EXPERIMENTAL COMPARISON OF PROBABILISTIC METHODS AND FUZZY SETS FOR DESIGNING UNDER UNCERTAINTY

by

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(ABSTRACT)

Recently, probabilistic methods have been used extensively to model uncertainty in many design optimization problems. An alternative approach for modeling uncertainties is fuzzy sets. Fuzzy sets usually require much less information than probabilistic methods and they rely on expert opinion. In principle, probability theory should work better in problems involving only random uncertainties, if sufficient information is available to model these uncertainties accurately. However, because such information is rarely available, probabilistic models rely on a number of assumptions regarding the magnitude of the uncertainties and their distributions and correlations. Moreover, modeling errors can introduce uncertainty in the predicted reliability of the system. Because of these assumptions and inaccuracies it is not clear if a design obtained from probabilistic optimization will actually be more reliable than a design obtained using fuzzy set optimization. Therefore, it is important to compare probabilistic methods and fuzzy sets and

determine the conditions under which each method provides more reliable designs. This research work aims to be a first step in that direction. The first objective is to understand how each approach maximizes reliability. The second objective is to experimentally compare designs obtained using each method.

A cantilevered truss structure is used as a test case. The truss is equipped with passive viscoelastic tuned dampers for vibration control. The structure is optimized by selecting locations for tuning masses added to the truss. The design requirement is that the acceleration at given points on the truss for a specified excitation be less than some upper limit. The properties of the dampers are the primary sources of uncertainty. They are described by their probability density functions in the probabilistic analysis. In the fuzzy set analysis, they are represented as fuzzy numbers.

Two pairs of alternate optimal designs are obtained from the probabilistic and the fuzzy set optimizations, respectively. The optimizations are performed using genetic algorithms. The probabilistic optimization minimizes the system probability of failure. Fuzzy set optimization minimizes the system possibility of failure. Problem parameters (e.g., upper limits on the acceleration) are selected in a way that the probabilities of failure of the alternate designs differ significantly, so that the difference can be measured with a relatively small number of experiments in the lab.

The main difference in the way each method maximizes safety is the following. Probabilistic optimization tries to reduce more the probabilities of failure of the modes that are easier to control. On the other hand, fuzzy set optimization tries to equalize the possibilities of failure of all failure modes.

These optimum probabilistic and fuzzy set designs are then compared in the laboratory. Twenty-nine realizations of each optimum design are tested and the failure rates are measured. The results confirm that, for the selected problems, probabilistic methods can provide designs that are significantly more reliable than designs obtained using fuzzy set methods.

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CHAPTER 1

INTRODUCTION

In this chapter, first, we explain the motivation that led us to this research project. Then, we review the literature on the methods for design under uncertainty. Following that, we contrast the two methods that will be the focus of this study (probabilistic methods and fuzzy sets). Next, we state the objectives of our work and the approach we followed. Finally, we present the outline of the dissertation.

1.1 Motivation

A designer often has to design a system in the presence of uncertainties in geometric, loading and material properties, as well as uncertainties in the operating environment. Uncertainties are often classified as *imprecision, modeling* and *random:*

1) Imprecision is vagueness in characterizing performance with terms such as good or unacceptable,

2) Modeling uncertainty is due to idealizations in modeling a system and simplifications in analyzing models to predict performance,

3) Random uncertainty reflects variations in the operating environment and the lack of control of a process by a designer.

There exist several methods for designing systems that must perform well in spite of uncertainties. These include safety factors, worst-case scenario, Taguchi methods, probabilistic methods and fuzzy sets.

In this study, we concentrate only on the treatment of random uncertainties. The most widely used methods for modeling random uncertainties are probability theory and fuzzy sets. In theory, probabilistic methods should be more effective for problems involving only random uncertainties, because they account for more information about these uncertainties than the other methods. However, probabilistic methods may require too much information. On the other hand, fuzzy set techniques require much less data than probabilistic techniques. The data is usually based on expert opinion.

It is important to develop guidelines that, depending on the amount of information on uncertainties and the accuracy of the predictive model used to solve a given problem, recommend what method to use. This will help designers efficiently design products that are less sensitive to uncertainties. However, different methods, including probabilistic methods, use approximate models of uncertainties and also approximate models for predicting the performance of a system. Consequently, it is not sufficient to use analysis to find out which method will lead to safer designs. Therefore, it is very important to compare methods for design under uncertainty experimentally. To our knowledge very little has been done in this direction. Experimental comparison is needed because the ultimate test is how well a method does in reality. This study on experimental comparison of probabilistic methods and fuzzy sets for designing under uncertainty aims to be a first step towards understanding how each method maximizes safety and developing guidelines on which method is better for a given problem.

1.2 Review of Methods for Designing Under Uncertainty

1.2.1 Approaches Using a Safety Factor or a Safety Margin

Traditionally, minimizing the risk of failure has been accomplished by using *safety factors* or *safety margins*. In this approach, the values defining limit conditions are either scaled by a given factor (safety factor approach), or shifted by a given amount (safety margin approach). The new values are then used as adjusted design requirements for the system. The values of the safety factors and safety margins are determined from experience and engineering judgment.

However, using safety factors may lead to inefficiencies (e.g., Yang, *et al.*, 1990), because they do not account effectively for the difference in magnitude of uncertainties in different properties. Furthermore, for some systems, even small variations in properties can lead to catastrophic reduction in performance, beyond what customary safety factors can compensate for (Elishakoff, 1983). Also, safety factors have to be specified by the designer for each failure mode and each component of the structure beforehand. Even if we try to calibrate safety factors so that they correspond to a required reliability level at the component level (e.g., Mischke, 1970), it is impractical to do the same in the system level.

Finally, there is a lack of experience and data to determine appropriate values of safety factors for new materials like composites (whose properties have large scatter). This can lead to very conservative attitudes towards those new technologies, which reduces or even eliminates the potential weight savings that could otherwise be achieved.

1.2.2 Methods that Rely on Worst Case Scenario

In view of all these difficulties, designers turned to methods that rely on the *worst case scenario* concept for improving safety and reducing sensitivity to errors. Several researchers, including Michael and Siddall (1981), Balling, *et al.* (1986), Chen, *et al.* (1984), and Bandler (1974), have incorporated *worst-case tolerances* into the design process. This approach uses lower and upper bounds for the possible variation of design variables. Then a "robust optimum" is determined, which is different from the "nominal optimum," and corresponds to the situation where all the variations occur simultaneously in the worst possible combination. Emch and Parkinson (1994), extended the method to accommodate tolerances not only for the design variables but for the other problem parameters as well.

Ben-Haim and Elishakoff (1990) proposed a method for finding the worst case scenario for problems where the space in which uncertain variables vary is convex. By exploiting the properties of the extremes of convex functions they found a closed form solution for the worst possible performance of a structure in several engineering problems involving uncertainties. Applications of convex modeling can be found in Elishakoff and Ben-Haim (1990), Ben-Haim and Elishakoff (1991), Elishakoff (1992), Elishakoff, *et al.* (1994a,b). In another publication, Elishakoff and Colombi (1993) combined probabilistic and convex models of uncertainty in random vibrations of structures.

However, methods that rely on worst case scenario can be too conservative because, when variations are independent of each other, it becomes very unlikely that they will simultaneously occur in the worst possible combination. Consequently, many researchers (e.g., Eggert, 1989 and Lewis, *et al.*, 1993), who have addressed the problem of incorporating statistically distributed tolerances into design optimization, allow the possibility of a small number of infeasible designs to achieve efficient designs.

Worst-case scenario methods had wide acceptance in the past because of their high degree of success in past design programs. Recently, however, besides keeping high performance and reliability levels, there is a demand for keeping costs low and minimizing weight. For that, worst-case scenario methods tend to become obsolete.

1.2.3 Taguchi Methods

Taguchi methodology (Taguchi, *et al.*, 1989, Bryne and Taguchi, 1987, Phadke, 1989, Otto and Antonsson, 1993) is an experimental approximation to minimizing variance in performance about a target for certain classes of problems. It integrates engineering insight with statistically designed experiments, in a simple cookbook manner. This experimental strategy is used to determine process conditions for achieving the target value and to identify variables that can be controlled to reduce variation in key performance characteristics.

Taguchi methods have become increasingly popular in industry. This is because of the renewed emphasis on product and service quality improvement as a means to attain competitive advantage. This method is also popular among designers, because it gives them the ability to increase the quality of a product through simple changes in the method by which they perform their usual design tasks.

An important disadvantage of Taguchi method is that it has no mechanism to handle design constraints. Otto and Antonsson (1993) have proposed extensions to the method, so that it can handle constraints, but this proposal has not been adopted by the industry yet.

1.2.4 Review of Probabilistic Methods

Probabilistic methods explicitly include uncertainties of the input parameters in the analysis. For given statistical data about geometry, loading and material properties, the statistics of the response quantities are evaluated using various analytical techniques. In problems involving optimization, the estimated system and/or component failure probability is used in the objective function or in the constraints of the optimization formulation.

The first direct reliability based formulation was presented by Charnes and Cooper (1958). They transformed the stochastic optimization problem into an equivalent deterministic formulation by linearly expanding the objective and constraints around the mean value of the random parameters. This approach was called *chance constrained programming (CCP)* technique. Hasofer and Lind (1974) introduced a refined version of the first-order reliability analysis by expanding the limit state functions around the most probable point (MPP) instead of the means of the random variables. This reduced the truncation errors due to the non-linearity of the response functions, but it increased the computational cost, because iterations are needed to locate the MPP. Since then, there has been a very large number of publications reporting optimal reliability-based designs in various engineering disciplines. A review of these publications can be found in Frangopol and Moses (1994).

However, probabilistic methods have not enjoyed great popularity in industry. One of the reasons is the long experience with the traditional safety factor approach. Another reason is that probabilistic design methods have a number of limitations:

1. In real life applications, there is rarely sufficient data to accurately estimate the statistics of the random parameters. This is particularly true at the tails of the distributions, because they correspond to extremely rare observations, which do not provide substantial evidence to support any particular choice of a distribution. On the other hand, some researchers (e.g., Moses and Stevenson, 1970 and Khalessi, *et al.*, 1994), maintain that it is more important to estimate accurately the standard deviations of uncertainties than to choose the

right distribution. Ben-Haim and Elishakoff (1990) and Fox and Safie (1992), showed that, in some cases, the predicted probability of failure is very sensitive to the choice of the probability distribution.

2. When approximate analytical techniques are used to evaluate the output scatter, the probability of failure can only be calculated by assuming a particular distribution for the output quantities. A common choice is the normal distribution. This choice is theoretically justified by the central-limit theorem, for linear problems in which the response is the sum of numerous random variables. However, for problems with pronounced non-linearities or with a small number of uncertain parameters dominating the output scatter, the distribution of the response quantities can be very different from normal. To avoid making an assumption on the distribution of the response quantities, some optimization methods are based on safety indices (a normalized measure of the distance from the nominal design point to the MPP). These methods are called *safety index optimization* (SIO). Examples of SIO can be found in Nikolaidis and Burdisso (1988) and Reddy, *et al.* (1993).

3. Modeling errors can greatly affect the predicted failure probability. Ben-Haim and Elishakoff (1990) present examples of simple structures, where the predicted failure rates largely depend on the theoretical assumptions used in the model, the effect of initial imperfections, etc. Ideally, modeling errors should be included in the probabilistic formulation as additional uncertainties. However, estimating the statistics of the modeling error is extremely difficult, because it requires data about analytical-experimental mismatch observed over a large number of systems of the same type. For example, Hasselman and Chrostowski (1990 and 1992) accumulated 22 sets of analysis/test data for dynamic analysis of conventional space structures. However, these results are applicable only for the particular type of analysis and structure.

Ponslet, *et al.* (1994) demonstrated experimentally the superiority of probabilistic optimization over deterministic optimization using safety factors. This study relied on complete knowledge of the statistical properties of the input random parameters. Also, modeling error was minimized by using experimental results to improve the analytical models, which may not be the case in real life design. Because of the above limitations, we can predict an idealized, *nominal* probability of failure, which can be significantly different from the actual one. It is not clear if probabilistic optimization, which relies on this nominal value, can yield better designs than other design methods.

1.2.5 Review of Studies on Fuzzy Sets

Including the uncertainties of the input parameters explicitly in the analysis make probabilistic methods a useful tool. Nevertheless, there are types of uncertainties that cannot be accounted by probability theory, because probability theory deals with events that are collections of outcomes of well-defined actions. These events must be subjected to repeatable testing and observations. Real-life problems are usually more complex than their corresponding mathematical models. Occasionally, we need to add a verbal explanation to the results obtained through models. The concept of *fuzzy sets* has been developed to deal with verbal information, that is usually meaningful but not clearly defined (imprecision).

The initial theory of fuzzy sets was formulated by Zadeh (1965). A fuzzy set is a set with boundaries that are not sharply defined. Later, Zadeh (1978) related the *theory of possibility* to the theory of fuzzy sets, by defining the concept of a *possibility distribution* as "a fuzzy restriction that acts as an elastic constraint on the values that may be assigned to a variable, called *fuzzy variable*." In the same publication, the author introduced the *possibility/probability consistency principle*. This principle constitutes a weak connection between possibility and probability for a variable that can be associated both with a possibility function and a probability function.

Fuzzy arithmetic is based on the *extension principle* (Zadeh, 1975), which permits the derivation of the membership functions of functions of fuzzy variables. Unfortunately, the extension principle is not trivial to implement directly. Researchers have proposed approaches such as the *α-cut approach* (Kaufmann and Gupta, 1985), and the *vertex method* (Dong and Shah, 1987), which permit the approximation of a fuzzy set as a collection of intervals using interval analysis concepts.

Fuzzy numbers and their associated arithmetic and calculus have been the subject of many publications and several textbooks (Dubois and Prade, 1980, Kaufmann and Gupta, 1985, Zimmermann, 1985) and will not be presented here.

Since their establishment, fuzzy sets have been used in a variety of engineering and other applications. Works by Wood, *et al.* (1990), Thurston and Carnahan (1990) and Buckley (1983) in multi-attribute decision making, Kubic and Stein (1988) in designing chemical engineering systems under random and modeling uncertainty, Allen, *et al.* (1992) in solving hierarchical design problems and Fang and Chen (1990) in geology are only a few examples.

In structural design, Brown (1979) applied fuzzy sets on structural safety assessment. He observed that the actual probability of failure of most structures appeared to be much higher than the value obtained if only objective information (statistics, probabilities) were taken into account. He concluded that subjective information (expert knowledge) was as important as objective information and had to be taken into account for safety measurement. Ayyub and Lai (1992) examined the effect of the vagueness in the perception of damage on structural reliability. Instead of using a crisp definition of failure they used a fuzzy definition of failure. They concluded that by introducing vagueness in the definition of failure, the resulting average probability of failure was larger than the probability of failure based on the crisp failure definition.

Fuzzy sets were used by Dong, *et al.* (1989) to model linguistic and qualitative factors, in evaluating the safety possibility of existing buildings. In the same publication the authors introduced the notion of *failure possibility* as a measure of structural safety. There were numerous other applications of fuzzy set theory in the past decade in civil engineering. An extensive review of relative publications can be found in Chou and Yuan (1993).

1.2.6 Studies Comparing Fuzzy Sets and Probability Theory

One of the biggest issues seems to be how fuzzy sets and possibility theory compare to probability theory. There have been several debates concerning whether probability theory or fuzzy analysis is the appropriate basis for addressing uncertainty. Recently, Laviolette and Seaman (1994) criticized the argument that fuzziness represents another type of uncertainty distinct from probability. In their conclusions, they emphasized that, although fuzzy set theory is applicable to some real life problems, probability theory provides a complete and uniquely optimal means for solving problems and managing uncertainty. Dubois and Prade (1994), Wilson (1994) and Klir (1994) disagreed with the statements made by Laviolette and Seaman, whereas Lindley (1994) fully agreed stating, "No one has provided me with an example of a problem that concerns personal uncertainty and has a more satisfactory solution outside the probability calculus than within it."

Other studies tried to establish some common theoretical ground for possibility and probability theories. Gaines (1978) established a common theoretical basis for probability and fuzzy logics. Natvig (1983) indicated that, at least in some applications, *possibility distribution* can be interpreted as a *family of probabilities*. Henkind and Harrison (1988) concluded that Bayesian calculus is well suited for applications where probabilities are known or can be acquired with a reasonable effort. They also concluded that fuzzy set calculus is well suited for applications where the evidence itself is fuzzy in nature. Fuzzy sets are also advantageous in situations of little information. Bordley (1989) concluded that there is a convergence between probability theory and possibility theory in *observer bias* situations. Dubois and Prade (1989 and 1993) tried to clarify some "classical misunderstandings" between fuzzy sets and probability. They also presented examples that "... should convince us that instead of considering probability and fuzzy sets as conflicting rivals, it sounds more promising to build bridges and take advantage of the enlarged framework for modeling uncertainty and vagueness they conjointly bring us to."

Chou and Yuan (1993) present such an approach, where fuzzy set and Bayesian theories are combined in evaluating the reliability of existing structures. Wood and Antonsson (1990) also combined the two theories in modeling imprecision and random uncertainty in preliminary engineering design. They used the notion of hybrid numbers (proposed by Kaufmann and Gupta, 1985). A hybrid number is similar to a complex number except that, instead of having a real and an imaginary part, a hybrid number has a fuzzy component (representing imprecision) and a random component (representing random uncertainty).

Vadde, *et al.* (1994), compared fuzzy sets and probabilistic optimization. They observed that the fuzzy set design was more conservative. Other analytical studies have compared methods for design under various types of uncertainty, but it is not always clear which method is better. Wood, *et al.* (1990) compared probability calculus and fuzzy sets for handling imprecision. They concluded that fuzzy sets were more suitable than probabilities in handling the imprecision aspect of uncertainty in design. Thurston and Carnahan (1990), compared fuzzy sets and utility analysis for multiple attribute design evaluation problems. They concluded that fuzzy set analysis is more useful and appropriate at very early stages of the preliminary design process and that utility analysis is more appropriate later in the design process when alternatives are more well-developed.

There is no consensus on how to treat modeling uncertainty. Most studies have used probability theory (e.g., Ang and Tang, 1984, Nikolaidis and Kaplan, 1991), but a few have used fuzzy sets (Kubic and Stein, 1988, Wood and Antonsson, 1990, and Wood, *et al.*, 1992).

Concerning random uncertainty, which is the focus of this study, most researchers (e.g., Henkind and Harrison, 1988, Wood, *et al.*, 1990) agree that it is better represented by probability theory, when there is sufficient statistical information about the random variables. However, when there is little information on the statistics of the random parameters, it might be better to use fuzzy sets. Chiang and Dong (1987), Hasselman, *et al.* (1994) and Dong, *et al.* (1987) compared analytically fuzzy sets and probabilities for deriving the uncertainty in the response of a system due to random uncertainties in loads or properties of the system. They concluded that fuzzy sets should be used in cases where there is little information about uncertainties, because fuzzy sets are simpler and more efficient than probabilistic methods. However, this conclusion was based on few simple examples where the predictions of fuzzy sets and probabilities were quite similar.

The following are three important issues in comparing probabilistic methods and fuzzy sets which, to our knowledge, have not been addressed:

1) How can we examine if fuzzy sets are better to model random uncertainty when little information is available,

2) How much information on random parameters is *little enough* to justify switching from probability theory to fuzzy sets for representing random uncertainty,

3) How can we compare experimentally *designs* obtained using these two approaches. This study will primarily focus on the third issue, that is experimentally compare designs obtained from fuzzy sets and probabilities.

1.3 Contrast Between Probabilistic Methods and Fuzzy Sets

In a problem where only random uncertainties are involved, if we have accurate models of these uncertainties, accurate models of a system, and a crisp definition of failure, we should expect that reliability based design will yield better designs than fuzzy set design, because it accounts for more information about uncertainties than all other methods for design under uncertainty. However, in many real life problems, probabilistic design may not be the best, because it requires too much information and in many cases it is sensitive to lack of information. This includes detailed information on the joint probability distributions of the random variables, as well as information on modeling and human errors. Because we rarely have all this information, we often need to guess or estimate it. For example, in most real life applications, very little is known about the correlation between the statistical properties of different variables, and it is common practice to assume that variables are uncorrelated. Design optimization can often exploit model weaknesses. Therefore, we may question the utility of reliability based optimization procedures, that rely on inaccurate models of uncertainties. Ben-Haim and Elishakoff (1990) and Fox and Safie (1992) showed that the effect of errors in the statistical models of random uncertainties on the calculated probability can be large.

Fuzzy-set methods also maximize reliability. Uncertainties are modeled using *membership functions* instead of probability distributions. A membership function typically varies between 0 and 1 and measures to what extent the argument of the function belongs to a particular set. A related concept to the membership function is that of the *possibility function*, which measures the degree to which it is possible for a variable to take a certain value. For example, assume that the nominal value of a length of a panel is 1 meter and experts say that the length can vary between 0.9 and 1.1 meters. The possibility function of the length may have a value of one between 0.99m and 1.01m and then taper linearly to zero at 0.9m and 1.1m. This is not so different from probability distributions. However, possibility functions are often constructed without precise information. They are often triangular or trapezoidal in shape. Also, rules for calculating the possibility function of the system response from the possibility functions of the system parameters are different from the rules used to calculate probability distributions. These rules tend to cater more to worst case scenarios because they reflect:

1) The fuzzier nature of the possibility functions.
2) The fact that the possibility of an event is always larger than the probability of this event (consistency principle, Zadeh, 1978).

The fuzzy set approach for structures has been found to provide more conservative (heavier) designs than probabilistic approach (Vadde, *et al.*, 1994). Because fuzzy set calculus assumes that models of uncertainty are approximate and produces more conservative results than probabilistic calculus, it might be better than probabilistic design for problem in which little information is available.

The definition of failure is usually crisp in the probabilistic design. A fuzzy set approach, however, can select a fuzzy definition of failure (e.g., Ayyub and Lai, 1992). That is, failure can have a membership function between zero and one expressing a transition between a totally acceptable and a totally unacceptable design.

From the above discussion we conclude that the relative merits of probabilistic designs and fuzzy set designs may depend on:

1) The amount of information available on the uncertainty.

2) How crisply failure is defined.

3) How accurate deterministic models used to predict the performance of a system are.

This dissertation is part of a study whose long term goal is to understand how these three factors affect each method and determine which method is better for a given problem.

1.4 Objective of the Present Study

This study intends to take a first step in the direction of comparing probabilistic and fuzzy set methods by:

1) Understanding how each method maximizes safety.

2) Finding problems for which fuzzy sets and probabilities yield alternate designs that have significantly different probabilities of failure, which can be measured with a small number of experiments in the lab.

1.5 Approach

In this study, we consider a problem where, using the same resources, we design two alternate systems whose properties are uncertain, to maximize safety. The properties of one design, called *probabilistic design*, are modeled as random variables. The properties of the alternate design, called *fuzzy set design*, are modeled as fuzzy variables. Using optimization we find an optimum probabilistic and an alternate optimum fuzzy set design. A large number of realizations are then tested in the lab to compare the actual probability of failure resulting from each alternate design approach.

Experimental study of the effects of uncertainty on safety requires building and testing many nominally identical designs and measuring how many fail. This can be difficult, if we have to build, compare and possibly destroy a large number of sample structures. Therefore, we need a low cost approach for the experimental comparison of methods for designing under uncertainty.

The key idea is to consider a structure with failure modes that do not imply the destruction of the structure or its members. The configuration of the structure must be such that will allow us to construct and test a large number of identical random samples of the same design at low cost.

We selected a dynamic system and defined failure as excessive vibration. We consider the optimum design of a small cantilevered truss structure equipped with tuned dampers for reducing the vibration amplitude. The truss is made of aluminum members and nodes. The dampers are made by hand from widely available plastic and viscoelastic materials. The performance of those dampers is controlled by tuning their natural frequency to a natural frequency of the truss. The variability of material properties and the manual fabrication induce uncertainties in the properties of the dampers.

During the optimizations, only the uncertainties in the properties of the dampers are considered, because they are much larger than the other uncertainties. We manufacture a large number of dampers, measure their properties and evaluate the statistics of the properties. The dampers are not perfectly tuned to the natural frequencies of the structure so the vibration amplitudes are high. The optimizer tries to correct the mistuning by adding tuning masses to the truss. If the dampers are undertuned (i.e., their natural frequencies are lower than those of the structure), the added masses reduce the natural frequencies of the truss thereby increasing the effectiveness of the dampers.

The optimization problem consists of finding the best locations of a maximum of 10 tuning masses on the truss. One damper per mode is used to control vibration modes 1 and 3 of the truss. Failure is defined as the event of the *peak dynamic acceleration* of the first or third mode (or both) exceeding given maximum allowable limits. We create two alternative formulations of the optimization problem. The probabilistic formulation minimizes the probability of failure while the fuzzy set formulation minimizes the possibility of failure. The same resources are available for both formulations (10 tuning masses).

