A Finite Element Model of Submarine Ground Water
Discharge to Tidal Estuarine Waters

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Dissertation submitted to the
Faculty of the Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY
IN
CIVIL ENGINEERING

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September 19, 1996
Blacksburg, Virginia

Keywords: Ground Water Discharge, Coastal Hydrology, Finite Element Model, Estuary
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A FINITE ELEMENT MODEL OF SUBMARINE GROUND WATER DISCHARGE
TO TIDAL ESTUARINE WATERS

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(ABSTRACT)

In the research presented here, a new ground water model, FEMCoast, was
developed to simulate ground water discharge to the intertidal zone of estuarine
systems. This research may be the first attempt to model the ground water
discharge process in a tidal estuarine system. The development of FEMCoast was
undertaken as no existing ground model was capable of directly simulating the
dynamic boundary conditions along the sediment water interface of the intertidal
zone. Reproducing the dynamic tidal boundary conditions along the sediment water
interface was determined to be essential to replicating the complex salinity
gradients observed in the ground water within the intertidal zone. Field data and
model results confirmed the presence of a region of ground water where an inverted
salinity gradient existed. In this region the concentration of salinity decreased with
depth from the ground surface. FEMCoast was also able to reproduce field data on
the movement of the near shore water table and ground water discharge rates and
patterns. However, the model was not able to replicate the short-term fluctuation in
the concentration of salinity within the aquifer due to changes in the concentration
of salinity within Cherrystone Inlet. It is believed that the inability to account for
the wave action of the tides within the intertidal zone is responsible for this
difficulty. The use of FEMCoast integrated with field studies provided a new method
to investigate ground water discharge to tidal estuarine systems.
Acknowledgments

I thank Dr. Gallagher, Dr. Dietrich, Dr. Dillaha, Dr. Loganathan, and Dr. Simmons for serving on my committee; Mr. Roger Buyrn for unlimited access to his farm and a willingness to assist in any manner required; Dr. Simmons for providing access to the Eastern Shore Laboratory Facility and Eduardo Miles and Howard Nippert for their hard work in installing monitoring wells.

I offer special thanks to Dr. Dan Gallagher and Dr. Andrea Dietrich who provided not only financial support but moral support during my stay in Blacksburg.

I thank Dr. William Reay for his assistance in the field, his discussions on the Eastern Shore, and for an enjoyment of the Eastern Shore.

I offer with deepest gratitude a special thanks to Ruby for her endless encouragement and support.

Funding for this research was provided by the National Science Foundation.
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Chapter 1

Introduction
The Algonquin Indians referred to the Chesapeake Bay as the “Chesepiooc” or “great shellfish bay” (White, 1989). The Bay deserved the name as oysters were once so abundant that the entire water volume of the Chesapeake Bay was filtered by the oyster population in several days removing large quantities of algae from the water column. Even today the Chesapeake Bay provides a bountiful harvest of seafood for the commercial fisherman. In 1986, 20 percent of the oyster harvest and 50 percent of the blue crab harvest of the United States were from the Bay (White, 1989). The Chesapeake Bay also provides abundant natural habitat for a wide variety of marine and avian life.

In addition, 14.7 million people make their home within the watershed of the Chesapeake Bay. The number of people is expected to increase to 17.4 million by 2020 (EPA 1995). The citizens of the watershed have benefited from the natural resources of the Bay in addition to the many recreational and commercial uses it affords. Unfortunately, pollutants generated by anthropogenic activities within the watershed enter the waters of the Bay on a daily basis. These pollutants originate from many sources including waste water treatment facilities, commercial facilities, surface runoff from lawns and agricultural fields, atmospheric deposition, and ground water discharge.

In the 1970’s it was recognized that pollutants were degrading water quality within the Bay. Research indicated that nutrient loading exceeding the assimilative capacity of the ecosystem was causing eutrophication within the Bay and its tributaries. Excessive algal blooms reduced sunlight levels to submerged aquatic
vegetation and depleted oxygen levels. In 1983, the Environmental Protection Agency (EPA) recommended several remediation strategies to improve water quality within the Bay. The EPA recommended reducing nitrogen and phosphorus inputs from waste water treatment facilities and non-point pollution sources. In 1987, the states of Virginia, Maryland, and Pennsylvania along with the District of Columbia signed an agreement to reduce the controllable amount of nitrogen and phosphorous entering the Bay by 40 percent by the year 2000. In 1992, the agreement was amended to include the goal of establishing watershed nutrient reduction targets for the Bay’s ten major tributaries.

**Formation of the Chesapeake Bay**

The birth of the Chesapeake Bay began approximately 20,000 years ago near the end of the Pleistocene Ice Age. Much of the Earth’s water was stored in massive glaciers. As a result sea level was 100 m lower than its current elevation. Glacial melt formed the head waters of the Susquehanna River which flowed through Maryland, Pennsylvania, Virginia and over the dry continental shelf. As the climate warmed the Susquehanna flowed with ever increasing volumes of glacial melt. The oceans progressed landward at the rate of 50 feet per year. Approximately 10,000 years ago the ocean reached to the current mouth of the Bay. During the next 7,000 years the rising sea level flooded the Susquehanna River valley and began to create the Chesapeake Bay. Approximately 3,000 years ago the rising sea stopped and the Chesapeake Bay as we know it today was created.
Statistics of the Chesapeake Bay

The following facts were obtained from Maryland Sea Grant (1995). The Chesapeake Bay is the largest estuary in North America. It was formed from the drowned river valley of the Susquehanna River. The Bay is 332 km in length from Havre de Grace, MD to Capes Henry and Charles, VA. There are over 150 tributary rivers and streams in the Bay watershed. The watershed has a drainage area of 165,800 km². The watershed consists of six states (New York, Pennsylvania, Delaware, Maryland, Virginia, and West Virginia) and the District of Columbia. The surface area of the Bay and its tributaries is 11,400 km² and has 12,870 km of shoreline. The Bay and its tributaries below the fall line contain 51 billion m³ of water with an average depth of 6.5 m.

Current State of the Bay

Based on much research, the current hypothesis is that the major pollution threat to the Bay is excessive nutrients with some localized areas threatened by toxic compounds (EPA, 1995). The general state of the Chesapeake Bay and its tributaries is summarized in the following paragraphs taken from The State of the Chesapeake Bay 1995 (EPA, 1995):

If the health of the Bay could be likened to that of a hospital patient, the doctor would report that the patient’s vital signs, such as living resources, habitat, and water quality, are stabilized and the patient is out of intensive care. Some vital signs, such as striped bass and Bay grasses, have improved dramatically, while a few, such as oysters, are in decline. Other vital signs are mixed but stable. Nutrients are being reduced, with phosphorus levels down considerably more than nitrogen levels, and dissolved oxygen remains steady. Overall, the patient still suffers stress from an expanding population and changing
land use, but it is on the road to recovery. Taken as a whole, the concentrated restoration and management effort begun ten years ago has produced tangible results -- a state of the Bay that is better today than when we started and that holds promise that the future will be brighter.

When taken as a whole, these results from the Bay's rivers, generally show very encouraging signs that the inputs of several important pollutants are either declining or leveling off after previously increasing trends. Point and nonpoint source controls of nutrients appear to be having an impact on the total phosphorus concentrations for a number of rivers. The phosphate detergent bans enacted in Maryland, Virginia, Pennsylvania, and the District of Columbia during the mid 1980's have clearly contributed to the lowering of phosphorus inputs from the rivers. Even nitrogen, which has only recently been targeted for load reductions is showing declines in parts of the Susquehanna and Patuxent Rivers.

**The Importance of Ground Water**

More than 50 percent of the total fresh water input to the Chesapeake Bay occurs as either base flow or direct discharge from shallow aquifers (EPA, 1993). Nevertheless, the impact of ground water discharge as a non-point source of nutrients and other pollutants to the Chesapeake Bay has not until recently been factored into strategies to improve water quality within the Bay. However, because of the comparatively slow movement of nitrogen within the ground water system and the potentially large reservoir of nitrogen within the ground water system, improvements in nitrogen levels within the Chesapeake Bay may not be seen for decades (USGS, 1995).

Ground water discharge is the movement of fresh ground water across the sediment water interface of a surface water. When this discharge occurs from a
coastal aquifer into an estuarine or marine environment the term submarine ground water discharge is often employed. Ground water discharge can occur in many settings ranging from localized submarine springs far offshore to diffuse discharge along the shoreline. As alluded to in the preceding paragraph, ground water discharge can be an advective transport mechanism for the movement of ground water contaminants into surface waters. Hubbert (1940), Glover (1959) and Cooper (1959) were among the first to describe the movement and discharge of fresh ground water along the fresh water/salt water interface in coastal aquifers. However, this research and much of the later research into coastal ground water was focused on understanding the movement of salt water into a fresh water coastal aquifer with regards to protecting the drinking water supply from salt water contamination. The environmental and ecological importance of ground water discharge were not considered.

In the 1960’s and 1970’s researchers focused on obtaining quantitative estimates of ground water discharge volumes to marine environments. Simmons and Netherton (1986) credit Kohout (1966) and Kohout and Koplinski (1967) as being among the earliest researchers to recognize the ecological importance of ground water discharge to marine ecosystems although the occurrence of fresh ground water discharge to marine environments was known well before 1966 (Zektzer et al., 1973). The research of Johannes (1980) highlighted the importance of ground water discharge as a transport mechanism for compounds, nitrates in Johannes’ research, through the sediment water interface and its ecological significance.
A comprehensive review of ground water discharge in coastal systems and the implications for the Chesapeake Bay is provided by Reay and Simmons (1992). With regards to the importance of ground water discharge to the Chesapeake Bay the authors state that “Because of the combination of elevated groundwater nitrogen levels, high groundwater discharge rates (which have been measured directly), and the extensive shoreline of the tidal Bay system, the potential is high for significant nutrient enrichment of water resources on a local and perhaps regional scale.”

Many field studies have been conducted within the Chesapeake Bay watershed to investigate ground water discharge. Field studies by Gallagher et al. (1996), Reay et al. (1992), MacIntyre et al. (1989) and Simmons (1988) have successfully measured ground water discharge rates along with nitrogen and pesticide fluxes into various waters of the Chesapeake Bay. However, field studies are only one method to investigate the ground water discharge process. Ground water modeling is a tool that when combined with field studies provides a method to gain a better understanding of the ground water discharge process.

**The Role of Ground Water Models**

Numerical ground water models have been utilized to study the fundamentals of the ground water flow and contaminant transport process, to predict ground water flow and contaminant transport pathways, and to evaluate remediation strategies for contaminated ground water. Ground water models can have similar roles in the study of ground water discharge. Indeed, such models have been utilized in studies of ground water discharge to fresh water lakes and streams.
(Munter and Anderson, 1981; Krabbenhof et al., 1990; Cherkauer and McKereghan, 1991; Cherkauer et al., 1992). Although researchers initial modeling of ground water discharge to fresh water lakes focused on estimating ground water discharge volumes, recently researchers have recognized the environmental aspects of ground water discharge as a transport mechanism for contaminants.

Similar modeling studies of ground water discharge to tidal estuarine and marine environments have not appeared in the research literature. Modeling ground water flow in the near shore zone of a tidal estuarine system requires the capability to capture the dynamic nature of ground water discharge along the intertidal zone and the density dependent flow along the salt water / fresh water transition zone. These two requirements preclude the use of currently existing ground water models.

**Research Objectives**

This document describes research to develop a ground water flow model of ground water discharge to tidal estuarine waters. The objectives of this research were:

1. To develop a numerical ground water model to simulate ground water discharge to tidal estuarine waters;
2. To conduct field studies to increase current understanding of the ground water discharge process and to collect data for model calibration; and
3. To apply the model to site specific conditions.
Development of the finite element ground water model FEMCoast is described in Chapter 2. The research discussed in this document focuses only on the ground water flow component of the ground water discharge process. However, FEMCoast is a complete contaminant transport model capable of simulating nonconservative contaminant transport within tidal coastal aquifers. The development of the nonconservative contaminant transport component of FEMCoast is only discussed briefly in Chapter 2. Field studies were conducted on Cherrystone Inlet on the Eastern Shore of Virginia. These field studies and along with the results are described in Chapter 3. Application of the model to site specific conditions measured during the field studies is described in Chapter 4.
References


Chapter 2

Development of a Finite Element Ground Water Model for Ground Water Discharge to Tidal Estuarine Waters
Introduction

Ground water has been and remains a vital source of fresh water for many coastal communities. Excessive withdrawal of fresh water from an aquifer will induce the landward movement of salt water into the aquifer making many ground water wells unsuitable for potable consumption. To protect these coastal aquifers many studies have investigated the interaction between the seaward flowing fresh ground water and the saline ground water at the shoreline.

Badon Ghyben and Herzberg conducted the first major quantitative analysis of the fresh water / salt water interface (Reilly and Goodman, 1985). In the late 1800's the two European scientists independently developed an equation to calculate the depth to the fresh water / salt water interface of an unconfined coastal aquifer. Modeled as an impermeable boundary, the interface separated the fresh water and salt water. The assumption of the interface as an impermeable boundary is termed the sharp interface approach.

The Ghyben-Herzberg equation is:

$$z = \frac{\rho_f}{\rho_s - \rho_f} h$$  \[2.1\]

where $z$ (L) is the depth of the interface below sealevel, $\rho_f$ (M/L^3) is the fresh water density, $\rho_s$ (M/L^3) is the salt water density, and $h$ (L) is the height of the water table above sea level. Based on static equilibrium conditions along the impermeable
boundary, the Ghyben-Herzberg equation is valid only if both the fresh water and salt water regions are stationary.

For typical values of density for fresh water (1000 kg / m³) and salt water (1025 kg / m³) the Ghyben-Herzberg equation states that the depth of the salt water interface below sea level, \( z \), is forty times the height of the water table above sea level, \( h \). As illustrated in Figure 2.1, a Ghyben-Herzberg system has a fresh water region that approaches a zero thickness at the shoreline. This would prevent the discharge of fresh ground water to the sea across the sediment water interface. A zero discharge of fresh ground water is not possible if the water table has a finite slope towards the shoreline. Despite the limitations of the Ghyben-Herzberg approach, the Ghyben-Herzberg equation does provide an accurate estimate under specific hydrogeological conditions (Bear, 1972 citing Bear and Dagan, 1962b).

The terms interface, zone of diffusion, zone of dispersion, and transition zone appear in the ground water literature somewhat interchangeably. Although a minor semantic distinction exists between the terms, the terms are considered synonymous. The particular term utilized in this discussion will be that selected by the cited author.

Nomitsu, Toyohara, and Kamimote (1927) were the first to consider in a qualitative sense the dynamic nature of the ground water flow system along the salt water / fresh water interface (Custudio, 1987). It was Hubbert (1940) however, who expanded upon the work of Ghyben and Herzberg to factor in the dynamic nature of
Figure 2.1 Schematic of the Ghyben-Herzberg Coastal Ground Water System.
the coastal ground water system. Hubert's (1940) work led to the development of an equation to predict the depth to the impermeable interface based on the fresh water head and salt water head on the interface under equilibrium conditions:

\[ z = \frac{\rho_f}{\rho_f - \rho_s} h_f - \frac{\rho_s}{\rho_f - \rho_s} h_s \]  

where \( z \) (L) is the elevation of the interface, \( h_f \) (L) is the fresh water head on the interface and \( h_s \) (L) is the salt water head on the interface. Hubert's equation is an improvement over the Ghyben-Herzberg equation in that it is valid under either a static or dynamic condition. Hubert's equation, however, continued to consider the interface as an impermeable boundary.

Discharge of fresh ground water from the aquifer was incorporated when Glover (1959) developed an equation to calculate the width of the fresh water discharge zone:

\[ x_o = -\frac{Q}{2\gamma K} \]  

where \( x_o \) (L) is the width of the discharge zone, \( Q \) (L²/T) is the fresh water flow per unit length of shoreline, \( \gamma \) (-) is the density difference ratio, \( K \) (L/T) is the hydraulic conductivity of the strata carrying the fresh water. Glover's equation was based on a static salt water body, steady-state flow conditions, and an impermeable boundary.
interface. In his original paper, Glover (1959) states that a zone of diffusion would alter the dynamic balance along the interface.

Cooper (1959) and Kohout (1960) addressed the dynamic balance along the fresh water / salt water interface in terms of a dispersive interface. The dispersive interface, or zone of diffusion, was an area between the fresh water and salt water with a salt concentration varying from fresh water to sea water. Cooper (1959) utilized field data collected from the Biscayne aquifer of southeastern Florida to demonstrate a seaward flow of diluted sea water occurred within the zone of diffusion. The sea water moved along the zone of diffusion and discharged through the sediment water interface along with the fresh ground water. Because of this large cyclic flow of diluted sea water Cooper (1959) hypothesized that molecular diffusion was an insufficient transport mechanism to maintain the substantial zone of diffusion observed in the Biscayne aquifer. The reciprocative movement of ground water by the tide was suggested to be a sufficiently large dispersive transport mechanism to maintain the zone of diffusion. Palmer (1927) and Wentworth (1948) are credited by Cooper (1959) for first considering that the reciprocative tidal motion aids in creating the zone of diffusion. Kohout (1960) utilized extensive field data on the Biscayne aquifer of southeastern Florida to confirm much of Cooper’s (1959) discussion.

Henry (1964) developed the first numerical model based on the dispersive interface approach. Unlike the sharp interface approach, the dispersive interface approach considered fresh water and salt water a single fluid with a continuous
concentration. This work pioneered the use of the advection-diffusion transport equation to account for salt transport. The problem solved, shown in Figure 2.2, is that of salt water intrusion into a hypothetical, homogeneous, isotropic confined aquifer with a steady-state flow of fresh ground water. The steady-state isoclines solved for by Henry are considered a benchmark solution for two-dimensional density-dependent salt water intrusion models.

Building on the work of Henry, researchers improved upon the dispersive interface approach for two-dimensional cross-sectional (profile) models. Pinder and Cooper (1970) developed a transient model to solve the coupled density-dependent flow equation and salt transport equation. The model utilized a finite difference procedure to solve the flow equation for pressure and the method of characteristics to solve the transport equation for salt concentration. Segol et al. (1975) utilized a "three equation scheme" to achieve a continuous velocity field to reduce numerical dispersion when solving the salt transport equation. This scheme was a Galerkin-finite element method to solve directly for not only pressure but also for the two velocity vectors at each node. Segol and Pinder (1976) utilized the model of Segol et al. (1975) to investigate the movement of the interface in response to a heavy rainfall.

Attempts were made to improve the accuracy of the model output when simulating narrow transition zones or long-term simulations. Frind (1982) utilized simple linear elements to reduce computational time for long-term transient
Boundary Conditions on sea water side (left side)

\[ C = 1 \text{ (sea water)} \]
\[ h = 0.0245 \text{ (1-z)} \]

initial Conditions

\[ h = 0 \]
\[ C = 0 \]

Figure 2.2. Description of Henry’s Problem for Salt Water Intrusion into a Confined Coastal Aquifer.
simulations. Frind was able to achieve a continuous velocity field by defining
velocity as elemental property at the element centroid. Voss (1987) developed a
modified Galerkin finite element method to simulate narrow transition zone within
large regional aquifers. Galeati et al., (1992) developed a Eulerian-Lagrangian
approach to achieve improved numerical stability and to reduced numerical
dispersion. Galeati et al., (1992) incorporated a water table boundary into the model
but did not allow for movement of the water table.

Although the dispersive interface approach is a more realistic representation
of a coastal ground water system, the sharp interface approach can provide accurate
simulations of the ground water system. The sharp interface approach is justified
when the width of the transition zone is small with respect to the thickness of the
aquifer (Fetter, 1972). Two versions of the sharp interface approach exist - the one
fluid model and the two fluid model. The two fluid model simultaneously solves the
coupled non-linear flow equations for fresh water and salt water. The one fluid
model assumes instantaneous adjustment of the salt water zone to changes in head
in the fresh water zone. This eliminates the salt water flow equation and simplifies
the solution. The one fluid model is most applicable when the aquifer is highly
permeable. If the aquifer is not highly permeable the one fluid model does not
capture the transient response of the aquifer as well as the two fluid model (Essaid,
1986)
Conceptual Model of Submarine Ground Water Discharge

Nearshore ground water flow in a coastal aquifer is a complex system characterized by variable density ground water, a tidally influenced fluctuating water table, and varying ground water discharge patterns within the intertidal zone. Development of many salt water intrusion models are based on a conceptual model of a ground water system with a constant water table elevation, a constant sea level, and a vertical ground water discharge boundary. These assumptions are valid for regional investigations of salt water intrusion in most coastal aquifers. However, these assumptions neglect the highly dynamic nature of the near shore ground water flow system.

Variation in the water table elevation from tidal activity and ground water recharge are typically only a fraction of the saturated thickness of the aquifer. For regional simulations the water table elevation is typically assumed constant with minimal error introduced into the solution. Common practice is to neglect the fluctuating sea level and to assign sea level as a fixed elevation, typically set to mean sea level (Urish and Ozbilgin, 1989).

Sinusoidal water table fluctuation are created in coastal aquifers by the action of diurnal tides (Yim and Mohsen, 1992). In addition, the fluctuating sea level induces the movement of sea water into and out of the sediment (Yim and Mohsen, 1992). The reciprocating motion of the tide will also enhance the dispersive transport mechanism in the near shore region of the aquifer. This dispersive transport mechanism maintains the transition zone between the fresh and salt
Figure 2.3. Comparison of Vertical Boundary Condition of Salt Water Intrusion Model (Bottom) to Sediment Water Interface Boundary (Top).
water (Cooper, 1959 citing Palmer, 1927 and Wentworth, 1948). To capture these dynamic processes the water table is modeled as a free surface boundary with the sea level modeled as a specified head boundary varying with both position and time.

Seaward flowing ground water is discharged into the sea through the sediment water interface. For the regional salt water intrusion problem an accurate conceptual representation of the discharge boundary is more important than incorporating the physical characteristics of the discharge zone. As shown in Figure 2.3 the sloping sediment water interface discharge boundary is often replaced with a vertical seaward boundary located near the shoreline. Both boundary representations in Figure 2.3 are conceptually valid. However, modeling the discharge boundary as a sloping sediment water interface better represents the true physical reality of the system.

