DISENTANGLING LOW-FREQUENCY VERSUS HIGH-FREQUENCY ECONOMIC RELATIONSHIPS VIA REGRESSION PARAMETER STABILITY TESTS

by

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(ABSTRACT)

This dissertation develops and applies new tools for distinguishing and disentangling high-frequency and low-frequency relationships among stationary economic time series. The new approach proposed here is a three-step procedure; the first step transforms the regression model in the time domain to a real-valued model in the frequency domain, which is functionally identical to an ordinary regression model, the only different being that "observations" of this model correspond to different frequencies rather than to different time periods. Consequently, in the second step, well established regression parameter stability tests are used to detect and assess the frequency dependence of relationships among economics time series. This new approach allows one to not only detect model misspecification of this type but also to correct it. In the third step, the results of the parameter stability across frequency tests is used to sensibly choose the best varying-parameter model in the frequency domain, which is then back-transformed to a time domain model and to be used for forecasting.

The empirical example (using macroeconomic data) presented in this dissertation shows that the back-transformed model that allows varying parameter across frequencies significantly improves the forecasting performance of the misspecified fixed-parameter model.

TO
THE MEMORY OF MY MOTHER

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List of Figures

List of Tables

Table of Contents

CHAPTER 1

OVERVIEW OF THE STUDY

1.1 Relationship to the Literature

The distinction between long-term and short-term economic relationships has been a topic of major concern and discussion in a number of areas of macroeconomic analysis for decades. It has accounted for significant differences in the styles and objectives of economic theorizing, as well as in policy making. In the literature, the notion of "long-term" has been variously associated with the concepts of equilibrium, stability, flexibility, systematic, certainty and low frequency. On the other hand, "short-term" connotes temporary, instability, inflexibility, non-systematic, uncertainty and high frequency.

The present study develops and applies new tools for distinguishing and disentangling high-frequency and low-frequency relationships among stationary economic time series. The following subsections briefly review concepts of "long-term" versus "short-term" relationships which have excited much attention in the macroeconomic literature, and provide extensive fertile ground for application of the new technique proposed here.

1.1.1 A Distinction of Importance in the Macroeconomics Literature

The distinction between long-term and short-term relationships was first introduced by Marshall (1920):

"Here we see an illustration of the almost universal law that the term Normal being taken to refer to a *short period* of time an increase in the amount demanded raises the normal supply price. .... But if we turn to consider the normal supply price with reference to a *long period* of time, we shall find that it is governed by a different set of causes, and with different results. .... For *short periods* the stock of appliances of production are practically fixed, but their employment varies with demand. .... In the *long periods* the flow of appliances for production is adjusted to the demand for the products of those appliances. .... Of course there is no hard and sharp line of division between "long" and "short" periods. Nature has drawn no such lines in the economic conditions of actual life; and in dealing with practical problems they are not wanted. "

A theoretical analysis of "long-term" versus "short-term" economic relationships was also provided by Keynes (1936), in his General Theory. In this theory he explains the prolonged high unemployment in capitalistic economies during the Great Depression. He pointed out that the reason that the Classical Market-Clearing Model fails to explain the situation during the Depression is because the model is true in the "long term," but is extremely unrealistic in the "short term". He turned away from the classical long-term method of analysis, positing sticky behavior for prices and wages in the short term. According to him, it is output, not prices, that adjusts to clear the market in the short term. The significance of the short-term versus the long-term distinction is also reflected in the view of monetarists. For example, they emphasize a long-term relationship between the stock of money and the price level, and a short-term relationship between the stock of money and real output.

The falling apart of the Phillips curve description of the relationship between the unemployment rate and the inflation rate is another example of the significance of the long-term versus the short-term distinction. The short-term Phillips curve shows the negative relationship, or tradeoff, between the unemployment rate and the inflation rate; however, the relationship breaks down in the long term. In contrast, the long-term Phillips curve is vertical in most treatments, leading to no tradeoff whatever; Friedman (1975) wrote in this regard

> "Suppose, to start with, the economy is at point E_0, with both prices and wages stable. Suppose something, say, a monetary expansion, ... produces a rise in prices and wages..., workers will initially interpret this as a rise in their real wage - because they still anticipate constant prices - and so will be willing to offer more labour, ie employment grows and unemployment falls. Employers may have the same anticipations as workers about the general price level, but they are more directly concerned about the price of the product they are producing and far better informed about that. They will initially interpret a rise in the demand for and price of their product as a rise in its relative prices and as implying a fall in the real wage rate they must pay measured in terms of their product. They will therefore be willing to hire more labour. ... . But as time passes, both employers and employees come to recognise that prices in general are rising. ... , they raise their estimate of anticipated rate of inflation, which reduces the rate of rise of anticipated real wages, ... , and leads you to slide down the curves back ultimately to the point E_0 There is thus a short-run 'trade-off ' between inflation and unemployment, but no long-run 'trade-off '."

To distinguish a short-term from a long-term Phillips curve, the original Phillips curve is usually modified to include expected inflation:

$$p = p^e + \delta(u - \bar{u}), \tag{1.1}$$

where p is the rate of inflation, p^e is the expected rate of inflation, and $(u - \bar{u})$ is the deviation of the unemployment rate from the "natural rate". Equation (1.1) is referred to as the "expectations-augmented Phillips curve", and it is considered a short-term Phillips curve. Based on the Friedman-Phelps view, which argues that the original Phillips curve shifts over time as employers and employees became used to and began to expect continuing inflation, Dornbusch and Fischer (1984) write[1]

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[1]The Friedman-Phelps view is expressed in Milton Friedman, "The Role of Monetary Policy," *American Economic Review*, March 1968 and Edmund Phelps, "Phillips Curves, Expectations of Inflation , and Optimal Unemployment Over Time," *Economica*, 1967.

"The short-run Phillips curve shows the relationship between the inflation and unemployment rates when the expected inflation rate is held constant. ... . The long-run Phillips curve describes the tradeoff, if any, between inflation and unemployment when the actual and expected inflation rates are equal."

In the long term, the actual and expected inflation rates are equal, and so $p = p^e$. Thus, in the long term, equation (1.1) implies that the actual unemployment rate is equal to the natural rate regardless of the rate of inflation. In other words, in the long term, there is no tradeoff between inflation and unemployment, and the long-term Phillips curve is a vertical line.

The Permanent Income Hypothesis, originated by Friedman (1957), provides another example of a theory in which the "short-term" versus "long-term" distinction is crucial. Friedman formulated this theory as follows:

$$c_p = k(i,w,u)y_p \tag{1.2}$$

$$y = y_p + y_t \tag{1.3}$$

$$c = c_p + c_t \tag{1.4}$$

$$\text{corr}(y_t y_p) = \text{corr}(c_t c_p) = \text{corr}(y_t c_t) = 0, \tag{1.5}$$

where c is the measured aggregate consumption, y is the measured aggregate disposable income, c_p and c_t are the permanent and the transitory components of aggregate consumption, and y_p and y_t are the permanent and the transitory components of aggregate disposable income. Friedman (1957) interprets the distinction between the permanent and the transitory components of income and consumption as follows:

> "The permanent component is interpreted as reflecting the effect of those factors that the unit regards as determining its capital value or wealth: the nonhuman wealth it owns; the personal attributes of the earners in the unit, such as their training, ability, personality; the attributes of the economic activity of the earners, such as the occupation followed, the location of the economic activity and so on. ... . The transitory component is to be interpreted as reflecting all "other" factors, factors that are likely to be treated by the unit affected as "accidental" or "chance" occurrences, ..., for example cyclical fluctuations in economic activity. ..., some of the factors producing transitory components of consumption are specific to particular consumer units, such as unusual sickness, a specially favorable opportunity to purchase and the like; others affect groups of consumer units in the same way, such as an unusually cold spell, a bountiful harvest, and the like."

