METHODOLOGIES FOR INTEGRATING TRAFFIC FLOW THEORY, ITS AND EVOLVING SURVEILLANCE TECHNOLOGIES
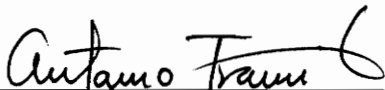
by

Do H. Nam

Dissertation submitted to the Faculty of the

Virginia Polytechnic Institute and State University

in partial fulfillment of the requirements for the degree of
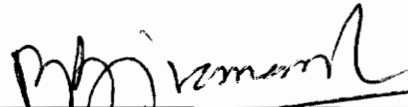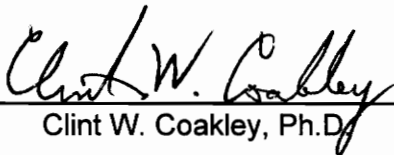
Doctor of Philosophy

in

Civil Engineering

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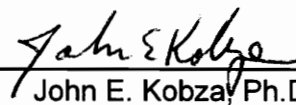
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December 8, 1995

Blacksburg, Virginia

Keywords: Dynamic Traffic Flow Model, Conservation of Vehicles, Freeway Diversion, Intelligent Transportation Systems, Travel Time Estimation, Delay Estimation

METHODOLOGIES FOR INTEGRATING TRAFFIC FLOW THEORY, *ITS* AND EVOLVING SURVEILLANCE TECHNOLOGIES

by

Do H. Nam

Donald R. Drew, Ph.D., Chairman

Civil Engineering

(ABSTRACT)

The purpose of this research is to develop methodologies for applying traffic flow theories to various ITS categories through the utilization of evolving surveillance technologies. This integration of theory, measurement and application has been overlooked since the advent of ITS because of the number of disciplines involved. In this context, the following illustrative methodologies are selected, developed and presented in this study:

- a methodology for automatic measurement of major spatial traffic variables for the present and the future implementation of various ITS functional areas, in general; and

- a methodology for real-time link and incident specific freeway diversion in conjunction with freeway incident management, in particular.

The first methodology includes the development of a dynamic flow model based on stochastic queuing theory and the principle of conservation of vehicles. An inductive

modeling approach adapted here utilizes geometric interpretations of cumulative arrival-departure diagrams which have been drawn directly from surveillance data. The advantages of this model are real-time applicability and transportability as well as ease of use. Analysis results show that the estimates are in qualitative and quantitative agreement with the empirical data measured at 30-second intervals. The analytical expression for link travel times satisfies traffic dynamics where the new form of the equation of conservation of vehicles has been derived. This methodology has potential applicable to automatic traffic control and automatic incident detection.

The methodology is then applied to freeway diversion in real-time in conjunction with freeway incident management. The proposed new form of the equation of conservation of vehicles is applied to detect recurring or non-recurring congestion analytically. The principle of conservation of vehicles is applied to develop the concept of progression and retrogression of incident domain, which turns out to be compatible with traditional shock wave traffic mechanism during incidents. The link and incident specific diversion methodology is achieved by using a delay diagram and volume-travel time curves, which can be plotted per link per incident. The use of such graphic aids makes problem solving much easier and clearer. The dynamic traffic flow model developed here can also be applied to estimate travel times during incidents as a function of time. The development of a computer program for freeway diversion concludes this research.

The fear of Lord is the beginning of knowledge,

but fools despise wisdom and disciplines. (Proverbs 1:7)

This dissertation is dedicated to

my parents, *Byung-Kyu Nam & Hyun-Soon Yang*

parents-in-law, *Yul-Hyo Ha & Seon-Ja You*

wife, *Jong H. Nam*

and

daughter, *Hannah*

for their endless love and support.

ACKNOWLEDGMENT

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APPENDIX

1-1 Background

Congestion on urban freeways has been diagnosed as the most serious traffic problem of large metropolitan areas in the country. A study by the Federal Highway Administration (Lindley 1989) illustrated the magnitude of the worsening problem. While freeway miles of new construction increased only by 10% from 1984 to 1987, vehicle-miles of travel increased by 22% and total delay of vehicle-hours by 61% respectively during the same period. This study estimated that in the year 2005, these figures would increase by 46% and fivefold respectively compared to those estimated in 1987. Another recent study indicated that the economic loss of congestion already cost the nation $100 billion in 1990 (IVHS AMERICA 1992).

While other developed countries launched similar programs such as PROMETHEUS and DRIVE in Europe and RACS and AMTICS in Japan in the late 1980s, the recognition of the worsening congestion problem laid out a vision what came to be called Intelligent Vehicle-Highway Systems (IVHS), which was defined as a system which applies advanced and evolving technologies of communications, control, electronics and computer hardware and software to the transportation system. The merging of evolving communications and electronics technologies and conventional

transportation practices has been accelerated by the Intermodal Surface Transportation Efficiency Act of 1991 which authorized over $600 million to ITS[1] research, development and operational tests.

Table 1.1 Urban freeway congestion statistics (source: Lindley 1989)

	1984	1987	2005
Freeway Miles	15,335	16,097	16,097
Vehicle-Miles of Travel[1]	276,600	337,400	492,500
Recurring Delay[2]	485	728	3,030
Non-Recurring Delay[2]	767	1,287	7,978
Total Delay[2]	1,252	2,015	11,008
Total Wasted Fuel[3]	1,378	2,206	11,638
Total User Costs[4]	9,200	50,500	88,200

[1] million vehicle-miles; [2] million vehicle-hours; [3] million gallons
[4] million dollars

1-2 Nature of Traffic Flow

Traffic flow is a comprehensive phenomena of interactions among the driver, the vehicle and the road which constantly changes over space and time. Such temporal and spatial fluctuations of traffic are usually described by traffic variables. Especially, flow (time headway), density (roadway occupancy) and speed are the so called three fundamental traffic variables at the macroscopic (microscopic) level. Table 1.2 summarizes the definitions of these variables at the microscopic and macroscopic levels.

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[1] IVHS was changed to ITS (Intelligent Transportation Systems) in 1994 and will be referred to as such throughout the text.

While flow or time headway is the easiest to measure, density is usually substituted by roadway occupancy due to its measurement difficulty. Speed has two averages: space-mean speed which is a harmonic mean of the speeds measured at a point; and spot speed which is the arithmetic mean of all the measured speeds. The average travel times are inversely proportional to space-mean speeds. While, flow is a quantitative measure of traffic performance at a particular point or along a particular route, density and speed are qualitative measures of traffic performance. The dimensional analysis leads to the general equation of traffic flow which shows that flow is the product of the density and space-mean speed.

Table 1.2 Definitions of the fundamental traffic variables

Traffic Variable	Definition
Microscopic level	
Time Headway	The time between successive arrivals
Roadway Occupancy	The ratio of presence-type detector's occupied time to the total observation time
Spot Speed	Instantaneous speed
Space-Mean Speed	The time rate of change of distance
Macroscopic level	
Flow	The number of vehicles passing a point (or a segment) during a specific period of time
Density	The number of vehicles per mile
Space-Mean Speed	The time rate of change of distance

1-3 Nature of Congestion

Congestion occurs when the demand exceeds the capacity on the macroscopic level. A queue consists of delayed vehicles and grows against the direction of traffic. Sometimes, it continues to grow and induces another traffic phenomenon: intersection or interchange blockage (spillback) which causes the wide spread of congestion through adjacent routes.

The cause of congestion can be categorized into three groups:

- Temporal surge in demand - recurring type congestion which usually happens in urban areas during peak-hours or in case of special events such as games;

- Transient reduction in capacity - non-recurring type congestion which is caused by incidents such as traffic signal failure, disabled vehicles, accidents and roadside maintenance activities; and

- Periodic interruption of service - the congestion which occurs at the pretimed signalized intersection.

As opposed to recurring type congestion of which spatial and temporal patterns are usually well defined, non-recurring congestion is highly random in terms of duration, frequency and location. Non-recurring type congestion accounted for 64% of the freeway congestion reported in 1987 and is expected to increase to over 70% by the year 2005 (Lindley 1989). As vehicle-miles of travel increases every year, more incidents are likely to happen during peak commuting hours.

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1-4 Evolving Surveillance Technologies

Accurate and reliable information about traffic characteristics is the intelligence for management and control of the traffic demand and the flow in real-time. Surveillance systems that measure traffic characteristics is the foundation of this information. Different surveillance technologies have been investigated to understand how they operate and measure traffic parameters (Kao 1991; Klein et al. 1993, Bernstein et al. 1993). Table 1.3 summarizes these technologies.

Most surveillance technologies are, however, limited to only providing point measurements such as flow and roadway occupancy. For instance, single inductive loop, ultrasonics and infrared based surveillance detects point data such as flow, roadway occupancy and spot speed. Although satellite-based global positioning system is good for wide-area vehicle tracking, it does not detect the fundamental traffic variables such as flow and density. Hence, it can be said that both spatial and temporal fluctuations of traffic flow are not detected by a single technology alone in real-time.

It is necessary to develop a methodology for integrating these technologies and appropriate traffic flow theories. Then, the features of the technologies actually contribute to data acquisition and subsequent data analysis depending on the requirement of the present and the future ITS applications.

Table 1.3 Summary of advanced surveillance technologies

Technology	Operation	Measurement[1]	Remark
Ultrasonics	Emit sound waves above human audible level	F,RO,P,QL	Subject to weather conditions
Active infrared	Emit a pulsed laser beam at higher frequencies	F,RO,VC,SS	Subject to weather conditions
Passive infrared	Measure an infrared energy emitted by objects	F,RO,P	Subject to weather conditions
Microwave radar	Similar to active infrared but emit at lower frequencies	F,P,SS	Health concern
Acoustics	Detect traffic sounds generated by a vehicle	F,P,VC	Subject to environment noise
Video image processing	On-line analysis of video images by signal processing hardware and software	F,RO,P,SS,	Extensive computational task, limited spatial coverage
Aerial video image processing	Analyze video images taken on the plane	F,D,QL	Wide-area coverage, subject to weather, not applicable in real-time
Inductive loops	Detect change in inductance indicating the presence or passage of a vehicle	F,RO,P,SS	Widely used, maintenance problem
Global positioning system	Measure the time taken for signals to travel from a set of at least four satellites to the receiver	TT,SMS	Wide-area vehicle tracking, reliable, DGPS increases accuracy
Automatic vehicle identification	Communicate between in-vehicle unit and roadside unit	F,D,VC,SMS, TT	Toll collection, privacy concern

[1]F = Flow; RO = Roadway Occupancy; P = Presence; QL = Queue Length; VC = Vehicle Classification, SS = Spot Speed; D = Density; TT = Travel Time and SMS = Space-Mean Speed

1-5 Problem Statements

With the advent of innovative ITS, it is required that the observation and the control of the traffic demand and the flow be performed in real-time with more accuracy and reliability than before. At the same time, advanced communications and electronics technologies do and will provide unprecedented tools for traffic surveillance and hence upgrade data acquisition quantitatively and qualitatively. Meanwhile, although there has been extensive research devoted to the development of traffic flow models over the past half century, not much has been directed towards the improvement of existing theories. The integration of theory, measurement and application has been overlooked since the advent of ITS and the information superhighway because of the number of disciplines involved. Therefore, there is a need to tie the state-of-the-art traffic flow theory to the present and the future ITS applications and emerging surveillance technologies. The objectives of the research are identified and defined in this context.

1-6 Objectives and Scope

The main objective of this research is to develop methodologies for applying traffic flow theories to various ITS categories through the utilization of evolving surveillance technologies. In this context, there are two equally important objectives of the research:

- To develop a methodology for automatic measurement of major spatial traffic variables for the present and the future implementation of various ITS functional areas, in general; and

- To apply the methodology to link and incident specific freeway diversion in real-time in conjunction with freeway incident detection and management, in particular.

The development of a dynamic traffic flow model for estimating traffic variables on freeways is limited to straight-pipe sections which do not include ramp junctions. Ramp volumes are assumed to be generally obtainable from the difference between two adjacent traffic counts. The example network used for the demonstration of the proposed freeway diversion procedures consists of a freeway and an arterial having intersections.

1-7 Research Tasks

Evolving communications and electronics technologies provide unprecedented tools for traffic surveillance. However, most of them are limited to only providing point measurements such as flows and occupancies. Both spatial and temporal fluctuations of traffic flow are not detected by a single technology alone in real-time. The details involved in this issue, identified as the first objective, are as follows:

- Complete literature review of traffic flow models and travel time estimation methods and identification of the deficiencies especially associated with real-time ITS application;

- Development of a methodology for a dynamic traffic flow model suitable for real-time ITS applications; and

- Verification of the model with empirical data.

The methodology mentioned above is then applied to freeway diversion in conjunction with freeway incident detection and management. This particular ITS application requires the following research tasks:

- Complete literature review of available delay analysis methods and theoretical comparison between the methods;

- Development of a method for estimating delay time during incidents using appropriate traffic flow theory;

- Development of a practical methodology for link and incident specific freeway diversion in real-time; and

- Development of a computer program for implementing the methodology.

1-8 Organization of Dissertation

The remainder of the dissertation is organized as follows: Chapter 2 presents the literature review on traffic flow models, travel time estimation methods and delay

analysis methods. Chapter 3 presents the development of a new dynamic traffic flow model and its verification with empirical data. The improvement of the traditional form of the equation of conservation of vehicles is also included. Chapter 4 deals with the development of dynamic analysis of delay. The mechanisms of progression and retrogression of congestion domain are explained in this chapter. Chapter 5 shows how a link and incident specific freeway diversion methodology is developed. Graphic aids like a delay diagram and volume-travel time curves are utilized to make problem solving much easier. Finally, Chapter 6 concludes the dissertation with findings from the research and suggestions for future study.

This research focuses on the improvement of understanding of dynamic characteristics of traffic flow and the limitation of available traffic flow models. The literature review has been concentrated on three subjects:

- traffic flow models;

- travel time estimation methods; and

- delay analysis methods, especially delays associated with incidents.

2-1 Traffic Flow Models

Traffic flow is a comprehensive stochastic phenomena of interactions among the driver, the vehicle, and the road. As mentioned before, there are three fundamental traffic variables: flow, density, and speed. The modeling of traffic flow in terms of these variables requires three equations (Ross 1988):

- The general equation of traffic stream where flow is the product of speed and density;

- The equation of conservation of vehicles where the difference between the number of vehicles entering a link and that leaving it during a given time interval corresponds

to the change in the number of vehicles traveling on the link (Lighthill and Whitham 1955); and

- The relationship between speed and density or between acceleration-density gradient.

Early research on traffic flow modeling was initiated in 1930s and mainly focused on the development of the speed-flow or speed-density relationship based on the observation of empirical data. The importance of the principle of conservation of vehicles was not emphasized until the development of high-speed digital computers and the growing tendency of use of computer simulation.

Depending on the level of flow description, traffic flow models can be divided into two categories: microscopic models and macroscopic models. On the microscopic level, traffic is described in terms of individual vehicle behavior. This modeling approach is also called the car-following modeling represented by response-stimulus relationships. Meanwhile, on the macroscopic level, traffic is described by the average behavior of a traffic stream in a local region. This approach is useful for understanding fundamental relationships of traffic variables. Macroscopic modeling technique can be further divided into traffic stream modeling and continuum modeling depending on the order of governing equations. Traffic stream models use speed-density relationship and continuum models employ acceleration-density gradient relationship respectively to represent average behavior of the traffic stream. Continuum models are further divided into first-order models and higher-order models.

2-1-1 Traffic stream models

Early research was focused on the development of flow-density-speed relationships based on the observation of empirical data. The basic structure of the model is defined by the first-order speed-density relationship:

$$u = u_e(k) \tag{2.1}$$

where $u_e(k)$ is the equilibrium relationship between speed and density.

Greenshields (1934), one of the early researchers of traffic flow, proposed a linear relationship between flow and speed. This model results in familiar parabola-shaped relationships between flow and density as well as between speed and flow:

$$u = u_f \left(1 - \frac{k}{k_j} \right) \tag{2.2}$$

where

  u_f = free-flow speed at which traffic is moving freely without interacting with other vehicles, and

  k_j = jam density at which concentration of vehicles is maximum.

Greenberg (1959) developed a logarithmic speed-density model, assuming that the traffic flow was equivalent to one-dimensional fluid state. It was later found that this model could be is related to one of the car-following models.

$$u = u_m \ln(\frac{k_j}{k})$$ (2.3)

where u_m is the speed at which flow reaches its capacity.

Underwood (1961) proposed another version of the traffic stream model:

$$u = u_f e^{-\left(\frac{k}{k_m}\right)}$$ (2.4)

where k_m is the density at which flow reaches its capacity.

Drew (1967) introduced an additional parameter n and proposed a family of traffic stream models:

$$u = u_f \left[1 - (\frac{k}{k_j})^{\frac{n+1}{2}}\right]$$ (2.5)

where

$n = 1$:   Greenshields' linear model;

$n = -1$:   exponential model; and

$n = 0$:   parabolic model.

Besides traffic stream models discussed above, there are numerous other models. For those developed in the past, excellent descriptions can be found in books (Gerlough and Huber 1975; May 1990; McShane and Roess 1990). Recently, the inverted V relationship between flow and concentration was proposed (Hall and Gunter 1986) and

further researched (Banks 1989). Figure 2.1 shows various speed-density hypotheses of traffic stream models.

2-1-2 Car following models

The study on the human behavior of car following started in the 1950s, two decades later than the first traffic stream model was introduced. The manual car-following theory assumed that the individual vehicle followed the vehicle ahead according to a certain headway control rule. A typical car following model is:

response = sensitivity $\times$ stimulus   (2.6)

where the response is the acceleration or deceleration of a vehicle in question, the stimulus involves the relative velocity between the vehicle in question and the vehicle ahead, the relative spacing between them, and the absolute velocity level. The sensitivity includes the driver's sensitivity and other factors like mechanical response delay.

Pipes (1953) initiated the development of the car following model by relating the velocity of a vehicle in question to the safe space headway. The minimum safe distance between the lead and the following vehicle was assumed equal the length of a vehicle every ten miles per hour of speed of the following car.

$$d_m = l_n(\frac{\dot{x}_{n+1}}{1.47 \times 10}) + l_n \qquad (2.7)$$

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where l_n is the length of a vehicle ahead.

Forbes (1958) improved the Pipes's safe distance model by incorporating driver's perception-reaction time necessary for perceiving the need to decelerate and applying the brakes. The time headway model he proposed is as follows:

$$h_m = \frac{l_n}{\dot{x}_{n+1}} + \Delta t \qquad (2.8)$$

where Δt is the driver's perception-reaction time.

General Motors (1959) launched extensive field studies and developed a series of modern forms of car-following models. The first model has a constant sensitivity term as follows:

$$\ddot{x}_{n+1}(t + \Delta t) = \alpha \; [\dot{x}_n(t) - \dot{x}_{n+1}(t) \;] \qquad (2.9)$$

where α is a constant sensitivity.

In the second model the space headway was added to the sensitivity term. The response of the following car was thus assumed to be inversely proportional to the space headway. The dimension of the constant sensitivity term became distance per time, i.e., velocity.

$$\ddot{x}_{n+1}(t + \Delta t) = \frac{\alpha}{x_n(t) - x_{n+1}(t)} \; [\dot{x}_n(t) - \dot{x}_{n+1}(t)] \qquad (2.10)$$

where α is the velocity of the vehicle in question (following vehicle).

The above second model was further refined. The driver of the following car would be more sensitive to the relative speed of the lead and the following car as the speed of the following car increases. In this model, the constant is dimensionless.

$$\ddot{x}_{n+1}(t + \Delta t) = \frac{\alpha \; [\dot{x}_{n+1}(t + \Delta t)]}{x_n(t) - x_{n+1}(t)} \; [\dot{x}_n(t) - \dot{x}_{n+1}(t)] \tag{2.11}$$

where α is a constant.

The generalization of sensitivity components was continued and the final model of the series had the following sensitivity components:

$$\ddot{x}_{n+1}(t + \Delta t) = \frac{\alpha_{l,m} \; [\dot{x}_{n+1}(t + \Delta t)]^m}{[x_n(t) - x_{n+1}(t)]^l} \; [\dot{x}_n(t) - \dot{x}_{n+1}(t)] \tag{2.12}$$

It is interesting to note that the car-following model in Eq. (2.12) for values $l = 0$ and $m = 2$ corresponds to the Greenberg's macroscopic traffic stream model in Eq. (2.3).

2-1-3 Continuum models

Lighthill and Whitham (1955) developed the famous "theory of kinematic waves" in which the behavior of traffic stream was considered as a continuum fluid motion as opposed to a few discrete vehicles. Defining shock wave as the movement of a boundary between two traffic states having different concentrations, the propagation of shock waves in traffic streams was very well documented. Richards (1956) proposed another version of the shock wave theory. The concept of conservation of vehicles was

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explained as the difference between the number of vehicles entering a link and those leaving it during the time interval corresponds to the change in the number of vehicles traveling on the link. These two independent researches were essentially the same, however, the former focused on the discontinuity of flow whereas the latter explained in terms of concentration discontinuity.

The concept of vehicle conservation led to the development of the equation of continuity. Considering traffic flow on a straight pipe section as a conserved system, the equation of continuity was derived:

$$\frac{\partial q}{\partial x} + \frac{\partial k}{\partial t} = 0 \tag{2.13}$$

It is important to note that the temporal change of flow rate $\partial q / \partial t$ was not properly considered in the original derivation of Eq. (2.13).

Continuum models refined the basic structure of the traffic stream models by incorporating time variable t and space variable x independently:

$$u = u_e(x,t) \tag{2.14}$$

Taking the total derivative of Eq. (2.14) gives

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} \tag{2.15}$$

where $\partial u / \partial t$ is the acceleration of the traffic stream as seen by an observer at a fixed point on the road.

Pipes (1969) derived the general form of acceleration of traffic:

$$\frac{du}{dt} = -k(\frac{du}{dk})^2 \frac{\partial k}{\partial x} \qquad (2.16)$$

where du/dt is the acceleration of an observer moving with the traffic stream.

Equating Eq. (2.15) with Eq. (2.16) gives the general form of the governing equation of a simple continuum model:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -v_0 \, k^n \frac{\partial k}{\partial x} \qquad \text{for } n = -1, 0,+1 \qquad (2.17)$$

where v_o = an anticipation parameter (mi/hr). Here, Greenshields' linear traffic stream model is obtained when $n=1$, an exponential model when $n=-1$, and a parabolic model when $n=0$. (For complete derivation, see Drew 1968). The physical interpretation of Eq. (2.17) is that the acceleration for the vehicles in the traffic stream changes with respect to drivers' anticipation of the traffic conditions ahead, i.e., the rate of change in density over distance $\partial k/\partial x$.

The first version of a high-order continuum model was proposed by Payne (1971). Using the car following theory, the relaxation term was additionally introduced to the equation of a simple continuum model by Eq. (2.9):

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{1}{T} [u_e(k)-u] - \frac{v_o^2}{k} \frac{\partial k}{\partial x} \qquad (2.18)$$

where T is the driver's reaction time. The implication of relaxation term in Eq. (2.18) is that the acceleration in the traffic stream changes with respect to the difference between

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the actual speed u and the desired equilibrium speed u_e over the reaction time T. It was reported that the convergence of the relaxation time was unrealistically slow under traffic jam conditions (Rathi et al 1987; Ross 1988).

Kuhne (1984) revised Payne's model by adding a viscosity term to address traffic friction:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{1}{T}\,[u_e(k)-u] - \frac{v_o^2}{k}\frac{\partial k}{\partial x} + v_c\frac{\partial^2 u}{\partial x^2} \tag{2.19}$$

where v_c is the viscosity coefficient. Traffic friction is caused by lane changing or weaving near entrance ramps. The viscosity term smears out sharp shocks in the case of sudden changes in traffic conditions and guarantees a continuous description of traffic patterns. Later, the numerical instability problem of the model was found (Lyrintzis et al. 1993).

Ross (1988) proposed another version of a high-order continuum model characterized by great simplicity.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{1}{T}\,[u_f - u] \qquad\qquad \text{if } k < k_j \tag{2.20}$$

$$\frac{\partial q}{\partial x} = 0 \qquad\qquad \text{if } k = k_j$$

where u_f is the free flow speed. Although this model had a much simpler expression, several weaknesses were reported (Newell 1989; Michalopoulos et al. 1993): Since the model did not have anticipation and friction effects, the acceleration rate was increasing

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as long as $k < k_j$. Also, the model assumed that when the front of a standing queue started to move, the entire queue moved at exactly same time.

Papageorgiou et al. (1989) proposed a higher-order model in the form:

$$u_j^{n+1} = u_j^n + \Delta t \xi \left[-u_j^n \frac{(u_j^n - u_{j-1}^n)}{\Delta x_j} - \frac{u_j^n - u_e(k_j^n)}{T} + \frac{v(k_{j+1}^n - k_j^n)}{\Delta x_j(k_j^n + \kappa)} \right] \qquad (2.21)$$

supplemented by

$$q_j = \alpha \, k_j \, u_j + (1-\alpha) \, k_{j+1} \, u_{j+1} \qquad (2.22)$$

where τ = a time delay.

Lyrintzis, Yi, Michalopoulos and Beskos (1993) proposed two higher-order models similar to Kuhne's model. The first semi-viscous model has density-dependent relaxation time T and friction term G:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \frac{\Phi}{T} [u_f(x) - u] - G - \alpha k^\beta \frac{\partial k}{\partial x} \qquad (2.23)$$

where the flag Φ was set to 1(0) if there was (not) a change in free flow speed from upstream to the current roadway section. The first term on the right-hand side of Eq. (2.23) represents relaxation and is given as:

$$T = t_o \left(1 + \frac{\gamma k}{k_j - \gamma k}\right) \qquad \text{for } 0 < \gamma < 1 \qquad (2.24)$$

where k_j = jam density. The reaction time T was formulated in such a way that it would increase when traffic became highly congested and it would decrease as traffic and

density decreased. Meanwhile, the second term on the right-hand side of Eq. (2.23) describes friction observed at entrance ramps. It was assumed to be a function of both traffic conditions and ramp volume entering or leaving the freeway:

$$G = \mu \, k^{\varepsilon} g(x,t) \tag{2.25}$$

where μ = a geometric parameter, ε = a dimensionless constant and $g(x,t)$ = the difference between ramp volume entering and leaving.

Another viscous model was developed by treating traffic flow as a viscous, compressive fluid mass. Its formulation is given as:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -\nu k^{\delta} \frac{\partial k}{\partial x} + \lambda k^{\varsigma} \frac{\partial^2 k}{\partial x^2} \tag{2.26}$$

where ν = anticipation parameter, λ = viscosity parameter and δ and ς are dimensionless constants. The second term on the right-hand side is the viscosity term and applicable to both ramp junctions and pipe sections (as opposed to semi-viscous model by Eq. 2.23).

The quantitative effectiveness of these models is discussed in Section 3.3 where a dynamic traffic model developed in this research has been tested with empirical traffic data.

I Greenshields

$u = u_f (1-k/k_j)$

u_f

k_j k

II 2 - Regime Linear

u_f

k_1 k_j k

III 3 - Regime Linear

u_f

k_2 k_3 k_j k

IV Greenberg

$u = u_m \ln(k_j/k)$

u_f

c

k_4 k_j k

V Underwood

$u = u_f e^{-k/k_m}$

u_f

k_m k

VI Edie

$u = u_f e^{-k/k_m}$

$u = u_m \ln(k_j/k)$

u_f

c

k_m k_j k

VII Bell Curve

$u = u_f e^{-\frac{1}{2}(k/k_m)^2}$

u_f

k

Sample Data Points

50
40
30
20
10
0

20   40   60   80   100   120   k
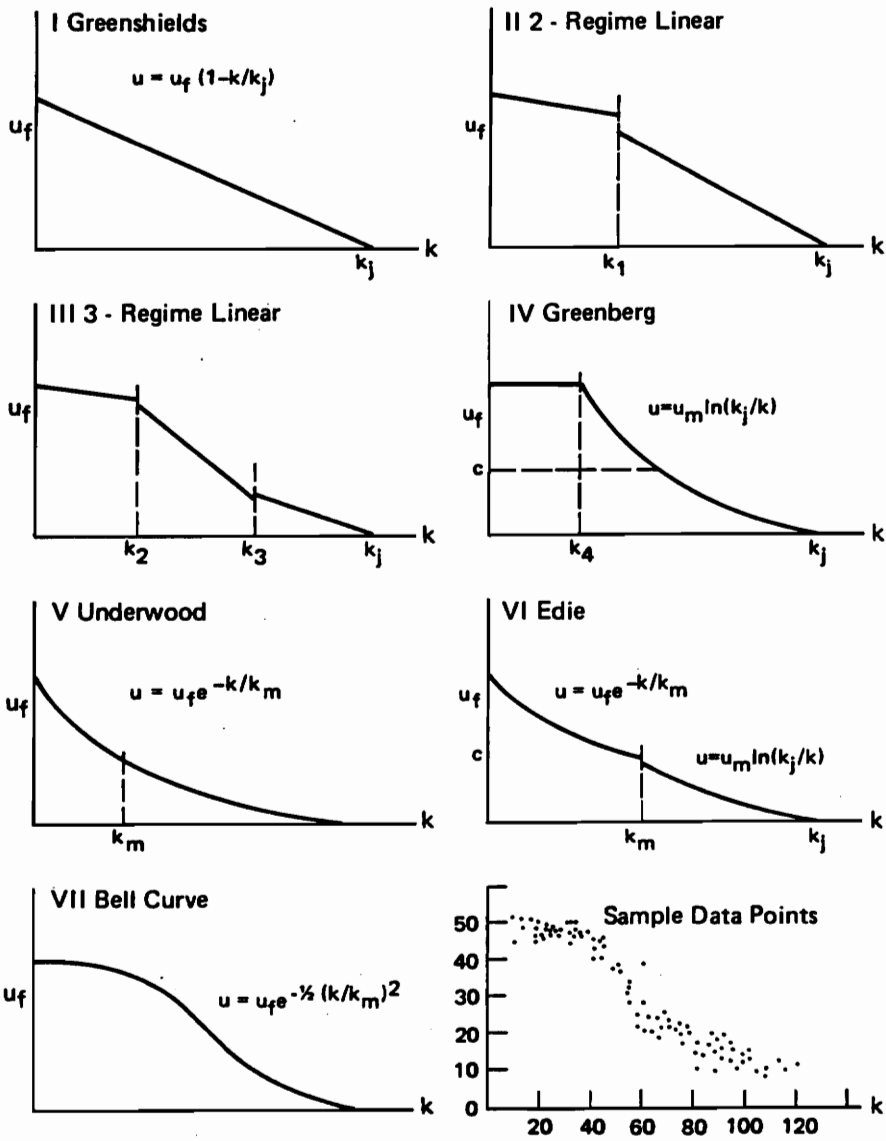
Figure 2.1   Speed-density hypotheses (source: Gerlough and Huber 1975)

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2-2 Travel Time Estimations

The estimation of travel times in an urban traffic network has played a fundamental role in transportation engineering. This estimation is a necessary step for the evaluation of proposed highway or transit facilities to serve present and future land uses. Most traditional methods such as use of link capacity functions deduce travel times from relating traffic variables and the static capacity of the road. This is inappropriate for representing dynamic characteristics of flow. With the advent of ITS, it is required that the estimation be performed in real-time with more accuracy and reliability than before. Considering its broad applications to ITS user services, the reliable and efficient estimation is pivotal to the success of ITS.

Typical relationships between travel time and other traffic variables are depicted in Figure 2.2. It is important to note the difference between demand volume which can be greater than capacity and measured volume which should be less than capacity.

