

Statistical Quality Control Techniques Using Multilevel Discrete Product Quality Measures

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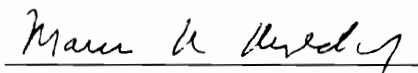
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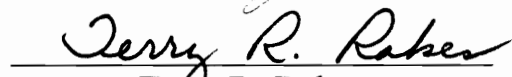
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STATISTICAL QUALITY CONTROL TECHNIQUES USING MULTILEVEL DISCRETE PRODUCT QUALITY MEASURES

by

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Industrial and Systems Engineering

(ABSTRACT)

Statistical quality control is the application of statistical methods to problems for which it is of interest to evaluate, establish, or verify the quality of a product. The two basic areas of statistical quality control that have received both the greatest attention in the literature and the widest acceptance in industry are acceptance sampling and statistical process control. In the majority of such techniques, a single characteristic of an item is used to describe its quality. In such cases, one of two basic types of product quality measures is typically used: attributes product quality measures and variables product quality measures. Variables product quality measures evaluate an item's quality by measuring its quality characteristic on a continuous scale. Attributes product quality measures assign a 0 to an item if its characteristic is conforming to some specification, and 1 if its characteristic is nonconforming.

Although attributes and variables product quality measures have many appropriate applications, there are many situations in which product quality is best described by classifying a single characteristic of the item using three or more discrete levels. A multilevel discrete product quality measure is a function that assigns a numerical value to such an item corresponding to the level in which it is classified.

Several acceptance sampling plans and control charts that incorporate the use of multilevel discrete product quality measures are defined here. In addition to the multilevel discrete product quality measure, each of the defined methods utilizes a quality value

function. A quality value function assigns a numerical value to an item based on the classification it receives from the multilevel discrete product quality measure. Each of the defined multilevel acceptance sampling plans and multilevel control charts is evaluated with respect to its probabilistic behavior. In addition, the problem of parameter selection and quality value function specification is addressed for each of the defined techniques. The cases considered are the 3-level case, the 4-level case, and the general j -level case.

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for Mom, Dad, and Wendy

CHAPTER 1: INTRODUCTION

Statistical quality control (SQC) is the application of statistical methods, such as hypothesis testing and experimental design, to industrial problems for which it is of interest to evaluate, establish, or verify the quality of a product. In such settings, the quality of a product is typically defined in terms of measurement or classification of one or more of the product's characteristics. The two basic areas of SQC that have received both the greatest attention in the literature and the widest acceptance in industry are acceptance sampling and statistical process control (SPC).

In most industrial processes, some of the items used as inputs to the process are received in lots from outside suppliers. Of these supplied items, some types may be sufficiently critical that the quality of each item in the lot is checked. In other words, lots of those items are subjected to 100% inspection. Other types of items may not be so critical. In addition, the testing of some types of items may be very costly or destructive. In such cases, acceptance sampling is often used to determine whether or not the quality of the items in the received lot is adequate. Acceptance sampling typically consists of taking a random sample of items from the lot, measuring or classifying one or more quality characteristics of each item in the sample, computing a sample statistic, and either accepting the lot for use in the process or rejecting the lot and returning it to the supplier. Since only a sample is taken, the danger exists of rejecting a lot with adequate quality (a Type I error) or accepting a lot that is inadequate (a Type II error).

Similarly, most industrial processes can be monitored by examining one or more of the quality characteristics of items produced by the process. If the process is studied sufficiently, it is usually possible to define the normal or in-control behavior of the process in terms of the probabilistic behavior of these characteristics. SPC is often used to monitor the behavior of a process and to indicate when the process has changed (in other words, when it has gone out of control). SPC typically consists of taking a random sample of finished items from the process at some regular interval in time, measuring or

classifying the quality characteristics of each item in the sample, computing a sample statistic, and either confirming that the process is in control or signaling that the process is out of control. If the process is determined to be in control, no action is taken before the next sampling point. However if a signal of an out-of-control situation occurs, the process is stopped and a search is conducted for the cause of the out-of-control behavior. If and when the cause is discovered, the process is corrected and then restarted. By far the most common implementation of SPC is the control chart. When implementing a control chart, the computed sample statistic is plotted on a chart. If the sample statistic falls within specified upper and lower control limits, the process is said to be in control. Otherwise a signal of an out-of-control situation is made. Again since only a sample is taken, the possibilities of not signaling when the process is out of control (a Type II error) or signaling when the process is in control (a false alarm or Type I error) do exist.

1.1 ATTRIBUTES AND VARIABLES STATISTICAL QUALITY CONTROL

In the majority of SQC techniques that are either utilized in industry or described in the literature, a single characteristic is used to describe product quality. In such cases, one of two basic types of product quality measures is typically used. These are referred to as variables product quality measures and attributes product quality measures. SQC techniques which incorporate the use of a variables product quality measure are referred to as variables SQC techniques. Likewise, SQC techniques which incorporate the use of an attributes product quality measure are referred to as attributes SQC techniques. In the case of variables SQC techniques, product quality is evaluated by measuring the quality characteristic of the product on a continuous scale. The quality characteristic is typically some physical tolerance placed on the product (such as length, weight, volume, etc.).

In attributes SQC techniques, the product quality is described using a binary variable. A product has quality measure 0 if its quality characteristic is conforming to some specifications and 1 if its quality characteristic is nonconforming. Attributes SQC

methods have the advantages of simpler inspection procedures and no required assumption regarding the distributional form of the quality characteristic (normal, Weibull, Poisson, etc.). Due to the increased precision of the product quality measure, variables SQC techniques typically require smaller sample sizes than corresponding attributes SQC techniques. However, some assumption must be made as to the distributional form of the quality characteristic. For both variables and attributes product quality measures, a wide variety of acceptance sampling and SPC techniques have been defined and evaluated in the literature and many have been widely implemented in industry.

It is worthwhile to note that some combined attributes-variables quality control methods have been developed. The typical implementation of these methods is to first evaluate lot quality or process behavior using an attributes product quality measure. If this information is inconclusive, further evaluation is performed using a variables product quality measure.

1.1.1 ATTRIBUTES AND VARIABLES ACCEPTANCE SAMPLING

Attributes acceptance sampling methods range from very basic concepts such as single sampling plans, double sampling plans, and multiple sampling plans (see Duncan[8] for a discussion of these methods), to more advanced strategies such as the Sequential Probability Ratio Test (SPRT) developed by Wald[25] and generalized sampling plans developed by Nachlas and Kim[15]. The U.S. Department of Defense MIL STD 105 series[22,23] is dedicated to the selection of single, double, and multiple sampling schemes.

Specific realizations of acceptance sampling methods (called acceptance sampling plans) are created by assigning values to the set of parameters that define the method. This can be done directly, but it is usually accomplished by specifying desired behavior of the Operating Characteristic (O.C.) function for the sampling plan. In acceptance sampling by attributes, the O.C. function, $P_a(p)$, is most often defined as the probability

of lot acceptance given p , the true underlying proportion of items in the lot that are nonconforming. The desired behavior of the O.C. function is usually specified in terms of an AQL (acceptable quality level) and a RQL (rejectable quality level). The AQL corresponds to the largest value of p that is satisfactory, and the RQL corresponds to the largest value of p that is tolerable. In order to complete the description of desired O.C. function behavior, a high probability of acceptance for the AQL is designated and a low probability of acceptance for the RQL is designated.

In all attributes acceptance sampling methods, a sample of items is taken from the received lot and lot disposition (lot acceptance or lot rejection) is based on the number of nonconforming items in the sample. However, the size of the sample may be either a predetermined constant or a random variable that is based on the classification of the items inspected. Thus another performance measure often associated with attributes acceptance sampling plans is the ASN (Average Sample Number) function. The ASN function, $ASN(p)$, gives the expected number of items inspected per lot given the underlying proportion nonconforming. ASN function behavior is sometimes considered in selecting an acceptance sampling plan. One additional related concept is curtailment. Curtailment refers to discontinuing inspection when the decision regarding lot disposition is implied by the items already inspected even if some portion of the specified sample is yet to be inspected.

Variables acceptance sampling techniques have also been studied in great detail. In almost all variables acceptance sampling methods, the quality characteristic for all items in a lot is assumed to be a normally distributed random variable (thus sample observations are assumed to be independent and identically distributed (IID) normal random variables). Variables acceptance sampling methods typically consist of taking a sample of items from the lot, measuring the quality characteristic of each item in the sample, computing the sample mean (and possibly the sample standard deviation), and making an appropriate decision regarding lot disposition. The O.C. function for a variables acceptance sampling

plan, $P_a(\mu, \sigma)$, designates the probability of lot acceptance given the underlying mean μ and variance σ^2 of the quality characteristic's distribution.

If the quality characteristic is subject to an upper specification limit U and a lower specification limit L (in other words, an item is conforming if its quality characteristic is between L and U), then variables acceptance sampling plans are designed to control the proportion nonconforming p , the proportion of items having a quality characteristic outside the specification limits. In such cases, sampling plan parameters are most often determined by specifying O.C. function behavior in a manner similar to that for attributes acceptance sampling plans. If such specification limits are not stated, then the variables acceptance sampling plans are designed to control the mean of the quality characteristic's distribution. Sampling plan parameters are still usually determined by specifying some desired O.C. function behavior. However in this case, the desired behavior is described by specifying a high probability of lot acceptance for the target value of the mean and a low probability of acceptance for an off-target mean value. Duncan[8] provides a thorough description of these types of acceptance sampling plans. Wald[25] also develops a variables SPRT. In addition, variables acceptance sampling techniques have been defined for controlling the variance of the quality characteristic's distribution and assuming other distributional forms for the quality characteristic.

1.1.2 ATTRIBUTES AND VARIABLES STATISTICAL PROCESS CONTROL

The most common implementation of attributes SPC is the p chart. At every sampling point, a sample of finished items taken from the process is inspected and the underlying proportion nonconforming is estimated by the proportion of nonconforming items in the sample. That estimate is then plotted on the control chart and compared to the upper and lower control limits. A slight variation of the p chart is the np chart, where the number of nonconforming items in the sample is charted.

Two related performance measures are commonly used to evaluate all control charts, the O.C. function and the ARL (Average Run Length) function. For attributes control charts, the O.C. function, $P_a(p)$, gives the probability that the sample statistic falls within the control limits given that the underlying proportion of the items produced that are nonconforming is p . The ARL function, $ARL(p)$, gives the expected number of samples until a signal given that the underlying proportion nonconforming is p . Parameters for attributes control charts are usually selected by specifying a large ARL function value for the in-control proportion nonconforming and a small ARL function value for a specific out-of-control proportion nonconforming. Montgomery[14] provides a detailed description of attributes SPC techniques.

Variables SPC techniques are much more extensive than attributes SPC techniques. In all variables SPC methods, at each sampling point, a sample of finished items is taken from the process and each item's quality characteristic is measured. As with variables acceptance sampling methods, variables SPC techniques are typically based on the assumption that the quality characteristic of finished item is normally distributed (thus sample observations are assumed to be IID normal random variables). If it is of interest to control the mean and the variance of the quality characteristic's distribution (referred to as the process mean and process variance), then the sample mean and one of (1) the sample range, (2) the sample standard deviation, and (3) the sample variance are computed. Then one control chart is used to monitor the process mean and another control chart is used to monitor the process variance. If it is only of interest to control the quality characteristic's mean, then only the sample mean is computed and only a single control chart is used. The most common variables control charts used for monitoring the process mean are the \bar{X} chart, the EWMA (Exponentially Weighted Moving Average) chart, and the CUSUM (Cumulative Sum) chart. The most common variables control charts used to monitor the variance are the R chart, the S chart, and the S^2 chart. The O.C. function for a variables control chart, $P_a(\mu, \sigma)$, gives the probability that the sample statistic falls within the

control limits given that the underlying values of the process mean and process variance are μ and σ^2 respectively. The ARL function for variables control charts, $ARL(\mu, \sigma)$, gives the expected number of samples until a signal given that the underlying values of the process mean and variance are μ and σ^2 respectively. For all variables control charts, control chart parameters are typically selected by specifying in-control ARL function behavior and ARL function behavior for a specific out-of-control situation (a large ARL function value is specified for the in-control case, and a small ARL function value is specified for the out-of-control case). Montgomery[14] and Duncan[8] also provide detailed descriptions of all of these control charts.

1.2 MULTILEVEL DISCRETE PRODUCT QUALITY MEASURES

Although attributes and variables product quality measures have many appropriate applications, there are many situations in which product quality is best described by classifying a single characteristic of the product using three or more discrete levels. Consider the following: (1) a food product may be classified as good, marginal, or bad depending on the concentration of harmful microorganisms in the product; (2) a four-level quality description is described in the MIL STD 105 series[22,23] (nondefective, minor defective, major defective, critical defective); (3) if a lot of 1000 items are arranged in order of increasing quality, then the resulting ordering is essentially an application of a 1000-level product quality measure. In fact, a variables product quality measure is one type of an infinite-level product quality measure (however the levels are not discrete) and an attributes quality measure is a 2-level product quality measure. In any case, multilevel discrete product quality measures fall somewhere between attributes and variables product quality measures.

Definition 1.1: A multilevel discrete product quality measure is a function which represents an item's quality by (1) classifying a single characteristic of the item using three or more discrete levels, and (2) assigning a numerical value to the item corresponding to the level in which it is classified.

Definition 1.2: A multilevel discrete product quality measure that uses j levels ($j > 2$) is called a j -level product quality measure.

The use of a multilevel discrete product quality measure has some attractive sampling properties. First, a multilevel discrete product quality measure provides increased precision over an attributes product quality measure without requiring exact measurement of the quality characteristic. Second, the infinitesimal difference between conforming and nonconforming items that sometimes exists when using either an attributes or a variables product quality measure is removed. Finally, there is no required assumption regarding the distributional form of the quality characteristic as there is when using a variables product quality measure.

The specific case in which product quality is described using three levels has been addressed in the literature but only in a manner that easily relates to existing attributes and variables quality control techniques (with one exception). All of these existing analyses are somewhat limited by their construction and assumptions. However, one acceptance sampling method which incorporates the use of a 3-level product quality measure has been widely recognized in the food processing industry. These techniques are reviewed in Chapter 2.

1.3 PROBLEM STATEMENT

The purpose of this research is to develop several acceptance sampling and SPC techniques (control charts) that incorporate the use of multilevel discrete product quality measures. Such techniques are referred to as multilevel acceptance sampling plans and multilevel control charts. The approach taken, which is quite different from any previous analysis, is first to characterize multilevel discrete product quality measures and to determine how to describe either lot quality (for acceptance sampling) or the state of a process (for SPC) in terms of the discrete quality levels. The next step is to define different types of quality value functions in terms of the discrete quality levels. A quality

value function assigns a numerical “quality value” to an item based on the classification it receives from the multilevel discrete product quality measure. For a given multilevel discrete product quality measure, any number of quality value functions can be selected for use in either multilevel acceptance sampling plans or multilevel control charts.

The development of a multilevel acceptance sampling technique begins with the selection of a multilevel discrete product quality measure and a corresponding quality value function. Having made those selections, the next step is the specification of a sampling strategy (sample size, when to terminate sampling, etc.) and a decision rule (for lot disposition). The O.C. function for the multilevel acceptance sampling method is constructed. O.C. function behavior is based on the quality of the lot as well as the quality value function used. Since any number of quality value functions can be specified, the potential for many alternative multilevel acceptance sampling techniques exists. Thus four alternative techniques are proposed and evaluated for the 3-level case and the 4-level case, and the general j -level case is considered as well.

Following the description and evaluation of multilevel acceptance sampling techniques, the problem of sampling plan parameter selection is addressed. For each of the proposed multilevel acceptance sampling techniques, methods are defined for selecting parameters for the sampling strategy and decision rule by specifying desired O.C. function behavior. In addition, the specification of a quality value function based on desired O.C. function behavior is considered.

The development of a multilevel control chart also begins with the selection of a multilevel discrete product quality measure and a corresponding quality value function. Having made those selections, the next step is the specification of a sampling strategy, the sample statistic to be plotted on the control chart, and the control limits for the control chart. The O.C. function and the ARL function for the multilevel SPC technique are constructed. O.C. function and ARL function behavior is based on the state of the process as well as the quality value function used. Since any number of quality value functions can be specified, the potential for many alternative multilevel SPC techniques exists. Thus

three alternative techniques are proposed and evaluated for the 3-level case and the 4-level case, and the general j -level case is considered as well.

Following the description and evaluation of multilevel control charts, the problem of control chart parameter selection is addressed. For each of the proposed multilevel SPC techniques, methods are defined for selecting parameters for the sampling strategy, the sample statistic, and the control limits by specifying desired ARL function behavior. In addition, the specification of a quality value function based on desired ARL function behavior is considered.

1.4 RESULTS AND CONCLUSIONS

In this research, a number of multilevel acceptance sampling plans are defined. For each of these defined plans, the O.C. function is constructed and used to develop methods for sampling plan parameter selection and quality value function specification. The parameter selection method allows a manufacturer to quickly select a multilevel acceptance sampling plan that closely meets desired O.C. function behavior. The quality value function specification method allows the manufacturer to construct a multilevel acceptance plan that closely meets as many as four or five O.C. function objectives. This is quite an improvement over existing methods which typically allow a manufacturer to specify only two O.C. function objectives.

A number of multilevel control charts are also defined. For each of these defined control charts, the O.C. function and ARL function are constructed and used to develop methods for control chart parameter selection and quality value function specification. The parameter selection method allows a manufacturer to select a multilevel control chart based on desired ARL function behavior. The quality value function specification method allows the manufacturer to construct a multilevel control chart based on as many as four or five ARL function objectives. This is quite an improvement over existing methods which typically allow a manufacturer to specify only two ARL function objectives.

The sampling plans, control charts, and the parameter selection and quality value function specification methods defined in this research are the beginnings of a new approach to multilevel statistical quality control. As these statistical quality control techniques are explored in further detail and extended with new concepts and methods, the result should be a set of tools that provide a manufacturer with greater flexibility in designing acceptance sampling plans and control charts.

CHAPTER 2: LITERATURE REVIEW

Before describing the research methodology, it is worthwhile to review existing analyses related to multilevel discrete product quality measures and their implementation in acceptance sampling and statistical process control. This review consists of describing (1) references to the possibility of applying such measures and (2) existing implementation of 3-level product quality measures.

2.1 REFERENCES TO MULTILEVEL DISCRETE PRODUCT QUALITY MEASURES

The most widely recognized reference to a multilevel discrete product quality measure is made in the U.S. Department of Defense MIL STD 105 series for attributes acceptance sampling. In MIL STD 105E[22], a defect is defined to be “any nonconformance of the unit of product with specified requirements.” Furthermore, three different types of defects are defined: critical defects, major defects, and minor defects. A critical defect is defined as “a defect that judgment and experience indicate would result in hazardous or unsafe conditions for individuals using, maintaining, or depending on the product, or a defect that judgment and experience indicate is likely to prevent performance of the tactical function of a major end item such as a ship, aircraft, tank, missile, or space vehicle.” A major defect is defined as “a defect, other than critical, that is likely to result in failure, or to reduce materially the usability of the unit of product for its intended purpose.” Finally, a minor defect is defined as “a defect that is not likely to reduce materially the usability of the unit of product for its intended purpose, or is a departure from established standards having little bearing on the effective use or operation of the unit.”

There are situations in both acceptance sampling and SPC in which the objective is to study the number of defects in a product (see Montgomery[14] or Duncan[8] for a description of these methods). However, this research has the more common objective of

studying the occurrence of nonconforming (or defective) items, or more specifically, the occurrence of different classes of nonconforming items. This objective is recognized in MIL STD 105E. A defective is defined as “a unit of product which contains one or more defects,” and three classes of defectives are defined: critical defectives, major defectives, and minor defectives. A critical defective is defined as “a unit of product which contains one or more critical defects and may contain major and/or minor defects.” A major defective is defined as “a unit of product which contains one or more major defects, and may also contain minor defects but contains no critical defect.” A minor defective is defined as “a unit of product which contains one or more minor defects but contains no critical or major defect.” Under such definitions, any unit of product could be classified in one of four classes: critical defective, major defective, minor defective, or nondefective. Such classification corresponds to implementing a 4-level product quality measure.

Although MIL STD 105E creates the framework for using a multilevel product quality measure, acceptance sampling schemes incorporating the use of a 4-level product quality measure are not provided. In fact, the standard proposes that each of the three classes of defectives should be treated as separate inspection problems. In other words, different quality specifications (in terms of an AQL) should be made for each class of defectives. As a result, implementation of MIL STD 105E results in one sampling scheme for critical defectives, one sampling scheme for major defectives, and one sampling scheme for minor defectives. It is worthwhile to note that this structure is also utilized in MIL STD 105D[23]. Wadsworth[24] develops a method for establishing an AQL for each class of defects when implementing MIL STD 105D. His method considers the number of types of defects in each class of defects. Liebesman[12] develops a method for computing the overall product AQL from the AQL for each of the individual classes of defectives. His method, however, is based on several restrictive assumptions, most notably that all classes of defectives are subjected to the same sampling scheme.

The first reference to a multilevel discrete product quality are the inspection procedures established by the U.S. Army Service Forces[1] during World War II. As

Dodge[7] describes, four classes of defects are also defined in these procedures: critical defects, major defects, minor defects and incidental defects. However, as with MIL STD 105E, each defect class is treated separately with respect to inspection.

Not all references to multilevel discrete product quality measures are based on defects. Some references are in fact much more general. In their discussion of the multinomial distribution, Shapiro and Zahedi[19] indicate that a logical application for a multinomial trial is “an inspection situation where a product is classified into three or more categories such as conforming, marginal, and nonconforming.” Bray et al[2] identify an identical classification but they go on to suggest other similar classifications such as “premium grade, regular grade, unacceptable grade” or “first quality, seconds, rejectables.” In addition, they suggest two industrial application of such classifications: the amount of toxins in foods and the amount of active ingredient in drugs. Newcombe and Allen[16] and the International Commission on Microbiological Specifications for Foods[10] (ICMSF) also suggest the application of a 3-level product quality measure to the microorganism count in a food sample. Clements[6] suggests that the 3-level product quality measure is appropriate for all inspection situations involving product safety.

2.2 STATISTICAL QUALITY CONTROL TECHNIQUES FOR THREE-LEVEL PRODUCT QUALITY MEASURES

The only multilevel discrete product quality measure to be addressed in the literature is the 3-level product quality measure. The earliest reference in which such a measure is studied is Bray et al[2]. They define a simple 3-level acceptance sampling method. In particular, they consider the situation in which product quality is a monotonically decreasing function of the quality characteristic of interest. Items having a quality characteristic less than or equal to m are classified as good, items having a quality characteristic in the range $(m, M]$ are classified as marginal, and items having a quality characteristic greater than M are classified as bad. They assume that the lot size for the product is infinite and accordingly, they describe the occurrence of items in the three

classifications using the multinomial distribution. They define p_0 to be the probability that any given item in the lot is good, p_1 to be the probability that any given item in the lot is marginal, and p_2 to be the probability that any given item in the lot is bad. However, they do not define these probabilities in terms of the underlying distribution of the quality characteristic. By avoiding such a definition, the authors are able to ignore their previous assumption regarding the behavior of the quality characteristic. The 3-level acceptance sampling technique they propose operates as follows.

Step 1: Sample and inspect n items. Classify each item as good, marginal, or bad.

Step 2: Let d_1 denote the number of marginal or bad items in the sample, and let d_2 denote the number of bad items in the sample.

Step 3: If $d_1 \leq c_1$ and $d_2 \leq c_2$ accept the lot. Otherwise, reject the lot.

At this point, the authors define the probability of acceptance for a lot having proportions (p_0, p_1, p_2) , the O.C. function, as

$$P_a(p_0, p_1, p_2) = \sum_{i=0}^{c_1-j} \sum_{j=0}^{c_2} \frac{n!}{i! j! (n-i-j)!} p_0^{n-i-j} p_1^i p_2^j \quad (2.1).$$

Note that equation (2.1) is nothing more than a cumulative multinomial probability. In addition, note that the O.C. function can be plotted as a surface in terms of two of the three defining probabilities, since the third probability is implied by the other two. As the authors indicate, the most obvious choices for these two probabilities are p_1 and p_2 .

Sampling plans of this type are thus defined by specifying n , c_1 , and c_2 . However, the authors feel that the most useful subset of sampling plans are those having $c_2 = 0$. In fact, this is the only case which they consider in any further detail. They construct the O.C. function in this case and provide several tables of O.C. function values for various selections of n and c_1 . In addition, they propose a heuristic procedure for selecting these sampling plan parameters in order to meet some requirements placed on the O.C. function. These requirements are specified by indicating the preferred acceptance probabilities for an

AQL and/or an RQL both of which correspond to some specific set of (p_0, p_1, p_2) . This approach to 3-level acceptance sampling is unlike the approach developed in this research. It is worthwhile to note that this 3-level acceptance sampling strategy has received wide recognition in the food processing industry (see ICMSF[10]).

Shah and Phatak[18] make several significant contributions to this 3-level acceptance sampling technique. They define semi-curtailment as the operation of this method under the added condition that inspection for any lot ceases when either of the rejection conditions are met. Under semi-curtailment, a set of four mutually exclusive and exhaustive stopping conditions are enumerated. For this 3-level acceptance sampling method with semi-curtailment, the authors construct the ASN function and the maximum likelihood estimators for p_1 and p_2 . The authors then define full-curtailment as the operation of this technique under the added condition that inspection for any plan ceases when (1) either of the rejection conditions are met, or (2) it becomes impossible for either of the rejection conditions to be met, in other words, lot acceptance is implied. Under full-curtailment, six mutually exclusive and exhaustive stopping conditions are enumerated. For full-curtailment, the authors also construct the ASN function and the maximum likelihood estimators for p_1 and p_2 . They then construct a numerical example which indicates that the savings in inspection using semi-curtailment is significant but additional savings for full-curtailment are marginal. However, they only consider one set of (n, c_1, c_2) . Since the conditions of full-curtailment are numerous and not always obvious, the authors also propose a graphical method that consists of constructing two plots. Both plots are similar to the plots used in the attributes SPRT (see Wald[25]) and are updated as each item is inspected. The two plots incorporate the full-curtailment conditions and thus indicate when inspection should cease and the appropriate decision concerning lot disposition.

In four papers, Clements[3,4,5,6] addresses several issues involving the 3-level acceptance sampling technique. Clements[6] discusses the applicability of this 3-level acceptance sampling technique for inspection problems involving product safety. He

concludes that the cases in which $c_2 = 0$ are the only relevant cases for inspection involving product safety. He also provides contour plots of the O.C. function (plots of $P_a(p_0, p_1, p_2)$) as a function of p_1 for several different values of p_2) for one particular 3-level acceptance sampling plan.

In a second paper, Clements[5] provides another example of O.C. function contour plots (in this case, plots of p_1 as a function of p_2 for several different values of $P_a(p_0, p_1, p_2)$) for a 3-level acceptance sampling plan. He then assumes that an item's quality classification is determined by comparing its quality characteristic to single-sided specification limits. From this, he describes the O.C. function behavior assuming the quality characteristic is normally distributed with varying mean and constant variance. He also examines O.C. function behavior under extreme violations of normality. In addition, Clements provides a partial set of 3-level acceptance sampling plans having O.C. curves that closely match the O.C. curves of some acceptance sampling plans recommended by MIL STD 105D.

In a third paper, Clements[4] demonstrates the O.C. function behavior if the quality characteristic is normally distributed with constant mean and varying variance. This behavior is significantly different than the behavior observed under the assumption of varying mean and constant variance. He indicates that by describing the O.C. function behavior under first the assumption of varying mean and constant variance and second the assumption of constant mean and varying variance, a reference is provided for studying the behavior of rejected lots. He then continues his description of constructing 3-level acceptance sampling plans that have similar O.C. function behavior to MIL STD 105D sampling plans. The key contribution of this paper is a closed-form set of equations that provide n , c_1 , and c_2 that approximately meet desired O.C. function behavior. The desired O.C. function behavior is quantified by the specification of six quantities:

- (1) AQM, acceptable quality level for marginal items,
- (2) AQD, acceptable quality level for defective items,
- (3) RQM, rejectable quality level for marginal items,
- (4) RQD, rejectable quality level for defective items,

- (5) α , desired probability of lot rejection when $p_1 = \text{AQM}$ and $p_2 = \text{AQD}$, and
- (6) β , desired probability of lot acceptance when $p_1 = \text{RQM}$ and $p_2 = \text{RQD}$.

The expressions for n , c_1 , and c_2 are constructed in a similar fashion to the expressions provided by Stephens[21] for selecting the parameters of single sampling plans. In a fourth paper, Clements[3] summarizes the results of his first three papers and compares 3-level acceptance sampling plans to narrow-limit gauging plans.

Narrow-limit gauging is similar to using a multilevel discrete product quality measure. Narrow-limit gauging refers to artificially establishing multiple sets of specification limits on a measurable quality characteristic. These limits are then used to classify the quality of an item using discrete levels instead of precisely measuring the characteristic of the item. The reason for using narrow-limit gauging is that gauged inspection is often cheaper than precise measurement of the quality characteristic, and the loss of statistical precision is not as severe as with attributes inspection. However, there are two main reasons why narrow-limit gauging differs from using multilevel discrete product quality measures. First, the discrete levels utilized by a multilevel discrete product quality measure are assumed to be the best way of classifying the quality of an item (in other words, they are real not artificial). Second, narrow-limit gauging assumes that an item has a measurable quality characteristic. No such assumption is required when using multilevel discrete product quality measures. See Montgomery[14] for a further description of narrow-limit gauging plans.

Newcombe and Allen[16] define a variables acceptance sampling method which incorporates the 3-level (conforming, marginal, nonconforming) product quality measure under the assumptions that (1) the quality characteristic is normally distributed with mean μ and variance σ^2 , and (2) product quality is a decreasing function of this characteristic. Their method begins with the definition of two upper specification limits, U_1 and U_2 , on the quality characteristic. An item is marginal if its quality characteristic is between U_1 and U_2 , and nonconforming if it exceeds U_2 . Otherwise an item is conforming. Newcombe and Allen point out that specification of (U_1, U_2) and (p_1, p_2) is equivalent to

specifying a unique normal distribution (Newcombe and Allen define p_1 and p_2 in an equivalent fashion to Bray et al). Their method operates as follows.

Step 1: Sample and measure the quality characteristic of n items.

Step 2: Compute the sample mean \bar{X} and the sample standard deviation S .

Step 3: If $\bar{X} + k_1 S \leq U_1$ and $\bar{X} + k_2 S \leq U_2$ accept the lot. Otherwise, reject the lot.

The authors construct the O.C. function for this method. Although denoted as $P_d(p_1, p_2)$, the O.C. function is expressed as a function of U_1 , U_2 , μ , and σ^2 . In addition, the authors propose a method for selecting n , k_1 , and k_2 based on desired O.C. function behavior. They also construct the maximum likelihood estimators for p_1 and p_2 .

As the authors indicate, the problem that they address could be approached using the 3-level acceptance sampling methods of Bray et al. They then demonstrate through the use of an example that their method, as compared to the method developed by Bray et al, yields a reduction in the required sample size as well as improved O.C. function behavior at points other than the ones used for parameter selection. However, these results cannot be obtained without assuming that the sample observations are IID normal random variables.

The 3-level product quality measure has received little attention in the SPC literature. Marcucci[13] considers a SPC situation in which items are classified as either minor nonconforming, major nonconforming, or conforming. He defines two general approaches for monitoring a process which produces such items. The first approach is a graphical application of statistical hypothesis testing. If the in-control proportions of each classification are known beforehand, he suggests taking a sample of n items from the process, classifying each item in the sample, and then computing the Pearson goodness-of-fit test statistic. The goodness-of-fit test statistic is based on the null hypothesis that the observed sample data is drawn from a multinomial population having the in-control proportions. The test statistic is then plotted on a control chart, with the upper control

limit (there is no lower control limit) corresponding to the appropriate critical value from the chi-square distribution. If the in-control proportions are unknown beforehand, he recommends collecting data during a known in-control period and comparing the sample data to this in-control data using a standard test for homogeneity. Again the test statistic is plotted on a control chart, and the control limit corresponds to the appropriate critical value from the chi-square distribution. Marcucci's second approach is designed for the situation in which it is of interest only to detect increases in the proportions of either minor or major nonconforming items. In this approach, he defines two individual control charts, one for minor nonconforming items and one for major nonconforming items. The use of each chart consists of comparing the number of minor (or major) nonconforming items in the sample to some upper control limit. He also proposes a heuristic procedure for selecting the upper control limits for the two charts. He does mention combining the two charts into a single control chart using a diagram similar to that defined by Snee[20] for displaying trinomial confidence intervals. He briefly discusses the run length properties of his proposed methods. However, he does not provide numerical examples. Marcucci's approaches to 3-level SPC are quite different from the approach taken in this research.

CHAPTER 3: METHODOLOGY

The objectives of this research are (1) to develop and evaluate several multilevel acceptance sampling techniques (realizations of these techniques are referred to as multilevel acceptance sampling plans), and (2) to develop and evaluate several multilevel SPC techniques (realizations of these techniques are referred to as multilevel control charts). The basic steps in the execution of the research have been described previously (see Section 1.3). A more complete description of the specific activities involved with each of those steps is provided here.

3.1 CHARACTERIZATION OF MULTILEVEL DISCRETE PRODUCT QUALITY MEASURES

Multilevel discrete product quality measures are applicable to those situations in which a product's quality can effectively be described by classifying a single characteristic of the product using three or more discrete levels. Specifically, a j -level product quality measure is applicable when the product's quality is described by classifying one of its characteristics using j levels. Let X denote the j -level product quality measure for a certain product. For a unit of product, $X = x_k$ if the unit of product is classified into level k , $k = 1, 2, \dots, j$. For example, suppose an item is classified as either conforming, marginal, or nonconforming. One possible 3-level product quality measure for this situation is

$$X = \begin{cases} 0 & \text{if the item is conforming} \\ 0.5 & \text{if the item is marginal} \\ 1 & \text{if the item is nonconforming} \end{cases} \quad (3.1).$$

Suppose some other item is classified using four levels: perfect, slightly flawed, severely flawed, critically flawed. For this case, a possible 4-level product quality measure is

$$X = \begin{cases} 1 & \text{if the item is perfect} \\ 2 & \text{if the item is slightly flawed} \\ 3 & \text{if the item is severely flawed} \\ 4 & \text{if the item is critically flawed} \end{cases} \quad (3.2).$$

It is true that any number of product quality measures can be defined for both of these examples. However, the primary function of a multilevel product quality measure is to provide an ordinal quality scale for items in a sample. In other words, let X_1 denote the value assigned by the multilevel product quality measure to some item, say item 1, and let X_2 denote the value assigned by the measure to some other item, say item 2 (both items are the same type of product). In this research, the following convention is followed. If $X_1 < X_2$, then item 1 has superior quality to item 2 (item 2 has inferior quality to item 1). If $X_1 = X_2$, then the two items have equivalent quality. Note however that the magnitude of the difference between X_1 and X_2 does not necessarily correspond to the magnitude of the difference in the two items' quality. For example, if $X_2 = bX_1$ ($b > 1$), it is not necessarily true that the quality of item 2 is b times worse than the quality of item 1. In fact, the difference in quality may be infinitesimal.

Having demonstrated simple examples of multilevel discrete product quality measures, the next step is to develop a method of describing the quality of a received lot (for the case of multilevel acceptance sampling) and the state of a process (for the case of multilevel SPC) in terms of the discrete quality levels. For multilevel acceptance sampling, the quality of a received lot can be described by the vector $\mathbf{p} = [p_1 p_2 \cdots p_j]$ where p_k denotes the proportion of items in the lot having classification k , $k = 1, 2, \dots, j$. Under such a description, the data obtained when sampling from the lot, the numbers of inspected items in each classification, is appropriately modeled using the multivariate hypergeometric distribution. If the number of items in the lot is assumed to be infinite (or at least very large with respect to the sample size), then the multinomial distribution is an appropriate model for sample data (see Bray et al[2] for a similar argument). For multilevel SPC, the state of the process can also be described by \mathbf{p} , however in this case p_k

denotes the probability that a produced item has classification k , $k = 1, 2, \dots, j$. As a result, the data obtained when sampling from the process is appropriately modeled using the multinomial distribution (this assumes that \mathbf{p} does not change while items are being sampled).

For both multilevel acceptance sampling and SPC, it may be the case that the quality characteristic of interest can be measured on a continuous scale and the classification of a unit of product is accomplished by comparing this measurement to some set of specifications. For example, the quality of a ball bearing may be determined by measuring its diameter, d . The ball bearing may then be classified as conforming, marginal, or nonconforming according to the following rule.

The ball bearing is conforming if $23 \text{ mm} \leq d \leq 27 \text{ mm}$.

The ball bearing is marginal if $21 \text{ mm} \leq d < 23 \text{ mm}$ or $27 \text{ mm} < d \leq 29 \text{ mm}$.

Otherwise, the ball bearing is nonconforming.

In such cases, it is sometimes possible to determine the probability distribution of the quality characteristic. Depending on the distribution of the quality characteristic, it may be possible to compute \mathbf{p} using the classification rule and the quality characteristic's distribution. If that is the case, the distribution of the quality characteristic is an equivalent description of the lot quality or the state of the process. Note, however, that this special case is not considered in this research.

3.2 QUALITY VALUE FUNCTIONS

Although a multilevel discrete product quality measure quantifies the quality of a unit of product, it does not necessarily do so in a manner that facilitates decision making regarding either lot disposition or process control signaling. Such quantification is accomplished through the use of quality value functions.

Definition 3.1: A quality value function, $V(X)$, is a function which assigns a numerical quality value to an item based on the numerical classification assigned to the item using a multilevel discrete product quality measure X .

Consider the 3-level product quality measure given in equation (3.1). Any number of quality value functions can be defined for this example. For instance, $V(X)$ could be defined in either of the following ways:

$$V(X) = X \tag{3.3},$$

or

$$V(X) = \begin{cases} 1 & \text{if } X = 0 \\ 10 & \text{if } X = 0.5 \\ 100 & \text{if } X = 1 \end{cases} \tag{3.4}.$$

Consider the 4-level product quality measure given in equation (3.2). For this example, two possibilities for $V(X)$ are given below:

$$V(X) = 2^X \tag{3.5},$$

or

$$V(X) = bX \text{ where } b \text{ is some constant} \tag{3.6}.$$

These four examples are only a small demonstration of the possibilities for quality value functions. As described in the following sections, the specific quality value function used depends on the problem of interest. More specifically, it depends on the emphasis that should be placed on items that fall into each of the quality classifications.

3.3 MULTILEVEL ACCEPTANCE SAMPLING

The development of multilevel SQC techniques begins with the development and evaluation of multilevel acceptance sampling techniques. Each technique is described in the general j -level case and then evaluated in the 3-level case and the 4-level case.

3.3.1 SAMPLING PLAN OPERATION AND LOT DISPOSITION

The development of a multilevel acceptance sampling technique begins with the selection of a multilevel discrete product quality measure and a quality value function.

These selections are based primarily on two factors: (1) how many levels are necessary to adequately describe the quality of an item, and (2) how significant are the differences between the quality levels. Suppose the quality of an item is adequately described in a fashion similar to the description implied in MIL STD 105E[22]: conforming, minor nonconforming, major nonconforming, and critical nonconforming. According to the MIL STD 105E definitions, a minor nonconformity does not result in a significant degradation of the product. A major nonconformity does significantly degrade the performance of the product. More importantly, a critical nonconformity results in a catastrophic failure of the product. The multilevel discrete product quality measure and the quality value function should portray these differences in the quality levels. For example, one combination of multilevel discrete product quality measure and quality value function might result in the following assignments:

$$V(x_1) = 1 \quad V(x_2) = 2 \quad V(x_3) = 5 \quad V(x_4) = 10 \quad (3.7).$$

Although this assignment reflects the general differences in quality levels, it does not emphasize the dramatic differences resulting from the use of items having the different classifications. A more appropriate combination of multilevel discrete product quality measure and quality value function might result in the following assignments:

$$V(x_1) = 1 \quad V(x_2) = 10 \quad V(x_3) = 100 \quad V(x_4) = 1000 \quad (3.8).$$

One key point to recognize is that a single multilevel product quality measure could have been used in both (3.7) and (3.8). In such a case, the differences in the quality value assignments results from differences in the quality value functions used. Thus a single multilevel discrete product quality measure is considered in this research. However, the sampling plan behavior resulting from the use of several quality value functions is investigated.

The next step in the development of a multilevel acceptance sampling technique is the selection of a sampling strategy. For a given combination of multilevel product quality measure and quality value function, a variety of sampling strategies can be implemented. However, the only sampling strategy considered in this research is single sampling. Single

sampling refers to taking a random sample of n items from the received lot and then inspecting and classifying each item in the sample. Single sampling with curtailment is not considered.

The final step in the development of a multilevel acceptance sampling technique is the selection of a decision rule. The decision rule indicates the sample statistic and criterion to be used for determining lot disposition. Some example decision rules are provided below. Let V_i denote the value assigned by the quality value function to item i in the sample, $i = 1, 2, \dots, n$. For multilevel acceptance sampling techniques utilizing single sampling, some possible decision rules are as follows.

$$\begin{aligned} &\text{If } \sum_{k=1}^n V_i \leq T \text{ accept the lot (} T \text{ is a sampling plan parameter).} \\ &\text{Otherwise reject the lot.} \end{aligned} \quad (3.9)$$

$$\text{If } \prod_{i=1}^n V_i \leq T \text{ accept the lot. Otherwise reject the lot.} \quad (3.10)$$

$$\text{If } \max_{1 \leq i \leq n} \{V_i\} < V(x_j) \text{ accept the lot. Otherwise reject the lot.} \quad (3.11)$$

The decision rule described in (3.9) is referred to as the summation rule. The summation rule is the only decision rule considered in this research.

Although a single multilevel discrete product quality measure and decision rule are considered, the numerous possibilities for the selection of a quality value function results in a number of possible multilevel acceptance sampling plans. Thus each choice for the quality value function represents an alternative multilevel acceptance sampling technique.

3.3.2 SAMPLING PLAN BEHAVIOR

The next stage of the multilevel acceptance sampling research effort is the evaluation of each of the proposed multilevel acceptance sampling techniques. The proposed techniques are evaluated on the basis of O.C. function behavior. For any multilevel acceptance sampling plan, the O.C. function, $P_a(\mathbf{p}) = P_a(p_1, p_2, \dots, p_j)$, is defined as the probability of lot acceptance given that the lot quality is \mathbf{p} . For each of the proposed multilevel acceptance sampling techniques, general expressions are constructed for the O.C. function using the multinomial model of sample data. Presentation of O.C. function behavior is also addressed.

3.3.3 SAMPLING PLAN PARAMETER SELECTION

Once a quality value function has been selected, specification of a multilevel acceptance sampling plan requires specifying a sample size n and a critical value T . A method is presented for selecting the parameters of a multilevel acceptance sampling plan by specifying desired O.C. function behavior. Since a multilevel acceptance sampling plan has two parameters, two points on the O.C. curve are specified. The sampling plan parameters are then selected such that the resulting O.C. curve passes through (or at least very close to) those two points. The sampling plan parameter selection method is then demonstrated and evaluated for each proposed technique through the use of numerical examples.

3.3.4 QUALITY VALUE FUNCTION SPECIFICATION

Suppose that in developing a multilevel acceptance sampling technique, the quality value function is not selected. All quality value functions can be described by j assignments: $V(x_k) = v_k, k = 1, 2, \dots, j$. The v_k can thus be thought of as j additional sampling plan parameters. In this stage of the research, the method for selecting sampling

plan parameters presented in the previous section is modified so that not only the original sampling plan parameters are selected but also the j assignments that determine the quality value function. This modified method requires the specification of as many as j additional points on the O.C. curve. Implementation of this revised method results in multilevel acceptance sampling plans that provide desired O.C. function behavior over a wider range of p

3.4 MULTILEVEL STATISTICAL PROCESS CONTROL

The next portion of the analysis of multilevel SQC techniques consists of the development and evaluation of multilevel SPC techniques. Each technique is described in the general j -level case and then evaluated in the 3-level case and the 4-level case.

3.4.1 CONTROL CHART OPERATION

The development of a multilevel SPC technique also begins with the selection of a multilevel discrete product quality measure and a quality value function. The considerations in selecting the combination of a multilevel discrete product quality measure and a quality value function for use in a multilevel acceptance sampling plan have been discussed previously. Those same considerations must be made in selecting a combination for use in a multilevel control chart. A single multilevel discrete product quality measure is considered in this research. However, the control chart behavior resulting from the use of several quality value functions is investigated.

The second step in the development of a multilevel SPC technique is the selection of a sampling strategy. The only sampling strategy considered is fixed sampling interval single sampling. At equal intervals in time, a sample of n finished items is taken from the process and each item in the sample is inspected and classified.

The third step in the basic development of a multilevel SPC technique is the selection of a sample statistic V^* , the value to be plotted on the control chart. Some example sample statistics are provided below. Let V_i denote the value assigned by the quality value function to item i in the sample, $i = 1, 2, \dots, n$. For multilevel control charts utilizing fixed sampling interval single sampling, some possible sample statistics are given below.

$$V^* = \sum_{i=1}^n V_i \quad (3.12)$$

$$V^* = \frac{1}{n} \sum_{i=1}^n V_i \quad (3.13)$$

$$V^* = \prod_{i=1}^n V_i . \quad (3.14)$$

The sample statistic described in equation (3.13) is referred to as the average quality value. The average quality value is the only sample statistic considered in this research.

The final step in the development of a multilevel SPC technique is the selection of control limits for the control chart. For each of the proposed multilevel SPC techniques, Shewhart control limits are constructed.

Although a single multilevel discrete product quality measure and sample statistic are considered, the numerous possibilities for the selection of a quality value function results in a number of possible multilevel control charts. Thus each choice for the quality value function represents an alternative multilevel control chart.

3.4.2 CONTROL CHART BEHAVIOR

The next stage of the multilevel SPC research effort is the evaluation of each of the proposed multilevel SPC techniques. The proposed control charts are evaluated based on ARL function behavior. For any multilevel control chart, the O.C. function, $P_a(\mathbf{p}) = P_a(p_1, p_2, \dots, p_j)$, is defined as the probability that the sample statistic falls within the control

limits given that the state of the process is \mathbf{p} . Since the average quality value only utilizes the information from the current sample, the run length of all the proposed multilevel control charts is geometrically distributed with probability $1 - P_a(\mathbf{p})$. Thus the ARL function for the proposed multilevel control charts is given by

$$ARL(\mathbf{p}) = ARL(p_1, p_2, \dots, p_j) = \frac{1}{1 - P_a(p_1, p_2, \dots, p_j)} \quad (3.15).$$

For each of the proposed multilevel SPC techniques, general expressions are constructed for the O.C. function which imply the corresponding expressions for the ARL function. Presentation of O.C. function and ARL function behavior is also addressed.

3.4.3 CONTROL CHART PARAMETER SELECTION

Specification of a multilevel control chart requires specifying a sample size n and a parameter for computing the control limits. For each proposed multilevel SPC technique, a method is presented for selecting the parameters of the control chart by specifying desired ARL function behavior. Since the proposed multilevel control charts have two parameters, two points on the ARL curve are specified. A method is presented for selecting control chart parameters such that the resulting ARL curve passes through (or at least very close to) those two points. The control chart parameter selection method for each technique is then demonstrated and evaluated through the use of numerical examples.

3.4.4 QUALITY VALUE FUNCTION SPECIFICATION

Suppose that in developing a multilevel SPC technique, the quality value function is not selected. All quality value functions can be described by j assignments: $V(x_k) = v_k$, $k = 1, 2, \dots, j$. The v_k can thus be thought of as j additional control chart parameters. In this stage of the research, the method for selecting control chart parameters presented in the previous section is modified so that not only the original control chart parameters are selected but also the j assignments that determine the quality value function. This

modified method requires the specification of as many as j additional points on the ARL curve. Implementation of this revised method results in multilevel control charts that provide desired ARL function behavior over a wider range of p .

CHAPTER 4: MULTILEVEL ACCEPTANCE SAMPLING

The first area of multilevel statistical quality control addressed in this research is multilevel acceptance sampling. The chapter begins with the definition of the multilevel acceptance sampling plans that are considered. The second section contains the development of the O.C. function for each of the defined multilevel acceptance sampling plans. In the third section, a method for selecting the parameters of a multilevel acceptance sampling plan is defined, demonstrated, and evaluated. The problem of quality value function specification is addressed in the fourth section. In each section, the topic of interest is motivated using the general j -level case followed by detailed discussions using the 3-level and 4-level cases.

4.1 MULTILEVEL ACCEPTANCE SAMPLING PLAN OPERATION AND LOT DISPOSITION

In the preceding chapters, the scenario for which multilevel acceptance sampling is most appropriate is defined. Suppose a manufacturer of some product receives a lot of N items, used as input to the production process, from some supplier. Suppose in addition, that the quality of these items is best described using j discrete levels where $j > 2$. In such a situation, the supplier may wish to utilize a multilevel acceptance sampling plan to evaluate the quality of the lot rather than automatically accepting the lot for use in the production process or performing 100% inspection.

The initial decision required for the implementation of a multilevel acceptance sampling plan is the selection of a multilevel discrete product quality measure X . In Chapter 3, several example multilevel discrete product quality measures are presented. However since the multilevel discrete product quality measure is used in conjunction with a quality value function, the choice of X is merely one of personal preference. In fact, there must be a 1 to 1 and onto mapping between any two multilevel discrete product quality measures (so long as the number of levels is finite and the numerical values

assigned by the measures are discrete and ordered). Thus in this research, a single multilevel discrete product quality measure is used. The multilevel discrete product quality measure used is $x_k = k$ for $k = 1, 2, \dots, j$ where x_k is the quality measure assigned to an item falling into quality classification k . For example, if a 3-level product quality measure is most appropriate (say items are either good, marginal, or bad), then $X = 1$ if an item falls into quality classification 1 (the item is good), $X = 2$ if an item falls into quality classification 2 (the item is marginal), and $X = 3$ if an item falls into quality classification 3 (the item is bad).

The next step in the implementation of a multilevel acceptance sampling plan is the selection of a quality value function. The manufacturer must specify v_k , $k = 1, 2, \dots, j$ where v_k is the quality value assigned to an item having a quality measure of x_k . Several quality value functions are considered in this research. Once a quality value function is specified, the resulting multilevel acceptance sampling plan can be implemented.

As mentioned in Chapter 3, all multilevel acceptance sampling plans considered in this research utilize single sampling. In this research, the summation rule is used to determine lot disposition. Thus a multilevel acceptance sampling plan operates as follows.

Step 1: Inspect a random sample of n items from the lot. Let $X_i =$ the quality measure of item i , and let $V_i =$ the quality value of item i .

Step 2: If $\sum_{i=1}^n V_i \leq T$, the lot is accepted. Otherwise, the lot is rejected.

The specific multilevel acceptance sampling plans considered in this research are defined in the two following sections.

4.1.1 3-level Acceptance Sampling Plans

In the preceding section, the general framework for multilevel acceptance sampling is established. This framework is applicable to the 3-level case. In this section, the specific quality value functions considered in the 3-level acceptance sampling portion of the research are defined. As mentioned earlier, the choice of the quality value function

3-AS-S-2(n, T, v)

Quality Value Function Conditions: $v > 1$

Step 1: Inspect a random sample of n items.

$$\text{Let } X_i = \begin{cases} 1 & \text{if item } i \text{ is conforming} \\ 2 & \text{if item } i \text{ is marginal} \\ 3 & \text{if item } i \text{ is nonconforming} \end{cases} \quad i = 1, 2, \dots, n.$$

$$\text{Let } V_i = \begin{cases} 1 & \text{if } X_i = 1 \\ v & \text{if } X_i = 2 \\ v^2 & \text{if } X_i = 3 \end{cases} \quad i = 1, 2, \dots, n.$$

Step 2: If $\sum_{i=1}^n V_i \leq T$, accept the lot.

Otherwise, reject the lot.

3-AS-S-3(n, T, v)

Quality Value Function Conditions: $v > 1$

Step 1: Inspect a random sample of n items.

$$\text{Let } X_i = \begin{cases} 1 & \text{if item } i \text{ is conforming} \\ 2 & \text{if item } i \text{ is marginal} \\ 3 & \text{if item } i \text{ is nonconforming} \end{cases} \quad i = 1, 2, \dots, n.$$

$$\text{Let } V_i = \begin{cases} v & \text{if } X_i = 1 \\ v^2 & \text{if } X_i = 2 \\ v^3 & \text{if } X_i = 3 \end{cases} \quad i = 1, 2, \dots, n.$$

Step 2: If $\sum_{i=1}^n V_i \leq T$, accept the lot.

Otherwise, reject the lot.

3-AS-S-G(n, T, v_1, v_2, v_3)

Note that this is the general 3-level case.

Quality Value Function Conditions: $0 \leq v_1 < v_2 < v_3$

Step 1: Inspect a random sample of n items.

$$\text{Let } X_i = \begin{cases} 1 & \text{if item } i \text{ is conforming} \\ 2 & \text{if item } i \text{ is marginal} \\ 3 & \text{if item } i \text{ is nonconforming} \end{cases} \quad i = 1, 2, \dots, n.$$

$$\text{Let } V_i = \begin{cases} v_1 & \text{if } X_i = 1 \\ v_2 & \text{if } X_i = 2 \\ v_3 & \text{if } X_i = 3 \end{cases} \quad i = 1, 2, \dots, n.$$

Step 2: If $\sum_{i=1}^n V_i \leq T$, accept the lot.

Otherwise, reject the lot.

4.1.2 4-level Acceptance Sampling Plans

In this section, the specific quality value functions considered in the 4-level acceptance sampling portion of the research are defined. As with the 3-level case, specification of the quality value function completes the definition of the acceptance sampling plan. Each 4-level acceptance sampling plan is given a reference code to be used in the remainder of this research. The form of the reference code is identical to the form used for 3-level acceptance sampling plans. To facilitate the discussions of these sampling plans, the following convention is established. When referring to 4-level acceptance sampling or 4-level SPC, the following descriptions are given to the four quality classifications: conforming, minor nonconforming, major nonconforming, critical nonconforming. The 4-level acceptance sampling plans considered are presented below.

