

**A DECISION MAKING TOOL FOR EVALUATING UNCERTAINTIES  
IN ELECTRIC POWER SYSTEM PLANNING**

by

**Ali Reza Osareh**

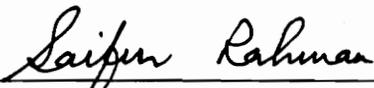
Dissertation Submitted to the Faculty of the  
Virginia Polytechnic Institute and State University  
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

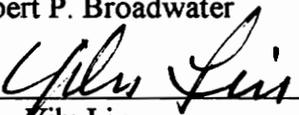
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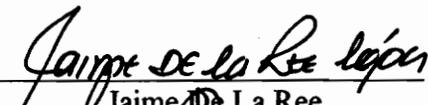
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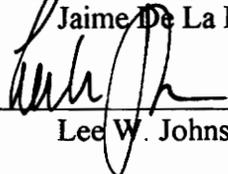
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# **A DECISION MAKING TOOL FOR EVALUATING UNCERTAINTIES IN ELECTRIC POWER SYSTEM PLANNING**

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(ABSTRACT)

Planning of today's electric utilities demand careful consideration of issues such as environment, demand-side management, non-utility generation, and new technologies which are subject to different constraints and uncertainties. Utilities have long developed and used models for their short and long-term planning, most of which are single purpose, large, data intensive, and do not fully account for uncertainties.

New techniques have emerged to deal with uncertainties in utility planning. Among them, the Analytic Hierarchy Process (AHP) has been more successful in assessing uncertainties, and found to be well structured and applicable to individual as well as group decision makers. However, the results of this method are merely point estimate values.

It is the objective of this research to identify a methodology which is capable of evaluating uncertainties with relative ease and accuracy without the need for a large volume of data and complicated software packages. The Analytic Hierarchy Process has been extended to estimate the variance of the error in judgments and therefore the confidence interval of values instead of point estimate values. A simulation study was

carried out to check the accuracy of error variance (QI) in confidence interval calculations. The results showed that QI has a linear relationships with the variance of weights.

The extended AHP method is applied to three case studies, including 1) Third party generation bidding evaluation criteria, 2) Identification and evaluation of different load management programs on utility peak reduction, and 3) Oil price prediction for electric utilities. This method promises to be an effective decision making tool for evaluating uncertainties in electric power system planning.

## **Acknowledgments**

I would like to express my appreciation and gratitude to those who helped and supported me during the course of my doctoral studies. First, the most gratitude is toward GOD almighty for his blessings on me without which I would not have been able to finish this task. Second, toward my advisor Professor Saifur Rahman for all his guidance, encouragements, support, and patience throughout the years of my study and research without which this work would not have been finished either.

I would like to extend my gratitude toward my committee members, Dr. Robert P. Broadwater, Dr. Yilu Liu, Dr. Jaime De La Ree, and Dr. Lee W. Johnson for their time, and serving on my committee and Dr. Ahmad Safaai Jazi for his comments and support.

Also my deepest gratitude is toward my wonderful family, specially my parents and brother who have stood by me through all the troubles during these years and supported me in any way which they could.

Finally I would like to thank all my friends, those who are back home and those in the US, specially at Virginia Tech and in the Energy Lab, for their encouragements and support.

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# CHAPTER I

## INTRODUCTION

Electric utility planning has changed significantly from the time when capacity expansion planning was based upon cost minimization techniques in an environment with relative certainty to the more complicated and uncertain world of today.

Planners in today's electric power industry must consider issues like non-utility generation, demand-side management, environmental effects, and new technologies, subject to constraints and uncertainties. There are two types of planning; the strategic planning which quantifies and determines all reasonable objectives, alternatives, and options by taking into consideration, the resources and constraints, and tell us where we should be going. And, the tactical planning which tells how we should get there. Electric utility planning in the not-too distant past was largely a matter of optimization among several alternatives. A utility first determined the probable future need for the supply of kilowatts of demand and kilowatt-hours of energy and then using computer simulations, determined its ability to meet such demand throughout its service area. Then, it chose, among several alternatives, the program for additional facilities that optimally met the load. Electric utilities were operating in an environment where:

- 1) Cost of fuel was constant or declining;
- 2) Inflation was low;
- 3) Load growth was high;
- 4) The real price of electricity declined; and
- 5) Utilities were financially sound.

In such an environment, strategic plans involved increasing the degree of inter-connection, selling as much energy as possible, building transmission and generation facilities ahead of demand, and encouraging the use of electricity. The tactical planning simply involved decision on where, when, and how much. This view of planning, however suffers from imprecision. Planning must include the commitment and the power to carry out the planned strategies, actions, or projects involving a significant amount of uncertainty.

## **1.1 Important Issues in Utility Planning**

The history of rising price of electricity in the US. during the past fifteen years and the financial failure of nuclear technology in this country have led to near break-down of the traditional regulatory process. Many planners are proposing more reliance on competitive market mechanisms to govern the industry. At the same time, regulators have moved to assert more control of the industry's planning process through "least-cost planning" and to mandate consideration of a wider range of options, including demand-side measures such as conservation and independent power producers. Furthermore, the world's energy markets have undergone significant structural changes in the last fifteen

years, destroying once-stable trends in cost, availability, and use of basic energy resources. Finally, growing public concern about environmental quality has restricted and will continue to restrict many technological options for electricity production. These issues have increased considerably the uncertainties under which electric power companies must make plans and decisions.

Utilities now must deal with short and long term policies for supplying electrical power. For instance, fuel planning decisions have become more complex and fuel costs directly affect a utility company's competitiveness. It accounts for 40-60% of the cost of electricity.

### **1.1.1 Independent Power Producers**

In the competitive environment in which US utilities are operating, competitive bidding is becoming an accepted means for obtaining resources to meet the utilities' forecast demand. The Public Utility Regulatory Policy Act (PURPA) of 1978 was the main force for the introduction of cogenerators and small power producers, the so called qualifying facilities (QF) as a new group of players in the electricity supply markets. Another class, the non-QF third party generators, and the independent power producers (IPP), also have brought widespread agreements that private power will be a major source of new supply of electricity.

Independent power producers must compete (with utilities and with each other) for a share of the power generation market. The generation market has become fiercely competitive. To obtain a long-term power sales agreement, IPPs often must win a competitive bid or successfully execute a competitive negotiation. In a typical bid, the

number of megawatts offered normally exceeds the number awarded by a ratio of 13 to 1, according to Naill [1]. Preparing a bid or negotiating a proposal has become expensive, especially since the purchasing utilities often award contracts only to bids with firm site plans, signed fuel contracts, and substantial progress in obtaining the necessary environmental permits. However, the development and implementation of electricity bidding systems is complicated. It is an issue with multiple attributes and some non-quantifiable factors which utilities must consider in their bidding process.

### **1.1.2 Demand-Side Management**

Demand-side management (DSM) is becoming an important part of the utility planning to meet the load by deferring the building of a new plant. Many utilities have found it in their economic interest to consider demand-side management programs along with supply-side options to meet future energy and load requirements. The process includes:

- 1) Identifying and screening DSM technologies;
- 2) Developing and screening DSM programs;
- 3) Treatment of uncertainty in selected programs; and
- 4) Integrating those programs with candidate resources on the supply side to develop a coherent plan.

Utility programs falling under the umbrella of DSM include load management, strategic conservation, time of use rates, and identification and promotion of customer generation. The utility benefits from DSM by inducing changes in the time pattern and

magnitude of electricity demand, which maximizes the productive and cost effective use of the utility's resources. Demand side alternatives are appropriate for consideration both by investor owned and public utilities. US peak demand savings by the year 2000 could be more than 60,000 MW. This will represent a saving to all utility customers of more than \$60 billion according to Keelin [2]. This program also has to be evaluated carefully and the uncertain issues be examined thoroughly.

### **1.1.3 Environmental Constraints**

The Clean Air Act Amendment (CAAA) of 1990 requires the Environmental Protection Agency (EPA) to implement programs to achieve and maintain air quality levels that protect public health and welfare and prevent significant deterioration of air quality. After four years of its passage, there still exist uncertainties over the technical, economic, and regulatory compliance issues. Electric utilities must consider environmental control as part of a business strategy. Decisions regarding investment must not only accommodate for demand growth and competitive forces and employ least-cost methods to meet generation needs, they must also consider the least-cost methods of complying with the Amendment, Torrens [3].

## **1.2 Utility Planning Environment**

Traditional electric power planning has usually considered single-purpose models. These models focus on production cost, capacity expansion, or financial planning. Since the 1973 oil crisis, power planning has become a multi-objective and multi-option problem

with ill-defined or uncertain parameters in which a variety of issues such as fuel price, fuel availability, non-utility generation activities, demand side management impact, environmental constraints, etc. must be considered.

Deterministic single-outcome models appear to be less effective in today's world of rapid and unexpected changes. Identifying uncertainties and incorporating them into resource planning is a major issue in every electric utility industry. For instance, in the UK., after privatization of electric industry, major changes have emerged in thinking on efficiency of electric supply and the methodology for its planning . Of the plans for new capacity laid down by the old Central Electricity Generation Board, which included two 900 MW coal-fired stations and four 1100 MW nuclear reactors, coal has been abandoned, and nuclear projects must wait another government review in 1994, Jeffs [4].

Therefore, planning for capacity expansion depends on many forecast parameters including load demand, plant cost, fuel cost, technological innovation, and environmental and regulatory requirements. These issues forces electric utilities to focus on critical parameters, consider the key elements of future uncertainty, and make the analyses acceptable and understandable to decision makers.

### **1.3 Scope of the Research**

Utility managers like methods and tools that not only are accurate, but also easy to use in making decisions under uncertainties. Unfortunately, what they get sometimes fails to meet these standards. Computer experts have long believed that bigger, and more complex models are the key to better planning. Most large models failed to anticipate major changes in an increasingly uncertain environment. Moreover, results often show

that more complexity and details do not always lead to greater accuracy, but tend to make a model difficult to understand and use by the decision makers, Cleary [5]. For a model to be of practical value, it should be versatile and applicable to diverse conditions of utility requirements. It should be capable of handling the complexity and the diversity of the problem and be simple and practical to be of operational value.

Therefore, models which could consider strategic planning questions as well as analyzing the uncertainties are required. These models may sacrifice the details of hour-by-hour production costing and the details on revenues, cash flows and expenses. They would focus on the utility's decision process along with the uncertainties involved in them. These difficulties suggest that selecting a resource plan necessarily calls for a considerable amount of judgments.

Therefore, the objective of this research is first to identify a methodology which is capable of evaluating uncertain issues with relative ease and accuracy without the need of large volume of data or complicated software packages. And then to implement the method to some of the problems facing electric utilities in their planning process.

In chapter two first we will review the issues and methods pertaining to multiple criteria decision making, then the issue of subjective judgments and probability assessment will be addressed. In the last section we will review the uncertain issues in electric utilities. Chapter three will discuss and evaluate the Analytical Hierarchy Process technique and its features such as consistency index (CI), quotient index (QI), and confidence interval will be discussed. Chapter four will compare different methods of dealing with uncertainty. These include probability methods, fuzzy sets, and Delphi technique. Chapter five will present techniques employed by electric utilities for dealing with uncertainties. In chapter six we will apply the technique to third party generation

bidding evaluation criteria, identification and evaluation of different load management programs on utility peak reduction, and oil price prediction for the electric utilities. Conclusions and recommendations for further research are presented in chapter seven.

## **CHAPTER II**

### **LITERATURE REVIEW**

The idea of decision making in general and in the area of power system planning in particular is the selection of the best alternative or an optimal combination of alternatives to satisfy the desired objective. The main sources of difficulty in achieving proper evaluation of the alternatives are: 1) Multiple attributes, which make it difficult to establish a common scale of comparison between alternatives, and 2) Uncertainty, which introduces the difficulty in predicting the results following an action.

This review is divided into three sections. The first part is a review of decision making with multiple criteria. Then, the issue of subjective judgments and probability in uncertain situations is discussed. In the last section, a review of uncertain issues in electric utilities is presented.

## 2.1 Multiple Criteria Decision Making (MCDM)

Decision making in real world deals with data and information which, in general, are imprecise, uncertain or fuzzy with multiple and conflicting criteria. For example, in a personal context, the job one chooses may depend upon its prestige, location, salary, advancement opportunities, working conditions, and so on. The car one buys may be characterized in terms of price, gas mileage, style, safety, comfort, etc.

In an academic context, a university's administrator's selection of the future structure of the university would be based on number of faculty, undergraduate and graduate enrollment, tuition, and new programs.

In a public context, the energy resource development plan for a community should be evaluated in terms of cost, reliability, protection, land use, and environment.

The problems of multiple criteria decision making (MCDM) are widely diverse. However, even with the diversity, all the problems which are considered share the following common characteristics, as mentioned by Hwang [7],

- (1) Multiple objectives/attributes: Each problem has multiple objectives/attributes. A decision maker must generate relevant objectives/attributes for each problem setting.
- (2) Conflict among criteria: Multiple criteria usually conflict with each other. For example, in designing a car, the objective of higher gas mileage might reduce the comfort rating due to the smaller passenger space.
- (3) Incommensurable units: Each objective/attribute has a different unit of measurement. In the car selection case, gas mileage is expressed by miles per

gallon, comfort is by cu ft, if it is measured by passenger space, cost is indicated by dollars, etc.

- (4) Design/Selection: Solutions to these problems are either to design the best alternative or to select the best one among previously specified finite alternatives. The MCDM process involves designing/searching for an alternative that is the most attractive overall criterion.

### 2.1.1 Types of MCDM Problems

The problems of MCDM can be broadly classified into two categories: 1) Multiple Attribute Decision Making (MADM), and 2) Multiple Objective Decision Making (MODM). In actual practice this classification is well fitted to the two facets of problem solving, MADM is for selection (evaluation), MODM is for design. This is a widely accepted classification, Cohon and Hwang [8,9]. Table 2.1 contrasts the features of these two classes.

- 1) The distinguishing feature of the MADM is that there is usually a limited (and countably small) number of predetermined alternatives. The alternatives have associated with them a level of achievement of the attributes (which may not necessarily be quantifiable) based on which the final decision is to be made. The final selection of the alternative is made based on inter and intra attribute comparisons.
- 2) Multiple objective decision making (MODM) is not associated with the problems where the alternatives are predetermined. The thrust of these models is to design the "best" alternative by considering the various interactions within the design

constraints which best satisfy the DM by way of attaining some acceptable levels of a set of some quantifiable objectives. The common characteristics of multiple objective decision making methods are that they possess: 1) a set of quantifiable objectives; 2) a set of well defined constraints; 3) a process of obtaining some tradeoff information, implicit or explicit, between the stated quantifiable objectives and also between stated or unstated non-quantifiable objectives. Literature on MODM methods and applications has been reviewed extensively by MacCrimmon [10].

**Table 2.1 Comparison of MADM and MODM features**

	MADM	MODM
Objective	Implicit	Explicit
Attribute	Explicit	Implicit
Constraint	Inactive (Incorporated into attributes)	Active
Alternative	Finite number, Discrete	Infinite number, Continuous
Interaction with DM	Not much	Mostly
Usage	Selection/Evaluation	Design

## 2.2 Multi Attribute Decision Making (MADM)

Although the effort to introduce the concept of multiple criteria into the normative decision making process started within the last two decades, the study on multiple criteria has a long tradition. The earlier works by researchers in many disciplines such as: management science, economics, applied statistics, and decision theory, are a good examples. They confront multiple criteria in quite different situations. Consequently, each area has developed method(s) for its own particular usage. For example

Decision theory: Maximum, prior probability, utility theory.

Economics: Pareto optimally, social welfare function, benefit/cost analysis.

Statistics: Multivariate regression, discrepancy analysis, factor analysis.

Their orientation and motivation are primarily to explain, rationalize, understand, or predict decision behavior and not to guide the decision making. If we define MADM in a narrow sense as decision aids to help a DM identify the best alternative that maximizes his/her satisfaction with respect to more than one attribute, many of the above approaches may not be directly used in multiple attribute decision making situations.

Churchman and his colleagues [11] were the first to treat a MADM problem (selecting business investment policy) formally using simple additive weighting method. Many potentially useful concepts/methods had been laid aside until MacCrimmon [12] reviewed the methods and applications of the multiple attribute decision making. He classified his collection of ten methods according to the number of dimensionality (single, intermediate, full) of attribute treatment. In his second review he added more methods

and grouped them according to their structure. The literature on MADM is treated in some of the MCDM reviews such as Nijkamp and Starr [13, 14].

There has been rapid theoretical development in multi-attribute utility theory (MAUT) which is a solution approach of MADM under uncertainty. It started with simple additive utility and has gone to quasi-additive (Multiplicative form) and multilinear utility function. There are several reviews on this field, Fishburn [15] and Keeney [16] have published a voluminous text basically on MAUT. But this method is one of the difficult approaches in MADM study, mainly due to its complexity in the assessment of utility function. Although the aim of most theoretical work in MAUT is the investigation of possibilities for simplifying the task of MAUT assessments, but there remains a certain amount of doubt concerning the practical usefulness of MAUT. A busy decision maker needs a technique which can be easily taught and used.

### **2.2.1 Classification of MADM Methods**

The decision makers' judgments vary in form and depth. One may not indicate his preferences at all, or may represent his preference through the form of attribute or alternative. The degree of judgment skill also varies. For instance, we may list the different preference information on attributes by the ascending order of complexity: standard level, ordinal, cardinal, and marginal rate of substitution. MADM methods are introduced to meet these various situational judgments. A classification of MADM methods as described by Hwang is shown in Figure 2.1. This has been made in three stages, stage I- the type of information (attribute or alternative or neither) needed from the DM; stage II- the salient feature of the information needed, and stage III- the major methods in any branch formed from stages I and II.

Before going into more details, some key concepts and notations will be defined so that the literature will have a unified notation of the most used terms. Then, some of the methods of stage III will be reviewed

## **Definitions**

The four words most used in MCDM literature are: criteria, goals, attributes, and, objectives. There are no universal definitions of these terms according to Keeney [16]. Some authors make distinctions in their usage while many use them interchangeably. We will make some distinctions among these words in terms of their usage.

Criteria: A criterion is a measure of effectiveness. It is the basis for evaluation. Criteria are emerging as a form of attributes or objectives in the actual problem setting.

Goals: Goals (synonymous with targets) are a priori values or levels of aspiration. These are to be either achieved or surpassed or not exceeded. Often we refer to them as constraints because they are designed to limit and restrict the alternative set. For example, the standard gas mileage, say 20 miles/gallon, set up by the federal government for 1980 models, is a constraint, whereas 30 miles/gallon may serve as a goal for the car manufacturer.

Attributes: Performance parameters, components, factors, characteristics, and properties are synonymous for attributes. An attribute should provide a means of evaluating the levels of an objective. Each alternative can be

characterized by a number of attributes (chosen by DM's conception of criteria), i.e., gas mileage, purchasing cost, horsepower, etc. of a car.

Objectives: An objective is something to be pursued to its fullest. For example, a power utility company wants to maximize sales while minimizing production cost or level of air pollution. An objective generally indicates the direction of change desired.

A MADM problem can be expressed in a matrix format. A decision matrix  $D$  is a  $(m \times n)$  matrix whose element  $x_{ij}$ 's indicate evaluation or value of alternative  $i$  ( $A_i$ ), with respect to attribute  $j$  ( $X_j$ ) as shown below,

$$D = \begin{matrix} & X_1 & X_2 & \cdots & X_n \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} & \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \end{matrix}$$

Decision matrix is also called goal achievement matrix.

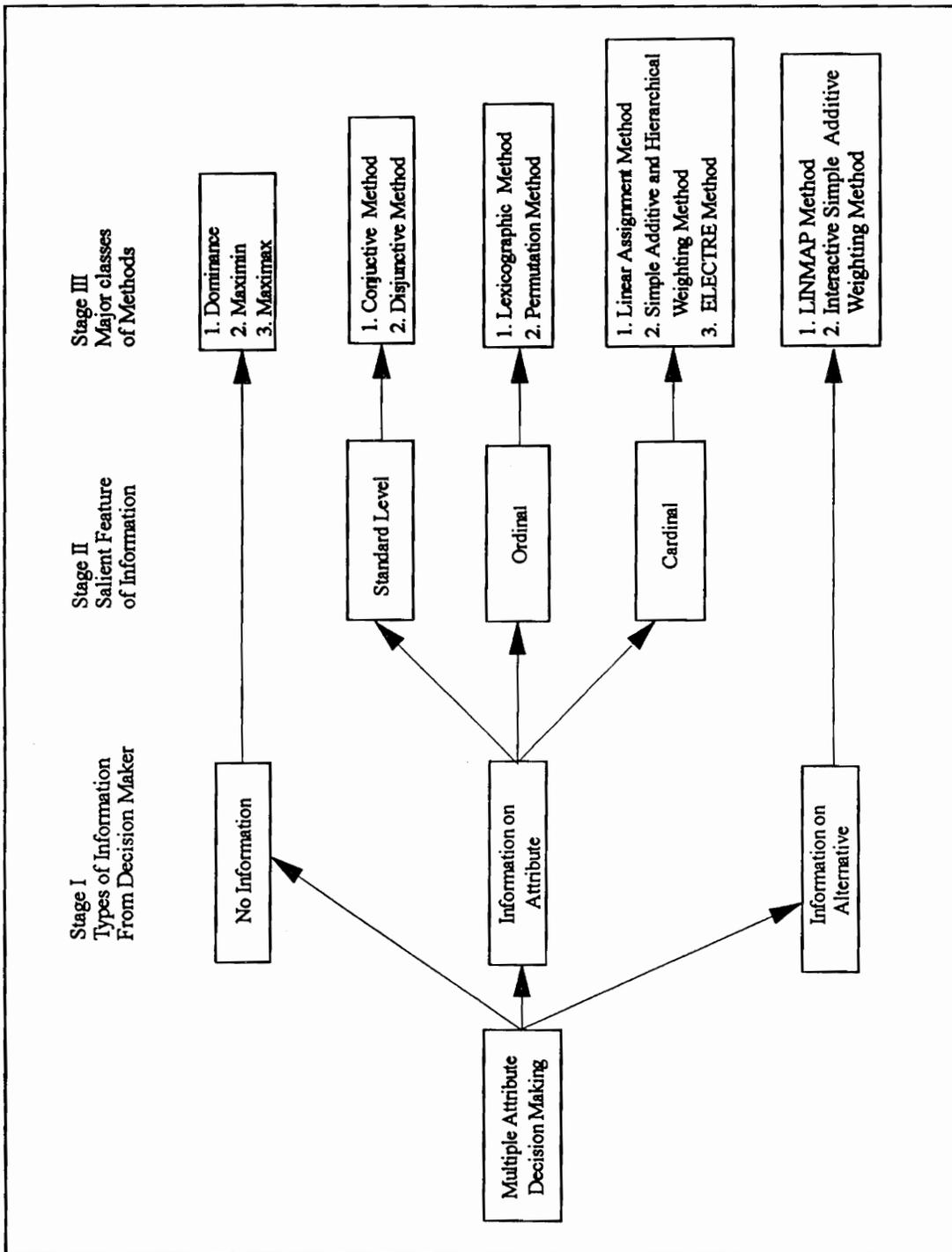


Figure 2.1 Classification of Methods for Multiple Attribute Decision Making

## **2.2.2 Methods for No Preference Information Given**

There are some classical decision rules such as dominance, maximin and maximax which are still fit for the MADM environment. They do not require the DM's preference information, and accordingly yield the objective (vs. subjective) solution. However, the right selection of these methods for the right situation is important, Yoon [17], (see Figure 2.1).

### **2.2.2.1 Dominance**

An alternative is dominated if there is another alternative which excels it in one or more attributes and equals it in the remainder. The number of alternatives can be reduced by eliminating the dominated ones. In other words, a set of alternatives is screened before the final choice is made.

This method does not require any assumption or any transformation of attributes. The procedure is to compare the first two alternatives and if one is dominated by the other, discard the dominated one. Next, compare the undiscarded alternatives with the third alternative and discard any dominated alternative. Then, introduce the fourth alternative and so on. After  $(m-1)$  stages, the nondominated set is determined. This nondominated set usually has multiple elements in it, hence the dominance method is mainly used for the initial filtering, Calpine [18].

### **2.2.2.2 Maximin**

An astronaut's life or death in the orbit may depend upon his worst vital organ, and a chain is only as strong as its weakest link. In this situation where the overall

performance of an alternative is determined by the weakest or poorest attribute, a DM would examine the attribute values for each alternative, note the lowest value for each alternative, and then select the alternative with the most acceptable value in its lowest attribute. It is the selection of the maximum (across alternatives) of the minimum (across attributes) values, or the maximin.

Under this procedure, only a single weakest attribute represents an alternative; all other (n-1) attributes for a particular alternative are ignored. If these lowest attribute values come from different attributes, as they often do, we may be basing our final choice on single values of attributes that differ from alternative to alternative. Therefore, the maximin method can be used only when inter-attribute values are comparable; that is, all attributes must be measured on a common scale; however, they need not be numerical. The alternative,  $A^+$ , is selected such that

$$A^+ = \{A_i \mid \max_i \min_j x_{ij}\}, \quad j = 1, 2, \dots, n \quad i = 1, 2, \dots, m$$

Where all  $x_{ij}$ 's are in a common scale.

This method utilizes only a small part of the available information in making a final choice—only one attribute per alternative. Thus, even if an alternative is clearly superior in all but one attribute which is below average, another alternative with only average on all attributes would be chosen over it. The maximin method, then, has some obvious shortcomings in decision making. What is appropriate for selecting a chain is not necessarily appropriate for general decision making, MacCrimmon [12].

### 2.2.2.3 Maximax

In contrast to the maximin method, the maximax method selects an alternate by its best attribute value rather than its worst. In this case, the highest attribute value for each alternative is identified, then these maximum values are compared in order to select the alternative with the largest such value.

Note that in this procedure, as with the maximin procedure, only the single strongest attribute represents an alternative; all other (n-1) attributes for the particular alternative are ignored; and it may evaluate different attributes in a final choice among alternatives. Therefore, as with the maximin method, the maximax method can be used only when all attributes are measured on a common scale. The alternative,  $A^+$ , is selected such that:

$$A^+ = \{A_i \mid \max_j x_{ij}\}, \quad j = 1, 2, \dots, n \quad i = 1, 2, \dots, m$$

The comparability assumptions and incompleteness properties of the maximax method do not make it a very useful technique for general decision making. However, just as the maximin method may have a domain in which it is quite reasonable, the maximax method may also be reasonable in some specific decision making situations. As an example, pro-football teams use the maximax procedure to draft players.

### 2.2.3 Methods When Information on Attributes Given

A DM may express his/her preference information either on attributes or on alternatives. Usually, the information on attributes is less demanding to assess than that

on alternatives. The majority of MADM methods require this kind of information to process inter- and intra-attribute comparisons.

The information can be expressed in various ways: 1) standard level of each attribute, 2) relative importance of each attribute by ordinal preference, 3) relative importance of each attribute by cardinal preference, and 4) marginal rate of substitution (MRS) between attributes.

### **a) When Standard Level of Information Given**

#### **2.2.3a.1 Conjunctive Method**

In the conjunctive method, the DM must supply the minimal attribute values (the cutoff values) acceptable for each of the attributes. The cutoff values given by the DM play the key role in eliminating the noncontender alternatives; if too high, none is left; if relatively low quite a few alternatives are left after filtering. Hence, increasing the minimal standard levels in an iterative way, we can sometimes narrow down the alternatives to a single choice. For instance,  $A_i$  is considered as an acceptable alternative only if

$$x_{ij} \geq x_j^0, \quad j = 1, 2, \dots, n$$

where  $x_j^0$  is the standard level of  $x_j$ . The conjunctive method is not usually used for selection of alternatives but rather for dichotomizing them into acceptable/unacceptable categories. Dawes [19] developed a way to set up the standards if the DM wants to dichotomize the alternatives.

Consider a set of  $n$  equally weighted independent attributes. Let

$r$  = the proportion of alternatives which are rejected,

$P_c$  = the probability that a randomly chosen alternative scores above the conjunctive cutting level.

Then

$$r = 1 - p_c^n \quad (2.1)$$

since the probability of being rejected is equal to one minus the probability of passing on all attributes. From equation (2.1),

$$p_c = (1 - r)^{1/n} \quad (2.2)$$

For example, suppose that a college evaluates applicants for admissions on each of four attributes-intellectual ability, academic ability, extracurricular activities, and character-and that it has applicants' scores on each of these attributes. Assume that these attributes are independent and that the college considers them all equally important. Suppose that the college wishes to accept one fifth of the applicants. Now we have

$$n = 4$$

$$r = 4/5$$

$$p_c = (1 - \frac{4}{5})^{1/4} = 0.67$$

Hence, the college must choose a cutting score for each attribute such that 67% of the applicants will place above this score. This cutting score is called the conjunctive cutting score.

### 2.2.3a.2 Disjunctive Method

A disjunctive method is one in which an alternative (or an individual) is evaluated on its greatest value (or talent) of an attribute. For example, professional football players are selected according to the disjunctive method; a player is selected because he can either pass exceptionally, or run exceptionally, or kick exceptionally, etc. A player's passing ability is irrelevant if he is chosen for his kicking ability. For instance:  $A_j$  is an acceptable alternative only if  $x_{ij} \geq x_j^o$ ,  $j = 1$  or  $2, \dots$  or  $n$

where  $x_j^o$  is a desirable level of  $x_j$ .

A disjunctive method guarantees selection of all individuals (candidates) with any extreme talent, while the conjunctive method guarantees rejection of all individuals with an extremely small talent.

For the disjunctive method, the probability of being rejected is equal to the probability of failing on all attributes, thus

$$r = (1 - p_d)^n \quad (2.3)$$

where  $r$  is the proportion of alternatives which are rejected, and  $P_d$  is the probability that a randomly chosen alternative scores above the disjunctive cutting level. From equation (2.3), we obtain

$$p_d = 1 - r^{1/n} \quad (2.4)$$

For example, in the problem of college evaluation of applicants, which was considered in the section on the conjunctive method, if the college uses a disjunctive method it will accept any applicant who scores above the cutting score on any attribute. Then we have,

$$\begin{aligned}n &= 4 \\r &= 4/5 \\p_d &= 1 - (4/5)^{1/4} = 0.05\end{aligned}$$

that is, the disjunctive probability,  $P_d$ , equals 0.05, hence the disjunctive cutting score for each attribute will be one such that 5% of the applicants score above it- in contrast to the conjunctive cutting score of 67%.

As with the conjunctive method, the disjunctive method does not require that the attribute information be in the numerical form, and it does not need information on the relative importance of the attributes.

## **b) When Ordinal Preferences Given**

### **2.2.3b.1 Lexicographic Method**

In some decision situations a single attribute seems to predominate. For example, "buy the cheapest" rule is that in which the price is the most important attribute to that DM. One way of treating this situation is to compare the alternatives on the most important attribute. If one alternative has a higher attribute value than any of the other alternatives, the alternative is chosen and the decision process ends. However, if some alternatives are tied on the most important attribute, the subset of tied alternatives is then compared with the next most important attribute. The process continues sequentially until a single alternative is chosen or until all  $n$  attributes have been considered. This method like maximin and maximax utilizes only a small part of the available information in making a final choice and because of this property it has received quite attention as a decision making technique in a number of areas, MacCrimmon [12].

### **2.2.3b.2 Permutation Method**

The permutation method uses Jaquet-Lagrange's successive permutations of all possible rankings of alternatives, as described by Paelinck [20]. The method consists of testing each possible ranking of the alternatives against all others. With  $m$  alternatives,  $m!$  permutation rankings are available. The method will identify the best ordering of the alternative rankings, then the dominating alternative. The method was originally developed for the cardinal preferences of attributes (i.e., a set of weights) given, but it is rather to be used for the ordinal preferences given.

The permutation method is a useful method due to its flexibility with regard to ordinal and cardinal rankings. A possible drawback of this method is the fact that, in the absence of a clear dominant alternative, rather complicated conditions for the values of weights may arise, particularly because numerical statements about ordinal weights are not easy to interpret. Also, with the increase of the number of alternatives the number of permutations increases drastically.

## **c) When Cardinal Preferences Given**

### **2.2.3c.1 Linear Assignment Method**

This method is based on a set of attribute rankings and weights. It features a linear process for attribute interaction and combination. In the process only ordinal data, rather than cardinal data, are used as the input. This information requirement is attractive in that we do not need to scale the qualitative attributes. But in spite of its apparent simplicity, this method fails to meet the linear requirement. This is to find an overall ranking which simultaneously uses all information contained in the attribute rankings rather than using this information sequentially as in the sum-of-ranks.

### 2.2.3c.2 Simple Additive and Hierarchical Weighting Method

Simple Additive Weighting method (SAW) is one of the widely used methods of MADM. The method is well summarized by MacCrimmon [12].

The DM assigns importance weights to each of the attributes which become the coefficients of the variables. To reflect the DM's marginal worth assessments within attributes, the DM also makes a numerical scaling of intra-attribute values. The DM can then obtain a total score for each alternative simply by multiplying the scale rating for each attribute value by the importance weight assigned to the attribute and then summing these products over all attributes. After the total scores are computed for each alternative, the alternative with the highest score (the highest weighted average) is the one prescribed to the DM.

Mathematically, simple additive weighting method can be stated as follows: Suppose the DM assigns a set of importance weights to the attributes,  $w = \{w_1, w_2, \dots, w_n\}$ . Then the most preferred alternative,  $A^*$ , is selected such that

$$A^* = \{A_i \mid \max_i \sum_{j=1}^n w_j x_{ij} / \sum_{j=1}^n w_j\} \quad (2.5)$$

where  $x_{ij}$  is the outcome of the  $i^{th}$  alternative about the  $j^{th}$  attribute with a numerically comparable scale. Usually the weights are normalized so that  $\sum_{j=1}^n w_j = 1$

This method uses all  $n$  attribute values of an alternative and uses the regular arithmetic operations, therefore, the values must be both numerical and comparable.

When the decision problem has a large number of attributes, say more than seven, it is easier to assess the set of weights using the hierarchical structure of the objectives. This will resemble Saaty's [21] approach in the sense that it is the vertical extension of the SAW.

### **2.2.3c.3 ELECTRE Method**

The ELECTRE method (ELimination et Choice Translating REality) which was originally introduced by Benayoun et al. [22], uses the concept of an "outranking relationship". The outranking relationship of  $A1 \rightarrow A2$  says that even though two alternatives 1 and 2 do not dominate each other mathematically (see Dominance), the DM accepts the risk of regarding A1 as almost surely better than A2. Through the successive assessments of the outranking relationships of the other alternatives, the dominated alternatives defined by the outranking relationships can be eliminated.

This method consists of a pairwise comparison of alternatives based on the degree to which evaluation of the alternatives and preference weights confirm or contradict the pairwise dominance relationships between alternatives. This method has several stages based on a 'concordance and discordance' set, hence it is also called concordance analysis by Nijkamp [13].

### **2.2.4 Methods When Information on Alternatives Given**

The methods in this class require that the DM be able to indicate his/her preference between two alternatives. This kind of information is far more demanding to assess than the information on attributes. LINMAP (LINear programming techniques for Multidimensional Analysis of Preference), Interactive Simple Additive method belong to this category. They take the pairwise preference information as an input which consists of a set of choices between pair of alternatives. It is then expected that the set may contain inconsistent elements. Each of these methods allows this inconsistency and tries to minimize it.

### 2.2.4.1 LINMAP Method

Srinivason and Shocker [23] developed LINMAP for assessing the weights as well as locating the ideal solution. In this method,  $m$  alternatives composed of  $n$  attributes are represented as  $m$  points in the  $n$ -dimensional space. A DM is assumed to have his/her ideal point that denotes the most preferred alternative location. Once the location of the ideal solution is decided, we can choose an alternative which has the shortest distance from the ideal solution. Given two alternatives, a DM is presumed to prefer an alternative which is closer to his ideal point. Then the weighted Euclidean distance,  $d_i$ , of the  $A_i$  from the ideal point is given by

$$d_i = \left[ \sum_{j=1}^n w_j (x_{ij} - x_j^*)^2 \right]^{1/2} \quad i = 1, 2, \dots, m \quad (2.6)$$

where  $x_j^*$  is the ideal value for the  $j^{\text{th}}$  attribute. The weights here take account of both the units in which each dimension is scaled and of the relative importance of each attribute to the DM.

The LINMAP procedure does not require that the set  $S$  consist of all  $m(m-1)/2$  paired comparison judgments from the DM. However, the set of weights obtained by LINMAP will be more reliable if the number of pairs in  $S$  is large. When the number of alternatives is greater than the number of attributes ( $m > n$ ), LINMAP gives the better fitting. But if number of attributes exceeds the number of alternatives ( $n > m$ ) it is hard to obtain reliable weights by LINMAP. The method does not require that the paired comparison judgment be transitive.

#### 2.2.4.2 Interactive Simple Additive Weighting Method

This method, presented by Zionts [24], introduces an interactive method for ranking alternatives subject to an initially unspecified linear utility function. The method is efficient in the number of paired judgments that must be made by the DM and leads to the identification of the final ranking of weights for the corresponding linear utility functions which would lead to this ranking.

Assume that the linear utility function (i.e. simple additive weighting) be adopted for the decision analysis, then an alternative  $p$  is preferred to  $q$  if,

$$\sum_j w_j x_{pj} > \sum_j w_j x_{qj} \quad (2.7)$$

or in the vector form,

$$w^T (x_p - x_q) > 0$$

where

$$w \in W = \{w \mid \sum_{i=1}^n w_i = 1, w_i > 0\}$$

This method introduces a way whereby the DM can progressively change his permutation of alternatives until approaches his desired one with associated  $w$ 's.

### **2.2.5 Some Remarks About MADM Methods**

The survey of MADM methods provides an overview of some classical and current methods, their characteristics, and their proper usage in the decision making process. These methods are useful in selecting an alternative from an explicit list of alternatives with multiple and conflicting attributes. As to which method(s) we should use, the selection of MADM method(s) itself is a kind of MADM problem. There is no specific choice rule, different methods are introduced for different decision situations.