2-2-1 Link capacity functions

By representing the traffic network by a set of links and nodes, the link capacity function mathematically relates link travel time to traffic variables on the link. Because of its simple yet consistent calculation, the use of link capacity function gets more favor especially in conjunction with dynamic traffic assignment. The formulation of link

capacity function can be divided into two categories: empirical approach and theoretical approach. The empirical approach observes empirical data and proposes mathematical function(s) which fit empirical data best. Meanwhile, the theoretical approach develops a link capacity function based on the accepted theory such as queuing theory. Here, Toronto function and the BPR are discussed as examples of the empirical approach and Davidson function as the example of the theoretical approach. The detailed description of most of link capacity functions can be found in Branston (1976) and Suh, Park and Kim (1990).

Irwin, Dodd, and Von Cube (1961) proposed a pair of link capacity function so called Toronto function. It had two straight line segments:

$$t = t_f + \alpha Q_p' + \alpha(q' - Q_p') \qquad\qquad \text{for } q' < Q_p' \qquad\qquad (2.27)$$

$$t = t_f + \beta Q_p' + \alpha(q' - Q_p') \qquad\qquad \text{for } q' \geq Q_p' \qquad\qquad (2.28)$$

where

t_f = free flow travel time;

q' = flow rate per lane; and

Q_p' = practical capacity per lane.

The BPR function (1964) is one of the most widely used link capacity functions and is given by:

$$t = t_0 \left[1 + \alpha \left(\frac{q}{q_m}\right)^{\beta}\right]$$

(2.29)

where

α = the additional increase in travel time taken at which flow is equal to the practical

capacity flow (the typical value of $\alpha = 0.15$) and

β = a model parameter, for which the value $\beta = 4.0$ is typically used.

Davidson (1966) derived a link capacity function from the queuing theory with the terms which can be determined empirically from traffic data. The travel time on a link is assumed to be the sum of two components: the time taken at which no congestion occurred and the waiting time due to the interference from other vehicles:

$$t = t_f + t_w$$

(2.30)

where

t_f = the free flow travel time per unit length of road, and

t_w = additional travel time due to the frequent interactions among drivers and

vehicles.

For the single-channel exponential arrival and departure queuing process (M/M/1, ∞, FIFO), the expected average waiting time $E(w)$ can be estimated as:

$$E(w) = \frac{1}{\mu(1-y)}$$

(2.31)

where

μ = service rate, and

$y = \lambda / \mu$, the ratio of the mean arrival rate to the mean service rate.

Converting these variables to traffic variables, $t_w = E(w)$ and $q_c = \mu$, gives

$$t_w \, q_c = \frac{1}{1-y} \tag{2.32}$$

A level of service parameter j was introduced to account for qualitative operational conditions on various road types. The parameter was defined as:

$$j = \frac{1}{t_w \, q_c} \tag{2.33}$$

Substituting Eq. (2.32) and Eq. (2.33) to Eq. (2.30) gives Davidson function:

$$t = t_f \frac{1-(1-j)y}{1-y} \tag{2.34}$$

The parameter j controls the shape of the curve. Boyce, Janson and Eash (1981) recommended the value of j to be 0.04 for uninterrupted freeway flow and 0.02 for interrupted flow. Blunden (1971) suggested different calibration results for j as tabulated in Table 2.1.

Table 2.1   Level of service parameters for various road types
              (source: Blunden 1971)

Roads	tt_f (min/mi.)	j	q_c (veh/hr/lane)
Multi-lane divided highway	0.8-1.0	0.0-0.2	2000
Multi-lane urban arterials	1.5-2.0	0.4-0.6	1800
Collector roads	2.0-3.0	1.0-1.5	< 1800

2-2-2 Recent development

The traditional deductive methods are inappropriate for real-time applications and transportability. Continuum traffic models which idealize traffic as continuos fluid based on the theory of shock waves requires a great amount of effort for calibration. Recently new attempts were made to estimate travel times directly from flow measurements which is desirable for ITS implementation. For instance, cross-correlation technique (Dailey 1993) used link flow measurements to determine the maximum correlation between continuous concentration signals generated from link flow measurements. Fewer number of traffic parameters are required to achieve the same goals. This statistical method, however, does not work well under congested traffic because the correlation disappears in such situations.

The modeling of traffic characteristics seems to be difficult without improving the existing knowledge in traffic dynamics. For the present and the future ITS applications, there is a need to refine the understanding of traffic dynamics and then to integrate it with

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surveillance systems for estimating traffic variables in real-time in an accurate and reliable manner. Hence, an inductive modeling approached based on appropriate traffic flow theories has been adapted in this research.
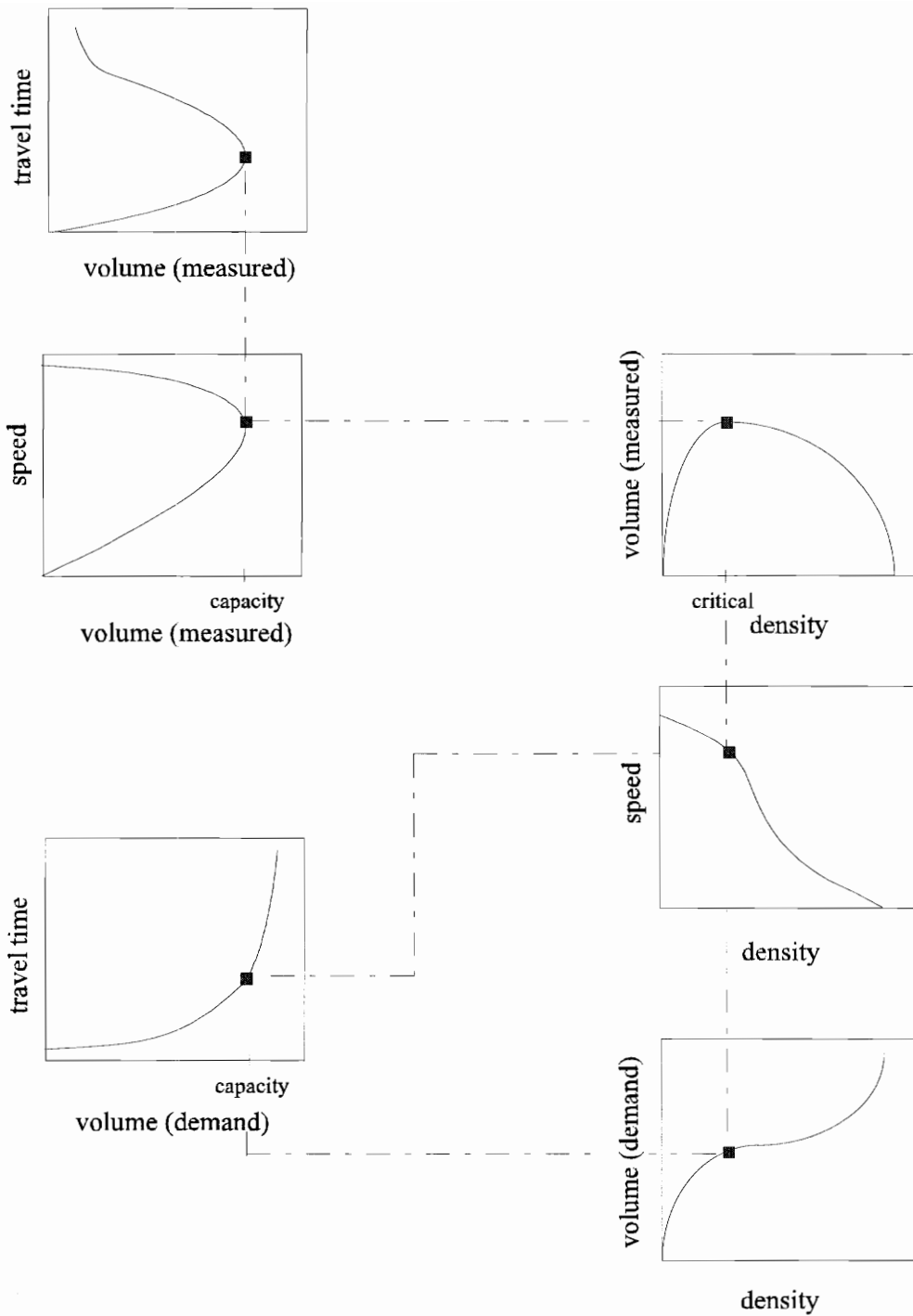
Figure 2.2   Typical travel time-flow-density-speed relationships

2-3 Delay Analysis Methods

In case of congestion caused when traffic demand exceeds the capacity, stochastic analytical methods are no longer applicable. Only two analytical methods are available: deterministic queuing method and shock wave analysis. Analytical simplifications are necessary for using these techniques for quantifying delays. The following assumptions are usually employed:

- demand and capacity queuing processes are deterministic, respectively;

- the arrival and service patterns are considered to be continuos rather than discrete; and

- flow and density relationship is deterministic.

There are a number of descriptive statistics that are used for describing the magnitude of congestion: These include: total vehicle-hours of delay; number of vehicles queued; average delay per vehicle; maximum delay; maximum number of vehicles queued; and maximum queue length.

The information on the maximum length during congestion is probably more significant than the average queue length from the operational point of view. Meanwhile, expected average delay is more meaningful to most travelers.

2-3-1 Deterministic queuing method

Deterministic queuing analysis utilizes cumulative demand-capacity curves to describe congestion. The area between the demand and capacity curves represents the total vehicle-hours of delay incurred. The longest abscissa between curves approximates the maximum individual delay and the longest ordinate denotes the maximum number of vehicles queued. The product of the number of vehicles queued and the average space headway between two adjacent vehicles in associated queuing state yields the expected queue length. It is convenient to define traffic intensity y as the ratio of the arrival rate (demand) to the service rate (capacity). This method is widely used because of its simplicity despite its underestimation of delays. The Highway Capacity Manual (1994) also adapted this method for estimating capacity of freeway work zone. Morales (1987) generalized deterministic queuing situations as shown in Figure 2.4.

2-3-1-1 Temporal reduction in capacity

The situation of temporal capacity reduction can arise from the closure of a lane or lanes due to accidents or roadside maintenance. Let $q(t)$ and $Q(t)$ be the intensity of the arrival and departure flows, respectively. Then, for the uniform deterministic queuing process, we have:

$$q(t) = q$$

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$$Q(t) \; = \; rQ \qquad\qquad\qquad\qquad \text{for } 0 < t \le t_1$$

$$ \; = \; Q \qquad\qquad\qquad\qquad\qquad \text{for } t > t_1$$

where

r = the capacity reduction factor due to blockage, and

t_1 = the duration of the blockage.

Referring to Figure 2.3(a), the number of vehicles in the queue at time t is estimated as:

$$n(t) \; = \; \int_0^{t_1}(q - rQ)dt \; + \; \int_{t_1}^{t}(q - Q)dt \qquad\qquad (2.35)$$

$$ \; = \; (q - rQ)t_1 \; + \; (q - Q)(t - t_1)$$

The maximum number of vehicles queued occurs at $t = t_1$:

$$n(t_1) \; = \; (q - rQ)t_1 \qquad\qquad\qquad\qquad (2.36)$$

Traffic restores to normal when the queued vehicles disappear. The duration of congestion is equal to:

$$t_2 \; = \; \frac{(Q - rQ)\, t_1}{Q - q} \qquad\qquad\qquad\qquad (2.37)$$

The total delay TD in vehicle-hours is

$$TD \; = \; \int_0^{t_2}[q(t) - Q(t)]\, t \; dt$$

$$= \frac{1}{2}[(q-rQ)t_1]\left[\frac{(Q-rQ)t_1}{Q-q}\right] \tag{2.38}$$

It can be shown that Eq. (2.38) equals:

$$TD = \frac{(1-r)(y-r)}{2(1-y)}Qt_1^2 \tag{2.39}$$

The average delay due to congestion t_d is obtained by dividing Eq. (2.39) by the number of vehicles delayed in the queue:

$$t_d = \frac{1}{2y}(y-r)t_1 \tag{2.40}$$

2-3-1-2 Transient surge in demand

Transient surge in demand can be observed on urban freeways during peak-hours or in case of special events such as games. Congestion occurs as soon as demand exceeds the capacity. The traffic intensities are:

$$Q(t) = Q$$

$$q(t) = pq \qquad\qquad \text{for } 0 < t \le t_1$$

$$= q \qquad\qquad \text{for } t > t_1$$

where

p = the overloading factor, and

t_1 = the duration of the peak demand.

Similarly, the number of vehicles in the queue at time t in Figure 2.3(b) is

$$n(t) = \int_0^{t_1} (pq - Q)dt + \int_{t_1}^{t} (q - Q)dt \qquad (2.41)$$

$$= (pq - Q)t_1 + (p - Q)(t - t_1)$$

The maximum number of vehicles in the system occurs at $t = t_1$:

$$n(t_1) = (pq - Q)t_1 \qquad (2.42)$$

The duration of congestion is the time when $n(t) = 0$:

$$t_2 = t_1 + \frac{(pq - Q)t_1}{Q - q} \qquad (2.43)$$

The total delay TD is:

$$TD = \int_0^{t_2} [q(t) - Q(t)] \, t \, dt$$

$$= \frac{1}{2}[(pq - Q)t_1] \left[t_1 + \frac{(pq - Q)t_1}{Q - q} \right]$$

$$= \frac{(p - 1)(py - 1)}{2(1 - y)} qt_1^2 \qquad (2.44)$$

The average delay is:

$$t_d = \frac{1}{2} (py - 1)t_1 \tag{2.45}$$

2-3-1-3 Periodic interruption of service

A queue of delayed vehicles can always be seen at signalized intersections. This is because of the periodic interruption of service, i.e., due to a red traffic light. In case of a pretimed traffic light, the arrival rate is:

$q(t) = q$

and the service rate is:

$Q(t) = 0$ for $0 < t \le (1-a)c$

     $= Q$ for $t > t_1$

where

     c = the signal cycle length;

     a = the proportion of green effective time, i.e., the ratio of the green time to the cycle length; and

     y = the degree of saturation, i.e., q/aQ.

For pulsed departure rate for one complete cycle:

$Q(t) = 0$ for $0 < t \le t_1$

     $= Q(t)$ for $t > t_1$

where

t_1 = the duration of the red and yellow time, i.e., $(1-a)c$, and

t_2 = one complete cycle length.

Similarly, the number of vehicles in the queue at time t is

$$n(t) = \int_0^{t_1} q\,dt + \int_{t_1}^{t} (q-Q)\,dt \tag{2.46}$$

$$= qt_1 + (q-Q)(t-t_1)$$

The maximum number of vehicles is when $t = t_1$:

$$n(t_1) = q(1-a)c \tag{2.47}$$

The duration of congestion is obtained when $n(t)$ is equal to zero:

$$t_2 = t_1 + \frac{qt_1}{Q-q}$$

$$= \frac{(1-a)c}{(1-ay)} \tag{2.48}$$

The total delay TD is:

$$TD = \int_0^{t_2} [q(t) - Q(t)]\, t\, dt$$

$$= \frac{1}{2}qt_1 \left[t_1 + \frac{qt_1}{Q-q} \right]$$

$$= \frac{(1-a)^2 c^2}{2(1-ay)} \tag{2.49}$$

The average delay per vehicle due to the red time is estimated by dividing the total delay Eq. (2.49) by the number of vehicles in the system.
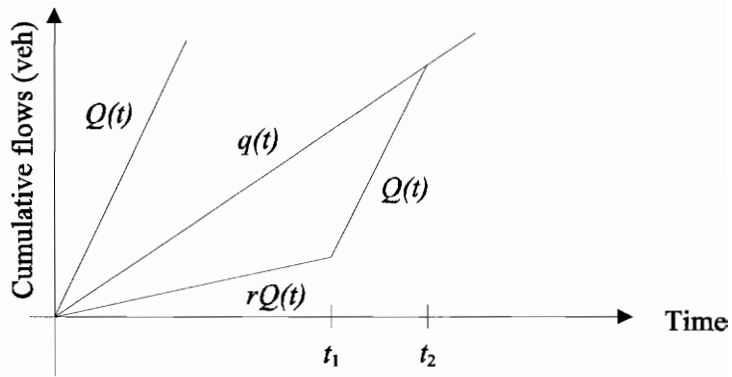
(a) transient reduction in capacity
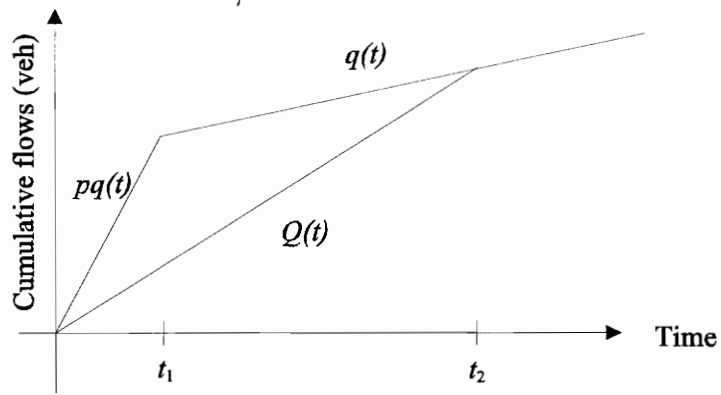
(b) temporal surge in demand
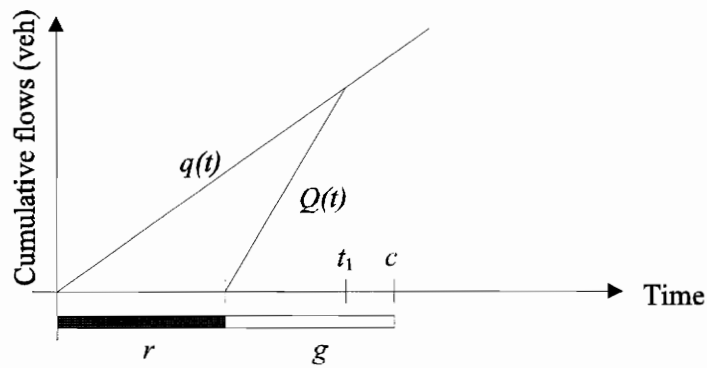
(c) periodic interruption of service

Figure 2.3   Deterministic queuing situations

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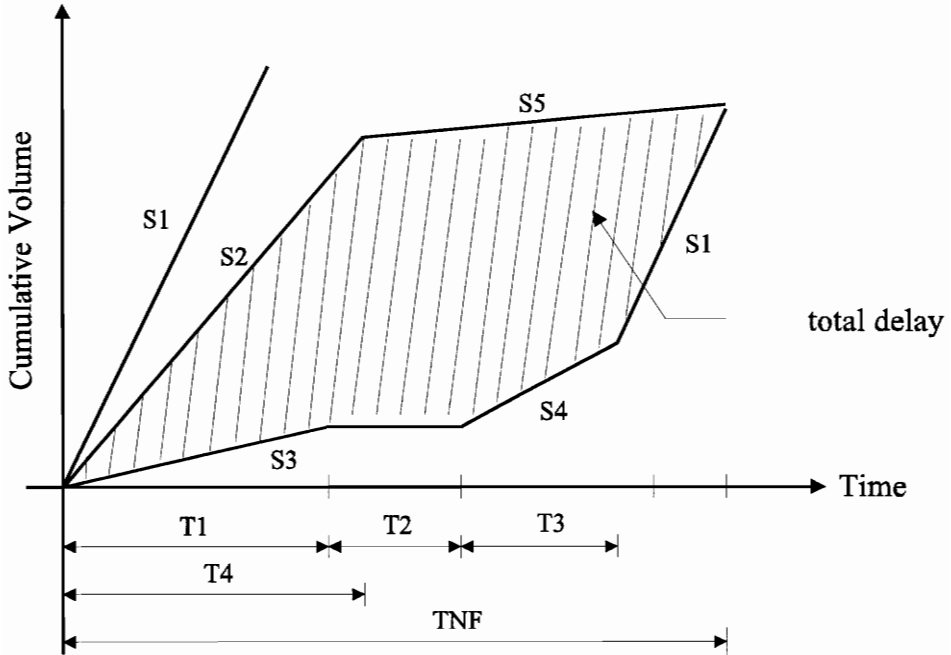
S1 = Capacity of the freeway
S2 = Initial demand flow
S3 = Incidnet flow
S4 = Adjusted incident flow
S5 = Revised demand flow
T1 = Detection time + Response time + Clearance time
T2 = Duration of total closure
T3 = Incident duration under adjusted incident flow
T4 = Elapsed time under initial demand flow
TNF = Total elapsed time until normal flow resumed

Figure 2.4   General delay condition diagram (source: Morales 1987)

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2-3-2 Shock wave method

Shock wave analysis (Lighthill and Whitham 1955; Richards 1956) estimates velocity as:

$$w = \frac{q_1 - q_2}{k_1 - k_2}$$

(2.46)

where subscripts 1 and 2 represent two traffic states having different densities and flows.

This method utilizes a time-space diagram to describe a time-space domain of congestion surrounded by shock waves. The total delay is obtained by multiplying the area and the density of the associated congestion state. The ordinate represents the length of the queue. A deterministic flow-density relationship is necessary for evaluating shock wave velocity.

Shock wave can travel along or against the direction of traffic or can be stagnant depending upon the interactions between two traffic streams having different flows and densities. Shock waves include:

- Frontal stagnant shock wave which is being stationary;

- Backward forming shock wave which travels against the direction of traffic causing increase in a time-space domain of congestion;

- Forward recovery shock wave which follows along the direction of traffic causing decrease in a time-space domain of congestion; and

- Backward recovery shock wave which can be seen at a signalized intersection when the traffic light turns green.

Shock wave analysis was applied to many traffic situations such as freeway incident delays (Messer, Dudek and Friebele 1973; Chow 1976; Al-Deek, Garib and Radwan 1995) and signalized intersections (Michalopoulos and Stephanopoulos 1977). Here, the literature review is limited to the application to freeway incident delays.

Messer, Dudek and Friebele (1973) established a framework for the analysis of delay caused by an incident using the shock wave theory. Various traffic states related to incidents were defined in terms of fundamental traffic variables as shown in Table 2.2 and Figure 2.4. Five different shock waves were observed during the incident and their velocities were estimated using Greenshields' linear traffic flow model. The hatched area is the area where vehicles were delayed and formed a queue. Note that a subscript d means traveling downstream and a subscript u means traveling upstream. Here, using the geometry, the maximum queue length l_{max} and the duration of delay T_2 can be written respectively as:

$$l_{max} = \frac{w_{u1} w_{u2}}{(w_{u1} - w_{u2})} T_1 \qquad\qquad (2.47)$$

$$T_2 = \frac{w_{u1} w_{u2}}{(w_{u1} - w_{u2})} T_1 + \frac{w_{u1}(w_{u2} + w_{d3})}{w_{d3}(w_{u2} - w_{u1})} T_1 \qquad\qquad (2.48)$$

where

T_i = duration of an incident.

Table 2.2   Dynamic traffic flow mechanisms during incidents

Event	Upstream	Downstream
Before incident	Normal flow $q(t)$ $k(t)$	Normal flow $q(t)$ $k(t)$
Incident occurs	Queuing flow $q_q(t)$ $k_q(t)$	Metered flow $q_m(t)$ $k_m(t)$
Incident cleared	Capacity flow $q_c(t)$ $k_c(t)$	Capacity flow $q_c(t)$ $k_c(t)$
Traffic recovered	Normal flow $q(t)$ $k(t)$	Normal flow $q(t)$ $k(t)$

Al-Deek, Garib and Radwan (1995) extended the earlier work of shock wave analysis and proposed a method suitable for a corridor with surveillance systems. Using shock wave theory, a time-space domain of an incident and freeway links within the domain were determined. Instead of using deterministic flow-density relationship, densities were estimated by the ratio of flow measurement and (spot) speed measurement. The longitudinal domain of each link was determined by the midpoint between adjacent detectors. The link travel time during the incident was estimated by dividing the link distance by the speed measurement. By referring to the historical link speeds, the total delay for each link was estimated as:

$$d_j^i = l_j \left(\frac{1}{u_j^i} - \frac{1}{u_j^{i,r}} \right) q_j^i \Delta t \qquad\qquad \text{for } u_j^i < u_j^{i,r} \qquad\qquad (2.49)$$

where

d_j^i = delay on link j during time interval i ;

l_j = length of link j;

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u_j^i = speed measurement on link j during time interval i ;

$u_j^{i,r}$ = historical speed on link j during time interval i ;

q_j^i = flow measurement measured at the midpoint on link j during time interval i ;

and

Δt = data aggregation rate or length of time interval i .

The total delay caused by an incident was:

$$TD = \sum_{i=1}^{m}\sum_{j=1}^{n} d_j^i \tag{2.50}$$

where

m = number of time intervals during a duration of delay, and

n = number of freeway links within a time-space domain of an incident .
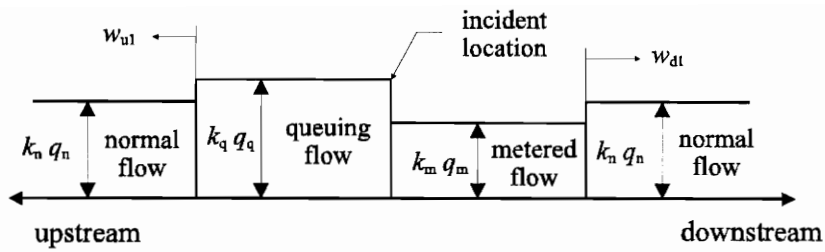
(a) different flow conditions until an incident is cleared
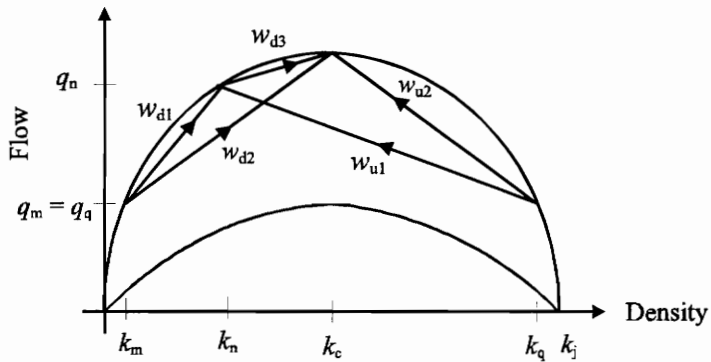
(b) shockwave velocities
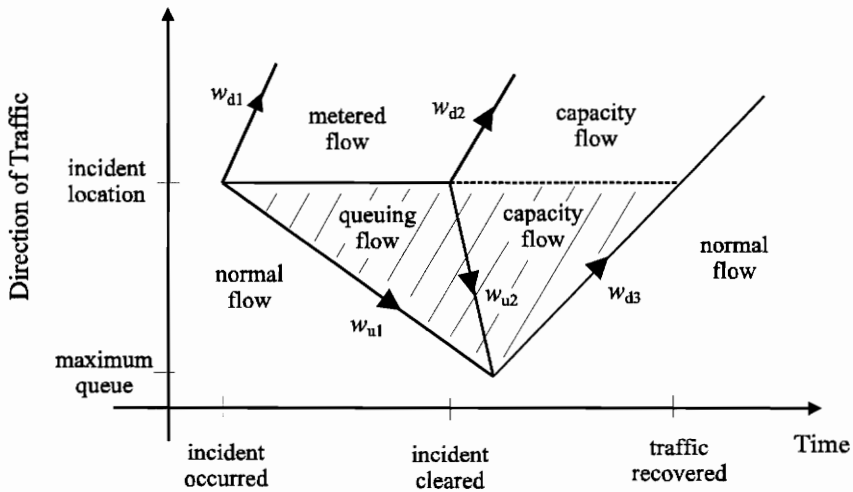
(c) time-space domain of incident

Figure 2.5   Shock wave analysis of incident conditions (source: Messer, Dudek and Friebele 1973)

3-1   Methodological Basis

As discussed in Chapter 1, spatial and temporal fluctuations of traffic can not be detected by a single surveillance technology alone in real-time. Although there has been a large amount of research devoted to the development of traffic flow models over the past half century, not much has been directed towards the improvement of existing theories. The time-dependent interrelationships of traffic characteristics are still not analytically well-defined. As a result, most of traditional traffic flow models reviewed in Chapter 2, though deductive in nature, are neither accurate nor efficient. Therefore, a methodology for automatic assessment of spatial traffic variables is still in need. Here, an inductive modeling approach based on stochastic queuing theory and the principle of conservation of vehicles is presented. Its advantages include real-time applicability and transportability. Figure 3.1 shows the block diagram for the proposed methodology and its application to various ITS user services.
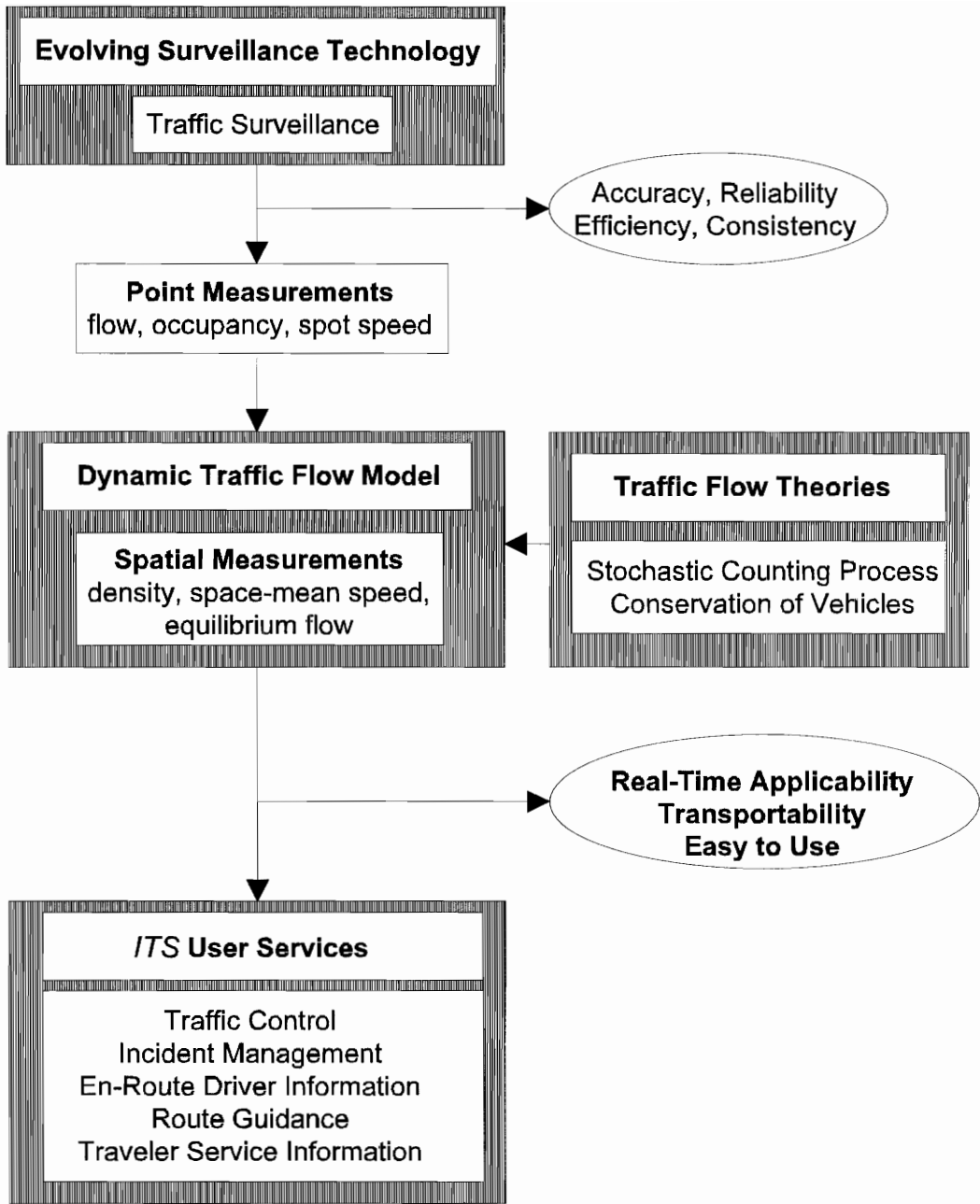
Figure 3.1   Block diagram of the proposed methodology for automatic traffic
              measurement in real-time and its application to various ITS user
              services

3-1-1 Stochastic queuing process

A stochastic process $\{n(t), \ t \in T\}$ in queuing theory is a collection of random variables, $n(t)$, indexed on time t (Newell 1989; Daigle 1992). Now consider a simple vehicle counting experiment: Two observers are located at an upstream and a downstream location. They count the cumulative number of vehicles passing the observation sites and record them with time at which they passed. The number of vehicle arrivals that have occurred at the upstream location by time t, $A(t)$, and the number of vehicle departures that have occurred at the downstream location by time t, $D(t)$, are random variables. Therefore, based on our earlier definition, this experiment can be defined as a stochastic process.

