

Three Essays on Middlemen in Intermediated Markets

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(ABSTRACT)

This dissertation comprises three essays on theoretical analysis of middlemen in intermediated markets. Chapter 1 gives a brief survey on the market intermediation literature and also briefly describes the subsequent chapters.

In Chapter 2 I study the role of horizontally differentiated middlemen in a bilateral search market in which heterogeneous agents of each group possess private information concerning the value of joint production. I focus on the effect of the middlemen on agents' search efforts and on pricing decisions by middlemen. In particular, I show that the middlemen intensify agents' search activities. I also provide an explanation for why middlemen often use asymmetric pricing for two groups in a market.

In Chapter 3 I study a model of platform competition when both indirect network effect and the desirability concerns of the agents are present. The desirability concerns are defined as the perceived quality of platforms. A platform with a higher proportion of high-type agents is regarded as a platform with a better quality. Under these circumstances, I derive conditions for the existence of equilibrium. In a dominant platform equilibrium, I show that some agents may not be served by the dominant platform. I also show that two platforms with different perceived quality may coexist in equilibrium. It suggests that endogenous market segmentation may arise in two-sided markets.

In chapter 4 I study the effort-maximizing contest rule when there is a positive externality between aggregate efforts and the contest audience: the audience is more willing to pay for watching a contest if each participating contestant expends more effort. The Tullock rent-seeking contest with endogenous entry is extended by incorporating the contest audience into the model. In order to fund the contest, the organizer with no budget has to collect fees from one or both of two groups. It is shown that the effort-maximizing contest rule under a positive externality attracts only two entrants and, in the unique subgame perfect equilibrium, the entrants are always subsidized regardless of the size of entry costs, and the audience pay a positive fee.

To My Parents and Family

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Chapter 1

Introduction

Many economic transactions and social interactions among groups of agents involve services provided by intermediaries whose roles are critical in the economy. For a better understanding of intermediaries, many works in the economic literature have been devoted to the theoretical development of a model of intermediation in various market settings. In particular, the literature on market intermediation has addressed many issues by trying to answer the following two questions: (1) why and how intermediaries exist, and (2) what intermediaries contribute to the economy. The key conclusion of the literature is that intermediaries exist in the market because they make transactions among buyers and sellers possible, or they add substantial value to these transactions. In other words, intermediaries reduce inherent inefficiencies of the market and enhance net gains from market interactions by offering various transaction models. When doing so, intermediaries are in a better position to make connections than other economic agents (e.g., buyers and sellers), and use their bridging positions in a number of ways.

The role of intermediaries differs across markets, and therefore the distinction among various types of intermediaries is important. The literature on market intermediation distinguishes four types of intermediaries: dealers, platform operators, certification intermediaries, and information gatekeepers. First, when intermediaries act as dealers (e.g., retailers, wholesalers, used car dealers), they buy goods or services from suppliers and resell them to buyers. The main function of dealers in the market is to operate a central place of exchange which provides a substitute for direct trade between suppliers and buyers, and to provide storage and financing between the time when sellers

want to sell and when buyers want to buy. In many decentralized markets, buyers and sellers have incomplete information about the other side of the market and search is costly. Therefore, trading in a decentralized way involves inefficiencies: uncertainties about the gains from trade and about successful matches. Dealers eliminate uncertainties by gathering information about supply and demand and assume equilibrium by adjusting bid-ask prices. Some market participants can obtain benefits from trading through intermediaries instead of directly. Many studies on market intermediation have been devoted to such intermediaries. See, for example, Gehrig (1993), Fingleton (1997), Spulber (1996, 2002), Rust and Hall (2003), Loertscher (2007), and Burani (2008).

The second type of intermediaries are platform operators who provide places where interactions among various groups of agents take place. This type of intermediary charges access and transaction fees for their services. For example, credit card companies (e.g., VISA, Mastercard) simply provide payment card systems and charge membership fees to their customers and transaction fees to stores. Other examples are videogame platforms (e.g., Playstation, Xbox 360), software operating systems (e.g., Windows 7, Mac OSX), and Yellow pages. In recent years, the two-sided market literature has dealt with platform operators' pricing decisions in competitive environments when indirect network effects are present. Instead of addressing the efficiency-enhancing role of intermediaries in the market, which is usually addressed in the market intermediation literature, the two-sided market literature tries to explain why platforms as intermediaries use 'asymmetric pricing' strategies for two groups of agents. Asymmetric pricing equilibrium arises in these environments because platforms have to use the 'divide-and-conquer' strategy in order to have both sides on board. See, for example, Caillaud and Jullien (2001, 2003), Rochet and Tirole (2003, 2006), and Armstrong (2006).

However, sometimes, it is difficult to implement transaction fees if a non-monetary interaction between two groups arises. In this case, intermediaries may simply charge access or membership fees only. For example, dating and marriage matchmakers (e.g., match.com) provide intermediation services between men and women by charging membership fees to their customers. Platform operators of this kind are often called matchmakers or brokers (e.g., real estate agents, insurance agents, stock brokers). The matchmaking role of intermediaries has been paid more attention in the traditional market intermediation literature than in the two-sided market literature. For example, Wolinsky (1987) and Yavas (1994) consider the role of matchmakers in bilateral search markets. In

their models, the key role of matchmakers is to shorten the time to match sellers with buyers by providing a better matching probability.

The third type of intermediaries are certification intermediaries who alleviate asymmetric information problems between firms and consumers. Since Akerlof's (1970) seminal model of a market for lemons, considerable efforts have been devoted to the study of "lemon problems." The market intermediation literature suggests that the introduction of certification intermediaries into markets, that are characterized by lemon problems and adverse selection, enhances efficiency by revealing information about a product's or a seller's reliability or quality. For example, Biglaiser (1993) suggests that a firm with reputation concern has an incentive to report the quality of goods to be traded in the market accurately as long as profits from investment in detecting quality are sufficiently high. In a dynamic setting, Biglaiser and Friedman (1994) address the role of certification intermediaries as guarantors of the product quality of their suppliers. The subsequent works to Biglaiser (1993) and Biglaiser and Friedman (1994) study the role of certification intermediaries in various market environments. See, for example, Biglaiser and Friedman (1999), Lizzeri (1999) and Hoppe and Ozdenoren (2005). While most authors address adverse selection in product markets including used-car markets, Hoppe and Ozdenoren (2005) study the role of certification intermediaries (e.g., university-industry technology transfer office, venture capitalist) between creators and users of new inventions (e.g., inventors and commercializing firms). In their model, they show that the intermediary helps to reduce inefficiency in a market for innovation by assessing the quality of new inventions.

The last type of intermediary is an information gatekeeper, who allows firms and consumers to transmit and acquire information about prices or the match value of products and services. For example, a simple reputation system employed by eBay allows both sellers and buyers to provide feedback after they complete each transaction. The feedback from market participants allows the other participants to make better-informed choices. Amazon marketplace provides a similar reputation mechanism, and book reviews on Amazon and Barnes and Noble are other examples. Kennes and Schiff (2007, 2008) study this type of information gatekeeper in search markets. They show that the provision of information by the information gatekeeper can improve welfare as long as information about the quality of match is sufficiently informative. Price comparison sites such as

Nextag.com and Shopper.com also act as information gatekeepers. Baye and Morgan (2001, 2002) consider a monopoly gatekeeper on the internet and show its effect on equilibrium price dispersion in a homogeneous product environment. See also Baye, Morgan, and Scholten (2006) for the survey of the price dispersion literature.

While the existing literature on market intermediation, as discussed above, provides a better understanding of the role of intermediaries, many questions still remain to be answered. In this dissertation, we provide extensions of the theoretical results of the market intermediation literature. The rest of this dissertation is organized as follows. In the following two chapters, chapter 2 and chapter 3, we study the role of oligopolistic market intermediaries (e.g., matchmakers) and their pricing strategies in specific market environments: Chapter 2 considers middlemen in costly-search markets, and chapter 3 addresses platforms in matching markets. In chapter 4, we discuss a contest organizer who serves both contestants and the contest audience.

In chapter 2, we study the role of horizontally differentiated middlemen in a bilateral search market in which heterogeneous agents of each group possess private information concerning the value of joint production. In earlier contributions by Rubinstein and Wolinsky (1987) and Yavas (1994), a monopoly matchmaker in a search market is viable because it provides a better matching probability. However, the introduction of new technology by intermediaries in search markets calls for modification of the existing models. For example, as Autor (2001) suggests, internet job-search engines (e.g., monster.com) are common and increase the efficiency of job-search markets. The efficiency stems from improvements in information and communication technology. In order to deal with this aspect, we develop a model of intermediaries that provide extra benefit to search market participants, based on Hotelling's model. And we interpret the provision of extra benefits as market environments (e.g., laws and regulations) or search environments of electronic marketplaces. Further, we try to explain why asymmetric pricing between two different groups of agents arises in duopoly settings. Our analysis provides interesting results. First, we show that the provision of extra benefits by matchmakers intensifies agents' search efforts by generating an increased expectation of the matching probability. Conventional wisdom in the literature is that the presence of a matchmaker in a search market reduces agents' search effort because it provides better matching. See, for example, Yavas (1994). In our model, the matchmakers can be viable by providing a bet-

ter search environment. Regarding the equilibrium pricing decision, our results suggest that the greater the differences between the reservation values of different types of agents, the greater is the asymmetry of pricing in equilibrium. In fact, our results provide another possible explanation for asymmetric pricing in two-sided markets. In the two-sided market literature, asymmetric pricing arises because platforms use the ‘divide-and-conquer’ strategy in order to keep both groups on board when indirect network effects are present. But chapter 2 shows a possibility that asymmetric pricing may stem from agents’ reservation values for joining the matchmakers. Indeed, this is what we often observe from pricing strategies by Internet job search engines: For example, both Monster.com and JOE (job openings for economist) charge a positive fee to firms and institutions, and jobseekers are served free of charge.

In chapter 3, we consider price competition between two *ex-ante identical* platforms when some agents of each group care about the types of agents from the other group, and there are indirect network effects. As discussed in Caillaud (2001, 2003), Rochet and Tirole (2003, 2006), and Armstrong (2006), the existing literature on two-sided markets helps to understand platforms’ asymmetric pricing when indirect network effects are present (e.g., payment card systems, videogame platforms, yellow pages). However, their models blur the picture when agents’ desirability concerns are taken into account. Therefore, chapter 3 complements previous works on two-sided markets by considering agents’ desirability concerns. Agents’ concerns about the types of agents from the other group are defined as the perceived quality of platforms: A platform with a higher proportion of high type agents is regarded as a platform with a better quality. When deciding an equilibrium pair of prices, platforms face a trade-off between network size and perceived quality because a larger network size ruins the perceived quality of the platform. As in Caillaud and Jullien (2003), we first show that only a single platform may be active in the market after price competition: A dominant platform equilibrium. Under some reasonable conditions, however, it is also shown that some agents may not be served in the dominant platform equilibrium. Interestingly, our analysis suggests the possibility of endogenous market segmentation in two-sided markets. In the model presented in chapter 3, price-settings by platforms endogenously determine the platforms’ perceived quality. As a consequence, the *ex-ante* identical platforms are able to differentiate themselves if they charge different prices. Essentially, the endogenous differentiation comes from agents’ expectations about

the proportion of a certain type of agents on each side, which are affected by price information. The signaling role of price is important in coordinating agents' expectations. In some earlier works on matchmakers, it was shown that the signaling role of price matters when matchmakers sort agents by type. However, indirect network effects are not considered in existing models. For example, Bloch and Ryder (2000) consider a two-sided search model and show that, if a matchmaker charges a uniform participation fee, only agents of higher quality participate in the matching procedure. And Damiano and Li (2007, 2008) consider a similar matchmaker's problem when agents' types are unobservable. Recently, Ambrus and Argenziano (2009) studied platforms' pricing decision in a two-sided market when two different types of agent are present. This is similar to our model. In their model, however, two types of agents are different because of their different valuations of the size of the network.

In the last chapter of this dissertation, chapter 4, we consider the effort-maximizing contest rule when (1) there is a *positive externality* between aggregate efforts expended by the participating contestants and the contest audience, and (2) the organizer has no budget to fund the contest. While this type of externality certainly exists in reality, it is not well addressed in the contest literature.¹

In order to deal with this problem, we consider a Tullock rent-seeking contest with endogenous entry, incorporating the contest audience into the model. Further, the organizer who initially has *no budget* is considered. In the contest literature, it is typically assumed that the organizers have enough budget to fund the contest. However, in chapter 4, we depart from this conventional assumption. By charging some fees to either one or both of two groups, the contestants and the audience, the organizer is able to fund the contest. As suggested in the market intermediation literature, the organizer is in a better position than other economic agents (e.g., the contestants and the contest audience) even though she has no budget. Our analysis suggests that the organizer without budget is able to design the effort-maximizing contest rule in conditions of a positive externality for participation. In the unique subgame equilibrium, the optimal contest rule always attracts only two entrants and the entrants are always subsidized, regardless of the size of entry costs, and the audience pay a positive fee. The result that the optimal contest rule attracts only

¹See examples discussed in chapter 4.

two potential contestants coincides with the result of Fu and Lu (2010), who consider a contest organizer with a fixed budget. However, our results differ from theirs in the following. First, we show that, when the positive externality is present, the contest participants are always subsidized regardless of the size of a fixed entry cost. Second, our analysis suggests that the organizer may achieve her goals without a budget, as long as she can utilize her bridging position in the market.

Chapter 2

Horizontally Differentiated Middlemen in Bilateral Search Markets

2.1 Introduction

There are many market situations where transactions are decentralized. These markets are often called *search markets*, agents on one side of a market try to match with agents on the other side by engaging in costly search. In such markets agents try to buy or sell a good if there are gains from trade, or try to produce a product jointly if they expect a positive surplus from matching. Examples include the real estate market, where property owners are matched with buyers; the labor market, where job-seekers are matched with jobs; the venture capital market, where start-ups are matched with financiers; technology transfer systems, where inventors are matched with commercializing firms; and business-to-business (B2B) electronic commerce, where buyers are looking for inputs and suppliers are seeking buyers for their goods and services.

In markets with these characteristics, intermediation often emerges as an alternative trading mechanism. The intermediation literature distinguishes two types of middlemen: Market makers and matchmakers.¹ Although both market makers and matchmakers improve the welfare of buyers

¹Some economists have studied middlemen in some other market settings: certification intermediaries and infor-

and sellers by reducing or eliminating the uncertainty associated with achieving a satisfactory match, they behave in different ways. When acting as market makers, middlemen operate a central place of exchange, gather information about supply and demand, and adjust their bid and ask prices to equalize supply and demand. When acting as matchmakers on the other hand, middlemen bring buyers and sellers together and match the two sides in order to facilitate transactions between agents. Indeed, the intermediation literature views matchmakers as an institution that reduces inherent inefficiencies of search markets.

The first type of middleman has been widely analyzed in the existing intermediation literature. Many researches focus on the situation where buyers and sellers have the option to choose whether to search directly for a trading partner or to trade through middlemen. See, for example, Gehrig (1993), Fingleton (1997), Spulber (1996, 2002), Rust and Hall (2003), Loertscher (2007), and Burani (2008). Middlemen in these models are viable since they provide immediacy of trade. That is, middlemen create markets by separating the market's supply decisions from its demand decisions. Standing between buyers and sellers and setting bid-ask prices, middlemen coordinate their transactions, and eliminate uncertainty in search market, for example, uncertainty about the gains from trade and the probability of a successful match.

Rubinstein and Wolinsky (1987) and Yavas (1994) study the second type of middlemen, matchmakers. In their models, the key role of middlemen is to shorten the time it takes to match sellers with buyers in bilateral search markets. While the two share a notion of how a (monopoly) middleman can be viable in a search market, they put different emphasis on the search intensity of the agents. In Yavas (1994), the presence of matchmaker affects the endogenously chosen search intensity of buyers and sellers. In Rubinstein and Wolinsky (1987), on the other hand, the search intensity level is fixed. In fact the middleman in both Rubinstein and Wolinsky (1987) and Yavas (1994) can be viable in the market because the middleman's probability of a match with an agent is higher than the agent's own probability of a match.

A recent literature on two-sided markets also deals with market intermediation, while viewing middlemen as platforms, with emphasis on indirect network effects. See, for example, Caillaud and Jullien (2001, 2003), Rochet and Tirole (2003, 2006), and Armstrong (2006). One of the typical
mation gatekeepers. In chapter 1, we provide a brief survey of the market intermediation literature. Also see Spulber (1999, 2009) for a comprehensive survey on market microstructure and intermediation.

features of the two-sided market literature is *asymmetric pricing*. In two-sided markets, asymmetric pricing prevails when platforms find it profitable to subsidize agents with less willingness to pay for joining platforms and charge high fees to agents with higher willingness to pay.² This happens when agents on one side care about the number of agents on the other side only because a higher number increases the probability of a good match—an indirect network effect.

The purpose of this chapter is to consider the role of competitive middlemen in a bilateral search market in which heterogeneous agents possess private information concerning the value of joint production. Specifically, we consider duopoly competition between *matchmakers* in a two-sided spatial model while explicitly taking into account the agents' participation constraints for joining the matchmakers, based on their reservation values. We focus on the equilibrium search effort of agents, the equilibrium probability of a match, and the equilibrium pair of prices set by duopolistic middlemen who provide extra benefits to the search market. Finally, we discuss the level of extra benefit that supports a specific market structure, given reservation values.

At the location of each search market, agents of each type can engage in random matching as in Gehrig (1993) and Loertscher (2007). However, our model differs from theirs in that middlemen act as matchmakers and provide extra benefits to the markets, which may enhance decentralized search activities. While in their models middlemen act as market makers and provide a centralized exchange mechanism. Also, we consider explicit search costs of agents, which costs represent the underlying inefficiencies of search markets.

In the existing literature, as discussed above, middlemen are viable since they reduce market inefficiencies by either a centralized exchange mechanism in the case of market makers or by reducing search costs (e.g., time) in the case of matchmakers. In this chapter, in contrast to earlier contributions in the middleman literature, we provide another explanation for how middlemen in search markets can be viable because of their provision of extra benefits. We interpret the provision of extra benefits by middlemen as affecting parameters of market environments such as the degree of institutional infrastructure (laws and regulations) or search environments of electronic marketplaces. In this regard, the most relevant examples of search markets are the venture capital market or technology transfer systems, where agents have concerns about contracts, disputes, and

²According to Caillaud and Jullien (2001, 2003), this strategy is called the 'divide-and-conquer' strategy.

intellectual property protection and B2B electronic commerce platforms, where buyers and sellers want to match with appropriate trading partners.

We formally consider the following two-stage game. In the first stage, each middleman after observing the reservation value of each type announces a pair of prices for his services and the provision of extra benefits to the search market, β . The extra benefits provided by the two middlemen are assumed to be fixed at some level. The level of extra benefits and a pair of prices affect agents' expectations, for example, expectations about the set of agents joining the same middleman, their search effort, and the probability of a match. In the second stage, after observing the level of extra benefits and a pair of prices, agents decide where to join and choose their search effort level based on their expectations. Alternatively, if joining neither middleman is profitable, agents can go to the factor markets and sell their endowment at a known price. This becomes their reservation value. At the end of the second stage, their expectations are fulfilled in equilibrium since all agents are assumed to be rational.

The following results are obtained. First we obtain the equilibrium search effort of each individual agent and show that how the provision of extra benefits by middlemen intensifies agents' search efforts, by generating a positive expectation about the probability of a match between different types of agents. It is also shown that an agent with higher expected matching surplus will expend more search effort than agents with lower expected matching surplus. However, an agent will expend less search effort if more agents of either the same type or the other type join the same middleman. This is because the expected matching surplus of each agent decreases as the number of agents (of the same or the other type) increases.

Then we discuss the equilibrium pair of prices charged by middlemen under different forms of competition – strong and weak competition, and local monopoly – and examine how the reservation values of agents –the agents' participation constraints – affect the equilibrium pricing decisions of middlemen.³ Our results suggest that the greater the differences between the reservation values of different types, the greater is asymmetric pricing between two different types of agents. In particular, when the differences are significantly large, agents with relatively high reservation value

³By strong competition, we mean that two middlemen compete for the agents of both types. On the other hand, weak competition implies a situation where no competition arises for one type of agents while both middlemen compete for the agents of the other type. The precise definitions of three types of competition will be presented in Section 2.4.

will be served for free under either strong competition or local monopoly. On the other hand, under weak competition, agents with lower reservation value are served for free because both middlemen compete for that type of agent, and it drives the price to zero. In fact, the asymmetric pricing result of the present model is quite similar to that of two-sided markets. The only difference comes from the source of market externalities: In our model, the market externality is associated with middlemen's demand functions, while in the two-sided market literature, the market externality is associated with indirect network effects.

Finally, we discuss the level of extra benefit that supports a specific market structure in equilibrium, given agents' reservation values. Specifically, we show the range of extra benefit when the difference between the reservation values of different types is small or large.

This chapter proceeds as follows. In Section 2.2, we develop the model and set up the two-stage game. In Section 2.3, we analyze the optimal search effort of agents. In Section 2.4, we derive the equilibrium pair of prices under different forms of competition – strong and weak competition and local monopoly. Then, in Section 2.5, we discuss the level of extra benefit that supports a specific market structure in equilibrium, given agents' reservation values. In Section 2.6, we conclude.

2.2 Model

Consider a bilateral search market in which two types of agents, $k \in \{a, b\}$, are endowed with a unit of input factor, and the agents of each type are *uniformly* distributed along a Hotelling line with measure one. In this market, in order to produce a single product, an agent of type a needs to be matched with an agent of type b , and vice versa. They use their endowments in production. In other words, there are technical interdependencies in the production process. Matching between agents requires a third party and search effort. We introduce two middlemen, $i \in \{1, 2\}$, located at the two ends of the Hotelling line: Middleman 1 is located at point 0 and 2 at point 1. The role of middlemen here is minimal in that middlemen only provide a marketplace with its extra benefits that facilitate searching among the agents, and they charge fees for their services.⁴

The agents have two options: They can either sell their endowments in factor markets or join

⁴The search market presented here is close to that of Yavas (1994). However, the present model is modified to deal with competition between middlemen. Also note that in Yavas (1994), the monopoly matchmaker is addressed and the role of middleman differs from ours.

one of the middlemen. If the agents go to factor markets, they sell their endowments at a known price w_k . Hence w_k is the reservation value for an agent of type k . If the agents choose to join the middlemen – that is, if their expected profit from joining the middlemen is higher than their reservation values – they get matched with some probability in the search market.^{5 6}

Once matched, the agents produce a single product jointly and share the surplus from the production equally.⁷ In order to determine the value of the output from the joint production, we first denote the value of each individual agent’s endowment in the respective search market by v_k^i , which differs across the agents. Specifically, the value of the endowment of an agent is characterized by her location on the Hotelling line.⁸⁹ If, for example, an agent of type a at $x \in [0, 1]$ joins middleman 1, the value of her endowment is given by $v_a^1 \equiv v - 2tx$.¹⁰ Otherwise – if she joins middleman 2 – the value of her endowment is given by $v_a^2 \equiv v - 2t(1 - x)$.¹¹ Similarly, we define the value of endowment of type b agents: If an agent at $y \in [0, 1]$ joins middleman 1, $v_b^1 \equiv v - 2ty$, and $v_b^2 \equiv v - 2t(1 - y)$ if she joins middleman 2.

