ADVANCED ANALYSIS OF ROTOR-BEARING SYSTEMS
FOR STABILITY AND RESPONSE

by

Krishnaswamy Ramesh

Dissertation submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY
in
MECHANICAL ENGINEERING

Approved:

Dr. R. G. Kirk (Chairman)

Dr. C. E. Knight

Dr. A. L. Wicks

Dr. R. H. Plaut

Dr. S. L. Hendricks

April 1996
Blacksburg, Virginia

Keywords: Stability, Response, Multi-level, Magnetic, Dampers, Animation
LD 5455
V856
1996
R3644
C.2
ADVANCED ANALYSIS OF Rotor-bearing Systems
For Stability and Response

by

Krishnaswamy Ramesh

Dr. R.G. Kirk, Chairman
Mechanical Engineering Department
Virginia Polytechnic Institute and State University

ABSTRACT

Rotor dynamics has become an integral part in the analysis and design of industrial turbomachinery. Rotor dynamics deals predominantly with the evaluation of the stability and damped critical speeds, and the response to an unbalance excitation, of turbomachinery. The majority of the industries which deal with rotor dynamics use the conventional and proven transfer matrix methods to solve the dynamics. However, the recent advances in computer technology and the distinct advantages of the finite element method make it an attractive tool to model complex rotor bearing systems.

This research has developed a PC-based finite element analysis program capable of modeling rotors supported not only on fluid film bearings, but also on Active Magnetic Bearings (AMB). Methods are described by which the non-synchronous bearing properties can be used to evaluate the stability of the rotors supported on AMB. The effect of sensor non-collocation on general elliptic orbit response and stability has also been studied, as compared to the circular response of the existing programs. A design procedure for the stability of rotors supported on squeeze film dampers has been outlined. The unbalance response of rotors supported on squeeze film dampers can be predicted using the new iterative solution method which accounts for the nonlinear behavior of the damper. Multilevel analysis, essential for systems such as aircraft jet engines and certain other
classes of turbomachinery, can be performed using this new computer program. A post-
processor for viewing/animating the damped mode-shapes and forced response of a rotor,
in 3-dimensions, has been developed. This ability to view the animated complex modes of
forward, backward, and mixed forward-backward whirl of the rotor adds a new dimension
in understanding the dynamics of rotating machinery.

With the increasing demand for more accurately predicting the dynamic response
and stability of high performance critical path turbomachinery, it is essential to develop
advanced capability computer programs. The new PC-based finite element program
developed in this research has the advanced capabilities required to model such complex
rotating machinery.
ACKNOWLEDGMENTS

I would sincerely like to thank Dr. R.G. Kirk for his support, assistance and invaluable guidance, without which this work would not have been possible.

I would like to acknowledge the support of my parents and thank them for giving me all the encouragement throughout my career.

In addition, I would also like to thank Dr. C.E. Knight, Dr. A.L. Wicks, Dr. R.H. Plaut and Dr. S.L. Hendricks for serving on my committee, and from whom I had the privilege of learning the best, during the course of the research. Thanks are due to Dr. B. S. Prabhu (Professor, Applied Mechanics Department, Indian Institute of Technology, Madras, India) for introducing me to the field of Rotor Dynamics and giving me the opportunity to work under his guidance for my Masters degree.

I would also like to extend my sincere thanks to Erik Swanson for the practical insight into the construction and operation of real-time test rigs, Sanjay Baheti for the intuitive feedback, and K.V.S. Raju, Farokh Kavarana and John Wang for their respective support.

Last but not the least I would like to extend my thanks to the Members of the Rotor Dynamics Laboratory Affiliates Group for their constant feedback.
Dedicated to my

father, mother, sister and dear wife
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>ii</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>iv</td>
</tr>
<tr>
<td>List of Figures</td>
<td>x</td>
</tr>
<tr>
<td>List of Tables</td>
<td>xvi</td>
</tr>
<tr>
<td>Nomenclature</td>
<td>xix</td>
</tr>
<tr>
<td>Chapter 1 INTRODUCTION AND LITERATURE REVIEW</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Literature Review</td>
<td>2</td>
</tr>
<tr>
<td>1.2.1 Finite Element Modeling in Rotor Dynamics</td>
<td>3</td>
</tr>
<tr>
<td>1.2.2 Stability and Response</td>
<td>6</td>
</tr>
<tr>
<td>1.2.3 Squeeze Film Dampers</td>
<td>9</td>
</tr>
<tr>
<td>1.2.4 Magnetic Bearings</td>
<td>11</td>
</tr>
<tr>
<td>1.3 Scope of the Dissertation</td>
<td>13</td>
</tr>
<tr>
<td>Chapter 2 DEVELOPMENT OF THE SYSTEM MODEL</td>
<td>15</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>15</td>
</tr>
<tr>
<td>2.2 Development of the Timoshenko Beam</td>
<td>18</td>
</tr>
<tr>
<td>Derivation of the K, M, C Matrices</td>
<td></td>
</tr>
<tr>
<td>2.2.1 The Rotor Element</td>
<td>18</td>
</tr>
<tr>
<td>2.2.2 Rigid Disk</td>
<td>22</td>
</tr>
<tr>
<td>2.2.3 Bearings/Seals</td>
<td>22</td>
</tr>
<tr>
<td>2.3 Stability Analysis</td>
<td>24</td>
</tr>
<tr>
<td>2.3.1 Method of Solution</td>
<td>24</td>
</tr>
<tr>
<td>2.3.2 Damped Natural Frequencies</td>
<td>27</td>
</tr>
<tr>
<td>2.3.3 Damped Modes</td>
<td>28</td>
</tr>
<tr>
<td>2.4 Unbalance Response Calculations</td>
<td>29</td>
</tr>
</tbody>
</table>
### Chapter 5 ANALYSES AND RESULTS OF SINGLE LEVEL ROTOR SYSTEMS

5.1 Analyses of Rotors Supported on Active Magnetic Bearings
   5.1.1 Stability Evaluation of a Multistage Compressor:
       Importance of Sensor Position in the Stability Predictions
   5.1.2 Effect of Support Flexibility on the Stability of AMB Supported Rotors
   5.1.2.1 Initial Verification for Stability with Flexible Pedestals
   5.1.3 Effect of Sensor Noncollocation on Unbalance Response

5.2 Analysis of Rotors Supported on Squeeze Film Dampers
   5.2.1 Introduction
   5.2.2 Stability Analysis
   5.2.3 Damping Number ($C_N$)
   5.2.4 Forced Response of a Rotor System Having a Squeeze Film Damper

5.3 Analysis of MultiStage Pump with InterStage Seals
   5.3.1 Stability Analysis
   5.3.2 Unbalance Response

### Chapter 6 ANALYSES AND RESULTS OF MULTILEVEL ROTOR SYSTEMS

Table of Contents
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Caption</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Typical Rotor Element Configuration</td>
<td>18</td>
</tr>
<tr>
<td>2.2</td>
<td>Flow of the Stability Program</td>
<td>26</td>
</tr>
<tr>
<td>2.3</td>
<td>Unbalance Mass ‘m’ at a Radius ‘e’</td>
<td>30</td>
</tr>
<tr>
<td>2.4</td>
<td>Flow Diagram of the Response Program</td>
<td>33</td>
</tr>
<tr>
<td>2.5</td>
<td>Squeeze Film Damper Configuration</td>
<td>35</td>
</tr>
<tr>
<td>2.6</td>
<td>Schematic Diagram of the Bearing, Damper and Pedestal</td>
<td>38</td>
</tr>
<tr>
<td>2.7</td>
<td>Assembled Stiffness (Mass/Damping) Matrix for the Common Approach</td>
<td>44</td>
</tr>
<tr>
<td>2.8</td>
<td>Assembled Stiffness (Mass/Damping) Matrix for the Modified Approach</td>
<td>45</td>
</tr>
<tr>
<td>2.9</td>
<td>Multilevel Rotor Model of the AMB Test Rig</td>
<td>46</td>
</tr>
<tr>
<td>2.10</td>
<td>Details of the Inboard and Outboard End of the Rotor</td>
<td>47</td>
</tr>
<tr>
<td>2.11</td>
<td>Model of the Seal as the ‘Second Level’</td>
<td>48</td>
</tr>
<tr>
<td>3.1</td>
<td>Schematic Diagram of a Magnetic Bearing</td>
<td>51</td>
</tr>
<tr>
<td>3.2</td>
<td>The Active Magnetic Bearing with the Control System</td>
<td>52</td>
</tr>
<tr>
<td>3.3</td>
<td>The Active Magnetic Bearing Terminologies</td>
<td>53</td>
</tr>
<tr>
<td>3.4</td>
<td>Schematic Diagram Showing the Location of the Bearing Stiffness in the</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>Stiffness Matrix</td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>The Frequency Dependent Bearing Characteristics</td>
<td>58</td>
</tr>
<tr>
<td>4.1</td>
<td>Jeffcott Rotor Model Used For the Stability Analysis</td>
<td>65</td>
</tr>
<tr>
<td>4.2</td>
<td>Simple Splitting of Inertias for 2 Sections of a Rotor Model</td>
<td>69</td>
</tr>
</tbody>
</table>

List of Figures x
<table>
<thead>
<tr>
<th>Figure</th>
<th>Caption</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.3</td>
<td>Inertia Split for Rotor Section ‘i’ by the Revised Method</td>
<td>70</td>
</tr>
<tr>
<td>4.4</td>
<td>Model of the 8 Stage Industrial Compressor Used for Stability Analysis</td>
<td>71</td>
</tr>
<tr>
<td>4.5</td>
<td>Model of the Uniform Rotor Used for Evaluation of Undamped Critical Speeds</td>
<td>75</td>
</tr>
<tr>
<td>5.1</td>
<td>Model of the 8 Stage Compressor Used for Stability Analysis (Importance of Sensor Positions)</td>
<td>82</td>
</tr>
<tr>
<td>5.2</td>
<td>Stability Plot for Synchronous Bearing Properties</td>
<td>85</td>
</tr>
<tr>
<td>5.3</td>
<td>Stability Plot for Nonsynchronous Bearing Properties</td>
<td>86</td>
</tr>
<tr>
<td>5.4</td>
<td>Uniform Rotor Model Used for the Stability Analysis (Pedestal Flexibility)</td>
<td>87</td>
</tr>
<tr>
<td>5.5</td>
<td>Plot of Stability vs. Pedestal Damping for Sensors Collocated</td>
<td>90</td>
</tr>
<tr>
<td>5.6</td>
<td>Plot of Stability vs. Pedestal Damping for Sensors 3 in. Outboard</td>
<td>91</td>
</tr>
<tr>
<td>5.7</td>
<td>Plot of Stability vs. Pedestal Damping for Sensors 3 in. Inboard</td>
<td>92</td>
</tr>
<tr>
<td>5.8</td>
<td>Plot of Stability vs. Pedestal Damping for Sensors 7 in. Inboard</td>
<td>93</td>
</tr>
<tr>
<td>5.9</td>
<td>Model of the 8 Stage Compressor Showing the Different Unbalance Locations Used for Response Analysis</td>
<td>96</td>
</tr>
<tr>
<td>5.10</td>
<td>Plot of Response vs. Running Speed at Bearing # 1 (Lower Damping)</td>
<td>101</td>
</tr>
<tr>
<td>5.11</td>
<td>Plot of Response vs. Running Speed at Midspan (Lower Damping)</td>
<td>102</td>
</tr>
<tr>
<td>5.12</td>
<td>Plot of Response vs. Running Speed at Bearing # 2 (Lower Damping)</td>
<td>103</td>
</tr>
<tr>
<td>5.13</td>
<td>Plot of Response vs. Running Speed at Sensor # 1 (Lower Damping)</td>
<td>104</td>
</tr>
<tr>
<td>5.14</td>
<td>Plot of Response vs. Running Speed at Sensor # 2 (Lower Damping)</td>
<td>105</td>
</tr>
<tr>
<td>5.15</td>
<td>Modeshapes for 4 Sensor Positions at the Respective First Critical Speeds (Lower Damping)</td>
<td>106</td>
</tr>
</tbody>
</table>

List of Figures xi
Figure | Caption                                                                 | Page |
---|---|---|
5.16 | Modeshapes for 4 Sensor Positions at the Respective Third Critical Speeds (Lower Damping) | 107  |
5.17 | Plot of Response vs. Running Speed at Bearing # 1 (Higher Damping) | 108  |
5.18 | Plot of Response vs. Running Speed at Midspan (Higher Damping) | 109  |
5.19 | Plot of Response vs. Running Speed at Bearing # 2 (Higher Damping) | 110  |
5.20 | Plot of Response vs. Running Speed at Sensor # 1 (Higher Damping) | 111  |
5.21 | Plot of Response vs. Running Speed at Sensor # 2 (Higher Damping) | 112  |
5.22 | Modeshapes for 4 Sensor Positions at the Respective First Critical Speeds (Higher Damping) | 113  |
5.23 | Model of the Rotor Supported on Squeeze Film Damper | 114  |
5.24 | Plot of Growth Factor vs. Retainer Stiffness for Different Values of Eccentricity Ratio | 122  |
5.25 | Plot of Growth Factor vs. Clearance Ratio for Different Values of Retainer Stiffness | 123  |
5.26 | Plot of Growth Factor vs. Retainer Stiffness for Different Values of Damper Eccentricity Ratio | 124  |
5.27 | Plot of Growth Factor vs. Clearance Ratio for Different Values of Damper Eccentricity Ratio | 125  |
5.28 | Plot of Growth Factor vs. Retainer Stiffness for Different Values of Clearance Ratio | 126  |
5.29 | Plot of Amplitude at Bearing #1 vs. Running Speed With and Without the Damper | 129  |
5.30 | Plot of Amplitude at Bearing #1 vs. Running Speed for the Two Damper Assumptions | 130  |
5.31 | Plot of Amplitude at Midspan vs. Running Speed | 131  |

List of Figures | xii
<table>
<thead>
<tr>
<th>Figure</th>
<th>Caption</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.32</td>
<td>Plot of Amplitude at Bearing #2 vs. Running Speed</td>
<td>132</td>
</tr>
<tr>
<td>5.33</td>
<td>Plot of Amplitude at Bearing #1 vs. Running Speed for Different Levels</td>
<td></td>
</tr>
<tr>
<td></td>
<td>of Different Levels of Unbalance at Midspan</td>
<td>133</td>
</tr>
<tr>
<td>5.34</td>
<td>Model of the MultiStage Pump</td>
<td>135</td>
</tr>
<tr>
<td>5.35</td>
<td>Modeshapes for the First Four Critical Speeds of the Pump</td>
<td>136</td>
</tr>
<tr>
<td>5.36</td>
<td>Response of the Pump for Unbalance at Midspan (Node # 12)</td>
<td>140</td>
</tr>
<tr>
<td>5.37</td>
<td>Response of the Pump for Unbalance at the Coupling (Node # 1)</td>
<td>141</td>
</tr>
<tr>
<td>6.1</td>
<td>Model of the MultiStage Pump with Bearings and Seals</td>
<td>143</td>
</tr>
<tr>
<td>6.2a</td>
<td>Schematic of the Seal in the Locked Configuration</td>
<td>144</td>
</tr>
<tr>
<td>6.2b</td>
<td>Schematic of the Seal in the Floating Configuration</td>
<td>144</td>
</tr>
<tr>
<td>6.3</td>
<td>Modeshapes for the First Two Critical Speeds for Locked Seal Case</td>
<td>146</td>
</tr>
<tr>
<td>6.4a</td>
<td>Plot of First Mode of Rotor and Floating Seal</td>
<td>147</td>
</tr>
<tr>
<td>6.4b</td>
<td>Plot of Second Mode of Rotor and Floating Seal</td>
<td>147</td>
</tr>
<tr>
<td>6.5</td>
<td>Model of the MultiStage Pump Used for MultiLevel Stability Analysis</td>
<td>154</td>
</tr>
<tr>
<td>6.6</td>
<td>Modeshapes for the First Four Critical Speeds of the Casing</td>
<td>157</td>
</tr>
<tr>
<td>6.7</td>
<td>Modeshapes for the First Four Critical Speeds of the Pump</td>
<td>158</td>
</tr>
<tr>
<td>6.8a</td>
<td>Modeshapes of the Casing for $N_{cr}$ # 1 and # 2</td>
<td>159</td>
</tr>
<tr>
<td>6.8b</td>
<td>Modeshapes of the Casing for $N_{cr}$ # 3 and # 4</td>
<td>159</td>
</tr>
<tr>
<td>6.8c</td>
<td>Modeshapes of the Casing for $N_{cr}$ # 5 and # 6</td>
<td>160</td>
</tr>
<tr>
<td>6.8d</td>
<td>Modeshapes of the Casing for $N_{cr}$ # 7 and # 8</td>
<td>160</td>
</tr>
<tr>
<td>6.9a</td>
<td>Modeshapes of the Pump for $N_{cr}$ # 1 and # 2</td>
<td>161</td>
</tr>
</tbody>
</table>

List of Figures: xiii
<table>
<thead>
<tr>
<th>Figure</th>
<th>Caption</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.9b</td>
<td>Modeshapes of the Pump for $N_\alpha$ # 3 and # 4</td>
<td>161</td>
</tr>
<tr>
<td>6.9c</td>
<td>Modeshapes of the Pump for $N_\alpha$ # 5 and # 6</td>
<td>162</td>
</tr>
<tr>
<td>6.9d</td>
<td>Modeshapes of the Pump for $N_\alpha$ # 7 and # 8</td>
<td>162</td>
</tr>
<tr>
<td>6.10a</td>
<td>Modeshapes for $N_\alpha$ # 1 and # 2 of the Pump-Casing System (with Seals)</td>
<td>165</td>
</tr>
<tr>
<td>6.10b</td>
<td>Modeshapes for $N_\alpha$ # 3 and # 4 of the Pump-Casing System (with Seals)</td>
<td>165</td>
</tr>
<tr>
<td>6.10c</td>
<td>Modeshapes for $N_\alpha$ # 5 and # 6 of the Pump-Casing System (with Seals)</td>
<td>166</td>
</tr>
<tr>
<td>6.10d</td>
<td>Modeshapes for $N_\alpha$ # 7 and # 8 of the Pump-Casing System (with Seals)</td>
<td>166</td>
</tr>
<tr>
<td>6.10e</td>
<td>Modeshapes for $N_\alpha$ # 9 and # 10 of the Pump-Casing System (with Seals)</td>
<td>167</td>
</tr>
<tr>
<td>6.10f</td>
<td>Modeshapes for $N_\alpha$ # 11 and # 12 of the Pump-Casing System (with Seals)</td>
<td>167</td>
</tr>
<tr>
<td>6.10g</td>
<td>Modeshapes for $N_\alpha$ # 13 and # 14 of the Pump-Casing System (with Seals)</td>
<td>168</td>
</tr>
<tr>
<td>6.10h</td>
<td>Modeshapes for $N_\alpha$ # 15 and # 16 of the Pump-Casing System (with Seals)</td>
<td>168</td>
</tr>
<tr>
<td>6.11a</td>
<td>Modeshapes for $N_\alpha$ # 1 and # 2 of the Pump-Casing System (without Seals)</td>
<td>171</td>
</tr>
<tr>
<td>6.11b</td>
<td>Modeshapes for $N_\alpha$ # 3 and # 4 of the Pump-Casing System (without Seals)</td>
<td>171</td>
</tr>
<tr>
<td>6.11c</td>
<td>Modeshapes for $N_\alpha$ # 5 and # 6 of the Pump-Casing System (without Seals)</td>
<td>172</td>
</tr>
<tr>
<td>6.11d</td>
<td>Modeshapes for $N_\alpha$ # 7 and # 8 of the Pump-Casing System (without Seals)</td>
<td>172</td>
</tr>
<tr>
<td>6.11e</td>
<td>Modeshapes for $N_\alpha$ # 9 and # 10 of the Pump-Casing System (without Seals)</td>
<td>173</td>
</tr>
<tr>
<td>6.11f</td>
<td>Modeshapes for $N_\alpha$ # 11 and # 12 of the Pump-Casing System (without Seals)</td>
<td>173</td>
</tr>
</tbody>
</table>

List of Figures
<table>
<thead>
<tr>
<th>Figure</th>
<th>Caption</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.11g</td>
<td>Modeshapes for $N_{cr}$ # 13 and # 14 of the Pump-Casing System (without Seals)</td>
<td>174</td>
</tr>
<tr>
<td>6.11h</td>
<td>Modeshapes for $N_{cr}$ # 15 and # 16 of the Pump-Casing System (without Seals)</td>
<td>174</td>
</tr>
<tr>
<td>7.1</td>
<td>Model of the Drum Rotor Supported on Active Magnetic Bearings</td>
<td>177</td>
</tr>
<tr>
<td>7.2</td>
<td>Plot of Stiffness vs. Frequency of the Magnetic Bearings</td>
<td>178</td>
</tr>
<tr>
<td>7.3</td>
<td>Plot of Damping vs. Frequency of the Magnetic Bearings</td>
<td>179</td>
</tr>
<tr>
<td>7.4</td>
<td>Modeshape for the First Critical Speed of the Drum Rotor</td>
<td>182</td>
</tr>
<tr>
<td>7.5</td>
<td>Modeshape for the Second Critical Speed of the Drum Rotor</td>
<td>183</td>
</tr>
<tr>
<td>7.6</td>
<td>Modeshape for the Third Critical Speed of the Drum Rotor</td>
<td>184</td>
</tr>
<tr>
<td>7.7</td>
<td>Modeshape for the Fourth Critical Speed of the Drum Rotor</td>
<td>185</td>
</tr>
<tr>
<td>7.8</td>
<td>Model of the Drum Rotor Showing the Unbalance Location</td>
<td>193</td>
</tr>
<tr>
<td>7.9</td>
<td>Response of Rotor for Unbalance at Outboard Disk</td>
<td>194</td>
</tr>
<tr>
<td>7.10</td>
<td>Response of Rotor for Unbalance at Inboard Disk</td>
<td>195</td>
</tr>
<tr>
<td>7.11</td>
<td>Response of Rotor for Unbalance at Rotor End of Coupling</td>
<td>196</td>
</tr>
<tr>
<td>7.12</td>
<td>Response of Rotor for Unbalance at Pinion End of Coupling</td>
<td>197</td>
</tr>
<tr>
<td>7.13</td>
<td>Response of Drum Rotor at 2000 RPM (Unbalance at Outboard Disk)</td>
<td>198</td>
</tr>
<tr>
<td>7.14</td>
<td>Response of Drum Rotor at 2500 RPM (Unbalance at Outboard Disk)</td>
<td>199</td>
</tr>
<tr>
<td>7.15</td>
<td>Response of Drum Rotor at 3500 RPM (Unbalance at Outboard Disk)</td>
<td>200</td>
</tr>
<tr>
<td>7.16</td>
<td>Response of Drum Rotor at 5000 RPM (Unbalance at Outboard Disk)</td>
<td>201</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Caption</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Expressions for Stiffness and Damping Properties of the Squeeze Film Damper</td>
<td>37</td>
</tr>
<tr>
<td>4.1a</td>
<td>Comparison of Damped Critical Speeds for different element numbers</td>
<td>66</td>
</tr>
<tr>
<td>4.1b</td>
<td>Comparison of Growth Factors for different element numbers</td>
<td>66</td>
</tr>
<tr>
<td>4.2a</td>
<td>Comparison of Damped Critical Speeds for the 3 Mass Matrix Formulations for the 8 Stage Compressor</td>
<td>72</td>
</tr>
<tr>
<td>4.2b</td>
<td>Comparison of Growth Factors for the 3 Mass Matrix Formulations for the 8 Stage Compressor</td>
<td>73</td>
</tr>
<tr>
<td>4.3</td>
<td>Data for the Uniform Rotor Model</td>
<td>76</td>
</tr>
<tr>
<td>4.4</td>
<td>Comparison of Results of Finite Element Program and Transfer Matrix Program (CRTMB2) for Undamped Critical Speeds</td>
<td>76</td>
</tr>
<tr>
<td>4.5</td>
<td>Comparison of Results of Finite Element Program and Transfer Matrix Program (ROBEST) for Damped Critical Speeds</td>
<td>78</td>
</tr>
<tr>
<td>4.6</td>
<td>Comparison of Stability Analysis for $Q = 1.7555 \times 10^4$ N/m</td>
<td>80</td>
</tr>
<tr>
<td>4.7</td>
<td>Comparison of Stability Analysis for $Q = 1.7555 \times 10^7$ N/m</td>
<td>80</td>
</tr>
<tr>
<td>5.1</td>
<td>Frequency Dependent Properties used for the Above Model</td>
<td>83</td>
</tr>
<tr>
<td>5.2</td>
<td>Comparison of Results of Finite Element Program and Transfer Matrix Program ROBEST</td>
<td>88</td>
</tr>
<tr>
<td>5.3</td>
<td>Frequency Dependent Bearing Properties</td>
<td>97</td>
</tr>
<tr>
<td>5.4a</td>
<td>First Critical Speeds for Different Sensor Positions (Lower Damping)</td>
<td>100</td>
</tr>
<tr>
<td>5.4b</td>
<td>Third Critical Speeds for Different Sensor Positions (Lower Damping)</td>
<td>100</td>
</tr>
<tr>
<td>5.5</td>
<td>Third Critical Speeds for Different Sensor Positions (Higher Damping)</td>
<td>100</td>
</tr>
<tr>
<td>5.6</td>
<td>Bearing Properties used for Analysis of the MultiStage Pump</td>
<td>137</td>
</tr>
<tr>
<td>Table</td>
<td>Caption</td>
<td>Page</td>
</tr>
<tr>
<td>-------</td>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td>5.7</td>
<td>Damped Critical Speeds and Growth Factors for the first 8 modes</td>
<td>137</td>
</tr>
<tr>
<td>6.1</td>
<td>Results of Stability Analysis for Locked Configuration of the Seal ((\varepsilon = 0.0001))</td>
<td>149</td>
</tr>
<tr>
<td>6.2</td>
<td>Results of Stability Analysis for Floating Configuration of the Seal ((\varepsilon = 0.0001))</td>
<td>149</td>
</tr>
<tr>
<td>6.3</td>
<td>Results of Stability Analysis for Locked Configuration of the Seal ((\varepsilon = 0.3))</td>
<td>150</td>
</tr>
<tr>
<td>6.4</td>
<td>Results of Stability Analysis for Floating Configuration of the Seal ((\varepsilon = 0.3))</td>
<td>150</td>
</tr>
<tr>
<td>6.5</td>
<td>Comparison of Results of Stability Analysis for Single and Multilevel Model of the AMB Rotor</td>
<td>152</td>
</tr>
<tr>
<td>6.6</td>
<td>Stability Analysis of Casing</td>
<td>155</td>
</tr>
<tr>
<td>6.7</td>
<td>Stability Analysis of the Pump with Bearings and InterStage Seals</td>
<td>156</td>
</tr>
<tr>
<td>6.8</td>
<td>Stability Analysis of Pump-Casing System (With Seals)</td>
<td>164</td>
</tr>
<tr>
<td>6.9</td>
<td>Stability Analysis of Pump-Casing System (Without Seals)</td>
<td>170</td>
</tr>
<tr>
<td>7.1</td>
<td>Synchronous Bearing Properties (Speed = 4000 RPM)</td>
<td>180</td>
</tr>
<tr>
<td>7.2</td>
<td>Results of Synchronous Stability Analysis of the AMB Rotor</td>
<td>181</td>
</tr>
<tr>
<td>7.3a</td>
<td>NonSynchronous Stability Analysis Damped Critical speeds (Sensors Noncollocated)</td>
<td>187</td>
</tr>
<tr>
<td>7.3b</td>
<td>NonSynchronous Stability Analysis Growth Factors (Sensors Noncollocated)</td>
<td>187</td>
</tr>
<tr>
<td>7.4a</td>
<td>NonSynchronous Stability Analysis Damped Critical speeds (Sensors Collocated)</td>
<td>188</td>
</tr>
<tr>
<td>7.4b</td>
<td>NonSynchronous Stability Analysis Growth Factors (Sensors Collocated)</td>
<td>188</td>
</tr>
<tr>
<td>Table</td>
<td>Caption</td>
<td>Page</td>
</tr>
<tr>
<td>-------</td>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td>7.5a</td>
<td>NonSynchronous Stability Analysis % Deviation from the “exact” Damped Critical speeds (Sensors Noncollocated)</td>
<td>189</td>
</tr>
<tr>
<td>7.5b</td>
<td>NonSynchronous Stability Analysis % Deviation from the “exact” Growth Factors (Sensors Noncollocated)</td>
<td>189</td>
</tr>
<tr>
<td>7.6a</td>
<td>NonSynchronous Stability Analysis % Deviation from the “exact” Damped Critical speeds (Sensors Collocated)</td>
<td>190</td>
</tr>
<tr>
<td>7.6b</td>
<td>NonSynchronous Stability Analysis % Deviation from the “exact” Growth Factors (Sensors Collocated)</td>
<td>190</td>
</tr>
</tbody>
</table>
NOMENCLATURE

' first derivative with respect to position
" second derivative with respect to position
· first derivative with respect to time
·· second derivative with respect to time
a Growth factor
h Fluid film thickness
i_o Steady state current
i_y Varying current in the Y-direction
i_z Varying current in the Z-direction
k Transverse shear form factor
k^B Bearing stiffness
k^R_{ij} (i,j)^{th} element of the element (rotor) stiffness matrix
\ell Length of the element
m mass
\{q\} Displacement vector
s axial distance along the rotor axis
t time
v_1, v_2 displacements of node 1 and node 2 of the rotor element in the Y-direction
w_1, w_2 displacements of node 1 and node 2 of the rotor element in the Z-direction
x, y, z Rotational reference frame coordinates
A Area of cross-section
[A] Stiffness matrix due to the axial load
[C] Damping matrix
C_r Radial clearance
C_d Diametrical clearance
C_N Damping number
E  Modulus of Elasticity
\{F\}  Total force vector
\{F_c\}  Force vector (Cosine components)
\{F_s\}  Force vector (Sine components)
\(G_o\)  Transfer function
\(G\)  Shear Modulus
\([G]\)  Gyroscopic matrix
I  Area moment of inertia
\(I_T\)  Transverse mass moment of inertia
\(I_p\)  Polar mass moment of inertia
\([K]\)  Stiffness matrix due to translation
L  Length of the journal
\([M]\)  Total mass matrix
\([M_T]\)  Mass matrix due to translation
\(N_{cr}\)  Critical speed
\([N]\)  Mass matrix due to rotation
P  Axial load
\(P_{vc},P_{vs}\)  Cosine and Sine components of the unbalance force in the y-direction
\(P_{vc},P_{vs}\)  Cosine and Sine components of the unbalance force in the z-direction
\(P_s\)  Supply pressure
\(\bar{P}\)  Pressure generated by the oil film
R  Radius of the journal
\(T_E\)  Kinetic energy
\(U_y\)  Unbalance force in Y-direction (mass*rotation speed*radius'2)
\(U_z\)  Unbalance force in Z-direction (mass*rotation speed*radius'2)
\(V_E\)  Potential energy
V  displacement in Y-direction
W  displacement in Z-direction

Nomenclature
\( V_{\beta}, W_{\beta} \)  \quad \text{Bending components of the displacement in Y, Z - directions respectively}

\( V_{\alpha}, W_{\epsilon} \)  \quad \text{Shear deformation of the displacement in Y, Z - directions respectively}

\( X, Y, Z \)  \quad \text{Fixed reference frame coordinates}

\( \alpha \)  \quad \text{Eigenvalue}

\( \delta \)  \quad \text{Log. decrement}

\( \delta W \)  \quad \text{Variational work done by nonconservative forces not included in the Potential work}

\( \varepsilon \)  \quad \text{Eccentricity ratio}

\( \mu \)  \quad \text{Viscosity}

\( \theta_{y1}, \theta_{y2} \)  \quad \text{Rotations of node 1 and node 2 of the rotor element in the Y-direction}

\( \theta_{z1}, \theta_{z2} \)  \quad \text{Rotations of node 1 and node 2 of the rotor element in the Z-direction}

\( \psi_i \)  \quad \text{Shape functions for bending}

\( \phi_i \)  \quad \text{Shape function for bending and shear deformation}

\( \nu \)  \quad \text{Poisson’s ratio}

\( \bar{\nu} \)  \quad \text{Non-dimensional quantity for element length}

\( \omega_d \)  \quad \text{Damped natural frequency}

\( B \)  \quad \text{Rotation in Y - direction}

\( \Gamma \)  \quad \text{Rotation in Z - direction}

\( \Phi \)  \quad \text{Shear deformation factor}

\( \Omega \)  \quad \text{Angular velocity (Rotational speed) in rad/s}
CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

1.1 Introduction

The goal in the modern petrochemical and aerospace industries has been towards higher operating speeds and higher power output. This growing trend necessitates that the power/weight ratio of the machinery be kept to a maximum. However this reduction in the weight of the machinery may result in a more flexible system which in turn, could lead to higher vibration amplitudes of the rotating machinery. Rotor dynamics involves in the calculation of response to unbalance excitation and prediction of stability of rotating machinery. In a broader sense, rotor dynamics also can involve the modeling of rotor-bearing systems for non-linear transient analysis. However, this research work involves only the steady-state analyses of rotor-bearing systems. The steady-state analysis results in : (a) the evaluation of the eigenvalues of the system and (b) the response of the system to an unbalance excitation. The real part of the eigenvalue gives the stability of the machinery, expressed as the growth factor or the logarithmic decrement, and the imaginary part of the eigenvalue gives the damped critical speed for each mode.

