PREDICTION OF THE RUNNING TORQUE OF INSTRUMENT BALL BEARINGS AT HIGH SPEED UNDER COMBINED RADIAL AND AXIAL LOADS

by

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I. INTRODUCTION

Recent trends for higher efficiency in mechanical components, and limited power sources such as in space applications, have necessitated reevaluation of the operational characteristics of many off-the-shelf components such as ball bearings. Present evaluation of instrument ball bearings is based on MIL-STD 206. Briefly, this procedure specifies that the friction torque of a ball bearing be measured at the outer race while the inner race is rotated at least one revolution clockwise and one revolution counter-clockwise at a speed not to exceed 0.5 rpm. The bearings are loaded with up to 400 grams axial load. This method gives much useful data about the torque characteristics of ball bearings at very low speeds. However, there is currently considerable discussion as to the merits of this method of evaluation when the bearings selected are for use in high speed applications with perhaps combined radial and axial load.

In order to obtain quantitative information on the running torque of ball bearings at high speed, it would be necessary to use some mathematical expression to project the required information from the standard low-speed test were the basic source of torque data. Unfortunately, there is presently no published expression which will predict the high-speed running torque for low-speed data. This renders the MIL-STD 206 test practically useless for the evaluation of high-speed ball bearing characteristics.

Another method of obtaining predictions of high-speed characteristics would be by use of analytical or empirical expressions based on common operational parameters such as bearing size, speed, lubricant, and equivalent load. There have been many such expressions or formulas put forth; however, almost without exception they have dealt with bearings having a bore of one-inch or larger. The result has been that such formulas have been grossly in error when applied to instrument-size ball bearings.

Accordingly, the purpose of this investigation was to obtain and study a sufficient quantity of data from high speed torque tests to permit the development of an operational design expression for predicting the running torque for various size oil-lubricated instrument bearings under combined radial and axial loads at various rotational speeds up to 40,000 rpm.

II. REVIEW OF LITERATURE

In studying the possible methods for predicting the running torque of ball bearings several empirical and analytical methods were reviewed.

A. Analytical Approach

Analytical methods were inherently complex because of the nature of friction encountered in ball bearings. Some of the major causes of friction in ball bearings as noted by Harris[1] are listed below.

1. Elastic hysteresis in rolling
2. Sliding due to deformation of contacting elements and bearing geometry
3. Spinning of rolling elements
4. Gyroscopic pivotal motion of rolling elements
5. Viscous friction due to lubricant action
6. Sliding between cage and rolling elements and between cage and bearing rings
7. Seal friction

Each of these factors will be discussed as they are related to friction in instrument ball bearings.

Because a ball contacting another curved surface tends to result in point contact unless the two radii of curvature are identical, the resulting stresses are quite high even for moderate loads. The result is that stresses in the balls and races of ball bearings are cyclically driven past the proportional limit and occasionally the elastic limit of the bearing material. This of course results in a hysteresis effect and a resulting energy loss. The energy source rotating the bearing must provide the energy to overcome this loss. For a ball contacting a spherical seat which approximates the conditions in a ball bearing, the maximum stress is given as follows:

$$P_{max} = 0.616 \sqrt[3]{PE^2 \left(\frac{d_2 - d_1}{d_2 d_1} \right)^2}$$

This expression from Timoshenko[2] was originally developed by Hertz and the calculated maximum stress is known as Hertzian stress. The quantities used in the above expression are:

P_{max} = maximum stress

P = applied load

E = modulus of elasticity of bearing material

d_1 = diameter of the ball

d_2 = diameter of the seat

The bearing race is not spherical but is defined by two different radii of curvature. Thus in practice, the above expression must be used with correction factors which take into account the elliptical shape of the area of contact between the ball and race under load.

The Hertzian stresses produce an area of deformation around the point of contact which is symmetrical about all axes of geometric symmetry. However, in the case of a ball rolling over a surface, the tangential forces required for rolling tend to produce a bulge ahead of the ball in the direction of rolling. One result is more surface contact along the forward portion of the ball and increased force necessary to roll the ball forward.

Another friction source associated with rolling over a deformed area is Reynolds' slip.[1] Reynolds' slip occurs when, for example, a rigid cylinder rolls on an elastic surface. The cylinder is observed to roll forward a distance less than the circumference of the cylinder. The difference between the two distances is the Reynolds' slip. The slip is believed to occur in the region of beginning and ending contact between the cylinder and the surface.

In considering other geometrical factors, it is observed that when the ball and race are deformed under load, their radii of curvature in the deformation zone are also changed. This gives rise to another source of friction known as Heathcote slip.[1] Because of the deformation of the ball and race, there is only one ball radius which correctly corresponds to pure rolling along the race. This single correct radius describes an arc of contact on the ball on each side of the axis of symmetry through a cross-section of the ball and race perpendicular to the direction of rolling. Thus, pure rolling occurs only along the two arcs and slipping must occur at all other points in the area of contact.

To this already complex slip problem, gyroscopic pivotal motion must be added. Since the balls, which are rotating about an axis through the balls, are also forced to follow a curved path in space as defined by the bearing races, there is a gyroscopic moment created which tends to spin the balls about an axis which is perpendicular to the raceways at the center of the contact area. This gyroscopic spinning is considerable at high speeds such as encountered in torque tests up to 40,000 rpm. The effect of gyroscopic spinning varies with speed and ball loading.

Lubrication is another factor which is relatively easy to consider if it were not for the change of ball loading around the raceway. Because the ball does travel around the raceway, it cycles between the maximum loaded condition and essentially a fully unloaded state when subjected to radial loading. Because of this cycling, a given ball is constantly changing its lubricating state between hydrodynamic and elastohydrodynamic lubrication. This also means that the viscosity of the lubricating oil is a significant parameter in determining the friction torque. Also associated with the lubrication problem is ball skidding. Ball skidding occurs when a lightly loaded ball does not rotate as fast as it is expected to and consequently skids rather than rolls across the bearing raceways.

Another major source of bearing friction occurs at the points of contact between the ball cage or retainer and the balls. It was shown experimentally by Mabie[3] that the type of retainer used can markedly affect the overall friction torque as well as the trend of the torque versus speed plot. The retainer friction is largely a function of contact area with the balls and type of lubricant.

Seal friction is rarely considered in bearing applications for the obvious reason that instrument bearings are rarely used with seals. Instrument bearings commonly have shields but these shields do not make contact between two moving parts and consequently do not contribute to the friction torque.

It can be seen from the preceeding discussion that because of the number and complexity of contributing factors, the analytical determination of running torque becomes quite involved.

B. Empirical Approach

Another approach to predicting bearing behavior is by empirical methods. Empirical methods often have the advantage that they are based on parameters which are most commonly known in design calculations.

One empirical expression suggested that there might be some set of simplified parameters for analyzing the ball bearings similar to the f versus Zn/p curves used for journal bearings. The parameters used above are:

f = coefficient of friction

Z = viscosity of the oil

p = load pressure on the projected bearing area

n = rotational speed

Work done by Barwell and Hughes[4] suggested that an analogous plot for ball bearings would show f versus ZUnd/W where:

f = coefficient of friction

Z = viscosity of the lubricating oil

U = pitch line velocity of the bearing

n = number of balls in the bearing

d = diameter of the balls in the bearing

W = equivalent bearing load

Their work was done on ball bearings with a 5-inch bore and showed f versus ZUnd/W to be approximately a straight line. In attempting to construct a similar plot for data collected by Mabie[3] and Clarke[5] it was found that the plot was essentially the same as a torque versus speed curve. The only modification was constants multiplied by the torque and speed data which served only to change the slope and not the overall curve which was not a straight line. In short, the simplified relations suggested by Barwell and Hughes did not hold for test results from R-3 (3/16" bore) instrument bearing data.

Another approach to predicting the performance of ball bearings at high speed would be by the use of empirically derived equations. According to Palmgren[6], some of the factors which affect the friction torque of ball bearings are:

1. Bearing design
2. Bearing dimensions
3. Bearing load and load distribution
4. Rotational speed
5. Quantity of lubricant in the bearing
6. Properties of the lubricant
    a) viscosity
    b) oiliness
    c) film strength

In considering the effects of many of these factors, Palmgren arrived at the following expression:

$$M_f = \sqrt[3]{vx^2nd_m^7} + f_fF^c$$

where:

M_f = total friction torque

v = kinematic viscosity of the oil

x = oiliness

n = rotational speed

d_m = mean or pitch diameter of bearing

f_f = coefficient based on bearing size and design

F = equivalent bearing load

c = exponent based on bearing type

Unfortunately, in practice this formula is very difficult to apply. Part of this difficulty lies in the coefficient f_f. Apparently this coefficient must be determined experimentally by the investigator for the particular bearings under test. Palmgren gives no indication of the order of magnitude of this coefficient. A second item of difficulty is the parameter x, oiliness. The Society of Automotive Engineers defines oiliness as "a term describing the differences in friction, greater than those attributable to viscosity alone, obtained with different lubricating oils in identical conditions." However, the SAE does not establish any method for quantitatively determining oiliness. Several different methods for determining oiliness are described by Kragelskii[7] but there is no way of relating any one method to another. This points up the nebulous nature of the term oiliness. In addition, manufacturers supplying oil to meet government requirements for instrument application do not mention oiliness in specifying the properties of their oil. These factors combine to make the utility of Palmgren's equation very questionable.

Elsewhere in his book, Palmgren states that it is not possible to make accurate calculations of friction torque under widely varying conditions and for different size bearings. Empirical formulas must be based on experimental test data and extrapolation of results is not safe.

Wilcock and Booser[8] noted that two bearing manufacturers suggested using the following expression for friction torque:

$$\tau = fRW$$

where:

τ = friction torque

f = coefficient of friction

R = bearing pitch radius

W = equivalent bearing load

Here it can readily be seen that this expression does not recognize any speed effect on friction torque. This does not agree with the experimental results of Mabie[3] and Clarke[5] on instrument size bearings.

Muzzoli[8] took quite a sophisticated approach to deriving a torque expression and arrived at the following equation:

$$\tau = 0.0868\ KV\left[n^{0.5}(2.6\ d^{0.2} - 3.33)N^{0.375}d^{-0.2} + \frac{(0.454W)^{m}}{72N^{0.73} + 150n(d-6)^{0.55}}\right]$$

In this equation, the symbols used represent the following:

τ = friction torque

K = constant based on type of lubrication

V = constant based on bearing design

n = number of balls in the bearing

d = diameter of balls in the bearing

N = rotational speed

W = equivalent bearing load

m = constant based on bearing design

Nevertheless, there are problems in using this formula for instrument bearings. First, in the term containing $(2.6\ d^{0.2} - 3.33)$, the d refers to the ball diameter measured in millimeters. Ball diameters in instrument bearings are sufficiently small that the above term becomes negative. Obviously the friction torque does not decrease with increasing speed. In the second term, the ball diameter is again needed for the part containing $(d - 6)^{0.55}$. Because ball diameters are less than 6 mm, it becomes necessary to raise a negative number to a fractional power which results in a mathematical answer which has no physical significance. Hence it can be seen that Muzzoli's carefully formulated expression is not valid in the instrument bearing range.

Considerable research has been conducted by Styri [9] for the SKF bearing company in Sweden. His work showed that while there was considerable change in the coefficient of friction with load changes, there was little change with changing speed. However, in relating coefficient of friction to bearing performance he noted two difficulties. One problem was that the coefficient of friction became infinite at zero load. Secondly, the friction coefficients depended on a bearing radius which was chosen arbitrarily. Because of these objections and difficulty in obtaining meaningful data utilizing coefficient of friction, he concluded that friction torque was a more suitable criterion for evaluating the performance of ball bearings.

From his tests, Styri developed the following expression for friction torque:

$$\tau = 1.50\times10^{-5} w(d_1+d_2)^2 N^{0.63}\mu + 3.24\times10^{-5} W^{1.5}$$

The symbols represent the following:

τ = friction torque

w = bearing width

d_1 = bearing bore

d_2 = bearing outside diameter

μ = coefficient of friction

N = rotational speed

W = equivalent bearing load

A comparison of the friction torque calculated by Styri's formula and the experimental data from Clarke's experiments is shown in Fig. 1. It is readily seen that the calculated results are much higher in value than the experimental results; in fact, higher by more than a factor of 10. Some of this might be explained by the fact that Styri assumed that the bearings would be running in a bath of oil with the level mid-way up on the bottom ball. This would lead to much higher friction torque estimates than actually experienced with the very lightly lubricated balls as is the case in instrument applications. However, his principle of separating the friction torque into two terms, one a dynamic term and one a static term, appears sound and should be applicable to instrument bearing work.

To test the accuracy of Styri's formula for the near static case, a comparison was made with experimental data by Mabie[3] for R-2 (1/8" bore) bearings at one revolution per minute with a 75 gram axial load. Experimental data showed a running torque of 817 mg-mm while Styri predicted a torque of 181 mg-mm. While not too far apart in absolute value, the two torques compare unfavorably on a ratio or percentage basis. Apparently Styri's equation is simply not valid in the instrument bearing range.

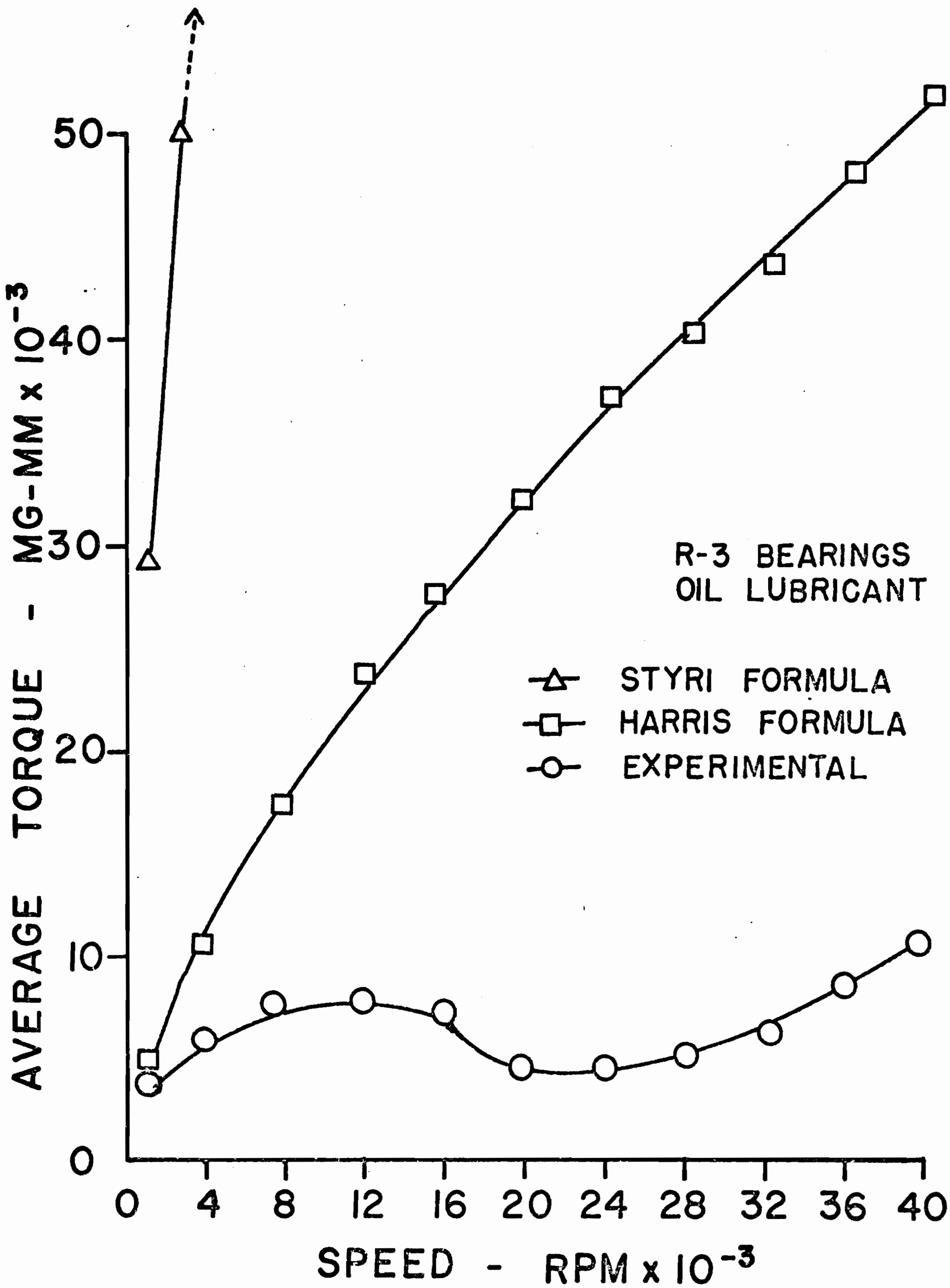

FIG. I ANALYTICAL VS. EXPERIMENTAL RUNNING TORQUE

Moore and Jones[10] did much work in determining the effect of oil flow rates on friction torque of ball bearings. Their data showed that the two most important factors affecting friction torque are bearing speed and bearing bore. And of these two, bore was found to be the more significant. It is appropriate to try to make similar comparisons for instrument bearings using the experimental data collected by Mabie[3]. Figure 2 shows a comparison of the running torques of R-2, R-3, and R-4 ball bearings versus rotational speed of the inner race. All of these bearings had ribbon retainers and were lubricated with MIL-L-6085A oil. For a size comparison, the R-2 bearing has a 1/8 inch bore, the R-3 bearing a 3/16 inch bore, and the R-4 bearing a 1/4 inch bore. The plot for the oil lubricated bearings shows that for a 50% increase in running speed such as from 20,000 rpm to 30,000 rpm, a 20 to 30% increase in running torque is noted. And for a 100% increase in speed such as from 20,000 to 40,000 rpm, a 60% increase in running torque is seen. Likewise, taking a given speed such as 30,000 rpm, it is seen that for a 50% increase in bore such as from a R-2 to a R-3 bearing, a 30% increase in torque is noted. And for a 100% increase in bore such as from R-2 to R-4, a 200% increase in running torque is observed. Figure 3 shows similar observations

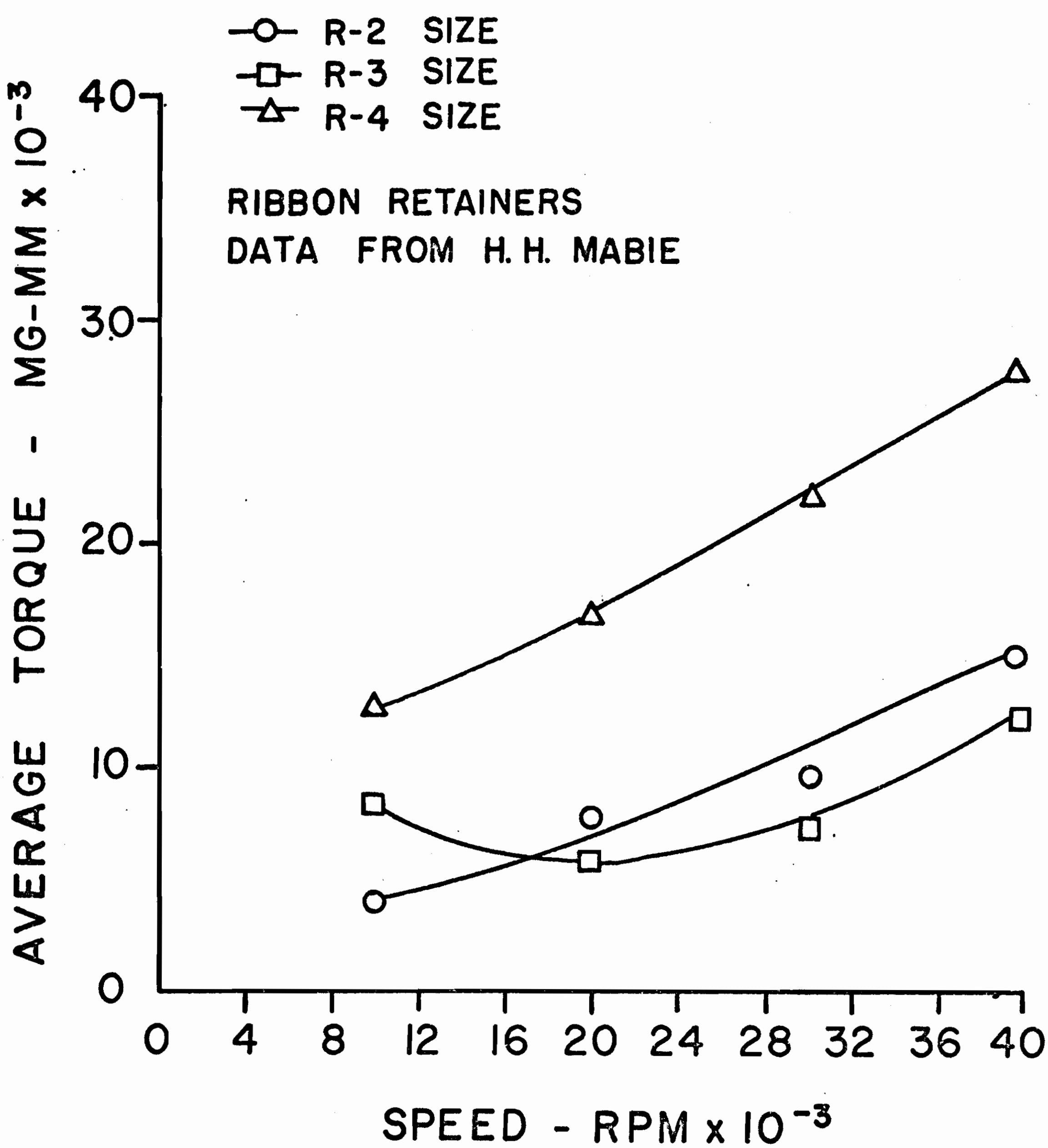

FIG. 2 SIZE EFFECT ON RUNNING TORQUE ON OIL LUBRICATED BALL BEARINGS

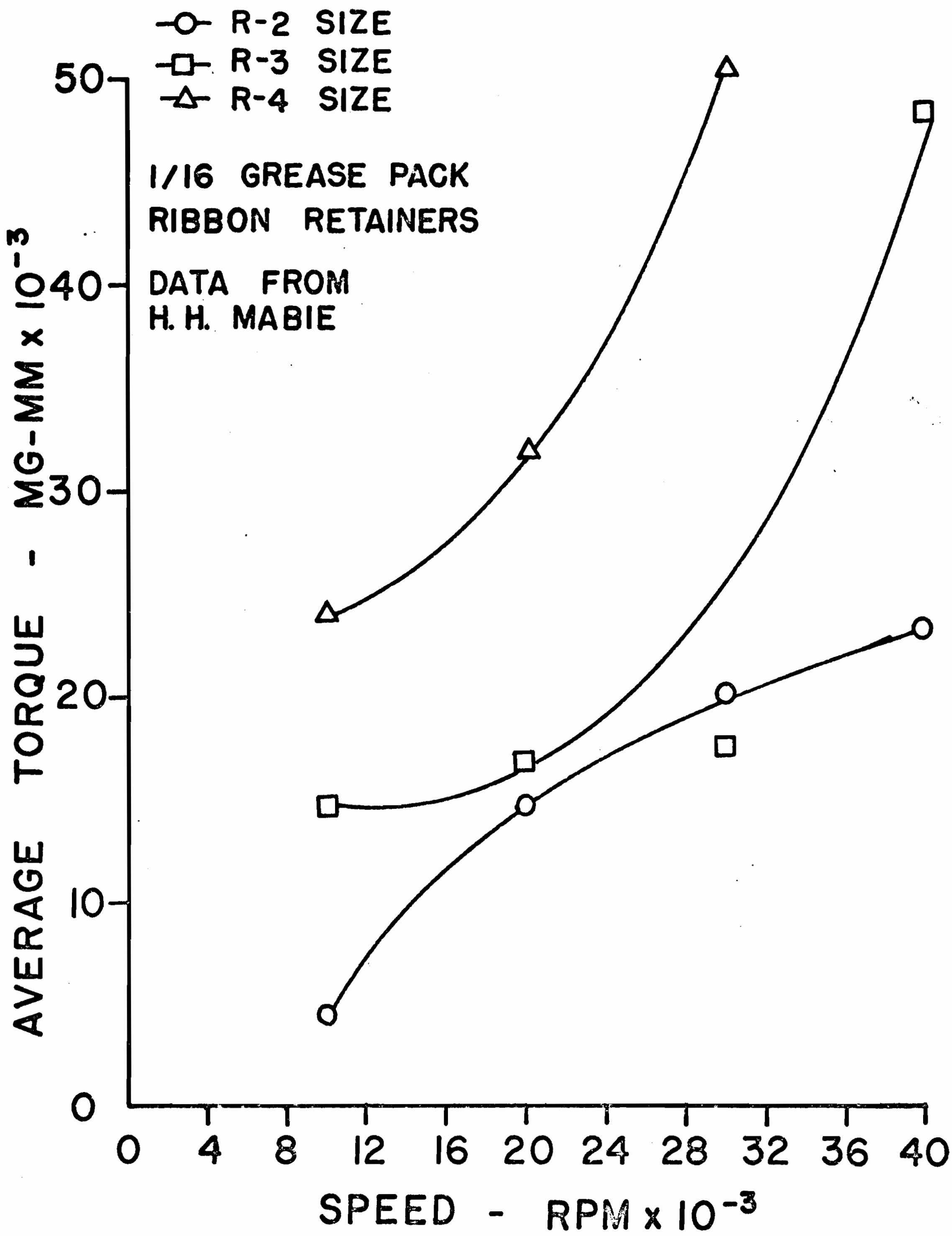

FIG. 3 SIZE EFFECT ON RUNNING TORQUE OF GREASE LUBRICATED BEARINGS

for grease lubricated bearings. A 50% increase in speed from 20,000 rpm to 30,000 rpm results in a 30 to 50% increase in running torque while a 100% speed increase results in torque increases of 60 to 200%. Considering changes in bore, a 50% increase in bore produces a 30 to 200% increase in running torque while a 100% increase in bore results in a 100 to 400% increase in running torque. From these observations, it appears that Moore and Jones' conclusions about the relative importance of speed and bore are correct in the instrument bearing size also. Their data showed also that the oil flow rate through the bearings had relatively little effect on friction torque. No data is available for comparison here since instrument ball bearings are nearly always lubricated with a small quantity of lubricant initially and with no further additions.

Some preliminary testing was done by Graneek and Wunsch[11] in England in conjunction with the construction of a high speed ball bearing tester. Their testing was done on oil lubricated 5-mm bore ball bearings which correspond approximately to R-3 instrument bearings. Their tests ran from near zero to 50,000 rpm with various axial and radial loads. It is interesting to observe that their torque versus speed plots showed a distinct dip in the curves in the 20,000 to 30,000 rpm region. The bulk of their remarks were concerned with the construction of the test equipment so little was mentioned about this unusual behavior. The same phenomenon was also noted in separate investigations by Mabie and Clarke. None of these investigators offer an explanation for this behavior and likewise none of the theorists have predicted the existence of this dip in the torque curve.

One of the most recent and comprehensive studies of the theoretical approach to bearing analysis is presented by Harris[1] of SKF Industries. In his book, Harris considers the factors contributing to bearing friction torque and derives the following empirical expression:

$$\tau = f_1 F_b d_m + 1.42\times10^{-5}(v_o n)^{2/3} d_m^3$$

where the symbols represent the following:

τ = total friction torque

f_1 = coefficient based on equivalent load, bearing load capacity, and bearing configuration

F_b = equivalent load

d_m = mean diameter of the bearing

v_o = kinematic viscosity of the oil

n = rotational speed

This expression was evaluated for several values of speed for a given loading and compared with the experimental results for R-3 bearings obtained by Clarke. The comparison is shown in Fig. 1. It is seen in this comparison that the empirical results are higher than the experimental results by an increasingly greater amount up to a factor of 5.

Also, there seemed to be a difference in the trends of the two curves under study. Harris assumed an oil mist continuously injected into the bearing as the mode of lubrication. This method would introduce considerably more lubricant to the bearing than would be the case with initial lubrication only. This may partially explain the high torques estimated by his empirical method.

As a further comparison, Harris' formula was compared with experimental results from tests on R-2 bearings run at approximately one revolution per minute. The tests showed an average running torque of 817 mg-mm. Harris' method predicted a running torque of 430 mg-mm. Again, the absolute difference in torques is not very large, but the percentage difference is quite significant. Thus it may be concluded that Harris' formula is not accurate or usable in the instrument ball bearing range.

Thus far, the discussion has served to point up the difficulties in predicting friction torque of instrument ball bearings at high speed by any published empirical methods. This indicates a need for the development of an analytical or empirical expression for use primarily in the instrument ball bearing range based on common design parameter inputs such as bearing size, radial and axial load, and rotational speed.

III. INVESTIGATION

A. Experimental Test Program

In considering the development of an expression to determine the high-speed running torque of instrument ball bearings, it was necessary to have a reasonably broad base of experimental data against which to compare the mathematical results. The size of this base determined the scope of the experimental investigation.

One of the factors considered important in this investigation was bearing size. For practical purposes it was thought best to use the three most common instrument bearing sizes which also gave a significant difference between sizes. The principal dimensions of the bearings tested are shown in Table 1.

Table 1 Bearing sizes

Designation	R-2	R-3	R-4
Outside Diameter (inches)	3/8	1/2	5/8
Bore (inches)	1/8	3/16	1/4

A relative size comparison of the three bearing sizes is shown in Fig. 4.

A secondary consideration in this selection was that other data was available on these size bearings which permitted some comparisons to be made. These comparisons were desirable to help validate the accuracy of the test

Fig. 4 Size comparison of Test Bearings

method used.

In an effort to reduce the number of unnecessary variables, it was decided to test bearings only with the same type of retainers. It had already been shown in work by Mabie that different type retainers alone could significantly affect bearing torques. To make the data more meaningful for design purposes, ribbon type retainers were chosen since this is the most prevalent type of retainer in precision applications.

In considering limits on the speed range, 1000 rpm was considered sufficiently low without duplicating other low speed studies. The upper limit was set at 40,000 rpm in deference to design considerations of the test equipment. Within these limits, reference speeds were selected approximately every 4000 rpm.

Only one lubricant was used in the test program. Greases were discarded because of their tendancy to produce erratic results. Also, oil is the accepted lubricant for instrument size ball bearings. The specific oil used was Univis P-38 conforming to MIL-L-6085A specifications.

One of the more difficult decisions made was the size of the loads to be placed on the bearings. It was thought desirable to use the same loads on all three sizes of bearings in order to facilitate comparison. Thus it was necessary to put a reasonably representative radial and axial load on the largest size bearing without overloading the smallest bearing and test rig. The loads selected are shown in the following table.

Table 2 Bearing Loads

Radial Load (grams)		50	100	200
Axial Load (grams)	0	50	100	200

The axial and radial loads were combined so that every axial load was applied with each radial load. The 0 gram radial load was omitted because of inherent limitations of the test apparatus.

B. Bearing Handling

One of the operations considered essential to obtaining accurate consistent data was proper bearing handling. In an effort to reduce the number of uncontrolled variables, it was decided to use the same set of sample bearings for all test loads. Six bearings of each of the three sizes were used for the entire test program. This of course necessitated cleaning and re-oiling each of the bearings before testing at a different loading.

The test bearings were of the shielded type so that it was necessary to remove the shields before cleaning. Each of the bearings was labelled and retained its identity throughout the test program.

The actual cleaning was a three-part program.

Initially, the bearings were cleaned with benzene while immersed in an ultrasonic cleaner. This operation was to remove the bulk of the remaining lubricant. Next, the bearings were ultrasonically cleaned in acetone. The acetone removed the last traces of lubricant as well as the benzene. In addition, the acetone left very little residue upon evaporating. The final cleaning was also accomplished ultrasonically but with a Freon solvent as the cleaning agent. The Freon was an excellent grease solvent and was noted for leaving no residue in the bearings. This was considered an effective method for removing all the lubricant from the bearings without introducing contaminants to the bearing races or balls.

It was next necessary to relubricate the bearings up to original specifications. The quantity of lubricant added was determined by averaging the quantity found in new bearings from the factory. The amount used is shown in Table 3.

Table 3 Quantity of Lubricant

Bearing Size	Lubricant Weight (grams)
R-2	0.0035
R-3	0.0076
R-4	0.0124

These quantities of lubricant were found to be approximately proportional to the dry weight of the bearings.

An analytical balance was used to weigh the bearings during cleaning and weighing operations. Lubrication was accomplished by adding oil to the races of the individual bearings with a micrometer syringe using a No. 27 hypodermic needle. The micrometer feature made it quite easy to regulate the oil quantities accurately.

At all times other than cleaning, lubricating, or testing, the bearings were kept in sealed glass containers to preclude the possibility of contamination. The above mentioned bearing handling equipment is illustrated in Fig. 5.

