population with mean μ_x and variance σ_x^2, will approach a normal distribution with mean μ_x and variance σ_x^2/n as the sample size n approaches infinity regardless of the distribution of the population. Cochran, however, points out that under certain conditions the inferences may not be valid. He states:

> From the study of theoretical distributions that are skewed and from the results of sampling experiments on actual skewed populations, some statements can be made about what usually happens to confidence probabilities when we sample from positively skewed populations. The sample size is assumed large enough so the distribution of $\bar{y}$ [sample means] shows some approach to normality.... The statements are as follows:
>
> 1. The frequency with which the assertion
>
> $$\bar{y} - 1.96s_{\bar{y}} < \bar{Y} < \bar{y} + 1.96s_{\bar{y}}$$
>
> is wrong is usually higher than 5%.
>
> 2. The frequency with which
>
> $$\bar{Y} > \bar{y} + 1.96s_{\bar{y}}$$
>
> is greater than 2.5%.
>
> 3. The frequency with which
>
> $$\bar{Y} < \bar{y} - 1.96s_{\bar{y}}$$
>
> is less than 2.5%.[10]

These statements have important implications for the auditor because formulae currently used for variables sampling plans for determination

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[10]William C. Cochran, Sampling Techniques (2d ed.; New York: John Wiley & Sons, Inc., 1963), p. 40.

of sample sizes are based on the central limit theorem. The question is, how large does the sample size have to be so that the normal approximation of the sampling distribution of means is good enough? In other words, how large does the sample size have to be so that the desired alpha and beta levels are attained under conditions of non-normality?

If the desired alpha and beta levels are not attained, the auditor is not effectively controlling the alpha and beta risks. Control of these two risks is discussed in Chapter II. The sample size formula used in this research project considers both of these sampling risks and is presented in the following section.

Sample Size Formula

The sample size formula used in this research study can be expressed as follows:

$$n = \frac{s^2 \, (Z_{\alpha/2} + Z_\beta)^2}{M^2}$$

where:

n = sample size,

s^2 = estimated population variance,

$Z_{\alpha/2}$ = the factor from the standard normal distribution necessary for the desired level of confidence,

Z_β = the factor from the standard normal distribution necessary for the desired power of the test where power = $1 - \beta$,

M = the desired precision which is equal to materiality when using this formula.

Since the population variance is unknown, it must be estimated. Accounting literature recognizes two methods for estimating the population variance.[11] One method suggests using the sample variance shown on prior year's working papers as an estimate of the current year's population variance. The other method requires a preliminary sample be taken. Telephone interviews with representatives of several large accounting firms indicate that the approach to such estimation varies from firm to firm.[12] Some firms indicated that they never take a preliminary sample, while two firms stated that preliminary samples were frequently used. This study uses a preliminary sample size of 30 units to make the estimate.

The earliest accounting source for the selected formula in the form given above was by Boatsman and Crooch in 1975.[13] The authors state, "To our knowledge, however, no undergraduate auditing texts

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[11]Herbert Arkin, Handbook of Sampling for Auditing and Accounting (2d ed.; New York: McGraw-Hill Book Company, Inc., 1974), p. 94.

[12]Personal telephone interviews were made with representatives of seven major CPA firms on June 23, 24, 27, and July 22, 1977. These firms prefer to remain anonymous.

[13]James R. Boatsman and G. Michael Crooch, "An Example of Controlling the Risk of a Type II Error for Substantive Tests in Auditing," The Accounting Review (July, 1975), p. 612.

contain a discussion of how this risk [beta] might be controlled."[14] However, at least two auditing texts published in 1976 utilize the formula, or a variation of it, and discuss the control of the beta risk.[15] The derivation and auditing implications of the sample size formula are provided in Chapter II.

Statement of the Problem

It has been suggested in the accounting literature that the MPU estimator and related procedures may not properly control the alpha and beta risks under conditions of non-normality. The question that arises is whether alpha and beta risk levels are controlled under conditions of non-normality when using the MPU estimator in conjunction with the sample size formula? Although the formula is designed to control the alpha and beta risk levels, it is hypothesized that under certain degrees of skewness and kurtosis the specified beta and alpha levels are not controlled when using the MPU estimator and the selected sample size formula.

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[14]Ibid., p. 610.

[15]Robertson, Auditing, p. 370; and Hermanson, et al., Auditing Theory and Practice, pp. 224-225.

Objectives of the Research

Basic objectives of this research are to provide evidence that a problem exists, and to provide recommendations that will lead to better auditing practices.

First, the study provides a basis for stating that a problem exists when applying normal statistical techniques to non-normal auditing populations. This should raise questions about the validity of many auditing procedures based on classical statistics.

Second, guidelines are developed for simultaneous control of alpha and beta risks when using the MPU estimator with certain non-normally distributed populations. The guidelines should improve the quality and validity of audit evidence.

Research Methodology

Information regarding the phases of this research project are given below. These different aspects include a review of relevant literature, a limited review of current practice, selection of an appropriate sample size formula, and an outline for the discussion of research design.

Review of Relevant Literature

For a review of relevant literature, several major sources were examined. These sources are Dissertation Abstracts International,[16]

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[16]Dissertation Abstracts International, Section A, The Humanities and Social Sciences (Ann Arbor, Michigan: University Microfilms, 1967-1977).

Accountants' Index,[17] Selected Bibliography,[18] auditing textbooks, and audit training manuals. Chapter III presents results of the literature review under the following categories: (1) historical aspects of statistical sampling within the audit environment, (2) support of statistical sampling and of mean-per-unit techniques, (3) limitation of mean-per-unit techniques, and (4) specification of variables necessary in variables estimation.

Limited Review of Current Practice

Seven major CPA firms were contacted concerning field use of mean-per-unit techniques. Information gained from telephone interviews with a statistical consultant or a director of accounting research for each firm is cited at appropriate points throughout the dissertation.

Selection of Sample Size Formula

Auditing and statistical textbooks, articles in professional journals, and audit training manuals were examined for various sample size formulas that might be considered for use in the study. The formula selected for the research provides for both the alpha and beta risk levels. Justification for selecting the formula is provided in Chapter II.

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[17] American Certified Public Accountants, Accountants' Index (New York: American Certified Public Accountants, 1962-1976).

[18] American Institute of Certified Public Accountants, Subcommittee on Statistical Sampling Bibliography Task Force, Selected Bibliography (October 31, 1976).

Summary of Research Design

A simulation study was conducted to provide a means for achieving the objectives of the research. A summary of the research design is presented in this section while details of the simulation procedures are described in Chapter IV.

A normally distributed population and twenty-one populations with J-shaped distributions[19] were used in the simulation procedures. The characteristics of these distributions ranged from zero degrees of skewness and 3.0081 degrees of kurtosis to 25.6089 and 37.7937 degrees of skewness and kurtosis, respectively. A unity variance for the population was utilized and is elaborated upon in Chapter IV. The curves of the twenty-two distributions are presented in Appendix II. Creation of the populations, as well as other details, is given in Chapter IV.

Simulation procedures were first applied to the normally distributed population. Utilizing the simulation computer program, the program procedures randomly selected the preliminary sample, computed the preliminary sample variance, and determined the sample size. In

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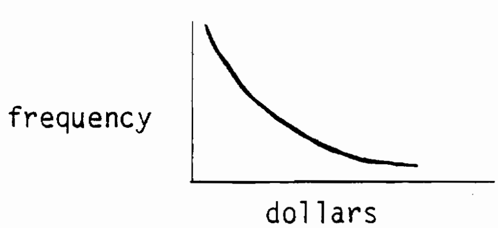
[19] The J-shaped distribution used in this study reflects a curve that is decreasing at decreasing rate and is of the following shape:



this study, the computed sample size was always larger than the preliminary sample size of 30.[20] Therefore, the additional items over and above the preliminary sample size were randomly selected from the population. Using all sample elements, a confidence interval around the sample mean was computed. From this information it was determined if a Type I or Type II error occurred.

Steps to determine whether a Type I or Type II error resulted were carried out 1,000 times and tabulations were made to account for the number of times each error occurred. The proportion of times a Type I error took place in the 1,000 applications was computed. If the alpha level used in the sample size formula is specified at 5%, a Type I error should occur approximately 5% of the time.

To account for Type II errors, two different and independent events were assumed to exist. First, an average booked account balance was assumed to be materially overstated from the true population mean. In this instance, beta errors are referred to as Type II errors on the upper side. In the second event, an average booked account balance was assumed to be materially understated from the true population mean. Errors in this situation are referred to as Type II errors on the lower side. The proportion of times a Type II error occurred was determined spearately for the upper and lower sides. Again, the proportion should approach 5% when the beta risk level is set at 5%.

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[20] An explanation for the computed sample size being larger than the preliminary sample size is given in Chapter IV.

Application of the above steps to one population constitutes one computer run of the simulation program. In other words, starting with the first preliminary sample and ending with computations for the proportions of sampling errors occurring in the 1,000 loops consists of one computer run.

Using the simulation program, identical procedures as those applied to the normally distribution population were carried out on the next population which deviated only a little from normality. The proportions of times Type I and Type II errors occurred were accounted for in this set of 1,000 applications. Again, identical procedures were applied to the next population which departed a little more from normality. These steps were repeated until all populations had been subjected to the simulation procedures using one particular combination of alpha and beta risk levels.

The cycles were repeated by starting over with the normally distributed population but with a new combination of alpha and beta levels. The cycles going from one population to the next population were repeated for the alpha and beta combinations given in Table 1. Since all levels of alpha and beta and their combinations could not be used in a study of this magnitude, the levels and combinations were subjectively selected based on recommendations found in accounting literature.[21]

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[21] Robert K. Elliott and J. R. Rogers, "Relating Statistical Sampling to Audit Objectives," Journal of Accountancy (July, 1972), pp. 46-55; Boatsman and Crooch, "An Example of Controlling," pp. 610-615; Robertson, Auditing, pp. 362-369; Hermanson et al., Auditing Theory, pp. 219-220; and Ernst & Ernst, Auditing Samples, p. 71.

Table 1

COMBINATIONS OF THE ALPHA AND BETA LEVELS USED IN THE STUDY

Levels	
Alpha	Beta
.20	.20
.15	.20
.10	.20
.08	.20
.05	.20
.01	.20
.001	.20
.20	.10
.10	.10
.20	.05
.05	.05

For each set of populations, going from normality to the distribution with the most skewness and kurtosis, trends in proportions of times each type of error occurred were identified for each combination of alpha and beta risk levels. Using these trends and simulation results, guidelines were developed so that specified alpha and beta levels could be simultaneously attained under certain conditions of non-normality when using the selected sample size formula and the MPU estimator. To investigate the validity of the guidelines, three new distributions with different size variances were created and subjected to simulation procedures described in this research project.

Criteria to Determine Non-attainment of a Sampling Risk

Whenever the proportion of times an error occurred was significantly greater than the specified sampling risk level, control of the risk was considered ineffective. In other words, the specified risk level was not attained. Significance was measured in terms of three standard errors of the proportion. The estimated standard error of a proportion is computed as follows:

$$\hat{\sigma}_p = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

where:

$\hat{\sigma}_p$ = estimator of the standard error of the proportion,

$\hat{p}$ = sample proportion,

n = sample size

In this case sample size, n, equals 1,000 for the number of loops performed.

If the proportions were greater by three standard errors than the specified risk level, the risk level was considered not attained. Using this criterion to determine the attainment, or lack of attainment of the sampling risk, the simulation results were compiled. These results are presented in the fifth chapter of this dissertation.

Outline of the Dissertation

The first chapter provides a foundation from which a statement of the problem was developed and presents the general methodology of the study. In Chapter II control of alpha and beta risks is discussed in relation to auditing implications. Chapter III provides a review of relevant literature from four different points of view. Design of the investigation is provided in the fourth chapter, while the fifth chapter presents research results and conclusions. Guidelines to control alpha and beta risk levels effectively for certain distributional assumptions are provided in the sixth chapter. The last chapter, Chapter VII, discusses implications for future research and gives a summary of research results which also includes limitations of the study.

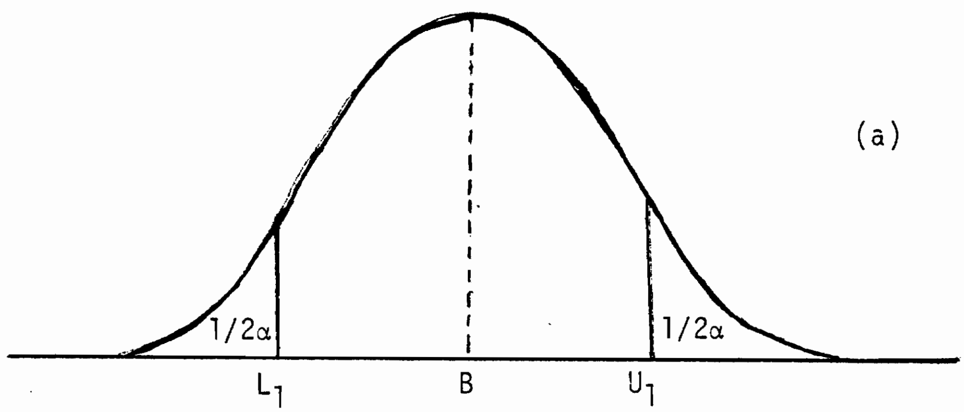
Chapter II

CONTROL OF ALPHA AND BETA RISKS AND ASSOCIATED AUDITING IMPLICATIONS

Control of alpha and beta risks and their associated auditing implications is discussed in this chapter. Control of the alpha and beta risks is discussed first. Then, two approaches described in accounting literature to control the beta risk are shown to be equivalent to one another. Next, a derivation of the formula selected for this study is given. The last section provides justification of the formula and discusses auditing implications associated with alpha and beta risks.

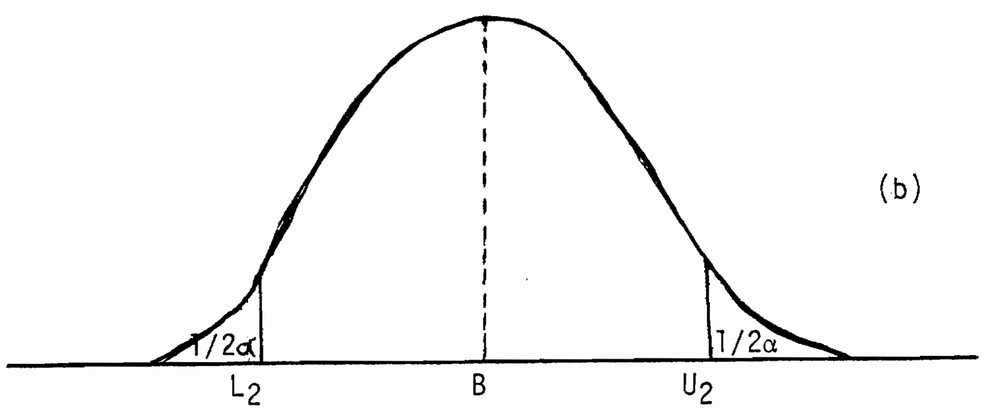
Control of the Alpha Risk

Control of the alpha risk is carried out by specifying a reliability level, R. The alpha risk level is the complement of the reliability level, $(1-R) = \alpha$. For example, when the reliability level is set at 90%, the alpha risk level is 10%. This is illustrated in Figure 1 (a). This curve reflects the distribution of statistical estimates for all possible samples of a given size n that can be selected from a given population. Consequently, a single sample estimate must come from some part of the curve. The sample estimate will support fair statement of the book value if the sample comes from



ALPHA RISK = 10%

RELIABILITY = 90%



ALPHA RISK = 5%

RELIABILITY = 95%

B = Book value (assumed true)

L_1, L_2 = Lower precision limits

U_1, U_2 = Upper precision limits

ILLUSTRATION OF ALPHA RISK

FIGURE 1

from the area between L_1 and U_1, the decision interval. The decision interval represents a 90% chance of obtaining a sample which will support fair statement of the book value. Or, there is a 10% chance that the sample will not come from that area and will not support fair statement.

Assuming the book value, B, to be true, one-half the decision interval from L_1 to B represents sampling precision. In other words, the decision interval consists of the book value plus and minus precision. Consequently, precision is a measure of how close the sample estimates correspond to the true population value for a specified reliability level. When the decision interval is referred to as the precision interval, it is important to remember that the actual amount of precision is one-half the precision interval.

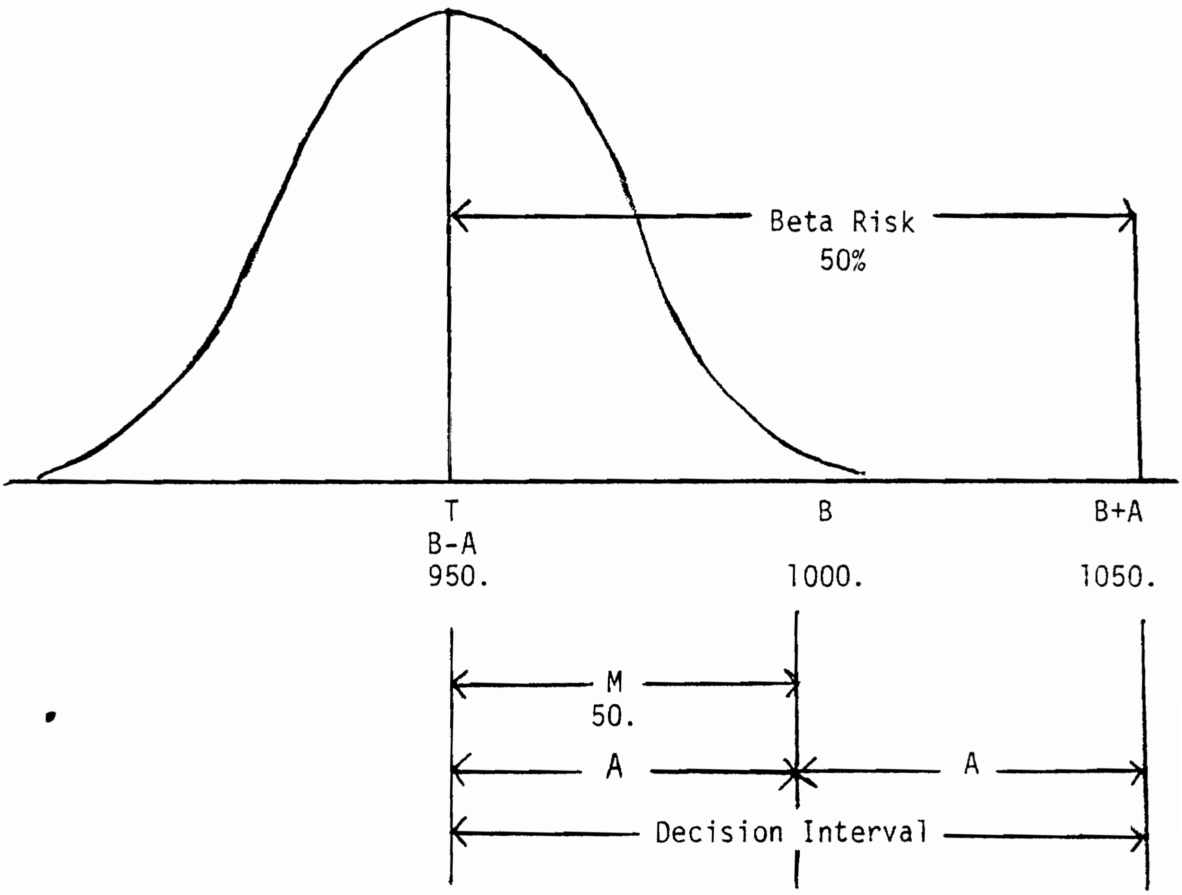
When the reliability level is raised, say from 90% to 95% and holding n constant, the decision interval becomes wider and, thus, precision is larger. This is illustrated in Figure 1 (a) and (b). The distance between L_2 and U_2 is greater than the distance between L_1 U_1. Consequently, the effect of decreasing the alpha risk level and holding n constant is to create a wider decision interval with less probability of rejecting a reasonably stated account balance. The level of alpha risk the auditor is willing to assume under different circumstances is reviewed and discussed in Chapter III.

Control of the Beta Risk

Control of the beta risk may be carried out by manipulating the ratio of precision to the minimum amount considered material. To explain why this manipulation controls beta risk, two different situations are illustrated. In the first situation, precision and materiality are equal. For the second situation, precision is equal to one-half materiality.

Figure 2 illustrates situation 1 where both materiality and precision equals \$50. The true value of the population equals \$950 and the book value is \$1,000 which is materially overstated by \$50. The decision interval will be from \$950 to \$1,050. Since the true value of \$950 is the center of the distribution, 50% of all possible samples drawn from this population will fall within the decision interval supporting fair statement of the materially overstated book value. Consequently, when precision and materiality are equivalent, the beta risk if 50%.

The second situation is illustrated in Figure 3 where precision equals one-half materiality. The same data are assumed as in situation 1 except for the amount of precision which is now \$25. Because precision is smaller in situation 2, the sample size will be larger than in situation 1. The increased sample size will cause the distribution to cluster more tightly around the true population value. Consequently, there will be very little area of the distribution that falls inside the decision interval which goes from the point



T = True population value = $950.

B = Book value = $1,000.

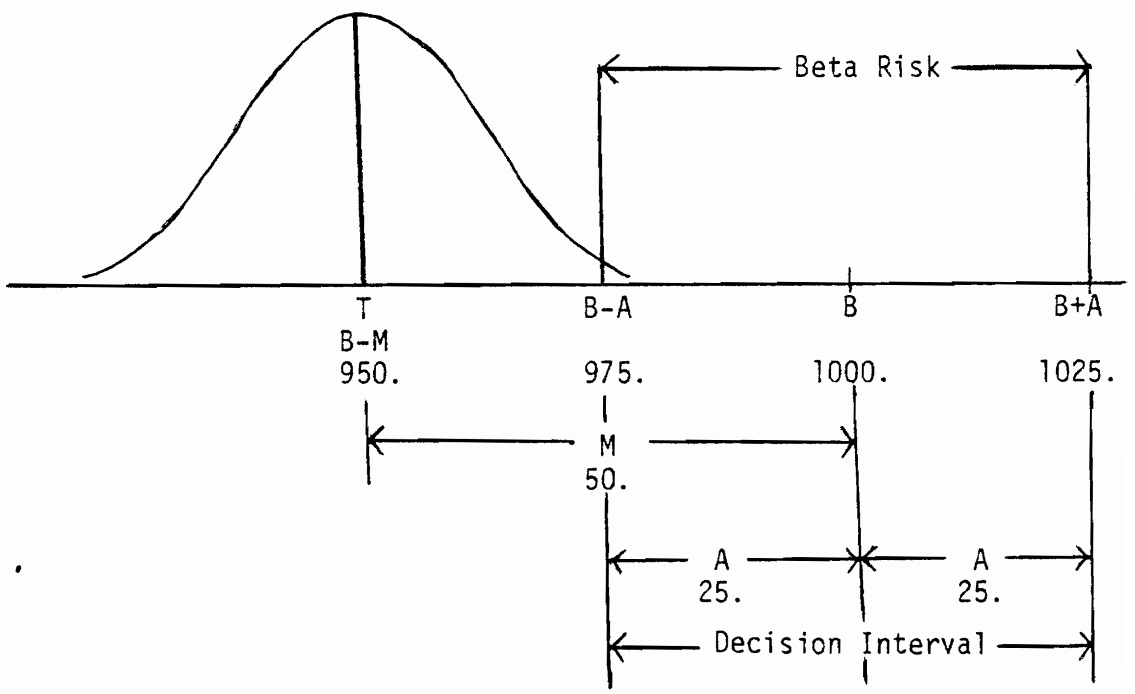
A = Precision = $50.

M = Materiality = $50.

ILLUSTRATION OF BETA RISK WHEN

PRECISION EQUALS MATERIALITY

FIGURE 2



T = True population value = $950.

B = Book value = $1,000.

A = Precision = $25.

M = Materiality = $50.

ILLUSTRATION OF BETA RISK WHEN

PRECISION EQUALS ½ MATERIALITY

FIGURE 3

represented by the book value minus precision to the point represented by the book value plus precision. In this case, the decision interval goes from $975 to $1,025. Therefore, the beta risk will be very small when precision equals one-half materiality.

From these two illustrations[1] it can be seen that by varying precision in proportion to materiality, the beta risk can be increased or decreased. Two different sources propose that this relationship between precision and materiality be expressed as follows:[2]

$$A = \frac{M}{1 + \frac{Z_\beta}{Z_{\alpha/2}}}$$

where:

A = amount of desired precision,

M = minimum amount considered material,

Z_β = factor from the standard normal distribution for the desired power of the test where power = $1 - \beta$,

$Z_{\alpha/2}$ = the factor from the standard normal distribution for the desired reliability level.

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[1]The illustrations for situations 1 and 2 were adapted from materials presented in Audit Sampling, 1976, by Ernst & Ernst, on pp. 63-65.

[2]Robert K. Elliott and J. R. Rogers, "Relating Statistical Sampling to Audit Objectives," Journal of Accountancy (July, 1972), pp. 48-49; and Ernst & Ernst, Auditing Sampling, pp. 70-72.

Using this expression of precision in the standard (traditional) sample size formula attempts to control both alpha and beta risk levels. The standard sample size formula for variables estimation is:

$$n = \frac{s^2 \, (Z_{\alpha/2})^2}{A^2}$$

where:

n = sample size,

s^2 = the sample variance as an estimate of the population variance,

$Z_{\alpha/2}$ = the factor from the normal table for the desired level of confidence,

A = amount of desired precision. (<u>Note</u>: historically, this precision was equated to materiality. It will no longer equal materiality when the above expression for precision is substituted in the formula.)

Under conditions of non-normality it has been suggested that control of the alpha and beta risks may not be effective. Implications that the auditor may not rely on the degree of assurance specified by his statistical estimating procedures are based on the idea that auditing populations may not be normally distributed. For many years accounting authors have suggested that accounting and auditing populations may be non-normal. The 1975 Neter-Loebbecke study empirically demonstrated the non-normality of four accounting populations.[3] Their investigation showed moderate to extreme positive skewness and moderate to extreme kurtosis.

When the above expression of precision, A, is substituted in the traditional sample size formula, the result is equivalent to the sample size formula used in this study. Equivalency of the two approaches to control the alpha and beta risks indicates that research results would be comparable using either approach. This equivalency is demonstrated in the next section.

Equivalency of Sample Size Formulae

The traditional sample size formula will be designated as n_1, while n_2 will represent the selected sample size formula. The remaining notation used here is the same as that given when the formulae were initially presented above.

$$n_1 = \frac{s^2 \, (Z_{\alpha/2})^2}{A^2}$$

To remove the squares from the term on the right:

$$\sqrt{n_1} = \frac{s \, (Z_{\alpha/2})}{A}$$

[3] John Neter and James K. Loebbecke, Auditing Research Monograph 2, Behavior of Major Statistical Estimators in Sampling Accounting Populations (New York: American Institute of Certified Public Accountants, Inc., 1975).

If:

$$A = \frac{M}{1 + \frac{Z_\beta}{Z_{\alpha/2}}}$$

Then:

$$\sqrt{n_1} = \frac{s\ (Z_{\alpha/2})}{\frac{M}{1 + \frac{Z_\beta}{Z_{\alpha/2}}}}$$

Manipulating the variables:

$$\sqrt{n_1} = \frac{s\ (Z_{\alpha/2}) \left(1 + \frac{Z_\beta}{Z_{\alpha/2}}\right)}{M}$$

$$\sqrt{n_1} = \frac{s\ (Z_{\alpha/2}) + s\ (Z_{\alpha/2})\ \frac{Z_\beta}{Z_{\alpha/2}}}{M}$$

$$\sqrt{n_1} = \frac{s\ (Z_{\alpha/2} + Z_\beta)}{M}$$

Squaring both sides:

$$n_1 = \frac{s^2\ (Z_{\alpha/2} + Z_\beta)^2}{M^2}$$

This formula for n_1 is identical to the sample size formula selected for this study as indicated below:

$$n_2 = \frac{s^2 \ (Z_{\alpha/2} + Z_\beta)^2}{M^2}$$

Formula n_2 appears to be less complex than the traditional formula with the substitution of the precision expression. Furthermore, the selected formula allows the auditor to express the desired amount of precision as equal to materiality. This advantage is reflected in the derivation of the selected sample size formula.

Derivation of the Sample Size Formula

When the auditor wants to determine the reasonableness of an account balance, he should consider (1) the probability of rejecting the book value when it is correct and (2) the probability of rejecting it when it is materially misstated. He must specifically state the amount he considers material in the circumstances. The amount specified as material should be the minimum amount by which the auditor would like to detect a misstatement. That is, the auditor would like to detect a misstatement if the true value of the account were equal to the book value plus or minus materiality.

If the book value is correct, the probability of rejecting it is equal to the alpha risk, α. If the book value is materially understated so that the true value of the population is equal to the book value plus materiality, the probability of rejecting the book value is $(1 - \beta)$. These two situations are illustrated in Figure 4.

Since the auditor wants to detect if there is a misstatement by the amount specified as material, the distance M in Figure 4 needs to be translated into statistical terms. Distance M in Figure 5 reflects the translation. The distance between BV and X can be represented by $Z_{\alpha/2}\ \sigma/\sqrt{n}$ which is the traditional plus and minus factor in computing confidence intervals where σ represents the population standard deviation. Assuming the true population value to be Q, the distance between X and Q can be represented by $Z_\beta\ \sigma/\sqrt{n}$. Therefore, the distance M, materiality, can be expressed as follows:

$$M = Z_{\alpha/2}\ \sigma / \sqrt{n} + Z_\beta\ \sigma / \sqrt{n}$$

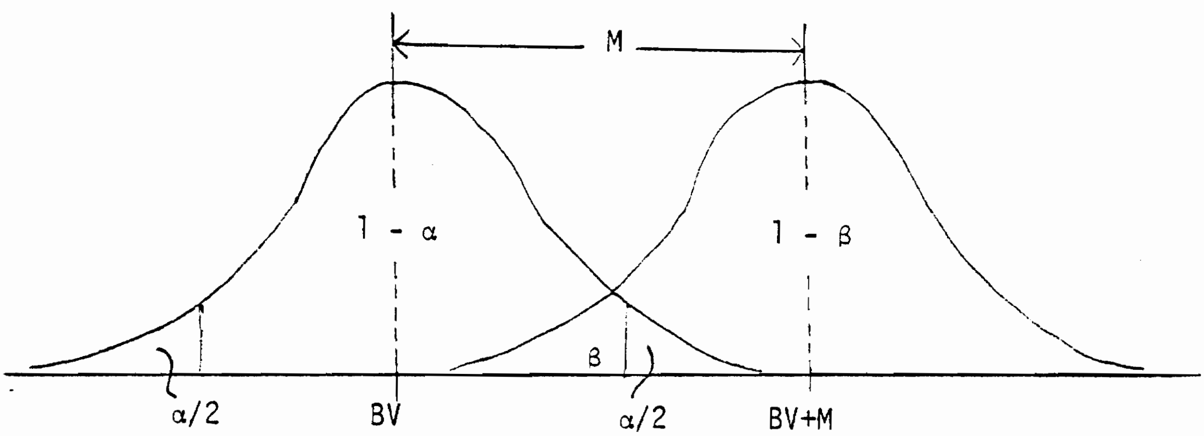
Multiplying through by $\sqrt{n}$ and dividing by M, the equation becomes

$$\sqrt{n} = \frac{Z_{\alpha/2}\sigma + Z_\beta\sigma}{M}$$

Squaring both sides so that:

$$n = \frac{\sigma^2 (Z_{\alpha/2} + Z_\beta)^2}{M^2}$$

Generally, the population variance, σ^2, is unknown. Therefore, an estimate of σ^2 must be used. The estimate of σ^2 is given by s^2 which is calculated from a sample taken from the population. When s^2 is



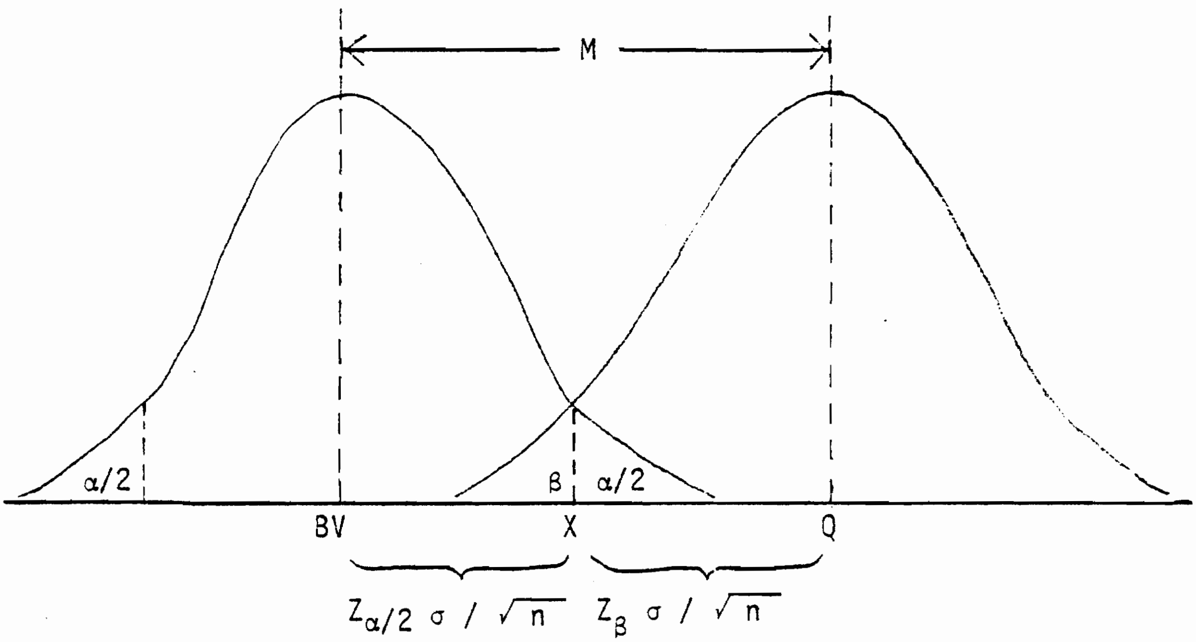
BV = Book Value

M = Materiality

PROBABILITY OF REJECTING

THE BOOK VALUE

FIGURE 4



BV = Book Value

M = Materiality

ILLUSTRATION FOR THE DERIVATION

OF SAMPLE SIZE FORMULA

FIGURE 5

substituted for σ^2 in the above formula, it is in the form used by this research project as shown below:

$$n = \frac{s^2(Z_{\alpha/2} + Z_\beta)^2}{M^2}$$

The foundation for auditing implications and justification of this formula is based on the fact that the formula is designed to control both the alpha risk and the beta risk.

Auditing Implications and Justification of the Formula

After the auditor has determined the sample size, selected and verified all the items in the sample, he will compute the achieved precision interval (confidence limits) for the sample statistic. If the account balance falls within the interval, the auditor will accept the account as reasonably stated. In this event it is assumed that no type of error has occurred and that the alpha and beta risk levels were effectively controlled.

There are two different types of events that could cause the account balance to fall outside the achieved precision interval statistically indicating rejection of the account balance. In either event, the auditor will extend his auditing procedures to determine the extent and cause of misstatement, if indeed there is a misstatement, or discover that a Type I error has occurred.

If the cost of additional sampling is relatively high, the auditor may want to set the alpha risk level at a low level initially, say 5%. Then, if the client is right, the auditor would have to extend his auditing procedures only 5% of the time. On the other hand, if additional sampling cost is relatively low, the auditor may want to set the alpha risk at a higher level, say 10%. The higher alpha risk level will result in a smaller sample size which will reduce the cost of the audit. But, even when the client is right, the probability that auditing procedures will have to be extended is increased to 10%. The auditor should want to consider the impact of extending auditing procedures in light of the cost of additional sampling.

If a Type I error is committed, the client may incur a larger auditing fee due to the extension of auditing tests. The commission of a Type I error, however, has no effect on the rendering of an opinion. Consequently, the auditor will specify the reliability level at which he wants to carry out his statistical procedures so that if the client is right, the probability of rejecting a reasonably stated account balance is consistent with the cost of additional auditing procedures.

If the account is materially misstated and the account balance falls inside the achieved precision interval, a Type II error occurs.

The commission of a Type II error is extremely hazardous to auditors and to third parties. A Type II error has overwhelming business and economic implications as well as auditing implications. For instance, there are many types of users who rely on information contained in audited statements to make decisions concerning financing and stock market investments. If a Type II error is not found in subsequent periods, there is no way to measure the detrimental effects of having relied on such erroneous information.

The auditing implications involve the auditor's responsibility to protect third parties from misleading and erroneous financial statements. If a Type II error occurs, the auditor does not have a sound basis for his audit opinion and has failed to protect third parties. Furthermore, in subsequent periods the discovery of a materially misstated financial statement may lead to lawsuits against the auditor and impair his prestige. Explicit costs of a lawsuit as a result from discovering a Type II error can be measured and determined. Implicit costs associated with loss of confidence, loss of prestige, and other similar detrimental effects are much more difficult to measure.

Chapter III

REVIEW OF RELEVANT LITERATURE

This chapter represents an assimilation of relevant material resulting from a review of literature related to variables estimation within the audit environment. This material is broken down as follows: (1) the historical development of statistical sampling in auditing, (2) literature documentation in support of statistical sampling and of mean-per-unit techniques, (3) limitations of mean-per-unit techniques, and (4) specification of the variables for variables estimation.

Historical Aspects

According to Aly and Duboff, statistical sampling in auditing was first proposed by Corman in 1933.[1] The American Institute of Certified Public Accountants, however, did not establish a committee on statistical sampling until 1956, and this committee did not make an official statement until February, 1962.[2] The special report released in

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[1]Hamdi F. Aly and Jack I. Duboff, "Statistical vs. Judgment Sampling: An Empirical Study of Auditing the Accounts Receivable of a Small Retail Store," The Accounting Review (January 1971), p. 119.

[2]Committee on Statistical Sampling, American Institute of Certified Public Accountants, "Statistically Sampling and the Independent Auditory," Journal of Accountancy (February 1962), pp. 60-62.

1962 contained two important authoritative positions. First, the Institute recognized that statistical sampling was permissible under generally accepted auditing standards (GAAS). Second, the auditor should understand the relationship between precision and reliability and recognize that these variables are auditing functions and must be based on judgment.

The second authoritative release, Statement on Auditing Procedure No. 33,[3] came in 1963. It emphasized that the application of statistical sampling to auditing populations provided measurements that were not available with judgmental sampling. The third pronouncement came in July, 1964, from the Institute's Committee on Statistical Sampling.[4] This pronouncement relates precision to materiality, and reliability to reasonableness of audit judgment. The next relevant pronouncement, "The Auditor's Study and Evaluation of Internal Control," was published in November, 1972,[5] and superseded Statement on Auditing Procedures No. 33. These pronouncements concerning statistical sampling

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[3]Committee on Auditing Procedures, American Institute of Certified Public Accountants, Auditing Standards and Procedures No. 33 (New York: American Institute of Certified Public Accountants, 1963).

[4]Committee on Statistical Sampling, American Institute of Certified Public Accountants, "Relationship of Statistical Sampling to Generally Accepted Auditing Standards," Journal of Accountancy (July, 1964), pp. 56-58.

[5]Committee on Auditing Procedures, American Institute of Certified Public Accountants, Statement on Auditing Procedure No. 54 (New York: American Institute of Certified Public Accountants, November, 1972). (Hereafter referred to as SAP 54.)

were codified in Appendices 320A and 320B in Statement on Auditing Standards No. 1 in 1973.[6]

The most recent publication of the Institute concerning statistical sampling is the Selected Bibliography by the Subcommittee on Statistical Sampling Bibliography Task Force.[7] Each article selected for the bibliography had to meet the following four criteria: (1) publication after 1966, (2) availability, (3) involves statistical sampling in auditing, and (4) has potential usefulness to help solve a specific problem. Numerous articles have been published on statistical sampling within the auditing environment. As indicated in the Selected Bibliography the subject matter covers a wide range of statistical sampling topics. The current research involves only one segment of statistical sampling within the audit environment--a variables estimation technique referred to as mean-per-unit estimation.

Literature Documentation in Support of Statistical Sampling

This section summarizes support for statistical sampling as provided by the professional accounting literature. In addition, an

[6]Committee on Auditing Procedures, American Institute of Certified Public Accountants, Statement on Auditing Standards 1 (New York: American Institute of Certified Public Accountants, 1973), pp. 36-54. (Hereafter referred to as SAS 1.)

[7]American Institute of Certified Puboic Accountants, Subcommittee on Statistical Sampling Bibliography Task Force, Selected Bibliography (New York: American Institute of Certified Public Accountants, October 31, 1976).

attempt is made to cite specific support for variables and for mean-per-unit techniques.

General Support of Statistical Sampling

The Special Report in 1962[8] recognized that statistical sampling enables the auditor to measure the amount of sampling error in the application of statistical techniques and that the auditor determines the amount of sampling error that is acceptable in given situations. Quantification of the sampling error can be a valuable auditing tool. In other words, the auditor can specify the degrees of alpha and beta risks he is willing to assume in the circumstances. Arkin, Elliott and Rogers have emphasized the advantages of risk quantification as opposed to non-quantification found in judgmental sampling.[9]

In addition to risk quantification, Arkin presented the following seven advantages in using statistical sampling:

> 1. The sample result is objective and defensible....
> 2. The method provides a means of advance estimation of sample size on the objective basis....
> 3. The method provides an estimate of the sampling error....
> 4. The statistical sampling approach may provide a more accurate method of drawing conclusions about a large mass of data than the examination of all the data....

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[8]Committee on Statistical Sampling, "Statistical Sampling and the Independent Auditor," p. 60.

[9]Herbert Arkin, Handbook of Sampling for Auditing and Accounting (2d ed.; New York: McGraw-Hill Book Company, Inc., 1974), p. 9 (Hereafter referred to as Handbook); and Robert K. Elliott and J. P. Rogers, "Relating Statistical Sampling to Audit Objectives," Journal of Accountancy (July, 1972), pp. 46-47.

5. Statistical sampling may save time and money....
6. Statistical samples may be combined and evaluated, even though accomplished by different auditors....
7. Objective evaluation of test results is possible.[10]

Hermanson et al. add to Arkins's list of advantages by pointing out that statistical sampling creates a situation where better advanced planning of the audit takes place, and that the auditor has a basis for expanding his audit procedures if the results of his statistical test indicate that such expansion is necessary in the circumstances.[11]

This documentation in support of statistical sampling pertains to all classical techniques including variables estimation. Variables estimation, however, has advantages peculiar to itself. These advantages are discussed in the next section.

Literature in Support of Variables Estimation

In auditing, attribute sampling is associated with compliance testing while variables sampling is associated with substantive testing. Attribute sampling involves the frequency of an event such as the rate of occurrence for a particular type of error within an audit population. On the other hand, variables sampling is concerned with the dollar amount of an audit population. Compliance testing, utilizing

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[10]Arkin, Handbook, pp. 9-11.

[11]Roger Hermanson et al., Auditing Theory and Practice (Homewood, Illinois: Richard D. Irwin, Inc., 1976), p. 237. (Hereafter referred to as Auditing Theory.)

attribute sampling, can be used to determine the degree of reliance placed on internal controls. If the compliance tests indicate strong internal controls, substantive testing may be restricted but not eliminated under generally accepted auditing standards.[12] Consequently, there is an inverse relationship between reliance placed on internal controls and substantive testing.[13] That is, strong internal controls indicate less substantive testing than when internal controls are weak. In applying this trade-off between the two types of testing, the extent of compliance testing should not be greater than what can be saved in substantive testing.[14] This same point concerning the relationship between substantive testing and compliance testing is brought out by Kinney.[15] He states,

> Only if compliance is found to be adequate and no unusual relationships are discovered which require additional tests of details [substantive testing] will the compliance test routes exhibit large cost savings over [substantive testing].[16]

Since substantive testing relies on variables sampling, the variables sampling plans and techniques are important to auditing

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[12]Committee on Auditing Procedures, SAS 1, Section 320B.33, p. 52.

[13]Ibid., p. 35.

[14]Robert K. Elliott and J. R. Rogers, "Relating Statistical Sampling to Audit Objectives," Journal of Accountancy (July, 1972), p. 49.

[15]William R. Kinney, Jr., "Decision Theory Aspects of Internal Control System Design/Compliance and Substantive Tests," Journal of Accounting Research: Supplement 1975, pp. 14-29.

[16]Ibid., p. 17.

if statistical sampling is used in the audit environment. Furthermore, when the circumstances are such that variables sampling can be used for substantive tests, certain authors and practitioners assert that "it is a more useful measurement device than attribute sampling because auditors are generally more interested in the monetary amount of errors than the frequency of errors.[17]

Literature in Support of Mean-per-unit Sampling Plans

Mean-per-unit techniques use only the audited value of a sampling unit rather than the difference or ratio between the booked value and the audited value. Consequently, this technique can be used when individual book amounts are not available at the time of the audit.[18] Another advantage peculiar to this estimator is that a population total is not required in its application.[19] And, unlike ratio and difference estimation techniques, the mean-per-unit estimator does not require a high error rate in the population.

> When no differences between individual book values and audited values are found in the sample, the technique is still a valid test of the compilation and totaling of the data from the point at which the sample is drawn.[20]

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[17]Alvin A. Arens and James K. Loebbecke, Auditing: An Integrated Approach (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1976), p. 345.

[18]Ernst & Ernst, Audit Sampling (Ernst & Ernst, 1976), p. 21.

[19]Arthur Young & Company, Auditing Manual ([New York]: Arthur Young & Company, 1973), p. 7.424.

[20]Ibid.

To use the difference and ratio estimators, each sample taken from the population should have a minimum of about 30 errors.[21] Another advantage of the MPU estimator is that it does not presuppose a particular error pattern. The ratio estimator will generally provide better sample reliability when the differences between the audited and book values are proportional to the book values. The difference estimator will be better when the differences are not related to the book values.[22] These two latter estimators also assume that the unaudited book values be correctly compiled and totaled because such errors will not be reflected when these two methods are used. This is not the case when using the MPU estimator as compilation errors will be disclosed with this technique.[23]

Consequently, under the circumstances mentioned above, the mean-per-unit techniques provide a potentially valuable tool for the auditor. However, the technique is not without limitations. These limitations are the topics of the following section.

Limitations of Mean-Per-Unit Techniques

There are two minor limitations that will be presented first before discussing the major disadvantage of mean-per-unit techniques.

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[21] Ernst & Ernst, Auditing Sampling, p. 34.

[22] Arkin, Handbook, p. 302

[23] Ibid., p. 203.

Since this technique focuses on the audited value of each sampling unit, negative confirmations for accounts receivable cannot be utilized in conjunction with a mean-per-unit sampling plan because:

> The auditor would, in effect, be asserting that all nonresponding accounts were correct--that the audited value and book value are equal. Mathematically, every sample item, whether responding or not, has equal statistical importance; therefore, even one incorrect nonresponding account would destroy any possible statistical conclusion, even though judgmentally the auditor might have reason to believe all errors on nonrespondent accounts would be immaterial in the aggregate.[24]

The second minor limitation concerns computerization. If the records are not computerized, it is tedious and time-consuming to carry out statistical computations manually. This limitation, however, is not unique to the mean-per-unit techniques but is applicable to most statistical sampling plans.

The major limitation of mean-per-unit sampling plans is the inefficiency of the technique compared to other variables estimators. That is, the mean-per-unit estimator will require a larger sample size than the difference or ratio estimators.[25] "This is because the standard deviation of the population of ratios or differences is

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[24] Hermanson et al., Auditing Theory, p. 237.

[25] Ernst & Ernst, Audit Sampling, p. 55; and Arkin, Handbook, pp. 190-203.

smaller than the standard deviation of the population of audited values."[26]

It appears, then, that mean-per-unit techniques are not always desirable because of the associated large sample sizes but in many instances it is the only variables estimating alternative.[27] To recapitulate, the MPU estimator can be used when no account balance exists or when individual book values are not available at the time of the audit. This technique can also be used when the population error pattern and rate are unknown. Furthermore, MPU techniques disclose compilation errors in the accounting population. The following section of this chapter reviews the variables necessary to determine sample size when using mean-per-unit techniques.

Specification of Variables for Mean-Per-Unit Techniques

In order to use mean-per-unit techniques, the sample size must be large enough for the sampling distribution to approximate normality so that valid statistical inferences can be made. The traditional sample size formula requires (1) an estimate of the population variance, (2) a given amount of desired precision, and (3) a specified degree of the alpha risk the auditor is willing to assume in the circumstances. In 1972, Elliott and Rogers developed a scheme where the beta risk

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[26] Ernst & Ernst, Audit Sampling, p. 37.

[27] Ibid., p. 21.

would also be a consideration in the computation of the sample size.[28] These two authors defined precision as a function of the minimum amount considered material, a degree of alpha risk, and a degree of beta risk. Although Elliott and Rogers' expression of precision was discussed in Chapter II along with control of the alpha and beta risks, a literature review of the variables used to determine sample size are now presented from a historical viewpoint.

Precision

Historically, the traditional sample size formula equates precision to materiality. Precision, then, is the amount above and below the sample mean that the auditor would like to know if the true mean lies for a given reliability level. For this reason some of the texts do not make a distinction between materiality and precision.[29] However, two training manuals and an auditing text make this distinction and point out the effects on the beta risk when

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[28] Elliott and Rogers, "Relating to Statistical Sampling," pp. 46-55.

[29] R. M. Cyert and Justin H. Davidson, Statistical Sampling for Accounting Information (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1962), p. 58; Meigs, Walter B., E. John Larsen and Robert F. Meigs. Principles of Auditing, 5th ed. Homewood, Ill.: Richard D. Irwin, Inc., 1973. pp. 249-250; DeFliese, Philip L., Kenneth P. Johnson, and Roderick K. Macleod. Montgomery's Auditing. New York: The Ronald Press Company, 1975, 9th ed. pp. 819-820; Willingham, John J. and D. R. Carmichael. Auditing Concepts and Methods, 2nd ed. New York: McGraw-Hill Book Company, 1975. pp. 164-165.

precision is varied in relation to materiality.[30] When using the traditional sample size formula and specifying precision equal to materiality, the beta risk would be 50% assuming misstatement equals materiality.[31]

As pointed out in Chapter II, precision can be adjusted as a percent of materiality to reflect the desired beta risk. However, "some auditors have compensated by simply letting precision equal one-half the amount considered material."[32] In this case, the beta risk would equal one-half the alpha risk. This rule is too restrictive and may not be a useful auditing decision rule for the two following reasons. When the beta risk equals one-half the alpha risk, the sample size may be much larger than necessary in the circumstances and the two-to-one relationship may not be the proper mix for the alpha and beta risks.[33] The risk levels for beta and alpha should be allowed to vary independently of one another. The amount of precision or materiality used is judgmental and dependent upon the

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[30]Ernst & Ernst, Audit Sampling, pp. 71-72; Arthur Young & Co., Auditing, "Appendix IV, Variables Sampling," p. 7.408; and Hermanson, et al., Auditing Theory, p. 225.

[31]James R. Boatsman and Michael G. Crooch, "An Example of Controlling the Risk of a Type II Error for Substantive Tests in Auditing," The Accounting Review (July, 1975), pp. 614-615; and Ernst & Ernst, Audit Sampling, p. 71.

[32]Elliott and Rogers, "Relating Statistical Sampling," p. 48.

[33]Ernst & Ernst, Audit Sampling, p. 48.

circumstances since there are no authoritative pronouncements to guide the auditor in setting these amounts.[34]

Alpha Risk

The 1964 "Special Report by the Committee on Statistical Sampling of the American Institute of Certified Public Accountants" does not discuss the alpha risk per se but discusses reliability levels.[35] Reliability is the complement of the alpha risk. This report advocates that reliability levels for substantive testing "vary inversely with the subjective reliance assigned to internal control and to any other auditing procedures or conditions relating to the particular matters to be tested by such samples."[36] This report recognizes that the auditor faces two different risks: (1) "that material errors will occur in the accounting process by which the financial statements are developed" and (2) "that any material errors that occur will not be detected in the auditor's examination."[37] The probability that a material error will occur (risk 1) is related to internal controls, and the probability that the material error will not be detected (risk 2) is related to

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[34]Committee on Auditing Procedures, SAS 1, pp. 36-38.

[35]Committee on Statistical Sampling, "Relationship of Statistical Sampling to Generally Accepted Auditing Standards," pp. 56-58.

[36]Ibid., p. 57.

[37]Ibid.

substantive tests and other auditing procedures. The alpha risk is not mentioned but it is implied. This authoritative position is carried to SAP 54[38] and SAS 1.[39]

SAP 54 and SAS 1, however, take the concept one step further by stating that the "combined risk of both of the related adverse events occurring jointly is the product of the respective individual risks, and the combined reliability is the complement of such combined risk."[40] These ideas are incorporated in a formula proposed in both appendices. The formula is:

$$S = 1 - \frac{1 - R}{1 - C}$$

where: S = reliability level for substantive tests,

R = desired combined reliability level,

C = reliance assigned to internal accounting control and other relevant factors.

From this formula, it can be seen that the reliability level for substantive tests, "S", is the complement of the alpha risk $1 - \alpha$.

It is important to note that there is no explicit recognition of either an alpha risk or a beta risk. The appendices in SAP 54

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38 Committee on Auditing Procedures, SAS 54, Appendix B, pp. 265-277.

39 Committee on Auditing Procedures, SAS 1, Section 230B.

40 SAP 54, Appendix B, paragraph 29; and SAS 1, Section 320B, paragraph 29.

and SAS 1, however, do make attempts to recognize the beta risk in the following statement:

> The risk that material errors will not be detected in the auditor's examination is measured by the complement of the reliability level used if the auditor compares the upper precision limit of monetary error to the amount he considers material.[41]

This is an attempt to control the beta risk through use of the reliability level. More current publications, however, have used the above formula, slightly modified, not to compute the reliability level, but to compute the amount of beta risk the auditor should assume in the circumstances.[42] (This is discussed in the following section of this chapter.) These more current sources look at the alpha risk in relation to the cost and feasibility of examining additional sample items. In other words, if additional sampling over and above the original sample is not feasible or the cost is prohibitive for additional sampling, the alpha risk should be set at a low level. These sources view unnecessary audit time and cost as the penalty for committing an alpha error. If internal controls are good or excellent, there is a higher likelihood that the account balance is reasonably stated and a higher alpha risk will reduce the sample size and audit cost only if an alpha error is not committed.[43]

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[41] SAP 54, Appendix B, paragraph 30; and SAS 1, Section 320B.30.

[42] Hermanson et al., Auditing Theory, pp. 221-224; and Ernst & Ernst, Audit Sampling, pp. 66-88.

[43] Hermanson, Auditing Theory, p. 220.

Laudeman, however, recommends an alpha risk of 20-30% if the cost of additional sampling is relatively low.[44] He further recommends that the alpha risk be reduced to a lower level in the event that the reasonableness of the account balance must be rejected. And if the account balance is to be adjusted based on the results of the statistical sample, both the alpha risk and the beta risk should be reduced to very low levels, 5% or less.[45] The reason Laudeman recommends the low levels for alpha and beta when the account balance will be based on the statistical results is due to the fact that the legal status of such balance has not been tested in the courts.

Beta Risk

One of the earliest articles to deal with the beta risk was Elliott and Rogers' 1972 publication.[46] These authors point out that the level of beta risk that an auditor should be willing to assume is a function of (1) reliance assigned to internal controls, (2) the subjective probability of management's disregard for internal controls, and (3) other auditing procedures. The level of acceptable beta risk should vary inversely with reliance assigned to internal controls and the subjective probability of management's nonadherence to internal

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[44]Max A. Laudeman, "Sampling Risks Associated with the Audit Environment" (unpublished Ph.D. dissertation, University of Arkansas, 1976), p. 119.

[45]Ibid.

[46]Elliott and Rogers, "Relating Statistical Sampling," pp. 46-55.

controls. And, the more confidence that an auditor can place on other auditing procedures allows a correspondingly higher beta risk. These three elements that influence the level of beta risk, according to Elliott and Rogers, are used by other and more current sources to determine the ultimate risk faced by those individuals who rely on the work of auditors.

First, ultimate risk must be defined. "This is the chance that a material error will occur in the accounting process <u>and</u> that the auditor's examination will not detect it."[47] The ultimate risk defined here and in other sources[48] is the same ultimate risk defined in <u>SAS 1</u>, Section 320A, paragraph .14 and .15. The non-authoritative sources, however, give an algebraic interpretation of the ultimate risk. The algebraic interpretation[49] is as follows:

$$\text{Ultimate Risk} = \beta\,(1 - C)\,(1 - SP)$$

where: β = beta risk,

C = reliance assigned to internal controls,

SP = reliance assigned to supplemental (other) audit procedures.

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[47] Ernst & Ernst, <u>Audit Sampling</u>, p. 67.

[48] See Hermanson <u>et al.</u>, <u>Auditing Theory</u>, pp. 221-222; Jack C. Robertson, <u>Auditing</u> (Dallas, Texas: Business Publications, Inc., 1976), pp. 367-369.

[49] The notation may vary between sources. No attempt is made to identify a particular notation with a particular source.

This algebraic expression is further manipulated so that all the variables are stated (known) except the degree of beta risk. Therefore, the expression can be solved to determine the level of beta risk that should be used in statistical procedures. The expression is:

$$\beta = \frac{(1 - CR)}{(1 - C)\ (1 - SP)}$$

where: 1 - CR = ultimate risk.

"CR" is the combined reliability alluded to in SAS 1, Section 320B.32-.35. Although it is not pointed out in the literature, it can be seen that the second term on the right hand side of the SAS 1 formula[50] is equivalent to the above beta expression. Consequently, the reliability level for substantive testing, and integrating SAS 1 and the current sources, becomes:

$$S = 1 - \beta$$

It should be noted at this point that there is at least one dissenting opinion concerning the idea of a combined reliability level. Broderick states that "[t]he average or overall risk is not the controlling factor. If a material accounting error has in fact occurred, the risk of failing to detect it is the specific risk assumed for

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[50] See page 52 for this formula.

the specific test designed to detect the specific type of error."[51]

In summary, current accounting literature proposes that an acceptable level for the alpha risk should be a function of the feasibility of additional sampling (in the event that the statistical test indicates rejection of the reasonableness of an account balance) rather than a function of the reliance assigned to internal accounting controls. This current literature further proposes that the beta risk level should be a function of the reliance assigned to internal accounting controls and the possibility of management's nonadherence to these controls and to the integrity of other auditing procedures.

