A STUDY ON ENDOGENOUS GROWTH MODELS AND TRADE

By

Spyros M. Lazaris

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APPROVED:

Yannis M. Ioannides

Yannis M. Ioannides, Chairman

Daniel A. Nuxoll

Richard Cothren

Catherine Eckel

Anya McGuirk

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Spyros M. Lazaris

Chairman: Yannis M. Ioannides

ABSTRACT

In this study we look at the effects on economic growth of the continuous introduction of intermediate goods. Our theoretical framework enhances the endogenous growth theory, by allowing for participation in international trade. Furthermore, we test the behavior of the proposed theoretical model with data provided from the Penn World Tables.

The theoretical part describes an economy in two different stages: first, we develop a model for a closed economy, and study the effects of the continuous introduction of durable goods. We show that the continuous introduction of durables causes sustained economic growth. We demonstrate that a more educated labor force contributes more to the growth of the economy. Second, we assume that trade takes place among similar countries. Both final and intermediate goods are traded, and we show that participation in international trade enhances the economic growth of this economy. We prove that with no restrictions to trade, countries continue to grow. Because we assumed that the countries were identical, we found that those countries that pursue the introduction of new durables will grow faster.

In the empirical part, we test the proposed theoretical model. We want to test how the economic growth of this economy, is affected by the measures of exports and imports.

We define the econometric method, and demonstrate empirically that there is a positive relation between the growth rate of an economy and the measures of exports and imports. Furthermore, the accumulation of physical capital has a positive effects on economic growth. We show that a positive relation exists between the available level of human capital and the growth rate of the economy. In our theoretical model, human capital is introduced indirectly through the efficiency of the labor force. We show that an educated labor force is preferred by the producers of final goods. We conclude that, other things being equal, poor countries that start with a relatively high level of human capital will grow faster.

We also test our statistical model against the assumptions of normality, linearity, homoskedasticity, and homogeneity. We demonstrate that our statistical specification does not violate any of these assumptions. Thus, we can call our model a well-specified statistical model.

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Chapter 1

INTRODUCTION

"Few problems are more fascinating, more important, or more neglected than the rates at which development proceeds in successive generations in different countries"[1] This statement by Wesley Clair Mitchell expresses clearly the attention that development was given in the 1930s and 1940s. Today, however, the problem of economic development has become one of the foremost problems that each country confronts.

Starting with Adam Smith, in *An Inquiry into the Nature and Causes of the Wealth of Nations* (1776), economic growth has been a major part of economic theory. Smith used the principle of the "division of labor" which led to an increase in the competence of workers, to a reduction in the time required to produce goods, and to the invention of better equipment. Therefore economic growth is the result of increased productivity. Specialization of labor could lead to an infinite self-enforced growth, but Smith pointed

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1 W. C. Mitchell, *Business Cycles,* NBER, New York, 1927, page 416.

out that growth is limited by the size of the country, the *"...soil and climate, and its situation with respect to other countries..."*

Later on, David Ricardo, in *Principles of Political Economy and Taxation* (1817), demonstrated a somewhat different approach. Ricardo considered the agricultural sector as the most important sector in an economic system. He assumed that population grew at a Malthusian pattern (exponential rate), that production exhibited diminishing returns to scale, and that profits are required for capital accumulation. Contrary to Smith's views about growth, Ricardo did not believe that growth could continue forever, but that, after a certain point, the growth rate will start declining. He assumed that the amount of cultivable land was a given, since there is only a limited amount of fertile land, and that population would continue to grow. If these assumptions hold, it is inevitable that diminishing returns in agriculture will take place.

Classical economists tried to discover the causes of long-run growth in national income and how this growth occurred. For them, economic growth was a "race" between population growth and exhaustion of resources on the one hand, and capital accumulation and expansion of the market on the other. The assumption of diminishing returns into capital formation led to a stationary state. Classical economists provided us with an illustration of an aggregate theory of economic development. For them, the fundamental feature of economic growth was capital formation. Their analysis gave us the factors that determine the pace of economic growth, such as factors of production (natural resources, capital, labor). A crucial contribution was the introduction of the notion of diminishing returns to scale, which dominated theory for the following years.

Modern economic growth theory began with the work of Domar (1946)[2] and Harrod (1948)[3] who, following different routes, concluded that the national savings rate (as a function of the income saved) should be equal to the product of the capital-output ratio and the growth rate of the labor force. Then the economy can steadily grow without appearances of labor shortages or labor surpluses.

In his well known 1956 paper, "A Contribution to the Theory of Economic Growth", Solow proposed the basis for the "neoclassical model of economic growth". He allowed for some degree of technological flexibility, which, instead of a unique feasible equilibrium path, produced a whole range of steady states. Furthermore, due to the assumption of diminishing returns to scale, the equilibrium growth rate was not proportional to the savings rate, but in fact was independent of it. Solow's approach dominated the economic growth theory for the coming years.

Arrow, "The Economic Implications of Learning by Doing" (1962), introduced the idea of learning by doing, and started the literature on endogenous economic growth. Arrow argues that the accumulation of learning is related to experience. The more experienced a person is, the more productive he becomes. He illustrated his argument using the airplane industry, where experience and increased productivity go hand in hand.

Interest in economic growth was renewed recently by the work of Romer (1986). Romer built on Arrow's ideas and expanded the literature on endogenous economic growth. In his 1986 paper Romer claimed that the learning-by-doing process is a possible source for sustained growth. He also explained (1990) that growth can be driven by technological change. This technological change is the product of investment that is

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2 E. V. Domar, "Capital Expansion, Rate of Growth, and Employment," *Econometrica*, **14**, 137-147.
3 R. F. Harrod, *Towards a Dynamic Economics*, Macmillan, London, 1948.

undertaken by profit-maximizing agents. Grossman and Helpman (1991) enhanced Romer's approach by introducing learning-by-doing to an open economy model. However, work on an open economy model has not been as widespread.

1.1. Motivation for this Study

Because of this renewed interest in economic growth theory, extensive work has been done on the closed economy case. It is interesting, though, to incorporate the ideas developed for the closed economy in a setup that allows for international trade. This idea is behind the motivation for this study. Specifically, we will explore the question of how an economy grows in a global environment when the ideas of endogenous growth are incorporated.

This thesis sets two main goals: first, to develop a theoretical framework that incorporates the ideas of endogenous growth literature into a model of an open economy, and second, to estimate this theoretical framework using available data sets. Both goals intend to show that the opening of an economy that grows endogenously to trade will lead to further enhancement of its growth. The theoretical part of this thesis is divided into two main sub-parts. Part one describes the case of a closed economy that grows endogenously, and provides the basic ground on which the case of an open economy is built. It assumes that there are two goods that this economy produces: final goods and intermediate goods. The intermediate goods are the products of research and development, while the final goods are made up of labor and intermediate goods. There are two production sectors,

one that produces the intermediate goods, and one that produces the final goods. The intermediate goods are products of research and development that use physical capital as production input. An important assumption here is that intermediates are produced in a non-competitive environment. The continuous introduction of intermediate goods leads the economy to grow over time. Part two presents the open economy, which participates in international trade on a global level, and intends to prove that participation in trade transactions further improves the economic growth of this country. The major assumptions are carried over from the closed economy case and are enhanced by assuming that there are two countries (home country and foreign country) with structures similar to the one described in the closed economy case. Each country produces intermediate and final goods that are traded. For reasons that are explained in Chapter 4, each country specializes in the production of specific intermediate goods. This analysis shows that economic growth is sustained as long as new intermediates are introduced in the world economy.

In the empirical section of the thesis, we lay out the statistical and econometric approaches used in this work. Our goal is to test the theoretical model developed in Chapters 3 and 4, and explore the question of how an economy is affected by opening up to trade. We also describe thoroughly the tests used to test the adequacy of the estimated model. Finally, the estimation results are presented. We can say that the theoretical arguments discussed in the theoretical part are generally supported by the data.

The key results of this study are summarized below:

1. Openness to trade promotes long run growth. There are two major effects taking place: first, integration of economies causes technological spillovers. Knowledge produced in different countries can be accessed by the producers of intermediate

goods in each country. This access causes the productivity of the intermediate goods sector to increase. Second, international spillovers of knowledge lead to more contacts among businessmen and lead eventually to more trade.

2. Trade implies that innovators in different countries participate in world markets with their innovations, making their incentives for pursuing new innovations higher, because of increased prospects of higher profits.

3. By opening an economy to trade, the decisions of investors are affected. In the first place, the incentive to invest more in R&D is higher, since the number of potential customers is bigger. On the other hand, because of international trade, the competition becomes more stiff, leading to a situation where it might become unprofitable to invest in research and technology.

1.2. Overview of the Thesis

The thesis is divided into two major parts: part one contains the theoretical framework, and part two contains the empirical application of part one. The remainder of the thesis is organized into seven additional chapters. Chapters 2 to 4 constitute the theoretical part, while Chapters 5 to 7 constitute the empirical part. Chapter 8 provides the conclusions of this work, states its limitations and suggests possible enhancements.

In more detail, Chapter 2 builds a simple model of endogenous economic growth, and provides the necessary review of the literature. The most important papers are

reviewed in this chapter, helping to establish the frontiers of the research on the subject and making clear directions for further inquiry.

Chapter 3 presents the model for the case of a closed economy. It builds on the theory originally proposed by Romer (1990), and shows clearly that for an economy that produces intermediate goods, endogenous growth can be achieved. This model lays the foundation that is needed to build the case of an open economy, presented in Chapter 4. In Chapter 4 the ability of countries to trade with each other enhances the results of the closed economy, and clearly shows that each country benefits under this setup, by participating in global trade.

Chapter 5 serves two major purposes: first, it lays out the statistical and econometric approaches used in the empirical estimation, and second, provides a thorough review of the literature on empirical applications of growth models. Chapter 6 provides the empirical specification of the theoretical model developed in Chapter 4. It details all the assumptions necessary to transform the theoretical model into an estimable one, and contains an extensive description of the data used in the empirical analysis.

Chapter 7 provides the results of estimating the empirical model defined in Chapter 6, and also presents misspecification testing for the empirical model, something that has not been done in the past. It presents in addition, some sensitivity analysis undertaken to check the robustness of the empirical model. Finally, Chapter 8 concludes by providing a summary of the results, by discussing possible limitations of the theory, and by proposing potential directions for further research.

Chapter 2

MAJOR PAPERS ON ENDOGENOUS GROWTH LITERATURE

Several recent papers have focused on the subject of endogenous economic growth. The key feature of these works is that they consider research and development a major factor in the production process. Traditional economic growth theory suggested that growth takes place as long as technological change is happening outside the model. In other words, technological change was necessary to sustain economic growth, but it is not modeled endogenously.

Romer (1986) introduced a new approach to modeling economic growth; he assumed that endogenous technological change takes place. Thus his model offers an alternative view of long-run growth. Specifically he proposed that:

a. The output level, in per capita terms, can grow boundlessly in a fully competitive equilibrium. Its rate of growth can be increasing monotonically over time.

b. The rate of investment and the rate of return to capital may increase rather than decrease with increases in the capital stock.

c. The level of output in per capita terms need not converge in different countries.

The above results did not depend on any kind of exogenously defined technological change, nor on differences between countries. Economic growth in this setup is driven by the continuous accumulation of knowledge. Even though knowledge is assumed to cause increasing returns in the production of products, knowledge itself is a product of research technology that exhibits decreasing returns to scale. Knowledge itself has two effects on the production process:

a. Investment in knowledge is a natural externality. In other words, if a firm produces new knowledge, this will benefit other firms too, since it is difficult to patent or keep concealed the existence of new knowledge.

b. Production of consumption goods as a function of the stock of knowledge exhibits increasing returns.

These three elements: externalities, increasing returns in the production of output, and decreasing returns in the production of new knowledge, specify a competitive equilibrium growth model. This equilibrium is not Pareto optimal, but the presence of externalities satisfies the existence of the equilibrium. The critical assumption is the increasing marginal productivity of knowledge. Several economists introduced research and development as an endogenous feature of growth models. The results showed that investing in research and development was paying off with higher growth rates. In this literature review we will refer to these works and their results.

2.1. A Model of Economic Growth

Let us start by introducing a typical neoclassical economic growth model. Assume that we have a closed economy, with an asset K_t called capital. Capital K_t does not describe physical capital only. It can easily extend to include other forms of capital, like human capital, in terms of knowledge, for example. The production technology in this economy is a standard Cobb-Douglas production function, with two inputs, capital K_t and labor L_t. Capital accumulates over time while labor does not.[1] Then the production function is given by the following equation:

$$Y_t = A L_t^{\alpha} K_t^{\beta} \tag{1}$$

where Y_t is the produced output, A is a technological parameter, and $\alpha, \beta > 0$ are constants.

We assume that capital changes over time. The change of capital is net investment. In a closed economy net investment equals savings minus depreciation, whereas savings is equal to the portion of production Y_t that is not consumed. On the other hand, depreciation is the portion of capital that is set aside for the time needed to replace any capital goods. Then if $\dot{K}_t$ defines the change in capital over time, s the savings rate, and δ the depreciation rate, we get:

$$\dot{K}_t = sY_t - \delta K_t, \tag{2}$$

---

1 Labor force can grow over time. Here we assume that population growth is exogenous.

and using (1), (2) becomes

$$\dot{K}_t = sAL_t^{\alpha} K_t^{\beta} - \delta K_t. \tag{3}$$

Assume now that the population size equals the size of the labor force. Moreover, that labor increases over time with an exogenous rate equal to n $(\dot{L}_t/L_t = n)$[2]. For simplicity purposes let us also define by

$$k_t = \frac{K_t}{L_t} \tag{4}$$

the ratio of capital to labor or the capital per worker. Then if we divide the production function given by (1) by L_t, we get:

$$y_t = AL_t^{\alpha-\beta-1} k_t^{\beta} \tag{5}$$

where y_t is the output per worker, and k_t is the capital per worker.

If we then take derivatives with respect to time on (4), we get:

$$\dot{k}_t = \dot{K}_t L_t - K_t \dot{L}_t, \tag{6}$$

and making use of (3), (5) becomes,

$$\dot{k}_t = sAK_t^{\beta} L_t^{\alpha+1} - (\delta + n) K_t L_t \tag{7}$$

or,

$$\dot{k}_t = sAk_t^{\beta} L_t^{\alpha+\beta-1} - (\delta + n) k_t. \tag{8}$$

---

2 Actually the population grows with a rate of n.

By dividing both sides of (8) by k_t, we define the ratio

$$\frac{\dot{k}_t}{k_t} = \gamma_k \tag{9}$$

as the growth rate of the capital per worker. Let us define as steady state the state at which all variables grow at the same pace. Then, at that state, γ_k is constant and independent of time. Using (9) into (8), and taking logarithms and derivatives with respect to time, we get:

$$0 = [\gamma_k(\beta - 1) + n(\alpha + \beta - 1)](k_t^{\beta-1} L_t^{\alpha+\beta-1}) \tag{10}$$

or,

$$0 = [\gamma_k(\beta - 1) + n(\alpha + \beta - 1)]. \tag{11}$$

If we make the assumption that Solow (1956) did about constant returns to scale of the production function, in capital and labor inputs ($\alpha + \beta = 1$), but decreasing returns to capital only ($\beta < 1$), (11) becomes

$$0 = \gamma_k(\beta - 1). \tag{12}$$

Because $\beta < 1$, and capital exhibits decreasing returns to scale, the only way that (12) should hold, is if the steady state growth rate is equal to zero, or

$$\gamma_k = 0. \tag{13}$$

Thus, in the neoclassical growth model the only possible steady state is reached if the growth rate of capital (and of the economy) is equal to zero. How does the neoclassical

growth theory explain the observed growth of economies? The answer is exogenous technological change which is represented by a time-dependent technological parameter A_t in the production function (1).[3]

Let us elaborate more on equation (11). Assume that constant returns to scale ($\alpha + \beta = 1$) prevail. If we then want the economy to exhibit positive growth rates, β should be equal to one ($\beta = 1$), an assumption that holds when capital exhibits constant returns. This assumption exists in the models of endogenous economic growth with constant returns, in the famous paper by Rebelo (1990).

A more interesting interpretation is the following. Assume that population growth is zero ($n = 0$). We can still experience economic growth with non reproducible inputs (labor, $\alpha > 0$) together with constant returns to capital $\beta = 1$. It is obvious that the latter one implies increasing returns to scale, since $\alpha + \beta > 1$, which leads to models of endogenous economic growth with increasing returns. The assumption of a production function that exhibits increasing returns to scale causes problems, since we cannot find a set of prices that supports a competitive equilibrium.

One way to get around this problem is to follow Marshall's view on increasing returns. Marshall introduced increasing returns on the aggregate level while firms faced constant returns to scale in their production. How can this happen? It can be done by assuming that spillovers exist in the economy. Each firm's decision affects other firms, but none of them considers these external effects. Each firm maximizes its profits without considering the effects on its production of the decisions that other firms make. Marshall introduced a production function with constant returns to scale and an extra factor that measured these externalities. Thus in the following production function

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3 It is also assumed that the growth rate of A_t is constant over time, and exogenously determined.

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha} \kappa_t^{\zeta}, \tag{14}$$

K_t is the private capital, while κ_t is the aggregate level of capital in this economy. Individual firms maximize their profits taking the value of κ_t as given and determined exogenously for them. This idea was used by Arrow (1962), and more recently by Romer (1986).

Another approach in solving the problem that increasing returns to scale in the production function cause is to drop the assumption of perfect competition. Let us assume that the economy follows the Chamberlinian approach (a monopolistic competition assumption). Then we can talk about growth models that involve knowledge as an input factor. Under these assumptions, the effects of research and development become very crucial, since research and development lead to the creation of knowledge. Work with these assumptions has be done by Grossman and Helpman (1991*a,b,c,d*).

2.2. Learning by Doing Models with Externalities (Spillovers)

In this section we will make a quick reference to the work by Arrow and Romer on learning by doing with spillovers. Arrow (1962)[4] introduced the idea of increasing returns to scale for the economy as a whole, while firms were experiencing constant returns to scale. It is obvious that to support a competitive equilibrium solution, the assumption of

4 "The Economic Implications of Learning by Doing", Review of Economic Studies, June 1962.

increasing returns being external to the firms is required. He argued that knowledge was related to experience. More formally the production function takes the form of (14),

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha} \kappa_t^{\zeta} \tag{15}$$

where the κ_t is the aggregate level of capital available in the economy, and $0 < \zeta < 1$.

In Arrow's formulation, growth is possible if and only if population continues to grow over time. The growth rate of per capita income increases directly with the rate of population increase, but goes to zero if population growth is zero. Arrow assumed that the aggregate elasticity of output with respect to capital and to aggregate capital (knowledge), was less than one (here, $\alpha + \zeta < 1$). Then it is obvious that a steady state still exists.

Romer (1986) based his work on Arrow's, but went further. He suggested that $\alpha + \zeta$ can equal one. He assumed that knowledge was a capital good with an increasing marginal product. A finite valued social optimum is guaranteed to exist since research technology exhibits diminishing returns to scale. Romer also mentioned the case that $\alpha + \zeta$ could be greater than one. For this case an explicit solution was not given, but phase plane analysis was used to study the equilibrium. In the model we are going to describe below, we will refer only to the cases of $\alpha + \zeta$ being less or equal to one.

Let us now briefly describe this model. The production function is a variation of equation (14). The production function refers first to any firm, and then to the aggregate level. Thus the production function becomes

$$Y_{it} = AK_{it}^{\alpha} L_{it}^{1-\alpha} \kappa_{it}^{\zeta} \tag{16}$$

where the subscript i denotes any firm i .

Furthermore assume that the number of firms in the economy is M. If M is big enough, each firm can take the size of aggregate capital κ_t as given. Following the assumptions stated above, one can assume that in reality

$$\kappa = \sum_{i=1}^{M} k_i = Mk . \tag{17}$$

Equation (17) says that the aggregate level of capital, or knowledge (in the case of Romer, 1986), is a function of the level of knowledge used by each specific firm. It is obvious, then, that the engine of growth in this model is the continuous increase of the aggregate stock of knowledge.

Aggregating then across all M firms, we get a production function similar to (14), or

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha} \kappa^{\zeta}, \tag{18}$$

where now $K_t = MK_{it}$, $L_t = ML_{it}$, while (17) still holds for κ. If then divide (18) by L_t, and define capital per worker as

$$k_t = \frac{K_t}{L_t}, \tag{19}$$

and output per worker as

$$y_t = \frac{Y_t}{L_t}, \tag{20}$$

equation (18) becomes

$$y_t = Ak_t^{\alpha} \kappa^{\zeta}. \tag{21}$$

Consumers share the same preferences. We use a model of an infinite lived household, with preferences given by the following the utility function:

$$U = \int_0^{\infty} e^{-\rho t} u(c_t)\,dt, \tag{22}$$

where c_t is the consumption level of each individual, and ρ is the constant rate of time preference ($\rho > 0$). Population is constant, and thus we represent utility function as

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}, \tag{23}$$

where σ^{-1} is the inter-temporal elasticity of substitution.[5] Then substituting (23) into (22), we get:

$$U = \int_0^{\infty} e^{-\rho t} \left[\frac{c_t^{1-\sigma} - 1}{1-\sigma} \right] dt. \tag{24}$$

Consumers face a inter-temporal budget constraint, which states that investment (new capital) equals to savings (output that is not consumed).[6] If, making use of (19)-(21), the budget constraint is given by

$$\dot{k}_t = y_t - c_t = A k_t^{\alpha} \kappa^{\zeta} - c_t. \tag{25}$$

Consumers seek to maximize their utility level as defined by (24) subject to the budget constraint (25), or

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5 It is assumed that there is a continuum of consumers on the interval [0,1].

6 Here it is assumed that population growth is zero, as Romer states in "Increasing Returns and Long-Run Growth", page 1021.

$$\underset{c_t}{\text{Max: }} U = \int_0^{\infty} e^{-\rho t}\left[\frac{c_t^{1-\sigma}-1}{1-\sigma}\right]dt \tag{26}$$

$$\text{s.t. } \dot{k}_t = A k_t^{\alpha}\kappa^{\zeta} - c_t. \tag{27}$$

To solve this maximization problem, we form the corresponding current value Hamiltonian (H_c)

$$H_c = \frac{c_t^{1-\sigma}-1}{1-\sigma} + \lambda_t (A k_t^{\alpha}\kappa^{\zeta} - c_t). \tag{28}$$

Take the first order conditions for the current value Hamiltonian H_c

$$\frac{\partial H_c}{\partial c_t} = 0 \Leftrightarrow c_t^{-\sigma} = \lambda_t, \tag{29}$$

and

$$\dot{\lambda}_t = \rho\lambda_t - \lambda_t A\alpha k_t^{\alpha-1}\kappa^{\zeta}. \tag{30}$$

Then taking time derivatives in (29), we get

$$-\sigma c_t^{-\sigma-1}\dot{c}_t = \dot{\lambda}_t, \tag{31}$$

and substituting (31) into (30), and making use of (29), we get

$$\gamma_c = \frac{\dot{c}_t}{c_t} = \frac{\alpha A k_t^{\alpha-1}\kappa^{\zeta} - \rho}{\sigma}, \tag{32}$$

where γ_c is the growth rate of consumption per capita.

Because the economy is closed, equilibrium in the capital market[7] requires that total capital stock equals the sum of individual capital stocks. Then it is obvious that

$$\kappa = L_t k_t \tag{33}$$

making use of (19). In this case, then, by substituting (33) into (32) we get the growth rate of consumption γ_c

$$\gamma_c = \frac{\alpha A k_t^{\alpha+\zeta-1} L_t^{\zeta} - \rho}{\sigma}. \tag{34}$$

Several points can be made by examining equation (34). First, it is obvious that if population size increases, this will cause consumption growth γ_c to be positive. Countries with big populations should experience bigger growth. Second, the question that we need to ask ourselves is if this model produces sustained growth, or in other words, when does $\gamma_c > 0$. The answer to this question depends on the assumptions we make about the sum $\alpha + \zeta$. Let us assume that $\alpha + \zeta < 1$. The model behaves like a neoclassical one, with the only difference being that now capital share is not α, but $\alpha + \zeta$. In this case the steady state implies that $\gamma_c = 0$. The last result shows that assuming increasing returns to scale does not necessarily suggest sustainable economic growth. So to achieve sustained economic growth, that is $\gamma_c > 0$, we need to assume that $\alpha + \zeta = 1$. Third, the growth rate of consumption depends on the size of the technological parameter A. The larger the parameter A is, the bigger the consumption growth. Clearly growth can be sustained as long as knowledge keeps increasing. This increase causes the growth rate of consumption to be different from zero (that is what the neoclassical model supports) as long as the economy experiences increasing returns to scale.

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7 We assume that capital includes physical capital, and the capital devoted to knowledge.

2.3. Models of Economic Growth with Imperfect Competition

In the previous section economic growth was sustained due to increasing returns to scale. Increasing returns arose from the accumulation of knowledge. Romer ("Endogenous Technological Change", 1990) introduced a model in which knowledge was accumulated through changes that took place in the human capital, and technology played the role of an input. According to Romer, technology is a nonrival, partially excludable good.

The argument of his paper was based on the following three points:

1. Technological change is related directly to economic growth. By definition, economic growth leads to technological change, and vice versa. Thus, Romer claimed, his model resembled Solow's (1956) model, with the addition of technological change. When technological change takes place, there is an extra incentive for capital accumulation, which drives the increase in the output.

2. In the model presented here, technological change was endogenous, rather than exogenous. It arose because individuals realized, and took advantage of, market incentives.

3. As long as knowledge is produced it can easily be used in the production process. For example, after a new type of robot used in the automotive industry is designed and produced, any extra units produced would occur with a standard fixed cost, since the research was carried out for the prototype.

Another of Romer's crucial assumptions was that firms operated in a non-competitive environment. He claimed that if the three points made above held, then there

would not be a competitive equilibrium. The only way to accommodate these three points was to assume a monopolistic competition framework.

We have to distinguish between scientific inputs, i.e., designs, and knowledge as input. A design for a new good is a nonrival input. By nonrival, we mean that this good has a property that limits no one from using it. Non rivalry is a characteristic of the technology itself. Conventional economic goods are both rivalrous and excludable. A good is excludable if its owner can prevent others from using it. As far as the design of a new good is concerned, we can say that is partially excludable. As long as someone holds a patent on a specific design, this makes the good excludable. But if his patent allows others to use the design, the good becomes partially excludable, since others are prohibited from copying the design.

If we then assume that we have a set of goods that are nonrival, but still excludable, this becomes an interesting case for the theory of economic growth. Among the three points listed above, the third point implies that knowledge is a nonrival input. On the other hand, the second point implies that individuals participate in the creation of new goods because the opportunities for profits are there, which implies that knowledge is at least partially an excludable good. Finally, the first point states that economic growth will be achieved, if an accumulation of partially excludable and nonrival inputs takes place.

It is obvious from the discussion so far, that there is a distinct difference between human capital and knowledge as inputs in the production process. Human capital as an input is rival and excludable.[8] The assumption of non-rivalry causes two important

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8 Romer gives as an example the ability for adding. If you can add, then you can not do simultaneously many other problems, and thus this characterizes human capital as a rival good. A design on the other hand is nonrival, since it can be copied.

implications. First, goods that are nonrival can be accumulated boundlessly, while the same cannot happen with human capital. As Romer stated

> *"...when a person dies, the skills are lost, but any nonrival good that this person produced, ..., lives on after the person is gone."*[9]

Second, by assuming that knowledge is a nonrival input, we are able to talk about knowledge spillovers that are nothing else than partial excludability.