The components of a typical rotor system, for example, a multi-stage compressor typical of an industrial application, are the impellers, disks, balance drum, bearings, seals and the shaft on which these are mounted. With the advent of modern technology, the
rotor system has become increasingly complex, and more powerful tools are required to analyze the rotor systems. For example, magnetic bearings are being used as an alternate means of supporting rotors. The analytical method used for modeling the rotors must be capable of including the characteristics of magnetic bearings -- viz., the positions of the sensors and the frequency dependent bearing characteristics.

The most common model used in rotor dynamics to study the basic lateral vibration of a rotating shaft is the Jeffcott rotor, named after H.H. Jeffcott. This model consists of a massless elastic rotor supported at the ends, and has a rigid disk located at the midspan. Jeffcott (1) concluded that the amplitude of the lateral vibration increases till the critical speed and then decreases after that point. Since then this simple but effective model has been used to understand the behavior of simple systems.

1.2 Literature Review

The dynamic analysis of a rotor-bearing systems involves deriving the equations of motion of the system, beginning with the physical model. The two most popular analytical methods used in rotor dynamics are the transfer matrix method and the finite element method.

The transfer matrix methods have been used quite extensively in rotor dynamics, well before the finite element method. In principle, the shaft is treated as a beam and the rotor system is divided into small beams. A relationship is obtained for the forces, moments, slopes and deflections of one end of the beam section in terms of the parameters at the other end. This set of variables forms the state variables for the rotor system. Thus, by progressively moving from one end of the rotor to the other, a relationship is obtained between the state variables at the two ends of the rotor. Appropriate boundary conditions
are used on the state variables and the system of equations is solved for the critical speeds and/or unbalance response.

1.2.1 Finite Element Modeling in Rotor Dynamics

The finite element method originated primarily from the analysis of structures, and dates back to the late 1950’s. With the advent of computers, the finite element method developed as a very powerful mathematical tool for design and analysis of not only structures, but also in the areas of fluid dynamics, heat transfer and structural dynamics. Huebner (2) summarizes the origin and applications of this powerful mathematical tool. Though this method has been in existence for quite long, its application in the area of rotor dynamics has been recent. In recent times, the finite element method has been used quite widely in rotor dynamics mainly due to the ability of the method to be able to model complex system relatively easily. The growth of the application of finite element method is partly due to the advent of computers.

In principle, the finite element method involves discretizing (or dividing) the shaft element into small beams and then deriving the stiffness and mass matrices for each beam section. The stiffness and mass matrices for the entire system are obtained by assembling the individual matrices and this results in the global stiffness and mass matrices. The equations of motion are obtained and solved for the critical speeds and/or response. Ruhl (3) compared some simple finite element formulations for rotor dynamics with the transfer matrix methods. Ruhl and Booker (4) outlined a numerical procedure for the stability analysis and unbalance response calculations based on distributed inertia and elasticity properties. This was based on an earlier work by Archer (5) who derived the mass and stiffness matrices for structural analysis. Ruhl and Booker (4) compared the discrete and distributed rotor representations and concluded that the discrete representation model was
conservative in the prediction of the instability threshold. Nelson and McVaugh (6) came up with a finite element formulation based on Ruhl's work. This model was based on a Bernoulli-Euler beam and included only the effects of translational and elastic bending energies. The above formulation modeled the shaft (rotor) as beam elements with 2 nodes, one at each end, and with 4 degrees of freedom per node. The 4 degrees of freedom were the two translations in the lateral directions and two respective rotations. Zorzi and Nelson (7,8) extended the above finite element model to include the effects of internal damping, and later the effect of axial torque. Two linear damping mechanisms, the internal viscous and hysteretic damping, were incorporated into the finite element model to study their effects on the stability of the rotor system.

However, if the shaft has a low length/diameter ratio, then the effects of shear deformations enter the picture and have to be included in the model. Beam elements modeled to include the effects of shear deformation and rotatory inertia are called as Timoshenko beams. This leads to the development of a few different types of Timoshenko beam elements. Davis et al. (9) derived the stiffness and consistent mass matrices based on the exact differential equations of an infinitesimal element in static equilibrium. Nickel and Secor (10) developed two Timoshenko beams elements, one with 7 degrees of freedom per element and the other with the number of degrees of freedom reduced to 4 using constraint equations. They also studied the convergence of the newly developed Timoshenko elements and compared the results to the other elements that were used. Thomas et al. (11) gave an overview and comparison of the different Timoshenko beam elements and also derived a new Timoshenko beam element with 3 degrees of freedom per node (in one lateral plane). The nodal variables were the transverse displacement $w$, the cross-section rotation $\theta$ and shear $\psi$. Cowper (12) had derived expressions for the shear
coefficient for a Timoshenko beam which is very widely used in Timoshenko beam formulations.

The most widely used Timoshenko beam element consists of a beam element with 2 nodes, one at each end of the beam, and 4 degrees of freedom per node (two lateral translations and two rotations). The effect of shear deformation is included in deriving the stiffness and mass matrices. Nelson (13) developed this Timoshenko beam to include the transverse shear deformation effects. This work also compared the finite element analysis with the classical closed form solution developed by Eshleman and Eubanks (14). Ö zgüven and Özkan (15) developed a computer program which modeled the rotor-bearing system including the effects of rotatory inertia, gyroscopic moments, internal viscous and hysteretic damping and transverse shear deformations.

All the above formulations considered the axial cross-section of the element to be cylindrical, which allows the area and inertia to be considered constant with respect to the length. The modern turbomachines have geometry which is usually far from being uniform as a function of the length. Usually the changes in cross-section are taken into account by modeling the rotor as an assemblage of stepped cylinders. However, modeling conical sections in the above manner may introduce error that may be quite large. Rouch and Kao (16) developed a linearly tapered Timoshenko beam element for use in the area of rotor dynamics. This was based on an earlier work by Thomas et al. (11), who had developed a Timoshenko beam element. Rouch and Kao extended the earlier formulation by including the gyroscopic effects and representing the area and inertia as second and fourth order polynomials as a function of the radius. To (17) developed closed form polynomial mass and stiffness expressions for a linearly tapered Timoshenko element using twelve degrees of freedom per element. Greenhill, Bickford and Nelson (18) extended the linearly tapered Timoshenko beam element to develop closed form polynomial expressions for element
matrices suitable for use in finite element based rotor dynamics programs. Nelson and Nataraj (19) developed an analytical procedure for modeling rotors with cracks, using the finite element method. The rotating assembly was modeled using finite rotating elements and the presence of a crack was taken into account by a rotating stiffness variation. The stiffness variation was a function of the rotor's bending curvature at the location of the crack, and was represented by a Fourier series expansion. The resulting nonlinear parametrically excited system was analyzed using a perturbation method along with an iteration procedure. Vest and Darlow (20) proposed a modified conical beam element. They suggested a modification in the conical beam element, to be used in rotor dynamics. The modification was made by altering the local value of the Young's modulus so that the element produces a bending flexibility corresponding to that obtained using three-dimensional finite element models.

1.2.2 Stability and Response

The increase in the performance requirements of high-speed rotating machinery poses the problem of designing the machinery capable of smooth operation under various conditions of speed and load. In most of these applications the design operating speed is often well beyond the rotor first critical speed, and under these conditions it is difficult to maintain a stable low-level amplitude of the vibration of the rotor. The most common cause of the vibration of the rotor is the unbalance present in the system. Apart from the fact that it would be practically impossible to perfectly balance a rotor, it would be equally difficult to maintain the balance condition at the same low level during the entire operating life of the machine. The unbalance results in a synchronous response, proportional to the running speed. Under certain high-speed operating conditions, above the first critical speed, such influences as internal rotor friction, hydrodynamic bearing and seal forces, and

1.0 Introduction and Literature Review
aerodynamic cross-coupling, as discussed by Alford (21), can lead to a destructive non-synchronous precessive whirl motion being developed in the rotor-bearing system.

One of the early significant contributors in the field of rotor dynamics, as far as the calculation of critical speeds and unbalance response is concerned, was by Jeffcott (1). Jeffcott developed the fundamentals of the dynamics response of this damped single-mass unbalanced rotor system, which later came to be known as the Jeffcott rotor. This analysis of the single-mass model showed that operating speeds above the first critical speed were possible and that a low level of vibration would be attained once the rotor had exceeded the first critical speed. The compressors and turbines were manufactured using the flexible rotor design concept in which the rotor was designed to operate above the first critical speed. But these developed severe operating difficulties which could not be explained by the basic Jeffcott model.

Newkirk and Kimball (22) were able to experimentally determine that the introduction of a flexible support system could greatly extend the stability threshold speed of the rotor system. They were working on the self-excited instability of compressors due to internal friction. Smith (23) verified Newkirk’s findings theoretically by enhancing the Jeffcott model with internal damping. Further researchers like Gunter (24,25), Tondl (26) and Dimentberg (27) showed that flexible damped supports may improve the stability characteristics of high-speed rotors. Lund and Sternlicht (28) showed that the forces transmitted could be significantly reduced by proper design of the bearing support system. Lund (29), one of the pioneers in the area of rotor dynamics, presented a theoretical analysis investigating the stability of a symmetrical, flexible rotor supported in journal bearings and concluded that the damping and flexibility of the bearing support raises the onset of instability. Lund and Orcutt (30) compared the analytical and experimental results of the unbalance response of a flexible rotor. Kirk and Gunter (31) presented an analysis
to determine the influence of flexible supports on the synchronous unbalance response of the single-mass Jeffcott rotor, and to optimize the support system characteristics, such that the rotor amplitude and the forces transmitted were minimized over a given speed range. Lund (32) described a method, analogous to the Myklestad-Prohl method, for calculating the threshold speed of instability and the damped critical speeds of a general flexible rotor in fluid-film journal bearings. Again Lund (33) came up with a modal method for calculating the response of a flexible rotor. Bansal and Kirk (34) came up with an analytical technique, based on the transfer matrix approach, to calculate the damped critical speeds and the instability threshold speed of multimass rotor-bearing systems, including pedestal flexibility. Choudhury et al. (35) studied the effect of fluid-film bearing damping on the lateral critical speeds of rotor-bearing systems. Barrett et al. (36) presented an approximate method for calculating the optimum bearing or support damping for multimass flexible rotors to minimize unbalance response and to maximize stability in the vicinity of the rotor first critical speed. This method was particularly applicable to bearings that have minimal cross-coupling effects such as the tilting pad bearings and squeeze film dampers. Murphy and Vance (37) proposed a new method for calculating the critical speeds and rotor dynamic stability of turbomachinery. This was based on deriving the characteristic polynomial for the given complex rotor-bearing system rather than the transfer matrix approach. This polynomial was used to calculate the critical speeds and the stability of the rotor-bearing system. Kim and David (38) proposed an improved method for the calculation of stability and damped critical speeds of rotor-bearing systems. The method was based on the transfer matrix-polynomial method. However, the process could find only some of the dominant eigenvalues. With the decrease in the time taken to solve a given problem using the modern computers, the finite element method is increasingly used.
to solve the huge eigenvalue problems either to obtain all the eigenvalues or to reduce the eigenvalue problem by the condensation methods.

1.2.3 Squeeze Film Dampers

Squeeze film dampers were introduced in the early 1960's (39). Research has been conducted in this area ever since, especially from the late seventies to the early eighties. One of the earliest experiments on a rotor supported on squeeze film damper was conducted by Cooper, who observed a "bistable" operation (or the "jump" phenomenon). This was the first indication of the non-linear behavior of the squeeze film damper. In 1972, White (40) was able to calculate the forces acting in the damper based on Reynolds equation. He studied theoretically and experimentally the dynamics of a rigid rotor having a squeeze film damper. Thomson and Anderson (41) measured the force transmitted for a squeeze film damper and compared results to short-bearing theory. Mohan and Hahn (42) also studied the dynamics of a rigid rotor on squeeze film dampers. They obtained the steady-state response based on short bearing theory and for the condition of synchronous circular whirl. They also did a parameteric study to determine the damper influence, the force transmitted and the maximum unbalance at which the damper was effective. Hibner (44) developed a transfer matrix method capable of predicting the vibratory response of complex multi-shaft gas turbine engines with non-linear viscous dampers. He had derived the damper coefficients from the Reynolds equation using the long bearing assumption. Vance and Kirkton (45) conducted an experimental investigation on squeeze film damper bearings and studied the dynamic force response for both centered and non-centered cases of the journal orbit in the annular clearance. They measured the pressure distribution for the above cases and numerically integrated to get the force components of the squeeze film. These were compared with both the long bearing and short bearing solutions.
Cunningham et al. (46) studied the influence of the damper supports on rotor amplitudes and forces transmitted and gave a design procedure which was based on a single mass flexible rotor. Simandiri and Hahn (47) studied the effect of pressurization on the vibration isolation capability of squeeze film bearings and stated that with increased pressurization, the likelihood of bistable operation could be reduced considerably.

Rabinowitz and Hahn (48,49,50,51) studied the steady-state orbits for a flexible rotor incorporating squeeze film dampers. They performed parametric studies to determine the regions of unacceptable behavior of rotors due to squeeze film dampers. They also did a stability analysis of the steady-state orbits they had obtained. These studies were conducted on both pressurized and unpressurized dampers. Gunter et al. (52) numerically studied the nonlinear response of aircraft engines incorporating squeeze film dampers and were able to show that the rotor exhibits the jump phenomenon and under unidirectional loading, subharmonic whirl motion may exist. Cookson and Kossa (53) studied, theoretically and experimentally, the dynamics of a rotor on uncentralized squeeze film dampers, and the same authors (54) investigated the experimental results with the three models of the squeeze film dampers. Szeri et al. (55) derived the linear force coefficients, which considered inertia of the fluid film, from the Navier-Stokes equation. Taylor and Kumar (56) did some investigations on the numerical techniques used to determine the response of a rigid rotor on squeeze film dampers and came up with a closed-form steady-state solution for a rigid rotor in squeeze film dampers by using a circular orbit. Cookson and Dainton (57) conducted investigations on the effect of the side-plate clearance of an uncentralized squeeze film damper. They observed that the influence of the side-plate clearance to reduce the amplitude of the central disk could be considerable if the clearance is less than the radial clearance.
McLean and Hahn (58) presented a technique for investigating the stability and the degree of damping in circular synchronous orbit equilibrium solutions. It involved the analysis of appropriate perturbation equations about the equilibrium solutions and is applicable to systems with several squeeze film dampers. San Andres and Vance (59) obtained the steady-state response of a rotor incorporating squeeze film dampers, including fluid inertia effects by using an averaged momentum approximation. El-Shafei (60) obtained a relatively faster algorithm for the steady-state unbalance response of a Jeffcott rotor incorporating short squeeze film dampers and executing circular centered whirl. The effects of fluid inertia were included in the model. He showed that the fluid inertia results in decreasing the possibility of jump response and also results in excitation of a second mode for the Jeffcott rotor.

Most of the work on squeeze film dampers has been in the area of response and bi-stable operation. Though squeeze film dampers have been used as an attractive means of supporting machinery, several undocumented cases, where the dampers have not been successful, suggest that a general design study could be helpful to understand how the system stability is influenced by the addition of a squeeze film damper.

1.2.4 Magnetic Bearings

The concept of suspending a rotating shaft in a magnetic field is relatively an old idea. But with the development in control systems, the magnetic bearings have become an attractive means of supporting rotating machinery. The conventional fluid film bearings require a continuous lubrication system for the supply and circulation of the oil. The absence of this lubrication system is one of the major advantages of magnetic bearings. Habermann and Liard (61) describe the principles of design and operation of the Actidyne magnetic bearing system. Typically, a magnetic bearing consists of a set of coils which
electromagnetically levitate the shaft. In an Active Magnetic Bearing (AMB), sensors are located in the magnetic bearings to sense the displacement, and this displacement is fed back to a control system which controls the input current to the bearing to produce a proportional force. This closed loop control system is used quite extensively in magnetic bearing technology. Most of the research in the area of magnetic bearings has been in the area of control systems. From the rotor dynamic point of view, the stability analysis requires the bearing stiffness and damping characteristics of the magnetic bearing. One of the characteristic features of the magnetic bearings is that the sensor is not located in the plane of the bearings. As a result of this sensor non-collocation, the force at the bearing location is proportional to the displacement sensed by the sensor. This non-collocation of the sensor has to be taken into account while performing stability analysis. Kirk et al. (62) studied the influence of the sensor location of the calculated critical speed of turbomachinery using the transfer matrix method. Rawal et al. (63) discussed the effect of sensor location on the forced response characteristics of rotors on AMB. This paper discussed the incorporation of the sensor locations using the transfer matrix method, and its effect on unbalance response.

One other typical characteristic of a magnetic bearing system is that the stiffness and damping characteristics of the bearing are dependent on the whirl frequency of the rotor, rather than the spin frequency (running speed of the rotor), which is a characteristic of the fluid film bearings. The analytical procedure used for the stability analysis must be able to evaluate the damped critical speeds and stability based on the whirl frequency dependent bearing parameters. The control system used for the magnetic bearings has its own dynamic characteristics. Again the analytical procedure must be able to model the dynamics of the control system and the coupled control-rotor system. Lewis et al. (64) discussed a method of modeling the control system dynamics and including it in the rotor
dynamic analysis, using the transfer matrix method. Barrett (65) described the modeling of magnetic supports using the transfer matrix method. Barrett et al. (66) discussed the modeling of non-collocated sensors of AMB supported rotors using the transfer matrix method. Barrett et al. (67) evaluated the stability of the rotor using the frequency dependent transfer functions of the magnetic bearing, and compared the results obtained with those obtained using constant bearing characteristics.

In the current research, two different methods of handling the frequency dependent bearing characteristics are implemented using the finite element method. A method of including the model of the transfer function of the control system into the dynamics of the rotor system is also derived.

1.3 Scope of the Dissertation

The thrust of the current research was in the area of modeling complex rotor systems for stability and response calculations. The intent of this research was to develop a PC based program for rotor dynamic analysis. Two different methods are proposed for the analysis of rotors supported on magnetic bearings. The magnetic bearing control system modeling has been incorporated into the analysis. The pedestal flexibility and its influence on the stability and response are studied. The importance of considering the sensor locations in stability and response are highlighted. A method for sub-synchronous stability evaluation is described.

Since most of the work done in the area of squeeze film dampers has considered the reducing the vibration and force transmitted, and also since the incorporation of the dampers has caused the rotors to become unstable, the stability of the rotors supported on dampers was studied. A design procedure is suggested for the stability analysis of turbomachinery supported on squeeze film dampers, introducing the concept of a
Damping Number. Most of the previous work done on the response of rotors supported on squeeze film dampers consider centered damper configuration. In this research, the non-linear behavior of the squeeze film damper is considered while calculating the forced response of the rotor system.

Multi-level rotor systems find their application in the vertical pumps and aero-engines. A typical multi-level aero-engine has two concentric shafts rotating at different speeds connected by inter-shaft bearings or inter-shaft squeeze film dampers. The current research also includes the methodology for modeling multi-level rotor systems, and also to model rotors connected by flexible couplings.

The additional capabilities of the above PC-based program are that it can account for pedestal flexibility, squeeze film dampers, forces transmitted to the foundation, and also sensor non-collocation in case of magnetic bearings. The program can model the rotor system based on distributed mass or lumped mass formulations.
CHAPTER 2
DEVELOPMENT OF THE SYSTEM MODEL

2.1 Introduction

In any analysis of a physical system, the first and primary step is to identify the system components and try to convert the physical system into a set of mathematical equations depending on the system and representing the system as closely as possible. The equations are subjected to the appropriate boundary conditions representing the physical conditions on the system and then solved for the respective results. In rotor dynamics, for the stability and response calculations the standard procedure is to derive the system equation of motion in the well known form,

\[ M \ddot{X} + C \dot{X} + K X = F \]  \hspace{1cm} (2.1)

This equation is then solved for the eigenvalues and eigenvectors (stability analysis) with the forcing function to be zero, and the unbalance response for the given unbalance force vector. This is the essence of rotor dynamics in a nutshell. The two methods used in rotor dynamics are the transfer matrix method and the finite element method. The transfer matrix method has been in use since the 1940's (68) and since the
mid-1960's for forced response to unbalance (69). The advent of computers has seen the increasing popularity of finite element methods. Conceptually the finite element method involves discretizing the system into smaller parts, then deriving the equations for the individual parts, and then assembling these equations to a global set representing the entire system.

The components of a rotor dynamic system fall into the following three broad categories. (i) the rotor element (shaft); (ii) the disks/impellers and (iii) the bearings, seals, squeeze film dampers, etc. The standard procedure used in rotor dynamics is to model the rotor sections as beam elements, the disks/impellers as discrete external mass elements having a mass and inertia property, and the bearings/seals as discrete elements having stiffness and damping.

In most of the beam element formulations the rotor section is represented by a beam with two nodes, and each node has 4 degrees of freedom. The degrees of freedom are the two lateral displacements and the respective rotations. In other words, the degrees of freedom associated with the axial displacement and rotation of the shaft are neglected in the formulations. Ruhl and Booker (4) developed a beam element for rotor dynamic analysis, based on Archer’s (5) work. This beam model considered only the translational and rotational energies -- the Euler-Bernoulli beam, and neglected the shear deformation and rotatory inertia effects. For many applications it is satisfactory to neglect the shear deformation and rotatory inertia of beams as explained by Archer (5). For long thin beams, and lowest modes, the inclusion of shear deformation and rotatory inertia does not
produce significant changes in the results. But when the beam section gets thicker in
diameter/width and shorter in length, significant errors are introduced if these effects are
neglected. Also the higher modes also deviate considerably if these effects are neglected.

The beam formulation that includes the effect of shear deformation and rotatory
inertia is called the Timoshenko beam, as derived by Timoshenko (70). A number of
Timoshenko beam finite elements have been derived as discussed by Thomas et al. (11).
These different elements can be divided into two categories: (a) simple beams having 2
nodes, one at each end, and with 4 degrees of freedom (2-translation, 2-rotation) per
node, and (b) complex beams with more than 4 degrees of freedom per node and/or more
than two nodes per element. However, in the finite element method, the size of the
problem increases with the number of degrees of freedom. The most commonly used
formulation is the Timoshenko beam with 2 nodes and 4-dof's per node. The effect of
shear deformation is included in the model. The derivation of the stiffness, mass and
gyroscopic matrices for the rotor element, modeled as a Timoshenko beam, are described
in the following sections (71,72,73). The derivations of the mass and damping matrices for
the rigid disks and the bearings are also discussed.
2.2 Development of the Timoshenko Beam -- derivation of the $K, M, C$ matrices

2.2.1 The Rotor element

A typical finite rotating shaft element is shown in Fig. 2.1 with the two reference systems - the $(XYZ)$ triad as the fixed reference frame with the $X$ axis coinciding with the undeformed rotor centerline, and the $(xyz)$ triad that rotates at a uniform rate $\omega$ about the $X$ axis. The element is considered initially straight and is modeled as a beam element with two nodes at each end. Each node has 4 degrees of freedom (dof) - $V, W$ the displacements in the $Y$ and $Z$ directions, respectively, and $B, \Gamma$ the rotations about the $Y$ and $Z$ directions. Thus each element is modeled with a total of 8 dofs. The symmetry of the rotor element about the axis of rotation is assumed.

![Diagram showing typical rotor element configuration](image.jpg)

Figure 2.1 Typical Rotor Element Configuration showing the Coordinate System

2.0 Development of the System Model
A typical cross-section of the rotor element, located at a distance ‘s’ from the left end, translates and rotates during the general motion of the element. The translation of the cross-section centerline neglecting axial motion is given by a set of two displacements \((V, W)\) which is made up of a bending component \((V_\beta, W_\beta)\) and a component due to shear deformation \((V_\sigma, W_\sigma)\). The rotations of the cross-section are given by the angles, defined as \(B = -\partial W/\partial s\) and \(\Gamma = \partial V/\partial s\). The translation of a typical point internal to the element is approximately given by the relation (13),

\[
\begin{bmatrix}
V(s,t) \\
W(s,t)
\end{bmatrix}
= 
\begin{bmatrix}
\psi_1 & 0 & 0 & \psi_2 & \psi_3 & 0 & 0 & \psi_4 \\
0 & \psi_1 & -\psi_2 & 0 & 0 & \psi_3 & -\psi_4 & 0
\end{bmatrix}
\{q(t)\}
\]

(2.2)

where,

\[
\{q(t)\} = \{v_1 \ w_1 \ \theta_{y1} \ \theta_{z1} \ v_2 \ w_2 \ \theta_{y2} \ \theta_{z2}\}^T
\]

The shape functions \(\psi_i(s)\) represent the static displacement modes associated with unit displacement of one of the end point coordinates with the rest constrained to zero. The shear deformation, given by the factor \(\Phi = 12EI/kGAi^2\), is incorporated in the shape function as given in Appendix. The factor ‘\(k\)’ is the transverse shear form factor and depends on the cross-section, given by Cowper (12).
The rotation is given by,

\[
\begin{bmatrix}
B(s,t) \\
\Gamma(t)
\end{bmatrix} = \begin{bmatrix}
0 & -\phi_1 & \phi_2 & 0 & 0 & -\phi_3 & \phi_4 & 0 \\
\phi_1 & 0 & 0 & \phi_2 & \phi_3 & 0 & 0 & \phi_4
\end{bmatrix} \{q(t)\}
\]

(2.3)

\[
\begin{bmatrix}
B(s,t) \\
\Gamma(t)
\end{bmatrix} = [\Phi(s)] \{q(t)\}
\]

The shape functions \(\phi_i(s)\) represent the static displacement modes associated with unit displacement of one of the end point coordinates with the rest constrained to zero.

The element equations are determined using Hamilton's principle which states that the definite integral,

\[
\bar{I} = \int_{t_1}^{t_2} (T_E - V_E + W) dt
\]

is stationary for the true path, with respect to any variation of the path between the two instants \(t_1\) and \(t_2\), provided that the path variation vanishes at the end points.

The equations of motion are determined by,

\[
\int_{t_1}^{t_2} [\delta(T_E - V_E) + \delta W] dt = 0
\]

(2.4)

where \(T_E\) = Kinetic energy, \(V_E\) = Potential energy, \(\delta W\) = Variational work done by the non conservative forces not accounted for in the potential energy.

2.0 Development of the System Model
Kinetic energy: The element kinetic energy consists of both the energy due to rotation and translation. The rotational energy includes terms due to spinning of the shaft.

\[ T_E = \frac{1}{2} \int_0^l \begin{bmatrix} \dot{\psi} \\ \dot{\omega} \end{bmatrix}^T \begin{bmatrix} \dot{\psi} \\ \dot{\omega} \end{bmatrix} ds + \frac{1}{2} \int_0^l \begin{bmatrix} \dot{B} \\ \dot{\Gamma} \end{bmatrix}^T \begin{bmatrix} \dot{B} \\ \dot{\Gamma} \end{bmatrix} ds + \frac{1}{2} \int_0^l I_p ds \dot{\phi}^2 - \phi \int_0^l I_p \dot{\Gamma}B \ ds \] (2.5)

Potential energy: The potential energy of the element consists of the elastic bending and shear energy and energy due to axial load.

\[ V_E = \frac{1}{2} \int_0^l EI \begin{bmatrix} V_B^T \\ W_B^T \end{bmatrix} \begin{bmatrix} V_B \\ W_B \end{bmatrix} ds + \frac{1}{2} \int_0^l kAG \begin{bmatrix} V_\sigma^T \\ W_\sigma^T \end{bmatrix} \begin{bmatrix} V_\sigma \\ W_\sigma \end{bmatrix} ds + \frac{1}{2} \int_0^l P \begin{bmatrix} V' \\ W' \end{bmatrix}^T \begin{bmatrix} V' \\ W' \end{bmatrix} ds \] (2.6)

Variational work: The variational work, assuming only unbalance, is due to the distributed unbalance force.

\[ \delta W = \int_0^l \begin{bmatrix} \delta V \\ \delta W \end{bmatrix}^T \begin{bmatrix} P_{vc} \\ P_{wc} \\ P_{vs} \\ P_{ws} \end{bmatrix} \begin{bmatrix} \cos \Omega t \\ \sin \Omega t \end{bmatrix} ds \] (2.7)

Substituting equations (2.2) and (2.3) into equations (2.5),(2.6) and (2.7), the integrals for the energies and variational work can be written as,

\[ T_E = \frac{1}{2} \{ \dot{q} \}^T \left( [M_T] + [N] \right) \{ \dot{q} \} - \dot{\phi} \{ \dot{q} \}^T \left[ H \right] \{ q \} + \frac{1}{2} I_p \dot{\phi}^2 \] (2.8)

\[ V_E = \frac{1}{2} \{ q \}^T \left( [K] - [A] \right) \{ q \} \] (2.9)

\[ \delta W = \{ \delta q \}^T \left( \{ Q_c \} \cos \Omega t + \{ Q_s \} \sin \Omega t \right) \] (2.10)
Using Hamilton's principle, and equations (2.8), (2.9) and (2.10), the equation of motion for the finite rotor element in matrix form can be written as,

\[
\begin{bmatrix} [M_T] + [N] \end{bmatrix}\{\ddot{q}\} - \dot{\phi}\begin{bmatrix} G \end{bmatrix}\{\dot{q}\} + \begin{bmatrix} [K] - [A] \end{bmatrix}\{\dot{q}\} = \begin{bmatrix} \{F_c\} \end{bmatrix}\cos\Omega t + \begin{bmatrix} \{F_s\} \end{bmatrix}\sin\Omega t
\] (2.11)

The matrices are given in the Appendix.