In this study we account for random uncertainties only. To minimize *modeling uncertainties* we use experimental results to improve the analytical models. Specifically, we use a correction equation to make the analytically predicted peak accelerations closely match experimental measurements (see Section 4.2.2). Because the experimental truss is available in the lab we can update that analytical-experimental correction at any time. However, in real life a designer has seldom the luxury of experimental measurements during the design phase. Instead, the designer has to rely on previous results obtained by others. To simulate this situation we use in our original analysis analytical-experimental correction equations that were estimated in a previous study (Ponslet, *et al.*, 1994) in the summer of 1994.

Because the design variables in this optimization problem (locations of added masses) are discrete, a genetic algorithm is used for the optimization. Detailed description of the algorithm can be found in Ponslet (1995). All probabilistic analyses are performed using Monte Carlo simulation. In the fuzzy set analysis, the membership function of the response is evaluated from the membership functions of the input parameters using the vertex method (Dong and Shah, 1987).

Note that there are several definitions of *failure possibility*. For example, Dong, *et al.* (1989) present three different definitions of failure possibility, depending on three different criteria of ranking two intervals that contain the load effect and the strength of a system. In this study, we define the failure possibility as being equal to the *possibility measure* of the interval containing all the values that correspond to structural failure. Zadeh (1978) defines the possibility measure $\pi(A)$ of a nonfuzzy subset A of a universe U as

$$\pi(A) \equiv \sup_{u \in A} \pi_x(u) \tag{1.1}$$

where $\pi_x(u)$ is the possibility distribution function associated with a variable X which takes values in U. This number, then, is interpreted as the possibility *that a value of X belongs to A*.

The optimal designs are compared analytically on the basis of their probabilities and possibilities of failure. To validate the results experimentally, 29 realizations of each design are tested in the laboratory. The experimental comparison is based on the probabilities of failure, which are evaluated by the number of failures out of the total number of realizations. To compare of the alternate probabilistic and fuzzy set designs using a relatively small number of experiments some problem parameters (such as the failure limits and the scatter of the natural frequencies of the dampers) have to be adjusted in a way that the difference in the probabilities of failure between the probabilistic and fuzzy set optimum designs is large. This procedure is along the lines of *anti-optimization* or *contrast maximization* (Haftka and Kao, 1990, Gangadharan, *et al.*, 1993, Van Wamelen, *et al.*, 1993).

1.6 Outline of Dissertation

In chapter 2, we briefly describe the hardware used in this study and the finite element models used to model it. In chapter 3, we present alternate probabilistic and fuzzy set approaches for designing a damped truss. The corresponding optimization problems are formulated and the probabilistic and fuzzy set analysis methods are described. Probabilistic analysis refers to the calculation of the probability of failure of a system when uncertainties in the parameters are described using their probability distributions. On the other hand, fuzzy set analysis refers to the calculation of the possibility of failure of a system when uncertainties in the parameters are described using their membership functions. Finally, we select design problems for which probabilistic and fuzzy set approach yield alternate designs that have considerably different failure probabilities.

In chapter 4, we compare analytically probabilistic and fuzzy set optimization results. Dampers whose natural frequencies have approximately the same probability distributions are constructed (the way these dampers are constructed is described in chapter 5). Once the dampers have been constructed they are measured again and the probability distributions of their properties are used to find a probabilistic and a fuzzy set optimum design. Then, the probabilities and possibilities of failure of the two designs are calculated and compared.

In chapter 5, we first explain how we obtained the 29 samples of dampers for the experimental validation. These 29 realizations are then tested in the laboratory and the number of failures observed is compared for the two designs. The measured relative frequencies of failure are also compared to the corresponding analytical predictions. Then some sources of error that can be responsible for the discrepancy between analytical and experimental failure probabilities are examined.

Chapter 6 contains the conclusions of the study and recommendations for future research.

Finally, a note. The word "we" used throughout this dissertation refers to the author and the three professors who closely participated in this project, Dr. H. Cudney, Dr. R.

Haftka and Dr. E. Nikolaidis. Part of the work presented in this dissertation is the fruit of collaboration between the author and Dr. E. Ponslet who received his Ph.D. in Aerospace Engineering in 1994. Dr. Ponslet, working towards his Ph.D., prepared most of the experimental setup and created the genetic optimization algorithm used in this study. He also designed the tuned dampers and constructed them with help from the author. Therefore, in Chapters 3 and 4 and in Section 5.1, the word “we” also includes the contribution by Dr. E. Ponslet.

CHAPTER 2

DESCRIPTION OF THE SELECTED PROBLEM

In this chapter we describe the truss structure, the tuned dampers and the tuning masses used in this study and the corresponding finite element models. Next, we describe the instrumentation used in our experiments. Finally, we give the definition of failure for our problem. The description is short and limited to what is necessary for the reader to follow the rest of the dissertation. Further detailed descriptions can be found in Ponslet (1995).

2.1 System Description

2.1.1 Truss Structure

The structure used in this study is shown in Fig. 2.1. It is a short, beam-like truss assembled from 30 tubular aluminum members with steel end fittings, connected through 12 spherical aluminum nodes. The truss is about 1 meter (40 in) long and weighs about 4.4 Kg (9.6 lb). The two middle bays of the truss are pyramids with a 0.254 m (10 in) square base. Two half bays attached to the ends of the middle section complete the structure. The 26 non-diagonal members are 0.254 m long from node center to node center, while the 4 diagonals are $\sqrt{2}$ times longer. Three nodes are attached to a base made of thick steel and aluminum plates mounted on the laboratory wall.

Figure 2.2 shows a plot of the magnitude of a measured frequency response function (FRF) from excitation force to response acceleration for that truss. The locations of the excitation and response measurements are shown in Fig. 3.2. The first three modes are well separated and clearly identified. Their natural frequencies are about 100, 130 and 193

Hz. Local bending modes in the members occur at frequencies 280 Hz and higher. The truss model neglects bending (see Section 2.2.1). Therefore, it is useful only for the first three modes. For this reason, only the first 3 modes of the structure will be considered.

The first and third modes of the truss are very lightly damped; their measured damping ratios are only 0.13% and 0.08%, respectively. The damping ratio of the second mode is significantly higher (1.05%). This is believed to be due to coupling with the dynamics of the wall.

2.1.2 Tuned Dampers

We designed and manufactured viscoelastic tuned dampers (Fig. 2.3) to reduce the dynamic response of the first and third modes of the truss. The dampers consist of symmetric cantilevered beams that carry adjustable tip inertias and are attached at their middle point to a node of the structure. The first bending mode of the beams is tuned to a natural frequency of the truss to maximize the energy absorbed by the tuned dampers.

Figure 2.4 is a schematic sketch of a tuned damper. The beam is a sandwich of 2 thin plastic sheets and an inner core of viscoelastic acrylic foam. This viscoelastic foam is available in the form of a 1.14 mm (0.045 in) thick, double sided self-adhesive tape. A plastic tip block is glued to one of the plastic layers at each end of the beam. A steel tuning screw is threaded into each of these blocks. This allows the fundamental frequency of the damper to be adjusted by moving the center of gravity of the tip masses in and out. A 6.35 mm (0.25 in) hole is drilled in the middle of the sandwich plate. The damper is attached to a node of the structure with a 6.35 mm (0.25 in) nylon screw passing through that hole.

Two slightly different versions of this design (we will refer to them as *type-1* and *type-3*) are used to target the first and the third natural frequencies of the structure. Type-1 dampers use 0.55 mm (1/48 in) thick plastic sheets and are 108 mm (4.25 in) long and 25.4 mm (1.0 in) wide. Type-3 dampers use 0.82 mm (1/32 in) thick plastic sheets and are 92.3 mm (3.75 in) long. Their width is the same as type-1 dampers. Type-1 dampers can be adjusted from about 98 Hz to about 116 Hz; the range for type-3 dampers is about 170 Hz to 204 Hz.

The mass of the dampers is very small compared to the mass of the structure. Both types weigh about 10 g, which represents about 0.2% of the total mass of the truss. Despite their size, these dampers provide very significant damping to the truss. Figure 2.5 shows the frequency response function of a tip node acceleration, before and after adding one type-3 damper to the structure in a quasi-optimal manner. That is, tuned to the frequency of mode 3 and located at the node and the direction that correspond to the largest acceleration amplitude of vibration in the third mode shape. The reduction in acceleration amplitude achieved in mode 3 is more than 25 dB (18 times smaller than it was).

Note that the natural frequency of the damper is the most important parameter to determine its effectiveness. The tuned damper does not significantly affect modes of the truss that are far from the tuned frequency of the damper. This implies that at least one damper per target mode is needed.

2.1.3 Tuning Masses

We used tuning masses that can be attached to the nodes of the structure to modify its natural frequencies. They consist of standard steel screws and nuts. These screws are compatible with the standard holes in the nodes of the structure so that they can be easily attached to the truss. Ten screws were selected and weighed. Their average mass is equal to 16.61 g with a sample coefficient of variation of about 0.4%. Even when all 10 tuning masses were added to the truss, the added mass represented less than 3.8% of the mass of the original truss.

2.2 Finite Element Models

2.2.1 Truss Structure

The finite element model of the truss is shown in Fig. 2.6. We define a global axis system the following way: positive z-axis is pointing vertically up, positive x-axis is orthogonal to and pointing away from the wall and positive y-axis completes the right-handed coordinate system.

Each member of the structure is modeled as a 6-degree of freedom, 3D rod finite element defined by its mass and complex stiffness $k(1+i\eta)$, where k is the stiffness and η is the loss factor of the member used to model inherent damping. The bending stiffness of the rod elements is zero. Consistent mass matrices are used to represent inertial properties of the members. Each node is modeled as an infinitely stiff concentrated mass equal to the measured mass of the physical node.

The values used in the model for the masses of the members and nodes and for the stiffnesses of the members are the mean values of series of measurements performed on a large number of members and nodes. Details on these measurements can be found in Ponslet (1995). The flexibility of the base (wall) is simulated in the finite element model of the structure with 9 springs with complex stiffnesses (3 per attached node, one normal to the base and two in the plane of the base), as shown in Fig. 2.6. With these support springs, the finite element model of the truss contains 36 degrees of freedom.

2.2.2 Tuned Dampers

The following simplifying assumptions are made about the dampers: First, the attachment point is assumed to move in pure translation, neglecting any rotation of the supporting node (note that the truss model does not include these rotations). Second, the two halves of the damper are assumed identical, so that their deflections are identical in magnitude and phase, and the contributions from the two halves can be superposed. Finally, the damper is assumed to deform only in its first bending mode (higher modes are neglected as well as torsion along the axis of the beam).

With these simplifications, a two degree of freedom (d.o.f.) model was devised for the tuned dampers. This simplified model is shown in Fig. 2.7. It consists of two masses (a base mass m_0 attached to the node and a tip mass m) that are connected with a spring of complex stiffness $k(1+i\eta)$. Four parameters are needed to completely define the damper model. The total mass m_T, the tip mass m, the natural frequency f_n, and the loss factor η representing the damping effect of the viscoelastic layer will be used in this study. Except for the total mass that can be measured on a scale, these parameters are not directly measurable, because of the simplifications used in the model. The remaining three parameters are identified by a three-parameter least-squares fit on the imaginary part of the measured transfer function from base acceleration to base force. Details on the experimental setup used for measuring the parameters of the tuned dampers are given in Ponslet (1995).

To include the tuned damper in the finite element model of the truss, we created a special 4 d.o.f. element. This element models the dynamics of the damper in the direction orthogonal to the sandwich beam, as well as the added mass effect in the other directions. Each damper adds one degree of freedom to the model of the structure. The remaining 3 d.o.f. correspond to the 3 components of displacement of the node to which the damper is attached and are shared with the existing model of the truss. With two dampers on the truss, the number of degrees of freedom increases from 36 to 38.

2.3 Instrumentation

To measure the dynamic response of the truss structure an excitation is provided by an electromagnetic shaker (Ling Dynamics, model 102) attached to the steel base plate and connected to node 6 of the truss through a stinger orthogonal to the wall and parallel to the x-axis (see Fig. 2.6). A piezoelectric load cell (PCB model 208B) measures the excitation force.

The shaker is driven by the signal generator of a Tektronix model 2630 FFT analyzer through a power amplifier (KEPCO, model BOP 50-2M). The response accelerations are measured at nodes 11 and 12 by subminiature piezoelectric accelerometers (PCB model 353B17). A sine dwell technique is used to measure the transfer functions in the frequency windows around modes 1 and 3. The locations of the excitation and the response measurement were selected so that the first 3 modes could be identified simultaneously. The data acquisition and FFT analysis were performed on the Tektronix model 2630, PC controlled analyzer.

To estimate the tip mass, the natural frequency, and the loss factor of a damper, the damper is attached to the moving coil of an electromagnetic shaker (MB Electronics, model EA1500) and the transfer function from base acceleration to base force is measured using a sine dwell technique. A 3 parameter least-squares fit is then performed using MATLAB (MATLAB User's Guide, 1992) on the imaginary part of the measured transfer function and provides estimates for m, f_n, and η.

2.4 Definition of Failure

In our study, failure is associated with excessive vibration. In structural analysis, the damping ratios associated with the vibration modes are often used as measures of the damping available to the structure. However, the use of tuned dampers produces pairs of very closely spaced natural modes. Around the natural frequency of a damped mode, the response of the truss corresponds to the superposition of these 2 modes (and of course smaller contributions from the other modes). The superposition of these two modes results in only one identifiable resonance peak in the response of the truss. Furthermore, the damping ratios associated with two closely spaced modes are very sensitive to small changes in the properties of the system. Under these conditions, the damping ratios of the individual modes are not a good measure of the damped response of the structure. Other measures are more appropriate.

For a given location of the excitation, one such measure is the largest magnitude (over a frequency range) of the transfer function between the excitation force and the response acceleration at specified points on the structure as illustrated in Fig. 2.8. This measure will be referred to as *peak acceleration.*

2.5 Summary of Chapter 2

In this chapter, we described of the hardware used in our study. We also presented the corresponding finite element models used in our analysis. Because, for the specific problem, the damping ratios are not a good measure of the damped response of the structure we introduced another measure, the peak acceleration. In the next chapter, we will present two alternative approaches for designing a damped truss subject to limits on the acceleration for a given dynamic excitation.

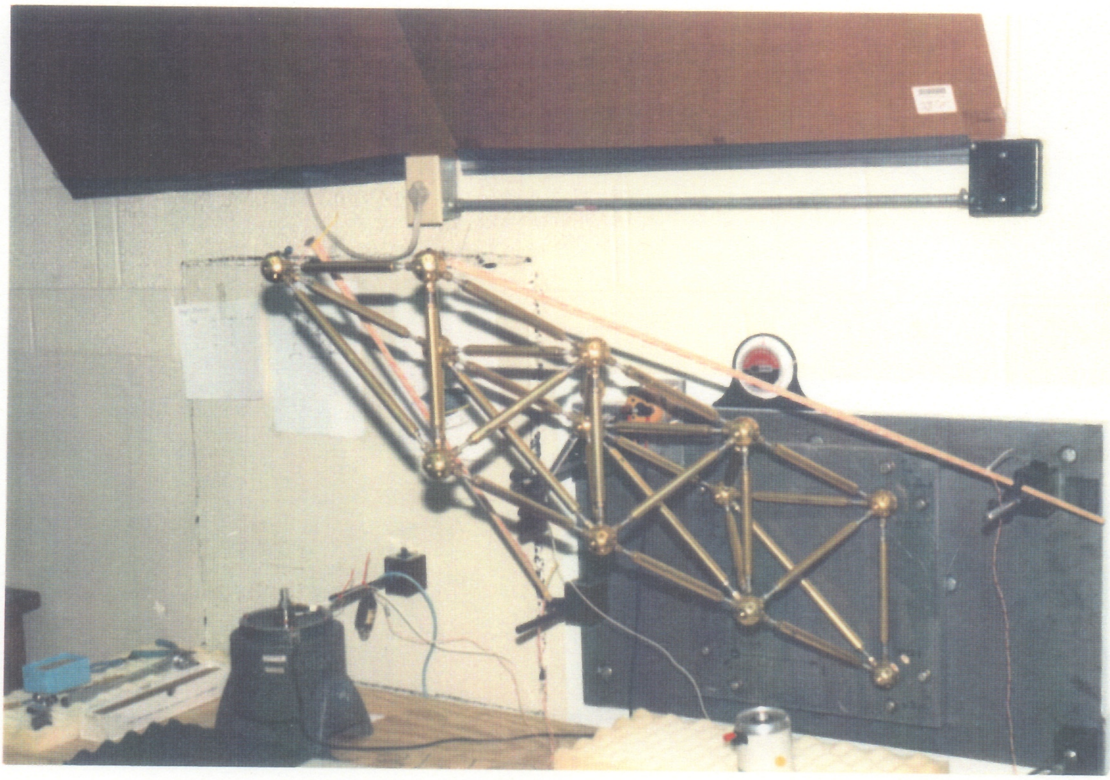

Figure 2.1. Laboratory truss.

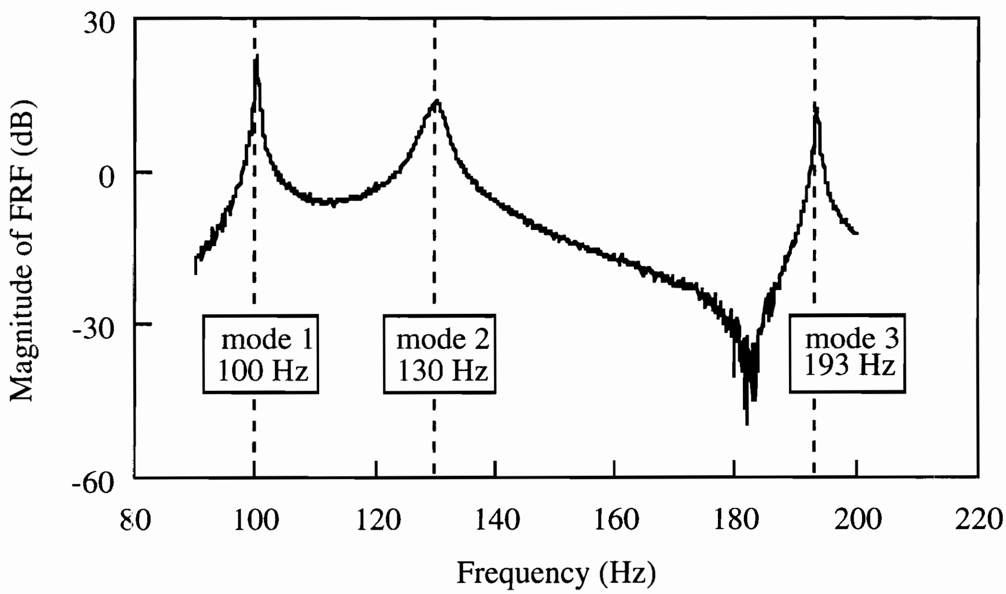

Figure 2.2. Measured frequency response function of the laboratory truss (magnitude of acceleration per unit force).

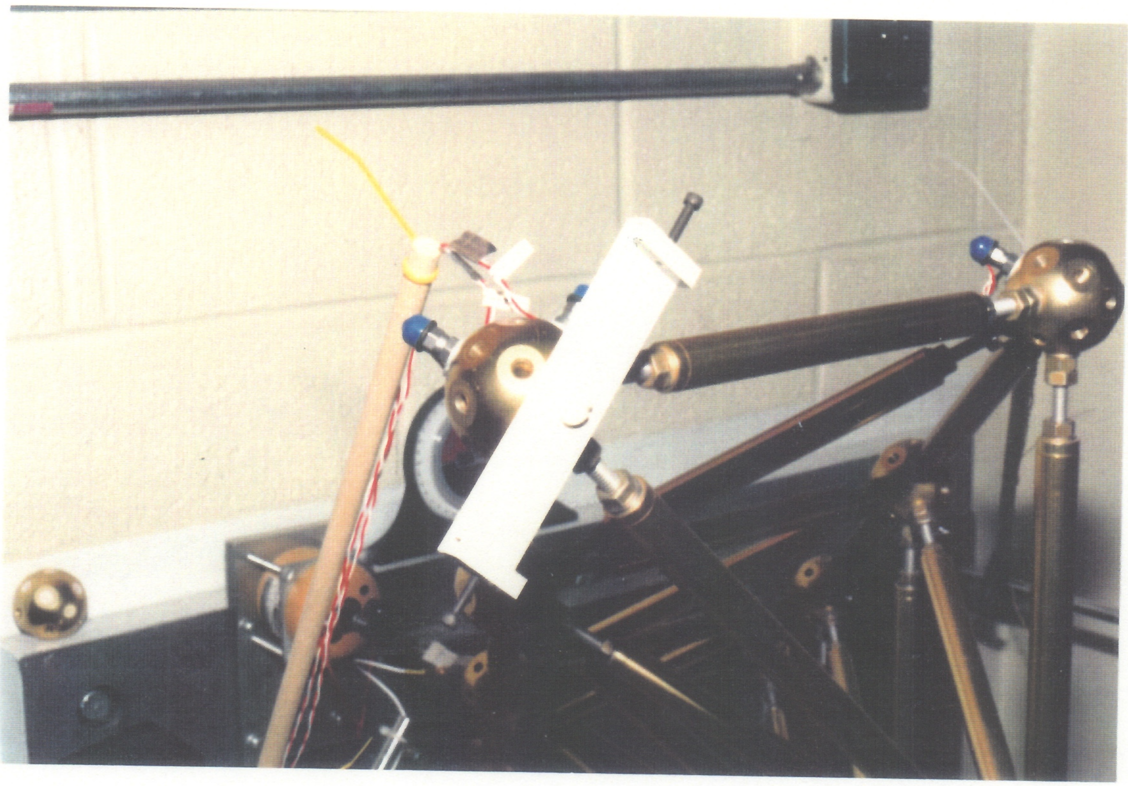

Figure 2.3. Tuned damper attached to the truss.

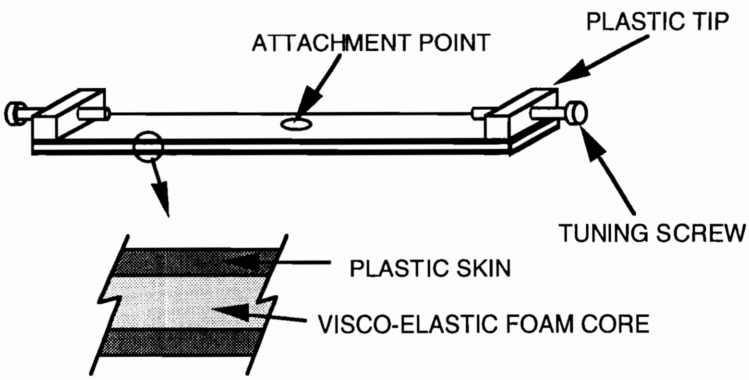

Figure 2.4. Design of the tuned damper.

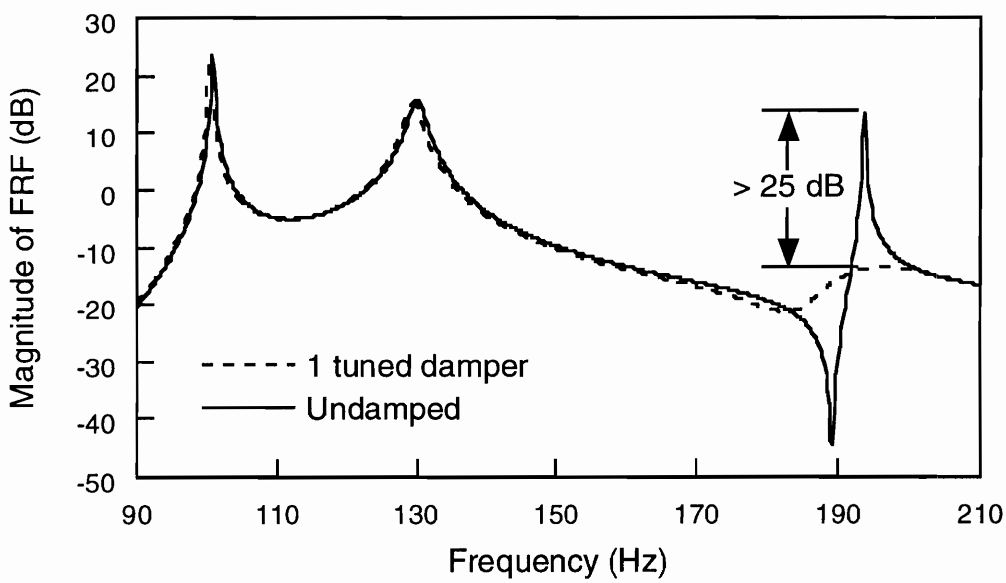

Figure 2.5. Effect of a type-3 tuned damper on the response of the laboratory truss.