2.3.1. Description of the Model

Assume an economy where labor, physical capital, and human capital are combined to produce the final good. Physical capital is measured in units of consumption goods, while labor for is equivalent to the population of the country. Human capital measures the cumulative effect of all activities, like education or on-the-job training. To include in the model both kinds of knowledge, the rival and the nonrival one, Romer defined H as the rival component of knowledge, and A as the nonrival technological component of knowledge. Each unit of knowledge corresponds to a design for a new good, and thus we can measure A by counting the number of designs.

This economy consists of three sectors. There is a research sector that uses as inputs human capital and knowledge to produce new knowledge, that is, new designs for durables. Second, the intermediate[10] goods sector that uses the designs produced at the

9 "Endogenous Technological Change", Journal of Political Economy, October 1990, page S75.
10 The terms intermediates and durables will be used interchangeably

research and development sector together with forgone output to produce durables necessary for the production of the final goods. Third, the final goods sector uses labor, human capital, and durables to produce final goods. The final output is either consumed or saved. Because population size is constant, the supply of labor is constant too. Furthermore, the stock of human capital available in this economy is fixed, as is the fraction of the human capital used in the production of the final good. Some other simplifying assumptions are also made. First, we assume that capital goods are produced in the same fashion as the final good, in separate sector. Thus, forgone consumption is equivalent to shifting resources from the final good production to the capital production. Second, we assumed earlier that research was human capital intensive. We extend the assumption by assuming that physical capital, as well as labor, can be used to produce research.

The characteristic of the production process of the final good is that it incorporates the use of durable goods. So if we index these goods by i, only a finite number of the designs have been invented and produced until today. Thus, if by

$$x = \{x\}_{i=1}^{\infty}$$

denote the list of all available durables used by a firm to produce the final good, there should be some value A, such that

$$x_i = 0 \quad \text{for all} \quad i \geq A.$$

In a sense, A represents the set of all available designs up to now. It is obvious then that A changes as new durables are invented. Assume a Cobb-Douglas type production function

$$Y(H_Y, L, x) = H_Y{}^{\alpha} L^{\beta} \int_0^{\infty} x(i)^{1-\alpha-\beta} di, \tag{35}$$

where Y is the final good produced, H_Y is the portion of human capital devoted to the production of the final good, L is the labor needed, and $x(i)$ is the *ith* durable good used in the production of the final good. Since (35) is homogeneous of degree one, the final goods sector can be represented by a single price-taking firm. This assumption does not hold for the case of the durables sector. Firms in the durables sector participate in a non-competitive environment. Furthermore there is a distinct firm i for each durable i produced. Once a firm owns a design, it can convert η units of final good into one durable unit of good i .

The firms that produce the durable goods, rent them out at a price $p(i)$. Since we assume those durables do not depreciate, the value of one unit of durable i is the discounted present value of the infinite stream of the rent that it generates. The physical capital evolves according to

$$\dot{K}(t) = Y(t) - C(t), \tag{36}$$

where $C(t)$ represents the consumption level at time t . It takes η units of consumption to create a unit of durable good, and thus the size of K is related to the number of durables used in the production of the final good. Specifically,

$$K = \eta \sum_{i=1}^{\infty} x(i) = \eta \sum_{i=1}^{A} x_i \tag{37}$$

since A is the maximum number of available durables at each time.

It is now time to define the process for accumulating new designs. Research and development use as their main input knowledge in terms of human capital. Everyone who

engages in research has access to already produced knowledge. Research that has been published becomes available for everyone who might want to use it in any future production process. Thus, we have here the non-rivalry of knowledge. In other words, all researchers have access to A if A denotes the body of knowledge produced up until today. Then, for all people working in the research sector, the aggregate stock of knowledge evolves according to

$$\dot{A} = \delta H_A A \tag{38}$$

where H_A is that portion of human capital devoted in the production of knowledge (research). Observing more carefully equation (38), we notice some interesting points. First, by increasing the size of human capital devoted to research, we end up producing more designs. Second, the larger the initial stock of knowledge, the bigger the expansion in the research sector. Thus, countries that start with more human capital can produce more new designs and thus increase the size of their knowledge. This argument is similar to the argument used extensively in the neoclassical growth theory, that big countries, in terms of population size, grow faster than small ones.

Notice that knowledge enters the production process in two ways. By having a newly produced design, we can use it in the production of the final good. On the other hand, this new design increases the stock of knowledge and thus increases the productivity of the research sector as (38) clearly shows.

Finally, some extra assumptions are required. We assume that the total stock of human capital H, equals the sum of its two components,

$$H = H_A + H_Y. \tag{39}$$

Also, let spot prices at any point in time to be measured in units of current output, and let r denote the interest rate. Define as P_A the price of a new design, and as w_H the rental rate per unit of human capital. Remember, though, that consumption goods are converted one to one into capital goods. This means that the spot price for capital is one with a rate of return equal to r. Following from equation (38), and because everyone can take advantage of the available knowledge, the relation between the price of a new design P_A, and the rental rate of human capital w_H, is given by

$$w_H = \delta A P_A. \tag{40}$$

Each firm in the production of a final good takes the price of a new design P_A as given. The same holds for the price of a capital good. When they start producing, they maximize their profits for given prices, and thus the price for each durable good $p(i)$ is determined. Then, each producer of final good faces the following problem:[11]

$$\underset{x}{\text{Max}} \int_0^{\infty} \left[H_Y{}^{\alpha} L^{\beta} x(i)^{1-\alpha-\beta} - p(i)x(i) \right] di. \tag{41}$$

Taking then the first order conditions on (41) we get

$$p(i) = (1-\alpha-\beta) H_Y{}^{\alpha} L^{\beta} x(i)^{-\alpha-\beta}, \tag{42}$$

which is the inverse demand function for durable goods.

The producers of the durables take the demand curve in (42), and maximize their profits if (42) is given. Each firm that produces durables, maximizes the difference between its revenues and the cost of using new designs

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11 In following the analysis x we drop the time subscripts, but we continue to assume that these variables are varying with time.

$$\pi = \max_{x} p(x)x - r\eta x \tag{43}$$

or if we substitute (42) into (43), we get

$$\pi = \max_{x} (1-\alpha-\beta) H_Y{}^{\alpha} L^{\beta} x^{1-\alpha-\beta} - r\eta x. \tag{44}$$

In equation (42) the rental income is given by $p(x)x$, while the term $r\eta x$ represents the interest cost on the ηx units of final good needed to produce x units of durables. It is obvious that each producer of durables maximizes (44) subject to (42), or

$$\pi = \max_{x} (1-\alpha-\beta) H_Y{}^{\alpha} L^{\beta} x^{1-\alpha-\beta} - r\eta x \tag{45}$$

$$\text{s.t. } x(i) = \left(\frac{(1-\alpha-\beta) H_Y{}^{\alpha} L^{\beta}}{p(i)} \right)^{\frac{1}{(\alpha+\beta)}}. \tag{46}$$

The producer has constant marginal cost facing a constant elasticity demand curve. Each producer of durables cannot discriminate in prices, and thus the price he charges for his durable is the same for all the producers of the final good. In addition, each producer of durables decides to enter the market for producing durables, if and only if the present value of his profits is equal to the cost of the initial investment in research and development $P_A(t)$. Then each of these producers enters the market for durables if at any period t the following is true:

$$\int_{t}^{\infty} e^{-\int_{t}^{\tau} r(s)ds} \pi(\tau) d\tau = P_A(t). \tag{47}$$

If we assume that the P_A is constant over time, take derivatives with respect to time, and solve using (47), we get

$$\pi(t) = r(t)P_A, \tag{48}$$

which states that any excess revenues should be sufficient to cover the interest rate on the cost of the initial investment in a design.

The analysis of consumers is the same as the one described by equations (24)-(32). If consumers are facing a fixed level of interest rate ($r_t = \bar{r}$) the growth rate of consumption is given by

$$\frac{\dot{c}_t}{c_t} = \frac{\bar{r} - \rho}{\sigma}. \tag{49}$$

2.3.2. Solution of the Model

It is clear from the analysis so far, that if A is constant over time, the model becomes similar to Solow's. The assumption that the number of intermediates increases over time becomes very crucial, because then the number of durables becomes the engine of growth. If we want to get a solution for a balanced path equilibrium, we need to derive a relation between the growth rate of output and the rate of return to investment. This, together with the relation between the interest rates and the growth rate coming from the preferences' problem, determines the rate of growth and the interest rate. Romer (1987) proved that along the balanced path, the ratio of K to A remained constant, which implied

that $\bar{x}$ was constant. Then, what is $\bar{x}$? If we solve the problem that each producer of durables faces, we find that the price he has to charge for his durables is given by

$$\bar{p} = \frac{r\eta}{(1-\alpha-\beta)}. \tag{50}$$

Then the profits corresponding to that price are

$$\pi = (\alpha+\beta)\bar{p}\bar{x}, \tag{51}$$

where $\bar{x}$ is the quantity of durables corresponding to the price $\bar{p}$, and is found by the demand for durables (46).

Looking at equation (38) we see that the productivity of human capital increases in proportion to A. Since the productivity of human capital increases with the same pace in both sectors that use human capital, then H_A and H_Y should remain constant if and only if P_A is constant. In other words, the wage rates paid in both sectors to human capital should be the same. In the final goods sector human capital is paid its marginal product, but since human capital receives all its income from the research sector, the wage earned there is equal to $P_A \delta A$. The wage rate has to be the same in both sectors as far as the human capital is concerned, so we have to choose that much H_Y, such that the following is true:

$$\begin{aligned} w_H = P_A \delta A &= MP_{Y_{H_Y}} \\ &= \alpha H_Y^{\alpha-1} L^{\beta} A \bar{x}^{1-\alpha-\beta}. \end{aligned} \tag{52}$$

The present value of profits should be equal to the price of the design P_A. Then in one period this is given by

$$P_A = \frac{1}{r}\pi = \frac{\alpha+\beta}{r}(1-\alpha-\beta)H_Y^{\alpha} L^{\beta} \bar{x}^{1-\alpha-\beta}. \tag{53}$$

Using the last expression into (52), and doing the calculations we get

$$H_Y = \frac{\alpha}{\delta(1-\alpha-\beta)(\alpha+\beta)} r. \tag{54}$$

If we assume that $\bar{r}$ is constant, then $\bar{x}$ is also constant, and the production function of the final goods (35), becomes

$$Y_t = H_Y{}^{\alpha} L^{\beta} \int_0^{\infty} \bar{x}^{1-\alpha-\beta} di = H_Y{}^{\alpha} L^{\beta} A \bar{x}^{1-\alpha-\beta}. \tag{55}$$

Making use of the consumption growth rate found earlier, and the fact that the ratio K/Y is constant, the growth rate is given by

$$g = \frac{\delta H - \Lambda\rho}{\sigma\Lambda + 1}, \tag{56}$$

where Λ is defined by the following:

$$\Lambda = \frac{\alpha}{(1-\alpha-\beta)(\alpha+\beta)}. \tag{57}$$

An interesting point about (56) is that if the interest rate is larger, the present discounted value of the net revenues will be lower. Also, if we devote less human capital to the research process, the rate of growth will be lower. Clearly an economy with larger stock of human capital will experience a larger growth rate. The latter suggests, then, that free international trade will lead to faster growth.

2.4. Expanding Product Variety Models

These types of models fall in the category of research and development growth models. Their basic assumption is similar to the Romer's model reviewed in the previous section. Research and development cause spillovers on the aggregate stock of knowledge. Research is considered as a typical economic good, which for its production requires resources and offers profit opportunities. For the purposes of this review we will refer to the book by Grossman and Helpman *Innovation and Growth in the Global Economy*.[12]

Grossman and Helpman divided product innovation in two major categories. If the new products provide the same services as the old ones but with better *quality*, then the expansion of products will cause a vertical relation among the products. On the other hand, if the new products added to the variety of those already in existence, then they would cause a horizontal relation among the products. Here we will refer extensively to the expansion of the varieties of the available products.

Their modeling depends on how technology as product is treated. If technology is considered a private product, economic growth will eventually stop . If, on the other hand, technology is treated as a public good (as in Romer 1990), endogenous innovation can be self-sustaining. In the next two sections we will refer to these two ways of treating technology, starting by assuming technology as a private good.

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12 Chapter 3 refers to expanding product varieties, and it is the one that is reviewed here.

2.4.1. Knowledge Treated as a Private Good

In this case, entrepreneurs invest resources for developing new goods. We will assume that the new varieties substitute imperfectly for existing ones, and that the innovators exploit monopolistic profits. We will start the model by describing in the next section the consumer's behavior and then the producer's behavior.

2.4.1.1. Consumers

Let us assume a representative household that seeks to maximize its utility level over time. Its inter-temporal preferences take the following form:

$$U_t = \int_t^{\infty} e^{-\rho(\tau - t)} \log D(\tau) d\tau, \tag{58}$$

where $D(\tau)$ is an index of consumption at time τ, and ρ is the discount rate. D represents the diversity of tastes of the household that leads to demand for differentiated products. Even though we can assume that the number of products extends to infinity, and index them by $j \in [0, \infty)$, at any moment in time only a finite subset of these varieties will be available to the consumers. Thus, the number of brands on the market is given by the interval $[0, n(t)]$, where $n(t)$ is a measure of the brands available to consumers that were

developed in the labs before time t. Then, if we adopt for D the specification used by Dixit and Stiglitz (1977), we have

$$D = \left[\int_0^n x(j)^{\alpha} dj \right]^{\frac{1}{\alpha}}, \tag{59}$$

with $0 < \alpha < 1$. It is easy to show that the elasticity of substitution is $\varepsilon = \frac{1}{1-\alpha} > 1$. Then, if we maximize (58) subject to the following budget constraint

$$\sum_j x(j) p(j) = E, \tag{60}$$

where $p(j)$ is the price for each variety of goods, and E is the expenditure level of each household, we get the following demand for the brand j:

$$x(j) = \frac{E\, p(j)^{-\varepsilon}}{\int_0^n p(j')^{1-\varepsilon} dj'}. \tag{61}$$

If we now assume that $D = E/p_D$, we can rewrite (58) as following:

$$U_t = \int_t^{\infty} e^{-\rho(\tau - t)} \left[\log E(\tau) - \log p_D(\tau) \right] d\tau. \tag{62}$$

Then maximizing (62) with respect to the budget constraint (60), each household chooses the composition of the level of spending so to maximize utility, and tries to optimize the time path of spending. The latter leads to the following condition with which spending evolves:

$$\frac{\dot{E}}{E} = r - \rho, \tag{63}$$

and if $E(t) = 1$ for all t, then (63) implies that

$$r = \rho, \quad \text{for all } t. \tag{64}$$

A question that needs to be answered concerns the way that households allocate their wealth across the available assets. It is implicitly assumed that there exists a market for consumption loans. Also, firms issue equities for financing their research projects, and households can purchase shares in profit making firms. Notice that there is no uncertainty in the modeling. Thus, stocks and bonds are perfect substitutes, which in equilibrium means that they yield the same rate of return.

2.4.1.2. Producers and Static Equilibrium

Producers participate in two production activities: they create the blueprints of the new intermediates, and they produce them. Let us assume that the up-front cost of designing new intermediates is fixed and independent of the number of durables N. Each variety is produced by a single firm, and all known varieties are produced with constant returns to scale production technology. Furthermore, let us assume that the economy is endowed with a single production factor, called labor. The technology specified above makes sure that the marginal cost at time t is always equal to the wage rate $w(t)$. Each of these producers takes the demand for intermediates given by (61) as given, and wants to maximize his profits:

$$\pi(j) = p(j)x(j) - wx(j) \tag{65}$$

at every time t, and where j represents a variety of the available intermediates. The above problem leads to an equalization of the price $p(j)$ to the wage rate w, $p(j) = w/\alpha$. By assuming a monetary equilibrium, all the varieties will be priced the same, or

$$p = \frac{w}{\alpha}. \tag{66}$$

If the demands for intermediates are symmetric and set the expenditures level equal to one ($E = 1$), the operating profits per variety are given by the following:

$$\pi = \frac{1-\alpha}{n}. \tag{67}$$

One can consider the profits defined previously to be only a part of the income that the owners of firms enjoy. We might want to consider the case for which equity holders expect capital gains or losses on their ownership shares. Thus, if by $v(t)$ we define the value of a claim to an infinite stream of profits that a firm receives at time t, then in equilibrium capital market requires that:

$$\pi + \dot{v} = r\,v. \tag{68}$$

Equation (68) relates the interest rate r with the total return to capital. Furthermore, if we assume that the stock market value of a firm equals the present discount value of its profits, then (68) becomes:

$$v(t) = \int_t^{\infty} e^{-[R(\tau)-R(t)]}\,\pi(\tau)\,d\tau, \tag{69}$$

where $R(t)$ is the cumulative discount factor applied to profits earned at time t.

Each entrepreneur devotes a finite amount of labor to the production of the intermediate goods. Firms can freely enter the production process of intermediates, and finance their up-front costs by issuing equities. Each producer who devotes l units of labor in the production of intermediates for time dt, can produce $dn = (l/a)\,dt$ new products with total cost of production $w\,l\,dt$. This process creates value for the producer $v(l/a)dt$, assuming that the value of each blueprint is v. Producers maximize their values v, and thus we get the following cases:

$$\begin{cases} \dfrac{v}{a} > w \text{ if } l \text{ is large} \\ \dfrac{v}{a} < w \text{ if } l = 0 \end{cases}, \tag{70}$$

where the first case does not arise in a general equilibrium, while the second one implies that there is no investment taking place in the research and development sector of the economy. Thus

$$wa \geq v \tag{71}$$

where the equality holds whenever $\dot{n} > 0$.

The last requirement for a static equilibrium is for the labor market to clear. The population supplies L units of labor that are used in the research and development sector. If the total number of new goods is given by $\dot{n}$, then total employment in R&D is given by $a\dot{n}$, where a is a productivity parameter of labor in the R&D sector. If we then set the aggregate expenditures equal to one, $E = 1$, and the price of differentiated products to p, each firm sells $1/np$ units. Then the labor market equilibrium is given by

$$a\dot{n} + \frac{1}{p} = L, \tag{72}$$

where the first part of the left hand represents the employment in the research and development sector, and the second the employment in the manufacturers' sector.

2.4.1.3. Dynamic Equilibrium

Looking at equation (69), we notice that there is a chance that no product development will take place, if an economy inherits a sufficient set of differentiated goods. So, if, for example, we assume that there is no research taking place, the labor market clearing condition requires that all the labor force is employed in the manufacturing sector. Using this and the hypothesis of free entry, we get the upper bound of the number of differentiated goods that can be introduced to an economy to be:

$$\bar{n} = \frac{(1-\alpha)L}{\alpha\rho a} \quad . \tag{73}$$

Suppose now, that the economy inherits a few intermediates. Then, the R&D sector exists, and there are firms that compete for the fixed labor supply. The equilibrium markups do not vary with n, as (66) shows, but the sales per brand decline. As a result, the profit rate falls over time, and eventually new inventions equate the profit rate to the discount rate, at which point any incentive for commercial R&D disappears.

To prove the last statements, let us assume for a minute that innovation takes place. Then $\dot{n} > 0$, and thus from (71) $wa = v$. Since labor employment is non negative in the

R&D sector, for the production of R&D to be profitable, the reward should be high. Then $\dot{n} > 0$ implies that $v > \bar{v}$, where

$$\bar{v} = \frac{\alpha a}{L}. \tag{74}$$

Another point is that when R&D takes place, the number of new innovations at any point in time is equal to the employment in research divided by the productivity parameter a. Then using (71), (66) and (72) we get

$$\dot{n} = \frac{L}{a} - \frac{\alpha}{v}. \tag{75}$$

Therefore the number of parameters incorporating (74) into (75) is given by

$$\dot{n} = \begin{cases} \frac{L}{a} - \frac{\alpha}{v} & \text{if } v > \bar{v} \\ 0 & \text{if } v \le \bar{v} \end{cases}. \tag{76}$$

Substituting then (64) and (67) into (68) we get the equation that defines the change in the value of a firm as a function of the number of new products and the current value of a blueprint, or

$$\dot{v} = \rho v - \frac{1-\alpha}{n}. \tag{77}$$

The last two equations define the dynamic equilibrium in this economy.

2.4.2. Public Knowledge Capital

In the preceding analysis preceded we treated knowledge as a private good. As a result of this assumption, economic growth eventually stops, since the economy reaches the maximum number of intermediates. There are, though, some very specific characteristics of knowledge that were neglected. In this analysis, Helpman and Grossman follow Romer (1990) in distinguishing between two types of research and development. First, the assumption that each new innovation leads to monopolistic profits to its innovator still holds. Second, each innovation contributes to the general stock of capital; thus we observe economies external to the firms, but internal to the economy. The latter assumption leads us to treat capital knowledge as a public input to R&D.

Assume now that the product innovation is given by

$$\dot{n} = \frac{L_n K_n}{a} \quad , \tag{78}$$

where K_n is the stock of knowledge capital, and L_n is the aggregate employment in R&D. The equation (78) is a production function for new designs. It is obvious that the link between research and general knowledge capital should be addressed. Therefore they assumed that each R&D project contributed to the stock of general knowledge, independently of the amount of R&D undertaken in the past. For simplicity purposes, let us set the proportionality factor of knowledge capital equal to one, or

$$K_n = n \, . \tag{79}$$

There are some major implications of the assumption that knowledge is a public good. Notice that R&D requires $a\dot{n}/K_n = a\dot{n}/n$ units of labor per time unit to develop $\dot{n}$ new products. Then the labor clearing condition (72) becomes

$$\frac{a\dot{n}}{n} + \frac{1}{p} = L. \tag{80}$$

Similarly the cost of the blueprints is now $wa/K_n = wa/n$, and thus the free entry condition (71) is now

$$\frac{wa}{n} \geq v \ , \tag{81}$$

where the equality holds whenever $\dot{n} > 0$. It is evident then that (75) becomes

$$\frac{\dot{n}}{n} = \begin{cases} \frac{L}{a} - \frac{\alpha}{vn} & \text{if } v > \frac{\alpha a}{nL} \\ 0 & \text{if } v \leq \frac{\alpha a}{nL} \end{cases}. \tag{82}$$

Then the equations (77) and (82) define the equilibrium dynamics.

In the proceeding analysis Grossman and Helpman assumed an economy in which innovation of new products takes place. If we assume that the economy in consideration inherits enough numbers of durables, then economic growth will eventually cease. On the other hand, if the economy starts with no durables, and we assume that knowledge creates side effects, then economic growth can be sustained.

Chapter 3

CLOSED ECONOMY

3.1. Introduction

Let us assume that there is a closed economy that produces two goods, one final good and one intermediate good.[1] The intermediate goods (also called from here on *durable* goods) are considered as variations of capital goods. Labor and physical capital are used as production inputs. We also assume that capital as input is called "raw", because it is not used directly in any production process. Consumers are the owners of capital in its raw form. In a sense, as in Romer (1990), the production technology introduced here disaggregates capital into an infinite number of different types of durables.

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1 The model is similar to the approach introduced by Romer (1987, 1990) and later on by Barro and Sala-i-Martin (1990). Similar modeling is used by Grossman and Helpman in several papers. Our model follows Romer (1990a), even though Barro and Sala-i-Martin use similar modeling.

Then the producers of intermediates take, or rather rent, raw capital from the consumers and transform it into intermediates (useful capital, i.e., capital that can be used in the production of the final good).[2] A key role in our model is played by human capital. We assume that the abundance of human capital helps to keep the costs of production low, and thus each producer will prefer the labor force to be educated, rather than illiterate.

3.2. Description of the Model

The final good is produced by using labor and intermediate goods. Labor is supplied inelastically by consumers. The producers of intermediate goods use raw capital to produce the durables. The raw capital is owned by the consumers, and they are the ones who supply it to the producers of durables. To get the right to participate in the market for durables, the producers of durables have to pay the cost of research and development (R&D). This cost can be viewed as the fee necessary to permit access to the production process of durables. Furthermore, we assume that the cost of R&D depends on the abundance of human capital. That is, the more human capital that is available, the lower the cost that each firm faces upon entering the market of durable goods. How does this assumption affect the decision to enter the market of durables? Each firm is better off with an educated labor force, since it will lower their entry cost into the market. Human capital becomes a factor in the production process even though it is introduced indirectly into the

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2 An example of this type of capital can be hard disks, or screen displays, used in the assembly of a computer.

model. To keep the analysis handy, we further assume that upon payment of this fee, producers face the marginal cost of producing durables, which is assumed to be constant. Once a firm has produced a new durable good, it gains an infinite patent on that good.[3]

Assume that consumers are endowed with the initial size of raw capital $K(0)$. Consumers lend raw capital to the producers of intermediate goods, so they own these firms, and payments to them, received from the producers of durables, can be considered as dividends. Consumers live for an infinite lifetime. Finally, the final good is produced by combining labor and durables in the production process. Thus the producers of the final goods hire labor from consumers and buy durables from the producers of intermediate goods.

Before we continue with the description of the model, there are two questions that require an answer about durable goods:

a. How many producers of durables can produce the same intermediate good?

b. Are all durables necessary for the production of the final good?

To answer the first question we assume that each producer of an intermediate good has an incentive to invest in the production of a specific durable good, if and only if there exists an opportunity for profits. Thus, if a specific variety of a durable good exists, a copier of the same variety will make zero profits, since it is assumed earlier that he faces a high entry cost.[4] It pays then for each producer to concentrate on the production of only one durable good.

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3 This is a variation of the work done that incorporates human capital in the production process, as in Romer (1990), and Grossman and Helpman (1991c).

4 Which means that he has to hire human capital to reinvent something that already exists, and is already in the market by another producer of durables.

As far as the second question goes, we assume that all varieties of durables contribute the same to the production of the final good, and thus if one of these is missing, the final good cannot be produced. The latter assumption verifies that all the durables will be purchased by the producers of the final good. Even though the new types of capital (durables) are different from the old ones, there are neither better nor worse.

Since the intermediates are types of capital,[5] their price is essentially the interest rate paid to the holders of capital (the consumers, in this case). It is also assumed that the same production function is applied to every durable good. A similar assumption holds for the initial R&D cost and for the marginal cost of producing the durables.

Utilizing all the assumptions mentioned so far we use a simple extension of the Cobb-Douglas production function to describe the production process. The technology for producing the final good Y_t uses labor L and intermediate inputs x_j with $j = 1,...,N$, where N is the number of varieties available at time t. Then, the production function for the final good, takes the following form:

$$Y_t = AL^{\alpha} \sum_{j=1}^{N} x_j^{1-\alpha} \tag{1}$$

where x_j are the intermediate inputs, L is the labor force used in the production of the final good, and A is a productivity parameter.[6]

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5 Even though intermediates are considered as types of capital, we assume that there is no depreciation of it. But even with depreciation being different from zero, the analysis that follows does not change.

6 Something that needs to be stretched out is the degree of substitutability among the different types of durables. Romer (1990) pointed out that, " *in the conventional specification of a production function, all capital goods are assumed to be perfect substitutes... But in (1) durables have additively separable effects on output...*"

It is obvious that even though potentially there are an infinite number of durables x_j that can be produced, we assume that at any moment in time only a finite number of these have been invented. In other words the upper end in the summation of (1) instead of being infinity, becomes $N (N < \infty)$. Since (1) is homogeneous of degree one, we can describe the final good sector by taking a representative price-taking firm. We cannot do the same with the producers of intermediate goods. There should be a distinct firm forever variety j of durable goods. If there is more than one firm that produces a specific durable good, then N varies continuously in time. There are two reasons for the increasing number of intermediates over time (which is the same as the increasing number of firms that produce intermediates):

(i) Each producer of an intermediate good realizes that potential profits exist as long as he can come up with a different intermediate.

(ii) New firms enter the production process of intermediates, seeking potential profits.