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[51] Arthur Andersen/University of Kansas Symposium on Auditing Problems, Contemporary Auditing Problems: Proceedings of the 1974 Arthur Andersen/University of Kansas Symposium on Auditing Problems, ed. Howard F. Stettler (Lawrence, School of Business, University of Kansas, 1974), p. 81.

Chapter IV

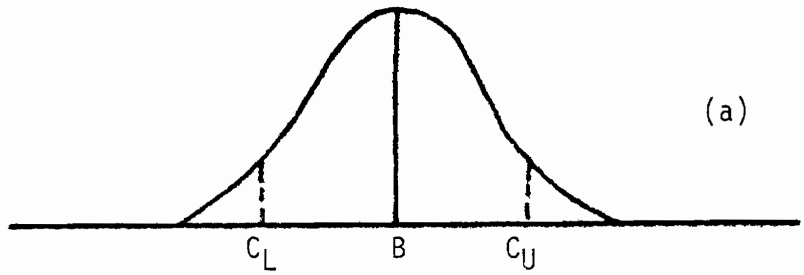
RESEARCH DESIGN

Procedures necessary to determine whether control of alpha and beta risks are effective under non-normal conditions when using the MPU estimator in conjunction with the sample size formula are presented in this chapter. The first section of this chapter discusses how simulation procedures determine whether Type I or Type II errors have occurred. Although the second two divisions are not integral parts of the simulation procedures, they are provided as justifications for using a unity variance and a standardized sample size in the investigation. The fourth section tells how the populations used in the research were created. With this background, simulation procedures are provided in the fifth section.

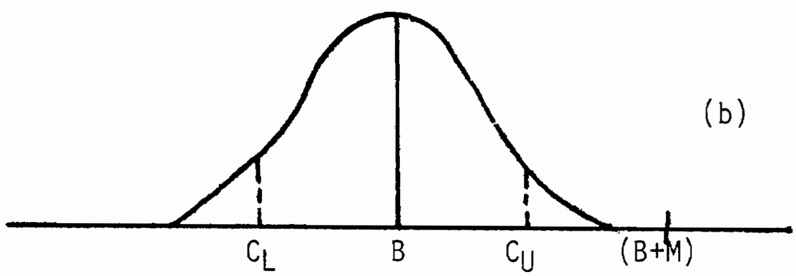
Determination of Type I and Type II Errors

In each of the 1,000 applications carried out during each computer run, three tests were made. That is, for each population and combination of alpha and beta risk levels, the simulation program performed three different tests 1,000 times. The first one tests for a Type I error while the next two are tests for Type II errors.

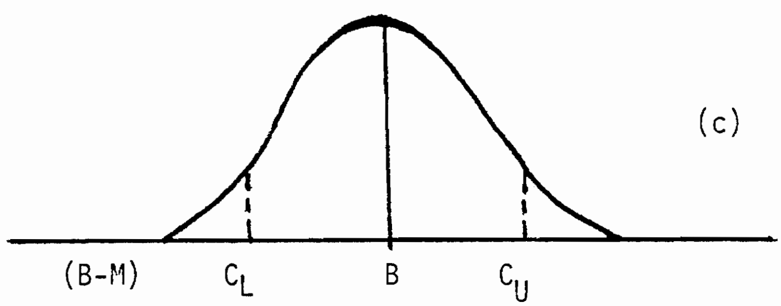
In testing for a Type I error, the population mean and average book value are identical. In other words, the book value is assumed true. In Figure 6, part (a), the population mean (average book value) is represented by "B." If the population



B = Book Value
Assumed true to test for Type I errors



(B+M) = Book Value
B = True Mean
To test for Type II errors when book value is overstated:
Type II errors--upper side



(B-M) = Book Value
B = True Mean
To test for Type II errors when book value is understated:
Type II errors--lower side

C_L = Lower confidence limit

C_U = Upper confidence limit

FIGURE 6

DESCRIPTION OF MEANS FOR TESTING

mean lies within the computed confidence interval around the sample mean, the confidence interval is considered acceptable such that a Type I error is committed. If the book value falls outside the confidence interval, a Type I error occurs.

To test for a Type II error, the book value can no longer be assumed true, but an alternative distribution must be assumed true. In this case, instead of moving the true distribution to a different location, it was assumed to remain at point "B" as illustrated in Figure 6, part (b). The book value was located at (B + M) which indicates that the book value was overstated by a material amount. If the book value, (B + M), fell inside the confidence interval just computed, the book value would be accepted as reasonably stated and a Type II error would have occurred. This was the second test and in the research it is considered as a test for Type II errors on the upper side.

The third test is similar to the second test in that the true distribution lies at point "B" as indicated in Figure 6, part (c). In this case, the book value is considered understated by a material amount and lies at point (B - M). The same test is carried out as above, except in this case it is referred to in the research as a Type II error or the lower side.

Justification of a Unity Variance

Support for a unity variance employs a mathematical model as well as data from simulation computer runs. There are two advantages to carrying out simulation procedures with a unity variance. First, the larger the variance used in computing a sample size, the larger the sample size. The larger the sample size, the more computer time necessary to carry out the simulation procedures. Therefore, by utilizing a variance equal to 1 instead of some larger number, it was possible to make many computer runs that otherwise would have been impractical to carry out. Second, research results would not have been significantly different whether a unity variance or a variance equal to 22,000 was used. Support for this statement is provided in a subsequent section. Consequently, research results will be valid for all degrees of variances found in populations. It will first be necessary to discuss the generation of random numbers by the computer program Purge3,[1] which is utilized in the current study. This computer program is presented in Appendix VI while its validation in relation to the current study is given in Appendix III.

Generation of Random Numbers

Purge3 uses Pearson's system of frequency curves to generate random numbers that follow a frequency distribution described by its first four moments.[2] The first four moments represent the mean,

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[1] The computer program Purge3 was developed by the Department of Statistics at Virginia Polytechnic Institute and State University.

[2] Using the first four moments of a frequency distribution it is possible to determine the characteristics of a family of twelve frequency distributions developed by Pearson. These distributions are known as Type I (beta distribution), Type II, Type III (gamma distribution), ..., Type X (exponential distribution), ..., Type XII.

variance, skewness, and kurtosis of a distribution. Definitions of these terms are given in Appendix I.

Purge3, using the first four moments of a distribution as input, generates as output random numbers that follow the distribution as described by the moments. Purge3 is written to generate these distributions with a mean of zero in which case the first moment is set at zero. The computer program, however, has the capability of generating a distribution around a positive mean when the positive figure is specified as the first moment.

When a distribution has a mean of zero, one-half the generated numbers are negative. Accounting populations do not have zero means and, if negative numbers exist in the population, they are generally separated from positive values for audit purposes. The generated negative random numbers cannot be thrown away using only the positive numbers because the character of the curve would change. Therefore, the curve must be shifted far enough to the right so that the distribution contains no negative numbers.

The shift can be done by defining the mean equal to the product of the absolute value of the minimum number and the standard deviation of the distribution. This can be represented symbolically by the following expression:

$$\bar{Y} = | t | s$$

where: $\overline{Y}$ = mean of the shifted distribution,

$|t|$ = absolute value of the minimum number of the distribution,

s = standard deviation of the distribution.

For example, the minimum value, mean, and standard deviation of a distribution are -5, 0, and 1, respectively. To shift this distribution to the right far enough so that no negative numbers appears in the population, the first moment for the shifted distribution must be specified as a 5. The distribution of random numbers generated by Purge3 will then center around a positive 5 rather than zero. The shift of the distribution with a standard deviation 1 and mean 0 to a distribution with a mean 5 and a standard deviation 1 is illustrated in Figure 7.

To shift any distribution as described above, it is necessary to know the minimum negative number and the standard deviation of the distribution. Purge3 is programmed to generate distributions with a variance of 1 in which case the second moment equals 1. Specification of the second moment can be changed to obtain a distribution with a variance other than 1 but as shown below, it was unnecessary to utilize this capability.[3] Consequently, the standard deviation for each distribution was automatically set at 1.

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[3]The capability to vary the population variance was utilized for the 5 simulation runs used in the research results to support a unity variance as discussed below.

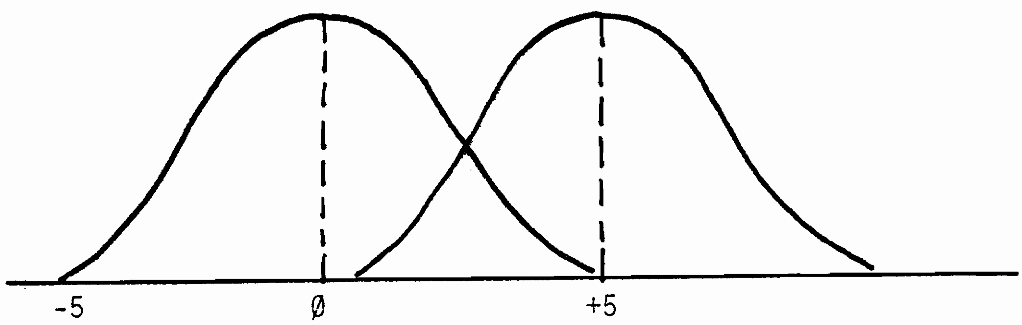

FIGURE 7

DISTRIBUTION WITH STANDARD DEVIATION 1 AND MEAN Ø
SHIFTED TO RIGHT TO OBTAIN A MEAN 5

It was necessary to determine the minimum negative number of each distribution used in the study before the shift took place. Procedures employed to find this number are given in the section, "Creation of Populations."

Mathematical Support for Unity Variance

To provide mathematical support for the unity variance, the sample size formula should first be reviewed.

$$n = \frac{s^2 \quad (Z_{\alpha/2} + Z_\beta)^2}{M^2}$$

where:

n = sample size,

s^2 = sample variance,

$Z_{\alpha/2}$ = factor from the standard normal distribution consistent with a specified realiability level,

Z_β = factor from the standard normal distribution consistent with the desired power of the test,

M = desired precision equal to materiality.

Materiality, although a judgmental factor, must be determined in a rational manner. Many times it is computed as a percent of the accounting variable under consideration. In this case, materiality can be expressed as a percent of the population mean or of an average account balance. For example, if 5% of the average account balance of $60 was considered material, materiality would equal $3. Expressed algebraically:

$$M = \%(\overline{Y})$$

where:

M = amount considered material,

$\%$ = percent considered material,

$\overline{y}$ = mean (average account balance).

As discussed above, the mean of a population with all positive values generated by Purge3 can be expressed as follows:

$$\overline{Y} = |\ t\ |\ s$$

Substituting this expression of the mean in the above materiality formula:

$$M = \%\ |\ t\ |\ s$$

This expression of materiality can now be substituted in the sample size formula so that:

$$n = \frac{s^2\ (Z_{\alpha/2} + Z_{\beta})^2}{\%^2\ |\ t\ |^2\ s^2}$$

It can be seen that the variance terms, s^2, cancel out. Furthermore, if the standard deviation of a population equals 1, then the mean of the population can be expressed as:

$$\overline{Y} = |\ t\ |$$

In other words, the mean is equal to the absolute value of the minimum negative number of a distribution. Substituting the mean for the minimum value, the sample size formula for a unity variance can be expressed as follows:

$$n = \frac{(Z_{\alpha/2} + Z_{\beta})^2}{(\% \overline{Y})^2}$$

where: $(\% \overline{Y})$ represents precision which is equal to materiality

Since the sample size formula can now be expressed with a unity population variance, the sample size for the investigation does not depend on the size of the variance. For this reason, populations with a unity variance were generated and used in the research. It is important to note at this point that employment of a unity variance does not affect the steps taken in the simulation computer program. The selected sample size formula will still be used and a preliminary sample will still be taken to estimate the population variance. The estimated variance, however, will fluctuate around 1 because any estimate of a population variance using a preliminary sample will fluctuate around the true variance.[4]

Simulation Results in Support of a Unity Variance

Five computer runs with different size variances were carried out in support of a unity variance. Results of these runs are given in

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[4]George W. Snedecor and William G. Cochran, Statistical Methods (6th ed.; Ames, Iowa: The Iowa State University Press, 1967), p. 46.

Table 2. Precision was set at 9% of the mean, which would be the maximum amount considered immaterial per item. Nine percent is the highest percent that can be considered immaterial in accordance with FASB Discussion Memorandum on "An Analysis of Issues Related to Criteria for Determining Materiality."[5] It was desirable to use the largest amount considered immaterial because of the efficiency gained in the computer runs. That is, the larger the amount of precision (due to the highest materiality percent), the smaller the sample size. Both alpha and beta levels were set at 20%. The runs were also made without standardization of the sample size. The average sample size for each of the five runs ranged from a size of 711 to a size of 798.[6] The standard errors of the averages ranged from 9.52 to 10.69.

The population variances for these five computer runs ranged from 3,732.21 to 22,470.24 as shown in the first column of Table 2. In the fourth column the proportion of times a Type I error was committed for each run is given. Columns five and six indicate, respectively, the proportion of times an Upper Type II error and a Lower Type II error was committed for each run. It can be seen that there are no

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[5]Financial Accounting Standards Board, FASB Discussion Memorandum, "An Analysis of Issues Related to Criteria for Determining Materiality," (Stamford, Connecticut: Financial Accounting Standards Board, March 21, 1975), pp. 156-157.

[6]The average sample size is an average over the 1,000 loops used in the simulation procedures. That is, for each loop a preliminary sample was taken and a sample size computed without regard to any standardization. The computed sample size was used in the procedures for that loop. Then at the end of the 1,000 loops an average size was determined.

TABLE 2

SIMULATION RESULTS USING DIFFERENT SIZE VARIANCES

			Commission of Errors		
				TYPE II	
Variance	Skewness	Kurtosis	TYPE I p*	UPPER p*	LOWER p*
3,732.21	5.52	10.01	.2110	.1760	.2550
5,618.18	5.56	10.07	.2130	.1920	.2540
7,534.80	5.60	10.12	.2030	.2270	.2560
9,433.30	5.62	10.13	.2110	.2000	.2640
22,470.24	5.63	10.19	.2080	.2010	.2600
Range of Standard Errors per column:	Low		.0127	.0120	.0138
	High		.0129	.0132	.0139

*Indicates the proportion of times an error was committed in the 1,000 loops carried out in the simulation procedures.

significant differences in the proportions within each type. Simulation results support the mathematical proof and, therefore, it is concluded that a unity variance can be used in the simulation procedures.

Standardization of Sample Size

This section explains the technical aspects and advantages of sample size standardization. Technical aspects involving mathematical procedures and research results will be discussed first.

Mathematical Procedures

The sample size formula with a unity variance can be reviewed on page 67. The denominator of this formula, ($\% \ \overline{Y}$), represents precision which is equal to materiality. For any given computer run of 1,000 loops, the desired combination of alpha and beta will be given and the population mean will be known. Therefore, in the sample size formula for any particular run, there are only two variables unspecified, the materiality percent and n. If n were set at a predetermined number, the formula could be solved for the materiality percent. Setting n at a predetermined number represents sample size standardization. The formula to solve for the materiality percent when n is predetermined is shown below:

$$\% = \frac{(Z_{\alpha/2} + Z_{\beta})}{\sqrt{n}\ \overline{Y}}$$

It can be seen that the materiality percent would change each time any variable on the right side would change. This means, in terms of simulation procedures for any combination of alpha and beta, that the percent would change each time the mean changes due to a change in the population. After going through a complete set of populations for a specific combination of alpha and beta, with the percent varying for each population, the steps are repeated with a new alpha and beta combination. Again, the materiality percent will change due to the combination change. Consequently, for each computer run of 1,000 cycles, the materiality percent was allowed to vary.

Computation of each materiality percent was manually performed and used as input for the computer program to determine the attainment or nonattainment of the sampling risks.

Letting the materiality percent vary to allow for sample size standardization does not alter the simulation program. In other words, the program would be the same whether the sample size was standardized or not because the only difference in program inputs between the two situations would be the materiality percent. If the sample size were not standardized, the percent would be constant from run to run. In the case of standardization, the manually computed materiality percent varies from run to run but since this was used as input to the simulation program, procedures within the program did not change.

A preliminary sample is still taken to estimate the unity population variance and to calculate the sample size using the selected sample size formula. The sample size determined in the simulation program will fluctuate around the predetermined size. The reason for the sample size fluctuation is due to the fluctuation in the estimate of the population variance around 1. That is, in the simulation program the preliminary sample estimate of the population variance will fluctuate around 1 and the sample size will fluctuate accordingly around the predetermined size.

The exact number for the predetermined sample size will be discussed in the section, "Simulation Procedures." Now, that the mathematical procedures and computer program implications have been explained, the effect of sample size standardization on simulation results will be discussed.

Research Implications

The effect of the decision to standardize the sample size, by letting the materiality percent vary, on simulation results is presented in Table 3. These results are presented in terms of three pairs. The runs were carried out on three different levels of skewness and kurtosis as indicated in columns three and four of Table 3. Each pair represents one run using a 5% materiality factor and a 9% materiality factor on the other run. It can be seen that with each pair and within each type of error, there are no significant differences between the proportions. There would have been a significant difference if the proportions had been more than three standard errors apart. Consequently, the simulation procedures can be carried out using a standardized sample size by letting the materiality percent vary without prejudice to the research results.

TABLE 3

SIMULATION RESULTS USING DIFFERENT DEGREES OF PRECISION

WHEN ALPHA = 20%, BETA = 20%

					Error		
						Type II	
Materiality Percent	Mean	Skewness	Kurtosis	Average Sample Size	Type I p*	Upper p*	Lower p*
5%	105.81	1.60	5.38	667	.195	.217	.188
9%	105.91	1.59	5.36	202	.199	.207	.219
5%	81.70	3.41	7.81	1040	.203	.185	.262
9%	81.77	3.42	7.82	320	.210	.190	.245
5%	53.66	5.51	10.01	2295	.203	.221	.230
9%	53.66	5.52	10.01	720	.211	.176	.255
Range of Standard Error per column							
		Low:			.0125	.0121	.0124
		High:			.0129	.0131	.0139

*Indicates the proportion of times an error occurred in the 1,000 loops carried out in the simulation procedure.

Advantages of Standardization

Advantages of sample size standardization include (1) reduction in computer time, (2) ability to generalize research results, and (3) the ability to control sample size so that guidelines could be developed for simultaneous attainment of desired alpha and beta levels. First, computer time can be utilized more effectively with a standardized sample size. For instance, using an alpha and beta combination of 20% each and a precision equal to 5% of a $2.50 mean, the sample size computed with the selected formula and a unity variance would be 289 units. If the sample size were predetermined, say at 150 elements, with a precision of 6.94% of the mean, computer time could be saved. Therefore, sample size standardization allows for more computer runs.

Furthermore, research results are generalizable as to the materiality percent. The simulation results will hold for any materiality percent and not just for one or two percentages. Control of the sample size enabled guidelines to be developed for the simultaneous attainment of a specified combination of alpha and beta for certain non-normal distributions. It is necessary to point out the effect of a standardized sample size on guideline development. The guidelines are based on sample sizes of 150, 450, and 1,000 units. If the simulation procedures did not control the sample sizes, it would have been impossible to develop such guidelines based on sample sizes. The standardized sample sizes of 150, 450, and 1,000 units are further discussed on page 91 in the section, "Simulation Procedures."

Creation of Populations

There are three major steps in creating the populations and verifying their shapes and characteristics. The first step involves determination of the minimum value and variance of a distribution to be used in scaling the moments. The second step consists of scaling the moments to get a proper distribution. The last step involves the preparation of a histogram to verify the J-shape of the distribution. These procedures are discussed below.

Determination of Minimum Value, Mean, and Variance of One Million Numbers

By utilizing Purge3 as a subroutine, a computer program was designed to determine the minimum value, mean, and variance of a population of one million elements.[7] (Computation of the mean was only incidental to the operations but the variance was necessary for a subsequent step.) For example, the first four original moments of a population could have been specified as 0.0, 1.0, 2.436, and 10.0, respectively. Purge3, using these four moments, created one million random numbers while the current program independently computed the minimum value, mean, and variance of the distribution. This step is represented by the first three blocks in Figure 8. Following through on the original moments given above, the minimum value, mean and

[7] See Appendix VII for this computer program. For its validation see Appendix IV.

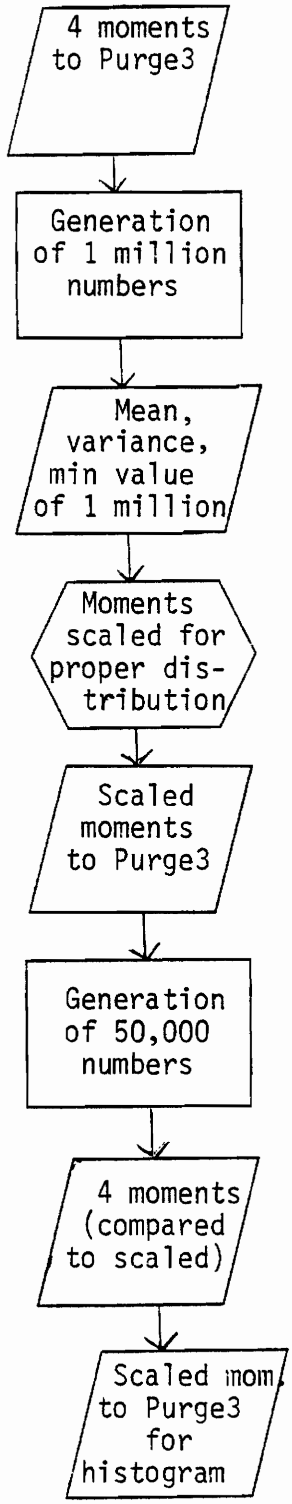

FIGURE 8

FLOWCHART TO CREATE POPULATIONS

variance of the million numbers were: -.64831185, .00124100, and .98950354, respectively.

Using the values determined in this step, the first four original moments were scaled to get a desired distribution as shown below.

Scaling of Original Four Moments

The first four moments of a distribution are related and dependent upon one another so that an operation on one requires similar operations on the other three. For instance, if the second moment is divided by the standard deviation squared, the first moment must be divided by the standard deviation, and the third and fourth moment must be divided by the third and fourth powers of the standard deviation, respectively. Purge3 generated the desired J-shpaed distribution with a unity variance using this technique. The scaling is demonstrated below using the original four moments, minimum value and variance of the one million numbers given above. The symbol, SD, represents the standard deviation.

MOMENT

First = | min value | / SD
= .64831185/.9947379
= .65174138

Second = original 2nd moment/SD^2
= 1.0/.98950354
= 1.01060780

Third = original 3rd moment/SD^3
= 2.436/.98429667
= 2.4748636

Fourth = original 4th moment/SD^4
= 10.00/.97911720
= 10.21328192

The scaling for all twenty-two distributions was carried out manually. This step is represented by the fourth block in Figure 8 on page 76. The four scaled moments were used as input to Purge3 for the generation of a J-shaped distribution with no negative numbers.

The scaling procedures were checked by using the scaled moments in a computer program utilizing Purge3. Purge3 was directed to generate 50,000 numbers. Since Purge3 automatically computes the first four moments of the distribution it generates, the output can be compared to the scaled moments. Fifty thousand numbers were arbitrarily selected because this procedure was only a check against the scaling procedures, and the reproduction of any size distribution using the scaled moments should approximate the moments of the population with one million elements. The moments of the distributions with 50,000 units were consistent with the manually computed scaled moments for each of the twenty-two populations used in the research. The fifth through the seventh blocks of Figure 8 illustrate this step.

Preparation of a Histogram

It was necessary to determine that distributions generated by the scaled moments were J-shaped. The only exception to this J-shape was the case of the normal distribution. The normal distribution was used because the MPU estimator and related auditing procedures are based on the normal approximation. A computer program, using the scaled moments as inputs and utilizing Purge3 as a subroutine to generate one million numbers, prepared a histogram of these numbers. See Appendix VIII for a listing of this computer program. Each histogram was visually checked to verify that it followed the desired shape. This step in the creation of the populations is represented by the eighth block in Figure 8. Also a software graphics package available at the Virginia Tech Computing Center was utilized to plot the histograms of the populations. See Appendix II for the plots of the twenty-two populations.

The J-shaped distribution characterizes auditing populations where there are many accounts with small book values and few accounts with large book values. It is, therefore, assumed that the generated populations describe real data sets. The actual first four moments for each of the twenty-two distributions are given in Table 4. When populations follow this type of distribution, kurtosis tends to become larger as skewness becomes larger. Simulation procedures were applied after the first four moments of the twenty-two populations used in the study were identified.

Simulation Procedures

The first part of this section provides more information on the simulation procedures outlined in Chapter I, while the second division discusses multiple sample sizes used in the simulation procedures. Standardized sample sizes of 150, 450, and 1,000 units are discussed in the third division.

Prior sections and chapters considered the characteristics and creation of the twenty-two populations each having one million elements. These populations were created using the first four scaled moments of a

Table 4

ACTUAL FIRST FOUR MOMENTS

OF 22 POPULATIONS USED IN SIMULATION PROCEDURES

Population Number	Moments			
	First	Second	Third	Fourth
1	10.00	.9917	.0000	3.0081
2	1.23	1.0014	.2511	1.9997
3	1.19	1.0023	1.8084	5.4554
4	1.04	1.0009	2.8648	6.6657
5	.66	1.0020	2.9869	5.0850
6	.65	1.0019	2.9913	4.9900
7	.72	1.0013	3.9112	6.9646
8	.62	1.0014	4.9679	7.9643
9	.55	1.0015	4.9872	7.3892
10	.65	1.0005	5.8274	9.8108
11	.59	1.0008	6.8529	10.8543
12	.51	1.0004	6.9285	9.9881
13	.59	1.0009	7.3251	11.7240
14	.53	1.0001	8.5233	12.8486
15	.50	1.0004	9.8138	14.6975
16	.47	.9992	11.4296	16.6631
17	.47	1.0121	13.2461	18.6029
18	.45	.9992	14.4238	21.3311
19	.43	.9995	16.0343	23.6213
20	.41	.9973	17.8738	26.0029
21	.35	.9984	21.0374	28.8256
22	.37	1.0004	25.6089	37.7937

distribution as inputs for Purge3 which generated the random numbers. Since Purge3 has reproductive capabilities associated with low computer cost, and because Purge3 generates the numbers randomly, the populations were not stored and used as data sets. Rather, the program utilized Purge3 as a subroutine to generate the random numbers as needed. Consequently, it was necessary to use the four scaled moments of each population as inputs for each computer run of the simulation program. The other inputs for each computer run, as well as basic steps performed in the program, are given below.

Expanded Simulation Procedures

A summary of the simulation plan was given in Chapter I. This section concentrates on details of the simulation. The block diagram, Figure 9, starting on page 83 is not a detailed program flowchart but a flowchart developed to provide an understanding of the procedures and steps taken in the simulation program. The computer program listing is given in Appendix IX and a validation of such program is provided in Appendix V. The following paragraphs provide a "walk" through of the block diagram given in Figure 9. Symbols used in the block diagram are:

$\bar{Y}$ = population mean,

$\bar{y}$ = sample mean,

% = materiality percent,

α = specified level of alpha risk,

β = specified level of beta risk,

s^2 = sample variance,

s = sample standard deviation,

i = 1,..., 4

n_i = sample size,

PS = preliminary sample of size 30,

$\subset$ = contained in,

$t_{\alpha/2}$ = factor from Student's t-distribution consistent with the specified alpha level,

df = degrees of freedom = n-1

CI = confidence interval,

LCL = lower confidence limit,

UCL = upper confidence limit,

PREC = desired precision,

SE = standard error,

RNS = random numbers.

Inputs necessary for each computer run are the materiality percent, combination of alpha and beta risks, and the four scaled moments which describe the population characteristics. Using these inputs, the program first computes the amount of desired precision as the product of the materiality percent and the population mean. It then selects a preliminary sample of size 30 to estimate the population variance. The estimated variance is then used to compute a sample size with the selected formula within the limitations of (1) the unity variance and (2) the standardized sample size. After computation of the sample size, multiples of the computed size were determined.

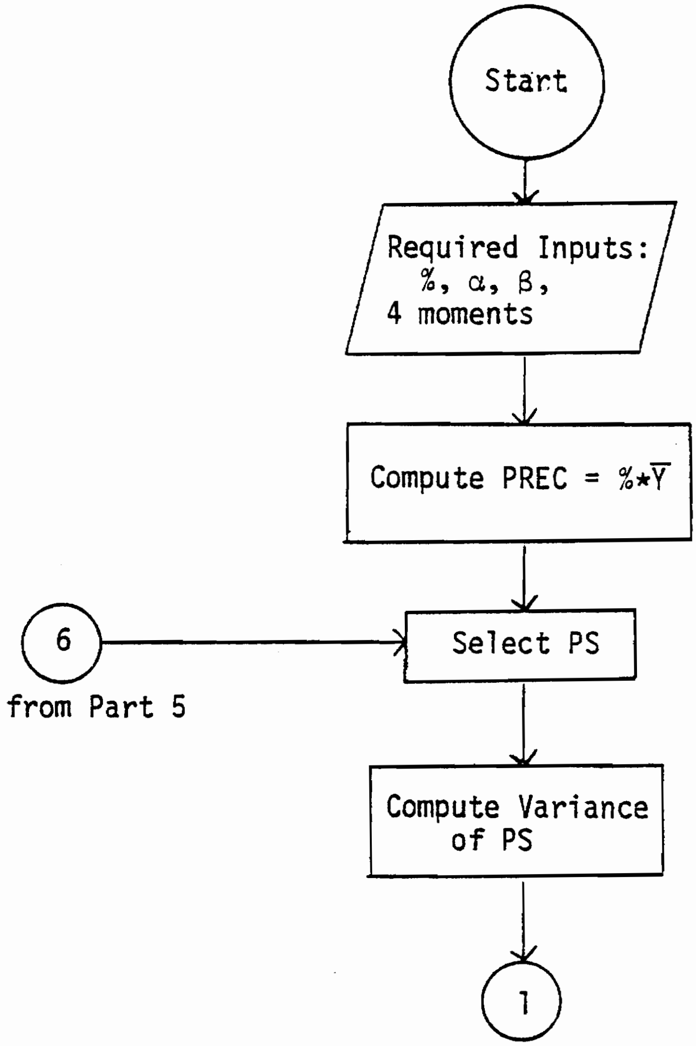

FIGURE 9
Part 1

BLOCK DIAGRAM - SIMULATION PROGRAM

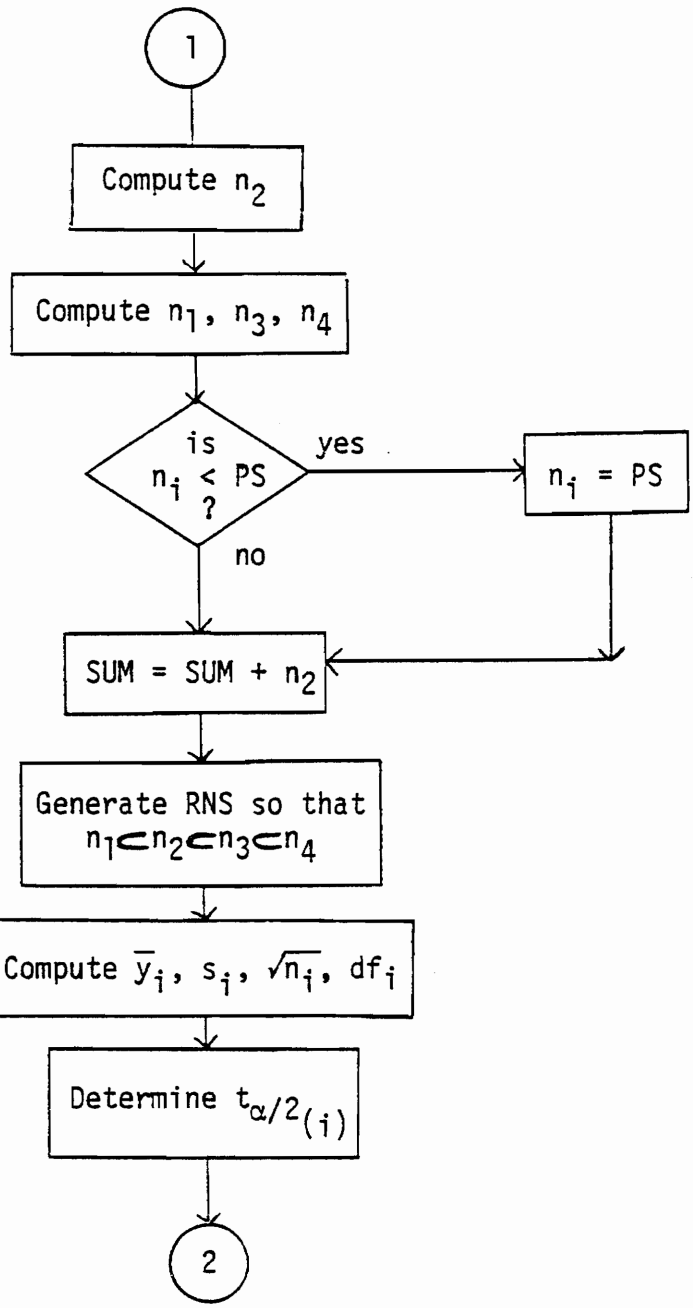

FIGURE 9
Part 2

BLOCK DIAGRAM - SIMULATION PROGRAM

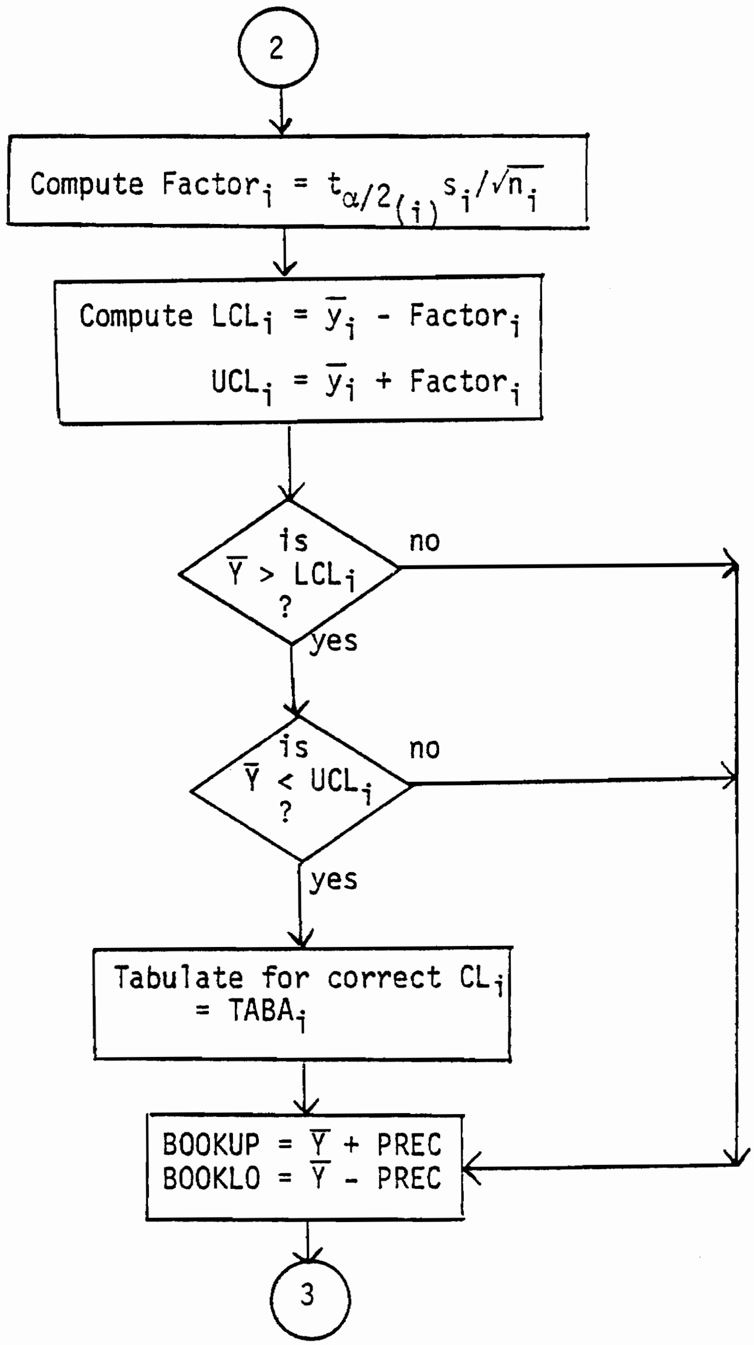

FIGURE 9
Part 3

BLOCK DIAGRAM - SIMULATION PROGRAM

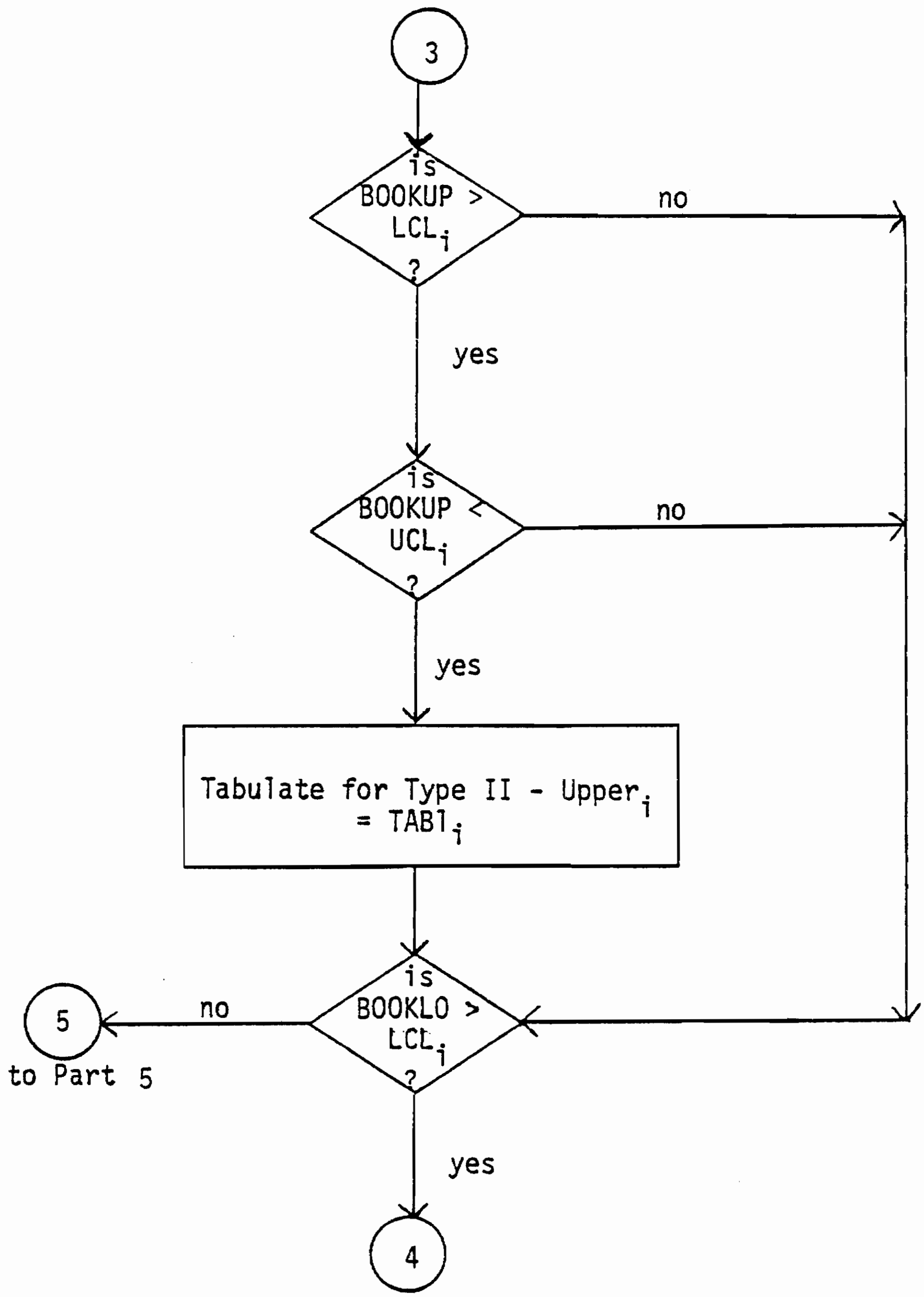

FIGURE 9
Part 4

BLOCK DIAGRAM - SIMULATION PROGRAM

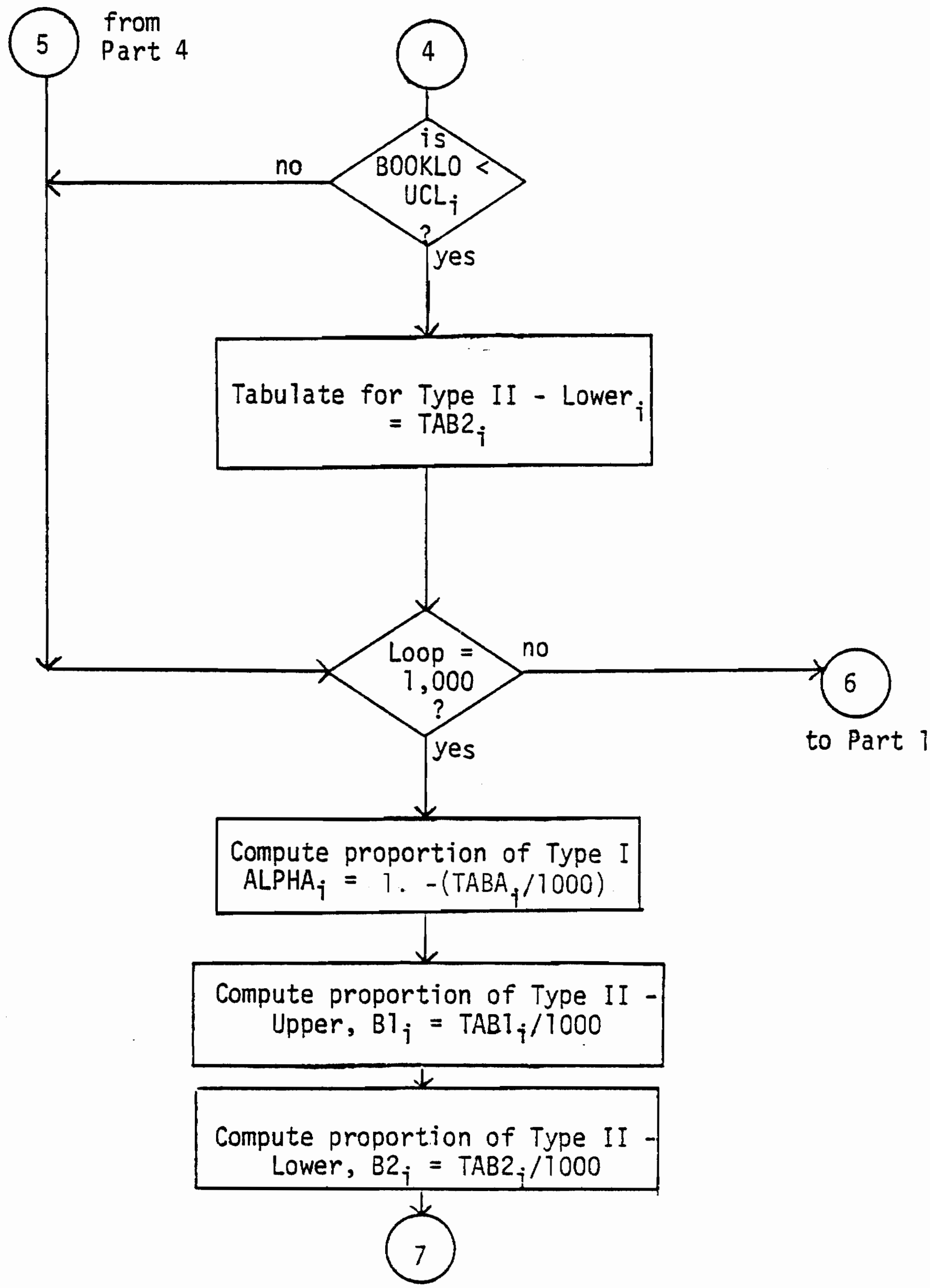

FIGURE 9
Part 5

BLOCK DIAGRAM - SIMULATION PROGRAM

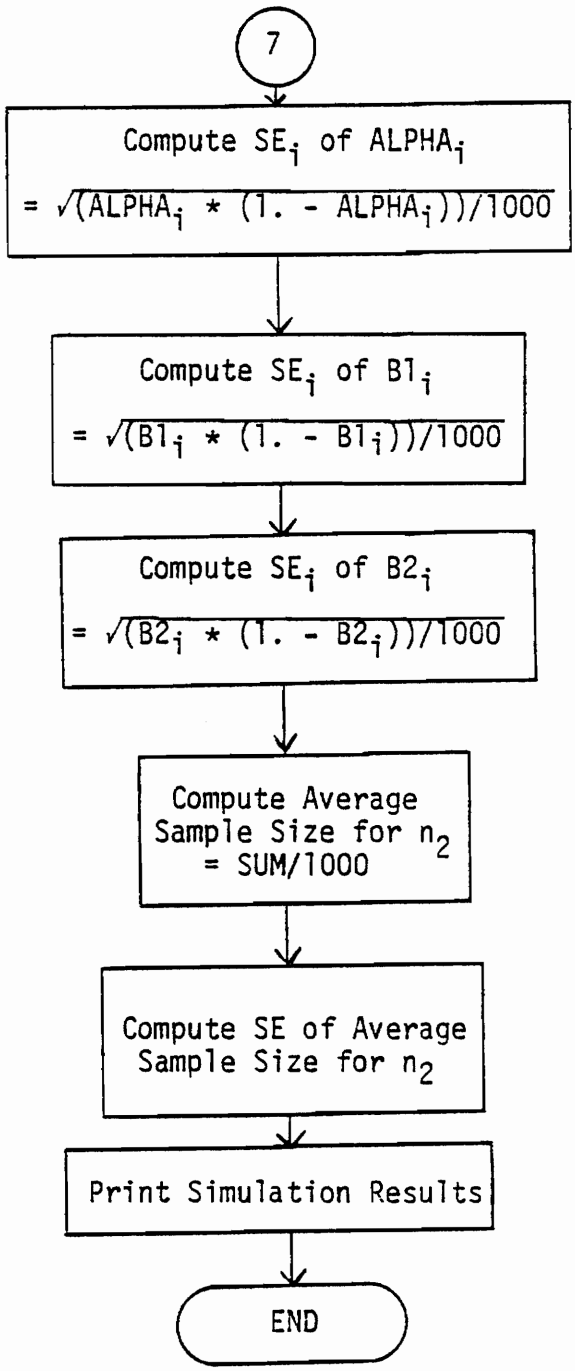

FIGURE 9
Part 6

BLOCK DIAGRAM - SIMULATION PROGRAM

Multiples were computed with factors of .5, 1.5, and 2.0 times the computed size. Having n symbolize a sample size and using n_2 to represent the sample size computed with the selected formula, the following notation is used:

$$n_1 = .5*n_2 \qquad\qquad n_3 = 1.5*n_2$$
$$n_2 = 1. \qquad\qquad n_4 = 2.0*n_2$$

If any sample size is less than the preliminary sample size, the computed size is disregarded and the preliminary size substituted in its place. However, this will probably not happen because the standardized sample size is at least five times larger than the preliminary sample size.

The sample size of n_2 is accounted for by summing its number of units into one counter so that at the end of 1,000 loops an average sample size can be determined.

Random numbers are generated so that the elements in n_1 are contained in n_2, and n_2 in n_3, and n_3 in n_4. The sample means, standard deviations, square root of the sample sizes, and the degrees of freedom are computed for all four samples. Using the appropriate degrees of freedom, the factor from Student's t-distribution consistent with the specified alpha level is determined. Using this factor, the attained precision--the plus and minus part of a confidence interval--is computed. The lower and upper confidence limits are then computed so that three tests can be performed.

The first test determines if the population mean falls inside the confidence interval. If so, a tabulation is made for a correct confidence interval. This test assumes that the population mean represents the average book value. The second test assumes the average book value, Bookup, to be the sum of the population average and desired precision. If this book value falls inside the confidence interval already computed, a Type II error on the upper side has occurred and a tabulation is made. The third test assumes the average book value, Booklo, to be the population mean minus the desired precision. If this book value falls inside the confidence interval, a Type II error on the lower side has occurred and a tabulation for this type of error is made.

The above steps, beginning with the selection of a preliminary sample, are repeated 1,000 times on one population with a given combination of alpha and beta risk levels. At the end of the 1,000 loops the proportions of times Type I and Type II errors occurred are calculated. The standard errors of the proportions are also determined. The average sample size for n_2 and its standard error are computed. The results of the computer run are then printed out.

Another computer run was started after changing the inputs to correspond with another population and/or combination of alpha and beta risks. These runs are discussed further in the last section of this chapter, "Standardized Sample Sizes."

Multiples of the Basic Sample Size

Multiples of .5, 1.5, and 2.0 times the basic sample size of n_2 are used in the simulation procedures to determine if the alpha and beta risk levels are attained using these sample sizes. In other words, if the sampling risks are not attained using the n_2 sample sizes, are they attained with the larger sample sizes? The one-half multiple was used to see if it were possible to attain the sampling risks with a sample size smaller than the basic n_2 size.

There are a few computer runs using a multiple of four times the basic size to determine if the sampling risks could be attained with this larger sample size. These runs are carried out on just a few selected distributions and selected combinations of alpha and beta risks to facilitate development of the guidelines presented in Chapter VI. The exact distributions and sampling risk levels using a multiple of four times the basic sample size are given in the next chapter.

Standardized Sample Sizes

Simulation procedures using a standardized sample size of 150 units are applied to selected populations for all combinations of alpha and beta risk levels. The mix of populations and alpha and beta combinations utilizing the 150 standardized sample size are given in the following chapter. Using these results, trends in the attainment of both sampling risks are identified for each alpha and beta combination.

Standardized sample sizes of 450 and 1,000 units are used in the simulation program with selected populations and combinations of alpha and beta to determine if similar trends can be identified when using these larger sample sizes. The populations and alpha and beta combinations utilizing standardized sample sizes of 450 and 1,000 units are identified in the next chapter. All of the simulation results, which are presented in Chapter V, were used to develop guidelines for the simultaneous attainment of specified alpha and beta risk levels under certain conditions of population non-normality. These guidelines are presented in Chapter VI.

Chapter V

RESEARCH RESULTS AND CONCLUSIONS

This chapter first synthesizes research results to this point and then provides detailed analyses of the alpha and beta factors. Although separate analyses are provided for the attainment of the alpha and beta levels, the separation is only a conceptual one as these two factors have an intrinsic interaction that can never be divorced. In fact, interaction between the attainment of the alpha and the beta risk levels is an integral part of the conclusions and is essential to understanding the recommendations made in the following chapter. Tables are presented at the end of the chapter while discussions of the results are given in the first part of the chapter.

General Conclusions

Table 5 reflects results of simulation runs where alpha and beta are set at 20% and the standardized (basic) sample size at 150 units. As the degree of non-normality increases, the discrepancy between the specified alpha level and the attained alpha level gets larger and larger. Column 3 of Table 5 reflects the trend concerning the discrepancy between the specified and the attained alpha levels. Although the point where the specified alpha level is no longer attained can be identified from the table, the trend appears to be the more important aspect. The trend appears more important because the point where the specified alpha level is no longer controlled changes when the variables which determine sample size change. For conditions set forth in Table 5 the specified alpha level is no longer attained when skewness and kurtosis are around 5 and 8 degrees, respectively. At this point the proportion of alpha errors is greater than 3 standard errors from the specified alpha level of 20%. Standard errors were computed using the formula given on page 20.

On the other hand when the client's book value is materially overstated above the true population value, the trend in the attainment of the specified beta level improves as the distributions become more skewed and more peaked. Column 4 of Table 5 reflects this trend concerning the attainment of the specified beta level.[1] A word of caution is advised here because this trend is affected by the interaction between the alpha and beta levels as sample sizes increase. This interaction is discussed at relevant points throughout the chapter.

Where the client's stated book value is materially below the true population, the trend reflects a larger and larger discrepancy

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[1]These conclusions are based on the commission of beta errors on the upper side as discussed previously.

between the specified and the attained beta levels as the degrees of skewness and kurtosis increase.[2] This trend is reflected in column 5 of Table 5.

These general conclusions on the three trends for the discrepancy between the specified alpha and beta levels and the attained levels are based on simulation procedures using a standardized sample size of 150 units. Simulation procedures were also carried out using standardized sample sizes of 450 and 1,000 units. These procedures using the larger standardized sample sizes were carried out allowing the materiality percent to vary with each of the 1,000 loops as discussed in the prior chapter. It should be pointed out that allowing the materiality percent to vary from run to run for a given combination of sampling risks and a standardized sample size provides for a constant amount of precision from run to run. However, when the standardized sample size is increased, the constant amount of precision from run to run will be smaller than under the smaller basic sample size. This is demonstrated in Table 6. Consequently, the effect of changing the standardized sample size on the trends is equivalent to changing precision. That is, as precision becomes smaller, the sample size gets larger and the effect of a smaller precision on the trends would be the same as increasing the standardized sample size.

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[2]These conclusions are based on the commission of beta errors on the lower side as discussed previously.

Another consequence of increasing the standardized sample size concerns the average book value when testing for beta errors. Since the average book value for upper beta errors is defined as the sum of the true population mean and the amount of desired precision, the average book value will be smaller with the smaller precision. The differences in the trends using a standardized sample size of 450 units as opposed to a 150 standardized size will now be discussed.

When using a 450 standardized sample size, the same alpha and lower beta trends prevail as under a sample size of 150 units except the trends are less pronounced. The mitigation of these trends with increased standardized sample sizes can be seen in columns 3 and 4, and in columns 7 and 8 of Table 7. Note in the two columns under Type I errors that the proportions of errors committed for a standardized sample size of 450 units are increasing at a slower rate than under the basic size of 150 units. The same inference can be made for the lower Type II errors.

Considering upper Type II errors, the decreasing proportion as skewness and kurtosis get larger is no longer apparent when using a 450 standardized sample size. This is reflected in column 6 of Table 7. Rather, the trend for the upper beta proportions, using the 450 sample size and disregarding the normal distribution, appears to be increasing. The behavior of this trend for the larger standardized sample size as compared to the trend for the smaller sample is probably caused by the interaction between two related phenomena.

First, since the increased standardized sample size creates a smaller book value for testing the upper beta level, the distance between the true population mean and the materially overstated book mean is smaller than with the smaller sample size. Furthermore, the increased sample size will cause the sampling distribution of means to more nearly approach the normal approximation. In other words, with smaller sample sizes, sampling distributions take on the characteristics of the original populations. Since the original population is J-shaped with a very small right tail, there are fewer potential samples from the right tail of the sampling distribution than when the sampling distribution approaches normality. Consequently, when more samples have the potential to come from the right tail of the sampling distribution due to the larger sample size, and with a shorter distance between the true population mean and the average book value, there is a higher probability that the confidence interval around the sample mean will include the materially overstated book value causing a Type II error.

Knowing these trends exist as population skewness and kurtosis get farther and farther from normality and as sample sizes get larger, four computer runs were made using a standardized sample size of 1,000 units to substantiate the existence of such trends. Only four runs using a 1,000 units sample size were made due to the computer cost for the larger sample sizes as compared to the smaller sample sizes. The four populations chosen for the runs are given in Table 8; the normal and the most skewed and kurtotic distributions were selected for the end points while the other two distributions are more equally spaced from the middle section of the gamut of populations.

The simulation results where alpha and beta equal 20% and with the standardized sample size at 1,000 units are presented in Table 8 along with parallel runs using the 150 unit standardized sample size. In column 4 of Table 8 it is reflected that the trend for the discrepancy between the alpha levels is still present but less pronounced with the larger sample size. Column 5 shows decreasing proportions for the upper beta level using the smaller standardized sample size while column 6 demonstrates increasing upper beta proportions for the larger sample size. Column 8 reflects increasing proportions for the lower beta risk level using the larger 1,000 standardized sample size but does not reflect a less pronounced trend.

The above general conclusions and trends are based on simulation procedures where alpha and beta are defined at 20% and for standardized sample sizes. Detailed analyses of alpha and beta risks will present conclusions and trends for the sampling risks defined at levels other than 20% and for sample sizes that are multiples of the standardized sample sizes.

Detailed Analyses for the Attainment of Alpha Risk Level

Evidence will first be presented to show that the attainment or lack of attainment, of the alpha level is independent of the specified beta level. However, the same inference cannot be made concerning the attainment of the beta level. This will be discussed in the section on the detailed analyses for the attainment of the beta risk level.

Proportions of Type I errors when alpha and beta are defined at 10% are equal to the proportions of Type I errors when alpha is still defined at 10% but beta at 20%. Table 9 reflects the results for alpha equal to 10% and with beta at 10% and 20%. The identical proportions hold for all multiples of the basic sample size. The same proportional inferences hold when alpha is defined at 20% and with beta at 20% or 10%; the identical proportions from these simulation procedures are presented in Table 10. Again, the proportional inferences hold for all sample size multiples.

Tables 9 and 10 also reflect that there is some improvement in the proportions of Type I errors committed between sample sizes that are 1.5 and 2.0 times the basic size of 150 units. The improvement, however, indicates that doubling the sample size will not always culminate in the attainment of the specified alpha level.

The data in Table 11 from all twenty-two distributions demonstrates (1) changes in Type I proportions for sample sizes that are .5, 1.0, 1.5 and 2.0 times the basic sample size of 150 units (horizontal analyses), and (2) <u>trends</u> in the attainment of the specified alpha level as the distributions get further and further from normality (vertical analyses). Horizontal analyses indicate that for sample sizes that are multiples of .5 and 1.0 the 150 basic size, the alpha level is attained until skewness is approximately 5 degrees and kurtosis 7 degrees. For the 1.5 multiple size, alpha is controlled until about 7 degrees of skewness and 11 degrees of kurtosis. Alpha is attained until approximately 9 and 13 degrees of skewness and kurtosis, respectively, for the 2.0 multiple sample size. Vertical analyses of 1.0, 1.5, and 2.0 multiples of the basic size support the statement that as the sample size increases the attained alpha level moves away from the specified alpha level at a slower rate.

Table 12 through 31 are presented (1) to give additional evidence of the trend for the increasing discrepancy between the specified and the attained alpha levels as the distributions get further and further from normality, and (2) to provide additional support for the statement that doubling the sample size does not always culminate in the attainment of the specified alpha level. These tables present data not only from the standardized sample sizes of 150, 450, and 1,000 units but also from sample sizes that are multiples of .5, 1.5, and 2.0 times the standardized sample size. Furthermore, several simulation runs were not made when most of the computer runs were initially carried out. Rather, these later runs were made to facilitate guideline development. For example, research results did not disclose a 5% attained alpha level using a 150 standardized sample size when skewness and kurtosis were approximately 16 and 24 degrees, respectively. In the development of the guidelines it was necessary to determine if an alpha level of 5% could be attained using a lower specified level of alpha. Two simulation runs were carried out for that degree of non-normality using specified alpha levels of 1% and .1%. As indicated in column 4 of Table 19, the 5% alpha level was attained using a specified alpha level of .1%. After presenting the remaining research results, development of the guidelines will be discussed in the following chapter.