A ground water model to investigate ground water discharge patterns and contaminant transport in a phreatic coastal aquifer requires an approach to account for movement of the water table, tidal impact on transport processes, and inclusion of the sloping sediment water interface discharge zone. The governing equations, boundary conditions, and matrix solution procedures utilized in FEMCoast are described in the following section.

**Governing Equations**

FEMCoast is a finite element model developed to simulate the transport of a contaminant in a variable density coastal ground water system. The model utilizes a variable density flow equation, Darcy’s Law, a mass transport equation for the
conservative solute salinity, and a linear equation of state relating salinity to fluid
density to simulate ground water flow. A contaminant mass transport equation is
utilized to simulate the movement of a nonconservative contaminant solute.
Development of the variable density fluid flow equation is shown in detail. The
equation is not as well known as the transport equations. The development of the
finite element equations is shown only for the fluid flow equation since similar
procedures are utilized to produce the finite element form of the transport equation.

**Density Dependent Fluid Flow**

Neglecting external sources and sinks the basic fluid continuity equation is:

\[
\frac{\partial (\rho n)}{\partial t} = -\text{div}(\rho q_i)
\]  

[2.4]

where \( \rho \) (M/L³) is fluid mass density, \( n \) (\( \cdot \)) is porosity, \( t \) (T) is time, and \( q_i \) (L/T) is
specific discharge.

The salt concentration in sea water is sufficient to create a density driven
flow in the region of the salt water / fresh water interface. Therefore, the density
term on left hand side of equation [2.4] is expanded to:

\[
\rho \frac{\partial n}{\partial t} + n \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial C} \frac{\partial C}{\partial t} = -\text{div}(\rho q_i)
\]  

[2.5]
to account for fluid density being both a function of pressure, \(P\) (M/LT\(^2\)), and relative salt concentration, \(C\) (-). Relative salt concentration is a dimensionless parameter equal to:

\[
C = \frac{C_{aq}}{C_{max}} 
\]

where \(C_{max}\) is equal to the concentration of sea water. Porosity can be considered a function of pressure.

For a variable density fluid Darcy’s Law is written in terms of pressure as:

\[
q_i = -\frac{k_{ij}}{\mu} \left( \frac{\partial P}{\partial x_j} + \rho g e_j \right) 
\]

where \(q_i\) (L/T) is the specific discharge, \(k_{ij}\) (L\(^2\)) is the permeability tensor, \(\mu\) (M/LT) is dynamic viscosity, \(g\) (L/T\(^2\)) is gravitational acceleration, and \(e_j\) (L) is the gravitational unit vector. The gravitational unit vector equals 1 in the \(z\) direction and 0 in all other directions.

Darcy’s Law, equation [2.7], is substituted into equation [2.5] to yield:

\[
\rho \left( \frac{\partial n}{\partial t} \frac{\partial P}{\partial t} + n \frac{\partial \rho}{\partial t} \frac{\partial P}{\partial t} + n \frac{\partial \rho}{\partial C} \frac{\partial C}{\partial t} \right) = \text{div} \left( \frac{k_{ij}}{\mu} \left( \frac{\partial P}{\partial x_j} + \rho g e_j \right) \right) 
\]
A reference fresh water head or equivalent fresh water head, \( h \) (L), is defined as:

\[
h = \frac{P}{\rho_{\text{ref}} g} + Y
\]  \[2.9\]

where \( \rho_{\text{ref}} \) (M/L^3) is the fresh water reference density, and \( Y \) (L) is the elevation above a datum. Temporal and spatial partial derivatives of pressure derived from Equation [2.9] are:

\[
\frac{\partial P}{\partial t} = \rho_f g \frac{\partial h}{\partial t}
\]

\[
\frac{\partial P}{\partial x_j} = \rho_f g \left( \frac{\partial h}{\partial x_j} - e_j \right)
\]

[2.10]

The partial derivatives in Equation [2.10] are substituted into Equation [2.8]:

\[
\left( \rho \frac{\partial n}{\partial P} + \rho \frac{\partial C}{\partial P} \right) \rho_f g \frac{\partial h}{\partial t} + n \rho \frac{\partial C}{\partial t} = \text{div} \left( \rho \frac{k_{ij}}{\mu} \left( \rho_f g \left( \frac{\partial h}{\partial x_j} - e_j \right) + \rho g e_j \right) \right)
\]

[2.11]

Matrix compressibility, \( \alpha \) (LT^2/M), and fluid compressibility, \( \beta \) (LT^3/M), are defined as:

\[
\alpha = \frac{1}{l-n} \frac{\partial n}{\partial P}
\]

\[
\beta = \frac{1}{\rho} \frac{\partial \rho}{\partial P}
\]

[2.12]

The compressibilities are substituted into the left hand side of Equation [2.11]:

\[27\]
\[
\rho_f \left[ \alpha (1-n) + \beta n \right] \frac{\partial h}{\partial t} + n \frac{\partial \rho}{\partial C} \frac{\partial C}{\partial t} = \text{div} \left( \frac{k_{ij}}{\mu} \left( \rho_f g \left( \frac{\partial h}{\partial x_j} - e_j \right) + \rho g e_j \right) \right)
\]

[2.13]

Specific storativity, \( S (1/L) \), is assumed to be independent of salt concentration and is defined in terms of fresh water density as:

\[
S = \rho_f g \left[ \alpha (1-n) + \beta n \right]
\]

[2.14]

Equation [2.13] is rewritten as:

\[
\rho_f S \frac{\partial h}{\partial t} + n \frac{\partial \rho}{\partial C} \frac{\partial C}{\partial t} = \text{div} \left( \frac{\rho g k_{ij}}{\mu} \rho_f \left( \frac{\partial h}{\partial x_j} + e_j \left( \frac{\rho}{\rho_f} - 1 \right) \right) \right)
\]

[2.15]

Variations in the dynamic viscosity due to changes in salinity have negligible influence on fluid flow when compared to the influence of density (Koch and Zhang, 1992). Therefore, the dynamic viscosity is considered independent of salinity and assigned a value equal to the dynamic viscosity of fresh water. Hydraulic conductivity, \( K_i \) (\( L/T \)), by definition is a function of fluid density and viscosity and, therefore, is a function of salinity. The natural heterogeneity of hydraulic conductivity will mask variations due to changes in salinity. Therefore, hydraulic conductivity is defined as:
\[ K_y = \frac{k_y \rho_f g}{\mu_f} \]  

An equation of state that is often used in salt water intrusion models to relate density to salt concentration is the linear equation:

\[ \rho = \rho_f + \gamma \rho_f C \]  

where \( \gamma \) is the density difference ratio defined as:

\[ \gamma = \frac{\rho_s - \rho_f}{\rho_f} \]  

Based on equation [2.17] the derivative \( \delta \rho / \delta C \) is:

\[ \frac{\partial \rho}{\partial C} = \gamma \rho_f \]  

In addition the following is obtained from equation [2.17]:

\[ \gamma C = \frac{\rho}{\rho_f} - 1 \]
The final form of the fluid continuity equation is obtained by substituting equations [2.16], [2.19], and [2.20] into equation [2.15]:

\[
\text{div} \left( \frac{\partial h}{\partial x_j} e_j + \gamma C \right) = \frac{\partial h}{\partial t} + n \gamma \frac{\partial C}{\partial t} \tag{2.21}
\]

To solve Equation [2.21] the salinity distribution within the aquifer is required. A contaminant transport equation is required in the flow submodel to calculate the salinity in the ground water. The transport equation for a conservative, non-sorbing solute is well documented (Bear, 1972; Fetter, 1993) and is written as:

\[
\frac{\partial C}{\partial t} = \frac{\partial}{\partial x_i} \left( D_{ij} \frac{\partial C}{\partial x_j} \right) - \frac{\partial (v_i C)}{\partial x_i} \tag{2.22}
\]

where \( \cdot \) is the relative salt concentration, \( D_{ij} [L^2/T] \) is the hydrodynamic dispersion tensor, \( v_i [L/T] \) is the interstitial fluid velocity. Relative salt concentration, \( C \), is defined as:

\[
C = \frac{C_{\text{act}}}{C_{\text{max}}} \tag{2.23}
\]

where \( C_{\text{act}} \) is the salt concentration in the water and \( C_{\text{max}} \) is the maximum salt concentration (sea water).
The hydrodynamic dispersion tensor, \( D_{\eta} \), in a two-dimensional x-z coordinate system is:

\[
D_{xx} = \alpha \frac{v_x^2}{v} + \alpha_t \frac{v_z^2}{v} + D_d T_{xx}
\]

\[
D_{zz} = \alpha_t \frac{v_z^2}{v} + \alpha \frac{v_x^2}{v} + D_d T_{zz}
\]

\[
D_{xz} = D_{zx} = (\alpha - \alpha_t) \frac{v_x v_z}{v}
\]

where \( \alpha_t \) (L) is the transverse dispersivity, \( \alpha \) (L) is the longitudinal dispersivity, \( D_d \) (L²/T) is the molecular diffusion coefficient, and \( T (\cdot) \) is the tortuosity tensor.

Equation [2.21] and Equation [2.22] are nonlinear coupled differential equations that constitute the density dependent flow model.

**Contaminant Transport**

The transport of a nonconservative contaminant is described by

\[
\frac{\partial C}{\partial t} = -\frac{\partial}{\partial x_i} (\bar{D}_{\eta} \frac{\partial C}{\partial x_j} \cdot \bar{v}_i \frac{\partial C}{\partial x_i} + \frac{q_x}{n} C_s + \sum_{k=1}^{N} R_k)
\]

where \( C \) (M/L³) is the concentration of solute in liquid phase, \( C_s \) (M/L³) is the concentration of the sources or sinks, \( q_x \) (1/T) volumetric flux of sources or sinks per
unit volume of aquifer, \( n \) in soil porosity, \( \Sigma R_k \) is the chemical reaction term. The chemical reaction term includes all sorption reactions and chemical reactions.

Sorption is modeled as an instantaneous equilibrium, linear process while all chemical degradation reactions are lumped into a single first-order irreversible reaction. The sorption and degradation processes are shown in Equation [2.26] respectively:

\[
\frac{-\rho_b}{n} K_d \frac{\partial C}{\partial t} - K_{rxn} \left(C + \frac{\rho_b}{n} C_s\right)
\]  

[2.26]

where \( K_d \) (\( L^3/M \)) is a linear distribution coefficient, \( K_{rxn} \), (1/T), is a first order rate constant. Substituting equation [2.26] into equation [2.25] yields

\[
R_f \frac{\partial C}{\partial t} = \frac{\partial}{\partial x_j} \left( D_j \frac{\partial C}{\partial x_j} \right) + \frac{q_s}{n} C_s - K_{rxn} \left(C + \frac{\rho_b}{n} K_d \right) 
\]  

[2.27]

where \( R_f \) is a retardation factor defined as:

\[
R_f = 1 + \frac{\rho_b}{n} K_d 
\]  

[2.28]

Equation [2.27] constitutes the contaminant transport model.

**Finite Element Formulation**
The Galerkin finite element method, a subset of the method of weighted residuals, was adopted to solve the fluid flow and transport equations. Although the method has been well documented (Istok, 1989), development of the finite element equations for the density dependent fluid flow equation, Equation [2.21], is shown in detail. Development of the finite element equations for the transport equations is a similar procedure and only the final system of equations are shown.

**Fluid Flow Equation**

The problem domain is discretized into a finite element mesh defined by a set of nodes and elements. An approximate or trial solution to Equation [2.21] is expressed as a series summation of the product of the head at each node, \( h_L(t) \), and the associated interpolation or basis functions, \( N_L(x,z) \):

\[
h(x,z,t) = \tilde{h}(x,z,t) = \sum_{L=1}^{N} h_L(t) N_L(x,z) \tag{2.30}
\]

where \( N \) is the number of nodes in the domain. The interpolation or basis functions, \( N_L \), define the solution over the entire domain as a function of the solution at the nodes.

When the approximate solution, Equation [2.30], is substituted into the differential equation, Equation [2.21], the differential equation does not equal zero.
The difference from zero is termed the residual. In the method of weighted residuals the weighted average of the residual over every node is forced to zero:

$$\int_D R(x,z) \, w_i(x,z) \, dA = 0 \quad [2.31]$$

where $R(x,z)$ is the residual and $w(x,z)$ are the weighting functions. For the Galerkin method the weighting functions, $w_i(x,z)$, are identical to the nodal interpolation functions, $N_{Le}(x,z)$ of Equation [2.30].

Written in Galerkin finite element form Equation [2.21] is:

$$\int_D \left[ \text{div} \left( K_p \left( \frac{\partial h}{\partial x_j} + e_j \gamma C \right) \right) - S_o \frac{\partial h}{\partial t} - n \gamma \frac{\partial C}{\partial t} \right] w_i \, dA = 0 \quad [2.32]$$

Equation [2.32] can be expanded to a two-dimension $(x,z)$ coordinate system parallel to the principal axes of hydraulic anisotropy:

$$\int_D \left[ K_{xx} \frac{\partial^2 h}{\partial x^2} + K_{xz} \frac{\partial^2 h}{\partial z^2} + e_j \gamma C \right] - S_o \frac{\partial h}{\partial t} - n \gamma \frac{\partial C}{\partial t} \right] N_{Le} \, dA = 0 \quad [2.33]$$

Since both the interpolation functions and the weighting functions are identical, the weighting functions in Equation [2.33] are replaced with the interpolation functions.
If the interpolation functions of Equation [2.30] are selected to be linear functions of $x$ and $z$ then the second order derivatives of $h$ in Equation [2.33] are not defined. Therefore, the second order derivatives in Equation [2.33] are reduced to first order by utilizing two-dimensional integration by parts (Wang and Anderson, 1982):

$$
\int_D \left( K_{xx} \frac{\partial^2 h}{\partial x^2} + K_{xz} \frac{\partial}{\partial x} \left( \frac{\partial h}{\partial z} + \gamma C^m \right) \right) N_L \ dA = \int_D \left( K_{xx} \frac{\partial h}{\partial x} \frac{\partial N_L}{\partial x} + K_{xz} \frac{\partial h}{\partial z} + \gamma C^m \right) \frac{\partial N_L}{\partial z} \ dA
$$

$$
+ \int_D \left( K_{xx} \frac{\partial h}{\partial x} n_x + K_{xz} \left( \frac{\partial h}{\partial z} + \gamma C^m \right) n_z \right) N_L \ dA
$$

[2.34]

where $\Gamma$ is the boundary of domain $D$, $\sigma$ is the distance along the boundary, and $n_x$ and $n_z$ are components of a unit vector outwardly normal to $\Gamma$, and $C^m$ is the mean concentration over an element. The flux normal to the boundary is given by the second term on the right hand side of Equation [2.34].

The right hand side of Equation [2.34] is substituted into Equation [2.33]

$$
\int_D \left( K_{xx} \frac{\partial h}{\partial x} \frac{\partial N_L}{\partial x} + K_{xz} \frac{\partial h}{\partial z} \frac{\partial N_L}{\partial z} \right) dA - \int_D S_0 \frac{\partial h}{\partial t} N_L \ dA
$$

$$
+ \int_D \left( K_{xx} \gamma C^m \frac{\partial N_L}{\partial z} \right) dA - \int_D n \gamma \frac{\partial C^m}{\partial t} N_L \ dA
$$

$$
+ \int_D \left( K_{xx} \frac{\partial h}{\partial x} n_x + K_{xz} \left( \frac{\partial h}{\partial z} + \gamma C \right) n_z \right) N_L \ dA = 0
$$

[2.37]
The integrals in Equation [2.37] are evaluated element by element with the results summed over the problem domain:

\[
\sum_e \int_e \left( K_{xx} \frac{\partial \hat{h}^e}{\partial x} \frac{\partial N^e_l}{\partial x} + K_{zz} \frac{\partial \hat{h}^e}{\partial z} \frac{\partial N^e_l}{\partial z} \right) dA + \sum_e \int_e S_{0} \frac{\partial \hat{h}^e}{\partial t} N^e_l dA \\
+ \sum_e \int_e \left( K_{x} \gamma C^e \frac{\partial N^e_l}{\partial z} \right) dA + \sum_e \int_e n \gamma \frac{\partial C^e}{\partial t} N^e_l dA \\
- \int_e \left( K_{xx} \frac{\partial \hat{h}^e}{\partial x} n_x + K_{zz} \left( \frac{\partial \hat{h}^e}{\partial z} + \gamma C^e \right) n_z \right) N^e_l d\sigma = 0
\]  

[2.37]

Equation [2.30] is applied at the element level:

\[
h(x,z,t) \approx \hat{h}^e(x,z,t) = \sum_{L=1}^{N_e} h^e_L(t) \ N^e_L(x,z)
\]  

[2.38]

where \( h_L \) and \( N_L \) are the head and interpolation functions for element \( e \), and \( N \) is the number of nodes in element \( e \). The following derivatives are obtained from Equation [2.38]:

\[
\frac{\partial \hat{h}^e}{\partial x} = \sum_{L=1}^{N_e} \frac{\partial N^e_L}{\partial x} h^e_L \\
\frac{\partial \hat{h}^e}{\partial z} = \sum_{L=1}^{N_e} \frac{\partial N^e_L}{\partial z} h^e_L
\]  

[2.39]

where \( N \) equals the number of nodes in element \( e \). When substituted into Equation [2.37]:

36
\[
\sum_{L=1}^{N} \left[ \sum_{e} \int_{e} \left( K_{xx} \frac{\partial N_x}{\partial x} \frac{\partial N_x}{\partial x} + K_{zz} \frac{\partial N_z}{\partial z} \frac{\partial N_z}{\partial z} \right) dA \right] \\
+ \sum_{e} \int_{e} \left( K_{xx} \gamma C^{m} \frac{\partial N_x}{\partial x} \right) dA \\
+ \sum_{e} \int_{e} n_{y} \gamma C^{m} \frac{\partial N_z}{\partial t} dA \\
- \int_{e} \left( K_{xx} \frac{\partial h}{\partial x} n_x + K_{zz} \left( \frac{\partial h}{\partial z} + \gamma C \right) n_z \right) N_L d\sigma
\] 

[2.40]

An additional term is added to Equation [2.40] to account for the phreatic surface. Two boundary conditions are satisfied simultaneously along the free surface (Neuman and Witherspoon, 1970; Huyakorn and Pinder, 1983):

\[
h(x, \xi, t) = \xi(x, t)
\]

\[
K_{xx} \frac{\partial h}{\partial x} n_x + K_{zz} \left( \frac{\partial h}{\partial z} + \gamma C \right) n_z = (I - S_{y} \frac{\partial \xi}{\partial t}) n_z
\]

[2.41]

where \( \xi (L) \) is the elevation of the free surface above a datum \( S_{y} \) (-) is the specific yield, and \( I \) (L/T) is the infiltration rate into the free surface. The first condition requires the total head at the free surface to be equal to the elevation of the free surface. The second condition written in Galerkin finite element form is:

\[
\sum_{F} \left[ \int_{F} \left( K_{xx} \frac{\partial h}{\partial x} + K_{zz} \left( \frac{\partial h}{\partial z} + \gamma C \right) \right) N_x dF \right] = \sum_{F} \int_{F} I N_x dF - \sum_{F} \int_{F} S_{y} \frac{\partial \xi}{\partial t} n_z dF
\]

[2.42]
Equation [2.42] is added to Equation [2.40] to create a specific yield term and an infiltration term:

\[
\sum_{L=1}^{N} n_t \left[ \sum_{e} \int_{e} \left( K_{xx} \frac{\partial N_L}{\partial x} \frac{\partial N_L}{\partial x} + K_{zz} \frac{\partial N_L}{\partial z} \frac{\partial N_L}{\partial z} \right) dA \right] \\
+ \sum \frac{\partial h^*}{\partial t} \left[ \sum_{e} \int_{e} S_n N_L dA \right] + \sum \frac{\partial h^*}{\partial t} \left[ \sum_{e} \int_{e} S_y N_L dA \right] \\
+ \sum_{e} \int_{e} \left( K_{xx} \gamma C_m \frac{\partial N_L}{\partial z} \right) dA \\
+ \sum_{e} \int_{e} n \gamma \frac{\partial C_m}{\partial t} N_L dA \\
- \int_{e} \left( K_{xx} \frac{\partial h^*}{\partial x} n_x + K_{zz} \left( \frac{\partial h^*}{\partial z} + \gamma C \right) n_z \right) N_L dA = 0
\]

[2.43]

With a fixed finite element grid the interpolations functions, \( N_L \), are a function of spatial location. However, to accommodate the movement of the water table a deformable finite element grid is incorporated. The interpolations functions, \( N_L \), become a function of spatial location and time. The temporal head derivatives are:

\[
\hat{h}^*(x,z,t) = \sum_{L=1}^{N} h^*_L(t) N^*_L(x,z,t) \\
\frac{\partial h}{\partial t} = N_i \frac{\partial h_i}{\partial t} + \frac{\partial N_i}{\partial t} h_i
\]

[2.44]

With a fixed finite element grid the second term of the derivative in Equation [2.44] is zero and Equation [2.44] is substituted directly into Equation [2.43].
However, for a deformable finite element grid the second term in Equation [2.44] is required. The following procedure was developed by Neumann and Witherspoon (1971). The total derivative of head is given by:

\[
\frac{dh}{dt} = \frac{\partial h}{\partial t} + \frac{\partial h}{\partial x_i} \frac{dx_i}{dt} \tag{2.45}
\]

Equation [2.45] is solved for the partial derivative of \( h \):

\[
\frac{\partial h}{\partial t} = \frac{dh}{dt} - \frac{\partial h}{\partial x_i} \frac{dx_i}{dt} \tag{2.46}
\]

In terms of nodal properties Equation [2.46] is:

\[
\frac{\partial h}{\partial t} = N_m \frac{dh}{dt} - (\frac{\partial N_m}{\partial x_i} h_m) (N_p \frac{dx_i}{dt}) \tag{2.47}
\]

For interior nodes of the mesh which exhibit no movement the second term on the right hand side of Equation [2.47] is zero. For boundary nodes on the free surface, Equation [2.47] is expressed in terms of the geometry of the nodal movement (Neumann and Witherspoon (1971)):

\[
\frac{\partial h}{\partial t} = N_m \frac{dh}{dt} (1 - \frac{\Delta h_e \cot \beta_m}{\Delta z_e}) \tag{2.48}
\]

39
where $\beta_m$ is the angle from horizontal in which node is moved. By allowing only vertical displacement of the nodes, $\beta_m = 90$. Equation [2.48] is simplified to:

$$\frac{\partial h}{\partial t} = N_m \frac{dh}{dt}$$  \[2.49\]

The temporal derivative of Equation [2.43] are replaced with Equation [2.49] to obtain the final finite element form of the governing differential equation:

$$\sum_{L=1}^{N} h_L \left[ \sum_{e} \int_{e} \left( K_{xx} \frac{\partial N_k}{\partial x} \frac{\partial N_k}{\partial x} + K_{zz} \frac{\partial N_k}{\partial z} \frac{\partial N_k}{\partial z} \right) dA \right] + \sum_{e} \frac{\partial h}{\partial t} \left[ \sum_{e} \int_{e} S_o N_k N_L dA \right] + \sum_{f} \frac{\partial h}{\partial t} \left[ \sum_{e} \int_{f} S_y N_k N_L dA \right] + \sum_{e} \int_{e} \left( \frac{\partial}{\partial t} h \left( C^m \frac{\partial N_k}{\partial x} \right) \right) dA + \sum_{e} \int_{e} n_{y} \frac{\partial C^m}{\partial t} N_k dA$$

$$- \int_{f} \left( K_{xx} \frac{\partial h}{\partial x} n_x + K_{zz} \left( \frac{\partial h}{\partial z} + \gamma C \right) n_z \right) N_k d\sigma = 0$$ \[2.50\]

Equation [2.50] is written in matrix notation as:

$$[\mathbf{K}] \{h\} + [\mathbf{S}] \left\{ \frac{\partial h}{\partial t} \right\} + \{G\} - \{F\} = 0$$ \[2.51\]
where \([K]\) is the global conductance matrix, \([S]\) is the global fluid mass matrix, \([G]\) is the global body force vector, \([I]\) is the global infiltration vector and \([F]\) is the global boundary flux vector.