Here we assume that new entries of firms occur, which is equivalent to new kinds of intermediates being produced. We do not consider the case of the quality improvements among the intermediates. In other words, we assume that some of the intermediates produced are necessary, and thus the newly produced intermediates are no better or worse than the ones that already exist. The assumptions about intermediates imply that they are all produced in equal amounts.

Treating then N_t as continuous, (1) becomes

$$Y_t = AL^{\alpha} \int_0^{N_t} x_j(t)^{1-\alpha} dj. \qquad (2)$$

This specification is due to Dixit and Stiglitz (1977) and has been used often when product differentiation is involved.[7]

Assume now that the available quantity of raw capital is K_t. This quantity is divided into many useful capital goods. As in Ethier (1982), total factor productivity increases when the number of available varieties of intermediates increases. Of course the latter implies that if we divide the available raw capital into an infinite number of varieties, we will get an infinite amount of output. To make the analysis meaningful, we assume that at any given moment the number of varieties is limited. A way to assume this is by imposing a cost for research and development. Producers of intermediates have to pay this entry fee (cost of R&D), which is dependent upon the level of human capital available in the country.

Clearly the market mechanism that supplies intermediates is not competitive. It is better to assume a monopolistic behavior of the producers of durables with free entrance, which better explains the previous set up. Recall that the producers of final goods buy all the available intermediates. In the model described in the following sections, the mechanisms that drives growth are the continuously introduced varieties of durables.[8]

Three types of agents participate in this economy: consumers (who consume the final goods, own capital, and provide labor), producers of the intermediate goods (who

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7 Romer (1987), using a dynamic infinite commodity version of Dixit-Stiglitz, introduced output as a function of all existing varieties of capital. A similar approach was used by Barro and Sala-i-Martin (1990).

8 Romer (1990) took a different approach here. He assumed that human capital was used in the production of the final good and in the production of durables. He further assumed that the growth rate of intermediates depended on the size of human capital devoted in the production of durable goods. In other words he assumed that the total stock of durables evolved according to $\dot{N}_t = \varepsilon H_N N_t$, where ε is a productivity parameter.

rent raw capital and transform it to durables), and producers of the final good (who use intermediates and labor to produce the final good).

In the discussion that follows, we show that the solution is characterized by exponential growth of the number of durables N_t, the size of the raw capital K_t, the size of the final good Y_t, and the abundance of human capital. Then, clearly, the model produces balanced growth paths. At equilibrium the interest rate r_t does not change over time but takes on a constant value. The solution of this problem will be determined by the relation between the growth rate of output, and the rate of return on investment. These are derived by solving the problems that the producers of final and intermediate goods face. Then by solving the consumers' problem, we will define the relation between the growth rate of consumption (which is the same as the growth rate of this economy), and the rate of interest.

The model is solved in three stages: we start with the producers of the final good by examining their behavior, then we examine the producers of the intermediate inputs, and finally we examine the behavior of the consumers.

3.3. Producers of the Final Good.

Let us assume that the number of firms producing the final good is big enough that they can be represented by a single aggregate price-taking firm. To work interchangeably with per capita and per firm quantities, we assume that there is a continuum of firms indexed on [0,1]. Thus each firm takes the price for each intermediate good as given. They

use the production technology given by (2), and rent each of the varieties of intermediate inputs x_j at rate (price) R_j. Assume that the price of the final good is equal to one. Each producer of the final good seeks to maximize his profits, or the present value of all future cash flows, by choosing the quantity of durables required for the production of the final good (labor is supplied inelastically by the consumers):

$$\underset{x_j(t)}{\text{Max}} \int_0^{\infty} e^{-rt}\left[Y_t - R_j(t)x_j(t) - w_t L\right]dt \tag{3}$$

$$\text{s.t. } Y_t = AL^{\alpha}\int_0^{N_t} x_j(t)^{1-\alpha}dj \tag{4}$$

where w_t is the wage rate paid to the workers of the final good sector.

From the first order conditions of the above we get:

$$AL^{\alpha}(1-\alpha)x_j^{-\alpha} = R_j,^{9} \tag{5}$$

which holds for any moment in time t, and that the marginal product of labor is equal to its price w_t,

$$A\alpha L^{\alpha-1}\int_0^{N_t}\left[x_j(t)^{1-\alpha} - R_j(t)x_j(t)\right]dj = w_t, \tag{6}$$

which is the inverse demand for labor by the producers of the final good.

Using (5) and solving for x_j, we get the demand for intermediate inputs

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9 The inverse demand for durables exhibits increasing returns to scale, since a doubling of the available durables causes R_j to increase (instead of remaining constant, as in the case of constant returns to scale).

$$x_j = L\left[\frac{A(1-\alpha)}{R_j}\right]^{1/\alpha} \tag{7}$$

where x_j is the amount of intermediates produced at any point in time t .

3.4. Producers of Intermediate Goods

It is obvious that if someone wants to derive the form of (2), he has to specify how x_j is related to capital K_t. We referred earlier to x_j's as durables which come from raw capital. The producers of durables use designs produced in the research and development sector.[10] For simplicity, we assume that each durable is produced with the same production technology. If K_t denotes the total amount of capital goods in existence owned by the consumers in this economy, then

$$K_t = \int_0^{N_t} x_j \, dj, \tag{8}$$

since all durables are produced by the same production process. This summation represents the cost of the total quantity of durable goods in terms of forgone consumption, since the price of the final good is by assumption equal to one.

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10 We assume that the research and development department are included in the firms that produce durables. In Romer (1990) the model specifies explicitly that human capital is used in the research and development process of new durables.

As mentioned earlier, in order to produce the intermediate inputs, producers have to employ raw capital. In order to enter the production process, they have to pay the cost of R&D. We assume that this cost depends on the abundance of human capital, and is expressed as human capital per units of output. Let us define this cost as β. Then

$$\beta = f(h_t) \tag{9}$$

where h_t defines the available human capital. We assumed earlier that the more educated the labor force, the lower the cost of entering the durables market, and thus $(\partial\beta/\partial h_t) < 0$. Then (9) can be written as

$$\beta = \frac{1}{h_t} \quad . \tag{10}$$

We also assume that the production of durables incorporates some kind of technology coming from the research and development sector. As a result, the new designs carry a price that represents at least the cost of creating new knowledge. Here, it is assumed that this price is incorporated into β. This allows each producer to develop a new variety of an intermediate good at constant marginal cost ϕ and to rent (sell) it at rate R_j. Specifically, producers of intermediate goods borrow raw capital K_t from the consumers, which is converted into intermediates x_j.[11]

Each producer takes the demand for intermediates provided by (7) as given and seeks to maximize his profits by choosing the price of the durables he uses, R_j. Then if we define his profits π (per unit of t) as

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11 Here we assume that the transformation of raw capital into durable goods takes place in the production process of the intermediate goods.

$$\pi = R_j x_j - r\phi x_j - \beta, \tag{11}$$

$R_j x_j$ represents the flow of income that the producers of intermediate goods earn by renting intermediates to the producers of the final good. The term $r\phi x_j$ is the interest cost on the ϕx_j units of output necessary to produce x_j units of durables.[12] Finally the term β is the initial cost for R&D, and it can be considered as an entrance fee to the production process of durables. Each producer of durables wants to maximize the size of his profits by taking into consideration that the producers of the final good demand as many durables as are defined by (7), or at any point in time t he faces the following problem:

$$\underset{R_j}{\text{Max:}}\ \pi = \underset{R_j}{\text{Max:}} R_j x_j - r\phi x_j - \beta \tag{12}$$

$$\text{s.t. } x_j = L\left[\frac{A(1-\alpha)}{R_j}\right]^{1/\alpha}. \tag{13}$$

Clearly the behavior of these producers is monopolistic. Furthermore by assuming that they cannot monitor the use of their products (durables), they cannot discriminate in prices either. As a result, the best thing for each firm is to charge its customers (i.e., producers of final good) the monopolistic price. Assume also that everyone in this economy can invest in R&D (i.e., pay the fee) and thus develop a new variety of intermediate goods. By allowing for free entry in this sector, profits should be equal to zero at any moment in time. The condition of zero profits implies that the amount of useful capital (raw capital transformed into durable goods) is fixed, and that if the demand of useful capital increases, it will be satisfied by the increase in the number of varieties.

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12 An underlying assumption (as in Romer, 1990) is that ϕ represents units of consumption that are forgone in order that one unit of durables be produced.

Thus it becomes obvious that the engine of growth in this model is the continuous introduction of new varieties of durable goods.

It was mentioned earlier that firms can enter freely into the production process of durable goods (the market for new designs and the production of new capital goods). This in fact means that the present value of profits π (as defined in (11)) at any point of time t should be equal to β, or[13]

$$\int_0^{\infty} e^{-\int_0^t r(s)ds} \pi(t)dt = \beta \tag{14}$$

For analytic purposes, we focus on a balanced growth equilibrium. Thus, we limit our attention to the steady state where interest rate $r(s)$ is equal to a constant value $\bar{r}$.

Solving the optimization problem defined by (12) and (13) and taking the first order conditions, we get

$$\frac{\partial \pi}{\partial R_j} = 0 \Leftrightarrow R_j = \frac{r\phi}{1-\alpha} \tag{15}$$

which gives the optimal rental rate (price) of intermediates.

Equation (15) says that the ratio of the present value of all future rental prices (incomes) R_j / r to the marginal cost ϕ equals a constant mark up. Since ϕ and α do not depend on j, the rental price R_j does not depend on j either, and thus the rental price is the same for all intermediate goods, since the interest rate gets the value $\bar{r}$ at the steady state. Thus (15) can be written as

$$\bar{R} = R_j = \frac{\bar{r}\phi}{1-\alpha} \Leftrightarrow \frac{R_j/\bar{r}}{\phi} = \frac{1}{1-\alpha} \quad . \tag{16}$$

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13 The inter temporal zero profit constraint comes from Grossman and Helpman (1989).

Using the result from (15) and (7) (which gives us the demand for intermediates, where x_j is defined as a function of R_j), we get

$$x_j = L\left[\frac{A(1-\alpha)^2}{\phi r}\right]^{1/\alpha}. \tag{17}$$

Note though that since $\overline{R}$ is constant in the steady state (as defined by (16), and L is exogenous to the model, x_j does not depend on time or j, and (17) becomes

$$\overline{x} = x_j = L\left[\frac{A(1-\alpha)^2}{\phi \overline{r}}\right]^{1/\alpha}. \tag{18}$$

Making use of the symmetry assumption and of (6), we can infer that

$$K_t = N_t \overline{x} \tag{19}$$

where $\overline{x}$ is given by (18). Romer (1990) used a simple graph to represent the behavior of intermediates. Figure 3.1 represents a rectangle with a height equal to $\overline{x}$ (since all intermediates produced up to time t are used at a level equal to $\overline{x}$), and a width equal to N_t. The area of the rectangle is equal to the total size of raw capital K_t. As we said earlier, overtime $\overline{x}$ remains the same, but the number of intermediates N_t grows exponentially.

From the zero profit condition (14), and if $R_j = \overline{R}$ and $x_j = \overline{x}$, we get:

$$\overline{x}\,(\overline{R}/\overline{r} - \phi) = \beta, \tag{20}$$

and substituting $\overline{R}$ for its equivalent from (15), we get

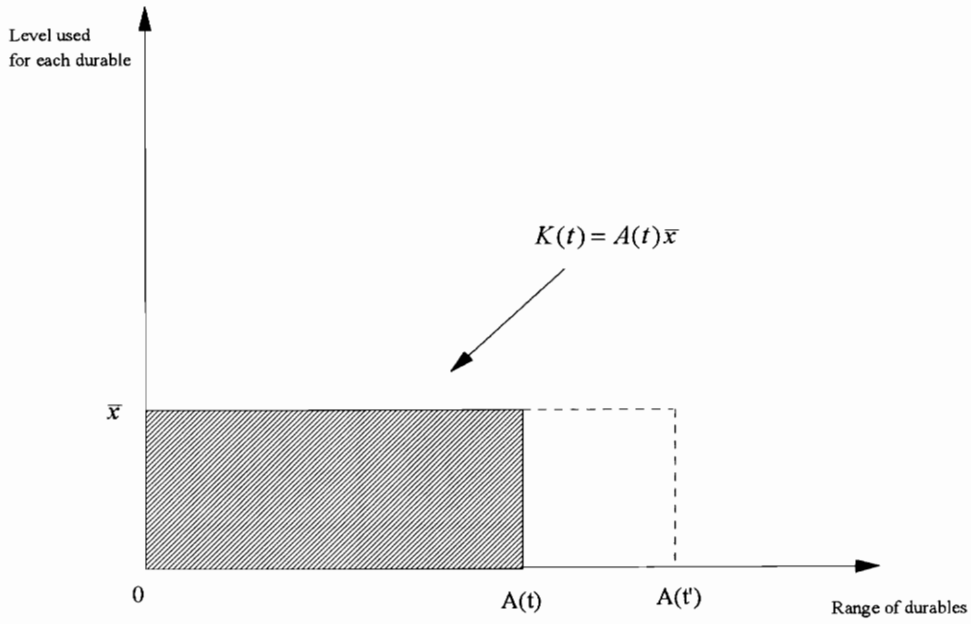

Figure 3.1.[14]

Intermediates Used in the Production at time t and t'.

14 This figure is similar to the one used by Romer (1990) in page S91.

$$\beta = \bar{x}\,(\phi/(1-\alpha)-\phi) = \bar{x}\,\phi\left[\frac{\alpha}{1-\alpha}\right]. \tag{21}$$

Combine now equations (18) and (20) to eliminate $\bar{x}$, which gives us a condition of the interest rate $\bar{r}$,

$$\bar{r} = A(1-\alpha)^{2-\alpha}\phi^{\alpha-1}\alpha^{\alpha}(L/\beta)^{\alpha}. \tag{22}$$

Equation (22) says that the interest rate $\bar{r}$ is equal to the private rate of return to investment. Also, the interest rate derived from the production side is independent of the growth rate. We know, though, that the private return to investment is equal to the marginal product of capital over the monopoly price of capital (which is given by equation (15)). Then (22) can be rewritten as

$$\bar{r} = \frac{(\partial Y/\partial x_j)}{\phi/(1-\alpha)}. \tag{23}$$

If we define the interest rate derived from the production side as r_{prod}, then (22) is rearranged as follows:

$$r_{\text{prod}} = \bar{r} = \frac{A\Theta L^{\alpha}}{\beta^{\alpha}\phi^{1-\alpha}} \tag{24}$$

where $\Theta = (1-\alpha)^{2-\alpha}\alpha^{\alpha}$. In the calculations to solve for the interest rate defined in the production side r_{prod}, we did not substitute β with its equivalent, as given in (11). The results do not change, though. The interest rate we calculated depends on the level of human capital, and thus the growth rate of the economy will depend on this level too, as the next sections will show. In the following section we will calculate the interest rate from the preferences' side, and where these two are equal ($r_{\text{prod}} = r_{\text{pref}}$), the interest rate for this economic system is defined.

3.5. Consumers

Individuals live forever. They consume the final good Y. They provide the labor for the production of Y and lend (sell) raw capital K_t to the producers of intermediate goods. They are endowed with one unit of raw capital $K(0)$ and they inelastically supply labor. Their income consists of the money collected by selling the raw capital, which is equal to real interest rate r_t times the quantity of raw capital K_t, plus the wage paid for their labor w_t. For simplicity, assume that the population growth and the rate of depreciation are zero. Since consumers are interested in the consumption of the final good Y, it is their behavior that determines the interest rate.

Each representative agent has time-separable preferences with a constant rate of time preference ρ. For convenience, assume that there is a continuum of identical indexed consumers on the interval $[0,1]$. The instantaneous utility function u exhibits constant relative risk aversion with a constant inter-temporal elasticity of substitution σ^{-1}. Also assume that $u(\cdot)$ is non-decreasing, strictly quasi-concave, and homogeneous of degree one in its arguments. Then the representative consumer, has the following utility function:

$$U = \int_0^\infty e^{-\rho t}\left[\frac{c_t^{1-\sigma}-1}{1-\sigma}\right]dt. \tag{25}$$

where c_t is the consumption level of the final good. Note that σ is the elasticity of the marginal utility of consumption, and thus σ^{-1} becomes the inter-temporal elasticity of substitution.

As mentioned earlier, the interest rate is independent of time, and thus the time subscript can be dropped. Then the budget constraint that consumers face is given by

$$\dot{K}_t = \tilde{r} K_t + w_t - c_t \tag{26}$$

where $\tilde{r}K_t + w_t$ is the income of the consumers at every point in time, which consists of the income coming from lending raw capital to the producers of durables, and from offering labor to the producers of the final good. The price of the final good is assumed to be one. Since x_j equals x_j (0) for all periods, then K_t can be written as

$$K_t = \int_0^{N_t} x_j \, dj = N_t \bar{x}, \tag{27}$$

where $\bar{x}$ is a constant stock of each of the varieties of capital good x_j. Each individual does not know that the wage rate is given by (6). What each consumer does is to choose the levels of consumption and the raw capital such that their utility level is maximized.

Thus individuals want to maximize their utility level (25) subject to their budget constraint (26), or

$$\begin{aligned} \underset{c_t}{\text{Max}} : U &= \int_0^{\infty} e^{-\rho t} \left[\frac{c_t^{1-\sigma} - 1}{1-\sigma} \right] dt \\ \text{s.t. } \dot{K}_t &= \tilde{r} K_t + w_t - c_t \end{aligned} \tag{28}$$

Solving this problem we need to form the current value Hamiltonian (H_c):[15]

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15 If $H = f(t,x,u) + \lambda_t \, g(t,x,u)$ is the Hamiltonian, then let's define as H_c the current value Hamiltonian. Thus $H_c = e^{\rho t} H$ and $m_t = e^{\rho t} \lambda_t$. Then if $H_c = f(t,x,u) + m_t \, g(t,x,u)$, the first order conditions are given by the following:

$$\partial H_c / \partial u = f_u + m g_u = 0$$

$$m' = \rho m - \partial H_c / \partial x = \rho m - f_x - m g_x.$$

$$H_c = \left[\frac{c_t^{1-\sigma} - 1}{1-\sigma} \right] + m_t \left(\tilde{r} K_t + w_t - c_t \right) \tag{29}$$

The first order conditions are then:

$$\frac{\partial H_c}{\partial c_t} = 0 \Leftrightarrow c_t^{-\sigma} = m_t \tag{30}$$

$$\dot{m}_t = \rho m_t - m_t \tilde{r} \Leftrightarrow \dot{m}_t = m_t (\rho - \tilde{r}) \Leftrightarrow \frac{\dot{m}_t}{m_t} = \rho - \tilde{r} \tag{31}$$

Taking derivatives with respect to time in (30), we get:

$$\dot{m}_t = -\sigma c_t^{-\sigma-1} \dot{c}_t, \tag{32}$$

and substituting (32) into (31) and doing the calculations, we get:

$$\gamma_c = \frac{\dot{c}_t}{c_t} = \frac{(\tilde{r} - \rho)}{\sigma}, \tag{33}$$

where γ_c is the growth rate of consumption, which is constant, since the interest rate is constant. Note that the interest rate is determined by the need of the country to equate supply and demand for capital goods, or to equate the domestic savings to the desired level of domestic investment.

Looking at (33), we can derive the interest rate resulting from the consumers' preferences to be

$$r_{\text{pref}} = \tilde{r} = \sigma g + \rho, \tag{34}$$

where $g = \gamma_c$. In Figure 3.2, we plot together (24) and (34) against the growth rate. It is obvious that r_{prod} does not depend on the growth rate, while r_{pref} is increasing in the rate of growth of output.

Equation (24) gives the interest rate of the production side, while equation (34) gives the interest rate of the preferences' side. The point where the lines of these two interest rates intersect defines the interest rate that prevails in the economy. Putting them together we can calculate the growth rate of the economy to be:

$$r_{\text{pref}} = r_{\text{prod}} \Rightarrow \frac{A\Theta L^{\alpha}}{\beta^{\alpha}\phi^{1-\alpha}} = \sigma g + \rho \Leftrightarrow$$
$$g = \frac{A\Theta L^{\alpha}}{\beta^{\alpha}\phi^{1-\alpha}} - \frac{\rho}{\sigma} \tag{35}$$

Looking at equation (35) for growth rate, we can see that g depends on the size of the labor force L. Furthermore, if the labor force increases, this will cause an increase in the growth rate. Since we already assumed that labor force is a proxy for the population size, the bigger the population, the faster the economy grows. The growth rate g depends also on the discount rate ρ. If ρ is high, that is, people's belief about tomorrow is pessimistic, then the growth rate is lower. On the other hand if people are optimistic about the future, the discount rate is small, and thus the growth rate is higher.

Another factor that affects the growth rate of the economy is the initial cost of entering the market for durables β. The higher the entrance cost, the lower the growth rate. This makes sense since a high entrance cost implies that much of the financial resources of the producers for durables is spent acquiring the right to participate in the market. We assumed earlier that the entrance cost depends on the abundance of human capital. That is, that the growth rate of the economy is positively related to the level of

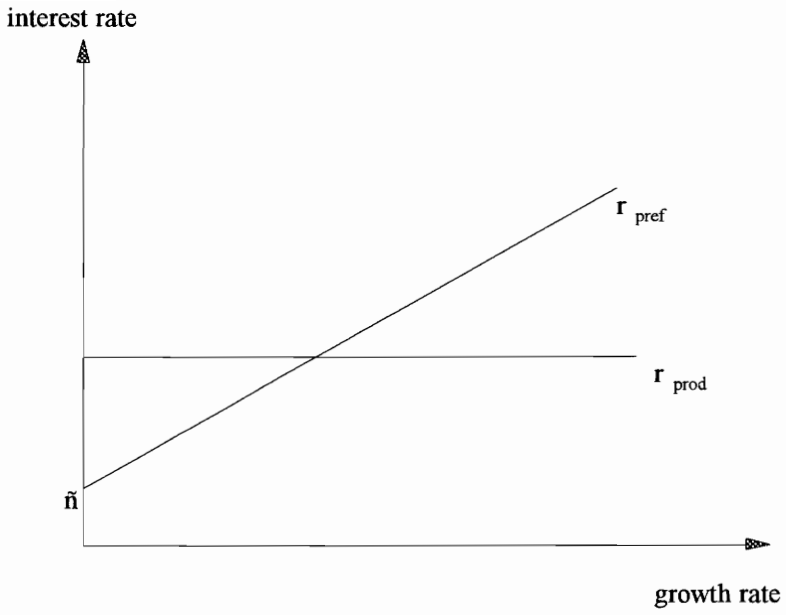

Figure 3.2.

Equilibrium Interest Rate.

human capital, something we should expect, since the more educated the labor force is, the lower the cost of a new intermediate good. Thus if the labor force is educated, it is easier to adjust to the new technologies that the production of the new intermediate good might require. Therefore, in our setup, the more educated the labor force, the more new intermediates that can be introduced.

Finally, a factor that plays a crucial role in the size of the growth rate is the technological parameter A. Equation (35) says that the higher the level of technological parameter, the higher the growth rate. Note though that A can be interpreted as a factor that measures how fast an economy incorporates new technologies. An economy with a high level of A means that the rate of absorbency is high and thus its growth rate should be higher.

Before we go on and calculate the growth rate of raw capital K_t, we need to look at the existence of balance growth paths. In order to do so, we will use a phase diagram. We know that the following always hold: $U'(c) > 0$, $U''(c) < 0$, and that the Inada conditions also hold (that is $U'(0) = \infty$, and $U'(\infty) = 0$).

To develop qualitative properties of the optimal solution, we construct a diagram in the nonnegative c-K plane. Consumption evolution is given by (33), while capital evolution is given by (26). Starting with the consumption side, $\dot{c}_t = 0$ only if $\tilde{r} = \rho$. If $\tilde{r} < \rho \Rightarrow \dot{c}_t < 0$, and if $\tilde{r} > \rho \Rightarrow \dot{c}_t > 0$. Thus the locus $\dot{c}_t = 0$ is a vertical line that corresponds to K_S, with K_S being the value of capital for which its marginal product is equal to $\tilde{r}$.

Looking now at the budget constraint (26), $\dot{K}_t = 0$, only if $c_t = \tilde{r}K_t + w_t$. Note that

$$\frac{\partial c_t}{\partial K_t} = \tilde{r} > 0, \text{ and } \frac{\partial^2 c_t}{\partial K_t^2} = 0,$$

which tells us that $\dot{K}_t = 0$ goes through (0,0). Take now a point (K_a, c_a) that satisfies $\dot{K}_t = 0$. Then, if we increase consumption c_a by λ, $\lambda > 0$, the point $(K_a, c_a + \lambda)$ is above the $\dot{K}_t = 0$ line. From the budget constraint (26), we get:

$$\dot{K}_t = \tilde{r}K_a + w_t - (c_a + \lambda) = -\lambda < 0,$$

which means that $\dot{K}_t$ is falling above the $\dot{K}_t = 0$ locus. The inverse happens below the $\dot{K}_t = 0$ locus. If we put these results in a *c*-*K* plane, we get Figure 3.3.

Is there a steady state level of per capita consumption and capital that can be sustained? Furthermore, is there a path towards this point? The answer to both questions is yes, as shown by the graph in Figure 3.4.

A steady state is defined as the point where the loci of consumption and capital intercept. Thus the (K_s, c_s) satisfies the previous condition. Now let us assume that there exists K_0, such that $K_0 < K_s$. Then the economy approaches the (K_s, c_s) from below, with both c and K rising monotonically from their initial values along the path ***a***. If the initial level of consumption were now too large (disaving), then capital would accumulate for a while but would eventually diminish, as is shown by the path ***b***. This is not a feasible solution since eventually capital will cross the vertical line and become negative ($K < 0$). On the other hand, if the level of consumption were very low (excess saving), then consumption would rise for a while and then decrease again. The latter is shown by the path ***c***. This solution is not a feasible one either. The reason is that one can increase the consumption level and find himself on the path ***d*** that leads towards (K_s, c_s) from above. Thus path ***a*** is the only feasible one if $K_0 < K_s$. For the opposite case, where $K_0 > K_s$, the optimal path is ***d***, for which consumption and capital are decreasing monotonically from above. Of course, for our analysis the case of consumption and capital increasing from below along the path ***a*** is more interesting.

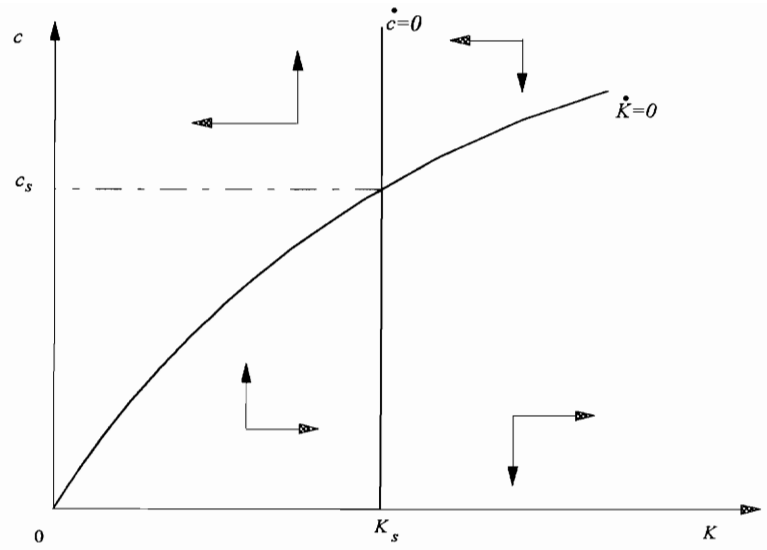

Figure 3.3.

Diagrammatic Analysis of $\dot{c}_t$ and $\dot{K}_t$.