For instance, when the DM does not have enough time and knowledge to examine the problem further for better decision, he may give the minimum acceptable level for attributes, therefore, the method of "conjunctive constraints" may be used. On the other hand if the DM wants to get a better solution with extra effort, and the alternatives are not screened then the method of "dominance" may be appropriate one to use. Therefore, for the best method to solve any problem at hand we should examine the problem carefully and ask question about the actors, objectives, attributes, and type of information available to us.

Some of the methods discussed in this review require information about the relative importance of each attribute or objective. It is usually given by a set of (priority) weights which is normalized to unity. Three widely used techniques: Weighted Least Square method, Eigenvector method, and Entropy method will be discussed in chapter three.

## **2.3 Uncertainty and Subjective Judgments in Attribute Values**

In the review of MADM methods it was assumed that the attribute values are known and the experts in the field are confident in their knowledge and that no imprecision exists in their judgment. However, in reality the information available to the decision maker is highly uncertain and decisions are made in an environment where facts and rules contain various kinds of vagueness and error.

There are several ways to represent this uncertainty. The simplest way is to avoid explicit representation of uncertainty and use expected values for each attribute and then treat the problem with certainty. A second and more computationally demanding procedure is to use an interval or range of values rather than a point estimate of attribute values. A third and most appealing way to account for uncertainty in attribute and alternative values is by assigning a probability value or distribution to them.

In the following we will discuss the methods of probability assignment for a single and group decision maker. A method of how to find an interval for these probability values will be discussed in chapter three.

### **2.3.1 Assessment by Individual Decision Maker**

#### **a) Point Estimation**

The simplest approach is to ask the DM what is his probability for an event. He may be asked to express his beliefs by stating a number between 0 and 1; or by a more visual response mode, for example dividing a line into two lengths corresponding to the relative probability of the two disjoint events.

A second procedure called the gamble or lottery technique was used by Savage [25] to obtain the subjective probability of the occurrence of single events. The DM is required to answer a series of yes-no questions phrased in terms of simple betting odds. His subjective probability for the occurrence of the event is then implied from the odds required to make him indifferent between two offered bets.

For such a method to be generally applicable there are some criteria which have to be satisfied; namely that there are only a few assessments to be made and that the DM has both the patience and the time to work through the procedure. However, the main criticism of the method is the introduction of gambling terminology. Much of the work in the past in the field of measurement of an individual's subjective probability has used this approach.

The concept of an "equivalent urn" by Raiffa, [26] has been used to quantify a DM's judgments concerning the chances of the occurrence of single events. The DM has to assess what proportional mix of balls bearing the various labels will make the urn equivalent to his feelings concerning the likely occurrence of the events. This simple indirect method is effective for assessing a small number of events, but the task becomes impossible for a large number of events. Then, the DM has to consider assessing a distribution.

## **b) Distribution Assessment**

In real world environment, it will be very rare that the decision analysis will require the probabilities of only a few events. Assessing the probability of the occurrence of the events will often, in practice, require the assessment of many possible values of some uncertain quantities. This problem becomes one of assessing a distribution. The DM must first decide how he wants his individual probabilities to be related to each other, after

which he can assess just a few probabilities and then construct a curve to represent his judgments concerning the whole continuum of possible values.

The methods that have been suggested for such assessments are: 1) probability and cumulative density functions, 2) direct judgmental curve fitting, and 3) the smoothing of historical data, which were discussed in details by Schlaifer [27].

### **2.3.1.1 Empirical Studies**

In practice, the assessor of probabilities is commonly not a statistician and is unfamiliar with the formal notions of probability, so that the methods to be used to obtain his subjective probability assessments have to be comprehensible to him. On the basis of this criterion the more elegant theoretical methods (i.e, judgmental curve fittings) have to be discarded in favor of a number of more basic approaches.

The main contribution to the measurement problem in practical contexts has been by Winkler [28]. After a pilot study, he selected four main assessment techniques for the measurement of unknown probabilities. They are as follows:

- (i) *Cumulative Distribution Function (CDF)*, the fractiles of the distribution are assessed and the CDF is graphed.
- (ii) *Hypothetical Future Samples (HFS)*, the effect of sample evidence on the decision-maker's assessments is considered.
- (iii) *Equivalent Prior Sample Information (EPS)*, prior judgments are expressed in the form of an equivalent prior sample.

(iv) *Probability Density Function (PDF)*, the points on the probability decision function are assessed by direct interrogation regarding relative density and relative areas from which the PDF is graphed.

Winkler reports that varying degrees of success were obtained from the use of direct methods (i, iv) on one hand and indirect methods (ii, iii) on the other. To reduce the inconsistencies he suggested a "feedback" session. The subject is presented with his assessments and, after a discussion, is asked to write down the "best" assessment.

### **2.3.1.2 Combining the Decisions**

When individuals of the group each provide a separate subjective probability distribution representing their judgment, a consensus problem arises in aggregating these distributions into a single input for the analysis.

#### **a) Linear Combination**

Winkler [29] suggests a number of aggregation schemes based on a linear combination of the individual distributions. The weights used in such a linear combination reflect the rationale that some individuals might be regarded as better assessors than others. In the event that all are believed equally able, the weights are made equal.

An alternative method, is applicable if the judgments of the individuals are sufficient to sample evidence of the same kind to be analyzed, then a natural-prior conjugate may be used to express the judgments. An intuitive explanation of natural-conjugate prior is to say that the prior information can be thought of as being equivalent to

sample information from the data-generating process. Each individual's distribution will be a member of the natural-conjugate family of distributions and the distributions can be combined in a manner similar to successive applications of Baye's theorem. These two different methods of aggregation produced varying results but it is the top decision maker responsibility for the final assessment of the single distribution and he should use those methods which simplify his problem and appear relevant to him.

### **b) Feedback and Reassessment**

The mathematical discussions of the group consensus problem assume that each member has already assessed their respective distributions. An alternative suggestion is to arrive at an agreement of opinion by encouraging each individual to reconsider his assessment, after presenting him with some feedback regarding the assessments of all the members of the group.

Two approaches have been suggested by Winkler [29]. Feedback and Reassessment (FR) and Group Reassessment (GR). The former method is to present the member with the anonymous assessments of his colleagues together with their self-rated ranks and to obtain from him his individual reassessment. The method is repeated until a convergence of opinion is obtained. The GR approach is to have the individuals meet as a group to be presented with the feedback and for them to discuss the matter in order to arrive at a single consensus.

Dalkey [30] criticizes such group discussions, believing them to be biased by the influence of dominant individuals, poor communication between members and by the distortion of individual judgments due to group pressure. He argues for the Delphi technique which is a variant of Winkler's FR method. The chosen members of the group are "experts" on the subject under consideration. They independently develop their

assessments, making explicit their underlying assumptions and sources of information and request any additional source which they feel would help refine their assessments. The feedback consists of the composite replies of their colleagues together with a list of underlying assumptions, and information requirements selected by the supervising experimenters.

### **2.3.2 Descriptive Decision Making**

The decision-maker's assessments of probability are always assumed to be those of a rational person. But It is a different situation when the decision-maker is represented by an individual or by an aggregate of individuals, it would be useful to include a section on the behavior of them.

#### **2.3.2.1 The Individual**

The results from empirical studies that are felt to be relevant are those of: objective/subjective probability comparisons, factors that affect the assessments, the consistency of these assessments and finally the way in which people process information for the revision of their probabilities in the presence of new information.

As a validation of the use of subjective probabilities, the emphasis of many of the studies was of the comparison between the measured assessments and the "true" objective probabilities. These latter quantities were in fact the "usual" or conventional probabilities assigned to the occurrence of well-known events; for example, the rolling of dice. The studies all took place in a laboratory setting using events with such well-known probabilities, but once-only events have no conventional probabilities and hence in a planning environment there is no basis for such comparisons according to Hampton [31].

The studies of an individual's behavior shows that the failure of subjectively assessed probability to equal the "objective" probability of comparable events is not a disturbing result for these assessments are taken as a decision-maker's beliefs based on his present knowledge and hence it cannot be a "correct" probability. However, the training of managers to help them understand the calculus of probability and some easier methods on probability assessments in real life decision problems must be considered.

### **2.3.2.2 The Group**

The effects of group interaction on decision-making have to be considered when the analysis is applied to a planning context because of the frequency which the "decision-maker" should aggregate the individual assessments.

After a group discussion usually a shift in risk preferences compared to those initially held by the individual members of the group will occur. This was observed first by Stoner [32], and Wallach *et al.*[33], when they employed a set of choice-dilemma questions to investigate the riskiness exhibited by groups after discussion compared to that shown by the individual members. A risk shift was found to have occurred taking the problem set as a whole. Later evidence by Brown [34], and Pruitt, however, shows that shifts occur towards caution after a group discussion of certain issues, suggests that the more general phenomenon of choice shift exists.

From the survey of group behavior it appears that the group opinion will differ from those expressed by its constituent members because of processes which take place during discussion, that lead individuals to revise their original recommendations.

Therefore, it is important that the decision makers be able to decompose a problem into separate factors, (such as a hierarchy) in order to reduce the total assessment problem into the assessment of individual components. The evidence of the literature suggests that

large influxes of information may not help the decision-maker very much; the necessity of being able to combine information and beliefs into a complex whole requires the understanding of probability fundamentals and a proper way of obtaining them. The Analytic Hierarchy Process method which will be discussed briefly here and in details in chapter three is a convenient and powerful technique in assisting both the individual and the group in assessment of subjective probabilities and measurement of the inconsistencies.

### **2.3.3 Analytic Hierarchy Process (AHP)**

Analytic hierarchy process is a multiple criteria decision-making method where a complicated problem can be first analyzed as an orderly hierarchy structure, and then ranked by its strengths. It is practical and adaptive, and has a wide range of applications in decision-making process under uncertainty. Some of its features are:

- 1) It will forecast and evaluate different kinds of decision-making problems according to the objectives. In particular, AHP satisfies, simulates and systematically integrates qualitative and quantitative information, making the forecast easier to justify and validate.
- 2) It helps to attain the objectives in the economic, social and technical dimensions by sorting them out in a hierarchy, finding out the relations between them, assigning weights according to the effects on the whole, and finally, synthesizing quantitative analyses to get a satisfying decision with multiple elements, objectives and criteria.

Therefore, a complicated problem can be analyzed through a successive hierarchy structure. After comparing the elements one by one, the relative importance of each element can be obtained, and then by synthesizing the priorities we can get a sequence of decisions that reflect the decision maker basic principles of analysis and judgments under uncertainty. The characteristics and features of this method and ways to improve its weaknesses will be discussed in chapter three.

## **2.4 Review of Uncertain Issues in Electric Utility Planning**

In the previous sections we discussed the issues in multiple criteria decision making and the methods of dealing with them under different situations by the individuals and the group. This section will address some of the main issues that electric utilities are facing in their decision making process. The methods for dealing with these uncertainties will be discussed in chapter five.

### **2.4.1 Load Growth**

Load-demand growth which is one of the key forecast parameters is influenced by many factors, including the national and local economies, energy prices, and energy conservation. The forecast of future load growth over the mid-term and long-term is subject to broad uncertainties. For instance, the load growth during the 1960's averaged 7% /year, 4.5% /year during the 1970's, and is predicted to grow at 2.5%/ year through the year 2000 in the US according to Stoll [35].

It is difficult for utilities to respond to a sudden high load-demand because of the long lead time associated with constructing capacity. For example, coal-fired power plants have lead times of 6 years on the average, and load management programs require several years to initiate customer participation. On the other hand it is important for the utilities to keep high quality of service with minimum loss of load probability (LOLP).

### **2.4.2 Fuel Cost**

Fuel cost is another key forecast with uncertainty. The price of oil constantly changes mainly due to instability of the countries in the Middle-East. Coal prices have been under a severe cost ceiling because of competition from low oil and gas prices prior to 1974, but later on with increase of oil and gas prices it went up, then in the 1980's its price was stable due to the oil and gas price trends. From the history of fuel price changes, future prices are likely to be cyclical in nature and very hard to predict, Shealy's [36]. According to director-general of the World Petroleum Congress [37], global oil supply and demand trends raise a distinct prospect of a third "price shock" in the future like the ones of 1973 and 1979-80. This is mainly due to 1) increasing global prosperity resulting from an international trend toward deregulation, privatization, and the opening of new markets in other countries, 2) the oil industry is in need of large amounts of capital to renew the refineries and transport systems. Low prices for oil have meant low rates of return, and an inability to spend adequately on infrastructure, and 3) the concentration of oil reserves among a few members of the OPEC members and the instability of the region.

A case study to quantify uncertain factors in prediction of oil prices for the year 2000 is shown in chapter six.

### **2.4.3 Environmental Issues**

Environmental concerns with electricity generation have technical, economic, and policy dimensions. Technical aspects include the extent and type of pollutant releases to

the environment, their pathways and impacts, and the feasibility and performance of technologies to control them.

The economic feasibility of technological solutions to environmental problems has also received considerable attention. This is true for all energy technologies, but much recent attention has been focused on coal. The Organization for Economic Co-operation and development (OECD) has published several reports concentrating on various control technologies and their feasibility for different plant types [38]. Policy issues associated with electricity generation are not independent of technical and economic aspects. Scientific research into environmental effects, engineering development with respect to technologies, and economic analysis of cost and benefits all affect the energy policy issue with many intangible factors which must be considered in the process.

#### **2.4.4 Adequacy of Future Supply**

Based on current capacity expansion plans of electric utilities, there is a high risk that supply will be inadequate to meet peak demand by the end of the century. Lazo [39] reports that the South-Eastern and South-Central areas of the US. (specially the SERC and SPP regions) are certain to face shortages by the year 2000 based on current plans. And throughout the central part of the country (MAAC, MAIN, MAPP, and ECAR) the risk of shortages by the end of century is in the range of 60 to 80 percent.

She emphasizes the need for methods to assess the risk of future electricity supply inadequacy and be able to:

- 1) Analyze the impact that a particular strategy under consideration will have on this risk,

- 2) Show the extent of demand reduction or supply expansion needed to bring the risks down to an acceptable level,
- 3) provide a meaningful answer when regulators or customers demand to know what a utility is doing about the future adequacy of electricity supply.

Supply is adequate only if the amount of generating capacity in place, and in working order, at the time of peak demand is at least as great as the peak load. This adequacy depends on three things: 1) on the level of peak demand, 2) on the amount of generating capacity (or other resources) that the utility has installed, 3) and on the portion of installed capacity that is actually available to serve load at the time of peak demand.

Each of these three factors is subject to considerable uncertainty, the more so the farther out into the future we are looking to. The traditional method of dealing with these uncertainties has been to look at a number of scenarios, such as "high growth, planned construction" or "low growth, construction delays." This type of analysis is all right, except that: 1) it does not say anything about how likely it is that any particular scenario or set of results will actually happen, 2) it side-steps the problem by asking whether you can get away with ignoring uncertainty, rather than giving you a means of taking uncertainty into account, and 3) it requires more calculations-one for every scenario-than an approach that deals with all the uncertainties at once.

#### **2.4.5 Installed Capacity**

Knowing exactly what generating units will be in commercial operation at any time in the future is the primary source of uncertainty over the future installed capacity. The degree of this uncertainty varies considerably among the various components of

supply, such as 1) existing units not yet retired, 2) new capacity additions, and 3) non-utility capacity such as cogenerations and IPP.

Considering such uncertainties utility planners must seek a robust resource plan, which will remain attractive regardless of uncertainty's effects. But providing robustness becomes very expensive and electric utilities are not willing to implement this option, Stillinger [40].

#### **2.4.6 Demand-Side Resources**

During the past several years, more utilities have prepared long-term resource plans that integrate demand-side programs into the utilities mix of energy and capacity resources in order to reduce the effects of uncertain elements in their plans. But utilities should carefully prepare and present their resource plans because the plans are very important both to the utility and to the public. A broad range of demand-side resources (both energy efficiency and load management) should be considered to balance the traditional emphasis on utility-owned power plants.

A major issue surrounding DSM involves customers who can experience increase in their rates because someone else is taking advantage of a DSM program. In evaluating DSM programs from a theoretical standpoint, one should consider the perspectives of: 1) the average rate payer, 2) the utility, 3) the DSM participant, and 4) the DSM non-participant with considerable amount of uncertainty attached to them.

#### **2.4.7 Supply-Side Resources**

The supply-side planning process is not as controversial as DSM, primarily because it is the direct descendent of traditional utility resource planning. Additionally,

supply-side alternatives are typically defined in terms of well-defined engineering parameters. For example, a coal unit of specific capacity will have standard heat rates, standard outage rates over its lifetime, and known maintenance requirements. There is relatively little guesswork involved, compared to the sociological nature of most DSM analysis, Hirst [41].

However, it is important to assess the possibility and consequences of higher than anticipated construction and operating costs caused by stricter environmental regulations and public opposition to construction of power plants and transmission lines.

#### **2.4.8 Integration of Demand and Supply Resources**

The selection of resource portfolios can be based on many different criteria (e.g., to minimize revenue requirements, capital costs, or average electricity prices; to ensure adequate reserve margins and the ability to meet high load growth; to reduce environmental effects of electricity production). The utility should clearly specify what criteria it used in selecting individual resources and choosing among alternative resource mixes, such as economic, financial, strategic, and reliability attributes.

The methods used to integrate supply and demand resources often involve linkages among several planning models which needs to be screened carefully. The screening process and criteria have important effects on the final mix of resources chosen for integration, Eto [42].

While many utilities take into account uncertainties about supply resources, few pay serious attention to uncertainties about DSM programs (mainly because of the models that utilities use for such analysis). New England Electric [43] has conducted probability analyses as a part of its resource planning to obtain an estimate of how certain

the utility can be that a given resource plan will meet future needs. The result was that DSM programs have an 80 percent chance of reducing peak demand by at least 400 MW in 1995. But, any approach for obtaining the probability estimates should show the level of confidence in them in order to make better decisions.

### **Concluding Remarks**

From this review we recognize the need for new techniques in solving complex problems under uncertainty. The techniques reviewed here mostly depend on the human users to estimate the probability values for a particular evidence or uncertain variable. For instance, in expert systems the expert's assessment of the situation under a given condition and assigning a probability estimate is a crucial part of the process. For electric utilities, techniques which are capable of handling the complexity and the diversity of the problems yet simple and practical to their decision makers are needed.

## CHAPTER III

### ANALYTIC HIERARCHY PROCESS APPROACH

To solve any complex problem, first we have to identify its components and the relations among them. Then, we should decompose the problem into simpler structures for analysis, and finally synthesize it together. This is the fundamental process for solving any complex problem. Our purpose is to develop a methodology for modeling unstructured and complex problems in order to be able to quantify uncertain issues that electric utilities are facing in their planning and decision making process.

The analytic hierarchy process (AHP) first introduced by Saaty [44], is a unique approach that employs a method based on pairwise comparisons to rank different alternatives or options for a problem which is formulated in hierarchical fashion.

In this approach, the problem at hand is decomposed into levels of factors or *elements*. Elements at a particular level of hierarchy are kept, to the extent possible, independent of, but comparable to, the elements at the same level. Elements at any level are directly related to, or influence, the elements at the level immediately below them. The strength of influence of elements at a particular level over those in the succeeding level is

measured by a procedure of paired comparisons. The procedure is repeated by moving downward along the hierarchy, computing the weights of each element at every level and using these to determine the *composite weights* for succeeding levels. The final set of weights relates to the alternatives under evaluation and gives measure of their overall relative importance.

Because the process requires that one make explicit the relationships within levels in the hierarchy through the pairwise comparison procedure, it can enhance creativity in identifying and evaluating the nature of the relationships involved in a particular problem.

An important difference between the AHP and other decision-making approaches is that the AHP requires the *simultaneous* use of data and judgment, as opposed to their sequential use in formal models. At the same time, there is a mathematical justification for the procedure of pairwise comparison and derivation of the weights as well as the consistency index for the judgments.

This approach, which is sufficiently general to use both in measurements and judgments, is described by Churchman [45] as:

"...It seems almost obvious that we cannot solve present-day major organizational problems simply by grinding through a mathematical model or a set of computer inputs and outputs. What we require besides is the design of better deliberation and judgment".

This method has been applied to different types of problems, ranging from the prediction of the outcomes of chess matches, to study of transportation system for Sudan, Saaty [46], and long range electric utility planning, Rahman [47]. This approach needs the knowledge of the experts in the field and some creativity in formulating a given problem as a hierarchy, but once this is done the method is easy to apply.

AHP consists of the following main steps:

1. A hierarchical formulation of the problem at hand;
2. A procedure for weighting each element at a particular level of the hierarchy, with regard to the "contribution" it makes to elements at the succeeding level of the hierarchy, by means of paired comparisons;
3. Repetition of process 2 until a "composite weight" is obtained for each of the "alternatives" represented as the final level of the hierarchy. This composite weight is an overall measure of importance for the particular element or alternative, and

These steps will become more clear in the context of theory discussed in the following sections and in applications presented in chapter six.

### **3.1 Decomposition of Complex Problems**

#### **3.1.1 Hierarchies**

A hierarchy is an abstraction of the structure of a system to study the functional interactions of its components and their impacts on the entire system. This abstraction can take several related forms, all of which essentially descend from an overall objective, down to sub-objectives, down further to forces which affect these sub-objectives, down to the people (actors) who influence these forces, and finally down to the strategies or alternatives. It is based on the assumption that the factors, which we have identified, can be grouped into disjoint sets, with the factors of one group (level) influencing the elements of only one other group and being influenced by the elements of only one other group. The attributes of one level are assumed to be independent of each other. If there is

dependence among them we should study independent and dependent elements separately and then combine the results.

The hierarchy could be either complete or semi-complete. It is complete when the elements in each level are criteria for all the elements in the level below. But it is semi-complete when not all elements in each level have to be criteria for the elements in the level below it.

Figures 3.1 and 3.2 show these two types of hierarchies. In Figure 3.1 the hierarchy consists of three levels: first level which is the main objective or goal of the plan where its priority is considered to be one. The second level has three sub-objectives and their priorities is derived from a matrix of pairwise comparisons with respect to the objective of the first level. The third level has four scenarios or alternatives. The idea is to determine the impact of factors in this level on the main objective through the intermediate second level.

In Figure 3.2 the hierarchy has one extra level which is the actors (level three) who will play an important role in the process and should be considered, and also not every actor affects each scenario nor does each objective affects every actor. This is the case of a semi-complete hierarchy. Again, the idea is to determine the priorities of the alternatives with respect to the overall objective. Here one must calculate the priority of each alternative by summing the product of the weights of its corresponding factors in the upper levels. This is the case for many real life situations and we will show an example concerning prediction of oil prices in chapter six.

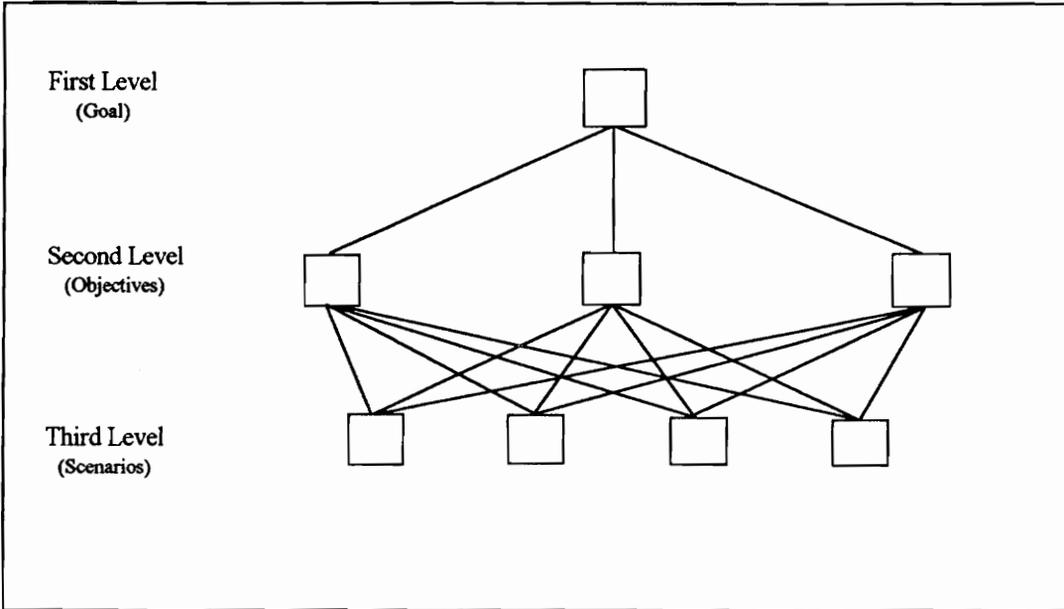


Figure 3.1 A complete hierarchy structure

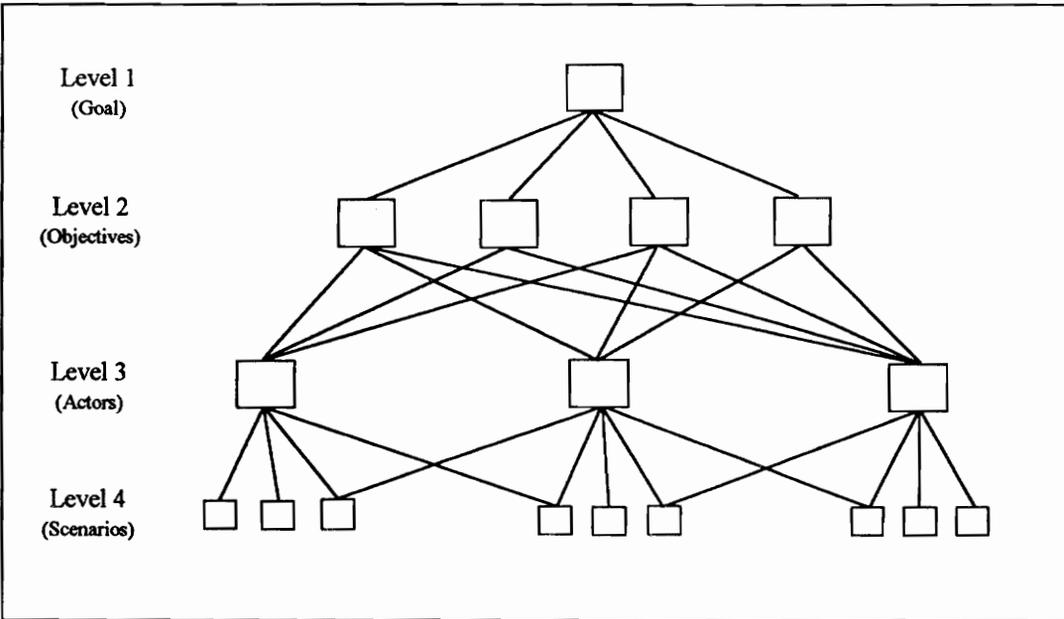


Figure 3.2 A Semi-complete hierarchy structure

### **3.1.2 Structuring Hierarchies**

Hierarchical decomposition and recomposition of complex systems appear to be a basic device by which the human mind copes with complexity. Thus, a vast array of complex, real-life problems lend themselves readily to hierarchical representations.

In practice, there is no set procedure or "ground rules" for generating the objectives, criteria, and alternatives to be included in a hierarchy. It is a matter of how much knowledge we have about the problem and what objectives we choose to decompose the problem.

One usually studies the literature for enrichment of ideas and, often by working with others, goes through a brainstorming session to list all concepts relevant to the problem without regard to relation or order. One attempts to keep in mind that the ultimate goals need to be identified at the top of the hierarchy; their sub-objectives immediately below, and so on, and in the last level the various possible outcomes (scenarios), (see Figures 3.1 and 3.2). Both cases require intelligence, knowledge and the ability to interact with others to benefit their understanding and experience in order to better identify the issues or attributes that contribute to the solution of the problem.

### **3.1.3 Forward and Backward Processes**

The usual approach to planning is to project forward what seems feasible or likely. The projected future is determined by the existing state of the system and by the persons or institutions that pursue certain objectives and implement certain policies to achieve their individual objectives. This process of estimating the likely future which is called forward planning can be represented by a hierarchy with the projected future on the top level. The

case study in chapter six for projecting oil prices for the year 2000 is an example of forward hierarchy and planning.

Sometimes we prefer to have a desired, rather than likely, future and work backward to determine the means to bring about such a future. The desired outcome is achieved by applying policies to influence the actors to remove obstacles in the way of this outcome. This normative process is called backward planning. For a better planning sometimes it is beneficial to combine the two processes by projecting the likely future first through forward planning. Then, a feasible and desired future is considered, and the necessary policies for achieving it are found through a backward process. These policies are then added to the set of existing policies to test their effect on a second projection of a likely future. This process is continued to obtain greater convergence, if possible, of the likely and the desired futures. The forward-backward planning is carried out within two limits, first it is fixed in the present by the actors and the available resources, then it is fixed in the future by the desired objectives.

In summary, hierarchies have the following advantages:

1. They are powerful methods for classification and decomposition used by the human brain-mind in ordering experience, observations and information.
2. They provide both an overall view of the complex relationships inherent in a problem and the detail of information on the structure and function of a system in the lower levels.
3. They are stable and flexible: stable in that small changes have small effect and flexible in that additions or subtractions to a well-structured hierarchy do not disrupt the performance.

### 3.2 Methods for Assessing the Impact of Attributes in the Hierarchy

A hierarchy, as described in the last section, is a close model of a real-life situation. It represents our analysis of the most important elements in the problem and of their relationships. However, this is not sufficient for our decision making or planning process. What is yet needed is a method to determine the impacts which various elements in one level have on the next higher level, so that we may compute the relative weight (priority) of the options or alternatives in the lowest level to the overall objective, and how accurate and consistent the judgments are.

Many methods have been proposed to derive the impact of the elements (weight) in a hierarchy such as right eigenvector and left eigenvector [48], arithmetic mean of the rows, logarithmic least squares (LLSM), least squares (LSM) [49], entropy method [50], to name a few.

Saaty [51] has shown that when the matrix of comparison is inconsistent, the principal right eigenvector is the "best" way to estimate the priority vector. Fichtner [52] has presented an axiomatic approach to decide which method is the "best". He has shown that both LLSM and EM fulfill all the axioms. Zahedi [53] has addressed estimation methods in recovering the true relative weights at one level of hierarchy based on their statistical accuracy and rank preservation properties. The estimation method included the eigenvector method, the mean transformation, the row geometric mean, and the simple row mean. She concluded that :

- 1) The most important factors in the estimation of relative weights comprise the probability distribution of error terms and type of input matrix.
- 2) The column geometric mean and the simple row average could be dropped from the

list of the estimators because they generally show the highest degree of sensitivity toward the distribution of error terms, and show in some cases very poor accuracy and rank preservation.

- 3) In the computation of the eigenvalue method, the "size" criterion performs exactly as well as the "convergence" criterion. It has the additional advantage of computational efficiency which becomes crucial in the case of a large number of elements.

### **3.2.1 Pairwise Comparison Technique**

The first step in establishing priorities examine of the attributes in a decision problem is to make pairwise comparisons- that is to compare the elements in pairs against a given criterion. For pairwise comparisons, a matrix is the preferred form because it offers a framework for testing consistency, obtaining additional information through making all possible comparisons, and analyzing the sensitivity of overall priorities to changes in the judgment. The matrix approach uniquely reflects the dual aspects of priorities; dominating and dominated.

To begin the comparison process, we should start at the top of the hierarchy to select the criterion that will be used for making the first comparison. Then from the level immediately below it, we consider two elements at a time and compare them according to the criterion.

To compare elements, we should ask questions like: how much stronger does this element contribute to, dominate, influence, satisfy, or benefit the objective than does the element with which it is being compared? The phrasing of the question is important. It

must reflect the proper relationship between the elements in one level with the property in the next higher level.

### 3.2.2 Judgment Matrix

In performing judgment matrices one is typically faced with a comparison of  $n(n-1)/2$  pairs of objects, if we denote the relative importance of  $i^{\text{th}}$  attribute with respect to  $j^{\text{th}}$  attribute by  $a_{ij}$ , then the relative importance of  $j^{\text{th}}$  attribute with respect to  $i^{\text{th}}$  attribute would be  $1/a_{ij}$ , and the importance of every attribute with itself ( $a_{ii}$ ) is equal to one.

For the entries of the matrix, we use numbers to represent the relative importance of one element with respect to another. Table 3.1 contains the scale for pairwise comparisons. It defines and explains the values 1 through 9 assigned to judgments for comparing pairs of like elements in each level of hierarchy.

The matrix obtained this way is called "reciprocal judgment matrix" and can be shown as:

$$[A] = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & a_{12} & \dots & a_{1n} \\ 1/a_{12} & 1 & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 1/a_{1n} & \dots & \dots & 1 \end{bmatrix}$$

This matrix translates to an eigenvalue problem. The Perron-Frobenius theory of positive matrices which is summarized by Gantmacher [54] ensures that a unique positive eigenvalue exists for matrices with only positive entries and the corresponding normalized eigenvector is the vector of relative weight of the attributes being compared. From this judgment matrix, the priority weights and subjective probabilities will be estimated when we will show the applications to hierarchy method.

## The Scale

The inconsistency in judgments calls for some kinds of scales for comparison which should at least satisfy the following requirements:

1. It should be possible to represent decision makers' differences in making comparisons. It should represent as much as possible all distinct shades of choices the decision makers may express.
2. If we denote the scale values by  $x_1, x_2, \dots, x_p$ , then

$$x_{i+1} - x_i = 1 \quad i = 1, 2, \dots, p-1$$

3. The decision maker must be aware of all gradations of the scale at the same time.

The third requirement is particularly important. Miller's [55] psychological experiments show that a decision maker cannot simultaneously compare more than seven ( $\pm 2$ ) objects without confusion. With a unit difference between values (Requirement 2) and assuming that  $x_1 = 1$  is the identity comparison, a reasonable scale would range from 1 to 9.

For estimating the scale ratio  $w_i / w_j$ , Saaty [44] gives an intensity scale of importance for comparisons and has broken down the importance ranks as shown in Table 3.1.

**Table 3.1 Scales and their descriptions**

Intensity of importance	Definition	Explanation
1	Equal importance	Two criteria contribute equally to the objective.
3	Weak importance of one over another	Experience and judgment slightly favor one criterion over another.
5	Essential or strong importance	Experience and judgment strongly favor one criterion over another.
7	Demonstrated importance	A criterion is strongly favored and its dominance is demonstrated in practice.
9	Absolute importance	The evidence favoring one criterion over another is of the highest possible order of affirmation.
2, 4, 6, 8	Intermediate values between the two adjacent judgments	When compromise is needed

### 3.2.2.1 Quasi-Reciprocal Judgment Matrices

In the pairwise comparison procedure, it is assumed that experts can assign values to  $n(n-1)/2$  elements in the judgment matrix. Harker [56] has given an extension of the approach which allows a decision maker to say "I don't know" or "I am not sure" to some of the questions asked. Therefore, a positive  $n \times n$  matrix  $A$  is quasi-reciprocal and can be

obtained if

$$a_{ij} \geq 0$$

and if

$$a_{ij} > 0, \text{ then } a_{ji} = \frac{1}{a_{ij}} \quad i, j = 1, 2, \dots, n$$

Let  $B$  be an  $n \times n$  matrix formed from the partially completed matrix  $A$ , then:

$$b_{ij} = \begin{cases} a_{ij}, & \text{if } a_{ij} \text{ is positive} \\ 0, & \text{otherwise} \end{cases}$$

$$b_{ii} = m_i, \text{ the number of unanswered questions in row } i$$

Then, the matrix  $A = I+B$  is primitive, i.e. there is an integer  $k > 1$  such that  $A^k$  is positive, Therefore, the solution of the eigenvalue problem for  $A$  can be considered as the priority of the alternatives under incomplete comparisons.

In the following we will discuss three techniques which have been widely used, 1) Eigenvector Method, 2) Weighted Least Square Method, and 3) Entropy Method.

### 3.2.3 Eigenvector Prioritization Method (EVM)

The eigenvector method, among several methods, has gained popularity due to its simplicity and ability to check the consistency of judgments, and analyzing the sensitivity of overall priorities to changes in each judgment. It centers around constructing a matrix of pairwise comparisons of activities. The entries of this matrix indicate the dominance of one activity over another with respect to a specific comparison criterion.

In applying this method, one is typically faced with a comparison of  $n$  objects in pairs according to their relative weights. Hence, we can represent these pairwise comparisons as a matrix:

$$A = \begin{bmatrix} w_1/w_1 & w_1/w_2 & \cdots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \cdots & w_2/w_n \\ \vdots & \vdots & \ddots & \vdots \\ w_n/w_1 & w_n/w_2 & \cdots & w_n/w_n \end{bmatrix}$$

It is clear that if we know the exact weights of the objects (normalized by the total weight of all the objects), then the following relation would hold:

$$Aw = nw \tag{3.1}$$

$$Aw = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \cdots & \frac{w_2}{w_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & \frac{w_n}{w_n} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = n \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

However, if we did not know the exact weights of the objects and had only the ratio scale of the elements, i.e.  $a_{ij} = w_i/w_j$ , we would have to solve Equation (3.1) for  $W$ . This equation can be transformed into:

$$(A - nI)w = 0 \tag{3.2}$$

We can easily recognize this as an eigenvalue problem. A problem for which a non-zero solution exists if and only if  $n$  is an eigenvalue of  $A$ . Two results in the theory of positive matrices are important in dealing with this problem:

- 1) The matrix  $A$  is of unit rank since every row is a constant multiple of the first row - as a result all the eigenvalues of  $A$ ,  $\lambda_i$   $i=1,2, \dots, n$  are zero except one; and
- 2) It is well known that the sum of the eigenvalues of a positive matrix is equal to the trace of that matrix (the sum of the diagonal elements). As a result, since  $a_{ii} = 1$  in reciprocal matrices:

$$\sum_{i=1}^n \lambda_i = \text{tr}(A) = n$$

For reciprocal matrices, as a result of (1) and (2), only one of the  $\lambda_i$  (which is referred to as  $\lambda_{\max}$ ) is equal to  $n$  and  $\lambda_i = 0$  for  $\lambda_i \neq \lambda_{\max}$ . Similarly,  $w$  is any normalized column of  $A$ . Therefore, by solving the eigenvalue problem we retrieve the vector of weights from the matrix of pairwise comparison.