C. Apparatus

Power Source

The power source selected to drive the bearings during the torque tests was a simple Pelton wheel type of air turbine. The design for the turbine had already proved itself to be a stable and reliable device in similar applications by Mabie and Clarke. A detailed drawing of this turbine is shown in Fig. 6. To significantly reduce the down time when changing bearing sizes, and to reduce the possibility of damage to the turbine shafts, three turbines were constructed. The three turbines were identical except for the ends of the shafts which were each a different size to accommodate the three sizes of bearings to be tested. Thus when switching to a different size of bearing, the entire turbine unit was replaced

Fig. 5 Bearing Handling Equipment

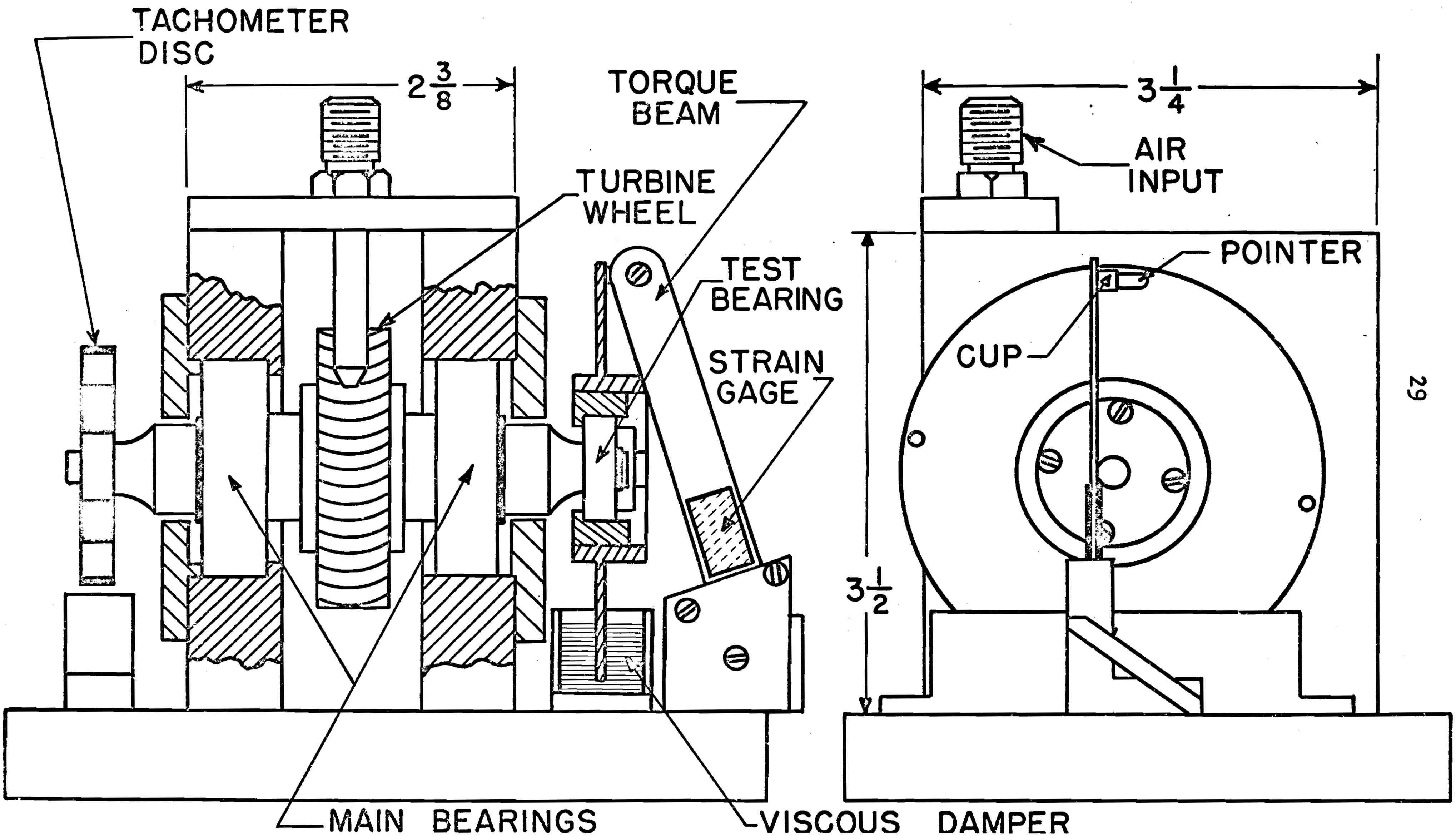

FIG. 6 HIGH SPEED TORQUE TESTER

with another unit with the proper size shaft.

Torque Sensor

The technique used to sense and measure the running torque was a method previously developed and proven in work by Clarke[5]. With this approach, the test bearing was mounted in a disk which was statically balanced so that it would have no preferred orientation. Thus, rotation of the inner race of the bearing could eventually cause rotation of the outer race and the attached disk. To restrain this rotation, the disk was equiped with an L-shaped conical tipped pointer mounted near the periphery of the disk and pointing in the direction of intended rotation. Fixed to the base of the test stand was a long thin beam. Attached to the free end of this cantilever beam was a small nylon cup, shaped to accept the nose of the pointer on the disk. The pointer and cup in operating position are shown in Fig. 7. Thus, the cantilever beam prevented the rotation of the disk while allowing some angular displacement to take place. It was decided that the maximum angular displacement that could be permitted was 5°. This is the generally accepted upper limit for which the sine and tangent are considered equal. It was further assumed that the maximum anticipated torque would be 50,000 mg-mm. It was known from the assumed disk diameter of three inches that the 5° limit would correspond to a linear displacement of

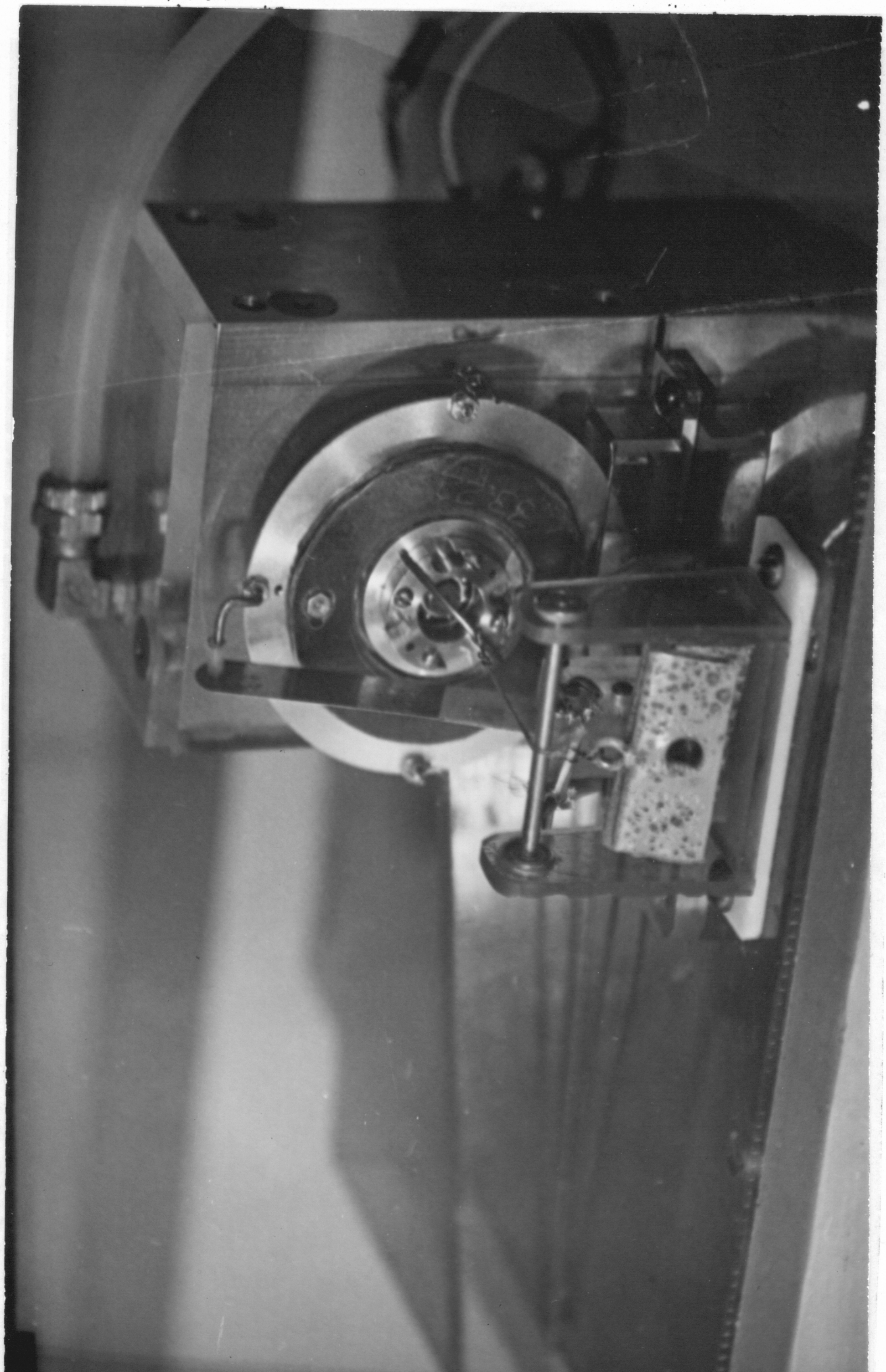

Fig. 7 Torque Tester and Test Turbine

approximately 1/8 inch at the circumference of the disk. From these considerations, a suitable beam was designed. The final design was a stainless steel beam with the dimensions 0.004 x 0.375 x 2.0 inches. A beam of these dimensions, subjected to the anticipated loads would experience a maximum of 4500 psi at the base of the beam.

Strain gages were mounted near the base of the beam with one gage on each flat side of the beam. The gages used were Baldwin SR-4 type C-7 gages. These gages had a short gage length which was considered desirable in order to take advantage of the relatively high strains which occur at or near the base of a cantilever beam. In addition, C-7 gages have a gage factor of approximately 3.59 which is relatively high. The high gage factor means that the gage will undergo a relatively high resistance change for a given amount of strain. Moreover, the gages had a resistance of approximately 499 ohms which is relatively large. This large resistance meant that for the one-volt maximum bridge used, there would be very little Joulian heating of the gages.

The gages were mounted to the beam with Duco cement. The cement bonding was tested with a very accurate bridge circuit. The gage resistances were measured before and after being subjected to bending. Since the gage resistances were identical before and after bending, it was determined that there were no residual stresses in the gages. Residual stresses would have indicated a poor bond to the beam. The sensitivity of the measuring bridge was 1 part in 50,000. The gage to ground resistance was also checked and found to be essentially infinite for both gages.

Disk Stabilizer

The running torque of ball bearings is not a constant value but rather a fluctuating torque superimposed on some average torque. This characteristic is undesirable when attempting to measure the average running torque particularly if the measuring device tries to follow the oscillations. In attempting to damp the oscillations of the torque disk caused by the torque fluctuations, the lower edge of the disk was immersed in a viscous fluid. The fluid selected for the damper was Dow Corning Fluid 200 which had a viscosity of 60,000 centistokes. This fluid was found to give the desired combination of effective damping and response to rotational acceleration for the torque disk. The stabilizer can be seen at the bottom of the disk in Fig. 8.

Speed Measurement

The rotational speed of the turbine wheel was measured by two methods. One method was also used to drive the X-Y plotter while the second method served as an independent

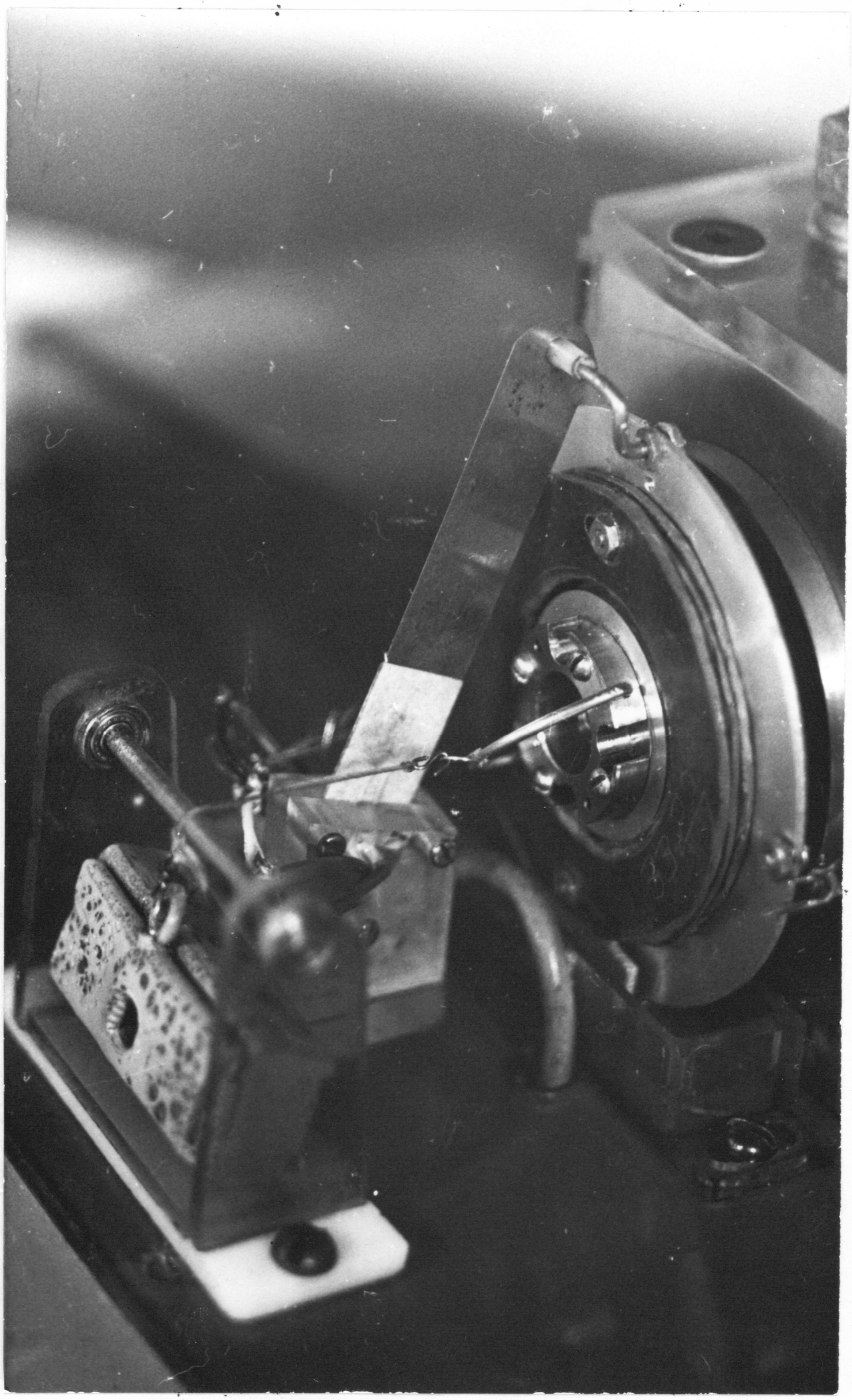

Fig. 8 Axial Loading Mechanism and Stabilizer

check on the first.

The first method employed a small magnetic pickup which was pulsed by ferrous inserts screwed into an aluminum disk. The output of the magnetic pickup was an a-c voltage which was directly proportional to the rotational speed of the disk which was rotating with the turbine shaft. The output voltage was used to drive the x-function of the X-Y plotter. The scale factor used was 4000 rpm per inch on the x-axis. Calibration was checked with a Strobotac.

The second measuring system depended on the same small aluminum disk mounted on the non-testing end of the turbine shaft. This disk is shown in Fig. 6. The circumference of the disk was marked with six light areas alternating with six dark areas. A small light was directed at the periphery of the disk and a photoelectric cell was positioned so as to detect the reflected beam from the light areas. The signal pulses from the photoelectric cell were the input to a Hewlett-Packard digital counter. The timing gate of the counter was set at one second. Thus with the counter registering six pulses per revolution for a period of one second, the speed in rpm was obtained by multiplying by 60 seconds and dividing by 6 pulses per revolution. The net result being that the counter readings simply had to be multiplied by ten to obtain the rotational speed in rpm. The tachometer circuitry is shown in Fig. 9 and an overall view of both speed measuring units is shown in Fig. 10.

Speed Regulation

One of the problems encountered in previous investigations by Mabie and Clarke was a uniform method for regulating and increasing the speed of the turbine. In an effort to remove this variable from the test program, an air control system was developed which accelerated the turbine from 0 to 40,000 rpm with approximately a linear increase in rotational speed with time. The final arrangement used accelerated the turbine to the maximum test speed in 110 seconds. A flow diagram for the system is shown in Fig. 11. The air at line pressure of 90 psi was allowed to flow into an 18 cubic-foot tank when the throttle valve was opened. Air which escaped from the tank was used to drive the turbine. However, due to the size of the tank, the pressure build-up in the tank was slow. This slow pressure build-up accounted for the slow uniform increase of rotational speed of the turbine. When the turbine speed reached 40,000 rpm, the throttle lever was pulled to the closed position. This lever movement stopped the flow of air to the damper tank and switched the flow of air from the tank to the turbine to a line vented to the atmosphere. The tank required approximately two minutes to bleed back to atmospheric pressure and the system was

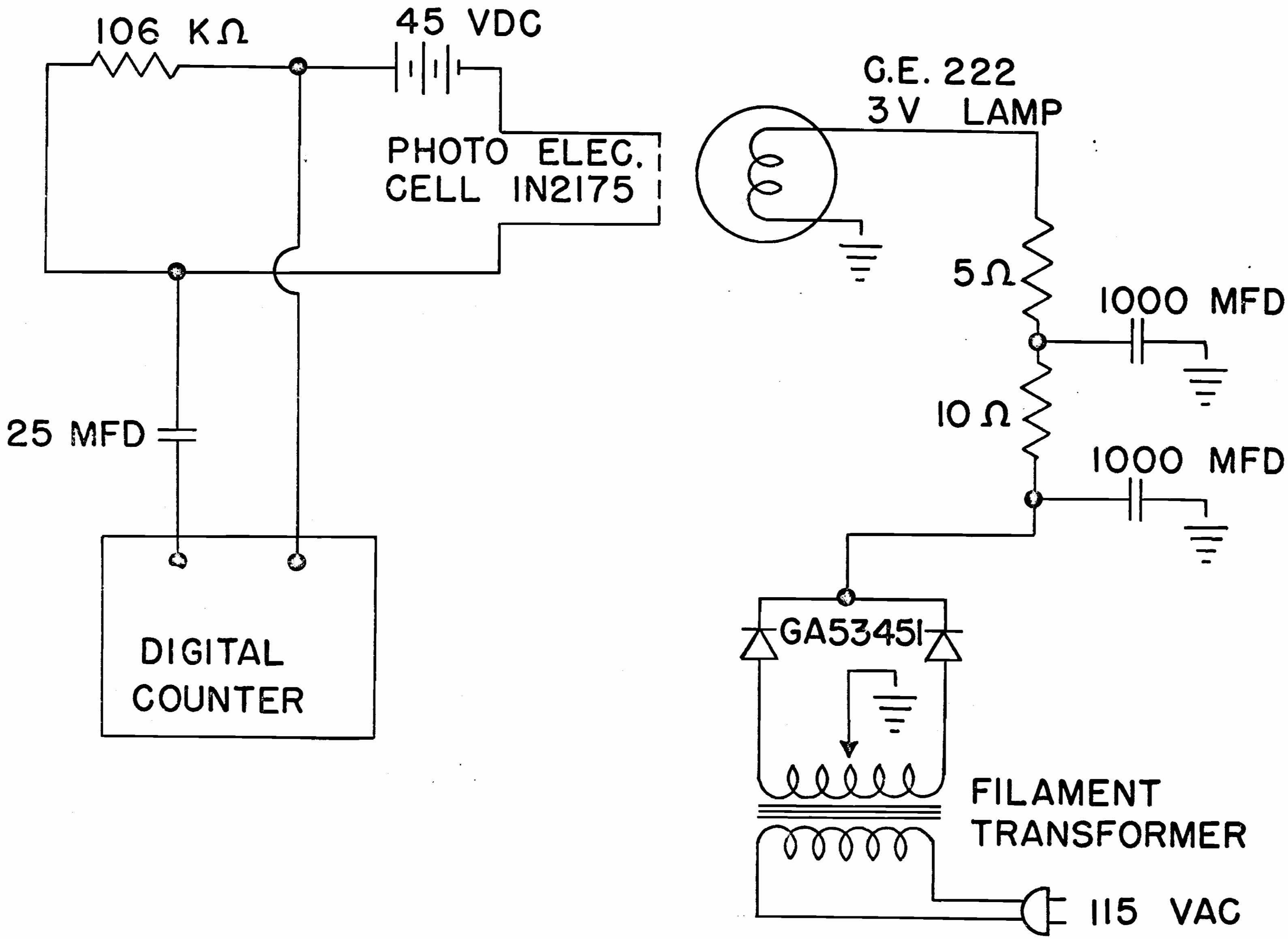

FIG. 9 TACHOMETER ELECTRICAL SCHEMATIC

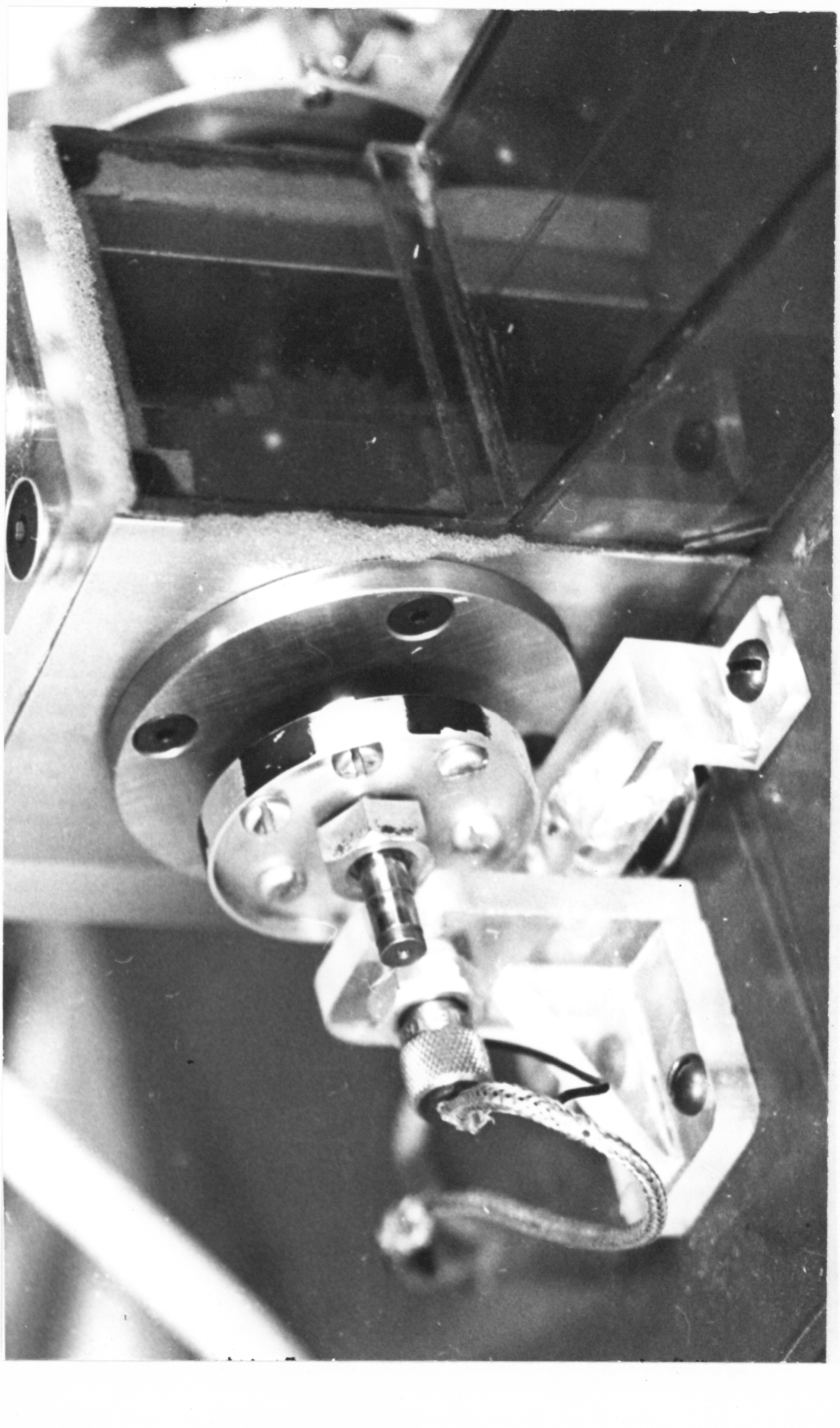

Fig. 10 Speed Measuring Units

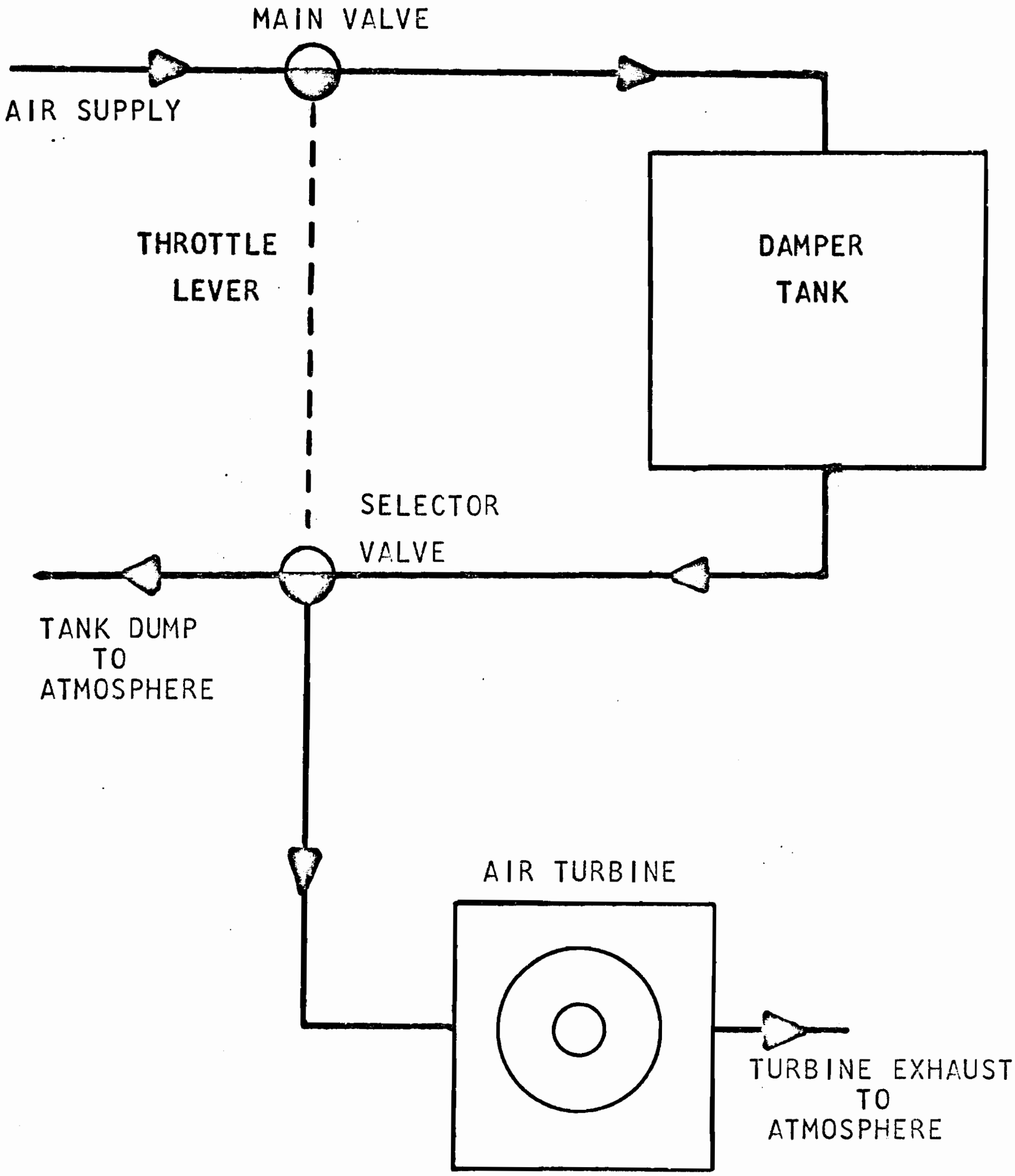

Fig. 11 Air Flow Diagram

ready for another test run. The arrangement of the actual regulating mechanism is shown in Fig. 12.

Strain Measurement

The strain measurement and subsequently the torque measurement was accomplished by means of a Bruel and Kjaer Model 1516 strain indicator. A four-arm bridge was used with the two active strain gages constituting two adjacent arms of the bridge. The common point between the two active gages was also connected to the shielding used on the strain gage leads. It was imperative that all leads be shielded and the beam be grounded since measurements were being made near the limit of sensitivity of the instrument. Preliminary investigation had shown that a Baldwin-Lima-Hamilton SR-4 Strain Indicator used with an oscilloscope was not capable of measuring the small resistance changes of the gages. The Bruel and Kjaer instrument, however, proved capable of producing a good indication of strain.

Torque Speed Records

A Moseley Autograph x-y plotter was used with the Bruel and Kjaer instrument to record the output from the strain amplifier. The strain output was recorded on the y-axis of the plotter. With the turbine speed recorded

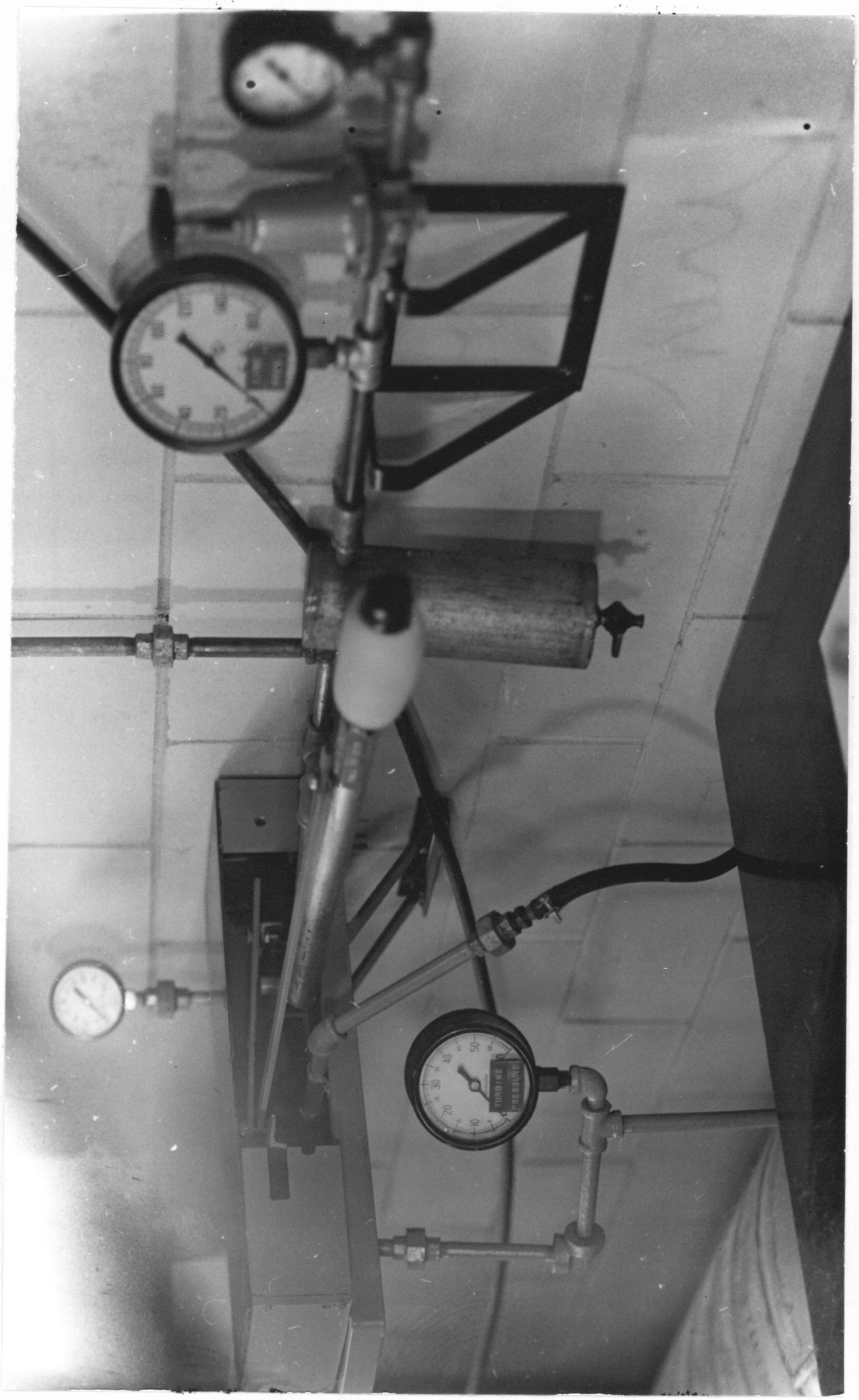

Fig. 12 Air Regulation Mechanism

simultaneously on the x-axis, the result was a continuous torque versus speed plot. The plots were made on 11 x 15 inch sheets of paper with the original runs on one side and re-runs on the reverse side with one sheet for each bearing and each load. This recording system facilitated later interpretation of the raw data. A sample plot with an overlying transparent scale is shown in Fig. 13.