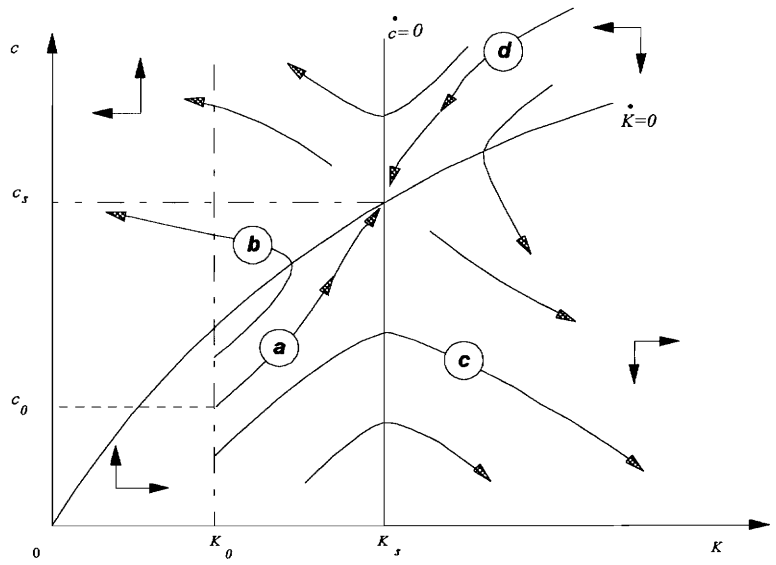

Figure 3.4.

Equilibrium Paths for $\dot{c}_t$ and $\dot{K}_t$.

Let us now calculate the growth rate of the raw capital K_t. To do so, we need to use the budget constraint of the consumers given by equation (26). Note that from equation (6) the wage rate w_t is proportional to the number of intermediates N_t. This means that as more new intermediates are introduced, the wage rate increases. It is safe to assume that the factor of proportionality is a positive number. We know though by using the following

$$K_t = \int_0^{N_t} x_j \, dj = N_t \bar{x},$$

that the number of durables available to this economy at each time period depends on the size of the raw capital. Thus we can consider the wage rate to be proportional to the size of raw capital, or $w_t = \xi K_t$, where $\xi > 0$. Then the budget constraint can be rewritten as follows:

$$\dot{K}_t = r K_t + \xi K_t - c_t \tag{36}$$

where the wage rate w_t has been substituted with ξK_t $(\xi > 0)$.

Then (36) becomes

$$\dot{K}_t - (r+\xi) K_t = -c_0 e^{\frac{r-\rho}{\sigma} t} \tag{37}$$

where (33) is used for defining the consumption with growth rate equal to γ_c.

The general solution of (37) gives the raw capital equation, or

$$K(t) = -\frac{c_0}{\frac{r-\rho}{\sigma} - (r+\xi)} e^{\frac{r-\rho}{\sigma} t} + \bar{c} e^{(r+\xi)t}. \tag{38}$$

To determine $\bar{c}$, we need to evaluate the function K at one point. Thus for the time $t = 0$ where $K(0) = K_0$, $\bar{c}$ becomes

$$\bar{c} = K_0 + \frac{c_0}{\frac{r-\rho}{\sigma} - (r+\xi)}. \tag{39}$$

Then the particular solution of (38) which satisfies $K(0) = K_0$ is

$$K(t) = K_0 + \left[e^{(r+\xi)t} - e^{\frac{r-\rho}{\sigma}t}\right] \frac{c_0}{\frac{r-\rho}{\sigma} - (r+\xi)}. \tag{40}$$

Before we continue with the calculation of the growth rate of raw capital, let us define Π as

$$\Pi = -\frac{1}{\frac{r-\rho}{\sigma} - (r+\xi)}. \tag{41}$$

Then (38) utilizing (41) becomes

$$K(t) = \Pi c_t + \bar{c}\, e^{(r+\xi)t}. \tag{42}$$

Taking now time derivatives on (42) we get

$$\dot{K}(t) = \left(\frac{r-\rho}{\sigma}\right) \Pi e^{\frac{r-\rho}{\sigma}t} + \bar{c}(r+\xi) e^{(r+\xi)t}. \tag{43}$$

Taking then the ratio of $\dot{K}_t / K_t$, we can talk about the growth rate of raw capital, or

$$\frac{\dot{K}_t}{K_t} = \frac{\frac{r-\rho}{\sigma} \Pi e^{\frac{r-\rho}{\sigma}t} + \bar{c}(r+\xi) e^{(r+\xi)t}}{\Pi e^{\frac{r-\rho}{\sigma}t} + \bar{c}\, e^{(r+\xi)t}}. \tag{44}$$

Earlier in the analysis a very critical assumption was made. We assumed that we would focus on balanced growth paths. Following this assumption, we know that the growth rate of capital should be equal to the growth rate of consumption, or

$$\frac{\dot{K}_t}{K_t} = \frac{r-\rho}{\sigma}. \tag{45}$$

Then, equating (44) with (45), we get the following condition, which should hold if the assumption about balanced growth stands:

$$(r+\xi) = \frac{r-\rho}{\sigma} \quad . \tag{46}$$

Several remarks can be made about the condition above. First, since it is assumed that the growth rate of this economy is driven by the number of durables introduced, it is obvious that $r > \rho$. In other words, people feel safe about tomorrow, and their discount rate does not exceed the interest rate. Second, if (46) stands as equality, this economy faces a balanced growth in which the growth rate of consumption equals the growth rate of raw capital, that is, the rate of introducing new intermediates. Third, if (46) stands as an inequality, there are two cases that we need to examine:

a. If $(r+\xi) > (r-\rho)/\sigma$, then this means that the growth rate of raw capital is larger than the growth rate of consumption. In other words, if the discount rate ρ is too large, today becomes more critical than tomorrow, and individuals care more about their utility level and less for their children's utility level. The question that arises then is the following: is this result feasible? And what will be the result of growth rates of raw capital larger than the consumption's growth rates? Since it is assumed that the economy is closed, eventually there will be no use for the excess durable goods that will be produced, because the producers of the final goods will not be

able to absorb the excess supply of durables. The only way for this economy to reach an equilibrium again would be for (44) to hold as an equality.

b. If $(r+\xi)<(r-\rho)/\sigma$, the growth rate of raw capital is less than the growth rate of consumption. The discount rate ρ is then too small, so that this economy is too patient. People care more for the well being of their children, and thus they discount less. Here we have the same question as before, the question of how feasible is this result. The answer is similar to the one before, that the higher consumption growth will force the producers of final goods to demand more durables and thus drive the growth rate of raw capital up.

Some extra remarks can be made regarding the level of interest rates. In equation (22) the interest rate was defined as a function of the parameters of the model, among which were the labor force size L, the marginal cost of producing durables ϕ, and the magnitude of the R&D cost β. It is obvious that an increase in the size of the labor force will cause the interest rate to increase, which will result in a larger growth rate. One way for the labor force to increase is by assuming the opening up of the economy to international trade. As for the cost of R&D, clearly the higher the β, with a scarcity of educated labor, the lower the growth rate. In addition, an unusually high marginal cost of producing durables will slow down the economy's growth.

In this chapter, the analysis was focused on a closed economy experiencing endogenous economic growth. We proved that as long as new durables are continuously introduced into the economic system, the economy will continue to grow. In the following chapter, we will enhance the model to incorporate the opening of the economy to international trade, and we will study the effects of international trade on the economic growth of the country.

Chapter 4

OPEN ECONOMY

In the analysis so far, we have focused on the case of a closed economy. We will continue by extending the basic model to an open economy case. Assume that there are two identical countries, home country H, and foreign country F. Each country produces two goods: manufactures and intermediates. In real life, each country produces more than two products, but here we will consider these two products (manufactures, intermediates) as broad categories of the several products that each country produces.

In this analysis, we continue building on the basic ideas of the closed economy case, as described in the previous chapter. We also assume that the symmetry assumption holds for our analysis. Thus, what stands for the home country is true for the foreign one, and vice versa. Furthermore, to distinguish between the different products, we use the star symbol (*) to refer to products used in the foreign country.

4.1. Specification of the Model

Each country produces two goods: a final homogeneous good, and an intermediate (durable) good. The final good is the manufactures. Manufactures are all goods that are produced by the industrial sector. This is a broad definition, but it helps to keep things in perspective. In this category fall goods like cars, refrigerators, computers, tv sets and similar items. On the other hand, intermediate goods are the capital goods necessary for the production of the manufactures. The same assumption used for a case of a closed economy holds here: intermediates are transformations of raw capital. In real life, the role of intermediates is more like that of bolts, computer chips, steel and similar things, less like that of capital goods. Generally, intermediate goods are all the goods necessary for producing a manufacture. It is obvious that without manufactures the need for intermediates vanishes.

To be more specific, let us assume that manufactures are produced by the combination of labor force and intermediate goods, as in the closed economy. Intermediates use for their production raw capital. Similar to the closed economy case, the owners of raw capital are the consumers, who lend capital to the producers of intermediates, and earn interest on that. In Figure 4.1 we give a graphical representation of the economic environment described above.

As Figure 4.1 shows, consumers are the owners of capital. They lend their capital to either domestic or foreign producers of intermediate goods, and they provide labor to the

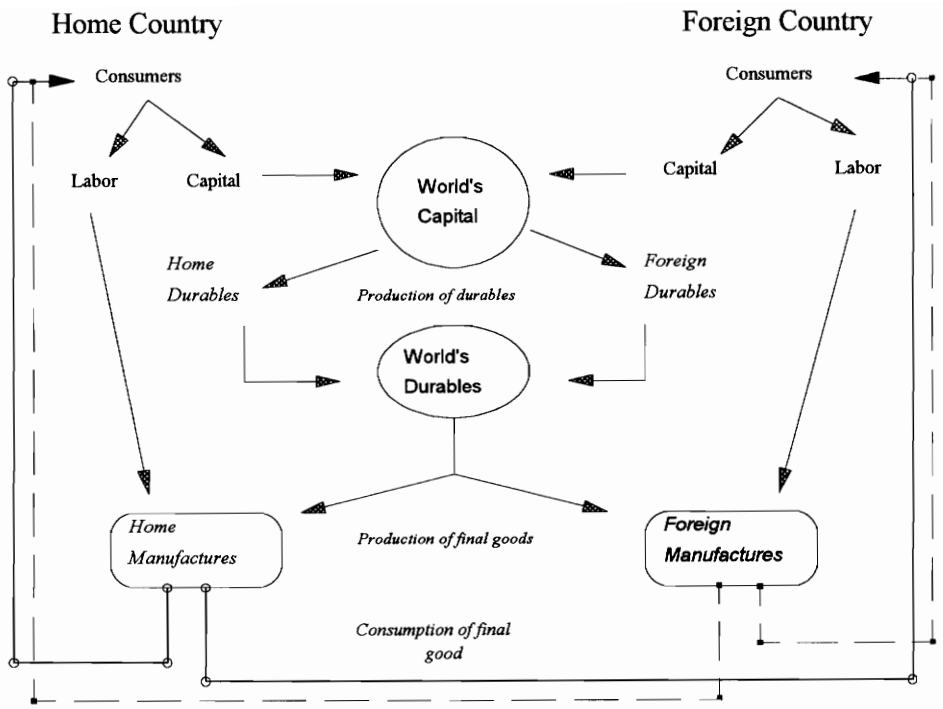

Figure 4.1.

Production and Trade between Home and Foreign Countries.

producers of the manufactures. On the other hand, the producers of intermediates borrow the necessary capital from consumers and transform it into durable goods. These goods are available to the producers of manufactures in both domestic and foreign countries. Finally, the manufactures are consumed by both domestic and foreign consumers. In the following sections we will examine more thoroughly the production of manufactures and durables, and the behavior of consumers.

4.2. Manufactures Sector

In our model, manufactures constitute the final good. The major difference between manufactures of the closed and the open economy models is that manufactures are tradable goods. Manufactures are made by employing labor and intermediate goods. The fact that manufactures are tradable goods poses some extra assumptions. First, we assume that the product called a manufacture is a homogeneous one. By assuming so, we avoid differences that consumers might think exist between the imported and the domestically produced goods. Second, there are no tariffs or other costs imposed on the imported goods. Thus, if there is a need for some of these to be imported, this importation can take place. Furthermore, we assume that there are no transportation costs for either manufactures or intermediates.

The producers of manufactures employ labor available in each country[1] and intermediate goods. The intermediates can be found either in the domestic market, or can be imported. Intermediates are made of capital, and can be considered the various types of capital necessary for the production process. In other words, technological progress arises from the invention of new types of durables (capital). This definition of intermediates is consistent with the case of a closed economy. We continue to assume that the capital available for transformation is called raw capital, and is owned by the consumers in both countries.

Let then j index the different types of capital goods called durables. Since in this model not only manufactures but also durables are traded, there is need for extra caution with the notation. Thus, let us define $d(j)$ as the durables that are produced and used in the home country, $d^*(j)$ as the home produced durables that are used in the foreign country, $f(j^*)$ as the durables produced and used in the foreign country, and $f^*(j^*)$ as the durables produced abroad and used in the home country. Each producer in the home country uses durables produced domestically or abroad and labor to produce the manufactures. Thus his production function is given by the following:

$$Y_t = BL^{\theta}\left[\int_0^{\infty} d(j)^{1-\theta}\,dj + \int_0^{\infty} f^*(j^*)^{1-\theta}\,dj^*\right] \tag{47}$$

with $0<\theta<1$ and, where Y_t is the domestic manufactures output, L is the labor force used in the production of manufactures, B is a technological parameter, $d(j)$ is the quantity of home made and used durables, and $f^*(j^*)$ is the quantity of foreign durables used in the home country.

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1 No labor mobility is assumed.

Since we assumed that the symmetry assumption prevails, a similar production function will be used by the foreign producers of manufactures:

$$Y_t^* = B^*(L^*)^{\theta}\left[\int_0^{\infty} f(j^*)^{1-\theta}\,dj^* + \int_0^{\infty} d^*(j)^{1-\theta}\,dj\right], \tag{48}$$

where everything stared denotes production in the foreign country, and $d^*(j)$ are the imported quantities of intermediates from the home country.

This type of production function is homogeneous of degree one in labor and durables respectively. Thus firms in this sector can be represented by a single price taking firm. There are always durables that could be used in the production of the manufactures, if they have been invented. Therefore, a durable j is invented if $j \le N$, where N is the maximum number of durables available up to that time. It is obvious that the upper limit of the integral in (47) is not ∞, but $N(t)$ at each date t. A similar assumption holds for the foreign-produced intermediates. However, what changes over time is not the production function but the set of inputs that are available. Then (47) becomes

$$Y_t = BL^{\theta}\left[\int_0^{N_t} d(j)^{1-\theta}\,dj + \int_0^{N_t^*} f^*(j^*)^{1-\theta}\,dj^*\right], \tag{49}$$

and similarly (48) becomes

$$Y_t^* = B^*(L^*)^{\theta}\left[\int_0^{N_t^*} f(j^*)^{1-\theta}\,dj^* + \int_0^{N_t} d^*(j)^{1-\theta}\,dj\right], \tag{50}$$

where N_t is the number of durables produced in the home country, and N_t^* is the number of durables produced in the foreign country.

This specification was used first by Dixit and Stiglitz (1977), when they wanted to model differentiated consumption goods. Ethier (1982) introduced a similar functional form where the modeled variables were differentiated inputs instead of differentiated consumption goods. In our model, we follow Ethier's approach, but we use only a single final good produced by intermediates.

Producers of manufactures rent raw capital, that is transformed into durable goods, from both domestic and foreign firms. Let us call the rental price they pay P_d for durables produced domestically, and P_f for durables produced in the foreign country and used by both domestic and foreign producers. Here for simplicity we assume that the price charged for the home made durables is the same in both countries, and that a similar assumption holds for the foreign country's durables too. This holds since we made the earlier assumption that there are neither tariffs nor transportation costs imposed on the traded goods. We also assumed that both countries not only produce intermediates, but also trade them. This poses the following question: why does home country manufacturing need foreign-made intermediates? We are considering the market of intermediates on a global level where domestic and foreign producers compete with each other. Therefore it is feasible for a new producer of durables to come in the market only if no one else produces the same good. In that sense, the strong underlying assumption is that there is an international patent protection.

As in the case of the closed economy model, it is assumed that the firms producing the manufactures are many and can be represented by a single aggregate price taking firm.[2] Therefore, each firm takes the price of the durables, domestic and imported, as given. Each firm in the home country uses the production technology given by (49), buying the

2 There is a continuum of firms indexed on [0,1].

domestic durables $d(j)$ at a price P_d, and the imported $f^*(j^*)$ at a price P_f. Each one of these producers seeks to maximize his profits, or the present value of the future cash flows, at any time t, by choosing the quantity of domestically and imported durables required for the production of the manufactures good:[3]

$$\underset{h(j),f^*(j^*)}{\text{Max}} \int_0^{\infty} e^{-rt}\left[Y_t - P_d d(j) - P_f f^*(j^*) - w_t L\right]dt \text{[4]} \tag{51}$$

$$\text{s.t.} \quad Y_t = BL^{\theta}\left[\int_0^{N_t} d(j)^{1-\theta}\,dj + \int_0^{N_t^*} f^*(j^*)^{1-\theta}\,dj^*\right] \quad . \tag{52}$$

The price of the manufactures is assumed to be the same in both countries and is assumed to play the role of a numeraire. Then, taking the first order conditions in (51), we get the inverse demand for domestically produced durables (for any time t):

$$BL^{\theta}(1-\theta)d(j)^{-\theta} = P_d, \tag{53}$$

the demand for imported durables:

$$BL^{\theta}(1-\theta)f^*(j^*)^{-\theta} = P_f, \tag{54}$$

and that price of labor w_t equals its marginal product,

$$B\theta L^{\theta-1}\left[\int_0^{N_t}\left(d(j)^{1-\theta} - P_d(t)d(j)\right)dj + \int_0^{N_t^*}\left(f^*(j^*)^{1-\theta} - P_f(t)f^*(j^*)\right)dj^*\right] = w_t \tag{55}$$

which gives the inverse demand function for labor in the manufactures section of the home country.

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3 We still assume that labor is inelastically supplied.

4 From now on we drop the time subscripts for $P_d, P_f, d(j)$ and $f^*(j^*)$.

Using (53) and (54), we can find the demand for domestic and imported durables by the domestic producers of manufacturing goods, respectively, at any point in time, or

$$d(j) = L\left(\frac{B(1-\theta)}{P_d}\right)^{1/\theta} \tag{56}$$

and

$$f^*(j^*) = L\left(\frac{B(1-\theta)}{P_f}\right)^{1/\theta}. \tag{57}$$

Obviously, by doing similar calculations we can find the demand for durables by the foreign producers of manufactures, or

$$f(j^*) = L^*\left(\frac{B^*(1-\theta)}{P_f}\right)^{1/\theta} \tag{58}$$

and

$$d^*(j) = L^*\left(\frac{B^*(1-\theta)}{P_d}\right)^{1/\theta} \quad . \tag{59}$$

Before we continue with the description of the production of intermediate goods, we need to mention a couple of things. First, (56) and (59) give us the demand for durables by the domestic and foreign producers of manufactures produced in the home country. By adding them we get the total demand (D_j) for home-produced durables, or

$$D_j = d(j) + d^*(j). \tag{60}$$

Making use of (56) and (59), (60) then becomes

$$D_j = \left(\frac{1-\theta}{P_d}\right)^{1/\theta}\left(LB^{1/\theta} + L^*B^{*1/\theta}\right). \tag{61}$$

Second, (57) and (58) give us, in a similar way, the demand for durables by foreign and domestic producers of manufactures produced in the foreign country. Then, if we define as F_j the total demand for durables produced abroad, we get

$$F_{j^*} = f(j^*) + f^*(j^*), \tag{62}$$

or, by substituting (57) and (58) into (62), we get

$$F_{j^*} = \left(\frac{1-\theta}{P_f}\right)^{1/\theta}\left(L^*B^{*1/\theta} + LB^{1/\theta}\right). \tag{63}$$

Looking at (61) and (63), we notice that both depend on the size of the labor force in the home country and abroad. Since we assumed earlier that labor size serves as a proxy of population, the larger the population, the bigger the demand for home- and foreign-produced durables. Furthermore, both depend on the productivity factors that we assumed were different in each country. Finally, they depend upon the price of durables, the higher the price, the lower the demand, and vice versa.

4.3. Producers of Intermediate Goods.

We use a model similar to the one used in the case of a closed economy. Durable goods are transformations of raw capital owned by the consumers. Since there is more than one country involved, there needs to be some modification of to the closed economy model. We continue to assume that all durables are produced with a production technology similar to that of the manufactures. For simplicity, we also assume that all the durables are produced by the same process. Then if K_t^W defines the total amount of capital goods available in the world economy, then

$$K_t^W = K_t + K_t^*. \tag{64}$$

where K_t is the total raw capital available in the hands of domestic consumers, and K_t^* is the total raw capital owned by foreigners.

There is an underlying assumption in the above expression . We assume that each producer of durables, domestic or foreign, can borrow raw capital from either domestic or foreign consumers. In other words, borrowing and lending capital is taking place between these countries. Furthermore, the total amount of raw capital available in the home country is given by the following equation,

$$K_t = \int_0^{N_t} \left[d(j) + d^*(j) \right] dj. \tag{65}$$

In a similar fashion, the total capital available to the producers of durables in the foreign country is given by the following equation,

$$K_t^* = \int_0^{N_t^*} \left[f(j^*) + f^*(j^*) \right] dj^* . \tag{66}$$

Another standard assumption for this type of model is that the market of intermediates is described by monopolistic competition. We continue assuming that the producers of durables have to pay the cost of research and development (R&D). In a fashion similar to the case of the closed economy, we assume that this cost depends on the abundance of human capital, and it is expressed as human capital per units of output. We continue defining this cost as β, which is expressed by (9) and (10), in Chapter 3. It still holds that the better the education of the labor force, the lower the cost of entering the durables market will be. In addition, we assume that after entering the durables market, each producer faces a constant marginal cost equal to ϕ.[5]

Following the assumptions stated above, let us describe what each producer of durables does in the home country. Everyone has the option to sell either in the domestic or in the foreign markets, or in both. They will prefer to sell in both markets, since in this way, they can maximize their profits. Since we assumed earlier that there are no tariffs, nor transportation costs, the price that one charges for each unit of the durable one produces will be the same for the domestic and the foreign market.[6]

Therefore, in the domestic market each durable's producer sells a quantity equal to $d(j)$, while in the foreign country the producer sells a quantity equal to $d^*(j)$, and charges both markets the same prices, equal to P_d. A critical assumption is that durable goods are considered as transformations of raw capital. This capital is owned by the

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5 We keep the same notation for both closed and open economy models.

6 An underlying assumption here is that each producer of durables produces a unique product. It is the uniqueness of each durable that makes domestically produced durables necessary in the production of the foreign manufacture goods, and vice versa.

consumers in both countries, and we make the assumption that there is perfect capital mobility.

The producers of durables pay the owners of capital a price, which is the interest rate that prevails, in this economic setup. What do the producers of durables want? They want to maximize their profits by choosing the price P_d of domestic durables, having in mind that the demand for durables is given by (61). Then if define the profits of a representative producer of durables as π_d (for every moment in time t), we get:

$$\pi_d = P_d\left(d(j)+d^*(j)\right)-\phi\left[r\left(d(j)+d^*(j)\right)+r^*\left(f(j^*)+f^*(j^*)\right)\right]-\beta. \qquad (67)$$

Equation (67) needs some clarification. Let us define as r the interest rate charged by the home consumers to the producers of intermediate goods. Similarly, by r^* we define the interest rate charged by the consumers in the foreign country. The expression $P_d(d(j)+d^*(j))$ is the revenue that the home producers of durables get when their product is sold to the domestic producers of manufactures, plus the revenues they get when their product is sold to the foreign country, because trade also takes place between intermediates.

On the cost side, the cost that each producer of intermediate goods faces is the cost of getting the raw capital. We assume that they borrow the capital from the consumers (domestic or foreigner), and pay them back interest rate. It is obvious, then, that some discussion about this interest rate has to take place. The fact that there is free flow of manufactures implies that the interest rates between the two countries will be the same ($r = r^*$). In this case, let $\tilde{r}$ define the common interest rate that consumers in both countries charge for their raw capital ($r = r^* = \tilde{r}$). Then $\tilde{r}\phi(d(j)+d^*(j)+f(j^*)+f^*(j^*))$ defines the interest cost of each domestic producer,

who borrows $d(j)+d^*(j)+f(j^*)+f^*(j^*)$ amount of raw capital from the domestic and/or the foreign market. Finally, ϕ is the constant marginal cost for producing intermediates, and β represents the initial cost of entering the market, which as stated earlier depends on the abundance of human capital.

The problem for each producer of durables at any point in time becomes:

$$\underset{P_d}{\text{Max:}}\ \pi_d = \underset{P_d}{\text{Max}}\ :P_d\left(d(j)+d^*(j)\right)-\tilde{r}\phi\left(d(j)+d^*(j)+f(j^*)+f^*(j^*)\right)-\beta \tag{68}$$

$$\begin{aligned} \text{s.t.}\quad & d(j)=L\left(\frac{B(1-\theta)}{P_d}\right)^{1/\theta} \\ \text{and}\quad & d^*(j)=L^*\left(\frac{B^*(1-\theta)}{P_d}\right)^{1/\theta}. \end{aligned} \tag{69}$$

By solving the optimization of the problem defined by (68) and (69), we get:

$$\frac{\partial \pi_d}{\partial P_d}=0 \Leftrightarrow P_d=\frac{\tilde{r}\phi}{1-\theta}, \tag{70}$$

which is the optimal price for the intermediates $d(j)$, $d^*(j)$ produced domestically and used by domestic and foreign producers.

By solving the symmetric problem of the foreign producers of durables, we derive a similar result to that of equation (70). If P_f defines the price of durables produced and sold within the foreign country and/or exported to home country, then P_f should be given by the following:

$$P_f=\frac{\tilde{r}\phi}{1-\theta}. \tag{71}$$

It is interesting to notice that $P_d = P_f$, that is, the price for domestically produced durables is the same as the price of durables imported. This, of course, depends on the assumptions made earlier about the interest rate, and therefore if P defines the price for durables internationally, we get:

$$P = P_d = P_f = \frac{\tilde{r}\phi}{1-\theta}. \tag{72}$$

There are some points to discuss about expression (72). It is already assumed that ϕ is constant. Then, since interest rate $\tilde{r}$ is constant too, (72) says that the price for durables, domestic or foreign, is constant in both countries. This result is similar to that of the closed economy, but here there are no extra assumptions necessary in order for this result to hold. The fact that we assume free capital mobility guarantees that the interest rate will be constant and time-independent.

Using expression (72), we can now rewrite the demands for durables as follows:

$$d_j = L\left(\frac{B(1-\theta)^2}{\tilde{r}\phi}\right)^{1/\theta} \tag{73}$$

for the demand of home-produced and used durables;

$$d_j^* = L^*\left(\frac{B^*(1-\theta)^2}{\tilde{r}\phi}\right)^{1/\theta} \tag{74}$$

for the demand of home-produced but used abroad durables;

$$f_{j^*} = L^*\left(\frac{B^*(1-\theta)^2}{\tilde{r}\phi}\right)^{1/\theta} \tag{75}$$

for the demand of foreign-produced and used durables; and

$$f_j^* = L\left(\frac{B(1-\theta)^2}{\tilde{r}\phi}\right)^{1/\theta} \tag{76}$$

for the demand of foreign-produced but home-used durables.