It can be noticed that if pairwise comparisons are completely consistent with one another, i.e.  $a_{ij} a_{jk} = a_{ik}$  (the 'strong' consistency property - transitivity) will hold. The reciprocal property mentioned earlier is referred to as the 'weak' consistency property  $a_{ji} = 1/a_{ij}$ . If a matrix is strongly consistent we can easily construct the entire matrix from a single row, e.g. the first row as:  $a_{jk} = a_{1k} / a_{1j}$  ( $a_{1j} \neq 0$ ).

Example:

If the following positive pairwise comparison matrix is given

$$A = \begin{bmatrix} 1 & \frac{1}{3} & \frac{1}{2} \\ 3 & 1 & 3 \\ 2 & \frac{1}{3} & 1 \end{bmatrix}$$

then set the determinant of  $(A - \lambda I)$  as zero. That is:

$$\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & \frac{1}{3} & \frac{1}{2} \\ 3 & 1-\lambda & 3 \\ 2 & \frac{1}{3} & 1-\lambda \end{vmatrix} = 0$$

The largest eigenvalue of  $A$ ,  $\lambda_{\max}$  is 3.053, and we have

$$\begin{bmatrix} -2.053 & \frac{1}{3} & \frac{1}{2} \\ 3 & -2.053 & 3 \\ 2 & \frac{1}{3} & -2.053 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = 0$$

The solution of the homogeneous system of linear equations gives (recall that  $\sum_{i=1}^3 w_i = 1$ )  
 $W^T = (0.1571, 0.5936, 0.2493)$ .

### 3.2.4 Weighted Least Square Method

A weighted least square method is proposed by Chu et al. [49] to obtain the weights. This method has the advantage that it involves the solution of a set of simultaneous linear algebraic equations.

Consider the elements  $a_{ij}$  of matrix  $A$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \cdots & \frac{w_2}{w_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & \frac{w_n}{w_n} \end{bmatrix}$$

It is desired to determine the weights such that,

$$a_{ij} \cong \frac{w_i}{w_j}$$

The weights can be obtained by solving the constrained optimization problem

$$\min z = \sum_{i=1}^n \sum_{j=1}^n (a_{ij} w_j - w_i)^2 \quad (3.3)$$

$$s.t. \quad \sum_{i=1}^n w_i = 1 \quad (3.4)$$

An additional constraint is that  $w_i > 0$ . However, it is conjectured that the above problem can be solved to obtain  $w_i > 0$  without this constraint.

In order to minimize  $z$ , the Lagrangian function is formed

$$L = \sum_{i=1}^n \sum_{j=1}^n (a_{ij} w_j - w_i)^2 + 2\lambda \left( \sum_{i=1}^n w_i - 1 \right) \quad (3.5)$$

where  $\lambda$  is the Lagrangian multiplier. Differentiating equation (3.5) with respect to  $w_1$ , the following set of equations is obtained:

$$\sum_{i=1}^n (a_{il} w_\ell - w_i) a_{il} - \sum_{j=1}^n (a_{lj} w_j - w_\ell) + \lambda = 0, \quad \ell = 1, 2, \dots, n \quad (3.6)$$

Equations (3.4) and (3.6) form a set of  $(n+1)$  non homogeneous linear equations with  $(n + 1)$  unknowns. For example, for  $n = 2$ , the equations are  $(a_{ij} = 1 \quad \forall i)$ :

$$\begin{aligned} (1 + a_{21}^2) w_1 - (a_{12} + a_{21}) w_2 + \lambda &= 0 \\ -(a_{21} + a_{12}) w_1 + (1 + a_{12}^2) w_2 + \lambda &= 0 \\ w_1 + w_2 &= 1 \end{aligned} \quad (3.7)$$

Given the coefficients  $a_{ij}$ , the above equations (3.7) can be solved for  $w_1$ ,  $w_2$ , and  $\lambda$ .

In general, equations (3.4) and (3.6) can be expressed in the matrix form

$$Bw = m \quad (3.8)$$

where

$$\begin{aligned} W &= (w_1, w_2, \dots, w_n, \lambda)^T \\ m &= (0, 0, \dots, 0, 1)^T \\ B &= (m+1) \times (n+1) \quad \text{matrix with elements } b_{ij} \\ b_{ii} &= (n-1) + \sum_{j=1}^n a_{ji}^2, \quad i, j = 1, \dots, n \\ b_{ij} &= -(a_{ij} + a_{ji}), \quad i, j = 1, \dots, n \\ b_{k, n+1} &= b_{n+1, k} = 1 \quad k = 1, \dots, n \\ b_{n+1, n+1} &= 0 \end{aligned}$$

Example:

Given the matrix  $A$  :

$$A = \begin{bmatrix} 1 & \frac{1}{3} & \frac{1}{2} \\ 3 & 1 & 3 \\ 2 & \frac{1}{3} & 1 \end{bmatrix}$$

For  $n = 3$ , equation (3.8) becomes:

$$\begin{aligned} (a_{21}^2 + a_{31}^2 + 2)w_1 - (a_{12} + a_{21})w_2 - (a_{13} + a_{31})w_3 + \lambda &= 0 \\ -(a_{21} + a_{12})w_1 + (a_{12}^2 + a_{32}^2 + 2)w_2 - (a_{23} + a_{32})w_3 + \lambda &= 0 \\ -(a_{31} + a_{13})w_1 - (a_{32} + a_{23})w_2 + (a_{13}^2 + a_{23}^2 + 2)w_3 + \lambda &= 0 \\ w_1 + w_2 + w_3 &= 1 \end{aligned}$$

Substituting values for  $a_{ij}$  into the above equations, we obtain

$$\begin{aligned} 15w_1 - \frac{10}{3}w_2 - \frac{5}{2}w_3 + \lambda &= 0 \\ -\frac{10}{3}w_1 + \frac{20}{9}w_2 - \frac{10}{3}w_3 + \lambda &= 0 \\ -\frac{5}{2}w_1 - \frac{10}{3}w_2 + \frac{45}{4}w_3 + \lambda &= 0 \\ w_1 + w_2 + w_3 &= 1 \end{aligned}$$

The solution is  $W^T = (0.1735, 0.6059, 0.2206)$ .

### 3.2.5 Entropy Method

When the entries of the decision matrix are known, instead of the Saaty's pairwise comparison matrix, the entropy method can be used for evaluating the weights.

According to Nijkamp [50] entropy has become an important concept in the science. In addition, entropy has a useful meaning in information theory, where it measures the expected information content of a certain message. Entropy in information theory is a criterion for the amount of uncertainty represented by a discrete probability distribution  $p_i$ , which agrees that a broad distribution represents more certainty than does a sharply peaked one. This measure of uncertainty is given by Shannon [57] as:

$$S(P_1, P_2, \dots, P_n) = -k \sum_{j=1}^n P_j \ln P_j \quad (3.9)$$

where  $k$  is a positive constant. Since this is just the expression for entropy as found in statistical mechanics, it is called the entropy of the probability distribution  $p_i$ ; hence the terms "entropy" and "uncertainty" are considered as synonymous. When all  $p_i$  are equal to each other for a given  $i$ ,  $p_i = 1/n$ ,  $S(p_1, \dots, p_n)$  assumes its maximum value.

The decision matrix for a set of alternatives contains a certain amount of information, entropy can be used as a tool in criteria evaluation. The entropy idea is particularly useful to investigate contrasts between sets of data. For example, a criterion does not function much when all the alternatives have the similar outcomes for that criterion. Further, if all the values are the same, we can eliminate the criterion.

Let the decision matrix  $D$  of  $m$  alternatives and  $n$  attributes be

$$D = \begin{matrix} & X_1 & X_2 & \dots & X_n \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} & \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix} \end{matrix}$$

The outcomes of attribute  $j$ ,  $p_{ij}$ , would be:

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}}, \quad \forall i, j \quad (3.10)$$

The entropy  $E_j$  of the set of outcomes of attributes  $j$  is:

$$E_j = -k \sum_{i=1}^m p_{ij} \ln p_{ij}, \quad \forall j \quad (3.11)$$

where  $k$  represents a constant:

$$k = \frac{1}{\ln(m)}$$

which guarantees that  $0 \leq E_j \leq 1$ .

The degree of diversification  $d_j$  of the information provided by the outcomes of attribute  $j$  can be defined as

$$d_j = 1 - E_j, \quad \forall j$$

If the DM has no reason to prefer one criterion over another, the Principle of Insufficient Reason by Starr [58] suggests that each one should be equally preferred. Then, the best weight set he can expect, instead of the equal weight, is

$$w_j = \frac{d_j}{\sum_{j=1}^n d_j}, \quad \forall j \tag{3.12}$$

For example, consider the following decision matrix of 4 alternatives and 6 attributes, by using equation (3.10) the  $P_{ij}$  for all  $i$  and  $j$  is:

$$P_{ij} = \begin{bmatrix} .235 & .187 & .253 & .255 & .250 & .346 \\ .294 & .337 & .227 & .302 & .150 & .192 \\ .212 & .250 & .265 & .209 & .350 & .269 \\ .258 & .225 & .253 & .232 & .250 & .192 \end{bmatrix}$$

The entropy of each attribute,  $E_j$ , the degree of diversification,  $d_j$ , and the normalized weight,  $w_j$ , are calculated. They are:

	x1	x2	x3	x4	x5	x6
$E_j$	0.994	0.982	0.998	0.993	0.970	0.9770
$d_j$	0.0054	0.0171	0.0011	0.0069	0.0297	0.0230
$w_j$	0.649	0.2055	0.0133	0.0829	0.3570	0.2764

The results indicate that the weight of importance is in the order of:

$$w_5 = 0.3570, w_6 = 0.2764, w_2 = 0.2055, w_4 = 0.0829, w_1 = 0.0649, w_3 = 0.0133.$$

### 3.3 Consistency Evaluation

In comparing different objects there are two kinds of measurements, absolute and relative. In absolute measurement objects are compared against a standard that has been developed mathematically or experimentally, while in relative measurement objects are compared in pairs with respect to a common attribute.

When the judgment is based on the exact measurement, that is, the weights  $w_1 \dots w_n$  are already known, then  $a_{ij} = w_i / w_j$  for  $i, j = 1, \dots, n$  and the judgment matrix is consistent.

When the judgment is not based on exact measurement but on subjective judgment, then  $a_{ij}$  will deviate from the ideal ratio of  $w_i / w_j$  and the judgment matrix is not perfectly consistent. This is true in real situations where people, despite their effort, are to some degree inconsistent and intransitive.

#### 3.3.1 Consistency Index (CI)

Since any small perturbation in a positive matrix entries implies similar perturbation in the eigenvalues, the eigenvalue problem for the inconsistent case becomes,

$$Aw' = \lambda_{\max} w'$$

The closer the  $\lambda_{\max}$  to  $n$ , the more consistent the judgments in  $A$ . Saaty [21] has established a consistency index (CI) as a measure of overall consistency of the judgments:

$$CI = \frac{\lambda_{\max} - n}{n - 1} \quad (3.13)$$

If  $\lambda_{\max} \neq n$ , then the other eigenvalues will be non-zero, and may be complex. This index is actually the average of the non-principal eigenvalues.

Due to the fact that any randomly formed judgment matrix will show some degree of consistency, a "Consistency Ratio" (CR) instead of CI, which is the ratio of the CI to the average CI of many randomly generated judgment matrices, is suggested by him. He claims that if CR is less than 10 percent, we will accept the judgments, otherwise we should revise our judgments.

It is worth nothing that judgments may not only violate the consistency relation, but also the transitivity property ( $a_{ij} a_{jk} = a_{ik}$ ) as well. As an example, basketball team A may win against team B which has won to a third team C; yet team A may lose against team C. Thus the team behavior is inconsistent. Despite internal transitivity requirement for any judgment, sometimes external factors like the place of the game or player's injury in case of the above example, will change the expectations. This is a fact which should be accepted in the formulation. May [59] has studied the idea that intransitivity among preferences may be a natural phenomenon and not a consequence of judgmental error or deviation. Therefore, the question is not "Are preferences transitive?" but "Under what conditions does transitivity fail?".

In many real-life situations due to the lack of data or imprecise information, it is only left to the experts' judgment to estimate the value of attributes or probability of occurrence; therefore, it is desired to have an interval estimation instead of a point estimates suggested by Saaty. Peper [60] and Rahman and Shrestha [61] have discussed the desirability of interval estimation instead of a point estimate. A model for the error in judgments along the variance of the errors will be discussed in the following. This will clear the way for calculating the variance of the priorities ( $w_j$ ) and consequently their likely range of variations (confidence intervals)

### 3.3.2 Appropriate Model for Errors in Judgments

The imprecision or uncertainty inherent in the judgment ratios could be considered by introducing an error factor ( $\epsilon_{ij}$ ) with each inconsistent judgment ratio  $a_{ij}$  such that  $a_{ij} = w_{ij} \times \epsilon_{ij}$ . The error factor will be equal to unity for a perfectly consistent matrix, and between zero and infinity for any inconsistent case. The range of error factor will increase when the judgment ratios are more inconsistent. This characteristic is satisfied by a non-negative distribution, such as Log-normal, Weibull, Beta, or Gamma distribution.

To obtain an appropriate distribution of a random phenomenon is to collect enough sample of the random variable and carry out a suitable statistical test. But there are situations that collection of data becomes infeasible or practically impossible. Under these conditions the selection of distribution may be based on a reasonable statistical justification such as Central Limit Theorem which suggest a normal distribution, or a Beta distribution for project completion time in CPM (Critical Path Method), or the Negative Exponential distribution in case of transition rates in Markov chain. Therefore, to model the judgment ratios as a random variable with a suitable distribution we should do the same.

Several possible distributions and their performance have been tested by Rahman and Shrestha [62]. Log-normal distribution was found to be a good candidate for the error model.

#### 3.3.2.1 Log-Normal Model

The log-normal error model which was chosen among several other non-negative distributions could be stated as:

$$\varepsilon_{ij} \approx \Lambda(0, \sigma_{ij}^2)$$

which is equivalent to

$$\ln \varepsilon_{ij} \approx N(0, \sigma_{ij}^2) \quad (3.14)$$

It can be shown from the above representation that the medium of the error factor  $\varepsilon_{ij}$  is equal to one and its range can be varied by a proper selection of  $\sigma_{ij}^2$  to represent imprecision in experts judgments.

If it were possible to generate  $n$  estimate of  $a_{ij}$ , then

$$(a_{ij})^n = (w_{ij})^n \prod_{k=1}^n \varepsilon_{ij}^k$$

so that

$$\ln(a_{ij}) = \ln(w_{ij}) + \frac{1}{n} \sum_{k=1}^n [\ln \varepsilon_{ij}^k] \quad (3.15)$$

By the central limit theorem the summation of the log errors can be approximated by a normal random variable with zero mean and certain variance ( $\sigma^2$ ), therefore:

$$\ln(a_{ij}) = \ln(w_{ij}) + \varepsilon_{ij}$$

$$\ln(a_{ij}) \approx N[\ln(w_{ij}), \sigma^2] \quad (3.16)$$

$$\text{and, } a_{ij} \approx \Lambda[\ln(w_{ij}), \sigma^2]$$

This provides a basis to show that  $a_{ij}$  also behaves like a log-normal variable with a median value of  $w_{ij}$  and the variance depending on the spread of the error term or the level of expertise.

### 3.3.3 Estimation of Variance of Computed Priorities

The conclusion that error in the expert judgments is log-normally distributed, helps to estimate the variance of computed priorities and therefore its likely range of variation. This will be more suitable for a variety of statistical treatment compared to the point estimates of the priorities given by the eigenvector method.

In this section a method to estimate the variance of the computed priorities is briefly discussed.

#### 3.3.3.1 Analysis

A good approximation to the eigenvector calculation is by taking the geometric mean of each row of the judgment matrix and then normalize it to obtain the priority vector. This can be expressed as,

$$w_i = \frac{\left[ \prod_{j=1}^n a_{ij} \right]^{\frac{1}{n}}}{\sum_{i=1}^n \left[ \prod_{j=1}^n a_{ij} \right]^{\frac{1}{n}}} \quad (3.17)$$

From the discussions found in [63], it can be shown that

$$\begin{aligned}
a_{ij} &\approx \Lambda(\ln p_{ij}, \sigma_{ij}^2), & p_{ij} \text{ is the actual priority value} \\
\prod_{j=1}^n a_{ij} &\approx \Lambda(\ln \prod_{j=1}^n p_{ij}, \sum_{j=1}^n \sigma_{ij}^2) \\
\left[ \prod_{j=1}^n a_{ij} \right]^{\frac{1}{n}} &\equiv \Lambda \left[ \ln \prod_{j=1}^n [p_{ij}]^{\frac{1}{n}}, \frac{n-1}{n^2} \sigma^2 \right] \\
\sum_{i=1}^n \left[ \prod_{j=1}^n a_{ij} \right]^{\frac{1}{n}} &\equiv \Lambda \left[ -\ln \prod_{i=1}^n [p_i], \frac{n-1}{n^2} \sigma^2 (\sum p_i^2 - k_1) \right] \tag{3.18}
\end{aligned}$$

Taking the ratio of (3.17) and (3.18), and taking into account the fact that they are not independent, because of the reciprocal nature of the matrix the distribution of computed priorities can be approximated by:

$$\begin{aligned}
w_i &\equiv \Lambda \left\{ \ln \left[ \prod_{i=1}^n p_i \prod_{j=1}^n [p_{ij}]^{\frac{1}{n}} \right], \frac{n-1}{n^2} \left( 1 + \sum_{i=1}^n p_i^2 - k_1 - k_2 \right) \sigma^2 \right\} \\
&\equiv N \left\{ p_i, \frac{n-1}{n^2} \left( 1 + \sum_{i=1}^n p_i^2 - k_1 - k_2 \right) \sigma^2 p_i^2 \right\} \tag{3.19}
\end{aligned}$$

The constants  $K_1$  and  $K_2$  can be expected to be dependent on  $n$  and  $p_i$ 's and are evaluated on the basis of simulation results. Substituting these values,

$$w_i \equiv N \left\{ p_i, \frac{n^2-1}{n^2} [\sum p_i^2 - p_i^2] \sigma^2 p_i^2 \right\} \tag{3.20}$$

If we can find a reasonable estimate of  $\sigma^2$ , and assume  $w_i$  to be the estimate of  $p_i$ , it can be reasonably expected that the computed priorities  $w_i$  will follow the student's t-distribution, therefore

$$w_i \equiv t_N \left\{ w_i, \frac{n^2 - 1}{n^2} [\sum w_i^2 - w_i^2] \hat{\sigma}^2 w_i^2 \right\} \quad (3.21)$$

Since the estimations are based on sample size  $m = n(n-1)/2$ , the degree of freedom for the above distribution can be reasonably assumed to be  $df = m-1$ .

The other unknown in equation (3.21) is the value of  $\hat{\sigma}^2$  which is an estimate of the error factor variability from the deviation between the consistent judgment ratio  $w_{ij} = w_i/w_j$  and the expert judgment ratio  $a_{ij}$ , since  $w_i$  are not mutually independent. The covariance terms become significant, then,

$$w_{ij} \equiv \Lambda \left[ \ln \frac{p_i}{p_j}, k\sigma^2 \right]$$

and since

$$a_{ij} \approx \Lambda \left[ \ln \frac{p_i}{p_j}, \sigma^2 \right]$$

then

$$\frac{a_{ij}}{w_{ij}} \equiv \Lambda \left[ 0, k\sigma^2 \right] \quad (3.22)$$

where the value of  $K$  has been found by simulation to be  $\frac{n-1}{n+1}$ . Therefore,

$$\ln \frac{a_{ij}}{w_{ij}} \equiv N \left[ 1, \frac{n-1}{n+1} \sigma^2 \right] = y_{ij} \quad (3.23)$$

The variance of the variable  $y_{ij}$  is proportional to the parameter  $\sigma^2$ , of the initial error factor. And since  $n(n-1)/2$  values of this variable is available for each judgment matrix, the variance of the actual error which is called Quotient Index (QI) can be estimated as

$$\hat{\sigma}^2 = QI = \frac{2}{(n-1)(n-2)} \sum_{i=1}^{n-1} \sum_{j=i+1}^n y_{ij}^2 \quad (3.24)$$

This index can be estimated directly from the judgment matrix and can serve as an estimate of the error variances associated with each judgment matrix, so that the range of variation of each priority can be calculated.

### 3.3.3.2 Confidence Intervals of Priorities

Based on the above justifications, the confidence interval for each priority ( $w_i$ ) can be estimated such that the actual priority is contained in the interval with a desired level of probability.

The confidence interval for each  $w_i$  with a probability of  $(1 - \alpha)$  100% is given by:

$$w_i - t_{m-1, \frac{\alpha}{2}} \hat{\sigma}_{w_i} \leq w_i \leq w_i + t_{m-1, \frac{\alpha}{2}} \hat{\sigma}_{w_i} \quad (3.25)$$

where

- $w_i$  the point estimate of the priorities
- $\hat{\sigma}_{w_i}$  standard deviation of  $w_i$
- $m - 1$  degree of freedom
- $\frac{\alpha}{2}$  the desired cutoff region

If the consistency in the judgments is perfect, the interval will collapse to a point estimate and QI will be equal to zero. But with increasing inconsistencies, the interval becomes wider and therefore the value of QI will increase.

The validity of the above models and the results of approximation analysis have been investigated by Shrestha [63] through simulation analysis of many cases of randomly generated matrices of different sizes. The mean value of the QI has been found to reproduce the error variance very accurately as shown in the Table 3.2. In the next section we will investigate the relationship between QI and variance of the priorities.

**Table 3.2 Comparison of error variance and mean Quotient Index (QI)**

Error Variance	Mean QI (n = 4)	Mean QI (n = 6)
0.05	0.0511	0.0505
0.10	0.9913	0.1018
0.15	0.1502	0.1498
0.20	0.2106	0.1998
0.25	0.2534	0.2642
0.30	0.3057	0.3071
0.35	0.3525	0.3444
0.40	0.3847	0.4153
0.50	0.5242	0.4917

### 3.3.4 Relationship between QI and Variance of the Priorities

To investigate the ability of QI in providing confidence intervals of priorities which contain the actual priorities, the simulation has been extended to examine the kind of relationship that exists between the variance of errors (QI) and the variance of priorities ( $\sigma_w^2$ ).

In situations when all the weights are equal, their variances are also equal and are proportional to QI. However, when the weights are not equal, which is true in most real cases, the variance of individual weights become different for the same value of QI and it is no longer easy to link QI to the variance of the weights. In these situations we are more interested in factors with the largest (or larger) weights since the factors with smaller weights have a lesser interest in the evaluation process by decision makers. Therefore, it may suffice to concentrate on the largest (or larger) weights in the simulation process.

To investigate this relationship we should first create some consistent matrices and then try to perturb them by some error numbers (log-normally distributed) and then analyze the behavior of QI and the variance of the weights.

Following are the steps for the simulation:

- 1) Choose the size ( $n$ ) of a set (or size of the judgment matrix);
- 2) Generate several sets of  $n$  uniform random numbers between 0 and 1;
- 3) Select one set of those random numbers such that the difference between them is not too large or too small in order to distinguish the largest number from the next largest one. For this study we chose 40% difference, This is the priority vector ( $w_j$ );

- 4) Normalize the selected set and take the ratio of  $w$ 's to form a consistent judgment matrix [CJM];
- 5) Generate random error numbers from a log-normal distribution equal to the number of cases in the simulation;
- 6) Multiply the error numbers with consistent matrix to create inconsistent judgment matrix [IJM];
- 7) Compute the priority weights of each IJM matrix, its corresponding QI, and variance of the weights based on the equations (3.17), (3.24), (3.21);
- 8) Repeat the procedure to obtain sufficient data;
- 9) Plot QI vs. variance of the highest weight and check if any relationship can be established between the two.

The above simulation procedure was carried out for fifty cases of different matrix sizes ( $n = 3, 4, 5$ ). The Mathematica package on NeXT computer was used to generate normally distributed random numbers and log-normally distributed numbers, and an interactive PC program developed at Virginia Tech. Energy Laboratory for analysis of matrices which were created by simulation.

For instance, the selected random number set (as the priority weights) for  $n=3$  is equal to:

$$U = \{0.06764, 0.9442, 0.6660\}$$

$$U_{\text{normalized}} = \{0.0403, 0.5627, 0.3969\}$$

And, the consistent judgment matrix [CJM] from the ratio of  $U_{\text{normalized}}$  is

$$\begin{bmatrix} 1 & 0.0716 & 0.1015 \\ 1/.0716 & 1 & 1.418 \\ 1/.1015 & 1/1.418 & 1 \end{bmatrix}$$

Multiplying the randomly generated error numbers of case# 1 [ $e = \{0.8584, 1.0838, 1.0452\}$ ] with the upper triangular elements of [CJM] matrix, the inconsistent judgment matrix [IJM] becomes

$$\begin{bmatrix} 1 & 0.0615 & 0.1100 \\ 1/.0615 & 1 & 1.4821 \\ 1/.1100 & 1/1.4821 & 1 \end{bmatrix}$$

Using the program, the corresponding weights and their variances, and the quotient index (QI) are:

$$w_1 = 0.0385, w_2 = 0.5885, w_3 = 0.3730$$

$$\text{var. } w_1 = 8.00\text{E-}06, \text{ var. } w_2 = 5.10\text{E-}04, \text{ var. } w_3 = 5.07\text{E-}04$$

$$\text{QI} = 0.0118$$

The detailed results are shown in Tables 3.3-3.6, and the corresponding plots in Figures 3.3-3.5 for  $n=3, 4,$  and  $5,$  respectively. It is observed for all cases that there is a linear relationship between QI and variance of the weights. The level of confidence intervals changes directly with the value of QI. Therefore, we can say that QI is a good indicator of variability of the weights.

**Table 3.3 Random error numbers and upper triangular elements of the inconsistent matrices for n=3**

Case No	e12	e13	e23	a12	a13	a23
1	0.8583	1.0837	1.0451	0.0615	0.1100	1.4821
2	0.9854	0.9242	1.0074	0.0706	0.0938	1.4286
3	0.8331	1.3297	0.7330	0.0597	0.1350	1.0395
4	1.3547	1.0965	1.0982	0.0970	0.1113	1.5573
5	0.8113	0.7260	0.6575	0.0581	0.0737	0.9324
6	1.1011	0.8188	1.2159	0.0788	0.0831	1.7242
7	1.1378	1.1171	1.0254	0.0815	0.1134	1.4540
8	0.6492	1.0168	1.2911	0.0465	0.1032	1.8308
9	0.6874	1.3124	1.0290	0.0492	0.1332	1.4592
10	1.1296	0.9814	1.0006	0.0809	0.0996	1.4189
11	0.9990	1.1773	1.6081	0.0715	0.1195	2.2804
12	1.3157	1.0857	0.9782	0.0942	0.1102	1.3871
13	0.9152	0.8482	1.1923	0.0655	0.0861	1.6908
14	0.8816	1.3812	1.0477	0.0631	0.1402	1.4857
15	0.9472	0.7684	1.1347	0.0678	0.0780	1.6091
16	1.0112	1.0718	0.7728	0.0724	0.1088	1.0959
17	0.9017	0.7106	0.7176	0.0646	0.0721	1.0176
18	1.2285	0.9339	1.2682	0.0880	0.0948	1.7984
19	0.9643	0.9052	0.9540	0.0690	0.0919	1.3529
20	1.2895	1.0000	1.0046	0.0923	0.1015	1.4246
21	0.8750	0.7423	1.0849	0.0627	0.0753	1.5385
22	1.0082	1.0048	1.1635	0.0722	0.1020	1.6499
23	0.8036	0.7891	0.8750	0.0575	0.0801	1.2408
24	1.1894	0.9888	0.8838	0.0852	0.1004	1.2533
25	0.7733	0.8449	1.0358	0.0554	0.0858	1.4689
26	0.9081	0.9488	0.9784	0.0650	0.0963	1.3875
27	1.2118	1.0691	0.8859	0.0868	0.1085	1.2562
28	0.9692	1.1805	1.0198	0.0694	0.1198	1.4461
29	0.8051	0.7363	0.7685	0.0576	0.0747	1.0898
30	0.7573	1.0114	1.4251	0.0542	0.1027	2.0208
31	0.5991	1.2909	0.6913	0.0429	0.1310	0.9802
32	1.1615	1.0425	1.3971	0.0832	0.1058	1.9811
33	1.1420	1.0039	0.9198	0.0818	0.1019	1.3044
34	0.9218	0.8654	0.8853	0.0660	0.0878	1.2554
35	0.9916	0.7377	1.0726	0.0710	0.0749	1.5210
36	1.1779	1.3944	0.8386	0.0843	0.1415	1.1891
37	0.9728	0.7594	0.6617	0.0697	0.0771	0.9383
38	1.2326	0.7578	1.1466	0.0883	0.0769	1.6260
39	1.3778	1.0671	1.0405	0.0987	0.1083	1.4755
40	1.0349	0.8666	0.6714	0.0741	0.0880	0.9522
41	1.2196	1.5466	1.0773	0.0873	0.1570	1.5276
42	0.7987	0.6671	0.8821	0.0572	0.0677	1.2509
43	1.1553	0.8744	0.7486	0.0827	0.0888	1.0616
44	0.7106	1.0943	0.8426	0.0509	0.1111	1.1948
45	0.9235	1.6142	2.1316	0.0661	0.1638	3.0226
46	0.7842	1.1120	0.9756	0.0562	0.1129	1.3835
47	0.9883	1.1490	0.9754	0.0708	0.1166	1.3832
48	1.2188	1.5803	1.1898	0.0873	0.1604	1.6872
49	0.6959	0.7127	1.4625	0.0498	0.0723	2.0739
50	0.6776	1.3505	0.9371	0.0485	0.1371	1.3289

**Table 3.4 weights, variances and QI of randomly generated matrices for n=3**

case No.	w1	w2	w3	var(w2)	QI
1	0.0385	0.5885	0.3730	5.10E-04	0.0118
2	0.0386	0.5599	0.4015	8.00E-05	0.0018
3	0.0425	0.5495	0.4080	9.09E-03	0.2013
4	0.0487	0.5560	0.3953	1.36E-03	0.0311
5	0.0317	0.4921	0.4763	1.55E-03	0.0316
6	0.0382	0.5714	0.3904	3.60E-03	0.0806
7	0.0452	0.5624	0.3925	2.80E-05	0.0006
8	0.0317	0.6403	0.3280	4.89E-04	0.0123
9	0.0374	0.6180	0.3447	5.20E-03	0.1273
10	0.0425	0.5506	0.4069	3.03E-04	0.0067
11	0.0416	0.6448	0.3136	1.19E-03	0.0322
12	0.0481	0.5399	0.4120	4.31E-04	0.0097
13	0.0354	0.5871	0.3776	9.31E-04	0.0211
14	0.0435	0.6021	0.3544	2.22E-03	0.0540
15	0.0345	0.5696	0.3959	1.71E-03	0.0375
16	0.0423	0.5258	0.4319	1.54E-03	0.0332
17	0.0330	0.4952	0.4718	1.39E-04	0.0028
18	0.0428	0.5768	0.3804	3.79E-03	0.0875
19	0.0379	0.5520	0.4101	4.00E-06	0.0001
20	0.0458	0.5406	0.4137	1.01E-03	0.0223
21	0.0327	0.5670	0.4003	9.43E-04	0.0205
22	0.0402	0.5859	0.3740	3.46E-04	0.0080
23	0.0326	0.5449	0.4225	2.12E-04	0.0045
24	0.0440	0.5269	0.4291	5.80E-05	0.0013
25	0.0327	0.5796	0.3877	4.20E-05	0.0009
26	0.0375	0.5645	0.3980	6.50E-05	0.0014
27	0.0460	0.5308	0.4232	8.15E-06	0.0000
28	0.0427	0.5795	0.3778	4.51E-04	0.0105
29	0.0317	0.5190	0.4494	4.90E-04	0.0101
30	0.0340	0.6416	0.3244	5.40E-05	0.0014
31	0.0356	0.5678	0.3966	1.96E-02	0.4304
32	0.0433	0.6035	0.3532	2.69E-03	0.0655
33	0.0433	0.5379	0.4188	3.20E-05	0.0007
34	0.0364	0.5409	0.4227	5.20E-05	0.0011
35	0.0348	0.5538	0.4114	2.07E-03	0.0446
36	0.0513	0.5422	0.4065	1.74E-03	0.0396
37	0.0354	0.4802	0.4845	4.37E-04	0.0090
38	0.0392	0.5467	0.4140	5.97E-03	0.1299
39	0.0487	0.5442	0.4071	1.29E-03	0.0292
40	0.0388	0.4865	0.4747	7.76E-04	0.0163
41	0.0538	0.5841	0.3621	3.61E-04	0.0089
42	0.0300	0.5346	0.4353	4.90E-05	0.0010
43	0.0411	0.4947	0.4642	2.00E-06	0.0001
44	0.0356	0.5724	0.3919	5.46E-03	0.1210
45	0.0437	0.7065	0.2497	3.75E-04	0.0132
46	0.0374	0.5876	0.3750	2.02E-03	0.0464
47	0.0427	0.5691	0.3882	4.46E-04	0.0101
48	0.0539	0.6003	0.3458	9.50E-05	0.0024
49	0.0279	0.6300	0.3421	1.76E-03	0.0424
50	0.0379	0.6070	0.3552	7.93E-03	0.1899

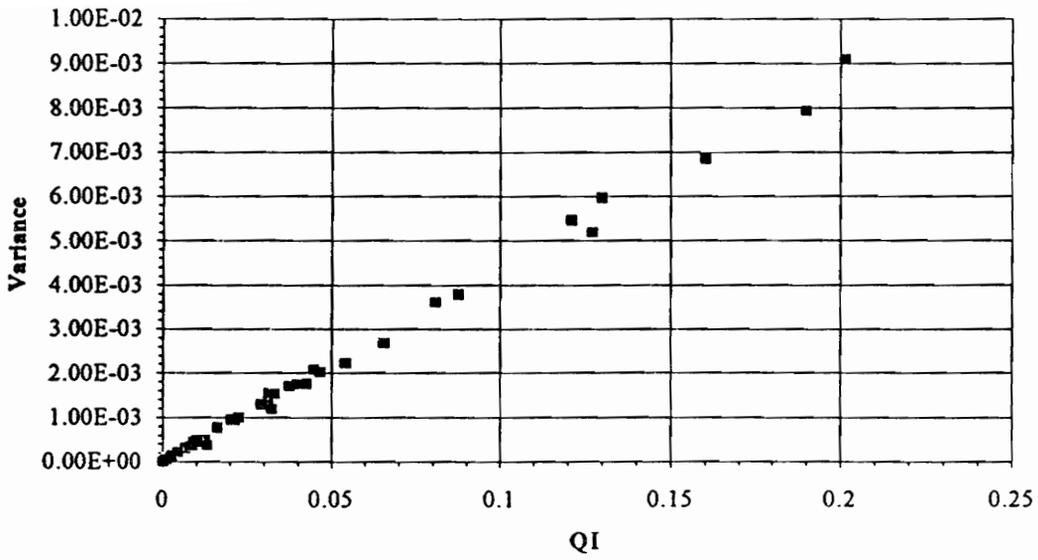


Figure 3.3 QI vs. Variance of the weights for n=3

**Table 3.5 weights, variances, and QI of randomly generated matrices for n=4**

Case No.	w1	w2	w3	w4	var. (w4)	QI
1	0.231	0.2602	0.0465	0.4623	4.89E-04	0.0198
2	0.2285	0.3032	0.047	0.4213	5.53E-04	0.0227
3	0.2465	0.3117	0.0491	0.3927	1.27E-03	0.0548
4	0.2037	0.3146	0.0443	0.4374	2.68E-04	0.0105
5	0.2297	0.2911	0.0375	0.4417	1.47E-03	0.0579
6	0.2179	0.3073	0.0494	0.4253	1.99E-04	0.0081
7	0.2633	0.3273	0.0521	0.3573	1.88E-03	0.0795
8	0.2282	0.3087	0.056	0.4071	9.80E-04	0.0419
9	0.2244	0.3214	0.0492	0.405	9.12E-04	0.038
10	0.2782	0.2954	0.0462	0.3802	1.36E-03	0.0602
11	0.2238	0.2354	0.0458	0.4949	1.89E-04	0.0077
12	0.2527	0.3137	0.055	0.3785	1.85E-03	0.0831
13	0.2724	0.2626	0.0446	0.4204	8.60E-05	0.0036
14	0.2614	0.2677	0.0461	0.4248	2.34E-03	0.0971
15	0.2344	0.2972	0.0501	0.4183	2.29E-03	0.0958
16	0.2039	0.3456	0.0371	0.4134	4.10E-04	0.0157
17	0.2393	0.3382	0.0541	0.3684	3.82E-04	0.0172
18	0.2432	0.2508	0.0499	0.4561	6.69E-04	0.0275
19	0.224	0.3112	0.0435	0.4213	1.08E-03	0.0436
20	0.2634	0.286	0.0525	0.3981	3.55E-04	0.0155
21	0.2313	0.2971	0.0448	0.4268	6.39E-04	0.026
22	0.2464	0.3051	0.0393	0.4092	1.31E-03	0.0537
23	0.2357	0.3058	0.0426	0.4159	9.28E-04	0.0379
24	0.225	0.2975	0.0442	0.4333	1.20E-03	0.0483
25	0.2355	0.2919	0.0487	0.4239	4.61E-04	0.0191
26	0.2158	0.2988	0.0435	0.4419	1.40E-03	0.0553
27	0.2148	0.3158	0.0481	0.4214	9.19E-04	0.0373
28	0.2356	0.2994	0.0415	0.4235	4.93E-04	0.0199
29	0.2543	0.2677	0.0514	0.4267	5.19E-04	0.0219
30	0.2349	0.2971	0.0439	0.4241	7.14E-04	0.0291
31	0.2238	0.2672	0.042	0.467	2.39E-04	0.0095
32	0.1964	0.3143	0.0385	0.4508	1.87E-03	0.0707
33	0.2199	0.2774	0.0412	0.4615	5.38E-04	0.0212
34	0.2343	0.2681	0.0476	0.4501	5.00E-05	0.0021
35	0.2177	0.3452	0.0423	0.3948	1.78E-03	0.0724
36	0.2003	0.3181	0.0499	0.4317	8.86E-04	0.0353
37	0.239	0.3193	0.0487	0.393	1.29E-03	0.055
38	0.2531	0.3177	0.0511	0.3781	7.04E-04	0.0313
39	0.2498	0.2963	0.0387	0.4152	6.97E-04	0.0284
40	0.2154	0.283	0.0501	0.4515	2.26E-03	0.0918
41	0.2427	0.2783	0.0544	0.4246	1.41E-03	0.0599
42	0.2534	0.3272	0.0415	0.3779	1.35E-03	0.0581
43	0.2238	0.3121	0.0466	0.4175	2.81E-03	0.115
44	0.251	0.2724	0.0422	0.4345	6.76E-04	0.0275
45	0.2556	0.2561	0.0405	0.4478	6.47E-04	0.026
46	0.2522	0.3044	0.0441	0.3992	6.04E-04	0.0255
47	0.2521	0.3067	0.051	0.3902	9.31E-04	0.0407
48	0.2072	0.3604	0.0506	0.3818	1.78E-03	0.0744
49	0.2243	0.3144	0.0472	0.4142	4.93E-04	0.0203
50	0.2312	0.3295	0.0482	0.391	1.97E-03	0.0837