2.2.2 Rigid disk

The rigid disk equation of motion is developed using a Lagrangian formulation. The kinetic energy of a typical rigid disk with mass center coincident with the elastic rotor centerline is given by,

\[
T_{d} = \frac{1}{2} \begin{bmatrix} \dot{\phi} \end{bmatrix}^T \begin{bmatrix} m_d & 0 \\
0 & m_d \end{bmatrix} \begin{bmatrix} \ddot{\phi} \\
\dot{\phi} \end{bmatrix} + \frac{1}{2} \begin{bmatrix} I_d \\
I_d \end{bmatrix}^T \begin{bmatrix} 0 & I_d \\
I_d & 0 \end{bmatrix} \begin{bmatrix} \dot{\phi} \\
\dot{\phi} \end{bmatrix} - \dot{\phi}\dot{\bar{\Gamma}}B_p
\]

The Lagrangian equation of motion for the rigid disk with constant spin speed, \( \dot{\phi} = \Omega \), can be written as,

\[
\begin{bmatrix} M_T^d \\ M_R^d \end{bmatrix}\{\ddot{q}^d\} - \Omega^T\begin{bmatrix} G^d \end{bmatrix}\{\dot{q}^d\} = \begin{bmatrix} F^d \end{bmatrix}
\] (2.12)

The matrices are given in the Appendix.

2.2.3 Bearings/Seals

The bearings/seals are assumed to follow the relation,
\[
\begin{bmatrix}
C^b
\end{bmatrix}
\begin{bmatrix}
q^b
\end{bmatrix}
+ \begin{bmatrix}
K^b
\end{bmatrix}
\begin{bmatrix}
q^b
\end{bmatrix}
= \begin{bmatrix}
F^b
\end{bmatrix}
\tag{2.13}
\]

where,
\[
\begin{bmatrix}
q^b
\end{bmatrix}
= \begin{bmatrix}
v
w
\end{bmatrix};
\begin{bmatrix}
K^b
\end{bmatrix}
= \begin{bmatrix}
k_{yy} & k_{yz}
k_{zy} & k_{zz}
\end{bmatrix};
\begin{bmatrix}
C^b
\end{bmatrix}
= \begin{bmatrix}
c_{yy} & c_{yz}
c_{zy} & c_{zz}
\end{bmatrix}
\]

Combining the equations for the finite rotor element (2.11), the rigid disks (2.12) and the bearings (2.13), the equation of motion for the rotor-bearing system can be assembled into the form,

\[
\begin{bmatrix}
M
\end{bmatrix}
\begin{bmatrix}
\ddot{q}
\end{bmatrix}
+ \begin{bmatrix}
C
\end{bmatrix}
\begin{bmatrix}
\dot{q}
\end{bmatrix}
+ \begin{bmatrix}
K
\end{bmatrix}
\begin{bmatrix}
q
\end{bmatrix}
= \begin{bmatrix}
F
\end{bmatrix}
\tag{2.14}
\]

where,
\[
\begin{bmatrix}
M
\end{bmatrix}
= \text{System mass matrix}
\]
\[
\begin{bmatrix}
C
\end{bmatrix}
= \text{System damping matrix}
\]
\[
\begin{bmatrix}
K
\end{bmatrix}
= \text{System stiffness matrix}
\]
\[
\begin{bmatrix}
F
\end{bmatrix}
= \text{System force vector.}
\]
2.3 Stability Analysis

2.3.1 Method of Solution

The above equation (2.14) represents the system equation of motion. This equation can be used for performing the stability analysis and the unbalance response of the rotor-bearing system. For the stability analysis, the second order system equation of motion is reduced to first order equations as follows,

Let \( \{r\} = \begin{bmatrix} \dot{q} \\ q \end{bmatrix} \) and \( \{S\} = \begin{bmatrix} 0 \\ F \end{bmatrix} \)

Hence,

\[
\begin{bmatrix} 0 & M \\ M & C \end{bmatrix} \{\dot{r}\} + \begin{bmatrix} -M & 0 \\ 0 & K \end{bmatrix} \{r\} = \{S\}
\]

Assuming the solution of the form, \( \{r\} = \{r_0\} e^{\alpha t} \), the eigenvalue problem is,

\[
\begin{bmatrix} 0 & I \\ -K^{-1}M & -K^{-1}C \end{bmatrix} \{r_0\} = \frac{1}{\alpha} \{r_0\}
\]

For solving the above Eigenvalue problem (given by equation (2.15)), the EISPACK routines BALANC, ELMHES, ELTRAN and HQR2 (74) are used. The output is in the form of the damped critical speeds and logarithmic decrement. The algorithm used for the eigenvalue problem is the HQR algorithm.

The flow of the stability program is as shown in Fig. 2.22. The input to the program is in the form of physical dimensions of the rotor, the bearing locations and the corresponding stiffness and damping characteristics (a total of 8), external mass and
inertia properties, sensor locations (in the case of magnetic bearings), the pedestal properties, the squeeze film dampers (if any) and speed dependent bearing properties. The program then evaluates the element mass, damping and stiffness matrices in turn assembling these into the respective global matrices. Once the global matrices are obtained, the program adds the bearing properties at the appropriate locations (accounting for the sensor positions in the case of rotors supported on active magnetic bearing).
In case a pedestal is present, the pedestal is modeled with additional degrees of freedom. In the case of stability analysis by the finite element method, the pedestal
properties cannot be reduced to an equivalent set of properties in series with the bearing. Hence the pedestal is modeled with additional degrees of freedom. Once the bearing, pedestal and squeeze film damper (if any) properties are accounted for in the respective global matrices, the equation of motion for the total system is complete. This is reduced to a first order form as explained above. The resulting eigenvalue problem (equation (2.15)) is solved using the EISPACK (74) routines. The output given by HQR2 is in the form of eigenvalues and eigenvectors. In general, the eigenvalues are complex and the real part is indicative of the stability of the system and the imaginary part gives the damped natural frequency. The mode shapes are calculated from the eigenvectors and are expressed in the form of an ellipse at the sections along the length of the rotor. The eigenvalues are expressed in the form of logarithmic decrement and damped critical speeds.

2.3.2 Damped Natural Frequencies

In the above analysis the inverse eigenvalue problem was solved. This evaluates the lowest frequencies first and hence any error propagation will affect only the higher modes. Typically, the lower frequencies are the most important modes of any rotor system. In the general case of a damped system, the eigenvalues are complex and appear in complex conjugate pairs. The eigenvalues have the general form, \( \lambda = a \pm i \omega_d \), where \( a = \) growth factor and \( \omega_d = \) the damped natural frequency (rad/s).

The rotor-system is said to be stable if the growth factor \( 'a' \) is negative. A positive growth factor indicates an exponentially increasingly unstable system. Another common
parameter used in turbomachinery industries to report the degree of stability is the logarithmic decrement. For a given eigenvalue, using the simple single mode theory, the logarithmic decrement is defined as, $\delta = -\frac{2\pi a}{\omega_d}$. In this case, the system is said to be stable if the logarithmic decrement is positive.

2.3.3 Damped Modes

The mode shapes of the rotor at the critical speeds are as important as the eigenvalues in the analysis of a rotor-bearing system. The eigenvectors give the respective mode shape of the damped critical speed. In general, the complex eigenvalue has a complex eigenvector associated with it. This results in a 3-dimensional mode shape of the rotor, i.e., the rotor rotates about its undeformed centerline in the resulting shape. The eigenvectors consist of complex coordinates in the two lateral directions, $Y_r$, $Y_i$, $Z_r$, and $Z_i$. These complex coordinates can be expressed in the form of an ellipse, with phase angles. The physical meaning of the ellipse is that it is the locus of the corresponding node as the rotor rotates about its center. The resulting motions in the $y$ and $z$ directions (lateral directions) are given by (32,82),

\[
\begin{align*}
y & = \text{Re}(Y) = \text{Re}\{e^{(a+i\omega_d)t} (Y_r + iY_i)\} \\
y & = \text{Re}\{e^{at} (Y_r \cos \omega dt - Y_i \sin \omega dt + iY_r \sin \omega dt + iY_i \cos \omega dt)\} \\
y & = e^{at} (Y_r \cos \omega dt - Y_i \sin \omega dt) \\
\text{Similarly,} \\
z & = e^{at} (Z_r \cos \omega dt - Z_i \sin \omega dt)
\end{align*}
\]

The elliptical parameters can be derived as follows (69).
\[ \text{var1} = Y_r^2 + Y_i^2 + Z_r^2 + Z_i^2 \]
\[ \text{var2} = Y_r^2 + Y_i^2 - Z_r^2 - Z_i^2 \]
\[ \text{var3} = Y_r^2 - Y_i^2 + Z_r^2 - Z_i^2 \]
\[ \text{var4} = Y_r Z_r + Y_i Z_i \]
\[ \text{var5} = Y_r Y_i + Z_r Z_i \]

\[ \text{semi\_major\_axis} = \sqrt{0.5[\text{var1} + \sqrt{(\text{var2}^2 + 4*\text{var4}^2)}]} \quad (2.16) \]
\[ \text{semi\_minor\_axis} = \sqrt{0.5[\text{var1} - \sqrt{(\text{var2}^2 + 4*\text{var4}^2)}]} \quad (2.17) \]
\[ \alpha = 0.5 \text{ arctan}(-2 \text{ var5} / \text{var3}) \]
\[ \beta = 0.5 \text{ arctan}(2 \text{ var4} / \text{var2}) \]

where, \( \alpha \) = Phase angle of the radius vector measured from the major axis, and \( \beta \) = angle of the positive y-axis to the major axis.

The whirl direction can be obtained from the following equations (82),

Forward whirl: if \[ \omega_d (Y_r Z_i - Y_i Z_r) \] > 0 \quad (2.18)
Backward whirl: if \[ \omega_d (Y_r Z_i - Y_i Z_r) \] < 0 \quad (2.19)

In general, rotor systems may experience a damped mode which is totally forward or totally backward, or a combination of both forward and backward whirl.

### 2.4 Unbalance Response Calculations

The unbalance force equation for the rotor system can be written as,

\[ \{ F_s^s \} = \left( \{ F_c^s \} \cos \Omega t + \{ F_s^s \} \sin \Omega t \right) \quad (2.20) \]
The \( F_y \) and \( F_z \) are the unbalance forces at the appropriate locations (nodes) on the rotor.

In general, the unbalance force in the \( y \) and \( z \) directions at any location (node) are given by,

\[
\begin{align*}
F_y &= U_y \cos \Omega t - U_z \sin \Omega t \\
F_z &= U_z \cos \Omega t + U_y \sin \Omega t
\end{align*}
\]  

(2.21)

where,

\( U_y = \) Unbalance force in the \( y \)-direction, and \( U_z = \) Unbalance force in the \( z \)-direction.

If an unbalance mass of \( m \) is placed at a radius \( e \) and at an angle \( \theta \) with respect to the fixed \( y \) axis (key-phasor) as shown in Fig. 4, then \( U_y = m\cdot e \cdot \Omega^2 \cos \theta \) and \( U_z = m\cdot e \cdot \Omega^2 \sin \theta \)

where \( \Omega \) is the spin (running) speed of the rotor.

\[ \begin{array}{c}
\Omega \\
y \\
\end{array} \]

\[ \begin{array}{c}
z \\
\end{array} \]

\[ \begin{array}{c}
\theta \\
e \\
m \\
\end{array} \]

\[ \begin{array}{c}
\end{array} \]

**Figure 2.3** Unbalance mass 'm' at a radius 'e''

Thus the steady state solution of the same form can be written as,

\[
\{ q^s \} = \left( \{ q_c^s \} \cos \Omega t + \{ q_s^s \} \sin \Omega t \right)
\]  

(2.22)
Substituting this in the system equation of motion, equation (2.14), the response equation can be written as,

\[
\begin{bmatrix}
(K_s^s - \Omega^2 M^s) & \Omega C_s^s \\
\Omega C_s^s & (K_s^s - \Omega^2 M^s)
\end{bmatrix}
\begin{bmatrix}
q_c^s \\
q_s^s
\end{bmatrix} =
\begin{bmatrix}
F_c^s \\
F_s^s
\end{bmatrix}
\]

(2.23)

The above equation is solved to obtain the unbalance response of the rotor-bearing system, using LU decomposition. The solution of the above system of equations results in the components of the displacements in the two directions given by \(Y_c\), \(Y_s\), \(Z_c\) and \(Z_s\). This results in a 3-dimensional mode shape of the rotor, i.e., the rotor rotates about its undeformed centerline in the resulting shape. These components can be expressed in the form of an ellipse, with the phase angles, similar to the eigenvectors, as discussed above.

The elliptical parameters can be obtained from the above equations by substituting \(Y_c\), \(Y_s\), \(Z_c\) and \(Z_s\) for \(Y_r\), \(Y_t\), \(Z_r\) and \(Z_t\), respectively. The resulting displacements in the y and z directions are given by,

\[
y = Y_c \cos \Omega t + Y_s \sin \Omega t
\]
\[
z = Z_c \cos \Omega t + Z_s \sin \Omega t
\]

(2.24)

The flow of the response program is as shown in Fig. 2.4. The input to the program consists of the dimensions of the rotor, the bearing locations and the associated stiffness and damping (a total of 8 characteristics), the external mass and inertia properties, sensor locations (in the case of magnetic bearings), the pedestal properties, the squeeze film dampers, the pedestal properties, and the unbalance mass and locations.

2.0 Development of the System Model
With these as input, the program evaluates the element mass, damping and stiffness matrices, in turn assembling these into the respective global matrices. This is similar to the stability program. The bearing and pedestal properties, which are in series, are reduced to a set of equivalent properties, as discussed in section 2.5, and these are added at the appropriate locations in the global matrices. The unbalance force vector is constructed and assuming synchronous response, the matrix for the response calculation is built. The responses at the nodes are solved for and the output is given in the form of elliptic parameters -- major axis and minor axis amplitudes with the phase angles. The post-processing consists of calculating the relative displacements of the bearing, and squeeze film dampers and pedestal (if the pedestals and dampers are present in the model), and then calculating the forces transmitted to each of the above components.
Figure 2.4 Flow Diagram of the Response Program
2.5 Squeeze Film Damper Model and Pedestal Flexibility

The squeeze film dampers, used quite extensively in aircraft engines, can be modeled by the program. Since the stiffness and damping of the squeeze film dampers are nonlinear functions of the eccentricity ratio, an iterative procedure is necessary to arrive at the equilibrium position for the given running speed. The squeeze film damper can be modeled using the long bearing or the short bearing assumption, with or without cavitation. An iterative procedure based on the eccentricity ratio is carried out. This is done till the specified tolerance is attained, which then gives the equilibrium position of the rotor. The program can evaluate the elliptic response of the damper and the output is in the form of the eccentricity ratio and the forces transmitted to the base.

The squeeze film damper consists essentially of an oil film in the annular space between the outer race of a rolling element bearing (prevented from rotating) and the support. Though the concept found its initial application in rotors supported on rolling element bearings, it has been widely used in rotors supported on journal bearings too. The journal ring is prevented from rotating by using anti-rotation devices such as pins. In theory, the squeeze film damper may be considered as a special case of the conventional journal bearing, with zero rotation of the journal. The well-known Reynolds equation, with appropriate boundary conditions, can best be used for the determination of the pressure distribution in the damper. The fluid-film forces can then be computed by integration of the fluid-film pressure distribution. A major problem which arises in the evaluation of the film forces is the choice of film cavitation model. Most of the linear
analysis work considers either a full-film \((2\pi)\) or half-film \((\pi)\) model. In the latter model, only the positive pressure region contributes to the total fluid-film forces. In most of the practical applications the film is close to a half-film model. Another model which is used for linear analysis is a linear combination of both the full and half-films.

Another assumption made for evaluating the fluid-film forces is based on the \(L/D\) ratio of the damper, where \(L\) is the length of the damper and \(D\) is the diameter of the journal. The classifications based on the \(L/D\) ratio are the long bearing assumption and the short bearing assumption. In most practical applications the length of the damper used is less than the diameter of the journal, and hence the short bearing solutions are widely used. The configuration of a squeeze film damper is as shown in Fig. 2.5.

![Figure 2.5 Squeeze Film Damper configuration](image)

2.0 Development of the System Model
The Reynolds equation for the short, plain journal bearing is given in rotating and
fixed coordinates by,

Fixed Coordinates:

\[
\frac{\partial}{\partial z} \left[ \frac{h^3}{6\mu} \frac{\partial P}{\partial z} \right] = \omega_j \frac{\partial h}{\partial \theta} + 2 \frac{\partial h}{\partial t} \quad (2.25)
\]

Rotating Coordinates:

\[
\frac{\partial}{\partial z} \left[ \frac{h^3}{6\mu} \frac{\partial \tilde{P}}{\partial z} \right] = (\omega_j - 2\phi) \frac{\partial h}{\partial \theta'} + 2 \frac{\partial h}{\partial t} \quad (2.26)
\]

where, \(\omega_j\) = journal speed. For a squeeze film damper, the spin \(\omega_j = 0.0\). Assuming steady
state synchronous precession of the journal about the bearing center and no axial
misalignments, the above equation (2.26) can be integrated in closed form to give the
damper forces, which in turn yield the damper stiffness and damping properties (40). The
closed form stiffness and damping equations used in the above finite element program are
given in the following Table 2.1. The effect of the supply pressure of the oil to the damper
has also been added.
Table 2.1 Expressions for Stiffness and Damping properties of the Squeeze Film Damper

<table>
<thead>
<tr>
<th>Film extent</th>
<th>Property</th>
<th>Long bearing assumption</th>
<th>Short bearing assumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>K</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Full</td>
<td>C</td>
<td>$\frac{R^3 \mu}{C_R} \frac{24\pi}{(2+\epsilon^2)\sqrt{1-\epsilon^2}}$</td>
<td>$\frac{R^3 \mu}{C_R} \frac{\pi}{3\sqrt[3]{(1-\epsilon^2)}}$</td>
</tr>
<tr>
<td>Cavitated</td>
<td>K</td>
<td>$\frac{R^3 \mu \omega}{C_R} \frac{24\epsilon}{(2+\epsilon^2)(1-\epsilon^2)}$</td>
<td>$\frac{R^3 \mu \omega}{C_R} \frac{2\epsilon}{(1-\epsilon^2)^2}$</td>
</tr>
<tr>
<td>Cavitated</td>
<td>C</td>
<td>$\frac{R^3 \mu}{C_R} \frac{12\pi}{(2+\epsilon^2)\sqrt{1-\epsilon^2}} + 2RLP_s$</td>
<td>$\frac{R^3 \mu}{C_R} \frac{\pi/2}{3\sqrt[3]{(1-\epsilon^2)}} + 2RLP_s$</td>
</tr>
</tbody>
</table>

The bearing with/without the squeeze film damper can be supported on flexible pedestals. In that case the pedestal stiffness, damping and mass properties have to be appropriately modeled and included in the equation of motion of the system. In general, a rotor model can consist of a squeeze film damper behind the bearing, and a pedestal behind the damper. The configuration of such a system is shown in the Fig. 2.6. The Bearing is designated with the subscript ‘b’, the Damper with the subscript ‘d’ and the Pedestal with the subscript ‘p’. The damper and pedestal are modeled as point masses and inertias, which are included in the system model.
2.5.1 Calculation of Displacements of Bearings/Dampers and Pedestals

Figure 2.6 Schematic diagram of the bearing, damper and pedestal.

As shown in the schematic diagram, Fig. 2.6, the rotor, in general, can be supported on bearings, squeeze film damper and flexible pedestals. In general, it is assumed that four stiffness and four damping (viz., two direct and two cross-coupled) characteristics are assumed to be present at each stage (i.e., the bearing, the damper and the pedestal). The equations of motion for the three-level system can be written as,

\[
K_{yy}(y_1 - y_2) + K_{yz}(z_1 - z_2) + C_{yz}(\dot{z}_1 - \dot{z}_2) + C_{yy}(\dot{y}_1 - \dot{y}_2) = F_{by} \\
K_{zz}(z_1 - z_2) + K_{zy}(y_1 - y_2) + C_{zy}(\dot{y}_1 - \dot{y}_2) + C_{yy}(\dot{z}_1 - \dot{z}_2) = F_{bz}
\]  

(2.27)  

(2.28)
\[ M_{2y} \ddot{y}_2 + (K_{yy1} + K_{yy2}) y_2 - K_{yy1} y_1 + (K_{yz1} + K_{yz2}) z_2 \\
- K_{yz1} z_1 - K_{yy2} y_3 - K_{yz2} z_3 - C_{yy1} \dot{y}_1 + (C_{yy1} + C_{yy2}) \dot{y}_2 \\
- C_{yy2} \dot{y}_3 - C_{yz1} \dot{z}_1 + (C_{yz1} + C_{yz2}) \dot{z}_2 - C_{yz2} \dot{z}_3 = 0 \] (2.29)

\[ M_{2z} \ddot{z}_2 + (K_{zz1} + K_{zz2}) z_2 - K_{zz1} z_1 + (K_{zy1} + K_{zy2}) y_2 \\
- K_{zy2} y_3 - K_{zz2} z_3 - K_{zy1} \dot{y}_1 - C_{zy1} \dot{y}_1 + (C_{zy1} + C_{zy2}) \dot{y}_2 \\
- C_{zy2} \dot{y}_3 - C_{zz1} \dot{z}_1 + (C_{zz1} + C_{zz2}) \dot{z}_2 - C_{zz2} \dot{z}_3 = 0 \] (2.30)

\[ M_{3y} \ddot{y}_3 - K_{yy2} y_2 + (K_{yy2} + K_{yy3}) y_3 - K_{yz2} z_2 + (K_{yz2} + K_{yz3}) z_3 \\
- C_{yy2} \dot{y}_2 + (C_{yy2} + C_{yy3}) \dot{y}_3 - C_{yz2} \dot{z}_2 + (C_{yz2} + C_{yz3}) \dot{z}_2 = 0 \] (2.31)

\[ M_{3z} \ddot{z}_3 - K_{zy2} y_2 + (K_{zy2} + K_{zy3}) y_3 - K_{zz2} z_2 + (K_{zz2} + K_{zz3}) z_3 \\
- C_{zy2} \dot{y}_2 + (C_{zy2} + C_{zy3}) \dot{y}_3 - C_{zz2} \dot{z}_2 + (C_{zz2} + C_{zz3}) \dot{z}_3 = 0 \] (2.32)

The subscripts 1,2,3 are for the bearing, the damper and the pedestal respectively. The above equations are solved by assuming the synchronous response of the form, \( y_i = Y_{ic} \cos \Omega t + Y_{is} \sin \Omega t \) and \( z_i = Z_{ic} \cos \Omega t + Z_{is} \sin \Omega t \) for \( i = 1,2,3 \). The expressions for \( y_i \) and \( z_i \) and its derivatives are substituted in the equations (2.27) through (2.32). The Cosine and Sine terms are collected and arranged in matrix form.

\[
\begin{bmatrix}
A & B & C \\
D & E & F
\end{bmatrix}_{(8 \times 12)}
\begin{bmatrix}
I \\
J
\end{bmatrix}_{(12 \times 1)} = \{0\}
\]
where,

\[
[A], [C] \text{ and } [E] \text{ are of the form,}
\]

\[
\begin{bmatrix}
-\Omega^2 C_{yzi} & -K_{yzi} & -\Omega C_{yzi} \\
K_{yzi} & -\Omega^2 C_{yzi} & -K_{yzi} \\
-\Omega C_{yzi} & -K_{yzi} & -\Omega C_{yzi} \\
\Omega C_{yzi} & -K_{yzi} & -\Omega C_{yzi}
\end{bmatrix}
\]

\(i = 1\) for \([A]\) and \(i = 2\) for \([C],[E]\)

and,

\[
[B] = \begin{bmatrix}
{[B]}_1 & {[B]}_2 & {[B]}_3 & {[B]}_4
\end{bmatrix}
\]

where,

\[
{[B]}_1 = \begin{bmatrix}
(K_{yzi} + K_{yzi} - \Omega^2 M_{2y}) \\
-\Omega(C_{yzi} + C_{yzi}) \\
(K_{yzi} + K_{yzi}) \\
-\Omega(C_{yzi} + C_{yzi})
\end{bmatrix}
\]

\[
{[B]}_2 = \begin{bmatrix}
\Omega(C_{yzi} + C_{yzi}) \\
(K_{yzi} + K_{yzi} - \Omega^2 M_{2y}) \\
\Omega(C_{yzi} + C_{yzi}) \\
(K_{yzi} + K_{yzi})
\end{bmatrix}
\]

\[
{[B]}_3 = \begin{bmatrix}
(K_{yzi} + K_{yzi}) \\
-\Omega(C_{yzi} + C_{yzi}) \\
(K_{yzi} + K_{yzi} - \Omega^2 M_{2y}) \\
-\Omega(C_{yzi} + C_{yzi})
\end{bmatrix}
\]

\[
{[B]}_4 = \begin{bmatrix}
\Omega(C_{yzi} + C_{yzi}) \\
(K_{yzi} + K_{yzi}) \\
\Omega(C_{yzi} + C_{yzi}) \\
(K_{yzi} + K_{yzi} - \Omega^2 M_{2y})
\end{bmatrix}
\]

and,

\[
[F] = \begin{bmatrix}
{[F]}_1 & {[F]}_2 & {[F]}_3 & {[F]}_4
\end{bmatrix}
\]

where,

\[
{[F]}_1 = \begin{bmatrix}
(K_{yzi} + K_{yzi} - \Omega^2 M_{2y}) \\
-\Omega(C_{yzi} + C_{yzi}) \\
(K_{yzi} + K_{yzi}) \\
-\Omega(C_{yzi} + C_{yzi})
\end{bmatrix}
\]

\[
{[F]}_2 = \begin{bmatrix}
\Omega(C_{yzi} + C_{yzi}) \\
(K_{yzi} + K_{yzi} - \Omega^2 M_{2y}) \\
\Omega(C_{yzi} + C_{yzi}) \\
(K_{yzi} + K_{yzi})
\end{bmatrix}
\]

\[
{[F]}_3 = \begin{bmatrix}
(K_{yzi} + K_{yzi}) \\
-\Omega(C_{yzi} + C_{yzi}) \\
(K_{yzi} + K_{yzi} - \Omega^2 M_{2y}) \\
-\Omega(C_{yzi} + C_{yzi})
\end{bmatrix}
\]

\[
{[F]}_4 = \begin{bmatrix}
\Omega(C_{yzi} + C_{yzi}) \\
(K_{yzi} + K_{yzi}) \\
\Omega(C_{yzi} + C_{yzi}) \\
(K_{yzi} + K_{yzi} - \Omega^2 M_{2y})
\end{bmatrix}
\]

2.0 Development of the System Model
and, \[ D = \begin{bmatrix} 0 \end{bmatrix} \]

\[ i \{ Y_{dc} \ Y_{ds} \ Z_{dc} \ Z_{ds} \}^T \]

\[ J \{ Y_{sc} \ Y_{ss} \ Z_{sc} \ Z_{ss} \}^T \]

\[ K \{ Y_{sc} \ Y_{ss} \ Z_{sc} \ Z_{ss} \}^T \]

The above matrix equation (2.33) is again used to obtain the motion of the damper and pedestal, once the motion of the rotor is known. The displacements, \( y_2, z_2 \) (the damper motion) and \( y_3, z_3 \) (the pedestal motion) are expressed in terms of \( y_1 \) and \( z_1 \) (the rotor motion) which are calculated by the program. This gives the displacements of the damper and pedestal.
2.6 Multi-level Rotor Systems

In aircraft engines, typically, multi-level rotors are designed with one rotor rotating inside another hollow rotor. The rotors can be supported by inter-shaft rolling element bearings in which both the inner and outer races rotate at different speeds or by inter-shaft squeeze film dampers. The analytical procedure must be capable of modeling the two rotor systems and to account for the inter-shaft stiffness and damping properties. A procedure is developed for performing stability and response analysis of multi-level systems. Applications can also be found in the areas of pump-casing systems which are used typically in submerged applications. The procedure also takes care of the node-numbering scheme to minimize the band-width. Instead of assembling the element equations for one system and then assembling the second system in a sequential numbering scheme, the current procedure alternates the node numbers so that the assembly produces a minimal band-width matrix. This application of multi-level systems allows the pedestal to be modeled as a second level and also makes it possible to evaluate the modes of the pedestal structure. Similarly, bearings and seals can be modeled as different levels and can be assumed to be floating or fixed to the pedestals.

The concept of modeling multi-level systems used in the current program is as described below. The general practice to model a multi-level system is to number all the nodes sequentially from left to right on one level and then repeat the same for the other levels. This method of numbering the nodes results in a matrix where the elements are present far away from the diagonal at the places where the rotors are interconnected, as
shown in Fig. 2.7. This may result in mathematical problems, especially for large rotor systems. In the current Finite Element program, the algorithm is written such that the bandwidth is minimized while assembling the matrices and the nodes are appropriately numbered. The schematic numbering and assembling process is shown in the Fig. 2.8.

2.6.1 Multi-level Modeling of Drum Rotor Supported on AMB for Stability Analysis

The rotor model of the drum rotor supported on AMB is shown in Fig. 2.9. This rotor was used to model the system as a multi-level rotor-bearing system. The model includes the coupling between the rotor and the driver motor as shown in the above Fig. 2.9. The rotor was assumed to be the first level and the landing sleeves as the second level. Though the length of the landing sleeve on the rotor is relatively small as compared to the length of the rotor, and the equivalent single level rotor model gives quite accurate results, this multi-level model was used as a case study to prove the multi-level capability of the finite element program. The details of the inboard and outboard ends of the rotor and the landing sleeves are shown in Fig. 2.10.
Common practice to handle multi-level systems:

Assembly of the element matrices for the above multi-level rotor system:

Figure 2.7 Assembled Stiffness (Mass/ Damping) matrix for the common approach
Algorithm of the current finite element program to handle multi-level systems:

![Diagram of multi-level rotor system]

- a: Rotor #1
- b: Rotor #2
- Inter-Connectivity
- Bearings to Ground

Assembly of the element matrices for the above multi-level rotor system:

![Assembled stiffness (mass/damping) matrix]

*Figure 2.8 Assembled Stiffness (Mass/Damping) matrix for the modified approach*

2.0 Development of the System Model
Input file name: t21rig.inp

Number of nodes/stations: 66
Number of bearings: 4
Total length of the rotor: 2.8805 m

Figure 2.9 Multi-Level Rotor Model of the AMB Test Rig
Figure 2.10 Details of the inboard and outboard end of the rotor

All dimensions in mm.
It be seen that the landing sleeve is in contact (interference fit) with the rotor (shaft) over only a small region. The landing sleeves at the two ends were modeled as shaft elements (on the second level) and the stability analysis of this 2-level system was performed.

