Prescribing Optimal Harvests in Forests

Containing Even-aged and Uneven-aged Stands

by

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(ABSTRACT)

Research in optimizing forest management has focused on single-stand problems to derive optimal harvest sequences in terms of residual basal area or residual stand structure for uneven-aged stands, and timing of pre-defined thinning treatments and clearcut harvests for even-aged stands. Recent research results provide various means of numerically deriving optimal management prescriptions for single-stand problems, thus considering all feasible solutions as opposed to considering only pre-defined harvest alternatives. However, forest-level problems involving aggregates of stands with similar management constraints are usually solved by evaluating pre-defined harvest sequences. Forest-level management optimization problems in which individual stands may be assigned to either even-aged or uneven-aged silvicultural systems have not been modelled. A dynamic forest management model is described that prescribes silvicultural treatments for stands within a multi-stand management unit. Results of an application of this approach to an Appalachian hardwood forest, comparisons of individual stand and whole forest optimal solutions, and efficiency of the solution algorithm are discussed.

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As the first member of the Miller family to achieve the doctoral level of higher education, I wish to thank my parents. Although my parents are no longer with us to share my success, I still owe them a debt of gratitude for whatever they did as parents to instill in me the tenacity to finish this project.

My wife, Nancy, and our five children, Stephen, Jeffrey, Elizabeth, Emily, and Mary have waited patiently for me to complete this journey, and I am grateful for their support. Now we can spend a few more hours together.

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CHAPTER 1: INTRODUCTION

After decades of research, numerous reliable techniques for optimizing management of individual timber stands are available. Approaches for solving stand-level problems have evolved from marginal analyses based on simple yield tables to sophisticated optimal control theoretic models which utilize powerful growth and yield simulators developed for major forest types. As a result, well-defined problems, such as optimal rotation age for stands with even-aged structure, or optimal residual stocking for stands with uneven-aged structure, can be solved efficiently with a variety of linear, nonlinear, or dynamic programming models.

A more challenging problem today is optimal management of several stands which may be affected in aggregate by forest-wide resource objectives. In the stand-level problem, key management constraints such as the silvicultural system are given, and the solution defines the optimal harvest strategy, as measured by an economic objective. In the forest-level problem, solutions assign individual stands to a particular silvicultural system and map out an optimal harvest strategy for all stands simultaneously. This dissertation addresses multi-stand problems where timber is assumed to be the sole market product, although nonmarket forest-wide objectives, such as esthetics or regular periodic yields, are included to examine their impact on management of individual stands.

The result of forest management planning is typically a series of scheduled activities or cutting prescriptions over a given time period. Although timber is one of many forest resources, often having a subordinate priority, management of the overstory trees has a dominant effect on production and quality of other forest resources. Resource objectives and practical constraints may have wide-ranging variation, yet most management prescriptions can be described in terms of how the residual tree distribution is controlled through cutting. In this study, a management model was constructed using control variables which represent management prescriptions that are described in terms of number of trees to cut in each size class over time. Therefore, multi-stand management prescriptions can be explicitly defined in terms of number of trees to cut and leave in each size class and stand over time to achieve landowner objectives.

Nontimber resources are of major importance in this dissertation primarily because forest-level management problems are defined by numerous conflicting resource constraints. Techniques for formulating resource constraints, as well as the impact of such constraints on economic performance, are discussed. Performance of feasible solutions is measured by net present value of the forest. Results of this study provide a means for examining unique combinations of resource constraints where field testing may be lacking or impractical. Solutions to such problems may be used to improve existing management guidelines or supplement expert system reliability where current recommendations may be suboptimal. Moreover, the resulting methodology provides a means for defining the cost or impact of stated nonmarket resource goals, thus allowing for intelligent modification of goals in the planning process.

Objectives and Justification

The primary objective of the research described in this dissertation was to formulate a framework for investigating multi-stand forest management problems. The key to achieving this objective was the construction of a model which numerically prescribes silvicultural treatments for individual stands within a multi-stand management unit over time to maximize net present forest value. The formulated model utilized a stage-structured stand growth simulator to account for individual stand development that result from variations in site productivity, species composition, and long-term silvicultural treatments. Equations embodying growth dynamics were nonlinear in the control variables, defined as the numbers of trees to be harvested in each size class in each stand over time. Due to nonlinearities in both the objective function and key management constraints, the model took the form of a nonlinear programming problem.

Prior multi-stand models evaluate a set of management alternatives by means of dynamic programming or linear programming to optimally assign cutting strategies among a group of stands. However, these approaches consider only alternatives explicitly defined in the model formulation, perhaps overlooking superior solutions. The model discussed in this dissertation is an extension of an optimal control model formulated to optimize management of a single stand which allows for continuous variation in cutting alternatives over time. While the original model (Haight 1987) is suitable for stands of either even-aged or uneven-aged structure, the present model assigns stands to a particular silvicultural system and prescribes cutting strategies for all stands simultaneously. As a result, both even-aged and uneven-aged stands may be recommended within a forest, depending on harvesting and market constraints, as well as any forest-level management constraints.

Prior models of forest-level management define solutions in very broad terms, such as basal area per acre to cut or timing of a complete harvest. Such broadly defined control variables mask stand density factors which are important in accounting for stand dynamics and in defining superior courses of action. By explicitly defining the size distribution of harvested products, the present model also accounts for product distribution and value at each time period. More explicit definition of treatments makes it possible to assess the impact of individual stand structures when scheduling woodflow, and planning for production of nonmarket outputs when numerous stands are managed in aggregate. Results of this study are versatile in that prescriptions can be generated subject to numerous landowner constraints, over any specific planning horizon, and include single or multiple stands as needed.

The method also allows for the use of multiple growth simulators, so that growth projections of individual stands can be made with the most appropriate simulator available for each stand. As new or improved simulators become available, management guidelines can be revised to reflect such advancements by repeating key analyses using updated model inputs. The general management model is modular in design, allowing for easy substitution of model functions and parameters. In addition, stand-level problems may be analyzed by formulating the special case where the forest contains one stand.

Analyses were relatively inexpensive, requiring an 80386 PC-compatible computer equipped with a fixed disk, 8 Mb of extended memory, and math coprocessor running under MS-DOS version 2.0 or higher. Preliminary analyses that involved less than five individual stands required about two hours of processing time per problem.

Overview

A review of forest management optimization methods is presented in Chapter 2. Early research solved static problems such as optimal stocking in an uneven-aged stand given cutting cycle, stand structure, and species composition using marginal analysis. As stage-structured stand growth simulators were formulated, research in optimizing stand management was aimed at modelling stand dynamics as linear or nonlinear constraints, allowing continuous variation in cutting strategies to be evaluated. As a result, problem solutions defined cutting strategies over time in terms of species and size classes to harvest over a given planning horizon to maximize economic objectives. More recent research has focused on nonlinear and dynamic programming techniques for solving management problems for stands with either even-aged or uneven-aged structures. A limited amount of research has dealt with the problem of converting a given stand to uneven-aged management through an optimal series of transition harvests. Interestingly, prior models do not determine which silvicultural system is better for a given stand. Instead, available techniques find optimal cutting strategies for stands in which the silvicultural system is given.

Chapter 2 also provides a summary of research results dealing with forest-level management problems. The forest-level problem is complicated by additional constraints involving harvest flow, environmental or esthetic requirements, or economic factors such as increasing returns to scale in the revenue function and downward sloping demand for timber products removed from the forest. While stand-level problems are designed to optimize management of an individual stand, constraints on forest-level problems link stands, causing decisions on a single stand to influence decisions on other stands. An important forest-level management problem is to define the optimal harvest strategy for a group of uneven-aged stands, or a forest made up of both even-aged and uneven-aged stands. Research in optimizing forest-level management has focused on forests made up of even-aged stands. Problem solutions indicate the number of acres to harvest, or which stands to harvest in each period. Specific intermediate even-aged treatments or partial regeneration cuts are included as pre-defined activities, thus precluding variation of treatments among stands. A numerical approach is needed whereby decision variables prescribe stand treatments, rather than choosing the best alternative from a pre-defined set.

In Chapter 3, a general forest-level management model is described. State variables are numbers of trees in each dbh class, species group, and stand at the beginning of each time period. Control variables are numbers of trees harvested in each time period, defining a harvest strategy over time. The objective function maximizes net present value of the initial stand, accounting for the discounted fixed cost of logging operations, the opportunity cost of residual stand stocking, and discounted values of periodic harvest revenues. FIBER, a growth and yield simulator for northern hardwoods and mixed species stands (Solomon, Hosmer, and Hayslett 1987) is formulated into model constraints to describe stand dynamics resulting from management decisions.

Model constraints which reflect various management constraints and market conditions are described in Chapter 4. Initially, constraints associated with stand-level problems such as minimum residual basal area, required cutting cycle, preferred species composition are described. Attention is then focused on forest-level constraints such as required woodflow or spatial allocation of clearcut operations over a group of stands. For a specific problem, model constraints are used to place lower and upper bounds on key variables to satisfy management goals. By properly formulating constraints, a feasible region is defined, within which resource and management goals are achieved. The problem is then reduced to determining an optimal harvest strategy from among all remaining feasible solutions. Program code for management constraints is provided in Appendix B.

Example problems are solved, and study results are presented in Chapter 5. In general, even-aged silvicultural systems required additional constraints which forced complete harvesting of residual stand stocking. In the absence of such constraints, optimal solutions promoted stands of uneven-aged structure through regular periodic harvests of financially mature growing stock. In the initial stages of testing, problem size was limited to $k=2$ stands, each with a unique initial stand structure, and $T=100$ years in the planning period. Notation used throughout the dissertation is defined in Appendix A.

Analyses focused on evaluating the impact of management constraints, growth simulator coefficients, and problem size. Management constraints such as esthetics, aggregate woodflow, and silvicultural systems were examined to define their relative impact on optimal solutions, holding other constraints constant. Finally, the number of stands and number of years in the planning period were increased to define the relationship between model size and solution time.

Management problems were formulated as a nonlinear programming problem and solved using GAMS/MINOS, a projected Lagrangian algorithm (Robinson 1972). Program code for the general model is provided in Appendix B, and a description of the solution algorithm is presented in Appendix C.

CHAPTER 2: REVIEW OF FOREST MANAGEMENT OPTIMIZATION

Research in optimizing forest management decisions for timber production is subdivided into stand-level problems where activities are determined for only one stand, and forest-level problems where activities are coordinated for several stands managed in the aggregate. The usual approach is to construct a model which includes constraints representing timber growth and yield capabilities of the stand or forest under management. These models are then enhanced by adding constraints designed to reflect other management goals such as regular periodic yields. Decision variables allow managers to define when and how harvests are to be conducted to meet preselected management constraints and optimize an economic objective, usually net present value or land expectation value. A related area of research focuses on refining computational methods to solve large problems more efficiently. Such studies evaluate solution techniques in order to identify algorithms which exploit model characteristics so that computation time is reduced, or convergence to an optimum point is more likely.

Stand-level Problems

Studies involving management of individual stands typically have taken silvicultural system, as given, and defined optimal cutting prescriptions within the broad definitions of that silvicultural system. The combination of resource constraints and silvical characteristics of desired species usually determine whether individual stands should be managed for even-aged or uneven-aged stand structures. Once the silvicultural system is selected, in broad terms, the problem is reduced to optimizing the sequence of harvests over the planning period to promote either even-aged or uneven-aged stand structures. For example, models were developed for stands targeted for even-aged structures to define optimal timing and intensity of thinnings and the optimal rotation age (Roise 1986b). Similarly, models were developed to define cutting cycle, transition harvests, and the optimal sequence of harvests over a given time period to promote an uneven-aged stand (Haight, Brodie, and Adams 1985). Several model structures are described below which demonstrate the development of stand-level optimization models.

Uneven-aged Structures

An important early study of a true selection practice to promote an uneven-aged stand structure, determined optimal residual board-foot volume, given tree size-class distribution, cutting cycle, and species composition (Duerr and Bond 1952). Marginal value growth percent over a 5-year period was estimated for each additional 1,000 board feet of growing stock. The objective was to equate marginal value growth percent with the alternative rate of return, essentially the same as maximizing land expectation value.

Adams and Ek (1974) modelled a similar management practice using nonlinear programming to determine optimal residual stand structure for a given residual basal area, cutting cycle, and species composition. Problem solutions define an optimal steady-state initial stand structure without regard to conversion from an existing stand structure. The objective function

$$VG = \sum_{i=1}^{MD+1} p_i \left(x_i(t+1) - x_i(t)\right) \qquad 2.1$$

where, $x_i(t)$ represents the number of trees in each size class i at the beginning of time period t, p_i is a vector of stumpage prices, and MD is the maximum dbh class, maximizes value growth (VG), the difference in stand value between periodic harvests. The model implicitly assures identical harvests over perpetual cutting cycles, so maximizing VG is the same as equating marginal value growth percent with the discount rate (Getz and Haight 1989).

The structural constraints require that basal area per acre BA remains constant at the beginning of each time period t,

$$\sum_{i=1}^{MD} b_i x_i(t) = BA \qquad 2.2$$

where b_i is the basal area per tree in dbh class i. Using an iterative approach, solutions were determined for a range of residual basal areas to identify both optimal stocking and stand structure. Sustained yield constraints

$$x_i(t+1) - x_i(t) \geq 0 \qquad 2.3$$

assure that periodic cuts do not exceed periodic growth. Other structural constraints

$$x_i(t) \geq 0 \qquad 2.4$$

require that state and control variables are nonnegative. Growth dynamics were expressed as nonlinear functions of $\mathbf{x}(t)$, the number of trees in each diameter class, such that ingrowth $ING(t)$, mortality $M_i(t)$, and upgrowth $U_i(t)$ to larger diameter classes were accounted for in deriving an optimal harvest and residual stand.

A total of $MD+1$ growth functions, generally defined as

$$x_i(t) = x_i(t-1) + U_{i-1}(t)(x(t)) - M_i(t)(x_i(t)) - U_i(t)(x_i(t)) \qquad 2.5$$

represent the influence of stand density on the movement of trees among size classes during a growth period and define the value of x_i at any time t. Equations 2.5 are substituted into equations 2.3 when solving the problem. Note that the Adams and Ek (1974) model defines the harvest as $x_i(t+1) - x_i(t)$, periodic growth between t and $t+1$.

Buongiorno and Michie (1980) used the same stand data to solve a similar linear programming problem with a fixed-coefficient matrix growth simulator, instead of a nonlinear simulator. A general expression of the linear growth model is

$$x(t+5) = G\big(x(t) - h(t)\big) + c \qquad 2.6$$

where $\mathbf{x}(t)$ represents the initial number of trees and $\mathbf{h}(t)$ represents harvest number of trees. A matrix of transition probabilities $\mathbf{G}$ is applied to the residual stand $\mathbf{x}(t)$-$\mathbf{h}(t)$ to determine growth, survival, and ingrowth c during a growth period, and consequently $\mathbf{x}(t+1)$, stand structure at the beginning of the next growth period. A problem with this formulation is that elements of **G** are fixed, computed prior to the analyses, and are not affected by stand density variables. Solomon and others (1987) built a similar growth model, but related transition probabilities to stand density and species composition for each growth period. The resulting two-stage matrix simulator is actually a system of nonlinear equations.

Martin (1982) also used the Adams and Ek (1974) growth model to optimize stocking of an uneven-aged stand, but reduced the decision space to two variables using a Weibull distribution function to describe stand structure. Instead of determining the optimal number of residual trees in each dbh class, the model determines the optimal scale and shape parameters of the Weibull distribution function, and the number of trees per acre that maximize land expectation value (LEV). Formulations employing a Weibull function to describe stand structure are particularly useful in reducing the decision space when multiple species are involved. The number of trees in a dbh class a given by

$$x_i(t)=\frac{N\left[\exp(-(\frac{i_{lower}}{wb})^{wc}) - \exp(-(\frac{i_{upper}}{wb})^{wc})\right]}{1 - \exp(-(\frac{MD}{wb})^{wc})} \qquad 2.7$$

where N equals the total number of trees per acre, wb is the scale parameter which determines the ratio of large trees to small trees, and wc is the shape parameter which causes the distribution to take on a reversed J shape if the value is between 0 and 1. The upper and lower bounds for each dbh class are given in the model, as is the largest allowable size class MD. For each time period t, the number of trees in each dbh class is simply a function of three decision variables N, wb, and wc. Preliminary analyses revealed that optimal solutions over a range of discount rates had wc parameter values equal to 1, resulting in a balanced size-class distribution characterized by a certain "q" value described by Meyer (1952). The decision space, therefore, could be reduced to only N and wb for each time period t. In employing the Weibull distribution function, equation 2.7 is substituted into equations 2.3 and 2.4. Equations 2.8 are included so that Weibull parameters are nonnegative.

$$wb > 0, \quad wc > 0, \quad N \geq 0 \qquad 2.8$$

Bare and Opalach (1987) used a Weibull distribution function to describe stand structure in conjunction with a species-dependent growth simulator to optimize residual stocking and species composition simultaneously. Bare and Opalach (1988) later attempted to verify Martin's (1982) results using a different nonlinear programming algorithm. Their 1988 study revealed that solutions derived using a Weibull distribution are very sensitive to maximum tree size constraints, and investment-efficient stand structures are not represented by a "q" factor, as described by Meyer (1952). Bailey and Dell (1973) discussed the Weibull distribution function in relation to stand structure, and Shifley and Lentz (1985) presented a method for estimating Weibull parameters for observed stand structures.