The time derivative in Equation [2.51] is approximated in finite difference form as:

\[
\frac{\partial h}{\partial t} = \frac{h_{t+\Delta t} - h_t}{\Delta t}
\]

[2.52]

where \(\Delta t\) is the time step, \(t+\Delta t\) is the time when the solution is sought, and \(\omega\) is the time weighting factor with a value between 0.0 - 1.0. Equation [2.51] written in a general time weighted form is:

\[
\begin{bmatrix}
\omega K_{t+\Delta t} + \left(\frac{\omega S_{t+\Delta t} + (l-\omega)S_t}{\Delta t}\right)
\end{bmatrix}h_{t+\Delta t} =
\begin{bmatrix}
\omega S_{t+\Delta t} + (l-\omega)S_t \Delta t \\
(l-\omega)K_t
\end{bmatrix} h_t - \omega V_{t+\Delta t} - (l-\omega)V
\]

[2.53]

where \(V\) is the sum of \([G]\), \([I]\), and \([F]\).

The selection of \(\omega\) for the fluid continuity equation is not critical and a fully implicit scheme provides satisfactory results with minimal computational effort (Frind, 1982). For a fully implicit scheme, \(\omega = 1\), Equation [2.53] becomes:

\[
41
\]
\[ K_{t+\Delta t} + \frac{S_{t+\Delta t}}{\Delta t} h_{t+\Delta t} = \left[ \frac{S_{t+\Delta t}}{\Delta t} \right] h_{t} + V_{t+\Delta t}. \]  

**Interpolation Functions**

Linear triangle elements were selected to define the finite element mesh. The elemental basis functions for a linear triangular element are a function of the coordinates for the vertices, i, j, and k, of the triangular element (Istok, 1989):

\[
N_i^e(x, z) = \frac{1}{2 A^e} (a_i + b_i x + c_i z)
\]

\[
N_j^e(x, z) = \frac{1}{2 A^e} (a_j + b_j x + c_j z)
\]

\[
N_k^e(x, z) = \frac{1}{2 A^e} (a_k + b_k x + c_k z)
\]

where \( a_i = x_j z_k - x_k z_j \)  
\( a_j = x_k z_i - x_i z_k \)  
\( a_k = x_i z_j - x_j z_i \)  
\( b_i = z_j - z_k \)  
\( b_j = z_k - z_i \)  
\( b_k = z_i - z_j \)  
\( c_i = x_k - x_i \)  
\( c_j = x_i - x_k \)  
\( c_k = x_j - x_i \)

\( A^e = \text{area of element} = 0.5^*[(x_i z_j - x_j z_i) + (x_k z_i - x_i z_k) + (x_j z_k - x_k z_j)] \)

The spatial derivatives of the linear interpolation functions are constants:
\[
\frac{\partial N_i}{\partial x} = \frac{b_i}{2 A^e} \quad \frac{\partial N_i}{\partial z} = \frac{c_i}{2 A^e}
\]

\[
\frac{\partial N_j}{\partial x} = \frac{b_j}{2 A^e} \quad \frac{\partial N_j}{\partial z} = \frac{c_j}{2 A^e} \quad [2.56]
\]

\[
\frac{\partial N_k}{\partial x} = \frac{b_k}{2 A^e} \quad \frac{\partial N_k}{\partial z} = \frac{c_k}{2 A^e}
\]

**Elemental Matrices**

The global matrices in Equation [2.56] are constructed by summation over the elemental matrices. The global conductance matrix \([K]\) in Equation [2.54] is constructed by summation over the elemental conductance matrices. The element conductance matrix in matrix integral form is (Istok, 1989):

\[
[K^e] = \int \begin{bmatrix}
\frac{\partial N_i}{\partial x} & \frac{\partial N_i}{\partial z} \\
\frac{\partial N_j}{\partial x} & \frac{\partial N_j}{\partial z} \\
\frac{\partial N_k}{\partial x} & \frac{\partial N_k}{\partial z}
\end{bmatrix} K_x \begin{bmatrix}
\frac{\partial N_i}{\partial x} & \frac{\partial N_j}{\partial x} & \frac{\partial N_k}{\partial x} \\
\frac{\partial N_i}{\partial z} & \frac{\partial N_j}{\partial z} & \frac{\partial N_k}{\partial z}
\end{bmatrix} dx \, dz
\]

\[
[2.57]
\]

where i, j, k are the vertices of the triangular element.

When the derivatives of the interpolation functions [2.56] are substituted into [2.57] and the matrix multiplication performed:
\[
\mathbf{K}^e = \frac{1}{4A^e} \begin{bmatrix}
K_x b_i b_i + K_z c_i c_i & K_x b_i b_j & K_x b_j b_k + K_z c_i c_k \\
K_x b_j b_i + K_z c_j c_i & K_x b_j b_j + K_z c_j c_j & K_x b_j b_k + K_z c_j c_k \\
K_x b_k b_i + K_z c_k c_i & K_x b_k b_j + K_z c_k c_j & K_x b_k b_k + K_z c_k c_k
\end{bmatrix}
\]

The element capacitance matrix, also called fluid mass or storage matrix, is composed of a specific storage component and a specific yield component. The specific storage component written in a matrix integral formulation is (Istok, 1989):

\[
\mathbf{S}^e = \int \begin{bmatrix} N_i^e \\ N_j^e \\ N_k^e \end{bmatrix} \begin{bmatrix} N_i^e & N_j^e & N_k^e \end{bmatrix} dx 
\]

To evaluate the integrals in equation [2.59] (Segerlind, 1984 cited by Istok, 1989):

\[
\int \begin{bmatrix} N_i^e \\ N_j^e \\ N_k^e \end{bmatrix} \begin{bmatrix} N_i^e & N_j^e & N_k^e \end{bmatrix} dA = \frac{a! b! c!}{(a+b+c+2)!} A^e
\]

The specific storage component of the elemental capacitance matrix is:

\[
\mathbf{S}^e = \frac{S^e}{A^e} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}
\]

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The specific storage component written in a matrix integral formulation is:

$$\left[ S^e \right] = \int \left[ \begin{array}{c} N^e_i \\ N^e_j \end{array} \right] \left[ \begin{array}{cc} S^e_i & N^e_i \\ N^e_j & N^e_j \end{array} \right] dL$$

[2.62]

To evaluate the integrals in equation [2.69]:

$$\int \left( N^e_i \right) \left( N^e_j \right) dx = \frac{a! \ b!}{(a+b+l)!} \left( x_i - x_j \right)$$

[2.63]

The specific storage component of the elemental capacitance matrix is:

$$\left[ S^e_{ij} \right] = \frac{S^e \ L^e}{6} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

[2.64]

The body force vector in equation [2.54] is composed of two terms:

$$\int_D \ K_{\pi} \ y \ C^m \ \frac{\partial N_L}{\partial z} \ dA \\ \int_D \ n \ y \ \frac{\partial C^m}{\partial t} \ N_L \ dA$$

[2.65]

Integration of Equation [2.73] yields:
\[
\frac{K_n \gamma C^n}{2} \left\{ \frac{x_k - x_j}{x_i - x_k} \right\}_{x_j - x_i} \quad n \gamma \frac{\partial C}{\partial t} \bigg|_n A^e \left[ \frac{l}{l} \right]
\]

[2.66]

where \( \delta C / \delta t \) is evaluated as \( \Delta C / \Delta t \) at the centroid of element \( e \) and lagged by one time step.

The infiltration term in equation [2.54] is:

\[
\int I N_L \, dF
\]

[2.67]

where \( F \) is the portion of the boundary over which \( I (L^T / T) \) is specified. Integration of [2.75] yields:

\[
\frac{I}{2L}
\]

[2.68]

where \( L \) (L) is the length of the element side through which infiltration enters.

**Transport Equation**

As with the fluid flow equation an approximate or trial solution to Equation [2.27] is expressed as a series summation of the product of the concentration at each node, \( C_L(t) \), and the associated interpolation or basis functions, \( N_L(x,z) \):
\[ C(x,z,t) = \hat{C}(x,z,t) = \sum_{L=1}^{N} C_L(t) N_L(x,z) \]  

where \( N \) is the number of nodes in the domain. The interpolation or basis functions, in equation [2.69] are the same functions defined in Equation [2.55]. The finite element equations will not be developed. The final finite element form of the governing differential equation is:

\[
\sum_{L=1}^{N} C_L \left( \sum_{e} \left( D_{xx} \frac{\partial N_L}{\partial x} \frac{\partial N_L}{\partial x} + D_{zz} \frac{\partial N_L}{\partial z} \frac{\partial N_L}{\partial z} + D_{xz} \frac{\partial N_L}{\partial x} \frac{\partial N_L}{\partial z} + + v_x \frac{\partial N_L}{\partial x} N_L + v_z \frac{\partial N_L}{\partial z} N_L + R_{mn} R_f N_L N_L dA \right) \right) \\
+ \sum \frac{\partial C}{\partial t} \left( \sum_{e} \left( R_f N_L N_L dA \right) \right) \\
- \int_{\Gamma} \left( D_{xx} \frac{\partial C}{\partial x} N_L n_x + D_{zz} \frac{\partial C}{\partial z} N_L n_z + D_{xz} \frac{\partial C}{\partial z} N_L n_x + + D_{zx} \frac{\partial C}{\partial x} N_L n_z \right) ds 
\]

Equation [2.70] written in matrix notation is:

\[
[R] [C] + [T] \left[ \frac{\partial C}{\partial t} \right] - [B] = 0
\]

where \([R]\) is the global advective/dispersive transport matrix, \([T]\) is the global solute mass matrix, \([B]\) is the global boundary flux vector. The time derivative in Equation [2.71] is approximated in finite difference form as:
\[
\frac{\partial}{\partial t} C = \frac{C_{t+\Delta t} - C_t}{\Delta t}
\]

[2.72]

where \( \Delta t \) is the time step, \( t + \Delta t \) is the time when solution sought. Written in a general time weighted form the matrix equation is:

\[
\begin{bmatrix}
\omega R_{t,\Delta t} + \left( \frac{\omega T_{t+\Delta t} + (l-\omega)T_t}{\Delta t} \right) \\
\frac{\omega T_{t+\Delta t} + (l-\omega)T_t}{\Delta t} - (l-\omega)R_t
\end{bmatrix} C_{t+\Delta t} = \\
\begin{bmatrix}
\omega R_{t,\Delta t} + \left( \frac{\omega T_{t+\Delta t} + (l-\omega)T_t}{\Delta t} \right) \\
\frac{\omega T_{t+\Delta t} + (l-\omega)T_t}{\Delta t} - (l-\omega)R_t
\end{bmatrix} C_t - \omega B_{t+\Delta t} - (l-\omega)B
\]

[2.73]

The selection of \( \omega \) for the contaminant transport equation is critical. Setting \( \omega \) to 0.5 results in improved accuracy but with some oscillatory behavior in the solution. Setting \( \omega \) to 1.0 improves stability but may lead to smearing of the concentration front.

**Elemental Matrices**

The global matrices in Equation [2.73] are constructed by summation over the elemental matrices. The global advective - dispersive transport matrix, \([R]\) in Equation [2.73] is constructed by summation over the elemental matrices. The element matrix in matrix integral form is (Istok, 1989):
\[
[R_e] = \int \begin{bmatrix}
\frac{\partial N_i^e}{\partial x} & \frac{\partial N_j^e}{\partial x} & \frac{\partial N_k^e}{\partial x} \\
\frac{\partial N_i^e}{\partial z} & \frac{\partial N_j^e}{\partial z} & \frac{\partial N_k^e}{\partial z} \\
\frac{\partial N_i^e}{\partial \xi} & \frac{\partial N_j^e}{\partial \xi} & \frac{\partial N_k^e}{\partial \xi}
\end{bmatrix}
\begin{bmatrix}
D_{x x} & D_{x \eta} & D_{x \eta} \\
D_{x \eta} & D_{\eta x} & D_{\eta \eta} \\
D_{x \eta} & D_{\eta \eta} & D_{\eta \eta}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial N_i^e}{\partial x} & \frac{\partial N_j^e}{\partial x} & \frac{\partial N_k^e}{\partial x} \\
\frac{\partial N_i^e}{\partial z} & \frac{\partial N_j^e}{\partial z} & \frac{\partial N_k^e}{\partial z} \\
\frac{\partial N_i^e}{\partial \xi} & \frac{\partial N_j^e}{\partial \xi} & \frac{\partial N_k^e}{\partial \xi}
\end{bmatrix}
dA
\]

\] + \int \begin{bmatrix}
N_i \\
N_j \\
N_k
\end{bmatrix}
V_x \begin{bmatrix}
\frac{\partial N_i^e}{\partial x} & \frac{\partial N_j^e}{\partial x} & \frac{\partial N_k^e}{\partial x} \\
\frac{\partial N_i^e}{\partial z} & \frac{\partial N_j^e}{\partial z} & \frac{\partial N_k^e}{\partial z} \\
\frac{\partial N_i^e}{\partial \xi} & \frac{\partial N_j^e}{\partial \xi} & \frac{\partial N_k^e}{\partial \xi}
\end{bmatrix}
\begin{bmatrix}
D_{x x} & D_{x \eta} & D_{x \eta} \\
D_{x \eta} & D_{\eta x} & D_{\eta \eta} \\
D_{x \eta} & D_{\eta \eta} & D_{\eta \eta}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial N_i^e}{\partial x} & \frac{\partial N_j^e}{\partial x} & \frac{\partial N_k^e}{\partial x} \\
\frac{\partial N_i^e}{\partial z} & \frac{\partial N_j^e}{\partial z} & \frac{\partial N_k^e}{\partial z} \\
\frac{\partial N_i^e}{\partial \xi} & \frac{\partial N_j^e}{\partial \xi} & \frac{\partial N_k^e}{\partial \xi}
\end{bmatrix}
dA

\]

\[R_f = K_{\text{rim}} \begin{bmatrix}
N_i & N_j & N_k
\end{bmatrix}
dA \tag{2.74}\]

Integration of Equation [2.82] results in:

\[
[R_e] = \frac{D_{x x}}{4 A^e} \begin{bmatrix}
b_i b_i & b_j b_j & b_k b_k \\
b_i b_i & b_j b_j & b_k b_k \\
b_i b_i & b_j b_j & b_k b_k
\end{bmatrix} + \frac{D_{x \eta}}{4 A^e} \begin{bmatrix}
c_i c_i & c_j c_j & c_k c_k \\
c_i c_i & c_j c_j & c_k c_k \\
c_i c_i & c_j c_j & c_k c_k
\end{bmatrix} + \frac{D_{x \eta}}{4 A^e} \begin{bmatrix}
c_i b_i & c_j b_j & c_k b_k \\
c_i b_i & c_j b_j & c_k b_k \\
c_i b_i & c_j b_j & c_k b_k
\end{bmatrix} + \frac{D_{x \eta}}{4 A^e} \begin{bmatrix}
c_i c_i & c_j c_j & c_k c_k \\
c_i c_i & c_j c_j & c_k c_k \\
c_i c_i & c_j c_j & c_k c_k
\end{bmatrix} + \frac{V_x}{6} \begin{bmatrix}
b_i b_i & b_j b_j & b_k b_k \\
b_i b_i & b_j b_j & b_k b_k \\
b_i b_i & b_j b_j & b_k b_k
\end{bmatrix} + \frac{V_x}{6} \begin{bmatrix}
c_i c_i & c_j c_j & c_k c_k \\
c_i c_i & c_j c_j & c_k c_k \\
c_i c_i & c_j c_j & c_k c_k
\end{bmatrix} + R_f \frac{K_{\text{rim}} A^e}{12} \begin{bmatrix}2 & 1 & 1 \\
1 & 2 & 1 \\
1 & 1 & 2\end{bmatrix} \tag{2.75}\]

The element solute mass matrix in matrix integral form is (Istok, 1989):

\[
[R_e] = \int \begin{bmatrix}
N_i^e \\
N_j^e \\
N_k^e 
\end{bmatrix}
[R_f] \begin{bmatrix}
N_i^e \\
N_j^e \\
N_k^e
\end{bmatrix}
dA \tag{2.76}\]
Integration of the elemental solute mass matrix results in:

\[
[r^e] = \frac{R_f}{l^2} A^e \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}
\]  
[2.77]

**Calculation of Ground Water Velocity**

Because of the selection of linear interpolation functions in the finite element method the ground water velocity, or Darcy flux, is defined as an elemental property. An elemental property is constant over an element. Velocity calculated from Darcy's law is:

\[
q_x = -K_{xx} \frac{\partial \hat{h}}{\partial x} \quad q_z = -K_{zz} \left( \frac{\partial \hat{h}}{\partial z} + \gamma \ C^m \right)
\]  
[2.78]

A standard technique is to replace the derivatives in Equation [2.78] by the definition given in Equation [2.39]:

\[
q_x = -K_{xx} \sum_{i=1}^{N} h_i \frac{\partial N_i}{\partial x} \quad q_z = -K_{zz} \left( \sum_{i=1}^{N} h_i \frac{\partial N_i}{\partial z} + \gamma \ C^m \right)
\]  
[2.79]

Equation [2.79] expanded in matrix form is:
\[ q_x = -K_{xx} \left[ \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} \frac{\partial N_k}{\partial x} \begin{bmatrix} h_i \\ h_j \\ h_k \end{bmatrix} \right] \quad q_z = -K_{zz} \left[ \frac{\partial N_i}{\partial z} \frac{\partial N_j}{\partial z} \frac{\partial N_k}{\partial z} \begin{bmatrix} h_i \\ h_j \\ h_k \end{bmatrix} \right] \]

where i, j, k are the three nodes of an element. This procedure typically results in a discontinuous velocity field.

An alternate technique to calculate a continuous velocity field was developed by Yeh (1981). The finite element method was applied to Darcy's law to generate a system of linear equations which when solved yielded the velocity at each node. The system of equations are:

\[ [S] \{q_x\} = \{Q_x\} \quad [S] \{q_z\} = \{Q_z\} \]
The elemental matrices are defined as:

\[
[S^e] = \int \begin{bmatrix} N_i^e \\ N_j^e \\ N_k^e \end{bmatrix} \begin{bmatrix} N_i^e & N_j^e & N_k^e \end{bmatrix} \, dA
\]

\[D_x^e = \int \begin{bmatrix} N_i \\ N_j \\ N_k \end{bmatrix} \left[ -K_{xx} \begin{bmatrix} \frac{\partial}{\partial x} N_i \\ \frac{\partial}{\partial x} N_j \\ \frac{\partial}{\partial x} N_k \end{bmatrix} \begin{bmatrix} h_i \\ h_j \\ h_k \end{bmatrix} \right] \]

\[D_z^e = \int \begin{bmatrix} N_i \\ N_j \\ N_k \end{bmatrix} \left[ -K_{zz} \begin{bmatrix} \frac{\partial}{\partial z} N_i \\ \frac{\partial}{\partial z} N_j \\ \frac{\partial}{\partial z} N_k \end{bmatrix} \begin{bmatrix} h_i \\ h_j \\ h_k \end{bmatrix} + \gamma C^{m} \right] \]

The method developed by Yeh (1981) required the solution of a set of simultaneous equations and is therefore more computationally intensive than the standard technique of calculating velocity as an elemental property. Both methods are available in FEMCoast.
**Iterative Solution Procedure**

The density dependent fluid flow model required the solution of two coupled nonlinear differential equations. An iterative solution scheme, the Picard iterative scheme, was adopted to partially decouple the equations (Frind, 1982; Galeati et al., 1992). The solution to the water table boundary required an additional iterative process internal to the decoupling iterative process. The nonconservative contaminant transport equation is solved independently of the salinity transport equation. Figure 2.4 is a flow diagram of the process.

At a time step the solution procedure begins with an initial estimate of the salinity concentrations and water table elevations based upon the previous time step. The nodal heads are calculated. Based on the boundary conditions in Equation (2.41) the heads for the nodes located on the water table are compared to the initial estimate of water table elevation. If the values do not agree within the convergence criteria a new estimate of the water table elevation is generated and new nodal head values are calculated. The iterative loop is repeated until convergence is achieved. The estimate of the salinity concentrations is not updated within this iterative loop. Upon convergence of the water table elevation the ground water velocity and
Figure 2.4. Schematic of Iterative Solution Process Utilized in FEMCoast to Solve Density-Dependent Flow Equation
dispersion coefficients are calculated. The nodal concentrations are calculated. The calculated concentrations are compared to the initial concentration estimates. If the values do not agree within the convergence criteria a new estimate of the concentration is generated and the iterative loop is repeated by solving for nodal heads.