The long-term relationship between c and y is given by equation (1.2). This equation states that k, the long-term marginal propensity to consume out of permanent income, depends on i the interest rate, w the ratio of nonhuman wealth to income, and u consumers' preferences for consumption versus additions to wealth . In part of his work, Friedman assumes $k \in (0,1]$; at other times, he assumes $k = 1$. The short-term relationship, on the other hand, is determined by equation (1.5), which states that the transitory component of consumption is uncorrelated with the transitory component of disposable income; this implies that the short-term marginal propensity to consume out of transitory income is zero. In other words, people gear their consumption behavior to their permanent or long-term income, not to their transitory or short-term income. This theory of consumption, therefore, hinges crucially on a very clear distinction between the short-term and the long-term relationships.

The Life Cycle Theory of Consumption and Saving of Modigliani (1966) also highlights the importance of this distinction. According to this theory, individuals plan their consumption and saving behavior so as to maximize expected (discounted) utility over their entire lifetime. As a result, they choose to smooth their planned consumption path. Ignoring discounting, initial wealth and bequests for simplicity, this implies that an individual who expects to earn income y_t, for $t = 1, \ldots, M$ and to consume c_t, for $t = 1, \ldots, N \geq M$, will choose to consume

$$c_t = \frac{\sum_{t=1}^{M} y_t}{N} \tag{1.6}$$

in period t. Thus, for a permanent increase in income, the marginal propensity to consume is M / N, which is close to one; whereas, in contrast, the marginal propensity to consume out of a temporary increase in income is only $1/ N$, which is close to zero.

Thus, both the Permanent Income and Life Cycle theories of consumption imply that the "short-term" relationship between consumption and income differs from the "long-term" relationship.

1.1.2 What About the Co-Integrated Model?

No discussion of long-term versus short-term relationships would be complete without a consideration of co-integrated relationships. The techniques developed in this dissertation are a complement to, not a substitute for, co-integration. The co-integrated model introduced by Granger (1981) is defined as

$$x_t = \beta y_t + \eta_t , \tag{1.7}$$

where the cointegrating parameter, β, is unique and significant, and the error term, η_t, is zero integrated or white, with a small variance. Equation (1.7) describes a stable relationship between x and y over time; it models a long-term or equilibrium relationship. The error term η_t can thus be referred to as the "equilibrium error" because it measures the extent to which the system is out of equilibrium. Engle and Granger (1991) write:

> "From a theoretical point of view, the power of economic equilibrium as an attractor should force different variables to move together in the long run even if not in the short run and even if they are individually non-stationary. From an empirical view, the observed high correlations among the levels of macroeconomic time series are characteristic of data clustering around a linear attractor and hence cointegrated."

The short-term relationship, on the other hand, is subsumed in the "error-correction model:"

$$\Delta x_t = \sum_{i=1}^{p} \alpha_i \Delta x_{t-i} + \sum_{j=1}^{q} \gamma_j \Delta y_{t-j} - \delta(x_{t-1} - \beta y_{t-1}) + \varepsilon_t, \quad \delta > 0, \qquad (1.8)$$

where $x_{t-1} - \beta y_{t-1}$ is actually the "equilibrium error" η_{t-1} given in equation (1.7)[2]. The error-correction mechanism is embodied in the parameter δ ; whereas the short-term relationship between x and y is quantified by the parameter γ. According to Engle and Granger (1987),

> "The idea is simply that a proportion of the disequilibrium from one period is corrected in the next period. For example, the change in price in one period may depend upon the degree of excess demand in the previous period. Such schemes can be derived as optimal behavior with some types of adjustment costs or incomplete information. .... For a two variable system a typical error correction model would relate the change in one variable to past equilibrium errors, as well as to past changes in both variables."

The co-integration framework is an important approach for analyzing "long-term" versus "short-term" relationships where the "long-term" relationship is taken to refer to an

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[2]Inclusion of additional terms in η_{t-2}, η_{t-3}, etc., are suppressed for clarity.

equilibrium relationship between levels of (integrated) time series, whereas "short-term" is taken to refer to the dis-equilibrium relationship between the (stationary) changes in the time series whereby the system moves toward equilibrium. The "long-term / short-term" distinction delineated in the co-integration framework does not, however, correspond to the "low-frequency relationship" versus "high-frequency relationship" distinction analyzed here.

A simple example will clarify this point. Presuming that the level of aggregate consumption C_t and the level of aggregate disposable income Y_t are co-integrated,

$$C_t = \beta Y_t + \eta_t, \tag{1.9}$$

where β is close to one, and η_t is stationary, white, and "small", and

$$c_t = \alpha + \gamma y_t - \delta(C_{t-1} - \beta Y_{t-1}) + \varepsilon_t , \tag{1.10}$$

with $\delta > 0$, $c_t \equiv C_t - C_{t-1}$, and $y_t \equiv Y_t - Y_{t-1}$. Equation (1.9) indicates that there is a strong "long-term" relationship between C_t and Y_t based on a stable "equilibrium" average propensity to save (APS) of $(1-\beta)$. One could therefore interprets β as " the long-term MPC." The short-term relationship between consumption and disposable income, however, is given by equation (1.10) which models the relationship between the stationary series c_t and y_t. The parameter γ can be interpreted as the "short-term MPC;" the third term in equation (1.10), involving δ, says that the current consumption c_t depends not only on the changes in disposable income y_t, but also on whether last period's saving rate (average propensity to save) was above or below its "equilibrium" value of $(1-\beta)$. Indeed, it is this term which makes the APS tends to return toward $(1-\beta)$; if $\delta \leq 0$, the "equilibrium" is unstable, and the levels of consumption and disposable income are not co-integrated.

The term γy_t does not, however, in any way distinguish whether y_t is perceived to be a (relatively) permanent or a (relatively) transitory change in disposable income. Therefore, one might expect from a consideration of the Life Cycle theory of consumption, that this error-correction model ought to be reformulated along the lines of

$$c_t = \alpha + \gamma^{perm} y_t^{perm} + \gamma^{temp} y_t^{temp} - \delta(C_{t-1} - \beta Y_{t-1}) + \varepsilon_t, \tag{1.11}$$

Where y_t^{perm} is the portion of the current change in Y_t that is expected to be "permanent," and y_t^{temp} is the portion that is expected to be "transitory." In term of the technique developed in this dissertation, the parameter γ in equation (1.10) is not in fact a stable parameter; rather, it is small "at high frequency" and large "at low frequency."

1.1.3 Existing techniques

Several techniques have been proposed in the literature to deal with the distinction between the long-term and the short-term economic relationships; they are the traditional Time Domain approach, the Band Spectral approach, the Filtered Data approach and the Geweke Frequency Decomposition approach.