Axial Loading Mechanism

The method for axially loading the bearings was a system previously proven in experimental work by Clarke. The system simply employed a dead weight on a string suspended across a ball-bearing mounted shaft with the bearing-cup cap restraining the opposite end of the string. A bar and ring attachment was used on the cup cap to assure that the axial load was applied directly through the center of the test bearing. A small V-groove was machined into the roller shaft to maintain alignment of the loading string. Although the loading system did introduce some small amount of resistance to rotation, it was of no consequence since the calibration system automatically compensated for irregularities due to a particular loading configuration. The axial loading mechanism is shown schematically in Fig. 14.

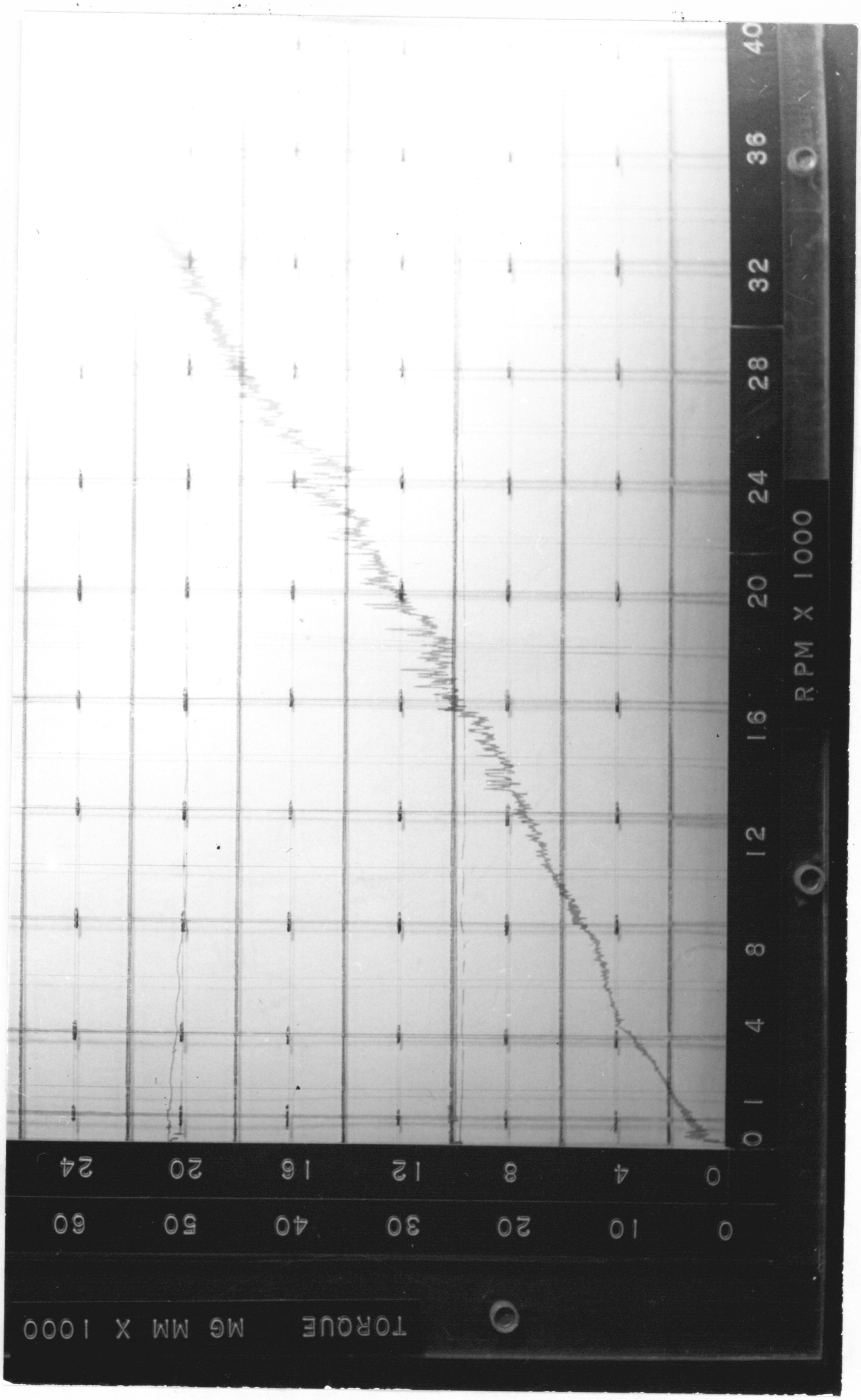

Fig. 13 Torque versus Speed Experimental Plot

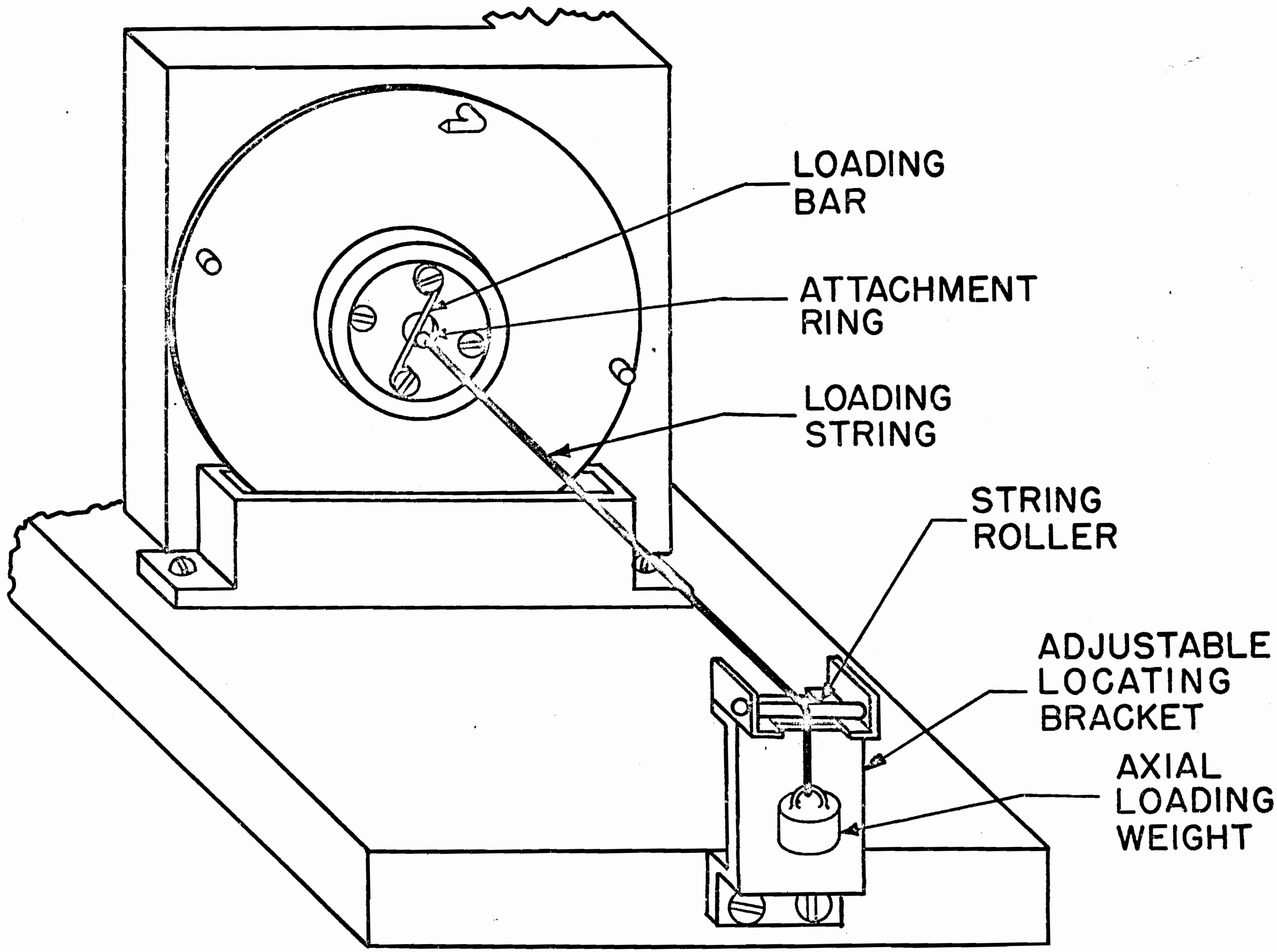

FIG. 14 AXIAL LOADING MECHANISM

Radial Loading Mechanism

The bearing cup assembly and torque disk assembly together weighed approximately 50 grams. This alone accounted for the lowest value of radial load placed on the test bearings. Higher values of radial load were accomplished by bolting carefully weighed and cut lead rings to the torque disk. It was necessary to use five rings to reach the maximum value of radial load which was 200 grams. This method proved quite satisfactory in use. The radial loading weights attached to the disk are shown in Fig. 8.

Safety Measures

The primary hazard considered was the possibility of a failure of the rotating parts at high rotational speeds. To best protect against human injury, it was decided to cover the entire turbine and torque sensing gear with a strong and absorbent hood. The hood case was constructed from 1/4 inch steel plates on both ends and the top. The sides were 1/8 inch steel plates. The steel gave the hood sufficient strength. Impact absorption was built in by lining the entire hood with 1/2 inch plywood. The sides of the protective hood were fitted with 4-inch lucite ports to permit observation of the test apparatus during operation. The hood was counterweighted with a pulley system to permit ease of handling. The overall test area and protective hood are shown in Fig. 15.

D. Procedure

Calibration

One of the most important phases of each test series was the calibration of the torque sensing unit. This was particularly necessary to compensate for any resistance introduced by changes in test loading.

Calibration was carried out by the use of dead weights hung on the periphery of the torque disk. A calibration weight was a specially made lead weight and hook of the proper weight so as to produce even torque forces on the disk. The torque values thus produced were 5000 mg-mm, 10,000 mg-mm, 20,000 mg-mm, and 30,000 mg-mm. The weights could be used individually or in combination, making it possible to calibrate in 5000 mg-mm intervals from 0 to 65,000 mg-mm. The torque disk with a calibration weight attached is shown in Fig. 16.

For the actual calibration, the test bearing was mounted in the torque disk and the assembly fitted on the turbine shaft. The torque sensing beam was carefully aligned with the pointer on the torque disk. A zero reading was taken with the pointer and cup just out of contact to assure a zero strain condition. A trace of this zero strain condition provided the x-axis for the plot. Next, the 5000 mg-mm calibration weight was hung

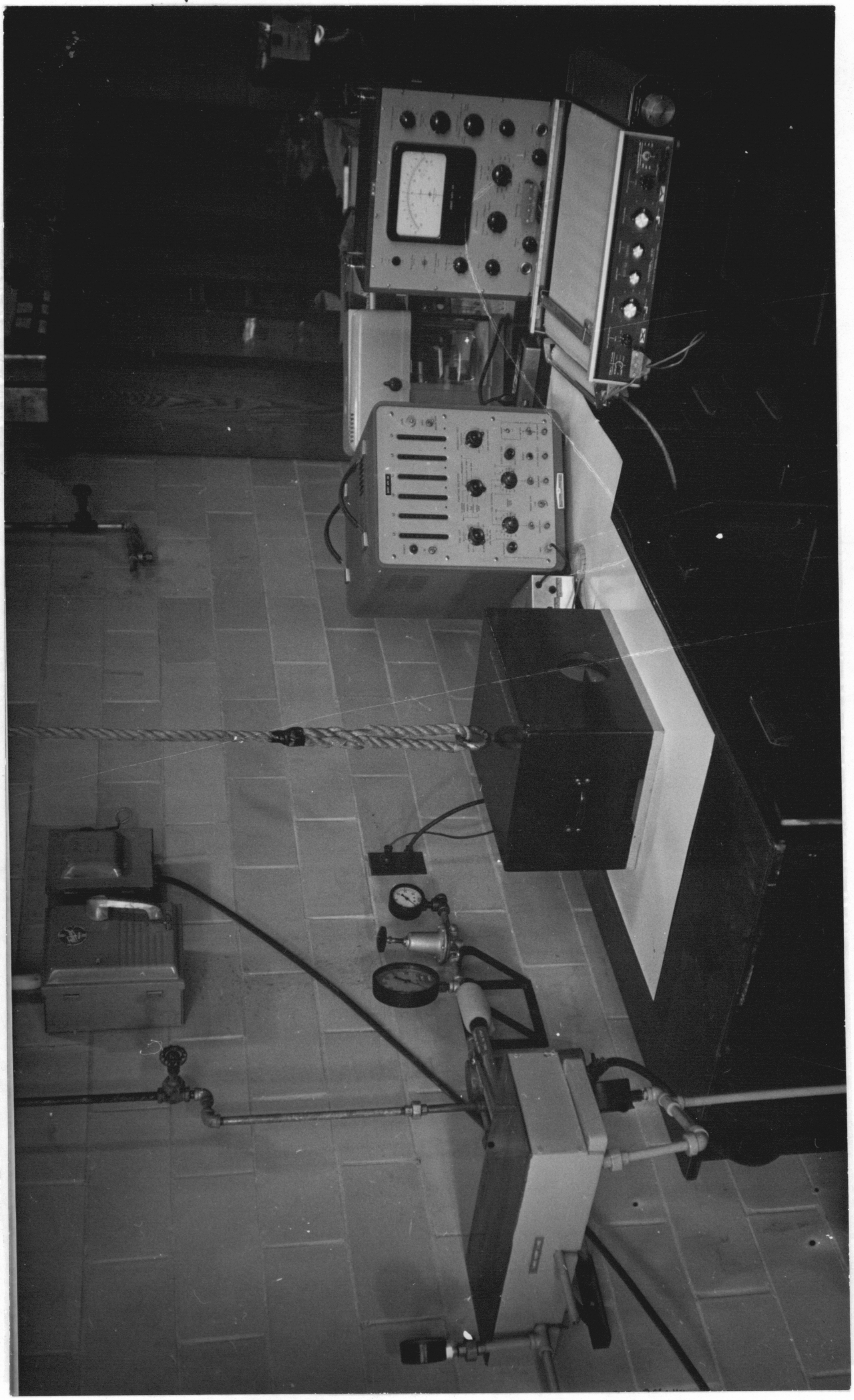

Fig. 15 Test Apparatus

Fig. 16 Calibration Weight in Use

on the calibration post. This produced a strain which was recorded on the y-axis. Additional weights were added and removed so as to increase the calibration torque in 5000 mg-mm increments up to the maximum value anticipated for the particular test. Finally, all the weights were removed and the zero reading rechecked. In no case were less than three points used for calibration and in several cases as many as ten points were used. In all calibration checks, the recorder showed the indicated torque to be a linear function of the actual torque. The question of the effective lever arm of the calibration weight changing during small angular displacements proved to be no problem. At the maximum designed deflection, the change of the effective lever arm increased by less than 0.0015 inches. This corresponds to a change of 0.01% which was at least one order of magnitude smaller than the recorder error alone. The calibration geometry is shown in Fig. 17. The primary advantage of this calibration system was that the calibration was carried out with all variables included for a given loading configuration thereby making the actual loading mechanisms less critical on the final results.

Testing

The speeds used for the evaluation of the data ranged between 1,000 and 40,000 rpm; however, the raw data from the

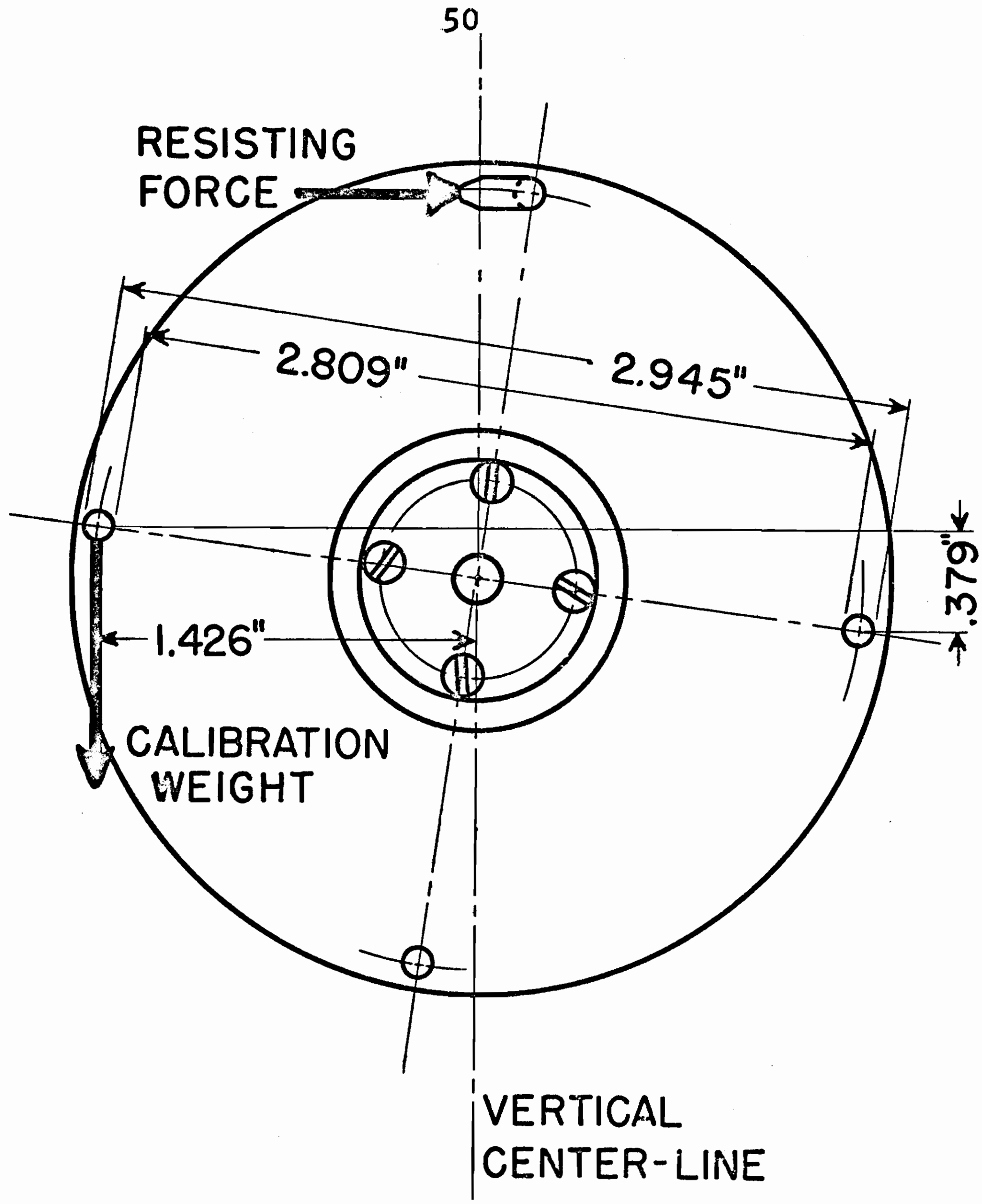

FIG. 17 CALIBRATION GEOMETRY

test runs began at 0 rpm. The lubrication criteria required that the bearings be run first with factory oil specifications and then re-run without re-oiling. In practice, all six bearings were tested in the original run before they were re-run. For each bearing size, the test program began with the lightest loading, 50 grams radial and 0 grams axial, and the load increased by adding axial loads up to 200 grams. The radial load was then increased and the same progression followed with the axial loading. Generally, there was a time period of one-half hour between the original run and the re-run for a given bearing at a specific loading. Of course, the bearings were cleaned and re-lubricated for each new load configuration.

When recording the test data, the same origin relative to the edge of the paper was used. This greatly facilitated evaluation of the data by means of a simple transparent overlay which had the appropriate axes and scales permanently inscribed. The overlay and a sample plot are shown in Fig. 13.

Because of identical test loadings and test speeds, some R-3 bearing data used in this investigation were obtained in tests by Edwards.[12] Data from this source included runs for 100 and 200 grams radial load and all axial loads for those two radial loads.

To ascertain whether there was a significant frictional heating effect, a small thermocouple was attached to the stationary outer race of the bearing during test runs with 200 grams radial and 200 grams axial load. One bearing of each size was tested up to a speed of 45,000 rpm. The most temperature rise recorded was 1.86° F for the R-4 bearing. This was considered to be negligible and no further consideration was given to heating effects.

E. List of Equipment

1. Strain Gage Apparatus
   Model 1515
   Serial No. 132912
   Bruel and Kjaer, Naerum, Denmark
   Use: To measure and amplify the strain signal from the torque sensor.

2. Moseley Autograph
   Model 2 D X-Y Plotter
   Serial No. 1217
   F.L. Moseley Company, Pasadena, California
   Use: To record and amplify the torque signal from the strain indicator simultaneously with the output signal from the magnetic speed pick-up.

3. Electronic Counter

   Model 521 A

   Serial No. 120-04376

   Hewlett-Packard, Palo Alto, California

   Use: To count and display the speed of the air turbine.

4. Gramatic Balance

   Model H 5

   Serial No. 113176

   Mettler Instrument Corp., Hightstown, New Jersey

   Use: To weigh the test bearings and the amount of lubricant added to the bearings.

5. Disontegrator System Forty

   Model G-40CIP

   Serial No. 26733

   Ultrasonic Industries Inc., Hicksville, N.Y.

   Use: To clean the test bearings.

6. Potentiometer

   Model 2745

   Serial No. P-3522

   Honeywell, Denver, Colorado

   Use: To measure test bearing temperatures.

IV. DATA AND RESULTS

The data and results are presented in three forms. The experimental data is presented in tabular form in Tables 4 through 39. The empirical values of running torque are listed in Tables 40 through 48. A graphical comparison of the experimental and empirical running torques is shown in Figures 18 through 53.

A. Experimental Test Results

Tables 4 through 39 list the average running torques from experimental tests on samples of six bearings as well as the sample standard deviation for the test sample at each reference speed. All of this information is given for the original run and the re-run for each test loading. The average torques for experimental values were calculated as the arithmetic mean of the torque values taken at each speed point.

$$\text{Average Torque} = \overline{X} = \frac{X_i}{N}$$

The sample standard deviations were calculated by normal statistical methods.[13]

$$\text{Sample Standard Deviation} = \sqrt{\frac{X_i^2}{N} - \overline{X}^2}$$

where: X_i = individual torque values

N = number of torque values summed

$\overline{X}$ = average torque value

Table 4

TORQUE CHARACTERISTICS FOR R-2 BEARINGS, 0.0035 GM. OIL

50 GM RADIAL LOAD

0 GM AXIAL LOAD

SPEED (RPM)	ORIGINAL RUN AVERAGE TORQUE (MG-MM)	ORIGINAL RUN SAMPLE STANDARD DEVIATION (MG-MM)	RE-RUN AVERAGE TORQUE (MG-MM)	RE-RUN SAMPLE STANDARD DEVIATION (MG-MM)
1000	1166	235	550	214
4000	2000	408	1583	448
8000	2916	837	2000	-0
12000	4250	989	3250	803
16000	7250	1406	5833	1343
20000	8416	2280	6333	2134
24000	9666	2134	7000	1732
28000	11500	2140	6833	1343
32000	14000	2081	7666	1374
36000	15500	1707	8666	1699
40000	16166	1771	10166	1674

Table 5

TORQUE CHARACTERISTICS FOR R-2 BEARINGS, 0.0035 GM. OIL

50 GM RADIAL LOAD

50 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	1666	552	550	111
4000	4250	1626	2833	1105
8000	7166	2114	4750	1145
12000	9166	1674	6000	1000
16000	9166	1863	6333	745
20000	10666	1885	7000	1000
24000	11666	3197	8000	1632
28000	13333	2981	8333	1885
32000	12166	3670	7833	897
36000	12333	3496	8333	745
40000	13333	2494	9833	1213

Table 6

TORQUE CHARACTERISTICS FOR R-2 BEARINGS, 0.0035 GM. OIL

50 GM RADIAL LOAD

100 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	1416	448	1500	816
4000	3666	745	4166	1462
8000	6166	2034	5833	1950
12000	8166	2339	7166	1863
16000	8166	2266	7166	1863
20000	8500	2140	7833	1572
24000	8833	2266	7833	1950
28000	10000	2000	8166	2793
32000	10666	2134	9000	2886
36000	11833	1674	10000	3000
40000	12000	2081	10666	2867

Table 7

TORQUE CHARACTERISTICS FOR R-2 BEARINGS, 0.0035 GM. OIL

50 GM RADIAL LOAD

200 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	3666	2134	3416	2422
4000	7166	2671	5166	2477
8000	9666	3349	6666	3197
12000	12000	3366	8166	2910
16000	13333	2624	9666	3197
20000	15166	2544	10666	3681
24000	14500	3685	11166	4099
28000	14500	2500	11000	2768
32000	13666	2981	10666	3197
36000	14000	3511	10833	3578
40000	15833	3077	13166	3578

Table 8

TORQUE CHARACTERISTICS FOR R-2 BEARINGS, 0.0035 GM. OIL

100 GM RADIAL LOAD

0 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	1416	448	1166	372
4000	2833	372	2833	687
8000	4500	500	3666	745
12000	6000	816	5000	577
16000	8000	1632	6166	372
20000	11000	1154	8166	687
24000	12500	1500	10666	1105
28000	15000	1914	12500	1118
32000	16666	2494	14500	2813
36000	18666	2748	17000	3958
40000	19833	3484	19000	4725

Table 9

TORQUE CHARACTERISTICS FOR R-2 BEARINGS, 0.0035 GM. OIL

100 GM RADIAL LOAD

50 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	2083	1426	2000	577
4000	3583	1592	4250	1108
8000	5916	1765	5916	1057
12000	8166	1950	7083	1169
16000	10250	2076	8000	1154
20000	12000	2466	9666	1213
24000	13833	3023	10750	1865
28000	16333	2687	13166	2544
32000	18750	2545	15500	3685
36000	21000	3055	18000	3872
40000	22833	3847	20333	4346

Table 10

TORQUE CHARACTERISTICS FOR R-2 BEARINGS, 0.0035 GM. OIL

100 GM RADIAL LOAD

100 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	2833	687	2050	1269
4000	5083	931	4833	1840
8000	8250	1282	7833	1771
12000	10833	2094	9750	1750
16000	11666	1699	10583	1693
20000	13416	2168	11500	2160
24000	15166	3023	12250	2765
28000	17000	2943	13500	2629
32000	14666	6523	15083	3790
36000	16333	7431	17333	3944
40000	20666	5055	19833	5047

Table 11

TORQUE CHARACTERISTICS FOR R-2 BEARINGS, 0.0035 GM. OIL

100 GM RADIAL LOAD

200 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	5916	1643	3750	1464
4000	8000	2160	6333	2134
8000	10333	2321	8583	2316
12000	13000	2236	11000	2000
16000	14083	2620	12333	2494
20000	15750	2479	15000	2380
24000	17333	2671	16166	2733
28000	19000	3651	18166	3184
32000	21666	4642	21000	4690
36000	22666	4149	22750	4356
40000	25000	3915	24500	4645

Table 12

TORQUE CHARACTERISTICS FOR R-2 BEARINGS, 0.0035 GM. OIL

200 GM RADIAL LOAD

0 GM AXIAL LOAD

SPEED (RPM)	ORIGINAL RUN AVERAGE TORQUE (MG-MM)	ORIGINAL RUN SAMPLE STANDARD DEVIATION (MG-MM)	RE-RUN AVERAGE TORQUE (MG-MM)	RE-RUN SAMPLE STANDARD DEVIATION (MG-MM)
1000	1333	471	1166	372
4000	2416	1304	2833	897
8000	4916	2168	5000	2449
12000	7166	3287	7166	2192
16000	9833	2967	9166	3287
20000	11166	3236	11166	3484
24000	13333	3636	12833	3933
28000	16666	3636	16166	4179
32000	19333	4606	18000	4434
36000	20833	5550	18833	4913
40000	22166	6517	20666	5878

Table 13

TORQUE CHARACTERISTICS FOR R-2 BEARINGS, 0.0035 GM. OIL

200 GM RADIAL LOAD

50 GM AXIAL LOAD

SPEED (RPM)	ORIGINAL RUN AVERAGE TORQUE (MG-MM)	ORIGINAL RUN SAMPLE STANDARD DEVIATION (MG-MM)	RE-RUN AVERAGE TORQUE (MG-MM)	RE-RUN SAMPLE STANDARD DEVIATION (MG-MM)
1000	2666	1105	2166	372
4000	4000	1414	4166	897
8000	5833	1674	6833	897
12000	7166	1863	9166	2034
16000	8833	4139	10000	4041
20000	11083	5495	11250	4723
24000	13083	5570	11750	4862
28000	13500	6048	12166	4801
32000	14250	6149	13666	5676
36000	16000	7280	15083	7014
40000	18000	8144	18166	8989

Table 14

TORQUE CHARACTERISTICS FOR R-2 BEARINGS, 0.0035 GM. OIL

200 GM RADIAL LOAD

100 GM AXIAL LOAD

SPEED (RPM)	ORIGINAL RUN AVERAGE TORQUE (MG-MM)	ORIGINAL RUN SAMPLE STANDARD DEVIATION (MG-MM)	RE-RUN AVERAGE TORQUE (MG-MM)	RE-RUN SAMPLE STANDARD DEVIATION (MG-MM)
1000	3166	897	2666	942
4000	4500	1500	5333	1972
8000	6500	2380	7750	3078
12000	8416	3114	9750	4180
16000	9666	4149	10166	5273
20000	11333	4570	11833	6039
24000	12166	4633	11916	6058
28000	12916	4419	12083	6085
32000	14416	3790	13500	6075
36000	17166	3933	15000	5859
40000	20333	4268	18333	6420

Table 15

TORQUE CHARACTERISTICS FOR R-2 BEARINGS, 0.0035 GM. OIL

200 GM RADIAL LOAD

200 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	4333	2748	2666	1699
4000	5833	2910	4833	2426
8000	9000	2886	8000	3316
12000	12333	3197	10000	3316
16000	14833	3184	12666	3771
20000	16916	4086	14666	4678
24000	18333	3944	17333	5343
28000	20166	3624	18500	6048
32000	21833	2266	21166	5756
36000	25500	2692	24666	6046
40000	30000	4618	29166	4775

Table 16

TORQUE CHARACTERISTICS FOR R-3 BEARINGS, 0.0076 GM. OIL

50 GM RADIAL LOAD

0 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	3150	627	1416	940
4000	6775	953	2233	1041
8000	7883	700	3200	1404
12000	7591	1068	3516	1041
16000	6875	1143	4966	980
20000	5858	704	5400	932
24000	5383	1330	5366	1367
28000	5841	1232	6050	892
32000	7658	636	6700	529
36000	8575	385	9733	286
40000	10066	293	12633	359

Table 17

TORQUE CHARACTERISTICS FOR R-3 BEARINGS, 0.0076 GM. OIL

50 GM RADIAL LOAD

50 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	3791	652	2716	252
4000	11662	649	4325	647
8000	17770	1350	5450	1602
12000	14183	1371	5575	1077
16000	9058	1224	7250	1282
20000	9687	529	8991	1156
24000	10925	781	10241	806
28000	10700	2240	10958	1326
32000	13833	1381	12758	859
36000	16166	1795	15125	314
40000	17683	1228	17358	958

Table 18

TORQUE CHARACTERISTICS FOR R-3 BEARINGS, 0.0076 GM. OIL

50 GM RADIAL LOAD

100 GM AXIAL LOAD

SPEED (RPM)	ORIGINAL RUN AVERAGE TORQUE (MG-MM)	ORIGINAL RUN SAMPLE STANDARD DEVIATION (MG-MM)	RE-RUN AVERAGE TORQUE (MG-MM)	RE-RUN SAMPLE STANDARD DEVIATION (MG-MM)
1000	5366	687	2233	471
4000	13150	1500	5800	911
8000	16766	2536	7583	813
12000	11000	692	7608	1058
16000	10500	629	8375	1077
20000	10591	465	9008	1178
24000	12708	667	10833	1605
28000	14058	1032	11233	2365
32000	15483	1958	12366	2547
36000	18208	1954	14525	2550
40000	20791	1944	17816	2218

Table 19

TORQUE CHARACTERISTICS FOR R-3 BEARINGS, 0.0076 GM. OIL

50 GM RADIAL LOAD

200 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	6333	584	5066	359
4000	15166	604	9916	628
8000	19100	3428	12100	781
12000	16033	769	13983	267
16000	15000	1032	15933	935
20000	15566	1353	15933	928
24000	16766	1502	16533	1463
28000	17466	2083	17983	977
32000	18500	1668	18083	1535
36000	21800	2262	21400	1148
40000	23566	1448	23366	1174