Assuming FIFO (first in, first out) queuing discipline, the characteristics of this stochastic process are:

- $A(t)$ and $D(t)$ are integers, non-negative and non-decreasing;

- for $t^* \in t$, $A(t^*) - D(t^*)$ is the number of vehicles traveling along the section and is non-negative;

- for $t^* < t$, $A(t) - A(t^*)$ is the number of events that occur in the interval $(t^*, t]$. The same holds good for $D(t) - D(t^*)$; and

- for the nth vehicle which arrives at the upstream location by time t' and leaves the downstream location by time t'', $t'' - t'$ is the travel time of the vehicle.

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3-1-2 Conservation of vehicles

The concept of the conservation of vehicles (Lighthill and Whitham 1955; Richards 1956) states that the difference between the number of vehicles entering the link and that leaving it during the time interval corresponds to the change in the number of vehicles traveling on the link. However, earlier work was developed assuming the same values of the inflow rate and the outflow rate at time t and $t + \Delta t$. In other words, temporal fluctuations of traffic flows were ignored and spatial fluctuations were partially considered. Therefore, the original formulation, as shown below, is not fully able to explain the changes in traffic which occurred during the time interval:

$$\frac{q(x_1,t+\Delta t)-q(x_2,t+\Delta t)}{\Delta x} = \frac{k(t+\Delta t)-k(t)}{\Delta t} \tag{3.1}$$

where

$q(x_1,t+\Delta t) = q(x_1,t)$,

$q(x_2,t+\Delta t) = q(x_2,t)$, and

$k(t)$ = the number of vehicles per unit distance at time t (density).

Now consider the generalized traffic conditions as shown in Figure 3.2. The mean inflow rate, $q(x_1,t)$ and the mean outflow rate, $q(x_2,t)$ are being measured at the upstream location x_1 and the downstream location x_2, respectively. Unlike in the earlier work, $q(x_1,t)$ and $q(x_2,t)$ are treated as independent variables in this research. Here, $q(x_1,t)$ represents the demand of traffic and $q(x_2,t)$ represents dynamic freeway

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capacity for a given $q(x_1, t)$. Total number of vehicles entering and exiting the link during Δt respectively are:

$$q(x_1, t + \Delta t)\ \Delta t \qquad\qquad (3.2)$$

$$q(x_2, t + \Delta t)\ \Delta t \qquad\qquad (3.3)$$

Under the principle of conservation of vehicles, the difference between Eqs. (3.2) and (3.3) equals the product of change in the density $k(t + \Delta t) - k(t)$ during the period Δt and the link distance Δx. Therefore,

$$[q(x_1, t + \Delta t) - q(x_2, t + \Delta t)]\ \Delta t\ =\ [k(t + \Delta t) - k(t)]\ \Delta x$$

Rearranging terms, the above equation can be written as:

$$\frac{q(x_1, t) - q(x_2, t)}{\Delta x} + \frac{q(x_1, t + \Delta t) - q(x_1, t)}{\Delta x} - \frac{q(x_2, t + \Delta t) - q(x_2, t)}{\Delta x}$$

$$=\ \frac{k(t + \Delta t) - k(t)}{\Delta t} \qquad\qquad (3.4)$$

The new form of the equation of conservation of vehicles has two additional terms of flows on the left-hand side as compared to Eq. (3.1). These are related to the change in the density which occurred during the period Δt on the right-hand side. Physical interpretations of these terms are as follows:

- $[q(x_1, t) - q(x_2, t)]/\Delta x$ represents initial condition at time t in conjunction with a density $k(t)$ at time t;

- $[q(x_1, t + \Delta t) - q(x_1, t)]/\Delta x$ represents changes occurred in demand flow during the interval $(t, t + \Delta t)$; and

- $[q(x_2, t + \Delta t) - q(x_2, t)]/\Delta x$ represents changes in dynamic freeway capacity during the interval $(t, t + \Delta t)$ for a given $q(x_1, t + \Delta t)$.

Since no assumptions are made during its derivation, Eq. (3.4) is self-explanatory. This equation has potential applicable to automatic freeway incident detection. If the changes in the number of vehicles on the link are analytically related to flow measurements, then temporal surge in demand (recurring congestion) or capacity reduction due to an incident (non-recurring congestion) or a combination of both can be detected analytically by tracking fluctuations in inflow and outflow measurements, respectively. The link densities can also be analytically obtained from flow measurements. This analytical flow-based algorithm is superior to the empirical occupancy-based algorithms such as California algorithm (Payne and Tignor 1978) which uses occupancy rates between two adjacent loop detectors. Theoretically speaking, there is no false alarm possibility and no need to calibrate thresholds. Another use of this equation lies in validating computer simulation results for their consistency. Also, it has been used in deriving the dynamic flow model as shown in the next section.
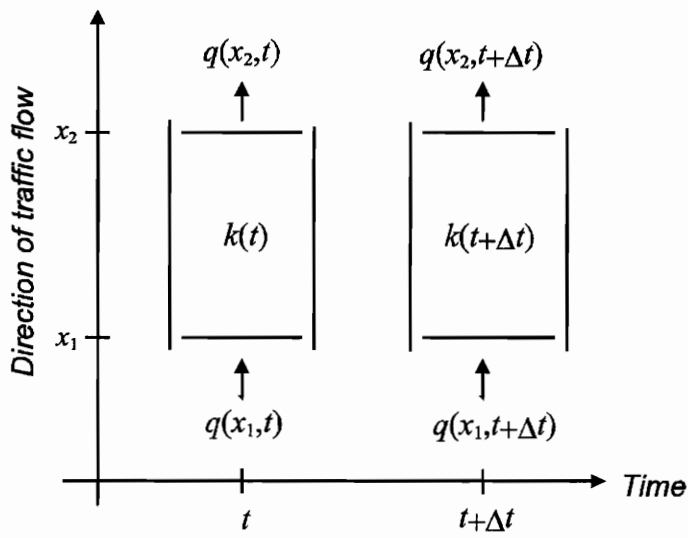
Figure 3.2   Generalized traffic conditions considered for the derivation of the equation of conservation of vehicles

3-2   Derivation of the Model

The aforementioned stochastic counting process is now applied to freeway traffic surveillance. For simplicity, as shown in Figure 3.2, a typical link with two detectors located at both ends is considered. The flow rates, $q(x_1,t)$ and $q(x_2,t)$ are being measured at the upstream location x_1 and the downstream location x_2. They are regularly aggregated at the interval Δt at the detector locations. It is reasonable to maintain the relationship between the link distance Δx and the aggregation rate Δt as follows:

$$\frac{\Delta x}{u_f} < \Delta t \le 5 - 10 \text{ min.} \tag{3.5}$$

where u_f is a free flow speed on the link.

Here, the cumulative flows $Q(x_1,t)$ and $Q(x_2,t)$ are equivalent to $A(t)$ and $D(t)$, respectively. The cumulative number of vehicles entering and leaving the link from time t_0 to time t_n correspond to Eq. (3.6) and (3.7), respectively.

$$
\begin{aligned}
Q(x_1,t_n) &= q(x_1,t_0) + q(x_1,t_0 + \Delta t) + q(x_1,t_0 + 2\Delta t) + \ldots\\
&= q(x_1,t_0) + q(x_1,t_1) + q(x_1,t_2) + \ldots\\
&= \sum_{i=0}^{n} q(x_1,t_i) \tag{3.6}
\end{aligned}
$$

Similarly,

$$Q(x_2, t_n) = \sum_{i=0}^{n} q(x_2, t_i) \tag{3.7}$$

The initial conditions are

$$Q(x_1, t_0) = 0 \tag{3.8}$$

$$Q(x_2, t_0) = -n(t_0) \leq 0 \tag{3.9}$$

where $n(t_0)$ is the number of vehicles which have entered and are traveling on the link just before the observation.

Here, $q(x_1, t_n)$ and $q(x_2, t_n)$ can be interpreted as the number of vehicles entering and leaving the link during the interval (t_{n-1}, t_n) as shown in Eq. (3.10) and (3.11), respectively.

$$Q(x_1, t_n) - Q(x_1, t_{n-1}) = q(x_1, t_n)\Delta t \geq 0 \tag{3.10}$$

$$Q(x_2, t_n) - Q(x_2, t_{n-1}) = q(x_2, t_n)\Delta t \geq 0 \tag{3.11}$$

By the analogy of a stochastic counting process and the principle of conservation of vehicles, it can be claimed that the cumulative number of vehicles leaving downstream can not exceed those arriving at upstream (equality holds if for the time Δt there are no arrivals and subsequently the link is empty). Therefore:

$$Q(x_1, t_n) \geq Q(x_2, t_n). \tag{3.12}$$

The number of vehicles traveling on the link at time t_n, $n(t_n)$, is obtained as:

$$n(t_n) = Q(x_1, t_n) - Q(x_2, t_n).$$

(3.13)

Then, the density at time t_n, $k(t_n)$, is given by

$$k(t_n) = \frac{Q(x_1, t_n) - Q(x_2, t_n)}{\Delta x}$$

(3.14)

Now, introduce an important variable in this derivation, $m(t_n)$, defined as the number of vehicles entering as well as exiting during the interval (t_{n-1}, t_n). It is estimated as:

$$m(t_n) = Q(x_2, t_n) - Q(x_1, t_{n-1})$$

(3.15)

Under normal traffic conditions, $m(t_n)$ is always positive. However, it can be either zero or negative if traffic is jammed so that no vehicles which entered the link previously have exited during the interval (t_{n-1}, t_n), i.e., traffic is conceptually under congested conditions subject to the link distance-data aggregation rate relationship as defined in Eq. (3.5). Thus, the derivation of the model can be divided into two categories depending on the polarity of $m(t_n)$: the first category is of normal flows in which $m(t_n)$ is positive and the second is of congested flows.

3-2-1 Under normal flows

Defining travel time as the time spent by the vehicle on the link, the total travel time for the vehicles entering and exiting during the interval (t_{n-1}, t_n) is schematically idealized as the hatched area in Figure 3.3. Note that the actual cumulative flow curves should be

step functions since the vehicle counting is integer valued. Analytically, the total travel time is equal to:

$$\frac{1}{2}\ [(t^{''}-t_{n-1})+(t_n-t^{'})]\ m(t_n) \tag{3.16}$$

where t' and t'' can be linearly interpolated respectively as:

$$t^{'}\ =\ t_{n-1}+\frac{Q(x_2,t_n)-Q(x_1,t_{n-1})}{q(x_1,t_n)} \tag{3.17}$$

$$t^{''}\ =\ t_n-\frac{Q(x_2,t_n)-Q(x_1,t_{n-1})}{q(x_2,t_n)} \tag{3.18}$$

The term $Q(x_2,t_n)-Q(x_1,t_{n-1})$ in Eqs. (3.17) and (3.18) can be manipulated as:

$$Q(x_2,t_n)-Q(x_1,t_{n-1})\ =\ Q(x_2,t_n)-Q(x_2,t_{n-1})+Q(x_2,t_{n-1})-Q(x_1,t_{n-1}) \tag{3.19}$$

Using Eqs. (3.11) and (3.14), it can be shown that Eq. (3.19) is equivalent to:

$$q(x_2,t_n)\Delta t\ +\ k(t_{n-1})\Delta x \tag{3.20}$$

Substitute Eq. (3.20) in Eqs. (3.17) and (3.18) respectively:

$$t''-t_{n-1}\ =\ \Delta t\ -\ \frac{q(x_2,t_n)\Delta t-k(t_{n-1})\Delta x}{q(x_2,t_n)}$$

$$=\ \frac{k(t_{n-1})\Delta x}{q(x_2,t_n)} \tag{3.21}$$

$$t_n-t'\ =\ \Delta t\ -\ \frac{q(x_2,t_n)\Delta t-k(t_{n-1})\Delta x}{q(x_1,t_n)}$$

$$=\ \frac{q(x_1,t_n)\Delta t-q(x_2,t_n)\Delta t+k(t_{n-1})\Delta x}{q(x_1,t_n)} \tag{3.22}$$

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Rewriting Eq. (3.4), we get:

$$\frac{q(x_1,t_n) - q(x_2,t_n)}{\Delta x} = \frac{k(t_n) - k(t_{n-1})}{\Delta t} \tag{3.23}$$

After substituting Eqs. (3.21) and (3.22) in Eq. (3.16), Eq. (3.23) has been utilized to get the simplified form of Eq. (3.16):

$$[(t^{"} - t_{n-1}) + (t_n - t^{'})]$$

$$= \left[\frac{k(t_{n-1})\Delta x}{q(x_2,t_n)} + \frac{q(x_1,t_n)\Delta t - q(x_2,t_n)\Delta t + k(t_{n-1})\Delta x}{q(x_1,t_n)} \right]$$

$$= \left[\frac{q(x_1,t_n)k(t_{n-1})\Delta x + q(x_1,t_n)q(x_2,t_n)\Delta t - q(x_2,t_n)q(x_2,t_n)\Delta t + q(x_2,t_n)k(t_{n-1})\Delta x}{q(x_1,t_n)q(x_2,t_n)} \right]$$

$$= \left[\frac{[q(x_1,t_n) + q(x_2,t_n)]k(t_{n-1})\Delta x + [q(x_1,t_n) - q(x_2,t_n)]q(x_2,t_n)\Delta t}{q(x_1,t_n)q(x_2,t_n)} \right]$$

$$= \left[\frac{[q(x_1,t_n) + q(x_2,t_n)]k(t_{n-1})\Delta x + [k(t_n) - k(t_{n-1})]q(x_2,t_n)\Delta x}{q(x_1,t_n)q(x_2,t_n)} \right]$$

$$= \frac{q(x_1,t_n)k(t_{n-1})\Delta x + q(x_2,t_n)k(t_n)\Delta x}{q(x_1,t_n)q(x_2,t_n)} \tag{3.24}$$

Therefore,

$$\frac{1}{2}[(t^{"} - t_{n-1}) + (t_n - t^{'})]\, m(t_n) = \frac{\Delta x}{2} \frac{q(x_1,t_n)k(t_{n-1}) + q(x_2,t_n)k(t_n)}{q(x_1,t_n)\, q(x_2,t_n)}\, m(t_n) \tag{3.25}$$

The average travel time during the interval (t_{n-1}, t_n), $tt(t_n)$, can be obtained by dividing Eq. (3.25) by Eq. (3.15):

$$tt(t_n) = \frac{\Delta x}{2} \frac{q(x_1,t_n)k(t_{n-1}) + q(x_2,t_n)k(t_n)}{q(x_1,t_n)\ q(x_2,t_n)} \quad (3.26)$$

The expression for the space-mean speed can be obtained by dividing the link distance Δx by the average travel time given in Eq. (3.26):

$$u(t_n) = \frac{2\ q(x_1,t_n)\ q(x_2,t_n)}{q(x_1,t_n)k(t_{n-1}) + q(x_2,t_n)k(t_n)} \quad (3.27)$$

The freeway travel time functions given by Eq. (3.27) have basically two independent traffic measurements: $q(x_1,t_n)$ and $q(x_2,t_n)$. Rewriting Eq. (3.27) using Eq. (3.4), we get:

$$tt(t_n) = \frac{[q(x_1,t_n)+q(x_2,t_n)]k(t_{n-1})\Delta x + [q(x_1,t_n)-q(x_2,t_n)]q(x_2,t_n)\Delta t}{2\ q(x_1,t_n)\ q(x_2,t_n)} \quad (3.28)$$

The relationships between the travel time and these two flow rates are found by differentiating Eq. (3.28) with respect to these variables. First:

$$\frac{\partial tt}{\partial q_1} = \frac{2q_1q_2[k(t_{n-1})\Delta x + q_2\Delta t] - 2q_2[k(t_{n-1})(q_1+q_2)\Delta x + q_2(q_1-q_2)\Delta t]}{(2q_1q_2)^2}$$

$$= \frac{q_2\Delta t - k(t_{n-1})\Delta x}{2q_1(q_2)^2} \quad (3.29)$$

where $q_1 = q(x_1,t_n)$ and $q_2 = q(x_2,t_n)$. Due to the precondition $Q(x_2,t_n) > Q(x_1,t_{n-1})$, $q_2\Delta t$ is always greater than $k(t_{n-1})\Delta x$. This always makes Eq. (3.29) positive, which means that as the traffic demand given by $q(x_1,t_n)$ increases, the travel time $tt(t_n)$ also increases. This can be better understood by referring to Figure 3.3.

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Second:

$$\frac{\partial tt}{\partial q_2} = \frac{2q_1q_2[k(t_{n-1})\Delta x + (q_1 - 2q_2)\Delta t] - 2q_1[k(t_{n-1})(q_1 + q_2)\Delta x + q_2(q_1 - q_2)\Delta t]}{(2q_1q_2)^2}$$

$$= -\frac{q_2^2\Delta t + k(t_{n-1})q_1\Delta x}{2q_1(q_2)^2} \qquad (3.30)$$

where $q_1 = q(x_1, t_n)$ and $q_2 = q(x_2, t_n)$. Since both the terms in the numerator and the term in the denominator are positive, Eq. (3.30) is always negative. This means that as the dynamic freeway capacity given by $q(x_2, t_n)$ improves, the travel time $tt(t_n)$ decreases.

3-2-2 Under congested flows

If the variable $m(t_n)$ becomes numerically zero or negative, then it describes the traffic situation, where there have been so many vehicles on the link before time t_{n-1}, that none of the vehicles which arrived during the interval (t_{n-1}, t_n) have exited the link during this interval. There are two versions of the model in this category. The earlier one (Drew and Nam 1995) assumed that the hatched area in Figure 3.4 approximated the total travel time under congested flow. The variable $m'(t_n)$, similar to $m(t_n)$ in Eq. (3.15), is defined as:

$$m'(t_n) = Q(x_1, t_{n-1}) - Q(x_2, t_n) \qquad (3.31)$$

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It can be shown that the schematic interpretation of the travel time as shown in Figure 3.4 gives exactly the same equation derived under normal flows. However, it was found that the precondition $Q(x_2,t_n) \le Q(x_1,t_{n-1})$ under congested flows made the relationship between travel time and inflow rate inversely proportional. It means that as the traffic demand $q(x_1,t_n)$ increases, the travel time $tt(t_n)$ decreases. This undesirable property can be observed graphically in Figure 3.4. As we increase traffic demand by rotating the demand curve $Q(x_1,t)$ counterclockwise, the total travel time represented by the hatched area decreases.

Thus, the revised version of the above model was developed. It is schematically presented in Figure 3.5. Here, the variable $m''(t_n)$ is similar to $m(t_n)$ in Eq. (3.15) and the total travel time is equal to:

$$\frac{1}{2} \left[(t'' - t_{n-1}) + (t' - t_n) \right] m''(t_n) \tag{3.32}$$

where

$$t' = t_{n-1} + \frac{k(t_{n-1})\Delta x}{q(x_2,t_n)}; \tag{3.33}$$

$$t'' = t_n + \frac{k(t_n)\Delta x}{q(x_2,t_n)} \tag{3.34}$$

$$m''(t_n) = Q(x_1,t_n) - Q(x_1,t_{n-1}). \tag{3.35}$$

After substituting Eq. (3.33) and (3.34) in Eq. (3.32), the travel time and space-mean speed during the interval (t_{n-1}, t_n) are respectively given as follows:

$$tt(t_n) = \frac{\Delta x}{2} \frac{k(t_{n-1}) + k(t_n)}{q(x_2, t_n)} \tag{3.36}$$

$$u(t_n) = \frac{2 \, q(x_2, t_n)}{k(t_{n-1}) + k(t_n)} \tag{3.37}$$

Rewriting the travel time function in Eq. (3.36) using Eq. (3.4). we get:

$$tt(t_n) = \frac{k(t_{n-1})\Delta x + [q(x_1, t_n) - q(x_2, t_n)]\Delta t}{2 \, q(x_2, t_n)} \tag{3.38}$$

The relationships between the travel time and the two flow rates can be found in a similar way:

$$\frac{\partial tt}{\partial q_1} = \frac{\Delta t}{2q_2} \tag{3.39}$$

where $q_1 = q(x_1, t_n)$ and $q_2 = q(x_2, t_n)$. Since both the terms in the numerator and the denominator are always positive, Eq. (3.39) is always positive. This means that as the traffic demand given by $q(x_1, t_n)$ increases, the travel time $tt(t_n)$ also increases.

$$\frac{\partial tt}{\partial q_2} = - \frac{k(t_{n-1})\Delta x + q_1 \Delta t}{2q_2} \tag{3.40}$$

where $q_1 = q(x_1, t_n)$ and $q_2 = q(x_2, t_n)$. Since both the terms in the numerator and the term in the denominator are positive, Eq. (3.40) is always negative. This means that as the dynamic freeway capacity given by $q(x_2, t_n)$ improves, the travel time $tt(t_n)$

decreases. It is clear that the new travel time function under congested flows given in Eq. (3.36) has desirable relationships with the two independent variables.

3-2-3   Equilibrium flow equation

The general equation of traffic stream states that flow is the product of speed and density. Since the fundamental traffic stream variables include spatial fluctuations, the interpretation of the flow in the equation should be equal to the equilibrium flow which represents the average number of vehicles passing the link during a specified period of time. It is also the product of space-mean speed and density. The estimation of the equilibrium flow is also useful for statistical measures of the traffic flow model.

So far, space-mean speeds (or equilibrium speeds) have been derived as seen Eqs. (3.27) and (3.37). The equilibrium density during the interval (t_{n-1}, t_n) is taken as the average of densities measured at the beginning and the end of the interval:

$$k_e(t_n) = \frac{k(t_{n-1}) + k(t_n)}{2}$$

(3.41)

The equilibrium flow can now be obtained as:

• Under normal flow

$$q_e(x, t_n) = k_e(t_n) u(x, t_n)$$

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$$= \frac{k(t_{n-1}) + k(t_n)}{2} \left[\frac{2 \, q(x_1,t_n) \, q(x_2,t_n)}{q(x_1,t_n)k(t_{n-1}) \, + \, q(x_2,t_n)k(t_n)} \right]$$

$$= \frac{q(x_1,t_n)q(x_2,t_n)[k(t_{n-1}) + k(t_n)]}{q(x_1,t_n)k(t_{n-1}) \, + \, q(x_2,t_n)k(t_n)} \tag{3.42}$$

- Under congested flow

$$q_e(x,t_n) \; = \; k_e(t_n)u(x,t_n)$$

$$= \frac{k(t_{n-1}) + k(t_n)}{2} \; \frac{2 \, q(x_2,t_n)}{k(t_{n-1}) \, + \, k(t_n)}$$

$$= q(x_2,t_n) \tag{3.43}$$

3-2-4 Travel time smoothing

Exponential averaging can be applied to estimate $TT(t_n)$, the final average travel time during the interval (t_{n-1},t_n). This numerical technique favors the most recent estimate by assigning the weight factors in the following fashion::

$$TT(t_{n-1}) \; = \alpha[tt(t_{n-1}) + (1-\alpha)tt(t_{n-2}) + \; ..... \; +(1-\alpha)^{n-1}tt(t_o)] \tag{3.44}$$

At the next time interval (t_{n-1},t_n):

$$TT(t_n) \; = \alpha[tt(t_n) + (1-\alpha)tt(t_{n-1}) + \; ..... \; +(1-\alpha)^n tt(t_0)] \tag{3.45}$$

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Above Eq. (3.45) can be written as the sum of a new term and the previous travel time $TT(t_n)$ multiplied by the exponential weighing factor by discarding the old value of the average:

$$TT(t_n) = \alpha \ tt(t_n) + (1-\alpha)TT(t_{n-1}) \tag{3.46}$$

Thus, it is only necessary to carry forward the single numerical value of the average and not the long sequence of earlier data. By rearranging terms, Eq. (3.46) can be written in the following fashion:

$$TT(t_n) = TT(t_{n-1}) + \alpha[tt(t_n) - TT(t_{n-1})] \tag{3.47}$$

Let $\alpha = \Delta t / T$ where Δt is the solution time interval and T is the smoothing time. There is a trade-off between the precision and the stability of the time-series estimation in selecting α. (For example, $\alpha = 1.0$ means that only the current estimate has been used. This gives the best correlation with the instantaneous measurement but experiences fluctuations in estimation). Then,

$$TT(t_n) = TT(t_{n-1}) + \frac{\Delta t}{T}[tt(t_n) - TT(t_{n-1})] \leq tt_f \tag{3.48}$$

where tt_f is the free-flow link travel time. Here, $tt(t_n)$ is given by Eq. (3.26).
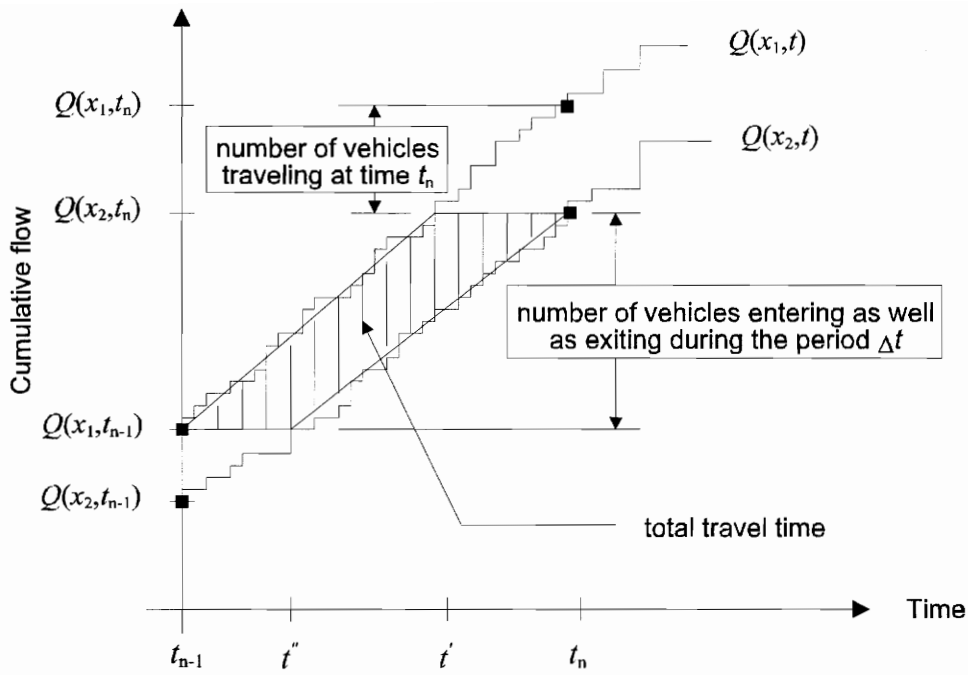
Figure 3.3   Schematic representation of the total travel time during the interval
(t_{n-1}, t_n) when $Q(x_2, t_n) > Q(x_1, t_{n-1})$
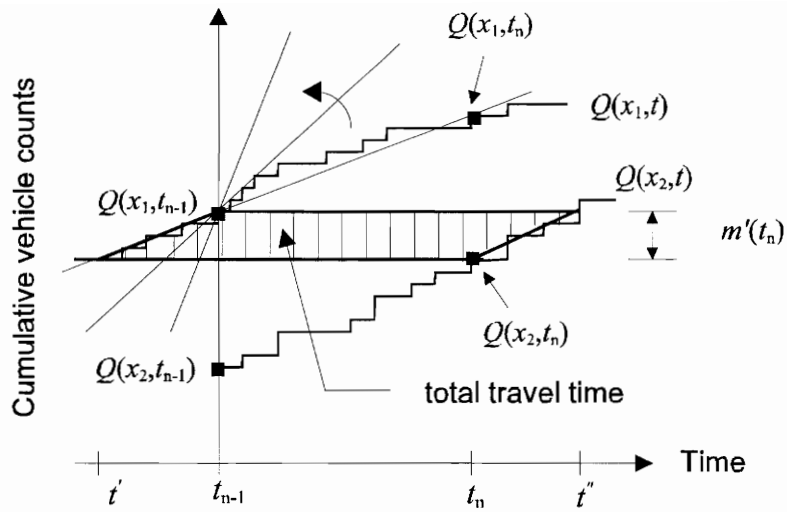
Figure 3.4   Schematic representation of the total travel time during the interval (t_{n-1}, t_n) when $Q(x_2, t_n) \leq Q(x_1, t_{n-1})$ and its inverse relationship with the inflow rate $q(x_1, t_n)$
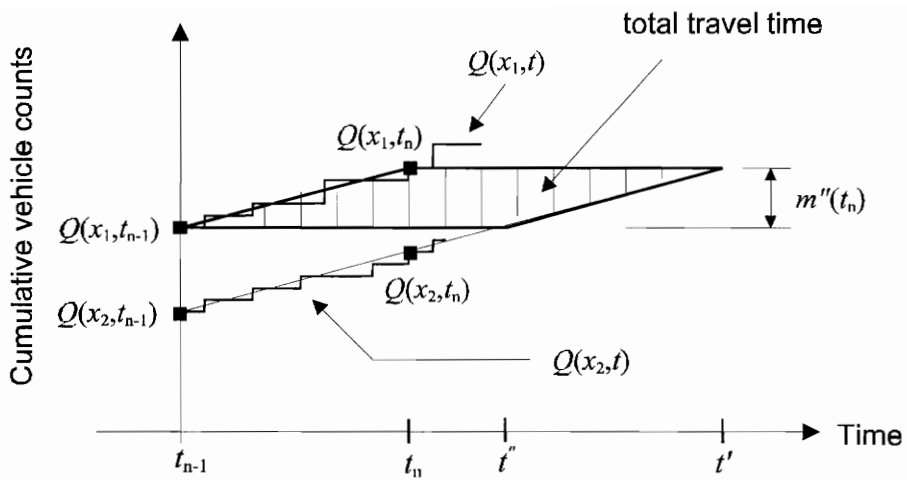
Figure 3.5   New schematic representation of the total travel time during the interval (t_{n-1}, t_n) when $Q(x_2, t_n) \le Q(x_1, t_{n-1})$

3-3   Empirical Evidence

The data used for validating the dynamic traffic model developed in this study was obtained using loop detectors during the morning peak-hours (5:00-10:00 a.m.) from March 14, 1994 to March 22, 1994 on a section of the Queen Elizabeth Way in Toronto, Canada. Raw data consisted of flows, roadway occupancies and spot speeds aggregated at an interval of thirty seconds for each lane of the mainstream. A schematic diagram of the study section is shown in Figure 3.6. For the analysis, the section of the freeway from Station 23 to Station 25 is assumed to be a single link. The data measured at Stations 23 and 25 during the morning peak-hours (6:00-10:00 a.m.) was selected and then aggregated for all lanes over an interval of two minutes. (Note that the recommended link distance-data aggregation rate as defined in Eq. (3.5) justifies the aggregation over one minute).