The traditional Time Domain approach, employed by Mack (1948), Modigliani (1949), Duesenbery (1952), Friedman (1957) and others, divides the variable of interest into the transitory and the permanent component; the permanent component of the variable is computed as weighted average of its past values. This computation is merely a smoothing process, and therefore cannot accurately measure the permanent component of the variable; this defect limits the effectiveness with which their approach can distinguish between long-term and short-term relationships.

The Band Spectral approach, initiated by Engle (1974), and the Filtered Data approach, employed by Lucas (1980), Summers (1983), Cochrance (1989), Lee (1994) and others, both attempt to distinguish short-term relationships from long-term relationships by "filtering out" bands of frequencies. In addition to requiring specialized software, these approaches suffer from the major drawback that the selection of the frequency bands is rather arbitrary.

The Geweke approach (1982) provides what initially appears to be an elegant way of quantifying how the strength of a dynamic linear relationship varies across frequencies. On closer examination, however, the frequency dependence computed by Geweke's analysis turns out to be a measure of the degree to which serially uncorrelated shocks in one time series are converted into more (or less) smoothly persistent fluctuations in the other series rather then a measure of how sensitive one series is to fluctuation in the other "at" a given frequency.

This study proposes a new approach which effectively delineates how a relationship between two variables (such as that between c_t and y_t in equation (1.11) above) depends on frequency. This new approach allows one to not only detect model misspecification of this type but also to correct it. Since this misspecification both invalidates the usual statistical inference machinery and erodes the model's forecasting effectiveness, correcting it can be expected to improve the quality of both inferences and forecasting.

1.1.4 Empirical Example

The advantage of the new technique proposed in this study in improving the statistical inference and forecasting effectiveness is illustrated below with an example from the time series literature. Jaditz and Sayers (1994) observe the paradoxical result that, while it is obvious that fluctuations in the monthly producer price index (PPI) Granger-cause fluctuations in the consumer price index (CPI) , a variety of models based on this premise fail miserably at post-sample forecasting.

In particular, Jaditz and Sayers identify and estimate a linear dynamic regression model relating the lagged growth rate in the PPI to the growth rate in the CPI :

$$CPI_t = \underset{(6.32)}{0.002} + \underset{(2.62)}{0.272}\, CPI_{t-1} + \underset{(2.32)}{0.104}\, PPI_{t-1} + \varepsilon_t \,; \quad R^2 = 0.25. \tag{1.12}$$

The estimated coefficient on lagged PPI (the t-statistic is given in parentheses) in this model is statistically significant and apparently stable over the sample period. Yet post-sample forecasts from this model are inferior to those of a univariate AR model for the growth rate of CPI. The technique developed here resolves this paradox and produces a model for the consumer price index fluctuations which yields a modest improvement in post-sample forecasting over the univariate AR model.

CHAPTER 2

A SURVEY ON THE SURROUNDING LITERATURE

2.1 Time-Domain Approach

Many economic relationships are expected to maintain themselves in the short term, and not necessarily in the long term and vice-versa. As a consequence, in discussions of economic theories as well as empirical research much attention has been devoted to the distinction between the short-term versus the long-term. Early attempts to empirically address the distinction between short-term and long-term relationships were basically done in the time domain, and are referred to here as the Time-Domain approaches. Mack (1948), Modigliani (1949), and Duesenberry (1952) are the pioneers who empirically investigated short-term versus long-term relationships between the time series of consumption and income using time-domain techniques.

Various terms have been given to "short-term" and "long-term", for example, "cyclical", "transitory", or "temporary" are equivalent to "short-term", whereas "secular", or "permanent" refer to "long-term". Modigliani refers "short-term" as "cyclical" and "long-term" as "secular" and defines them as

> "By the secular movement of income we mean a movement that carries real income per capita above the highest level reached in any preceding year; by cyclical movement we mean any movement, whether upward or downward, that leaves real income per capita below the highest previous peak. These definitions may be conveniently given in symbolic terms. Let Y_t denote real income in the year t and Y_t^o denote the highest real income per capita realized in any year preceding t; the change in income between the year t and the year (t+1) will be called cyclical, if both Y_t and $Y_{t+1} < Y_t^o = Y_{t+1}^o$; otherwise, it will be called secular. "

Modigliani and Duesenberry estimate permanent income y_t^p as a weighted average of the lagged income y_{t-1}, current income y_t, and the highest level of income previously experienced y_t^o :

$$y_t^p = w_1 y_t^o + w_2 y_t + w_3 y_{t-1} \tag{2.1}$$

$$w_1 + w_2 + w_3 = 1,$$

and formulate the regression equation of consumption-income as

$$c_t = \alpha + \beta w_1 y_t^o + \beta w_2 y_t + \beta w_3 y_{t-1} + \epsilon_t . \tag{2.2}$$

Mack uses a slightly different form from (2.2), namely

$$c_t = \alpha + \beta y_t + \gamma (y_t - y_{t-1}) + \varepsilon_t . \tag{2.3}$$

According to Friedman(1957), the consumption-income equation that he proposed in the Permanent Income Hypothesis (equation (1.3)) can be shown to be equivalent to that of Modigliani-Duesenberry and Mack's. However he disagrees with their technique of estimating the permanent income:

> "On this interpretation, the incomes of prior years enter into the functions as a means of estimating permanent income. Judge from this point of view, the Modigliani-Duesenberry and Mack functions are questionable in several respects. In the first place, they estimate permanent income as the average of two or at most three years, yet it seems plausible that permanent income should be estimated from a longer period. More importantly, this is not an issue that should be decided a priori; the data themselves should dictate the appropriate number of years. In the second place, the use of the highest previous income seems rather arbitrary. For example , it might lead to use of a different year according to the form of the data - one year, say, for per capita deflated data, another for aggregates in current prices. It seems rather arbitrary, too, that the same weight should be attached to the highest previous income regardless of how many years separate it from the current year."

He argues that the estimate of the permanent income at time T should be a weighted average of a series of its past values, and that the weighting pattern should be exponential and declining in earlier values:

$$y^{P}(T) = \beta \int_{-\infty}^{T} e^{(\beta - \alpha)(t - T)} y(t)\, dt. \tag{2.4}$$

The corresponding aggregate consumption function is

$$c(T) = k\beta \int_{-\infty}^{T} e^{(\beta - \alpha)(t - T)} y(t)\, dt, \tag{2.5}$$

where c is aggregate or per capita consumption and y is aggregate or per capita income, T is the current time, k, α , and β are the parameters of the function. According to Friedman (1957),

> "k is to be interpreted as the ratio of permanent consumption to permanent income, α as the secular rate of growth of income, and β as the damping coefficient which describes the process of forming estimates of expected or permanent income from current and past measured income; the higher β, the more rapidly the weights decline as one go back in time, and the shorter the average lag between permanent income and the incomes averaged."

In practice, the integral in equation (2.4) and (2.5) is converted into a summation of annual terms and a limited number of terms are retained.

This technique, however, was still unsatisfactory; the long-term relationship between consumption and income, in this context, is just the relationship between the smoothed consumption and the smoothed income.

2.2 Band Spectral Analysis

A number of later studies attempted to improve the empirical analysis of short-term versus long-term economic relationships via frequency decomposition. In these approaches the distinction between the short-term versus the long-term relationship is expressed in the frequency domain by mapping long-term relationships to low-frequency relationships and short-term relationships to high-frequency relationships.