Tables 12 through 19 present data from simulation procedures using a basic sample size of 150 units with Tables 12 through 16 reflecting beta at 20% and alpha at 15%, 8%, 5%, 1%, and .1%, respectively. Tables 17 and 18 reflect simulation results where beta was specified at 5% and alpha at 20% and 5%. Table 19 presents the results of two simulation runs where beta was set at 5% while alpha was first specified at 1% and then at .1%.

Tables 20 through 29 reflect simulation results using a standardized sample size of 450 units. Tables 20 through 25 present data where beta was set at 20% while alpha was specified at 15%, 10%, 8%, 5%, 1% and .1%, respectively. Tables 26 and 27 use beta at 10% and alpha at 20% and 10%, respectively. Tables 28 and 29 show simulation results from beta specified at 5% while alpha was set at 20% and 5%, respectively.

Table 30 reflects simulation results defining alpha and beta at 20% and using a standardized sample size of 1,000 units. Table 31 presents results of four simulation runs with a standardized sample size of 1,000 units; beta was set equal to 20% in all four runs, while alpha was set at 15% in two runs and at 10% in the other two runs.

Using a multiple of 4 times the basic sample size of 150 units, simulations procedures were applied to selected distributions. The data from these runs are provided in Table 32. The selected distributions were made on a basis involving the attainment of the beta level on the lower side. This will be discussed in detail under the analyses for the attainment of the lower beta risk level. For the simulation runs represented in Table 32, there was only one case where the specified alpha level was attained. This was in the case where the alpha and beta levels were specified at 20% and 5%, respectively, and skewness was approximately at nine degrees while kurtosis was about thirteen degrees.

In summary, the analyses for attainment of alpha levels reflect several important findings. First, the attainment of the alpha level does not depend on the level specified for beta. Second, as skewness and kurtosis get farther and farther from normality, the discrepancy between the attained alpha level and the specified alpha level gets larger and larger. With larger sample sizes, however, this trend is less pronounced. These conclusions result from vertical analyses of the alpha factor. Horizontal analyses of the simulation results indicate that doubling, or even quadrupling, the sample size for a given degree of skewness and a given degree of kurtosis does not always culminate in the attainment of the specified alpha level. A system has been developed, however, in Chapter VI where a specified alpha level can be attained. The alpha guidelines are integrated with beta guidelines so that specified levels of alpha and beta can be attained simultaneously.

Detailed Analyses for the Attainment of the Upper Beta Risk Level

The attainment of the upper and lower beta risk levels depend not only on the size of the sample in relation to skewness and kurtosis but also on the specified alpha level. In the prior section, Tables 9 and 10 demonstrated that the proportions for the alpha errors were identical whether beta was specified at 10% or at 20%. This is not the case for the beta risk level. Tables 33 through 36 give the upper beta proportions which are the counterparts to the alpha proportions in Tables 9, 10, and 11. For example, Tables 33 and 35 demonstrate that when beta and alpha equal 10% the upper Type II proportions are not identical to the proportions when beta still equals 10% but with alpha at 20%. The same proportional inferences can also be made when beta equals 20% and with alpha at 10% and 20% as shown in Tables 34 and 36. Table 36 also reflects data from all twenty-two populations which are the corresponding beta proportions to the alpha proportions given in Table 11.

Tables 33 through 36 reflect that the upper beta level was attained for all degrees of skewness and kurtosis for the average sample size of 150 units and with alpha and beta combinations involving 10% and 20%. These tables also support the general conclusion that as non-normality increases, proportions become smaller for sample sizes that are 1.0 times the 150 standardized sample size. On the other hand, the vertical trends for the multiples of 1.5 and 2.0 times the standardized sample size show increasing beta proportions. The horizontal analyses show that the proportions are decreasing as the sample size gets larger.

Tables 37 through 57 reflect the remaining data on the proportions for upper Type II errors and are coordinated with Tables 12 through 32 for the alpha levels. The data in Tables 37 through 57 are presented, first, to give additional evidence that for a given distribution, the beta proportions decrease with increased multiples of the standardized sample size (horizontal analyses). Second, the data reflect the interaction between the alpha and beta factors which is discussed below. As a consequence of the interaction between the alpha and beta levels, the attainment of the beta risk level depends not only on skewness and kurtosis, and sample size, but also on the level of alpha used in the test.

It is reflected in Tables 33 through 42 that the beta levels were attained in all instances for the 1.0 multiple sample size. Of course, since the beta errors reflect decreasing proportions with the increased multiples horizontally, the beta level will be more than attained. That is, the proportions will be smaller than the specified beta level. In more detail, the beta level was attained for all degrees of skewness and kurtosis using a standardized sample size of 150 units when beta was defined at 10% and alpha at 10% and 20%, and when beta was set at 20% and with alpha set at the levels of 20%, 15%, 10%, 8%, 5%, 1%, .1%, and when beta and alpha were set at 5% and 20%, respectively. There is one exception for the multiplicative factor of 1.0 and this is where skewness and kurtosis equals 0.000 and 3.0081, respectively, and when alpha and beta are defined at 10% each (Table 33). In this instance, subtracting three standard errors, .0324, from the .1360 proportion yields .1036 which is slightly larger than the target proportion of 10%.

However, when alpha and beta are both defined at 5%, Table 43, the upper beta level is not always controlled. Using a basic sample size of 150 units (multiplicative factor of 1.0), the upper beta level is attained until skewness and kurtosis is approximately 16 and 24 degrees, respectively. For this degree of non-normality, the upper beta level is not attained with the 1.0 multiple of the 150 standardized sample size. However, using the sample size 1.5 times the basic size of 150 units, the upper beta level is attained for this degree, or higher degrees, of non-normality. In Table 44, it can be seen that when skewness and kurtosis approximate 16 and 24 degrees, respectively, and when alpha is defined at 1% or .1%, the beta level of 5% is not attained unless 2 times the 150 basic sample size is used.

With the larger standardized size of 450 units, Table 45 reflects that the 20% beta level is attained with the 1.0 factor sample sizes for all degrees of skewness and kurtosis and when alpha is defined at 15%. When using the basic 450 sample size (multiplicative factor of 1.0) and with alpha defined at 10%, 8%, 5%, 1%, and .1%, the beta level of 20% is not always attained as discussed below.

Data in Tables 46, 47, and 48 reflect that the upper beta proportions for the 1.0 factor are more than 3 standard errors away from the target proportion of 20% when skewness is at about 21 degrees or more. However, the target proportion of 20% can be attained with a sample size of 1.5 times the 450 standardized size. In these tables alpha is defined at 10%, 8%, and 5%, respectively. When alpha is defined at 1% and .1% and with skewness at approximately 13 degrees or more, the beta level of 20% is not attained. This is reflected in Tables 49 and 50. Again, the 20% beta level is attained when using a sample size of 1.5 times the 450 basic sample size.

When beta is defined at 10% and alpha at 20% (Table 51) for distributions with skewness equal to or greater than approximately 16 degrees, the proportions for the basic size of 450 units for the upper beta level are greater than three standard errors from the target proportion of 10%. The 10% beta level can be attained with the sample size multiple of 1.5. In Table 52 where alpha and beta are set at 10%, upper beta proportions for the sample size factor of 1.0 are greater than 3 standard errors from the target proportion of 10% except in the case where skewness is about 9 and kurtosis about 13 degrees. In this one case the proportion is only 2.14 standard errors from the 10% target proportion. In all of the other cases the specified beta level of 10% can be attained using sample sizes of 1.5 times the basic size of 450 units.

When beta is defined at 5% and with alpha at 20% and 5% (Tables 53 and 54), the specified beta proportion of 5% is not attained for the 1.0 sample size factor when skewness is equal to or greater than approximately 6 degrees. An exception occurred where alpha and beta were 20% and 5%, respectively, and skewness equaled 8.5233 (Table 53); the specified beta level of 5% was attained since the attained 7% proportion was within 2.5 standard errors of the target proportion. In all of the other cases, however, the data reflect that the beta level of 5% was attained using 1.5 times the 450 standardized sample size for degrees of skewness greater than about 6 but less than 16 degrees. For degrees of skewness equal to or greater than about 16, the sample size of 2 times the basic 450 size must be used to attain the 5% beta level when alpha is set at 20% or 5%.

When beta and alpha are specified at 20%, and with the factor of 1.0 times the standardized sample size of 1,000 units, the upper beta level is not attained for degrees of skewness equal to or greater than approximately 16 degrees as shown in Table 55. However, a 20% target proportion for upper beta was attained for all degrees of skewness equal to or greater than 16 degrees when using the sample size of 1.5 times the basic size of 1,000 units and with alpha set at 20%.

Table 56 demonstrates that when beta is identified at 20% and alpha at 15% or 10%, a standardized sample size of 1,000 units will not allow attainment of the beta level for a distribution with about 21 degrees or more of skewness and 29 degrees or more of kurtosis. If the sample size is increased to 1.5 times the basic size of 1,000 units, a beta of 20% will be attained under these conditions.

The last table concerning upper sided beta errors, Table 57, reflects results of simulation runs where a multiple of 4 times the basic sample size of 150 units was used. The selected distributions were made on a basis involving the attainment of the lower beta risk level. This selection is discussed under the analyses of the attainment of the lower beta risk level. The proportions, for the upper beta risk level using an average sample size of 4 times the 150 basic size and for the selected distributions and combinations of alpha and beta levels, are well under the target proportions. In other words, this sample size is much too large in these instances and the attained beta level is extremely small compared to the specified beta level.

In summary of the analyses for control of the upper beta levels, the simulation results reflect the following important findings. All of these findings are related and cannot be completely discussed as separate and distinct conclusions. The attainment of the upper beta risk level depends on (1) the amount of skewness and kurtosis, (2) the sample size, and (3) the level of alpha used in the test.

The attainment of the upper beta risk level appears achievable by increasing the sample size when the population distributions are positively skewed as defined in this study. The increase in sample size involves either a factor of 1.5 or 2.0 times the basic sample size and is dependent upon the alpha level used in the test and the degrees of skewness and kurtosis found in the population. A system has been developed in Chapter VI to be used as guidelines in the attainment of the upper beta risk level while attaining a specified alpha level.

Detailed Analyses for the Attainment of the Lower Beta Risk Level

The simulation results for the lower beta risk levels are different from the results for the upper beta levels due to the positive skewness of the populations. As stated under the section, "General Conclusions," simulation results reflect a trend where the discrepancy between the specified and the attained beta levels gets larger and larger as the population distribution moves away from normality. This trend was demonstrated in column 5 of Table 5. The trend is not only seen for the basic sample size of 150 units but also for the standardized sizes of 450 and 1,000 units as seen in Tables 7 and 8, respectively. This vertical trend, where the discrepancy between the target and the attained lower beta levels gets larger as the distributions have more and more skewness and kurtosis, is also seen in the simulation results for the sample sizes which are multiples of 1.5 and 2.0 of the basic sample size. Tables 58 through 69 show results using the 150 standardized size while Tables 70 through 79 reflect results for a standardized size of 450 units, and Tables 80 and 81 give results for a basic size of 1,000 units. The last table, Table 82, provides for data using a multiple of 4 times the basic sample size of 150 units. Tables 58 through 82 are coordinated with tables providing proportional data for Type I and upper Type II errors. Table 83 coordinates table numbers for the cross-referencing of proportional data for sampling errors.

There is a horizontal trend which is seen consistently throughout the simulation results for the lower beta level. For a given distribution, going from the sample size of .5 times to 2.0 times the basic size, the proportions of lower beta errors decrease. In fact, the horizontal trend is such that the specified beta level can be attained under most situations by using a multiple of 1.5 or 2.0 times the basic sample size. In those cases where the beta level was not attained with at least a doubling of the basic size, simulation runs were made quadrupling the basic sample size of 150. Table 82 demonstrates not only the proportion of lower beta errors using a multiple of 4 times the 150 basic sample size but also the combinations of alpha and beta levels used in conjunction with specific degrees of skewness and kurtosis. Comparing the proportions given in Table 82 with the specified beta level, it is seen that in all cases the specified level was attained by quadrupling the 150 standardized sample size.

The attainment of the lower beta level, like the upper beta level, depends on the level of alpha used in the test as well as the sample size and population degrees of skewness and kurtosis. The following two graphs (Figures 10 and 11) demonstrate the relationship between the degrees of skewness and kurtosis, the alpha level attained, the the necessary sample size factor in order to attain a lower beta level of 20%. The "x-axis" on these graphs is based on the product of skewness and kurtosis found in the population. The reason for using the product of these two factors is to coordinate these graphs with graphs developed in Chapter VI; this point will be elaborated upon therein. Figure 10 is based on a standardized sample size of 150 units while Figure 11 is based on a 450 standardized sample size. The lines dividing the multiplicative factors for sample sizes are not definitive lines but are made to show approximate divisions between the sample size factors. The divisions cannot be definitive because the simulation results indicate trends and deal with only a few points within the axes representing the product of skewness and kurtosis and the alpha levels attained in the simulation runs.

In summary, analyses for the attainment of lower beta levels demonstrate the following findings. The attainment of the lower beta level depends on (1) the amount of skewness and kurtosis found in

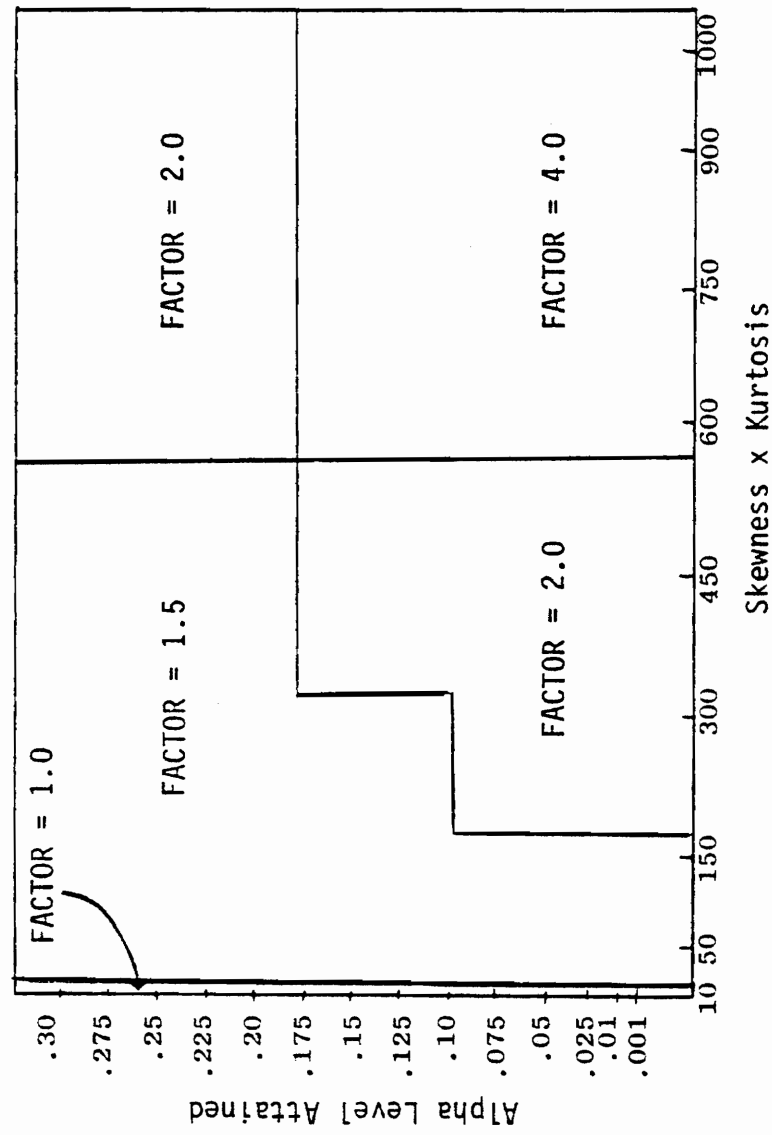

FIGURE 10
ATTAINMENT OF 20% LOWER BETA LEVEL
WHEN n = 150

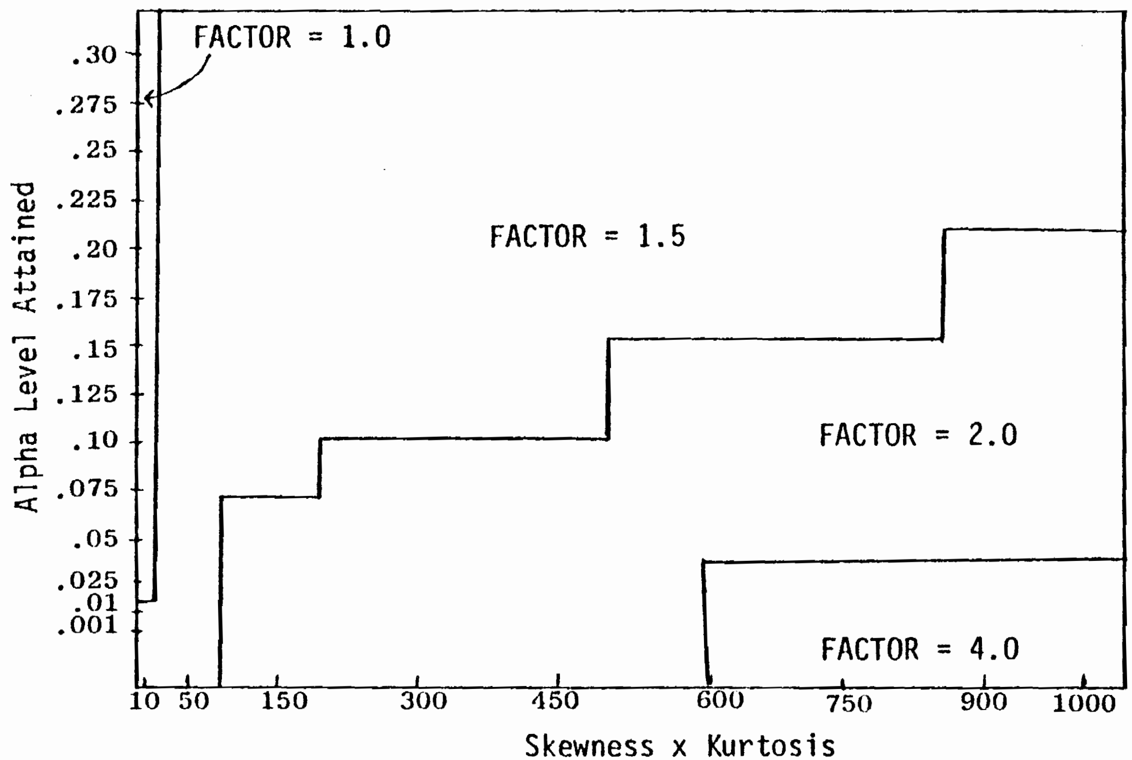

FIGURE 11

ATTAINMENT OF 20% LOWER BETA LEVEL
WHEN n = 450

the population, (2) the sample size, and (3) the level of alpha used in the test. A vertical trend reflects an increasing discrepancy between the attained and specified lower beta levels as skewness and kurtosis get larger. The horizontal trend for a given distribution shows that as sample sizes increase, the proportions for the beta errors decrease. The simulation results demonstrate that the specified beta level can be attained by using a factor of 1.5, 2.0, or 4.0 times the basic sample size dependent upon the degrees of skewness and kurtosis found in the population and upon the alpha level used in the test. Guidelines have been developed in Chapter VI for the attainment of the lower beta risk level. Lower beta guidelines are integrated with alpha guidelines so that the two specified risk levels can be simultaneously achieved.

TABLE 5

PROPORTIONS OF TIMES TYPE I AND TYPE II ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN ALPHA = 20%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Errors for		
			Type II	
Skewness	Kurtosis	Type I	Upper	Lower
.0000	3.0081	.2040	.2310	.2060
.2511	1.9997	.1950	.2100	.1900
1.8084	5.4554	.2000	.1970	.2220
2.8648	6.6657	.2300	.1860	.2460
2.9869	5.0850	.2450	.2180	.2390
2.9913	4.9900	.2290	.1970	.2210
3.9112	6.9646	.2230	.1730	.2390
4.9679	7.9643	.2480	.1590	.2600
4.9872	7.3892	.2490	.1860	.2560
5.8274	9.8108	.2580	.1780	.2560
6.8529	10.8543	.2550	.1760	.2770
6.9285	9.9881	.2680	.1720	.2810
7.3251	11.7240	.2630	.1600	.2750
8.5233	12.8486	.2640	.1520	.2760
9.8138	14.6975	.3000	.1460	.2930
11.4296	16.6631	.3130	.1570	.3020
13.2461	18.6029	.2890	.1540	.2950
14.4238	21.3311	.3240	.1810	.3080
16.0343	23.6213	.3220	.2030	.2970
17.8738	26.0029	.3620	.1570	.3180
21.0374	28.8256	.3820	.1540	.3370
25.6089	37.7937	.3820	.1560	.3520
Range of standard errors per column				
Low		.0125	.0112	.0124
High		.0154	.0133	.0151

TABLE 6

EFFECT OF CHANGING MATERIALITY

PERCENT WHEN ALPHA = 10% and BETA = 10%

Mean	Skewness	Kurtosis	150 Standardized Sample Size Precision		450 Standardized Sample Size Precision	
			%	Amount	%	Amount
1.19	1.8084	5.4554	.20	.24	.12	.14
.65	5.8274	9.8108	.37	.24	.21	.14
.53	8.5233	12.8486	.45	.24	.26	.14
.43	16.0343	23.6213	.56	.24	.32	.14
.37	25.6089	37.7937	.65	.24	.37	.14

TABLE 7

PROPORTIONS OF TIMES TYPE I AND TYPE II
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 20%, BETA = 20% AND FOR BASIC SAMPLE SIZES = 150 and 450

		Proportions of Errors for					
		Type I		Type II			
				Upper		Lower	
Skewness	Kurtosis	150*	450*	150*	450*	150*	450*
.0000	3.0081	.2040	.1910	.2310	.2130	.2060	.2020
4.9872	7.3892	.2490	.2190	.1860	.1890	.2560	.2730
6.9285	9.9881	.2680	.2670	.1720	.2290	.2810	.2640
13.2461	18.6029	.2890	.2500	.1540	.1970	.2950	.3020
21.0374	28.8256	.3820	.3070	.1540	.2240	.3370	.2820
25.6089	37.7937	.3820	.2980	.1560	.2270	.3520	.3000
Range of standard errors per column							
Low		.0127	.0124	.0114	.0124	.0128	.0127
High		.0154	.0146	.0133	.0133	.0151	.0145

* The data in the table represent the proportion of times an error was committed in the 1,000 loops carried out in the simulation procedures using the standardized sample size indicated at the top of the column.

TABLE 8

PROPORTIONS OF TIMES TYPE I AND TYPE II
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 20%, BETA = 20% AND FOR BASIC SAMPLE SIZES = 150 and 1,000

		Proportions of Errors for					
		Type I		Type II			
				Upper		Lower	
Skewness	Kurtosis	150*	1,000*	150	1,000*	150*	1,000*
.0000	3.0081	.2040	.1770	.2310	.1780	.2060	.2100
8.5233	12.8486	.2640	.2070	.1520	.1930	.2760	.3100
16.0343	23.6213	.3220	.2140	.2030	.2720	.2970	.2860
25.6089	37.7937	.3820	.2770	.1560	.2520	.3520	.3630
Range of standard errors per column							
Low		.0127	.0121	.0114	.0121	.0128	.0129
High		.0154	.0142	.0133	.0141	.0151	.0152

* The data in the table represents the proportion of times an error was committed in the 1,000 loops carried out in the simulation procedures using the standardized sample size indicated at the top of the column.

TABLE 9

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 10%, BETA = 10% AND 20%,[1] BASIC SAMPLE SIZE = 150

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
.0000	3.0081	.1210	.0980	.1200	.1120
1.8084	5.4554	.1350	.1180	.1020	.1100
5.8274	9.8108	.1720	.1470	.1490	.1260
8.5233	12.8486	.1950	.1860	.1690	.1650
13.2461	18.6029	.1970	.2040	.1950	.1850
16.0343	23.6213	.2030	.2420	.2170	.1990
25.6089	37.7937	.2450	.3040	.2660	.2530
Range of standard errors per column					
Low		.0103	.0094	.0096	.0099
High		.0136	.0145	.0140	.0137

[1] Proportions for Type I errors when beta = 10% are identical to proportions for Type I errors when beta = 20%.

TABLE 10

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 20%, BETA = 10% AND 20%,[1] BASIC SAMPLE SIZE = 150

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.2040	.1930	.1830	.1880
1.8084	5.4554	.2320	.2090	.2080	.1860
5.8274	9.8108	.2710	.2580	.2210	.2090
8.5233	12.8486	.3070	.2640	.2510	.2490
13.2461	18.6029	.3010	.2890	.2680	.2700
16.0343	23.6213	.3010	.3220	.3000	.2900
25.6089	37.7937	.3310	.3820	.3490	.3430
Range of standard errors per column					
Low		.0127	.0125	.0122	.0124
High		.0149	.0154	.0151	.0150

[1] Proportions for Type I errors when beta = 10% are identical to proportions for Type I errors when beta = 20%.

TABLE 11

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 20%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.2180	.2040	.2180	.2180
.2511	1.9997	.2120	.1950	.1920	.1990
1.8084	5.4554	.2290	.2000	.2050	.1950
2.8648	6.6657	.2470	.2300	.2260	.2250
2.9869	5.0850	.2400	.2450	.2200	.2010
2.9913	4.9900	.2410	.2290	.2120	.2010
3.9112	6.9646	.2630	.2230	.2280	.2080
4.9679	7.9643	.2860	.2480	.2110	.2210
4.9872	7.3892	.2540	.2490	.2190	.2190
5.8274	9.8108	.2710	.2580	.2210	.2090
6.8529	10.8543	.2790	.2550	.2470	.2350
6.9285	9.9881	.2980	.2680	.2680	.2330
7.3251	11.7240	.2520	.2630	.2340	.2150
8.5233	12.8486	.3070	.2640	.2510	.2490
9.8138	14.6975	.3100	.3000	.2720	.2570
11.4296	16.6631	.3160	.3130	.2910	.2610
13.2461	18.6029	.3010	.2890	.2680	.2700
14.4238	21.3311	.3260	.3240	.2540	.2430
16.0343	23.6213	.3010	.3220	.3000	.2900
17.8738	26.0029	.3410	.3620	.3120	.2850
21.0374	28.8256	.3350	.3820	.3510	.3060
25.6089	37.7937	.3310	.3820	.3490	.3430
Range of standard errors per column					
Low		.0129	.0125	.0125	.0125
High		.0150	.0154	.0151	.0150

TABLE 12

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 15%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
.0000	3.0081	.1800	.1520	.1670	.1660
1.8084	5.4554	.1850	.1600	.1530	.1480
5.8274	9.8108	.2280	.2090	.1870	.1700
8.5233	12.8486	.2790	.2260	.2140	.2170
13.2461	18.6029	.2570	.2560	.2330	.2280
25.6089	37.7937	.2910	.3510	.3020	.3000
Range of standard errors per column					
Low		.0121	.0114	.0114	.0112
High		.0144	.0151	.0145	.0145

TABLE 13

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 8%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
.0000	3.0081	.1050	.0790	.0960	.0930
1.8084	5.4554	.1210	.0920	.0830	.0870
5.8274	9.8108	.1580	.1320	.1270	.1080
8.5233	12.8486	.1760	.1690	.1580	.1430
13.2461	18.6029	.1810	.1920	.1760	.1660
25.6089	37.7937	.2270	.2940	.2540	.2410
Range of standard errors per column					
Low		.0097	.0085	.0087	.0089
High		.0132	.0144	.0138	.0135

TABLE 14

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 5%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
.0000	3.0081	.0530	.0490	.0490	.0590
1.8084	5.4554	.0850	.0630	.0590	.0610
5.8274	9.8108	.1200	.0960	.0910	.0660
8.5233	12.8486	.1310	.1360	.1200	.0980
13.2461	18.6029	.1310	.1710	.1490	.1350
21.0374	28.8256	.1750	.2250	.2270	.1910
25.6089	37.7937	.1840	.2450	.2310	.2050
Range of standard errors per column					
Low		.0071	.0068	.0068	.0075
High		.0123	.0136	.0133	.0128

TABLE 15

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 1%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
.0000	3.0081	.0110	.0140	.0120	.0060
1.8084	5.4554	.0330	.0170	.0160	.0130
5.8274	9.8108	.0510	.0420	.0340	.0270
8.5233	12.8486	.0630	.0880	.0590	.0550
13.2461	18.6029	.0740	.1030	.1030	.0760
25.6089	37.7937	.1340	.1540	.1680	.1630
Range of standard errors per column					
Low		.0033	.0037	.0034	.0024
High		.0108	.0114	.0118	.0117

TABLE 16

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = .1%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
8.5233	12.8486	.0310	.0470	.0390	.0280
13.2461	18.6029	.0450	.0490	.0740	.0510
17.8738	26.0029	.0780	.0810	.0970	.0930
21.0374	28.8256	.0910	.0950	.1060	.1280
25.6089	37.7937	.0920	.0960	.1060	.1260
Range of standard errors per column					
Low		.0055	.0067	.0061	.0052
High		.0091	.0093	.0097	.0106

TABLE 17

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 20%, BETA = 5%, BASIC SAMPLE SIZE = 150

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
.0000	3.0081	.2160	.2050	.2100	.2150
1.8084	5.4554	.2290	.2000	.2050	.1950
5.8274	9.8108	.2710	.2580	.2210	.2090
8.5233	12.8486	.3070	.2640	.2510	.2490
13.2461	18.6029	.3010	.2890	.2680	.2700
25.6089	37.7937	.3310	.3820	.3490	.3430
Range of standard errors per column					
Low		.0130	.0126	.0129	.0125
High		.0149	.0154	.0151	.0150

TABLE 18

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 5%, BETA = 5%, BASIC SAMPLE SIZE = 150

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
.0000	3.0081	.0530	.0490	.0490	.0591
1.8084	5.4554	.0890	.0690	.0570	.0590
5.8274	9.8108	.1200	.0960	.0910	.0660
8.5233	12.8486	.1310	.1360	.1200	.0980
13.2461	18.6029	.1310	.1710	.1490	.1350
16.0343	23.6213	.1470	.1950	.1730	.1530
25.6089	37.7937	.1840	.2450	.2310	.2050
Range of standard errors per column					
Low		.0071	.0068	.0068	.0075
High		.0123	.0136	.0133	.0128

TABLE 19

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 1% AND .1%, BETA = 5%, BASIC SAMPLE SIZE = 150

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
*16.0343	23.6213	.0940	.1190	.1220	.1040
**16.0343	23.6213	.0630	.0680	.0810	.0750
Range of standard errors per column					
Low		.0077	.0080	.0086	.0083
High		.0092	.0102	.0103	.0097

* Alpha = 1%.

** Alpha = .1%.

TABLE 20

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 15%, BETA = 20%, BASIC SAMPLE SIZE = 450

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
8.5233	12.8486	.2140	.1910	.1880	.1890
13.2461	18.6029	.2380	.1930	.1800	.1750
16.0343	23.6213	.2680	.1970	.1750	.1660
21.0374	28.8256	.3060	.2620	.2170	.2020
Range of standard errors per column					
Low		.0130	.0124	.0121	.0118
High		.0146	.0139	.1300	.0270

TABLE 21

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 10%, BETA = 20%, BASIC SAMPLE SIZE = 450

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
13.2461	18.6029	.2140	.1550	.1350	.1170
21.0374	28.8256	.2700	.2100	.1700	.1600
25.6089	37.7937	.2430	.2130	.1670	.1680
Range of standard errors per column					
Low		.0130	.0114	.0108	.0102
High		.0140	.0129	.0119	.0118

TABLE 22

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 8%, BETA = 20%, BASIC SAMPLE SIZE = 450

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
1.8084	5.4554	.0670	.0740	.0660	.0870
5.8274	9.8108	.1130	.0800	.0820	.0940
13.2461	18.6029	.1820	.1320	.1060	.1020
21.0374	28.8256	.2460	.1950	.1530	.1410
25.6089	37.7937	.2290	.1970	.1560	.1490
Range of standard errors per column					
Low		.0079	.0083	.0079	.0089
High		.0133	.0126	.0115	.0113

TABLE 23

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 5%, BETA = 20%, BASIC SAMPLE SIZE = 450

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
1.8084	5.4554	.0460	.0520	.0470	.0480
8.5233	12.8486	.1150	.0830	.0830	.0820
21.0374	28.8256	.2190	.1630	.1320	.1150
Range of standard errors per column					
Low		.0066	.0070	.0067	.0068
High		.0131	.0117	.0107	.0101

TABLE 24

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 1%, BETA = 20%, BASIC SAMPLE SIZE = 450

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
1.8084	5.4554	.0130	.0070	.0080	.0080
8.5233	12.8486	.0610	.0420	.0350	.0330
13.2461	18.6029	.1070	.0590	.0410	.0360
17.8738	26.0029	.1500	.0860	.0630	.0430
21.0374	28.8256	.1670	.1180	.0860	.0680
25.6089	37.7937	.1410	.1190	.0940	.0760
Range of standard errors per column					
Low		.0036	.0026	.0028	.0028
High		.0118	.0102	.0092	.0084

TABLE 25

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = .1%, BETA = 20%, BASIC SAMPLE SIZE = 450

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
13.2461	18.6029	.0790	.0440	.0230	.0180
16.0343	23.6213	.0650	.0390	.0260	.0170
21.0374	28.8256	.1120	.0970	.0710	.0560
25.6089	37.7937	.0950	.0970	.0670	.0500
Range of standard errors per column					
Low		.0078	.0061	.0047	.0041
High		.0100	.0094	.0081	.0073

TABLE 26

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 20%, BETA = 10%, BASIC SAMPLE SIZE = 450

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
1.8084	5.4554	.1990	.1860	.1970	.1950
5.8274	9.8108	.2410	.2070	.1980	.2110
8.5233	12.8486	.2550	.2300	.2390	.2420
16.0343	23.6213	.2990	.2470	.2390	.2310
25.6089	37.7937	.3440	.2980	.2580	.2560
Range of standard errors per column					
Low		.0126	.0123	.0126	.0125
High		.0150	.0145	.0138	.0138

TABLE 27

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 10%, BETA = 10%, BASIC SAMPLE SIZE = 450

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
1.8084	5.4554	.0870	.0890	.0890	.1030
5.8274	9.8108	.1330	.0990	.1050	.1120
8.5233	12.8486	.1660	.1370	.1440	.1300
16.0343	23.6213	.2180	.1600	.1220	.1250
25.6089	37.7937	.2510	.2110	.1680	.1660
Range of standard errors per column					
Low		.0089	.0090	.0090	.0096
High		.0137	.0129	.0118	.0118

TABLE 28

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 20%, BETA = 5%, BASIC SAMPLE SIZE = 450

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
1.8084	5.4554	.1990	.1860	.1970	.1950
5.8274	9.8108	.2410	.2070	.1980	.2120
8.5233	12.8486	.2550	.2300	.2390	.2420
13.2461	18.6029	.3000	.2500	.2300	.2150
Range of standard errors per column					
Low		.0126	.0123	.0126	.0125
High		.0145	.0137	.0133	.0135

TABLE 29

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 5%, BETA = 5%, BASIC SAMPLE SIZE = 450

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
.0000	3.0081	.0430	.0460	.0380	.0390
1.8084	5.4554	.0560	.0450	.0440	.0530
5.8274	9.8108	.0820	.0540	.0580	.0580
8.5233	12.8486	.1150	.0830	.0830	.0820
13.2461	18.6029	.1580	.0990	.0750	.0680
16.0343	23.6213	.1590	.1060	.0770	.0760
25.6089	37.7937	.2060	.1680	.1270	.1250
Range of standard errors per column					
Low		.0064	.0066	.0060	.0061
High		.0128	.0118	.0105	.0105

TABLE 30

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 20%, BETA = 20%, BASIC SAMPLE SIZE = 1,000

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.1790	.1770	.1660	.1580
8.5233	12.8486	.2330	.2070	.2050	.2090
16.0343	23.6213	.2290	.2140	.2050	.2010
25.6089	37.7937	.3290	.2770	.2510	.2380
Range of standard errors per column					
Low		.0121	.0121	.0118	.0115
High		.0149	.0142	.0137	.0135

TABLE 31

PROPORTIONS OF TIMES TYPE I
ERRORS OCCURRED IN SIMULATION PROCEDURES WHEN
ALPHA = 15% AND 10%, BETA = 20%, BASIC SAMPLE SIZE = 1,000

		Proportions of Type I Errors			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
*21.0374	28.8256	.2150	.1900	.1810	.1700
*25.6089	37.7937	.2830	.2250	.1930	.1880
**21.0374	28.8256	.1670	.1410	.1220	.1240
**25.6089	37.7937	.2030	.1530	.1420	.1410
Range of standard errors per column					
Low		.0118	.0110	.0103	.0104
High		.0142	.0132	.0122	.0124

* Alpha = 15%.

** Alpha = 10%.

TABLE 32

PROPORTIONS OF TIMES TYPE I ERRORS OCCURRED
IN SIMULATION PROCEDURES FOR SELECTED LEVELS OF ALPHA AND BETA,
AND FOR SAMPLE SIZE = TO 4 TIMES BASIC SAMPLE SIZE OF 150

Levels of				Type I Errors	
Alpha	Beta	Skewness	Kurtosis	%*	SE**
.05	.20	21.0374	28.8256	.1360	.0108
.05	.20	25.6089	37.7937	.1370	.0109
.01	.20	21.0374	28.8256	.0980	.0094
.01	.20	25.6089	37.7937	.0840	.0088
.001	.20	25.6089	37.7937	.0570	.0073
.20	.10	13.2461	18.6029	.2620	.0139
.20	.10	25.6089	37.7937	.2740	.0141
.10	.10	13.2461	18.6029	.1500	.0113
.10	.10	16.0343	23.6213	.1510	.0113
.10	.10	25.6089	37.7937	.1770	.0121
.20	.05	8.5233	12.8486	.2250	.0132
.20	.05	13.2461	18.6029	.2620	.0139
.20	.05	25.6089	37.7937	.2740	.0141
.05	.05	8.5233	12.8486	.0850	.0088
.05	.05	13.2461	18.6029	.0940	.0092
.05	.05	25.6089	37.7937	.1370	.0109

* Indicates proportions of times Type I errors occurred in the 1,000 loops carried out in the simulation procedures.

** Indicates standard errors for the proportions.

TABLE 33

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 10%, BETA = 10%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.3770	.1360	.0530	.0180
1.8084	5.4554	.3030	.1120	.0340	.0190
5.8274	9.8108	.2630	.1050	.0490	.0270
8.5233	12.8486	.2330	.0930	.0430	.0270
13.2461	18.6029	.2180	.0950	.0490	.0400
16.0343	23.6213	.2100	.1260	.0710	.0480
25.6089	37.7937	.1930	.1010	.0790	.0670
Range of standard errors per column					
Low		.0125	.0095	.0057	.0042
High		.0153	.0108	.0085	.0079

TABLE 34

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 10%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.4960	.2260	.1130	.0540
1.8084	5.4554	.4180	.2010	.0960	.0480
5.8274	9.8108	.3520	.1860	.0930	.0600
8.5233	12.8486	.3280	.1590	.0830	.0520
13.2461	18.6029	.2920	.1650	.0920	.0640
16.0343	23.6213	.2840	.2190	.1460	.1040
25.6089	37.7937	.2680	.1710	.1300	.1050
Range of standard errors per column					
Low		.0140	.0119	.0091	.0068
High		.0580	.0132	.0106	.0097

TABLE 35

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 20%, BETA = 10%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.3340	.1300	.0580	.0220
1.8084	5.4554	.2640	.1030	.0350	.0160
5.8274	9.8108	.2330	.0990	.0440	.0250
8.5233	12.8486	.2050	.0830	.0410	.0240
13.2461	18.6029	.1920	.0860	.0420	.0350
16.0343	23.6213	.1770	.1240	.0790	.0540
25.6089	37.7937	.1670	.0880	.0760	.0610
Range of standard errors per column					
Low		.0118	.0087	.0058	.0040
High		.0149	.0106	.0085	.0076

TABLE 36

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 20%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.4320	.2310	.1320	.0630
.2511	1.9997	.4130	.2100	.0970	.0490
1.8084	5.4554	.3760	.1970	.0970	.0590
2.8648	6.6657	.3640	.1860	.1050	.0670
2.9869	5.0850	.3770	.2180	.1000	.0430
2.9913	4.9900	.3510	.1970	.0900	.0430
3.9112	6.9646	.3500	.1730	.0920	.0510
4.9679	7.9643	.3180	.1590	.0960	.0510
4.9872	7.3892	.3250	.1860	.1030	.0560
5.8274	9.8108	.3180	.1780	.1010	.0610
6.8529	10.8543	.3220	.1760	.1010	.0560
6.9285	9.9881	.3220	.1720	.1170	.0580
7.3251	11.7240	.3000	.1600	.0940	.0590
8.5233	12.8486	.2950	.1520	.0880	.0540
9.8138	14.6975	.2700	.1460	.0860	.0590
11.4296	16.6631	.2760	.1570	.1190	.0700
13.2461	18.6029	.2670	.1540	.0890	.0660
14.4238	21.3311	.2700	.1810	.1210	.0810
16.0343	23.6213	.2590	.2030	.1470	.0980
17.8738	26.0029	.2230	.1570	.1200	.0830
21.0374	28.8256	.2350	.1540	.1190	.0930
25.6089	37.7937	.2460	.1560	.1230	.1030
Range of standard errors per column					
Low		.0132	.0112	.0090	.0068
High		.0157	.0133	.0104	.0096

TABLE 37

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 15%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.4650	.2260	.1170	.0590
1.8084	5.4554	.3860	.1990	.0980	.0570
5.8274	9.8108	.3300	.1800	.0940	.0590
8.5233	12.8486	.3060	.1530	.0860	.0510
13.2461	18.6029	.2740	.1570	.0910	.0630
25.6089	37.7937	.2520	.1560	.1250	.1030
Range of standard errors per column					
Low		.0137	.0114	.0089	.0070
High		.0158	.0132	.0105	.0096

TABLE 38

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 8%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.4980	.2250	.1130	.0470
1.8084	5.4554	.4280	.2090	.0910	.0450
5.8274	9.8108	.3550	.1830	.0920	.0590
8.5233	12.8486	.3280	.1570	.0820	.0510
13.2461	18.6029	.2890	.1640	.0930	.0660
25.6089	37.7937	.2650	.1690	.1310	.1030
Range of standard errors per column					
Low		.0140	.0115	.0087	.0060
High		.0158	.0132	.0107	.0096

TABLE 39

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 5%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.5350	.2300	.1090	.0410
1.8084	5.4554	.4680	.2140	.0890	.0420
5.8274	9.8108	.3910	.1940	.0990	.0570
8.5233	12.8486	.3550	.1670	.0840	.0520
13.2461	18.6029	.3110	.1700	.0960	.0680
21.0374	28.8256	.2580	.1700	.1230	.1020
25.6089	37.7937	.2860	.1810	.1340	.1070
Range of standard errors per column					
Low		.0138	.0118	.0088	.0063
High		.0158	.0133	.0108	.0098

TABLE 40

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 1%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.6160	.2390	.0930	.0320
1.8084	5.4554	.5470	.2180	.0860	.0360
5.8274	9.8108	.4480	.2110	.0980	.0600
8.5233	12.8486	.3950	.1780	.0970	.0520
13.2461	18.6029	.3500	.1880	.1040	.0750
25.6089	37.7937	.3140	.2010	.1500	.1220
Range of standard errors per column					
Low		.0147	.0121	.0089	.0056
High		.0157	.0135	.0113	.0103

TABLE 41

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = .1%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
8.5233	12.8486	.4620	.2050	.1030	.0610
13.2461	18.6029	.4060	.2090	.1150	.0820
17.8738	26.0029	.3620	.2190	.1590	.1150
21.0374	28.8256	.3260	.2050	.1470	.1160
25.6089	37.7937	.3620	.2210	.1600	.1350
Range of standard errors per column					
Low		.0148	.0128	.0096	.0076
High		.0158	.0131	.0116	.0108

TABLE 42

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 20%, BETA = 5%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.2430	.0710	.0270	.0090
1.8084	5.4554	.1830	.0570	.0160	.0050
5.8274	9.8108	.1700	.0630	.0260	.0130
8.5233	12.8486	.1580	.0520	.0290	.0170
13.2461	18.6029	.1240	.0590	.0230	.0180
25.6089	37.7937	.1200	.0650	.0540	.0390
Range of standard errors per column					
Low		.0103	.0070	.0040	.0022
High		.0136	.0081	.0071	.0061

TABLE 43

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 5%, BETA = 5%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.3290	.0780	.0250	.0070
1.8084	5.4554	.2570	.0620	.0130	.0060
5.8274	9.8108	.2230	.0750	.0270	.0160
8.5233	12.8486	.2030	.0650	.0330	.0220
13.2461	18.6029	.1780	.0730	.0350	.0240
16.0343	23.6213	.1690	.1090	.0630	.0400
25.6089	37.7937	.1550	.0820	.0620	.0500
Range of standard errors per column					
Low		.0114	.0076	.0036	.0026
High		.0149	.0099	.0077	.0069

TABLE 44

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 1% AND .1%, BETA = 5%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
*16.0343	23.6213	.2090	.1350	.0810	.0510
**16.0343	23.6213	.2650	.1650	.0970	.0620
Range of standard errors per column					
Low		.0129	.0108	.0086	.0070
High		.0140	.0117	.0094	.0076

* Alpha = 1%.

** Alpha = .1%.

TABLE 45

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 15%, BETA = 20%, BASIC SAMPLE SIZE = 450

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
8.5233	12.8486	.3640	.1760	.1190	.0830
13.2461	18.6029	.3580	.2020	.1400	.1070
16.0343	23.6213	.3620	.2570	.1670	.1380
21.0374	28.8256	.3360	.2350	.1800	.1430
Range of standard errors per column					
Low		.0149	.0120	.0102	.0087
High		.0152	.0138	.0121	.0111

TABLE 46

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 10%, BETA = 20%, BASIC SAMPLE SIZE = 450

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
13.2461	18.6029	.3880	.2270	.1440	.1090
21.0374	28.8256	.3520	.2430	.1870	.1440
25.6089	37.7937	.3700	.2460	.1740	.1380
Range of standard errors per column					
Low		.0151	.0132	.0111	.0099
High		.0154	.0136	.0123	.0111

TABLE 47

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 8%, BETA = 20%, BASIC SAMPLE SIZE = 450

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
1.8084	5.4554	.4750	.2370	.1220	.0590
5.8274	9.8108	.4370	.2510	.1450	.1010
13.2461	18.6029	.3950	.2210	.1510	.1050
21.0374	28.8256	.3530	.2470	.1890	.1480
25.6089	37.7937	.3730	.2480	.1780	.1380
Range of standard errors per column					
Low		.0153	.0131	.0103	.0075
High		.0158	.0137	.0124	.0112

TABLE 48

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 5%, BETA = 20%, BASIC SAMPLE SIZE = 450

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
1.8084	5.4554	.4850	.2380	.1110	.0540
8.5233	12.8486	.4110	.1930	.1230	.0880
21.0374	28.8256	.3700	.2590	.1970	.1510
Range of standard errors per column					
Low		.0153	.0125	.0099	.0071
High		.0158	.0139	.0126	.0113

TABLE 49

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 1%, BETA = 20%, BASIC SAMPLE SIZE = 450

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
1.8084	5.4554	.5430	.2330	.0990	.0560
8.5233	12.8486	.4750	.2210	.1330	.0930
13.2461	18.6029	.4630	.2530	.1700	.1240
17.8738	26.0029	.4090	.2920	.2160	.1540
21.0374	28.8256	.4130	.2840	.2160	.1590
25.6089	37.7937	.4300	.2860	.2120	.1600
Range of standard errors per column					
Low		.0155	.0131	.0094	.0073
High		.0158	.0144	.0129	.0116

TABLE 50

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = .1%, BETA = 20%, BASIC SAMPLE SIZE = 450

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
13.2461	18.6029	.5150	.2970	.1890	.1350
16.0343	23.6213	.5120	.3450	.2360	.1680
21.0374	28.8256	.4580	.3130	.2360	.1830
25.6089	37.7937	.4680	.3290	.2260	.1710
Range of standard errors per column					
Low		.0158	.0144	.0124	.0108
High		.0158	.0150	.0134	.0122

TABLE 51

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 20%, BETA = 10%, BASIC SAMPLE SIZE = 450

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
1.8084	5.4554	.3260	.1280	.0670	.0270
5.8274	9.8108	.2870	.1350	.0680	.0350
8.5233	12.8486	.2450	.1040	.0570	.0490
16.0343	23.6213	.2820	.1520	.1160	.0730
25.6089	37.7937	.2660	.1530	.1040	.0800
Range of standard errors per column					
Low		.0136	.0097	.0079	.0051
High		.0148	.0114	.0101	.0086

TABLE 52

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 10%, BETA = 10%, BASIC SAMPLE SIZE = 450

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
1.8084	5.4554	.3720	.1370	.0650	.0250
5.8274	9.8108	.3250	.1460	.0730	.0370
8.5233	12.8486	.2780	.1220	.0650	.0510
16.0343	23.6213	.3030	.1750	.1190	.0870
25.6089	37.7937	.2830	.1740	.1130	.0900
Range of standard errors per column					
Low		.0142	.0103	.0078	.0049
High		.0153	.0120	.0102	.0090

TABLE 53

PROPORTIONS OF TIMES TYPE II ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES WHEN ALPHA = 20%, BETA = 5%, BASIC SAMPLE SIZE = 450

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
1.8084	5.4554	.2440	.0690	.0270	.0100
5.8274	9.8108	.2090	.0840	.0360	.0170
8.5233	12.8486	.1770	.0700	.0340	.0250
13.2461	18.6029	.1990	.0950	.0570	.0380
Range of standard errors per column					
Low		.0121	.0080	.0051	.0031
High		.0136	.0093	.0073	.0060

TABLE 54

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 5%, BETA = 5%, BASIC SAMPLE SIZE = 450

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.2810	.0720	.0120	.0010
1.8084	5.4554	.3110	.0690	.0270	.0120
5.8274	9.8108	.2810	.1020	.0430	.0190
8.5233	12.8486	.2430	.0940	.0450	.0320
13.2461	18.6029	.2670	.1150	.0740	.0470
16.0343	23.6213	.2800	.1390	.0970	.0610
25.6089	37.7937	.2480	.1450	.0940	.0720
Range of standard errors per column					
Low		.0136	.0080	.0034	.0010
High		.0146	.0111	.0094	.0082

TABLE 55

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 20%, BETA = 20%, BASIC SAMPLE SIZE = 1,000

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.3870	.1780	.0870	.0400
8.5233	12.8486	.3340	.1930	.1110	.0730
16.0343	23.6213	.4070	.2720	.1720	.1330
25.6089	37.7937	.3390	.2520	.2160	.1840
Range of standard errors per column					
Low		.0149	.0121	.0089	.0062
High		.0155	.0141	.0130	.0123

TABLE 56

PROPORTIONS OF TIMES TYPE II
ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 15% AND 10%, BETA = 20%, BASIC SAMPLE SIZE = 1,000

		Proportions of Type II Errors-Upper Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
*21.0374	28.8256	.3960	.2530	.1750	.1440
*25.6089	37.7937	.3580	.2650	.2240	.1890
**21.0374	28.8256	.4180	.2720	.1810	.1490
**25.6089	37.7937	.3850	.2800	.2240	.1950
Range of standard errors per column					
Low		.0152	.0137	.0120	.0111
High		.0155	.0142	.0132	.0125

* Alpha = 15%.

** Alpha = 10%.

TABLE 57

PROPORTIONS OF TIMES TYPE II ERRORS-UPPER SIDE OCCURRED IN SIMULATION PROCEDURES FOR SELECTED LEVELS OF ALPHA AND BETA, AND FOR SAMPLE SIZE = TO 4 TIMES BASIC SAMPLE SIZE OF 150

Levels				Type II Errors-Upper Side	
Alpha	Beta	Skewness	Kurtosis	%*	SE**
.05	.20	21.0374	28.8256	.0400	.0062
.05	.20	25.6089	37.7937	.0570	.0073
.01	.20	21.0374	28.8256	.0450	.0066
.01	.20	25.6089	37.7937	.0610	.0076
.001	.20	25.6089	37.7937	.0680	.0080
.20	.10	13.2461	18.6029	.0120	.0034
.20	.10	25.6089	37.7937	.0300	.0054
.10	.10	13.2461	18.6029	.0130	.0036
.10	.10	16.0343	23.6213	.0210	.0045
.10	.10	25.6089	37.7937	.0330	.0056
.20	.05	8.5233	12.8486	.0030	.0017
.20	.05	13.2461	18.6029	.0060	.0024
.20	.05	25.6089	37.7937	.0210	.0045
.05	.05	8.5233	12.8486	.0030	.0017
.05	.05	13.2461	18.6029	.0110	.0033
.05	.05	25.6089	37.7937	.0240	.0048

* Indicates proportions of times Type II errors-upper side occurred in the 1,000 loops carried out in the simulation procedures.

** Indicates standard errors for the proportions.

TABLE 58

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 10%, BETA = 10%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.3330	.0990	.0320	.0170
1.8084	5.4554	.3640	.1500	.0780	.0370
5.8274	9.8108	.4120	.1880	.0990	.0630
8.5233	12.8486	.4310	.2330	.1400	.0930
13.2461	18.6029	.4160	.2450	.1720	.1320
16.0343	23.6213	.4290	.2590	.1920	.1600
25.6089	37.7937	.4750	.3280	.2310	.1950
Range of standard errors per column					
Low		.0149	.0094	.0056	.0041
High		.0158	.0148	.0133	.0125

TABLE 59

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 10%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.4300	.2020	.0800	.0530
1.8084	5.4554	.4830	.2340	.1220	.0660
5.8274	9.8108	.5120	.2790	.1590	.1020
8.5233	12.8486	.5130	.3130	.1970	.1590
13.2461	18.6029	.5240	.3190	.2210	.1670
16.0343	23.6213	.5240	.3280	.2390	.1890
25.6089	37.7937	.5560	.3990	.3080	.2330
Range of standard errors per column					
Low		.0157	.0127	.0086	.0071
High		.0158	.0155	.0146	.0134

TABLE 60

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 20%, BETA = 10%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.2770	.0960	.0350	.0190
1.8084	5.4554	.3250	.1330	.0730	.0380
5.8274	9.8108	.3520	.1700	.0910	.0600
8.5233	12.8486	.3970	.2160	.1330	.0960
13.2461	18.6029	.3790	.2210	.1510	.1110
16.0343	23.6213	.3600	.2260	.1670	.1340
25.6089	37.7937	.4180	.2930	.2130	.1790
Range of standard errors per column					
Low		.0142	.0093	.0058	.0043
High		.0156	.0144	.0129	.0121

TABLE 61

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 20%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.4190	.2060	.0950	.0570
.2511	1.9997	.4070	.1900	.0960	.0480
1.8084	5.4554	.4180	.2220	.1350	.0790
2.8648	6.6657	.4390	.2460	.1520	.0840
2.9869	5.0850	.4270	.2390	.1350	.0830
2.9913	4.9900	.4300	.2210	.1330	.0770
3.9112	6.9646	.4350	.2390	.1600	.1130
4.9679	7.9643	.4690	.2600	.1670	.1210
4.9872	7.3892	.4430	.2560	.1610	.1130
5.8274	9.8108	.4560	.2560	.1500	.1050
6.8529	10.8543	.4530	.2770	.2020	.1400
6.9285	9.9881	.4740	.2810	.1920	.1410
7.3251	11.7240	.4650	.2750	.1830	.1460
8.5233	12.8486	.4590	.2760	.1860	.1580
9.8138	14.6975	.4880	.2930	.1970	.1570
11.4296	16.6631	.4860	.3020	.2150	.1470
13.2461	18.6029	.4650	.2950	.1930	.1560
14.4238	21.3311	.4840	.3080	.1980	.1430
16.0343	23.6213	.4710	.2970	.2150	.1740
17.8738	26.0029	.4680	.3180	.2350	.1780
21.0374	28.8256	.4800	.3370	.2490	.1900
25.6089	37.7937	.4890	.3520	.2670	.1860
Range of standard errors per column					
Low		.0155	.0124	.0093	.0068
High		.0158	.0151	.0140	.0163

TABLE 62

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 15%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.4090	.1970	.0860	.0570
1.8084	5.4554	.4390	.2210	.1340	.0750
5.8274	9.8108	.4770	.2620	.1560	.1050
8.5233	12.8486	.4750	.2960	.1910	.1560
13.2461	18.6029	.4780	.2990	.2080	.1560
25.6089	37.7937	.5120	.3710	.2780	.2040
Range of standard errors per column					
Low		.0155	.0126	.0089	.0073
High		.0158	.0153	.0142	.0127

TABLE 63

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 8%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.4350	.2010	.0780	.0510
1.8084	5.4554	.4860	.2300	.1280	.0710
5.8274	9.8108	.5140	.2760	.1570	.0980
8.5233	12.8486	.5160	.3140	.1910	.1570
13.2461	18.6029	.5240	.3200	.2230	.1650
25.6089	37.7937	.5650	.4000	.3090	.2380
Range of standard errors per column					
Low		.0157	.0127	.0085	.0070
High		.0158	.0155	.0146	.0135

TABLE 64

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 5%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.4790	.2030	.0750	.0460
1.8084	5.4554	.5260	.2400	.1250	.0660
5.8274	9.8108	.5640	.2940	.1640	.1000
8.5233	12.8486	.5440	.3320	.2030	.1570
13.2461	18.6029	.5650	.3410	.2340	.1770
21.0374	28.8256	.5740	.3950	.2950	.2340
25.6089	37.7937	.5970	.4200	.3210	.2480
Range of standard errors per column					
Low		.0155	.0127	.0083	.0066
High		.0158	.0156	.0148	.0137

TABLE 65

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 1%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.5610	.2110	.0680	.0290
1.8084	5.4554	.6100	.2540	.1190	.0550
5.8274	9.8108	.6370	.3100	.1670	.1020
8.5233	12.8486	.6150	.3530	.2230	.1540
13.2461	18.6029	.6260	.3680	.2440	.1830
25.6089	37.7937	.6740	.4460	.3350	.2570
Range of standard errors per column					
Low		.0148	.0129	.0080	.0053
High		.0157	.0157	.0149	.0138

TABLE 66

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = .1%, BETA = 20%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
8.5233	12.8486	.6900	.3850	.2340	.1540
13.2461	18.6029	.6930	.4060	.2590	.1870
17.8738	26.0029	.7220	.4340	.3230	.2530
21.0374	28.8256	.7060	.4530	.3510	.2790
25.6089	37.7937	.7550	.4850	.3480	.2710
Range of standard errors per column					
Low		.0136	.0154	.0134	.0114
High		.0146	.0158	.0151	.0141

TABLE 67

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 20%, BETA = 5%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.2020	.0470	.0140	.0090
1.8084	5.4554	.2470	.0920	.0440	.0210
5.8274	9.8108	.2710	.1120	.0590	.0390
8.5233	12.8486	.3110	.1550	.0990	.0660
13.2461	18.6029	.2950	.1740	.1220	.0880
25.6089	37.7937	.3320	.2380	.1870	.1480
Range of standard errors per column					
Low		.0127	.0067	.0037	.0030
High		.0149	.0135	.0123	.0112

TABLE 68

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 5%, BETA = 5%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.2740	.0560	.0030	.0070
1.8084	5.4554	.3300	.1050	.0430	.0180
5.8274	9.8108	.3580	.1360	.0660	.0450
8.5233	12.8486	.3920	.1940	.1200	.0820
13.2461	18.6029	.3750	.2000	.1340	.1010
16.0343	23.6213	.3820	.2240	.1510	.1210
25.6089	37.7937	.4040	.2460	.1870	.1430
Range of standard errors per column					
Low		.0141	.0073	.0036	.0026
High		.0155	.0136	.0123	.0111

TABLE 69

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 1% AND .1%, BETA = 5%, BASIC SAMPLE SIZE = 150

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
*16.0343	23.6213	.4600	.2460	.1560	.1250
**16.0343	23.6213	.5630	.2820	.1780	.1230
Range of standard errors per column					
Low		.0157	.0136	.0115	.0104
High		.0158	.0142	.0121	.0105

* Alpha = 1%.