Time-series plots of flow, roadway occupancy and speed measurements for the data sets are presented in Figure 3.7 through 3.10. Typical scatter curves are also shown in Figure 3.11. After studying the figures, the following observations were made:

- The daily traffic behavior measured at two stations shows similar patterns over the study period in general;

- Each data set illustrates temporal and spatial fluctuations of traffic; and

- The measured spot speeds (occupancies) at upstream Station 23 are lower (higher) than those at downstream Station 25 at most times. A possible reason could be the effect of drivers reacting to downstream traffic conditions which is known as

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anticipation. There is a heavy demand near Station 23 while there is an additional lane just after Station 25.

While the daily traffic behavior measured at two stations seems to be relatively homogenous over the period and thus the data appeared to be reasonably consistent, flows at the downstream station were consistently underestimated within -4%. (See Table 3.1). Therefore, a volume adjustment factor is calculated for each hour for each data set based on the difference of cumulative flows at the end of each hour. Here, two data sets, 0314 and 0315, are considered for further analysis. All flow measurements at the downstream station are then multiplied by these volume adjustment factors (see Table 3.2).

Table 3.1   Flow measurements at the end of the study period and corresponding error

| Data Set | Cumulative Counts (veh) | | Discrepancy |
	Station 23	Station 25	(%)
0314	7727	7520	-2.75
0315	7618	7398	-2.97
0316	7207	6949	-3.71
0317	7711	7424	-3.87
0321	7964	7739	-2.91
0322	8030	7843	-2.38

Table 3.2 Calculated volume adjustment factors

Data Set	Study Hour (a.m.)			
	6:00 - 7:00	7:00 - 8:00	8:00 - 9:00	9:00 - 10:00
0314	1.020	1.015	1.025	1.045
0315	1.025	1.020	1.025	1.035

Two scenarios were defined for each data set (see Table 3.3). A travel time smoothing parameter α is selected to investigate the stability of the proposed algorithm. Here, α = 1 means that only current estimate has been used. This gives the best correlation with the instantaneous measurement but experiences fluctuations in estimation.

Table 3.3 Summary of analysis scenarios

Scenario	1	2	3	4
Data set	0314	0314	0315	0315
α	0.40	1.00	0.50	1.00

A computer program has been developed using MATLAB to implement the algorithm. Verification results show that the estimates are in quantitative and qualitative agreement with the measured data. First, it can be assumed that the data measured at the upstream and the downstream locations serves as the lower and the upper bound of the true traffic characteristics respectively due to the observed anticipation. Time-series profiles of the estimated space-mean speeds for the link closely follow those of the measured spot speeds and stay within the bounds for all scenarios (see Figures 3.12

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and 3.13). Congestion parameter $m(t)$ indicates the efficiency of traffic stream during the study hours (see Figure 3.14). When there is a severe congestion, it is analytically detected and represented by its negative values (see Figure 3.15). The estimated density, one of spatial variables, is compared with the point measurements of occupancies (see Figure 3.16). Scatter curves of the estimated speed-flow-density relationship clearly display the variation of the traffic characteristics on the link over time (see Figures 3.17 through 3.19). Finally, link travel times are illustrated as a function of time (see Figures 3.20 and 3.21).

For the evaluation of quantitative effectiveness, the following models are included:

- The original formulation of conservation of vehicles (Model 1)

$$q(x_2, t_n) = q(x_1, t_{n-1}) + \frac{\Delta x}{\Delta t}[k(t_n) - k(t_{n-1})]$$

- The new form of the equation of conservation of vehicles (Model 2)

$$q(x_2, t_n) = q(x_1, t_n) + \frac{\Delta x}{\Delta t}[k(t_n) - k(t_{n-1})]$$

- The dynamic traffic flow model with $\alpha = 0.4$ (Model 3) and $\alpha = 1.0$ (Model 4)

$$q_e(t_n) = = \frac{q(x_1, t_n)q(x_2, t_n)[k(t_{n-1}) + k(t_n)]}{q(x_1, t_n)k(t_{n-1}) + q(x_2, t_n)k(t_n)} \qquad \text{if } Q(x_2, t_n) > Q(x_1, t_{n-1})$$

$$q_e(t_n) = q(x_2, t_n) \qquad \text{if } Q(x_2, t_n) \le Q(x_1, t_{n-1})$$

Models 1 and 2 estimate flow rates at the downstream location and these estimates are compared to the flow measurements at the location. Meanwhile, Models 3 and 4

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estimate spatial equilibrium flows which represent the number of vehicles passing a link during a specific period of time. It is therefore assumed that the true value of equilibrium flow is very close to the average of two upstream and downstream flow measurements. The following statistical measures are calculated: Mean Absolute Error (MAE) and percentage of relative MAE (MAE %). They are defined as:

$$MAE = \frac{\sum |measured - estimated|}{N} \tag{3.33}$$

$$MAE(\%) = \sum \frac{|measured - estimated|}{measured} \frac{100}{N} \tag{3.34}$$

where N is the number of measurements.

Here, each data set is aggregated twice over two minutes and five minutes. As shown in Table 3.4, the new formulation of equation of conservation of vehicles (Model 2) is much better than the original equation (Model 1). Also, as expected, the analyses with using only current estimate (Model 4) give better results compared to the analyses with using time-series estimation (Model 3). This shows a trade-off between the precision and the stability of the time-series estimation in selecting α .

These results are very satisfactory in terms of accuracy compared to other continuum models. Lyrintzis, Yi, Michalopoulos and Beskos (1993) used five-minute data for the evaluation of the models. Their research included (see Section 2.1 for the description of individual models):

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- the original equation of conservation of vehicles previously described as Model 1 but in different formulation

$$k_j^{n+1} = k_j^n + \frac{\Delta t}{\Delta x}[q_{j-1}^n - q_j^n]$$

- Payne's model

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{1}{T}[u_e(k) - u] - \frac{v_o^2}{k}\frac{\partial k}{\partial x}$$

- Papageorgiou's model

$$u_j^{n+1} = u_j^n + \Delta t \xi \left[-u_j^n \frac{(u_j^n - u_{j-1}^n)}{\Delta x_j} - \frac{u_j^n - u_e(k_j^n)}{T} + \frac{v(k_{j+1}^n - k_j^n)}{\Delta x_j(k_j^n + \kappa)}\right]$$

$$q_j = \alpha \, k_j \, u_j + (1-\alpha) \, k_{j+1} \, u_{j+1}$$

- their semi-viscous model

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{\Phi}{T}[u_f(x) - u] - G - \alpha k^\beta \frac{\partial k}{\partial x}$$

- their viscous models

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -vk^\delta \frac{\partial k}{\partial x} + \lambda k^\varsigma \frac{\partial^2 k}{\partial x^2}$$

The results are shown in Table 3.5. It is interesting to note that the numerical implementation of Model 1 yields the MAE(%) error of 15.26% as opposed to the average of 3.0% in this research. Hence, in this research, the principle of conservation of vehicles (Model 1) has been properly implemented and has been further improved to Model 2.

Table 3.4 Quantitative effectiveness analysis of flow rate estimation

Model	1	2	3	4
Data Set 0314				
MAE[a]	61.7501	35.2662	82.1488	6.5173
MAE (%)	3.3813	1.9150	4.2997	0.3530
MAE[b]	49.3840	24.9866	103.9146	2.5560
MAE (%)	2.6371	1.3650	5.3099	0.1469
Data Set 0315				
MAE[a]	55.2461	33.5370	109.3833	48.6147
MAE (%)	3.0295	1.9099	6.5388	3.2796
MAE[b]	53.9147	25.6299	83.3091	4.5540
MAE (%)	2.9550	1.4142	4.4130	0.2513
[a] veh/2-min/lane, [b] veh/5-min/lane				

Table 3.5 Error indices for flow rate (source: Lyrintzis, Yi, Michalopoulos and Beskos 1993)

Model	Conservation (Model 1)	Payne	Papageorgiou	Lyrintzis et al.[b]	Lyrintzis et al.[c]
MAE[a]	82	59	46	23	40
MAE (%)	15.26	8.10	7.84	4.46	7.23
[a] veh/5-min, [b] semi-viscous model, [c] viscous model					
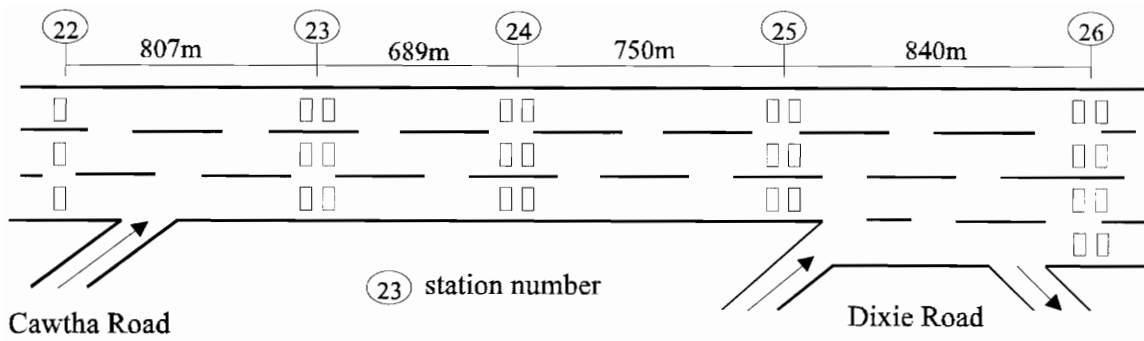
Figure 3.6   Schematic diagram of the study section of the Queen Elizabeth Way in Toronto, Canada

Flow and Occupancy Measurements (031494)
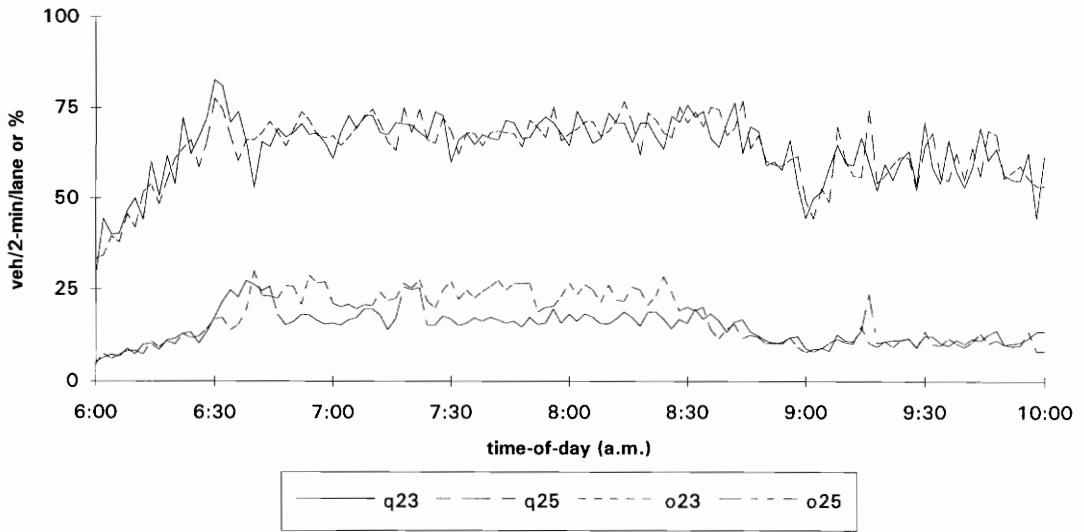
Speed Measurements (031494)
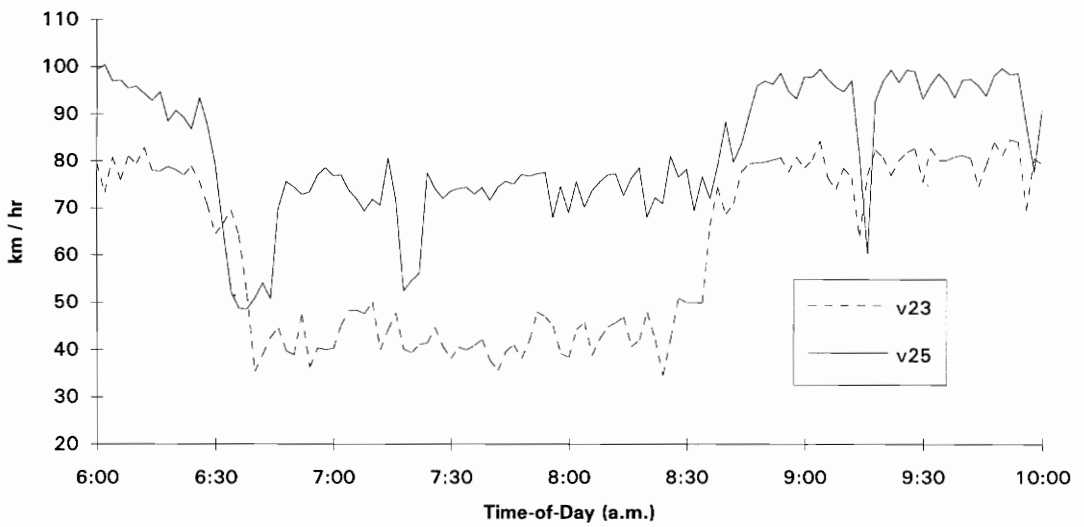
Figure 3.7   Time-series plots of flow, roadway occupancy and speed measurements of data set 0314

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Flow and Occupancy Measurements (031594)

Speed Measurements (031594)
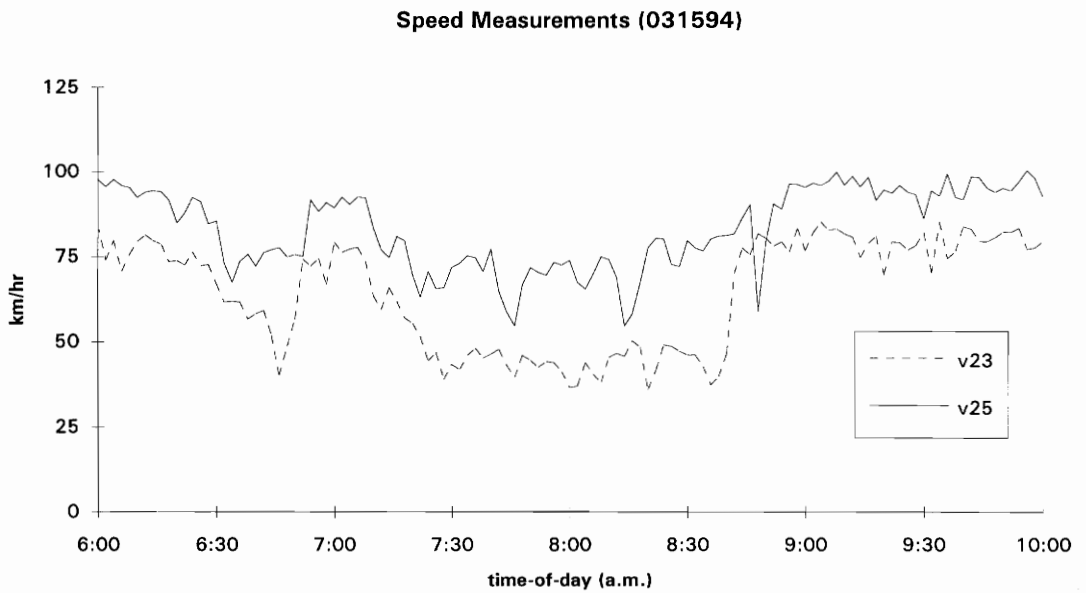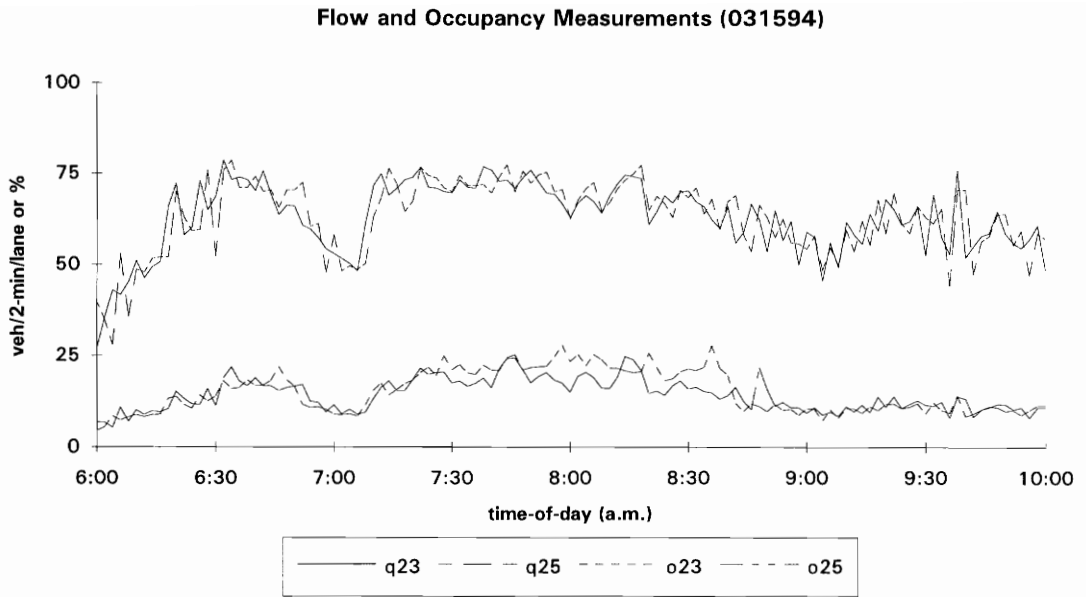
Figure 3.8   Time-series plots of flow, roadway occupancy and speed measurements of data set 0315

Flow and Occupancy Measurements (302194)

Speed Measurements (032194)
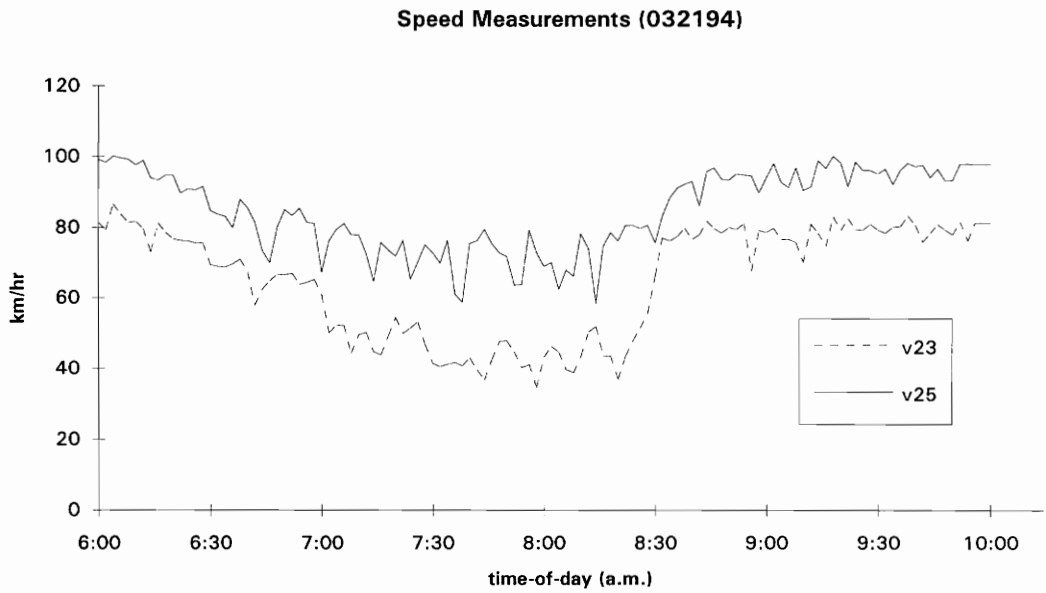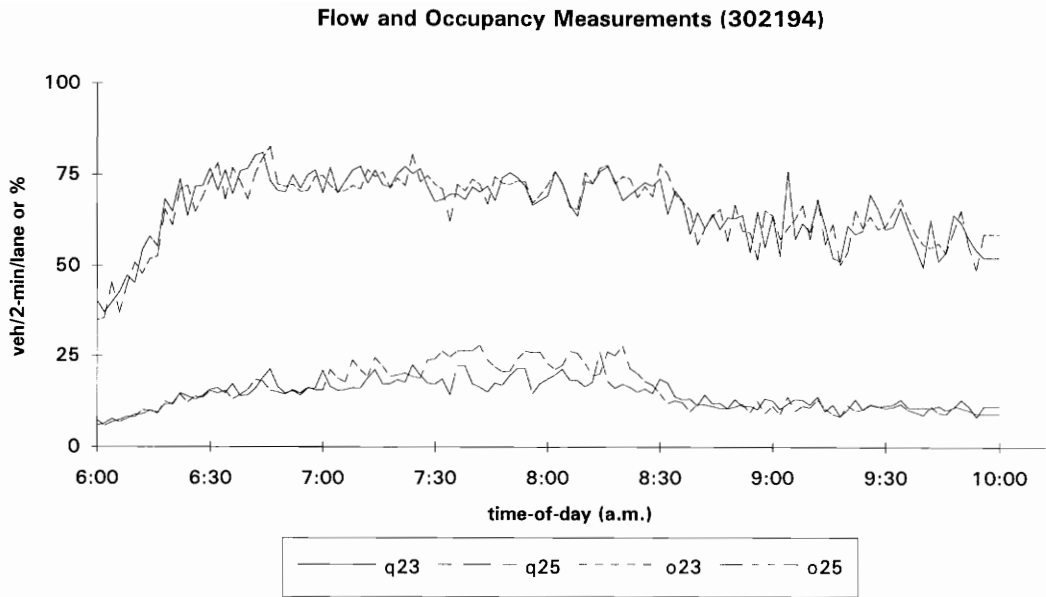
Figure 3.9   Time-series plots of flow, roadway occupancy and speed measurements of data set 0321

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Flow and Occupancy Measurements (032294)

Speed Measurements (302294)
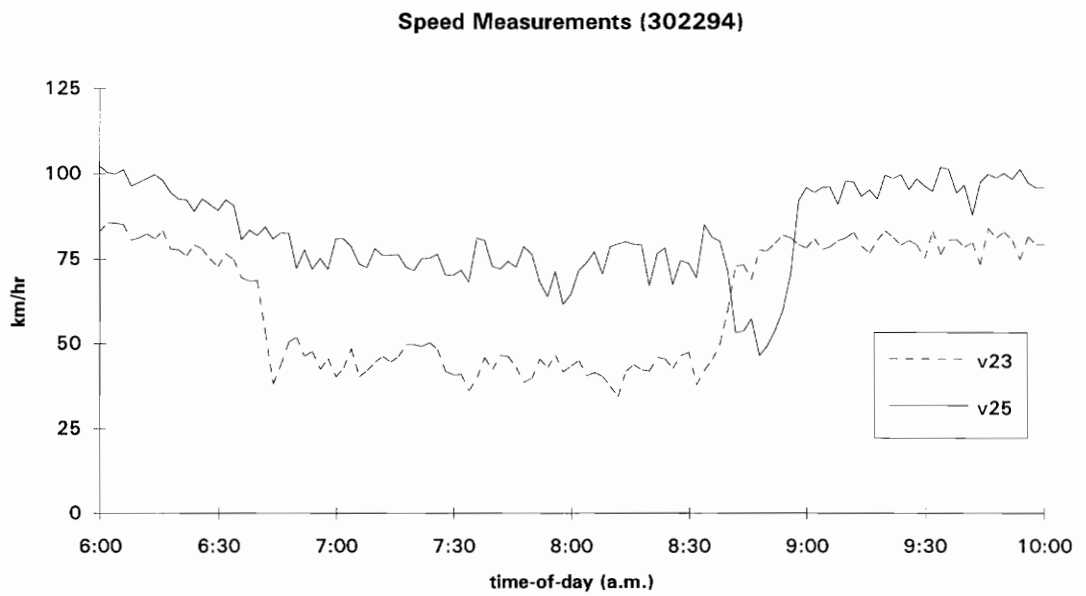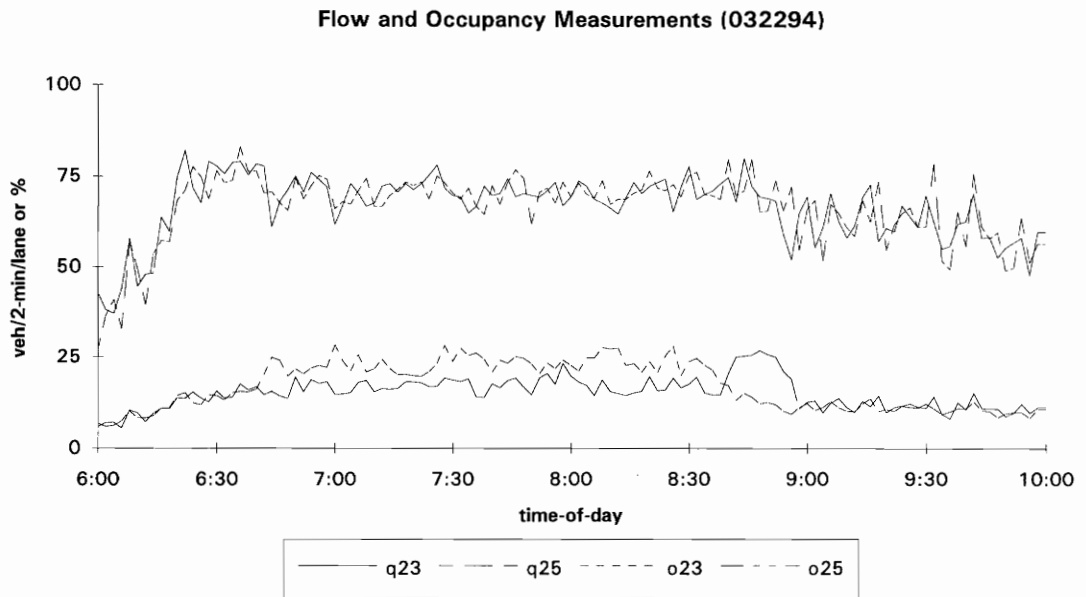
Figure 3.10   Time-series plots of flow, roadway occupancy and speed measurements
of data set 0322

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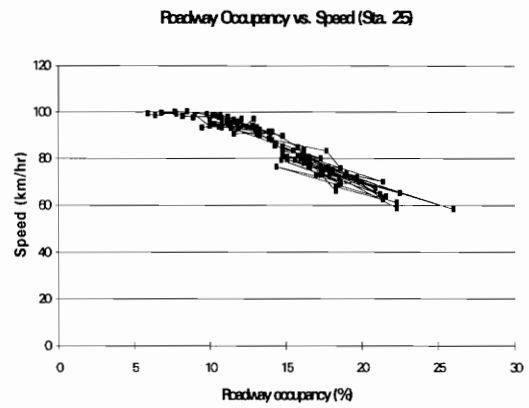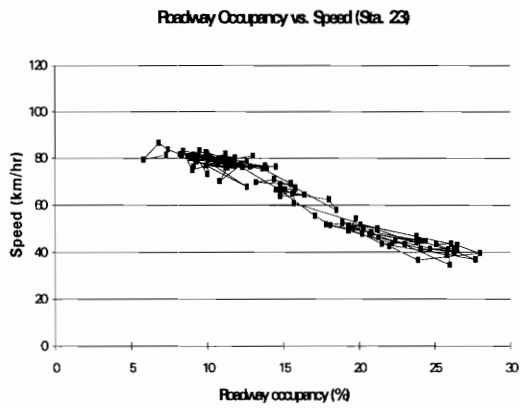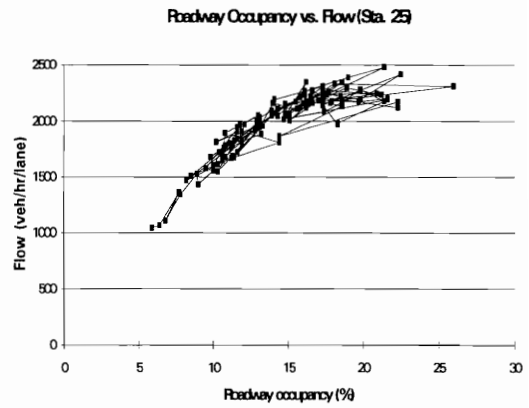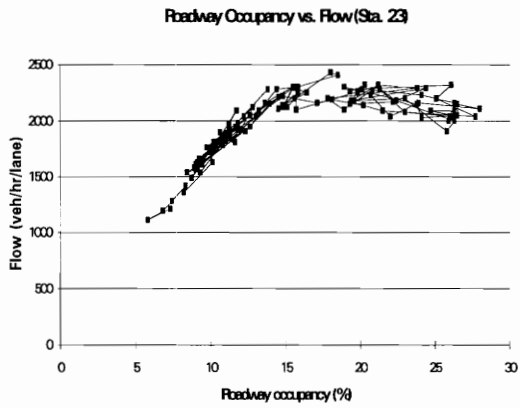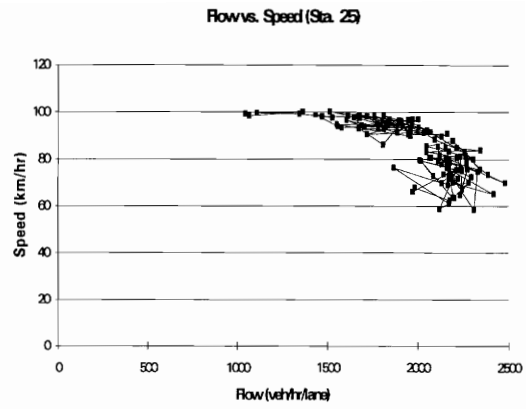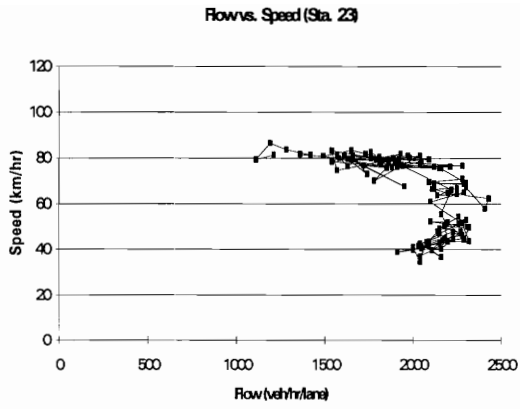
Figure 3.11   Scatter curves of the data set 0321

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Measured and Estimated Speeds (0314)
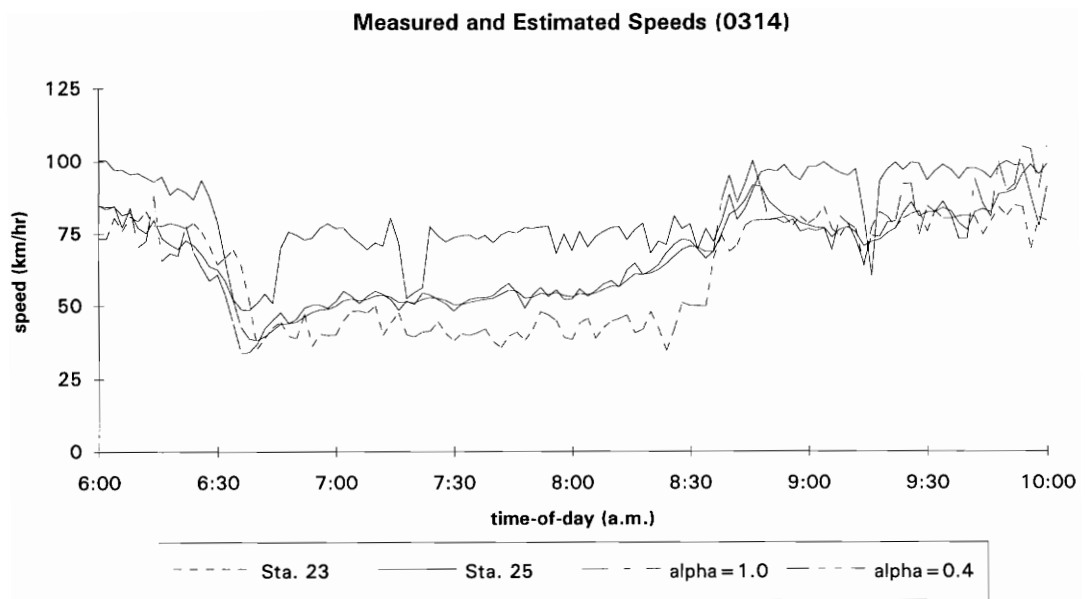
Figure 3.12   Time-series plot of measured spot speeds and estimated space-mean speeds - data set 0314