The idea of regression analysis in the frequency domain was first proposed by Hannan (1963). Suppose we have a standard regression model in the time domain based on N observations:

$$Y = X\beta + u, \tag{2.6}$$

where $u \sim N(0, \sigma^2 I)$ is the disturbance vector. The model in (2.6) can be analyzed in the frequency domain by applying a finite Fourier transformation to the dependent variable and the set of independent variables. This transformation generates a set of N observations, indexed now by frequency, rather than by time. Regressions based on the transformed set of observations are known as spectral regressions.

The important property of the spectral regression related to this discussion is that it allows one to run regressions on subsets of the N frequencies. The technique of running spectral regressions on subsets of frequencies is referred to as the Band Spectral Regression and has been extensively discussed by Engle (1974,1978,1980). The short-term and the long-term versions of a relationship can be separated by isolating the high and the low frequency bands. If W is a unitary transformation matrix which consists of the typical element $w_{p,q} = e^{i\frac{2\pi}{N}pq}$; $t, s \in [0, N-1]$, then the finite Fourier transformation of (2.6) in the frequency domain is

$$\tilde{Y} = \tilde{X}\beta + \tilde{u}, \tag{2.7}$$

where $\tilde{Y} = WY$, $\tilde{X} = WX$, and $\tilde{u} = Wu$ is a complex-valued spherical disturbance vector[3]. As shown in Engle (1974), the OLS estimator of β in (2.7) is BLUE and can be written as

$$\hat{\beta} = \left[\sum_{k=0}^{N-1}\hat{f}_{xx}(\theta_k)\right]^{-1}\sum_{k=0}^{N-1}\hat{f}_{xy}(\theta_k), \tag{2.8}$$

where $\theta_k = 2\pi k / N$, for $k \in [\,1, N\,]$, denotes the frequency. $\hat{f}_{xx}$ is a matrix of cross-periodograms at each frequency and $\hat{f}_{xy}$ is a vector of cross periodograms. Equation (2.8) is a full-spectral estimator since it is based on all the frequencies. If only a subset of the frequencies is of interest, the irrelevant frequencies can be omitted by premultiplying (2.8) with Z, an $N \times N$ matrix with zeros everywhere except at the diagonal entries corresponding to the included frequencies. Thus (2.7) becomes

$$Z\tilde{Y} = Z\tilde{X}\beta + Z\tilde{u}. \tag{2.9}$$

A BLUE estimator of β in (2.9), given by Engle (1974), is

[3]See Engle(1974) for the case of non-spherical disturbances.

$$\hat{\beta}^{*} = \left[\sum_{f\in f^{*}} \hat{f_{xx}}(\theta_k)\right]^{-1} \sum_{f\in f^{*}} \hat{f_{xy}}(\theta_k), \tag{2.10}$$

where f^{*} denotes the subset of the frequencies of interest. However, Engle did not carry out regressions on the Fourier-transformed representations because regression software does not allow complex data. Instead, he back-transformed the processed data ($Z\widetilde{Y}$ and $Z\widetilde{X}$) into the time domain by taking an inverse Fourier transformation and then applied OLS. He has shown that the inverse Fourier transformation does not affect the efficiency of the method[4].

The band spectral technique has been applied by a number of researchers including Espasa and Sargan (1977), Garbers (1987), Siklos (1988), Gamber (1988), Nachane and Chrissanthaki (1991), and Thoma (1992).

The major problem with band spectral analysis is that the decision at what bands to include (or delete) is rather arbitrary.

2.3 The Filtered Data Approach

Besides the band spectral analysis, several researchers have carried out regressions on filtered data to analyze short-term versus long-term relationships between economic variables. Various types of filters have been used to eliminate unwanted frequencies from the data set before running regressions.

2.3.1 The Spectral-Window Filter

[4]See Engle(1974,1978,1980) for a detailed discussion and examples.

A particular frequency band component $\{ x_t^* \}$ can be obtained by passing the series under study $\{ x_t \}$ through an appropriate filter that passes only the frequencies in a specific range and highly attenuates all other frequencies. For example, a two-sided moving average filter d(L) can be used to obtain

$$x_t^* = \sum_{J=-\infty}^{\infty} d_j x_{t-j} = d(L)x_t \, , \tag{2.11}$$

such that the spectral density of $\{x_t^*\}$ vanishes outside a specified range. The values of d_j's can be computed using the inverse Fouriers transformation given in Sargent (1979). For example, if the passband lies between frequencies a and b, then

$$d_j = \frac{1}{\pi}\left(\frac{\sin(aj)}{j} - \frac{\sin(bj)}{j} \right) \, , \tag{2.12}$$

for $j = -\infty, \ldots, -1, 1, \ldots, \infty$ and $d_0 = (b\text{-}a)/\pi^2$. The two-sided moving-average filter given by (2.12) is referred to as the spectral-window filter. This filter was used by Cochrane (1989) to investigate the short-term relationship between the money growth and the interest rate; in this way he finds a negative short-term relationship between the money growth and the interest rate.

2.3.2 The Low-Pass Exponential Filter

Long-term relationships between time series can also be isolated from short-term relationships by passing the series through a low-pass filter that eliminates high frequencies. Lucas (1980) used a low-pass filter to confirm the long-term relationship among the money growth, the inflation rate and the nominal interest rate. He posited a two-sided exponentially weighted filter given by

$$\{x_t(\beta)\} = \left\{ \frac{(1-\beta)}{(1+\beta)} \sum_{k=-\infty}^{\infty} \beta^{|k|} x_{t-k} \right\}, \tag{2.13}$$

where $0 < \beta < 1$ and x_t is a time series. This filter has a Fourier-transformation:

$$\beta(e^{-iw}) = \frac{(1-\beta)^2}{(1+\beta^2-2\beta\cos w)}, \tag{2.14}$$

which, with relatively large β, concentrates most of its power at low frequencies. Such a filter is referred to as a "low pass" filter. Whiteman (1984) shows that Lucas' method is a robust technique for computing the sums of lag coefficients from a distributed-lag regression of the inflation and the nominal interest rates on the money growth[5]. Summers (1983) used the same technique to investigate the long-run Fisher effect.

2.3.3 Time-Domain Chebyshev Filter

Lee (1994) argues that it is not appropriate to use two-sided filters (involving both lags and leads) in the analysis of Granger causal relationships, since these require a well-defined timing of the accumulation of information. Lee proposes one-sided time-domain Chebyshev filters instead. A typical time-domain low-pass Chebyshev filter can be

[5]See Lucas(1980) and Whiteman(1984) for details in the Lucas' calculation of the sums of the distributed lag coefficients in the regressions.

represented by the equation[6]

$$y_t^L = 1.73963 y_{t-1}^L - 0.78048 y_{t-2}^L + 0.00964(x_t + 2x_{t-1} + x_{t-2}). \qquad (2.15)$$

The filtered series $\{y_t^L\}$ represents the components of the input series $\{x_t\}$ whose cyclical periods are longer than three years. On the other hand, the time-domain-high-pass-Chebyshev filter is given by

$$y_t^H = 1.46641 y_{t-1}^H - 0.61219 y_{t-2}^H + 0.72660(x_t - 2x_{t-1} + x_{t-2}). \qquad (2.16)$$

The filtered series $\{y_t^H\}$ represents the components of the input series $\{x_t\}$ whose cyclical periods are shorter than one year. Lee (1994) uses a time-domain low-pass Chebyshev filter to investigate the impact of the anticipated money growth on the real economic activity. His results support the IS-LM framework but refute the neutrality proposition. He also examines the impact of the unanticipated money growth on the real economic activity by utilizing the high-pass filtered data. His findings show that an unanticipated money growth predicts a strengthening of the real economic activity over short horizons, but a weakening one over long horizons.