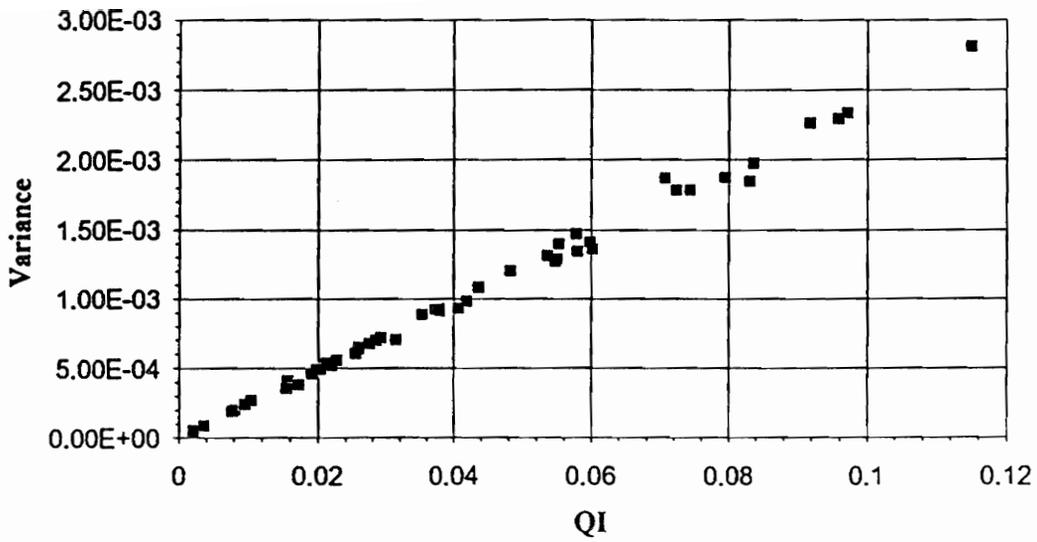


Figure 3.4 QI vs. Variance of the weights for n=4

**Table 3.6 weights, variances, and QI of randomly generated matrices for n=5**

Case No.	w1	w2	w3	w4	w5	var (w1)	QI
1	0.4123	0.2012	0.2644	0.1196	0.0024	2.57E-04	0.0126
2	0.3945	0.2052	0.2632	0.1349	0.0022	6.05E-04	0.0312
3	0.3698	0.2388	0.2824	0.1064	0.0027	1.85E-03	0.0953
4	0.4012	0.2238	0.2444	0.128	0.0026	1.11E-03	0.0569
5	0.4208	0.1916	0.2758	0.1097	0.0022	8.07E-04	0.0380
6	0.4017	0.1944	0.2869	0.1145	0.0026	2.37E-03	0.1148
7	0.394	0.2059	0.2809	0.1169	0.0024	3.32E-04	0.0165
8	0.3563	0.238	0.2835	0.12	0.0023	1.34E-03	0.0724
9	0.3821	0.2372	0.2728	0.1059	0.0021	2.62E-04	0.0132
10	0.4209	0.228	0.248	0.1011	0.002	3.81E-04	0.0181
11	0.3412	0.2308	0.2937	0.1319	0.0025	1.62E-03	0.0924
12	0.3482	0.2497	0.2737	0.1264	0.0021	6.98E-04	0.0391
13	0.3453	0.2127	0.2919	0.1481	0.002	7.19E-04	0.0412
14	0.3613	0.2275	0.2849	0.1242	0.0022	4.91E-04	0.0264
15	0.3656	0.2214	0.2952	0.1156	0.0023	7.44E-04	0.0388
16	0.3864	0.2318	0.2567	0.1231	0.002	3.47E-04	0.0180
17	0.3725	0.2131	0.2652	0.1465	0.0027	7.14E-04	0.0391
18	0.3731	0.1968	0.3213	0.1069	0.0019	7.17E-04	0.0350
19	0.3956	0.2109	0.2679	0.1233	0.0022	9.93E-04	0.0503
20	0.3898	0.1967	0.2692	0.1415	0.0029	1.25E-03	0.0652
21	0.3788	0.2117	0.298	0.1095	0.002	1.82E-03	0.0907
22	0.3703	0.2022	0.2718	0.1532	0.0025	3.47E-04	0.0191
23	0.3491	0.2055	0.3261	0.1172	0.0021	9.72E-04	0.0512
24	0.3293	0.2334	0.3106	0.1245	0.0022	5.96E-04	0.0344
25	0.3842	0.2117	0.2745	0.1274	0.0021	3.32E-04	0.0172
26	0.4038	0.2194	0.2378	0.1365	0.0025	2.54E-04	0.0131
27	0.375	0.2358	0.261	0.1261	0.0022	1.66E-03	0.0813
28	0.3414	0.2584	0.2917	0.1061	0.0024	5.01E-04	0.0275
29	0.3645	0.1986	0.3091	0.1252	0.0026	7.45E-04	0.0388
30	0.3671	0.1988	0.2841	0.1481	0.0019	1.67E-03	0.0905
31	0.3654	0.2238	0.2783	0.1303	0.0023	9.25E-04	0.0499
32	0.4078	0.2047	0.2648	0.1204	0.0023	3.78E-04	0.0187
33	0.3495	0.2346	0.2817	0.1318	0.0023	1.03E-03	0.0580
34	0.4011	0.2123	0.2596	0.1249	0.0021	4.17E-04	0.0211
35	0.3665	0.2307	0.2732	0.1272	0.0024	9.16E-04	0.0493
36	0.4116	0.2121	0.2523	0.1216	0.0024	9.28E-04	0.0462
37	0.4052	0.2182	0.2483	0.1259	0.0024	5.78E-04	0.0293
38	0.4018	0.2361	0.2378	0.1222	0.0022	1.34E-03	0.0681
39	0.415	0.2142	0.2578	0.1109	0.0021	2.52E-04	0.0122
40	0.372	0.2107	0.2959	0.1192	0.0022	9.32E-04	0.0480
41	0.4037	0.202	0.2623	0.1296	0.0023	5.80E-04	0.0293
42	0.3291	0.2571	0.2813	0.1304	0.0021	2.53E-04	0.0150
43	0.3618	0.2068	0.2979	0.1313	0.0022	1.32E-04	0.0071
44	0.4097	0.2174	0.2362	0.1344	0.0023	4.50E-04	0.0230
45	0.3529	0.2428	0.2761	0.1257	0.0025	4.04E-04	0.0224
46	0.3808	0.1848	0.2856	0.1464	0.0023	1.91E-04	0.0100
47	0.3841	0.234	0.2659	0.1139	0.002	1.26E-04	0.0064
48	0.3982	0.1858	0.2906	0.1228	0.0026	6.35E-04	0.0311
49	0.3995	0.2185	0.256	0.1235	0.0025	5.59E-04	0.0284
50	0.3768	0.1899	0.2988	0.1323	0.0022	6.36E-04	0.0327

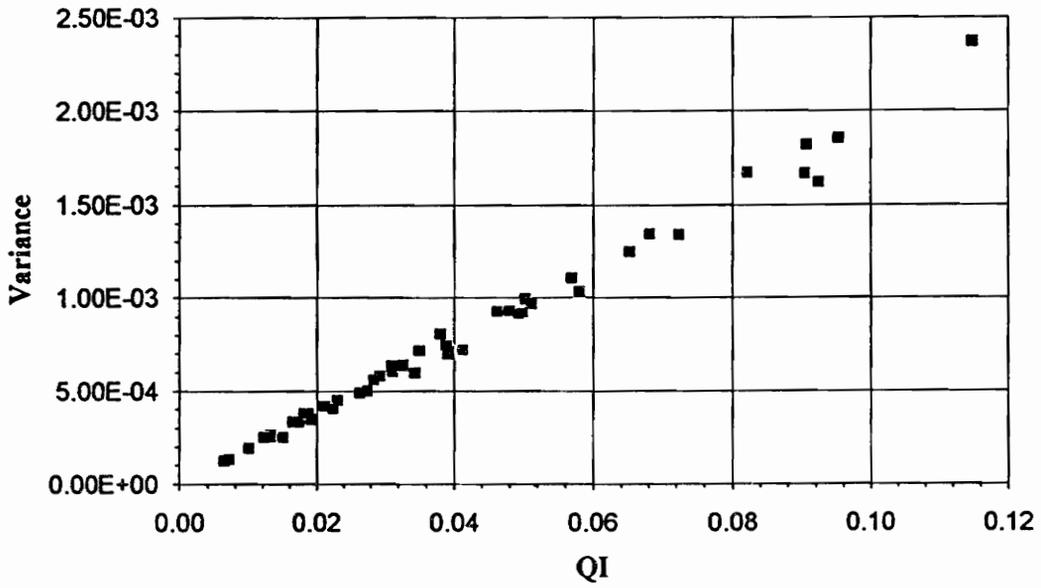


Figure 3.5 QI vs. Variance of the weights for n=5

### 3.3.5 Calculation of Composite Priorities and Their Variances

Since in many real-life situations the decision making problems are complex, their hierarchical structure is also more than two level. Therefore, we should extend the previous discussions on the simple hierarchy to multi-level hierarchies, that is to find the composite priorities and their variances from those of an individual judgment matrices.

To calculate the priorities of attributes in a lower level with respect to any higher level involves the summation of products of the lower level priority with the related ones on the upper level. Therefore, based on the earlier discussions, the priorities for two successive levels  $i$  and  $j$  can be represented as,

$$\begin{aligned} u_i, w_i &\equiv N(p_{i,j}, \sigma_{i,j}^2) \\ &\equiv \Lambda(\ln p_{i,j}, \delta_{ij}^2) \quad \text{for } i = 1, \dots, n \end{aligned}$$

where

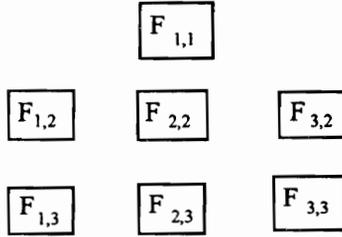
$p_{i,j}$  's are the priorities (weights).

$$\begin{aligned} \sigma_{i,j}^2 &= \delta_{i,j}^2, p_{i,j}^2 \\ \delta_{i,j}^2 &= \frac{n^2 - 1}{n^2} [\sum p_{i,j}^2 - p_{i,j}^2] \sigma^2 \end{aligned}$$

The variances  $\delta_i^2, \delta_j^2$  in the above expressions can be estimated from the knowledge of  $p_i$ , and  $p_j$  and the quotient indices  $QI_i$ , and  $QI_j$ . According to Shrestha [63], due to the log-normal property of  $u_i$  and  $w_j$  their combination is,

$$\begin{aligned}
 u_i, w_j &\cong \Lambda \left[ (\ln p_i + \ln p_j), (\delta_i^2 + \delta_j^2) \right] \\
 &\cong N \left[ p_i p_j, (p_i^2 \sigma_j^2 + p_j^2 \sigma_i^2) \right]
 \end{aligned}
 \tag{3.26}$$

This will give a convenient expression to complete both the mean and variance of products of the priorities computed for successive levels of the hierarchy. For example, for a three level hierarchy with three factors on levels two and three, we will have,



the priority vector for  $F_{1,3}$  with respect to  $F_{1,1}$  is,

$$p_{1,3}^{1,1} = (p_{1,3}^{1,2} p_{1,2}^{1,1}) + (p_{1,3}^{2,2} p_{2,2}^{1,1}) + (p_{1,3}^{3,2} p_{3,2}^{1,1})$$

and the variance of composite priority for  $F_{1,3}$  based on the normality assumption of the composite priorities can be approximated by the following expression,

$$(\sigma_{1,3}^{1,1})^2 = (p_{1,3}^{1,2})^2 (\sigma_{1,2}^{1,1})^2 + (p_{1,3}^{2,2})^2 (\sigma_{2,2}^{1,1})^2 + (p_{1,3}^{3,2})^2 (\sigma_{3,2}^{1,1})^2$$

and in general,

$$p_{k,n}^{1,1} = \sum_{i=1}^m p_{k,n}^{i,n-1} \times p_{i,n-1}^{1,1}
 \tag{3.27}$$

$$(\sigma_{k,n}^{1,1})^2 = \sum_{i=1}^{m_{n-1}} (p_{k,n}^{i,n-1})^2 (\sigma_{i,n-1}^{1,1})^2 \quad (3.28)$$

These expressions consist of the product of the priority and variance of factor  $k$  in level  $n$  with respect to the top level ( $F_{1,1}$ ).

The results of the simulations to show the accuracy of the expressions (3.27) and (3.28) for computing composite priorities and variances indicate that,

- The mean and the variance of composite priorities can be estimated with reasonable accuracy on the basis of the knowledge of weights and variances computed for each individual judgment matrix.
- The variations in the composite priorities can be specified in terms of confidence intervals by the knowledge of their variances.

Example:

Following is an example of a three-level hierarchy judgment matrices along with their computed priority weights ( $w_j$ ) and confidence intervals for each level and the overall composite priorities,

2 1 3

1.0000 3.0000 5.0000

.3333 1.0000 4.0000

.2000 .2500 1.0000

WEIGHTS (W)= .6267 .2797 .0936

LAMDA(MAX)= 3.086, CI.= .0429, CR.= .0739, QI.= .2555

CONFIDENCE INTERVALS OF THE WEIGHTS:

.3695 <W< .8839

.0331 <W< .5263

.0042 <W< .1830

3 1 3

1.0000 6.0000 3.0000

.1667 1.0000 .2500

.3333 4.0000 1.0000

WEIGHTS (W)= .6442 .0852 .2706

LAMDA(MAX)= 3.054, CI.= .0268, CR.= .0462, QI= .1602

CONFIDENCE INTERVALS OF THE WEIGHTS:

.4429 <W< .8456

.0196 <W< .1508

.0769 <W< .4643

3 2 3

1.0000 .3333 5.0000

3.0000 1.0000 7.0000

.2000 .1429 1.0000

WEIGHTS (W)= .2790 .6491 .0719

LAMDA(MAX)= 3.065, CI.= .0324, CR.= .0559, QI= .1936

CONFIDENCE INTERVALS OF THE WEIGHTS ARE :

.0583 <W< .4996

.4226 <W< .8756

.0104 <W< .1335

3 3 3

1.0000 8.0000 4.0000

.1250 1.0000 .2000

.2500 5.0000 1.0000

WEIGHTS (W)= .6986 .0643 .2370

LAMDA(MAX)= 3.094, CI.= .0470, CR.= .0810, QI.= .2799

CONFIDENCE INTERVALS OF THE WEIGHTS:

.4487 <W< .9485

.0000 <W< .1335

.0000 <W< .4792

\*\*\*\*\*

COMPOSITE PRIORITIES FOR LEVEL 3

.5472 .2410 .2119

\*\*\*\*\*

CONFIDENCE INTERVALS FOR THE COMPOSITE PRIORITIES OF LEVEL 3:

.4048 <W< .6896

.1652 <W< .3168

.0872 <W< .3366

### 3.4 Revising Judgments

When the consistency index is high the best way to reduce it to an acceptable level is to ask the expert(s) to revise his/her judgments. But since it is hard to revise all the judgments and sometimes it leads to even more inconsistency, we need to know which two comparisons in a row are farther from consistency (transitivity).

One method is to form the matrix of priority ratios  $w_i/w_j$  and consider the matrix of absolute differences  $|(a_{ij} - (w_i/w_j))|$ , or take the percentage deviations between them  $|(a_{ij} - (w_i/w_j))/ w_{ij}|$  and attempt to revise the judgment on the element(s) or row sums with the largest such differences.

The second approach is to form the root mean square deviation using the rows of  $a_{ij}$  and  $w_{ij}$  and revise the judgments for the row with the largest value. The reason for this is that generally one tends to be uncertain about how an attribute relates to all others in a row rather than to a single one. The procedure can then be repeated to note improvement in the consistency,

$$\max_i \sum_{j=1}^n \sqrt{\left(\frac{a_{ij} - w_{ij}}{n}\right)^2}$$

Although the above expressions will indicate where the inconsistency is coming from, but revising the judgments by repetition of the process must not be considered the solution to the problem. Consistency is only the necessary condition to reach at a reasonable results. Therefore, one should naturally improve the judgments by obtaining more knowledge or consultation with other experts.

### **3.4.1 Consensus on the Judgments**

Consensus means improving confidence in the priority values by using several judges to bring the results in line with majority preferences. The process of obtaining consensus can be used to persuade people that their interests are taken into consideration.

In seeking consensus, it is preferable that the experts interact. A well informed person can effect substantial change in the beliefs of another person who has less information. The debate should help bring judgments closer together. Such a debate would assist in providing information to apply the method of priority assignment to the judges themselves.

The factors affecting judgment may be: relative intelligence, years of experience, past record, depth of knowledge, experience in related fields, personal involvement in the issue at stake, and so on. If we have high confidence in the judgment of these people, the priority derived is used to weight the final priority result derived from the judgment of each individual and an overall weighted priority is then obtained in the usual way. On the other hand, if we have low confidence in the judgments provided by the experts, we can use the geometric mean (like in AHP) of their individual judgments as they appear in each of the comparison matrices.

How to represent group judgment in a satisfactory way when people's experiences and judgments differ, and whose opinions should be taken more seriously and why, is a major problem in conflict analysis and decision making process..

It seems possible that an idea developed and evaluated by one group should be handed over to another group for further debate and change in judgment. But the end results may still have wide variability in the solution. One cannot arbitrate the priorities by using the judgments of a favored group over others. In other words, discovering a

convenient and workable mathematical framework for a problem does not automatically solve its social intricacies. One of the main contributions of hierarchical analysis is in the structuring of the problem from the start jointly by the conflicting groups rather than by a third party, and then do the bargaining through the numerical entries.

## **Concluding Remarks**

Analytic Hierarchy Process is based on axioms of pairwise comparisons, a valid scale to translate judgment to numbers, and hierarchic composition by weighting and adding. A number of theorems have been developed which make the AHP a mathematically viable approach for deriving ratio scales in solving multi-objective problems under uncertainty. Designing an analytic hierarchy-like the structuring of a problem by any other method necessitates a substantial knowledge of the system in question. A strong aspect of the AHP is that the knowledge of individuals who supply judgments for the pairwise comparisons usually also play a prominent role in specifying the hierarchy.

This approach has many applications in a variety of fields, including: plan to allocate energy to industries, designing a transport system, planning the future of a corporation and measuring the impact of environmental factors on its development, and design of future scenarios for higher education. In chapter six we will show the benefits of this methodology in planning issues of electric utilities.

## **CHAPTER IV**

### **COMPARATIVE ANALYSIS OF METHODS DEALING WITH UNCERTAINTY**

After the analysis of AHP method in chapter three and its capability to handle complex problems under uncertainty it would be beneficial to compare this method with some other methods in use today in order to gain an in depth understanding of these approaches and how well they perform in real-world problems.

In the following sections, we will first discuss the sources of uncertainty and then the alternative methods of dealing with uncertainty. These include subjective probability methods, Fuzzy Set, and Delphi techniques.

#### **Sources of Uncertainty**

There are several sources of uncertainty in any decision making process. They could be listed as:

- 1) Uncertainty within the knowledge domain. This could be in the following forms:
  - The existing state of knowledge in a domain may be imperfect and incomplete.

- The information/knowledge may be vague by nature.
  - The evaluation method of the knowledge may contain errors.
- 2) Uncertainty related to the expert or the decision maker. If the knowledge of the experts or the data contains uncertainty, it will lead to a different rule and causes inconsistency in the decision.
  - 3) Uncertainty in the data, which could be due to method of collection or the equipments

Therefore, the assumption that the expert or DM is certain and 100 percent correct in their knowledge and that no vagueness exists in the domain is not true in the real world. Knowledge could be vague; experts could be uncertain or even wrong; and the user may not have the ideal data for the problem. This situation calls for some kind of explicit representation of uncertainty and techniques to analyze them properly, Zahedi [64].

#### 4.1 Probability Theory

Probability theory has long been used to deal with the concept of uncertainty. It is based on three axioms:

- 1) The probability  $P(h)$  is non-negative and less than or equal to 1, that is,

$$0 \leq p(h) \leq 1$$

- 2) The probabilities of all possible events sum to 1,

$$\sum_i p(h_i) = 1$$

- 3) The probability that any one of the K events will happen is the sum of their probabilities,

$$p(h_1 \cup h_2 \cup \dots \cup h_k) = \sum_{i=1}^k p(h_i)$$

where  $h_i$ 's are the simple events that are mutually exclusive and exhaustive. That is, only one of the  $h_i$ 's could happen or be true at any one time, and together they represent all possible events. Another important aspect of probability theory is that the probability of an event (or a hypothesis) and its complement should sum to 1. that is,

$$p(h) + p(\bar{h}) = 1$$

#### 4.1.1 Types of Probability

There are two interpretations of probability:

- 1) The *frequentist* or *objective* probability
- 2) The *Bayesian* or *subjective* probability

In the *frequentist* or *objective* interpretation of probability,  $p(h)$  represents the actual or theoretical frequency of hypothesis being true. This is commonly ascribed to Venn [65]. The underlying assumption in the frequentist approach is that  $h$  is an event or hypothesis that occurs repeatedly, and we can measure its frequency by observing the number of times it has been true ( $n$ ) in  $N$  observations  $f = n/N$  represents the estimate of the true probability of  $h$ . The larger the value of  $N$ , the closer  $f$  would be to the true probability of  $h$ . In other words, in the frequentist approach to probability, we assume a

true probability exists out there, and we can get an estimate of it by using the frequency approach.

In the *subjective* or *Bayesian* approach to probability,  $p(h)$  is representing the degree of belief that  $h$  may be true. This is the case for many real life situations.

De Finetti, et al [66] believe that there is no need to assume that the probability of some events has a uniquely determinable value. His detailed work has formed the basis of the subjective or 'personalistic' approach to probability. It represents the extent to which a coherent person believes a statement is true, based on the information available to him at the time. To be coherent, the DM is required to make his assessments consistent with each other, such that fundamental contradictions (intransitivity) do not exist among them.

While the basic formulas of probability are the same in both objective and subjective school of thought, the collection of data, the application of the formulas, and the interpretation of the results differ in the two approaches. In the following we will discuss some of the prominent methods in subjective probability assessment.

#### **4.1.2 Bayesian Approach**

The subjective or Bayesian approach to probability which goes back to Thomas Bayes's work in 1763, treats  $p(h)$  as the degree of belief that hypothesis  $h$  may be true. This belief is expressed by the decision maker, or the expert. In the subjective approach, the expert does not assume that a true value for  $p(h)$  necessarily exists. Rather, he/she revises his/her belief to represent a better understanding of the situation

When the expert has uncertainty about an event or when a system has uncertainty, it is common to refer to the antecedent as evidence and the consequent as hypothesis. It tells us that when a rule contains uncertainty, consequent is in the form of a hypothesis,

which receives its support from the fact or evidence in its conditions. Baye's theorem [67, 68] provides a method of updating the probability of a hypothesis on the basis of an evidence.

The probability of concluding the hypothesis ( $h_i$ ), given the evidence ( $e$ ), has the form of a conditional probability:

which is defined by:

$$p(h_i|e) = \frac{p(h_i \cap e)}{p(e)} \quad (4.1)$$

and

$$p(e|h_i) = \frac{p(h_i \cap e)}{p(h_i)} \quad (4.2)$$

Therefore,

$$p(h_i \cap e) = p(h_i|e)p(e) = P(e|h_i)p(h_i) \quad (4.3)$$

where  $p(h_i \cap e)$  is the probability that  $h$ , and  $e$  occurs together.

therefore, the Bayes formula will be:

$$p(h|e) = \frac{p(e|h)p(h)}{p(e)} \quad (4.4)$$

where:

$p(h)$  = prior probability of  $h$

$p(e|h)$  = probability of  $e$  given  $h$

$p(e)$  = probability of the event  $e$

$p(h|e)$  = posterior probability

If a problem has a number of rules that have  $e$  in their antecedent, with different consequents,  $h_1, \dots, h_n$ , then the probability of the event  $e$  is the sum of intersections of  $e$  with all possible hypotheses, as:

$$\begin{aligned}
 p(e) &= \sum_{j=1}^n p(e \cap h_j) \\
 &= \sum p(e|h_j)p(h_j)
 \end{aligned}
 \tag{4.5}$$

where the denominator of (4.4) could be replaced by (4.5)

This gives a good method to revise probabilities when more evidences are available, but it becomes more complex when there are more than one  $e$  in the rule. In that case, we have the following expression:

$$p(h_i|e_1, \dots, e_k) = \frac{p[(e_1, \dots, e_k)|h_i] p(h_i)}{\sum_{j=1}^n p(h_j) p[(e_1, \dots, e_k)|h_j]}
 \tag{4.6}$$

It is clear that the formula (4.6) requires a number of complex conditional probabilities and also is dependent on the knowledge of the expert, since he should know the interdependencies of the pieces of evidence and the hypotheses, as well as their probabilities.

#### 4.1.2.1 Problems with Bayes' Method

Although the procedure has a strong foundation in probability theory, but it is clear that the formula requires a number of complex conditional probabilities which makes it even more complex in most real-world applications. To reduce the complexity, two assumptions are made in the process. First, it is assumed that the evidences are conditionally independent, which is not satisfied in many applications. Second, the hypotheses ( $h_j$ ) should be mutually exclusive and exhaustive, which means that having certain belief to a hypothesis implies commitment of the remaining belief to its negation,

$[p(h|\bar{e}) = p(h|e)-1]$ . This is a contradiction of the opinion that evidence in favor of a hypothesis should not be used as evidence against the same hypothesis. This was the experience of the developers of MYCIN Shortliffe [69] who encountered the physician experts' objection that lack of a positive test result does not impact the hypothesis. The relatively new methods by Pearl, Dempster and Shafer deal with these problems.

Another problem with Bayes method is that probability approach doesn't distinguish between uncertainty inherent within the knowledge, and the uncertainty due to errors and lack of complete knowledge.

#### 4.1.3 Pearl's Bayesian-Based Method

Since the real world problems have many rules, and require too many probabilities by the Bayesian approach, Pearl [70] introduced the idea that the hypothesis of interest can be organized in a tree hierarchy. In this hierarchy, every node in the tree represents a subset  $S$  of individual hypotheses ( $h_i$ 's). The evidence  $e$  impacts the hypothesis within the set  $S$ . The expert should only assess the impact of the evidence on the most directly affected set  $S$ , not on its individual hypotheses. Therefore, the expert must provide the prior probabilities  $p(h_i)$  and the likelihood ratios:

$$\lambda_s = \frac{p(e|s)}{p(e|\bar{s})}$$

where  $p(e|s)$  means the probability of the evidence  $e$  being true given that the hypotheses in set  $S$  are true, and  $p(e|\bar{s})$  is the probability that the same evidence is true given that the hypotheses outside set  $S$  are true.  $S$  and  $\bar{S}$  are mutually exclusive and within each set only one hypothesis could become true.

The likelihood ratio ( $\lambda_s$ ) calculates the odds of observing the evidence  $e$  when one of the hypotheses in  $S$  is true. When the odds are favorable to  $S$ ,  $\lambda_s$  is greater than 1, otherwise it is less than 1. Each hypothesis in  $S$  gets a weight  $w_i = \lambda_s$ ,

If the user inputs the presence of evidence  $e$ , then its impact on the hypotheses of  $S$  and the posterior probabilities are:

$$p(h_i|e) = \alpha_s w_i p(h_i) \quad i = 1, 2, \dots, n$$

where  $n$  is the number of hypotheses, and  $\alpha_s$  is the normalizing factor which has the form

$$\alpha_s = \left[ \sum_{j=1}^n w_j (h_j) \right]^{-1}$$

Pearl's Bayesian-based method simplifies to a great extent the computational requirements of the problem. But it demands a careful analysis of the structure of the hypotheses by the expert.

#### 4.1.4 Dempster-Shafer (D-S) Theory of Evidence

Dempster [71] in 1967 and later Shafer [72] in 1976 developed a theory in which the probabilities are expressed in the form of an interval and the probabilities  $p(h)$  and  $p(\bar{h})$  do not have to add to 1, as requires by an axiom in probability theory.

This method specifies a *frame of discernment*  $\theta$  which is the collection of mutually exclusive and exhaustive set of hypotheses. Unlike in probability theory, this approach supports hypotheses which are any subset of  $\theta$ . The maximum number of possible hypothesis is  $2^\theta$ , including the null hypotheses set  $\phi$ . This is called the *power set* ( $\Theta$ ).

The impact of an evidence on a hypothesis is affected by a function called *basic probability assignment* [bpa]  $m$ , which maps the power set ( $\Theta$ ) to the interval  $[0,1]$  such that

$$m(\phi) = 0$$

$$\sum_{h_i \in \Theta} m(h_i) = 1$$

In other words, the  $m$  value for the empty set is zero, and its sum should be equal to 1 for all possible subsets in  $\Theta$ . This is termed the measure of belief  $m(h)$ , which is the belief committed exclusively to  $h$  only and does not include the beliefs committed to any subset of  $h$ . Therefore, the remaining belief does not go to the negation of the hypothesis supported by the evidence.

Belief function  $Bel(h)$  is a measure of the total amount of belief in  $h$ :

$$Bel(h) = \sum_{h \subset H} m(h)$$

The belief interval is defined as  $1 - Bel(h) - Bel(\bar{h})$ , which represents the balance of belief that is neither assigned to  $h$  nor to its negation. This can be considered as the level of uncertainty present with respect to the hypothesis and regardless of the evidence.

The D-S theory addresses a number of concerns regarding the use of probability theory:

- The number of probability values ( $m$ ) is limited, because the hypotheses are grouped into focal elements or relevant sets. This is similar to the sets in the pearl approach.

- The probability value of an element when the evidence is lacking is not necessarily one minus that when the evidence is present. The likelihood ratio in Pearl's method has a similar property.
- The D-S theory is not based on probability theory. The hypothesis that form the problem are mutually exclusive.
- Neither the D-S theory nor the Pearl's method distinguished between the ambiguity and inconsistency inherent in the domain as opposed to the errors caused by the incomplete or erroneous knowledge of the expert or the user.

#### 4.1.5 Certainty Factors (CF) Method

Shortliffe and Buchanan [73] developed the concept of certainty factor in 1975 to overcome the short-comings of Bayesian method. They observed the necessity of updating beliefs with partial evidence and criticized the extensive requirements of the frequentist approach to probability theory and the requirement that the hypothesis could be revised even when evidence is lacking, in the form of:

$$p(h|\bar{e}) = 1 - p(h|e)$$

In this approach, the system has measures for belief and disbelief where a new piece of evidence changes this measure, according to the following formula:

$$MB(h|e) = \begin{cases} 1 & \text{if } p(h) = 1 \\ \frac{\max[p(h|e), p(h)] - p(h)}{\max[1, 0] - p(h)} & \text{otherwise} \end{cases} \quad (4.7)$$

$$MD(h|e) = \begin{cases} 1 & \text{if } p(h) = 0 \\ \frac{\min[p(h|e), p(h)] - p(h)}{\min[1, 0] - p(h)} & \text{otherwise} \end{cases} \quad (4.8)$$

and

$$CF(h|e) = MB(h|e) - MD(h|e) \quad (4.9)$$

where:  $MB(h|e)$  is the measure of increased belief in hypothesis  $h$  based on the evidence  $e$

$MD(h|e)$  is the measure of increased disbelief in hypothesis  $h$  based on the evidence  $e$

Based on the contention that a single piece of evidence should not both confirm as well as disconfirm a hypothesis,  $MB$  and  $MD$  are defined in terms of prior probabilities and likelihoods such that,

if  $MB(h|e) > 0$ , then  $MD(h|e) = 0$

and if  $MD(h|e) > 0$ , then  $MB(h|e) = 0$

the certainty factor ranges from -1 to 1. A negative certainty factor means that evidence does not support the hypothesis.

In this approach,  $CF(h|e) \neq 1 - CF(\bar{h}|e)$ , which means that if  $e$  confirms  $h$  with a strength of  $CF$ , it does not confirm  $\bar{h}$  with a strength of  $1 - CF$ . In fact,  $CF(h|e) + CF(\bar{h}|e) = 0$ , which says that evidence that confirms a hypothesis, disconfirms the negation of the hypothesis to an equal extent.

For combining rules and evidences, Shortliffe and Buchanan have suggested some guidelines which are mostly ad hoc and use the properties like minimum and maximum value similar to those used in fuzzy sets.

#### 4.1.5.1 Problems with Certainty Factors

The Certainty Factors (CF) are successfully used in MYCIN program. However, there are some problems with this method. First, it is an ad hoc approach, which may result in unexpeted or unacceptable conclusions.

For example, assume that we have the following belief values from the expert:

$$\begin{array}{ll} p(h1) = .90 & p(h1|e) = .95 \\ p(h2) = .10 & p(h2|e) = .70 \end{array}$$

Using the formulas (4.7) and (4.8) for the measure of the belief and disbelief, the certainty factors for the first and second hypotheses are:

$$\begin{array}{l} CF(h1|e) = .50 \\ CF(h2|e) = .86 \end{array}$$

which is an unexpected result, because the certainty factor of  $h_1$  is far below  $h_2$ , while the probabilities of  $h_1$  are far higher than those of  $h_2$  before and after observing the evidence  $e$ . Second, neither CF nor MB or MD is a measure of probability. MB's and MD's obey some axioms of probability but are not derived from a population sample of any kind and therefore, cannot be given a statistical interpretation. They merely allow the system to grade the hypothesis according to their strength of support.

#### 4.1.5.2 Comparison with AHP Method

The knowledge in hierarchies, relations and qualitative judgments in AHP have inherent associations with expertise in rule-based Expert Systems (ES) such as MYCIN. Following are some of the similarities and differences between these two methods.

- 1) AHP is applicable to high level descriptions involving a great number of uncertain factors, while MYCIN is good for detailed and casual descriptions.
- 2) Elements in the AHP involve relatively comprehensive and complex concepts, so the casual relations drastically reduced to such relations as dominate, hierarchical level of concepts, etc. As a result, if the answer to the problem can also be described by similar complex concepts, then we might only need to consider all general concepts with uncertainty and their roughly hierarchical relations. In contrast, elements in MYCIN generally involve smaller and simpler concepts for micro descriptions, among which casual relations play an important role. Nevertheless, dominance contribution, and importance in the AHP are a natural extension of intensity of the evidence supporting the successor in IF-THEN rules in MYCIN. But relations of AND, OR are so vague that they are included in dominate. Therefore, IF-THEN rules can be applied to represent relations among elements in high level decision problems as long as there are some measurements.
- (3) Consistency Index (C.I) in the AHP makes the inference of IF-THENS consistent to formal logic so as to reduce conflicts in MYCIN. Manual or run-time test of conflicts may cause some problems. So, the AHP seems to be advantageous to MYCIN in acquiring high level knowledge, Zhu[74].

Table 4.1 shows the summary of characteristics of the two methods.

**Table 4.1 Comparison of Certainty Factors (CF) with AHP method**

Characteristics	AHP	CF
elements	goal, actors, criteria, scenario	fact, reason, consequence, conclusion, evidence, hypothesis
mono-relation	dominance, feedback, contribution, importance	IF-THEN rules
multi-relation	hierarchy, positive reciprocal matrices	AND, OR, CF model of approximate reasoning
problem	finding priority, weights	diagnose, inference from evidenced and CF
information needed	structuring hierarchy, making judgments by pairwise comparison	giving all facts, rules and CFs
measurements	relative and absolute ratio scale	casual relation, probability, experience
consistency	CR., QI, and confidence interval	-
cost of solving problems	low	high

## 4.2 Possibility Theory

Possibility is quite different from probability, in that the probability shows the frequency of a hypothesis or the strength of belief in its occurrence. Possibility, on the other hand, is the extent of feasibility of the hypothesis, even though it has not yet occurred in reality. Possibility values vary between 0 and 1.

The necessity of a hypothesis is defined as one minus the possibility of its negation. For example, assume that the possibility of a good credit rating is 0.95, and the possibility of not a good credit rating is 0.3. The necessity of a good credit rating is  $1 - 0.3 = 0.7$ . A redundancy has the possibility of 1 and the necessity of 1. A contradiction has the possibility of 0, and the necessity of 0.

The possibility measure does not require that the possibility values for a given universe add up to 1. The only requirement is that the possibility value must be between 0 and 1.