We now define the value of the output from the joint production, which is the sum of the value of endowment of each *matched* agent. More precisely, if an agent of type a at $x \in [0, 1]$ is matched with an agent b at $y \in [0, 1]$ via middleman 1. Then we assume that the total surplus from the joint production is given by $v_a^1 + v_b^1$, which will differ across all possible matches. For each *matched* agent, the surplus from the production is given by $\frac{v_a^1 + v_b^1}{2} = v - (tx + ty)$ (where $\frac{v}{t} > 2$) because of the equal-sharing assumption.¹² (hereafter referred to as “matching surplus”)

⁵Discussions about matching probability will be presented in the following section.

⁶Note that we do not explicitly model how any participating agents search for agents of the other type. In this chapter, agents’ expected payoff from joining the search market run by the middlemen affects their participation decision.

⁷The equal-sharing assumption is quite common in the intermediation literature. For example, see Yavas (1994), Loertscher (2007), and Damiano and Li (2008)

⁸In other words, her distances from the middlemen

⁹In order to allow agents’ heterogeneity in terms of the value of their endowment, we used a linear transportation cost. Much like the model of Loertscher (2007), we interpret v_k^i as the net value of endowment in each market. In Loertscher (2007), the net valuation of a buyer at x , who join the search market, for a seller’s product is defined by $v_c \equiv v - tx$. Indeed, both the present model and Loertscher (2007) deal with a class of modified Hotelling *vertical-differentiation* model in which two ideal points at 0 and 1 exist on the $[0, 1]$ interval. On this account, in the present chapter, the different location among agents implies the different values of the agents’ endowments in the respective market.

¹⁰The term $2tx$ can be interpreted as the differences in the value of endowment between the highest one and others in the market.

¹¹It implies that the value of an agent’s endowment is different in two marketplaces.

¹²Obviously, one may argue that a pair of matched agents will split the gross value created from the matching, which is $2v$, since the travel costs are sunk once they join either of middlemen. In general, this argument is plausible if an agent travels to the market. However, in the present model, the value of an agent’s endowment in the marketplace

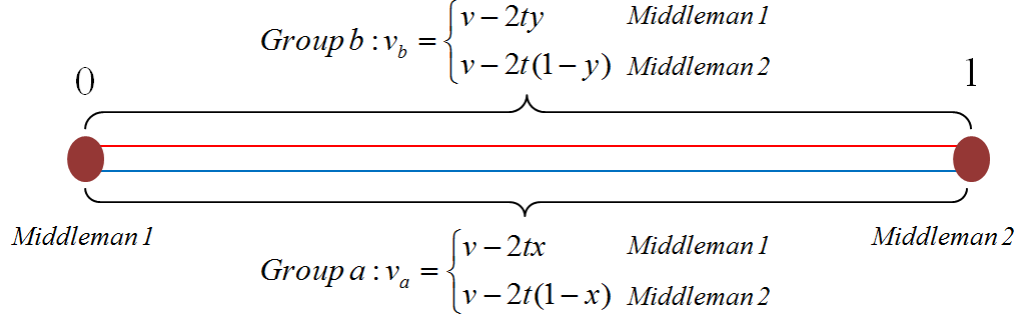


Figure 2.1: Summary of the Model

Furthermore, the value of an agent's endowment is private information.¹³ Each agent knows only the value of her own endowment and the distribution of the other side when she joins middlemen. For an agent of type k , when joining middleman i , the location of an agent of the other type $j \neq k$ is a random draw from the probability distribution $F_j^i(\cdot)$. For the sake of symmetry, we will only consider the agents who join middleman 1. Let $F_a^1(x)$ be the *uniform* distribution of agents of type a who join middleman 1, where $F_a^1(\cdot)$ is defined on the interval $[0, x_1]$, the *market segment of middleman 1*. Similarly, the distribution of agents of type b joining middleman 1, $F_b^1(y)$, is defined on the interval $[0, y_1]$.¹⁴ Then the *expected* matching surplus of an agent of type a at location $x'_1 \in [0, x_1]$ is:

$$ES_a^1(x'_1) = \int_0^1 [v - tx'_1 - ty] dF_b^1(y) = \int_0^{\hat{y}_1} [v - tx'_1 - ty] \frac{1}{\hat{y}_1} dy = v - tx'_1 - \frac{1}{2}t\hat{y}_1$$

where " \hat{y}_1 " is the *expected* location of marginal agent of type b joining middleman 1. Similarly, consider an agent of type b at location $y'_1 \in [0, y_1]$, her *expected* matching surplus is:

$$ES_b^1(y'_1) = \int_0^1 [v - tx - ty'_1] dF_a^1(x) = \int_0^{\hat{x}_1} [v - tx - ty'_1] \frac{1}{\hat{x}_1} dx = v - \frac{1}{2}t\hat{x}_1 - ty'_1$$

where " \hat{x}_1 " is the *expected* location of marginal agent of type a joining middleman 1.

Finally, we incorporate an agent's search effort into the model and present the *expected* utilities of agents of type k , V_i^k , when joining middleman i . Let $e_k : [0, \bar{z}] \rightarrow [0, 1]$ represent an agent's search effort, where \bar{z} denotes the location of a marginal agent who joins middleman i . The search

is different from the other marketplace. I am grateful to Professor Hans Haller for pointing out this issue.

¹³In other words, the location of the agents is private information.

¹⁴We implicitly assumed that $0 < x_1 \leq 1$ and $0 < y_1 \leq 1$. That is, a positive number of agents of each type join middleman 1.

cost is assumed to have the form $\frac{1}{2}ce_k^2$, where the parameter $c > 0$ represents the inherent inefficiency of the search market. The search cost function is assumed to be the same across all agents and common knowledge. Next, let $e_k\hat{e}_j$ be the probability of a match, where e_k and \hat{e}_j are her own search effort and the *expected* search effort of the other type, respectively. This implies that the matching surplus is conditional on joint search effort. Moreover, since the location of the agents is private information, an agent chooses her search effort level based on *expectations* about the search effort of agents of the other type j . The expected utility of agent of type a , when joining middleman 1, is given by:

$$V_1^a(x'_1) = e_a\hat{e}_b(v - tx'_1 - \frac{1}{2}t\hat{y}_1) + e_aB - \frac{1}{2}ce_a^2 - P_1^a$$

where B is extra benefit provided by the middleman and P_1^a is a price charged to agents of type a . Similarly, the expected utility for an agent of type b , when she joins middleman 1, is:

$$V_1^b(y'_1) = \hat{e}_ae_b(v - \frac{1}{2}t\hat{x}_1 - ty'_1) + e_bB - \frac{1}{2}ce_b^2 - P_1^b$$

where P_1^b is a price charged to agents of type b . We adopt the following assumption on the search cost.¹⁵

Assumption. Let $B = 0$. Then the search market fails to exist for sufficiently large c . That is, the inherent inefficiency of the search market deters agents from joining the market in the absence of the middlemen.

The main role of the middlemen is to provide extra benefit B so as to establish a viable search market.¹⁶ The provision of extra benefit is essential in this market because it helps to reduce the costs of search for the other type of agent. The higher B , in the present model, implies the better institutional infrastructure or searching environment. We interpret the term e_kB as the level of utilization of infrastructure or searching environment by agents. In the rest of this chapter, we assume that the level of extra benefit available to both middlemen is fixed at some level. We further assume that both middlemen do not use the extra benefit as their strategic variable.¹⁷

¹⁵Obviously, this assumption is strong. It is possible that, in the absence of middlemen, some agents join a search market as long as their expected utilities exceed the search costs. However, we adopt this assumption in order to highlight the role of middlemen in search markets in a clear way.

¹⁶The role of middlemen here is similar to that of Armstrong and Vickers (2001) and Armstrong (2006) in which firms (or platforms) supply utility directly to consumers.

¹⁷The strategic choice of B by middlemen will be an interesting question. However, it is beyond the scope of this

Middleman i charges a price P_i^k to agents of type k for providing the service. We assume there is no cost involved in market establishments. Then the profit of the middleman i is:

$$\begin{cases} \Pi_1 = (P_1^a + P_1^b) \text{Min}\{x_1, y_1\} & \text{for middleman 1} \\ \Pi_2 = (P_2^a + P_2^b) \text{Min}\{1 - x_2, 1 - y_2\} & \text{for middleman 2} \end{cases}$$

where x_1 (and $1 - x_2$) and y_1 (and $1 - y_2$) are the number of agents of type a and b , respectively, that middleman i attracts.¹⁸ The middlemen choose a price pair P_i^a and P_i^b to maximize Π_i .

Formally, we consider the following two-stage game. In the first stage, each middleman who provides extra benefits to the market announces a pair of prices P_i^a and P_i^b . In the second stage of the game, after observing the extra benefit and the prices, the agents may join either of middlemen and expend search effort. Alternatively, they can sell their endowments in the factor market, and their selling price becomes their reservation value, w_k . So, they will join the search market and expend search effort only if $V_i^k \geq w_k$. It is worth noting that, in evaluating her expected utility from joining a middleman, each agent forms expectations *rationally* about the distribution of the other type $F_j(\cdot)$ and accordingly their expected average effort level \hat{e}_j . Based on her expectation, the agent chooses search effort level e_k to maximize her expected profit. Note also that the search efforts are strategically dependent. Since all agents are assumed to be rational, they have the same expectations, and these will be fulfilled in equilibrium. We employ perfect Bayesian equilibrium as the solution concept.

2.3 Optimal Search Effort

Let us first analyze the optimal search effort of the agents after observing the prices announced in the first stage, (P_i^a, P_i^b) , and the level of extra benefit. In order to do so, in this section, we assume for a moment that agents of type a at $x'_1 \in [0, x_1]$ and agents of type b at $y'_1 \in [0, y_1]$ join middleman 1.¹⁹ Since the location of the agents is private information, each agent forms an expectation about the market segments of middleman 1, $[0, \hat{x}_1]$ and $[0, \hat{y}_1]$, and the expected search effort of the other type \hat{e}_j . Given a non-negative expected search effort of type b agents, $\hat{e}_b \geq 0$,

chapter to study a case in which middlemen compete in both prices and the level of extra benefit.

¹⁸In equilibrium $x_1 \leq x_2$ and $y_1 \leq y_2$ will be hold.

¹⁹The participation decisions of agents will be presented in Section 2.4.

and the expected market segment of middleman 1, $[0, \hat{y}_1]$, the agent of type a at $x'_1 \in [0, x_1]$ solves

$$\max_{e_a} V_1^a(e_a, x'_1, \hat{e}_b, \hat{y}_1) = e_a \hat{e}_b (v - tx'_1 - \frac{1}{2}t\hat{y}_1) + e_a B - \frac{1}{2}ce_a^2 - P_1^a \quad (2.1)$$

And the optimal search effort for agent x'_1 is characterized by the following first-order condition:

$$e_a = \frac{\hat{e}_b}{c} (v - tx'_1 - \frac{1}{2}t\hat{y}_1) + \frac{B}{c} \quad (2.2)$$

Similarly, given her expectation about the search effort of type a agents, $\hat{e}_a \geq 0$, and the expected market segment, $[0, \hat{x}_1]$, the agent of type b at $y'_1 \in [0, y_1]$ chooses e_b that maximizes her expected utility:

$$\max_{e_b} V_1^b(e_b, y'_1, \hat{e}_a, \hat{x}_1) = \hat{e}_a e_b (v - \frac{1}{2}t\hat{x}_1 - ty'_1) + e_b B - \frac{1}{2}ce_b^2 - P_1^b \quad (2.3)$$

The first-order condition for maximizing the expected utility of agent y'_1 yields:

$$e_b = \frac{\hat{e}_a}{c} (v - \frac{1}{2}t\hat{x}_1 - ty'_1) + \frac{B}{c} \quad (2.4)$$

Conditions (2.2) and (2.4) say that, given a search effort of the other type, $\hat{e}_j \geq 0$, each agent chooses an effort level that equates her expected marginal gross payoff to her marginal cost of search. An agent's expected utility function is strictly concave in her search effort level. Thus the second-order condition for maximizing $V_i^k(\cdot)$ is satisfied. The provision of extra benefit B plays an important role in enhancing an agent's search effort. Since all agents are rational and $B > 0$, they always expect that agents of the other type expend a positive search effort.²⁰ In other words, it is a best response for any participating agents to expend a positive search effort as long as $B > 0$.

By integrating over the interval $[0, x_1]$, we obtain the average search effort of type a agents given \hat{e}_b :

$$\bar{e}_a = \frac{\hat{e}_b}{c} \int_0^{x_1} (v - tx - \frac{1}{2}t\hat{y}_1) \frac{1}{x_1} dx + \frac{B}{c} = \frac{\hat{e}_b}{c} (v - \frac{1}{2}tx_1 - \frac{1}{2}t\hat{y}_1) + \frac{B}{c} \quad (2.5)$$

²⁰Note that when the expected search effort of agents of the other type is zero, that is, $\hat{e}_j = 0$, each agent chooses $e_k = \frac{B}{c} > 0$.

Similarly, the average search effort of type b agents given \hat{e}_a is:

$$\bar{e}_b = \frac{\hat{e}_a}{c} \int_0^{y_1} (v - \frac{1}{2}t\hat{x}_1 - ty) \frac{1}{y_1} dy + \frac{B}{c} = \frac{\hat{e}_a}{c} (v - \frac{1}{2}t\hat{x}_1 - \frac{1}{2}ty_1) + \frac{B}{c} \quad (2.6)$$

Now we impose a rationality requirement on agents' expectations – that is, the expected search effort is equal to the average search effort $\hat{e}_k = \bar{e}_k$. By substituting equation $\hat{e}_k = \bar{e}_k$ in equations (2.5) and (2.6), and solving the system of two equations, we obtain:

$$\bar{e}_a = \hat{e}_a = \frac{B}{c - (v - \frac{1}{2}t\hat{x}_1 - \frac{1}{2}ty_1)} = \frac{B}{c - (v - \frac{1}{2}tx_1 - \frac{1}{2}ty_1)} \quad (2.7)$$

$$\bar{e}_b = \hat{e}_b = \frac{B}{c - (v - \frac{1}{2}tx_1 - \frac{1}{2}t\hat{y}_1)} = \frac{B}{c - (v - \frac{1}{2}tx_1 - \frac{1}{2}ty_1)} \quad (2.8)$$

where the second equalities in both (2.7) and (2.8) hold because the agents' expectations about the location of the marginal agents are fulfilled in equilibrium, $\hat{x}_1 = x_1$ and $\hat{y}_1 = y_1$. In order for \bar{e}_k to belong to $[0, 1]$, it is assumed that $c > v + B$. For the rest of the analysis, we also assume that $c > v > 2t$.

Substituting equations (2.7) and (2.8) into (2.2) and (2.4), respectively, we obtain the optimal search effort of individual agent. The optimal search effort of agents of type a at $x'_1 \in [0, x_1]$, as a function of their location, is given by:

$$e_a^* = \frac{B}{c} \left[\frac{c + \frac{1}{2}tx_1 - tx'_1}{c - (v - \frac{1}{2}tx_1 - \frac{1}{2}ty_1)} \right] = \bar{e}_b \left[\frac{c + \frac{1}{2}tx_1 - tx'_1}{c} \right] \quad (2.9)$$

where the second equality follows from (2.8).

Similarly, for agents of type b at $y'_1 \in [0, y_1]$, the optimal search effort is:

$$e_b^* = \frac{B}{c} \left[\frac{c + \frac{1}{2}ty_1 - ty'_1}{c - (v - \frac{1}{2}tx_1 - \frac{1}{2}ty_1)} \right] = \bar{e}_a \left[\frac{c + \frac{1}{2}ty_1 - ty'_1}{c} \right] \quad (2.10)$$

where the second equality follows from (2.7). In order for e_k^* to belong to $[0, 1]$, it is assumed that $B \leq (c - v + \frac{t}{2}) \left[\frac{c}{c + \frac{t}{4}} \right]$.

The following proposition summarizes relations among the optimal search effort level of agents, agents' locations, and the level of extra benefit B .

Proposition 2.1. *Let $B > 0$ and consider an agent of type k for which $V_k^i \geq w_k$ (e.g., after*

observing prices from the first-stage of the game, agent of type k joins one of the middlemen). Then in the second stage of the game, she will always choose a positive search effort $e_k^* > 0$. Furthermore, e_k^* is increasing in B , e_a^* is strictly decreasing in x'_1 , x_1 , and y_1 , and e_b^* is strictly decreasing in y'_1 , x_1 , and y_1 .

Proof. The first part of Proposition 2.1 is straightforward since $\bar{e}_b > 0$ when $B > 0$, and $(c + \frac{1}{2}tx_1 - tx'_1)/c > 0$ because of the assumption $c > v > 2t$. For the second part, we can easily check the signs of $\partial e_a^*/\partial x'_1 < 0$, $\partial e_a^*/\partial x_1 < 0$, and $\partial e_a^*/\partial y_1 < 0$ from equation (2.9). The proof for e_b^* is similar to the proof for e_a^* and is therefore omitted. \square

The first part of Proposition 2.1 emphasizes the role of extra benefit B . Suppose, for a moment, that $B = 0$ and all agents expect $\hat{e}_j = 0$. Then they will always expend zero search effort since the choice of search effort is strategically dependent.²¹ In this case, an agent would expend a positive search effort only if the expected search effort of the other type is positive. In the present model, $B > 0$ ensures that any participating agent will choose a positive search effort when forming her expectations.

The second part of Proposition 2.1 implies that an agent with higher expected matching surplus will expend more search effort than agents with lower expected matching surplus. That is, for an agent at $x'_1 < x''_1$, $e_a^*(x'_1) > e_a^*(x''_1)$ since $ES_a^1(x'_1) > ES_a^1(x''_1)$. Accordingly, the expected utility of each agent from joining the middleman $V_a^1(x'_1)$ decreases as x'_1 increases. Further, an agent tends to reduce her search effort if more agents join the same middleman. We can explain this as follows. First each agent anticipates that, if more agents of the other type join the same middleman, the expected matching surplus will decrease, and therefore she would expend less search effort. Second, if more agents of the same type join the same middleman, this will decrease the expected matching surplus of agents of the other type. Therefore, agents of the other type will expend less search effort, hence less the average effort level, which in turn reduces the search effort of agents on the same side. As will become clear from the next section, middlemen have to consider these results when they make pricing decisions.

²¹Following the terminology of Bulow, Geanakoplos, and Klemperer (1985), the agents' effort level decisions are strategically complements since an increase in search effort of an agent raises the marginal payoff of agents of the other type.

We close this section by representing the probability of a match of individual agent $e_k \hat{e}_j$. This probability follows immediately from equations (2.7) and (2.9), and from equations (2.8) and (2.10). For agents of type a at $x'_1 \in [0, x_1]$, the probability of a match is given by:

$$e_a^* \bar{e}_b = \bar{e}_b^2 \left[\frac{c + \frac{1}{2} t x_1 - t x'_1}{c} \right] \quad (2.11)$$

And for agents of type b at $y'_1 \in [0, y_1]$, the probability of a match is:

$$e_b^* \bar{e}_a = \bar{e}_a^2 \left[\frac{c + \frac{1}{2} t y_1 - t y'_1}{c} \right] \quad (2.12)$$

In this section, we characterized the optimal search effort of the agents. Much like the model of Yavas (1994), the optimal search effort is characterized by valuations about the matching surplus of agents. However, in his model, agents' choices of search effort is strategically independent. Here we analyzed a case when the choices are strategically dependent. Moreover, the role of the middleman in his model is different from ours: In his model, the middleman with monopoly power provides a better matching technology (e.g, a higher probability of a match); in our model, two competitive middlemen provide an extra benefit that facilitates searching so as to make a search market viable.²²

2.4 Price Competition

In this section we analyze the price game between two horizontally differentiated middlemen. We first show that each middleman always balances the number of agents on both sides when he charges fees for his services – that is, middleman i chooses a price pair such that $x_i = y_i$. Then we obtain the equilibrium price pair chosen by the profit-maximizing middlemen under different forms of competition.

The rest of the game proceeds as follows: In the first stage, the middlemen announce their prices simultaneously and independently, anticipating that a pair of prices (P_i^a, P_i^b) will affect agents' expectations about utilities from the search market participation – more specifically, the expected utility of the participating agents from the second-stage of the game. Then, agents also

²²In fact, the present model provides one possible explanation for how middlemen can provide the better matching technology in search markets. In the existing literature, it is usually assumed that middlemen provide a better matching probability which is exogenously given.

form their expectations taking as given the prices that have been announced – that is, as discussed in the previous section, all agents *rationally* anticipate the market segment of each middleman and compute the matching surplus, the probability of a match, and the expected utility from joining the middleman – and choose which middleman to join only if $V_k^i(\cdot) \geq w_k$. After joining one of the middlemen, they choose their search effort based on their expectations. Otherwise they go to factor markets, and sell their endowment at w_k . In this section, we compute an equilibrium pair of prices by assuming that $B > 0$ is fixed at some level, while we allow different values of w_k .

Before studying the price competition between the two middlemen, we first present the following Lemma that allows us to simplify our analysis.

Lemma 2.1. *Consider an equilibrium in which a middleman announce a pair of prices (P_i^{a*}, P_i^{b*}) and attracts positive number of agents of each type. Then the number of agents attracted of each type are the same.*

Proof. See the Appendix. □

Lemma 2.1 implies that any participating agent is always matched with an agent of the other type as long as she joins the middleman. Lemma 2.1 rules out uncertainty due to a mismatch – that is, $x_1/y_1=1$ always holds at equilibrium. In our model, the source of uncertainty arises from agents' locations, and consequently their search effort and the probability of a match.