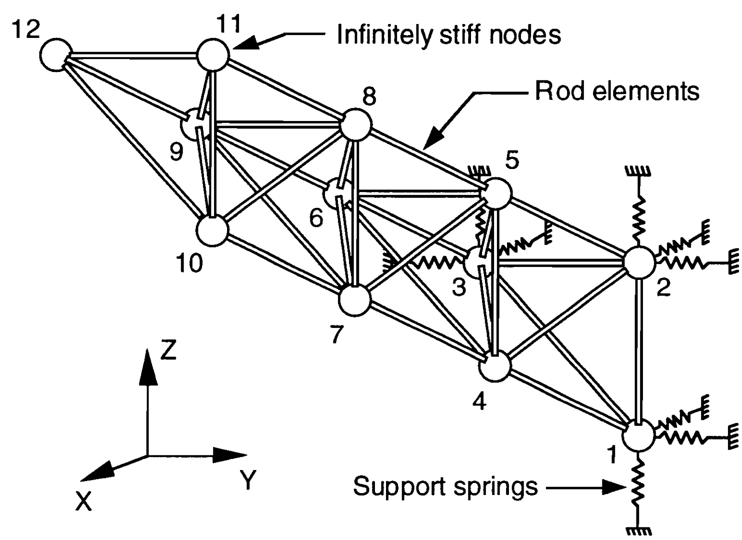

Figure 2.6. Finite element model of the laboratory truss.

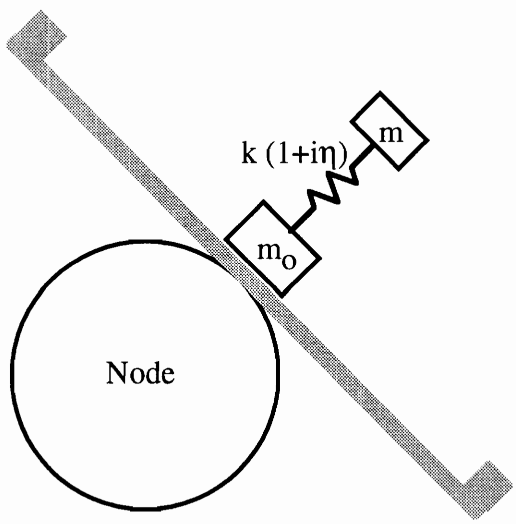

Figure 2.7. Tuned damper finite element.

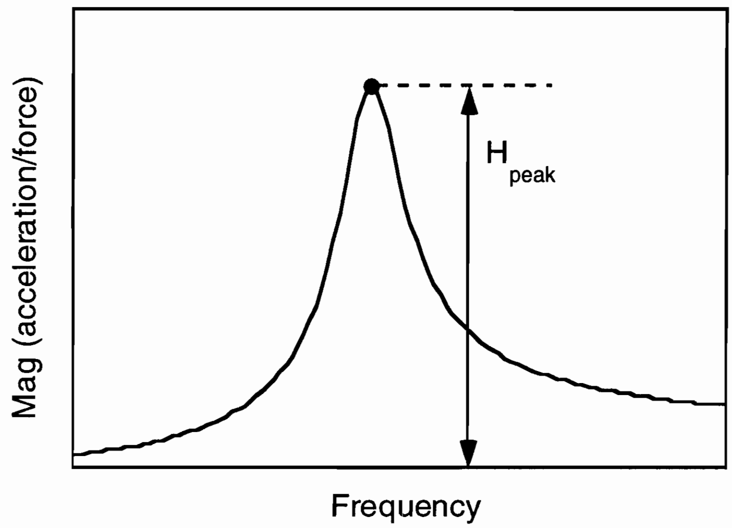

Figure 2.8. Peak acceleration of a damped mode. (From Ponslet, 1995).

CHAPTER 3

PROBLEM FORMULATION AND SOLUTION TECHNIQUES

In this chapter we design a damped truss subject to limits on the response (acceleration) to a given dynamic excitation. Two alternative design optimization procedures are presented; a probabilistic and a fuzzy set approach. The probabilistic optimization minimizes the *probability* of exceeding given acceleration limits, while fuzzy set optimization minimizes the corresponding *possibility*. Note that the same resources (e.g., mass of system, dampers) are used in both formulations.

In the first section of this chapter, we present a general procedure for designing a damped truss structure. Then, we formulate the probabilistic and fuzzy set optimizations for finding the safest design for given resources. Next, we describe the methods used for assessing safety in the probabilistic and fuzzy set optimizations. Following that, we explain how we selected a problem that provided large difference in the probabilities of failure between probabilistic and fuzzy set optimum designs. Finally, we present an approximate solution technique for estimating the frequency response function, that we used to reduce computational cost during the optimizations.

3.1 Probabilistic and Fuzzy Set Approaches

The design requirements are expressed as upper limits on the acceleration at given points on the structure and within prescribed frequency ranges, for a given excitation. These requirements can be formulated as a series of upper limits $A_{lim}^{(m)}$ on the acceleration at prescribed locations within a series of n_m frequency "windows" as illustrated in Fig. 3.1.

We assume that the system fails if acceleration exceeds these limits. The locations, frequency content and amplitudes of the excitation forces are assumed to be specified.

Note that the locations on the structure where acceleration limits are imposed need not be the same in each frequency window. This is the case for example when sensitive devices are attached at given locations on a structure; each device could be sensitive to a particular range of frequencies.

Design variables **x** can include design parameters of the damping devices and the truss itself, as well as the locations of the damping devices on the structure. In the actual structures, some or all design variables as well as other parameters **p** of the model are uncertain. In the probabilistic approach they are considered random variables. In the fuzzy set approach they are represented as fuzzy variables.

The design problem consists of bringing the peak acceleration $A_{peak}^{(m)}$ in each window m below the prescribed limit $A_{lim}^{(m)}$ by adjusting the design variables between bounds x_l and x_u, while satisfying r resource limits $g_i \le 0, \quad i = 1,\dots,r$ (for example, limit on the total weight of the structure, limit on gains in active systems, etc.).

If the uncertainties are represented by their probability distributions, the probability of failure can be computed using a model of the truss and minimized using optimization. A **probabilistic** optimization that minimizes the probability that any of the acceleration amplitudes will exceed the corresponding limits can then be formulated as follows:

$$\begin{array}{lll}
\textbf{Find} & \bar{\mathbf{x}} \quad \text{to} & (3.1) \\
\textbf{Minimize} & P_f = P\left(\bigcup_{m=1}^{n_m} [A_{peak}^{(m)}(\mathbf{x},\mathbf{p}) \ge A_{lim}^{(m)}]\right) & \\
\textbf{such that} & \mathbf{x}_l \le \bar{\mathbf{x}} \le \mathbf{x}_u & \\
\textbf{and} & g_i(\bar{\mathbf{x}},\bar{\mathbf{p}}) \le 0, \quad i = 1,\dots,r &
\end{array}$$

where $\overline{\mathbf{x}}$ and $\overline{\mathbf{p}}$ we denote the mean values of the design and the other uncertain variables respectively. **U**[•] denotes the union of events.

An alternate formulation can be produced if each uncertain variable is represented as a fuzzy variable. In this case we can obtain, using the vertex method (Dong and Shah, 1987), a membership function of the structural response. Then, optimization can be used to minimize the possibility that the peak acceleration will exceed the limits $A_{lim}^{(m)}$ in the frequency windows of interest. A **fuzzy set** optimization problem can be formulated as follows:

$$
\begin{array}{lll}
\textbf{Find} & \overline{\mathbf{x}} \quad \text{to} & (3.2) \\
\textbf{Minimize} & \Pi_f = \Pi(\bigcup_{m=1}^{n_m} [A_{peak}^{(m)}(\mathbf{x},\mathbf{p}) \geq A_{lim}^{(m)}]) & \\
\textbf{such that} & \mathbf{x}_l \leq \overline{\mathbf{x}} \leq \mathbf{x}_u & \\
\textbf{and} & g_i(\overline{\mathbf{x}},\overline{\mathbf{p}}) \leq 0, \quad i=1,\ldots,r &
\end{array}
$$

where Π stands for possibility. To compare designs obtained by each approach fairly, the same resources are used in both formulations.

3.2 Problem Description

In chapter 2, we presented the truss and dampers that we used in this study. There, we also mentioned that the damping of the second mode of the truss is much larger than that of modes 1 and 3, probably because of coupling with the dynamics of the support plates and wall. These dynamics are not included in our model. Hence, we will ignore mode 2 and we will try to control only modes 1 and 3. For simplicity, only one tuned damper will be used for each controlled mode. Therefore, a total of two dampers will be added to the truss. One type-1 damper to control the first mode and one type-3 damper to control the third mode.

3.2.1 Uncertainties

We neglected uncertainties in the truss elements because we found experimentally that these uncertainties are negligible compared to uncertainties in the properties of the dampers. Therefore, we only considered the latter uncertainties. The probability distributions of the properties of the dampers were estimated using measurements. Because uncertainties in the truss elements were negligible, we used the same truss in all the experiments. That allowed us to complete the experiments quickly.

3.2.2 Design Requirements

The locations and directions of the excitation and the response measurements used in this study are shown in Fig. 3.2. The locations of the response measurements ($u^{(1)}$ and $u^{(3)}$) were chosen at the nodes and in the directions that correspond to the largest acceleration amplitudes in the first and third modes, respectively, in the original undamped truss.

The excitation is assumed to have a flat spectrum and a unit amplitude. This allows us to use transfer functions from excitation force to response accelerations as a substitute for the accelerations themselves. The transfer functions $H^{(1)}$ and $H^{(3)}$ corresponding to the two measurement points are defined as:

$$H^{(1)}(i\omega) = \frac{\ddot{u}^{(1)}(i\omega)}{f(i\omega)} \tag{3.3}$$

$$H^{(3)}(i\omega) = \frac{\ddot{u}^{(3)}(i\omega)}{f(i\omega)} \tag{3.4}$$

where $u^{(1)}$ and $u^{(3)}$ are displacements, f is the excitation force, ω is the frequency, $i = \sqrt{-1}$, and $(\cdot)$ denotes differentiation with respect to time.

The upper limits on $H^{(1)}$ and $H^{(3)}$ in two frequency windows, covering the two modes of interest (mode 1 and mode 3) are $H_{lim}^{(1)}$ and $H_{lim}^{(3)}$, respectively. This is illustrated in Fig. 3.3. The limits $H_{lim}^{(1)}$ and $H_{lim}^{(3)}$ are selected in a way that the difference in probabilities of failure between probabilistic and fuzzy set approaches is large. This is explained in detail in section 3.5.

3.2.3 Optimization Scenario

We consider the following scenario: the truss has been designed with one damper for each mode (modes 1 and 3 only) to limit dynamic response accelerations. The locations of the dampers have been determined experimentally to be the most effective for each mode of the undamped truss. A large number of trusses and dampers have been manufactured and samples have been tested. The tests have revealed a significant mistuning of the dampers that will result in poor overall performance of the damped structure. We assume that the dampers cannot be modified to improve their tuning. However, *tuning masses* can be easily added to the nodes of the truss to modify its natural frequencies and improve tuning. These tuning masses have been described in Section 2.1.3. They have a fixed magnitude (16.6 g) and, in order to limit the added weight, a maximum of 10 masses can be used. The problem consists of optimally redesigning the system by adding tuning masses to ensure satisfactory performance.

3.2.4 Formulation of the Probabilistic and Fuzzy Set Optimizations

As explained in Section 3.2.2, the design requirements consist of upper limits on the peak acceleration of modes 1 and 3. The probabilistic and fuzzy set optimizations use the same design variables: the locations of a <u>maximum</u> of 10 tuning masses on the structure.

To reduce the CPU time needed for the optimizations, we restricted the candidate locations for the masses to nodes 7 to 12. Note that these are the most effective locations for the masses for altering the frequencies of modes 1 and 3.

The general probabilistic formulation of equation (3.1) can then be applied to the particular problem as follows:

Find	Tuning Masses Locations to	(3.5)
Minimize	$P_f = P(H^{(1)}_{peak} \geq H^{(1)}_{lim} \quad \text{OR} \quad H^{(3)}_{peak} \geq H^{(3)}_{lim})$	
such that	number of tuning masses ≤ 10	

The corresponding fuzzy set formulation minimizes the possibility that the acceleration will exceed any of the two acceleration limits $H^{(1)}_{lim}$ and $H^{(3)}_{lim}$ (we will call it *failure possibility*):

Find	Tuning Masses Locations to	(3.6)
Minimize	$\Pi_f = \Pi(H^{(1)}_{peak} \geq H^{(1)}_{lim} \quad \text{OR} \quad H^{(3)}_{peak} \geq H^{(3)}_{lim})$	
such that	number of tuning masses ≤ 10	

Note that according to the definition of the possibility of a union of events (Zadeh, 1978):

$$\Pi(H^{(1)}_{peak} \geq H^{(1)}_{lim} \text{ OR } H^{(3)}_{peak} \geq H^{(3)}_{lim}) = \max[\Pi(H^{(1)}_{peak} \geq H^{(1)}_{lim}), \Pi(H^{(3)}_{peak} \geq H^{(3)}_{lim})] \quad (3.7)$$

3.2.5 Optimization Using a Genetic Algorithm

The probabilistic and fuzzy set optimizations use the same 10 design variables: the locations of a maximum of 10 tuning masses. The locations of the masses are restricted to the nodes of the truss so that all 10 design variables are discrete. For this reason we use a genetic algorithm for the optimizations. The same algorithm is used for the probabilistic and fuzzy set cases; only the objective functions differ.

The genetic algorithm uses the three classical genetic operators (selection, crossover, and mutation) and an elitist strategy, where the best individual of a population is always cloned into the next generation. The selection uses the ranking technique, where the probability of selecting an individual is proportional to one plus the population size minus its rank in the population. Details on the genetic optimization algorithm can be found in Ponslet (1995).

3.3 Probabilistic Analysis

For the probabilistic optimization, we need to evaluate repeatedly the system probability of failure. Failure occurs when the peak acceleration at given points on the structure exceeds a prescribed limit value (see Section 3.2.2 for details). The vibration amplitude (or acceleration) of a structure near resonance is highly non-linear in the structural parameters. For that reason, a mean value based, first order covariance propagation method cannot be used (Hasselman, *et al.*, 1994). In addition, because of the complex, non-linear relationship between the parameters of a tuned damper and the magnitude of the peak response of a truss equipped with that tuned damper, failure can occur at both tails of the damper distribution. This means that we have multiple "most probable failure points," (MPFP). This is illustrated in Fig. 3.4, which shows a hypothetical distribution of the natural frequency of a tuned damper. The other curve in the figure shows the peak acceleration of a structural mode as a function of the damper natural frequency. The total failure probability is the sum of the probability of failure of the two MPFP. However, a second moment method may grossly underestimate the probability of failure, because it finds only one MPFP. There is no guarantee that a second moment method will find the MPFP that has the largest probability of failure. Furthermore, there are situations where the failure probabilities of both MPFP must be taken into account. Therefore, although second moment methods are faster, we used Monte Carlo simulation to evaluate the probabilities of failure.

In general, Monte Carlo simulation is computationally expensive and can be used only when the cost for one analysis is small. In our study the problem is relatively simple and we used an approximate solution technique (see Section 3.6.2) to reduce computational cost. That makes Monte Carlo simulation affordable. For more complicated problems, other methods such as *integrated analysis and design* (e.g., Maglaras and Nikolaidis, 1990) and methods using *response surface polynomials* to estimate the response of the structure for many values of the random variables and the design variables (e.g., Fox, 1993) can be used.

To evaluate the required sample size, N, we used the formula (Melchers, 1987):

$$N = \frac{1-P}{P\, COV_P^2} \tag{3.8}$$

where P is the anticipated probability of failure and COV_P the desired coefficient of variation (defined as the standard deviation divided by the mean) of the probability of failure. In this study, we work with probabilities of failure of the order of 0.1 and we accept a coefficient of variation of 0.1. Substituting these values in the above formula we find that the minimum value of N must be 900. We chose a sample size of 1000. The standard deviation in the evaluated probabilities of failure associated with that sample size is then (Melchers, 1987):

$$\sigma_P = \sqrt{\frac{P(1-P)}{1000}} \tag{3.9}$$

Because the total mass of the dampers has very little scatter, it was assumed deterministic in the simulations. The 3 remaining parameters -- natural frequency (f), tip mass (m) and loss factor (η) -- of each damper were found to be approximately normally distributed random variables (see Section 4.1). Their mean values, standard deviations and correlation coefficients were estimated from tests.

3.4 Fuzzy Set Analysis

The objective of the fuzzy set analysis is to evaluate the system possibility of failure. For the fuzzy set analysis we use the same uncertain parameters as for the probabilistic analysis. These are the natural frequency (f), the tip mass (m) and the loss factor (η) of each type of dampers. Each uncertain parameter is represented as a fuzzy variable. One problem that appears when using fuzzy sets is the determination of membership functions of the fuzzy parameters. A review of methods for practical estimation of membership functions can be found in Dubois and Prade (1980). However, there is no method giving the appropriate type of membership function for given information. Nevertheless, in many practical applications -- e.g., Dong, *et al.* (1987), Wood and Antonsson (1989), Dong, *et al.* (1989) - there is a strong trend to use triangular membership functions to represent fuzzy sets. Triangular fuzzy numbers represent a linear transition from a zero level to a maximum value of membership. We decided to use triangular membership functions because they involve few parameters and they are easy to manipulate using the vertex method.

The next question is what values to use for the apex and the left and right ends of the triangular fuzzy number. In other words, how should we use the available statistical data to generate the membership functions. We adopted the following approach. The apex of the triangle corresponds to the statistical mean of the uncertain parameter. The points where the membership function becomes zero correspond to $\pm 3\sigma$ (where σ is the standard deviation of the uncertain variable). This is illustrated in Fig. 3.5. This membership function is consistent with the following scenario. We ask an expert who has manufactured and measured a large number of tuned dampers to give three values for an uncertain parameter--the most common one and a range which contains all possible values the variable can take. Because the uncertain parameters in our study are approximately normal (see Section 4.1), it is reasonable to assume that the expert would give a range that is centered about the mean and extends plus and minus three standard deviations from the mean. For a normally distributed variable, 99.74% of its values are contained in the interval between $m-3\sigma$ and $m+3\sigma$, where m is the mean and σ the standard deviation. Note also, that in this study we will be dealing with probabilities of failure in the range of 20-30%. Therefore, we are not interested in extremely rare events. However, in a study involving small probabilities of failure, where rare events are important, a wider interval for the membership function might be needed.

When some information is given in both probabilistic and deterministic terms, the probability - possibility *consistency principle* must be satisfied (Zadeh, 1978). This principle states that the possibility of an event is always greater than or equal to its probability. Figure 3.6 shows the probability density function of a standard normal variable (*i.e.*, with zero mean and unit standard deviation) and the membership function that was created the way we described in the last paragraph. For any $x_o \geq 0$, any combination of events of the form $\{x \geq x_o \text{ or } x \leq x_l \leq -x_o\}$ has by definition (Zadeh, 1978) a possibility equal to the maximum value of the membership function for all $\{x \geq x_o\}$ and $\{x \leq x_l\}$. This value is equal to $\mu(x_0)$. From all the above combinations of events, the one with the highest probability is $\{x \geq x_o \text{ or } x \leq -x_o\}$. Therefore, for the consistency principle to be satisfied,

$$\mu(x_0) = \mu(-x_0) \geq P(x \leq -x_0) + P(x \geq x_0) = 2P(x \geq x_0), \quad \text{for every } x_0 \geq 0 \tag{3.10}$$

where μ stands for value of the membership function and P stands for probability.
The above inequality holds for:

$$-2.994 \leq x_0 \leq 2.994 \tag{3.11}$$

This range covers 99.7% of the values of a normally distributed variable. Note again, that in this study we are not interested in the tails of the distributions, due to the magnitude of probabilities we are trying to measure.

To obtain the response from the input parameters we use the *vertex method* (Dong and Shah, 1987) with 5 α-cuts at 0, 0.25, 0.50, 0.75 and 1. We use linear interpolation to obtain membership values other than those corresponding to the above α-cuts.

A typical membership function of the response is presented in Fig. 3.7. In the fuzzy set optimization, we want to minimize the possibility that the acceleration will exceed the peak acceleration limit. If we consider one failure mode this possibility is by definition (Zadeh, 1978),

$$\Pi(H_{peak} \geq H_{lim}) = \max\left[\mu_R(H_{peak})\right] \tag{3.12}$$

where μ_R stands for the membership function of the response and the maximum is for all values of H_{peak} that exceed H_{lim}. Furthermore, it makes sense to expect the acceleration limit to the right of the apex of the membership function of the response, because we want the possibility of survival of the structure to be 1. If the opposite were true, it would mean that the structure would fail for the nominal values of the parameters. If the acceleration limit is to the right of the apex then the possibility of failure is equal simply to the value of the membership function of the response at the failure limit. With this in mind, Eq. (3.7) can be rewritten as follows:

$$\Pi(H_{peak}^{(1)} \geq H_{lim}^{(1)} \text{OR } H_{peak}^{(3)} \geq H_{lim}^{(3)}) = \max\left[\mu_R(H_{lim}^{(1)}), \mu_R(H_{lim}^{(3)})\right] \tag{3.13}$$

Note however that two response values correspond to each membership value between 0 and 1. One lies to the left and one to the right of the apex. To guarantee that the optimizer will move to the correct direction we modify the membership function in Fig. 3.7 as follows (see Fig. 3.8). We keep the part of the curve to the right of the apex as is. We replace the part of the membership function to the left of the apex by linearly extrapolating, using points A and B (Fig. 3.8) as the basis for the extrapolation. The new curve is not a valid membership function because its highest value exceeds 1 but by using it in optimization, we eliminate the possibility of getting a fake optimum design, which fails if the fuzzy variables became equal to their nominal values. We will refer to the modified curve as *pseudo-membership function*. Using the above modification we do not need to calculate analytically the part of the possibility function of the response that lies to the left of the apex.

In general, the vertex method requires calculation of the minimum and maximum values of the peak acceleration for each α-cut. This means that it requires solution of 2 optimization problems for each α-cut. Fortunately, in this study, the computational cost can be greatly simplified, because the maximum peak acceleration corresponds to one of the vertices of each α-cut. This is explained next.

Figures 3.9, 3.10 and 3.11 show the effect of the tip mass, the tuning ratio (ratio of damper natural frequency and structure natural frequency) and the loss factor, respectively, for a type-3 damper on the peak acceleration of mode 3. The curves of a type-1 damper look similar. We observe that all these curves have no maxima other than the extreme points. That means that no maximization is needed for the vertex method. In addition, because we do not need to evaluate the part of the response to the left of the apex, no minimization is needed either. Therefore, we only need to calculate the response at the vertices. Because there are 3 uncertain parameters, there are $2^3 = 8$ vertices in each α-cut. So, we need 4x8 = 32 function evaluations for the four α-cuts plus one function evaluation for the α=1 cut (apex). That makes a total of 33 function evaluations for each mode. This number is much smaller than the number of function evaluations that would be required if optimizations were needed for each α-cut.

3.5. Selecting a Problem for Experimental Validation

One of the goals of this study is to measure experimentally the difference in reliability between probabilistic and fuzzy set approaches. Measuring probabilities of failure in the laboratory is time consuming because it requires to measure many realizations of a design. Moreover, measuring very small probabilities or very small differences in probabilities of failure would require a prohibitively large number of experiments. For this reason, we need to identify a design problem that produces alternative designs whose probabilities of failure differ significantly.

There are a number of problem parameters that we can vary to achieve the increased contrast. In our study, we examined the effect of the following parameters:

-- the mean values of the natural frequencies of the two types of dampers,

-- the scatter in the natural frequencies of each type of dampers,

-- the failure limits. Note that the failure limits can be different for each mode.

We can also take advantage of the difference in the way each approach (probabilistic or fuzzy set) maximizes safety. In a problem with two failure modes, the fuzzy set approach tries to minimize the maximum of the individual failure possibilities (see Eq. 3.7). This means that, unless a constraint does not allow to make the possibilities of failure of failure of modes 1 and 3 equal, at the optimum, these possibilities must be equal. In other words, fuzzy set optimization does not pay any attention to the cost and/or scatter differences between the two modes. The probabilistic approach on the other hand tries to minimize the system failure probability (see Eq. 3.5), which is a combination of both failure modes. Because of this, at the probabilistic optimum, the failure probabilities of the two individual failure modes are not necessarily equal. Instead, they depend on the relative cost of controlling each mode or the relative scatter of the two modes. If the costs of controlling the two failure modes are the same but the scatters are different, the probabilistic optimization will provide a larger safety margin to the large-scatter failure mode. If the scatters are similar but the costs of controlling the different failure modes are different, then the probabilistic optimization will provide a larger safety margin to the cheaper mode.