Table 20

TORQUE CHARACTERISTICS FOR R-3 BEARINGS, 0.0076 GM. OIL

100 GM RADIAL LOAD

0 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	2733	280	900	387
4000	6816	343	2283	1301
8000	10333	692	3100	1131
12000	10650	1907	3333	851
16000	6150	1054	3666	628
20000	5883	841	4566	634
24000	7800	525	7366	555
28000	9550	1090	9366	1004
32000	11583	895	11150	848
36000	12816	474	12383	1026
40000	13733	743	13433	558

Table 21

TORQUE CHARACTERISTICS FOR R-3 BEARINGS, 0.0076 GM. OIL

100 GM RADIAL LOAD

50 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	4716	517	2700	957
4000	11433	1282	4950	512
8000	16883	1005	6916	338
12000	11983	1442	7516	429
16000	9416	1952	8700	1029
20000	9133	438	9566	926
24000	10733	1054	11933	1411
28000	13600	1394	14400	1290
32000	15766	1341	16200	1024
36000	16483	1435	16916	773
40000	17450	1229	17183	821

Table 22

TORQUE CHARACTERISTICS FOR R-3 BEARINGS, 0.0076 GM. OIL

100 GM RADIAL LOAD

100 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	5316	302	2666	414
4000	11000	1238	4766	767
8000	16400	2402	6533	729
12000	12133	1299	7450	793
16000	9566	760	8050	828
20000	10016	589	9083	633
24000	12566	1277	11833	1133
28000	15716	2391	15616	1337
32000	18283	2519	17800	781
36000	21700	2235	20733	1081
40000	26833	1478	25933	1205

Table 23

TORQUE CHARACTERISTICS FOR R-3 BEARINGS, 0.0076 GM. OIL

100 GM RADIAL LOAD

200 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	6200	472	2433	508
4000	13983	895	6016	548
8000	19425	2842	8225	463
12000	16816	1257	10808	808
16000	17100	1589	13416	587
20000	16558	1280	14050	869
24000	16954	1182	14316	982
28000	18300	1754	16408	626
32000	19766	1348	17433	590
36000	22408	1258	19283	498
40000	23016	893	20858	591

Table 24

TORQUE CHARACTERISTICS FOR R-3 BEARINGS, 0.0076 GM. OIL

200 GM RADIAL LOAD

0 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	3663	719	2225	695
4000	8866	349	3930	731
8000	12416	712	5025	622
12000	10400	1083	5905	613
16000	7875	516	7178	785
20000	8441	394	8561	845
24000	9483	233	9933	861
28000	10566	228	10708	864
32000	10791	458	10716	755
36000	10700	378	10983	483
40000	10533	659	10758	589

Table 25

TORQUE CHARACTERISTICS FOR R-3 BEARINGS, 0.0076 GM. OIL

200 GM RADIAL LOAD

50 GM AXIAL LOAD

SPEED (RPM)	ORIGINAL RUN AVERAGE TORQUE (MG-MM)	ORIGINAL RUN SAMPLE STANDARD DEVIATION (MG-MM)	RE-RUN AVERAGE TORQUE (MG-MM)	RE-RUN SAMPLE STANDARD DEVIATION (MG-MM)
1000	4300	391	3016	285
4000	8566	531	4866	521
8000	12566	377	6016	477
12000	13450	1157	7166	696
16000	9550	381	8150	694
20000	10166	679	9916	1086
24000	11700	571	11700	1112
28000	12283	380	12450	937
32000	12750	593	12633	760
36000	13600	685	13250	818
40000	14000	723	14183	539

Table 26

TORQUE CHARACTERISTICS FOR R-3 BEARINGS, 0.0076 GM. OIL

200 GM RADIAL LOAD

100 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	4883	323	3366	749
4000	11833	874	6333	1263
8000	14450	859	8183	1612
12000	13600	1665	10183	1330
16000	11450	1341	11383	1288
20000	12316	367	12433	1930
24000	13916	511	14366	1293
28000	14883	669	15433	825
32000	16883	942	16200	704
36000	18100	1093	17066	1030
40000	19183	1169	18300	991

Table 27

TORQUE CHARACTERISTICS FOR R-3 BEARINGS, 0.0076 GM. OIL

200 GM RADIAL LOAD

200 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	4933	997	3616	601
4000	13666	829	9000	1562
8000	16083	2002	12116	1693
12000	16666	1267	14316	1508
16000	14833	495	15150	1390
20000	15716	1010	15916	1688
24000	17100	1445	17816	1849
28000	18200	1653	18750	1945
32000	19883	2172	19650	2269
36000	21233	2752	21183	2041
40000	22000	2773	21966	2189

Table 28

TORQUE CHARACTERISTICS FOR R-4 BEARINGS, 0.0124 GM. OIL

50 GM RADIAL LOAD

0 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	3333	471	3500	763
4000	6333	1795	5833	1462
8000	11500	1707	9166	1462
12000	14500	3403	10166	2114
16000	18833	4561	12500	3354
20000	21666	3144	14833	1572
24000	18666	1490	15000	1414
28000	18333	1374	14500	1607
32000	17666	2134	14500	1607
36000	17666	2134	15666	2687
40000	20500	5090	18666	2748

Table 29

TORQUE CHARACTERISTICS FOR R-4 BEARINGS, 0.0124 GM. OIL

50 GM RADIAL LOAD

50 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	4166	372	4000	816
4000	11333	2211	9500	1979
8000	18500	2986	13500	3149
12000	20833	2608	16000	3000
16000	21166	2544	16333	2981
20000	21333	3986	16333	4027
24000	21500	4462	17000	4618
28000	20500	3730	16833	4740
32000	19500	4681	16833	4775
36000	19000	4725	16166	2608
40000	20333	5343	17333	3496

Table 30

TORQUE CHARACTERISTICS FOR R-4 BEARINGS, 0.0124 GM. OIL

50 GM RADIAL LOAD

100 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	5000	1632	4333	1795
4000	14000	3696	11833	1067
8000	21333	5527	19500	2432
12000	25333	5280	22333	2624
16000	26166	3760	23666	3299
20000	24833	4179	24000	3958
24000	24000	4163	24000	4434
28000	24166	4179	24000	4434
32000	23833	3933	23666	4533
36000	23833	3184	23333	4988
40000	24666	2426	23666	5217

Table 31

TORQUE CHARACTERISTICS FOR R-4 BEARINGS, 0.0124 GM. OIL

50 GM RADIAL LOAD

200 GM AXIAL LOAD

SPEED (RPM)	ORIGINAL RUN AVERAGE TORQUE (MG-MM)	ORIGINAL RUN SAMPLE STANDARD DEVIATION (MG-MM)	RE-RUN AVERAGE TORQUE (MG-MM)	RE-RUN SAMPLE STANDARD DEVIATION (MG-MM)
1000	6333	2134	6666	1374
4000	16333	5249	15500	3149
8000	26000	6298	24500	4153
12000	30500	4072	29666	3901
16000	32166	3387	31000	3415
20000	30833	3435	31166	4017
24000	30166	3287	31000	3958
28000	30166	3287	32000	4760
32000	30833	3184	32000	4760
36000	30333	2867	31500	4310
40000	30833	3184	31666	4533

Table 32

TORQUE CHARACTERISTICS FOR R-4 BEARINGS, 0.0124 GM. OIL

100 GM RADIAL LOAD

0 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	3333	1247	2166	1067
4000	8833	2608	4000	2236
8000	15666	3858	7833	3847
12000	20666	5705	11166	4879
16000	24833	5241	16833	5785
20000	26166	5727	21166	5428
24000	27333	5821	24166	5013
28000	27666	5185	27833	4099
32000	30333	2624	31000	2828
36000	31333	3349	31166	2910
40000	31666	3399	31833	3131

Table 33

TORQUE CHARACTERISTICS FOR R-4 BEARINGS, 0.0124 GM. OIL

100 GM RADIAL LOAD

50 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	5666	1699	5166	2034
4000	11833	2608	10500	3041
8000	17333	5990	16000	4358
12000	20333	5120	18166	4879
16000	25333	3901	23666	2624
20000	26666	3543	25666	3299
24000	27500	4609	27500	5377
28000	28500	4031	29666	6446
32000	29833	3670	31166	5241
36000	30500	3253	31833	5112
40000	30500	3253	31166	3933

Table 34

TORQUE CHARACTERISTICS FOR R-4 BEARINGS, 0.0124 GM. OIL

100 GM RADIAL LOAD

100 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	5833	1674	5833	1674
4000	13500	3989	11333	2924
8000	22500	3593	18500	4031
12000	28166	4740	23500	2929
16000	32500	4193	27833	1863
20000	35166	5047	29500	4609
24000	35333	4714	31000	5416
28000	37333	4887	33000	4582
32000	37833	4669	34666	4422
36000	37666	4714	35000	3741
40000	37833	4669	35333	3543

Table 35

TORQUE CHARACTERISTICS FOR R-4 BEARINGS, 0.0124 GM. OIL

100 GM RADIAL LOAD

200 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	6833	1343	5500	2140
4000	15333	1795	11000	2236
8000	27166	3670	16166	1674
12000	34666	2981	22500	3774
16000	40166	3760	29666	4346
20000	44000	3915	29166	3131
24000	43333	5647	33166	3847
28000	44333	5088	37833	3670
32000	45500	5852	40833	3435
36000	43500	6677	41666	2981
40000	42000	8266	41500	3862

Table 36

TORQUE CHARACTERISTICS FOR R-4 BEARINGS, 0.0124 GM. OIL

200 GM RADIAL LOAD

0 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	4500	1258	4166	2852
4000	9333	1374	10833	3287
8000	17500	1384	15500	3730
12000	24666	1598	19666	3448
16000	29500	3304	22833	3531
20000	26500	2629	23666	3399
24000	25166	2910	24166	4099
28000	24666	3144	23500	3905
32000	25500	2986	23166	4297
36000	26166	3184	25500	5377
40000	28500	2565	27833	5639

Table 37

TORQUE CHARACTERISTICS FOR R-4 BEARINGS, 0.0124 GM. OIL

200 GM RADIAL LOAD

50 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	6000	2236	5000	1527
4000	14166	3236	11333	3681
8000	24000	3000	20500	3593
12000	29500	3201	26333	4346
16000	33333	4109	26833	3847
20000	32166	4058	27666	5055
24000	32000	3915	29833	7581
28000	31500	3989	27833	13981
32000	31500	3774	31333	7695
36000	32333	2867	31166	7335
40000	32333	3197	31166	6938

Table 38

TORQUE CHARACTERISTICS FOR R-4 BEARINGS, 0.0124 GM. OIL

200 GM RADIAL LOAD

100 GM AXIAL LOAD

SPEED (RPM)	ORIGINAL RUN AVERAGE TORQUE (MG-MM)	ORIGINAL RUN SAMPLE STANDARD DEVIATION (MG-MM)	RE-RUN AVERAGE TORQUE (MG-MM)	RE-RUN SAMPLE STANDARD DEVIATION (MG-MM)
1000	6166	1771	4666	942
4000	16500	2753	12500	2291
8000	26166	3131	19500	4031
12000	29166	3890	23833	4258
16000	34166	2477	28000	1825
20000	35166	2671	29000	2708
24000	35000	2380	30333	1885
28000	35833	2910	31000	2000
32000	35333	2748	32000	1732
36000	36166	2192	31833	1950
40000	36333	1972	32333	2426

Table 39

TORQUE CHARACTERISTICS FOR R-4 BEARINGS, 0.0124 GM. OIL

200 GM RADIAL LOAD

200 GM AXIAL LOAD

	ORIGINAL RUN		RE-RUN	
SPEED (RPM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)	AVERAGE TORQUE (MG-MM)	SAMPLE STANDARD DEVIATION (MG-MM)
1000	7166	3077	7500	4856
4000	13333	5312	11833	4412
8000	26166	5698	19666	4606
12000	34000	6298	27166	8354
16000	36833	5871	32666	6896
20000	38333	5763	30666	13337
24000	39166	7104	39833	8858
28000	41666	6394	40666	8711
32000	40500	5283	40666	8595
36000	39833	5047	41166	8687
40000	41666	5185	43000	8225

B. Empirical Results

The torque values calculated by empirical means are listed in Tables 40 through 48. Each table is for a given bearing size and radial load and all combinations of axial load. The speeds at which values were calculated correspond to the same points where experimental data was tabulated.

The empirical values were calculated form the following expression:

$$\begin{aligned} T = N^{1/4} \Big[& (4569 - 28030d_m + 45327d_m^2) \\ & + (-1.33 + 0.03R - 6.7\times10^{-5}R^2)(746 - 3906d_m + 5482d_m^2) \\ & + (-167 + 1072d_m)(0.025A - 0.00005A^2) \Big] \\ + N^2 \Big[& (-27 + 201d_m - 313d_m^2) \\ & + (-1.33 + 0.03R - 6.7\times10^{-5}R^2)(18.14 - 109d_m + 170d_m^2) \\ & + (-0.17 + 2.68d_m)(0.025A - 0.00005A^2) \Big] \times10^{-6} \end{aligned}$$

where: N = rotational speed (rpm)

d_m = mean bearing diameter (inches)

A = axial load (grams)

R = radial load (grams)

T = running torque (milligram-millimeters)

The development of the above expression is covered in the Discussion of Results.

TABLE 40

PREDICTED TORQUE CHARACTERISTICS FOR R-2 BEARINGS

50 GM RADIAL LOAD

AXIAL LOAD

SPEED RPM	0 GM	50 GM	100 GM	200 GM
1000	2223	2862	3360	3928
4000	3198	4110	4820	5631
8000	3969	5079	5943	6930
12000	4662	5933	6920	8050
16000	5384	6806	7912	9176
20000	6170	7746	8972	10373
24000	7039	8777	10129	11674
28000	7999	9910	11396	13095
32000	9059	11154	12784	14647
36000	10220	12515	14299	16338
40000	11488	13995	15944	18173

TABLE 41

PREDICTED TORQUE CHARACTERISTICS FOR R-2 BEARINGS

100 GM RADIAL LOAD

AXIAL LOAD

SPEED RPM	0 GM	50 GM	100 GM	200 GM
1000	2853	3493	3990	4559
4000	4111	5024	5734	6545
8000	5123	6234	7098	8085
12000	6051	7321	8309	9438
16000	7029	8450	9556	10820
20000	8105	9681	10907	12308
24000	9301	11039	12391	13936
28000	10631	12542	14028	15726
32000	12102	14198	15828	17691
36000	13720	16014	17798	19837
40000	15487	17994	19944	22172

TABLE 42

PREDICTED TORQUE CHARACTERISTICS FOR R-2 BEARINGS

200 GM RADIAL LOAD

AXIAL LOAD

SPEED RPM	0 GM	50 GM	100 GM	200 GM
1000	3479	4119	4616	5184
4000	5018	5931	6640	7452
8000	6269	7380	8243	9231
12000	7428	8699	9687	10816
16000	8661	10083	11189	12453
20000	10025	11601	12827	14228
24000	11547	13285	14637	16182
28000	13243	15154	16640	18338
32000	15123	17218	18848	20711
36000	17192	19487	21271	23310
40000	19457	21964	23914	26142

TABLE 43

PREDICTED TORQUE CHARACTERISTICS FOR R-3 BEARINGS

50 GM RADIAL LOAD

AXIAL LOAD

SPEED RPM	0 GM	50 GM	100 GM	200 GM
1000	1634	2909	3901	5034
4000	2385	4201	5613	7227
8000	3066	5264	6973	8926
12000	3767	6261	8200	10417
16000	4566	7331	9482	11940
20000	5489	8522	10881	13577
24000	6549	9856	12429	15369
28000	7754	11348	14143	17338
32000	9108	13004	16034	19498
36000	10614	14830	18110	21858
40000	12274	16831	20375	24426

TABLE 44

PREDICTED TORQUE CHARACTERISTICS FOR R-3 BEARINGS

100 GM RADIAL LOAD

AXIAL LOAD

SPEED RPM	0 GM	50 GM	100 GM	200 GM
1000	1921	3196	4188	5322
4000	2802	4618	6031	7645
8000	3597	5794	7503	9456
12000	4410	6903	8843	11059
16000	5333	8098	10249	12707
20000	6398	9431	11790	14486
24000	7619	10927	13499	16439
28000	9006	12600	15396	18591
32000	10564	14461	17491	20955
36000	12296	16513	19793	23541
40000	14206	18763	22307	26358

TABLE 45

PREDICTED TORQUE CHARACTERISTICS FOR R-3 BEARINGS

200 GM RADIAL LOAD

AXIAL LOAD

SPEED RPM	0 GM	50 GM	100 GM	200 GM
1000	2206	3481	4473	5607
4000	3217	5033	6445	8059
8000	4123	6321	8030	9983
12000	5047	7541	9481	11697
16000	6094	8859	11010	13468
20000	7300	10333	12692	15388
24000	8682	11989	14561	17501
28000	10250	13844	16639	19834
32000	12010	15906	18937	22400
36000	13967	18184	21463	25212
40000	16123	20680	24224	28275

TABLE 46

PREDICTED TORQUE CHARACTERISTICS FOR R-4 BEARINGS

50 GM RADIAL LOAD

AXIAL LOAD

SPEED RPM	0 GM	50 GM	100 GM	200 GM
1000	5523	7434	8921	10620
4000	7825	10545	12661	15079
8000	9353	12638	15193	18113
12000	10426	14144	17036	20342
16000	11308	15418	18615	22268
20000	12091	16582	20076	24068
24000	12818	17696	21490	25827
28000	13513	18792	22898	27591
32000	14193	19892	24324	29390
36000	14867	21008	25785	31244
40000	15542	22152	27292	33167

TABLE 47

PREDICTED TORQUE CHARACTERISTICS FOR R-4 BEARINGS

100 GM RADIAL LOAD

AXIAL LOAD

SPEED RPM	0 GM	50 GM	100 GM	200 GM
1000	6010	7922	9409	11108
4000	8559	11279	13394	15812
8000	10359	13644	16199	19120
12000	11758	15476	18368	21674
16000	13042	17152	20348	24001
20000	14310	18801	22294	26287
24000	15609	20488	24282	28618
28000	16968	22247	26353	31045
32000	18401	24100	28533	33599
36000	19921	26063	30840	36299
40000	21536	28145	33286	39161

TABLE 4B

PREDICTED TORQUE CHARACTERISTICS FOR R-4 BEARINGS

200 GM RADIAL LOAD

AXIAL LOAD

SPEED RPM	0 GM	50 GM	100 GM	200 GM
1000	6494	8406	9893	11592
4000	9286	12006	14122	16540
8000	11357	14643	17198	20118
12000	13080	16796	19690	22995
16000	14762	18872	22068	25722
20000	16512	21003	24497	28489
24000	18380	23258	27053	31389
28000	20396	25675	29781	34474
32000	22578	28277	32710	37775
36000	24938	31080	35896	41315
40000	27484	34094	39234	45109

C. Graphical Comparison of Results

Figures 18 through 53 show a graphical comparison of the experimental test results and the empirically computed results. Each figure is for a given bearing size and test loading. Along with the average experimental test values, the sample standard deviations are also shown as short lines essentially perpendicular to the average torque curve. Only the re-run data from the experimental tests are used in these figures.