Transition Harvests

The preceding research results developed the techniques for defining an optimal steady-state uneven-aged stand harvested at regular periodic intervals. Methods for defining the optimal conversion from an initial stand to a target steady-state stand were discussed by Adams and Ek (1974). They presented an example which is similar to regulating a forest made up of even-aged stands by the end of a certain conversion period, as done by Nautiyal and Pearse (1967).

Haight and others (1985) used a control theory model to maximize net present value of an uneven-aged stand over a 150-year planning period (not LEV as done in previous work). Instead of defining an optimal balanced uneven-aged stand and an optimal conversion to it, this approach allows the user to define optimal partial harvests for a finite planning horizon without achieving a specified target residual stand. This work also employed the Adams and Ek (1974) growth model. The study demonstrated that a dynamic model determined a better solution (higher NPV) than that defined earlier through static optimization.

Haight (1985) then compared dynamic and static models for stands with uneven-aged structures. The dynamic model determined both optimal transition and steady-state management regimes. He found that the initial stand affects the length and harvest pattern of the transition period, but not the steady state solution. Steady state solutions found by static analyses were inferior to those found using similar dynamic models. Moreover, the study also showed that even-aged and uneven-aged silvicultural systems should not be compared based on steady state solutions alone, because decision criteria are sensitive to the initial stand and activities in the transition period.

Haight (1987) described a general investment model for stand management. Steady states which maximize LEV do not maximize NPV of the transition period plus subsequent steady state management except when the stand begins in the steady state. Results of this study support using NPV rather than LEV over a long planning horizon for comparing stand management alternatives. A detailed discussion of the general investment model and solution techniques was presented by Getz and Haight (1989).

Even-aged Structures

Stand-level management problems involving even-aged stand structures with nonlinear objectives or constraints were first solved with dynamic programming algorithms (Brodie, Adams, and Kao 1978; Brodie and Kao 1979). These approaches used aggregate yield functions and later studies utilized stage-class growth functions and single-tree simulators (Haight et al. 1985). Optimal size-class thinnings in even-aged stands were also found by nonlinear programming formulations for a stage-class growth model (Haight 1987) and a single-tree simulator (Roise 1986a).

Research in optimizing even-aged silviculture at the stand level included a nonlinear-integer programming model presented by Bullard, Sherali, and Klemperer (1985). This study employed a heuristic random search algorithm to estimate optimal solutions for two species groups and four diameter classes. This approach estimated optimal thinning regime and rotation age simultaneously. Roise (1986b) solved an unconstrained nonlinear programming problem with a direct search algorithm to optimize thinning a one-species even-aged stand. Haight and others (1985) employed a dynamic programming model to optimize thinning and harvesting in lodgepole pine stands.

Forest-level Problems

Research in optimizing forest-level management has focused on forests made up of even-aged stands. Decision variables indicate the number of acres to clearcut in each period of the planning horizon. Simple models do not include intermediate commercial harvests. Forest-level problems cannot be solved by simply optimizing the management of individual stands. The forest-level problem is complicated by additional constraints involving harvest flow, environmental or esthetic requirements, or economic factors such as increasing returns to scale in the revenue function which make management decisions interrelated among multiple stands (Getz and Haight 1989). Many forest-level harvest scheduling problems were modelled as constrained linear programming problems using stage-structured growth models. A few key examples illustrate the simplicity of earlier models.

Nautiyal and Pearse (1967) formulated a linear model to regulate a forest of even-aged stands within a given transition period P. The underlying management goal was to provide for an even flow of wood from a group of stands. The model was simplified by assuming all stands would be even-aged, with site quality and optimal rotation age R the same for each stand, and that regeneration is established immediately following clearcutting. Thus, stand volume and value per acre V_a are simply a function of stand age a. Once conversion is complete, the forest has a uniform age-class distribution, and an equal volume and acreage are harvested each time period. Although quite simple, this formulation was the first to include woodflow constraints to link stands together to form a forest-level management model. Using this model, they found that forest value increased as the transition period was lengthened and that forest value with an equilibrium-endpoint (a flexible age distribution) was greater than or equal to a fixed-endpoint (equal acres in each age class) problem. In general, the objective function value increased as constraints were relaxed, indicating that conversion to a normal forest within a given period is suboptimal.

The control variable in Nautiyal and Pearse's model is $A_a(t)$, the number of acres to clearcut in time period t, age class a. The objective function

$$NPV = \sum_{a=1}^{R+P}\sum_{t=1}^{P}\frac{V_a\,A_a(t)}{(1+r)^t} \qquad 2.9$$

where R is rotation age, and P is the transition period, simply maximizes NPV, the sum of harvest revenues $V_aA_a(t)$ discounted by r.

Primary structural constraints limit the number of acres available for harvest in each time period. For age classes where $a<t$, or classes that have been harvested since the beginning of the planning period, the constraints take the form:

$$A_a(t) \leq \sum_{m=1}^{R+P} A_m(t-a) - A_1(t-a+1) - A_2(t-a+2) - \ldots - A_{a-1}(t-1) \qquad 2.10$$

The first term on the right side of equation 2.10 is the acreage that was cut a years ago in year t-a, and could now be a years old. The remaining right side terms are acres cut in year t-a that were cut again prior to year t. For age classes where $a \geq t$, or classes harvested before the beginning of the planning period, the primary structural constraints are:

$$A_a(t) \leq n_{a-t+1} - A_{a-t+1}(1) - A_{a-t+2}(2) - \ldots - A_{a-1}(t-1) \qquad 2.11$$

In equation 2.11, n_{a-t+1} is the number of acres in age-class a-$t+1$ at the beginning of the planning period. When $a \geq t$, the number of acres available for harvest at time t that are age a, is simply n_{a-t+1} minus any of those particular acres that were harvested prior to period t. There are $P(R+P)$ primary structural constraints in the form of equation 2.10 or 2.11.

Other constraints are added to ensure a uniform age-class distribution after P years. For each age a where $R < P+1$, these constraints take the form:

$$\sum_{k=1}^{R+P} A_k(P-a+1) - A_1(P-a+2) - A_2(P-a+3) - \ldots - A_{a-1}(P) = EA \qquad 2.12$$

Equation 2.12 requires that the sum of all acres that could be age a minus any that were subsequently cut must equal a constant acreage EA, representing equal acreage defined as total acres divided by rotation age. If $R \geq P+1$, then the constraint for year R takes the form:

$$n_{R-P} - A_{R-P}(1) - A_{R-P+1}(2) - \ldots - A_{R-1}(P) = EA \qquad 2.13$$

There are R constraints using either equations 2.12 or 2.13, depending on the relationship between R and P, to ensure uniform age class distribution after P years.

Finally, acres that are R years or older must be cut at the end of the planning period P to complete the conversion. If $R<P$, all acres that are R years old in year p must be harvested as defined by:

$$A_R(P) = \sum_{m=1}^{R+P} A_m(P-R) - A_1(P-R+1) - A_2(P-R+2) - \ldots - A_{R-1}(P-1) \qquad 2.14$$

Also, if $R \geq P$ then all acres R or more years old must be cut as defined by:

$$A_R(P) = n_{R-P+1} - A_{R-P+1}(1) - A_{R-P+2}(2) - \ldots - A_{R-1}(P-1) \qquad 2.15$$

There are a total of $P+1$ constraints to cut all acres R years or older, and a total of $R+P+1$ to bring about a uniform age-class distribution after P years. Non-negativity constraints, $P(N+P)$ inequalities in the form of equation 2.16, are added to complete the linear programming problem.

$$A_a(t) \geq 0 \qquad 2.16$$

Again, Nautiyal and Pearse (1967) determined that present forest value increased as the transition period was lengthened, essentially meaning that conversion to a normal forest is suboptimal. Instead, optimal management (subject to their model structure) was conversion to an equilibrium age-class distribution, not necessarily uniform, over an infinite time horizon.

Although recognized as a pioneering step in the development of harvest scheduling techniques, a major shortcoming of this approach is that it does not generate management schedules for individual stands within the forest over time. Instead, $A_a(t)$ defines number of acres to cut in year t, but the solution does not preclude breaking existing stands into numerous smaller stands, each cut at different ages. Furthermore, the model structure only allows for complete regeneration harvests. Partial harvests, such as intermediate commercial thinnings or selection practices to promote uneven-aged stand structures are not defined in the model. Consequently, objectives such as maintaining a minimum desired residual basal area for esthetics may not be included in the model. Johnson and Scheurman (1977) referred to Nautiyal and Pearse's formulation as "Model II." In Model II, once acres receive a regeneration harvest, they are assigned to a new age class and may be assigned one or more subsequent regeneration harvest periods. For each assignment there is a pre-defined contribution to the objective function value.

An alternative structure, referred to as "Model I" by Johnson and Scheurman (1977), assigns acres to prespecified harvest schedules that describe the treatments to be applied throughout the planning horizon. Unlike the Nautiyal and Pearse (1967) formulation, the Model I formulation retains the identity of individual acres assigned to a particular harvest strategy, making it possible to track management activities in a stand over time. The control variables are $A_{l,q}$, acres in management unit l that are assigned to harvest sequence q. The objective function

$$NPV = \sum_{t=1}^{P}\sum_{l=1}^{L}\sum_{q=1}^{Q_l} \frac{V_{l,q}(t)\, A_{l,q}}{(1 + r)^t} + \frac{V_{l,q}(P)}{(1 + r)^P} \qquad 2.17$$

maximizes NPV, the sum of harvest revenues $V_{l,q}(t)A_{l,q}$, where $V_{l,q}(t)$ is the harvest value per acre in year t, from management unit l, harvest sequence q.

The value of ending inventory is $V_{l,q}(P)$, which defines the residual stand value in year P, stand l, harvest sequence q. Area constraints defined by

$$\sum_{q=1}^{Q_l} A_{l,q} = n_l \qquad 2.18$$

require that the sum of all acres in management unit l assigned to one of the harvest sequences q, must equal n_l, the total acres in management unit l. Thus, all acres are assigned to a management sequence.

In Model I, individual acres present at the beginning of the planning period are assigned to a harvest sequence throughout the planning horizon. For every assignment there is a pre-defined contribution to the objective function value. The problem solution is simply the optimal set of assignments.

Berck (1976) introduced an alternative formulation, called Model III, which effectively *grows* individual acres within the forest. Recall that Adams and Ek (1974) used $x_i(t)$ to represent the number of trees in size class i at time t, and within each growth period the number of trees in a size class changed due to upgrowth, mortality, and harvests. Stand growth for Model III is represented by

$$y_{a+1}(t+1) = y_a(t) - c_a(t) \qquad 2.19$$

$$y_{MA}(t+1) = y_{MA}(t) + y_{MA-1}(t) - c_{MA}(t) - c_{MA-1}(t) \qquad 2.20$$

$$y_1(t+1) = \sum_{a=1}^{MA} c_a(t) \qquad 2.21$$

where $y_a(t)$ represents the number of acres in age class a at time t, and $c_a(t)$ represents the number of acres clearcut from age class a at time t. Equation 2.19 states that uncut acres at time t become one year older each year. Acres which are equal to or greater than a certain maximum age MA, can be aggregated into a single age class, as defined in equation 2.20. In equation 2.21, all acres clearcut in a given year are placed into age class 1 in the following year.

The objective function for Model III

$$NPV = \sum_{t=1}^{T} \sum_{a=1}^{MA} \delta^t \; V_a \; c_a(t) \qquad 2.22$$

where δ is the discount factor $(1/(1+r))$, sums the discounted value of all harvests using value per acre as a function of stand age, given by V_a. The model may be formulated as a linear programming problem and solved rather easily. The solution defines the optimal number of acres to clearcut in each age class over time.

Binary search techniques offer a numerical alternative for approximating optimal time paths for several control variables. Lyon and Sedjo (1983) described a search procedure to find optimal harvest levels (acres to clearcut) over a planning period, given initial number of acres in each age class and functions describing stand growth. An initial feasible time path for each control variable is calculated as a guess, and a search direction is determined based on derivatives of the objective function with respect to the control variables. This information is used to adjust the control variable values up or down to improve the solution. Large problems may be decomposed into a series of subproblems to determine equalities and inequalities that exist at the optimal solution. Solutions to the subproblems are then used to determine a numerical solution to the larger problem that lies very close to the optimal solution. This approach is useful for solving long-term timber supply problems which maximize net surplus as a surrogate for profit maximization.

The above formulations of forest-level management problems are based on even-aged stand structures. Stand value is a function of stand age, thus simplifying the analyses and leading to solutions which define the timing of regeneration harvests involving only clearcutting. Alternative silvicultural treatments involving intermediate thinnings or uneven-aged management can be included as additional activities in the linear programming problem. Intermediate commercial thinnings may be included as part of a particular harvest sequence, whereby thinning intensities and contribution to the objective function are determined before the problem is solved. This approach is limited, because only pre-specified alternative activities are considered. The global optimal solution may involve a silvicultural treatment not specified in the problem formulation. A numerical approach is needed whereby decision variables prescribe stand treatments, rather than choosing the best alternative from a predefined set.

An important forest-level management problem is to define the optimal harvest strategy for a forest composed of uneven-aged stands or a forest made up of both even-aged and uneven-aged stands. Getz and Haight (1989) suggested that decomposition of linear programming problems, as done by Hoganson and Rose (1984), would allow either even-aged or uneven-aged silviculture for an existing stand, and optimal solutions to forest-level problems could thus include both regeneration systems. However, published results of forest-level management optimization problems in which individual stands may be managed by either harvesting system were not found.

Solution Techniques

For uneven-aged stand-level problems, optimal timing of size-class harvests were found by linear programming (Buongiorno and Michie 1980) and nonlinear programming (Adams and Ek 1974). In general, growth equations developed for managed stands are nonlinear, so nonlinear optimization methods provide a mechanism for solving a wider variety of dynamic forest management problems. Bazaraa and Shetty (1979) discussed numerous algorithms and variations for solving nonlinear programming problems.

Adams and Ek (1974) optimized uneven-aged stand management using a gradient projection algorithm. This method involves projecting the negative gradient of the objective function in the null space of the equality constraints and binding inequality constraints. In the presence of nonlinear constraints, a correction move is required to achieve feasibility after moving along the projected gradient. The line search map for this algorithm is not closed, thus a proof of convergence is not available (Bazaraa and Shetty 1979).

Getz and Haight (1989) described gradient and penalty function methods for solving discrete-time optimal control problems, effectively treated as nonlinear programming problems. Haight and others (1985) solved a dynamic uneven-aged stand management problem using a gradient-based algorithm called method of steepest descent. From a beginning point, the search direction is determined as simply the negative gradient of a linear approximation of the objective function. This method performs well early in the procedure, but can lead to relatively inefficient zigzagging as a stationary point is approached.

Roise (1986a) compared the efficiency of three nonlinear programming techniques: simplex method of Nelder and Meade, Hooke and Jeeves method, Powell's method, and discrete dynamic programming to solve an unconstrained nonlinear program for optimizing management of stands with even-aged structures. The simplex method of Nelder and Meade compares functional values at the extreme points of a simplex, continually eliminating the worst point until an optimum is found. Hooke and Jeeves method employs coordinate directions and directions created by adjacent evaluation points to guide the numerical search. Powell's method optimizes quadratic functions in at most n steps if the search follows conjugate directions of the function's Hessian matrix. Near the vicinity of the optimal point, a general function can be represented by its quadratic approximation, thus this method is useful for nonquadratic functions as well. These methods differ from previous methods discussed in that they use functional evaluations without considering derivatives. Roise (1986a) found that coordinate and conjugate direction methods converged faster than the simplex method.

Many algorithms have been developed for solving nonlinear programming problems. This study utilized the projected Lagrangian algorithm (Appendix C) discussed by Robinson (1972). A later study could investigate the relative efficiency of alternative solution techniques.