The only means of controlling the vibration in this study is the addition of tuning masses. Adding masses to a truss structure reduces its natural frequencies. Therefore, it improves the response of a structure equipped with tuned dampers only for those modes for which the corresponding tuned dampers are *undertuned* -- i.e., their natural frequency is smaller than that of the corresponding structural mode. On the other hand, when the dampers are *overtuned* (i.e., their natural frequency is larger than that of the corresponding structural mode), adding masses can only worsen the structural response of the respective mode.

If both types of dampers were undertuned, both approaches would simply add masses to improve both modes. On the other hand, if both types of dampers were overtuned neither approach would add any tuning mass, since any added mass would deteriorate both modes. So it makes sense to make one type of dampers undertuned and the other type overtuned. Then we can achieve different compromises between improving one mode and deteriorating the other for the two approaches.

Added masses generally affect high frequency modes more than low frequency modes. Consequently, it is easier to control the natural frequency of the third mode of the truss with tuning masses than the first mode. It is also desirable to have less scatter in the natural frequency of the undertuned type of dampers so that addition of tuning masses will improve one mode more than it will worsen the other. Taking into account all these facts, we can find a problem in which probabilistic design will decrease significantly the probability of failure. We select a design problem in which type-1 dampers are overtuned with a relatively large scatter and type-3 dampers are undertuned with a relatively small scatter.

The failure limits for each mode are selected in such a way that without any tuning masses on the structure the possibilities of failure of modes 1 and 3 are approximately equal. That design is then the fuzzy set optimum, because any added mass will increase the possibility of failure of mode 1, thus increasing the system possibility of failure. Of course, at the same time the possibility of failure of mode 3 will decrease, but this will not be taken into account, because fuzzy set approach looks only at the maximum of the two possibilities of failure. Consequently, for problems which have two failure modes where every change in the design variables results in improvement of one failure mode and deterioration of the other, the optimality condition for fuzzy set optimization is to have equal possibilities of failure for the two failure modes. As a result, at the fuzzy set optimum the possibilities of failure of the two failure modes will be equal, unless a constraint does not allow them to be. Probabilistic optimization however will add tuning masses on the structure to reduce the failure probability of mode 3. Of course, at the same time the failure probability of mode 1 will increase, but at a slower rate than the rate of decrease of mode 3 because mode 1 is affected less by added masses and it has larger scatter. Note that it is not necessary to change the mean values of the dampers' natural frequencies (i.e., the amount of mistuning) because we can always equalize the membership functions of modes 1 and 3 at any desirable level, simply by adjusting the failure limits. Also, due to the way that our membership functions were created (see Section 3.4), equal possibilities of failure for each mode correspond to almost equal failure probabilities for each mode. This is particular to our problem and it is not a general characteristic of membership functions.

Using the above procedure we identified the following two design problems that give large differences in the probabilities of failure between probabilistic and fuzzy set optimizations. Type-1 dampers should have an average natural frequency of 110 Hz (overtuned) and a coefficient of variation of about 1.6%. Type-3 should have a mean natural frequency of about 182 Hz (undertuned) and a coefficient of variation of about 0.8%. Table 3.1 summarizes the damper properties. In the first case (we will refer to it as *case 1*), the failure limits should be 4.80 m/s^{2}N for mode 1 and 6.60 m/s^{2}N for mode 3. The possibilities of failure with no tuning masses on the structure are 0.88 for mode 1 and 0.86 for mode 3. Note that the two possibilities are almost equal. In the second case (we will refer to it as *case 2*), the failure limits should be 5.00 m/s^{2}N for mode 1 and 6.75 m/s^{2}N for mode 3. The possibilities of failure with no tuning masses on the structure are 0.82 for mode 1 and 0.79 for mode 3. Again the two possibilities are almost equal.

Table 3.1. Damper properties.

	Truss natural frequency (Hz)	Average damper natural frequency (Hz)	COV of damper natural frequency
Mode 1	100	110	1.6%
Mode 3	193	182	0.8%

The two cases described above correspond to different levels of failure possibility of the fuzzy optimum design. Because the analysis is cheap we decided to examine these two cases to obtain greater confidence in the results.

3.6 Approximate Solution Technique

3.6.1 Evaluation of Peak Acceleration

As mentioned earlier (Section 3.2.2), the design requirements limit the largest acceleration at specific nodes under given excitation and within prescribed frequency windows. These peak accelerations are evaluated in the frequency domain using the following numerical procedure: the frequency response function (FRF) from excitation force to response acceleration is computed using the mode superposition method, which is described below. The magnitude of the FRF is evaluated numerically with a relatively coarse step size within the frequency window of interest. The peak is first located approximately using a simple slope-reversal search (the slope is approximated using forward differences). The location of the peak and the corresponding value of the FRF are then refined by second order interpolation between the approximate peak and the 2 neighboring points in the FRF.

3.6.2 2-mode Approximation

The mode superposition method mentioned above requires the first few eigenvalues and eigenvectors of the truss with tuning masses and dampers. Obtaining exact eigenvectors and eigenvalues for a truss with two dampers requires solving a generalized complex eigenproblem with 38 degrees of freedom.

To reduce computational expenses, we use an approximate 2-mode model, that is valid when no more than 1 damper is used for each structural mode. It uses a reduced basis made of one mode (either first or third, depending on the frequency of the peak) of the truss without damper plus one Ritz vector. The Ritz vector contains zeros everywhere except at the d.o.f. associated with the damper. An analytical expression for the frequency response function from excitation force to response acceleration is easily derived for the resulting 2 d.o.f. model and is used instead of the numerical eigensolution and mode superposition method. This analysis has to be repeated with a different basis for each mode of interest. It is about 50 times faster than the full analysis. The associated error is about 10%. Detailed description of the full analysis and the 2-mode approximation can be found in Ponslet (1995).

All analyses in the probabilistic and fuzzy set optimizations are performed using this 2-mode approximation and approximate mode shapes. The approximate mode shapes are obtained from the mode shapes of the original truss (without dampers or tuning masses), using a first order correction for the effect of the tuning masses (Ponslet, 1995).

A complete probabilistic analysis (1000 point Monte Carlo simulation) uses about 12 seconds of CPU time with the 2-mode approximation and about 500 seconds with the full analysis (using an IBM 3090 in vectorized mode). A complete fuzzy set analysis uses about

1 second of CPU time with the 2-mode approximation and about 45 seconds with the full analysis.

3.7 Summary of Chapter 3

In this chapter we presented two alternative approaches for design of a damped truss. One based on probability theory and one based on fuzzy set theory. We used Monte Carlo simulation for the probabilistic analysis and the vertex method for the fuzzy set analysis. Then, we selected problem parameters that led to large difference in the probabilities of failure between probabilistic and fuzzy set optimum designs. Finally, we presented an approximate solution technique that allows us to evaluate the structural response with a much smaller computational cost than the full order analysis.

In the next chapter, we will present the analytically obtained optimization results for the two approaches and we will compare the corresponding optimum designs.

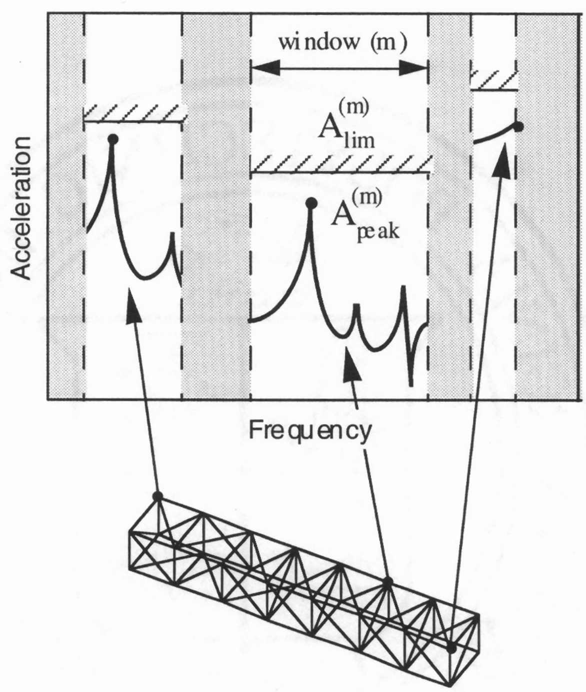

Figure 3.1. Performance requirements for a damped truss.

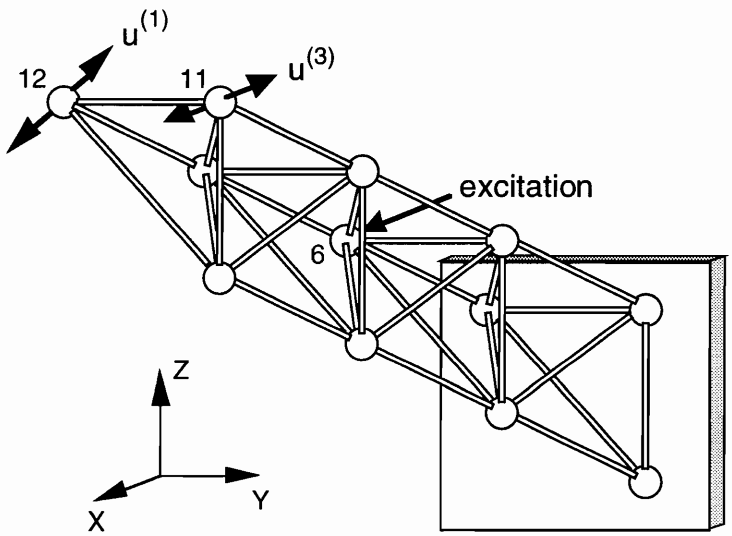

Figure 3.2. Locations of excitation and response measurements on the laboratory truss.

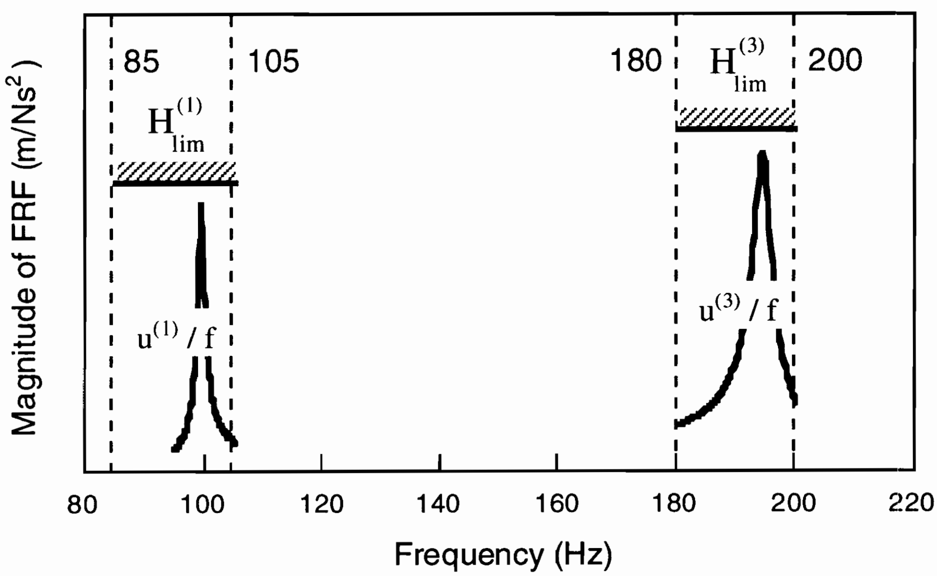

Figure 3.3. Frequency windows and amplitude limits.

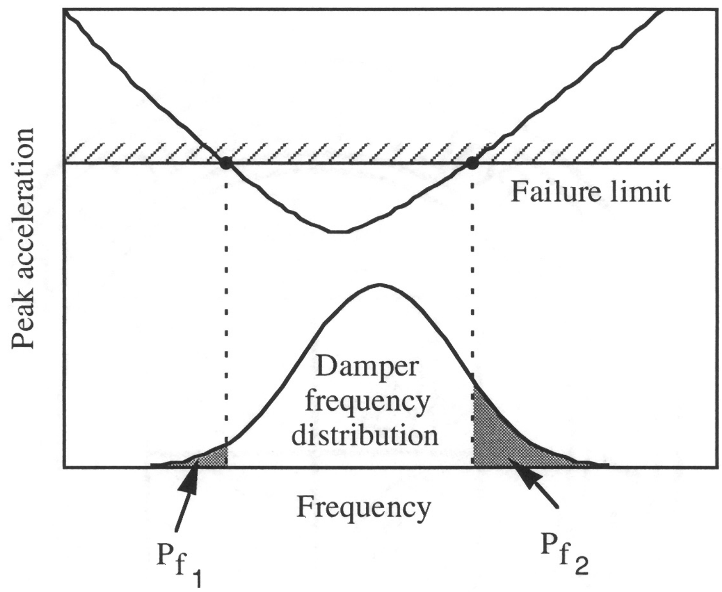

Figure 3.4. Existence of multiple failure regions for a truss equipped with tuned dampers with random parameters. Note: $P_f = P_{f1} + P_{f2}$.

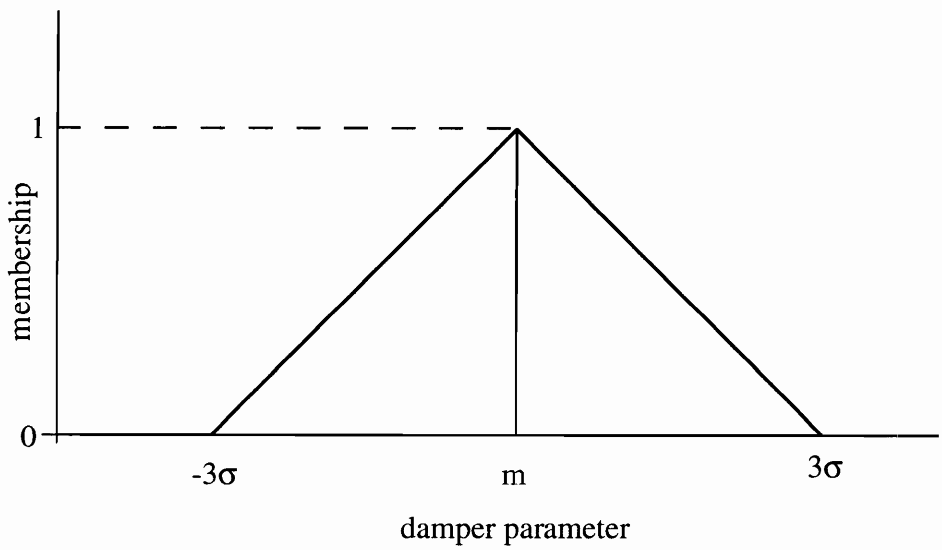

Figure 3.5. Fuzzy set representation of an uncertain damper parameter (m, s are the statistical mean and standard deviation respectively).

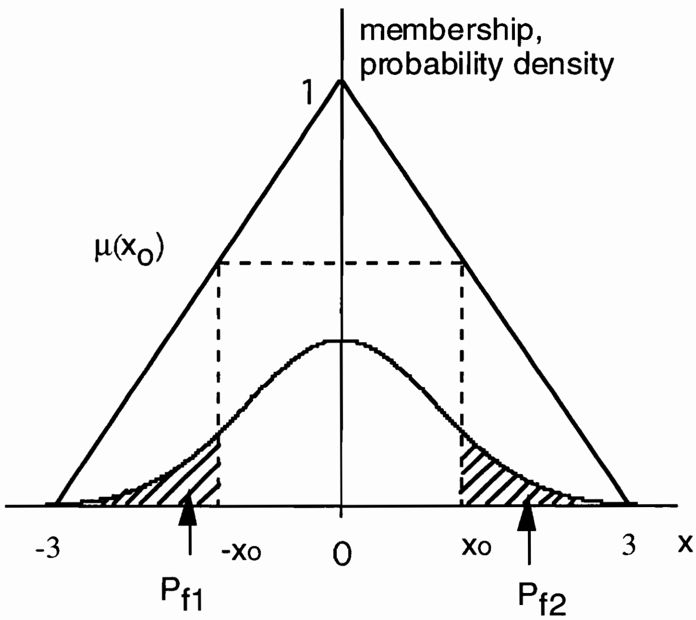

Figure 3.6. Validation of consistency principle:
$$P(x \le x_0 \cup x \ge x_0) \le \Pi(x \le x_0 \cup x \ge x_0) = \mu(x_0) \Leftrightarrow P_{f_1} + P_{f_2} \le \mu(x_0)$$

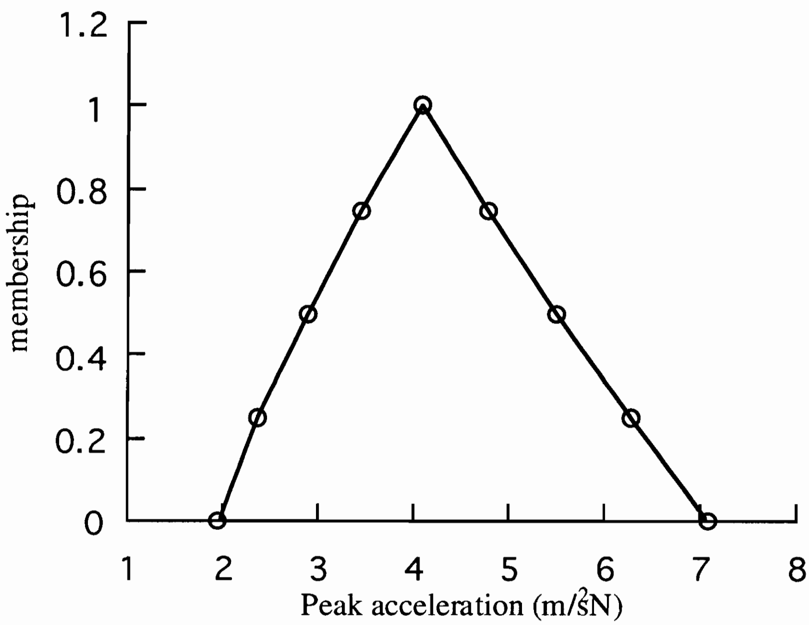

Figure 3.7. Typical membership function of the response for the laboratory truss.

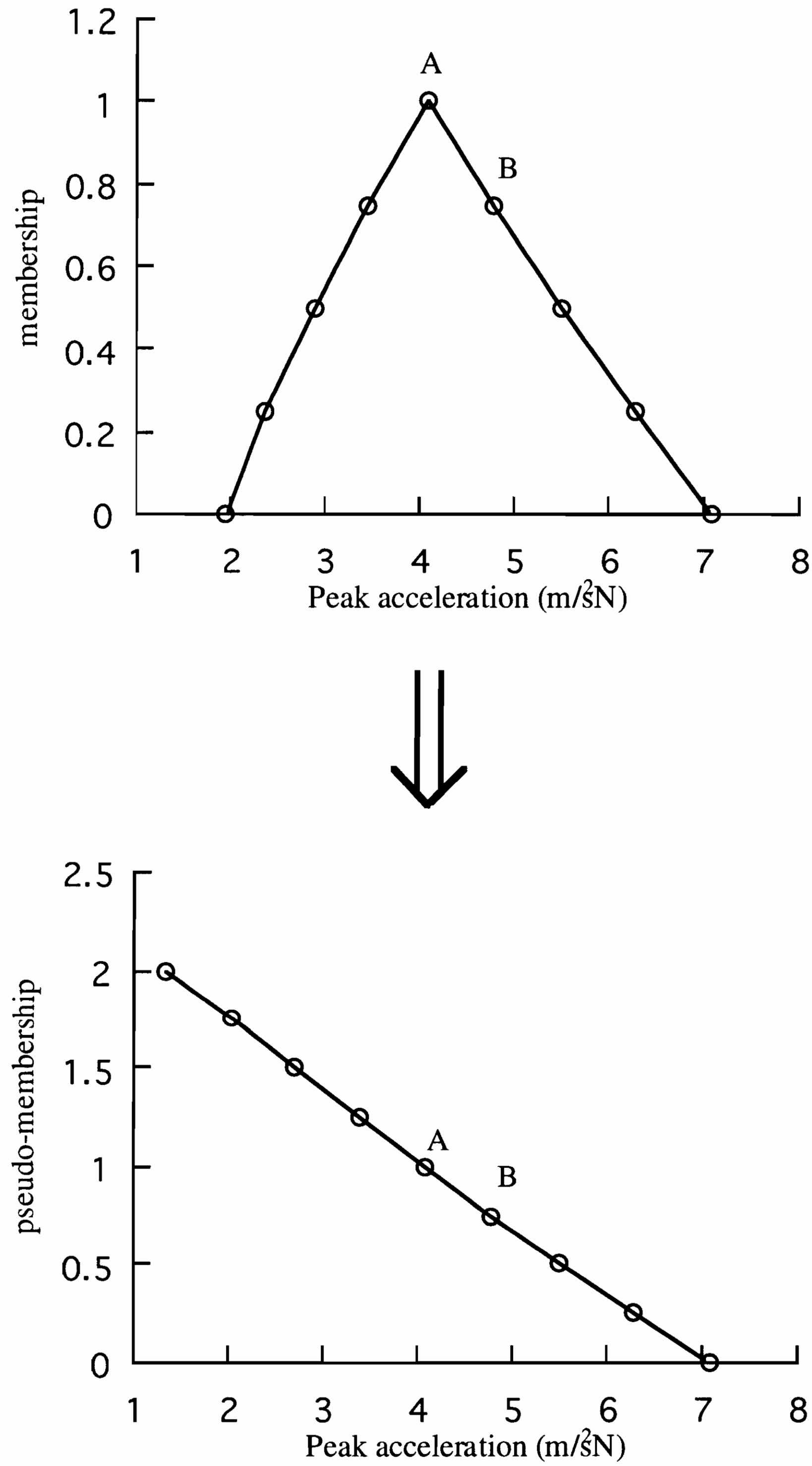

Figure 3.8. Modification of the membership function of the response.

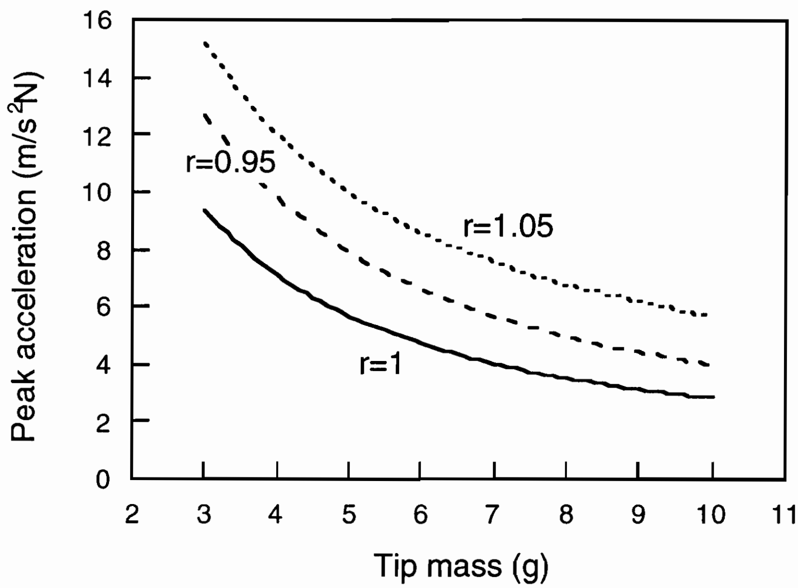

Figure 3.9. Effect of a type-3 damper tip mass on the peak acceleration of the third mode of the laboratory truss. The three curves correspond to different values of the tuning ratio (r). (From Ponslet, 1995).

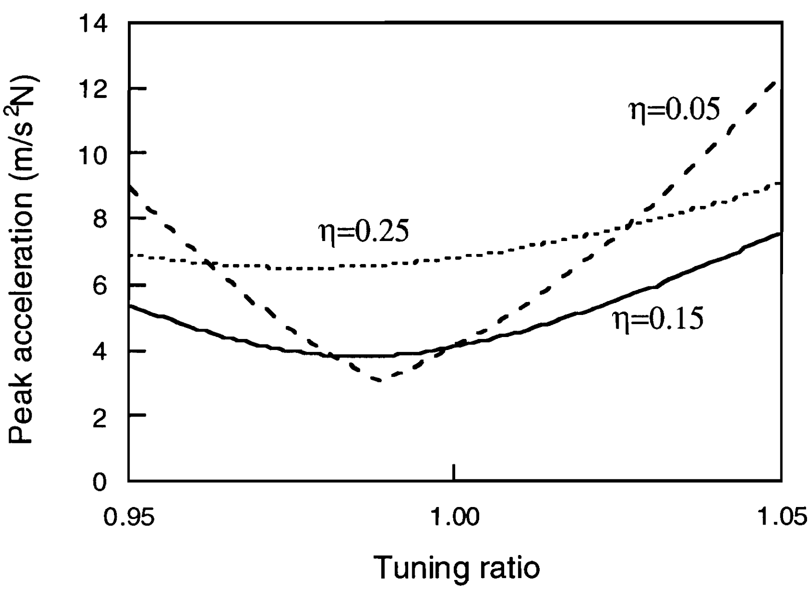

Figure 3.10. Effect of a type-3 damper tuning ratio on the peak acceleration of the third mode of the laboratory truss. The three curves correspond to different values of the loss factor (η). (From Ponslet, 1995).