By substituting the result from Lemma 2.1, $z_1 \equiv x_1 = y_1$ in equations (2.7) and (2.8), we rewrite the average search effort of each side:

$$\bar{e}_{k_1} = \frac{B}{c - v + tz_1}$$

where subscript k_1 denotes agents of type k joining middleman 1. Accordingly, for the marginal agent of each type, x_1 and y_1 , the optimal search effort is given by:

$$e_{k_1}^* = \frac{B}{c} \left[\frac{c - \frac{1}{2}tz_1}{c - v + tz_1} \right] = \bar{e}_{k_1} \left[\frac{c - \frac{1}{2}tz_1}{c} \right]$$

and the probability of a match is:

$$e_1^* \equiv e_{k_1}^* \bar{e}_{j_1} = \bar{e}_{k_1}^2 \left[\frac{c - \frac{1}{2}tz_1}{c} \right] \text{ where } j_1 \neq k_1 \in \{a, b\}$$

For notational simplicity, we define $e_1^* \equiv e_{k_1}^* \bar{e}_{j_1}$, the probability of a match for the marginal agents of each type who join middleman 1. In order to study price competition between the middlemen, we also present the optimal search effort and the probability of a match for the agents participating middleman 2. In the view of the symmetry of the problem, the probability of a match for the marginal agents of each type, $z_2 \equiv x_2 = y_2$, who join middleman 2 is:

$$e_2^* \equiv e_{k_2}^* \bar{e}_{j_2} = \bar{e}_{k_2}^2 \left[\frac{c - \frac{1}{2}t(1 - z_2)}{c} \right]$$

where $e_{k_2}^* = \bar{e}_{k_2} \left[\frac{c - \frac{1}{2}t(1 - z_2)}{c} \right]$ and $\bar{e}_{j_2} = [B/c - v + t(1 - z_2)]$.

We now proceed to the next step of the analysis, the price competition between the two middlemen. We begin by looking at the participation decision of the agents. Note that it is possible that some of the agents might not join either middleman because agents' participation decisions depend on whether their expected utility from joining the middleman is greater or less than their reservation value w_k . For that reason, it is important to distinguish between the agents who join the middleman and the agents who go to factor markets. Hence it is useful to define the potential market shares of the two middlemen.²³ For middleman 1, the potential market share is:²⁴

$$S_{1k}(P_1^k) \equiv \begin{cases} \{x_1 \in [0, 1] \mid e_1^*(v - tx_1 - \frac{1}{2}ty_1) + e_{a_1}^*B - \frac{1}{2}ce_{a_1}^{*2} - P_1^a \geq w_a\} & \text{for type } a \\ \{y_1 \in [0, 1] \mid e_1^*(v - \frac{1}{2}tx_1 - ty_1) + e_{b_1}^*B - \frac{1}{2}ce_{b_1}^{*2} - P_1^b \geq w_b\} & \text{for type } b \end{cases}$$

Similarly, the potential market share for middleman 2 is:

$$S_{2k}(P_2^k) \equiv \begin{cases} \{x_2 \in [0, 1] \mid e_2^*[v - t(1 - x_2) - \frac{1}{2}t(1 - y_2)] + e_{a_2}^*B - \frac{1}{2}ce_{a_2}^{*2} - P_2^a \geq w_a\} & \text{for type } a \\ \{y_2 \in [0, 1] \mid e_2^*[v - \frac{1}{2}t(1 - x_2) - t(1 - y_2)] + e_{b_2}^*B - \frac{1}{2}ce_{b_2}^{*2} - P_2^b \geq w_b\} & \text{for type } b \end{cases}$$

where $S_{ik}(P_i^k)$ denotes the potential market share of middleman $i \in \{1, 2\}$ for agents of type $k \in \{a, b\}$ at price P_i^k . In words, the potential market share for middleman i is the set of agents for whom the participation constraint, $V_i^k \geq w_k$ is satisfied.

²³The notion of potential market share is often used in the spatial model literature. See, for example, Gabszewicz and Thisse (1986) and Raatle and Webers (1998).

²⁴By Lemma 2.1, $v - tx_1 - \frac{1}{2}ty_1 = v - \frac{3}{2}tz_1$ and $v - t(1 - x_2) - \frac{1}{2}t(1 - y_2) = v - \frac{3}{2}t(1 - z_2)$ hold at equilibrium. We didn't simplify terms in order to show the decision of the marginal agent explicitly.

Now we distinguish three different types of competition, by using the notion of the potential market share: Strong competition, weak competition, and local monopoly. In order to obtain the equilibrium price pair (P_i^{a*}, P_i^{b*}) , we first impose some conditions on agents' participation constraints and thereby ensure that a certain type of competition will arise as an equilibrium outcome. Then, if it exists, we derive an equilibrium price pair subject to the constraints. Note that three types of competition, as equilibrium outcomes, depend crucially on agents' expected profit from joining a middleman and on their reservation value, w_k . To simplify our notation, we let $M_i^k \equiv e_i^*v + e_{k_i}^*B - \frac{1}{2}ce_{k_i}^{*2}$ for $i \in \{1, 2\}$ hereafter.

Strong Competition

Definition 2.1. An equilibrium pair of prices (P_i^{a*}, P_i^{b*}) for $i \in \{1, 2\}$ results in a *strong competition* if the potential market shares of the two middlemen at (P_i^{a*}, P_i^{b*}) have a nonempty intersection for both types of agents, that is $S_{ik}(\cdot) \cap S_{hk}(\cdot) \neq \emptyset$ for $k \in \{a, b\}$, $h \neq i \in \{1, 2\}$.

The definition implies that, for sufficiently high values of V_i^k or low values of w_k , strong competition arises if there are some agents who always prefer to join one of the middlemen instead of going to factor markets. In such a case, their choice of a middleman crucially depends on the fees charged by the middlemen. More specifically, the marginal agent x_1 will choose either middleman 1 or 2 instead of going to factor markets if the following two participation constraints (2.13) and (2.14) hold simultaneously:

$$M_1^a - e_{a_1}^*tx_1 - \frac{1}{2}e_{a_1}^*ty_1 - P_1^a \geq w_a \quad (2.13)$$

where $M_1^a \equiv e_1^*v + e_{a_1}^*B - \frac{1}{2}ce_{a_1}^{*2}$. The formula above is simply the participation constraint for agents such that $x_1 \leq \frac{M_1^a - w_a - P_1^a}{e_{a_1}^*t} - \frac{y_1}{2}$, who prefer to join middleman 1 rather than go to factor markets. They also prefer to join middleman 2 rather than go to factor markets if:

$$M_2^a - e_{a_2}^*t \left[1 - \left(\frac{M_1^a - w_a - P_1^a}{e_{a_1}^*t} - \frac{y_1}{2} \right) \right] - \frac{1}{2}e_{a_2}^*t(1 - y_1) - P_2^a \geq w_a$$

By rearranging terms, we obtain:

$$\frac{e_{a_2}^*}{e_{a_1}^*} M_1^a + M_2^a - \left(1 + \frac{e_{a_2}^*}{e_{a_1}^*}\right) w_a - \frac{3}{2} e_{a_2}^* t \geq \frac{e_{a_2}^*}{e_{a_1}^*} P_1^a + P_2^a \quad (2.14)$$

The potential market shares of the two middlemen for agents of type a have a nonempty intersection if the condition (2.14) holds.

Similarly, the marginal agent y_1 will choose middleman 1 if:

$$M_1^b - \frac{1}{2} e_{b_1}^* t x_1 - e_{b_1}^* t y_1 - P_1^b \geq w_b \quad (2.15)$$

And the potential market shares of two middlemen for agents of type b have a nonempty intersection if the following holds:

$$\frac{e_{b_2}^*}{e_{b_1}^*} M_1^b + M_2^b - \left(1 + \frac{e_{b_2}^*}{e_{b_1}^*}\right) w_b - \frac{3}{2} e_{b_2}^* t \geq \frac{e_{b_2}^*}{e_{b_1}^*} P_1^b + P_2^b \quad (2.16)$$

According to Definition 2.1, strong competition occurs if there exists an equilibrium pair of prices that satisfies conditions (2.13), (2.14), (2.15), and (2.16) – a nonempty intersection for both types agents exists. In fact, strong competition is quite similar to one of the standard Hotelling competition results, where a market is fully covered and the two firms share the market equally.

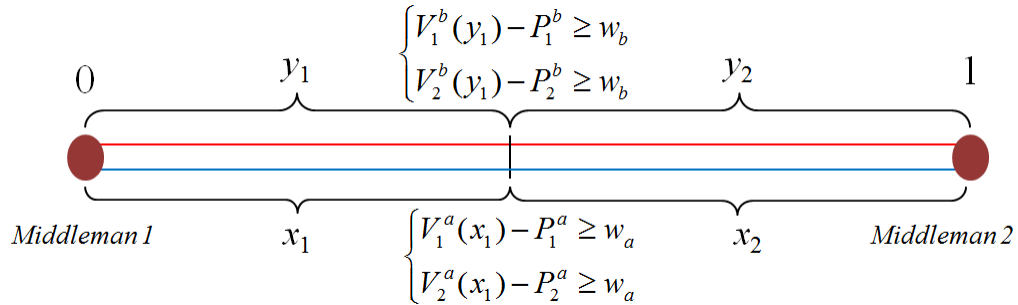


Figure 2.2: Strong Competition

Now we will look at a price equilibrium under strong competition. In order to do so, we identify the ‘marginal agent’ of each type who is indifferent between joining middlemen 1 and 2 for a given pair of prices, and we compute her expected profit. The marginal agent x_1 is such that $V_1^a(x_1) = V_2^a(x_1)$. Solving for x_1 , we obtain:

$$x_1^* = \frac{(P_2^a - P_1^a) + (M_1^a - M_2^a) + \frac{3}{2}e_{a_2}^* t}{(e_{a_1}^* + e_{a_2}^*)t} - \frac{y_1}{2}$$

Similarly, for the marginal agent of type b , we obtain:

$$y_1^* = \frac{(P_2^b - P_1^b) + (M_1^b - M_2^b) + \frac{3}{2}e_{b_2}^* t}{(e_{b_1}^* + e_{b_2}^*)t} - \frac{x_1}{2}$$

Then middleman 1 solves the following constrained-maximization problem, if there exists an equilibrium price pair under strong competition:

$$\begin{aligned} \max_{P_1^a, P_1^b} \Pi_1 &= (P_1^a + P_1^b) \text{Min}\{x_1^*, y_1^*\} \\ \text{s.t.} & \quad (2.13), (2.14), (2.15) \text{ and } (2.16) \end{aligned}$$

The results are given in the following proposition.

Proposition 2.2. *Let $w_k \leq \frac{e^*}{2}(c - \frac{t}{4})$ and $\frac{c}{t} > \frac{1}{4}$ for $k \in \{a, b\}$. Then, under strong competition, there exist four possible pairs of equilibrium prices (P_a^*, P_b^*) . In each equilibrium, half of the agents of each type join middleman 1 and the rest join middleman 2, and the middlemen charge:*

(1) *a single equilibrium pair of prices $(\frac{e^*}{2}(c - \frac{t}{4}) - w_a, \frac{e^*}{2}(c - \frac{t}{4}) - w_b)$ if $w_a + w_b \in [e^*(c - \frac{13}{4}t), e^*(c - \frac{1}{4}t)]$ and $\frac{c}{t} > \frac{13}{4}$*

(2) *if $w_a + w_b \leq e^*(c - \frac{13}{4}t)$ and $\frac{c}{t} > \frac{13}{4}$, there exist three continua of equilibria:*

(a) *$(p^a, 3e^*t - p^a)$ where $p^a \in [0, \frac{e^*}{2}(c - \frac{t}{4}) - w_a] \cap [w_b - \frac{e^*}{2}(c - \frac{25}{4}t), 3e^*t]$,*

(b) *$(0, p^b)$ where $p^b \in [3e^*t, \frac{e^*}{2}(c - \frac{t}{4}) - w_b]$ if $w_a \leq \frac{e^*}{2}(c - \frac{t}{4})$, $w_b \leq \frac{e^*}{2}(c - \frac{25}{4}t)$ and $\frac{c}{t} > \frac{25}{4}$,*

(c) *$(p^a, 0)$ where $p^a \in [3e^*t, \frac{e^*}{2}(c - \frac{t}{4}) - w_a]$ if $w_a \leq \frac{e^*}{2}(c - \frac{25}{4}t)$, $w_b \leq \frac{e^*}{2}(c - \frac{t}{4})$ and $\frac{c}{t} > \frac{25}{4}$,*

where $e^ = e_k^* \bar{e}_j^*$ denotes the equilibrium probability of a match for a marginal agent at $\frac{1}{2}$, and e_k^* and \bar{e}_j^* are given by $\bar{e}_j^* \left[\frac{c-t/4}{c} \right]$ and $\frac{B}{c-v+t/2}$, respectively.*

Proof. See the Appendix. □

Let us discuss Proposition 2.2. First note that, for the marginal agent of each type indifferent between joining two middlemen, the gross expected utility is given by $\frac{e^*}{2}(c - \frac{t}{4})$. And this gross expected utility is high enough and reservation values are sufficiently low to ensure that strong competition arises. When the middlemen decide fees for their services, they have to take into account

agents' reservation values and the market externality associated with their demand function. For the values of $w_a + w_b \in [e^*(c - \frac{13}{4}t), e^*(c - \frac{1}{4}t)]$, both middlemen charge a pair of prices such that the marginal agent of each type obtains zero profit in equilibrium. In fact, this result is similar to standard Hotelling duopoly except that the middlemen have to balance 'two-sides' of the market. However, for the values of $w_a + w_b \leq e^*(c - \frac{13}{4}t)$, the middlemen choose a pair of prices such that any participating agents of one type obtain higher utilities than participating agents of the other type. Agents of one type with reservation value $w_k \leq e^*(c - \frac{25}{4}t)$ are served for free, while the other type of agents, with reservation value $w_j \leq e^*(c - \frac{1}{4}t)$, are charged a positive fee.

Weak Competition

We now move on to the case of weak competition and compute equilibrium prices. Formally, we define weak competition as follows.

Definition 2.2. At an equilibrium pair of prices (P_i^{a*}, P_i^{b*}) for $i \in \{1, 2\}$, *weak competition* arises if the potential market shares of the two middlemen at (P_i^{a*}, P_i^{b*}) have a nonempty intersection for one of the two types of agents and for the other the intersection is either a point or is empty, that is $S_{ik}(\cdot) \neq \emptyset$, $S_{ik}(\cdot) \cap S_{hk}(\cdot) \neq \emptyset$ and $|S_{ij}(\cdot) \cap S_{hj}(\cdot)| = 0$ or 1 for $j \neq k \in \{a, b\}$, and $h \neq i \in \{1, 2\}$.

The definition implies that weak competition emerges when no competition arises for one type of agents, while both middlemen compete for agents of the other type. Without loss generality, in this subsection we assume that two middlemen compete only for agents of type a . Specifically, the following condition holds for agents of type a :

$$\frac{e_{a2}^*}{e_{a1}^*} M_1^a + M_2^a - \left(1 + \frac{e_{a2}^*}{e_{a1}^*}\right) w_a - \frac{3}{2} e_{a2}^* t \geq \frac{e_{a2}^*}{e_{a1}^*} P_1^a + P_2^a$$

As under strong competition, the potential market shares of the two middlemen for agents of type a have nonempty intersection. In other words, the marginal agent x_1 will choose either middleman 1 or 2 rather than go to factor markets. However, agents of type b , under weak competition, will choose only their proximate middleman, otherwise they will go to factor markets. More specifically, the marginal agent y_1 will choose middleman 1 if:

$$M_1^b - e_{b1}^* t x_1 - \frac{1}{2} e_{b1}^* t y_1 - P_1^b \geq w_b$$

And the potential market share of middleman 2 for agents of type b has either a point or empty intersection if the following holds:

$$\frac{e_{b_2}^*}{e_{b_1}^*} M_1^b + M_2^b - \left(1 + \frac{e_{b_2}^*}{e_{b_1}^*}\right) w_b - \frac{3}{2} e_{b_2}^* t \leq \frac{e_{b_2}^*}{e_{b_1}^*} P_1^b + P_2^b \quad (2.17)$$

In words, under weak competition, agents of type b , who may join middleman 1, have no incentive to join middleman 2. Analogously, the rest of agents of type b who may join middleman 2 have no incentive to join middleman 1. In summary, weak competition occurs if there exists an equilibrium pair of prices that satisfies conditions (2.13), (2.14), (2.15), and (2.17).

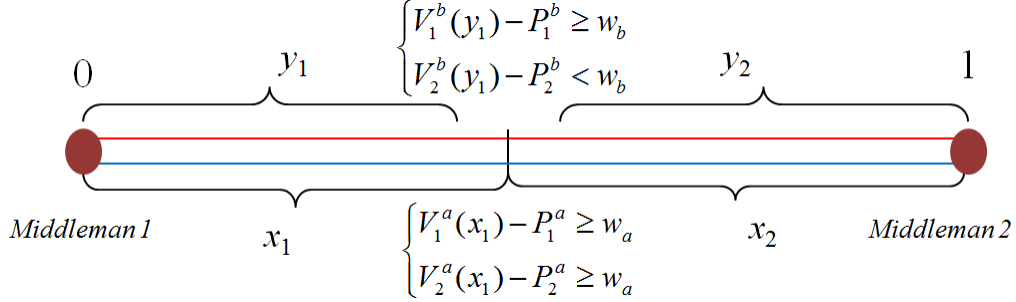


Figure 2.3: Weak Competition

Now we compute equilibrium price pairs under weak competition. Since competition arises for agents of type a , we compute the marginal agent of type a such that $V_1^a(x_1) = V_2^a(x_2)$, given a pair of prices and rational expectation. Solving for x_1 , we obtain:

$$x_1^* = \frac{(P_2^a - P_1^a) + (M_1^a - M_2^a) + \frac{3}{2} e_{a_2}^* t}{(e_{a_1}^* + e_{a_2}^*)t} - \frac{y_1}{2}$$

For agents of type b , the demand faced by middleman 1 is given by:

$$y_1^* = \frac{M_1^b - w_b - P_1^b}{e_{b_1}^* t} - \frac{x_1}{2}$$

Now the following constrained-maximization problem is faced by middleman 1 under weak competition:

$$\begin{aligned} \underset{P_1^a, P_1^b}{Max} \Pi_1 &= (P_1^a + P_1^b) \text{Min}\{x_1^*, y_1^*\} \\ \text{s.t.} & \quad (2.13), (2.14), (2.15), \text{ and } (2.17) \end{aligned}$$

We summarize the results in the following proposition.

Proposition 2.3. *Let $w_k \leq \frac{e^*}{2}(c - \frac{t}{4})$, $w_k \neq w_j$, and $\frac{c}{t} > \frac{1}{4}$ for $j \neq k \in \{a, b\}$. Then, under weak competition, there exist three possible pairs of equilibrium prices (P^{a*}, P^{b*}) . In equilibrium, half of the agents of each type join middleman 1, the rest join middleman 2, and the middlemen charge:*

(1) *a unique equilibrium pair of prices $(\frac{e^*}{2}(c - \frac{t}{4}) - w_a, \frac{e^*}{2}(c - \frac{t}{4}) - w_b)$ if $w_a + w_b \in [e^*(c - \frac{5}{2}t), e^*(c - \frac{1}{4}t)]$ and $\frac{c}{t} > \frac{5}{2}$*

(2) *If $w_a + w_b \leq e^*(c - \frac{5}{2}t)$ and $\frac{c}{t} > \frac{5}{2}$, there exists a unique equilibrium:*

(a) *$(w_b - \frac{e^*}{2}(c - \frac{19}{4}t), \frac{e^*}{2}(c - \frac{t}{4}) - w_b)$ if $w_a \leq \frac{e^*}{2}(c - \frac{19}{4}t)$ and $w_b \in [\frac{e^*}{2}(c - \frac{19}{4}t), \frac{e^*}{2}(c - \frac{1}{4}t)]$,*

(b) *$(0, \frac{e^*}{2}(c - \frac{t}{4}) - w_b)$ if $w_b \leq \frac{e^*}{2}(c - \frac{19}{4}t)$*

where $e^ = e_k^* \bar{e}_j^*$ denotes the equilibrium probability of a match for the marginal agent at $\frac{1}{2}$, and e_k^* and \bar{e}_j^* are given by $\bar{e}_j^* \left[\frac{c-t/4}{c} \right]$ and $\frac{B}{c-v-t/2}$, respectively.*

Proof. See the Appendix. □

Under weak competition, the gross expected utility for the marginal agents is given by $\frac{e^*}{2}(c - \frac{t}{4})$, which is the same as in strong competition. It happens since the two middlemen cover the entire market and share the market equally even though competition for agents of type b doesn't arise.²⁵ The intuition for this result is as follows. First, for agents of type a , both middlemen compete for the indifferent agent as in strong competition, which results in the market share of each middleman for agent of type a being $\frac{1}{2}$. Next, consider pricing decision, and therefore the market coverage by middleman i for agents of type b . Given $x_1 = x_2 = \frac{1}{2}$, if middleman i sets a price for agents of type b , P_i^b , that admits less than half of agents, the market demand faced by middleman i will be less than $\frac{1}{2}$ since $\min\{x_i = \frac{1}{2}, y_1\} = \min\{x_i = \frac{1}{2}, 1 - y_2\} < \frac{1}{2}$. However, as shown in Lemma 2.1, it is not profitable for middleman i under weak competition to do so. In fact the full market coverage for agents of type b is not due to competition for agents of type b , but due to the market externality associated with the demand function.

We now proceed to discussion of determining the equilibrium price pair according to different values of the reservation value. First note that, under weak competition, the marginal agent of type b always obtains zero utility in equilibrium because middleman i exercises the monopoly

²⁵In fact, this result immediately follows from Lemma 2.1.

power over this type of agent. However, the price charged to agents of type a , after competition, differs according to the values of w_a and w_b . For the values of $w_a + w_b \in [e^*(c - \frac{5}{2}t), e^*(c - \frac{1}{4}t)]$, much like in strong competition, the two middlemen charge a pair of prices such that the marginal agent of each type obtains zero utility in equilibrium. For the values of $w_a + w_b \leq e^*(c - \frac{5}{2}t)$, however, both middlemen choose a price for agents of type a such that, in equilibrium, the marginal agent of type a can enjoy a positive utility if $w_a < w_b$. This implies that competition for agents of type a drives fees down if $w_a < \frac{e^*}{2}(c - \frac{19}{4}t)$ and $w_b \in [\frac{e^*}{2}(c - \frac{19}{4}t), \frac{e^*}{2}(c - \frac{1}{4}t)]$. Agents of type a are served for free if the reservation value of agents of type b is less than $\frac{e^*}{2}(c - \frac{19}{4}t)$.