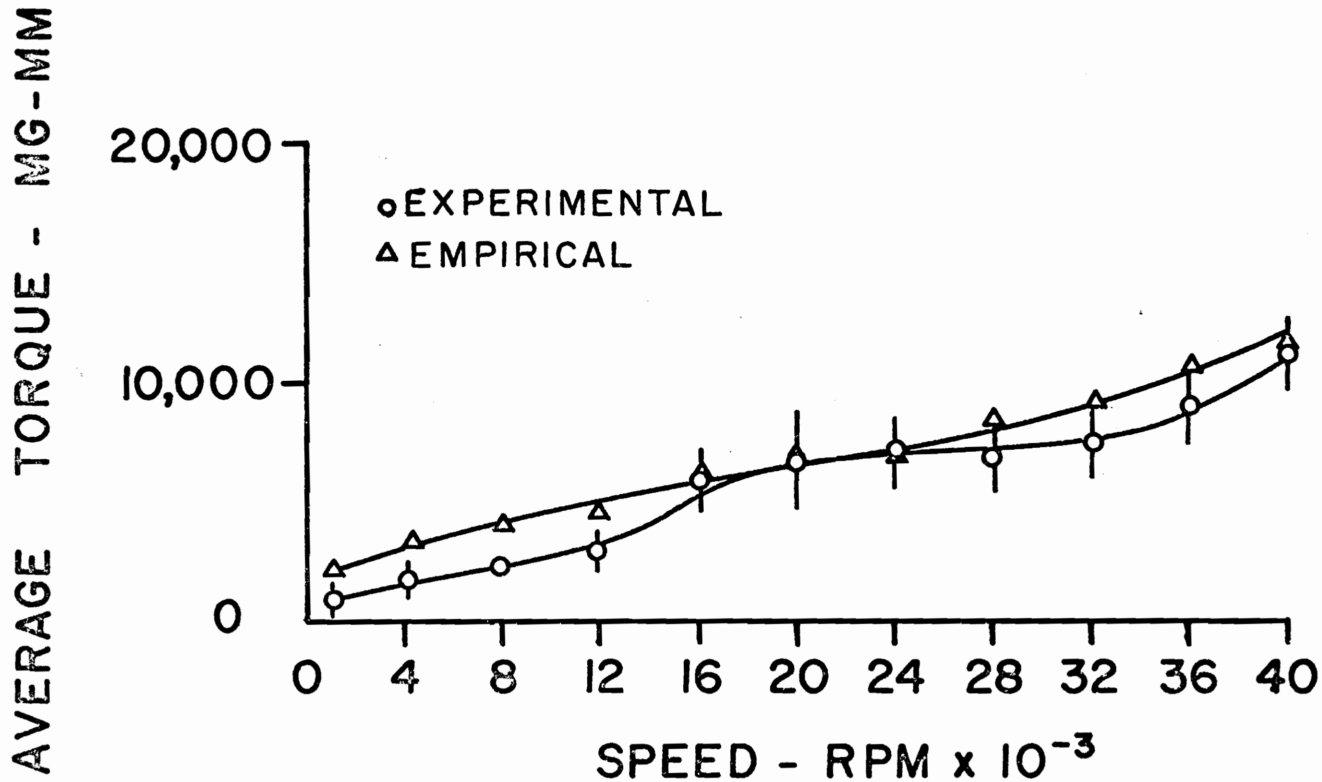

FIG. 18 COMPARISON FOR R-2
50 RADIAL, 0 AXIAL

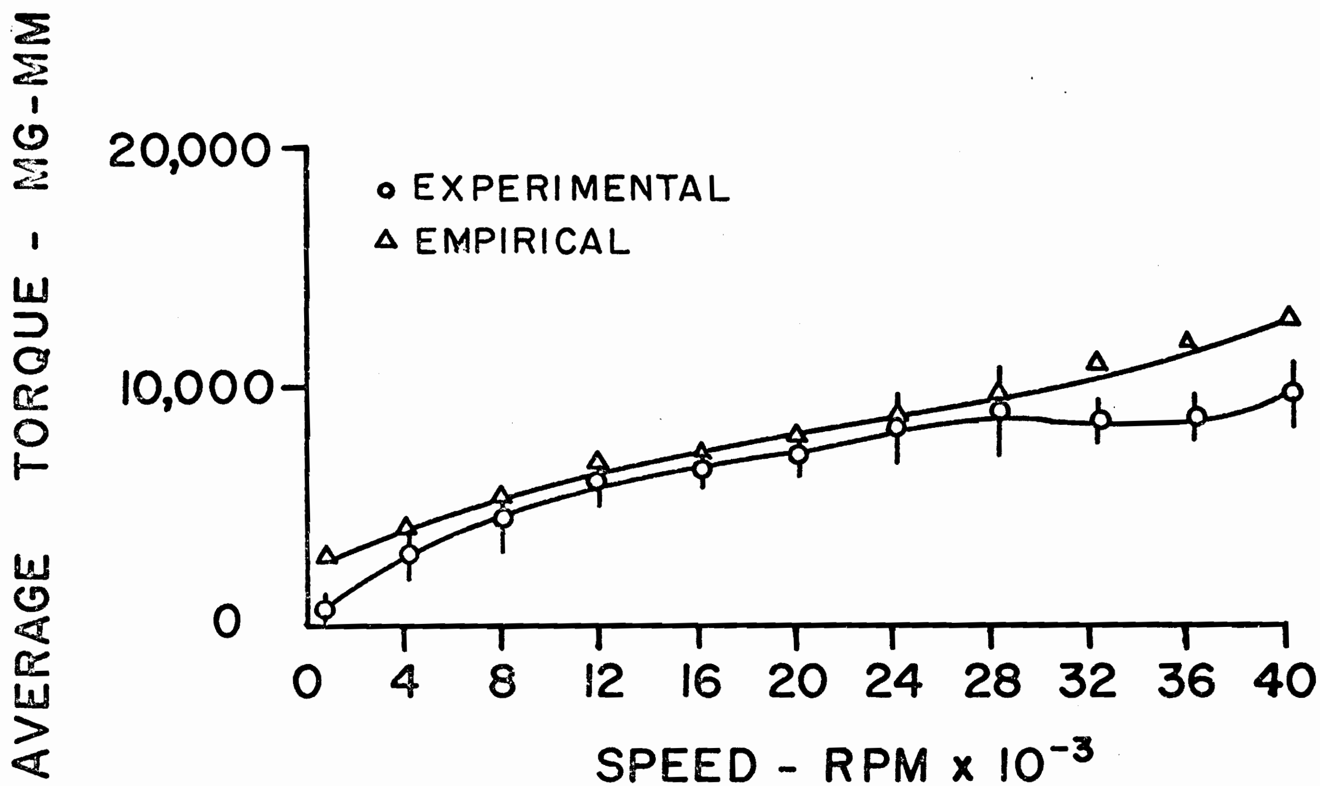

FIG. 19 COMPARISON FOR R-2

50 RADIAL, 50 AXIAL

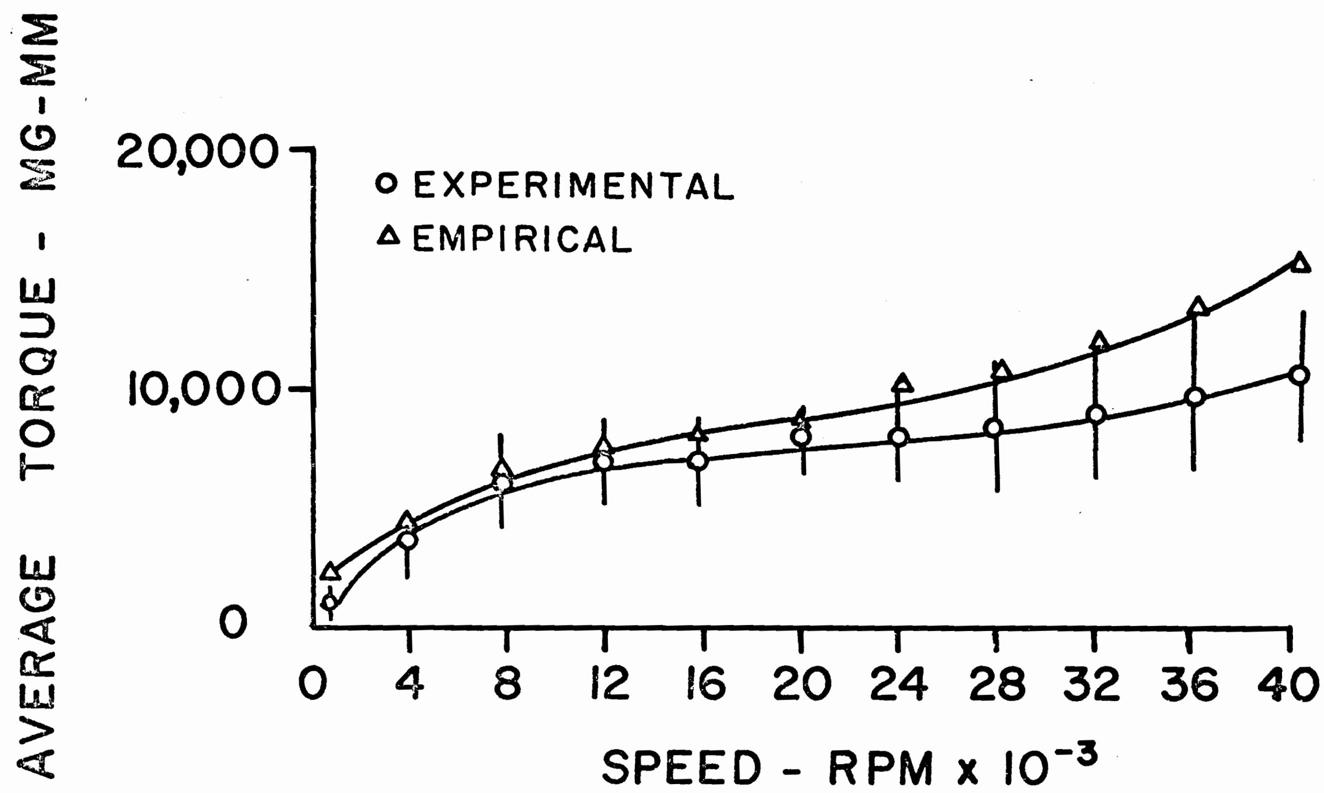

FIG. 20 COMPARISON FOR R-2
50 RADIAL, 100 AXIAL

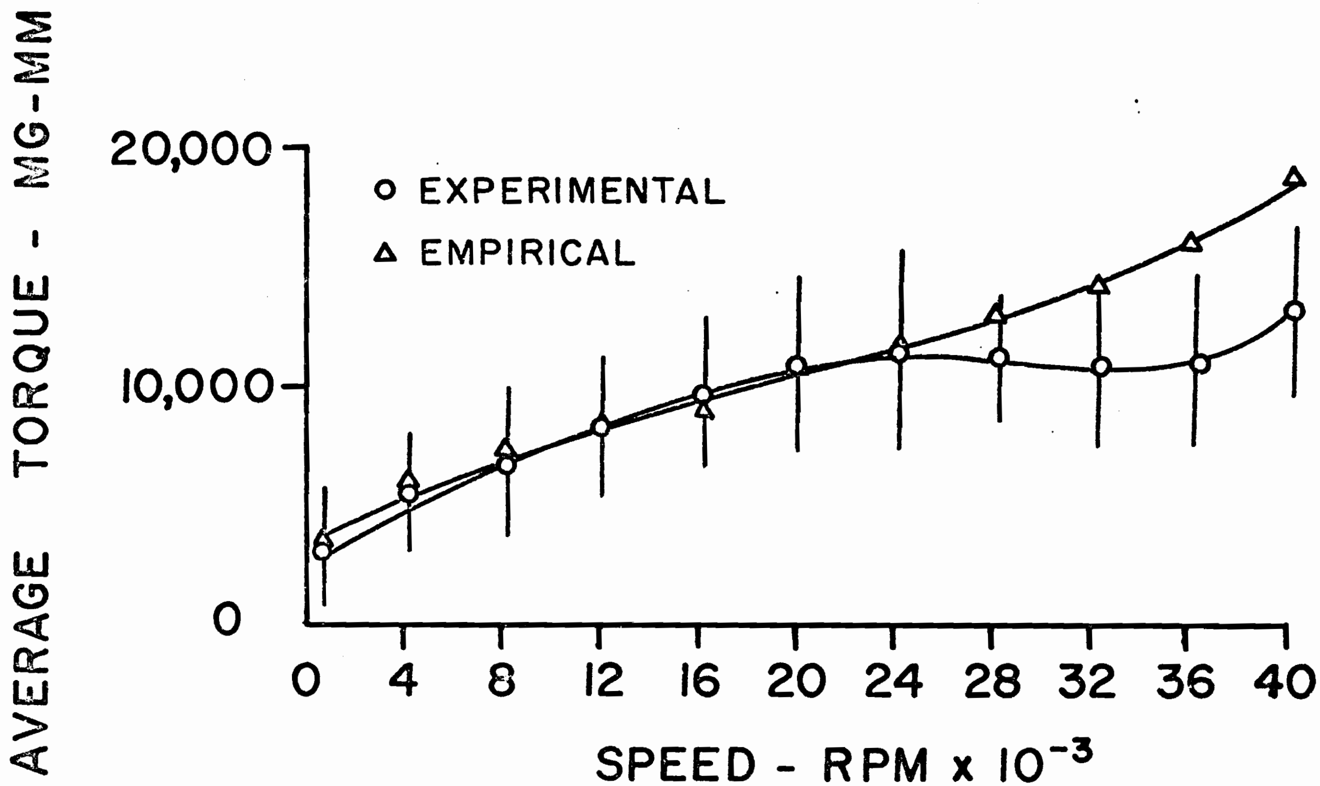

FIG. 21 COMPARISON FOR, R-2
50 RADIAL, 200 AXIAL

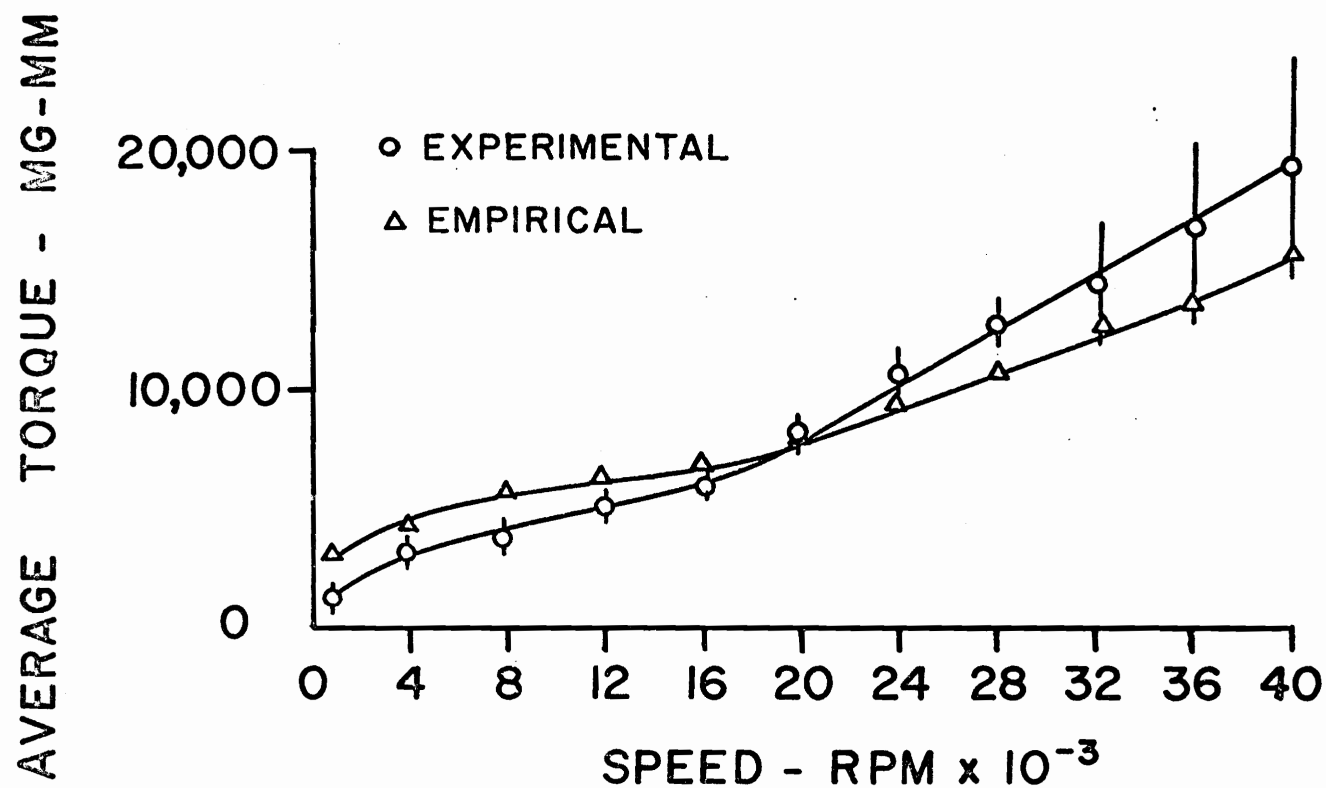

FIG. 22 COMPARISON FOR R-2

100 RADIAL, 0 AXIAL

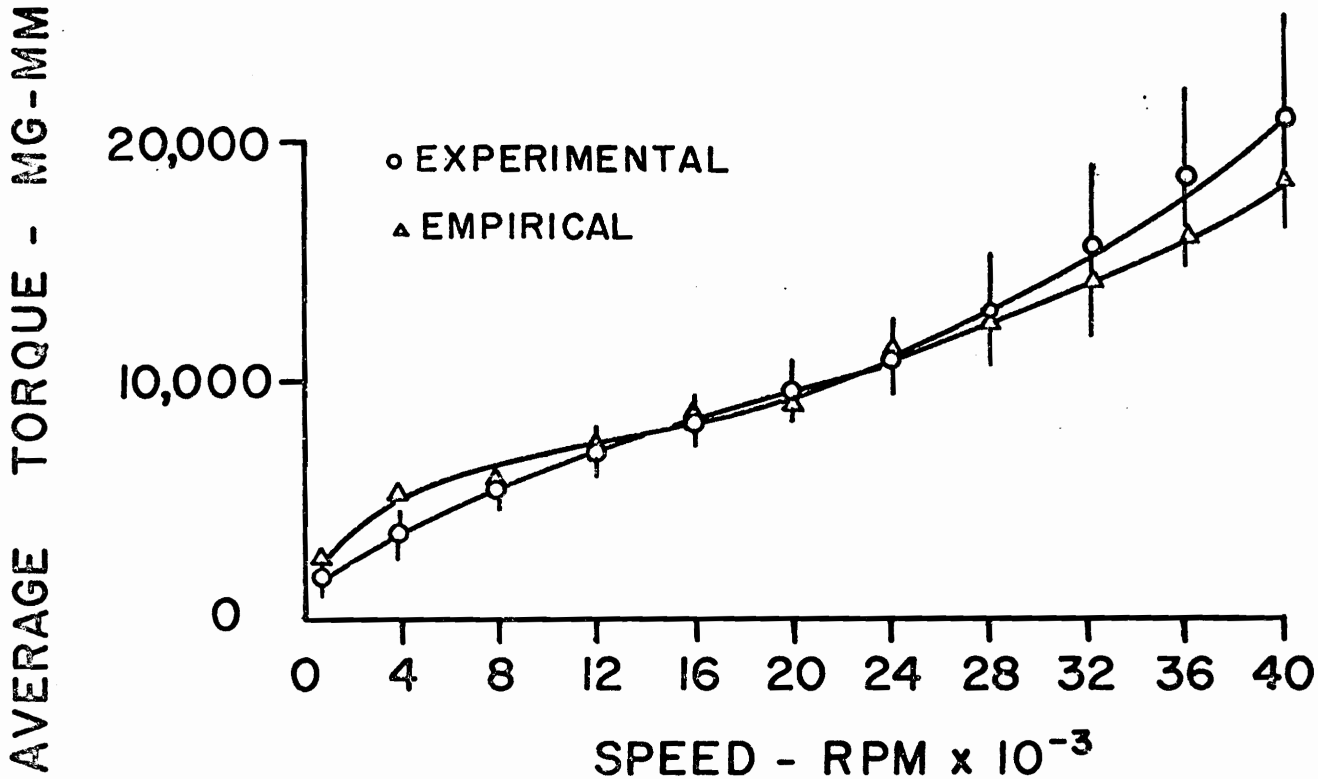

FIG. 23 COMPARISON FOR R-2
100 RADIAL, 50 AXIAL

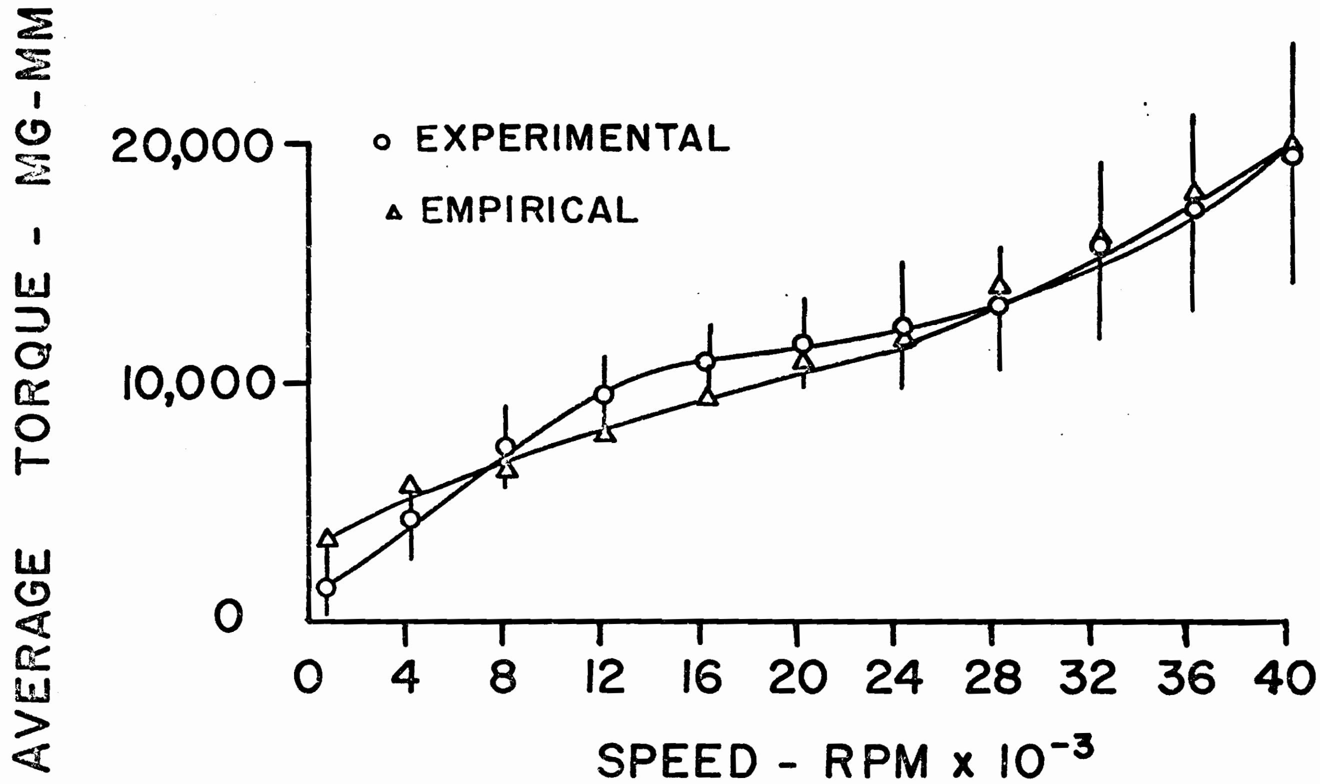

FIG. 24 COMPARISON FOR R-2
100 RADIAL, 100 AXIAL

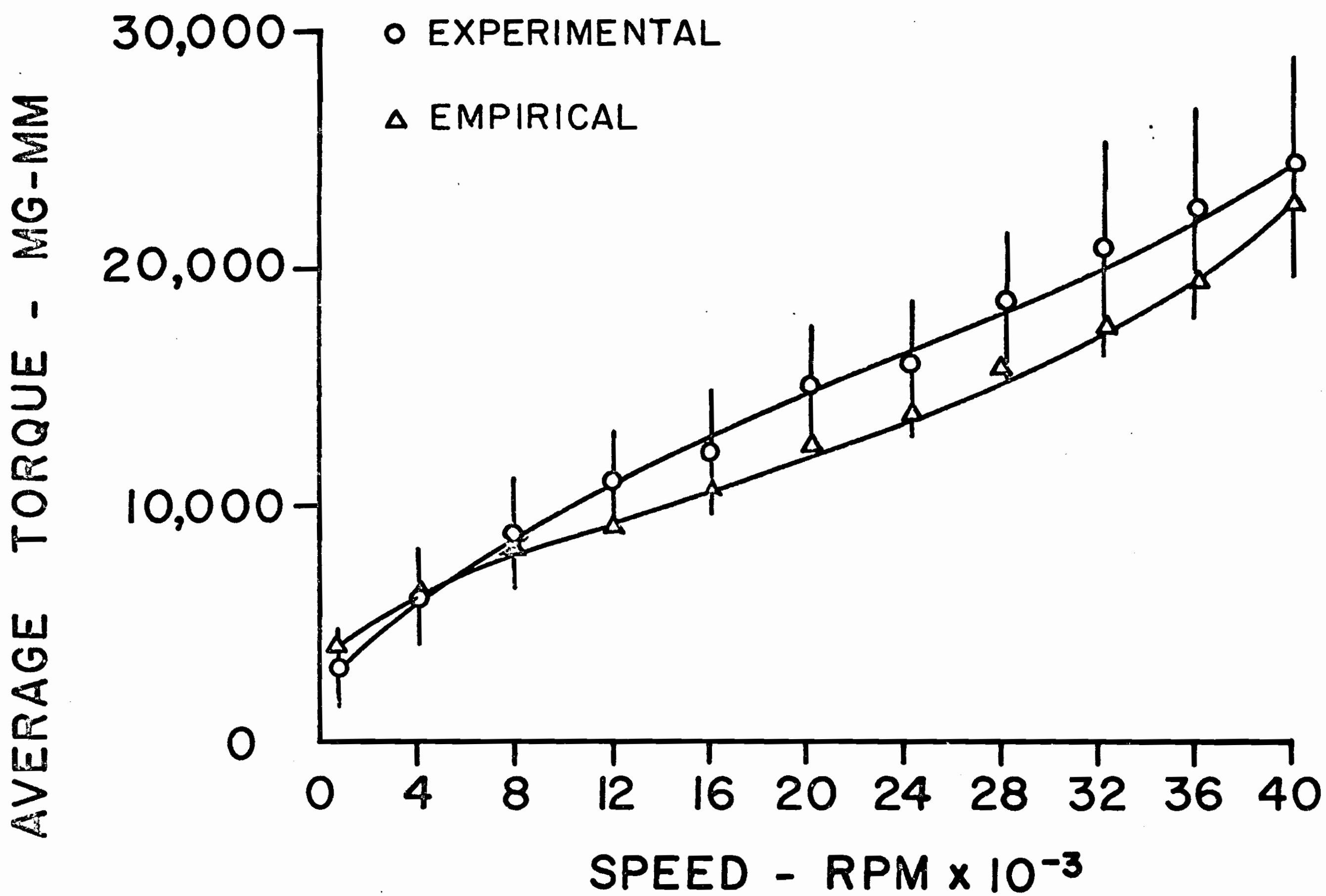

FIG. 25 COMPARISON FOR R-2
100 RADIAL, 200 AXIAL

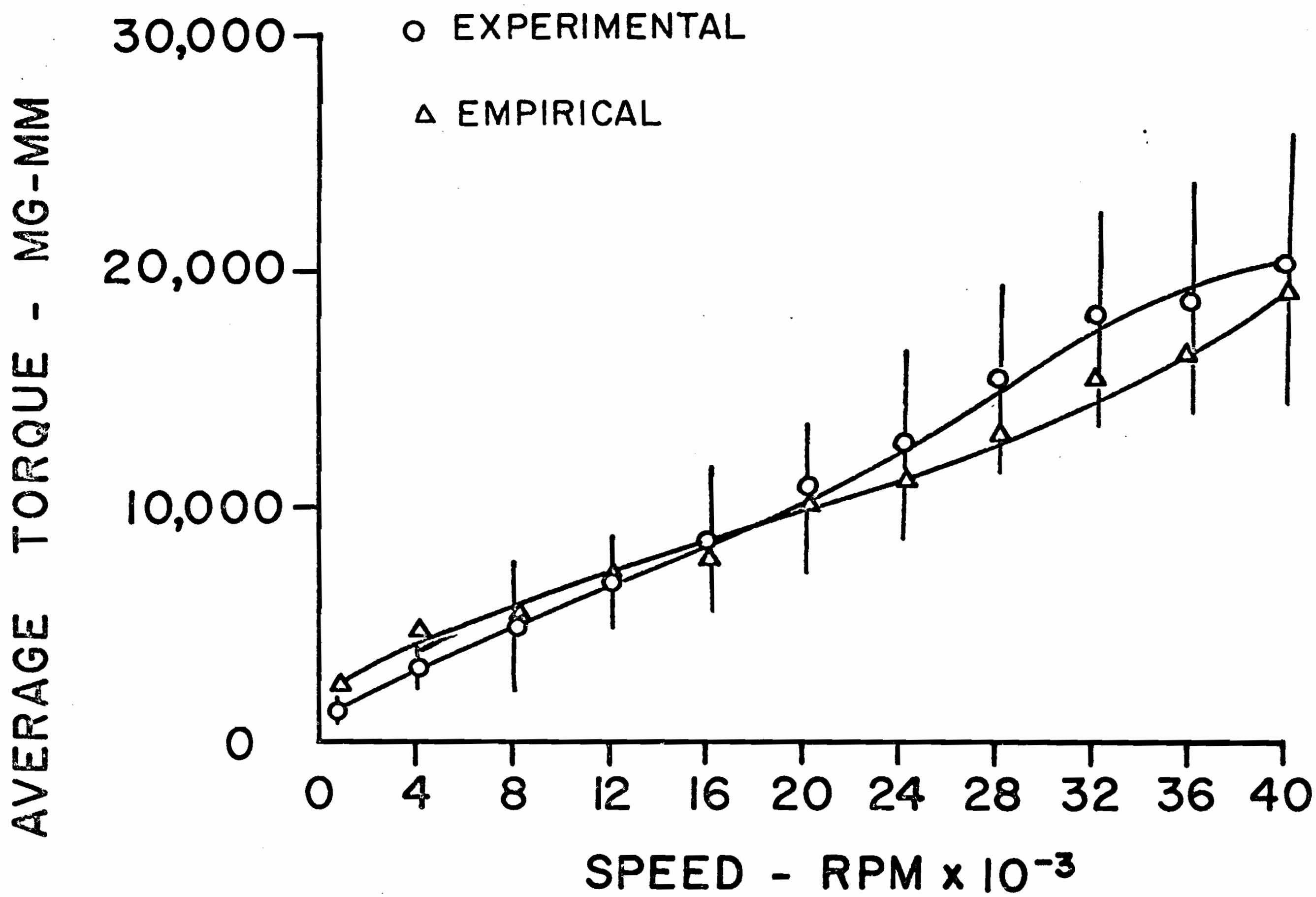

FIG. 26 COMPARISON FOR R-2
200 RADIAL, 0 AXIAL

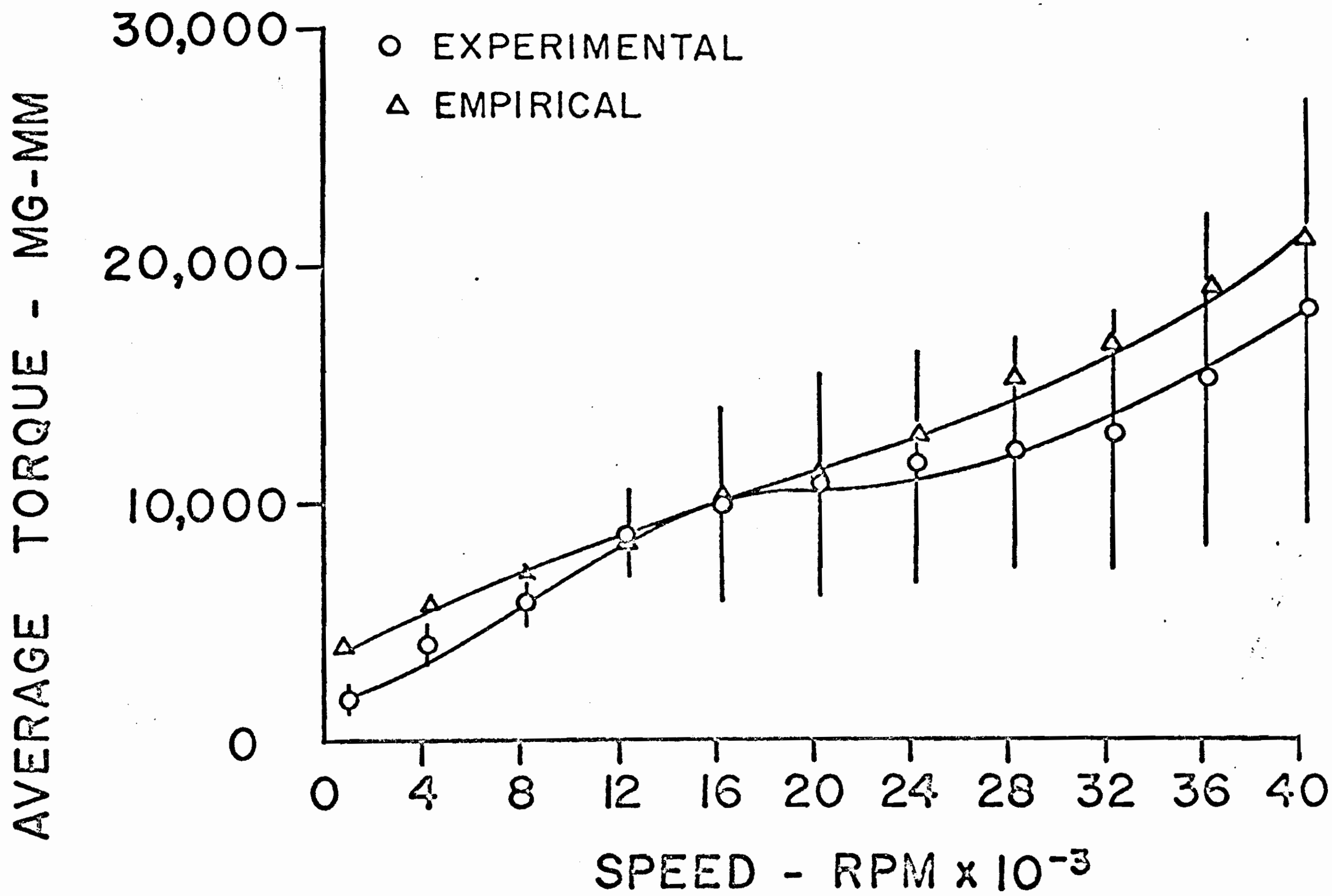

FIG. 27 COMPARISON FOR R-2
200 RADIAL, 50 AXIAL

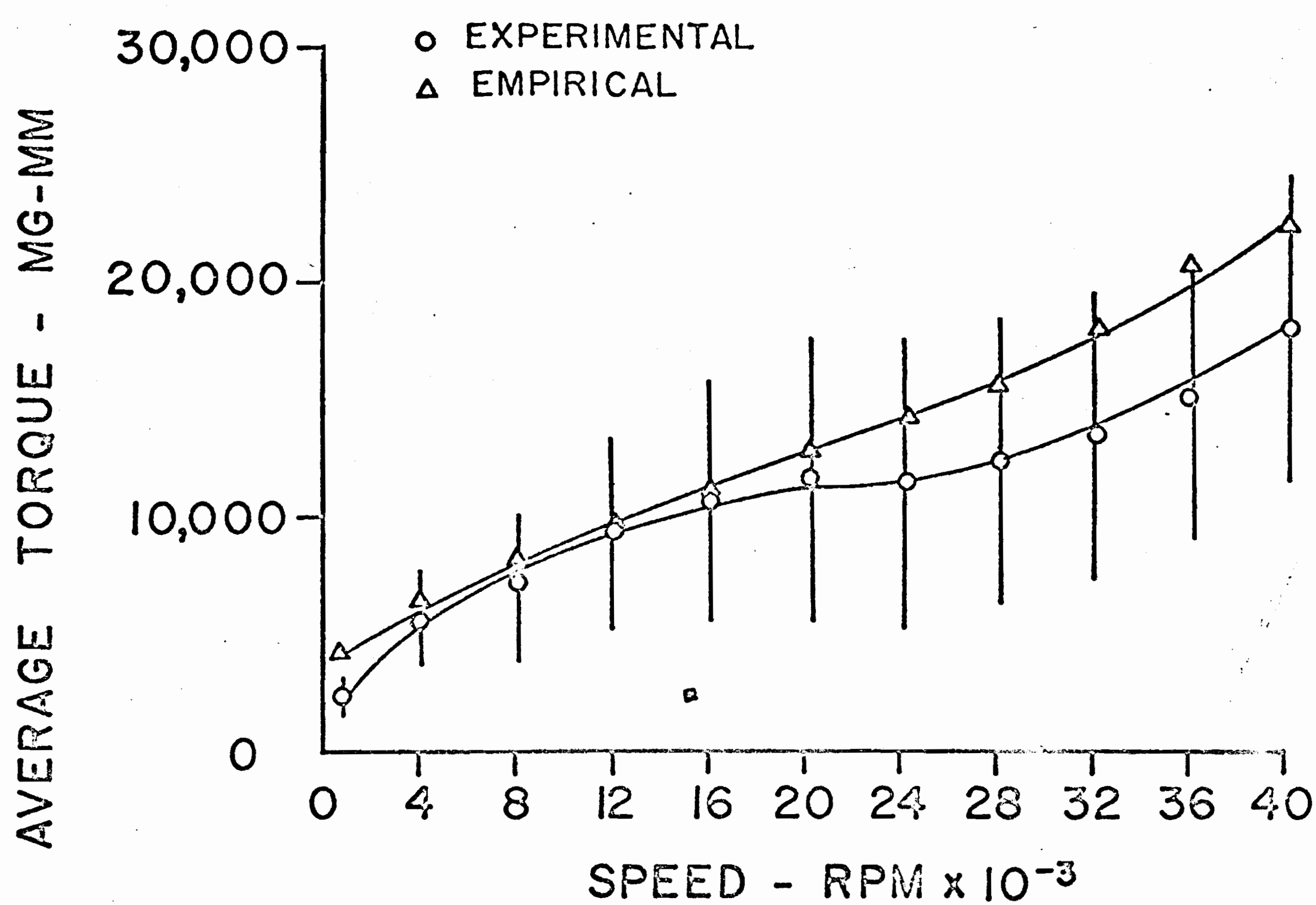

FIG. 28 COMPARISON FOR R-2
200 RADIAL, 100 AXIAL

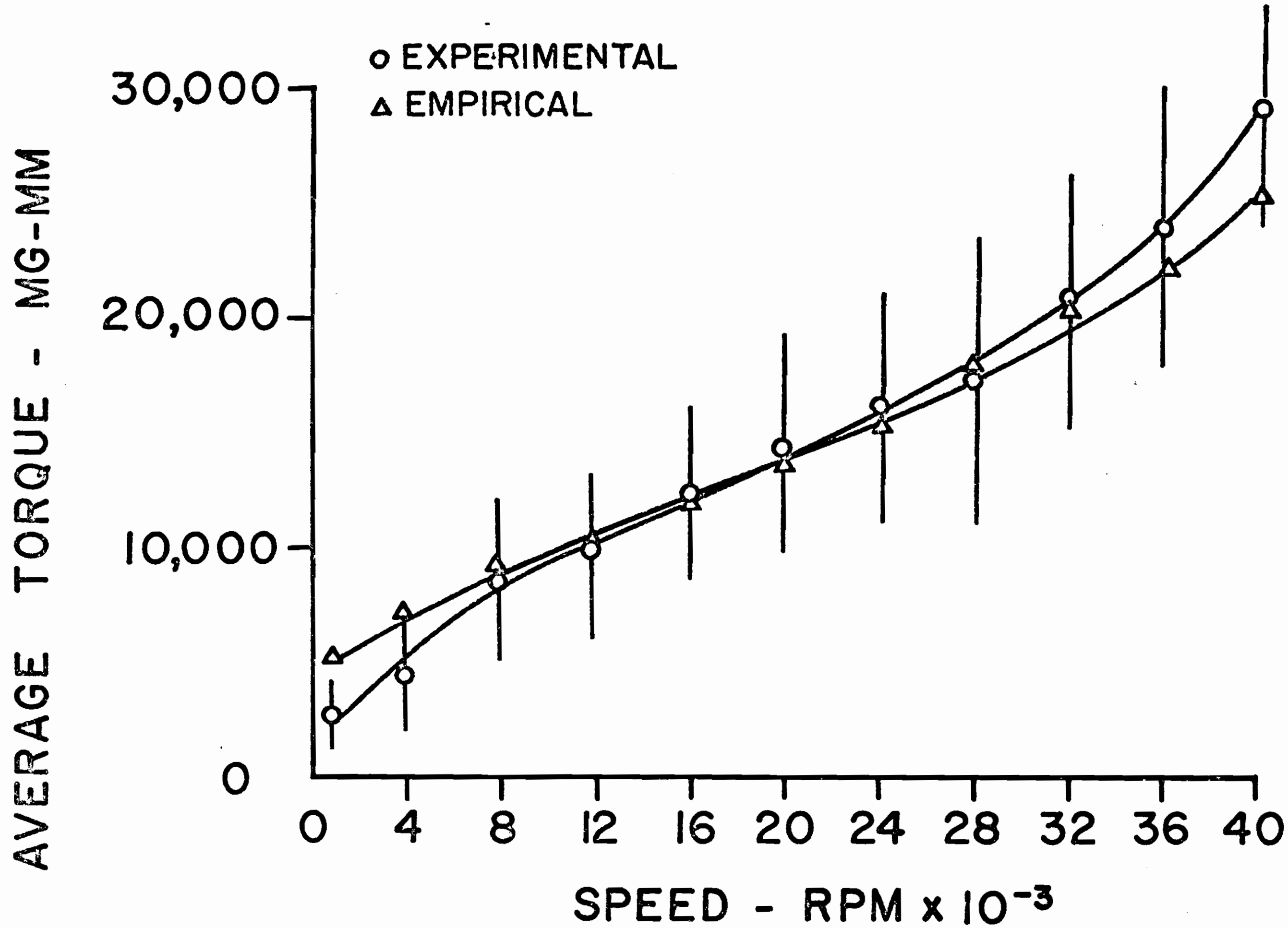

FIG. 29 COMPARISON FOR R-2
200 RADIAL, 200 AXIAL

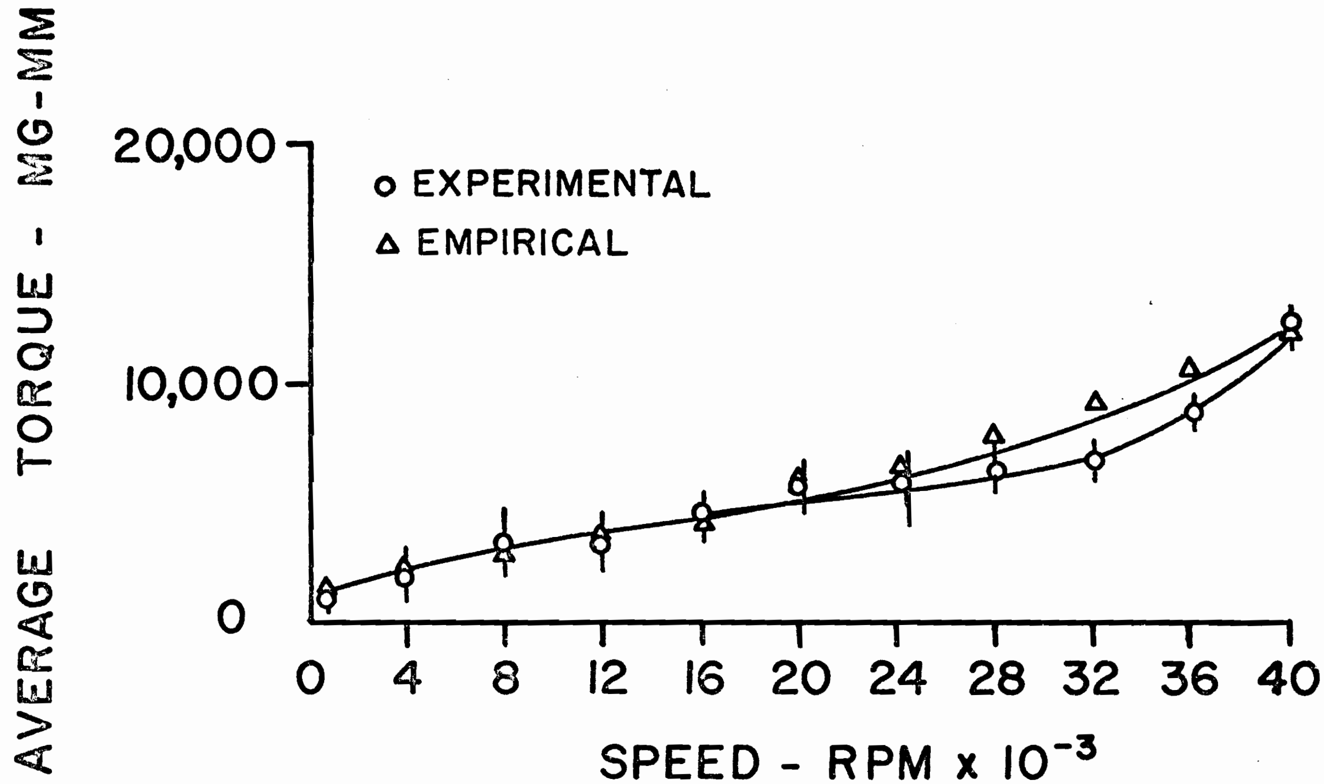

FIG. 30 COMPARISON FOR R-3
50 RADIAL, 0 AXIAL

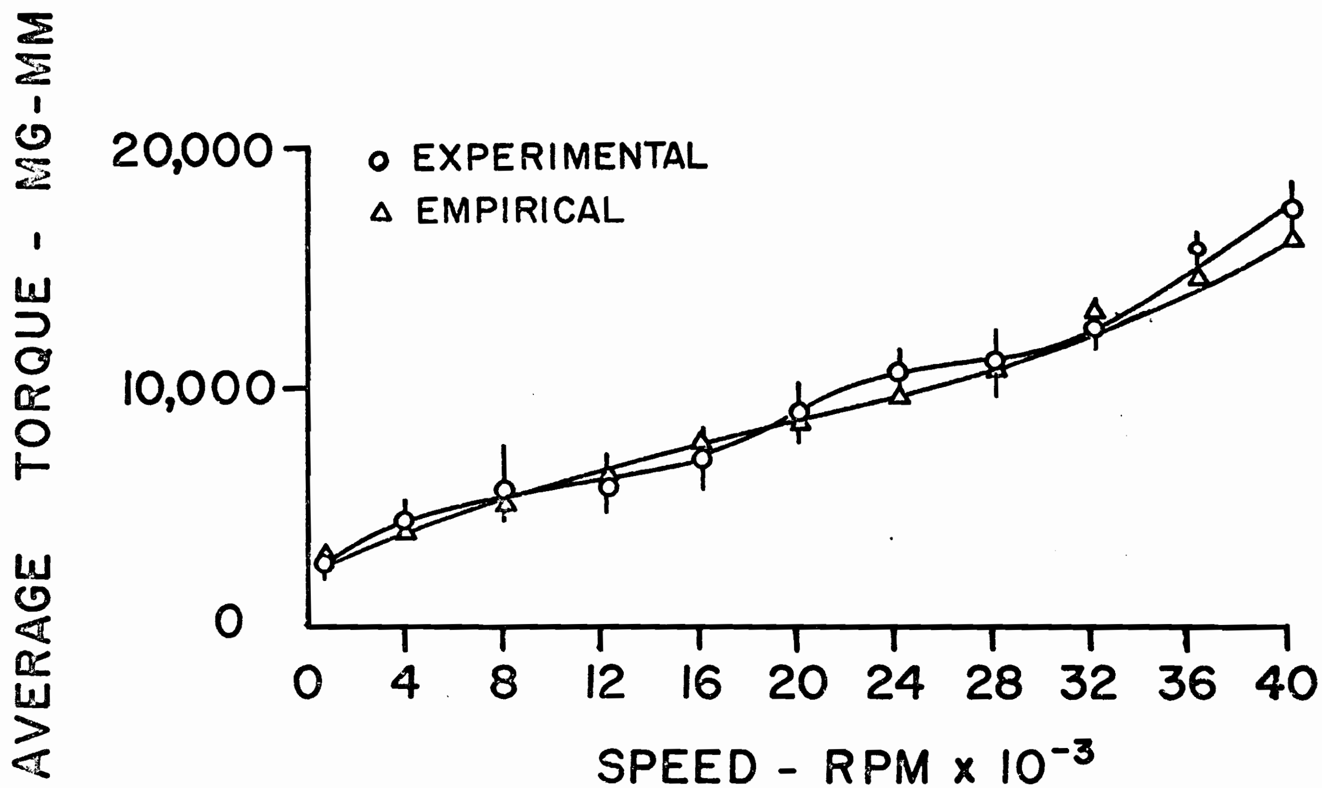

FIG. 31 COMPARISON FOR R-3
50 RADIAL, 50 AXIAL

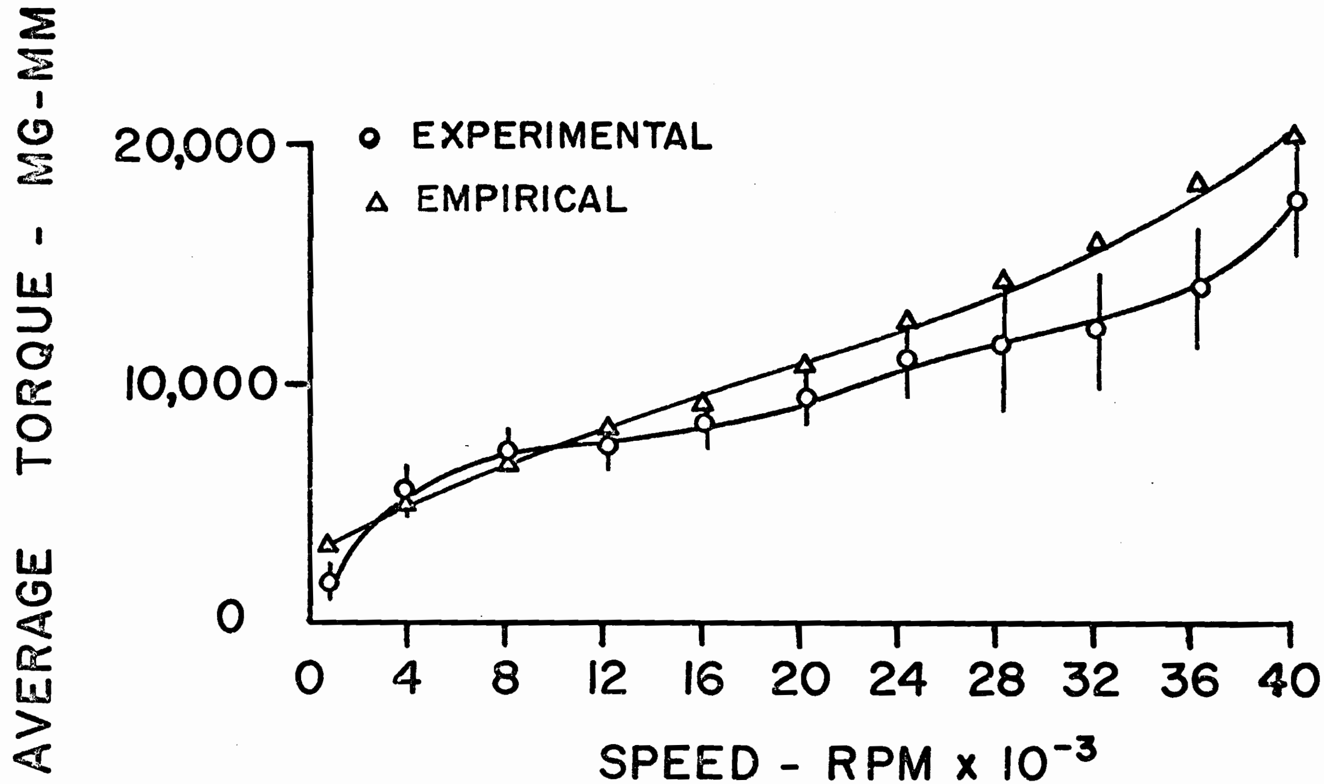

FIG. 32 COMPARISON FOR R-3
50 RADIAL, 100 AXIAL

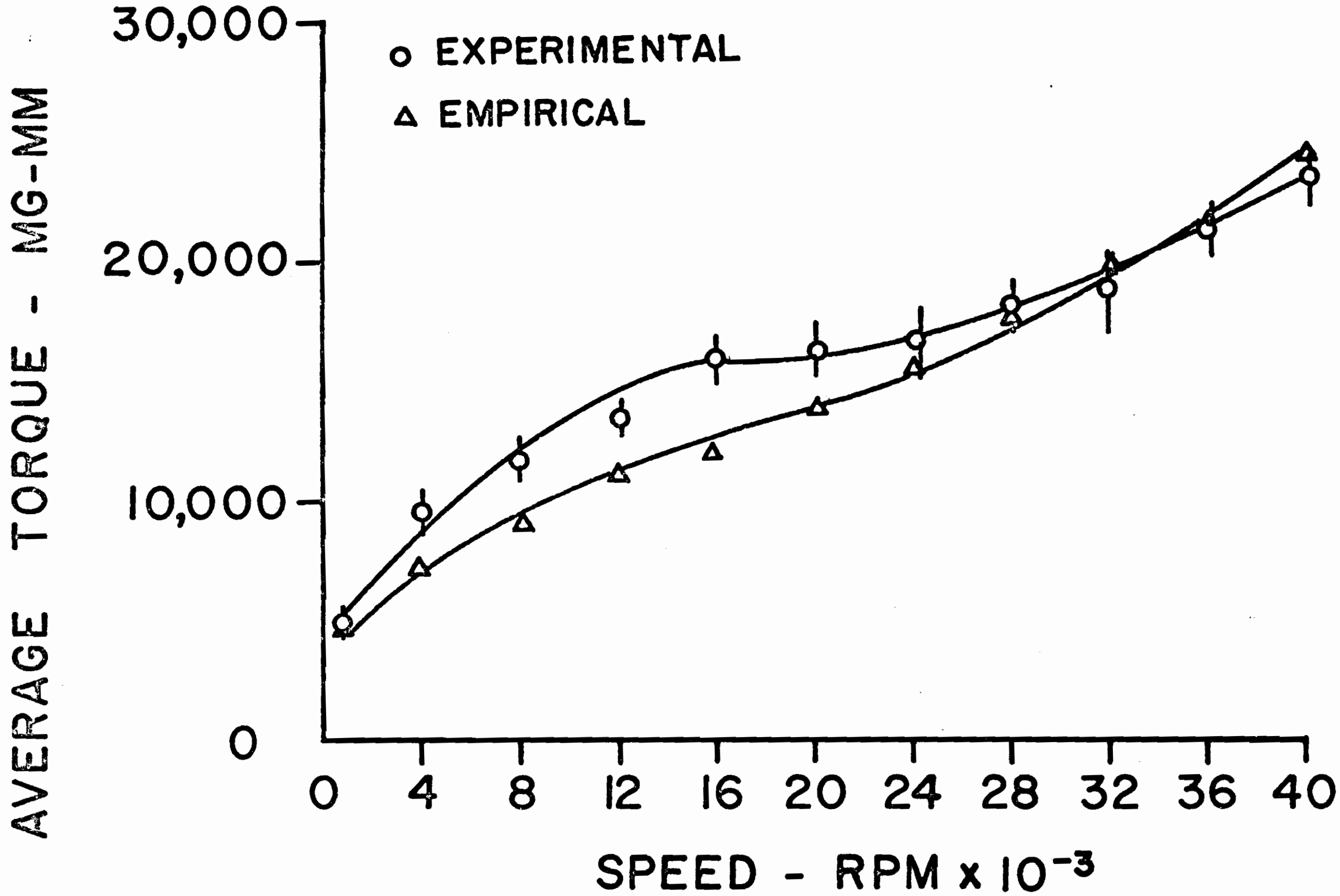

FIG. 33 COMPARISON FOR R-3
50 RADIAL, 200 AXIAL

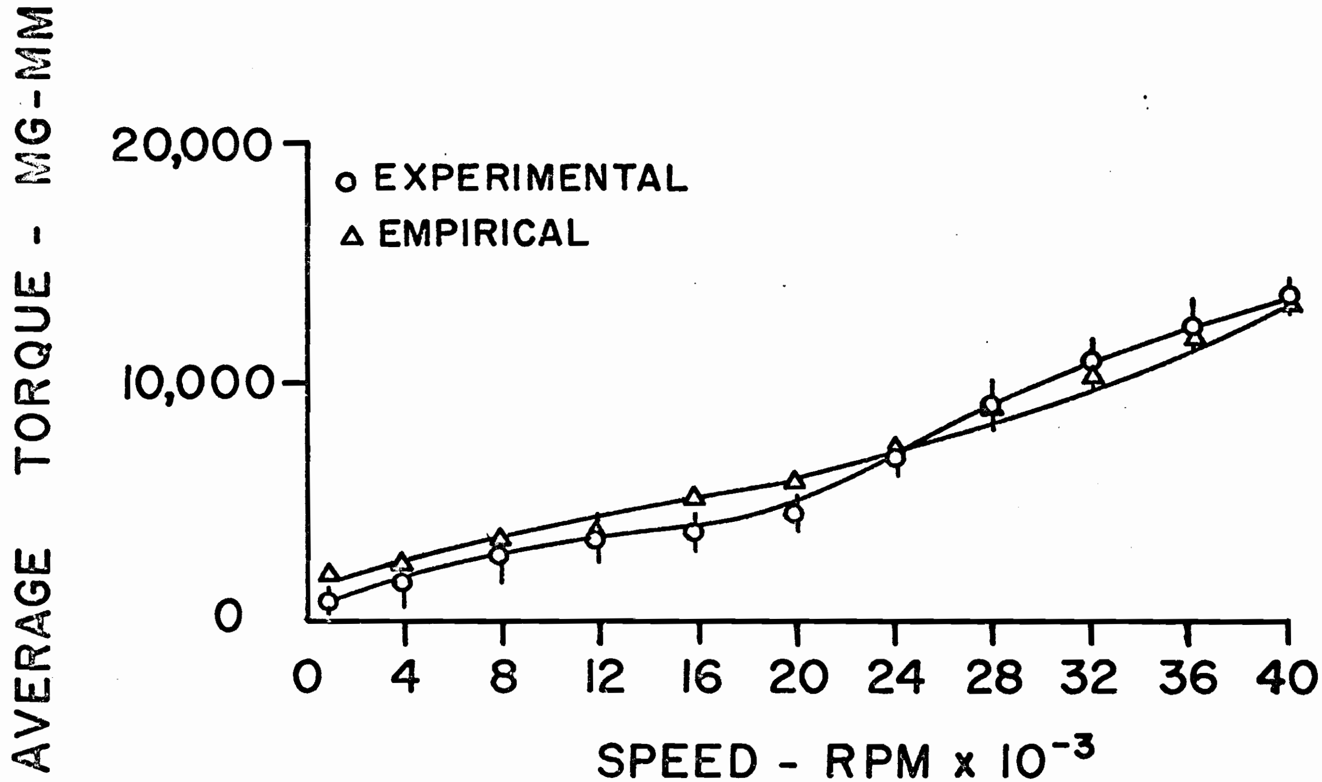

FIG. 34 COMPARISON FOR R-3
100 RADIAL, 0 AXIAL

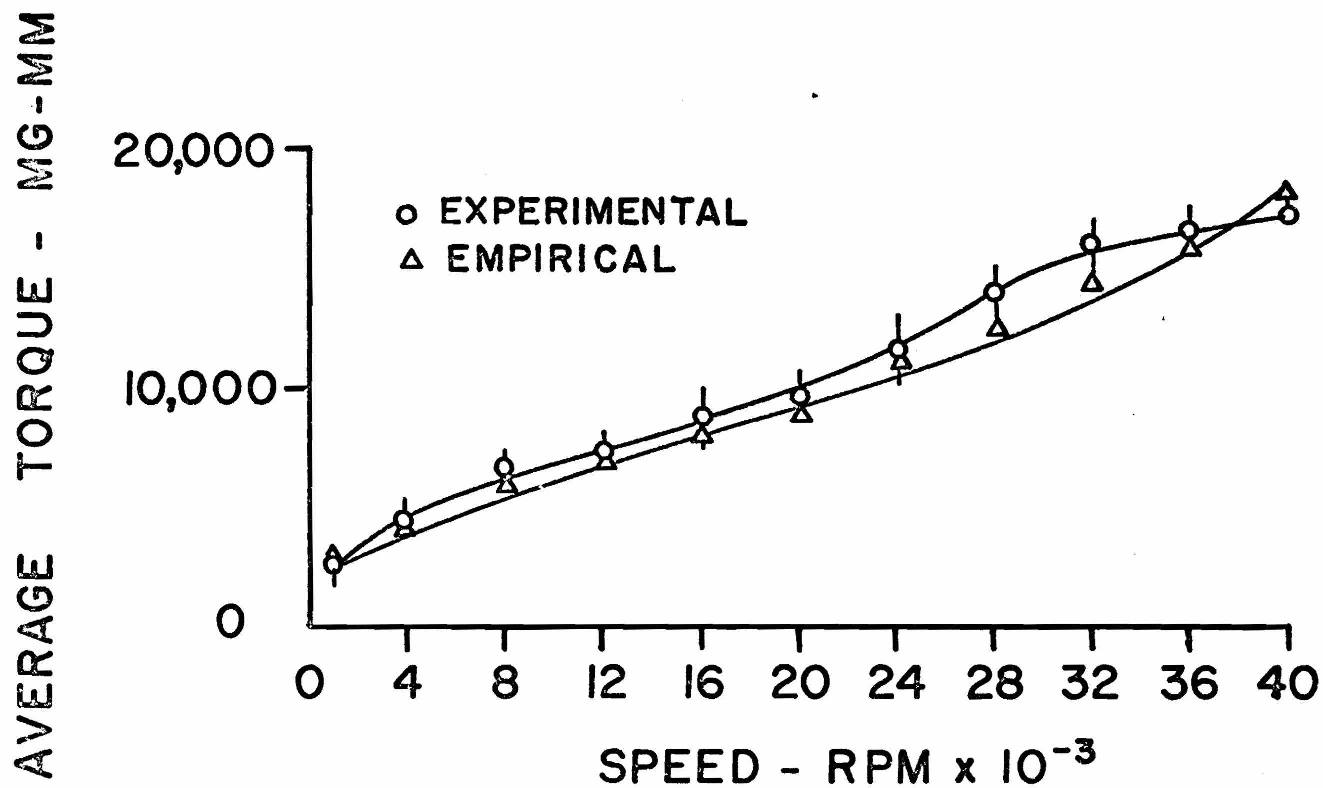

FIG. 35 COMPARISON FOR R-3
100 RADIAL, 50 AXIAL

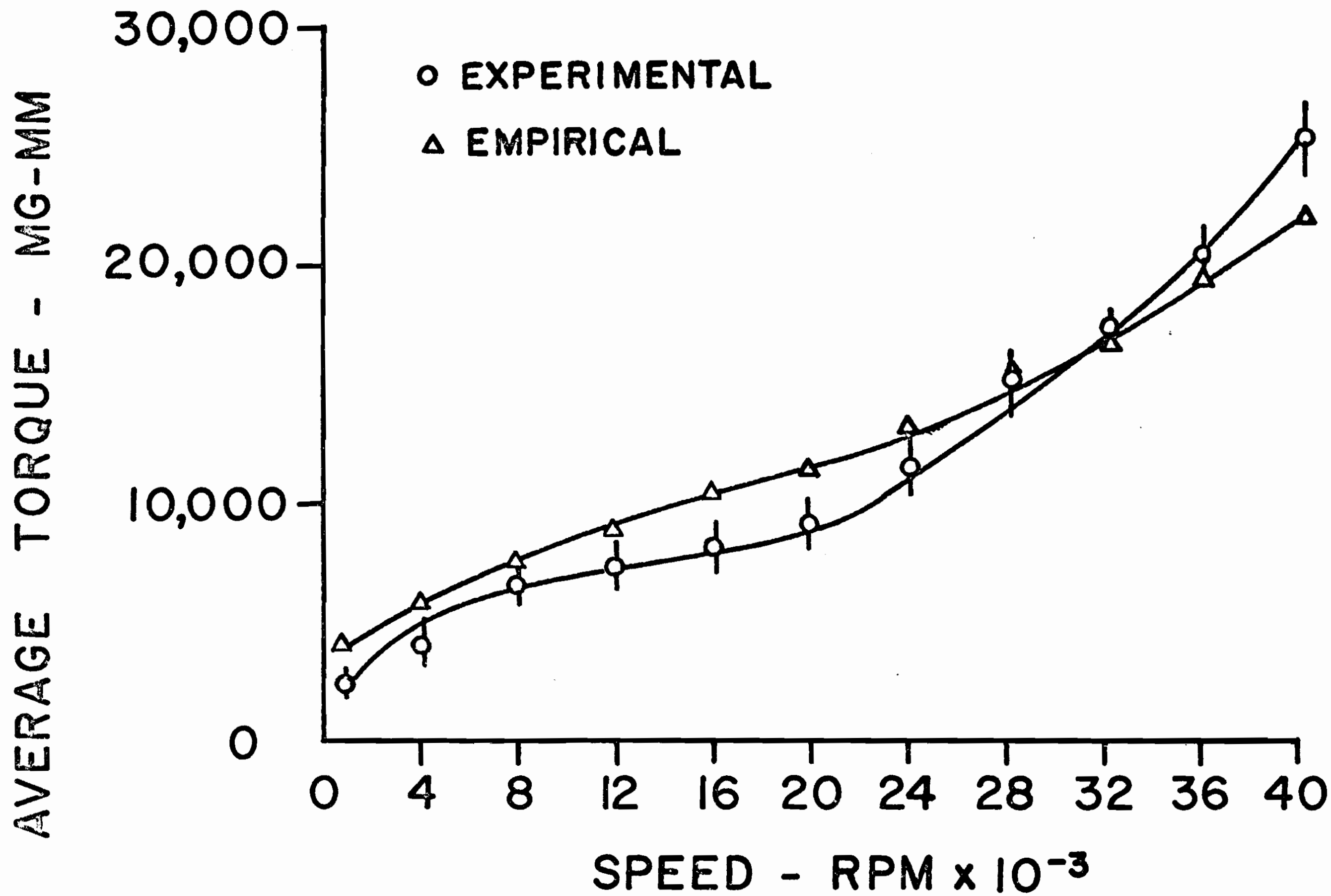

FIG. 36 COMPARISON FOR R-3
100 RADIAL, 100 AXIAL

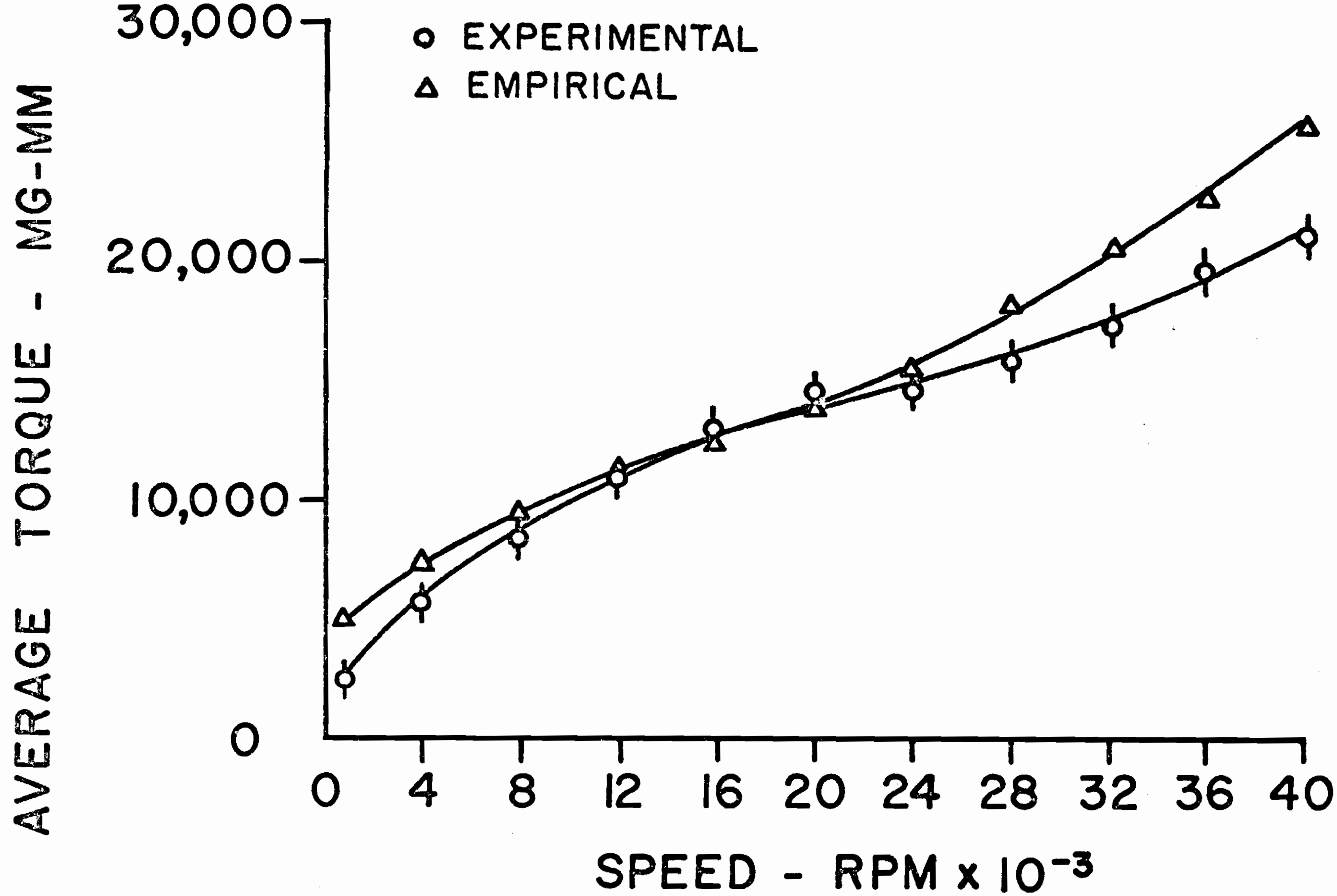

FIG. 37 COMPARISON FOR R-3
100 RADIAL, 200 AXIAL

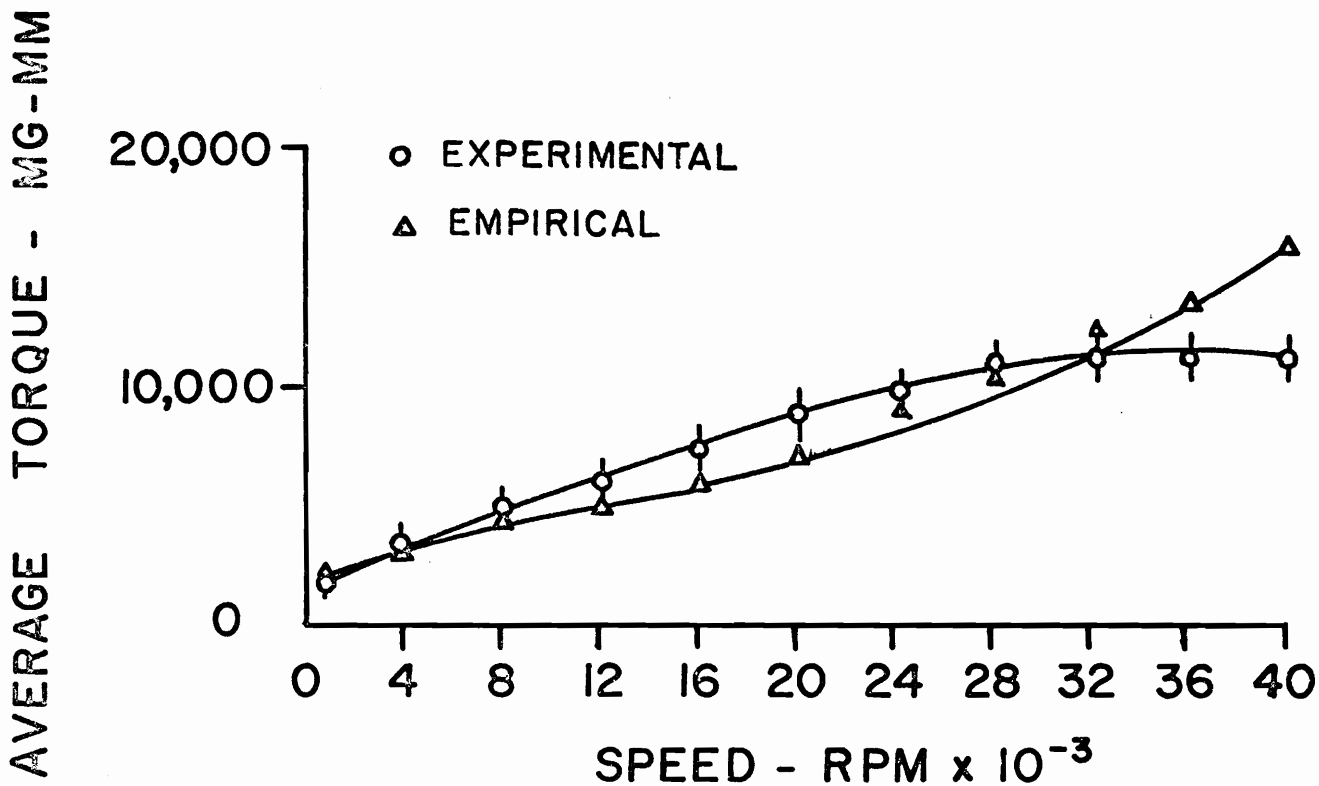

FIG. 38 COMPARISON FOR R-3
200 RADIAL, 0 AXIAL

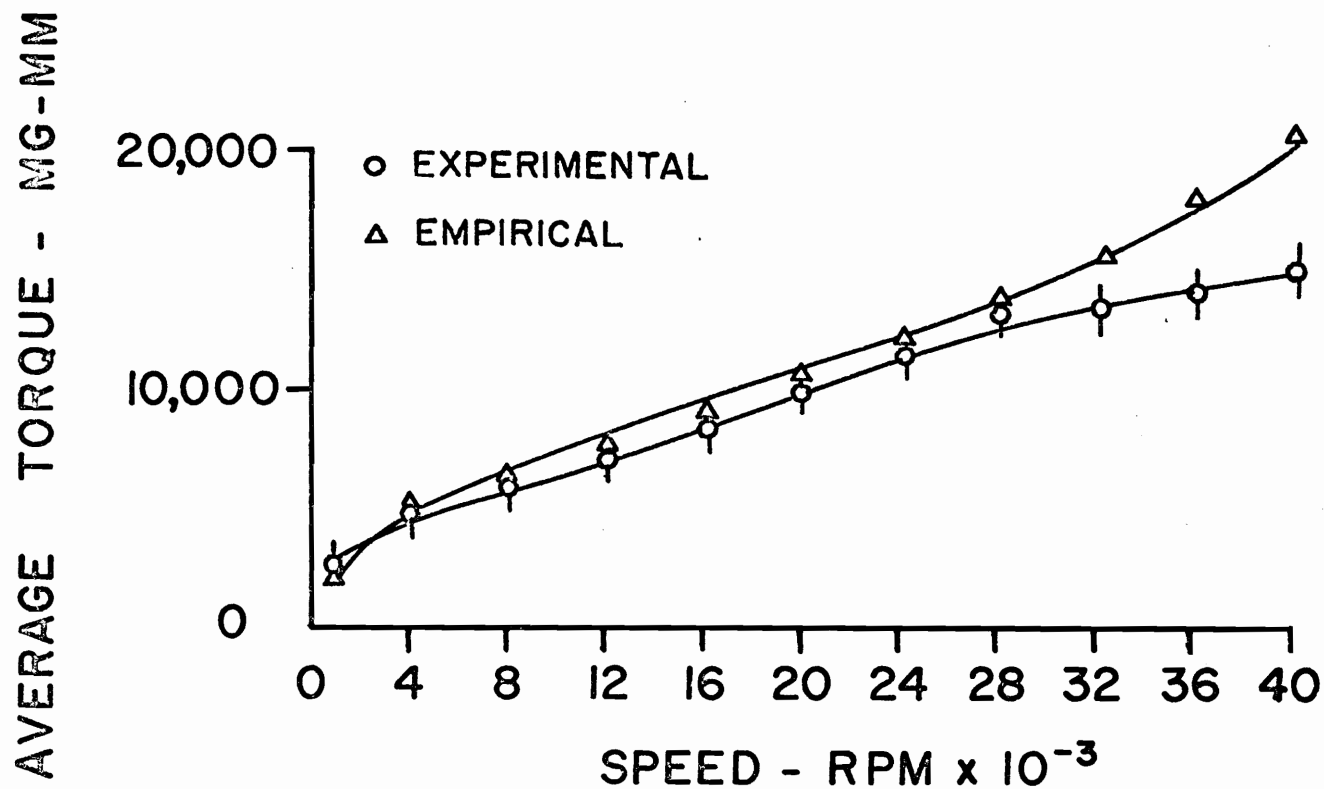

FIG. 39 COMPARISON FOR R-3
200 RADIAL, 50 AXIAL

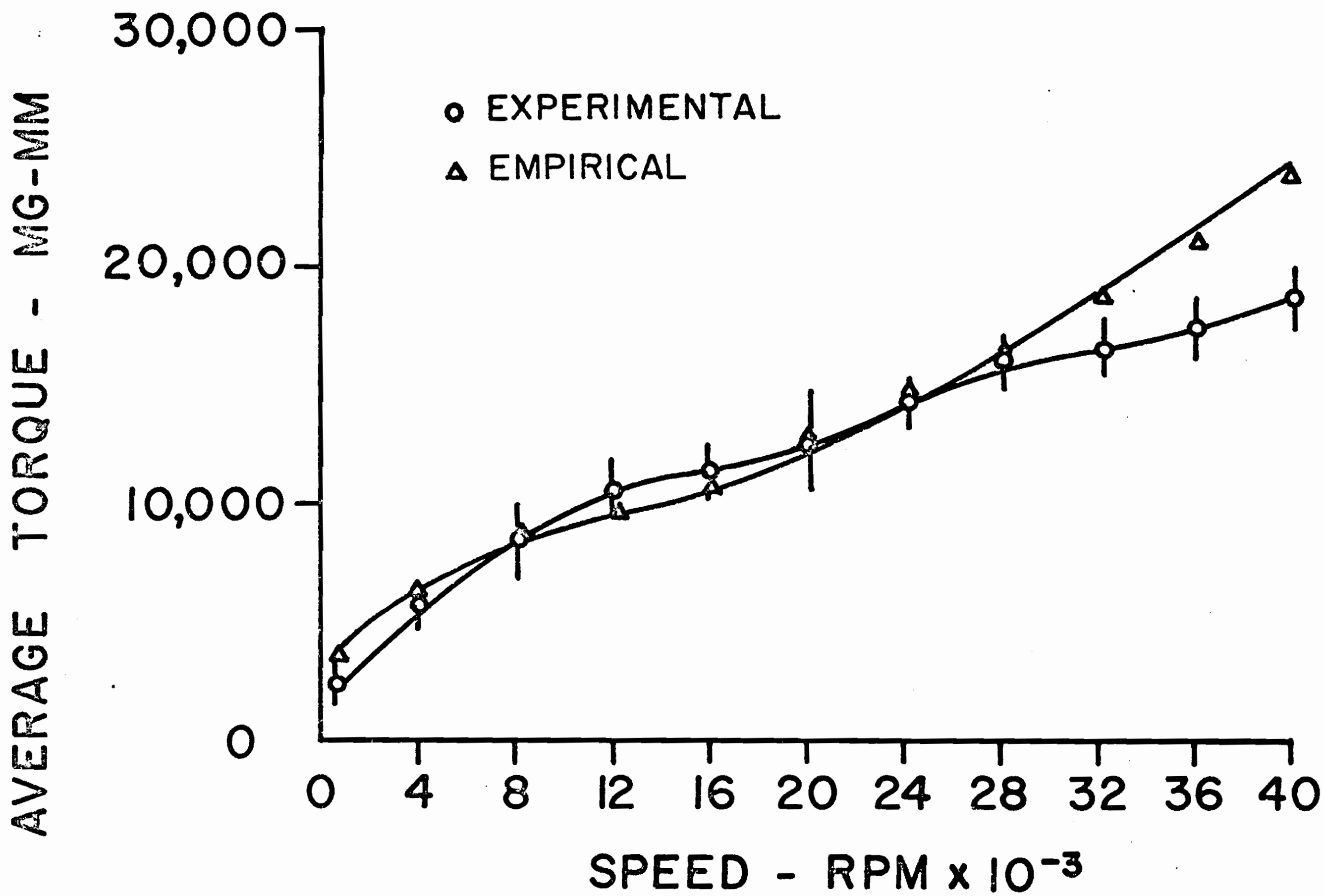

FIG. 40 COMPARISON FOR R-3
200 RADIAL, 100 AXIAL

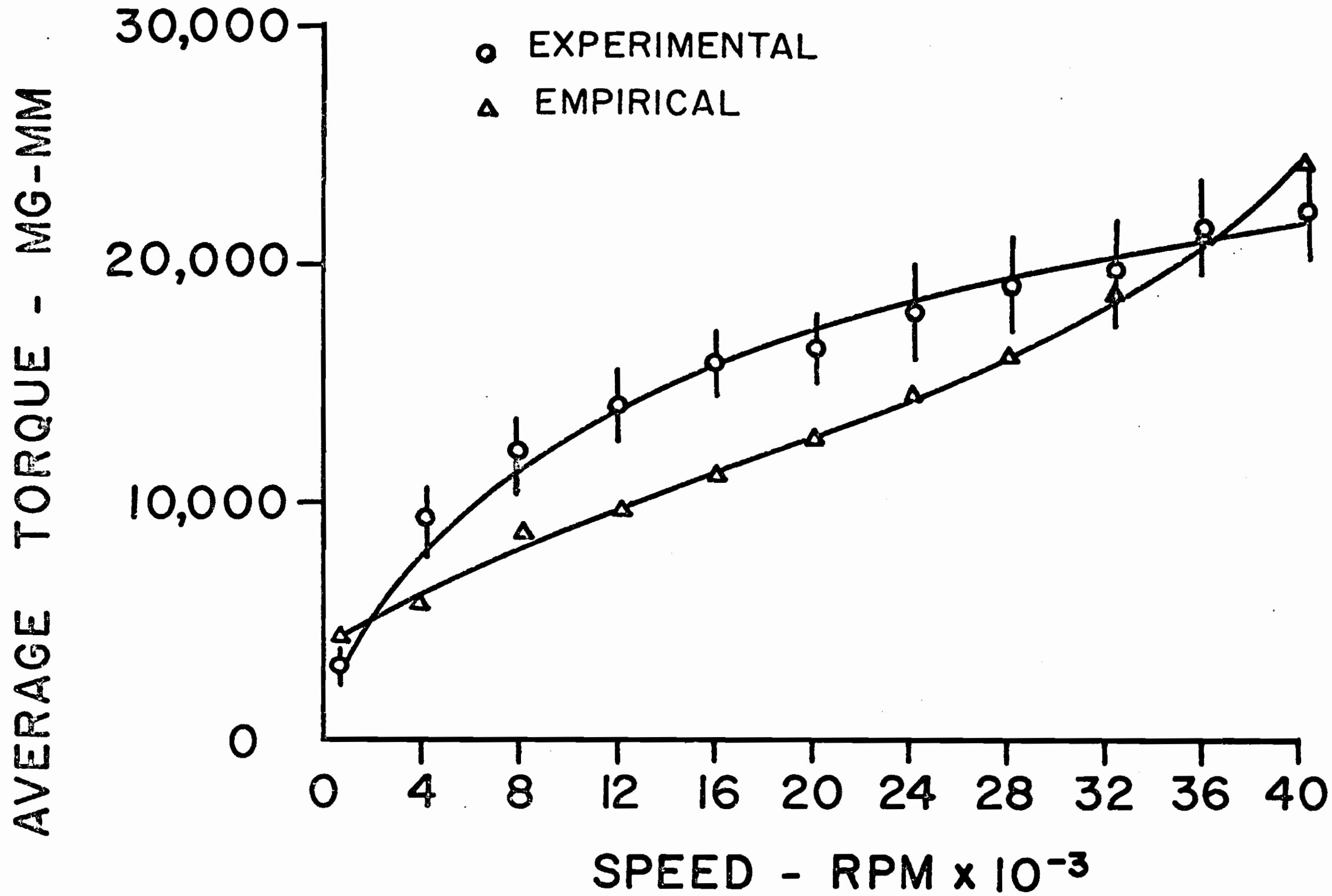

FIG. 41 COMPARISON FOR R-3
200 RADIAL, 200 AXIAL

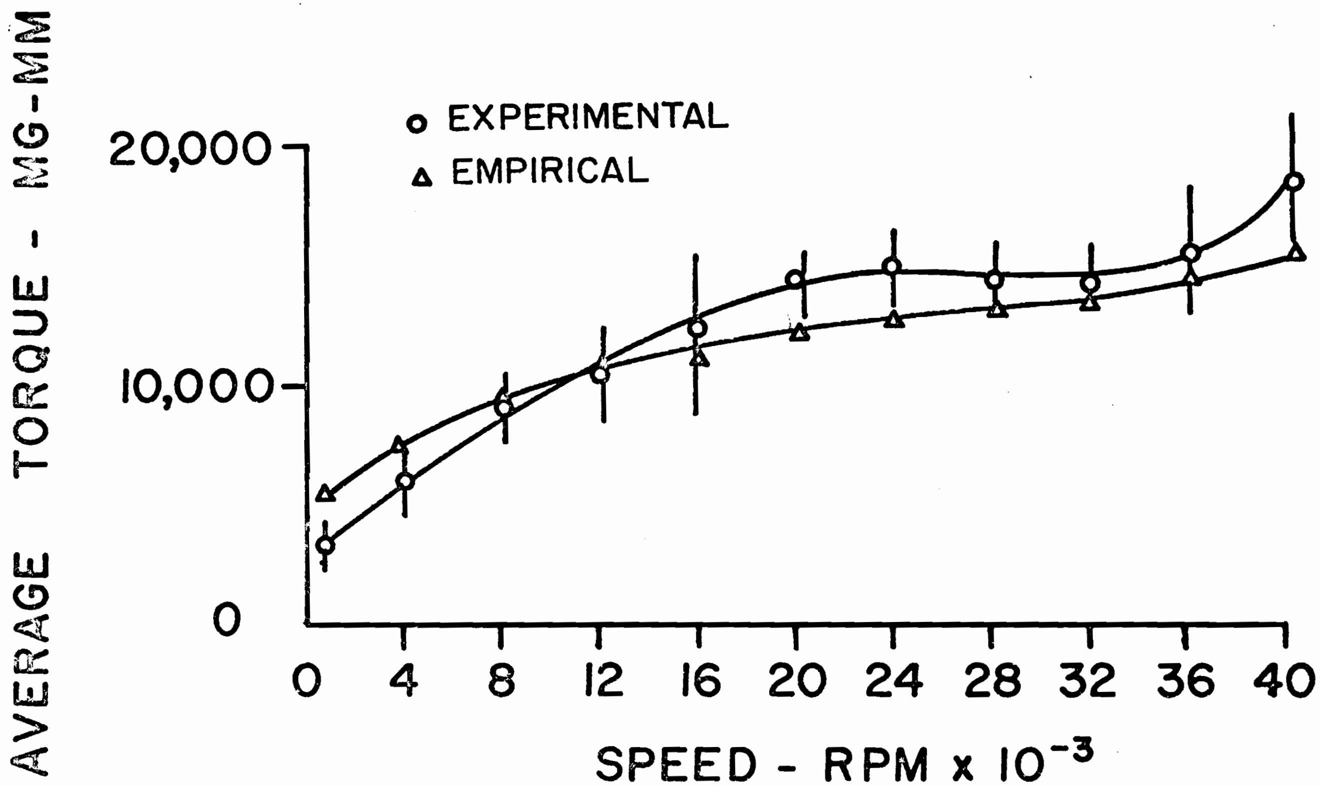

FIG. 42 COMPARISON FOR R-4
50 RADIAL, 0 AXIAL

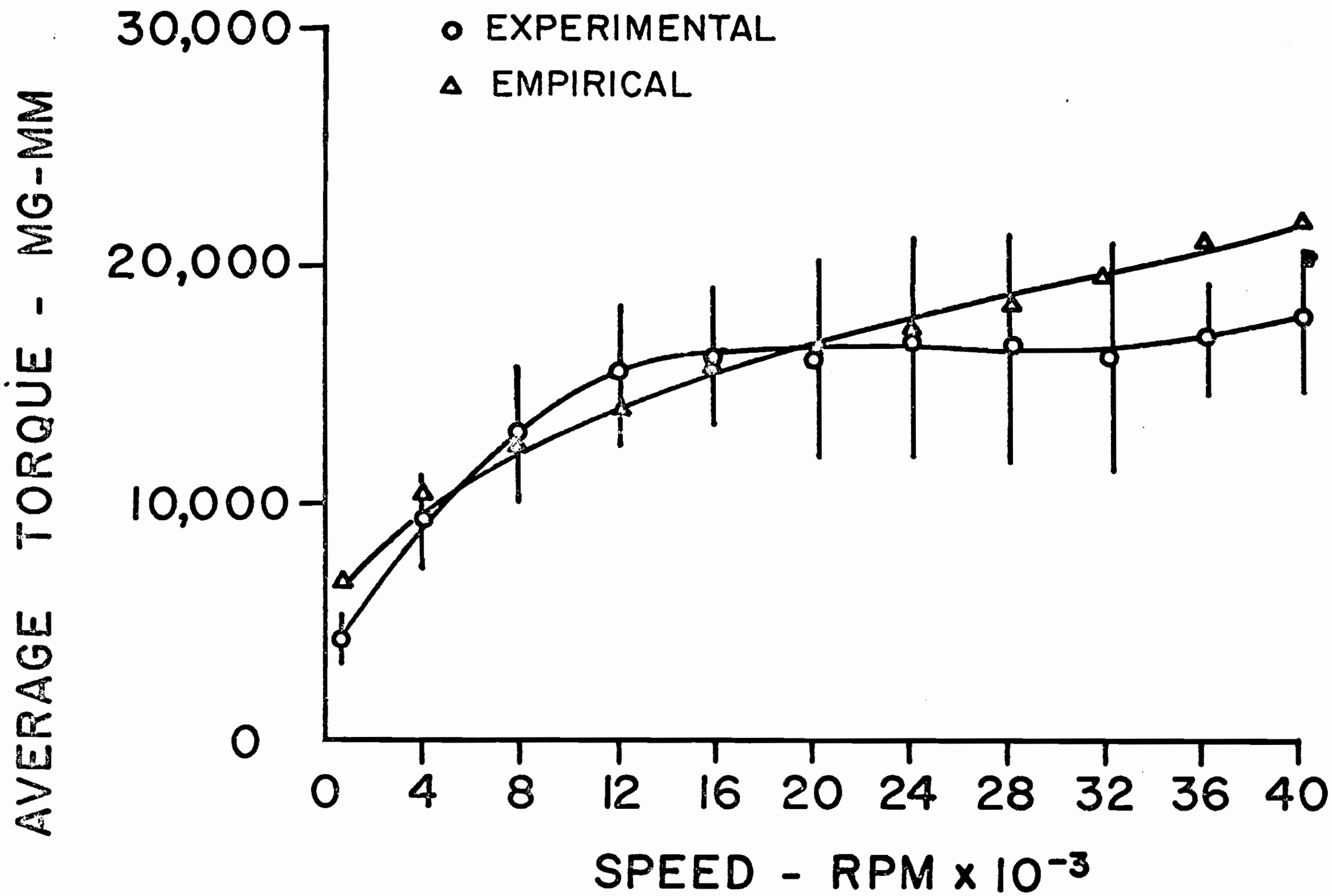

FIG. 4.3 COMPARISON FOR R-4
50 RADIAL, 50 AXIAL

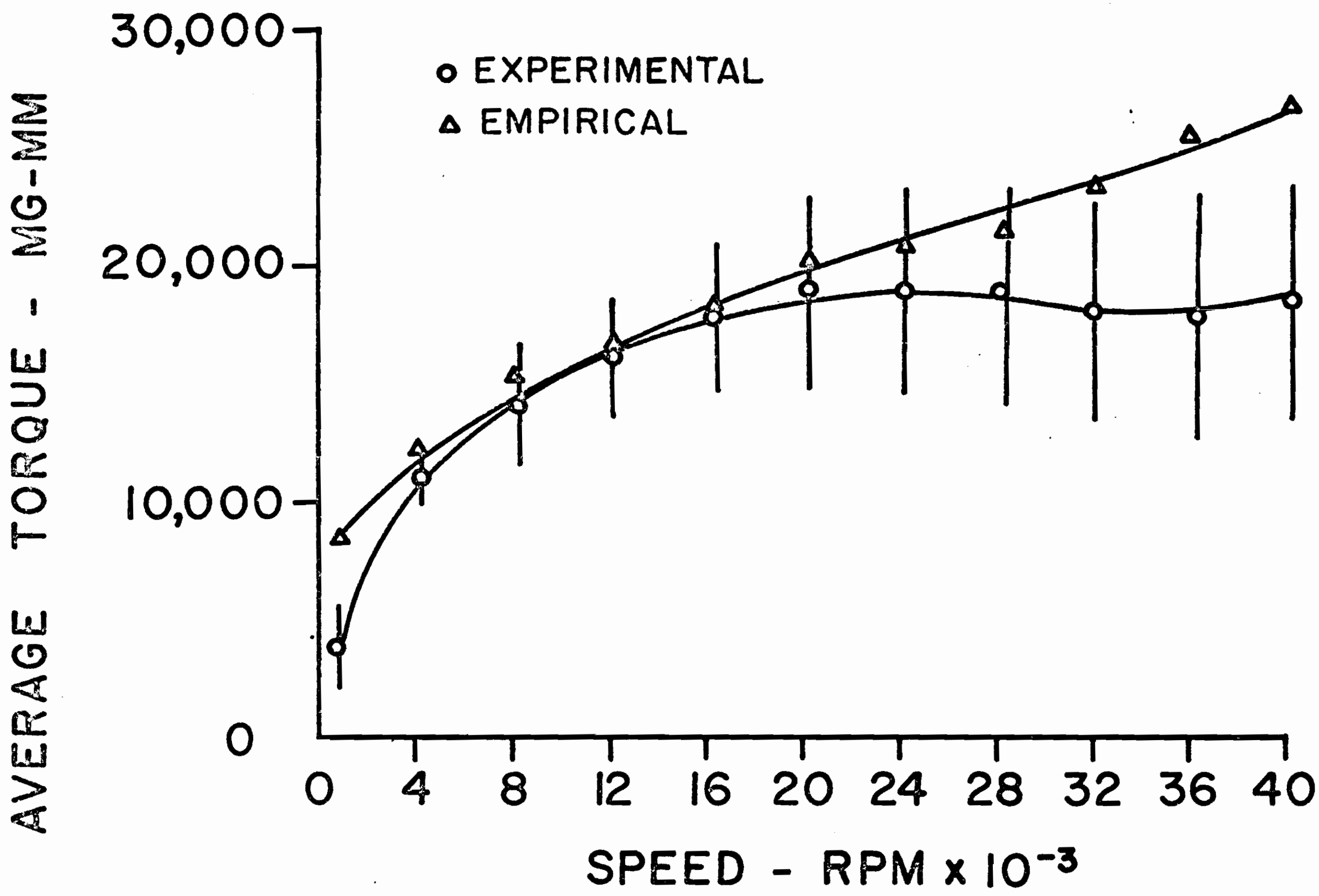

FIG. 44 COMPARISON FOR R-4
50 RADIAL, 100 AXIAL

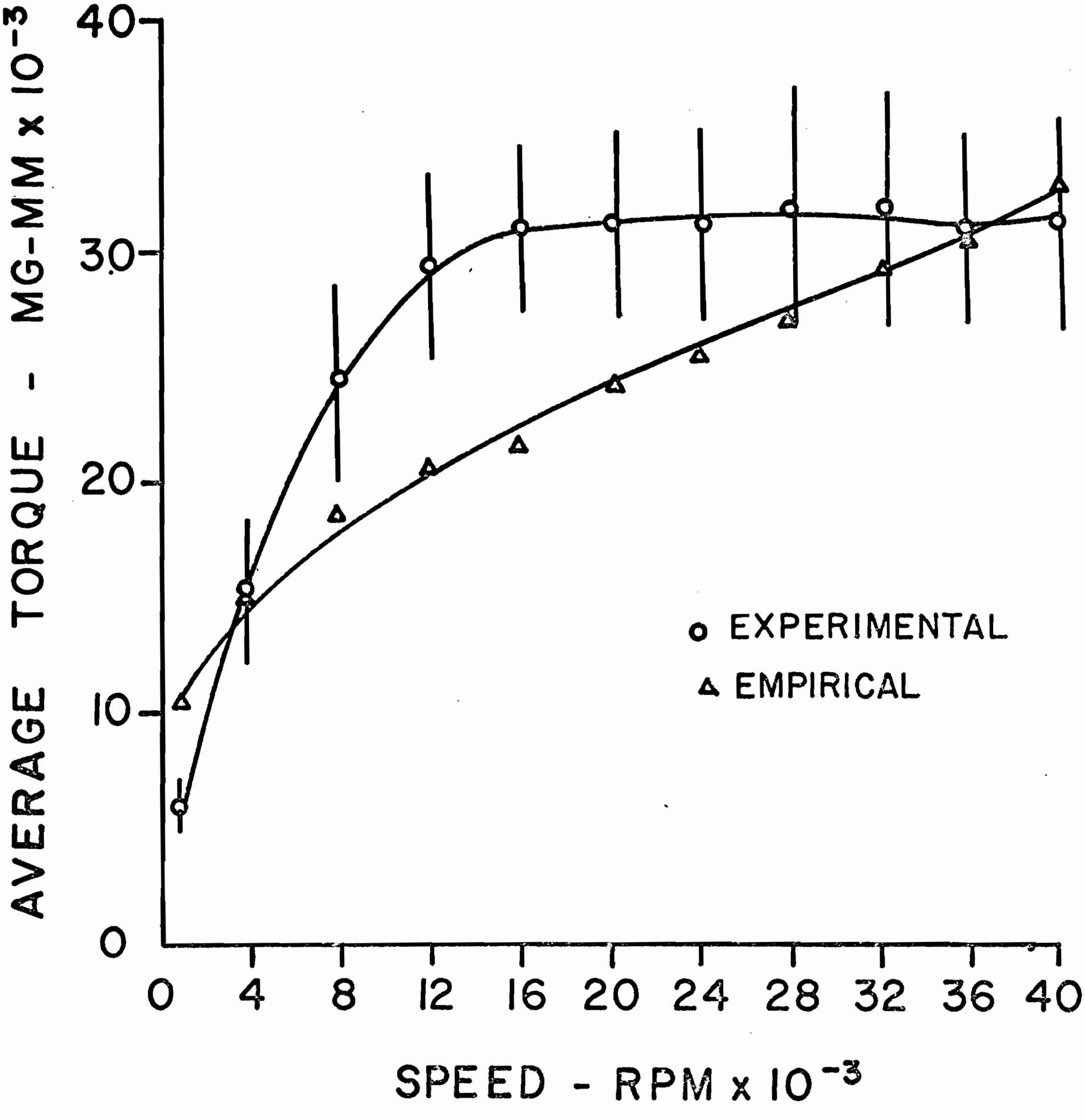

FIG. 45 COMPARISON FOR R-4
50 RADIAL, 200 AXIAL

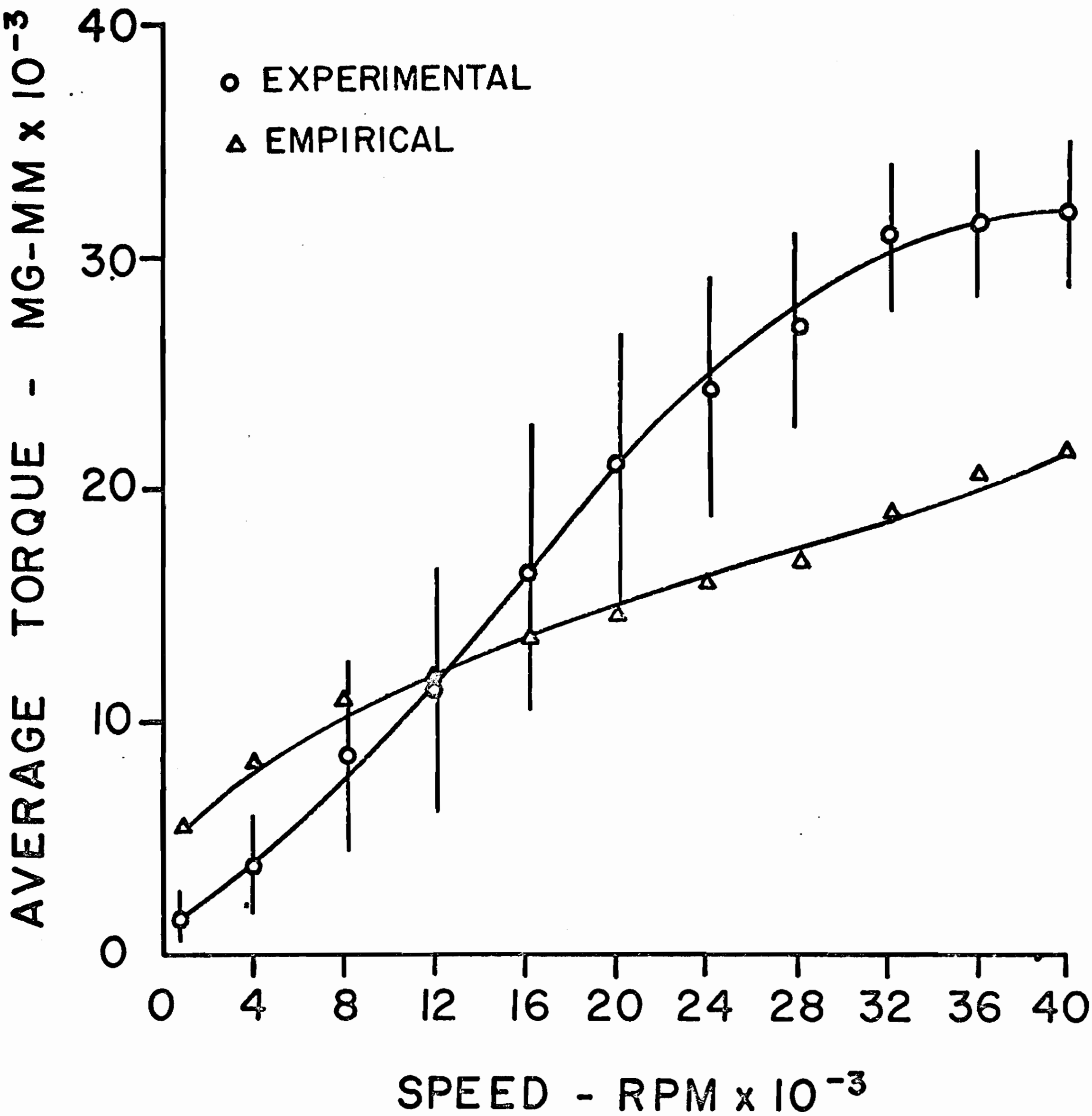

FIG. 46 COMPARISON FOR R-4
100 RADIAL, 0 AXIAL

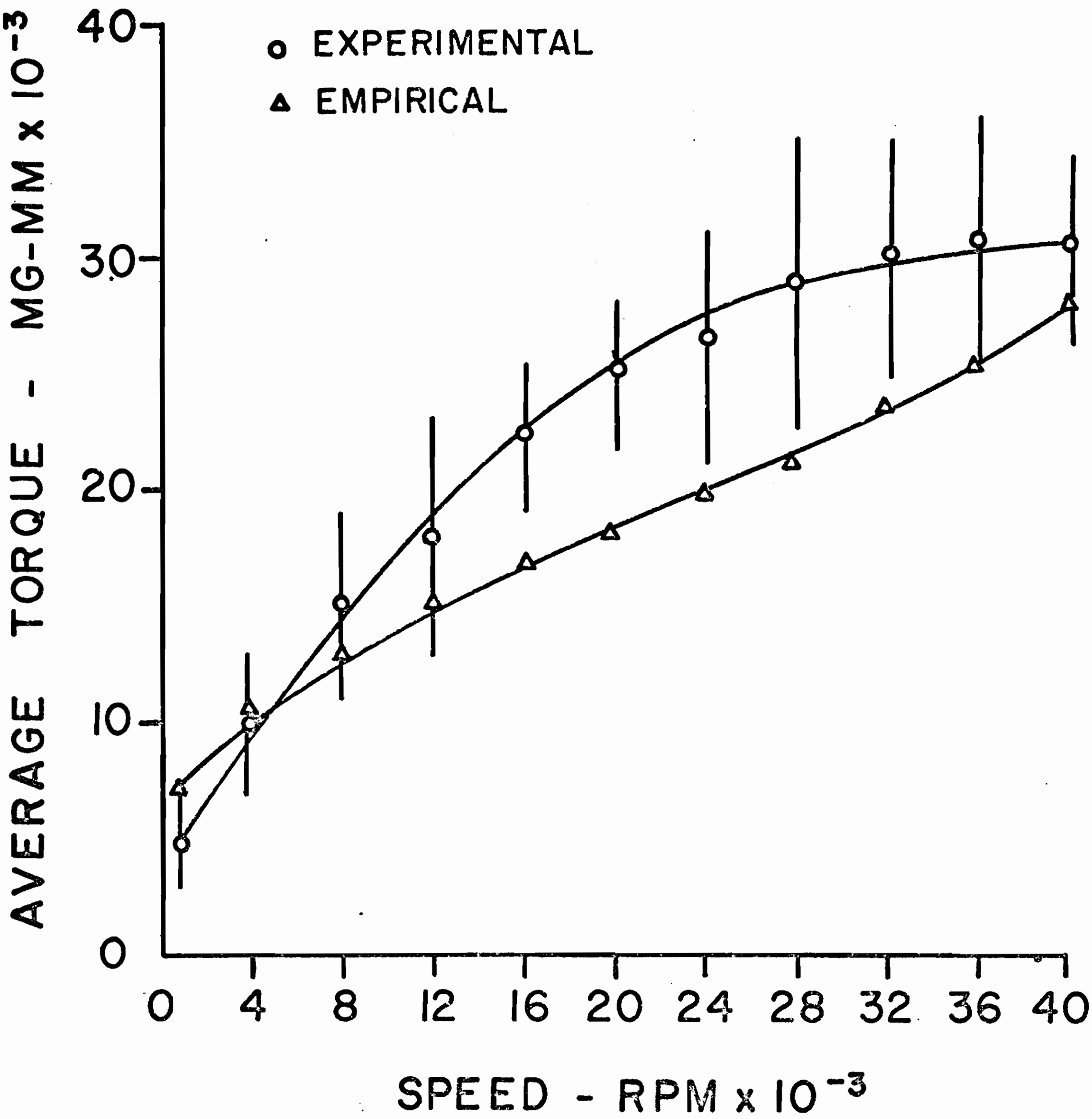

FIG. 47 COMPARISON FOR R-4
100 RADIAL, 50 AXIAL

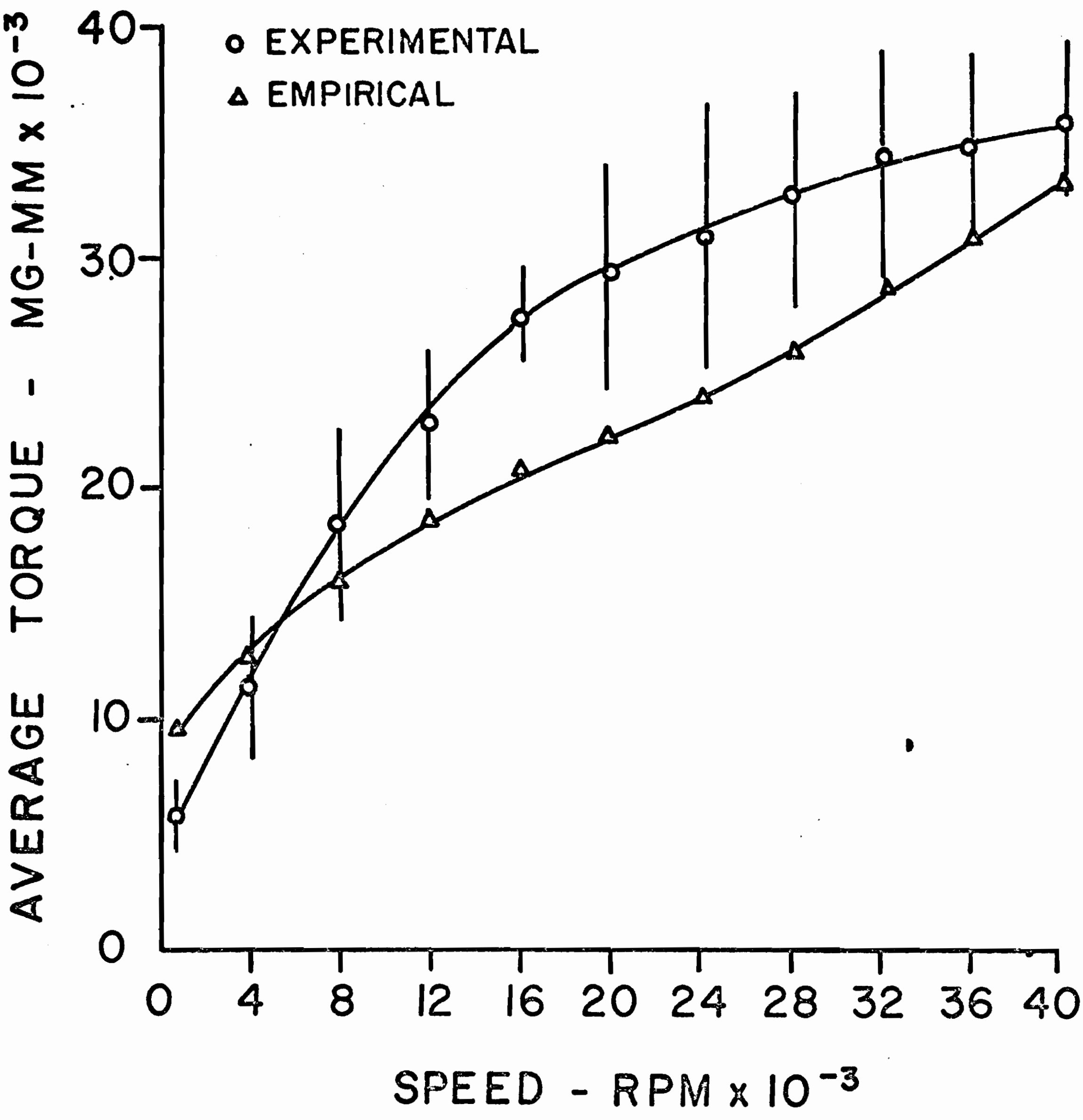

FIG. 48 COMPARISON FOR R-4
100 RADIAL, 100 AXIAL

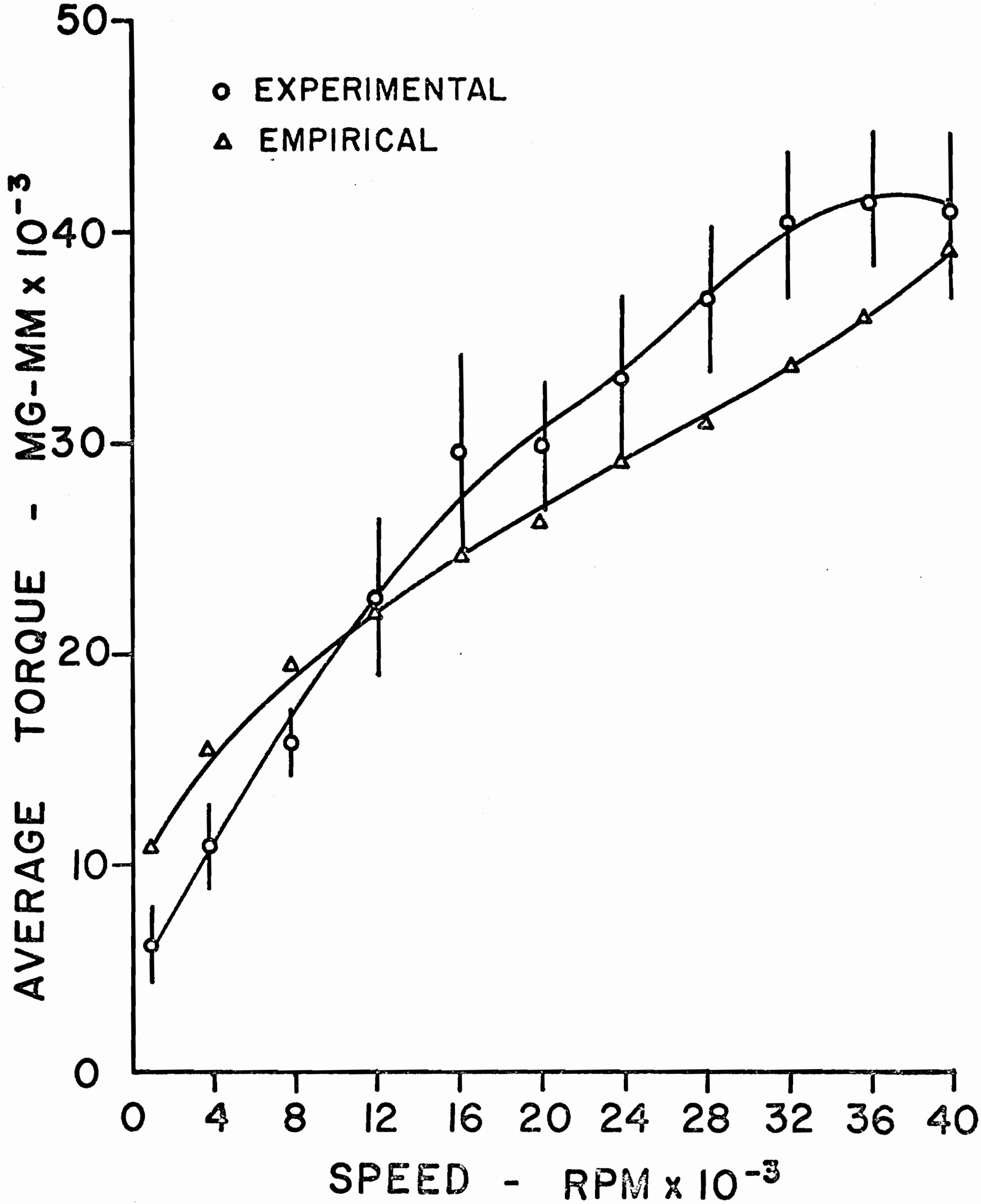

FIG. 49 COMPARISON FOR R-4
100 RADIAL, 200 AXIAL

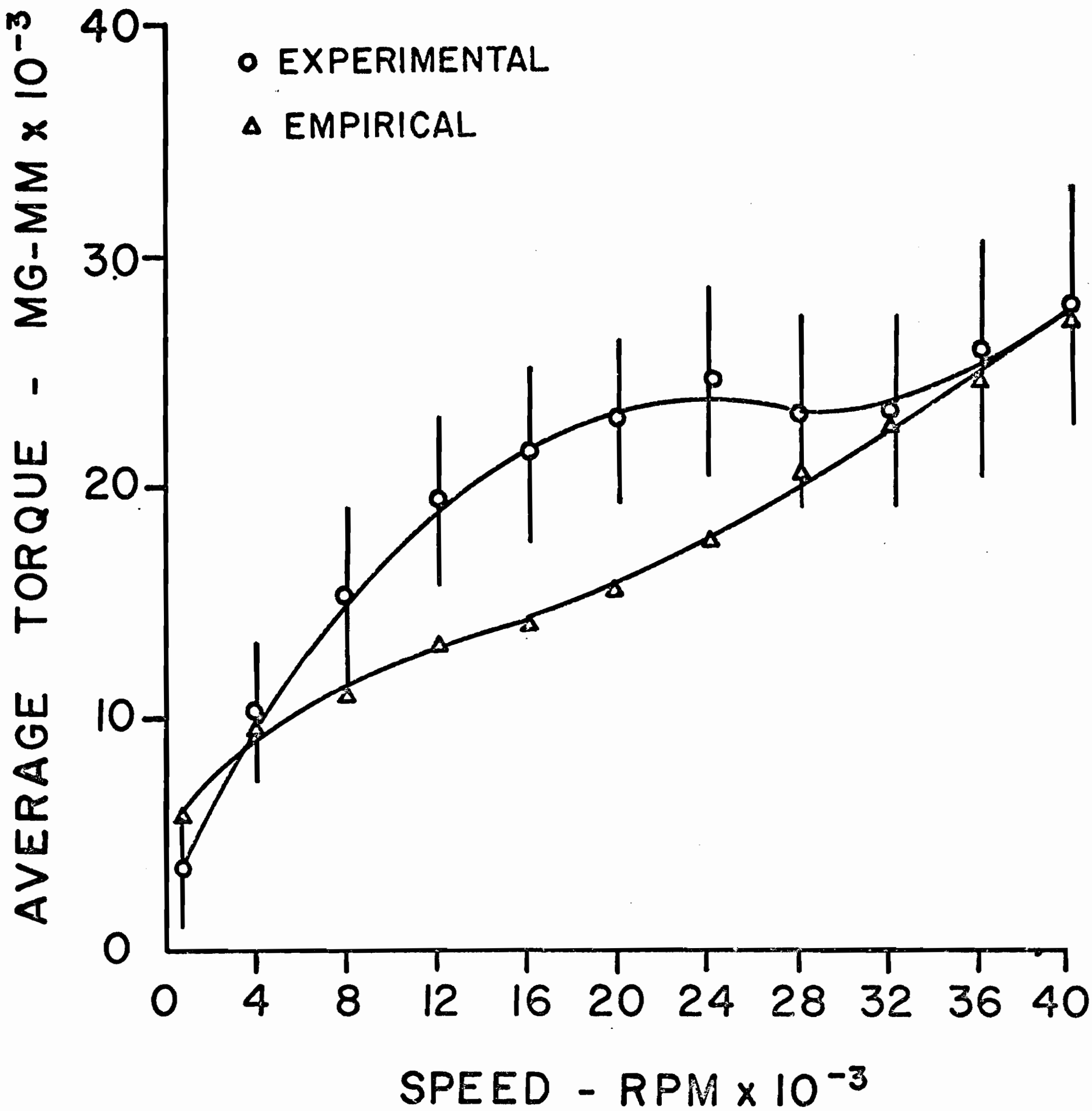

FIG. 50 COMPARISON FOR R-4
200 RADIAL, 0 AXIAL

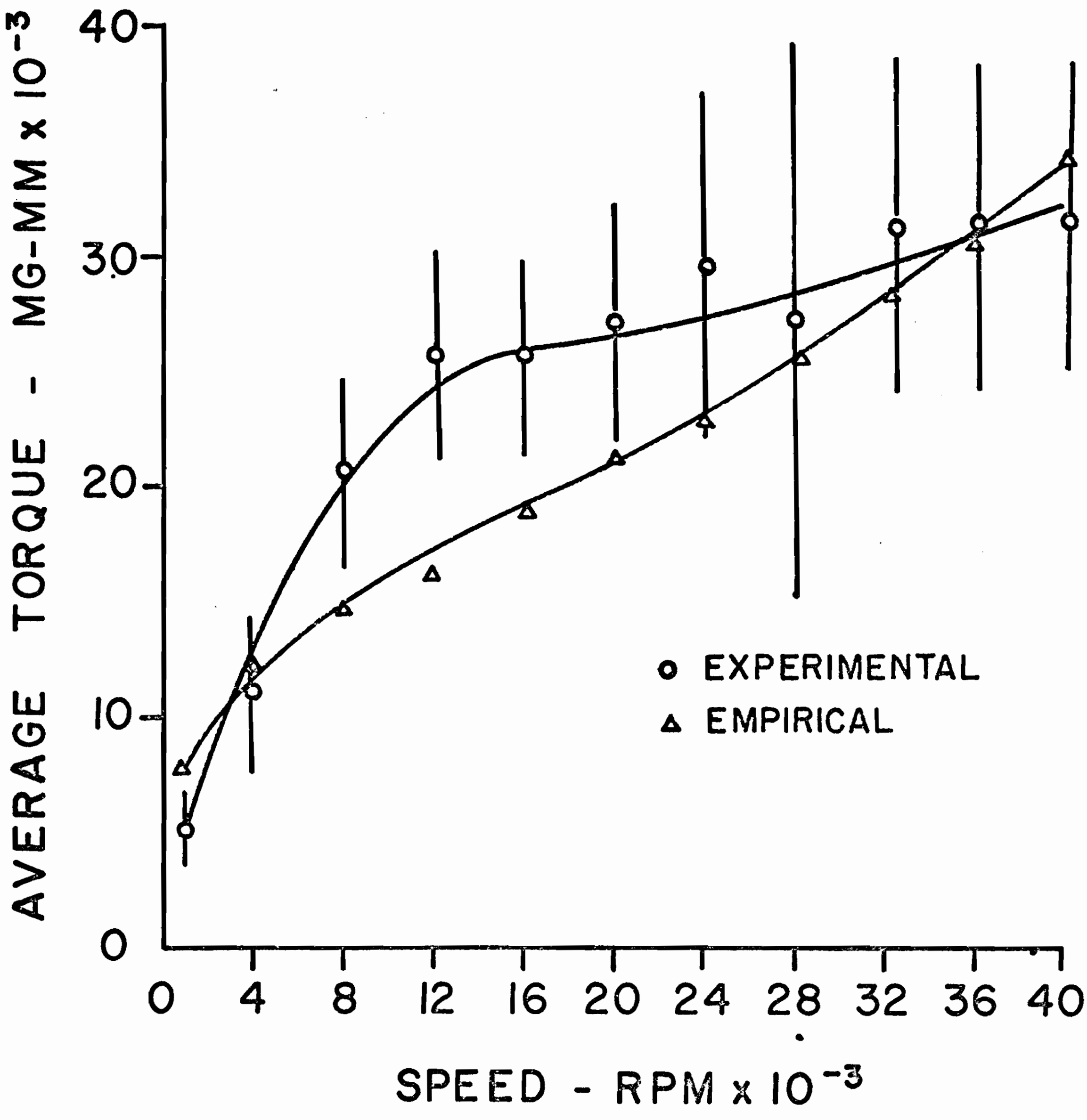

FIG. 51 COMPARISON FOR R-4
200 RADIAL, 50 AXIAL

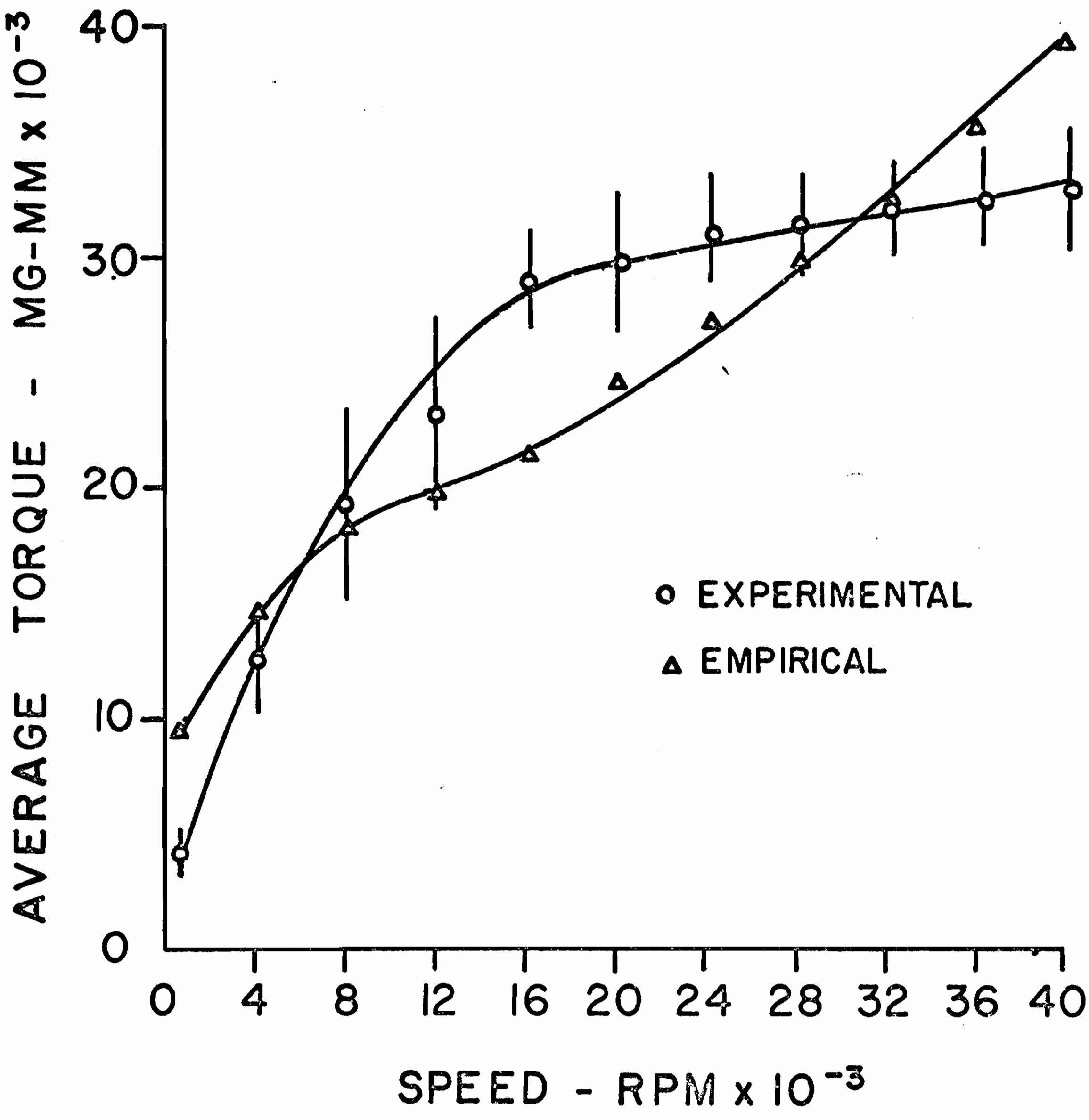

FIG. 52 COMPARISON FOR R-4
200 RADIAL, 100 AXIAL

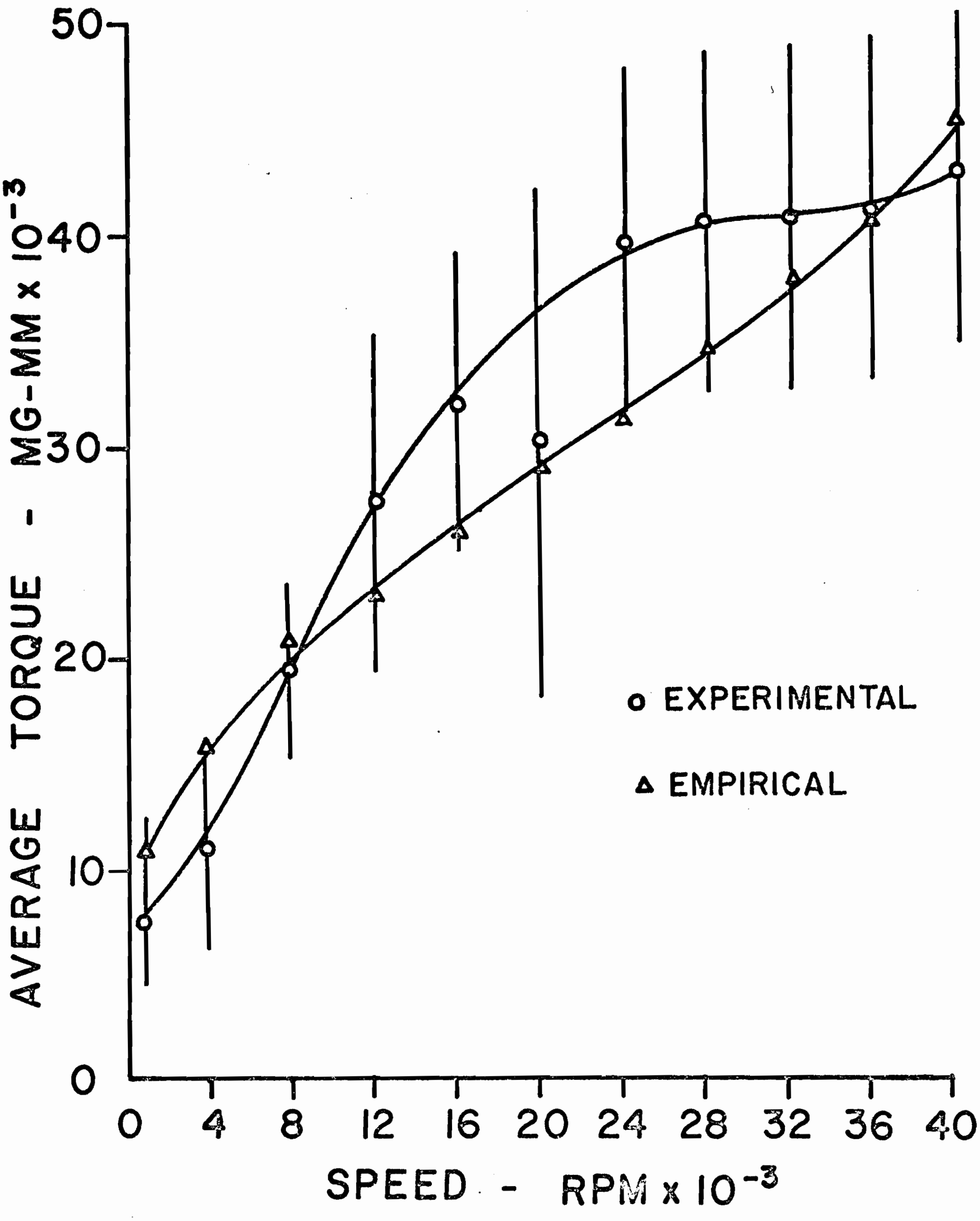

FIG. 53 COMPARISON FOR R-4
200 RADIAL, 200 AXIAL

V. DISCUSSION OF RESULTS

In the planning stages of this investigation, the primary consideration was the development of an analytical expression, if feasible, and comparison of the predicted values with experimental results. As mentioned in the Review of Literature, the factors considered were:

1. Elastic hysteresis in rolling
2. Sliding due to deformation of contacting elements and bearing geometry
3. Spinning of rolling elements
4. Gyroscopic pivotal motion of rolling elements
5. Viscous friction due to lubricant action
6. Sliding between cage and rolling elements and between cage and bearing rings
7. Seal friction

These factors and their relation to running torque will be discussed individually.

The elastic hysteresis due to rolling is traced to the cyclic stressing of the ball and raceways as the balls rotate about their own axes and the main axis of the bearing. The maximum stress a given ball is subjected to is dependent upon its position around the bearing since for a radially loaded bearing, the top ball is carrying the greatest load while the bottom ball may be completely unstressed. The point around the bearing where a ball begins to be loaded or becomes completely unloaded is highly dependent on the radial play in the individual bearing. This requires that considerable geometrical data be available for each individual bearing before any calculations may be made concerning the stress states of the balls. The result of these difficulties is that it is not feasible in design calculations to determine the stress states of the balls and consequently it is not possible to make a determination of losses due to a hysteresis effect.

The same problem mentioned above of determining the exact stress state of a given ball at each position around the bearing also carries over into attempts to determine the area of contact between the ball and raceway or the ball "footprint". The previously discussed Reynolds' slip corresponds to sliding due to deformation of the contacting elements. Heathcote's slip corresponds to sliding due to bearing geometry and spinning of the rolling elements. Friction losses due to these two types of slip are very difficult to determine and are highly theoretical in nature. Consequently when the information concerning the stress state and subsequent area of deformation is not known with any degree of certainty, losses due to these sources cannot be calculated.