4-AS-S-1(n, T, v_1, v_2)

Quality Value Function Conditions: $0 < v_1 < v_2 < 1$

Step 1: Inspect a random sample of n items.

$$\text{Let } X_i = \begin{cases} 1 & \text{if item } i \text{ is conforming} \\ 2 & \text{if item } i \text{ is minor nonconforming} \\ 3 & \text{if item } i \text{ is major nonconforming} \\ 4 & \text{if item } i \text{ is critical nonconforming} \end{cases} \quad i = 1, 2, \dots, n.$$

$$\text{Let } V_i = \begin{cases} 0 & \text{if } X_i = 1 \\ v_1 & \text{if } X_i = 2 \\ v_2 & \text{if } X_i = 3 \\ 1 & \text{if } X_i = 4 \end{cases} \quad i = 1, 2, \dots, n.$$

Step 2: If $\sum_{i=1}^n V_i \leq T$, accept the lot.

Otherwise, reject the lot.

4-AS-S-2(n, T, v)

Quality Value Function Conditions: $v > 1$

Step 1: Inspect a random sample of n items.

$$\text{Let } X_i = \begin{cases} 1 & \text{if item } i \text{ is conforming} \\ 2 & \text{if item } i \text{ is minor nonconforming} \\ 3 & \text{if item } i \text{ is major nonconforming} \\ 4 & \text{if item } i \text{ is critical nonconforming} \end{cases} \quad i = 1, 2, \dots, n.$$

$$\text{Let } V_i = \begin{cases} 1 & \text{if } X_i = 1 \\ v & \text{if } X_i = 2 \\ v^2 & \text{if } X_i = 3 \\ v^3 & \text{if } X_i = 4 \end{cases} \quad i = 1, 2, \dots, n.$$

Step 2: If $\sum_{i=1}^n V_i \leq T$, accept the lot.

Otherwise, reject the lot.

4-AS-S-3(n, T, v)

Quality Value Function Conditions: $v > 1$

Step 1: Inspect a random sample of n items.

$$\text{Let } X_i = \begin{cases} 1 & \text{if item } i \text{ is conforming} \\ 2 & \text{if item } i \text{ is minor nonconforming} \\ 3 & \text{if item } i \text{ is major nonconforming} \\ 4 & \text{if item } i \text{ is critical nonconforming} \end{cases} \quad i = 1, 2, \dots, n.$$

$$\text{Let } V_i = \begin{cases} v & \text{if } X_i = 1 \\ v^2 & \text{if } X_i = 2 \\ v^3 & \text{if } X_i = 3 \\ v^4 & \text{if } X_i = 4 \end{cases} \quad i = 1, 2, \dots, n.$$

Step 2: If $\sum_{i=1}^n V_i \leq T$, accept the lot.

Otherwise, reject the lot.

4-AS-S-G(n, T, v_1, v_2, v_3, v_4)

Note that this is the general 4-level case.

Quality Value Function Conditions: $0 \leq v_1 < v_2 < v_3 < v_4$

Step 1: Inspect a random sample of n items.

$$\text{Let } X_i = \begin{cases} 1 & \text{if item } i \text{ is conforming} \\ 2 & \text{if item } i \text{ is minor nonconforming} \\ 3 & \text{if item } i \text{ is major nonconforming} \\ 4 & \text{if item } i \text{ is critical nonconforming} \end{cases} \quad i = 1, 2, \dots, n.$$

$$\text{Let } V_i = \begin{cases} v_1 & \text{if } X_i = 1 \\ v_2 & \text{if } X_i = 2 \\ v_3 & \text{if } X_i = 3 \\ v_4 & \text{if } X_i = 4 \end{cases} \quad i = 1, 2, \dots, n.$$

Step 2: If $\sum_{i=1}^n V_i \leq T$, accept the lot.

Otherwise, reject the lot.

4.2 MULTILEVEL ACCEPTANCE SAMPLING PLAN O.C. FUNCTION BEHAVIOR

The most common measure used for evaluating the behavior of an acceptance sampling plan is the O.C. function. In this section, the O.C. functions for the multilevel acceptance sampling plans defined in section 4.1 are constructed. The O.C. function for the general j -level case is considered first. Then the O.C. function for each of the defined 3-level and 4-level acceptance sampling plans is constructed.

Before the construction of the O.C. function for the general j -level case, it is worthwhile to recall two previously mentioned considerations. Recall first that the quality of a received lot is described by $\mathbf{p} = [p_1 \ p_2 \ \dots \ p_j]$ where p_k represents the proportion of items in the lot that fall into quality classification k , $k = 1, 2, \dots, j$. Second, recall that the multinomial model of sample data is assumed. Thus for each item in the lot, $p_k = P(\text{the item falls into quality classification } k)$, $k = 1, 2, \dots, j$.

Let n denote the sample size for the multilevel acceptance sampling plan, and let N_k denote the number of items in the sample that fall into quality classification k , $k = 1, 2, \dots, j$. Since the multinomial model of sample data is assumed

$$P(N_1 = n_1, N_2 = n_2, \dots, N_j = n_j) = \begin{cases} \frac{n!}{n_1! n_2! \dots n_j!} p_1^{n_1} p_2^{n_2} \dots p_j^{n_j} & \text{if } \sum_{k=1}^j n_k = n \\ 0 & \text{otherwise} \end{cases} \quad (4.1)$$

(see Walpole and Myers[26]). Let $m(n_1, n_2, \dots, n_j, p_1, p_2, \dots, p_j) = P(N_1 = n_1, N_2 = n_2, \dots, N_j = n_j)$. Recall that the O.C. function is given by $P_a(\mathbf{p})$, where $P_a(\mathbf{p})$ denotes the probability that a lot is accepted given that the quality of the lot is \mathbf{p} . The acceptance criterion can be restated in terms of N_1, N_2, \dots, N_j and the assignments made by the quality value function v_1, v_2, \dots, v_j . In particular, a lot is accepted if

$$\sum_{k=1}^j N_k v_k \leq T \quad (4.2).$$

Equation (4.2) can be used to complete the construction of the O.C. function. For specified values of n and T , let

$$m_a(n_1, n_2, \dots, n_j, p_1, p_2, \dots, p_j) = \begin{cases} m(n_1, n_2, \dots, n_j, p_1, p_2, \dots, p_j) & \text{if } \sum_{k=1}^j n_k v_k \leq T \\ 0 & \text{otherwise} \end{cases} \quad (4.3).$$

This leads to the result that

$$P_a(p_1, p_2, \dots, p_j) = \sum_{n_1=0}^n \sum_{n_2=0}^{n-n_1} \dots \sum_{n_j=0}^{n-n_1-n_2-\dots-n_{j-1}} m_a(n_1, n_2, \dots, n_j, p_1, p_2, \dots, p_j) \quad (4.4).$$

However, this expression can be simplified by recognizing that $N_1 + N_2 + \dots + N_j = n$. Accounting for this fact leads to the following expression for the O.C. function for multilevel acceptance sampling plans.

$$P_a(p_1, p_2, \dots, p_j) = \sum_{n_1=0}^n \sum_{n_2=0}^{n-n_1} \dots \sum_{n_{j-1}=0}^{n-n_1-n_2-\dots-n_{j-2}} m_a(n_1, n_2, \dots, n_{j-1}, n-n_1-n_2-\dots-n_{j-1}, p_1, p_2, \dots, p_j) \quad (4.5).$$

Unfortunately, equation (4.5) is the simplest representation of the O.C. function. The random variable

$$\sum_{k=1}^j N_k v_k$$

is what drives the O.C. function. If $v_k = k$, for $k = 1, 2, \dots, j$, then Jacobson and Morrice[11] have developed a recursive approach to computing the probability mass function of this random variable. However if some other quality value function is used, the conditional multinomial expression above must be used. Even in the special case considered by Jacobson and Morrice, the conditional multinomial expression is easier to use.

4.2.1 O.C. Function Behavior of 3-level Acceptance Sampling Plans

If the development used in the preceding section is followed, the O.C. function for each of the 3-level acceptance sampling plans defined in section 4.1.1 can be constructed. In this case, the quality of a lot is described by $\mathbf{p} = [p_1 \ p_2 \ p_3]$ where p_1 denotes the proportion of items in the lot that are conforming, p_2 denotes the proportion of items in the lot that are marginal, and p_3 denotes the proportion of items in the lot that are nonconforming. For a specified sample size n , N_1 denotes the number of sample items that are conforming, N_2 denotes the number of sample items that are marginal, and N_3 denotes the number of sample items that are nonconforming. Using the assumed multinomial model,

$$m(n_1, n_2, n_3, p_1, p_2, p_3) = \begin{cases} \frac{n!}{n_1! n_2! n_3!} p_1^{n_1} p_2^{n_2} p_3^{n_3} & \text{if } n_1 + n_2 + n_3 = n \\ 0 & \text{otherwise} \end{cases} \quad (4.6).$$

Thus given values for n and T , each O.C. function can be constructed as follows.

3-AS-S-1(n, T, v)

Acceptance Criterion: $N_2v + N_3 \leq T$

$$m_a(n_1, n_2, n_3, p_1, p_2, p_3) = \begin{cases} m(n_1, n_2, n_3, p_1, p_2, p_3) & \text{if } n_2v + n_3 \leq T \\ 0 & \text{otherwise} \end{cases}$$

3-AS-S-2(n, T, v)

Acceptance Criterion: $N_1 + N_2v + N_3v^2 \leq T$

$$m_a(n_1, n_2, n_3, p_1, p_2, p_3) = \begin{cases} m(n_1, n_2, n_3, p_1, p_2, p_3) & \text{if } n_1 + n_2v + n_3v^2 \leq T \\ 0 & \text{otherwise} \end{cases}$$

3-AS-S-3(n, T, v)

Acceptance Criterion: $N_1v + N_2v^2 + N_3v^3 \leq T$

$$m_a(n_1, n_2, n_3, p_1, p_2, p_3) = \begin{cases} m(n_1, n_2, n_3, p_1, p_2, p_3) & \text{if } n_1v + n_2v^2 + n_3v^3 \leq T \\ 0 & \text{otherwise} \end{cases}$$

3-AS-S-G($n, T, \nu_1, \nu_2, \nu_3$)

Acceptance Criterion: $N_1\nu_1 + N_2\nu_2 + N_3\nu_3 \leq T$

$$m_a(n_1, n_2, n_3, p_1, p_2, p_3) = \begin{cases} m(n_1, n_2, n_3, p_1, p_2, p_3) & \text{if } n_1\nu_1 + n_2\nu_2 + n_3\nu_3 \leq T \\ 0 & \text{otherwise} \end{cases}$$

Having constructed the conditional multinomial expressions, the O.C. function for all the 3-level acceptance sampling plans can be written as

$$P_a(p_1, p_2, p_3) = \sum_{n_1=0}^n \sum_{n_2=0}^{n-n_1} m_a(n_1, n_2, n-n_1-n_2, p_1, p_2, p_3) \quad (4.7).$$

Computation of O.C. function values for any of the 3-level acceptance sampling plans is quite straightforward. However, presentation of the O.C. function behavior in a meaningful way is not so straightforward. There are three useful methods of presenting O.C. function behavior of 3-level acceptance sampling plans: (1) tables, (2) 3-dimensional O.C. curves, and (3) contour plots of O.C. function values. Examples of each of these methods are provided for the 3-AS-S-1(30,1.5,0.5) acceptance sampling plan. In Table 4.1, a table of O.C. function values for this plan is given. In Figure 4.1, the 3-dimensional O.C. curve for the sampling plan is given. Note that the values on the x -axis correspond to values of p_2 and the values on the y -axis correspond to the values of p_3 . Since p_1 is implied by the specification of p_2 and p_3 ($p_1 = 1 - p_2 - p_3$), this is sufficient for describing the complete O.C. function. In Figure 4.2, a contour plot of O.C. function values for the sampling plan is given. As with Figure 4.1, the values on the x -axis correspond to values of p_2 and the values on the y -axis correspond to the values of p_3 . Each of these methods has its own advantages and disadvantages. The tables provide specific O.C. function values, but they provide only a limited description of overall O.C. function behavior. The 3-dimensional O.C. curve does not provide specific O.C. function values, but it does provide a basic description of overall O.C. function behavior. The contour plot can be used to identify some specific O.C. function values (however, not as easily as the tables), and to provide a basic description of overall O.C. function behavior (but perhaps not as clearly as the 3-dimensional O.C. curves).

Table 4.1
O.C. Function Values for the 3-AS-S-1(30,1.5,0.5) Acceptance Sampling Plan

p_1	p_2	p_3	$P_a(p_1, p_2, p_3)$
.995	.0035	.0015	.9988
.995	.0025	.0025	.9972
.99	.0075	.0025	.9959
.99	.005	.005	.9889
.975	.02	.005	.9727
.975	.015	.01	.9471
.975	.0125	.0125	.9322
.96	.035	.005	.9375
.96	.025	.015	.8730
.96	.02	.02	.8384
.95	.04	.01	.8669
.95	.03	.02	.7982
.95	.025	.025	.7634
.925	.065	.01	.7378
.925	.05	.025	.6380
.925	.0375	.0375	.5640
.9	.09	.01	.5812
.9	.075	.025	.4967
.9	.05	.05	.3839
.85	.125	.025	.2471
.85	.075	.075	.1468
.8	.15	.05	.0752
.8	.1	.1	.0456
.75	.175	.075	.0190
.75	.125	.125	.0118

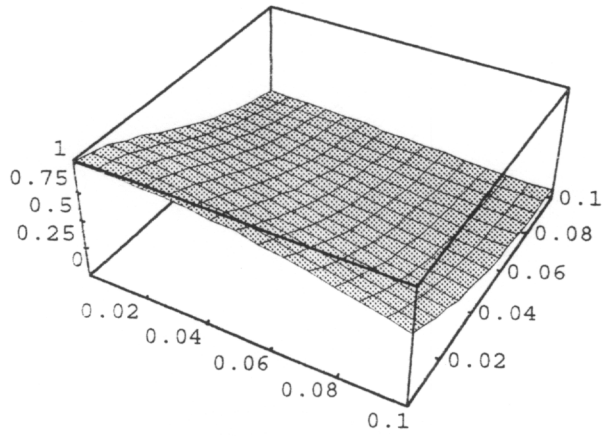


Figure 4.1
O.C. Curve for the 3-AS-S-1(30,1.5,0.5) Acceptance Sampling Plan

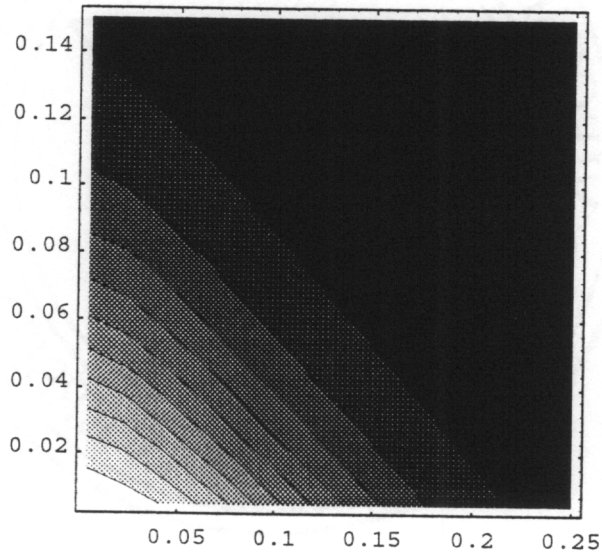


Figure 4.2
Contour Plot of O.C. Function Values for the 3-AS-S-1(30,1.5,0.5) Acceptance Sampling Plan

4.2.2 O.C. Function Behavior of 4-level Acceptance Sampling Plans

In this section, the O.C. function for each of the 4-level acceptance sampling plans defined in section 4.1.2 is constructed. In the 4-level case, the quality of a received lot is given by $\mathbf{p} = [p_1 \ p_2 \ p_3 \ p_4]$ where p_1 denotes the proportion of items in the lot that are conforming, p_2 denotes the proportion of items in the lot that are minor nonconforming, p_3 denotes the proportion of items in the lot that are major nonconforming, and p_4 denotes the proportion of items in the lot that are critical nonconforming. For a specified sample size n , N_1 denotes the number of sample items that are conforming, N_2 denotes the number of sample items that are minor nonconforming, N_3 denotes the number of sample items that are major nonconforming, and N_4 denotes the number of sample items that are critical nonconforming. Using the assumed multinomial model,

$$m(n_1, n_2, n_3, n_4, p_1, p_2, p_3, p_4) \quad (4.8)$$

$$= \begin{cases} \frac{n!}{n_1! n_2! n_3! n_4!} p_1^{n_1} p_2^{n_2} p_3^{n_3} p_4^{n_4} & \text{if } n_1 + n_2 + n_3 + n_4 = n \\ 0 & \text{otherwise} \end{cases}$$

Given values for n and T , each O.C. function can be constructed as follows.

4-AS-S-1(n, T, v_1, v_2)

Acceptance Criterion: $N_2 v_1 + N_3 v_2 + N_4 \leq T$

$$m_a(n_1, n_2, n_3, n_4, p_1, p_2, p_3, p_4)$$

$$= \begin{cases} m(n_1, n_2, n_3, n_4, p_1, p_2, p_3, p_4) & \text{if } n_2 v_1 + n_3 v_2 + n_4 \leq T \\ 0 & \text{otherwise} \end{cases}$$

4-AS-S-2(n, T, v)

Acceptance Criterion: $N_1 + N_2v + N_3v^2 + N_4v^3 \leq T$

$$m_a(n_1, n_2, n_3, n_4, p_1, p_2, p_3, p_4) = \begin{cases} m(n_1, n_2, n_3, n_4, p_1, p_2, p_3, p_4) & \text{if } n_1 + n_2v + n_3v^2 + n_4v^3 \leq T \\ 0 & \text{otherwise} \end{cases}$$

4-AS-S-3(n, T, v)

Acceptance Criterion: $N_1v + N_2v^2 + N_3v^3 + N_4v^4 \leq T$

$$m_a(n_1, n_2, n_3, n_4, p_1, p_2, p_3, p_4) = \begin{cases} m(n_1, n_2, n_3, n_4, p_1, p_2, p_3, p_4) & \text{if } n_1v + n_2v^2 + n_3v^3 + n_4v^4 \leq T \\ 0 & \text{otherwise} \end{cases}$$

4-AS-S-G(n, T, v_1, v_2, v_3)

Acceptance Criterion: $N_1v_1 + N_2v_2 + N_3v_3 + N_4v_4 \leq T$

$$m_a(n_1, n_2, n_3, n_4, p_1, p_2, p_3, p_4) = \begin{cases} m(n_1, n_2, n_3, n_4, p_1, p_2, p_3, p_4) & \text{if } n_1v_1 + n_2v_2 + n_3v_3 + n_4v_4 \leq T \\ 0 & \text{otherwise} \end{cases}$$

Having constructed the conditional multinomial expressions, the O.C. function for all the 4-level acceptance sampling plans can be written as

$$P_a(p_1, p_2, p_3, p_4) = \sum_{n_1=0}^n \sum_{n_2=0}^{n-n_1} \sum_{n_3=0}^{n-n_1-n_2} m_a(n_1, n_2, n_3, n-n_1-n_2-n_3, p_1, p_2, p_3, p_4) \quad (4.9).$$

Computation of this O.C. function for any of the 4-level acceptance sampling plans is quite straightforward. However unlike 3-level acceptance sampling plan, presentation of the O.C. function behavior for a 4-level acceptance sampling plan in a meaningful way is limited to tables.

4.2.3 Computing the O.C. Function for 3-level and 4-level Acceptance Sampling Plans

As was discovered during the initial stages of the research, the current form of the O.C. function is computationally inefficient. In this section, the computational efficiency of the current form of the O.C. function is discussed, and a new equivalent form is presented which is much more efficient.

The original form of the O.C. function for multilevel acceptance sampling plans defined here is given by

$$P_a(p_1, p_2, \dots, p_j) = \sum_{n_1=0}^n \sum_{n_2=0}^{n-n_1} \cdots \sum_{n_j=0}^{n-n_1-n_2-\dots-n_{j-1}} m_a(n_1, n_2, \dots, n_j, p_1, p_2, \dots, p_j) \quad (4.10).$$

It is easy to recognize the computational inefficiency of this form of the O.C. function. For a given value of n , $(n+1)^j$ evaluations of the function m_a are required. However, a simplified form of the O.C. function is possible. This simplification is based on the fact that the number of observed sample items in each category, N_1, N_2, \dots, N_j must sum to the sample size n . The resulting final form of the O.C. function is given by

$$P_a(p_1, p_2, \dots, p_j) = \sum_{n_1=0}^n \sum_{n_2=0}^{n-n_1} \cdots \sum_{n_{j-1}=0}^{n-n_1-n_2-\dots-n_{j-2}} m_a(n_1, n_2, \dots, n_{j-1}, n-n_1-n_2-\dots-n_{j-1}, p_1, p_2, \dots, p_j) \quad (4.11).$$

In the 3-level case, this form of the O.C. function is given by

$$P_a(p_1, p_2, p_3) = \sum_{n_1=0}^n \sum_{n_2=0}^{n-n_1} m_a(n_1, n_2, n-n_1-n_2, p_1, p_2, p_3) \quad (4.12),$$

and in the 4-level case, it is given by

$$P_a(p_1, p_2, p_3, p_4) = \sum_{n_1=0}^n \sum_{n_2=0}^{n-n_1} \sum_{n_3=0}^{n-n_1-n_2} m_a(n_1, n_2, n_3, n-n_1-n_2-n_3, p_1, p_2, p_3, p_4) \quad (4.13).$$

Although this form of the O.C. is more efficient than the previous, it is still inefficient for relatively large sample sizes. In the 3-level case, computing the O.C. function for a plan having sample size n requires $(1/2)(n + 1)(n + 2)$ evaluations of the function m_a . For a sample size as small as 50, 1326 evaluations of m_a are required. Although this is a vast improvement over the 132,651 evaluations required by the original form, it is not efficient enough to allow for extensive study of O.C. function behavior.

The computational inefficiency of the current O.C. function expression lies in the order of summation (N_1 followed by N_2 , followed by N_3). By taking advantage of the acceptance criterion for the defined 3-level acceptance sampling plans, a more efficient form can be created. Recall the general acceptance criterion for the defined 3-level acceptance sampling plans:

$$N_1v_1 + N_2v_2 + N_3v_3 \leq T \quad (4.14).$$

The new form of the O.C. function is constructed by considering the implications of the acceptance criterion for the allowed number of observed nonconforming and marginal sample items. For nonconforming items in the sample, the acceptance criterion implies that

$$N_3v_3 \leq T \quad (4.15).$$

Thus in order for a lot to be accepted, the number of nonconforming items in the sample must fall below some value. Specifically,

$$N_3 \leq \frac{T}{v_3} \quad (4.16).$$

Since N_3 is an integer and cannot be larger than the sample size, a threshold value for N_3 can be defined. This threshold value is

$$N_3^{\max} = \min\left(n, \left\lfloor \frac{T}{v_3} \right\rfloor\right) \quad (4.17).$$

Given that $N_3 = n_3$, a limit can also be placed on the number of marginal sample items by recognizing that

$$N_2v_2 + n_3v_3 \leq T \quad (4.18),$$

which can be rewritten as

$$N_2 \leq \frac{T - n_3 v_3}{v_2} \quad (4.19).$$

Since N_2 is an integer and the sum of N_2 and N_3 cannot exceed the sample size, a threshold value for N_2 can be defined. This threshold value is

$$N_2^{\max} = \min\left(n - n_3, \left\lfloor \frac{T - n_3 v_3}{v_2} \right\rfloor\right) \quad (4.20).$$

These thresholds can be used to define a new form for the O.C. function. For the 3-level case, this revised version of the O.C. function is given by

$$P_a(p_1, p_2, p_3) = \sum_{n_3=0}^{N_3^{\max}} \sum_{n_2=0}^{N_2^{\max}} m_a(n - n_2 - n_3, n_2, n_3, p_1, p_2, p_3) \quad (4.21).$$

These thresholds can be described specifically for each of the defined types of 3-level acceptance sampling plans.

3-AS-S-1(n, T, v)

$$N_3^{\max} = \min\left(n, \lfloor T \rfloor\right)$$

$$N_2^{\max} = \min\left(n - n_3, \left\lfloor \frac{T - n_3}{v} \right\rfloor\right)$$

3-AS-S-2(n, T, v)

$$N_3^{\max} = \min\left(n, \left\lfloor \frac{T}{v^2} \right\rfloor\right)$$

$$N_2^{\max} = \min\left(n - n_3, \left\lfloor \frac{T - n_3 v_2}{v} \right\rfloor\right)$$

3-AS-S-3(n, T, v)

$$N_3^{\max} = \min\left(n, \left\lfloor \frac{T}{v^3} \right\rfloor\right)$$

$$N_2^{\max} = \min\left(n - n_3, \left\lfloor \frac{T - n_3 v^3}{v^2} \right\rfloor\right)$$

The improvement in computational efficiency resulting from revising the form of the O.C. function can be demonstrated using several examples. The improvements obtained in these examples are presented in Table 4.2.

Table 4.2
Examples of Achieved Savings in 3-Level O.C. Function Computation

Sampling Plan	Evaluations of m_a Required by Equation (4.10)	Evaluations of m_a Required by Equation (4.12)	Evaluations of m_a Required by Equation (4.21)
3-AS-S-1(91,3.25,0.25)	778688	4278	32
3-AS-S-1(63,5,0.5)	262144	2080	36
3-AS-S-1(52,4.5,0.75)	148877	1431	20
3-AS-S-2(81,90,2)	551368	3403	552
3-AS-S-2(124,270,5)	1953125	7875	330
3-AS-S-2(64,240,10)	274625	2145	45
3-AS-S-3(140,322,2)	2803221	10011	1681
3-AS-S-3(41,405,5)	74088	903	38
3-AS-S-3(78,2880,10)	493039	3160	57

In the 4-level case, computing the O.C. function for a plan having sample size n requires

$$\sum_{k=1}^{n+1} (1/2)j(j+1) \tag{4.22}$$

evaluations of the function m_a . For a sample size of 50, 23,426 evaluations of m_a are required. Again this is a vast improvement over the 6,765,201 evaluations required by the

original form, but it is not efficient enough to allow for extensive study of O.C. function behavior.

By taking advantage of the acceptance criterion for the defined 4-level acceptance sampling plans, a more efficient form of the O.C. function can be created. Recall the general acceptance criterion for the defined 4-level acceptance sampling plans:

$$N_1v_1 + N_2v_2 + N_3v_3 + N_4v_4 \leq T \quad (4.23).$$

The new form of the O.C. function is constructed by considering the implications of the acceptance criterion for the allowed number of observed critical, major, and minor nonconforming sample items. For critical nonconforming items in the sample, the acceptance criterion implies that

$$N_4v_4 \leq T \quad (4.24).$$

Thus in order for a lot to be accepted, the number of critical nonconforming items in the sample must fall below some value. Specifically,

$$N_4 \leq \frac{T}{v_4} \quad (4.25).$$

Since N_4 is an integer and cannot be larger than the sample size, a threshold value for N_4 can be defined. This threshold value is

$$N_4^{\max} = \min\left(n, \left\lfloor \frac{T}{v_4} \right\rfloor\right) \quad (4.26).$$

Given that $N_4 = n_4$, a limit can also be placed on the number of major nonconforming sample items by recognizing that

$$N_3v_3 + n_4v_4 \leq T \quad (4.27),$$

which can be rewritten as

$$N_3 \leq \frac{T - n_4v_4}{v_3} \quad (4.28).$$

Since N_3 is an integer and the sum of N_3 and N_4 cannot exceed the sample size, a threshold value for N_3 can be defined. This threshold value is

$$N_3^{\max} = \min\left(n - n_4, \left\lfloor \frac{T - n_4 v_4}{v_3} \right\rfloor\right) \quad (4.29).$$

Given that $N_3 = n_3$ and $N_4 = n_4$, a limit can also be placed on the number of minor nonconforming sample items by recognizing that

$$N_2 v_2 + n_3 v_3 + n_4 v_4 \leq T \quad (4.30),$$

which can be rewritten as

$$N_2 \leq \frac{T - n_4 v_4 - n_3 v_3}{v_2} \quad (4.31).$$

Since N_2 is an integer and the sum of N_2 , N_3 and N_4 cannot exceed the sample size, a threshold value for N_2 can be defined. This threshold value is

$$N_2^{\max} = \min\left(n - n_3 - n_4, \left\lfloor \frac{T - n_4 v_4 - n_3 v_3}{v_2} \right\rfloor\right) \quad (4.32).$$

These thresholds can be used to define a new form of the O.C. function. For the 4-level case, this revised version of the O.C. function is given by

$$P_a(p_1, p_2, p_3, p_4) = \sum_{n_4=0}^{N_4^{\max}} \sum_{n_3=0}^{N_3^{\max}} \sum_{n_2=0}^{N_2^{\max}} m_a(n - n_2 - n_3 - n_4, n_2, n_3, n_4, p_1, p_2, p_3, p_4) \quad (4.33).$$

These thresholds can be described specifically for each of the defined types of 4-level acceptance sampling plans.

4-AS-S-1(n, T, v_1, v_2)

$$N_4^{\max} = \min(n, \lfloor T \rfloor)$$

$$N_3^{\max} = \min\left(n - n_4, \left\lfloor \frac{T - n_4}{v_2} \right\rfloor\right)$$

$$N_2^{\max} = \min\left(n - n_3 - n_4, \left\lfloor \frac{T - n_4 - n_3 v_2}{v_1} \right\rfloor\right)$$

4-AS-S-2(n, T, v)

$$N_4^{\max} = \min\left(n, \left\lfloor \frac{T}{v^3} \right\rfloor\right)$$

$$N_3^{\max} = \min\left(n - n_4, \left\lfloor \frac{T - n_4 v^3}{v^2} \right\rfloor\right)$$

$$N_2^{\max} = \min\left(n - n_3 - n_4, \left\lfloor \frac{T - n_4 v^3 - n_3 v^2}{v} \right\rfloor\right)$$

4-AS-S-3(n, T, v)

$$N_4^{\max} = \min\left(n, \left\lfloor \frac{T}{v^4} \right\rfloor\right)$$

$$N_3^{\max} = \min\left(n - n_4, \left\lfloor \frac{T - n_4 v^4}{v^3} \right\rfloor\right)$$

$$N_2^{\max} = \min\left(n - n_3 - n_4, \left\lfloor \frac{T - n_4 v^4 - n_3 v^3}{v^2} \right\rfloor\right)$$

The improvement in computational efficiency resulting from revising the form of the O.C. function can be demonstrated using several examples. The improvements obtained in these examples are presented in Table 4.3.

Table 4.3
Examples of Achieved Savings in 4-Level O.C. Function Computation

Sampling Plan	Evaluations of m_a Required by Equation (4.10)	Evaluations of m_a Required by Equation (4.12)	Evaluations of m_a Required by Equation (4.33)
4-AS-S-1(37,2.75,0.25,0.5)	2085136	9880	68
4-AS-S-1(20,2.5,0.25,0.75)	194481	1771	41
4-AS-S-1(8,1.25,0.5,0.75)	6561	165	6
4-AS-S-2(34,50,2)	1500625	7770	504
4-AS-S-2(59,525,5)	12960000	37820	1975
4-AS-S-2(121,5220,10)	221533456	310124	14529

4.3 MULTILEVEL ACCEPTANCE SAMPLING PLAN PARAMETER SELECTION

In many practical situations, the manufacturer knows the type of acceptance sampling plan that they wish to use. However they typically do not know what parameter values to use. In the case of multilevel acceptance sampling, the manufacturer may know the quality value function (i.e. the multilevel acceptance sampling plan) that they wish to use (e.g. 3-AS-S-1($n, T, 0.5$) or 4-AS-S-2($n, T, 2$)), but they do not necessarily know the values of n and T that they wish to use. In this section, a method is defined for selecting n and T based on some desired O.C. function behavior. The methods are then demonstrated and evaluated using the 3-level and 4-level acceptance sampling plans defined in section 4.1.

The problem of parameter selection is not unique to multilevel acceptance sampling. The problem exists for all types of acceptance sampling plans. In most instances, parameter selection methods are defined using some specified desirable O.C. function behavior. Duncan[8] provides a thorough discussion of such methods for attributes and variables acceptance sampling plans. In general, if an acceptance sampling plan has b parameters, b points on the O.C. curve are specified. This results in a system of b equations in b unknowns. The system of equations is then solved for the unknown sampling plan parameters. Since multilevel acceptance sampling plans have two parameters, n and T , two points on the O.C. curve must be specified.

The first of these two points utilizes what is known as the acceptable quality level (AQL). The AQL is described in terms of the vector $\mathbf{p}_1 = [p_{11} p_{12} \dots p_{1j}]$. \mathbf{p}_1 represents proportions for the quality classifications that the manufacturer finds to be acceptable. One common interpretation of the AQL is that \mathbf{p}_1 corresponds to the target proportions for the j quality levels. Thus a high probability of acceptance (≥ 0.9) is specified for \mathbf{p}_1 . In other words, $P_a(\mathbf{p}_1) = 1 - \alpha$, where α is the probability that a lot having quality $\mathbf{p} = \mathbf{p}_1$ is rejected. α is often referred to as the probability of a Type I error or the producer's risk.

The second of these two points utilizes what is known as the rejectable quality level (RQL). The RQL is described by the vector $\mathbf{p}_2 = [p_{21} p_{22} \dots p_{2j}]$. \mathbf{p}_2 represents a “lower bound” on the quality of received lots. Any lot having quality worse than \mathbf{p}_2 is unacceptable, thus a low probability of acceptance (≤ 0.1) is specified for \mathbf{p}_2 . Specifically, $P_a(\mathbf{p}_2) = \beta$, where β is the probability that a lot having quality $\mathbf{p} = \mathbf{p}_2$ is accepted. β is often referred to as the probability of a Type II error or the consumer’s risk.

The supplier and the manufacturer must agree on choices for \mathbf{p}_1 , \mathbf{p}_2 , α , and β . Having agreed upon these values, the resulting system of two equations and two unknowns is given by

$$\sum_{n_1=0}^n \sum_{n_2=0}^{n-n_1} \dots \sum_{n_{j-1}=0}^{n-n_1-n_2-\dots-n_{j-2}} m_a(n_1, n_2, \dots, n_{j-1}, n-n_1-n_2-\dots-n_{j-1}, p_{11}, p_{12}, \dots, p_{1j}) = 1 - \alpha \quad (4.44),$$

and

$$\sum_{n_1=0}^n \sum_{n_2=0}^{n-n_1} \dots \sum_{n_{j-1}=0}^{n-n_1-n_2-\dots-n_{j-2}} m_a(n_1, n_2, \dots, n_{j-1}, n-n_1-n_2-\dots-n_{j-1}, p_{21}, p_{22}, \dots, p_{2j}) = \beta \quad (4.45).$$

Solution to this set of equations for n and T must be done numerically, and is quite tedious. Thus an approximate method is defined here.

Recall that the acceptance criterion for multilevel acceptance sampling plans is

$\sum_{i=1}^n V_i \leq T$. Thus the O.C. function can be rewritten as

$$P_a(p_1, p_2, \dots, p_j) = P\left(\sum_{i=1}^n V_i \leq T \mid p_1, p_2, \dots, p_j\right) \quad (4.46).$$

Advantage can be taken of equation (4.46) by considering the following (see Ross[17]).

$$E(V_i) = \sum_{k=1}^j v_k p_k = \mu_v \quad (4.47)$$

$$E(V_i^2) = \sum_{k=1}^j v_k^2 p_k \quad (4.48)$$

$$\text{Var}(V_i) = \sum_{k=1}^j v_k^2 p_k - \mu_V^2 = \sigma_V^2 \quad (4.49)$$

Since V_1, V_2, \dots, V_n is a sequence of independent and identically distributed (IID) random variables with mean μ_V and variance σ_V^2 , then by the Central Limit Theorem (see Ross[17])

$$\frac{\sum_{i=1}^n V_i - n\mu_V}{\sigma_V \sqrt{n}}$$

converges to the standard normal random variable (a normal random variable having mean 0 and variance 1) as $n \rightarrow \infty$. So by the Central Limit Theorem,

$$P_a(p_1, p_2, \dots, p_j) = P\left(\sum_{i=1}^n V_i \leq T \mid p_1, p_2, \dots, p_j\right) \cong P\left(Z \leq \frac{T - n\mu_V}{\sigma_V \sqrt{n}}\right) \quad (4.50)$$

where Z represents the standard normal random variable. Let μ_1 denote the expected value of V_i when $\mathbf{p} = \mathbf{p}_1$, and let μ_2 denote the expected value of V_i when $\mathbf{p} = \mathbf{p}_2$. Let σ_1 denote the standard deviation of V_i when $\mathbf{p} = \mathbf{p}_1$, and let σ_2 denote the standard deviation of V_i when $\mathbf{p} = \mathbf{p}_2$. Then

$$\mu_1 = \sum_{k=1}^j v_k p_{1k} \quad (4.51),$$

$$\sigma_1 = \sqrt{\sum_{k=1}^j v_k^2 p_{1k} - \mu_1^2} \quad (4.52),$$

$$\mu_2 = \sum_{k=1}^j v_k p_{2k} \quad (4.53),$$

and

$$\sigma_2 = \sqrt{\sum_{k=1}^j v_k^2 p_{2k} - \mu_2^2} \quad (4.54).$$

Applying these results to the two specified points on the O.C. curve yields the following equations (ignoring the approximation for now).

$$P_a(p_{11}, p_{12}, \dots, p_{1j}) = P\left(Z \leq \frac{T - n\mu_1}{\sigma_1 \sqrt{n}}\right) = 1 - \alpha \quad (4.55)$$

$$P_a(p_{21}, p_{22}, \dots, p_{2j}) = P\left(Z \leq \frac{T - n\mu_2}{\sigma_2 \sqrt{n}}\right) = \beta \quad (4.56)$$

These two equations reduce to the following:

$$(4.55) \Rightarrow Z_{1-\alpha} = \frac{T - n\mu_1}{\sigma_1 \sqrt{n}} \quad (4.57)$$

$$(4.56) \Rightarrow Z_{\beta} = \frac{T - n\mu_2}{\sigma_2 \sqrt{n}} \quad (4.58)$$

where Z_{γ} is a real number such that $P(Z \leq Z_{\gamma}) = \gamma$. Each of these equations can be solved for T .

$$(4.57) \Rightarrow T = n\mu_1 + Z_{1-\alpha} \sigma_1 \sqrt{n} \quad (4.59)$$

$$(4.58) \Rightarrow T = n\mu_2 + Z_{\beta} \sigma_2 \sqrt{n} \quad (4.60)$$

Equating these two expressions for T results in an expression that can be used to find n .

$$n\mu_1 + Z_{1-\alpha} \sigma_1 \sqrt{n} = n\mu_2 + Z_{\beta} \sigma_2 \sqrt{n}$$

$$n(\mu_2 - \mu_1) = \sqrt{n}(Z_{1-\alpha} \sigma_1 - Z_{\beta} \sigma_2)$$

$$\sqrt{n} = \left(\frac{Z_{1-\alpha} \sigma_1 - Z_{\beta} \sigma_2}{\mu_2 - \mu_1} \right)$$

$$n = \left(\frac{Z_{1-\alpha} \sigma_1 - Z_{\beta} \sigma_2}{\mu_2 - \mu_1} \right)^2 \quad (4.61)$$

As with most parameter selection methods for acceptance sampling plans, the decision must be made at this point to either fix α or fix β . Having found n , either of the two equations for T may be used. If the equation in terms of p_1 and α , equation (4.59), is used, it is said that α is “fixed.” If the equation in terms of p_2 and β , equation (4.60), is used, then it is said that β is “fixed.” Again this choice must be agreed upon by the

supplier of the items and the manufacturer. Thus the equations to be used for selecting T are

$$(4.59) \Rightarrow \text{fix } \alpha: T = n\mu_1 + Z_{1-\alpha}\sigma_1\sqrt{n} \quad (4.62)$$

$$(4.60) \Rightarrow \text{fix } \beta: T = n\mu_2 + Z_\beta\sigma_2\sqrt{n} \quad (4.63).$$

A procedure is needed for evaluating the capability of the method. In other words, do the parameters selected provide the O.C. function behavior specified by \mathbf{p}_1 , \mathbf{p}_2 , α , and β ? This question can be answered by using the selected values of n and T to compute the adjusted value of α , α' , and the adjusted value of β , β' . This is accomplished by using the selected values of n and T to compute the resulting values of the O.C. function at \mathbf{p}_1 and at \mathbf{p}_2 . So,

$$\alpha' = 1 - P_a(p_{11}, p_{12}, \dots, p_{1j}) \quad (4.64),$$

and

$$\beta' = P_a(p_{21}, p_{22}, \dots, p_{2j}) \quad (4.65).$$

Once the adjusted values of α and β have been computed, they can be compared to the specified target values of α and β to see how well the parameter selection method performed.

The method for selecting n and T is now demonstrated and evaluated for each of the defined 3-level and 4-level acceptance sampling plans.

4.3.1 Parameter Selection for 3-level Acceptance Sampling Plans

In the preceding section, the method for selecting n and T for a multilevel acceptance sampling is defined. In this section, the details required for implementing this method for the 3-level acceptance sampling plans defined in Section 4.1.1 are provided. The method requires that $\mathbf{p}_1 = [p_{11} \ p_{12} \ p_{13}]$, $\mathbf{p}_2 = [p_{21} \ p_{22} \ p_{23}]$, α , and β be specified.

For each of the defined 3-level acceptance sampling plans, expressions for μ_1 , μ_2 , σ_1 , and σ_2 are provided below.

3-AS-S-1(n, T, v)

$$E(V_i) = vp_2 + p_3$$

$$E(V_i^2) = v^2 p_2 + p_3$$

$$\mu_1 = vp_{12} + p_{13}$$

$$\mu_2 = vp_{22} + p_{23}$$

$$\sigma_1 = \sqrt{v^2 p_{12} + p_{13} - (vp_{12} + p_{13})^2}$$

$$\sigma_2 = \sqrt{v^2 p_{22} + p_{23} - (vp_{22} + p_{23})^2}$$

3-AS-S-2(n, T, v)

$$E(V_i) = p_1 + vp_2 + v^2 p_3$$

$$E(V_i^2) = p_1 + v^2 p_2 + v^4 p_3$$

$$\mu_1 = p_{11} + vp_{12} + v^2 p_{13}$$

$$\mu_2 = p_{21} + vp_{22} + v^2 p_{23}$$

$$\sigma_1 = \sqrt{p_{11} + v^2 p_{12} + v^4 p_{13} - (p_{11} + vp_{12} + v^2 p_{13})^2}$$

$$\sigma_2 = \sqrt{p_{21} + v^2 p_{22} + v^4 p_{23} - (p_{21} + vp_{22} + v^2 p_{23})^2}$$

3-AS-S-3(n, T, v)

$$E(V_i) = vp_1 + v^2 p_2 + v^3 p_3$$

$$E(V_i^2) = v^2 p_1 + v^4 p_2 + v^6 p_3$$

$$\mu_1 = vp_{11} + v^2 p_{12} + v^3 p_{13}$$

$$\mu_2 = vp_{21} + v^2 p_{22} + v^3 p_{23}$$

$$\sigma_1 = \sqrt{v^2 p_{11} + v^4 p_{12} + v^6 p_{13} - (vp_{11} + v^2 p_{12} + v^3 p_{13})^2}$$

$$\sigma_2 = \sqrt{v^2 p_{21} + v^4 p_{22} + v^6 p_{23} - (vp_{21} + v^2 p_{22} + v^3 p_{23})^2}$$

3-AS-S-G(n, T, v_1, v_2, v_3)

$$E(V_i) = v_1 p_1 + v_2 p_2 + v_3 p_3$$

$$E(V_i^2) = v_1^2 p_1 + v_2^2 p_2 + v_3^2 p_3$$

$$\mu_1 = v_1 p_{11} + v_2 p_{12} + v_3 p_{13}$$

$$\mu_2 = v_1 p_{21} + v_2 p_{22} + v_3 p_{23}$$

$$\sigma_1 = \sqrt{v_1^2 p_{11} + v_2^2 p_{12} + v_3^2 p_{13} - (v_1 p_{11} + v_2 p_{12} + v_3 p_{13})^2}$$

$$\sigma_2 = \sqrt{v_1^2 p_{21} + v_2^2 p_{22} + v_3^2 p_{23} - (v_1 p_{21} + v_2 p_{22} + v_3 p_{23})^2}$$

Appendix 1.1 contains the results obtained from implementing the method of parameter selection for the 3-AS-S-1(n, T, v) acceptance sampling plan. For the given examples, the following information is provided: v , p_1 , p_2 , α , β , the selected values of n and T , α' , and β' (for both fix α and fix β). Appendices 1.2 and 1.3 contain similar results for the 3-AS-S-2(n, T, v) acceptance sampling plan and the 3-AS-S-3(n, T, v) acceptance sampling plan respectively.

By studying the results presented in Appendices 1.1, 1.2, and 1.3, several conclusions can be made regarding the approximate parameter selection method as it relates to the defined 3-level acceptance sampling plans. The first conclusion is that the 3-AS-S-2(n, T, v) and the 3-AS-S-3(n, T, v) plans are equivalent. For every example considered for the 3-AS-S-2(n, T, v) and 3-AS-S-3(n, T, v) sampling plans, identical adjusted values of α and β are obtained. This result leads to the following more general conclusion.

Theorem 4.1: Let $V_1(X)$ be a quality value function defined on a 3-level discrete product quality measure X , and let $V_2(X)$ be a quality value function defined on X such that $V_2(X) = bV_1(X)$ for all X where b is some positive constant. Then a multilevel acceptance sampling plan (utilizing single sampling and the summation decision rule) using $V_1(X)$ and having parameters n and $T = T_1$ is equivalent to a multilevel acceptance sampling plan (utilizing single sampling and the summation decision rule) using $V_2(X)$ and having parameters n and $T = T_2 = bT_1$.

Proof: Consider a multilevel acceptance sampling plan (utilizing single sampling and the summation decision rule) using $V_1(X)$ and having parameters n and $T = T_1$.

Let V_{1i} denote the quality value of the i^{th} item in a sample.

The O.C. function for this sampling plan is given by

$$P_a(p_1, p_2, \dots, p_j) = P\left(\sum_{i=1}^n V_{1i} \leq T_1 \mid p_1, p_2, \dots, p_j\right).$$

Consider a multilevel acceptance sampling plan (utilizing single sampling and the summation decision rule) using $V_2(X) = bV_1(X)$ (for all X) and having parameters n and $T = T_2 = bT_1$.

Let V_{2i} denote the quality value of the i^{th} item in a sample.

The O.C. function for this sampling plan is given by

$$P_a(p_1, p_2, \dots, p_j) = P\left(\sum_{i=1}^n V_{2i} \leq T_2 \mid p_1, p_2, \dots, p_j\right).$$

Using the defined relationship between the two plans, this second O.C. function can be rewritten as

$$P_a(p_1, p_2, \dots, p_j) = P\left(\sum_{i=1}^n bV_{1i} \leq bT_1 \mid p_1, p_2, \dots, p_j\right).$$

Simplification of this expression results in the following.

$$P_a(p_1, p_2, \dots, p_j) = P\left(b \sum_{i=1}^n V_{1i} \leq bT_1 \mid p_1, p_2, \dots, p_j\right)$$

$$P_a(p_1, p_2, \dots, p_j) = P\left(\sum_{i=1}^n V_{1i} \leq T_1 \mid p_1, p_2, \dots, p_j\right)$$

Thus the two plans are equivalent.

This result implies that it is no longer necessary to consider the 3-AS-S-3(n, T, v) and 4-AS-S-3(n, T, v) sampling plans. In addition, note that a similar theorem could be defined for the product decision rule given in equation (3.10).

A second conclusion is that there is little difference between the fix α and fix β values of T for the defined 3-level acceptance sampling plans. For the 384 examples considered, 345 (90.3%) result in identical T values for the fix α and fix β cases. For those examples that have different values of T for the fix α and fix β cases, the difference between the two T values is very small. As a result, only the fix β cases are considered in additional detail.

A third conclusion is that the Central Limit Theorem approximation for the O.C. function works very well for the defined 3-level acceptance sampling plans. For the 384 examples presented in Appendices 1.1 and 1.2, the average percent error in the adjusted value of α is 22.1%, and the average percent error in the achieved value of β is 16.4%. These values are computed by averaging the percent error for α and the percent error for β for each example. These values are computed using the following formulae.

$$\text{percent error for } \alpha = \frac{|\alpha' - \alpha|}{\alpha} \times 100\% \quad (4.66)$$

$$\text{percent error for } \beta = \frac{|\beta' - \beta|}{\beta} \times 100\% \quad (4.67)$$

Similar results are obtained if the fix α cases are considered. For the 192 3-AS-S-1(n, T, v) examples, the average percent error in the achieved value of α is 20.6%, and the average percent error in the achieved value of β is 15.4%. For the 192 3-AS-S-2(n, T, v) examples, the average percent error in the achieved value of α is 23.6%, and the average percent error in the achieved value of β is 17.4%.

A fourth conclusion is that, for the defined 3-level acceptance sampling plans, the parameter selection method tends to favor the manufacturer more than the supplier. For the 384 fix β examples, 80 (20.8%) result in an adjusted value of α that is less than or equal to α and 336 (87.5%) result in an adjusted value of β that is less than or equal to β .

It is very important to stress that corresponding results for the fix α cases tend to more strongly favor the manufacturer. For the 192 3-AS-S-1(n, T, v) examples, 47 achieve the specified value of α and 159 achieve the specified value of β . For the 192 3-AS-S-2(n, T, v) examples, 33 achieve the specified value of α and 177 achieve the specified value of β .

4.3.2 Parameter Selection for 4-level Acceptance Sampling Plans

In this section, the details required for implementing the parameter selection method for the 4-level acceptance sampling plans defined in Section 4.1.2 are provided. The method requires that $\mathbf{p}_1 = [p_{11} \ p_{12} \ p_{13} \ p_{14}]$, $\mathbf{p}_2 = [p_{21} \ p_{22} \ p_{23} \ p_{24}]$, α , and β be specified.

For each of the defined 4-level acceptance sampling plans, expressions for μ_1 , μ_2 , σ_1 , and σ_2 are provided.

4-AS-S-1(n, T, v_1, v_2)

$$E(V_i) = v_1 p_2 + v_2 p_3 + p_4$$

$$E(V_i^2) = v_1^2 p_2 + v_2^2 p_3 + p_4$$

$$\mu_1 = v_1 p_{12} + v_2 p_{13} + p_{14}$$

$$\mu_2 = v_1 p_{22} + v_2 p_{23} + p_{24}$$

$$\sigma_1 = \sqrt{v_1^2 p_{12} + v_2^2 p_{13} + p_{14} - (v_1 p_{12} + v_2 p_{13} + p_{14})^2}$$

$$\sigma_2 = \sqrt{v_1^2 p_{22} + v_2^2 p_{23} + p_{24} - (v_1 p_{22} + v_2 p_{23} + p_{24})^2}$$

4-AS-S-2(n, T, v)

$$E(V_i) = p_1 + vp_2 + v^2 p_3 + v^3 p_4$$

$$E(V_i^2) = p_1 + v^2 p_2 + v^4 p_3 + v^6 p_4$$

$$\mu_1 = p_{11} + vp_{12} + v^2 p_{13} + v^3 p_{14}$$

$$\mu_2 = p_{21} + vp_{22} + v^2 p_{23} + v^3 p_{24}$$

$$\sigma_1 = \sqrt{p_{11} + v^2 p_{12} + v^4 p_{13} + v^6 p_{14} - (p_{11} + vp_{12} + v^2 p_{13} + v^3 p_{14})^2}$$

$$\sigma_2 = \sqrt{p_{21} + v^2 p_{22} + v^4 p_{23} + v^6 p_{24} - (p_{21} + vp_{22} + v^2 p_{23} + v^3 p_{24})^2}$$

4-AS-S-G(n, T, v_1, v_2, v_3, v_4)

$$E(V_i) = v_1 p_1 + v_2 p_2 + v_3 p_3 + v_4 p_4$$

$$E(V_i^2) = v_1^2 p_1 + v_2^2 p_2 + v_3^2 p_3 + v_4^2 p_4$$

$$\mu_1 = v_1 p_{11} + v_2 p_{12} + v_3 p_{13} + v_4 p_{14}$$

$$\mu_2 = v_1 p_{21} + v_2 p_{22} + v_3 p_{23} + v_4 p_{24}$$

$$\sigma_1 = \sqrt{v_1^2 p_{11} + v_2^2 p_{12} + v_3^2 p_{13} + v_4^2 p_{14} - (v_1 p_{11} + v_2 p_{12} + v_3 p_{13} + v_4 p_{14})^2}$$

$$\sigma_2 = \sqrt{v_1^2 p_{21} + v_2^2 p_{22} + v_3^2 p_{23} + v_4^2 p_{24} - (v_1 p_{21} + v_2 p_{22} + v_3 p_{23} + v_4 p_{24})^2}$$

Appendix 2.1 contains a sample of the results obtained from implementing the approximate method of parameter selection for the 4-AS-S-1(n, T, v_1, v_2) acceptance sampling plan. For the given examples, the following information is provided: $v_1, v_2, p_1, p_2, \alpha, \beta$, the selected values of n and T , α' , and β' (for both fix α and fix β). Appendix 2.2 contains similar results for the 4-AS-S-2(n, T, v) acceptance sampling plan.

By studying the results of the examples, including those presented in Appendices 2.1 and 2.2, several conclusions can be made regarding the approximate parameter selection method as it relates to the defined 4-level acceptance sampling plans. A first conclusion is that there is little difference between the fix α and fix β values of T for the defined 4-level acceptance sampling plans. For the 1536 examples considered (of which a subset is presented in Appendices 2.1 and 2.2), 998 (65%) result in identical T values for the fix α and fix β cases. For those examples that have different values of T for the fix α

and fix β cases, the difference between the two T values is very small. As a result, only the fix β cases are considered in additional detail.

A second conclusion is that the Central Limit Theorem approximation for the O.C. function works very well for the defined 4-level acceptance sampling plans. For the 1536 examples, the average percent error in the adjusted value of α is 18.9%, and the average percent error in the achieved value of β is 15.9%. Similar results are obtained if the fix α cases are considered. For the 768 4-AS-S-1(n, T, v_1, v_2) examples, the average percent error in the achieved value of α is 16.3%, and the average percent error in the achieved value of β is 13.1%. For the 768 4-AS-S-2(n, T, v) examples, the average percent error in the achieved value of α is 21.4%, and the average percent error in the achieved value of β is 18.7%.