An extrapolation formula (Huyakorn et al., 1987) is utilized to provide the initial concentration estimates for a new time step:

\[
\{C\}^{k+1} = \{C\}^k \quad k = 1
\]

\[
\{C\}^{k+1} = \{C\}^k + \left(\frac{\{C\}^k - \{C\}^{k-1}}{2} \right) \frac{\Delta t^{k-1}}{t^{k-1}} \quad k = 2
\]

\[
\{C\}^{k+1} = \{C\}^k + \left(\frac{\{C\}^k - \{C\}^{k-1}}{\log\left(\frac{t^{k+1}}{t^k}\right)} \right) \frac{\log\left(\frac{t^k}{t^{k-1}}\right)}{\log\left(\frac{t^{k+1}}{t^k}\right)} \quad k > 2
\]

An underrelaxation formula (Huyakorn et al., 1987) was incorporated into the iterative scheme to update head and concentration estimates:

\[
\{p\}^{i+1} = (1 - \alpha) \{p\}^i + \alpha \{p\}^{i-1}
\]
where \( p \) is either head or concentration, \( i \) is the iteration and \( \alpha \) is an iteration dependent underrelaxation factor valued between 0 and 1. The underrelaxation factor was calculated as:

\[
\alpha = \frac{3 + \varepsilon}{|3 + \varepsilon|} \quad \varepsilon \geq -1
\]

\[
\alpha = \frac{0.5}{\varepsilon} \quad \varepsilon < -1
\] [2.87]

where \( \varepsilon \) is an iteration parameter defined as:

\[
\varepsilon = 1 \quad i = 0
\]

\[
\varepsilon = \frac{\Delta P_i}{\varepsilon_{i-1} \Delta P_{i-1}} \quad i > 0
\] [2.88]

where \( \Delta P \) is the largest absolute change in either head or concentration between iterations.

**Matrix Solver**

Implementation of the finite element method yields a set of simultaneous algebraic equations of the form:

\[
[M]\{x\} = \{b\}
\] [2.89]
where $[M]$ is a known square matrix of order $n$, $\{x\}$ is a column vector of $n$ unknowns, and $\{b\}$ is a known column vector of $n$ components where $n$ equals the number of nodes in the domain. In the fluid flow equation $[M]$ is a symmetric banded matrix and in the transport equation $[M]$ is a nonsymmetric banded matrix. The matrix equations were solved with Cholesky's method (Johnson and Reiss, 1982). As a result of the deformable grid (at the water table boundary) the decomposition of $[M]$ must be performed at each iteration.

**Boundary Conditions Along the Intertidal Zone**

**Fluid Flow**

Figure 2.5 is a schematic of the ground water flow system showing the various boundary segments. Point D on the sediment water interface of the intertidal zone will fluctuate from high tide to low tide. To accommodate this fluctuation the sediment water interface is assigned two time and space variant boundary conditions. The unexposed section, D-F, is assigned as a time variant fixed head boundary condition. The hydraulic head is set equal to the elevation plus the density corrected height of the overlying water column. The exposed section above point D is assigned to the free surface boundary, C-D. The hydraulic head of boundary D-F and the relative length of D-F and C-D fluctuated as the tidal elevation fluctuated between high tide and low tide. The tidal boundary conditions along the intertidal zone are updated at each time step.
Salt Transport Boundary Conditions

The sediment water interface boundary (D-F) was assigned as a fixed-concentration boundary when flow was directed inward across the sediment water interface. This condition simulated the flow of sea water into the aquifer and therefore a relative concentration of one (sea water) was given to the boundary. This boundary condition implies that ground water discharge has negligible impact on the water column salt concentration. Although ground water discharge may alter the water column salt concentration, this boundary condition was selected for practical necessity as any changes in the water column salt concentration are unknown. Conversely, a zero dispersive flux boundary was assigned to the sediment water interface boundary (D-F) when flow was directed outward across the sediment water interface. This condition simulated ground water discharge and allowed for the advective movement of salt from the aquifer. The boundary conditions were updated at each time step.

Contaminant Transport Boundary Conditions

The sediment water interface boundary (D-F) was assigned as a fixed-concentration boundary when flow was directed inward across the sediment water interface. This condition simulated the flow of sea water into the aquifer and therefore a concentration of zero (uncontaminated water) was given to the boundary. The contaminant transport model is written in terms of concentration and not relative concentration. Conversely, a zero dispersive flux boundary when flow was directed outward across the sediment water interface. This condition implied ground
Figure 2.5  Schematic of Ground Water Flow System Indicating the Various Boundary Segments.
water discharge and allowed for the advective movement of contaminant from the aquifer.

Summary

A finite element ground water model was developed to simulate ground water discharge and contaminant transport to tidal estuarine environments. The model is unique in its capability to handle the complex boundary conditions that exist along the sediment water interface of the intertidal zone. Developed to investigate ground water discharge from an unconfined coastal aquifer the model is capable of simulating density dependent flow and incorporates a deformable finite element grid so that the fluctuating water table can be directly simulated.
References


Chapter 3

Field Measurements of Ground Water Discharge to a Tidal Estuarine Environment
Introduction

Ground water discharge, once considered an insignificant process, has been found to be an important transport mechanism for the movement of contaminants into many types of surface waters. Ground water discharge was the source of septic system derived nitrogen to marine and coastal embayments (Giblin and Gaines, 1990; Weiskel and Howes, 1992). Nitrogen and phosphorous, originating from upland vegetated marsh sediments, were transported into a tidal creek by ground water discharge (Whiting and Childers, 1989). Nitrate, originating from a wide range of human activities, was transported into a coastal lagoon (Johannes and Hearn, 1985) and to Lake Michigan and Green Bay (Cherkauer et al., 1992) by ground water discharge. Ground water discharge supplied agricultural pesticides and nutrients to the tidal surface waters of Virginia's coastal plain (Gallagher et al., 1996). Volatile organic compounds, originating from an industrial landfill, entered a tidal pond through ground water discharge (Vroblesky et al., 1991). Besides being a transport mechanism for contaminants from natural and anthropogenic sources, ground water discharge can decrease near-shore surface salinity concentrations in coastal environments (Johannes and Hearn, 1985), alter near-shore ecological processes (Johannes, 1980), and impact the chemical budget of fresh water lakes (Cherkauer et al., 1992).

The quality of the ground water discharge is generally indicative of upland ground water quality and land use practices (Simmons et al., 1992). Based on hydrogeological characteristics of the ground water flow system, contaminants can
have flow paths that require several years or longer travel time before entering surface waters through ground water discharge. Therefore, surface waters whose dominant contaminant transport pathway is ground water discharge may not see immediate significant improvement in water quality as a result of the implementation of best management practices (BMPs). Ironically, BMPs which reduce surface runoff in favor of increased infiltration may increase the negative impact of ground water discharge on water quality (Dillaha, 1990). In addition, the volume of even a shallow aquifer is many times greater than the average annual ground water discharge volume and thus the aquifer can remain a loading source for many years (Staver and Brinsfield, 1991).

Field studies have confirmed that nutrients and pesticides are transported to the Chesapeake Bay and its tributaries by ground water discharge (Simmons, 1989; Gallagher et al., 1996). These contaminants originate from certain agricultural land use practices and improperly operating on-site waste disposal systems (MacIntyre et al., 1989; Libelo et al., 1991). Fresh ground water entering the Chesapeake Bay as base flow to tidal and nontidal tributaries and as direct discharge has been estimated to constitute over 50 percent of the mean fresh water discharge to the Bay (EPA, 1993). The amount of inorganic nitrogen entering the Chesapeake Bay and its tributaries through ground water discharge has been estimated to be of an equal magnitude to other nonpoint sources (Libelo et al., 1991).

As with other nonpoint sources of pollution to the Chesapeake Bay, certain regions within the watershed are more vulnerable to contaminant transport by ground water discharge than other areas. The Eastern Shore of Virginia is
characterized by a relatively flat topography, a shallow water table and permeable sandy soils. Agricultural land use is extensive and often borders the shoreline of adjacent surface waters. These conditions make the tidal creeks and inlets of the Eastern Shore vulnerable to contamination by ground water discharge.

This section describes the results of an integrated field study of ground water discharge from an unconfined coastal aquifer to a small coastal inlet on the Eastern Shore of Virginia. The objectives of the field study were:

1) to determine the hydrogeological characteristics of the ground water system and the near shore environment
2) to identify ground water discharge patterns in the near shore environment, and
3) to characterize the behavior of the near shore ground water system with regards to salinity patterns and water table fluctuation.

**Description of Study Site**

Research was conducted along the shoreline edge of an agricultural field bordering Cherrystone Inlet on the Eastern Shore of Virginia, see Figure 3.1. The site was selected because it exhibits characteristics favorable to contaminant transport by ground water discharge (shallow water table, flat topography, permeable soils and adjacent agriculture practices) and has been used in previous studies of ground water discharge (Reay et al., 1992, Gallagher et al., 1996).
Figure 3.1. Map of Chesapeake Bay Showing Location of Study Site on Cherrystone Inlet.
The Eastern Shore of Virginia, composed of Northampton and Accomac counties, comprises 286,296 ha of the southern tip of the Delmarva Peninsula. The area is the eastern part of Virginia's Coastal Plain physiographic province. A relatively narrow strip of land, the Eastern Shore is 12.9 km wide in Accomac county and 9.7 km wide in Northampton county and has a maximum elevation of 15.2 m above mean sea level (Fennema and Newton, 1982). The Atlantic shoreline is composed of extensive vegetated tidal marshes and barrier islands while the Chesapeake Bay shoreline is comprised of meandering tidal creeks and long irregular shaped necks. Annual precipitation on the Eastern Shore is 108.5 cm in Painter, VA (approximately 30 km north of the study site) and 103.6 cm in Cheriton, VA (approximately 3.7 km southeast of the study site) (Cobb and Smith, 1989) of which 21.6 cm to 38.1 cm recharges the Columbia aquifer (Richardson, 1992). Land use for the Eastern Shore is estimated at 40% agricultural, 50% forested, 8.5% pasture, and 1.5% urban (US EPA, 1988).

Cherrystone Inlet is a 640 ha polyhaline system located on the Chesapeake Bay side of Northampton county. The Inlet has a maximum depth of four meters and water salinity of approximately 20 ppth (Reay et al., 1992). Tides are semidiurnal with a range of 0.70 m and a spring tide range of 0.85 m (NOAA, 1993).
Hydrogeologic Framework of the Coastal Plain

The Coastal Plain physiographic province of Virginia extends from Virginia’s Eastern Shore westward to the Piedmont physiographic province. The border between the two provinces, known as the fall line, occurs where most streams have rapids or falls as the landscape transforms from the consolidated metamorphic rocks of the Piedmont to the unconsolidated sedimentary rocks of the Coastal Plain. A generally seaward sloping land surface, the Coastal Plain province has a very low topographic relief. The ground water flow system is a multi-aquifer system composed of an eastward-thickening wedge of unconsolidated sand, silt, clay and calcareous shell fragments (Harsh and Laczniak, 1990).

The Eastern Shore is underlain by a series of aquifers and confining layers that define a local and regional ground water flow system, see Figure 3.2. The local flow system consists of the Columbia aquifer and the Yorktown-Eastover aquifer system. The Columbia aquifer is unconfined throughout the Eastern Shore. The Yorktown-Eastover aquifer system, composed of the Upper, Middle, and Lower Yorktown-Eastover aquifers, is confined throughout the Eastern Shore. Beneath the Lower Yorktown-Eastover aquifer lies a series of aquifers and confining units that contain water with chloride concentrations greater than 250 mg/L (Richardson, 1992) which limits their use as potable water sources.

Ground water flow in the shallow, unconfined Columbia is generally laterally from the ground water divide, located approximately at the center of the peninsula,
Figure 3.2. Aquifers and Confining Units Comprising the Local and Regional Aquifer System of the Eastern Shore of Virginia. Figure taken from Richardson, 1992.
towards discharge sites along streams, ponds, marshes, the Atlantic Ocean, and the Chesapeake Bay. The Columbia aquifer is composed of Pleistocene sediments ranging from very fine silty sands to very coarse and gravely clean sands, with thin, discontinuous, clay and silt lenses (Richardson, 1992). The saturated thickness of the Columbia aquifer ranges from 4.6 m to 24.4 m generally increasing in thickness in an east to west direction (Harsh and Laczniak, 1990). A silt-clay confining layer ranging in thickness from 7.9 m at the Southern tip of the Eastern Shore to 33.2 m at the northern end separates the Columbia aquifer from the Upper Yorktown-Eastover aquifer system. Calcareous shell fragments at the lower end of the Columbia aquifer indicate the beginning of the confining layer (Richardson, 1992).

**Hydrogeologic Framework of the Study Site**

The immediate upland soil at the study site is classified as a well drained Bojac fine sandy loam with a 0 - 2 % slope with moderately rapid permeability in the subsoil and rapid permeability in the substratum (Cobb and Smith, 1989). Depth to the water table is approximately 2 meters. Depth to the top of the upper Yorktown-Eastover confining unit has been reported to be less than 6 m below sea level near the study site (Richardson, 1992). However, based on data obtained during the installation of deeper monitoring wells in the Columbia aquifer the top of the confining unit may be located between 8.5 m and 11.8 m below sea level (Reay et al., 1996).

Estimates of hydrogeologic parameters for the Columbia aquifer are scarce. Based on pump tests Cushing et al., (1973) estimated the specific yield of the
Columbia aquifer to be 0.15 (Harsh and Laczniak citing, 1990). Fennema and Newton (1982) calculated the transmissivity to be 30,000 gpd/ft based on specific capacity data from a large irrigation well in Northampton county. This yields a hydraulic conductivity of 145 cm/hr based on an aquifer thickness of 10.7 m. Based on slug tests performed on monitoring wells in Northampton County, the hydraulic conductivity of the upper sediments of the Columbia aquifer range from 0.8 to 37.5 cm/day (Reay et al., 1996). The calibration of a model of the regional ground water flow system of the Coastal Plain of Virginia resulted in a horizontal hydraulic conductivity of 23 cm/hr for the Columbia aquifer (Harsh and Laczniak, 1990). Laboratory tests on cores from the confining unit indicate a vertical hydraulic conductivity of $7.49 \times 10^{-4}$ cm/hr to $4.95 \times 10^{-3}$ cm/hr (Harsh and Laczniak, 1990). Vertical hydraulic conductivity of near surface sediment cores collected in Cherrystone Inlet decreased from $13.40$ cm/hr to $0.97$ cm/hr over a 50 m sampling transect into Cherrystone Inlet (Reay et al., 1992). Horizontal hydraulic gradients within the Columbia aquifer were measured at 0.001 to 0.005 (Nippert, 1994). Mean horizontal hydraulic gradients at the study site were reported to be 0.003 while shoreline hydraulic gradients were reported as large as 0.023 (Reay et al., 1992).

**Field Methods**

To investigate the two-dimensional flow patterns near the shoreline a series of monitoring wells, piezometers, and seepage meters were installed along a transect normal to the shoreline, see Figure 3.3. The sampling transect extended approximately 30 m into Cherrystone Inlet and approximately 400 m upland from
Figure 3.3. Schematic of Study Site Showing Location and Depth of Ground Water Monitoring Wells and Piezometers.
the shoreline. A datum to reference the position of sampling locations was defined as the location along the beach with an elevation of mean local sea level. This location is referred to as the shoreline.

**Monitoring Wells**

To measure water table elevation, vertical head variations and salinity within the aquifer, monitoring wells were installed along the transect in a nested configuration. Shallow water table wells were hand augured while deeper wells were water jet drilled. The wells were constructed of either 3.81 cm or 5.08 cm diameter schedule 40 polyvinyl chloride (PVC) casing with 0.30 m of 0.025 centimeter slotted PVC well screen. The well annulus around the well screen was backfilled with gravel. The remainder of the well annulus was backfilled with native sand and sealed with granular bentonite.

Distances from the shoreline and the well screen elevations are given in Table 3.1. Elevations are to the midpoint of the well screen except where noted and are referenced to mean local sea level. Well clusters were installed at the shoreline, 10.2 m inland, and 36.7 m inland. A single well was installed at 401.3 m inland from the shoreline. Additional single wells were later installed at 2.8 m, 17.7 m and at 27.2 m inland from the shoreline. Additional wells later installed along a second parallel transect were used only to determine the ground water flow direction at the site. The second well transect was located 47.5 m away with single wells installed at 17.7m, 27.2m, and 36.7 m from the shoreline. All single wells in both transects were installed to specifically measure water table elevation.
Water levels were measured with an electronic water probe to an accuracy of 0.30 cm. Prior to sampling the monitoring wells for salinity, the wells were purged a minimum of three well volumes or until dry with a peristaltic pump. Salinity was measured with a Beckman Industrial Model RS-10 induction salinometer with a precision of 0.003 parts per thousandth (ppt).

Table 3.1  Elevations and Distances from the Shoreline for Monitoring Well Network at Cherrystone Inlet.

<table>
<thead>
<tr>
<th>Well Cluster</th>
<th># of Wells</th>
<th>Elevation (mean local sea level)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shoreline</td>
<td>3</td>
<td>- 0.65 m, - 3.60 m, - 6.17m</td>
</tr>
<tr>
<td>2.8 m inland</td>
<td>1</td>
<td>N/A</td>
</tr>
<tr>
<td>10.2 m inland</td>
<td>3</td>
<td>- 0.49 m, - 3.78 m, - 6.83 m</td>
</tr>
<tr>
<td>17.7 m inland</td>
<td>1</td>
<td>-1.20 m (to bottom of well screen)</td>
</tr>
<tr>
<td>27.2 m inland</td>
<td>1</td>
<td>-1.10 m (to bottom of well screen)</td>
</tr>
<tr>
<td>36.7 m inland</td>
<td>2</td>
<td>- 0.44 m, -3.49 m</td>
</tr>
<tr>
<td>401.3 m inland</td>
<td>1</td>
<td>-0.23 m (to bottom of well screen)</td>
</tr>
</tbody>
</table>

**Piezometers**

First described by Warnick (1957) to measure water loss in irrigation canals (Woessner and Sullivan, 1984), the minipiezometer is an inexpensive and simple device that when used in conjunction with a manometer can measure head differential between surface water and ground water. Piezometers were installed in a nested configuration to depths of 50 cm, 75 cm, 100 cm, and 150 cm at distances of 4.8 m, 9.8 m, 19.8 m, and 29.8 m from the shoreline.

Piezometers were constructed of 1.3 cm diameter rigid tubing. One end of the tube was sealed, drilled with small holes, and wrapped with nylon mesh to create a 10 cm screen interval. The piezometers were hand driven to the desired depth as
described by Woessner and Sullivan (1984). The piezometer was attached by tubing to one end of a differential manometer following the basic design of Winter et al. (1988). The other end of the differential manometer was attached to the bottom of a stilling well installed at each piezometer station. The stilling well reduced the impact of wave action on the manometer readings.

The piezometers provided a method to examine the variation in ground water discharge rates over a tidal cycle. Based on Darcy's law the head differential measured across the sediment water interface is proportional to the discharge across the sediment water interface. If the hydraulic conductivity of the sediments is known the discharge rate can be calculated.

**Seepage Meters**

First used by Israelson and Reive (1944) to measure water loss in irrigation canals (Woessner and Sullivan, 1984), the seepage meter is an inexpensive and simple device to directly measure ground water discharge rates and to collect ground water discharge for chemical analysis. Although some variations in design exist (Lee and Hynes, 1977; Lock and John, 1978; Cherkauer and McBride, 1988) the basic design and operating principles as described by Lee (1977) remain unchanged.

The seepage meter consisted of a Nalgene cylinder (0.20 m in depth, 0.55 m in diameter, 0.25 m² in surface area) sealed on one end with a Plexiglas plate. A collection bag is attached to a drilled hole in the Plexiglas plate. The collection bag was connected to the Plexiglas plate by a rubber stopper and flexible tubing. The
collection bag was either a 3.75 liter volume Teflon gas sampling bag (Fisher, Raleigh, NC) or a 4.0 liter polyethylene bag (Forestry Supply, Jackson, MS).

During low tide, seepage meters were placed at known distances from the shoreline. Seepage meters were pushed into the sediment until a 5.0 cm head space remained. This resulted in approximately 12.5 liters of free head space within the seepage meter. After the flood tide completely submerged the seepage meter, a collection bag was attached to the seepage meter. The sampling bags were attached in their natural expanded state to minimize the short term anomalous flux of water into the bag (Shaw and Prepas, 1989). The sampling period was ended near ebb tide when the seepage meter and bag were no longer completely submerged. At the end of the sampling period the collection bags were removed, the volume of water in the bags measured and samples of the seepage taken for the measurement of salinity. Sampling time ranged from 2.4 hours to 5.1 hrs with the seepage meters placed shoreward having the shortest sampling times.

Seepage meters were utilized in an attempt to measure the variation in ground water discharge rates over a tidal cycle. A sampling time of 0.5 hours was used. The short sampling time produced discharge measurements that were very erratic. Disturbance of the seepage meters during the frequent removal and replacement of the collections bags is believed to have produced the erratic results. This method was abandoned based on the preliminary results.

The volume of fresh ground water discharge was calculated from a mass balance on the salinity within the seepage meter, Equation [3.1], assuming complete
mixing occurred within the seepage meter and the fresh ground water discharge has zero salinity.

\[
V_{fw} = \frac{V_{sm} (S_o - S_i) + V_b (S_{sw} - S_i)}{S_{sw}}
\]  

[3.1]

where \( V_{fw} \) is the volume of fresh water, \( V_{sm} \) is the volume of seepage meter headspace, \( V_b \) is volume of discharge in the seepage meter collection bag, \( S_o \) is the initial salinity concentration in seepage meter, \( S_{sw} \) is the salinity concentration of the salt water component of seepage, \( S_i \) is the salinity concentration in the seepage meter and collection bladder at time \( i \).

Equation [3.1] is generally simplified by assuming the salinity concentration in the water initially enclosed in the seepage meter is equal to the salinity concentration in the salt water fraction of the discharge, \( S_o = S_{sw} \). With this assumption Equation [3.1] becomes:

\[
V_{fw} = (V_{sm} + V_b) \frac{(S_o - S_i)}{S_o}
\]  

[3.2]

Estimates of fresh water discharge based on Equation [3.2] are most sensitive to the headspace in the seepage meter. All other parameters in Equation [3.2] can be simply measured with a greater degree of accuracy than the seepage meter headspace. Therefore, installation of the seepage meters must be carefully
done to ensure that reliable measurements of the headspace are obtained. In the upper region of the intertidal zone seepage meter placement was not possible due to the presence of the coastal grass *Spartina alterniflora*.