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Measured and Estimated Speeds (0315)
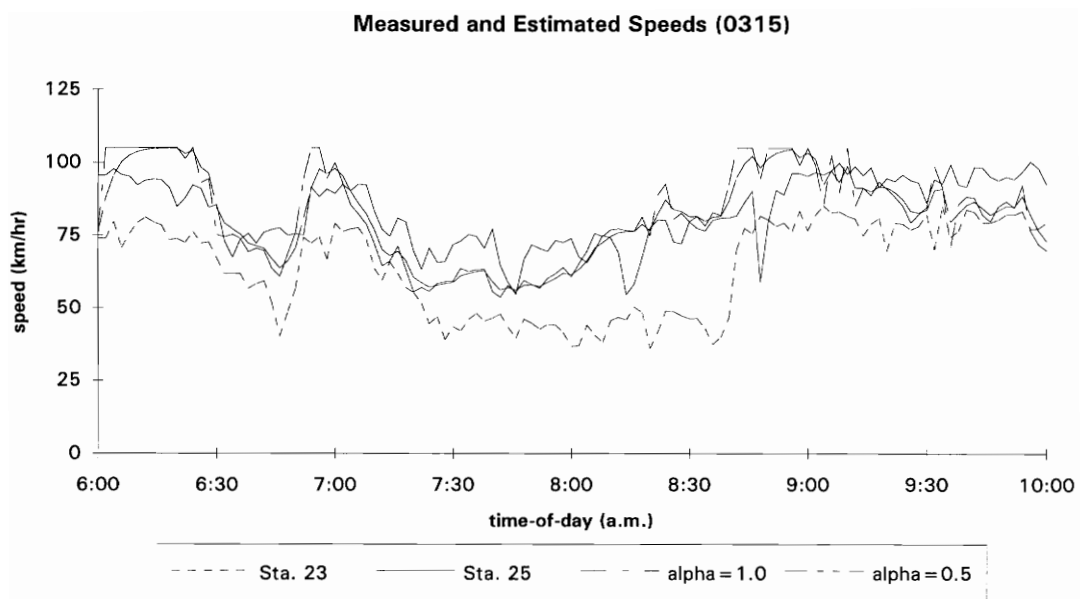
Figure 3.13   Time-series plot of measured spot speeds and estimated space-mean speeds - data set 0315

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Congestion Parameter, m(t)
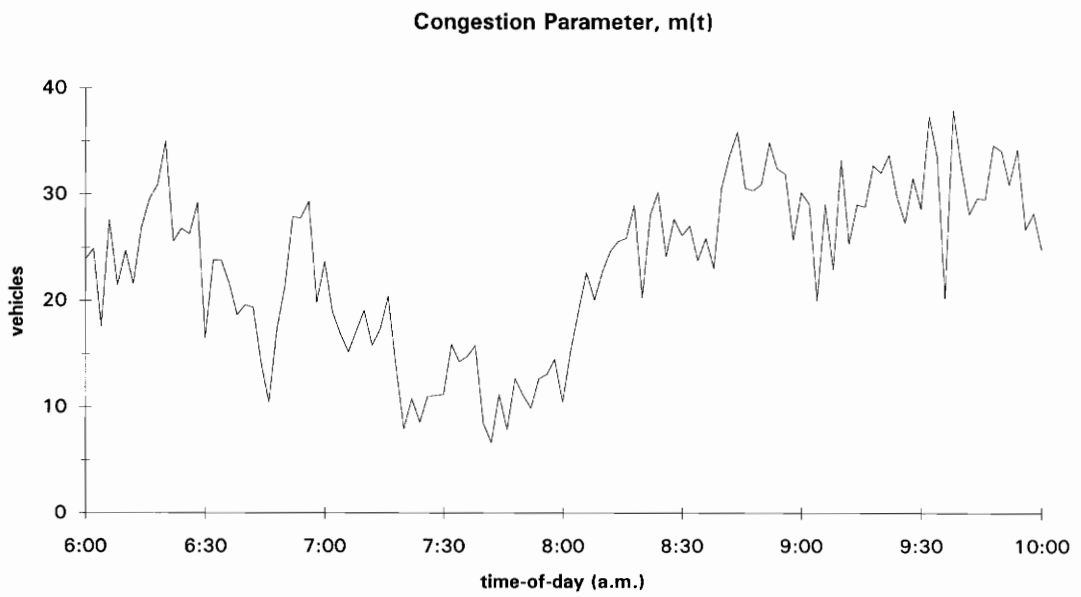
Figure 3.14   Analysis results of congestion parameter showing the efficiency of the study section - data set 0315

Congestion Parameter, m(t)
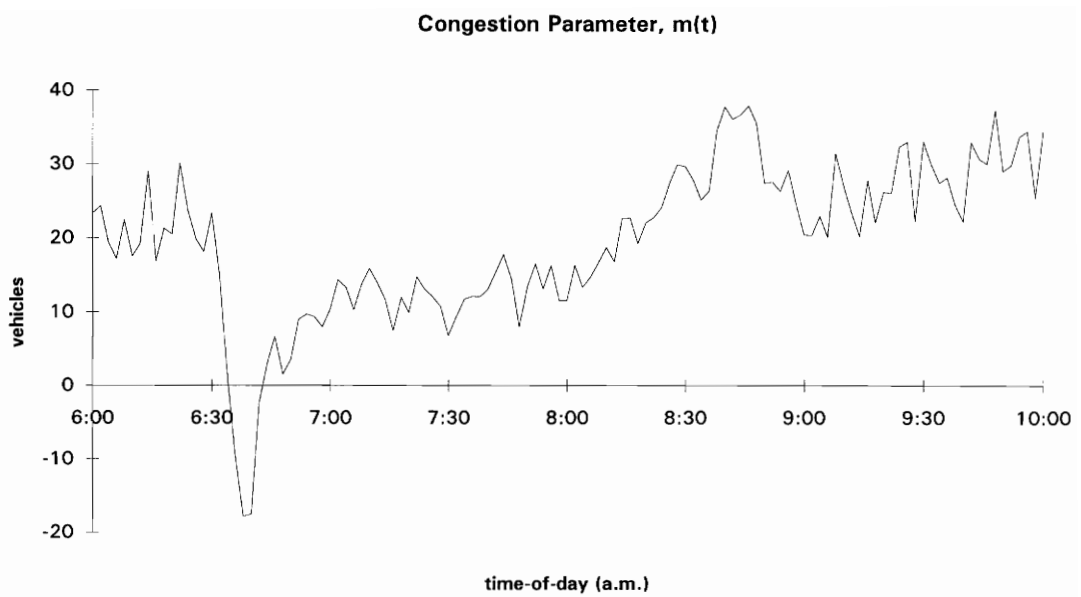
Figure 3.15   Analysis results of congestion parameter showing the efficiency of the study section - data set 0314

Comparison of Measured Roadway Occupancies and Estimated Density
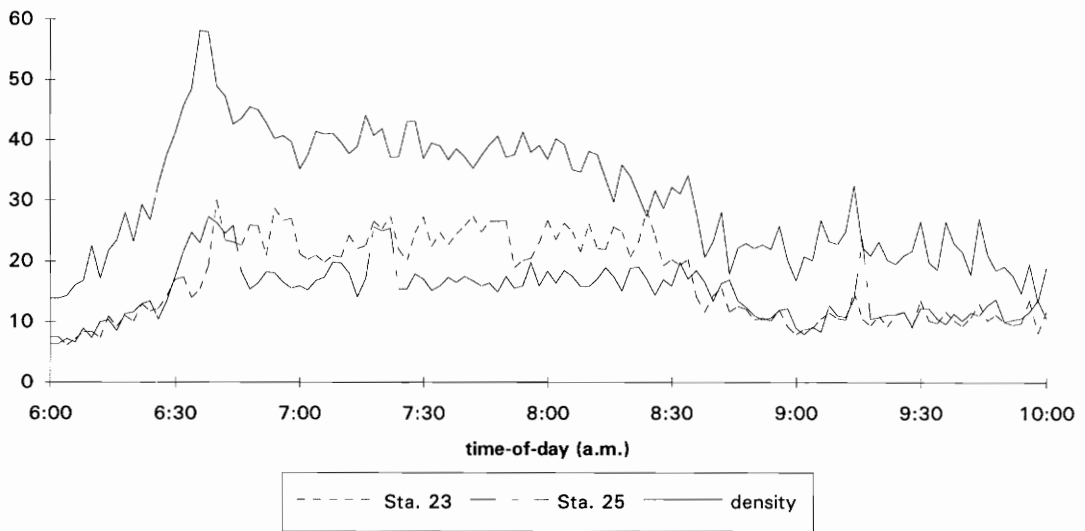
Figure 3.16   Comparison of estimated density vs. measured roadway occupancies -
data set 0314

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Scatter Plot of Density vs. Speed
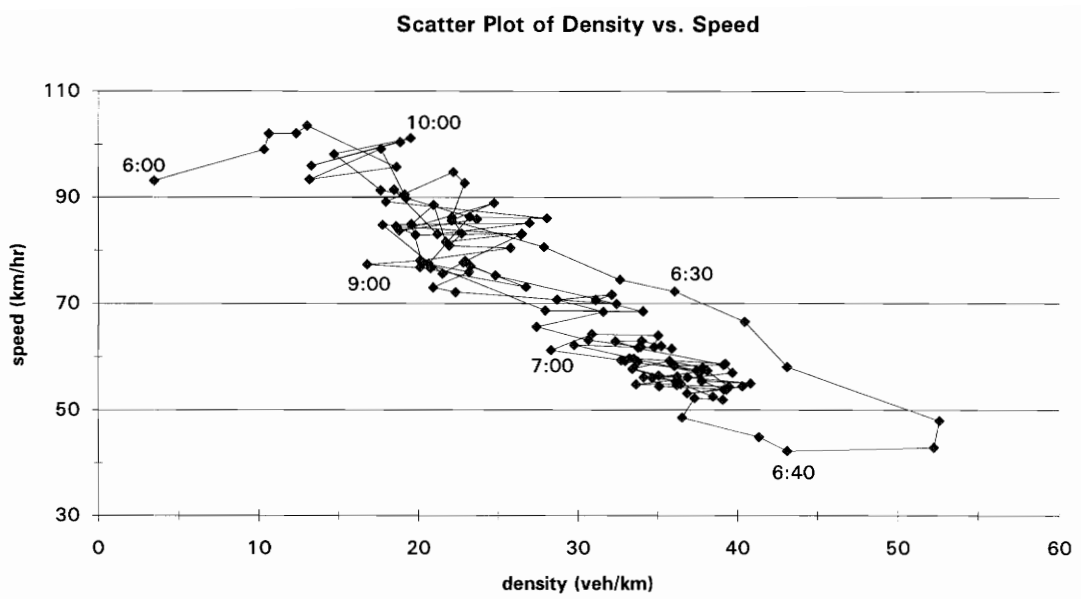
Figure 3.17   Analysis results of density and space-mean speed relationship - data set 0314

Scatter Plot of Flow vs. Speed
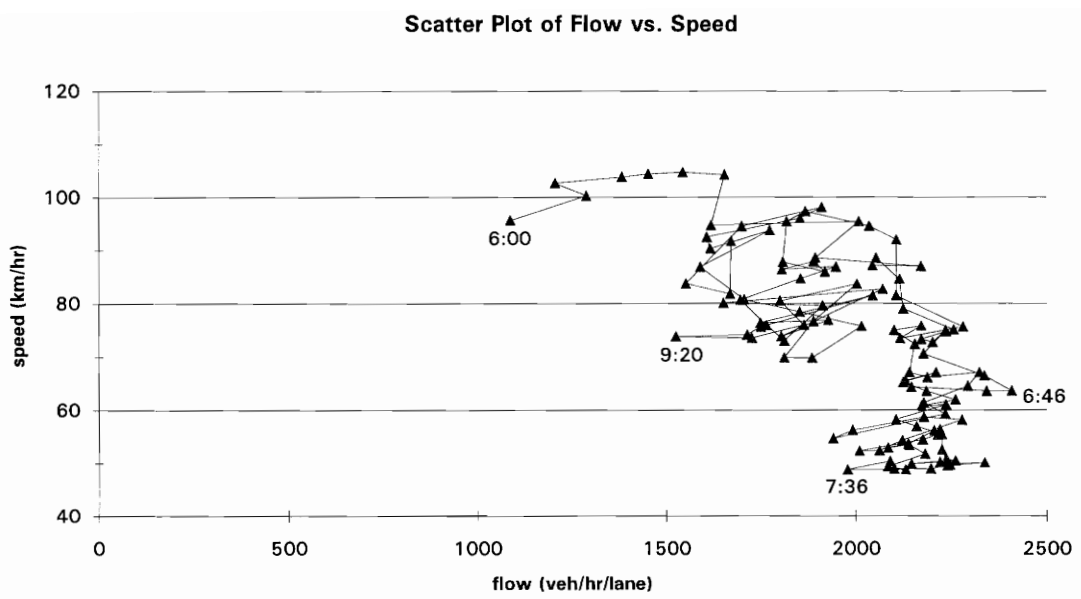
Figure 3.18   Analysis results of space-mean speed and equilibrium flow relationship - data set 0321
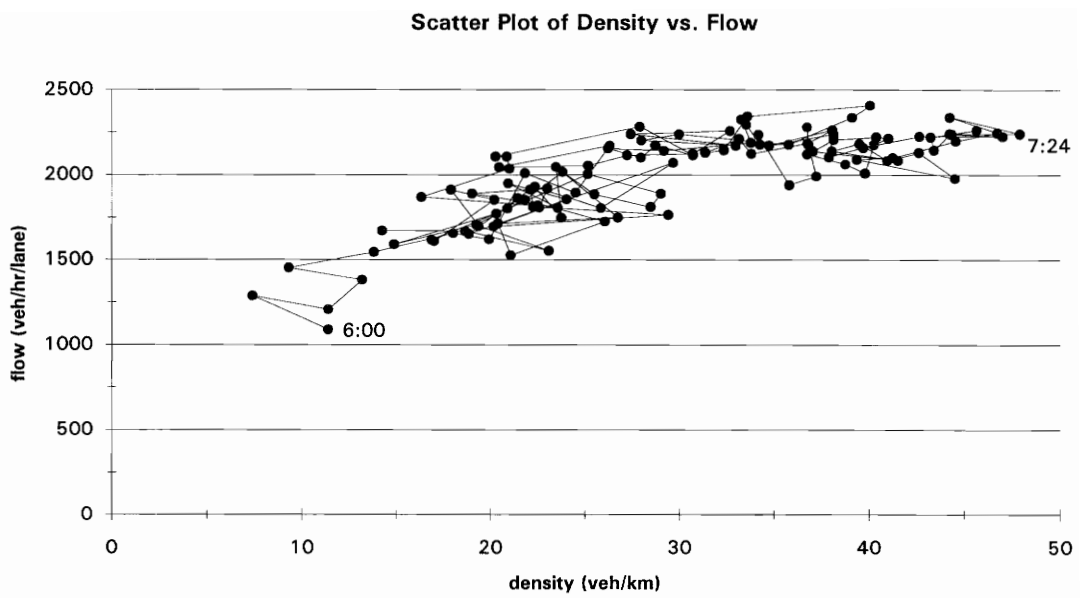
Figure 3.19   Analysis results of density and equilibrium flow relationship - data set
0321

Estimated Link Travel Times (031494)
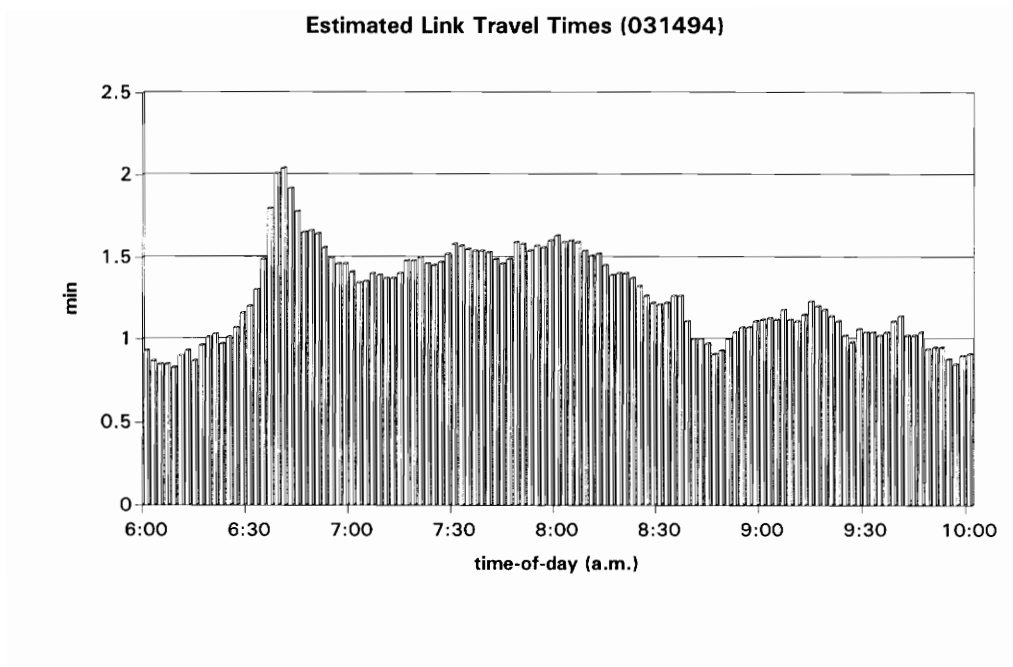
Figure 3.20   Estimation of travel times as a function of time - data set 0314

Estimated Link Travel Times (031594)
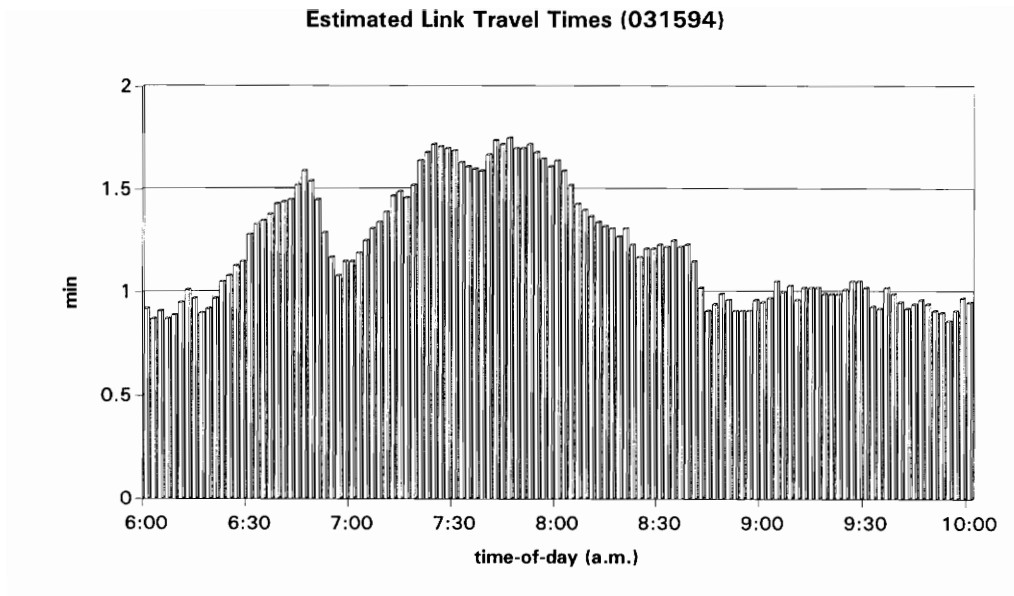
Figure 3.21   Estimation of travel times as a function of time - data set 0315

Comparison of Measured and Estimated Flow (0314)
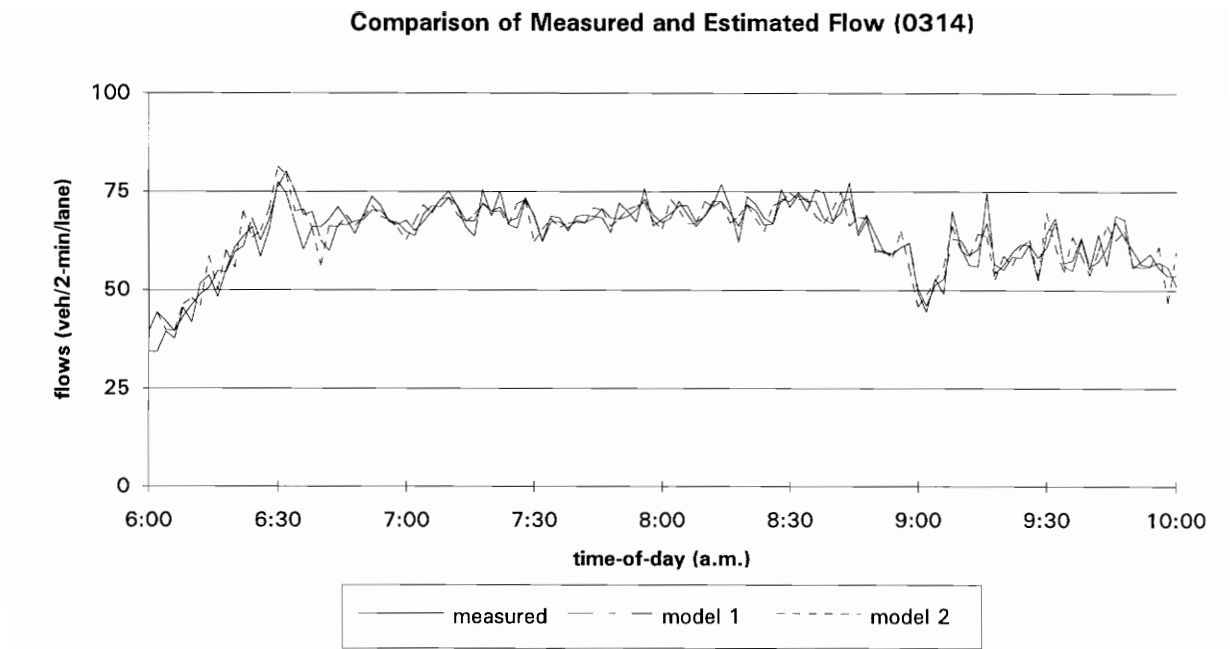
Figure 3.22   Comparison of the estimated flows vs. measured flows at Station 25 - data set 0314

Comparison of Measured and Estimated Equilibrium Flow (0314)
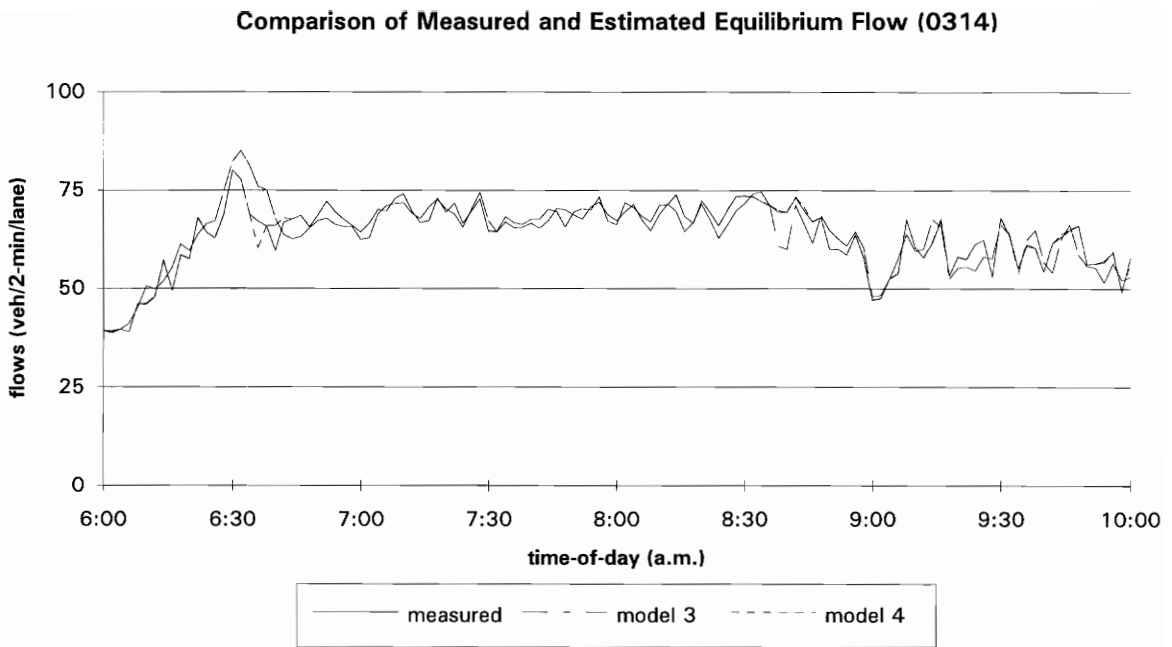
Figure 3.23   Comparison of the estimated equilibrium flows vs. measured average flows - data set 0314

Comparison of Measured and Estimated Flow (0315)
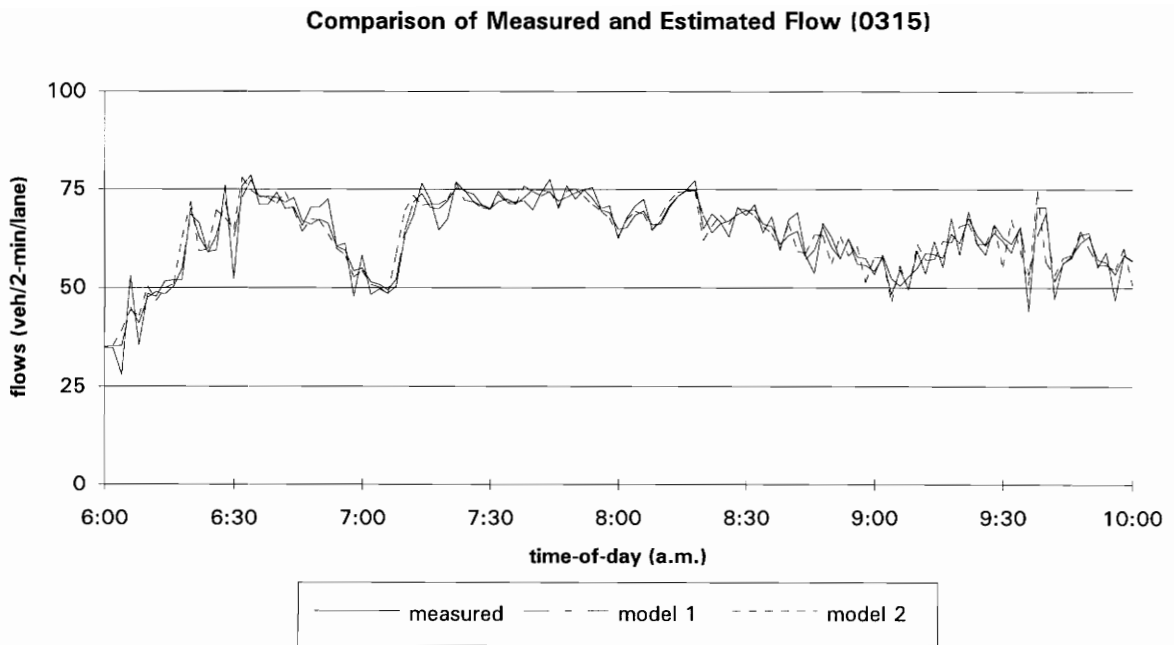
Figure 3.24   Comparison of the estimated and measured flows at Stattion 25 - data set 0315

Comparison of Measured and Estimated Equilibrium Flow (0315)
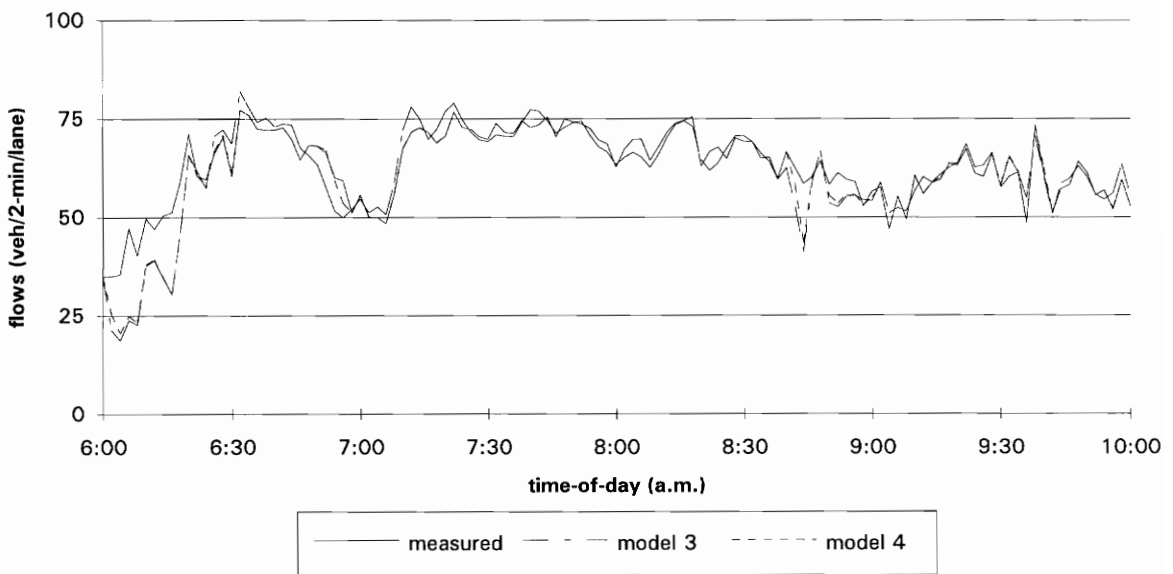
Figure 3.25   Comparison of the estimated equilibrium flows vs. measured average flows - data set 0315

4-1 Methodological Basis

Congestion occurs when the demand exceeds the capacity. Stochastic queuing methods can no longer be applied because the traffic density, a ratio of demand to capacity, exceeds unit value. Deterministic queuing analysis and shock wave analysis, which are the applicable methods, are reviewed in Chapter 2. It is an accepted fact that shock wave analysis can simulate real traffic behavior especially in the vicinity of a bottleneck. Despite its theoretical attraction, this method is not widely used. The reason behind this could be its complexity or no clear aids. Instead, deterministic queuing analysis is popular due to its simplicity. The Highway Capacity Manual (1994) also adapts this method for estimating delay in highway work zones.

The differences between these two methods have not been well researched. Chow (1976) compared both methods by computing total vehicle-hours of delay using the two methods. His rigorous mathematical derivation showed that the two methods yielded the same results if traffic mechanisms during incidents were such that the density of traffic demand, usually called normal traffic state, was assumed to be constant during the incident. Al-Deek, Garib and Radwan (1995) referred to the above conclusion without reservation in their recent paper. However, it was later found that the inappropriate

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assumption of the equal progression or retrogression rate of a time-space domain of congestion led to this conclusion.