CHAPTER 3: MULTI-STAND MANAGEMENT MODEL

Forest managers need detailed cutting prescriptions which account for the interaction of size, species, and other product value factors when planning harvests in individual stands. Cutting guides broadly defined by target residual basal area or volume per acre do not adequately describe how key economic factors such as size and species should be controlled to optimize management. Therefore, a basic requirement for this type of timber oriented optimization model is that resulting prescriptions must define the number of trees to remove in each size class and species group over time. In this chapter, a general model is described which can be used to optimize harvest schedules for a range of resource objectives.

Haight (1987) presented a model for uneven-aged management which allows the user to determine the optimal cut and residual diameter distributions over a sequence of periodic harvests in a single stand. A more general version of the model, presented later by Getz and Haight (1989), is a discrete-time optimal control formulation that defines harvest sequences in terms of the number of trees to cut and leave in each size class. The state variable $x_{i,j,k}(t)$ defines the number of trees per acre at the beginning of time t, in size class i, in species group j, in stand k. The control variable $h_{i,j,k}(t)$ represents the number of trees per acre to harvest at time t, in size class i, in species group j, in stand k. The control vector $\mathbf{h}(t)$ could be broken down into roundwood products such as pulpwood, sawtimber, or veneer. For this discussion, however, $\mathbf{h}(t)$ represents only the harvest of merchantable sawtimber, commercial species 11.0 inches dbh and larger.

The Objective Function

Optimization models include an objective function which simply measures the desirability of a particular solution. The solution algorithm systematically adjusts the level of the decision variables until no other solution has a higher objective function value. In this case, the objective function:

$$Max \;\; NPV = \sum_{i=1}^{MD} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{t=1}^{T} \delta^t \, p_{i,j,k}(t) \, h_{i,j,k}(t) - \sum_{t=1}^{T} \sum_{k=1}^{K} \delta^t \, fc_k(t) \qquad 3.1$$

maximizes the present value of all harvests starting with an initial forest $\mathbf{x}(0)=\mathbf{x}_0$. The first term in equation 3.1 sums the product of price $p_{i,j,k}$ times number of trees harvested $h_{i,j,k}$, and discounts each harvest revenue to time $t=0$, where δ is equal to the discount factor $1/(1+r)$, and r is a positive annual discount rate. Parameters MD, J, K, and T represent the maximum diameter class, number of species groups, number of stands, and number of time periods, respectively. The second term of equation 3.1 is a fixed cost $fc_k(t)$ associated with each harvest discounted to $t=0$. This general formulation can be expanded to include variable harvest cost, value of ending inventory, and planting costs.

Stand Growth Constraints

Stand growth constraints take the general form

$$\mathrm{x}(0) = \mathrm{x}_0 \qquad 3.2$$

$$\mathrm{x}(t+1) = G(\mathrm{x}(t),\mathrm{h}(t)) + F(\mathrm{x}(t),\mathrm{h}(t)) \qquad t = 0,1,2,\ldots,T \qquad 3.3$$

The number of trees at the beginning of the planning period $\mathbf{x}(0)$ is given as $\mathbf{x}_0$ in equation 3.2. In subsequent time periods, the initial number of trees $\mathbf{x}(t+1)$ is defined by two components given in equation 3.3. The first term $G(\mathbf{x}(t), \mathbf{h}(t))$ is a function which estimates growth of the residual stand from the previous period. The second term $F(\mathbf{x}(t), \mathbf{h}(t))$ is a function which estimates ingrowth into the smallest dbh class from the previous period.

Growth and yield simulators applicable to central hardwood forests can be formulated for use in a model of this type that is designed to optimize stand management. Transformed equations from the simulator take the form of equations 3.2 and 3.3 and serve as model constraints which define the dynamics of stand development brought about by harvesting decisions. In large models, representing forests with varying cover types, equations from several distinct simulators can be combined to account for individual stand growth.

Feasibility and Nonnegativity Constraints

Nonnegativity and feasibility constraints are defined by:

$$\begin{aligned} h(t) &\geq 0 \\ x(t) &\geq 0 \end{aligned} \qquad t=0,1,2,\ldots,T \tag{3.4}$$

$$x_0 - h(0) \geq 0 \qquad t=0 \tag{3.5}$$

$$x(t) - h(t) \geq 0 \qquad t=1,2,\ldots,T \tag{3.6}$$

Equation 3.4 assures that all initial stands and harvests are nonnegative, and equation 3.5 assures that the initial harvest does not exceed stand stocking at time $t=0$, the initial stand state. Equation 3.6 assures that the number of trees harvested does not exceed the initial number of trees in the stand at any time period t. Management constraints to achieve specific objectives can be added to this group of equations to further define feasibility.

Formulating Growth Constraints

In this study, stand growth constraints corresponding to equation 3.3 were derived from FIBER, a two-stage matrix stand growth simulator originally developed for spruce-fir and northern hardwood types in the Northeast (Solomon, Hosmer, and Hayslett 1987). Growth functions in FIBER were developed from empirical studies to provide a mathematical framework for estimating growth dynamics that may result from management decisions. In general, stand growth is projected with a stage structured model which transforms the residual stand vector $\mathbf{x}(t)$-$\mathbf{h}(t)$ to a new initial stand vector $\mathbf{x}(t+1)$ in the future using a transition matrix whose elements are nonlinear functions of $\mathbf{x}(t)$ and $\mathbf{h}(t)$. These nonlinear growth functions estimate the probability that trees in the initial stand die, survive and grow into a larger dbh class, or survive without growing into a larger size class.

Basics of FIBER

Regression equations in FIBER employ initial basal area in stand k ($IBA_k(t)$), residual basal area ($RBA_k(t)$), diameter class midpoint (D_i), proportion of total basal area comprising hardwood species ($PH_k(t)$), and proportion of total basal area comprising species j ($PS_{j,k}(t)$) at time t to predict transition and mortality probabilities over a 5-year period. For example, $u_{i,j,k}(t)$, the probability of a tree in dbh class i, species j, stand k

growing into dbh class $i+1$ during a growth period beginning at time t is given by

$$u_{i,j,k}(t) = \beta_{u,j} \; (IBA_k(t),\; RBA_k(t),\; PH_k(t),\; D_i) \qquad 3.7$$

where $\beta_{u,j}$ are estimated upgrowth coefficients for each species group j. Similarly, a transition probability is computed for $a_{i,j,k}(t)$, survival but no upgrowth by:

$$a_{i,j,k}(t) = \beta_{a,j} \; (IBA_k(t),\; RBA_k(t),\; PH_k(t),\; D_i) \qquad 3.8$$

and ingrowth into the smallest size class, in this case 6 inches dbh, is given by:

$$f_{6,j,k}(t) = \beta_{f,j} \; (RBA_k(t),\; PH_k(t),\; PS_{j,k}(t)) \qquad 3.9$$

Mortality for the growth period is estimated implicitly as $1-a_{i,j,k}(t)-u_{i,j,k}(t)$. Transition probabilities are then placed into a matrix $\mathbf{G}_{j,k}(t)$ which is applied to a residual stand vector $\mathbf{x}_{j,k}(t)-\mathbf{h}_{j,k}(t)$ plus ingrowth defined by $f_{6,j,k}$ to generate a stand vector $\mathbf{x}_{j,k}(t+5)$ by

$$\mathrm{x}_{j,k}(t+5) = \mathrm{G}_{j,k} \; (\mathrm{x}_{j,k}(t) - \mathrm{h}_{j,k}(t)) + f_{6,j,k} \qquad 3.10$$

Note that stand growth is dependent on harvest decisions as they affect species composition and residual basal area in each time period.

In summary, FIBER performs the growth simulation in two stages. In the first stage, elements of $\mathbf{G}(t)$ and $\mathbf{F}(t)$ are computed based on number of trees in each size class and species group before and after a harvest in each stand. In the second stage, a linear operation is performed according to equation 3.10 to estimate a new initial stand structure $\mathbf{x}(t+5)$ 5 years in the future. At each 5-year interval, the two-stage procedure is repeated so that growth is always a function of stand density and species composition at the beginning of each time period.

FIBER in Nonlinear Programming

For nonlinear programming problems, growth functions from FIBER are formulated as constraints which describe stand growth dynamics, one growth constraint for each combination of diameter class i, species group j, and stand k. For example, the general constraint for $x_{18j,k}(t+5)$, the number of trees in the 18-inch dbh class in species j, stand k, at time $(t+5)$ is derived according to equation 3.11.

$$x_{18j,k}(t+5) = a_{18j,k}(t)\,(x_{18j,k}(t) - h_{18j,k}(t)) + u_{16j,k}(t)\,(x_{16j,k}(t) - h_{16j,k}(t)) \tag{3.11}$$

Coefficients $a_{18j,k}$ and $u_{16j,k}$ are probabilities for surviving residual trees at time period t remaining in the 18-inch dbh class or growing up from the 16-inch class, respectively, during the growth period t to $t+5$. Equation 3.11 appears to be a linear operation, but transition probabilities are functions of $\mathbf{x}(t)$ and $\mathbf{h}(t)$ through the initial and residual basal area as shown in equations 3.12 and 3.13. Similar to the first stage of FIBER, transition probabilities for each diameter class i, each species group j, and each stand k are estimated by equations 3.12 and 3.13, plus ingrowth by equation 3.14, where β is a vector of regression coefficients estimated from growth data. There are 13 sets of regression coefficients in FIBER, one set for each species group.

$$a_{i,j,k}(t) = \beta_{0,a,j} + \beta_{1,a,j} IBA_k(t) + \beta_{2,a,j} RBA_k(t) + \beta_{3,a,j} D_i + \beta_{4,a,j} PH_k(t) + \beta_{5,a,j} D_i^2 + \beta_{6,a,j} RBA_k^2(t) \qquad 3.12$$

$$u_{i,j,k}(t) = \beta_{0,u,j} + \beta_{1,u,j} IBA_k(t) + \beta_{2,u,j} RBA_k(t) + \beta_{3,u,j} D_i + \beta_{4,u,j} PH_k(t) + \beta_{5,u,j} D_i^2 + \beta_{6,u,j} RBA_k^2(t) \qquad 3.13$$

$$f_{1,j,k} = \beta_{0,f,j} + \beta_{1,f,j} RBA_k(t) + \beta_{2,f,j} PH_k(t) + \beta_{3,f,j} PS_{j,k}(t) \qquad 3.14$$

Measures of stand stocking (RBA_k, IBA_k, PH_k, and PS_k) are functions of $\mathbf{x}(t)$ and $\mathbf{h}(t)$, computed as inner products of basal area at the midpoint of each dbh class b_i and vectors $\mathbf{x}(t)$ and $\mathbf{h}(t)$ as shown in equations 3.15-3.18. Diameter classes range from 6 inches dbh to *MD*, the selected maximum diameter, although 30 inches is the maximum recommended dbh for the FIBER growth model. In equations 3.15 and 3.16, $IBA_k(t)$ and $RBA_k(t)$ include all species in a particular stand k, thus the species group, subscript j, was dropped for simplicity. In equation 3.17, *hd* represents hardwood species.

$$IBA_k(t) = 0.196\ x_{6,k}(t) + 0.349\ x_{8,k}(t) + \ldots + b_{MD}\ x_{MD,k}(t) \qquad 3.15$$

$$RBA_k(t) = 0.196\ (x_{6,k}(t) - h_{6,k}(t)) + \ldots + b_{MD,k}\ (x_{MD,k}(t) - h_{MD,k}(t)) \qquad 3.16$$

$$PH_k(t) = \frac{0.196\ x_{6,hd,k}(t) + 0.349\ x_{8,hd,k}(t) + \ldots + b_{MD}\ x_{MD,hd,k}(t)}{RBA_k(t)} \qquad 3.17$$

$$PS_{j,k}(t) = \frac{0.196\ x_{6,j,k}(t) + 0.349\ x_{8,j,k}(t) + \ldots + b_{MD}\ x_{MD,j,k}(t)}{RBA_k(t)} \qquad 3.18$$

In formulating explicit growth constraints, equations 3.15-3.18 are substituted into equations 3.12-3.14, and then for each diameter class, equations 3.12-3.14 are substituted into a general form of equation 3.11. A complete generalized growth constraint defined in equation 3.19 is constructed for each size class i, species group j, in each stand k. Basal area measured at the midpoint of each dbh class is represented by b_i. Ingrowth defined in equation 3.14 is added to the smallest dbh class in each time period t. Note that much of the nonlinearity associated with the model is attributed to squared terms involving decision variables in the growth constraints.

In practice, equation 3.19 must be expanded for each species group j and stand k included in the analyses. In the general form presented in equation 3.19, each initial stand variable $\mathbf{x}(t)$ and harvest control variable $\mathbf{h}(t)$ is indexed only by i for dbh class, which is appropriate in the special case where there is only one stand containing a single species. In more practical applications, the NLP would contain growth constraints to represent all diameter classes, species groups, and stands included in the problem. FIBER can accommodate combinations of 13 species groups (Solomon, Hosmer, and Hayslett 1987).

$$
\begin{aligned}
x_i(t+1) = {} & (\beta_{0,a} + \beta_{1,a}\ [b_6 x_6(t) + \ldots + b_{MD}\ x_{MD}(t)] \\
& + \beta_{2,a}[b_6(x_6(t)-h_6(t)) + \ldots + b_{MD}(x_{MD}(t)-h_{MD}(t))] + \beta_{3,a} D_i \\
& + \beta_{4,a}[b_6 x_{6,hd}(t) + \ldots + b_{MD} x_{MD,hd}(t)] \ / \ [b_6(x_6(t)-h_6(t)) + \ldots + b_{md}(x_{MD}(t)-h_{MD}(t))] \\
& + \beta_{5,a} D_i^2 \\
& + \beta_{6,a}[b_6(x_6(t)-h_6(t)) + \ldots + b_{MD}(x_{MD}(t)-h_{MD}(t))]^2\ )\ (x_i(t)-h_i(t)) \\
& + (\ \beta_{0,u} + \beta_{1,u}\ [b_6 x_6(t) + \ldots + b_{MD} x_{MD}(t)] \\
& + \beta_{2,u}\ [b_6(x_6(t)-h_6(t)) + \ldots + b_{MD}(x_{MD}(t)-h_{MD}(t))] + \beta_{3,u} D_{i-1} \\
& + \beta_{4,u}[b_6 x_{6,hd}(t) + \ldots + b_{MD} x_{MD,hd}(t)] \ / \ [b_6(x_6(t)-h_6(t)) + \ldots + b_{MD}(x_{MD}(t)-h_{MD}(t))] \\
& + \beta_{5,u} D_{i-1}{}^2 \\
& + \beta_{6,u}\ [b_6(x_6(t)-h_6(t)) + \ldots + b_{MD}(x_{MD}(t)-h_{MD}(t))]^2\ )\ (x_{i-1}(t)-h_{i-1}(t))
\end{aligned}
\qquad 3.19
$$

CHAPTER 4: RESOURCE AND MANAGEMENT CONSTRAINTS

For particular management situations, a specialized multi-stand model can be custom built by incorporating appropriate stand structure constraints into the basic model. For example, constraints may be added to achieve desired species composition or stocking level in a particular stand. Also, total woodflow or spatial allocation of clearcut operations can be formulated as functions of residual number of trees, $\mathbf{x}_k$-$\mathbf{h}_k$, and/or harvest number of trees, $\mathbf{h}_k$, from a group of stands to assure that aggregate objectives are met.

With appropriate constraints in place, problem solutions maximize net present value of the forest, subject to other formulated resource constraints. In this chapter, important model constraints are defined in general terms and discussed as they relate to specific management problems. Examples are given to demonstrate the nature of stand-level and forest-level problems, each characterized by unique combinations of management constraints included in the formulation.

Single-stand Constraints

Model constraints can be stated such that bounds are placed on decision variables for a single stand. In the context of a multi-stand problem, single-stand constraints may affect how other stands are managed, but in building the model, such constraints need only affect decision variables in a particular stand. A good example is where esthetic goals call for a minimum level of residual stocking in large trees for a given stand k. In equation 4.1, the sum of residual basal area in *visually desirable* trees, defined as v inches dbh and larger, in stand k must be greater than or equal to the minimum visual basal area VBA.

$$\sum_{i=v_k}^{MD} \sum_{j=1}^{J} b_i(x_{i,j,k}(t) - h_{i,j,k}(t)) \geq VBA_k(t) \qquad 4.1$$

Other management goals can be incorporated into the model with constraints similar to equation 4.1. For instance, minimum stocking levels could be used to provide for wildlife habitat or even color of fall foliage by controlling the size distribution and species composition of residual growing stock. Note that equation 4.1 is indexed by time period t such that residual basal area levels can also remain constant or fluctuate according to management goals.

Single-stand constraints are usually guided by previous research results pertaining to particular forest resources. For example, parameters used in equation 4.1 would be based on established guidelines for visual impact, squirrel habitat, seed production, sunlight, and so on, that can be quantified in terms of residual tree stocking. The forest manager may evaluate opportunities available in each stand, and then set constraint parameters such that forest management is optimized subject to requirements in individual stands.