Spinning due to gyroscopic motion is caused by forcing a rotating element to follow a curved path in space. In a ball bearing, this causes a spinning torque about an axis perpendicular to the raceways at the center of contact with the ball. The gyroscopic spinning may be considerable at high speeds; however, because of the unknown state of restraint of the ball as it travels around the race, the spinning and friction effect of gyroscopic spinning cannot be computed.

Lubrication is another factor which is relatively easy to consider if it were not for the change of ball loading around the raceway. Because of the ball load cycling, a given ball is constantly cycling between hydrodynamic and elastohydrodynamic lubrication. This also means that the viscosity of the lubricating oil is constantly varying in the region of a given ball as it is known that the viscosity of oil increases with pressure at a fixed temperature. However, the increase in pressure is known to raise the local temperature which decreases the viscosity of the oil.[1] The net effect of these two phenomena is not known for the microstates existing in the region of a ball under stress. Frictional heating was determined not to be a significant factor in the test run for this investigation by operating the loaded bearings at high speed and monitoring the temperature with small thermocouples.

Another lubrication problem encountered when a ball is lightly loaded is skidding. Skidding can be corrected by making the outer race slightly out of round thus artificially loading the balls. However, this technique is not used in instrument bearing manufacture.

A major source of bearing friction occurs at the points of contact between the balls and the retainer. In practice, retainers are stamped from thin gage sheet stock and crimped into place during final assembly of the bearing. The consequence of this is that it is very difficult to determine precisely the ball retainer clearances without disassembly which of course destroys the sample. Determination of pertinent friction parameters from this cause is rendered useless by production methods.

Seal friction need not be considered since instrument bearings rarely use seals. Instrument bearings usually have shields but these shields do not make contact between two moving parts and consequently do not contribute to the friction torque.

In considering the analytical approach to an expression for running torque of instrument ball bearings, it may be seen that the primary reason that an ideal approach does not apply is that manufacturing tolerances keep the bearing from being an ideal component. That is to say that the tolerances introduce errors which show up in radial play and areas of contact which in turn prevent ideal analytical determination of the torque contributing factors.

Because of the failure of an analytical approach to provide an expression for the running torque , an empirical approach was tried. The development of an empirical relationship is difficult to describe in distinct steps since to a large degree it depends on interpretation of trends in the data which can possibly vary from one investigator to another. In the development of the torque relationship finally used here, speed was considered to be the common variable and hence the variable which had to be determined first. Although not present in every torque speed plot, there was a definite trend for the curve to rise rapidly, level off, and then continue rising again so as to form a type of dip in the 15,000 to 30,000 rpm range. To account for this dip, it was thought necessary to use two speed terms. The first speed term was raised to a fractional power to account for the initial rise in the curve. The second term was raised to a power greater than unity to represent the secondary rise in the torque plot. The terms finally arrived at, $N^{1/4}$ and N^2, were the result of considerable trial and error experimentation with combinations of exponents.

The next most common factor in the experimental results was the bearing loading. This was divided into radial load and axial load. At this point, the general solution was felt to take the form:

$$T = N^{1/4}\left[f_1(d_m) + f_2(d_m,R) + f_3(d_m,A)\right] + N^2\left[f_4(d_m) + f_5(d_m,R) + f_6(d_m,A)\right]$$

where:
- T = running torque
- d_m = mean diameter
- R = radial load
- A = axial load
- f_i = arbitrary function

It was possible to partially isolate the radial load effects by starting with the zero axial load condition which was assumed to cause the terms with axial load drop out of the equation. Isolation of the radial load effect resulted in the following terms:

$$f_2(d_m,R) = (-1.33+0.03R-6.7\times10^{-5}R^2)(746-3906d_m+5482d_m^2)$$

and $$f_5(d_m,R) = (-1.33+0.03R-6.7\times10^{-5}R^2)(18.14-109d_m+170d_m^2)$$

These two terms were slightly more complex than originally anticipated.

The next step was to add in the effect of axial load. This was simplified slightly since the effect of radial load was already partially established. The axial load was found to add the following terms:

$$f_3(d_m,A) = (-167 + 1072d_m)(0.025A - 0.00005A^2)$$

and $$f_6(d_m,A) = (-0.17 + 2.68d_m)(0.025A - 0.00005A^2)$$

Again, the resulting expressions were more complex than anticipated in the general formula. The additional complexity is attributed to the interaction between radial load or axial load and bearing size.