Local Monopoly

Let us now turn to the local monopoly case. In the cases we have examined above – strong and weak competition – competition between the middlemen results in full market coverage for both types of agents. In the case of local monopoly, in contrast, each middleman exercises monopoly power over his proximate agents of both types, and therefore, under certain conditions, some of agents may not be served by either middleman. Formally, local monopoly is defined as follows.

Definition 2.3. At an equilibrium pair of prices (P_i^{a*}, P_i^{b*}) for $i \in \{1, 2\}$, local monopoly arises if the potential market shares of the two middlemen at (P_i^{a*}, P_i^{b*}) have an intersection which is either a point or empty, that is $S_{ik}(\cdot) \neq \emptyset$ and $|S_{ik}(\cdot) \cap S_{jk}(\cdot)| = 0$ or 1 for $k \in \{a, b\}$, and $h \neq i \in \{1, 2\}$.

The definition implies that, for sufficiently low values of V_i^k or high values of w_k , local monopoly may arise if there are some agents who may choose either their proximate middleman or factor market. More specifically, the marginal agent x_1 will join middleman 1 if the following two conditions hold simultaneously:

$$M_1^a - e_{a_1}^* t x_1 - \frac{1}{2} e_{a_1}^* t y_1 - P_1^a \geq w_a$$

and

$$\frac{e_{a_2}^*}{e_{a_1}^*} M_1^a + M_2^a - \left(1 + \frac{e_{a_2}^*}{e_{a_1}^*}\right) w_a - \frac{3}{2} e_{a_2}^* t \leq \frac{e_{a_2}^*}{e_{a_1}^*} P_1^a + P_2^a \quad (2.18)$$

Otherwise they will go to factor markets rather than join middleman 2. The potential market

shares of the two middlemen for agents of type a have a point or an empty intersection if condition (2.18) holds.

Similarly, the marginal agent y_1 will join middleman 1 if:

$$M_1^b - e_{b_1}^* t x_1 - \frac{1}{2} e_{b_1}^* t y_1 - P_1^b \geq w_b$$

and

$$\frac{e_{b_2}^*}{e_{b_1}^*} M_1^b + M_2^b - \left(1 + \frac{e_{b_2}^*}{e_{b_1}^*}\right) w_b - \frac{3}{2} e_{b_2}^* t \leq \frac{e_{b_2}^*}{e_{b_1}^*} P_1^b + P_2^b \quad (2.19)$$

According to Definition 2.3, local monopoly arises if there exists an equilibrium pair of prices that satisfies conditions (2.13), (2.15), (2.18), and (2.19).

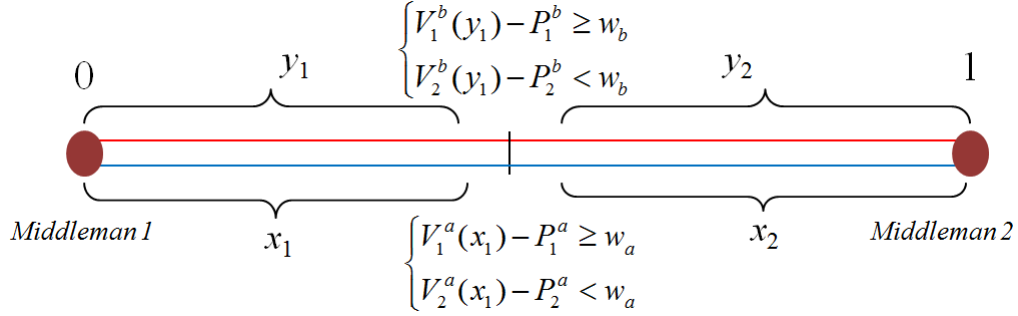


Figure 2.4: Local Monopoly

We now proceed to the computation of equilibrium pair of prices under local monopoly. For agents of type a , the demand faced by middleman 1 is given by:

$$x_1^* = \frac{M_a^1 - w_a - P_1^a}{e_{a_1}^* t} - \frac{y_1}{2}$$

Similarly, for agents of type b , we obtain:

$$y_1^* = \frac{M_b^1 - w_b - P_1^b}{e_{b_1}^* t} - \frac{x_1}{2}$$

Middleman 1 solves the following constrained-maximization problem under local monopoly.

$$\begin{aligned} \underset{P_1^a, P_1^b}{Max} \Pi_1 &= (P_1^a + P_1^b) \text{Min}\{x_1^*, y_1^*\} \\ \text{s.t.} & \quad (2.13), (2.15), (2.18), \text{ and } (2.19) \end{aligned}$$

We summarize the results in the following proposition.

Proposition 2.4. *Let $M^* = \frac{e^*}{2}(c + \frac{5}{2}tx^*) = \frac{e^*}{2}(c + \frac{5}{2}ty^*)$, where $x^*, y^* \leq \frac{1}{2}$ denote the equilibrium number of agents. Then, under local monopoly, there exist four possible pairs of equilibrium prices (P^{a*}, P^{b*}) .*

(1) *If $w_k \leq \frac{e^*}{2}(c - \frac{t}{4})$, $w_a + w_b \leq e^*(c - \frac{7}{4}t)$ and $\frac{c}{t} > \frac{7}{4}$ then, in equilibrium, $x^* = y^* = \frac{1}{2}$ and a unique equilibrium price pair is given by $(\frac{e^*}{2}(c - \frac{t}{4}) - w_a, \frac{e^*}{2}(c - \frac{t}{4}) - w_b)$.*

(2) *If $w_a + w_b \geq 2M^* - 3e^*t$ and $3w_b - 2M^* \leq w_a \leq \frac{2M^* + w_b}{3}$ then, in equilibrium, agents $x^* = y^* = \frac{(M^* - (w_a + w_b)/2)}{3e^*t} \leq \frac{1}{2}$ will join their proximate middleman i , and the rest will go to the factor market. The unique equilibrium price pair is given by $(\frac{M^*}{2} + \frac{1}{4}(w_b - 3w_a), \frac{M^*}{2} + \frac{1}{4}(w_a - 3w_b))$*

(3) *If $w_k \geq \frac{2M^* + w_j}{3}$, $w_k \geq M^* - \frac{3}{4}e^*t$ and $\frac{M^*}{et} > \frac{3}{4}$ then, in equilibrium, agents $x^* = y^* = \frac{2}{3} \left(\frac{M^* - w_k}{e^*t} \right) \leq \frac{1}{2}$ will join their proximate middleman i , and the rest will go to the factor market. Further, agents with high reservation value (w_k) are served for free, while those with low reservation value (w_j) are charged a positive fee $w_k - w_j$.*

Proof. See the Appendix □

In Proposition 2.4, $e^* = e_k^* \bar{e}_j^*$ denotes the equilibrium probability of a match for a marginal agent at $x^* = y^* \leq \frac{1}{2}$, and e_k^* and \bar{e}_j^* are given by $\bar{e}_j^* \left[\frac{c - tx^*/2}{c} \right] = \bar{e}_j^* \left[\frac{c - ty^*/2}{c} \right]$ and $\frac{B}{c - v + tx^*} = \frac{B}{c - v + ty^*}$, respectively. Let us discuss the proposition stated above. First note that the gross expected utility for the marginal agents is given by $\frac{e^*}{2}(c - \frac{t}{2}x^*)$. When $x^* = y^* = \frac{1}{2}$ – that is, each middleman covers half of the market for each type – the gross expected utility for the marginal agent of each type is given by $\frac{e^*}{2}(c - \frac{t}{4})$, which is the same as in both strong and weak competition. However, the explanation for this result differs from the other two forms of competition. In fact, it happens not due to the consequence of competition but due to the consequence of local monopolistic behavior of each middleman. Therefore, it is also possible that both middlemen serve less than half of the agents of each type in some other cases. In particular, when w_k exceeds a certain threshold – that is, when reservation values are relatively high – it would not be profitable for each middlemen to serve up to the agent at $\frac{1}{2}$. In this case, serving less than half of the agents of each type is more profitable since $\frac{e^*}{2}(c - \frac{t}{2}x^*)$ is strictly decreasing in x^* and it allows each middleman to charge higher prices on both sides. In fact, the results (2), (3) and (4) of Proposition 2.4 show that both

middlemen have an incentive to do so under certain ranges of parameters.

We now turn to the determination of equilibrium price pairs by the middlemen. For the values of $w_a + w_b \in [0, e^*(c - \frac{7}{4}t)]$, the two middlemen charge a pair of prices such that the marginal agent of each type at $\frac{1}{2}$ obtains zero utility in equilibrium. However, for the values of $w_a + w_b > 2M^* - 3e^*t$ – that is, reservation values are relatively high compared to case (1) – both middlemen set a pair of prices such that $(\frac{M^*}{2} + \frac{1}{4}(w_b - 3w_a), \frac{M^*}{2} + \frac{1}{4}(w_a - 3w_b))$, and serve less than half of the agents. In fact, both middlemen find that serving less than half of the agents is more profitable since they are less willing to pay for joining middlemen when their reservation value is high. The more interesting result arises when the reservation value of one type is sufficiently higher than the reservation value of the other type. In such a case, the type of agents with higher reservation value is served for free, while the other type is charged a positive fee. It is worth emphasizing that this arises solely due to the market externality associated with the demand function, while under weak competition, competition for agents of one type, as well as the market externality, drives fees to zero.

2.5 An equilibrium market structure under different B

Up to now, we obtained the equilibrium pairs of prices under three different forms of competition: strong competition, weak competition, and local monopoly. Our results suggest that the middlemen charge a positive price for both types of agents when the difference between reservation values is relatively small: In the extreme case, where the reservation values are the same across types, the middlemen charge the same price to both types of agents. (*Symmetric pricing*) On the other hand, the middlemen tend to subsidize one type of agent when the difference between reservation values are sufficiently large: In both strong competition and local monopoly, agents with high reservation values are charged zero price. However, under weak competition, agents with low reservation value are served for free because the two middlemen compete for that type of agents, and this drives fees to zero. (*Asymmetric pricing*)

Based upon the results obtained in the previous section, we look at the equilibrium market structure under different values of B – the extra benefit provided by the middlemen – while keeping other parameters being fixed. Specifically, we are going to discuss the levels of extra

benefit that support a specific market structure given w_k . Allowing different B s between the two middlemen and discussing an equilibrium level of B would be an interesting question. However, it goes beyond the scope of the present chapter. For ease of exposition, we adopt the following assumptions, in order to contrast two extreme cases:

1. *Symmetric pricing*: The reservation values for agents of both types are the same. That is, $w_a = w_b = w^* > 0$.
2. *Asymmetric pricing*: The reservation value for agents of type a is given by $w_a = \Delta w^* > 0$ while the reservation value for agents of type b is given by $w_b = 0$.

Symmetric Pricing

We first consider the case where $w_a = w_b = w^*$. From the results from the previous section, one can easily observe that, as long as the reservation value for agents of both types are the same, the equilibrium price for the agents of both types is given by $P^{a*} = P^{b*} = \frac{e^*}{2}(c - \frac{t}{4}) - w^*$ under strong competition and under local monopoly.²⁶ We now determine the range of the level of B that supports a particular type of competition in equilibrium, when reservation values for the two types of agent are the same.

Proposition 2.5. *Let $w_a = w_b = w^*$ and $\frac{c}{t} > \frac{13}{4}$. In equilibrium, two different forms of competition will be supported by some values of B while keeping all other parameters fixed.*

- (a) *Strong competition is supported by $B \in \left[\frac{(c-v+t/2)}{(c-t/4)} \sqrt{2cw^*}, \frac{(c-v+t/2)}{\sqrt{(c-t/4)(c-13t/4)}} \sqrt{2cw^*} \right]$,*
- (b) *Local monopoly is supported by $B \in \left[\frac{(c-v+t/2)}{\sqrt{(c-t/4)(c-7t/4)}} \sqrt{2cw^*}, (c-v + \frac{t}{2}) \left[\frac{c}{c+t/4} \right] \right]$,*

In both cases, the middleman i charges $P_i^{k} = \frac{e^*}{2}(c - \frac{t}{4}) - w^*$ and the profits of middleman i are given by $\pi_i = \frac{1}{2c} \left[\frac{B(c-t/4)}{c-v+t/2} \right]^2 - w^*$.*

Proof. See the Appendix □

Asymmetric Pricing

We now turn to the case in which $w_a = \Delta w^* > 0$ and $w_b = 0$. From the results in the previous section, we observe that the middlemen tend to subsidize one type of agent when the difference

²⁶See Proposition 2.2 (1) and Proposition 2.4 (1). Note that weak competition can not occur when $w_a = w_b$.

between reservation values is sufficiently large (e.g., Δw^* is sufficiently large). Table 2.1 summarizes the equilibrium price pairs and profits of middleman i under different types of competition.

Table 2.1: The equilibrium price pair and profit of middleman i when $w_a = \Delta w^*$ and $w_b = 0$

	Strong Competition	Weak Competition	Local Monopoly
(P_i^{a*}, P_i^{b*})	$(0, p^b)$	$(\frac{e^*}{2}(c - \frac{t}{4}), 0)$	$(0, \Delta w^*)$
π_i^*	$\frac{p^b}{2}$	$\frac{e^*}{4}(c - \frac{t}{4})$	$\frac{\Delta w^*}{2}$

where $p^b \in [3e^*t, \frac{e^*}{2}(c - \frac{t}{4})]$ for strong competition and $e^* = \bar{e}^2 \left[\frac{c-t/4}{c} \right] = \left[\frac{B}{c-v+t/2} \right]^2 \left[\frac{c-t/4}{c} \right]$.

Note that agents of type b under weak competition are served for free because competition between the two middlemen drives fees to zero. We now determine the range of the level of B that supports a particular type of competition in equilibrium when the difference between reservation values is sufficiently large.²⁷

Proposition 2.6. *Let $w_a = \Delta w^* > 0$, $w_b = 0$ and $\frac{c}{t} > \frac{1}{4}$. In equilibrium, three forms of competition will be supported by some values of B while keeping all other parameters fixed.*

- (a) *Local monopoly is supported by $B \in (0, \frac{(c-v+t/2)}{\sqrt{(c-t/4)(c+5t/4)}} \sqrt{3c\Delta w^*}]$*
- (b) *Weak (or Strong) competition is supported by $B \in \left[\frac{(c-v+t/2)}{(c-t/4)} \sqrt{2c\Delta w^*}, \frac{c(c-v+t/2)}{(c+t/4)} \right]$*

Proof. See the Appendix □

2.6 Conclusion

This chapter addresses the role of horizontally differentiated middlemen in a bilateral search market in which heterogeneous agents of each group possess private information concerning the value of joint production. In contrast to the existing literature on intermediation, we model middlemen who provide extra benefit to search market participants. We first study how the provision of extra benefit by middlemen affects agents' search effort and their expectations about matching probabilities. Specifically, we have shown that the extra benefit provided by the middlemen intensifies the search effort of market participants and increases the probability of a match between the two types of agents.

²⁷Under weak and strong competition, the results coincide just because we assumed that $p^b = \frac{e^*}{2}(c - \frac{t}{4})$ under strong competition.

We then discuss the equilibrium price pairs under different types of competition, while explicitly taking into account agents' participation constraints, based on their reservation values. Much like the results in the two-sided market literature, our results suggest that the greater the difference between the reservation values of different types, the greater is the asymmetry of pricing in equilibrium. However, the source of externality of our model differs from that of other models of two-sided markets. In our model, the market externality is associated with middlemen's demand functions, while, in other models in the two-sided market literature, the market externality is associated with indirect network effects.

Finally, we discuss the level of extra benefit that supports a specific market structure in equilibrium, given agents' reservation values. Specifically, we show the range of extra benefit when the difference between the reservation values of different types is small or large.

2.7 Appendix

Proof of Lemma 2.1.

Proof. Without loss generality, suppose that $x_1 \leq y_1$. In this case, the total demand faced by middleman 1 is given by $\min[x_1, y_1] = x_1$. If the middleman increases the fee for agents of type b , P_1^b , this induces less participation of agents of type b – that is, it induces a decrease in y_1 , but it still increases the middleman’s profit, since the total demand is unchanged as long as $x_1 \leq y_1$. Further, decrease in y_1 increases the expected matching surplus and \bar{e}_b , which induces more agents of type a to trade with the middleman – that is, it induces an increase in x_1 . Analogously, next assume that $x_1 \geq y_1$. Then the total demand is $\min[x_1, y_1] = y_1$. If the middleman increases the fee for agents of type a , P_1^a , this induces less participation of agents of type a but increases the middleman’s profit, as long as $x_1 \geq y_1$. Further, decrease in x_1 increases the expected matching surplus and \bar{e}_a , which induces more participation by agents of type b – that is, it increases in y_1 . Thus the middleman should balance both sides in equilibrium. \square

Proof of Proposition 2.2. (Strong Competition)

Proof. Consider the maximization problem of middlemen 1 under strong competition.

$$\max_{P_1^a, P_1^b} \Pi_1 = (P_1^a + P_1^b) \text{Min}\{x_1^*, y_1^*\}$$

where $x_1^* = \frac{(P_2^a - P_1^a) + (M_1^a - M_2^a) + \frac{3}{2}e_{a_2}^* t}{(e_{a_1}^* + e_{a_2}^*)t} - \frac{y_1}{2}$ and $y_1^* = \frac{(P_2^b - P_1^b) + (M_1^b - M_2^b) + \frac{3}{2}e_{b_2}^* t}{(e_{b_1}^* + e_{b_2}^*)t} - \frac{x_1}{2}$. Under strong competition, the maximization problem is constrained by the following constraint set.

$$M_1^a - e_{a_1}^* t x_1 - \frac{1}{2} e_{a_1}^* t y_1 - P_1^a \geq w_a, \quad \frac{e_{a_2}^*}{e_{a_1}^*} M_1^a + M_2^a - \left(1 + \frac{e_{a_2}^*}{e_{a_1}^*}\right) w_a - \frac{3}{2} e_{a_2}^* t \geq \frac{e_{a_2}^*}{e_{a_1}^*} P_1^a + P_2^a$$

$$M_1^b - \frac{1}{2} e_{b_1}^* t x_1 - e_{b_1}^* t y_1 - P_1^b \geq w_b, \quad \frac{e_{b_2}^*}{e_{b_1}^*} M_1^b + M_2^b - \left(1 + \frac{e_{b_2}^*}{e_{b_1}^*}\right) w_b - \frac{3}{2} e_{b_2}^* t \geq \frac{e_{b_2}^*}{e_{b_1}^*} P_1^b + P_2^b$$

By Lemma 2.1, since middleman i always balances two sides, we can substitute $P_1^b = P_1^a - P_2^a + P_2^b$ into a maximization problem. Then we construct the Lagrangian for middleman 1 to obtain a profit-maximizing pair of prices. After solving the Lagrangian, we get (P_1^{a*}, P_1^{b*}) :

- (1) $(p^a, 3e^*t - p^a)$ if $p^a \in [0, \frac{e^*}{2}(c - \frac{t}{4}) - w_a]$, $p^a \in [w_b - \frac{e^*}{2}(c - \frac{25}{4}t), 3e^*t]$
(2) $(0, p^b)$ if $p^b \in [3e^*t, \frac{e^*}{2}(c - \frac{t}{4}) - w_b]$
(3) $(p^a, 0)$ if $p^a \in [3e^*t, \frac{e^*}{2}(c - \frac{t}{4}) - w_a]$
(4) $(\frac{e^*}{2}(c - \frac{t}{4}) - w_a, \frac{e^*}{2}(c - \frac{t}{4}) - w_b)$ if $w_a + w_b \geq e^*(c - \frac{13}{4}t)$

where $w_k \leq \frac{e^*}{2}(c - \frac{t}{4})$

In order to confirm that these solutions are indeed equilibrium pairs of prices, we need to check whether middlemen have an incentive to deviate. One can easily show that middlemen here don't have such incentives. Due to symmetry, it is sufficient to present equilibrium pair of prices for middleman 1. Therefore, the derivations for middleman 2 are omitted in the rest of proofs. \square

Proof of Proposition 2.3. (Weak Competition)

Proof. Consider the profit maximization problem of middleman 1 under weak competition.

$$\max_{P_1^a, P_1^b} \Pi_1 = (P_1^a + P_1^b) \text{Min}\{x_1^*, y_1^*\}$$

where $x_1^* = \frac{(P_2^a - P_1^a) + (M_1^a - M_2^a) + \frac{3}{2}e_{a_2}^*t}{(e_{a_1}^* + e_{a_2}^*)t} - \frac{y_1}{2}$ and $y_1^* = \frac{M_1^b - w_b - P_1^b}{e_{b_1}^*t} - \frac{x_1}{2}$. Under weak competition, the maximization problem is constrained by the following constraint set.

$$M_1^a - e_{a_1}^*tx_1 - \frac{1}{2}e_{a_1}^*ty_1 - P_1^a \geq w_a, \quad \frac{e_{a_2}^*}{e_{a_1}^*}M_1^a + M_2^a - \left(1 + \frac{e_{a_2}^*}{e_{a_1}^*}\right)w_a - \frac{3}{2}e_{a_2}^*t \geq \frac{e_{a_2}^*}{e_{a_1}^*}P_1^a + P_2^a$$

$$M_1^b - \frac{1}{2}e_{b_1}^*tx_1 - e_{b_1}^*ty_1 - P_1^b \geq w_b, \quad \frac{e_{b_2}^*}{e_{b_1}^*}M_1^b + M_2^b - \left(1 + \frac{e_{b_2}^*}{e_{b_1}^*}\right)w_b - \frac{3}{2}e_{b_2}^*t \leq \frac{e_{b_2}^*}{e_{b_1}^*}P_1^b + P_2^b$$

By Lemma 2.1, since middleman i always balances two sides, we can substitute $P_1^b = M_1^b - w_b - \frac{3}{4}e_{b_2}^*t - \frac{1}{2}(P_2^a - P_1^a)$ into a maximization problem. Then we construct the Lagrangian for middleman 1 to obtain the profit-maximizing pair of prices. After solving the Lagrangian, we get (P_1^{a*}, P_1^{b*}) :

- (1) $(w_b - \frac{e^*}{2}(c - \frac{19}{4}t), \frac{e^*}{2}(c - \frac{t}{4}) - w_b)$ if $w_a + w_b \leq e^*(c - 4t)$, $w_b \in [\frac{e^*}{2}(c - \frac{19}{4}t), \frac{e^*}{2}(c - \frac{1}{4}t)]$
(2) $(w_b - \frac{e^*}{2}(c - \frac{19}{4}t), \frac{e^*}{2}(c - \frac{t}{4}) - w_b)$ if $w_a + w_b \geq e^*(c - 4t)$, $w_k \leq \frac{e^*}{2}(c - \frac{t}{4})$
(3) $(0, \frac{e^*}{2}(c - \frac{t}{4}) - w_b)$ if $w_a \leq \frac{e^*}{2}(c - \frac{t}{4})$, $w_b \leq \frac{e^*}{2}(c - \frac{19}{4}t)$

where $w_k \leq \frac{e^*}{2}(c - \frac{t}{4})$.