Here, dynamic mechanisms of delay, progression and retrogression, are analyzed using the fundamental concept of conservation of vehicles. The analytically derived expressions of these mechanisms are self-explanatory and show the differences between the two methods. These expressions are useful for estimating delay times, the number of vehicles in a queue and the associated queue length. With these estimates, the use of a delay diagram, similar to a cumulative demand-capacity curves, is proposed to easily visualize the dynamic mechanism. Figure 4.1 illustrates the block diagram of the proposed dynamic analysis of delay and its potential applications to various ITS user services.
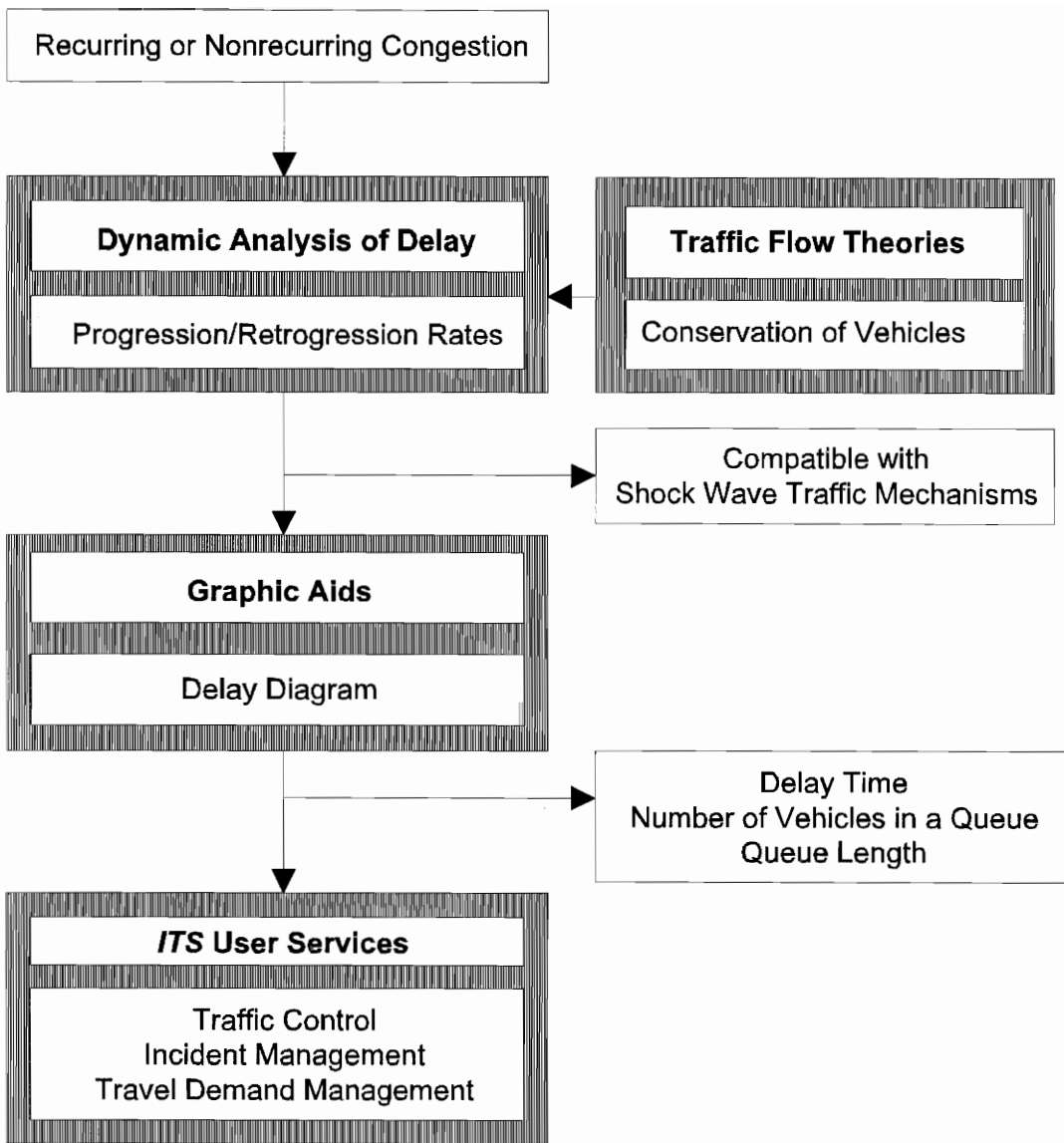
Figure 4.1   Block diagram of the proposed dynamic analysis of delay and its application to various ITS user services

4-2 Progression/Retrogression of Congestion Domain

Consider the movement of two traffic streams having different flows and densities. They are separated by the moving front edge w_u. Such traffic phenomenon can be observed upstream of an incident location or in a vicinity of a bottleneck. Traffic states of the two traffic streams are assumed remaining constant during the period of progression or retrogression. For convenience, the time variable t is ignored here.

First, the time-space domain of the congested domain growing against the direction of traffic is considered. The two traffic streams considered here are: normal flow which represents the demand; and queuing flow in which traffic is jammed due to congestion. As depicted in Figure 4.2, the longitudinal length of queuing flow and normal flow are l_1 and l_2 at time t_{n-1}. After the time Δt, domains of these traffic states have changed to l_3 and l_4. In other words, the congestion domain has grown by $l_3 - l_1$ during the period Δt.

Now, define a progression rate as the derivative of the number of vehicles queued in a congestion domain $n(t)$ with respect to time t. The principle of conservation of vehicles is then applied to derive the progression rate dn/dt as follows: The difference between number of vehicles entering and exiting during the period Δt is:

$$(q_n - q_q)\ \Delta t \tag{4.1}$$

The change in density is given as:

$$(k_q l_3 + k_n l_4) - (k_q l_1 + k_n l_2)$$

$$= k_q (l_3 - l_1) + k_n (l_4 - l_2)$$

$$= k_q (l_3 - l_1) - k_n (l_3 - l_1)$$

$$= (k_q - k_n)(l_3 - l_1) \tag{4.2}$$

By the principle of conservation of vehicles, Eq. (4.1) and (4.2) are identical:

$$(q_n - q_q) \Delta t = (k_q - k_n)(l_3 - l_1) \tag{4.3}$$

Rewriting Eq. (4.3):

$$\frac{(l_1 - l_3)}{\Delta t} = \frac{(q_q - q_n)}{(k_q - k_n)} < 0 \tag{4.4}$$

Here, $(l_1 - l_3)/\Delta t$ is the velocity of w_u. Since the term in the numerator is positive and the term in the denominator is negative, Eq. (4.4) is always negative. This means the shock wave w_u travels along the direction of progression, i.e., upstream. Therefore, it is so called a backward forming shock wave.

Since the increment in the number of vehicles included in a queue during the interval $(t - \Delta t, \ t)$ equals $k_q (l_1 - l_3)$, the expression for the progression rate can be obtained from Eq. (4.4) as:

$$\frac{dn}{dt} = - w_u k_q$$

$$= (q_n - q_q) - w_u k_n \tag{4.5}$$

Here, the first two terms in a parenthesis on the right-hand side represent the effect of differences of flow rates between the two traffic streams. The third term represents the additional effect due to the presence of progression of congested domain. This is not included in the deterministic queuing theory. It implies that the deterministic queuing theory underrates a progression of a congested domain and thus underestimates the overall magnitude of delay.

The retrogression rate of congested domain can also be obtained in a similar manner. Here, the time-space of congested domain is shrinking as the incident is cleared or the demand decreases below the capacity of the roadway. The two traffic stream considered here are: normal flow which represents the demand; and capacity flow in which traffic moves at the maximum rate. The congested domain is decreasing along the direction of traffic as illustrated in Figure 4.3. Using the principle of conservation of vehicles again, we have:

$$(q_n - q_c) \; \Delta t \;\; = \;\; (k_n - k_c) \; (l_3 - l_1) \tag{4.6}$$

The velocity of w_d, the forward recovery shock wave, is estimated as:

$$\frac{l_1 - l_3}{\Delta t} \;\; = \;\; \frac{q_n - q_c}{k_n - k_c} \; > 0 \tag{4.7}$$

Here, $(l_1 - l_3)/\Delta t$ is the velocity of w_d. Since the term in the numerator is positive and the term in the denominator is also positive, Eq. (4.4) is always positive. This means the shock wave w_d travels along the direction of retrogression, i.e., downstream. Hence, it is so called a forward recovery shock wave.

The rate of retrogression is obtained in a similar fashion:

$$-\frac{dn}{dt} = w_d k_c$$

$$= (q_c - q_n) + w_d k_n \tag{4.8}$$

The first two terms in a parenthesis on the right-hand side of the above equation show the effect of the differences of flow rates between two traffic streams. The last term on the right-hand side represents the additional effect due to the presence of retrogression of congested domain. Again, this effect is not available in the deterministic queuing theory. This implies that the deterministic queuing theory also underestimates the recovery rate of a congestion domain.

The analysis of dynamic mechanisms of delay, progression and retrogression of congested domain, clearly demonstrates the differences between the two delay analysis methods which has not been clearly explained. These analytical derivations are then applied to develop a delay diagram to schematically represent total delay in vehicle-hours.
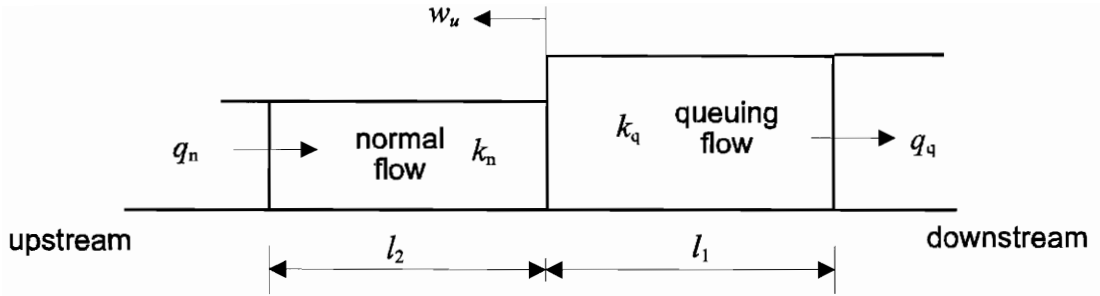
(a) traffic condition at time $t\text{-}\Delta t$
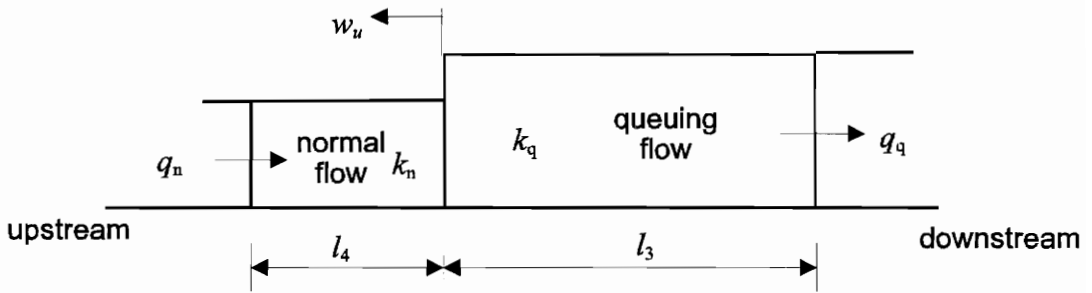
(b) traffic condition at time t

Figure 4.2   Progression of a congestion domain
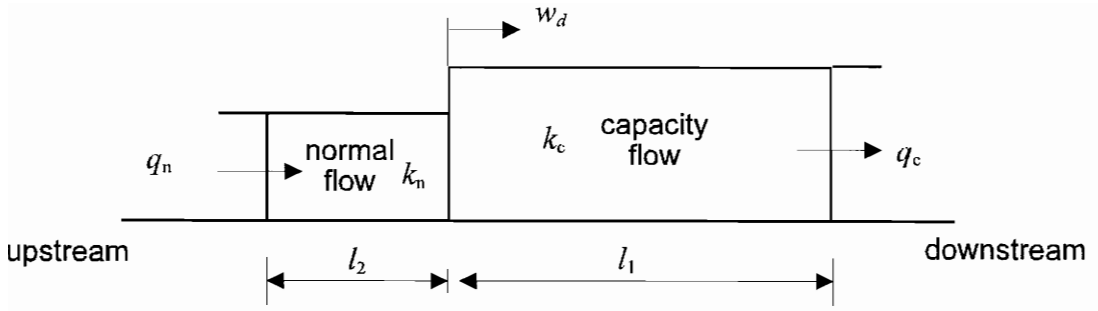
(a) traffic condition at time $t\text{-}\Delta t$
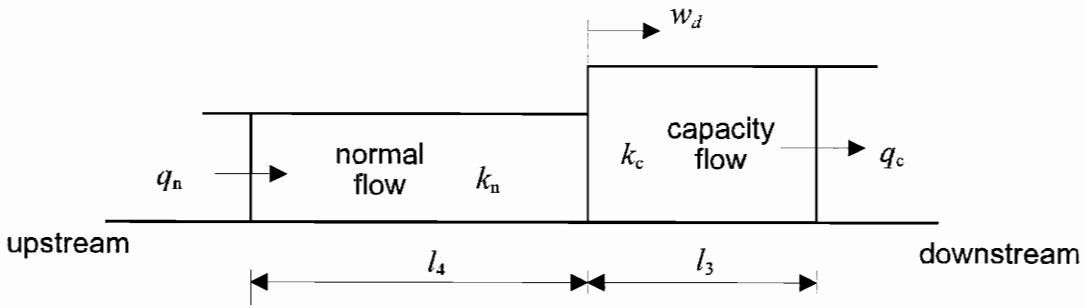
(b) traffic condition at time t

Figure 4.3   Retrogression of a congestion domain

4-3 Delay Diagrams

Using the derivation of progression and retrogression rates, a delay diagram has been developed. It is similar to the deterministic demand-capacity diagram and satisfies the theoretical requirements of the dynamic mechanisms of a congestion domain. The delay diagram is accompanied with a familiar time-space diagram of the domain. Here, two delay diagrams are presented: one for delay analysis due to temporal surge in demand and the other for delay due to transient reduction in capacity. Both the cases are frequently faced and have different dynamic mechanisms of delay.

In case of temporal surge in demand a delay diagram in Figure 4.4 and a time-space diagram in Figure 4.5 can be applied. Congestion starts to build up as soon as the demand exceeds the capacity and continues to grow until time T_1 at which the demand becomes less than the capacity. A queue of delayed vehicles disappears at time T_2. The area enclosed by the modified demand curve and the capacity curve represents the total delay in vehicle-hours. The traffic states within the congestion domain remain homogeneous throughout the duration of delay as jammed conditions. The duration of delay T_2 is much longer than the one predicted by deterministic queuing analysis. A time-space diagram of delay domain is illustrated in Figure 4.5. Here, two moving shock waves can be observed: a backward forming shock wave w_u and a forward recovery shock wave w_d:

$$w_u = \frac{q_c - q_d}{k_c - k_d} < 0 \tag{4.9}$$

where q_c and k_c are for capacity flow and q_d and k_d are for demand flow which exceeds the capacity.

$$w_d = \frac{q_c - q_d'}{k_c - k_d'} > 0 \tag{4.10}$$

where q_d' and k_d' are for reduced demand flow after peak-hour demand. Using the geometry, a maximum queue length l_{max} and duration of delay T_2 can be written respectively as:

$$l_{max} = -w_u T_1 \tag{4.11}$$

$$T_2 = T_1 + \frac{l_{max}}{w_d} \tag{4.12}$$

The total delay TD is estimated as:

$$TD = \int_0^{T_2} n(t)\ dt$$

$$= \frac{T_2 Q_1 - T_1 Q_2}{2} \tag{4.13}$$

where

$$Q_1 = (q_d - q_c)T_1 + (-w_u k_n)T_1 \tag{4.14}$$

$$Q_2 = q_c T_2. \tag{4.15}$$

In case of delay caused by an incident, the delay diagram shown in Figure 4.6 can be utilized. The capacity of the roadway is reduced due to the incident and a queue

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develops along with the backward forming shock wave w_u. At time T_1 the incident is cleared completely and capacity of the road recovers to its full capacity. At the same time, a backward recovery shock wave w'_u develops and travels against the direction of traffic. It prevents the incident domain from progressing further by intersecting w_u at time T_2. The queue reaches its maximum length at that time. The traffic flow states within the domain of incident are bisected by w'_u as illustrated in Figure 4.7. Now, the domain of the incident retrogresses with a rate of forward recovery shock wave w_d and the queue finally disappears at time T_3. The maximum travel time can occur either at time T_1 or T_2 depending on the interacting traffic situations. Here, two backward shock waves and one forward shock wave are observed:

$$w_u = \frac{q_d - q'_c}{k_d - k_j} < 0 \tag{4.16}$$

where q'_c and k_j are for reduced capacity flow and q_d and k_d are for demand flow which exceeds the reduced capacity.

$$w'_u = \frac{q_c - q'_c}{k_c - k_j} < 0 \tag{4.17}$$

where q_c and k_c are for capacity flow.

$$w_d = \frac{q_c - q_d}{k_c - k_d} > 0 \tag{4.18}$$

where q_c and k_c are for capacity flow and q_d and k_d are for demand flow.

Using geometry, the maximum queue length l_{max}, the time T_2 that queue reaches maximum and the duration of delay T_3 can be written respectively as:

$$l_{max} = \frac{w_u w_u'}{(w_u - w_u')} T_1 \tag{4.19}$$

$$T_2 = \frac{w_u'}{w_u' - w_u} T_1 \tag{4.20}$$

$$T_3 = \frac{w_u'(w_u - w_d)}{w_d(w_u - w_u')} T_1 \tag{4.21}$$

The total delay TD is estimated as:

$$TD = \int_0^{T_3} n(t)\ dt$$

$$= \frac{T_1 Q_3 + T_3 Q_2 - T_3 Q_1 - T_2 Q_3}{2} \tag{4.22}$$

where

$$Q_1 = q_c' T_1 \tag{4.23}$$

$$Q_2 = l_{max} k_c + (T_2 - T_1)q_c + Q_1 \tag{4.24}$$

$$Q_1 = (T_3 - T_1)q_c + Q_1. \tag{4.25}$$

The use of a delay diagram and a time-space diagram helps understand the dynamic mechanism of traffic flow during congestion period. Time-dependent information about the number of vehicles in a queue, the time at which the maximum queue occurs, the maximum travel time, etc. can be easily visualized.

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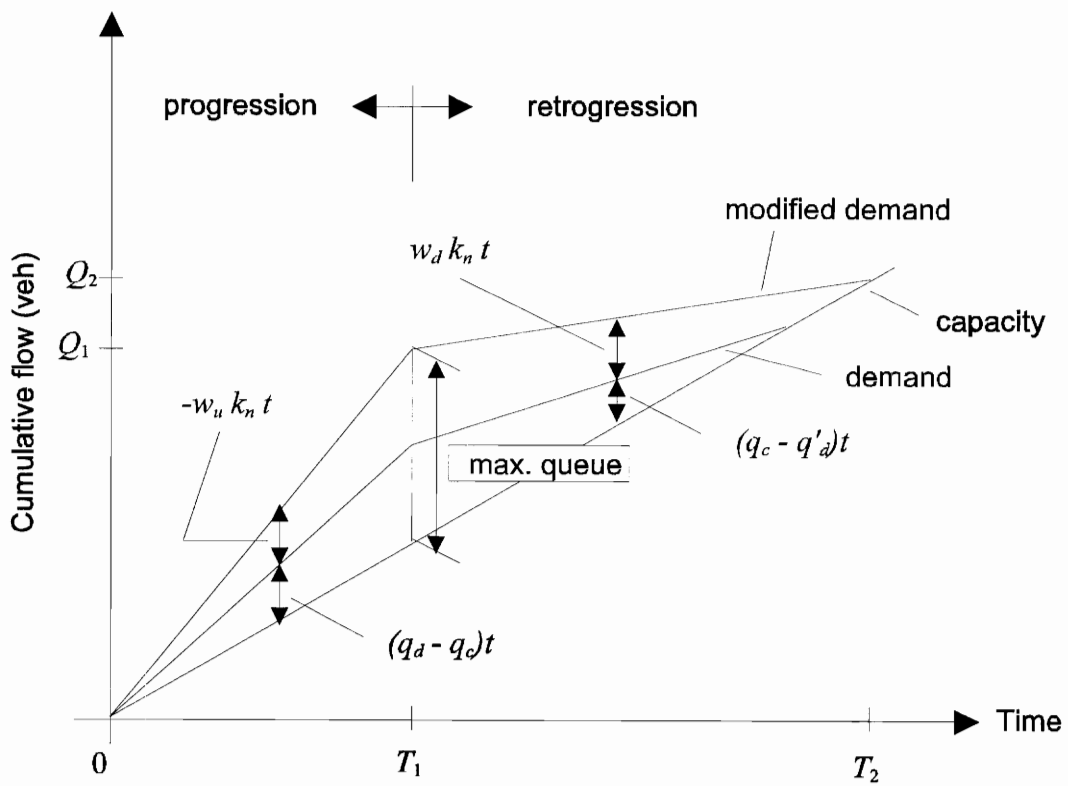
Figure 4.4   Proposed delay diagram in case of temporal surge in demand
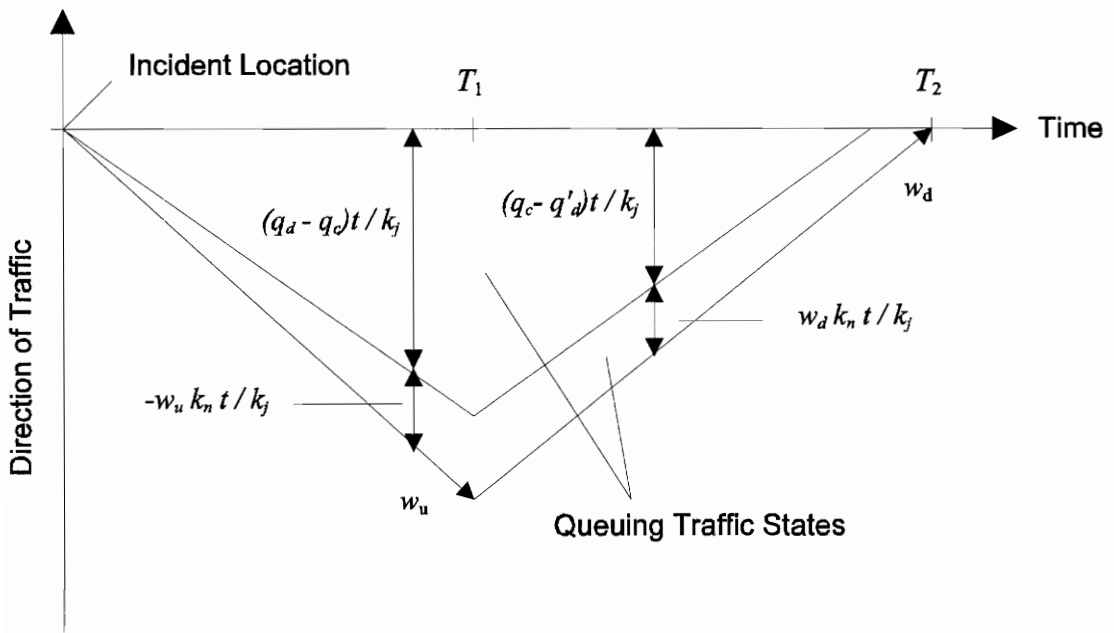
Figure 4.5   Time-space diagram of a delay domain in case of temporal surge
in demand
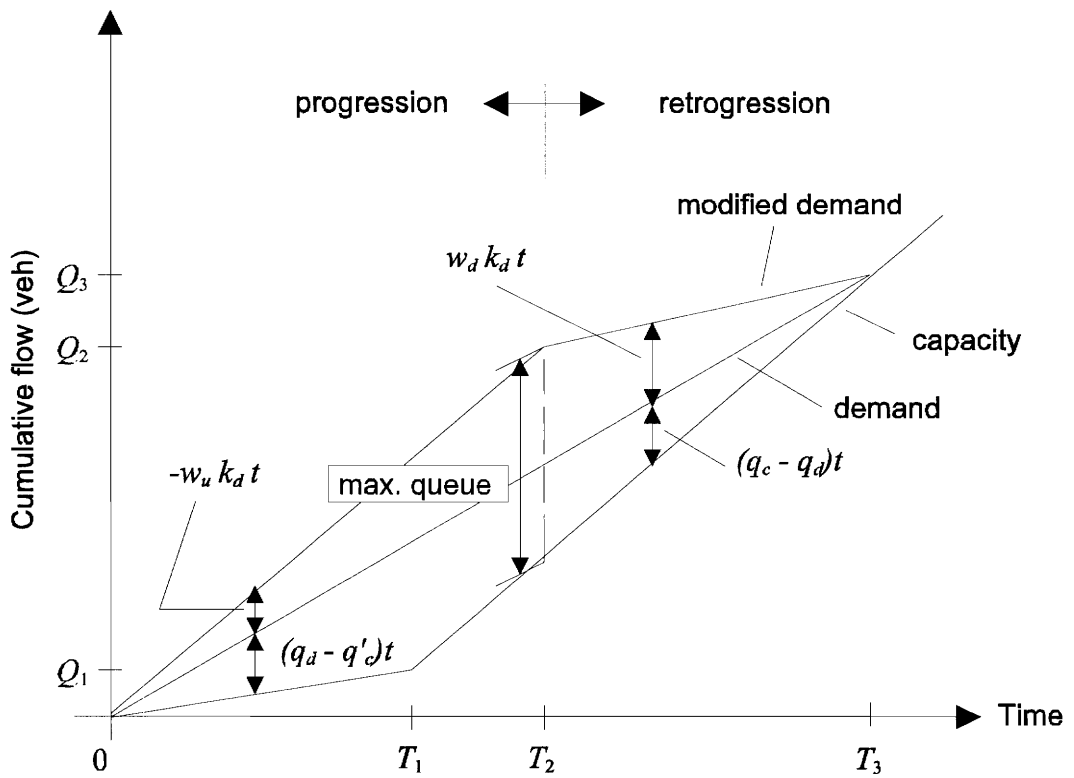
Figure 4.6   Proposed delay diagram in case of temporal reduction in capacity
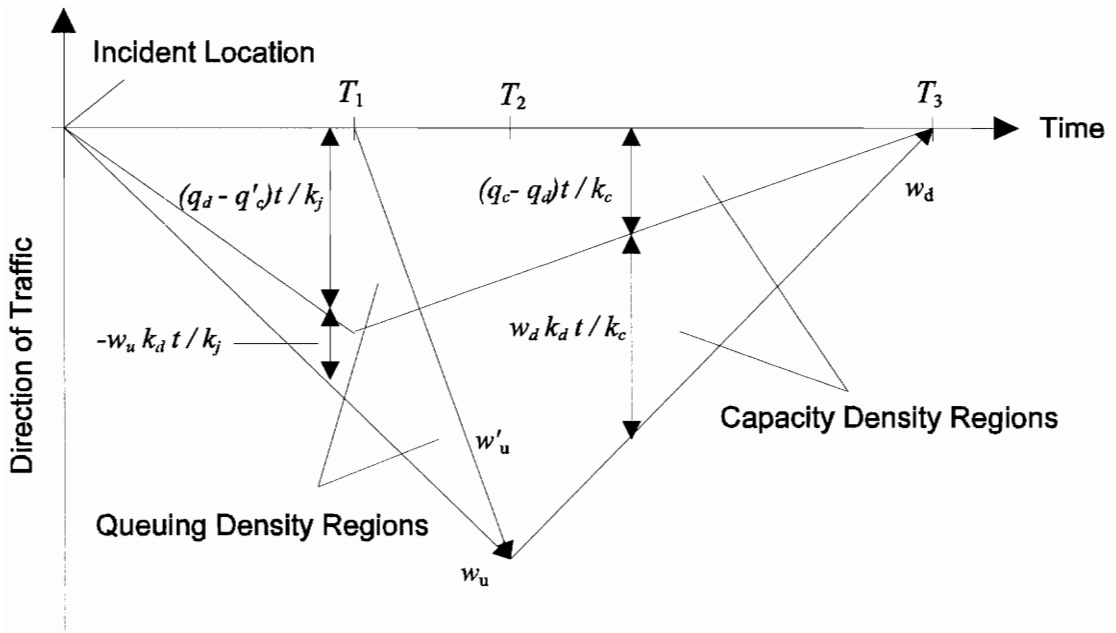
Figure 4.7   Time-space diagram of a delay domain in case of temporal reduction in capacity

Diversion has been accepted as an effective method that can reduce delays during incidents. The process of diversion involves the selection of the alternate routes to bypass the link on which the incident occurs. This is currently done off-line and is not responsive to each incident case. The volumes on these preselected routes on that particular day are also ignored. The widely accepted deterministic queuing technique significantly underestimates the magnitude of delays caused by incidents. Considering the volume of traffic flow on the freeways, the current practice leads to a considerable discrepancy. An improved methodology is needed to determine in real-time whether diversion is required in response to an incident. Also required is the volume on the freeway that needs to be diverted. The development of a methodology for link and incident specific freeway diversion is presented in this chapter.

5-1 Methodological Basis

A diversion strategy developed in this study follows the principle of a user-optimized equilibrium which requires that all the feasible paths between an origin and a destination have the same travel time (Wardrop 1952). Here, two travel times are considered: freeway travel time and arterial travel time. Freeway travel time is defined as the time

taken to travel along the freeway from one interchange (origin) to other interchange (destination) and it includes the delay times caused by incidents. On the other hand, the arterial travel time is the time taken to travel along the alternative route from the interchange (origin) to the other interchange (destination). A diversion is required if the expected freeway travel time under incidents is greater than the arterial travel time. The volume that has to be diverted is determined based on the equilibrium travel times between the routes. Figure 5.1 illustrates the block diagram of a link and incident specific freeway diversion methodology.
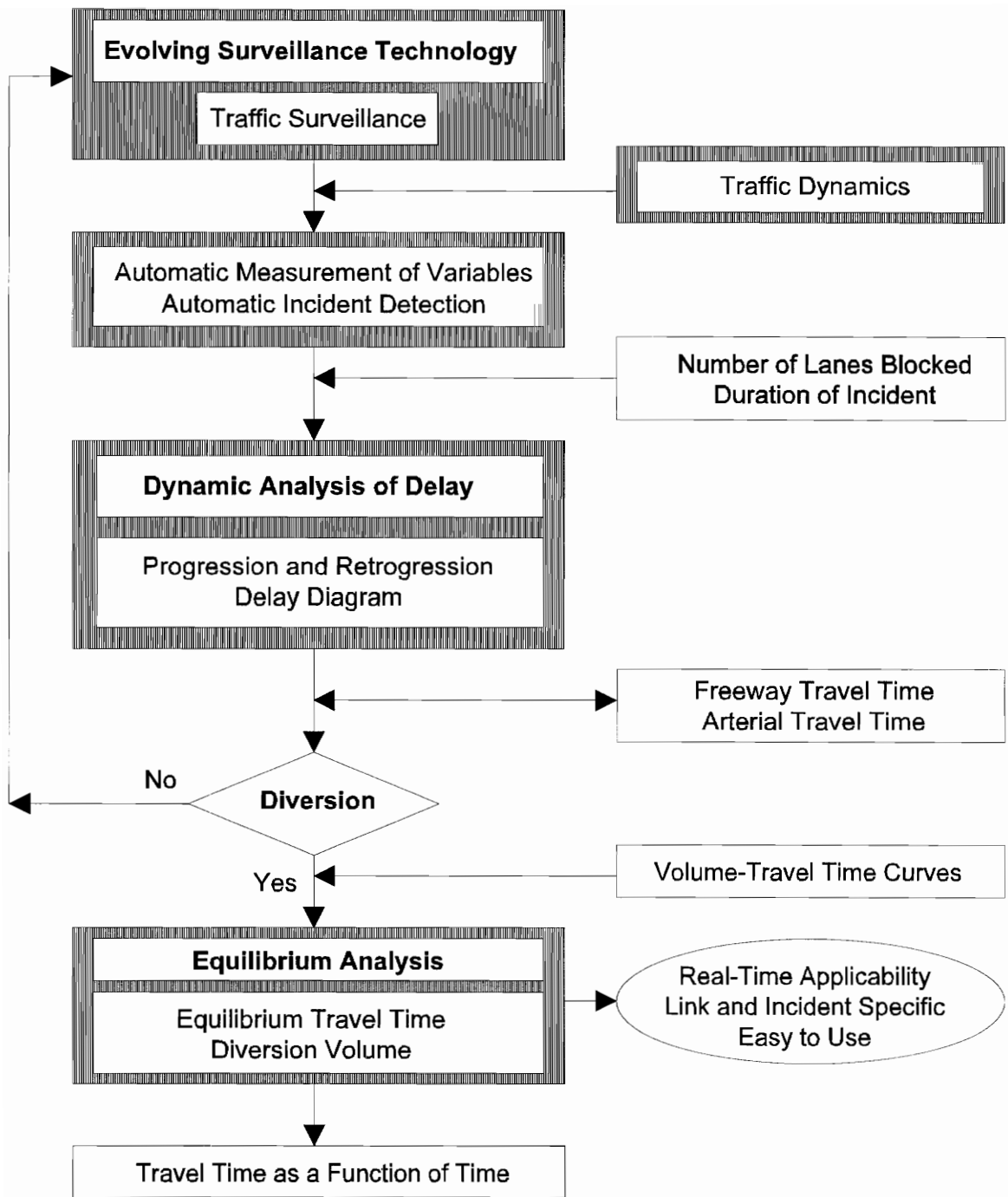
Figure 5.1   Block diagram of the proposed methodology for link and incident specific
               freeway diversion in real-time

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5-2 Development of Freeway Diversion Strategy

As illustrated in Figure 5.1, the development of link and incident specific freeway diversion strategy involves the following sequential steps after the incident is detected and verified:

- estimation of total delay,

- estimation of freeway and alternative route travel times,

- determination of the need for diversion; and

- equilibrium analysis if necessary.