** Alpha = .1%.

TABLE 70

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 15%, BETA = 20%, BASIC SAMPLE SIZE = 450

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
8.5233	12.8486	.4760	.3060	.2310	.1780
13.2461	18.6029	.4370	.3050	.2280	.1810
16.0343	23.6213	.4360	.2890	.2420	.1820
21.0374	28.8256	.4270	.2960	.2250	.1920
Range of standard errors per column					
Low		.0156	.0143	.0132	.0121
High		.0158	.0146	.0135	.0125

TABLE 71

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 10%, BETA = 20%, BASIC SAMPLE SIZE = 450

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
13.2461	18.6029	.4600	.3220	.2480	.1960
21.0374	28.8256	.4750	.3190	.2390	.2000
25.6089	37.7937	.4970	.3290	.2550	.2160
Range of standard errors per column					
Low		.0158	.0147	.0135	.0126
High		.0158	.0149	.0138	.0130

TABLE 72

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 8%, BETA = 20%, BASIC SAMPLE SIZE = 450

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
1.8084	5.4554	.4680	.2260	.1110	.0520
5.8274	9.8108	.4950	.2630	.1490	.0950
13.2461	18.6029	.4800	.3310	.2410	.1850
21.0374	28.8256	.4940	.3230	.2410	.1990
25.6089	37.7937	.5110	.3360	.2570	.2210
Range of standard errors per column					
Low		.0158	.0132	.0099	.0070
High		.0158	.0149	.0138	.0131

TABLE 73

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 5%, BETA = 20%, BASIC SAMPLE SIZE = 450

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
1.8084	5.4554	.5050	.2280	.1020	.0460
8.5233	12.8486	.5510	.3330	.2380	.1760
21.0374	28.8256	.5310	.3430	.2530	.2060
Range of standard errors per column					
Low		.0157	.0133	.0096	.0066
High		.0158	.0150	.0137	.0128

TABLE 74

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 1%, BETA = 20%, BASIC SAMPLE SIZE = 450

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
1.8084	5.4554	.5660	.2550	.1070	.0340
8.5233	12.8486	.6130	.3630	.2560	.1730
13.2461	18.6029	.6260	.3750	.2680	.1960
17.8738	26.0029	.6430	.4400	.3400	.2730
21.0374	28.8256	.6090	.3960	.2890	.2190
25.6089	37.7937	.6140	.4150	.3090	.2450
Range of standard errors per column					
Low		.0153	.0138	.0098	.0057
High		.0157	.0157	.0146	.0141

TABLE 75

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = .1%, BETA = 20%, BASIC SAMPLE SIZE = 450

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
13.2461	18.6029	.6700	.4120	.2800	.2080
16.0343	23.6213	.6660	.4150	.2970	.2220
21.0374	28.8256	.6770	.4610	.3400	.2650
25.6089	37.7937	.6760	.4650	.3570	.2790
Range of standard errors per column					
Low		.0148	.0156	.0142	.0128
High		.0149	.0158	.0152	.0142

TABLE 76

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 20%, BETA = 10%, BASIC SAMPLE SIZE = 450

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
1.8084	5.4554	.3020	.1260	.0490	.0270
5.8274	9.8108	.3360	.1590	.0930	.0630
8.5233	12.8486	.3740	.2160	.1500	.1140
16.0343	23.6213	.3280	.2240	.1570	.1120
25.6089	37.7937	.3540	.2290	.1740	.1380
Range of standard errors per column					
Low		.0145	.0105	.0068	.0051
High		.0151	.0133	.0120	.0109

TABLE 77

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 10%, BETA = 10%, BASIC SAMPLE SIZE = 450

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
1.8084	5.4554	.3530	.1340	.0490	.0260
5.8274	9.8108	.3830	.1700	.0950	.0640
8.5233	12.8486	.4170	.2320	.1600	.1170
16.0343	23.6213	.4010	.2440	.1800	.1230
25.6089	37.7937	.4250	.2680	.2060	.1630
Range of standard errors per column					
Low		.0151	.0108	.0068	.0050
High		.0156	.0140	.0128	.0117

TABLE 78

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 20%, BETA = 5%, BASIC SAMPLE SIZE = 450

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
1.8084	5.4554	.2440	.0690	.0210	.0120
5.8274	9.8108	.2590	.1110	.0630	.0380
8.5233	12.8486	.3070	.1640	.1080	.0830
13.2461	18.6029	.2880	.1590	.1220	.0910
Range of standard errors per column					
Low		.0136	.0080	.0045	.0034
High		.0146	.0116	.0103	.0091

TABLE 79

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 5%, BETA = 5%, BASIC SAMPLE SIZE = 450

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.2880	.0550	.0150	.0030
1.8084	5.4554	.3070	.0920	.0240	.0100
5.8274	9.8108	.3500	.1370	.0710	.0460
8.5233	12.8486	.3900	.2030	.1270	.0890
13.2461	18.6029	.3910	.2100	.1490	.1150
16.0343	23.6213	.3920	.2290	.1540	.0940
25.6089	37.7937	.4280	.2660	.1790	.1320
Range of standard errors per column					
Low		.0143	.0072	.0038	.0017
High		.0156	.0140	.0121	.0107

TABLE 80

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 20%, BETA = 20%, BASIC SAMPLE SIZE = 1,000

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
0.0000	3.0081	.4330	.2100	.1060	.0530
8.5233	12.8486	.4800	.3100	.2230	.1520
16.0343	23.6213	.4360	.2860	.2110	.1720
25.6089	37.7937	.4570	.3630	.2970	.2400
Range of standard errors per column					
Low		.0157	.0129	.0097	.0071
High		.0158	.0152	.0144	.0135

TABLE 81

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES
WHEN ALPHA = 15% AND 10%, BETA = 20%, BASIC SAMPLE SIZE = 1,000

		Proportions of Type II Errors-Lower Side			
		Multiplicative Factors of Basic Sample Size			
Skewness	Kurtosis	.5	1.0	1.5	2.0
*21.0374	28.8256	.4220	.3030	.2430	.2030
*25.6089	37.7937	.4880	.3870	.3020	.2560
**21.0374	28.8256	.4500	.3190	.2500	.2070
**25.6089	37.7937	.5200	.3930	.3020	.2460
Range of standard errors per column					
Low		.0156	.0145	.0136	.0127
High		.0158	.0154	.0145	.0136

* Alpha = 15%.

** Alpha = 10%.

TABLE 82

PROPORTIONS OF TIMES TYPE II
ERRORS-LOWER SIDE OCCURRED IN SIMULATION PROCEDURES FOR SELECTED LEVELS
OF ALPHA AND BETA, AND FOR SAMPLE SIZE = TO 4 TIMES BASIC SAMPLE SIZE OF 150

Levels of						Type II Errors-Lower Side	
Alpha	Beta	Mean	Variance	Skewness	Kurtosis	%*	SE**
.05	.20	.35	1.0016	21.0374	28.8256	.1340	.0108
.05	.20	.37	1.0042	25.6089	37.7937	.1350	.0108
.01	.20	.35	1.0016	21.0374	28.8256	.1430	.0111
.01	.20	.37	1.0042	25.6089	37.7937	.1500	.0113
.001	.20	.37	1.0042	25.6089	37.7937	.1460	.0112
.20	.10	.47	1.0147	13.2461	18.6029	.0590	.0075
.20	.10	.37	1.0042	25.6089	37.7937	.0900	.0090
.10	.10	.47	1.0148	13.2461	18.6029	.0610	.0076
.10	.10	.43	1.0030	16.0343	23.6213	.0510	.0070
.10	.10	.37	1.0042	25.6089	37.7937	.0990	.0094
.20	.05	.53	1.0018	8.5233	12.8486	.0240	.0048
.20	.05	.47	1.0147	13.2461	18.6029	.0340	.0057
.20	.05	.37	1.0042	25.6089	37.7937	.0600	.0075
.05	.05	.53	1.0018	8.5233	12.8486	.0320	.0056
.05	.05	.47	1.0147	13.2461	18.6029	.0510	.0070
.05	.05	.37	1.0042	25.6089	37.7937	.0630	.0077

* Indicates proportions of times Type II errors-lower side occurred in 1,000 loops carried out in the simulation procedures.

** Indicates standard errors for the proportions.

Table 83

COORDINATING TABLE FOR CROSS-REFERENCE

OF TABLE NUMBERS FOR PROPORTIONAL DATA OF SAMPLING ERRORS

Standardized Sample Size	Level of Alpha	Level of Beta	Table Numbers For Type I	Type II Upper	Type II Lower
150	.10	.10	9	33	58
	.10	.20	9	34	59
	.20	.10	10	35	60
	.20	.20	11	36	61
	.15	.20	12	37	62
	.08	.20	13	38	63
	.05	.20	14	39	64
	.01	.20	15	40	65
	.001	.20	16	41	66
	.20	.05	17	42	67
	.05	.05	18	43	68
	.01	.05	19	44	69
450	.15	.20	20	45	70
	.10	.20	21	46	71
	.08	.20	22	47	72
	.05	.20	23	48	73
	.01	.20	24	49	74
	.001	.20	25	50	75
	.20	.10	26	51	76
	.10	.10	27	52	77
	.20	.05	28	53	78
	.05	.05	29	54	79
1000	.20	.20	30	55	80
	.10	.20	31	56	81
	.15	.20	31	56	81
150	{ 4 times 150 for selected levels of α and β }		32	57	82

Chapter VI

DEVELOPMENT AND TESTING

GUIDELINES FOR ALPHA AND BETA ATTAINMENT

This chapter presents guidelines for the attainment of specified alpha and beta levels. Development and interpretation of the alpha guidelines are discussed first and are followed by a presentation of the beta guidelines. Then, cases are utilized to test and illustrate the use of such guidelines. The last narrative section of the chapter discusses guideline implementation. Graphs for the guidelines are provided at the end of the chapter while discussions of the guidelines are presented in the first part of the chapter.

Alpha Guidelines

Development

In developing alpha guidelines a graph was constructed where the x-axis represents the product of population skewness and kurtosis while the y-axis provides for the level of alpha that should actually be used in carrying out statistical procedures. An example of these two axis can be seen in Figure 12.

Next, it was necessary to assemble all simulation runs for a given population which utilized the 150 standardized sample size. Two columns were prepared for each run. One column listed the proportion (the attained alpha level) while in the other column the specified alpha level (the level actually used in the test) was recorded. Then, the column of attained levels was searched for a 5% proportion. When this proportion was located, the percent in the corresponding column indicated the alpha level actually used. For example, the actual alpha level used could have been .1%. Consequently, for that particular degree of non-normality and sample size, a desired alpha level of 5% could be attained by using an empirical alpha level of .1%. Using this data, the point representing (1) the product of skewness and kurtosis for the given population and (2) the actual alpha level used in the test was plotted on the graph described above.

The next assemblage of all simulation runs using the 150 standardized sample size for the second population was searched for an attained alpha level of 5%. Again, the parallel column indicated the level of alpha actually used in the test and another point on the graph was plotted. These procedures were then repeated for all populations. After considering all of the populations, the points on the graph were connected to yield a curve representing a desired alpha level of 5%. This curve is shown in Figure 12.

Starting over again with the first group of all simulation runs for a given population, the steps were repeated to determine what empirical alpha levels are necessary to attain a 10% alpha level. This group of points forms the curve for a desired alpha level of 10% which is also shown in Figure 12. To obtain the curve for a desired alpha level of 20%, the procedures were again repeated.

It was then necessary to assemble all simulation runs for a given population which used a standardized sample size of 450 units. The steps carried out above using the 150 standardized sample size were applied to the simulation runs using the standardized size of 450 units. The application of the above procedures to these simulation runs yielded the desired 5%, 10% and 20% alpha level curves shown in Figure 13 for the 450 standardized sample size.

Again, procedures were carried out for the standardized sample size of 1,000 units. However, due to the limited number of simulation runs utilizing the 1,000 standardized smaple size, the only curve that could be constructed was for the desired alpha level of 20%, which is shown in Figure 14.

A composite graph of all the curves found in Figures 12, 13 and 14 is provided in Figure 15. The composite graph, however, reflects the curves before smoothing; these curves are based on specific points with linear lines connecting the points. The curves are smoothed in the guidelines for the following reason. When a curve bends sharply at a specific point, there is an implication that at that precise point the alpha level to be used in the test changes. Since the number of populations used in the research is limited in comparison to the infinite number of possible populations, such a precise point should not be implied. Consequently, the curves were smoothed as indicated in Figures 12, 13 and 14.

Interpretation

Alpha guidelines are based on desired alpha levels of 5%, 10%, and 20%. To use the guidelines the auditor must first estimate the population skewness and kurtosis. These estimates are then used as input to the guidelines. The alpha guidelines also depend on the "original" sample size. The original sample size is computed using (1) the formula specified by this study, (2) the desired alpha and beta levels, (3) the desired amount of per unit precision, and (4) the estimated population variance. Using this sample size, the desired level of alpha, and the product of skewness and kurtosis as input, the guidelines indicate a level of alpha to be used in the test so that the desired alpha level can be attained.

For example, if the auditor wants to test at a 20% alpha level and if the original sample size equals 150 elements, the alpha guidelines for n = 150 should be used to determine the level of alpha to be used in the test. On Figure 12 locate on the horizontal axis an assumed 610 degrees which represent the product of skewness and kurtosis. Now move an imaginary straight line vertically up the graph until the imaginary line bisects the curve for the desired alpha level of 20%. Continue by moving an imaginary line horizontally to the left until the y-axis is reached. The y-axis indicates the level of alpha to be used in the MPU sampling procedures. In this case, an alpha level of 5% is recommended. In other words, the auditor will now use a 5% alpha level as though he had originally set out to use this level. The sample size must be recomputed using the recommended alpha level of 5% rather than the desired 20% level. The new sample size should be used in carrying out the statistical sampling procedures subject to the recommendations set forth by the beta guidelines. A 5% alpha level should also be used in computing the confidence interval around the sample mean. Under these conditions, the auditor can expect to attain an alpha level of 20%. The 5% alpha level actually used in the statistical procedures has been referred to as either the alpha level used in the test or the empirical alpha level.

If the "original" sample size is 450 units, alpha guidelines given in Figure 13 should be used. If the size is 1,000 units, Figure 14 should be used to determine the alpha level to be used in the test. The alpha levels to be used in the test range from .1% to 20%. For those levels of alpha not reflected in a Student's t-table, the corresponding $Z_{\alpha/2}$ factor for the level of alpha may be located in a normal table so long as the sample size is greater than 120 units.

The x-axis of all the guidelines is based on the products of skewness and kurtosis from zero degrees to 1,000 degrees. Since the research involves both population skewness and kurtosis, the guidelines should also be based on skewness and kurtosis, rather than on one variable for the basis of the x-axis. After using the alpha guidelines to determine the level of alpha that should be used in the test, the auditor should consult the beta guidelines to identify procedures necessary to attain the specified beta level.

Beta Guidelines

Development

Development of beta guidelines utilized graphs with axes similar to those used in the alpha guideline graphs. The x-axis represents the product of population skewness and kurtosis while the y-axis symbolizes the level of alpha to be used in the test. The y-axis in some of the beta guidelines, however, do not cover a wide range of alpha levels to be used in the test because of the limited number of runs in some instances. This will be commented upon in subsequent discussions.

Construction and use of the beta graphs, however, are not identical to the alpha graphs. It was necessary to assemble all simulation runs using a 150 standardized sample size and where beta was specified at 20%. These runs supplied the data necessary to construct Figure 16. First, the runs were subgrouped according to populations. That is, all of the runs applied to the normal population were subgrouped together. Then, the runs applied to the next population were assembled together. This type of subgrouping was continued for all populations so that each group contained only those runs utilizing a 150 standardized sample size and beta specified at 20%.

Now, starting with the normal population, the run specifying alpha equal to 20% was investigated to determine the sample size multiple (factor) necessary to attain the upper beta level of 20%. When an upper beta proportion indicated 20%, or was within three standard errors, the corresponding sample size factor was plotted on the graph for (1) that product of skewness and kurtosis and (2) that level of alpha used in the test. Still keeping within the runs applied to the normal population, the simulation run utilizing a 15% alpha level was scrutinized to determine the sample size factor that yields an attained beta level of 20%. This factor was then plotted on the graph. Then, the simulation run where alpha equals 10% was examined so that the sample size factor necessary to attain a 20% beta level could be plotted. These steps were repeated on simulation runs for alpha equal to 8%, 5%, 1% and .1%. This completed the procedures applied to the normal population.

Next, results of simulation runs carried out on the next population were investigated. The sample size factors necessary to yield an attained beta level of 20% for each level of alpha used in the test are plotted on the graph for this degree of non-normality. These procedures were repeated until all populations had been considered. In Figure 16, however, the beta level was always attained with a sample size factor of 1.0. This is not the case, however, for all levels of beta.

To construct Figure 17 the procedures described above in the formation of Figure 16 were again carried out. However, in the assemblage of computer runs using a 150 standardized sample size, the specified upper beta level was changed to 10%. There were a limited number of simulation runs utilizing a 150 standardized sample size for beta equal to 10%. Consequently, the y-axis on this particular beta guideline reflects only two levels of alpha to be used in the test, 20% and 10%.

The formation of the next upper beta guideline, Figure 18, provides for beta equal to 5% when utilizing a basic sample size of 150 units. Alpha levels to be used in the test, in this instance, are restricted to 20% and 5% due to the limited number of computer runs.

Figures 19, 20, and 21 were constructed in the same manner described above except these latter upper beta guidelines utilize a 450 standardized sample size. In fact, Figures 19, 20, and 21 for a standardized sample size of 450 units correspond to graphs in Figures 16, 17, and 18 which utilize a 150 standardized sample size.

The last upper beta guideline, Figure 22, specifies a 1,000 basic sample size and beta equal to 20%. Guidelines using other levels of beta in conjunction with a standardized sample size of 1,000 units were not developed due to the limited number of simulation runs utilizing this standardized size.

The lower beta guidelines, Figures 23 through 29, are parallel to Figures 16 through 22 for the upper beta guidelines. Procedures used to develop the upper beta guidelines were repeated for construction of the lower beta guidelines.

Interpretation

The upper and lower beta guidelines are integrated with the alpha guidelines so that the auditor can expect to attain the specified alpha level and one of the beta levels simultaneously. The guidelines for the upper and lower beta risk levels, however, are not integrated with one another. In the field the auditor will determine which beta risk level (upper or lower) is appropriate in the circumstances. If the auditor wishes to guard against (1) rejecting a reasonably stated book value, and (2) accepting a materially overstated book amount, the guidelines for the alpha level and the upper beta level should be followed. On the other hand, if the auditor wants to guard against accepting a materially understated book amount, the guidelines for the lower beta level, as well as the alpha guidelines, should be followed.

The beta guidelines consist of two sets of block diagrams. As stated previously, one set, Figures 16 through 22, represents the upper beta guidelines while the other set represents the lower beta guidelines, Figures 23 through 29. The appropriate graph depends on the sample size and the desired upper or lower beta level. For example, assume that the auditor had previously determined from the alpha guidelines that an empirical 5% alpha level was necessary to attain a desired level of 20%. Using the 5% recommended alpha level and a desired beta level of 20%, the recalculated sample size was 450 units. If the auditor is testing for possible overstatement of the account balance, Figure 19 should be consulted because this graph represents upper beta guidelines for beta equal to 20% and for a 450 unit sample size. Now, assuming the product of population skewness and kurtosis at 610 degrees, locate such point on the x-axis of Figure 19. Extend an imaginary perpendicular line upward from the x-axis. Then, locate the empirical alpha level of 5% on the y-axis and extend a second imaginary line horizontally to the right. At the intersection of these two imaginary lines, the graph will indicate the sample size factor that should be applied to the recomputed sample size of 450 units. In this instance, the beta guidelines indicate that a sample size factor of 1.5 should be applied to the 450 sample size to yield a sample size of 675 units. Consequently, to attain 20% alpha and beta levels under the stated non-normal conditions, the auditor should carry out his MPU sampling procedures using a 675 unit sample size and a 5% alpha level in computing the confidence intervals.

However, when the sample size factor is applied to the recomputed sample size, the resulting size may cause the attained alpha level to be lower than the desired alpha level. In the following section of this chapter examples illustrate when the resulting sample size would be larger than needed for the attainment of the desired alpha level. When the sample size appears to be too large, the alpha level recommended by the guidelines must be adjusted upward. That is, if the alpha level to be used in the test is .1%, it should now be increased to 1%. In similar cases, the empirical alpha level should be adjusted from 1% to 3%, or from 3% to 5%, or from 5% to 8%, or from 8% to 10%. When the recommended alpha level has been adjusted upward, it is necessary to recalculate the sample size again using the increased alpha level and the desired beta level. The beta guidelines must also be consulted again to determine the final sample size to be used in the MPU procedures. This final sample size and increased alpha level should enable the auditor to simultaneously attain the specified levels of alpha and beta.

Before illustrating the use of the alpha and beta guidelines, it should be pointed out that the lines dividing the sample size factors within the guideline graphs are only estimates. The dividing lines are estimates because the distributions used in the study represent only a small fraction of the infinite number of possible distributions.

Illustrations of Guidelines

Illustrations on the utilization of alpha and beta guidelines are carried out using three new populations. The creation of these three new distributions was carried out in the same manner as the creation of distributions used in the research simulation runs. These illustrative distributions have the same J-shape as the research distributions but they do not have unity variances. Rather, the illustrative populations have variances ranging from 887.2667 to 4,991.4125 and degrees of skewness and kurtosis from 13.7062 to 26.4263 and from 19.9830 to 38.1070, respectively. It was believed that these characteristics may approximate real auditing populations as evidenced by the distributions provided in Auditing Research Monograph 2.[1] This will be elaborated upon in more detail in the last chapter of the

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[1]John Neter and James K. Loebbecke, Auditing Research Monograph 2, Behavior of Major Statistical Estimators in Sampling Accounting Populations (New York: American Institute of Certified Public Accountants, Inc., 1975). Hereafter referred to as Behavior of Statistical Estimators.

dissertation. To make sure that the illustrative distributions follow the desired shape, they were subjected to the same scrutiny as the research populations. The plots of the histograms representing the three populations, Figures 52, 53, and 54 are presented in Appendix II along with the plots of the twenty-two research distributions.

After creating the illustrative populations, MPU sampling techniques were carried out using the same simulation procedures as those used on the research distributions except for the following procedures. A unity variance and standardization of the sample sizes were not used in the illustrative cases but the alpha and beta guidelines were utilized.

After the desired alpha and beta levels were specified, a sample size was computed from the formula using the illustrative population's standard deviation. Simulation procedures were first carried out without any recognition of the guidelines to determine whether control of the alpha and beta risks was effective. Then, the computed sample size was used as input to the alpha guidelines to determine the level of alpha that should be used in the test. This level of alpha was then used as input to the beta guidelines to determine the sample size factor. If application of this factor caused the sample size to be large enough so that the attained alpha level would possibly be lower than the desired alpha level, the empirical alpha level was increased. Then, another sample size was computed using the increased alpha level. Again, the beta guidelines were consulted and a new sample size factor determined. Simulation procedures were then carried out to determine whether the desired alpha and beta levels were attained when the alpha and beta guidelines were utilized. These procedures carried out on the three illustrative populations show that the specified alpha and beta levels are controlled.

Illustrative Population #1

The characteristics of this infinite population are as follows: mean = 85.2343, variance = 3940.4581, skewness = 13.7062 degrees, and kurtosis = 19.9830 degrees.

Case No. 1. The levels of alpha and beta are both specified at 20%. With precision equal to 9% of the mean, the calculated sample size is 302 units. Without utilizing the guidelines, simulation procedures produce an attained alpha level of 27.9% which is 5.56 standard errors away from the specified level of 20%. The upper beta level, however, is attained at 20.1%. The lower beta level is not attained; the proportion of lower beta errors is 26.9% which is 4.93 standard errors from the desired 20% level. The standard errors for these three proportions are .0142, .0127, and .0140, respectively. The average sample size for this simulation run is 304 units.

To obtain the desired alpha and beta levels simultaneously, the guidelines are consulted. Since the computed sample size using the desired alpha and beta levels of 20% is 302 units, the alpha guidelines for n=450 (Figure 13) are used to determine the level of alpha that should be used in the MPU procedures. Using the product of skewness and kurtosis of 274 degrees, the guidelines indicate that an alpha level of 15% be used in the test so that a 20% alpha level will be attained. Recalculating the sample size using the recommended 15% alpha level and the desired 20% beta level, the sample size becomes 348 units. The upper beta guidelines (Figure 19) for n = 450 and for an empirical alpha of 15% show that the sample size does not have to be increased for the potential attainment of an upper beta level of 20%. Simulation results show the attained alpha and beta levels to be 22.5% and 20.4%, respectively, with standard errors of .0132 and .0127. Consequently, the alpha and beta levels are attained simultaneously. The average sample size for this simulation run is 353 units.

The lower beta guidelines for n = 450 (Figure 26) indicate that the sample size should be increased 1.5 times to a size of 522 units (348 x 1.5). Since the calculated sample size of 348 units is less than the 450 sample size upon which the guidelines are based, the increase in sample size to 522 units will probably not reduce the attained alpha level below the desired level of 20%.[2] Simulation procedures for an average sample size of 529 units reflect that the attained alpha and beta levels are 20.9% and 22.3% with standard errors of .0129 and .0132, respectively. Therefore, both sampling risks are effectively controlled when the guidelines are followed.

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[2]The probable reduction in the attained alpha level below the desired level due to an increased sample size is demonstrated in Case No. 6.

Case No. 2. Using the same population as in Case No. 1 and keeping everything constant except for changing the desired alpha level to 10%, the following results are obtained. The calculated sample size using alpha at 10% and beta at 20% is 414 units. Simulation results without using the guidelines reflect (1) an attained alpha level of 14% which is 3.64 standard errors from the desired alpha level of 10%, (2) an attained upper beta level of 24.8% which is 3.50 standard errors from the 20% desired level, and (3) an attained lower beta level of 31.9% which is 8.09 standard errors away from the desired beta level of 20%. The average sample size for these simulation runs is 423 elements.

Applying the alpha guidelines for n = 450 (Figure 13) to the above situation, the level of alpha that should be used in the test is 5%. The recalculated sample size using a 5% alpha level is 526 units. The upper beta guidelines for n = 450 (Figure 19) indicate that the sample size should be 1.5 times 526 units, or 789 units. Since the calculated size of 526 units is larger than the 450 sample size upon which the upper beta guidelines are based, the increase in sample size to 789 units would probably reduce the attained alpha level below the desired level of 10%. Therefore, the recommended alpha level to be used in the test is increased from 5% to 8%. The recalculated sample size using the adjusted alpha level becomes 450 elements. The upper beta guidelines specify a sample size of 1.5 times 450, or 675 elements. The simulation run using an 8% alpha level and having an average sample size of 677 units show an attained alpha level of

12.3% which is 2.21 standard errors from the desired 10% alpha level. And, the attained upper beta level is 17.4% with a standard error of .0120.

Referring to the recomputed sample size of 526 units from using the recommended alpha level of 5% and the 20% desired beta level, the lower beta guidelines for n = 450 (Figure 26) indicate that the sample size should be doubled to 1,052 elements. Again, the increase in sample size will probably reduce the attained level below the desired alpha level. Consequently, the recommended 5% alpha level is increased to 8% which creates a new sample size of 450 units. The lower beta guidelines specify a sample size of 2.0 times 450, or 900 units. The simulation results reflect an attained alpha level of 11.6% which is 1.58 standard errors from the desired level and an attained lower beta level of 18.6% with a standard error of .0123. The average sample size for this simulation run is 903 units.

These results reflect that the specified alpha and beta risk levels are simultaneously attained when the guidelines are utilized.

Illustrative Population #2

The characteristics of this infinite population are: mean = 94.2445, variance = 4,991.4125, skewness = 22.1512 degrees, and kurtosis = 29.8134 degrees.

Case No. 3. The alpha and beta levels are both specified at 20%. Using the population standard deviation of 70.65 and precision equal to 9% of the mean, the sample size per formula is 313 elements. Without utilizing the guidelines, the simulation run with an average sample size of 314 units reflect an attained alpha level of 33.5% which is 9.06 standard errors from the desired alpha level of 20%. The upper beta level is attained with a proportion of .2220 which has a standard error of .0131. The lower beta level is not attained; the proportion of beta errors on the lower side is .2510 which is 3.7226 standard errors from the desired 20% level. To attain the alpha and beta levels simultaneously, the guidelines are consulted and the simulation results, having utilized the guidelines, are discussed below.

Since the product of skewness and kurtosis equals 660 degrees, the alpha guidelines for n = 450 (Figure 13) specify the alpha level to be used in the test at 8%. The recalculated sample size using the recommended alpha level is 467 elements. The upper and lower beta guidelines for n = 450 (Figures 19 and 26) specify a sample size of 1.5 and 2.0 times the recalculated sample size, respectively. Since sample sizes of 701 and 934 units will probably lower the attained alpha level below the desired level, the alpha level to be used in the test should be increased from 8% to 10%. Another recalculation of the sample size is 429 units. As determined from the upper beta guidelines for n = 450 (Figure 19), it is unnecessary to increase the sample size of 429 units in order to attain the desired alpha and upper beta levels. The simulation results having an average sample size of 453 elements reflect that the attained alpha level is 20.4% with a standard error of .0127 and that the attained beta level is 22.3% which is 1.74 standard errors away from the desired 20%

upper beta level. Therefore, the specified alpha and upper beta levels are controlled simultaneously.

Consulting the lower beta guidelines for n = 450 (Figure 26), it is determined that by using the adjusted 10% alpha level, the sample size should be 2 times the recalculated size of 429 units, or 858 units. The simulation results, having an average sample size of 906 units, demonstrate that both the alpha and lower beta levels are attained simultaneously. The proportion of alpha errors is 16.2% with a standard error of .0117 and the proportion of lower beta errors is 20.7 with a standard error of .0128.

Case No. 4. This illustration uses the same distribution described in Case No. 3 and holds everything constant except for the desired alpha level which is changed to 10%. The calculated sample size using precision equal to 9% of the mean is 429 units. The simulation results without consulting the guidelines indicate ineffective control of the alpha level and lower beta level but there is effective control of the upper beta level. The sampling proportions are as follows: for alpha errors -- 20.4%, for upper beta errors -- 22.3%, and for lower beta errors -- 30.3%. The standard errors for the proportions are .0127, .0132, and .0145, respectively.

Consulting the guidelines so that the specified alpha and beta levels can be attained simultaneously, it is determined from Figure 13 that the empirical alpha level should be 1%. The recalculated sample size using a 1% alpha level is 811 elements. In this instance, the beta guidelines for n = 450 are used because the beta guidelines for n = 1,000 do not include an empirical alpha level of 1%. Since the original computed sample size of 429 units is compatible with the alpha guidelines based on a 450 sample size, and because the recalculated sample size of 811 units falls between the sample sizes of 450 and 1,000 units, it was felt that the beta guidelines based on a 450 sample size could possibly be used. Upper beta guidelines for n = 450 (Figure 19) reflect that the sample size should be increased 1.5 times the 811 units, or to 1,217 elements. Lower beta guidelines for n = 450 (Figure 26) recommend quadrupling the 811 units to a sample size of 3,244 elements. The sample sizes of 1,217 and 3,244 for the upper and lower beta levels, respectively, would probably cause the attained alpha level to be lower than the specified alpha level. Therefore, the level of alpha to be used in the test should be increased from 1% to 3% which causes the recalculated sample size to be decreased from 811 to 629 units. The upper beta guidelines for n = 450 indicate that the sample size should be increased to 940 elements which is 1.5 times the calculated size. The simulation procedures demonstrate that both the desired 10% alpha level and 20% beta level are attained with the proportions being 9.4% and 20%, respectively. The standard error for the alpha proportion is .0092 and for the beta proportion the standard error is .0126.

The lower beta guidelines, however, indicate that the calculated sample size of 629 units would have to be increased 4 times to 2,516 units. Again, this latter sample size would probably cause the attained alpha level to be lower than the desired level. Therefore, if the auditor wants to attain an alpha level of 10% and a lower beta level of 20% at the same time, the alpha level should be increased for the second time from 3% to 5%. The recalculated sample size using the new adjusted alpha level becomes 545 units. The lower beta guidelines for n = 450 indicate that 2 times the recalculated sample size, or 1,090 units, would be required. The simulation results for an average sample size of 1,068 elements demonstrate that the attained alpha level is 12.1% which is 2.04 standard errors from the desired 10% alpha level. The attained lower beta level is 23.8% which is 2.81 standard errors from the desired 20% lower beta level.

Consequently, in Case No. 4 the empirical alpha level had to be adjusted from 1% to 3% for the simultaneous attainment of the 10% alpha level and a 20% upper beta level. For the lower beta level, however, the empirical alpha level had to be adjusted a second time to 5% for the simultaneous control of the alpha and beta risks.

Case No. 5. This case also uses Population #2. In this case the alpha level is specified at 20% and the beta level at 10%. The calculated sample size is 456 units. The alpha guidelines (Figure 13) indicate an empirical alpha level of 10% which creates a recomputed sample size of 594 elements. The upper beta guidelines for n = 450 (Figure 20) indicate that the sample size should be increased to 1.5 times 594, or to 891 units. The lower beta guidelines (Figure 27) indicate that the sample size should be 2,376, or 4 times 594. These increased sample sizes would probably cause the attained alpha level to be lower than necessary. Therefore, the alpha level to be used in the test should be increased to 15%. The recomputed sample size using an alpha level of 15% is 514 units. There are no guidelines, however, for beta specified at 10% with an empirical alpha of 15%. There are guidelines, however, for a 10% beta level to be used in conjunction with empirical alpha levels of 20% and 10% (Figures 17, 20, 24, 27). Upper beta guidelines for n = 450 and for beta equal to 10% (Figure 20) indicate that the sample size should be increased 1.5 times when the empirical alpha level is 10% or 20%. Therefore, it is assumed that for a 15% alpha level, the calculated sample size of 514 units should be increased 1.5 times, or to 711 units. The simulation results having an average sample size of 768 units demonstrate that the 20% alpha level and the 10% upper beta level are attained simultaneously. The proportion for the alpha errors is 21.2% with a standard error of .0129 and the proportion for the upper beta errors is 12.3% which is 2.21 standard errors away from the desired 10% upper beta level.

The interpretation of what sample size factor should be used for the simultaneous attainment of the alpha and lower beta levels is not as clear-cut as under the guidelines for the upper beta level.

The lower beta guidelines for beta equal to 10% and n = 450 (Figure 27) and for using a 20% empirical alpha level indicate that the calculated sample size should be doubled. If, however, a 10% empirical alpha level is used, the guidelines indicate that the calculated sample size should be quadrupled. Consequently, in a practical situation, the guidelines do not indicate whether the sample size should be doubled or quadrupled. For this study, however, simulation runs are made having an average sample size of 1,024 (2 times the calculated size of 514 units) and an average size of 2,047 (4 times the calculated size of 514 units). The doubled sample size results reflect that the attained alpha level is 19.8% with a standard error of .0126 but the lower beta risk is not effectively controlled. The lower beta proportion is 15.3% which is 4.65 standard errors away from the 10% desired level. The quadrupled sample size, however, reflects simultaneous attainment of the alpha and lower beta levels. The proportion for the alpha errors is 18.5% with a standard error of .0123 and the proportion for the lower beta errors is 8.3% with a standard error of .0087. This latter example indicates possible need for more extensive guidelines.

Illustrative Population #3

The characteristics of this infinite population are: mean = 100.3494, variance = 887.2667, skewness = 26.4263 degrees, and kurtosis = 38.1070 degrees.

Case No. 6. The alpha and beta levels are both specified at 20%. Only the upper beta risk level is considered in this case.

Using the population standard deviation of 29.79 and precision equal to 5% of the mean, the original sample size is 159 units. Without consulting the guidelines, the simulation run having an average sample size of 157 elements reflect that the proportion of alpha errors is 40.2% with a standard error of .0155. The upper beta risk level, however, is more than attained since the proportion is 14.8% with a standard error of .0112. The guidelines are consulted to find the necessary procedures for the simultaneous control of the alpha and upper beta risk levels.

Using the product of skewness and kurtosis equal to 1,000 degrees, the alpha guidelines for n = 150 (Figure 12) recommend a 3% empirical alpha level for the potential attainment of the desired 20% level. The recomputed sample size is 320 units. The upper beta guidelines for n = 450 indicate that the sample size should be increased 1.5 times, or to 480 units. In addition to the increase in sample size due to the beta recommendation, jumping from guidelines based on sample sizes of 150 units to those based on sample sizes of 450 elements when the original sample size is 159 units, the increased sample size will probably cause the attained alpha level to be lower than the desired 20% alpha level. In this instance simulation results are presented to demonstrate the effect of the increased sample size on the attained alpha level. After these results are presented, the case continues in the usual manner. Simulation results having an average sample size of 442 units reflect an attained alpha level of 14.9% which is 4.51 standard errors below the desired 20% level. The upper beta level is, however, effectively controlled as the attained level is 19.9%.

Since the increased sample size causes the alpha level to be over-attained, the empirical alpha level is increased from 3% to 5%. The recalculated sample size is 277 units. The upper beta guidelines for n = 150 (Figure 16) indicate that the sample size does not have to be increased for the potential attainment of the upper beta risk level. Simulation results having an average sample size of 281 units reflect that the proportion for alpha errors is 21.5% with a standard error of .0130 and that the proportion for upper beta errors is 19.6% with a standard error of .0126. Consequently, the specified alpha and beta levels are attained simultaneously when the alpha and upper beta guidelines are followed.

Case No. 7. This case uses the same population distribution as in Case No. 6 and holds everything constant except the specified alpha level which is changed from 20% to 10%. Using the population standard deviation of 29.79 and precision equal to 5% of the population mean, the sample size is calculated at 218 units. The simulation results having an average sample size of 219 elements show the proportion of alpha errors to be 27.3% which is 12.06 standard errors from the desired 10% alpha level. The proportion for the upper beta level is 20.2% with a standard error of .0127. The lower beta level, however, is not attained as this proportion is 36.0% with a standard error of .0152. To determine the procedures to be used to attain the alpha and beta levels simultaneaously, the guidelines are consulted.

Using the product of skewness and kurtosis equal to 1,000 degrees, the alpha guidelines for n = 150 (Figure 12) recommend an alpha level to be used in the test of .1%. The recalculated sample size becomes 602 units. The upper beta guidelines for n = 450 indicate the sample size should be increased 1.5 times, or to 903 units. The lower beta guidelines for n = 450 show that the size should be 2,408 units which is 4 times the calculated size of 602 units. The increased size for both the upper and lower levels of beta would probably culminate in an attained alpha level lower than the desired 10% level. Therefore, the empirical alpha level is increased from .1% to 1%. The recalculated sample size is 412 units.

The upper beta guidelines for n = 450 indicate that the 412 sample size should be increased 1.5 times, or to 618 units. The simulation results demonstrate that the attained alpha level is 8.9% with a standard error of .0090 while the attained upper beta level is 20.9% with a standard error of .0129. Therefore, the alpha and upper beta levels are controlled with an average sample size of 625 elements.

The lower beta guidelines for n = 450 show that the calculated sample size of 412 units should be increased 4 times, or to 1,648 units. This increased sample size, as well as jumping from guidelines based on a 150 unit sample size to guidelines based on a sample size of 450 units, will more than likely cause the attained alpha level to be below the specified level although the empirical alpha level has already been adjusted upward. The alpha level should be adjusted upward again from 1% to 3%. The recalculated sample size is now 320 units. The lower beta guidelines for n = 450 still recommend that the sample size be increased 4 times, or to 1,280 elements. The simulation procedures demonstrate that with an average sample size of 1,283 units, the proportion of alpha errors is 8.4% and the proportion of lower beta errors is 15.2%. The standard errors for these proportions are .0088 and .0114, respectively. Thus, the alpha risk level of 10% is attained while the lower 20% beta risk level is slightly overachieved.

The above cases demonstrate that utilization of the alpha and beta guidelines culminate in the simultaneous attainment of alpha and beta levels. Control of alpha and beta risks should be effective when the guidelines are utilized under conditions simulated in the research. Problems that may be encountered in implementing the guidelines are discussed in the following section.

Implementation of Guidelines

If an auditor would like to determine the reasonableness of an account balance, or to estimate such balance using the MPU estimator, the alpha and beta guidelines should be used provided the following four conditions are met. First, the alpha and beta levels desired by the auditor must be compatible with the levels used in the guidelines. Second, the computation of the first sample size should approach one of the standardized sample sizes used in guideline development. Third, the auditing population should approximate the type of J-shaped distribution used in the research, and fourth, the product of skewness and kurtosis must be within the range specified in the guidelines.

The first two conditions should not present implementation problems provided the original sample size approximates the sample size used in the guideline development and that the sampling risk levels within the guidelines are compatible with the levels desired by the auditor. The third and fourth conditions discussed above, however, may present problems for the auditor. Currently, the auditor could utilize a computer program to determine the shape of a distribution and to compute the skewness and kurtosis of the population. Potentially, the auditor could develop his own program or could possibly purchase a program from a software dealer.

Implementation of the guidelines can also be integrated with a sampling plan where the auditor stratifies the auditing population into two strata. On the upper stratum 100% sampling could be used while the alpha and beta guidelines could be utilized on the lower stratum for simultaneous control of the alpha and beta risks.

Relationships between the necessary conditions for guideline implementation, research limitations, and future research are discussed in the last chapter of the dissertation.