In order to confirm that these solutions are indeed equilibrium pairs of prices, we need to check whether middlemen have an incentive to deviate. One can easily show that middlemen here don't have such incentives. \square

Proof of Proposition 2.4. (Local Monopoly)

Proof. Consider the profit maximization problem of middleman 1 under local competition.

$$\max_{P_1^a, P_1^b} \Pi_1 = (P_1^a + P_1^b) \text{Min}\{x_1^*, y_1^*\}$$

where $x_1^* = \frac{M_1^a - w_a - P_1^a}{e_{a_1}^* t} - \frac{y_1}{2}$ and $y_1^* = \frac{M_1^b - w_b - P_1^b}{e_{b_1}^* t} - \frac{x_1}{2}$. Under local monopoly, the maximization problem is constrained by the following constraint set.

$$M_1^a - e_{a_1}^* t x_1 - \frac{1}{2} e_{a_1}^* t y_1 - P_1^a \geq w_a, \quad \frac{e_{a_2}^*}{e_{a_1}^*} M_1^a + M_2^a - \left(1 + \frac{e_{a_2}^*}{e_{a_1}^*}\right) w_a - \frac{3}{2} e_{a_2}^* t \leq \frac{e_{a_2}^*}{e_{a_1}^*} P_1^a + P_2^a$$

$$M_1^b - \frac{1}{2} e_{b_1}^* t x_1 - e_{b_1}^* t y_1 - P_1^b \geq w_b, \quad \frac{e_{b_2}^*}{e_{b_1}^*} M_1^b + M_2^b - \left(1 + \frac{e_{b_2}^*}{e_{b_1}^*}\right) w_b - \frac{3}{2} e_{b_2}^* t \leq \frac{e_{b_2}^*}{e_{b_1}^*} P_1^b + P_2^b$$

By Lemma 2.1, since middleman i always balances two sides, we can substitute $P_1^b = w_a - w_b + P_1^a$ into a maximization problem. Then we construct the Lagrangian for middleman 1 to obtain the profit-maximizing pair of prices. After solving the Lagrangian, we get (P_1^{a*}, P_1^{b*}) :

- | | |
|--|--|
| (1) $(\frac{e^*}{2}(c - \frac{t}{4}) - w_a, \frac{e^*}{2}(c - \frac{t}{4}) - w_b)$ | if $w_k \leq \frac{e^*}{2}(c - \frac{t}{4}), w_a + w_b \leq e^*(c - \frac{7}{4}t)$ |
| (2) $(\frac{M^*}{2} + \frac{1}{4}(w_b - 3w_a), \frac{M^*}{2} + \frac{1}{4}(w_a - 3w_b))$ | if $w_a + w_b \geq 2M^* - 3e^*t, 3w_b - 2M^* \leq w_a \leq \frac{2M^* + w_b}{3}$ |
| (3) $(0, w_a - w_b)$ | if $w_a \geq \frac{2M^* + w_b}{3}, w_a \geq M^* - \frac{3}{4}e^*t$ |
| (4) $(w_b - w_a, 0)$ | if $w_b \geq \frac{2M^* + w_a}{3}, w_b \geq M^* - \frac{3}{4}e^*t$ |

In order to confirm that these solutions are indeed equilibrium pairs of prices, we need to check whether middlemen have an incentive to deviate. One can easily show that middlemen here don't have such incentives. \square

Proofs of Proposition 2.5 and 2.6

Proof. The result of Proposition 2.5 can be derived by using the results of first part of Propositions 2.2, and 2.4. Given c , t , $w_a = w_b = w^*$, and $x^* = y^* = \frac{1}{2}$, $e^* = \frac{B^2}{(c-v+t/2)^2} \left[\frac{c-t/4}{c} \right]$, we obtain the results stated in Proposition 2.5 by rearranging terms. The result of Proposition 2.6 can be derived in similar way. By using the results of (2)(b) of Proposition 2.2 and 2.3, and (3) of Proposition 2.4, we rearrange terms and obtain the results stated in Proposition 2.6. \square

Chapter 3

Endogenous Platform Differentiation under Bertrand Competition

3.1 Introduction

Many economic transactions and social interactions among different groups of agents involve intermediation services provided by firms and intermediaries. An intermediary is an economic agent who provides assistance to agents in meeting their needs and transactions. As Spulber (1999) points out, intermediaries often engage in matchmaking by allocating agents to coordinate transactions. For instance, dating and marriage matchmakers (e.g., match.com) provide intermediation services between men and women. Other examples of intermediaries include job agencies, where match job seekers with employers; real-estate agencies, who facilitate transactions between property-owners and buyers; payment card systems with merchants and cardholders; shopping malls with stores and shoppers, and so on.

In the two-sided market literature, intermediaries are often referred to as platforms. The focus of the literature is on platforms' pricing decisions in competitive environments when the market exhibits indirect network effects. In two-sided markets, indirect network effects are present because an agent's utility from joining a platform increases when more agents from the other side of the market join the same platform. Indeed, indirect network effects imply that the more agents on each side, the more potential transactions between merchants and buyers in the payment card

systems; the higher chances of successful deal in the real-estate markets; and the better matches in the marriage markets. When making pricing decisions, the platforms need to internalize indirect network effects.

Since Caillaud and Jullien (2001, 2003), Rochet and Tirole (2003, 2006) and Armstrong (2006), it is well understood that a group with less willingness-to-pay may be subsidized in two-sided markets because platforms use the ‘*divide-and-conquer strategy*’ to deal with the ‘*chicken-and-egg*’ problem: For the success of its business, a platform has to assure one group that the other group will join the same platform. The platform achieves its goal by subsidizing one group (*divide*) and charging a positive fee to the other group (*conquer*). The subsidy is given to the group with less willingness-to-pay.

While indirect network effects may be well suited to describe some two-sided markets where two distinct groups care only about the number of people on the other side of the same platform (e.g., payment card systems, videogame platforms, yellow pages). However, in other two-sided markets (e.g., dating and marriage matchmaking), there may be some people in each group who want to interact with a certain type of people on the other side. In other words, some agents in two-sided markets may have desirability concerns about agents on the other side. These concerns may affect agents’ decisions in addition to indirect network effects. The distribution of agents among platforms will be affected by both the desirability concerns and indirect network effects. Indeed, it implies that a platform’s perceived quality might be endogenously determined by hosting a certain type of agent.

The following two examples will illustrate some situations of two-sided markets that we have in mind. First, consider the medieval history of Europe. Matchmakers played an important role in arranging marriages at those times. As discussed above, more men would provide a better chance of successful matching for women, and vice versa if both groups join the same matchmaker. Further, there might be some type of people (e.g., royal families) who often regarded marriage as a tool of maintaining their status. From their point of view, the desirability concerns (e.g., wealth and power of the potential spouses) mattered as well. When making pricing decisions in such a circumstance, matchmakers had to take account of both the number of people and the type of people in each

group.¹

The second example is related to tax-competition between two local jurisdictions which levy lump-sum taxes from their residents. Consider two groups with two different types (e.g., skilled and unskilled workers, and high- and low-tech firms). And suppose they need to choose one of the places to get jobs or hire workers. In this case, the two jurisdictions can be regarded as revenue-maximizing marketplaces which compete for hosting more workers and firms. If the skilled workers have a higher chance of being matched with the high-tech firms, they have incentives to join a marketplace with more high-tech firms. The same reasoning holds for the high-tech firms. With the existing models in the two-sided market literature, both examples cannot be well addressed because they do not explicitly take into account both forces, indirect network effects and the desirability concerns.

The main purpose of this chapter is to explore the possibility of endogenous market segmentation in two-sided markets. In our model, price-setting by platforms endogenously determines the perceived quality of platforms: A platform with a higher proportion of high type agents is regarded a platform with better quality. This opens a possibility that *ex-ante* identical platforms differentiate themselves when they charge different prices. Essentially the endogenous differentiation comes from agents' expectations about the proportion of a certain type of agent on each side. The signaling role of price plays an important role in coordinating agents' expectations.

For example, when a platform sets a higher pair of prices in order to host *high* types only, the agents form expectations that only high types of both groups will join that platform. And given this, *low* types will divert themselves into the other platform if it charges less. Under reasonable conditions, the agents' expectations will be fulfilled, and the two platforms may coexist at equilibrium. Therefore the market is divided by two *ex-ante* identical platforms and, after all, the platforms differentiate themselves by only hosting a certain type of agent: Endogenous market segmentation arises. The underlying driving force that affects the platforms' pricing decisions stems from an agent's valuation of the desirability of a certain type of agent from the other side relative to market size. In fact, the monopoly benchmark intuitively illustrates this point. If high types on both sides have high enough willingness-to-pay for the perceived quality, the monopoly platform

¹“Careful selection of a spouse was important to maintain the royal status of a family: Depending on the law of the land in question, if a prince or king was to marry a commoner who had no royal blood, even if the first-born was acknowledged as a son of a sovereign, he might not be able to claim any of the royal status of his father. Throughout history, members of a royal family who are not granted a royal title rarely have much power.” *from Wikipedia*.

will charge high prices on both sides and therefore only host high types. Otherwise it will charge low prices and serve all agents to maximize its profit.

This chapter contributes to the literature on two-sided markets. Caillaud and Jullien (2001, 2003), Rochet and Tirole (2003, 2006) and Armstrong (2006) share the basic idea of two-sided markets that explains how the presence of indirect network effects may alter platforms' pricing decisions. Other works in this literature apply to specific industries.² While the prior work provides a better understanding of two-sided markets, there have been few considerations regarding market segmentation in two-sided markets. The few exceptions are Bloch and Ryder (2000), Gabszewicz and Wauthy (2004), Damiano and Li (2007, 2008), and Ambrus and Argenziano (2009).

Bloch and Ryder (2000) consider a two-sided search model to show that, if a matchmaker charges a uniform participation fee, only agents of higher quality participate in the matching procedure. Damiano and Li (2007, 2008) also consider a similar matchmaker's problem when agents' types are unobservable. Though these three papers share the same idea with ours, the signaling role of prices, they do not take into account the indirect network externalities - which is of importance in this chapter. Ambrus and Argenziano (2009) study platforms' pricing decisions in a two-sided market when two different types of agents are present. Their consideration of agent heterogeneity differs from ours in that, in their work, each type has a different valuation of (indirect) network size. However, in our model, each type has a different valuation of the desirability of a certain type of agent. Finally, Gabszewicz and Wauthy (2004) provide an explanation how platforms can differentiate themselves when the expected quality of the platform by one group can be endogenously determined by the size of the network on the other side of the market. Their model is suited well for their study of exhibition centers. However, their model blurs the picture when applied to our examples (e.g., matchmakers or local jurisdictions).

In this regard, the present chapter complements the previous works in the two-sided market literature. The most important differences between the present chapter and the existing literature are the consequence of price competition and the equilibrium distribution of agents that may arise. In particular, in contrast to Caillaud and Jullien (2003), we show the possibility that, in a dominant platform equilibrium, the market may not be fully covered in equilibrium. It happens when the

²See, for example, Roson (2005) and references therein.

dominant platform charges a relatively high membership fee. It is also shown that two platforms with different perceived quality may coexist in equilibrium. In other words, endogenous market segmentation by *ex-ante* identical platforms may arise as an equilibrium outcome. This kind of equilibrium occurs when one platform charges a high membership fee and the other charges a low fee.

The chapter is organized as follows. In Section 3.2, we set up the model. In Section 3.3, we consider the case of the monopoly platform. In Section 3.4, we study platform competition in a duopoly. In Section 3.5, we conclude.

3.2 Model

We build on the framework of Caillaud and Jullien (2001, 2003). However we introduce an explicit utility functional form in order to incorporate endogenous vertical differentiation into our model. The model features two platforms indexed by $i \in \{A, B\}$, which intermediate two distinct groups of agents indexed by $k \in \{1, 2\}$. Each agent group consists of two types indexed by $t \in \{H, L\}$. For convenience, we denote the H -type agents from group k by H_k and the L -type agents from group k by L_k . Each platform provides intermediation services between two groups and charges membership fees for its services which are denoted by P_i^k . This intermediation conveys two characteristics for each side of the market: (1) The perceived quality of platforms (*endogenous quality*), and (2) the number of agents on the other side of the same platform (*indirect network effects*). Once the platforms charge membership fees for their services, then the perceived quality of each platform is endogenously determined by hosting a specific type of agent. Throughout this chapter we assume that an agent from each group can find a match from the other group *only* via platforms.

Agents' Utility Functions

Each side $k \in \{1, 2\}$ has N agents who may be of two types: High type (H_k) with measure $x \in (0, 1)$ and low type (L_k) with measure $(1 - x)$. The utility U_k^t that t -type agents on side k enjoys depends on the perceived quality of platform i by a group k , q_i^k , and the total number of agents on the other side of the platform, N_i^{-k} . The quality, q_i^k , is determined by the number of high type relative to the total number of agents from the other side joining the same platform. Within each group, the

only difference between two types of agents is their willingness to pay for the perceived quality, denoted by θ_t . It is also assumed that both types of agents within a group, H_k and L_k , have the same valuations of indirect network effects. The net utility that an agent can enjoy from joining platform i has the following form:³

$$U_k^t = \theta_t q_i^k + \alpha N_i^{-k} - P_i^k \text{ for } t \in \{H, L\} \text{ and } k \in \{1, 2\} \quad (3.1)$$

In particular, the quality of platform i perceived by a group k takes value $q_i^k = 0$ when the platform only serves low types. And it becomes $q_i^k = 1$ when platform i only hosts high types. If the platform serves both types, the perceived platform's quality is given by $q_i^k = x$. And the indirect network parameter, α , can be interpreted as the benefit from interacting with the other side, and we assume that $\alpha = 1$.⁴

Throughout this chapter, we only consider a case with exclusive intermediation services. In other words, a single-homing is only considered in the present chapter.⁵ Therefore, we can define the set of choices available to k -agents as follows:

$$S_k \in \{A, B\}$$

This means k -agents can choose either platform A or B based on their expected utility from joining a platform. Let $D = \{N_k^A, N_k^B\}_{k=1,2}$ denote the distribution of agents across the platforms with N_k^i being the number of k -agents who register with platform i .

For the rest of our analysis, we impose some restrictions on the parameters. For willingness to pay for quality θ_t , we assume L -types have willingness to pay such that $\theta_L^k \in (0, 1)$. For H -types, we assume their willingness to pay, θ_H^k , is greater than one and proportional to θ_L^k such that $\theta_H^k = a\theta_L^k$ where $a > \frac{1}{\theta_L^k}$. The assumptions above describe a situation where low types put more weight on the network size or choice variety of the other side rather than the expected gains from

³The utility functional form about product quality was first introduced by Mussa and Rosen (1978).

⁴In Ambrus and Argenziano (2009), their distinction between two types of agents only refers to the reservation value of consumers. For example, given the network size of the other side, high type agents have more willingness to pay than low types. However, in this chapter, we explicitly consider the quality in terms of the number of high types on the other side of the platform.

⁵The term 'single-homing' is often used in the two-sided market literature. It means the an agent can choose at most one platform. For example, a reader subscribes to at most one newspaper.

the perceived quality. In contrast, high types care more about the perceived quality of platforms in terms of the number of high type agents on the other side. For tractability, we assume that each group has symmetric preferences, $U_1^t = U_2^t$, in the duopoly case. However, in the monopoly case, we will relax this assumption.

Platforms

There are two ex-ante identical platforms, $i \in \{A, B\}$, operating in a market. Platforms can only charge membership fees, $P = \{(P_i^1, P_i^2)\}_{i=A, B}$, to each side of the market. The underlying reason is that we consider a situation where transactions do not give rise to physical or monetary exchanges. In pure informational intermediation or pure matching, transaction fees are difficult to implement. When transactions do not involve trade and monetary exchange, they may also be difficult or costly to monitor, or else it may be costly to prove that they actually took place through the platforms' intermediation services. We assume that the platforms can not discriminate in prices within a group and compete *à la* Bertrand. However, we leave open the possibility where pricing could differ between groups. It is important to notice that we do not restrict registration prices to be nonnegative. In fact, one group can get subsidized from the platforms whenever profitable.⁶

Agents will join neither platform if they expect to obtain a negative surplus. We thus restrict ourselves to prices such that an

$$\text{agent's participation constraint: } \theta_t q_i^k + N_i^{-k} - P_i^k \geq 0$$

holds. For a given platform i , we will focus on the pricing decision by platform i , $P_i = (P_i^1, P_i^2)$ under the agent's participation constraint. The profits of platform i are given by

$$\Pi_i = \sum_{k=1,2} P_i^k N_i^k(P_i, P_{-i}) \tag{3.2}$$

⁶Subsidizing or tipping one group is quite well known feature in the two-sided market literature. See Caillaud and Jullien (2001, 2003), Rochet and Tirole (2003, 2006) and Armstrong (2006), for example.

Timing and Equilibrium

The timing of the game is as follows. In the first stage, both platforms set prices P_i simultaneously and non-cooperatively. The resulting price system becomes common knowledge to the agents. In the second stage, the agents simultaneously choose which platform to join, yielding the distribution of the agents across the platforms.

Definition 3.1. (Caillaud and Jullien (2003)) A distribution of agents D is an *equilibrium distribution* for a price system P if for all $s \in S_k$:

$$U_k^t(P, s, D) \geq U_k^t(P, s', D).$$

A market allocation is a mapping $D(\cdot)$ that associates to each price P an equilibrium distribution of agents $D(P)$.

In words, if some k -agent registers with i , then he must be as well off as if he registered instead with the other platform $j \neq i$. Note that, as a function of prices $P = (P_i, P_j)$, $N_i^1(P) + N_i^2(P)$ determines the total demand faced by the platform i . There can be multiple market allocations but we only study some cases where agents play pure strategies.⁷ Here it is worth to mention that prices are viewed as a signal of quality by agents. In our model, the “quality” of the intermediation service depends on the number of high type agents relative to the total number of agents on board. Therefore each agent expects that a platform has low quality if it charges relatively low prices. This is not the case in Caillaud and Jullien (2003). A market allocation $D(\cdot)$ generates a price-setting game among platforms, where payoff functions are given by $\Pi_i(P, D(P))$. An equilibrium is then defined as follows:

Definition 3.2. (Caillaud and Jullien (2001)) An equilibrium is a pair $(P^*, D(\cdot))$ where (i) $D(\cdot)$ is a market allocation and (ii) P^* is a Nash equilibrium of the pricing game induced by the market allocation $D(\cdot)$.

Under this definition, an equilibrium describes how agents choose among the platforms at the second stage of the game and presents a set of prices charged by platforms. The allocation of the

⁷In markets with network externalities, the possibility of the existence of multiple equilibrium is quite common. For more discussion, see Farrell and Saloner (1985).

agents corresponds to a demand addressed to each platform. Once demand is characterized, the first stage amounts to a classical Bertrand price setting game.

As discussed in Caillaud and Jullien (2001, 2003), it is convenient to interpret this equilibrium concept as a rational expectation equilibrium where, following the choice of a price system P , each agent has an expectation about how all agents will allocate among the different platforms, and, in equilibrium, the expectations of all agents are common and fulfilled. In particular, this interpretation helps us to explain why, for a given price system, there may exist several market allocations. In a dominant platform equilibrium where only one platform is active, it will be argued that common pessimistic beliefs about the number of users registering with one intermediary may indeed prevent any agent from registering with this intermediary, thereby justifying these pessimistic beliefs. However, in a case where both platforms have some positive market share, we can not use this argument. Instead we interpret price as a signal of quality of platform.

3.3 Monopoly Platform

As a benchmark, we first consider the case of a monopoly platform. For the sake of generality, we relax our assumption that θ_H is the same across groups. We denote the quality coefficient for group 2 by γ_t where $t \in \{H, L\}$: For H_2 , $\gamma_H = b \cdot \gamma_L$ where $b > 1/\gamma_L$. We assume that b is proportional to a such that $b = a \cdot c$ where $c > 0$.⁸ However, we still assume that $\theta_L = \gamma_L$. Therefore, group 2 agents have the following utility function:

$$U_2^t = \gamma_t q_i^k + N_i^{-k} - P_i^k$$

We assume that the other model components presented in Section 3.2 remain the same. The monopoly platform i decides membership fees, $P = (P^1, P^2)$, which become common knowledge for agents on both sides of the market. Agents of both groups then simultaneously decide whether to join the platform or not. Note that, due to a common knowledge of prices, a modification of P_i may affect group 1 agents' expectations on the decisions of group 2 agents, and vice versa.⁹

⁸If $c \geq 1$, then $b \geq a$ and it implies $\theta_H \leq \gamma_H$. And if $1 > c > 0$, then $b < a$ and implies $\theta_H > \gamma_H$.

⁹This is what is called the 'Chicken & Egg' problem in the two-sided market literature. To serve one side, platform has to serve the other side, and vice versa.

The market allocation for group 1 agents is determined by both their beliefs about group 2 agents' allocation, N_i^2 , and their perceived platform quality. More generally, t -type agents from group k will join platform i only if:

$$\begin{cases} U_1^t = \theta_t q_i^k + N_i^{-k} - P_i^k \geq 0 \text{ for } t \in \{H, L\} \\ U_2^t = \gamma_t q_i^k + N_i^{-k} - P_i^k \geq 0 \text{ for } t \in \{H, L\} \end{cases} \quad (3.3)$$

In equilibrium, the agents' expectations should be correct. For any prices such that $U_k^t \geq 0$, there always exists a market allocation with either $N_i^{-k} = xN$ when only high types are served or with $N_i^{-k} = N$ when both types are served. Then the perceived quality of platform i by the other side is endogenously determined: $q_i^k = 1$ and $q_i^k = x$, respectively. The monopoly platform which provides intermediation services seeks to maximize its profit by either hosting more agents from both sides or charging higher prices only for high types. In other words, the platform faces a trade-off between the network size and the perceived quality because a larger network size ruins the perceived quality of platform.