The filtered data approach suffers from the same disadvantages as the band spectral analysis, in that the bands used are arbitrarily determined.

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[6]See Lee(1994) for the construction procedure of the filter.

2.4 The Geweke Approach

Geweke (1982) proposes a statistical measure to quantify the intensity of causal relationships between variables across various frequencies. The measure is constructed as a function of the spectral density of the variable under study.

2.4.1 Decomposing Intensity of Response (Not Relationship) Across Frequencies

For two stationary and purely nondeterministic time series x and y, Geweke defines $F_{x,y}$ as the linear relationship between x and y, $F_{x\to y}$ as the linear causality from x to y, $F_{y\to x}$ as the linear causality from y to x, and $F_{x\cdot y}$ as the instantaneous linear causality between x and y, in the time domain. Geweke derived a decomposition of the linear relationship between x and y as

$$F_{x,y} = F_{x\to y} + F_{y\to x} + F_{x\cdot y} . \tag{2.17}$$

Measures of linear dependence and causality between multiple time series are constructed on a canonical form that consists of four sets of linear projections of x_t and y on the following different information sets[7].

I. The linear projections of x_t on $\{ x_j : j < t \}$ and of y_t on $\{ y_j : j < t \}$ are

$$x_t = \sum_{j=1}^{\infty} E_{1j} x_{t-j} + u_{1t}, \qquad \text{var}\,(u_{1t}) = \Sigma_1 \tag{2.18}$$

[7]See Geweke(1982) for the construction of the canonical form.

and

$$y_t = \sum_{j=1}^{\infty} G_{1j} y_{t-j} + v_{1t}, \qquad \text{var}(v_{1t}) = \mathrm{T}_1 , \tag{2.19}$$

where u_{1t} and v_{1t} are disturbance vectors that are each serially uncorrelated but may be correlated with each other contemporaneously and at various leads and lags.

II. The linear projections of x_t and y_t on $\{ x_j : j < t \} \mathrm{U} \{ y_j : j < t \}$ are

$$x_t = \sum_{j=1}^{\infty} E_{2j} x_{t-j} + \sum_{j=1}^{\infty} F_{2j} y_{t-j} + u_{2t}, \qquad \text{var}(u_{2t}) = \Sigma_2 , \tag{2.20}$$

and

$$y_t = \sum_{j=1}^{\infty} G_{2j} y_{t-j} + \sum_{j=1}^{\infty} H_{2j} x_{t-j} + v_{2t}, \qquad \text{var}(v_{2t}) = \mathrm{T}_2 , \tag{2.21}$$

where the disturbance vectors u_{2t} and v_{2t} are each serially uncorrelated. They are also uncorrelated with lags of x_t and y , so they can only be correlated with each other contemporaneously.

$$\gamma = \text{var}\begin{pmatrix} u_{2t} \\ v_{2t} \end{pmatrix} = \begin{bmatrix} \Sigma_2 & C \\ C' & \mathrm{T}_2 \end{bmatrix}. \tag{2.22}$$

III. The projection of x_t on $\{ x_j : j < t \} \mathrm{U} \{ y_j : j \le t \}$ is

$$x_t = \sum_{j=1}^{\infty} E_{3j} x_{t-j} + \sum_{j=0}^{\infty} F_{3j} y_{t-j} + u_{3t} , \quad \text{var}(u_{3t}) = \Sigma_3 , \tag{2.23}$$

and the projection of y_t on $\{ y_j : j < t \} \mathrm{U} \{ x_j : j \le t \}$ is

$$y_t = \sum_{j=1}^{\infty} G_{3j} y_{t-j} + \sum_{j=0}^{\infty} H_{3j} x_{t-j} + v_{3t}, \; \mathrm{var}(v_{3t}) = \mathrm{T}_3 . \tag{2.24}$$

IV. The projection of x_t on $\{ x_j : j < t \} \mathrm{U} \{ y_j : -\infty < j < \infty \}$ is

$$x_t = \sum_{j=1}^{\infty} E_{4j} x_{t-j} + \sum_{j=-\infty}^{\infty} F_{4j} y_{t-j} + u_{4t}, \; \mathrm{var}(u_{4t}) = \Sigma_4 , \tag{2.25}$$

and the projection of y_t on $\{ y_j : j < t \} \mathrm{U} \{ x_j : -\infty < j < \infty \}$ is

$$y_t = \sum_{j=1}^{\infty} G_{4j} y_{t-j} + \sum_{j=-\infty}^{\infty} H_{4j} x_{t-j} + v_{4t}, \qquad \mathrm{var}(v_{4t}) = \mathrm{T}_4 . \tag{2.26}$$

Measures of linear dependence and feedback in the time domain are given by:

(I) $\quad F_{x \to y} = \ln (| \mathrm{T}_1 | / | \mathrm{T}_2 |)$, (2.27)

(ii) $\quad F_{y \to x} = \ln (| \Sigma_1 | / | \Sigma_2 |)$, (2.28)

(iii) $\quad F_{x \cdot y} = \ln (| \mathrm{T}_2 | \cdot | \Sigma_2 | / | \gamma |)$, (2.29)

(iv) $\quad F_{x,y} = \ln (| \Sigma_1 | \cdot | \mathrm{T}_1 | / | \gamma |)$. (2.30)

Geweke defines two nonnegative functions $f_{x \to y}(\omega)$ and $f_{y \to x}(\omega)$ as the decomposition of $F_{x \to y}$ and $F_{y \to x}$ by frequency:

$$F_{x \to y} = \frac{1}{2\pi} \int_{-\pi}^{\pi} f_{x \to y}(\omega) \, d\omega \tag{2.31}$$

and

$$F_{y \to x} = \frac{1}{2\pi} \int_{-\pi}^{\pi} f_{y \to x}(\omega) \, d\omega . \tag{2.32}$$

The construction of $f_{y \to x}(\omega)$ is based on the system:

$$\begin{bmatrix} E_2(L) & F_2(L) \\ H_3(L) & G_3(L) \end{bmatrix} \begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} u_{2t} \\ v_{3t} \end{pmatrix}, \tag{2.33}$$

which is constructed from (2.20) and (2.24). Under the assumption of invertibility, the moving-average representation of this system can be written as

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{bmatrix} E^2(L) & F^2(L) \\ H^3(L) & G^3(L) \end{bmatrix} \begin{pmatrix} u_{2t} \\ v_{3t} \end{pmatrix}. \tag{2.34}$$

The corresponding decomposition of the spectral density $S_x(\omega)$ is given by

$$S_x(\omega) = E_w^{\ 2}(\omega)\Sigma_2 E_w^{\ 2}(\omega)' + F_w^{\ 2}(\omega)\mathrm{T}_3 F_w^{\ 2}(\omega)' , \tag{2.35}$$

where $E_w^{\ 2}(\omega)$ and $F_w^{\ 2}(\omega)$ are the Fourier transformations of $E^2(\mathrm{L})$ and $F^2(\mathrm{L})$, and the prime (') denotes conjugation as well as transposition. The measure of linear causality from y to x at frequency ω is given by

$$f_{y\to x}(\omega) = \ln(|S_x(\omega)| \,/\, |E_w^{\ 2}(\omega)\Sigma_2 E_w^{\ 2}(\omega)'|). \tag{2.36}$$