**Tidal Elevations**

A stilling well and tide staff were installed in Cherrystone Inlet to obtain tidal elevations.

**Sediment Characteristics and Aquifer Parameters**

Surface sediments from the intertidal zone were collected for grain size analysis. The sample consisted of approximately 100 ml of sample collected from the top 5 cm of sediment. Each 100 ml sample was thoroughly mixed from which a sub-sample was taken for analysis. Grain sizes were determined by wet sieve and pipette analysis (Folk, 1980). The Wentworth grain size scale was used to separate sediment into gravel, sand, fine sand, silt and clay fractions.

A 5.4 m sediment core was obtained from the sandy region of the intertidal zone. The core was collected by impact driving a 5.08 cm PVC pipe (Starr and Ingleton, 1992) into the sediment. The process was stopped when the PVC pipe ceased to easily progress through the sediment. Core compaction was less than 5 cm. The core was removed by longitudinally cutting the PVC pipe. Samples were removed from the core for grain size analysis.
Permeability tests and specific yield measurements were conducted on sediment cores collected from the sandy sediment near the shoreline well and further offshore in the finer textured sediment. Sediment cores were obtained by pushing a 5.08 cm plastic core liner to the desired depth into the sediment. Permeabilities were measured using the full core. Core length ranged from 22 cm to 70 cm. The core liners were placed directly into a permeameter to minimize core disturbance. Saturated hydraulic conductivity was calculated based on the falling-head method (Klute, 1965). To match field conditions, hydraulic heads utilized in the tests ranged from 2 cm to 8 cm with ambient water from Cherrystone Inlet being used to conduct all permeability tests.

Specific yields were measured using a simple free drainage procedure (Johnson et al., 1962) and a moisture content procedure (Johnson et al., 1962). Specific yield measurements can vary significantly depending on the specifics of the test procedure. The two procedures were selected for their simplicity and applicability to a coastal environment.

The free drainage procedure measured the volume of water that drained from a core of known volume. Sediment cores were obtained in the same manner as previously described and ranged in length from 15 cm to 36 cm. Core lengths were selected as to yield a representative specific yield for the sediment layer. Ambient Cherrystone Inlet water was passed through each core for four to six hours to fully saturate the core. Specific yield values were based on a drainage time of six hours. The drainage time was selected to equal the approximate average time between the
ebb tide and saturation of the sediments by the next flood tide. Specific yield ($S_y$) was calculated as:

$$S_y = \frac{V_w}{V_c} \tag{3.3}$$

where $V_w$ is the volume of water drained in six hours and $V_c$ is the total volume of the sample core over which drainage occurred.

The moisture content procedure was conducted by measuring the volumetric water content of a saturated core and the volumetric water content of a core collected at low tide. The two sediment cores were collected from within a two foot radius and to the same depth. Volumetric water content was determined by weighing a known volume of core, drying the core for 24 hours at $105^\circ C$, and weighing the dried core. The porosity of the sediment is equal to the volumetric water content of the saturated core. Specific yield ($S_y$) was calculated as:

$$S_y = \frac{Vol_{sat} - Vol_{low}}{Vol_{sat}} \tag{3.4}$$

where $Vol_{sat}$ is the volumetric water content of the saturated core and $Vol_{low}$ is the volumetric water content of the core collected at low tide. Volumetric water content, $Vol$, is calculated as:
\[ V_{ol} = \frac{W_i - W_D}{V \rho_f} \]  

where \( W_i \) is the initial weight of the sample, \( W_D \) is the dry weight of the sample, \( \rho_f \) is the fresh water density, and \( V \) is the volume of the core.

Slug tests were conducted at the 10.2 m and 36.7 m monitoring wells. Horizontal hydraulic conductivity was calculated using the procedure of Bouwer and Rice (1976) to account for unconfined aquifer and partial penetration of the aquifer by the well. Vertical flow conditions were minimized by selecting the farthest upland wells. Prior to the start of the slug test the initial head in the monitoring well was measured. Water was pumped from the well to lower the head by a minimum of 3 m. Recovery of the well was measured with an electronic water probe until the well recovered to within several percent of its initial head.

**Results and Discussion**

**Sediment Characteristics of the Ground Water Discharge Zone**

The near shore region was characterized by an area of well drained sandy sediment located in the upper intertidal zone and an area of finer textured sediment extending into Cherrystone Inlet. Unlike the upper intertidal sandy sediment, little interstitial water appeared to drain from the finer textured sediment region during tidal exposure. The transition from the sandy sediment to the finer textured sediment began at approximately 5.2 m from the shoreline. The intertidal zone
ranged from approximately 1 m landward of shoreline to 10 m seaward of shoreline. The two regions were characterized by differing slopes - 6.7% for the upper sandy sediment and 0.9% for the lower intertidal finer texture sediment.

Grain size properties of the surface sediment samples collected from the intertidal zone are given in Table 3.2. Both sediments are classified as a sand. The upper intertidal sediments were 88% sand, 11% fine sand with less than 1% silt and clay while the lower intertidal sediments were 39% sand, 51% fine sand with 8% silt and clay.

Table 3.2  Grain Size Properties of Surface Sediments from Intertidal Zone.

<table>
<thead>
<tr>
<th></th>
<th>% Gravel</th>
<th>% Sand</th>
<th>% Fine Sand</th>
<th>% Silt</th>
<th>% Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Upper Intertidal</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample 1</td>
<td>0.21</td>
<td>88.98</td>
<td>10.16</td>
<td>0.01</td>
<td>0.63</td>
</tr>
<tr>
<td>Sample 2</td>
<td>0.27</td>
<td>87.98</td>
<td>10.87</td>
<td>0.11</td>
<td>0.77</td>
</tr>
<tr>
<td>Sample 3</td>
<td>0.15</td>
<td>88.48</td>
<td>10.61</td>
<td>0.08</td>
<td>0.69</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>0.21</td>
<td>88.48</td>
<td>10.55</td>
<td>0.07</td>
<td>0.70</td>
</tr>
<tr>
<td><strong>Lower Intertidal</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample 1</td>
<td>0.27</td>
<td>42.16</td>
<td>49.95</td>
<td>3.60</td>
<td>4.02</td>
</tr>
<tr>
<td>Sample 2</td>
<td>0.28</td>
<td>38.92</td>
<td>53.00</td>
<td>3.47</td>
<td>4.32</td>
</tr>
<tr>
<td>Sample 3</td>
<td>0.11</td>
<td>36.45</td>
<td>53.55</td>
<td>4.59</td>
<td>5.31</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>0.22</td>
<td>39.18</td>
<td>52.17</td>
<td>3.89</td>
<td>4.55</td>
</tr>
</tbody>
</table>

Vertical hydraulic conductivity measured from the sediment cores are shown in Table 3.3. Hydraulic conductivity for the upper intertidal sediment ranged from 13.59 cm/hr to 44.90 cm/hr while the lower intertidal sediment hydraulic conductivity varied from 3.28 to 7.57 cm/hr. The vertical hydraulic conductivity of the sediments decrease with distance from the shoreline. This pattern was also observed by Reay et al. (1992).
Table 3.3  Vertical Hydraulic Conductivity of Surface Sediments in the Intertidal Zone.

<table>
<thead>
<tr>
<th>Location of Core</th>
<th>$K_{sat}$ (cm/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6 m landward - upper intertidal</td>
<td>13.7</td>
</tr>
<tr>
<td>3.0 m past shoreline - upper intertidal</td>
<td>27.7</td>
</tr>
<tr>
<td>10.0 m past shoreline - upper intertidal</td>
<td>13.6, 44.9</td>
</tr>
<tr>
<td>13.0 m past shoreline - upper intertidal</td>
<td>36.9</td>
</tr>
<tr>
<td>15.2 m past shoreline - lower intertidal</td>
<td>7.6</td>
</tr>
<tr>
<td>31.1 m past shoreline - lower intertidal</td>
<td>3.3, 7.9</td>
</tr>
</tbody>
</table>

During tidal exposure little interstitial water appeared to drain from the finer textured sediment of the lower intertidal zone. The finer textured sediment are located in the part of the intertidal zone termed the zone of resurgence. Although exposed, this region can remain relatively saturated with water draining from the higher elevations on the beach (Thurman and Webber, 1984). When exposed, the sediments in lower intertidal zone exhibited many small areas, approximately 15 cm or less in diameter, of pooled water.

**Hydrogeological Parameters of the Columbia Aquifer**

Horizontal hydraulic conductivity from slug tests conducted on two of the monitoring wells were 1.40 cm/hr at the 10.2 m monitoring well and 5.12 cm /hr at the 36.7 m monitoring well. Hydraulic conductivity for a sand can range from 0.36 cm /hr to 3600 cm /hr (Freeze and Cherry, 1979).

Specific yields based on the free drainage procedure averaged 0.026 for the well drained sandy sediments of the upper intertidal zone and 0.004 for the finer textured sediments in the lower intertidal zone. Specific yields based on the
moisture content procedure were very erratic and were not used. Typical values for specific yield in a fine sand range from 0.10 to 0.28 and for a medium sand from 0.15 to 0.32 (Johnson, 1967). However, these values represent long-term specific yield measurements. The low value of the specific yield measurements based on the free drainage procedure and the moisture content procedure are representative of short-term specific yield measurements. Nwankwor et al. (1984) found that due to delayed drainage in a shallow sand aquifer short-term specific yield measurements were significantly lower than long-term specific yield measurements. Long-term specific yield was estimated to be 0.30 based on a laboratory drainage curve. A volume-balance analysis of pump test data resulted in specific yield measurements which varied from 0.02 at 15 minutes to 0.07 at 3 hours to 0.25 at 64.5 hours. The time required to reach long-term specific yield increases considerably with decreasing mean grain size (Johnson, 1967 and Stallman, 1971). As was the case, the short-term specific yield of the well-drained sandy sediments should be considerably higher than the finer textured sediment.

Table 3.4 gives the grain size analysis of the core taken from the top 5.4 m of the Columbia aquifer. The particle size of the substrate decreased from a 93% sand / 5% fine sand mixture in the top 20 cm to a 60% sand / 36% fine sand mixture at 180 cm. Below 180 cm the sand fractions varied but averaged 41% sand and 56% fine sand. Average silt and clay content were 1.8% silt (1.6% standard deviation) and 1.95% clay (1.2% standard deviation). The silt and clay zones were relatively narrow regions dispersed throughout the column with the most significant region in the 230 cm to 345 cm depth range. No significant clay lenses were found. The
occurrence of a shallow clay lens extending offshore would act to confine the aquifer and cause ground water to discharge farther offshore than would be expected (Bokuniewicz, 1980).
Table 3.4  Grain Size Properties of Sediment Core from Columbia Aquifer Taken From Shoreline of Cherrystone Inlet.

<table>
<thead>
<tr>
<th>Sample</th>
<th>% Gravel</th>
<th>% Sand</th>
<th>% Fine Sand</th>
<th>% Silt</th>
<th>% Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 20 cm</td>
<td>0.14</td>
<td>92.79</td>
<td>4.90</td>
<td>1.71</td>
<td>0.46</td>
</tr>
<tr>
<td>35 - 65 cm</td>
<td>0.00</td>
<td>94.67</td>
<td>4.50</td>
<td>0.41</td>
<td>0.41</td>
</tr>
<tr>
<td>65 - 85 cm</td>
<td>0.00</td>
<td>91.51</td>
<td>5.44</td>
<td>0.92</td>
<td>2.13</td>
</tr>
<tr>
<td>90 - 105 cm</td>
<td>0.00</td>
<td>82.96</td>
<td>9.98</td>
<td>2.68</td>
<td>4.38</td>
</tr>
<tr>
<td>105 - 130 cm</td>
<td>9.10</td>
<td>81.46</td>
<td>5.81</td>
<td>1.30</td>
<td>2.33</td>
</tr>
<tr>
<td>130 - 150 cm</td>
<td>4.60</td>
<td>77.18</td>
<td>13.95</td>
<td>1.22</td>
<td>3.05</td>
</tr>
<tr>
<td>150 - 180 cm</td>
<td>0.18</td>
<td>60.35</td>
<td>35.81</td>
<td>1.98</td>
<td>1.68</td>
</tr>
<tr>
<td>180 - 210 cm</td>
<td>0.00</td>
<td>31.53</td>
<td>67.55</td>
<td>0.00</td>
<td>0.92</td>
</tr>
<tr>
<td>230 - 250 cm</td>
<td>0.00</td>
<td>23.41</td>
<td>72.51</td>
<td>1.48</td>
<td>2.59</td>
</tr>
<tr>
<td>260 - 270 cm</td>
<td>0.08</td>
<td>33.94</td>
<td>59.42</td>
<td>5.99</td>
<td>0.57</td>
</tr>
<tr>
<td>290 cm</td>
<td>0.23</td>
<td>24.67</td>
<td>66.88</td>
<td>4.93</td>
<td>3.29</td>
</tr>
<tr>
<td>291 - 304 cm</td>
<td>0.00</td>
<td>50.19</td>
<td>45.95</td>
<td>0.46</td>
<td>3.49</td>
</tr>
<tr>
<td>305 - 345 cm</td>
<td>0.00</td>
<td>31.65</td>
<td>61.29</td>
<td>4.01</td>
<td>3.05</td>
</tr>
<tr>
<td>345 - 370 cm</td>
<td>0.00</td>
<td>43.91</td>
<td>52.74</td>
<td>2.01</td>
<td>1.34</td>
</tr>
<tr>
<td>370 - 385 cm</td>
<td>0.15</td>
<td>80.38</td>
<td>18.44</td>
<td>0.00</td>
<td>1.03</td>
</tr>
<tr>
<td>385 - 402 cm</td>
<td>0.00</td>
<td>35.56</td>
<td>63.57</td>
<td>0.87</td>
<td>0.00</td>
</tr>
<tr>
<td>430 - 440 cm</td>
<td>0.00</td>
<td>66.43</td>
<td>30.94</td>
<td>0.44</td>
<td>2.19</td>
</tr>
<tr>
<td>446 - 462 cm</td>
<td>0.00</td>
<td>59.44</td>
<td>38.87</td>
<td>0.00</td>
<td>1.70</td>
</tr>
<tr>
<td>470 - 480 cm</td>
<td>0.00</td>
<td>41.15</td>
<td>55.98</td>
<td>2.36</td>
<td>0.51</td>
</tr>
<tr>
<td>500 - 510 cm</td>
<td>0.00</td>
<td>24.78</td>
<td>71.33</td>
<td>1.06</td>
<td>2.83</td>
</tr>
<tr>
<td>530 - 540 cm</td>
<td>0.00</td>
<td>21.35</td>
<td>74.77</td>
<td>2.99</td>
<td>0.90</td>
</tr>
</tbody>
</table>
Near Shore Hydraulic Head Variations

Movement of the near-shore water table over a tidal cycle is shown in Figure 3.4. Tides create a fluctuating hydraulic head condition on the submarine outcrop of the unconfined aquifer that creates pressure waves that travel inland from the shore (Serfes, 1991). These pressure waves cause the inland ground water levels and hydraulic gradients to continuously fluctuate. As shown in Figures 3.4 the water table fluctuations are increasingly dampened and lagged behind the tide with distance inland. Water table fluctuations are not as sinusoidal as the fluctuation of tidal elevation. In response to tidal influence the water table will tend to rise faster during flood tide than decline during ebb tide (Emery and Foster, 1948; Staver and Brinsfield, 1990). Nielsen (1990) described the sloping beach face as a “highly nonlinear filter which causes the water table to rise abruptly and drop off slowly compared to the near-sinusoidal tide which drives it.”

The fluctuation of the near-shore water table elevation caused by tidal activity provides a simple and convenient method to estimate the horizontal hydraulic conductivity of a coastal aquifer (Urish and Ozbilgin, 1989; Pandit et al., 1991). The tidal damping equation often used is (Ferris, 1963):

\[ h = 2 h_0 \exp \left[ - \frac{x}{t_c K b} \left( \frac{\pi S}{t_c K b} \right)^{0.5} \right] \]  \hspace{1cm} [3.6]

where \( h \) is the coastal ground water fluctuation at a distance \( x \) from the shoreline, \( S \) is the storativity (or specific yield for an unconfined aquifer), \( K \) is the horizontal
Figure 3.4. Near Shore Water Table Fluctuation Measured In Monitoring Wells Located 0.0 m, 10.2 m, and 401.3 m from the Shoreline (Top Graph) and 17.7 m, 27.2 m, 36.7 m, and 401.3m from the Shoreline (Bottom Graph). Tidal Elevation is Shown for Reference. Sample date was July 17, 1994.
hydraulic conductivity, $h_o$ is the tidal amplitude, $t_o$ is the tidal period, $b$ is the thickness of the aquifer and $t$ is time. Equation [3.6] is based on the assumption of a vertical seaward boundary, horizontal flow conditions and a semi-infinite confined aquifer. If the water table movement is a small fraction of the aquifer thickness Equation [3.6] can be applied to an unconfined aquifer (Fetter, 1994). However, because of the vertical seaward boundary assumption, Equation [3.6] does not account for the skewed sinusoidal pattern of the water table fluctuations that are shown in Figures 3.4a and 3.4b.

In addition to tidal damping, the time lag between high or low tide and the arrival of the associated wave at a particular distance inland, can also be used to estimate the horizontal hydraulic conductivity of a coastal aquifer. The time lag equation is (Ferris, 1963):

$$K = \frac{x^2 S t_o b}{4 \pi t_l^2} \quad [3.7]$$

where $t_l$ is the time lag to the arrival of the high or low tide wave.

Estimates of specific yield and horizontal hydraulic conductivity were obtained by performing a non-linear regression analysis on Equations [3.6] and [3.7]. The results are shown in Figure 3.5 along with the field data. The regression of the time lag equation, Figure 3.5 (Top Graph), yielded a hydraulic conductivity of 0.72 cm/hr and a specific yield of 0.011 while the tidal damping equation, Figure 3.5 (Bottom Graph), yielded a hydraulic conductivity of 0.19 cm/hr with a specific yield.
Figure 3.5. Plots of Field Data and Results of Non-Linear Regression for Time Lag Equation (Top Graph) and Tidal Damping Equation (Bottom Graph).
of 0.00073. Equations [3.6] and [3.7] are a convenient method to estimate the hydraulic conductivity or specific yield of a coastal aquifer. The above values are compared to the field measured values in Table 3.5. Researchers, however, have found Equation [3.6] to give erratic results for an unconfined sand and gravel aquifer even with the analysis of 58 consecutive tidal cycles (Millham and Howes, 1995).

<table>
<thead>
<tr>
<th>Hydraulic Conductivity (cm/hr)</th>
<th>Time Lag Equation</th>
<th>Tidal Damping Equation</th>
<th>Field Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Yield (°)</td>
<td>0.011</td>
<td>0.00073</td>
<td>0.026</td>
</tr>
</tbody>
</table>

|                          | 0.72              | 0.19                   | 1.40, 5.12     |

**Horizontal Hydraulic Gradients**

The near shore ground water flow patterns at the study site are shown in Figure 3.6. Hydraulic gradients were calculated with the method described by Pinder et al. (1981) utilizing the hydraulic heads measured in the upland wells along the parallel transects. Ground water flow direction was nearly perpendicular to the shore. The direction varied slightly over ebb tide from near high tide conditions, Figure 3.6 (Top Graph), to near low tide conditions, Figure 3.6 (Bottom Graph). The ground water flow direction was slightly skewed from perpendicular to the shoreline. This was more predominant at near low tide conditions.
Figure 3.6. Near Shore Ground Water Flow Patterns (September 30, 1994)
The regional horizontal hydraulic gradient was defined as the hydraulic gradient measured at a distance from the shoreline as to exhibit no influence from tidal activity. The regional gradient was calculated from water table measurements at the 401 m upland well and the 36.7 m upland well. Although the 401 m well exhibited no water table fluctuation, a slight fluctuation in the water table existed at 36.7 m upland well. Therefore, the regional gradients shown in Table 3.6 include a regional gradient calculated from the low and high water table elevations at the 36.7 m well. The average of the two values given in Table 3.6 can be taken as the regional gradient.

Table 3.6 Regional Horizontal Hydraulic Gradients Adjacent to Cherrystone Inlet

<table>
<thead>
<tr>
<th>Sample Date</th>
<th>Regional Gradient (Low)</th>
<th>Regional Gradient (High)</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 31, 1993</td>
<td>$4.68 \times 10^{-4}$</td>
<td>$5.60 \times 10^{-4}$</td>
</tr>
<tr>
<td>October 21, 1993</td>
<td>$2.68 \times 10^{-4}$</td>
<td>$3.85 \times 10^{-4}$</td>
</tr>
<tr>
<td>May 24, 1994</td>
<td>$5.01 \times 10^{-4}$</td>
<td>$6.02 \times 10^{-4}$</td>
</tr>
<tr>
<td>May 25, 1994</td>
<td>$5.18 \times 10^{-4}$</td>
<td>$5.77 \times 10^{-4}$</td>
</tr>
<tr>
<td>July 17, 1994</td>
<td>$4.51 \times 10^{-4}$</td>
<td>$5.43 \times 10^{-4}$</td>
</tr>
<tr>
<td>October 01, 1994</td>
<td>$4.43 \times 10^{-4}$</td>
<td>$5.02 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

The regional gradient varied from $3.27 \times 10^{-4}$ to $5.51 \times 10^{-4}$. Based on the maximum regional hydraulic gradient of $5.51 \times 10^{-4}$, an aquifer thickness of 10 m, and a hydraulic conductivity of 5 cm/hr the upland ground water flow towards the shoreline is $6.6 \times 10^{-3}$ m$^3$/day/m of shoreline.
Table 3.7 gives the depth to the water table depth at the 401.3 m monitoring well corresponding to each regional gradient given in Table 3.6. The data is insufficient to indicate any seasonal trend in depth to the water table.