Proposition 3.1. *Let $A_1 \equiv 2(1-x^2)\frac{N}{\theta_L} + 2x$, $A_2 \equiv 2(1-x)x\frac{N}{\theta_L} + 1$ and suppose there is a monopoly platform in the market. Then:*

(1) *If $a \geq \max[\frac{A_1}{x(1+c)}, \frac{A_2}{cx+(1-x)x}, \frac{A_2}{cx(1-x)+x}]$ then the platform charges $(P^1, P^2) = (\theta_H + xN, \gamma_H + xN)$ and only hosts H -type agents from both sides.*

(2) *If $a \in [\frac{A_1-A_2}{cx^2}, \frac{A_2}{cx(1-x)+x}]$ and $c > 1$ then the platform serves all agents of group 1 by charging $P^1 = \theta_L + xN$ and only H -types from group 2 by charging $P^2 = x\gamma_H + N$.*

(3) *If $a \in [\frac{A_1-A_2}{x^2}, \frac{A_2}{cx(1-x)+x}]$ and $c \in (0, 1]$ then the platform serves only H -types of group 1 by charging $P^1 = x\theta_H + N$ and all agents of group 2 by charging $P^2 = \theta_L + xN$.*

(4) *If $a < \min[\frac{A_1}{x(1+c)}, \frac{A_1-A_2}{x^2}, \frac{A_1-A_2}{cx^2}]$ then the platform serves the entire market by charging $(P^1, P^2) = (x\theta_L + N, x\gamma_L + N)$.*

Proof. See the Appendix. □

When both θ_H and γ_H are high enough, the profit-maximizing monopoly platform sets higher prices in order to serve only high types. In this case, the monopoly platform extracts the entire utilities from H -types. However, if the market size is large enough relative to both θ_H and γ_H ,

the platform will try to cover the entire market by charging $P^k = x\theta_L + N = x\gamma_L + N$. Therefore, H -types can enjoy positive net utilities ($U_k^H > 0$) whereas L -types obtain zero utility from joining the platform. In cases in between, the monopoly platform targets H -types on one side and both types on the other side.

Similar results can be found in Bloch and Ryder (2000), Damiano and Li (2005), and Ambrus and Argenziano (2009). Bloch and Ryder (2000) consider a two-sided search model to show that if a matchmaker charges a uniform participation fee, only agents of higher quality participate in the matching procedure. Damiano and Li (2005) consider participants who care about the identities of other participants. However, neither paper takes into account the indirect network externality which is of importance in this chapter. In contrast to those two papers, Ambrus and Argenziano (2009) study the pricing decision of a monopoly network provider when two different type of agents exist. The main difference between Ambrus and Argenziano (2009) and our work is that, in their study, two types of agents are different because each type has different weight on the indirect network effect, which is exogenously given. In contrast, in our study, each type of agent has different valuation of the perceived quality, which is endogenously determined.

3.4 Price Competition between Platforms

In this section we assume that the intermediation services are exclusive. In other words, the agents are allowed to join at most one platform, either A or B . This restriction applies to situations where agents cannot join both platforms at the same time due to physical, contractual or legal limitations. For example, both firms and workers may choose at most one city for their workplace.

The timing of the game is the following. In the first stage each platform sets a pair of prices simultaneously and noncooperatively. In the second stage the agents observe prices, infer the quality of each platform and decide which platform to join, if any. We solve this game by backward induction. In the second stage, after observing prices announced by the platforms, every agent has to decide whether to join or not, then choose either A or B if they decide to join. This results in an equilibrium agents distribution, $D(P)$, for a corresponding price system P . Next, in the first stage, we study the platforms' pricing decisions under some possible equilibrium agents distributions.

In the following discussion, first we will search for some conditions which constitute Nash Equilibrium in the second stage of the game. Because there could be many possible equilibrium agent distributions, we will characterize some of them where agents play pure strategies. Next, under the conditions from stage 2, we will discuss possible market outcomes when two platforms compete in prices. In fact, we will show that there are two forms of market outcomes: (1) A dominant platform equilibrium where only one platform is active even though there are no fixed costs of entry for the other platform, and (2) a market segmentation equilibrium where both platforms have positive market shares. In the remainder of this chapter, for tractability, we assume that each group has a symmetric preference such that $U_1^t = U_2^t$ where $t \in \{H, L\}$.

3.4.1 Distribution of Agents: The Second Stage

At the second stage of the game, each agent decides whether to join platform i or j according to the agent's payoff given by (3.1). We assume that the strategy of not participating in any platform yields zero payoff for the agents. For any given price system P , t -type agents from group k will join platform i if and only if:

$$U_k^t(i; P) \geq \max\{U_k^t(j; P), 0\} \text{ for } j \neq i \quad (3.4)$$

The agents may hence face one of following possible situations: (1) A single active platform, and (2) two active platforms. As in the monopoly platform, a single active platform may serve the entire market or only high types according to prices charged by the platforms. Also it is possible that a single platform only hosts H -types from one side and both types from the other. We will discuss next whether these configurations are sustainable or not. In case of two active platforms, three possible outcomes may arise: (1) Both platforms serve both types and share the market equally, (2) both platforms serve only high type of agent and share the market equally, and (3) one platform only serves H -types and the other serves L -types. In what follows, we discuss in detail more under what conditions these possible outcomes may arise.

It is worthwhile to note that the market allocation suggested above is about agents' decisions according to prices announced by the platforms. Therefore, we only care about the equilibrium distribution of the agents from the second stage. In other words, we will find platform i 's pricing

decision under a equilibrium distribution condition. Platform j 's possibility of entering the market will be discussed below.

3.4.2 Dominant Platform Equilibrium: Single Active Platform

For the existence of a dominant platform equilibrium, there must be a price system which supports such market allocations even if there are two potential platforms in the market. In this section, we examine which of the three possible agents distributions can be supported as an equilibrium outcome. A single dominant platform equilibrium can arise if there exists no pricing strategy that allows platform j to capture a positive market share on both groups at non-negative profits. And from Definition 3.2, an equilibrium price system must be sustained by a specific market allocation. In other words, a dominant platform equilibrium is sustainable as long as k -agents have a “bad expectation” market allocation against platform j such that $N_j^k = 0$ and $N_i^k > 0$. “Bad expectations” are formed if every agent expects not to get as high utility from platform j as from platform i . As long as all agents have such expectations, they will only choose platform i since in equilibrium expectations are common and fulfilled.¹⁰ For platform j , in order to escape from the curse of “pessimistic expectation”, it has to give benefits to some of agents on platform i to change their expectations against platform j . However, in equilibrium, such an attempt by platform j will not be successful if a dominant platform equilibrium exists. That is, even after any deviation in prices P'_j by platform j , agents still coordinate on an equilibrium distribution with zero market share for j , $N_j^k(P_i, P'_j) = 0$.

Divide and Conquer Strategy¹¹

Intuitively, many of our results can be understood by platform j 's pricing strategy, which is called the “Divide and Conquer Strategy” (hereafter DC-strategy). In response to a price P_i charged by platform i , platform j has to use a pricing strategy which enables it to escape from the resulting market allocation against platform j . That is, platform j has to change pessimistic beliefs against j for some agents at platform i by offering a good price to some agents on platform i . Note, first, that for a price system $P = (P_A^1, P_A^2, P_B^1, P_B^2)$, there exists a “bad expectation” market allocation

¹⁰This argument was first used by Caillaud and Jullien (2003) in the two-sided market literature.

¹¹For more general discussions, see Caillaud and Jullien (2001, 2003). This paragraph is a summary.

against j , that is, $N_j^k(P) = 0$ and $N_i^k(P) > 0$, as long as:

$$\theta_t q_i^k + N_i^{-k} - P_i^k \geq -P_j^k$$

Under the pessimistic belief about platform j , there is no way for j to gain a positive market share by charging membership fee such that the above condition holds for both $k = 1, 2$, since every agent expects the others to join platform i rather than platform j . Therefore, under the pessimistic belief, platform j must adopt the DC-strategy to get a positive market share. In other words, it has to subsidize one group of agents (*divide*) and charge a positive fee to the other group of agents in order to extract benefits from them (*conquer*).

Let us explain why. First, platform j must target one group at platform i , say group-1, and subsidize them so that the agents can obtain more utilities from joining platform j rather than platform i . That is, platform j tries to serve group-1 agents by subsidizing them by $P_j^1 + \varepsilon_j^1$ and the following holds¹²:

$$-P_j^1 = \theta_t q_i^t + N_i^{-k} - P_i^k + \varepsilon_j^1$$

Then, the distribution of group-1 agents is changed to $N_j^1 > 0$, and platform j can escape the curse of pessimistic agents' beliefs and, therefore, it has market power over the group-2 agents. Now it charges a positive membership fee P_j^2 to group-2 agents and will try to make up losses from group-1 agents under the following condition:

$$U_2^t = \theta_t q_j^t + N_j^1 - P_j^2 \geq 0$$

Since group-2 agents now rationally expect the agents from group 1 to join platform j , they will also join platform j as long as they can enjoy a positive benefit. Therefore, platform j can choose a membership fee for agents from group 2 under this constraint. From platform i 's perspective, in order to deter platform j 's entry in the market, platform i 's pricing strategy must be designed so that no such DC-strategy for platform j is profitable. Throughout the rest of this chapter, we often use the DC-strategy to identify the existence of equilibrium. The next two propositions summarize results when only a single platform is active in the market.

¹² ε_j^k denotes a subsidy to group k by platform j .

Market is Fully Covered

In this case, both groups are served by a single platform i because the agents of both groups form pessimistic expectations against platform j . Since the platform hosts both types of the agents, the market demand of each side faced by platform i is N and the perceived quality is given by $q_i^k = x$. This distribution is sustainable if a price system satisfies a condition such that:

$$x\theta_L + N - P_i^k \geq -P_j^k \quad (3.5)$$

In other words, every agent has a “bad expectation” market allocation against platform j . Under this constraint, if there exists a pricing strategy for platform i to deter platform j ’s entry in the market, a dominant platform equilibrium exists.

Proposition 3.2. *There exists a dominant platform equilibrium in which only one platform survives and serves the entire market when charging $P_i^k = \theta_L x + N$ to one group and $P_i^{-k} = -\varepsilon$ to the other. The resulting profit is $\Pi_i = (x\theta_L + N)N$ and $\Pi_j = 0$.*

Proof. See the Appendix. □

Proposition 3.2 mirrors the result of Caillaud and Jullien (2001, 2003). To deter a deviation from the equilibrium price, platform i has to set prices such that the DC-strategy is not profitable for platform j . One outcome is that one group is subsidized and the other pay a positive membership fee, $x\theta_L + N$. In this case, the group with subsidy enjoys the maximum utility whereas the other group obtain zero utility from joining the platform i . It is also important to note that, by using price as an instrument, a platform can deter the other’s entry even though no fixed cost is considered in the model.¹³

¹³This result is quite similar to Sutton’s theory for market structure and concentration as shaped by endogenous barriers to entry. An industry with endogenous sunk cost (ESC) is one in which the strategic decision whether to sink certain costs is key to determining the firm’s competitive position. In consumer product industries, the ESC are marketing effort to build brand names, generally called “advertising” although they obviously include other activities. The competitive benefit delivered by these ESC is brand preference by consumers. For the overview of Sutton’s theory, we refer to Bresnahan (1992) who provides a good summary of Sutton’s endogenous sunk cost theory.

Market is Not Fully Covered

Even though the market is not fully covered by a single platform, there is still a possibility that only one platform is active in the market by deterring the other's entry. In the following, we discuss a situation where the market is not fully covered by a single active platform. We'll first show that when a single active platform tries to serve H -types only, an equilibrium does not exist. However, it is still possible that a single active platform does not cover the entire market in equilibrium: The platform serves all agents from one group and high type agents from the other.

First, suppose that H -types have pessimistic expectations against platform j and L -types are indifferent between two platforms (e.g., they only get zero utility from joining either platform). If platform i charges a price so as to host only H -type agents from both groups, the market demands faced by platform i are given by xN , and the the perceived quality is $q_i^k = 1$. This agents' distribution is sustainable if there is a price system such that $\theta_H + xN - P_i^k > -P_j^k$, $P_i^k \in (\theta_L + xN, \theta_H + xN]$ and L -types decide not to join both platforms. However, such price system does not exist. Let us explain why. Suppose, for example, L -type agents from group 1 (L_1) are subsidized with ε by platform j . Then, it means that $N_j^1 = (1 - x)N > 0$. Now, platform j can exercise a market power over L -type agents from group 2 (L_2) and make a positive profit. Therefore, there is no equilibrium price system which allows a single platform to be active in the market by serving H -types only.

Now, suppose that platform i only hosts H -type of agent from group 1 (H_1) and both type of agents from group 2 (H_2 and L_2). It happens in a case where both groups have bad expectations against platform j and platform i decides to charge a higher price on group 1. The corresponding market demand will be $N_i^1 = xN$ and $N_i^2 = N$, and the quality perceived by k -agents will be $q_i^1 = x$ and $q_i^2 = 1$, respectively. More precisely, this distribution can be sustained if there is a price system such that:

$$\begin{cases} x\theta_H + N - P_i^1 > -P_j^1 \text{ and } P_i^1 \in (x\theta_L + N, x\theta_H + N] \\ \theta_L + xN - P_j^2 > -P_j^2 \text{ and } P_i^2 \in (-\infty, \theta_L + xN] \end{cases} \quad (3.6)$$

Under this constraint, if there exists a pricing strategy for platform i to deter platform j 's

entry in the market, a dominant platform equilibrium exists. In fact, we can think of two possible strategies for platform j : (1) The DC-strategy where platform j subsidizes either group 1 or group 2, and charges unsubsidized group to recover its loss, and (2) a 'market niche' strategy where platform j takes unserved agent (e.g., L -type agents of group 1, L_1) and charges a price to make profits from group 2. However, as shown below, platform j can not make a positive profit when platform i can make enough profit from H -type agents of group 1 (H_1). In fact, under these circumstances, platform i can set a pair of prices such that both strategies are not profitable for platform j .

Proposition 3.3. *Suppose a market is not fully covered by a single platform (for example, L -types from group-1 are not served). Let $x^* \equiv \max \left[\frac{1}{2} \left(1 - \frac{\theta_L}{N} \right), \frac{1}{\sqrt{a}} \right]$ and $x_* \equiv \min \left[\frac{1}{2} \left(1 - \frac{\theta_L}{N} \right), \frac{1}{\sqrt{a}} \right]$. Then:*

(1) *If $a > 1/x^2$ and $x \in [x^*, 1)$ then there exists a dominant-platform equilibrium where platform i sets prices $P_i^1 = (\theta_L + xN)/x$ and $P_i^2 = -\varepsilon$ and platform j is inactive. The resulting profit is $\Pi_i = (\theta_L + xN)N$ and $\Pi_j = 0$*

(2) *If $a < 1/x^2$ and $x \in (0, x_*)$ then a dominant-platform equilibrium does not exist.*

Proof. See the Appendix. □

This result is explained as follows. First, note that platform i has to subsidize group 2 in order for platform j not to use the market niche strategy. Because if platform j decides to subsidize unserved agents (i.e. L -types from group 1, L_1), it can have L -type agents with $N_j^1 = (1 - x)N$ and have some market power over group 2. To make such a strategy unsuccessful for platform j , platform i has to subsidize group 2 with $-P_i^2 = (1 - x)N - P_j^2 + \varepsilon_i^2$. Then, platform i can charge a higher membership fee to H -type agents from group-1. This allows platform i to recover its loss from group 2 and make a profit. However, platform i 's pricing ability over H -types from group 1 is constrained by the DC-strategy. Besides the market niche strategy, if θ_H is high enough, platform j has an incentive to use the DC-strategy by subsidizing group 2 and make a profit from H -types from group 1 (H_1). Therefore, platform i has to set a price such that the DC-strategy is not profitable for platform j . In sum, when θ_H is high enough, platform i can set a price to make a profit so that the market niche strategy is not profitable for platform j . However, it cannot extract

all utilities from H_1 because of the DC-strategy. On the other hand, if θ_H is not high enough, platform i 's ability to increase P_i^1 is limited and, in fact, there is no pricing strategy which yields a profit for platform i .

3.4.3 Market Segmentation Equilibrium: Two Active Platforms

In this section we examine cases where both platforms are active in the market and serve all agents. First, we start from a case in which both platforms charge the same price and share the market equally. Then we will discuss a market segmentation equilibrium where one platform serves only high types and the other serves only low types.

Market Segmentation Equilibrium under the Symmetric Pricing

For two platform to be active in the market, there must be a price system that supports such a market allocation. Let us consider a candidate equilibrium with a price system $P = (P_i, P_j)$ and a market allocation such that $N_i^k > 0$ and $N_j^k > 0$ from the second stage of the game. If both charge the same prices, both types of agents must be indifferent between both platforms. That is:

$$U_k^t(i; P) = U_k^t(j; P) > 0 \text{ for } k \in \{1, 2\}, t \in \{H, L\}, \text{ and } j \neq i$$

In what follows, we assume that both platforms split half of the market on both sides when they charge the same price, which is typically assumed in Bertrand competition. Depending on the pricing decisions by both platforms, there are two possible cases: (1) Both platforms try to serve only high types in the market, and (2) they serve both types and cover the entire market. For the first case, they will equally share the market for high types with $xN/2$. And the perceived quality by both sides will be $q_i^k = q_j^k = 1$. And this distribution can be sustained if:

$$P_i^k \in (\theta_L + \frac{xN}{2}, \theta_H + \frac{xN}{2}] \tag{3.7}$$

For the second case, the market demands faced by each platform become $N_i^k = N_j^k = \frac{N}{2}$ and the perceived qualities are given by $q_i^k = q_j^k = x$. This distribution can be sustained if there is a price system such that:

$$P_i^k \in (-\infty, \theta_L x + \frac{N}{2}] \quad (3.8)$$

Now let us consider price competition between the two platforms. Given the distributions of the agents, a deviation by platform i to prices P_i' must give rise to a market allocation such that it takes the entire market. But this only holds when both platforms cover the entire market.

Proposition 3.4. *Suppose both platforms charge the same prices and share the market equally.*

(1) *If the market is not fully covered by the two platforms (e.g., both try to serve high type agents only), the resulting agent distribution is not sustainable.*

(2) *If the market is fully covered by the two platforms then there exists a unique equilibrium in which both platforms charge $P_i = P_j = 0$ and obtain $\Pi_i = \Pi_j = 0$.*

Proof. See the Appendix. □

The first part of Proposition 3.4 holds because price competition between the platforms yields a Bertrand type result. In other words, it forces both platforms to set a price equal to the marginal cost which is zero in our model. However, in order to serve only H -type agents, an equilibrium pair of prices should be high enough so that L -type of agents are not allowed to either platforms. Indeed, a price system which satisfies such conditions does not exist. For the second part of Proposition 3.4, there exists an equilibrium because their pricing strategies are not constrained below. Therefore, both platforms stay in the market by charging zero price to both groups.

Market Segmentation Equilibrium under Asymmetric Pricing

Another possible equilibrium, when both platforms are active in the market, is a situation where one platform serves only high types and the other serves only low types. Suppose platform i serves H -type agents of both groups and platform j serves the rest of agents of both groups. Then, the market demand faced by platform i is $N_i^k = xN$ and the perceived quality of platform i is given by $q_i^k = 1$. For platform j , the market demand is given by $N_j^k = (1 - x)N$ and the perceived quality is $q_j^k = 0$. Under these circumstances, H -type agents of each group will join platform i if the following two conditions are satisfied:

$$\begin{cases} \theta_H - (1-x)N - P_i^k \geq (1-x)N - P_j^k & (a) \\ \theta_L + xN < P_i^k \leq \theta_H + xN & (b) \end{cases} \quad (3.9)$$

For L -types, they will join platform j if the following holds

$$\begin{cases} (1-x)N - P_j^k \geq \theta_L + (1-x)N - P_i^k & (a) \\ P_j^k \leq (1-x)N & (b) \end{cases} \quad (3.10)$$

Conditions (3.9) and (3.11) ensure that H -types will join platform i and L -types will join platform j in the second stage as long as they are jointly satisfied. Unlike a dominant platform equilibrium, each type of agent has a ‘bad expectation’ against neither platform because $N_i^k > 0$ and $N_j^k > 0$. This distribution occurs because each platform charges prices to host only a specific type of agent. In other words, prices provide information about what type of agent will join a specific platform. Suppose, for example, platform i decides to charge a higher price and platform j decides to charge a lower price. In this case, H -type agents of each group will assume that they can meet the same type of the other group if they join platform i . For L -types, even though they prefer a larger network size, they will join platform j because they can enjoy more utility at the sacrifice of the perceived quality of platform i , θ_L . A question to be answered is whether this distribution is sustainable after price competition between the two platforms. As shown below, there is a price system which supports such an equilibrium under certain conditions. However, an equilibrium price system may vary depending on the parameters a , θ_L , x , and N . Because the profit-maximizing platforms face trade-off between the perceived quality and the network size. Our main task is to characterize the set of conditions which ensures a market segmentation equilibrium.

Let us think of possible strategies which platforms can use under the given market configuration. First, note that undercutting the rivals price is always a dominated strategy for both platforms, because this strategy always results in zero price and zero profit under the given market configuration. For example, suppose platform i decides to undercut prices to capture the whole market. In such a case, there is fierce price competition between the platforms and they will earn zero profit. However, when they just serve agents that they are already serving, they can have at least zero

profits. Anticipating this, both platforms will never use such a pricing strategy. In other words, they know that they can have at least zero profit if they are using the other pricing strategies.