Combining the radial and axial load terms and adding a size term, the final expression became:

$$\begin{aligned} T = {} & N^{1/4} \Big[(4569 - 28030d_m + 45327d_m^2) \\ & + (-1.33 + 0.03R - 6.7\times10^{-5}R^2)(746 - 3906d_m + 5482d_m^2) \\ & + (-167 + 1072d_m)(0.025A - 0.00005A^2)\Big] \\ & + N^2 \Big[(-27 + 201d_m - 313d_m^2) \\ & + (-1.33 + 0.03R - 6.7\times10^{-5}R^2)(18.14 - 109d_m + 170d_m^2) \\ & + (-0.17 + 2.68d_m)(0.025A - 0.00005A^2)\Big]\times10^{-6} \end{aligned}$$

When an empirical expression is developed in this manner it is difficult to justify all of the terms in purely analytical arguments. Nevertheless, some explanation should be offered whenever possible.

By noting the trends in the experimental data, the plateau in the curve in the mid-speed range should be apparent in most cases. It is thought that the initial rapid rise of the torque is attributable to the balls ploughing through the lubricant which tends to remain in the bearing races. However, after the bearing attains sufficient speed, the restoring forces acting on the lubricant are not sufficient to permit it to flow back into its original place in the bearing race before the next ball passes this point in the race. Thus, rapid ball movement prevents the lubricant from collecting in the races and also causes the lubricant to be splashed against the dust shields. From this point on, the primary contribution to the total running torque is probably due to viscoelastic forces, microslip, gyroscopic spinning, windage, and other purely frictional forces. The behavior of original run bearings as compared to re-run bearings lends some credence to this theory. The original run bearings in general have a more pronounced initial hump in the torque curve. This could be associated with the fresh lubricant being concentrated in the bearing races. The re-run bearings have already had the lubricant forced out of the races during the original run and possibly splashed up on the dust shields thereby accounting for less lubricant being accumulated in the races. Less oil accumulation in the races could account for lower ploughing forces in the lower speed ranges and hence a lower and less pronounced torque hump.

In relating these physical theories to the empirical expression for torque, the $N^{1/4}$ term represents the lubricant buildup in the races and is predominant below 20,000 rpm. Likewise, the N^2 term represents the purely frictional forces and drag terms. This speed term becomes dominant above 20,000 rpm.

Comparisons are made between empirically calculated running torques and experimental running torques in Figures 18 through 53. Those comparisons for R-2 size bearings are made in Figures 18 through 29. By looking through these plots, it can be seen that the empirical values correspond closely to the average experimental values. In fact, the empirical curve falls well within the range of the sample standard deviations in most cases. However, there is a tendency for the analytical curve to predict slightly higher running torques than evidenced by the experimental data up to approximately 8000 rpm. Some of this error is attributed to the compromises necessary to allow the curve to fit better elsewhere and with other size bearings.

Similar comparisons for R-3 bearings are shown in

Figures 30 through 41. For this size bearing, the empirical predictions compare quite favorably with the experimental results. The empirical values occasionally differ from the experimental values; however, these differences seem to occur in a random fashion and are small in magnitude. The random variations are acceptable.

The comparative data for the R-4 bearings is shown in Figures 42 through 53. The empirical and experimental results differ more for R-4 bearings than either of the other two sizes. It should be noted, however, that the sample standard deviations are larger for the R-4 bearings than for R-2 and R-3 bearings and that the empirical torques generally fall within the envelope of the sample standard deviations. Most of the differences encountered seem to be in the mid speed range where the empirical values are consistantly low. The differences in this region should be attributed to efforts to make the empirical expression agree with data in the high and low speed ranges as well as other sizes of bearings. In examining the relatively large sample standard deviations, bearing contamination might be considered. The same cleaning and lubricating procedures were used for all three size bearings. If there was some possibility of contamination occurring, it should have occurred with all three sizes and the smaller bearings should have been affected most. This was not the case as may be seen by the experimental results.

From the comparisons of torque values for all three sizes of bearings, the empirical and experimental results appear to be in good agreement. In most cases, any differences between the two values of running torque seem to be essentially random in nature. This would seem to imply that any differences observed would be due to statistical variation of individual samples rather than a problem with the empirical equation developed here.