2.6.2 Modeling Floating Seals/Bearings

Seals that are mounted on rotors can be modeled using the multi-level Finite Element program as a second level, thereby providing the ability to evaluate the seal frequencies and the stability of the rotor-system. The seal can be modeled as floating or with different (constant) support properties. The schematic model of the seal is as shown in Fig. 2.11.

![Diagram of a rotor with seal/Pedestal highlighted.]

Figure 2.11 Model of the seal as the 'second-level'
The seal can be modeled as entirely floating by removing the stiffness to the ground completely. This method of analysis has been used to study the effect of axially grooved seals on the stability of a multi-stage compressor.
CHAPTER 3
MODELING OF ROTORS SUPPORTED ON
ACTIVE MAGNETIC BEARINGS

3.1 Introduction

The most common means of supporting turbomachinery is by conventional fluid film bearings. The fluid film bearings operate on the principle of hydrodynamic pressure generated by the fluid film trapped between the rotating shaft (journal) and the bearing. This hydrodynamic pressure supports the rotating shaft. The lubrication system forms a major bulk of the accessory used for the fluid film bearings. This gives rise to a untidy environment and also the maintenance of such lubricating systems poses additional problems. From the operational point of view, it would be ideal if the friction between the rotating shaft and the bearing is reduced as close to zero as possible. This gives rise to the idea of supporting the rotating shaft in air, in other words levitating the rotor. The suspension of the rotating shaft of machinery, in a magnetic field without any mechanical contact and without lubrication, is an old idea. Passive magnetic suspension was used as
early as 1842. This later developed to active magnetic suspension systems where the control system was closed looped.

The concept on which the magnetic bearing operates is magnetic levitation. The rotating shaft is supported by a set of electro-magnets (generally four in number). A schematic diagram of the magnetic bearing is shown in Fig. 3.1. The shaft (rotor) at the center is levitated by the four electro-magnets surrounding it. The electro-magnets are energized by currents supplied by the amplifier of the control system.

Figure 3.1. Schematic diagram of a Magnetic Bearing
Figure 3.2. The Active Magnetic Bearing with the Control System

The active magnetic bearing (AMB) consists of an electromagnet that is energized by the current supplied by the amplifier as shown in Fig. 3.2. The electronic system typically consists of the displacement sensor, the controller and the amplifier. The highly accurate sensors monitor the position of the rotating shaft continuously. The current supplied to the electromagnets are proportional to the displacement of the rotor, measured by the sensor. The displacement detected by the sensor is compared to the reference signal and the change in current is conditioned. This conditioned current drives the power amplifier which suitably amplifies the current. After amplification, currents are induced in the windings of the electromagnets of the stator and the magnetic forces produced serve to restore the rotor to the desired position so that equilibrium is achieved.
3.2 Sensor Noncollocation

In active magnetic bearings the sensors measure the relative displacement of the rotor with respect to the support, and this results in a feedback of a proportional current to the magnetic bearings. The bearing unit of a magnetic bearing consists of the bearing and sensor built into the unit. Mainly, due to manufacturing constraints, the centerline of the sensor does not coincide with the centerline of the magnetic bearing. Thus there is a definite axial distance between the bearing and sensor centerlines. Such a sensor is said to be 'non-collocated' with respect to the bearings. If the bearing and sensor positions are the same, then the sensor is said to be 'collocated'. The sensor non-collocation is schematically shown in Fig. 3.3.

![Diagram showing sensor positions]

**Figure 3.3 Schematic diagram showing the Sensor positions**
Also associated with the magnetic bearing terminology are the sensor positions with respect to the bearings. When the sensor is located towards the rotor center (or in other words, away from the ends of the rotor) with respect to the bearing centerline, then the sensor is said to be located inboard. When the sensor is located away from the rotor center (or in other words, towards the ends of the rotor) with respect to the bearing centerline, then the sensors are said to be located outboard.

The non-collocated positions of the sensor make it slightly cumbersome to implement in the conventional transfer matrix method. Suppose the bearing centerline is located at the $i^{th}$ node and the sensor is located at the $j^{th}$ node (say, inboard). Since the properties are carried from one station (or, node) to another through transfer matrices, the transfer matrix method requires the displacement to be known at the $i^{th}$ node while performing the calculations. But since this displacement is dependent on the displacement at the $j^{th}$ node, an assumption is made initially and this is iterated between the $i^{th}$ and the $j^{th}$ node, till the solution converges. However, in the finite element method it is relatively easy to take care of sensor noncollocation. With the bearing at the $i^{th}$ node and the sensor at the $j^{th}$ node, the bearing stiffness gets added to the $(i,j)^{th}$ element of the system stiffness matrix. A similar procedure is adopted to account for the bearing damping, in the system damping matrix. In case the sensor is collocated, then the bearing stiffness gets added to the $(i,i)^{th}$ element. This process of implementing the bearing stiffness for a sample problem is shown in Fig. 3.4.
3.3 Forces in the Magnetic Bearing

The resulting current in the electromagnet gives the necessary stiffness and damping properties to the magnetic bearing. For stable operation, the control system is tuned appropriately. To analyze the dynamics of the rotor, i.e., to obtain the critical speeds and the mode shapes, the equation of motion of the system has to be solved. To write the governing equations of the magnetic bearing, the forces developed in the magnetic bearing are written as follows. The linearized relationship between the sensor measurement and the magnetic bearing forces can be written as (76),

\[
F_y = -k_{py}y - k_{iy}i_y \quad \text{(3.1a)}
\]
\[
F_z = -k_{pz}z - k_{iz}i_z \quad \text{(3.1b)}
\]

where,
- \(y, z\) = the shaft displacements from equilibrium, in the two lateral directions,
- \(i_y, i_z\) = coil perturbation currents in the \(y\) and \(z\) directions,
- \(k_{py}, k_{pz}\) = open loop gain in the \(y\) and \(z\) directions,
- \(k_{iy}, k_{iz}\) = current gain in the \(y\) and \(z\) directions,

\[
k_p = \frac{-2\mu_0 AN^2 i_o^2}{h_s^3}; \quad k_i = \frac{-2\mu_0 AN^2 i_o}{h_s^2}.
\]
where,

\[ A = \text{pole face area}, \quad \mu_0 = \text{permeability of free space}, \]
\[ N = \text{number of coil turns}, \quad i_o = \text{steady state coil current}, \]
\[ h_s = \text{nominal clearance}. \]

The overall transfer function for a magnetic bearing configuration can be written as the product of the individual transfer functions of the displacement sensor, the controller (typically PID) and the power electronics used. This overall transfer function can be expressed as a ratio of polynomials.

\[ G_o = G_p \cdot G_c \cdot G_s \]
\[ G_o = \frac{\alpha_m s^m + \alpha_{m-1} s^{m-1} + \alpha_{m-2} s^{m-2} + \ldots \ldots \alpha_1 s + \alpha_0}{s^n + \beta_{n-1} s^{n-1} + \beta_{n-2} s^{n-2} + \ldots \ldots \beta_1 s + \beta_0} \] (3.2)

The above transfer function relates the coil perturbation current (in the force equation) to the shaft motion as seen by the sensor.

\[ i_y = G_y y \]
\[ i_z = G_z z \]

Substituting these equations into the force equations (3.1a and 3.1b),

\[ F_y = -k_{py} y - k_{iy} G_y y \] (3.3a)
\[ F_z = -k_{pz} z - k_{iz} G_z z \] (3.3b)

The above force equations can now be used in deriving the equations of motion for the active magnetic bearing system.
3.4 Modeling the Magnetic Bearing Forces in the System Equation of Motion

The functions $G_y$ and $G_z$ contain the frequency and are hence frequency dependent parameters. The characteristics of the conventional fluid film journal bearings, i.e., the stiffness and damping, are predominantly running speed dependent. However, in magnetic bearings, the stiffness and damping characteristics are predominantly dependent on the whirl frequency (i.e., the frequency of lateral vibration) rather than the running speed. Hence, this type of stability evaluation is known as nonsynchronous stability analysis. Since, in the stability analysis, the whirl frequency is not known apriori, an iterative procedure is required to obtain the “exact” damped critical speeds and stability parameters, for the given mode.

The stability of the rotors supported on magnetic bearings can be determined by three methods. The following are the three methods implemented into the finite element procedure while modeling rotors supported on AMB.

3.4.1 Method 1: Table of Bearing Characteristics Known Apriori

This method is applicable when the frequency dependent bearing characteristics are known. The bearing stiffness and damping are given over a range of frequencies as shown in Fig. 3.5.
\[ \text{Frequency (rpm)} \quad \text{Stiffness (N/m)} \quad \text{Damping (N-s/m)} \]

<table>
<thead>
<tr>
<th>Frequency (rpm)</th>
<th>Stiffness (N/m)</th>
<th>Damping (N-s/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>2.1e7</td>
<td>1.5e4</td>
</tr>
<tr>
<td>1500</td>
<td>2.4e7</td>
<td>1.55e4</td>
</tr>
<tr>
<td>2000</td>
<td>2.6e7</td>
<td>1.62e4</td>
</tr>
<tr>
<td>2500</td>
<td>2.8e7</td>
<td>1.8e4</td>
</tr>
</tbody>
</table>

**Figure 3.5 The Frequency Dependent Bearing Characteristics**

The stability analysis is performed for each mode, i.e., the iterative procedure described above is performed for each mode to obtain the "exact" frequency for the given mode. For the first stability run, the stiffness and damping characteristics corresponding to the running speed are selected. For the successive iterations, the newly obtained frequency (for the given mode) is used and the corresponding bearing characteristics are calculated. The Lagrangian 4-point interpolation scheme is used to curve fit the given bearing characteristics. The newly calculated stiffness and damping are used once again to perform the stability analysis. The convergence is checked at the end of each stability run, by comparing the previously obtained critical speed and the corresponding newly obtained critical speed.

\[
\text{\% Current Tolerance} = \left( \frac{\text{Newly obtained } N_{cr} - \text{Previously obtained } N_{cr}}{\text{Newly obtained } N_{cr}} \right) \times 100
\]
The current tolerance is checked against the tolerance to be obtained. If the current tolerance is less than the to-be-obtained-tolerance level, the calculations are stopped, else the iterations continue till the required tolerance level is reached. The following flow diagram shown in Fig. 3.6 shows the iterative procedure used in the nonsynchronous stability evaluation.

![Flow Diagram](image)

Figure 3.6 Flow Diagram of the Iterative Procedure for the Stability Evaluation

3.4.2 Method 2: The Overall Transfer Function is Known Apriori

The above Method 1 is useful only if the table of frequency dependent bearing characteristics is known beforehand. Usually the magnetic vendor supplies the table of
properties after the tuning of the magnetic bearings. This method is very straightforward and can be applied and used conveniently for the stability evaluation. Another method to model the magnetic bearings is to obtain the bearing characteristics directly from the transfer function.

The overall transfer function of the controller, the sensor and the power amplifier can be expressed as a ratio of polynomials, given by equation (3.4).

\[
G = G_p G_c G_s
\]

\[
G = \frac{\alpha_m s^m + \alpha_m \cdot 1s^{m-1} + \alpha_m \cdot 2s^{m-2} + \ldots \ldots + \alpha_1 s + \alpha_0}{s^n + \beta_{n-1} s^{n-1} + \beta_{n-2} s^{n-2} + \ldots \ldots + \beta_1 s + \beta_0}
\]  \hspace{1cm} (3.4)

The bearing stiffness and damping characteristics for a given frequency can be written from the complex transfer function as,

Stiffness, \( K = \text{Re}(G) \)

Damping, \( C = \text{Imag.}(G)/\omega \)

where,

'\(G' is the overall transfer function, and '\(\omega' is the whirl frequency.

The above method necessitates the knowledge of the transfer function beforehand. This is equivalent to the Method 1, but generates the magnetic bearing characteristics internally while performing the calculations. The iteration procedure described in section 3.3.1 is still used to calculate the "exact" damped critical speeds for the required mode.
3.4.3 Method 3: Modeling the Transfer Function explicitly into the Equation of Motion of the System for Stability Analysis

The above two methods do not model the control system's dynamics, i.e., they do not include the poles and zeros of the transfer function explicitly into the system equation of motion, though the effect of the zeros and poles are included in the model. However, it is possible that the control system may have poles that are unstable. This makes the rotor-bearing system unstable. Hence it is equally important to include the dynamics of the controller into the equation of motion of the system, and also to include the coupling of the control system and the rotor system.

The overall transfer function of the controller, the sensor and the power amplifier can be expressed as a ratio of polynomials, given by equation (3.5).

\[ G_0 = G_p G_c G_s \]

\[ G_0 = \frac{\alpha_m s^m + \alpha_{m-1} s^{m-1} + \alpha_{m-2} s^{m-2} + \ldots + \alpha_1 s + \alpha_0}{s^n + \beta_{n-1} s^{n-1} + \beta_{n-2} s^{n-2} + \ldots + \beta_1 s + \beta_0} \] (3.5)

where,

\( \alpha_m, \alpha_{m-1}, \alpha_{m-2}, \ldots, \alpha_1, \alpha_0 = \) coefficients of the numerator of the transfer function

\( \beta_{n-1}, \beta_{n-2}, \beta_{n-3}, \ldots, \beta_1, \beta_0 = \) coefficients of the denominator of the transfer function

\( s = \) frequency.
It can be assumed that the number of poles must be greater than the number of zeros, i.e., $n > m$.

This transfer function is a relation between the displacement of the rotor sensed by the sensor to the current fed back to the magnetic bearing. This can be written as,

$$\frac{i_y}{y_s} = \frac{\alpha_m s^m + \alpha_{m-1} s^{m-1} + \ldots + \alpha_1 s + \alpha_0}{s^n + \beta_{n-1} s^{n-1} + \ldots + \beta_1 s + \beta_0} \tag{3.6}$$

where,

$i_y =$ current fed back to the magnetic bearing;

$y_s =$ displacement of the rotor sensed by the sensor.

The magnetic bearing forces can be written as,

$F_y = -k_{pv} y - k_{iy} G_y y$

$F_z = -k_{pz} z - k_{iz} G_z z$

The original equation of motion of just the rotor system can be written as,

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{0\} \tag{3.7}$$

where $\{u\} =$ displacement vector of the rotor

The modified equation of motion can be written as,
\[
\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \ddot{u} \\ i \end{bmatrix} + \begin{bmatrix} C & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \dot{u} \\ i \end{bmatrix} + \begin{bmatrix} K & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} u \\ i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\] (3.8)

where, the 'i' and its derivatives are the new state variables introduced by the control system.

\[
[k_{22}] = \begin{bmatrix} 0 & \ldots & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ \vdots & \ddots & \ddots \\ \beta_0 & \beta_2 & \ldots & \beta_{n-2} \end{bmatrix}, \quad [k_{21}] = \begin{bmatrix} 0 & \ldots & 0 \\ \vdots & \ddots & \ddots \\ -\alpha_0 & -\alpha_2 & \ldots \end{bmatrix}
\]

\[
[k_{12}] = \begin{bmatrix} k_i & 0 & \ldots \\ \vdots & \ddots & \ddots \\ 0 & \ldots \end{bmatrix}, \quad [c_{12}] = [0], \quad [c_{21}] = \begin{bmatrix} 0 & \ldots & 0 \\ \vdots & \ddots & \ddots \\ -\alpha_1 & -\alpha_3 & \ldots \end{bmatrix}
\]

\[
[c_{22}] = \begin{bmatrix} 0 & \ldots & 0 \\ \vdots & \ddots & \ddots \\ \beta_1 & \beta_3 & \ldots & \beta_n \end{bmatrix}
\]

The equation (3.8) is the system equation of motion, which now includes the dynamics of the control system explicitly, and also the coupling of the control system to the rotor system is accounted for. The equation is solved for the stability of the rotor-bearing system.

3.0 Modeling of Rotors Supported on Active Magnetic Bearings 63
CHAPTER 4

INITIAL VERIFICATION

4.1 Convergence of Solutions

The finite element modeling procedure is generally checked for convergence to verify the procedure. The most common method of verifying the convergence of the solutions is to decrease the "mesh" size, in effect increasing the number of elements. In this case, the elements are 1-D beam elements. To verify the convergence, the l/d ratio of the sections is decreased and the problem is solved each time to make sure the solutions converge monotonically.

As the first step in this test, a Jeffcott rotor, viz., a uniform rotor with a mass at midspan and supported by two bearings at each end (77), was used as the case study. The rotor shown in Fig. 4.1 consists of a uniform diameter shaft of 0.0254 m (1.0 in.) diameter and 0.508 m (20.0 in.) in length, supported by two fluid film bearings at both the ends. The number of elements used to model the rotor was varied from 4 elements to 40 elements. The results of the stability analysis, viz., the growth factors and the damped
critical speeds for the first 8 modes are compared for the different element sizes in Table 4.1a and Table 4.1b.

\[ \text{Figure 4.1 Jeffcott Rotor Model Used for the Stability Analysis} \]

Data used for the above model:

\( K_b = 1.756 \times 10^6 \text{ N/m (10,000 lb/in.)} \)
\( C_b = 17.56 \text{ N-s/m (0.1 lb-s/in.)} \)
\( M = 2.5258 \text{ kg. (5.55676 lbs)} \)
\( I_t = 2.546 \times 10^{-3} \text{ kg-m}^2 (8.682 \text{ lb-in}^2) \)
\( I_p = 5.092 \times 10^{-3} \text{ kg-m}^2 (17.364 \text{ lb-in}^2) \)
Modulus of Elasticity = \( 2.0 \times 10^{11} \text{ N/m}^2 (28.9376 \times 10^6 \text{ lb/in}^2) \)
Poisson's Ratio = 0.25
Density = 7850.0 \text{ kg/m}^3 (0.283 \text{ lb/in}^3)
Running Speed of the rotor = 1000 rpm

4.0 Initial Verification
### Table 4.1a Comparison of Damped Critical Speeds for Different Number of Elements (With Shear Deformation and Rotatory Inertia)

<table>
<thead>
<tr>
<th>Mode #</th>
<th>4 Elements</th>
<th>8 Elements</th>
<th>10 Elements</th>
<th>20 Elements</th>
<th>40 Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 fw</td>
<td>5055.2</td>
<td>5055.2</td>
<td>5055.2</td>
<td>5055.1</td>
<td>5055.1</td>
</tr>
<tr>
<td>1 bw</td>
<td>5054.9</td>
<td>5054.8</td>
<td>5054.8</td>
<td>5054.8</td>
<td>5054.8</td>
</tr>
<tr>
<td>2 fw</td>
<td>19318.9</td>
<td>19303.9</td>
<td>19303.1</td>
<td>19302.2</td>
<td>19301.8</td>
</tr>
<tr>
<td>2 bw</td>
<td>19516.9</td>
<td>19501.3</td>
<td>19500.4</td>
<td>19499.4</td>
<td>19499.0</td>
</tr>
<tr>
<td>3 bw</td>
<td>31328.2</td>
<td>31268.9</td>
<td>31265.2</td>
<td>31261.3</td>
<td>31216.7</td>
</tr>
<tr>
<td>3 fw</td>
<td>31334.8</td>
<td>31275.7</td>
<td>31272.0</td>
<td>91268.1</td>
<td>31260.1</td>
</tr>
<tr>
<td>4 fw</td>
<td>58395.8</td>
<td>58225.7</td>
<td>58213.0</td>
<td>58198.7</td>
<td>58193.2</td>
</tr>
<tr>
<td>4 bw</td>
<td>59330.9</td>
<td>59151.5</td>
<td>59137.9</td>
<td>59122.8</td>
<td>59116.9</td>
</tr>
</tbody>
</table>

### Table 4.1b Comparison of Growth Factors for Different Number of Elements (With Shear Deformation and Rotatory Inertia)

<table>
<thead>
<tr>
<th>Mode #</th>
<th>4 Elements</th>
<th>8 Elements</th>
<th>10 Elements</th>
<th>20 Elements</th>
<th>40 Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 fw</td>
<td>-0.4866</td>
<td>-0.4866</td>
<td>-0.4866</td>
<td>-0.4866</td>
<td>-0.4866</td>
</tr>
<tr>
<td>1 bw</td>
<td>-0.4864</td>
<td>-0.4864</td>
<td>-0.4864</td>
<td>-0.4864</td>
<td>-0.4864</td>
</tr>
<tr>
<td>2 bw</td>
<td>-17.2427</td>
<td>-17.1881</td>
<td>-17.1849</td>
<td>-17.1816</td>
<td>-17.1805</td>
</tr>
<tr>
<td>4 fw</td>
<td>-26.1118</td>
<td>-25.6827</td>
<td>-25.6535</td>
<td>-25.6231</td>
<td>-25.6164</td>
</tr>
</tbody>
</table>

4.0 Initial Verification
4.2 Modeling the Rotor Using Different Mass Matrix Formulations

The two most common methods of formulating the element matrices (mass, stiffness and damping matrices) are the distributed property method and the lumped parameter method (78,79). The distributed property method involves integrating the property of a small elemental section over the length of the beam element. This results in a matrix that is not diagonal, but contains off diagonal terms. This method gives relatively more accurate results, but requires more computer storage space. For some simple problems/models, the distributed property matrices may not be required. The simpler method of lumped parameters can be used in such cases. This method involves lumping the properties of the elements at the nodes. In the case of a beam element with two nodes at each end of the element, the properties are divided between the two nodes. This results in diagonal element matrices, i.e., the mass, stiffness and damping matrices are diagonal. The diagonal matrices not only decrease the amount of computer storage but also cut down on the executing time of the program. However, care must be taken in selecting the type of formulation, as it very much depends on the type of problem being analyzed.

The following study was made to compare the results of three different types of mass matrix formulations. The three formulations being studied for the stability analysis are the distributed mass formulation, the lumped mass formulation with simple splitting (explanation follows) and the lumped mass formulation with revised splitting (explanation follows).
4.2.1 Distributed Mass Matrix Formulation

The shape functions used for the derivation of the distributed mass matrix are as given in the Appendix. Writing out the kinetic energy of the beam element as explained in section 2.2.1, given by equations (2.5), the mass matrix for the rotor element can be written as,

\[
\begin{bmatrix}
m_{11} & 0 & 0 & \cdots & 0 & 0 \\
0 & m_{22} & 0 & \cdots & 0 & 0 \\
0 & 0 & m_{33} & \cdots & 0 & 0 \\
m_{41} & 0 & 0 & m_{44} & \cdots & 0 \\
m_{51} & 0 & 0 & m_{55} & \cdots & 0 \\
m_{61} & m_{62} & m_{63} & 0 & \cdots & m_{66} \\
m_{71} & m_{72} & m_{73} & 0 & \cdots & m_{77} \\
m_{81} & m_{82} & m_{83} & 0 & \cdots & m_{88}
\end{bmatrix}
\]

4.2.2 Lumped Mass formulation : Simple splitting

In the lumped mass formulation, as the name indicates, the properties, i.e., the mass, the transverse moment of inertia and the polar moment of inertia are split between the two ends of the rotor element as shown in Fig. 4.2. This results in a diagonal mass matrix given by,

\[
\begin{bmatrix}
\frac{1}{2}(m_{i-1} + m_i) & 0 & 0 & \cdots & 0 \\
0 & \frac{1}{2}(m_{i-1} + m_i) & 0 & \cdots & 0 \\
0 & 0 & \frac{1}{2}(I_{r_{i-1}} + I_{r_i}) & \cdots & 0 \\
0 & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots & \frac{1}{2}(I_{r_{i-1}} + I_{r_i})
\end{bmatrix}
\]

4.0 Initial Verification 68
The simple splitting resulting in the above matrix is shown schematically in Fig. 4.2, in which a rotor element of diameter ‘D’ is modeled by lumped parameters.

\[
\begin{align*}
\frac{1}{2} (m_i + m_{i+1}) & \quad \frac{1}{2} (m_{i+1} + m_i) \\
\frac{1}{2} (I_{p_i} + I_{p_{i+1}}) & \quad \frac{1}{2} (I_{p_i+1} + I_{p_i}) \\
\frac{1}{2} (I_{T_i} + I_{T_{i+1}}) & \quad \frac{1}{2} (I_{T_{i+1}} + I_{T_i}) \\
\end{align*}
\]

Figure 4.2 Simple splitting of inertias for 2 sections of a rotor model

4.2.3 Lumped Parameter Formulation: Revised Splitting

The above method discussed in section 4.2.2 involves the simple splitting of inertias. However, when the rigid body \( I_T \) of the rotor is considered, the simple splitting does not maintain the proper transverse mass moment of inertia about the rotor system’s center of gravity due to the moving of the point masses to the ends of the respective sections (80). To keep the rigid body transverse mass moment of inertia the same, a value of \( \frac{1}{8} m_i l_i^2 \) has to be subtracted from the transverse moment of inertia after moving the calculated inertias to the ends of the section. This gives accuracy on the lower modes of the rotor-bearing system, which is of most concern in the design and operation of turbomachinery. Applying this method to the mass matrix formulation, the mass matrix for each node of an element is given by,
\[
\begin{bmatrix}
\frac{1}{2}(m_{i-1} + m_i) & 0 & 0 & 0 \\
0 & \frac{1}{2}(m_{i-1} + m_i) & 0 & 0 \\
0 & 0 & \frac{1}{2}(I_{T_{i-1}} + I_{T_i}) & 0 \\
0 & 0 & 0 & \frac{1}{2}(I_{T_{i-1}} + I_{T_i}) \\
\end{bmatrix}
\]

where,

\[I_{T_i}^l = I_{T_i} - \frac{1}{8} m_i l_i^2\]

The revised splitting resulting in the above matrix is shown schematically in Fig. 4.3, in which a rotor element of diameter 'D' is modeled by lumped parameters.

![Diagram](image)

**Figure 4.3 Inertia split for rotor section ‘i’ by the revised method**

The 8-stage industrial compressor shown in Fig. 4.4 was modeled using the above mass matrix formulations. The results of the stability analysis of the three cases are shown in Tables 4.2a and 4.2b. As can be observed from the Tables 4.2a and 4.2b, in general the lumped parameter formulation by the revised method gives relatively better results as compared to the lumped formulation by the simple splitting method for the 4.0 Initial Verification
Description: Stability Analysis of Multi-Stage Compressor supported on 4 bearings; Model used to compare different mass matrix formulations

Number of stations (nodes) : 35
Bearings at (from left): 8 15 21 33

Total Length of Rotor: 2.3791 m
(93.67 in.)

Maximum Diameter: 0.1042 m
(7.26 in.)

Press any key to continue

Figure 4.4 Model of the 8-Stage Industrial Compressor used for the Stability Analysis
lower modes of the rotor system, as indicated by the % deviation of the respective formulations.

**Table 4.2a Comparison of the Damped Critical Speeds for the 3 Mass Matrix Formulations for the 8-Stage Compressor**

<table>
<thead>
<tr>
<th>Mode</th>
<th>( N_{cr1} ) (rpm)</th>
<th>( N_{cr2} ) (rpm)</th>
<th>( N_{cr3} ) (rpm)</th>
<th>% difference1</th>
<th>% difference2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 bw</td>
<td>3073.0</td>
<td>3072.0</td>
<td>3073.0</td>
<td>-0.0325</td>
<td>0.0</td>
</tr>
<tr>
<td>1 fw</td>
<td>3294.0</td>
<td>3294.0</td>
<td>3298.0</td>
<td>0.0</td>
<td>0.1214</td>
</tr>
<tr>
<td>2 bw</td>
<td>5366.0</td>
<td>5337.0</td>
<td>5362.0</td>
<td>-0.5404</td>
<td>-0.0745</td>
</tr>
<tr>
<td>2 fw</td>
<td>6886.0</td>
<td>6840.0</td>
<td>6879.0</td>
<td>-0.6680</td>
<td>-0.1017</td>
</tr>
<tr>
<td>3 bw</td>
<td>9432.0</td>
<td>9368.0</td>
<td>9424.0</td>
<td>-0.6785</td>
<td>-0.0848</td>
</tr>
<tr>
<td>3 fw</td>
<td>12042.0</td>
<td>11943.0</td>
<td>11986.0</td>
<td>-0.8221</td>
<td>-0.4650</td>
</tr>
<tr>
<td>4 bw</td>
<td>17360.0</td>
<td>17249.0</td>
<td>17627.0</td>
<td>-0.6394</td>
<td>1.5380</td>
</tr>
<tr>
<td>4 fw</td>
<td>17837.0</td>
<td>17708.0</td>
<td>18073.0</td>
<td>-0.7232</td>
<td>1.3231</td>
</tr>
</tbody>
</table>

fw = forward whirl  
bw = backward whirl  
\( N_{cr1} \) = Damped critical speed using distributed mass parameters (finite element solution)  
\( N_{cr2} \) = Damped critical speed using lumped mass parameters (Simple splitting: transfer matrix solution)  
\( N_{cr3} \) = Damped critical speed using lumped mass parameters (Revised splitting: transfer matrix solution)  
% difference1 = 100 * (\( N_{cr2} \) - \( N_{cr1} \))/\( N_{cr1} \)  
% difference2 = 100 * (\( N_{cr3} \) - \( N_{cr1} \))/\( N_{cr1} \)
### Table 4.2b Comparison of the Growth Factors for the 3 Mass Matrix Formulations for the 8-Stage Compressor

<table>
<thead>
<tr>
<th>Mode</th>
<th>( N_{cr1} ) (rpm)</th>
<th>( N_{cr2} ) (rpm)</th>
<th>( N_{cr3} ) (rpm)</th>
<th>% difference1</th>
<th>% difference2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 bw</td>
<td>-60.7395</td>
<td>-60.7459</td>
<td>-60.9767</td>
<td>0.0105</td>
<td>0.3905</td>
</tr>
<tr>
<td>1 fw</td>
<td>-32.8171</td>
<td>-32.8216</td>
<td>-33.1192</td>
<td>0.0137</td>
<td>0.9206</td>
</tr>
<tr>
<td>2 bw</td>
<td>-313.5504</td>
<td>-308.0853</td>
<td>-312.4930</td>
<td>-1.7430</td>
<td>-0.3372</td>
</tr>
<tr>
<td>2 fw</td>
<td>-315.7110</td>
<td>-310.3231</td>
<td>-314.3780</td>
<td>-1.7066</td>
<td>-0.4222</td>
</tr>
<tr>
<td>3 bw</td>
<td>-542.2741</td>
<td>-534.1275</td>
<td>-539.9896</td>
<td>-1.5023</td>
<td>-0.4213</td>
</tr>
<tr>
<td>3 fw</td>
<td>-590.1705</td>
<td>-581.6239</td>
<td>-588.6553</td>
<td>-1.4482</td>
<td>-0.2567</td>
</tr>
<tr>
<td>4 bw</td>
<td>-315.4692</td>
<td>-306.9600</td>
<td>-315.6816</td>
<td>-2.6973</td>
<td>0.0673</td>
</tr>
<tr>
<td>4 fw</td>
<td>-246.7198</td>
<td>-239.7260</td>
<td>-247.7680</td>
<td>-2.8347</td>
<td>0.4249</td>
</tr>
</tbody>
</table>

fw = forward whirl

bw = backward whirl

\( N_{cr1} \) = Damped critical speed using distributed mass parameters (finite element solution)

\( N_{cr2} \) = Damped critical speed using lumped mass parameters (Simple splitting : transfer matrix solution)

\( N_{cr3} \) = Damped critical speed using lumped mass parameters (Revised splitting : transfer matrix solution)

\[
\text{% difference1} = 100 \times \frac{(N_{cr2} - N_{cr1})}{N_{cr1}}
\]

\[
\text{% difference2} = 100 \times \frac{(N_{cr3} - N_{cr1})}{N_{cr1}}
\]
4.3 Initial Verification of the Finite Element Program

With the development of any new program, the first step is to verify the results of the program by comparing it with the results obtained by previously well established programs that are being used. In this case, the initial verification of the newly developed finite element program was accomplished by comparing the results of the stability analysis for the rotor shown in Fig. 4.5 with the results obtained from the transfer matrix program.

