OPTIMAL TENURE CHOICE AND COLLUSIVE BEHAVIOR IN CONTRACT NEGOTIATION MODELS

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(ABSTRACT)

The assumption of a purely self-interested supervisor in a three-tier hierarchy (a principal-supervisor-agent framework) gives rise to the possibility of supervisor-agent collusion which lowers the principal's profits. It has also been shown that the transfer of information in side contract negotiations between the supervisor and the agent may hinder collusion and maintain high principal profits. In chapter 2, I show that imposing "credible" updating of type beliefs during negotiations can guarantee one of two outcomes that are Pareto superior for the supervisor-agent coalition. I further refine the equilibria by endogenizing the decision of who makes the side contract proposal, and a unique collusive equilibrium results. In allowing the principal to form a collusion-proof incentive contract, I find that the only plausible solution is for the principal to ignore the supervisor. It is clear that there is no value at all to the principal in hiring a self-interested supervisor. This casts doubt on the validity of the assumption that the supervisor is self-interested, and I discuss some possible alternatives.

Chapter 3 studies job matching inefficiencies under two-sided uncertainty. I examine these inefficiencies in a setting of a single-stage, simultaneous-offer bargaining situation, where the applicant does not know his productivity with the firm, and the employer does not know the applicant's reservation wage. I compare linear bid strategy
equilibria between the cases where the applicant is uninformed of his productivity and where he is informed. I find that the payoff to the applicant is higher if he is informed. He is thus willing to collude with an informed person within the firm, paying him up to the difference in payoffs to obtain his productivity information. It is noteworthy that the collusive equilibrium is always more efficient than the non-collusive equilibrium, and that most types of employer prefer the applicant to have the knowledge. In the cases that the employer does not wish the applicant to possess the information, I examine possible reward schemes for the employer to use to deter collusion. I find, however, that a successful reward scheme is too costly to the employer, and coalition formation always occurs in equilibrium.

Chapter 4 studies the strategic choice of job tenures to maximize lifetime earnings. A worker's salary typically increases with tenure, and the possible net starting salary at a new job depends on such factors as search costs, training period duration, rate of human capital accumulation, and experience. The worker thus wishes to choose appropriate tenures considering the levels of these factors for the industry in which he works. I set up a general framework for the problem, and solve using specific functional forms for salary increments and the new starting salary. I find that these factors are important in determining the optimal number of jobs to work, and the optimal distribution of tenures among the jobs. It is easy to see how variations in these factors across industries can help explain variations in turnover rates and tenure choices of individuals at different points in their working lifetimes. Also, we see how realistic variations in these values over the course of a worker's lifetime yield results consistent with empirical findings.
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Chapter 1

An Overview and the Surrounding Literature

1. Information Gathering and Side Contracting

The value of information gathering prior to contract formation and negotiations is quite common. Parties engaging in these acts will often willingly incur costs in attaining information about other parties to improve their payoffs. An individual buying a car may purchase published information on dealer invoices to have a better understanding of what the dealer will accept, an employer may hire an industrial engineer to determine workers' efforts and efficiency to improve incentive contracting, or the government may attain information on production costs before negotiating a defense contract with a private company. All of these examples are similar in that parties incur costs to improve their position.

The quest of information to improve a party's payoff may, by design, hurt another party's payoff. There have been several recent research papers addressing the possibility of side-contracting, or collusion, instigated by the potentially injured parties to prevent other parties from attaining such information. A common feature of chapters 2 and 3 of this dissertation is collusion in the presence of asymmetric information. Game theory and its applications are, by and large, divided into two major fields. Strategic games mainly deal with single-minded strategic behavior, whereas cooperative games emphasize coalition formation to coordinate behavior. Labor unionization is an example in which the benefits of cooperation and coordination are realized.
Pure cooperation, however, may not be realistic. Indeed, most of social choice theory, in particular mechanism design, contains both strategic and cooperative elements. Hybrid concepts like Aumann's strong equilibrium and coalition-proof equilibrium amount Collusion is an economic, political, or social phenomenon that entails both strategic and cooperative behavior. It means that a group of agents, or a coalition, forms a strategic alliance against the rest of the world in an effort to increase the welfare of the participants. It has been shown that in the context of cooperative games, collusion can backfire. Harsanyi (1977) and Haller (1994) show that collusion can reduce the coalition's bargaining and voting power, respectively.

Chapters 2 and 3 of this dissertation fit into a broader framework of triangular or multilateral business relationships, where participants may contemplate side-contracting and double-crossing. The following section briefly discusses some of the related literature. I then present an overview of chapters 2 and 3, describe how these contributions fit into the general framework, and describe the implications in an applied setting.

1.1. The Surrounding Literature

Tirole (1986) and (1990) studies a situation where an employer (a principal) wishes to hire a worker (an agent) to perform a productive act. To attain information on the agent's costs, the principal considers hiring an auditor (a supervisor) to perform the task. The supervisor is assumed to be self-interested in the sense that he always pursues the highest expected monetary payoff regardless of the consequences of his actions on others. Tirole shows that it is possible, in equilibrium, for the agent to offer a bribe, or side contract, deterring the supervisor from truthfully reporting any information he has learned. For example, if the government is considering a defense contract with a private company and hires auditors to determine costs, the company may have an incentive to
bribe the auditors to report a cost of production higher than it actually is. This way, the government will feel obliged to offer a higher price for the products.

Even though there are potential gains to side-contracting, Felli (1990) shows that with the presence informational asymmetries, information learned through the negotiation of side-contracts can prevent collusion from occurring. An example in Tirole (1990) shows that this may indeed be the case. In the example, there exist equilibria in which collusion, though Pareto superior for the colluding parties, does not occur. The absence of collusion keeps the resulting payoff to the principal relatively high. Since in equilibrium collusion may not exist, the principal realizes some value to the existence of the self-interested supervisor.

1.2. An Outline of the Contribution of Chapter 2

Chapter 2 of this dissertation considers a situation in the above framework. In this chapter, I study the possible formation of side-contracts between an agent and a purely self-interested supervisor, and the potential for the principal to form collusion-proof contracts. During contract and side contract negotiations, information about parties' types is transferred through offers made. I assume that all parties use this information to update their beliefs about other parties' types in a credible manner. I find that under the circumstances and parameters delineated in the model, successful side-contracting occurs in equilibrium. Under this equilibrium, there is no value at all to employing the supervisor. Furthermore, there is no way a principal can improve her payoff with a collusion-proof contract. The only plausible option that is collusion-free is to eliminate the supervisor altogether. If there are no exogenous costs to hiring and employing the supervisor, and the principal's value function does not include the utilities of the supervisor and the agent, the principal is indifferent between the collusive outcome and the outcome with no

Chapter 1. An Overview and the Surrounding Literature 3
supervisor. If there are any costs associated with employing the supervisor, then the
principal strictly prefers not to employ him.

The results are contradictory to what we typically observe. The government
would not pay high fees to auditors if it expected that they would accept bribes. In a more
modest setting, an employer would expect her supervisors to resist accepting bribes from
the workers. The contradiction stems from the assumption that the supervisor is purely
self-interested. Since we do observe widespread use of supervisors and auditors, the
results of chapter 2 cast doubt on the validity of the assumption that people always act in
their own self-interest. I suggest that future models should incorporate the fact that
auditors may act with loyalty or professional integrity and reject bribe and kickback
proposals, even if there is no possibility of detection.

A more realistic assumption may be for a principal to assess a probability that the
supervisor is self-interested. This way, the principal may or may not choose to guard
against side contracting, and if she does, it may need only be to offer minor incentives.
This seems to be a more plausible assumption since it allows for some value to the
existence of a supervisor or auditor. Also, the principal may choose to hire an external
auditor, who is costly but completely honest, to monitor the supervisor or the agent.

Kofman and Lawarree (1993) study this situation with the extensions suggested
above. They assume that there is a known probability that the internal supervisor is self-
interested, and there exists the option to hire an honest outside auditor. Also, the principal
can institute an incentive and punishment contract to deter collusion, where the
punishments are monetary and paid from the supervisor to the principal. Results are that
in equilibrium, the external auditor is hired on a random basis to monitor the supervisor.
In addition, the principal offers an incentive contract to internal parties that may,
depending on the parameters, allow collusion. Important results are that given the more realistic assumptions of potentially honest internal supervisors and the availability of honest external auditors, there is some value to the presence of the internal supervisors, even if collusion does occur.

1.3. An Outline of the Contribution of Chapter 3

Chapter 3 considers a similar situation, but where the seller of a productive act seeks the value to the buyer. The context I use is a job applicant negotiating salary with a potential employer. I consider the value of the existence of an informed third party who is willing to accept payment in return for providing the applicant with the employer's willingness-to-pay. An obvious example of this situation occurs when a professional actor or athlete hires an agent to negotiate the terms of his contract. The agent has more time and resources to determine the worth of the applicant to the employer, and is thus in a better position to negotiate a higher salary. It appears that there is value to these types of workers of having an agent since they are willing to pay 20% to 30% of the resulting salary for their services.

Rarely do we witness, however, agents serving lower-paid workers in this manner. In chapter 3, I examine the reasons why this is so by finding the benefits of an applicant acquiring his productivity information prior to negotiations. Chatterjee and Samuelson (1983) study a one-shot, simultaneous-offer bargaining situation with two-sided incomplete information, and equilibrium results with varying preciseness of information. They find that the split of the surplus from trade favors the party who is better informed. I consider a similar bargaining situation where an employer and an applicant negotiate salary, and determine the optimal bidding strategies for both parties under two-sided incomplete information. Next, I determine the equilibrium if the applicant is better
informed of his value to the employer. This is a general solution, and from it I determine the outcome if the applicant is fully informed. Results, not surprisingly, are that the expected salary gain to the applicant from the interview is higher when he is informed than when he is not. In equilibrium, the applicant will thus pay a positive amount to an informed third party for the productivity information.

It is interesting to note, however, that most types of employer receive higher expected profits if the applicant is informed. The reason is that even though the expected salary if the applicant is informed is higher, the probability that a match is made increases enough to override the higher salary. An employer would be particularly interested in efficient matching if the costs of conducting the interview are high. If an applicant knows that he will not accept the most an employer is willing to pay, he will cancel the interview. This outcome is in the interest of all parties because there is a zero probability of a match.

The possibility of this outcome depends crucially on the assumption that the acquisition of information is relatively costless for the third party. To justify this assumption, I assume that this party is a supervisor with the firm, and learns the applicant's productivity costlessly by reading his application and resume. Since the third party is internal, I allow the principal the option to offer incentive contracts to prevent the dissemination of the information.

As stated, most types of employer prefer that the applicant has the information. For those types who do not, I consider the provision of a profit-sharing contract to the supervisor to induce him to reject offers from the applicant. I find that the incentives necessary for the supervisor to reject the offer are higher than the principal is willing to pay. In equilibrium, therefore, information transfer does occur, and the principal allows it either because it is in her interest or because it is too costly to prevent.
The results of chapter 3 state that there is value to an applicant of having his productivity information, and he is willing to pay for it. The remaining question then is why we don't see widespread existence of this type of market. To answer this, I offer several possible explanations.

As stated above, it may be an exaggeration to assume that individuals are purely self-interested. An internal worker at a firm may costlessly obtain productivity information, but he may not be willing to sell it to the applicant. Reasons, as stated, may be that a worker has a sense of professional integrity or loyalty to his employer, or he fears possible reputational effects. Therefore, if the informed worker is unwilling to participate, the applicant may not have another costless option.

If there are no internal sources, the applicant would need to either find the information himself or hire an external agent with more time and resources to perform the task. Either way, expenses in obtaining the information may be higher than the resulting benefits. For low-paying jobs, the possible benefits are also necessarily low. This may explain why we tend to see this behavior in higher-paying occupations. The gains there from having productivity information are large enough to justify paying high prices for the services of an agent.

Another reason for the absence of these markets may be that even if the applicant could obtain the information through an external source, the information may not be accurate. Since an external source is, in essence, making an estimate, an employer will have some power in denying its accuracy. This has the effect of decreasing the value of the information, possibly below its price.
2. Job Tenure, Mobility, and Earnings

Chapter 4 of this dissertation examines a separate problem of lifetime earnings maximization through strategic job tenure choice. It is widely accepted that workers' salaries increase with tenure at a particular job. It is also common that workers voluntarily leave their jobs in search of new ones. In this instance, the new job must have some characteristic that the old job did not. The worker may not have liked his boss, grew tired of his occupation, or wished to relocate. Certainly, pecuniary factors may also play an important role one's decision to switch jobs. If the worker receives a high-paying alternative offer, he may choose to pursue that option strictly to increase his earnings.

An earnings maximizing worker thus has a decision to make: When should he leave his job for a new one? If at his current job his salary increases in tenure, he must be offered a sufficient starting salary, net of the costs of switching jobs, to entice him to accept. More broadly, if the worker has a finite working lifetime and it is, to some degree, costly to switch jobs, he needs to make some decision as to which opportunities to accept over the course of his entire lifetime. The problem becomes that of lifetime earnings maximization by choosing the ideal number of jobs to work and the length of time to stay at each of them.

2.1. The Surrounding Literature

The surrounding literature concerning effects of tenure and job mobility on earnings is quite extensive. Mincer (1986), Abraham and Farber (1987), Mortensen (1988), and Topel (1991), among many others, find that wages do increase (albeit to varying degrees) with tenure. Results generally indicate that wages are increasing and concave due to the diminishing marginal productivity of workers. Gibbons and Murphy
(1992) state that efficiency wages may induce employers to offer linear salaries in tenure despite diminishing marginal productivity.

Other research has addressed the effect of job mobility on earnings. Mincer and Jovanovic (1981), Mincer (1986), and Topel (1986) find that there are positive gains in salary from switching jobs when a worker is young. However, switching when the worker is older can carry small or negative increments in salary. The authors cite several reasons for the declining of opportunities with age, among which is the fact that older workers tend to desire non-pecuniary benefits such as job security, low stress work, and pleasant work environments. Mincer (1986) and van den Berg (1992) find that the costs associated with switching jobs increase as workers get older. This of course has the effect of lowering the net benefits from switching. Bartel and Borjas (1981) show that wage changes between jobs are significant when a worker switches strictly for pecuniary reasons.

Much of this research goes on to empirically demonstrate turnover tendencies and the effect of the above findings on tenure choices and turnover. Stoikov and Raimon (1968) find that quit rates are negatively correlated with the rate of increase in wages. Furthermore, they state that the rate of increase in wages depends partly on a firm's tendency to make competing offers if its employees receive outside offers. Mincer and Jovanovic (1981) and Topel (1991) present evidence that turnover is typically higher for younger workers than for older workers. Akerlof, Rose, and Yellen (1988) find evidence that job mobility is negatively correlated with unemployment. The reason given is that there are more beneficial outside opportunities if unemployment is low. Their paper, Mincer and Jovanovic (1981), and Topel (1991) find that quit rates tend to decline as tenure increases.
2.2. An Outline of the Contribution of Chapter 4

The purpose of chapter 4 is to provide a theoretical framework that incorporates the above findings into a single model. I consider a situation where a worker is seeking strictly to maximize his lifetime earnings. He does so by choosing the optimal tenure at each of his jobs over the course of his lifetime, and the optimal number of jobs to work given the tenure distribution. Since both specific and transferable human capital tend to increase with tenure, I assume that both the worker's salary at his current job and the possible starting salary if he switches tend to increase. It must be the case that if a worker always takes the highest paying job available at the time of the switch, the starting salaries must increase at a faster rate than the worker's current salary if he is ever to switch at all. I explain how the worker's raises are an increasing function of his rate of accumulation of human capital and of the firm's willingness to make competing offers, or are simply an exogenously given percentage, as is the case in many state or federal positions. The worker's starting salary at a new job is an increasing function of his rate of accumulation of transferable human capital and a decreasing function of the unemployment rate.

Results of chapter 4 are that if raises, starting salary increments, and costs of job search are fixed throughout the worker's lifetime, he should work relatively long tenures early in life and shorter tenures late in life. This is due to the fact that it is beneficial to acquire large amounts of human capital early in life which can be used throughout the rest of the worker's lifetime. These findings contradict the stated results in Mincer and Jovanovic (1981) and Topel (1991) that turnover decreases over the typical worker's lifetime. I offer two explanations for this contradiction. Gibbons and Murphy (1992) find that career concerns are higher for younger workers than for older workers. This indicates that the rate of accumulation of human capital tends to be higher early in life than later in life. Results in chapter 4 show that if the rate is higher for younger workers, early
tenures will become shorter. This is because less time is required to accumulate sufficient human capital before switching jobs.

A second reason for the contradiction is due to the assumption of fixed costs of switching jobs. If we assume results consistent with Mincer (1986) and van den Berg (1992) that costs of switching increase as workers get older, the model does predict that turnover will be lower later in life as the net benefits from switching jobs lessen.

Results from the model also suggest how the overall rate of turnover (defined as the number of jobs the worker chooses over the course of his fixed working lifetime) depends on raises, starting salaries, and costs of switching. As a worker's wages increase quickly, the model predicts that turnover is lower; a result consistent with findings in Stoikov and Raimon (1968). That is, if a firm is willing to make competing offers or if the rate of specific human capital accumulation is high relative to the rate of transferable human capital, the quit rate is lower.

If the worker experiences high pay raise opportunities from switching, the model predicts that turnover is higher. If a worker is adept at accumulating transferable human capital or if the unemployment rate is low, he will have more outside opportunities. In these instances, if the worker seeks to maximize earnings, he should take advantage of these opportunities relatively often in life, and turnover is thus higher.

I also discuss in chapter 4 some possible variations in the functional forms of the raises and starting salaries. In particular, I discuss the possibility of quadratic starting salaries. This functional form implies that a worker's potential starting salary at a new job increases in tenure, and then eventually begins to fall. This may be true if a worker becomes too specific in skills (they need to be re-trained) or sends a signal of "overstaying his welcome" at jobs. The model suggests that if the worker's marginal accumulation of
transferable human capital diminishes at an increasing rate, possibly becoming negative, a worker will change jobs more quickly. That is, he should take advantage of gains from switching before they disappear. This theory may be an interpretation of the stated results that quit rates decline with tenure. Quadratic starting salary functions suggest that eventually, starting salaries will begin to fall. As they do, the likelihood that a worker will switch for any reason will fall also.

The last section of chapter 4 discusses some possible extensions to the model. Adjustments can be made to incorporate the fact that firms may offer incentives to induce workers to lengthen tenures. Many firms, particularly in the public sector, offer pensions and retirement benefits. These benefits are typically payable only if the worker stays a specified length of time. Implicitly, the worker is thus penalized for leaving early. Ippolito (1987) offers this as an explanation of low quit rates in federal jobs.