A third conclusion is that, for the defined 4-level acceptance sampling plans, the parameter selection method tends to favor the manufacturer more than the supplier. For the 1536 fix β examples, 328 (21.4%) result in an adjusted value of α that is less than or equal to α and 1336 (87%) result in an adjusted value of β that is less than or equal to β . It is very important to stress that corresponding results for the fix α cases tend to more strongly favor the manufacturer. For the 768 4-AS-S-1(n, T, v_1, v_2) examples, 210 achieve the specified value of α and 590 achieve the specified value of β . For the 768 4-AS-S-2(n, T, v) examples, 118 achieve the specified value of α and 746 achieve the specified value of β .

4.4 QUALITY VALUE FUNCTION SPECIFICATION

In this section, the problem of quality value function specification is addressed. In section 4.3, the problem of selecting n and T is considered. In that case, it is assumed that the manufacturer knows the type of multilevel acceptance sampling plan they wish to use, as well as the numerical values to be assigned to items by the quality value function. In this section, it is assumed that the manufacturer only knows the type of multilevel

acceptance sampling plan that they wish to use. For example, the manufacturer may wish to use a 4-AS-S-2(n, T, v) acceptance sampling plan, but they may not know what values of n , T , and v to use. In this section, a method is developed for selecting the parameters and specifying the quality value function for a multilevel acceptance sampling plan.

Suppose the quality value function requires the specification of b values (in the acceptance sampling plans defined in section 4.1, b is either one, two, three, or four). If $b + 2$ points on the O.C. curve are specified, then the result is a set of $b + 2$ equations in $b + 2$ unknowns (the quality value function, n , and T). The quality value function specification method defined here uses the Central Limit Theorem approximation to the O.C. function that is used in the parameter selection method to approximately solve this set of equations.

The quality value function specification method is described for each of the defined multilevel acceptance sampling plans in the sections that follow.

4.4.1 Quality Value Function Specification for 3-Level Acceptance Sampling Plans

Consider first the 3-AS-S-1(n, T, v) and the 3-AS-S-2(n, T, v) plans. To use either of these sampling plans, values of n , T , and v must be specified. In section 4.3.1, an approximate method is developed for selecting n and T . This method requires the specification of two points on the O.C. curve, summarized by \mathbf{p}_1 , \mathbf{p}_2 , α , and β . This method is extended to address quality value function specification.

The extension to the method begins with the specification of a third point on the O.C. curve, summarized by $\mathbf{p}_3 = [p_{31} \ p_{32} \ p_{33}]$ and γ , where $P_a(\mathbf{p}_3) = \gamma$. Let μ_3 denote the expected value of V_i when $\mathbf{p} = \mathbf{p}_3$, and let σ_3 denote the standard deviation of V_i when $\mathbf{p} = \mathbf{p}_3$. This additional specification results in the following system of three equations and three unknowns.

$$P_a(p_{11}, p_{12}, p_{13}) = P\left(Z \leq \frac{T - n\mu_1}{\sigma_1 \sqrt{n}}\right) = 1 - \alpha \quad (4.68)$$

$$P_a(p_{21}, p_{22}, p_{23}) = P\left(Z \leq \frac{T - n\mu_2}{\sigma_2 \sqrt{n}}\right) = \beta \quad (4.69)$$

$$P_a(p_{31}, p_{32}, p_{33}) = P\left(Z \leq \frac{T - n\mu_3}{\sigma_3 \sqrt{n}}\right) = \gamma \quad (4.70)$$

This series of equations can be reduced as follows.

$$(4.68) \Rightarrow Z_{1-\alpha} = \frac{T - n\mu_1}{\sigma_1 \sqrt{n}} \quad (4.71)$$

$$(4.69) \Rightarrow Z_\beta = \frac{T - n\mu_2}{\sigma_2 \sqrt{n}} \quad (4.72)$$

$$(4.70) \Rightarrow Z_\gamma = \frac{T - n\mu_3}{\sigma_3 \sqrt{n}} \quad (4.73)$$

Each of these equations can be solved for T .

$$(4.71) \Rightarrow T = n\mu_1 + Z_{1-\alpha} \sigma_1 \sqrt{n} \quad (4.74)$$

$$(4.72) \Rightarrow T = n\mu_2 + Z_\beta \sigma_2 \sqrt{n} \quad (4.75)$$

$$(4.73) \Rightarrow T = n\mu_3 + Z_\gamma \sigma_3 \sqrt{n} \quad (4.76)$$

At this point, three options exist. Any of the three expressions for T can be substituted into the other two. If equation (4.74) is used, the result is as follows.

$$(4.75) \Rightarrow n\mu_1 + Z_{1-\alpha} \sigma_1 \sqrt{n} = n\mu_2 + Z_\beta \sigma_2 \sqrt{n} \quad (4.77)$$

$$(4.76) \Rightarrow n\mu_1 + Z_{1-\alpha} \sigma_1 \sqrt{n} = n\mu_3 + Z_\gamma \sigma_3 \sqrt{n} \quad (4.78)$$

Each of these expressions can be solved for n .

$$(4.77) \Rightarrow n = \left(\frac{Z_{1-\alpha} \sigma_1 - Z_\beta \sigma_2}{\mu_2 - \mu_1} \right)^2 \quad (4.79)$$

$$(4.78) \Rightarrow n = \left(\frac{Z_{1-\alpha} \sigma_1 - Z_\gamma \sigma_3}{\mu_3 - \mu_1} \right)^2 \quad (4.80)$$

Equating these expressions yields the following.

$$\left(\frac{Z_{1-\alpha}\sigma_1 - Z_\beta\sigma_2}{\mu_2 - \mu_1}\right)^2 = \left(\frac{Z_{1-\alpha}\sigma_1 - Z_\gamma\sigma_3}{\mu_3 - \mu_1}\right)^2$$

$$\left(\frac{Z_{1-\alpha}\sigma_1 - Z_\beta\sigma_2}{\mu_2 - \mu_1}\right)^2 - \left(\frac{Z_{1-\alpha}\sigma_1 - Z_\gamma\sigma_3}{\mu_3 - \mu_1}\right)^2 = 0 \quad (4.81)$$

The left-hand-side of equation (4.81) can be reduced to a polynomial function of v . Numerical evidence proves that this polynomial function does not always provide a feasible solution for v . In some case, the roots of the polynomial function are imaginary and in some cases, the roots are real but do not satisfy the quality value function conditions stated in section 4.1.1. This occurs when the three points specified cannot all be met using the desired type of 3-level acceptance sampling plan. This type of occurrence is demonstrated in the following examples.

Example 4.1: Suppose a consumer wishes to use a 3-AS-S-1(n, T, v) sampling plan. In order to select values for n , T , and v , the consumer has specified the following points on the O.C. curve.

$$P_a(0.945, 0.04, 0.015) = 0.9$$

$$P_a(0.81, 0.125, 0.065) = 0.05$$

$$P_a(0.88, 0.08, 0.04) = 0.5$$

If these specified values of p_1 , p_2 , p_3 , α , β , and γ are used, no solution to equation (4.81) can be found such that $0 < v < 1$.

To investigate this further, the parameter selection method defined in section 4.3 is implemented for $v = 0.1, 0.2, \dots, 0.9$. $P_a(0.88, 0.08, 0.04)$ for the resulting sampling plans can be computed. The results of this investigation are as follows.

ν	n	T	$P_a(0.88,0.08,0.04)$
0.1	92	3.2	0.2807
0.2	73	3	0.3052
0.3	62	3	0.3013
0.4	56	3.1	0.3024
0.5	52	3.2	0.3114
0.6	50	3.5	0.3303
0.7	49	3.7	0.3055
0.8	48	4	0.3252
0.9	48	4.3	0.3022

It appears that if a value of γ in the range (0.28,0.33) had been specified a feasible value of ν would have been found. However, $\gamma = 0.5$ appears to be infeasible.

Example 4.2: Suppose a consumer wishes to use a 3-AS-S-2(n, T, ν) sampling plan. In order to select values for n , T , and ν , the consumer has specified the following points on the O.C. curve.

$$P_a(0.95, 0.03, 0.02) = 0.95$$

$$P_a(0.83, 0.1, 0.07) = 0.1$$

$$P_a(0.92, 0.05, 0.03) = 0.35$$

If these specified values of p_1 , p_2 , p_3 , α , β , and γ are used, no solution to equation (4.81) can be found such that $\nu > 1$.

To investigate this further, the parameter selection method defined in section 4.3 is implemented for several values of ν . $P_a(0.92, 0.05, 0.03)$ for the resulting sampling plans can be computed. The results of this investigation are as follows.

ν	n	T	$P_a(0.92, 0.05, 0.03)$
1.5	60	64.5	0.7498
2	64	75	0.7376
3	72	104	0.7742
5	83	184	0.7605
10	98	545	0.7684
15	105	1150	0.7890
25	112	3115	0.7534
100	121	50000	0.7018

It appears that if a value of γ in the range (0.70,0.79) had been specified a feasible value of ν would have been found. However, $\gamma = 0.35$ appears to be infeasible.

The quality value function specification method for the 3-AS-S-1(n, T, ν) and the 3-AS-S-2(n, T, ν) works as follows. $p_1, p_2, p_3, \alpha, \beta,$ and γ are specified. The polynomial function given in equation (4.81) is solved for ν . If the expression does not provide a feasible solution for ν , then p_3 and γ must be “re-specified.” Once a feasible solution for ν has been found, the parameter selection method defined in section 4.2 is used to find n and T . The capability of the method is evaluated by computing the adjusted values of $\alpha, \beta,$ and γ , and comparing them to the specified values. This method is demonstrated in the following examples.

Example 4.3: Suppose a consumer wishes to use a 3-AS-S-1(n, T, ν) sampling plan. In order to select values for $n, T,$ and ν , the consumer has specified the following points on the O.C. curve.

$$\begin{aligned} P_a(0.945, 0.04, 0.015) &= 0.9 \\ P_a(0.81, 0.125, 0.065) &= 0.05 \\ P_a(0.88, 0.08, 0.04) &= 0.31 \end{aligned}$$

Equation (4.81) is solved and $\nu = 0.9045$.

The parameter selection method is implemented using $\nu = 0.9045$, and the selected parameter values are $n = 48$ and $T = 4.3$.

The capability of the method is evaluated by computing the O.C. function values at the three specified points. The results are as follows.

$$\begin{aligned} P_a(0.945, 0.04, 0.015) &= 0.8774 \\ P_a(0.81, 0.125, 0.065) &= 0.0359 \\ P_a(0.88, 0.08, 0.04) &= 0.3022 \end{aligned}$$

Example 4.4: Suppose a consumer wishes to use a 3-AS-S-2(n, T, v) sampling plan. In order to select values for n , T , and v , the consumer has specified the following points on the O.C. curve.

$$\begin{aligned} P_a(0.95, 0.03, 0.02) &= 0.95 \\ P_a(0.83, 0.1, 0.07) &= 0.1 \\ P_a(0.92, 0.05, 0.03) &= 0.75 \end{aligned}$$

Equation (4.81) is solved and $v = 13.2293$.

The parameter selection method is implemented using $v = 0.9045$, and the selected parameter values are $n = 103$ and $T = 908.532$.

The capability of the method is evaluated by computing the O.C. function values at the three specified points. The results are as follows.

$$\begin{aligned} P_a(0.945, 0.04, 0.015) &= 0.9433 \\ P_a(0.81, 0.125, 0.065) &= 0.0848 \\ P_a(0.88, 0.08, 0.04) &= 0.7901 \end{aligned}$$

Examples 4.1 through 4.4 are representative of the quality value function specification process for all the defined multilevel acceptance sampling plans and multilevel control charts.

4.4.2 Quality Value Function Specification for the General 3-Level Case

Suppose the manufacturer does not even know which type of 3-level acceptance sampling plan to use. In this case, the manufacturer may wish to select the quality value function and the parameters for a 3-AS-S-G(n, T, v_1, v_2, v_3) acceptance sampling plan. To use this type of sampling plan, values of n , T , v_1 , v_2 , and v_3 must be specified. The quality value function selection method defined in section 4.4.1 is extended to consider this situation. The extension begins with the presentation and proof of the following theorem.

Theorem 4.2: Every 3-level acceptance sampling plan (utilizing single sampling and the summation decision rule) is equivalent to either a 3-AS-S-1(n, T, v) sampling plan or a 3-AS-S-G($n, T, 1, v_2, v_3$) sampling plan.

Proof: Consider a 3-AS-S-G(n, T, v_1, v_2, v_3) sampling plan having $n = n_1$ and $T = T_1$ such that $v_1 = 0$. Call this sampling plan #1, and let $V_1(X)$ represent the quality value function utilized by this sampling plan.

Let $v = v_2 / v_3$, $n_2 = n_1$, and $T_2 = T_1 / v_3$.

Consider the 3-AS-S-1($0, v, 1$) sampling plan having $n = n_2$ and $T = T_2$. Call this sampling plan #2, and let $V_2(X)$ represent the quality value function utilized by this sampling plan.

Note that $V_1(X) = bV_2(X)$, and $T_1 = bT_2$, where $b = v_3$.

By Theorem 4.1, sampling plan #1 and sampling plan #2 are equivalent.

Consider a 3-AS-S-G(n, T, v_1, v_2, v_3) sampling plan having $n = n_1$ and $T = T_1$ such that $v_1 > 0$. Call this sampling plan #1, and let $V_1(X)$ represent the quality value function utilized by this sampling plan.

Let $v_{2g} = v_2 / v_1$, $v_{3g} = v_3 / v_1$, $n_2 = n_1$, and $T_2 = T_1 / v_1$.

Consider the 3-AS-S-G($n, T, 1, v_{2g}, v_{3g}$) sampling plan having $n = n_2$ and $T = T_2$. Call this sampling plan #2, and let $V_2(X)$ represent the quality value function utilized by this sampling plan.

Note that $V_1(X) = bV_2(X)$, and $T_1 = bT_2$, where $b = v_1$.

By Theorem 4.1, sampling plan #1 and sampling plan #2 are equivalent.

As a result of Theorem 4.2, all that is necessary to complete the quality value specification method for 3-level acceptance sampling plans is to extend the method defined in section 4.4.1 to the 3-AS-S-G($n, T, 1, v_2, v_3$) sampling plan. In addition, note that a similar theorem could be defined for the product decision rule.

The extension to the quality value function specification method defined in section 4.4.1 begins with the specification of an additional point on the O.C. curve, summarized by $\mathbf{p}_4 = [p_{41} p_{42} p_{43}]$ and η , where $P_a(\mathbf{p}_4) = \eta$. Let μ_4 denote the expected value of V_i when $\mathbf{p} = \mathbf{p}_4$, and let σ_4 denote the standard deviation of V_i when $\mathbf{p} = \mathbf{p}_4$. This additional specification results in the following system of four equations in four unknowns.

$$P_a(p_{11}, p_{12}, p_{13}) = P\left(Z \leq \frac{T - n\mu_1}{\sigma_1 \sqrt{n}}\right) = 1 - \alpha \quad (4.82)$$

$$P_a(p_{21}, p_{22}, p_{23}) = P\left(Z \leq \frac{T - n\mu_2}{\sigma_2 \sqrt{n}}\right) = \beta \quad (4.83)$$

$$P_a(p_{31}, p_{32}, p_{33}) = P\left(Z \leq \frac{T - n\mu_3}{\sigma_3 \sqrt{n}}\right) = \gamma \quad (4.84)$$

$$P_a(p_{41}, p_{42}, p_{43}) = P\left(Z \leq \frac{T - n\mu_4}{\sigma_4 \sqrt{n}}\right) = \eta \quad (4.85)$$

This series of equations can be reduced as follows.

$$(4.82) \Rightarrow Z_{1-\alpha} = \frac{T - n\mu_1}{\sigma_1 \sqrt{n}} \quad (4.86)$$

$$(4.83) \Rightarrow Z_\beta = \frac{T - n\mu_2}{\sigma_2 \sqrt{n}} \quad (4.87)$$

$$(4.84) \Rightarrow Z_\gamma = \frac{T - n\mu_3}{\sigma_3 \sqrt{n}} \quad (4.88)$$

$$(4.85) \Rightarrow Z_\eta = \frac{T - n\mu_4}{\sigma_4 \sqrt{n}} \quad (4.89)$$

Each of these equations can be solved for T .

$$(4.86) \Rightarrow T = n\mu_1 + Z_{1-\alpha} \sigma_1 \sqrt{n} \quad (4.90)$$

$$(4.87) \Rightarrow T = n\mu_2 + Z_\beta \sigma_2 \sqrt{n} \quad (4.91)$$

$$(4.88) \Rightarrow T = n\mu_3 + Z_\gamma \sigma_3 \sqrt{n} \quad (4.92)$$

$$(4.89) \Rightarrow T = n\mu_4 + Z_\eta \sigma_4 \sqrt{n} \quad (4.93)$$

If one of the four expressions for T is substituted into the other three equations, the result is three equations for n . If equation (4.90) is used, then the following expressions for n are obtained.

$$(4.91) \Rightarrow n = \left(\frac{Z_{1-\alpha}\sigma_1 - Z_\beta\sigma_2}{\mu_2 - \mu_1} \right)^2 \quad (4.94)$$

$$(4.92) \Rightarrow n = \left(\frac{Z_{1-\alpha}\sigma_1 - Z_\gamma\sigma_3}{\mu_3 - \mu_1} \right)^2 \quad (4.95)$$

$$(4.93) \Rightarrow n = \left(\frac{Z_{1-\alpha}\sigma_1 - Z_\eta\sigma_4}{\mu_4 - \mu_1} \right)^2 \quad (4.96)$$

If one of the three expressions for n is substituted into the other three, the result is two expressions which can be rewritten as polynomial functions of v_2 , and v_3 . If equation (4.94) is used, then the following expressions are obtained.

$$(4.95) \Rightarrow \left(\frac{Z_{1-\alpha}\sigma_1 - Z_\beta\sigma_2}{\mu_2 - \mu_1} \right)^2 - \left(\frac{Z_{1-\alpha}\sigma_1 - Z_\gamma\sigma_3}{\mu_3 - \mu_1} \right)^2 = 0 \quad (4.97)$$

$$(4.96) \Rightarrow \left(\frac{Z_{1-\alpha}\sigma_1 - Z_\beta\sigma_2}{\mu_2 - \mu_1} \right)^2 - \left(\frac{Z_{1-\alpha}\sigma_1 - Z_\eta\sigma_4}{\mu_4 - \mu_1} \right)^2 = 0 \quad (4.98)$$

The quality value function specification method for the 3-AS-S-G($n, T, 1, v_2, v_3$) sampling plan works as follows. $p_1, p_2, p_3, p_4, \alpha, \beta, \gamma$, and η are specified. The polynomial functions given in equations (4.97) and (4.98) are simultaneously solved for v_2 and v_3 . If the expression does not provide a feasible solution for v_2 and v_3 ($v_3 > v_2 > 1$), then p_3, p_4, γ and η must be “re-specified.” Once a feasible solution for v_2 and v_3 has been found, the parameter selection method defined in section 4.2 is used to find n and T . The capability of the method is evaluated by computing the adjusted values of α, β, γ , and η , and comparing them to the specified values. It is worthwhile to note that finding a feasible solution for v_2 and v_3 can be very time-consuming.

4.4.3 Quality Value Function Specification for 4-Level Acceptance Sampling Plans

Consider first the 4-AS-S-2(n, T, v) sampling plan. To use this sampling plan, values of n, T , and v must be specified. In section 4.4.1, a method is developed for

selecting n , T , and v for the 3-AS-S-1(n, T, v) and the 3-AS-S-2(n, T, v) sampling plans. This method is directly applicable to the 4-AS-S-2(n, T, v) sampling plan.

Consider next the 4-AS-S-1(n, T, v_1, v_2) sampling plan. To use this sampling plan, values of n , T , v_1 , and v_2 must be specified. In section 4.4.2, a method is developed for selecting n , T , v_2 , and v_3 for the 3-AS-S-G($n, T, 1, v_2, v_3$) sampling plan. This method is directly applicable to the 4-AS-S-1(n, T, v_1, v_2) sampling plan with one exception. In this case, equations (4.97) and (4.98) are polynomial functions of v_1 and v_2 .

4.4.4 Quality Value Function Specification for the General 4-Level Case

Suppose the manufacturer does not even know which type of 4-level acceptance sampling plan to use. In this case, the manufacturer may wish to select the quality value function and the parameters for a 4-AS-S-G(n, T, v_1, v_2, v_3, v_4) acceptance sampling plan. To use this type of sampling plan, values of n , T , v_1 , v_2 , v_3 , and v_4 must be specified. The general 3-level quality value function selection method defined in section 4.4.2 is extended to the 4-level case. The extension begins with the presentation of a corollary to Theorem 4.2.

Corollary 4.1: Every 4-level acceptance sampling plan (utilizing single sampling and the summation decision rule) is equivalent to either a 4-AS-S-1(n, T, v_1, v_2) sampling plan or a 4-AS-S-G($n, T, 1, v_2, v_3, v_4$) sampling plan.

The extension to the method defined in section 4.4.2 begins with the specification of five points on the O.C. curve, summarized by $\mathbf{p}_1 = [p_{11} p_{12} p_{13} p_{14}]$, $\mathbf{p}_2 = [p_{21} p_{22} p_{23} p_{24}]$, $\mathbf{p}_3 = [p_{31} p_{32} p_{33} p_{34}]$, $\mathbf{p}_4 = [p_{41} p_{42} p_{43} p_{44}]$, $\mathbf{p}_5 = [p_{51} p_{52} p_{53} p_{54}]$, α , β , γ , η , and φ , where $P_d(\mathbf{p}_1) = 1 - \alpha$, $P_d(\mathbf{p}_2) = \beta$, $P_d(\mathbf{p}_3) = \gamma$, $P_d(\mathbf{p}_4) = \eta$, and $P_d(\mathbf{p}_5) = \varphi$. Let $\mu_1, \mu_2, \mu_3, \mu_4$, and μ_5 denote the expected value of V_i when $\mathbf{p} = \mathbf{p}_1$, $\mathbf{p} = \mathbf{p}_2$, $\mathbf{p} = \mathbf{p}_3$, $\mathbf{p} = \mathbf{p}_4$, and $\mathbf{p} = \mathbf{p}_5$, respectively, and let $\sigma_1, \sigma_2, \sigma_3, \sigma_4$, and σ_5 denote the standard deviation of V_i when $\mathbf{p} = \mathbf{p}_1$, $\mathbf{p} = \mathbf{p}_2$, $\mathbf{p} = \mathbf{p}_3$, $\mathbf{p} = \mathbf{p}_4$, and $\mathbf{p} = \mathbf{p}_5$, respectively. This specification results in the following system of five equations in five unknowns.

$$P_a(p_{11}, p_{12}, p_{13}, p_{14}) = P\left(Z \leq \frac{T - n\mu_1}{\sigma_1 \sqrt{n}}\right) = 1 - \alpha \quad (4.99)$$

$$P_a(p_{21}, p_{22}, p_{23}, p_{24}) = P\left(Z \leq \frac{T - n\mu_2}{\sigma_2 \sqrt{n}}\right) = \beta \quad (4.100)$$

$$P_a(p_{31}, p_{32}, p_{33}, p_{34}) = P\left(Z \leq \frac{T - n\mu_3}{\sigma_3 \sqrt{n}}\right) = \gamma \quad (4.101)$$

$$P_a(p_{41}, p_{42}, p_{43}, p_{44}) = P\left(Z \leq \frac{T - n\mu_4}{\sigma_4 \sqrt{n}}\right) = \eta \quad (4.102)$$

$$P_a(p_{51}, p_{52}, p_{53}, p_{54}) = P\left(Z \leq \frac{T - n\mu_5}{\sigma_5 \sqrt{n}}\right) = \phi \quad (4.103)$$

This series of equations can be reduced as follows.

$$(4.99) \Rightarrow Z_{1-\alpha} = \frac{T - n\mu_1}{\sigma_1 \sqrt{n}} \quad (4.104)$$

$$(4.100) \Rightarrow Z_\beta = \frac{T - n\mu_2}{\sigma_2 \sqrt{n}} \quad (4.105)$$

$$(4.101) \Rightarrow Z_\gamma = \frac{T - n\mu_3}{\sigma_3 \sqrt{n}} \quad (4.106)$$

$$(4.102) \Rightarrow Z_\eta = \frac{T - n\mu_4}{\sigma_4 \sqrt{n}} \quad (4.107)$$

$$(4.103) \Rightarrow Z_\phi = \frac{T - n\mu_5}{\sigma_5 \sqrt{n}} \quad (4.108)$$

Each of these equations can be solved for T .

$$(4.104) \Rightarrow T = n\mu_1 + Z_{1-\alpha} \sigma_1 \sqrt{n} \quad (4.109)$$

$$(4.105) \Rightarrow T = n\mu_2 + Z_\beta \sigma_2 \sqrt{n} \quad (4.110)$$

$$(4.106) \Rightarrow T = n\mu_3 + Z_\gamma \sigma_3 \sqrt{n} \quad (4.111)$$

$$(4.107) \Rightarrow T = n\mu_4 + Z_\eta \sigma_4 \sqrt{n} \quad (4.112)$$

$$(4.108) \Rightarrow T = n\mu_5 + Z_\varphi\sigma_5\sqrt{n} \quad (4.113)$$

If one of the five expressions for T is substituted into the other four equations, the result is four equations for n . If equation (4.109) is used, then the following expressions for n are obtained.

$$(4.110) \Rightarrow n = \left(\frac{Z_{1-\alpha}\sigma_1 - Z_\beta\sigma_2}{\mu_2 - \mu_1} \right)^2 \quad (4.114)$$

$$(4.111) \Rightarrow n = \left(\frac{Z_{1-\alpha}\sigma_1 - Z_\gamma\sigma_3}{\mu_3 - \mu_1} \right)^2 \quad (4.115)$$

$$(4.112) \Rightarrow n = \left(\frac{Z_{1-\alpha}\sigma_1 - Z_\eta\sigma_4}{\mu_4 - \mu_1} \right)^2 \quad (4.116)$$

$$(4.113) \Rightarrow n = \left(\frac{Z_{1-\alpha}\sigma_1 - Z_\varphi\sigma_5}{\mu_5 - \mu_1} \right)^2 \quad (4.117)$$

If one of the four expressions for n is substituted into the other three, the result is three expressions which can be rewritten as polynomial functions of v_2 , v_3 , and v_4 . If equation (4.114) is used, then the following expressions are obtained.

$$(4.115) \Rightarrow \left(\frac{Z_{1-\alpha}\sigma_1 - Z_\beta\sigma_2}{\mu_2 - \mu_1} \right)^2 - \left(\frac{Z_{1-\alpha}\sigma_1 - Z_\gamma\sigma_3}{\mu_3 - \mu_1} \right)^2 = 0 \quad (4.118)$$

$$(4.116) \Rightarrow \left(\frac{Z_{1-\alpha}\sigma_1 - Z_\beta\sigma_2}{\mu_2 - \mu_1} \right)^2 - \left(\frac{Z_{1-\alpha}\sigma_1 - Z_\eta\sigma_4}{\mu_4 - \mu_1} \right)^2 = 0 \quad (4.119)$$

$$(4.117) \Rightarrow \left(\frac{Z_{1-\alpha}\sigma_1 - Z_\beta\sigma_2}{\mu_2 - \mu_1} \right)^2 - \left(\frac{Z_{1-\alpha}\sigma_1 - Z_\varphi\sigma_5}{\mu_5 - \mu_1} \right)^2 = 0 \quad (4.120)$$

The quality value function specification method for the 4-AS-S-G($n, T, 1, v_2, v_3, v_4$) sampling plan works as follows. $p_1, p_2, p_3, p_4, p_5, \alpha, \beta, \gamma, \eta,$ and φ are specified. The polynomial functions given in equations (4.118), (4.119), and (4.120) are simultaneously solved for $v_2, v_3,$ and v_4 . If the expression does not provide a feasible solution for $v_2, v_3,$ and v_4 ($v_4 > v_3 > v_2 > 1$), then $p_3, p_4, p_5, \gamma, \eta,$ and φ must be “re-specified.” Once a feasible solution for

v_2 , v_3 , and v_4 has been found, the parameter selection method defined in section 4.3 is used to find n and T . The capability of the method is evaluated by computing the adjusted values of α , β , γ , η , and φ , and comparing them to the specified values. It is worthwhile to note that finding a feasible solution for v_2 , v_3 , and v_4 can be very time-consuming.

CHAPTER 5: MULTILEVEL STATISTICAL PROCESS CONTROL

The second area of multilevel statistical quality control addressed in this research is multilevel statistical process control. The chapter begins with the definition of the multilevel control charts that are considered. The second section contains the development and evaluation of the O.C. function and the ARL function for each of the defined multilevel control charts. In the third section, a method for selecting the parameters of a multilevel control chart is defined, demonstrated, and evaluated. The problem of quality value function specification is addressed in the fourth section. In each section, the topic of interest is motivated using the general j -level case followed by detailed discussions using the 3-level and 4-level cases.

5.1 MULTILEVEL CONTROL CHART OPERATION AND SIGNALING RULES

In the preceding chapters, the scenario for which multilevel statistical process control is most appropriate is defined. Suppose a manufacturer is interested in monitoring the quality of a product generated by some part of a production process. Suppose in addition, that the quality of this product is best described using three or more discrete levels. In such a situation, the supplier may wish to utilize a multilevel control chart to monitor the status of the process.

As with multilevel acceptance sampling plans, the initial decision required for implementing a multilevel control chart is the selection of a multilevel discrete product quality measure X . As mentioned previously, the multilevel discrete product quality measure used in this research is $x_k = k$, $k = 1, 2, \dots, j$. The next step in the implementation of a multilevel control chart is the selection of a quality value function. The quality value functions considered for multilevel SPC are the same as those considered for multilevel

acceptance sampling. Once a quality value function is specified, the resulting multilevel control chart can be implemented.

As described in Chapter 3, the sampling strategy utilized in the multilevel control charts addressed here is fixed sampling interval single sampling. The sample statistic considered in this research is the average quality value. The computation of this sample statistic proceeds as follows.

At sample point t , inspect a random sample of n finished items from the process. Let X_{it} = the quality measure of item i , and let V_{it} = the quality value of item i .

Compute the sample statistic
$$\bar{V}_t = \sum_{i=1}^n \frac{V_{it}}{n}.$$

The sample statistic is then plotted on the control chart (see Figure 5.1). If the value of the sample statistic lies between the lower and upper control limits, LCL_V and UCL_V , then the process is concluded to be in a state of statistical control. Otherwise, a signal of an out-of-control situation is made, and a search for the cause of the out-of-control condition is initiated. Note that CL refers to the center line of the control chart.

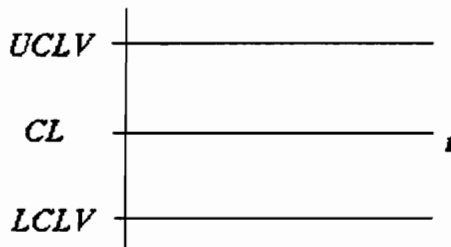


Figure 5.1
Layout of a Multilevel Control Chart

The multilevel control charts considered in this research are Shewhart control charts. The implications for computing the control limits for the charts are as follows.

Recall that in a statistical process control setting, the state of the process (as it relates to product quality) is assumed to be described by $\mathbf{p} = [p_1 p_2 \dots p_j]$ where p_k represents the probability that a produced item falls into quality classification k , $k = 1, 2, \dots, j$. The process is said to be in a state of statistical control if $\mathbf{p} = \mathbf{p}_0 = [p_{01} p_{02} \dots p_{0j}]$. It is commonly assumed that $p_{01}, p_{02}, \dots, p_{0j}$ represent target proportions for each of the quality classifications. Let μ_{V_0} denote the expected value of V_{ii} when the process is in control, and let σ_{V_0} denote the standard deviation of V_{ii} when the process is in control.

Then

$$\mu_{V_0} = \sum_{k=1}^j v_k p_{0k} \quad (5.1),$$

$$\sigma_{V_0} = \sqrt{\sum_{k=1}^j v_k^2 p_{0k} - \mu_{V_0}^2} \quad (5.2),$$

$$E(\bar{V}_i) = \mu_{V_0} \quad (5.3),$$

and

$$\sqrt{Var(\bar{V}_i)} = \frac{\sigma_{V_0}}{\sqrt{n}} \quad (5.4).$$

Thus, the control limits for the multilevel control charts considered here are given by

$$UCL_V = \mu_{V_0} + k_V \frac{\sigma_{V_0}}{\sqrt{n}} \quad (5.5),$$

and

$$LCL_V = \mu_{V_0} - k_V \frac{\sigma_{V_0}}{\sqrt{n}} \quad (5.6)$$

where k_V is some positive constant. Note that if the lower control limit is negative, then it is automatically set to zero and no sample points can fall below it. In addition, note that $CL = \mu_{V_0}$.

An example multilevel control chart is presented in Figure 5.1. Note that two types of signals may occur when using such a control chart. The first signal is caused by a sample statistic falling above the upper control limit. When this type of signal occurs, the

conclusion is made that the quality of the process has degraded. Thus an attempt is made to find and remove the cause of the degradation in quality. The second signal is caused by a sample statistic falling below the lower control limit. This type of signal leads to the conclusion that the quality of the process has improved. If this improvement is real, then the manufacturer does not wish to remove the cause, but they do wish to discover, understand, and benefit from the cause.

5.1.1 3-level Control Charts

In the preceding section, the general framework for multilevel statistical process control is established. This framework is applicable to the 3-level case. In this section, the specific quality value functions considered in the 3-level statistical process control portion of the research are defined. As mentioned earlier, the choice of the quality value function completes the definition of the multilevel control chart. Thus each quality value function (or type of 3-level control chart) is given a reference code to be used in the remainder of this research. This code is of the form j -SPC-AVG-Y(\bullet) where j indicates the number of levels used by the multilevel discrete product quality measure, SPC indicates that it is a Statistical Process Control chart, AVG indicates that the AVeraGe quality value sample statistic is used, Y is a reference number or letter, and the control chart parameters and quality value function assignments are specified within the parentheses. When referring to 3-level control charts, the following descriptions are given to the three quality classifications: conforming, marginal, nonconforming. The state of the process is defined by $\mathbf{p} = [p_1 \ p_2 \ p_3]$ where p_1 represents the probability that a produced item is conforming, p_2 represents the probability that a produced item is marginal, and p_3 represents the probability that a produced item is nonconforming. The process is said to be in a state of statistical control when $\mathbf{p} = \mathbf{p}_\bullet = [p_{01} \ p_{02} \ p_{03}]$. The 3-level control charts considered are presented below.

3-SPC-AVG-1(n, k, ν)

Quality Value Function Conditions: $0 < \nu < 1$

$$X_n = \begin{cases} 1 & \text{if item } i \text{ is conforming} \\ 2 & \text{if item } i \text{ is marginal} \\ 3 & \text{if item } i \text{ is nonconforming} \end{cases} \quad i = 1, 2, \dots, n$$

$$V_n = \begin{cases} 0 & \text{if } X_n = 1 \\ \nu & \text{if } X_n = 2 \\ 1 & \text{if } X_n = 3 \end{cases} \quad i = 1, 2, \dots, n$$

$$\mu_{\nu 0} = \nu p_{02} + p_{03}$$
$$\sigma_{\nu 0} = \sqrt{\nu^2 p_{02} + p_{03} - (\nu p_{02} + p_{03})^2}$$

3-SPC-AVG-2(n, k, ν)

Quality Value Function Conditions: $\nu > 1$

$$X_n = \begin{cases} 1 & \text{if item } i \text{ is conforming} \\ 2 & \text{if item } i \text{ is marginal} \\ 3 & \text{if item } i \text{ is nonconforming} \end{cases} \quad i = 1, 2, \dots, n$$

$$V_n = \begin{cases} 1 & \text{if } X_n = 1 \\ \nu & \text{if } X_n = 2 \\ \nu^2 & \text{if } X_n = 3 \end{cases} \quad i = 1, 2, \dots, n$$

$$\mu_{\nu 0} = p_{01} + \nu p_{02} + \nu^2 p_{03}$$
$$\sigma_{\nu 0} = \sqrt{p_{01} + \nu^2 p_{02} + \nu^4 p_{03} - (p_{01} + \nu p_{02} + \nu^2 p_{03})^2}$$

3-SPC-AVG-G(n, k, v_1, v_2, v_3)

Note that this is the general 3-level case.

Quality Value Function Conditions: $0 \leq v_1 < v_2 < v_3$

$$X_{ii} = \begin{cases} 1 & \text{if item } i \text{ is conforming} \\ 2 & \text{if item } i \text{ is marginal} \\ 3 & \text{if item } i \text{ is nonconforming} \end{cases} \quad i = 1, 2, \dots, n$$

$$V_{ii} = \begin{cases} v_1 & \text{if } X_{ii} = 1 \\ v_2 & \text{if } X_{ii} = 2 \\ v_3 & \text{if } X_{ii} = 3 \end{cases} \quad i = 1, 2, \dots, n$$

$$\mu_{v_0} = v_1 p_{01} + v_2 p_{02} + v_3 p_{03}$$
$$\sigma_{v_0} = \sqrt{v_1^2 p_{01} + v_2^2 p_{02} + v_3^2 p_{03} - (v_1 p_{01} + v_2 p_{02} + v_3 p_{03})^2}$$

5.1.2 4-level Control Charts

The general framework for multilevel statistical process control is also applicable to the 4-level case. In this section, the specific quality value functions considered in the 4-level statistical process control portion of the research are defined. The choice of the quality value function completes the definition of the multilevel control chart, and thus each quality value function (or type of 4-level control chart) is given a reference code to be used in the remainder of this research. The form of the reference code is identical to the form used for 3-level control charts. When referring to 4-level control charts, the following descriptions are given to the four quality classifications: conforming, minor nonconforming, major nonconforming, and critical nonconforming. The state of the process is defined by $\mathbf{p} = [p_1 p_2 p_3 p_4]$ where p_1 represents the probability that a produced item is conforming, p_2 represents the probability that a produced item is minor nonconforming, p_3 represents the probability that a produced item is major nonconforming, and p_4 represents the probability that a produced item is critical

nonconforming. The process is said to be in a state of statistical control when $\mathbf{p} = \mathbf{p}_0 = [p_{01} p_{02} p_{03} p_{04}]$. The 4-level control charts considered are presented below.

4-SPC-AVG-1(n, k, v_1, v_2)

Quality Value Function Conditions: $0 < v_1 < v_2 < 1$

$$X_n = \begin{cases} 1 & \text{if item } i \text{ is conforming} \\ 2 & \text{if item } i \text{ is minor nonconforming} \\ 3 & \text{if item } i \text{ is major nonconforming} \\ 4 & \text{if item } i \text{ is critical nonconforming} \end{cases} \quad i = 1, 2, \dots, n$$

$$V_n = \begin{cases} 0 & \text{if } X_n = 1 \\ v_1 & \text{if } X_n = 2 \\ v_2 & \text{if } X_n = 3 \\ 1 & \text{if } X_n = 4 \end{cases} \quad i = 1, 2, \dots, n$$

$$\mu_{v_0} = v_1 p_{02} + v_2 p_{03} + p_{04}$$

$$\sigma_{v_0} = \sqrt{v_1^2 p_{02} + v_2^2 p_{03} + p_{04} - (v_1 p_{02} + v_2 p_{03} + p_{04})^2}$$

4-SPC-AVG-2(n, k, v)

Quality Value Function Conditions: $v > 1$

$$X_n = \begin{cases} 1 & \text{if item } i \text{ is conforming} \\ 2 & \text{if item } i \text{ is minor nonconforming} \\ 3 & \text{if item } i \text{ is major nonconforming} \\ 4 & \text{if item } i \text{ is critical nonconforming} \end{cases} \quad i = 1, 2, \dots, n$$

$$V_n = \begin{cases} 1 & \text{if } X_n = 1 \\ v & \text{if } X_n = 2 \\ v^2 & \text{if } X_n = 3 \\ v^3 & \text{if } X_n = 4 \end{cases} \quad i = 1, 2, \dots, n$$

$$\mu_{v_0} = p_{01} + v p_{02} + v^2 p_{03} + v^3 p_{04}$$

$$\sigma_{v_0} = \sqrt{p_{01} + v^2 p_{02} + v^4 p_{03} + v^6 p_{04} - (p_{01} + v p_{02} + v^2 p_{03} + v^3 p_{04})^2}$$

4-SPC-AVG-G(n, k, v_1, v_2, v_3)

Note that this is the general 4-level case.

Quality Value Function Conditions: $0 \leq v_1 < v_2 < v_3 < v_4$

$$X_n = \begin{cases} 1 & \text{if item } i \text{ is conforming} \\ 2 & \text{if item } i \text{ is minor nonconforming} \\ 3 & \text{if item } i \text{ is major nonconforming} \\ 4 & \text{if item } i \text{ is critical nonconforming} \end{cases} \quad i = 1, 2, \dots, n$$

$$V_n = \begin{cases} v_1 & \text{if } X_n = 1 \\ v_2 & \text{if } X_n = 2 \\ v_3 & \text{if } X_n = 3 \\ v_4 & \text{if } X_n = 4 \end{cases} \quad i = 1, 2, \dots, n$$

$$\mu_{V_0} = v_1 p_{01} + v_2 p_{02} + v_3 p_{03} + v_4 p_{04}$$

$$\sigma_{V_0} = \sqrt{v_1^2 p_{01} + v_2^2 p_{02} + v_3^2 p_{03} + v_4^2 p_{04} - (v_1 p_{01} + v_2 p_{02} + v_3 p_{03} + v_4 p_{04})^2}$$

5.2 MULTILEVEL CONTROL CHART O.C. FUNCTION AND ARL FUNCTION BEHAVIOR

The most common measure used for evaluating the behavior of a fixed sampling interval control chart is the ARL function. For control charts (such as those considered in this research) having sample statistics that utilize only the information from the current sample, the ARL function can easily be expressed in terms of the O.C. function. In this section, the O.C. function and ARL function for the multilevel control charts defined in section 5.1 are constructed and evaluated. The O.C. and ARL functions for the general j -level case are considered first. Then the O.C. and ARL functions for each of the defined 3-level and 4-level control charts are constructed.

Before the construction of the O.C. and ARL functions for the general j -level case, it is worthwhile to recall that the state of a process is described by $\mathbf{p} = [p_1 \ p_2 \ \dots \ p_j]$ where p_k represents the probability that a produced item falls into quality classification k , $k = 1, 2,$

... , j . Let n denote the sample size for the multilevel control chart, and let N_k denote the number of items in the sample that fall into quality classification k , $k = 1, 2, \dots, j$. Then

$$P(N_1 = n_1, N_2 = n_2, \dots, N_j = n_j) = \begin{cases} \frac{n!}{n_1! n_2! \dots n_j!} p_1^{n_1} p_2^{n_2} \dots p_j^{n_j} & \text{if } \sum_{k=1}^j n_k = n \\ 0 & \text{otherwise} \end{cases} \quad (5.7).$$

Let $m(n_1, n_2, \dots, n_j, p_1, p_2, \dots, p_j) = P(N_1 = n_1, N_2 = n_2, \dots, N_j = n_j)$. Recall that the O.C. function is given by $P_a(\mathbf{p})$, where $P_a(\mathbf{p})$ denotes the probability that the sample statistic falls within the control limits given that the state of the process is \mathbf{p} . The signaling rule can be restated in terms of N_1, N_2, \dots, N_j and the assignments made by the quality value function v_1, v_2, \dots, v_j . In particular, no signal is made if

$$LCL_v \leq \frac{\sum_{k=1}^j N_k v_k}{n} \leq UCL_v \quad (5.8)$$

or equivalently if

$$nLCL_v \leq \sum_{k=1}^j N_k v_k \leq nUCL_v \quad (5.9).$$

This relationship can be used to complete the construction of the O.C. function. For specified values of n , k_v , and \mathbf{p}_0 let

$$m_a(n_1, n_2, \dots, n_j, p_1, p_2, \dots, p_j) = \begin{cases} m(n_1, n_2, \dots, n_j, p_1, p_2, \dots, p_j) & \text{if } nLCL_v \leq \sum_{k=1}^j n_k v_k \leq nUCL_v \\ 0 & \text{otherwise} \end{cases} \quad (5.10).$$

This leads to the result that

$$P_a(p_1, p_2, \dots, p_j) = \sum_{n_1=0}^n \sum_{n_2=0}^n \dots \sum_{n_j=0}^n m_a(n_1, n_2, \dots, n_j, p_1, p_2, \dots, p_j) \quad (5.11).$$

However, this expression can be simplified by recognizing that $N_1 + N_2 + \dots + N_j = n$. Accounting for this fact leads to the following expression for the O.C. function for multilevel control charts.

$$P_a(p_1, p_2, \dots, p_j) = \sum_{n_1=0}^n \sum_{n_2=0}^{n-n_1} \dots \sum_{n_{j-1}=0}^{n-n_1-n_2-\dots-n_{j-2}} m_a(n_1, n_2, \dots, n_{j-1}, n - n_1 - n_2 - \dots - n_{j-1}, p_1, p_2, \dots, p_j) \quad (5.12).$$

The ARL function, $ARL(\mathbf{p})$, denotes the expected number of samples until a signal given that the state of the process is constant at \mathbf{p} . Since the multilevel control charts considered in this research have sample statistics that only utilize the information from the current sample, the number of samples until a signal given that the state of the process is constant at \mathbf{p} is a geometric random variable with probability, $1 - P_a(\mathbf{p})$. Thus the ARL function is given by

$$ARL(p_1, p_2, \dots, p_j) = \frac{1}{1 - P_a(p_1, p_2, \dots, p_j)} \quad (5.13).$$

Thus specification of the O.C. function is sufficient for describing O.C. and ARL function behavior.

5.2.1 O.C. Function and ARL Function Behavior of 3-level Control Charts

If the development used in the preceding section is followed, the O.C. function and ARL function for each of the 3-level control charts defined in section 5.1.1 can be constructed. In this case, the state of the process is described by $\mathbf{p} = [p_1 \ p_2 \ p_3]$ where p_1 denotes the probability that a produced item is conforming, p_2 denotes the probability that a produced item is marginal, and p_3 denotes the probability that a produced item is nonconforming. For a specified sample size n , N_1 denotes the number of sample items that are conforming, N_2 denotes the number of sample items that are marginal, and N_3 denotes

the number of sample items that are nonconforming. Since the sample data is appropriately modeled using the multinomial distribution,

$$m(n_1, n_2, n_3, p_1, p_2, p_3) = \begin{cases} \frac{n!}{n_1! n_2! n_3!} p_1^{n_1} p_2^{n_2} p_3^{n_3} & \text{if } n_1 + n_2 + n_3 = n \\ 0 & \text{otherwise} \end{cases} \quad (5.14).$$

Thus given values for n , k_v , and \mathbf{p}_0 , each O.C. function and ARL function can be constructed as follows.

3-SPC-AVG-1(n, k_v, v)

Signaling Rule: no signal if $nLCL_v \leq N_2v + N_3 \leq nUCL_v$

$$m_a(n_1, n_2, n_3, p_1, p_2, p_3) = \begin{cases} m(n_1, n_2, n_3, p_1, p_2, p_3) & \text{if } nLCL_v \leq n_2v + n_3 \leq nUCL_v \\ 0 & \text{otherwise} \end{cases}$$

3-SPC-AVG-2(n, k_v, v)

Signaling Rule: no signal if $nLCL_v \leq N_1 + N_2v + N_3v^2 \leq nUCL_v$

$$m_a(n_1, n_2, n_3, p_1, p_2, p_3) = \begin{cases} m(n_1, n_2, n_3, p_1, p_2, p_3) & \text{if } nLCL_v \leq n_1 + n_2v + n_3v^2 \leq nUCL_v \\ 0 & \text{otherwise} \end{cases}$$

3-SPC-AVG-G(n, k_v, v_1, v_2, v_3)

Signaling Rule: no signal if $nLCL_v \leq N_1v_1 + N_2v_2 + N_3v_3 \leq nUCL_v$

$$m_a(n_1, n_2, n_3, p_1, p_2, p_3) = \begin{cases} m(n_1, n_2, n_3, p_1, p_2, p_3) & \text{if } nLCL_v \leq n_1v_1 + n_2v_2 + n_3v_3 \leq nUCL_v \\ 0 & \text{otherwise} \end{cases}$$

Having constructed the conditional multinomial expressions, the O.C. function for all the 3-level control charts can be written as

$$P_a(p_1, p_2, p_3) = \sum_{n_1=0}^n \sum_{n_2=0}^{n-n_1} m_a(n_1, n_2, n-n_1-n_2, p_1, p_2, p_3) \quad (5.15),$$

and the ARL function can be written as

$$ARL(p_1, p_2, p_3) = \frac{1}{1 - P_a(p_1, p_2, p_3)} \quad (5.16).$$

Computation of O.C. and ARL function values for any of the 3-level control charts is quite straightforward. However, presentation of this behavior in a meaningful way is not so straightforward. As with 3-level acceptance sampling plans, there are three useful methods of presenting the O.C. function (and thus ARL function) behavior of 3-level control charts: (1) tables, (2) 3-dimensional O.C. (or ARL) curves, and (3) contour plots of O.C. (or ARL) function values. Examples of each of these methods are provided for the 3-SPC-AVG-1(46,2.5758,0.25) control chart having $\mathbf{p}_0 = [0.97 \ 0.02 \ 0.01]$. In Table 5.1, a table of O.C. function and ARL function values for this control chart is given. In Figure 5.2, the 3-dimensional O.C. curve for the control chart is given. Note that the values on the x -axis correspond to values of p_2 and the values on the y -axis correspond to the values of p_3 . Since p_1 is implied by the specification of p_2 and p_3 ($p_1 = 1 - p_2 - p_3$), this is sufficient for describing the complete O.C. function. In Figure 5.3, the 3-dimensional ARL curve for the control chart is given. In Figure 5.4, a contour plot of O.C. function values for the control chart is given. As with Figure 5.2, the values on the x -axis correspond to values of p_2 and the values on the y -axis correspond to the values of p_3 . In Figure 5.5, a contour plot of ARL function values for the control chart is given. Each of these methods has its own advantages and disadvantages. The tables provide specific O.C. (or ARL) function values, but they provide only a limited description of overall behavior. The 3-dimensional curves do not provide specific values, but they do provide a basic description of overall control chart behavior. The contour plots can be used to identify some specific values (however, not as easily as the tables), and to provide a basic description of overall control chart behavior (but perhaps not as clearly as the 3-dimensional O.C. or ARL curves).

Table 5.1
O.C. and ARL Function Values for the 3-SPC-AVG-1(46,2.5758,0.25) Control Chart Having $p_0 = [0.97 \ 0.02 \ 0.01]$

p_2	p_3	$P_a(p_1, p_2, p_3)$	$ARL(p_1, p_2, p_3)$
0.02	0.01	0.9850	66.8
0.04	0.01	0.9709	34.4
0.04	0.02	0.8894	9.0
0.04	0.03	0.7734	4.4
0.06	0.01	0.9502	20.1
0.06	0.02	0.8418	6.3
0.06	0.03	0.7075	3.4
0.06	0.04	0.5706	2.3
0.06	0.05	0.4450	1.8
0.08	0.01	0.9219	12.8
0.08	0.02	0.7885	4.7
0.08	0.03	0.6402	2.8
0.08	0.04	0.4998	2.0
0.08	0.05	0.3782	1.6
0.10	0.01	0.8820	8.5
0.10	0.02	0.7272	3.7
0.10	0.03	0.5710	2.3
0.10	0.04	0.4323	1.8
0.10	0.05	0.3181	1.5

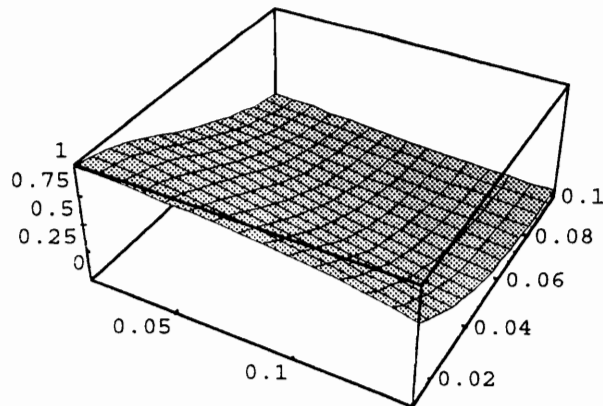


Figure 5.2
O.C. Curve for the 3-SPC-AVG-1(46,2.5758,0.25) Control Chart Having $p_0 = [0.97 \ 0.02 \ 0.01]$

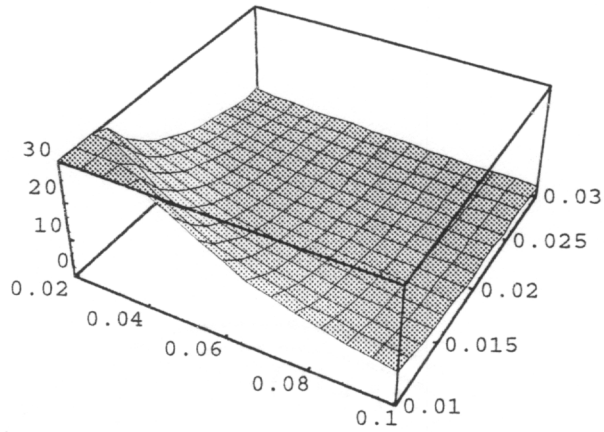


Figure 5.3
ARL Curve for the 3-SPC-AVG-1(46,2.5758,0.25) Control Chart Having
 $p_0 = [0.97 \ 0.02 \ 0.01]$

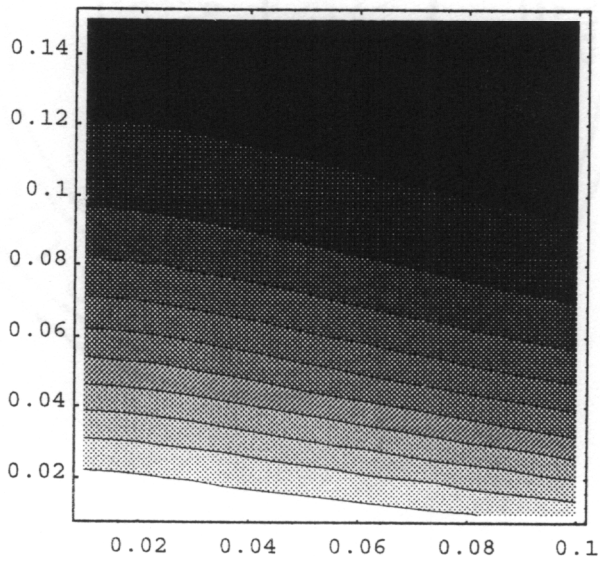


Figure 5.4
Contour Plot of O.C. Function Values for the 3-SPC-AVG-1(46,2.5758,0.25) Control
Chart Having $p_0 = [0.97 \ 0.02 \ 0.01]$

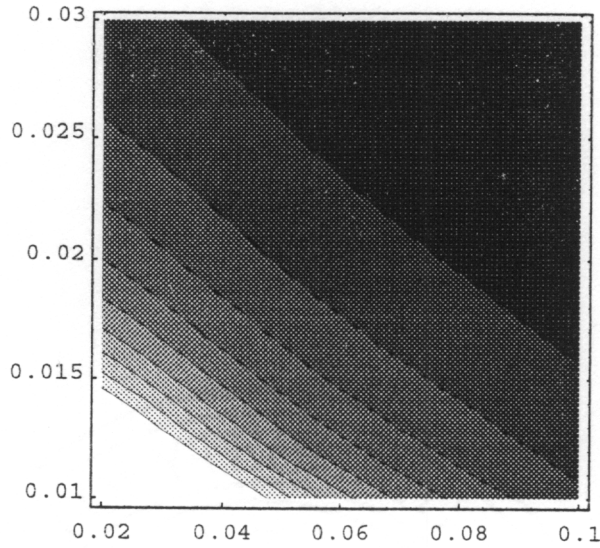


Figure 5.5
Contour Plot of ARL Function Values for the 3-SPC-AVG-1(46,2.5758,0.25) Control Chart Having $p_0 = [0.97 \ 0.02 \ 0.01]$

5.2.2 O.C. Function and ARL Function Behavior for 4-level Control Charts

In this section, the O.C. function and ARL function for each of the 4-level control charts defined in section 5.1.2 is constructed. In the 4-level case, the state of the process is given by $\mathbf{p} = [p_1 \ p_2 \ p_3 \ p_4]$ where p_1 denotes the probability that a produced item is conforming, p_2 denotes the probability that a produced item is minor nonconforming, p_3 denotes the probability that a produced item is major nonconforming, and p_4 denotes the probability that a produced item is critical nonconforming. For a specified sample size n , N_1 denotes the number of sample items that are conforming, N_2 denotes the number of sample items that are minor nonconforming, N_3 denotes the number of sample items that are major nonconforming, and N_4 denotes the number of sample items that are critical nonconforming. Using the multinomial model of sample data,

$$\begin{aligned}
& m(n_1, n_2, n_3, n_4, p_1, p_2, p_3, p_4) \\
& = \begin{cases} \frac{n!}{n_1! n_2! n_3! n_4!} p_1^{n_1} p_2^{n_2} p_3^{n_3} p_4^{n_4} & \text{if } n_1 + n_2 + n_3 + n_4 = n \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{5.17}.$$

Given values for n , k_v , and \mathbf{p}_0 , each O.C. function and ARL function can be constructed as follows.