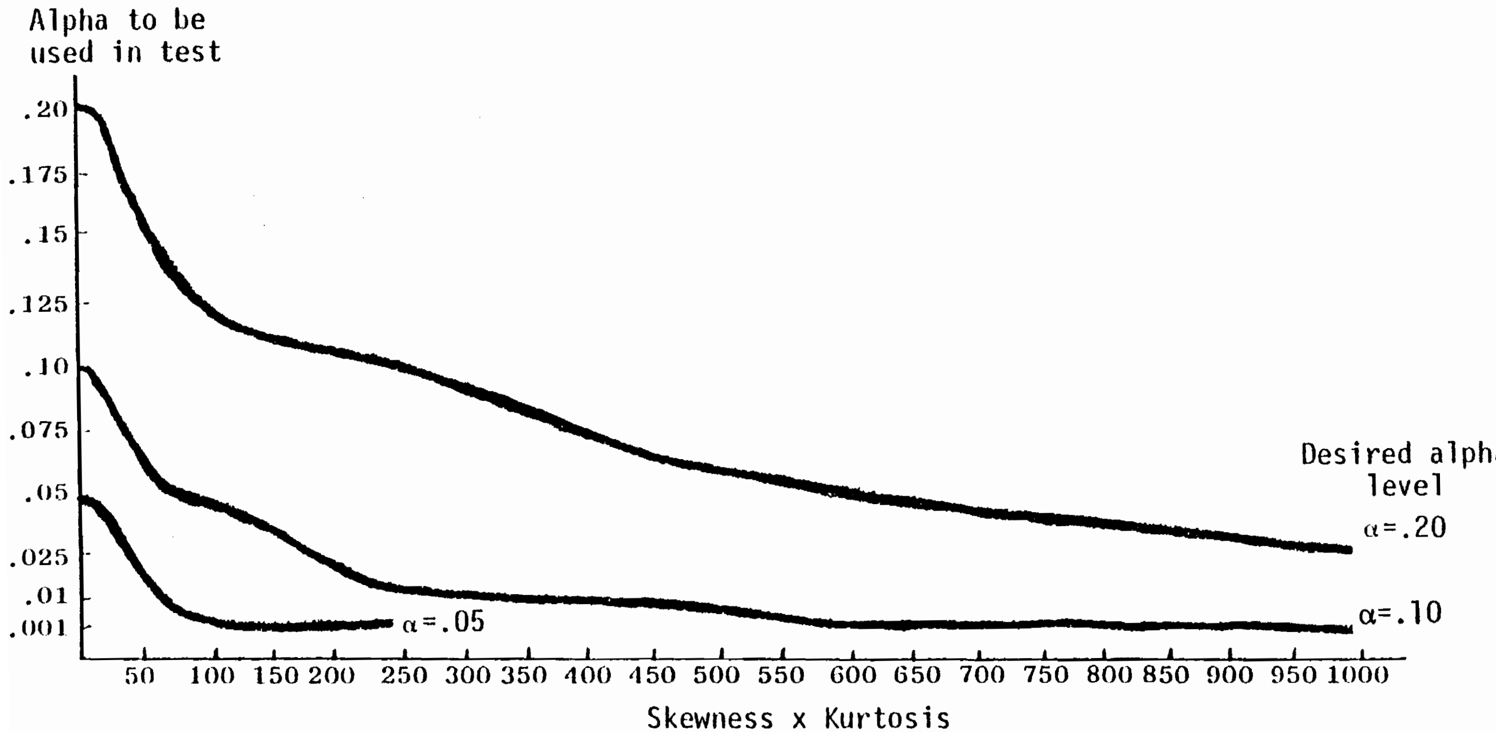

FIGURE 12

<u>ALPHA GUIDELINES FOR n = 150</u>

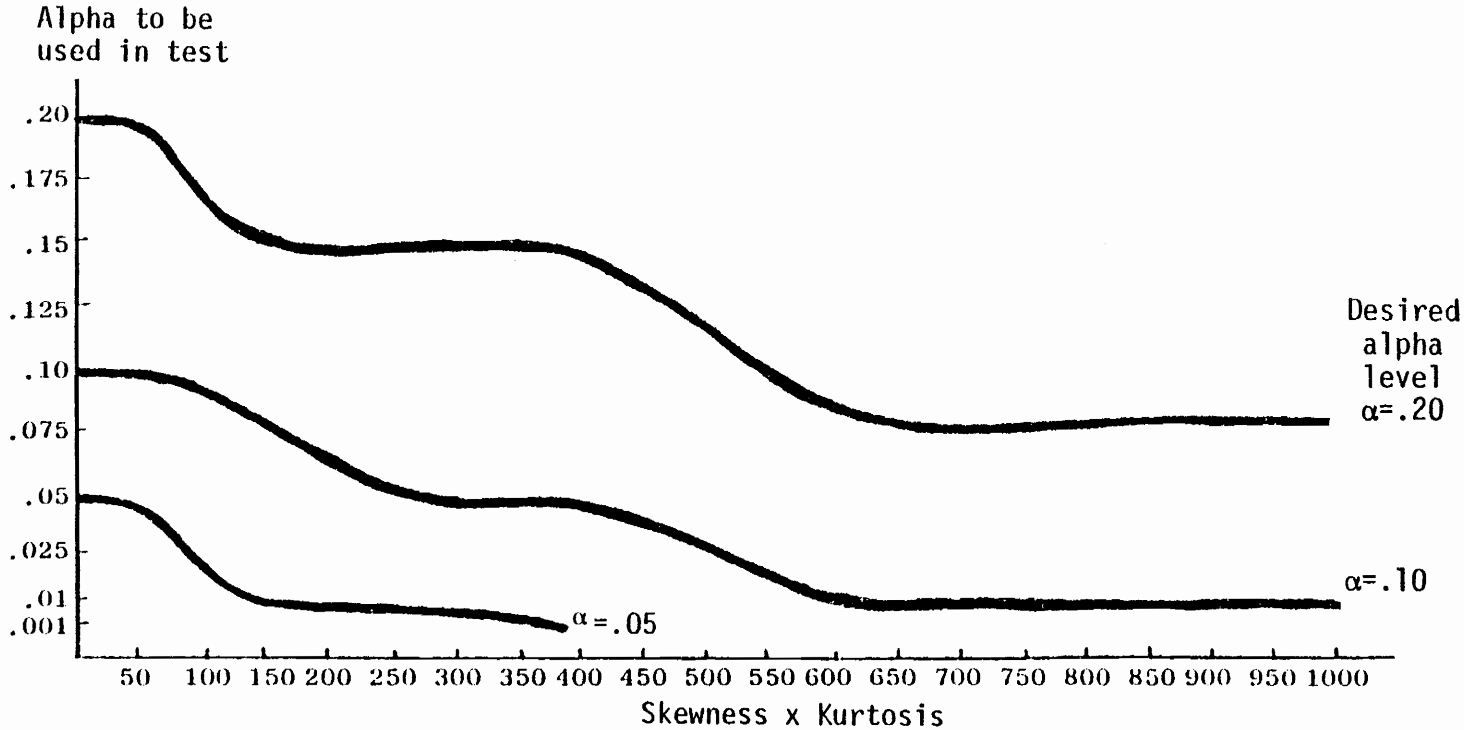

FIGURE 13

ALPHA GUIDELINES FOR n = 450

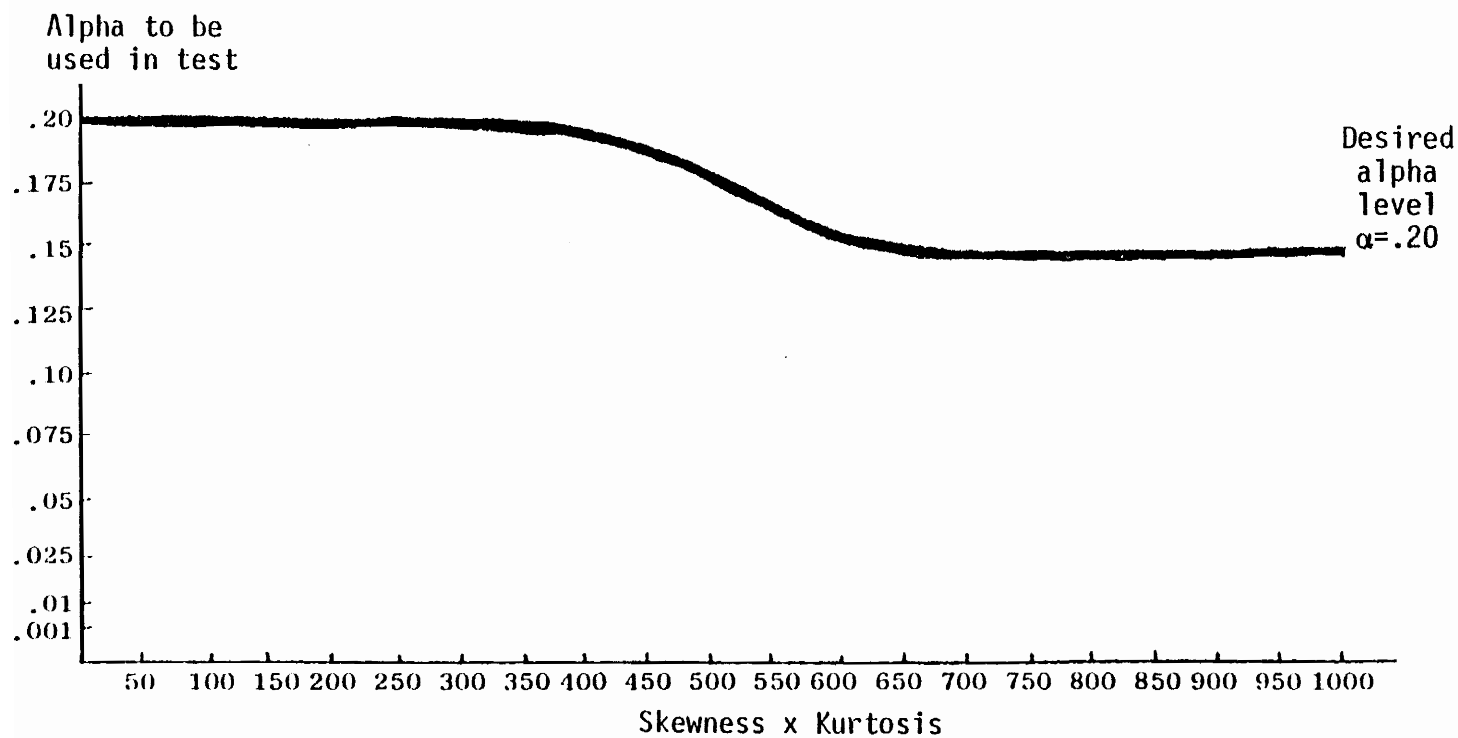

FIGURE 14

ALPHA GUIDELINES FOR n = 1000

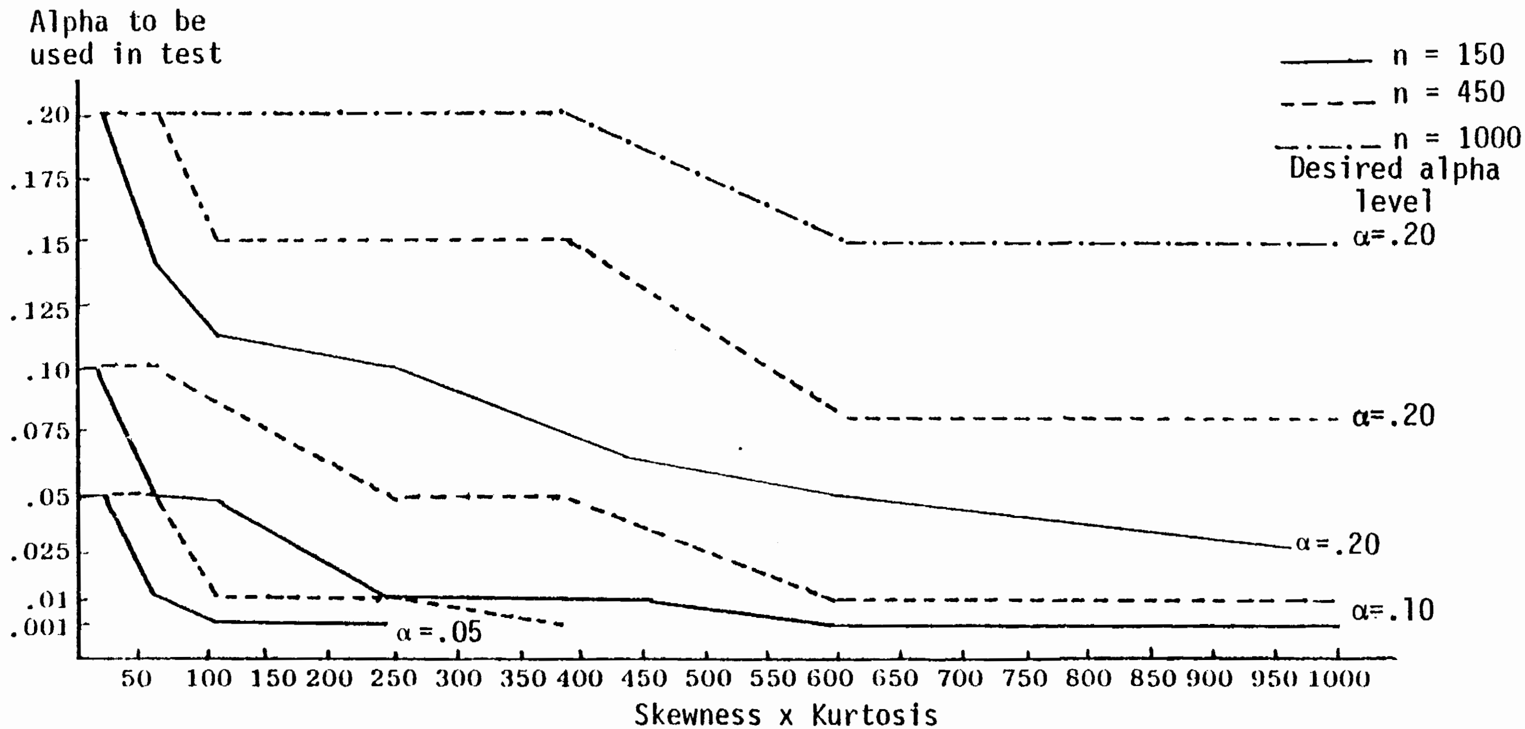

FIGURE 15

<u>COMPOSITE GRAPH</u>

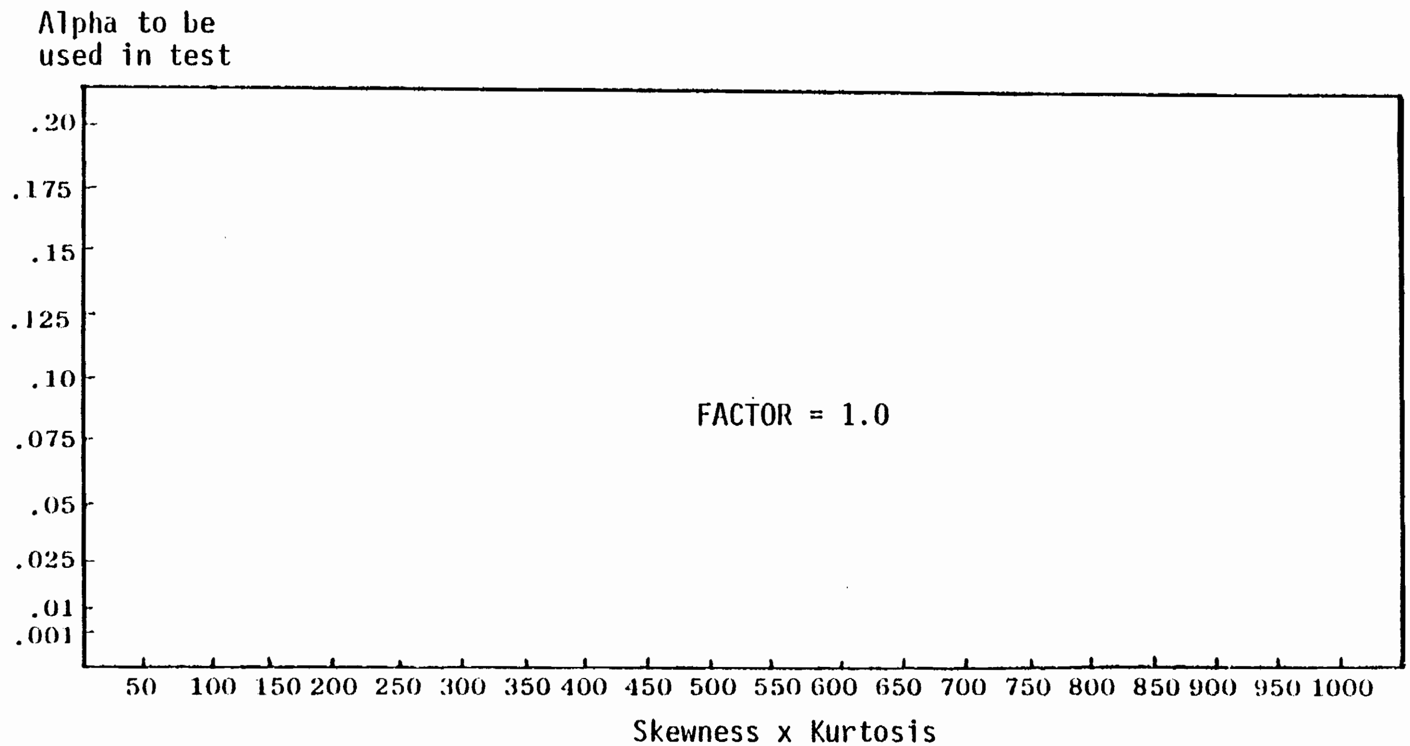

FIGURE 16

UPPER BETA GUIDELINES FOR
β = .20 AND n = 150

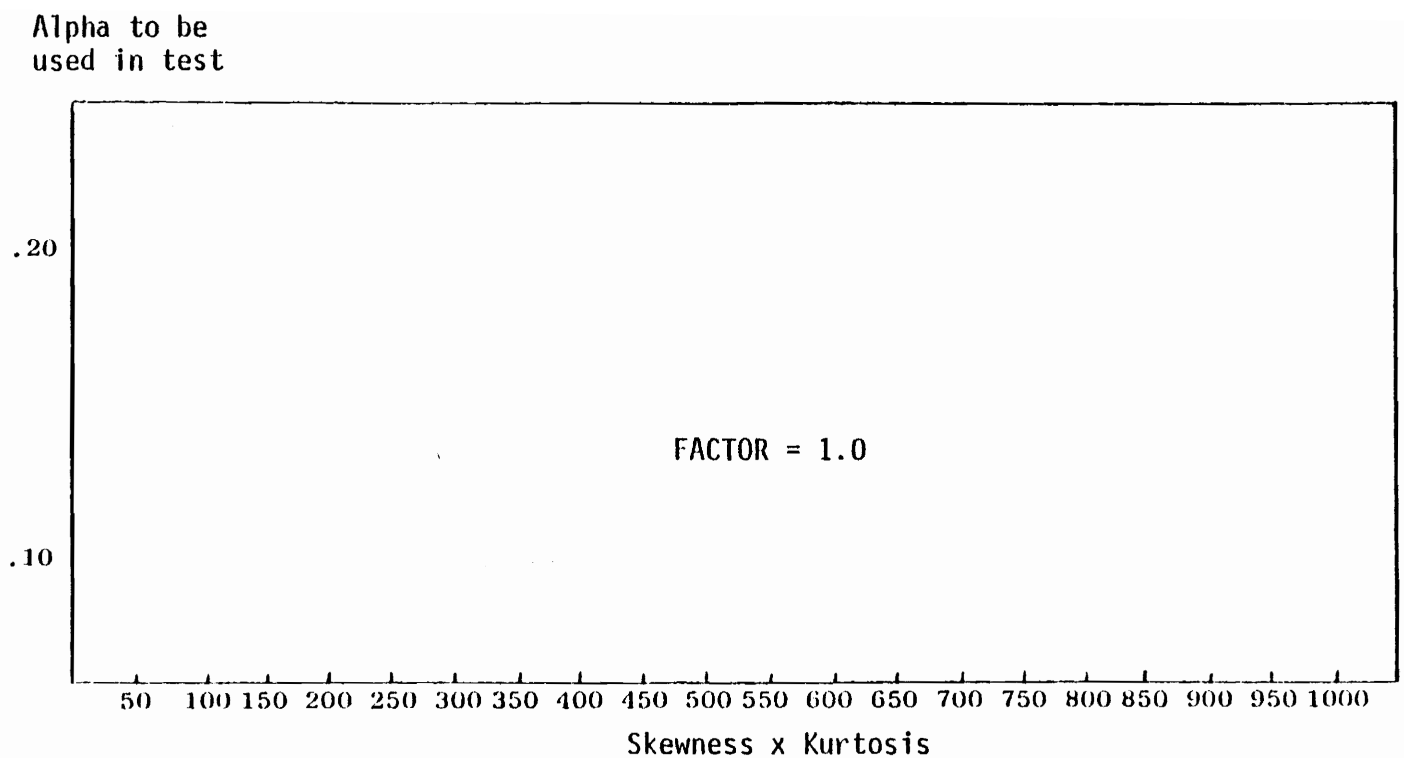

FIGURE 17

<u>UPPER BETA GUIDELINES FOR</u>
<u>β = .10 AND n = 150</u>

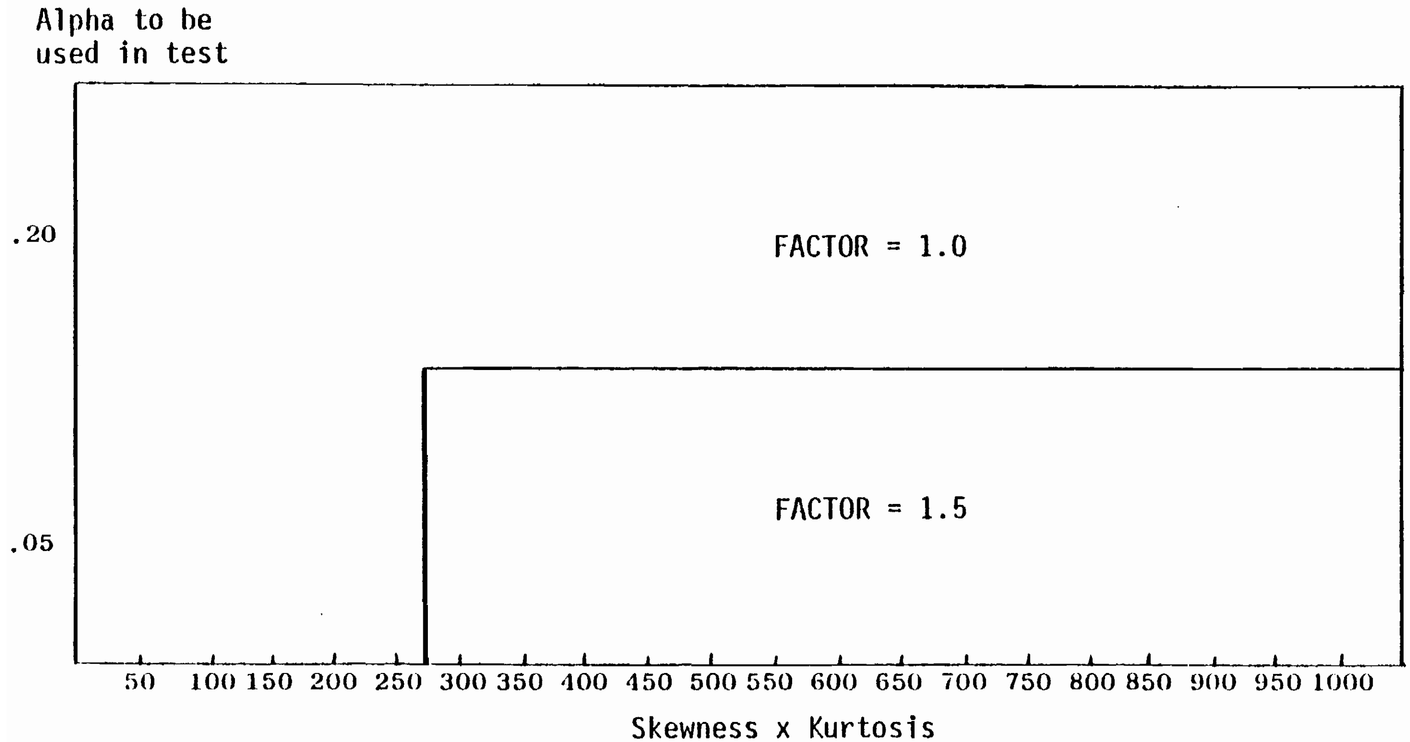

FIGURE 18

UPPER BETA GUIDELINES FOR
β = .05 AND n = 150

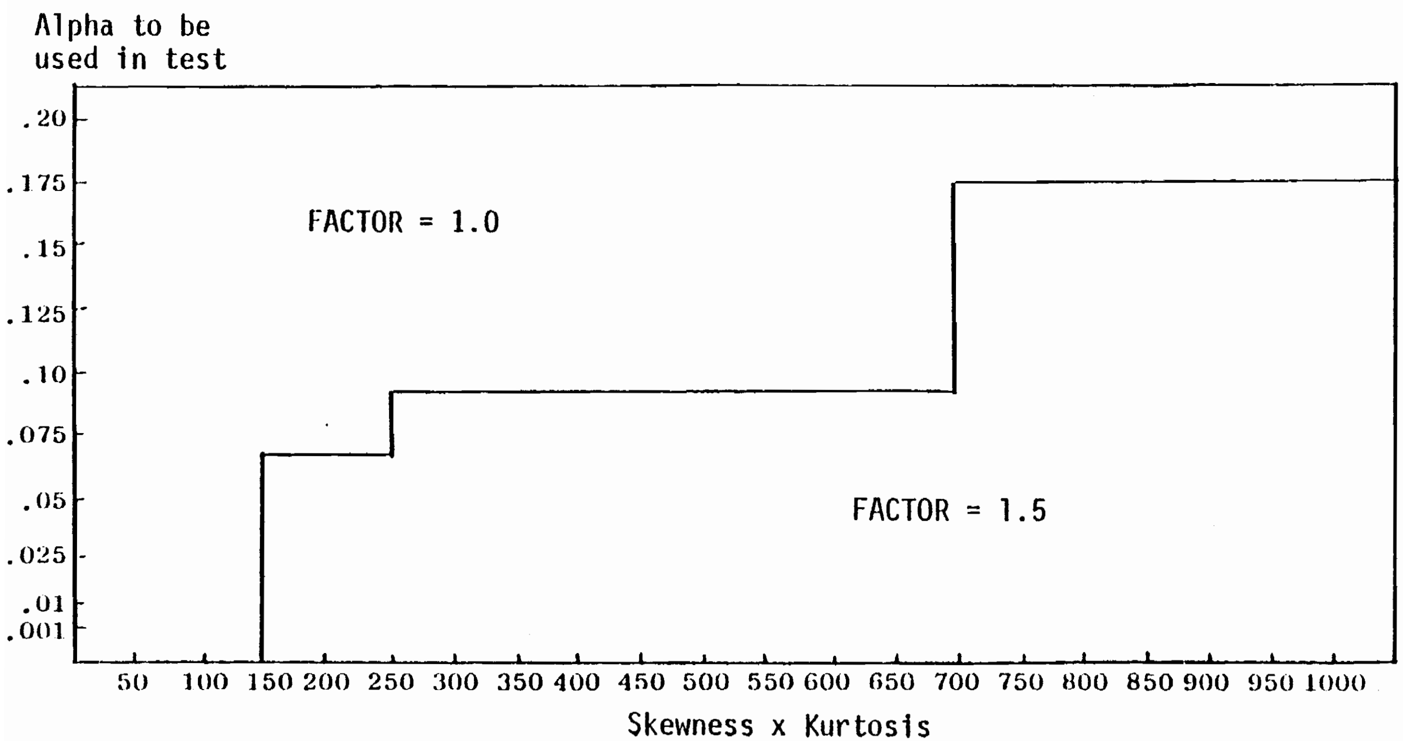

FIGURE 19

UPPER BETA GUIDELINES FOR

β = .20 AND n = 450

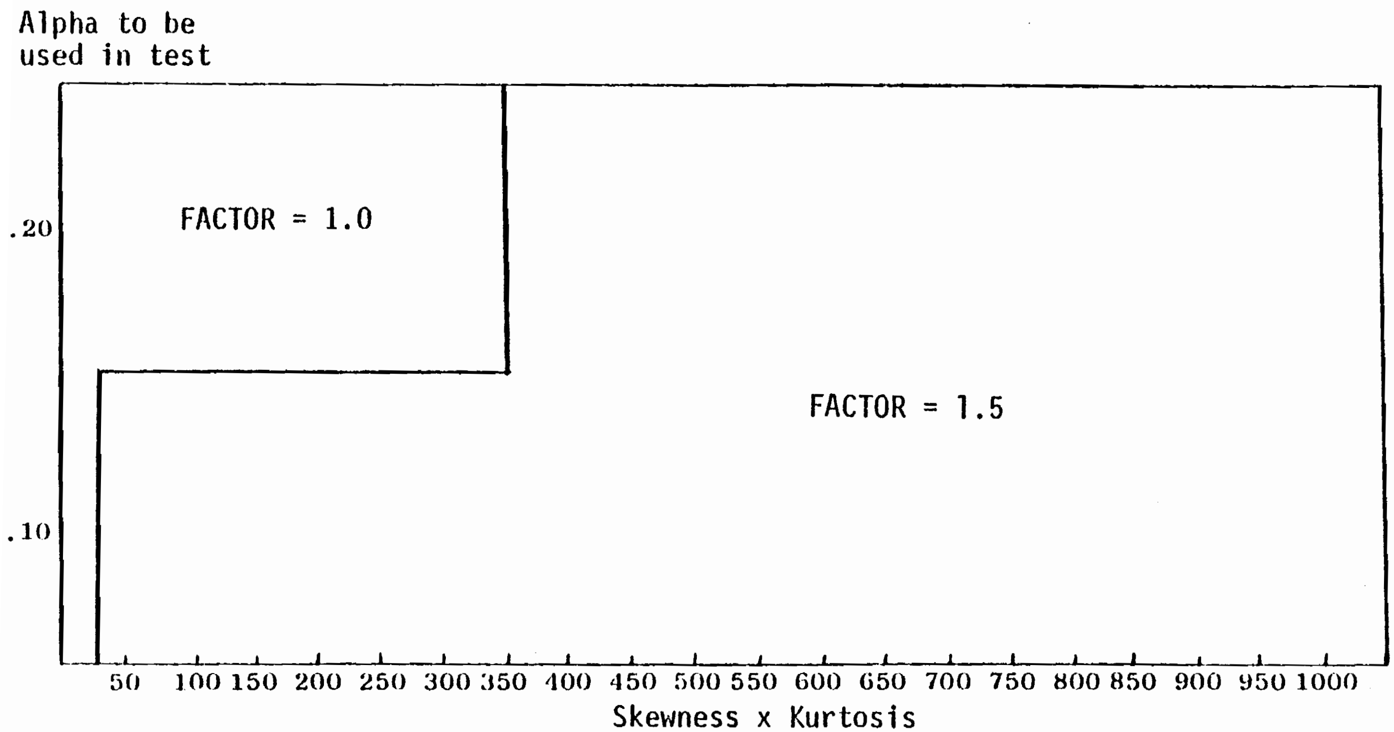

FIGURE 20

UPPER BETA GUIDELINES FOR

β = .10 AND n = 450

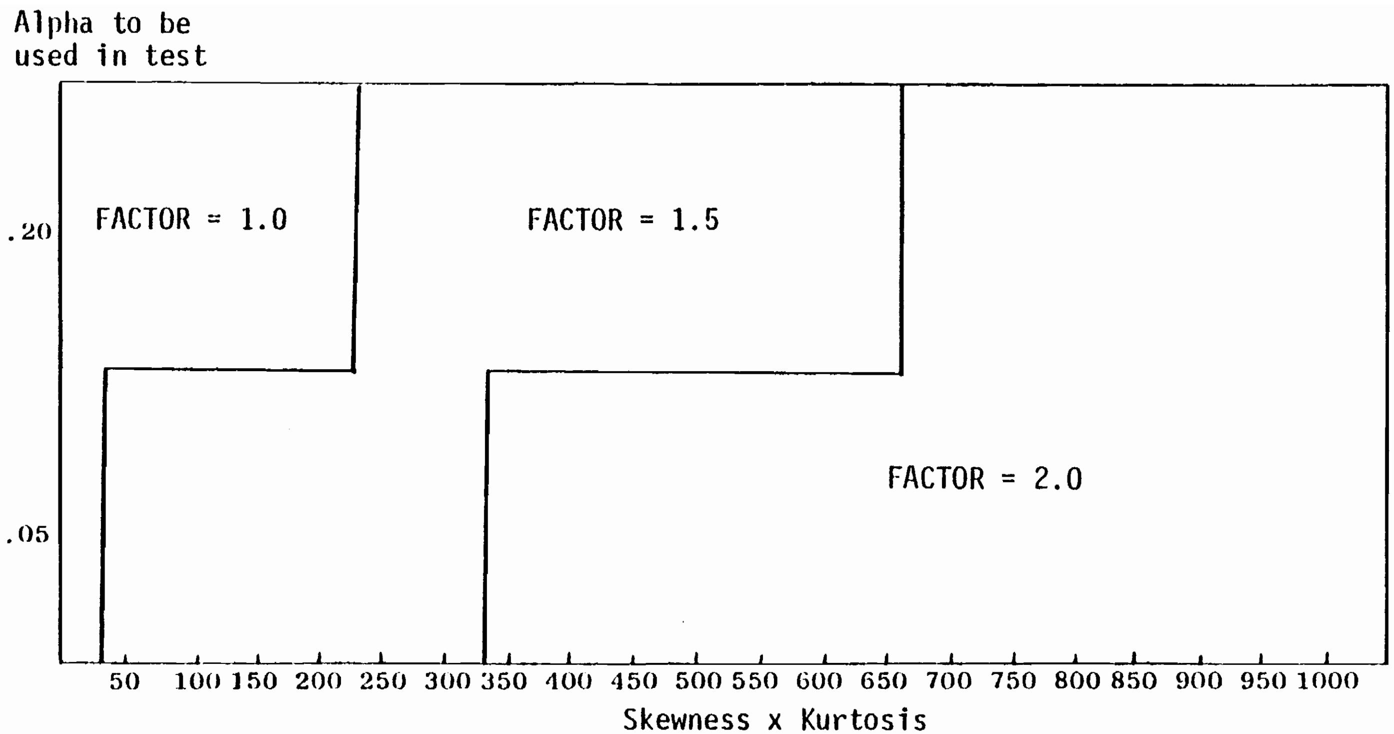

FIGURE 21

UPPER BETA GUIDELINES FOR
β = .05 AND n = 450

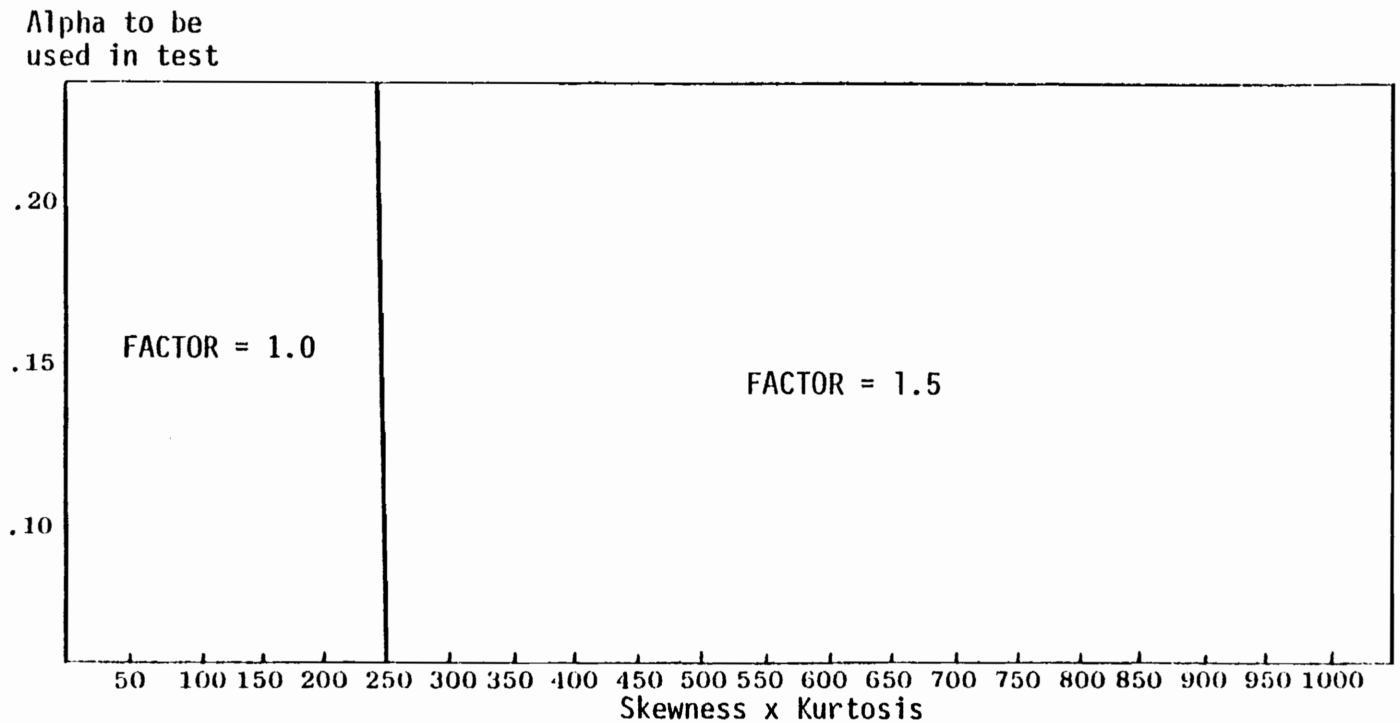

FIGURE 22

UPPER BETA GUIDELINES FOR

β = .20 AND n = 1000

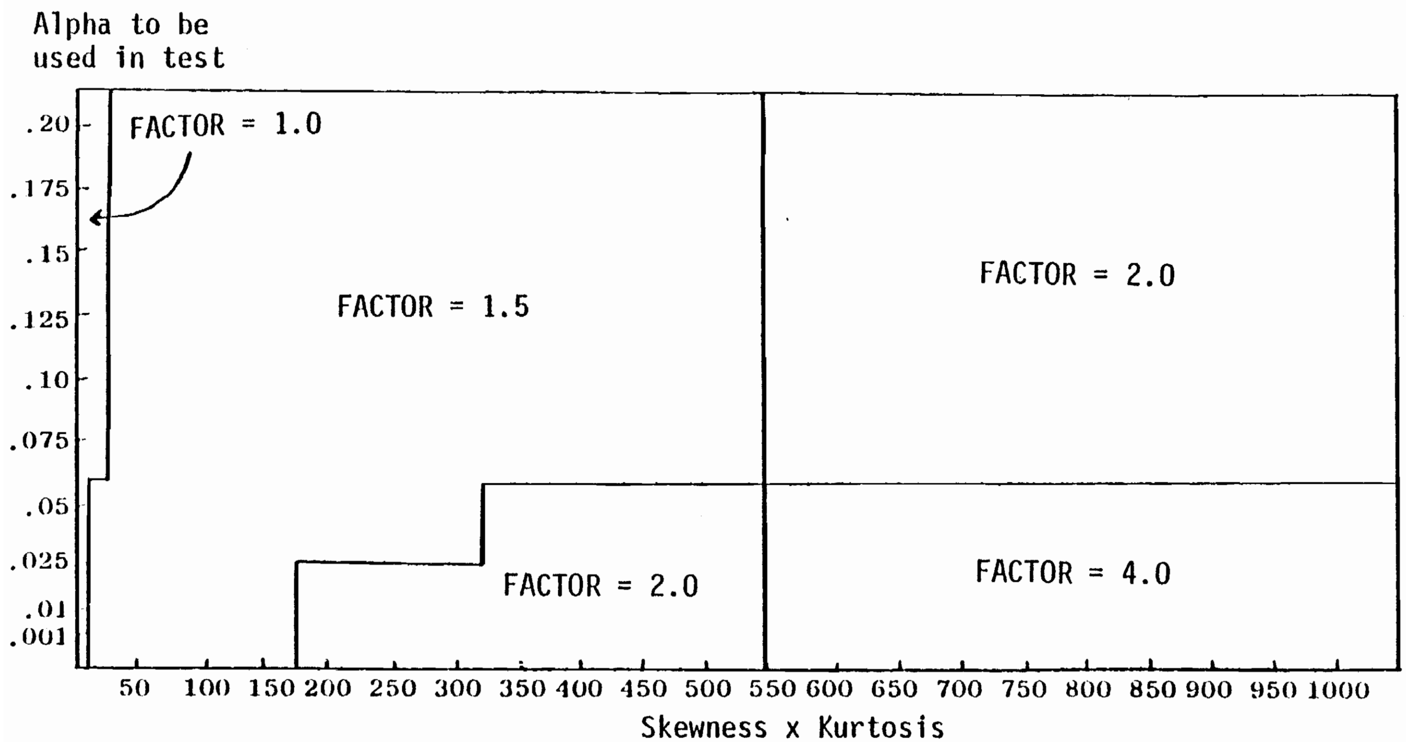

FIGURE 23

LOWER BETA GUIDELINES FOR

β = .20 AND n = 150

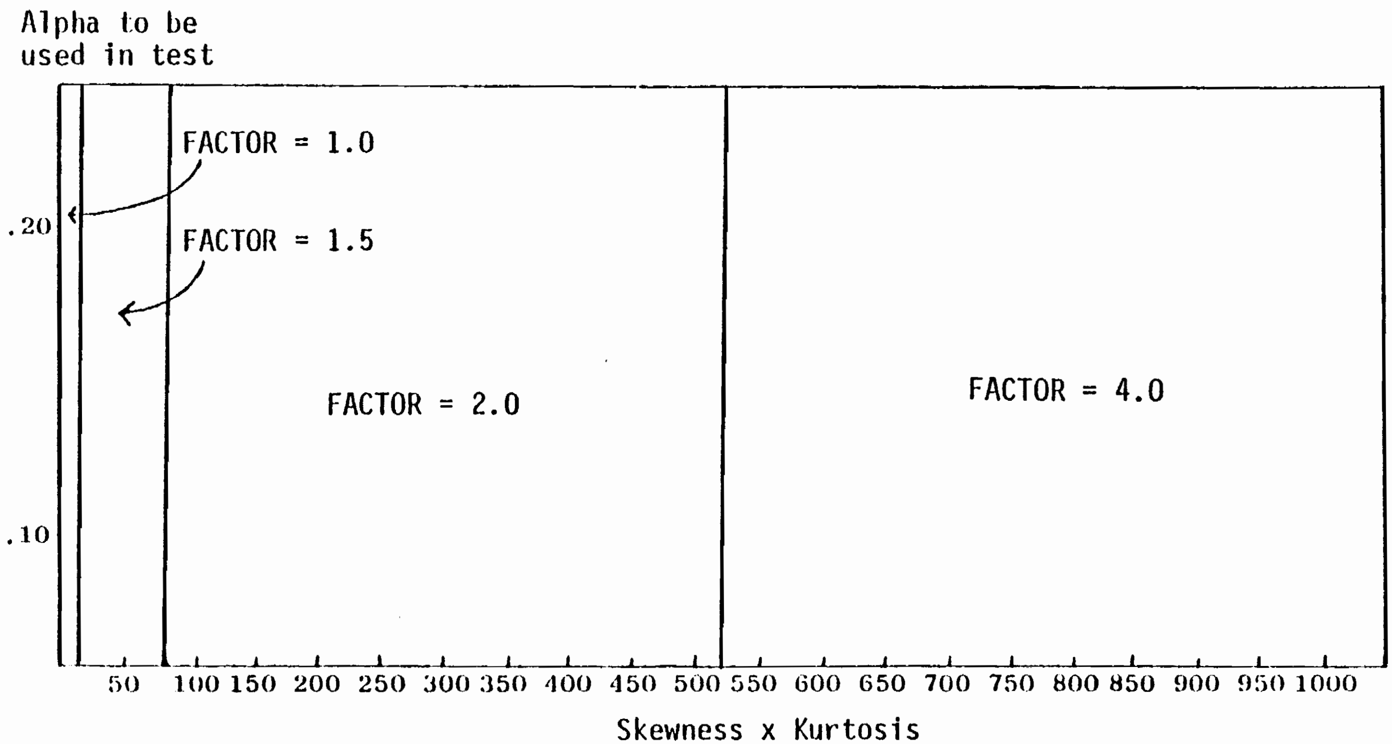

FIGURE 24

<u>LOWER BETA GUIDELINES FOR</u>
<u>β = .10 AND n = 150</u>

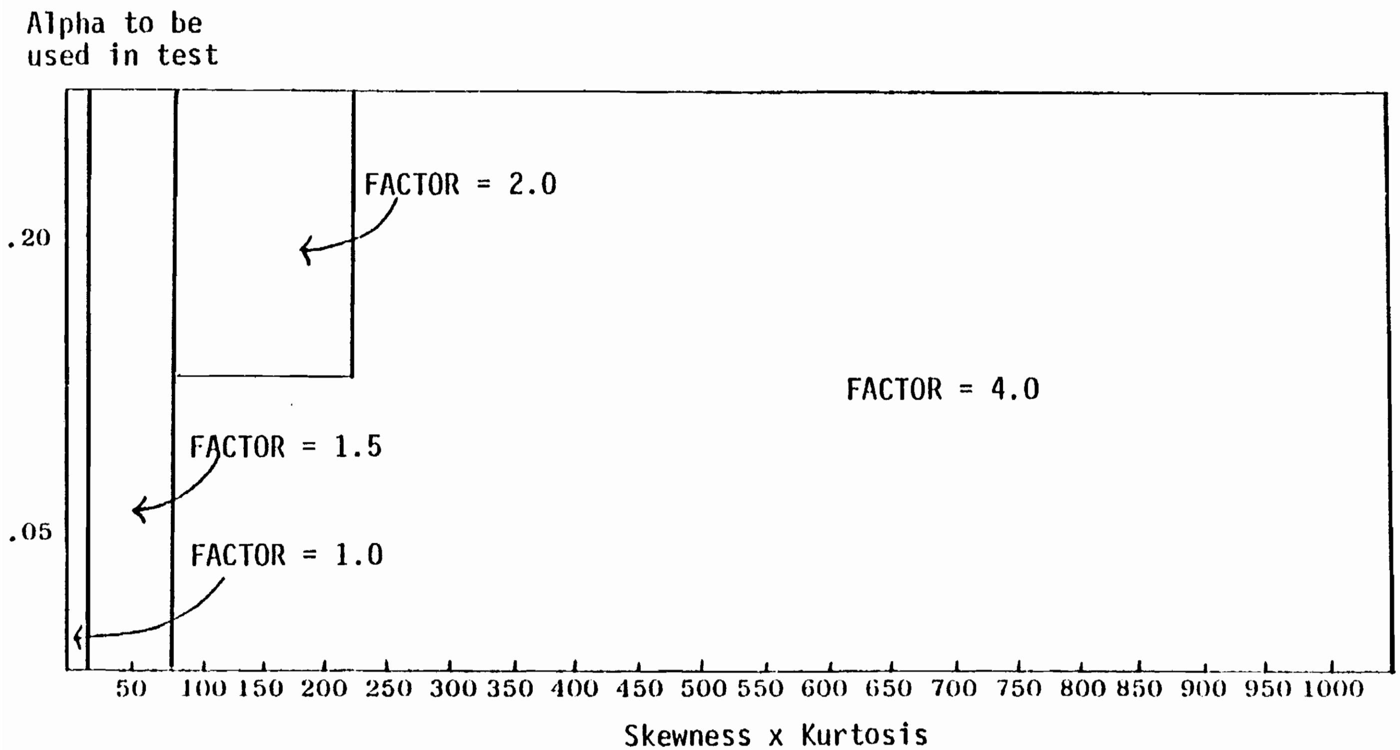

FIGURE 25

<u>LOWER BETA GUIDELINES FOR</u>

<u>β = .05 AND n = 150</u>

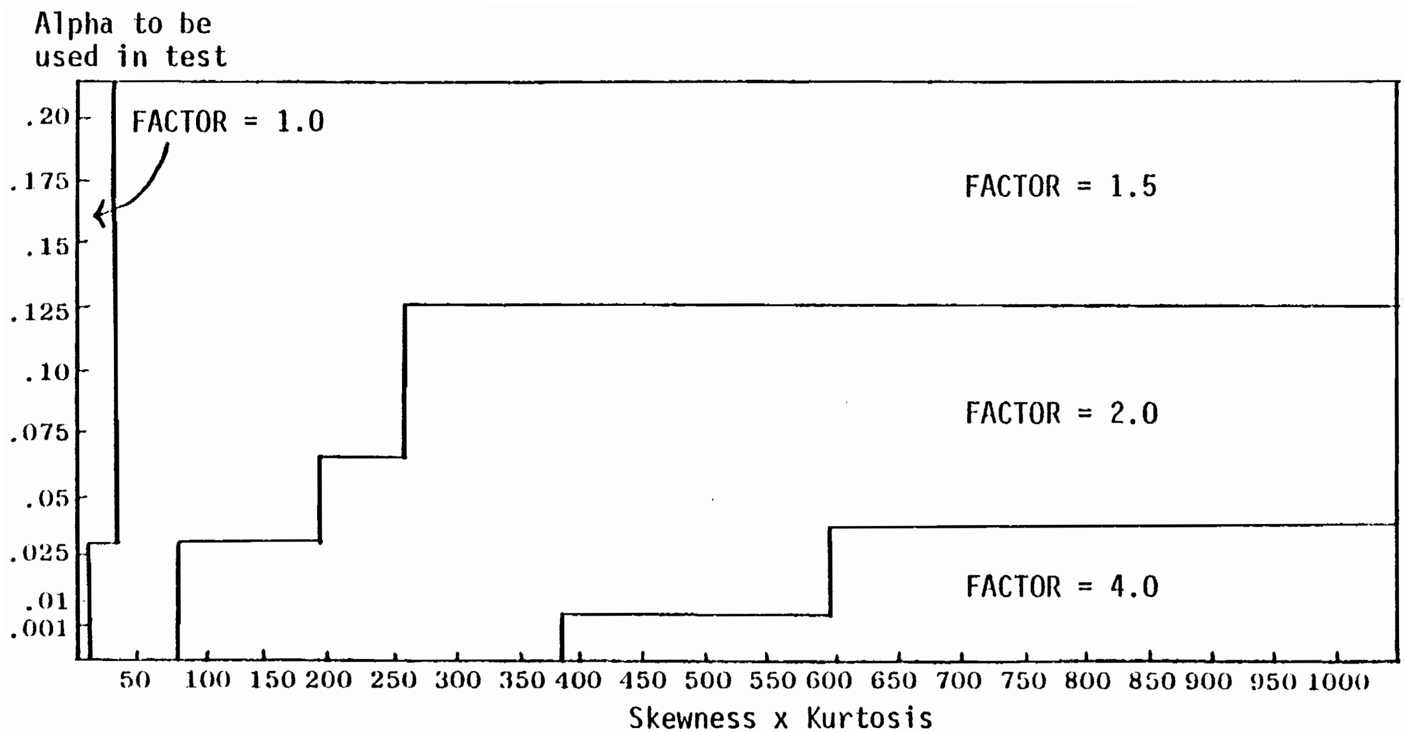

FIGURE 26

LOWER BETA GUIDELINES FOR
β = .20 AND n = 450

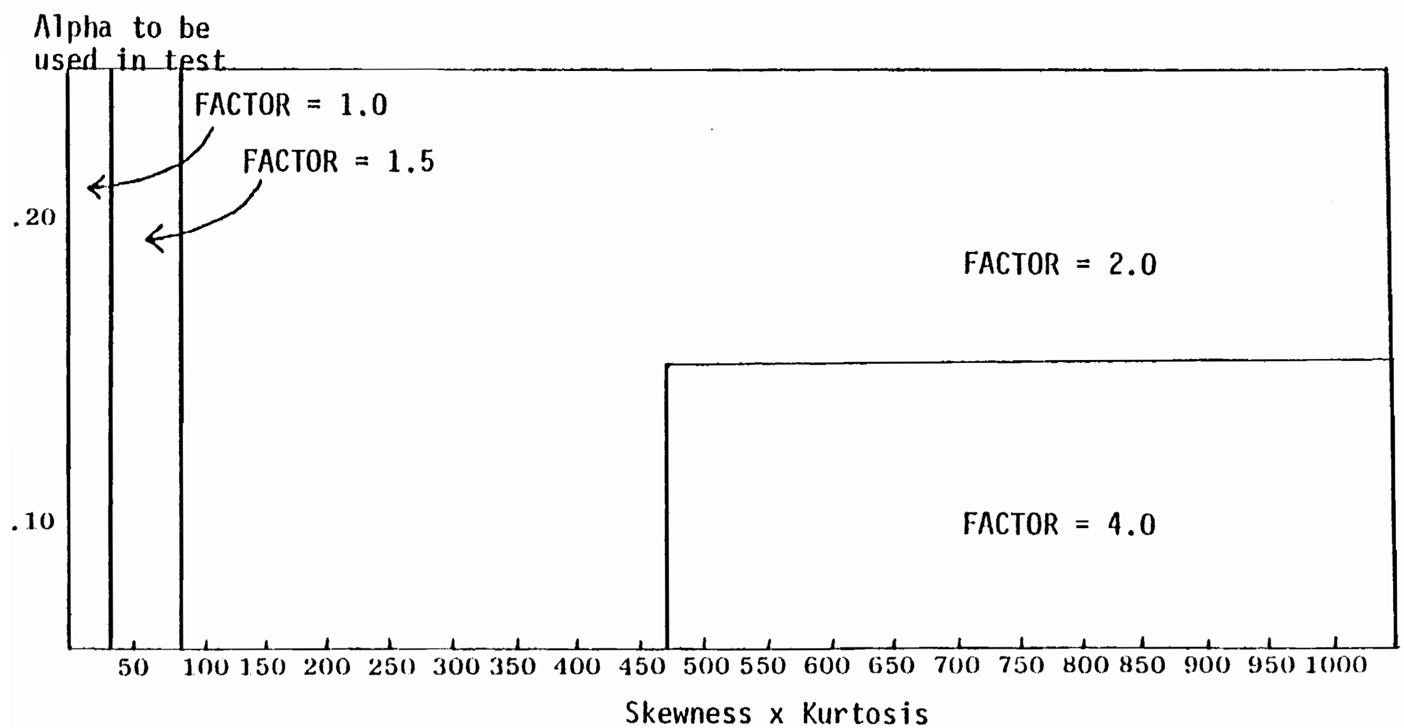

FIGURE 27

<u>LOWER BETA GUIDELINES FOR</u>

<u>β = .10 AND n =450</u>

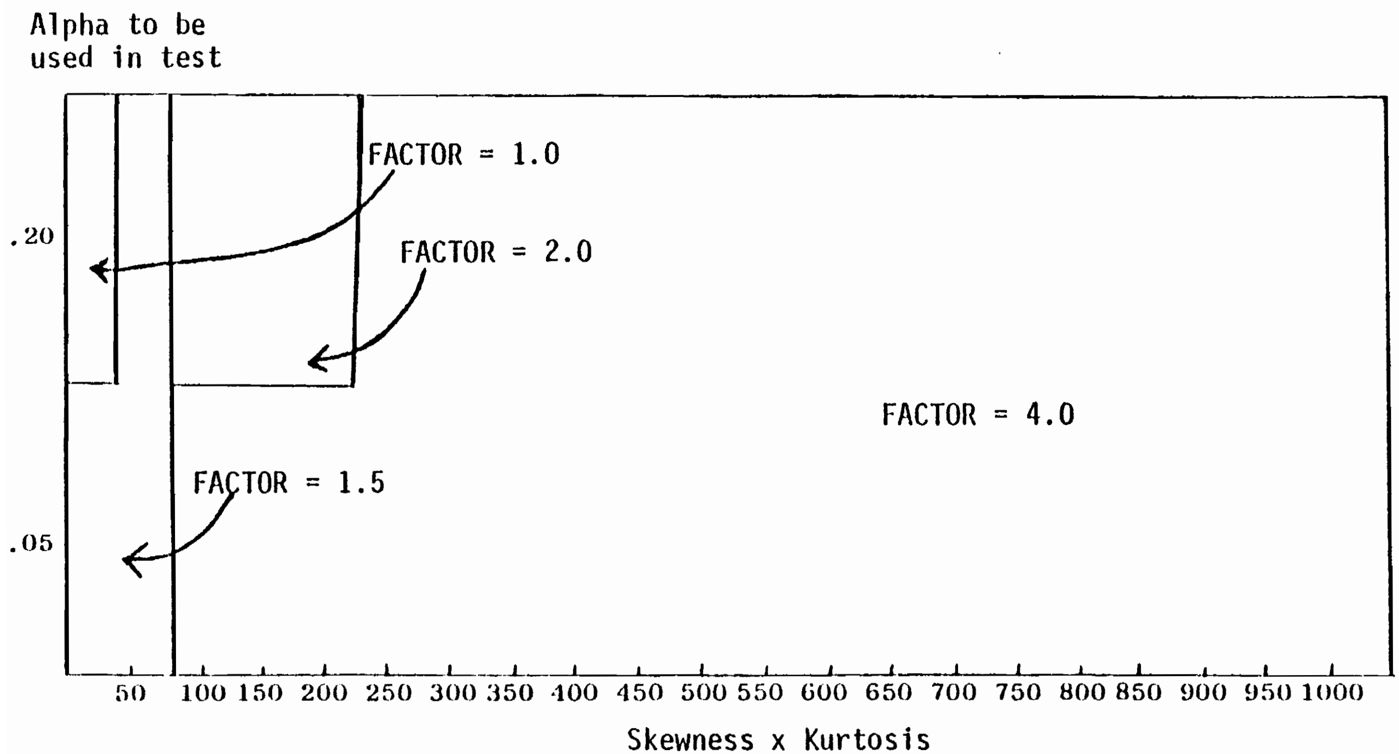

FIGURE 28

<u>LOWER BETA GUIDELINES FOR</u>
<u>β = .05 AND n = 450</u>

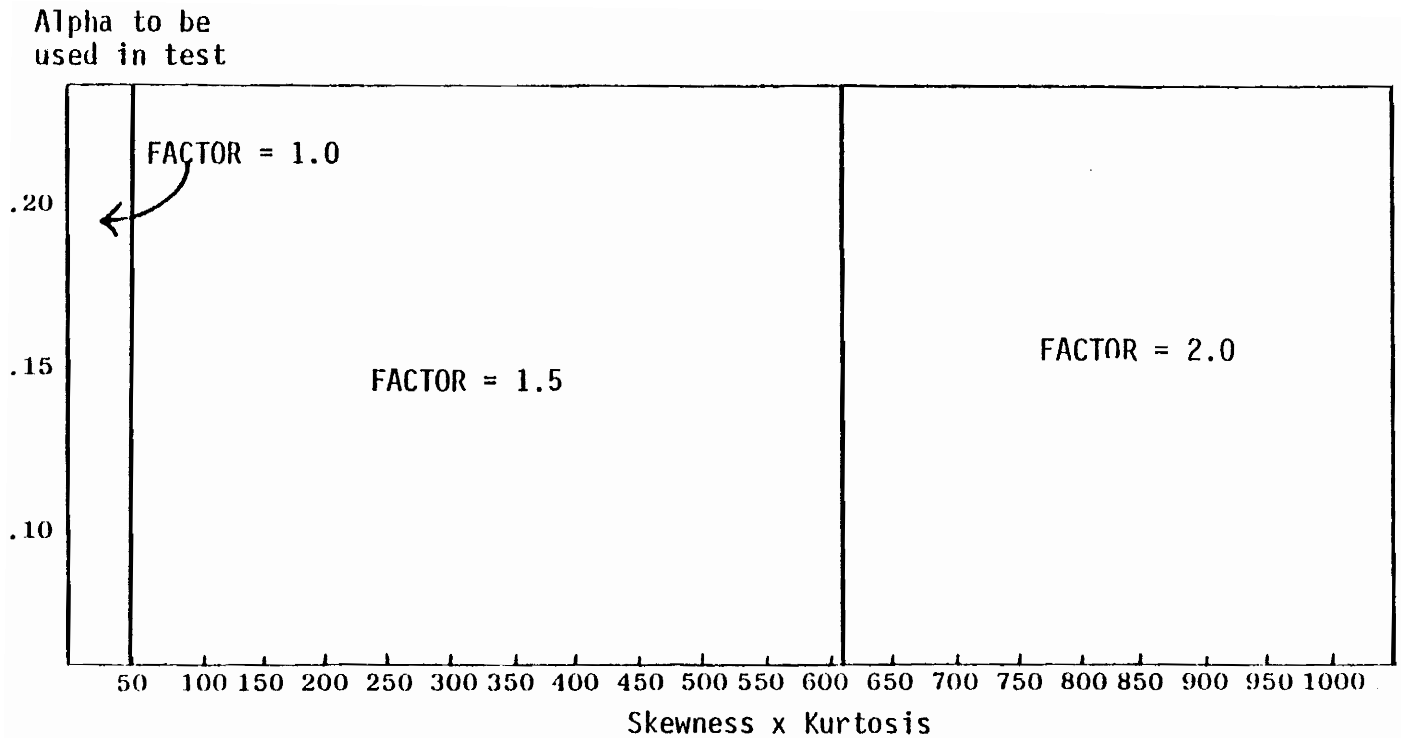

FIGURE 29

LOWER BETA GUIDELINES FOR

β = .20 AND n = 1000

Chapter VII

SUMMARY, CONCLUSIONS, AND IMPLICATIONS FOR FUTURE RESEARCH

Summaries of the investigation and of research conclusions are presented in this chapter. Limitations of the study are included in the summary of the investigation. Finally, implications for future research are discussed.

Summary of Investigation

It was hypothesized that under certain conditions of non-normality, alpha and beta risks are not effectively controlled when using the MPU estimator in conjunction with the sample size formula used in this study. A simulation study was carried out to determine the validity of this statement.

Although a normal distribution was considered, the simulation study utilized J-shaped distributions whose skewness and kurtosis ranged from .2511 and 1.9997 degrees to 25.6089 and 37.7937 degrees, respectively. On each population, computerized simulation procedures were carried out to determine how many times Type I and Type II errors occurred in 1,000 applications of the MPU sampling technique. A sample size formula which accounts for both the alpha and beta risks was used in the procedures to determine if control of the two sampling risks was effective under the specified conditions of non-normality. Control of a sampling risk was considered effective if the proportion of times the error occurred in the 1,000 applications equaled the specified alpha or beta level. Control was also considered effective if the specified level was within plus and minus three standard errors of the proportion. Furthermore, the beta risk considered both possible overstatement and understatement of the population. When the stated book value is materially overstated, errors are referred to as Type II on the upper side. Type II errors on the lower side refer to the situation where the stated book value is understated.

For a given combination of alpha and beta levels, the MPU sampling technique was applied 1,000 times to each population starting with the normal distribution and ending with the population having the highest degree of skewness and kurtosis. These procedures yield a series of proportions for Type I and Type II errors which form the basis of the vertical trends referred to in the research conclusions. In addition to investigating control of the alpha and beta levels using the sample size as computed from the formula, the same simulation procedures were also carried out using sample sizes that were .5, 1.5 and 2.0 times the formula-produced sample size.

After completing these simulation procedures on a series of populations using a certain combination of alpha and beta, the procedures were repeated for a different set of alpha and beta levels.

The proportions for the sampling errors occurring in the stimulation procedures provided a foundation for the development of alpha and beta guidelines. The guidelines are structured so that specified alpha and beta levels can be simultaneously attained under the conditions of non-normality simulated in the research and when using the research sample size formula in conjunction with the MPU estimator.

Limitations of the Study

Some of the limitations center around the distributions used in the research. The characteristics of the populations, with the exception of the normal population, were J-shaped distributions within a specified range of skewness and kurtosis. The study focused on this type of distribution because these characteristics describe real auditing populations and because no other types of distributions have been identified as characteristic of auditing populations.

Furthermore, infinite populations were created as opposed to finite distributions. Infinite populations were used so that research results would be generalizable to all populations having those characteristics defined in the study.

Three limitations of the study involve economic factors as well as time constraints of the researcher. Because computer time and funds, and researcher time, are limited resources, the study had to be kept within reasonable limits. First, only twenty-two populations were used out of an infinite number of possible distributions. It was felt that twenty-two populations would be sufficient to carry out the research procedures, particularly in view of the fact that the range in degrees of skewness and kurtosis covered the desired amount of non-normality. Second, alpha and beta levels are restricted to eleven combinations. Although the levels used in the research appear compatible with levels suggested in the literature, there are many more potential combinations of alpha and beta risk levels. Third, only three standardized sample sizes were used in the investigation and guideline development. Furthermore, there was limited involvement of one sample size.

With the possible addition of more alpha-beta combinations and/or sample sizes and/or populations, the potential number of simulation runs and resulting sets of data would expand at a very fast rate. Computer time and funds, as well as researcher time, would increase accordingly with such additional variables.

Some of the limitations of the study suggest topics for future research. Before discussing these suggestions, a summary of the research conclusions is provided.

Summary of Research Conclusions

The research results demonstrate that specified alpha and beta levels are not always attained when using the MPU estimator under certain conditions of non-normality. The results indicate particular trends in the attainment of the three risk levels--alpha, upper beta, lower beta--as the populations become more skewed and more peaked. Comments concerning these trends are made in the following sections.

Alpha Conclusions and Recommendations

As population distributions get farther and farther from normality, the discrepancy between the specified and attained alpha levels becomes larger and larger. As the sample sizes get bigger, however, the trend becomes less pronounced. The results also demonstrate that doubling the sample size, and in some instances quadrupling the sample size, does not always culminate in the attainment of the specified alpha level. Furthermore, the attainment of the alpha level is independent of the level specified for beta.

From these research results concerning the attainment of the alpha level, alpha guidelines were developed. The guidelines use as inputs (1) the product of the population skewness and kurtosis, (2) the desired alpha level and (3) the sample size. The guidelines determine the level of alpha to be used in the test so that the effective alpha level is consistent with the specified alpha level. Generally, the alpha guidelines will recommend an empirical alpha level that is lower than the desired level.

Upper Beta Conclusions and Recommendations

The vertical and horizontal trends in the upper beta proportions depend on the sample size. When the standardized sample size of 150 units is utilized, the vertical trend for the 1.0 multiple sample size shows improvement in the proportions of upper beta errors as the distributions become more and more skewed and peaked. When the 450 standardized sample size is utilized, the vertical trend for the sample size of 1.0 times the basic size reflect increasing proportions as the distributions move away from normality. However, when considering sample sizes that are 1.5 and 2.0 times the standardized sizes, the vertical trends reflect increasing proportions. Horizontally, going from a 1.0 multiple sample size to a 1.5 multiple and to a 2.0 multiple, the proportions of beta errors improve.

The simulation results reflect that the upper beta level can be attained by increasing the sample size by factors of 1.5 or 2.0 times. The upper beta guidelines indicate a sample size factor to be applied to the sample size computed from the formula after considering the alpha guidelines.

The upper beta guidelines use as inputs (1) the desired beta level, (2) sample size, (3) the product of population skewness and kurtosis, and (4) the level of alpha to be used in the test. If the sample size is increased too much when applying the recommended beta factor, the attained alpha level may be smaller than the desired alpha level. To remedy this situation the recommended empirical alpha level is then raised, thereby reducing the sample size. The beta guidelines are consulted again for the "new" factor to be applied to the sample size. These procedures and the final sample size are designed to attain a specified level of alpha and beta simultaneously. The lower beta guidelines follow the same procedures and techniques as the upper beta guidelines. The trends for the proportions of lower beta errors, however, differ from the trends for the upper beta errors.

Lower Beta Conclusions and Recommendations

The vertical trends demonstrate increasing proportions for the lower beta errors as the population distributions have more and more skewness and kurtosis. This type of trend is reflected in the simulation results for all the standardized sample sizes of 150, 450, and 1,000 units. The horizontal trends, as sample sizes increase for a given distribution, reflect improvement in the proportions of lower beta errors. Dependent upon the degrees of population skewness and kurtosis and upon the level of alpha used in the test, simulation results demonstrate that the specified beta level can be attained by increasing the sample size by a factor of 1.5, 2.0, or 4.0 times. Although the trends for the lower beta proportions differ from the trends for the upper beta proportions, the procedures for using the guidelines are the same.

Implications for Future Research

Implications for future research involves two different areas. First, research results should raise questions concerning statistical sampling techniques used in auditing where underlying populations have non-normal distributions and where the sampling techniques are based on normality. It appears that investigations into those areas are warranted.

Second, limitations of the study and conditions for implementation suggest areas for future research.

Since the research and guidelines utilize a limited number of alpha and beta levels and their combinations, the levels may not meet the needs of an auditor in certain circumstances. Therefore, future research could be carried out using additional risk levels for alpha and beta.

Subsequent research could utilize sample sizes other than the sample sizes used in the research. In a practical situation the closer the first sample size approximates one of the sample sizes used in the guidelines, the more effective will be guideline utilization.

Since the current research utilized J-shaped distributions, successful implementation of the guidelines necessitates that auditing populations have similar characteristics. Although four populations were empirically identified as having this type of distribution, it may be that additional types of distributions in the audit environment need to be identified so that ensuing research includes such distributions.

Furthermore, future research could attempt to simplify identification of the shape and characteristics of auditing populations. Currently, the auditor could use a computer program to determine population skewness and kurtosis. Perhaps subsequent research could focus on a preliminary sample, or some other scheme, to estimate population skewness and kurtosis.

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APPENDIX I

Definition of Terms

APPENDIX I

Definition of Terms

Alpha risk: The alpha risk, α, is the probability of erroneously rejecting an auditing population when the population parameter is properly stated. The alpha risk can be set at a level that appears appropriate in the circumstances. The confidence level (reliability level) is defined as $(1 - \alpha)$.

Attribute sampling: Attribute sampling is used to estimate the frequency of occurrence of some event and answers the question how many. This type of sampling uses mutually exclusive categories where the event of interest falls into only one category. Consequently, the techniques lend themselves to proportional analyses.

Beta risk: The beta risk, β, is the probability of erroneously accepting an auditing population when the population parameter is materially misstated. The beta risk can be set at a level that appears appropriate in the circumstances.

Kurtosis: The absolute measure of kurtosis in a population is given by the fourth moment about the mean. The fourth moment can be expressed as

$$M_4 = \frac{\sum_{i=1}^{N} (X - \mu)^4}{N}$$

where X = individual observation,

μ = population mean,

N = number of observations in the population.

The relative measure of kurtosis is the ratio of the fourth moment to the second moment squared and is generally referred to as β_2 (beta two). This relative measure of kurtosis can be expressed as

$$\beta_2 = \frac{M_4}{M_2^2}$$

For a normally distributed population, β_2 will equal three. A peaked distribution will show a β_2 value greater than three. A flat topped or "U-shaped" distribution will show a value less than three.

<u>Mean</u>: The mean is an average of a set of numbers obtained by dividing the sum of the numbers by the quantity in the set. The mean of a population can be expressed as

$$\mu = \frac{\sum_{i=1}^{N} X}{N}$$

Precision: Sampling precision is a "measure of closeness of a sample estimate to the corresponding population characteristic. It is a probabilistic measure in that the precision of an estimate... can only be made for a specified reliability."[1] The precision of a sample average can be expressed as follows:

$$P = Z_{\alpha/2} s / \sqrt{n}$$

where P = amount of precision,

$Z_{\alpha/2}$ = a factor determined by the confidence level and found in a normal table,

s = standard deviation of the sample as an estimate of the population standard deviation,

n = sample size.

Then, the precision interval (confidence interval) of a sample mean goes from $(\bar{y}-P)$ to $(\bar{y}+P)$ when $\bar{y}$ represents the sample mean. The probability that the precision interval will include the true population mean will be the confidence level with which the statistical procedures are carried out.

Reliability: Reliability is the confidence with which we can state that the proportion of sample means, from all possible samples of the same size from the given population, will lie within the

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[1] Ernst & Ernst, Audit Sampling, 1976, p. 179.

confidence (precision) interval. This reliability level is set by the auditor depending upon the circumstances.

Sampling distribution: A sampling distribution is a probability distribution of a statistic computed from samples of the same size taken one at a time from a specified population.

Skewness: An absolute measure of skewness in a population is given by the third moment about the mean. The third moment can be expressed as

$$M_3 = \frac{\sum_{i=1}^{N} (X - \mu)^3}{N}$$

The third moment represents the arithmetic mean of the deviations about the mean cubed. The current research will use a relative measure of skewness which is the ratio of the third moment squared to the second moment cubed and is generally referred to as β_1 (beta one). This relative measure of skewness can be expressed as

$$\beta_1 = \frac{M_3^2}{M_2^3}$$

If β_1 = zero, the population distribution will be symetrical. If β_1 is a positive figure the distribution will be positively skewed. If β_1 is negative the distribution will be negatively skewed.

Variables sampling: Variables sampling is used to estimate an average or total amount, and answers the question how much. This type of estimation provides the auditor with a confidence interval for the average or total value rather than a point estimate for the population. It is often used to estimate the reasonableness of an account balance, to estimate an unknown account balance, or it can be used to estimate the average or total amount of certain errors in the population.