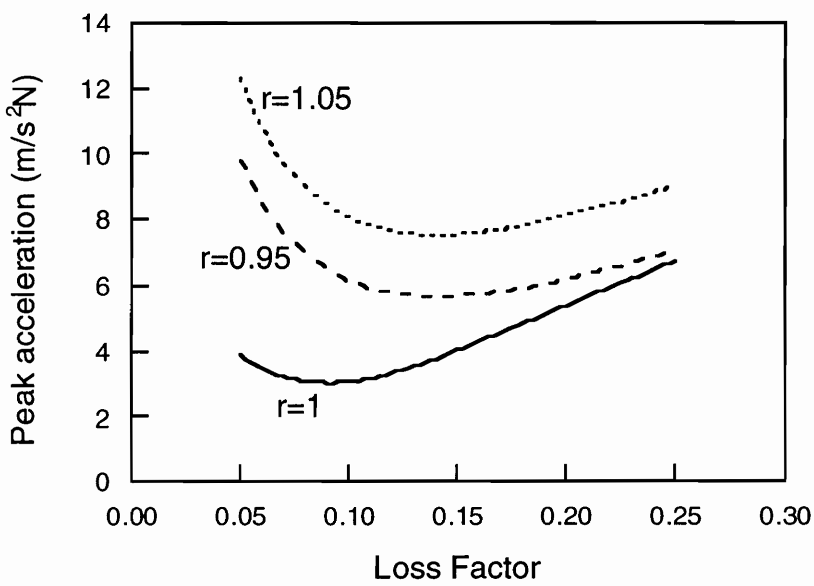

Figure 3.11. Effect of a type-3 damper loss factor on the peak acceleration of the third mode of the laboratory truss. The three curves correspond to different values of the tuning ratio (r). (From Ponslet, 1995).

CHAPTER 4

ANALYTICAL COMPARISON OF PROBABILISTIC AND FUZZY SET OPTIMIZATION

In the previous chapter, we presented a probabilistic and a fuzzy set approach for designing a damped truss. We also formulated the corresponding optimization problems. In Section 3.5, we identified two pairs of designs problems that give large differences in the probabilities of failure obtained using the two approaches.

In this chapter, we first describe how we manufactured the sample of dampers used to compare the probabilistic and the fuzzy set designs. Then, we explain how we calibrated the analytical models used to predict the response of the truss. Finally, we present the results of the probabilistic and fuzzy set optimizations. The results of the two approaches are compared on the basis of the resulting probabilities and possibilities of failure. In the next chapter, we will compare the alternative designs experimentally. The experimental comparison will be based on the number of failures out of a number of realizations of each design.

The flow-chart of Fig. 4.1, summarizes the approach for comparing the two methods both analytically and experimentally. Note that some of the results presented in this chapter are experimental. These results include the statistics of the dampers, presented in Section 4.1, and the calibration of analytical models, presented in Section 4.2.

4.1. Creating a Sample of Dampers that Have Desired Natural Frequencies, and Estimating Their Statistics (Experimental)

We manufactured 29 dampers of each type and measured their properties. Because the dampers use a viscoelastic foam, their characteristics depend strongly on temperature. The effect of temperature changes was measured to be about 9% per ^{o}C on the loss factor, -0.9% per ^{o}C on the natural frequency, and 0.7% per ^{o}C on the identified tip masses. Note that, as mentioned in Section 2.2.2, these three parameters are not directly measurable but are estimated using regression. Therefore, parameters such as the damper tip mass can also be affected by temperature. To reduce this temperature effect, we use a temperature stabilization system that maintains an average temperature of 24.4^{o}C (76^{o}F) with a rapid oscillation of 0.8^{o}C (1.5^{o}F). The period of that oscillation is about 15 minutes. All measurements are repeated 3 times and averaged to reduce the effect of the small temperature oscillation and other measurement errors. The average of these measurements is used to evaluate the mean values, standard deviations and correlation coefficients of the parameters. These statistics are listed in tables 4.1 and 4.2.

Table 4.1. Type-1 dampers, statistics of parameters (sample of 29).

Parameter	m (g)	f_n (Hz)	η	m_T (g)
Mean	6.942	105.08	0.11934	10.811
Standard dev.	0.108	3.1950	0.00681	0
COV (%)	1.55	3.00	5.71	0
Correl. Coeff.				
m	1.000	0.804	0.729	
f_n	0.804	1.000	0.491	
η	0.729	0.491	1.000	

Table 4.2. Type-3 dampers, statistics of parameters (sample of 29).

Parameter	m *(g)*	f_n *(Hz)*	η	m_T *(g)*
Mean	7.613	198.08	0.14722	11.528
Standard dev.	0.070	2.1750	0.00812	0
COV (%)	0.91	1.10	5.52	0
Correl. Coeff.				
m	1.000	0.218	0.692	
f_n	0.218	1.000	0.055	
η	0.692	0.055	1.000	

The mean values and standard deviations of the dampers were different than those determined in Section 3.5, that maximize the contrast between probabilistic and fuzzy set designs. For the natural frequency, which most affects the performance of the dampers, we created a sample corresponding to the desired distribution the following way. We sampled the probability distributions of the dampers' natural frequencies with a uniform step size in probability. Then, we estimated the number of turns of the screws needed to realize these desired distributions. The dampers were then adjusted and their properties were measured again.

Note that the samples of dampers are not random; rather, they are discretized representations of the desired distribution of the natural frequency. The above procedure, which is along the lines of *stratified sampling* (see, McKay, *et al.*, 1979), will be explained in chapter 5.

After the adjustments of the tip screws, the dampers were measured again three times and their properties were averaged. The statistics of the dampers whose screws were adjusted are listed in Tables 4.3 and 4.4 for damper types 1 and 3, respectively. The mean value and coefficient of variation of the natural frequencies of these dampers are close to the corresponding values found in Section 3.5 (Table 3.1). These statistics should lead to large differences in the failure probability of the fuzzy set and probabilistic designs.

Table 4.3. Type-1 dampers after adjustment of tuning screws: statistics of parameters (sample of 29).

Parameter	m (g)	f_n (Hz)	η	m_T (g)
Mean	7.033	110.43	0.11903	10.811
Standard dev.	0.131	1.7520	0.00715	0
COV (%)	1.86	1.59	6.01	0
Correl. Coeff.				
m	1.000	0.682	0.648	
f_n	0.682	1.000	0.247	
η	0.648	0.247	1.000	

Table 4.4. Type-3 dampers after adjustment of tuning screws: statistics of parameters (sample of 29).

Parameter	m (g)	f_n (Hz)	η	m_T (g)
Mean	7.366	181.71	0.13710	11.528
Standard dev.	0.093	1.2314	0.00628	0
COV (%)	1.26	0.68	4.58	0
Correl. Coeff.				
m	1.000	0.333	0.760	
f_n	0.333	1.000	0.079	
η	0.760	0.079	1.000	

Figures 4.2 (a, b and c) show normal probability plots of the measured distributions of tip masses, natural frequencies and loss factors of the 29 dampers of type-1, after

adjustment of the tuning screws. The same plots for type-3 dampers are given in Fig. 4.3 (a, b and c). The scale used on the probability axes of these plots is such that a perfectly normally distributed sample would lie on a straight line. The figures show that the normalcy assumption used in the statistical analysis appears to be justified, for the natural frequency of type-1 and type-3 dampers and for the tip mass and the loss factor of type-3 dampers. For the tip mass and the loss factor of type-1 dampers, it is not clear from Fig. 4.2 whether the normalcy assumption is justified or not.

To quantify the consistency of each damper parameter distribution with the normal distribution, we performed the *chi-square test* (Walpole and Myers, 1972) for all the damper parameters. This test determines if, based on the sample of a random variable, we should suspect the hypothesis that the random variable follows a particular probability distribution. This test requires that the data be classified in k mutually exclusive groups, where the observed frequency of occurrence for the ith group is f_i^o. Based on a theoretical distribution, the expected frequency of occurrence for the ith group is f_i^e. It is recommended that f_i^e be at least equal to 5. Then we evaluate the quantity:

$$\chi^2 = \sum_{i=1}^{k} \frac{(f_i^o - f_i^e)^2}{f_i^e} \tag{4.1}$$

This quantity follows approximately the chi-square distribution with *k-n-1* degrees of freedom (d.o.f). Factor n is the number of quantities obtained from the observed data. In this case, the mean and the standard deviation of the observed data were required to find the expected frequencies. Consequently, n=2. Then, we select a probability level α ($0<\alpha<1$) and, using tables of the chi- square distribution, compute the value χ_α^2, which will not be exceeded by χ^2 with a probability α:

$$\text{Prob}[\chi^2 < \chi_\alpha^2 \,/\, \chi^2 \text{ is chi} - \text{square}] = \alpha, \text{ with } \text{k} - \text{n} - 1 \text{ d.o.f.} \tag{4.2}$$

where $\text{Prob}[\chi^2 < \chi^2_\alpha / \chi^2 \text{ is chi-square}]$ is the conditional probability of χ^2 being less than χ^2_α, given that χ^2 follows the chi-square distribution.

If even for a large value of α, say 95%, $\chi^2 > \chi^2_\alpha$ we should suspect the hypothesis that χ^2 follows the specified distribution, because the event $\chi^2 > \chi^2_\alpha$ is unlikely to occur. In this case the hypothesis that a sample is statistically consistent with a theoretical distribution is rejected. Otherwise, there is no reason to suspect that the hypothesis is wrong on the basis of the test results. The outcome of the test depends on the value of α, the higher the value of α the easier to pass the test. Typical values of α range between 0.9 and 0.99. In this study, we used a 95% probability level.

Table 4.5. Type-1 dampers. Chi-square test results -- probability level α=0.95.

Parameter	Tip mass	Natural frequency	Loss factor
Number of groups (k)	5	5	5
Degrees of freedom (k-3)	2	2	2
χ^2	8.76	0.38	2.30
χ^2_α	5.99	5.99	5.99

The results of the chi-square test for each damper parameter are listed in Tables 4.5 and 4.6 for damper types 1 and 3 respectively. We observe that in all cases $\chi^2 < \chi^2_\alpha$, except for the tip mass of type-1 dampers. Note, however, that for accurate results chi-square test requires a larger number of samples than the 29 we used (about 100 samples minimum). The margin between χ^2 and the critical value χ^2_α is large for all parameters but the tip mass of type-1 damper. Consequently, there is no reason to suspect the normalcy assumption for the remaining type-1 and type-3 damper parameters. For simplicity and because the tip mass does not affect the response significantly, we assumed that its marginal probability distribution is normal as well.

Table 4.6. Type-3 dampers. Chi-square test results - probability level α=0.95.

Parameter	Tip mass	Natural frequency	Loss factor
Number of groups (k)	5	5	5
Degrees of freedom (k-3)	2	2	2
χ^2	2.42	1.13	2.32
χ^2_α	5.99	5.99	5.99

4.2. Validation and Calibration of Analytical Models

4.2.1 Structural Model Refinement

As mentioned earlier in Section 3.2.1, the truss is considered deterministic. The stiffnesses and loss factors of the support springs (see Fig. 2.6) contained in the finite element model as well as the loss factor of the truss members (assumed to be the same for all members) have not been measured directly. Instead, they have been identified by fitting the analytical natural frequencies and damping ratios of the first 3 modes of the original truss (without tuning masses or dampers) to measured values. The resulting model predicts exactly the first and third natural frequencies and damping ratios.

4.2.2. Calibration of Peak Acceleration

We performed two series of tests on a few type-1 and type-3 dampers, chosen among the 29 available to cover the whole range of the parameter distributions. The parameters of each damper were measured 3 times and averaged to reduce experimental errors and temperature effects. The resulting sets of parameters were used to analytically predict the peak of the frequency response curve of the original truss (no tuning masses) with a damper at the location and in the direction corresponding to the largest amplitude in its target mode. The peak acceleration amplitudes were then measured. Again, the measurements were repeated 3 times and averaged.

These experimental results are plotted versus the corresponding analytical predictions in Fig. 4.4 and 4.5 for modes 1 and 3, respectively. The analytical values in these figures were obtained using the 2-mode approximation. The coefficient of correlation between analysis and experiment is equal to 98.4% for type-1 and 99.8% for type-3. This indicates that the analytical models and the measurements are precise.

Straight lines have been fitted to the data in each plot. These lines indicate a systematic mismatch between experimental and analytical results (the dotted lines in the figures represent an ideal one to one correspondence between analysis and experiment). The equations of these fitted lines, which are shown in Fig. 4.4 and 4.5, were incorporated into the analysis to correct the mismatch. Note that this analytical-experimental comparison was made during the summer of 1994 (Ponslet, *et al.*, 1994), while the experimental comparison of probabilistic and fuzzy set optimum designs, described in Chapter 5, was done during the summer of 1995. Because of this difference the regression equations might not be accurate.

4.3. Optimization Results

Using the data of Tables 4.3 and 4.4 for the statistics of the uncertain parameters we optimized the truss according to the probabilistic formulation in Eq. (3.5). We also generated membership functions for the same uncertain parameters as described in Section

3.4. Using these membership functions we performed the fuzzy set optimization according to the formulation in Eq. (3.6). The objective function has been modified using Eq. (3.13). During the optimizations, all analyses were performed using the 2-mode approximation (Section 3.6.2). We evaluated the probabilities of failure in each case using a Monte-Carlo simulation with 1000 samples.

We examined two cases. In the first (case 1), the failure limits were 4.80 m/s^2N for mode 1 and 6.60 m/s^2N for mode 3. In the second (case 2), the failure limits were 5.00 m/s^2N for mode 1 and 6.75 m/s^2N for mode 3. These limits were found in Section 3.5 and correspond to a problem that is favorable to probabilistic optimization. Type-1 and type-3 dampers were attached to nodes 12 and 11, respectively, in both cases. The designs are compared on the basis of their probabilities and possibilities of failure.

4.3.1 Case 1 - Failure limits 4.80 m/s^2N (mode 1) and 6.60 m/s^2N (mode 3)

The fuzzy set optimum design does not use any tuning masses, because this satisfies the optimality condition for the fuzzy optimum, which stipulates that the possibility of failure of the two modes are equal. The estimated probabilities and possibilities of failure of each mode and the system are listed in Table 4.7. Note that the possibilities of failure are not exactly equal but only approximately so, because this is a discrete optimization problem. Also listed in Table 4.7 are the standard deviations of the estimated failure probabilities, due to finite sample size used in Monte Carlo simulation. The standard deviations were calculated using Eq. (3.9). The failure possibilities of the two modes are almost equal. This is because fuzzy set optimization tries to minimize the maximum of the failure possibilities of the two modes. The system possibility of failure is 0.881 and is equal to the maximum of the individual failure possibilities. Also, the probabilities of failure of each mode are almost equal at the optimum. The system probability of failure is 49.7%.

Table 4.7. Case 1 - Fuzzy set optimum, probabilities and possibilities of failure.

	Mode 1	Mode 3	System
Failure possibility	0.881	0.856	0.881
Failure probability, P_f (%)	29.4	29.8	49.7
Standard deviation in P_f (%)	1.4	1.4	1.6

The probabilistic optimum design uses 3 tuning masses, attached to node 7 of the truss. The probabilities and possibilities of failure of the probabilistic optimum are presented in Table 4.8. The system possibility of failure is 0.916, which is larger than that of the fuzzy optimum. On the other hand, the system probability of failure is 34.6%, which means that the probability of failure of the probabilistic design is 15.1% lower that the probability of failure of the fuzzy set design. Note that each approach is safer by its own criterion.

Table 4.8. Case 1 - Probabilistic optimum, probabilities and possibilities of failure.

	Mode 1	Mode 3	System
Failure possibility	0.918	0.386	0.916
Failure probability, P_f (%)	34.2	1.1	34.6
Standard deviation in P_f (%)	1.5	0.3	1.5

Figures 4.6 and 4.7 compare the two alternate designs. Figure 4.6 shows the membership functions of the response of the two failure modes for each design. Only the

part of the membership function lying to the right of the apex is shown in that figure. Figure 4.7 contains the histograms of the peak acceleration of the two failure modes for each design. The dashed vertical line in each plot in Fig. 4.6 and 4.7 represents the corresponding failure limit. Figures 4.6 and 4.7 provide insight into the way the probabilistic and fuzzy set approaches maximize safety. The fuzzy set approach equalizes the failure possibilities of the two modes (in this particular problem, this means that it also equalizes the failure probabilities of the two modes). The probabilistic approach, on the other hand, uses the tuning masses at locations that affect more the mode that is easier to control (mode 3 in this case). As a result, the probability of failure of mode 3 is drastically reduced (from 29.8% to 1.1%), while mode 1 degrades only slightly (from 29.4% to 34.2%) (Fig. 4.7). This results in a significant reduction in the system failure probability.

4.3.2 Case 2 - Failure limits 5.00 m/s^2N (mode 1) and 6.75 m/s^2N (mode 3)

Both fuzzy and probabilistic optimum designs are identical to those in case 1. The probabilities and possibilities of failure and the standard deviations of the failure probabilities of the fuzzy optimum design are listed in Table 4.9. Table 4.10 shows the corresponding results for the probabilistic design. In this case, the difference in probability of failure between the two approaches is 12.3%.

Table 4.9. Case 2 - Fuzzy set optimum, probabilities and possibilities of failure.

	Mode 1	Mode 3	System
Failure possibility	0.816	0.792	0.816
Failure probability, P_f (%)	20.8	23.2	38.2
Standard deviation in P_f (%)	1.3	1.3	1.5

Table 4.10. Case 2 - Probabilistic optimum, probabilities and possibilities of failure.

	Mode 1	Mode 3	System
Failure possibility	0.853	0.329	0.853
Failure probability, P_f (%)	25.6	0.6	25.9
Standard deviation in P_f (%)	1.4	0.2	1.4

In figures 4.8 and 4.9 we compare the two alternative designs. Figure 4.8 presents the membership values of the response of the two failure modes for each design. Figure 4.9 presents the histograms of the peak acceleration of the two failure modes for each design. The dashed vertical line in each plot in Fig. 4.8 and 4.9 represents the corresponding failure limit. Note that these two figures are similar to Fig. 4.6 and 4.7, respectively. The conclusions on how each approach maximizes safety are the same as in the previous section.

4.4. Summary of Chapter 4

In this chapter, we obtained probabilistic and fuzzy set optimum designs for two cases involving different failure limits. The problems selected were favorable for the probabilistic optimization. The conclusions were similar for both cases. The important difference between the two design methods is that the fuzzy set approach does not consider what failure mode is easier to control - it simply tries to equalize the possibilities of failure of each failure mode. This is because we have assumed that the possibility of the union of events is equal to the maximum of the possibilities of these events, which is the most common definition used in fuzzy set calculus. On the other hand, the probabilistic approach tries to reduce the failure probability of the failure mode that is easier to control.

Of course, one can use a different definition of the possibility of the union of events, such as the sum of the possibilities of these events. In this case, there would still be problems for which the probabilistic optimum would have considerably lower failure probability than the fuzzy one. An example of such a problem is the case where the natural frequencies of type-1 and type-3 dampers are highly correlated. Because the above definition of the possibility of the union of events neglects correlation between modes, probabilistic design would still be better.

The above conclusions on the differences of the two approaches should be independent of the metric of failure used by the fuzzy set approach. For example, if instead of the membership function of the maximum allowable acceleration, we had used the area under the membership curve to the right of the maximum allowable acceleration as the metric of failure, our conclusions would still hold. Indeed, the fuzzy set approach would try to equalize the areas under the membership curves for the two failure modes, which means it would still not consider how easy it is to control each mode.

The most important conclusion of this chapter is that under ideal conditions, where we have sufficient information about random uncertainties, accurate models for predicting the response of a structure and failure is crisp, probabilistic methods can yield significantly better designs than fuzzy sets, because they take more information into account than fuzzy sets.

In the following chapter we will test the optimum designs experimentally. The objective is to determine whether, in practice, the probabilistic optimum designs are actually better than their fuzzy set counterparts.

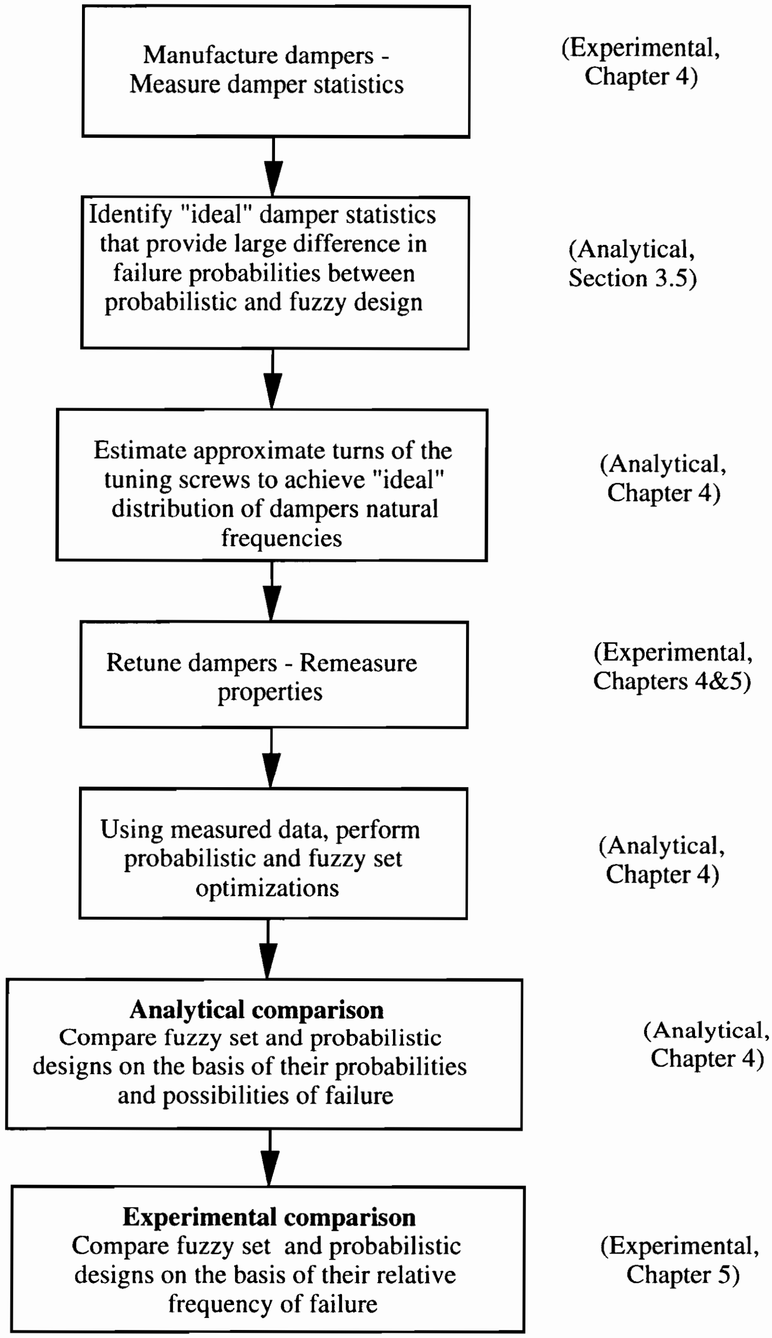

Figure 4.1 . Flow-chart of approach for comparing probabilistic and fuzzy set approaches. (Note: Each task is labeled as analytical or experimental. The chapter which describes each task is also specified).