Local product markets may also influence minimum commercial harvest levels in individual stands. In central Appalachian hardwoods, minimum acceptable commercial harvest volume averages 2500 board feet per acre. This type of market-driven constraint affects all stands within the forest, but formulation of the constraint is based on variable values within individual stands. Harvest volume $hv_k(t)$

$$hv_k(t) = \sum_{i=1}^{MD} \sum_{j=1}^{J} v_{i,j} \, h_{i,j,k}(t) \qquad 4.2$$

is simply the sum of individual tree volumes $v_{i,j}$ harvested at time t.

In nonlinear programming problems, a penalty variable $PEN_k(t)$ can be defined as a function of number of harvest trees, $h_{i,j,k}(t)$, and used to constrain harvest volume $hv_k(t)$ to take on acceptable values. For example,

$$PEN_k(t) = \frac{1}{1+\exp(10 - 100 \sum_{i=1}^{MD} \sum_{j=1}^{J} h_{i,j,k}(t))} \qquad 4.3$$

$$PEN_k(t)\ (2500 - hv_k(t)) \leq 0 \qquad 4.4$$

define the penalty $PEN_k(t)$ as a logistic function of $\mathbf{h}_k(t)$ which takes on a value near zero if no trees are harvested or a value near 1 if the number of harvested trees per acre is greater than a small positive value (e.g., greater than or equal to one tree per acre). Equation 4.4 is then included in the problem formulation to require either no harvest or harvest volume in excess of a local market minimum. Note that equation 4.4 is satisfied when $hv_k(t) \leq 0$ *board feet per acre* or when $hv_k(t) \geq 2500$ *board feet per acre.*

Many additional stand-level constraints may be imposed to achieve management objectives in individual stands. For example, administrative objectives such as frequency of harvest may be achieved by constraining harvests to equal zero in particular time periods. Habitat may be controlled by requiring residual stand structures to lie between bounds conducive to particular wildlife species. So long as desired stand conditions can be expressed in terms of initial and harvest number of trees per acre at desired time periods, this formulation will allow the user to analyze a wide variety of optimization problems.

Forest-level Constraints

The characteristic feature of a multi-stand model is that certain management constraints link together decisions made on each of the individual stands included in the analyses. For example, forest managers may require that woodflow from the entire forest equal periodic volume growth and remain relatively stable over the planning period. However, optimal management of individual stands, in the absence of woodflow restrictions, may lead to unacceptable fluctuations in harvest volume. To assure a stable woodflow, bounds may be placed on the sum of harvests from all stands. As a result, harvest decisions among individual stands are interdependent, such that optimal forest management is not a simple summation of optimal stand management. Woodflow *WF(t)* is defined by:

$$WF(t) = \sum_{k=1}^{K} hv_k(t) ac_k \qquad 4.5$$

as the sum of per acre harvest volumes $hv_k(t)$ times the number of acres ac_k from each stand k in each time period t.

Woodflow may be constrained by placing upper and lower bounds on allowable fluctuations in *WF(t)*. In equation 4.6 and 4.7 *wf* represents the allowable deviation as a proportion of average woodflow over the planning period *P*. This formulation does not constrain total woodflow throughout the planning period. Total woodflow will depend on growth capabilities of the stand and any other constraints that affect harvests.

$$WF(t) \leq (1+wf) \frac{\sum_{t=0}^{P} WF(t)}{P} \qquad 4.6$$

$$WF(t) \geq (1-wf) \frac{\sum_{t=0}^{P} WF(t)}{P} \qquad 4.7$$

Spatial allocation of harvest operations is another important forest-level constraint which serves to link together management decisions on individual stands. Esthetic, recreation, habitat, or other environmental factors may require that adjacent stands not be harvested less that *sp* years apart. The simplified equation

$$hv_k(t)\ hv_{k+1}(t) + hv_k(t)\ hv_{k+1}(t+1) + \ldots + hv_k(t)\ hv_{k+1}(t+sp) = 0 \qquad 4.8$$

prevents cutting in adjacent stands k and $k+1$ in less than *sp* years (Roise 1990).

Other forms of forest-level management factors may be accounted for by including appropriate constraints in the problem formulation. For example, product demand, representing the influence of total harvest volume on product prices within a time period, may also be formulated as functions of $\mathbf{h}(t)$. One possible formulation:

$$Max\ \ NPV = \sum_{i=1}^{MD} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{t=1}^{T} \delta^t\ PAF(t) p_{i,j,k}(t)\, h_{i,j,k}(t) - \sum_{t=1}^{T} \sum_{k=1}^{K} \delta^t fc_k(t) \qquad 4.9$$

is to include a price adjustment factor *PAF(t)*, a function of total volume harvested, in the objective function to lower product prices as harvest volume increases as shown in equation 4.9. Instead of computing revenues based on a static price vector, $p_{i,j,k}(t)$, harvest value is the product of two factors, $PAF(t)$ *and* $p_{i,j,k}(t)$, thus accounting for local demand. This type of formulation is appropriate only for unusually large forest management problems where harvest volume is great enough to influence local market prices.

Although this model is flexible as a result of decision variables controlling numbers of trees in each size class and species group, problem dimensions increase greatly as additional stands or time periods are included. The remaining chapters in this dissertation deal with performance of the multi-stand management model presented in Chapters 3 and 4 and the impact of increasing problem size on the solution procedure.

CHAPTER 5: MODEL PERFORMANCE AND RESULTS

Performance of the multi-stand model was evaluated on a series of selected forest management problems, each designed to test a unique aspect of the model. Initial problems were formulated to test performance of the growth equations adapted from FIBER (Solomon et al. 1987). Representative problems were then formulated for individual stands to evaluate performance of the solution algorithm in dealing with unconstrained harvesting, specific non-timber residual stand goals, and harvest scheduling restrictions in separable multi-stand problems. Following validation of model capabilities, well-defined management problems such as optimal even-age or uneven-age management, given an initial stand structure, were solved. Forest-level constraints were then imposed to analyze nonseparable problems, those in which optimal stand management prescriptions are influenced by activities on other stands. Finally, the effect of problem dimensionality was evaluated by extending the planning period and increasing the number of stands.

Stand Dynamics

The initial series of problems simply required the model to project individual stand growth, with harvests constrained to equal zero, $\mathbf{h}(t) = 0$, over a 100-year planning horizon. Initial stand vectors $\mathbf{x}_k(t)$ represented given stand structures at time $t=0$, thus providing insight into the effect of initial stand structure on performance of stand growth constraints defined in equations 3.9.

Initial stand vectors were taken from selection and uncut stands on the Fernow Experimental Forest near Parsons, West Virginia. Site index (base age 50 years) for northern red oak on example stands ranged from 64 to 78. Projections made by the nonlinear programming (NLP) model were compared to actual stand development and to projections made directly by FIBER software. The primary purpose of these comparisons was to verify that coefficients from FIBER were properly formulated in the NLP model. A secondary purpose was to assess the applicability of FIBER equations for use in central Appalachian stands. In practical applications, particularly for the development of management guidelines, the accuracy of growth model projections is a critical concern. In this study, however, highest priority was afforded to developing a compact management model.

Example stands included a 55-year-old, second-growth mixed hardwood stand on site index 64 for northern red oak. A 100 percent inventory from this 12.5-acre control stand was taken in 1964, and again in 1974, and 1984, allowing for comparison with 10-year growth projections from FIBER and the multi-stand management model.

Initial performance tests compared actual and projected stand structure and basal area stocking. Basal area projections from the nonlinear programming (NLP) formulation varied less than 7 percent from direct FIBER projections. Projected stand development varied from observed stand development less than 5 percent at 10 years and less than 8 percent at 20 years (Table 5.1). Direct FIBER projections were obtained from a modified algorithm that provided a 2-inch stand structure needed for this comparison (Marquis 1990). Discrepancies in stand structure projections are due to conversions from 1-inch stand structures generated by FIBER to 2-inch stand structures. Results indicated that modelling the simulator as a system of nonlinear equations predicted as well as the two-stage matrix approach used by the FIBER program. Also, predictions from either formulation exhibited the degree of compatibility with central Appalachian hardwoods needed to test the capabilities of the multi-stand management model.

Table 5.1. Stand growth projections for a 55-year-old mixed hardwood stand on site index 64.

		10-year			20-year		
Dbh	Initial	Actual	NLP	FIBER	Actual	NLP	FIBER
	-------------------number of stems/acre--------------------						
6	77.1	57.1	52.3	42.7	45.5	40.6	30.6
8	38.6	40.1	49.3	53.5	31.5	45.8	45.9
10	20.7	25.3	28.6	30.5	25.7	37.7	41.2
12	10.2	14.3	14.7	16.5	17.8	24.0	23.4
14	7.7	9.1	9.2	9.4	13.0	13.9	12.8
16	6.1	7.6	6.7	7.0	9.0	8.7	8.1
18	4.6	4.9	4.9	5.8	7.0	6.3	6.2
20	3.1	3.3	3.6	3.9	3.6	4.6	4.6
22	1.8	2.7	2.3	2.4	3.0	3.0	3.0
24	1.7	1.7	1.4	1.7	2.0	1.9	1.9
26	1.4	1.1	1.1	1.4	1.7	1.4	1.4
28	0	1.0	0.4	0.4	1.1	0.6	0.6
	-------------------------ft^2/acre-------------------------------						
Basal area	96.8	108.6	106.0	113.8	123.7	133.2	130.5

Separable Problems

In the absence of forest-level constraints, which link together optimal cutting strategies for a group of stands, management goals in one stand should not influence optimal management of other stands in the forest. Such problems are described as separable. Moreover, optimal management of a group of such stands is found by simply optimizing management in each stand, treating each stand as if the others do not exist. As a further test on model performance, three separable problems were solved for a one-stand and a two-stand forest to determine if the solution algorithm would properly derive identical optimal solutions for each stand. The first case included only growth and non-negativity constraints, which represents the most flexible problem with no management constraints imposed. Throughout the remainder of the dissertation, this formulation is referred to as the "basic problem." In the second case, a constraint was added to the basic problem to require the residual stand structure to conform with a particular management objective in each time period. In the third case, an operational constraint was added to the basic problem to require harvests to occur only in certain time periods.

Recall that the model was designed to determine the optimal sequence of cutting prescriptions over the planning horizon to maximize net present value subject to growth and user-defined management constraints. Model constraints are simply mathematical representations of biological factors and management constraints. A wide range of concerns can be addressed in this formulation, if they can be expressed in terms of number of cut and leave trees per acre.

The Basic Problem: No Management Constraints

In the first case, no management constraints were included in the problem. Harvests were permitted in any time period and at any volume level, subject only to nonlinear stand dynamics and non-negativity described in Chapter 4. Optimal cutting strategy was not altered by increasing the number of stands. A relatively heavy harvest was made in the first and last periods, with light cuts made every 5 years in the interim (Table 5.2). The model correctly determined identical optimal cutting strategies for the one-stand and two-stand cases.

Results from the unconstrained model also shed light on logical aspects of the model. The first harvest removed all trees 16 inches dbh and larger, an indication that such trees are financially mature according to the price function defined in the model. Product prices were derived from tree value conversion standards (DeBald and Dale 1991), whose values are based in part on grade. Trees 16 inches dbh and larger qualify for the most valuable grade, allowing large increases in value from 14 to 16 inches dbh, but comparatively less value increase beyond 16 inches. The optimal cutting strategy simply applied a 16-inch diameter limit until the final period, when all merchantable volume was removed. Understandably, the objective function sensed no future periods beyond 100 years, so all remaining potential revenue was taken at that time by harvesting all merchantable trees.

Table 5.2. Optimal cutting prescriptions for the basic problem -- only growth and non-negativity constraints included.

		Cutting prescription by time period					
Dbh	Initial	t=0	t=5	t=25	t=50	t=75	t=100
	trees/acre	----------------- cut trees/acre ---------------------					
6	77.1	-	-	-	-	-	-
8	38.6	-	-	-	-	-	-
10	20.7	-	-	-	-	-	-
12	10.2	-	-	-	-	-	20.8
14	7.7	-	-	-	-	-	21.5
16	6.1	6.1	3.0	5.5	9.5	9.6	8.3
18	4.6	4.6	-	-	-	-	-
20	3.1	3.1	-	-	-	-	-
22	1.8	1.8	-	-	-	-	-
24	1.7	1.7	-	-	-	-	-
26	1.4	1.4	-	-	-	-	-
28	-	-	-	-	-	-	-
		----------board feet (Int. 1/4-inch rule)/acre------------					
harvest volume		5952	541	993	1705	1721	4465

Results were not consistent with existing management guidelines which take into account operational and administrative considerations. For example, interim harvests removed volumes as low as 541 board feet per acre, well below minimum harvests required by timber buyers in most central Appalachian hardwood sawtimber markets. A more practical approach is to lessen the frequency of partial harvests to a 10- or 20-year interval. An alternative model structure could also be employed to penalize the objective function for low volumes or to make the price vectors $\mathbf{p}(t)$ functions of product volume and size. As harvest volume increases, efficiency is improved and administrative and logging costs are reduced. As a result, the landowner usually realizes a higher stumpage price. In this test case, however, the model is not able to quantify such increased efficiency in terms of increased net present value from higher volumes or less frequent harvests.

Residual Stand Constraints

One approach to achieving certain management goals is to constrain the residual stand structure, such that optimal harvests are made subject to quantifiable limits. For example, landowners may have stand goals that relate to esthetics, requiring a minimum residual stocking of large trees. In another test of the multi-stand management model, residual basal area in trees 14 inches dbh and larger was required to exceed 14 ft^2 per acre in a particular stand k as defined by

$$\sum_{i=14}^{MD} \sum_{j=1}^{J} b_i(x_{i,j,k}(t) - h_{i,j,k}(t)) \geq 14 \qquad 5.1$$

Equation 5.1 imposed a "big tree" constraint, requiring a minimum basal area in large trees to meet a particular landowner goal, while the basic problem described previously had no such restriction. As expected, *NPV* was reduced compared to the unconstrained case, and optimal cutting strategy was not influenced by increasing the number of stands. So long as the inputs for each stand k are the same in a multi-stand problem, the optimal cutting prescription is the same for each stand (Table 5.3).

The residual basal area constraint was met by retaining some 16-inch trees in addition to 14-inch residual trees that were cut in the unconstrained case. In the two-stand case, only one stand was required to meet the esthetic residual basal area level. Optimal cutting strategy for the stand with the esthetics constraint was identical to the one-stand case. Cutting strategy for the other stand was identical to the unconstrained case in the previous example.

Although this test case was very simple in structure, the results indicated that management constraints may be applied to particular stands without influencing optimality in companion stands if stand management problems are separable. However, if forest-level constraints are imposed, whereby optimal levels of control variables in several stands are interdependent, constraints on one stand may affect optimality on other stands included in the problem.

Table 5.3. Optimal cutting prescriptions for stand with minimum of 14 ft^2 of residual basal area in trees 14 inches dbh and larger in each time period.

		Cutting prescription by time period					
Dbh	Initial	t=0	t=5	t=25	t=50	t=75	t=100
	trees/acre	--------------------cut trees/acre-----------------------					
6	77.1	-	-	-	-	-	-
8	38.6	-	-	-	-	-	-
10	20.7	-	-	-	-	-	-
12	10.2	-	-	-	-	-	20.9
14	7.7	-	-	-	-	-	8.6
16	6.1	2.0	1.7	5.4	9.4	9.6	8.3
18	4.6	4.6	1.5	-	-	-	-
20	3.1	3.1	-	-	-	-	-
22	1.8	1.9	-	-	-	-	-
24	1.7	1.7	-	-	-	-	-
26	1.4	1.4	-	-	-	-	-
28	-	-	-	-	-	-	-
		--------board feet (Int. 1/4-inch rule)/acre---------					
harvest volume		5208	730	975	1695	1728	3442

Operational Constraints

Another category of problem constraints can be imposed to represent desired administrative controls on one or more stands. For example, such constraints may be used to control the frequency of commercial harvest operations, length of rotation, timing of thinnings, or other planning objectives. The harvest vector $\mathbf{h}(t)$ is simply set to zero for the desired years, or in the case of clearcutting, requiring the harvest to equal the initial stand $\mathbf{h}(t) = \mathbf{x}(t)$ has the effect of assuring a complete harvest in year t. With such constraints in place, optimal cutting strategies are defined subject to desired timing restrictions imposed by management.