Each step is further discussed in the next sections.

5-2-1 Estimation of total delay

The enclosed area between the two curves, the capacity curve and the modified demand curve, represents total delays (see Figure 5.2). The shape of the area is different from that of demand-capacity diagram because of the dynamic characteristics of traffic flow mechanisms during incidents. As discussed in Section 4.3 during the development of the delay diagram, the total delay TD can be estimated as:

$$TD = \frac{1}{2} \left(T_1 Q_3 + T_3 Q_2 - T_3 Q_1 - T_2 Q_3 \right)$$

(5.1)

where T_1 = duration of an incident; T_2 = the time elapsed from the incident occurrence to a time that queue reaches maximum length; T_3 = duration of delay caused by the incident; and Q_i = cumulative flows associated with time T_i.

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The total delay given in Eq. (5.1) can be estimated analytically if the following two pieces of information are available: the number of lanes blocked and duration of incident. The fraction of section capacity available under incidents, known as incident flow q_c', can be analytically related to the capacity of the roadway using the number of lanes blocked (Lindley 1987). For example, as tabulated in Table 5.1, a six-lane freeway with three lanes blocked has an incident flow equivalent to 25% of total capacity until the incident is cleared, i.e., $q_c' = 0.25q_c$. The disablement of shoulder has less effect on the available capacity of freeway. Meanwhile, the duration of delay can also be predicted based on historical incident cases. The application of knowledge-based expert systems to the prediction has been explored (Ritchie and Prosser 1990). Therefore, it is assumed that these two pieces of information are available in this study.

Table 5.1 Fraction of freeway section capacity available under incidents (source: Lindley 1987)

Lanes[1]	Shoulder		Lanes blocked		
	disablement	accident	one	two	three
2	0.95	0.81	0.35	0.00	0.00
3	0.99	0.83	0.49	0.17	0.00
4	0.99	0.85	0.58	0.25	0.13
5	0.99	0.87	0.65	0.40	0.20
6	0.99	0.89	0.71	0.50	0.25
7	0.99	0.91	0.75	0.57	0.36
8	0.99	0.93	0.78	0.63	0.41

[1]per each direction

5-2-2 Estimation of travel times

After quantifying the total delay caused by the incident, estimated freeway and arterial travel times can be obtained in the following manner. Let L and $l(t)$ be the length of a freeway link and a queue formed at time t, respectively. Then, freeway travel time, $TT^f(t)$, is the sum of the following two components:

$$TT^f(t) = TT_n^f(t) + TT_q^f(t) \tag{5.2}$$

where

$TT_n^f(t)$ = travel time taken to travel the distance $L - l(t)$, and

$TT_q^f(t)$ = travel time taken to travel the distance $l(t)$ while in the queue.

Here, the expected average of $TT_q^f(t)$ under deterministic queuing conditions is equal to the ratio of the total delay to the total number of vehicles in the queue:

$$TT_q^f = \frac{1}{2Q_3}\left(T_1Q_3 + T_3Q_2 - T_3Q_1 - T_2Q_3\right) \tag{5.3}$$

and the expected average queue length is:

$$l = \frac{l_{max}}{2} \tag{5.4}$$

Meanwhile, the other components, $TT_n^f(t)$ and $TT^a(t)$, are not subjected to queuing conditions. These travel times can be obtained using link capacity functions like Davidson function which has a level-of-service factor in its formula determined

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empirically from traffic data (see Section 2.2). Figure 5.3 shows the various shapes of Davidson function for different j factors with the famous BPR function. It is recommended that the whole link length be considered to estimate $TT_n^f(t)$. The reason behind this conservative approach is to account for possible human behaviors observed near the incident zone. For instance, the driver tends to slow down reacting to incident conditions ahead. This is known as anticipation.

Thus, the expected freeway travel time and arterial travel time are respectively:

$$TT^f = TT_f^f \frac{1-(1-j^f)y^f}{1-y^f} + \frac{1}{2Q_3}\left(T_1Q_3 + T_3Q_2 - T_3Q_1 - T_2Q_3\right) \tag{5.5}$$

$$TT^a = TT_f^a \frac{1-(1-j^a)y^a}{1-y^a} \tag{5.6}$$

where

$TT_f^f = tt_f^f L$, the free flow travel time on the freeway,

TT_f^a = the free flow travel time on arterial,

j = a level-of-service factor, and

y = traffic intensity, the ratio of traffic volume to the capacity of the roadway.

A diversion strategy will be feasible for reducing current freeway congestion if the expected freeway travel time given by Eq. (5.5) is greater than arterial travel time given by Eq. (5.6).

5-2-3 Equilibrium analysis

There are two basic principles of network equilibrium analysis (Wardrop 1952):

- User-optimized approach in which the drivers choose their routes independently in their own best interests in response to the traffic conditions resulting from the choice of others; and

- System-optimized approach in which the drivers cooperate with the system in their choice of routes so as to maximize the capacity of the roadway.

The principle of a user-optimized equilibrium requires that all feasible paths between an origin and a destination have the same travel time assuming that all the drivers have an identical perception of travel cost. This principle can be implemented to the traditional volume-travel time curves. In Figure 5.4, the following curves are plotted:

- arterial travel time (TT^a) curve as defined in Eq. (5.6),

- freeway travel time (TT^f) curve under normal conditions which is obtained without considering delay caused by the incident,

- freeway travel time $(TT_n^f + TT_q^f)$ curve under incident conditions which is obtained by superimposing the expected delay (TT_q^f) onto freeway travel time (TT^f),

- travel time $(TT^a + TT_n^f + TT_q^f)$ curve of the combined facilities under incident conditions which is obtained by adding volumes of two travel time curves, $TT^a(V)$ and $TT_n^f(V) + TT_q^f(V)$, for every value of travel time.

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The travel time curve of the combined facilities under normal conditions, $TT^a(V) + TT^f(V)$, can be obtained in a similar way.

Once these travel time-volume curves are plotted, the user-optimized equilibrium travel time TT_e can be easily determined by reading the travel time of the curve $TT^a(V) + TT_n^f(V) + TT_q^f$ for the current demand $V^a + V^f$. The user-optimized travel volume on the alternative route is the volume of the travel time curve $TT^a(V)$ corresponding to TT_e. Similarly, freeway volume is the volume of $TT_n^f(V) + TT_q^f(V)$ associated with TT_e. Diversion volume, which will be assigned from the freeway to the alternate route, is the difference between the current freeway volume and the newly estimated freeway volume $V^f(TT_e)$.

$$T_3 \, \theta_3 - \frac{1}{2} (T_3 - T_1)(\theta_3 - \theta_1)$$
$$- (T_3 - T_1) \theta_2 - \frac{1}{2} T_1 \theta_1$$
$$- \frac{1}{2} T_2 \theta_2 - T_2 (\theta_3 - \theta_2)$$
$$- \frac{1}{2} (T_3 - T_2)(\theta_3 - \theta_2)$$
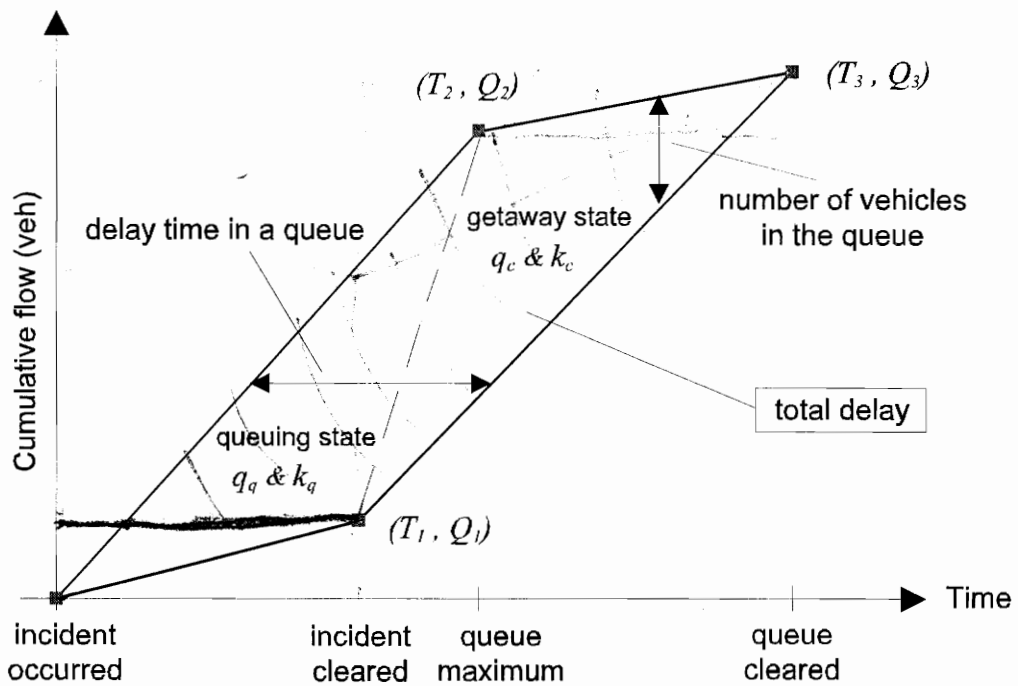
Figure 5.2   A delay diagram for delay analysis caused by an incident

$$\frac{1}{2} T_3 Q_3 - \frac{1}{2} T_1 \theta_1 - (T_3 - T_1) \theta_1$$

Normalized Travel Times
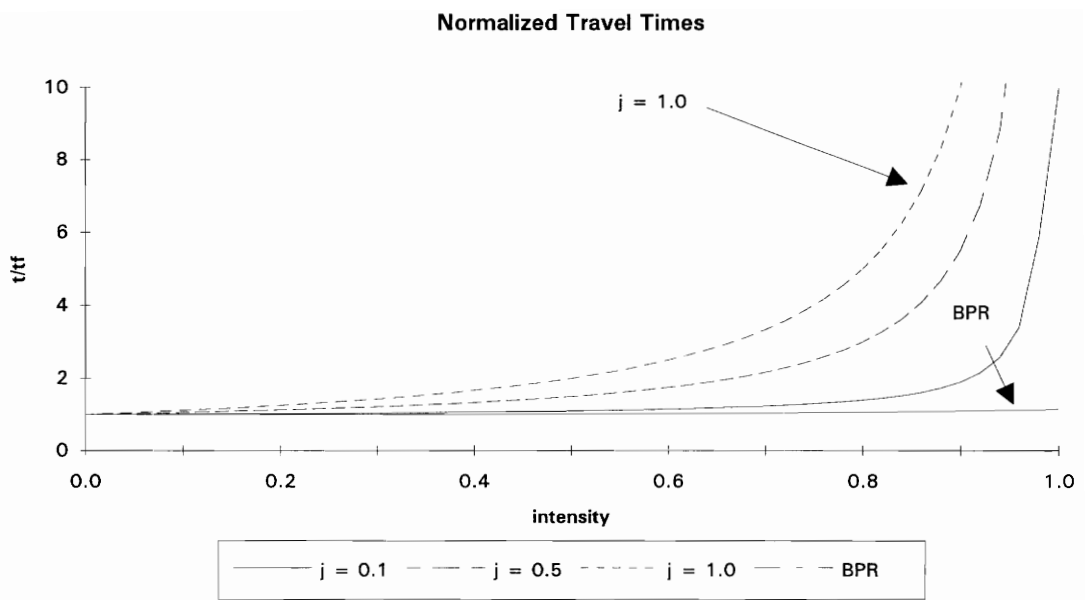
Figure 5.3   Davidson travel time curves for various level of service factors and BPR function
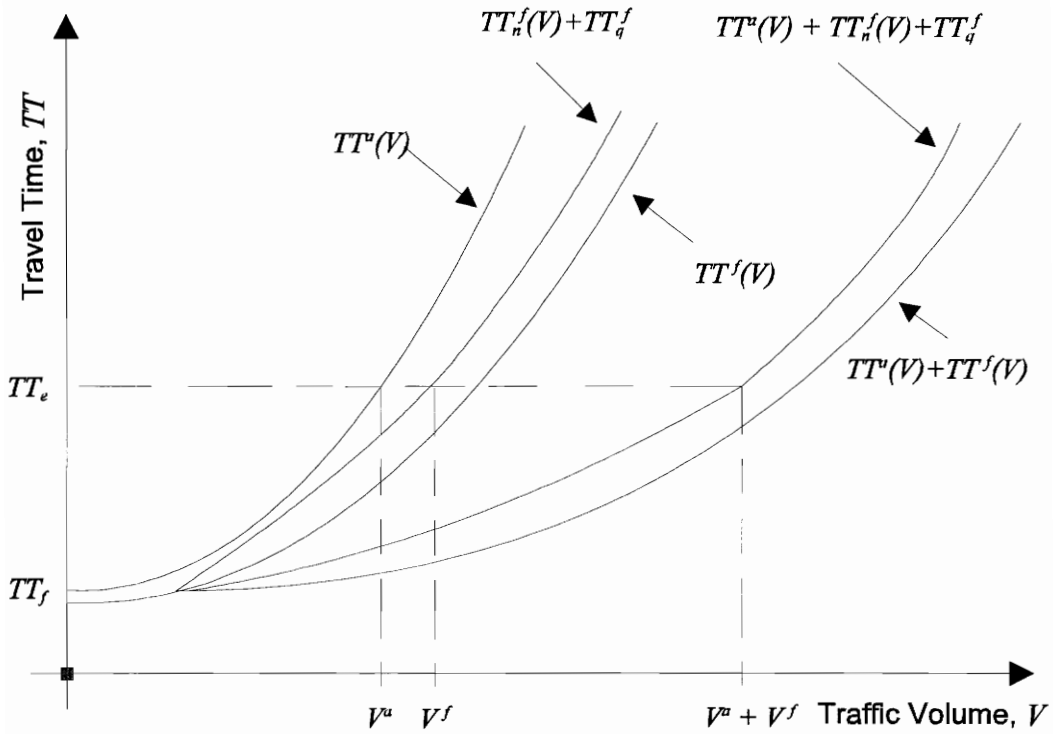
Figure 5.4   Determination of equilibrium travel time and diversion volumes
using volume-travel time curves

5-3 Example Problem

Consider an idealized network which consists of a freeway and alternate route as shown in Figure 5.6. The freeway has three lanes in each direction while the alternate route is a four-lane (two in each direction) urban arterial. The distances between the interchanges is 15 miles on the freeway and 12 miles on the alternate route. The lane capacities and free flow speeds on the alternate routes are three-fourths of those in the freeway. The level of service on the two facilities are those typical of a freeway and arterial. Other parameter values and assumptions are as follows:

- freeway capacity flow, q_c = 6000 veh/hr/direction and k_c = 60 veh/mi; $\Rightarrow$ u_{jam} ~ 107.85 ?

- freeway queuing density, k_j = 102.5 veh/mi;

  ? ?

- freeway free flow speed, u_f = 80 mph;

- level of service factor, j^f = 0.15 and j^a = 0.35;

- freeway traffic demand, q_d = 5000 veh/hr and k_d = 38.5 veh/mi; and $\Rightarrow$ u_{jam} = 33,000

- arterial traffic demand, q_d = 1500 veh/hr/direction. Also, u_{jam} = $k_c \times 2$ = 6 × 2 ...

An accident has occurred just upstream of Interchange 2; three vehicles are involved in the accident and the middle lane is currently blocked. After querying the historical incident database, the duration of incident, T_1, is estimated to be about 30 min. A prompt decision has to be made whether diversion is necessary and, if yes, what volume should be diverted to the alternate route at Interchange 1.

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A computer program has been developed using MATLAB to demonstrate the real-time applicability of the proposed methodology. The general purpose program MATLAB has been selected over other program languages like C or FORTRAN simply because it has much more built-in functions especially for matrix computations and graphics. The source code of the program is provided in Appendix I. For the purpose of illustration, simulation results are presented in a question-answer manner:

1) Shock wave velocity

$$w_u = \frac{\Delta q}{\Delta k} = \frac{(5000 - 0.49 \times 6000)/3}{38.5 - 102.5} = -10.729 \text{ (mph)}$$

$$w'_u = \frac{\Delta q}{\Delta k} = \frac{(0.49 \times 6000 - 6000)/3}{102.5 - 60.0} = -24.0 \text{ (mph)}$$

$$w_d = \frac{\Delta q}{\Delta k} = \frac{(6000 - 5000)/3}{60.0 - 38.5} = 15.504 \text{ (mph)}$$

2) Progression and retrogression rates

$$\frac{dn}{dt} = (q_n - q_q) - w_u k_n = (5000 - 0.49 \times 6000) - (-10.729)(38.5)(3) = 3299.2 \text{ (vph)}$$

$$-\frac{dn}{dt} = (q_c - q_n) + w_d k_n = (6000 - 5000) + (15.504)(38.5)(3) = 2790.7 \text{ (vph)}$$

3) Total delay of veh-hours

$$T_1 = 0.5 \text{ (given)}$$

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$$T_2 = \frac{w'_u}{w'_u - w_u} T_1 = \frac{(-24.0)}{(-24.0 + 10.729)}(0.5) = 0.904 \ \text{(hr)}$$

$$T_3 = \frac{w'_u(w_u - w_d)}{w_d(w_u - w'_u)} T_1 = \frac{(-24.0)(-10.729 - 15.504)}{(15.504)(-10.729 + 24.0)}(0.5) = 1.530 \ \text{(hr)}$$

$$Q_1 = q'_c T_1 = (0.49 \times 6000)(0.5) = 1470 \ \text{(veh)}$$

$$l_{max} = \frac{w_u w'_u}{(w_u - w'_u)} T_1 = \frac{(-10.729)(-24.0)}{(-10.729 + 24.0)}(0.5) = 9.701 \ \text{(miles)}$$

$$Q_2 = l_{max} k_c + (T_2 - T_1) q_c + Q_1$$

$$= (9.701)(60.0)(3) + (0.904 - 0.5)(6000) + 1470$$

$$= 5640.18 \ \text{(veh)}$$

$$Q_3 = (T_3 - T_1) q_c + Q_1 = (1.530 - 0.5)(6000) + 1470 = 7650 \ \text{(veh)}$$

$$TD = \frac{1}{2}(T_1 Q_3 + T_3 Q_2 - T_3 Q_1 - T_2 Q_3)$$

$$= \frac{(0.5)(7650) + (1.53)(5640.18) - (1.53)(1470) - (0.904)(7650)}{2}$$

$$= 1645.2 \ \text{(veh-hrs)}$$

4) Table of queue length and number of vehicles in a queue

	Time	Deterministic queuing		Proposed methodology	
	(hr)	n (veh)	l (mile)	n (veh)	l (mile)
$0.5T_1$	0.25	515	1.675	824.8	2.682
T_1	0.50	1030	3.350	1649.6	5.365
T_2	0.904	626	3.478	1746.2	9.699
$T_2 + .5(T_3 - T_2)$	1.217	313	1.739	872.7	4.853

These values have been computed as shown below:

$$(5000 - 0.49 \times 6000)(0.25) = 515$$

$$(5000 - 0.49 \times 6000)(0.50) = 1030$$

$$(5000)(0.904) - (6000)(0.904 - 0.50) - 1470 = 626$$

$$(5000)(1.217) - (6000)(1.217 - 0.50) - 1470 = 313$$

$$\frac{515}{(102.5)(3)} = 1.675$$

$$\frac{1030}{(102.5)(3)} = 3.350$$

$$\frac{626}{(60.0)(3)} = 3.478$$

$$\frac{313}{(60.0)(3)} = 1.739$$

$$(3299.2)(0.25) = 824.8$$

$$(3299.2)(0.50) = 1649.6$$

$$5640.18 - [1470 + (6000)(0.904 - 0.5)] = 1746.2$$

$$1746.2 - (2790.7)(1.217 - 0.904) = 872.69$$

$$(10.729)(0.25) = 2.682$$

$$(10.729)(0.50) = 5.365$$

$$(10.729)(0.904) = 9.699$$

$$(15.504)(1.530 - 1.217) = 4.853$$

5) Freeway and arterial times

$$TT_f^f = \frac{15}{80}(60) = 11.25 \text{ (min)}$$

$$y^f = \frac{5000}{6000} = 0.833$$

$$TT^f = (11.25)\frac{1-(1-0.15)(0.833)}{(1-0.833)} + \frac{1645.2}{7650}(60) = 19.667 + 12.904 = 32.571 \text{ (min)}$$

$$TT_f^a = \frac{12}{(0.75 \times 80)}(60) = 12.0 \text{ (min)}$$

$$y^f = \frac{1500}{(0.75 \times 2000 \times 2)} = 0.50$$

$$TT^a = (12.0)\frac{1-(1-0.35)(0.50)}{(1-0.50)} = 16.20 \text{ (min)}$$

Diversion is required since $TT^f > TT^a$.

6) Volume-travel time curves

Volume-Travel Time Curves
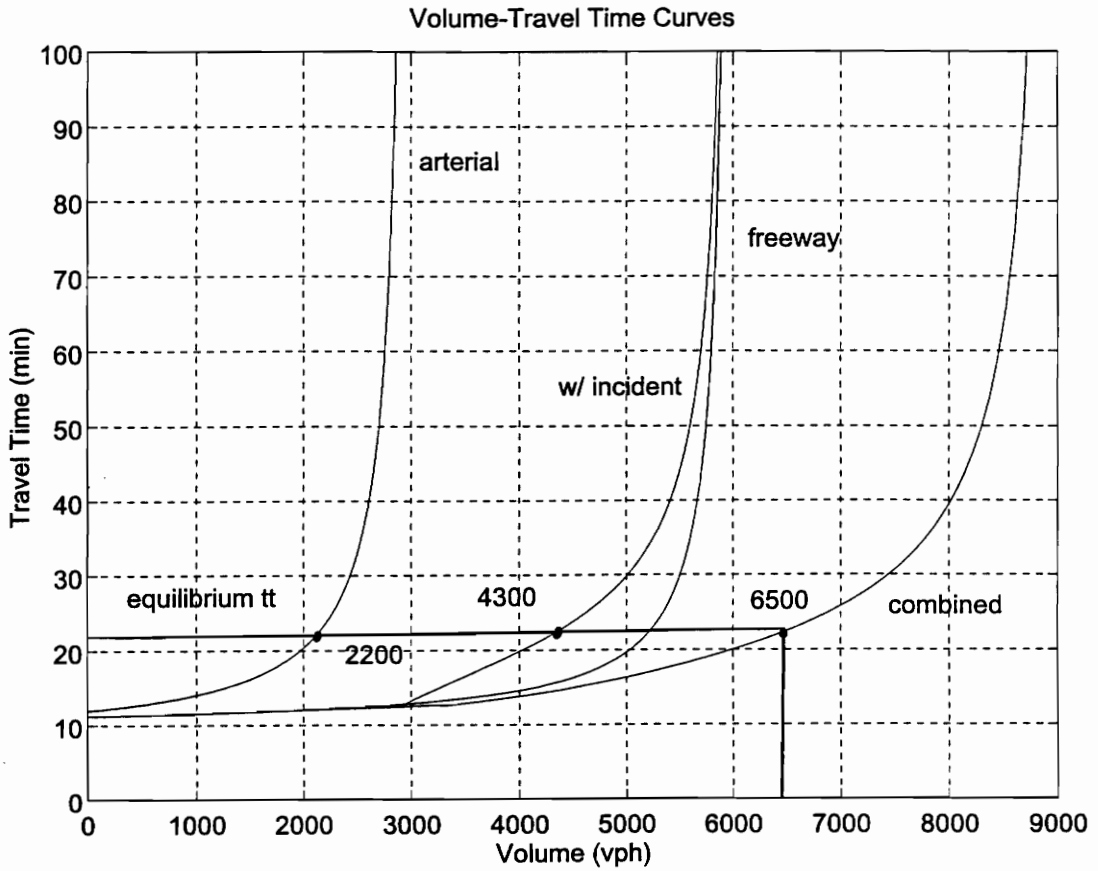
Figure 5.5   Example solution: equilibrium travel time and diversion volume

7) Equilibrium analysis

	Freeway	Alternative
Before diversion		
Traffic volume (vph)	5000	1500
Travel time (min.)	32.57	16.20
After diversion		
Traffic volume (vph)	4300	2200
Travel time (min.)	23.55	23.55

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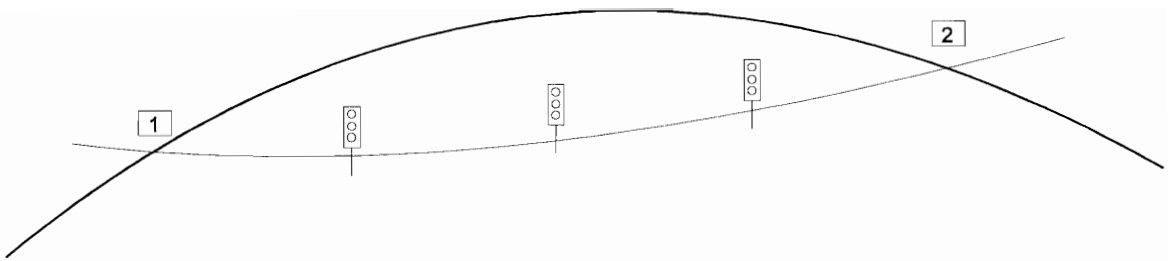
Figure 5.6   Example network: a freeway and an arterial with signalized intersections

CHAPTER VI

CONCLUSION

6-1 Summary of the Research

This research presents the development of methodologies for applying traffic flow theories to various ITS categories through the utilization of evolving surveillance technologies. This integration of theory, measurement and application has been overlooked since the advent of ITS because of the number of disciplines involved. In this context, the following illustrative methodologies are selected, developed and presented:

- a methodology for automatic measurement of major spatial traffic variables for the present and the future implementation of various ITS functional areas in general; and

- a methodology for real-time link and incident specific freeway diversion in conjunction with freeway incident management in particular.

The first methodology includes the development of a dynamic flow model based on stochastic queuing theory and the principle of conservation of vehicles. It is shown that the counting of vehicle arrivals and departures is a stochastic random process. The characteristics of the process are then applied to a freeway traffic surveillance. Meanwhile, it is found that the original formulation of the equation of conservation of vehicles is not fully able to describe the changes in traffic incurred during the time

interval. The new form of the equation is derived based on the principle. The new form of the equation has two additional terms of flows on the left-hand side and these are related to the change in the density which occurred during the period on the right-hand side. This equation is self-explanatory since no assumptions are made during its derivation. It is potentially applicable to automatic incident detection. Another important application lies in validating computer simulation results or observed data for their consistency and accuracy.

An inductive modeling approach adapted here utilizes geometric interpretations of cumulative arrival-departure diagrams which have been drawn directly from surveillance data. The advantages of this model are real-time applicability and transportability as well as ease of use. The derivation of the model is conceptually divided into two categories depending on the number of vehicles entering as well as exiting during the aggregation interval, i.e., under normal flows and under congested flows. The relationships between the travel time function and independent variables are found mathematically satisfactory. The analytical expressions of the model satisfy traffic dynamics where the new form of the equation of conservation of vehicles has been derived. Exponential averaging is introduced to increase stability of the time-series estimation of travel times. This numerical technique favors the most recent estimate by manipulating respective weight factors. The numerical study includes:

- The original formulation of conservation of vehicles (Model 1)
- The new form of the equation of conservation of vehicles (Model 2)

- The dynamic traffic flow model with exponential averaging (Model 3) and without exponential averaging (Model 4)

For the evaluation of quantitative effectiveness of the model, the empirical data measured at 30-second intervals is used. The analysis results show that the new formulation of conservation of equation (Model 2) is much better than the original equation (Model 1). Also, as expected, the analyses while using only the current estimate (Model 4) give better results compared to the analyses while using time-series estimation (Model 3). This shows that there is a trade-off between the precision and the stability of the time-series estimation in selecting the smoothing time. These results are also compared to the analysis of other state-of-the-art continuum models reported recently. Since different data sets were utilized for the two studies, no absolute comparisons can be made. However, the comparison of MAE(%) clearly shows that the inductive model developed in this research (Model 4) provides much better accuracy than the state-of-the-art continuum models. Also, it has real-time applicability, transportability and the ease of use.

The methodology is then applied to freeway diversion in real-time in conjunction with freeway incident management which has been accepted as an effective method that can reduce delays during incidents. The principle of conservation of vehicles is applied to the conceptual development of progression and retrogression of congestion domain. The analytical expressions show the additional effect due to the dynamic mechanisms of progression and retrogression in addition to the differences between the demand and

the reduced capacity due to the incident. It helps to clear the decade-long uncertainty in the theoretical differences between the two applicable methods; deterministic queuing analysis and shock wave analysis. It is found that deterministic queuing analysis significantly underestimates the overall magnitude of congestion in terms of total delay, maximum queue length, etc. The link and incident specific diversion methodology is achieved by using a delay diagram and volume-travel time curves which can be plotted per link per incident. The use of such graphic aids makes problem solving much easier and clearer. The principle of a user-optimized equilibrium is selected in determining an equilibrium travel time and implemented to the volume-travel time curves. For illustration, an example problem is selected and solved numerically. The development of a computer program using MATLAB for freeway diversion concludes this research.

6-2 Traffic Dynamics

Traditional deterministic traffic models have not been very successful in simulating the observed dynamic characteristics of traffic. A possible reason behind this is the high uncertainty in the behavior of traffic which makes modeling difficult. For example, human behaviors such as anticipation (the effect of drivers reacting to downstream traffic conditions) and relaxation (the effect of drivers adjusting to the equilibrium traffic conditions) are difficult to model adequately especially in real-time. As a result, continuum models which attempt to describe the acceleration of the traffic stream in terms of the aforementioned human factors are deductive in nature and require a great

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deal of calibration effort. Without improving the existing knowledge in traffic dynamics and reforming existing methods, technology-oriented transportation systems such as ITS can not succeed.

The proposed inductive method has real-time applicability, transportability and the ease of use. In conclusion, the integration of traffic theories, surveillance technologies and ITS is the key in solving traffic problems we face today and will face in the future.