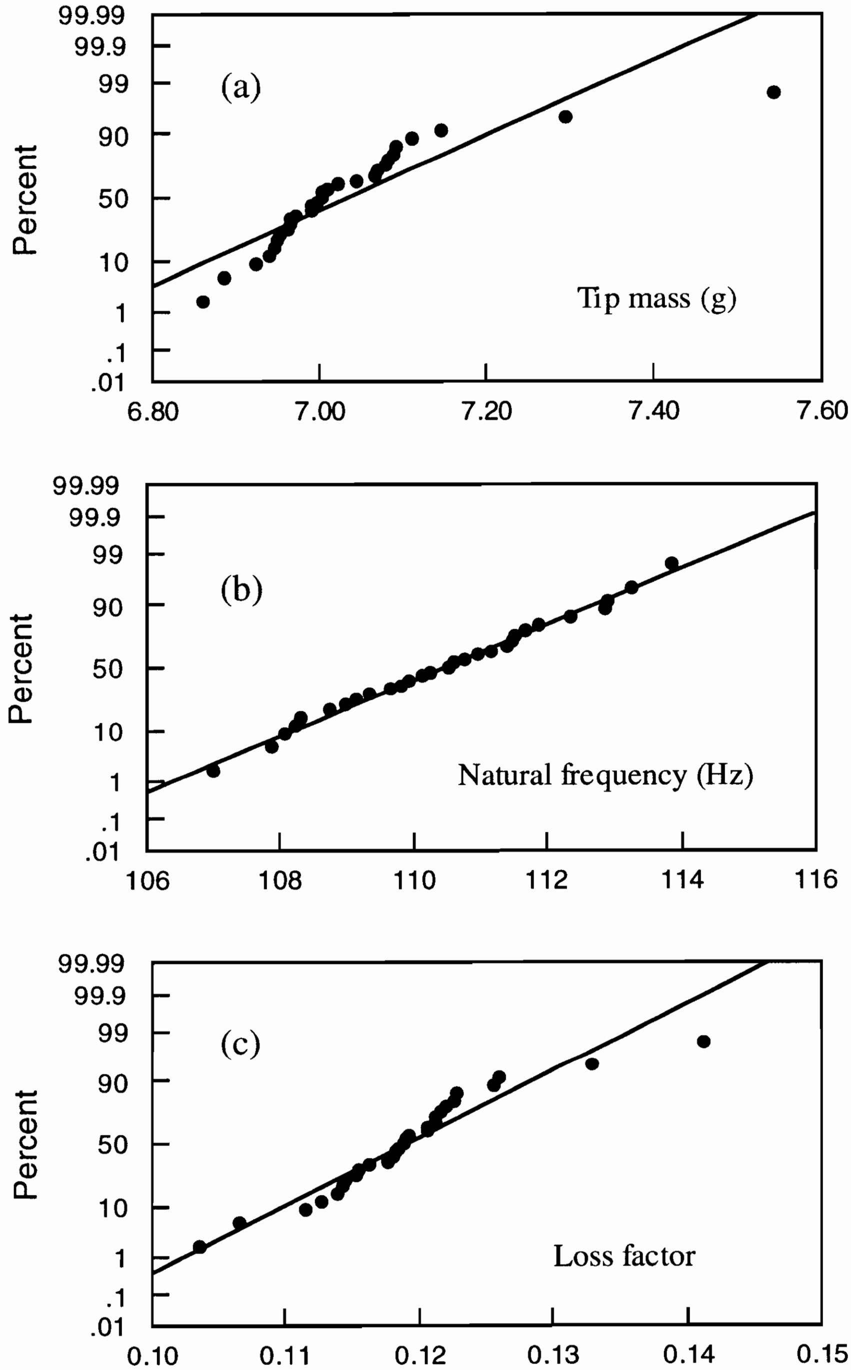

Figure 4.2. Type-1 dampers, distribution of parameters: (a) tip mass, (b) natural frequency, (c) loss factor.

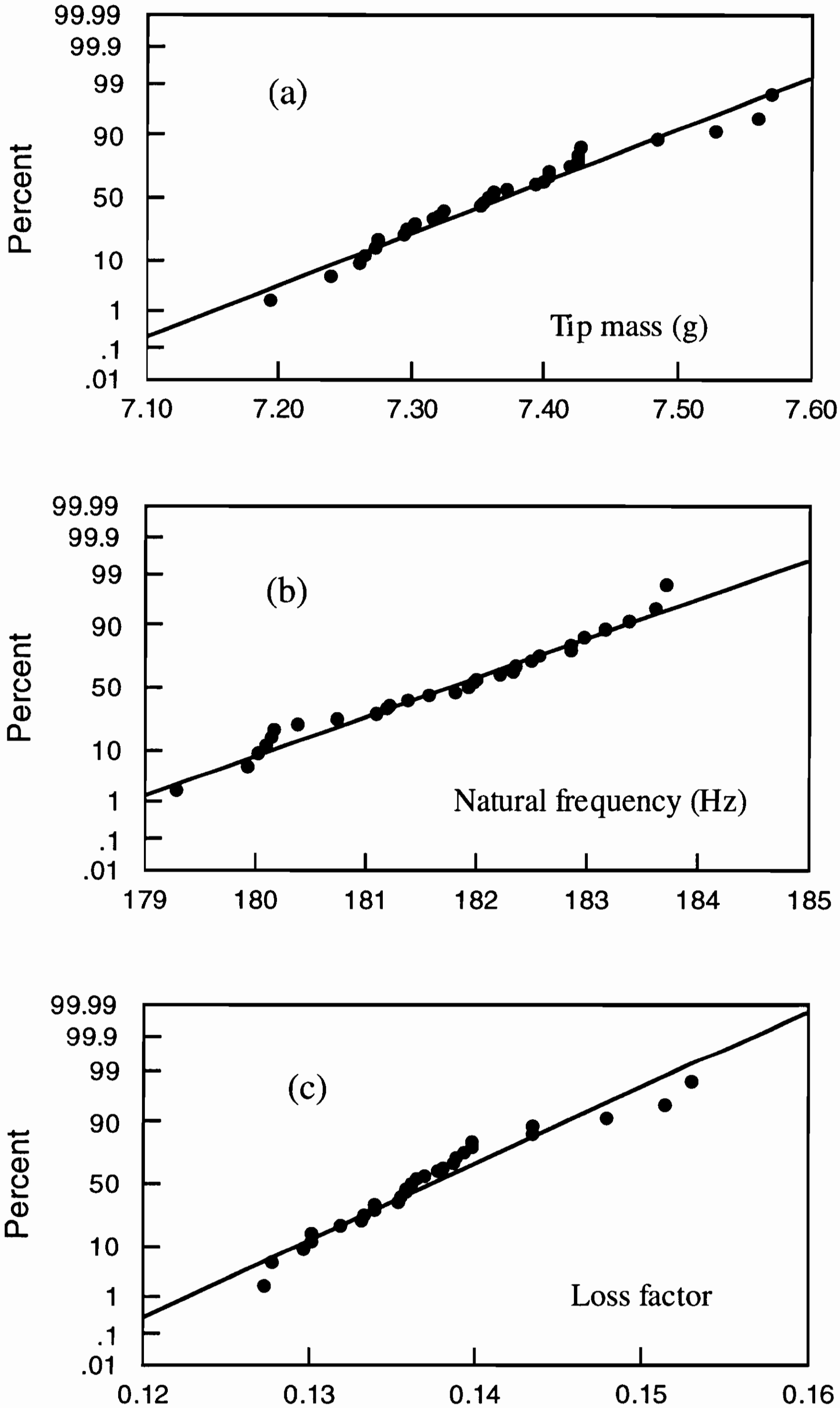

Figure 4.3. Type-3 dampers, distribution of parameters: (a) tip mass, (b) natural frequency, (c) loss factor.

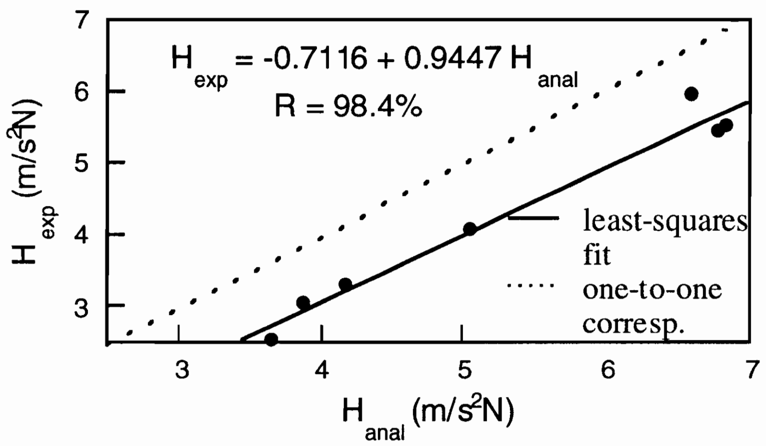

Figure 4.4. Mode 1, analytical-experimental correlation (2-mode approximation).

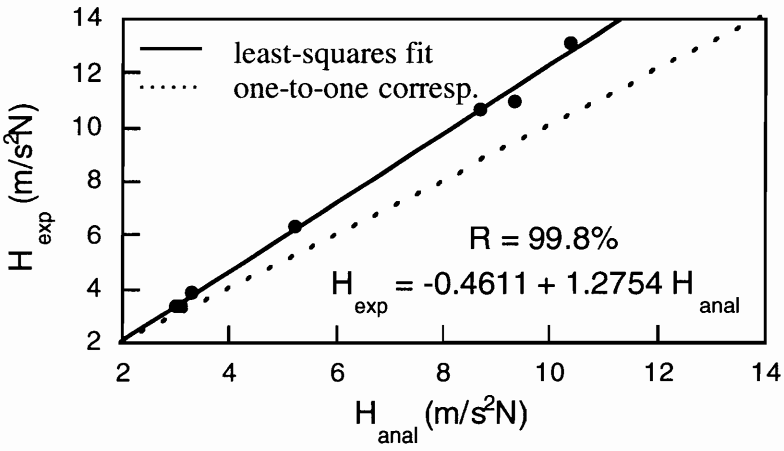

Figure 4.5. Mode 3, analytical-experimental correlation (2-mode approximation).

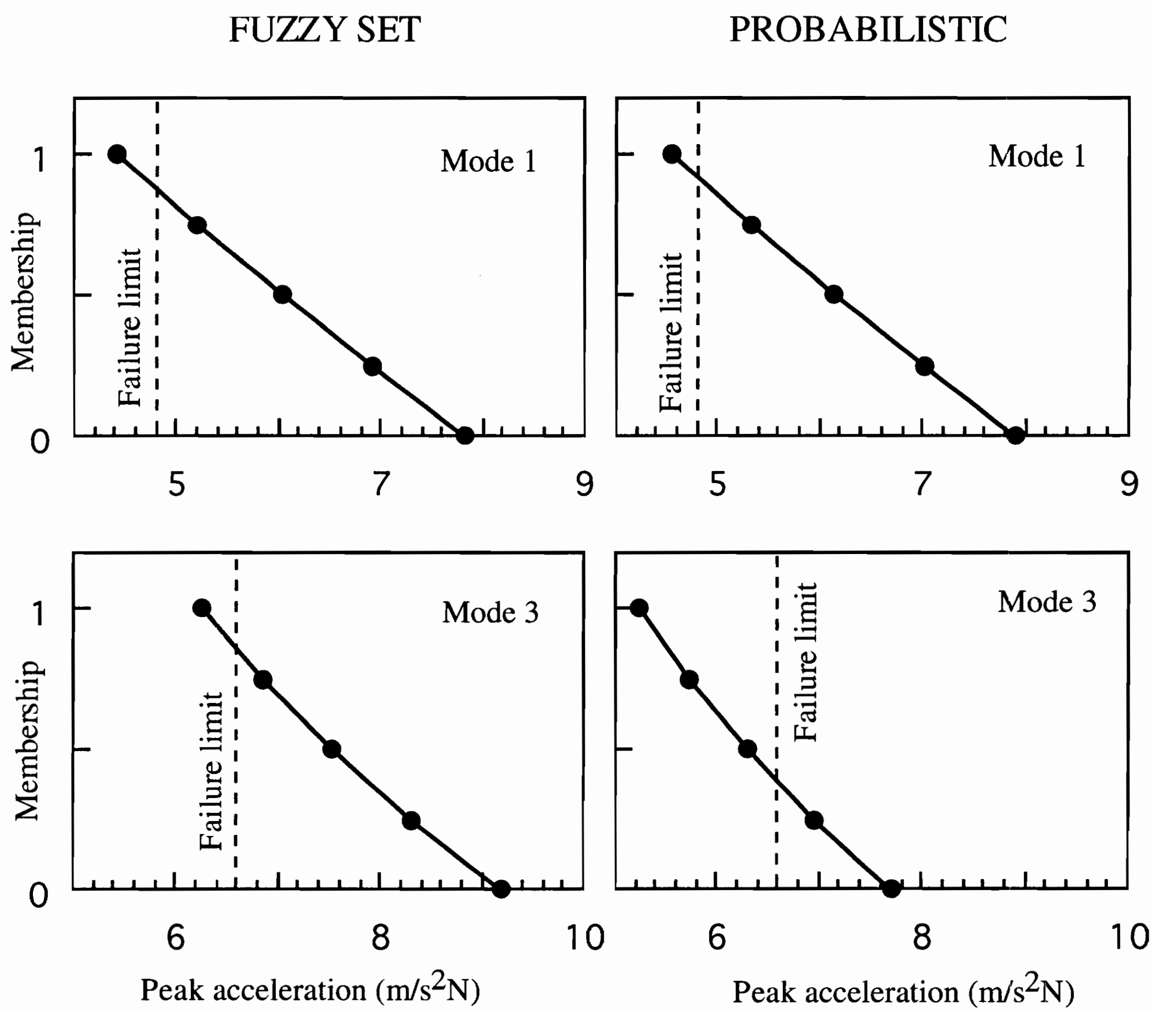

Figure 4.6. Case 1 - Fuzzy set based and probabilistic optimum designs. Membership functions of the peak acceleration.

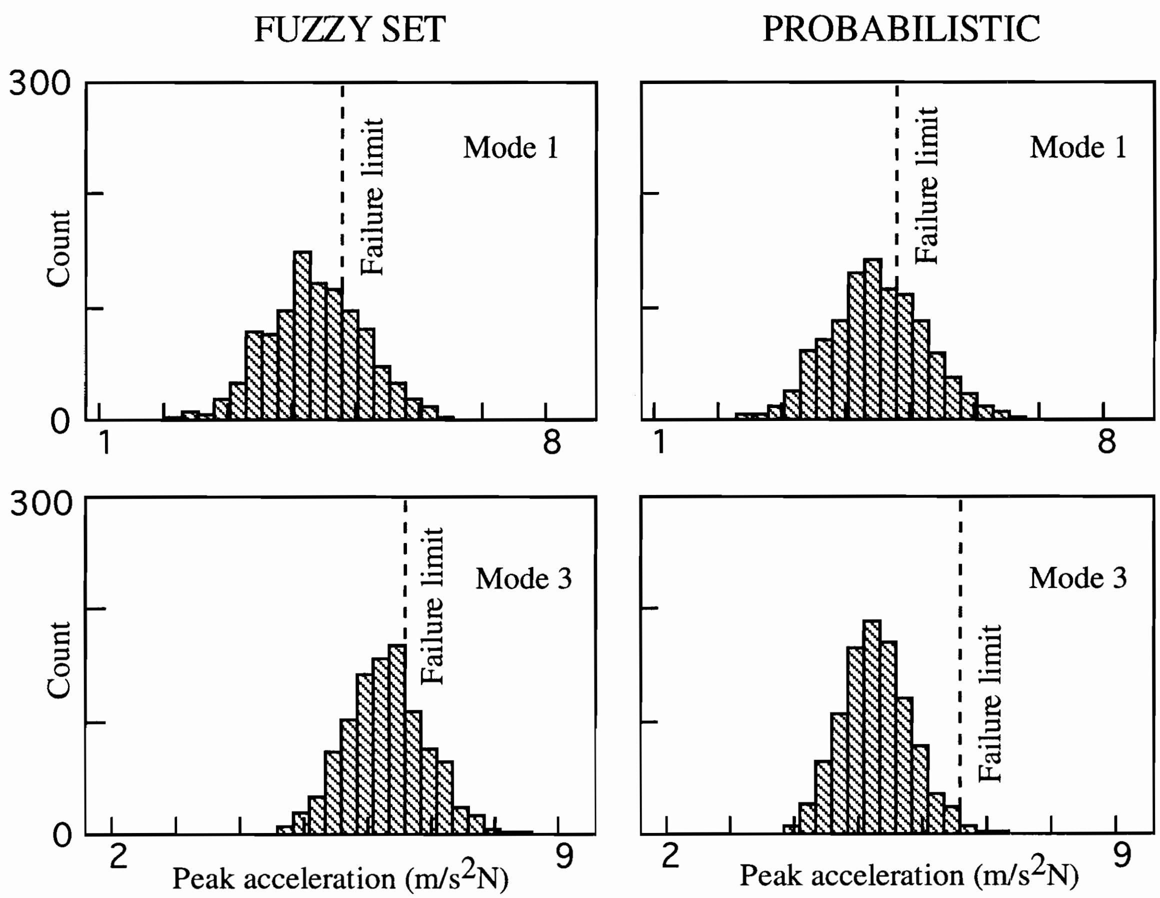

Figure 4.7. Case 1 - Fuzzy set and probabilistic optimum designs. Distributions of peak acceleration (Monte Carlo simulations with 1000 points).

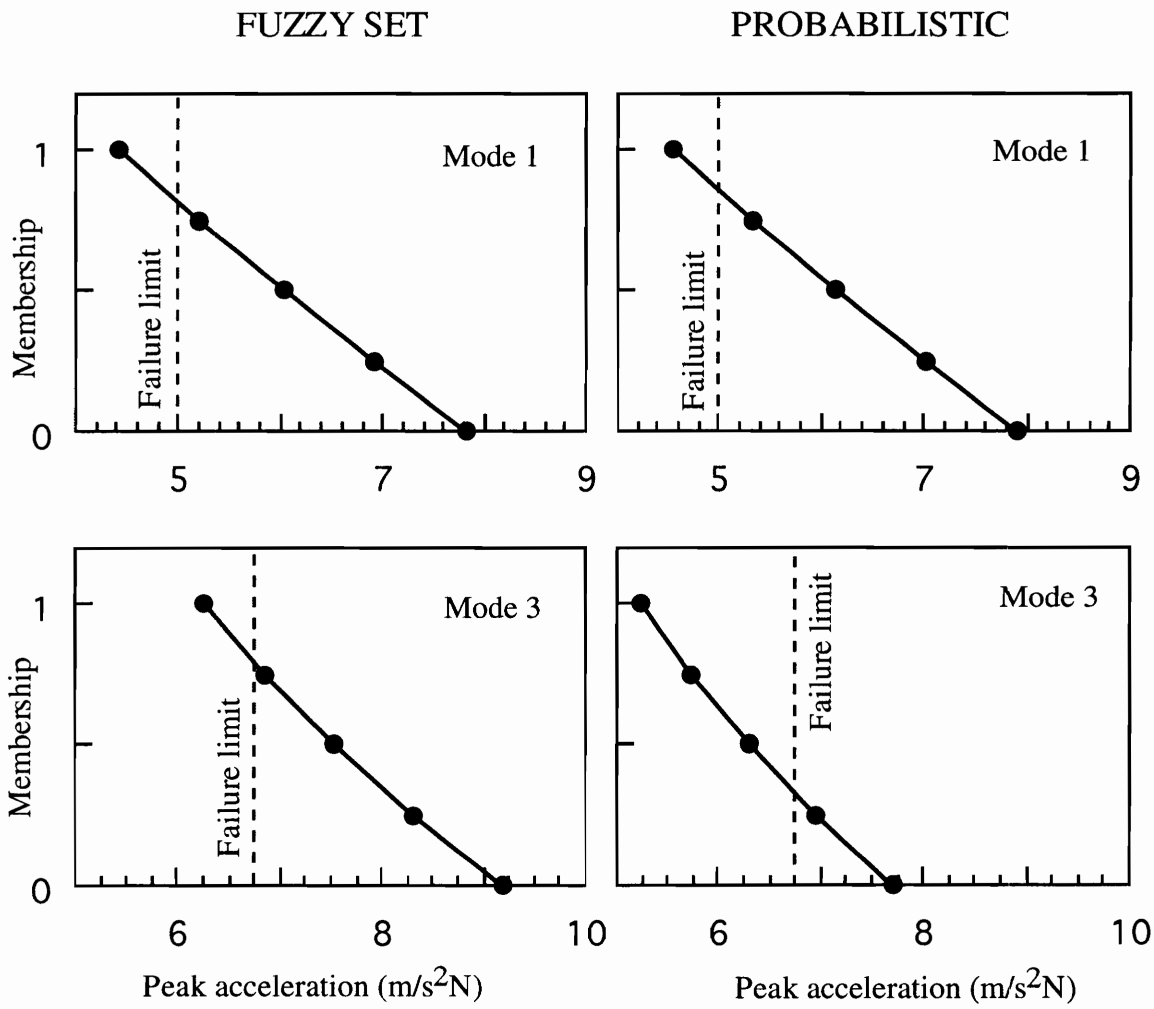

Figure 4.8. Case 2 - Fuzzy set and probabilistic optimum designs. Membership functions of the peak acceleration.

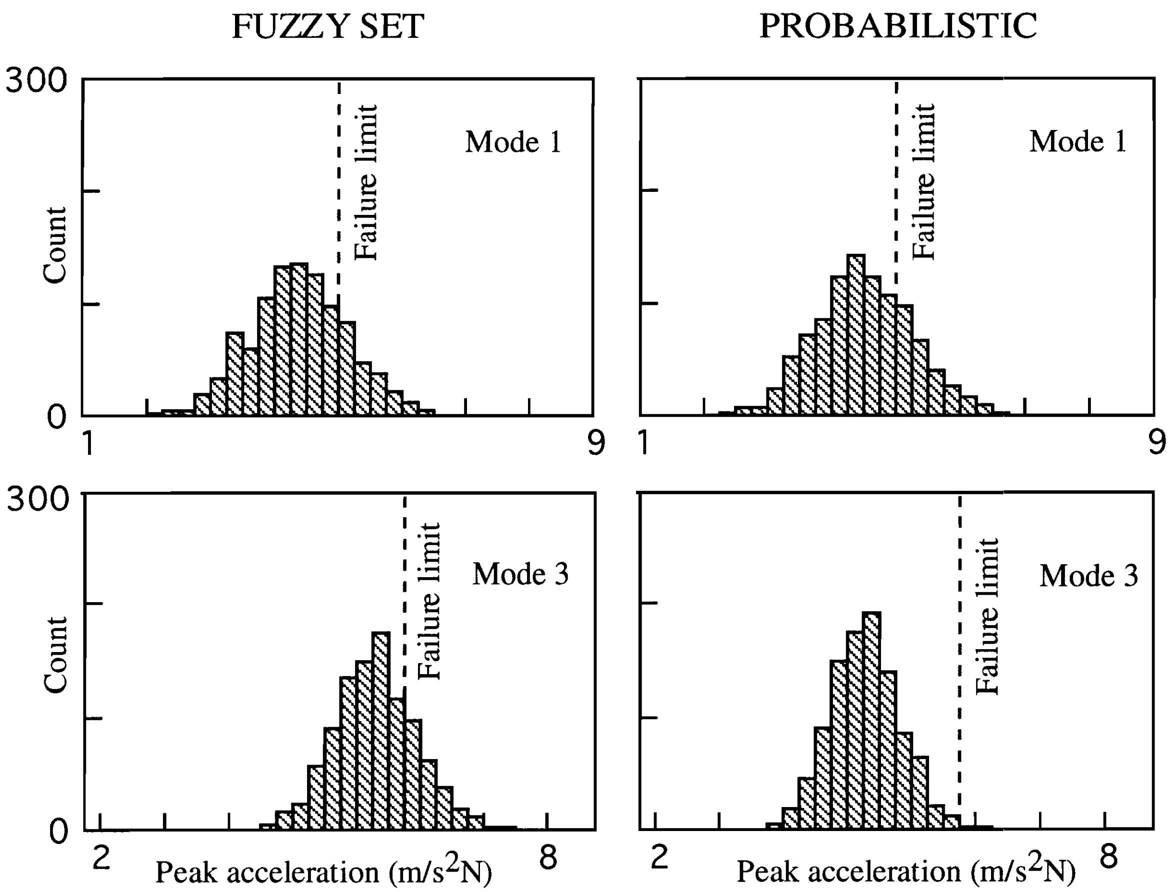

Figure 4.9. Case 2 - Fuzzy set and probabilistic optimum designs. Distributions of peak acceleration (Monte Carlo simulations with 1000 points).

CHAPTER 5

EXPERIMENTAL COMPARISON OF PROBABILISTIC AND FUZZY SET OPTIMUM DESIGNS

In the previous chapter, we found analytically two pairs of probabilistic and fuzzy set optimum designs. In both cases, the probabilistic optimum designs had lower probability of failure than their fuzzy set counterparts. However, this comparison was based on analytically predicted probabilities of failure. Due to modeling errors and incomplete knowledge of the probability distributions, the predicted probabilities of failure can be significantly different from the actual ones (e.g., Ben-Haim and Elishakoff, 1990). Therefore, we need to verify experimentally that the probabilistic designs are better despite modeling and other errors.

First, we describe the procedure that we used to obtain a sample of dampers that would reduce the error due to the randomness of the sample. Then, we compare experimentally the probabilities of failure of the optimum designs that were obtained in the previous chapter. Due to shortage in viscoelastic foam we constructed only 29 nominally identical realizations of both the probabilistic and the fuzzy set optimum designs. The comparison is based on the number of realizations that fail out of the total number of 29 realizations. Following that, we estimate the error in the experimentally measured failure probabilities. We also compare the experimental to the analytical results and we discuss some sources of error that might have affected the experimental results, thus explaining the differences between analytical and experimental results.