The model may also be formulated such that a worker maximizes lifetime utility rather than income. This type of model would be more flexible because it allows for the consideration of non-pecuniary benefits in a worker's decision to switch jobs. Workers may switch to improve work environments, to move closer to (or further from) family, or to obtain less stressful jobs. Also, the worker may consider putting high effort into accumulating human capital. This would have the effect of increasing earnings, but also increasing the disutility from effort. A model incorporating all of these factors could provide a theoretical framework to more thoroughly explain turnover and tenure distributions.
REFERENCES


*Chapter 1. An Overview and the Surrounding Literature*


Chapter 2

Credible Beliefs, Side Contracts Negotiations, and Collusion in a Three-Tier Hierarchy

1. Introduction

This paper addresses collusion in the framework of triangular or multilateral business relationships where participants may contemplate side contracting and double crossing. Collusion means that within a group of agents, a coalition forms a strategic alliance against the rest of the world. Obvious examples are military alliances and trading blocks, collusion among firms at the expense of consumers or a government procurement agency, or collusion between a domestic firm and the government against foreign competitors.

Recent research has addressed the possibility of supervisor-agent coalition formation within a three-tier hierarchy, where a principal may wish to monitor an agent and hire a supervisor to perform the task. The supervisor often is assumed to be purely self-interested, and is willing to accept a payment from the agent in return for misreporting his findings. The principal has the option to create collusion-proof contracts to deter the supervisor's misbehavior. The focus of this paper is to examine the validity of the assumption that the supervisor is purely self-interested. I show with the aid of an example that if parties update their beliefs in a credible manner, a self-interested supervisor may be of no value to the principal. If this is the case, the role of a self-interested auditor,

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1 By self-interested, I mean that the supervisor always seeks the highest expected monetary payoff, regardless of the consequences on others.
in a more general sense, is nullified. Since we generally think of an auditor or supervisor as having some value, these findings may suggest that they cannot be purely self-interested. People in positions to take bribes and kickbacks may refuse even if there is no chance of being discovered. Clearly, there are non-monetary reasons like loyalty, professional integrity, and reputation that prevent this behavior. This should thus be taken into consideration when addressing such a situation.

To illustrate the problem, this paper studies the instance where the agent knows his own type, but the self-interested supervisor, in observing the agent, does not gain clear information about the type. The supervisor may choose to report the probability he is correct, to make no report at all, or to guess the agent's type. It certainly may be the case that even the report of high probability of the agent's type is potentially damaging to the agent. There may be room for the agent and supervisor to form a side contract where the agent pays the supervisor to conceal the information from the principal.

Felli (1990) shows that these informational asymmetries may hinder the possibility of collusion between the supervisor and the agent even when the parties can gain from doing so. That is, the outcome may not be "coalition interim efficient" (Holmstrom-Myerson (1983)). The structure of the game may be such that even though the supervisor and the agent have a chance to negotiate a side contract, the coalition interim efficient (henceforth CIE) outcome does not occur in equilibrium. In this instance, the principal does not experience losses from collusion and may acquire benefits from employing the self-interested supervisor.

This paper addresses the role of information transfer with credible belief updating in such side contract negotiations and its effect on the value of employing self-interested

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2 An outcome is coalition interim efficient if it is Pareto efficient for the coalition parties and it is brought about by a contract made by the coalition parties at a stage when they have private information.
supervisor. In particular, I look at its effect on the possibility of forming side contracts, on the terms of the side contract formed, and on the bargaining power of the negotiating parties. The principal, in anticipating the results, can better understand the collusive nature of the coalition, and will thus know better how to address the problem. This may cause her to reevaluate the incentive structure in obtaining information, and it casts doubt on the value to the principal of a self-interested supervisor.

There are three main sections in this paper. In section 2, I consider an example in the spirit of Tirole (1992) that illustrates a possible equilibrium with no credibility restrictions. The example illustrates results from Felli (1990) that state that informational asymmetries may prevent colluding parties from realizing gains from trade due to information transferred during the negotiation process.

In the example, I consider two cases. The first is where the agent has an exogenously given right to make the side contract offer to the supervisor. In this case, there are sustainable equilibria that are not CIE. I find, however, that by imposing certain credibility restrictions on the updating of type beliefs, non-CIE equilibria are eliminated, and a unique equilibrium is found when the agent makes the proposal. The second case is where the supervisor makes the proposal. In this case there is a different, unique equilibrium that is also CIE. The determination of which equilibrium actually occurs is a question of who, the supervisor or the agent, has the right to make the proposal.

In section 2.4, I find a unique CIE equilibrium by endogenizing the decision of who makes the proposal. When the supervisor and the agent are allowed to negotiate, credible updating of type beliefs through bargaining eliminates one equilibrium, and leaves only the outcome where the supervisor proposes the side contract and acquires nearly all the gains.
In light of the developments to this point, the principal knows that if collusion is allowed, it will occur and stems from the supervisor, who acquires all the gains. Section 2.5 considers the effect of manipulating the supervisor's incentives and punishments in response. I modify an infinite punishment scheme to an incentive compatible balance between the supervisor's rewards and punishments. I also consider the use of rewarding the supervisor for different reports and the option of not using his report at all. Results are that the CIE outcome or an equally efficient contract occurs. The only choice the principal has is to whom the gains should go: the supervisor or the agent. In either case, the principal's payoff is equivalent to eliminating the supervisor completely. It is the case that if parties update beliefs in a credible manner, a self-interested supervisor is of no value to the principal.

2. The Model

In the model, there are three risk neutral parties; a principal (P), a supervisor (S), and an agent (A). The principal is considered the buyer of the agent's product, and lacks either the time or knowledge to supervise the agent during production. The supervisor's role is to collect information to help the principal control the agent, and the agent performs a productive action for the principal.

The setup is a simplified model in the framework of an example from Tirole (1992). This example is considered because it illustrates the role and the actions of a self-interested supervisor. Consider a three-player, finite-stage game where A chooses to supply either 0 or 1 unit of a good to P. The agent's privately-known cost of supplying one unit is denoted by \( c \), which takes one of two values: the low cost of \( c_L \) with probability
$p_c$, or the high cost of $c_h$ with probability $1-p_L$. Define $\Delta c$ as $c_h-c_L > 0$. The levels of these costs and their probabilities are common knowledge.

The principal offers wage $w$ to the agent for the good, and the agent produces if $w \geq c$. If the agent produces, the principal gets gross surplus $G$. If the agent does not produce, gross surplus is 0. For his services, the supervisor receives payment $r$ from the principal.\(^3\)

The payoffs to the agent, supervisor, and principal are defined as $V_A$, $V_S$, and $V_P$ respectively, where

$$V_A = w-c, \quad V_S = r, \quad V_P = G-w-r.$$  

Each party's reservation value is assumed to be 0.

P cannot observe A's costs. Thus P wishes to hire S to observe A in hopes of gaining this information. In supervising the agent, there are two possible outcomes:

1. The supervisor receives a signal. This outcome is denoted $\sigma = \bar{p}_L$.
2. The supervisor receives no signal. This outcome is denoted $\sigma = \phi$.

If $\sigma = \bar{p}_L > p_L$, S learns there is a high probability $\bar{p}_L$ that A is low-cost. The outcome $\sigma$ is common knowledge between S and A.

It is assumed that P will only offer $c_L$ and risk A rejecting the transaction if P believes A is low-cost with probability $\bar{p}_L$. That is to say

$$G-c_h < \bar{p}_L (G-c_L)$$  \hspace{1cm} (1)

and

---

\(^3\) The reader can refer to the list of notation provided after the appendix.

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\[ G-c_H > p_L(G-c_L) \]  

where the left hand sides of the equations represent the value to P if she offers \( c_H \), and the right hand sides represent the value from offering \( c_L \) when she believes A is low-cost with probabilities \( \bar{p}_L \) and \( p_L \) respectively.

The agent of course always prefers a higher wage. However, the more information P has about A's costs, the greater the chance that she will offer him exactly his cost of producing, thus giving him no profits. The agent's problem then is to try to keep P from attaining the information, and he can only accomplish this through S. He may thus wish to pay S to not reveal to P the information he has learned. The principal wishes to maximize her payoff by choosing a scheme to get S's information, and then offering the appropriate wage to A for the good. The supervisor's problem, then, is simply to decide whether to accept A's payment or P's salary.

The supervisor can make one of three potential reports to the principal. He can make a verifiable report of \( \bar{p}_L \) that A is low-cost with probability \( \bar{p}_L \). He can report \( \phi \) stating that he has not received a signal. If this report is made, by Bayes' rule P's posterior beliefs of \( c_L \) fall to \( p_L \). The inequality given by (2) still holds with new beliefs \( p_L \). If he chooses, the supervisor can also report \( c_L \) stating that A is low-cost. These last two reports are non-verifiable. In brief, if \( \sigma = \bar{p}_L \), then the report must be an element from the set \( \{ \bar{p}_L, \phi, c_L \} \), and if \( \sigma = \phi \), then the report must be an element from the set \( \{ \phi, c_L \} \). S makes his report to P, and A makes the truthful announcement of his type.

---

4 See appendix for explicit derivation of \( p_L \), showing that \( p_L < p_L \).

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The timing is as follows:

1. A learns $c$, $S$ and A learn $\sigma$. It is common knowledge that they have this information.

2. $P$ offers a grand contract to $S$ and $A$. This contract specifies $w$ and $r$ as functions of $S$'s report and the agent's declaration of his costs.\(^5\)

3. $A$ and $S$ can form a binding coalition where either $A$ or $S$ makes a take-it-or-leave-it offer\(^6\) specifying payment as a function $t(report,c)$ from $A$ to $S$ in return for $S$ not revealing $\sigma=\overline{p}_L$.\(^7\) Note that the most $A$ would offer $S$ is $\Delta c$ because it's at best a question of receiving $w=c_H$ instead of $w=c_L$ from $P$.

4. All contracts are implemented.

2.1. The Grand Contract

Assume the principal constructs the grand contract in the following way:

If $c = c_L$:

- With a report of $c_L$, the wage offer is $c_L$. The agent accepts, and $r=\Delta c$.
- With a report of $\overline{p}_L$, the wage offer is $c_L$. The agent accepts, and $r=0$.
- With a report of $\phi$, the wage offer is $c_H$. The agent accepts, and $r=0$.

If $c = c_H$:

- With a report of $c_L$, the wage offer is $c_L$. The agent rejects, and $r=-\infty$.
- With a report of $\overline{p}_L$, the wage offer is $c_L$. The agent rejects, and $r=0$.
- With a report of $\phi$, the wage offer is $c_H$. The agent accepts, and $r=0$.

---

\(^5\) There is a trivial acceptance stage where $A$ and $S$ agree to the contract proposed by $P$. $P$ would not offer a contract which is not welfare improving for herself, and $A$ and $S$ would not accept the contract if they expect negative utility from it. Thus we can limit our study to individually rational contracts.

\(^6\) The decision of who makes the offer is addressed in section 2.4.

\(^7\) Theorem 6 in Tirole (1990) states that coalitions other than the S/A coalition are not binding. Thus we need only consider this case.
The principal pays \( r=\Delta c \) to deter \( S \) from accepting a side contract from \( A \) and to report truthfully. The infinite punishment simply implies that the punishment is big enough to deter \( S \) from reporting false information. Lastly, \( r=0 \) in the event that the report equals \( \phi \) because this report is of no value to the principal.

### 2.2. Side Contract Options: The Agent Makes the Proposal

The principal proposes the above grand contract, and all parties sign it. Assume it is the agent who has the option to make a take-it-or-leave-it proposal to \( S \). Note that a low-cost agent will only attempt a positive payment when \( S \) receives \( \sigma=\bar{p}_L \) (he wants to conceal this information). Also, a high-cost agent will never offer a positive payment because he never has anything to gain.\(^8\) The agent thus chooses a payment that is conditioned on his true type (which is revealed later) so as to reveal no information. In effect, the agent states "If I am high-cost, I will pay \( t(\phi,c_H) \) to \( S \) for a report of \( \phi \), and if I am low-cost I will pay \( t(\phi,c_L) \)."\(^9\) Again, \( t(\phi,c_H) \) should always be 0, and \( t(\phi,c_L) \) can be at most \( \Delta c \).

Tirole describes a no-collusion equilibrium resulting from this contract. Suppose \( \sigma=\bar{p}_L \). Both types of the agent pool to offer the null side contract \( t(\cdot,\cdot)=0 \). This means that both types of the agent propose the side contract stating, "No matter what my type is, I will pay 0 to \( S \) regardless of his report", and any other offer is interpreted by \( S \) as coming from a low-cost agent. The supervisor then reports truthfully, \( \bar{p}_L \), \( P \) offers \( w=c_L \), and \( A \) and \( S \) each net 0. A high-cost agent can never gain a positive amount, and therefore has no incentive to deviate. A low-cost agent cannot gain from deviating either.

---

\(^8\) A high-cost agent nets 0 utility if \( w=c_H \) and gets his reservation utility of 0 if \( w=c_L \).

\(^9\) It is necessary to assume that the act of making this proposal is effortless to ensure that such an offer cannot be used as a signal of a low-cost agent.

---

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In the event he makes an out-of-equilibrium proposal specifying positive payment if he is low-cost, S reports $c_L$. Since S is being truthful, his reward is $\Delta c$, which is enough to override any payment that a low-cost agent is willing to make. P offers $c_L$, and A gets 0 payoff again.

When $\sigma=\phi$, A never has any reason to offer any positive amount because P will offer $w=c_H$ which is the best A can do.

This outcome is not interim efficient for S and A. When $\sigma=\overline{p}_L$, both S and A net 0. However, if a low-cost agent paid S some positive amount $t \in (0,\Delta c)$ and S reported $\phi$, they would both improve their payoffs. I now show that there exist equilibria that can sustain this outcome.

2.2.1. Coalition Interim Efficient Outcome

Tirole discusses an equilibrium that achieves coalition interim efficiency. When $\sigma=\overline{p}_L$, both types of A offer a side contract stipulating that a high-cost agent pays $t(\phi,c_H)=0$ and a low-cost agent pays $t(\phi,c_L)=\hat{t}>0$ in return for a report $\phi$. Again, any other offer is interpreted as coming from type $c_L$. Since both types make the offer, S can make no inferences on A's type, accepts the offer, and reports $\phi$. This outcome is CIE because S and A have achieved the potential gains from colluding.

2.2.2. Imposing Credible Type Belief Updating

I now wish to address the non-CIE equilibria above in the context of a credible belief structure similar to the Intuitive Criterion employed in Cho-Kreps (1987). In their example, there is an equilibrium sustainable by a particular probability assessment on the types of players. The equilibrium breaks down when one player makes an out-of-

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equilibrium move to signal his type. The second player assesses what type of player could gain by making such a move, sees that it can only be one type, and changes her beliefs accordingly.

In this example, the adjustment moves in the opposite direction. The equilibria are held intact when player 2 assumes only one type of player 1 would make an out-of-equilibrium offer. However, when such an out-of-equilibrium offer is made, player 1 is not signaling his type, but is forcing player 2 to update his beliefs in a credible manner. He is taking advantage of the fact that the out-of-equilibrium offer may have come from either type. The example is as follows:

Consider the following figure depicting the null contract equilibrium given in section 3.1. The agent starts at one of the two open nodes in the center, depending on his type. A priori the supervisor believes the applicant is high cost with probability 1-\(\bar{p}_L\), and low cost with probability \(\bar{p}_L\). The agent can either move left and offer the null contract, or move right and choose an offer equal to \(t(\phi, c_L) = 0\), \(t(\phi, c_R) = t > 0\) (denoted simply as \(t\)). The supervisor makes the next move by choosing his report from \(\{\bar{p}_L, \phi, c_L\}\). \((\pi, 1-\pi)\) are the probabilities the supervisor assigns to type \(c_L\) and \(c_R\) respectively when the null contract is offered, and \((\rho, 1-\rho)\) are the associated probabilities when the \(t\) contract is offered. The first entry in the parentheses is the agent's payoff, the second is the supervisor's.
The non-CIE equilibrium states that both types of agent offer the null contract, \( \pi = \pi_L \), and the supervisor reports \( \pi_L \). If the out-of-equilibrium offer \( t \) is made, the supervisor threatens to assess \( \rho \) equal to one. If \( \rho = 1 \), the null contract is indeed an equilibrium because there is no risk to S in reporting \( c_L \).

Suppose, however, that A makes the out-of-equilibrium offer \( t \). At this point, should S still assume \( \rho = 1 \) and report \( c_L \)? Note that the high-cost agent is indifferent between making this proposal and the equilibrium proposal; his payoff is always zero. It would thus have been incentive compatible for a high-cost agent to have made it. Even though the chances may be very small that a high-cost agent chose this particular out-of-equilibrium offer, the credibility restriction states that S will not assess \( \rho \) equal to one when the offer is made.
Note that if a high-cost agent had made the offer, and S reported $c_\ell$, he is infinitely punished. This implies that if S assigns any positive weight to A's type being $c_H$ (if $\rho<1$), he will not report $c_\ell$ for fear of being punished. Therefore, given this out-of-equilibrium offer, S should not report $c_\ell$ and risk $r=-\infty$. Knowing this, a low-cost agent can exploit S's fear of punishment and offer any out-of-equilibrium contract specifying $t(\phi,c_\ell)=0$. A now has the freedom to choose $t(\phi,c_\ell)$ as small as he wishes (i.e. he will pick $t(\phi,c_\ell)=t=$ the smallest unit of currency). The supervisor will accept the offer and report $\phi$ because he prefers the chance for $t$ over nothing.

It is easy to see at this point that the CIE equilibrium offer given by Tirole, where $t(\phi,c_H)=0$, $t(\phi,c_\ell)=\hat{t}$, also collapses to the new equilibrium offer. Substituting the $\hat{t}$ strategy into the figure for the null strategy and adjusting the payoffs accordingly will yield the same results when a value less than one is assigned to $\rho$.

I have shown thus far that if A has the right to make the take-it-or-leave-it offer to S, the only plausible equilibrium achieves coalition interim efficiency where the agent acquires virtually all the gains.

2.3. Side Contract Options: The Supervisor Makes the Proposal

A second sustainable equilibrium, discussed by Tirole, occurs when the supervisor makes the take-it-or-leave-it offer. Suppose S makes the proposal to A specifying $t(\phi,c_H)$ and $t(\phi,c_\ell)$. S now has the power to extract virtually all the gains. As long as A acquires any positive expected payoff from accepting this contract, he will do so. The supervisor proposes $t(\phi,c_H)=0$ and $t(\phi,c_\ell)=\Delta c-t$. The agent accepts this offer, and S acquires virtually all the gains. We do not need to consider credibility restrictions in this instance because

---

10 The literal interpretation of $\infty$ need not be made if the high-cost agent randomizes $t(\phi,c_\ell)$, giving equal weight for each number of units of currency on the interval $(0,\Delta c)$.
there can be no information transfer through the proposal or through the agent's acceptance.