Symmetrically, the system constructed from (2.23) and (2.21) is

$$\begin{bmatrix} E_3(L) & F_3(L) \\ H_2(L) & G_2(L) \end{bmatrix} \begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} u_{3t} \\ v_{2t} \end{pmatrix}, \tag{2.37}$$

which can be inverted to obtain

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{bmatrix} E^3(L) & F^3(L) \\ H^2(L) & G^2(L) \end{bmatrix} \begin{pmatrix} u_{3t} \\ v_{2t} \end{pmatrix} . \tag{2.38}$$

The measure of linear causality from x to y at frequency ω is defined as

$$f_{x\to y}(\omega) = \ln (|S_y(\omega)| \;/\; |G_w^{\,2}(\omega)\mathrm{T}_2 G_w^{\,2}(\omega)'|) . \tag{2.39}$$

Point estimates of $f_{x\to y}(\omega)$ and $f_{y\to x}(\omega)$ for various values of ω can be obtained from the ordinary least squares estimates of the finite versions of (2.18) through (2.21). By equating low frequency with long term and high frequency with short term, the strength of long or short-term causal relationship can be determined. Geweke's computation, using this methodology, shows that variations in the money supply do not Granger cause variations in the real output in the long term, but do Granger cause variations of the price level in the long term. His result also shows the evidence of short-term causality from the real output to the money supply, which can be interpreted as an example of the counter-cyclical monetary policy.

The Geweke Approach appears to provide a statistical way of quantifying the strength of causal relationships through frequency decomposition. In fact, however, his approach merely quantifies the degree to which the system converts uncorrelated shocks to one series into more (or less) smooth responses by the other variable. As a consequence, the frequency dependence of his measure has little bearing on how the intensity of the relationship varies across frequencies.

This is demonstrated below via two counter examples based on a simple bivariate model. The first counter example (section 2.4.2) demonstrates that the frequency dependence of the Geweke measure depends crucially on a "nuisance parameter" in the model. In the second counter example (section 2.4.3), the model is modified to explicitly introduce frequency domain variation in the parameter linking x_t and lagged y_t . The estimated results using data generated from this model show that Geweke's measure can easily fail to detect this actual frequency dependence.

2.4.2 Counter Example I - Spurious Frequency Dependence of *f*(ω)

This example shows how the frequency dependence of Geweke's measure is dominated by a "nuisance parameter".

Lemma 1:

If x_t and y_t are the time series generated from a simple Geweke's bivariate model:

$$\begin{aligned} x_t &= (1 + \phi B)u_t + \rho v_{t-1} \\ y_t &= v_t \\ \begin{pmatrix} u_t \\ v_t \end{pmatrix} &\sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_u^2 & 0 \\ 0 & \sigma_v^2 \end{pmatrix}\right] \end{aligned} \tag{2.40}$$

where B is the lag operator, u_t and v_t are the respective innovations for the time series x_t and y_t. Then the Geweke measure of frequency dependent causal relationship running from y_t to x_t is

$$f_{y \to x}(\omega) \;=\; \frac{1}{2}\ln\left\{\frac{[\sigma_u^2(1 + \phi^2 + 2\phi\cos(\omega)) + \rho^2\sigma_u^2\sigma_v^2]^2}{[\sigma_u^2(1 + \phi^2 + 2\phi\cos(\omega))]^2}\right\}, \tag{2.41}$$

and for all $\rho \neq 0$,

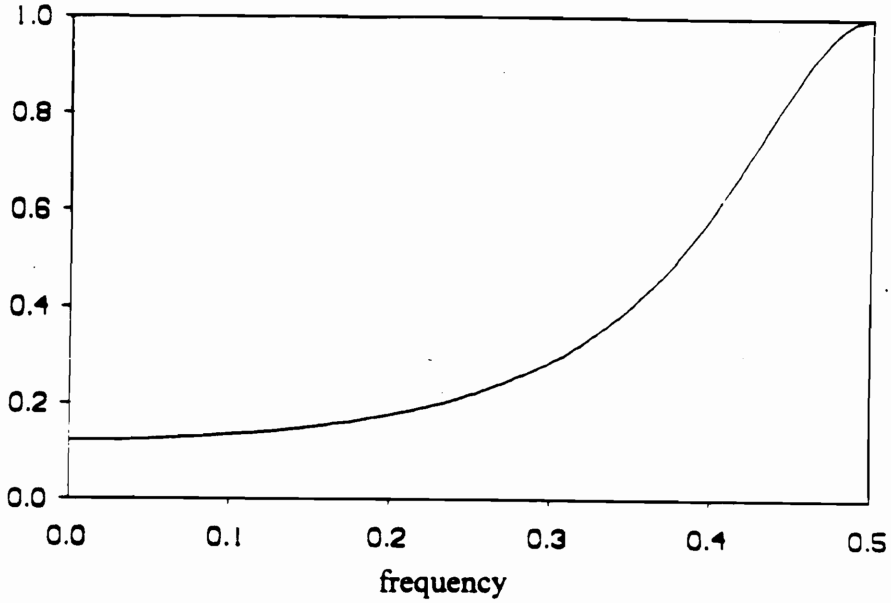
$$\begin{aligned} &f_{y \to x}(0) \to 0\,,\; f_{y \to x}(\pi) \to \infty\,,\; \text{as}\;\; \phi \to 1, \;\text{and} \\ &f_{y \to x}(0) \to \infty\,,\; f_{y \to x}(\pi) \to 0\,,\; \text{as}\;\; \phi \to -1. \end{aligned} \tag{2.42}$$

Proof: given in Appendix A.

In Figure 1, the Geweke measure of frequency dependent relationship between y_t and x_t is plotted for the case of $\phi > 0$ and $\phi < 0$. For both cases, σ_u^2 and σ_v^2 are taken to be 1. The first graph in Figure 1 is plotted by setting $\phi = .6$ and $\rho = .6$, whereas the second graph is plotted by setting $\phi = -.6$ and $\rho = .6$. It is clear that the measure $f_{y \to x}(\omega)$ is interfered by the serial correlation of the x_t series. Evidently, the frequency dependence of Geweke's measure is actually measuring the degree to which innovation in y_t cause smooth, persistent fluctuations (low frequencies dominal) or rough, persistent fluctuations (high frequencies dominal) in x_t, not the degree to which the intensity of the relationship between x_t and y_t varies across frequencies.