5.1 Creating a Sample of Dampers to Reduce Uncertainty in Experimentally Measured Failure Probability

In Section 3.5, we described the procedure used to obtain large differences in the probabilities of failure between probabilistic and fuzzy set approaches. In Section 4.1, we mentioned that in order to approximate this scenario we should obtain the desired distribution at least for the natural frequencies of the dampers. This can be achieved by adjusting the tuning screws of the 29 dampers of each type. In this section, we describe how this adjustment was performed to reduce the error due to randomness of the sample.

The classical technique to experimentally evaluate probabilities of failure is to pick a random sample of n realizations, test them, and count the number of failures. There are two sources of error associated with this technique. The first is a resolution error due to the limited size of the sample. We cannot distinguish between probabilities of failure that differ less than a minimum limit. The other type of error is related to the randomness of the sample and also depends on the sample size. If a probability of failure P is estimated from a random sample of size n, the standard deviation of P is given by Eq. (3.9), which we rewrite here for a sample n:

$$\sigma_P = \sqrt{\frac{P(1-P)}{n}} \tag{5.1}$$

There is very little interaction between the two modes of the structure. Indeed, a type-1 damper has no measurable effect on the response of mode 3 and vice versa. As a result, we cannot create more than 29 independent combinations of type-1 and type-3 dampers, using the 29 type-1 and type-3 dampers available. If we used 29 random realizations of dampers, the standard deviation of P due to the randomness in the sample would obscure the difference in probabilities of failure. For example, if the failure probability of a design were P=30%, its standard deviation would be 8.5%.

Equation 5.1 shows that we need prohibitively large samples to reduce the uncertainty in P enough to allow meaningful measurements of the probability differences between the two approaches. For example, let us estimate the required number of samples we would need in case 2, where the difference in failure probability between the alternate probabilistic and fuzzy designs is 12.3% (see Section 4.3.2). Let us assume that the difference in probabilities of failure between the two alternate designs, $\Delta P = P_f - P_p$, is normally distributed. P_f is the probability of failure of the fuzzy set design and P_p is the probability of failure of the probabilistic design. The mean and standard deviation of ΔP are:

$$\text{Mean: } \overline{\Delta P} = \overline{P_f} - \overline{P_p} \tag{5.2}$$

$$\text{Standard deviation: } \sigma_{\Delta P} = \sqrt{\sigma_{P_f}^2 + \sigma_{P_p}^2} \tag{5.3}$$

$\overline{P_f}$ and $\overline{P_p}$ are the mean values and σ_{P_f} and σ_{P_p} are the standard deviations of P_f and P_p, respectively. By substituting numerical values in Eq. (5.1), (5.2) and (5.3) we get $\overline{\Delta P} = 0.123$ and $\sigma_{\Delta P} = \dfrac{0.6542}{\sqrt{n}}$. To determine whether the probabilistic design is safer than the fuzzy set design with enough confidence, it is reasonable to require that the mean of ΔP be equal to two standard deviations of ΔP (that corresponds to a probability of 97.72% confidence level that the probabilistic design is safer than the fuzzy set design). Using Eq. (5.1), we find that n should be 113. It is too expensive to construct and test so many dampers.

Sampling uncertainty can be reduced if we use a non-random sample. Since we need to adjust the damper tuning screws to obtain the desired distributions of the natural frequencies of the dampers anyway (see Section 4.1 and Figure 4.1), we can do it in such a way that the resulting uncertainty in the measured probabilities is reduced. This can be achieved by distributing the natural frequencies of the dampers uniformly along the probability axis of the probability distribution function of a normal sample. The process is illustrated in Fig. 5.1. The natural frequencies of the dampers are given on the horizontal axis and the cumulative probability distribution is given on the vertical axis. The solid line represents a normal distribution of the natural frequencies with the desired mean and standard deviation. We first distributed the natural frequencies of the 29 dampers of the sample uniformly along the probability axis. The first damper natural frequency has a probability of non-exceedance of 100/(29x2)%. The natural frequencies of the rest of the dampers correspond to probabilities of non exceedance obtained by incrementing the previous probability by 100/29%. Comparing those values with the measured natural frequencies of the 29 dampers, we obtained a desired change in natural frequency for each damper. Using measured sensitivity derivatives of the natural frequency with respect to the number of turns on the tuning screws, these changes in natural frequency were translated into adjustments of the tip screws.

We applied the above tailoring procedure on both types of dampers. The dampers were measured again after the adjustment. Each damper was measured three times and the parameters were averaged through the three measurements to reduce the effect of temperature. The resulting parameters after the averaging procedure were used to estimate the means, standard deviations, coefficients of variation and correlation coefficients between the parameters. These values were presented in Tables 4.3 and 4.4.

5.2 Experimental Results

In the previous chapter we found probabilistic and fuzzy set optimum designs for two different problems (cases 1 and 2). In case 1, the peak acceleration limits were 4.80 m/s^2N and 6.60 m/s^2N for modes 1 and 3, respectively. In case 2 the failure limits were 5.00 m/s^2N and 6.75 m/s^2N for modes 1 and 3, respectively. We prepared 29 sets of two dampers by pairing randomly the 29 dampers of each type (all dampers were used only once). These 29 sets are used to create 29 realizations of each design. Each realization is tested 3 times in the laboratory and the measured peak acceleration amplitudes are averaged through the 3 measurements. This averaging is intended to reduce the effect of measurement errors and temperature changes. For each design, the probabilities of failure are estimated from these 29 values by counting the number of designs that vibrate more than the prescribed maximum levels. Note that in both cases studied, the fuzzy set optimum designs have no tuning masses on the truss. Also, both probabilistic optimum designs have three tuning masses at node 7 of the truss. The only difference between the two cases are the failure limits. So, we can perform only one set of experiments and estimate the probabilities of failure in both cases, counting failures for both pairs of peak acceleration limits.

With a sample of 29 measurements, the resolution of the probability measurement is about 3.45%. This resolution is sufficient to measure the differences in the failure probabilities between probabilistic and fuzzy set designs. Indeed, the difference in the analytical predictions of the probability of failure between the two approaches is 15.1% for case 1 (see Tables 4.7 and 4.8) and 12.3% for case 2 (see Tables 4.9 and 4.10).

The results of the experiment for case 1 are listed in Table 5.1. The experimental probabilities of failure of the two designs are compared to predicted values from Monte

Carlo simulations using 1000 points and the 2-mode approximation. In parentheses are the number of failures out of the 29 realizations. Similar results are presented in Table 5.2 for case 2. We observe that the experiments verify that the probabilistic design is superior over the fuzzy set optimum design, as was predicted by the analysis.

Table 5.1. Case 1 - Fuzzy set and probabilistic optimum designs: Comparison of experimental (sample of 29) and analytical probabilities of failure (Monte Carlo, sample of 1000, 2-mode approximation).

	Fuzzy set optimum		Probabilistic optimum	
	Analytical	Experimental	Analytical	Experimental
Mode 1 (%)	29.4	34.5 (10/29)	34.2	34.5 (10/29)
Mode 3 (%)	29.8	48.3 (14/29)	1.1	10.3 (3/29)
System (%)	49.7	65.5 (19/29)	34.6	44.8 (13/29)

Table 5.2. Case 2 - Fuzzy set and probabilistic optimum designs: Comparison of experimental (sample of 29) and analytical probabilities of failure (Monte Carlo, sample of 1000, 2-mode approximation).

	Fuzzy set optimum		Probabilistic optimum	
	Analytical	Experimental	Analytical	Experimental
Mode 1 (%)	20.8	27.6 (8/29)	25.6	31.0 (9/29)
Mode 3 (%)	23.2	41.4 (12/29)	0.6	10.3 (3/29)
System (%)	38.2	62.1 (18/29)	25.9	41.4 (12/29)

Figures 5.2 (a and b) show the cumulative probability distribution curves of the peak acceleration amplitudes of the fuzzy set design for modes 1 and 3, respectively. The same plots are shown in Fig. 5.3 (a and b) for the probabilistic design. The vertical solid lines

correspond to the peak acceleration limits for case 1. The vertical dashed lines correspond to the peak acceleration limits for case 2.

Table 5.3 lists the differences in the failure probabilities obtained analytically and experimentally for the two design approaches and the two cases examined. In both cases, the experimental probabilities of failure are higher than the analytical ones for both failure modes. In both cases, for the fuzzy set design, the analysis predicted almost equal probabilities of failure for the two failure modes. However, the experiment gave a much higher probability of failure for mode 3. In the probabilistic design, the increase in probability of failure between analysis and experiment was again larger for mode 3. However, the increase in the failure probability of mode 3 was smaller for the probabilistic design than for the fuzzy set design.

Table 5.3. Differences between analytical and experimental failure probabilities.

	Case 1		Case 2	
	Fuzzy set	Probabilistic	Fuzzy set	Probabilistic
Mode 1 (%)	5.1	0.3	6.8	5.4
Mode 3 (%)	18.5	9.2	18.2	9.7

The above results can be explained based on the following observations. The fuzzy set designs and the probabilistic designs have very close probabilities of failure in mode 1. However, in mode 3, the probabilistic designs have much smaller failure probabilities than their fuzzy set counterparts, because mode 3 has a smaller scatter than mode 1, so it is easier to reduce the probability of failure of mode 3 by slightly reducing the mean acceleration amplitude. Because of the sharpness of the distribution (see Fig. 4.7, 4.9, 5.2

and 5.3) the probability of failure of mode 3 is more sensitive than that of mode 1 to modeling errors and other unmodeled uncertainties. Moreover, the probability of failure of the fuzzy optimum is more sensitive to modeling errors which result in a shift of the entire probability density function of the acceleration amplitudes to the right (see Fig. 4.7 and 4.9). As a result, the discrepancy between analytical and experimental results is larger for the probability of failure of the fuzzy set design for mode 3.

In this study, modeling and other errors during the experiment affected both approaches in the same direction because they led to underestimation of the acceleration. As a result, the experimental distributions of the accelerations are shifted to the right relative to the analytical distributions (Fig. 5.2 and 5.3). Because of this systematic shift, the analytical predictions in terms of the relative reliability of each method were verified experimentally. We do not know if this would be true if the errors affected each approach in a different direction. In Section 5.4, we will examine sources of error that can explain the difference between analytical predictions and experimental results.

Although the experiment validated that the probabilistic optimum design is safer than the fuzzy set optimum design, it also showed that if there is little information or large modeling errors it might be better to use fuzzy design. To see this, let us assume that the maximum allowable failure probability was 0.35. Then, according to the results in Tables 4.8 and 4.10, probabilistic design could be judged acceptable, whereas in reality it would be unacceptable. Fuzzy set analysis, due to its conservatism, could have protected us from accepting such an unsafe design, because it would have shown that its failure possibility is almost one. Although this is not as precise as failure probability, there is a clear implication that the obtained designs are likely to fail. Therefore, if a designer used fuzzy set optimization and assessed the risk of failure using the possibility of failure, it is possible that he or she would conclude that the truss must be redesigned to increase reliability using more resources (e.g., more dampers).

5.3 Estimation of Error in Measured Failure Probabilities

In this section, we will provide estimates for two types of error:

1) *Experimental error*, which is due to non-ideal conditions during the experiment (i.e., temperature variation, small differences in the orientation of the damper on the truss, etc.).

2) *Resolution error.*

Note that, these two types of error are independent. They cannot be combined because we do not know the probability distribution of the resolution error.

5.3.1 Estimation of Experimental Error

Experimental error is due to lack of repeatability. To accurately estimate the experimental error we would have to determine the statistics of the measured failure probabilities. This could be impractical, because it requires repeating the whole experiment (i.e., measuring the response of the truss three times for each of the 29 dampers) many times and calculate the statistics of the obtained failure probabilities. In this section, we will provide a crude estimate of the error of the measured failure probabilities using Monte Carlo simulation and the existing experimental measurements.

We assume that the measured structural response, R_{m_i}, when damper i is attached to the structure can be expressed as:

$$R_{m_i} = R_i + e_i \tag{5.4}$$

where R_i is the response of the damper that we would measure if there were no experimental error and e_i is the experimental error (including the effect of temperature variation). We assume that the mean value of the error term is zero and that the error is normally distributed. R_i is constant for a given damper, because it depends only on the damper properties. The mean and the variance of R_{m_i} are:

$$\text{Mean: } \overline{R_{m_i}} = R_i + \overline{e_i} = R_i \tag{5.5}$$

$$\text{Variance: } \sigma^2_{Rm_i} = \sigma^2_{R_i} + \sigma^2_{e_i} = \sigma^2_{e_i} \tag{5.6}$$

We have three structural response measurements for each damper. The average of these three measurements provides an estimate of R_i while their variance provides an estimate of $\sigma^2_{e_i}$. Note, that these estimates are very crude because they were obtained using only three sample points. We observed that $\sigma^2_{e_i}$ was independent from R_i and that it varied significantly from damper to damper. Assuming that e_i is the same for all dampers we obtained a very crude approximation of its variance, σ^2_e, by averaging the variances of the 29 measurements, i.e.,

$$\sigma^2_e = \frac{1}{29}\sum_{i=1}^{29}\sigma^2_{e_i} \tag{5.7}$$

We can simulate the experimental procedure for estimating the probability of failure using Monte Carlo as follows. For each damper, we assume that the average of three measurements is equal to R_i. For each damper, we generate three sample values of the error, e_i, following the distribution $N(0, \sigma_e)$. We then add these values to R_i, which yields three sample values of the measured response of the truss, when this damper is attached to the truss. We average these three responses as we do in the experiment. Then we compare the average value of the truss response for each damper to the corresponding failure limit and estimate the probability of failure by counting how many of the 29 realizations fail. We repeat the whole procedure 1000 times and we calculate the statistics of the failure probabilities. The standard deviations of the failure probabilities are listed in Table 5.4 for both designs and both cases 1 and 2.

Table 5.4. Standard deviation of failure probability (%) due to experimental error, obtained from Monte Carlo simulation with 1000 repetitions.

	Case 1		Case 2	
	Fuzzy set	Probabilistic	Fuzzy set	Probabilistic
Mode 1 (%)	4.8	5.2	4.2	4.1
Mode 3 (%)	4.3	3.0	4.1	2.6
System (%)	3.9	5.0	4.0	4.6

Assuming again (as we did in Section 5.1) that the difference in failure probabilities of the alternate designs, ΔP, is normally distributed, from Eq. (5.3) we get that the standard deviations, $\sigma_{\Delta P}$, are 6.34% and 6.10% for cases 1 and 2, respectively. In case 1, $\Delta P = 20.7\% = 3.26\sigma_{\Delta P}$. This corresponds to a probability of 99.94% that the failure probability of the probabilistic design is smaller than that of the fuzzy set design. In case 2, $\Delta P = 20.7\% = 3.39\sigma_{\Delta P}$. The corresponding confidence level is 99.97%. These results provide satisfactory confidence in the experimental results.

5.3.2 Estimation of Resolution Error

As explained in Section 5.1, the sample of dampers used in the experiments was not random but it was obtained by sampling the distributions of the natural frequencies of the dampers with a uniform step size in probability. Figures 5.4 (a and b) are plots of the peak acceleration amplitude versus the natural frequency of type-1 and type-3 dampers, respectively, for the range of damper frequencies used in this study. Two designs are examined. One with no tuning masses on the structure (fuzzy set optimum) and one with 3 tuning masses at node 7 of the truss (probabilistic optimum). The remaining two damper parameters are assumed constant, equal to their mean values. The relation between the peak acceleration and the natural frequency is monotonic for the range examined in this study. Therefore, if the natural frequency is sampled with a uniform step size in probability then the response (peak acceleration) is also sampled with a uniform step size in probability.

By neglecting the experimental error (discussed in Section 5.3.1) and the error due to randomness of the sample, the only error left is a resolution error due to the limited size of the sample. This type of error is illustrated in Fig. 5.5. This figure shows part of the cumulative probability distribution plot of the response. There are 20 realizations whose peak acceleration is less than that corresponding to point B. Therefore, if the failure limit lies between points A and B of Fig. 5.5, we will always measure 9 failures out of 29 samples (i.e., there are 9 realizations whose peak acceleration is higher than the failure limit). This translates into a probability of failure of 31.03% (9/29). But, as it can be seen from Fig. 5.5, the actual probability of failure can be anywhere in the range from 29.31% to 32.76% or equivalently 31.03% plus-minus half the resolution. The resolution is approximately 3.45%. So, if the probability of failure estimated from an experiment is p% the actual probability of failure can be between (p-1.725)% and (p+1.725)%. Note that the above error analysis is approximate because it does not take into account the fact that the other two damper parameters, besides the natural frequency, are not sampled with a uniform step size in probability. However, their effect on the response is much smaller than the effect of the natural frequency. Note that the resolution error is quite smaller than the error due to randomness in the sample (see Section 5.1).

5.4 Sources of Error in Analytically Predicted Probabilities of Failure of Optimum Designs

In Section 5.2, we observed a discrepancy between the analytical predictions and the experimental results. In this section, we will examine the effect of two sources of error on the analytically predicted failure probabilities:

a) Errors in calibration of peak acceleration.

b) Difference in the average temperature when we measured damper properties and when we measured the response of the truss.

Using the exact model instead of the calibrated 2-mode approximation we concluded that the modeling error due to the 2-mode approximation is negligible compared to the above two errors.

5.4.1 Calibration of Peak Acceleration

In Section 4.2.2, we mentioned that we corrected the analytical predictions of the peak acceleration amplitudes using a linear regression equation. This correction, however, was determined using measurements taken about one year prior to the experiments described in the previous section. To assess the effect of this discrepancy we took new measurements on a number of dampers to determine a new correction equation and used this equation to correct the structural response predictions obtained from the analysis.

We followed the procedure described in Section 4.2.2. The experimental results are plotted versus the corresponding analytical predictions in Fig. 5.6 and 5.7 for modes 1 and 3, respectively. The coefficient of correlation between analysis and experiment is equal to 97.3% for mode 1 and 98.2% for mode 3. The equations of the straight lines fitted to the data of each plot are shown in Fig. 5.6 and 5.7. These equations replace the old ones (obtained in Section 4.2.2) in the analysis. Note that the difference between the old and new regression equations used to calibrate the response is considerable for both modes 1 and 3.

5.4.2 Difference in the Average Temperature When Damper Properties and Structural Response Were Measured

Because the dampers are made of viscoelastic material, their properties are sensitive to temperature. The lab temperature during the structural response measurements was on the average approximately 0.7 ^{O}F higher than the temperature when the properties of the dampers were measured. Because these properties were used in the analysis, the temperature variation can be responsible for the discrepancy between analysis and experiment.

We estimated the effect of the above temperature difference the following way. From tests we determined the sensitivity of each parameter with respect to temperature. Table 5.5 lists these sensitivities for each damper parameter. Using these sensitivities and linear extrapolation we corrected the mean values of the damper parameters as shown in Table 5.6. For simplicity, we assume that, because the temperature variation is small, the standard deviations and the correlation coefficients of the damper properties are not affected.

Table 5.5. Sensitivities of damper parameters with respect to temperature.

	Tip mass (kg/^{O}F)	Natural frequency (Hz/^{O}F)	Loss factor (1/^{O}F)
Type-1 dampers	2.88×10^{-5}	-0.41	5.20×10^{-3}
Type-3 dampers	2.95×10^{-5}	-0.98	6.73×10^{-3}

Table 5.6. Correction for temperature. Mean values of effective damper properties.

	Tip mass (g)	Natural frequency (Hz)	Loss factor
Type-1 dampers	7.050	110.15	0.12267
Type-3 dampers	7.390	181.02	0.14181

5.4.3 Estimation of Failure Probabilities Using Corrected Properties Due to Temperature and Updated Regression Equation for Correction of Analytical Predictions

We generated 1000 new samples of each type of dampers using the mean values of the damper parameters in Table 5.6. Using these samples and the new analytical-experimental correction as determined in Section 5.4.1, we calculated the probabilities of failure for the fuzzy set and the probabilistic optimum designs by Monte Carlo simulation. The new probabilities of failure are listed in Tables 5.7 and 5.8 for cases 1 and 2, respectively. The differences between these values and the experimental ones are presented in Table 5.9. It is observed, that the difference between the predicted and measured failure probability of mode 3 of the fuzzy set design is reduced by about 75% in case 1 and by about 60% in case 2 compared to the values of Table 5.3. The difference in mode 3 of the probabilistic optimum design is also reduced by about 35% in case 1 and 20% in case 2. Mode 1 was not affected significantly, because it has larger scatter, and therefore, the corresponding failure probability is not sensitive to small variations in the mean values of the damper parameters.

Table 5.7. Case 1 - Fuzzy set and probabilistic optimum designs. Analytical probabilities of failure (Monte Carlo, sample of 1000, 2-mode approximation, new analytical-experimental correction, damper properties corrected for temperature).

	Fuzzy set optimum	Probabilistic optimum
Mode 1 (%)	29.2	35.1
Mode 3 (%)	43.8	4.2
System (%)	59.8	37.9

Table 5.8. Case 2 - Fuzzy set and probabilistic optimum designs. Analytical probabilities of failure (Monte Carlo, sample of 1000, 2-mode approximation, new analytical-experimental correction, damper properties corrected for temperature).

	Fuzzy set optimum	Probabilistic optimum
Mode 1 (%)	20.6	24.1
Mode 3 (%)	34.3	2.4
System (%)	48.6	25.8

Table 5.9. Differences between corrected analytical and experimental failure probabilities.

	Case 1		Case 2	
	Fuzzy set	Probabilistic	Fuzzy set	Probabilistic
Mode 1 (%)	5.3	-0.6	7.0	6.9
Mode 3 (%)	4.5	6.1	7.1	7.9

In case 1, the differences between analytical and experimental failure probabilities for mode 1 (both approaches) and mode 3 (fuzzy set approach only) are either smaller or slightly higher than one standard deviation of the experimental error (see Table 5.4). This means that experimental error can be responsible for the largest part of the discrepancy between analysis and experiment. However, in case 2 and in mode 3 of the probabilistic optimum of case 1 the differences between analytical and experimental failure probabilities are considerably larger than one standard deviation of the corresponding experimental error. Thus, in these situations experimental error is most likely a small part of the discrepancy between analysis and experiment. This is not unreasonable because experimental error is only one component of the total discrepancy between analysis and experiment. One should also keep in mind that the standard deviations of the experimental error evaluated in Section 5.3.1 were approximate.

Figures 5.8 (a and b) show the cumulative probability curves of the peak acceleration for modes 1 and 3 respectively of the fuzzy set design. The same plots are shown in Fig. 5.9 (a and b) for the probabilistic design. The corrections brought the analytical curves of mode 3 closer to the experiment, without significantly affecting mode 1. This implies that the discrepancy in the average temperature during the experiments and the error in the calibration equation were responsible for the larger portion of the discrepancy between analytical and experimental failure probabilities.

Note that when we took the measurements to determine the new correction equation (see Section 5.4.1) the temperature was the same as when the damper properties were measured. Consequently, the correction equations do not account for any part of the error due to change in the average temperature when the damper properties were measured and when the structural response measurements were taken.

5.5 Summary of Chapter 5

In this chapter, we tested in the laboratory the two fuzzy set optimum designs and two probabilistic optimum designs found in Chapter 4. We created the sample of dampers in such a way that the sampling uncertainty of the experiment was reduced. The experiments verified the superiority of the probabilistic over the fuzzy set design. Note that a design problem which was favorable to the probabilistic approach was selected. On the other hand, the example considered demonstrated that it might be better to use fuzzy sets when very little information is available or when large modeling errors are present.

We observed a considerable discrepancy between the analytically predicted and the experimentally measured failure probabilities. We identified two sources of error. The errors in the regression equation used to correct the analytical peak acceleration and the difference in the average temperature when the damper properties were measured and when the experiments were conducted. Accounting for these two factors in the analysis significantly reduced the difference between analytically predicted and experimentally measured failure probabilities.

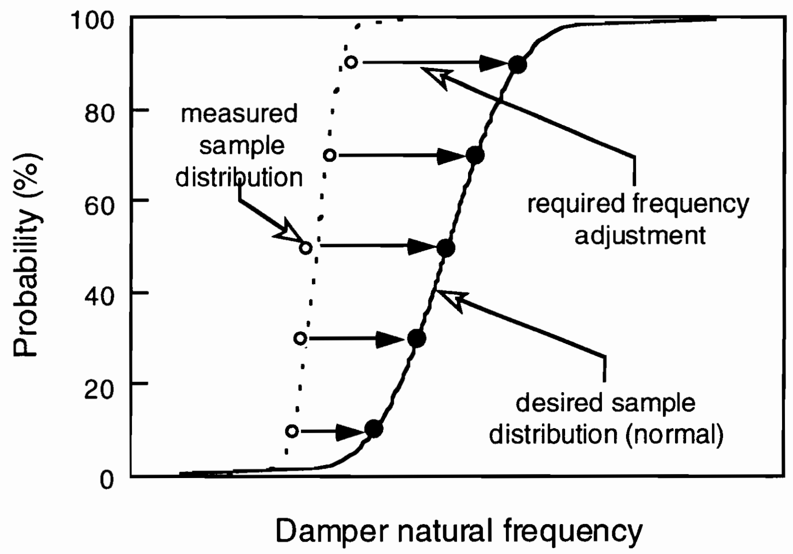

Figure 5.1. Creation of an "ideal" sample of dampers. (From Ponslet, 1995).