Variance: The variance of a population is a measure of dispersion or variability among the members. It is an average of the square of the differences between a member and the population mean. The variance of a population may be expressed as follows:

$$\sigma^2 = \frac{\sum_{i=1}^{N} (X - \mu)^2}{N}$$

APPENDIX II

Distributions of the Research and Demonstration Populations

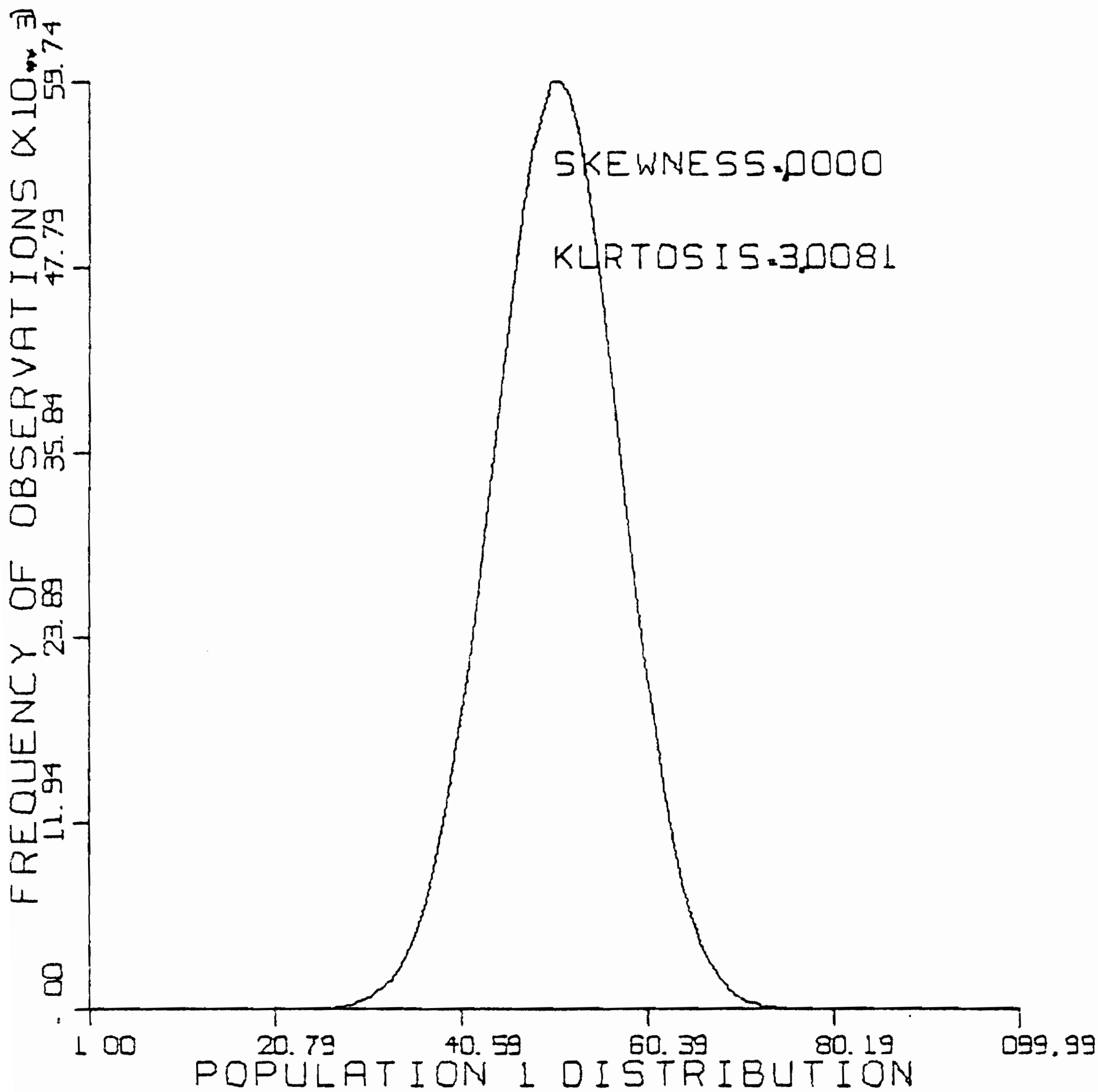


FIGURE 30

RESEARCH DISTRIBUTION 1

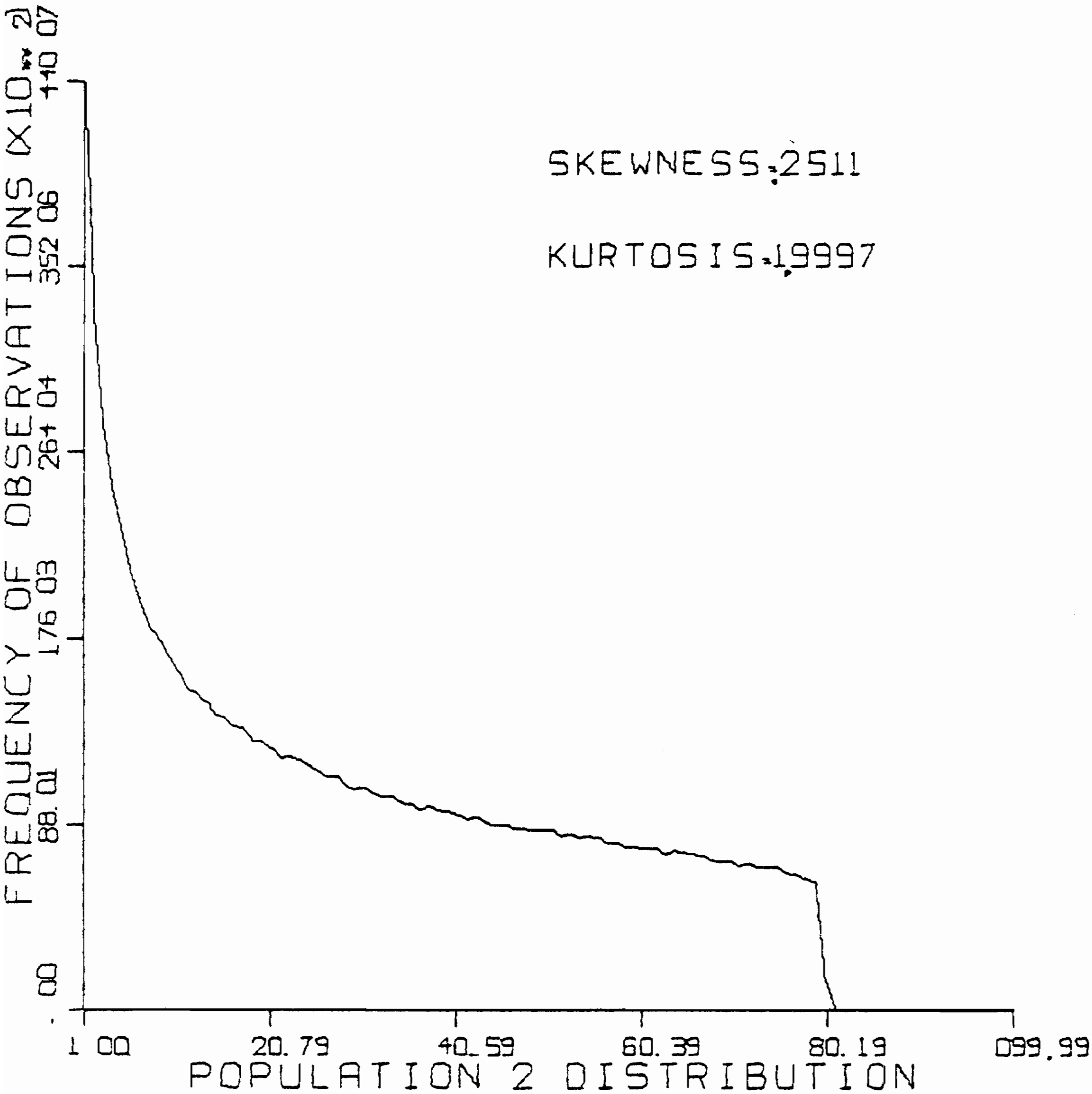


FIGURE 31

RESEARCH DISTRIBUTION 2

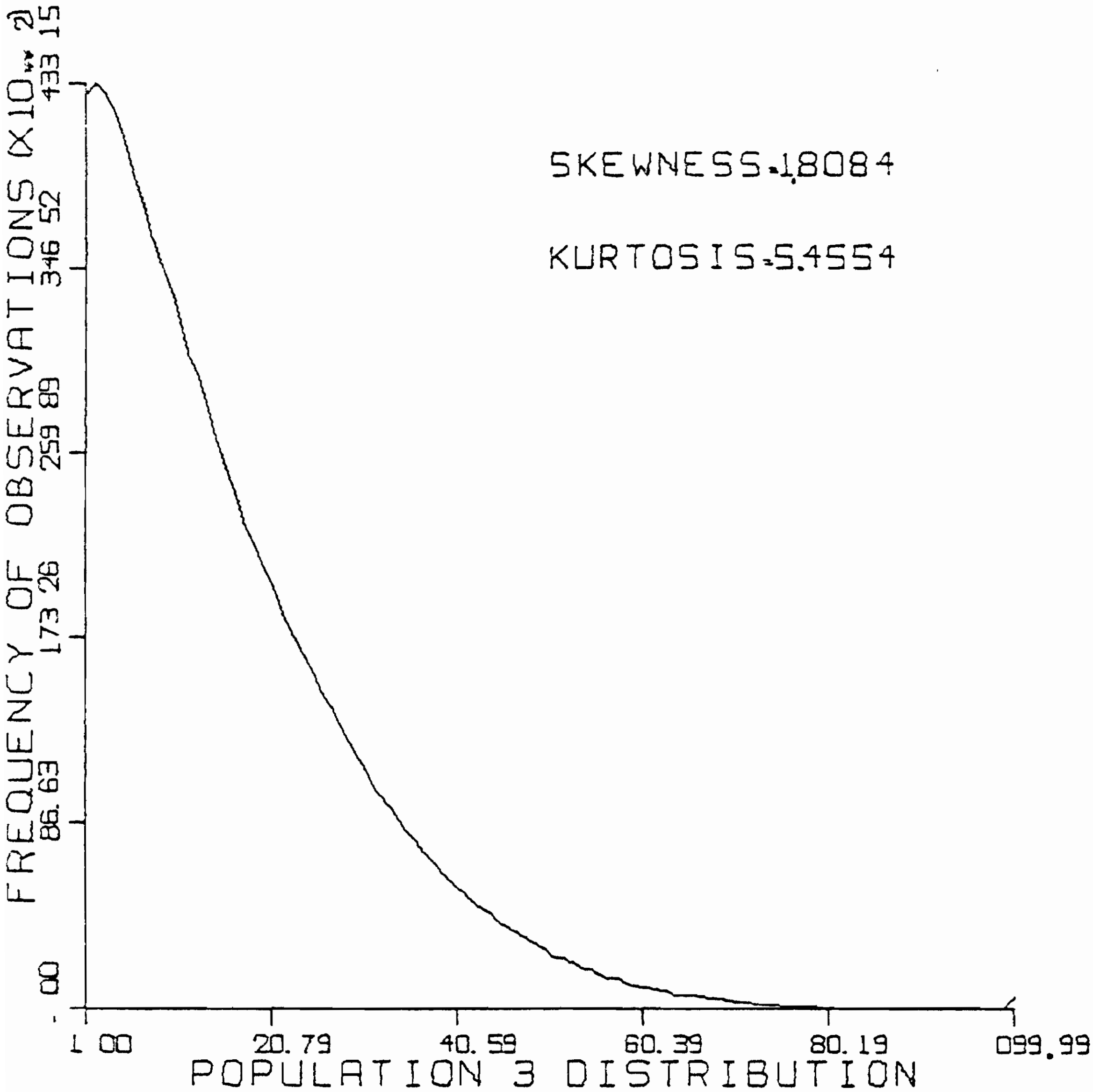


FIGURE 32

RESEARCH DISTRIBUTION 3

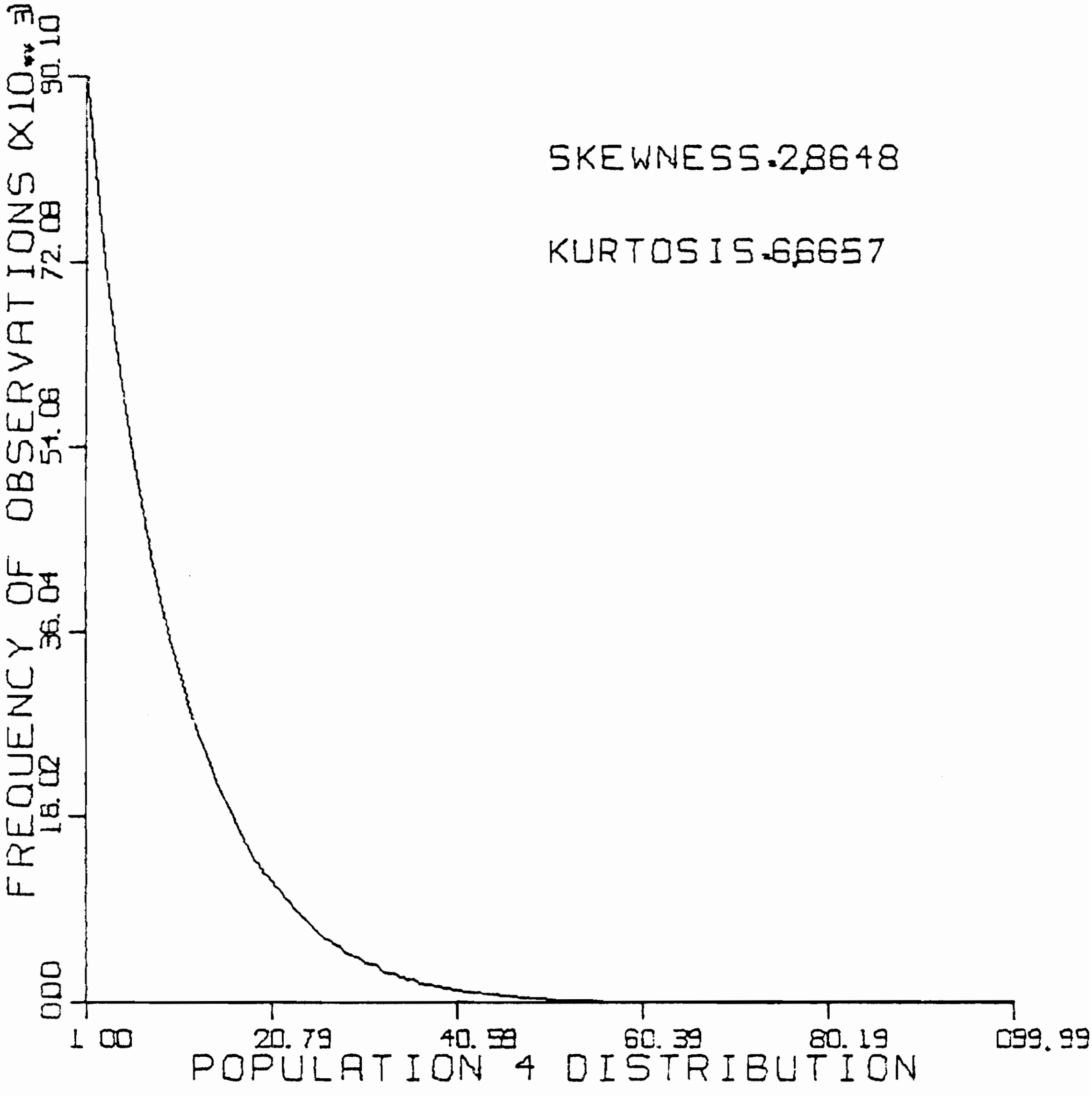


FIGURE 33

RESEARCH DISTRIBUTION 4

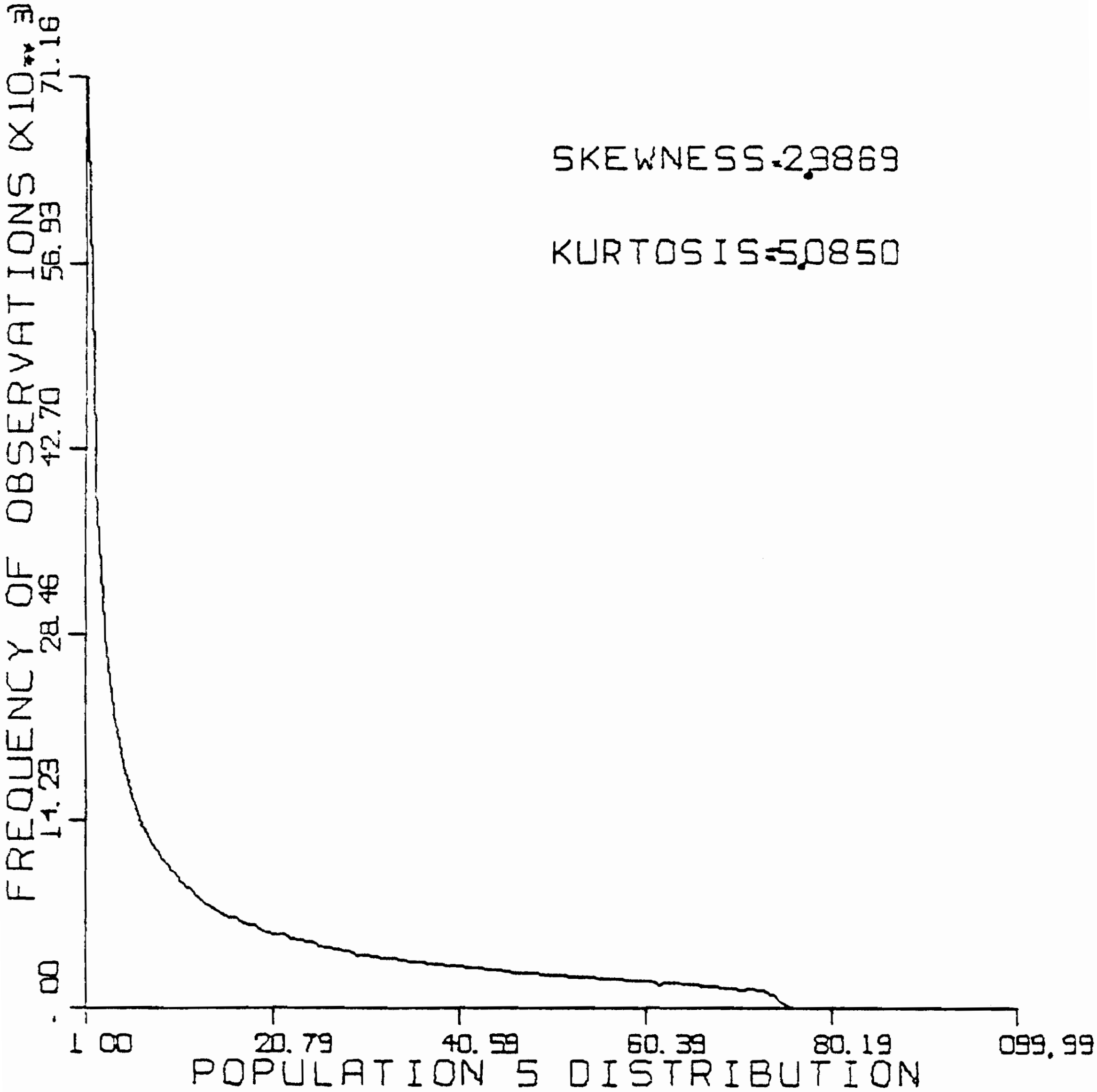


FIGURE 34

RESEARCH DISTRIBUTION 5

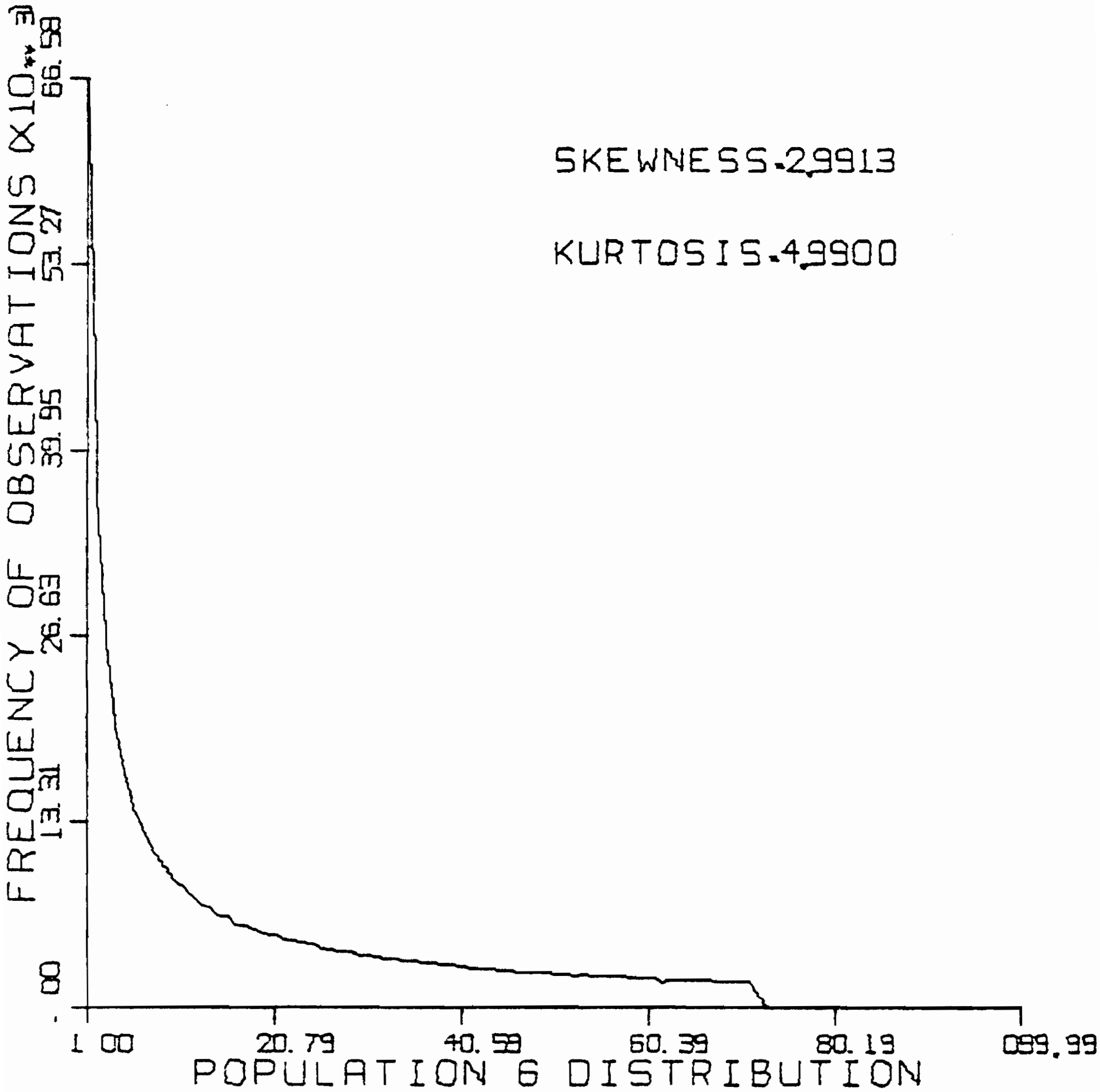


FIGURE 35

RESEARCH DISTRIBUTION 6

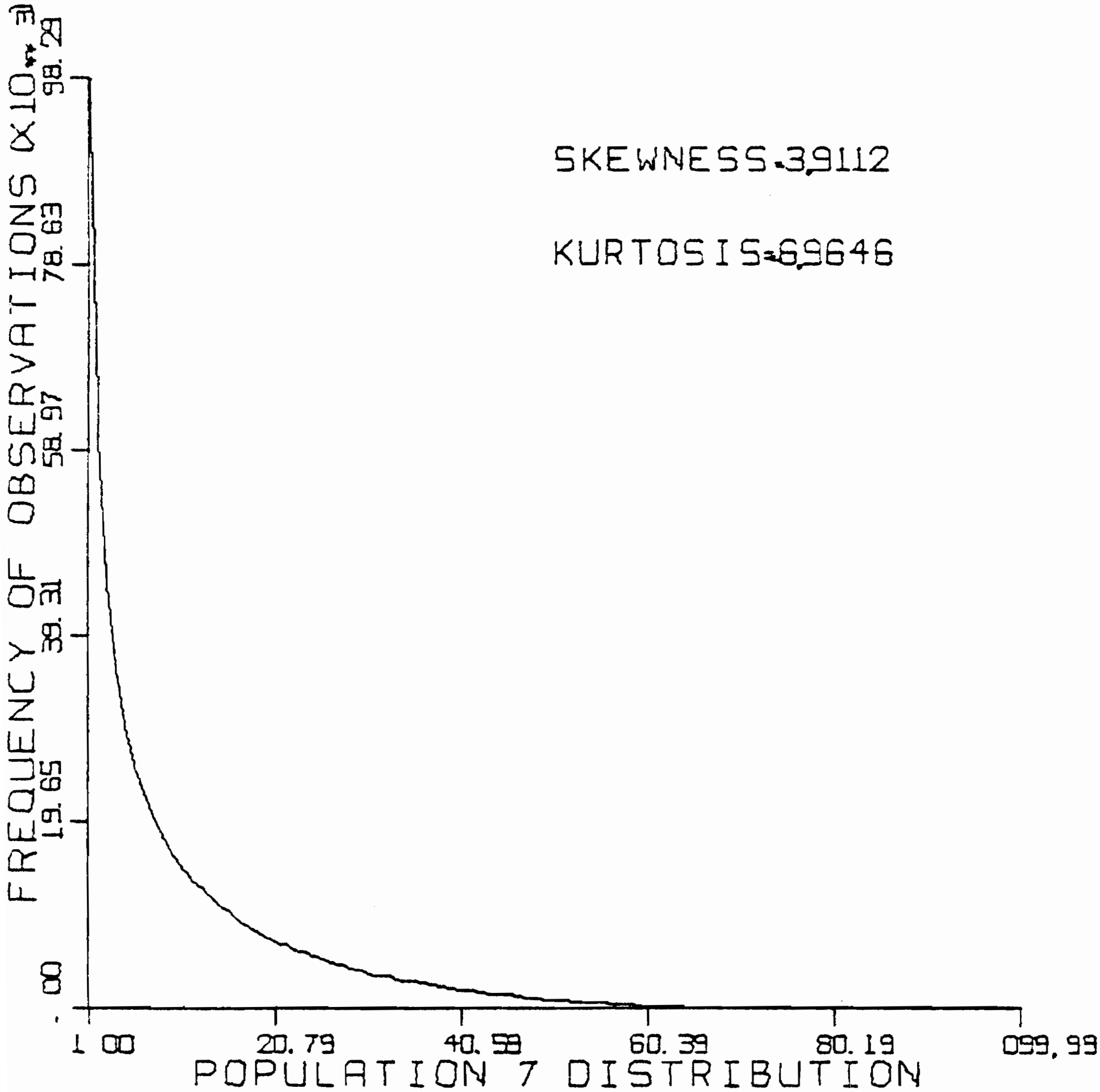


FIGURE 36

RESEARCH DISTRIBUTION 7

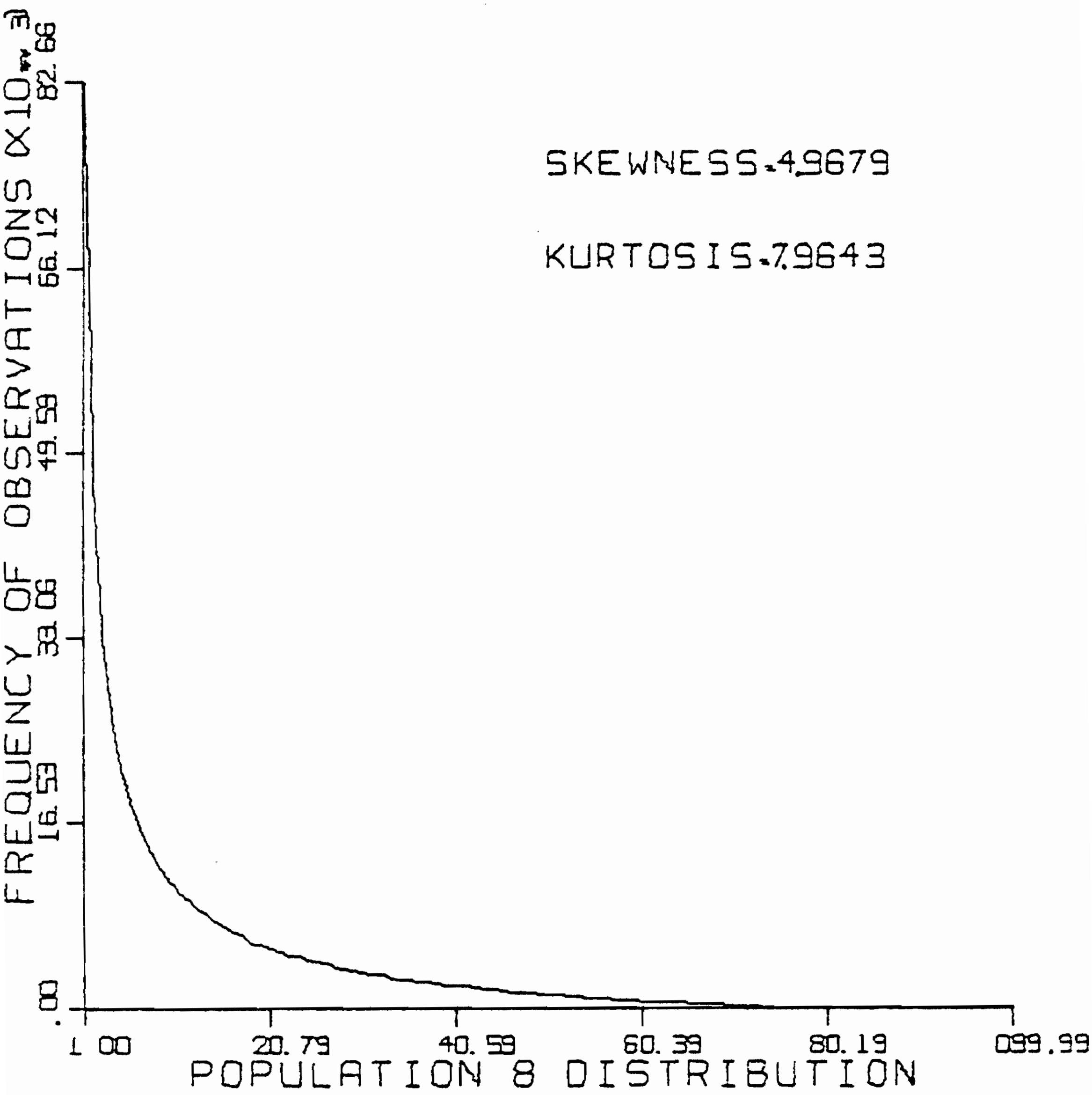


FIGURE 37

RESEARCH DISTRIBUTION 8

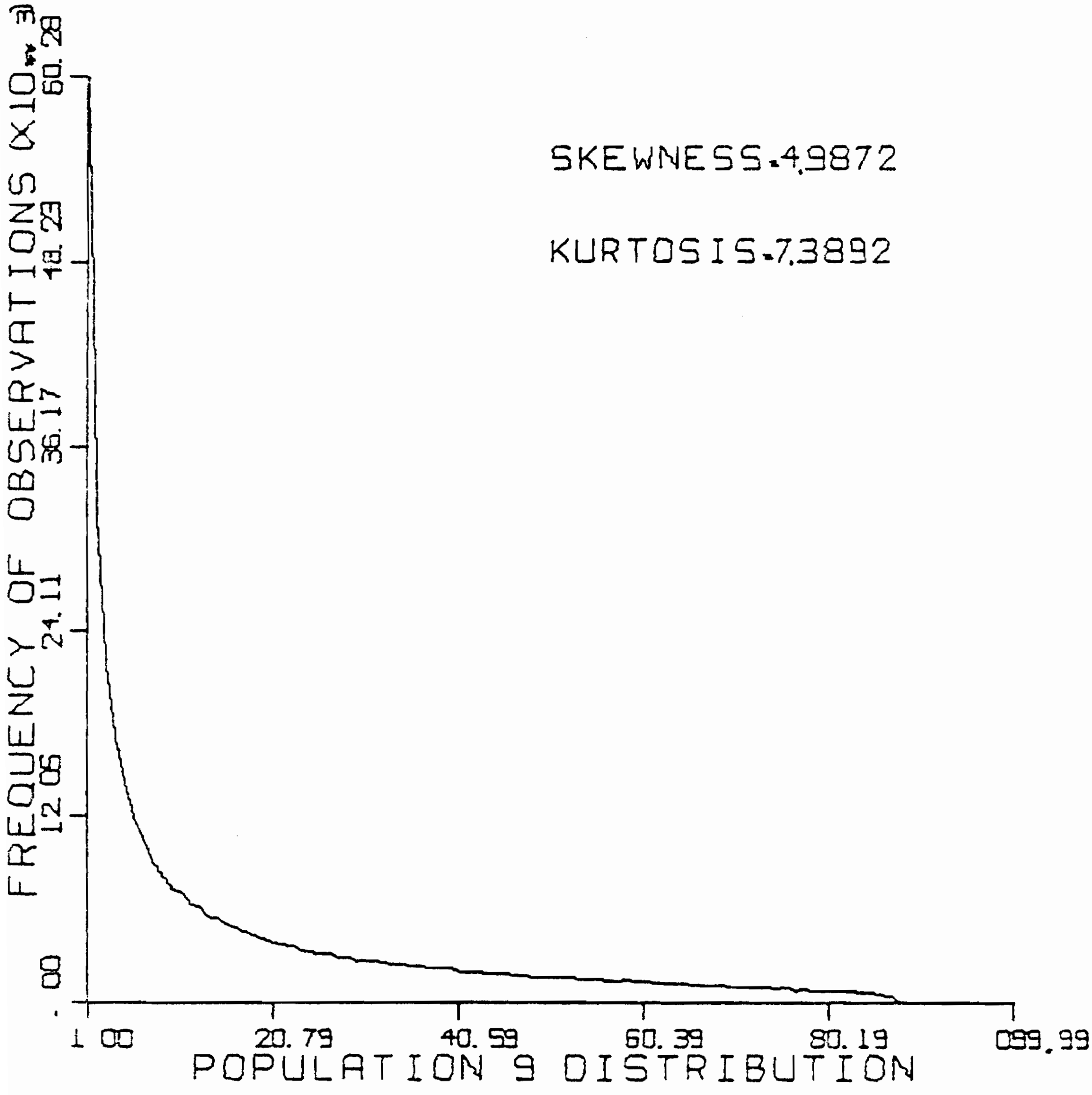


FIGURE 38

RESEARCH DISTRIBUTION 9

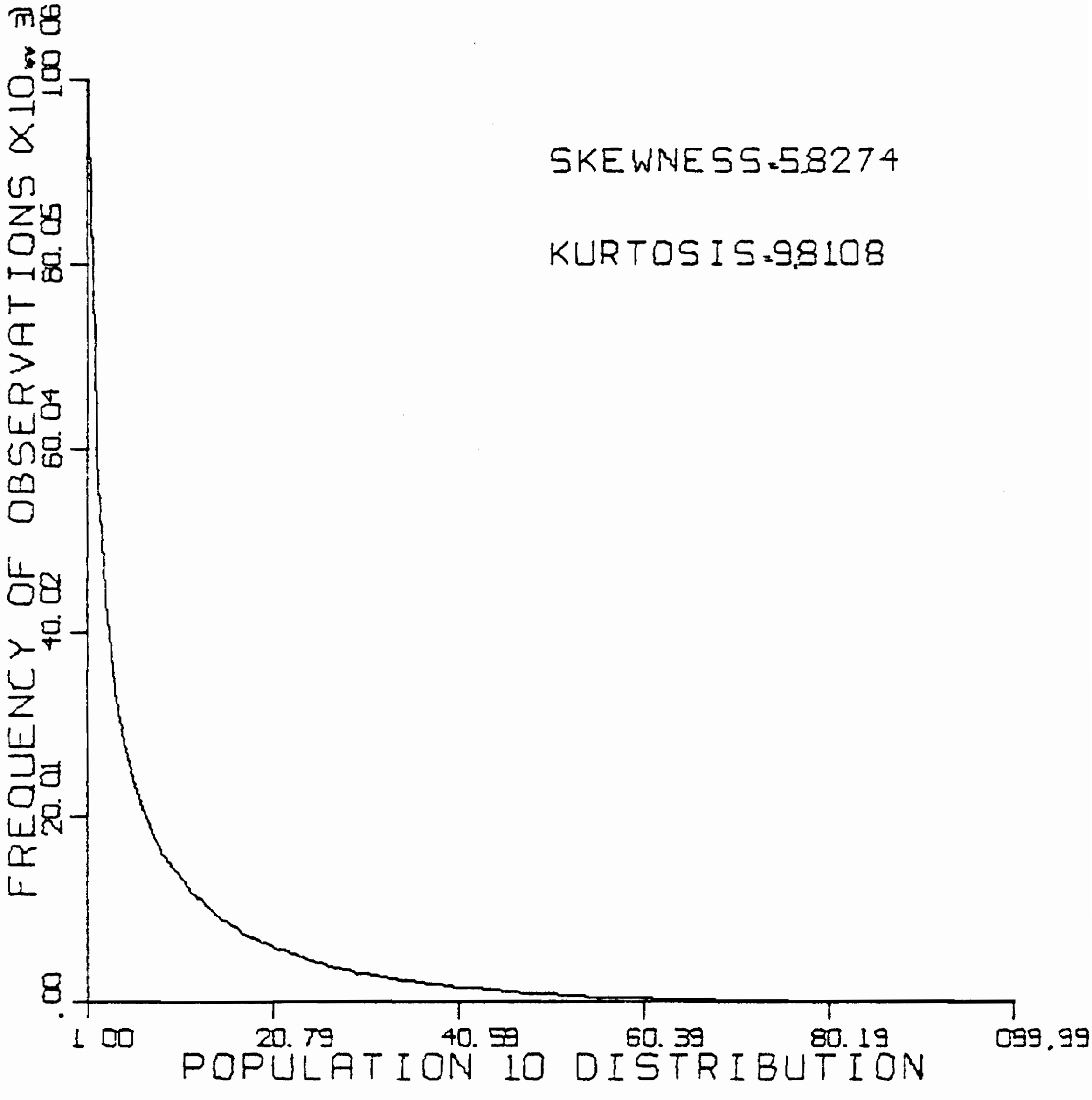


FIGURE 39

RESEARCH DISTRIBUTION 10

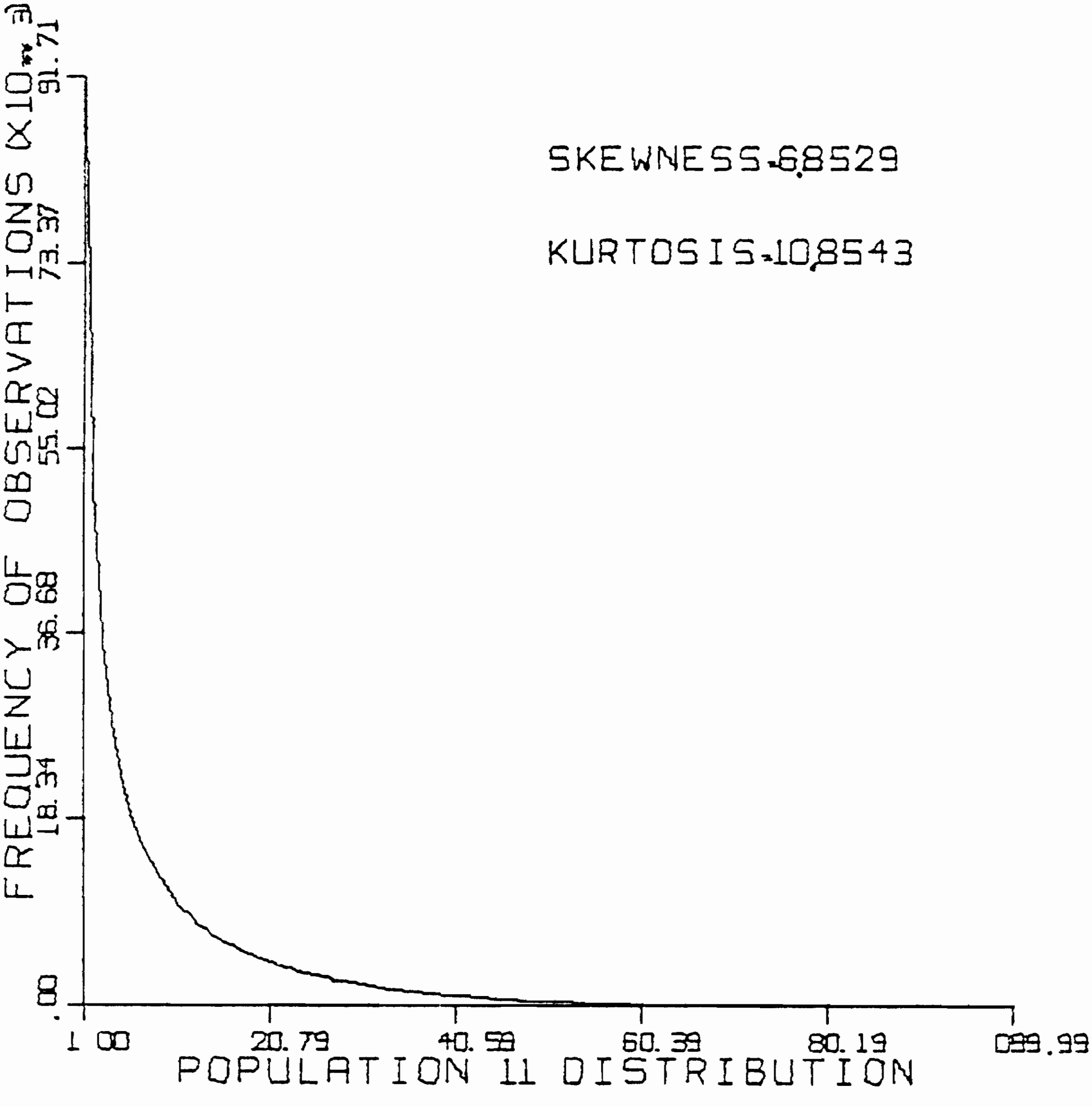


FIGURE 40

RESEARCH DISTRIBUTION 11

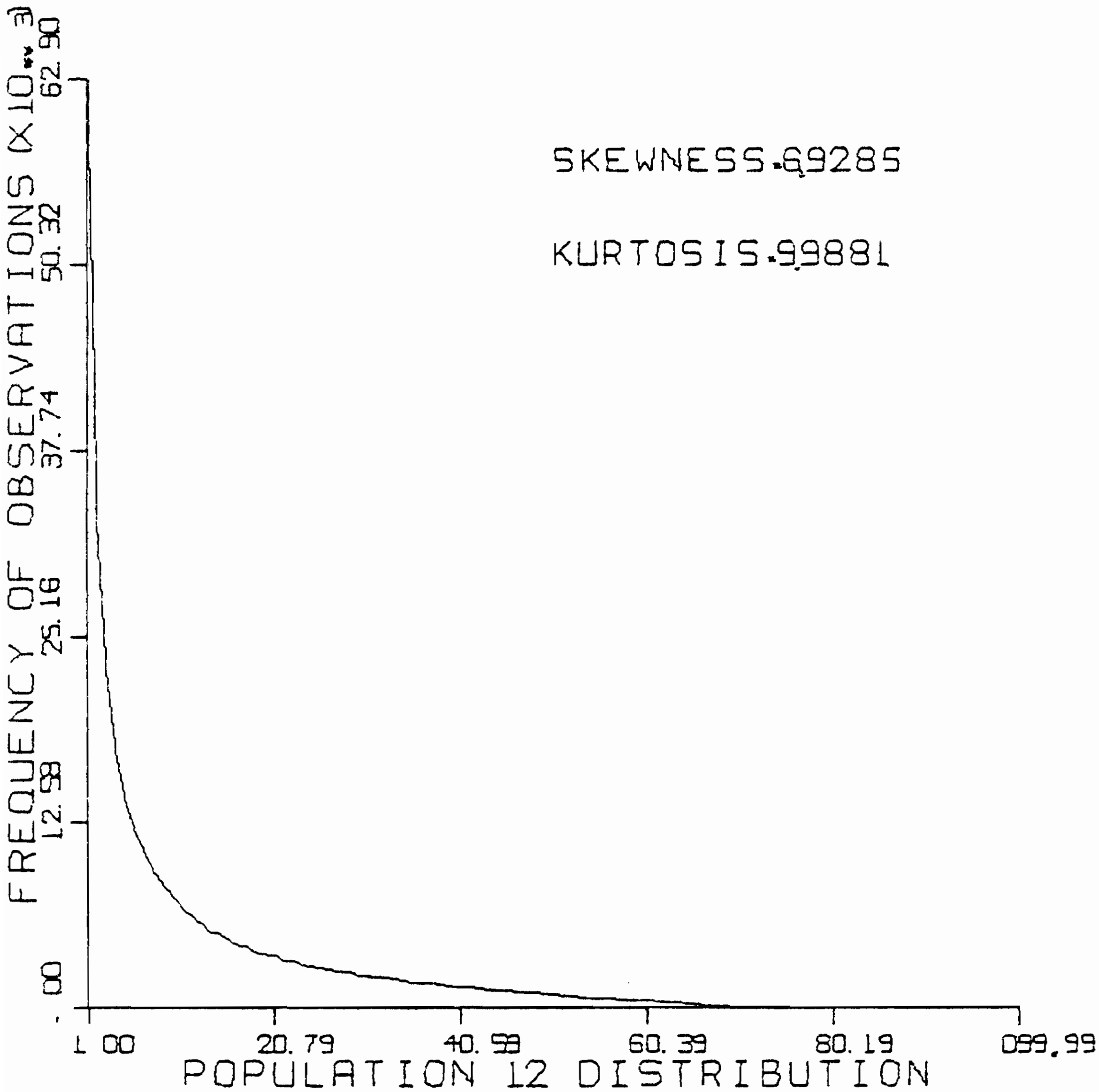


FIGURE 41

RESEARCH DISTRIBUTION 12

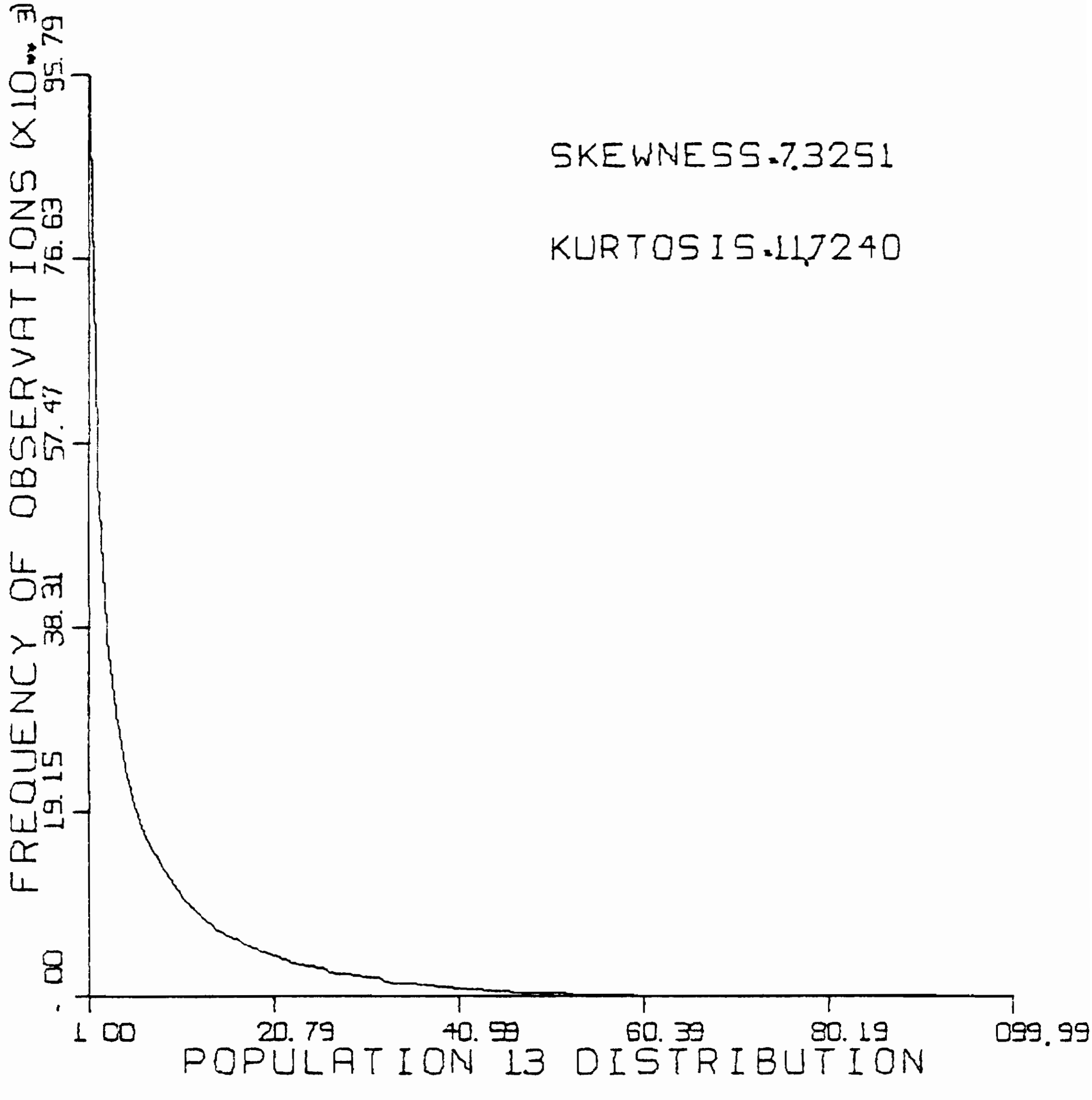


FIGURE 42

RESEARCH DISTRIBUTION 13

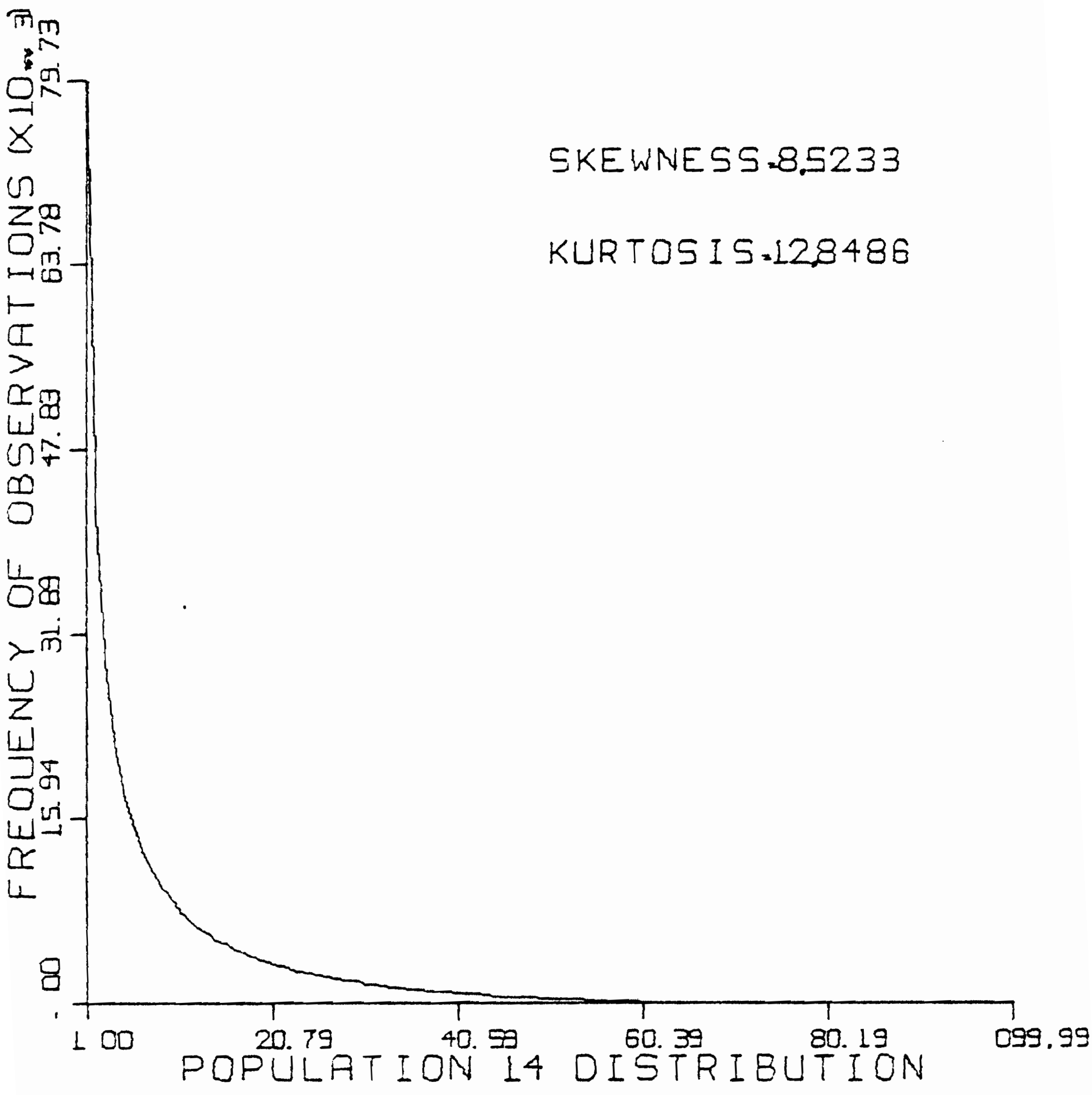


FIGURE 43

RESEARCH DISTRIBUTION 14

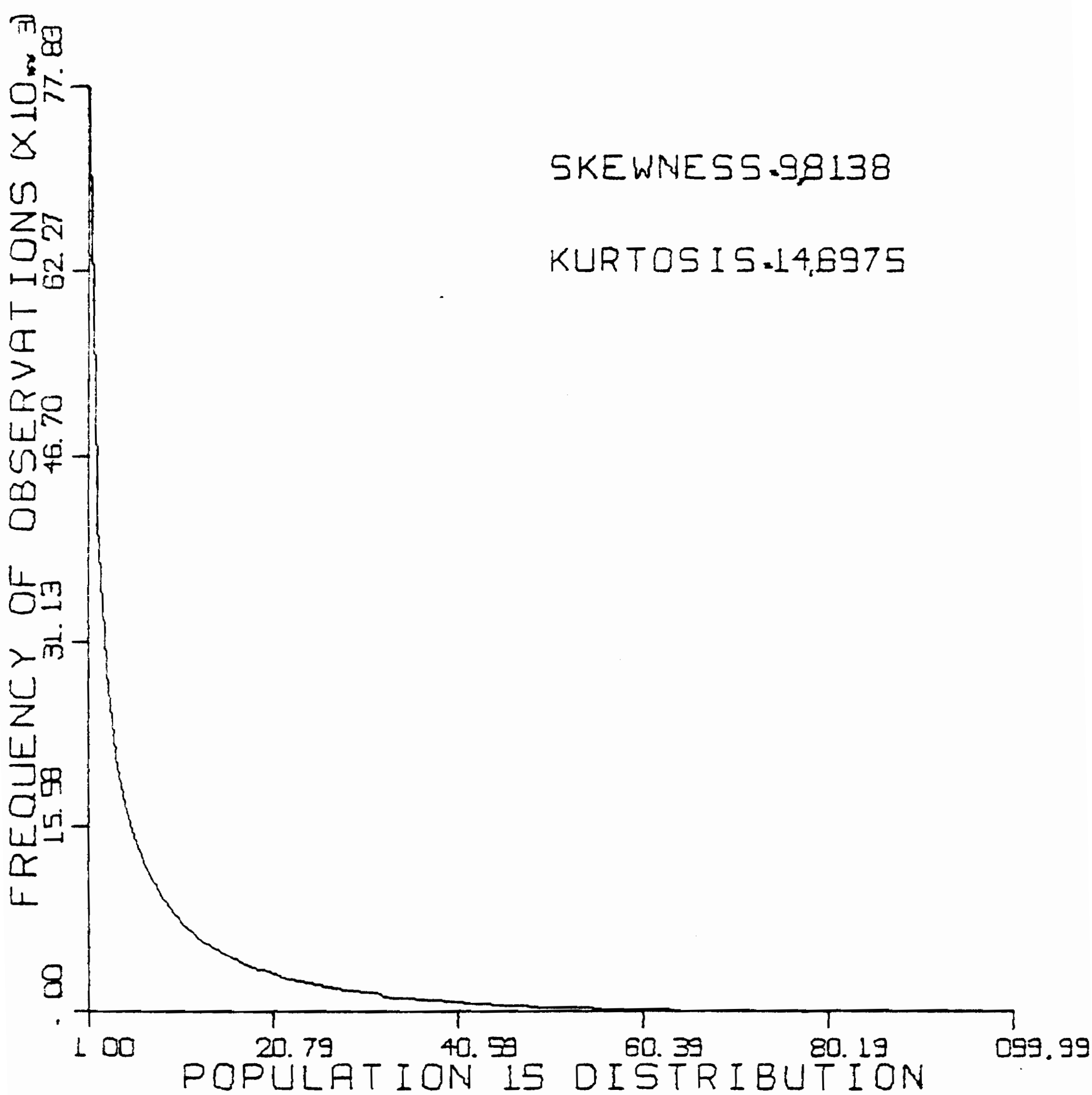


FIGURE 44

RESEARCH DISTRIBUTION 15

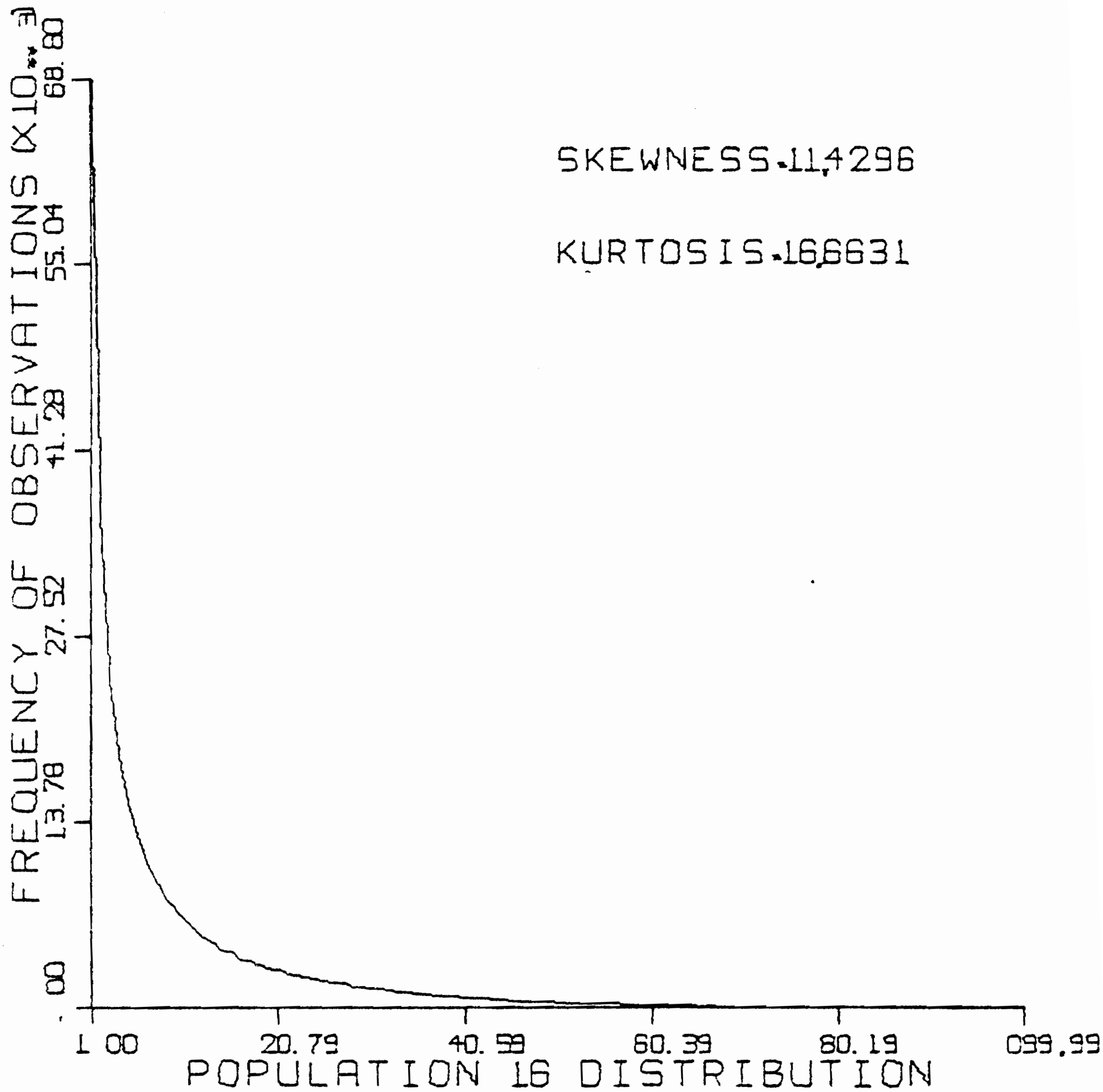


FIGURE 45

RESEARCH DISTRIBUTION 16

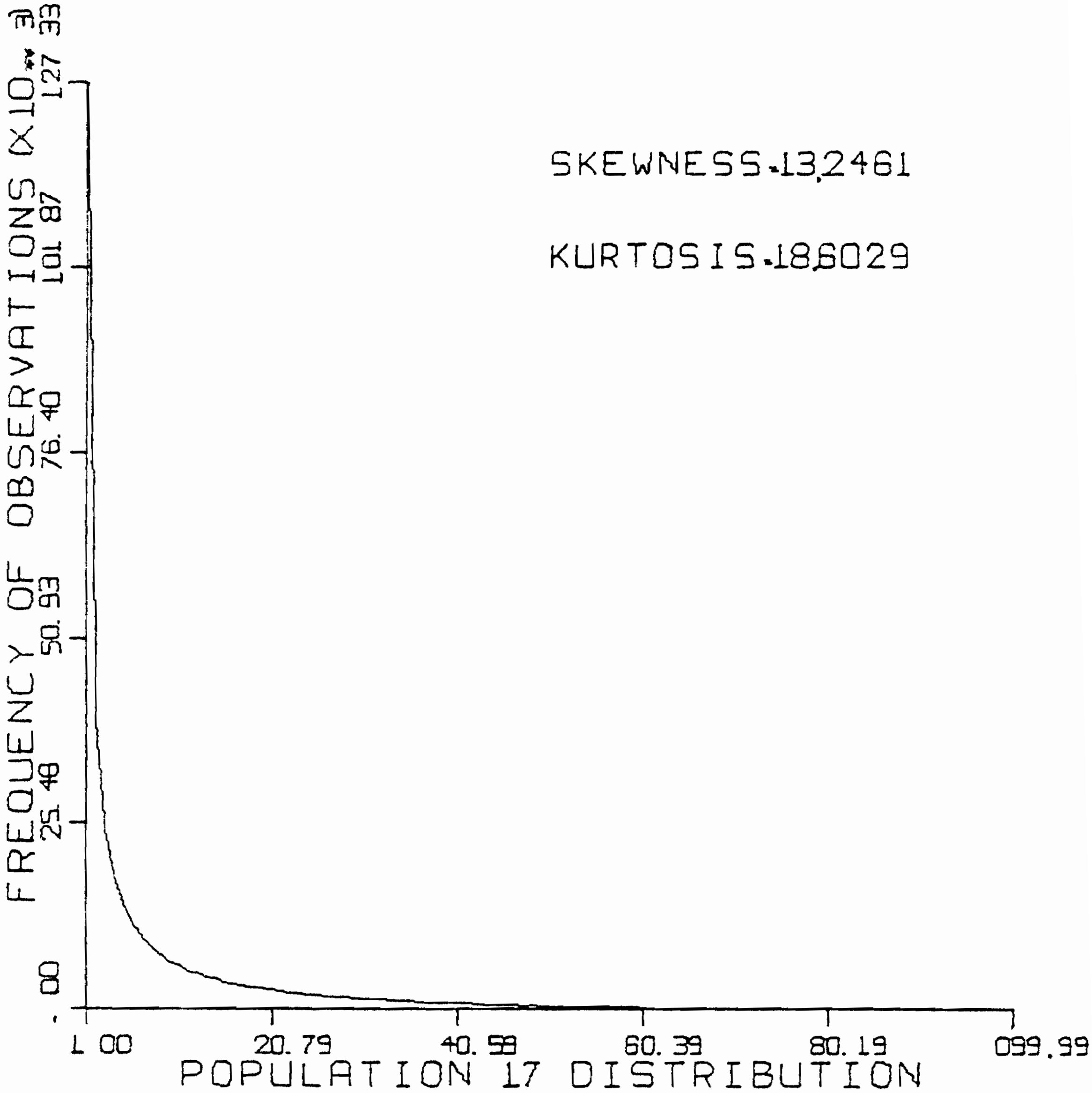


FIGURE 46

RESEARCH DISTRIBUTION 17

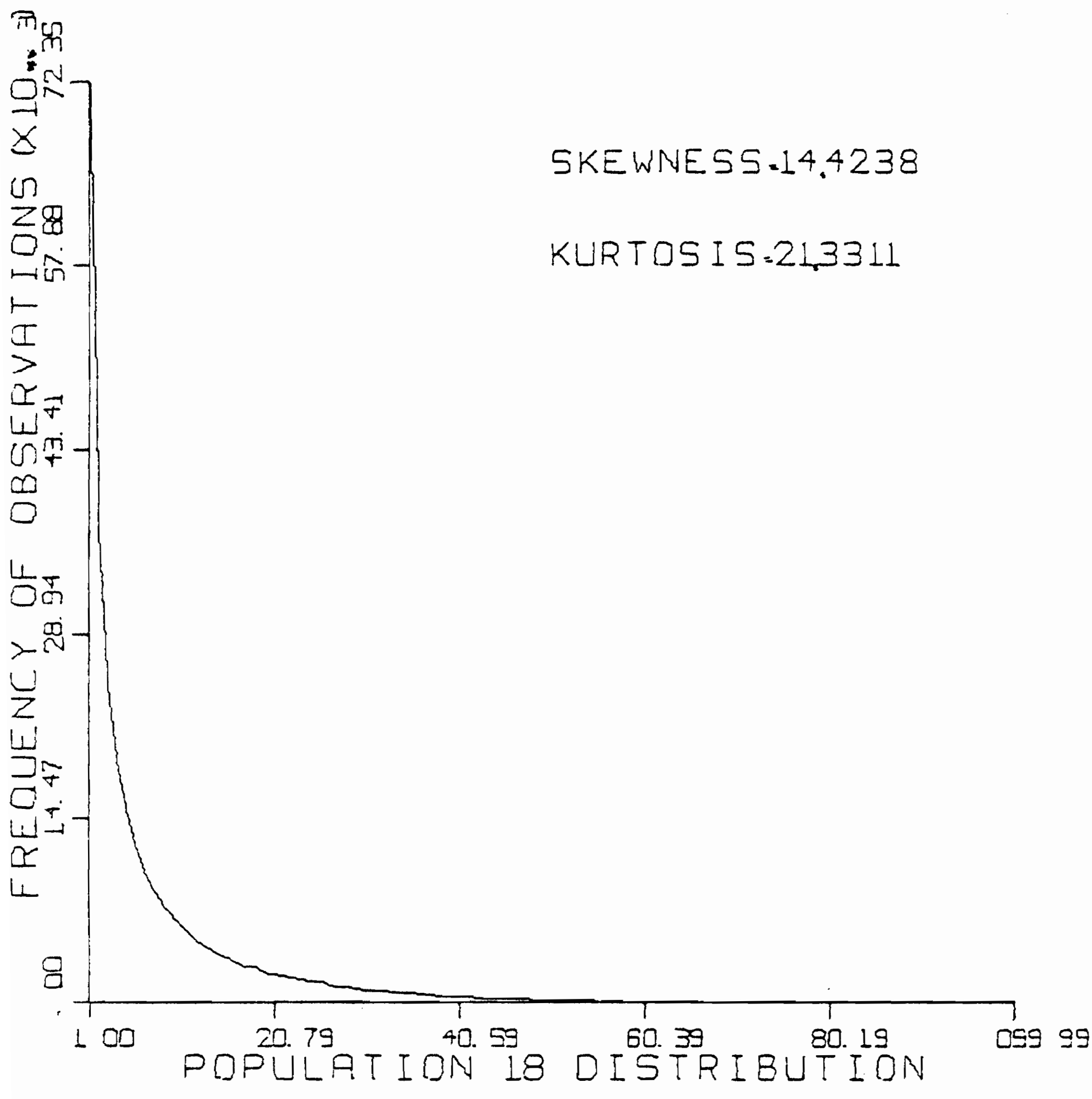


FIGURE 47

RESEARCH DISTRIBUTION 18

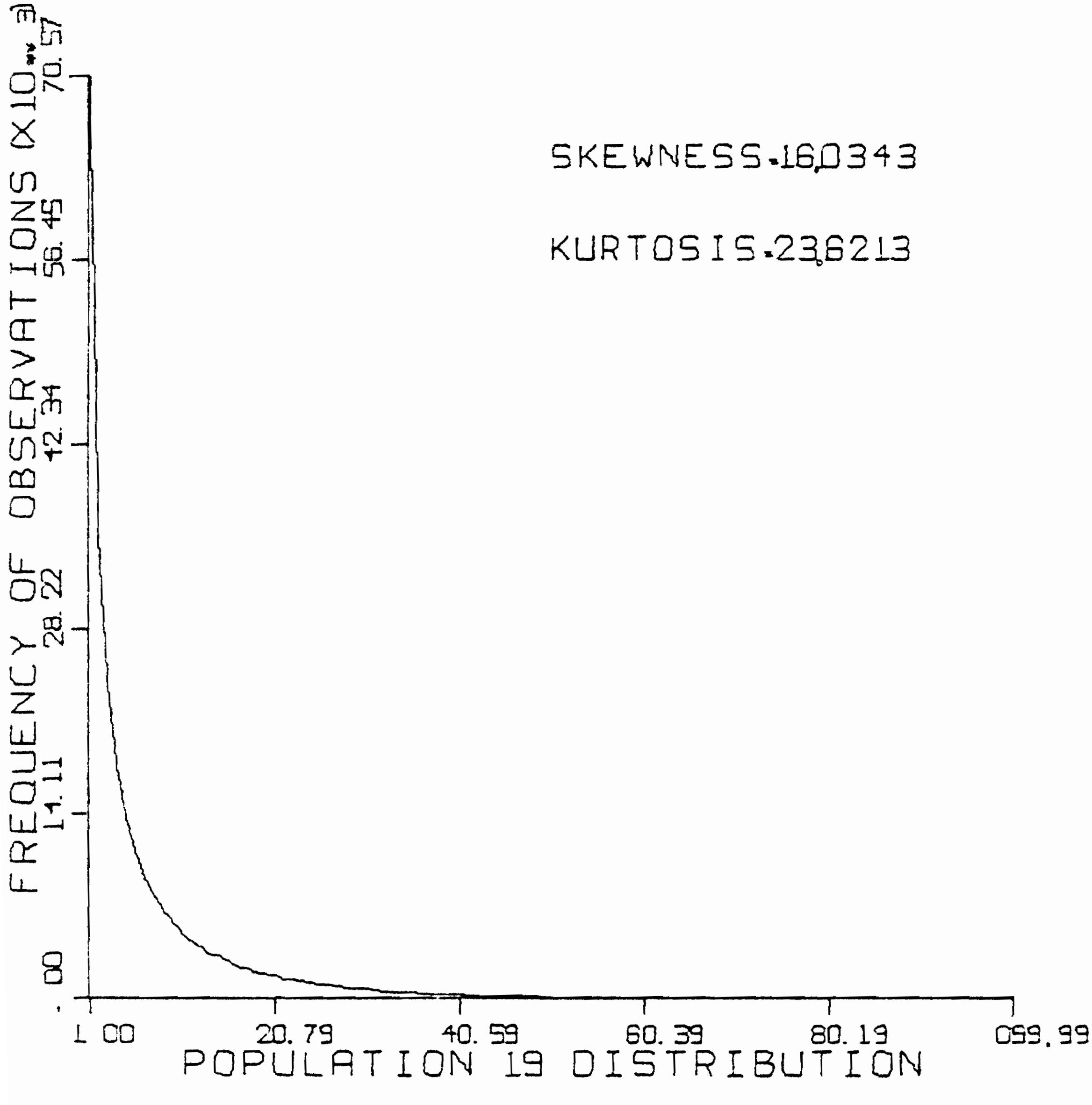


FIGURE 48

RESEARCH DISTRIBUTION 19

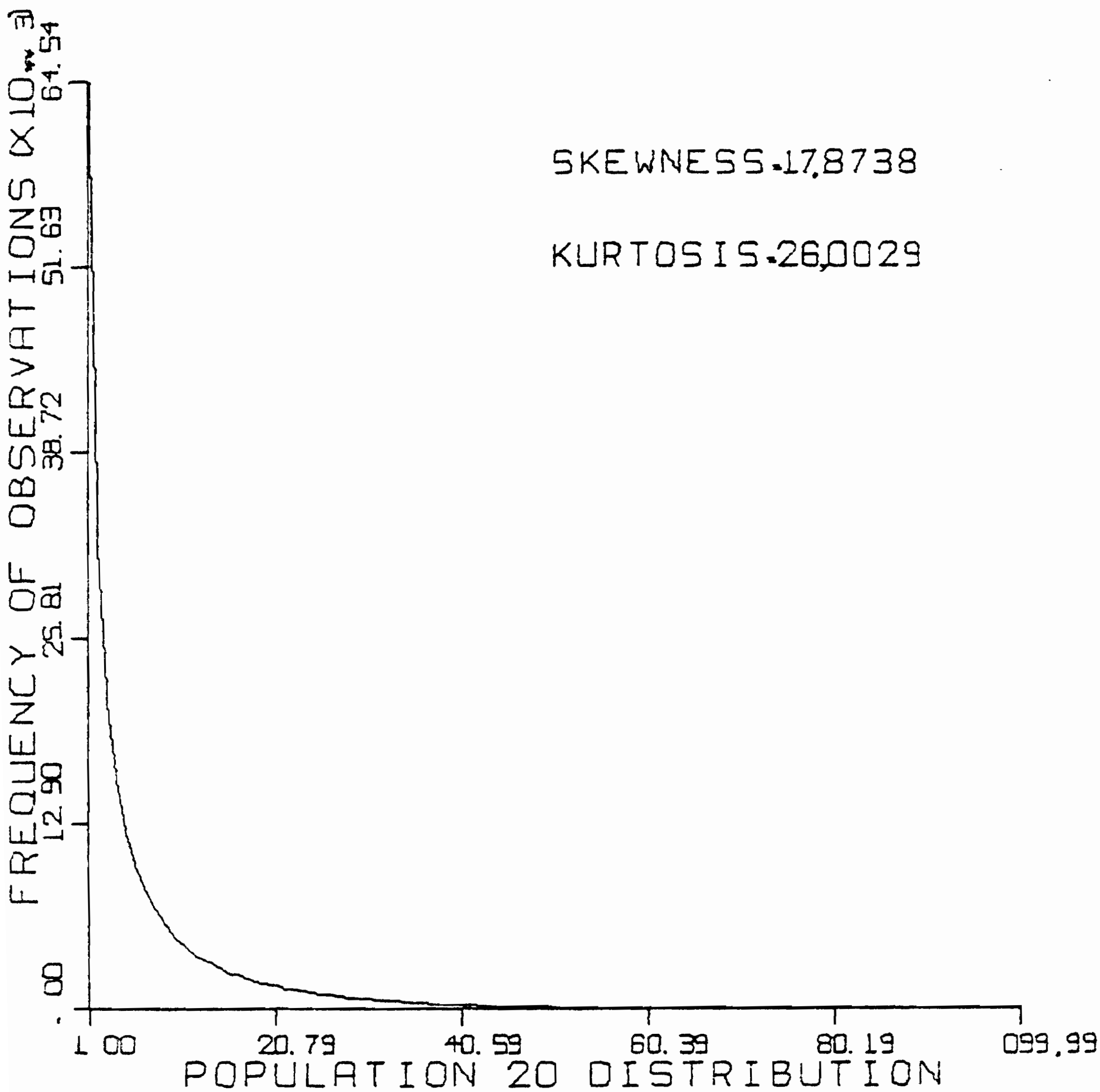


FIGURE 49

RESEARCH DISTRIBUTION 20

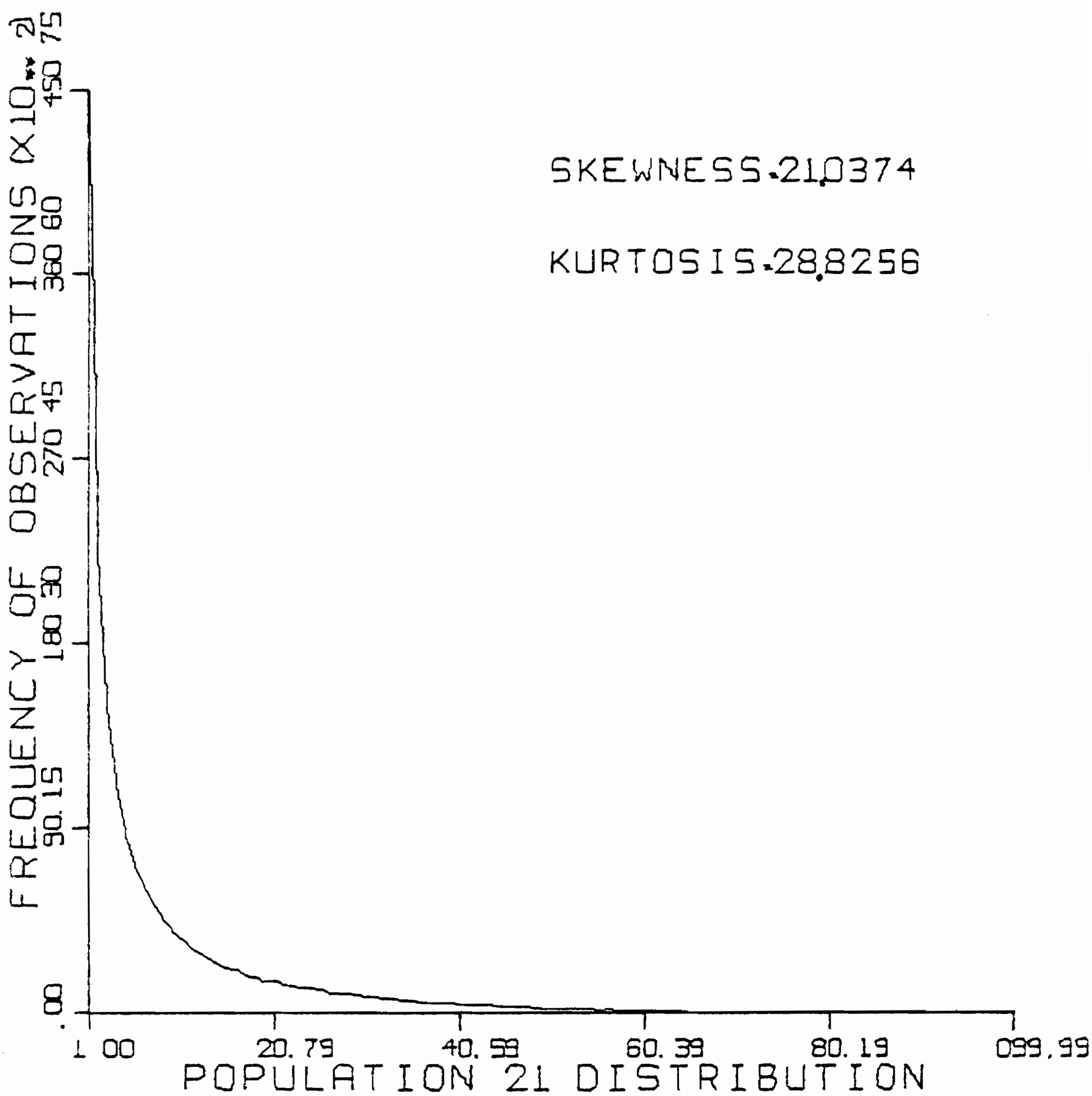


FIGURE 50

RESEARCH DISTRIBUTION 21

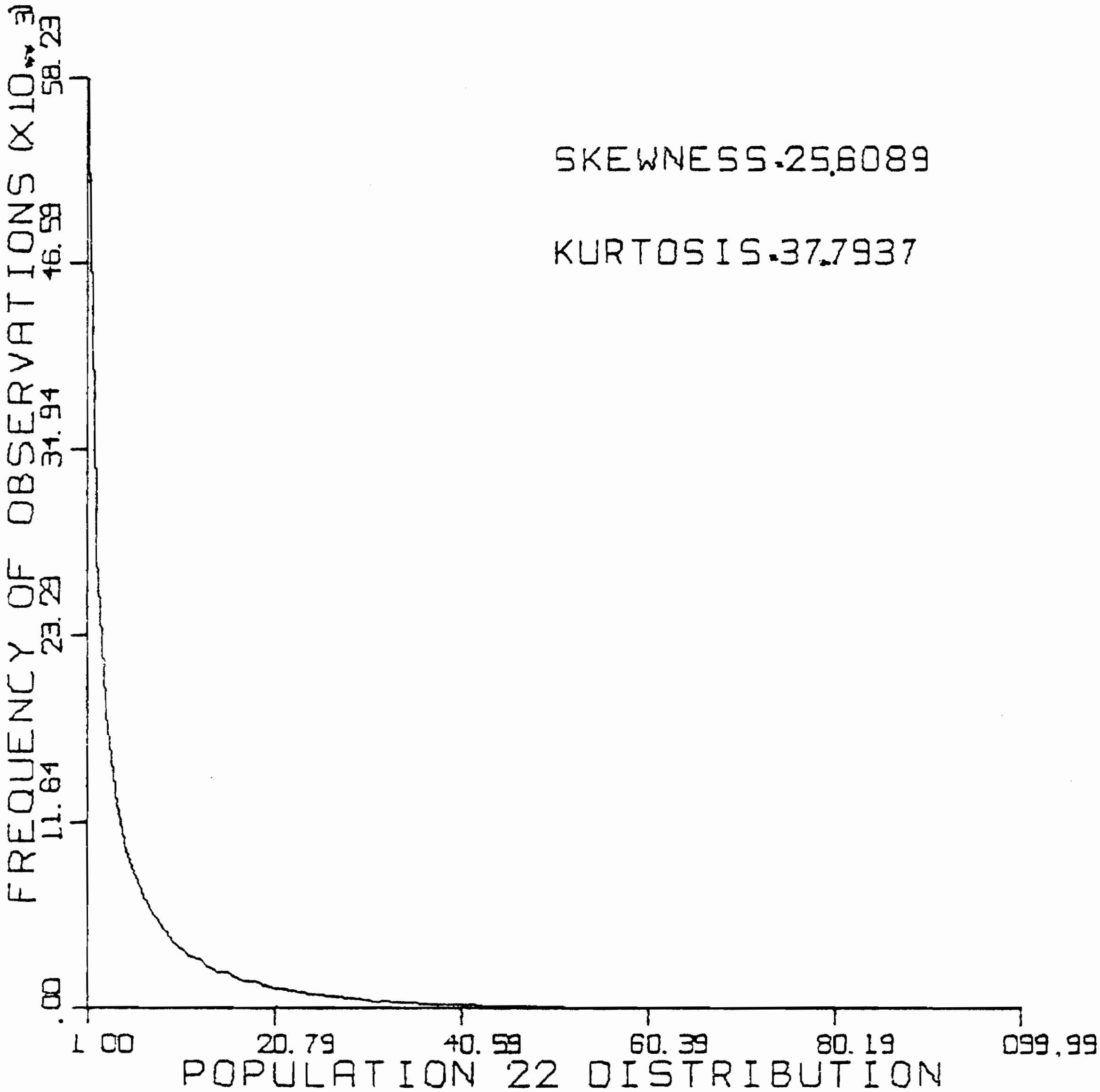


FIGURE 51

RESEARCH DISTRIBUTION 22

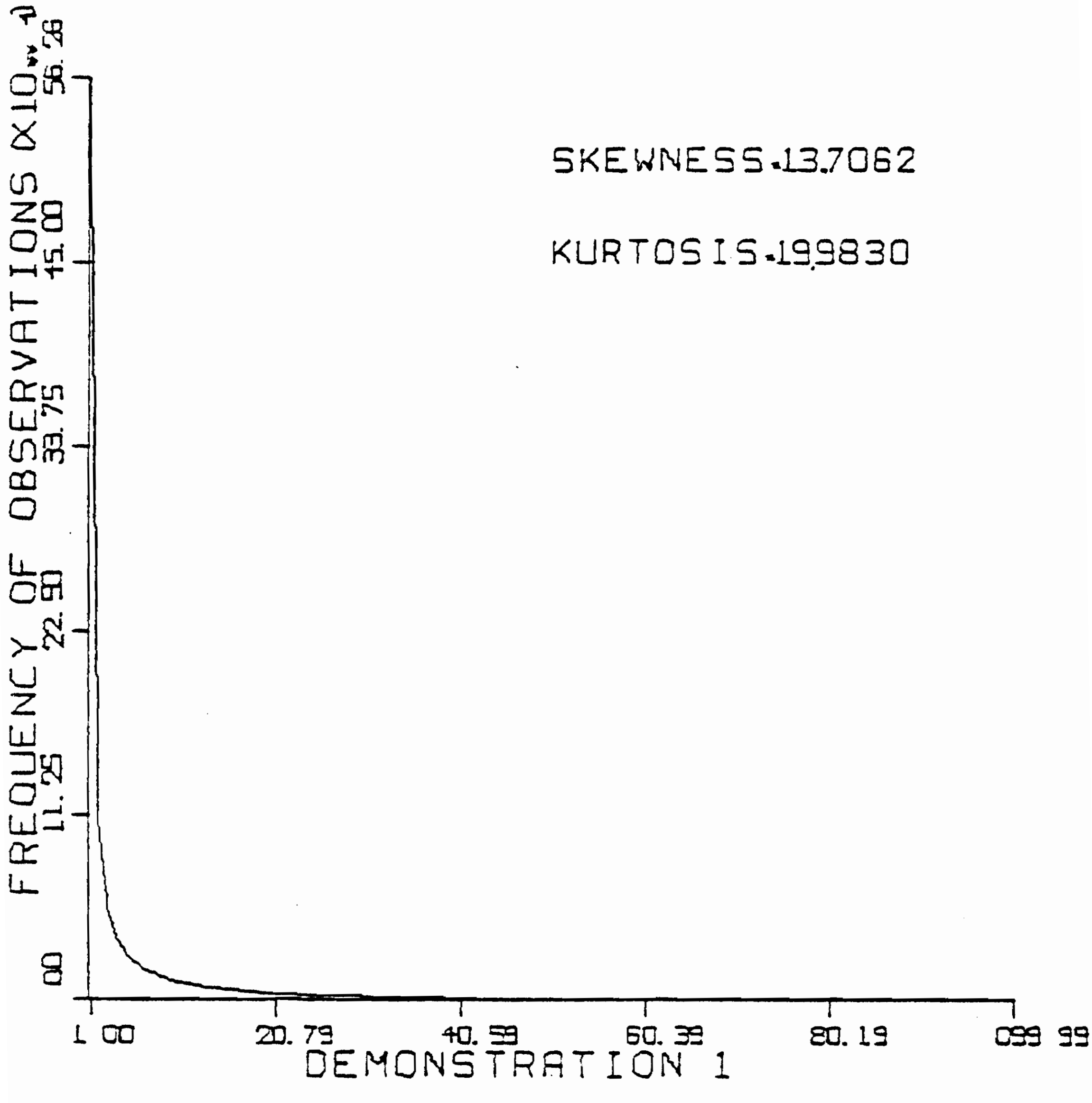


FIGURE 52

ILLUSTRATIVE DISTRIBUTION 1

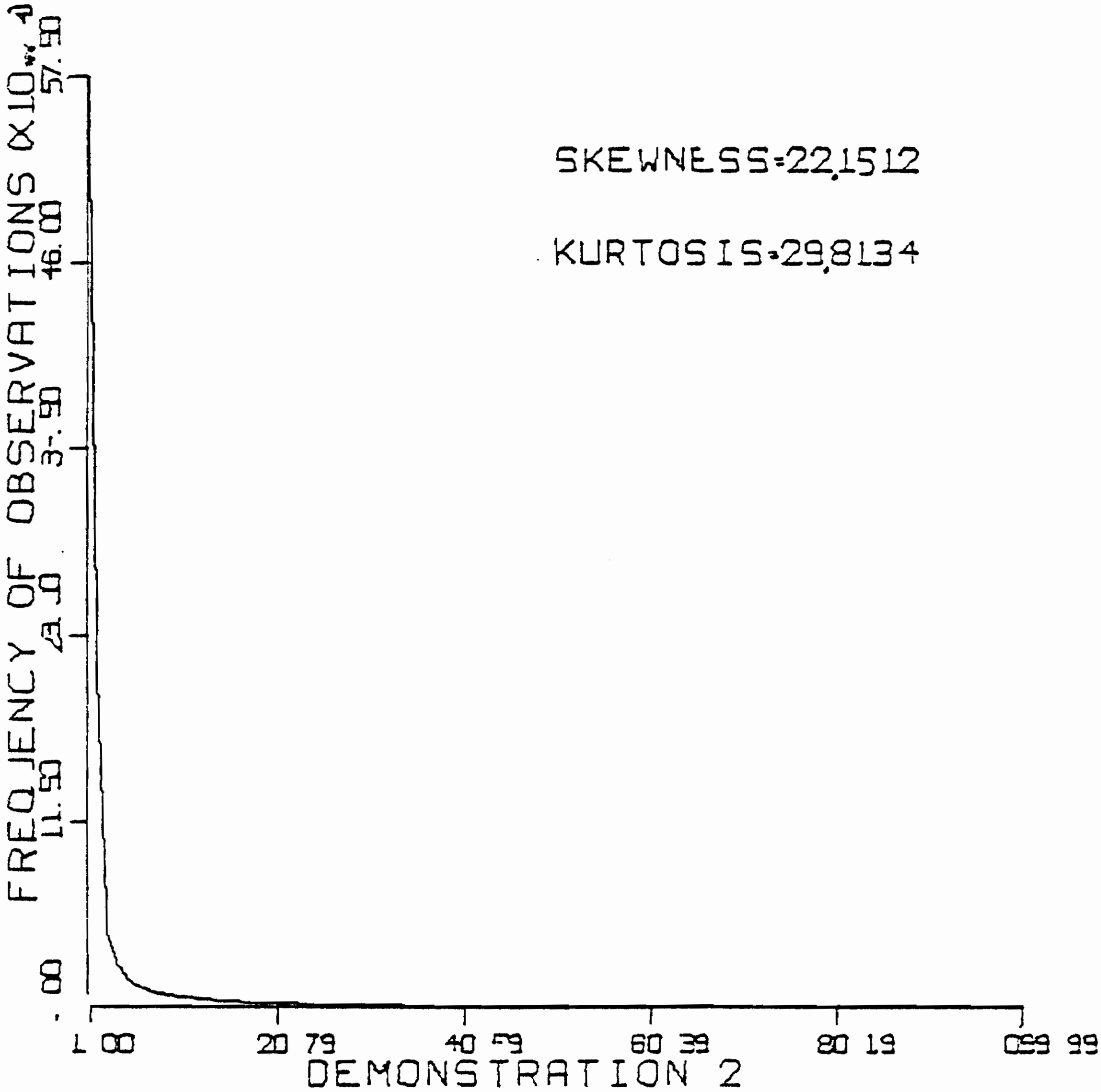


FIGURE 53

ILLUSTRATIVE DISTRIBUTION 2

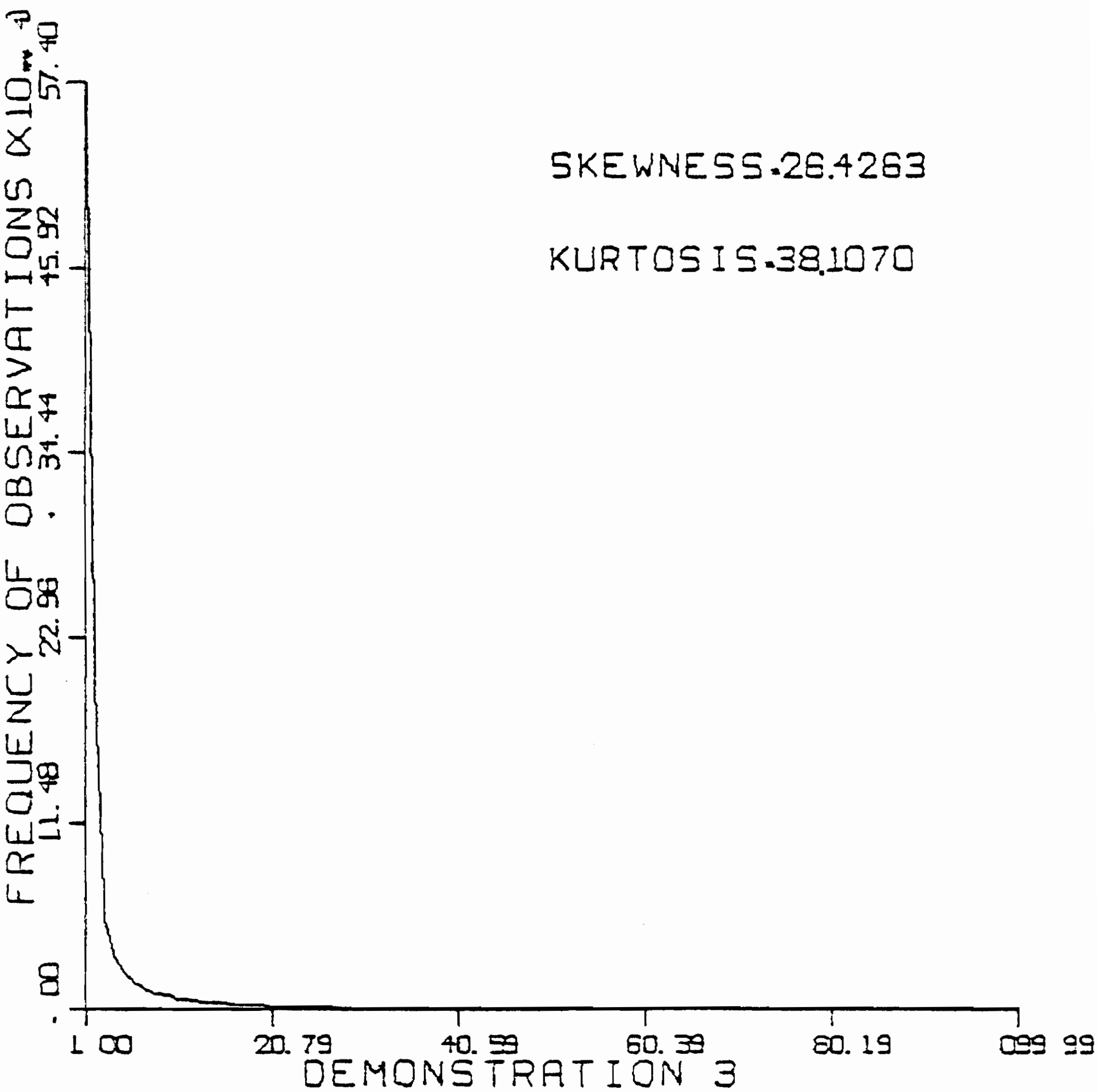


FIGURE 54

ILLUSTRATIVE DISTRIBUTION 3

APPENDIX III

Validation of Purge3 Computer Program in Relation to Current Study

APPENDIX III

Validation of Purge3 Computer Program in Relation to Current Study

Purge3 was developed by the Department of Statistics at Virginia Polytechnic Institute and State University. The program is provided in Appendix VI. The program creates random numbers using as input the first four moments of a distribution. Procedures used to validate Purge3 for this study are given below and are summarized in Figure 55. Using the four moments of 200.00, 1.0, 0.0, and 3.0, the program was directed to generate 400 random numbers. These 400 random numbers were stored on magnetic tape and also punched on IBM cards. An independent computer program was designed to pick up these 400 random numbers from the tape and to compute their first four moments. The 400 stored data points on tape were also used as input to Purge3 which also computes the four moments from the data points. The 400 data points stored on cards were also run through Purge3. In all instances the computed first four moments were consistent as shown in Table 84. Therefore, it was accepted that once the random numbers were generated, Purge3 correctly computed the first four moments of a data set. Purge3 was used to generate random numbers and to compute the first four moments of the created data set. The simulation computer program to determine the attainment, or lack of attainment, of the alpha and beta levels used Purge3 as a subroutine to generate random numbers which represent auditing populations. The simulation program

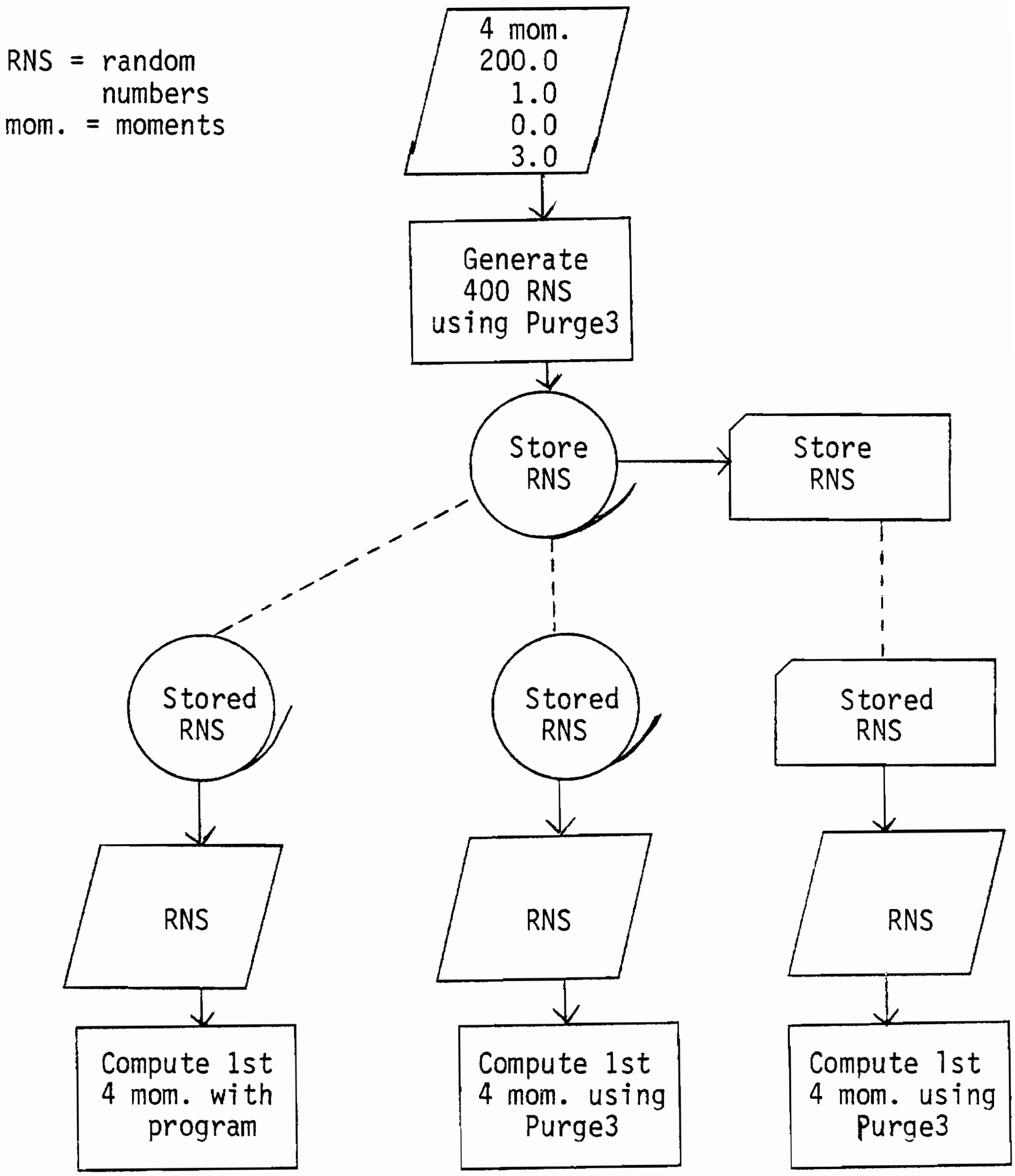


FIGURE 55

PROCEDURES FOR PURGE3 VALIDATION FOR CURRENT STUDY

Table 84

FIRST FOUR MOMENTS OF 400 RANDOM NUMBERS
USED IN COMPUTER PROGRAM VALIDATION

Moment	Independent Program	Purge3 from Tape	Purge3 from cards
Mean (1st)	199.97	199.98	199.98
Variance (2nd)	.9837	.9812	.9812
3rd	-.1356	-.1565	-.1565
4th	2.9893	2.9862	2.9862

which utilized Purge3 was validated independently of Purge3. This validation is discussed in Appendix V.

APPENDIX IV

Validation of Computer Program to Determine Mean, Variance, and Minimum Value of One Million Numbers

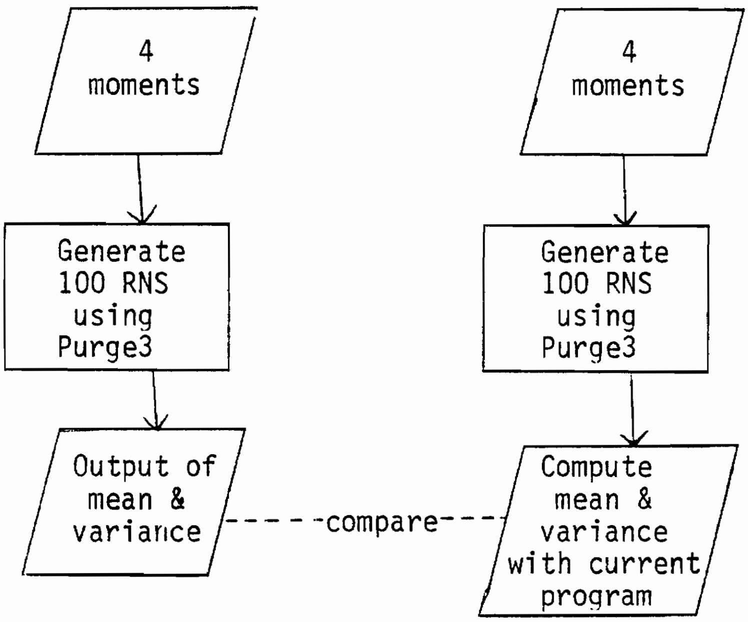
APPENDIX IV

Validation of Computer Program to Determine Mean, Variance, and Minimum Value of One Million Numbers

The following procedures were used to validate the computer program to determine the minimum value, mean, and variance of one million numbers. Part of the procedures is summarized in Figure 56. The computer program is provided in Appendix VII.