To test the effect of harvest frequency constraints, $\mathbf{h}(t)$ was constrained to zero except in 20-year intervals from $t=0$ through $t=100$ in the analysis period. In the unconstrained case, cutting was permitted every 5 years. Reducing the frequency of harvests lowered *NPV* compared to the unconstrained case involving only growth and nonnegativity restrictions. Again, optimal harvest strategies were not affected by increasing the number of stands in the problem. In the absence of forest-level factors which link stand harvests together, optimal cutting prescription for each stand are independent of other stands. As a result, optimal cutting strategy was identical for each stand. Optimality took the form of a 16-inch dbh diameter limit harvest repeated every 20 years (Table 5.4). Sawlog volume ranged from 3,000 to 7,400 board feet per acre up to the final period where all 10,000 board feet of merchantable volume was removed.

The solution to the 20-year cutting cycle problem is similar to the unconstrained case. At each harvest, all financially mature trees are removed, and in the last period, where the model does not consider future values, all merchantable trees are removed.

Table 5.4. Optimal cutting prescription for 20-year cutting cycle over a 100-year planning horizon.

		Cutting prescription by time period.					
Dbh	Initial	t=0	t=20	t=40	t=60	t=80	t=100
	trees/acre	----------------------cut trees/acre---------------------					
6	77.1	-	-	-	-	-	-
8	38.6	-	-	-	-	-	-
10	20.7	-	-	-	-	-	-
12	10.2	-	-	-	-	-	21.3
14	7.7	-	-	4.5	-	-	22.1
16	6.1	6.1	8.7	16.9	20.2	20.7	18.5
18	4.6	4.6	3.8	6.6	8.5	9.9	9.4
20	3.1	3.1	1.0	1.4	1.9	2.4	2.4
22	1.8	1.8	0.1	0.1	0.1	0.2	0.2
24	1.7	1.7	-	-	-	-	-
26	1.4	1.4	-	-	-	-	-
28	-	-	-	-	-	-	-
	-------------board feet (Int. 1/4-inch rule)/acre------------						
harvest volume		5,952	3,009	5,793	6,743	7,404	9,976

Summary of Preliminary Problem Solutions

Preliminary problem solutions validated growth projections and, to some degree, economic logic in the performance of the basic management model. Growth equations derived from the FIBER growth and yield model were shown to adequately represent stand dynamics as functions of stand density variables before and after each periodic harvest. The projected Lagrangian algorithm was capable of optimizing a harvest strategy over a 100-year planning with only growth and non-negativity constraints in place. Preliminary analyses further demonstrated that constraints placed on residual stand density to represent specific management objectives could be formulated within separable, single or multi-stand problems. Finally, arbitrary administrative constraints to control the frequency or intensity of harvests had the effect of reducing the feasible region, resulting in relatively lower *NPV* values compared to the unconstrained case, and further demonstrated the capability of the solution algorithm to converge for a variety of problems formulations. Problem solution time ranged from 1 to 2 hours on a PC-compatible 386/20 MHz processor. In general, solution time increased with the number of constraints included in the problem, but time requirements were increased most dramatically as the number of stands increased.

Applications of the management model were then extended to more realistic stand-level problems involving even-age and uneven-age silviculture. In both cases, the procedure was to first establish a target harvest strategy given the growth coefficients for a particular site quality and species composition (Getz and Haight 1989). Then the problem was reformulated to define an optimal transition harvest strategy to manage an existing stand structure until the target harvest strategy is achieved. Two general problems were solved: optimal transition from an existing stand to an optimal series of perpetual rotations for even-age silviculture and optimal transition from an existing stand to an optimal series of periodic partial harvests for uneven-age silviculture.

Optimizing Even-age Silviculture

In the single-stand case, where the initial stand structure is given as $\mathbf{x}(0)=\mathbf{x}_0$, it was possible to derive the optimal R period rotation based on productivity of the site as represented in the growth equations, plus the optimal prescription for managing the existing stand until it is harvested in conversion year *CY* and a series of perpetual optimal rotations is begun. In practice, the optimal rotation problem is independent of the conversion problem. The first phase is the simple problem of maximizing the present value of a perpetual series of identical rotations starting with bare ground defined by:

$$Max\ Q_R = \frac{\sum_{i=1}^{MD}\sum_{j=1}^{J}\sum_{t=0}^{R}\delta^t\, p_{i,j}(t)\ h_{i,j}(t)}{1-\delta^R} \qquad 5.2$$

For this example, data from a 50-year-old even-aged Appalachian hardwood stand on site index 70 were used to represent stand development following a clearcut regeneration harvest. In the absence of a reliable hardwood regeneration model, it was assumed that a complete harvest any time beyond age 50 would repeatedly result in a stand structure similar to the example stand 50 years following harvest (Table 5.5). The assumed stand structure at age 50 following a clearcut varies by site quality, land use history, genetics, and other key factors. In deriving a beginning point for analyzing the even-age problem, care must be taken to choose a stand structure which is most representative of stands growing on a particular site. In this case, data were obtained from a second-growth stand typical of stands throughout the surrounding area.

The optimal rotation age R^* was found by solving equation 5.2 using a range of rotation ages from $R=50$ to $R=90$, and comparing values of Q_R. A simple management problem was formulated which included equation 5.2 as the objective function, growth constraints appropriate for the example stand, and constraints which required a complete harvest in a particular time period. It is possible to evaluate the influence of intermediate thinnings on optimal rotation length by relaxing constraints on $\mathbf{h}(t)$. For this example, however, it was sufficient to demonstrate the technique for the *no thinning* case. Q_R was maximized at $R^*=80$ *years* (Table 5.5).

The second phase of the even-age silviculture problem involves maximizing the present value of a sequence of harvests needed to manage an existing stand until the optimal sequence of rotations is begun. The general problem is defined by:

$$Max\ NPV = \sum_{i=1}^{MD} \sum_{j=1}^{J} \sum_{t=0}^{CY} \delta^t\ p_{i,j}(t)\ h_{i,j}(t)\ +\ \delta^{CY}\ Q_R. \qquad 5.3$$

The first term in equation 5.3 sums the present value of harvest revenues during the conversion period. In the example problem, harvests were constrained to $\mathbf{h}(t)=0$ except in conversion year CY, representing the simplest case where conversion is completed in one harvest at time $t=CY$. The second term in equation 5.3 is the present value of the optimal sequence of rotations obtained in the first phase of the solution. The second-phase problem was solved for $CY=0$ to $CY=40$ in 5-year intervals and the even-age silviculture problem solution was obtained by comparing *NPV* for each conversion year. The optimal conversion strategy was to allow the stand to continue to grow until time $t=30$ when the example stand is 80 years old (Table 5.5). As in the first phase analysis, it is also possible to analyze the effect of intermediate harvests on the conversion strategy.

While the optimal sequence of perpetual rotations can be solved independent of the conversion problem, the optimal conversion harvest strategy depends on the initial stand structure $\mathbf{x}(t=0)$ and the value of the optimal rotation strategy as shown in equation 5.3. In general, dense or mature stands require immediate conversion as a result of their relatively inefficient growth. On the other hand, young stands or stands capable of thrifty growth due to reduced stocking require longer periods before conversion to the optimal sequence of perpetual rotations.

The general problem of optimizing even-age silviculture for a single stand is the sum of a two-phase analysis given by:

$$Max \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{CY} \delta^{t}\, p_{i,j}(t)\, h_{i,j}(t) + \delta^{CY}\left[\frac{\sum_{i+1}^{I}\sum_{j=1}^{J}\sum_{t=CY+1}^{CY+R} \delta^{t}\, p_{i,j}(t)\, h_{i,j}(t)}{1-\delta^{R}}\right] \tag{5.4}$$

Site productivity, as defined by growth constraint coefficients, is the major determinant of the optimal sequence of perpetual rotations optimized in the second term. The optimal sequence of perpetual rotations is independent of the conversion strategy. Note that the optimal conversion strategy, represented in the first term, is dependent on both the initial stand structure and the optimal perpetual management strategy.

Table 5.5. Stand structure, land expectation value (LEV), NPV including conversion and discounted LEV for perpetual rotations, and harvest volume by rotation age.

	Rotation age (years)				
Dbh	50	60	70	80	90
	--------------------number of trees/acre--------------------				
6	56.0	38.6	33.0	30.3	28.0
8	37.1	41.8	37.1	32.4	28.4
10	23.3	32.0	35.2	33.7	30.3
12	13.0	20.3	26.7	29.8	29.5
14	6.3	11.5	17.1	21.9	24.6
16	3.0	5.8	9.7	13.8	17.4
18	1.6	2.8	4.9	7.6	10.6
20	0.8	1.4	2.4	3.8	5.6
22	0.7	0.8	1.2	1.8	2.7
24	-	0.4	0.6	0.8	1.2
26	-	0.1	0.2	0.3	0.5
28	-	-	-	0.1	0.2
	----------------------$/acre------------------------				
LEV	11	24	32	34	32
NPV	101	179	225	239	225
	--------board feet/acre (Int. 1/4-inch rule)----------				
Periodic harvest	2853	5040	7856	13960	16628

Optimizing Uneven-age Silviculture

The uneven-age silviculture problem, given an initial stand structure $\mathbf{x}(0)=\mathbf{x}_0$, involves defining a harvest strategy which optimizes transition to an investment-efficient steady state. Similar to the even-age management problem, the uneven-age silviculture problem is solved in two phases. The first phase derives the optimal investment-efficient steady state, $\mathbf{x}^*$, $\mathbf{h}^*$ which provides a perpetual periodic yield. The second phase derives the optimal transition harvest schedule to achieve the desired steady state within a specified transition period *tr*.

The concept of investment efficiency was proposed by Duerr and Bond (1952) and simply equates the marginal value growth percentage with the discount rate. The investment-efficient steady state is determined by the objective function:

$$Max \ \frac{\sum_{i=1}^{I}\sum_{j=1}^{J}\delta^{t} \ \ p_{i,j}(t=cc)h_{i,j}(t=cc)}{1-\delta^{t}} - \sum_{i=1}^{I}\sum_{j=1}^{J} \ p_{i,j}(0)(x_{i,j}(0)-h_{i,j}(0)) \qquad 5.5$$

The first term is the discounted value of all future harvests, removed at regular intervals equal to the cutting cycle *cc* in perpetuity. The second term is the value of the residual steady state stand, viewed as an opportunity cost of the invested capital to produce all future harvests. Equation 5.5 is solved subject to a steady state condition,

$$x^* = G(x^* - h^*) + F(x^* - h^*) \qquad 5.6$$

which requires each harvest $\mathbf{h}^*$ to equal periodic growth. Equation 5.6 also requires that periodic harvests are identical, and such harvests result in identical residual stands $\mathbf{x}^*$-$\mathbf{h}^*$ in perpetuity.

The first phase of the uneven-age management problem does not consider the effect of an initial stand structure. Variables $\mathbf{x}(t)$ and $\mathbf{h}(t)$, representing the initial stand and harvest at time t, respectively, are permitted to take on any non-negative value, subject to growth constraints and sustained yield constraints (equation 5.6) to solve for the investment-efficient steady state. The only time periods relevant to the investment-efficient solution are $t=0$ and $t=cc$, the length of one cutting cycle. The denominator in the first term of equation 5.5 accounts for an unending series of identical harvests in the future. The optimal cutting cycle is found by solving for the investment-efficient steady state over a range of cutting cycles and comparing the objective function value of the solutions.

Without a fixed cost associated with each harvest, *NPV* declines as the cutting cycle is lengthened. In practice, however, a sale preparation or administrative cost must be incurred, thus reducing the present value of short cutting cycles to a greater extent than longer cutting cycles. For the example problem based on growth coefficients representing site index 70, *NPV* was maximized at a 10-year cutting cycle, $cc=10$ (Table 5.6).

Once the investment-efficient steady state and optimal cutting cycle have been determined, the remaining problem is to optimize the transition strategy from the initial stand $\mathbf{x}(0)=\mathbf{x}_0$ to the target steady state $\mathbf{x}^*$, $\mathbf{h}^*$ within the optimal transition period tr^*. The optimal transition strategy is also found by solving the problem using a range of transition periods tr and comparing the resulting present values. The objective function of the transition harvest problem is defined by:

$$Max \quad \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=0}^{tr}\delta^{t}\,p_{i,j}(t)h_{i,j}(t) + \delta^{tr}\,\frac{\sum_{i=1}^{I}\sum_{j=1}^{J}\delta^{cc}\;p_{i,j}h_{i,j}^{*}}{1-\delta^{cc}} - \delta^{tr}\sum_{i=1}^{I}\sum_{j=1}^{J}\;p_{i,j}(x_{i,j}^{*}-h_{i,j}^{*}) \qquad 5.7$$

The first term in equation 5.7 is the sum of harvest revenues throughout the transition period, discounted to time $t=0$. The second and third terms represent the present value of subsequent perpetual harvests and the capital investment of the residual stand at the investment-efficient steady state, respectively. Constraints defined in equations 5.8 and 5.9 are included in the transition problem to require the residual stand in year tr and the harvest in year $tr+cc$ to equal the investment-efficient steady state $\mathbf{x}^*$-$\mathbf{h}^*$ and $\mathbf{h}^*$, respectively.

$$x_{i,j}(tr) - h_{i,j}(tr) = x^* - h^* \qquad 5.8$$

$$h_{i,j}(tr+cc) = h^* \qquad 5.9$$

In general, present values increase asymptotically as the transition period is lengthened (Getz and Haight 1989). Lengthening the transition period has the effect of relaxing the time constraint implicit in constraints 5.8 and 5.9. However, increases in present value are slight beyond eight cutting cycles (Figure 5.1). Field trials have shown that conversions of second growth hardwoods in Appalachian forests require about five periodic harvests to reach a target stand structure. Conservative cuts are required in the first two or three periodic harvests to avoid overcutting and causing interruptions in woodflow in future harvests.

Table 5.6. Investment-efficient steady state residual stand structure and periodic harvest by cutting cycle length.

	Cutting cycle (years)							
	5		10		15		20	
Dbh	Residual	Cut	Residual	Cut	Residual	Cut	Residual	Cut
	---number of trees/acre---							
6	23.3	-	24.2	-	25.1	-	25.9	-
8	20.7	-	21.4	-	22.0	-	22.4	-
10	19.1	-	19.8	-	20.3	-	20.6	-
12	18.0	-	18.7	-	19.2	-	19.5	-
14	17.3	-	17.9	-	18.4	-	18.7	-
16	-	6.9	-	11.0	-	13.5	-	15.1
18	-	-	-	2.6	-	5.5	-	8.0
20	-	-	-	-	-	0.8	-	2.2
22	-	-	-	-	-	-	-	0.2
	---board feet/acre (Int. 1/4-inch rule)---							
harvest volume		1243		2698		4260		5852
	---$/acre---							
NPV		433		441		427		402

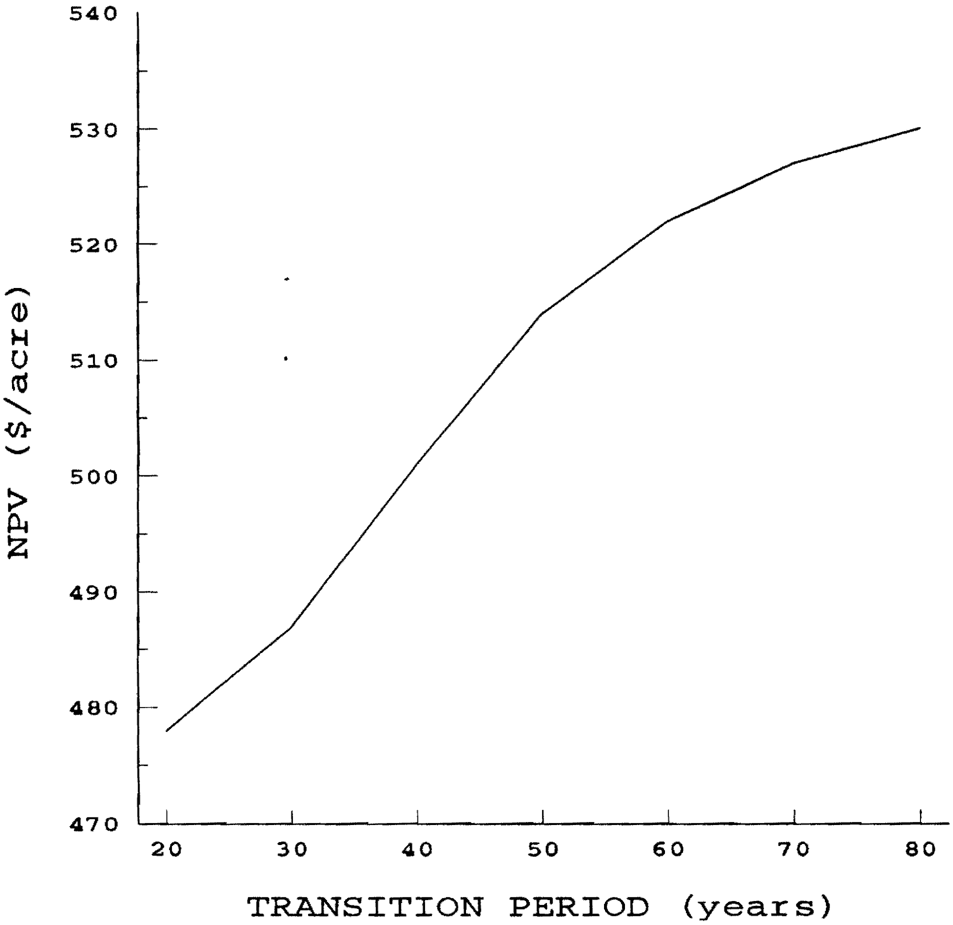

Figure 5.1. Net present value (NPV) related to transition period for the investment-efficient 10-year cutting cycle.