A final note for the producers of intermediate goods is necessary. We assume that firms can enter freely into this market, as long as they undertake the entrance cost β. The latter implies that the present value of profits π_d as defined by (67), should be zero, or the present value of profits should be equal to the initial cost of entering the market β:

$$\int_t^{\infty} e^{-\int_t^s \tilde{r}(\tau)d\tau} \pi_d(s)ds = \beta. \tag{77}$$

Differentiating (77) with respect to time, we get the following:

$$\pi_d(t) - \tilde{r}(t)\int_0^{\infty} e^{-\int_t^s \tilde{r}(\tau)d\tau} \pi_d(s)ds = 0. \tag{78}$$

And substituting (77) into (78), we get an expression for the interest rate:

$$\tilde{r}(t) = \frac{\pi_d(t)}{\beta}. \tag{79}$$

Substituting the expression of $\pi_d(t)$ in (79), we get

$$\tilde{r}(t) = \frac{PD(j) - \beta}{\phi(D(j) + F(j^*))}^{7}. \tag{80}$$

Since $P_d = P_f = P$, then, as long as the interest rate is constant over time, $D = F$. Substituting (72) into (80) and using $D = F$, (80) becomes:

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7 From now on we drop the j and j^* from D and F respectively.

$$\tilde{r} = \frac{(1-\theta)\beta}{\phi D(2\theta-1)}. \tag{81}$$

The only condition for (81) which gives the expression for the interest rate from the production side, is that $(2\theta-1)>0$, that is $\theta > 0.5$.

4.4. Consumers.

Finally, we need to describe the consumers' side of this economy. As we mentioned earlier, the consumers in this model play a very important role. They are the ones who provide labor inelastically in both countries, and they buy the final good manufactures, produced either domestically or abroad. Furthermore, they own the raw capital necessary to produce the intermediate goods, which they lend to their country's producers of durables. As in the case of a closed economy, we assume that they live forever. It is also assumed that they are endowed with one unit of raw capital $K(0)$. Their earnings consist of the money collected by lending their capital, that is the interest rate r_t times the amount of raw capital K_t^i, ($i = d, f$ for home and foreign country respectively), and the wages they collect for the labor they provide, w_t.

Consumers in this model share the same homothetical preferences. Thus, each representative agent has time-separable preferences with a constant rate of time preference ρ. Assume also that all individuals form a continuum (they are indexed over the interval [0,1]). As for the instantaneous utility function, it exhibits constant relative risk aversion,

with a constant intertemporal elasticity of substitution σ^{-1}. Then, for each representative consumer, the utility function u has the following form:

$$U = \int_0^{\infty} e^{-\rho t} \frac{c_t^{1-\sigma} - 1}{1-\sigma} dt \tag{82}$$

These consumers face a budget constraint given by the following equation:

$$\dot{K}_t + \dot{K}_t^* = r_t(K_t + K_t^*) + w_t - c_t{}^8. \tag{83}$$

The budget constraint is different from the closed economy case because the available capital to the producers of durables is equals the total world capital. If we substitute $K_t + K_t^*$ with Z_t, then $\dot{Z}_t = \dot{K}_t + \dot{K}_t^*$ and the budget constraint becomes

$$\dot{Z}_t = r_t Z_t + w_t - c_t \quad . \tag{84}$$

Assuming for the time being that the interest rate is treated as constant by the consumers, we can solve the maximization problem defined by (82) and (84) by forming a current-value Hamiltonian. The solution of this maximization problem gives us the following expression for the growth rate of consumption:

$$\gamma_c = \frac{\dot{c}_t}{c_t} = \frac{(r - \rho)}{\sigma}. \tag{85}$$

Finally, we need to calculate the growth rate of the economy. If we substitute (61) into (81) and assume that $P_d = P$, we get:

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8 Two things need some clarification here: first, the consumption c_t is assumed to include both domestic and foreign manufacturers. Second, instead of using K_t which is the "raw" capital available by the consumers to the producers of intermediates, we can assume that consumers hold debt instruments, and for convenience that all firms are 100% debt-financed. In any case the budget constraint will be given by (83).

$$\tilde{r} = r_{\text{prod}} = \frac{\left[(1-\theta)^{2-\theta}\Xi(2\theta-1)\right]^{1/(1-\theta)}}{\phi\beta^{\theta/(1-\theta)}}, \tag{86}$$

where $\Xi = LB^{1/\theta} + L^{*}B^{*1/\theta}$.

Equation (86) gives the expression of the return to investment as it is derived from the intermediates sector (production side). On the other hand, the interest rate derived from the consumer preferences is given by:

$$r_{\text{pref}} = \sigma g + \rho. \tag{87}$$

It is required that the rate of return to investment be equal to the interest rate determined by preferences, or

$$r_{\text{pref}} = r_{\text{prod}} \quad . \tag{88}$$

Finally, if we equate (86) to (87) and solve for the growth rate g, we get:

$$g = \frac{1}{\sigma}\left[\frac{\left[(1-\theta)^{2-\theta}\Xi(2\theta-1)\right]^{1/(1-\theta)}}{\phi\beta^{\theta/(1-\theta)}} - \rho\right] \quad . \tag{89}$$

Equation (89) is the growth rate of the open economy. It is obvious that the higher the β, the lower the growth rate of the economy. Since β is inversely related to the available human capital in this country, a high β means a very low level of human capital, which leads to lower economic growth. Also the factor Ξ is positively related to the growth rate of the economy. These observations explain two things: first, since the major component of Ξ is labor, the more available labor in the country, the larger the growth rate of the economy. Second, since Ξ incorporates the labor supply of the foreign country, the economy grows faster when it participates in the international trade arena.

Chapter 5

ECONOMETRIC APPROACH

In the previous chapters we discussed the effects of intermediate goods in a model of economic growth. It is now time to see how the model behaves with real data. In this chapter we will refer to the statistical approach we will use, and to econometric work already done in the area of economic growth.

Specifically, this chapter consists of two major sections: section one describes the approach that we will use for our econometric analysis. It lays out the foundation of our econometric approach, and makes clear why misspecification testing is essential in econometric applications. Furthermore, it provides us with an analytical explanation of all the tests we will use in our empirical approach to test the theory proposed in chapters three and four. Section two provides a review of the econometric work done in the area of economic growth. It also refers to the empirical work incorporating exports and economic growth, and lays the groundwork for our econometric approach.

5.1. Statistical Approach

In our econometric analysis we follow the Probabilistic Reduction Approach, as defined by Spanos (1986, 1993*a*, 1993*b*). Here we refer briefly to this method and describe the various tests necessary to ensure an adequate statistical model.

The Probabilistic Reduction Approach (PRA) distinguishes between the *actual mechanism*, the *theoretical model* and the *statistical model*. The *actual mechanism* that gives rise to the observed data, is the real world that we want to approximate. The *theoretical model* is the mechanism that provides us with an approximation of the real world, while the *statistical model* is a summarization of the sample information in the observed data. It is a set of probabilistic assumptions that relates the observable random variables underlying the data chosen to the theoretical model.

The task that each researcher faces is to choose that statistical model which *adequately* describes the information in the data. This principle of statistical adequacy is the foundation of the PRA. The researcher has to distinguish between the theoretical and the statistical model. The statistical model provides us with a summarization of the data on hand. As Spanos (1993b) states,

> *"...this approach raises two questions:*
> *a. How is data chosen, and*
> *b. What type of summarization is chosen."*

The type of data used in the research is determined by the theory we test. On the other hand, the choice of a statistical model among all the possible summarizations is based on both theoretical and probabilistic grounds. The researcher chooses a particular statistical model as long as it enables him to respond to the questions posed by the theory. Once statistical adequacy is established, the theoretical and the statistical models are

related in an attempt to give empirical meaning to the theoretical one, and theoretical meaning to the statistical one. The latter constitutes the main idea of this approach.

In an attempt to show how the assumptions of the Probabilistic Reduction work, let us consider the linear regression model, $y_t = \boldsymbol{b}' \boldsymbol{x}_t + \varepsilon_t$, with $t = 1,2,\ldots,T$, where y_t and ε_t are $T \times 1$, and $\boldsymbol{b}$ and $\boldsymbol{x}_t$ are $\mathrm{K} \times 1$. The probability assumptions underlying the linear regression model are given by the following:

I. Probability model:

1. The distribution of y_t conditioned on $\mathbf{X}_t$ follows the normal distribution, i.e., $g(y_t/\mathbf{X}_t;\theta) \sim$ normal, where $\theta \equiv (\boldsymbol{b},\sigma^2)$ are the statistical parameters of interest, with $\boldsymbol{b} = \sum_{22}^{-1}\sigma_{21}$, $\sigma^2 = \sigma_{11} - \sigma_{12}\sum_{22}^{-1}\sigma_{21}$, and $\sum_{22} = \mathrm{Cov}\,(\mathbf{X}_t)$, $\sigma_{21} = \mathrm{Cov}\,(\mathbf{X}_t, y_t)$, $\sigma_{11} = \mathrm{Var}\,(y_t)$.
2. The conditional mean is linear with respect to the independent variable, $\mathrm{E}\,(y_t/\mathbf{X}_t = \boldsymbol{x}_t) = \boldsymbol{b}'\boldsymbol{x}_t$.
3. Homoskedasticity: The conditional variance $\mathrm{Var}(y_t/\mathbf{X}_t = \boldsymbol{x}_t)$ is independent of $\boldsymbol{x}_t$.
4. The parameters $\theta \equiv (\boldsymbol{b},\sigma^2)$, conditional mean and variance, do not vary with t .

II. Sampling Model:

5. $\boldsymbol{y} \equiv (y_1, y_2,\ldots, y_T)'$ is an independent sample sequentially drawn from $g(y_t/\mathbf{X}_t;\theta)$, $t = 1,2,\ldots,T$.

III. Statistical Mechanism:

6. Weak Exogeneity: $\mathbf{X}_t$ is weakly exogenous with respect to θ. The marginal distribution of $\mathbf{X}_t$ does not contain any relevant information about the estimation of θ, and thus can be ignored.
7. Rank $(\mathbf{X}) = k + 1$, where k is the number of regressors (no perfect collinearity).

If assumptions 1-7 hold, the OLS estimators of $\boldsymbol{b}$ and σ^2 are BLU estimators (best, linear and unbiased). Assumptions 1-5 can be tested directly, and if any of these is violated all finite and most asymptotic tests of $\boldsymbol{b}$ and σ^2 are invalid. In the following section we discuss the misspecification tests we will use through out this work.

5.1.1. Misspecification Tests

A question someone might ask is why do we need to check for misspecified econometric models? What a researcher should do is form an econometric model that best describes the theory he has in mind, and then observe how this model reacts with the data. So where does misspecification come in? What will happen if we do not check the econometric model for any violations of the assumptions used to build it? When a researcher constructs an econometric model, he or she makes several assumptions about the characteristics of this econometric model. If any of these assumptions is violated, statistical difficulties will result. Misspecification testing is required to test the researcher's empirical applications.

Let us now present the consequences of a misspecified statistical model. First, let us assume that the assumption of independence is violated (assumption 5 in the previous section). The estimators $\hat{\boldsymbol{b}}$ and s^2 are biased and inconsistent estimators of $\boldsymbol{b}$ and σ^2. Furthermore, the *t*-tests and the *F*-tests are no longer valid. Second, assume now that the normality assumption (assumption 1) is violated. Then any results based on the distributions of $\hat{\boldsymbol{b}}$ and s^2 are not valid for a finite sample of size T. Furthermore, if T goes to infinity, the results based on the asymptotic distribution of $\hat{\boldsymbol{b}}$ are valid, but this does not hold for the results based on the asymptotic distribution of s^2. Third, if the assumption of linearity (assumption 2) is violated, then $\hat{\boldsymbol{b}}$ and s^2 are biased and inconsistent estimators of $\boldsymbol{b}$ and σ^2. Finally, if the assumption of homoskedasticity (assumption 3) is invalid, then $\hat{\boldsymbol{b}}$ is an unbiased but inefficient estimator of $\boldsymbol{b}$, while s^2 does not estimate anything of interest. In addition, all the *t*-tests and the *F*-tests are no longer valid. It is obvious then why someone should seek a statistical adequate model. This way the researcher is able to make inferences about the relevance of his theory.

There are several tests proposed for assumptions 1-5. The tests that we present and use here throughout this work are chosen because of their generality, and because in most of the cases they do not require explicitly stating the alternative hypotheses.

Starting with the test used to test departures from *normality*, we use the test proposed by D'Agostino *et al.* (1990). This test is based on the standardized third and forth moments of the residuals. We also used the kurtosis test of Anscombe and Glynn, and the joint test of D'Agostino and Pearson (1973) based on the sum of the third and forth moments. For more on these tests, please refer to Appendix B.

To test for any departures from linearity, we use two tests. The first is a RESET type test in which we use second and third order powers of the model's fitted values ($\hat{y}$). In the auxiliary regression

$$\hat{u}_t = \boldsymbol{b}' \mathbf{X}_t + \boldsymbol{\Gamma}' \boldsymbol{\Psi}_t + v_t,$$

we use an F-statistic to test the significance of Γ, where $\boldsymbol{\Psi}_t \equiv (\hat{y}_t^2)$ for a RESET2 test, and $\boldsymbol{\Psi}_t \equiv (\hat{y}_t^2, \hat{y}_t^3)$ for a RESET3 test. Second, we use the test suggested by Spanos (1986). It is a test similar to a RESET test, but uses a Kolmogorov-Gabor polynomial expansion (KG2). For a second order KG test, $\boldsymbol{\Psi}_t$ includes $x_{it} x_{jt}$, $i \geq j$, $i, j = 1, 2, \ldots, k+1$.

Finally, to test for any violations of the homoskedasticity assumption, we also use a RESET type test and a White test. The RESET test is similar to the one used for violations of linearity. The White test is a Kolmogorov-Gabor type test. The auxiliary regression used by both tests is the following:

$$\hat{u}_t = \boldsymbol{b}' \mathbf{X}_t + \boldsymbol{\Delta}' \boldsymbol{\Psi}_t + v_t,$$

by which we test for the significance of Δ, where for the RESET2 test $\boldsymbol{\Psi}_t \equiv (\hat{y}_t^2)$, and for the White test includes a KG2 polynomial in x's.

5.2. Review of Empirical Work on Endogenous Economic Growth

Empirical work on economic growth is not something new. There is a significant amount of literature on it, and for quite some time the center of this work was Solow's residual. Solow, in his pioneer work (1957), determined growth of output as a function of labor and capital inputs. Output growth reflects growth in both labor productivity and the labor force. Empirical application of his theory should ideally provide us with results that are consistent with his model. Applications of the Solow model showed clearly that there was a significant portion of economic growth that could not be explained by either growth of labor or growth of capital. Solow (1957) suggested a decomposition of the economy's growth rate into the growth rate of capital, the growth rate of labor, and the Solow residual that measures that part of growth not captured by the growth rate of capital and the growth rate of labor. Studies that incorporated the notion of a Solow residual showed that the growth rate of output (in per capita terms) depends positively on the growth rate of the capital to output ratio and to the Solow residual. Furthermore, Solow (1957) pointed out that the differences between his theoretical model and the empirical results were due to technological changes that took place. Technological change takes place during the business cycle and in most occasions leads to an improved production process. In a sense, Solow's residual captures the effects of technological change on the growth rate of the economy.

Romer (1987) took the discussion about the Solow residual to another dimension. He argued that there was no need to continue explaining shifts of the production function as a result of technological change. He also added that the assumptions of perfect

competition and no externalities were limitations of the theory proposed by Solow. He assumed (1987) that firms experience externalities that are external to them, but internal to the economy. Assume, for example, that a firm creates a new patent. This firm gains the rights to profit from this specific patent, but other firms in the economy will also benefit from the new discovery. The latter constitutes an externality for the rest of the firms, but is an internal effect for the economy as a whole. The model he proposed (1987) allowed for increasing returns at the aggregate level. He also claimed that the

> *"...conventional growth analysis ...underestimates the role of capital accumulation in growth."*[1]

Furthermore, the

> *"...true elasticity of output with respect to capital changes might be greater than the share of capital in the total income, because of positive externalities associated with investment".*[2]

In the same paper (1987) he presented two models, each of which generated capital elasticity of output bigger than one third. In the first model he needed to explain the strong positive association (aggregate data) between the Solow residual and the growth of capital stock. He assumed that there existed a large positive externality in the formation of capital. Each firm uses labor and physical capital as production inputs in a standard Cobb-Douglas production function. The positive externality was due to the effects that the spillovers had on the production of each firm, since every firm is affected by the improvements and enhancements that take place in the rest of the economy. Thus the production function of each firm includes an extra factor to capture this positive

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1 "Crazy Explanations for the Productivity Slowdown", *NBER Macroeconomics Annual*, 1987, page 165.

2 Same as in footnote 1, page 166.

externality. Romer (1987) suggested that the social marginal product of capital is perhaps twice or even three times its private marginal product. Thus, the equilibrium level of investment should be much lower than its social level. In the second model, he introduced knowledge in the form of ideas used to produce intermediate goods. He stated that this model

> *"...behaves as if there is a positive externality associated with the range of inputs".*[3]

Assuming a usual production function $Y = \Gamma(t)K^{\alpha}L^{\beta}$ with $\alpha + \beta = 1$, and $\Gamma(t)$ expressing technological change, Romer concluded that if we allow for simultaneous effects on capital and labor, the estimated coefficient of labor will still be small, unless we assume exogenous technological change. He also mentioned (1987) the convergence of growth rates. Neoclassical theory predicts that the poor in physical capital countries will eventually catch up, since the higher interest rates attract foreign investment and lead to higher savings. The data Romer used[4] suggested that there was no evidence of growth rates' convergence.

On the subject of convergence of growth rates, Barro (1991) focused on both domestic and international cases. For the international case, he stated that the hypothesis of convergence was not consistent with the cross-country data he used.[5] In the same analysis he used human capital in a similar fashion to that of Romer (1990). Barro (1991) supported the idea that countries with an abundance in human capital experience a much higher rate of introducing new goods and thus grow faster than others. Similarly, Becker, Murphy, and Tamura (1990) assumed that the return of human capital increases and

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3 Romer (1987), page 175.
4 The data are from Madisson (1982).
5 Data mainly from Penn World Table mark 4.

benefits from the spillovers caused by human capital. An increase in the amount of available human capital leads to higher rates of investment in both physical and human capitals.

Barro (1991) used data for 98 countries for the period 1960-1985, and stated that the growth rate of real per capita gross domestic product (GDP) is positively related to initial human capital, (proxied by the 1960 school enrollment rate). Furthermore, the growth rate of real per capita GDP is negatively correlated with the level of GDP for 1960. He stated that the latter indicated a conditional convergence that took place for the years 1960 to 1985. He also found that countries with a higher human capital level had lower fertility rates and higher ratios of physical capital investment to GDP. In addition, a poor country tends to grow faster as long as its human capital level is more than its physical capital level.

Barro and Sala-i-Martin (1992) wanted to examine if any forces would lead poor countries to grow faster than richer ones. Their theoretical framework employed a neoclassical growth model to study the convergence across the 48 contiguous states of the United States. They found that convergence was present in the case of the 48 states. Their results supported the contention that poorer areas grow faster than richer ones, whether restrictions are opposed or not.[6] For reasons of comparison they repeated the same exercise with 98 countries using data from Penn World Table 4. They found that convergence held there too, but only if they held constant variables such as initial school enrollment rates and the ratio of government consumption to GDP.[7] Furthermore, if technologies are the same, the introduction of a global economy will speed the

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6 A requirement for these results was the use of long sample sets.

7 If they make the same assumptions as in the case of the American states, then the convergence assumption fails.

convergence rate for output but will slow down the convergence rate for income. Even though they admitted that a slow convergence rate was not addressed directly by the theory, they explained it as due to imperfections in the physical capital's market.

Benhabib and Jovanovic (1991) looked at Romer (1987) and used a model similar to Romer's (1987) to conclude that there was no support that capital-related externalities were present. They stated that the results were different because Romer faced simultaneity problems when

> *"...he estimates a production function in which capital and labor are endogenous and correlated with the disturbance to the production function."*[8]

Romer (1987) stated that there might be a simultaneity problem, and recognized the business cycle as a source for this disturbance. He attempted to remove this disturbance by filtering out high frequencies with long run averages. Benhabib and Jovanovic (1991) calculated the correlations between inputs and disturbance, and concluded that even for long-run data the correlation was high enough to explain the high empirical elasticity of output with respect to capital. Furthermore, the positive association between knowledge shocks and capital investments seemed to explain a portion of the variance in the growth rate of different countries. They showed that, if a country is assumed to have a constant returns to scale production function, there was consistency of variation in the growth rates among countries. They concluded that there was no evidence in the data to support the hypothesis that beneficial spillovers from physical capital exist. Likewise, the Solow model, without externalities but with stochastic shocks to knowledge, accords with long-run data on labor, capital and output, and on the micro evidence on knowledge spillovers.

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8 Benhabib and Jovanovic (1991), page 85.

Recently, Mankiw, Romer and Weil (1992) reexamined the Solow growth model and argued that its predictions are, to an approximation, consistent with the evidence. Solow assumed a standard production function that exhibits decreasing returns to capital, with exogenous rates of saving and population growth, and proved that savings and population growth determine the steady state level of income. A country with a high rate of savings is considered rich if compared to a country with a lower rate of savings, and vice versa. Similarly, the faster the population grows, the poorer the country becomes.

Mankiw *et al.* (1992), in an effort to explain the international differences in income per capita, used an augmented version of the Solow model by adding human capital as production input. Their model assumed that a constant portion of output s is invested, and that capital depreciates in a rate given by δ. Thus the steady state income per capita expression is given by the following:

$$\ln\left(\frac{Y_t}{L_t}\right) = \ln A_0 + gt + \frac{\alpha}{1-\alpha}\ln(s) - \frac{\alpha}{1-\alpha}\ln(n+g+\delta), \qquad (1)$$

where $0<\alpha<1$, Y_t is the output, K_t is capital, L_t is labor, A_0 is the level of technology, and g and n are the growth rates of technology and labor respectively. By assuming that the level of technology A_0 is subject to a country specific shock ε, they estimated the following transformation of (1):

$$\ln\left(\frac{Y}{L}\right) = a + \frac{\alpha}{1-\alpha}\ln(s) - \frac{\alpha}{1-\alpha}\ln(n+g+\delta) + \varepsilon. \qquad (2)$$

They estimated equation (2) both with and without imposing the restriction that the coefficients on $\ln(s)$ and $\ln(n+g+\delta)$ were equal in size and opposite in sign. They

concluded that the constraint cannot be rejected by the data they used.[9] Furthermore, the differences among population and savings growth accounted for a large fraction of the variation in income per capita among countries.

They also estimated the expanding version of the Solow model, by adding to (2) the share of human capital to income. Specifically, they also estimated the following:

$$\ln\left(\frac{Y}{L}\right) = \ln A(0) + gt - \frac{\alpha+\beta}{1-\alpha-\beta}\ln(n+g+\delta) + \frac{\alpha}{1-\alpha-\beta}\ln(s_k) + \frac{\beta}{1-\alpha-\beta}\ln(s_h) \quad (3)$$

where, as before, α is the physical capital's share of income, β is the human capital's share of income, s_k is the portion of income devoted to investment for physical capital, and s_h is the portion of income devoted to investment for human capital. They showed that human capital, proxied by the school enrollment rate, was positively related to the level of income. They concluded that adding human capital improved the performance of the traditional Solow model. The main points of their work were: first, the elasticity of income with respect to physical capital does not differ from the capital's share in income. Second, the accumulation of physical capital has a bigger impact on income per capita than the Solow model predicts. Third, population growth has bigger impact on income per capita than the Solow model predicts. And fourth, in contrast to the models with endogenous growth, Mankiw *et al.* (1992) predicted that countries with similar technologies, rates of accumulation, and population growth will converge in income per capita. This convergence occurs with rates slower than the ones that the Solow model predicted.

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9 The data set used in their analysis can be found in the Appendix of their 1992 paper in *QJE*, p.436.

Blomström *et al.* (1992) tested whether factors found significant in studies that included all type of countries, played the same roles if developing countries were studied. They also attempted to measure the acquisition of knowledge through international contacts. To do so, they used the inflow of direct investment capital, and the import of machinery and transport equipment. These two variables were intended to measure the potential for technology transfer. Both variables did not seem to have much of an impact, but the inflow of foreign direct investment had a significant positive effect on income growth, which was stronger for the high-income, developing countries and rather weak for the poor ones.

Something that became apparent by reviewing this literature is that human capital plays a special role in all studies of economic growth. In Lucas (1988) human capital was the engine of economic growth, while Romer (1990) used human capital as the key input to the research sector that produced new ideas or products that led to technological progress. Barro (1991) used school enrollment rates as proxies to human capital. He showed that a country's growth rate was positively related to the initial level of human capital. Something that became apparent from these studies was the need for a better data set on human capital. Recently, Barro and Lee (1993*b*) introduced a data set based on school enrollment ratios that tried to make up for inadequate data on human capital. Their data set is based on educational attainment for 129 countries over five year periods from 1960 to 1985.

5.2.1. Exports and Economic Growth

The idea that international trade can be used as an engine for economic growth is very old and goes back to Adam Smith. It assumes that a country is better off when it participates in trading activities, and, as Ricardo put it, sells those goods for which it has a comparative advantage. However, the idea of opening an economy to international trade was not that popular for most of this century. The twentieth century is the century when underdeveloped countries got a chance to grow, and there were several economists who suggested that protectionism could be one way out of their economic misery.

Therefore, there were no major studies done in this area until the early seventies. Even then, these studies were focused on the results of tariffs and other economic policies and restrictions, and the importance of trade was considered a secondary subject. None of these studies had tackled the question of how trade orientation affects economic performance. One could argue that among the difficulties that these studies faced, was the absence from the traditional growth theory of an adequate explanation that could link national domestic policies with long run equilibrium growth. The development of the endogenous growth theory gave the necessary framework needed to move in this direction.

Michaely (1977) was among the first researchers who used a relatively large sample of cross-country data to analyze econometrically the relationship between trade orientation and economic growth. He used a sample of 41 countries for 1950-1973, and examined whether the rate of exports growth is associated with GDP growth. In order to do so, he used simple rank correlations and found that the Spearman rank coefficient was

significantly positive for the sample as a whole. Furthermore, to avoid deceptive results that come from the fact that exports are components of GDP, he used the rate of growth of the exports shares of GDP. On a similar track, Balassa (1978) used pooled data for 11 countries for the period 1960-1973 and found a positive correlation coefficient between different measures of exports growth and output growth. Criticism of these results included the facts that looking at the correlation coefficient ignored other factors that affect economic growth, that there was no clear distinction between endogenous and exogenous variables, and finally that there was no theoretical background for their econometric work.

Feder (1983) presented a different approach that tried to go beyond the simple rank correlation studies. He assumed that the economy consisted of two sectors: one that produces export goods, and one that produces domestic goods. Thus, the two sectors correspond to the two distinct markets that exist in any country. There is no one national production function, and the output of the domestic sector is dependent on the volume of exports produced. With this formulation Feder wanted to capture the effects of efficient management, better techniques, and other beneficial things created in the export sector that represent positive externalities for the domestic sector.