2.4. Endogenizing The Decision of Who Makes the Proposal

The restrictions introduced in section 2 eliminated all but two equilibria. Clearly, the agent prefers the equilibrium where he makes the offer, and the supervisor prefers to make the offer himself. One might ask at this point how it is determined who has this right. One approach is to have this decision determined exogenously, but a problem exists of who should make the proposal and why.

A seemingly more plausible method is to endogenize the decision using type belief updating. In this section, I do this by allowing the supervisor and agent to negotiate the proposal. Tirole (1986) briefly mentions bargaining to determine the distribution of gains, and makes two assumptions on the bargaining process under symmetric information:

A1. The supervisor and agent choose a side contract which is Pareto optimal for these parties.

A2. Each of these two parties can guarantee itself the no-side contract (the null contract) outcome.

And the bargaining solution is trivial: a fifty-fifty split of the surplus. In the present case, we assume that these assumptions hold, but under asymmetric information.

Assume the act of bargaining includes time discounting. For every period the negotiations proceed beyond the first offer, payoffs are discounted because bargaining takes time and causes (minimal) disutility - enough that an individual with nothing to gain will not choose to bargain. It is clear then that the supervisor has all the power in negotiating who makes the offer.

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If $S$ and $A$ discuss who makes the offer, $S$ will demand that he have the right. We know that a high-cost agent will not attempt to bargain because he has nothing to gain. The supervisor knows that any attempt by $A$ to negotiate a different probability is unmistakable evidence that he is low-cost, and he would then report $c_L$ for the reward. Clearly a low-cost agent will not choose to bargain. Since the null contract results if $A$ simply refuses the bargaining proposal, a low-cost agent will not choose this option either. The agent accepts this demand, and the supervisor makes the side contract proposal. $S$ nets $\Delta c - t$, and $A$ nets $t$.

We have shown thus far that given $P$'s original mechanism, interim efficiency is always achieved, and the supervisor acquires virtually all the gains. This outcome, however, is restrictive in that it does not allow the principal to attempt to deter collusion. In the following section, I allow the principal to foresee this development, and to manipulate the terms of the contract in attempt to induce a more favorable outcome.

### 2.5. Collusion-Proofness Possibilities

The payoff to the principal is clearly affected by the coalition. Her expected payoff, conditional on $\sigma = \overline{p}_L$, if this collusion outcome persists is:

$$G - c_H$$

which is necessarily lower than if the collusion did not take place.

The only strategies I allow the principal to improve her profit are those embodied in the contract she offers. More precisely, I only allow her to manipulate the reward
structure, the punishment structure, or the use of potential information. The plausible strategies she can take, then, are as follows:

1. The principal can choose to ignore the supervisor.
2. The principal can offer a reward to the supervisor for a report \( \bar{p}_L \).
3. The principal can adjust the supervisor's reward and punishment for a report \( c_L \).

Note that when comparing any of these new strategies with the previous collusion outcome or with each other, it is necessary only to compare the payoffs to \( P \) when \( \sigma = \bar{p}_L \). The reason being that if \( \sigma = \phi \), the agent will never make any positive offer, and \( P \) will always offer \( w = c_H \). This paired with the fact that the probability of \( \sigma = \phi \) never changes implies that the expected payoff in this instance is always the same.

1) **The principal can choose to ignore the supervisor.**

A collusion-proof strategy the principal can use is to make it known that she will not listen to \( S \). If \( P \) chooses to ignore \( S \), she acquires no information. \( P \)'s beliefs of \( c_L \) remain at \( p_L \), she offers \( w = c_H \), and pays \( r = 0 \). If \( P \) ignores \( S \), the agent then acquires the entire surplus (\( \Delta c \)), and will not accept a proposal from the supervisor. The expected payoff to \( P \) when \( \sigma = \bar{p}_L \) and she does not accept \( S \)'s signal is:

\[
G - c_H.
\]

---

11 Clearly, there are possibilities other than those listed; i.e. \( P \) could institute a punishment for truthful reports. However, it is obvious that such strategies are not acceptable because they are contradictory to the goal of the principal.
2) The principal can offer a reward to the supervisor for a report $\bar{p}_L$.

Suppose P proposes a contract that specifies a reward of $\Delta c$ for a report $\bar{p}_L$ and no reward for a report of $c_L$. We can still assume punishment for a false report of $c_L$, but it is not binding. Also, there need not be a punishment for a false report of $\bar{p}_L$ due to the assumption that this report is verifiable. The agent then can no longer improve on the principal's offer because the supervisor is guaranteed the reward. Thus, when $\sigma = \bar{p}_L$, the supervisor will always report $\bar{p}_L$ and acquire $\Delta c$. This strategy is collusion-proof, and the principal's expected payoff is:

$$\bar{p}_L \cdot (G - c_L) - \Delta c.$$  \hspace{1cm} (5)

3) The principal can adjust the supervisor's reward and punishment for a report $c_L$.

The goal of the principal in using this strategy is to lower the punishment and raise the reward enough that the supervisor is willing to risk the report $c_L$ when $\sigma = \bar{p}_L$. The expected payoff to the supervisor of making the report should also dominate any possible side contract.

Let $x$ be the new, finite punishment, and $r$ is the reward for a truthful report $c_L$. S will report $c_L$ when $\sigma = \bar{p}_L$ if:

$$(1 - \bar{p}_L)(-x) + \bar{p}_L \cdot r \geq \bar{p}_L \cdot \Delta c.$$  \hspace{1cm} (6)

P also wishes to prevent S from reporting $c_L$ when $\sigma = \phi$. That is:

$$(1 - p_L)(-x) + p_L \cdot r \leq 0.$$  \hspace{1cm} (7)
The optimal choices $P$ can make satisfying these constraints are\(^{12}\):

$$
    x = \left( \frac{p_i}{p_i - \bar{p}_i} \right) \cdot p_L \Delta c \quad \text{and} \quad r = \Delta c + \left( \frac{1 - \bar{p}_L}{\bar{p}_L} \right) \cdot p_L \Delta c
$$

(8)

to yield payoff

$$
    \bar{p}_L \cdot (G - c_L) - \bar{p}_L \Delta c + \left( \frac{p_i}{p_i - \bar{p}_i} \right) \cdot p_L \Delta c \left( \bar{p}_L - 1 \right).
$$

(9)

This strategy is also collusion-proof because condition (6) guarantees that $A$ cannot beat $P$'s offer.

### 3. Implications and Extensions

The principal now has an expected payoff for each strategy available to her. Clearly, the payoff from the original contract is equivalent to ignoring the supervisor. The only difference between the two outcomes is who acquires the gains; the supervisor or the agent. This payoff to the supervisor dominates the payoffs from the other strategies. A simple comparison shows that

$$
    G - c_H > \bar{p}_L (G - c_L) - \Delta c
$$

and

$$
    G - c_H > \bar{p}_L \cdot (G - c_L) - \bar{p}_L \Delta c + \left( \frac{p_i}{p_i - \bar{p}_i} \right) \cdot p_L \Delta c \left( \bar{p}_L - 1 \right).
$$

The principal therefore does not wish to pursue variations in the incentive scheme. Even though the infinite punishment prevents seemingly beneficial risk-taking on behalf of the supervisor, balanced incentive schemes and rewards for reports of probabilities do not improve the principal's payoffs because the risk of rejection is too high. Even though

\(^{12}\) See appendix.

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there may be large gross gains from preventing collusion, the incentives required to adequately prevent collusion must necessarily be high also. (In this case, they are the same.)

The implications are that if the supervisor is purely self-interested, there is no way the principal can prevent losses from the threat of collusion. If, given credible belief updating, the principal wishes to prevent collusion, the optimal method is to eliminate the supervisor. There is the option to hire an external auditor who may be both honest and expensive. A problem with this is that his information may not be better than the internal supervisor's. This would be similar to the principal's strategy of rewarding a report of high probability. She would thus need to reevaluate her payoffs given the external auditor's fee and the accuracy of his information. In any case, the absence of an honest internal supervisor precludes the principal from a first-best payoff and ineffectuates the role of the supervisor. A more plausible situation that may justify the role of the supervisor is for the principal to assess a probability that the supervisor is self-interested. This way, the principal's payoff may increase, and she may realize a benefit from the supervisor. Since we do see widespread use of internal supervisors, the assumption of a purely self-interested supervisor seems to be an exaggerated assumption.

Kofman and Lawarree (1993) address this situation with the extensions suggested above. They assume that there is a known probability that the internal supervisor is self-interested, and there exists the option to hire an honest outside auditor. Also, the principal can institute an incentive and punishment contract to deter collusion, where the punishments are monetary and paid from the supervisor to the principal. Results are that in equilibrium, the external auditor is hired on a random basis to monitor the supervisor. In addition, the principal offers an incentive contract to internal parties that may, depending on the parameters, allow collusion. Important results are that given the more
realistic assumptions of potentially honest internal supervisors and the availability of honest external auditors, there is some value to the presence of the internal supervisors, even if collusion does occur.

4. Concluding Remarks

This paper has addressed the usefulness of a self-interested supervisor in the presence of credible belief updating. It has been shown that informational asymmetries may serve to prevent coalition formation and thus benefit the principal. This paper showed, however, that credible belief updating can guarantee collusion. Three issues in coalition formation under asymmetric information were studied in a three-tier hierarchy. First, the implementation of a credible belief structure eliminated all non-CIE equilibria, leaving two possible equilibria. One equilibrium resulted when the agent had the right to make a take-it-or-leave-it side contract proposal, and the other resulted when the supervisor had the right. This paper then endogenized the decision of who makes the proposal, and a unique collusion equilibrium resulted where the supervisor gained the right through negotiations.

The final result of this paper came in allowing the principal to institute her choice of collusion-proof contracts. We saw that in allowing the principal to change only the incentives of the original contract, she was able to prevent collusion, but could not improve her payoff. The only plausible collusion-proof scheme was for the principal to simply eliminate the supervisor. In this case, the principal's payoff was equivalent to allowing collusion. The only difference between the two outcomes is who, the supervisor or the agent, acquires the gains. It is clear that there is no benefit at all to hiring a self-interested supervisor. The assumption that the supervisor acts this way may therefore be

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problematic. Most people value things other than monetary payoffs. A supervisor may have a sense of loyalty or professional integrity which prevents him from accepting side contracts. It may also be the case that the agent is unwilling to offer a bribe. Consequently, a more realistic assumption may be to assess probabilities that the supervisor acts in a self-interested manner and that the agent is willing to offer a bribe. This could result in benefits to hiring internal supervisors, and more realistic results.

It is clear that these results are restrictive in that the principal was not allowed strategies outside the incentive structure delineated by Tirole's example. It does, however, provide groundwork for extensions to other strategies that are available. Since it was shown that in the collusion equilibrium the supervisor will make the side contract proposal, extensions beyond the incentive structure perhaps should focus more on the supervisor than the agent. The principal may wish to consider hiring an (honest) outside auditor to monitor the supervisor and institute a new incentive scheme accordingly. The principal may also wish to hire a costly outside auditor instead of the supervisor, and collusion then would no longer be an issue. These options, among others, are open to the principal, and could also be considered in this context.
1) Show that the outcome $\sigma = \phi$ causes posterior beliefs of type $c_L$ to fall.

Define the following probabilities:

$$
pr(c = c_L) = p_L \quad pr(c = c_H) = 1 - p_L
$$

$$
pr(\sigma = \bar{p}_L / c = c_L) = \zeta \quad pr(\sigma = \phi / c = c_L) = 1 - \zeta
$$

$$
pr(\sigma = \bar{p}_H / c = c_H) = \alpha \quad pr(\sigma = \phi / c = c_H) = 1 - \alpha
$$

$$
pr(c = c_L / \sigma = \bar{p}_L) = \bar{p}_L \quad pr(c = c_H / \sigma = \bar{p}_L) = 1 - \bar{p}_L \quad (\bar{p}_L > p_L)
$$

$$
pr(c = c_L / \sigma = \phi) = \bar{p}_L
$$

We wish to show that $pr(c = c_L / \sigma = \phi) < p_L$.

That is

$$
p_L > \bar{p}_L = \frac{pr(c = c_L) \cdot pr(\sigma = \phi / c = c_L)}{pr(\sigma = \phi)} = \frac{p_L (1 - \zeta)}{p_L (1 - \zeta) + (1 - p_L) (1 - \alpha)}.
$$

To find $(1 - \alpha)$:

$$
\bar{p}_L = \frac{pr(c = c_L) \cdot pr(\sigma = \bar{p}_L / c = c_L)}{pr(\sigma = \phi)} = \frac{p_L \zeta}{p_L \zeta + (1 - p_L) \alpha}.
$$

solving for $\alpha$ yields:

$$
\alpha = \frac{p_L \zeta (1 - \bar{p}_L)}{(1 - p_L) \bar{p}_L}.
$$

and plugging $1 - \alpha$ into $\bar{p}_L$ and simplifying yields:

$$
\bar{p}_L = \frac{p_L (1 - \zeta)}{1 - p_L \zeta - \frac{p_L \zeta (1 - \bar{p}_L)}{\bar{p}_L}}.
$$
Now to show that $p_L > p_L^*$,

$$\Rightarrow 1 > \frac{1 - \zeta}{1 - p_L \zeta - p_L (1 - p_L) \frac{p_L}{p_L}} \Rightarrow p_L (1 - p_L) < 1 - p_L,$$

and we know that $\frac{p_L}{\bar{p}_L} < 1$ and $(1 - \bar{p}_L) < (1 - p_L)$. \hfill $\square$

2) P’s payoff from adjusting the reward and punishment:

Suppose P raises the reward and lowers the punishment enough that S is induced to report $c_L$ when $\sigma = \bar{p}_L$ regardless of A’s offer. That is:

$$(1 - \bar{p}_L)(-x) + \bar{p}_L r \geq \bar{p}_L \cdot \Delta c$$

Note that if this condition holds then A cannot override P’s proposal, regardless of his offer. Note also that beliefs remain at $\bar{p}_L$ because in section 5 we showed that S makes the side contract proposal, and no information is revealed.

We can assume equality here because P wants to give S just enough to induce him to report. That is:

$$(1 - \bar{p}_L)(-x) + \bar{p}_L r = \bar{p}_L \cdot \Delta c.$$  

Also, P does not want S to report $c_L$ when $\sigma = \phi$ because S’s report then conveys no information. This condition states that:

$$p_L r + (1 - p_L)(-x) \leq 0,$$

$\Box$

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* and ** hold when

\[ r = \Delta c + x \left( \frac{1 - \bar{p}_L}{\bar{p}_L} \right) \]  
(from *),

and then

\[ p_L \cdot \left[ \Delta c + x \left( \frac{1 - \bar{p}_L}{\bar{p}_L} \right) \right] + \left( 1 - p_L \right) (-x) \leq 0 \]  
(from **),

which yields

\[ x \cdot \left( \frac{p_L - \bar{p}_L}{\bar{p}_L} \right) \leq - p_L \cdot \Delta c. \]

We know that \( \bar{p}_L \succ p_L \), thus

\[ x \geq \left( \frac{p_L - \bar{p}_L}{\bar{p}_L - p_L} \right) \cdot p_L \Delta c. \]  
(***)

P's payoff when \( \sigma = \bar{p}_L \) is

\[ \bar{p}_L \cdot (G - c_L - r). \]

Substituting for \( r \):

\[ = \bar{p}_L \cdot (G - c_L) - \bar{p}_L \cdot \left[ \Delta c + x \left( \frac{1 - \bar{p}_L}{\bar{p}_L} \right) \right] \]

\[ = \bar{p}_L \cdot (G - c_L) - \bar{p}_L \Delta c + x \cdot (\bar{p}_L - 1) \]

We now maximize this term with respect to \( x \) subject to (***). Notice that the coefficient of \( x \) is always negative. We therefore choose \( x \) as small as possible. P thus chooses

\[ x = \left( \frac{p_L - \bar{p}_L}{\bar{p}_L - p_L} \right) \cdot p_L \Delta c \quad \text{and} \quad r = \Delta c + \left( \frac{1 - \bar{p}_L}{\bar{p}_L - p_L} \right) \cdot p_L \Delta c, \]

and P's expected payoff from choosing this strategy is

\[ \bar{p}_L \cdot (G - c_L) - \bar{p}_L \Delta c + \left[ \left( \frac{p_L - \bar{p}_L}{\bar{p}_L - p_L} \right) \cdot p_L \Delta c \right] (\bar{p}_L - 1). \]
NOTATION

A  ≡ the agent
α  ≡ probability the supervisor receives the signal  \( \bar{p}_L \) if the agent is high-cost
\( c \)  ≡ the agent's privately known cost of supplying one unit of output
\( c_L \)  ≡ cost of supplying one unit of output if the agent is low-cost
\( c_H \)  ≡ cost of supplying one unit of output if the agent is high-cost
\( \Delta c \)  ≡ \( c_H - c_L \)
CIE  ≡ coalition interim efficient
\( \phi \)  ≡ no signal is received by the supervisor
G  ≡ gross surplus to the principal if the agent produces
\( p_L \)  ≡ prior probability that the agent is low-cost
\( \bar{p}_L \)  ≡ signal to the supervisor of high probability that the agent is low-cost
\( \bar{p}_L \)  ≡ posterior probability that the agent is low-cost if \( \sigma = \phi \)
P  ≡ the principal
\( \pi \)  ≡ probability assessed to a low-cost agent when an equilibrium move is made
\( \rho \)  ≡ probability assessed to a low-cost agent when an out-of-equilibrium move is made
r  ≡ the supervisor's reward paid by the principal
S  ≡ the supervisor
\( \sigma \)  ≡ signal the supervisor receives on the agent's costs as a result of supervising the agent
\( t(\text{report}, c) \)  ≡ specification of payment from the agent to the supervisor when they collude
\( t \)  ≡ smallest unit of currency
w  ≡ transfer from the principal to the agent for the good produced
\( x \)  ≡ punishment imposed on the supervisor for a report of false information
\( \zeta \)  ≡ probability that the supervisor receives the signal \( \bar{p}_L \) if the agent is low-cost

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Chapter 3

Wage Negotiations Under Two-Sided Incomplete Information with the Possibility of Side Contracts

1. Introduction

In the recent literature concerning the employer-employee relationship, the worker usually has private information about his own productivity or costs, and the employer uses a scheme to try to determine this information. Tirole (1986) and (1992) and Frascatore (1993) among others address this situation in the context of an agent with private cost information. The wage paid to the worker is typically some function of his expected productivity or cost. The worker always wishes to send a signal of high productivity or high cost to the employer in hopes of realizing a high wage. The employer of course wishes to pay as low a wage as possible while keeping the worker from accepting a job elsewhere.

In this paper I examine a reverse situation where the employer knows the productivity of a job applicant, but the applicant does not. This requires that the employer can observe the talent of an applicant. Examples might be a clothing firm hiring a model or a professional sports team hiring an athlete. The talent of the model is easily observed through photos or previous modeling jobs, and an athlete through his past personal statistics. However, the applicant's value to a specific firm is not discernible by the applicant. I shall assume that the employer has this information; i.e., the clothing firm may know that a particular model would be very suitable for advertising a new line of apparel.
and the owner of a sports team will know the team's depth at a particular position, and the 
subsequent need for another player.