4-SPC-AVG-1(n, k_v, v_1, v_2)

Signaling Rule: no signal if $nLCL_v \leq N_2 v_1 + N_3 v_2 + N_4 \leq nUCL_v$

$$m_a(n_1, n_2, n_3, n_4, p_1, p_2, p_3, p_4)$$

$$= \begin{cases} m(n_1, n_2, n_3, n_4, p_1, p_2, p_3, p_4) & \text{if } nLCL_v \leq n_2 v_1 + n_3 v_2 + n_4 \leq nUCL_v \\ 0 & \text{otherwise} \end{cases}$$

4-SPC-AVG-2(n, k_v, v)

Signaling Rule: no signal if $nLCL_v \leq N_1 + N_2 v + N_3 v^2 + N_4 v^3 \leq nUCL_v$

$$m_a(n_1, n_2, n_3, n_4, p_1, p_2, p_3, p_4)$$

$$= \begin{cases} m(n_1, n_2, n_3, n_4, p_1, p_2, p_3, p_4) & \text{if } nLCL_v \leq n_1 + n_2 v + n_3 v^2 + n_4 v^3 \leq nUCL_v \\ 0 & \text{otherwise} \end{cases}$$

4-SPC-AVG-G(n, k_v, v_1, v_2, v_3)

Signaling Rule: no signal if $nLCL_v \leq N_1 v_1 + N_2 v_2 + N_3 v_3 + N_4 v_4 \leq nUCL_v$

$$m_a(n_1, n_2, n_3, n_4, p_1, p_2, p_3, p_4)$$

$$= \begin{cases} m(n_1, n_2, n_3, n_4, p_1, p_2, p_3, p_4) & \text{if } nLCL_v \leq n_1 v_1 + n_2 v_2 + n_3 v_3 + n_4 v_4 \leq nUCL_v \\ 0 & \text{otherwise} \end{cases}$$

Having constructed the conditional multinomial expressions, the O.C. function for all the 4-level control charts can be written as

$$P_a(p_1, p_2, p_3, p_4) = \sum_{n_1=0}^n \sum_{n_2=0}^{n-n_1} \sum_{n_3=0}^{n-n_1-n_2} m_a(n_1, n_2, n_3, n-n_1-n_2-n_3, p_1, p_2, p_3, p_4) \quad (5.18),$$

and the ARL function can be written as

$$ARL(p_1, p_2, p_3, p_4) = \frac{1}{1 - P_a(p_1, p_2, p_3, p_4)} \quad (5.19).$$

Computation of the O.C. function and ARL function for any of the 4-level control charts is quite straightforward. However unlike 3-level control charts, presentation of the O.C. and ARL function behavior for a 4-level control charts in a meaningful way is limited to tables.

5.2.3 Computing the O.C. Function for 3-level and 4-level Control Charts

The current form of the O.C. function (and thus the ARL function) for multilevel control charts is computationally inefficient. In this section, the computational efficiency of the current form of the O.C. function is discussed, and a new equivalent form is presented which is much more efficient.

The original form of the O.C. function for the multilevel control charts defined here is given by

$$P_a(p_1, p_2, \dots, p_j) = \sum_{n_1=0}^n \sum_{n_2=0}^n \dots \sum_{n_j=0}^n m_a(n_1, n_2, \dots, n_j, p_1, p_2, \dots, p_j) \quad (5.20).$$

It is easy to recognize the computational inefficiency of this form of the O.C. function. For a given value of n , $(n+1)^j$ evaluations of the function m_a are required. However, a simplified form of the O.C. function is presented. This simplification is based on the fact that the number of observed sample items in each category, N_1, N_2, \dots, N_j must sum to the sample size n . The resulting final form of the O.C. function is given by

$$\begin{aligned}
P_a(p_1, p_2, \dots, p_j) &= \sum_{n_1=0}^n \sum_{n_2=0}^{n-n_1} \dots \sum_{n_{j-1}=0}^{n-n_1-n_2-\dots-n_{j-2}} m_a(n_1, n_2, \dots, n_{j-1}, n-n_1-n_2-\dots-n_{j-1}, p_1, p_2, \dots, p_j)
\end{aligned} \tag{5.21}$$

In the 3-level case, this form of the O.C. function is given by

$$P_a(p_1, p_2, p_3) = \sum_{n_1=0}^n \sum_{n_2=0}^{n-n_1} m_a(n_1, n_2, n-n_1-n_2, p_1, p_2, p_3) \tag{5.22},$$

and in the 4-level case, it is given by

$$P_a(p_1, p_2, p_3, p_4) = \sum_{n_1=0}^n \sum_{n_2=0}^{n-n_1} \sum_{n_3=0}^{n-n_1-n_2} m_a(n_1, n_2, n_3, n-n_1-n_2-n_3, p_1, p_2, p_3, p_4) \tag{5.23}.$$

Although this form of the O.C. is more efficient than the previous, it is still inefficient for relatively large sample sizes. In the 3-level case, computing the O.C. function for a control chart having sample size n requires $(1/2)(n+1)(n+2)$ evaluations of the function m_a . The computational inefficiency of the current O.C. function expression lies in the order of summation (N_1 followed by N_2 , followed by N_3). By taking advantage of the signaling rule for the defined 3-level control charts, a more efficient form can be created. Recall the general signal rule for the defined 3-level control charts:

$$\text{no signal if } nLCL_v \leq N_1v_1 + N_2v_2 + N_3v_3 \leq nUCL_v \tag{5.24}.$$

The new form of the O.C. function is constructed by considering the implications of the signaling rule for the allowed number of observed nonconforming and marginal sample items. For nonconforming items in the sample, the signaling rule implies that

$$N_3v_3 \leq nUCL_v \tag{5.25}.$$

Thus in order for their not to be a signal, the number of nonconforming items in the sample must fall below some value. Specifically,

$$N_3 \leq \frac{nUCL_v}{v_3} \tag{5.26}.$$

Since N_3 is an integer and cannot be larger than the sample size, a threshold value for N_3 can be defined. This threshold value is

$$N_3^{\max} = \min\left(n, \left\lfloor \frac{nUCL_V}{v_3} \right\rfloor\right) \quad (5.27).$$

Given that $N_3 = n_3$, a limit can also be placed on the number of marginal sample items by recognizing that

$$N_2v_2 + n_3v_3 \leq nUCL_V \quad (5.28),$$

which can be rewritten as

$$N_2 \leq \frac{nUCL_V - n_3v_3}{v_2} \quad (5.29).$$

Since N_2 is an integer and the sum of N_2 and N_3 cannot exceed the sample size, a threshold value for N_2 can be defined. This threshold value is

$$N_2^{\max} = \min\left(n - n_3, \left\lfloor \frac{nUCL_V - n_3v_3}{v_2} \right\rfloor\right) \quad (5.30).$$

These thresholds can be used to define a new form of the O.C. function. For the 3-level case, this revised version of the O.C. function is given by

$$P_a(p_1, p_2, p_3) = \sum_{n_3=0}^{N_3^{\max}} \sum_{n_2=0}^{N_2^{\max}} m_a(n - n_2 - n_3, n_2, n_3, p_1, p_2, p_3) \quad (5.31).$$

These thresholds can be described specifically for each of the defined types of 3-level control charts.

3-SPC-AVG-1(n, k_V, v)

$$N_3^{\max} = \min\left(n, \lfloor nUCL_V \rfloor\right)$$

$$N_2^{\max} = \min\left(n - n_3, \left\lfloor \frac{nUCL_V - n_3}{v} \right\rfloor\right)$$

3-SPC-AVG-2(n, k, v, v)

$$N_3^{\max} = \min\left(n, \left\lfloor \frac{nUCL_v}{v^2} \right\rfloor\right)$$

$$N_2^{\max} = \min\left(n - n_3, \left\lfloor \frac{nUCL_v - n_3 v_2}{v} \right\rfloor\right)$$

The improvement in computational efficiency resulting from revising the form of the O.C. function can be demonstrated using several examples. The improvements obtained in these examples are presented in Table 5.2.

Table 5.2
Examples of Achieved Savings in 3-Level O.C. Function Computation

Control Chart	Evaluations of m_a Required by Equation (5.20)	Evaluations of m_a Required by Equation (5.22)	Evaluations of m_a Required by Equation (5.31)
3-SPC-AVG-1(34,2.5758,0.25) $\mathbf{p}_0 = [0.975 \ 0.02 \ 0.005]$	42875	630	12
3-SPC-AVG-1(63,2.8782,0.5) $\mathbf{p}_0 = [0.965 \ 0.03 \ 0.005]$	262144	2080	16
3-SPC-AVG-1(22,2.8782,0.75) $\mathbf{p}_0 = [0.975 \ 0.02 \ 0.005]$	12167	276	3
3-SPC-AVG-2(44,2.5758,2) $\mathbf{p}_0 = [0.965 \ 0.03 \ 0.005]$	91125	1035	182
3-SPC-AVG-2(94,2.8782,5) $\mathbf{p}_0 = [0.965 \ 0.03 \ 0.005]$	857375	4560	133
3-SPC-AVG-1(162,2.8782,10) $\mathbf{p}_0 = [0.96 \ 0.03 \ 0.01]$	4330747	13366	312

In the 4-level case, computing the O.C. function for a control chart having sample size n requires

$$\sum_{k=1}^{n+1} (1/2)j(j+1) \tag{5.32}$$

evaluations of the function m_a . Again by taking advantage of the signaling rule for the defined 4-level control charts, a more efficient form of the O.C. function can be created

Recall the general signaling rule for the defined 4-level control charts:

$$\text{no signal if } nLCL_V \leq N_1v_1 + N_2v_2 + N_3v_3 + N_4v_4 \leq nUCL_V \quad (5.33).$$

The new form of the O.C. function is constructed by considering the implications of the signaling rule for the allowed number of observed critical, major, and minor nonconforming sample items. For critical nonconforming items in the sample, the signaling rule implies that

$$N_4v_4 \leq nUCL_V \quad (5.34).$$

Thus in order for their not to be a signal, the number of critical nonconforming items in the sample must fall below some value. Specifically,

$$N_4 \leq \frac{nUCL_V}{v_4} \quad (5.35).$$

Since N_4 is an integer and cannot be larger than the sample size, a threshold value for N_4 can be defined. This threshold value is

$$N_4^{\max} = \min\left(n, \left\lfloor \frac{nUCL_V}{v_4} \right\rfloor\right) \quad (5.36).$$

Given that $N_4 = n_4$, a limit can also be placed on the number of major nonconforming sample items by recognizing that

$$N_3v_3 + n_4v_4 \leq nUCL_V \quad (5.37),$$

which can be rewritten as

$$N_3 \leq \frac{nUCL_V - n_4v_4}{v_3} \quad (5.38).$$

Since N_3 is an integer and the sum of N_3 and N_4 cannot exceed the sample size, a threshold value for N_3 can be defined. This threshold value is

$$N_3^{\max} = \min\left(n - n_4, \left\lfloor \frac{nUCL_V - n_4v_4}{v_3} \right\rfloor\right) \quad (5.39).$$

Given that $N_3 = n_3$ and $N_4 = n_4$, a limit can also be placed on the number of minor nonconforming sample items by recognizing that

$$N_2 v_2 + n_3 v_3 + n_4 v_4 \leq n UCL_V \quad (5.40),$$

which can be rewritten as

$$N_2 \leq \frac{n UCL_V - n_4 v_4 - n_3 v_3}{v_2} \quad (5.41).$$

Since N_2 is an integer and the sum of N_2 , N_3 , and N_4 cannot exceed the sample size, a threshold value for N_2 can be defined. This threshold value is

$$N_2^{\max} = \min \left(n - n_3 - n_4, \left\lfloor \frac{n UCL_V - n_4 v_4 - n_3 v_3}{v_2} \right\rfloor \right) \quad (5.42).$$

These thresholds can be used to define a new form of the O.C. function. For the 4-level case, this revised version of the O.C. function is given by

$$P_a(p_1, p_2, p_3, p_4) = \sum_{n_4=0}^{N_4^{\max}} \sum_{n_3=0}^{N_3^{\max}} \sum_{n_2=0}^{N_2^{\max}} m_a(n - n_2 - n_3 - n_4, n_2, n_3, n_4, p_1, p_2, p_3, p_4) \quad (5.43).$$

These thresholds can be described specifically for each of the defined types of 4-level control charts.

4-SPC-AVG-1(n, k_V, v_1, v_2)

$$N_4^{\max} = \min(n, \lfloor n UCL_V \rfloor)$$

$$N_3^{\max} = \min \left(n - n_4, \left\lfloor \frac{n UCL_V - n_4}{v_2} \right\rfloor \right)$$

$$N_2^{\max} = \min \left(n - n_3 - n_4, \left\lfloor \frac{n UCL_V - n_4 - n_3 v_2}{v_1} \right\rfloor \right)$$

4-SPC-AVG-2(n, k_v, v)

$$N_4^{\max} = \min\left(n, \left\lfloor \frac{nUCL_v}{v^3} \right\rfloor\right)$$

$$N_3^{\max} = \min\left(n - n_4, \left\lfloor \frac{nUCL_v - n_4v^3}{v^2} \right\rfloor\right)$$

$$N_2^{\max} = \min\left(n - n_3 - n_4, \left\lfloor \frac{nUCL_v - n_4v^3 - n_3v^2}{v} \right\rfloor\right)$$

The improvement in computational efficiency resulting from revising the form of the O.C. function can be demonstrated using several examples. The improvements obtained in these examples are presented in Table 5.3.

Table 5.3
Examples of Achieved Savings in 4-Level O.C. Function Computation

Control Chart	Evaluations of m_a Required by Equation (5.20)	Evaluations of m_a Required by Equation (5.23)	Evaluations of m_a Required by Equation (5.43)
4-SPC-AVG-1(18,2.5758,0.25,0.5) $\mathbf{p}_0 = [0.93 \ 0.045 \ 0.02 \ 0.005]$	130321	1330	20
4-SPC-AVG-1(77,2.8782,0.25,0.75) $\mathbf{p}_0 = [0.915 \ 0.06 \ 0.02 \ 0.005]$	37015056	82160	319
4-SPC-AVG-1(28,2.8782,0.5,0.75) $\mathbf{p}_0 = [0.905 \ 0.06 \ 0.03 \ 0.005]$	707281	4495	72
4-SPC-AVG-2(48,2.5758,2) $\mathbf{p}_0 = [0.915 \ 0.06 \ 0.02 \ 0.005]$	5764801	20825	1050
4-SPC-AVG-2(172,2.8782,5) $\mathbf{p}_0 = [0.91 \ 0.06 \ 0.02 \ 0.01]$	895745041	877975	12920
4-SPC-AVG-1(2,2.8782,10) $\mathbf{p}_0 = [0.93 \ 0.045 \ 0.02 \ 0.005]$	81	10	6

5.3 MULTILEVEL CONTROL CHART PARAMETER SELECTION

In many practical situations, the manufacturer knows the type of control chart that they wish to use. In addition, it is assumed that the in-control state of the process (\mathbf{p}_\bullet) is known. However the manufacturer typically does not know what parameter values to use. In the case of multilevel SPC, the manufacturer may know the quality value function (i.e. the multilevel control chart) that they wish to use (e.g. 3-SPC-AVG-1($n, k_\nu, 0.5$) or 4-SPC-AVG-2($n, k_\nu, 2$)), but they do not necessarily know the values of n and k_ν that they wish to use. In this section, a method is defined for selecting n and k_ν based on some desired ARL function behavior. The methods are then demonstrated using the 3-level and 4-level control charts defined in section 5.1.

The problem of parameter selection is not unique to multilevel control charts. The problem exists for all types of control charts. In most instances, parameter selection methods are defined using some specified desirable ARL function behavior. Duncan[8] provides a thorough discussion of such methods for attributes and variables control charts. In general, if a control chart has b parameters, b ARL function objectives are specified. This results in a system of b equations in b unknowns. The system of equations is then solved for the unknown control chart parameters. Since multilevel control charts have two parameters, n and k_ν , two ARL function objectives must be specified.

The first specification involves the in-control state of the process \mathbf{p}_\bullet . The manufacturer must specify some acceptable rate of false alarms (signals when the process is actually in-control). Thus the manufacturer specifies

$$ARL(\mathbf{p}_\bullet) = 1 / \alpha \tag{5.44}$$

where

$$1 - \alpha = P_d(\mathbf{p}_\bullet) \tag{5.45}$$

Note that α denotes the probability of a false alarm given that the process is in control.

The implications of this ARL specification can be determined by considering the O.C. function in additional detail. The O.C. function for the defined multilevel control charts can be expressed in terms of the signaling rule. Specifically,

$$P_a(p_1, p_2, \dots, p_j) = P(LCL_V \leq \bar{V}_t \leq UCL_V | p_1, p_2, \dots, p_j) \quad (5.46).$$

Note also that

$$\bar{V}_t = \sum_{j=1}^n \frac{V_{jt}}{n} = \frac{1}{n} \sum_{j=1}^n V_{jt} \quad (5.47).$$

Thus,

$$P_a(p_1, p_2, \dots, p_j) = P\left(nLCL_V \leq \sum_{j=1}^n V_{jt} \leq nUCL_V | p_1, p_2, \dots, p_j\right) \quad (5.48).$$

Let μ_V denote the expected value of V_{it} and let σ_V denote the standard deviation of V_{it} . These values can also be expressed in terms of the in-control mean and standard deviation of V_{it} :

$$\mu_V = \delta_1 \mu_{V0} \quad (5.49),$$

$$\sigma_V = \delta_2 \sigma_{V0} \quad (5.50).$$

Note that if the process is in control then $\delta_1 = \delta_2 = 1$.

The Central Limit Theorem approximation used in developing the parameter selection methods for multilevel acceptance sampling plans can also be applied here. Since $V_{t1}, V_{t2}, \dots, V_{tn}$ are assumed to be IID random variables, then as $n \rightarrow \infty$

$$P_a(p_1, p_2, \dots, p_j) \cong P\left(\frac{nLCL_V - n\mu_V}{\sigma_V \sqrt{n}} \leq Z \leq \frac{nUCL_V - n\mu_V}{\sigma_V \sqrt{n}}\right) \quad (5.51).$$

Ignoring the approximation for now, this expression can be simplified as follows.

$$P_a(p_1, p_2, \dots, p_j) = P\left(\frac{\sqrt{n}(LCL_V - \mu_V)}{\sigma_V} \leq Z \leq \frac{\sqrt{n}(UCL_V - \mu_V)}{\sigma_V}\right)$$

$$P_a(p_1, p_2, \dots, p_j) = P \left(\frac{\sqrt{n} \left(\mu_{v0} - k_v \frac{\sigma_{v0}}{\sqrt{n}} \right)}{\sigma_v} \leq Z \leq \frac{\sqrt{n} \left(\mu_{v0} + k_v \frac{\sigma_{v0}}{\sqrt{n}} \right)}{\sigma_v} \right)$$

$$P_a(p_1, p_2, \dots, p_j) = P \left(\frac{\sqrt{n} \left(\mu_{v0} - k_v \frac{\sigma_{v0}}{\sqrt{n}} - \delta_1 \mu_{v0} \right)}{\delta_2 \sigma_{v0}} \leq Z \leq \frac{\sqrt{n} \left(\mu_{v0} + k_v \frac{\sigma_{v0}}{\sqrt{n}} - \delta_1 \mu_{v0} \right)}{\delta_2 \sigma_{v0}} \right)$$

$$P_a(p_1, p_2, \dots, p_j) = P \left(\frac{\sqrt{n} \mu_{v0} (1 - \delta_1) - k_v}{\delta_2 \sigma_{v0}} \leq Z \leq \frac{\sqrt{n} \mu_{v0} (1 - \delta_1) + k_v}{\delta_2 \sigma_{v0}} \right) \quad (5.52)$$

If the process is in control ($\delta_1 = \delta_2 = 1$), then

$$P_a(p_{01}, p_{02}, \dots, p_{0j}) = P(-k_v \leq Z \leq k_v) \quad (5.53)$$

The in-control ARL specification can now be used to select a value for k_v .

$$P_a(p_{01}, p_{02}, \dots, p_{0j}) = P(-k_v \leq Z \leq k_v) = 1 - \alpha$$

$$k_v = Z_{1-\alpha/2} \quad (5.54)$$

The second ARL specification is made for some likely out-of-control condition.

The state of the process in this out-of-control condition is denoted by $\mathbf{p}_c = [p_{c1} p_{c2} \dots p_{cj}]$.

The manufacturer must specify an upper bound for the ARL in this out-of-control condition. Specifically, the manufacturer must specify

$$ARL(\mathbf{p}_c) \leq \frac{1}{1 - \beta} \quad (5.55)$$

which implies that

$$P_a(\mathbf{p}_c) \leq \beta \quad (5.56)$$

This second ARL specification is then used to select a value for n . The specification requires that

$$P_a(p_{c1}, p_{c2}, \dots, p_{cj}) = P\left(\frac{\sqrt{n}\mu_{v0}(1-\delta_1)}{\delta_2\sigma_{v0}} - \frac{k_v}{\delta_2} \leq Z \leq \frac{\sqrt{n}\mu_{v0}(1-\delta_1)}{\delta_2\sigma_{v0}} + \frac{k_v}{\delta_2}\right) \leq \beta \quad (5.57).$$

Given \mathbf{p}_0 and \mathbf{p}_c , μ_{v0} , σ_{v0} , δ_1 and δ_2 can be calculated, and k_v has been selected using the first ARL specification. Thus n is the only unknown in equation (5.57). The smallest value of n that meets the second ARL specification is then selected.

The selected values of n and k_v are then used to construct the control limits for the multilevel control chart. However, if the lower control limit is zero, then the values of n and k_v must be selected again, this time assuming that the control chart is one-sided. In this case the ARL specifications can be expressed as

$$P_a(p_{01}, p_{02}, \dots, p_{0j}) = P(Z \leq k_v) = 1 - \alpha \quad (5.58),$$

and

$$P_a(p_{c1}, p_{c2}, \dots, p_{cj}) = P\left(Z \leq \frac{\sqrt{n}\mu_{v0}(1-\delta_1)}{\delta_2\sigma_{v0}} + \frac{k_v}{\delta_2}\right) \leq \beta \quad (5.59).$$

The values of k_v and n can then be computed using

$$k_v = Z_{1-\alpha} \quad (5.60),$$

and

$$n = \left[\left(Z_\beta - \frac{k_v}{\delta_2} \right)^2 \left(\frac{\delta_2\sigma_{v0}}{\mu_{v0}(1-\delta_1)} \right)^2 \right] \quad (5.61).$$

The revised values of n and k_v are used to compute the upper control limit, and the lower control limit is set to zero.

A method is needed for evaluating the “goodness” of the multilevel control chart resulting from the parameter selection method defined above. The method used is similar to the method used for evaluating the multilevel acceptance sampling plan parameter

selection method defined in Chapter 4. The resulting control limits are used to compute $ARL(\mathbf{p}_\bullet)$ and $ARL(\mathbf{p}_c)$. The resulting values are then compared to the values specified by the manufacturer.

5.3.1 Parameter Selection for 3-level Control Charts

In the preceding section, the method for selecting n and k_v for a multilevel control chart is defined. In this section, the details required for implementing this method for the 3-level control charts defined in Section 5.1.1 are provided. The method requires that $\mathbf{p}_\bullet = [p_{01} p_{02} p_{03}]$ and $\mathbf{p}_c = [p_{c1} p_{c2} p_{c3}]$ be specified, and that ARL function objectives for each of these process states be specified.

For each of the defined 3-level control charts, expressions for δ_1 and δ_2 are provided below.

3-SPC-AVG-1(n, k_v, v)

$$\delta_1 = \frac{vp_{c2} + p_{c3}}{vp_{02} + p_{03}}$$

$$\delta_2 = \sqrt{\frac{v^2 p_{c2} + p_{c3} - (vp_{c2} + p_{c3})^2}{v^2 p_{02} + p_{03} - (vp_{02} + p_{03})^2}}$$

3-SPC-AVG-2(n, k_v, v)

$$\delta_1 = \frac{p_{c1} + vp_{c2} + v^2 p_{c3}}{p_{01} + vp_{02} + v^2 p_{03}}$$

$$\delta_2 = \sqrt{\frac{p_{c1} + v^2 p_{c2} + v^4 p_{c3} - (p_{c1} + vp_{c2} + v^2 p_{c3})^2}{p_{01} + v^2 p_{02} + v^4 p_{03} - (p_{01} + vp_{02} + v^2 p_{03})^2}}$$

3-SPC-AVG-G(n, k_V, v_1, v_2, v_3)

$$\delta_1 = \frac{v_1 p_{c1} + v_2 p_{c2} + v_3 p_{c3}}{v_1 p_{01} + v_2 p_{02} + v_3 p_{03}}$$

$$\delta_2 = \sqrt{\frac{v_1^2 p_{c1} v_2^2 p_{c2} + v_3^2 p_{c3} - (v_1 p_{c1} + v_2 p_{c2} + v_3 p_{c3})^2}{v_1^2 p_{01} + v_2^2 p_{02} + v_3^2 p_{03} - (v_1 p_{01} + v_2 p_{02} + v_3 p_{03})^2}}$$

Appendix 3.1 contains the results obtained from implementing the parameter selection method for the 3-SPC-AVG-1(n, k_V, v) control chart. For the given examples, the following information is provided: v , \mathbf{p}_0 , \mathbf{p}_c , the specified values of $ARL(\mathbf{p}_0)$ and $ARL(\mathbf{p}_c)$, the selected values of n and k_V , the resulting control limits and center line, and the achieved values of $ARL(\mathbf{p}_0)$ and $ARL(\mathbf{p}_c)$. Appendix 3.2 contains similar results for the 3-SPC-AVG-2(n, k_V, v) control chart.

By studying the results of the examples, several conclusions can be made regarding the approximate parameter selection method as it relates to the defined 3-level control charts. The first conclusion is that the vast majority of the 3-level control charts selected are one-sided (in other words, $LCL_V = 0$). Of the 384 examples, 320 (82.9%) have $LCL_V = 0$.

The second conclusion is that the Central Limit Theorem approximation does not work as well for the 3-level control charts as it did for 3-level acceptance sampling plans. The average percent error in the achieved value of $ARL(\mathbf{p}_0)$ is 64.7% for the 192 example 3-SPC-AVG-1(n, k_V, v) control charts, and 67.2% for the 192 example 3-SPC-AVG-2(n, k_V, v) control charts. However, the approximation does work better for the out-of-control case. The average percent error in the achieved value of $ARL(\mathbf{p}_c)$ for the 192 example 3-SPC-AVG-1(n, k_V, v) control charts is 9.2%. The average percent error in the achieved value of $ARL(\mathbf{p}_c)$ for the 192 example 3-SPC-AVG-2(n, k_V, v) control charts is 14.7%. This indicates that the approximation to the O.C. function degrades near the extreme tails of the standard normal distribution.

The third and final conclusion is that the parameter selection method does not provide 3-level control charts that meet the ARL specifications established by the manufacturer. Of the 384 examples, only 1 satisfies the in-control ARL objective. Only 85 satisfy the out-of-control ARL objective.

5.3.2 Parameter Selection for 4-level Control Charts

In this section, the details required for implementing the parameter selection method for the 4-level control charts defined in Section 5.1.2 are provided. The method requires that $\mathbf{p}_o = [p_{o1} p_{o2} p_{o3} p_{o4}]$ and $\mathbf{p}_c = [p_{c1} p_{c2} p_{c3} p_{c4}]$ be specified, and that ARL specifications for each of these process states be made.

For each of the defined 4-level control charts, expressions for δ_1 and δ_2 are provided

4-SPC-AVG-1(n, k, v_1, v_2)

$$\delta_1 = \frac{v_1 p_{c2} + v_2 p_{c3} + p_{c4}}{v_1 p_{o2} + v_2 p_{o3} + p_{o4}}$$

$$\delta_2 = \sqrt{\frac{v_1^2 p_{c2} + v_2^2 p_{c3} + p_{c4} - (v_1 p_{c2} + v_2 p_{c3} + p_{c4})^2}{v_1^2 p_{o2} + v_2^2 p_{o3} + p_{o4} - (v_1 p_{o2} + v_2 p_{o3} + p_{o4})^2}}$$

4-SPC-AVG-2(n, k, v, v)

$$\delta_1 = \frac{p_{c1} + v p_{c2} + v^2 p_{c3} + v^3 p_{c4}}{p_{o1} + v p_{o2} + v^2 p_{o3} + v^3 p_{o4}}$$

$$\delta_2 = \sqrt{\frac{p_{c1} + v^2 p_{c2} + v^4 p_{c3} + v^6 p_{c4} - (p_{c1} + v p_{c2} + v^2 p_{c3} + v^3 p_{c4})^2}{p_{o1} + v^2 p_{o2} + v^4 p_{o3} + v^6 p_{o4} - (p_{o1} + v p_{o2} + v^2 p_{o3} + v^3 p_{o4})^2}}$$

4-SPC-AVG-G($n, k_v, v_1, v_2, v_3, v_4$)

$$\delta_1 = \frac{v_1 p_{c1} + v_2 p_{c2} + v_3 p_{c3} + v_4 p_{c4}}{v_1 p_{o1} + v_2 p_{o2} + v_3 p_{o3} + v_4 p_{o4}}$$

$$\delta_2 = \sqrt{\frac{v_1^2 p_{c1} v_2^2 p_{c2} + v_3^2 p_{c3} + v_4^2 p_{c4} - (v_1 p_{c1} + v_2 p_{c2} + v_3 p_{c3} + v_4 p_{c4})^2}{v_1^2 p_{o1} + v_2^2 p_{o2} + v_3^2 p_{o3} + v_4^2 p_{o4} - (v_1 p_{o1} + v_2 p_{o2} + v_3 p_{o3} + v_4 p_{o4})^2}}$$

Appendix 4.1 contains a sample of the results obtained from implementing the parameter selection method for the 4-SPC-AVG-1(n, k_v, v_1, v_2) control chart. For each of the given examples, the following information is provided: $v_1, v_2, \mathbf{p}_o, \mathbf{p}_c$, the specified values of $ARL(\mathbf{p}_o)$ and $ARL(\mathbf{p}_c)$, the selected values of n and k_v , the resulting control limits and center line, and the achieved values of $ARL(\mathbf{p}_o)$ and $ARL(\mathbf{p}_c)$. Appendix 4.2 contains similar results for the 4-SPC-AVG-2(n, k_v, v) control chart.

By studying the results of the examples, including those presented in Appendices 4.1 and 4.2, several conclusions can be made regarding the approximate parameter selection method as it relates to the defined 4-level control charts. The first conclusion is that the vast majority of the 4-level control charts selected are one-sided (in other words, $LCL_v = 0$). Of the 1536 examples, 1486 (96.7%) have $LCL_v = 0$.

The second conclusion is that the Central Limit Theorem approximation does not work as well for the 4-level control charts as it did for 4-level acceptance sampling plans. The average percent error in the achieved value of $ARL(\mathbf{p}_o)$ is 72.8% for the 768 example 4-SPC-AVG-1(n, k_v, v_1, v_2) control charts, and 76.5% for the 768 example 4-SPC-AVG-2(n, k_v, v) control charts. However, the approximation does work better for the out-of-control case. The average percent error in the achieved value of $ARL(\mathbf{p}_c)$ for the 768 example 4-SPC-AVG-1(n, k_v, v_1, v_2) control charts is 7.4%. The average percent error in the achieved value of $ARL(\mathbf{p}_c)$ for the 192 example 4-SPC-AVG-2(n, k_v, v) control charts is 33.9%. As with the 3-level case, this indicates that the approximation to the O.C. function degrades near the extreme tails of the standard normal distribution.

The third and final conclusion is that the parameter selection method does not provide 4-level control charts that meet the ARL objectives established by the manufacturer. Of the 1536 examples, only 32 satisfy the in-control ARL objective. Only 396 satisfy the out-of-control ARL objective.

5.4 QUALITY VALUE FUNCTION SPECIFICATION

In this section, the problem of quality value function specification is addressed. In section 5.3, the problem of selecting n and k_v is considered. In that case, it is assumed that the manufacturer knows the type of multilevel control chart they wish to use, as well as the numerical values to be assigned to items by the quality value function. In this section, it is assumed that the manufacturer only knows the type of multilevel control chart that they wish to use. For example, the manufacturer may wish to use a 4-SPC-AVG- $2(n, k_v, v)$ control chart, but they may not know what values of n , k_v , and v to use. In this section, a method is developed for selecting the parameters and specifying the quality value function for a multilevel control chart.

Suppose the quality value function requires the specification of b values (in the control charts defined in section 5.1, b is either one, two, three, or four). If $b + 2$ points on the ARL curve are specified, then the result is a set of $b + 2$ equations in $b + 2$ unknowns (the quality value function, n , and k_v). The quality value function specification method defined here uses the Central Limit Theorem approximation to the O.C. function that is used in the parameter selection method to approximately solve this set of equations.

The quality value function specification method is described for each of the defined multilevel control charts in the sections that follow.

5.4.1 Quality Value Function Specification for 3-Level Control Charts

Consider first the 3-SPC-AVG-1(n, k_v, v) and the 3-SPC-AVG-2(n, k_v, v) control charts. To use either of these control charts, values of n , k_v , and v must be specified. In section 5.3.1, an approximate method is developed for selecting n and k_v . This method requires the specification of two points on the ARL curve, summarized by \mathbf{p}_0 , \mathbf{p}_c , $ARL(\mathbf{p}_0) = 1/\alpha$, and $ARL(\mathbf{p}_c) = 1/(1 - \beta)$. This method is extended to address quality value function specification. In section 5.3, examples of the multilevel control chart parameter selection method are discussed. In the vast majority of these examples, the resulting control chart is one-sided ($LCL_v = 0$). Thus the quality value function specification method defined here assumes that the control chart is one-sided.

The extension to the method begins with the specification of a third point on the ARL curve, summarized by $\mathbf{p}_3 = [p_{31} \ p_{32} \ p_{33}]$ and $ARL(\mathbf{p}_3)$, where $ARL(\mathbf{p}_3) = 1/(1 - \gamma)$. This additional specification results in the following system of three equations in three unknowns:

$$P_a(p_{01}, p_{02}, p_{03}) = P(Z \leq k_v) = 1 - \alpha \quad (5.62),$$

$$P_a(p_{c1}, p_{c2}, p_{c3}) = P\left(Z \leq \frac{\sqrt{n}\mu_{v0}(1 - \delta_{c1})}{\delta_{c2}\sigma_{v0}} + \frac{k_v}{\delta_{c2}}\right) = \beta \quad (5.63),$$

and

$$P_a(p_{31}, p_{32}, p_{33}) = P\left(Z \leq \frac{\sqrt{n}\mu_{v0}(1 - \delta_{31})}{\delta_{32}\sigma_{v0}} + \frac{k_v}{\delta_{32}}\right) = \gamma \quad (5.64),$$

where $\delta_{c1} = \delta_1$ given that $\mathbf{p} = \mathbf{p}_c$, $\delta_{c2} = \delta_2$ given that $\mathbf{p} = \mathbf{p}_c$, $\delta_{31} = \delta_1$ given that $\mathbf{p} = \mathbf{p}_3$, and $\delta_{32} = \delta_2$ given that $\mathbf{p} = \mathbf{p}_3$. This series of equations can be reduced as follows.

$$(5.62) \Rightarrow k_v = Z_{1-\alpha} \quad (5.65)$$

$$(5.63) \Rightarrow n = \left(Z_\beta - \frac{k_v}{\delta_{c2}}\right)^2 \left(\frac{\delta_{c2}\sigma_{v0}}{\mu_{v0}(1 - \delta_{c1})}\right)^2 \quad (5.66)$$

$$(5.64) \Rightarrow n = \left(Z_\gamma - \frac{k_\nu}{\delta_{32}} \right)^2 \left(\frac{\delta_{32} \sigma_{\nu 0}}{\mu_{\nu 0} (1 - \delta_{31})} \right)^2 \quad (5.67)$$

Equation (5.65) is used to find k_ν . Equations (5.66) and (5.67) are equated and the resulting equation,

$$\left(Z_\beta - \frac{k_\nu}{\delta_{c2}} \right)^2 \left(\frac{\delta_{c2} \sigma_{\nu 0}}{\mu_{\nu 0} (1 - \delta_{c1})} \right)^2 - \left(Z_\gamma - \frac{k_\nu}{\delta_{32}} \right)^2 \left(\frac{\delta_{32} \sigma_{\nu 0}}{\mu_{\nu 0} (1 - \delta_{31})} \right)^2 = 0 \quad (5.68),$$

is a polynomial function of ν .

As with the polynomial functions used in the quality value function specification methods for multilevel acceptance sampling plans, the polynomial function given in equation (5.68) does not always provide a feasible solution for ν . Thus the quality value function specification method for the 3-SPC-AVG-1(n, k_ν, ν) control chart and the 3-SPC-AVG-2(n, k_ν, ν) control chart works as follows. \mathbf{p}_\bullet , \mathbf{p}_c , \mathbf{p}_3 , $ARL(\mathbf{p}_\bullet)$, $ARL(\mathbf{p}_c)$, and $ARL(\mathbf{p}_3)$ are specified. Equation (5.65) is used to find k_ν , and then equation (5.68) is solved for ν . If a feasible solution for ν is not found, then \mathbf{p}_3 and $ARL(\mathbf{p}_3)$ must be “re-specified.” Once a feasible solution for ν has been found, equation (5.66) is used to find n , and the upper control limit of the chart is computed. The capability of the method is evaluated by computing the values of $ARL(\mathbf{p}_\bullet)$, $ARL(\mathbf{p}_c)$, and $ARL(\mathbf{p}_3)$ and comparing them to the specified target values.

5.4.2 Quality Value Function Specification for the General 3-Level Case

Suppose the manufacturer does not even know which type of 3-level control chart to use. In this case, the manufacturer may wish to select the quality value function and the parameters for a 3-SPC-AVG-G($n, k_\nu, \nu_1, \nu_2, \nu_3$) control chart. To use this type of control chart, values of n , k_ν , ν_1 , ν_2 , and ν_3 must be specified. The quality value function specification method defined in section 5.4.1 is extended to consider this situation. The extension begins with the presentation of two theorems corresponding to the two theorems presented for 3-level acceptance sampling plans.

Theorem 5.1: Let $V_1(X)$ be a quality value function defined on a 3-level discrete product quality measure X , and let $V_2(X)$ be a quality value function defined on X such that $V_2(X) = bV_1(X)$ for all X where b is some positive constant. Then a multilevel control chart (utilizing fixed sampling interval single sampling and the average quality value sample statistic) using $V_1(X)$ and having parameters n and k_v and control limits $UCL_v = UCL_{v1}$ and $LCL_v = LCL_{v1}$ is equivalent to a multilevel control chart (utilizing fixed sampling interval single sampling and the average quality value sample statistics) using $V_2(X)$ and having parameters n and k_v and control limits $UCL_v = UCL_{v2} = bUCL_{v1}$ and $LCL_v = LCL_{v2} = bLCL_{v1}$.

Theorem 5.2: Every 3-level control chart (utilizing fixed sampling interval single sampling and the average quality value sample statistic) is equivalent to either a 3-SPC-AVG-1(n, k_v, v) control chart or a 3-SPC-AVG-G($n, k_v, 1, v_2, v_3$) control chart.

Note that similar theorems could be defined for the product sample statistic given in equation (3.14). As of result of Theorem 5.2, all that is necessary to complete the quality value function specification method for 3-level control charts is to extend the method defined in section 5.4.1 to the 3-SPC-AVG-G($n, k_v, 1, v_2, v_3$) control chart.

The extension to the quality value function specification method defined in section 5.4.1 begins with the specification of an additional point on the ARL curve, summarized by $\mathbf{p}_4 = [p_{41} \ p_{42} \ p_{43}]$ and $ARL(\mathbf{p}_4)$, where $ARL(\mathbf{p}_4) = 1/(1 - \eta)$. This additional specification results in the following system of four equations in four unknowns:

$$P_a(p_{01}, p_{02}, p_{03}) = P(Z \leq k_v) = 1 - \alpha \quad (5.69),$$

$$P_a(p_{e1}, p_{e2}, p_{e3}) = P\left(Z \leq \frac{\sqrt{n}\mu_{v0}(1 - \delta_{e1})}{\delta_{e2}\sigma_{v0}} + \frac{k_v}{\delta_{e2}}\right) = \beta \quad (5.70),$$

$$P_a(p_{s1}, p_{s2}, p_{s3}) = P\left(Z \leq \frac{\sqrt{n}\mu_{v0}(1 - \delta_{s1})}{\delta_{s2}\sigma_{v0}} + \frac{k_v}{\delta_{s2}}\right) = \gamma \quad (5.71),$$

and,

$$P_a(p_{41}, p_{42}, p_{43}) = P\left(Z \leq \frac{\sqrt{n}\mu_{v0}(1 - \delta_{41})}{\delta_{42}\sigma_{v0}} + \frac{k_v}{\delta_{42}}\right) = \eta \quad (5.72),$$

where $\delta_{41} = \delta_1$ given that $\mathbf{p} = \mathbf{p}_4$, and $\delta_{42} = \delta_2$ given that $\mathbf{p} = \mathbf{p}_4$. This series of equations can be reduced as follows.

$$(5.69) \Rightarrow k_V = Z_{1-\alpha} \quad (5.73)$$

$$(5.70) \Rightarrow n = \left(Z_\beta - \frac{k_V}{\delta_{c2}} \right)^2 \left(\frac{\delta_{c2} \sigma_{V0}}{\mu_{V0}(1-\delta_{c1})} \right)^2 \quad (5.74)$$

$$(5.71) \Rightarrow n = \left(Z_\gamma - \frac{k_V}{\delta_{32}} \right)^2 \left(\frac{\delta_{32} \sigma_{V0}}{\mu_{V0}(1-\delta_{31})} \right)^2 \quad (5.75)$$

$$(5.72) \Rightarrow n = \left(Z_\eta - \frac{k_V}{\delta_{42}} \right)^2 \left(\frac{\delta_{42} \sigma_{V0}}{\mu_{V0}(1-\delta_{41})} \right)^2 \quad (5.76)$$

Equation (5.73) is used to find k_V . If one of the three expressions for n is substituted into the other two, the result is two expressions which are polynomial functions of v_2 and v_3 . If equation (5.74) is used, then the following expressions are obtained.

$$(5.75) \Rightarrow \left(Z_\beta - \frac{k_V}{\delta_{c2}} \right)^2 \left(\frac{\delta_{c2} \sigma_{V0}}{\mu_{V0}(1-\delta_{c1})} \right)^2 - \left(Z_\gamma - \frac{k_V}{\delta_{32}} \right)^2 \left(\frac{\delta_{32} \sigma_{V0}}{\mu_{V0}(1-\delta_{31})} \right)^2 = 0 \quad (5.77)$$

$$(5.76) \Rightarrow \left(Z_\beta - \frac{k_V}{\delta_{c2}} \right)^2 \left(\frac{\delta_{c2} \sigma_{V0}}{\mu_{V0}(1-\delta_{c1})} \right)^2 - \left(Z_\eta - \frac{k_V}{\delta_{42}} \right)^2 \left(\frac{\delta_{42} \sigma_{V0}}{\mu_{V0}(1-\delta_{41})} \right)^2 = 0 \quad (5.78)$$

The quality value function specification method for the 3-SPC-AVG-G($n, k_V, 1, v_2, v_3$) control chart works as follows. $\mathbf{p}_0, \mathbf{p}_c, \mathbf{p}_3, \mathbf{p}_4, ARL(\mathbf{p}_0), ARL(\mathbf{p}_c), ARL(\mathbf{p}_3)$, and $ARL(\mathbf{p}_4)$ are specified. First, equation (5.73) is used to find k_V . The polynomial functions given in equations (5.77) and (5.78) are then simultaneously solved for v_2 and v_3 . If the expression does not provide a feasible solution for v_2 and v_3 ($v_3 > v_2 > 1$), then $\mathbf{p}_3, \mathbf{p}_4, ARL(\mathbf{p}_3)$, and $ARL(\mathbf{p}_4)$ must be “re-specified.” Once a feasible solution for v_2 and v_3 has been found, equation (5.74) is used to find n , and the upper control limit of the chart is computed. The capability of the method is evaluated by computing the values of $ARL(\mathbf{p}_0)$,

$ARL(\mathbf{p}_c)$, $ARL(\mathbf{p}_3)$, and $ARL(\mathbf{p}_4)$ and comparing them to the specified target values. It is worthwhile to note that finding a feasible solution for v_2 and v_3 can be very time-consuming.

5.4.3 Quality Value Function Specification for 4-Level Control Charts

Consider first the 4-SPC-AVG-2(n, k_V, v) control chart. To use this control chart, values of n , k_V , and v must be specified. In section 5.4.1, a method is developed for selecting n , k_V , and v for the 3-SPC-AVG-1(n, k_V, v) and the 3-SPC-AVG-2(n, k_V, v) control charts. This method is directly applicable to the 4-SPC-AVG-2(n, k_V, v) control chart.

Consider next the 4-SPC-AVG-1(n, k_V, v_1, v_2) control chart. To use this control chart, values of n , k_V , v_1 , and v_2 must be specified. In section 5.4.2, a method is developed for selecting n , k_V , v_2 , and v_3 for the 3-AS-S-G($n, k_V, 1, v_2, v_3$) control chart. This method is directly applicable to the 4-SPC-AVG-1(n, k_V, v_1, v_2) control chart with one exception. In this case, equations (5.77) and (5.78) are polynomial functions of v_1 and v_2 .

5.4.4 Quality Value Function Specification for the General 4-Level Case

Suppose the manufacturer does not even know which type of 4-level control chart to use. In this case, the manufacturer may wish to select the quality value function and the parameters for a 4-SPC-AVG-G($n, k_V, v_1, v_2, v_3, v_4$) control chart. To use this type of control chart, values of n , k_V , v_1 , v_2 , v_3 , and v_4 must be specified. The quality value function specification method defined in section 5.4.2 is extended to the 4-level case. The extension begins with the presentation of a corollary to Theorem 5.2.

Corollary 5.1: Every 4-level control chart (utilizing fixed sampling interval single sampling and the average quality value sample statistic) is equivalent to either a 4-SPC-AVG-1(n, k_V, v_1, v_2) control chart or a 4-SPC-AVG-G($n, k_V, 1, v_2, v_3, v_4$) control chart.

The extension to the quality value function specification method defined in section 5.4.2 begins with the specification of five points on the ARL curve, summarized by $\mathbf{p}_\bullet = [p_{01} p_{02} p_{03} p_{04}]$, $\mathbf{p}_c = [p_{c1} p_{c2} p_{c3} p_{c4}]$, $\mathbf{p}_3 = [p_{31} p_{32} p_{33} p_{34}]$, $\mathbf{p}_4 = [p_{41} p_{42} p_{43} p_{44}]$, $\mathbf{p}_5 = [p_{51} p_{52} p_{53} p_{54}]$, $ARL(\mathbf{p}_\bullet)$, $ARL(\mathbf{p}_c)$, $ARL(\mathbf{p}_3)$, $ARL(\mathbf{p}_4)$, and $ARL(\mathbf{p}_5)$, where $ARL(\mathbf{p}_\bullet) = 1/\alpha$, $ARL(\mathbf{p}_c) = 1/(1 - \beta)$, $ARL(\mathbf{p}_3) = 1/(1 - \gamma)$, $ARL(\mathbf{p}_4) = 1/(1 - \eta)$, and $ARL(\mathbf{p}_5) = 1/(1 - \varphi)$. This additional specification results in the following system of five equations in five unknowns:

$$P_a(p_{01}, p_{02}, p_{03}, p_{04}) = P(Z \leq k_v) = 1 - \alpha \quad (5.79),$$

$$P_a(p_{c1}, p_{c2}, p_{c3}, p_{c4}) = P\left(Z \leq \frac{\sqrt{n}\mu_{v0}(1 - \delta_{c1})}{\delta_{c2}\sigma_{v0}} + \frac{k_v}{\delta_{c2}}\right) = \beta \quad (5.80),$$

$$P_a(p_{31}, p_{32}, p_{33}, p_{34}) = P\left(Z \leq \frac{\sqrt{n}\mu_{v0}(1 - \delta_{31})}{\delta_{32}\sigma_{v0}} + \frac{k_v}{\delta_{32}}\right) = \gamma \quad (5.81),$$

$$P_a(p_{41}, p_{42}, p_{43}, p_{44}) = P\left(Z \leq \frac{\sqrt{n}\mu_{v0}(1 - \delta_{41})}{\delta_{42}\sigma_{v0}} + \frac{k_v}{\delta_{42}}\right) = \eta \quad (5.82),$$

and

$$P_a(p_{51}, p_{52}, p_{53}, p_{54}) = P\left(Z \leq \frac{\sqrt{n}\mu_{v0}(1 - \delta_{51})}{\delta_{52}\sigma_{v0}} + \frac{k_v}{\delta_{52}}\right) = \varphi \quad (5.83),$$

where $\delta_{c1} = \delta_1$ given that $\mathbf{p} = \mathbf{p}_c$, $\delta_{c2} = \delta_2$ given that $\mathbf{p} = \mathbf{p}_c$, $\delta_{31} = \delta_1$ given that $\mathbf{p} = \mathbf{p}_3$, $\delta_{32} = \delta_2$ given that $\mathbf{p} = \mathbf{p}_3$, $\delta_{41} = \delta_1$ given that $\mathbf{p} = \mathbf{p}_4$, $\delta_{42} = \delta_2$ given that $\mathbf{p} = \mathbf{p}_4$, $\delta_{51} = \delta_1$ given that $\mathbf{p} = \mathbf{p}_5$, and $\delta_{52} = \delta_2$ given that $\mathbf{p} = \mathbf{p}_5$. This series of equations can be reduced as follows.

$$(5.79) \Rightarrow k_v = Z_{1-\alpha} \quad (5.84)$$

$$(5.80) \Rightarrow n = \left(Z_\beta - \frac{k_v}{\delta_{c2}}\right)^2 \left(\frac{\delta_{c2}\sigma_{v0}}{\mu_{v0}(1 - \delta_{c1})}\right)^2 \quad (5.85)$$

$$(5.81) \Rightarrow n = \left(Z_\gamma - \frac{k_v}{\delta_{32}}\right)^2 \left(\frac{\delta_{32}\sigma_{v0}}{\mu_{v0}(1 - \delta_{31})}\right)^2 \quad (5.86)$$

$$(5.82) \Rightarrow n = \left(Z_\eta - \frac{k_V}{\delta_{42}} \right)^2 \left(\frac{\delta_{42} \sigma_{V0}}{\mu_{V0}(1-\delta_{41})} \right)^2 \quad (5.87)$$

$$(5.83) \Rightarrow n = \left(Z_\phi - \frac{k_V}{\delta_{52}} \right)^2 \left(\frac{\delta_{52} \sigma_{V0}}{\mu_{V0}(1-\delta_{51})} \right)^2 \quad (5.88)$$

Equation (5.84) is used to find k_V . If one of the four expressions for n is substituted into the other three, the result is three expressions which are polynomial functions of v_2 , v_3 , and v_4 . If equation (5.85) is used, then the following expressions are obtained.

$$(5.86) \Rightarrow \left(Z_\beta - \frac{k_V}{\delta_{c2}} \right)^2 \left(\frac{\delta_{c2} \sigma_{V0}}{\mu_{V0}(1-\delta_{c1})} \right)^2 - \left(Z_\gamma - \frac{k_V}{\delta_{32}} \right)^2 \left(\frac{\delta_{32} \sigma_{V0}}{\mu_{V0}(1-\delta_{31})} \right)^2 = 0 \quad (5.89)$$

$$(5.87) \Rightarrow \left(Z_\beta - \frac{k_V}{\delta_{c2}} \right)^2 \left(\frac{\delta_{c2} \sigma_{V0}}{\mu_{V0}(1-\delta_{c1})} \right)^2 - \left(Z_\eta - \frac{k_V}{\delta_{42}} \right)^2 \left(\frac{\delta_{42} \sigma_{V0}}{\mu_{V0}(1-\delta_{41})} \right)^2 = 0 \quad (5.90)$$

$$(5.88) \Rightarrow \left(Z_\beta - \frac{k_V}{\delta_{c2}} \right)^2 \left(\frac{\delta_{c2} \sigma_{V0}}{\mu_{V0}(1-\delta_{c1})} \right)^2 - \left(Z_\phi - \frac{k_V}{\delta_{52}} \right)^2 \left(\frac{\delta_{52} \sigma_{V0}}{\mu_{V0}(1-\delta_{51})} \right)^2 = 0 \quad (5.91)$$

The quality value function specification method for the 4-SPC-AVG- $G(n, k_V, 1, v_2, v_3, v_4)$ control chart works as follows. \mathbf{p}_0 , \mathbf{p}_c , \mathbf{p}_3 , \mathbf{p}_4 , \mathbf{p}_5 , $ARL(\mathbf{p}_0)$, $ARL(\mathbf{p}_c)$, $ARL(\mathbf{p}_3)$, $ARL(\mathbf{p}_4)$, and $ARL(\mathbf{p}_5)$ are specified. First, equation (5.84) is used to find k_V . The polynomial functions given in equations (5.89), (5.90), and (5.91) are then simultaneously solved for v_2 , v_3 and v_4 . If the expression does not provide a feasible solution for v_2 , v_3 , and v_4 ($v_4 > v_3 > v_2 > 1$), then \mathbf{p}_3 , \mathbf{p}_4 , \mathbf{p}_5 , $ARL(\mathbf{p}_3)$, $ARL(\mathbf{p}_4)$, and $ARL(\mathbf{p}_5)$ must be “re-specified.” Once a feasible solution for v_2 , v_3 , and v_4 has been found, equation (5.85) is used to find n , and the upper control limit of the chart is computed. The capability of the method is evaluated by computing the values of $ARL(\mathbf{p}_0)$, $ARL(\mathbf{p}_c)$, $ARL(\mathbf{p}_3)$, $ARL(\mathbf{p}_4)$, and $ARL(\mathbf{p}_5)$ and comparing them to the specified target values. It is

worthwhile to note that finding a feasible solution for v_2 , v_3 , and v_4 can be very time-consuming.