The formulation of the model assumed that there were two sectors in the economy, the export sector (X) and the domestic sector (D). The production functions for the two sectors are:

$$X_t = G(K_t^x, L_t^x) \tag{4}$$

$$D_t = F(K_t^d, L_t^d, X_t) \tag{5}$$

where K^i and L^i are the physical capital and labor used in the i sector $(i = X,F)$. Furthermore, Feder assumed that productivity was different between the two sectors. Then if we denote by subscripts the partial derivatives, we get:

$$\left(\frac{G_k}{F_k}\right)=\left(\frac{G_l}{F_l}\right)=1+\delta \tag{6}$$

where G_k, F_k and G_l, F_l are the marginal productivities of capital and labor respectively for both sectors. The parameter δ represents the difference in productivity between the two sectors. Also, the fact that exports were included in (5), helped to capture the externalities that Feder introduced into the domestic sector. Aggregate GDP (y) is given by the sum of exports and domestic sectors. Taking time derivatives on (4) and (5) and assuming that the marginal productivity of labor is a linear combination of the average product of labor, [10]

$$F_l = \beta(y/L), \tag{7}$$

Feder derived the following equation that described the growth rate of GDP:

$$\dot{y}/y = \alpha(I/y)+\beta(\dot{L}/L)+\left[F_x+\frac{\delta}{1+\delta}\right](\dot{X}/X)(X/y), \tag{8}$$

where (I/y) is the investment ratio, $\alpha = F_k$, and β is assumed constant.

Transforming (8), Feder estimated the following equation:

$$\dot{y}/y = \alpha(I/y)+\beta(\dot{L}/L)+\gamma\left[(\dot{X}/X)(X/y)\right], \tag{9}$$

10 This assumption is the only restriction that the production function should satisfy.

where γ characterizes the differential productivities of factors in the two sectors of the economy. For estimating (9), he used a sample set of 31 countries for the period of 1964-1973, and tested whether the coefficient of $(\dot{X}/X)(X/y)$ is positive or not. He obtained the following results:

$$\begin{array}{l}(\dot{y}/y) = 0.002 + 0.178\,(I/y) + 0.747\,(\dot{L}/L) + 0.422\,(\dot{X}/X)(X/y) \\ \qquad\quad (0.180)\ \ (3.542) \qquad\quad (2.862) \qquad\quad (5.454) \\ \qquad\quad \overline{R}^2 = 0.689\end{array} \quad , \qquad (10)$$

where the *t*-statistics are in parentheses. Looking at (10), Feder inferred that the results provide a strong support of the hypothesis that marginal factor productivities are higher in the export sector as compared to the domestic sector.

Following Feder's results, several papers appeared that tried to expand the analysis in a different direction. Balassa (1985) used exports as a production input and compared results for a sample of 11 countries in 1960-1973, with results of a sample of 43 countries for 1973-1979 that were adversely affected by the 1973 oil shock. He found that the coefficient of $(\dot{X}/X)$ was higher in the 1973-1979 period than in the period before. On a similar track, Ram (1985) also used a production function that had exports as a production input, and for 73 countries for 1960-1970 and 1970-1977 found that the coefficient of $(\dot{X}/X)$ was significantly positive. Kohli and Singh (1989), using a model similar to Feder's, estimated the production function for a 41-country sample, for 1960-1970 and 1970-1981. They also found the coefficient on $(\dot{X}/X)$ to be positive and significant.

5.3. The Issue of Convergence

Convergence is an issue that comes up every time someone discusses economic growth. Starting with Solow (1957), economic theory has suggested that, in the long-run, all countries should experience the same level of per capita product and productivity. Economies may differ in the capital-per-labor ratio, the level of available technology, the savings rate, even in the rate of depreciation and population growth, and still end up at the same steady state of income level. In that framework, even the countries that start with a lower capital-to-labor ratio will eventually catch up with the rest of the world. In addition, the less-developed countries will grow faster than the more developed if the assumptions stated earlier hold.

The subject of convergence became a very interesting issue for empirical research. It was apparent that the new theory of economic growth was about to touch this issue, too. There are several recent papers on this issue, including Barro and Sala-i-Martin (1991, 1992), Mankiw *et al.* (1992), and Helliwell and Chung (1992). The main question asked is what conditions enable a country to enter a sustained long-run growth that will lead to convergence toward productivity and income levels. Helliwell and Chung (1992) found that there was evidence of international convergence among the industrialized countries. if better quality data were available, the same evidence could be supported for the less-developed countries. Similarly, Barro and Sala-i-Martin (1992) found that convergence in per capita terms held for the United States for the period of 1840 to 1988. They stated that for long samples

> *"...poor states tend to grow faster in per capita terms than rich states even if we do not hold constant any variables other than the initial per capita income or product."*[11]

Sala-i-Martin proposed (1990a) a way of incorporating convergence into a regression analysis. Here we briefly discuss his method, and we incorporate it in our analysis. Let us start with a usual neoclassical growth model, for which output is produced as a function of per capita capital ($k_t = K_t/L$)

$$Y_t = f(k_t). \tag{11}$$

This economy faces the following resource constraint:

$$\dot{k}_t = f(k_t) - c_t, \tag{12}$$

assuming that there is neither population growth, nor depreciation of physical capital k_t. Furthermore, assuming that consumers have the usual Ramsey type preferences, we get the following growth rate for consumption:

$$\frac{\dot{c}_t}{c_t} = \frac{f'(k_t) - \rho}{\sigma}, \tag{13}$$

where σ^{-1} is the inter temporal elasticity of substitution for the consumers' utility, and ρ is the discount rate. If then we linearize equations (12) and (13) around the steady state and solve this system of equations, we get the convergence equation:

$$\frac{(\ln y_t - \ln y_0)}{t} = a - \vartheta \ln y_0. \tag{14}$$

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11 Barro and Sala-i-Martin (1992), page 245.

Equation (14) states that if all the assumptions made earlier hold, the regression of growth on the log of the initial level of income has a negative coefficient. In other words, poor countries tend to grow faster than rich ones.

In the following chapter we describe our econometric specification. We use for our theoretical framework the models for closed and open economy cases discussed in earlier chapters. Our analysis is based on the assumption that an economy that produces intermediate goods will benefit by opening up to trade opportunities. It became clear that human capital should be included in an econometric analysis of economic growth. The discussion of the last section made also clear that in our econometric model we need to incorporate the value of GDP of a given year, so as to test for the convergence hypothesis.

Chapter 6

SPECIFICATION OF THE EMPIRICAL ANALYSIS

In the previous chapter we referred to the econometric process that we will follow. We also briefly discussed the work that has been done in the area of economic growth, with production functions that incorporate labor, physical capital and human capital as production inputs. In this chapter we lay out the model we will estimate, and describe the data sets that we will use for this estimation.

The question that is of interest is how economic growth is affected and in which direction (better or worse), when an economy is opened to trade. This is not the first time that have economists posed this question, but in our case the differentiating factor is the assumption that not only traditional production inputs are used (i.e. physical capital and labor), but also human capital, which becomes an input in the production process. Furthermore, as our theoretical model suggests, the continuously introduced intermediate goods drive the economic growth of the country. It is apparent, then, that exports and

imports will play a significant role in this exercise. The following sections provide the framework of the econometric analysis and describe the data set used for our modeling.

6.1. Framework of the Econometric Analysis

This section refers explicitly to the framework of our analysis. In order to transform our theoretical model to an estimable model, we follow an approach similar to the one Feder (1983) introduced.

Let us assume that each economy consists of two sectors: the final goods sector, and the intermediate goods sector. Both intermediate (N_t) and final goods (Q_t) are considered tradable. The final goods sector uses intermediates and labor as production inputs, while the intermediate goods sector uses physical and human capital. The production function for the final good sector is given by:

$$Q_t = F(L_t, N_t^d, X_t), \tag{1}$$

where X, and L are exports and labor respectively, and N_t^d is the intermediate goods used for producing domestic final goods.

In a similar fashion, the production function of the intermediate goods is given by:

$$N_t = G(K_t, h_t, M_t) \tag{2}$$

where K, and h are physical capital and human capital respectively, and M is imports. It is assumed, as in Feder (1983), that exports have positive externalities on the production of final goods. In addition, let us assume that imports play a similar role to exports and also have positive externalities in the production of intermediate goods. This assumption is justifiable since, according to the theory presented in earlier chapters, it is not profitable for a firm that produces intermediate goods to produce a good that is already available in another country.[1] For example, it does not pay for a firm abroad to market a computer processor similar to the ones that Intel already has in the market. Furthermore, human capital is used in the production of the intermediate goods in conjunction with physical capital.

In the analysis so far, an important assumption is made. The production of intermediate goods is divided into two portions, one that is used in the production of final goods in the home country, and another that is exported for use in foreign countries. Then, if we assume that aggregate GDP (y) is the sum of the production of final and intermediate goods, what we really mean is that in the calculation of GDP we incorporate the production of final goods and that portion of intermediate goods used for exports. Thus, we get:

$$y_t = Q_t + N_t^e, \tag{3}$$

where N_t^e is the intermediate goods exported in the foreign countries.

Differentiating equations (1) and (2) yields

$$\dot{Q}_t = F_{L_t}\dot{L}_t + F_{N_t^d}\dot{N}_t^d + F_{X_t}\dot{X}_t \tag{4}$$

---

1 We continue assuming that there are neither differences in the world prices, nor any kind of trade restrictions, such as tariffs, quotas etc.

$$\dot{N}_t = G_{K_t}\dot{K}_t + G_{h_t}\dot{h}_t + G_{M_t}\dot{M}_t. \tag{5}$$

Differentiating (3), we get:

$$\dot{y}_t = \dot{Q}_t + \dot{N}_t^e. \tag{6}$$

Earlier we assumed that the production of intermediates is the sum of those used in the production of domestic final goods and those that were exported. In other words,

$$N_t = N_t^d + N_t^e. \tag{7}$$

Taking time derivatives on (7), and then substituting into (6), together with (4) and (5), we get:

$$\dot{y}_t = F_{L_t}\dot{L}_t + (F_{N_t^d} - 1)\dot{N}_t^d + F_{X_t}\dot{X}_t + G_{K_t}\dot{K}_t + G_{h_t}\dot{h}_t + G_{M_t}\dot{M}_t. \tag{8}$$

Let us also assume that the marginal productivity of labor is a linear combination of the average product of labor, similar to Feder (1983), or

$$F_L = \beta(y/L). \tag{9}$$

Thus, the marginal productivity of labor, F_L, is substituted by β. In addition, we assumed earlier that intermediates are made mainly of raw physical capital. Thus here we assume that $\dot{N}_t$ can be approximated by $\dot{K}_t$. Applying these assumptions to (8), we get:

$$\dot{y}_t = F_{L_t}\dot{L}_t + (F_{N_t^d} - 1 + G_{K_t})\dot{K}_t + F_{X_t}\dot{X}_t + G_h\dot{h}_t + G_{M_t}\dot{M}_t. \tag{10}$$

Technically, equation (10) needs to be divided by the level of y_t to represent the growth rate of *GDP* as a function of the growth rates of labor, physical and human capital,

exports and imports. In order to do so, we need to make a few adjustments. These adjustments are associated with the availability of data for specific variables. In the section that follows, we explicitly describe the data used in this analysis.

6.2. Data

In this analysis two different data sets were combined. Most of the data comes from the data set put together by Summers and Heston, known as the Penn World Table (PWT). In this analysis, we use the most recent version, which is the Mark 5.5 (PWT5.5). It is based on the preceding version of Mark 5, for which more information can be found in Summers and Heston (1991). Penn World Table has a unique feature, in that its expenditure entries are denominated by using a common currency in a common set of prices. As a result international comparisons are possible, both between countries and over time.

PWT5.5 presents several time series on expenditures of various types (like private consumption expenditures, governmental expenditures, capital stock measures, etc.) and on relative prices (purchasing power parity). It covers 150 countries for the period 1950-1990. From all these countries we eventually used 79, based on the data we had for those 79 countries. A list of all available variables in PWT5 and PWT5.5 can be found in the Appendix, Table A.1. Table A.2 lists the countries we used in our analysis, together with the abbreviations used by the World Bank.

The series used from PWT5.5 are the Real Gross Domestic Product (RGDP) in 1985 prices, the population, investment, and labor data. The data set contains figures on real income, private and governmental expenditures, investment, and population for most countries. Special attention is required for the quality of the provided data. The quality of the data has to do with the way that the data are collected and/or reported. The period of time for PWT5.5 spans from 1950 to 1990, and during this time many new countries appeared in the world scene. Countries that had been colonies or parts of other countries obtained the right to become independent. This fact alone might create difficulties for the researcher, as far as the quality of the reported data is concerned. As a result the PWT5.5 does not provide the same detail for all the variables we are interested in. Although the significance of the PWT5.5 is not reduced by these problems, it is the researcher's responsibility to choose from those the countries that provide a better quality of data. Summers and Heston (1991) recognized this problem and suggested a way of rating the quality of the data. They provided a scale from A to D, where A stands for a very good quality of data and D for data of rather poor quality. In this analysis we used mostly countries that had a quality grade of better than C, but our data set does include some countries whose data quality is graded D. A detailed list of all countries used in this analysis and their corresponding data quality grade is provided in the Appendix A, Table A.3.

PWT5.5 does not provide us with data on capital formation. On the other hand, PWT5 provides some data on capital formation from 1980 to 1988 for a few countries. Unfortunately this variable has been dropped for the PWT5.5, so we will have to approximate the capital formation. In the empirical work on an aggregate level, it is customary to approximate capital formation with investment data. Specifically, we used the investment-to-income share as an approximation for capital formation.

Our analysis focuses on the effects of openness to international trade on economic growth. This focus requires availability of export and import data. PWT5.5 does not directly provide figures on exports. Instead the variable *OPEN* is given which, once transformed, gives back the figures for exports and imports. The variable *OPEN* is the sum of the current priced exports and imports divided by gross domestic product in current prices (*CGDP*).[2] There is a minor problem with respect to the export and import data. The way that *OPEN* is calculated incorporates two different normalizations. As a result, the data that we can recover using the formulas provided by Summers and Heston (1991)[3] can potentially give us even negative figures for imports. We decided to solve this problem by using the raw data for exports and imports instead of the ones that are derived from the formulas provided in Summers and Heston.[4]

PWT5.5 provides data on the labor force participation rate instead of data on the labor force. Summers and Heston (1991) provide the following formula, which is used to derive the labor force data:

$$\left(\text{Labor Force Participation Rate}\right) = \frac{\text{RGDP}}{\text{RGDPW}}\ ^{5},$$

where *RGDP* is the gross domestic product per capita in 1985 prices, and *RGDPW* is the real GDP per worker.

The data used in this study cover the period 1960 to 1989. In this data set there are neither centrally planned economies nor-oil-producing countries. The reason for eliminating the former has to do with the credibility of the reported data, and for the latter

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2 Summers and Heston (1991) provide the necessary recovery formulas for exports and imports.
3 Summer and Heston (1991), page 347.
4 We want to thank Dan Nuxoll for giving us access to these data.
5 Summers and Heston (1991), page 348.

with the fact that the reported GDP figures represent extraction of existing resources rather than value added.[6] Specifically the sample set consists of 17 African countries, 14 Asian countries, 20 European countries, 24 American (North and South) countries, and 4 Oceanic ones.

Finally, we need data for human capital. This task requires an approximation. The available data do not explicitly measure human capital. A way to approximate it would be to use educational enrollment rates. If there were data available on educational enrollment rates for the 1960-1989 period, growth rates could be calculated and used to approximate the growth rates of human capital. The fact is that the data set in Barro and Lee (1993) does not provide enough years to work out growth rates. Actually, these data state enrollment rates for every five year starting with 1960. We decided to use as proxy the enrollment rate for the year 1960.

There are educational data for 79 countries. The Barro and Lee (1993) data set is rich since it provides not only enrollment rates for those who completed some educational level, but also provides the rates for those who attained any kind of education for this specific year. This is very important, since in most of the countries the rate of completed education is significantly smaller than the rate of attained education, always compared at the same educational level. The data set provides enrollment rates for three basic levels of education, *primary, secondary,* and *higher*. It reports data over five year period from 1960 to 1985, for 129 countries. This was the main reason we were forced to consider only the value of educational enrollment rates for the year 1960. As we said earlier, our

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6 Oil-producing countries are Bahrain, Gabon, Iran, Iraq, Kuwait, Oman, Saudi Arabia and United Arab Emirates. Lesotho is also excluded since the sum of private and government expenditures exceed GDP, which suggests that a critical portion of its population works abroad. The same assumption is made in Mankiw *et al.* (1992).

data set examines 79 countries that belong to the intersection of the data set based on PWT5.5 and the one based on Barro-Lee data set.

In the previous chapter we referred to the issue of convergence. Since it is customary for empirical growth theory to check for convergence, we incorporated convergence in the model. Specifically, we used the logarithm of the value of GDP for the year 1960, and we expect the estimated coefficient to be negative, denoting that some type of convergence has taken place among countries over the years.

Before we present the estimation results in the following chapter, it is interesting to look at Figure 6.1. It presents the growth rates of the real GDP for the seven largest countries of OECD. One thing is very clear from this graph: the rate of growth of these seven countries is converging to a common value as we get closer to 1989. A characteristic example is Japan's growth rate of GDP, which even though it is much lower than the United States' rate in 1960, it is very close to the growth rate of U.S. by 1989. It is also obvious that the U.S. started much higher than the other countries, probably because of the abundance of labor and physical capital that the U.S. enjoyed after the second World War. In contrast, Europe and Japan had to rebuild the damages of loss of infrastructure, caused by World War II, and could only concentrate on economic growth much later than did the U.S. The countries presented in this figure show a convergence pattern towards the growth rate of the United States.

Figure 6.2. is similar to Figure 6.1, but adds the average real per-capita GDP for African and Latin American countries, those that are among the 79 countries of our sample set. It is interesting to note that the average growth rate for both groups from is much lower than the growth rate of the seven countries, of Figure 6.1. Thus, we need to treat the countries that belong to these two groups more cautiously. A way to do so is to

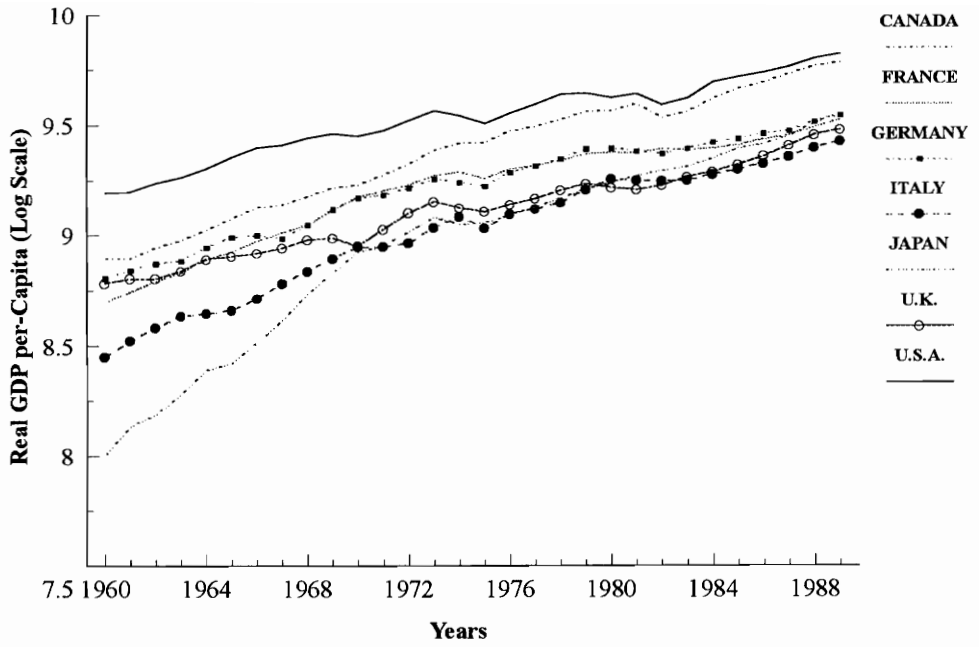

Figure 6.1.

Real per-capita GDP in the Seven Largest OECD Countries 1960-1989

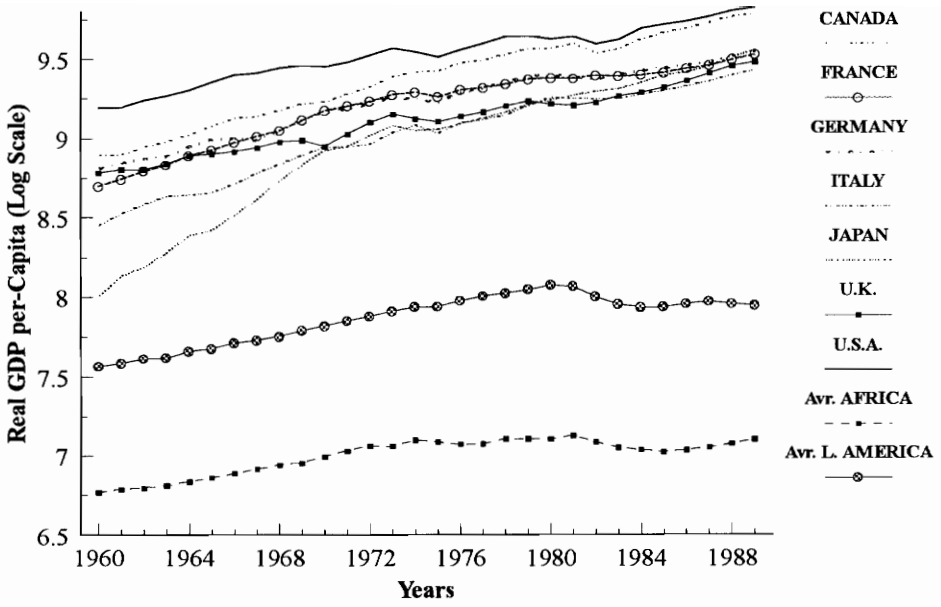

Figure 6.2.

Comparison between the Real per-Capita GDP of the 7 Countries versus the Average of Africa and Latin America for 1960-1989

introduce two dummy variables that take the value "one", when a country is African or Latin American, respectively, and take the value "zero" otherwise.

6.3. The Model of Estimation

In the previous section of this chapter we referred to some necessary adjustments needed to be made in equation (10). We mentioned earlier that instead of capital formation data, we will use investment-to-income shares. Also, the human capital growth rate is approximated by the educational enrollment rate of the year 1960 for the primary and secondary level. Putting all these assumptions into (10), we get:

$$\dot{y}_t = F_{L_t}\dot{L}_t + (F_{N_t^d} - 1 + G_{K_t})I + F_{X_t}\dot{X}_t + G_h h_{60} + G_{M_t}\dot{M}_t. \qquad (11)$$

Now, dividing (11) by y_t and making use of (9), we get the growth rate of the GDP $(\dot{y}_t/y_t)$, or

$$\frac{\dot{y}}{y} = \alpha\frac{I}{y} + \beta\frac{\dot{L}}{L} + \gamma\frac{\dot{X}}{X}\frac{X}{y} + \delta\frac{\dot{M}}{M}\frac{M}{y} + \varepsilon h_{60}, \qquad (12)$$

where $\alpha = (F_{N_t^d} - 1 + G_{K_t})$, and $\varepsilon = G_h/y$.

If we now incorporate into equation (12) the dummies for African and Latin American countries , the value of GDP for 1960, and split the educational variable h_{60} into primary and secondary education enrollment rates, we get the equation we estimate for the growth rate of the economy, or

$$\frac{\dot{y}}{y} = \alpha\frac{I}{y} + \beta\frac{\dot{L}}{L} + \gamma\frac{\dot{X}}{X}\frac{X}{y} + \delta\frac{\dot{M}}{M}\frac{M}{y} + \varepsilon_1 Pr60 + \varepsilon_2 Se60 + \zeta y_{60} + \eta Afr + \theta LAm + e_t \quad . \quad (13)$$

Notice that there is not a clean effect of exports (imports) growth, since the variable is the product of the growth rate times the share of exports (imports) in GDP. In the next chapter we present the results of estimating (13).

Chapter 7

ESTIMATION RESULTS

The previous chapter provided us with the necessary framework for the econometric analysis. It laid out the framework that is based on the theory we proposed in the earlier chapters, and it described in detail the data used in this analysis. The next section presents the estimation results and is followed by the section on misspecification testing. In this section we test for departures from assumptions such as normality, linearity, and homoskedasticity. This section is important because, in the empirical work on economic growth, no one else has considered the effects that a misspecified model might have on the results. Finally, we close the chapter with an attempt to test the robustness of the empirical model.

7.1. Econometric Results

We estimated the following equation:

$$\frac{\dot{y}}{y} = \alpha\frac{I}{y} + \beta\frac{\dot{L}}{L} + \gamma\frac{\dot{X}}{X}\frac{X}{y} + \delta\frac{\dot{M}}{M}\frac{M}{y} + \varepsilon_1 Pr60 + \varepsilon_2 Se60 + \zeta y_{60} + \eta Afr + \theta LAm + e_t \quad , \quad (1)$$

where $\dot{y}/y$ is the growth rate of GDP, I/y is the investment to income share, $\dot{L}/L$ is the labor growth rate, $\frac{\dot{X}}{X}\frac{X}{y}$ and $\frac{\dot{M}}{M}\frac{M}{y}$ are the measures of export and import growth rates respectively, $Pr60$ and $Se60$ are the enrollment rates in primary and secondary educational levels for the year 1960, y_{60} is the value of GDP for the year 1960, and Afr and LAm are dummies for the African and Latin American countries respectively.

Before continuing with the presentation of the estimation results, let us present some figures that show the relationship between growth rate of GDP and investment ratios, exports and imports respectively. Figure 7.1 shows the relation between the average investment to income ratio and the average per-capita aggregate growth rate of GDP for the 79 countries and for the period 1960 to 1989. There is clearly a positive relation that totally supports the theoretical results that suggest that the more abundant in physical capital a country is, the faster it grows. This finding accords not only with the original Solow model, but with more recent models, such as Lucas (1988), Barro (1991), Mankiw *et al* (1992).

Figure 7.2 is very similar to Figure 6.1 of chapter 6. Instead of presenting the growth rates of GDP for all 79 countries converging to a common growth rate, it

represents the negative relation between the average per-capita aggregate growth rate of GDP and the level of GDP for the year 1960. This graph supports the idea that poorer countries grow faster than richer ones, all other things being equal.

In Figures 7.3 and 7.4 the relation between the real average per-capita aggregate growth rate of GDP and the measure of exports and imports is presented. Note that the estimated variable is not the growth rate of exports (or imports) but the multiplication of the growth rate times the share of exports (imports) of the real GDP. Both figures show clearly that the relationship is positive, something that accords with our theory, which suggests that the opening of an economy that produces intermediate and final goods will lead to economic expansion.

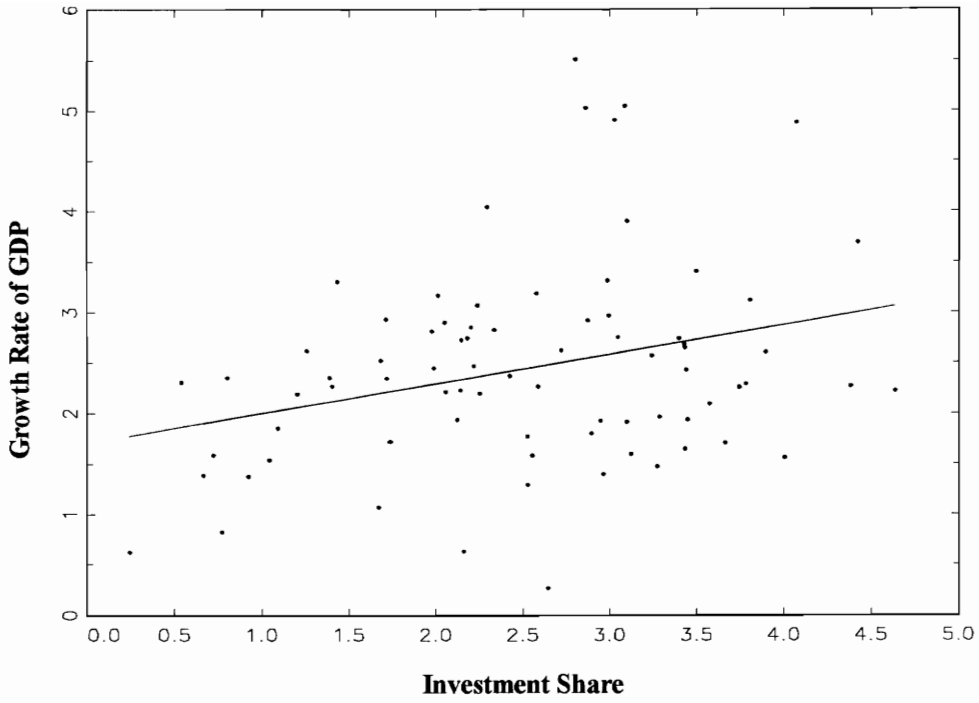

Figure 7.1.