4.3.1 Evaluation of Undamped Critical Speeds: Synchronous Stability Evaluation

The uniform rotor model shown in Fig. 4.5, and whose dimensions are given in Table 4.3 in terms of the length and diameter of the rotor sections, was selected for the undamped analysis. The transfer matrix undamped critical speed program is called CRTMB2 (81), which has been used in the Turbomachinery industry quite effectively for several years. The rotor is a uniform shaft of 0.0508 m (2.0 in.) in diameter and 0.9592 m (37.76 in.) in length, supported on bearings with an in-span length of 0.756 m (29.76 in.). A disk of mass 19.3977 kg. was assumed to be located at mid-span. The results of the comparison of the undamped critical speeds are given in Table 4.4. The results for the first three forward modes are compared and the results are quite encouraging with the difference typically less than 1%. The movement of the critical speeds, as influenced by the sensor position, is in total agreement with the previously observed trends reported in (62).
Description: Stability Analysis of Uniform Rotor supported on 2 bearings; Synchronous Undamped Critical Speed Comparison with CRTMB2

Number of stations (nodes) : 15
Bearings at (from left) : 3 13
Ext.Mass at (from left) : 8

Total Length of Rotor : 0.8552 m
(37.76 in.)

Maximum Diameter : 0.0509 m
(2.00 in.)

--- Outside dimension; 'B' - Bearings
--- Inside dimension; 'M' - Ext.Mass

Press any key to continue

Figure 4.5 Model of Uniform Rotor used for the Evaluation Undamped Critical Speeds
Table 4.3 Data for the uniform model used for the initial verification

<table>
<thead>
<tr>
<th>Section</th>
<th>Length (m)</th>
<th>Diameter (m)</th>
<th>Section</th>
<th>Length (m)</th>
<th>Diameter (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0508</td>
<td>0.0508</td>
<td>8</td>
<td>0.0782</td>
<td>0.0508</td>
</tr>
<tr>
<td>2</td>
<td>0.0508</td>
<td>0.0508</td>
<td>9</td>
<td>0.1016</td>
<td>0.0508</td>
</tr>
<tr>
<td>3</td>
<td>0.0508</td>
<td>0.0508</td>
<td>10</td>
<td>0.1016</td>
<td>0.0508</td>
</tr>
<tr>
<td>4</td>
<td>0.0508</td>
<td>0.0508</td>
<td>11</td>
<td>0.0408</td>
<td>0.0508</td>
</tr>
<tr>
<td>5</td>
<td>0.1016</td>
<td>0.0508</td>
<td>12</td>
<td>0.0508</td>
<td>0.0508</td>
</tr>
<tr>
<td>6</td>
<td>0.1016</td>
<td>0.0508</td>
<td>13</td>
<td>0.0508</td>
<td>0.0508</td>
</tr>
<tr>
<td>7</td>
<td>0.0782</td>
<td>0.0508</td>
<td>14</td>
<td>0.0508</td>
<td>0.0508</td>
</tr>
</tbody>
</table>

Table 4.4 Comparison of Results of FE program (current) and Transfer Matrix program (Synchronous Undamped Critical Speeds)

<table>
<thead>
<tr>
<th>Cases</th>
<th>Critical Speeds (rpm)</th>
<th>Transfer Matrix Solution</th>
<th>Finite Element Solution</th>
<th>% difference with respect to Transfer Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collocated</td>
<td>First</td>
<td>2657.3</td>
<td>2630.0</td>
<td>-1.027</td>
</tr>
<tr>
<td></td>
<td>Second</td>
<td>6455.5</td>
<td>6420.0</td>
<td>-0.550</td>
</tr>
<tr>
<td></td>
<td>Third</td>
<td>12330.5</td>
<td>12320.9</td>
<td>-0.085</td>
</tr>
<tr>
<td>Sensor located 4 in.</td>
<td>First</td>
<td>2510.2</td>
<td>2564.0</td>
<td>2.143</td>
</tr>
<tr>
<td>Outboard</td>
<td>Second</td>
<td>7269.7</td>
<td>7230.0</td>
<td>-0.546</td>
</tr>
<tr>
<td></td>
<td>Third</td>
<td>13076.7</td>
<td>13130.0</td>
<td>0.408</td>
</tr>
<tr>
<td>Sensor located 2 in.</td>
<td>First</td>
<td>2580.4</td>
<td>2615.0</td>
<td>1.341</td>
</tr>
<tr>
<td>Outboard</td>
<td>Second</td>
<td>6873.7</td>
<td>6820.0</td>
<td>-0.781</td>
</tr>
<tr>
<td></td>
<td>Third</td>
<td>12707.9</td>
<td>12715.0</td>
<td>0.056</td>
</tr>
<tr>
<td>Sensor located 2 in.</td>
<td>First</td>
<td>2742.5</td>
<td>2715.0</td>
<td>-1.003</td>
</tr>
<tr>
<td>Inboard</td>
<td>Second</td>
<td>6013.9</td>
<td>6000.0</td>
<td>-0.231</td>
</tr>
<tr>
<td></td>
<td>Third</td>
<td>11950.3</td>
<td>12000.0</td>
<td>0.416</td>
</tr>
<tr>
<td>Sensor located 4 in.</td>
<td>First</td>
<td>2833.3</td>
<td>2842.0</td>
<td>0.307</td>
</tr>
<tr>
<td>Inboard</td>
<td>Second</td>
<td>5536.5</td>
<td>5485.0</td>
<td>-0.930</td>
</tr>
<tr>
<td></td>
<td>Third</td>
<td>11572.8</td>
<td>11640.0</td>
<td>0.581</td>
</tr>
</tbody>
</table>
4.3.2 Evaluation of Damped Critical Speeds: Non-Synchronous Stability Evaluation

The evaluation of complex non-synchronous eigenvalues is required for stability evaluation of turbomachinery. An existing computer program, ROBEST (82), was used to verify the gyroscopic and damped critical speed evaluation of the new program. This comparison considers only sensor collocation. The results of the uniform rotor (shown in Fig. 4.5) for a lightly damped condition is given in Table 4.5, for rotor speeds of 1000.0 rpm and 20000.0 rpm. The percent deviation is typically less than 1.5% which gives added confidence in the finite element program.

Data used for the model:

Point mass added at mid-span of the rotor 19.3977 kg.
Polar Moment of Inertia of the mid-span mass 0.1627 kg-m²
Diametrical Moment of Inertia of the mid-span mass 0.0855 kg-m²
Stiffness of the bearings 1.7555 x 10⁹ N/m²
Damping of the bearings 17555.0 N-s/m²
Table 4.5 Comparison of Damped Critical Speeds -- Non-Synchronous Stability Analysis

<table>
<thead>
<tr>
<th>Running Speed (RPM)</th>
<th>Critical Speeds (RPM)</th>
<th>% difference with respect to Transfer Matrix Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Transfer Matrix Solution</td>
<td>Finite Element Solution</td>
</tr>
<tr>
<td>1000.0</td>
<td>3432.4</td>
<td>3437.3</td>
</tr>
<tr>
<td></td>
<td>3433.4</td>
<td>3438.3</td>
</tr>
<tr>
<td></td>
<td>20151.0</td>
<td>20116.1</td>
</tr>
<tr>
<td></td>
<td>20952.0</td>
<td>20917.4</td>
</tr>
<tr>
<td></td>
<td>42872.0</td>
<td>43497.2</td>
</tr>
<tr>
<td></td>
<td>42888.0</td>
<td>53514.5</td>
</tr>
<tr>
<td></td>
<td>53548.0</td>
<td>54318.9</td>
</tr>
<tr>
<td></td>
<td>54492.0</td>
<td>55240.8</td>
</tr>
<tr>
<td>20000.0</td>
<td>3423.5</td>
<td>3428.3</td>
</tr>
<tr>
<td></td>
<td>3442.3</td>
<td>3447.3</td>
</tr>
<tr>
<td></td>
<td>13534.0</td>
<td>13530.9</td>
</tr>
<tr>
<td></td>
<td>27989.0</td>
<td>28053.7</td>
</tr>
<tr>
<td></td>
<td>42721.0</td>
<td>43332.5</td>
</tr>
<tr>
<td></td>
<td>43040.0</td>
<td>43679.8</td>
</tr>
<tr>
<td></td>
<td>47629.0</td>
<td>48471.0</td>
</tr>
<tr>
<td></td>
<td>67789.0</td>
<td>68158.5</td>
</tr>
</tbody>
</table>

4.0 Initial Verification
4.4 Verification with Dresser-Rand's Finite Element Program

The multi-stage compressor shown in Fig. 4.4 was modeled for stability analysis. The results of the stability analysis (damped critical speeds and growth factors) were compared with the results obtained by the Finite Element Program being used at Dresser-Rand, Turbo Division. This program has been independently developed and has been in use at Dresser-Rand (Steam Turbine Division, Wellsville, NY) for the stability and response analysis of the turbomachinery (83). The results of the stability analysis for two cases of aerodynamic cross-coupling at midspan are shown in Tables 4.6 and 4.7. The % difference for the first four modes with respect to the Dresser-Rand results are shown in the tables.

Table 4.6 shows the results of the stability analysis for an aerodynamic cross-coupling $Q = 1.7555 \times 10^5$ N/m at midspan. Table 4.7 shows the results of the stability analysis for an aerodynamic cross-coupling $Q = 1.7555 \times 10^7$ N/m at midspan.
Table 4.6 Comparison of Stability Analysis for \( Q = 1.7555 \times 10^5 \) N/m

<table>
<thead>
<tr>
<th>Mode #</th>
<th>Damped Critical Speed (Hz)</th>
<th>Growth Factors (sec(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DR</td>
<td>FE</td>
</tr>
<tr>
<td>1b</td>
<td>50.93</td>
<td>50.93</td>
</tr>
<tr>
<td>1f</td>
<td>54.78</td>
<td>54.78</td>
</tr>
<tr>
<td>2b</td>
<td>89.40</td>
<td>89.40</td>
</tr>
<tr>
<td>2f</td>
<td>114.70</td>
<td>114.70</td>
</tr>
<tr>
<td>3b</td>
<td>156.52</td>
<td>156.57</td>
</tr>
<tr>
<td>3f</td>
<td>201.33</td>
<td>201.33</td>
</tr>
<tr>
<td>4b</td>
<td>280.40</td>
<td>280.38</td>
</tr>
<tr>
<td>4f</td>
<td>289.50</td>
<td>289.57</td>
</tr>
</tbody>
</table>

Table 4.7 Comparison of Stability Analysis for \( Q = 1.7555 \times 10^7 \) N/m

<table>
<thead>
<tr>
<th>Mode #</th>
<th>Damped Critical Speed (Hz)</th>
<th>Growth Factors (sec(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DR</td>
<td>FE</td>
</tr>
<tr>
<td>1b</td>
<td>53.50</td>
<td>53.50</td>
</tr>
<tr>
<td>1f</td>
<td>55.21</td>
<td>55.21</td>
</tr>
<tr>
<td>2b</td>
<td>89.39</td>
<td>89.40</td>
</tr>
<tr>
<td>2f</td>
<td>114.70</td>
<td>114.70</td>
</tr>
<tr>
<td>3b</td>
<td>156.58</td>
<td>156.62</td>
</tr>
<tr>
<td>3f</td>
<td>201.25</td>
<td>201.25</td>
</tr>
<tr>
<td>4b</td>
<td>280.59</td>
<td>280.58</td>
</tr>
<tr>
<td>4f</td>
<td>289.31</td>
<td>289.38</td>
</tr>
</tbody>
</table>

DR_FE: Finite Element Program used at Dresser-Rand

FE: Finite Element Program developed in the current research

\% difference = 100*(FE - DR_FE)/DR_FE

4.6 Initial Verification
CHAPTER 5

ANALYSES AND RESULTS OF SINGLE LEVEL ROTOR SYSTEMS

5.1 Analyses of Rotors Supported on Active Magnetic Bearings

The stability analysis and unbalance response calculations were performed for industrial multi-stage compressors. The importance of magnetic bearing sensor positions, along the length of the rotor, was initially established.

5.1.1 Stability Evaluation of a Multi-stage Compressor:

Importance of Sensor Position in the Stability Predictions

The model of an eight-stage compressor (62), shown in Fig. 5.1, was used for the stability analysis. The compressor was modeled using 19 stations with lumped parameters (i.e., mass, transverse inertia and polar inertia). The design speed of the compressor was 14,000 rpm. The mode of concern for instability of high-speed turbomachinery is typically the first or the lowest mode. Table 5.1 gives the frequency dependent bearing properties used for the above compressor.
Description: Stability Analysis of the B-stage Compressor supported on AMB
Sensor Colocated; Lower Damping Properties:

Number of stations (nodes): 17
Bearings at (from left): 3 15
Sensors at (from left): 3 15

Press any key to continue.

Figure 5.1 Model of 8-Stage Compressor used for Stability Analysis (Importance of Sensor Positions)
To study the effect of the non-synchronous bearing properties and also the sensor positions with respect to the bearing centerline, an aerodynamic cross-coupling, represented as cross-coupled stiffness \((K_{yz} = -K_{zy}, K_{yy} = K_{zz} = K)\) was applied at midspan. The aerodynamic cross-coupling acts as a destabilizing mechanism and drives the rotor unstable.

**Table 5.1 Frequency dependent bearing properties used for the above compressor model**

<table>
<thead>
<tr>
<th>Speed (RPM)</th>
<th>Stiffness (N/m)</th>
<th>Damping (N-s/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000.0</td>
<td>(1.949 \times 10^7)</td>
<td>(3.089 \times 10^3)</td>
</tr>
<tr>
<td>2555.6</td>
<td>(2.142 \times 10^7)</td>
<td>(1.229 \times 10^4)</td>
</tr>
<tr>
<td>4111.1</td>
<td>(2.229 \times 10^7)</td>
<td>(2.019 \times 10^4)</td>
</tr>
<tr>
<td>5666.7</td>
<td>(2.405 \times 10^7)</td>
<td>(2.423 \times 10^4)</td>
</tr>
<tr>
<td>7222.2</td>
<td>(2.774 \times 10^7)</td>
<td>(2.967 \times 10^4)</td>
</tr>
<tr>
<td>8777.8</td>
<td>(3.195 \times 10^7)</td>
<td>(3.388 \times 10^4)</td>
</tr>
<tr>
<td>10333.3</td>
<td>(3.652 \times 10^7)</td>
<td>(3.581 \times 10^4)</td>
</tr>
<tr>
<td>11888.9</td>
<td>(4.126 \times 10^7)</td>
<td>(3.458 \times 10^4)</td>
</tr>
<tr>
<td>13444.4</td>
<td>(4.564 \times 10^7)</td>
<td>(2.879 \times 10^4)</td>
</tr>
<tr>
<td>15000.0</td>
<td>(4.951 \times 10^7)</td>
<td>(1.773 \times 10^4)</td>
</tr>
</tbody>
</table>

Initially the stability analysis was performed for synchronous bearing properties, i.e., properties corresponding to the running speed of 14,000 rpm. The sensor positions were varied from the collocated position to 0.1778 m (7 in.) inboard. Fig. 5.2 shows the plot of stability, in terms of log. decrement vs. the cross-coupling stiffness at mid-span.

5.0 Analyses and Results of Single Level Rotor Systems
Typically, the first forward mode is of more concern for the stability of turbomachinery. Hence the logarithmic decrement of the first forward mode is plotted for stability. The stability is shown for four different sensor locations: collocated, 0.0762 m (3 in.) outboard, 0.0762 m (3 in.) inboard and 0.1778 m (7 in.) inboard. It can be seen that as the sensors are pulled inboard, the stability of the rotor drops. At 0.1778 m (7 in.) inboard position, the rotor is inherently unstable, i.e., unstable even at zero cross-coupling. Increase in mid-span cross-coupling worsens the stability of the forward mode. However, the stability of the first backward mode improves, and eventually the rotor becomes stable.

Figure 5.3 shows the stability plot for non-synchronous bearing properties, i.e., the bearing properties now used are those corresponding to the first forward mode. Comparing the plots in Fig. 5.2 and Fig. 5.3, it can be seen that the non-synchronous bearing properties markedly improve the stability of the rotor system. The stability for all the above 4 sensor positions have improved and it can be seen that at zero cross-coupling the rotor is stable! In fact, with the sensors located at 0.1778 m (7 in.) inboard, the rotor can now withstand a cross-coupling of approximately $4.5 \times 10^6$ N/m before the first forward mode goes unstable. The reduced stiffness at the lower whirl frequency has increased the stability such that the 0.1778 m (7 in.) inboard sensor location mode change does not drive the system unstable at zero cross-coupling. The influence of the sensor position is shown to be of great importance by these results. Hence for accurate prediction of rotor stability for active magnetic bearing machinery, the effects of sensor position must be considered and also the non-synchronous bearing properties have to taken into account.
Figure 5.2 Stability Plot for Synchronous Bearing Properties
Figure 5.3 Stability Plot for Nonsynchronous Bearing Properties
5.1.2 Effect of Support Flexibility on the Stability of AMB Supported Rotors

5.1.2.1 Initial Verification for Stability with Flexible Pedestals

The pedestals that support the rotors are generally assumed to be rigid for the stability and response calculations. In reality, there exists some amount of flexibility and structural damping due to the support structure. To study the effect of flexible pedestals on the stability, the 8-stage compressor shown above in Fig. 5.1 was used. The pedestal behind bearing #1 was assumed to be flexible, i.e., the pedestal was assumed to have stiffness and damping. Before evaluating the stability of the multi-stage compressor, an initial verification was performed to compare the results of the finite element program with that of the well established transfer matrix program, ROBEST (82). The uniform rotor model shown in Fig. 5.4 was used for the analysis. Table 5.2 shows the comparison of the results between the finite element program and transfer matrix program, ROBEST (82). The results show a very close agreement.

![Uniform Rotor Model Diagram](image)

**Figure 5.4 Uniform Rotor Model used for the Stability Analysis (Pedestal Flexibility)**
Data used for the above model:

K_b (Bearing Stiffness) = 1.756 \times 10^6 \text{ N/m (10,000 lb/in.)}

C_b (Bearing Damping) = 17.56 \text{ N-s/m (0.1 lb-s/in.)}

M = 2.5258 \text{ kg. (5.55676 lbs)}

I_i = 2.546 \times 10^{-3} \text{ kg-m}^2 (8.682 \text{ lb-in}^2)

I_p = 5.092 \times 10^{-3} \text{ kg-m}^2 (17.364 \text{ lb-in}^2)

Modulus of Elasticity = 2.0 \times 10^{11} \text{ N/m}^2 (28.9376 \times 10^6 \text{ lb/in2})

Poisson’s Ratio = 0.25

Density = 7850.0 \text{ kg/m}^3 (0.283 \text{ lb/in}^3)

Running Speed of the rotor = 14,000 \text{ rpm}

### Table 5.2 Comparison of Results of Finite Element Program with Transfer matrix Program ROBEST

<table>
<thead>
<tr>
<th>Critical Speeds (RPM)</th>
<th>Results of the finite element program</th>
<th>Results of ROBEST</th>
<th>% difference*</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Backward</td>
<td>9812.98</td>
<td>9871.32</td>
<td>0.59</td>
</tr>
<tr>
<td>First Forward</td>
<td>9864.58</td>
<td>9878.36</td>
<td>0.14</td>
</tr>
<tr>
<td>Second Backward</td>
<td>33818.26</td>
<td>34026.00</td>
<td>0.61</td>
</tr>
<tr>
<td>Second Forward</td>
<td>33857.13</td>
<td>34033.05</td>
<td>0.63</td>
</tr>
</tbody>
</table>

*% difference = \frac{\text{Results of ROBEST} - \text{Results of finite element program}}{\text{Results of ROBEST}}
With this initial verification established, the 8-stage compressor was studied. The model of the 8-stage compressor is as shown in Fig. 5.1, and the frequency dependent bearing characteristics are given in Table 5.1. The running speed of the rotor was 14,000 rpm. The synchronous bearing stiffness and damping properties corresponding to 14,000 rpm were used for the stability analysis. The bearings were located at 0.3048 m (12 in.) and 1.5748 m (62 in.) from the left end of the rotor. A flexible support was introduced behind the bearing at 0.3048 m (12 in.) from the left end. The stability of the rotor was evaluated over a range of pedestal damping ($C_p$) for 4 different cases of the pedestal stiffness ($K_p$). This was repeated for different position of the sensors.

To study the effect of the pedestal properties on the stability of the rotor, the rotor was made unstable by applying an aerodynamic cross-coupling (similar to section 5.1) at mid-span, $K_{xy} = -K_{yx} = 1.7555 \times 10^6$ N/m. Four cases of pedestal stiffness were selected for the analysis: a relatively soft pedestal ($8.7775 \times 10^6$ N/m); a moderately stiff pedestal ($1.7555 \times 10^7$ N/m); a relatively stiff pedestal ($3.511 \times 10^7$ N/m) and a relatively rigid pedestal ($7.022 \times 10^3$ N-s/m). The pedestal damping was varied from a relatively low value ($1.7555 \times 10^3$ N-s/m) to a high value ($1.7555 \times 10^6$ N-s/m). The stability of the rotor was evaluated at each value of the damping. The stability of the first forward mode was plotted for the above cases for different sensor positions and the results are as shown in Figs. 5.5 - 5.18, in the form of logarithmic decrement vs. pedestal damping, for different values of pedestal stiffness.
Figure 5.5 Plot of Stability vs. Pedestal Damping for Sensors Collocated
Figure 5.6 Plot of Stability vs. Pedestal Damping for Sensors 3 in. Outboard
Figure 5.7 Plot of Stability vs. Pedestal Damping for Sensors 3 in. Inboard

5.0 Analyses and Results of Single Level Rotor Systems
Figure 5.8 Plot of Stability vs. Pedestal Damping for Sensors 7 in. Inboard
Figure 5.5 shows the stability for different pedestal stiffnesses for collocated sensors, for increasing pedestal damping. Similarly, Figs. 5.6 and 5.7 show the stability for the sensors located at 0.0762 m (3 in.) outboard and inboard, respectively. In the previous analysis (see Fig. 5.2) it was shown that the rotor was stable for an aerodynamic cross-coupling of $1.7555 \times 10^6$ N/m when the sensors were located at 0.0762 m (3 in.) inboard and outboard. It can be seen from Figs. 5.5, 5.6 and 5.7 that for a given pedestal stiffness, increasing the pedestal damping increases the stability of the first forward mode till an optimum damping is reached. Any further increase in pedestal damping drops the stability of the rotor to that of a rigid pedestal stability value. This suggests that an optimum pedestal damping exists for a given pedestal stiffness, such that the stability is maximized. Also it can be seen that an increase in pedestal damping increases the stability till an optimum pedestal stiffness is reached. Further increase in pedestal stiffness drops the stability. This suggests that there also exists an optimum pedestal stiffness for maximum stability, for a given pedestal damping.

It can be seen from Fig. 5.5 and Fig. 5.6 that for certain values of pedestal stiffness and damping, the rotor might actually go unstable. This is indicated by a negative logarithmic decrement obtained for very low values of the pedestal damping. However, the system becomes stable with an increase in damping. Hence, care has to be taken in introducing a flexible pedestal. Both the pedestal stiffness and damping have to be considered while performing the stability analysis. It can be concluded from the above
Figs. 5.5, 5.6 and 5.7 that maximum stability of the rotor can be obtained by a suitable combination of the pedestal stiffness and damping.

Figure 5.8 shows the plot of logarithmic decrement vs. pedestal damping for the sensors located 0.1778 m (7 in.) inboard. It is seen that for the above ranges of the pedestal stiffness, an increase in pedestal damping does not improve the stability of the rotor system. This unstable behavior of the rotor at 0.1778 m (7 in.) inboard position of the sensors can be confirmed from Fig. 5.2, in the previous analysis. This shows that an inherently unstable rotor (due to sensor noncollocation) cannot be stabilized by introducing a flexible support.

5.1.3 Effect of Sensor Noncollocation on Unbalance Response

It was shown in section 5.1 that the locations of the sensors on a rotor, supported on active magnetic bearings, played an important role in the stability of the system. This section deals with the study of the influence of the sensor positions on the unbalance response of a rotor supported on AMB. The 8-stage compressor shown in Fig. 5.9 was used for the case study. The rotor is supported on magnetic bearings at locations 0.3048m (Node #3) and 1.5747m (Node #15) from the left end, as shown in the figure. To study the influence of the sensor positions, 4 different sensor positions were considered (shown in Fig. 5.9) -- 0.0762m (3.0 in.) outboard, collocated, 0.1143m (4.5 in.) inboard and 0.1778m (7.0 in.) inboard. An unbalance mass of $3.6 \times 10^{-4}$ kg-m (0.5 oz-in.) was
Figure 5.9 Model of the 8-Stage Compressor Showing the Different Unbalance Locations used for Response Analysis
considered to be acting at midspan. The frequency dependent bearing characteristics are given in Table 5.3. Two sets of damping cases were considered -- the relatively lightly damped case and the relatively heavily damped case (5 times the lightly damped case). The results are given in terms of the response at the given locations as a function of the running speed of the rotor. The running speed was varied from 0 - 10,000 rpm. The results for the lower damping case are given in Figs. 5.10 to 5.14. The results for the higher damping case are given in Figs. 5.17 to 5.22.

Table 5.3 Frequency Dependent Bearing Properties Used for the Compressor Model

<table>
<thead>
<tr>
<th>Speed (RPM)</th>
<th>Stiffness (N/m)</th>
<th>Damping (N-s/m) (Lower Damping)</th>
<th>Damping (N-s/m) (Higher Damping)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000.0</td>
<td>$1.949 \times 10^7$</td>
<td>$3.089 \times 10^4$</td>
<td>$1.544 \times 10^4$</td>
</tr>
<tr>
<td>2555.6</td>
<td>$2.142 \times 10^7$</td>
<td>$1.229 \times 10^4$</td>
<td>$6.145 \times 10^4$</td>
</tr>
<tr>
<td>4111.1</td>
<td>$2.229 \times 10^7$</td>
<td>$2.019 \times 10^4$</td>
<td>$1.009 \times 10^5$</td>
</tr>
<tr>
<td>5666.7</td>
<td>$2.405 \times 10^7$</td>
<td>$2.423 \times 10^4$</td>
<td>$1.212 \times 10^5$</td>
</tr>
<tr>
<td>7222.2</td>
<td>$2.774 \times 10^7$</td>
<td>$2.967 \times 10^4$</td>
<td>$1.484 \times 10^5$</td>
</tr>
<tr>
<td>8777.8</td>
<td>$3.195 \times 10^7$</td>
<td>$3.388 \times 10^4$</td>
<td>$3.388 \times 10^4$</td>
</tr>
<tr>
<td>10333.3</td>
<td>$3.652 \times 10^7$</td>
<td>$3.581 \times 10^4$</td>
<td>$3.581 \times 10^4$</td>
</tr>
<tr>
<td>11888.9</td>
<td>$4.126 \times 10^7$</td>
<td>$3.458 \times 10^4$</td>
<td>$3.458 \times 10^4$</td>
</tr>
<tr>
<td>13444.4</td>
<td>$4.564 \times 10^7$</td>
<td>$2.879 \times 10^4$</td>
<td>$2.879 \times 10^4$</td>
</tr>
<tr>
<td>15000.0</td>
<td>$4.951 \times 10^7$</td>
<td>$1.773 \times 10^4$</td>
<td>$1.773 \times 10^4$</td>
</tr>
</tbody>
</table>

Figures 5.10 - 5.12 show the plots of the response at bearing #1, midspan and bearing #2 as a function of the running speed of the rotor, respectively. It can be seen that
as the sensors are moved from the outboard position (0.0762m) to the inboard position (0.1778m), the first critical speed increases. That is, the peak response speed increases from 3150 rpm for the outboard sensor position to 3575 rpm for the inboard position of the sensor (at 0.1143m). This is due to the fact that moving the sensors inboard is in effect moving away from the point of minimum response of the first mode. This increases the stiffness for the first mode and hence the first critical speed is increased. Similarly, moving the sensor inboard is in effect moving towards the point of minimum response for the third mode. This has the effect of decreasing the stiffness and hence the third critical drops as the sensors are moved inboard. It can also be noted from the plots that the response at the first criticals for the sensors' locations, as they are moved inboard, drops and the response at the third critical increases. From Fig. 5.11, the response at the midspan increases to a relatively high magnitude when the sensors are at 0.1778m (7.0 in.) inboard. This is an indication of the system going unstable or near instability. Also it can be seen that the first critical speed is not very pronounced for the 0.1778m (7.0 in.) inboard position. This can be verified from the mode shapes for the first and third critical speeds as shown in Figs. 5.15 and 5.16, respectively. It can be seen from these figures that the first and third modes are essentially the same, i.e., the first and third modes coalesce as the sensors are moved inboard to the 0.1778m (7.0 in.) position. The shifts in the first and third frequencies with the sensor positions, for the lower damping case, are given in Table 5.4a and Table 5.4b.