CHAPTER 6: CONCLUSIONS AND FUTURE DIRECTIONS

In this research, a number of multilevel acceptance sampling plans and multilevel control charts are developed. In this chapter, the advantages and disadvantages of these methods are discussed and future research areas involving these techniques are described.

6.1 MULTILEVEL ACCEPTANCE SAMPLING

In this research, a number of multilevel acceptance sampling plans are defined. For each of these defined plans, the O.C. function is constructed and used to develop approximate methods (based on the Central Limit Theorem) for sampling plan parameter selection and quality value function specification. The parameter selection method allows a manufacturer to quickly select a multilevel acceptance sampling plan that closely meets desired O.C. function behavior. The quality value function specification method is not as efficient. However, when successful, this method allows the manufacturer to construct a multilevel acceptance sampling plan that closely meets as many as four O.C. function objectives (in the 3-level case -- five in the 4-level case). This is quite an improvement over existing methods which typically allow a manufacturer to specify two O.C. function objectives.

All the defined multilevel acceptance sampling plans utilize the summation decision rule. Theorem 4.2 and Corollary 4.1 indicate that the entire set of such sampling plans can be limited to two specific cases ($3\text{-AS-S-1}(n, T, \nu)$ and $3\text{-AS-S-G}(n, T, 1, \nu_2, \nu_3)$ in the 3-level case -- $4\text{-AS-S-1}(n, T, \nu_1, \nu_2)$ and $4\text{-AS-S-G}(n, T, 1, \nu_2, \nu_3, \nu_4)$ in the 4-level case). As a result, future study of the types of multilevel acceptance sampling plans defined in this research can be limited to these two cases. Such future study might include additional numerical study of the defined parameter selection and quality value function specification methods, or the development of other methods (numerical or analytical, exact or approximate) for

solving the sets of simultaneous equations resulting from the specification of O.C. function objectives.

In addition to further study of the defined methods, there is also the possibility of the development and evaluation of other types of multilevel acceptance sampling plans. For example, any number of alternative decision rules can be explored. For each new decision rule, a number of quality value functions may be considered and, for each quality value function, parameter selection and quality value function specification methods must be developed. In addition, sampling strategies other than single sampling may be considered.

6.2 MULTILEVEL STATISTICAL PROCESS CONTROL

In this research, a number of multilevel control charts are defined. For each of these defined control charts, the O.C. function and ARL function are constructed and used to develop approximate methods (based on the Central Limit Theorem) for control chart parameter selection and quality value function specification. The parameter selection method allows a manufacturer to quickly select a multilevel control chart based on desired ARL function behavior. The quality value function specification method is not as efficient. However, when successful, this method allows the manufacturer to construct a multilevel control chart based on as many as four ARL function objectives (in the 3-level case -- five in the 4-level case). This is quite an improvement over existing methods which typically allow a manufacturer to specify two ARL function objectives. However, the Central Limit Theorem approximation is not sufficiently accurate for specified in-control ARL function behavior. In addition, the quality value function specification method assumes that the control chart should be one-sided. In order to achieve the full benefits of the defined control charts, an alternative method (numerical or analytical, exact or approximate) must be developed for overcoming these shortcomings.

All the defined multilevel control charts utilize the average quality value sample statistic. Theorem 5.2 and Corollary 5.1 indicate that the entire set of such control charts can be limited to two specific cases (3-SPC-AVG-1(n, k_v, v) and 3-SPC-AVG-G($n, k_v, 1, v_2, v_3$) in the 3-level case -- 4-SPC-AVG-1(n, k_v, v_1, v_2) and 4-SPC-AVG-G($n, k_v, 1, v_2, v_3, v_4$) in the 4-level case). As a result, future study of the types of multilevel control charts defined in this research can be limited to these two cases.

In addition to further study of the defined methods, there is also the possibility of the development and evaluation of other types of multilevel control charts. For example, any number of alternative sample statistics can be explored. For each new sample statistic, a number of quality value functions may be considered and, for each quality value function, parameter selection and quality value function specification methods must be developed. In addition, sampling strategies other than fixed sampling interval single sampling may be considered.

6.3 SUMMARY

The quality of a large number of products, particularly those in the food and chemical industries, can easily be evaluated using a multilevel discrete product quality measure. The sampling plans, control charts, and the parameter selection and quality value function specification methods defined in this research are the beginnings of a new approach to multilevel statistical quality control. As these statistical quality control techniques are explored in further detail and extended with new concepts and methods, the result should be a set of tools that provide a manufacturer with greater flexibility in designing acceptance sampling plans and control charts.

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APPENDIX 1

PARAMETER SELECTION EXAMPLES FOR 3-LEVEL ACCEPTANCE SAMPLING PLANS

APPENDIX 1.1

PARAMETER SELECTION EXAMPLES FOR THE 3-AS-S-1(n, T, v) ACCEPTANCE SAMPLING PLAN

Table A1.1.1 - Parameter Selection Examples for the 3-AS-S-1($n, T, 0.25$) Acceptance Sampling Plan having $\alpha = 0.05$ and $\beta = 0.05$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	91	3.28	0.0602	0.0409	3.29	0.0602	0.0409
0.03	0.01	0.15	0.05	62	2.48	0.0844	0.0288	2.48	0.0844	0.0288
0.03	0.01	0.10	0.08	52	2.19	0.0843	0.0321	2.21	0.0843	0.0321
0.03	0.01	0.15	0.08	40	1.82	0.0688	0.0366	1.84	0.0688	0.0366
0.05	0.01	0.10	0.05	112	4.48	0.0734	0.0338	4.48	0.0734	0.0338
0.05	0.01	0.15	0.05	74	3.25	0.0563	0.0431	3.26	0.0563	0.0431
0.05	0.01	0.10	0.08	60	2.78	0.0588	0.0408	2.81	0.0588	0.0408
0.05	0.01	0.15	0.08	45	2.25	0.0584	0.0433	2.25	0.0584	0.0433
0.03	0.02	0.10	0.05	165	7.61	0.0606	0.0409	7.62	0.0606	0.0409
0.03	0.02	0.15	0.05	105	5.34	0.0618	0.0399	5.35	0.0618	0.0399
0.03	0.02	0.10	0.08	80	4.34	0.0637	0.0398	4.36	0.0637	0.0398
0.03	0.02	0.15	0.08	59	3.46	0.0754	0.0338	3.46	0.0754	0.0338
0.05	0.02	0.10	0.05	210	10.37	0.0593	0.0416	10.39	0.0593	0.0416
0.05	0.02	0.15	0.05	127	6.88	0.0624	0.0392	6.90	0.0624	0.0392
0.05	0.02	0.10	0.08	93	5.38	0.0638	0.0387	5.41	0.0638	0.0387
0.05	0.02	0.15	0.08	68	4.22	0.0733	0.0329	4.26	0.0547	0.0445

Table A1.1.2 - Parameter Selection Examples for the 3-AS-S-1($n, T, 0.25$) Acceptance Sampling Plan having $\alpha = 0.05$ and $\beta = 0.10$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	66	2.59	0.0645	0.0919	2.61	0.0645	0.0919
0.03	0.01	0.15	0.05	45	1.97	0.0873	0.0738	1.98	0.0873	0.0738
0.03	0.01	0.10	0.08	37	1.72	0.0774	0.0756	1.75	0.0774	0.0756
0.03	0.01	0.15	0.08	29	1.46	0.0796	0.0695	1.51	0.0453	0.1048
0.05	0.01	0.10	0.05	82	3.52	0.0574	0.0999	3.54	0.0574	0.0999
0.05	0.01	0.15	0.05	54	2.57	0.0641	0.0946	2.58	0.0641	0.0946
0.05	0.01	0.10	0.08	43	2.18	0.0787	0.0816	2.21	0.0787	0.0816
0.05	0.01	0.15	0.08	32	1.77	0.0570	0.1047	1.77	0.0570	0.1047
0.03	0.02	0.10	0.05	124	6.07	0.0595	0.0959	6.09	0.0595	0.0959
0.03	0.02	0.15	0.05	79	4.30	0.0608	0.0969	4.32	0.0608	0.0969
0.03	0.02	0.10	0.08	59	3.46	0.0754	0.0813	3.50	0.0754	0.0813
0.03	0.02	0.15	0.08	44	2.80	0.0641	0.0958	2.83	0.0641	0.0958
0.05	0.02	0.10	0.05	158	8.21	0.0654	0.0873	8.23	0.0654	0.0873
0.05	0.02	0.15	0.05	95	5.47	0.0701	0.0842	5.47	0.0701	0.0842
0.05	0.02	0.10	0.08	68	4.22	0.0733	0.0830	4.24	0.0733	0.0830
0.05	0.02	0.15	0.08	50	3.35	0.0666	0.0909	3.38	0.0666	0.0909

Table A1.1.3 - Parameter Selection Examples for the 3-AS-S-1($n, T, 0.25$) Acceptance Sampling Plan having $\alpha = 0.10$ and $\beta = 0.05$

P_{12}	P_{13}	P_{22}	P_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	79	2.61	0.1074	0.0350	2.64	0.1074	0.0350
0.03	0.01	0.15	0.05	54	1.96	0.1263	0.0271	1.98	0.1263	0.0271
0.03	0.01	0.10	0.08	46	1.74	0.1233	0.0270	1.77	0.0913	0.0418
0.03	0.01	0.15	0.08	35	1.43	0.1168	0.0291	1.44	0.1168	0.0291
0.05	0.01	0.10	0.05	96	3.57	0.1022	0.0398	3.57	0.1022	0.0398
0.05	0.01	0.15	0.05	64	2.59	0.1062	0.0360	2.61	0.1062	0.0360
0.05	0.01	0.10	0.08	53	2.24	0.1316	0.0288	2.28	0.0916	0.0415
0.05	0.01	0.15	0.08	40	1.81	0.0971	0.0366	1.84	0.0971	0.0366
0.03	0.02	0.10	0.05	138	5.98	0.1177	0.0349	6.00	0.0952	0.0449
0.03	0.02	0.15	0.05	88	4.17	0.1146	0.0345	4.19	0.1146	0.0345
0.03	0.02	0.10	0.08	68	3.41	0.1155	0.0362	3.42	0.1155	0.0362
0.03	0.02	0.15	0.08	51	2.73	0.1183	0.0298	2.76	0.0941	0.0432
0.05	0.02	0.10	0.05	174	8.17	0.1099	0.0396	8.17	0.1099	0.0396
0.05	0.02	0.15	0.05	106	5.41	0.1109	0.0369	5.42	0.1109	0.0369
0.05	0.02	0.10	0.08	79	4.26	0.0965	0.0433	4.29	0.0965	0.0433
0.05	0.02	0.15	0.08	58	3.33	0.1062	0.0380	3.37	0.1062	0.0380

Table A1.1.4 - Parameter Selection Examples for the 3-AS-S-1($n, T, 0.25$) Acceptance Sampling Plan having $\alpha = 0.10$ and $\beta = 0.10$

P_{12}	P_{13}	P_{22}	P_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	55	1.98	0.1310	0.0752	1.99	0.1310	0.0752
0.03	0.01	0.15	0.05	38	1.51	0.0820	0.1019	1.53	0.0820	0.1019
0.03	0.01	0.10	0.08	32	1.34	0.0976	0.0866	1.37	0.0976	0.0866
0.03	0.01	0.15	0.08	25	1.13	0.1290	0.0800	1.18	0.1290	0.0800
0.05	0.01	0.10	0.05	68	2.72	0.1260	0.0798	2.72	0.1260	0.0798
0.05	0.01	0.15	0.05	45	1.98	0.1280	0.0738	1.98	0.1280	0.0738
0.05	0.01	0.10	0.08	37	1.71	0.1248	0.0756	1.75	0.1248	0.0756
0.05	0.01	0.15	0.08	28	1.39	0.1188	0.0798	1.43	0.1188	0.0798
0.03	0.02	0.10	0.05	100	4.61	0.1074	0.0929	4.62	0.1074	0.0929
0.03	0.02	0.15	0.05	64	3.25	0.1316	0.0754	3.27	0.0966	0.1028
0.03	0.02	0.10	0.08	49	2.65	0.1066	0.0846	2.68	0.1066	0.0846
0.03	0.02	0.15	0.08	36	2.11	0.1192	0.0916	2.12	0.1192	0.0916
0.05	0.02	0.10	0.05	127	6.27	0.0984	0.1010	6.28	0.0984	0.1010
0.05	0.02	0.15	0.05	77	4.17	0.1142	0.0861	4.18	0.1142	0.0861
0.05	0.02	0.10	0.08	56	3.24	0.1268	0.0805	3.25	0.1268	0.0805
0.05	0.02	0.15	0.08	41	2.55	0.0978	0.0983	2.56	0.0978	0.0983

Table A1.1.5 - Parameter Selection Examples for the 3-AS-S-1($n, T, 0.5$) Acceptance Sampling Plan having $\alpha = 0.05$ and $\beta = 0.05$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	72	3.61	0.0532	0.0449	3.64	0.0532	0.0449
0.03	0.01	0.15	0.05	43	2.48	0.0960	0.0261	2.48	0.0960	0.0261
0.03	0.01	0.10	0.08	45	2.56	0.0487	0.0485	2.57	0.0487	0.0485
0.03	0.01	0.15	0.08	31	1.96	0.1013	0.0242	2.01	0.0399	0.0556
0.05	0.01	0.10	0.05	103	6.04	0.0476	0.0504	6.04	0.0476	0.0504
0.05	0.01	0.15	0.05	58	3.86	0.0727	0.0335	3.89	0.0727	0.0335
0.05	0.01	0.10	0.08	59	3.91	0.0776	0.0328	3.92	0.0776	0.0328
0.05	0.01	0.15	0.08	39	2.86	0.0774	0.0316	2.90	0.0774	0.0316
0.03	0.02	0.10	0.05	112	6.74	0.0604	0.0413	6.76	0.0604	0.0413
0.03	0.02	0.15	0.05	62	4.27	0.0652	0.0393	4.28	0.0652	0.0393
0.03	0.02	0.10	0.08	64	4.37	0.0733	0.0338	4.41	0.0733	0.0338
0.03	0.02	0.15	0.08	42	3.20	0.0611	0.0399	3.25	0.0611	0.0399
0.05	0.02	0.10	0.05	166	11.17	0.0545	0.0452	11.20	0.0545	0.0452
0.05	0.02	0.15	0.05	83	6.35	0.0672	0.0378	6.36	0.0672	0.0378
0.05	0.02	0.10	0.08	84	6.41	0.0712	0.0350	6.45	0.0712	0.0350
0.05	0.02	0.15	0.08	52	4.41	0.0762	0.0331	4.43	0.0762	0.0331

Table A1.1.6 - Parameter Selection Examples for the 3-AS-S-1($n, T, 0.5$) Acceptance Sampling Plan having $\alpha = 0.05$ and $\beta = 0.10$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	52	2.84	0.0756	0.0818	2.84	0.0756	0.0818
0.03	0.01	0.15	0.05	32	2.01	0.0436	0.1187	2.06	0.0436	0.1187
0.03	0.01	0.10	0.08	33	2.05	0.0475	0.1094	2.10	0.0475	0.1094
0.03	0.01	0.15	0.08	22	1.55	0.0469	0.1180	1.57	0.0469	0.1180
0.05	0.01	0.10	0.05	76	4.75	0.0616	0.0932	4.75	0.0616	0.0932
0.05	0.01	0.15	0.05	43	3.08	0.0489	0.1074	3.12	0.0489	0.1074
0.05	0.01	0.10	0.08	43	3.08	0.0489	0.1090	3.10	0.0489	0.1090
0.05	0.01	0.15	0.08	28	2.25	0.0663	0.0899	2.27	0.0663	0.0899
0.03	0.02	0.10	0.05	84	5.38	0.0715	0.0815	5.41	0.0715	0.0815
0.03	0.02	0.15	0.05	46	3.42	0.0813	0.0757	3.42	0.0813	0.0757
0.03	0.02	0.10	0.08	47	3.47	0.0868	0.0707	3.50	0.0448	0.1187
0.03	0.02	0.15	0.08	31	2.57	0.0512	0.1079	2.62	0.0512	0.1079
0.05	0.02	0.10	0.05	125	8.84	0.0635	0.0887	8.85	0.0635	0.0887
0.05	0.02	0.15	0.05	63	5.11	0.0528	0.1035	5.15	0.0528	0.1035
0.05	0.02	0.10	0.08	62	5.05	0.0490	0.1101	5.06	0.0490	0.1101
0.05	0.02	0.15	0.08	39	3.55	0.0483	0.1097	3.60	0.0483	0.1097

Table A1.1.7 - Parameter Selection Examples for the 3-AS-S-1($n, T, 0.5$) Acceptance Sampling Plan having $\alpha = 0.10$ and $\beta = 0.05$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	62	2.86	0.1251	0.0310	2.90	0.1251	0.0310
0.03	0.01	0.15	0.05	37	1.94	0.1470	0.0261	1.94	0.1470	0.0261
0.03	0.01	0.10	0.08	39	2.01	0.0745	0.0521	2.02	0.0745	0.0521
0.03	0.01	0.15	0.08	27	1.54	0.0748	0.0502	1.57	0.0748	0.0502
0.05	0.01	0.10	0.05	88	4.83	0.1160	0.0357	4.87	0.1160	0.0357
0.05	0.01	0.15	0.05	49	3.02	0.0784	0.0526	3.04	0.0784	0.0526
0.05	0.01	0.10	0.08	51	3.12	0.0900	0.0448	3.14	0.0900	0.0448
0.05	0.01	0.15	0.08	34	2.28	0.1133	0.0336	2.34	0.1133	0.0336
0.03	0.02	0.10	0.05	94	5.30	0.1124	0.0374	5.33	0.1124	0.0374
0.03	0.02	0.15	0.05	52	3.32	0.1173	0.0360	3.32	0.1173	0.0360
0.03	0.02	0.10	0.08	54	3.42	0.1307	0.0315	3.43	0.1307	0.0315
0.03	0.02	0.15	0.08	36	2.51	0.0784	0.0510	2.56	0.0784	0.0510
0.05	0.02	0.10	0.05	137	8.78	0.1079	0.0413	8.79	0.1079	0.0413
0.05	0.02	0.15	0.05	70	5.02	0.0847	0.0510	5.06	0.0847	0.0510
0.05	0.02	0.10	0.08	71	5.08	0.0900	0.0471	5.12	0.0900	0.0471
0.05	0.02	0.15	0.08	44	3.46	0.1385	0.0294	3.48	0.1385	0.0294

Table A1.1.8 - Parameter Selection Examples for the 3-AS-S-1($n, T, 0.5$) Acceptance Sampling Plan having $\alpha = 0.10$ and $\beta = 0.10$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	44	2.20	0.1017	0.0927	2.23	0.1017	0.0927
0.03	0.01	0.15	0.05	27	1.54	0.0748	0.1151	1.59	0.0748	0.1151
0.03	0.01	0.10	0.08	28	1.58	0.0811	0.1070	1.63	0.0811	0.1070
0.03	0.01	0.15	0.08	19	1.20	0.0982	0.0918	1.24	0.0982	0.0918
0.05	0.01	0.10	0.05	63	3.69	0.0992	0.0967	3.71	0.0992	0.0967
0.05	0.01	0.15	0.05	35	2.33	0.1223	0.0803	2.34	0.1223	0.0803
0.05	0.01	0.10	0.08	36	2.38	0.1317	0.0760	2.40	0.1317	0.0760
0.05	0.01	0.15	0.08	24	1.76	0.1099	0.0845	1.80	0.1099	0.0845
0.03	0.02	0.10	0.05	68	4.09	0.0912	0.1068	4.11	0.0912	0.1068
0.03	0.02	0.15	0.05	38	2.61	0.0910	0.1066	2.63	0.0910	0.1066
0.03	0.02	0.10	0.08	39	2.66	0.0977	0.0981	2.69	0.0977	0.0981
0.03	0.02	0.15	0.08	25	1.91	0.1439	0.0712	1.92	0.1439	0.0712
0.05	0.02	0.10	0.05	101	6.79	0.1098	0.0895	6.82	0.1098	0.0895
0.05	0.02	0.15	0.05	51	3.89	0.1259	0.0769	3.92	0.1259	0.0769
0.05	0.02	0.10	0.08	51	3.89	0.1259	0.0790	3.91	0.1259	0.0790
0.05	0.02	0.15	0.08	32	2.71	0.1022	0.0933	2.74	0.1022	0.0933

Table A1.1.9 - Parameter Selection Examples for the 3-AS-S-1($n, T, 0.75$) Acceptance Sampling Plan having $\alpha = 0.05$ and $\beta = 0.05$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	68	4.39	0.0630	0.0406	4.42	0.0630	0.0406
0.03	0.01	0.15	0.05	39	2.92	0.0716	0.0328	2.96	0.0716	0.0328
0.03	0.01	0.10	0.08	45	3.24	0.0821	0.0320	3.28	0.0519	0.0459
0.03	0.01	0.15	0.08	29	2.37	0.0741	0.0372	2.42	0.0741	0.0372
0.05	0.01	0.10	0.05	109	8.43	0.0626	0.0411	8.46	0.0626	0.0411
0.05	0.01	0.15	0.05	55	4.92	0.0627	0.0422	4.93	0.0627	0.0422
0.05	0.01	0.10	0.08	65	5.60	0.0580	0.0421	5.63	0.0580	0.0421
0.05	0.01	0.15	0.08	39	3.80	0.0604	0.0423	3.84	0.0604	0.0423
0.03	0.02	0.10	0.05	95	7.04	0.0559	0.0456	7.05	0.0559	0.0456
0.03	0.02	0.15	0.05	50	4.30	0.0587	0.0450	4.30	0.0587	0.0450
0.03	0.02	0.10	0.08	59	4.87	0.0640	0.0400	4.91	0.0640	0.0400
0.03	0.02	0.15	0.08	36	3.38	0.0698	0.0430	3.41	0.0698	0.0430
0.05	0.02	0.10	0.05	157	13.39	0.0579	0.0435	13.42	0.0579	0.0435
0.05	0.02	0.15	0.05	72	7.09	0.0588	0.0444	7.11	0.0588	0.0444
0.05	0.02	0.10	0.08	86	8.17	0.0622	0.0407	8.22	0.0622	0.0407
0.05	0.02	0.15	0.08	48	5.17	0.0642	0.0423	5.17	0.0642	0.0423

Table A1.1.10 - Parameter Selection Examples for the 3-AS-S-1($n, T, 0.75$) Acceptance Sampling Plan having $\alpha = 0.05$ and $\beta = 0.10$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	50	3.49	0.0726	0.0852	3.52	0.0535	0.1048
0.03	0.01	0.15	0.05	28	2.31	0.0680	0.1029	2.32	0.0680	0.1029
0.03	0.01	0.10	0.08	33	2.59	0.0576	0.0964	2.65	0.0576	0.0964
0.03	0.01	0.15	0.08	21	1.89	0.0594	0.0981	1.95	0.0594	0.0981
0.05	0.01	0.10	0.05	81	6.65	0.0615	0.0939	6.65	0.0615	0.0939
0.05	0.01	0.15	0.05	41	3.94	0.0721	0.0847	3.97	0.0721	0.0847
0.05	0.01	0.10	0.08	48	4.44	0.0695	0.0895	4.47	0.0695	0.0895
0.05	0.01	0.15	0.08	29	3.06	0.0615	0.0906	3.12	0.0615	0.0906
0.03	0.02	0.10	0.05	71	5.61	0.0613	0.0944	5.62	0.0613	0.0944
0.03	0.02	0.15	0.05	37	3.45	0.0758	0.0944	3.45	0.0758	0.0944
0.03	0.02	0.10	0.08	43	3.85	0.0616	0.0956	3.85	0.0616	0.0956
0.03	0.02	0.15	0.08	27	2.75	0.0813	0.0832	2.82	0.0506	0.0991
0.05	0.02	0.10	0.05	119	10.64	0.0590	0.0939	10.67	0.0590	0.0939
0.05	0.02	0.15	0.05	54	5.66	0.0650	0.0930	5.68	0.0650	0.0930
0.05	0.02	0.10	0.08	64	6.47	0.0669	0.0882	6.49	0.0669	0.0882
0.05	0.02	0.15	0.08	36	4.16	0.0669	0.0914	4.19	0.0669	0.0914

**Table A1.1.11 - Parameter Selection Examples for the 3-AS-S-1($n, T, 0.75$)
Acceptance Sampling Plan having $\alpha = 0.10$ and $\beta = 0.05$**

P_{12}	P_{13}	P_{22}	P_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	58	3.45	0.1134	0.0384	3.48	0.1134	0.0384
0.03	0.01	0.15	0.05	33	2.26	0.1009	0.0496	2.26	0.1009	0.0496
0.03	0.01	0.10	0.08	39	2.55	0.0905	0.0442	2.60	0.0905	0.0442
0.03	0.01	0.15	0.08	25	1.84	0.0882	0.0465	1.88	0.0882	0.0465
0.05	0.01	0.10	0.05	91	6.64	0.1070	0.0422	6.65	0.1070	0.0422
0.05	0.01	0.15	0.05	47	3.90	0.1141	0.0386	3.93	0.1141	0.0386
0.05	0.01	0.10	0.08	55	4.41	0.1151	0.0404	4.44	0.1151	0.0404
0.05	0.01	0.15	0.08	33	2.96	0.1344	0.0354	2.98	0.1344	0.0354
0.03	0.02	0.10	0.05	80	5.55	0.1003	0.0442	5.57	0.1003	0.0442
0.03	0.02	0.15	0.05	43	3.40	0.1170	0.0420	3.44	0.1170	0.0420
0.03	0.02	0.10	0.08	50	3.82	0.1020	0.0417	3.85	0.1020	0.0417
0.03	0.02	0.15	0.08	31	2.65	0.1142	0.0418	2.70	0.1142	0.0418
0.05	0.02	0.10	0.05	129	10.50	0.1138	0.0404	10.50	0.0969	0.0481
0.05	0.02	0.15	0.05	60	5.55	0.1015	0.0468	5.56	0.1015	0.0468
0.05	0.02	0.10	0.08	72	6.44	0.1136	0.0393	6.48	0.1136	0.0393
0.05	0.02	0.15	0.08	41	4.09	0.1051	0.0432	4.13	0.1051	0.0432

**Table A1.1.12 - Parameter Selection Examples for the 3-AS-S-1($n, T, 0.75$)
Acceptance Sampling Plan having $\alpha = 0.10$ and $\beta = 0.10$**

P_{12}	P_{13}	P_{22}	P_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	41	2.65	0.1030	0.0985	2.65	0.1030	0.0985
0.03	0.01	0.15	0.05	24	1.79	0.0805	0.1094	1.84	0.0805	0.1094
0.03	0.01	0.10	0.08	27	1.95	0.1045	0.0965	1.95	0.1045	0.0965
0.03	0.01	0.15	0.08	18	1.46	0.1607	0.0577	1.52	0.0890	0.1103
0.05	0.01	0.10	0.05	66	5.11	0.1065	0.0972	5.12	0.1065	0.0972
0.05	0.01	0.15	0.05	34	3.03	0.0993	0.0991	3.07	0.0993	0.0991
0.05	0.01	0.10	0.08	40	3.44	0.1064	0.0845	3.48	0.1064	0.0845
0.05	0.01	0.15	0.08	24	2.33	0.1028	0.0901	2.38	0.1028	0.0901
0.03	0.02	0.10	0.05	58	4.29	0.0991	0.1018	4.31	0.0991	0.1018
0.03	0.02	0.15	0.05	31	2.65	0.1142	0.0961	2.69	0.1142	0.0961
0.03	0.02	0.10	0.08	36	2.97	0.1142	0.0854	3.00	0.0944	0.1009
0.03	0.02	0.15	0.08	22	2.06	0.0948	0.0897	2.09	0.0948	0.0897
0.05	0.02	0.10	0.05	95	8.11	0.1046	0.0963	8.11	0.1046	0.0963
0.05	0.02	0.15	0.05	44	4.33	0.1003	0.0958	4.36	0.1003	0.0958
0.05	0.02	0.10	0.08	52	4.95	0.1163	0.0867	4.96	0.1163	0.0867
0.05	0.02	0.15	0.08	30	3.21	0.1290	0.0773	3.27	0.0875	0.1104

APPENDIX 1.2

PARAMETER SELECTION EXAMPLES FOR THE 3-AS-S-2(n, T, v) ACCEPTANCE SAMPLING PLAN

Table A1.2.1 - Parameter Selection Examples for the 3-AS-S-2($n, T, 2$) Acceptance Sampling Plan having $\alpha = 0.05$ and $\beta = 0.05$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	81	90.9	0.0804	0.0309	90.9	0.0804	0.0309
0.03	0.01	0.15	0.05	53	60.3	0.0606	0.0390	60.3	0.0606	0.0390
0.03	0.01	0.10	0.08	49	55.9	0.0869	0.0285	56.0	0.0869	0.0285
0.03	0.01	0.15	0.08	36	41.5	0.0733	0.0319	41.7	0.0733	0.0319
0.05	0.01	0.10	0.05	106	120.7	0.0675	0.0368	120.7	0.0675	0.0368
0.05	0.01	0.15	0.05	66	76.2	0.0555	0.0423	76.3	0.0555	0.0423
0.05	0.01	0.10	0.08	59	68.3	0.0609	0.0389	68.5	0.0609	0.0389
0.05	0.01	0.15	0.08	42	49.3	0.0594	0.0399	49.3	0.0594	0.0399
0.03	0.02	0.10	0.05	140	161.3	0.0586	0.0421	161.4	0.0586	0.0421
0.03	0.02	0.15	0.05	84	98.3	0.0601	0.0404	98.4	0.0601	0.0404
0.03	0.02	0.10	0.08	72	84.8	0.0719	0.0357	84.8	0.0719	0.0357
0.03	0.02	0.15	0.08	51	60.9	0.0801	0.0313	60.9	0.0801	0.0313
0.05	0.02	0.10	0.05	188	219.2	0.0548	0.0452	219.3	0.0548	0.0452
0.05	0.02	0.15	0.05	105	124.4	0.0601	0.0412	124.5	0.0601	0.0412
0.05	0.02	0.10	0.08	88	104.9	0.0717	0.0348	105.0	0.0717	0.0348
0.05	0.02	0.15	0.08	60	72.5	0.0659	0.0378	72.6	0.0659	0.0378

Table A1.2.2 - Parameter Selection Examples for the 3-AS-S-2($n, T, 2$) Acceptance Sampling Plan having $\alpha = 0.05$ and $\beta = 0.10$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	59	66.9	0.0819	0.0767	66.9	0.0819	0.0767
0.03	0.01	0.15	0.05	38	43.7	0.0826	0.0769	43.8	0.0826	0.0769
0.03	0.01	0.10	0.08	35	40.4	0.0689	0.0889	40.5	0.0689	0.0889
0.03	0.01	0.15	0.08	25	29.3	0.0586	0.0977	29.3	0.0586	0.0977
0.05	0.01	0.10	0.05	78	89.5	0.0661	0.0887	89.6	0.0661	0.0887
0.05	0.01	0.15	0.05	48	56.0	0.0511	0.1094	56.1	0.0511	0.1094
0.05	0.01	0.10	0.08	42	49.3	0.0594	0.0982	49.3	0.0594	0.0982
0.05	0.01	0.15	0.08	30	35.7	0.0788	0.0793	35.8	0.0788	0.0793
0.03	0.02	0.10	0.05	105	122.0	0.0534	0.1050	122.1	0.0534	0.1050
0.03	0.02	0.15	0.05	63	74.5	0.0664	0.0871	74.6	0.0664	0.0871
0.03	0.02	0.10	0.08	53	63.2	0.0565	0.1027	63.2	0.0565	0.1027
0.03	0.02	0.15	0.08	38	46.0	0.0831	0.0693	46.1	0.0536	0.1065
0.05	0.02	0.10	0.05	142	166.8	0.0653	0.0867	166.8	0.0653	0.0867
0.05	0.02	0.15	0.05	79	94.5	0.0635	0.0909	94.6	0.0635	0.0909
0.05	0.02	0.10	0.08	65	78.3	0.0602	0.0952	78.4	0.0602	0.0952
0.05	0.02	0.15	0.08	44	53.9	0.0813	0.0769	54.0	0.0813	0.0769

Table A1.2.3 - Parameter Selection Examples for the 3-AS-S-2($n, T, 2$) Acceptance Sampling Plan having $\alpha = 0.10$ and $\beta = 0.05$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	70	77.9	0.1293	0.0296	77.9	0.1293	0.0296
0.03	0.01	0.15	0.05	46	51.7	0.1246	0.0276	51.8	0.1246	0.0276
0.03	0.01	0.10	0.08	43	48.4	0.1079	0.0344	48.6	0.1079	0.0344
0.03	0.01	0.15	0.08	31	35.3	0.0928	0.0379	35.3	0.0928	0.0379
0.05	0.01	0.10	0.05	91	102.7	0.1190	0.0338	102.8	0.1190	0.0338
0.05	0.01	0.15	0.05	56	64.0	0.1374	0.0272	64.0	0.0849	0.0470
0.05	0.01	0.10	0.08	51	58.4	0.1056	0.0374	58.5	0.1056	0.0374
0.05	0.01	0.15	0.08	37	42.8	0.1301	0.0272	43.0	0.1301	0.0272
0.03	0.02	0.10	0.05	117	133.8	0.1161	0.0356	133.8	0.1161	0.0356
0.03	0.02	0.15	0.05	70	81.1	0.0945	0.0442	81.2	0.0945	0.0442
0.03	0.02	0.10	0.08	62	72.1	0.0940	0.0434	72.2	0.0940	0.0434
0.03	0.02	0.15	0.08	44	51.8	0.1213	0.0300	51.9	0.1213	0.0300
0.05	0.02	0.10	0.05	156	180.6	0.1098	0.0397	180.7	0.1098	0.0397
0.05	0.02	0.15	0.05	88	103.3	0.1006	0.0415	103.4	0.1006	0.0415
0.05	0.02	0.10	0.08	75	88.4	0.1056	0.0392	88.5	0.1056	0.0392
0.05	0.02	0.15	0.08	51	60.9	0.1274	0.0313	60.9	0.1274	0.0313

Table A1.2.4 - Parameter Selection Examples for the 3-AS-S-2($n, T, 2$) Acceptance Sampling Plan having $\alpha = 0.10$ and $\beta = 0.10$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	50	56.1	0.0912	0.1035	56.2	0.0912	0.1035
0.03	0.01	0.15	0.05	32	36.4	0.0992	0.0913	36.4	0.0992	0.0913
0.03	0.01	0.10	0.08	30	34.2	0.0867	0.0982	34.3	0.0867	0.0982
0.03	0.01	0.15	0.08	22	25.4	0.1082	0.0882	25.5	0.1082	0.0882
0.05	0.01	0.10	0.05	65	74.0	0.1345	0.0729	74.0	0.0854	0.1084
0.05	0.01	0.15	0.05	40	46.2	0.0915	0.1027	46.2	0.0915	0.1027
0.05	0.01	0.10	0.08	36	41.7	0.1220	0.0793	41.8	0.1220	0.0793
0.05	0.01	0.15	0.08	26	30.5	0.1078	0.0839	30.6	0.1078	0.0839
0.03	0.02	0.10	0.05	85	98.0	0.1265	0.0777	98.0	0.0903	0.1083
0.03	0.02	0.15	0.05	51	59.7	0.1195	0.0806	59.8	0.1195	0.0806
0.03	0.02	0.10	0.08	44	51.8	0.1213	0.0799	51.8	0.1213	0.0799
0.03	0.02	0.15	0.08	31	37.0	0.1487	0.0685	37.0	0.0895	0.1115
0.05	0.02	0.10	0.05	114	132.9	0.1209	0.0824	132.9	0.1209	0.0824
0.05	0.02	0.15	0.05	64	75.8	0.1223	0.0794	75.9	0.1223	0.0794
0.05	0.02	0.10	0.08	53	63.2	0.0963	0.1027	63.2	0.0963	0.1027
0.05	0.02	0.15	0.08	37	44.7	0.1210	0.0791	44.8	0.1210	0.0791

Table A1.2.5 - Parameter Selection Examples for the 3-AS-S-2($n, T, 5$) Acceptance Sampling Plan having $\alpha = 0.05$ and $\beta = 0.05$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	106	186.0	0.0647	0.0403	186.2	0.0647	0.0403
0.03	0.01	0.15	0.05	79	143.6	0.0572	0.0374	143.9	0.0572	0.0374
0.03	0.01	0.10	0.08	57	108.2	0.0950	0.0341	108.5	0.0950	0.0341
0.03	0.01	0.15	0.08	47	91.8	0.0812	0.0361	92.2	0.0812	0.0361
0.05	0.01	0.10	0.05	123	223.1	0.0596	0.0405	223.5	0.0596	0.0405
0.05	0.01	0.15	0.05	90	169.0	0.0717	0.0329	169.5	0.0717	0.0329
0.05	0.01	0.10	0.08	63	123.7	0.0651	0.0371	124.3	0.0651	0.0371
0.05	0.01	0.15	0.08	51	103.1	0.0714	0.0404	103.3	0.0714	0.0404
0.03	0.02	0.10	0.05	205	408.3	0.0637	0.0387	408.7	0.0637	0.0387
0.03	0.02	0.15	0.05	143	295.9	0.0621	0.0404	296.4	0.0621	0.0404
0.03	0.02	0.10	0.08	90	197.2	0.0704	0.0374	197.3	0.0704	0.0374
0.03	0.02	0.15	0.08	72	162.8	0.0636	0.0367	162.8	0.0636	0.0367
0.05	0.02	0.10	0.05	244	498.4	0.0607	0.0408	498.8	0.0607	0.0408
0.05	0.02	0.15	0.05	165	350.0	0.0594	0.0414	350.3	0.0594	0.0414
0.05	0.02	0.10	0.08	100	224.6	0.0600	0.0415	225.0	0.0600	0.0415
0.05	0.02	0.15	0.08	79	183.1	0.0626	0.0421	183.2	0.0626	0.0421

Table A1.2.6 - Parameter Selection Examples for the 3-AS-S-2($n, T, 5$) Acceptance Sampling Plan having $\alpha = 0.05$ and $\beta = 0.10$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	77	140.4	0.0689	0.0815	140.9	0.0689	0.0815
0.03	0.01	0.15	0.05	57	108.2	0.0950	0.0801	108.4	0.0950	0.0801
0.03	0.01	0.10	0.08	41	81.8	0.0653	0.0977	82.8	0.0653	0.0977
0.03	0.01	0.15	0.08	33	68.2	0.0602	0.0750	68.4	0.0602	0.0750
0.05	0.01	0.10	0.05	89	167.3	0.0692	0.0886	167.6	0.0692	0.0886
0.05	0.01	0.15	0.05	65	127.1	0.0716	0.0883	127.4	0.0716	0.0883
0.05	0.01	0.10	0.08	45	92.6	0.0812	0.0837	93.5	0.0695	0.0985
0.05	0.01	0.15	0.08	36	76.7	0.0580	0.0958	76.8	0.0580	0.0958
0.03	0.02	0.10	0.05	154	316.0	0.0625	0.0919	316.5	0.0625	0.0919
0.03	0.02	0.15	0.05	107	229.2	0.0665	0.0902	229.5	0.0665	0.0902
0.03	0.02	0.10	0.08	66	151.2	0.0564	0.0929	151.4	0.0564	0.0929
0.03	0.02	0.15	0.08	53	125.7	0.0756	0.0954	126.0	0.0756	0.0954
0.05	0.02	0.10	0.05	183	384.1	0.0593	0.0958	384.3	0.0593	0.0958
0.05	0.02	0.15	0.05	124	271.4	0.0665	0.0870	271.7	0.0665	0.0870
0.05	0.02	0.10	0.08	73	171.0	0.0657	0.0927	171.1	0.0657	0.0927
0.05	0.02	0.15	0.08	58	140.6	0.0730	0.0893	140.8	0.0730	0.0893

Table A1.2.7 - Parameter Selection Examples for the 3-AS-S-2($n, T, 5$) Acceptance Sampling Plan having $\alpha = 0.10$ and $\beta = 0.05$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	92	155.5	0.1147	0.0318	156.0	0.1147	0.0318
0.03	0.01	0.15	0.05	68	118.6	0.1344	0.0310	118.7	0.1344	0.0310
0.03	0.01	0.10	0.08	51	92.0	0.0979	0.0363	92.9	0.0979	0.0363
0.03	0.01	0.15	0.08	42	77.7	0.1016	0.0260	78.6	0.0759	0.0349
0.05	0.01	0.10	0.05	106	185.9	0.1194	0.0323	186.2	0.0996	0.0403
0.05	0.01	0.15	0.05	77	139.3	0.1168	0.0338	139.3	0.1168	0.0338
0.05	0.01	0.10	0.08	55	103.2	0.1053	0.0410	103.3	0.1053	0.0410
0.05	0.01	0.15	0.08	45	86.5	0.0948	0.0334	86.8	0.0948	0.0334
0.03	0.02	0.10	0.05	171	330.8	0.1158	0.0369	331.1	0.1006	0.0430
0.03	0.02	0.15	0.05	119	238.1	0.1091	0.0358	238.3	0.1091	0.0358
0.03	0.02	0.10	0.08	77	161.6	0.0940	0.0404	161.8	0.0940	0.0404
0.03	0.02	0.15	0.08	62	133.6	0.1298	0.0323	134.2	0.1134	0.0406
0.05	0.02	0.10	0.05	203	403.9	0.1035	0.0422	404.1	0.1035	0.0422
0.05	0.02	0.15	0.05	138	283.7	0.1068	0.0388	284.2	0.1068	0.0388
0.05	0.02	0.10	0.08	85	183.5	0.1093	0.0378	183.6	0.1093	0.0378
0.05	0.02	0.15	0.08	68	150.6	0.1192	0.0352	151.3	0.1192	0.0352

Table A1.2.8 - Parameter Selection Examples for the 3-AS-S-2($n, T, 5$) Acceptance Sampling Plan having $\alpha = 0.10$ and $\beta = 0.10$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	65	113.9	0.1232	0.0942	114.5	0.1232	0.0942
0.03	0.01	0.15	0.05	48	87.2	0.0995	0.0770	87.5	0.0995	0.0770
0.03	0.01	0.10	0.08	35	66.3	0.1164	0.0693	66.9	0.1164	0.0693
0.03	0.01	0.15	0.08	29	56.5	0.1604	0.0744	57.3	0.0795	0.0912
0.05	0.01	0.10	0.05	75	136.0	0.1086	0.0917	136.4	0.1086	0.0917
0.05	0.01	0.15	0.05	55	103.2	0.1053	0.0943	103.7	0.1053	0.0943
0.05	0.01	0.10	0.08	38	74.7	0.0856	0.0999	74.8	0.0856	0.0999
0.05	0.01	0.15	0.08	31	62.6	0.1420	0.0730	62.8	0.1420	0.0730
0.03	0.02	0.10	0.05	124	247.1	0.1127	0.0889	247.1	0.1127	0.0889
0.03	0.02	0.15	0.05	87	180.0	0.1042	0.0936	180.4	0.1042	0.0936
0.03	0.02	0.10	0.08	55	120.4	0.1024	0.0923	120.8	0.1024	0.0923
0.03	0.02	0.15	0.08	44	99.4	0.1177	0.0845	99.7	0.1177	0.0845
0.05	0.02	0.10	0.05	148	302.3	0.1083	0.0916	302.5	0.1083	0.0916
0.05	0.02	0.15	0.05	100	212.1	0.1020	0.0985	212.2	0.1020	0.0985
0.05	0.02	0.10	0.08	61	136.9	0.1255	0.0847	137.4	0.1077	0.0994
0.05	0.02	0.15	0.08	48	111.2	0.1081	0.0841	111.3	0.1081	0.0841

Table A1.2.9 - Parameter Selection Examples for the 3-AS-S-2($n, T, 10$) Acceptance Sampling Plan having $\alpha = 0.05$ and $\beta = 0.05$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	128	474.3	0.0578	0.0376	476.0	0.0578	0.0376
0.03	0.01	0.15	0.05	106	407.9	0.0879	0.0331	409.6	0.0879	0.0331
0.03	0.01	0.10	0.08	64	275.5	0.0875	0.0317	276.8	0.0875	0.0317
0.03	0.01	0.15	0.08	57	252.3	0.1114	0.0386	254.4	0.1114	0.0386
0.05	0.01	0.10	0.05	139	533.1	0.0611	0.0345	534.5	0.0611	0.0345
0.05	0.01	0.15	0.05	114	453.8	0.0805	0.0350	454.5	0.0805	0.0350
0.05	0.01	0.10	0.08	68	301.5	0.0813	0.0274	305.2	0.0592	0.0323
0.05	0.01	0.15	0.08	60	273.8	0.1013	0.0364	276.6	0.0784	0.0388
0.03	0.02	0.10	0.05	263	1225.7	0.0632	0.0402	1227.0	0.0570	0.0428
0.03	0.02	0.15	0.05	207	1001.9	0.0593	0.0412	1002.2	0.0593	0.0412
0.03	0.02	0.10	0.08	104	571.3	0.0594	0.0351	572.9	0.0586	0.0394
0.03	0.02	0.15	0.08	91	514.0	0.0731	0.0370	516.4	0.0566	0.0417
0.05	0.02	0.10	0.05	292	1393.2	0.0605	0.0420	1394.8	0.0605	0.0420
0.05	0.02	0.15	0.05	227	1123.9	0.0615	0.0407	1124.1	0.0615	0.0407
0.05	0.02	0.10	0.08	110	617.7	0.0696	0.0423	619.1	0.0696	0.0423
0.05	0.02	0.15	0.08	96	553.8	0.0597	0.0384	556.4	0.0521	0.0413

Table A1.2.10 - Parameter Selection Examples for the 3-AS-S-2($n, T, 10$) Acceptance Sampling Plan having $\alpha = 0.05$ and $\beta = 0.10$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	92	364.8	0.0656	0.0919	365.4	0.0656	0.0919
0.03	0.01	0.15	0.05	76	314.3	0.0510	0.0922	314.5	0.0510	0.0922
0.03	0.01	0.10	0.08	45	211.4	0.0747	0.1062	212.1	0.0747	0.1062
0.03	0.01	0.15	0.08	40	193.8	0.0608	0.1030	194.8	0.0608	0.1030
0.05	0.01	0.10	0.05	100	408.5	0.0772	0.0997	409.0	0.0772	0.0997
0.05	0.01	0.15	0.05	82	349.0	0.0567	0.0832	349.0	0.0567	0.0832
0.05	0.01	0.10	0.08	48	231.1	0.0834	0.0924	234.0	0.0834	0.0924
0.05	0.01	0.15	0.08	42	209.1	0.0664	0.1017	210.0	0.0664	0.1017
0.03	0.02	0.10	0.05	197	961.3	0.0588	0.0910	962.0	0.0533	0.0975
0.03	0.02	0.15	0.05	156	792.7	0.0656	0.0908	794.6	0.0656	0.0908
0.03	0.02	0.10	0.08	76	446.4	0.0663	0.1009	447.7	0.0663	0.1009
0.03	0.02	0.15	0.08	66	400.3	0.0479	0.0924	400.3	0.0479	0.0924
0.05	0.02	0.10	0.05	219	1090.4	0.0652	0.0913	1091.7	0.0652	0.0913
0.05	0.02	0.15	0.05	171	886.2	0.0603	0.0926	887.9	0.0603	0.0926
0.05	0.02	0.10	0.08	81	484.1	0.0787	0.0944	487.1	0.0745	0.0985
0.05	0.02	0.15	0.08	70	431.9	0.0542	0.0866	433.1	0.0542	0.0866

Table A1.2.11 - Parameter Selection Examples for the 3-AS-S-2($n, T, 10$) Acceptance Sampling Plan having $\alpha = 0.10$ and $\beta = 0.05$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	111	385.1	0.1019	0.0330	387.1	0.1019	0.0330
0.03	0.01	0.15	0.05	92	330.1	0.0887	0.0366	332.3	0.0887	0.0366
0.03	0.01	0.10	0.08	57	225.0	0.1115	0.0403	227.8	0.1115	0.0403
0.03	0.01	0.15	0.08	50	203.1	0.0896	0.0352	203.5	0.0896	0.0352
0.05	0.01	0.10	0.05	120	433.2	0.1197	0.0376	433.9	0.1197	0.0376
0.05	0.01	0.15	0.05	99	369.1	0.0879	0.0362	370.7	0.0879	0.0362
0.05	0.01	0.10	0.08	60	245.7	0.1215	0.0382	248.7	0.1215	0.0382
0.05	0.01	0.15	0.08	53	222.6	0.1001	0.0326	225.2	0.0990	0.0409
0.03	0.02	0.10	0.05	219	975.5	0.1000	0.0416	975.9	0.1000	0.0416
0.03	0.02	0.15	0.05	173	796.7	0.1168	0.0402	797.6	0.1168	0.0402
0.03	0.02	0.10	0.08	89	457.4	0.1043	0.0350	459.2	0.1038	0.0409
0.03	0.02	0.15	0.08	78	410.9	0.0979	0.0372	413.9	0.0822	0.0403
0.05	0.02	0.10	0.05	243	1111.9	0.1100	0.0408	1112.3	0.1100	0.0408
0.05	0.02	0.15	0.05	190	897.8	0.1027	0.0389	899.4	0.1027	0.0389
0.05	0.02	0.10	0.08	94	495.6	0.1197	0.0416	496.8	0.1197	0.0416
0.05	0.02	0.15	0.08	82	443.0	0.0894	0.0369	445.2	0.0894	0.0369

Table A1.2.12 - Parameter Selection Examples for the 3-AS-S-2($n, T, 10$) Acceptance Sampling Plan having $\alpha = 0.10$ and $\beta = 0.10$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	78	288.8	0.1381	0.0894	290.5	0.1381	0.0894
0.03	0.01	0.15	0.05	64	246.6	0.1346	0.0949	246.7	0.1346	0.0949
0.03	0.01	0.10	0.08	39	167.7	0.0654	0.0907	169.1	0.0654	0.0907
0.03	0.01	0.15	0.08	35	154.5	0.0685	0.0652	157.4	0.0685	0.0652
0.05	0.01	0.10	0.05	84	322.4	0.1135	0.0798	322.4	0.1135	0.0798
0.05	0.01	0.15	0.05	70	278.0	0.1387	0.0999	280.4	0.1387	0.0999
0.05	0.01	0.10	0.08	41	182.1	0.0771	0.0979	183.3	0.0771	0.0979
0.05	0.01	0.15	0.08	36	164.7	0.0753	0.0738	164.8	0.0753	0.0738
0.03	0.02	0.10	0.05	160	745.4	0.1043	0.0937	746.8	0.1043	0.0937
0.03	0.02	0.15	0.05	126	609.5	0.1101	0.0919	610.4	0.1101	0.0919
0.03	0.02	0.10	0.08	63	346.2	0.1321	0.1018	346.8	0.1321	0.1018
0.03	0.02	0.15	0.08	55	310.9	0.0980	0.0838	311.5	0.0980	0.0838
0.05	0.02	0.10	0.05	177	844.7	0.1070	0.0969	845.2	0.1070	0.0969
0.05	0.02	0.15	0.05	138	683.1	0.1171	0.0902	683.6	0.1171	0.0902
0.05	0.02	0.10	0.08	67	376.0	0.1343	0.0876	377.6	0.1343	0.0876
0.05	0.02	0.15	0.08	58	334.9	0.1106	0.0902	335.5	0.1106	0.0902