This of course poses a problem for the applicant. He does not know his value to 
the firm, and therefore does not know the highest wage the employer is willing to pay. An 
applicant who demands a wage that is too high is not hired. An applicant who accepts a 
wage lower than his actual productivity fails to achieve a first-best settlement. This type 
of situation is addressed in Chatterjee and Samuelson (1983) and in the research 
symposium, JET (1989). Equilibria are defined, but the possibility of information 
gathering is not considered. In this paper, I find the effects on the efficiency of negotiation 
if parties are allowed means to gather information.

Clearly, having knowledge of his productivity to a specific firm is valuable to the 
applicant. For some high-wage jobs, such as those mentioned above, it may be 
worthwhile for the applicant to hire an agent to determine his worth to the firm. The 
applicant can use the information from the agent as bargaining power in negotiating a 
wage.

For low-wage jobs, an applicant may not be willing to hire outside help to 
determine his productivity with a firm. He may, however, seek internal help. The workers 
already employed by the firm may know the value of an applicant. Hence this applicant 
might appeal to current workers for this information.¹³ Since the applicant expects to 
benefit from having this information, he is willing to offer a kickback for it. This kickback

¹³ The informed party does not have to be a worker at the firm. He merely has to be a person, perhaps an 
employment agency, with inside information and who is under the influence of the employer.

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induces the worker to reveal productivity information, and this coalition improves its welfare.\textsuperscript{14}

Intuitively, employers should try to deter such behavior. Typically, employers discourage workers from telling applicants about adverse work conditions, or the actual need for the applicant. A worker's revelation of harsh work conditions to an applicant may result in the applicant demanding a higher wage or refusing the job. Thus the employer encourages her workers to make the job sound as pleasant as possible. Along the same lines, the employer may attempt to prevent the workers from revealing to the applicant his value to the firm. The applicant's problem then is to get this information from the workers, while the employer may wish to devise a scheme to prevent the leakage of such information.

It is interesting to note that the employer is not always inclined to deter collusion. In fact, most types of employer prefer collusion under certain circumstances. The reason is that without collusion, neither the applicant nor the employer knows the other's reservation wage, and consequently they do not know each other's equilibrium bids. As a result, there is a significant probability that the applicant will bid too high, and the employer will bid too low. A deal may not be struck even if there are gains from trade, and inefficiencies result. If the applicant knows his productivity (and thus the employer's equilibrium bid), the probability of an agreement increases. It may in fact increase enough to override the losses to the employer from a higher equilibrium wage.

This paper addresses a bargaining situation where a job applicant and an employer negotiate a wage. There is initially two-sided incomplete information in that the employer does not know the applicant's reservation wage and the applicant does not know his

\textsuperscript{14} I use the "kickback" simply to determine the value of the information. It can be thought of as a fee for any institution providing a similar service.

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productivity at the job, and thus the maximum wage an employer is willing to offer. There is an existing worker at the firm who knows the applicant's productivity, and he is assumed to be self-interested. That is, the worker is willing to accept a kickback from the applicant in return for the knowledge of the applicant's productivity. The paper is arranged as follows:

A simultaneous, single-offer bargaining situation between the employer and the applicant is examined in section 2. In the model, it is common knowledge that neither the employer nor the applicant has any other prospective interviews. As a benchmark, I assume collusion is not an option, and an equilibrium is given under these circumstances.

Coalition formation is allowed in section 3, and the collusive equilibrium through wage negotiations is described. The applicant should, in learning the employer's reservation wage, expect a greater wage outcome. The kickback offered to the worker should be some function of the expected gain to the applicant in receiving the information. A new equilibrium is found which may be strictly Pareto superior for all parties. If it is, this outcome will persist. If not, the employer may wish to prevent the information transfer.

In section 4 the employer attempts to deter the collusive behavior through incentive contracts. In particular, the employer can institute a profit-sharing scheme, where existing workers get a cut of the surplus from a new worker's production. The result in this situation is that side contract deterrence is too costly for the employer. In equilibrium, the employer always allows collusion, a side contract is formed, and a more efficient outcome results.
2. The Model

In this model, consider an industry with many similar, but not identical, firms. There are three risk neutral parties: an employer, who is the owner of a firm, a supervisor who works for the employer, and a qualified job applicant. The employer wishes to fill a job opening, and is considering the applicant. (Assume this is the only applicant at this point.) The productivity of a qualified worker among firms in this industry is uniformly distributed on the interval \([0,1]\), the actual value depending on the firm. That is, if a worker has productivity \(w^* \in [0,1]\) at a particular firm, the employer gets gross surplus \(w^*\) from employing this worker. This interval is common knowledge among all potential workers and employers in the industry.

The applicant submits his resume and all relevant materials, so the employer and the supervisor know both the talent of the applicant and the characteristics of the job vacancy. It follows then that each also knows the productivity, \(w\), of the applicant. The applicant does not know the specific needs of the firm, and thus only knows that his productivity is uniformly distributed on the interval \([0,1]\).

At this point, assume the applicant has an exogenously given reservation wage denoted by \(c\). This can be thought of as the applicant's current salary, or his best offer from other employers in the industry. His only outside opportunities are with employers in the same industry, so \(c\) also lies on the interval \([0,1]\).\(^{15}\) The applicant has this offer in writing and can present it as evidence of his reservation wage if he chooses. The employer and the supervisor know only that \(c\) is uniformly distributed on the interval. Naturally, the employer would like to pay the applicant as low a wage as possible, and the applicant

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\(^{15}\) For simplicity, assume the applicant has no plaas for future interviews. This assumption can be relaxed to include the case where the applicant’s reservation wage includes expected gains from future interviews.
wishes to be paid a wage equal to his productivity at the job. That is, the employer wishes to pay \( c \), and the applicant wishes to be paid \( w \) (assuming \( w \geq c \)). The employer's only objective is to maximize profits from hiring the applicant, the applicant wants a wage as high as possible beyond \( c \), and the supervisor is interested in obtaining the highest possible payment from either the employer or the applicant.

**The Negotiations Procedure**

The steps are as follows:

**Job Application**  The applicant submits his job application and resume. The employer and the supervisor review the materials and determine the applicant's productivity.\(^{16}\)

**Collusion Deterrence**  The employer and the supervisor may arrange a reward/punishment scheme to deter the supervisor from colluding with the applicant. This scheme will only be signed if it is incentive compatible and individually rational for both parties.

**Side Contract Formation**  The supervisor and the applicant may meet to form an individually rational, incentive compatible side contract where the applicant pays some kickback to the supervisor in return for productivity information.

**Interview**  The employer interviews the applicant, and wage negotiations take place.

**Contract Implementation**  All contracts are carried out.

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\(^{16}\) Assume the employer needs the supervisor's input to determine the applicant's productivity, so that the supervisor needs to see the materials the applicant has sent to the employer.
As a benchmark, suppose there is no possibility of collusion (i.e. the Collusion Deterrence and Side Contract Formation stages are skipped). The employer and the applicant negotiate a wage when each has private information on his or her reservation wage. I shall use the wage negotiations procedure as introduced by Chatterjee and Samuelson (1983). The negotiations take place as follows: During the interview, the employer and the applicant use a single-stage sealed-bid mechanism, where the employer and the applicant simultaneously make sealed bids \( g(w) \) and \( f(c) \) respectively. If \( g(w) \geq f(c) \), the applicant is hired and a wage equal to \( \frac{1}{2}(g(w)+f(c)) \) is realized. If \( g(w) < f(c) \), both parties cease negotiations and the applicant is dismissed with no offer. In this case, the conflict outcome results. The employer gets her reservation payoff of 0 (all parties know this), and the applicant gets his reservation payoff of \( c \) (again, the employer and supervisor know only that \( c \in [0,1] \)).

To define the properties of equilibrium bid strategies, it is useful to state the following theorem from Chatterjee and Samuelson (1983):

**THEOREM 1.** Under the sealed bid offer bargaining rule, the equilibrium bargaining strategies of the buyer and seller are increasing in their respective reservation wages.

The proof is given by the authors.

### 2.1. The General Bargaining Case

Define \( w \) as the applicant's productivity with the employer. As stated above, \( g(w) \) depicts the wage proposal of the employer when the applicant has productivity \( w \) at her firm, and \( f(c) \) is the wage proposal of the applicant with reservation wage \( c \). Individual
rationality must be satisfied for both parties by the restrictions $g(w) \leq w$ and $f(c) \geq c$. This
assures that neither party can get a negative payoff from the deal.

The applicant wishes to maximize his wage beyond $c$. The applicant's surplus is
defined by

$$F = \int_{c, f(c) \leq g(w)} \left[ \frac{1}{2} (f(c) + g(w)) - c \right] dw.$$ 

The applicant chooses the optimal wage proposal $a^*$ given that

1. his productivity $w$ is uniformly distributed on the interval $[0,1]$, 
2. the employer's wage proposal is the function $g(w)$ of the applicant's productivity, and 
3. if $g(w) \geq a^*$, the realized wage equals $\frac{1}{2} (g(w) + a^*)$

and determines the proposal as follows:

$$a^* = \arg \max_d \int_{c, a \leq g(w)} \left[ \frac{1}{2} (a + g(w)) - c \right] dw;$$

the expected gain to the applicant when bidding $a^*$ is

$$F^* = \int_{a \leq g(w)} \left[ \frac{1}{2} (a^* + g(w)) - c \right] dw.$$ 

The applicant's decision rule is thus

$$f(c) = \begin{cases} a^* & \text{if } F^* \geq 0, \\ \in [c, 1] & \text{if } F^* = 0. \end{cases}$$

When $F^* = 0$, note that no matter what wage proposal the applicant makes, he
cannot gain a wage greater than $c$, given that the employer has used decision rule $g$. He
therefore makes any proposal in $[c,1]$.

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The decision rule for the employer is determined in a similar fashion. She wishes to maximize the surplus acquired from hiring the applicant. The employer's surplus is defined by

\[ G = \int_{w, z (w) \geq f(c)} \left[ w - \frac{1}{2} (g(w) + f(c)) \right] dc. \]

The employer chooses the optimal wage proposal \( b^* \) given that

1. the applicant's reservation wage \( c \) is uniformly distributed on the interval \([0, 1]\),
2. the applicant's wage proposal is a function \( f(c) \) of his reservation wage, and
3. if \( b^* \geq f(c) \), the realized wage equals \( \frac{1}{2} (b^* + f(c)) \)

and determines the proposal as follows:

\[ b^* \in \arg \max_b \int_{w, z (w) \geq f(c)} \left[ w - \frac{1}{2} (b + f(c)) \right] dc; \]

the expected net surplus to the employer when bidding \( b^* \) is

\[ G^* = \int_{b \geq f(c)} \left[ w - \frac{1}{2} (b^* + f(c)) \right] dc. \]

The employer's decision rule is thus

\[ g(w) = \begin{cases} 
  b^* & \text{if } G^* \geq 0, \\
  \in [0, w] & \text{if } G^* = 0.
\end{cases} \]

Again, \( G^* = 0 \) states that no matter what wage proposal the employer makes, she cannot agree on a wage below \( w \) given that the applicant has used decision rule \( f \). She therefore makes any proposal in \([0, w]\).

Given the above equilibrium, when the applicant bids \( a^* \), he expects a gain of \( F^* \). When the employer bids \( b^* \), she expects a wage gain of \( G^* \). In order to see the effect of

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the possibility of collusion, it is necessary to characterize the solution with and without collusion.

2.1.1. The Linear Solution

As shown in Chatterjee and Samuelson (1983) and Leininger, Linhart, and Radner (1989) among others, there is an infinite number of equilibria in this problem. Any equilibrium strategies satisfying the first order linked differential equations resulting from maximizing the expected gain functions $F$ and $G$ are possible. Linear strategies are included in this set. Other equilibria include an infinite number of step function equilibria.

In order to make a valid comparison between collusive and non-collusive bargaining outcomes, it must be assumed that all parties consistently use the same type of strategies. Because there are an infinite number of equilibria, it might seem very bold indeed to assume that any particular type of equilibrium strategies is consistently used. However, results in Myerson and Satterthwaite (1983) and Radner and Schotter (1989) offer some justification of the use of linear strategies.

Myerson and Satterthwaite show in a single-stage simultaneous-offer bargaining situation with identical priors that the use of linear strategies yields the most efficient outcome given a 50-50 split of the surplus. The term "most efficient" is used because the authors show that a fully efficient outcome, where a deal is always struck if possible, cannot occur. Furthermore, Radner and Schotter show in an experimental setting that parties in this bargaining situation do tend to use linear strategies. In fact, the strategies they use converge surprisingly close to the linear strategies predicted by the theory. For these reasons, I shall compare among cases using linear strategies.
I shall first restate from Chatterjee and Samuelson (1983) a solution to the above problem when both parties use a linear strategy. That is, the applicant assumes the employer chooses the bid

\[ b = \alpha w + \beta \]

and the applicant chooses the optimal bid defined by

\[ a^* = \arg \max_a \int_0^\infty \left[ \frac{1}{2} (a + \alpha w + \beta) - c \right] dw \]

which yields

\[ a^* = \frac{\alpha + \beta + c}{3}. \quad (1) \]

Simultaneously, in response to the general linear strategy of the applicant

\[ a = \mu c + \rho \]

the employer chooses the bid

\[ b^* = \arg \max_b \int_0^b \left[ w - \frac{1}{2} (b + \mu c + \rho) \right] dc \]

which yields

\[ b^* = \frac{2w + \rho}{3}. \]

Taking \( \alpha = \frac{2}{3}, \beta = \frac{2}{3}, \) substituting into (1), and solving for \( \rho \) yields

\[ a^* = \frac{2c}{3} + \frac{1}{4} \quad \text{and} \quad b^* = \frac{8w + 1}{12}. \]

The parties' decision rules are thus

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\[ f(c) = \begin{cases} \frac{2c}{3} + \frac{1}{4} & \text{if } c \leq \frac{3}{4}, \\ \epsilon [c, 1] & \text{if } c \geq \frac{3}{4} \end{cases} \]

\[ g(w) = \begin{cases} \frac{8w + 1}{12} & \text{if } w \geq \frac{1}{4}, \\ \epsilon [0, w] & \text{if } w \leq \frac{1}{4} \end{cases} \]

and the resulting wage equals

\[
\frac{1}{2} \left( \frac{2c}{3} + \frac{1}{4} + \frac{8w + 1}{12} \right) = \frac{2c + 2w + 1}{6}
\]

if \( c \leq w - \frac{1}{4} \), and there is no deal if \( c \geq w - \frac{1}{4} \).

2.1.2. Type Participation

There is an intuitive problem with this equilibrium. Consider the following definition:

**Definition:** Define a bargaining equilibrium to be type-anticipatory if given the equilibrium strategies of the participating types, no type of participant in the negotiations has a zero chance of striking a deal, and no type not participating has a positive chance of striking a deal if he engages. That is, no type of participant wishes to drop out of negotiations, and no non-participant wishes to engage. Furthermore, it must be the case that common knowledge among all parties that only certain types will participate will not change the bid strategies.

The equilibrium above is not type-anticipatory. This is because in equilibrium, the highest bid an employer can make is \( \frac{3}{4} \) (at \( w = 1 \)), and the lowest bid an applicant can make is \( \frac{1}{4} \) (at \( c = 0 \)). This is common knowledge prior to negotiations. Therefore, any applicant with \( c > \frac{3}{4} \) and any employer with \( w < \frac{1}{4} \) know there is no hope of striking a deal at the interview. If the interview causes the slightest disutility, then these types will not interview. A new equilibrium must be found where the participants are applicants of types \( c \in [0, \frac{1}{4}] \), and employers of types \( w \in [\frac{1}{4}, 1] \). It must be shown then that these type sets

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will not change further, i.e., each new equilibrium does not enlarge the sets of types who wish to drop out.

**THEOREM 2.** The equilibrium characterized by the linear strategies \( a^* = \frac{2c}{3} + \frac{1}{4} \) and \( b^* = \frac{8w+1}{12} \) with participants of type \( c \in [0, \frac{1}{4}] \) and \( w \in [\frac{1}{4}, 1] \) is type-anticipatory.

Proof:

The problem changes from the previous case in the prior distribution of the participants. At the interview, the employer knows \( c \in [0, \frac{1}{4}] \) and \( w \in [\frac{1}{4}, 1] \). The density functions thus change from \( \frac{1}{4} \) to \( \frac{4}{3} \). The applicant's problem is then to choose

\[
a^* = \arg \max_a \int_{-\beta}^{\beta} \left[ \frac{1}{2}(a + \alpha w + \beta) - c \right] \left( \frac{4}{3} \right) dw
\]

which yields

\[
a^* = \frac{\alpha + \beta + c}{3}.
\]

The employer's problem becomes

\[
b^* = \arg \max_b \int_0^b \left[ w - \frac{1}{2} (b + \mu c + \rho) \right] \left( \frac{4}{3} \right) dc
\]

which yields

\[
b^* = \frac{2w + \rho}{3}.
\]

The solutions to these yield

\[
a^* = \frac{2c}{3} + \frac{1}{4} \quad \text{and} \quad b^* = \frac{8w + 1}{12} \tag{2}
\]

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which is identical to the outcome above.

Again, the highest an employer can bid is \( \frac{3}{4} \), and the lowest an applicant can bid is \( \frac{1}{4} \). Therefore, only applicants of type \( c \in [0, \frac{1}{4}] \) and employers of type \( w \in [\frac{1}{4}, 1] \) have a positive chance of striking a deal. The remaining types cannot strike a deal given these strategies, and thus will not attend. This equilibrium is thus type-anticipatory.\(^{17}\)

The difference between this type-anticipatory equilibrium and the previous equilibrium is that applicants of type \( c \in (\frac{1}{4}, 1] \) and employers of type \( w \in [0, \frac{1}{4}) \) do not interview. In the previous outcome, these types attend the negotiations, but make unacceptable bids knowing that a deal cannot be struck. The strategies for the participating types do not change, and in essence the outcome and payoffs for all types do not change. I shall, however, assume for intuitive reasons that this new outcome persists. This is more appealing because workers with high paying jobs should be less apt to search, and a firm should be reluctant to incur the cost of interviewing low-productivity applicants.

3. Wage Negotiations Under Asymmetric Information

The applicant may benefit from information gathering. If he has a potential source of information, he needs to consider the monetary value of the information. In doing so, it is necessary to determine the equilibrium when the applicant possesses the information.

**Theorem 3.** In single-stage, simultaneous-offer wage negotiations where the applicant knows \( w \), the employer's prior becomes \( c \) uniformly distributed on \( [0, \frac{3w}{4}] \).

\(^{17}\) There are other type-anticipatory equilibria with smaller sets of participants. I will, however, use this equilibrium because it represents the "first cut" of participants.