Using the original first four moments of 0.0, 1.0, 1.7320509, and 5.0 as inputs to Purge3, 100 random numbers were generated. Since Purge3 was previously validated, the mean and variance of the distribution as computed by the current program could be compared to the mean and variance as derived by Purge3. A mean and variance of .00081375 and .99836003, respectively, were computed by Purge3 when using the above original four moments. The program being validated independently produced the same mean and variance.

To validate the computation of the minimum number of the distribution, a program prepared by the Department of Statistics at Virginia Polytechnic Institute and State University was utilized. The minimum number as derived by this program for the distribution described above was a negative .64334744. The minimum value of the distribution as computed by the current program was a negative .64346925.



RNS = random numbers

FIGURE 56

PROCEDURES FOR VALIDATION OF PROGRAM
TO DETERMINE MEAN, VARIANCE AND
MINIMUM VALUE OF ONE MILLION NUMBERS

Since comparisons of the mean, variance, and minimum value derived by independent procedures was consistent with the program output, the computer program was accepted as free from error.

APPENDIX V

Validation of Computer

Program to Determine Proportions of

Times Type I and Type II Errors are Committed

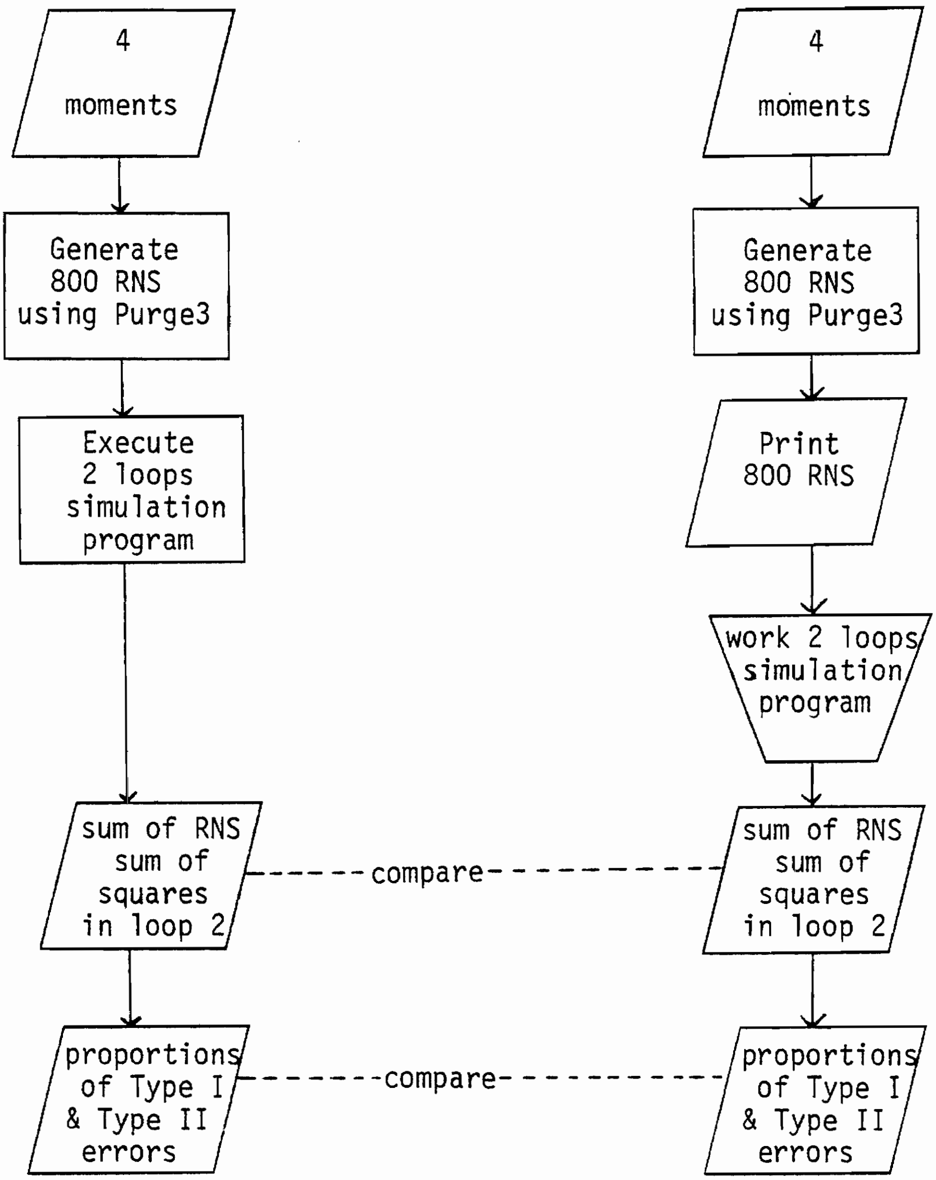
APPENDIX V

Validation of Computer Program to Determine Proportions of Times Type I and Type II Errors are Committed

This program computed the proportions of times Type I and Type II errors were committed in the 1,000 loops of each computer run and has been referred to as the simulation program. Procedures used to validate this program are described below while the computer program itself is provided in Appendix IX.

Two different distributions, A and B, were used in this program validation. The steps carried out on the first distribution are summarized in Figure 57. The basic plan for validation was to compute independently the proportions of times sampling errors occurred and to compare these proportions with the output of the simulation program. The simulation program was executed for two loops using Purge3 as a subroutine to generate 800 random numbers which follow distribution A. In addition to the regular output of the simulation program, the sum and the sum of squares of the random numbers used in the second loop were printed.

In an independent program, Purge3 was used to generate and to print the 800 random numbers in distribution A. The mean, variance, third and fourth moments, of this distribution are 99.81, 97.37, 0.03, 3.03 respectively. Using the print out of random numbers, identical procedures used in the simulation program were carried out manually. The sum and sum of squares for the random numbers used



RNS = random numbers

FIGURE 57

PROCEDURES FOR VALIDATION OF SIMULATION PROGRAM USING DISTRIBUTION A in the second loop were compared to the computer output. The manual computations produced 26,992.33 and 2,707,762.80 for the sum and sum of squares, respectively, while the computer output gave 26,992.04 and 2,707,718.00.

The proportions for errors computed under both methods were also compared. There were no discrepancies between these proportions. Table 85 provides the proportions of sampling errors as well as the sample sizes used in both loops. In the first loop, the sample that is a .5 multiple of n_2, should be 48 units. The preliminary sample size of 50 units was substituted because the program has a built in feature that guarantees no sample size smaller than the preliminary sample size. This is also seen in loop two, where the computed sample size was 40 elements which was changed to 50 elements. The simulation procedures used a preliminary sample size of 30 rather than 50 units. This does not invalidate these procedures because the program is written to facilitate an easy change in the size of the preliminary sample. The program uses "RPS = 50." for the preliminary sample size stated in real terms and provides that "IPS = RPS" for the size stated in integer form. Therefore, only the "RPS = 50." need be changed for a change in the preliminary sample size.

A second distribution was used to carry out similar validation procedures as described above. A summary of the steps carried out on distribution B is provided in Figure 58. The mean, variance, third and fourth moment of this distribution are 47.93, 4,692.01, 7.46 and 11.49, respectively. Again, the simulation program was executed

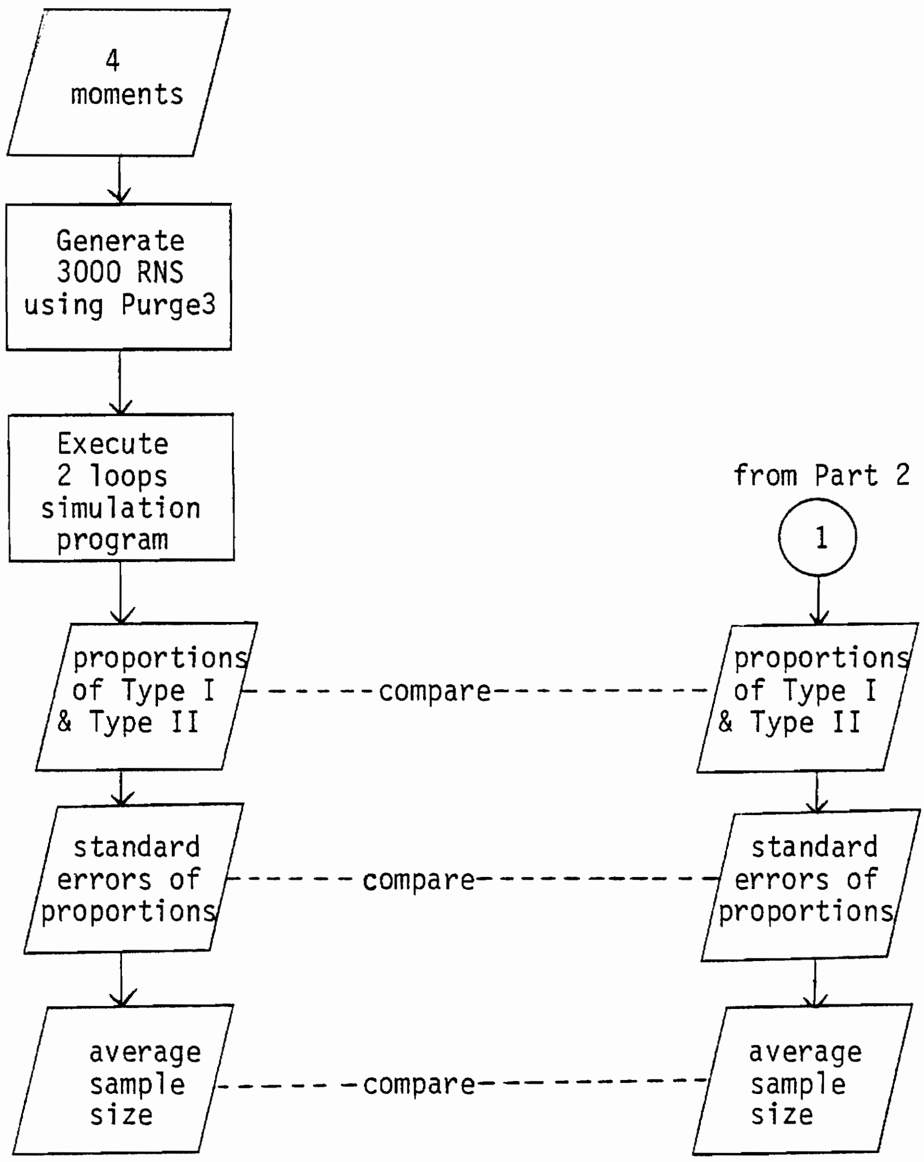
Table 85

SIMULATION RESULTS IN VALIDATION

USING DISTRIBUTION A

	n_1^*	n_2^*	n_3^*	n_4^*
Type I - Error	.5	.5	0.0	0.0
Type II - Upper	0.0	0.0	0.0	0.0
Type II - Lower	0.0	.5	0.0	0.0
Sample Sizes				
Loop 1	50	96	144	192
Loop 2	50	50	60	80

*n_1, n_2, n_3, n_4 = sample sizes .5, 1.0, 1.5, and 2.0 times the computed size, respectively.



RNS = random numbers

FIGURE 58
Part 1

PROCEDURES FOR VALIDATION OF SIMULATION PROGRAM USING DISTRIBUTION B

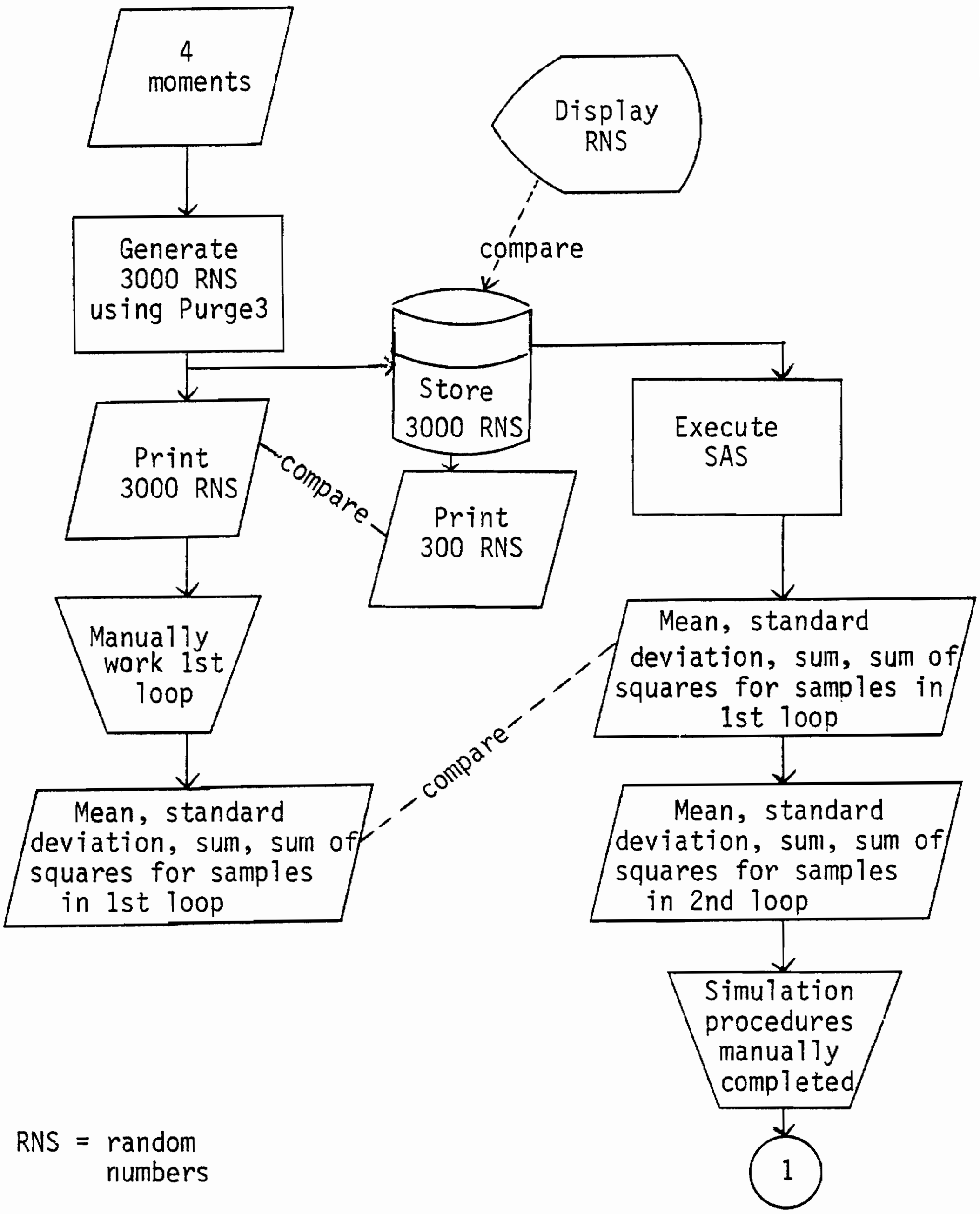


FIGURE 58
Part 2

PROCEDURES FOR VALIDATION OF SIMULATION PROGRAM USING DISTRIBUTION B for two loops using Purge3 to generate 3,000 random numbers. At this time, the standard errors of the proportions and the average sample size were added to the output of the simulation program and used to compare with the independently produced figures as described below.