Nonseparable Multi-Stand Problems

Nonseparable problems include key management constraints which make optimal harvest strategies on individual stands interrelated with activities occurring on other stands. The constraints serve to link activities on stands included in the analyses based on how each stand contributes to a forest-level goal or market factor. For example, a woodflow constraint was imposed on a series of problems to require total harvest volume, from all stands combined, to remain within selected bounds over the planning period. The constraint was expressed in terms of an acceptable range within which total volume harvested from the forest could fluctuate as defined in

$$\sum_{i=1}^{MD}\sum_{j=1}^{J}\sum_{k=1}^{K} v_{i,j,k}\, h_{i,j,k}(t)\ ac_k \leq (1+wf)\ \frac{\sum_{i=1}^{MD}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T} v_{i,j,k}\, h_{i,j,k}(t)\ ac_k}{T} \tag{5.10}$$

$$\sum_{i=1}^{MD}\sum_{j=1}^{J}\sum_{k=1}^{K} v_{i,j,k}\, h_{i,j,k}(t)\ ac_k \geq (1-wf)\ \frac{\sum_{i=1}^{MD}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T} v_{i,j,k}\, h_{i,j,k}(t)\ ac_k}{T} \tag{5.11}$$

Terms on the left side simply sum harvest volume from all stands in a particular time period t. Terms on the right side specify the allowable percentage *(wf)* deviation from the average harvest determined as the total volume harvested in all time periods divided by the number of cutting periods. Note that the level of woodflow is endogenous to model because constraints on volume harvested are functions of $\mathbf{h}(t)$, instead of arbitrary bounds set by the user.

Example Problem

A simple problem was formulated to test woodflow constraints on single-stand and multi-stand problems. Woodflow constraints similar to equations 5.10 and 5.11 were added to the basic problem discussed above where harvests are permitted in any time period subject only to nonlinear growth and non-negativity constraints. Woodflow was allowed to fluctuate by 25 percent *($wf=0.25$)* of average harvest volume over 21 time periods *($T=100$ years)*. In the single-stand problem, the addition of woodflow constraints reduced *NPV* from \$1027 to \$957 per acre, a result of shrinking the feasible decision space for $\mathbf{h}(t)$. The problem was then expanded to include two identical stands, where woodflow in each period was equal to the sum of harvests from both stands. *NPV* per acre increased to \$958 per acre in the two-stand problem.

The slight increase in the objective function value reflects a relaxation of harvest constraints at the stand level, allowing greater fluctuation in harvest for a single stand while balancing the forest-level woodflow constraint with harvests from the accompanying stand. In effect, harvests from individual stands may be timed more efficiently in the two-stand case compared to the single-stand case where the woodflow constraint is more restrictive.

For a single stand with no management constraints, optimal harvests begin with a heavy cut, before values are subject to discounting, followed by a gradual increase in harvest volume over the next 50 years (Figure 5.2). When woodflow constraints were imposed on the single-stand case, the initial cut was greatly reduced compared to the unconstrained problem. For two stands, where the total woodflow from both stands is constrained, fluctuations in harvest volume within each stand are greater than in the single-stand case. However, the average woodflow from both stands shows variation which is nearly identical to the single-stand, constrained case. Although difficult to detect in Figure 5.2, average woodflow in the two-stand case is slightly greater than woodflow in the single-stand case. The two-stand case is less restrictive on individual stands, allowing fluctuations in harvest volume for each stand which would be infeasible in the single-stand problem. As a result of the expanded feasible decision space, a slightly greater average woodflow and *NPV* are possible in the two-stand case.

In the two-stand example, the forest-level constraint had little impact on *NPV*, although optimal cutting prescriptions were changed dramatically on individual stands to meet the woodflow constraint. The nonseparable management problem example was expanded to four identical stands to determine if additional stands would have the effect of further relaxing the woodflow constraint on individual stands and increasing *NPV* compared to the two-stand case. Results indicated that woodflow constraints continued to become less binding on individual stands, increasing *NPV* to $959 per acre in the four-stand case.

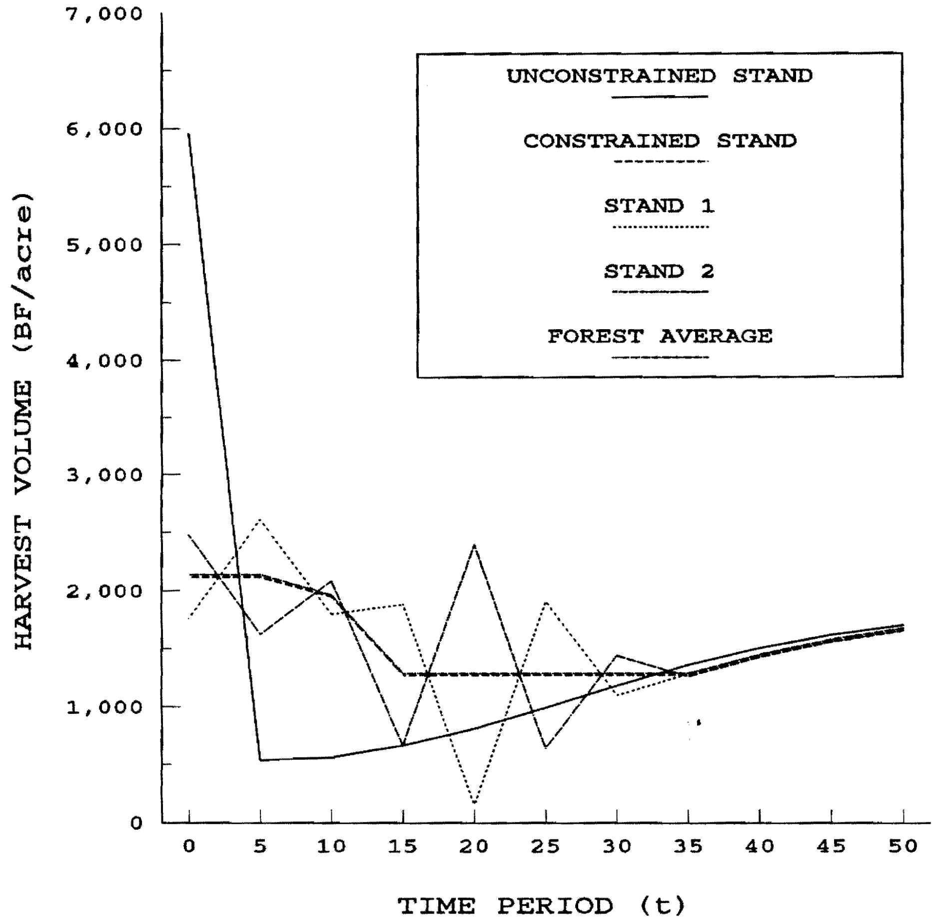

Figure 5.2. Fluctuation in harvest volume: a stand with and without a woodflow constraint, two identical stands with aggregate woodflow constraint, and average woodflow from both stands.

To further expose the effect of forest-level constraints, another example problem was formulated for a four-stand forest in which each stand had a unique initial stand structure. Previous examples evaluated forest problems where initial stand structures were identical. Two cases were compared to show the difference between separable and nonseparable multi-stand problems for four unique stands. In the first case, woodflow constraints were placed on each stand such that optimal harvests were independent, thus representing a separable problem. The initial stand structure in one stand was identical to the single-stand case discussed above. As expected, the resulting optimal cutting prescription for this stand was also identical to the single-stand case. In the second case, woodflow constraints were placed on the total volume removed from all four stands combined, thus representing a nonseparable problem.

NPV was $929 per acre for the nonseparable formulation, compared to $883 per acre for the separable formulation with woodflow constraints. Higher *NPV* for the nonseparable case was due to greater flexibility in harvests on individual stands. Note that the nonseparable formulation removed greater volume per acre than the separable formulation in the first three harvests, taking advantage of lower discount factors in early time periods (Figure 5.3). Wide variation in harvest volume for individual stands is possible in the nonseparable case (Figure 5.4). However, relatively tight restrictions on individual stands render such fluctuations in harvests infeasible in the separable case.

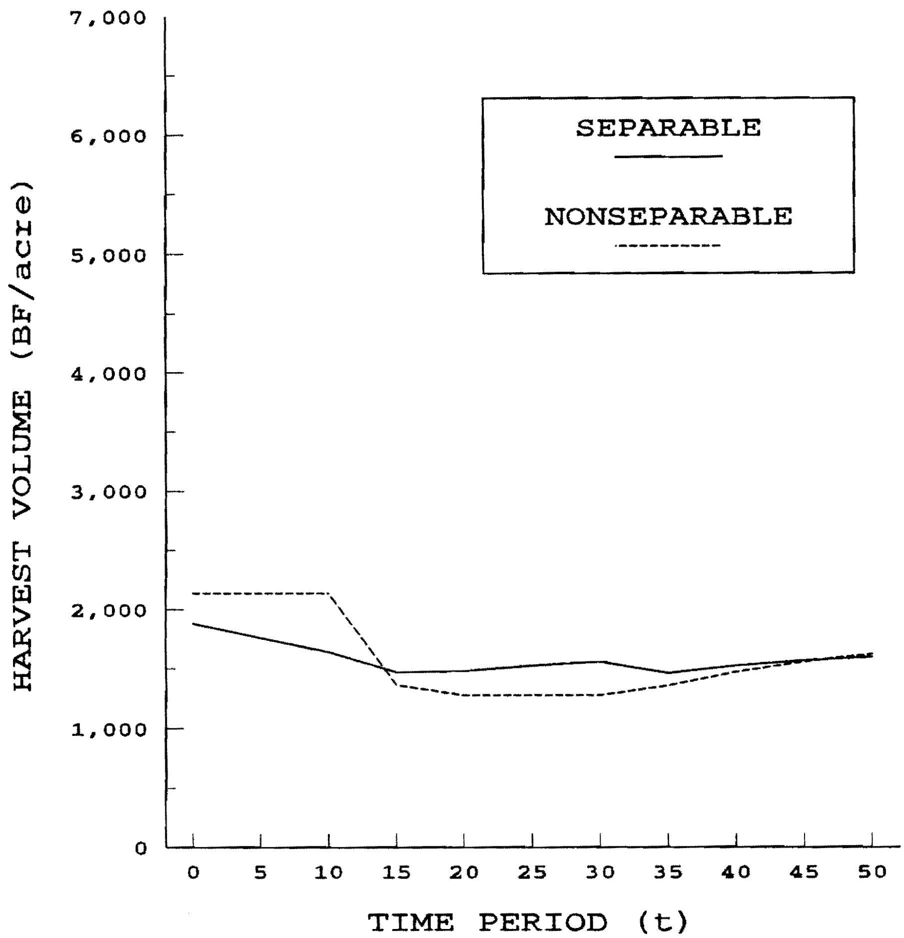

Figure 5.3. Average harvest volume for four unique initial stands, comparing a separable and nonseparable woodflow constraint formulation.

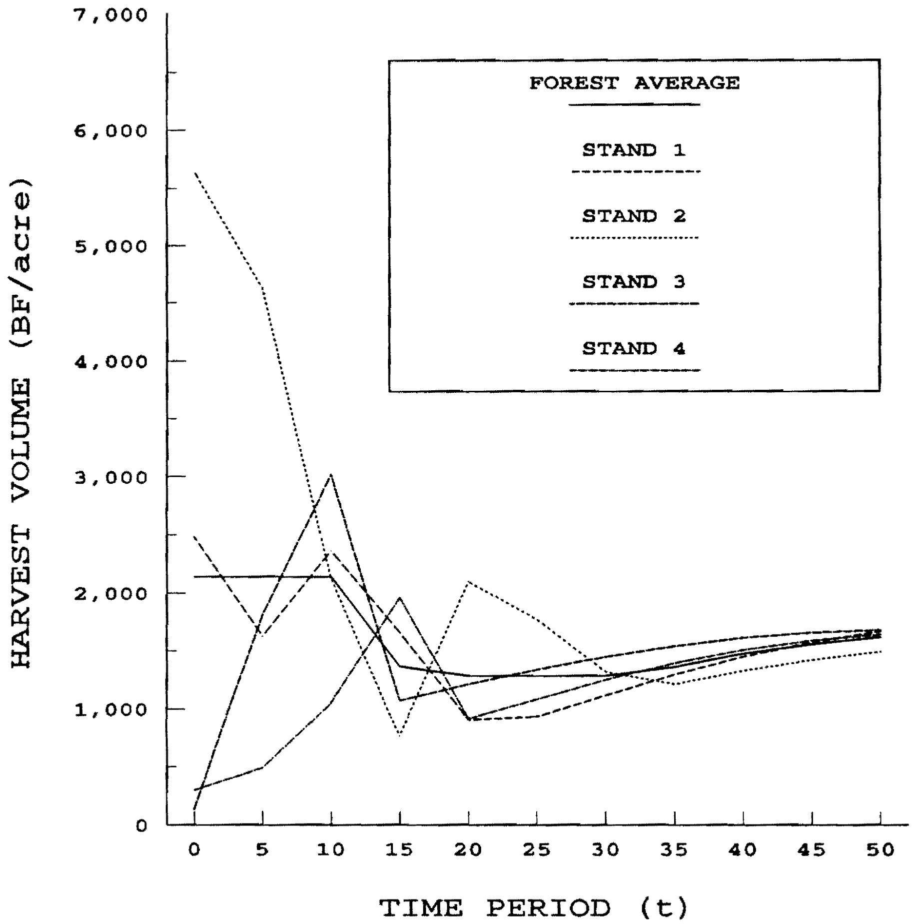

Figure 5.4. Harvest volume for four unique initial stands and average harvest volume for all stands under a nonseparable woodflow constraint formulation.

Combining Silviculture Systems

In multi-stand problems, individual stands may be managed using distinct silvicultural systems. Some stands may be assigned to uneven-aged silviculture involving relatively frequent partial harvests, while other stands may be assigned to even-aged silviculture involving clearcutting which is repeated at a given rotation age. Earlier in Chapter 5, it was shown that the optimal rotation age and optimal sequence of transition harvests could be found for the single-stand case.

Forest-level constraints such as the woodflow restriction discussed above can affect the optimal solution to even-aged silviculture on an individual stand. The woodflow constraint requires harvesting decisions on the even-aged stand to be made subject to concurrent harvests in other stands. To demonstrate how even-aged and uneven-aged silviculture can be optimized on the same forest, subject to a forest-level constraint, one stand in the four-stand case was constrained to even-aged silviculture, while the others continued to be managed using periodic partial harvests.

Similar to the single-stand case, the multi-stand problem was solved using a range of rotation ages, and optimal solutions were compared to determine the optimal rotation age for the even-aged stand and the optimal harvest schedules for the remaining uneven-aged stands. In the single-stand case *NPV* was maximized at rotation age $R=80$ years. In the presence of woodflow constraints for the four-stand case, *NPV* was maximized at $R=70$

years. One reason for this result is that a longer rotation results in greater volume removed at each harvest of the even-aged stand. However, due to the woodflow constraint, higher harvest volume for the even-aged stand requires reduced harvests in the uneven-aged stands to satisfy the forest-level volume limits. At $R=70$, an optimum balance of harvest volume is achieved. The benefits of increased harvests from the uneven-aged stands outweigh the benefit of lengthening rotation age in the even-aged stand beyond 70 years.

Average woodflow was also affected by adding the even-aged stand. Note that the additional constraints for the even-aged stand resulted in lower average woodflow over time (Figure 5.5). In years when the even-aged stand is harvested, average woodflow from the forest increases as shown for time period 20. However, harvests in the accompanying uneven-aged stands fluctuate in early transition periods as an equilibrium harvest level is achieved (Figure 5.6). The initial stand structure of individual stands within the forest will determine early transition harvests. As the forest-level problem is enlarged, as was shown in previous example problems, management constraints which require some stands to be even-aged will become less binding on individual stands. With additional stands, fluctuations in harvest volume for uneven-aged stands would be less dependent on large periodic harvests in the even-aged stands. Stands become less responsive to harvests in other stands because individual clearcuts make up a smaller, less influential portion of the total woodflow from the forest.