APPENDIX 1.3

PARAMETER SELECTION EXAMPLES FOR THE 3-AS-S-3(n, T, v) ACCEPTANCE SAMPLING PLAN

Table A1.3.1 - Parameter Selection Examples for the 3-AS-S-3($n, T, 2$) Acceptance Sampling Plan having $\alpha = 0.05$ and $\beta = 0.05$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	81	181.8	0.0804	0.0309	181.8	0.0804	0.0309
0.03	0.01	0.15	0.05	53	120.5	0.0606	0.0390	120.7	0.0606	0.0390
0.03	0.01	0.10	0.08	49	111.7	0.0869	0.0285	112.0	0.0869	0.0285
0.03	0.01	0.15	0.08	36	83.1	0.0733	0.0319	83.4	0.0733	0.0319
0.05	0.01	0.10	0.05	106	241.3	0.0675	0.0368	241.4	0.0675	0.0368
0.05	0.01	0.15	0.05	66	152.3	0.0555	0.0423	152.5	0.0555	0.0423
0.05	0.01	0.10	0.08	59	136.7	0.0609	0.0389	136.9	0.0609	0.0389
0.05	0.01	0.15	0.08	42	98.5	0.0594	0.0399	98.7	0.0594	0.0399
0.03	0.02	0.10	0.05	140	322.7	0.0586	0.0421	322.8	0.0586	0.0421
0.03	0.02	0.15	0.05	84	196.7	0.0601	0.0404	196.9	0.0601	0.0404
0.03	0.02	0.10	0.08	72	169.5	0.0719	0.0357	169.5	0.0719	0.0357
0.03	0.02	0.15	0.08	51	121.7	0.0801	0.0313	121.9	0.0801	0.0313
0.05	0.02	0.10	0.05	188	438.4	0.0548	0.0452	438.5	0.0548	0.0452
0.05	0.02	0.15	0.05	105	248.8	0.0601	0.0412	248.9	0.0601	0.0412
0.05	0.02	0.10	0.08	88	209.8	0.0717	0.0348	209.9	0.0717	0.0348
0.05	0.02	0.15	0.08	60	145.1	0.0659	0.0378	145.2	0.0659	0.0378

Table A1.3.2 - Parameter Selection Examples for the 3-AS-S-3($n, T, 2$) Acceptance Sampling Plan having $\alpha = 0.05$ and $\beta = 0.10$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	59	133.7	0.0819	0.0767	133.8	0.0819	0.0767
0.03	0.01	0.15	0.05	38	87.5	0.0826	0.0769	87.5	0.0826	0.0769
0.03	0.01	0.10	0.08	35	80.8	0.0689	0.0889	81.1	0.0689	0.0889
0.03	0.01	0.15	0.08	25	58.6	0.0586	0.0977	58.6	0.0586	0.0977
0.05	0.01	0.10	0.05	78	179.1	0.0661	0.0887	179.2	0.0661	0.0887
0.05	0.01	0.15	0.05	48	112.0	0.0511	0.1094	112.1	0.0511	0.1094
0.05	0.01	0.10	0.08	42	98.5	0.0594	0.0982	98.6	0.0594	0.0982
0.05	0.01	0.15	0.08	30	71.4	0.0788	0.0793	71.5	0.0788	0.0793
0.03	0.02	0.10	0.05	105	244.0	0.0534	0.1050	244.2	0.0534	0.1050
0.03	0.02	0.15	0.05	63	149.1	0.0664	0.0871	149.3	0.0664	0.0871
0.03	0.02	0.10	0.08	53	126.3	0.0565	0.1027	126.4	0.0565	0.1027
0.03	0.02	0.15	0.08	38	92.0	0.0831	0.0693	92.3	0.0536	0.1065
0.05	0.02	0.10	0.05	142	333.5	0.0653	0.0867	333.7	0.0653	0.0867
0.05	0.02	0.15	0.05	79	189.0	0.0635	0.0909	189.1	0.0635	0.0909
0.05	0.02	0.10	0.08	65	156.7	0.0602	0.0952	156.9	0.0602	0.0952
0.05	0.02	0.15	0.08	44	107.9	0.0813	0.0769	107.9	0.0813	0.0769

Table A1.3.3 - Parameter Selection Examples for the 3-AS-S-3($n, T, 2$) Acceptance Sampling Plan having $\alpha = 0.10$ and $\beta = 0.05$

P_{12}	P_{13}	P_{22}	P_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	70	155.7	0.1293	0.0296	155.8	0.1293	0.0296
0.03	0.01	0.15	0.05	46	103.5	0.1246	0.0276	103.7	0.1246	0.0276
0.03	0.01	0.10	0.08	43	96.9	0.1079	0.0344	97.1	0.1079	0.0344
0.03	0.01	0.15	0.08	31	70.6	0.0928	0.0379	70.7	0.0928	0.0379
0.05	0.01	0.10	0.05	91	205.5	0.1190	0.0338	205.6	0.1190	0.0338
0.05	0.01	0.15	0.05	56	128.0	0.1374	0.0272	128.0	0.0849	0.0470
0.05	0.01	0.10	0.08	51	116.9	0.1056	0.0374	117.0	0.1056	0.0374
0.05	0.01	0.15	0.08	37	85.6	0.1301	0.0272	85.9	0.1301	0.0272
0.03	0.02	0.10	0.05	117	267.5	0.1161	0.0356	267.7	0.1161	0.0356
0.03	0.02	0.15	0.05	70	162.2	0.0945	0.0442	162.3	0.0945	0.0442
0.03	0.02	0.10	0.08	62	144.2	0.0940	0.0434	144.4	0.0940	0.0434
0.03	0.02	0.15	0.08	44	103.6	0.1213	0.0300	103.8	0.1213	0.0300
0.05	0.02	0.10	0.05	156	361.3	0.1098	0.0397	361.3	0.1098	0.0397
0.05	0.02	0.15	0.05	88	206.6	0.1006	0.0415	206.8	0.1006	0.0415
0.05	0.02	0.10	0.08	75	176.9	0.1056	0.0392	177.1	0.1056	0.0392
0.05	0.02	0.15	0.08	51	121.8	0.1274	0.0313	121.9	0.1274	0.0313

Table A1.3.4 - Parameter Selection Examples for the 3-AS-S-3($n, T, 2$) Acceptance Sampling Plan having $\alpha = 0.10$ and $\beta = 0.10$

P_{12}	P_{13}	P_{22}	P_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	50	112.2	0.0912	0.1035	112.3	0.0912	0.1035
0.03	0.01	0.15	0.05	32	72.8	0.0992	0.0913	72.8	0.0992	0.0913
0.03	0.01	0.10	0.08	30	68.4	0.0867	0.0982	68.6	0.0867	0.0982
0.03	0.01	0.15	0.08	22	50.7	0.1082	0.0882	51.0	0.1082	0.0882
0.05	0.01	0.10	0.05	65	148.0	0.1345	0.0729	148.1	0.0854	0.1084
0.05	0.01	0.15	0.05	40	92.3	0.0915	0.1027	92.4	0.0915	0.1027
0.05	0.01	0.10	0.08	36	83.4	0.1220	0.0793	83.6	0.1220	0.0793
0.05	0.01	0.15	0.08	26	60.9	0.1078	0.0839	61.2	0.1078	0.0839
0.03	0.02	0.10	0.05	85	195.9	0.1265	0.0777	196.0	0.0903	0.1083
0.03	0.02	0.15	0.05	51	119.4	0.1195	0.0806	119.5	0.1195	0.0806
0.03	0.02	0.10	0.08	44	103.6	0.1213	0.0799	103.7	0.1213	0.0799
0.03	0.02	0.15	0.08	31	74.0	0.1487	0.0685	74.1	0.0895	0.1115
0.05	0.02	0.10	0.05	114	265.9	0.1209	0.0824	265.9	0.1209	0.0824
0.05	0.02	0.15	0.05	64	151.7	0.1223	0.0794	151.8	0.1223	0.0794
0.05	0.02	0.10	0.08	53	126.4	0.0963	0.1027	126.4	0.0963	0.1027
0.05	0.02	0.15	0.08	37	89.4	0.1210	0.0791	89.7	0.1210	0.0791

Table A1.3.5 - Parameter Selection Examples for the 3-AS-S-3($n, T, 5$) Acceptance Sampling Plan having $\alpha = 0.05$ and $\beta = 0.05$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	106	930.1	0.0647	0.0403	931.2	0.0647	0.0403
0.03	0.01	0.15	0.05	79	717.9	0.0572	0.0374	719.5	0.0572	0.0374
0.03	0.01	0.10	0.08	57	541.1	0.0950	0.0341	542.4	0.0950	0.0341
0.03	0.01	0.15	0.08	47	459.0	0.0812	0.0361	461.2	0.0812	0.0361
0.05	0.01	0.10	0.05	123	1115.7	0.0596	0.0405	1117.7	0.0596	0.0405
0.05	0.01	0.15	0.05	90	844.9	0.0717	0.0329	847.4	0.0717	0.0329
0.05	0.01	0.10	0.08	63	618.3	0.0651	0.0371	621.3	0.0651	0.0371
0.05	0.01	0.15	0.08	51	515.4	0.0714	0.0404	516.4	0.0714	0.0404
0.03	0.02	0.10	0.05	205	2041.8	0.0637	0.0387	2043.7	0.0637	0.0387
0.03	0.02	0.15	0.05	143	1479.5	0.0621	0.0404	1482.0	0.0621	0.0404
0.03	0.02	0.10	0.08	90	986.2	0.0704	0.0374	986.6	0.0704	0.0374
0.03	0.02	0.15	0.08	72	814.1	0.0636	0.0367	814.2	0.0636	0.0367
0.05	0.02	0.10	0.05	244	2492.0	0.0607	0.0408	2494.2	0.0607	0.0408
0.05	0.02	0.15	0.05	165	1749.8	0.0594	0.0414	1751.4	0.0594	0.0414
0.05	0.02	0.10	0.08	100	1123.2	0.0600	0.0415	1125.1	0.0600	0.0415
0.05	0.02	0.15	0.08	79	915.3	0.0626	0.0421	915.9	0.0626	0.0421

Table A1.3.6 - Parameter Selection Examples for the 3-AS-S-3($n, T, 5$) Acceptance Sampling Plan having $\alpha = 0.05$ and $\beta = 0.10$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	77	702.0	0.0689	0.0815	704.3	0.0689	0.0815
0.03	0.01	0.15	0.05	57	541.1	0.0950	0.0801	542.2	0.0950	0.0801
0.03	0.01	0.10	0.08	41	409.0	0.0653	0.0977	413.8	0.0653	0.0977
0.03	0.01	0.15	0.08	33	341.2	0.0602	0.0750	341.9	0.0602	0.0750
0.05	0.01	0.10	0.05	89	836.6	0.0692	0.0886	838.0	0.0692	0.0886
0.05	0.01	0.15	0.05	65	635.3	0.0716	0.0883	636.8	0.0716	0.0883
0.05	0.01	0.10	0.08	45	463.2	0.0812	0.0837	467.4	0.0695	0.0985
0.05	0.01	0.15	0.08	36	383.7	0.0580	0.0958	384.1	0.0580	0.0958
0.03	0.02	0.10	0.05	154	1580.2	0.0625	0.0919	1582.4	0.0625	0.0919
0.03	0.02	0.15	0.05	107	1146.3	0.0665	0.0902	1147.5	0.0665	0.0902
0.03	0.02	0.10	0.08	66	756.0	0.0564	0.0929	757.0	0.0564	0.0929
0.03	0.02	0.15	0.08	53	628.3	0.0756	0.0954	630.0	0.0756	0.0954
0.05	0.02	0.10	0.05	183	1920.3	0.0593	0.0958	1921.6	0.0593	0.0958
0.05	0.02	0.15	0.05	124	1357.0	0.0665	0.0870	1358.7	0.0665	0.0870
0.05	0.02	0.10	0.08	73	855.2	0.0657	0.0927	855.7	0.0657	0.0927
0.05	0.02	0.15	0.08	58	702.9	0.0730	0.0893	704.1	0.0730	0.0893

Table A1.3.7 - Parameter Selection Examples for the 3-AS-S-3($n,T,5$) Acceptance Sampling Plan having $\alpha = 0.10$ and $\beta = 0.05$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	92	777.5	0.1147	0.0318	779.8	0.1147	0.0318
0.03	0.01	0.15	0.05	68	593.0	0.1344	0.0310	593.4	0.1344	0.0310
0.03	0.01	0.10	0.08	51	459.9	0.0979	0.0363	464.6	0.0979	0.0363
0.03	0.01	0.15	0.08	42	388.3	0.1016	0.0260	393.2	0.0759	0.0349
0.05	0.01	0.10	0.05	106	929.7	0.1194	0.0323	931.2	0.0996	0.0403
0.05	0.01	0.15	0.05	77	696.3	0.1168	0.0338	696.4	0.1168	0.0338
0.05	0.01	0.10	0.08	55	515.9	0.1053	0.0410	516.3	0.1053	0.0410
0.05	0.01	0.15	0.08	45	432.5	0.0948	0.0334	433.9	0.0948	0.0334
0.03	0.02	0.10	0.05	171	1653.9	0.1158	0.0369	1655.6	0.1006	0.0430
0.03	0.02	0.15	0.05	119	1190.5	0.1091	0.0358	1191.6	0.1091	0.0358
0.03	0.02	0.10	0.08	77	807.8	0.0940	0.0404	808.8	0.0940	0.0404
0.03	0.02	0.15	0.08	62	668.1	0.1298	0.0323	670.9	0.1134	0.0406
0.05	0.02	0.10	0.05	203	2019.6	0.1035	0.0422	2020.7	0.1035	0.0422
0.05	0.02	0.15	0.05	138	1418.4	0.1068	0.0388	1421.1	0.1068	0.0388
0.05	0.02	0.10	0.08	85	917.4	0.1093	0.0378	917.9	0.1093	0.0378
0.05	0.02	0.15	0.08	68	753.2	0.1192	0.0352	756.6	0.1192	0.0352

Table A1.3.8 - Parameter Selection Examples for the 3-AS-S-3($n,T,5$) Acceptance Sampling Plan having $\alpha = 0.10$ and $\beta = 0.10$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	65	569.7	0.1232	0.0942	572.4	0.1232	0.0942
0.03	0.01	0.15	0.05	48	436.1	0.0995	0.0770	437.3	0.0995	0.0770
0.03	0.01	0.10	0.08	35	331.7	0.1164	0.0693	334.5	0.1164	0.0693
0.03	0.01	0.15	0.08	29	282.5	0.1604	0.0744	286.4	0.0795	0.0912
0.05	0.01	0.10	0.05	75	680.0	0.1086	0.0917	682.2	0.1086	0.0917
0.05	0.01	0.15	0.05	55	515.9	0.1053	0.0943	518.7	0.1053	0.0943
0.05	0.01	0.10	0.08	38	373.3	0.0856	0.0999	373.9	0.0856	0.0999
0.05	0.01	0.15	0.08	31	313.2	0.1420	0.0730	314.0	0.1420	0.0730
0.03	0.02	0.10	0.05	124	1235.4	0.1127	0.0889	1235.5	0.1127	0.0889
0.03	0.02	0.15	0.05	87	899.9	0.1042	0.0936	902.0	0.1042	0.0936
0.03	0.02	0.10	0.08	55	602.1	0.1024	0.0923	603.9	0.1024	0.0923
0.03	0.02	0.15	0.08	44	497.0	0.1177	0.0845	498.5	0.1177	0.0845
0.05	0.02	0.10	0.05	148	1511.6	0.1083	0.0916	1512.7	0.1083	0.0916
0.05	0.02	0.15	0.05	100	1060.7	0.1020	0.0985	1061.2	0.1020	0.0985
0.05	0.02	0.10	0.08	61	684.7	0.1255	0.0847	687.1	0.1077	0.0994
0.05	0.02	0.15	0.08	48	556.1	0.1081	0.0841	556.6	0.1081	0.0841

Table A1.3.9 - Parameter Selection Examples for the 3-AS-S-3($n, T, 10$) Acceptance Sampling Plan having $\alpha = 0.05$ and $\beta = 0.05$

P_{12}	P_{13}	P_{22}	P_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	128	4743.0	0.0578	0.0376	4759.7	0.0578	0.0376
0.03	0.01	0.15	0.05	106	4079.3	0.0879	0.0331	4096.0	0.0879	0.0331
0.03	0.01	0.10	0.08	64	2754.7	0.0875	0.0317	2767.7	0.0875	0.0317
0.03	0.01	0.15	0.08	57	2522.9	0.1114	0.0386	2544.2	0.1114	0.0386
0.05	0.01	0.10	0.05	139	5330.7	0.0611	0.0345	5344.6	0.0611	0.0345
0.05	0.01	0.15	0.05	114	4537.7	0.0805	0.0350	4545.1	0.0805	0.0350
0.05	0.01	0.10	0.08	68	3015.5	0.0813	0.0274	3052.3	0.0592	0.0323
0.05	0.01	0.15	0.08	60	2738.0	0.1013	0.0364	2766.4	0.0784	0.0388
0.03	0.02	0.10	0.05	263	12257.0	0.0632	0.0402	12270.0	0.0570	0.0428
0.03	0.02	0.15	0.05	207	10018.5	0.0593	0.0412	10021.5	0.0593	0.0412
0.03	0.02	0.10	0.08	104	5712.7	0.0594	0.0351	5729.4	0.0586	0.0394
0.03	0.02	0.15	0.08	91	5139.5	0.0731	0.0370	5163.9	0.0566	0.0417
0.05	0.02	0.10	0.05	292	13932.2	0.0605	0.0420	13948.0	0.0605	0.0420
0.05	0.02	0.15	0.05	227	11239.3	0.0615	0.0407	11241.3	0.0615	0.0407
0.05	0.02	0.10	0.08	110	6176.9	0.0696	0.0423	6191.1	0.0696	0.0423
0.05	0.02	0.15	0.08	96	5538.5	0.0597	0.0384	5564.0	0.0521	0.0413

Table A1.3.10 - Parameter Selection Examples for the 3-AS-S-3($n, T, 10$) Acceptance Sampling Plan having $\alpha = 0.05$ and $\beta = 0.10$

P_{12}	P_{13}	P_{22}	P_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	92	3647.8	0.0656	0.0919	3654.4	0.0656	0.0919
0.03	0.01	0.15	0.05	76	3143.3	0.0510	0.0922	3145.3	0.0510	0.0922
0.03	0.01	0.10	0.08	45	2114.1	0.0747	0.1062	2121.2	0.0747	0.1062
0.03	0.01	0.15	0.08	40	1938.3	0.0608	0.1030	1947.8	0.0608	0.1030
0.05	0.01	0.10	0.05	100	4084.7	0.0772	0.0997	4089.7	0.0772	0.0997
0.05	0.01	0.15	0.05	82	3490.2	0.0567	0.0832	3490.2	0.0567	0.0832
0.05	0.01	0.10	0.08	48	2310.7	0.0834	0.0924	2340.5	0.0834	0.0924
0.05	0.01	0.15	0.08	42	2090.7	0.0664	0.1017	2099.9	0.0664	0.1017
0.03	0.02	0.10	0.05	197	9613.0	0.0588	0.0910	9620.2	0.0533	0.0975
0.03	0.02	0.15	0.05	156	7927.0	0.0656	0.0908	7945.6	0.0656	0.0908
0.03	0.02	0.10	0.08	76	4464.1	0.0663	0.1009	4477.1	0.0663	0.1009
0.03	0.02	0.15	0.08	66	4003.3	0.0479	0.0924	4003.4	0.0479	0.0924
0.05	0.02	0.10	0.05	219	10903.5	0.0652	0.0913	10916.6	0.0652	0.0913
0.05	0.02	0.15	0.05	171	8862.5	0.0603	0.0926	8878.9	0.0603	0.0926
0.05	0.02	0.10	0.08	81	4841.1	0.0787	0.0944	4871.4	0.0745	0.0985
0.05	0.02	0.15	0.08	70	4318.6	0.0542	0.0866	4331.4	0.0542	0.0866

Table A1.3.11 - Parameter Selection Examples for the 3-AS-S-3($n, T, 10$) Acceptance Sampling Plan having $\alpha = 0.10$ and $\beta = 0.05$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	111	3851.0	0.1019	0.0330	3870.9	0.1019	0.0330
0.03	0.01	0.15	0.05	92	3301.4	0.0887	0.0366	3323.0	0.0887	0.0366
0.03	0.01	0.10	0.08	57	2250.2	0.1115	0.0403	2278.2	0.1115	0.0403
0.03	0.01	0.15	0.08	50	2031.0	0.0896	0.0352	2035.2	0.0896	0.0352
0.05	0.01	0.10	0.05	120	4331.8	0.1197	0.0376	4339.0	0.1197	0.0376
0.05	0.01	0.15	0.05	99	3690.6	0.0879	0.0362	3707.3	0.0879	0.0362
0.05	0.01	0.10	0.08	60	2456.6	0.1215	0.0382	2486.6	0.1215	0.0382
0.05	0.01	0.15	0.08	53	2226.1	0.1001	0.0326	2251.7	0.0990	0.0409
0.03	0.02	0.10	0.05	219	9754.9	0.1000	0.0416	9758.6	0.1000	0.0416
0.03	0.02	0.15	0.05	173	7966.6	0.1168	0.0402	7976.2	0.1168	0.0402
0.03	0.02	0.10	0.08	89	4573.8	0.1043	0.0350	4592.3	0.1038	0.0409
0.03	0.02	0.15	0.08	78	4109.0	0.0979	0.0372	4139.0	0.0822	0.0403
0.05	0.02	0.10	0.05	243	11118.6	0.1100	0.0408	11122.7	0.1100	0.0408
0.05	0.02	0.15	0.05	190	8978.5	0.1027	0.0389	8994.0	0.1027	0.0389
0.05	0.02	0.10	0.08	94	4955.6	0.1197	0.0416	4968.4	0.1197	0.0416
0.05	0.02	0.15	0.08	82	4429.7	0.0894	0.0369	4451.7	0.0894	0.0369

Table A1.3.12 - Parameter Selection Examples for the 3-AS-S-3($n, T, 10$) Acceptance Sampling Plan having $\alpha = 0.10$ and $\beta = 0.10$

p_{12}	p_{13}	p_{22}	p_{23}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.03	0.01	0.10	0.05	78	2888.1	0.1381	0.0894	2905.1	0.1381	0.0894
0.03	0.01	0.15	0.05	64	2465.7	0.1346	0.0949	2467.1	0.1346	0.0949
0.03	0.01	0.10	0.08	39	1677.1	0.0654	0.0907	1690.7	0.0654	0.0907
0.03	0.01	0.15	0.08	35	1544.8	0.0685	0.0652	1573.9	0.0685	0.0652
0.05	0.01	0.10	0.05	84	3224.1	0.1135	0.0798	3224.1	0.1135	0.0798
0.05	0.01	0.15	0.05	70	2780.2	0.1387	0.0999	2804.1	0.1387	0.0999
0.05	0.01	0.10	0.08	41	1820.9	0.0771	0.0979	1832.9	0.0771	0.0979
0.05	0.01	0.15	0.08	36	1647.3	0.0753	0.0738	1647.9	0.0753	0.0738
0.03	0.02	0.10	0.05	160	7454.3	0.1043	0.0937	7468.4	0.1043	0.0937
0.03	0.02	0.15	0.05	126	6095.5	0.1101	0.0919	6104.3	0.1101	0.0919
0.03	0.02	0.10	0.08	63	3462.1	0.1321	0.1018	3467.8	0.1321	0.1018
0.03	0.02	0.15	0.08	55	3109.2	0.0980	0.0838	3115.5	0.0980	0.0838
0.05	0.02	0.10	0.05	177	8446.9	0.1070	0.0969	8452.1	0.1070	0.0969
0.05	0.02	0.15	0.05	138	6831.2	0.1171	0.0902	6836.3	0.1171	0.0902
0.05	0.02	0.10	0.08	67	3759.8	0.1343	0.0876	3775.6	0.1343	0.0876
0.05	0.02	0.15	0.08	58	3349.4	0.1106	0.0902	3355.4	0.1106	0.0902

APPENDIX 2

PARAMETER SELECTION EXAMPLES FOR 4-LEVEL ACCEPTANCE SAMPLING PLANS

APPENDIX 2.1

PARAMETER SELECTION EXAMPLES FOR THE 4-AS-S-1(n, T, ν_1, ν_2) ACCEPTANCE SAMPLING PLAN

**Table A2.1.1 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.25, 0.5$)
Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.05$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	37	2.99	0.0792	0.0305	3.00	0.0792	0.0305
0.30	0.10	0.05	24	2.16	0.0729	0.0321	2.18	0.0729	0.0321
0.20	0.15	0.05	26	2.29	0.0576	0.0411	2.34	0.0576	0.0411
0.30	0.15	0.05	18	1.75	0.0534	0.0440	1.80	0.0534	0.0440
0.20	0.10	0.08	27	2.36	0.0652	0.0382	2.37	0.0652	0.0382
0.30	0.10	0.08	19	1.82	0.0622	0.0383	1.85	0.0622	0.0383
0.20	0.15	0.08	20	1.89	0.0717	0.0343	1.92	0.0717	0.0343
0.30	0.15	0.08	15	1.54	0.0573	0.0376	1.61	0.0573	0.0376

**Table A2.1.2 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.25, 0.5$)
Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.05$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	82	8.48	0.0656	0.0372	8.50	0.0509	0.0485
0.30	0.10	0.05	47	5.36	0.0606	0.0399	5.39	0.0606	0.0399
0.20	0.15	0.05	50	5.64	0.0622	0.0388	5.69	0.0622	0.0388
0.30	0.15	0.05	32	3.95	0.0708	0.0321	4.01	0.0488	0.0501
0.20	0.10	0.08	51	5.73	0.0694	0.0355	5.75	0.0510	0.0481
0.30	0.10	0.08	33	4.05	0.0564	0.0441	4.06	0.0564	0.0441
0.20	0.15	0.08	35	4.24	0.0739	0.0323	4.29	0.0517	0.0468
0.30	0.15	0.08	24	3.16	0.0690	0.0349	3.19	0.0690	0.0349

**Table A2.1.3 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.25, 0.5$)
Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.05$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	28	2.42	0.0733	0.0774	2.47	0.0733	0.0774
0.30	0.10	0.05	18	1.75	0.0534	0.1073	1.78	0.0534	0.1073
0.20	0.15	0.05	19	1.82	0.0622	0.0947	1.85	0.0622	0.0947
0.30	0.15	0.05	13	1.39	0.0763	0.0812	1.41	0.0763	0.0812
0.20	0.10	0.08	20	1.89	0.0717	0.0829	1.93	0.0717	0.0829
0.30	0.10	0.08	14	1.47	0.0898	0.0670	1.50	0.0898	0.0670
0.20	0.15	0.08	15	1.54	0.0573	0.0951	1.60	0.0573	0.0951
0.30	0.15	0.08	11	1.24	0.0981	0.0558	1.30	0.0524	0.1059

**Table A2.1.4 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.25, 0.5$)
Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.05$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	62	6.72	0.0672	0.0851	6.73	0.0672	0.0851
0.30	0.10	0.05	36	4.34	0.0593	0.0939	4.37	0.0593	0.0939
0.20	0.15	0.05	38	4.53	0.0543	0.1008	4.57	0.0543	0.1008
0.30	0.15	0.05	24	3.16	0.0690	0.0839	3.18	0.0690	0.0839
0.20	0.10	0.08	38	4.53	0.0543	0.1038	4.54	0.0543	0.1038
0.30	0.10	0.08	25	3.26	0.0538	0.1043	3.29	0.0538	0.1043
0.20	0.15	0.08	26	3.36	0.0632	0.0915	3.39	0.0632	0.0915
0.30	0.15	0.08	18	2.54	0.0573	0.1015	2.57	0.0573	0.1015

**Table A2.1.5 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.25, 0.5$)
Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.10$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	32	2.39	0.1114	0.0334	2.43	0.1114	0.0334
0.30	0.10	0.05	21	1.72	0.1329	0.0226	1.78	0.0820	0.0442
0.20	0.15	0.05	22	1.79	0.0930	0.0422	1.82	0.0930	0.0422
0.30	0.15	0.05	15	1.34	0.1043	0.0365	1.36	0.1043	0.0365
0.20	0.10	0.08	24	1.91	0.1172	0.0301	1.97	0.1172	0.0301
0.30	0.10	0.08	17	1.47	0.1362	0.0214	1.55	0.0791	0.0423
0.20	0.15	0.08	17	1.47	0.1362	0.0284	1.47	0.1362	0.0284
0.30	0.15	0.08	13	1.21	0.1346	0.0214	1.28	0.0763	0.0458

**Table A2.1.6 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.25, 0.5$)
Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.10$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	68	6.70	0.1145	0.0363	6.74	0.1145	0.0363
0.30	0.10	0.05	39	4.20	0.1181	0.0324	4.24	0.1181	0.0324
0.20	0.15	0.05	41	4.38	0.1079	0.0396	4.40	0.1079	0.0396
0.30	0.15	0.05	26	3.02	0.0932	0.0466	3.04	0.0932	0.0466
0.20	0.10	0.08	43	4.56	0.0986	0.0425	4.59	0.0986	0.0425
0.30	0.10	0.08	28	3.21	0.1217	0.0308	3.25	0.0850	0.0483
0.20	0.15	0.08	29	3.30	0.0973	0.0442	3.32	0.0973	0.0442
0.30	0.15	0.08	20	2.45	0.1244	0.0309	2.48	0.1244	0.0309

**Table A2.1.7 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.25, 0.5$)
Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.10$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	23	1.85	0.1047	0.0884	1.88	0.1047	0.0884
0.30	0.10	0.05	15	1.34	0.1043	0.0865	1.38	0.1043	0.0865
0.20	0.15	0.05	16	1.41	0.1198	0.0760	1.45	0.1198	0.0760
0.30	0.15	0.05	11	1.07	0.0981	0.0920	1.11	0.0981	0.0920
0.20	0.10	0.08	17	1.47	0.1362	0.0673	1.52	0.0791	0.1094
0.30	0.10	0.08	12	1.14	0.1158	0.0744	1.19	0.1158	0.0744
0.20	0.15	0.08	12	1.14	0.1158	0.0863	1.14	0.1158	0.0863
0.30	0.15	0.08	9	0.93	0.1361	0.0680	0.96	0.1361	0.0680

**Table A2.1.8 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.25, 0.5$)
Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.10$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	50	5.17	0.1119	0.0870	5.19	0.1119	0.0870
0.30	0.10	0.05	29	3.30	0.0973	0.0969	3.34	0.0973	0.0969
0.20	0.15	0.05	30	3.39	0.1106	0.0897	3.40	0.1106	0.0897
0.30	0.15	0.05	19	2.36	0.1066	0.0938	2.36	0.1066	0.0938
0.20	0.10	0.08	31	3.48	0.1249	0.0789	3.50	0.1249	0.0789
0.30	0.10	0.08	20	2.45	0.1244	0.0785	2.46	0.1244	0.0785
0.20	0.15	0.08	21	2.55	0.0969	0.1021	2.56	0.0969	0.1021
0.30	0.15	0.08	15	1.96	0.1293	0.0682	2.02	0.0816	0.1122

**Table A2.1.9 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.25, 0.75$)
Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.05$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	38	3.62	0.0641	0.0398	3.64	0.0641	0.0398
0.30	0.10	0.05	26	2.72	0.0788	0.0308	2.75	0.0517	0.0487
0.20	0.15	0.05	25	2.64	0.0698	0.0369	2.69	0.0698	0.0369
0.30	0.15	0.05	18	2.08	0.0630	0.0418	2.10	0.0630	0.0418
0.20	0.10	0.08	28	2.88	0.0666	0.0391	2.90	0.0666	0.0391
0.30	0.10	0.08	20	2.25	0.0844	0.0305	2.25	0.0844	0.0305
0.20	0.15	0.08	20	2.25	0.0844	0.0289	2.32	0.0541	0.0450
0.30	0.15	0.08	15	1.83	0.0652	0.0396	1.88	0.0652	0.0396

**Table A2.1.10 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.25, 0.75$)
Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.05$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	84	10.22	0.0630	0.0404	10.22	0.0630	0.0404
0.30	0.10	0.05	51	6.77	0.0537	0.0482	6.78	0.0537	0.0482
0.20	0.15	0.05	47	6.34	0.0580	0.0431	6.39	0.0580	0.0431
0.30	0.15	0.05	32	4.67	0.0660	0.0383	4.71	0.0660	0.0383
0.20	0.10	0.08	54	7.09	0.0578	0.0436	7.12	0.0578	0.0436
0.30	0.10	0.08	36	5.12	0.0616	0.0423	5.13	0.0616	0.0423
0.20	0.15	0.08	34	4.90	0.0638	0.0398	4.95	0.0638	0.0398
0.30	0.15	0.08	25	3.86	0.0620	0.0389	3.94	0.0620	0.0389

**Table A2.1.11 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.25, 0.75$)
Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.05$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	28	2.88	0.0666	0.0919	2.89	0.0666	0.0919
0.30	0.10	0.05	19	2.17	0.0733	0.0853	2.17	0.0733	0.0853
0.20	0.15	0.05	18	2.08	0.0630	0.0991	2.09	0.0630	0.0991
0.30	0.15	0.05	14	1.74	0.0939	0.0623	1.83	0.0545	0.1011
0.20	0.10	0.08	21	2.33	0.0626	0.0921	2.39	0.0626	0.0921
0.30	0.10	0.08	15	1.83	0.0652	0.0947	1.86	0.0652	0.0947
0.20	0.15	0.08	14	1.74	0.0939	0.0771	1.75	0.0939	0.0771
0.30	0.15	0.08	11	1.48	0.0880	0.0695	1.52	0.0551	0.1142

**Table A2.1.12 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.25, 0.75$)
Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.05$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	64	8.15	0.0606	0.0933	8.16	0.0606	0.0933
0.30	0.10	0.05	39	5.46	0.0677	0.0857	5.47	0.0677	0.0857
0.20	0.15	0.05	35	5.01	0.0536	0.1068	5.02	0.0536	0.1068
0.30	0.15	0.05	24	3.74	0.0751	0.0807	3.75	0.0751	0.0807
0.20	0.10	0.08	41	5.68	0.0651	0.0868	5.72	0.0651	0.0868
0.30	0.10	0.08	28	4.21	0.0705	0.0790	4.27	0.0501	0.1070
0.20	0.15	0.08	25	3.86	0.0620	0.0974	3.86	0.0620	0.0974
0.30	0.15	0.08	19	3.13	0.0657	0.0840	3.22	0.0657	0.0840

**Table A2.1.13 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.25, 0.75$)
Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.10$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	32	2.83	0.1028	0.0421	2.84	0.1028	0.0421
0.30	0.10	0.05	22	2.12	0.1089	0.0362	2.15	0.1089	0.0362
0.20	0.15	0.05	21	2.05	0.0963	0.0452	2.06	0.0963	0.0452
0.30	0.15	0.05	16	1.67	0.1247	0.0292	1.75	0.0768	0.0509
0.20	0.10	0.08	24	2.26	0.0924	0.0456	2.29	0.0924	0.0456
0.30	0.10	0.08	18	1.82	0.1027	0.0356	1.91	0.1027	0.0356
0.20	0.15	0.08	17	1.75	0.1414	0.0290	1.79	0.0893	0.0469
0.30	0.15	0.08	13	1.43	0.1229	0.0293	1.50	0.1229	0.0293

**Table A2.1.14 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.25, 0.75$)
Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.10$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	69	8.01	0.0962	0.0481	8.02	0.0962	0.0481
0.30	0.10	0.05	42	5.28	0.0965	0.0469	5.29	0.0965	0.0469
0.20	0.15	0.05	39	4.96	0.1177	0.0363	5.01	0.0899	0.0495
0.30	0.15	0.05	27	3.68	0.1162	0.0344	3.75	0.0846	0.0508
0.20	0.10	0.08	45	5.59	0.1029	0.0430	5.62	0.1029	0.0430
0.30	0.10	0.08	30	4.01	0.0936	0.0482	4.02	0.0936	0.0482
0.20	0.15	0.08	28	3.79	0.0974	0.0474	3.80	0.0974	0.0474
0.30	0.15	0.08	21	3.01	0.0933	0.0439	3.10	0.0933	0.0439

**Table A2.1.15 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.25, 0.75$)
Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.10$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	23	2.19	0.1222	0.0824	2.20	0.1222	0.0824
0.30	0.10	0.05	16	1.67	0.1247	0.0781	1.71	0.1247	0.0781
0.20	0.15	0.05	15	1.59	0.1089	0.0971	1.60	0.1089	0.0971
0.30	0.15	0.05	11	1.27	0.0880	0.1103	1.29	0.0880	0.1103
0.20	0.10	0.08	17	1.75	0.1414	0.0745	1.76	0.0893	0.1114
0.30	0.10	0.08	13	1.43	0.1229	0.0694	1.51	0.0799	0.1132
0.20	0.15	0.08	12	1.35	0.1049	0.0935	1.38	0.1049	0.0935
0.30	0.15	0.08	9	1.10	0.0990	0.0932	1.12	0.0990	0.0932

**Table A2.1.16 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.25, 0.75$)
Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.10$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	51	6.20	0.1147	0.0880	6.21	0.1147	0.0880
0.30	0.10	0.05	31	4.11	0.1065	0.0949	4.12	0.1065	0.0949
0.20	0.15	0.05	29	3.90	0.1112	0.0861	3.96	0.1112	0.0861
0.30	0.15	0.05	20	2.90	0.1136	0.0825	2.97	0.1136	0.0825
0.20	0.10	0.08	33	4.33	0.1021	0.0965	4.36	0.1021	0.0965
0.30	0.10	0.08	22	3.13	0.1091	0.0923	3.14	0.1091	0.0923
0.20	0.15	0.08	21	3.01	0.0933	0.1018	3.07	0.0933	0.1018
0.30	0.15	0.08	15	2.32	0.1011	0.0981	2.35	0.1011	0.0981

**Table A2.1.17 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.5, 0.75$)
Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.05$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	31	3.92	0.0665	0.0373	4.00	0.0665	0.0373
0.30	0.10	0.05	18	2.61	0.0634	0.0417	2.68	0.0634	0.0417
0.20	0.15	0.05	21	2.92	0.0697	0.0397	2.95	0.0697	0.0397
0.30	0.15	0.05	13	2.06	0.0592	0.0508	2.08	0.0592	0.0508
0.20	0.10	0.08	24	3.23	0.0748	0.0335	3.31	0.0504	0.0477
0.30	0.10	0.08	15	2.28	0.0560	0.0429	2.38	0.0560	0.0429
0.20	0.15	0.08	17	2.50	0.0523	0.0515	2.54	0.0523	0.0515
0.30	0.15	0.08	11	1.83	0.0630	0.0467	1.86	0.0630	0.0467

**Table A2.1.18 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.5, 0.75$)
Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.05$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	65	10.28	0.0522	0.0485	10.32	0.0522	0.0485
0.30	0.10	0.05	32	5.75	0.0689	0.0384	5.78	0.0510	0.0524
0.20	0.15	0.05	39	6.74	0.0667	0.0405	6.75	0.0667	0.0405
0.30	0.15	0.05	22	4.28	0.0534	0.0527	4.30	0.0534	0.0527
0.20	0.10	0.08	45	7.58	0.0552	0.0454	7.63	0.0552	0.0454
0.30	0.10	0.08	25	4.73	0.0694	0.0368	4.80	0.0496	0.0512
0.20	0.15	0.08	30	5.46	0.0663	0.0379	5.54	0.0486	0.0509
0.30	0.15	0.08	18	3.67	0.0662	0.0406	3.73	0.0662	0.0406

**Table A2.1.19 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.5, 0.75$)
Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.05$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	23	3.13	0.0641	0.0902	3.18	0.0641	0.0902
0.30	0.10	0.05	13	2.06	0.0592	0.1090	2.07	0.0592	0.1090
0.20	0.15	0.05	16	2.39	0.0685	0.0860	2.46	0.0685	0.0860
0.30	0.15	0.05	10	1.71	0.0826	0.0769	1.77	0.0486	0.1175
0.20	0.10	0.08	18	2.61	0.0634	0.0882	2.70	0.0634	0.0882
0.30	0.10	0.08	11	1.83	0.0630	0.0903	1.89	0.0630	0.0903
0.20	0.15	0.08	13	2.06	0.0592	0.0951	2.14	0.0592	0.0951
0.30	0.15	0.08	8	1.47	0.0905	0.0782	1.47	0.0905	0.0782

**Table A2.1.20 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.5, 0.75$)
Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.05$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	50	8.26	0.0515	0.1032	8.30	0.0515	0.1032
0.30	0.10	0.05	25	4.73	0.0694	0.0804	4.79	0.0496	0.1075
0.20	0.15	0.05	30	5.46	0.0663	0.0869	5.49	0.0663	0.0869
0.30	0.15	0.05	17	3.51	0.0521	0.1097	3.55	0.0521	0.1097
0.20	0.10	0.08	34	6.03	0.0532	0.1026	6.07	0.0532	0.1026
0.30	0.10	0.08	19	3.82	0.0571	0.0976	3.87	0.0571	0.0976
0.20	0.15	0.08	23	4.43	0.0657	0.0831	4.52	0.0463	0.1095
0.30	0.15	0.08	14	3.03	0.0542	0.1001	3.12	0.0542	0.1001

**Table A2.1.21 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.5, 0.75$)
Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.10$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	26	3.06	0.0989	0.0436	3.12	0.0989	0.0436
0.30	0.10	0.05	15	2.00	0.0888	0.0527	2.05	0.0888	0.0527
0.20	0.15	0.05	18	2.30	0.0974	0.0458	2.35	0.0974	0.0458
0.30	0.15	0.05	11	1.59	0.1032	0.0482	1.61	0.1032	0.0482
0.20	0.10	0.08	20	2.49	0.1311	0.0326	2.54	0.0889	0.0500
0.30	0.10	0.08	12	1.70	0.1256	0.0411	1.70	0.1256	0.0411
0.20	0.15	0.08	15	2.00	0.0888	0.0465	2.10	0.0888	0.0465
0.30	0.15	0.08	10	1.48	0.1416	0.0257	1.61	0.0826	0.0497

**Table A2.1.22 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.5, 0.75$)
Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.10$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	53	8.01	0.0952	0.0498	8.03	0.0952	0.0498
0.30	0.10	0.05	26	4.42	0.1122	0.0429	4.42	0.1122	0.0429
0.20	0.15	0.05	33	5.37	0.1061	0.0419	5.45	0.1061	0.0419
0.30	0.15	0.05	18	3.28	0.0953	0.0533	3.29	0.0953	0.0533
0.20	0.10	0.08	37	5.91	0.1098	0.0416	5.95	0.1098	0.0416
0.30	0.10	0.08	21	3.71	0.1205	0.0350	3.81	0.0872	0.0500
0.20	0.15	0.08	25	4.28	0.0951	0.0467	4.36	0.0951	0.0467
0.30	0.15	0.08	15	2.84	0.1040	0.0441	2.91	0.1040	0.0441

**Table A2.1.23 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.5, 0.75$)
Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.10$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	19	2.40	0.1137	0.0844	2.46	0.1137	0.0844
0.30	0.10	0.05	11	1.59	0.1032	0.0994	1.64	0.1032	0.0994
0.20	0.15	0.05	13	1.80	0.0973	0.1022	1.84	0.0973	0.1022
0.30	0.15	0.05	8	1.26	0.0905	0.1139	1.29	0.0905	0.1139
0.20	0.10	0.08	15	2.00	0.0888	0.1018	2.10	0.0888	0.1018
0.30	0.10	0.08	9	1.37	0.1150	0.0814	1.42	0.1150	0.0814
0.20	0.15	0.08	11	1.59	0.1032	0.0879	1.69	0.1032	0.0879
0.30	0.15	0.08	7	1.15	0.1176	0.0884	1.21	0.1176	0.0884

**Table A2.1.24 - Parameter Selection Examples for the 4-AS-S-1($n, T, 0.5, 0.75$)
Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.10$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	40	6.31	0.0992	0.0980	6.37	0.0992	0.0980
0.30	0.10	0.05	20	3.57	0.1004	0.0967	3.64	0.1004	0.0967
0.20	0.15	0.05	24	4.14	0.1088	0.0930	4.17	0.1088	0.0930
0.30	0.15	0.05	14	2.69	0.1209	0.0789	2.77	0.0824	0.1147
0.20	0.10	0.08	27	4.55	0.0984	0.1033	4.57	0.0984	0.1033
0.30	0.10	0.08	15	2.84	0.1040	0.0973	2.87	0.1040	0.0973
0.20	0.15	0.08	18	3.28	0.0953	0.1055	3.31	0.0953	0.1055
0.30	0.15	0.08	11	2.24	0.1319	0.0771	2.28	0.0870	0.1140

APPENDIX 2.2

PARAMETER SELECTION EXAMPLES FOR THE 4-AS-S-2(n, T, v) ACCEPTANCE SAMPLING PLAN

**Table A2.2.1 - Parameter Selection Examples for the 4-AS-S-2($n, T, 2$) Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.05$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	47	67.8	0.0712	0.0321	68.0	0.0712	0.0321
0.30	0.10	0.05	34	50.3	0.0655	0.0347	50.4	0.0655	0.0347
0.20	0.15	0.05	32	47.6	0.0707	0.0317	47.9	0.0707	0.0317
0.30	0.15	0.05	24	36.6	0.0729	0.0304	36.7	0.0729	0.0304
0.20	0.10	0.08	33	48.9	0.0774	0.0288	49.3	0.0598	0.0378
0.30	0.10	0.08	25	38.0	0.0810	0.0282	38.0	0.0622	0.0392
0.20	0.15	0.08	24	36.6	0.0729	0.0311	36.9	0.0729	0.0311
0.30	0.15	0.08	19	29.7	0.0788	0.0292	29.8	0.0788	0.0292

**Table A2.2.2 - Parameter Selection Examples for the 4-AS-S-2($n, T, 2$) Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.05$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	105	165.7	0.0619	0.0392	165.9	0.0619	0.0392
0.30	0.10	0.05	70	113.5	0.0613	0.0382	113.7	0.0613	0.0382
0.20	0.15	0.05	62	101.4	0.0612	0.0400	101.6	0.0612	0.0400
0.30	0.15	0.05	45	75.5	0.0642	0.0364	75.7	0.0642	0.0364
0.20	0.10	0.08	60	98.4	0.0610	0.0407	98.5	0.0610	0.0407
0.30	0.10	0.08	45	75.5	0.0642	0.0367	75.8	0.0642	0.0367
0.20	0.15	0.08	41	69.4	0.0628	0.0394	69.5	0.0628	0.0394
0.30	0.15	0.08	32	55.4	0.0647	0.0365	55.7	0.0647	0.0365

**Table A2.2.3 - Parameter Selection Examples for the 4-AS-S-2($n, T, 2$) Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.05$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	34	50.3	0.0655	0.0929	50.3	0.0655	0.0929
0.30	0.10	0.05	25	38.0	0.0810	0.0720	38.1	0.0622	0.0971
0.20	0.15	0.05	23	35.2	0.0653	0.0924	35.4	0.0653	0.0924
0.30	0.15	0.05	18	28.3	0.0696	0.0849	28.6	0.0696	0.0849
0.20	0.10	0.08	23	35.2	0.0653	0.0951	35.2	0.0653	0.0951
0.30	0.10	0.08	18	28.3	0.0696	0.0934	28.3	0.0696	0.0934
0.20	0.15	0.08	17	26.9	0.0874	0.0773	27.0	0.0874	0.0773
0.30	0.15	0.08	14	22.6	0.0785	0.0760	22.9	0.0785	0.0760

**Table A2.2.4 - Parameter Selection Examples for the 4-AS-S-2($n, T, 2$) Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.05$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	79	127.0	0.0658	0.0874	127.0	0.0658	0.0874
0.30	0.10	0.05	53	87.7	0.0659	0.0859	87.9	0.0659	0.0859
0.20	0.15	0.05	47	78.6	0.0647	0.0883	78.8	0.0647	0.0883
0.30	0.15	0.05	34	58.5	0.0661	0.0874	58.7	0.0661	0.0874
0.20	0.10	0.08	45	75.5	0.0642	0.0896	75.7	0.0642	0.0896
0.30	0.10	0.08	33	57.0	0.0728	0.0834	57.0	0.0728	0.0834
0.20	0.15	0.08	31	53.8	0.0714	0.0801	54.2	0.0572	0.0974
0.30	0.15	0.08	24	42.8	0.0734	0.0789	43.1	0.0573	0.0996

**Table A2.2.5 - Parameter Selection Examples for the 4-AS-S-2($n, T, 2$) Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.10$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	40	56.3	0.1067	0.0348	56.5	0.1067	0.0348
0.30	0.10	0.05	29	41.8	0.1176	0.0274	41.9	0.1176	0.0274
0.20	0.15	0.05	27	39.1	0.0984	0.0377	39.2	0.0984	0.0377
0.30	0.15	0.05	21	31.0	0.0988	0.0342	31.3	0.0988	0.0342
0.20	0.10	0.08	28	40.4	0.1078	0.0345	40.5	0.1078	0.0345
0.30	0.10	0.08	22	32.4	0.1095	0.0319	32.5	0.1095	0.0319
0.20	0.15	0.08	21	31.0	0.0988	0.0362	31.3	0.0988	0.0362
0.30	0.15	0.08	17	25.6	0.1154	0.0258	26.0	0.1154	0.0258

**Table A2.2.6 - Parameter Selection Examples for the 4-AS-S-2($n, T, 2$) Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.10$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	87	135.0	0.1134	0.0364	135.2	0.0992	0.0430
0.30	0.10	0.05	58	92.1	0.1008	0.0397	92.3	0.1008	0.0397
0.20	0.15	0.05	51	81.6	0.1106	0.0380	81.7	0.1106	0.0380
0.30	0.15	0.05	37	60.6	0.1109	0.0352	60.7	0.1109	0.0352
0.20	0.10	0.08	51	81.6	0.1106	0.0367	81.9	0.1106	0.0367
0.30	0.10	0.08	38	62.1	0.1011	0.0386	62.4	0.1011	0.0386
0.20	0.15	0.08	35	57.5	0.1105	0.0353	57.9	0.1105	0.0353
0.30	0.15	0.08	27	45.3	0.1068	0.0357	45.6	0.1068	0.0357

**Table A2.2.7 - Parameter Selection Examples for the 4-AS-S-2($n, T, 2$) Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.10$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	29	41.8	0.1176	0.0762	42.0	0.0919	0.0992
0.30	0.10	0.05	21	31.0	0.0988	0.0941	31.2	0.0988	0.0941
0.20	0.15	0.05	19	28.3	0.1093	0.0923	28.3	0.1093	0.0923
0.30	0.15	0.05	15	22.8	0.1331	0.0666	23.1	0.0902	0.0998
0.20	0.10	0.08	20	29.7	0.1211	0.0798	29.9	0.1211	0.0798
0.30	0.10	0.08	16	24.2	0.1025	0.0856	24.6	0.1025	0.0856
0.20	0.15	0.08	15	22.8	0.1331	0.0700	23.2	0.0902	0.0969
0.30	0.15	0.08	12	18.7	0.1404	0.0708	19.0	0.0913	0.1043

**Table A2.2.8 - Parameter Selection Examples for the 4-AS-S-2($n, T, 2$) Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.10$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	64	101.0	0.1159	0.0836	101.2	0.0995	0.0979
0.30	0.10	0.05	42	68.1	0.1017	0.0979	68.1	0.1017	0.0979
0.20	0.15	0.05	38	62.1	0.1011	0.0953	62.3	0.1011	0.0953
0.30	0.15	0.05	27	45.3	0.1068	0.0927	45.4	0.1068	0.0927
0.20	0.10	0.08	37	60.6	0.1109	0.0868	60.8	0.1109	0.0868
0.30	0.10	0.08	27	45.3	0.1068	0.0930	45.4	0.1068	0.0930
0.20	0.15	0.08	25	42.3	0.1058	0.0930	42.4	0.1058	0.0930
0.30	0.15	0.08	20	34.5	0.1125	0.0800	34.9	0.1125	0.0800

**Table A2.2.9 - Parameter Selection Examples for the 4-AS-S-2($n, T, 5$) Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.05$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	86	475.8	0.0673	0.0343	479.1	0.0647	0.0359
0.30	0.10	0.05	76	431.6	0.0668	0.0332	432.0	0.0636	0.0351
0.20	0.15	0.05	63	372.9	0.0744	0.0322	374.2	0.0744	0.0322
0.30	0.15	0.05	57	345.2	0.0767	0.0322	346.4	0.0767	0.0322
0.20	0.10	0.08	50	312.4	0.0822	0.0333	317.0	0.0783	0.0353
0.30	0.10	0.08	46	293.3	0.0810	0.0308	296.3	0.0798	0.0328
0.20	0.15	0.08	40	264.1	0.0672	0.0327	265.5	0.0672	0.0327
0.30	0.15	0.08	38	254.2	0.0627	0.0303	260.4	0.0618	0.0322

**Table A2.2.10 - Parameter Selection Examples for the 4-AS-S-2($n, T, 5$) Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.05$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	195	1402.3	0.0612	0.0406	1402.7	0.0612	0.0406
0.30	0.10	0.05	168	1235.6	0.0621	0.0395	1236.3	0.0604	0.0407
0.20	0.15	0.05	131	1002.9	0.0635	0.0378	1005.4	0.0616	0.0391
0.30	0.15	0.05	115	900.4	0.0632	0.0385	900.5	0.0632	0.0385
0.20	0.10	0.08	88	723.6	0.0658	0.0380	726.5	0.0634	0.0394
0.30	0.10	0.08	80	670.1	0.0668	0.0381	671.5	0.0668	0.0381
0.20	0.15	0.08	68	588.6	0.0636	0.0386	588.9	0.0636	0.0386
0.30	0.15	0.08	63	554.1	0.0658	0.0360	556.4	0.0627	0.0376

**Table A2.2.11 - Parameter Selection Examples for the 4-AS-S-2($n, T, 5$) Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.05$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	62	368.3	0.0768	0.0879	370.1	0.0709	0.0920
0.30	0.10	0.05	55	335.9	0.0819	0.0873	336.3	0.0819	0.0873
0.20	0.15	0.05	46	293.3	0.0810	0.0814	296.2	0.0798	0.0858
0.30	0.15	0.05	42	273.9	0.0723	0.0798	278.2	0.0693	0.0886
0.20	0.10	0.08	35	239.2	0.0589	0.0864	240.6	0.0589	0.0864
0.30	0.10	0.08	33	229.1	0.0590	0.0781	233.9	0.0553	0.0825
0.20	0.15	0.08	29	208.5	0.0650	0.0783	213.9	0.0493	0.0852
0.30	0.15	0.08	27	198.0	0.0690	0.0813	203.1	0.0545	0.0890