Possibility theory covers a vast area and has counterparts in many methods of approximate reasoning. For example, possibility has an interpretation similar to the theory of evidence. We can have the evidential interval for possibility, with the necessity measure at its lower end and possibility at its upper limit. To make this point simple, assume that we denote necessity as: "Bel" and possibility as: "Pls", then the evidential interval is  $[Bel(h), Pls(h)]$  for the hypothesis  $h$ . Now, assume we have  $h_1$  and  $h_2$  with the intervals  $[Bel(h_1), Pls(h_1)]$  and  $[Bel(h_2), Pls(h_2)]$ . Then we have

$$Bel(h_1 \cap h_2) = \min [Bel(h_1), Bel(h_2)]$$

$$Pls(h_1 \cup h_2) = \max [Pls(h_1), Pls(h_2)]$$

which provides a method for combining belief values. (For further details between the possibility measure and other methods, see Klir and Folger[75]).

### 4.3 Fuzzy Set Theory

Due to the recent attention to fuzzy set theory and logic in practical and decision making problems, we will spend a little more on the concepts and properties of this theory and then its role in practice and decision making will be discussed. At the end we will compare it with AHP method.

In 1965 Zadeh [76] introduced the fuzzy set theory as a mathematical theory of Vagueness. Fuzzy theory forms the link between the precise nature of mathematical models and the imprecise nature of reality. The theory was based on the notion that key elements in human thinking and human decision making are based not on numbers but fuzzy sets. Zadeh has argued that the study of systems involving humans are often unsuccessful, primarily, because the methodology used needs very high level of precision in measuring variables and in describing how they are related. He emphasized that these levels of precision are often neither obtained nor necessary for effective decision making. To overcome the needs for precision, he stated the need for a fuzzy approach in the area of decision making.

However, some of the studies on the applications of fuzzy set theory appear to have misinterpreted Zadeh's interpretation of the mathematical aspects of this theory. According to Watson [77] imprecision and uncertainty are distinct qualities, which ought to be modeled in different ways, the former using fuzzy set theory and the latter using probability theory. Thus, fuzzy set theory should not be viewed as an alternative to probability theory, but a parallel calculus for the representation of a distinct, though related, phenomenon.

### 4.3.1 Basic Definitions

In mathematics, sets are used to formally represent a concept. For instance the "integer numbers which are greater than 4 and smaller than 10" may be represented by the set  $A = \{5,6,7,8,9\}$  or by its characteristic function  $\phi_A = X \rightarrow \{0,1\}$ . Here  $X$  is the universe of discourse (the set of integer numbers),  $\phi_A(x) = 0$  means that  $x$  does not belong to set  $A$ , while  $\phi_A(x) = 1$  means that  $x$  belongs to it.

Some difficulty arises when we want to use set theory to characterize vague concepts, say "number more or less equal to 7" which do not present a clear-cut (crisp) differentiation between the elements belonging and not belonging to the set.

Zadeh suggested the replacement of the characteristic function by the so-called "membership function,  $\mu_A$ " which associates with each element of the universe  $X$  its grade of membership in a fuzzy set  $A$ , belonging to the interval  $[0,1]$ . Thus,  $\mu_A(x) = 0$  means that  $x$  does not belong to  $A$ ,  $\mu_A(x) = 1$  means that  $x$  belongs to  $A$ , while  $0 < \mu_A(x) < 1$  means that  $x$  partially belongs to  $A$ . The following definition can now be given:

A fuzzy set  $A$  in a universe of discourse  $X = \{x\}$ ,  $A \subseteq X$  is defined as the set of pairs.

$$A = \{(\mu_A(x), x)\}, \quad x \in X \quad (4.10)$$

where  $\mu_A$  is the membership function of  $A$  and  $\mu_A(x)$  is called the grade of membership of  $x \in X$  in  $A$ .

### 4.3.2 Determination of Fuzzy Memberships

The most crucial step in building a fuzzy evaluation model is to determine the fuzzy memberships. The determination of fuzzy memberships for subjective factors differs from that for objective factors, as explained below:

#### 4.3.2.1 Subjective Factors

To determine fuzzy memberships for subjective factors, a statistical method proposed by Li, et al. [78] can be used. There are four elements involved in a fuzzy statistical model: (1) a set  $U$ ; (2) a certain number of test  $u_k$  in  $U$ ; (3) a moveable (definable) subset  $A$  of  $U$ ; and (4) the condition  $S$ , which is associated with both objective and subjective information to determine the fuzzy grades. For a certain situation  $S$ , if  $u_k$  test are conducted, then the fuzzy membership is calculated by a simple ratio:

$$\mu = \frac{\text{number of } u_k \in A}{\text{total number of } u_k} \quad (4.11)$$

where  $\mu$  represent the fuzzy membership of the  $u_k$  tests contributing to subset A.

#### 4.3.2.2 Objective Factors

To determine fuzzy membership for objective factors, one must select a type of distribution appropriate for the application. A few standard distributions, which are suitable for objective fuzzy membership calculations include, the normal distribution, the half-decrease distribution, the triangular distribution, and the trapezoidal distribution, as

discussed by Kaufmann [79]. The user should, on the basis of experience, select the type of distribution that best represents the actual situation, and then use the following expressions to assign the fuzzy membership values.

The normal distribution (Figure 4.1), is the best fit for many applications and its fuzzy membership function  $\mu(x)$  defined by the mean  $a$  and the range  $b$  is:

$$\mu(x) = \exp \left[ - \left( \frac{4(x-a)}{b} \right)^2 \right] \quad (4.12)$$

The half-decrease distribution (Figure 4.2), is appropriate for representing information such as cost, since lower cost is always preferred and low values have high memberships. Its membership function  $\mu(x)$  is:

$$\mu(x) = \begin{cases} 1 & (0 \leq x < a) \\ \frac{1}{2} - \frac{1}{2} \left[ \sin \left[ \left( x - \frac{a+b}{2} \right) \left( \frac{2}{\pi} \right) \right] \right] & (a \leq x < b) \\ 0 & (b \leq x) \end{cases} \quad (4.13)$$

The triangular distribution (Figure 4.3), is useful for situations where there is a single optimal value ( $a_2$ ) and there is not a normal distribution. Retail price may be represented by this distribution, since consumers may have negative reactions to prices both above and below some optimum price. It can be defined by a triplet ( $a_1 a_2 a_3$ ), and its  $\mu(x)$  is:

$$\mu(x) = \begin{cases} 0 & (x < a_1) \\ \frac{x - a_1}{a_2 - a_1} & (a_1 \leq x < a_2) \\ \frac{a_3 - x}{a_3 - a_2} & (a_2 \leq x < a_3) \\ 0 & (a_3 \leq x) \end{cases} \quad (4.14)$$

A trapezoidal distribution (Figure 4.4) is similar to a triangular distribution except that the trapezoidal function allows for a range of optimal values (between  $a_2$  and  $a_3$ ) Its membership function  $\mu(x)$  can be defined as:

$$\mu(x) = \begin{cases} 0 & (x < a_1) \\ \frac{x - a_1}{a_2 - a_1} & (a_1 \leq x < a_2) \\ 1 & (a_2 \leq x < a_3) \\ \frac{a_4 - x}{a_4 - a_3} & (a_3 \leq x < a_4) \\ 0 & (a_4 \leq x) \end{cases} \quad (4.15)$$

Selection of the best distribution is largely based on an understanding of the problem and on experience, but an exemplification method by Turksen [80] which requires the provision of several discrete points on the reference axis, can help to select the most suitable function.

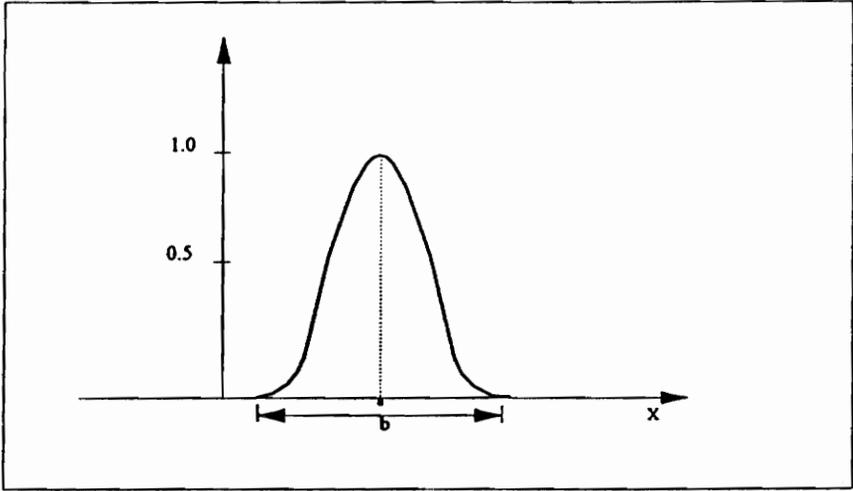


Figure 4.1 Normal Fuzzy Membership Distribution

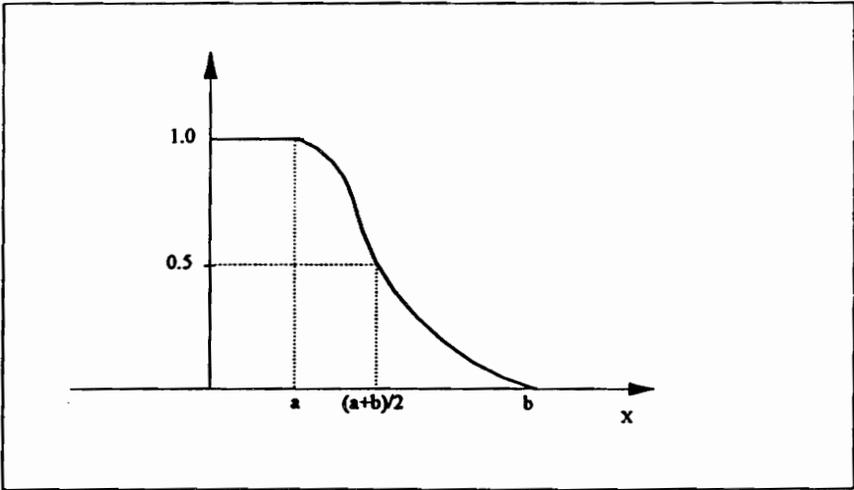


Figure 4.2 Half-Decrease Fuzzy Membership Distribution

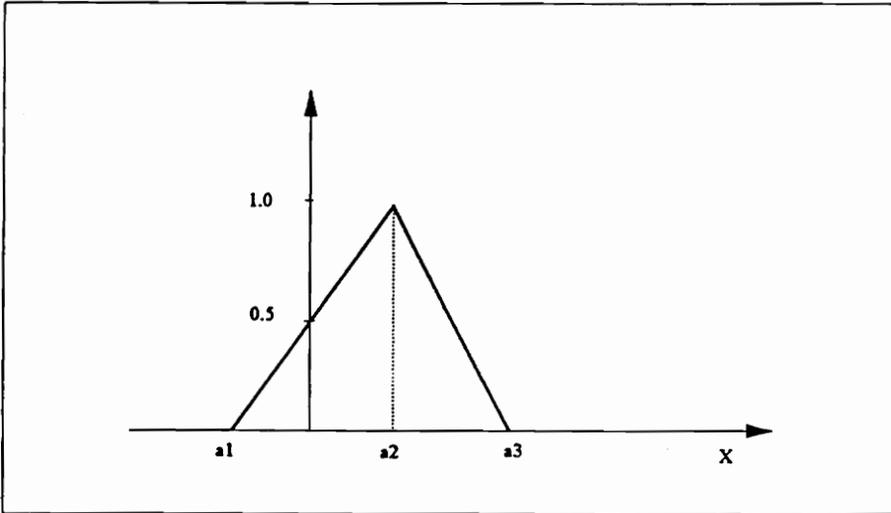


Figure 4.3 Triangular Fuzzy Membership Distribution

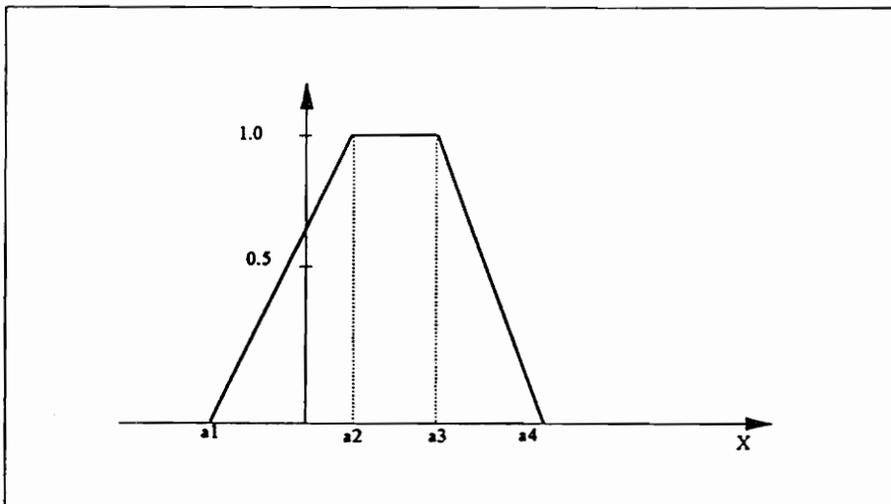


Figure 4.4 Trapezoidal Fuzzy Membership Distribution

### 4.3.3 Some Basic Operations of Fuzzy Sets

The axioms of fuzzy set theory are developed analogous to probability theory. Some of the rules of this theory developed by Bellman and Zadeh [81] are:

1. Intersection: The degree to which  $x$  belongs both to  $A$  and to  $B$  is equal to the smaller of the individual degrees of membership:

$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x)) \quad \forall x \in E$$

2. Union: The degree to which  $x$  belongs either to  $A$  or to  $B$  is equal to the larger of the individual degrees of membership:

$$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x)) \quad \forall x \in E$$

3. Complement: The degree to which  $x$  belongs to (not  $A$ ) is one minus the degree to which  $x$  belongs to  $A$ :

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x) \quad \forall x \in E$$

4. Algebraic Sum: The algebraic sum of  $A$  and  $B$  is denoted by  $A \oplus B$ , and is defined by:

$$\mu_{A \oplus B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x) \quad \forall x \in E$$

### **4.3.4 Applications of Fuzzy Logic**

Since the expansion of fuzzy set theory by Zadeh, the world has gained enough mathematical theory to work with the concept of fuzzy sets in theory as well as in practice.

#### **4.3.4.1 Industry and Commercial**

The Industrial process control units, Climate control for buildings and houses, Focusing mechanism in cameras and camcorders, Automobile automatic break systems, transmission and Cruise control among hundreds of other industrial and commercial applications developed in Japan, illustrate several important points. First, in some cases, fuzzy systems can be added to existing control systems with substantial savings on very modest computers, second, conflicting rules can be rationally resolved, third, saving in energy usage, and fourth, very complex control functions can be generated using a few simple rules. For instance, the camcorder focusing system uses only seven inputs as variables and produces one output, the focusing of the lens, or the climate control for a buildings uses 25 rules for cooling and 25 rules for heating only, Schwartz [82].

#### **4.3.4.2 In Decision Making**

There has also been much research work in applying fuzzy reasoning to decision making. We will discuss some of the prominent works in this area.

Forsyth [83] used Hacker's guide to fuzzy logic to discuss reasoning when the problem data or the rules of inference are not 100% reliable.

In order to combine non-integer truth-values, Hacker's guide defines the AND, OR, and NOT operators in the following manner when used with fuzzy decision-making rules:

$X1 \text{ AND } X2 = \text{Min. } (X1, X2)$  (i.e. choose the smaller value)

$X1 \text{ OR } X2 = \text{Max. } (X1, X2)$  (i.e. choose the larger value)

$\text{NOT } X1 = 1 - X1$  (i.e. the inverse)

Thus, pieces of evidence can be combined in a rigorous and consistent manner. This was how fuzzy logic (inexact reasoning) was used successfully in decision support systems like REVEAL developed by Sprague [84], and MYCIN by Shortliffe [69]. Forsyth however, noted a weak point in fuzzy logic; this being its mapping or membership function. There was no reliable way to determine whenever the mapping function should be a straight line or a curve. Sprague solved this problem by developing REVEAL in such a way that it allows for easy modification of various mapping functions and thus enabling the user to find out experimentally the precise shape of the required curve.

#### **4.3.5 Linguistic Approach**

Yager [85] has discussed the usefulness of fuzzy set to decision making using the linguistic approach. He stressed that since most communication between analyst and the decision maker takes place in the linguistic media, the linguistic approach has the advantage of not restricting the decision maker in expressing his information and allowing him free play of his institution. The decision maker could then express, to the best of his ability, in a natural and technical language, the types of tradeoffs and relationships he feels exists between the objectives. These tradeoffs are purely the decision maker's own

subjective information. The analyst would then try using the linguistic structure available in fuzzy sets to model the relationships expressed linguistically by the decision maker. This would involve using the connections, "AND", "OR", "NOT" and the logical, IF\_THEN, operation. Although it is highly unlikely that the analyst would be able to exactly reproduce the decision makers linguistic model, but he could at least obtain a number of models which would approximate the information relayed to him by the decision-maker. The decision maker then decides the degree to which each model satisfies his ideal model. However, one problem exists when using linguistic fuzzy sets, if a fuzzy set has only numerical grades of membership (type I) the best solution is the one with the highest grade of membership, (the biggest number). The same idea holds for fuzzy sets of type II, except that, in general, the problem of finding the biggest fuzzy subset of  $[0,1]$  is more difficult.

Laarhaven and Pedrycz [86] presented a method involved displaying ratios which express the relative significance of each pair of factors in a matrix from which suitable weights can be extracted. Since the ratios are essentially fuzzy, they express the opinion of a decision-maker on the importance of a pair of factors. The decision makers are then asked to express their opinions in fuzzy numbers with triangular membership functions. This method was applied at two distinct levels: first to find fuzzy weights for the decision criteria, and second to find fuzzy weights for the alternatives under each of the decision criteria. By using a suitable combination of the results, they obtained fuzzy scores for their alternatives, as well as their sensitivities. With those fuzzy scores, the decision makers should be able to make a choice for the preferred alternatives.

### 4.3.6 Comparison with AHP Method

AHP technique may be thought of as a method to give meaning to problems involving fuzziness. The various levels of the hierarchy provide priority values with regard to multiple purposes or objectives. The pairwise comparison of all elements in the set is a better estimation of membership degree which is called priority vector  $w$ .

The good thing about AHP method is that we can calculate the consistency of the judgment scales, and by assigning a distribution to the weights, confidence interval of the priorities could be found which tells us the variations in the degree of membership. This will enable us to revise or retire our judgments if there is an inconsistency.

Some of the other advantages and disadvantages of the two methods are listed in Table 4.2:

**Table 4.2 Comparison of Fuzzy Set theory with AHP method**

	Fuzzy Set	AHP
Advantages	<ul style="list-style-type: none"> <li>- Can handle problems with many number of alternatives, fuzzy goals, and constraints.</li> <li>- Will account for non-linearity through the membership function.</li> <li>- Expands the usefulness and applicability of some proven operations research tools.</li> <li>- May be used to select an alternative from an optimal set of alternatives.</li> <li>- It is applied to a more quantitative type of problems with more alternatives.</li> </ul>	<ul style="list-style-type: none"> <li>- Capable of quantifying intangible attributes.</li> <li>- Obtains membership degree, and subjective probability estimates easier when dealing with uncertainty.</li> <li>- Allows for a decision structure with multiple levels.</li> <li>- Contains internal consistency check.</li> <li>- Allows DM to communicate her/his decision process more clearly and concisely.</li> <li>- Easy to understand and use.</li> </ul>
Disadvantages	<ul style="list-style-type: none"> <li>- Obtaining membership functions is difficult.</li> <li>- Can not handle problems with large number of intangible attributes.</li> </ul>	<ul style="list-style-type: none"> <li>- Constraints on attributes cannot be represented.</li> <li>- Can not handle large numbers of alternatives or attributes easily.</li> </ul>

## 4.4 Delphi Technique

This method which was invented by Helmer [87] is a technique for eliciting expert judgments about the events that are not quantifiable.

There are two types of Delphi technique, the conventional and modified Delphi . The conventional Delphi processes are characterized by anonymity and controlled feedback. The anonymity of panel members is preserved by physical separation and by the use of carefully constructed questionnaires or other communication procedures such as on-line computer communication. Therefore, the members of a conventional Delphi panel do not debate their responses to questions posed by the planners. The purpose of maintaining anonymity and physical separation is to avoid several potential pitfalls of group decision making. The most frequent one is "Ash Effect" a phenomenon of group behavior that tends to encourage the domination of the group by one or more persons who can exert more influence due to their position within the organizational hierarchy.

But through a controlled feedback the aggregate group response to a question is communicated to each panel member. The purpose of the feedback is to allow respondents to see how their judgments are compared with those of others on the panel without knowing who provided individual responses. The feedback is controlled because participants are offered only limited information on the responses of their colleagues. That is, they are shown the average or the mean response of the other panel forecasts by removing outliers from the aggregate response. Controlled feedback following two or more rounds of questioning provides a foundation for group learning and an opportunity for panel members to modify their judgments in light of the response of others.

#### **4.4.1 Modified Delphi**

Some of the shortcomings of traditional Delphi methods have been addressed by a modified version called "Policy Delphi" which seeks to maximize conflict by eliciting the judgments of informed advocates instead of the judgments of impartial experts. Thus, an attempt is made to include the interests and values of stakeholders or those who will be affected by the outcome of a plan. The problem of artificial consensus is addressed by a modified version of controlled feedback in which measures of group response accentuate conflict or polarization instead of central tendency. Moreover, in policy Delphi participants have the opportunity to exchange ideas through limited interaction and debate. Policy Delphi improves upon the procedural weaknesses of the traditional method, yet it remains a rather awkward and time consuming procedure.

#### **4.4.2 Problems with Delphi Method**

The Delphi process represents one of the first formalized methods for obtaining and aggregating group judgments. In the early 1960s it stood apart as a unique and creative method of analyzing problems for which no good theory existed and for which there was insufficient data for the formulation of a new theory. However, experience with the technique has revealed several problems. The designers of the method wished to maximize divergence by including a multi-disciplinary dimension. Yet, by including anonymity and controlled feedback and statistical group response, the designers unwittingly incorporated an element of "artificial consensus". Thus, what appears on the surface to be an interdisciplinary approach to forecasting and planning may in fact, degenerate into an arbitrary consensus that fails to capture the diversity of knowledge, values, and beliefs held by all participants. This arrangement tends to exclude information

and new insights that could be generated and tested in the course of reasoned debate and argument.

Traditional Delphi techniques are incomplete because they fail to incorporate elements of dynamic interaction among participants on the panel. Instead, alternative appraisals of the problem are formulated individually and secretly. Following each round of a Delphi process, new questionnaires must be devised on the basis of information obtained from respondents in the previous round and be tested for reliability and validity. This process places significant demands on the time and resources of both the Delphi panel members and the group moderators. Thus, to a large extent, the conclusions reached by a Delphi panel are shaped or predetermined by the questions that are asked.

#### **4.4.3 Basic Differences with AHP Method**

1) Anonymous versus group discussion:

In Delphi each member of the group responds anonymously to a previously prepared questionnaire to avoid disproportionate influence of strong personalities. In hierarchies the criteria and judgments are established mostly by an open group process.

2) Feedback versus dynamic discussion

In Delphi there must be a review of the questionnaire results, and adjustments are requested again on an anonymous basis. In hierarchies dynamic discussion is used while constructing the hierarchy and providing judgments by mutual agreement and revision of views. People attempt to present their arguments openly.

### 3) Questionnaire versus hierarchy structure as a basis for judgments

In Delphi, the design of the questionnaire implies the choice of the variables involved by the person who creates the questionnaire. In hierarchies the group decides on the variables which have any effect on the judgment to be made. Initially all variables suggested are accepted. Later in the procedure some might be ignored due to low priority assigned to them by the group.

### 4) Statistical and quantitative analysis versus qualitative analysis

The Delphi method requires numerical responses which are to be analyzed statistically as a basis for the next round. In hierarchies the judgments involve absolute numbers from 1 to 9 reflecting qualitative judgment on pairwise comparison and used as a part of a rigorous derivation of an estimate for an underlying ratio scale. Consistency is an important criterion as a necessary condition to valid scaling of reality.

In both cases the process of analyzing the problem improves the quality of judgments, but the hierarchy method breaks down the judgment into its elementary components, and therefore better fits the human cognitive style. Another important issue here is that the group determines the important set of variables, and therefore has better confidence in the relevance of their judgments. This procedure is helpful in diminishing disagreements in an open dynamic fashion. As a short and simple procedure with highly effective results, its many practitioners have recommended its use in planning and in making forecasts as a reflection of the beliefs of the participants, Saaty [44].

## Concluding Remarks

The theory of probability has gone through subtle criticisms in its interpretation and application. The frequentist theory where the probabilities are based on the observed frequencies has restricted its application. On the other hand, the subjective probability, which is a measure of personal belief, has broadened its application in areas like decision analysis and operations research. The Bayesian technique of revising the probabilities and obtaining posterior probabilities from the conditional probability is perfectly suited to the framework of these applications.

The requirements of many probability assessments in real world cases and the simplifying assumptions are: 1) conditionally independent evidences, and 2) mutually exclusive and collectively exhaustive set of hypotheses. These have made the theory too strict to be practical and therefore not widely accepted. The simplifying assumptions were mistakenly taken as requirements of such systems. In addition, the intractable quantitative numerical approach of Bayes' technique did not match the informal qualitative and symbolic inference requirements sought by other methods, like artificial intelligence (AI). The common objections to the method of probability is listed in Henrion [88].

Due to these perceived inadequacy or inability of probability, the automated reasoning in AI and fuzzy theory which are not coherent, and uses the probability concepts in a loose fashion, have gained more popularity. This popularity is mainly due to two factors, 1) The simple If -Then type of procedure with a certainty factor proved very convenient in applications, 2) the ability to add or remove rules from the knowledge bases without modifying other rules, made it very flexible. But these advantages are only the result of independency assumptions implicit in these systems, which makes them incapable of handling complex problems with dependency and uncertainty.

As a result, once again the emphasis has been on probability as the only satisfactory representation of uncertainty in complex problems. Methods such as: Inference Net, Causal Trees, Belief Networks, Influence Diagrams, and Hierarchical methods such as AHP are the topics currently discussed in the literature [89].

Although each method has its proponents, no clear consensus exists in the choice of the best method, and no method provides answers to all issues in solving the real problems under uncertainty, but the AHP theory has the advantage of producing subjective probabilities with limited available information under uncertainty.

## **CHAPTER V**

### **TECHNIQUES FOR ELECTRIC UTILITIES TO DEAL WITH UNCERTAINTY**

During the past 15 years, the environment in which utilities operate has changed dramatically. The cost to construct and operate traditional central station power plants have increased dramatically, natural gas and oil prices increased and decreased many times since 1973 oil crises, the load growth has slowed substantially, environmental protection requirements have increased, non-utility sources of supply have emerged, the use of conservation and load management programs as resources has become important, and the concerns of state regulatory commissions have changed.

Because of all these changes uncertainty is a critical element of utility analysis, planning, and decision making. New terms such as risk, diversity, flexibility, resource portfolio, and hedging are frequently discussed among utility planners. But, appropriate methods to define and value these terms in a way understandable to the decision makers and regulators have not fully developed.

In this chapter we will address both analytical and non-analytical methods available to the utilities in coping with uncertainty.

The basic objectives of electric utility system resource planning are to:

- Meet the projected loads with an adequate level of service reliability,
- Minimize the future revenue requirements and the cost of electricity to customers, and,
- Comply with environmental regulations

In the past, the number of resource options available for system expansion was relatively small and it was generally possible to forecast load growth and operating costs quite accurately based on past records. However in today's planning process uncertainty is one of the important element for the electric utilities. A set of decisions that is highly favorable under one set of assumptions may be a poor choice under a different set of conditions. In recent years forecasting of load growth, cost and availability of fuels among other issues have become very difficult due to many uncertainties.

## **5.1 Analytical Methods to Deal with Uncertainty**

To plan for an uncertain future, assumptions must be made concerning the values of key factors affecting a utility's ability to serve its customers and shareholders. These key factors include external influences, such as the cost of fuel and growth in the number of the customers, and internal factors, such as the type and availability of new generating facilities. The value of these factors can not be a single point estimate but rather a range of possible values.

While consideration of a range of possible values for key parameters can reduce the likelihood that a utility will make a catastrophic planning error, the need remains for reliable data and estimates on important variables.

Because of this issue, utilities have been forced to develop or sponsor different planning methodologies and computer programs to support decision making in utility planning. Three categories of this work are described below.

- Simulation/Optimization tools like WASP, EGEAS, PROMOD, RISKMIN, [90] and a variety of other corporate models. Some of these programs simulate a power system or a utility. Others optimize plans, typically by minimizing present worth of revenue requirements, sometimes recognizing technological constraints.
- Probabilistic decision support methods which typically build a decision tree whose nodes represent decisions or uncertainties. An objective function consisting of a weighting of a variety of attributes (revenue requirements, capital costs, loss of load probability, etc.) is defined. The approaches seek a set of decisions which minimizes the objective function.
- Vector optimization methods such as describing function method (SMARTE) simultaneously evaluates a number of objectives or attributes and finds the plan which represent the best trade off among them. This method has been developed as a deterministic tool and uncertainty has never been adequately incorporated into it.

But these methods have some limitations in them as recognized by utilities. The optimization/simulation methods can only optimize in terms of one objective. However, they are very powerful and refined tools for simulating a particular plan given specific external conditions. The probabilistic decision analysis methods require that attributes of different importance and dimensions be combined into a single objective function, which many utility planners are uncomfortable with. And the vector optimization methods have not been developed to include uncertainty.

In an interview with 14 organizations, where ten of them were utilities, about the methods they use to treat uncertainty, Hirst [91] reports that all of them fit into one of the major categories: Scenario Analysis, Sensitivity Analysis, Portfolio Analysis, and Probabilistic Analysis. These methods among some other non-analytical methods are discussed in the following.

### **5.1.1 Scenario Analysis**

Scenario analysis like backward process creates the alternative futures which are internally consistent combinations of key uncertain factors, such as fuel prices, and load growth. Then from a various options identified in the different scenarios, a utility will choose which action to initiate in the near future. This method allows the planner to anticipate a broad range of plausible and internally consistent futures and to understand the nature of the factors that determine which scenario will actually happen.

Southern California Edison [92] for example, developed a base case scenario, assuming the continuation of present trends, and 11 alternative futures. These include economic bust, high fuel cost, and expanded environmentalism, all resulting in lower-than baseline resource requirements; and economic boom, electrification, and generation shutdown, all requiring substantial new resources. The company selected five key elements (existing oil and gas units, transmission network, purchased power, energy management, and new generating resources) and arranged them like building blocks to accommodate any scenario that might develop.

The distinguishing feature of scenario analysis is that alternative visions of the future are created first, and then appropriate combinations of resources are identified that best fit each future. This method allows its users to anticipate a broad range of plausible

and internally consistent futures and to understand the nature of the underlying factors that determine which future will actually occur.

Scenario analysis relies less on computer models and requires more communication and interdepartmental discussions, therefore easier to communicate to the management.

### **5.1.2 Sensitivity Analysis**

In this method a preferred combination of options or plan is first identified. Then, different values are assumed for important factors (e.g., natural gas price and economic growth) and the performance of the original plan is examined against the variations to these factors. This procedure allows the analyst to see which factors trigger the biggest changes in plan performance and which options are most sensitive to changes, unlike the scenario analysis where supply and demand side options that perform well under different conditions are attractive to planners.

The Southern Company [93] practices sensitivity analysis in their planning process. A capacity expansion model, the Generation Mix Planning Package (GMP), was used to create an optimum expansion plan using base-case assumptions about future load growth and other key items. Once the preferred mix of resource options was identified, an alternative mix of resources was developed in recognition of the volatility of oil and gas prices. The alternative mix eliminated all new generators using these fuels. The two resource plans were then compared to each other under four different circumstances: base-case conditions, high oil and gas prices, high load growth, and lower-than-expected availability of generating units (Table 5.1). In each case, the relevant assumptions were input to GMP, and construction & operating cost (expressed as present worth of revenue requirements) were calculated. Additional sensitivity cases were run with GMP assuming

substitution of coal or combined-cycle units for some combustion turbine and different costs of capital. In all cases, the original base-mix plan was shown to have the lowest cost. In those instances where costs are significantly higher under some of the sensitivity cases, utilities must decide if a different mix of resources exposes the utility to less potential damage and provides more flexibility to respond to change.

**Table 5.1 Comparison of optimum (base case) expansion plan and alternative plan from Southern Company's plan**

Scenario	Base mix		Alternative mix	
	PWRR*	% difference from baseline	PWRR	% difference from baseline
Baseline	85,600	-	96,090	-
High Oil/Gas	89,520	4.6	96,090	0.0
High Load Growth	96,170	12.3	103,150	7.3
Low Availabilities	86,830	1.4	96,350	0.3

\* Present Worth of Revenue Requirements for each scenario, expressed in millions of 1996 dollars, for the 20 years study period, [93].

### 5.1.3 Probabilistic Analysis

In this approach, probabilities are assigned to different values of key parameters, either by levels such as high, medium, and low, or by a continuous probability distribution. These probabilities are based on the Judgment of utility experts, or an extrapolations of historical data. Outcomes are then identified that are associated with the different combinations of values for the key factors. This method is similar to sensitivity analysis, in that the effect on important outcomes that results from varying specific parameters can be observed. The main differences from sensitivity analysis are that the probabilities associated with the various outcomes are identified and that the correlations among these uncertainties are explicitly considered.

In utility planning, probabilistic analysis is most commonly used to identify future load growth or the costs associated with different plans under different conditions. As a simple example, consider that system costs are determined primarily by oil prices and interest rates and that a model has been developed that translates any combination of values for these two variables into a dollar figure for system costs. If we assign three different values (low, medium, and high), then we have nine possible combinations, each resulting in a different outcome. This can be shown on a decision tree, with each branch representing a unique combination of events. Since each input was assigned a probability by the analyst, each outcome also has an associated probability, which is the product of individual probabilities of the two events. In practice, decision trees can get very complicated as more variables are added and makes the probability assignment a difficult task.

Different approaches are used to make sense out of the often large set of outcomes developed through probabilistic analysis. A single "expected value" can be calculated which is the weighted average of all the individual outcomes. Cumulative probability distribution is another method where the experts estimate the probabilities of achieving targeted levels of system performance in all major areas.

This method is used by many utilities. For example, New England Electric System [94] uses in-house experts to estimate the probabilities of achieving targeted levels of system performance in all major areas (e.g., alternative-energy development, availability of new generating facilities). These estimates were expressed as cumulative probability curves, so utility decision makers could see how likely they were to meet various levels of power production. This information was then used to develop a realistic plan for meeting customer demand.

The main problem with this approach is the assignment of probability values to different factors in the analysis, specially when there is not enough data and information available to the planners. The proposed method for calculating the probability values and their confidence interval is a valuable tool in these situations.

#### **5.1.4 Portfolio Analysis**

In this method two or more plans are identified, each of which meets different corporate goals. Then a sensitivity and/or probabilistic analysis is applied to them and the performance of each plan is compared to the others. The most robust plans could then be selected.

Northeast Utilities [95] identified several different objectives and selected different resource portfolios matched to each one. Some of the objectives specified were:

- Minimizing electricity cost over the long run;
- Minimizing dependence on oil;
- Emphasizing conservation and load-management programs;
- Life extension and repowering of existing fossil plants;

- Using small generation alternatives, such as renewable resources, cogeneration plants, and small power production facilities.

The resource-mix portfolios developed in response to each objective were then tested against each other using sensitivity analysis.

Potomac Electric Power Company [96] illustrate another application of portfolio analysis. The company described eight alternative plans, each utilizing varying amounts of different resources, such as combustion turbines, steam-cycle generators, pulverized-coal units, and coal-gasification plants. Sensitivity analysis was then performed on each of the plans, different key factors, and examining each plan through multiple runs of production-cost and revenue requirements models. The analysis revealed that the plan that is least expensive under base-case assumptions does not provide the flexibility needed to adapt to uncertain future conditions.

This method as mentioned earlier is used in conjunction with other analytical techniques which requires probability values based on either subjective estimates or cumulative distribution function if there is enough data.

### **5.1.5 Trade-Off Analysis**

This method which has been in practice for more than a decade compares different pairs of plans and identifies a decision set containing plans that do not significantly dominate each other. Simple trade-off analysis can be used to eliminate relative risk and uncertainty. But it becomes complex when more than two trade-off considered.

This method consists of four steps:

- Formulating the problem properly, in terms of options, uncertainties, and attributes,
- Developing a large data base,
- Trade-off analysis, and

- Risk analysis.

For development of data base we must run simulation programs like production cost many times which may not be possible or too costly. High order piece-wise linear interpolation can be used to expand a small data base to a large one, but Dorfner [97] found that linear interpolation did not give sufficiently accurate results for their reliability analysis. This method has been employed by Stons & Webster [98] in developing RISKMIN program.

### **5.1.6 Interval Math Approach**

This method provides a useful tool in determining the effects of uncertainty in the parameters by assuming an upper and lower limits on the uncertainties when there is not enough data for a probability distributions. Schweppe has addressed this approach in his book titled "Uncertain Dynamic Systems" and in his last paper [99] but without any example. Later some applications of the method has emerged in general and in power system planning such as: calculating value of service (reliability) in utilities by Broadwater [100]. Confidence intervals can not be calculated in this approach since there is no probability distributions.

## **5.2 Non-Analytical Methods to Deal with Uncertainty**

Following are some of the non-analytical methods available for the utility planners to deal with uncertainty:

### **5.2.1 Short-Term Planning**

A simple way to make decisions is to ignore the uncertainties associated with the external conditions and the resources themselves. This means that a utility might pursue its base-case plan with the belief that this is the most likely event of the future or a utility might focus on the actions it should take during the next few years rather than worrying about a long term plan. Such a short-term planning may be justified by increasing competition that electric utilities facing today.