5.0 Analyses and Results of Single Level Rotor Systems
The response of the compressor rotor for the relatively higher damping case are given in Figs. 5.17 to 5.21. These figures show the response of the rotor at different sensor locations. As seen from these figures, the response plots have only one peak! The peak corresponds to the third critical which can be verified from the mode-shape plot in Fig. 5.22. With the increase in damping, the first mode is completely damped. Figures 5.17, 5.18 and 5.19 give the response of the rotor at the bearing #1, midspan and bearing #2, respectively. From Fig. 5.18, it can be seen that the response at midspan increases as the sensors are moved from the outboard position to the inboard position. With the sensors at the 0.1778m (7.0 in.) inboard position, the response at midspan peaks up to a relatively very high value. This again indicates an unstable, or close to unstable, behavior of the rotor system. Figure 5.22 shows the mode shape of the rotor for the higher damping case. The shifts in frequencies with the sensor positions, for the higher damping case, are given in Table 5.5.
Table 5.4a First Critical Speeds for Different Sensor Positions (Lower Damping)

<table>
<thead>
<tr>
<th>#</th>
<th>Sensor Position</th>
<th>First Peak Speed (RPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0762m Outboard</td>
<td>3150</td>
</tr>
<tr>
<td>2</td>
<td>Collocated</td>
<td>3280</td>
</tr>
<tr>
<td>3</td>
<td>0.1143m Inboard</td>
<td>3575</td>
</tr>
<tr>
<td>4</td>
<td>0.1778m Inboard</td>
<td>3775</td>
</tr>
</tbody>
</table>

Table 5.4b Third Critical Speeds for Different Sensor Positions (Lower Damping)

<table>
<thead>
<tr>
<th>#</th>
<th>Sensor Position</th>
<th>First Peak Speed (RPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0762m Outboard</td>
<td>6475</td>
</tr>
<tr>
<td>2</td>
<td>Collocated</td>
<td>6175</td>
</tr>
<tr>
<td>3</td>
<td>0.1143m Inboard</td>
<td>5725</td>
</tr>
<tr>
<td>4</td>
<td>0.1778m Inboard</td>
<td>5525</td>
</tr>
</tbody>
</table>

Table 5.5 Third Critical Speeds for Different Sensor Positions (Higher Damping)

<table>
<thead>
<tr>
<th>#</th>
<th>Sensor Position</th>
<th>First Peak Speed* (RPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0762m Outboard</td>
<td>4550</td>
</tr>
<tr>
<td>2</td>
<td>Collocated</td>
<td>4925</td>
</tr>
<tr>
<td>3</td>
<td>0.1143m Inboard</td>
<td>5525</td>
</tr>
<tr>
<td>4</td>
<td>0.1778m Inboard</td>
<td>5875</td>
</tr>
</tbody>
</table>

* The first critical speed is completely damped out for this set of damping properties, and the first peak corresponds to the third critical
Figure 5.10 Plot of Response vs. Running Speed at Bearing #1 (Lower damping case)
Figure 5.11 Plot of Response vs. Running Speed at Midspan
(Lower damping case)
Figure 5.12 Plot of Response vs. Running Speed at Bearing #2
(Lower damping case)
Figure 5.13 Plot of Response vs. Running Speed at Sensor #1 (Lower damping case)
Figure 5.14 Plot of Response vs. Running Speed at Sensor #2
(Lower damping case)
Figure 5.15 Mode-shapes for 4 Sensor Positions at the Respective First Critical Speed (Lower Damping)
Figure 5.16 Mode-shapes for 4 Sensor Positions at the Respectively Third Critical Speed (Lower Damping)
Figure 5.17 Plot of Response vs. Running Speed at Bearing #1 (Higher damping case)
Figure 5.18 Plot of Response vs. Running Speed at Midspan (Higher damping case)
Figure 5.19 Plot of Response vs. Running Speed at Bearing #2
(Higher damping case)
Figure 5.29 Plot of Response vs. Running Speed at Sensor #1 (Higher damping case)
Figure 5.21 Plot of Response vs. Running Speed at Sensor #2
(Higher damping case)
5.0 Analyses and Results of Single Level Rotor Systems
Description: Stability Analysis of Multi-Stage Compressor supported on 2 Fluid Film Bearings; Squeeze Film Damper Behind Bearing #1 (from Left):

Number of stations (nodes): 32
Bearings at (from left): B 28
Damper at (from left): 5
Aero_Cross at (from left): 16

Stability Program

File: MODEL.S

Total Length of Rotor: 1.8796 m
(74.00 in.)

Maximum Diameter: 0.1778 m
(7.00 in.)

Outside dimension: 'B' - Bearings 'A' - Aero_Cross
Inside dimension: 'D' - Damper

Press any key to continue.

Figure 5.23 Model of the Rotor supported on Squeeze Film Damper
5.2 Stability Analysis of Rotors Supported on Squeeze Film Dampers

5.2.1 Introduction

The ability of a squeeze film damper (SFD) to attenuate the amplitude of vibration and to decrease the dynamic forces transmitted to the support system makes it an attractive means of supporting turbomachinery. However, adding a squeeze film damper can significantly affect the stability of the turbomachinery. The squeeze film bearing has been subject to a considerable amount of investigation since the 1960's and has been used practically, as documented by Mohan and Hahn (42) and Rabinowitz and Hahn (48 to 51). However, several undocumented cases where the dampers have not been successful suggests that a general design study could be helpful to understand how the system stability is influenced by the addition of a squeeze film damper.

The parameters considered in the design of a squeeze film damper are the clearance ratio (the ratio of the diametrical clearance space to the diameter of the damper ring), the width of the land, the number of lands, the eccentricity ratio (the ratio of the radial eccentricity to the radius of the damper ring) and the retainer stiffness. The retainer spring supports the damper to prevent the rotation of the damper ring. In some designs, anti-rotation pins, or dogs, are used to prevent the rotation of the damper ring. Each of these parameters influences the characteristics of the squeeze film damper, i.e., the stiffness and damping properties. These in turn govern the stability of the rotor which is supported on squeeze film dampers. Different configurations can be obtained by varying each of these parameters. Stability analysis has to be performed for each of these
configurations, which makes it cumbersome due to the number of calculations involved. A
design procedure is described for evaluating the stability of rotors supported on squeeze
film dampers. To decrease the number of stability evaluations, the concept of a Damping
Number will be introduced, which can be used as a parameter for the stability evaluation
of the turbomachinery for the different configurations of the squeeze film damper. Given a
stability evaluation of the rotor for a particular design of the squeeze film damper, the
analysis shows that the results are applicable to other possible configurations of the
damper. The different configurations can be obtained by changing the clearance ratio, land
width and/or the number of lands.

Closed form solutions have been obtained by White (1972) for the stiffness and
damping of squeeze film dampers from the Reynolds equation based on short-bearing and
long-bearing approximations for both cavitated and uncavitated film extent. These closed
form solutions are used for the following analysis.

5.2.2 Stability Analysis

The model used for the design study was an eight-stage compressor, typical of the
process and gas transmission industries. The rotor shaft, shown in Fig. 5.23, was modeled
with 32 nodes (stations) and is supported by two fluid film bearings and has a design speed
of 233.33 Hz (14000 rpm). To study the effect of adding a squeeze film damper behind
the plain (non-thrust) bearing, the rotor was made initially unstable (the first forward
mode) by applying an aerodynamic cross-coupling at midspan \((K_{yz} = - K_{zy} = 9.655 \times 10^6\)
N/m). This resulted in a growth factor of about 62, equivalent to a logarithmic decrement of -0.7580.

A single land squeeze film damper with land width 76.2 mm (3 in.) and a diameter 177.8 mm (7 in.) was assumed to be located behind one of the bearings (the bearing at the left end of the rotor). The damper was assumed to be supported by a retainer of potentially varying stiffness. Initially the damper was assumed to be operating with an eccentricity ratio of 0.1. The shaft diameter at the bearing location was 101.6 mm (4.0 in.), giving an L/D of 0.428. The short bearing, cavitated film approximation was used for obtaining the properties of the damper. The stability evaluation was performed by varying the clearance, eccentricity and retainer stiffness. The results are shown in Figs. 5.24 to 5.28.

Figure 5.24 shows the plot of the growth factor (for the first forward mode) vs. the retainer stiffness for different clearances of the squeeze film. It can be seen from the plot that for very high values of the retainer stiffness, the stability approaches that of a rotor on a rigid pedestal. For low values of the clearances, i.e., 2.5 mm/m, the rotor is unstable initially and remains unstable with increase in the retainer stiffness. However, as the clearance is increased, i.e., for 2.75, 3.0, 3.25, and 3.38 mm/m, the stability improves to an appreciable extent. A clearance of 3.25 mm/m shows the existence of an optimum retainer stiffness for maximum stability. As the clearances are increased further, the stability decreases drastically and the rotor becomes highly unstable, as shown by the clearance of the 15 mm/m line. The results of Fig. 5.24 are replotted in Fig. 5.25, where
the stability is plotted as a function of the clearance ratio for different values of the retainer stiffness. It is quite evident that for a given retainer stiffness, there exists an optimum value for the clearance of the squeeze film.

The clearance ratio of 3.25 mm/m was selected, and the eccentricity ratios were varied. Figure 5.26 shows the plot of stability as a function of the retainer stiffness for different operating eccentricities. From the plot, it can be seen that the low eccentricity ratio of 0.1 seems to be better with regards to stability. As the operating eccentricity increases, the stability drops and eventually the rotor becomes unstable (as shown for ε = 0.5). Figure 5.27 shows the stability as a function of the clearance ratio, for different eccentricity ratios. The plot also shows the existence of an optimum value for the clearance ratio.

5.2.3 Damping Number ($C_N$)

All the above cases were evaluated with certain operating conditions, viz., the eccentricity ratio, the clearances, the land width and the diameter of the squeeze film damper. The stiffness of the squeeze film, assuming a cavitating short bearing film, is given by,

$$K = \frac{2R \mu L^3}{c_r^3} \frac{\varepsilon}{(1 - \varepsilon^2)^3}$$

(5.1)
where,

\[ c_r = \text{radial clearance}, \quad R = \text{radius of the damper}, \quad \omega = \text{speed, (rads./sec)} \]

\[ L = \text{width of the land}, \quad \varepsilon = \text{eccentricity ratio}, \quad \mu = \text{viscosity}, \]

Assuming the above quantities to be in consistent units, it is possible to define a parameter called the Damping Number \((C_N)\). For the general case of a single land damper,

\[ C_N = \frac{\mu \omega D L^3}{c_r^3} \quad (5.2) \]

To obtain the same stiffness and damping for a multi-land damper, with \(n\) equal lands, each of width \(L_i\) (m), and with \(c_{d_i}\) (mm) as the diametrical clearance and a diameter \(D\) (m), the equivalent Damping Number can be written as,

\[ C_N = \frac{8 \times 10^9 n \mu \omega D (L_i / D)^3}{(c_{d_i} / D)^3} \quad (5.3) \]

The usefulness and application of the Damping Number \((C_N)\), given in equation (5.3), is explained in the following discussion. Consider the plot of growth factor vs. the retainer stiffness for, say, a clearance ratio of 3.25 mm/m, with a damper radius of 177.8 mm (7 in.) and a single land 76.2 mm (3 in.) wide. To obtain the same stability for a different \(L/D\) ratio, the new clearance ratio can be calculated using the Damping Number. For example, if the same damper stiffness as in the above configuration is desired in a
damper with 2 lands, each 38.1 mm (1.5 in.) wide, the new clearance can be calculated as explained below.

Let,

\( L_1, L_2 = \text{land widths of the first and the second damper respectively, (m)} \)

\( n_1, n_2 = \text{number of lands on the first and the second damper respectively,} \)

\( c_{d1}, c_{d2} = \text{diametrical clearance of the first and the second damper, (mm)} \)

\( D_1, D_2 = \text{diameter of the first and the second damper, (m)} \)

Assuming the same effective viscosity and operating speed,

\[
\frac{n_1 D_1 L_1^3}{c_{d1}^3} = \frac{n_2 D_2 L_2^3}{c_{d2}^3} \quad (5.4)
\]

For the above example, \( L_2 = L_1/2 \) and \( n_1 = 1, n_2 = 2 \) and \( D_1 = D_2 \). Using these values, in equation (5.4), we obtain the new equivalent clearance ratio to be 2.0 mm/m.

This gives an example of the usefulness of the Damping Number. Hence, for a given rotor configuration, if one set of plots of stability vs. retainer stiffness (as in Fig. 5.24) is obtained, the Damping Number can be used to study the different configurations obtained by varying the length and number of lands, and the clearances, from the same plot. Figure 5.28 shows the plot of stability vs. the retainer stiffness for the squeeze film damper with 2 lands each of width 38.1 mm (1.5 in.) and with an eccentricity ratio of 0.1. Comparing Figs. 5.24 and 5.28, it can be seen that the 3.25 mm/m clearance on the 76.2 mm (3 in.)
The results generated for the stability of the rotor supported on a squeeze film damper with a single land of 76.2 mm (3 in.) wide are applicable to other configurations of the damper, and need not be generated again. For the above example, where the new damper had 2 lands each of width 38.1 mm (1.5 in.), the lowest point on the plot in Fig. 5.24 would shift to 2 mm/m. The results could be correlated as follows. The lowest point on Fig. 5.25 would shift to the new clearance of 2.0 mm/m. In Fig. 5.27, the lowest point would again shift to the new clearance. Figure 5.28 would remain the same, for the clearance of 2.0 mm/m. This shows that the plots generated for one configuration could be used for other configurations of the squeeze film damper.
Figure 5.24 Plot of Growth Factor vs. Retainer Stiffness for Different Values of Clearance Ratio
Figure 5.25 Plot of Growth Factor vs. Clearance Ratio for Different Values of Retainer Stiffness
Figure 5.26 Plot of Growth Factor vs. Retainer Stiffness for Different Values of Damper Eccentricity Ratio
Figure 5.27 Plot of Growth factor vs. Clearance Ratio for Different Values of Damper Eccentricity Ratio

Retainer stiffness = $1.755 \times 10^6$ N/m
Figure 5.28 Plot of Growth Factor vs. Retainer Stiffness for Different Values of Clearance Ratio.

Land width = 38.1 mm (1.5 in.)
No. of lands = 2; Damper Ecc. ratio = 0.1
5.2.4 Forced Response of a Rotor System Having a Squeeze Film Damper

The main reasons for using a squeeze film damper on a rotor are to improve the dynamic stability and also to reduce the forces transmitted to the foundation. Generally the squeeze film damper ring is centered by a retainer spring. For linearized stability evaluation this is an acceptable assumption. While evaluating the forced response of a rotor having a squeeze film damper, the damper ring is usually assumed to be centralized in the clearance space through the entire running speed range. During the actual running condition the damper does not remain in the centered position and the damper ring tends to find its own eccentric position. This non-linear behavior of the damper should be taken into account while performing the response calculations. The current finite element program can predict the response using an iterative solution procedure to account for the non-linear damper behavior.

To demonstrate the effect of the non-linear behavior, the uniform rotor shown in Fig. 5.29 was modeled using the finite element program. The unbalance mass was assumed to be located at midspan. The squeeze film damper was assumed to be located behind the bearing #1 (see Fig. 5.29). To see the effect of damper on the response, the response was calculated for a centered damper configuration and also by considering the non-linear behavior of the damper. Figure 5.30 shows the response at bearing #1 for the cases, without the damper, with damper and centered assumption and with damper and non-linear damper. As shown in the Fig. 5.30, it can be seen that the response without the damper is extremely high. Figure 5.31 shows the comparison of the response at bearing #1
for the two different bearing assumptions. It can be seen that the non-linear damper behavior results in a shift in the peak response speed and also results in a higher response. It is interesting to note that the damper starts at a low eccentricity and goes to about 0.54 eccentricity while going through the critical. A similar trend is seen at midspan as shown in Fig. 5.32. A comparison of the response at the two bearings is shown in Fig. 5.33. The bearing #2 does not have a squeeze film damper and it can be seen that the response at this bearing is less than the one with the damper.

Figure 5.34 shows the response for different level of unbalance at midspan. The calculations were performed using the non-linear damper behavior. It can be noted from the Fig. 5.34 that not only does the peak response speed shift, but also the maximum response while passing through the critical is highly non-linear with changes in the level of unbalance. The eccentricity of the damper ring also increases non-linearly with the increase in the level of unbalance. The above analysis shows that the non-linear damper behavior leads to a totally different response, as compared to the centralized damper assumption. Hence, it is important to consider this non-linear behavior of the squeeze film damper, while evaluating the response of the rotor system.
Figure 5.29 Plot of Amplitude at Bearing #1 vs. Running Speed
With and Without the Damper
(Unbalance at Midspan = 2.5 oz-in)
Figure 5.30 Plot of Amplitude at Bearing #1 vs. Running Speed for the Two Damper Assumptions
(Unbalance at Midspan = 2.5 oz-in)
Figure 5.31 Plot of Amplitude at Midspan vs. Running Speed
(Unbalance at Midspan = 2.5 oz-in)
Figure 5.32 Plot of Amplitude at Bearing #2 vs. Running Speed
(Unbalance at Midspan = 2.5 oz-in)
Figure 5.33 Plot of Amplitude at Bearing #1 vs. Running Speed for Different Levels of Unbalance at Midspan (Non-linear Squeeze Film Damper Behavior)
5.3 Analysis of Multi-Stage Pump with Inter-Stage Seals

5.3.1 Stability Analysis

The Multi-Stage industrial pump shown in Fig. 5.34 was modeled with 29 stations (28 elements). The rotor is supported on two fluid-film bearings (at nodes #3 and # 23 from the left end) as shown in the figure. The pump has 9 stages and the stages are shown by the external mass locations (Node # 8 to Node # 16) in Fig. 5.34. The left end (indicated by 'L' on Fig. 5.34) is the coupling end of the pump. The Stability Analysis was performed to evaluate the damped critical speeds and the stability of the rotor system. The running speed of the rotor was 7600 rpm and the corresponding bearing characteristics are given in Table 5.6.

The first 8 damped critical speeds along with the growth factors and logarithmic decrements are given in Table 5.7. The mode shapes for the first 4 damped critical speeds are shown in Fig. 5.35. The first mode at 2483 RPM is the common bending mode. The second mode at 2765 RPM appears to be excited by the bearings, indicated by predominant motion at the bearing ends compared to the rest of the rotor. The third mode is again a bending mode of the rotor at 9093 RPM. The higher modes show the higher bending modes of the rotor-system. From Table 5.7, it can be seen that the first forward mode at 2483 RPM has an extremely small logarithmic decrement but is still stable.
Description: Multi-Stage Pump supported on 2 fluid film bearings:
Model for single level Stability Analysis — Basic Model

Number of stations (nodes): 29
Bearings at (from left): 3 23
Ext.Mass at (from left): 1 5 8 9 10 11 12 13 14 15 16 17 21 29 29

Figure 5.34 Model of the Multi-Stage Pump
5.0 Analyses and Results of Single Level Rotor Systems
<table>
<thead>
<tr>
<th>Node #</th>
<th>Bearing Stiffness (N/m)</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$K_{yy}$</td>
<td>$K_{yz}$</td>
<td>$K_{zy}$</td>
<td>$K_{zz}$</td>
</tr>
<tr>
<td>3</td>
<td>0.7285E+08</td>
<td>0.1791E+08</td>
<td>-.7356E+08</td>
<td>0.9954E+08</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>0.7285E+08</td>
<td>0.1791E+08</td>
<td>-.7356E+08</td>
<td>0.9954E+08</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Node #</th>
<th>Bearing Damping (N-s/m)</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$C_{yy}$</td>
<td>$C_{yz}$</td>
<td>$C_{zy}$</td>
<td>$C_{zz}$</td>
</tr>
<tr>
<td>3</td>
<td>0.1066E+06</td>
<td>-.7759E+05</td>
<td>-.7408E+05</td>
<td>0.2563E+06</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>0.1066E+06</td>
<td>-.7759E+05</td>
<td>-.7408E+05</td>
<td>0.2563E+06</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.7 Damped Critical speeds and Growth Factors for the first 8 Modes

<table>
<thead>
<tr>
<th>Mode #</th>
<th>Growth Factor (sec&lt;sup&gt;-1&lt;/sup&gt;)</th>
<th>Damped Critical speed (Hz)</th>
<th>(rpm)</th>
<th>Logarithmic decrement</th>
<th>Whirl&lt;sup&gt;*&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.6337</td>
<td>40.905</td>
<td>2454.33</td>
<td>0.0399</td>
<td>MBW</td>
</tr>
<tr>
<td>2</td>
<td>-0.0393</td>
<td>41.395</td>
<td>2483.72</td>
<td>0.0009</td>
<td>FW</td>
</tr>
<tr>
<td>3</td>
<td>-860.2765</td>
<td>40.657</td>
<td>2439.42</td>
<td>21.1593</td>
<td>FW</td>
</tr>
<tr>
<td>4</td>
<td>-743.1473</td>
<td>46.091</td>
<td>2765.48</td>
<td>16.1234</td>
<td>FW</td>
</tr>
<tr>
<td>5</td>
<td>-1.8628</td>
<td>145.660</td>
<td>8739.58</td>
<td>0.0128</td>
<td>BW</td>
</tr>
<tr>
<td>6</td>
<td>-2.9827</td>
<td>151.552</td>
<td>9093.15</td>
<td>0.0197</td>
<td>FW</td>
</tr>
<tr>
<td>7</td>
<td>-262.9341</td>
<td>224.204</td>
<td>15452.22</td>
<td>1.1727</td>
<td>FW</td>
</tr>
<tr>
<td>8</td>
<td>-57.7590</td>
<td>230.961</td>
<td>13857.67</td>
<td>0.2501</td>
<td>MBW</td>
</tr>
</tbody>
</table>

* FW : Forward Whirl; BW : Backward Whirl; MBW : Mostly Backward Whirl
5.3.2 Unbalance Response

The unbalance response calculations of the multi-stage pump are shown in Fig. 5.34. The responses to an unbalance of $3.6 \times 10^{-4}$ kg-m (0.5 oz-in.) at midspan (Node # 12) and also at the coupling (Node # 1) were studied over a speed range of 500 rpm to 10,000 rpm. The response mode shapes for 4 different speeds over the above speed range, for the two unbalance cases, are shown in Figs. 5.36 and 5.37.

For the unbalance at mid-span (Node # 12), the unbalance force dominates the response of the rotor, as seen from Fig. 5.36. The response at mid-span is maximum for running speeds ranging from 500 rpm to 5000 rpm. Also, for a running speed of 500 rpm, it can be seen from the phase plot that the response at midspan is in-phase with the unbalance force. As the speed increases to 2500 rpm, the phase between the response at midspan and the unbalance force (also at mid-span) goes through 90° and becomes almost 180° at 2500 rpm, indicating that the rotor has gone through the first critical speed. However, the phase plots get complicated for real rotor systems, partly due to multi-degrees of freedom and also due to increased damping. The phase shift through 90° occurs close to 2500 rpm which is in the range of the first critical speed. This can be verified from the mode shape for the first critical speed as shown in Fig. 5.35. Also, the response of the rotor at 9000 rpm looks like the second bending mode of the rotor (see mode shape at 9093 rpm -- Fig. 5.35). At this speed, the response at mid-span is almost minimum and the two ends of the rotor have relatively more activity.
The coupling is located at the end indicated by 'L' in Fig. 5.34. The unbalance at the couplings is the most common type of unbalance excitation on turbomachinery. To study the effect of coupling unbalance, an unbalance of $3.6 \times 10^{-4}$ kg-m (0.5 oz-in) was placed at Node #1. The response was studied over a speed range of 500 rpm to 10000 rpm. The response for the coupling unbalance for four different speeds are as shown in Fig. 5.37. For low running speeds of around 500 rpm, the unbalance mass at the coupling end dominates the response shape of the rotor, as seen by the relatively higher response at the coupling end. The response at 2500 rpm is again close to the mode shape at the first bending critical speed of the rotor (comparing Figs. 5.37 and 5.35). Similarly, the response of the rotor at 9000 rpm is similar to the second bending mode shape of the rotor.
5.0 Analyses and Results of Single Level Rotor Systems
Figure 5.37 Response of the Pump for Unbalance at the Coupling (Node # 1)
CHAPTER 6

ANALYSES AND RESULTS OF MULTI LEVEL ROTOR SYSTEMS

6.1 Multi-level Modeling of Seals

The seals and bearings form integral parts of any turbomachinery. While performing the stability analysis of such rotor systems, it is important to consider the mass effects of the seals and bearings also. That is, the mass and geometry of the seals and bearings must be included in the model. In most common applications, the properties of the seals and bearings are considered at their respective locations. This results in a model in which the seal/bearing is considered to be locked in position to the support structure. However it has been found in some applications (84) that the seal is not completely locked but is indeed floating. Such systems can be modeled using the multi-level procedure included in the finite element program.

An 8-stage industrial compressor, shown in Fig. 6.1, was considered for the stability analysis. The seal positions and dimensions are as shown in the figure. Figures 6.2a and 6.2b show the schematic model of the seal when it is modeled as locked and
Description: Stability Analysis of Multi-Stage Compressor supported on 2 Fluid Film Bearings and 2 Seals

Number of stations (nodes): 36
Bearings at (from left): 5 32
Seals at (from left): 8 29

Total Length of Rotor: 1.8796 m
(74.00 in.)

Maximum Diameter: 0.1778 m
(7.00 in.)

--- Outside dimension: 'B' - Bearings
----- Inside dimension: 'S' - Seals

Figure 6.1 Model of the Multi-Stage Pump with Bearings and Seals
floating, respectively. The stability analysis of the above two configurations was performed for different eccentricities of the seal, different seal types and different running speeds of the rotor. Results will be presented for four cases in the following discussion.

![Schematic of the Seal in the Locked Configuration](image)

**Figure 6.2a Schematic of the Seal in the Locked Configuration**

![Schematic of the Seal in the Floating Configuration](image)

**Figure 6.2b Schematic of the Seal in the Floating Configuration**

The properties of the seal (the stiffness and damping coefficients) were evaluated for the specified configuration using a finite element program (85) and used for the stability analysis. The results of the stability analysis for four different cases are shown in
Tables 6.1 to 6.4, in the form of Growth Factors and Damped Critical Speeds for the first 8 modes of the rotor system. Table 6.1 gives the results for the seal with straight grooves, a running speed of the rotor of 10,000 rpm, an operating eccentricity of the seal of 0.0001 and the seal in the locked configuration. Figure 6.3 shows the mode shape (the normalized values of the semi-major axis amplitude) of the rotor and the seal, when the seal is locked. It can be seen from the mode shape that the lowest frequency at 4800 RPM corresponds to the first mode of the rotor.

Table 6.2 gives the results for the same seal geometry and eccentricity ratio, but with the seal in the floating configuration. As can be seen from the above two sets of results, the modeling of the seal as a floating element introduces an additional frequency at 5704 RPM. As expected, this is the damped frequency of the seal. This fact can be ascertained by looking at the mode shape of the rotor and the seal at this frequency, as shown in Figs. 6.4a and 6.4b. The normalized value of the semi-major axis amplitude is again plotted, both for the rotor and the floating seal. The seal properties do not influence the rotor mode shapes and the damped rotor frequencies a great amount, as can be seen from Table 6.2. The first rotor mode is at 4557 RPM, which is 5.06% less than the first rotor mode obtained when the seal is locked.
Figure 6.3 Mode shapes for the First Two Critical Speeds for Locked Seal Case
Seal is locked; Running speed = 10,000 RPM; Seal Ecc. = 0.0001
Figure 6.4a Plot of First Mode of Rotor and Floating Seal

Figure 6.4b Plot of Second Mode of Rotor and Floating Seal

6.0 Analyses and Results of Multi-Level Rotor Systems
However, the growth factors which give a measure of the stability of the rotor system are relatively more influenced by the modeling of the seal as locked or floating. The influence of the type of modeling on the stability can be seen in Table 6.3 and Table 6.4. The Tables 6.3 and 6.4 give the results of the stability analysis of the rotor system, with the seal in the locked and floating configuration, respectively, but the operating eccentricity of the seal is 0.3. The locked seal configuration indicates that the first mode of the rotor system, at 4847 RPM, is close to instability, given by a growth factor of -1.37. But still the rotor is stable in this mode as indicated by the negative growth factor.