For large forest-level problems involving perhaps dozens of stands, any number of constraints may be placed on individual stands, subgroups of stands, or the entire forest. However, for a given forest size, the addition of constraints brings about a reduction in the feasible decision space for the stands involved and an associated decrease in *NPV.* In this example, adding constraints to model even-aged silviculture on one stand reduced *NPV* from $957 to $775 per acre compared to the four-stand problem in which woodflow constraints affected only uneven-aged stands.

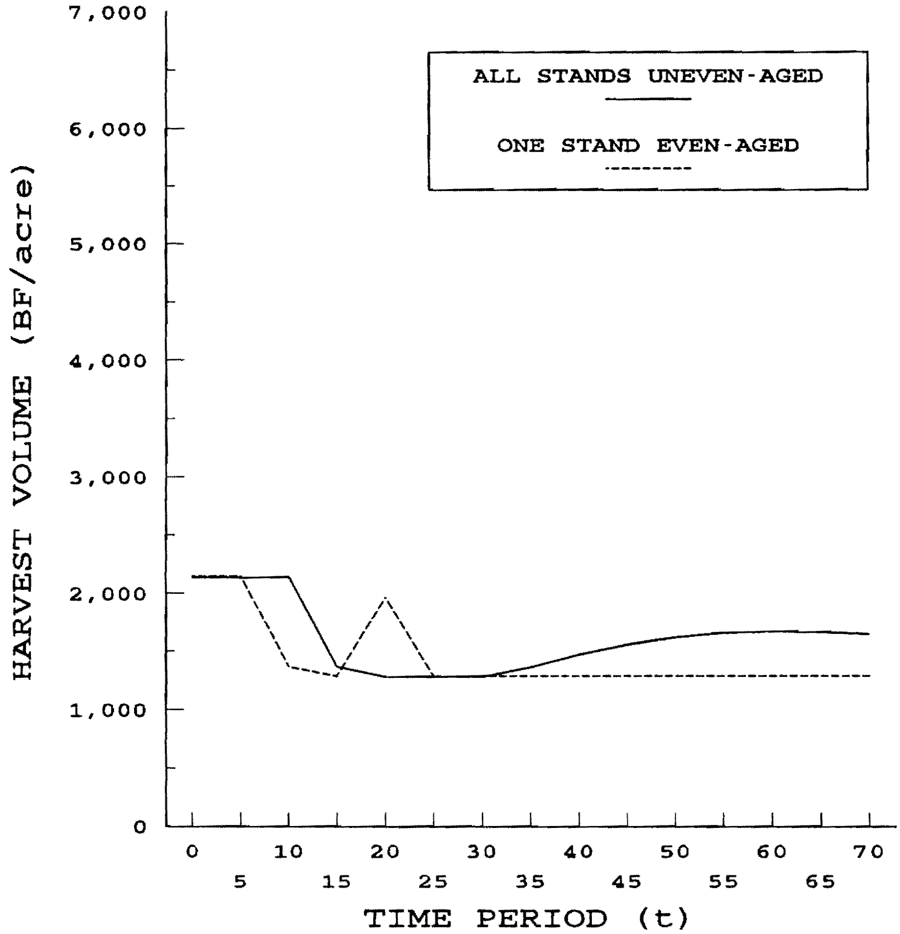

Figure 5.5 Average harvest volume for a four-stand forest with an aggregate woodflow constraint, comparing a forest with and without an even-aged stand.

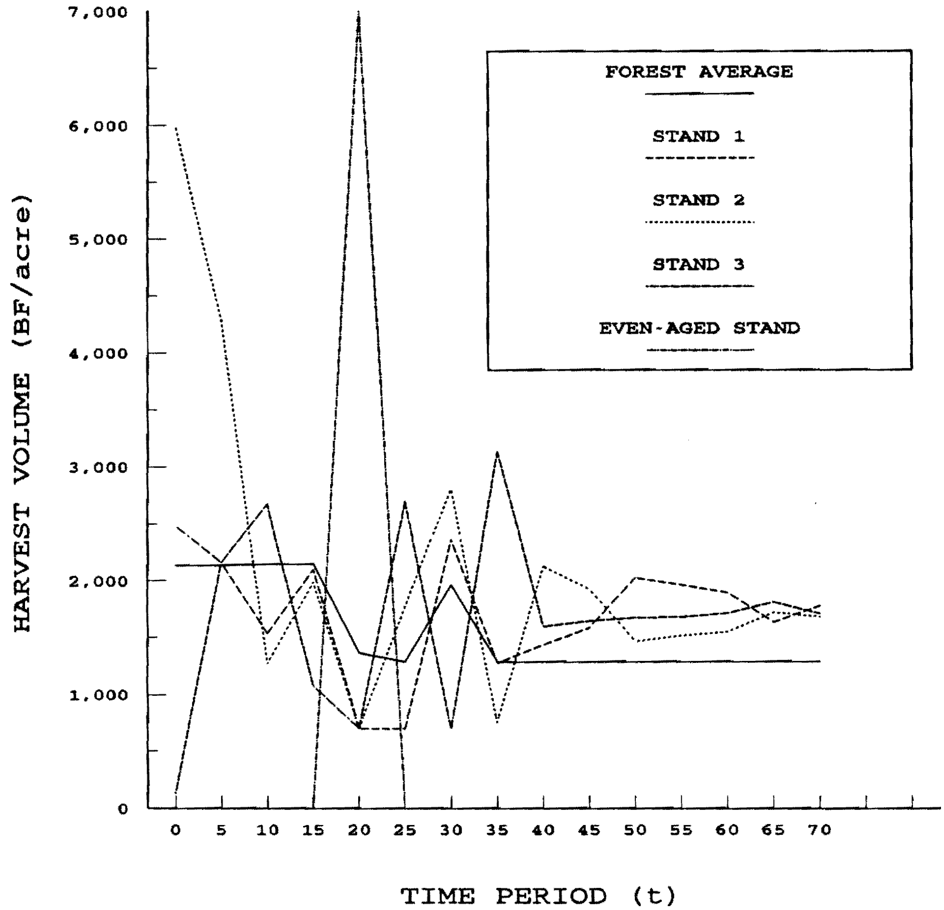

Figure 5.6 Harvest volume for three uneven-aged stands, one even-aged stand, and average harvest volume for all stands under a nonseparable, aggregate woodflow constraint.

CHAPTER 6: CONCLUSIONS AND RECOMMENDATIONS

The multi-stand management model described in this dissertation prescribes optimal stand treatments in terms of the number of trees per acre to harvest in each stand over a specified planning horizon. The objective function maximizes net present value of all harvests, subject to user-supplied growth and management constraints defined in terms of initial and residual stand structure. Due to the structure of the stand growth model employed (Solomon et al. 1987), harvest decisions may occur at intervals of 5 years within a stand included in the analysis. Problem size depends on number of tree size classes, species groups, stands, and time periods. In general the model is designed to analyze multi-stand problems, although stand-level problems may be solved by formulating the special case where only one stand is included.

This chapter is used to clarify potential uses of the multi-stand model and describe necessary data and their availability. The discussion then focuses on efficiency of the solution algorithm, problem size limits, and particular problems encountered using the GAMS/MINOS solver. Finally, recommendations are provided for further research in optimizing multi-stand management problems.

Potential Uses of the Model

Problem solutions define a detailed harvest strategy over time, including marking guidelines for individual stands, volume estimates based on factors supplied by the user, and other outputs which can be expressed as a function of initial and harvest stand structures. The multi-stand model may be used to evaluate the impact of management constraints for a stand or subgroup of stands within the whole forest. This may be accomplished by evaluating the reduced cost or dual values associated with the binding constraints in individual stands. The user may focus on a particular resource constraint to quantify tradeoffs associated with management goals, thus providing a basis for setting or adjusting resource priorities. The impact of constraints may also be evaluated by varying the level of constraints and comparing problem solutions in terms of resource outputs, such as harvest volume or quality, as well as comparing objective function values.

In multi-stand problems, optimal harvest schedules may be used to plan other management activities among various stands (for example, road construction and maintenance) to coordinate with planned harvests. This would facilitate estimating manpower requirements and expenditures over time that are not formulated within the model itself. Results may also be used to analyze the effect of forest organization and stand acreage when establishing individual management units. By comparing solutions from various alternatives, the forest could be subdivided and harvest practices assigned so that overall efficiency is improved.

At the stand level, traditional management guidelines may be tested by formulating representative problems and optimizing harvests for user-supplied inputs. This aspect of the model is useful in developing expert system models, and updating recommendations for existing artificial intelligence modules. Because the general formulation is based on number of trees per acre in species groups and size classes, many resource problems (habitat, diversity, etc.), which can be quantified as a function of these units may be investigated. In addition, the objective function and constraints may be constructed from transformations of the state and control variables described to model specific problems of interest to the user. The formulation is extremely flexible, although its usefulness can be limited by lack of reliable inputs, as discussed in the next section.

Required Data and Availability

Key inputs to the multi-stand model include a stand growth model which accounts for a variety of species groups and site productivity classes. It is preferable to utilize a growth model which estimates growth as a function of stand density as expressed by initial and residual stand structures. Density dependent models appropriately represent the dynamic nature of a managed forest, particularly when planning periods involve long rotations and/or perpetual cutting cycles. As formulated, the model presented here is based on 5-year growth intervals, providing harvest schedules for any user-defined planning period.

Several suitable growth models are available for eastern hardwood forest types (e.g., Hilt 1985; Teck 1990; Marquis and Ernst 1992). For use in nonlinear programming applications, each model must be converted from its existing format into formulations which conform to general mathematical structures required by solution algorithms. In some cases, conversion may be difficult because estimated prediction equations contained in the growth model software have not been published (Marquis and Ernst 1992).

Another key input to the general management model is a reliable system for valuation of outputs so that performance of feasible alternatives may be measured and compared. The price function used for this dissertation was based on the simplifying assumption that harvesting economies and product quality were accounted for in a stumpage price for each tree size and species. In many practical cases, this assumption is valid, particularly when harvest volumes show little fluctuation among individual stands. For instance, cost curves are relatively flat for merchantable harvests from 5 to 15 Mbf per acre using ground skidding equipment, and the resulting stumpage price per unit of volume is roughly constant within this range as well.

The model could be made more sophisticated by incorporating harvest cost functions to account for slight differences in average product size, terrain, and skidding and hauling distance in valuing outputs. Another enhancement would address individual tree quality. The basic management model would remain intact, while these suggested refinements would simply improve the reliability of price estimates $\mathbf{p}(t)$ used in the objective function.

Solution Efficiency and Dimension Limits

Solution times are dependent on problem size and the relative speed of the computer used to run the GAMS/MINOS solver. In the early stages of this study, a 386/20MHz PC-based processor was used to solve problems involving up to 15 dbh classes, 4 stands, and 31 time periods representing a 150-year planning period. Solution times often exceeded 10 hours for problems involving nearly 5,000 variables. Similar-sized problems were solved in the latter stages of the study on 486/50MHz processors, where solution time was reduced to approximately 2 hours.

For large problems, GAMS provides a means of saving work files to facilitate evaluation of intermediate solutions. In addition, GAM/MINOS options statements can be used to increase the speed of the optimization by adjusting feasibility and optimality tolerances, as well as linesearch tolerances to reduce the number of iterations. Options may be changed between intermediate runs to aid in the speed and accuracy of major iterations.

Problems analyzed in this study were small compared to the size limits that affect FORTRAN-based solution algorithms. GAMS/MINOS is capable of solving problems with 32,767 rows, columns, and nonlinear nonzero elements, and over 2 billion nonzero elements. Problems involving 4 stands and 31 time periods were about one-tenth of the maximum size for the systems used. Dimensions increase more quickly as species groups and stands are added, compared to adding dbh classes or time periods. In most cases, two or three species groups will adequately represent eastern hardwood forest problems in local areas. Thus, the number of stands in a multi-stand problem is the limiting factor. Based on performance of GAMS/MINOS and the hardware used in this study, it is possible to model problems involving up to 20 stands, 2 species groups, size classes ranging from 6 to 30 inches dbh, and a 150-year planning period.

Further Research

This study has shown that optimization models based on individual dbh classes and species groups provide great latitude in the kinds of timber and nontimber resource problems that may be analyzed. Additional study is needed to reduce the size of such problems without losing flexibility. Bare and Opalach (1988) used a Weibull distribution function to represent stand structures, and reduced the decision space to 2 variables for each species group and stand combination. Equation 2.7 may be substituted into equation 3.19 to achieve this result. However, program code presented in Appendix C must be rewritten to accommodate the Weibull formulation.

Certain aspects of this study were more amenable to mixed integer programming than the nonlinear programming format employed. For instance, in Appalachian forests minimum harvest constraints usually require that harvest volume either equal zero or exceed 2500 board feet per acre to attract a local buyer. In modelling these conditions in the nonlinear programming format, disjoint feasible regions are created, often resulting in the trivial solution where all harvests equal zero. In the mixed integer programming format, variables may be assigned values of zero or one, thus providing a means of solving problems with such conditions. However, current technology would require the basic management model to be linearized to exploit the capabilities of mixed integer programming. Additional research would be needed to quantify the tradeoffs associated with linearizing the model employed in this study. Moreover, further research is needed to model and solve highly nonlinear problems in which disjoint feasible regions are needed to represent real conditions.

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Appendix A. NOTATION

This appendix summarizes notation used in chapters 2-5. Symbols are listed in the order in which they are introduced in the text. The following general rules are used for notation.

-- Bold capital letters represent matrices.

-- Bold small letters represent vectors.

-- Italic capital letters represent functions or scalar parameters.

-- Italic small letters represent variables or vector elements.

Chapter 2

$VG \equiv$ value growth, increase in stand value during a growth period.

$MD \equiv$ maximum diameter class.

$p_i \equiv$ stumpage price (dollars) per tree in diameter-class i.

$x_i(t) \equiv$ initial number of trees in diameter-class i at time t.

$b_i \equiv$ basal area (square feet) per tree in diameter-class i.

$BA \equiv$ basal area per acre.

$ING_{i=1}(t)$ ≡ ingrowth, number of trees growing into the smallest diameter-class i=1 during a growth period beginning at time t.

$M_i(t)$ ≡ mortality, number of trees in diameter-class i that die during a growth period beginning at time t.

$U_i(t)$ ≡ upgrowth, number of trees in diameter-class i that grow into diameter-class i+1 during a growth period beginning at time t.

$\mathbf{x}(t) \equiv (x_1(t), x_2(t), \ldots, x_{MD}(t))$.

$h_i(t)$ ≡ number of trees harvested from diameter-class i at time t.

$\mathbf{h}(t) \equiv (h_1(t), h_2(t), \ldots, h_{MD}(t))$.

G ≡ a matrix of transition probabilities; elements are probability estimates for ingrowth, mortality, and upgrowth over a 5-year period.

c ≡ a column vector containing a single regression coefficient β_0 as the first element; used to estimate the number of trees growing into the smallest diameter-class over a 5-year period.

N ≡ total number of trees per acre in diameter-classes i to MD.

wb ≡ Weibull b parameter, scale parameter which defines the ratio of large to small trees.

wc ≡ Weibull c parameter, shape parameter which causes the distribution to take on a reversed J-shape if the value is between 0 and 1.

i_{lower}, i_{upper} ≡ lower and upper bounds for diameter-class i.

P ≡ planning period (years).

R ≡ rotation age (years).

V_a ≡ total stumpage value (dollars) per acre at age a.

$A_a(t)$ ≡ number of acres to clearcut from age-class a at time t.

m ≡ age class (years)

$n \equiv$ number of acres at the beginning of the planning period.

$n_l \equiv$ number of acres in management unit l.

$r \equiv$ discount rate.

$EA \equiv$ equal area (acres); total forest area divided by rotation age.

$A_{l,q} \equiv$ number of acres in management unit l assigned to harvest sequence q.

$V_{l,q}(t) \equiv$ harvest value (dollars) per acre from management unit l assigned to harvest sequence q at time t.

$L \equiv$ number of management units.

$Q_l \equiv$ number of possible regeneration harvest sequences over the planning period for management l.

$A_l \equiv$ number of acres in management unit l.

Chapter 3

$x_{i,j,k}(t) \equiv$ initial number of trees in diameter-class i, species group j, stand k, at time t.

$h_{i,j,k}(t) \equiv$ number of trees harvested from diameter-class i, species group j, stand k, at time t.

$MD \equiv$ maximum diameter class.

$J \equiv$ number of species groups.

$K \equiv$ number of stands.

$T \equiv$ number of time periods.

$\mathbf{x}_{j,k}(t) \equiv (x_{1j,k}(t), x_{2j,k}(t), \ldots, x_{MDj,k}(t))$.