Table 3.7  Depth to Water Table from Ground Surface for 401.3 m Monitoring Well

<table>
<thead>
<tr>
<th>Sample Date</th>
<th>Depth to Water Table (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 31, 1993</td>
<td>1.87</td>
</tr>
<tr>
<td>October 21, 1993</td>
<td>1.92</td>
</tr>
<tr>
<td>May 24, 1994</td>
<td>1.73</td>
</tr>
<tr>
<td>May 25, 1994</td>
<td>1.74</td>
</tr>
<tr>
<td>July 17, 1994</td>
<td>1.94</td>
</tr>
<tr>
<td>October 01, 1994</td>
<td>1.88</td>
</tr>
</tbody>
</table>

Figure 3.7 shows the variation of the near shore horizontal gradient over the tidal cycle calculated from the water table elevation measured at several monitoring well pairs. The near shore horizontal hydraulic gradient continually changed in magnitude and reversed direction over the tidal cycle. The near shore hydraulic gradient differed significantly with distance from the shoreline. The hydraulic gradient was higher near the shore (shoreline well - 17.7 m well) ranging from +0.130 to -0.139 than farther inland (27.7 m well - 36.7 m well) where the hydraulic gradient ranged from $+2.6 \times 10^{-3}$ to $-9.6 \times 10^{-3}$. 
Figure 3.7. Fluctuation in Near Shore Horizontal Hydraulic Gradient over a Tidal Cycle. A positive gradient indicates flow towards the shoreline.
**Vertical Hydraulic Gradients**

Field measured heads should not be directly used to calculate vertical or horizontal hydraulic gradients in a variable density ground water system (Luschynski, 1961). Field measured heads should be converted to either a transformed head or an equivalent fresh water head. Equivalent fresh water head is defined as the level of fresh water in a well needed to balance the pressure at the screened interval of the well and is written as:

\[ h_f = \frac{P}{\rho_f g} + Z \]  \[3.8\]

where \( P \) is the fluid pressure, \( g \) is the gravitational acceleration, \( Z \) is the elevation above a datum, and \( \rho_f \) is fresh water density. Field measured head is converted to equivalent fresh water head (\( h_f \)) by:

\[ h_f = \frac{P}{\rho_f} h - Z \left( \frac{\rho - \rho_f}{\rho_f} \right) \]  \[3.9\]

where \( \rho \) is the density of the ground water at the point the head is being measured. Equation [3.9] is accurate only if the monitoring well contains water of a constant and equal density to that of the ground water at the point being measured. Vertical ground water velocity is calculated from equivalent fresh water head by:
\[ V_z = -K_z \left[ \frac{\partial h_f}{\partial z} + C \left( \frac{\rho_s - \rho_f}{\rho_f} \right) \right] \]  

[3.10]

where \( \rho \) is the salt water density, \( V_z \) is the vertical Darcy velocity, \( C \) is relative salt concentration, and \( K_z \) is the vertical hydraulic conductivity. The relative salt concentration, \( C \), in Equation [3.10] is the average salt concentration over the distance the gradient is defined.

Only contours of the transformed head should be used to derive flow directions in the transition zone (Huyakorn et al., 1987). Transformed head, \( h_t \), is defined as:

\[ h_t = h_f + \left( \frac{\rho_s - \rho_f}{\rho_f} \right) C Z \]  

[3.11]

Figure 3.8 shows the field measured head, equivalent fresh water head, and transformed head from the three shoreline monitoring wells. There is a minimal difference in the values of the field measured heads and the transformed heads. The small differences between the two values are the result of the small changes in elevation encountered in the Columbia aquifer. Based on the plot of transformed head, Figure 3.18, a downward vertical hydraulic gradient existed for approximately four hours surrounding high tide. Surface water entered the sediments during this recharge period. Although not as predominant as the downward gradient, an
upward gradient existed before and after recharge period. The period of time the
gradient was upwards does not indicate ground water discharge. A portion of that
time the sediments were exposed and the upward gradient represented the
movement of the water table towards the sediment water interface.

**Fresh Water / Salt Water Interface**

Salinity concentrations measured in the monitoring wells are shown for two
sampling dates in Table 3.8 and Table 3.9. The transition zone is narrow, decreasing
in concentration from greater than 24 ppth salinity to less than 0.5 ppth salinity
over 36.7 m. A very slight landward movement of the transition zone at high tide is
indicated by the increase in the salinity concentration at the two deeper shoreline
wells.

Table 3.8 Concentration of Salinity in Monitoring Wells for July 31, 1993.

<table>
<thead>
<tr>
<th>Well Location</th>
<th>Low Tide Salinity (ppth)</th>
<th>High Tide Salinity (ppth)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance Elevation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shoreline -0.65 m</td>
<td>22.013</td>
<td>22.237</td>
</tr>
<tr>
<td>Shoreline -3.60 m</td>
<td>16.283</td>
<td>17.416</td>
</tr>
<tr>
<td>Shoreline -6.17 m</td>
<td>6.679</td>
<td>8.095</td>
</tr>
<tr>
<td>10.2 m -0.49 m</td>
<td>0.463</td>
<td>0.451</td>
</tr>
<tr>
<td>10.2 m -3.78 m</td>
<td>0.523</td>
<td>0.613</td>
</tr>
<tr>
<td>10.2 m -6.83 m</td>
<td>0.054</td>
<td>3.328</td>
</tr>
<tr>
<td>36.7 m -0.44 m</td>
<td>0.129</td>
<td>0.129</td>
</tr>
<tr>
<td>36.7 m -3.49 m</td>
<td>0.309</td>
<td>0.312</td>
</tr>
<tr>
<td>401.3 m water table</td>
<td>0.072</td>
<td>0.077</td>
</tr>
<tr>
<td>Cherrystone Inlet Water</td>
<td>22.801</td>
<td>22.801</td>
</tr>
</tbody>
</table>
Table 3.9 Concentration of Salinity in Monitoring Wells for October 21, 1993.

<table>
<thead>
<tr>
<th>Well Location</th>
<th>Low Tide Salinity (ppth)</th>
<th>High Tide Salinity (ppth)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance Elevation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shoreline -0.65 m</td>
<td>24.343</td>
<td>24.330</td>
</tr>
<tr>
<td>Shoreline -3.60 m</td>
<td>21.868</td>
<td>22.321</td>
</tr>
<tr>
<td>Shoreline -6.17 m</td>
<td>17.752</td>
<td>18.218</td>
</tr>
<tr>
<td>10.2 m -0.49 m</td>
<td>0.321</td>
<td>0.305</td>
</tr>
<tr>
<td>10.2 m -3.78 m</td>
<td>1.411</td>
<td>1.486</td>
</tr>
<tr>
<td>10.2 m -6.83 m</td>
<td>2.536</td>
<td>2.596</td>
</tr>
<tr>
<td>36.7 m -0.44 m</td>
<td>0.120</td>
<td>0.124</td>
</tr>
<tr>
<td>36.7 m -3.49 m</td>
<td>0.309</td>
<td>0.322</td>
</tr>
<tr>
<td>401.3 m water table</td>
<td>0.157</td>
<td>0.154</td>
</tr>
<tr>
<td>Cherrystone Inlet Water</td>
<td>24.577</td>
<td>24.579</td>
</tr>
</tbody>
</table>

Neglecting tidal influences, the density difference between fresh water and salt water should create a concentration gradient within the transition zone of a coastal aquifer such that at any horizontal distance along the transition zone the salinity concentration would increase with depth from the water table. This pattern is created because of the density difference between the two fluids and salt water and fresh water tend to be immiscible fluids. Seaward flowing less dense fresh water would flow over the intruding wedge of denser salt water creating a transition zone between the two fluids. However, as shown in Table 3.10, such conditions do not exist within the transition zone at the shoreline. The salinity measurements from the three monitoring wells at the shoreline indicate the concentration gradient within the transition zone is reversed from the expected pattern.

An unstable salinity gradient exists where less dense, less saline ground water is overlaid by a region of more dense, more saline ground water. At all three
depths of the shoreline well cluster the salinity decreased with increasing depth from the water table. The inverted salinity gradient was consistently observed at the shoreline with the exception of three out of eleven sampling dates. On those three dates, the salinity measured in the shallow and mid-depth wells decreased with depth. However, the salinity measured in the deep well increased over the salinity measured in the mid-depth well. The inverted salinity gradient was predicted by computer modeling of the near shore ground water system.

Table 3.10  Concentration of Salinity from the Shoreline Monitoring Well Cluster. The dates marked with an asterisk indicate those sampling dates showing a pocketing of the fresh water region.

<table>
<thead>
<tr>
<th>Sample Date</th>
<th>-0.65 m Well Salinity (ppth)</th>
<th>-3.60 m Well Salinity (ppth)</th>
<th>-6.17 m Well Salinity (ppth)</th>
<th>Ambient Cherrystone Salinity (ppth)</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 08, 1993</td>
<td>17.860</td>
<td>13.800</td>
<td>12.210</td>
<td>N/A</td>
</tr>
<tr>
<td>July 31, 1993</td>
<td>22.013</td>
<td>16.283</td>
<td>8.095</td>
<td>22.801</td>
</tr>
<tr>
<td>May 24, 1994 *</td>
<td>16.214</td>
<td>2.803</td>
<td>11.816</td>
<td>18.155</td>
</tr>
<tr>
<td>July 10, 1994</td>
<td>20.591</td>
<td>12.000</td>
<td>11.476</td>
<td>N/A</td>
</tr>
<tr>
<td>Sept. 8, 1994</td>
<td>22.237</td>
<td>17.654</td>
<td>10.301</td>
<td>N/A</td>
</tr>
</tbody>
</table>

For the salinity gradient observed at the shoreline monitoring wells to consistently exist the unstable pattern must be regenerated on a frequent basis. The proposed mechanism for the creation of this salinity gradient is the tide. Although the tidal process has been cited as a mechanism that would create a more disperse
transition zone (Fetter, 1994), the author is unaware of any references within the ground water literature that describe the salinity pattern observed at the study site.

The inverted salinity gradient occurs as a result of the flood tide saturating the intertidal sediments with high salinity sea water. Fresh ground water flowing upward along the fresh water/salt water interface does not flush the sediments of this high salinity water. After many tidal cycles a region is created within the intertidal zone where the salinity gradient within transition zone is from high salinity to low salinity with increasing depth from the sediment water interface.

The depth to which this inverted salinity gradient created by the tide penetrates the aquifer will depend on the depth of the aquifer and the relative location of the salt water fresh water interface. The depth to which the inverted gradient penetrates into the Columbia aquifer at the study site can not be determined from current field data. The inverted salinity gradient present in the ground water within the intertidal zone will result in a ground water discharge with a lower percentage of fresh ground water than would occur in a non-tidal system. No data are available to compare the tidal system studied to a similar non-tidal system.

The concentration of salinity in the transition zone varied significantly throughout the study period. The salinity measured in the shoreline monitoring wells fluctuated from 16.21 ppth to 24.34 ppth salinity in the shallow monitoring well, 2.80 ppth to 22.32 ppth salinity in the mid-level monitoring well, and 6.68 ppth to 19.99 ppth in the deep monitoring well.

The variation in the concentration of salinity within the transition zone is caused by either a shift in the location of the transition zone in response to an
increased or decreased regional hydraulic gradient or the variation in the
ccentration of salinity in Cherrystone Inlet. The concentration of salinity within
the landward edge of the transition zone is given in Table 3.11 for two values of the
regional hydraulic gradient. The transition zone shifted slightly seaward due to the
increased regional hydraulic gradient. This shift of the landward edge of the
transition zone can not be translated directly to the seaward edge as the
concentration gradients within the transition zone are not expected to be uniform.
The salinity concentration of Cherrystone Inlet varied from 18.16 ppth to 24.58 ppth
salinity. The variation in salinity measured in the shallow and mid-depth
monitoring well at the shoreline tended to follow the variation in the salinity
measured in Cherrystone Inlet. However, changes in the concentration of salinity in
the mid-depth well were often more drastic. Both a shift in the location of the
transition zone in response to a change in the regional hydraulic gradient and the
variation in the concentration of salinity of Cherrystone Inlet are believed to
explain the salinity variation within the region of the transition zone located in the
intertidal zone.
Table 3.11  Salinity Measurements in Monitoring Wells Indicating Movement of Fresh Water / Salt Water Transition Zone in Response to Increased Regional Hydraulic Gradient.

<table>
<thead>
<tr>
<th>Sample Date</th>
<th>October 21, 1993</th>
<th>May 24, 1994</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regional Hydraulic Gradient</td>
<td>$3.27 \times 10^{-4}$</td>
<td>$5.51 \times 10^{-4}$</td>
</tr>
<tr>
<td>Well Location</td>
<td>Well Depth</td>
<td>Salinity (ppth)</td>
</tr>
<tr>
<td>10.2 m</td>
<td>-0.49 m</td>
<td>0.305</td>
</tr>
<tr>
<td></td>
<td>-3.78 m</td>
<td>1.486</td>
</tr>
<tr>
<td></td>
<td>-6.83 m</td>
<td>2.569</td>
</tr>
<tr>
<td>36.7 m</td>
<td>-0.44 m</td>
<td>0.124</td>
</tr>
<tr>
<td></td>
<td>-3.49 m</td>
<td>0.322</td>
</tr>
</tbody>
</table>

**Ground Water Discharge Rates**

Ground water discharge rates are shown in Figure 3.9. The discharge rates represent discharge rates averaged over the sampling interval. Discharge rates ranged from 3.3 L/m²hr to 0.0 L/m²hr. Discharge rates exhibited an exponential decrease with distance from the shore. This characteristic exponential decrease with distance from the shore has been detected in both fresh water (Lock and John, 1978; Lee et al., 1980) and marine environments (Bokuniewicz, 1980) although the trend of an exponential decrease is not universal (Cherkauer and Nader, 1989; Shaw and Prepas, 1990). Results from seepage meters can vary significantly even between meters placed as close as one meter apart (Shaw and Prepas, 1990a). Heterogeneity of the sediment characteristics (Zimmermann, 1991), variation in sediment thickness (Cherkauer and Nader, 1989), and the presence of benthic invertebrates (Zimmerman et al., 1985) have been cited as explanations for the large variation in
ground water discharge rates. This trend of significant variation in measured
ground water discharge as measured with seepage meters was frequent in the field
study as evident from the large standard deviations shown in Figure 3.9.

Ground water discharge from the study site was a mixture of fresh ground
water and sea water. Total ground water discharge rates and fresh water discharge
rates are given in Table 3.12. Fresh water discharge rates were calculated from
Equation [3.2]. The fresh water discharge rate averaged over the sampled discharge
zone was 0.14 liter / m² hr. Fresh water constituted 5 % to 32 % of the total
discharge. However, the fresh water discharge rates should be considered only an
order of magnitude estimate at best.

Table 3.12  Comparison of Total Ground Water Discharge and Fresh Water

<table>
<thead>
<tr>
<th>Distance Offshore (m)</th>
<th>Total Discharge Volume (mL)</th>
<th>Discharge Rate (L/m²hr)</th>
<th>Volume Fresh Water (mL)</th>
<th>Fresh Water Discharge Rate (L/m² hr)</th>
<th>Percent Fresh Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.7</td>
<td>1730</td>
<td>2.85</td>
<td>180</td>
<td>0.30</td>
<td>10.5</td>
</tr>
<tr>
<td>4.8</td>
<td>1130</td>
<td>1.06</td>
<td>55</td>
<td>0.05</td>
<td>4.7</td>
</tr>
<tr>
<td>9.8</td>
<td>430</td>
<td>0.37</td>
<td>137</td>
<td>0.12</td>
<td>32.4</td>
</tr>
<tr>
<td>9.8</td>
<td>720</td>
<td>0.62</td>
<td>176</td>
<td>0.15</td>
<td>24.2</td>
</tr>
<tr>
<td>9.8</td>
<td>520</td>
<td>0.44</td>
<td>83</td>
<td>0.07</td>
<td>15.9</td>
</tr>
<tr>
<td>9.8</td>
<td>890</td>
<td>0.76</td>
<td>109</td>
<td>0.09</td>
<td>11.8</td>
</tr>
<tr>
<td>19.8</td>
<td>200</td>
<td>0.16</td>
<td>31</td>
<td>0.03</td>
<td>18.8</td>
</tr>
</tbody>
</table>
Figure 3.8. Field Measured, Equivalent Fresh Water, and Transformed Heads from Shoreline Monitoring Wells on July 17, 1994.
Figure 3.9. Seepage Meters Results of Ground Water Discharge Rates with Distance from Shoreline. Error bars represent one standard deviation.
Data shown is compilation of all sampling dates.
Head Differential Across Sediment Water Interface

Hydraulic head differences between the sediment and surface water at various distances from the shoreline are shown in Figures 3.10 and 3.11. Maximum head differentials were measured at the 4.8 m offshore piezometer. This piezometer was located in the transition region from the well drained sandy sediments to the finer textured sediments. Maximum discharge rates were also measured at this location with seepage meters (see Figure 3.9). The head differentials at the 4.8 m offshore piezometer were inversely correlated with tidal elevation. Hydraulic head differences, and therefore the discharge rate, at the 4.8 m offshore piezometer increased by a factor of six to eight during the ebb tide.

Farther offshore, as measured by the 9.8 m offshore piezometer, the hydraulic head differences were greatly reduced yet showed the same trend of increasing head differences during the ebb tide. Even farther offshore, at the 14.8 m offshore piezometer, negative hydraulic head differences were measured indicating the flow of surface water into the sediments. The reduced hydraulic gradient in conjunction with the lower permeability of the finer textured sediments account for the lower discharge rates measured in the offshore sediments.

As shown in Figures 3.10 and 3.11, the head differential across the sediment water interface increased as the height of the overlying water column decreased during ebb tide. The maximum head differentials at the 4.8 m offshore piezometer were measured as the edge of the ebb tide approached the piezometer location. For the May 24, 1994 data, Figure 3.10, this occurred approximately 4.5 hours past high
Figure 3.10. Hydraulic Head Differences Measured by 100 cm Deep Piezometers in the Intertidal Zone on May 24, 1994.
Figure 3.11. Hydraulic Head Differences Measured by 100 cm Deep Piezometers in the Intertidal Zone on July 17, 1994.
tide while for the July 17, 1994 data, Figure 3.11, this occurred approximately at low tide. The inverse relationship between tide height and head differential across the sediment-water interface was also noticeable at the 9.8 m offshore piezometer. Head differentials across the sediment-water interface in the six meters of intertidal zone above the first piezometer station were not measured.

The maximum head differential, and therefore the maximum ground water discharge rate, at the 4.8 m offshore piezometers occurred when the overlying water column was at its minimum height. On May 24, 1994, Figure 3.10, this occurred approximately 1.5 hours before low tide. The low tide water mark for May 24 was approximately 4.3 m past the piezometers (9.1 m offshore). On July 17, 1994, Figure 3.11, maximum ground water discharge rate occurred approximately at low tide. The low tide water mark for July 17, 1994 was approximately 1.0 m past the piezometers (5.8 m offshore). Within the intertidal zone, the time of maximum discharge did not necessarily coincide with low tide. When low tide mark extended into the finer textured sediments, as was the case for the May 24 data, maximum ground water discharge occurred before low tide. However, the location within the intertidal zone where maximum ground water discharge rates occur remained unchanged with the variations in the tidal range.

In a non-tidal system the aerial extent of the ground water discharge zone and hydrologic conditions are relatively constant on a daily basis. The maximum ground water discharge volume would occur in the same region where maximum ground water discharge rates are measured. However, in a tidal system where ground water discharge occurs in an intertidal zone, the ground water discharge
zone and hydrologic conditions vary with the tide. Maximum ground water discharge volume may not necessarily occur in the same region where maximum ground water discharge rates are measured. The zone of maximum ground water discharge is a function of ground water discharge and time over which ground water discharge occurs. This should be considered when discussing ground water discharge from an environmental and ecological viewpoint.

**Conclusions**

This research was a field study of the near shore ground water system and of ground water discharge to a tidal estuarine system. This field study confirmed several previously known characteristics of ground water discharge. Ground water discharge from the Columbia aquifer was focused near the shore. Ground water discharge rates decreased rapidly with distance offshore and were inversely related to the tidal elevation.

Specific to this field study, maximum ground water discharge rates occurred within the intertidal zone and were found to increase by a factor of six to eight during ebb tide. The hydraulic gradient as measured at the water table fluctuated over a tidal cycle with a reversal of the hydraulic gradient towards land occurring under high tide conditions. Significantly higher hydraulic gradients were measured near the shoreline.

The salinity gradient within the salt water/fresh water transition zone was found to be significantly different than the uniform wedge like appearance generally discussed in the ground water literature. A region of the fresh water/salt water
transition zone within the intertidal zone exhibited a salinity gradient inverted from the conventional gradient of an increased salinity concentration with depth from the water table. This region of inverted concentration gradient appears to be maintained by the saturation of the sediments in the intertidal zone during flood tide.
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Environmental Protection Agency *Chesapeake Bay Groundwater Toxics Loading Workshop Proceedings*. April 15-16, 1992 Annapolis, Maryland CBP/TRS 96/93 July 1993


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CHAPTER 4

MODELING GROUND WATER DISCHARGE TO TIDAL ESTUARINE ENVIRONMENTS
Introduction

Coastal aquifers are a vital source of fresh ground water for many coastal communities. Beginning with the work of Badon Ghyben and Herzberg in the late 1800's many researchers have investigated the interaction between fresh ground water and saline ground water along the coast. Much of this research has involved the development and application of numerical ground water models to predict the movement of salt water in response to withdrawals of fresh ground water. This research was necessary to prevent the excessive movement of salt water into the fresh water aquifer which would result in the loss of a potable ground water supply and the need for costly remedial or treatment strategies.

In the late 1960's, researchers began to investigate the movement of fresh ground water from coastal aquifers into estuarine and marine systems. This process, termed submarine ground water discharge, has been shown to be an advective transport mechanism for anthropogenic contaminants originating from upland sources (Johannes and Hearn, 1985; Giblin and Gaines, 1990; Vroblesky et al., 1991). This advective transport of ground water contaminants has also been observed in fresh water systems (Cherkauer and Hensel, 1986; Shaw and Prepas, 1990; Cherkauer et al., 1992). Research of submarine ground water discharge has consisted mainly of field studies (Macintyre et al., 1989; Reay et al., 1992, Gallagher et al. 1996). Although numerical ground water flow and transport models have been utilized to study ground water discharge in fresh water systems (Munter and Anderson, 1982; Krabbenhoft et al., 1990; Cherkauer and McKereghan, 1991;
Cherkauer et al., 1992) the author is unaware of any studies specifically undertaken to model ground water discharge in the intertidal zone of estuarine or marine systems.