When the seal is modeled as floating, the additional frequency introduced as a result of this configuration at 5600 RPM makes the system unstable! This frequency, which corresponds to the damped frequency of the seal, has a growth factor of 2.06. The rotor by itself is stable, and the seal is unstable. But the rotor system is considered to be unstable if either the seal or the rotor is unstable. This set of results shows the importance of considering the seal geometry and including the seal into the model of the rotor system, while evaluating the stability of the system.
Table 6.1 Results of Stability Analysis for Locked Seal Configuration
(Seal Eccentricity = 0.0001; Running Speed = 10,000 RPM)

<table>
<thead>
<tr>
<th>Mode #</th>
<th>Growth Factor (sec^{-1})</th>
<th>Damped Critical Speed (Hz)</th>
<th>Damped Critical Speed (RPM)</th>
<th>Logarithmic decrement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-80.71</td>
<td>80.00</td>
<td>4800.0</td>
<td>1.009</td>
</tr>
<tr>
<td>2</td>
<td>-2.65</td>
<td>81.01</td>
<td>4860.8</td>
<td>0.025</td>
</tr>
<tr>
<td>3</td>
<td>-318.86</td>
<td>189.52</td>
<td>11371.5</td>
<td>1.682</td>
</tr>
<tr>
<td>4</td>
<td>-253.91</td>
<td>192.25</td>
<td>11535.2</td>
<td>1.321</td>
</tr>
<tr>
<td>5</td>
<td>-342.56</td>
<td>230.44</td>
<td>13826.4</td>
<td>1.487</td>
</tr>
<tr>
<td>6</td>
<td>-269.7</td>
<td>258.11</td>
<td>15486.9</td>
<td>1.045</td>
</tr>
<tr>
<td>7</td>
<td>-796.02</td>
<td>223.09</td>
<td>13385.3</td>
<td>3.568</td>
</tr>
<tr>
<td>8</td>
<td>-921.64</td>
<td>228.28</td>
<td>13697.1</td>
<td>4.037</td>
</tr>
</tbody>
</table>

Table 6.2 Results of Stability Analysis for Floating Seal Configuration
(Seal Eccentricity = 0.0001; Running Speed = 10,000 RPM)

<table>
<thead>
<tr>
<th>Mode #</th>
<th>Growth Factor (sec^{-1})</th>
<th>Damped Critical Speed (Hz)</th>
<th>Damped Critical Speed (RPM)</th>
<th>Logarithmic decrement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-6.76</td>
<td>75.96</td>
<td>4557.9</td>
<td>0.089</td>
</tr>
<tr>
<td>2</td>
<td>-7.89</td>
<td>79.53</td>
<td>4772.1</td>
<td>0.099</td>
</tr>
<tr>
<td>3</td>
<td>-12.06</td>
<td>95.08</td>
<td>5704.8</td>
<td>0.127</td>
</tr>
<tr>
<td>4</td>
<td>-13.58</td>
<td>95.15</td>
<td>5708.9</td>
<td>0.143</td>
</tr>
<tr>
<td>5</td>
<td>-271.99</td>
<td>184.27</td>
<td>11056.2</td>
<td>1.476</td>
</tr>
<tr>
<td>6</td>
<td>-307.38</td>
<td>186.81</td>
<td>11208.9</td>
<td>1.645</td>
</tr>
<tr>
<td>7</td>
<td>-676.36</td>
<td>215.67</td>
<td>12940.5</td>
<td>3.136</td>
</tr>
<tr>
<td>8</td>
<td>-695.67</td>
<td>221.17</td>
<td>13270.3</td>
<td>3.145</td>
</tr>
</tbody>
</table>
Table 6.3 Results of Stability Analysis for Locked Seal Configuration
(Seal Eccentricity = 0.3; Running Speed = 10,000 RPM)

<table>
<thead>
<tr>
<th>Mode #</th>
<th>Growth Factor (sec⁻¹)</th>
<th>Damped Critical Speed (Hz)</th>
<th>Damped Critical Speed (RPM)</th>
<th>Logarithmic decrement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-97.95</td>
<td>80.24</td>
<td>4814.3</td>
<td>1.221</td>
</tr>
<tr>
<td>2</td>
<td>-1.37</td>
<td>80.78</td>
<td>4847.1</td>
<td>0.017</td>
</tr>
<tr>
<td>3</td>
<td>-320.84</td>
<td>190.20</td>
<td>11411.9</td>
<td>1.687</td>
</tr>
<tr>
<td>4</td>
<td>-244.91</td>
<td>193.61</td>
<td>11616.9</td>
<td>1.265</td>
</tr>
<tr>
<td>5</td>
<td>-372.95</td>
<td>227.59</td>
<td>13655.4</td>
<td>1.639</td>
</tr>
<tr>
<td>6</td>
<td>-281.62</td>
<td>255.63</td>
<td>15337.6</td>
<td>1.102</td>
</tr>
<tr>
<td>7</td>
<td>-824.14</td>
<td>222.53</td>
<td>13351.8</td>
<td>3.703</td>
</tr>
<tr>
<td>8</td>
<td>-1000.09</td>
<td>225.86</td>
<td>13551.5</td>
<td>4.428</td>
</tr>
</tbody>
</table>

Table 6.4 Results of Stability Analysis for Floating Seal Configuration
(Seal Eccentricity = 0.3; Running Speed = 10,000 RPM)

<table>
<thead>
<tr>
<th>Mode #</th>
<th>Growth Factor (sec⁻¹)</th>
<th>Damped Critical Speed (Hz)</th>
<th>Damped Critical Speed (RPM)</th>
<th>Logarithmic decrement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-6.74</td>
<td>75.96</td>
<td>4557.9</td>
<td>0.089</td>
</tr>
<tr>
<td>2</td>
<td>-7.93</td>
<td>79.56</td>
<td>4773.7</td>
<td>0.099</td>
</tr>
<tr>
<td>3</td>
<td>2.06</td>
<td>93.34</td>
<td>5600.3</td>
<td>-0.022</td>
</tr>
<tr>
<td>4</td>
<td>0.72</td>
<td>93.37</td>
<td>5602.2</td>
<td>-0.008</td>
</tr>
<tr>
<td>5</td>
<td>-271.90</td>
<td>184.26</td>
<td>11055.7</td>
<td>1.476</td>
</tr>
<tr>
<td>6</td>
<td>-307.35</td>
<td>186.77</td>
<td>11205.9</td>
<td>1.646</td>
</tr>
<tr>
<td>7</td>
<td>-676.76</td>
<td>215.79</td>
<td>12947.4</td>
<td>3.132</td>
</tr>
<tr>
<td>8</td>
<td>-694.17</td>
<td>221.11</td>
<td>13266.76</td>
<td>3.139</td>
</tr>
</tbody>
</table>

6.0 Analyses and Results of Multi-Level Rotor Systems
6.2 Multi-level Modeling of Drum Rotor supported on AMB for Stability Analysis

The rotor model of the drum rotor supported on AMB is as shown in Fig. 2.9. This rotor was used to model the system as multi-levels (the rotor being the first level and the landing sleeves being the second level). The rotor was originally modeled with just one level. The landing sleeve, which is relatively small in this case, was modeled as lumped masses at the bearing locations. The results of the stability analysis of the multi-level model were compared to those of the single level rotor system. Though the landing sleeve in the rotor is relatively small and the equivalent single level rotor model gives quite accurate results, this model was used as a case study to prove the multi-level capability of the finite element program. This drum rotor is currently being used to study the drop of the rotor on the auxiliary bearings at the Virginia Tech Rotor Dynamics Laboratory. The details of the inboard and outboard ends of the rotor are as shown in Fig. 2.10. It can be seen that the landing sleeve is in contact (interference fit) with the rotor (shaft) over only a small portion. The landing sleeves at the two ends were modeled as shaft elements (on the second level) and the stability analysis of this 2-level system was performed. The results shown in Table 6.5 indicate good correlation on the first two modes. The frequencies introduced by this additional level appear further down in the frequency table (not shown), well beyond the first 12 modes of the single level rotor system. This is true since the contact was modeled with high values of stiffness and the landing sleeves are relatively short and rigid.
Table 6.5 Comparison of the Results of Stability Analysis for Single and Multi-level Models

<table>
<thead>
<tr>
<th>Mode</th>
<th>Single level rotor system</th>
<th>2-level rotor system</th>
<th>Exp. results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Growth Factor (1/sec)</td>
<td>Ncr (RPM)</td>
<td>Growth Factor (1/sec)</td>
</tr>
<tr>
<td>1</td>
<td>-78.74</td>
<td>2859</td>
<td>-76.28</td>
</tr>
<tr>
<td>2</td>
<td>-109.57</td>
<td>3239</td>
<td>-121.83</td>
</tr>
<tr>
<td>3</td>
<td>-20.63</td>
<td>6741</td>
<td>-21.31</td>
</tr>
</tbody>
</table>
6.3 Multi-level Analysis of Pump-Casing Systems

6.3.1 Stability Analysis of Pump and Casing as Separate Systems

In industrial applications, one of the most common type of turbomachinery which lends itself to multi-level modeling for rotor dynamic analysis is the multi-stage pump. The pump rotor which carries the multiple stages is typically about 0.1016 m - 0.1270 m (4.0 in. - 5.0 in.) in diameter. A casing surrounds the rotor and the stages, and is about 1.016 m (40.0 in.) in inner diameter and about 0.381 m - 0.0508 m (1.5 in. - 2.0 in.) thick. This casing is generally not modeled for the stability analysis and is assumed rigid. In actual practice, the casing does have some flexibility and does affect not only the damped natural frequencies of the system, but also the stability of the entire system. Generally, the casing modes are relatively higher than the rotor modes.

To study the effect of the casing modes on the damped critical speeds and the stability of the pump-casing system, the pump-casing was modeled using the multi-level capability of the finite element program. The pump (rotor) was modeled as the first level. The pump, shown in Fig. 6.5, has 9 intermediate stages and has a balance piston as shown in the figure at node # 17. The pump is supported by two fluid-film bearings. The 9 intermediate stages have seals (indicated by the symbol ‘S’) at the respective locations. The seals are modeled by their stiffness and damping properties at the respective locations along the length of the rotor.
Description: Rotor model of the pump with Balance drum — 1x clearance
Modified Pump Model With Seals; Single Level Rotor Model

Number of stations (nodes): 29
Bearings at (from left): 3 23
Seals at (from left): 8 9 10 11 12 13 14 15 16
Balance-Dr at (from left): 17

---

Outside dimension: 'B' - Bearings  'D' - Balance-Dr
Inside dimension: 'S' - Seals

Figure 6.5 Model of the Multi-Stage Pump used for Multi-level Stability Analysis
The pump with the bearings and seals was analyzed first to obtain the damped critical speeds and growth factors. The casing was analyzed separately to obtain the damped critical speeds and growth factors. Next, the pump and the casing were coupled to form the multi-level pump-casing system and stability analysis of this multi-level system was performed to see the influence of the coupled system on the frequencies and mode shapes. The frequencies and growth factors for the casing and pump analyzed separately are given in Tables 6.6 and 6.7.

Table 6.6 Stability Analysis of Casing Supported on Bearings at the Ends

<table>
<thead>
<tr>
<th>Mode #</th>
<th>Growth Factor (sec(^{-1}))</th>
<th>Critical Speed (Hz)</th>
<th>Critical Speed (rpm)</th>
<th>Logarithmic decrement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-17.49</td>
<td>214.65</td>
<td>12879.16</td>
<td>0.08</td>
</tr>
<tr>
<td>2</td>
<td>-17.49</td>
<td>214.64</td>
<td>12878.51</td>
<td>0.08</td>
</tr>
<tr>
<td>3</td>
<td>-64.39</td>
<td>388.35</td>
<td>23301.13</td>
<td>0.16</td>
</tr>
<tr>
<td>4</td>
<td>-64.52</td>
<td>389.55</td>
<td>23373.16</td>
<td>0.16</td>
</tr>
<tr>
<td>5</td>
<td>-69.35</td>
<td>1066.80</td>
<td>64008.11</td>
<td>0.06</td>
</tr>
<tr>
<td>6</td>
<td>-69.36</td>
<td>1066.20</td>
<td>63967.74</td>
<td>0.06</td>
</tr>
<tr>
<td>7</td>
<td>-63.38</td>
<td>1674.77</td>
<td>100486.38</td>
<td>0.04</td>
</tr>
<tr>
<td>8</td>
<td>-63.25</td>
<td>1672.83</td>
<td>100370.12</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Table 6.7 Stability Analysis of Pump with 9 Impeller Seals

<table>
<thead>
<tr>
<th>Mode #</th>
<th>Growth Factor (sec^{-1})</th>
<th>Critical Speed (Hz)</th>
<th>Critical Speed (rpm)</th>
<th>Logarithmic decrement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-854.70</td>
<td>32.38</td>
<td>1942.74</td>
<td>26.40</td>
</tr>
<tr>
<td>2</td>
<td>-864.74</td>
<td>42.33</td>
<td>2539.63</td>
<td>20.43</td>
</tr>
<tr>
<td>3</td>
<td>-182.05</td>
<td>65.46</td>
<td>3927.40</td>
<td>2.78</td>
</tr>
<tr>
<td>4</td>
<td>-49388.89</td>
<td>80.84</td>
<td>4850.27</td>
<td>610.96</td>
</tr>
<tr>
<td>5</td>
<td>-108.81</td>
<td>135.77</td>
<td>8145.98</td>
<td>0.80</td>
</tr>
<tr>
<td>6</td>
<td>-260.88</td>
<td>139.19</td>
<td>8351.71</td>
<td>1.87</td>
</tr>
<tr>
<td>7</td>
<td>-426.67</td>
<td>208.95</td>
<td>12537.12</td>
<td>2.04</td>
</tr>
<tr>
<td>8</td>
<td>-148.56</td>
<td>226.99</td>
<td>13619.25</td>
<td>0.65</td>
</tr>
<tr>
<td>9</td>
<td>-306.79</td>
<td>235.07</td>
<td>14104.32</td>
<td>1.30</td>
</tr>
<tr>
<td>10</td>
<td>-117.63</td>
<td>263.99</td>
<td>15839.22</td>
<td>0.44</td>
</tr>
<tr>
<td>11</td>
<td>-721.53</td>
<td>400.10</td>
<td>24006.20</td>
<td>1.80</td>
</tr>
<tr>
<td>12</td>
<td>-325.69</td>
<td>424.49</td>
<td>25469.24</td>
<td>0.77</td>
</tr>
</tbody>
</table>

The mode shapes of the first four damped critical speeds of the casing and pump, analyzed separately, are shown in Fig. 6.6 and Fig. 6.7, respectively. The normalized semi-major axis amplitudes for the first 4 pairs of damped frequencies are plotted in Figs. 6.8 (a-d) and 6.9 (a-d), for the casing and the pump. As seen from Table 6.6 and Figs. 6.8a - 6.8d, the frequencies of the casing are very much higher than those for the first 6 modes of the pump (Table 6.7). The first two modes of the pump are seen to be the rigid modes of the casing, i.e., the cylindrical and the conical mode, respectively. The bending mode of the casing is at 64008 rpm, shown in Fig. 6.8c. The mode shapes of the first four frequencies of the pump analyzed separately are shown in Figs. 6.9 (a-d).
Figure 6.6: Modeshapes for the First Four Critical Speeds of the Casing

6.0 Analyses and Results of Multi-Level Rotor Systems
Figure 6.7 Modeshapes for the First Four Critical Speeds of the Pump
Figure 6.8a Mode-shapes of the Casing for $N_{cr}$ #1 and #2

Figure 6.8b Mode-shapes of the Casing for $N_{cr}$ #3 and #4
Figure 6.8c Mode-shapes of the Casing for $N_c$ # 5 and # 6

Figure 6.8d Mode-shapes of the Casing for $N_c$ # 7 and # 8
Figure 6.9a Mode-shapes of the Pump for $N_{cr} \# 1$ and $\# 2$

Figure 6.9b Mode-shapes of the Pump for $N_{cr} \# 3$ and $\# 4$
Figure 6.9c Mode-shapes of the Pump for $N_{cr}^\#5$ and $N_{cr}^\#6$

Figure 6.9d Mode-shapes of the Pump for $N_{cr}^\#7$ and $N_{cr}^\#8$
6.3.2 Stability Analysis of Pump-Casing as 2-level System
(With 9 Inter-Stage Seals)

The results of the stability analysis of the pump-casing system are shown in Tables 6.8 and 6.9. Comparing the results from Table 6.7 and Table 6.8, it can be seen that the first 6 frequencies are not altered much relative to the higher modes. That is, the effect of the casing is not felt much in the first 6 modes of the pump-casing system. However, in general the stability is slightly reduced, indicated by the drop in growth factors. The effect of the casing can be seen from the plot of mode shapes of the 2-level system, plotted in Figs. 6.10 (a-h). The normalized semi-major axis amplitudes are plotted against the axis of the rotor for the first 16 modes of the dual level rotor system. From Figs. 6.10a and 6.10b, it is seen that the casing is essentially 'stationary' with respect to the pump mode shape. However at higher frequencies, as seen from Figs. 6.10c, the casing starts to move, though relatively much less than the pump. At the frequency of around 12000 rpm, the casing undergoes its first mode, seen in Figs. 6.10d. Figures 6.10e and 6.10f are essentially higher modes of the pump. The second mode of the casing is shown in Figs. 6.10g and 6.10h. Also observed from the above figures are that the pump undergoes modes with high bending. This analysis shows that the casing, which might be relatively rigid when compared to the pump, has to be considered in the complete analysis of such pump-casing systems. The casing not only affects the frequencies of the system but also affects the growth factors of the system.
Table 6.8 Stability Analysis of 2-level Level Pump-Casing system with 9 Impeller Seals

<table>
<thead>
<tr>
<th>Mode #</th>
<th>Growth Factor (sec⁻¹)</th>
<th>Critical Speed (Hz)</th>
<th>Critical Speed (rpm)</th>
<th>Logarithmic decrement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-851.82</td>
<td>32.51</td>
<td>1950.38</td>
<td>26.20</td>
</tr>
<tr>
<td>2</td>
<td>-851.95</td>
<td>42.19</td>
<td>2531.51</td>
<td>20.19</td>
</tr>
<tr>
<td>3</td>
<td>-182.09</td>
<td>65.48</td>
<td>3928.62</td>
<td>2.78</td>
</tr>
<tr>
<td>4</td>
<td>-60378.94</td>
<td>83.68</td>
<td>5020.64</td>
<td>721.57</td>
</tr>
<tr>
<td>5</td>
<td>-103.93</td>
<td>134.27</td>
<td>8056.20</td>
<td>0.77</td>
</tr>
<tr>
<td>6</td>
<td>-250.25</td>
<td>138.01</td>
<td>8280.50</td>
<td>1.81</td>
</tr>
<tr>
<td>7</td>
<td>-36.76</td>
<td>198.54</td>
<td>11912.57</td>
<td>0.18</td>
</tr>
<tr>
<td>8</td>
<td>-102.99</td>
<td>201.10</td>
<td>12066.31</td>
<td>0.51</td>
</tr>
<tr>
<td>9</td>
<td>-412.02</td>
<td>216.97</td>
<td>13018.06</td>
<td>1.90</td>
</tr>
<tr>
<td>10</td>
<td>-227.14</td>
<td>236.38</td>
<td>14182.61</td>
<td>0.64</td>
</tr>
<tr>
<td>11</td>
<td>-156.29</td>
<td>243.08</td>
<td>14584.57</td>
<td>0.64</td>
</tr>
<tr>
<td>12</td>
<td>-105.73</td>
<td>260.38</td>
<td>15622.76</td>
<td>0.41</td>
</tr>
</tbody>
</table>
Figure 6.10a Mode Shapes for Nt, #1, #2 for the Pump-Casing System

Figure 6.10b Mode Shapes for Nt, #3, #4 for the Pump-Casing System

6.0 Analyses and Results of Multi-Level Rotor Systems
Figure 6.10c Mode Shapes for $N_{\alpha}$ # 5, # 6 for the Pump-Casing System

Figure 6.10d Mode Shapes for $N_{\alpha}$ # 7, # 8 for the Pump-Casing System

6.0 Analyses and Results of Multi-Level Rotor Systems
Figure 6.10c Mode Shapes for \( N_e \) # 9, # 10 for the Pump-Casing System

Figure 6.10f Mode Shapes for \( N_e \) # 11, # 12 for the Pump-Casing System

6.0 Analyses
Figure 6.10g Mode Shapes for $N_{cr}$ #13, #14 for the Pump-Casing System

Figure 6.10h Mode Shapes for $N_{cr}$ #15, #16 for the Pump-Casing System
6.3.3 Stability Analysis of Pump-Casing as 2-level System
(Without Inter-Stage Seals)

To study the effect of the inter-stage seals on the stability, the stability of the above multi-stage pump without the 9 inter-stage seals is analyzed. A similar multi-level stability analysis was performed as described in section 6.3.2 and the results were obtained in terms of damped critical speeds, growth factors and mode shapes. The frequencies and the growth factors are shown in Table 6.9. Comparing the results in Table 6.8 and Table 6.9, it can be seen that the first 8 modes drop when the seals are removed. The growth factors vary by small amounts, decreasing in some cases and increasing for some modes. The most significant change is observed in the 5th and 6th modes, where the frequencies drop from around 8000 rpm to 5200 rpm. It can be concluded that these are the frequencies primarily governed by the inter-stage seals. The mode shapes for the first 16 frequencies are shown in Figs. 6.11a to 6.11h. The mode shapes of Figs. 6.10 (a-h) and those of Figs. 6.11 (a-h) are plotted to the same scale to make it easier to compare the effect of the seals on the mode shapes. Comparing the first two modes, i.e., Fig. 6.10a and Fig. 6.11a, the mode shapes are almost the same and the seals do not influence the first two modes much. The shapes of the rotor for the 3rd and 4th critical speeds are different as seen in Figs. 6.10b and 6.11b. The inter-stage seals seem to hold down the rotor along the midspan length of the rotor. This again is true for the higher modes also, which is seen by comparing Figs. 6.10c - 6.10h with Figs. 6.11c - 6.11h. It can be seen from this analysis that the seals do play a role in determining the frequencies and the mode shapes of the rotor system being analyzed.

6.0 Analyses and Results of Multi-Level Rotor Systems
Table 6.9 Stability Analysis of 2-level Level Pump-Casing System without Inter-Stage Impeller Seals

<table>
<thead>
<tr>
<th>Mode #</th>
<th>Growth Factor (sec(^{-1}))</th>
<th>Critical Speed (Hz)</th>
<th>Critical Speed (rpm)</th>
<th>Logarithmic decrement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-853.41</td>
<td>31.99</td>
<td>1919.32</td>
<td>26.68</td>
</tr>
<tr>
<td>2</td>
<td>-857.99</td>
<td>41.81</td>
<td>2508.41</td>
<td>20.52</td>
</tr>
<tr>
<td>3</td>
<td>-133.44</td>
<td>70.53</td>
<td>4232.10</td>
<td>1.89</td>
</tr>
<tr>
<td>4</td>
<td>-60378.92</td>
<td>83.68</td>
<td>5020.72</td>
<td>721.56</td>
</tr>
<tr>
<td>5</td>
<td>-50.97</td>
<td>88.24</td>
<td>5294.13</td>
<td>0.58</td>
</tr>
<tr>
<td>6</td>
<td>-15.53</td>
<td>92.61</td>
<td>5556.86</td>
<td>0.17</td>
</tr>
<tr>
<td>7</td>
<td>-34.08</td>
<td>197.82</td>
<td>11869.27</td>
<td>0.17</td>
</tr>
<tr>
<td>8</td>
<td>-128.97</td>
<td>202.27</td>
<td>12136.38</td>
<td>0.64</td>
</tr>
<tr>
<td>9</td>
<td>-325.47</td>
<td>225.99</td>
<td>13559.76</td>
<td>1.44</td>
</tr>
<tr>
<td>10</td>
<td>-164.36</td>
<td>228.33</td>
<td>13699.93</td>
<td>0.72</td>
</tr>
<tr>
<td>11</td>
<td>-45.50</td>
<td>236.33</td>
<td>14179.76</td>
<td>0.19</td>
</tr>
<tr>
<td>12</td>
<td>-102.65</td>
<td>261.94</td>
<td>15716.35</td>
<td>0.39</td>
</tr>
</tbody>
</table>
Figure 6.11a Mode Shapes for $N_n$ #1, #2 for the Pump-Casing System

Figure 6.11b Mode Shapes for $N_n$ #3, #4 for the Pump-Casing System
Figure 6.11 Mode Shapes for $N_\alpha$, #5, #6 for the Pump-Casing System

Figure 6.11d Mode Shapes for $N_\alpha$, #7, #8 for the Pump-Casing System

6.0 Analyses and Results of Multi-Level Rotor Systems
Figure 6.11e Mode Shape for $N_e$ #9, #10 for Pump-Casing System

Figure 6.11f Mode Shape for $N_e$ #11, #12 for Pump-Casing System

6.0 Analyses and Results of Multi-Level Rotor Systems
Figure 6.11g Mode Shape for $N_e$ # 13, # 14 for Pump-Casing System

Figure 6.11h Mode Shape for $N_e$ # 15, # 16 for Pump-Casing System

6.0 Analyses and Results of Multi-Level Rotor Systems
CHAPTER 7

ANALYSIS OF ACTIVE MAGNETIC BEARING TEST RIG

7.1 Introduction - Non-Synchronous Stability Evaluation

In the conventional fluid film bearings, the bearing characteristics change with the running speed of the rotor. The characteristics, viz., the stiffness and damping, are predominantly dependent on the spin (or running) speed of the rotor and much less on the whirl frequency (or the frequency of lateral vibration of the rotor). But in the Magnetic Bearings the sensor-controllers are so designed that the bearing properties depend on the whirl frequency rather than the running speed. This necessitates that for the stability analysis, the stability be evaluated using the bearing properties corresponding to the whirl speed of the rotor for the given mode. For example, to arrive at the first damped critical speed, the characteristics corresponding to the first whirl frequency have to be used in the program. Since the whirl frequency is not known apriori, this necessitates an iterative process, in which the properties corresponding to an initial speed (usually the running speed) is picked and then the stability is evaluated. Then the characteristics corresponding
to the newly obtained first (or the required) whirl frequency are used for further calculations. The above procedure can also be called non-synchronous stability evaluation, since the bearing characteristics do not correspond to the running, or synchronous, speed.

7.2 Stability Analysis of the AMB Drum Rotor at the Virginia Tech Rotor Dynamics Laboratory

The rotor model of the drum rotor supported on active magnetic bearings is shown in Fig. 7.1. The drum type rotor is typical of aero-engine applications. The drum rotor is hollow in cross-section at the mid-span. The two magnetic bearings that support the rotor have sensors located about 0.1778 m (7.0 in.) outboard at each end. The magnetic bearings at both ends of the rotor are inclined at 45° to the vertical. The bearings were tuned to a set of bearing characteristics, i.e., the stiffness and damping properties, which are shown in Figs. 7.2 and 7.3, respectively. The rotor was modeled together with the coupling/spacer tube and the pinion shaft, which drives the rotor. The entire train was modeled using the finite element program with 39 elements. The pinion shaft is supported on two fluid film 3-axial groove bearings. The stiffness properties of the fluid film bearings were calculated using a separate program (85) and were used in the stability analysis. The properties of the fluid film bearings were calculated almost to be constant over the speed range of interest. Both synchronous and non-synchronous stability evaluation were performed.
Figure 7.1 Model of the Drum Rotor supported on Active Magnetic Bearings
Figure 7.2 Plot of Stiffness vs. Frequency of the Magnetic Bearings
Figure 7.3 Plot of Damping vs. Frequency of the Magnetic Bearings
7.2.1 Synchronous Stability Evaluation

The initial stability evaluation was performed at the running speed of 4000 rpm, the design speed of the drum rotor, and synchronous bearing characteristics were assumed, i.e., the bearing stiffness and damping were selected at 4000 rpm. The bearing properties of the two magnetic bearings and the two fluid film bearings are given in Table 7.1. The results of the synchronous stability evaluation are shown in Table 7.2, given by the growth factors and the first 8 critical speeds. The mode shapes for the first 4 damped critical speeds are shown in Figs. 7.4 to 7.7.

Table 7.1 Synchronous Bearing Characteristics (Speed = 4000 RPM)

<table>
<thead>
<tr>
<th>Location (Node #)</th>
<th>$K_{sy}$ (N/m)</th>
<th>$K_{yz}$ (N/m)</th>
<th>$K_{zy}$ (N/m)</th>
<th>$K_{zz}$ (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$7.186 \times 10^6$</td>
<td>0.0</td>
<td>0.0</td>
<td>$7.186 \times 10^6$</td>
</tr>
<tr>
<td>13</td>
<td>$7.186 \times 10^6$</td>
<td>0.0</td>
<td>0.0</td>
<td>$7.186 \times 10^6$</td>
</tr>
<tr>
<td>33</td>
<td>$3.511 \times 10^8$</td>
<td>0.0</td>
<td>0.0</td>
<td>$3.511 \times 10^8$</td>
</tr>
<tr>
<td>36</td>
<td>$3.511 \times 10^8$</td>
<td>0.0</td>
<td>0.0</td>
<td>$3.511 \times 10^8$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Location (Node #)</th>
<th>$C_{sy}$ (N-s/m)</th>
<th>$C_{yz}$ (N-s/m)</th>
<th>$C_{zy}$ (N-s/m)</th>
<th>$C_{zz}$ (N-s/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$9.207 \times 10^6$</td>
<td>0.0</td>
<td>0.0</td>
<td>$9.207 \times 10^6$</td>
</tr>
<tr>
<td>13</td>
<td>$9.207 \times 10^6$</td>
<td>0.0</td>
<td>0.0</td>
<td>$9.207 \times 10^6$</td>
</tr>
<tr>
<td>33</td>
<td>$3.511 \times 10^8$</td>
<td>0.0</td>
<td>0.0</td>
<td>$3.511 \times 10^8$</td>
</tr>
<tr>
<td>36</td>
<td>$3.511 \times 10^8$</td>
<td>0.0</td>
<td>0.0</td>
<td>$3.511 \times 10^8$</td>
</tr>
<tr>
<td>Mode #</td>
<td>Growth Factor (sec(^{-1}))</td>
<td>Damped Critical Speed (Hz)</td>
<td>Damped Critical Speed (RPM)</td>
<td>Logarithmic decrement</td>
</tr>
<tr>
<td>-------</td>
<td>-----------------------------</td>
<td>---------------------------</td>
<td>-----------------------------</td>
<td>-----------------------</td>
</tr>
<tr>
<td>1</td>
<td>-65.46</td>
<td>50.99</td>
<td>3059</td>
<td>1.284</td>
</tr>
<tr>
<td>2</td>
<td>-65.51</td>
<td>51.02</td>
<td>3061</td>
<td>1.284</td>
</tr>
<tr>
<td>3</td>
<td>-85.14</td>
<td>58.79</td>
<td>3527</td>
<td>1.448</td>
</tr>
<tr>
<td>4</td>
<td>-86.72</td>
<td>60.43</td>
<td>3625</td>
<td>1.435</td>
</tr>
<tr>
<td>5</td>
<td>-18.25</td>
<td>110.98</td>
<td>6658</td>
<td>0.164</td>
</tr>
<tr>
<td>6</td>
<td>-20.17</td>
<td>112.50</td>
<td>6750</td>
<td>0.179</td>
</tr>
<tr>
<td>7</td>
<td>-0.95</td>
<td>197.26</td>
<td>11835</td>
<td>0.005</td>
</tr>
<tr>
<td>8</td>
<td>-1.01</td>
<td>199.89</td>
<td>11993</td>
<td>0.005</td>
</tr>
</tbody>
</table>
Figure 7.4 Mode-shape for the First Critical Speed of the Drum Rotor
Figure 7.5: Mode-shape for the Second Critical Speed of the Drum Rotor

7.0 Analysis of Active Magnetic Bearing Test Rig
Figure 7.6 Mode-shape for the Third Critical Speed of the Drum Rotor
Figure 7.7 Mode-shape for the Fourth Critical Speed of the Drum Rotor
7.2.2 Non-Synchronous Stability Evaluation

The rotor model of the Drum rotor supported on Active Magnetic Bearings is shown in Fig. 7.1. The following Tables 7.3 (a-b) show the results of the non-synchronous stability evaluation for the first eight modes of the Rotor supported on AMB, which is the test stand rotor at the Virginia Tech Rotor Dynamics Laboratory. Tables 7.4 (a-b) show the % deviation from the exact values. Tables 7.3a, 7.3b and 7.4a, 7.4b are the results for noncollocated sensor positions and Tables 7.5a, 7.5b and Tables 7.6a, 7.6b are for collocated sensor positions.

It is seen that while evaluating the lower modes for the corresponding non-synchronous bearing properties, the higher modes are not much affected, as can be seen in the upper triangle of the matrix. But the lower modes are seen to be more sensitive while evaluating the higher modes, as can be seen in the lower triangle of the matrix. This is because the lower modes (the first four modes) are predominantly more sensitive to the bearing stiffness and damping properties, whereas the higher modes, i.e., the bending modes, are more sensitive to the bending of the rotor.