### **5.2.2 Delaying Action**

Another way to deal with uncertainty is to delay decisions as long as possible. It might allow enough time for additional information to become available to the utility where it might reduce the uncertainties associated with certain decisions and give the utility more confidence about its preferred resource strategy. For example a utility studying construction of a new coal-fired power plant might wait a couple of years and hoping to learn more about the costs of compliance with possible new federal clean-air requirements.

### **5.2.3 Selling Risks to Others**

In some situations, it might be wise to sell certain risks to other parties who are better able to manage them. One of the attractions of utility auctions for supply and demand resources is that the risks associated with nonperformance are shifted largely to the non-utility supplier. For example, a utility that contracts with an independent power producer (IPP) to purchase the output of the IPP's power plant include in the contract penalties if the IPP plant is brought online late or if the plant's forced outage rate is higher than anticipated.

### **5.2.4 Preparation of Many Alternative Plans**

In this approach a utility by assessing alternative future conditions beforehand, is prepared for certain contingencies which may evolve. So the need for future planning is reduced. Such a strategy is costly and will not work well if the plans include long -term commitments that prevent alternative actions.

### **5.2.5 Flexibility and Robustness**

One possible solution to the problem of uncertainty in power system planning is the robustness of the system to withstand the external impacts. This approach has been practiced in the past, however at the present time the amplitude and the number of the possible impacts is such that the cost of a robust system becomes unavoidable.

Another solution more adapted to the present time is to introduce flexibility within the system. From the planner's point of view a flexible system is one which will be able to adapt quickly to any external changes with little difficulty and at low cost. For instance utilities try to avoid construction of large baseload plants because of their long

construction times and high capital cost. On the other hand, they favor combustion turbines because they take only a few years to build, are inexpensive and small, and can be easily converted to combined-cycle units.

This approach although is very attractive to many utilities but has a major unresolved issue which is the identification of the extra costs involved in risk premium or insurance policy.

### 5.3 Computer Models for Uncertainty Evaluation

Due to the complexity of the relationships among key parameters in the utilities, the rapid advancement of computer technology both in hardware and software, and the ability to provide rapid feedback to the decision makers, have made the computer models an important part of the decision making process. Following is the description of some of the commonly used models.

MUFCAP, a mainframe package for multi-attribute decision analysis, was developed by Sicherman [101]. It has the ability to structure utility functions and then assess them. It also allows the use of both scalar and vector attributes, thereby permitting the combining of individual attributes to facilitate independence assumptions.

IMAP developed by Deutsch and Malmberg [102] is a microcomputer package which was created to be used by a single decision maker analyzing a problem under certainty. It contains interactive independence testing, interactive value function assessment, and evaluation of decision alternatives capabilities. Graphics, automated computation and record-keeping are other qualities of IMAP.

Two microcomputer packages, DECISION and SMART, were developed by Jones [103]. Algorithms for independence testing and parameter estimation for both the continuous and discrete attributes were also developed. Both packages were designed based on these algorithms. DECISION, an additive value function package, differed from SMART based on the technique developed by Edwards [104] in two aspects: 1) DECISION requires independent testing be carried out to ensure that the necessary independence is present among all the attributes, while SMART assumes that necessary independence is already present, and 2) Decision uses the mid-point splitting method

which requires the use of the two extreme values of an attribute to determine weights, while SMART uses a single anchor point for the same purpose.

Decision Programming Language (DPL) is a software developed by ADA Decision Systems [105] for decision analysis. It is a combination of decision tree and influence diagram approach where problems are specified in an english like language created from the vocabulary commonly used in decision analysis. Although, the DPL algorithm works to internally reduce the size of the problem by exploiting its structural properties, but it has limitation on model size like other modeling packages.

The RISKMIN computer program provides a management tool for evaluating alternative resource plans while assessing the sensitivity of the plans to uncertainties. This program which was developed by Stone & Webster [98] for EPRI analyzes the trade offs between multiple and conflicting objectives, identifies robust plans, and performs sensitivity and risk analyses.

This program finds conditional decision sets of plans for all selected futures. From these sets, the percentage of futures supporting each plan is determined as a measure of robustness and the plan is ranked by this percentage. If probabilities associated with the futures are used, probabilities of futures supporting each plan are determined in addition to percentages. One problem with this method is how to determine the percentage or probability values.

The Interdependence Data Analysis (IDA) developed at Virginia Tech [106] is a method which estimates the attributes of the target scenarios from the boundary conditions generated by the limited base cases when complete information is not available, or exhaustive evaluation of all alternatives becomes time consuming and expensive. This method has been applied to different cases in utility planning, including the case for Consolidated Edison Company of New York, [107].

Apart from trying to computerize decision support models, the outcome of a decision-making session can depend largely on the interaction between decision maker and the analyst or computer, the way answers are elicited from the decision maker, and the cognitive style. The programs used in decision making should be more human-like and respond to the decision maker in a more natural manner instead of making the decision maker to learn more complicated programs every time.

### **Concluding Remarks**

The review and discussions in this chapter show that considerable amount of progress has been made during the past few years in the development and application of the methods to treat uncertainty in planning and in decision making. However, more improvements are needed in the modeling aspects which will focus more on the utility's decision process and less on the simulation details. Although sensitivity, scenario, and portfolio analyses can provide valuable insights concerning resource alternatives, they suffer from a critical failing. These approaches use a computer model that simulates the operation of the utility for 20 to 30 years. Because the model's complete set of inputs is specified before the model is run, the utility's decisions are made at one time for the full simulation period. In sensitivity and portfolio analyses, decisions are all made before uncertainties are resolved. In scenario analyses, decisions are made after uncertainties are resolved. Thus, these approaches do not permit incremental and dynamic decision making and are, therefore, not very appealing to actual decision making.

The following is a summary of what the utilities have said in their interviews about uncertainty in planning,

1. Uncertainty is a critical factor that must be considered in utility planning and decision making. Planning only for the base case is too risky.
2. Utilities should monitor changes in their external environment, especially changes in the energy use patterns and preferences of the customers. They also should monitor the costs and performance of their energy resources.
3. Planning should be on going because the environment in which utilities operate is changing rapidly. Formal plans should be revised and published regularly.
4. Utilities should use a variety of methods and data sources in their planning analysis because no one analytical method is best in all cases for treating uncertainty in resource planning.

## **CHAPTER VI**

### **APPLICATIONS OF THE UNCERTAINTY EVALUATION TECHNIQUE**

The goal in this research has been to find a technique which could help decision makers, specially those in electric utilities, in their decisions under many unknowns and uncertainties. The Analytic Hierarchy Process (AHP) was found to be a good candidate for this purpose based on the analysis presented in chapter three, and its comparative analysis with other methods in chapter four.

Through a review of uncertain issues in electric utilities we realize that the utility planning environment has changed dramatically over the last few years. Utilities now need to include a wider variety of resources to meet the load and operating requirements. For instance, because of the instability of oil prices and the high capital cost of a new coal or nuclear power plant, they have to compare the cost and benefits of DSM programs against those options. The emergence of independent power producers (IPP) and non-utility generators (NUG) along with the growing attention and concern for environmental issues such as depletion of scarce resources, contamination and waste disposal, and acid rain have made the planning process more difficult with many uncertain issues to be considered.

In this chapter we will apply the technique to three of the issues mentioned above

concerning the electric utilities in order to show the strength and usefulness of the method in this area. The first case will analyze the third party generation bid evaluation process. The second case demonstrates how to estimate the impact of integrated load management on utility peak load reduction. And the third case study would examine the prediction of oil prices for future planning of electric utilities.

## **Case Study No. 1**

### **6.1 Third Party Generation Bidding Evaluation Criteria**

The independent power producers (IPP) and third party generation (TPG) have become significant alternatives to fulfill electric utility requirements here in US and abroad. This is evident by the increasing number of requests for proposals (RFP) issued by many utilities. In 1989, 17 utilities issued RFP's for a total of over 5000 MW capacity, Burr [108]. This is a substantial increase compared to 1987 when the total request was 729 MW from 4 utilities. Also, in 1989 Ontario Hydro issued an RFP with no limit on capacity, Marier [109]. This trend continues to grow in the 90's. For example, Virginia Power and North Carolina Power in 1993 have executed contracts with 77 non-utility generations which 61 of them are providing 2867 MW of summer capacity and the remaining 16 are projects under development which are expected to provide an additional 655 MW of summer capacity by the summer of 1998 [110]. Some estimates indicate that 30% to 40 % of new generating capacity will be built by IPP's over the next decade, Brown [111].

Other countries specially those in the Asia/Pacific region are also working hard to develop a regulatory and economic framework to support private power development. For instance, India, Indonesia, and Malaysia are trying to attract foreign investment in private power to meet an expected demand of 30 GW, 12GW, and 8.3 GW of new capacity during the next five years respectively, Burr [112].

As a result of this increase in demand for Non-Utility Generation, utilities now must examine many different offers through a bidding process for selecting proper projects to best serve the utilities overall interest.

Wide range of criteria have been adopted by different utilities depending on the regulatory environment and the issues important to them. For instance, a simple criterion based solely on price was adopted by California Public Utilities Commission (CPUC). Virginia Power uses both price and non-price criteria, and Central Maine Power (CMP) developed an elaborate evaluation process based on variety indices like capacity index, security index, and price index to judge various aspects of a bid. Different utilities assign different weights to criteria; for instance, Virginia Power gives 70% to price and 30% to non price factors, Boston Edison gives 90% and 10%, and New Jersey Board of Public Utilities has proposed 55% and 45% weight respectively, Meade [113]. Outside the US, other issues like local laws and regulations, cultural differences, and business practices make the task more complicated.

Therefore, it is important to clearly define the goals, objectives, and criteria set to measure them to facilitate a proper evaluation. However, it is most often the case that the goal consists of many imprecise and mutually conflicting objectives (e.g. best option at the least cost) or other factors related to system performance, operation and control, which cannot be ignored in third party generation acquisition which are qualitative in nature and are difficult to specify in precise quantitative terms. More detailed discussions of the factors are given later.

Many approaches have been suggested for this purpose using elaborate theoretical models such as one by Kahn, et al. [114]. But these models require many parameters to be specified which are difficult from a practical standpoint. The difficulty arises partly because of the fact that different utility system characteristics has different requirements. Therefore, for a model to be of practical value it should be versatile, and applicable to diverse conditions of utility requirements. It should be capable of handling the complexity and the diversity of the problem and be simple and practical to be of operational value.

In this case study, which comprises of two phases, we will first show how AHP method could be utilized to quantify the degrees of importance of the factors in the TPG acquisition from Virginia Power's RFP report [115] and then how to use these information in evaluation of the offers (bids).

### **Phase 1: Evaluation of Criteria**

The first stage consists of expressing the requirements and objectives related to each TPG acquisition according to the utility's need. We will present a structured mechanism to extract indices from the judgments of the experts and management of the company. This can be carried out to the extent to which the utility is willing to specify the criteria of evaluation beforehand.

#### **6.1.1 The Structure**

The framework for analysis is shown in Figure 6.1.1. Since major decisions are seldom made by a single individual, we have structured the hierarchy to incorporate different perspectives of the important actors in level two. They are, planning department, operations department and the state commission who will provide their judgments regarding the objectives and criteria. Next level are the major objectives where the criteria for bid evaluation are based on. These will be discussed in the next section. In the last level the two criteria, price and non-price factors which their priorities need to be found are listed.

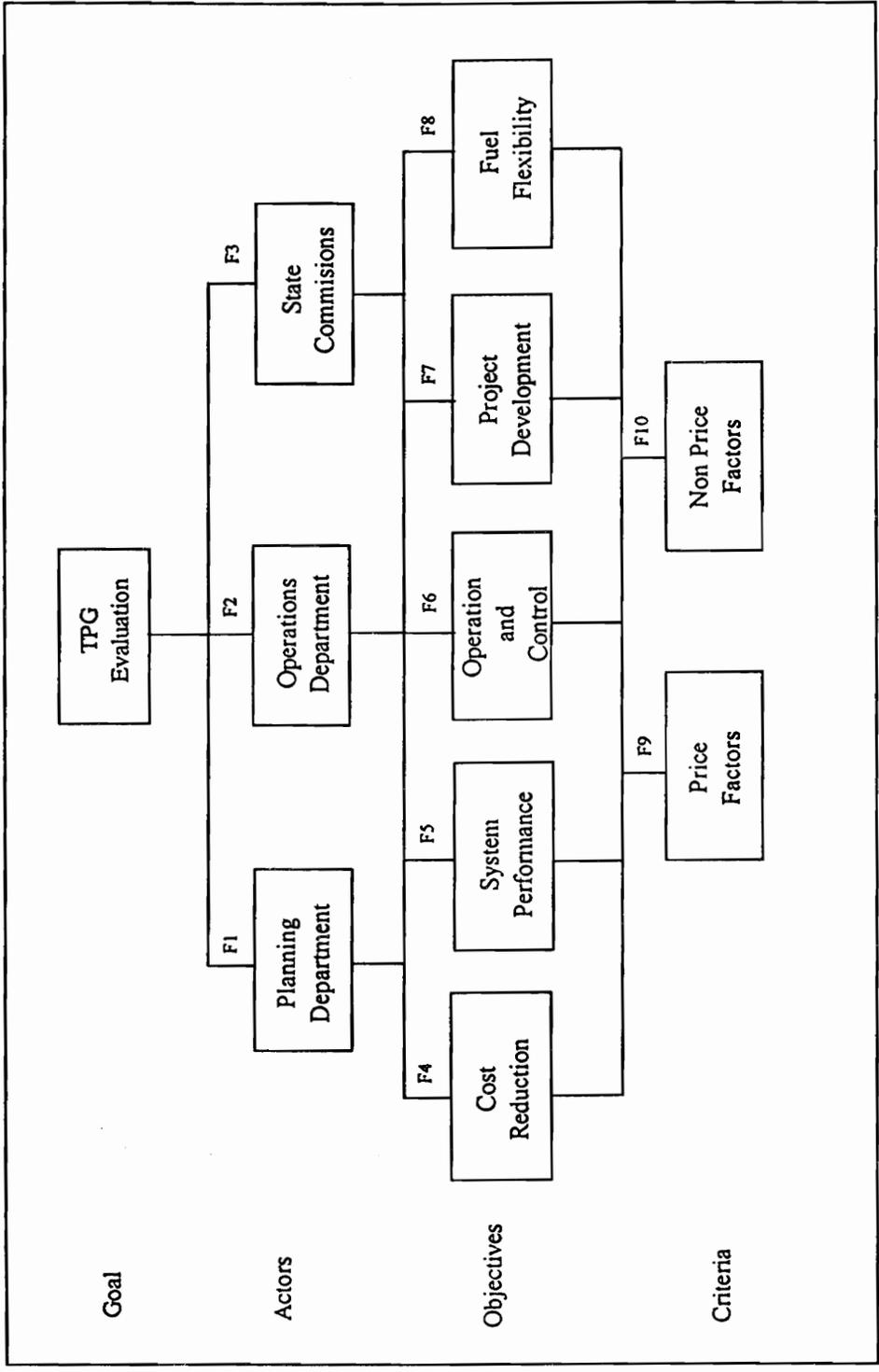


Figure 6.1.1 Hierarchy Structure for Third Party Generation Bidding Evaluation Criteria

## **6.1.2 Objectives**

The criteria for bid evaluation should be based on the objectives set for the proposed acquisition. These might be categorized under the following five headings.

### **1. Cost:**

The basic reason for acquiring TPG is the expected cost reduction made possible by the arrangement, and minimizing the utility's long term financial risk. The whole rationale behind all source bidding is to give the lower cost producers of energy/power an opportunity to sell their low cost energy and serve the utility customer at the least possible cost. Therefore, reduction in cost will remain the major objective in seeking third party generation capacity. Some of the important factors related to this objective are: energy cost, capacity cost, and transmission connection cost.

### **2. System Performance:**

The savings provided by third party generations (TPG) are not without a compromise. With added non utility generation (NUG), it becomes more difficult to monitor system performance. The utility is no longer in complete control of the technical details of these plants. The utility is concerned about the possible effects on reliability, transmission loading, etc. This aspect stays relatively harmless for smaller size of NUGs, or at their low penetration levels, but will become increasingly significant as the size and number of NUGs grow. Some of the important factors related to this objective are: technical characteristics, configuration of the system, and environmental impacts.

### **3. Operation and Control:**

This is another aspect which becomes very significant with increasing proportion of NUGs. The dispatch ability of NUG units and their control becomes very important for the optimal operation of the system. Without proper operational features, it may not be possible to realize the expected savings from NUGs. Some of the important factors related to this objective are: dispatchability, load flow, and stability of the system.

### **4. Project Development:**

Even when a proposed project meets the standards set for the objectives listed above, it may be difficult to convert them to reality. While acquiring capacity from a third party, bidder qualifications become vitally important for the success of the project. Therefore, it becomes essential to ascertain that the bidder is sufficiently qualified, that he has enough experience or enough resources, etc. This aspect may be critical in the evaluation of bids, specially during their initial screening. Some of the important factors related to this objective are: bidder qualification and experience, location of the IPP site, obtaining permit, and viability (e.g., financial security, technical and human resources).

### **5. Fuel Flexibility:**

Steady availability of fuel is a concern for the long term operation of a plant. It arises from the possible changes in fuel supply. This aspect was evident in the Virginia Power's RFP in its preference for the abundant and locally available coal. Some of the important factors related to this objective are: fuel type, fuel availability, and fuel storage.

### **6.1.3 Criteria**

In evaluating and comparing proposals, utilities will consider the cost of receiving the energy and capacity to be the dominant selection criteria. However, because the overall value is also desired, the non-price factors also should be considered. Some of the important factors are discussed below.

#### **6.1.3.1 Price Factors**

##### **1) Prices of Energy, Capacity and Variable O&M**

Bidders should clearly state the prices for energy, capacity, and variable O&M in a way that makes it possible to determine the total cost to the utility and its customers of the proposal for each year of the contract term.

##### **2) Term of Contract**

The utility prefers contracts that cover 25 years from the commercial operations date, but differing contract lengths will be considered with the possibility that differing terms may yield benefits to the company.

##### **3) Dispatch**

Dispatch is a complex topic which includes factors such as the range of minimum and maximum operation, minimum time necessary between operating cycles, the amount of time needed to reduce to "minimum load" and to "no load", and the amount of time needed to increase to "minimum load" and "maximum load".

#### **4) Timing/In-Service Date**

The company will select proposals which offer the best means of meeting its power supply requirements. Payments for dependable capacity will commence only on the first day of the Summer or Winter demonstration period following the commercial operations date. Failure to meet promised scheduled commercial operations dates may result in developer being liable to compensate the company for damages under the agreement.

#### **5) Interconnection Costs**

Bidders for facilities inside the company's control area should not include interconnection costs in their proposal pricing. Bidders should also note that the agreement assumes that the step-up transformer will be owned by the operator. The company has determined that these costs should be direct costs to the company rather than unknown adjustments included in the capacity payments.

### **6.1.3.2 Non-Price Factors**

#### **1) Viability of the Project**

The company will assess the overall project viability by evaluating the following:

- 1a) Level of Development: An appropriate level of development will provide the utility with a greater confidence in project viability, and the rating of the proposal will be increased accordingly.
- 1b) Financial Status of the Bidder: This refers to the bidders and not to affiliated entity companies, unless the parent or affiliated entity company fully guarantees all obligations of the bidder.

- 1c) Experience: The company will consider the bidder's prior experience with constructing, financing and operating power production facilities and the relevance of that experience to the technology proposed by the Bidder. This includes both favorable and unfavorable experience.

## **2) Fuel and Fuel Diversity**

- 2a) Stable Prices: The company favors projects using fuel with stable prices and assured supplies, specifically solid fuels (coal, coal waste, wood) and those with no "fuel" costs (such as hydroelectric and municipal solid waste).
- 2b) Availability: Use of fuels from Virginia or North Carolina may be considered favorably for facilities located in these states and within the company's control area.
- 2c) Multi-Fuel Facilities: The company prefers a mix of fuel types providing generation for its system to avoid undue reliance on any particular fuel. The company also prefers multi-fuel capable facilities for the flexibility they provide in future fuel markets

## **3) Other Factors**

- 3a) Dispatchability: The ability to dispatch a generation resource has an economic and an operating effect on the company. The operating effect of dispatchability will also be considered in the final evaluation of all proposals. If all other factors are equal, the company prefers projects which are fully dispatchable because of the operating benefits and flexibility offered by a fully dispatchable generating resource.

- 3b) Location: The location of a generation resource has certain operating and planning effects on the company's system. Proximity to transmission facilities and the company's load centers will be considered in the evaluation of proposals. In addition, location can be important if coupled with societal economic benefits to the industries and people of such locations.
- 3c) Technical Description of Facility: Bidders should identify and describe major equipments (e.g., turbine, generator, auxiliaries, pollution control equipment, fuel handling, waste disposal, etc.) and its vendor, performance characteristics such as heat rate, nameplate rating, and partial load performance information.
- 3d) Siting: Specific site information including maps and charts must be provided by the bidders. Statement of whether site is owned, leased, or under option, and whether the site is properly zoned is also required from the bidders. No contract will be awarded to a bidder unless the site is owned or is contingent under option to purchase.
- 3e) Permits, Licenses, and Regulatory Approvals: All the required federal, state and local permits, licenses, and regulatory approvals and the status of each, including time needed to perform required monitoring or modeling are necessary before evaluation.

#### 6.1.4 Procedure

After listing the major objectives and factors related to acquiring third party generation, we should present a formal question regarding different elements of hierarchy in a pairwise comparison fashion to extract indices from perceived requirements reflected in the expert judgments. The kind of questions to be asked are in the form of:

- Which actor is more involved ( or concerned ) in IPP evaluation?
- Which objective is more important to each individual actor?
- Which objectives belong to the price criteria and which ones to non-price criteria?

The next step is to obtain the judgment matrices comparing the involvement of different actors with respect to TPG evaluation, the objectives with respect to actors and the criteria with objectives. The judgment matrices are shown in Appendix A. It should be noted that the data to extract such pairwise comparisons are not easily available, and those are the judgments of the experts who were actually involved in this process.

For instance matrix 3-1-5 in Appendix A:

$$\begin{bmatrix} 1 & 5 & 5 & 3 & 3 \\ 1/5 & 1 & 1 & 2 & 1/2 \\ 1/5 & 1 & 1 & 1 & 1/3 \\ 1/3 & 1/2 & 1 & 1 & 2 \\ 1/3 & 2 & 3 & 1/2 & 1 \end{bmatrix}$$

is based on the judgment that cost reduction is strongly more important than system performance and operation & control with respect to planning department. But it is only slightly more important than project development and fuel flexibility. System performance is equally important as operation and control, very slightly more important than project

development, and very slightly less important than fuel flexibility. Operation and control are equally important as project development but slightly less important than fuel flexibility. Finally, project development is slightly more important than fuel flexibility.

The ratios in matrices 4-1-2 through 4-5-2, comparing price and non-price factors with respect to each objective, are based on the judgment that :

- Cost reduction is dominantly a price factor,
- System operation is very slightly less price factor than non price factor,
- Operation & control is very slightly less price factor than non price factor,
- Project development is slightly more price factor than non price factor, and
- Fuel flexibility has a higher price factor than non price factor.

### 6.1.5 Discussion of Results

After studying the report and the assignment of scales to the factors in each level of hierarchy we observed the following results:

In level two a pairwise comparison between the role of each department in third party generation showed that, the Planning department (F1) involvements on TPG is 63.70%, the Operations department (F2) 25.83%, and the State Commission (F3) 10.47%. Then the objectives of the utilities for IPP evaluation in level 3 were evaluated with respect to each actor and judgment matrices were formed to calculate the priority vector showing the importance of each objective, (see Appendix A for details). These weights are then multiplied by the weights in level 2 to obtain the intermediate composite weights in level 3. Table 6.1.2. shows the results when cost reduction (F4) has the highest priority, 36.07% followed by system performance (F5) 18.6%, operation and control (F6) is third with the value of 17.48%, Fuel flexibility (F8) is ranked fourth with 15.17%, and finally project development (F7) with the value of 12.68%.

In the last level the importance of the criteria in level 5 were calculated against each objective from the judgments of the experts. These results were then multiplied by composite weights in level 3 in order to obtain the overall composite priority for the two criteria. Based on the results shown in Table 6.1.3, price factor is almost twice more important than non-price factor with the weight of 64.32% and 35.68% respectively.

Based on the available information and rational judgments the weights of price and non-price factors are found to be close to those used in actual evaluation of the bids by Virginia Power which were 70% and 30% respectively. However, it should be noted that the hierarchy and the factors considered in this case were not the full representation of the problem. We tried mostly to show how this method could be useful and effective in this kind of issues faced by electric utilities.

**Table 6.1.1 Weights and variations of actors in level 2 w.r.t. the goal**

Goal	Weights	Variations
F1	0.6370	± 0.1653
F2	0.2583	± 0.1553
F3	0.1047	± 0.0670

**Table 6.1.2 Weights and the variations of objectives in level 3 w.r.t. actors and the corresponding composite weights**

	Weights w.r.t. F1	Weights w.r.t. F2	Weights w.r.t. F3	Composite Weights	Variations
F4	0.4612	0.0470	0.5229	0.3607	±0.0902
F5	0.1266	0.3143	0.2307	0.1860	±0.0617
F6	0.0946	0.4170	0.0648	0.1748	±0.0538
F7	0.1465	0.0796	0.1238	0.1268	±0.0546
F8	0.1711	0.1420	0.0578	0.1517	±0.0641

**Table 6.1.3 Weights of criteria in level 4 w.r.t. factors in level 3 and the corresponding composite weights**

	Weights w.r.t. F4	Weights w.r.t. F5	Weights w.r.t. F6	Weights w.r.t. F7	Weights w.r.t. F8	Composite Weights	Variations
F9	0.9000	0.3333	0.3333	0.6667	0.7500	0.6432	±0.0407
F10	0.1000	0.6667	0.6667	0.3333	0.2500	0.3568	±0.0354

## **Phase II: Evaluation of Bids:**

The second stage consists of the evaluation of the NUG offers using the information (and the indices derived from them) provided in the offer. This should be carried out in accordance with the requirements specified in phase I. Since enough data for the analysis were not available in the report we will show only the evaluation procedure and two possible hierarchies for the evaluation.

One of the possible structures for the evaluation of the proposals is shown in Figure 6.1.2. In this hierarchy utilities can compare different proposals with respect to the price and non-price factors based on the information provided by the bidders. Similarly, the procedure can be applied to compare directly the offers using their relative standing against the objectives as shown in Figure 6.1.3. In the latter case they need a more detailed hierarchy for the evaluation.

It may be noted that the preliminary screening, which basically checks the adherence to minimal requirements stated in RFPs, seldom poses any difficulty. For example, determining whether a proposal was submitted by the specified time or whether the proposal is accompanied by required fees, is not the source of difficulty in the evaluation process. Therefore, this aspect of evaluation is not of much concern for this procedure. After these preliminary screenings, the final comparison is reduced to a few best offers which make the task of comparison much easier.

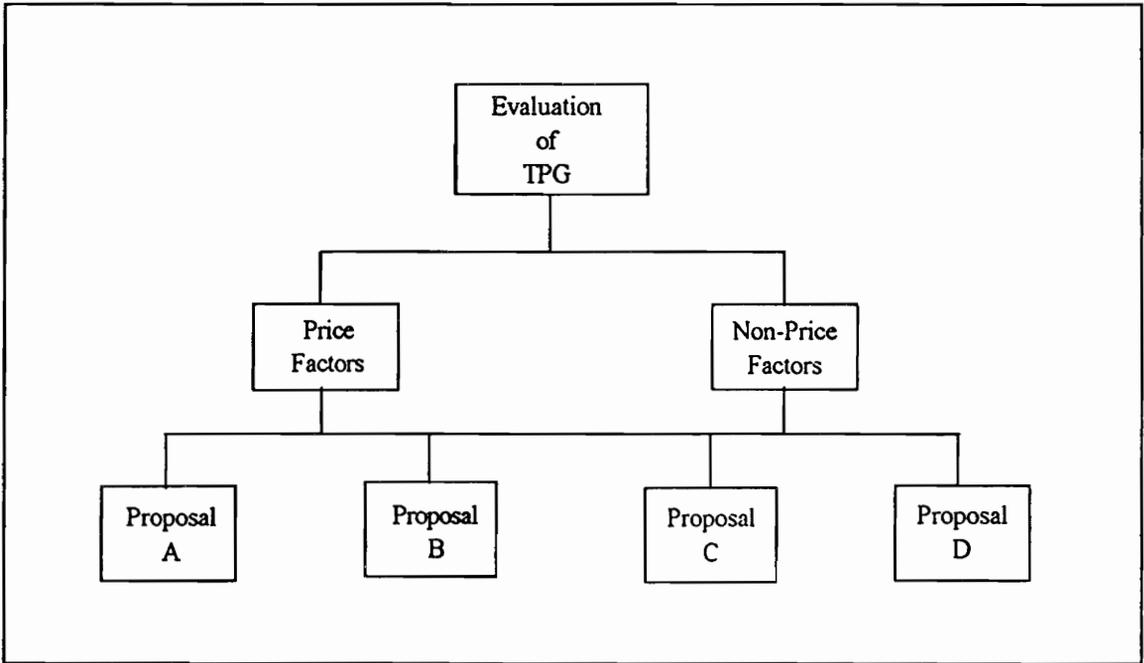


Figure 6.1.2 Hierarchy Structure for Evaluation of Proposals Using Price and Non-Price Factors

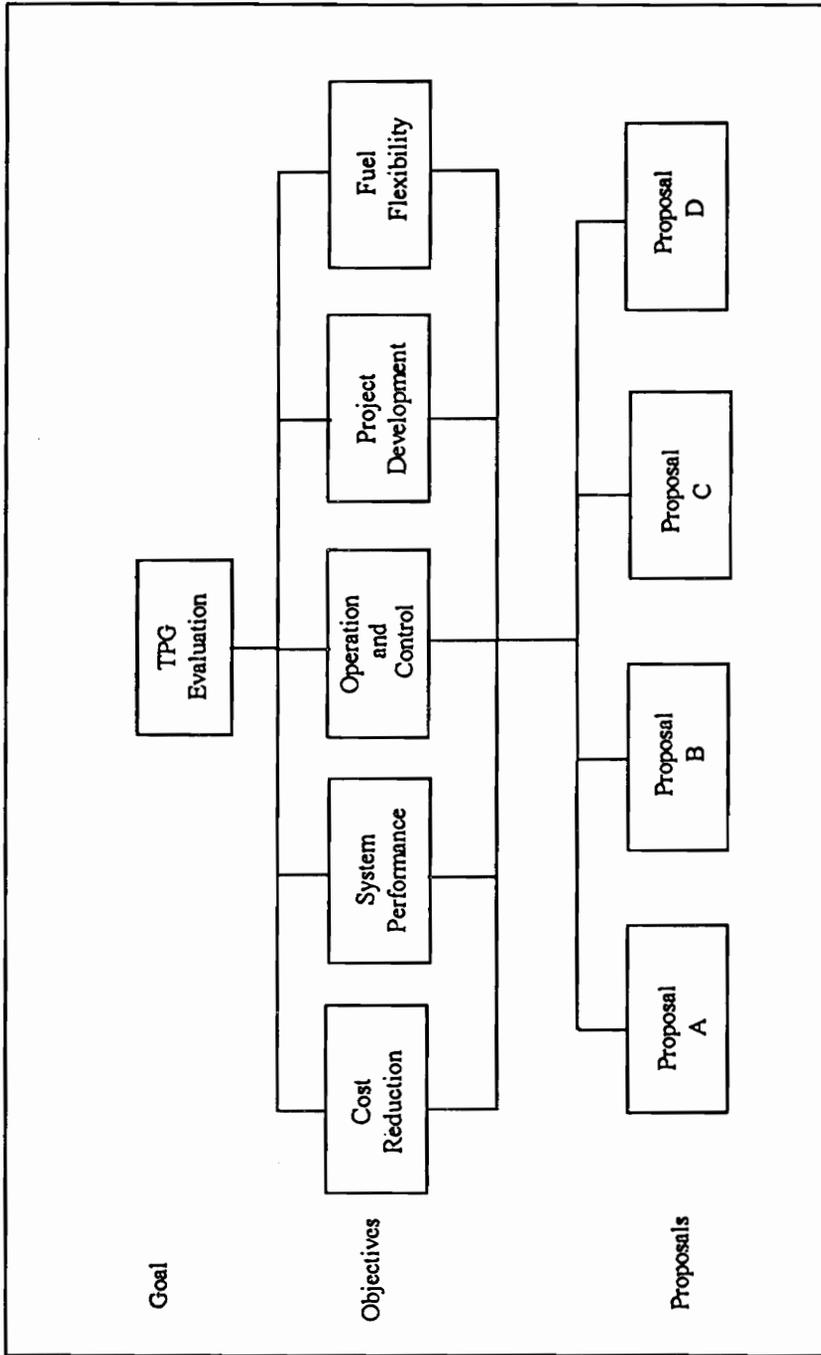


Figure 6.1.3 Hierarchy Structure for Third Party Generation Bidding Evaluation Using the Objectives

The evaluation using price and non-price composition of the offers will require the judgment ratios comparing two offers at a time with respect to each of these factors. One way of generating these ratios will be to estimate the measures of total price for each offer and take the ratios of appropriate pairs to form the ratios. Under these conditions, the judgment matrix will be consistent. However, it may be difficult to obtain such price measures. Then expert opinion has to be used to obtain these ratios on the basis of the information provided in the offers.

This example shows the general outline of the approach to emphasize the value of the AHP technique in the evaluation process of third party generation acquisition. It provides a flexible procedural method that can be adjusted to varying conditions of individual utility company. This can be utilized to quantify the criteria for evaluations according to the requirements of the company. The accuracy of this technique will depend, however, on the ability to collect proper input data which reflect the reality of the situation. The evaluation process can be elaborated by extending the hierarchies to represent more sub-factors under each basic factor.

## **Case Study No. 2**

### **6.2 Identification and Evaluation of Different Load Management Programs on Utility Peak Reduction**

The electric utilities constantly examine alternative options for improving the load factor, in order to meet the increasing demand for electric energy with minimum possible new generation. However, only selected alternatives can be deployed in an area with optimal results depending on the type and mix of utility and the consumer. In this example we will show how the utility planners can identify and prioritize possible alternatives and issues based on their experts' judgment and to evaluate their impact on reducing the peak load. This approach will be useful to decision makers in identifying focus areas before going into a full-scale analysis.

Over the last few years, utilities have gone beyond the relatively one-sided task of forecasting the load and supplying power to incorporating demand side management (DSM) and alternate energy programs as part of an integrated resource planning exercise. Peak-load reduction and load-factor improvement using DSM are now well established and are integral part of utility operations impacting both the utilities and consumers. The utility's main objective is to reduce costs by inducing changes in the time pattern and magnitude of electricity demand, thus deferring the need to operate peaking units or bring new and expensive ones on-line. Load management, energy conservation, and alternative pricing are among the strategies widely utilized in effecting DSM [116]. Load management modifies the system load shape via direct load control, the creation of

off-peak loads, and the shifting of peak time loads to off-peak hours. Conservation is mainly directed towards improving end-use appliance efficiencies and thermal envelopes of buildings. Alternative pricing is based on time-of-use, interruptible or real-time rates and are assumed to convey pricing signals to the consumer commensurate with the real economic costs of production. Alternate energy sources and fuel conversion technologies such as solar water heating, photovoltaics, wind turbines, small hydro as well as cogeneration based on diesel engines and fuel cells are being re-evaluated by utilities and non-utility producers alike to assist in load management objectives.

Assessing which alternatives are best suited for a given utility is not an easy task. The choice is further complicated since the attractiveness of alternatives is influenced strongly by utility specific factors, such as the generating mix, expected load growth, pollution control requirements, system reliability, load factor, and rate structures. A key step in evaluating DSM alternatives is deciding what overall objectives are to be met by DSM. This step can be accomplished in a hierarchical fashion as follows; i) establishing strategic utility objectives, such as reducing future capacity requirements, satisfying the customer, and assuring sound financial performance, ii) setting tactical and operational objectives such as time-of-use rate, and direct load control and, iii) determining load shape objectives (i.e., peak clipping, load shifting).

Such an approach would enable system planners to capture the interactions and interrelationship between the alternatives as well as understanding the uncertainties involved.

This study utilizes the AHP method to compute a priority vector ranking of a sample of alternatives available to the utility for peak demand reduction as shown in the hierarchy (Figure 6.2.1). These are: electricity pricing, utility DSM, advertising and

promotion, consumer DSM, alternative sources (e.g., cogeneration and renewable sources) and consumer education. The impacts of each of these alternatives on the goal of peak load reduction and on the objectives of the actors in the decision-making process that drive this goal, are discussed in the following sections.

## **6.2.1 Alternatives**

### **a) Electricity Pricing**

Alternative electricity pricing is a method used by utilities for indirectly modifying the load shape. It involves the use of rate structures to provide price signals and elicit a desired market response. A survey made for EPRI [117] indicates utility objectives for load shape modification: peak clipping (71%), load shifting (50%), valley filling and strategic conservation (29%), and strategic load growth (21%). Three types of rates are being used: interruptible rates, wherein customers agree to the temporary curtailment of electricity in exchange for a reduced demand charge schedule; time-of-use programs, wherein the utility bills higher rates during peak periods and lower rates for off-peak periods; and real-time pricing wherein customer bills are based on actual hourly production costs.

The EPRI survey indicated average interruptible customer load ranging from 25 kW to 5.5 MW. Utilities with time-of-use rates reported peak to off-peak load shifts ranging from 280 watts to 1.5 MW per customer. The report also indicated limited customer response to real-time pricing. Relative impacts on the daily load curve were not reported for other utilities.

The above numbers show that the load impacts are case-specific to the utility and are highly dependent on its size and on the type of customers being served. Unfortunately,

it may not be worthwhile or possible to conduct a detailed impact analysis because of this variability and the lack of data.