**Table A2.2.12 - Parameter Selection Examples for the 4-AS-S-2($n, T, 5$) Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.05$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	147	1104.2	0.0617	0.0929	1105.8	0.0617	0.0929
0.30	0.10	0.05	127	977.4	0.0628	0.0909	980.0	0.0609	0.0934
0.20	0.15	0.05	98	789.7	0.0647	0.0903	790.3	0.0626	0.0933
0.30	0.15	0.05	87	717.0	0.0649	0.0898	719.0	0.0625	0.0929
0.20	0.10	0.08	65	567.9	0.0650	0.0881	572.2	0.0626	0.0909
0.30	0.10	0.08	59	526.2	0.0683	0.0901	528.8	0.0647	0.0932
0.20	0.15	0.08	50	462.5	0.0708	0.0913	463.5	0.0708	0.0913
0.30	0.15	0.08	46	433.6	0.0702	0.0887	434.2	0.0676	0.0921

**Table A2.2.13 - Parameter Selection Examples for the 4-AS-S-2($n, T, 5$) Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.10$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	74	382.3	0.1170	0.0335	384.4	0.1170	0.0335
0.30	0.10	0.05	66	348.4	0.1251	0.0307	350.5	0.1188	0.0326
0.20	0.15	0.05	55	301.1	0.1137	0.0298	304.8	0.1121	0.0318
0.30	0.15	0.05	49	274.7	0.1022	0.0307	275.0	0.1022	0.0307
0.20	0.10	0.08	44	252.5	0.0930	0.0307	256.3	0.0881	0.0327
0.30	0.10	0.08	41	238.9	0.0962	0.0288	244.5	0.0911	0.0304
0.20	0.15	0.08	36	216.1	0.0948	0.0308	222.0	0.0838	0.0325
0.30	0.15	0.08	33	202.1	0.1056	0.0323	204.7	0.1056	0.0323

**Table A2.2.14 - Parameter Selection Examples for the 4-AS-S-2($n, T, 5$) Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.10$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	163	1121.3	0.1076	0.0393	1123.5	0.1049	0.0406
0.30	0.10	0.05	140	983.0	0.1084	0.0384	984.3	0.1054	0.0397
0.20	0.15	0.05	109	793.5	0.1068	0.0379	795.3	0.1068	0.0379
0.30	0.15	0.05	96	712.7	0.1090	0.0370	713.9	0.1090	0.0370
0.20	0.10	0.08	75	579.9	0.1071	0.0379	582.5	0.1071	0.0379
0.30	0.10	0.08	68	534.9	0.1133	0.0372	535.5	0.1133	0.0372
0.20	0.15	0.08	58	469.6	0.1148	0.0358	470.3	0.1109	0.0375
0.30	0.15	0.08	54	443.1	0.1079	0.0351	447.2	0.1045	0.0368

**Table A2.2.15 - Parameter Selection Examples for the 4-AS-S-2($n, T, 5$) Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.10$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	52	288.0	0.1085	0.0814	289.2	0.1055	0.0859
0.30	0.10	0.05	47	265.9	0.0984	0.0797	269.4	0.0960	0.0836
0.20	0.15	0.05	39	229.8	0.0934	0.0819	233.8	0.0892	0.0865
0.30	0.15	0.05	35	211.4	0.1017	0.0884	213.9	0.1017	0.0884
0.20	0.10	0.08	30	187.9	0.1132	0.0799	188.8	0.1132	0.0799
0.30	0.10	0.08	28	178.4	0.1416	0.0833	180.7	0.1190	0.0862
0.20	0.15	0.08	25	163.8	0.1372	0.0845	169.1	0.1290	0.0940
0.30	0.15	0.08	23	154.0	0.1581	0.0912	157.3	0.1328	0.0962

**Table A2.2.16 - Parameter Selection Examples for the 4-AS-S-2($n, T, 5$) Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.10$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	119	855.1	0.1067	0.0926	857.0	0.1067	0.0926
0.30	0.10	0.05	102	750.2	0.1074	0.0927	750.7	0.1074	0.0927
0.20	0.15	0.05	80	611.9	0.1093	0.0869	614.9	0.1059	0.0901
0.30	0.15	0.05	70	547.8	0.1104	0.0897	548.5	0.1104	0.0897
0.20	0.10	0.08	54	443.1	0.1079	0.0883	447.5	0.1045	0.0919
0.30	0.10	0.08	49	409.7	0.1031	0.0876	412.6	0.1031	0.0876
0.20	0.15	0.08	42	362.2	0.1074	0.0895	366.2	0.1028	0.0934
0.30	0.15	0.08	38	334.6	0.1185	0.0925	334.8	0.1185	0.0925

**Table A2.2.17 - Parameter Selection Examples for the 4-AS-S-2($n, T, 10$) Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.05$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	119	3539.0	0.0719	0.0379	3561.4	0.0694	0.0388
0.30	0.10	0.05	114	3427.8	0.0801	0.0360	3437.6	0.0794	0.0365
0.20	0.15	0.05	97	3042.9	0.0742	0.0356	3047.0	0.0742	0.0356
0.30	0.15	0.05	94	2973.8	0.0691	0.0359	2987.6	0.0690	0.0367
0.20	0.10	0.08	61	2180.8	0.1022	0.0367	2192.1	0.0965	0.0369
0.30	0.10	0.08	60	2155.5	0.1047	0.0379	2185.7	0.0928	0.0386
0.20	0.15	0.08	54	2002.3	0.1018	0.0370	2025.4	0.1018	0.0388
0.30	0.15	0.08	53	1976.4	0.0987	0.0343	2009.5	0.0987	0.0382

**Table A2.2.18 - Parameter Selection Examples for the 4-AS-S-2($n, T, 10$) Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.05$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	270	11046.4	0.0590	0.0417	11050.1	0.0590	0.0417
0.30	0.10	0.05	257	10604.9	0.0614	0.0414	10616.2	0.0606	0.0420
0.20	0.15	0.05	208	8917.9	0.0612	0.0406	8920.6	0.0610	0.0409
0.30	0.15	0.05	199	8603.5	0.0591	0.0402	8609.8	0.0587	0.0406
0.20	0.10	0.08	105	5188.3	0.0623	0.0388	5188.6	0.0623	0.0388
0.30	0.10	0.08	103	5111.9	0.0596	0.0363	5144.4	0.0590	0.0377
0.20	0.15	0.08	91	4648.9	0.0658	0.0395	4668.5	0.0622	0.0404
0.30	0.15	0.08	89	4570.9	0.0693	0.0373	4604.8	0.0648	0.0390

**Table A2.2.19 - Parameter Selection Examples for the 4-AS-S-2($n, T, 10$) Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.05$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	86	2787.3	0.0577	0.0800	2808.8	0.0574	0.0817
0.30	0.10	0.05	82	2692.9	0.0543	0.0826	2695.4	0.0543	0.0826
0.20	0.15	0.05	70	2404.0	0.0637	0.0970	2408.7	0.0637	0.0970
0.30	0.15	0.05	68	2354.9	0.0691	0.0952	2370.9	0.0688	0.0966
0.20	0.10	0.08	43	1711.1	0.0690	0.1066	1720.8	0.0690	0.1088
0.30	0.10	0.08	42	1683.8	0.0663	0.1065	1694.9	0.0662	0.1077
0.20	0.15	0.08	38	1573.2	0.0558	0.0790	1587.9	0.0557	0.0833
0.30	0.15	0.08	37	1545.1	0.0537	0.0768	1554.1	0.0534	0.0796

**Table A2.2.20 - Parameter Selection Examples for the 4-AS-S-2($n, T, 10$) Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.05$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	203	8743.4	0.0597	0.0938	8755.4	0.0589	0.0949
0.30	0.10	0.05	193	8392.9	0.0603	0.0927	8405.7	0.0598	0.0933
0.20	0.15	0.05	156	7075.7	0.0630	0.0930	7077.3	0.0621	0.0936
0.30	0.15	0.05	150	6858.5	0.0669	0.0924	6878.1	0.0653	0.0937
0.20	0.10	0.08	77	4096.4	0.0694	0.0981	4105.0	0.0693	0.0993
0.30	0.10	0.08	75	4016.2	0.0648	0.0933	4030.1	0.0645	0.0953
0.20	0.15	0.08	67	3691.7	0.0560	0.0871	3722.4	0.0520	0.0890
0.30	0.15	0.08	65	3609.6	0.0576	0.0916	3631.3	0.0527	0.0933

**Table A2.2.21 - Parameter Selection Examples for the 4-AS-S-2($n, T, 10$) Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.10$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	103	2809.6	0.0917	0.0327	2830.2	0.0908	0.0335
0.30	0.10	0.05	99	2725.6	0.0905	0.0348	2743.7	0.0883	0.0353
0.20	0.15	0.05	84	2405.7	0.1198	0.0338	2411.9	0.1165	0.0345
0.30	0.15	0.05	81	2340.7	0.1187	0.0326	2344.7	0.1187	0.0326
0.20	0.10	0.08	54	1734.1	0.1022	0.0382	1748.2	0.1022	0.0405
0.30	0.10	0.08	53	1710.7	0.0990	0.0363	1737.1	0.0990	0.0389
0.20	0.15	0.08	48	1592.4	0.0845	0.0258	1625.5	0.0843	0.0282
0.30	0.15	0.08	47	1568.5	0.0822	0.0248	1605.8	0.0812	0.0274

**Table A2.2.22 - Parameter Selection Examples for the 4-AS-S-2($n, T, 10$) Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.10$ and $\beta = 0.05$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	225	8741.1	0.1038	0.0412	8741.9	0.1038	0.0412
0.30	0.10	0.05	214	8379.0	0.1084	0.0405	8383.6	0.1084	0.0405
0.20	0.15	0.05	174	7046.6	0.1118	0.0393	7061.0	0.1101	0.0400
0.30	0.15	0.05	166	6776.6	0.1125	0.0389	6784.5	0.1118	0.0392
0.20	0.10	0.08	90	4125.1	0.1087	0.0394	4135.9	0.1085	0.0400
0.30	0.10	0.08	88	4052.3	0.1034	0.0361	4081.3	0.1025	0.0376
0.20	0.15	0.08	78	3685.0	0.0967	0.0381	3712.5	0.0945	0.0392
0.30	0.15	0.08	76	3610.8	0.1045	0.0390	3640.5	0.0938	0.0404

**Table A2.2.23 - Parameter Selection Examples for the 4-AS-S-2($n, T, 10$) Acceptance Sampling Plan having $p_{12} = 0.07, p_{13} = 0.03, p_{14} = 0.01,$
 $\alpha = 0.10$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	72	2143.1	0.1486	0.0994	2150.9	0.1486	0.0994
0.30	0.10	0.05	69	2076.3	0.1512	0.1019	2077.3	0.1512	0.1019
0.20	0.15	0.05	59	1849.9	0.1181	0.0779	1855.4	0.1181	0.0799
0.30	0.15	0.05	57	1803.8	0.1115	0.0757	1810.5	0.1115	0.0757
0.20	0.10	0.08	37	1323.1	0.0773	0.0735	1328.8	0.0773	0.0761
0.30	0.10	0.08	36	1297.8	0.0730	0.0652	1299.4	0.0730	0.0652
0.20	0.15	0.08	33	1221.2	0.1035	0.0674	1244.4	0.0885	0.0679
0.30	0.15	0.08	32	1195.3	0.0974	0.0700	1208.0	0.0973	0.0703

**Table A2.2.24 - Parameter Selection Examples for the 4-AS-S-2($n, T, 10$) Acceptance Sampling Plan having $p_{12} = 0.10, p_{13} = 0.05, p_{14} = 0.02,$
 $\alpha = 0.10$ and $\beta = 0.10$**

p_{22}	p_{23}	p_{24}	n	T fix α	α adjusted fix α	β adjusted fix α	T fix β	α adjusted fix β	β adjusted fix β
0.20	0.10	0.05	164	6709.0	0.1105	0.0916	6713.0	0.1105	0.0916
0.30	0.10	0.05	156	6437.3	0.1035	0.0933	6444.0	0.1030	0.0939
0.20	0.15	0.05	127	5439.2	0.1117	0.0925	5455.8	0.1109	0.0939
0.30	0.15	0.05	121	5229.6	0.1023	0.0922	5237.6	0.1019	0.0931
0.20	0.10	0.08	64	3159.4	0.1329	0.0970	3168.2	0.1328	0.0976
0.30	0.10	0.08	62	3083.1	0.1276	0.1018	3085.2	0.1276	0.1018
0.20	0.15	0.08	55	2812.7	0.0991	0.0844	2816.1	0.0991	0.0844
0.30	0.15	0.08	54	2773.7	0.0963	0.0800	2793.3	0.0955	0.0826

APPENDIX 3

PARAMETER SELECTION EXAMPLES FOR 3-LEVEL CONTROL CHARTS

APPENDIX 3.1

PARAMETER SELECTION EXAMPLES FOR THE 3-SPC-AVG-1(n, k_V, ν) CONTROL CHART

**Table A3.1.1 - Parameter Selection Examples for the 3-SPC-AVG-1($n, k_V, 0.25$)
Control Chart having $ARL(p_0) = 200$ $ARL(p_c) \leq 2$**

p_{02}	p_{03}	p_{c2}	p_{c3}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.5758	182	0.0250	0.0100	0.0000	71.3	2.2
0.020	0.005	0.060	0.015	2.5758	103	0.0299	0.0100	0.0000	56.2	2.2
0.020	0.005	0.040	0.030	2.5758	46	0.0398	0.0100	0.0000	40.5	2.3
0.020	0.005	0.060	0.030	2.5758	34	0.0446	0.0100	0.0000	59.9	2.5
0.030	0.005	0.040	0.015	2.5758	286	0.0250	0.0125	0.0000	75.7	2.1
0.030	0.005	0.060	0.015	2.5758	146	0.0300	0.0125	0.0000	62.9	2.1
0.030	0.005	0.040	0.030	2.5758	59	0.0400	0.0125	0.0000	46.2	2.2
0.030	0.005	0.060	0.030	2.5758	43	0.0447	0.0125	0.0000	38.1	2.1
0.020	0.010	0.040	0.015	2.8070	869	0.0250	0.0150	0.0050	162.6	2.0
0.020	0.010	0.060	0.015	2.5758	326	0.0300	0.0150	0.0000	96.7	2.2
0.020	0.010	0.040	0.030	2.5758	118	0.0399	0.0150	0.0000	57.0	2.0
0.020	0.010	0.060	0.030	2.5758	82	0.0449	0.0150	0.0000	55.0	2.1
0.030	0.010	0.040	0.015	2.8070	1621	0.0250	0.0175	0.0100	200.0	2.1
0.030	0.010	0.060	0.015	2.8070	584	0.0300	0.0175	0.0050	184.0	2.1
0.030	0.010	0.040	0.030	2.5758	152	0.0400	0.0175	0.0000	77.1	2.2
0.030	0.010	0.060	0.030	2.5758	102	0.0449	0.0175	0.0000	67.3	2.2

**Table A3.1.2 - Parameter Selection Examples for the 3-SPC-AVG-1($n, k_V, 0.25$)
Control Chart having $ARL(p_0) = 200$ $ARL(p_c) \leq 5$**

p_{02}	p_{03}	p_{c2}	p_{c3}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.5758	39	0.0423	0.0100	0.0000	43.6	5.5
0.020	0.005	0.060	0.015	2.5758	21	0.0541	0.0100	0.0000	27.3	4.7
0.020	0.005	0.040	0.030	2.5758	4	0.1110	0.0100	0.0000	45.1	8.1
0.020	0.005	0.060	0.030	2.5758	3	0.1266	0.0100	0.0000	62.1	10.3
0.030	0.005	0.040	0.015	2.5758	67	0.0383	0.0125	0.0000	55.3	5.6
0.030	0.005	0.060	0.015	2.5758	32	0.0498	0.0125	0.0000	49.3	5.8
0.030	0.005	0.040	0.030	2.5758	6	0.0987	0.0125	0.0000	33.2	5.9
0.030	0.005	0.060	0.030	2.5758	4	0.1181	0.0125	0.0000	40.0	7.5
0.020	0.010	0.040	0.015	2.5758	260	0.0318	0.0150	0.0000	92.8	5.5
0.020	0.010	0.060	0.015	2.5758	111	0.0407	0.0150	0.0000	75.0	5.7
0.020	0.010	0.040	0.030	2.5758	25	0.0691	0.0150	0.0000	35.5	5.0
0.020	0.010	0.060	0.030	2.5758	17	0.0806	0.0150	0.0000	55.0	5.8
0.030	0.010	0.040	0.015	2.8070	660	0.0293	0.0175	0.0057	178.0	5.2
0.030	0.010	0.060	0.015	2.5758	174	0.0385	0.0175	0.0000	68.3	4.9
0.030	0.010	0.040	0.030	2.5758	33	0.0657	0.0175	0.0000	34.8	4.7
0.030	0.010	0.060	0.030	2.5758	22	0.0766	0.0175	0.0000	40.6	5.3

**Table A3.1.3 - Parameter Selection Examples for the 3-SPC-AVG-1($n, k_V, 0.25$)
Control Chart having $ARL(p_0) = 500$ $ARL(p_c) \leq 2$**

p_{02}	p_{03}	p_{c2}	p_{c3}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.8782	227	0.0250	0.0100	0.0000	107.8	2.1
0.020	0.005	0.060	0.015	2.8782	128	0.0300	0.0100	0.0000	99.1	2.2
0.020	0.005	0.040	0.030	2.8782	57	0.0399	0.0100	0.0000	80.8	2.3
0.020	0.005	0.060	0.030	2.8782	42	0.0448	0.0100	0.0000	49.0	2.2
0.030	0.005	0.040	0.015	2.8782	357	0.0250	0.0125	0.0000	142.4	2.1
0.030	0.005	0.060	0.015	2.8782	182	0.0300	0.0125	0.0000	100.8	2.0
0.030	0.005	0.040	0.030	2.8782	74	0.0399	0.0125	0.0000	67.8	2.1
0.030	0.005	0.060	0.030	2.8782	53	0.0449	0.0125	0.0000	64.2	2.2
0.020	0.010	0.040	0.015	3.0902	1053	0.0250	0.0150	0.0050	392.0	2.1
0.020	0.010	0.060	0.015	2.8782	406	0.0300	0.0150	0.0000	164.5	2.0
0.020	0.010	0.040	0.030	2.8782	147	0.0399	0.0150	0.0000	119.6	2.1
0.020	0.010	0.060	0.030	2.8782	102	0.0449	0.0150	0.0000	110.6	2.2
0.030	0.010	0.040	0.015	3.0902	1965	0.0250	0.0175	0.0100	438.6	2.0
0.030	0.010	0.060	0.015	3.0902	708	0.0300	0.0175	0.0050	322.5	2.0
0.030	0.010	0.040	0.030	2.8782	190	0.0400	0.0175	0.0000	143.3	2.1
0.030	0.010	0.060	0.030	2.8782	127	0.0450	0.0175	0.0000	102.2	2.0

**Table A3.1.4 - Parameter Selection Examples for the 3-SPC-AVG-1($n, k_V, 0.25$)
Control Chart having $ARL(p_0) = 500$ $ARL(p_c) \leq 5$**

p_{02}	p_{03}	p_{c2}	p_{c3}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.8782	61	0.0389	0.0100	0.0000	65.6	5.1
0.020	0.005	0.060	0.015	2.8782	33	0.0493	0.0100	0.0000	64.2	5.4
0.020	0.005	0.040	0.030	2.8782	7	0.0953	0.0100	0.0000	28.8	5.2
0.020	0.005	0.060	0.030	2.8782	5	0.1109	0.0100	0.0000	40.3	7.0
0.030	0.005	0.040	0.015	2.8782	103	0.0357	0.0125	0.0000	80.6	4.9
0.030	0.005	0.060	0.015	2.8782	50	0.0459	0.0125	0.0000	76.9	5.8
0.030	0.005	0.040	0.030	2.8782	11	0.0836	0.0125	0.0000	18.6	3.5
0.030	0.005	0.060	0.030	2.8782	8	0.0959	0.0125	0.0000	25.4	4.6
0.020	0.010	0.040	0.015	2.8782	373	0.0306	0.0150	0.0000	159.0	5.0
0.020	0.010	0.060	0.015	2.8782	161	0.0388	0.0150	0.0000	103.2	4.7
0.020	0.010	0.040	0.030	2.8782	39	0.0634	0.0150	0.0000	63.4	4.7
0.020	0.010	0.060	0.030	2.8782	26	0.0743	0.0150	0.0000	35.7	4.8
0.030	0.010	0.040	0.015	3.0902	885	0.0287	0.0175	0.0063	375.9	5.1
0.030	0.010	0.060	0.015	2.8782	249	0.0371	0.0175	0.0000	132.4	4.8
0.030	0.010	0.040	0.030	2.8782	52	0.0604	0.0175	0.0000	72.6	5.0
0.030	0.010	0.060	0.030	2.8782	34	0.0706	0.0175	0.0000	66.1	5.0

Table A3.1.5 - Parameter Selection Examples for the 3-SPC-AVG-1($n, k_v, 0.5$) Control Chart having $ARL(p_0) = 200$ $ARL(p_c) \leq 2$

p_{02}	p_{03}	p_{c2}	p_{c3}	k_v	n	UCL_v	CL	LCL_v	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.5758	163	0.0349	0.0150	0.0000	85.1	2.1
0.020	0.005	0.060	0.015	2.5758	73	0.0448	0.0150	0.0000	59.2	2.1
0.020	0.005	0.040	0.030	2.5758	53	0.0500	0.0150	0.0000	69.2	2.3
0.020	0.005	0.060	0.030	2.5758	33	0.0593	0.0150	0.0000	29.0	1.9
0.030	0.005	0.040	0.015	2.8070	424	0.0350	0.0200	0.0050	157.3	2.0
0.030	0.005	0.060	0.015	2.5758	129	0.0449	0.0200	0.0000	76.5	2.0
0.030	0.005	0.040	0.030	2.5758	90	0.0499	0.0200	0.0000	49.6	1.9
0.030	0.005	0.060	0.030	2.5758	51	0.0597	0.0200	0.0000	96.9	2.4
0.020	0.010	0.040	0.015	2.8070	512	0.0350	0.0200	0.0050	149.8	2.0
0.020	0.010	0.060	0.015	2.5758	155	0.0450	0.0200	0.0000	62.0	1.9
0.020	0.010	0.040	0.030	2.5758	108	0.0499	0.0200	0.0000	60.2	2.0
0.020	0.010	0.060	0.030	2.5758	61	0.0599	0.0200	0.0000	73.8	2.2
0.030	0.010	0.040	0.015	2.8070	1330	0.0350	0.0250	0.0150	218.6	2.1
0.030	0.010	0.060	0.015	2.8070	333	0.0450	0.0250	0.0050	138.2	1.9
0.030	0.010	0.040	0.030	2.5758	180	0.0499	0.0250	0.0000	70.5	1.9
0.030	0.010	0.060	0.030	2.5758	92	0.0599	0.0250	0.0000	110.7	2.3

Table A3.1.6 - Parameter Selection Examples for the 3-SPC-AVG-1($n, k_v, 0.5$) Control Chart having $ARL(p_0) = 200$ $ARL(p_c) \leq 5$

p_{02}	p_{03}	p_{c2}	p_{c3}	k_v	n	UCL_v	CL	LCL_v	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.5758	39	0.0558	0.0150	0.0000	60.4	5.8
0.020	0.005	0.060	0.015	2.5758	15	0.0808	0.0150	0.0000	44.1	5.7
0.020	0.005	0.040	0.030	2.5758	7	0.1113	0.0150	0.0000	23.7	4.6
0.020	0.005	0.060	0.030	2.5758	4	0.1423	0.0150	0.0000	45.1	7.5
0.030	0.005	0.040	0.015	2.5758	105	0.0477	0.0200	0.0000	117.2	6.3
0.030	0.005	0.060	0.015	2.5758	33	0.0693	0.0200	0.0000	50.0	5.0
0.030	0.005	0.040	0.030	2.5758	17	0.0887	0.0200	0.0000	90.7	7.3
0.030	0.005	0.060	0.030	2.5758	8	0.1202	0.0200	0.0000	16.4	3.5
0.020	0.010	0.040	0.015	2.5758	147	0.0457	0.0200	0.0000	84.9	5.3
0.020	0.010	0.060	0.015	2.5758	47	0.0654	0.0200	0.0000	80.3	6.3
0.020	0.010	0.040	0.030	2.5758	25	0.0822	0.0200	0.0000	74.9	6.7
0.020	0.010	0.060	0.030	2.5758	13	0.1063	0.0200	0.0000	29.4	4.4
0.030	0.010	0.040	0.015	2.8070	552	0.0405	0.0250	0.0095	167.9	4.9
0.030	0.010	0.060	0.015	2.5758	94	0.0595	0.0250	0.0000	97.8	5.8
0.030	0.010	0.040	0.030	2.5758	48	0.0733	0.0250	0.0000	97.8	6.4
0.030	0.010	0.060	0.030	2.5758	22	0.0963	0.0250	0.0000	67.8	6.2

**Table A3.1.7 - Parameter Selection Examples for the 3-SPC-AVG-1($n, k_V, 0.5$)
Control Chart having $ARL(p_0) = 500$ $ARL(p_c) \leq 2$**

p_{02}	p_{03}	p_{c2}	p_{c3}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.8782	203	0.0350	0.0150	0.0000	189.2	2.2
0.020	0.005	0.060	0.015	2.8782	90	0.0450	0.0150	0.0000	162.4	2.4
0.020	0.005	0.040	0.030	2.8782	67	0.0498	0.0150	0.0000	82.8	2.0
0.020	0.005	0.060	0.030	2.8782	40	0.0600	0.0150	0.0000	56.0	2.0
0.030	0.005	0.040	0.015	3.0902	514	0.0350	0.0200	0.0050	297.8	1.9
0.030	0.005	0.060	0.015	2.8782	161	0.0450	0.0200	0.0000	164.5	2.1
0.030	0.005	0.040	0.030	2.8782	112	0.0499	0.0200	0.0000	185.1	2.2
0.030	0.005	0.060	0.030	2.8782	63	0.0599	0.0200	0.0000	107.7	2.1
0.020	0.010	0.040	0.015	3.0902	620	0.0350	0.0200	0.0050	392.7	2.1
0.020	0.010	0.060	0.015	2.8782	194	0.0450	0.0200	0.0000	166.9	2.1
0.020	0.010	0.040	0.030	2.8782	135	0.0499	0.0200	0.0000	143.1	2.1
0.020	0.010	0.060	0.030	2.8782	76	0.0599	0.0200	0.0000	158.3	2.3
0.030	0.010	0.040	0.015	3.0902	1612	0.0350	0.0250	0.0150	422.7	2.0
0.030	0.010	0.060	0.015	3.0902	403	0.0450	0.0250	0.0050	401.0	2.1
0.030	0.010	0.040	0.030	2.8782	224	0.0500	0.0250	0.0000	200.3	2.1
0.030	0.010	0.060	0.030	2.8782	115	0.0599	0.0250	0.0000	122.6	2.0

**Table A3.1.8 - Parameter Selection Examples for the 3-SPC-AVG-1($n, k_V, 0.5$)
Control Chart having $ARL(p_0) = 500$ $ARL(p_c) \leq 5$**

p_{02}	p_{03}	p_{c2}	p_{c3}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.8782	60	0.0517	0.0150	0.0000	128.6	6.1
0.020	0.005	0.060	0.015	2.8782	23	0.0743	0.0150	0.0000	70.3	5.7
0.020	0.005	0.040	0.030	2.8782	13	0.0939	0.0150	0.0000	58.9	5.8
0.020	0.005	0.060	0.030	2.8782	7	0.1226	0.0150	0.0000	23.7	4.1
0.030	0.005	0.040	0.015	2.8782	156	0.0453	0.0200	0.0000	208.4	5.8
0.030	0.005	0.060	0.015	2.8782	50	0.0648	0.0200	0.0000	105.4	5.2
0.030	0.005	0.040	0.030	2.8782	27	0.0809	0.0200	0.0000	95.1	5.6
0.030	0.005	0.060	0.030	2.8782	14	0.1046	0.0200	0.0000	32.3	3.9
0.020	0.010	0.040	0.015	2.8782	212	0.0439	0.0200	0.0000	156.5	4.9
0.020	0.010	0.060	0.015	2.8782	69	0.0619	0.0200	0.0000	104.9	5.1
0.020	0.010	0.040	0.030	2.8782	38	0.0764	0.0200	0.0000	61.3	4.5
0.020	0.010	0.060	0.030	2.8782	20	0.0978	0.0200	0.0000	37.7	4.0
0.030	0.010	0.040	0.015	3.0902	738	0.0398	0.0250	0.0102	379.0	4.9
0.030	0.010	0.060	0.015	2.8782	136	0.0571	0.0250	0.0000	156.0	5.1
0.030	0.010	0.040	0.030	2.8782	72	0.0691	0.0250	0.0000	85.8	4.3
0.030	0.010	0.060	0.030	2.8782	34	0.0891	0.0250	0.0000	148.0	6.5

Table A3.1.9 - Parameter Selection Examples for the 3-SPC-AVG-1($n, k_v, 0.75$) Control Chart having $ARL(p_0) = 200$ $ARL(p_c) \leq 2$

p_{02}	p_{03}	p_{c2}	p_{c3}	k_v	n	UCL_v	CL	LCL_v	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.5758	169	0.0449	0.0200	0.0000	89.1	2.1
0.020	0.005	0.060	0.015	2.5758	66	0.0599	0.0200	0.0000	53.2	2.0
0.020	0.005	0.040	0.030	2.5758	66	0.0599	0.0200	0.0000	53.2	2.0
0.020	0.005	0.060	0.030	2.5758	35	0.0748	0.0200	0.0000	63.5	2.2
0.030	0.005	0.040	0.015	2.8070	544	0.0450	0.0275	0.0100	176.3	2.0
0.030	0.005	0.060	0.015	2.5758	133	0.0600	0.0275	0.0000	80.6	2.0
0.030	0.005	0.040	0.030	2.5758	133	0.0600	0.0275	0.0000	80.6	2.0
0.030	0.005	0.060	0.030	2.5758	63	0.0747	0.0275	0.0000	61.0	2.0
0.020	0.010	0.040	0.015	2.8070	407	0.0450	0.0250	0.0050	187.7	2.1
0.020	0.010	0.060	0.015	2.5758	112	0.0600	0.0250	0.0000	71.6	2.0
0.020	0.010	0.040	0.030	2.5758	112	0.0600	0.0250	0.0000	71.6	2.0
0.020	0.010	0.060	0.030	2.5758	55	0.0749	0.0250	0.0000	62.7	2.1
0.030	0.010	0.040	0.015	2.8070	1302	0.0450	0.0325	0.0200	194.2	2.0
0.030	0.010	0.060	0.015	2.8070	270	0.0599	0.0325	0.0051	171.2	2.0
0.030	0.010	0.040	0.030	2.8070	269	0.0600	0.0325	0.0050	177.7	2.1
0.030	0.010	0.060	0.030	2.5758	95	0.0750	0.0325	0.0000	84.5	2.1

Table A3.1.10 - Parameter Selection Examples for the 3-SPC-AVG-1($n, k_v, 0.75$) Control Chart having $ARL(p_0) = 200$ $ARL(p_c) \leq 5$

p_{02}	p_{03}	p_{c2}	p_{c3}	k_v	n	UCL_v	CL	LCL_v	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.5758	44	0.0689	0.0200	0.0000	62.2	5.4
0.020	0.005	0.060	0.015	2.5758	14	0.1067	0.0200	0.0000	21.5	3.5
0.020	0.005	0.040	0.030	2.5758	12	0.1136	0.0200	0.0000	28.6	4.9
0.020	0.005	0.060	0.030	2.5758	5	0.1650	0.0200	0.0000	35.0	5.9
0.030	0.005	0.040	0.015	2.5758	153	0.0578	0.0275	0.0000	98.2	5.2
0.030	0.005	0.060	0.015	2.5758	37	0.0890	0.0275	0.0000	82.8	5.9
0.030	0.005	0.040	0.030	2.5758	34	0.0917	0.0275	0.0000	57.9	5.0
0.030	0.005	0.060	0.030	2.5758	14	0.1275	0.0275	0.0000	76.0	6.6
0.020	0.010	0.040	0.015	2.5758	112	0.0600	0.0250	0.0000	71.6	4.8
0.020	0.010	0.060	0.015	2.5758	30	0.0925	0.0250	0.0000	73.1	5.4
0.020	0.010	0.040	0.030	2.5758	28	0.0949	0.0250	0.0000	49.9	5.0
0.020	0.010	0.060	0.030	2.5758	12	0.1318	0.0250	0.0000	34.3	4.9
0.030	0.010	0.040	0.015	2.8070	548	0.0518	0.0325	0.0132	183.6	5.1
0.030	0.010	0.060	0.015	2.5758	74	0.0806	0.0325	0.0000	70.9	4.8
0.030	0.010	0.040	0.030	2.5758	69	0.0823	0.0325	0.0000	70.3	4.9
0.030	0.010	0.060	0.030	2.5758	25	0.1153	0.0325	0.0000	57.3	5.2

**Table A3.1.11 - Parameter Selection Examples for the 3-SPC-AVG-1($n, k_V, 0.75$)
Control Chart having $ARL(p_0) = 500$ $ARL(p_c) \leq 2$**

p_{02}	p_{03}	p_{c2}	p_{c3}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.8782	211	0.0449	0.0200	0.0000	151.2	2.0
0.020	0.005	0.060	0.015	2.8782	83	0.0598	0.0200	0.0000	106.7	2.0
0.020	0.005	0.040	0.030	2.8782	83	0.0598	0.0200	0.0000	106.7	2.0
0.020	0.005	0.060	0.030	2.8782	44	0.0746	0.0200	0.0000	122.0	2.2
0.030	0.005	0.040	0.015	3.0902	659	0.0450	0.0275	0.0100	421.0	2.0
0.030	0.005	0.060	0.015	2.8782	166	0.0600	0.0275	0.0000	164.0	2.0
0.030	0.005	0.040	0.030	2.8782	166	0.0600	0.0275	0.0000	164.0	2.0
0.030	0.005	0.060	0.030	2.8782	78	0.0749	0.0275	0.0000	145.6	2.1
0.020	0.010	0.040	0.015	3.0902	493	0.0450	0.0250	0.0050	375.6	2.0
0.020	0.010	0.060	0.015	2.8782	140	0.0599	0.0250	0.0000	155.6	2.1
0.020	0.010	0.040	0.030	2.8782	140	0.0599	0.0250	0.0000	155.6	2.1
0.020	0.010	0.060	0.030	2.8782	69	0.0748	0.0250	0.0000	114.4	2.1
0.030	0.010	0.040	0.015	3.0902	1578	0.0450	0.0325	0.0200	485.0	2.1
0.030	0.010	0.060	0.015	3.0902	327	0.0600	0.0325	0.0050	385.1	2.1
0.030	0.010	0.040	0.030	3.0902	327	0.0600	0.0325	0.0050	385.1	2.1
0.030	0.010	0.060	0.030	2.8782	119	0.0749	0.0325	0.0000	161.2	2.0

**Table A3.1.12 - Parameter Selection Examples for the 3-SPC-AVG-1($n, k_V, 0.75$)
Control Chart having $ARL(p_0) = 500$ $ARL(p_c) \leq 5$**

p_{02}	p_{03}	p_{c2}	p_{c3}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.8782	67	0.0643	0.0200	0.0000	128.8	5.5
0.020	0.005	0.060	0.015	2.8782	22	0.0973	0.0200	0.0000	59.3	4.4
0.020	0.005	0.040	0.030	2.8782	20	0.1010	0.0200	0.0000	77.2	6.2
0.020	0.005	0.060	0.030	2.8782	9	0.1408	0.0200	0.0000	50.0	5.2
0.030	0.005	0.040	0.015	2.8782	221	0.0556	0.0275	0.0000	216.4	5.3
0.030	0.005	0.060	0.015	2.8782	55	0.0839	0.0275	0.0000	115.5	5.0
0.030	0.005	0.040	0.030	2.8782	52	0.0855	0.0275	0.0000	99.8	4.8
0.030	0.005	0.060	0.030	2.8782	21	0.1188	0.0275	0.0000	59.9	4.2
0.020	0.010	0.040	0.015	2.8782	163	0.0574	0.0250	0.0000	172.7	5.1
0.020	0.010	0.060	0.015	2.8782	45	0.0866	0.0250	0.0000	97.8	4.9
0.020	0.010	0.040	0.030	2.8782	43	0.0880	0.0250	0.0000	118.0	5.6
0.020	0.010	0.060	0.030	2.8782	18	0.1224	0.0250	0.0000	63.6	4.6
0.030	0.010	0.040	0.015	3.0902	732	0.0509	0.0325	0.0141	415.7	4.9
0.030	0.010	0.060	0.015	2.8782	107	0.0772	0.0325	0.0000	189.0	5.4
0.030	0.010	0.040	0.030	2.8782	101	0.0785	0.0325	0.0000	147.9	4.9
0.030	0.010	0.060	0.030	2.8782	38	0.1075	0.0325	0.0000	115.5	5.2

APPENDIX 3.2

PARAMETER SELECTION EXAMPLES FOR THE 3-SPC-AVG-2(n, k, ν, ν) CONTROL CHART

Table A3.2.1 - Parameter Selection Examples for the 3-SPC-AVG-2($n, k_v, 2$) Control Chart having $ARL(p_0) = 200$ $ARL(p_c) \leq 2$

p_{02}	p_{03}	p_{c2}	p_{c3}	k_v	n	UCL_v	CL	LCL_v	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.5758	170	1.0849	1.0350	0.0000	70.4	2.1
0.020	0.005	0.060	0.015	2.5758	87	1.1047	1.0350	0.0000	70.0	2.3
0.020	0.005	0.040	0.030	2.5758	47	1.1299	1.0350	0.0000	58.1	2.4
0.020	0.005	0.060	0.030	2.5758	32	1.1500	1.0350	0.0000	34.3	2.1
0.030	0.005	0.040	0.015	2.5758	303	1.0850	1.0450	0.0000	79.7	2.0
0.030	0.005	0.060	0.015	2.5758	135	1.1049	1.0450	0.0000	84.1	2.2
0.030	0.005	0.040	0.030	2.5758	68	1.1294	1.0450	0.0000	45.5	2.0
0.030	0.005	0.060	0.030	2.5758	44	1.1499	1.0450	0.0000	44.1	2.1
0.020	0.010	0.040	0.015	2.8070	692	1.0850	1.0500	1.0150	159.7	2.0
0.020	0.010	0.060	0.015	2.5758	236	1.1050	1.0500	0.0000	75.2	2.0
0.020	0.010	0.040	0.030	2.5758	112	1.1298	1.0500	0.0000	65.5	2.1
0.020	0.010	0.060	0.030	2.5758	72	1.1495	1.0500	0.0000	50.2	2.0
0.030	0.010	0.040	0.015	2.8070	1468	1.0850	1.0600	1.0350	179.9	2.0
0.030	0.010	0.060	0.015	2.8070	453	1.1050	1.0600	1.0150	165.2	2.1
0.030	0.010	0.040	0.030	2.5758	158	1.1299	1.0600	0.0000	77.7	2.1
0.030	0.010	0.060	0.030	2.5758	96	1.1497	1.0600	0.0000	72.2	2.2

Table A3.2.2 - Parameter Selection Examples for the 3-SPC-AVG-2($n, k_v, 2$) Control Chart having $ARL(p_0) = 200$ $ARL(p_c) \leq 5$

p_{02}	p_{03}	p_{c2}	p_{c3}	k_v	n	UCL_v	CL	LCL_v	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.5758	38	1.1405	1.0350	0.0000	46.1	5.7
0.020	0.005	0.060	0.015	2.5758	18	1.1883	1.0350	0.0000	35.6	5.5
0.020	0.005	0.040	0.030	2.5758	5	1.3259	1.0350	0.0000	35.0	6.5
0.020	0.005	0.060	0.030	2.5758	3	1.4106	1.0350	0.0000	62.1	10.3
0.030	0.005	0.040	0.015	2.5758	78	1.1238	1.0450	0.0000	52.6	4.9
0.030	0.005	0.060	0.015	2.5758	32	1.1680	1.0450	0.0000	48.7	5.6
0.030	0.005	0.040	0.030	2.5758	8	1.2910	1.0450	0.0000	24.6	4.6
0.030	0.005	0.060	0.030	2.5758	5	1.3562	1.0450	0.0000	30.2	5.9
0.020	0.010	0.040	0.015	2.5758	204	1.1091	1.0500	0.0000	88.1	5.4
0.020	0.010	0.060	0.015	2.5758	78	1.1456	1.0500	0.0000	64.5	5.4
0.020	0.010	0.040	0.030	2.5758	24	1.2224	1.0500	0.0000	38.6	5.4
0.020	0.010	0.060	0.030	2.5758	15	1.2681	1.0500	0.0000	72.7	7.3
0.030	0.010	0.040	0.015	2.8070	601	1.0991	1.0600	1.0209	173.3	5.1
0.030	0.010	0.060	0.015	2.5758	133	1.1362	1.0600	0.0000	89.4	5.7
0.030	0.010	0.040	0.030	2.5758	37	1.2045	1.0600	0.0000	48.5	5.2
0.030	0.010	0.060	0.030	2.5758	21	1.2518	1.0600	0.0000	45.0	5.8

Table A3.2.3 - Parameter Selection Examples for the 3-SPC-AVG-2($n, k_V, 2$) Control Chart having $ARL(p_0) = 500$ $ARL(p_c) \leq 2$

p_{02}	p_{03}	p_{c2}	p_{c3}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.8782	212	1.0849	1.0350	0.0000	163.6	2.3
0.020	0.005	0.060	0.015	2.8782	108	1.1049	1.0350	0.0000	106.5	2.2
0.020	0.005	0.040	0.030	2.8782	59	1.1296	1.0350	0.0000	67.0	2.1
0.020	0.005	0.060	0.030	2.8782	40	1.1499	1.0350	0.0000	40.9	2.0
0.030	0.005	0.040	0.015	3.0902	436	1.0850	1.0450	1.0050	357.0	2.2
0.030	0.005	0.060	0.015	2.8782	168	1.1050	1.0450	0.0000	120.9	2.1
0.030	0.005	0.040	0.030	2.8782	84	1.1298	1.0450	0.0000	74.6	2.0
0.030	0.005	0.060	0.030	2.8782	55	1.1498	1.0450	0.0000	99.9	2.3
0.020	0.010	0.040	0.015	3.0902	839	1.0850	1.0500	1.0150	397.6	2.1
0.020	0.010	0.060	0.015	2.8782	295	1.1049	1.0500	0.0000	139.3	2.0
0.020	0.010	0.040	0.030	2.8782	140	1.1298	1.0500	0.0000	145.9	2.2
0.020	0.010	0.060	0.030	2.8782	90	1.1495	1.0500	0.0000	106.3	2.1
0.030	0.010	0.040	0.015	3.0902	1779	1.0850	1.0600	1.0350	459.4	2.1
0.030	0.010	0.060	0.015	3.0902	549	1.1050	1.0600	1.0150	336.6	2.0
0.030	0.010	0.040	0.030	2.8782	197	1.1300	1.0600	0.0000	146.4	2.1
0.030	0.010	0.060	0.030	2.8782	120	1.1496	1.0600	0.0000	100.4	2.0

Table A3.2.4 - Parameter Selection Examples for the 3-SPC-AVG-2($n, k_V, 2$) Control Chart having $ARL(p_0) = 500$ $ARL(p_c) \leq 5$

p_{02}	p_{03}	p_{c2}	p_{c3}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.8782	59	1.1296	1.0350	0.0000	67.0	4.9
0.020	0.005	0.060	0.015	2.8782	28	1.1724	1.0350	0.0000	46.9	4.5
0.020	0.005	0.040	0.030	2.8782	9	1.2773	1.0350	0.0000	22.4	4.1
0.020	0.005	0.060	0.030	2.8782	6	1.3317	1.0350	0.0000	28.4	4.8
0.030	0.005	0.040	0.015	2.8782	118	1.1166	1.0450	0.0000	96.7	4.8
0.030	0.005	0.060	0.015	2.8782	49	1.1561	1.0450	0.0000	69.0	4.9
0.030	0.005	0.040	0.030	2.8782	15	1.2458	1.0450	0.0000	36.1	4.9
0.030	0.005	0.060	0.030	2.8782	9	1.3042	1.0450	0.0000	21.7	4.0
0.020	0.010	0.040	0.015	2.8782	293	1.1051	1.0500	0.0000	147.2	4.8
0.020	0.010	0.060	0.015	2.8782	113	1.1388	1.0500	0.0000	103.4	4.9
0.020	0.010	0.040	0.030	2.8782	38	1.2031	1.0500	0.0000	67.7	4.9
0.020	0.010	0.060	0.030	2.8782	23	1.2468	1.0500	0.0000	42.1	4.9
0.030	0.010	0.040	0.015	3.0902	806	1.0971	1.0600	1.0229	409.5	5.2
0.030	0.010	0.060	0.015	2.8782	191	1.1311	1.0600	0.0000	185.2	5.7
0.030	0.010	0.040	0.030	2.8782	57	1.1901	1.0600	0.0000	75.0	4.7
0.030	0.010	0.060	0.030	2.8782	33	1.2309	1.0600	0.0000	69.8	5.0

Table A3.2.5 - Parameter Selection Examples for the 3-SPC-AVG-2($n, k_v, 5$) Control Chart having $ARL(p_0) = 200$ $ARL(p_c) \leq 2$

p_{02}	p_{03}	p_{c2}	p_{c3}	k_v	n	UCL_v	CL	LCL_v	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.5758	205	1.5198	1.2000	0.0000	55.8	2.1
0.020	0.005	0.060	0.015	2.5758	132	1.5985	1.2000	0.0000	43.2	2.1
0.020	0.005	0.040	0.030	2.5758	46	1.8751	1.2000	0.0000	44.0	2.4
0.020	0.005	0.060	0.030	2.5758	37	1.9528	1.2000	0.0000	49.3	2.2
0.030	0.005	0.040	0.015	2.5758	280	1.5197	1.2400	0.0000	70.7	2.1
0.030	0.005	0.060	0.015	2.5758	170	1.5990	1.2400	0.0000	60.7	2.1
0.030	0.005	0.040	0.030	2.5758	54	1.8770	1.2400	0.0000	32.1	2.0
0.030	0.005	0.060	0.030	2.5758	43	1.9538	1.2400	0.0000	47.2	2.4
0.020	0.010	0.040	0.015	2.8070	1178	1.5200	1.3200	1.1200	184.8	2.1
0.020	0.010	0.060	0.015	2.8070	601	1.6000	1.3200	1.0401	167.5	2.1
0.020	0.010	0.040	0.030	2.5758	127	1.8788	1.3200	0.0000	60.7	2.0
0.020	0.010	0.060	0.030	2.5758	97	1.9594	1.3200	0.0000	57.7	2.2
0.030	0.010	0.040	0.015	2.8070	1881	1.5200	1.3600	1.2000	184.3	2.0
0.030	0.010	0.060	0.015	2.8070	836	1.6000	1.3600	1.1200	172.4	2.1
0.030	0.010	0.040	0.030	2.5758	150	1.8799	1.3600	0.0000	58.5	2.0
0.030	0.010	0.060	0.030	2.5758	113	1.9590	1.3600	0.0000	66.8	2.2

Table A3.2.6 - Parameter Selection Examples for the 3-SPC-AVG-2($n, k_v, 5$) Control Chart having $ARL(p_0) = 200$ $ARL(p_c) \leq 5$

p_{02}	p_{03}	p_{c2}	p_{c3}	k_v	n	UCL_v	CL	LCL_v	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.5758	42	1.9065	1.2000	0.0000	49.0	6.3
0.020	0.005	0.060	0.015	2.5758	26	2.0980	1.2000	0.0000	56.3	5.6
0.020	0.005	0.040	0.030	2.5758	3	3.8436	1.2000	0.0000	67.0	11.4
0.020	0.005	0.060	0.030	2.5758	3	3.8436	1.2000	0.0000	67.0	11.4
0.030	0.005	0.040	0.015	2.5758	60	1.8443	1.2400	0.0000	31.9	4.6
0.030	0.005	0.060	0.015	2.5758	35	2.0312	1.2400	0.0000	62.2	6.9
0.030	0.005	0.040	0.030	2.5758	4	3.5805	1.2400	0.0000	50.1	8.7
0.030	0.005	0.060	0.030	2.5758	3	3.9425	1.2400	0.0000	66.9	11.4
0.020	0.010	0.040	0.015	2.5758	356	1.6538	1.3200	0.0000	89.8	5.3
0.020	0.010	0.060	0.015	2.5758	179	1.7907	1.3200	0.0000	75.1	5.2
0.020	0.010	0.040	0.030	2.5758	26	2.5551	1.3200	0.0000	36.0	5.4
0.020	0.010	0.060	0.030	2.5758	20	2.7282	1.3200	0.0000	55.9	6.4
0.030	0.010	0.040	0.015	2.8070	761	1.6115	1.3600	1.1085	173.0	5.1
0.030	0.010	0.060	0.015	2.5758	252	1.7611	1.3600	0.0000	71.9	4.9
0.030	0.010	0.040	0.030	2.5758	31	2.5036	1.3600	0.0000	26.0	4.2
0.030	0.010	0.060	0.030	2.5758	23	2.6877	1.3600	0.0000	44.0	6.0

Table A3.2.7 - Parameter Selection Examples for the 3-SPC-AVG-2($n, k_V, 5$) Control Chart having $ARL(p_0) = 500$ $ARL(p_c) \leq 2$

p_{02}	p_{03}	p_{c2}	p_{c3}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.8782	256	1.5198	1.2000	0.0000	113.2	2.2
0.020	0.005	0.060	0.015	2.8782	164	1.5995	1.2000	0.0000	89.7	2.1
0.020	0.005	0.040	0.030	2.8782	57	1.8777	1.2000	0.0000	42.8	2.0
0.020	0.005	0.060	0.030	2.8782	46	1.9544	1.2000	0.0000	44.0	2.2
0.030	0.005	0.040	0.015	2.8782	349	1.5200	1.2400	0.0000	133.5	2.1
0.030	0.005	0.060	0.015	2.8782	212	1.5992	1.2400	0.0000	99.2	2.1
0.030	0.005	0.040	0.030	2.8782	67	1.8790	1.2400	0.0000	58.1	2.1
0.030	0.005	0.060	0.030	2.8782	53	1.9584	1.2400	0.0000	42.2	2.1
0.020	0.010	0.040	0.015	3.0902	1428	1.5199	1.3200	1.1201	393.0	2.0
0.020	0.010	0.060	0.015	3.0902	729	1.5998	1.3200	1.0402	337.3	2.1
0.020	0.010	0.040	0.030	2.8782	158	1.8798	1.3200	0.0000	114.8	2.1
0.020	0.010	0.060	0.030	2.8782	121	1.9597	1.3200	0.0000	118.0	2.2
0.030	0.010	0.040	0.015	3.0902	2280	1.5200	1.3600	1.2000	432.6	2.0
0.030	0.010	0.060	0.015	3.0902	1014	1.5999	1.3600	1.1201	390.1	2.1
0.030	0.010	0.040	0.030	2.8782	188	1.8789	1.3600	0.0000	130.1	2.1
0.030	0.010	0.060	0.030	2.8782	141	1.9592	1.3600	0.0000	99.9	2.1

Table A3.2.8 - Parameter Selection Examples for the 3-SPC-AVG-2($n, k_V, 5$) Control Chart having $ARL(p_0) = 500$ $ARL(p_c) \leq 5$

p_{02}	p_{03}	p_{c2}	p_{c3}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.8782	66	1.8298	1.2000	0.0000	52.9	4.7
0.020	0.005	0.060	0.015	2.8782	41	1.9990	1.2000	0.0000	54.9	6.3
0.020	0.005	0.040	0.030	2.8782	6	3.2887	1.2000	0.0000	33.7	6.0
0.020	0.005	0.060	0.030	2.8782	5	3.4881	1.2000	0.0000	40.4	7.1
0.030	0.005	0.040	0.015	2.8782	94	1.7795	1.2400	0.0000	78.3	5.4
0.030	0.005	0.060	0.015	2.8782	56	1.9389	1.2400	0.0000	58.1	5.3
0.030	0.005	0.040	0.030	2.8782	8	3.0892	1.2400	0.0000	25.4	4.6
0.030	0.005	0.060	0.030	2.8782	6	3.3753	1.2400	0.0000	33.7	6.0
0.020	0.010	0.040	0.015	3.0902	633	1.6203	1.3200	1.0197	328.6	5.2
0.020	0.010	0.060	0.015	2.8782	256	1.7598	1.3200	0.0000	137.7	5.1
0.020	0.010	0.040	0.030	2.8782	41	2.4190	1.3200	0.0000	95.6	5.8
0.020	0.010	0.060	0.030	2.8782	31	2.5839	1.3200	0.0000	52.2	4.7
0.030	0.010	0.040	0.015	3.0902	1021	1.5991	1.3600	1.1209	354.1	5.0
0.030	0.010	0.060	0.015	2.8782	361	1.7345	1.3600	0.0000	168.0	5.2
0.030	0.010	0.040	0.030	2.8782	49	2.3764	1.3600	0.0000	71.0	5.2
0.030	0.010	0.060	0.030	2.8782	37	2.5296	1.3600	0.0000	98.1	5.7