Details of the rotor model:

(a) Rotor modeled with 39 elements (40 nodes)

(b) Accuracy for the iterations for the non-synchronous stability evaluation = 0.10 %
### Table 7.3a. Damped Critical Speeds (Sensors Noncollocated)

<table>
<thead>
<tr>
<th>1st Mode</th>
<th>2nd Mode</th>
<th>3rd Mode</th>
<th>4th Mode</th>
<th>5th Mode</th>
<th>6th Mode</th>
<th>7th Mode</th>
<th>8th Mode</th>
<th>Converged to %</th>
</tr>
</thead>
<tbody>
<tr>
<td>2816</td>
<td>2820</td>
<td>3156</td>
<td>3263</td>
<td>6625</td>
<td>6713</td>
<td>11834</td>
<td>11993</td>
<td>0.0377</td>
</tr>
<tr>
<td>2812</td>
<td>2816</td>
<td>3149</td>
<td>3257</td>
<td>6623</td>
<td>6710</td>
<td>11834</td>
<td>11993</td>
<td>0.0384</td>
</tr>
<tr>
<td>2934</td>
<td>2939</td>
<td>3285</td>
<td>3392</td>
<td>6639</td>
<td>6728</td>
<td>11835</td>
<td>11993</td>
<td>0.0788</td>
</tr>
<tr>
<td>2969</td>
<td>2974</td>
<td>3324</td>
<td>3431</td>
<td>6644</td>
<td>6734</td>
<td>11835</td>
<td>11993</td>
<td>0.0484</td>
</tr>
<tr>
<td>3416</td>
<td>3420</td>
<td>3826</td>
<td>3924</td>
<td>6730</td>
<td>6829</td>
<td>11836</td>
<td>11994</td>
<td>0.0216</td>
</tr>
<tr>
<td>3425</td>
<td>3430</td>
<td>3835</td>
<td>3932</td>
<td>6733</td>
<td>6813</td>
<td>11836</td>
<td>11995</td>
<td>0.0260</td>
</tr>
<tr>
<td>3670</td>
<td>3677</td>
<td>4261</td>
<td>4343</td>
<td>6792</td>
<td>6896</td>
<td>11838</td>
<td>11996</td>
<td>0.0220</td>
</tr>
<tr>
<td>3676</td>
<td>3683</td>
<td>4273</td>
<td>4355</td>
<td>6793</td>
<td>6898</td>
<td>11838</td>
<td>11996</td>
<td>0.0228</td>
</tr>
</tbody>
</table>

### Table 7.3b. Growth Factors (Sensors Noncollocated)

<table>
<thead>
<tr>
<th>1st Mode</th>
<th>2nd Mode</th>
<th>3rd Mode</th>
<th>4th Mode</th>
<th>5th Mode</th>
<th>6th Mode</th>
<th>7th Mode</th>
<th>8th Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>-80.52</td>
<td>-80.60</td>
<td>-108.61</td>
<td>-111.61</td>
<td>-17.33</td>
<td>-19.18</td>
<td>-1.69</td>
<td>-1.80</td>
</tr>
<tr>
<td>-81.93</td>
<td>-82.00</td>
<td>-110.61</td>
<td>-113.68</td>
<td>-17.56</td>
<td>-19.43</td>
<td>-1.72</td>
<td>-1.83</td>
</tr>
<tr>
<td>-77.50</td>
<td>-77.55</td>
<td>-104.07</td>
<td>-106.70</td>
<td>-17.47</td>
<td>-19.35</td>
<td>-1.64</td>
<td>-1.75</td>
</tr>
<tr>
<td>-75.51</td>
<td>-75.56</td>
<td>-101.28</td>
<td>-103.75</td>
<td>-17.31</td>
<td>-19.17</td>
<td>-1.61</td>
<td>-1.71</td>
</tr>
<tr>
<td>-45.11</td>
<td>-44.57</td>
<td>-65.85</td>
<td>-66.24</td>
<td>-13.21</td>
<td>-14.67</td>
<td>-1.09</td>
<td>-1.16</td>
</tr>
<tr>
<td>-44.19</td>
<td>-43.61</td>
<td>-65.04</td>
<td>-65.37</td>
<td>-12.99</td>
<td>-14.44</td>
<td>-1.08</td>
<td>-1.14</td>
</tr>
<tr>
<td>-30.30</td>
<td>-30.74</td>
<td>-50.38</td>
<td>-50.28</td>
<td>-12.47</td>
<td>-13.82</td>
<td>-0.835</td>
<td>-0.886</td>
</tr>
<tr>
<td>-29.92</td>
<td>-30.35</td>
<td>-49.97</td>
<td>-49.86</td>
<td>-12.42</td>
<td>-13.76</td>
<td>-0.828</td>
<td>-0.879</td>
</tr>
</tbody>
</table>

7.0 Analysis of Active Magnetic Bearing Test Rig

187
Table 7.4a. Damped Critical Speeds (Sensors Collocated)

<table>
<thead>
<tr>
<th>1st Mode</th>
<th>2nd Mode</th>
<th>3rd Mode</th>
<th>4th Mode</th>
<th>5th Mode</th>
<th>6th Mode</th>
<th>7th Mode</th>
<th>8th Mode</th>
<th>Converged to %</th>
</tr>
</thead>
<tbody>
<tr>
<td>2786</td>
<td>2789</td>
<td>3213</td>
<td>3321</td>
<td>6620</td>
<td>6707</td>
<td>11836</td>
<td>11995</td>
<td>0.0351</td>
</tr>
<tr>
<td>2787</td>
<td>2790</td>
<td>3215</td>
<td>3322</td>
<td>6620</td>
<td>6707</td>
<td>11836</td>
<td>11995</td>
<td>0.0357</td>
</tr>
<tr>
<td>2938</td>
<td>2941</td>
<td>3390</td>
<td>3497</td>
<td>6640</td>
<td>6729</td>
<td>11837</td>
<td>11995</td>
<td>0.0620</td>
</tr>
<tr>
<td>2970</td>
<td>2973</td>
<td>3428</td>
<td>3534</td>
<td>6646</td>
<td>6735</td>
<td>11837</td>
<td>11996</td>
<td>0.0312</td>
</tr>
<tr>
<td>3381</td>
<td>3384</td>
<td>3914</td>
<td>4013</td>
<td>6724</td>
<td>6822</td>
<td>11839</td>
<td>11998</td>
<td>0.0194</td>
</tr>
<tr>
<td>3390</td>
<td>3392</td>
<td>3923</td>
<td>4021</td>
<td>6726</td>
<td>6824</td>
<td>11839</td>
<td>11998</td>
<td>0.0235</td>
</tr>
<tr>
<td>3634</td>
<td>3639</td>
<td>4354</td>
<td>4439</td>
<td>6778</td>
<td>6882</td>
<td>11841</td>
<td>12000</td>
<td>0.0329</td>
</tr>
<tr>
<td>3640</td>
<td>3645</td>
<td>4366</td>
<td>4451</td>
<td>6779</td>
<td>6883</td>
<td>11842</td>
<td>12000</td>
<td>0.0343</td>
</tr>
</tbody>
</table>

Table 7.4b. Growth Factors (Sensors Collocated)

<table>
<thead>
<tr>
<th>1st Mode</th>
<th>2nd Mode</th>
<th>3rd Mode</th>
<th>4th Mode</th>
<th>5th Mode</th>
<th>6th Mode</th>
<th>7th Mode</th>
<th>8th Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>-81.03</td>
<td>-80.95</td>
<td>-110.95</td>
<td>-114.00</td>
<td>-17.25</td>
<td>-19.10</td>
<td>-1.70</td>
<td>-1.81</td>
</tr>
<tr>
<td>-80.93</td>
<td>-81.01</td>
<td>-110.92</td>
<td>-113.97</td>
<td>-17.26</td>
<td>-19.11</td>
<td>-1.70</td>
<td>-1.81</td>
</tr>
<tr>
<td>-74.90</td>
<td>-74.96</td>
<td>-102.30</td>
<td>-104.79</td>
<td>-17.03</td>
<td>-18.88</td>
<td>-1.60</td>
<td>-1.70</td>
</tr>
<tr>
<td>-73.03</td>
<td>-73.08</td>
<td>-99.74</td>
<td>-102.09</td>
<td>-16.87</td>
<td>-18.70</td>
<td>-1.56</td>
<td>-1.66</td>
</tr>
<tr>
<td>-44.06</td>
<td>-44.43</td>
<td>-65.92</td>
<td>-66.48</td>
<td>-12.79</td>
<td>-14.25</td>
<td>-1.08</td>
<td>-1.14</td>
</tr>
<tr>
<td>-43.16</td>
<td>-43.55</td>
<td>-65.08</td>
<td>-65.60</td>
<td>-12.57</td>
<td>-14.01</td>
<td>-1.06</td>
<td>-1.13</td>
</tr>
<tr>
<td>-29.97</td>
<td>-30.33</td>
<td>-50.34</td>
<td>-50.31</td>
<td>-11.94</td>
<td>-13.29</td>
<td>-0.821</td>
<td>-0.871</td>
</tr>
<tr>
<td>-29.59</td>
<td>-29.96</td>
<td>-49.93</td>
<td>-49.88</td>
<td>-11.88</td>
<td>-13.23</td>
<td>-0.815</td>
<td>-0.864</td>
</tr>
</tbody>
</table>

7.0 Analysis of Active Magnetic Bearing Test Rig
Table 7.5a. Damped Critical Speeds (Sensors Noncollocated)
(% deviation from the “exact” value)

<table>
<thead>
<tr>
<th>Mode</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
<th>7th</th>
<th>8th</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.0</td>
<td>0.14</td>
<td>-3.93</td>
<td>-4.89</td>
<td>-1.56</td>
<td>-1.73</td>
<td>-0.03</td>
<td>-0.02</td>
<td></td>
</tr>
<tr>
<td>-0.14</td>
<td>0.0</td>
<td>-4.14</td>
<td>-5.07</td>
<td>-1.59</td>
<td>-1.77</td>
<td>-0.03</td>
<td>-0.02</td>
<td></td>
</tr>
<tr>
<td>4.19</td>
<td>4.37</td>
<td>0.0</td>
<td>-1.14</td>
<td>-1.35</td>
<td>-1.51</td>
<td>-0.02</td>
<td>-0.02</td>
<td></td>
</tr>
<tr>
<td>5.43</td>
<td>5.61</td>
<td>1.19</td>
<td>0.0</td>
<td>-1.38</td>
<td>-1.42</td>
<td>-0.02</td>
<td>-0.02</td>
<td></td>
</tr>
<tr>
<td>21.31</td>
<td>21.45</td>
<td>16.47</td>
<td>14.37</td>
<td>0.0</td>
<td>-0.03</td>
<td>-0.02</td>
<td>-0.02</td>
<td></td>
</tr>
<tr>
<td>21.63</td>
<td>21.80</td>
<td>16.74</td>
<td>14.60</td>
<td>0.04</td>
<td>0.0</td>
<td>-0.02</td>
<td>-0.02</td>
<td></td>
</tr>
<tr>
<td>30.33</td>
<td>30.57</td>
<td>29.71</td>
<td>26.58</td>
<td>0.92</td>
<td>0.95</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>30.54</td>
<td>30.79</td>
<td>30.08</td>
<td>26.93</td>
<td>0.94</td>
<td>0.98</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.5b. Growth Factors (Sensors Noncollocated)
(% deviation from the “exact” value)

<table>
<thead>
<tr>
<th>Mode</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
<th>7th</th>
<th>8th</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.0</td>
<td>-1.71</td>
<td>4.36</td>
<td>7.57</td>
<td>31.19</td>
<td>32.82</td>
<td>102.39</td>
<td>104.78</td>
<td></td>
</tr>
<tr>
<td>1.75</td>
<td>0.0</td>
<td>6.28</td>
<td>9.57</td>
<td>32.93</td>
<td>34.56</td>
<td>105.99</td>
<td>108.19</td>
<td></td>
</tr>
<tr>
<td>-3.75</td>
<td>-5.43</td>
<td>0.0</td>
<td>2.84</td>
<td>32.25</td>
<td>34.00</td>
<td>96.41</td>
<td>99.09</td>
<td></td>
</tr>
<tr>
<td>-6.22</td>
<td>-7.85</td>
<td>-2.68</td>
<td>0.0</td>
<td>31.04</td>
<td>32.76</td>
<td>92.81</td>
<td>94.54</td>
<td></td>
</tr>
<tr>
<td>-43.98</td>
<td>-45.65</td>
<td>-36.72</td>
<td>-36.15</td>
<td>0.0</td>
<td>1.59</td>
<td>30.54</td>
<td>31.97</td>
<td></td>
</tr>
<tr>
<td>-45.12</td>
<td>-46.82</td>
<td>-37.50</td>
<td>-36.99</td>
<td>-1.66</td>
<td>0.0</td>
<td>29.34</td>
<td>29.69</td>
<td></td>
</tr>
<tr>
<td>-62.37</td>
<td>-62.51</td>
<td>-51.59</td>
<td>-51.54</td>
<td>-5.60</td>
<td>-4.29</td>
<td>0.0</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>-62.84</td>
<td>-62.99</td>
<td>-51.98</td>
<td>-51.94</td>
<td>-5.98</td>
<td>-4.71</td>
<td>-0.84</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>
Table 7.6a. Damped Critical Speeds (Sensors Collocated) (% deviation from the “exact” value)

<table>
<thead>
<tr>
<th>1st Mode</th>
<th>2nd Mode</th>
<th>3rd Mode</th>
<th>4th Mode</th>
<th>5th Mode</th>
<th>6th Mode</th>
<th>7th Mode</th>
<th>8th Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>-0.03</td>
<td>5.22</td>
<td>-6.03</td>
<td>-1.55</td>
<td>-1.71</td>
<td>-0.04</td>
<td>-0.04</td>
</tr>
<tr>
<td>0.03</td>
<td>0.0</td>
<td>-5.16</td>
<td>-5.99</td>
<td>-1.55</td>
<td>-1.71</td>
<td>-0.04</td>
<td>-0.04</td>
</tr>
<tr>
<td>5.45</td>
<td>5.41</td>
<td>0.0</td>
<td>-1.05</td>
<td>-1.25</td>
<td>-1.39</td>
<td>-0.03</td>
<td>-0.04</td>
</tr>
<tr>
<td>6.60</td>
<td>6.56</td>
<td>1.12</td>
<td>0.0</td>
<td>-1.16</td>
<td>-1.30</td>
<td>-0.03</td>
<td>-0.03</td>
</tr>
<tr>
<td>21.36</td>
<td>21.29</td>
<td>15.46</td>
<td>13.55</td>
<td>0.0</td>
<td>-0.03</td>
<td>-0.02</td>
<td>-0.02</td>
</tr>
<tr>
<td>21.68</td>
<td>21.58</td>
<td>15.72</td>
<td>13.78</td>
<td>0.03</td>
<td>0.0</td>
<td>-0.02</td>
<td>-0.02</td>
</tr>
<tr>
<td>30.44</td>
<td>30.43</td>
<td>28.44</td>
<td>25.61</td>
<td>0.80</td>
<td>0.85</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>30.65</td>
<td>30.64</td>
<td>28.79</td>
<td>25.95</td>
<td>0.82</td>
<td>0.86</td>
<td>0.01</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 7.6b. Growth Factors (Sensors Noncollocated) (% deviation from the “exact” value)

<table>
<thead>
<tr>
<th>1st Mode</th>
<th>2nd Mode</th>
<th>3rd Mode</th>
<th>4th Mode</th>
<th>5th Mode</th>
<th>6th Mode</th>
<th>7th Mode</th>
<th>8th Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>-0.07</td>
<td>8.45</td>
<td>11.67</td>
<td>34.87</td>
<td>36.33</td>
<td>107.06</td>
<td>109.49</td>
</tr>
<tr>
<td>-0.12</td>
<td>0.0</td>
<td>8.43</td>
<td>11.64</td>
<td>34.95</td>
<td>36.40</td>
<td>107.06</td>
<td>109.49</td>
</tr>
<tr>
<td>-7.56</td>
<td>-7.47</td>
<td>0.0</td>
<td>2.64</td>
<td>33.15</td>
<td>34.76</td>
<td>94.88</td>
<td>96.76</td>
</tr>
<tr>
<td>-9.87</td>
<td>-9.79</td>
<td>-2.50</td>
<td>0.0</td>
<td>31.90</td>
<td>33.48</td>
<td>90.01</td>
<td>92.13</td>
</tr>
<tr>
<td>-45.62</td>
<td>-45.15</td>
<td>-35.56</td>
<td>-34.88</td>
<td>0.0</td>
<td>1.71</td>
<td>31.55</td>
<td>31.94</td>
</tr>
<tr>
<td>-46.73</td>
<td>-46.24</td>
<td>-36.38</td>
<td>-35.74</td>
<td>-1.72</td>
<td>0.0</td>
<td>29.11</td>
<td>30.79</td>
</tr>
<tr>
<td>-63.01</td>
<td>-62.56</td>
<td>-50.79</td>
<td>-50.72</td>
<td>-6.64</td>
<td>-5.14</td>
<td>0.0</td>
<td>0.81</td>
</tr>
<tr>
<td>-63.48</td>
<td>-63.02</td>
<td>-51.19</td>
<td>-51.14</td>
<td>-7.11</td>
<td>-5.57</td>
<td>-0.73</td>
<td>0.0</td>
</tr>
</tbody>
</table>

7.0 Analysis of Active Magnetic Bearing Test Rig 190
7.3 Unbalance Response Analysis of the AMB Drum Rotor

The Drum rotor supported on magnetic bearings was analyzed for unbalance response for different unbalance magnitudes and locations. The unbalance masses, in the form of small bolts of known weights, can be placed on the two disks on either side of the hollow drum (at the mid-span location). The disks have 30 holes equally spaced along a circumference of radius of 0.127 m (5.0 in.). Also, unbalance weights can be placed at 4 equally spaced holes at a radius of 0.0635 m (2.5 in.) at the ends of the coupling -- one end is the rotor end and the other end is the pinion end. Experimentally, the running speed of the rotor was increased to a maximum speed of 4800 rpm and then the drive motor was switched off, allowing the rotor to coast down on the magnetic bearings. During the coast down the rotor essentially rotates on the film of air, levitated by the magnetic bearings. The readings of the probes at 4 different locations along the length of the rotor (2 sensor positions, 1 near the coupling end of the rotor and 1 near the pinion end of the coupling) were recorded on an ADRE Data Acquisition System.

The rotor system was modeled using the finite element program and the model is shown in Fig. 7.8, with the unbalance weight at the outboard disk location. The response calculations were performed for different locations of the unbalance weights -- (1) 0.0046 kg. at 0.127m at the outboard disk; (2) 0.0046 kg. at 0.127m at the inboard disk; (3) 0.0031 kg. at 0.0635 m at the rotor end of the coupling and (4) 0.00476 kg. at 0.0635 m at the pinion end of the coupling. The plots of the response for the above set of unbalance weights at the respective locations are shown in Figs. 7.9 - 7.12. The responses of rotor at
the four different running speeds (2000 rpm, 2500 rpm, 3500 rpm and 5000 rpm), for the unbalance weight of 4.66g on the outboard disk are shown in Figs. 7.13 - 7.16. From the response plots shown in Figs. 7.9 to 7.12, it can be observed that the coupling end of the rotor is relatively very much sensitive to the above unbalance locations. There are no sharp peaks in the plots, which may be due to the distributed weight on the system. Unbalance in the pinion end of the coupling does not have much effect on the response of the rotor at the given four locations. The 3-D response plots shown in Figs. 7.13 to 7.16 for the unbalance at the outboard disk location give a good picture of the response along the length of the rotor.
Description: Response of the Drum-type rotor — Magnetic Bearing Test Rig
Unbalance of 4.66g @ 5in. @ hole #1; Disk #1 (Outboard)

Number of stations (nodes): 40
Bearings at (from left): 5 13 33 36
Sensors at (from left): 6 12
Unbalance at (from left): 7

Figure 7.8 Model of the Drum Rotor showing the Unbalance Location
Figure 7.9 Response of Rotor for Unbalance at Outboard Disk
Unbalance of 0.00466 kgs at 0.127 m (5.0 in.) radius
Figure 7.10 Response of Rotor for Unbalance at Inboard Disk
Unbalance of 0.00466 kgs. at 0.127m (5.0 in.) radius
Figure 7.11 Response of Rotor for Unbalance at Rotor end of Coupling
Unbalance of 0.0031 kgs. at 0.0635m (5.0 in.) radius
Figure 7.12 Response of Rotor for Unbalance at Pinion end of Coupling
Unbalance of 0.00476 kgs. at 0.0635m (5.0 in.) radius
Figure 7.13 Response of the Drum Rotor at 2000 RPM for Unbalance on Outboard Disk
Figure 7.14 Response of the Drum Rotor at 2500 RPM for Unbalance on Outboard Disk
Figure 7.15 Response of the Drum Rotor at 3500 RPM for Unbalance on Outboard Disk
Figure 7.16 Response of the Drum Rotor at 5000 RPM for Unbalance on Outboard Disk
CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

A PC based Finite Element Program has been developed for performing the stability analysis and unbalance response of complex rotor-bearing systems. The previously existing state-of-the-art capabilities included in the new finite element program are as follows:

- evaluation of the matrices based on distributed mass and stiffness properties;
- includes effects of bending stiffness, rotatory inertia, shear deformation, internal damping and axial load.

The new state-of-the-art capabilities included in the program are the following:

- The program can model multi-level rotor systems considering general elliptic motion for both stability and forced response to unbalance.
- The program can evaluate the influence of non-linear squeeze film dampers for the forced response analysis, including general elliptic motion for multi-level flexible pedestals.
• The Finite Element Program can model rotors supported on magnetic bearings including the sensor-noncollocation and non-synchronous bearing characteristics that have not been previously accounted for by the finite element rotor dynamic programs.

• The program also has the option to include the transfer function governing the feedback control system of the AMB, into the rotor model for stability evaluation.

• The program post-processor includes a newly developed unique capability to show the damped mode shapes of the complex eigenvectors generated by the stability analysis.

• The post-processor also has the ability to animate the damped forced response mode shape with the added feature to show the response orbit at any two selected rotor stations.

The conclusions obtained from the current research work can be cited as follows.

1. The sensor positions play a major role in the stability and response of the turbomachinery supported on AMB. For proper stability evaluation and response calculations the sensor noncollocation has to be considered.

2. The frequency dependent bearing characteristics should be used for proper stability analysis prediction for rotors supported on AMB. Lower stiffness with moderate to high damping was shown (section 5.1.1) to prove to increase the system instability threshold for resistance to aerodynamic excitation (or labyrinth seal excitation).

8.0 Conclusions and Recommendations

203
3. While evaluating the response of rotors supported on AMB, the peak response frequencies of the rotor system changes with change in the sensor position. In section 5.1.3 it was shown that the first critical frequency increases as the sensors are moved inboard and the third critical frequency drops as the sensors are moved inboard.

4. For rotors mounted on AMB and flexible pedestals, optimum values of pedestal damping and pedestal stiffness can be obtained such that the stability is maximum. However, if the rotor is inherently unstable due to sensor non-colllocation, the rotor cannot be made stable by introducing a flexible pedestal.

5. For rotors supported on squeeze film dampers, the design procedure described in section 5.2 can be adopted to obtain different configurations of the damper, and the stability analysis needs to be performed only for one configuration.

6. The clearance of the squeeze film damper and the retainer stiffness can be optimized to obtain maximum stability. However, care should be taken in designing the damper, since the stability is very sensitive to the changes in the clearances, operating eccentricity ratio, and non-linear hardening retainer stiffness characteristics.

7. The non-linear behavior of the squeeze film damper has to be considered while evaluating the response of rotors. The results obtained by the above procedure can be entirely different from the more commonly assumed centralized damper configuration.

8. The newly developed PC based finite element program can successfully model multi-level rotors, viz., rotor-casing systems, rotor-floating seal systems and rotor-pedestal systems.

8.0 Conclusions and Recommendations
9. While modeling pumps, the casing is not always "rigid" as it is more commonly modeled. The flexibility of the casing influences not only the damped critical speeds of the rotor, but also plays a significant role on the growth factors (stability values). Generally the casing frequencies are relatively higher than the first 4-6 modes of the pump rotor, but its influence is seen on the higher modes and frequencies. Hence for a complete analysis of such systems, both the rotor and the casing should be modeled and studied.

10. The floating seals can lead to unstable system behavior, depending on the seal characteristics. The rotor might be inherently stable, and the floating seal may become unstable. So it is important to model the seal and include it in the system equation of motion, to evaluate the dynamics of the seal and its influence on the dynamics (stability and damped critical frequencies) of the rotor.
Recommendations

A PC-based finite element program has been successfully developed to model linear rotor stability and unbalance response analysis. However, the following recommendations can be considered for further development.

- The fluid-film bearing characteristics also include non-linear terms, which might be significant in evaluating the stability of the rotor-bearing system. The modeling can be modified to include such non-linearities in the bearing characteristics.

- For rotors supported on AMB, the equation of motion is linearized while deriving the forces of the magnetic bearing. The linearity is good as long as the motions of the rotor are confined to small values, which is sufficient for initial stability analysis. However, if relatively large rotor displacements are encountered, the forces are highly non-linear and a suitable algorithm has to be developed to incorporate these non-linear forces.

- When large-diameter hollow rotors are modeled, the “shell modes” of the hollow rotor are excited and have to be considered. The rotor model must be capable of modeling the shell structure.

- In some applications of turbomachinery, the thrust loads are quite significant. This gives rise to axial motion, and hence axial frequencies also. Suitable modifications can be made to include the axial frequencies and to study the coupling between the axial and the transverse vibrations of the turbomachinery.
REFERENCES


47. Simandiri, S., and Hahn, E.J., "Effect of Pressurization on the Vibration Isolation


References 211


References


APPENDIX

Shape Functions:

\[ \psi_i (s) = \frac{1}{1 + \Phi} [\alpha_i(s) + \Phi \beta_i(s)] \quad i = 1, 2, 3, 4 \]

\[ \alpha_1 = (1 - 3\bar{v}^2 + 2\bar{v}^3) \quad \beta_1 = (1 - \bar{v}) \]
\[ \alpha_2 = \ell(\bar{v} - 2\bar{v}^2 + \bar{v}^3) \quad \beta_2 = \frac{\ell}{2} (\bar{v} - \bar{v}^2) \]
\[ \alpha_3 = (3\bar{v}^2 - 2\bar{v}^3) \quad \beta_3 = \bar{v} \]
\[ \alpha_4 = \ell(-\bar{v}^2 + \bar{v}^3) \quad \beta_4 = \frac{\ell}{2} (-\bar{v} + \bar{v}^2) \]

where, \( \bar{v} = \frac{s}{\ell} ; \Phi = \frac{12EI}{kAG\ell^2} \)

\[ \phi_i (s) = \frac{1}{1 + \Phi} [\varepsilon_i(s) + \Phi \delta_i(s)] \quad i = 1, 2, 3, 4 \]
\[ \varepsilon_1 = \frac{1}{\ell}(6\bar{v}^2 - 6\bar{v}) \quad \delta_1 = 0 \]
\[ \varepsilon_2 = (1 - 4\bar{v} + 3\bar{v}^2) \quad \delta_2 = (1 - \bar{v}) \]
\[ \varepsilon_3 = \frac{1}{\ell}(-6\bar{v}^2 + 6\bar{v}) \quad \delta_3 = 0 \]
\[ \varepsilon_4 = (3\bar{v}^2 - 2\bar{v}) \quad \delta_4 = \bar{v} \]

where, \( \bar{v} = \frac{s}{\ell} ; \Phi = \frac{12EI}{kAG\ell^2} \)
1. Element Matrices for the Rotor

A. Stiffness Matrix \([K]\)

\[
[K] = \frac{EI}{\ell^3(1+\Phi^2)} \begin{bmatrix}
12 & \text{SYM.} \\
0 & 12 \\
0 & -6\ell (4+\Phi)\ell^2 \\
6\ell & 0 & 0 & (4+\Phi)\ell^2 \\
-12 & 0 & 0 & -6\ell & 12 \\
0 & -12 & 6\ell & 0 & 0 & 12 \\
0 & -6\ell (2-\Phi)\ell^2 & 0 & 0 & 6\ell & (4+\Phi)\ell^2 \\
6\ell & 0 & 0 & (2-\Phi)\ell^2 & -6\ell & 0 & 0 & (4+\Phi)\ell^2 \\
\end{bmatrix}
\]

B. Translational Mass Matrix \([M]\)

\[
[M] = \frac{m\ell}{420(1+\Phi^2)} \begin{bmatrix}
m_1 & \text{SYM.} \\
0 & m_1 \\
0 & -m_3 & m_2 \\
m_3 & 0 & 0 & m_2 \\
m_4 & 0 & 0 & m_5 & m_1 \\
0 & m_4 & -m_5 & 0 & 0 & m_1 \\
0 & m_5 & -m_6 & 0 & 0 & m_3 & m_2 \\
-m_5 & 0 & 0 & -m_6 & -m_3 & 0 & 0 & m_2 \\
\end{bmatrix}
\]

where,

\[
m_1 = (156 + 294\Phi + 140\Phi^2)\ell \\
m_2 = (4 + 7\Phi + 3.5\Phi^2)\ell^2 \\
m_3 = (22 + 38.5\Phi + 17.5\Phi^2)\ell \\
m_4 = (54 + 126\Phi + 70\Phi^2) \\
m_5 = (13 + 31.5\Phi + 17.5\Phi^2)\ell \\
m_6 = (3 + 7\Phi + 3.5\Phi^2)\ell^2
\]
C. Rotational Mass Matrix $[N]$

$$[N] = \frac{mr^2}{120\ell(1+\Phi)^2} \begin{bmatrix}
  n_1 & & & & & \\
  0 & n_1 & & & & \\
  0 & -n_3 & n_2 & & & \\
  n_3 & 0 & 0 & n_2 & & \\
  -n_1 & 0 & 0 & -n_3 & n_1 & \\
  0 & -n_1 & n_3 & 0 & 0 & n_1 \\
  0 & -n_3 & -n_4 & 0 & 0 & n_3 & n_2 \\
  n_3 & 0 & 0 & -n_4 & -n_3 & 0 & 0 & n_2
\end{bmatrix}$$

where,

$$n_1 = 36$$

$$n_2 = (4 + 5\Phi + 10\Phi^2)\ell^2$$

$$n_3 = (3 - 15\Phi)\ell$$

$$n_4 = (1 + 5\Phi - 5\Phi^2)\ell^2$$
D. Gyroscopic Matrix [G]

\[
[G] = \frac{2mr^2}{120\ell(1+\Phi)^2}
\begin{bmatrix}
0 & \text{Skew Sym.} \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \\
& \
\end{bmatrix}
\]

where,

\[
g_1 = 36
\]
\[
g_2 = (3 - 15\Phi)\ell
\]
\[
g_3 = (4 + 5\Phi + 10\Phi^2)\ell^2
\]
\[
g_4 = (1 + 5\Phi - 5\Phi^2)\ell^2
\]

E. Axial Load Stiffness Matrix

\[
[A] = \frac{P}{30\ell(1+\Phi)^2}
\begin{bmatrix}
a_1 & \text{SYM.} \\
0 & a_1 \\
0 & a_3 \\
a_3 & 0 & a_2 \\
-a_1 & 0 & 0 & -a_3 & a_1 \\
0 & -a_1 & a_3 & 0 & 0 & a_1 \\
0 & -a_3 & -a_4 & 0 & 0 & a_3 & a_2 \\
a_3 & 0 & -a_4 & -a_3 & 0 & 0 & a_2
\end{bmatrix}
\]

where,

\[
a_1 = (36 + 60\Phi + 30\Phi^2)
\]
\[
a_2 = (4 + 5\Phi + 2.5\Phi^2)\ell^2
\]
\[
a_3 = 3\ell
\]
\[
a_4 = (1 + 5\Phi + 2.5\Phi^2)\ell^2
\]
2. Matrices for the Rigid disk

A. Translational Mass Matrix

$$[M_T^d] = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

B. Rotational Mass Matrix

$$[M_R^d] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I_T & 0 \\ 0 & 0 & 0 & I_T \end{bmatrix}$$

C. Gyroscopic Matrix

$$[G^d] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I_p & 0 \\ 0 & 0 & -I_p & 0 \end{bmatrix}$$
VITA

Krishnaswamy (Krish) Ramesh was born November 3, 1965 in Bangalore, India. He graduated from Rashtriya Vidyalaya College of Engineering, Bangalore in Mechanical Engineering. He did his Master's at Indian Institute of Technology, Madras in Industrial Tribology. His thesis work was in the area of finite element method applied to Rotor Dynamics. He later worked for Tata Consulting Engineers, Bangalore, one of the pioneers in Engineering Consultants in India. He was at the R & D Department, working on engineering analysis & graphics programs. After 2 years of work experience, in pursuit of higher education, he joined Virginia Polytechnic Institute & State University in the Fall of 1991, and continued his research in the area of Rotor Dynamics. He did his doctoral work at the Rotor Dynamics Laboratory under the guidance of Dr. R.G. Kirk.