Instead of undercutting prices, both platforms will try to use the DC-strategy so as to attract one side and make a profit from the other group. However, the role of subsidizing one side is different from that of establishing a dominant platform equilibrium. In a dominant platform equilibrium, a platform uses the DC-strategy to change ‘bad expectations’ against it. However, when both platforms are active, a platform can use the DC-strategy to have market power over the other side. For example, suppose platform i decides to cover the whole market instead of only serving H -types in the market. In such a case, platform i holds one side hostage and extracts all utility that the other side can enjoy. However, this strategy is costly because platform i has to promise one side that it will always achieve higher utilities than the other. Otherwise it is always possible that some agents leave platform i and join platform j whenever platform j yields more utility. Therefore each platform has to compare a profit from using the DC-strategy to a profit from only serving either H -or L -types. If there is a equilibrium price system, the DC-strategy should not be profitable for both platform i and j and it should support an equilibrium distribution from the stage 2 where H -types join platform i and L -types join platform j . In summary, for platform i , serving both types of agents instead of serving only H -types should not be profitable. In other words, platform i will not use the DC-strategy if:

$$\Pi_i = (P_i^1 + P_i^2)xN \geq -((1-x)N - P_j^1 + \varepsilon_i^1)N + (P_j^2 - \varepsilon_i^2)N \quad (3.11)$$

And for platform j , serving both types of agents instead of serving only L -types should not be profitable such that:

$$\Pi_j = (P_j^1 + P_j^2)(1-x)N \geq -(\theta_H + xN - P_i^1 + \varepsilon_j^1)N + (P_i^2 - \varepsilon_j^2)N \quad (3.12)$$

Both conditions above are non-profitability conditions for both platforms and these conditions hold as long as:

$$\begin{cases} P_j \leq xP_i + (1-x)N \\ [P_i - (\theta_H + xN)]/(1-x) \leq P_j \end{cases} \quad (3.13)$$

where $P_i \equiv P_i^1 + P_i^2$ and $P_j \equiv P_j^1 + P_j^2$. If there exists a price system which satisfies conditions (3.9), (3.10), and (3.13), there is a market segmentation equilibrium. Hereafter, we characterize conditions for such an equilibrium. We assume that $x \in (0, \frac{1}{2}(1 - \frac{\theta_L}{N}))$ and $P_i^k \geq P_j^k$. Depending on the value of a , we will consider four different cases: (1) a is low, (2) and (3) a is neither low nor high, and (4) a is high. Each case clarifies how both θ_H and the network size affect platforms' pricing decisions under price competition. For notational convenience, let $a_1 \equiv \frac{N}{\theta_L}(1-2x)$, $a_2 \equiv \frac{N}{\theta_L}(1-x - \frac{x^2}{1-2x+2x^2})$, $a_3 \equiv \frac{N}{\theta_L} \frac{2(1-x)(1-x+x^2)}{1-x+2x^2}$, and $a_4 \equiv \frac{N}{\theta_L} \frac{(1-2x)(1-x+x^2)}{x}$.

Proposition 3.5. *Consider a market in which both platforms are active, where one platform serves only high types and the other serves only low types. Further assume that $x \in (0, \frac{1}{2}(1 - \frac{\theta_L}{N}))$. Then:*

- (1) if $a \in (a_1, a_2]$ then the two platforms set prices such that $P_i^{k*} = \theta_H + xN$ and $\sum P_j^{k*} = 2x(\theta_H + xN) + (1-x)N$
- (2) if $a \in (a_2, a_3]$ then the two platforms set prices such that $\sum P_i^{k*} = \frac{2(\theta_H + xN)}{1-x} - N$ and $\sum P_j^{k*} = \frac{2x(\theta_H + xN)}{1-x} + (1-2x)N$
- (3) if $a \in (a_3, a_4]$ then the two platforms set prices such that $\sum P_i^{k*} = N + \frac{\theta_H}{1-x+x^2}$ and $\sum P_j^{k*} = N + \frac{x\theta_H}{1-x+x^2}$
- (4) if $a \in (a_4, \infty)$ then the two platforms set prices such that $\sum P_i^{k*} = \frac{(1-x)N}{x}$ and $P_j^{k*} = (1-x)N$

Proof. See the Appendix. □

The intuition for these results can be described as follows. First consider two polar cases, (1) and (4). When a is low, platform i has an incentive to use the DC-strategy to capture the entire market. Knowing this, platform j should set a price such that the DC-strategy would not be profitable for platform i . This results in the condition $\sum P_j^{k*} = 2x(\theta_H + xN) + (1-x)N < 2(1-x)N$. Therefore, platform j cannot set a price that extracts all utilities from serving L -types. Because of platform j 's pricing strategy, platform i seeks to maximize its profit by only serving H -types in the market. The same reasoning applies when a is high. Platform j has now an incentive to use the DC-strategy

to capture the entire market. To deter this action, platform i has to reduce its prices and platform j cannot use the DC-strategy. Thus we have $\sum P_i^{k*} = \frac{(1-x)N}{x} < 2(\theta_H + xN)$. Instead of following the DC-strategy, platform j seeks to maximize its profit under the given market distribution. Now let us consider a case where a is neither low or high as in case (2) and (3). In both cases, both platforms have incentives to use the DC-strategy to capture the whole market. Anticipating this, both platforms have to reduce their prices so that the DC-strategy is not profitable for the rival. The only difference between (2) and (3) is that, for case (2), the condition (3.10)-(a) imposes a restriction on both platforms' pricing choice, whereas for case (3), the equilibrium price is solely determined by the non-profitability condition.

3.5 Conclusion

In this chapter, we study a model of platform competition when agents care not only about the indirect network externalities but also about the types of agents who are supposed to interact with each other. Typically, in the two-sided market literature, agents are assumed to only care about the network size of the opposite side. This chapter characterizes and derives conditions for the existence of equilibrium outcomes when agents care not only about indirect network externalities but also about the types of agents who are supposed to interact. The assumption that agents also care about agent's type allows us to derive new results on platform competition, which are not addressed in existing the two-sided market literature.

We find that a monopoly platform may only serve H -types, suggesting that there is a trade-off between the market size and the willingness to pay for quality. This trade-off also plays an important role in duopoly cases. In contrast to Caillaud and Jullien (2003), it is shown that for the duopoly case only a single platform can be active in the market even if some agents are not served. However, our framework nests the result from Caillaud and Jullien: When the market is fully covered by a single platform, there exist a dominant platform equilibrium.

Most interestingly, we show that when t -type agents have an optimistic expectation with respect to one platform and not with respect to the other, there exists an equilibrium where one platform serves only H -types and the other only serves L -types. When both platforms charge the same

price and share the market equally, we show that both platforms end up with receiving zero profit because of fierce price competition.

3.6 Appendix

Proof of Proposition 3.1.

Proof. In fact four possible market allocations may arise: (1) Only high types are served, (2) both types are served, and (3) only high type agents of one group and both type of agents of the other group. The monopoly platform faces the following simple profit maximization problem under the participation constraints such that $U_k^t \geq 0$.

$$\Pi_i = \sum_{k=1,2} P_i^k N_i^k$$

under

$$\begin{cases} U_1^t = \theta_t q_i^k + N_i^{-k} - P_i^k \geq 0 \text{ for } t \in \{H, L\} \\ U_2^t = \gamma_t q_i^k + N_i^{-k} - P_i^k \geq 0 \text{ for } t \in \{H, L\} \end{cases}$$

Therefore, we need to find a set of conditions which gives the highest profits according to four different market configurations.

Only H -types join the platform: H -types will join the platform as long as $U_k^H \geq 0$. Under this condition, the perceived quality of platform is $q_i^k = 1$ and market demand on each side is xN . Therefore the platform can charge $(P^1, P^2) = (\theta_H + xN, \gamma_H + xN)$. The resulting profit is $\Pi_H = N^2[2x^2 + ax(1+c)\frac{\theta_L}{N}]$.

H -type from group-1 and both type from group-2 join the platform: Unlike the previous case, the perceived quality by group-1 becomes $q^1 = x$ because by charging lower price to group-2, both types join the platform and market demand becomes N . Under this condition, the platform can charge $(P^1, P^2) = (x\theta_H + N, \theta_L + xN)$. The resulting profit is $\Pi_1 = N^2[2x + (1 + ax^2)\frac{\theta_L}{N}]$

Both type from group-1 and H -type from group-2 join the platform: This case is exactly the opposite case of the previous one. Under this configuration, the platform can charge $(P^1, P^2) = (\theta_L + xN, x\gamma_H + N)$ and the profit is $\Pi_2 = N^2[2x + (1 + acx^2)\frac{\theta_L}{N}]$

Both types join the platform: Since both types join the platform, the perceived quality of platform i is $q_i^k = x$ and market demand on each side is N . Therefore the platform can charge

$P_i^k = x\theta_L + N = x\gamma_L + N$. The resulting profit is $\Pi_B = N^2[2 + 2\frac{\theta_L}{N}]$.

First note that when $c \in (0, 1]$, $\Pi_1 \geq \Pi_2$ always holds. Under this condition, we need to identify some conditions such that $\Pi_H \geq \Pi_1$ and $\Pi_H \geq \Pi_B$. $\Pi_H \geq \Pi_1$ holds as long as $a \geq \frac{1}{x(1+c)}[2(1-x^2)\frac{N}{\theta_L} + 2x] = \frac{A_1}{x(1+c)}$. And $\Pi_H \geq \Pi_B$ holds as long as $a \geq \frac{1}{cx+(1-x)x}[2(1-x)x\frac{N}{\theta_L} + 1] = \frac{A_2}{cx+(1-x)x}$. Therefore if $a \geq \max[\frac{A_1}{x(1+c)}, \frac{A_2}{cx+(1-x)x}]$ and $c \in (0, 1]$, Π_H is the highest profit. Similarly one can easily show that what conditions guarantee the highest profit in the other cases. \square

Proof of Proposition 3.2

Proof. To escape from pessimistic expectations against platform j , it has to subsidize k -group, say group-1, with $-P_j^1 = \theta_L x + N - P_i^1 + \varepsilon_j^1$. By subsidizing group-1, now, platform j has a market power over group-2 because $N_j^2 > 0$. Therefore platform j will charge group-2 side with $P_j^2 = P_i^2 - \varepsilon_j^2$. From i 's perspective, it has to set a price such that platform j 's pricing strategy (DC-strategy) is not profitable:

$$\Pi_j = -(\theta_L x + N - P_i^1 + \varepsilon_j^1)N + (P_i^2 - \varepsilon_j^2)N \leq 0$$

This condition holds as long as $P_i^1 + P_i^2 \leq \theta_L x + N + \varepsilon_j^1 + \varepsilon_j^2$. Therefore, platform i maximizes its profits under this constraint and gets a positive profit. \square

Proof of Proposition 3.3.

Proof. First we will characterize a condition where the DC-strategy is not profitable for platform j . Then, we will characterize a condition where a niche strategy is not profitable for platform j . At equilibrium both conditions should be satisfied under agents' distribution.

First let us look at DC-strategy. There are two possibilities where platform j uses the DC-strategy: i) Subsidizing a group with high type only (e.g., Group-1), and ii) subsidizing a group with both types (e.g., Group-2). To deter platform j 's entry, both strategies should not be profitable.

If platform j decides to subsidize group-1 side with $\theta_H x + N - P_i^1 + \varepsilon_j^1$ and charges group-2 side with $P_j^2 = P_i^2 - \varepsilon_j^2$, platform i has to set a price so that such a pricing strategy is not profitable for platform j :

$$\Pi_j = -(\theta_H x + N - P_i^1 + \varepsilon_j^1)xN + (P_i^2 - \varepsilon_j^2)N \leq 0$$

This condition holds as long as $xP_i^1 + P_i^2 \leq x(\theta_H x + N + \varepsilon_j^1) + \varepsilon_j^2$.

If platform i decides to subsidize group-2 side with $\theta_L + xN - P_i^2 + \varepsilon_j^2$ and charges group-1 side with $P_j^1 = P_i^1 - \varepsilon_j^1$. Platform i has to set a price so that such a pricing strategy is not profitable for platform j :

$$\Pi_j = (P_i^1 - \varepsilon_j^1)xN - (\theta_L + xN - P_i^2 + \varepsilon_j^2)N \leq 0$$

This condition holds as long as $xP_i^1 + P_i^2 \leq \theta_L + xN + x\varepsilon_j^1 + \varepsilon_j^2$.

Now let consider a niche strategy by platform j . Platform j can make L -type agents from group-2 to join its platform by subsidizing them with ε . By doing so, it can try to make a profit by charging a positive price whenever possible. However, it turns out that platform can not make any profit from group-1. Because platform i anticipates that platform j will use a niche strategy, it will subsidize group-1 and, therefore, they remain at platform i . And platform i charges a price to maximize its profit under the constraints by DC-strategy. In other words, platform i should always guarantee that for group-2, it will always yield higher utility than platform j and at the same time this pricing strategy should be profitable for platform i . Note that when $N_j^1 > 0$, agents from group-2 will compare their utilities from joining platform i to that from platform j . Therefore, platform i has to subsidize group-2 with $-P_i^1 = (1-x)N - P_j^2 + \varepsilon_i^2$ such that:

$$\Pi_i = -N[(1-x)N - P_j^2 + \varepsilon_i^2] + xN \cdot P_i^1 \geq 0$$

Because $P_j^2 \in (0, (1-x)N]$ should be always satisfied, $\Pi_i \geq 0$ always holds as long as $P_i^1 \geq \frac{(1-x)}{x}N$.

Now we need to check whether there is a price system which satisfies the three conditions above: 1) $xP_i^1 + P_i^2 \leq x(\theta_H x + N)$, 2) $xP_i^1 + P_i^2 \leq \theta_L + xN$, and 3) $P_i^1 \geq \frac{(1-x)}{x}N$. First, when $a < 1/x^2$, $\theta_H x + N < (\theta_L + xN)/x$ is always satisfied. However, one can easily show that $\frac{(1-x)}{x}N > \theta_H x + N$ when $x \in (0, \frac{1}{2}(1 - \frac{\theta_L}{N}))$ and $a < 1/x^2$. Therefore, there is no price system which supports a dominant firm equilibrium. However, when $a > 1/x^2$, $\theta_H x + N > (\theta_L + xN)/x$ is always satisfied.

Also when $x \in (\frac{1}{2}(1 - \frac{\theta_L}{N}), 1)$, $\frac{(1-x)}{x}N < (\theta_L + xN)/x$ is always hold. Therefore platform i sets $P_i^1 = (\theta_L + xN)/x$ and subsidizes group-2 with ε . \square

Proof of Proposition 3.4.

Proof. Two possible cases may arise when both platforms charge equal prices: i) Both platforms serve only high type, and ii) both platforms serve the entire market. Let (P_i^*, P_j^*) be a price system that constitutes an equilibrium. If this is an equilibrium, then any deviation from either platform i or j should not be profitable. Also note that undercutting price, by platform i , will allow i to have the entire market. That is $N_i^k = xN$ (when both serves only high type) or $N_i^k = N$ (when both serves the whole market) .

When both platforms serve only high type, undercutting by platform j , for example $P_j^1 = P_i^1 - \varepsilon_j^1$ and $P_j^2 = P_i^2 - \varepsilon_j^2$ should not be profitable. That is:

$$\Pi_j(P_j^*) = P_j^1 \frac{xN}{2} + P_j^2 \frac{xN}{2} \geq \Pi_j(P_j^1) = (P_i^1 - \varepsilon_j^1)xN + (P_i^2 - \varepsilon_j^2)xN$$

This condition holds as long as $P_j^1 + P_j^2 \leq 2(\varepsilon_j^1 + \varepsilon_j^2)$. However, $P_j^k \in (\theta_L + \frac{xN}{2}, \theta_H + \frac{xN}{2}]$ must be satisfied in order to serve only high types. Therefore there is no such a price system that constitute an equilibrium in this case.

When both platforms serve the entire market, undercutting by platform j , for example $P_j^1 = P_i^1 - \varepsilon_j^1$ and $P_j^2 = P_i^2 - \varepsilon_j^2$ should not be profitable. That is:

$$\Pi_j(P_j^*) = P_j^1 \frac{xN}{2} + P_j^2 \frac{xN}{2} \geq \Pi_j(P_j^1) = (P_i^1 - \varepsilon_j^1)N + (P_i^2 - \varepsilon_j^2)N$$

This condition holds as long as $P_j^1 + P_j^2 \leq 2(\varepsilon_j^1 + \varepsilon_j^2)$. Since there is no lower bound for price, the resulting equilibrium price system is $P = 0$. \square

Proof of Proposition 3.5.

Proof. Recall that in order to characterize equilibrium under a given market configuration where H -types join platform i and L -types join platform j , the following conditions are needed.

- (1) A condition for H -types to join platform i :

$$\begin{cases} P_i - P_j \leq 2[\theta_H - (1 - 2x)N] & (a) \\ P_i \leq 2(\theta_H + xN) & (b) \end{cases}$$

(2) A condition for L -types to join platform j :

$$\begin{cases} P_i - P_j \geq 2[\theta_L - (1 - 2x)N] & (a) \\ P_j \leq 2(1 - x)N & (b) \end{cases}$$

(3) Non-profitability condition for platform i :

$$P_j \leq xP_i + (1 - x)N$$

(4) Non-profitability condition for platform j :

$$[P_i - (\theta_H + xN)]/(1 - x) \leq P_j$$

To identify an equilibrium price, we need to characterize parameters which satisfy conditions (1) to (4) above.¹⁴ And let $a_1 = \frac{N}{\theta_L}(1 - 2x)$, $a_2 = \frac{N}{\theta_L}(1 - x - \frac{x^2}{1 - 2x + 2x^2})$, $a_3 = \frac{N}{\theta_L} \frac{2(1-x)(1-x+x^2)}{1-x+2x^2}$, and $a_4 = \frac{N}{\theta_L} \frac{(1-2x)(1-x+x^2)}{x}$ as in section 4.3. In the following, we will look at a case when $x \in (0, \frac{1}{2}(1 - \frac{\theta_L}{N}))$. Before we prove the proposition, it is worth to note that the condition (2a) holds as long as $P_i^k \geq P_j^k$ and N is large enough. In the rest of the proof, we will find an equilibrium price on the $P_i P_j$ -plane.

Case 1) When $a \in (a_1, a_2]$, the conditions (1b) and (3) only matter. First note that the solution of the two equations from (1a) and (3), $P_i = \frac{2(\theta_H + xN)}{1-x} - N$ and $P_j = \frac{2x(\theta_H + xN)}{1-x} + (1 - 2x)N$, violates the condition (1b) because $P_i = \frac{2(\theta_H + xN)}{1-x} - N > 2(\theta_H + xN)$. And the solution of the two equations from (3) and (4), $P_i = N + \frac{\theta_H}{1-x+x^2}$ and $P_j = N + \frac{x\theta_H}{1-x+x^2}$, satisfies $2(\theta_H + xN) < N + \frac{\theta_H}{1-x+x^2}$ and $N + \frac{x\theta_H}{1-x+x^2} < 2(1 - x)N$. Therefore, the conditions (1b) and (3) determine an equilibrium price system such that $P_i^{k*} = \theta_H + xN$ and $\sum P_j^{k*} = 2x(\theta_H + xN) + (1 - x)N$.

Case 2) When $a \in (a_2, a_3]$, the conditions (1a) and (3) only matter. In this case, the solution of the two equations from (1a) and (3) satisfies the conditions (1b) and (2b): $P_i = \frac{2(\theta_H + xN)}{1-x} - N <$

¹⁴We ignore ε because it is neglectable.

$2(\theta_H + xN)$ and $P_j = \frac{2x(\theta_H + xN)}{1-x} + (1 - 2x)N < 2(1 - x)N$. And the solution of the two equation from (3) and (4) satisfies $N + \frac{\theta_H}{1-x+x^2} < 2(\theta_H + xN)$ and $N + \frac{x\theta_H}{1-x+x^2} < 2(1 - x)N$. Therefore, the conditions (1a) and (3) determine an equilibrium price system such that $\sum P_i^{k*} = \frac{2(\theta_H + xN)}{1-x} - N$ and $\sum P_j^{k*} = \frac{2x(\theta_H + xN)}{1-x} + (1 - 2x)N$

Case 3) When $a \in (a_3, a_4]$, the conditions (3) and (4) matter. In this case, the condition (1a) becomes an irrelevant restriction because the solution of equations from (3) and (4), $\sum P_i^{k*} = N + \frac{\theta_H}{1-x+x^2}$ and $\sum P_j^{k*} = N + \frac{x\theta_H}{1-x+x^2}$, always satisfies the rest of the conditions.

Case 4) When $a \in (a_4, \infty)$, the conditions (2b) and (3) matter. Because the solution from the two equations (3) and (4) violates condition (2b) such that $N + \frac{x\theta_H}{1-x+x^2} > 2(1 - x)N$. Therefore the conditions (2b) and (3) determine an equilibrium price system such that $\sum P_i^{k*} = \frac{(1-x)N}{x}$ and $P_j^{k*} = (1 - x)N$. □

Chapter 4

A Contest Organizer Meets a Two-Sided Market

4.1 Introduction

Since Tullock's (1967, 1980) seminal works, the theory of contests has been widely analyzed in order to explain many economic and social phenomena including marketing, R&D contests, sports contests, electoral competition, and so on. While addressing a variety of important issues, most of papers in the literature have focused on how changes in the parameters (e.g.: the number of contestants and their valuation for the winning prize), which describe a specific contest situation, affect the equilibrium outcome. Further, many questions have been addressed by assuming that the size of the prize is exogenously given.¹ In some situations, however, where the organizer has enough budget to fund the contest, the assumption is reasonable. In other situations in which the organizer has no budget modification of the existing contest models is required.

The focus of this chapter is to consider the problem of the effort-maximizing-contest organizer when she has no money to fund a contest.² In order to deal with this problem, we incorporate a contest audience into the classical Tullock rent-seeking contest, with endogenous entry based on

¹Since Tullock (1967), many contest models have been studied by researchers under a fixed prize assumption: See, for example, Rosen (1986), Dixit (1987), Nitzan (1991, 1994), Baik (1994, 2004), Konrad (2000, 2007).

²Many previous works in the contest literature have focused on designing the effort-maximizing contest. See, for example, Baye, Kovenock, and de Vries (1993), Gradstein and Konrad (1999), Rosen (1986) Fu and Lu (2010) and Siegel (2009).

Fu and Lu (2010) and Morgan, Orzen and Sefton (2010). The organizer in this chapter acts as an intermediary between the contestants and the audience: She organizes the contest and presents it to the audience. With the introduction of the audience, the cross-group externality between contestants and the audience arises. In particular, this chapter considers a situation with a positive externality between aggregate efforts expended by the participating contestants and the audience: The audience are more willing to pay for watching a contest if each participating contestant expends more efforts, and contestants are more willing to expend effort if they anticipate a higher prize.