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and a type-anticipatory equilibrium exists where any type of employer participates and bids $b_i^* = \frac{3w}{4}$, and only an applicant of type $c \in [0, \frac{3w}{4}]$ participates and also bids $a_i^* = \frac{3w}{4}$.

Proof:

As in Chatterjee and Samuelson (1983), it is first necessary to compute the equilibrium when the applicant's priors of his productivity change. Strategies with an informed applicant will be denoted with a subscript $I$.

Suppose the applicant believes his productivity is uniformly distributed on the interval $[w, \bar{w}]$, where $0 \leq w \leq \bar{w} \leq 1$. The applicant must then solve

$$a_i^* = \arg\max_a \left[ \int_{-\beta}^\beta \left( \frac{1}{2} (a + \alpha w + \beta) - c \right) \frac{1}{\bar{w}-w} dw \right]$$

which yields

$$a_i^* = \frac{\alpha \bar{w} + \beta + 2c}{3}.$$ 

This strategy, however, is conditional on the minimum bid of the employer. The applicant will never bid a wage lower than $g_i(w)$. Therefore, there will be a bid mass point at $g_i(w)$ for all values of $c$ such that $a_i^* \leq g_i(w)$. This will be treated formally in the employer's optimization problem.

The employer's problem changes slightly in the setup. She knows the applicant will not bid less than $g_i(w)$. Her problem therefore is to find

$$b_i^* = \arg\max_b \left[ \int_0^\infty \left( w - \frac{1}{2} (b + \alpha w + \beta) \right) dc + \int_{-\beta}^\beta \left( w - \frac{1}{2} (b + \mu c + \rho) \right) dc \right];$$

solving yields

$$b_i^* = \frac{2w}{3} + \frac{\rho}{3},$$

which is identical to the employer's previous strategy.

Solving for $\rho$, $\alpha$, and $\beta$ yields the following results:

$$a_i^* = \frac{2c}{3} + \frac{w}{4} \quad \text{and} \quad b_i^* = \frac{2w}{3} + \frac{\bar{w}}{12}. \quad (3)$$

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The applicant's equilibrium strategies are thus as follows:

$$f_1(c) = \begin{cases} 
\frac{2w}{3} + \frac{w}{12} & \text{if } c \in \left[0, w - \frac{w}{4}\right], \\
\frac{2c}{3} + \frac{w}{4} & \text{if } c \in \left[w - \frac{w}{4}, \frac{3w}{4}\right].
\end{cases}$$

and the employer's strategy is always

$$g_1(w) = \frac{2w}{3} + \frac{w}{12}.$$ 

The assumption that the applicant gains perfect information simply requires $w = \bar{w} = w$. With perfect information, the equilibrium bids become

$$d^*_i = b^*_i = \frac{3w}{4}$$ (4)

for any type of employer and an applicant of type $c \in \left[0, \frac{3w}{4}\right]$.

This equilibrium is type-anticipatory because given the strategies it is not possible for an applicant of type $c \in \left[\frac{3w}{4}, 1\right]$ to strike a deal, and an applicant of type $c \in \left[0, \frac{3w}{4}\right]$ and any type of employer has a positive chance to strike a deal. Also, even with the smaller set of types, it is easy to see that a unilateral change in strategy will not benefit either party because they both bid the same amount. Therefore, strategies remain unchanged.

\[\square\]

3.1. Gains From Information

The applicant can learn how much he benefits from obtaining this information. This will be important to the applicant when deciding the price he will pay for it.

If the applicant learns $w$, he knows the value of the information because he knows his new bid, and he knows what he would have bid without the information. There are two possible scenarios for the applicant:
Case 1. If $c \in [0, w_1]$ an agreement is reached with an uninformed applicant, but his bid is not as high as possible. The gain to the applicant of having the information in this instance is the wage with the information minus the wage without the information, or:

$$b_i^* - \frac{1}{2}(b^* + c^*)$$

$$= \frac{5w}{12} - \frac{c}{3} - \frac{1}{6}.$$

Case 2. If $c \in [w_1, \frac{3w}{4}]$ the uninformed applicant bids too high, and a deal is not struck. The gain to the applicant from having the information in this instance is the wage with information minus the value of no deal (the reservation wage):

$$b_i^* - c$$

$$= \frac{3w}{4} - c.$$

The next step is for the applicant and the informed third party (the supervisor) to negotiate the kickback.

3.2. Side Contract Negotiations

Suppose the supervisor and the applicant are free to arrange a meeting before the interview to form a (enforceable) side contract. The supervisor is assumed to be self-interested in the sense that he is willing to accept payments from the applicant in return for disclosing the applicant's productivity with no regard for the consequences on others. It must be common knowledge at this meeting that the employer is not attempting to interfere.
Any side contract signed by the applicant and the supervisor should be individually rational for both parties. These constraints are as follows:

Denote the kickback paid to the supervisor by $k$. Individual rationality for the applicant requires that the payment for the information cannot be greater than its worth. That is:

$$k \leq \frac{5w}{12} - \frac{c}{3} - \frac{1}{6}$$

if case 1 holds,

$$k \leq \frac{3w}{4} - c$$

if case 2 holds, and

$$k \leq 0$$

if no deal is struck.

Individual rationality for the supervisor simply states that $k \geq 0$ because there is no deterrence to forming a side contract.

The contract should induce honest reporting of all necessary information. The following terms satisfy both of the constraints above and allow room for negotiating a kickback:

The kickback is to be paid by the applicant to the supervisor if and only if the actual wage is agreed upon. The supervisor reveals $w$ to the applicant by providing hard evidence of the applicant's productivity, and the applicant simultaneously reveals $c$ to the

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supervisor by providing documentation of his salary or of his highest offer. The parties then decide the terms of the kickback. If no deal is struck at the interview then must be zero to satisfy both individual rationality constraints. If a deal is struck, the kickback should be some function of the gain from the information. The size of the kickback should be in the region delineated by either Case 1 or Case 2 above. Which case applies is common knowledge after the revelation of and because the equilibrium strategies are known.

**Theorem 4.** To induce truthful type reporting in single-stage, simultaneous-offer side contract negotiations with two-sided uncertainty, the size of the kickback is increasing in the perceived gains from the side contract, and if there are no gains, the kickback is zero.

The proof is in two parts:

1. Truthful reporting of by the supervisor is always assumed because of the assumption that the report of is hard information.

2. Truthful reporting of by the applicant:

Define as the applicant's report of his reservation wage, and is the supervisor's truthful report of the productivity, which is taken as given by the applicant. Define the kickback function as either or , depending on which case persists. To say the kickback is increasing in the perceived gains implies that

\[
\frac{dk(\frac{3w}{12} - \frac{c}{3} - \frac{1}{6})}{d(\frac{3w}{12} - \frac{c}{3} - \frac{1}{6})} > 0 \quad \text{or} \quad \frac{dk(\frac{3w}{4} - c^*)}{d(\frac{3w}{4} - c^*)} > 0.
\]

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18 If , we can assume the applicant does not attend the interview because there is no possibility of a gain in salary, and . We can thus restrict the study to the case .

19 This assumption can be relaxed so that the supervisor's report is non-verifiable, and therefore becomes a strategic variable. This becomes important when the employer institutes a profit-share scheme to deter collusion.

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Since the applicant treats \( w \) as given, this condition is equivalent to
\[
\frac{dk\left(\frac{5w}{12} - c - \frac{1}{6}\right)}{dc^*} < 0 \quad \text{or} \quad \frac{dk\left(\frac{3w}{4} - c^*\right)}{dc^*} < 0.
\]

By assumption the applicant cannot forge false offers. He may, however, have more than one offer. The kickback function should induce the applicant to report his highest offer (or his current salary if that is highest), defined as \( c \). Since the applicant wishes to minimize the kickback, he wants to choose \( c^* \) to minimize \( k(\cdot) \). He therefore should choose \( c^* \) as high as possible; that is, \( c^* = c \).

Thus, to induce the applicant to report his highest offer and the supervisor to report \( w \), the kickback must be an increasing function of the gains, and \( k(0) = 0 \).

At this point, the size of the surplus is known, and each party has equal bargaining power. In negotiating the kickback, the natural outcome is the Nash Bargaining Solution where the applicant pays the supervisor one-half of the gains. Identifying the two cases again:

**Case 1.** If \( c \in [0, w - \frac{1}{3}) \), \[ k = \frac{1}{2}\left(\frac{5w}{12} - \frac{c}{3} - \frac{1}{6}\right). \]

**Case 2.** If \( c \in \left[w - \frac{1}{3}, \frac{3w}{4}\right] \), \[ k = \frac{1}{2}\left(\frac{3w}{4} - c\right). \]

Note that information gathering does improve efficiency. Before the coalition was allowed, a simple calculation shows that the equilibrium expected total surplus was \( \frac{9}{32} \) out of a possible \( \frac{1}{2} \). When collusion is allowed, the expected total surplus increases to \( \frac{11}{16} \).

There is clearly a societal gain from information gathering. There are, however, still possible gains which are not realized. If \( c \in \left(\frac{3w}{4}, w\right] \) there is room for agreement, but an

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agreement will not be reached because the employer bids too low. Even though the applicant is informed and the total surplus has improved, inefficiencies still exist.

An outcome has thus been defined where collusion occurs in equilibrium. If collusion is allowed, both the applicant and the supervisor benefit from the side contract by the amount of the kickback, and the outcome itself is more efficient. Intuition suggests that the expected profits to the employer from the deal are subsequently decreased. A simple profit check shows that this usually is not the case.

4. The Employer's Expected Profits

CLAIM: Most types of employer earn higher profits in the collusive equilibrium than in the non-collusive equilibrium. Also, it is too costly for any type of employer to deter collusion.

This is demonstrated in the following sections.

Assume now it is common knowledge that the employer has the ability to institute collusion-prevention schemes. The employer assesses the value of instituting such a scheme by comparing her expected profit when she allows collusion and when she does not. When there is no collusion, the expected profits from hiring the applicant are the probability the applicant is hired $\times$ the expected profit if he is hired. That is:

$$E(\text{profits/no collusion}) = \left( w - \frac{1}{4}\right)(w - \frac{w + e}{3} - \frac{1}{6})$$

where

$$E(\text{c/there is a deal}) = \frac{w}{2} - \frac{1}{8}.$$

This yields expected profits of
\[
\frac{w^2}{2} - \frac{w}{4} - \frac{1}{32}.
\]

When there is collusion, by the same method we see that

\[
E(\text{profits/collusion}) = \left(\frac{3w}{4}\right)(w - \frac{3w}{4})
= \frac{3w^2}{16}.
\]

The employer prefers the collusive outcome if

\[
\frac{3w^2}{16} \geq \frac{w^2}{2} - \frac{w}{4} - \frac{1}{32}
\]

which is true when

\[w \in [0, .91].\]

A somewhat surprising result is that an employer usually prefers an informed applicant. Clearly, employers of type \(w \in [0, \frac{1}{4}]\) prefer an informed applicant because in the non-collusive equilibrium these types have no possibility of a deal. Under the collusive equilibrium, there is a positive chance of a positive payoff. Also, for all types of employers, there is always a greater probability of a deal with an informed applicant. In fact, if \(w \in [0, .91]\), the collusive outcome is strictly preferred by all parties. This may lead to the question "Why doesn't the employer give the applicant the information herself if it makes everyone better off?" The answer is that there is a subtle difference in having the supervisor make the report. The strategies used above depend on the fact that the productivity is not discussed at the interview. If the employer reveals the true productivity at the interview, the applicant can claim his reservation wage is just a bit less, and so on.

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The strategies change to disadvantage the employer. Also, the applicant cannot make his knowledge known at the interview for other reasons that will be discussed shortly.

If in fact \( w \in [0, .91] \), the employer allows the side contract. When it comes time to negotiate the side contract, the applicant learns that \( w \in [0, .91] \) before negotiating the kickback, and it is common knowledge the employer is not attempting to interfere. The supervisor thus does not gain any bargaining power, and the Nash Bargaining Solution stated in section 3.1 again results. The a priori expected net gains/payoffs are as follows:

\[
\begin{array}{|c|c|}
\hline
& c \in [0, w^{\frac{1}{4}}) & c \in [w^{\frac{1}{4}}, \frac{3w}{4}] \\
\hline
\text{Employer} & \frac{3w^2}{16} & \frac{3w^2}{16} \\
\hline
\text{Supervisor} & \frac{1}{2} \left( \frac{5w^2}{12} - \frac{13w}{48} - \frac{cw}{3} + \frac{c}{12} + \frac{1}{24} \right) & \frac{1}{2} \left( \frac{3w}{16} - \frac{3w^2}{16} + \frac{cw}{4} - \frac{c}{4} \right) \\
\hline
\text{Applicant} & \frac{1}{2} \left( \frac{5w^2}{12} - \frac{13w}{48} - \frac{cw}{3} + \frac{c}{12} + \frac{1}{24} \right) & \frac{1}{2} \left( \frac{3w}{16} - \frac{3w^2}{16} + \frac{cw}{4} - \frac{c}{4} \right) \\
\hline
\end{array}
\]

4.1. Collusion Proofness

Consider now the case when \( w \in (.91, 1] \). In this case the employer wishes to deter collusion between the supervisor and the applicant. That is, she wants somehow to keep the applicant from learning his productivity. Again, any scheme which the employer proposes must be both individually rational and incentive compatible for all parties involved.
The Punishment Scheme

There are essentially two methods available to the employer to prevent collusion (barring outside help). The first is a punishment scheme. If it is clear to the employer that the supervisor has disclosed the information, she punishes him by some monetary payment or she fires him. An apparent problem with this scheme is how to trigger the punishment. The applicant may never tell the employer about the side contract. Therefore she has no proof of collusion other than a suspicious wage proposal. The employer can attempt to punish the supervisor as the applicant's actual wage gets higher, but individual rationality may be threatened. The supervisor has no control over the applicant's reservation wage, and a high wage may occur even without collusion. Moreover, the supervisor can easily shade his report to the applicant slightly below the actual value to avoid punishment and obtain a nearly maximum kickback. It becomes very difficult to institute an acceptable punishment scheme, and for these reasons I shall discard this scheme and examine only an incentive scheme.\(^{20}\)

The Incentive Scheme

With an incentive scheme, the employer attempts to avoid side contracts by offering a reward for not colluding. In employing a reward scheme, the employer creates a mechanism such that the size of the reward increases as the size of the potential kickback increases, and is always large enough to override any kickback the applicant is willing to offer.

\(^{20}\) The applicant may wish to claim the side contract at the interview to improve his bargaining power. This however would be a clear signal to the employer that collusion occurred. A punishment scheme could then work. The supervisor would thus need to include in the side contract an enforceable stipulation that the applicant not reveal this information at the interview.
With collusion, the applicant agrees to pay the supervisor a kickback of one-half the surplus. In order to override the kickback, the employer ensures the supervisor a payment of at least this amount. (Assume that if the supervisor is indifferent between the two offers, he accepts the employer's offer.) Note that a flat salary has no effect. If the supervisor is paid the reward regardless of his actions, there is no incentive for him to not agree to a side contract with the applicant. Therefore, the size of the payment must be a function of the profits.

Because productivity is fixed, the employer can only increase profits by paying a lower wage to the applicant. Therefore, the employer's reward to the supervisor should be a decreasing function of the applicant's wage. She could thus institute a profit-sharing scheme based on the profits earned from employing the applicant. In this fashion, the supervisor always wants the applicant's wage to be as small as possible.

An assumption I have made is that the supervisor's report of the productivity is verifiable. Because it is verifiable, he cannot make strategic reports to increase the employer's profits, and thus his share. Therefore, given the employer's profit-share scheme, the supervisor chooses to make the truthful report \( w \), or no report at all.\(^{21}\) (NOTE: There is nothing to prevent a supervisor from signing both a profit-share agreement and a side contract. The employer needs to consider this possibility in her scheme.)

\(^{21}\) An extension could be made where the report is non-verifiable, and is thus a strategic choice variable. With a profit-share scheme, the supervisor chooses the report to manipulate the applicant's bid to increase profits. In pure strategies, however, the applicant can simply take the inverse of the report to determine the true productivity. Mixed strategies thus need to be used, and there are likely many equilibria. The mixed strategy report may then result in mixed bid strategies. (Broman (1989) discusses the use of mixed strategies in a bilateral monopoly situation under certainty.) Since this is neither crucial for nor within the framework of the study, I shall leave this as a possible future variation.

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If the supervisor chooses to make the report, the side contract is signed and enforceable (he cannot back out). Therefore, he considers his actions before learning \( c \). The employer thus makes her profit-share scheme based on an expected value of \( c \).

The Profit-Share Scheme

As stated above, the scheme must specify the payment to the supervisor as an increasing function of the profits. Define this function as \( p(\pi) \) where \( \pi \) denotes the profits. If the employer institutes a scheme, the supervisor always accepts it and then decides whether to form a side contract. This is because the supervisor always has the option to do both. The condition the employer must satisfy to successfully prevent collusion is as follows:

Define \( \pi_N \) as the expected profits the employer receives when there is no collusion, and \( \pi_C \) as the employer's expected profits with the side contract. The condition states that

\[
p(\pi_N) \geq p(\pi_C) + \text{expected kickback}
\]

which means the supervisor is to be paid more from the profits without collusion than he can be paid from both the profits with collusion and the kickback.

A profit check shows, however, that the employer is never willing to use a profit-share scheme. To deter collusion, the employer will of course never pay more than the losses occurring from it. The most, then, that the employer is willing to pay to deter the collusion is \( \pi_N - \pi_C \). The least the supervisor will take to not collude is the expected kickback (in the instance that \( p(\pi_C) = 0 \)). I can now show that the amount of the kickback is greater than the maximum amount the employer will pay for all feasible values of \( w \).

Collusion deterrence is possible if
\[ \pi_w - \pi_c > \text{expected kickback} \]

\[ \Rightarrow \frac{w^2}{2} - \frac{w}{4} - \frac{1}{32} - \frac{3w^2}{16} > \frac{9w^2}{64} - \frac{w}{8} + \frac{1}{32} \]

\[ \Rightarrow -11w^2 + 8w + 4 > 0 \]

and the solutions are approximately

\[ w < -0.34 \quad \text{and} \quad w > 1.07. \]

Since \( w \in [0,1] \), there are no values of \( w \) for which it is worthwhile for the employer to attempt to deter collusion.

**The Final Type-Anticipatory Equilibrium**

The final type-anticipatory equilibrium with the possibility of collusion is thus characterized as follows:

1. The employer chooses to interview any type of applicant, and only an applicant of type \( c \in [0, \frac{3}{4}] \) chooses to apply.