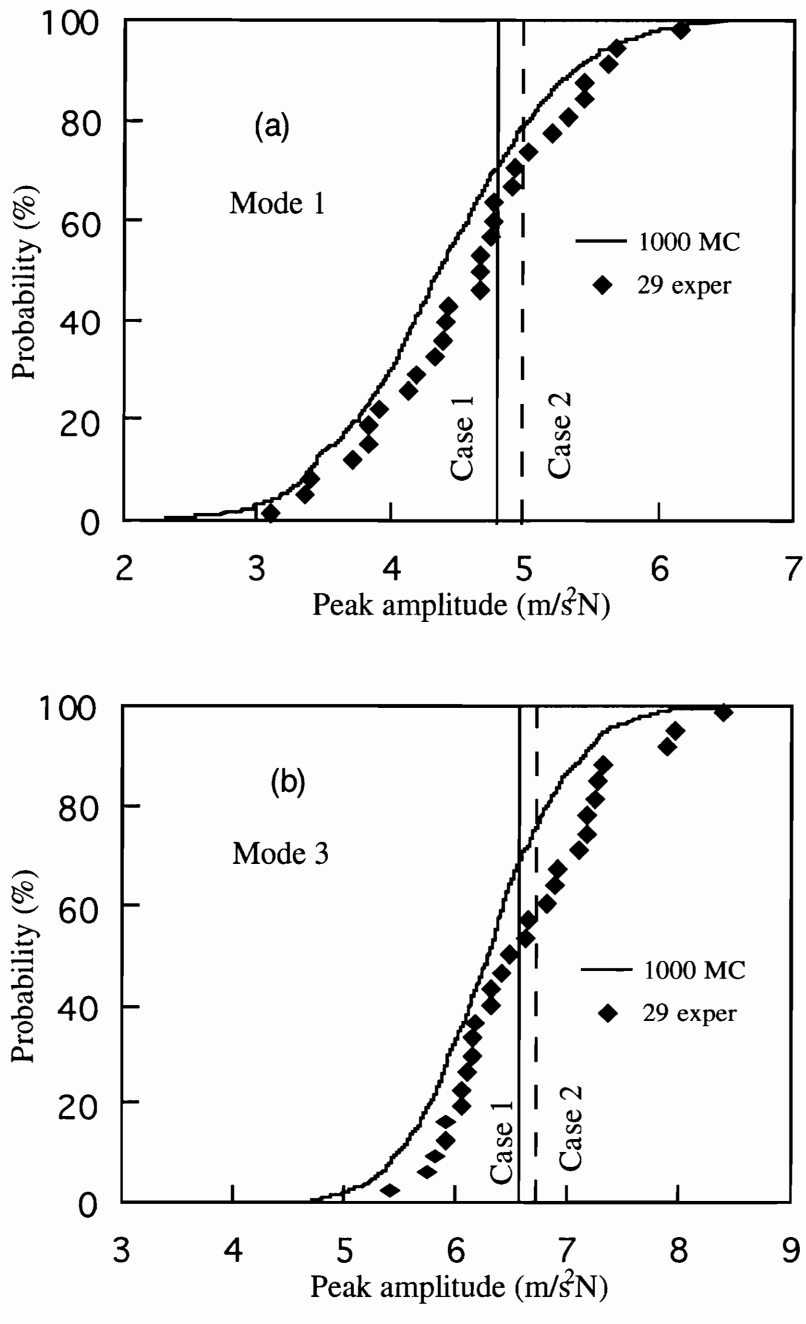

Figure 5.2. Fuzzy set design, cumulative probability distributions from analysis (Monte Carlo simulation-1000 samples) and experiment (29 samples): (a) Mode 1, (b) Mode 3.

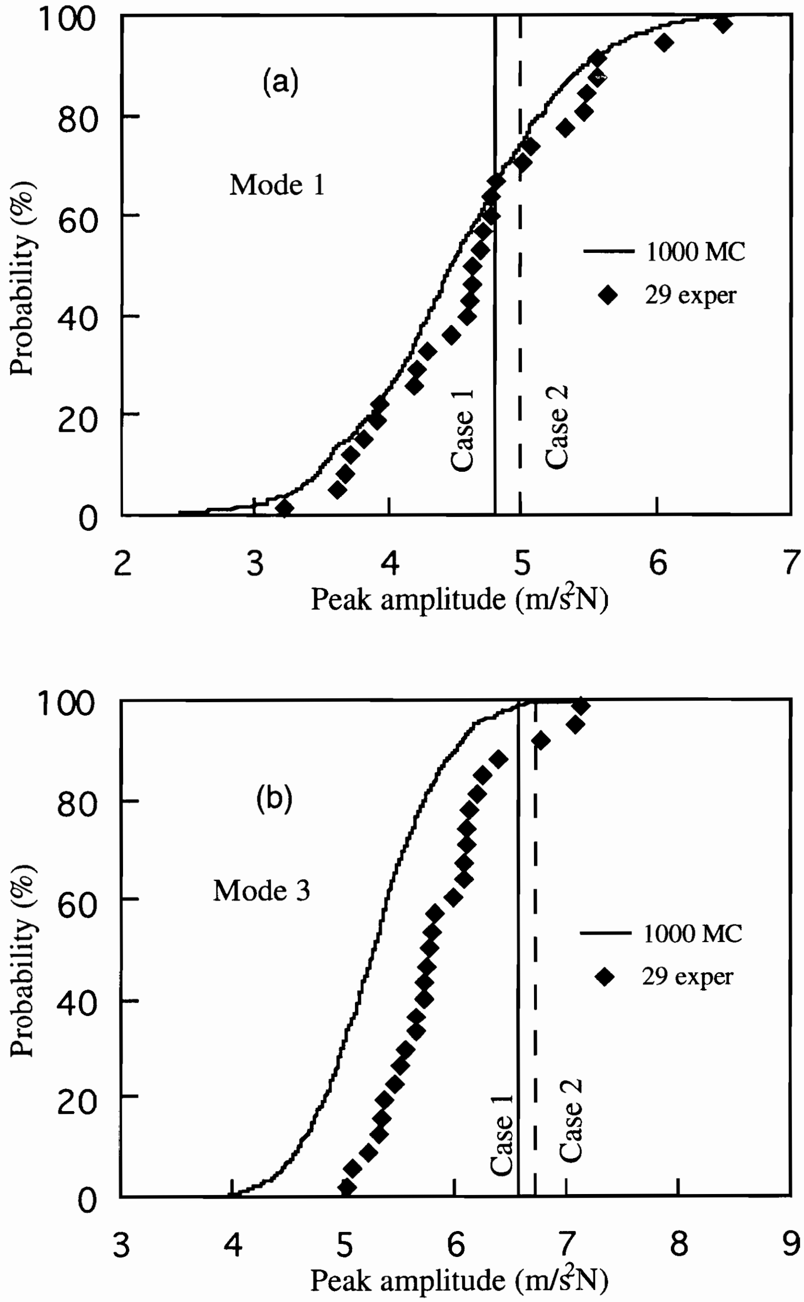

Figure 5.3. Probabilistic design, cumulative probability distributions from analysis (Monte Carlo simulation-1000 samples) and experiment (29 samples): (a) Mode 1, (b) Mode 3.

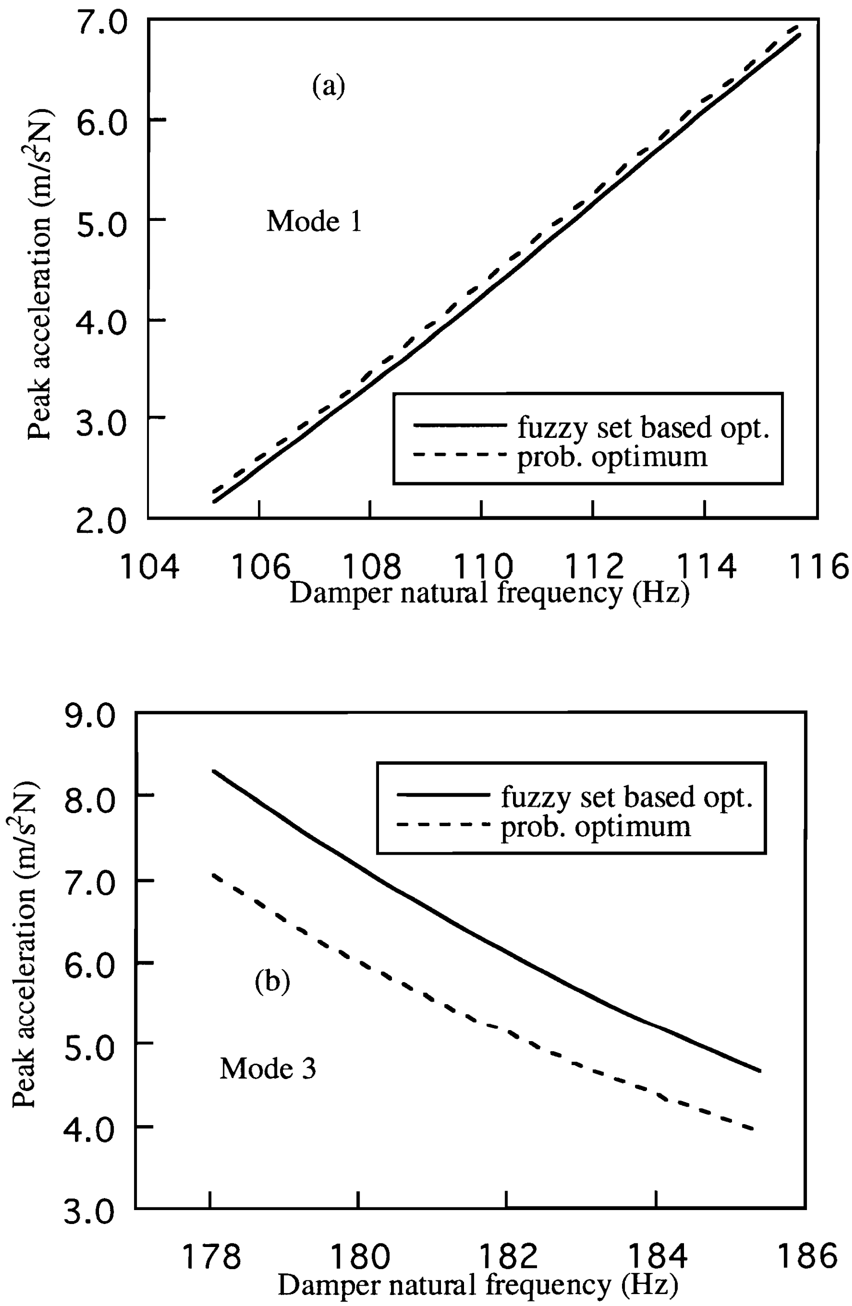

Figure 5.4. Peak acceleration vs. damper natural frequency: (a) Mode 1, (b) Mode 3.

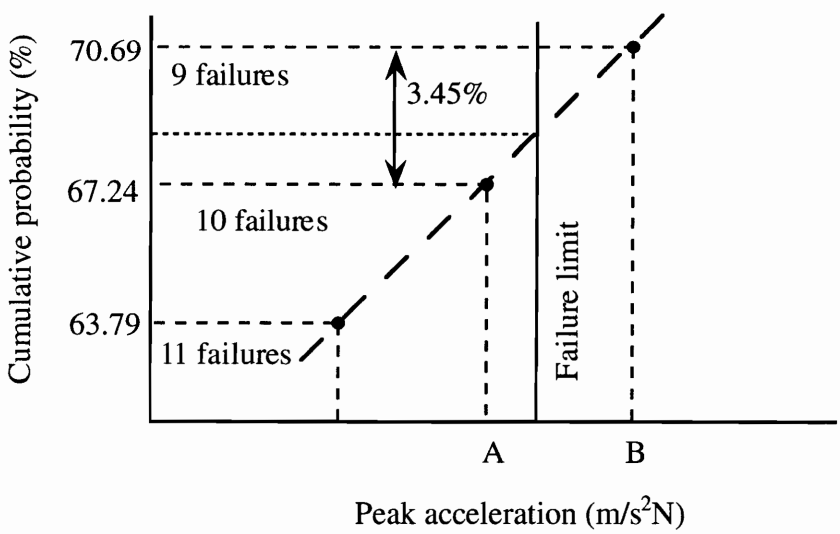

Figure 5.5. Resolution error of measured failure probabilities.

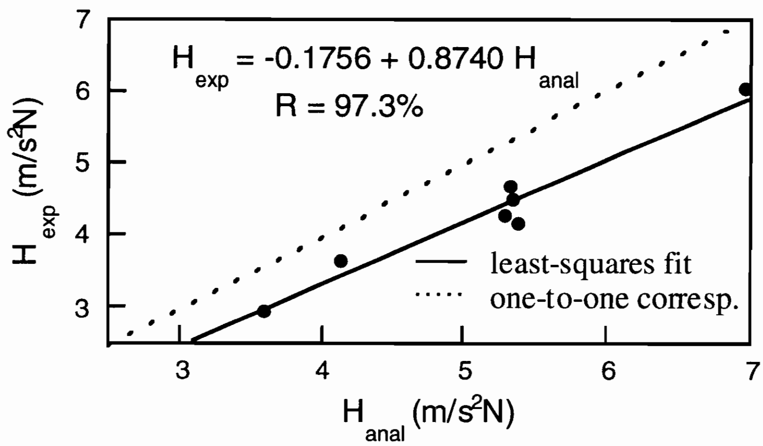

Figure 5.6. Mode 1, analytical-experimental correlation (2-mode approximation).

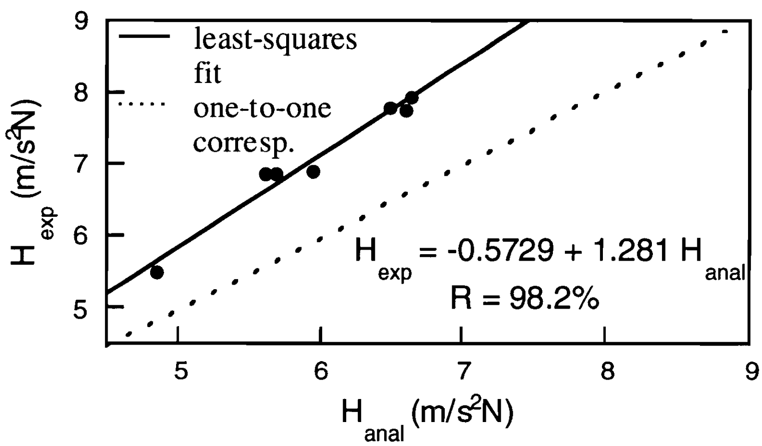

Figure 5.7. Mode 3, analytical-experimental correlation (2-mode approximation).

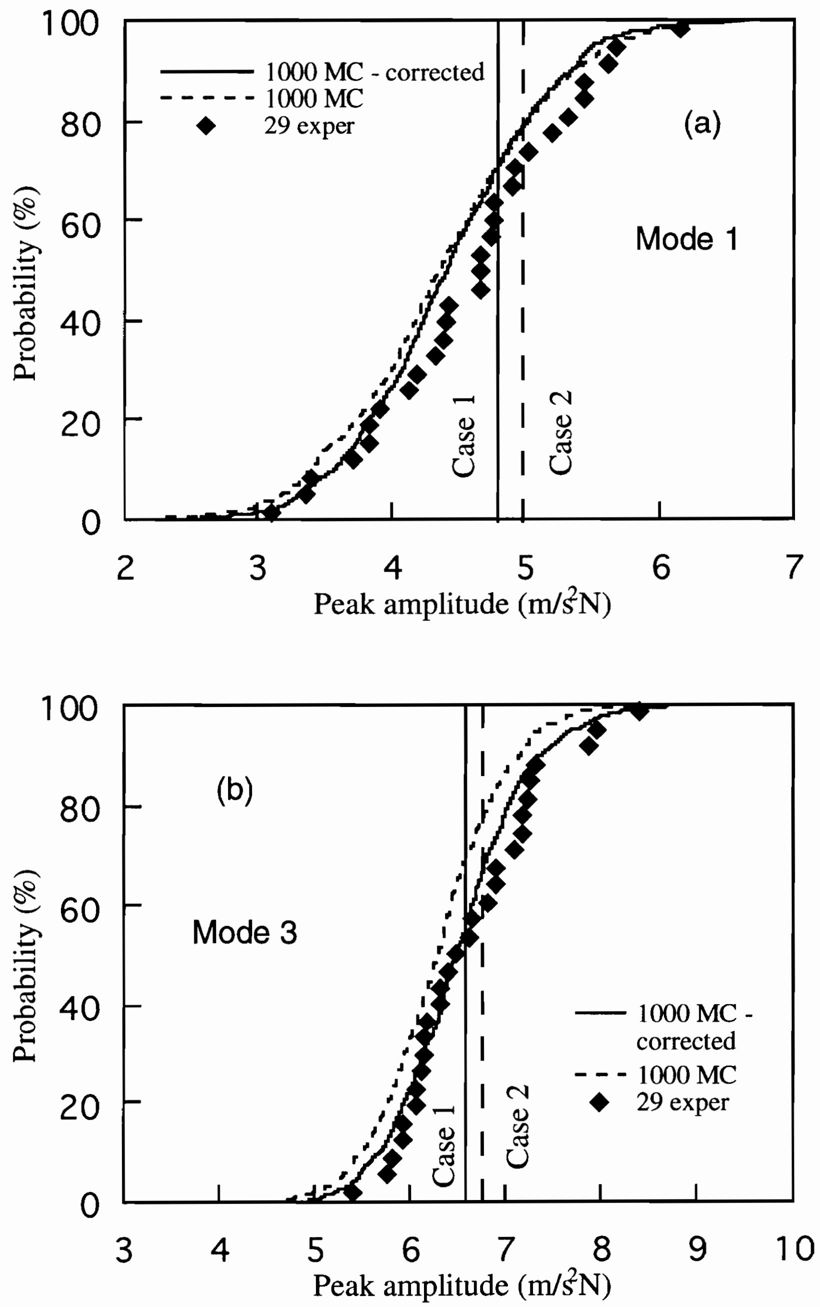

Figure 5.8. Fuzzy set design, cumulative probability distributions from analysis (Monte Carlo simulation-1000 samples, original and corrected sample) and experiment (29 samples): (a) Mode 1, (b) Mode 3.

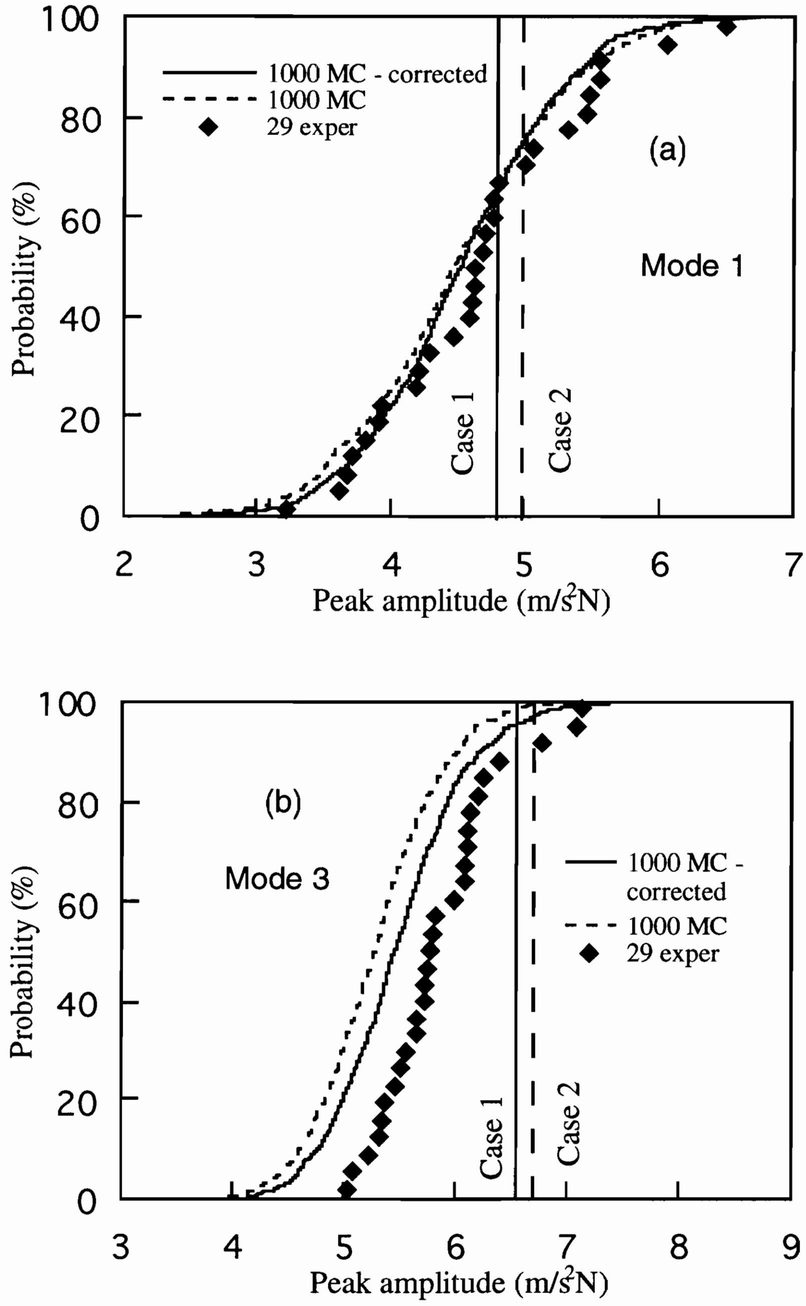

Figure 5.9. Probabilistic design, cumulative probability distributions from analysis (Monte Carlo simulation-1000 samples, original and corrected sample) and experiment (29 samples): (a) Mode 1, (b) Mode 3.

CHAPTER 6

CONCLUSIONS AND FUTURE WORK

The primary objectives of this work were to understand the differences in the way that probabilistic methods and fuzzy set methods maximize reliability and experimentally test optimal designs obtained using each approach. To this end, the research has provided us with the following conclusions and key observations:

1) Probabilistic optimization tries to reduce the probabilities of failure of the modes that are easier to control, in order to minimize the system failure probability. This is because the system probability of failure is related to the sum of the probabilities of failure of the individual failure modes.

2) Fuzzy set optimization does not consider which failure mode is easier to control but simply tries to equalize the possibilities of failure of all failure modes. This is because, according to the most commonly used rule for calculating the possibility of failure of the union of failure events, this possibility is equal to the maximum of the possibilities of failure of the individual failure events.

3) We were able to apply probabilistic and fuzzy set optimization to a practical design problem and compare the failure probabilities of the optimum designs. We used a modular structure and a non-destructive definition of failure. Also, the same resources were available for both approaches.

4) Analytical results showed that when we have sufficient information about uncertainties, accurate models for predicting the response of a structure and crisp definition of failure probabilistic methods can yield significantly better designs than fuzzy sets.

5) For a problem selected to provide large contrast in the probabilities of failure of the two approaches, the higher reliability of the probabilistic approach, predicted by the analysis, was verified experimentally.

6) There was considerable discrepancy between the experimentally measured rates of failure and the analytically predicted values. Two possible sources of such a discrepancy were examined: errors in the regression equation used to calibrate the analytically calculated peak acceleration and an average temperature shift between the time that the damper properties were measured and the structural response was measured. Accounting for these two factors in the analysis brought the analytical results significantly closer to the experiment.

Although in the experimental comparison the improved reliability of the probabilistic design was confirmed for the problem studied, one should keep in mind that this problem relied on complete knowledge of the statistical characteristics of the random parameters. Also, modeling error had been minimized. However, designers rarely have the luxury of complete information about uncertainties and accurate models for predicting the response of a system. Therefore, further research is needed to determine which approach is better to use when there is limited statistical information and large modeling errors are present. These are the situations where fuzzy sets could be better because they are more conservative than probabilistic methods.

This study was also based on a crisp definition of failure. That means that there is an abrupt transition from a completely acceptable to a completely unacceptable design. However, in real life there are situations in which there are designs that can be satisfactory to a certain degree and unsatisfactory to another degree. Fuzzy sets can handle this fuzziness in the definition of failure. Further research should investigate the effect of such imprecision in the definition of failure on the efficacy of the fuzzy set method relative to other design methods.

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