Ground water flow in the near shore region of coastal aquifers differs in two important aspects from fresh ground water systems. The near shore flow pattern in a coastal aquifer is generally dominated by variable density ground water and the daily fluctuations of the tide. The variable density ground water requires the use of a density-dependent flow model similar to a salinity intrusion model. However, the tide creates a dynamic discharge boundary along the sediment water interface of the intertidal zone that is generally not accounted for in salt water intrusion models. This paper describes the development and application of a density dependent ground water flow model that incorporates the dynamic tidal boundary conditions that exist along the intertidal zone.

**Model Development**

A two-dimensional finite element ground water flow and contaminant transport model, termed FEMCoast, was developed to simulate ground water discharge and contaminant transport to the near-shore zone of an estuarine or marine system. The model is a density-dependent flow model incorporating a dynamic tidal boundary condition along the sediment water interface of the intertidal zone. The model domain is a two-dimensional vertical plane normal to the shoreline.
The model solves the two-dimensional equation governing the flow of a variable density fluid:

\[
\text{div} \left( K_{ij} \left( \frac{\partial h}{\partial x_j} + e_j \gamma C \right) \right) = S_o \frac{\partial h}{\partial t} + n \gamma \frac{\partial C}{\partial t} \tag{4.1}
\]

where \( K_{ij} \) (L/T) is the hydraulic conductivity tensor, \( h \) (L) is the equivalent fresh water head, \( x_j \) (L) are Cartesian coordinates, \( e_j \) (-) is the gravitational unit vector, \( \gamma \) (-) is the density difference ratio, \( C \) (-) is the relative salt concentration, \( n \) (-) is porosity, \( S_o \) (1/L) is the specific storage, and \( t \) (T) is time. Relative salt concentration, \( C \), and density difference ratio, \( \gamma \), are defined as:

\[
C = \frac{C_{act}}{C_{max}} \quad \quad \gamma = \frac{\rho_s - \rho_{ref}}{\rho_{ref}} \tag{4.2}
\]

where \( C_{act} \) (M/L^3) is the salt concentration in the water, \( C_{max} \) (M/L^3) is the maximum salt concentration, \textit{i.e.} ambient surface water, \( \rho_{ref} \) (M/L^3) is the reference density, and \( \rho_s \) (M/L^3) is the density corresponding to the maximum salt concentration. The reference density is taken to be that of fresh water. Equivalent fresh water head, \( h \), is defined as:

\[
h = \frac{P}{\rho_{ref} g} + Z \tag{4.3}
\]
where \( P \, (M/LT^2) \) is pressure, \( g \, (L/T^2) \) is the acceleration due to gravity, and \( Z \, (L) \) is the elevation above a referenced datum.

The model must also solve the conservative advective-dispersive transport equation for salt:

\[
\frac{\partial C}{\partial t} = \frac{\partial}{\partial x_i} \left( D_{ij} \frac{\partial C}{\partial x_j} \right) - v_i \frac{\partial C}{\partial x_i}
\]

\[ [4.4] \]

where \( D_{ij} \, (L^2/T) \) is the hydrodynamic dispersion tensor, \( v_i \, (L/T) \) is the interstitial fluid velocity.

Darcy's Law written in terms of equivalent freshwater head is:

\[
q_i = -K_{iy} \left( \frac{\partial h}{\partial x_j} + \gamma C \right)
\]

\[ [4.5] \]

where \( q_i \, (L/T) \) is the specific discharge or Darcy velocity and \( K_{ij} \, (L/T) \) is the freshwater hydraulic conductivity.

Equations [4.1] and [4.4] constitute a coupled nonlinear system of equations. A flow chart of the two iterative solutions schemes utilized to solve these equations is given in Figure 4.1. The first iterative solution scheme was utilized by Frind (1982) to solve the density dependent flow equations for a confined aquifer. At a given time step an estimate of the salinity distribution within the aquifer is made.
Figure 4.1  Flow Chart of Iterative Solution Scheme Utilized to Solve Density Dependent Flow Equations.
based on the salinity distribution from the previous time step. Based on this salinity
distribution the flow equation, Equation [4.1], is solved for the nodal head values.

However, to solve the flow equation, Equation ([4.1], a second iterative
solution scheme was required because the location of the free surface (water table)
boundary is unknown at each time step. An iterative solution scheme developed by
Neumann and Witherspoon (1971) for a free surface boundary was adapted for this
purpose. The method utilized an expandable finite element grid to track the
movement of the free surface boundary. For the current time step, an estimate of
the elevation of the water table boundary is made based on the water table
elevation from the previous time step. The flow equation, Equation [4.1], is solved
for the nodal head values. A comparison between the solved nodal head values and
the assumed elevations of the nodes at the water table is conducted. At the water
table hydrostatic pressure is zero and the hydraulic head is equal to the elevation.
Modifications are made to the estimate of the water table elevation until
convergence of the two values to a prespecified criterion is achieved. The updated
elevation of the water table nodes is simply taken to be the most recent hydraulic
head solution.

Elemental ground water velocities and dispersion coefficients are calculated
from the hydraulic head values. The transport equation, Equation [4.4], is then
solved for the salinity nodal concentration. A comparison between the solved nodal
concentration values and the assumed salinity distribution is conducted. If
necessary, the entire process is repeated until convergence is achieved.
Although not discussed in this paper, FEMCoast does contain a contaminant transport model. Solution of the density-dependent flow model is not linked to the contaminant transport model. Therefore, these two equations can be solved sequentially. The contaminant transport model solves the advective-dispersive transport equation:

\[ R_f \frac{\partial C}{\partial t} = \frac{\partial}{\partial x_i} \left( D_{ij} \frac{\partial C}{\partial x_j} \right) - \frac{\partial}{\partial x_i} \left( v_i C \right) + \frac{q_s}{n} C_s - R_f K_m C \]  \[4.6\]

where \( C \) (\( M/L^3 \)) is the concentration of solute in the liquid phase, \( C_s \) (\( M/L^3 \)) is the dissolved solute concentration at the source/sink, \( q_s \) (\( T^{-1} \)) is the volumetric flux of water per unit volume of aquifer for a source or sink, \( K_m \) (\( T^{-1} \)) is the first order rate constant and \( R_f (\cdot) \) is the retardation factor. Based on the assumptions of sorption following a linear isotherm and instantaneous equilibrium, the retardation factor is defined as:

\[ R_f = 1 + \frac{\rho_b}{n} K_d \]  \[4.7\]

where \( \rho_b \) (\( M/L^3 \)) is the bulk density of the porous medium and \( K_d \) (\( L^3/M \)) is the distribution coefficient.
Comparison to Henry's Problem

Henry (1964) published the first steady-state two-dimensional solution to the salt water intrusion process that considered the effects of dispersion and density-dependent flow. Henry's model solved for the salinity contours in a hypothetical, homogeneous, isotropic, confined coastal aquifer. Henry's problem is often a standard for code intercomparison between salt water intrusion models (Frind, 1982; Galeati et al., 1992). Henry's problem was utilized to verify the density-dependent flow capabilities of FEMCoast.

The flow system described by Henry is shown in Figure 4.2. The original boundary conditions imposed on the system were an impermeable upper and lower boundary, a constant freshwater flux on the upland boundary, and a vertical specified head seaward boundary with a specified concentration equal to sea water. To better simulate the physical reality of the system, Segol et al., (1975) replaced the specified concentration seaward boundary condition with a mixed boundary condition where the top 0.2 m of the boundary was a zero dispersive flux boundary and the lower 0.8 m was a specified concentration boundary. The zero dispersive flux boundary allowed for the advective mass transport out of the system and for the dilution of seawater outside the exit boundary. This approach to the seaward boundary was adopted by Frind (1982).

Sherif et al. (1988) later modified the mixed boundary condition of Frind (1982) by adjusting the length of the zero dispersive flux boundary throughout the
simulation. By determining the direction of flow in each element along the boundary the lengths of the two boundaries were adjusted during the simulation. When the direction of the velocity was out of the domain, a zero dispersive flux condition was set for that element. When the direction of the velocity was into the domain a specified concentration boundary was set for that element. This approach to the seaward boundary was adopted by Galeati et al. (1992). A similar approach was adopted in FEMCoast to handle the boundary conditions along the sediment water interface.

A comparison of the solution to Henry’s problem from FEMCoast’s with the solutions from the previously developed models of Frind (1982) and Galeati et al. (1992) is shewn in Figure 4.3. The position of the salinity isochlors along the bottom of the aquifer after 100 minutes is compared in the bottom graph of Figure 4.3. The position of the 0.50 salinity isochlor is compared in the top graph of Figure 4.3. The deviation from Henry’s solution in the top graph of Figure 4.3 by all three numerical codes is a result of the modification of the transport boundary condition along the seaward edge of the domain. The FEMCoast solution compares favorably with the previously published solutions to Henry’s problem.

Figure 4.4 is a vector plot of interstitial velocity and relative salinity contours of the FEMCoast solution to Henry’s problem. The transition zone in the form of the intruding salt water wedge is clearly evident. The general salinity pattern of the transition zone is representative of the salinity pattern exhibited from the majority of published modeling studies. At any horizontal distance relative salinity increases with depth. The upward flow of the fresh ground water along the salt water wedge is
also evident. Recirculation of the salt water along the transition zone as described by Kohout (1960) is also clearly shown.
\[ q(z) = 0 \quad \frac{\partial C}{\partial z} = 0 \]

\[ K = 1.0 \times 10^{-2} \text{ m/sec} \]
\[ n = 0.35 \]
\[ D = 6.6 \times 10^{-6} \text{ m}^2/\text{sec} \]
\[ q = 6.6 \times 10^{-5} \text{ m/sec} \]
\[ C = 0 \text{ (fresh water)} \]

Boundary Conditions on sea water side (left side)
\[ C = 1 \text{ (sea water)} \]
\[ h = 0.0245 (1-z) \]

Initial Conditions
\[ h = 0 \]
\[ C = 0 \]

Figure 4.2 Hypothetical Flow System of Salt Water Intrusion Process Solved by Henry.
Figure 4.3  Comparison of Solutions to Henry's Problem for FEMCoast, Frind (1982) and Galeati (1992). Top graph shows the location of the 0.50 relative concentration isochlor for all three solutions. Bottom graph shows the concentration profile along the bottom of the aquifer for all three solutions.
Figure 4.4  Vector Plot of Interstitial Velocity and Relative Concentration Contours from the FEMCoast Steady-State Solution to Henry's Problem.
Boundary Conditions for Ground Water Discharge

FEMCoast was developed to model ground water discharge from shallow unconfined aquifers to the near shore intertidal zone of marine or estuarine systems. The boundary conditions required for such a system are discussed in this section. The dynamic nature of ground water discharge in the intertidal zone required an approach similar to the approach developed by Sherif et al. (1988) to model the seaward boundary of Henry's problem.

Fluid Flow Boundary Conditions

Figure 4.5 is a schematic of the ground water flow system showing the various boundary segments. Boundary A-B, the confining layer of the aquifer, was assigned as a no-flow boundary. Boundary B-C, the upland boundary for all simulations, was assigned as a time variant fixed head boundary. This boundary is not a physical hydrogeologic boundary, such as the impermeable confining layer, but a selectively chosen hydraulic boundary. The upland boundary hydraulic head was varied to approximate the seasonal variation in the regional water table elevation from the seasonal variation in recharge. The seaward boundary (A-F) could be assigned as either a no-flow boundary or a time variant fixed head boundary. The distance offshore of the seaward boundary minimized its influence on the solution and therefore the boundary was treated as the simpler no-flow boundary. Point D on the sediment water interface of the intertidal zone will fluctuate from high tide to low tide. To accommodate this fluctuation the sediment
Figure 4.5  Schematic of the Ground Water Flow System Indicating the Various Boundary Segments.
water interface was assigned two time and space variant boundary conditions. The unexposed section, D-F, was assigned as a time variant fixed head boundary condition. The hydraulic head was set equal to the elevation plus the density corrected height of the overlying water column. The exposed section above point D was assigned to the free surface boundary, C-D. The hydraulic head of boundary D-F and the relative length of D-F and C-D fluctuated as the tidal elevation fluctuated between high tide and low tide. The tidal boundary conditions along the intertidal zone were updated at each time step.

If the tide was not simulated the boundary conditions along the intertidal zone were spatially and temporally constant and the dynamic tidal boundary was not required. Boundary D-F was assigned as a fixed head boundary condition while boundary C-D was a free surface boundary. These boundary conditions were applicable for non-tidal systems.

**Salt Transport Boundary Conditions**

Boundary A-B, the confining layer of the aquifer, was assigned as a zero dispersive flux boundary. The upland boundary (B-C) and the seaward boundary (A-F) were considered as fixed-concentration boundaries. A concentration of zero (fresh water) was assigned to the upland boundary and a concentration of one (salt water) to the seaward boundary. Boundary C-D, the free surface or water table boundary, was assigned a zero-dispersive flux boundary. There is no movement of salt across the water table boundary. The sediment water interface boundary (D-F) was assigned as a fixed-concentration boundary when flow was directed inward across
the sediment water interface. This condition simulated the flow of sea water into the aquifer and therefore a relative concentration of one (sea water) was given to the boundary. This boundary condition implies that ground water discharge has negligible impact on the water column salt concentration. Conversely, a zero dispersive flux boundary was assigned to the sediment water interface boundary (D-F) when flow was directed outward across the sediment water interface. This condition simulated ground water discharge and allowed for the advective movement of salt from the aquifer. The boundary conditions were updated at each time step.

**Initial Conditions**

Initial concentration and hydraulic head within the aquifer were obtained from a steady-state non-tidal simulation of the salt water intrusion process into an initially fresh water aquifer. The tidal elevation remained constant at the approximate elevation of mid-tide. The simulation was performed for a time period of 450 years. The length of the simulation was arbitrary but was sufficient to achieve steady state conditions. Simulations incorporating the tidal boundary condition based on this simulation were conducted for a sufficient length of time to minimize the impact of the initial conditions within the aquifer.
Density-Dependent Flow and Tidal Boundary Conditions

A series of six simulations were performed to investigate the necessity of simulating the density-dependent flow and tidal boundary conditions that occur in the near-shore zone of an estuarine system. The objective was to determine if these conditions had a significant impact on the ground water discharge pattern, near-shore ground water flow pattern, and salinity distribution along the transition zone. The dimensions of the ground water system are shown in Figure 4.6. The point noted as the shoreline is assigned a distance of 0.0 m. A negative distance denotes distance from the shoreline into the surface water. Elevations are referenced to the confining layer. For all simulations the aquifer was modeled as a homogeneous, anisotropic system with a hydraulic conductivity of 7.6 cm/hr and a porosity of 0.35. These values were selected as representative of an unconfined coastal aquifer within the Coastal Plain of Virginia. The surface water salinity and reference salinity were 20 parts per thousands (ppt).

Simulations 1 and 2 were a comparison of the effect of density-dependent flow on the ground water discharge pattern and near-shore ground water flow pattern. Simulation 1 was of a non-tidal fresh water system while simulation 2 was of a non-tidal estuarine system. For both systems the upland fresh water boundary was a constant head boundary set equal to 9.14 m while the shoreline boundary was a constant head boundary with the surface water elevation set equal to 8.78 m.
Figure 4.6  Schematic of Ground Water System Showing Dimensions of Ground Water System.
A comparison of the ground water discharge patterns for the non-tidal fresh water system and non-tidal estuarine system, simulations 1 and 2 respectively, is shown in Figure 4.7. The discharge values are summarized in Table 4.1. Ground water discharge from the non-tidal fresh water system, simulation 1, was focused near shore and exhibited an exponential decay over a discharge zone of approximately 55 m. There was no recharge of surface water across the sediment water interface into the aquifer. Ground water discharge from the non-tidal estuarine system, simulation 2, was focused extremely close to the shoreline. The discharge zone was approximately 2 m. Beyond the 2 m discharge zone the aquifer was recharged with sea water. Figure 4.8 is a vector plot of the ground water Darcy velocities overlaid on a contour plot of the relative concentration of salinity in the near shore region of the estuarine system, simulation 2. The intruding wedge of denser seawater in the estuarine system caused the ground water discharge to be more focused near the shoreline. The vector plot, Figure 4.8, shows a cyclic flow of sea water along the transition zone. This cyclic flow pattern was first hypothesized by Cooper (1959) and Kohout (1960).

The ground water discharge across the sediment/water interface of the non-tidal fresh water system, simulation 1, originated from the upland fresh water boundary. However, for the non-tidal estuarine system, simulation 2, the ground water discharge across the sediment/water interface was a mixture of fresh ground water originating from the upland boundary and recirculated sea water. Fresh ground water constituted 50.5 percent of the total discharge. Although the upland and shoreline boundaries for both systems were identical the non-tidal fresh water
system had a slightly larger regional gradient. This resulted in a 17% increase in fresh ground water discharge from the non-tidal fresh water system. Since both systems were non-tidal there were no temporal variations in the discharge rates and the discharge rates represent steady-state values.

Table 4.1  Summary of Ground Water Discharge from a Non-Tidal Fresh Water System and a Non-Tidal Estuarine System.

<table>
<thead>
<tr>
<th></th>
<th>Simulation 1 Fresh Water System</th>
<th>Simulation 2 Estuarine System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regional Gradient</td>
<td>$8.88 \times 10^{-4}$</td>
<td>$7.61 \times 10^{-4}$</td>
</tr>
<tr>
<td>freshwater Flow</td>
<td>$2.02$ L/hr</td>
<td>$1.72$ L/hr</td>
</tr>
<tr>
<td>Total Discharge</td>
<td>$2.02$ L/hr</td>
<td>$3.40$ L/hr</td>
</tr>
<tr>
<td>% Fresh Ground Water in Discharge</td>
<td>$100%$</td>
<td>$50.5%$</td>
</tr>
</tbody>
</table>

Figure 4.9 shows a vector plot of the elemental Darcy velocities overlaid on a contour plot of the hydraulic head distribution of the near shore region for the non-tidal fresh water system of simulation 1 and the non-tidal estuarine system of simulation 2. The hydraulic head values contoured in Figure 4.9 represent equivalent fresh water heads. In a variable density ground water system equivalent fresh water head contours can not be used to derive flow direction (Huyakorn et al., 1987).

Simulation 3, 4, 5, and 6 were of an estuarine system. Simulations 3, 4, and 5 were of a non-tidal system and simulation 6 was of a tidal system. The dispersivity parameter in the transport equation was varied in simulations 3, 4, and 5 to compare to the dynamic tidal boundary condition utilized in simulation 6. The
reciprocating motion of the tide is a dispersive mechanism that creates a transition zone between the fresh and saline ground water. If the dispersivity parameter of the transport equation can reproduce the dispersive effect of the tide, the tide will not be required to be directly simulated. For simulations 3, 4, and 5 the upland fresh water boundary was a constant head boundary set equal to 9.30 m while the shoreline boundary was a constant head boundary with the surface water elevation set equal to 8.78 m.

Contour plots of the relative concentration of salinity for the non-tidal estuarine system from simulations 3 and 5 are shown in Figure 4.10. The ground water discharge rates for the non-tidal system from simulations 3, 4, and 5 are summarized in Table 4.2. Although the width of the transition zone increased with increased values of dispersivity, the general contour pattern within the transition zone for the non-tidal estuarine system was not significantly altered with increased values of the dispersivity parameter. The contour pattern is similar to the contour pattern obtained for Henry's problem shown in Figure 4.4. The increase in the width of the transition zone was more significant along the top of the aquifer. The rate of total ground water discharge across the sediment water interface of the non-tidal estuarine system increased as the value of dispersivity increased although the rate of fresh ground water discharge remained constant.
Table 4.2  Summary of Ground Water Discharge from a Non-Tidal Estuarine System with Increased Dispersivity Parameter.

<table>
<thead>
<tr>
<th></th>
<th>Simulation 3</th>
<th>Simulation 4</th>
<th>Simulation 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha_L=0.10$  $\alpha_T=0.025$</td>
<td>$\alpha_L=1.0$  $\alpha_T=0.25$</td>
<td>$\alpha_L=10.0$  $\alpha_T=2.5$</td>
</tr>
<tr>
<td>Fresh Water Flow</td>
<td>2.57 L/hr</td>
<td>2.57 L/hr</td>
<td>2.58 L/hr</td>
</tr>
<tr>
<td>Total Discharge</td>
<td>3.61 L/hr</td>
<td>3.66 L/hr</td>
<td>3.88 L/hr</td>
</tr>
<tr>
<td>% Fresh Water</td>
<td>71.2 %</td>
<td>70.0 %</td>
<td>66.5 %</td>
</tr>
</tbody>
</table>

Simulation 6 utilized the dynamic tidal boundary condition and density-dependent flow. The upland fresh water boundary was a constant head boundary set equal to 9.30 m while the shoreline boundary was modeled as a dynamic tidal boundary with a mean tide range of 0.73 m and tidal period of 12.8 hours.

Figure 4.11 is a contour plot of the relative concentration of salinity within the tidal estuarine system. The general contour pattern within the transition zone is significantly different from the contour pattern of the non-tidal estuarine system shown in Figure 4.10. The transition zone in the tidal estuarine system is significantly broader than the transition zones of the non-tidal estuarine system. More significant than the width of the transition zone is the general difference in the contour patterns shown in Figures 4.10 and 4.11. The tidal estuarine system, Figure 4.11, exhibits a more complex contour pattern near the shoreline. The contour patterns of the non-tidal estuarine system, Figure 4.11, exhibit an inverted concentration gradient in the intertidal zone. The inverted concentration gradient is a gradient of increasing relative concentration (salinity) with depth from the water table. This region with the inverted concentration gradient occurs within the intertidal zone and to a depth of approximately 4 to 5 meters below the sediment.
water interface. Below this depth and landward the concentration gradient and contour pattern return to the standard salt water wedge shape and gradient depicted in the ground water literature.