2. The employer does not choose to implement a profit-share scheme to deter collusion.

3. The applicant and the supervisor form a side contract stipulating the kickback as \( k = \frac{1}{2} \left( \frac{5c}{12} - \frac{5}{6} - \frac{1}{6} \right) \) if \( c \in [0, w-\frac{1}{4}] \), and \( k = \frac{1}{2} \left( \frac{3w}{4} - c \right) \) if \( c \in [w-\frac{1}{4}, \frac{3w}{4}] \). The supervisor and the applicant then simultaneously reveal \( w \) and \( c \) respectively. If \( c > \frac{3w}{4} \), there is no kickback and the applicant does not interview.

4. If the interview occurs, the employer and the applicant each bid \( \frac{3w}{4} \), and the applicant is hired at that wage.
Again, the expected surplus from this final equilibrium increases to $\frac{5}{14}$ from $\frac{9}{32}$ with this collusive equilibrium, but still does not reach the fully efficient surplus of $\frac{1}{3}$.

5. Concluding Remarks

This paper considers the possibilities and benefits of information gathering within an organization prior to wage negotiations. We saw that in single-stage, simultaneous-offer wage negotiations, side contracts occur between the applicant and someone within the firm. In this situation, we have assumed that the employer cannot use a punishment scheme to deter this behavior because collusive action is difficult to prove. Also, the employer chooses not to offer a reward to prevent collusion because an effective reward is too costly. Indeed, most types of employer, in particular those for whom the applicant is not exceedingly productive, prefer the applicant to have the information prior to negotiations. The intuition is that while information increases the equilibrium wage, the probability of an agreement becomes high enough to override the increased wage. For very high productivity workers, the employer prefers that the applicant not have the information because the probability of an agreement is already quite high. Even though in this region the employer prefers the applicant be uninformed, the gains from preventing collusion are not large enough to justify a reward scheme. Therefore, collusion always occurs in equilibrium.

Another result of this paper is that in equilibrium, certain types do not attend the interview. I define this type of equilibrium as type-anticipatory. An applicant with a very high reservation wage and an employer for whom the applicant is not very productive will not attend the interview if no side contract is formed. This is also a fairly intuitive result. Any party with very little to gain does not go through the trouble of seeking a match. One
result of side contract formation is that every type of employer wishes to interview, thus
improving efficiency. An extension which can be done is to assume a significant disutility
or cost from job or candidate search. The result will not surprisingly be that fewer types
will search for jobs and attend interviews, causing larger inefficiencies in the market, but
will enlarge the set of types of employer who prefer an informed applicant.

Even though information gathering usually serves to benefit all parties and always
increases efficiency, inefficiencies still result. These inefficiencies result when \( c \in (\frac{3w}{4}, w] \).
In this region, there is of course room for a deal, but in equilibrium the applicant does not
attend the interview because, given the employer's strategy, there is no possibility of a
deal. I offer two possibilities in attempt to address these inefficiencies. First, if \( c \in (\frac{3w}{4}, w] \),
the applicant can bring proof of his reservation wage to the interview and reveal it
to the employer. A deal could then be struck. The problem of course is then changed,
and a new type of equilibrium would result, but it may serve to decrease losses. Also, the
presentation of the applicant's reservation wage is evidence of collusion, and the employer
can institute a punishment scheme if she chooses.

Another possibility which may eliminate inefficiencies is allowing the applicant to
claim his knowledge at the interview. This possibility was precluded in this paper because
such a claim puts the employer at a disadvantage during the negotiations. The employer
can prevent such a claim by punishing the supervisor, who in turn bars such a claim in the
side contract.

A method typically used in interviewing that allows the applicant to claim
knowledge of his productivity is employment of outside help in the form of a talent agent.
There is no way an employer can prevent this from occurring, and it is legal, so the
applicant can use this information. There are, however, some problems with this method.

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First, a talent agent's services may be prohibitively costly. Also, his assessment of an applicant's productivity with a particular firm is, after all, an estimate. In this model, the reason an applicant's productivity varies among firms is not because of the applicant himself, but because of the unique needs of the firm. It is unlikely then that any outsider would be able to determine the exact productivity of the applicant. The employer would thus have some power in denying the applicant's information, and the value of the information would decrease. Lastly, the information may help the applicant when $c \in \left(3 \frac{1}{4}, w\right]$, but the applicant would likely use the information for any relationship between $c$ and $w$. The problem changes, and any equilibrium in the current setting is no longer valid.

These are a few suggestions for dealing with the remaining inefficiencies. However, the focus of the paper is to show that information gathering can improve efficiency in wage negotiations, and usually results in Pareto improvements for all parties. In such instances, from a social standpoint, information gathering should be encouraged to improve the chances of job matching. Various other extensions can be made allowing for multiperiod and alternating offer negotiations, or a dynamic setting where expected gains from future interviews and collusive behavior will factor into an applicant's reservation wage. These topics may be interesting to pursue to test the robustness of the results.
REFERENCES


Chapter 3. *Wage Negotiations Under Two-Sided Incomplete Information with the Possibility of Side Contracts*
Chapter 4
Choosing Optimal Job Tenures

1. Introduction

There are many factors that affect the length of time a worker stays at a particular job. Certainly non-pecuniary factors such as unexpected job disutility, spousal relocation, and costs of family relocation, among others, play a large role in one's decision to switch jobs. It may also be the case that a worker finds a new job strictly to increase his or her salary or gross pecuniary benefits. The size of the possible increase in salary can be instrumental in explaining when an individual decides to switch jobs. In this paper I use a theoretical framework to study the effect of salary increments on the optimal length of time to stay with a job before switching to a new one.

In many industries, particularly in the public sector, jobs carry specified increases in salary.\(^ {22} \) A significant amount of research has been done showing that wages increase with tenure at a decreasing rate.\(^ {23} \) In these industries, significant salary increments can only be achieved by switching jobs within the industry. Consider a state employee hired at a modest salary. This type of person often is locked into standard incremental salary increases, which I shall call the raise function. If this person earns a good reputation through hard, productive work, he may be offered higher salaries by other institutions. By

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\(^ {22} \) Gibbons and Murphy (1992) state that the wage increments for a particular job may be relatively constant in tenure despite diminishing marginal productivity. An older worker may be paid a wage higher than the value of his marginal product in an effort both to give younger workers something to look forward to and as an efficiency wage to induce higher effort.

\(^ {23} \) The literature here is quite large, but specific papers include Mincer (1986), Mortensen (1988), Abraham and Farber (1987), Lazear (1981), Topel (1991), and Gibbons and Murphy (1992).
the same token, it is possible that this person can switch jobs too early and not earn a salary leap. He may even take a cut in salary. This person would like to maximize his earnings over the course of his lifetime by performing well at his jobs and voluntarily switching at the proper time.24

In this paper I study the optimal length of time for a worker to remain at each of a specified number of jobs throughout his working lifetime taking into consideration the raise function and the size of the salary leaps between jobs. The raise function and the salary leaps are exogenously given, but I discuss the determinants of these values and how they may be endogenized in a competitive labor market. Certain factors such as search costs, marginal productivity, training costs, hiring costs, and the rates of general and specific human capital accumulation affect these values, and I consider how variations affect the results.

Given the optimal tenures for a specified number of jobs, I consider the ideal number of jobs for the worker over the course of his lifetime. This number is also a function of the raises and salary leaps. We also see how the above factors affect the number of jobs a worker chooses in his lifetime. The model suggests how these factors impact the industry turnover rate.

I start in section 2 with a general setup of the model and characterize the solutions. I then consider an example in section 2.1 where the worker faces linear raise and starting salary functions. In solving for the optimal tenures for a specific number of jobs, I find that given the constant linear functions, the worker should stay the longest at his first job, and have progressively shorter tenures throughout his lifetime. The variance of the job tenures depends on the coefficients of the raises and leaps. Next, I find a unique solution

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24 The possibility of layoffs is discussed later.

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for the optimal number of jobs as a function of the raises and leaps. This solution suggests how the above factors affect industry turnover. Section 2.1.3 discusses the intuitive implications of the results.

In section 2.2 I compare the results of the example with empirical findings. Results that workers experience gains from switching jobs is fairly consistent with the empirical results, but the model contradicts findings that turnover is high early in life and low later in life, and workers typically experience decreases in salary later in life. I show that this is due to the assumption of constant raise and starting salary functions, and allowing these functions to vary realistically with time yields consistent results.

This paper treats as constant the raise function and the rate of increase of the salary leaps. It may be the case that these functions are not linear or can change over the course of a worker's lifetime, and section 2.3 discusses the intuition and possible results of a quadratic starting salary function. Section 2.5 discusses some possible extensions of the model. It is also conceivable that the worker can change his behavior to alter these values. I discuss possible ways that a worker can affect his rate of human capital accumulation to increase his lifetime earnings. Another extension would be to put earnings into the worker's utility function. In doing this, it is possible to include other factors, such as job disutility, in the worker's job tenure decision. I briefly mention some possible results of these extensions.

2. The Model

Consider a worker with a total working lifetime of T. He chooses the number of jobs to work in his lifetime and the length of time to stay at each. Bartel and Borjas (1981) classify quits into three categories:
1. Personal or family non-market reasons.
2. Reasons of dissatisfaction with working conditions.
3. Job-related wage maximization reasons.

This paper ignores type (1) and (2) quits and isolates the study on the benefits of turnover on job-related wage maximization.25

At any particular job, the worker's salary increases with tenure. Furthermore, if he starts a new job, the starting salary is some function of his accumulated experience and tenures. If the worker is to change jobs, it should be the case that his starting salary can be lower than, equal to, or higher than his ending salary at the previous job. Given the worker's current salary, the starting salary function, and \( T \), the worker must choose the number of jobs to work and the tenure at each job to maximize his lifetime income.

In general, define

\[
\text{salary rate at job } i = S_i(\tau_i)
\]

where \( \tau_i \) equals the length of time the worker has been at job \( i \).26 This takes the form

\[
S_i(\tau_i) = s_i + f(\tau_i)
\]

(1)

where \( s_i \) equals the worker's starting salary at job \( i \) and \( f(\tau_i) \) defines the raise function.27

\( S_i(\tau_i) \) is a density function as it represents the worker's salary rate at the specified point in

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25 Akerlof, Rose, and Yellen (1988) state that a large proportion of quits are due to job switches that involve no spell of unemployment. The reader can assume this case, or can consider a subsequent spell of unemployment as a cost associated with switching jobs.

26 Altonji and Shakotko (1987) and Akerlof, Rose and Yellen (1988) state that workers also consider fringe benefits when changing jobs for pecuniary reasons. The reader may therefore consider \( S_i(\tau_i) \) as a total compensation function that includes both salary and fringe benefits.

27 Mortensen (1988) studies the case where job and starting salaries are random variables. I assume here that the worker is able to predict and respond to specific functional forms to precisely analyze incentives.

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time. The worker's best possible starting salary at job \( i \) is some function of his acquired human capital through work experience. I define this as

\[
s_i = g\left(\sum_{j=1}^{i-1} \tau_j\right),
\]

and the starting salary at the worker's first job is fixed at \( s_1 \). Here, \( \sum_{j=1}^{i-1} \tau_j \) equals the worker's total job tenure through experience prior to job \( i \). From (1) and (2) we get

\[
S_i(\tau_1, \tau_2, \ldots, \tau_i) = g\left(\sum_{j=1}^{i-1} \tau_j\right) + f(\tau_i). \tag{3}
\]

If this worker is to work \( n \) jobs in his lifetime, his problem is to maximize his lifetime earnings function:

\[
\max_{\tau_1, \tau_2, \ldots, \tau_n} \left( \int_0^{\tau_1} (s_i + f(\theta))d\theta + \sum_{i=2}^{n} \int_0^{\tau_i} \left[ g\left(\sum_{j=1}^{i-1} \tau_j\right) + f(\theta) \right] d\theta \right). \tag{4}
\]

where \( \tau_1 + \tau_2 + \ldots + \tau_n = T \).

If \( f(0) = 0 \) (there is no raise without some positive experience) and \( f(\cdot) = F(\cdot) \), we get

\[
\max_{\tau_1, \tau_2, \ldots, \tau_n} \left( \tau_1 s_1 + F(\tau_1) + \sum_{i=2}^{n} \left( \tau_i g\left(\sum_{j=1}^{i-1} \tau_j\right) + F(\tau_i) \right) \right). \tag{5}
\]

The problem can be solved either by taking the \( n \) partial derivatives and solving simultaneously or by backward induction. A general solution can be demonstrated by considering the last two jobs of the individual's lifetime. The problem becomes

\[
\max_{\tau_{n-1}, \tau_n} \left( \tau_{n-1} s_{n-1} + F(\tau_{n-1}) + \tau_n g(\tau_{n-1}) + F(\tau_n) \right). \tag{6}
\]
where $\tau_{n-1} + \tau_n = T_2$. ($T_i$ denotes the total time left to work the last $i$ jobs.) At this point $s_{n-1}$ is not a choice variable and is treated as fixed. Substituting $\tau_n = T_2 - \tau_{n-1}$ into (6), we get

$$\max_{\tau_{n-1}} \left( \tau_{n-1} s_{n-1} + F(\tau_{n-1}) + (T_2 - \tau_{n-1}) g(\tau_{n-1}) + F(T_2 - \tau_{n-1}) \right).$$

Differentiating yields

$$s_{n-1} + f(\tau_{n-1}) + F_{\tau_n} (T_2 - \tau_{n-1}) + (T_2 - \tau_{n-1}) g_{\tau_n} (\tau_{n-1}) - g(\tau_{n-1}) = 0. \tag{7}$$

Solving for $\tau_{n-1}$ yields the optimal tenure $\tau^*_{n-1}$. It follows that $\tau^*_n = T_2 - \tau^*_{n-1}$.

Since this generalizes the solution for the last two jobs, it is easy to expand to the last three jobs, where $\tau_{n-2} + \tau_{n-1} + \tau_n = T_3$, and $\tau_{n-1}$ and $\tau_n$ are defined as above. By backward induction, we can choose and solve for any number of stages.

Given the solutions for the optimal tenures of $n$ jobs, $\tau^*_1, \tau^*_2, \ldots, \tau^*_n$, the worker chooses the optimal $n$. The worker's lifetime earnings are

$$\tilde{\tau}_{n+1}^* + F(\tilde{\tau}_1^*) + \sum_{i=2}^n \left( \tau^*_i g \left( \sum_{j=1}^{i-1} \tau^*_j \right) + F\left( \tau^*_i \right) \right). \tag{8}$$

The worker then chooses $n$ to maximize this function. The result is his optimal number of jobs to work in his lifetime, defined as $n^*$.

Solutions in this general form, however, do not yield intuitive results. For this reason, I now consider an example with specific functional forms to find more informative results.

**Chapter 4. Choosing Optimal Job Tenures**
2.1. An Example

As an example, assume that the worker's salary increases linearly in time. A worker who has worked time \( \tau_i \) at his \( i^{th} \) job earns a salary of

\[
S_i(\tau_i) = s_i + k\tau_i
\]

where \( s_i \) equals the worker's starting salary at job \( i \), and \( k\tau_i \) equals the raise function.

I argue that if \( \tau_i \rightarrow 0 \), then \( s_{i+1} < s_i \) should hold. This property implies that a worker cannot increase his salary by immediately leaving his job for another. This must be true for several reasons:

Firstly, if at \( \tau = 0 \) it is the case that \( s_{i+1} > s_i \) and if costs from job search are sufficiently low, the worker will never stay a positive length of time at any job. A worker can always obtain a higher salary by immediately leaving his job for another.

Secondly, even if there are costs to changing jobs that prevent a worker from immediately moving, a firm will not wish to pay a high salary to a worker it regards as fickle. There are training costs involved with hiring a worker and if the firm expects the worker to leave within a short period of time, it will not earn a positive marginal profit from the worker if it offers a high salary. Therefore, the very act of working short tenures should have the effect of lowering a worker's starting salary.

Lastly, we can assume that the worker always takes the highest paying job. This way, even if the costs of job search are zero, there is never a higher paying job for the worker until he gains some experience. This ensures that \( s_{i+1} < s_i \), and the worker will stay a positive length of time at every job.

Assume then that the starting salary at job \( i \) is given by the function

Chapter 4. Choosing Optimal Job Tenures
\[ s_i = s_{i-1} - c + r \tau_{i-1} \]  

where \( c \) is some positive value and \( r > k \). Note that the worker's starting salary at his next job increases linearly and at a faster rate than the salary of his current job.\(^{28}\) This implies that the worker cannot increase his salary from immediately switching jobs, but he can increase his salary after a sufficient tenure. The value \( c \) depends on such things as costs of job search and relocating, training costs, hiring costs, and job specificity. The costs of job search must be subtracted from the worker's new salary to find his net starting salary. If training and hiring costs are high, the starting salary is typically lowered to accommodate them. The value of \( c \) should thus increase as training and hiring costs increase. Also, the more skill specific the industry is, the higher \( c \) should be (see Mincer and Jovanovic (1981) and Bartel and Borjas (1981)). Clearly, a worker's experience will mean less if his skills are firm specific. His productivity will thus diminish if he switches jobs, translating to a longer training period, and thus a lower starting salary.\(^{29}\)

Since \( s_{i-1} = s_{i-2} - c + r \tau_{i-2} \), \( s_{i-2} = s_{i-3} - c + r \tau_{i-3} \), and so on, (5) can be rewritten as

\[ s_i = s_1 - (i-1)c + \sum_{j=1}^{i-1} r \tau_j \]  

Here we can see that the starting salary at job \( i \) is a function of the individual's total work experience.

The value of \( k \) is influenced by the returns to specific human capital and firm tendency to make competing offers. As productivity increases due to an increased rate of specific human capital accumulation, the value of \( k \) tends to increase. Also, industries tend

\(^{28}\) Implications of the case when \( r < k \) are discussed in section 2.2.

\(^{29}\) I assume here that \( c \) is constant over all firms and throughout the worker's lifetime. Later, I consider the case where \( c \) changes as the worker grows older.
to vary in the tendency to make competing offers. (See Stoikov and Raimon (1968) and Burton and Parker (1969).) For example, firms in the fast-food industry rarely make competing offers; raises are fairly standard with tenure. For professional sports teams, however, making competing offers to keep players is standard practice. The more competing offers a firm makes, the higher is the value of \( k \). More generally, \( k \) may simply be an exogenously given raise coefficient. For example, a state government may restrict the raises of state employees to a small percentage interval. In this case, the worker's immediate employer has little freedom in determining the amount of the raise regardless of the worker's productivity. Typical values for \( k \) can be found in the references in the introduction.

The value of \( r \) depends on the rate of general human capital accumulation through experience and tenure. It is a function of the worker's rate of learning transferable skills and the rate of unemployment. As the worker's rate of learning transferable skills increases, so does the level of productivity he acquires in time. As productivity increases, the starting salary should also increase.\(^{30}\) For now I assume that \( r \) is fixed for each worker, but I later discuss the implications of allowing \( r \) to be a function of effort. Also, as unemployment is low, there are more opportunities for a worker searching for a new job. This will have the effect, on average, of increasing \( r \).