In summary, an analytical approach to predicting the running torque was attempted and it was determined that it was not possible to develop an analytical formula because of the random variables introduced by manufacturing methods and tolerances. Since there was a wealth of experimental data obtained in tests on R-2, R-3, and R-4 bearings with several different load configurations, it was decided to develop an empirical expression to represent the running torque within the limits of size, speed, and load of the experimental data. The resulting expression is shown elsewhere in this discussion. The computer was programmed to calculate an empirical value at each speed where there was a known experimental value. A comparison of the empirical and experimental values was made to show the validity of the empirical expression. This completed the objective set forth at the beginning of the investigation of developing a suitable design expression for predicting the running torque of instrument ball bearings at least within specific limits of size, load, and speed.

VI. CONCLUSIONS

From the results of the subject investigation, the following conclusions can be drawn:

1. There is sufficient uniformity of behavior of ball bearings to permit an empirical expression to describe this torque versus speed characteristic.
2. The empirical equation developed in this investigation accurately represents the torque versus speed behavior of ball bearings ranging in size from 1/8 inch bore to 1/4 inch bore loaded with radial loads between 50 and 200 grams and axial loads between 0 and 200 grams and all combinations thereof over a speed range of 1000 to 40,000 rpm, at ambient temperatures.
3. Manufacturing methods and production tolerances of instrument ball bearings are not accurate enough at this time to permit the development of an analytical expression to predict the running torque of these bearings. In addition there is little known about the interaction and relative magnitudes of the friction inducing components.

VII. RECOMMENDATIONS

1. It is recommended that larger values of radial load and axial load be applied during testing in an effort to extend the useful range of the expression developed herein.
2. It is recommended that a test device be constructed and operated to obtain data with no radial load on the test bearing to provide data which was previously lacking in this area.

VII. BIBLIOGRAPHY

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The author was born in Washington, D.C. on December 15, 1942. He attended Parochial primary school and Arlington County Public Schools for his secondary education, graduating from Wakefield High School in June, 1960.

He entered Virginia Polytechnic Institute in September of 1960 as a civilian student on a partial academic scholarship. While working toward the Bachelor of Science Degree in Mechanical Engineering, he was employed by NASA in Greenbelt, Maryland, during the three summer vacations. He is a member of Pi Tau Sigma, Tau Beta Pi, and Phi Kappa Phi honorary fraternities. He was also an officer of Omicron Delta Kappa leadership fraternity and chairman of the student section of ASME. In his senior year he was a member of the German Club and elected to Who's Who of American Colleges and Universities. He was graduated in June, 1964.

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George E. Clarke III

George E. Clarke III

"Predicting the Running Torque of Instrument Ball Bearings at High Speed under Combined Radial and Axial Loads"

George E. Clarke

ABSTRACT

The purpose of this investigation was to develop an expression to represent the torque versus speed behavior of instrument ball bearings between 1000 and 40,000 rpm with various combinations of radial and axial load ranging between 0 and 200 grams. Because of the lack of experimental data for instrument bearings over any range of speeds, loads and sizes, it was necessary to construct a suitable bearing tester and accumulate the required data. The testers used were based on previous work by H.H. Mabie at Sandia Corporation and G.E. Clarke at V.P.I. The driving source was a small air turbine developed by Mabie which performed smoothly and reliably between 0 and 50,000 rpm. The torque measuring system employed strain gages on a very small beam which was used to sense forces on the stationary outer race of the bearing while the inner race was driven at speed. Each test was conducted from 0 to 40,000 rpm. The radial load took on values of 50, 100, and 200 grams. The axial load was 0, 50, 100, and 200 grams. All combinations of these loads were used for each size bearing. The sizes tested were R-2, R-3, R-4. Six bearings of each size were

used with all six bearings of each size undergoing the same test program in order to yield statistically reasonable averages.

Investigation of analytical methods of predicting the running torque indicated that production tolerances of ball bearings rendered such an approach impractical. This led to the development of an empirical expression to predict the running torque within the same range of sizes, loads, and speeds for which experimental test data was obtained. Such an empirical expression was successfully developed and the resulting torque predictions compared with the experimental values of torque. The empirical expression proved capable of predicting the running torques within the envelope of the sample standard deviations for a given bearing size and loading in most cases.

During the investigation of supplementary topics, it was determined that frictional heating was insignificant during the conduct of the torque tests which had a duration of approximately two minutes. All tests were at ambient temperature.

All tests conducted were with oil lubricant and ribbon retainer ball bearings. There was no evidence that the empirical expression for friction torque developed here was valid when extrapolated beyond the limits of size, load, and speed used in its development.