#### **b) Utility DSM**

Utility DSM alternatives refer to those options available to the utility for direct control of the load. The more common appliance targets for direct load control are thermal loads (water heaters and air conditioners), although fixed cycle loads are also compatible with load control strategies. Direct load control has the advantage of "dispatchability" compared with other available options.

The EPRI survey cited earlier [117] indicates average peak demand reduction ranging between 0.45 kW and 1.53 kW per water heater, and between 0.2 kW and 1.95 kW per air conditioner.

#### **c) Customer DSM**

Customer DSM alternatives refer to those DSM strategies that are not under direct control of the utility. Among these are conservation programs that employ building envelope improvements (weatherization), efficient space conditioning equipment, heat storage, efficient lighting and control systems, and efficient appliances. In addition to these traditional strategies, there is a trend towards the increased use of heat recovery and solar energy (passive solar heating or solar-assisted heat pumps).

Building envelope improvements have resulted in savings ranging from 2 to 31 kW per participant, high efficiency air conditioners from 0.7 to 28.3 kW per participant, and efficient lighting and control systems from 0.115 to 64 kW. Demand reductions of 0.6 kW per participant for heat recovery and solar assisted equipment, and 3 kW per participant for efficient motors have been reported [118].

#### **d) Alternative Sources**

The most prominent alternative technologies available are cogeneration, small hydroelectric, wind and photovoltaics. Cogenerators, by their number and diversity, provide a more or less constant supply of power to the grid, although their availability still exhibits large standard deviations [119]. The output of hydroelectric non-utility generators depend on the natural flow of water or on the water release schedules. The wind turbine design parameters for cut-in speed and furling velocity, together with the characteristics of the wind regime, cause power swings which have to be addressed by both system planners and operators. The power output of photovoltaics varies seasonally, although the effects of the minute-to-minute variations in cloud cover can be minimized by dispersing solar arrays. All of these point to the fact that there is considerable uncertainty in the output of independent power producers.

#### **e) Consumer Education/ Advertising and Promotion**

These two options were deemed to have little impact unless associated with other peak load reduction alternatives. This observation is reflected in the results of the pairwise comparisons conducted among the alternatives.

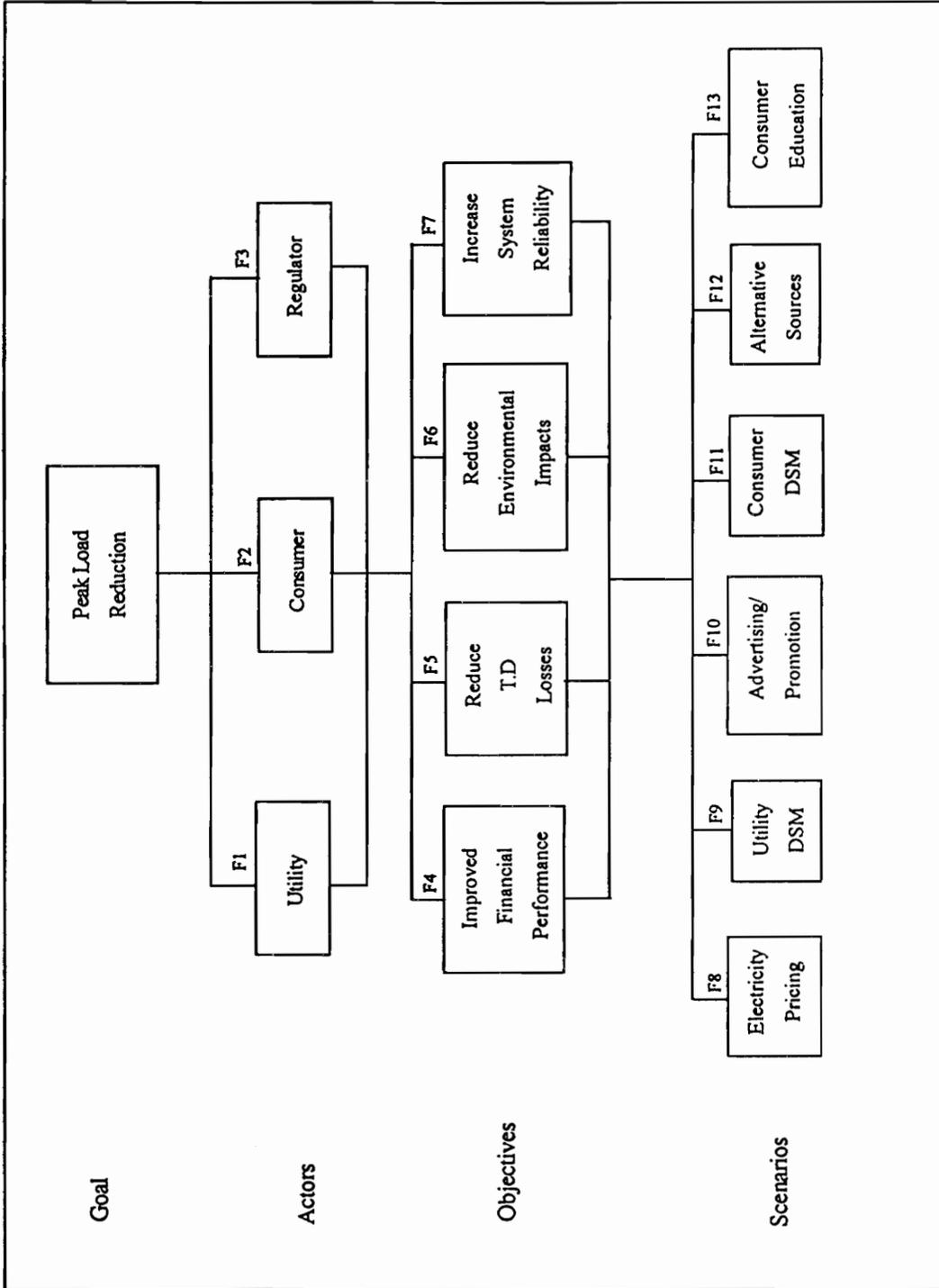


Figure 6.2.1 Hierarchy Structure for Utility Peak Reduction

## **6.2.2 Objectives**

### **a) Improving Financial Performance**

Reducing the peak demand on the system via DSM minimizes the costs associated with operating peaking units (start-up and shutdowns, ramping, fuel, and maintenance). Peak reducing DSM alternatives can be financially and economically attractive to implement, thus obviating the need for new and costly generation and transmission capacities. Most of the alternative pricing schemes adopted in load management are usually designed to reflect economic costs of production and are therefore beneficial to both the utility and the consumer. On the other hand, some schemes such as time-of-use rates and demand charge require expensive control and metering equipment, thereby affecting utility finances [120].

### **b) Transmission and Distribution System**

The transmission and distribution system accounts for more than half of the total capital expense, more than 60% of system losses and more than 80% of all outages. It is therefore necessary that peak reduction programs consider their impacts on the transmission and distribution system. Depending on the coincidence of peak loads, customer locations and transmission line configurations, proper planning of resource options can help to defer the addition of transmission line capacity, increase line utilization and minimize average losses.

### **c) Environmental Emissions**

Reduction of peak load requirements and increasing the load factor permits the efficient operation of the system. However, this can also result in shifting generation from

low-emission gas peaking plants to high-emission base coal plants. Supplying this peak load with coal-fired cogeneration sources will have similar impacts. Conservation and the use of renewable sources, on the other hand, are expected to meet the goal with harmless environmental effects.

#### **d) System Reliability**

Reduction of the system peak defers capacity additions necessary to meet reliability requirements. Accurate probabilistic models have been proposed for the assessment of these capacity benefits [121]. Data for these models are, however, usually unavailable, a fact that is further complicated by the high variability of the value that customers attach to reliability. A survey conducted on 27 U.S utilities showed estimates ranging from \$0.05/kWh to \$10/kWh for residential customers, \$2/kWh to \$35/kWh for commercial customers, and \$3/kWh to \$53/kWh for industrial customers [122].

### **6.2.3 The Actors**

There are usually three main actors involved in utility issues. The utilities themselves, the consumers and the regulators, each with different objectives and roles. Utilities want lower cost and higher rate of return, the consumers want low rate and reliable supply of electricity, and the regulators are concerned about the environment, the price of electricity to the customers and the overall performance of the utilities.

#### 6.2.4 Procedure

we should evaluate the relative importance of different alternatives for utility load reduction discussed in previous sections by comparison of factors in each level with respect to its upper level factors. Their relationships are defined by answering the following questions:

- i. What is the relative concern of each actor in reducing the peak load?
- ii. To what extent are the actors driven by each objective?
- iii. What is the impact of each scenario option on the objectives?

Using pairwise comparisons on each level of the hierarchy for the questions listed above yields the judgment matrices shown in Appendix A. For example matrix 2-1-3 shown below, which is between level 2 and the goal, indicates that utilities have higher concern (3 times) than consumers for load reduction and much more (5 times) than regulators, and consumers also have higher concern than regulators.

$$\begin{bmatrix} 1 & 3 & 5 \\ 1/3 & 1 & 3 \\ 1/5 & 1/3 & 1 \end{bmatrix}$$

### 6.2.5 Discussion of Results

The first step of analysis was to determine the relative importance of each actor in reducing the peak load which is shown in Table 6.2.1. The utility role in reducing the peak was found to be first with 63.7% weight followed by consumers with 25.83% and last was the regulators with 10.47%.

$$[w_1 = 0.6370, w_2 = 0.2583, w_3 = 0.1047]$$

For each actor, pairwise comparisons are conducted on the extent to which it is driven by the objectives on level 3. This yields a vector of weights for each actor. Multiplying these vectors by the corresponding weights obtained for the actors and taking their sums yields the composite weights of Table 6.2.2. For the impacts of the objectives on the focus, increasing system reliability had the highest weight 40.88%, followed by improving financial performance 27.58%, the environmental impact was third with 17.54% and reduction of transmission and distribution losses had the least impact, 14.01%.

A similar procedure yields the final composite priorities for the alternatives impacts in level 4 on the peak load reduction in level 1. It shows that the most significant impact can be expected from alternative sources with the value of 34.73%, utility DSM is second with 25% weight, electricity pricing third with 18.33%, consumer DSM fourth with 12.09%, advertising and promotion and consumer education are the least important factors effecting the peak load reduction with weights of 5.16% and 4.70% respectively.

This case study shows another possible application of the method in evaluating the impacts of alternative programs on peak load reduction when there is not enough data available for a detailed analysis or there is uncertainty on decision variables.

A technique for estimating the impact of each alternative on the peak reduction when additional data are available is discussed by Rahman, *et al* [123].

**Table 6.2.1 Priority values of the actors w.r.t. peak load reduction (goal)**

Actors	Weights
F1	0.6370
F2	0.2583
F3	0.1047

**Table 6.2.2 Priority values of the objectives w.r.t. actors and the composite weights**

Objective s	Weights w.r.t. F1	Weights w.r.t. F2	Weights w.r.t F3	Composite Weights	Vriations
F4	0.3899	0.0837	0.0553	0.2758	$\pm 0.050$
F5	0.1523	0.1189	0.1175	0.1401	$\pm 0.0280$
F6	0.0679	0.2824	0.5650	0.1754	$\pm 0.0285$
F7	0.3899	0.5149	0.2622	0.4088	$\pm 0.0563$

**Table 6.2.3 Priority values of the alternatives w.r.t. the objectives and the overall composite priorities of the hierarchy**

Alternatives	Weights w.r.t. F4	Weights w.r.t. F5	Weights w.r.t. F6	Weights w.r.t. F7	Overall Composite Weights	Variations
F8	0.4610	0.1220	0.0961	0.0545	0.1833	±0.0361
F9	0.2185	0.2500	0.3111	0.2449	0.2499	±0.0440
F10	0.0359	0.0563	0.0726	0.0515	0.0516	±0.0101
F11	0.0930	0.0967	0.1549	0.1333	0.1209	±0.0236
F12	0.1526	0.4201	0.3224	0.4642	0.3473	±0.0425
F13	0.0390	0.0549	0.0429	0.0515	0.0470	±0.0092

## **Case Study No. 3**

### **6.3 Prediction of Oil Price for Electric Utilities**

The price of oil, one of the world's major energy resources, has fluctuated many times during the last 15 years. A barrel of light crude oil went from 2 dollars a barrel in 1972 to 34 dollars in 1981, resulting in profound economic, political, and social consequences both for consumers and producers. This includes electric utilities which depend on oil as a fuel for generating electricity.

However, higher oil prices decreased world demand for oil, increased energy conservation activities, fuel substitution, and expanded investments in oil exploration and production in non-OPEC countries. As a result, in 1983, for the first time OPEC reduced its official price to 29 dollars a barrel and established production quotas for its members. In 1986, the price war brought the price to 10 dollars a barrel at spot market. Then again in 1987 OPEC increased the prices to 17 dollars a barrel and in 1990 after the Iraqi invasion of Kuwait it jumped to about 40 dollars and currently it sells for 18 dollars a barrel, Teel [124].

Predictions of oil price made by major oil companies, different agencies for the Department of Energy, and by electric utilities themselves are mainly based on quantitative factors such as economic growth rate, demand and supply for oil, etc. But in today's world, oil market economics and politics are interwoven, and political decisions increasingly influence the levels of oil production, and prices. Therefore, factors which are not quantifiable must be considered in the process.

### 6.3.1 Hierarchy Structure

In this case study, we will show how the AHP method can be used as a tool by electric utilities to quantify intangible factors based on the experts knowledge and judgments and include them in their prediction equations.

In this analysis five main factors and twenty two subfactors which will affect the price of oil over the period are considered. They are grouped into two clusters: economic-technological cluster and political cluster. Economic-technological factors were further subdivided according to three levels of intensity: high (vigorous), medium (moderate), and low (restrained). Two of the four political factors (F18, F19) were further decomposed into three subfactors. Notice that not all the elements in third and fourth levels are related to the elements of the second level. This is the case of "incomplete hierarchy structure", shown in Figure 6.3.1.

Although the factors and judgments used here are based on experience of the experts in the field and used for actual price predictions, Gholamnezhad [125], but we consider this example to be more of an illustration of the methodology.

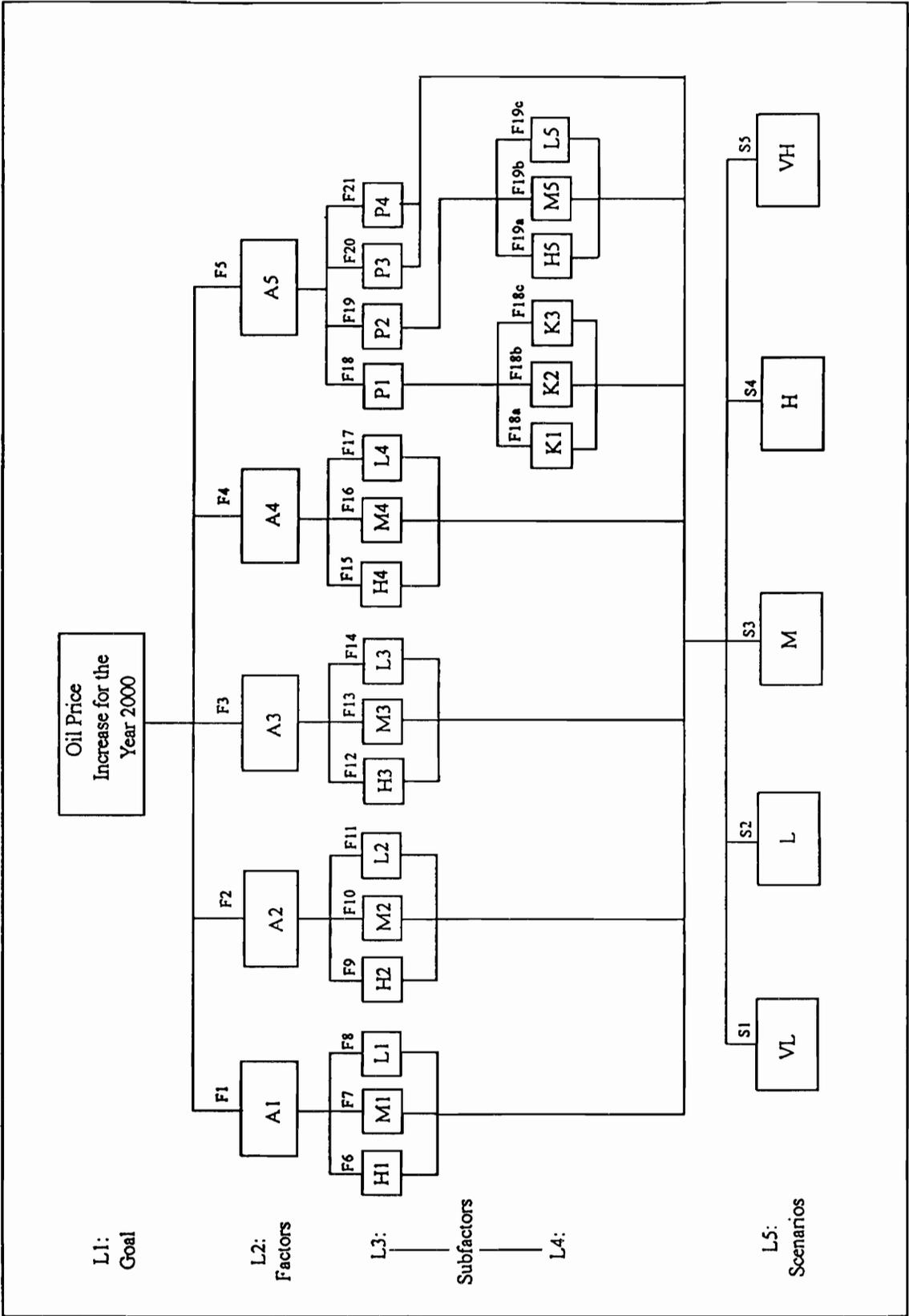


Figure 6.3.1 Oil Price Increase Prediction Model (Partial Hierarchy Structure)

**Legend:**

Level 2:

- A1: World Oil Consumption Increase (F1)
- A2: World Excess Oil Production Capacity (F2)
- A3: Oil Discovery Rate (F3)
- A4: Development of Alternative Energy Sources (F4)
- A5: Political Factors (F5)

Level 3:

- |                |                  |                   |                           |
|----------------|------------------|-------------------|---------------------------|
| H1: 4% /y (F6) | H2: 15% /y (F9)  | H3: 20 bb/y (F12) | H4: High Growth (F15)     |
| M1: 2% /y (F7) | M2: 10% /y (F10) | M3: 10 bb/y (F13) | M4: Moderate Growth (F16) |
| L1: 1% /y (F8) | L2: 5% /y (F11)  | L3: 5 bb/y (F14)  | L4: Low Growth (F17)      |

- P1: Instability in the Persian Gulf Region (F18)
- P2: Big Economic Powers Competition (F19)
- P3: Russian Influence In the Middle East (F20)
- P4: Continuation of Arab-Israeli Conflict (F21)

Level 4:

- |                                              |                                 |
|----------------------------------------------|---------------------------------|
| K1: Social Strain within Countries (F18a)    | H5: High Competition (F19a)     |
| K2: Tension between Individual States (F18b) | M5: Moderate Competition (F19b) |
| K3: Common Market in the Region (F18c)       | L5: Low Competition (F19c)      |

Level 5:

- VL: Very Low (5%) (S1)
- L: Low (15%) (S2)
- M: Moderate (40%) (S3)
- H: High (80%) (S4)
- VH: Very High (140%) (S5)

## **6.3.2 Factors Affecting Future Oil Prices**

### **1) Economic-Technological Factors (F1-F4)**

#### **F1: *World Oil Consumption Increase***

Demand for oil continues to grow despite conservation and other energy saving programs by some industrialized countries. According to International Energy Agency (IEA) world oil consumption is expected to rise 800,000 b/d until the end of 1994, and will jump to one million b/d in the first quarter of 1995. The OECD demand in first quarter of 1995 will rise 1.2% from first quarter of 1994, [126].

#### **F2: *World Excess Production Capacity***

The world's excess production capacity has been more than 10 million barrels per day (mb/d), two-thirds of which was from the Middle East in mid 80's, [125]. At this level of excess capacity, only large oil producers could affect oil prices significantly by fluctuating their production levels. However, this trend has changed in the 90's. According to Teel [124] supply and demand are tight and there is little, if any, excess capacity right now. This will also give the smaller producers the power to rise the price of oil by cutting back their production.

#### **F3: *Oil Discovery Rate***

Before 1970, oil discovery rates were higher than oil production rates, about 20 to 30 bb/y. Therefore, the volume of the world's discovered reserves was increasing. But since the early 1970s, oil discovery has declined steadily while production rates have

increased continuously. This downward trend for discovery rates is predicted to continue in the future. For instance, the number of wells drilling in Europe has dropped 23% between 1992-93, and in the Far East by 6.2%, [127].

#### **F4: *Development of Alternative Energy Sources***

A substantial amount of oil could be replaced by synthetic fuels from large coal, oil shale, and tar sand reserves and from biomass resources. Most of the oil used in the residential and commercial sectors could be replaced by natural gas, electricity, and renewable sources. Solar energy, particularly solar heating will become more popular in the future. In US geothermal and solar energies are expected to replace 27 MTOE (million of tons of oil equivalent) in 1995 and 39.6 MTOE by the year 2000. In JAPAN they are expected to replace 4.2 and 23 MTOE for the same period respectively [128].

But because of the large capital requirements, and environmental constraints, these alternatives are not expected to make a significant contribution in the near future.

## **2) Political Factors (F5)**

Political factors play a very important role in the world oil market. The 1973 oil crisis, the Iraqi invasions of Iran (1980) and Kuwait (1990) and the Soviet Union disintegration (1991) have demonstrated the significance of political factors in the supply, demand, and price of oil.

#### **F18: *Instability in the Persian Gulf Region***

The region that will continue to be of extreme importance in the future supply and price of oil is the Middle East, particularly the Persian Gulf states. The Persian Gulf is

surrounded by a number of major oil-exporting countries including Iran, Kuwait, Saudi Arabia, Iraq, Qatar, and United Arab Emirates. These countries, altogether account for nearly half of the world's total reserves [127].

Stability of the Persian Gulf itself depends on several other factors, particularly the social strains due to rapid economic development, industrialization, and unstable political systems. Another factor to be considered is the possibility of continued disorder in some of the countries in the region.

**F19: *Competition of Big Economic Powers***

This factor represents all major economic powers as they individually and collectively work to enhance their influence in the world and to secure their domestic economic needs. Trade agreements like NAFTA (North American Free Trade Agreement), and the one between EU (European Union) countries are a few examples of such activities.

**F20: *Russian Influence in the Middle East***

The political and economical influences of Russia after disintegration of the former Soviet Union and formation of other independent states, have dramatically decreased in the Middle East. And, because of their internal political and social problems, they are not a major exporter of oil at the present time..

**F21: *Continuation of the Arab-Israeli Conflict***

The continuation of Arab-Israeli conflict and long delays in resolving the conflict

has resulted in disruption of the flow of oil to the rest of the world.

Although recent peace talks with some Arab countries have reduced the tensions between them and kept the price of oil low, but this may change anytime by political and social uprisings in those countries.

### **6.3.3 Procedure:**

After the factors were identified and put in the proper levels of the hierarchy, the following steps are required to predict future price of oil using AHP:

1. Compute the relative weights of the factors (F1,...,F5) in level two by pairwise comparison according to their effectiveness in increasing the price of oil (see legend of figure for their definitions).
2. For each of the factors in level two, compute the relative likelihood of its corresponding subfactors. For example, for the period under consideration, we ask the following questions:
  - a. Which of the three levels of oil consumption increase is most likely: 4 percent, 2 percent, or 1 percent per year? and so on.
  - b. Which of the three levels of excess production capacity is most likely: 15 percent, 10 percent, or 5 percent above production level?
  - c. Which of the three rates of oil discovery is more likely: 20 billion barrels/year (bb/y), 10 bb/y, or 5 bb/y?
  - d. Which of the three levels of development in alternative energy resources is most likely: vigorous, moderate, or restrained?

- e. Which of the four political factors would have the greatest influence in determining future oil prices: 1) degree of instability in the Persian Gulf region (F18), 2) big economic powers competition (F19), 3) Russian Influence in the Middle East (F20), or 4) intensity of the Arab-Israeli conflict (F21)?
3. For instability in the Persian Gulf region, compute the relative importance of its three subfactors; namely, social strains within countries (K1), tension between individual states (K2), and common market in the region (K3).
  4. For the big economic powers competition, which of the three levels of intensity is most likely for the period under consideration: high (H5), medium (M5), or low (L5)?
  5. Compute the composite weights for each subfactor and select subfactors with high relative weights.
  6. Compute the relative likelihood for each level of price increase for each selected subfactor.
  7. Compute the composite weights of the levels of price increase (level 5). The result will be a set of numbers representing the likelihood of each price increase.
  8. Compute the expected price increase by multiplying each price increase level by its corresponding likelihood.

### 6.3.4 Discussion of Results

Implementing the above procedures the following results are observed.

The column of weights in Table 6.3.1 indicates, as one would expect, that political factors (F5) have by far the dominant influence on oil prices. The next most important factor is the increase in oil consumption (F1), which will take place mainly in the developing countries. The third important factor is the development of alternate energy sources other than oil (F4), which is taking place mostly in developed countries such as Japan and the US. due to the cost and the environmental restrictions on fossil fuels. The fourth ranking factor, the oil discovery rate (F3), and the world excess oil production capacity (F2) which is in the fifth rank have the least influence on the price increase during this time period. See Appendix A for judgment matrices and their details.

**Table 6.3.1 Weights and the variations of factors in level 2 with respect to the goal**

Main Factors	F1	F2	F3	F4	F5	Weights
F1	1	7	5	2	1/5	0.2088
F2	1/7	1	1/5	1/7	1/9	0.0281
F3	1/5	5	1	1/3	1/7	0.0690
F4	1/2	7	3	1	1/5	0.1389
F5	5	9	7	5	1	0.5552

Table 6.3.2 shows the composite weights of the subfactors in levels 3 and 4 with respect to level 2. Notice that those factors with highest weights have been selected in each group and then normalized to 1 to represent probability values. In the political factors group, the social strain within the countries of the Persian Gulf region (F22) contributes most to the instability of the region (F18) with an estimated value of 73%. The continuous concern of the western world for the stability of this area is a good validation of this outcome. The big economic powers (F19) in the second position have a relatively strong influence on escalating the price of oil as a result of: a) competing to get the quantity they need by paying more and b) attempting to improve their political status in the world. The continuation of the Arab-Israeli conflict (F21) is ranked third which is true with the recent peace negotiations but may change if any change of government happens in any of those countries. The least weight in this group is given to the Russian influence in the Middle-East (F20), which reflects the current situation. After the collapse of communism in the former Soviet Union and disintegration of the country into different independent countries and their internal economic problems, Russian and other independent countries have little influence in the region. This may also change in the future, but for the period of our study is very unlikely.

**Table 6.3.2 Composite weights of levels 2,3,4 and the selected weights**

Factors	Composite Weights	Selected Weights	Normalized Values	Variations
F1   F6	$(.209) (.105) = 0.0219$			
F1   F7	$(.209) (.637) = 0.1331$	0.1331	0.1998	$\pm 0.0298$
F1   F8	$(.209) (.258) = 0.0539$			
F2   F9	$(.0281) (.131) = 0.0037$			
F2   F10	$(.0281) (.661) = 0.0185$	0.0185	0.0277	$\pm 0.0043$
F2   F11	$(.0281) (.208) = 0.0058$			
F3   F12	$(.069) (.081) = 0.0056$			
F3   F13	$(.069) (.188) = 0.0130$			
F3   F14	$(.069) (.731) = 0.0504$	0.0504	0.0756	$\pm 0.0107$
F4   F15	$(.139) (.105) = 0.0146$			
F4   F16	$(.139) (.637) = 0.0885$	0.0885	0.1329	$\pm 0.0197$
F4   F17	$(.139) (.258) = 0.0358$			
F5   F18, F18a	$(.555) (.558) (.731) = 0.2264$	0.2264	0.3400	$\pm 0.0571$
F5   F18, F18b	$(.555) (.558) (.188) = 0.0582$			
F5   F18, F18c	$(.555) (.558) (.081) = 0.0251$			
F5   F19, F19a	$(.555) (.253) (.139) = 0.0195$			
F5   F19, F19b	$(.555) (.253) (.528) = 0.0741$	0.0741	0.1113	$\pm 0.0152$
F5   F19, F19c	$(.555) (.253) (.332) = 0.0466$			
F5   F20	$(.555) (.054) = 0.0302$			
F5   F21	$(.555) (.135) = 0.075$	0.075	0.1126	$\pm 0.060$
<b>Sum</b>	<b>1.0000</b>	<b>0.6659</b>	<b>1.0000</b>	

Table 6.3.3 shows the result of the analysis and the assigned values to each scenario (percentage of oil price increase) in level 5 with respect to selected factors in levels 3 and 4. These values are based on the judgment of the experts and the calculation of priority vector.

**Table 6.3.3 Weight of scenarios with respect to selected factors of levels 3 and 4**

Selected Factors of Level 3 and 4	Price Increase Scenarios				
	S1 (5%)	S2 (15%)	S3 (40%)	S4 (80%)	S5 (160%)
F7	0.5561	0.2603	0.0966	0.0553	0.0319
F10	0.4890	0.3133	0.1092	0.0599	0.0285
F14	0.0383	0.0734	0.4203	0.2939	0.1741
F16	0.0610	0.0982	0.4682	0.2455	0.1270
F18a	0.0372	0.0847	0.1541	0.4547	0.2693
F19b	0.0443	0.0840	0.4045	0.2894	0.1778
F21	0.0385	0.0859	0.3978	0.3184	0.1594

The overall composite priority which shows the oil price increase probability for each scenario is then calculated based on the following procedure:

**Probability of price increase =**

$$\begin{aligned}
 & [(F1/F7) (Si/F7)] + [(F2/F10) (Si/F10)] + [(F3/F14) (Si/F14)] + [(F4/F16) \\
 & (Si/F16)] + [(F5/F18, F18a) (Si/F18, F18a)] + [(F5/F19, F19b) (Si/F19, F19b)] + \\
 & [(F5/F21) (Si/F21)] \qquad \qquad \qquad (6.1)
 \end{aligned}$$

For example, if we substitute the values from Tables 6.3.2 and 6.3.3 in equation 6.1 for the first scenario (S1), we will get:

$$[(0.1998) (0.5561)] + [(0.0277) (0.4890)] + [(0.0756) (0.0383)] + [(0.1329) (0.0610)] + [(0.340) (0.0372)] + [(0.1113) (0.0443)] + [(0.1126) (0.0385)] = 0.1575$$

Therefore, based on these estimated values, the overall expected price increase will be:

$$5\% (0.1575) + 15\% (0.1271) + 40\% (0.2585) + 80\% (0.2902) + 160\% (0.1665) = 59.56\%$$

The rest of the scenarios will follow the same formula (6.1) but with different values of  $S_i$ . Table 6.3.4 shows the overall probability values for each price increase scenario for the year 2000.

**Table 6.3.4 Overall probability estimates of the price increase scenarios for the year 2000**

Scenarios		Percentage of Increase	Probability of Increase
i	$S_i$		
1	S1	Very Low (5%)	0.1575
2	S2	Low (15%)	0.1271
3	S3	Medium (40%)	0.2585
4	S4	High (80%)	0.2902
5	S5	Very High (160%)	0.1665

## **CHAPTER VII**

### **CONCLUSIONS AND RECOMMENDATIONS**

The work in this dissertation has focused on "identifying" and "applying" a proper model for the uncertainties which electric utilities are facing in their planning process. Utilities long have developed different models for their short and long-term planning. Most of these models are single purpose, large, data intensive, and intended for detailed analysis of specific options and not for strategic analysis (e.g. screening or uncertainty).

New techniques have emerged to deal with uncertain issues in utility planning. Four widely used approaches include: sensitivity analysis, scenario analysis, portfolio analysis, and probabilistic analysis. It was found that probabilistic methods are practiced more than the others. The main problem with the probabilistic methods is the assignment of probability values, specially when not enough data are available and subjective judgments must be made.

Several methods have been suggested for subjective probability estimation such as Bayesian approach, Dempster-Shafer method, and Certainty Factors. The comparative analysis of these methods against others showed that the requirements of many probability assessments in real world cases and the assumption of independency among variables have caused them to be less practical.

The Analytic Hierarchy Process was found to be a well structured and practical approach in assessing uncertain factors and assisting both the individual and the group in assigning subjective probability values based on their judgments. This method has been extended further to estimate the variance of the error in judgments and therefore calculate the confidence interval of probability values rather than the point estimate values. A simulation study was conducted to check the accuracy of error variance (QI) in confidence interval calculations. The results showed that QI has a linear relationship with the variance of the weights and therefore is a good index in interval estimations.

The benefits of this method in evaluating uncertainties for the electric utilities were demonstrated in three major planning issues. First, we have demonstrated how utility planners can quantify the price and non-price criteria for third party generation proposals, considering different objectives and actors in the evaluation process. The results showed 64.3 percent to price factors and 35.7 percent to non-price factors which was very close to 70 percent and 30 percent estimation by Virginia Power. Then two hierarchy structure and a procedure for evaluation of Non-Utility Generation (NUG) bid offers were shown using the information obtained in phase I.

In second case we have showed how the utility planners can identify and prioritize different alternatives based on the experts' judgments in a hierarchy fashion in order to reduce the peak load when there is not enough data available for detailed analysis. The final results indicate that most significant impact on peak reduction can be expected from alternative sources such as cogeneration, hydro, wind, and photovoltaics.

And, in third case we examined the application of the technique to prediction of oil prices for the electric utilities by quantification of many intangible factors involved in the analysis. The hierarchy structure for this case was different from the first two cases. It is a semi-complete hierarchy due to many factors both economical and political in the problem

which may not necessarily related to each other. Based on the information available the final results indicated that probability of 40 percent (medium) oil price increase for the year 2000 is 25.8 percent and for 80 percent (high) increase would be 29 percent. Other scenarios had lower probability values.

It was concluded that the improved Analytic Hierarchy Process method, can provide a flexible procedural method that can be adjusted to varying conditions of individual utility company and therefore a useful tool for the planners and decision makers in the evaluation, quantification, and integration of uncertain factors in their planning process.

## **Recommendations**

This technique might be extended further both in theory and application. In theory, 1) other error models for judgments may be considered for the analysis. 2) AHP could be combined with expert systems or fuzzy sets to address decision problems at different levels from the very detailed micro level to the macro qualitative and quantitative analyses. This may be used in problems such as MYCIN.

In applications, besides the three cases discussed earlier, this technique could be applied to other cases of interest to utilities including: 1) quantification of uncertain factors in load forecasting and rate design. 2) selection of the demand-side management programs for different customers and, 3) selection of different power plants under the new environmental constraints.

## REFERENCES

1. Naill, R. F., Dudley, W. E., " IPP leveraged Financing: Unfair Advantages," Public Utility Fortnightly, PP. 15-19, January 15, 1992.
2. Keelin, T. W., Gellings, C.W., "Impact of Demand-Side Management on Future Customer Electric Demand", EPRI Report EM-4815-SR, October 1986.
3. Torrens, J. M., "Strategic Implications of the 1990 Clean Air Act Amendments: SO<sub>2</sub> Control," Environmental Update, EPRI Report, Vol. 6, No. 2, p. 7, November 1992.
4. Jeffs, E. "British Energy Situation After Privatization", International Turbo machinery, Vol. 32, No. 4, pp. 60-72., May 1991.
5. Cleary, P., "Planning and Forecasting Under Uncertainty", EPRI Journal, p. 47, December 1989.
- ✓7. Hwang, G. H., Yoon, K., "Multiple Attribute Decision Making, Methods and Applications," Springer-Verlag, New York, 1981.
8. Cohon, J. L., "Multiobjective Programming and Planning," Academic Criteria Press, New York, 1978.
9. Hwang, C. L., Masud, M., in Collaboration with S. R. Paidy and K. Yoon, "Multiple Objective Decision Making-Method and Applications: A State-of-the-art Survey," Springer-Verlag, Berlin/Heidelberg/New York, 1979.
10. MacCrimmon, K. R., "An Overview of Multiple Objective Decision Making," in Reference to: Cochrane, J. L. 1973, pp. 18-44, 1974.
11. Churchman, C. W., Ackoff, R. L., Arnoff, E. L., "Introduction to Operations Research," Wiley, New York, 1957.
12. MacCrimmon, K. R., "Decision Making Among Multiple-Attribute Alternatives: A Survey and Consolidated Approach," Rand Memorandum, RM-4823-ARPA, 1968.

13. Nijkamp, P., "Reflections on Gravity and Entropy Models," Regional Science and Urban Economics, Vol. 5, No. 2, pp. 203-225, 1975.
14. Starr, M. K., Zeleny, M., "MCDM State and Future of the Arts," North Holland, New York, pp. 5-29, 1977.
15. Fishburn, P. C., "A Survey of Multiattribute/Multicriteria Evaluation Theories," in Reference to: Zionts, S. 1978, pp. 181-224, 1978.
16. Keeney, R. L., Raiffa, H., "Decision with Multiple Objectives: Preferences and Tradeoffs," Wiley, New York, 1976.
17. Yoon, K., "Systems Selection by Multiple Attribute Decision Making," Ph.D. Dissertation, Kansas State University, Manhattan, Kansas, 1980.
18. Calpine, H. C., and Golding, A., "Some Properties of Pareto-Optimal Choices in Decision Problems," OMEGA, Vol. 4, No. 2., pp. 141-147, 1976.
19. Dawes, R. M. "Social Selection Based on Multi-dimensional Criteria," Journal of Abnormal and Social Psychology, Vol. 68, No.1, pp. 104-109, 1964.
20. Paclinck, J. H., "Qualitative Multiple Criteria Analysis, Environmental Protection and Multi-regional Development," Papers of Regional Science Association, Vol. 4, No.1, pp. 21-30, 1973.
21. Saaty, T. L., "A Scaling Method for Priorities in Hierarchical Structures," Journal of Mathematical Psychology, Vol.15, No. 3, pp. 234-281, 1977.
22. Benayoun, R., Roy, B., Sussman, N. "Manual de Programme Electra," Note de Synthese et Formation, No. 25, Direction Scientifique, Paris, 1966.
23. Srinivasan, V., Shocker, A. D., "Linear Programming Techniques for Multidimensional Analysis of Preference," Psychometrika, Vol. 38, No. 3, pp. 337-369, 1973.
24. Zionts, S., "Multiple Criteria Problem Solving," Springer-Verlag, NY., 1978.
25. Savage, L. J. "The Foundations of Statistics Reconsidered." Studies in Subjective Probabilities, Proceedings of the fourth Berkeley Symposium on Math. New York 1960.
26. Raiffa, H., "Decision Analysis: Introductory Lectures on Choices Under Uncertainty," Massachusetts, Addison-Wesley. 1968.