An independent computer program generated 3,000 random numbers following the B distribution. In addition to the printing of the 3,000 numbers, the program stored the numbers as a data set on a disk pack. Using the random number print out, procedures identical to the steps in the simulation program were manually carried out for one loop only. In determining if any errors of Type I or II occurred in this first loop, it was necessary to compute the mean, standard deviation, sum and sum of squares for the random numbers used in the samples of the first loop. These statistics were compared with the corresponding values produced by a program using the Statistical Analysis System (SAS). The program using SAS picked up the random numbers from the disk. To verify that the stored data set was identical to the 3,000 random number print out, two procedures were carried out. First, an independent program picked up and printed 300 numbers of the stored data set which was compared with the 3,000 number print out. Second, the data set stored on disk pack was linked to by using a cathoderay tube and visually comparing the random numbers with the print out of 3,000 random numbers. These comparisons were consistent in every instant. Since the SAS program produced identical results compared to the manually produced values of the first loop, the SAS program was used to produce the mean, standard deviation, sum and sum of squares for samples in the second loop. Using these values, simulation procedures were manually completed to determine the proportions for Type I and Type II errors committed in these two loops. The standard errors of the proportions and the average sample size were also manually computed. The results of the simulation program were consistent with the manually computed values. These results are given in Table 86.

Since the validation procedures produced consistent results, the computer program was accepted as free from error.

Table 86

SIMULATION RESULTS IN VALIDATION

USING DISTRIBUTION B

	n_1^*	n_2^*	n_3^*	n_4^*
Type I - Error	0.0	.5	.5	.5
Type II - Upper	1.0	1.0	1.0	1.0
Type II - Lower	.5	0.0	0.0	1.0
Standard Errors of Proportions for:				
Type I	0.0	.3536	.3536	.3536
Type II - Upper	0.0	0.0	0.0	0.0
Type II - Lower	.3536	0.0	0.0	0.0
Average Sample Size	345	690	1,035	1,380
Sample Sizes for:				
Loop 1	177	355	532	710
Loop 2	512	1,025	1,537	2,050

*n_1, n_2, n_3, n_4 = sample sizes .5, 1.0, 1.5, and 2.0 times the computed size, respectively.

APPENDIX VI

Purge3 Computer Program

```
      SUBROUTINE PURGE3(MKX,MKZ)
C      UNIVERSAL RANDOM DISTRIBUTION GENERATOR
      DOUBLE PRECISION A,AN,X,Z,W,FA,FAN
      COMMON/Z7/APRXM,APRXV
      COMMON/VPI001/IX/VPI002/KX
      DIMENSION  SKUNK(5), FRM(12)
      DIMENSION FM(18),FMH(5),S(2641),KU(101),FA(4),FAN(4)
      COMMON/MCK1/AVE(2),CM2(2),CM3(2),CM4 (2),BETA1,BETA2,SKAPPA
      COMMON/MCK2/TOM(100),LIMIT
      COMMON/MCK4/N(2),EL(2),H(2),K2U(2,101),SS(2,2641)
      COMMON/Z2/FA/Z3/FAN/Z4/FCM2,FCM3,FCM4
      COMMON/Z5/FBETA1,FBETA2,FKAPPA
      DIMENSION CN(8),TN(8),A(4),AN(4),X(30),U(100),VAL(100),NA(100)
      DIMENSION  BOT(100),SID(5)
      EQUIVALENCE (S,O),(S(101),VAL),(S(201),NA),(S(301),BOT),(S(401),SI
     1D),(S(406),BOX),(S(407),PIP),(S(408),TMP),(X,KU)
      EQUIVALENCE (A,KU(61)),(AN,KU(69)),(CN,KU(77)),(TN,KU(85)),(FRM,
     1KU(93))
      REAL LOGF,LGAMF,NORMAL
      DATA     FMH(1)/4H(4X, /
      DATA     FMH(2)/4H4F17 /
      DATA     FMH(3)/4H.8,6 /
      DATA     FMH(4)/4HX,I2 /
      DATA     FMH(5)/4H)    /
      DATA     BLANK/4H     /
      DATA     ASTRIC/4H*    /
C                                        FUNCTIONS DESCRIBING EACH EQ
      ATANF(P)=ATAN(P)
      COSF(P)=COS(P)
      EXPF(P)=EXP(P)
      LOGF(P)=ALOG(P)
      LGAMF(P)=ALGAMA(P)
```

```
      SQRTF(P)=SQRT(P)
      ABSF(P)=ABS(P)
      FLOATF(L)=FLOAT(L)
      SIGNF(P,Y)=SIGN(P,Y)
      PEARIF (Y0,B1,B2,D1,D2,P)=Y0*(1.+P/B1)**D1*(1.-P/B2)**D2
C
      PEARIV (Y0,B,V,R,D,P)=EXPF(Y0-D*LOGF(1.+(P/B-V/R)**2)-V*ATANF(P/B-
     2V/R))
      NORMAL (Y0,C,P)=Y0*EXPF(-P**2/C)
      PEARVI (Y0,B1,B2,D1,D2,P)=EXPF(Y0+D2*LOGF(1.+P/B2)-D1*LOGF(1.+P/B1
     2))
       PERIII (Y0,G,P,A,U)=Y0*EXPF(P*LOGF(1.+U/A)-G*U)
      NORM = 0.0
      N1=MKX
      N2=MKZ
      N3=1
      IF(N2.EQ.7) N3=2
      IF(N1.EQ.2) GO TO 696
  695 LT=0
      LIMIT=100
C
C
      NT1=0
       NOUTP=6
C
C
C                          CHANGE NEXT 2 CARDS FOR SPECIAL TAPE CONFIGURATI
       NINTP=5
C
       L2=NINTP
       L3=NOUTP
 9831 CALL OVERFL(J)
```

```
      IF(N1 .EQ. 3) GO TO 604
C
C                                                    READ *CONTROL CARD
 9030  READ          (L2,1)IDENT,NUMB,LIMIT,NORM,ITT,NOUT
 1     FORMAT (3X,I3,3X,I6,3X,I6,3X,I3,3X,I2,3X,I2)
      IF(IDENT - 999) 604,605,604
604   L2=NINTP
      L3=NOUTP
C
       WRITE           (L3,9022)
9022  FORMAT (1H1,  92X,22HDISTRIBUTION GENERATOR)
      DO 600 I=1,2641
600    S(I)=0.
      IF(N1.NE.3) GO TO 2030
      N1=1
      GO TO 2081
C                                                    DETERMINE OPTIONS
C                                                    TAPES
2080   IF (ITT) 2,2,3
 2     ITT=L2
 3     IF (NOUT) 4,4,5
 4     NOUT=L3
C                                                    TYPE OF INPUT
 5     IF(IDENT) 40,8,6
C                                                    DATA POINT
 6     READ          (L2,7)(FM(I),I=1,18)
 7    FORMAT(18A4)
       GO TO 10
C                                                    BUILT IN FORMAT
 8    CONTINUE
        FM(1) = FMH(1)
        FM(2) = FMH(2)
```

```
       FM(3) = FMH(3)
       FM(4) = FMH(4)
       FM(5) = FMH(5)
      IDENT=4
C                                                      NUMBER OF DP CARDS GIVEN
C                                                      OR CALCULATED
 10   IF (NUMB) 11,11,12
 11   NUMB=32000
 12   M=0
C                                                      INITIALIZE SUMS
      AN(1)=0.
      AN(2)=0.
      AN(3)=0.
      AN(4)=0.
C                                                      READ DATA POINT CARDS
 13   READ          (ITT,FM)(X(I),I=1,IDENT),LST
      IF (LST) 14,14,20
 14   M=M+IDENT
C                                                      CALCULATE SUMS
 15   DO 17 I=1,IDENT
     X(I)=(X(I)-APRXM)/SQRT(APRXV)
      AN(1)=AN(1)+X(I)
     Z=X(I)*X(I)
      AN(2)=AN(2)+Z
     Z=Z*X(I)
      AN(3)=AN(3)+Z
   17 AN(4)=AN(4)+X(I)*Z
     CALL OVERFL(J3K)
     GO TO (780,16,16),J3K
  780 LIMIT=0
 16   NUMB=NUMB-IDENT
C                                                      DETERMINE OF DP CARDS
```

```
      IF (NUMB) 25,25,13
 20   M=M-IDENT+LST
      GO TO 27
 25   M=M+NUMB
C                                          HOW MANY OP
 27   FN=M
     IF(LIMIT.EQ.0) GO TO 910
C                                          FORM MOMENTS
      DO  30  K=1,4
   30 A(K)=AN(K)/FN
     Z=A(1)*A(1)
     CM2(N3)=A(2)-Z
      W=A(1)*A(2)
      Z=Z*A(1)
     CM3(N3)=A(3)-3.*W       +2.*Z
     CM4(N3)=A(4)-4.*A(1)*A(3)+6.*A(1)*W       -3.*A(1)*Z
     Z=SQRT(APRXV)
     CM2(N3)=CM2(N3)*Z**2
     CM3(N3)=CM3(N3)*Z**3
     CM4(N3)=CM4(N3)*Z**4
     AVE(N3)=A(1)*Z+APRXM
     RCM2 =SQRT(CM2(N3))
C
C                                          CALCULATE COEFF FOR PEARSON
C
   35 TP = CM2(N3)*CM2(N3)
     BETA1 =CM3(N3) **2/(CM2(N3)* TP)
     BETA2 = CM4(N3)/TP
C    DETERMINE IF BETA1 AND BETA2 ADMISSIBLE
     IF(((BETA1-BETA2+1.)/(-1.414213562).LT.0).OR.(BETA1.LT.0.).OR.
    1((15.*BETA1-8.*BETA2+36.)/(-17.).GE.0.)) GO TO 750
      DOP=0.
```

```
      SK1=BETA1*(BETA2+3.)**2
      SK2=4.*(4.*BETA2-3.*BETA1)*(2.*BETA2-3.*BETA1-6.)
      IF (SK1) 351,621,351
 351  IF (SK2) 352,353,352
 352  SKAPPA=SK1/SK2
      GO TO 45
  621 SKAPPA=0.
      GO TO 45
 353  SKAPPA=(10.**10)**10
       IF (BETA1-4.) 8008,8007,8008
 8007  WRITE           (L3,8006)
8006  FORMAT(1H ,35X,17HSUBCLASS TYPE TEN)
       DOP=1.
       GO TO 45
 8008  WRITE           (L3,8009)
 8009 FORMAT(1H ,35X,19HSUBCLASS TYPE THREE)
       DOP=1.
      GO TO 45
C                                          READ MOMENTS CARD IF NECCESAR
   40 READ  (ITT,9023) AVE(N3),CM2(N3),CM3(N3),CM4(N3)
 9023  FORMAT (4X,4F17.8)
 2081 RCM2 = SQRT(CM2(N3))
      GO TO 35
C                                          DETERMINE MAIN TYPE
C                                                 OR FORCE FIT
 45    IF (NORM-1) 451,50,452
 452   IF (NORM-6) 71,111,90
 451   IF (DOP) 453,453,9400
 453   IF (SKAPPA)50,8001,8002
 8001  BB3=BETA2-3.0
       IF(BB3.LT. .0001 .AND. BB3 .GT. -.0001) GO TO 90
      IF (BETA2-3.)    8004,90,8003
```

```
 8002  IF (SKAPPA-1.) 71,8005,111
 8004  WRITE              (L3,8010)
 8010 FORMAT(1H ,35X,17HSUBCLASS TYPE TWO)
       GO TO 50
 8003  WRITE              (L3,8011)
 8011 FORMAT(1H ,35X,19HSUBCLASS TYPE SEVEN)
       GO TO 71
 8005  WRITE              (L3,8012)
 8012 FORMAT(1H ,35X,18HSUBCLASS TYPE FIVE)
       GO TO 111
C
C                                            TYPE ONE CONSTANTS
C
  50  WRITE              (L3,51)
  51  FORMAT(1H ,30X,22HPEARSON CURVE TYPE ONE)
 511  R=6.* (BETA2-BETA1-1.)/(6.+3.*BETA1-2.*BETA2)
      TP=           R*(R+2.)*SQRTF(BETA1/(BETA1*(R+2.)**2+16.*(R+1.)))
      RP=.5*(R-2.+TP)
      RM=.5*(R-2.-TP)
   53 IF (CM3(N3))57,56,56
  56  D2=RP
      D1=RM
      GO TO 58
  57  D1=RP
      D2=RM
  58  SCOTT=.5*RCM2       *SQRTF(BETA1*(R+2.)**2+16.*(R+1.))
      B2=SCOTT*(D2+1.)/(D2+D1+2.)
      B1=SCOTT-B2
      IF((D1+1.)*(D2+1.)) 903,903,582
 582  IF (B1+B2) 583,906,583
 583  Y0=SIGNF(1.0,B1+B2)*EXPF(D1*LOGF(D1+1.)-LOGF(ABSF(B1+B2))-LGAMF(D1
     2+1.)+D2*LOGF(D2+1.)-LGAMF(D2+1.)+LGAMF(D1+D2+2.)-(D1+D2)*LOGF(D1+D
```

```
     32+2.))
 585  WRITE            (L3,581)D1,D2,B1,B2,YO
 581  FORMAT (4HOM1=,1PE12.5,4H M2=,1PE12.5,4H A1=,1PE12.5,4H A2=,1PE12.
     25,4H YE=,1PE12.5)
      V1=B1
      V2=B2
      V3=D1
      V4=D2
C                                              JAB IS FOR INTEGRATION BRANCH
      JAB=1
      IF(B2    ) 60,906,63
 60   IF (B1) 61,61,906
C                                              RANGES
   61 H(N3)=(-B1-B2)/5120.
      UPPER=-B1
      EL (N3) =B2
      GO TO 145
 63   IF (B1) 906,64,64
   64 H(N3)=(B2+B1)/5120.
      UPPER=B2
      EL(N3)=-B1
      GO TO 145
C
C                                              TYPE FOUR CONSTANTS
C
  71  WRITE            (L3,72)
  72  FORMAT(1H ,30X,23HPEARSON CURVE TYPE FOUR)
  73  R=6.*(BETA2-BETA1-1.)/(2.*BETA2-3.*BETA1-6.)
  74  D=.5*(R+2.)
  75  V=R*(R-2.)*SQRTF(BETA1)/SQRTF(16.*(R-1.)-BETA1*(R-2.)**2)
 76   B=SQRTF(CM2(N3)/16.)*SQRTF(16.*(R-1.)-BETA1*(R-2.)**2)
      IF(CM3(N3)) 77,783,783
```

```
 783    V=-V
  77   PHI=ATANF(V/R)
  78   TOP=     ((COSF(PHI))**2/(3.*R)-1./(12.*R)-PHI*V)
        Y0=LOGF( .3989423)-LOGF(B)+.5*LOGF(R)+TOP-(R+1.)*LOGF(COSF(PHI))
        IF (Y0-38.9) 782,782,784
  782   Y1=EXPF(Y0)
        GO TO 785
  784   Y1=.99999999D 38
  785 WRITE            (L3,781)R,D,V,B,Y1
 781   FORMAT (3H0R=,1PE12.5,4H  M=,1PE12.5,4H  V=,1PE12.5,3H A=,1PE12.5,
     24H Y0=,1PE12.5)
       JAB=2
       H(N3)=RCM2 /512.
       UPPER=5.* RCM2
       EL (N3)=-UPPER
       V1=B
       V2=V
       V3=R
       V4=D
       GO TO 145
C
C                                        NORMAL CURVE CONSTANTS
C
  90   WRITE            (L3,91)
  91   FORMAT(1H ,30X,29HTHIS IS A NORMAL DISTRIBUTION)
   92 C = CM2(N3)*2.
       Y0=1./SQRTF(6.283185 *CM2(N3))
       WRITE            (L3,1101)C,Y0
 1101 FORMAT (3H0C=,1PE12.5,4H Y0=,1PE12.5)
       JAB=3
       H(N3)=RCM2 /512.
       UPPER=5.*RCM2
```

```
      EL (N3) = -UPPER
      V1=C
      GO TO 145
C
C                                              TYPE SIX CONSTANTS
C
 111  WRITE            (L3,112)
 112  FORMAT(1H ,30X,22HPEARSON CURVE TYPE SIX)
 113  R=6.*(BETA2-BETA1-1.)/(6.+3.*BETA1-2.*BETA2)
      CAT=.5*RCM2      *SQRTF(BETA1*(R+2.)**2+16.*(R+1.))
      IF(CM3(N3)) 9836,9837,9837
9836  CAT=-CAT
9837  TERM=R*(R+2.)/2.*SQRTF(BETA1/(BETA1*(R+2.)**2+16.*(R+1.)))
      D2=(R-2.)/2.+TERM
      D1=-((R-2.)/2.-TERM)
      TP=              ((D1-1.)-(D2+1.))
      B1=CAT*(D1-1.)/TP
      B2=CAT*(D2+1.)/TP
      CAC1=1.
      Y0=(D2*LOGF(D2+1.)-LGAMF(D2+1.)-LOGF(CAT)+(D1-D2)*LOGF(D1-D2-2.)-D
     21*LOGF(D1-1.)+LGAMF(D1)-LGAMF(D1-D2-1.))+LOGF(CAC1)
      Y1=EXPF(Y0)
      WRITE            (L3,1134)D2,D1,B1,B2,Y1
1134  FORMAT (4H0Q2=,1PE12.5,4H Q1=,1PE12.5,4H A1=,1PE12.5,4H A2=,1PE12.
     25,4H YE=,1PE12.5)
      JAB=4
      V1=B1
      V2=B2
      V3=D1
      V4=D2
      IF (B2   ) 115,906,120
 115  H(N3)=(-B2+5.*RCM2    )/5120.
```

```
       UPPER=-B2
      EL (N3) =-5.*RCM2
      GO TO 145
 120  UPPER=5.*RCM2
      H(N3)=(B2+UPPER    )/5120.
      EL (N3) =-B2
       GO TO 145
C
C                                                    TYPE THREE CONSTANTS
C
 9400 G=2.*CM2(N3) /CM3(N3)
       P=4./BETA1-1.
       A1=(P+1.)/G
       YO=G*(P+1.)**P/EXPF(P+1.+LGAMF(P+1.))
       YO=ABSF(YO)
       WRITE           (L3,9401)G,P,A1,YO
 9401  FORMAT (3H0G=,1PE12.5,3H P=,1PE12.5,3H A=,1PE12.5,4H YF=,1PE12.5)
       JAB=5
       V1=G
       V2=P
       V3=A1
       IF (A1) 9402,9403,9403
 9402 H(N3) =(-A1+5.*RCM2 )/5120.
       UPPER=-A1
      EL(N3)=-5.*RCM2
       GO TO 145
 9403  UPPER=5.*RCM2
      H(N3)=(A1+UPPER      )/5120.
      EL(N3)=-A1
C                                                    INITIALIZE OUTPUT SUMS
C                                                    GENERATION
C                                                    BRANCH TO INTEGRATE
```

```
 950   FORMAT(1H )
145    V0=Y0
       V6=EL(N3)
       V7=UPPER
951    DO 146 KK=1,4
 146   FAN(KK)=0.
        IF (LIMIT) 148,148,147
147    S(1)=0.
 153    GO TO (200,250,300,350,460),JAB
C
C                                                  TYPE ONE INTEG
C
  200 IF (D1) 2054,2055,2055
 2055 IF (D2) 2056,2057,2057
 2057 EXVAL=EL(N3)
      ADD1=EL(N3)+H(N3)
      TP = H(N3)+H(N3)
      XVAL = EL(N3)+TP
      I=2
 205  SKUNK(1)=PEARIF(Y0,B1,B2,D1,D2,EXVAL)
 2051 SKUNK(2)=PEARIF(Y0,B1,B2,D1,D2,ADD1)
 2052 SKUNK(3)=PEARIF(Y0,B1,B2,D1,D2,XVAL)
 2053 S(I)=S(I-1)+SKUNK(1)+4.*SKUNK(2)+SKUNK(3)
      IF (XVAL+TP-UPPER)210,210,385
 210  EXVAL=XVAL
      ADD1 = XVAL + H(N3)
      XVAL=XVAL+TP
      I=I+1
      GO TO 205
C
C                                                  TYPE ONE L SHAPE
 2054 IF (D2) 7001,3001,3001
```

```
 3001 H(N3)=(UPPER-EL(N3))/5121.
      EL(N3)=EL(N3)+H(N3)
      EXVAL=EL(N3)
      ADD1=EL(N3)+H(N3)
      TP=H(N3)+H(N3)
      XVAL = EL(N3)+TP
      S(2)=0.
      I=3
 3002 SKUNK(1)=PEARIF(YO,B1,B2,D1,D2,EXVAL)
      SKUNK(2)=PEARIF(YO,B1,B2,D1,D2,ADD1)
      SKUNK(3)=PEARIF(YO,B1,B2,D1,D2,XVAL)
      S(I)=S(I-1)+SKUNK(1)+4.*SKUNK(2)+SKUNK(3)
      IF (XVAL+TP-UPPER)3003,3003,3004
 3003 EXVAL=XVAL
      ADD1=XVAL+H(N3)
      XVAL=XVAL+TP
      I=I+1
      GO TO 3002
 3004 S(2)=3./H(N3)-S(I)
 4005 N(N3)=I
      KI=N(N3)
      DO 4006 I=3,KI
 4006 S(I)=S(I)+S(2)
      I=N(N3)
      GO TO 385
C
C                                             TYPE ONE U SHAPE
 7001 H(N3)=(UPPER-EL(N3))/5122.
      EL(N3)=EL(N3)+H(N3)
      UPPER=UPPER-H(N3)
      EXVAL=EL(N3)
      ADD1=EL(N3)+H(N3)
```

```
      TP=  2*H(N3)
      XVAL=EL(N3)+TP
       I=2
 7002 SKUNK(1)=PEARIF(YO,B1,B2,D1,D2,EXVAL)
      SKUNK(2)=PEARIF(YO,B1,B2,D1,D2,ADD1)
      SKUNK(3)=PEARIF(YO,B1,B2,D1,D2,XVAL)
      S(I)=S(I-1)+SKUNK(1)+4.*SKUNK(2)+SKUNK(3)
       IF  (XVAL+TP-UPPER)7003,7003,385
 7003 EXVAL=XVAL
      ADD1=XVAL+H(N3)
      XVAL=XVAL+TP
      I=I+1
      GO TO  7002
C
C                                           TYPE ONE J SHAPE
 2056 H(N3)  =  (UPPER-EL(N3))/5121.
      UPPER=UPPER-H(N3)
      EXVAL=EL(N3)
 4002 SKUNK(1)=PEARIF(YO,B1,B2,D1,D2,EXVAL)
      ADD1=EL(N3)+H(N3)
      TP=  2*H(N3)
      XVAL=EL(N3)+TP
      I=2
      SKUNK(2)=PEARIF(YO,B1,B2,D1,D2,ADD1)
      SKUNK(3)=PEARIF(YO,B1,B2,D1,D2,XVAL)
      S(I)=S(I-1)+SKUNK(1)+4.*SKUNK(2)+SKUNK(3)
      IF  (XVAL+TP-UPPER)4003,4003,385
 4003 EXVAL=XVAL
      ADD1=XVAL+H(N3)
      XVAL=XVAL+2.*H(N3)
      I=I+1
      GO TO  4002
```

```
C
C                                                   TYPE FOUR INTEG
  250 EXVAL=EL(N3)
      ADD1=EL(N3)+H(N3)
      TP= 2*H(N3)
      XVAL=EL(N3)+TP
      I=2
  255 SKUNK(1)=PEARIV  (YO,B,V,R,D,EXVAL)
      SKUNK(2)=PEARIV  (YO,B,V,R,D,ADD1)
      SKUNK(3)=PEARIV  (YO,B,V,R,D,XVAL)
      S(I)=S(I-1)+SKUNK(1)+4.*SKUNK(2)+SKUNK(3)
      IF (XVAL-UPPER) 260,385,385
  260 EXVAL=XVAL
      ADD1=XVAL+H(N3)
      XVAL=XVAL+TP
      I=I+1
      GO TO 255
C
C                                                   NORMAL INTEG
  300 TP= 2*H(N3)
      EXVAL=EL(N3)-TP
      ADD1=EL(N3)-H(N3)
      XVAL=EL(N3)
      I=2
  305 SKUNK(1)=NORMAL  (YO,C,EXVAL)
      SKUNK(2)=NORMAL  (YO,C,ADD1)
      SKUNK(3)=NORMAL  (YO,C,XVAL)
      S(I)=S(I-1)+SKUNK(1)+4.*SKUNK(2)+SKUNK(3)
      IF (XVAL-UPPER) 310,385,385
  310 EXVAL=XVAL
      ADD1=XVAL+H(N3)
      XVAL=XVAL+TP
```

```
      I=I+1
      GO TO 305
C
C                                              TYPE SIX  INTRG
  350 EXVAL=EL(N3)
      ADD1=EL(N3)+H(N3)
      TP=  2*H(N3)
      XVAL=EL(N3)+TP
      I=2
 3551 SKUNK(1)=0
      GO TO 3553
 3552 SKUNK(1)=PEARVI  (Y0,B1,B2,D1,D2,EXVAL)
 3553 SKUNK(2)=PEARVI  (Y0,B1,B2,D1,D2,ADD1)
      SKUNK(3)=PEARVI  (Y0,B1,B2,D1,D2,XVAL)
 3559 S(I)=S(I-1)+SKUNK(1)+4.*SKUNK(2)+SKUNK(3)
      IF (XVAL-UPPER) 360,385,385
 360  EXVAL=XVAL
      ADD1=XVAL+H(N3)
      XVAL=XVAL+TP
      I=I+1
      GO TO 3552
C
C                                              TYPE THREE  INTEG
  460 EXVAL=EL(N3)
      ADD1=EL(N3)+H(N3)
      TP=  2*H(N3)
      XVAL=EL(N3)+TP
       I=2
       SKUNK(1)=0
       GO TO 462
 461   SKUNK(1)=PERIII  (Y0,G,P,A1,EXVAL)
 462   SKUNK(2)=PERIII  (Y0,G,P,A1,ADD1)
```

```
       SKUNK(3)=PERIII (YO,G,P,A1,XVAL)
       S(I)=S(I-1)+SKUNK(1)+4.*SKUNK(2)+SKUNK(3)
       IF (XVAL-UPPER) 463,385,385
 463   EXVAL=XVAL
      ADD1=XVAL+H(N3)
       XVAL=XVAL+TP
       I=I+1
       GO TO 461
  385 A2B=H(N3)/3.
  682 N(N3)=I+1
      N33=N(N3)
  678 DO  3851 I=1,N33
 3851 S(I)=S(I)*A2B
      LL=N(N3)
      S(LL+1)=1.
C
C                                   RANDOM UNIT CALCULATION
 9301 KU(1)=1
C
      A1=0.
      I=1
      DO 675 J=2,100
      A1=A1+.01
      M=I
      KKI=N(N3)
      DO 676 I=M,KKI
      IF(S(I)-A1)676,675,675
676   CONTINUE
  675 KU(J)=I
      KU(101)=N(N3)
C
      CALL OVERFL(J3K)
```

```
696    DO 393 K=1,LIMIT
       RND=RDM(1.0)
       RR=    (100.*RND) + 1.
       NR=RR
  671  M=KU(NR)
       KKI=N(N3)
       DO 387 I=M,KKI
       IF(S(I)-RND) 387,390,388
387    CONTINUE
388    BUT=FLOAT(I+I-2)-2.*(S(I)-RND)/(S(I)-S(I-1))
C
        GO TO 391
  390  BUT=I+I-2
  391  TOM(K)=EL(N3)+BUT*H(N3)+AVE(N3)
       IF(N2.EQ.5) GO TO 393
683     DO 685 KK=1,4
       Z=SQRT(CM2(N3))
685    FAN(KK)=FAN(KK)+((TOM(K)-AVE(N3))/Z)**KK
393     CONTINUE
C
       LT=LT+LIMIT
       GO TO  (952,400,953,400,952,401,401),N2
952    RETURN
  401  KKI=N(N3)
       DO 720 MM=1,KKI
  720  SS(N3,MM) = S(MM)
       DO 730 MM=1,101
  730  K2U(N3,MM) = KU(MM)
       GO TO 400
C                                                CALC OUTPUT MOMENTS
  953  WRITE(7,954) N(N3),EL(N3),H(N3),AVE(N3),CM2(N3),CM3(N3),CM4(N3)
  954  FORMAT(I4,4F18.8/2F18.8)
```

```
      WRITE(7,955) (KU(I),I=1,101)
  955 FORMAT(20I4)
      KN=N(N3)
      WRITE(7,956) (S(I),I=1,KN)
  956 FORMAT(8F10.8)
  400 IF(FAN(2).EQ.0.) GO TO 148
      TP=FLOAT(LT)
      DO 405 KK=1,4
  405 FA(KK)=FAN(KK)/TP
      FCM2 =FA(2)-FA(1)**2
      FCM3 =FA(3)-3.*FA(1)*FA(2)+2.*FA(1)**3
      FCM4 =FA(4)-4.*FA(1)*FA(3)+6.*FA(1)**2*FA(2)-3.*FA(1)**4
      FCM2=FCM2*Z**2
      FCM3=FCM3*Z**3
      FCM4=FCM4*Z**4
      FA(1)=FA(1)*Z+AVE(N3)
      FBETA1=FCM3**2/FCM2**3
      FBETA2=FCM4/FCM2**2
      FKAPPA=FBETA1*(FBETA2+3.)**2/(4.*(4.*FBETA2-3.*FBETA1)*
     2             (2.*FBETA2-3.*FBETA1-6.))
      CALL OVERFL(J3K)
      IF(J3K .EQ. 1) WRITE(6,922)
  922 FORMAT(6H OVRFL)
C                                        WRITE BOTH SETS OF MOMENTS
      WRITE            (L3,410) LT
 410  FORMAT(1H0,27X,32HMOMENTS        FROM ORIGINAL DATA,6X,
     219HFROM GENERATED DATA,6H    N=,I6)
  407 WRITE(L3,411) AVE(N3),FA(1),CM2(N3),FCM2,CM3(N3),FCM3,CM4(N3),
     1 FCM4,
     2             BETA1,FBETA1,BETA2,FBETA2,SKAPPA,FKAPPA
 411  FORMAT (35X,4HMEAN,3X,F16.8,4X,F18.8 /31X,8HVARIANCE,3X,F18.8,4X,
     2             F18.8 /34X,5HMU(3),3X,F18.8,4X,F18.8 /34X,5HMU(4),
```

```
     3                 3X,F18.8,4X,F18.8 /33X,6HBETA 1,3X,F18.8,4X,F18.8/
     4                 33X,6HBETA 2,3X,F18.8,4X,F18.8 /34X,5HKAPPA,3X,
     5                 F18.8,4X,F18.8)
C
C
C
C
681    IF(N2.EQ.4) GO TO 680
  412  RETURN
C
C                                                     NO GENERATIN WRITE OUT
 148    WRITE              (L3,149)
 149    FORMAT(1H0,51X,17HMOMENTS FROM DATA  )
       WRITE(L3,150) AVE(N3),CM2(N3),CM3(N3),CM4(N3),BETA1,BETA2,
      1 SKAPPA
 150    FORMAT (54X,4HMEAN,4X,F18.8 /50X,8HVARIANCE,4X,F18.8 /53X,5HMU(3)
      2   ,4X,F18.8 /53X,5HMU(4),4X,F18.8 /52X,6HBETA 1,4X,F18.8 /52X,6HB
      3ETA 2,4X,F18.8 /53X,5HKAPPA,4X,F18.8)
       IF(N2 .EQ. 4) GO TO 680
       RETURN
C                 GRAPH ROUTINE
680    WRITE(L3,950)
       DO 80 I=1,100
       O(I) = BLANK
 80    CONTINUE
       DO 81 I=1,5
  81   SID(I)=0.
       BOX=(V7-V6)/99.
       Q=V6+BOX
       PIP=0.
       SMAX=0.
       SMIN = 16.**62
```

```
      DO 82 I=2,99
      GO TO (83,84,85,86,87),JAB
83    VAL(I)=PEARIF(V0,V1,V2,V3,V4,Q)
      GO TO 88
84    VAL(I)=PEARIV(V0,V1,V2,V3,V4,Q)
      GO TO 88
85    VAL(I)=NORMAL(V0,V1,Q)
      GO TO 88
86    VAL(I)=PEARVI(V0,V1,V2,V3,V4,Q)
      GO TO 88
87    VAL(I)=PERIII(V0,V1,V2,V3,Q)
 88   BOT(I)=Q+AVE(N3)
      Q=Q+BOX
      IF(VAL(I)-SMAX) 93,93,89
89    SMAX=VAL(I)
93    IF(VAL(I)-SMIN) 94,82,82
94    SMIN=VAL(I)
82    CONTINUE
      N(N3)=0.
      DO 95 K=1,5
      E=8*(6-K)
      SID(K)=SMIN+E/39.*(SMAX-SMIN)
      DO 95 J=1,8
      N(N3)=N(N3)+1
      NA(1)=1
      NA(2)=1
      M1=0
      DO 96 I=2,99
      TMP   =(VAL(I)-SMIN)/(SMAX-SMIN)*39.+1.
      L=TMP
      L=41-L
      IF(N(N3).NE.L) GO TO 96
```

```
      O(I) = ASTRIC
      MI=MI+1
      NA(MI)=I
   96 CONTINUE
      GO TO (97,98,98,98,98,98,98,98),J
   97 WRITE (L3,99) SID(K),(O(I),I=1,100)
   99 FORMAT(2X,E9.2,2H +,100A1)
      GO TO 65
   98 WRITE(L3,66) (O(I),I=1,100)
   66 FORMAT(13X,100A1)
   65 IF(MI .LE. 0 )    MI = 1
      DO 95 I=1,MI
      L=NA(I)
      O(L) = BLANK
   95 CONTINUE
C
C
      WRITE             (L3,67)(BOT(I),I=10,90,10)
   67 FORMAT ( /13X,9(9X,1H+)/20X,9F10.3)
      RETURN
C                                          ERROR NOTIFICATION
  750 WRITE(L3,751) BETA1,BETA2
  751 FORMAT(3X,2F18.8,29H ILLEGAL VALUES FOR B1 AND B2)
      GO TO 983
  903 WRITE             (L3,904)D1,D2
  904 FORMAT (3X,2F18.8,29H ILLEGAL  VALUES FOR M1 AND M2)
       GO TO 983
  906 WRITE             (L3,907)B1,B2
  907 FORMAT (3X,2F18.8,29H ILLEGAL  VALUES FOR A1 AND A2)
       GO TO 983
  910  WRITE             (L3,911)
  911  FORMAT (3X,33HTOO MUCH AND / OR TOO LARGE DATUM)
```

```
C                                                              SKIP              TO NXT PASS
  983 LIMIT=0
      RETURN
  605 STOP
      END
      FUNCTION RDM(X)
      COMMON / VPI001/ IX
      IY = IX*65539
      IF ( IY ) 1,2,2
1     IY = IY+ 2147483647 +1
2     YFL = IY
      RDM = YFL * .4656613E-9
      IX = IY
      RETURN
      END
      FUNCTION RNOR (X)
      COMMON /VPI002/ IX
      A = 0.0
      DO 3 I=1,12
      IY = IX*65539
      IF(IY) 1,2,2
    1 IY = IY + 2147483647 + 1
    2 YFL = IY
      IX = IY
    3 A = A + YFL * .4656613 E-9
      RNOR = A-6.0
 919  RETURN
       END
```

APPENDIX VII

Computer Program to Determine Mean, Variance, and Minimum Value of One Million Numbers

```
PROGRAM TO COMPUTE MEAN, VARIANCE, AND MINIMUM VALUE
OF ONE MILLION NUMBERS
      REAL * 8 SUM,SSQR
      REAL * 8 FAN,FA
      REAL * 8 VAR,TRMEAN
      COMMON/Z7/APRXM,APRXV
      COMMON/VPI001/IX/VPI002/KX
      COMMON/MCK1/AVE(2),CM2(2),CM3(2),CM4(2),BETA1,BETA2,SKAPPA
      COMMON/MCK2/TOM(100),LIMIT
      APRXM=0.00
      APRXV=1.000
      IX=65549
      AVE(1)=0.0
      CM2(1)=1.00000
      CM3(1)=1.2247449
      CM4(1)=4.450000
      SSQR=0.0
      SUM=0.0
      AMIN=100.00
      CALL PURGE3 (3,5)
      DO 400 J=1,100
      IF(TOM(J).LT.AMIN) AMIN=TOM(J)
      SUM=SUM+TOM(J)
      SSQR=SSQR+(TOM(J)**2)
400   CONTINUE
      DO 300 K=1,9999
      CALL PURGE3 (2,5)
      DO 500 J=1,100
      IF(TOM(J).LT.AMIN) AMIN=TOM(J)
      SUM=SUM+TOM(J)
      SSQR=SSQR+(TOM(J)**2)
500   CONTINUE
```

```
300    CONTINUE
       SUMSQ=SUM**2
       VAR=(SSQR-(SUMSQ/1000000.))/1000000.
       TRMEAN=SUM/1000000.
       WRITE(6,600) TRMEAN
600    FORMAT('0','TRUE MEAN = ',F60.8)
       WRITE(6,601) VAR
601    FORMAT('0','TRUE VARIANCE = ',F60.8)
       WRITE(6,700) AMIN
700    FORMAT('0','MINIMUM VALUE =',F60.8)
       STOP
       END
```

APPENDIX VIII

Computer Program to Prepare Histogram

of One Million Numbers

```
PROGRAM TO PREPARE PRINTED HISTOGRAM OF 1 MILLION NUMBERS
      REAL * 8 FAN
      REAL * 8 FA
      REAL * 8 REAL,DELTA
      DIMENSION IPDF(100)
      COMMON/VPI001/IX/VPI002/KX
      COMMON/MCK1/AVE(2),CM2(2),CM3(2),CM4(2),BETA1,BETA2,SKAPPA
      COMMON/MCK2/TUM(100),LIMIT
      IX=65549
C     MOMENTS FOR RGB 30
      TUAVE=1.19
      DELTA=(6.00+TUAVE)/100.
C     DELTA IS BASED ON 6 STANDARD DEVIATIONS
      AVE(1)=1.18972513
      CM2(1)=.97706807
      CM3(1)=1.18285871
      CM4(1)=4.24824603
      DO 300 K=1,100
      IPDF(K)=0
300   CONTINUE
      CALL PURGE3 (3,5)
      DO 400 J=1,100
      REAL=TOM(J)/DELTA
      ANO=REAL+.5
      I=ANO
      IF(I.GT.100) I=100
      IPDF(I)=IPDF(I)+1
400   CONTINUE
      DO 500 J=1,9999
      CALL PURGE3 (2,5)
      DO 600 L=1,100
      REAL=TOM(L)/DELTA
```

```
      AND=REAL+.5
      I=AND
      IF(I.GT.100) I=100
      IPDF(I)=IPDF(I)+1
600   CONTINUE
500   CONTINUE
      WRITE (7,701)
701   FORMAT ('RGB 30')
      WRITE (7,700) (IPDF(I),I=1,100)
700   FORMAT (I7)
      WRITE (6,801)
801   FORMAT ('0','RGB 30')
      WRITE (6,800) (IPDF(I),I=1,100)
800   FORMAT ('0',10I10)
      STOP
      END
```

APPENDIX IX

Computer Program to Determine Proportions

of Times Type I and Type II

Errors are Committed

```
PROGRAM TO DETERMINE THE PROPORTION OF TIMES AN ALPHA ERROR OR
A BETA ERROR WAS COMMITTED IN THE 1,000 LOOPS OF THE SIMULATION
PROCEDURES.
          REAL * 8 FAN
          REAL * 8 FA
          DIMENSION TABA(4),TAB1(4),TAB2(4),SSS(5),NS(5),NN(4),SEBS(4)
          DIMENSION ALPHA(4),B1MY(4),B2MY(4),FA(4),FAN(4),SEA(4),SEB1(4)
          COMMON/Z7/APRXM,APRXV
          COMMON/VPI001/IX/VPI002/KX
          COMMON/MCK1/AVE(2),CM2(2),CM3(2),CM4(2),BETA1,BETA2,SKAPPA
          COMMON/MCK2/TUM(100),LIMIT
          COMMON/MCK4/N(2),EL(2),H(2),K2U(2,101),SS(2,2641)
          COMMON/Z2/FA/Z3/FAN/Z4/FCM2,FCM3,FCM4
          COMMON/Z5/FBETA1,FBETA2,FKAPPA
          APRXM=1.19
          APRXV=1.000
          LOOPS=1000
          DLOOPS=LOOPS
          AVE(1)=1.18972513
          CM2(1)=.97706807
          CM3(1)=1.18285871
          CM4(1)=4.24824603
          TUAVE=1.19
          PREC=.14573436*TUAVE
          ZALPHA=1.960
          ZBETA=.842
C         ZALPHA=1.282 FOR 2 SIDED TEST WHERE ALPHA=20%
C         ZALPHA=1.439 FOR 2 SIDED TEST WHERE ALPHA=15%
C         ZALPHA=1.645 FOR 2 SIDED TEST WHERE ALPHA=10%
C         ZALPHA=1.751 FOR 2 SIDED TEST WHERE ALPHA=8%
C         ZALPHA=1.960 FOR 2 SIDED TEST WHERE ALPHA=5%
C         ZALPHA=2.576 FOR 2 SIDED TEST WHERE ALPHA=1%
```

```
C       ZALPHA=3.291 FOR 2 SIDED TEST WHERE ALPHA=.1%
C       ZBETA=.842 MEANS BETA = .20 FOR 1 SIDED TEST
C       ZBETA=1.282 FOR 1 SIDED TEST WHERE BETA=10%
C       ZBETA=1.645 FOR 1 SIDED TEST WHERE BETA=5%
        IX=65549
        RPS=30.
        IPS=RPS
        DO 21 J=1,4
        TAB1(J)=0.0
        TAB2(J)=0.0
        TABA(J)=0.0
21      CONTINUE
        SSTOT=0.0
        SSSSQR=0.0
        CALL PURGE3 (3,1)
        COUNT=1.
        MO=1
        DO 1000 M=1,LOOPS
        TOT=0.0
        SSQRS=0.0
        MOR=MO+(IPS-1)
        DO 130 J=MO,MOR
        IF(J.EQ.1) GO TO 120
        L=MOD(J,100)
        IF(L.EQ.1) CALL PURGE3(2,1)
        IF(L.EQ.1) COUNT=COUNT+1.
        IF(L.EQ.0) L=100
C       MODULAR DIVISION TO CALL RNDM AT END OF EACH 100 NOS.
        GO TO 125
120     L=1
125     TOT=TOT+TOM(L)
        SSQRS=SSQRS+(TOM(L)**2)
```

```
130     CONTINUE
        MD=MOR+1
        TOTSQ=TOT**2
        VAR=(SSQRS-(TOTSQ/RPS))/(RPS-1.)
        S=SQRT(VAR)
        AVER=TOT/RPS
        SQSS=SQRT(RPS)
C       STATS ON PRELIMINARY SAMPLE
C       BEGIN COMPUTATION OF SAMPLE SIZES
C       BELOW PUTS REAL SAMPLE SIZE IN COL 3 OF ARRAY SS
        SSS(3)=(((S*(ZALPHA+ZBETA))/PREC)**2)
        NS(3)=SSS(3)+.5
        SSS(3)=NS(3)
        SSS(1)=RPS
        SSS(2)=SSS(3)*.5
        SSS(4)=SSS(3)*1.5
        SSS(5)=SSS(3)*2.0
C       ABOVE PUTS MULTIPLE OF SAMPLE SIZE IN ARRAY SSS
C       MULTIPLES ARE .5,1.5,AND 2.0 FOR COLUMNS 2,4,5 RESPECTIVELY
        DO 200 J=1,5
        NS(J)=SSS(J)
200     CONTINUE
C       ABOVE LOOP PUTS SAMPLE SIZE IN INTEGER FORM IN ARRAY NS
        DO 300 J=1,5
        IF(NS(J).LT.IPS) NS(J)=IPS
300     CONTINUE
C       ABOVE LOOP GUARANTEES NO SAMPLE SIZE LESS THAN PREL SAMPLE  SIZE
        DO 220 J=1,5
        SSS(J)=NS(J)
220     CONTINUE
C       ABOVE LOOP CHANGES INTEGER SIZE BACK TO DECIMAL
        SSTOT=SSTOT+SSS(3)
```

```
      SSSSQR=SSSSQR+(SSS(3)**2)
C     SSTOT=SUM OF SAMPLE SIZES FROM FORMULA
C     SSSSQR=SUM OF SQUARES OF SAMPLE SIZES
      DO 351 J=2,5
      NN(J-1)=(NS(J)-NS(J-1))-1
351   CONTINUE
C     ABOVE LOOP COMPUTES NN ARRAY OF DIFFERENCES
C     HOW MANY MORE MINUS 1
      DO 400 K=1,4
      IF(NN(K).LT..00001) GO TO 705
      MOR=MO+NN(K)
      DO 420 J=MO,MOR
      L=MOD(J,100)
      IF(L.EQ.1) CALL PURGE3(2,1)
      IF(L.EQ.1) COUNT=COUNT+1.
      IF(L.EQ.0) L=100
      IF(L.GT.1) GO TO 419
419   TOT=TOT+TOM(L)
      SSQRS=SSQRS+(TOM(L)**2)
420   CONTINUE
C     ABOVE LOOP SUMS UNITS AND COMPUTES SUM OF SQUARES
      MO=MOR+1
      TOTSQ=TOT**2
      VAR=(SSQRS-(TOTSQ/SSS(K+1)))/(SSS(K+1)-1.)
      S=SQRT(VAR)
      AVER=TOT/SSS(K+1)
C     SQSS=SQUARE ROOT OF SAMPLE SIZE
      SQSS=SQRT(SSS(K+1))
705   DF=NS(K+1)-1
      IF(ZALPHA.EQ.1.282) GO TO 600
      IF(ZALPHA.EQ.1.645) GO TO 2000
      IF(ZALPHA.EQ.1.960) GO TO 4000
```

```
      IF(ZALPHA.FQ.1.439) GO TO 5000
      IF(ZALPHA.EQ.1.751) GO TO 6000
      IF(ZALPHA.EQ.2.576) GO TO 7000
      IF(ZALPHA.EQ.3.291) GO TO 7050
7050  IF(DF.LT.60.) GO TO 7060
      IF(DF.LT.120.) GO TO 7061
      IF(DF.EQ.120.) GO TO 7062
      TVALUE=3.291
      GO TO 670
7060  TVALUE=3.551
      GO TO 670
7061  TVALUE=3.460
      GO TO 670
7062  TVALUE=3.73
      GO TO 670
4000  IF(DF.LT.60.) GO TO 680
      IF(DF.LT.120.) GO TO 690
      IF(DF.EQ.120.) GO TO 695
      TVALUE=1.960
      GO TO 670
680   TVALUE=2.021
      GO TO 670
690   TVALUE=2.000
      GO TO 670
695   TVALUE=1.980
      GO TO 670
600   IF(DF.LT.60.) GO TO 630
      IF(DF.LT.120.) GO TO 640
      IF(DF.EQ.120.) GO TO 650
      TVALUE=1.282
      GO TO 670
630   TVALUE=1.303
```

```
      GO TO 670
640   TVALUE=1.296
      GO TO 670
650   TVALUE=1.289
      GO TO 670
2000  IF(DF.LT.60.) GO TO 2001
      IF(DF.LT.120.) GO TO 2002
      IF(DF.EQ.120.) GO TO 2003
      TVALUE=1.645
      GO TO 670
2001  TVALUE=1.684
      GO TO 670
2002  TVALUE=1.671
      GO TO 670
2003  TVALUE=1.658
      GO TO 670
5000  TVALUE=1.439
      GO TO 670
6000  TVALUE=1.751
      GO TO 670
7000  IF(DF.LT.60.) GO TO 7001
      IF(DF.LT.120.) GO TO 7002
      IF(DF.EQ.120.) GO TO 7003
      TVALUE=2.576
      GO TO 670
7001  TVALUE=2.704
      GO TO 670
7002  TVALUE=2.660
      GO TO 670
7003  TVALUE=2.617
670   CONTINUE
      FACTOR=TVALUE*(S/SQSS)
```

```
      CIU=AVER+FACTOR
      CIL=AVER-FACTOR
      IF(TUAVE.GT.CIL.AND.TUAVE.LT.CIU) TABA(K)=TABA(K)+1
C     TABA COLLECTS NO. OF TIMES AVE(1) IS INSIDE CONFIDENCE INTERVAL
C     NOW DETERMINE NO. OF TIMES TYPE II ERROR IS COMMITTED
      BOOKUP=TUAVE+PREC
      BOOKLO=TUAVE-PREC
      IF(BOOKUP.GT.CIL.AND.BOOKUP.LT.CIU) TAB1(K)=TAB1(K)+1.
      IF(BOOKLO.GT.CIL.AND.BOOKLO.LT.CIU) TAB2(K)=TAB2(K)+1.
C     TAB1 ACCOUNTS FOR TYPE II ERROR ON UPPER SIDE
C     TAB2 ACCOUNTS FOR TYPE II ERROR ON LOWER SIDE
400   CONTINUE
1000  CONTINUE
      DO 800 J=1,4
      ALPHA(J)=1.-(TABA(J)/DLOOPS)
      B1MY(J)=TAB1(J)/DLOOPS
      B2MY(J)=TAB2(J)/DLOOPS
800   CONTINUE
C     ALPHA(J)=1-ATTAINED RELIABILITY LEVEL
C     B1MY(J)=PROPORTION OF TIMES TYPE II ERROR COMMITTED ON UPPER SIDE
C     B2MY(J)=PROPORTION OF TIMES TYPE II ERROR COMMITTED ON LOWER SIDE
      CALL PURGE3(2,4)
      DO 805 J=1,4
      SEO=(ALPHA(J)*(1.-ALPHA(J)))/DLOOPS
      SEA(J)=SQRT(SEO)
      SEO=(B1MY(J)*(1.-B1MY(J)))/DLOOPS
      SEB1(J)=SQRT(SEO)
      SEO=(B2MY(J)*(1.-B2MY(J)))/DLOOPS
      SEB2(J)=SQRT(SEO)
805   CONTINUE
      TOTNUM=COUNT*100.
      STOTSQ=SSTOT**2
```

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      SSVAR=(SSSSQR-(STOTSQ/DLOOPS))/(DLOOPS-1.)
      STDV=SQRT(SSVAR)
      SSAVER=SSTOT/DLOOPS
      SSSRT=SQRT(DLOOPS)
      SSSE=STDV/SSSRT
C     THE FOLLOWING STATEMENTS PRINT THE RESULTS
      IF(ZALPHA.EQ.1.282) GO TO 870
      IF(ZALPHA.EQ.1.645) GO TO 3000
      IF(ZALPHA.EQ.1.960) GO TO 3020
      IF(ZALPHA.EQ.1.439) GO TO 3025
      IF(ZALPHA.EQ.1.751) GO TO 3030
      IF(ZALPHA.EQ.2.576) GO TO 3040
      IF(ZALPHA.EQ.3.291) GO TO 4050
3025  WRITE (6,3026)
3026  FORMAT ('0','ALPHA LEVEL = 15%')
      GO TO 872
3030  WRITE (6,3031)
3031  FORMAT ('0','ALPHA LEVEL = 8%')
      GO TO 872
3040  WRITE (6,3041)
3041  FORMAT ('0','ALPHA LEVEL = 1%')
      GO TO 872
3020  WRITE (6,860)
860   FORMAT('0','ALPHA LEVEL = 5%')
      GO TO 872
3000  WRITE(6,3001)
3001  FORMAT('0','ALPHA LEVEL=10%')
      GO TO 872
4050  WRITE (6,4051)
4051  FORMAT ('0','ALPHA LEVEL = .1%')
      GO TO 872
870   WRITE (6,871)
```

```
871     FORMAT('0','ALPHA LEVEL = 20%')
872     IF(ZBETA.EQ..842) GO TO 874
        IF(ZBETA.EQ.1.282) GO TO 3005
        WRITE (6,873)
873     FORMAT('0','BETA LEVEL = 5%')
        GO TO 876
3005    WRITE(6,3006)
3006    FORMAT ('0','BETA LEVEL=10%')
        GO TO 876
874     WRITE(6,875)
875     FORMAT('0','BETA LEVEL = 20%')
876     WRITE (6,877) PREC
877     FORMAT ('0','PRECISION = ',F30.8)
880     CONTINUE
        WRITE (6,1025) TOTNUM
1025    FORMAT('-','TOTAL GENERATED NUMBERS,N,= ',F30.0)
        WRITE (6,1016)
1016    FORMAT('-','NO. OF CORRECT CONFIDENCE INTERVALS')
        WRITE (6,890) (TABA(J),J=1,4)
890     FORMAT(' ','TABA= ',4F20.4)
        WRITE(6,1017)
1017    FORMAT('-','NO. OF TIMES TYPE II ERROR COMMITTED ON UPPER SIDE')
        WRITE (6,891) (TAB1(J),J=1,4)
891     FORMAT(' ','TAB1= ',4F20.4)
        WRITE (6,1018)
1018    FORMAT('-','NO. OF TIMES TYPE II ERROR COMMITTED ON LOWER SIDE')
        WRITE(6,892) (TAB2(J),J=1,4)
892     FORMAT(' ','TAB2= ',4F20.4)
        WRITE (6,980)
980     FORMAT('-','TYPE I ERROR COMMITTED')
        WRITE(6,981) (ALPHA(J),J=1,4)
981     FORMAT('0',26X,4F20.4)
```

```
      WRITE(6,1019) (SEA(J),J=1,4)
1019  FORMAT('0','STANDARD ERROR',12X,4F20.4)
      WRITE(6,982)
982   FORMAT('-','TYPE II ERROR ON UPPER SIDE')
      WRITE (6,983) (B1MY(J),J=1,4)
983   FORMAT ('0',26X,4F20.4)
      WRITE (6,1020) (SEB1(J),J=1,4)
1020  FORMAT('0','STANDARD ERROR',12X,4F20.4)
      WRITE (6,984)
984   FORMAT('-','TYPE II ERROR ON LOWER SIDE')
      WRITE (6,985) (B2MY(J),J=1,4)
985   FORMAT('0',26X,4F20.4)
      WRITE (6,1021) (SEB2(J),J=1,4)
1021  FORMAT('0','STANDARD ERROR',12X,4F20.4)
      WRITE (6,1013) SSAVER
1013  FORMAT('-','AVERAGE SAMPLE SIZE= ',F30.2)
      WRITE (6,1014) STDV
1014  FORMAT('-','STANDARD DEVIATION OF SAMPLE SIZES= ',F30.2)
      WRITE (6,1015) SSSE
1015  FORMAT('-','STANDARD ERROR OF SAMPLE SIZES= ',F30.2)
      STOP
      END
```

VITA

Ann B. Pushkin is currently employed by Northern Illinois University as an assistant professor of Accountancy. Before entering the Ph.D. program at Virginia Polytechnic Institute and State University in September, 1974, Mrs. Pushkin had served on the faculties of West Virginia University, Radford College, New River Community College, and Virginia Polytechnic Institute and State University.

During the period from January, 1972, through August, 1973, Mrs. Pushkin was employed by Leamon E. Simmons, C.P.A., in Christiansburg, Virginia. She received her Certified Public Accountant's certificate in January, 1970, from the State of West Virginia and in October, 1972, she received her Virginia certificate by reciprocity.

Mrs. Pushkin's first two years of undergraduate work was completed at West Virginia University. After transferring to Virginia Polytechnic Institute and State University in September, 1964, she completed her Bachelor of Science in Business with a major in accounting with high honors. After completing the B.S. degree she obtained a Master of Science with a major in accounting from Virginia Polytechnic Institute. The degrees were awarded in 1966 and 1968, respectively. The Doctor of Philosophy was completed in

Memberships are maintained in the American Institute of Certified Public Accountants, American Accounting Association, and the Institute of Internal Auditors. Mrs. Pushkin is also a member of Beta Alpha Psi, Beta Gamma Sigma, and Phi Kappa Phi.

Mrs. Pushkin is the former Ann Brannon of Clarksburg, West Virginia. She was born to Richard Garland Brannon and Blondena Boggess Brannon on January 15, 1935, in Spencer, West Virginia.

Ann B. Pushkin

AN INVESTIGATION OF THE VALIDITY OF AUDITING PROCEDURES USED IN MEAN-PER-UNIT SAMPLING PLANS

by

Ann B. Pushkin

(ABSTRACT)

Information from accounting literature indicates that the mean-per-unit (MPU) estimator may not properly control the alpha and beta risks under conditions of non-normality. This study concerns the effect of non-normal distributions on the validity of existing auditing procedures used in MPU sampling plans. The following question was specifically addressed: Are alpha and beta risk levels effectively controlled under conditions of non-normality when using the MPU estimator in conjunction with the sample size formula?

One objective of the study was to provide evidence that a potential problem exists when using traditional statistical sampling techniques with non-normal auditing populations. Another objective of the research was to provide recommendations that would lead to better auditing practices involving the MPU estimator.

A simulation study using positively skewed J-shaped distributions was conducted to provide a means for achieving the objectives of the research. The distributions used in the study ranged in degrees of skewness and kurtosis from 0.0 and 3.0081 to 25.6089 and 37.7937, respectively. Simulation procedures considered a

two-sided alpha level, an upper beta level concerning the potential acceptance of a materially overstated account, and a lower beta level for the potential acceptance of an account that is materially understated.

Simulation results indicate that specified alpha and beta levels are not always attained when using the MPU estimator under conditions of non-normality. Guidelines were then developed for simultaneous control of alpha and beta risk levels under the non-normal conditions simulated in the research. Case studies were utilized to test and illustrate use of the guidelines. Results of the study are limited, however, because implementation of the guidelines requires a positively skewed audit population reflected by a J-shaped distribution within the degrees of skewness and kurtosis simulated in the study.