$\mathbf{h}_{j,k}(t) \equiv (h_{ij,k}(t), h_{2j,k}(t), \ldots, h_{MDj,k}(t))$.

$p_{i,j,k}(t)$ ≡ stumpage price (dollars) per tree in diameter-class i, species group j, stand k, at time t.

A_k ≡ number of acres in stand k.

r ≡ discount rate.

δ ≡ $1/(1+r)$; a one-period discount factor.

$IBA_k(t)$ ≡ initial basal area (square feet) per acre in stand k at time t.

$RBA_k(t)$ ≡ residual basal area (square feet) per acre in stand k at time t.

D_i ≡ midpoint of diameter-class i.

$PH_k(t)$ ≡ proportion of total basal area comprising hardwood species in stand k at time t.

$PS_{j,k}(t)$ ≡ proportion of total basal area comprising species group j in stand k at time t.

$u_{i,j,k}(t)$ ≡ estimated proportion of trees in diameter-class i species group j stand k at time t that survive and grow into diameter-class i+1 during a 5-year growing period.

$a_{i,j,k}(t)$ ≡ estimated proportion of trees in diameter-class i species group j stand k at time t that survive and remain in diameter-class i during a 5-year growing period.

G ≡ a function which estimates survival, growth, and mortality based on initial and residual number of trees at each 5-year period.

F ≡ a function which estimates ingrowth into the smallest diameter-class based on initial and residual number of trees at each 5-year period.

$\mathbf{G}$ ≡ a matrix containing estimated transition probabilities; elements are $a_{i,j,k}(t)$ and $u_{i,j,k}(t)$.

$f_{1,j,k}(t)$ ≡ an estimate of the number of trees growing into the smallest diameter-class in species group j stand k at time t.

β ≡ a matrix of estimated regression coefficients used in functions G and F.

Chapter 4

$x_{i,j,k}(t)$ ≡ initial number of trees in diameter-class i, species group j, stand k, at time t.

$h_{i,j,k}(t)$ ≡ number of trees harvested from diameter-class i, species group j, stand k, at time t.

$\mathbf{x}_{j,k}(t) \equiv (x_{1,j,k}(t), x_{2,j,k}(t), \ldots, x_{MD,j,k}(t))$.

$\mathbf{h}_{j,k}(t) \equiv (h_{1,j,k}(t), h_{2,j,k}(t), \ldots, h_{MD,j,k}(t))$.

m_k ≡ minimum diameter-class considered to be visually desirable in stand k; used to define feasibility associated with esthetic objectives.

MD ≡ maximum diameter class.

J ≡ number of species groups.

K ≡ number of stands.

T ≡ number of time periods.

b_i ≡ basal area (square feet) per tree in diameter-class i.

$VBA_k(t)$ ≡ minimum basal area (square feet) per acre in stand k comprising trees equal to or larger than $i=m_k$.

$v_{i,j,k}$ ≡ board foot volume (International 1/4-inch rule) per tree in diameter-class i species group j stand k.

$hv_k(t)$ ≡ harvest volume; board foot volume (International 1/4-inch rule) harvested from stand k at time t.

$PEN_k(t)$ ≡ penalty variable; a logistic function of $\mathbf{h}(t)$ with values from 0 to 1.

$P \equiv$ planning period.

$WF(t) \equiv$ woodflow; the sum of harvest volume $hv_k(t)$ from each stand k at time t.

$wf \equiv$ allowable percent deviation in woodflow.

$ac_k \equiv$ number of acres in stand k.

$hyr_k \equiv$ harvest year; the year in which stand k is harvested.

$sp \equiv$ spatial allocation period; the minimum number of years between harvest years for adjacent stands.

Chapter 5

$CY \equiv$ conversion year; the time period in which a perpetual series of rotations is begun.

$R^* \equiv$ optimal rotation length (years).

$\mathbf{x}^* \equiv$ the optimal initial stand structure for an uneven-aged stand.

$\mathbf{h}^* \equiv$ the optimal periodic harvest for an uneven-aged stand.

$\mathbf{cc}^* \equiv$ the optimal cutting cycle; years between periodic harvests.

$\mathbf{tr}^* \equiv$ the optimal transition period; years to achieve the optimal uneven-aged stand.

$b_i \equiv$ basal area (square feet) per tree in diameter class i.

$MD \equiv$ maximum diameter class.

$J \equiv$ number of species groups.

$T \equiv$ number of time periods.

$Q_R \equiv$ present value at stand age=0 of a perpetual series of *R-year* rotations.

$r \equiv$ discount rate.

$\delta \equiv 1/(1+r)$; a one-period discount rate.

$x_{i,j,k}(t) \equiv$ initial number of trees in diameter-class i, species group j, stand k, at time t.

$h_{i,j,k}(t) \equiv$ number of trees harvested from diameter-class i, species group j, stand k, at time t.

$p_{i,j,k}(t) \equiv$ stumpage price (dollars) per tree in diameter-class i, species group j, stand k, at time t.

Appendix B. PROGRAM CODE

```
$TITLE RESULTS OF MODEL TESTS
$OFFSYMXREF OFFSYMLIST
*
*     PART OF PHD DISSERTATION BY GARY W. MILLER
*
*     THIS RUN IS CALLED FLOW2S.  TWO STANDS WITH GIVEN
*     INITIAL STAND STRUCTURES.  ONE SET OF GROWTH COEFFICIENTS.
*     HARVESTING AT ANY TIME, VOLUME MAY NOT FLUCTUATE MORE
*     THAN WF PERCENT FROM AVERAGE HARVEST.
*
*     10/30/92.
*
*     THIS SECTION DEFINES SETS
 SETS
    I    DIAMETER CLASSES /6,8,10,12,14,16,18,20,22,24,26,28/
    T    TIME PERIODS / T0,T5,T10,T15,T20,T25,T30,T35,T40,T45,T50,T55,T60,
                   T65,T70,T75,T80,T85,T90,T95,T100/
    R     COEFF ROWS / 0 * 7 /
    C     COEFF COLUMNS / 1, 2, 3 /
    K     BEGINNING STAND /S1, S2/;
*
*      THIS SECTION DEFINES PARAMETERS
 PARAMETER
*    MIDPOINT DIAMETERS ARE USED TO COMPUTE TREE BASAL AREAS

     D(I)      MIDPOINT DIAMETERS
            / 6 6.0,8 8.0,10 10.0,12 12.0,14 14.0,
              16 16.0,18 18.0,20 20.0,22 22.0,24 24.0,
              26 26.0,28 28.0/
```

```
    V(I)      INTERNATIONAL BF VOLUME PER TREE
              / 6 0,8 0,10 0,12 60,14 80,
                16 180,18 280,20 350,22 430,24 510,
                26 600,28 700/
    AC(K)      STAND ACRES
              /S1 50, S2 50/
*
*
TABLE      P(I,K)      DELIVERED PRICES PER TREE
                  S1      S2
          6    -1.0    -1.0
          8    -1.0    -1.0
         10    -1.0    -1.0
         12      .063    .063
         14     5.29    5.29
         16    15.38   15.38
         18    24.61   24.61
         20    35.88   35.88
         22    50.18   50.18
         24    67.49   67.49
         26    87.12   87.12
         28   109.71  109.71;
*
PARAMETER
    YRS(T)    DISCOUNT PERIOD
              /T0 0,T5 5,T10 10,T15 15,T20 20,T25 25,T30 30,
              T35 35,T40 40,T45 45,T50 50,T55 55,T60 60,T65 65,
              T70 70,T75 75,T80 80,T85 85,T90 90,T95 95,T100 100/
*
    BA(I)     BASAL AREA PER TREE
    FACTOR(T) PRESENT VALUE FACTOR
    PERC(T,K)   PERCENT HARDWOODS
    PS(T,K)     PERCENT SPECIES;
    SCALAR RR REQUIRED DISCOUNT RATE /.04/;
    SCALAR WF ALLOWABLE WOODFLOW CHANGE /.25/;
    BA(I) = .005454 * SQR(D(I));
    PERC(T,K) = 1.0;
    PS(T,K) = 1.0;
    FACTOR(T) = 1$(ORD(T) EQ 1) + ((1+RR)**YRS(T))$(ORD(T) GT 1);
    SCALAR FCACRE  FIXED COST PER CUT /100/;
*
```

```
*    THE FOLLOWING COEFFICIENTS ARE FROM FIBER
 TABLE      A(R,C,K)     REGRESSION COEFFICIENTS
                  1.S1          2.S1          3.S1
          0       .3446         .4997          2.6186
          1      -.000785      -.000215          0
          2       .006395      -.004305        -.02633
          3      -.0095         .0207           0
          4       .0717        -.0316          2.1191
          5       .000335      -.000785          0
          6      -.0000221      .0000153         0
          7         0              0          5.8985

          +         1.S2          2.S2          3.S2
          0       .3446         .4997          2.6186
          1      -.000785      -.000215          0
          2       .006395      -.004305        -.02633
          3      -.0095         .0207           0
          4       .0717        -.0316          2.1191
          5       .000335      -.000785          0
          6      -.0000221      .0000153         0
          7         0              0          5.8985;
*

*   THE USER CAN SPECIFY THE INITIAL STAND USING THE FOLLOWING
*   TABLE
*   PLUS UPPER AND LOWER BOUNDS ON X AT TIME=0 DEFINED BELOW
  TABLE     INITVALX(I,K)  INITIAL STAND VALUES FOR X
                 S1     S2
          6      77.1   77.1
          8      38.6   38.6
         10      20.7   20.7
         12      10.2   10.2
         14       7.7    7.7
         16       6.1    6.1
         18       4.6    4.6
         20       3.1    3.1
         22       1.8    1.8
         24       1.7    1.7
         26       1.4    1.4
         28        0      0;
```

```
* STARTING VALUES USED TO INITIATE OPTIMIZATION
 TABLE     STARTX(I,K)  INITIAL VALUES FOR X
                 S1   S2
        6        20   20
        8        17   17
       10        16   16
       12        16   16
       14        16   16
       16        16   16
       18        14   14
       20        11   11
       22         8    8
       24         5    5
       26         2    2
       28         1    1;

*       THIS SECTION DEFINES VARIABLES
*
*
 POSITIVE VARIABLES
    X(I,T,K)  INITIAL TREES
    H(I,T,K)  HARVEST TREES
    IBA(T,K)  INITIAL BASAL AREA
    RBA(T,K)  RESIDUAL BASAL AREA
    RES(I,T,K)  RESIDUAL TREES
    HVOL(T,K)  CUT VOLUME
    TOTCUT    TOTAL HARVEST VOLUME
    ING(T,K)  INGROWTH;
*
 VARIABLE
    NPV     NET PRESENT VALUE OBJ;
*
*       THIS SECTION DEFINES EQUATIONS
 EQUATIONS
    RESID(I,T,K)   RESIDUAL TREES
    ISBA(T,K)      INITIAL STAND BA
    RSBA(T,K)      RESIDUAL STAND BA
    KING(T,K)      K INGROWTH
    GROWTH(I,T,K)  GROWTH CONSTRAINTS
```

```
    CUT(T,K)       VOLUME CUT
    FLOW1(T)        WOODFLOW
    FLOW2(T)        WOODDLOW
*
*
    TOTCUT1         TOTAL VOLUME HARVESTED
    NETPV          NET PRESENT VALUE OBJ;

    RESID(I,T,K)..  RES(I,T,K) =E= X(I,T,K) - H(I,T,K);
    ISBA(T,K)..     IBA(T,K) =E= SUM(I, BA(I) * X(I,T,K));
    RSBA(T,K)..      RBA(T,K) =E= SUM(I, BA(I) * (X(I,T,K)-H(I,T,K)));
    KING(T,K)..      ING(T,K) =E= A("0","3",K) + A("2","3",K)*RBA(T,K)
                   + A("4","3",K)*PERC(T,K) + A("7","3",K)*PS(T,K);
*
*
*

    GROWTH(I,T+1,K).. X(I,T+1,K) =E= ( (A("0","1",K)
                   + A("1","1",K)*IBA(T,K)
                   + A("2","1",K)*RBA(T,K) + A("3","1",K)*D(I)
                   + A("4","1",K)*PERC(T,K) + A("5","1",K)*SQR(D(I))
                   + A("6","1",K)*SQR(RBA(T,K))) * RES(I,T,K)
                   + (A("0","2",K) + A("1","2",K)*IBA(T,K)
                   + A("2","2",K)*RBA(T,K) + A("3","2",K)*D(I-1)
                   + A("4","2",K)*PERC(T,K) + A("5","2",K)*SQR(D(I-1))
                   + A("6","2",K)*SQR(RBA(T,K))) * RES(I-1,T,K)
                   + ING(T,K)$(ORD(I) EQ 1));
*
     CUT(T,K)..    HVOL(T,K) =E= SUM(I,V(I)*H(I,T,K));
     TOTCUT1..       TOTCUT =E= SUM((T,K), HVOL(T,K));
     FLOW1(T)..     SUM(K, HVOL(T,K)) =L= (1+WF)*(TOTCUT/21);
     FLOW2(T)..     SUM(K, HVOL(T,K)) =G= (1-WF)*(TOTCUT/21);
*
     NETPV..        NPV =E=(SUM((I,T,K), P(I,K)*H(I,T,K)/FACTOR(T)));
*
 MODEL  STAND1 /ALL/;
 STAND1.OPTFILE=2;

      X.FX(I,"T0",K) = INITVALX(I,K);
      X.L(I,T,K) = STARTX(I,K);
```

```
OPTION ITERLIM = 7000;
OPTION RESLIM = 7000;
OPTION LIMROW = 0;
OPTION LIMCOL = 0;
SOLVE STAND1 USING NLP MAXIMIZING NPV;
DISPLAY  X.L, H.L, RES.L, RBA.L, HVOL.L, TOTCUT.L;
```

Appendix C. SOLUTION ALGORITHM

The basic model was formulated as a sizable nonlinear programming problem and coded in the General Algebraic Modeling System (GAMS), a FORTRAN-based equation generator capable of representing complex models in compact form (Brooke, Kendrick, and Meeraus 1988). For problems with nonlinear constraints, the GAMS/MINOS optimizer employs a projected Lagrangian algorithm (Robinson 1972). The general expression of a nonlinear programming problem is shown in equations C.1 through C.4.

$$\min \quad F(x) + c^T x + d^T y \tag{C.1}$$

$$s.t. \quad f(x) + A_1 y \geq b_1 \tag{C.2}$$

$$A_2 x + A_3 y \geq b_2 \tag{C.3}$$

$$l \leq \begin{pmatrix} x \\ y \end{pmatrix} \leq u \tag{C.4}$$

Vectors c, d, b_1, b_2, l, u, and matrices A_1, A_2, A_3 are constant, $F(x)$ is a smooth scalar function, and $f(x)$ is a vector of smooth functions. Components of x are the nonlinear variables and components of y are the linear variables. The projected Lagrangian algorithm solves the original problem using linear approximations of $f(x)$, given by equation C.5, where $J(x_k)$ is the Jacobian matrix evaluated at x_k. At the start of each major iteration the estimated nonlinear variables x_k and estimates of their dual variables λ_k are used to construct a linearized version shown in equations C.6 through C.9.

$$\tilde{f}(x,x_k) = f(x_k) + J(x_k)(x-x_k) \tag{C.5}$$

$$\min \quad F(x) + c^T x + d^T y - \lambda^T_k(f - \tilde{f}) + \frac{1}{2}\rho\,(f - \tilde{f})^T(f - \tilde{f}) \tag{C.6}$$

$$s.t. \quad \tilde{f} + A_1 y \geq b_1 \tag{C.7}$$

$$A_2 x + A_3 y \geq b_2 \tag{C.8}$$

$$l \leq \begin{pmatrix} x \\ y \end{pmatrix} \leq u \tag{C.9}$$

In the linearized subproblem, the scalar ρ is a penalty parameter used to construct a quadratic penalty function which aids in convergence. Program options in GAMS/MINOS allow ρ to be increased for highly nonlinear problems such as the multi-stand model in this study.

Once the original problem has been linearized at the start of each *major* iteration k, GAMS/MINOS uses a reduced gradient algorithm (Wolfe 1962) to minimize equation 6 subject to 7 through 9 through a sequence of *minor* iterations. From a starting point defined by the previous iteration, a search direction is defined and the objective function is minimized along that direction.

GAMS/MINOS also allows the user to adjust the linesearch tolerance and optimality tolerance for the reduced gradient procedure. Once the reduced gradient algorithm finds an optimal solution to a given linearly constrained subproblem, the original problem is evaluated at the solution value x_k to test for feasibility of nonlinear constraints and optimality. Major iterations continue unless limited by program options until nonlinear feasibility and optimality are achieved, usually at about the same time (Brooke, Kendrick, and Meeraus 1988).

Vita

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