Relation between Average per-Capita Aggregate Growth Rate of Real GDP and Average Investment to Income Ratio

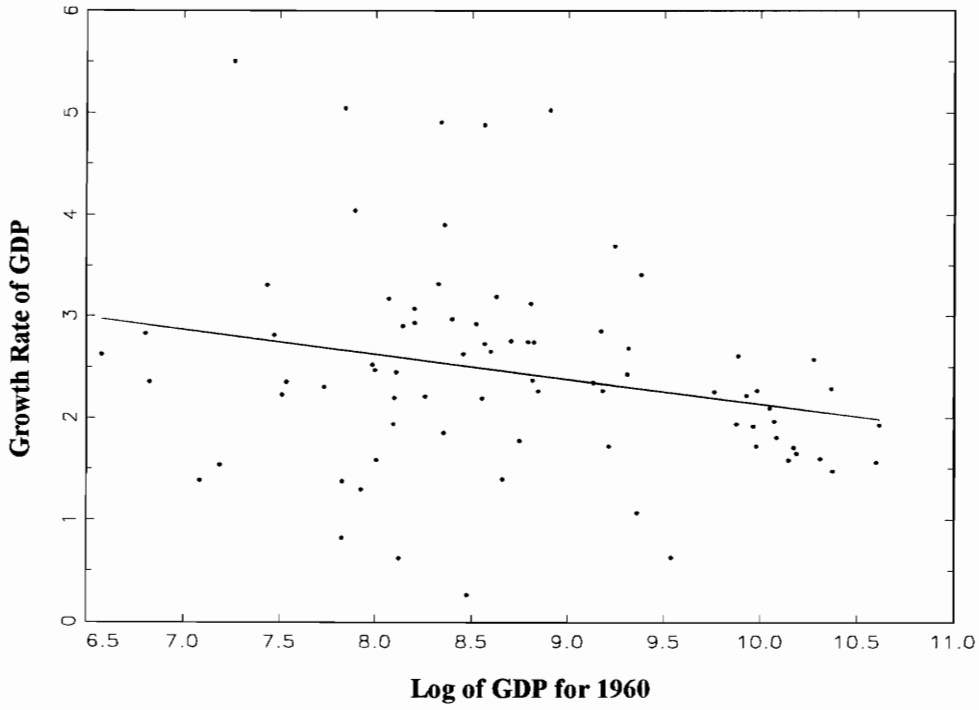

Figure 7.2.

Relation between the Average per-Capita Aggregate Growth Rate of Real GDP and the GDP Value for the Year 1960

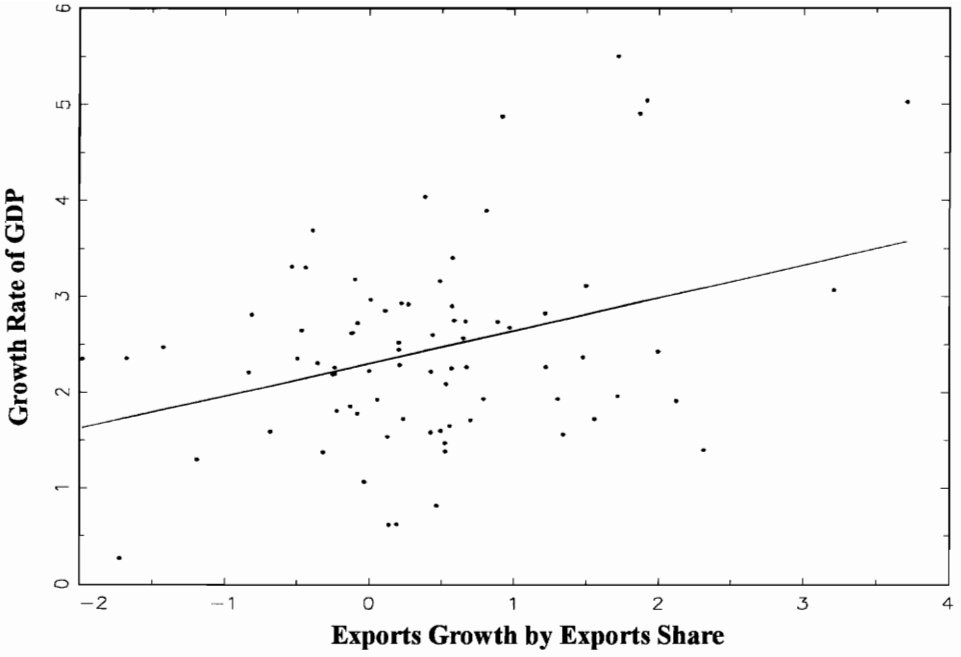

Figure 7.3.

Relation between the Average per-Capita Aggregate Growth Rate of Real GDP and Exports Measure

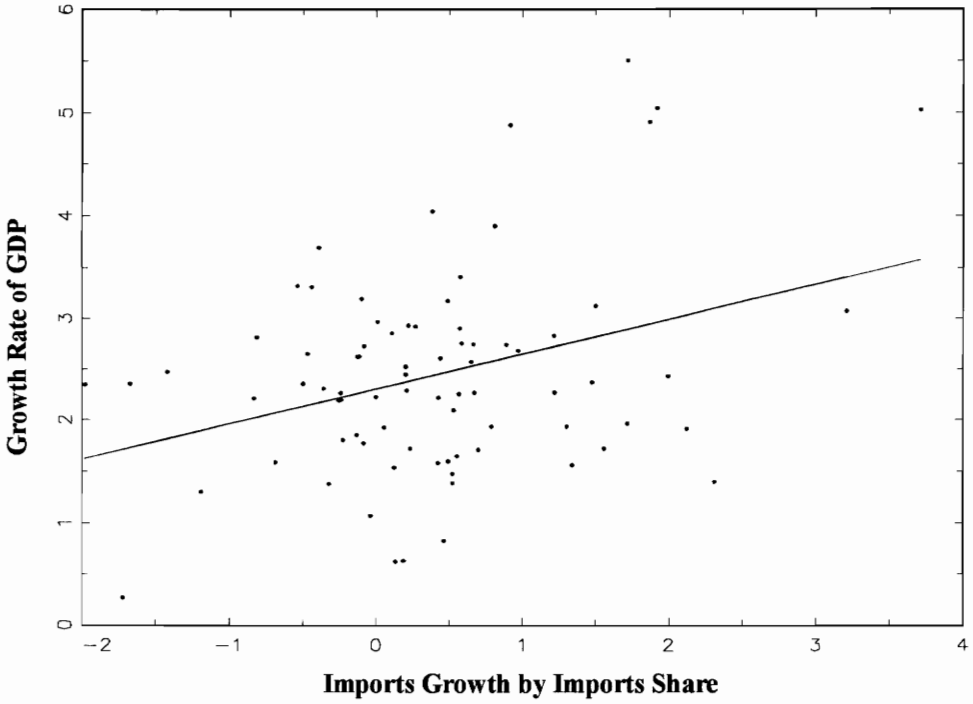

Figure 7.4.

Relation between the Average per-Capita Aggregate Growth Rate of Real GDP and Imports Measure

To estimate (1), we used the data set that was described in chapter six. The dependent variable is the aggregate real growth rate of GDP per-capita (*AGDP*). It is calculated by taking the logarithms of the real GDP data for the years 1960-1989. Then to calculate the growth rate, we subtract the logarithm of the GDP value of 1989 from the logarithm of the 1960 value, and we get the 30-year growth rate for the specific country. We divide the growth rate by the number of years, in our case 30, to get the annual growth rate. To this annual growth rate we add the annual population growth rate, that is calculated the same way, and we get the annual aggregate growth rate.

The growth rate of labor is calculated similarly to the growth rate of GDP, while for the average investment share to physical capital we take the average of investment share from PWT5.5, *CI*, and divide it by the number of years. The education enrollment rates are the values that Barro-Lee (1993) report for the year 1960 in their data set. Finally Africa and Latin America are dummies that take the value of 1 when the country is an African or a Latin American one, and zero otherwise.

Table 1 presents the results of estimating equation (1). The coefficient of investment to income ratio is significantly different from zero, and the same holds for the coefficient of the labor growth rate. Theory predicts that an increase in the available labor force will cause a positive effect on the economic growth of the country, and the same holds for a change in the physical capital stock. The coefficients of primary and secondary education enrollment rates for 1960 are positive, but not significant at the 5% level. Still the sign is the one that the theory supports, that is, the more human capital that is available, the better the economic growth a country experiences. Furthermore, the coefficient for export measures is positive but not significant, while the coefficient for import measure is positive and almost significant at the 20% level. Thus the theoretical framework proposed in the early chapters of this thesis is supported by the data.

TABLE 1

ESTIMATION of EQUATION (1)
Dependent variable: ln *RGDP* per-capita in 1989

Number of Observations	79
Degrees of Freedom	69
CONSTANT	0.0823
	(*0.0180*)
Average I/y	0.1009
	(*0.0245*)
Annual $\dot{L}/L$	0.9201
	(*0.1888*)
Primary 1960	0.00051
	(*0.0154*)
Secondary 1960	0.0121
	(*0.0326*)
$(\dot{X}/X)(X/y)$	0.2449
	(*0.1754*)
$(\dot{M}/M)(M/y)$	0.4420
	(*0.3442*)
$\ln GDP_{1960}$	-0.0105
	(*0.0027*)
Africa	-0.0094
	(*0.0043*)
Latin America	-0.0079
	(*0.0036*)
R^2	0.6369
s.s.e.	0.0105

Note. Standard errors are in parentheses (*italics*) under the parameter estimates. The investment figures are averages for the period 1960-1989.

It is also interesting to see graphically how exports and imports together affect the growth rate of GDP. In order to do so, we created a new variable called TRADE. Using the coefficients for the measures of exports and imports, from the estimation of (1), TRADE equals to $0.245(\dot{X}/X)(X/y)$ $+0.442(\dot{M}/M)(M/y)$. Figure 7.5 represents the relation of TRADE to the growth rate of GDP. It is also obvious that the relation of this variable to the per-capita aggregate growth rate of GDP is positive, something that is expected, since a country that participates in international trade gains from importing goods that are not produced domestically, while exporting goods that are produced by domestic producers.

Another issue that needs our attention is that the coefficient of the logarithm of the value of GDP for 1960 is negative, and significant at the 5% level. This result verifies that the convergence hypothesis holds, that the poorer the country the faster it grows (with everything else held constant). In addition, the coefficients for the dummies Africa and Latin America are also negative and significantly different from zero. This should be expected, since as Figure 6.2 clearly shows, the performance of the African and Latin American countries is much lower than the rest of the world.

We mentioned earlier that as proxy for the human capital we used the enrollment rates in the primary and secondary levels of education for the year 1960. It is obvious that this approximation depends heavily on the quality of the available data. It is observed that in many cases and in several countries the principals have the tendency to inflate the number of students registered in their institutions. There are several reasons for these actions, but the most common is the extra funding that is allocated to schools with more students. The researcher has to be aware of this problem and must always question the quality of this type of data. As an alternative, someone could use years of enrollment versus enrollment rates, but the Barro-Lee data set provides data for very few countries,

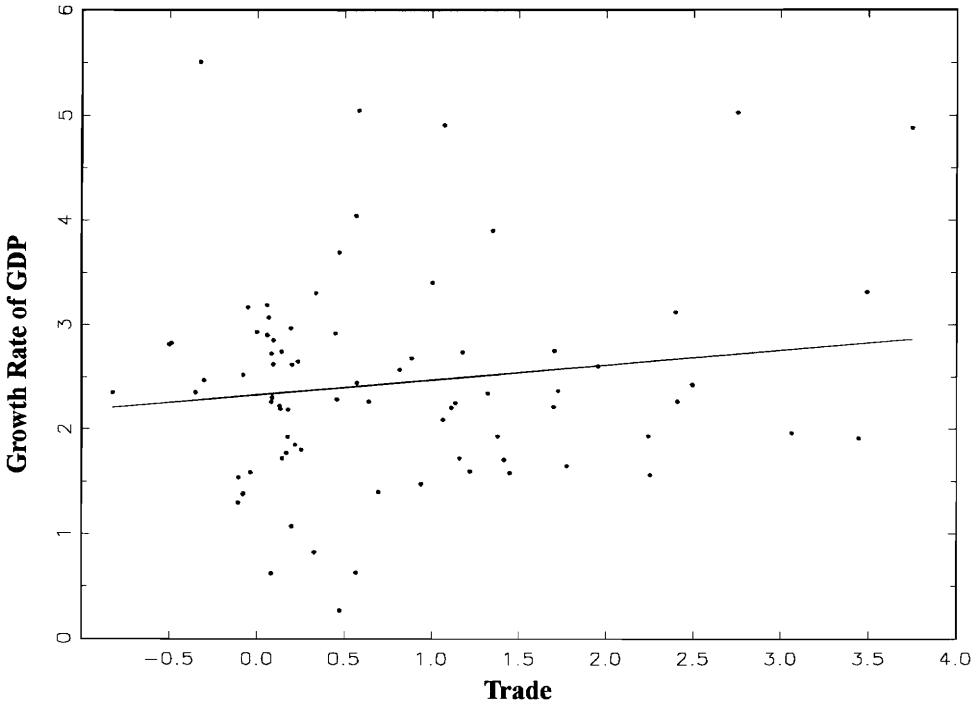

Figure 7.5.

Relation between the Average per-Capita Aggregate Growth Rate of Real GDP and the variable TRADE, where TRADE = $0.245(\dot{X}/X)(X/y)+0.442(\dot{M}/M)(M/y)$

which makes decent econometric modeling rather difficult.

Thus, we wanted to see if the inclusion of the primary school enrollment rates for 1970 will improve the explanatory power of equation (1). We added the new regressor *PR70* into (1), and we report the results in Table 10. The results are very similar to the one of Table 1. We can infer, then, that there is no significant change in the quality of the data for the school enrollment between 1960 and 1970.

The estimation of equation (1) poses an extra challenge. The inclusion of the variables exports and imports as independent implies that there is no dependency among these two variables and the aggregate growth rate of the economy. In the accounting equation of GDP, exports and imports are included as explanatory variables, something that makes us suspect a possible simultaneity problem. Furthermore, one can argue that if the size of an economy increases, it will affect the size of exports and imports of this particular economy. It is then apparent that we need to treat equation (1) more carefully. In order to address this problem, we will use the method of instrumental variables.

The instrumental variables method is useful when we suspect that the regressor is contemporaneously correlated with the disturbance. In general, several variables can be used as instruments. In our case, as instruments we will use the investment-to-income ratio (*AvgCI*), the labor growth rate ($\dot{L}/L$), the primary and secondary education enrollment rates for 1960 (*PR60, SE60*), the value of GDP in 1960 (*GDP60*), and finally the dummies for Africa and Latin America countries (*Afr, Lam*). The fact that we use as instruments all the independent variables but the ones that we suspect for simultaneity, make the method of instrumental variables equivalent to two-stage least-squares (2SLS) estimation. Thus, we regress the growth rates of exports (imports) on the instruments

TABLE 2

ESTIMATION of EQUATION (1) INCLUDING *PR70*
Dependent variable: ln *RGDP* per-capita in 1989

Number of Observations	79
Degrees of Freedom	68
CONSTANT	0.0824
	(*0.0182*)
Average I/y	0.1002
	(*0.0251*)
Annual $\dot{L}/L$	0.9260
	(*0.1949*)
Primary 1960	0.004
	(*0.0173*)
Secondary 1960	0.0130
	(*0.0334*)
Primary 1970	0.0025
	(*0.0182*)
$(\dot{X}/X)(X/y)$	0.2470
	(*0.1774*)
$(\dot{M}/M)(M/y)$	0.4327
	(*0.3532*)
$\ln GDP_{1960}$	-0.0106
	(*0.0027*)
Africa	-0.0093
	(*0.0043*)
Latin America	-0.0078
	(*0.0037*)
R^2	0.6370
s.s.e.	0.0106

Note. Standard errors are in parentheses (*italics*) under the parameter estimates. The investment figures are averages for the period 1960-1989.

picked earlier and save the fitted values of these growth rates. Then for estimating equation (1), we use the fitted values of the growth rates, instead of their original values, and report the results in Table 3.[1]

Table 3 presents the results of estimating equation (1) using two stage least squares. As in Table 1, the coefficients of constant, average investment to income ratio, and the annual growth rate of labor, are positive and significant at the 5% level. Furthermore, the coefficients of the average investment-to-income ratio in both estimations are similar in size to the one reported in Table 1, something that is also true for the coefficients of the labor growth rate. Thus similar explanations apply, that an increase in the average investment-to-income ratio causes the growth rate of the economy to increase. Similarly, an increase in the available labor force causes a positive effect on the economic growth of a country. In a similar pattern, the results for the educational variables are also similar to that of Table 1. Additionally, the coefficients of primary and secondary education enrollment rates for the year 1960 are still insignificant. They have a positive sign, which is what the theory predicts; that is, the more educated the labor force is, the better the economic growth for the country.

The results for the measures of exports and imports differ from the one presented in Table 1. Both coefficients of exports and imports have a positive sign. We can argue that the opening of a country to international trade has positive effects on the growth of the economy. There is a critical difference between the estimation results presented in the two tables. The coefficient exports measured with the 2SLS estimation is now positive but not significantly different from zero. Its value, though, is slightly larger than the one reported in Table 1. On the other hand, the coefficient of imports measured is still not

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[1] The results for the first stage are reported in Appendix C.

TABLE 3

ESTIMATION of EQUATION (1) USING 2SLS

Dependent variable: ln *RGDP* per-capita in 1989

Number of Observations	79
Degrees of Freedom	69
CONSTANT	0.0932
	(*0.0233*)
Average I/y	0.1262
	(*0.0314*)
Annual $\dot{L}/L$	0.9269
	(*0.2093*)
Primary 1960	0.002
	(*0.0179*)
Secondary 1960	0.0145
	(*0.0367*)
$(\dot{X}/X)(X/y)$	0.3027
	(*0.6242*)
$(\dot{M}/M)(M/y)$	1.3644
	(*1.2173*)
ln GDP_{1960}	-0.0123
	(*0.0036*)
Africa	-0.0115
	(*0.0054*)
Latin America	-0.0078
	(*0.0043*)
R^2	0.5671
s.s.e.	0.0115

Note. Standard errors are in parentheses (*italics*) under the parameter estimates. The investment figures are averages for the period 1960-1989.

significant, but its value is, for 2SLS, much larger that the one reported in Table 1 (1.3644 vs. 0.4327).

Finally, the coefficient of the value of GDP for the year 1960 is also negative and significant at 5% level, something that proves that the convergence hypothesis is supported by the data. In a similar fashion, the coefficients of the dummies Latin America and Africa, are negative and also significantly different from zero. Furthermore, their values are very close to the ones reported in Table 1.

For the same reasons reported earlier in this section, we wanted to check whether the explanatory power of our model will be improved by the inclusion of the variable *PR70* (the enrollment rate of primary education for the year 1970). The results of a 2SLS estimation are the results presented in Table 4. These results are very similar to those of Table 3. Thus, we can still infer that there is no reason to believe that a change in the quality of the human capital data took place between the years 1960 and 1970.

Barro (1991) and Mankiw *et al* (1992) have also included human capital data in their growth estimation models. Both concluded that the human capital coefficients are significantly different from zero. In our case, we were not able to prove that human capital coefficients are significantly different from zero. We want to test if both coefficients of human capital, as represented in our estimation by the enrollment rates, are jointly different from zero. Specifically the following hypothesis is of interest:

$$H_0: \varepsilon_1 = \varepsilon_2 = 0 \text{ vs. } H_1: \text{either } \varepsilon_1 \text{ or } \varepsilon_2 \neq 0,$$

where $\varepsilon_1, \varepsilon_2$ are the coefficients of primary and secondary education enrollment rates for 1960, from equation (1). In order to do so, we used an F-type test, which is given by the following general formula:

TABLE 4

ESTIMATION of EQUATION (1) USING 2SLS and INCLUDING *PR70*

Dependent variable: ln *RGDP* per-capita in 1989	
Number of Observations	79
Degrees of Freedom	68
CONSTANT	0.0883
	(*0.0237*)
Average I/y	0.1185
	(*0.324*)
Annual $\dot{L}/L$	0.9089
	(*0.2137*)
Primary 1960	0.0027
	(*0.0191*)
Secondary 1960	0.0124
	(*0.0376*)
Primary 1970	0.0059
	(*0.0206*)
$(\dot{X}/X)(X/y)$	0.0357
	(*0.6364*)
$(\dot{M}/M)(M/y)$	0.7410
	(*1.1963*)
$\ln GDP_{1960}$	-0.0115
	(*0.0037*)
Africa	-0.0105
	(*0.0055*)
Latin America	-0.0079
	(*0.0044*)
R^2	0.5647
s.s.e.	0.0116

Note. Standard errors are in parentheses (*italics*) under the parameter estimates. The investment figures are averages for the period 1960-1989.

$$FT = \frac{RRSS - URSS}{URSS}\left(\frac{T - k^*}{m}\right), \tag{2}$$

where *RRSS* is the Restricted Residual Sum of Squares, *URSS* is the Unrestricted Residual Sum of Squares, T is the sample size, k^* is the number of parameters in the unrestricted equation, and m is the number of restrictions. The *RRSS* comes from estimating equation (1) without the variables for human capital, while the *URSS* comes from estimating equation (1) which includes the variables for human capital. In our case, the sample size T is 79, the number of parameters in the unrestricted equation k^* is 10, and the number of restrictions m is 2. Calculating the value of the statistic given by (2), we get FT=0.121. Given that the value of the F distribution at 5% significance level for 2 degrees of freedom for the numerator and 69 for the denominator is $c_a \approx 3.10$, the null hypothesis is not rejected. Thus, we cannot infer that both coefficients of human capital are different from zero.

It is also interesting to test whether both the dummies Latin America and Africa are different from zero. We said earlier that each one of these is significant at the 5% level. Now we want to test the hypothesis

$$H_0: \eta = \theta = 0 \text{ vs. } H_1: \text{either } \eta \text{ or } \theta \neq 0, \tag{3}$$

where η, θ are the coefficients of Africa and Latin America, respectively, in equation (1). We will use the same test statistic given by (2). Since we are again testing for 2 variables, the parameters k^*, m, and T are the same as in the previous test. In this case the FT=2.6131, while, as before, $c_a \approx 3.10$. Here again, we fail strongly to reject the null hypothesis, that together the coefficients of the dummies are not significantly different from zero.

Finally, we want to perform a similar type of test for the coefficients of exports and imports. In this case the formulation of the hypothesis is the same, that is, we want to test whether

$$H_0: \gamma = \delta = 0 \text{ vs. } H_1: \text{either } \gamma \text{ or } \delta \neq 0, \tag{4}$$

where γ, δ are the coefficients of the measures of exports and imports, respectively. Using the same test statistic as in the last two cases, we get FT=0.971, a much smaller figure than $c_a \approx 3.10$. Thus we fail to reject the null hypothesis, that both coefficients of exports and imports are simultaneously not significantly different from zero.

Several explanations can be given for these results. First, as far as the results for human capital are concerned, the quality of the data set used for human capital can be questioned. Not because we failed to reject the hypotheses' tests reported earlier, but because, as Barro and Lee (1993) admitted, their collection of data on human capital hides potential problems because the reported data might not be true. Second, the data set does not cover the entire period from 1960 to 1989 but reports figures for every five years, and thus does not allow us to use growth rates. And third, the results about exports and imports might be improved if we had data on capital accumulation available.

7.2. Misspecification Testing

In the previous section we presented the results of estimating equation (1). The estimates given in Tables 1-4 are produced by assuming that the underlying probability

model is the normal one, that the mean is a linear function of the independent variables, that the variance of the dependent variable σ^2, is homoskedastic (free of any association with the independent variables of the model), and that the parameters are homogeneous with respect to a specific order. Moreover, a normality assumption implies linearity and homoskedasticity. In order to see if these assumptions hold true for our model, we need to perform several misspecification tests.

Misspecification testing is not very common in the empirical work done on economic growth. Several authors have acknowledged that their estimations are based on the assumptions stated above, but in the literature so far, there were no attempts to check for misspecified models. As mentioned in chapter 5, a misspecified model is of little value, since all its results are based on false assumptions. In the case of cross section data, homoskedasticity is a very crucial assumption. Thus, even though no one tests explicitly for any departures from it, they state[2] that we should keep in mind that their results might violate the homoskedasticity assumption. Barro (1991)[3] mentions the potential problem of heteroskedastic variance, and he proposes, instead of the usual t-values, the use of White's standard errors, or as they are known, the Homoskedasticity Corrected Standard Errors (HCSE). This procedure does not solve the problem, since the normality assumption is assumed to hold for HCSE to be of any use. In the following sub-sections we provide the results of the misspecification testing for the assumptions of normality, linearity, homoskedasticity, and homogeneity for both models that we estimated in the previous section of this chapter.

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2 Barro (1991), Mankiw *et al* (1992).

3 *Op cit.* page 414.

7.2.1. Normality

We start our analysis of the misspecification tests by checking whether the assumption of normality holds. Chapter 5 discusses the consequences of violating the normality assumption. For our testing purposes we use the test proposed by D'Agostino and Pearson, a description of which can be found in Appendix B.

Table 5 reports the results of the misspecification tests for normality. It is divided into two groups: the group on the top refers to the estimation of equation (1) with and without the variable *PR70*. Apparently the assumption of normality is accepted for both models. Furthermore, since we did not observe too many differences between the two, both should face the same problems, if any. The group at the bottom of Table 5 refers to the estimation of equation (1) using instrumental variables, also with and without the variable *PR70*. For these models the assumption of normality is accepted too. Of course for the model that includes the *PR70* and incorporates the 2SLS method, the acceptance of the normality assumption is on the border.

Table 5 also reports the results for the skewness and kurtosis coefficients. It is known that the normal distribution is characterized by the skewness and kurtosis coefficients. In the case of a normal distribution, the skewness coefficient should be equal to zero, while the kurtosis should be equal to 3. These tests reported in Table 5 can be used to test the following two hypotheses: first, whether the skewness coefficient is significantly different from zero, and second, whether the kurtosis coefficient is significantly different from 3. Looking at the Table 5, we see that the skewness coefficient

TABLE 5

TESTING DEPARTURES FROM NORMALITY for BOTH MODELS

Model	D'Agostino-Pearson	D'Agostino Skewness	D'Agostino Kurtosis
	Estimation of Equation (1)		
Equation (1)	4.5722	0.2454	2.1241
	(0.1017)	(0.4031)	(0.0168)
Equation (1)	4.7747	0.3065	2.1635
with *PR70*	(0.0919)	(0.3796)	(0.0153)
	Estimation of Equation (1) by Using 2SLS Method		
Equation (1)	3.4810	1.8549	0.2006
	(0.1754)	(0.0318)	(0.4205)
Equation (1)	6.1159	2.4646	0.2045
with *PR70*	(0.0470)	(0.0069)	(0.4190)

Note. Probability values in parentheses.

is not significantly different from zero, while for the kurtosis coefficient, we can say that it is significantly different from 3, as far as the top of Table 5 is concerned. In the case of estimating (1) using 2SLS, the reverse results hold. Now, the kurtosis coefficient is not significantly different from three, while the skewness coefficient is significantly different from zero.