27. Schlaiffer, R., "Analysis of Decisions Under Uncertainty," New York, McGraw-Hill, 1979.
28. Winkler, R. L., "The assessment of Prior Distributions in Bayesian Analysis," Journal of American Statistical Association, No. 62, pp. 1103-1120, 1967.
29. Winkler, R. L., "The Consensus of Subjective Probability Distributions," Mathematical Science, 15, No. 2, pp. 61-75, 1968.
30. Dalkey, N., "An Experimental Study of Group Opinion," Futures, No. 3, pp. 408-426, September 1969.
31. Hampton, J. M., Moore, P. G., and Thomas H., "Subjective Probability and its Measurement," Journal of Statistical Society, No. 136, Part I, 1973.
32. Stoner, J. A. F., "A Comparison of Individual and Group Decisions Involving Risk," Masters Thesis, MIT., Sloan School of Management, 1961.
33. Wallach, M. A., Kogan, N., Bem, D. J., "Group Influence on Individual Risk-taking," Journal of Social Psychology, Vol. 65, pp. 75-86, 1962.
34. Brown, R., "Social Psychology," New York, Free Press of Glencone, 1965.
35. Stoll, H. G., "Least Cost Electric Utility Planning," Wiley & Sons, New York, 1989.
36. Shealy, M. T., "A Review of Energy Forecast: What Have We Learned?," Proceeding of the Eleventh Annual North America Conference of the International Association of Energy Economists, Washington, DC., pp. 353-362, October 1989.
37. Tempest, P., " Third Price Shock of Oil," Wind Energy Weekly, December, 1993.
38. "Coal, Environmental Issues & Remedies," Organization for Economic Co-operation and Development (OECD), Paris, October, 1983.
39. Lazo, S. L., "Risk Assessment Applied to Future Electricity Supply Adequacy," Public Utility Fortnightly, pp. 20-29, September 28, 1988.
40. Stillinger, W., "Assessing Resource Adequacy in New England," Proceedings of the Workshop on Resource Planning Under Uncertainty for Electric Power System, Stanford University, January 1989.

41. Hirst, E., "Assessing Integrated Resource Plans Prepared by Electric Utilities," Oak Ridge National Laboratory, ORNL/CON-298, February 1990.
42. Eto, J., "An Overview of Integrated Resource Planning Model," LBL-28842, Lawrence Berkeley Laboratory, Berkeley, CA., 1989.
43. New England Electric, "Conservation and load Management Annual Report," West borough, MA., May 1989.
44. Saaty, T. L., "The Analytic Hierarchy Process," McGraw Hill, New York, 1980.
45. Churchman, C., Eisenberg, H., B., "Deliberation and Judgment, Chapter 3, in Shelley II and Bryan, "Human Judgments and Optimality," Wiley, New York, 1969.
46. Saaty, T. L., "Scenarios and Priorities in Transport Planning: Application to the Sudan," Transportation Research, Vol. 11, No. 5, October 1977.
47. Rahman, S., "Analytic Hierarchy Process and Electricity Supply System Planning," IFAC 10th Triennial World Congress, Munich, Germany, pp. 165-169., 1987.
48. Johnson, C. R., "Right-Left Asymmetry in an Eigenvector ranking Procedure," Journal of Mathematical Psychology, Vol., 19, No. 1, pp. 61-64, 1979.
49. Chu, A. T., Kalaba, R. E., Spingarn, K., "A Comparison of Two Methods for Determining the Weight of Belonging to Fuzzy Sets," Journal of Optimization Theory and Applications, Vol. 27, No. 4, pp. 531-538, 1979.
50. Nijkamp, P., "Stochastic Quantitative and Qualitative Multicriteria Analysis for Environmental Design," Papers of the Regional Science Association, Vol. 39, pp. 175-199, 1977.
51. Saaty, T. L., "Concepts, Theory, and Techniques; Rank Generation, Preservation and Reversal in the Analytical Hierarchy Process," Decision Science, Vol. 18, pp. 157-177., 1987.
52. Fichtner, J., "On Deriving Priority Vectors From Matrices of Pairwise Comparisons," Socio-Economic Planning Science, Vol. 20, No. 6, pp. 347-354, 1986.
53. Zahedi, F., "A Simulation Study of Estimation Methods in the Analytical Hierarchy Process," Socio-Economic Planning Science, Vol. 20, No. 6, pp. 347-354., 1986.

54. Gantmacher, F. R., "Applications of the Theory of Matrices," Interscience, New York. 1960.
55. Miller, G. A., "The Magical Number Seven Plus Or Minus Two: Some Limits on Our Capacity for Processing Information," The Psychological Review, Vol. 63, pp. 81-97, March 1956.
56. Harker, P. T., "Incomplete Pairwise Comparisons in the Analytic Hierarchy Process" Mathematical Modeling, Vol. 9, No. 11, pp. 837-848, 1987
57. Shannon, C. E., Weaver, W., "The Mathematical Theory of Communication," University of Illinois Press, Urbana, III., 1974.
58. Starr, M. K., Greenwood, L. H, "Normative Generation of Alternatives With Multiple Criteria Evaluation," in reference to "Multiple Criteria Decision Making" North Holland, New York, 1977.
59. May, K. O., "Intransitivity, Utility and the Aggregation of Preference Patterns," Econometrica, Vol. 22, No. 1, January 1954.
60. Peper, G. L., Walicki, J. S., "In exact Reasoning in Expert Systems," Proceedings of IEEE International Conference on Industrial Electronics, Control and Instrumentation, Cambridge, Mass, Vol. SPIE-857, pp. 802-809, November, 1987.
61. Rahman, S., and Shrestha, G., "A Confidence Interval Based Technique for Evaluating Generation Planning Alternatives," Proceedings of Conference on Decision Support Method for Electric Power Industry, Cambridge, MA, May 1990.
62. Rahman, S., Shrestha, G., "A Statistical Representation of Imprecision in Expert Judgments," International Journal of Approximate Reasoning, January 1991.
63. Shrestha, G., "Formulation and Analysis of A Probabilistic Uncertainty Evaluation Technique," Ph.D. Dissertation, Virginia Polytechnic Institute and State University. 1990.
64. Zahedi, F., "Intelligent Systems for Business, Expert systems with Neural networks," Wadsworth publishing, 1993.
65. Venn, J., "The Logic Of Chance", 3rd edition, Macmillan, London, 1888.
66. De Finetti, B., "Foresight: its logical laws, its subjective sources. In studies in subjective probability," Translated by H. E. Kyburg and H. E. Smoker, eds. New York: Wiley, 1964.

67. Cohen, P. R., "Heuristic Reasoning About Uncertainty: An Artificial Intelligence Approach," Pittman Advanced Publishing Program, Boston, 1985.
68. Kanal, L. M., Lemmer, J. F., "Uncertainty in Artificial Intelligence," North Holland, 1986.
69. Shortliffe, E. H., "Computer Based Medical Consultations: MYCIN," New York, American Elsevier Publishing Co. Inc., 1976.
70. Pearl, J., "On Evidential Reasoning in a Hierarchy of Hypotheses," Artificial Intelligence, Vol. 28, pp. 9-15. 1986.
71. Dempster, A. P., "Upper and Lower Probabilities Induced By Multi-Valued Mapping," Annals of Mathematical Statistics, Vol. 38, pp. 325-329. 1967.
72. Shafer, G., "A Mathematical Theory of Evidence", Princeton University Press, Princeton, NJ, 1976.
73. Shortliffe, E. H., Buchanan, B. G., "A Method of Inexact Reasoning," Mathematical Biosciences, Vol. 23, pp. 351-379. 1975.
74. Zhu, S., Wang, H., "The Analytic Hierarchy Process and Artificial Intelligence: A Comparative study," Proceedings of Analytic Hierarchy Process, pp. 303-315 Pittsburg, PA., 1991.
75. Klir, G. J., and Folger, T. A., "Fuzzy Sets, Uncertainty, and Information," Prentice Hall, Englewood Cliffs, NJ., 1988.
76. Zadeh, L. A., Fuzzy Sets, Information and Control, Vol. 8, pp. 338-353. 1965.
77. Watson, S. R., Weiss, J., and Donnell, M., "Fuzzy Decision Analysis," IEEE Transactions on System, Man and Cybernetics, Vol. SMC-9, No. 1, pp. 1-9, 1979.
78. Li, Z., Chen, Z., and Li, J., "A Model of Weather Forecast by Fuzzy Grade Statistics," Fuzzy Sets and Systems, Vol. 26, No. 257, 1988.
79. Kaufmann A., Gupta, M., "Fuzzy Mathematical Models in Engineering and Management Science," North Holland Press, New York, 1988.
80. Turksen, I. B., "Measurements of Membership Functions in Applications of Fuzzy Set theory in Human Factors," Edited by Karwowski and Mital, Elsevier, New York, 1986.

81. Bellman, R. E., Zadeh, L. A., "Decision-Making in Fuzzy Environment," Management Science, Vol. 17, No. 4, pp. B141-B164, 1970.
82. Schwartz, T. J., "Fuzzy Systems in the Real World," AI Expert, Vol. 5, No. 8, pp. 29-36, August 1990.
83. Forsyth, R., "Fuzzy Reasoning Systems," Expert Systems, pp. 51-61, 1984.
84. Sprague, R. H., "Framework for the Development of Decision Support Systems," Management Science Information Quarterly, Vol. 4, No. 4, 1980.
85. Yager, R., "A Foundation for a Theory of Possibility," Journal of Cybernetics, Vol. 10, pp. 177-204, 1980.
86. Laarhoven, P. J., Pedrycz, W., "A Fuzzy Extension of Saaty's Priority Theory," Fuzzy Sets and Systems, Vol. 14, pp. 229-241, 1983.
87. Helmer, O., "Analysis of the Future: The Delphi Method," Santa Monica, Rand Corporation, 1969.
88. Henrion, M., "Uncertainty in Artificial Intelligence: Is Probability Epistemologically and Heuristically Adequate?," Expert Systems and Expert Judgment, Springer-Verlag, 1987.
89. Shafer, G., "Probability Judgment in Artificial Intelligence and Expert Systems," Statistical Science, Vol. 2, No. 1, pp. 3-44, 1987.
90. "An Approach to Risk Evaluation in Electric Resource Planning," EPRI Report No. EL-5851, Projects 2537-1, -2, Final Report, August 1988.
91. Hirst, E., and Schweitzer, M., "Uncertainty in Long-Term Resource Planning for Electrical Utilities," Oak Ridge National Laboratory, ORNL/CON-272 December 1988.
92. Southern California Edison, "Strategies for an Uncertain Future," Rosemead, CA, March 1988.
93. Southern Company, "1986 Annual Planning Hearing: Generation Expansion Planning Document," Atlanta, GA, September 1986.
94. New England Electric System, "NEESPLAN II, 1988: Effective Planning in an Era of Uncertainty," West borough, MA., 1988.

95. Northeast Utilities, "Integrated Demand/Supply Planning 1983-1985," Hartford, December 1985.
96. Potomac Electric Power Company, "The 1987 Energy Plan," Washington, DC., 1987.
97. Dorfner, P., Crousillat, E., Alvarado, P., and Merrill, H. M., "Conflicting Objectives and Risk in Power System Planning," IEEE Transaction on Power Systems, Vol. 8, No. 3, August 1993.
98. RISKMIN, "An Approach to Risk Evaluation in Electric Resource Planning," Stone & Webster Management Consulting, Inc., August 1988.
99. Burke, W., Merrill, H., Schweppe, F., Lovell, B., McCoy, M., and Monohon, S. "Trade Off Methods in system planning," IEEE Transactions on Power System, Vol. 3, pp. 1284-1290, August 1988.
100. Broadwater, R., P., and Shaalan, H., E., "A Bound Conserving Interval Mathematics Approach to Electric Utility Economic Evaluation," Proceedings of the 1991 North American Power Symposium, Carbondale, IL., October, 1991.
101. Sicherman, A., "Assessing and Analyzing Preferences Concerning Multiple Objectives: An Interactive Computer Program," Behavioral Science, Vol. 21, pp. 173-182, 1976.
102. Deutsch, S. J., Malmborg, and Charles, J., "IMAP: An Interactive computer Package for the Assessment of Multiattribute Value Functions," IEEE Transactions on Systems, Man and Cybernetics, Vol. SMC-13, No. 3, pp. 417-421, 1983.
103. Jones, J., E., "Microcomputer Programming Package for the Assessment of Multiattribute Value Functions," M.S. Thesis, Virginia Tech., 1985.
104. Edwards, W., "Use of Multiattribute Utility Measurement for Social Decision Making: Conflicting Objectives in Decisions," John Wiley and Sons, pp. 247-276, 1977.
105. "Decision Programming Language (DPL)," ADA Decision Systems, Menlo Park, CA., 1990.
106. Rahman, S., Shrestha, G., "A Technique to Incorporate New Information in Evaluating Generation Alternatives," IEEE Transaction. on Power Systems, Vol. 7, No. 2, pp. 900-906, May 1992,

107. "Strategic Planning for Electric Energy in the 1980's for New York City and Westchester County," MIT. Energy Laboratory Technical Report, Cambridge, MA, 1981.
108. Burr, M., "Bidding Expands to Public Power," Independent Energy, Vol. 20, No 2, pp. 85-68, February 1990.
109. Marier, D., "A Banner Year," Independent Energy, Vol. 19, No. 9, pp. 14-20, November 1989.
110. "Long Rang Forecast of Load and Resources, 1994-2113," Virginia Power and North Carolina Power Report, 1994.
111. Brown, R., "Projecting Capacity," Independent Power, pp. 22-23, January 1989.
112. Burr, M. T., "Mobilizing Private Power," Independent Energy, Vol. 24, No. 6, pp. 10-33, July/August 1994.
113. Meade, M., "The Evolution of Competitive Bidding," Independent energy, Vol. 19, No 6, pp. 22-26, July/August 1990.
114. Kahn, E.P., Stoff, S., and Berman, D. "Optimization Methods for Evaluating Competitive Bids from Private Producers," Presented at IEEE Summer Meeting, July 15-17, 1990.
115. "Request for Proposal for Power Purchases," Virginia Power and North Carolina Power Report, August, 1989.
116. "Control Strategies for Load Management," EPRI EM-3882, Project 2035-4, Final Report, July 1985.
117. "1987 Survey of Commercial-Sector Demand-Side Management Programs," EPRI Report CU-6294, Research Project 2884-1, Final Report, March 1989.
118. Yau, T. S., et al., "Demand-Side Management Impact on the Transmission and Distribution System," IEEE Transactions on Power Systems, Vol. 5, No. 2, pp. 506 - 512. May 1990.
119. "Small Cogeneration System Costs and Performance," EPRI EM-5954, Project 1276-27, Final Report, August 1988.
120. "Demand Side Management. Volume 3: Technology Alternatives and Market Implementation Methods," EPRI EA/EM-3597, Vol. 3, Project 2381-4, Final

Report, December. 1984.

121. "Study of the Effect of Load Management on Generating-System Reliability," EPRI EA-3575, Project 1955-3, Final Report, July 1984.
122. Sanghvi, A. P., "Power System Reliability Practices in North America," IEEE Transaction on Power systems, Vol. 6, No. 4, pp. 1485-1492, November 1991.
123. Rahman S., Osareh A., de Castro A., and Teklu Y., "Value of Integrated Load Management, Conservation and Solar Applications in Utility Peak Reduction," Proceedings of NTUA/IEEE Joint International Power Conference, September, 1993
124. Teel, M. E., "What's Happening in Drilling," World Oil Journal, p.25, July 1994.
125. Gholamnezhad, A. H, Saaty, T. L., "Oil Prices 1985 and 1990," Energy Systems and Policy, Vol. 5, No. 4, 1981.
126. "The Surging World Economy Continues to Fuel the Locomotive of Growing Oil Demand," International Energy Agency, Oil and Gas Journal, August 15, 1994.
127. "Forecast of 1994 World Drilling," World Oil Journal, p.24, August 1994
128. "Electricity Supply in the OECD Countries," International Energy Agency (IEA) Publications, Paris, 1992.

## Appendix A

### PAIRWISE COMPARISON MATRICES AND DETAILED RESULTS FOR THE CASE STUDIES

Following are the comparison matrices and the detailed results of the analysis for all levels of the hierarchy of the case studies. The three digit number on the top of each judgment matrix represents, 1) the level of the hierarchy, 2) the factor being compared with in the upper level, and 3) the size of the matrix respectively. The weight of the factors and the composite weights along with consistency ratio (CR), quotient index (QI) and the confidence intervals are shown below each judgment matrix.

#### Case No. 1, (TPG)

2 1 3

1.0000 3.0000 5.0000

.3333 1.0000 3.0000

.2000 .3333 1.0000

WEIGHTS (W)= .6370 .2583 .1047

LAMDA(MAX)= 3.039 CI.= .0193 CR.= .0332

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI)= .1152

CONFIDENCE INTERVALS OF THE WEIGHTS:

.4717 <W< .8023

.1030 <W< .4135

.0377 <W< .1718

3 1 5

1.0000 5.0000 5.0000 3.0000 3.0000

.2000 1.0000 1.0000 2.0000 .5000

.2000 1.0000 1.0000 1.0000 .3333

.3333 .5000 1.0000 1.0000 2.0000

.3333 2.0000 3.0000 .5000 1.0000

WEIGHTS (W)= .4612 .1266 .0946 .1465 .1711

LAMDA(MAX)= 5.465 CI.= .1161 CR.= .1037

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI)= .3707

CONFIDENCE INTERVALS OF THE WEIGHTS:

.3224 <W< .5999

.0544 <W< .1989

.0399 <W< .1493

.0637 <W< .2292

.0759 <W< .2664

3 2 5

1.0000 .2000 .2000 .3333 .2500

5.0000 1.0000 .5000 3.0000 5.0000

5.0000 2.0000 1.0000 7.0000 3.0000

3.0000 .3333 .1429 1.0000 .3333

4.0000 .2000 .3333 3.0000 1.0000

WEIGHTS (W)= .0470 .3143 .4170 .0796 .1420

LAMDA(MAX)= 5.430 CI.= .1074 CR.= .0959

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI)= .3470

CONFIDENCE INTERVALS OF THE WEIGHTS:

.0198 <W< .0742

.1647 <W< .4640

.2595 <W< .5746

.0339 <W< .1254

.0623 <W< .2216

3 3 5

1.0000 5.0000 4.0000 5.0000 5.0000

.2000 1.0000 4.0000 3.0000 5.0000

.2500 .2500 1.0000 .3333 1.0000

.2000 .3333 3.0000 1.0000 3.0000

.2000 .2000 1.0000 .3333 1.0000

WEIGHTS (W)= .5229 .2307 .0648 .1238 .0578

LAMDA(MAX)= 5.439 CI.= .1097 CR.= .0979

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI)= .3533

CONFIDENCE INTERVALS OF THE WEIGHTS:

.3689 <W< .6768  
.0967 <W< .3648  
.0241 <W< .1054  
.0474 <W< .2001  
.0215 <W< .0942

\*\*\*\*\*

COMPOSITE PRIORITIES FOR LEVEL 3

.3607 .1860 .1748 .1268 .1517

\*\*\*\*\*

\*\* CONFIDENCE INTERVALS FOR THE COMPOSITE PRIORITIES OF LEVEL 3

.2705 <W< .4508  
.1243 <W< .2477  
.1210 <W< .2285  
.0722 <W< .1814  
.0876 <W< .2159

4 1 2

1.0000 9.0000  
.1111 1.0000

WEIGHTS (W)= .9000 .1000

LAMDA(MAX)= 2.000 CI.= .0000 CR.= .0000

4 2 2

1.0000 .5000  
2.0000 1.0000

WEIGHTS (W)= .3333 .6667

LAMDA(MAX)= 2.000 CI.= .0000 CR.= .0000

4 3 2

1.0000 .5000

2.0000 1.0000

WEIGHTS (W)= .3333 .6667

LAMDA(MAX)= 2.000 CI.= .0000 CR.= .0000

4 4 2

1.0000 2.0000

.5000 1.0000

WEIGHTS (W)= .6667 .3333

LAMDA(MAX)= 2.000 CI.= .0000 CR.= .0000

4 5 2

1.0000 3.0000

.3333 1.0000

WEIGHTS (W)= .7500 .2500

LAMDA(MAX)= 2.000 CI.= .0000 CR.= .0000

\*\*\*\*\*  
 COMPOSITE PRIORITIES FOR LEVEL 4  
 .6432 .3568  
 \*\*\*\*\*

\*\* CONFIDENCE INTERVALS FOR THE COMPOSITE PRIORITIES OF LEVEL 4  
 .6025 <W< .6839  
 .3214 <W< .3922

CONSISTENCY RATIO OF THE HIERARCHY (C.R.H.)= .0779

**Case No. 2, (DSM)**

2 1 3

1.0000 3.0000 5.0000

.3333 1.0000 3.0000

.2000 .3333 1.0000

WEIGHTS (W)= .6370 .2583 .1047

LAMDA(MAX)= 3.039 CI.= .0193 CR.= .0332

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI)= .1152

CONFIDENCE INTERVALS OF THE WEIGHTS ARE :

.4711 <W< .8028

.1025 <W< .4141

.0375 <W< .1720

3 1 4

1.0000 3.0000 5.0000 1.0000

.3333 1.0000 3.0000 .3333

.2000 .3333 1.0000 .2000

1.0000 3.0000 5.0000 1.0000

WEIGHTS (W)= .3899 .1524 .0679 .3899

LAMDA(MAX)= 4.043 CI.= .0145 CR.= .0161

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI)= .0577

CONFIDENCE INTERVALS OF THE WEIGHTS ARE :

.3124 <W< .4674  
.1127 <W< .1920  
.0497 <W< .0861  
.3124 <W< .4674

3 2 4

1.0000 .5000 .3333 .2000  
2.0000 1.0000 .3333 .2000  
3.0000 3.0000 1.0000 .5000  
5.0000 5.0000 2.0000 1.0000

WEIGHTS (W)= .0837 .1190 .2824 .5149

LAMDA(MAX)= 4.065 CI.= .0216 CR.= .0240

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI)= .0859

CONFIDENCE INTERVALS OF THE WEIGHTS ARE :

.0550 <W< .1124  
.0786 <W< .1593  
.1960 <W< .3688  
.4214 <W< .6084

3 3 4

1.0000 .3333 .1429 .2000  
3.0000 1.0000 .2000 .3333  
7.0000 5.0000 1.0000 3.0000  
5.0000 3.0000 .3333 1.0000

WEIGHTS (W)= .0553 .1175 .5650 .2622

LAMDA(MAX)= 4.117 CI.= .0390 CR.= .0433

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI)= .1545

CONFIDENCE INTERVALS OF THE WEIGHTS ARE :

.0284 <W< .0822

.0612 <W< .1738

.4382 <W< .6918

.1456 <W< .3788

\*\*\*\*\*

COMPOSITE PRIORITIES FOR LEVEL 3

.2758 .1401 .1754 .4088

\*\*\*\*\*

CONFIDENCE INTERVALS FOR THE COMPOSITE PRIORITIES OF LEVEL 3

.2258 <W< .3258

.1121 <W< .1680

.1469 <W< .2038

.3525 <W< .4651

4 1 6

1.0000 5.0000 7.0000 5.0000 3.0000 7.0000

.2000 1.0000 5.0000 3.0000 3.0000 5.0000

.1429 .2000 1.0000 .2000 .2000 1.0000

.2000 .3333 5.0000 1.0000 .3333 3.0000

.3333 .3333 5.0000 3.0000 1.0000 4.0000

.1429 .2000 1.0000 .3333 .2500 1.0000

WEIGHTS (W)= .4610 .2185 .0359 .0930 .1526 .0390

LAMDA(MAX)= 6.508 CI= .1016 CR= .0819

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI)= .2929

**CONFIDENCE INTERVALS OF THE WEIGHTS ARE :**

.3366 <W< .5855  
.1164 <W< .3206  
.0176 <W< .0542  
.0462 <W< .1398  
.0779 <W< .2274  
.0191 <W< .0588

4 2 6

1.0000 .3333 3.0000 2.0000 .2000 2.0000  
3.0000 1.0000 4.0000 3.0000 .5000 4.0000  
.3333 .2500 1.0000 .3333 .2500 1.0000  
.5000 .3333 3.0000 1.0000 .2000 2.0000  
5.0000 2.0000 4.0000 5.0000 1.0000 7.0000  
.5000 .2500 1.0000 .5000 .1429 1.0000

WEIGHTS (W)= .1220 .2500 .0563 .0967 .4201 .0549

LAMDA(MAX)= 6.244 CI= .0489 CR= .0394

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI)= .1438

**CONFIDENCE INTERVALS OF THE WEIGHTS ARE :**

.0814 <W< .1625  
.1751 <W< .3249  
.0372 <W< .0754  
.0642 <W< .1292  
.3358 <W< .5044  
.0363 <W< .0736

4 3 6

1.0000 .2000 3.0000 .3333 .2000 3.0000  
5.0000 1.0000 3.0000 3.0000 1.0000 5.0000  
.3333 .3333 1.0000 .3333 .2500 3.0000  
3.0000 .3333 3.0000 1.0000 .3333 3.0000  
5.0000 1.0000 4.0000 3.0000 1.0000 5.0000  
.3333 .2000 .3333 .3333 .2000 1.0000

WEIGHTS (W)= .0961 .3111 .0726 .1549 .3224 .0429

LAMDA(MAX)= 6.464 CI= .0927 CR= .0748  
ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI)= .2705

CONFIDENCE INTERVALS OF THE WEIGHTS ARE :

.0543 <W< .1379  
.2044 <W< .4179  
.0408 <W< .1045  
.0897 <W< .2200  
.2146 <W< .4302  
.0240 <W< .0619

4 4 6

1.0000 .1667 1.0000 .3333 .2000 1.0000  
6.0000 1.0000 5.0000 2.0000 .3333 5.0000  
1.0000 .2000 1.0000 .3333 .1429 1.0000  
3.0000 .5000 3.0000 1.0000 .2000 3.0000  
5.0000 3.0000 7.0000 5.0000 1.0000 7.0000  
1.0000 .2000 1.0000 .3333 .1429 1.0000

WEIGHTS (W)= .0545 .2449 .0515 .1333 .4642 .0515

LAMDA(MAX)= 6.159 CI.= .0318 CR.= .0256

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI)= .0941

CONFIDENCE INTERVALS OF THE WEIGHTS ARE :

.0386 <W< .0704  
.1808 <W< .3090  
.0365 <W< .0665  
.0955 <W< .1712  
.3917 <W< .5368  
.0365 <W< .0665

\*\*\*\*\*  
COMPOSITE PRIORITIES FOR LEVEL 4  
.1833 .2499 .0516 .1209 .3473 .0470  
\*\*\*\*\*

CONFIDENCE INTERVALS FOR THE COMPOSITE PRIORITIES OF LEVEL 4

.1472 <W< .2195  
.2059 <W< .2940  
.0415 <W< .0616  
.0973 <W< .1444  
.3048 <W< .3897  
.0378 <W< .0563

CONSISTENCY RATIO OF THE HIERARCHY (C.R.H.)= .0376

**Case No. 3, (Oil Prices )**

2 1 5

1.0000 7.0000 5.0000 2.0000 .2000  
.1429 1.0000 .2000 .1429 .1111  
.2000 5.0000 1.0000 .3333 .1429  
.5000 7.0000 3.0000 1.0000 .2000  
5.0000 9.0000 7.0000 5.0000 1.0000

WEIGHTS (W)= .2088 .0281 .0690 .1389 .5552

LAMDA(MAX)= 5.4573 C.I.= .1143 C.R.= .1021

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI) : .368

CONFIDENCE INTERVALS OF THE WEIGHTS:

.0775 <W< .3401  
.0093 <W< .0468  
.0232 <W< .1149  
.0484 <W< .2294  
.3969 <W< .7135

3 1 3

1.0000 .2000 .3333  
5.0000 1.0000 3.0000  
3.0000 .3333 1.0000

WEIGHTS (W)= .1047 .6370 .2583

LAMDA(MAX)= 3.0385 C.I.= .0193 C.R.= .0332

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI) : .115

**CONFIDENCE INTERVALS OF THE WEIGHTS:**

.0377 <W< .1718

.4717 <W< .8023

.1030 <W< .4135

3 2 3

1.0000 .2500 .5000

4.0000 1.0000 4.0000

2.0000 .2500 1.0000

WEIGHTS (W)= .1311 .6608 .2081

LAMDA(MAX)= 3.0536 C.I.= .0268 C.R.= .0462

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI) : .160

**CONFIDENCE INTERVALS OF THE WEIGHTS:**

.0314 <W< .2308

.4823 <W< .8392

.0542 <W< .3621

3 3 3

1.0000 .3333 .1429

3.0000 1.0000 .2000

7.0000 5.0000 1.0000

WEIGHTS (W)= .0810 .1884 .7306

LAMDA(MAX)= 3.0649 C.I.= .0324 C.R.= .0559

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI) : .194

**CONFIDENCE INTERVALS OF THE WEIGHTS:**

.0070 <W< .1550

.0206 <W< .3562

.5492 <W< .9121

3 4 3

1.0000 .2000 .3333

5.0000 1.0000 3.0000

3.0000 .3333 1.0000

WEIGHTS (W)= .1047 .6370 .2583

LAMDA(MAX)= 3.0385 C.I.= .0193 C.R.= .0332

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI) : .115

CONFIDENCE INTERVALS OF THE WEIGHTS ARE :

.0375 <W< .1720

.4711 <W< .8028

.1025 <W< .4141

3 5 4

1.0000 4.0000 7.0000 3.0000

.2500 1.0000 5.0000 3.0000

.1429 .2000 1.0000 .3333

.3333 .3333 3.0000 1.0000

WEIGHTS (W)= .5579 .2531 .0544 .1346

LAMDA(MAX)= 4.2209 C.I.= .0736 C.R.= .0818

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI) : .289

CONFIDENCE INTERVALS OF THE WEIGHTS:

.3873 <W< .7285

.1002 <W< .4061

.0186 <W< .0902

.0478 <W< .2214

\*\*\*\*\*

COMPOSITE PRIORITIES FOR LEVEL 3

.0219 .1330 .0539 .0037 .0186 .0058 .0056 .0130  
.0505 .0145 .0885 .0359 .3097 .1405 .0302 .0747

\*\*\*\*\*

\*\* CONFIDENCE INTERVALS FOR THE COMPOSITE PRIORITIES OF LEVEL 3

.0098 <W< .0339  
.1033 <W< .1626  
.0261 <W< .0818  
.0013 <W< .0061  
.0142 <W< .0229  
.0021 <W< .0096  
.0012 <W< .0100  
.0031 <W< .0229  
.0397 <W< .0612  
.0065 <W< .0225  
.0688 <W< .1082  
.0174 <W< .0544  
.1922 <W< .4273  
.0352 <W< .2459  
.0056 <W< .0548  
.0150 <W< .1345

4 18 3

1.0000 5.0000 7.0000

.2000 1.0000 3.0000

.1429 .3333 1.0000

WEIGHTS (W)= .7306 .1884 .0810

LAMDA(MAX)= 3.0649 C.I.= .0324 C.R.= .0559

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI) : .194

CONFIDENCE INTERVALS OF THE WEIGHTS:

.5492 <W< .9121  
.0206 <W< .3562  
.0070 <W< .1550

4 19 3

1.0000 .3333 .3333

3.0000 1.0000 2.0000

3.0000 .5000 1.0000

WEIGHTS (W)= .1396 .5278 .3325

LAMDA(MAX)= 3.0536 C.I.= .0268 C.R.= .0462

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI) : .160

CONFIDENCE INTERVALS OF THE WEIGHTS:

.0437 <W< .2356

.3181 <W< .7376

.1325 <W< .5325

5 7 5

1.0000 5.0000 6.0000 7.0000 8.0000

.2000 1.0000 5.0000 6.0000 8.0000

.1667 .2000 1.0000 3.0000 4.0000

.1429 .1667 .3333 1.0000 3.0000

.1250 .1250 .2500 .3333 1.0000

WEIGHTS (W)= .5561 .2603 .0966 .0553 .0319

LAMDA(MAX)= 5.5361 C.I.= .1340 C.R.= .1197

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI) : .429

**CONFIDENCE INTERVALS OF THE WEIGHTS:**

.3697 <W< .7424  
.0863 <W< .4342  
.0264 <W< .1667  
.0148 <W< .0957  
.0085 <W< .0553

5 10 5

1.0000 3.0000 5.0000 7.0000 9.0000

.3333 1.0000 5.0000 7.0000 9.0000

.2000 .2000 1.0000 3.0000 5.0000

.1429 .1429 .3333 1.0000 4.0000

.1111 .1111 .2000 .2500 1.0000

WEIGHTS (W)= .4890 .3133 .1092 .0599 .0285

LAMDA(MAX)= 5.4209 C.I.= .1052 C.R.= .0940

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI) : .341

**CONFIDENCE INTERVALS OF THE WEIGHTS:**

.3155 <W< .6626  
.1472 <W< .4794  
.0423 <W< .1761  
.0228 <W< .0971  
.0107 <W< .0462

5 14 5

1.0000 .3333 .1429 .2000 .1429

3.0000 1.0000 .2500 .2000 .2500

7.0000 4.0000 1.0000 2.0000 4.0000

5.0000 5.0000 .5000 1.0000 3.0000

7.0000 4.0000 .2500 .3333 1.0000

WEIGHTS (W)= .0383 .0734 .4203 .2939 .1741

LAMDA(MAX)= 5.3917 C.I.= .0979 C.R.= .0874

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI) : .320

CONFIDENCE INTERVALS OF THE WEIGHTS:

.0170 <W< .0596

.0329 <W< .1139

.2701 <W< .5705

.1558 <W< .4321

.0822 <W< .2660

5 16 5

1.0000 .3333 .2500 .2500 .5000

3.0000 1.0000 .2000 .5000 .3333

4.0000 5.0000 1.0000 3.0000 5.0000

4.0000 2.0000 .3333 1.0000 4.0000

2.0000 3.0000 .2000 .2500 1.0000

WEIGHTS (W)= .0610 .0982 .4682 .2455 .1270

LAMDA(MAX)= 5.4999 C.I.= .1250 C.R.= .1116

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI) : .400

CONFIDENCE INTERVALS OF THE WEIGHTS:

.0227 <W< .0994

.0372 <W< .1592

.3088 <W< .6276

.1064 <W< .3846

.0489 <W< .2052

5 18a 5

1.0000 .3333 .2000 .1429 .1429  
3.0000 1.0000 .5000 .2500 .2500  
5.0000 2.0000 1.0000 .5000 .3333  
7.0000 4.0000 2.0000 1.0000 4.0000  
7.0000 4.0000 3.0000 .2500 1.0000

WEIGHTS (W)= .0372 .0847 .1541 .4547 .2693

LAMDA(MAX)= 5.3649 C.I.= .0912 C.R.= .0814

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI) : .293

CONFIDENCE INTERVALS OF THE WEIGHTS:

.0171 <W< .0574  
.0393 <W< .1301  
.0737 <W< .2345  
.3116 <W< .5978  
.1413 <W< .3973

5 19b 5

1.0000 .3333 .1667 .2000 .2500  
3.0000 1.0000 .3333 .2500 .2000  
6.0000 3.0000 1.0000 2.0000 4.0000  
5.0000 4.0000 .5000 1.0000 3.0000  
4.0000 5.0000 .2500 .3333 1.0000

WEIGHTS (W)= .0443 .0840 .4045 .2894 .1778

LAMDA(MAX)= 5.4689 C.I.= .1172 C.R.= .1047

ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI) : .377

**CONFIDENCE INTERVALS OF THE WEIGHTS:**

.0181 <W< .0704  
.0349 <W< .1332  
.2471 <W< .5619  
.1451 <W< .4337  
.0785 <W< .2772

5 21 5

1.0000 .3333 .1667 .1667 .1429

3.0000 1.0000 .3333 .2500 .3333

6.0000 3.0000 1.0000 2.0000 4.0000

6.0000 4.0000 .5000 1.0000 4.0000

7.0000 3.0000 .2500 .2500 1.0000

**WEIGHTS (W)=** .0385 .0859 .3978 .3184 .1594

**LAMDA(MAX)=** 5.4327 **C.I.=** .1082 **C.R.=** .0966

**ESTIMAT OF ACTUAL VARIANCE OF ERROR (QI) :** .352

**CONFIDENCE INTERVALS OF THE WEIGHTS:**

.0163 <W< .0607  
.0369 <W< .1349  
.2416 <W< .5539  
.1695 <W< .4673  
.0714 <W< .2474

## VITA

Ali Reza Osareh was born in Isfahan, Iran in September 28, 1959. After his high school he came to US to continue his education. He received his B.S. degree in Electrical Engineering from University of Colorado at Denver in 1983, and his M.S. in Electrical Engineering in 1985 with concentration in power electronics.

He has been working in the electrical engineering department at Virginia Tech as graduate research assistant and graduate teaching assistant since 1988. His areas of interest include power system planning and operation, evaluation of DSM, and alternate energy sources, and uncertainty analysis for electric utilities. He has published several papers in these fields.

A handwritten signature in cursive script that reads "Ali R. Osareh" followed by a horizontal line.