Table A3.2.9 - Parameter Selection Examples for the 3-SPC-AVG-2($n, k_V, 10$) Control Chart having $ARL(p_0) = 200$ $ARL(p_c) \leq 2$

p_{02}	p_{03}	p_{c2}	p_{c3}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.5758	244	2.8430	1.6750	0.0000	56.7	2.0
0.020	0.005	0.060	0.015	2.5758	183	3.0237	1.6750	0.0000	68.9	2.1
0.020	0.005	0.040	0.030	2.5758	48	4.3084	1.6750	0.0000	41.3	2.4
0.020	0.005	0.060	0.030	2.5758	42	4.4902	1.6750	0.0000	53.0	2.7
0.030	0.005	0.040	0.015	2.5758	290	2.8436	1.7650	0.0000	63.2	2.2
0.030	0.005	0.060	0.015	2.5758	213	3.0236	1.7650	0.0000	48.1	2.2
0.030	0.005	0.040	0.030	2.5758	52	4.3122	1.7650	0.0000	35.6	2.2
0.030	0.005	0.060	0.030	2.5758	45	4.5031	1.7650	0.0000	46.5	2.5
0.020	0.010	0.040	0.015	2.8070	1700	2.8449	2.1700	1.4951	184.0	2.1
0.020	0.010	0.060	0.015	2.8070	1060	3.0246	2.1700	1.3154	173.3	2.1
0.020	0.010	0.040	0.030	2.5758	140	4.3280	2.1700	0.0000	71.4	2.2
0.020	0.010	0.060	0.030	2.5758	120	4.5009	2.1700	0.0000	55.4	2.1
0.030	0.010	0.040	0.015	2.8070	2276	2.8450	2.2600	1.6750	187.8	2.0
0.030	0.010	0.060	0.015	2.8070	1331	3.0250	2.2600	1.4950	179.2	2.1
0.030	0.010	0.040	0.030	2.5758	154	4.3237	2.2600	0.0000	52.5	2.0
0.030	0.010	0.060	0.030	2.5758	130	4.5062	2.2600	0.0000	76.4	2.1

Table A3.2.10 - Parameter Selection Examples for the 3-SPC-AVG-2($n, k_V, 10$) Control Chart having $ARL(p_0) = 200$ $ARL(p_c) \leq 5$

p_{02}	p_{03}	p_{c2}	p_{c3}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.5758	48	4.3084	1.6750	0.0000	41.3	6.1
0.020	0.005	0.060	0.015	2.5758	36	4.7158	1.6750	0.0000	67.3	6.5
0.020	0.005	0.040	0.030	2.5758	3	12.2086	1.6750	0.0000	67.0	11.5
0.020	0.005	0.060	0.030	2.5758	2	14.5759	1.6750	0.0000	100.3	16.9
0.030	0.005	0.040	0.015	2.5758	58	4.1768	1.7650	0.0000	29.1	4.6
0.030	0.005	0.060	0.015	2.5758	42	4.5992	1.7650	0.0000	52.4	7.0
0.030	0.005	0.040	0.030	2.5758	3	12.3698	1.7650	0.0000	67.0	11.5
0.030	0.005	0.060	0.030	2.5758	3	12.3698	1.7650	0.0000	67.0	11.5
0.020	0.010	0.040	0.015	2.8070	682	3.2355	2.1700	1.1045	167.0	5.1
0.020	0.010	0.060	0.015	2.5758	321	3.5951	2.1700	0.0000	71.7	5.1
0.020	0.010	0.040	0.030	2.5758	28	6.9954	2.1700	0.0000	31.4	4.9
0.020	0.010	0.060	0.030	2.5758	24	7.3820	2.1700	0.0000	41.9	6.2
0.030	0.010	0.040	0.015	2.8070	917	3.1816	2.2600	1.3384	172.8	5.1
0.030	0.010	0.060	0.015	2.8070	534	3.4677	2.2600	1.0523	149.8	5.1
0.030	0.010	0.040	0.030	2.5758	31	6.8597	2.2600	0.0000	26.0	4.2
0.030	0.010	0.060	0.030	2.5758	26	7.2826	2.2600	0.0000	36.1	5.5

Table A3.2.11 - Parameter Selection Examples for the 3-SPC-AVG-2($n, k_v, 10$) Control Chart having $ARL(p_0) = 500$ $ARL(p_c) \leq 2$

p_{02}	p_{03}	p_{c2}	p_{c3}	k_v	n	UCL_v	CL	LCL_v	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.8782	304	2.8442	1.6750	0.0000	119.6	2.1
0.020	0.005	0.060	0.015	2.8782	229	3.0222	1.6750	0.0000	122.1	2.1
0.020	0.005	0.040	0.030	2.8782	59	4.3291	1.6750	0.0000	28.2	1.9
0.020	0.005	0.060	0.030	2.8782	52	4.5021	1.6750	0.0000	35.6	2.1
0.030	0.005	0.040	0.015	2.8782	362	2.8437	1.7650	0.0000	116.0	2.1
0.030	0.005	0.060	0.015	2.8782	266	3.0234	1.7650	0.0000	93.3	2.1
0.030	0.005	0.040	0.030	2.8782	65	4.3107	1.7650	0.0000	38.5	2.0
0.030	0.005	0.060	0.030	2.8782	56	4.5076	1.7650	0.0000	31.0	2.0
0.020	0.010	0.040	0.015	3.0902	2060	2.8449	2.1700	1.4951	420.8	2.1
0.020	0.010	0.060	0.015	3.0902	1284	3.0249	2.1700	1.3151	369.7	2.0
0.020	0.010	0.040	0.030	2.8782	175	4.3267	2.1700	0.0000	113.5	2.2
0.020	0.010	0.060	0.030	2.8782	149	4.5073	2.1700	0.0000	117.0	2.1
0.030	0.010	0.040	0.015	3.0902	2759	2.8449	2.2600	1.6751	437.4	2.0
0.030	0.010	0.060	0.015	3.0902	1614	3.0248	2.2600	1.4952	409.8	2.1
0.030	0.010	0.040	0.030	2.8782	192	4.3252	2.2600	0.0000	99.4	2.0
0.030	0.010	0.060	0.030	2.8782	162	4.5083	2.2600	0.0000	142.8	2.1

Table A3.2.12 - Parameter Selection Examples for the 3-SPC-AVG-2($n, k_v, 10$) Control Chart having $ARL(p_0) = 500$ $ARL(p_c) \leq 5$

p_{02}	p_{03}	p_{c2}	p_{c3}	k_v	n	UCL_v	CL	LCL_v	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.020	0.005	0.040	0.015	2.8782	76	4.0135	1.6750	0.0000	102.5	5.5
0.020	0.005	0.060	0.015	2.8782	57	4.3752	1.6750	0.0000	30.1	4.7
0.020	0.005	0.040	0.030	2.8782	6	9.9976	1.6750	0.0000	33.8	6.0
0.020	0.005	0.060	0.030	2.8782	5	10.7919	1.6750	0.0000	40.4	7.1
0.030	0.005	0.040	0.015	2.8782	92	3.9048	1.7650	0.0000	85.7	6.0
0.030	0.005	0.060	0.015	2.8782	67	4.2724	1.7650	0.0000	58.2	4.5
0.030	0.005	0.040	0.030	2.8782	7	9.5223	1.7650	0.0000	29.0	5.2
0.030	0.005	0.060	0.030	2.8782	6	10.1438	1.7650	0.0000	33.8	6.0
0.020	0.010	0.040	0.015	3.0902	917	3.1816	2.1700	1.1584	348.0	5.1
0.020	0.010	0.060	0.015	2.8782	459	3.5017	2.1700	0.0000	154.5	5.1
0.020	0.010	0.040	0.030	2.8782	44	6.4711	2.1700	0.0000	101.5	6.6
0.020	0.010	0.060	0.030	2.8782	38	6.7982	2.1700	0.0000	121.2	5.4
0.030	0.010	0.040	0.015	3.0902	1231	3.1357	2.2600	1.3843	385.9	5.1
0.030	0.010	0.060	0.015	3.0902	718	3.4066	2.2600	1.1134	307.8	5.0
0.030	0.010	0.040	0.030	2.8782	49	6.3480	2.2600	0.0000	76.4	5.5
0.030	0.010	0.060	0.030	2.8782	41	6.7291	2.2600	0.0000	119.4	6.9

APPENDIX 4

PARAMETER SELECTION EXAMPLES FOR 4-LEVEL CONTROL CHARTS

APPENDIX 4.1

PARAMETER SELECTION EXAMPLES FOR THE 4-SPC-AVG-1(n, k_1, ν_1, ν_2) CONTROL CHART

**Table A4.1.1 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.25, 0.5$) Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005,$
 $ARL(p_0) = 200$ and $ARL(p_c) \leq 2$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.5758	48	0.0672	0.0263	0.0000	51.0	2.0
0.120	0.050	0.020	2.5758	34	0.0749	0.0263	0.0000	68.3	2.3
0.090	0.075	0.020	2.5758	28	0.0798	0.0263	0.0000	40.0	2.0
0.120	0.075	0.020	2.5758	22	0.0867	0.0263	0.0000	44.2	2.1
0.090	0.050	0.040	2.5758	22	0.0867	0.0263	0.0000	44.2	2.1
0.120	0.050	0.040	2.5758	18	0.0931	0.0263	0.0000	40.3	2.0
0.090	0.075	0.040	2.5758	15	0.0995	0.0263	0.0000	29.4	2.0
0.120	0.075	0.040	2.5758	13	0.1049	0.0263	0.0000	41.0	2.2

**Table A4.1.2 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.25, 0.5$) Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010,$
 $ARL(p_0) = 200$ and $ARL(p_c) \leq 2$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8070	205	0.0675	0.0400	0.0125	174.4	2.1
0.120	0.050	0.020	2.8070	127	0.0749	0.0400	0.0051	177.4	2.2
0.090	0.075	0.020	2.5758	82	0.0799	0.0400	0.0000	94.6	2.2
0.120	0.075	0.020	2.5758	58	0.0874	0.0400	0.0000	83.7	2.2
0.090	0.050	0.040	2.5758	58	0.0874	0.0400	0.0000	83.7	2.2
0.120	0.050	0.040	2.5758	44	0.0944	0.0400	0.0000	66.0	2.1
0.090	0.075	0.040	2.5758	37	0.0994	0.0400	0.0000	59.7	2.0
0.120	0.075	0.040	2.5758	29	0.1071	0.0400	0.0000	63.9	2.1

**Table A4.1.3 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.25, 0.5$) Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005,$
 $ARL(p_0) = 200$ and $ARL(p_c) \leq 5$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.5758	10	0.1159	0.0263	0.0000	34.0	5.0
0.120	0.050	0.020	2.5758	7	0.1334	0.0263	0.0000	21.9	4.6
0.090	0.075	0.020	2.5758	5	0.1531	0.0263	0.0000	33.7	6.5
0.120	0.075	0.020	2.5758	4	0.1681	0.0263	0.0000	31.0	5.3
0.090	0.050	0.040	2.5758	3	0.1900	0.0263	0.0000	46.9	6.8
0.120	0.050	0.040	2.5758	2	0.2268	0.0263	0.0000	19.5	5.4
0.090	0.075	0.040	2.5758	2	0.2268	0.0263	0.0000	19.5	4.4
0.120	0.075	0.040	2.5758	2	0.2268	0.0263	0.0000	19.5	4.3

Table A4.1.4 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.25, 0.5$) Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010, ARL(p_0) = 200$ and $ARL(p_c) \leq 5$

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.5758	57	0.0878	0.0400	0.0000	93.2	5.7
0.120	0.050	0.020	2.5758	35	0.1010	0.0400	0.0000	77.9	5.7
0.090	0.075	0.020	2.5758	25	0.1122	0.0400	0.0000	68.0	5.6
0.120	0.075	0.020	2.5758	17	0.1276	0.0400	0.0000	46.0	4.9
0.090	0.050	0.040	2.5758	14	0.1365	0.0400	0.0000	42.6	4.8
0.120	0.050	0.040	2.5758	10	0.1542	0.0400	0.0000	58.2	6.1
0.090	0.075	0.040	2.5758	8	0.1677	0.0400	0.0000	42.6	5.3
0.120	0.075	0.040	2.5758	6	0.1874	0.0400	0.0000	37.7	5.3

Table A4.1.5 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.25, 0.5$) Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005, ARL(p_0) = 500$ and $ARL(p_c) \leq 2$

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8782	60	0.0672	0.0263	0.0000	141.3	2.2
0.120	0.050	0.020	2.8782	43	0.0746	0.0263	0.0000	83.0	2.0
0.090	0.075	0.020	2.8782	35	0.0798	0.0263	0.0000	112.4	2.3
0.120	0.075	0.020	2.8782	27	0.0872	0.0263	0.0000	86.0	2.2
0.090	0.050	0.040	2.8782	27	0.0872	0.0263	0.0000	86.0	2.2
0.120	0.050	0.040	2.8782	22	0.0938	0.0263	0.0000	89.7	2.2
0.090	0.075	0.040	2.8782	19	0.0990	0.0263	0.0000	68.5	2.1
0.120	0.075	0.040	2.8782	16	0.1055	0.0263	0.0000	55.9	2.0

Table A4.1.6 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.25, 0.5$) Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010, ARL(p_0) = 500$ and $ARL(p_c) \leq 2$

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	3.0902	249	0.0675	0.0400	0.0125	395.2	2.1
0.120	0.050	0.020	3.0902	154	0.0749	0.0400	0.0051	348.6	2.1
0.090	0.075	0.020	2.8782	102	0.0799	0.0400	0.0000	159.5	2.0
0.120	0.075	0.020	2.8782	73	0.0872	0.0400	0.0000	149.0	2.1
0.090	0.050	0.040	2.8782	73	0.0872	0.0400	0.0000	149.0	2.1
0.120	0.050	0.040	2.8782	54	0.0949	0.0400	0.0000	130.7	2.1
0.090	0.075	0.040	2.8782	46	0.0995	0.0400	0.0000	134.6	2.1
0.120	0.075	0.040	2.8782	36	0.1072	0.0400	0.0000	113.9	2.1

**Table A4.1.7 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.25, 0.5$)
Control Chart having $p_{e2} = 0.045, p_{e3} = 0.020, p_{e4} = 0.005,$
 $ARL(p_0) = 500$ and $ARL(p_c) \leq 5$**

p_{e2}	p_{e3}	p_{e4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8782	16	0.1055	0.0263	0.0000	55.9	4.7
0.120	0.050	0.020	2.8782	11	0.1218	0.0263	0.0000	60.7	5.6
0.090	0.075	0.020	2.8782	9	0.1319	0.0263	0.0000	42.3	4.6
0.120	0.075	0.020	2.8782	7	0.1460	0.0263	0.0000	72.1	6.4
0.090	0.050	0.040	2.8782	5	0.1680	0.0263	0.0000	33.7	4.7
0.120	0.050	0.040	2.8782	4	0.1847	0.0263	0.0000	31.0	4.5
0.090	0.075	0.040	2.8782	3	0.2092	0.0263	0.0000	46.9	6.0
0.120	0.075	0.040	2.8782	3	0.2092	0.0263	0.0000	46.9	5.7

**Table A4.1.8 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.25, 0.5$)
Control Chart having $p_{e2} = 0.060, p_{e3} = 0.030, p_{e4} = 0.010,$
 $ARL(p_0) = 500$ and $ARL(p_c) \leq 5$**

p_{e2}	p_{e3}	p_{e4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8782	83	0.0843	0.0400	0.0000	127.5	4.6
0.120	0.050	0.020	2.8782	50	0.0971	0.0400	0.0000	132.1	5.3
0.090	0.075	0.020	2.8782	36	0.1072	0.0400	0.0000	113.9	5.2
0.120	0.075	0.020	2.8782	26	0.1191	0.0400	0.0000	102.9	5.2
0.090	0.050	0.040	2.8782	21	0.1280	0.0400	0.0000	75.5	4.8
0.120	0.050	0.040	2.8782	16	0.1409	0.0400	0.0000	106.9	5.9
0.090	0.075	0.040	2.8782	12	0.1565	0.0400	0.0000	66.1	5.1
0.120	0.075	0.040	2.8782	10	0.1676	0.0400	0.0000	58.2	4.7

**Table A4.1.9 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.25, 0.75$)
Control Chart having $p_{e2} = 0.045, p_{e3} = 0.020, p_{e4} = 0.005,$
 $ARL(p_0) = 200$ and $ARL(p_c) \leq 2$**

p_{e2}	p_{e3}	p_{e4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.5758	51	0.0798	0.0313	0.0000	75.2	2.2
0.120	0.050	0.020	2.5758	38	0.0874	0.0313	0.0000	68.2	2.2
0.090	0.075	0.020	2.5758	27	0.0979	0.0313	0.0000	52.2	2.1
0.120	0.075	0.020	2.5758	22	0.1051	0.0313	0.0000	58.8	2.2
0.090	0.050	0.040	2.5758	26	0.0992	0.0313	0.0000	59.6	2.2
0.120	0.050	0.040	2.5758	21	0.1068	0.0313	0.0000	39.3	2.0
0.090	0.075	0.040	2.5758	16	0.1179	0.0313	0.0000	45.6	2.1
0.120	0.075	0.040	2.5758	14	0.1238	0.0313	0.0000	30.5	1.9

**Table A4.1.10 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.25, 0.75$) Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010,$
 $ARL(p_0) = 200$ and $ARL(p_c) \leq 2$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8070	212	0.0800	0.0475	0.0150	166.9	2.0
0.120	0.050	0.020	2.8070	140	0.0875	0.0475	0.0075	144.8	2.0
0.090	0.075	0.020	2.5758	72	0.0986	0.0475	0.0000	89.2	2.1
0.120	0.075	0.020	2.5758	55	0.1060	0.0475	0.0000	85.6	2.1
0.090	0.050	0.040	2.5758	69	0.0997	0.0475	0.0000	84.5	2.1
0.120	0.050	0.040	2.5758	53	0.1071	0.0475	0.0000	73.1	2.0
0.090	0.075	0.040	2.5758	38	0.1179	0.0475	0.0000	59.7	2.0
0.120	0.075	0.040	2.5758	31	0.1254	0.0475	0.0000	65.6	2.1

**Table A4.1.11 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.25, 0.75$) Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005,$
 $ARL(p_0) = 200$ and $ARL(p_c) \leq 5$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.5758	12	0.1312	0.0313	0.0000	43.6	5.3
0.120	0.050	0.020	2.5758	9	0.1467	0.0313	0.0000	44.4	5.8
0.090	0.075	0.020	2.5758	5	0.1862	0.0313	0.0000	22.7	4.4
0.120	0.075	0.020	2.5758	4	0.2045	0.0313	0.0000	31.3	5.4
0.090	0.050	0.040	2.5758	4	0.2045	0.0313	0.0000	31.3	4.9
0.120	0.050	0.040	2.5758	4	0.2045	0.0313	0.0000	31.3	4.6
0.090	0.075	0.040	2.5758	2	0.2762	0.0313	0.0000	20.3	4.6
0.120	0.075	0.040	2.5758	2	0.2762	0.0313	0.0000	20.3	4.6

**Table A4.1.12 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.25, 0.75$) Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010,$
 $ARL(p_0) = 200$ and $ARL(p_c) \leq 5$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.5758	60	0.1035	0.0475	0.0000	72.3	4.8
0.120	0.050	0.020	2.5758	39	0.1170	0.0475	0.0000	80.3	5.4
0.090	0.075	0.020	2.5758	21	0.1422	0.0475	0.0000	47.3	4.5
0.120	0.075	0.020	2.5758	16	0.1560	0.0475	0.0000	40.6	4.4
0.090	0.050	0.040	2.5758	18	0.1498	0.0475	0.0000	47.2	4.7
0.120	0.050	0.040	2.5758	14	0.1635	0.0475	0.0000	62.4	5.7
0.090	0.075	0.040	2.5758	9	0.1921	0.0475	0.0000	30.2	4.1
0.120	0.075	0.040	2.5758	7	0.2115	0.0475	0.0000	30.5	4.5

**Table A4.1.13 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.25, 0.75$) Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005,$
 $ARL(p_0) = 500$ and $ARL(p_c) \leq 2$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8782	64	0.0796	0.0313	0.0000	133.7	2.1
0.120	0.050	0.020	2.8782	48	0.0871	0.0313	0.0000	101.4	2.0
0.090	0.075	0.020	2.8782	33	0.0986	0.0313	0.0000	125.1	2.3
0.120	0.075	0.020	2.8782	27	0.1057	0.0313	0.0000	93.7	2.1
0.090	0.050	0.040	2.8782	32	0.0997	0.0313	0.0000	83.2	2.0
0.120	0.050	0.040	2.8782	26	0.1072	0.0313	0.0000	108.6	2.3
0.090	0.075	0.040	2.8782	20	0.1178	0.0313	0.0000	80.2	2.1
0.120	0.075	0.040	2.8782	17	0.1251	0.0313	0.0000	75.2	2.1

**Table A4.1.14 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.25, 0.75$) Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010,$
 $ARL(p_0) = 500$ and $ARL(p_c) \leq 2$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	3.0902	257	0.0800	0.0475	0.0150	415.6	2.1
0.120	0.050	0.020	3.0902	170	0.0874	0.0475	0.0076	352.6	2.1
0.090	0.075	0.020	2.8782	90	0.0986	0.0475	0.0000	172.9	2.1
0.120	0.075	0.020	2.8782	69	0.1059	0.0475	0.0000	174.3	2.1
0.090	0.050	0.040	2.8782	86	0.0998	0.0475	0.0000	180.6	2.1
0.120	0.050	0.040	2.8782	66	0.1072	0.0475	0.0000	166.8	2.1
0.090	0.075	0.040	2.8782	47	0.1182	0.0475	0.0000	152.4	2.1
0.120	0.075	0.040	2.8782	38	0.1261	0.0475	0.0000	144.7	2.2

**Table A4.1.15 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.25, 0.75$) Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005,$
 $ARL(p_0) = 500$ and $ARL(p_c) \leq 5$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8782	18	0.1225	0.0313	0.0000	63.0	4.7
0.120	0.050	0.020	2.8782	13	0.1386	0.0313	0.0000	82.2	5.8
0.090	0.075	0.020	2.8782	8	0.1681	0.0313	0.0000	57.1	5.2
0.120	0.075	0.020	2.8782	6	0.1893	0.0313	0.0000	61.0	5.6
0.090	0.050	0.040	2.8782	7	0.1775	0.0313	0.0000	43.7	4.5
0.120	0.050	0.040	2.8782	6	0.1893	0.0313	0.0000	61.0	5.1
0.090	0.075	0.040	2.8782	4	0.2248	0.0313	0.0000	31.3	4.2
0.120	0.075	0.040	2.8782	3	0.2547	0.0313	0.0000	47.1	5.7

**Table A4.1.16 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.25, 0.75$) Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010,$
 $ARL(p_0) = 500$ and $ARL(p_c) \leq 5$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8782	87	0.0995	0.0475	0.0000	164.0	5.0
0.120	0.050	0.020	2.8782	57	0.1117	0.0475	0.0000	147.8	5.1
0.090	0.075	0.020	2.8782	31	0.1346	0.0475	0.0000	104.9	4.8
0.120	0.075	0.020	2.8782	23	0.1486	0.0475	0.0000	92.0	4.9
0.090	0.050	0.040	2.8782	27	0.1408	0.0475	0.0000	126.3	5.5
0.120	0.050	0.040	2.8782	21	0.1533	0.0475	0.0000	80.1	4.6
0.090	0.075	0.040	2.8782	13	0.1820	0.0475	0.0000	79.6	5.2
0.120	0.075	0.040	2.8782	11	0.1937	0.0475	0.0000	77.5	5.1

**Table A4.1.17 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.5, 0.75$) Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005,$
 $ARL(p_0) = 200$ and $ARL(p_c) \leq 2$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.5758	48	0.1021	0.0425	0.0000	76.6	2.0
0.120	0.050	0.020	2.5758	31	0.1167	0.0425	0.0000	71.4	2.1
0.090	0.075	0.020	2.5758	28	0.1205	0.0425	0.0000	68.0	2.1
0.120	0.075	0.020	2.5758	20	0.1348	0.0425	0.0000	51.8	2.0
0.090	0.050	0.040	2.5758	27	0.1220	0.0425	0.0000	79.9	2.2
0.120	0.050	0.040	2.5758	19	0.1372	0.0425	0.0000	62.6	2.1
0.090	0.075	0.040	2.5758	18	0.1398	0.0425	0.0000	76.6	2.2
0.120	0.075	0.040	2.5758	14	0.1528	0.0425	0.0000	52.2	2.0

**Table A4.1.18 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.5, 0.75$) Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010,$
 $ARL(p_0) = 200$ and $ARL(p_c) \leq 2$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8070	187	0.1025	0.0625	0.0225	183.7	2.0
0.120	0.050	0.020	2.8070	99	0.1175	0.0625	0.0075	174.7	2.0
0.090	0.075	0.020	2.5758	73	0.1212	0.0625	0.0000	104.1	2.1
0.120	0.075	0.020	2.5758	47	0.1357	0.0625	0.0000	89.6	2.0
0.090	0.050	0.040	2.5758	70	0.1225	0.0625	0.0000	106.7	2.1
0.120	0.050	0.040	2.5758	45	0.1373	0.0625	0.0000	81.9	2.0
0.090	0.075	0.040	2.5758	41	0.1409	0.0625	0.0000	101.7	2.2
0.120	0.075	0.040	2.5758	29	0.1557	0.0625	0.0000	96.0	2.2

Table A4.1.19 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.5, 0.75$) Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005, ARL(p_0) = 200$ and $ARL(p_c) \leq 5$

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.5758	12	0.1617	0.0425	0.0000	43.0	4.7
0.120	0.050	0.020	2.5758	8	0.1885	0.0425	0.0000	65.7	6.0
0.090	0.075	0.020	2.5758	6	0.2111	0.0425	0.0000	55.9	6.1
0.120	0.075	0.020	2.5758	4	0.2489	0.0425	0.0000	23.4	4.1
0.090	0.050	0.040	2.5758	6	0.2111	0.0425	0.0000	55.9	5.6
0.120	0.050	0.040	2.5758	4	0.2489	0.0425	0.0000	23.4	3.6
0.090	0.075	0.040	2.5758	3	0.2809	0.0425	0.0000	37.1	5.4
0.120	0.075	0.040	2.5758	3	0.2809	0.0425	0.0000	37.1	4.8

Table A4.1.20 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.5, 0.75$) Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010, ARL(p_0) = 200$ and $ARL(p_c) \leq 5$

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.5758	55	0.1302	0.0625	0.0000	89.4	4.9
0.120	0.050	0.020	2.5758	28	0.1574	0.0625	0.0000	73.6	4.9
0.090	0.075	0.020	2.5758	23	0.1672	0.0625	0.0000	77.9	5.2
0.120	0.075	0.020	2.5758	14	0.1966	0.0625	0.0000	83.5	5.9
0.090	0.050	0.040	2.5758	21	0.1720	0.0625	0.0000	73.2	5.1
0.120	0.050	0.040	2.5758	13	0.2017	0.0625	0.0000	61.8	5.1
0.090	0.075	0.040	2.5758	11	0.2138	0.0625	0.0000	61.3	5.2
0.120	0.075	0.040	2.5758	8	0.2400	0.0625	0.0000	46.0	4.5

Table A4.1.21 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.5, 0.75$) Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005, ARL(p_0) = 500$ and $ARL(p_c) \leq 2$

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8782	60	0.1021	0.0425	0.0000	157.4	2.1
0.120	0.050	0.020	2.8782	38	0.1173	0.0425	0.0000	112.2	2.0
0.090	0.075	0.020	2.8782	35	0.1205	0.0425	0.0000	106.4	2.0
0.120	0.075	0.020	2.8782	25	0.1348	0.0425	0.0000	113.0	2.1
0.090	0.050	0.040	2.8782	34	0.1216	0.0425	0.0000	123.8	2.1
0.120	0.050	0.040	2.8782	24	0.1367	0.0425	0.0000	136.3	2.2
0.090	0.075	0.040	2.8782	22	0.1409	0.0425	0.0000	112.5	2.1
0.120	0.075	0.040	2.8782	17	0.1544	0.0425	0.0000	95.1	2.1

**Table A4.1.22 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.5, 0.75$) Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010,$
 $ARL(p_0) = 500$ and $ARL(p_c) \leq 2$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	3.0902	227	0.1025	0.0625	0.0225	468.2	2.1
0.120	0.050	0.020	3.0902	120	0.1175	0.0625	0.0075	391.0	2.1
0.090	0.075	0.020	2.8782	92	0.1210	0.0625	0.0000	210.0	2.0
0.120	0.075	0.020	2.8782	58	0.1361	0.0625	0.0000	176.2	2.0
0.090	0.050	0.040	2.8782	88	0.1223	0.0625	0.0000	240.8	2.1
0.120	0.050	0.040	2.8782	56	0.1374	0.0625	0.0000	161.3	2.0
0.090	0.075	0.040	2.8782	51	0.1410	0.0625	0.0000	156.6	2.0
0.120	0.075	0.040	2.8782	36	0.1560	0.0625	0.0000	156.2	2.1

**Table A4.1.23 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.5, 0.75$) Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005,$
 $ARL(p_0) = 500$ and $ARL(p_c) \leq 5$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8782	19	0.1483	0.0425	0.0000	114.3	5.3
0.120	0.050	0.020	2.8782	11	0.1816	0.0425	0.0000	55.4	4.3
0.090	0.075	0.020	2.8782	10	0.1884	0.0425	0.0000	73.4	4.8
0.120	0.075	0.020	2.8782	7	0.2169	0.0425	0.0000	94.4	5.8
0.090	0.050	0.040	2.8782	9	0.1963	0.0425	0.0000	101.0	5.7
0.120	0.050	0.040	2.8782	6	0.2308	0.0425	0.0000	55.9	4.7
0.090	0.075	0.040	2.8782	5	0.2488	0.0425	0.0000	38.2	4.3
0.120	0.075	0.040	2.8782	4	0.2732	0.0425	0.0000	61.7	5.4

**Table A4.1.24 - Parameter Selection Examples for the 4-SPC-AVG-1($n, k_V, 0.5, 0.75$) Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010,$
 $ARL(p_0) = 500$ and $ARL(p_c) \leq 5$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8782	79	0.1256	0.0625	0.0000	189.1	4.9
0.120	0.050	0.020	2.8782	40	0.1512	0.0625	0.0000	182.3	5.4
0.090	0.075	0.020	2.8782	34	0.1587	0.0625	0.0000	144.9	4.9
0.120	0.075	0.020	2.8782	21	0.1849	0.0625	0.0000	122.8	4.9
0.090	0.050	0.040	2.8782	31	0.1632	0.0625	0.0000	163.9	5.3
0.120	0.050	0.040	2.8782	19	0.1912	0.0625	0.0000	118.1	5.0
0.090	0.075	0.040	2.8782	17	0.1985	0.0625	0.0000	114.7	4.9
0.120	0.075	0.040	2.8782	12	0.2244	0.0625	0.0000	83.8	4.5

APPENDIX 4.2

PARAMETER SELECTION EXAMPLES FOR THE 4-SPC-AVG-2(n, k_v, v) CONTROL CHART

**Table A4.2.1 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 2$) Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005,$
 $ARL(p_0) = 200$ and $ARL(p_c) \leq 2$**

p_{e2}	p_{e3}	p_{e4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.5758	52	1.3797	1.1400	0.0000	50.4	2.1
0.120	0.050	0.020	2.5758	41	1.4100	1.1400	0.0000	44.7	2.1
0.090	0.075	0.020	2.5758	31	1.4505	1.1400	0.0000	37.7	2.0
0.120	0.075	0.020	2.5758	26	1.4790	1.1400	0.0000	44.5	2.2
0.090	0.050	0.040	2.5758	21	1.5172	1.1400	0.0000	34.6	2.1
0.120	0.050	0.040	2.5758	18	1.5475	1.1400	0.0000	30.3	2.1
0.090	0.075	0.040	2.5758	15	1.5863	1.1400	0.0000	33.0	2.1
0.120	0.075	0.040	2.5758	13	1.6195	1.1400	0.0000	45.0	2.4

**Table A4.2.2 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 2$) Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010,$
 $ARL(p_0) = 200$ and $ARL(p_c) \leq 2$**

p_{e2}	p_{e3}	p_{e4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8070	238	1.3798	1.2200	1.0602	160.9	2.1
0.120	0.050	0.020	2.8070	169	1.4097	1.2200	1.0303	152.0	2.1
0.090	0.075	0.020	2.5758	93	1.4546	1.2200	0.0000	80.0	2.1
0.120	0.075	0.020	2.5758	73	1.4848	1.2200	0.0000	72.5	2.1
0.090	0.050	0.040	2.5758	57	1.5197	1.2200	0.0000	63.3	2.1
0.120	0.050	0.040	2.5758	48	1.5466	1.2200	0.0000	66.4	2.2
0.090	0.075	0.040	2.5758	37	1.5920	1.2200	0.0000	50.3	2.1
0.120	0.075	0.040	2.5758	32	1.6200	1.2200	0.0000	48.9	2.1

**Table A4.2.3 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 2$) Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005,$
 $ARL(p_0) = 200$ and $ARL(p_c) \leq 5$**

p_{e2}	p_{e3}	p_{e4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.5758	10	1.6867	1.1400	0.0000	18.4	4.4
0.120	0.050	0.020	2.5758	8	1.7512	1.1400	0.0000	23.7	5.5
0.090	0.075	0.020	2.5758	5	1.9131	1.1400	0.0000	33.8	6.5
0.120	0.075	0.020	2.5758	4	2.0043	1.1400	0.0000	44.2	8.5
0.090	0.050	0.040	2.5758	2	2.3624	1.1400	0.0000	20.3	5.8
0.120	0.050	0.040	2.5758	2	2.3624	1.1400	0.0000	20.3	5.8
0.090	0.075	0.040	2.5758	1	2.8687	1.1400	0.0000	40.0	8.7
0.120	0.075	0.040	2.5758	1	2.8687	1.1400	0.0000	40.0	8.7

**Table A4.2.4 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 2$) Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010,$
 $ARL(p_0) = 200$ and $ARL(p_c) \leq 5$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.5758	64	1.5028	1.2200	0.0000	73.4	5.4
0.120	0.050	0.020	2.5758	45	1.5573	1.2200	0.0000	67.9	5.5
0.090	0.075	0.020	2.5758	27	1.6554	1.2200	0.0000	47.1	5.0
0.120	0.075	0.020	2.5758	22	1.7024	1.2200	0.0000	48.6	5.2
0.090	0.050	0.040	2.5758	12	1.8732	1.2200	0.0000	41.3	5.3
0.120	0.050	0.040	2.5758	10	1.9355	1.2200	0.0000	35.1	5.3
0.090	0.075	0.040	2.5758	7	2.0752	1.2200	0.0000	31.2	5.2
0.120	0.075	0.040	2.5758	6	2.1437	1.2200	0.0000	16.2	4.1

**Table A4.2.5 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 2$) Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005,$
 $ARL(p_0) = 500$ and $ARL(p_c) \leq 2$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8782	65	1.3796	1.1400	0.0000	90.5	2.1
0.120	0.050	0.020	2.8782	52	1.4079	1.1400	0.0000	95.3	2.2
0.090	0.075	0.020	2.8782	38	1.4533	1.1400	0.0000	82.2	2.2
0.120	0.075	0.020	2.8782	32	1.4815	1.1400	0.0000	73.2	2.2
0.090	0.050	0.040	2.8782	26	1.5188	1.1400	0.0000	61.8	2.2
0.120	0.050	0.040	2.8782	23	1.5428	1.1400	0.0000	62.2	2.2
0.090	0.075	0.040	2.8782	19	1.5831	1.1400	0.0000	76.0	2.3
0.120	0.075	0.040	2.8782	16	1.6229	1.1400	0.0000	38.8	2.1

**Table A4.2.6 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 2$) Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010,$
 $ARL(p_0) = 500$ and $ARL(p_c) \leq 2$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	3.0902	288	1.3800	1.2200	1.0601	333.1	2.1
0.120	0.050	0.020	3.0902	205	1.4096	1.2200	1.0304	267.3	2.0
0.090	0.075	0.020	2.8782	116	1.4547	1.2200	0.0000	139.6	2.1
0.120	0.075	0.020	2.8782	92	1.4836	1.2200	0.0000	134.7	2.1
0.090	0.050	0.040	2.8782	72	1.5180	1.2200	0.0000	128.1	2.1
0.120	0.050	0.040	2.8782	59	1.5491	1.2200	0.0000	115.0	2.1
0.090	0.075	0.040	2.8782	46	1.5928	1.2200	0.0000	107.5	2.2
0.120	0.075	0.040	2.8782	39	1.6248	1.2200	0.0000	97.1	2.2

**Table A4.2.7 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 2$) Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005,$
 $ARL(p_0) = 500$ and $ARL(p_c) \leq 5$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8782	16	1.6229	1.1400	0.0000	38.8	4.6
0.120	0.050	0.020	2.8782	13	1.6757	1.1400	0.0000	45.0	4.8
0.090	0.075	0.020	2.8782	8	1.8229	1.1400	0.0000	23.7	4.8
0.120	0.075	0.020	2.8782	7	1.8701	1.1400	0.0000	27.5	5.5
0.090	0.050	0.040	2.8782	4	2.1058	1.1400	0.0000	44.2	6.0
0.120	0.050	0.040	2.8782	3	2.2552	1.1400	0.0000	47.1	6.5
0.090	0.075	0.040	2.8782	2	2.5058	1.1400	0.0000	82.1	10.3
0.120	0.075	0.040	2.8782	2	2.5058	1.1400	0.0000	82.1	9.8

**Table A4.2.8 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 2$) Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010,$
 $ARL(p_0) = 500$ and $ARL(p_c) \leq 5$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8782	93	1.4822	1.2200	0.0000	124.9	4.9
0.120	0.050	0.020	2.8782	66	1.5312	1.2200	0.0000	131.3	5.4
0.090	0.075	0.020	2.8782	40	1.6197	1.2200	0.0000	86.6	4.9
0.120	0.075	0.020	2.8782	32	1.6669	1.2200	0.0000	90.6	5.3
0.090	0.050	0.040	2.8782	18	1.8159	1.2200	0.0000	61.7	5.0
0.120	0.050	0.040	2.8782	15	1.8728	1.2200	0.0000	66.1	5.6
0.090	0.075	0.040	2.8782	11	1.9823	1.2200	0.0000	51.5	4.9
0.120	0.075	0.040	2.8782	10	2.0195	1.2200	0.0000	66.0	5.5

**Table A4.2.9 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 5$) Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005,$
 $ARL(p_0) = 200$ and $ARL(p_c) \leq 2$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.5758	77	5.0256	2.2800	0.0000	44.9	2.2
0.120	0.050	0.020	2.5758	70	5.1596	2.2800	0.0000	39.2	2.2
0.090	0.075	0.020	2.5758	52	5.6210	2.2800	0.0000	34.6	2.3
0.120	0.075	0.020	2.5758	48	5.7574	2.2800	0.0000	39.3	2.3
0.090	0.050	0.040	2.5758	22	7.4165	2.2800	0.0000	25.0	2.1
0.120	0.050	0.040	2.5758	21	7.5373	2.2800	0.0000	26.7	2.1
0.090	0.075	0.040	2.5758	18	7.9586	2.2800	0.0000	16.6	2.0
0.120	0.075	0.040	2.5758	17	8.1232	2.2800	0.0000	12.2	2.0

**Table A4.2.10 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 5$)
Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010,$
 $ARL(p_0) = 200$ and $ARL(p_c) \leq 2$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8070	390	5.0377	3.2000	1.3623	145.5	2.1
0.120	0.050	0.020	2.8070	343	5.1596	3.2000	1.2404	139.3	2.1
0.090	0.075	0.020	2.5758	187	5.6354	3.2000	0.0000	72.7	2.1
0.120	0.075	0.020	2.5758	170	5.7542	3.2000	0.0000	73.1	2.1
0.090	0.050	0.040	2.5758	60	7.4994	3.2000	0.0000	47.4	2.2
0.120	0.050	0.040	2.5758	57	7.6111	3.2000	0.0000	48.0	2.2
0.090	0.075	0.040	2.5758	46	8.1103	3.2000	0.0000	52.3	2.1
0.120	0.075	0.040	2.5758	44	8.2206	3.2000	0.0000	53.7	2.1

**Table A4.2.11 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 5$)
Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005,$
 $ARL(p_0) = 200$ and $ARL(p_c) \leq 5$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.5758	11	9.5441	2.2800	0.0000	18.6	5.0
0.120	0.050	0.020	2.5758	10	9.8986	2.2800	0.0000	20.4	5.4
0.090	0.075	0.020	2.5758	7	11.3860	2.2800	0.0000	29.0	7.3
0.120	0.075	0.020	2.5758	7	11.3860	2.2800	0.0000	29.0	7.3
0.090	0.050	0.040	2.5758	1	26.3721	2.2800	0.0000	200.0	25.0
0.120	0.050	0.040	2.5758	1	26.3721	2.2800	0.0000	200.0	25.0
0.090	0.075	0.040	2.5758	1	26.3721	2.2800	0.0000	200.0	25.0
0.120	0.075	0.040	2.5758	1	26.3721	2.2800	0.0000	200.0	25.0

**Table A4.2.12 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 5$)
Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010,$
 $ARL(p_0) = 200$ and $ARL(p_c) \leq 5$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.5758	98	6.5641	3.2000	0.0000	59.1	5.3
0.120	0.050	0.020	2.5758	87	6.7705	3.2000	0.0000	60.0	5.1
0.090	0.075	0.020	2.5758	55	7.6906	3.2000	0.0000	49.6	5.2
0.120	0.075	0.020	2.5758	50	7.9098	3.2000	0.0000	55.1	5.2
0.090	0.050	0.040	2.5758	9	14.3010	3.2000	0.0000	11.6	3.3
0.120	0.050	0.040	2.5758	9	14.3010	3.2000	0.0000	11.6	3.3
0.090	0.075	0.040	2.5758	7	15.7873	3.2000	0.0000	14.7	4.0
0.120	0.075	0.040	2.5758	7	15.7873	3.2000	0.0000	14.7	4.0

**Table A4.2.13 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 5$)
Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005,$
 $ARL(p_0) = 500$ and $ARL(p_c) \leq 2$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8782	96	5.0275	2.2800	0.0000	72.9	2.2
0.120	0.050	0.020	2.8782	88	5.1497	2.2800	0.0000	79.4	2.2
0.090	0.075	0.020	2.8782	65	5.6190	2.2800	0.0000	61.4	2.2
0.120	0.075	0.020	2.8782	60	5.7554	2.2800	0.0000	57.0	2.3
0.090	0.050	0.040	2.8782	27	7.4607	2.2800	0.0000	61.2	2.3
0.120	0.050	0.040	2.8782	26	7.5594	2.2800	0.0000	56.2	2.3
0.090	0.075	0.040	2.8782	22	8.0193	2.2800	0.0000	54.1	2.1
0.120	0.075	0.040	2.8782	21	8.1544	2.2800	0.0000	38.4	2.0

**Table A4.2.14 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 5$)
Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010,$
 $ARL(p_0) = 500$ and $ARL(p_c) \leq 2$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	3.0902	472	5.0390	3.2000	1.3610	291.9	2.1
0.120	0.050	0.020	3.0902	416	5.1589	3.2000	1.2411	277.6	2.1
0.090	0.075	0.020	2.8782	233	5.6378	3.2000	0.0000	142.3	2.1
0.120	0.075	0.020	2.8782	212	5.7557	3.2000	0.0000	137.9	2.1
0.090	0.050	0.040	2.8782	75	7.4969	3.2000	0.0000	93.9	2.1
0.120	0.050	0.040	2.8782	71	7.6162	3.2000	0.0000	93.8	2.1
0.090	0.075	0.040	2.8782	58	8.0862	3.2000	0.0000	69.7	2.1
0.120	0.075	0.040	2.8782	55	8.2177	3.2000	0.0000	66.5	2.2

**Table A4.2.15 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 5$)
Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005,$
 $ARL(p_0) = 500$ and $ARL(p_c) \leq 5$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8782	19	8.4559	2.2800	0.0000	32.6	4.6
0.120	0.050	0.020	2.8782	17	8.8090	2.2800	0.0000	37.1	4.5
0.090	0.075	0.020	2.8782	12	10.0511	2.2800	0.0000	17.1	4.6
0.120	0.075	0.020	2.8782	12	10.0511	2.2800	0.0000	17.1	4.6
0.090	0.050	0.040	2.8782	2	21.3152	2.2800	0.0000	96.4	12.4
0.120	0.050	0.040	2.8782	2	21.3152	2.2800	0.0000	96.4	12.4
0.090	0.075	0.040	2.8782	2	21.3152	2.2800	0.0000	96.4	11.9
0.120	0.075	0.040	2.8782	2	21.3152	2.2800	0.0000	96.4	11.9

Table A4.2.16 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 5$) Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010, ARL(p_0) = 500$ and $ARL(p_c) \leq 5$

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8782	145	6.2903	3.2000	0.0000	114.4	5.1
0.120	0.050	0.020	2.8782	128	6.4891	3.2000	0.0000	108.0	5.1
0.090	0.075	0.020	2.8782	81	7.3347	3.2000	0.0000	94.2	5.2
0.120	0.075	0.020	2.8782	74	7.5258	3.2000	0.0000	93.1	5.1
0.090	0.050	0.040	2.8782	15	12.8081	3.2000	0.0000	83.8	6.9
0.120	0.050	0.040	2.8782	14	13.1453	3.2000	0.0000	64.7	6.5
0.090	0.075	0.040	2.8782	12	13.9421	3.2000	0.0000	71.4	5.7
0.120	0.075	0.040	2.8782	11	14.4198	3.2000	0.0000	31.7	4.3

Table A4.2.17 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 10$) Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005, ARL(p_0) = 200$ and $ARL(p_c) \leq 2$

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.5758	102	26.6566	8.3800	0.0000	62.8	2.2
0.120	0.050	0.020	2.5758	99	26.9315	8.3800	0.0000	67.5	2.2
0.090	0.075	0.020	2.5758	79	29.1474	8.3800	0.0000	41.3	2.1
0.120	0.075	0.020	2.5758	77	29.4154	8.3800	0.0000	34.0	2.2
0.090	0.050	0.040	2.5758	24	46.0582	8.3800	0.0000	21.6	1.9
0.120	0.050	0.040	2.5758	23	46.8686	8.3800	0.0000	23.3	2.0
0.090	0.075	0.040	2.5758	21	48.6597	8.3800	0.0000	13.3	1.8
0.120	0.075	0.040	2.5758	21	48.6597	8.3800	0.0000	13.3	1.7

Table A4.2.18 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 10$) Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010, ARL(p_0) = 200$ and $ARL(p_c) \leq 2$

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8070	531	26.7402	14.5000	2.2598	149.9	2.1
0.120	0.050	0.020	2.8070	509	27.0019	14.5000	1.9981	150.9	2.1
0.090	0.075	0.020	2.5758	310	29.2003	14.5000	0.0000	77.7	2.1
0.120	0.075	0.020	2.5758	299	29.4682	14.5000	0.0000	81.5	2.1
0.090	0.050	0.040	2.5758	65	46.6033	14.5000	0.0000	36.2	2.1
0.120	0.050	0.040	2.5758	64	46.8531	14.5000	0.0000	37.7	2.1
0.090	0.075	0.040	2.5758	56	49.0869	14.5000	0.0000	53.2	2.4
0.120	0.075	0.040	2.5758	55	49.3999	14.5000	0.0000	55.8	2.4

**Table A4.2.19 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 10$)
Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005,$
 $ARL(p_0) = 200$ and $ARL(p_c) \leq 5$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.5758	13	59.5746	8.3800	0.0000	15.9	4.3
0.120	0.050	0.020	2.5758	13	59.5746	8.3800	0.0000	15.9	4.3
0.090	0.075	0.020	2.5758	10	66.7508	8.3800	0.0000	20.5	5.5
0.120	0.075	0.020	2.5758	10	66.7508	8.3800	0.0000	20.5	5.5
0.090	0.050	0.040	2.5758	1	192.9650	8.3800	0.0000	200.0	25.0
0.120	0.050	0.040	2.5758	1	192.9650	8.3800	0.0000	200.0	25.0
0.090	0.075	0.040	2.5758	1	192.9650	8.3800	0.0000	200.0	25.0
0.120	0.075	0.040	2.5758	1	192.9650	8.3800	0.0000	200.0	25.0

**Table A4.2.20 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 10$)
Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010,$
 $ARL(p_0) = 200$ and $ARL(p_c) \leq 5$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.5758	132	37.0278	14.5000	0.0000	76.1	5.1
0.120	0.050	0.020	2.5758	127	37.4670	14.5000	0.0000	71.0	4.9
0.090	0.075	0.020	2.5758	91	41.6322	14.5000	0.0000	69.8	5.1
0.120	0.075	0.020	2.5758	88	42.0908	14.5000	0.0000	71.9	4.9
0.090	0.050	0.040	2.5758	9	100.7750	14.5000	0.0000	11.6	3.3
0.120	0.050	0.040	2.5758	9	100.7750	14.5000	0.0000	11.6	3.3
0.090	0.075	0.040	2.5758	8	106.0080	14.5000	0.0000	12.9	3.6
0.120	0.075	0.040	2.5758	8	106.0080	14.5000	0.0000	12.9	3.6

**Table A4.2.21 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 10$)
Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005,$
 $ARL(p_0) = 500$ and $ARL(p_c) \leq 2$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8782	127	26.6817	8.3800	0.0000	65.2	2.1
0.120	0.050	0.020	2.8782	123	26.9769	8.3800	0.0000	55.5	2.2
0.090	0.075	0.020	2.8782	98	29.2144	8.3800	0.0000	74.5	2.2
0.120	0.075	0.020	2.8782	96	29.4303	8.3800	0.0000	78.4	2.1
0.090	0.050	0.040	2.8782	29	46.6797	8.3800	0.0000	104.0	2.8
0.120	0.050	0.040	2.8782	29	46.6797	8.3800	0.0000	104.0	2.8
0.090	0.075	0.040	2.8782	26	48.8290	8.3800	0.0000	110.5	2.5
0.120	0.075	0.040	2.8782	26	48.8290	8.3800	0.0000	110.5	2.5

**Table A4.2.22 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 10$) Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010,$
 $ARL(p_0) = 500$ and $ARL(p_c) \leq 2$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	3.0902	644	26.7359	14.5000	2.2641	308.5	2.1
0.120	0.050	0.020	3.0902	617	27.0008	14.5000	1.9992	299.5	2.1
0.090	0.075	0.020	2.8782	387	29.2011	14.5000	0.0000	157.4	2.1
0.120	0.075	0.020	2.8782	373	29.4744	14.5000	0.0000	162.4	2.1
0.090	0.050	0.040	2.8782	81	46.6337	14.5000	0.0000	106.3	2.3
0.120	0.050	0.040	2.8782	80	46.8340	14.5000	0.0000	110.9	2.3
0.090	0.075	0.040	2.8782	70	49.0665	14.5000	0.0000	102.8	2.0
0.120	0.075	0.040	2.8782	69	49.3160	14.5000	0.0000	93.4	2.0

**Table A4.2.23 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 10$) Control Chart having $p_{02} = 0.045, p_{03} = 0.020, p_{04} = 0.005,$
 $ARL(p_0) = 500$ and $ARL(p_c) \leq 5$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8782	23	51.3861	8.3800	0.0000	75.4	5.9
0.120	0.050	0.020	2.8782	23	51.3861	8.3800	0.0000	75.4	5.9
0.090	0.075	0.020	2.8782	18	56.9936	8.3800	0.0000	16.6	3.4
0.120	0.075	0.020	2.8782	18	56.9936	8.3800	0.0000	16.6	3.3
0.090	0.050	0.040	2.8782	2	154.2210	8.3800	0.0000	100.3	12.8
0.120	0.050	0.040	2.8782	2	154.2210	8.3800	0.0000	100.3	12.8
0.090	0.075	0.040	2.8782	2	154.2210	8.3800	0.0000	100.3	12.8
0.120	0.075	0.040	2.8782	2	154.2210	8.3800	0.0000	100.3	12.8

**Table A4.2.24 - Parameter Selection Examples for the 4-SPC-AVG-2($n, k_V, 10$) Control Chart having $p_{02} = 0.060, p_{03} = 0.030, p_{04} = 0.010,$
 $ARL(p_0) = 500$ and $ARL(p_c) \leq 5$**

p_{c2}	p_{c3}	p_{c4}	k_V	n	UCL_V	CL	LCL_V	$ARL(p_0)$ adjusted	$ARL(p_c)$ adjusted
0.090	0.050	0.020	2.8782	195	35.2103	14.5000	0.0000	115.3	5.1
0.120	0.050	0.020	2.8782	187	35.6487	14.5000	0.0000	103.5	5.3
0.090	0.075	0.020	2.8782	135	39.3907	14.5000	0.0000	88.8	5.2
0.120	0.075	0.020	2.8782	130	39.8648	14.5000	0.0000	97.4	5.1
0.090	0.050	0.040	2.8782	15	89.1721	14.5000	0.0000	102.7	8.2
0.120	0.050	0.040	2.8782	15	89.1721	14.5000	0.0000	102.7	8.2
0.090	0.075	0.040	2.8782	13	94.7107	14.5000	0.0000	126.1	8.6
0.120	0.075	0.040	2.8782	13	94.7107	14.5000	0.0000	126.1	8.3

VITA

Charles Richard Cassady was born August 21, 1970, in Martinsville, Virginia. He graduated as valedictorian from Bassett High School in Bassett, Virginia in June, 1988. In August, 1992, he received a Bachelor of Science summa cum laude in Industrial and Systems Engineering (ISE) from Virginia Polytechnic Institute and State University (Virginia Tech) in Blacksburg, Virginia. While completing his bachelor's degree, he was inducted into Tau Beta Pi, Alpha Pi Mu, Kappa Theta Epsilon, Phi Kappa Phi, Phi Eta Sigma, and the Golden Key National Honor society. In addition, he participated in the Cooperative Education Program as an employee of the Management Systems Department of Carilion Health System in Roanoke, Virginia.

In December, 1993, he received a Master of Science in ISE also from Virginia Tech, and in August, 1996, he will receive his Doctor of Philosophy in ISE from Virginia Tech. During his graduate study, he served the ISE department as a graduate teaching assistant and as an instructor, teaching ten undergraduate courses in statistical quality control, industrial quality control, probabilistic operations research, and discrete-event computer simulation. He also received the 1994-1995 American Society for Quality Control (ASQC) Reliability Division Scholarship and the 1994-1995 Institute of Industrial Engineers (IIE) Gilbreth Memorial Fellowship. In addition, he worked/volunteered as a consultant for Elizabeth Arden, Co. and Meridium, Inc. both in Roanoke, Virginia. His thesis research area was availability modeling, and his dissertation research area was statistical quality control. Beginning August 16, 1996, he will be an Assistant Professor in the Department of Industrial Engineering at Mississippi State University in Mississippi State, Mississippi.

Richard's primary research interests are in the applications of probability, statistics, simulation, and optimization to problems involving product quality and reliability. He is a member of the Institute of Industrial Engineers, the Institute for Operations Research and the Management Sciences, and the American Society for Quality Control.