7.2.2. Linearity

Next in the misspecification testing comes the assumption of linearity. It refers to the conditional mean, which we assume to be a linear combination of the independent variables of the model. We will use both the Kolmogorov-Gabor polynomial of *n*-th degree, and the usual *RESET* test. The results are reported in Table 6, which is divided in two parts, similar to Table 5. As we can see, both models accept the hypothesis that the conditional mean is a linear function of the independent variables.

7.2.3. Homoskedasticity

We also need to check for any departures from the homoskedasticity assumption. Under this assumption the variance of the dependent variable σ^2 does not depend on the

TABLE 6

TESTING DEPARTURES FROM LINEARITY for BOTH MODELS

Model	KOLMOGOROV-GABOR Test of Order 2	*RESET* Test of Order 2
	F-Statistic	*F*-Statistic
	Estimation of Equation (1)	
Equation (1)	0.9831	1.5369
	(0.5213)	(0.2193)
Equation (1)	0.9117	1.2418
with *PR70*	(0.5213)	(0.2691)
	Estimation of Equation (2) by Using 2SLS Method	
Equation (1)	1.5808	2.5406
	(0.0913)	(0.1156)
Equation (1)	2.7991	3.7002
with *PR70*	(0.068)	(0.5864)

Note. Probability values in parentheses.

variations of the independent variables. In those cases where the homoskedasticity assumption is violated, the best way to tackle the problem is by modeling the variance.

Table 7 presents the results of the misspecification testing for any departures from the homoskedasticity assumption. Clearly both models rather strongly accept this assumption.

7.2.4. Homogeneity

From the assumptions described in Chapter 5, we have tested so far for any possible departures from normality, linearity, and homoskedasticity. The assumption we have not tested is homogeneity. The homogeneity assumption implies invariance of the parameters with respect to a specific order. For example, in the case of time series, the homogeneity assumption takes the form of time-independence. In our case, the data set consists of cross-sectional data. Thus a way to check for any violations of this assumption would be to drop the first and last observations from our sample, and run the same regressions as before, to test whether the results change significantly. For our sample this means that we need to drop the first and the last country of the sample of 79 countries. The countries dropped from the sample are Algeria and Papua-New Guinea. Then we run the same regressions as before, and report the results in Table 8.

As we can see from Table 8, the results for the smaller sample set are very close to those for the original set. The coefficients of investment-to-income ratio and labor growth rate are positive and significant at the 5% level. Similarly the coefficients of human capital

TABLE 7

TESTING DEPARTURES FROM HOMOSKEDASTICITY for BOTH MODELS

Model	*RESET* Test of Order 2
	F-Statistic
Estimation of Equation (1)	
Equation (1)	0.8611
	(0.3563)
Equation (1) with *PR70*	0.2865
	(0.5940)
Estimation of Equation (1) by Using Method	
Equation (1)	2.5447
	(0.1148)
Equation (1) with *PR70*	1.8870
	(0.1735)

Note. Probability values in parentheses.

TABLE 8

ESTIMATION of EQUATION (1) USING 2SLS for 77 COUNTRIES
Dependent variable: ln *RGDP* per-capita in 1989

Number of Observations	77
Degrees of Freedom	67
CONSTANT	0.0972
	(*0.0243*)
Average I/y	0.1265
	(*0.0334*)
Annual $\dot{L}/L$	0.9249
	(*0.2192*)
Primary 1960	0.0035
	(*0.0174*)
Secondary 1960	0.0132
	(*0.0393*)
$(\dot{X}/X)(X/y)$	0.4501
	(*0.6984*)
$(\dot{M}/M)(M/y)$	1.6229
	(*1.2896*)
$\ln GDP_{1960}$	-0.0127
	(*0.0038*)
Africa	-0.013
	(*0.0052*)
Latin America	-0.0082
	(*0.0045*)
R^2	0.5801
s.s.e.	0.0115

Note. Standard errors are in parentheses (*italics*) under the parameter estimates. The investment figures are averages for the period 1960-1989.

are also positive , but are still insignificant, as in the case of the whole sample. The coefficient of GDP for 1960 is also negative and significant, while the same holds for the dummies of African and Latin American countries. The fact, however, that we dropped two countries with a generally lower level of educational attendance, helped increase slightly the coefficient of primary education. To conclude, we infer that the results provided by the smaller sample set are not inconsistent with the comments we made earlier with respect to our empirical model.

7.3. Robustness

Before we conclude this work, we want to check how strong or fragile are our empirical results. Testing for model robustness allows researchers to observe any changes that happen to the results when the conditional information set slightly changes. In order to do this, the data set is enhanced so as to contain new variables that other researchers proved need to be in a growth-estimating model. Levine and Renelt (1992) examine whether empirical models used in the growth literature are robust or not, with regard to small changes in the list of right hand variables.[4]

To check for the robustness of our model, we enhance the information set by including such variables as the government's expenditures share to GDP (*AvgG*), and two variables that try to capture political instability, called assassinations (*ASSAS*), and

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4 Levine and Renelt (1992) use extreme-bounds analysis, pioneered by Leamer, to test several empirical growth models for robustness. It is not our intention to present a thorough analysis of this type here.

revolutions per coups (*REVC*). The average share of governmental expenditures to income *AvgG* is from PWT5.5, while the variables for political instability are from the data set used by Barro (1991).[5] The variable *REVC* is the number of revolutions and coups per year, while the *ASSAS* is the number of assassinations per million population per year. We incorporate these variables one at a time, and all together, and perform a 2SLS estimation, with the results reported in Tables 9 and 10.

Another approach that we use is to sequentially drop the countries that belong to either Africa or Latin America. Then the size of our sample set varies from 57, when no Latin American countries are included, to 62 when no African countries are included. We also perform a 2SLS estimation on these two sub-samples and report the results in Table 10.

By including the variable *AvgG*, the coefficients of exports and imports are now smaller than before. Still they are not significant, and the same holds for the coefficients of human capital. The effect of governmental expenditures on economic growth is positive and significant at the 10% level.[6] The coefficients of GDP for 1960, and the dummies for African and Latin American countries are also negative and significant. Including in the model the variable *ASSAS*, the values of the primary and secondary education enrollment rates increase a little, but they are still insignificant. The inclusion in the last regression of the variable *REVC* causes the coefficient of exports to get a negative sign that was also observed in Barro (1991) and Levine and Renelt (1992).

The most interesting results are provided by the regressions that drop either the Latin American or the African countries. As Table 10 reports, the exclusion of Latin American countries causes the values of educational attainment to jump. The coefficient of

5 Barro (1991) pages 407-443.
6 Similar results are reported in Barro (1991).

the primary education enrollment rate increases by more than four times, while the coefficient of the secondary education rate goes up by almost 50%. These results should be expected since, by excluding these countries, the quality of the provided data drastically improves. Similar results are given by the regression that drops the African countries. Furthermore, the coefficients of export and import measures are also larger than before, for the regressions that drop either the Latin American or the African countries. These results provide us with an indication that our model is somewhat robust. The fact that there are differences when we drop specific countries might have to do with the nature and the quality of the provided data.

TABLE 9

CHECKING for ROBUSTNESS of EQUATION (1)
Dependent variable: ln *RGDP* per-capita in 1989

Independent Variables	Estimation of (1)	Estimation of (1) including *AvgG*	Estimation of (1) including *AvgG, ASSAS*
CONSTANT	0.0932	0.0669	0.0699
	(*0.0233*)	(*0.0259*)	(*0.026*)
Average I/y	0.1262	0.1177	0.1256
	(*0.0314*)	(*0.0319*)	(*0.033*)
Annual $\dot{L}/L$	0.9269	0.8017	0.8391
	(*0.2093*)	(*0.217*)	(*0.2207*)
Primary 1960	0.002	0.0034	0.0044
	(*0.0179*)	(*0.017*)	(*0.017*)
Secondary 1960	0.0145	0.0123	0.0194
	(*0.0367*)	(*0.0374*)	(*0.038*)
$(\dot{X}/X)(X/y)$	0.3027	0.0751	0.199
	(*0.6242*)	(*0.6636*)	(*0.6597*)
$(\dot{M}/M)(M/y)$	1.3644	0.8912	1.2622
	(*1.2173*)	(*1.2673*)	(*1.2609*)
Average G/y		0.0605	0.0625
		(*0.0337*)	(*0.0337*)
ASSAS			0.0028
			(*0.003*)
REVC			
ln GDP_{1960}	-0.0123	-0.0099	-0.0109
	(*0.0036*)	(*0.0036*)	(*0.0038*)
Africa	-0.0115	-0.0091	-0.0084
	(*0.0054*)	(*0.0055*)	(*0.0056*)
Latin America	-0.0078	-0.0034	-0.0030
	(*0.0043*)	(*0.0055*)	(*0.0054*)

TABLE 9
(CONTINUED)

CHECKING for ROBUSTNESS of EQUATION (1)
Dependent variable: ln *RGDP* per-capita in 1989

Independent Variables	Estimation of (1)	Estimation of (1) including *AvgG*	Estimation of (1) including *AvgG, ASSAS*
Number of Observations	79	77*	77
Degrees of Freedom	69	66	65
R^2	0.5671	0.56	0.5662
s.s.e.	0.0115	0.0115	0.0115

Note. Standard errors are in parentheses(*italics*) under the parameter estimates. The investment and governmental figures are averages for the period 1960-1989.

* The sample size for the variables used by Barro (1991) was 77 countries.

TABLE 10

CHECKING for ROBUSTNESS of EQUATION (1)
Dependent variable: ln *RGDP* per-capita in 1989

Independent Variables	Estimation of (1) including *AvgG, ASSAS, REVC*	Estimation of (1) without Latin America	Estimation of (1) without Africa
CONSTANT	0.0738	0.1003	0.1081
	(*0.0267*)	(*0.024*)	(*0.0284*)
Average I/y	0.1264	0.1304	0.1033
	(*0.0323*)	(*0.0342*)	(*0.0309*)
Annual $\dot{L}/L$	0.8463	0.8251	0.9160
	(*0.2217*)	(*0.2862*)	(*0.2019*)
Primary 1960	0.0039	0.0182	0.0067
	(*0.0171*)	(*0.0215*)	(*0.0167*)
Secondary 1960	0.0207	0.0309	0.0305
	(*0.0381*)	(*0.0416*)	(*0.0387*)
$(\dot{X}/X)(X/y)$	-0.3027	0.4291	0.3773
	(*0.6144*)	(*0.4716*)	(*0.6871*)
$(\dot{M}/M)(M/y)$	1.4453	0.2161	1.5347
	(*1.2190*)	(*1.0294*)	(*1.1951*)
Average G/y	0.0638		
	(*0.034*)		
ASSAS	0.0035		
	(*0.0033*)		
REVC	-0.0033		
	(*0.0074*)		
ln GDP_{1960}	-0.0114	-0.0140	-0.0139
	(*0.0038*)	(*0.0037*)	(*0.0042*)
Africa	-0.0089	-0.0062	
	(*0.0056*)	(*0.0058*)	

TABLE 10
(CONTINUED)

CHECKING for ROBUSTNESS of EQUATION (1)
Dependent variable: ln *RGDP* per-capita in 1989

Independent Variables	Estimation of (1) including *AvgG, ASSAS, REVC*	Estimation of (1) without Latin America	Estimation of (1) without Africa
Latin America	-0.0026		-0.0081
	(*0.0055*)		(*0.0042*)
Number of Observations	77*	57**	62**
Degrees of Freedom	64	48	53
R^2	0.5681	0.6105	0.6344
s.s.e.	0.0116	0.0114	0.0105

Note. Standard errors are in parentheses (*italics*) under the parameter estimates. The investment and governmental figures are averages for the period 1960-1989.

* The sample size for the variables used by Barro (1991) was 77 countries.

** The samples without the Latin American (African) countries are a subset of the original sample of 79 countries.

Chapter 8

CONCLUSIONS

This work studied the effects on economic growth of the continuous introduction of intermediate goods. We developed a theoretical framework that enhanced the endogenous growth theory, by allowing for participation in international trade. Furthermore, we tested the behavior of the proposed theoretical model with data provided mainly from the Penn World Tables.

In the theoretical part, we described an economy in two different stages: first, we developed a model for a closed economy with a production process a la Romer, and studied the effects of the continuous introduction of durable goods. We showed that the continuous introduction of durables is responsible for sustained economic growth. Furthermore, we showed that a more educated labor force contributes more to the growth of the economy. Second, we enhanced the model of a closed economy by assuming that trade can take place among similar countries. Both final and intermediate goods are

traded, and we showed that participation in international trade further enhances the economic growth of this type of economy. Our intention was to focus on the effects of opening up an economy to trade, and thus we did not consider the effects of trade policies and/or trade barriers. Furthermore, we proved that as long as there are no restrictions to trade, countries can continue to grow. Our theoretical assumptions were strict enough, and thus we did not attempt to make direct comparisons between countries. Because we assumed that the countries participating in our theoretical model were identical, we found that those countries that pursue the introduction of new durables will grow faster.

In the empirical part of this thesis, we tested the proposed theoretical model. Specifically, we wanted to test how the economic growth of an economy, as described by our theoretical model, is affected by the measures of exports and imports. First, we defined the econometric method, used throughout this thesis. Then, we showed empirically that there is a positive relation between the growth rate of an economy and the measures of exports and imports. Furthermore, the accumulation of physical capital was shown to have positive effects on economic growth. We used enrollment rates for primary and secondary educational levels for the year 1960 to approximate human capital. The estimations showed that a positive relation exists between the available level of human capital and the growth rate of the economy. In our theoretical model, human capital was introduced indirectly through the efficiency of the labor force. We showed that an educated labor force is preferred by the producers of final goods. Obviously, the more educated the labor force, the easier it is to apply new techniques, and thus the more efficient the production of new durables becomes. Therefore we conclude that, all other things being equal, poor countries that start with a relatively high level of human capital will grow faster.

An important portion of the empirical work is dedicated to misspecification testing. To our knowledge, this has not been done before in any type of applied work in the area of economic growth (in either neoclassical or endogenous growth theory). We tested our statistical model against the assumptions of normality (the underlined distribution is normal), linearity (the conditional mean is a linear combination of the independent variables), homoskedasticity (the variance of the dependent variable does not depend on variations of the independent variables), and homogeneity (invariance of the parameters with respect to a specific order). We demonstrated that our statistical specification did not violate any of these four assumptions. Thus, we can call our model a well-specified statistical model.

The work presented in this thesis has some limitations. On the theoretical side, it might become interesting if a government were introduced into the modeling. Furthermore, the model needs to consider the different trade policies that countries apply, and to incorporate them into the proposed theory. Thus a model that considers the rates of tariffs, quotas, etc., will enhance the power of the proposed theory. Another limitation of the present work is the poor quality of the educational data. The approximation of human capital with enrollment rates depends heavily on the quality of the reported data. Even though the Barro-Lee data set is the largest and most complete data set available today, it faces integrity problems, because it depends on sources of questionable quality. Another aspect of the limitations that enrollment rates present, is that with these rates it becomes difficult to distinguish the standards of the education provided. Obviously, comparing two different educational systems requiring students to spend the same time in school, the better one will stand out. We hope that a revised version of this data set will be available soon, because this is something that will really help to improve empirical testing on this subject. Furthermore, we look forward to a new release of the Penn World Table that will

provide us with extended data on durables' capital, which will be a better proxy for intermediate goods (instead of the investment-to-income ratio used today).

Future research should be extended to the effects of tariffs, and of taxes, on economic growth in a global system. Additional attention should be given to different tax systems and to what effects they have on economic growth. Moreover, attention should be given to the different qualities of for education, to the number of children, and to different tariff policies that influence economic growth.

Appendix A

TABLE A.1

VARIABLE LIST of PWT5 and PWT5.5

Variable Name	Definition	Variables used in our analysis	PWT5.5 variable
POP*	Population	✓	✓
RGDPCH[a]	Real GDP per capita (1985 international prices; Chain index)	✓	✓
C	Real Consumption (% of RGDPCH; 1985 international prices)		✓
I	Real Gross Domestic Investment (Private and Public) (% of RGDPCH; 1985 international prices)		✓
G	Real Government (Public Consumption) (% of RGDPCH; 1985 international prices)		✓
RGDP[b]*	Real GDP per capita (1985 international prices; Laspeyres Index)	✓	✓
RGDPTT[c]	Real GDP per capita adjusted for changes in the terms of trade (1985 international prices for domestic absorption; current prices for Net Foreign Balance)		✓

TABLE A.1
(CONTINUED)

Variable Name	Definition	Variables used in our analysis	PWT5.5 variable
Y	CGDPj/CGDPUS (current international prices)		✓
CGDP	Real GDP per capita (current international prices)	✓	✓
CC	Real Consumption (% of CGDP; current international prices)	✓	✓
*CI**	Real Gross Domestic Investment (Private and Public) (% of CGDP, current international prices)	✓	✓
CG	Real Government (Public Consumption) (% of CGDP, current international prices)	✓	✓
P	Price level of GDP (%) (PPP of GDP)/(Exchange Rate)[d]		✓
PC	Price level of Consumption (%) (PPP of Consumption)/Exchange Rate)[d]		✓
PI	Price level of Investment (%) (PPP of Investment) /(Exchange Rate)[d]		✓
PG	Price level of Government (%) (PPP of Government) /(Exchange Rate)[d]		✓
ExR	Exchange Rate[d]		✓
RGDPEA	Real GDP per equivalent adult (1985 international prices)		✓
RGDPW*	Real GDP per worker (1985 international prices)	✓	✓
KapW	Capital stock per worker (1985 international prices) (Kap1w + Kap2w + Kap3w)		
Kap1w	Producers Durables (% of KapW) (1985 international prices)		

TABLE A.1
(CONTINUED)

Variable Name	Definition	Variables used in our analysis	PWT5.5 variable
OPEN*	Openness (Exports + Imports)/CGDP) (current international prices)	✓	✓
RGNP	Real Gross National Product (% of CGDP) (1985 international prices)		
IPri	Gross Domestic Private Investment (% of GDI in current international prices)		
AGDP	Annual Gross Domestic Product	✓	
ASSAS	Number of assassinations per million of population per year (Barro (1991))	✓	
Lam	Dummy for Latin American countries	✓	
Afr	Dummy for African countries	✓	
PR60	Enrollment rate in Primary Education for 1960 (Barro-Lee (1993))	✓	
SE60	Enrollment rate in Secondary Education for 1960 (Barro-Lee (1993))	✓	
PR70	Enrollment rate in Primary Education for 1970 (Barro-Lee (1993))	✓	
REVC	Number of revolutions and coups per year (Barro (1991))	✓	

Source: Summers and Heston (1991).

Note. Variables marked with * were used for the purposes of our estimation.

[a] RGDP2 in PWT4

[b] RGDP1 in PWT4

[c] RGDP3 in PWT4

[d] PPP: Purchasing Power Parity relative to the US dollar. Exchange Rate relative to the US dollar.

[e] Note that Residential Construction is not included in KapW.

TABLE A.2
COUNTRY LIST, SAMPLE SETS and QUALITY RANK [a]

Countries	World Bank Abbreviation	Countries in Barro (1991)	Quality Rank
Africa			
Algeria	DZA	✓	D
Ghana	GHA	✓	D
Kenya	KEN	✓	C
Malawi	MWI	✓	D+
Mauritius	MUS	✓	D+
Mozambique	MOZ		C-
Niger	NER		D
Senegal	SEN	✓	C-
South Africa	ZAF	✓	C-
Swaziland	SWZ	✓	D+
Togo	TGO	✓	D
Tunisia	TUN	✓	C-
Uganda	UGA	✓	D
Zaire	ZAR	✓	D
Zambia	ZMB	✓	D+
Zimbambwe	ZWE	✓	
Asia			
Bangladesh.	BGD	✓	D
Hong Kong	HKG	✓	B-
India	IND	✓	C
Israel	ISR	✓	B
Japan	JPN	✓	A
Republic of Korea	KOR	✓	B-
Malaysia	MYS	✓	C
Myanmar (Burma)	BUR	✓	D
Pakistan	PAK	✓	C-
Philippines	PHL	✓	C
Singapore	SGP	✓	C

TABLE A.2
(CONTINUED)

Countries	World Bank Abbreviation	Countries in Barro (1991)	Quality Rank
Sri Lanka	LKA	✓	C-
Taiwan	OAN	✓	D-
Thailand	THA	✓	C-
Europe			
Austria	AUT	✓	A-
Belgium	BEL	✓	A
Cyprus	CYP	✓	C
Denmark	DNK	✓	A-
Finland	FIN	✓	A-
France	FRA	✓	A
Federal Republic of Germany	DEU	✓	A
Greece	GRC	✓	A-
Iceland	ISL	✓	B+
Ireland	IRL	✓	A-
Italy	ITA	✓	A
Malta	MLT	✓	C
Netherlands	NLD	✓	A
Norway	NOR	✓	A-
Portugal	PRT	✓	A-
Spain	ESP	✓	A-
Sweden	SWE	✓	A-
Switzerland	CHE	✓	B+
Turkey	TUR	✓	C
United Kingdom	GBR	✓	A
Central and North America			
Barbados	BRB	✓	C
Canada	CAN	✓	A-
Costa Rica	CRI	✓	C
Dominican Republic	DOM	✓	C

TABLE A.2
(CONTINUED)

Countries	World Bank Abbreviation	Countries in Barro (1991)	Quality Rank
El Salvador	SLV	✓	C
Guatemala	GTM	✓	C
Haiti	HTI	✓	D
Honduras	HND	✓	C
Jamaica	JAM	✓	C
Mexico	MEX	✓	C
Trinidad and Tobago	TTO	✓	C
United States	USA	✓	A
South America			
Argentina	ARG	✓	C
Bolivia	BOL	✓	C
Brazil	BRA	✓	C-
Chile	CHL	✓	C
Colombia	COL	✓	C
Equador	ECU	✓	C
Guyana	GUY	✓	D
Paraguay	PRY	✓	C
Peru	PER	✓	C
Uruguay	URY	✓	C-
Venezuela	VEN	✓	C
Oceania			
Australia	AUS	✓	A-
Fiji	FJI	✓	D
New Zealand	NZL	✓	A-
Papua New Guinea	PNG	✓	D

[a] *Source*: Penn World Table 5, Summers and Heston (1991).

Appendix B

NORMALITY TESTS[1]

The normality test is based on the standardized third and forth moments of the residuals, $\sqrt{b_1} = m_3 / \sqrt{m_2^3}$ and $b_2 = m_4 / m_2^2$, where $m_r = \frac{1}{T}\sum_{t=1}^{T} \hat{u}_t^r$, and $r = 2,3,4$. The skewness test statistic by D'Agostino is the following:

$$\delta \ln\left[Y/\alpha + \sqrt{1+(y/\alpha)^2} \right], \text{ where}$$

$$\delta = 1/\sqrt{\ln(W)}, \quad Y = \sqrt{b_1}\sqrt{\frac{(T+1)(T+3)}{6(T-2)}}, \quad \alpha = \sqrt{\frac{2}{(W^2-1)}}, \quad W^2 = -1 + \sqrt{2\left[\beta_2\sqrt{b_1} - 1\right]},$$

$$\text{and } \beta_2\sqrt{b_1} = \frac{3(T^2+27T-70)(T+1)(T+3)}{(T-2)(T+5)(T+7)(T+9)}.$$

This test statistic is distributed approximately by the standard normal distribution.

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1 This appendix is based on the intuition and notes given to me by Anya McGuirk.

An other test used in the misspecification tests section is the kurtosis test of Anscombe and Glynn. This is given by the following:

$$\left[\left(1-\frac{2}{9\nu}\right)-\left(\frac{1-2/\nu}{1+x\sqrt{2/(\nu-4)}}\right)^{1/3}\right]\left(\frac{2}{9\nu}\right)^{-1/2}, \text{ where}$$

$\nu = 6+\frac{8}{\sqrt{\beta_1 b_2}}\left[\frac{2}{\sqrt{\beta_1 b_2}}+\sqrt{1+\frac{4}{\beta_1 b_2}}\right]$, $\sqrt{\beta_1 b_1} = \frac{6(T^2-5T+2)}{(T+7)(T+9)}\sqrt{\frac{6(T+3)(T+5)}{T(T-2)(T-3)}}$, $x = \frac{b_2 - E[b_2]}{\sqrt{Var[b_2]}}$, $E[b_2] = \frac{3(T-1)}{T+1}$, and $Var[b_2] = \frac{24T(T-2)(T-3)}{(T+1)^2(T+3)(T+5)}$. This is also distributed approximately by the standard normal.

Finally, the joint test for normality suggested by D'Agostino and Pearson (1973) is based on the sum of the third and forth moments, or $K^2 = X^2(\sqrt{b_1}) + X^2 b_2$, where $X(\sqrt{b_1})$ and $X b_2$ are standardized normal deviates and the test statistic is distributed approximately by the chi-squared distribution with two degrees of freedom.

Appendix C

RESULTS OF THE FIRST STAGE FOR THE 2SLS OF (1)

TABLE C.1

FIRST STAGE of the 2SLS for EQUATION (1)		
Dependent variable :	$(\dot{X}/X)(X/y)$	$(\dot{M}/M)(M/y)$
Number of Observations	79	79
Degrees of Freedom	71	71
CONSTANT	0.0323	-0.0246
	(0.0390)	*(0.0343)*
Average I/y	0.0500	0.0089
	(0.0534)	*(0.0462)*
Annual $\dot{L}/L$	0.3971	0.4081
	(0.4280)	*(0.364)*
Primary 1960	0.00406	0.0048
	(0.0341)	*(0.0296)*
Secondary 1960	-0.0170	-0.059
	(0.0723)	*(0.0627)*
$\ln GDP_{1960}$	-0.0016	0.0068
	(0.0059)	*(0.0051)*
Africa	-0.0194	-0.0117
	(0.0092)	*(0.0069)*
Latin America	-0.0081	-0.011

TABLE C.2
(CONTINUED)

Dependent variable :	$(\dot{X}/X)(X/y)$	$(\dot{M}/M)(M/y)$
	(0.0079)	*(0.008)*
R^2	0.0986	0.1146
s.s.e.	0.0235	0.0203

Note. Standard errors are in parentheses (*italics*) under the parameter estimates. The investment figures are averages for the period 1960-1989.

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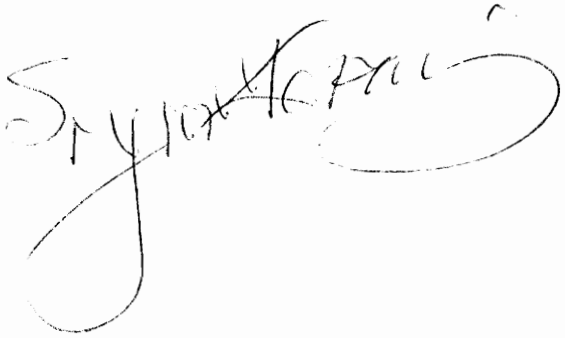
Vita

Spyros Lazaris was born in Athens Greece, on April 14, 1964. He is the first son of two, of Erofili and Michalis Lazaris. He attended the public schools in Athens and in September of 1983, he entered the Athens School of Economics, in Athens Greece. He graduated first in his class on June 1987, with a BA in Economics. For the academic year 1987-1988 he attended the graduate program of Economics at North Carolina State University. In August 1988, he transferred to Virginia Polytechnic Institute and State University, where he enrolled in the Economics program. He got his Master's degree in Economics in May 1990, and his Ph.D. degree in Economics in December 1993.