6-3 Topics for Further Research

This research concentrates on refining the existing knowledge in traffic dynamics and integrating this knowledge with surveillance systems. While the proposed inductive dynamic model is superior to traditional deductive models, it still possesses undesirable properties. It is formulated in such a way that densities are obtained using cumulative flow measurements and then used for estimating velocities and travel times. Accuracy of the estimation can be constantly degraded since small errors in flow measurements cumulatively affect the current estimation. (A volume adjustment factor has been therefore used in Section 3.3). The proposed methodology works under a controlled environment like computer simulation. It is necessary to have a feedback routine for calibrating the error in flow measurement under real world environment and real-time applications.

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The second methodology presented in this research is for freeway diversion in real-time. It includes the conceptual development of dynamic traffic mechanisms during incidents, progression and retrogression of congested domain. Although the analytical derivation has theoretical importance, it is still based on simplified traffic conditions. Shock wave velocity is estimated under the assumption of constant traffic states. In reality, traffic fluctuates over time. It would have been advantageous to test the delay estimation function using empirical traffic data measured during incident conditions. (In fact, such data is available on the Internet. The Freeway Service Patrol Project on I-880 in Hayward, California contains around 1200 incident data. However, it does not have ramp volume measurements and the accuracy of flow measurements is questionable).

As the vehicle-miles of travel increase, more incidents are likely to occur during peak commuting hours. It becomes important to detect incidents analytically rather than empirically. Therefore, the new form of the equation of conservation of vehicles, which has the potential of automatic incident detection, can be tested for real world situations.

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```
%========================================================================
%      tte.m - LINK TRAVEL TIME ESTIMATION
%
%      Estimates freeway link travel times directly from link flow
%      measurements. Last modification: Nov. 6, 1995
%========================================================================

echo off

%----------------------------------------------------------
%      define analysis variables
%----------------------------------------------------------

dt = 2.0;                          % analysis time interval in min.
duration = 60*4;                   % duration of analysis in min.
time = 0:dt:duration;              % clock time in min.
TIME = time/60+6;                  % time of day in hours
N = duration/dt+1;                 % number of analysis data points
alpha = 0.4;                       % exponential averaging


%----------------------------------------------------------
%      describe a study section
%----------------------------------------------------------

length = .69+.75;                  % link distances in km
lanes = 3;                         % number of lanes
vf = 105;                          % free flow speed (km/hr)


%----------------------------------------------------------
%      read data file
%----------------------------------------------------------

data_dt = 0.5;                     % data aggregation internal in min.
n = duration/data_dt;              % number of empirical data points
stations = 3;                      % number of nodes

fid = fopen ('0314.txt');
a = fscanf (fid,'%d %d %d %d %d %d %d %d %d %d %d %d %d %d %d %d %d %d
%d %d %d %d %d',inf);
fclose (fid);

for i=1:n

     for j=1:stations

q_data(i,j) = (a(69*(i-1)+11+(j-1)*3) + a(69*(i-1)+12+(j-1)*3) +
a(69*(i-1)+13+(j-1)*3))/lanes;
```

```
o_data(i,j) = (a(69*(i-1)+34+(j-1)*3) + a(69*(i-1)+35+(j-1)*3) +
a(69*(i-1)+36+(j-1)*3))/lanes;
v_data(i,j) = (a(69*(i-1)+57+(j-1)*3) + a(69*(i-1)+58+(j-1)*3) +
a(69*(i-1)+59+(j-1)*3))/lanes;

        end
end


step = dt/data_dt;
for i=2:N

        sum1 = 0; sum2 = 0; sum3 = 0;
        sum4 = 0; sum5 = 0; sum6 = 0;
        sum7 = 0; sum8 = 0; sum9 = 0;
        for j = 1:step

                sum1 = q_data((i-1)*step-j+1,1) + sum1;
                sum2 = q_data((i-1)*step-j+1,2) + sum2;
                sum3 = q_data((i-1)*step-j+1,3) + sum3;

                sum4 = o_data((i-1)*step-j+1,1) + sum4;
                sum5 = o_data((i-1)*step-j+1,2) + sum5;
                sum6 = o_data((i-1)*step-j+1,3) + sum6;

                sum7 = v_data((i-1)*step-j+1,1) + sum7;
                sum8 = v_data((i-1)*step-j+1,2) + sum8;
                sum9 = v_data((i-1)*step-j+1,3) + sum9;

        end

        q23(i) = sum1; q24(i) = sum2; q25(i) = sum3;
        o23(i) = sum4/step; o24(i) = sum5/step; o25(i) = sum6/step;
        v23(i) = sum7/step; v24(i) = sum8/step; v25(i) = sum9/step;

end


q23(1) = q23(2); q24(1) = q24(2); q25(1) = q25(2);
o23(1) = o23(2); o24(1) = o24(2); o25(1) = o25(2);
v23(1) = v23(2); v24(1) = v24(2); v25(1) = v25(2);



%-----------------------------------------------------------
%       volume correction factors
%-----------------------------------------------------------

for i=1:N
        if (i < (60/dt+1))
                q25(i) = q25(i) * 1.020;
        elseif (i < (2*60/dt+1))
                q25(i) = q25(i) * 1.015;
        elseif (i < (3*60/dt+1))
                q25(i) = q25(i) * 1.025;
        else
                q25(i) = q25(i) * 1.045;
```

```
        end
end


%-----------------------------------------------------------
%       get cumulative flows
%-----------------------------------------------------------

n(1) = 20;                          % initial condition
qq23(1) =   0;
qq25(1) = -n(1);

qq23 = q23;                         % dummy variables
qq25 = q25;                         % dummy variables

Q23 =   cumsum(qq23);               % cumulative flows
Q25 =   cumsum(qq25);               % cumulative flows


%-----------------------------------------------------------
%       calculate the number of vehicles traveling and density
%-----------------------------------------------------------

k(1) = n(1) / length;

        for i=2:N
n(i) = Q23(i) - Q25(i);             % number of vehicles on the link
k(i) = n(i) / length;               % density
        end


%-----------------------------------------------------------
%       analyze flow fluctuations
%-----------------------------------------------------------

        for i = 2:N
dq(i) = q23(i) - q25(i);            % differences between flow rates
dk(i) = k(i) - k(i-1);             % fluctuation in density
        end

dq(1) = dq(2); dk(2) = dk(1);


%-----------------------------------------------------------
%       get the number of vehicles entered and exited the link
%-----------------------------------------------------------

        for i= 2:N
m(i) = Q25(i) - Q23(i-1);           % congestion parameter
        end

m(1) = m(2) - 1;                    % initial condition


%-----------------------------------------------------------
%       travel time estimation
%-----------------------------------------------------------
```

```
v(1)  = (q23(1)+q25(1))/2*(60/dt)/k(1);   % initial conditions

      if (v(1) > vf)
v(1) = vf;
      end

tt(1) = length/v(1)*60;                    % initial travel time in min.

TT(1) = tt(1);                             % initial conditions
V(1) = length/TT(1)*60;

for i=2:N

          if (m(i) > 0)
      v(i) = 2*q23(i)*q25(i) / (q23(i)*k(i-1)+q25(i)*k(i)) * 60 / dt;

                if (v(i) > vf)
                    v(i) = vf;
                end
          else

      v(i) = 2*q25(i) / (k(i-1)+k(i)) * 60 / dt;

          end

      tt(i) = length/v(i)*60;              % travel time in min.


      TT(i) = TT(i-1) + alpha*(tt(i)-TT(i-1));
      V(i) = length/TT(i)*60;

end


%------------------------------------------------------------------
%      equilibrium flows
%------------------------------------------------------------------

ke(1) = k(1);
      for i=2:N
ke(i) = (k(i-1)+k(i))/2;                   % equilibrium density
      end

q25_est1(1) = q25(1);
qe_est1(1) = (q25_est1(1)+q23(1))/2;
      for i=2:N
q25_est1(i) = -dk(i)/dt*length + q23(i-1);  % estimated q25 flow
      end

q25_est2(1) = q25(1);
qe_est2(1) = (q25_est2(1)+q23(1))/2;
      for i=2:N
q25_est2(i) = -dk(i)/dt*length + q23(i);    % estimated q25 flow
      end

qe_est3 = ke .* V ./ (60/dt);              % equilibrium flow (veh/5-min)
```

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```
qe_est4 = ke .* v ./ (60/dt);              % equilibrium flow (veh/5-min)
qe_data = (q23+q25)./ 2;                    % empirical data (veh/5-min)


%-------------------------------------------------------------------
%      statistical error analysis
%-------------------------------------------------------------------
%      error1 = equation of conservation of vehicles
%      error2 = new equation of conservation of vehicles
%      error3 = analysis w/  exponential averaging
%      error4 = analysis w/o exponential averaging
%-------------------------------------------------------------------

A = abs(qe_est1 - qe_data);
B = A./q25;
sum_error1 = cumsum(A);
sum_error2 = cumsum(B);
sum_error1(1) = 0;
sum_error2(1) = 0;
mae(1) = sum_error1(N)/N*(60/dt);
maep(1) = sum_error2(N)/N*100;

C = abs(qe_est2 - qe_data);
D = C./q25;
sum_error3 = cumsum(C);
sum_error4 = cumsum(D);
sum_error3(1) = 0;
sum_error4(1) = 0;
mae(2) = sum_error3(N)/N*(60/dt);
maep(2) = sum_error4(N)/N*100;

E = abs(qe_est3 - qe_data);                 % estimation with averaging
F = E ./ qe_data;
sum_error5 = cumsum(E);
sum_error6 = cumsum(F);
sum_error5(1) = 0;
sum_error6(1) = 0;
mae(3) = sum_error5(N)/N*(60/dt);
maep(3) = sum_error6(N)/N*100;

G = abs(qe_est4 - qe_data);                 % estimation w/o averaging
H = G ./ qe_data;
sum_error7 = cumsum(G);
sum_error8 = cumsum(H);
sum_error7(1) = 0;
sum_error8(1) = 0;
mae(4) = sum_error7(N)/N*(60/dt);
maep(4) = sum_error8(N)/N*100;


%-------------------------------------------------------------------
%      print results
%-------------------------------------------------------------------

fid = fopen ('tte2.out','w');

        for i=1:N
```

```
fprintf (fid, '%6.2f %6.2f %6.2f %6.2f %6.2f %6.2f %6.2f %6.2f %6.2f
%6.2f %6.2f %6.2f %6.2f %6.2f %6.2f\n',
q23(i),o23(i),v23(i),q25(i),o25(i),v25(i),k(i),m(i),v(i),V(i),qe_data(i)
,qe_est1(i),qe_est2(i),qe_est3(i),qe_est4(i));
     end

fclose (fid);


%----------------------------------------------------------
%      plot   curves
%----------------------------------------------------------

plot (TIME,q23,'-',TIME,q23,'go',TIME,q25,'-',TIME,q25,'go');
xlabel ('Time-of-Day (hour)');
ylabel ('Flow (2-min rate)');
title ('Flow Rate Histogram');
pause

plot (TIME,Q23,'-',TIME,Q23,'go',TIME,Q25,'--',TIME,Q25,'go');
xlabel ('Time-of-Day (hour)');
ylabel ('Cumulative no. of Vehicles');
title ('Cumulative Flows');
pause

plot (TIME,q23,'-',TIME,q23,'go',TIME,q25,'--',TIME,q25,'go',TIME,k,'-
',TIME,k,'go',TIME,m,'-',TIME,m,'go');
xlabel ('Time-of-Day (hour)');
ylabel ('Flow (30-sec rate)');
title ('Flow Rate Histogram');
pause

%----------------- end of program ----------------------------------
```

```
%=====================================================================
%       diversion.m - FREEWAY DIVERSION ANALYSIS
%
%       Estimates delay times during incidents and determines equilibrium
%       freeway travel time an diversion volumes. Last modification:
%       October 5, 1995
%=====================================================================

echo off


%---------------------------------------------------------------------
% SYSTEM VARIABLES
%
%       TIME      = time elapsed after simulation starts (min)
%        dt       = the interval of time between TIME.J and TIME.K
%       length    = duration of simulation run (min)
%       interval  = data aggregation interval (min)
%       n         = vector dimension (length/dt+1)
%
%---------------------------------------------------------------------



%-----------------------------------------------------------
% define simulation variables
%-----------------------------------------------------------

length = 120;                          % duration of simulation run (min)
dt=1;                                  % time increment (min)
n = length/dt+1;                       % time vector size
TIME = 0:dt:length;                    % time vector (min)


%-----------------------------------------------------------
% define roadway characteristics
%-----------------------------------------------------------

distance(1) = 15;                      % freeway length (mile)
distance(2) = 12;                      % arterial length (mile)

lanecapacity(1) = 2000;
lanecapacity(2) = 1500;

lanes(1) = 3;
lanes(2) = 2;

ffspd(1) = 80;
ffspd(2) = 60;

capacity = lanes .* lanecapacity;      % freeway capacity (veh/hr)
unitfftt = 60 ./ ffspd;                % freeway free flow travel time
                                       % (min/mile)


%---------------------------------------------------------------------
% traffic conditions on freeways and arterials
%---------------------------------------------------------------------
```

```
normalflow(1) = 5000;                % traffic demand on freeway (veh/hr)
normalflow(2) = 2000;                % traffic demand on arterials

los(1) = 0.15;                       % LOS parameter of freeway
los(2) = 0.35;

y = normalflow ./ capacity;

unitttn = unitfftt .* (1 - (1 - los) .* y) ./ (1 - y);
                                     % travel time under traffic demand
                                     % (unit time/mile)
fftt = unitfftt .* distance;         % free flow travel times (min)


%---------------------------------------------------------------------
% read incident condition
%---------------------------------------------------------------------

lanesblocked = 1;

detection = 5;
response  = 5;
clearance = 20;

incidentduration  = (detection + response + clearance) / 60;
                                     % incident duration (hr)
residualcapacity = 0.49 * capacity(1);   % freeway capacity after
                                     % incident occurs


%---------------------------------------------------------------------
% determine the need for diversion based on expected average freeway
% travel times
%---------------------------------------------------------------------

[Ettq0,queuing_density,capacity_density] = queuing (residualcapacity,
incidentduration, unitttn, lanes, distance, normalflow, capacity, n,
dt);


Ett0(1) = unitttn(1) * distance(1) + Ettq0;
Ett0(2) = unitttn(2) * distance(2);


%---------------------------------------------------------------------
% estimate average freeway travel times for traffic demand volumes while
% reducing traffic demands
%---------------------------------------------------------------------

[Ettq,demand,m,k] = fwytt (residualcapacity, incidentduration, unitfftt,
distance, normalflow, capacity, queuing_density,capacity_density,los,
lanes, n, dt);
```

```
%--------------------------------------------------------------------
% get equilibrium travel time curves
%--------------------------------------------------------------------


[diversionvolume] = equitt (fftt, Ettq, demand, los, capacity, dt, n, m,
k);


%------------------ end of main program ----------------------------




%===================================================================
% FUNCTION: fwytt.M
%     DATE: September 29, 1995
%  PURPOSE: This program estimates expected freeway travel times during
%           the incident while reducing demand volumes by dv
%===================================================================


function [Ettq,demand,m,k] = fwytt (residualcapacity, incidentduration,
unitfftt, distance, normalflow, capacity,
queuing_density,capacity_density,los, lanes, n, dt);

%------------------------------------------------------------------
% Receiving variables from main program
%
%     lanesblocked = number of freeway lanes blocked due to the incident
%     incidentduration = duration of the incident (hr)
%     fwylanes = number of freeway lanes in each direction
%     fwydist = distance on freeway from an origin to a destination
%           (mile)
%     normalflow = freeway travel demand (veh/hr)
%     fwycap  = total capacity on freeways (veh/hr)
%     n = TIME vector size
%     dt = time increment (min)
%
% Passing variables to main program
%
%     Ett = expected average travel time (min)
%     ttq = travel time in a queue (min)
%      queuelength = queue length vector (mile)
%------------------------------------------------------------------


%----------------------------------------------------------
% change notation
%----------------------------------------------------------

incident_flow = residualcapacity;
T1 = incidentduration;                     % duration of incident (hr)


%------------------------------------------------------------------
% define traffic demand volume
%------------------------------------------------------------------
```

```
fftt = unitfftt .* distance;
dv0 = capacity(1) / (n-1);

    for i=1:n
demand(i) = dv0 * (i-1);
    end


m = round ((incident_flow - rem(incident_flow,dv0)) / dv0);
k = round (capacity(1) / dv0);

dv = round((capacity(1) - incident_flow) / (k-m));
                                %demand volume increment

    for i=m+1:k
demand(i) = demand(m) + dv * (i-m);
    end


%-------------------------------------------------------------------
% draw time-space diagram for volumes greater than fwyvol0 and less than
% normal flow
%-------------------------------------------------------------------

for i=m+1:k

    queuing_flow = incident_flow;    % queuing flow conditions (veh/hr)
    capacity_flow = capacity(1);     % capacity flow after incident is
                                     % cleared

    demand_density(i) = 60 - (3800 - 1.7*demand(i)/lanes(1))^0.5;

    wu1(i) = (demand(i) - queuing_flow) / (demand_density(i) -
queuing_density) / lanes(1);
    wu2 = (queuing_flow - capacity_flow) / (queuing_density -
capacity_density) / lanes(1);
    wd(i) = (demand(i) - capacity_flow) / (demand_density(i) -
capacity_density) / lanes(1);

    queue_max(i) = wu1(i) * wu2 / (wu1(i) - wu2) * T1;
                                % maximum queue length (mile)
    T2(i) = wu2 / (wu2 - wu1(i)) * T1;
                                % the time that queue reaches
                                % max (hr)
    T3(i) = wu2 * (wu1(i) - wd(i)) / wd(i) / (wu1(i) - wu2) * T1;
                                % the time queue disappears (hr)


    Q1 = incident_flow * T1;
    Q2(i) = Q1 + capacity_flow * (T2(i)-T1) + queue_max(i) *
capacity_density * lanes(1);
    Q3(i) = Q1 + capacity_flow * (T3(i)-T1);
    T4(i) = Q1 / (demand(i) - wu1(i) * demand_density(i) * lanes(1));
```

```
        TD(i) = (T1*Q3(i) + T3(i)*Q2(i) - T3(i)*Q1 - T2(i)*Q3(i)) / 2;
        % total-delay (veh-hrs)
        Ettq(i) = TD(i) / Q3(i) * 60;
                                    % expeced average travel time during
                                    % queuing conditions (min)

end


for i=k+1:n

        Ettq(i) = 0;
        queue_max(i) = 0;

end


plot (demand,Ettq)
xlabel ('Volume (min)');
ylabel ('Freeway travel time in a queue (min)');
pause


%------------------- end of function program -------------------------



%=====================================================================
%       Function equitt.m
%
%       Determines equilibrium travel times during incidents.
%=====================================================================

function [diversionvol] = equitt (fftt, Ettq, demand, los, capacity, dt,
n, m, k)


%---------------------------------------------------------------------
% find demand volume on freeway which causes travel time equal to
% arterial free flow travel time
%---------------------------------------------------------------------


dv = round(capacity(1) / (n-1));            %demand volume increment
vol = round(capacity(1) * (fftt(2) - fftt(1)) / (fftt(2) - (1-los(1)) *
fftt(1)));
                                            % freeway volume at arterial
                                            % free flow travel time
p = (vol - rem(vol,dv)) / dv;


%--------------------------------------------------------------
% get travel time curves for freeway and arterials
%--------------------------------------------------------------


for i=1:n
```

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```
        rho1(i) = demand(i) / capacity(1);
        rho2(i) = demand(i) / capacity(2);

            if (rho1(i) >= 0.99) rho1(i) = 0.99; end
            if (rho2(i) >= 0.98) rho2(i) = 0.98; end


        tt1(i) = fftt(1) * (1 - (1-los(1)) * rho1(i)) / (1 - rho1(i));
                            % freeway travel times (min)
        tt2(i) = fftt(2) * (1 - (1-los(2)) * rho2(i)) / (1 - rho2(i));
                            % arterial travel times (min)


            if (tt1(i) >= 200) tt1(i) = 200; end
            if (tt2(i) >= 200) tt2(i) = 200; end


        Ett1(i) = tt1(i) + Ettq(i);   % expected freeway travel
                                      % times under varing traffic demand
end


plot (demand,tt1,demand,tt2,demand,Ett1)
xlabel ('Volume (veh/hr)');
ylabel ('Travel time (min)');
axis ([0 capacity(1) 0 100])
pause


%-----------------------------------------------------------
% get travel time curve for the network
%-----------------------------------------------------------


for i=1:p

     fwyvol(i) = round(capacity(1)*((tt1(i) - fftt(1)) / (tt1(i) - (1 -
los(1))*fftt(1))));
     artvol(i) = 0;
     combvol(i) = fwyvol(i);

end


for i=p+1:n

     fwyvol(i) = round(capacity(1)*((tt1(i) - fftt(1)) / (tt1(i) - (1 -
los(1))*fftt(1))));
     artvol(i) = round(capacity(2)*((tt1(i) - fftt(2)) / (tt1(i) - (1 -
los(2)) * fftt(2))));
     combvol(i) = fwyvol(i) + artvol(i);

end


capacity(3) = capacity(1) + capacity(2);
```

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```
plot (demand,tt1,demand,tt2,combvol,tt1)
xlabel ('Volume (veh/hr)');
ylabel ('Travel time (min)');
axis ([0 capacity(3) 0 100])
grid
pause




%-------------------------------------------------------------
% find equilibrium travel time
%-------------------------------------------------------------


for i=1:p

     equitt(i)  = tt1(i);
     equivol(i) = fwyvol(i);

end


for i=p+1:m

     equitt(i) = tt1(i);
     fwyvols(i) = round(capacity(1)*((equitt(i) - fftt(1)) / (equitt(i)
- (1 - los(1))*fftt(1))));
     artvols = round(capacity(2)*((equitt(i) - fftt(2)) / (equitt(i) -
(1 - los(2))*fftt(2))));
     equivol(i) = artvols+fwyvols(i);

end



for i=m+1:k

     equitt(i) = Ett1(i);
     fwyvols(i) = demand(i);
     artvols = round(capacity(2)*((equitt(i) - fftt(2)) / (equitt(i) -
(1 - los(2))*fftt(2))));
     equivol(i) = artvols+fwyvols(i);
     diversionvol(i) = fwyvols(i) - round(capacity(1)*((equitt(i) -
fftt(1)) / (equitt(i) - (1 -los(1))*fftt(1))));


end


for i=k+1:n

     equitt(i) = tt1(i);
     fwyvols(i) = round(capacity(1)*((equitt(i) - fftt(1)) / (equitt(i)
- (1 - los(1))*fftt(1))));
     artvols = round(capacity(2)*((equitt(i) - fftt(2)) / (equitt(i) -
(1 - los(2))*fftt(2))));
     equivol(i) = artvols+fwyvols(i);
```

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```
        diversionvol(i) = 0;

end


plot (demand,tt1,demand,tt2,demand,Ett1,combvol,tt1,equivol,equitt);
axis ([0 capacity(3) 0 100]);
xlabel ('Volume (vph)');
ylabel ('Travel Time (min)');
grid
pause




%-------------------------------------------------------------------
% print output data
%-------------------------------------------------------------------

fid = fopen ('equilib.out','w');

fprintf(fid,'                          w/o Incident
w/ incident              Equilibrium      \n');
fprintf(fid,'  Volume       Art TT        Fwy TT          Comb. Cap
Fwy TT          Volume        TT       \n');
fprintf(fid,'  (vph)        (min)         (min)           (vph)
(vph)        (min)     \n');
fprintf(fid,'-----------------------------------------------------
------------------\n');

     for i=1:n

fprintf (fid,'%10d %10.2f %10.2f %10d %10.2f %10d %10.2f\n', demand(i),
tt2(i), tt1(i), combvol(i), Ett1(i),equivol(i), equitt(i));

     end


fclose (fid);

%------------------ end of function program -------------------------




%=====================================================================
% FUNCTION: QUEUING.M
%      DATE: September 29, 1995
%   PURPOSE: This program estimates freeway travel times during an
%            incident at given freeway traffic demand.
%=====================================================================


function [Ettq,queuing_density,capacity_density]  = queuing
(residualcapacity, incidentduration, unitttn, lanes, distance,
normalflow, capacity, n, dt)


%-------------------------------------------------------------------
```

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```
% Receiving variables from main program
%
%      lanesblocked = number of freeway lanes blocked due to the incident
%      incidentduration = duration of the incident (hr)
%      fwylanes = number of freeway lanes in each direction
%      fwydist = distance on freeway from an origin to a destination
%           (mile)
%      normalflow = freeway travel demand (veh/hr)
%      fwycap  = total capacity on freeways (veh/hr)
%        n = TIME vector size
%        dt = time increment (min)
%
% Passing variables to main program
%
%      Ettq = expected average travel time in a queue (min)
%      queue_max = maximum queue length (mile)
%-----------------------------------------------------------------------



%-------------------------------------------------------------
% change notation
%-------------------------------------------------------------


T1 = incidentduration;                     % duration of incident (hr)
incident_flow = residualcapacity;
normal_flow = normalflow(1);

TIME = 0:dt/60:(n-1)/60;                    % time vecor (hr)



%-------------------------------------------------------------
% Draw time-space diagram
%-------------------------------------------------------------

queuing_flow = incident_flow;              % queuing flow veh/hr)
capacity_flow = capacity(1);               % capacity flow after incident
                                           % is cleared

normal_density = 38.5;
queuing_density = 102.5;                    % density under queuing
conditions (veh/mi)
capacity_density = 60;                      % density under capacity flow
conditions

wu1 = (normal_flow - queuing_flow) / (normal_density - queuing_density)
/ lanes(1);
wu2 = (queuing_flow - capacity_flow) / (queuing_density -
capacity_density) / lanes(1);
wd = (normal_flow - capacity_flow) / (normal_density - capacity_density)
/ lanes(1);

queue_max = wu1 * wu2 / (wu1 - wu2) * T1  % maximum queue length (mile)
T2 = wu2 / (wu2 - wu1) * T1               % the time that queue reaches
                                          % max (hr)
T3 = wu2 * (wu1 - wd) / wd / (wu1 - wu2) * T1
```

```
                                              % the time queue removed (hr)

%------------------------------------------------------------
% Draw time-delay diagram
%------------------------------------------------------------

Q1 = incident_flow * T1
Q2 = Q1 + capacity_flow * (T2-T1) + queue_max * capacity_density *
lanes(1)
Q3 = Q1 + capacity_flow * (T3-T1)
T4 = Q1 / (normal_flow - wu1 * normal_density * lanes(1));

TD = (T1*Q3 + T3*Q2 - T3*Q1 - T2*Q3) / 2;
                                    % total-delay (veh-hrs)
Ettq = TD / Q3 * 60;                % expeced average travel time during
                                    % queuing conditions (min)



%------------------------------------------------------------
% Estimate travel times
%------------------------------------------------------------

queue_in(1) = 0;                    % cumulative number of vehicles in a
                                    % queue (veh)
queue_out(1) = 0;                   % cumulative number of vehicles
                                    % discharged from a queue (veh)
queue_veh(1) = 0;                   % number of vehicles in a queue
                                    % (veh)
queuelength(1) = 0;                 % queue length (mile)
ttq(1) = 0;                         % travel time in a queue (min)


m = round (T3 * 60);
for i=2:m

      if (TIME(i) <= T4)

            queue_in(i)  = (normal_flow - wu1 * normal_density *
lanes(1)) * TIME(i);
            queue_out(i) = incident_flow * TIME(i);
            queue_veh(i) = queue_in(i) - queue_out(i);
            queuelength(i) = -wu1 * TIME(i);

            t1 = queue_in(i) / incident_flow;
            ttq(i) = (t1 - TIME(i)) * 60;
                            % travel time during queuing conditions
                            % (min)

      elseif (TIME(i) <= T1)

            queue_in(i)  = (normal_flow - wu1 * normal_density *
lanes(1)) * TIME(i);
            queue_out(i) = incident_flow * TIME(i);
            queue_veh(i) = queue_in(i) - queue_out(i);
            queuelength(i) = -wu1 * TIME(i);

            t2 = T1 + (queue_in(i) - Q1) / capacity_flow;
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```
                ttq(i) = (t2 - TIME(i)) * 60;
                                    % travel time during queuing conditions
                                    % (min)

        elseif (TIME(i) <= T2)

                queue_in(i)  = (normal_flow - wu1 * normal_density *
lanes(1)) * TIME(i);
                queue_out(i) = Q1 + capacity_flow * (TIME(i) - T1);
                queue_veh(i) = queue_in(i) - queue_out(i);
                queuelength(i) = -wu1 * TIME(i);

                t2 = T1 + (queue_in(i) - Q1) / capacity_flow;
                ttq(i) = (t2 - TIME(i)) * 60;
                                    % travel time during queuing conditions
                                    % (min)

        else

                queue_in(i)  = Q2 + (normal_flow - wd * normal_density *
lanes(1)) * (TIME(i) - T2);
                queue_out(i) = Q1 + capacity_flow * (TIME(i) - T1);
                queue_veh(i) = queue_in(i) - queue_out(i);
                queuelength(i) = queue_veh(i) / capacity_density / lanes(1);

                t3 = T1 + (queue_in(i) - Q1) / capacity_flow;
                ttq(i) = (t3 - TIME(i)) * 60;
                                    % travel time during queuing conditions
                                    % (min)

        end
end


for i=m+1:n

        queue_in(i) = 0;
        queue_out(i) = 0;
        queue_veh(i) = 0;
        queuelength(i) = 0;
        ttq(i) = 0;
end



for i=1:n
        ttn(i) = unitttn(1) * distance(1);
        tt(i) = ttn(i) + ttq(i);                % freeway travel times (min)
end


plot (TIME,ttq)
xlabel ('Time after incident occurs (hr)');
ylabel ('Delay time in a queue (min)');
pause
```

```
plot (TIME,queue_in,TIME,queue_out)
xlabel ('Time after incident occurs (hr)');
ylabel ('Cumulative vehicles in a queue (veh)');
pause




plot (TIME,queue_in-queue_out,TIME,queuelength.*100)
xlabel ('Time after incident occurs (hr)');
ylabel ('Cumulative vehicles in a queue (veh)');
pause


%-----------------------------------------------------------------------
% print output data
%-----------------------------------------------------------------------

fid = fopen ('queuing.out','w');

fprintf(fid,'  TIME        Veh in         Veh out       Veh in Queue
Queue          TTq          TTn           TT  \n');
fprintf(fid,'  (min)      (veh)        (mile)              (veh)
(min)          (vph)          (min)              \n');
fprintf(fid,'-----------------------------------------------------------
-----------------------------------------------\n');

       for i=1:n

fprintf (fid,'%8d %10.2f %10.2f %10.2f %10.2f %10.2f %10.2f %10.2f\n',
i, queue_in(i), queue_out(i), queue_veh(i), queuelength(i), ttq(i),
ttn(i), tt(i));

       end


fclose (fid);


%------------------ end of function program -------------------------
```

Do H. Nam was born in Seoul, Korea on August 3, 1960. After receiving his Bachelor's degree of Science in Civil Engineering at Hanyang University in Seoul, Korea in March 1983, he attended the University of Iowa and earned a Master's degree of Science in Civil Engineering in August 1985. He came back to Korea after practicing professional skills in designing transportation infrastructure in US. He became computer application specialist at Intergraph Korea where his work involved designing transportation facilities and lecturing on state-of-the-art computer-aided-design.

He crossed the Pacific again in July 1992 to attend Virginia Tech. On December 8, 1995 he defended his dissertation and earned a degree of Doctor of Philosophy in Civil Engineering with an emphasis on Transportation. He has published several papers including one in the Journal of Transportation Engineering.

He wants to be a professor and continue research on traffic flow modeling, simulation software development, freeway operations and ITS applications.

Do H. Nam