Density-dependent flow and tidal boundary conditions significantly alter the ground water discharge pattern, near-shore ground water flow pattern, and salinity distribution along the transition zone. Density dependent flow significantly increased the total discharge across the sediment water interface and slightly varied the regional gradient resulting in decreased fresh water flow across the sediment water interface. The salinity pattern within the intertidal zone created by the dynamic tidal boundary condition could not be replicated by altering the dispersivity parameter of the transport equation.
Figure 4.7  Steady-State Ground Water Discharge Rate with Distance Offshore for a Non-Tidal Fresh Water System and a Non-Tidal Estuarine System.
Figure 4.8  Vector Plot of Steady-State Darcy Velocities Overlaid on Relative Concentration Contours for Non-Tidal Estuarine System.
Figure 4.9  Vector Plot of Elemental Darcy Velocities and Hydraulic Head Distribution for Non-Tidal Fresh Water System (Top Graph) and Non-Tidal Estuarine System (Bottom Graph).
Figure 4.10  Contour Plot of Relative Concentration for Non-Tidal Estuarine System. Top graph: $\alpha_L = 0.10$ ft, $\alpha_T = 0.025$ ft. Bottom graph: $\alpha_L = 10.0$ ft, $\alpha_T = 2.5$ ft.
Figure 4.11  Contour Plot of Relative Concentration in Tidal Estuarine System.
Sensitivity Analysis

A sensitivity analysis was performed to determine the effect of specific yield, hydraulic conductivity, and the ratio of horizontal to vertical hydraulic conductivity on the fluctuation of the water table. A normalized sensitivity, $S$, was calculated for each parameter:

$$S_K = K \frac{\partial s}{\partial K}$$  \[4.6\]

where $S_K$ (L) is the normalized sensitivity of hydraulic conductivity, $K$ (L/T) is the hydraulic conductivity, $s$ (L) is the water table fluctuation. Water table fluctuation was calculated as the difference between the maximum elevation and minimum elevation of the water table corresponding to the high and low tide of a specific tidal period.

Water table fluctuation at various distances inland for a range of values of specific yield, hydraulic conductivity, and hydraulic conductivity ratio are shown in Figure 4.12. Normalized sensitivity for each parameter is shown in Figure 4.13. The base values for the sensitivity calculations were a specific yield of 0.025, a hydraulic conductivity of 7.6 cm/hr and a ratio of 1.0 for hydraulic conductivity anisotropy.
Figure 4.12  Graph of Water Table Fluctuation at Various Distances Inland from the Shoreline for Various Values of Specific Yield, Horizontal Hydraulic Conductivity, and Hydraulic Conductivity Ratio.
Figure 4.13  Normalized Sensitivity for Water Table Fluctuation with Respect to Specific Yield, Horizontal Hydraulic Conductivity, and the Ratio of Horizontal to Vertical Hydraulic Conductivity.
Based on the sensitivity plots in Figure 4.13, horizontal hydraulic conductivity has the most influence on fluctuation of the water table. In addition, fluctuation of the water table is most sensitive to the various parameters at approximately 11 m inland from the shoreline. Therefore, the data from a monitoring well located at 10.2 m inland was used for primary calibration of the flow model. Data from monitoring wells located at 17.7 m, 27.2 m, and 36.7 m were also used for model calibration.

A sensitivity analysis was performed to determine the effect of isotropic dispersivity and anisotropic dispersivity on the salinity distribution within the aquifer. No formal sensitivity parameter was calculated for dispersivity. Contour plots of the relative concentration of salinity for isotropic dispersivity values of 0.15 m, 0.30 m, and 1.5 m are shown in Figure 4.14. The salinity distribution is sensitive to dispersivity. Increasing the dispersivity increased the width of the transition zone while decreasing the wedge-like appearance of the transition zone.

The tidal influence on the salinity contours is evident at all three isotropic dispersivity values. However, increased isotropic dispersivity did alter the salinity variation with elevation along the shoreline. Figure 4.15 shows the variation in relative concentration with elevation at the shoreline (distance = 0.0). At the lower values of isotropic dispersivity, salinity decreased with depth from the surface in the upper section of the aquifer and increased with depth in the lower section of the aquifer. However, at an isotropic dispersivity of 1.5 m the salinity did not increase in the lower section of the aquifer.
Contour plots of the relative concentration of salinity for anisotropic
dispersivity ratios of values of 1.0 ($\alpha_L = 0.30 \text{ m}$, $\alpha_L = 0.30 \text{ m}$) and 10 ($\alpha_L = 0.30 \text{ m}$, $\alpha_L$
$= 0.030 \text{ m}$) are shown in Figure 4.16.
Figure 4.14. Contour Plots of Relative Concentration of Salinity for Isotropic Dispersivity Values of 0.15 m (Top Graph), 0.30 m (Middle Graph), and 1.5 m (Bottom Graph). Reference Salinity is 20 ppth.
Figure 4.15  Variation in Relative Concentration of Salinity with Elevation at the Shoreline (Distance = 0.0 m) for Isotropic Dispersivity Values of 0.15 m, 0.30 m, and 1.5 m. Reference Salinity is 20 ppth.
Figure 4.16  Contour Plots of Relative Concentration of Salinity for Anisotropic Dispersivity Ratios of 1.0 ($\alpha_L = 0.30$ m, $\alpha_L = 0.30$ m) (Top Graph) and 10 ($\alpha_L = 0.30$ m, $\alpha_L = 0.030$ m) (Bottom Graph). Reference Salinity is 20 ppth.
Model Calibration

Manual trial-and-error calibration of the model was performed to field data collected on July 17, 1994 at a field site on Cherrystone Inlet. Field data was collected along the shoreline edge of an agricultural field bordering Cherrystone Inlet on the Chesapeake Bay side of the Eastern Shore of Virginia. This site was selected because it exhibits characteristics favorable to contaminant transport by ground water discharge (shallow water table, flat topography, permeable soils and adjacent agriculture practices) and has been used in previous studies of ground water discharge (Reay et al., 1992, Gallagher et al., 1996).

The Eastern Shore of Virginia comprises 286,296 ha of the southern tip of the Delmarva Peninsula and is the eastern part of Virginia's Coastal Plain physiographic province. The Atlantic shoreline is composed of extensive vegetated tidal marshes and barrier islands while the Chesapeake Bay shoreline is comprised of meandering tidal creeks and long irregular shaped necks. Land use for the Eastern Shore is estimated at 40 % agricultural, 50 % forested, 8.5 % pasture, and 1.5 % urban (US EPA, 1988).

The Eastern Shore is underlain by a series of aquifers and confining layers that define a local and regional ground water flow system. The local flow system consists of the unconfined Columbia aquifer and the confined Yorktown-Eastover aquifer system. Depth to the water table is approximately 2 meters while depth to the top of the upper Yorktown-Eastover confining unit may be between 8.5 m and 11.8 m below sea level (Reay et al., 1996).
The field data consisted of water table elevations, ground water discharge, and ground water salinity measurements. This data was collected from a series of monitoring wells, piezometers, and seepage meters were installed along a transect normal to the shoreline. The sampling transect extended approximately 30 m into Cherrystone Inlet and approximately 400 m upland from the shoreline.

The salinity in Cherrystone Inlet and the ground water in the intertidal zone fluctuated significantly over a short period of time. Table 4.3 gives the salinity in Cherrystone Inlet and in the ground water within the intertidal zone over a 54 day period. During the entire field study measurements of the near shore salinity in Cherrystone Inlet varied from 15.7 ppth to 24.6 ppth. Although the model is capable of accounting for a variable salinity boundary condition along the sediment water interface, the calibration procedure for the transport model did not account for this fluctuation in the salinity of Cherrystone Inlet. Sufficient long term field data on the near shore salinity of Cherrystone Inlet were not available. A concentration of salinity equal to 20 ppth was selected as a representable value for the model calibration period.

Calibration of the flow sub-model was performed first although an iterative calibration process between the flow model and salt transport model was required for calibration.

**Flow Model**

The flow model was calibrated to match the water table fluctuation measured in four monitoring wells over twelve hours. The time domain for model calibration
simulations was approximately six months, i.e. the initial time for the simulation was January 1, 1994. Tidal data for the tidal boundary were obtained from published NOAA tide tables. The upland fresh water boundary was assigned as a constant head boundary set equal to the water table elevation measured on July 17, 1994. Although the water table elevation at the upland boundary fluctuated throughout the simulated time period, the boundary was fixed throughout the simulation due to insufficient field data. This approach had minimal impact on the calibration procedure.

Table 4.3  Salinity (ppth) Measured at Monitoring Wells at Cherrystone Inlet Field Site for Three Sampling Dates.

<table>
<thead>
<tr>
<th>Well Location</th>
<th>Well Elevation</th>
<th>June 9, 1993</th>
<th>July 8, 1993</th>
<th>July 31, 1993</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 m Inland</td>
<td>-0.65 m</td>
<td>16.42</td>
<td>17.86</td>
<td>22.24</td>
</tr>
<tr>
<td>0.0 m Inland</td>
<td>-3.60 m</td>
<td>12.17</td>
<td>13.80</td>
<td>17.42</td>
</tr>
<tr>
<td>0.0 m Inland</td>
<td>-6.17 m</td>
<td>9.37</td>
<td>12.21</td>
<td>6.68</td>
</tr>
<tr>
<td>10.2 m Inland</td>
<td>-0.49 m</td>
<td>N/A</td>
<td>N/A</td>
<td>0.46</td>
</tr>
<tr>
<td>10.2 m Inland</td>
<td>-3.78 m</td>
<td>1.58</td>
<td>1.39</td>
<td>0.52</td>
</tr>
<tr>
<td>10.2 m Inland</td>
<td>-6.83 m</td>
<td>4.35</td>
<td>6.23</td>
<td>0.05</td>
</tr>
<tr>
<td>Ambient Cherrystone</td>
<td></td>
<td>18.68</td>
<td>N/A</td>
<td>22.80</td>
</tr>
</tbody>
</table>

Hydraulic conductivity and specific yield were varied to achieve the best match to the field data. The aquifer was modeled as a homogeneous system with the exception of the surficial sediments within the intertidal zone. The hydraulic
conductivity of these sediments had minimal influence on calibration of the flow model with respect to water table fluctuation. Therefore, the hydraulic conductivity of this region remained constant during the calibration procedure. Figure 4.17 shows the calibrated water table elevations and field measured water table elevations at the four upland monitoring wells selected for model calibration.

Calibrated water table elevations and fluctuation were in good agreement with field data. For the calibrated simulation the root mean square error between observed and predicted water table elevations at the 10.2 m well was 0.014 m. Water table fluctuation over the calibration period was 0.21 m. Predicted water table elevations at the 10.2 m well were above observed values at high tide and the following ebb tide. Tide elevations and times of high and low tide obtained from published NOAA tide tables were not corrected for actual field measurements.

Predicted water table elevations for the three wells further upland were consistently above observed elevations although the maximum absolute difference between observed and predicted water table elevations was only 0.020 m. The predicted and observed patterns of water table fluctuation were in good agreement. The predicted regional gradient was 4.90 x 10^{-4} while the observed regional gradient was 4.97 x 10^{-4}.

**Salinity Transport Model**

The salt transport model was calibrated to salinity measurements taken at three wells at the shoreline and at three wells 10.2 m inland from the shoreline. Dispersivity was the primary parameter utilized to calibrate the salinity transport
model. Table 4.4 shows the calibrated salinity and field measured salinity at the six monitoring well locations while the salinity contours are shown in Figure 4.18. Predicted salinity at the shoreline well cluster were slightly below observed values. However, the observed trend in salinity variation with depth was the same as the predicted trend. Similar results were seen at the 10.2 m well cluster. The calibrated model parameters are given in Table 4.5.

<table>
<thead>
<tr>
<th>Well Location, Elevation</th>
<th>June 17, 1994</th>
<th>FEMCoast</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 m Inland, -0.65 m</td>
<td>9.74</td>
<td>19.65</td>
</tr>
<tr>
<td>0.0 m Inland, -3.60 m</td>
<td>9.90</td>
<td>8.10</td>
</tr>
<tr>
<td>0.0 m Inland, -6.17 m</td>
<td>10.42</td>
<td>8.27</td>
</tr>
<tr>
<td>10.2 m Inland, -0.49 m</td>
<td>0.14</td>
<td>0.75</td>
</tr>
<tr>
<td>10.2 m Inland, -3.78 m</td>
<td>0.34</td>
<td>0.90</td>
</tr>
<tr>
<td>10.2 m Inland, -6.83 m</td>
<td>2.05</td>
<td>1.38</td>
</tr>
<tr>
<td>Ambient Cherrystone</td>
<td>20.23</td>
<td>20.00</td>
</tr>
</tbody>
</table>

The calibrated model was able to replicated the inverted salinity gradient measured within the intertidal zone. Unfortunately, there were no additional monitoring wells located within the intertidal zone. The three monitoring wells located within the intertidal zone do not provide sufficient data to describe the transition zone with certainty.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_x$</td>
<td>15.2 cm/hr</td>
</tr>
<tr>
<td>$K_z$</td>
<td>15.2 cm/hr</td>
</tr>
<tr>
<td>$S_z$</td>
<td>0.025</td>
</tr>
<tr>
<td>$n$</td>
<td>0.40</td>
</tr>
<tr>
<td>$\alpha_L$</td>
<td>0.31 m</td>
</tr>
<tr>
<td>$\alpha_T$</td>
<td>0.31 m</td>
</tr>
<tr>
<td>$K$ (Intertidal Zone Sediments)</td>
<td></td>
</tr>
<tr>
<td>0 - 5 m offshore</td>
<td>15.2 cm/hr</td>
</tr>
<tr>
<td>5 - 10 m offshore</td>
<td>7.6 cm/hr</td>
</tr>
<tr>
<td>10 - 15 m offshore</td>
<td>3.0 cm/hr</td>
</tr>
<tr>
<td>15 m - beyond</td>
<td>1.5 cm/hr</td>
</tr>
<tr>
<td>Upland Boundary</td>
<td>9.28 m</td>
</tr>
<tr>
<td>(Hydraulic Head)</td>
<td></td>
</tr>
</tbody>
</table>

The boundary conditions along the sediment water interface of the intertidal zone do not account for effects of wave action. Within the model, the movement of the tide along the intertidal zone proceeds in incremental steps based on the size of the time step. As the water level increases with the flood tide, the water table is adjusted to the each new tide location. The concentration of the water in the newly saturated sediments are given the concentration of the water below the newly
saturated sediments from previous time step. In this manner the concentration of the sediments change as if the sediments are saturated from below.

In reality the wave action saturates the sediments ahead of the actual tide location. The sediments are saturated from above with water having the same salinity as the surface water. As the concentration of salinity within the surface water changes the wave action aids to introduce this salinity change into the aquifer. This accounts for the rapidly changing concentration of salinity measured in the monitoring wells at the shoreline as shown in Table 4.3.

An attempt to model the salinity fluctuation within the aquifer as a result of the salinity fluctuation within Cherrystone Inlet was unsuccessful. The predicted concentration of salinity within the aquifer did not change as rapidly as the observed data shown in Table 4.3. However, when the simulation was continued for an additional period of time the predicted concentration of salinity within the aquifer approached the observed data. It is thought that the inability to account for the wave action is responsible for this unsuccessful model simulation. This weakness in the model must be corrected before short term transient simulations can be effectively simulated.
Figure 4.17  Comparison of Water Table Elevations from Calibrated Model and Field Data. Top Graph - Monitoring Well 10.2 m Inland from Shoreline. Bottom Graph - Monitoring Wells Located Further Upland.
Figure 4.18  Contour Plot of Salinity from Calibrated Model.
Ground Water Discharge

Temporal and spatial variation in the ground water discharge rates from the calibrated simulation are shown in Figure 4.19 and Figure 4.20. Rates ($L/m^2/hr$) for total ground water discharge, a mixture of fresh ground water and recirculated sea water, are shown in contours in Figure 4.19 while rates ($L/m^2/hr$) for fresh ground water discharge are shown in Figure 4.20. The contour plots show discharge rates with distance from the shoreline over a tidal period. The dotted curve is the distance of the tide from the shoreline. Maximum discharge rates for both total and fresh discharge occurred at low tide. The low tide for this tidal period was relatively high and therefore the maximum discharge rates occurred in the higher permeability sediments of the intertidal zone. Figure 4.21 shows the spatial distribution of total and fresh ground water discharge ($L/m^2$) across the sediment water interface over a tidal cycle and the percent fresh water in the total discharge. Maximum discharge volumes of both total and fresh water discharge occurred approximately 6 m from the shoreline within the higher permeability sediments of the intertidal zone.

Based on a mass balance fresh water input into the system during the tidal period (12.5 hours) was 1.3 liters. This represents the volume of fresh ground water flowing through a one foot width of the upper boundary. Fresh water discharge across the sediment water interface for the same time period was 1.2 liters which represents an 8.3 percent mass balance error. Fresh water discharge rates across the sediment water interface were calculated by multiplying the total discharge rate at a node by the average salinity of the element. Total discharge across the sediment
water interface was 19.4 liters. Therefore, fresh ground water constituted on average 6.2 percent of the total discharge across the sediment water interface.

Figure 4.22 shows the average total ground water discharge rates along with field measured discharge rates. The field measured ground water discharge rates are the average and standard deviation of ground water discharge rates measured over several sampling dates. Observed ground water discharge rates measured with seepage meters varied from 0.07 to 3.82 L/m²/hr. Predicted total ground water discharge rates were within this observed range.

Figure 4.23 is a series of vector plots showing the ground water flow pattern over a tidal cycle from high tide to high tide. The plot of the low tide condition has the salinity contours overlaid on the velocity vectors. The plots show the reciprocating motion of the ground water along the transition zone caused by the tide.
Figure 4.19  Variation in Total Ground Water Discharge Rates (L/m² hr) with Distance from the Shoreline Over a Tidal Period. Results from Calibrated Simulation.
Figure 4.20 Variation in Fresh Ground Water Discharge Rates (L/m²/hr) with Distance from the Shoreline Over a Tidal Period. Results are from Calibration Simulation.
Figure 4.21  Total and Fresh Ground Water Discharge (L/m²) for a Tidal Period. Results are from calibration simulation.
Figure 4.22  Average Total Ground Water Discharge Rates (L/m² hr) for a Tidal Period. Results are from calibration simulation.
Figure 4.23  Series of Vector Plots Showing Ground Water Flow Pattern over a Tidal Period.
Figure 4.23  Series of Vector Plots Showing Ground Water Flow Pattern over a Tidal Period.
Figure 4.23  Series of Vector Plots Showing Ground Water Flow Pattern over a Tidal Period.
Figure 4.23  Series of Vector Plots Showing Ground Water Flow Pattern over a Tidal Period.
Near Shore Clay Lens

Simmons et al., (1990) describe an effort to measure ground water discharge to an inlet of the Chesapeake Bay. Elevated nitrate levels in the ground water were measured from monitoring wells installed in the adjacent upland agricultural field. However, seepage meters placed in the low tide region failed to indicate the presence of fresh ground water discharge. Further investigation indicated the presence of a clay layer in the near shore intertidal region. When seepage meters were placed further offshore past the edge of the clay lens fresh ground water discharge was measured. The clay layer was a confining layer that restricted the discharge of ground water in the intertidal zone. Instead fresh ground water traveled beneath the confining clay layer and discharged further offshore.

To investigate the impact of a clay lens an estuarine system with a clay lens extending 23 m offshore was modeled. The hydraulic conductivity of the clay lens was 4.2 x 10⁻⁷ cm/sec. All other model parameters were identical to the calibrated model data set. The effect of the clay lens on the salinity pattern of the estuarine system is shown in Figure 4.24 The transition zone receded seaward under the clay lens. Figure 4.25 shows the flow path of the ground water under the clay lens. The fresh ground water discharge pattern is shown in Figure 4.26. As was shown by the field work of Simmons et al. (1990) the ground water discharge occurred at the end of the clay lens. The discharge rates were higher and the discharge zone more compact when compared to the system with no clay lens (see Figure 4.20) although the fresh water discharge volumes were identical in both systems.
Figure 4.24  Comparison of the Effect of a Near Shore Clay Lens on the Ground Water Salinity Distribution for Estuarine System.

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Figure 4.25  Vector Plot of Ground Water Flow Paths Under Near Shore Clay Lens of A Tidal Estuarine System.
Figure 4.26  Fresh Ground Water Discharge Rates (L/m²·hr) for Estuarine System with Near Shore Clay Lens.
Seasonal Discharge Patterns

Fresh ground water discharge rates vary with the seasonal variation in ground water recharge rates. Although this relationship has been noted in the literature (Staver and Brinsfield, 1990), no published field data showing this relationship was found. This seasonal pattern would be difficult to measure directly as fresh ground water often constitutes less than ten percent of the total ground water discharge. In addition, the significant variation in ground water discharge rates over a short distances would make reproducible measurements difficult. The salt water component of the total ground water discharge should not vary significantly due to the seasonal variation in the fresh water discharge rates. However, increased fresh ground water discharge rates will shift the transition zone slightly sea ward due to the elevated upland hydraulic head.

To investigate the seasonal variation in discharge in an estuarine system the upland boundary of the ground water system was increased from 9.28 m to 9.43 m. This increase raised the regional gradient from $4.90 \times 10^{-4}$ to $1.0 \times 10^{-3}$. All other model parameters were identical to the calibrated model data set. The effect of the elevated water table on the salinity pattern of the estuarine system is shown in Figure 4.27. The increased fresh water flow shifted the transition zone seaward. The shift was more significant at the landward edge of the transition zone. Figure 4.28 shows the fresh ground water discharge zone. The ground water discharge zone was expanded with higher ground water discharge rates.
Figure 4.27  Variation in Salinity Pattern Resulting from Seasonal Variation in Recharge Rates.
Figure 4.28  Fresh Ground Water Discharge Pattern from Elevated Seasonal Recharge Rates.
Conclusions

The results of this study show that the ground water model FEMCoast can simulate ground water discharge to tidal estuarine systems. The model was able to reproduce the movement of the water table in the near shore zone of the coastal aquifer. Predicted ground water discharge rates compared to the observed ground water discharge rates. Both observed and predicted ground water discharge rates decreased rapidly with distance offshore.

The dynamic tidal boundary conditions along the sediment water interface were essential to replicating the complex salinity gradients observed within the intertidal zone. The salinity gradient in the ground water within the intertidal zone exhibited an inverted concentration gradient where the concentration of salinity decreased with depth from the sediment water interface. This concentration gradient is maintained by the action of the flood tide. The model was able to reproduce this inverted salient gradient in the ground water within the intertidal zone.

However, the model was not able to replicate the short-term fluctuation in the concentration of salinity within the aquifer. It is believed that the inability to account for the wave action of the tides within the intertidal zone is to account for this difficulty.
References


Vita

Michael A. Robinson received a B.S. in Chemical Engineering from West Virginia Institute of Technology in 1985, a M.S. in Environmental Engineering from Virginia Polytechnic Institute in 1990, and a Ph.D. in Civil Engineering from Virginia Polytechnic Institute in 1996.

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