Geweke $f_{y-x}(\omega)$ Measure

$\phi = .6$ and $\rho = .6$



$\phi = -.6$ and $\rho = .6$

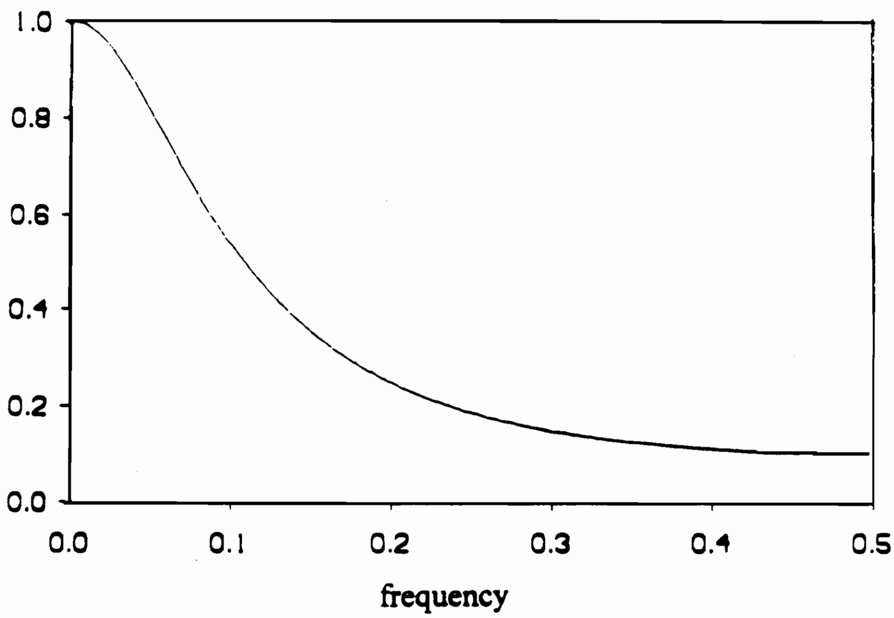


Figure 1
Geweke's Measure of Frequency Dependence

2.4.3 Counter Example II - Failure of $f(\omega)$ to Detect Actual Frequency Dependence

In this example, the simple model of (2.40) is modified to explicitly introduce actual frequency dependence in the relationship between x_t and y_t. The data set generated from this model is used to show that the Geweke measure $f_{y-x}(\omega)$ can easily fail to detect not only the frequency dependence in the $x_t - y_t$ relationship, but even the existence of any relationship at all.

Again consider the simple bivariate model of equation (2.40):

$$
\begin{aligned}
x_t &= (1 + \phi B)u_t + \rho v_{t-1} \\
y_t &= v_t \\
\begin{pmatrix} u_t \\ v_t \end{pmatrix} &\sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_u^2 & 0 \\ 0 & \sigma_v^2 \end{pmatrix}\right]
\end{aligned}
\tag{2.43}
$$

But - in order to now allow the parameter ρ (formerly fixed) to vary with frequency - replace v_{t-1} in equation (2.43) by $Q_{0,t-1}, Q_{1,t-1}, \ldots, Q_{\frac{m}{2},t-1}$ defined by

$$
Q_{j,t-1} = \begin{cases} \frac{1}{m}\sum_{l=0}^{m-1} v_{l+t-(m-1)}, & \text{for } j = 0 \\ \frac{2}{m}(-1)^{m-2}\sum_{l=0}^{m-1} (-1)^{-l}\cos\left[\frac{2\pi}{m}(2+l)(\frac{m}{2}-j)\right] v_{l+t-(m-1)}, & \text{for } j = 1,\ldots,(\frac{m}{2}-1) \\ \frac{1}{m}(-1)^{m-2}\sum_{l=0}^{m-1} (-1)^{-l}\, v_{l+t-(m-1)}, & \text{for } j = \frac{m}{2}. \end{cases}
\tag{2.44}
$$

They are just the finite Fourier transformation of the data block $\{v_{t-1}, v_{t-2}, \ldots, v_{t-m}\}$ at frequency $2\pi\frac{j}{m}$, algebraically transformed to yield $\frac{m}{2}+1$ real-valued components[8].

Now the model becomes

$$\begin{aligned} x_t &= (1 + \phi B)u_t + \sum_{j=0}^{\frac{m}{2}} r_j Q_{j,t-1} \\ y_t &= v_t, \end{aligned} \qquad (2.45)$$

where the variation (over j) in the r_j's quantifies the frequency variation in the relationship between x_t and $y_{t-1} = v_{t-1}$.

Lemma 2:

Given the definition of $Q_{j,t-1}$ in equation (2.44),

$$\sum_{j=0}^{\frac{m}{2}} Q_{j,t-1} = v_{t-1}, \qquad (2.46)$$

so that model (2.45) reduces to model (2.43) when the r_j 's are all equal.

Proof: Given in Appendix C.

Using the result from Lemma 2, observations on x_t and y_t were generated using $m = 12$, $\phi = .6$, $\sigma_u^2 = 1$, $\sigma_v^2 = 1$, and

Case I : $r_j = .6$, for $j = 0, 1, 2, ..., 6$

Case II : $r_j = 0$, for $j = 0, 1, 2, ..., 4$.
$r_j = .6$, for $j = 5, 6$.

[8] See Appendix B for derivation.

Case I corresponds to Geweke's assumption of fixed parameter. The Box-Jenkins estimates of the Geweke model (2.53) and the Frequency Block model (2.54) using this generated data set are given, respectively, in Table 1 and Table 2. Note that, with this fairly large sample, $\hat{\rho}$ is correctly recovered as .6 in Table 1, and the $\hat{r}_0, \hat{r}_1, \ldots, \hat{r}_6$, in Table 2, are all insignificantly different from .6 as one would expect.

Table 1
Box-Jenkins Estimates of the Geweke Model (Case I)

Dependent Variable X - Estimation by Box-Jenkins
Iterations Taken 6
Usable Observations 499 Degrees of Freedom 497
Centered R**2 0.396520 R Bar **2 0.395306
Uncentered R**2 0.397180 T x R**2 198.193
Mean of Dependent Variable -0.041195149
Std Error of Dependent Variable 1.246800854
Standard Error of Estimate 0.969538218
Sum of Squared Residuals 467.18216504
Durbin-Watson Statistic 1.979819

Variable	Coeff	Std Error	T-Stat	Signif
1. MA{1}	0.5703631244	0.0368762408	15.46695	0.00000000
2. v{1}	0.5837007299	0.0359278345	16.24648	0.00000000

Table 2
Box-Jenkins Estimates of the Frequency Block Model (Case I)

Dependent Variable X - Estimation by Box-Jenkins
Iterations Taken 5
Usable Observations 499 Degrees of Freedom 491
Centered R**2 0.398123 R Bar **2 0.389543
Uncentered R**2 0.398781 T x R**2 198.992
Mean of Dependent Variable -0.041195149
Std Error of Dependent Variable 1.246800854
Standard Error of Estimate 0.974147558
Sum of Squared Residuals 465.94106087
Durbin-Watson Statistic 1.982082

Variable	Coeff	Std Error	T-Stat	Signif
1. MA{1}	0.5702121740	0.0371183138	15.36202	0.00000000
2. Q0{1}	0.5590387311	0.2218541417	2.80422	0.00093075
3. Q1{1}	0.5866294439	0.1652018748	3.55099	0.00042076
4. Q2{1}	0.6007925885	0.1506525616	3.98793	0.00007679
5. Q3{1}	0.5873953431	0.1202175217	4.88610	0.00000139
6. Q4{1}	0.5514870607	0.0890695212	6.19165	0.00000000
7. Q5{1}	0.6060233029	0.0689888777	8.78436	0.00000000
8. Q6{1}	0.5845481231	0.0644435859	9.07070	0.00000000

Case II corresponds to a relationship which is non existent at low frequencies. The Box-Jenkins estimates of the Geweke model (2.53) and the correctly specified Frequency Block model are given, respectively, in Table 3 and Table 4.