2.1.1. The Setup

The worker has a total working lifetime of \( T \). \( \tau_{i,n} \) represents the tenure of job \( i \) if the worker works \( n \) jobs over the course of \( T \). If this worker is to work \( n \) jobs in his lifetime, he should choose his job tenures to maximize his lifetime earnings function.

\(^{30}\) The worker's rate of learning can be conveyed from one firm to another through the worker's resume, letters of recommendation, and known accomplishments.

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\[
\max_{\tau_{1,n}, \tau_{2,n}, \ldots, \tau_{n,n}} \sum_{i=1}^{n} \int_{0}^{\tau_{i,n}} (s_i + k\theta) \, d\theta
\]

where \( \tau_{1,n} + \tau_{2,n} + \ldots + \tau_{n,n} = T \): Inputting the starting salary function (9), the problem becomes

\[
\max_{\tau_{1,n}, \tau_{2,n}, \ldots, \tau_{n,n}} \left( \int_{0}^{\tau_{i,n}} (s_i + k\theta) \, d\theta + \sum_{j=2}^{n} \int_{0}^{\tau_{j,n}} (s_{j-1} - c + r\tau_{j-1,n} + k\theta) \, d\theta \right). \tag{11}
\]

Substituting (10) into (11) yields

\[
\max_{\tau_{1,n}, \tau_{2,n}, \ldots, \tau_{n,n}} \left( \int_{0}^{\tau_{i,n}} (s_i + k\theta) \, d\theta + \sum_{j=2}^{n} \int_{0}^{\tau_{j,n}} \left( s_i - (i-1)c + k\theta + \sum_{j=1}^{i-1} r\tau_{j,n} \right) \, d\theta \right), \tag{12}
\]

and integrating yields a general form of the lifetime earnings function:

\[
\max_{\tau_{1,n}, \tau_{2,n}, \ldots, \tau_{n,n}} \left( \tau_{1,n} \left( s_i + \frac{k\tau_{i,n}}{2} \right) + \sum_{i=2}^{n} \tau_{i,n} \left( s_i - (i-1)c + \frac{k\tau_{i,n}}{2} + \sum_{j=1}^{i-1} r\tau_{j,n} \right) \right). \tag{13}
\]

The following graph illustrates the problem. The vertical axis measures the starting and actual salaries. The horizontal axis measures time. The solid lines on the graph depict the actual salary of the worker when he works tenures \( \tau_1, \tau_2, \tau_3, \) and \( \tau_4 \) over a working lifetime \( T \). (The second subscripts have been dropped in the figure for simplicity.) The vertical leaps on the salary line represent salary increments from switching jobs. The sizes of the vertical leaps are determined by the starting salary functions, depicted by the dashed lines.

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The worker wishes to maximize the area under the salary line by choosing \( \tau_1, \tau_2, \tau_3, \) and \( \tau_4. \)

The problem is solved for \( n=2 \) and then generalized. Consider two successive jobs the individual works; job 1 and job 2. As previously stated, \( s_i \) is assumed fixed. There are two jobs to work and \( T_2 \) equals the time in which to work these two jobs. The worker will

\[
\max_{\tau_{1,2}, \tau_{2,2}} \left( \tau_{1,2} \left( s_1 + \frac{k \tau_{1,2}}{2} \right) + \tau_{2,2} \left( s_1 - c + r \tau_{1,2} + \frac{k \tau_{2,2}}{2} \right) \right). \tag{14}
\]

Since \( \tau_{2,2} = T_2 - \tau_{1,2} \), this becomes

\[
\max_{\tau_{1,2}} \left( \tau_{1,2} \left( s_1 + \frac{k \tau_{1,2}}{2} \right) + \left( T_2 - \tau_{1,2} \right) \left( s_1 - c + r \tau_{1,2} + \frac{k \left( T_2 - \tau_{1,2} \right)}{2} \right) \right), \tag{15}
\]

and solving yields

\[
\tau_{1,2}^* = \frac{T_2}{2} + \frac{c}{2(r-k)}; \quad \tau_{2,2}^* = \frac{T_2}{2} - \frac{c}{2(r-k)} \tag{16}\]

\textit{Chapter 4. Choosing Optimal Job Tenures}
Generalizing this technique, we find that the optimal tenure of job $i$ of $n$ total jobs equals\(^{31}\)

$$
\tau^*_{i,n} = \frac{T}{n} + \frac{c(n - 2i + 1)}{2(r - k)}.
$$

(17)

2.1.2 Finding The Optimal $n$

At this point the worker knows the optimal length of time to work each of $n$ jobs over the course of his lifetime. In this section I allow the worker to choose the optimal $n$ given this solution. The worker's maximum lifetime earnings when working $n$ jobs, found by inserting the worker's optimal tenures (18) into his lifetime earning function (13), equals

$$
\left( T + \frac{c(n-1)}{2(r-k)} \right) \left[ \frac{1}{n} \right] + \frac{k}{2} \left( T + \frac{c(n-1)}{2(r-k)} \right) + \sum_{i=2}^{n} \left[ \frac{T}{n} + \frac{c(n+1-2i)}{2(r-k)} \right] \left[ \frac{1}{n} \right] - \frac{n}{2} \left[ \frac{T}{n} + \frac{c(n+1-2i)}{2(r-k)} \right] + \sum_{j=1}^{n} \left[ \frac{T}{n} + \frac{c(n+1-2j)}{2(r-k)} \right] \right)
$$

This term collapses to\(^3{32}\)

$$
\frac{s_i T^2 (k + mn - r)}{2n} + \frac{c^2 (n^3 - n)}{24(r - k)} - \frac{c T (n - 1)}{2}.
$$

(18)

Differentiating (18) with respect to $n$ yields

$$
\frac{-3c^2 n^4}{4(r - k)} + \frac{c^2 n^2}{4(r - k)} + cTn^2 + T^2 (r - k) = 0,
$$

and solving for $n$,\(^{33}\)

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\(^{31}\) See appendix for derivation.

\(^{32}\) See appendix for derivation.

\(^{33}\) See appendix for derivation.

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\[ n^* = \left( \frac{1}{6} + \frac{2T(r-k)}{c} + \sqrt{\frac{1}{36} + \frac{2T(r-k)}{3c}} \right)^{\frac{1}{2}} \]  

It is thus the case that \( n^* \) represents the number of jobs that maximizes lifetime earnings. In the case that \( n^* \) is not an integer, the worker need only compare the lifetime earnings for \( n = \) the nearest integer greater than and less than \( n^* \), and choose the \( n \) representing the higher lifetime earnings.

2.1.3. Interpreting the Results

It is interesting to note from (17) that the optimal tenure at each job decreases over the course of the worker's lifetime. The reason seems to be that if the raise and starting salary coefficients (\( k \) and \( r \)) are constant, the worker should stay longer at early jobs to attain large raises from switching, which then "carry over" for the rest of his lifetime (see Bartel and Borjas (1981)). In other words, it is good to build up a large amount of human capital early in life so the worker can better make use of it at future jobs. Note that as the costs associated with switching (\( c \)) increase, the worker stays longer at early jobs and shorter at later jobs. This is because as these costs increase, so does the initial pay cut from switching jobs. It takes longer to achieve the large early salary increases, and early tenures must rise.

As the starting salary coefficient increases, the optimal tenure at early jobs tends to fall. This is because as it increases, a shorter tenure is required to attain a large enough salary increase in the early jobs. Embedded in the starting salary coefficient is the worker's rate of transferable human capital accumulation. A worker who rapidly accumulates human capital is a quick learner and an increasingly productive worker. If human capital increases quickly, the worker can attain large enough salary increments in shorter periods of time. This type of worker can be highly productive in a short period of time.

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Consequently, an employer is willing to hire this type of person even if he has a record of short tenures.

As the raise coefficient increases, the relative tenure at early jobs increases. This is true because as this coefficient gets larger, holding $c$ and $r$ fixed, the benefits from switching jobs at any particular time lessen. It therefore takes longer during the early jobs to acquire a large enough salary leap.

From (19) we can see how the factors affect turnover in the model. As search costs, training costs, hiring costs, and job specificity increase, the optimal number of jobs tends to fall. This is a fairly intuitive result in the sense that if search costs are high, the worker is less likely to search. Also, if training periods and hiring costs are high, employers will shun a worker with a reputation for short tenures. They should in turn offer lower salaries to these types. Increased job specificity implies that a worker has fewer skills to bring to a new job, and this is reflected by a lower starting salary. This in turn necessitates, on average, longer tenures to achieve sufficient leaps from switching, and thus fewer jobs in total.

Job turnover increases in the starting salary coefficient. The explanation is similar to that above. A worker with a high $r$ can gain from switching jobs in a shorter time. This would indicate that workers with high rates of transferable human capital accumulation have higher turnover rates. Also, since $r$ and unemployment are negatively correlated, turnover should increase as unemployment drops; a result consistent with findings in Akerlof, Rose, and Yellen (1988).

Lastly, if a worker enjoys a high raise coefficient due to a high rate of specific human capital accumulation or as an exogenous occurrence, his salary increases quickly at each job he works. This implies that there is less to gain from finding a new job, and job
turnover tends to fall. Results in Parsons (1972) state that a high rate of specific human capital results in lower turnover. A high \( k \) may also be the result of the firm's tendency to make competing offers. Stoikov and Raimon (1968) study factors that affect industry quit rates. The paper states "... all employers, unionized or not, are under pressure to keep pace with changes in area or industry wage levels. Failure to do so invites ... higher quit rates." The paper also demonstrates a significant negative relation between percentage salary increments and industry quit rates. Industries that characteristically make competing offers (possibly due to high search and hiring costs after losing an employee) have higher values of \( k \) relative to \( r \). The present model, consistent with these results, predicts lower turnover in this instance.

These results suggest, in this case, how salary structure can affect both job turnover within an industry, and the allocation of time to each job over the course of a worker's lifetime.

2.2 Comparisons With Previous Results

Topel (1986), Abraham and Farber (1987), and Mincer and Jovanovic (1981) find that on average, workers who stay on the job earn more than workers who move more frequently. There is, however, a bias in that workers with high salaries and good job matches are more likely to stay on the job. This example suggests that workers with a high raise coefficient exhibit a lower rate of turnover. Indeed, the model states that a worker whose salary increases faster than his starting salary \((k \geq r)\) will never switch for pecuniary reasons. The bias is thus implicit in \( n^* \). Workers for whom \( r \) is significantly higher than \( k \) and for whom \( c \) is relatively low can certainly gain from switching. Mincer and Jovanovic (1981), Mincer (1986), and Topel (1986) state that gains from switching are positive for young workers and decline, perhaps becoming negative, as the worker.
gets older. Much of this may be due to early investments in transferable human capital (a decreasing $r$ is discussed shortly) and to the fact that workers are laid off and do not switch by choice; a condition I do not formally allow in the model. Abraham and Farber (1987) state that if productivity is common knowledge, highly productive workers can receive wage gains from switching, and low-productivity workers cannot, and are thus more likely to stay. This helps explain this paper’s results that a worker who quickly accumulates transferable human capital is more mobile. Akerlof, Rose, and Yellen (1988) find that many quits involve low to negative wage changes, but carry significant increases in fringe benefits. Also, at least 30% of all quits occur when a worker receives a better offer and most job related quits involve an increase in overall labor income. Altonji and Shakotko (1987) find that most wage growth results from total market experience rather than from tenure. Finally, Bartel and Borjas (1981) show that wage gains are highest (and significant) for workers who quit for job-related reasons.

Results in the example contradict previous empirical results in two ways: Typically, turnover is higher for younger workers than for older workers (see Topel (1991) and Mincer and Jovanovic (1981)), and earnings tend to drop as a worker gets older. The reason for these contradictory results is that the example assumes constant levels of $c$, $k$, and $r$ throughout the worker’s lifetime. Empirical evidence suggests that this is not the case, and allowing for variations consistent with the findings yields more plausible results.

We saw that an overall increase in $r$ shortens tenures early in life and lengthens tenures late in life. Furthermore, Gibbons and Murphy (1992) state that career concerns are stronger for a worker further from retirement. A worker further from retirement is thus willing to take more costly unobservable actions to influence market belief. This implies that younger workers are more willing to put high effort into achieving transferable

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human capital than older workers. If we assume that $r$ is thus higher for younger workers than for older workers, turnover will be significantly higher for the younger workers so that they may gain many large early salary increments.

It may also be the case that the costs from switching change over the course of the worker's lifetime. Mincer (1986) and van den Berg (1992) state that older workers need to search longer for new jobs than younger workers. This will have the effect of increased search costs for older workers. The latter paper states that workers aged 50 experience search costs three times as large as a worker aged 25. We can see from equation (19) that these increased costs will result in lower turnover if we restrict $T$ to the remaining working lifetime of an older worker.34

Another consideration may be the rate at which the worker discounts future income. After casual consideration, it would seem that a high discount rate may lead to higher turnover rates and shorter relative tenures early in life. This may be due to the fact that a worker with a high discount rate would value a small increase in salary after a short tenure rather than waiting a long period of time for a large increase.

In addressing the quadratic lifetime earnings, note that if $r$ eventually falls below $k$ and an older worker faces a positive probability of losing his job, he may experience (unwanted) pay cuts. Also, Bartel and Borjas (1981) state that older workers are more likely to seek new jobs for non-pecuniary reasons and sacrifice salary for these advantages. These factors help explain in this model the presence of quadratic lifetime earnings.

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34 Deere (1987) states that turnover is inefficient if job training is costly.

Chapter 4. Choosing Optimal Job Tenures
2.3. Variations in the Functional Form

The condition in section 2.1 stipulates that a worker cannot achieve a raise by immediately switching jobs. This condition should hold true under any circumstances, but there is no reason why the starting salary at the next job must increase linearly in time. One can imagine a starting salary that increases to a point, and then begins to fall if the worker stays too long at his job. This is possible if the worker exhibits diminishing marginal productivity and raises are constant. In this instance, it may only be a matter of time before the employer begins to earn a loss from employing this worker. A worker like this can send a signal of "overstaying his welcome" at jobs, and this may result in a lower starting salary to compensate.

In this case, the starting salary function may take a parabolic form. The following graph illustrates this situation where salaries increase linearly by the coefficient $k$, and starting salaries are quadratic. The solid lines depict the worker's actual salary when he works tenures $\tau_1$, $\tau_2$, $\tau_3$, and $\tau_4$ over working lifetime $T$. The sizes of the vertical leaps are determined by the starting salary functions of jobs 2, 3, and 4, $s_2(\cdot)$, $s_3(\cdot)$, and $s_4(\cdot)$ respectively, depicted by the dashed lines.
Figure 2.
*Lifetime Earnings with Quadratic Starting Salaries*

Certain results are apparent from observing the figure. Firstly, a worker will always switch jobs at a point where he can achieve a raise, and where the slope of the starting salary function is greater than or equal to $k$. If the slope is less than $k$, the worker has waited too long. His starting salary function is increasing at a slower rate than his salary would if he takes a new job.

Also, as a worker's rate of diminishing transferable marginal productivity increases, the starting salary function will get "thinner". This is because the worker will more quickly lose his value relative to his salary. Consequently, the worker is forced to leave his job quickly or experience a pay cut at his new job. He needs to send a signal that he will leave his new job at the appropriate time. This will have the effect of increasing turnover.

The peak of the starting salary function may also vary. This may be a function of the capacity of human capital accumulation at that particular job. It would seem that the
higher the peak (holding the shape constant), the longer the individual should stay at his job. This simply means there is more to gain from working longer at his current job, and he should reap the benefits. Turnover is thus lower.

Mincer and Jovanovic (1981), Topel (1991), and Akerlof, Rose, and Yellen (1988) show that quit rates decline as tenure increases. This type of starting salary function may suggest why. Even if the peak of the starting salary function does not exceed the salary, a worker may wish to switch jobs for non-pecuniary reasons. If he were to switch, he should do so at the point when the starting salary is closest to his current salary to minimize his pay cut. Indeed, if he has missed this opportunity or has decided late in his tenure that he wishes to switch, the losses from switching become greater as tenure gets longer, and thus the worker becomes less likely to switch.

2.4. Extensions

One can imagine other factors that may affect the shapes of the raise and starting salary functions, and manipulating these factors can suggest how changes in industries with different characteristics affect tenures and turnovers. Workers in different industries may exhibit different characteristics, and it may also be possible for a worker to change his own characteristics. I have mentioned how a worker might achieve a high \( r \) through hard work. This of course is not costless. The worker may have to work longer hours and experience a higher level of disutility. He may also choose to invest in specific human capital to improve \( k \). The worker considers earnings as utility, and he maximizes his lifetime utility with respect to the level of effort he puts into \( r \) and \( k \).\(^{35}\) It may be interesting to note how the worker's effort varies over the course of his lifetime given

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\(^{35}\) Gibbons and Murphy (1992) consider the problem of wage earnings as a function of effort and disutility.

*Chapter 4. Choosing Optimal Job Tenures*
possible variations in his value of leisure. If for instance a worker values leisure more as he grows older, his \( r \) and \( k \) will subsequently decrease. Also, as stated in the introduction, we may witness an increasing and concave raise function. Here, raises decline in tenure. This is another possibility that can be explored in the context of this model.

A firm may also offer incentive schemes that induce workers to lengthen tenures. Federal jobs carry fairly standard salary increments with tenure, and the practice of making competing offers is fairly restricted. We do, however, see very low turnover rates in federal jobs. Ippolito (1987) suggests that these low quit rates are due to the presence of high pensions and retirement benefits. These benefits depend on the length of time the worker stays at the job and are payable only if the worker stays a specified tenure. They are penalized for early quits, and turnover is thus significantly lower.

3. Concluding Remarks

This paper considered a labor market situation where a worker strategically chooses job tenures to maximize his lifetime earnings. A general framework was first described, and assumptions were then made on the form of the salary increment and starting salary functions. Under a constant linear increment and starting salary setting, we saw that increased search costs, training period duration, hiring costs, and job specificity have the effect of decreasing the rate of job turnover and increasing the relative tenure of jobs early in life. This was due to the resulting costs of switching and the difficulty in acquiring transferable human capital, which in turn results in fewer pay raise opportunities. Also, job turnover increases in the rate of human capital acquisition, and relative tenure at early jobs tends to fall. This is due to the fact that increased human capital provides more job opportunities, and higher salaries from switching.
Section 2.2 discussed the model's relation to the existing literature. We saw that there are differing results on the extent of salary leaps from switching jobs. However, most research generally agrees that those who stay on the job are doing so because they are earning a high salary. This is consistent with this paper's results that high raises \((k)\) decrease turnover. Also, most of the literature agrees that there are at least some gains to be made from switching, especially if the reason for the switch is to increase salary. The results of this paper suggest the circumstances under which moving is beneficial, and those under which it is not.