The cross-group externalities have been well addressed in the two-sided market literature in the content of platforms' pricing decisions.³ Unfortunately, however, this type of externality has not been considered in the contest literature, even though we often observe it in reality. For example, consider contest organizers that host and TV stations that broadcast international piano competitions such as the International Chopin Piano Competition and the Van Cliburn International Piano Competition.⁴ In these cases, it is reasonable to assume that the audience is willing to bear more disutility from watching advertisements when the competitions become more interesting. And broadcasters in these circumstances can increase their revenues by selling more advertisements. It further implies that the contest organizers can allocate more funds for the contests because the broadcasters are willing to pay more for rights to air the competition programs.⁵ To this end, it becomes important for the contest organizers to design an effort-maximizing rule under the cross group externality: That is, when the cross group externality between aggregate efforts and the total time spent by the audience exists. Similarly, World Cups and Olympic Games share the same feature because the audience spend more time on watching TV as the games become more interesting. It is also worth to note that, under these circumstances, the contestants' efforts are productive. Because each contestant's private effort in pursuit of the winning prize increases the audience's willingness to pay for watching the contest and, therefore, the organizer can potentially

³One of the main research questions in the two-sided market literature is to explain why platforms (e.g., credit card companies) charge different prices to different groups when the cross group externalities exist and matter between two groups (e.g., merchants and buyers). See, for example, Caillaud and Jullien (2003), Rochet and Tirole (2003, 2006), and Armstrong (2006).

⁴I am grateful to Professor Hans Haller for suggesting this example.

⁵Studying interactions between the contest organizers and the broadcasters might be interesting question. However, in this chapter, we restrict our attention to an effort-maximizing contest rule when the cross group externality is present.

appropriate more funds for the contest.⁶

There are two papers closely related to ours: Chung (1996) and Fu and Lu (2010). In Chung (1996), while his focus is more on the social efficiency issues of the contest, he studies contests in which the size of the prize increases with the contestants' aggregate efforts. His model is well-suited for explaining, for example, lottery-contests, because the size of the prize increases with the amount of money on which people spend (e.g., players' aggregate efforts) when some portion of ticket sales are added to the prize. In this regard, efforts, in his model, are also productive, but his model is different from ours in that our model has a link that relates aggregate efforts to the size of the prize: the contest audience.

Fu and Lu (2010) address the effort-maximizing contest design problem with endogenous entry, which is similar to ours. Since in their setting, the organizer has a fixed budget, the problem of the organizer becomes an allocation problem: the determination of the prize and an entry fee (or subsidy) under the budget constraint. In the present model, however, the organizer initially has *no budget* to fund the contest. By charging some fees to either one or both of two groups, the contestants and the audience, the organizer is able to fund the contest. Since the positive externality exists, the pricing decision as well as the prize determination by the organizer is *a priori* undetermined.

This chapter is organized as follows. In Section 4.2, we set out a simple contest model that consists of a pool of potential contestants, the contest audience and the organizer. Following the spirit of Fu and Lu (2010) and Morgan, Orzen and Sefton (2010), endogenous entry by potential contestants is considered in a three-stage game. In the first stage, the organizer announces a contest rule that consists of fees charged to each group and the size of the prize. In the second stage, potential contestants make entry decisions. In the third stage, all participating contestants expend their efforts to win the prize. In Section 4.3, we consider the effort-maximizing contest organizer. Our analysis suggests that the audience's valuation for aggregate efforts is critical for the existence of optimally designed contest rules. In Section 4.4, we discuss our results.

⁶Another example could be *American Idol*, a television singing contest although it should be regarded as a profit-maximizing contest rather than a effort-maximizing one. In fact, the success of *American Idol* critically depends on both aggregate efforts by the contestants and the audience's willingness to watch the show.

4.2 Model

Consider a market for a contest with a contest organizer, $N(\geq 3)$ potential contestants and a contest audience with size m . The organizer has to appropriate funds for the contest from other sources because she initially has no budget. The organizer in the present model acts as an intermediary between the contestants and the contest audience: She organizes the contest for the contestants and presents it to the audience.

The organizer can charge either one or both of two groups for her services. Further, if necessary, one group may be subsidized by the organizer.⁷ Total revenue from either one or both of two groups becomes the source of her funding for the contest. Let s denote either an entry subsidy when $s > 0$ or an entry fee when $s \leq 0$ is charged to each participating contestant. And let T denote the entire audience's total amount of time spent on watching the contest and $p \geq 0$ represent a disutility per a unit time (e.g., the nuisance costs of advertisement per a unit time) incurred, measured in dollars.⁸ Therefore, the total disutility incurred by the entire audience is given by Tp . We assume that this total disutility becomes the organizer's revenue from the audience.⁹¹⁰ Net revenue from organizing the contest is given by

$$\Pi = -ns + Tp - V \tag{4.1}$$

where n denotes the actual number of entrants out of N potential contestants and V represents a prize determined by the organizer. A priori, we don't know which group will be charged or how

⁷In the two-sided market literature, platform operators have an incentive to do so because they must have two groups on board for their business.

⁸In fact, we follow the spirit of Armstrong (2006). In his model, a platform in a two-sided market is considered as an intermediary that offers utilities to each group (e.g., u_1 and u_2) rather than prices (e.g., p_1 and p_2). Then, the implicit price for each group is expressed in terms of utility: $p_i = f(u_i)$ for a decreasing function $f(\cdot)$. And the platform chooses the profit-maximizing level of utility.

⁹Therefore, the present model implicitly assumes that the organizer sells time slots to advertisers in order to fund the contest. Because we do not consider interactions between the contest organizer and the broadcaster, to some extent, the organizer in the present model can be seen as a broadcaster who sets the contest and sells time slots to advertisers.

¹⁰The assumption that the nuisance costs of advertisement become the revenues of the organizer (or the broadcaster) is often employed in the media economics literature. See, for example, Anderson and Coate (2005) and Kind, Nilssen, and Sørsgard (2005). The intuition behind this assumption is that the broadcaster parlays nuisance costs into advertising revenues: The broadcaster chooses the optimal level of advertising and an advertising price in advertising markets after considering the viewers' nuisance costs. We refer to Anderson and Gabszewicz (2006) which provides discussions about the advertising and the media industry. Also see Bagwell (2007) which provides a comprehensive survey of the advertising literature in economics.

much they will be charged, or the size of the prize, that will be awarded. The determination of s , p and V by the effort-maximizing organizer are the key questions of the present model.

Contest Audience

Consider the contest audience with size m , each with quasi-linear preferences and maximizing the net benefit function $u(t) - pt$ with $u(t) = \alpha t \sum_j^n e_j - \beta t^2/2$, where $\alpha > 0$, $\beta > 0$, and $t \equiv T/m$ represents each audience's time spent on watching the contest. It implies that each audience values a contest with more aggregate effort $\sum_j^n e_j$, which is the sum of each participating contestant's effort e_j . This gives rise to the inverse demand function given by

$$p(T) = \alpha \sum_{j=1}^n e_j - \beta T \quad (4.2)$$

Contests with Endogenous Entry

For the contestant side, we consider a Tullock rent-seeking contest with endogenous entry, in which the participating contestants compete with each other to win a prize V .¹¹ In order to determine contestants' entry decision, we follow the simple contest model by Fu and Lu (2010) and Morgan, Orzen and Sefton (2010).

There is a pool of potential contestants consisting of a fixed number of identical risk-neutral individuals, denoted by $N(\geq 3)$. They observe V , p and s announced by the organizer, and make a decision on whether or not to participate in the contest. Each individual makes a decision *sequentially*, and his decision is known to the public. In other words, each potential contestant can observe the number of competitors in the contest, regardless of his decision. Further a contestant's entry entails a fixed cost $c > 0$, which is assumed to be the same across all contestants and common knowledge.¹²

If $n \geq 2$ contestants have entered the contest and the other $N - n \geq 0$ have decided not to, all participating contestants simultaneously expend irreversible effort to win the prize. Let $e_i \geq 0$ denote the effort level expended by a contestant i and let $\sum_j^n e_j$ represent the total effort level

¹¹For the contestant side, we use the same notation as in Fu and Lu (2010) in order to compare our results with theirs.

¹²For more detailed descriptions of endogenous entry in contests, we refer to Morgan, Orzen and Sefton (2010).

expended by all participating contestants. The probability of winning for contestant i is given by the following logit form:

$$p_i(e_1, \dots, e_n) = \begin{cases} e_i / \sum_{j=1}^n e_j & \text{if } \max\{e_1, \dots, e_n\} > 0 \\ 1/n & \text{otherwise} \end{cases}$$

And player i 's payoff function can be written as

$$\Gamma_i = \left(\frac{e_i}{\sum_{j=1}^n e_j} \right) V - e_i + s - c \quad (4.3)$$

Timing of the Game

We formally consider the following three-stage game. In the first stage, the contest organizer announces a contest rule that consists of the size of prize V and a pair of prices s and p charged to each group. The rule specifies how much each participating contestant will be awarded if he wins the contest, how much he will be subsidized (or charged) when joining the contest and how much the contest audience will be charged. In the second stage, after observing the contest rule, each potential contestant decides whether or not to join the contest and the audience calculate their watching time based on their expectations of $\sum e_i$, and they determine whether they are willing to watch the contest. In the third stage, all participating contestants simultaneously expend effort to win the prize, provided that more than one potential contestant joins the contest. Otherwise, when fewer than two contestants participate in the contest: (1) The contest is closed without winner when no one joins the contest or (2) when there is only one participating contestant, he receives the prize regardless of his effort. At the end of the third stage, all audience's expectations are fulfilled in equilibrium: We employ a subgame-perfect equilibrium as the solution concept.

Preliminaries: The Expected Payoffs of the Entrants

In order to determine the optimal contest rule, we first calculate the expected payoffs of the participating contestants. Since we are solving this game by backward induction, for the moment we assume that a positive number of the potential contestants $n \geq 1$ have entered the contest.¹³ Using

¹³The exact number of the participating contestants is determined in the first-stage of the game.

standard procedure from the contest literature, we can obtain the following expected payoff for participating contestant i

$$\Gamma_i = \begin{cases} V + s - c & \text{if } n = 1 \\ \frac{V}{n^2} + s - c & \text{if } n \geq 2 \end{cases} \quad (4.4)$$

where a fixed entry cost $c < V + s$ is assumed in order to guarantee that at least one contestant joins the contest. And the equilibrium effort levels and total effort levels in this subgame are given by

$$e_i = \frac{n-1}{n^2}V$$

and

$$\sum e_i = \frac{n-1}{n}V,$$

respectively. Further it is well known that all participating contestants under sequential entry obtain zero expected payoffs in any equilibrium. That is, $\Gamma_i = \frac{V}{n^2} + s - c = 0$ holds in the equilibrium when $n \geq 2$. This is known as the zero expected utility condition.¹⁴ In fact this is the analog of the zero profit condition in the oligopoly literature.¹⁵ Using this condition, we derive an entry subsidy (or entry fee) as a function of c , V , and n :

$$s = c - \frac{V}{n^2} \stackrel{\leq}{\geq} 0 \quad (4.5)$$

Finally we impose a feasibility condition in order to ensure that the organizer makes no loss from setting the contest.

$$0 \leq \Pi \leq Tp - ns - V \quad (4.6)$$

In other words, this condition says that the amount of the prize cannot exceed the total revenues from organizing the contest.

¹⁴See, for example, Morgan et al. (2010) and Fu and Lu (2010).

¹⁵See, for example, Mankiw and Whinston (1986) for the free-entry equilibrium number of firms.

4.3 The Effort-maximizing-contest Organizer

In this section, we consider the contest organizer who wants to maximize the total effort level $\sum_j^n e_j$ expended by all the participating contestants.¹⁶ First, consider a contest rule that attracts only one contestant. Under the effort-maximizing motive, this rule is not optimal because the participating contestant has no incentive to make effort. Therefore, in the following analysis, we consider a contest rule that attracts more than one entrant ($2 \leq n \leq N$) as a candidate for the effort-maximizing contest rule.

Lemma 4.1. *In the effort-maximizing contest, the organizer exhausts all revenues from organizing the contest. That is, $\Pi^* = 0$ and $V^* = T^*p^* - n^*s^*$.*

Proof. See the Appendix □

Lemma 4.1 provides a useful condition for obtaining the optimal contest rule. By substituting equations (4.2) and (4.4) into the result of Lemma 4.1, we obtain

$$V = Tp - ns = T\left(\alpha \sum_{j=1}^n e_j - \beta T\right) - n \left(c - \frac{V}{n^2}\right) \quad (4.7)$$

By rearranging equation (4.7), we have

$$\left(1 - \frac{1}{n}\right) V = \frac{1}{\alpha T - 1} \left(\frac{\beta}{m} T^2 + nc\right) \quad (4.8)$$

Now, we consider the organizer's objective function. That is,

$$\text{Max} \sum_{j=1}^n e_j = \left(1 - \frac{1}{n}\right) V$$

Note that this objective function is obtained from the third-stage of the game. By substituting equation (4.8) into the objective function, we can rewrite the organizer's objective function as follows:

¹⁶In reality, a contest organizer may have various objectives including the profit-maximizing motive. However, we leave this topic for future research.

$$\text{Max}_{n,T} \sum_{j=1}^n e_j = \frac{1}{\alpha T - 1} \left(\frac{\beta}{m} T^2 + nc \right) \quad (4.9)$$

Equation (4.9) says that the effort-maximizing contest rule can be obtained by choosing the optimal number of the contestants n and the optimal watching time T .¹⁷ If more than one potential entrant joins the contest, the first-order condition for maximizing the total effort level, $\sum_{j=1}^n e_j$, is

$$\frac{\partial \sum_{j=1}^n e_j}{\partial T} = \frac{1}{(\alpha T - 1)^2} \left(\frac{2\beta}{m} T(\alpha T - 1) - \alpha \left(\frac{2\beta}{m} T^2 + nc \right) \right) = 0. \quad (4.10)$$

Solving for the first-order condition (4.10) for T yields

$$T = \frac{\alpha m}{2\beta} \left(1 - \frac{1}{n} \right) V$$

and the second-order condition $\frac{\partial^2 \sum_{j=1}^n e_j}{\partial T^2} < 0$ is satisfied as long as $\alpha T < 1$ holds in the equilibrium.

For the rest of analysis, we assume that $\alpha T < 1$. And the solution $T = \frac{\alpha m}{2\beta} \left(1 - \frac{1}{n} \right) V$ implies that, for each audience, a disutility level per a unit time is given by¹⁸

$$p = \frac{\alpha}{2} \left(1 - \frac{1}{n} \right) V$$

Combining the two results, total revenues from the audience becomes

$$Tp = \left(\frac{\alpha m}{2\beta} \left(1 - \frac{1}{n} \right) V \right) \left(\frac{\alpha}{2} \left(1 - \frac{1}{n} \right) V \right) > 0$$

This implies that the organizer has a positive revenue from the audience as long as the optimal contest rule exists.¹⁹

Now we discuss the optimal number of entrants. Due to the assumption $\alpha T < 1$, the first-order condition for the optimal number of entrants is given by

¹⁷Strictly speaking, the contest organizer decides the optimal entry fee/subsidy s and the optimal disutility levels p . The optimal number of the contestants n and the optimal watching time T are automatically determined once the organizer optimally chooses s and p .

¹⁸It is derived from the audience demand function.

¹⁹If $\frac{\partial \sum_{j=1}^n e_j}{\partial T} < 0$, $T^* = 0$ and $\sum_{j=1}^n e_j = -nc < 0$. And if $\frac{\partial \sum_{j=1}^n e_j}{\partial T} > 0$, $p^* = 0$. However, in this case, $\Pi = -nc + \frac{V}{n} - V < 0$ (for $n \geq 1$) and it violates the zero profit condition suggested in Lemma 4.1.

$$\frac{\partial \sum_{j=1}^n e_j}{\partial n} = \frac{c}{\alpha T - 1} < 0 \quad (4.11)$$

This happens because the organizer's objective function (4.9) is strictly decreasing with $n \geq 2$ when $\alpha T < 1$. Using the results from both equation (4.10) and equation (4.11), we establish the following Proposition.

Proposition 4.1. *Under the effort-maximizing motive, the contest organizer designs a contest rule that attracts only two potential contestants (e.g., $n^* = 2$) by subsidizing the entrants $s^* = c - \frac{V^*}{4} < 0$ and charging the audience $p^* = \frac{\alpha}{4} V^* > 0$, resulting in $T^* = \frac{\alpha m}{4\beta} V^*$. The organizer awards the prize $V^* = \frac{4\beta}{\alpha^2 m} \left(1 + \sqrt{1 + \frac{2\alpha^2 mc}{\beta}}\right)$ to the winner and the effort-maximizing contest rule induces the total effort of $\sum_{j=1}^{n=2} e_j^* = \frac{V^*}{2} = \frac{2\beta}{\alpha^2 m} \left(1 + \sqrt{1 + \frac{2\alpha^2 mc}{\beta}}\right)$.*

Proof. See the Appendix. □

Proposition 4.1 characterizes the subgame perfect equilibrium of the three-stage game. It says that the total effort level is maximized by attracting only two contestants. Further, the equilibrium entry subsidy s^* should be large enough to cover contestants' fixed entry costs. The organizer with no budget has to collect money from the audience in order to fund the contest.

In fact, this result is quite similar to the result of Fu and Lu (2010), which also concludes that the effort-maximizing contest rule attracts only two contestants when the organizer has a fixed budget to fund the contest. However, our result differs from theirs in two ways. First, the entrants in the present model are always subsidized by the organizer in equilibrium, regardless of the size of entry costs. In particular, even when $c = 0$, the participating contestants are subsidized. However, in Fu and Lu (2010), the fees charged to the entrants depend on the size of fixed entry costs: If the entry costs are relatively small, the organizer subsidizes the entrants, who would otherwise be charged positive entry fees. Second, the condition for the existence of the equilibrium is more restrictive since the assumption $\alpha T^* < 1$ is required. This suggests that audience's valuation of aggregate effort is critical for the existence of optimally designed contest rule. This assumption is somewhat related to the assumption introduced in Chung (1996) for the existence of a unique equilibrium, because in Chung (1996), it is assumed that a winning prize $R(\sum e_i)$ has decreasing

returns to scale with respect to aggregate effort levels $\sum e_i$.²⁰

4.4 Conclusion

This chapter studies the effort-maximizing contest rule when (1) the positive externality between aggregate efforts and the contest audience exists, and (2) the organizer has no budget to fund the contest. Our analysis suggests that the audience's valuation for aggregate effort is critical for the existence of an effort-maximizing contest rule. Indeed, a unique equilibrium exists when the marginal utility with respect to aggregate efforts is less than one: That is, when $\alpha T^* < 1$ holds.

With regard to the optimal contest design, the presence of fixed entry costs which potential contestants bear plays less important a role when the cross-group externality is present in the model. By charging a positive fee to the audience, the organizer with no budget always finds it worthwhile to subsidize the participating contestants regardless of the size of fixed entry costs. Only the amount of subsidy will be different under different values of fixed costs.

For the optimal number of contestants, in Fu and Lu (2010), the organizer does not have to set the contest rule that attracts only two contestants when potential contestants bear zero entry cost. In fact, in their model, the organizer under a budget constraint faces the allocation problem between entry subsidies and the prize when the contestants bear fixed entry costs. Therefore, when there is no fixed entry cost, the organizer can award a higher prize to the contestants and therefore induce more entrants, in order to maximize aggregate efforts. However, in the present chapter, we show that the effort-maximizing contest rule always attracts only two contestants even when no entry cost is considered.

For future research, we believe that it will be interesting to study profit-maximizing contest rules, because we often find that contest organizers have a profit-maximizing motive. For example, many television networks recently broadcast singing or dancing contest programs in the U.S., and they make a large profit. From the broadcasters' perspective, designing the profit-maximizing contest rule will be important for the success of their programs. It is also an interesting question for academic research.

²⁰That is, $R' > 0$ and $R'' < 0$ are satisfied.

4.5 Appendix

Proof of Lemma 4.1.

Proof. For the proof of Lemma 4.1, we follow the approach used in Lemma 5 by Fu and Lu (2010). Under the effort-maximizing motive, the optimal contest rule always attracts more than one potential contestant. Therefore, there are two possible cases for the equilibrium number of contestants. Further, suppose that there exists an optimal contest rule such that $\Pi^* = T^*p^* - n^*s^* - V^* > 0$.

Case 1: Suppose $n^*(p^*, s^*, V^*) = N$ holds in the equilibrium. If the organizer decides to use $\Pi^*(> 0)$ to add to the prize, she can induce larger aggregate efforts without changing the number of the participating contestants.

Case 2: Suppose $2 \leq n^*(p^*, s^*, V^*) < N$ holds in the equilibrium. Since $\Gamma_i^* = \frac{V^*}{n^{*2}} + s^* - c = 0$ holds in the equilibrium, it implies that $n^* = \lfloor n \rfloor = \sqrt{\left| \frac{V^*}{c-s^*} \right|}$ and $\lfloor n \rfloor < n^* + 1$. Now suppose that the organizer increases the size of the prize by ε such that $\lfloor n \rfloor < n^* + 1$. Then this contest rule can induce larger aggregate efforts while keeping the number of the participating contestants being constant. \square

Proof of Proposition 4.1.

Proof. Since equation (4.9) is strictly decreasing with n , it is only necessary to show that the feasibility condition is satisfied in equilibrium. That is, $\Pi^* = T^*p^* - n^*s^* - V^* = 0$. Substituting the equilibrium results $n^* = 2$, $s^* = c - \frac{V^*}{4}$, $p^* = \frac{\alpha}{4}V^*$ and $T^* = \frac{\alpha m}{4\beta}V^*$ into $\Pi^* = T^*p^* - n^*s^* - V^* = 0$, we obtain

$$\frac{\alpha^2 m}{4\beta} \left(1 - \frac{1}{2}\right)^2 V^{*2} - \left(1 - \frac{1}{2}\right) V^* - 2c = 0$$

Solving this equation, we have $V^* = \frac{4\beta}{\alpha^2 m} \left(1 + \sqrt{1 + \frac{2\alpha^2 mc}{\beta}}\right)$. Further, it implies that the organizer sets the prize such that $c = \frac{\alpha^2 m}{32\beta} V^{*2} - \frac{V^*}{4}$. Substituting this into equation (4.5), we have $s^* = c - \frac{V^*}{4} = \frac{\alpha^2 m}{32\beta} V^{*2} - \frac{V^*}{2}$. When $\alpha T^* = \frac{\alpha^2 m}{4\beta} V^* < 1$, which we have assumed in Section 4.3, $s^* = \frac{V^*}{2} \left(\frac{\alpha^2 m}{16\beta} V^{*2} - 1\right) < 0$. This means that, under the effort-maximizing motive, the organizer always subsidizes the entrants. \square

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