The results of the example, however, are in contrast with some known truths of career tenure choice and earnings. Typically, younger workers exhibit higher turnover than older workers. However, empirical findings mentioned state that a worker's rate of transferable human capital accumulation \((r)\) declines in age, and costs of job search \((c)\) increase. The model predicts that if these conditions hold, turnover early in life increases and turnover late in life decreases. Standard empirical findings also state that earnings are quadratic in age. We saw that if \(r\) falls below \(k\) late in life, and a worker is laid off or leaves for non-pecuniary reasons, he may experience wage cuts.

Different industries may exhibit distinct functional forms. Section 2.3 outlined the case where starting salary functions are parabolic. It appears that an increasing rate of diminishing marginal productivity has the effect of increasing job turnover. Also, increased capacity of human capital accumulation should have the effect of decreasing job turnover because there are more benefits from a worker remaining at his current job to build large salary leaps. The presence of this type of starting salary function may also suggest why quit rates decrease in tenure.

Chapter 4. Choosing Optimal Job Tenures
Other extensions include analyzing different functional forms of raise and starting salary functions. Also, since many industries are free to offer any salary deemed appropriate, competitive bidding for workers and high pension benefits can affect job turnover as it becomes more attractive to the worker to remain with the same firm. A next step might be to examine the problem in the form of utility maximization. This would allow including other non-pecuniary factors such as job disutility in the worker's decision set. This would strengthen the model's flexibility in integrating different factors in determining tenure variation and turnover.
APPENDIX

1. The optimal tenures for the first two jobs in \( n \) total jobs are given by

\[
\hat{\tau}_{1,n} = \frac{T_2}{2} + \frac{c}{2(r-k)} \quad \hat{\tau}_{2,n} = \frac{T_2}{2} - \frac{c}{2(r-k)}
\]

where \( \hat{\tau}_{1,n} + \hat{\tau}_{2,n} = T_2 \) = the length of time to work the first two jobs. In equilibrium,

\[
\hat{\tau}_{1,n} = \frac{\hat{\tau}_{1,n} + \hat{\tau}_{2,n}}{2} + \frac{c}{2(r-k)}, \quad \hat{\tau}_{2,n} = \frac{\hat{\tau}_{1,n} + \hat{\tau}_{2,n}}{2} - \frac{c}{2(r-k)}.
\]

Solving for \( \hat{\tau}_{2,n} \), we get

\[
\hat{\tau}_{2,n} = \hat{\tau}_{1,n} - \frac{c}{(r-k)}.
\]

By the same token, the relation between the second and third jobs equals

\[
\hat{\tau}_{3,n} = \hat{\tau}_{2,n} - \frac{c}{(r-k)},
\]

and therefore

\[
\hat{\tau}_{3,n} = \hat{\tau}_{1,n} - \frac{2c}{(r-k)}.
\]

Generalizing, we see that

\[
\hat{\tau}_{i,n} = \hat{\tau}_{1,n} - \frac{(i-1)c}{(r-k)}. \tag{1A}
\]

Since \( \sum_{i=1}^{n} \hat{\tau}_{i,n} = T \), it follows from (1A) that

\[
\sum_{i=1}^{n} \left( \hat{\tau}_{i,n} - \frac{(i-1)c}{(r-k)} \right) = T,
\]

\[
\sum_{i=1}^{n} \hat{\tau}_{i,n} - \frac{ic}{(r-k)} + \sum_{i=1}^{n} \frac{c}{(r-k)} = T,
\]

\[
n\hat{\tau}_{i,n} - \frac{c}{(r-k)} \sum_{i=1}^{n} i + \frac{nc}{(r-k)} = T,
\]
\[ n \tau_{i,n}^* = \frac{(n^2 + n)c}{2(r - k)} + \frac{nc}{(r - k)} = T, \]
\[ n \tau_{i,n} = T + \frac{n^2 c - nc}{2(r - k)}, \]
\[ \tau_{i,n} = \frac{T}{n} + \frac{(n-1)c}{2(r - k)}. \]

and from \((IA)\),
\[ \tau_{i,n} = \frac{T}{n} + \frac{(n-1)c}{2(r - k)} \cdot \frac{(i-1)c}{2(r - k)}. \]

From this, we get
\[ \tau_{i,n}^* = \frac{T}{n} + \frac{(n-2i+1)c}{2(r - k)}. \]

2. The optimized lifetime earnings function is given by
\[
\left( \frac{T}{n} + \frac{c(n-1)}{2(r-k)} \right) \left( s_i + k \left( \frac{T}{n} + \frac{c(n-1)}{2(r-k)} \right) \right) + \sum_{i=1}^{n} \left( \frac{T}{n} + \frac{c(n+1-2i)}{2(r-k)} \right) \left( s_i - (i-1)c + k \left( \frac{T}{n} + \frac{c(n+1-2i)}{2(r-k)} \right) \right) + \sum_{j=1}^{n} \left( \frac{T}{n} + \frac{c(n+1-2j)}{2(r-k)} \right) \right). 
\]

Expanded, this term equals
\[
\frac{A}{n} = \left( \frac{T}{n} + \frac{c(n-1)}{2(r-k)} \right) \left( s_i + k \left( \frac{T}{n} + \frac{c(n-1)}{2(r-k)} \right) \right) + \sum_{i=1}^{n} \left( \frac{T}{n} + \frac{c(n+1-2i)}{2(r-k)} \right) \left( s_i - (i-1)c + k \left( \frac{T}{n} + \frac{c(n+1-2i)}{2(r-k)} \right) \right) + \sum_{j=1}^{n} \left( \frac{T}{n} + \frac{c(n+1-2j)}{2(r-k)} \right) \right). 
\]

\[
B = \sum_{i=2}^{n} \left( \frac{T}{n} + \frac{c(n+1-2i)}{2(r-k)} \right) \left( \frac{T}{n} + \frac{c(n+1-2i)}{2(r-k)} \right) + \sum_{i=2}^{n} \left( \frac{T}{n} + \frac{c(n+1-2i)}{2(r-k)} \right) \left( \frac{T}{n} + \frac{c(n+1-2i)}{2(r-k)} \right) 
\]

\[
D = \sum_{i=2}^{n} \left( \frac{T}{n} + \frac{c(n+1-2i)}{2(r-k)} \right), \quad E = \sum_{i=2}^{n} \left( \frac{T}{n} + \frac{c(n+1-2i)}{2(r-k)} \right) 
\]

\[
F = \sum_{i=2}^{n} \left( \frac{T}{n} + \frac{c(n+1-2i)}{2(r-k)} \right) \sum_{i=1}^{n} \left( \frac{T}{n} + \frac{c(n+1-2j)}{2(r-k)} \right) \right). 
\]
The solution is broken down into the parts depicted above. Note that

\[ \sum_{i=2}^{n} i = \frac{n^2}{2} + \frac{n}{2} - 1, \quad \sum_{i=2}^{n} i^2 = \frac{n^3}{3} + \frac{n^2}{2} + n - 1, \quad \sum_{i=2}^{n} i^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4} - 1. \]

A)

\[
\frac{s_i T}{n} + \frac{kT^2}{n^2} + \frac{ckT}{4(r-k)} + \frac{ckn}{2(r-k)} - \frac{c^2 kn^2}{8(r-k)} - \frac{c^2 kn}{8(r-k)}
\]

\[
= s_i \frac{T}{n} + \frac{c(n-1)}{2(r-k)} + \frac{kT^2}{2n^2} + \frac{ckT}{2(r-k)} - \frac{ckn}{2(r-k)} + \frac{c^2 kn^2}{8(r-k)^2} - \frac{c^2 kn}{8(r-k)^2}
\]

\[= s_i \frac{T}{n} + \frac{(n-1) c s_i}{2(r-k)} + \frac{kT^2}{2n^2} + \frac{ckT}{2(r-k)} - \frac{ckn}{2(r-k)} + \frac{c^2 kn^2}{8(r-k)^2} - \frac{c^2 kn}{8(r-k)^2}. \quad (A^*) \]

B)

\[ \sum_{i=2}^{n} i = \sum_{i=2}^{n} \left( \frac{T}{n} + \frac{c(n+1-2i)}{2(r-k)} \right) \]

\[= \sum_{i=2}^{n} s_i \frac{T}{n} + \sum_{i=2}^{n} \frac{c s_i}{2(r-k)} + \sum_{i=2}^{n} \frac{c s_i n}{2(r-k)} - \frac{c s_i}{(r-k)} \sum_{i=2}^{n} i \]

\[= \frac{(n-1) s_i T}{n} + \frac{c s_i n^2}{2(r-k)} - \frac{c s_i n}{2(r-k)} + \frac{c s_i n}{2(r-k)} - \frac{c s_i n^2}{2(r-k)} - \frac{c s_i n}{2(r-k)} + \frac{c s_i}{(r-k)} \]

\[= \left( \frac{n-1}{n} \right) s_i T - (n-1) \frac{c s_i}{2(r-k)}. \quad (B^*) \]

C)

\[= -\sum_{i=2}^{n} c(i-1) \left( \frac{T}{n} + \frac{c(n+1-2i)}{2(r-k)} \right) \]

\[= -\sum_{i=2}^{n} c \left( i - \frac{c i^2}{2(r-k)} \right) - \sum_{i=2}^{n} \frac{c^2}{2(r-k)} \sum_{i=2}^{n} \frac{i^2}{2} - \sum_{i=2}^{n} \frac{c^2}{2(r-k)} \sum_{i=2}^{n} \frac{i}{n} + \sum_{i=2}^{n} \frac{c^2}{2(r-k)} - \frac{c^2}{(r-k)} \sum_{i=2}^{n} i \]

\[= -\frac{c T n^2}{2} + \frac{c T}{4(r-k)} - \frac{c T n^3}{2(r-k)} + \frac{c T n^2}{4(r-k)} - \frac{c T n^3}{4(r-k)} + \frac{c^2}{2(r-k)} + \frac{c^2}{6(r-k)} + \frac{c^2}{2(r-k)} \]

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\[ + \frac{c^2 n^3}{3(r-k)} - \frac{c^2}{(r-k)} + cT - \frac{cT}{n} + \frac{c^2 n^2}{2(r-k)} - \frac{c^2 n}{2(r-k)} + \frac{c^2 n}{2(r-k)} - \frac{c^2 n}{2(r-k)} - \frac{c^2 n}{2(r-k)} + \frac{c^2}{(r-k)} \]
\[ = - \frac{cT n}{2} + \frac{cT}{2} + \frac{c^2 n^3}{12(r-k)} - \frac{c^2 n}{12(r-k)}. \]  
(C*)

D)
\[ \sum_{i=2}^{n} \frac{kT \left( T + \frac{c(n+1-2i)}{2(r-k)} \right)}{2n} \]
\[ = \sum_{i=2}^{n} \frac{kT^2}{2n^2} + \sum_{i=2}^{n} \frac{ckT}{4(r-k)} + \sum_{i=2}^{n} \frac{ckT}{4(n(r-k))} - \frac{ckT}{2n(r-k)} \sum_{i=2}^{n} \]
\[ = \frac{kT^2}{2n} - \frac{kT^2}{2n^2} + \frac{ckT}{4(r-k)} - \frac{ckT}{4(n(r-k))} - \frac{ckT}{4(r-k)} - \frac{ckT}{4(r-k)} + \frac{ckT}{4(n(r-k))} + \frac{ckT}{n(r-k)} \]
\[ (D*) \]

E)
\[ \sum_{i=2}^{n} \frac{kc(n+1-2i)(T + \frac{c(n+1-2i)}{2(r-k)})}{4(r-k)} \]
\[ = \sum_{i=2}^{n} \frac{ckT}{4(r-k)} + \sum_{i=2}^{n} \frac{ckT}{4(n(r-k))} - \frac{ckT}{2n(r-k)} \sum_{i=2}^{n} \frac{c^2 kn^2}{8(r-k)^2} + \sum_{i=2}^{n} \frac{c^2 kn}{8(r-k)^2} = \frac{c^2 kn}{4(r-k)} \sum_{i=2}^{n} \]
\[ + \sum_{i=2}^{n} \frac{c^2 kn}{8(r-k)^2} + \sum_{i=2}^{n} \frac{c^2 k}{8(r-k)^2} \sum_{i=2}^{n} \frac{c^2 kn}{8(r-k)^2} + \frac{c^2 k}{8(r-k)^2} \sum_{i=2}^{n} \sum_{i=2}^{n} \frac{c^2 k}{8(r-k)^2} + \frac{c^2 k}{8(r-k)^2} \sum_{i=2}^{n} \]
\[ = \frac{ckT}{4(r-k)} - \frac{ckT}{4(r-k)} - \frac{ckT}{4(n(r-k))} - \frac{ckT}{4(r-k)} + \frac{c^2 kn}{8(r-k)^2} + \frac{c^2 k}{8(r-k)^2} + \frac{c^2 kn}{8(r-k)^2} + \frac{c^2 k}{8(r-k)^2} \]
\[ - \frac{c^2 kn}{8(r-k)^2} - \frac{c^2 kn}{8(r-k)^2} + \frac{c^2 kn}{8(r-k)^2} + \frac{c^2 k}{8(r-k)^2} + \frac{c^2 kn}{8(r-k)^2} + \frac{c^2 k}{8(r-k)^2} \]
\[ = \frac{ckT}{4n(r-k)} - \frac{ckT}{4(r-k)} - \frac{c^2 kn}{8(r-k)^2} + \frac{c^2 k}{8(r-k)^2} + \frac{5c^2 kn}{24(r-k)^2} + \frac{c^2 kn^3}{24(r-k)^2}. \]  
(E*)

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\[
F) \quad \sum_{i=2}^{n} \left( \frac{T}{n} + \frac{c(n+1-2i)}{2(r-k)} \right) \sum_{j=1}^{i-1} r \left( \frac{T}{n} + \frac{c(n+1-2j)}{2(r-k)} \right)
\]

and

\[
\sum_{j=1}^{i-1} r \left( \frac{T}{n} + \frac{c(n+1-2j)}{2(r-k)} \right) = \sum_{j=1}^{i-1} \frac{rt}{n} + \sum_{j=1}^{i-1} \frac{crn}{2(r-k)} + \sum_{j=1}^{i-1} \frac{cri}{(r-k)} - \frac{cri}{(r-k)} \sum_{j=1}^{i-1} j
\]

\[
= \frac{rTi}{n} + \frac{rT}{2(r-k)} + \frac{crni}{2(r-k)} - \frac{crn}{2(r-k)} + \frac{cri}{2(r-k)} - \frac{cr}{2(r-k)} - \frac{cri^2}{2(r-k)}.
\]

Substituting this term back in,

\[
= \sum_{i=2}^{n} \left( \frac{T}{n} + \frac{c(n+1-2i)}{2(r-k)} \right) \left( \frac{rTi}{n} + \frac{rT}{n} + \frac{crni}{2(r-k)} - \frac{crn}{2(r-k)} + \frac{cri}{2(r-k)} - \frac{cri}{2(r-k)} - \frac{cri^2}{2(r-k)} \right)
\]

\[
= \frac{rT^2}{n^2} \sum_{i=2}^{n} i - \frac{crT}{2(r-k)} \sum_{i=2}^{n} \frac{ciri}{2(r-k)} - \frac{crT}{2(r-k)} \sum_{i=2}^{n} \frac{ciri^2}{2(r-k) + 2(r-k)} - \frac{cri}{2(r-k)} \sum_{i=2}^{n} \frac{cri}{2(r-k)} - \frac{cr}{2(r-k)} \sum_{i=2}^{n} \frac{cri}{2(r-k)} - \frac{cri}{2(r-k)} \sum_{i=2}^{n} \frac{cri^2}{2(r-k)}
\]

\[
+ \frac{crT}{2n(r-k)} \sum_{i=2}^{n} i - \frac{crT}{2n(r-k)} \sum_{i=2}^{n} \frac{ciri}{2(r-k)} - \frac{cri}{2(r-k)} \sum_{i=2}^{n} \frac{cri}{2(r-k)} - \frac{cri}{2(r-k)} \sum_{i=2}^{n} \frac{cri^2}{2(r-k)} + \frac{cri}{2(r-k)} \sum_{i=2}^{n} \frac{cri}{2(r-k)}
\]

\[
= \frac{rT^2}{2} + \frac{rT^2}{2n} + \frac{rT^2}{n} - \frac{rT^2}{2} + \frac{crTn^2}{2(r-k)} + \frac{crTn^2}{2(r-k)} - \frac{crT}{(r-k)} - \frac{crT}{(r-k)} + \frac{crT}{(r-k)}
\]

\[
+ \frac{5crTn}{4(r-k)} - \frac{5crTn}{2n(r-k)} + \frac{5crTn}{(r-k)} - \frac{crT}{n(r-k)} + \frac{crT}{4(r-k)} - \frac{3crTn}{2(r-k)} + \frac{3crTn}{2n(r-k)}
\]

\[
+ \frac{c^2r}{4(r-k)^2} \left( \frac{n^4}{2} + \frac{n^3}{2} - n^2 - n^2 + n^2 - \frac{5n^3}{2} - \frac{5n^2}{2} - 5n - 2n^2 + 2n - \frac{n^2}{2} - n^4 + 3n + 2n^2
\]

\[
+ \frac{2n}{4} - 4 - n + 1 - \frac{5n}{6} - \frac{5n^3}{2} - \frac{5n^3}{3} + 5 + \frac{n^4}{2} + n^3 + \frac{n^3}{2} - 2 \right)
\]

\[
= \frac{rT^2(n-1)}{2n} - \frac{c^2r(n^3-n)}{24(r-k)^2}. \quad (F^*)
\]
Adding $(A^*)$ through $(F^*)$ and simplifying yields

\[
s_T + \frac{T^2(k + rn - r)}{2n} + \frac{c^2(n^3 - n)}{24(r - k)} - \frac{cT(n - 1)}{2}.
\]

3. Differentiating (18) with respect to $n$ and setting equal to zero yields

\[
\frac{c^2 n^4}{4(r - k)} - \frac{c^2 n^2}{12(r - k)} - cTn^2 + T^2 (r - k) = 0.
\]

Taking $x = n^2$,

\[
x^2 \left( \frac{c^2}{4(r - k)} \right) - x \left( \frac{c^2}{12(r - k)} + cT \right) + T^2 (r - k) = 0
\]

and solving yields

\[
x = \frac{1}{6} + \frac{2T(r - k)}{c} \pm \sqrt{\frac{1}{36} + \frac{2T(r - k)}{3c}}.
\]

Substituting back for $n$,

\[
n = \pm \left( \frac{1}{6} + \frac{2T(r - k)}{c} + \sqrt{\frac{1}{36} + \frac{2T(r - k)}{3c}} \right)^{1/2}.
\]

Since $n$ cannot be negative, only the positive roots are possible, and the maximum is represented by

\[
n^* = \left( \frac{1}{6} + \frac{2T(r - k)}{c} + \sqrt{\frac{1}{36} + \frac{2T(r - k)}{3c}} \right)^{1/2}.
\]
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Vitae

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