Table 3
Box-Jenkins Estimates of the Geweke Model (Case II)

Dependent Variable X - Estimation by Box-Jenkins
Iterations Taken 7
Usable Observations 499 Degrees of Freedom 497
Centered R**2 0.289938 R Bar **2 0.288509
Uncentered R**2 0.289981 T x R**2 144.701
Mean of Dependent Variable -0.009167000
Std Error of Dependent Variable 1.178026759
Standard Error of Estimate 0.993664534
Sum of Squared Residuals 490.72249593
Durbin-Watson Statistic 2.041130

Variable	Coeff	Std Error	T-Stat	Signif
1. MA{1}	0.621831690	0.035597593	17.46836	0.00000000
2. v{1}	-0.018524219	0.035622113	-0.52002	0.60328103

Table 4
Box-Jenkins Estimates of the Frequency Block Model (Case II)

Dependent Variable X - Estimation by Box-Jenkins
Iterations Taken 6
Usable Observations 499 Degrees of Freedom 491
Centered R**2 0.323943 R Bar **2 0.314305
Uncentered R**2 0.323984 T x R**2 161.668
Mean of Dependent Variable -0.009167000
Std Error of Dependent Variable 1.178026759
Standard Error of Estimate 0.975485330
Sum of Squared Residuals 467.22167009
Durbin-Watson Statistic 2.005622

Variable	Coeff	Std Error	T-Stat	Signif
1. MA{1}	0.614337094	0.035694407	17.21102	0.00000000
2. Q0{1}	-0.066681627	0.061740193	-1.08004	0.28065640
3. Q1{1}	0.001969020	0.065018132	0.03028	0.97585272
4. Q2{1}	0.003665291	0.087987252	0.04166	0.96678901
5. Q3{1}	-0.065545399	0.128306861	-0.51085	0.60968657
6. Q4{1}	-0.060150838	0.155399085	-0.38707	0.69886964
7. Q5{1}	0.578497405	0.170031014	3.40231	0.00072279
8. Q6{1}	0.569734280	0.219092022	2.60043	0.00959122

Note that Geweke's approach would, in this instance, find no significant relationship between lagged y_t and x_t at all, regardless of frequency; whereas the correctly specified Frequency Block regression model clearly shows that there is a significant relationship between x_t and y_{t-1} (at high frequencies).

CHAPTER 3

BLOCK-WISE-FREQUENCY-DEPENDENT-REGRESSION APPROACH

3.1 Transforming a Time Domain Linear Regression Model into a Real-Valued Regression Model in the Frequency Domain

The Block-Wise-Frequency-Dependent-Regression (BWFDR) Approach proposed by this study is a three-step procedure to assess low-frequency versus high-frequency relationships between economic time series. The first step transforms the time domain linear regression model of interest into a real-valued regression model in the frequency domain. This frequency domain model "looks" like an ordinary regression equation, the only different being that "observations" now correspond to different frequencies rather than to different time periods. Consequently, in the second step, well-established methods in the literature are used to test the stability of the regression parameters across the "observations." In a third step, the resulting "picture" of the parameter stability across frequencies is used to sensibly choose frequency bands over which it is reasonable to assume that the parameter are constant. This leads to a time domain model allowing for frequency dependence which can be estimated and used to forecast.

One way to analyze short-term (high-frequency) versus long-term (low-frequency) relationships between time series is by transforming the model of interest from the time domain into the frequency domain. Engle (1974) constructed a complex-valued regression in the frequency domain by pre-multiplying the usual multivariate linear regression model in the time domain,

$$Y = X\beta + \varepsilon ; \quad \varepsilon \sim \mathrm{N}(\vec{0}, \sigma^2 \mathrm{I}_n), \tag{3.1}$$

by the unitary matrix W, which consists of the typical element:

$$w_{ts} = e^{i\frac{2\pi}{n}ts} ; \quad t,s \in [0,n-1]. \tag{3.2}$$

The resulting complex-valued model in the frequency domain is

$$\tilde{Y} = \tilde{X}\beta + \tilde{\varepsilon} ; \quad \tilde{\varepsilon} \sim \mathrm{N}(\vec{0}, \sigma^2 \mathrm{I}_n), \tag{3.3}$$

where

$$\tilde{Y} = WY, \ \tilde{X} = WX, \text{ and } \tilde{\varepsilon} = W\varepsilon . \tag{3.4}$$

Since the rows of W apply a complex finite Fourier transformation to any column vector they multiply, the n rows of equation (3.3) correspond to the n frequencies: 0, $2\pi(1/n)$, $2\pi(2/n)$, ..., $2\pi(n\text{-}2/n)$, $2\pi(n\text{-}1/n)$. In other words, the observations in this equation are indexed by frequency, and it is referred to as a complex-valued regression in the frequency domain. The transformed (complex-valued) disturbance vector $\tilde{\varepsilon}$ has the same distribution as the disturbance vector ε in (3.1) because W is a unitary matrix[9].

Engle's formulation of complex-valued regression in the frequency domain, however,

---

[9]That is $WW' = \mathrm{I}$, where W' is the complex conjugate of the transpose of W.

has several drawbacks. The first one is that the high frequencies above the Nyquist folding frequency, π, in equation (3.3), are merely the mirror images, or the complex conjugates, of "observations" below the folding frequency, so that the highest frequencies (close to 2π) actually refer to low frequency phenomenon; clearly this dramatically complicates the interpretation of this equation. The second disadvantage is that all the regressors, except for the first and the $n/2$ observation, are complex-valued, so a special estimation routine must be obtained before running any regression on this model. One can avoid running regressions with complex regressors by eliminating designated frequencies and then back-transforming the complex-valued model to the time domain. One might, for example, omit the high-frequency observations to assess the long-term economic relationships. Engle (1974) employed this method, the band spectral regression, to test the Permanent Income Hypothesis. This method gives no indication as to which frequencies are, in some sense, "reasonable" to omit, however.

3.1.1 Harvey's Real-Valued Regression Model in the Frequency Domain

Harvey (1978) proposed a way to circumvent the complications of the complex-valued regression model in the frequency domain by constructing a real-valued regression model in the frequency domain. Consider the standard linear regression model given by (3.1); a real-valued frequency-indexed model can be obtained by pre-multiplying (3.1) by an alternative unitary transformation matrix A :

$$AY = AX\beta + A\varepsilon \tag{3.5}$$

or

$$Y^{*} = X^{*}\beta + \varepsilon^{*}, \quad \varepsilon^{*} \sim \mathrm{N}(\vec{0}, \sigma^2 \mathrm{I}_n), \tag{3.6}$$

where $Y^{*} = AY$, and so on, and the typical element of A, $a_{t,s}$, is

$$a_{t,s} = \begin{cases} \left(\frac{1}{n}\right)^{\frac{1}{2}}, & \text{for } t = 1; \\ \left(\frac{2}{n}\right)^{\frac{1}{2}} \cos\left[\frac{\pi t(s-1)}{n}\right], & \text{for } t = 2, 4, 6, \ldots, (n-2) \text{ or } (n-1); \\ \left(\frac{2}{n}\right)^{\frac{1}{2}} \sin\left[\frac{\pi (t-1)(s-1)}{n}\right], & \text{for } t = 3, 5, 7, \ldots, (n-1) \text{ or } n; \\ \left(\frac{1}{n}\right)^{\frac{1}{2}} (-1)^{s+1}, & \text{for } t = n \text{ and } n \text{ is even}, \quad s = 1, \ldots, n. \end{cases} \tag{3.7}$$

This transformation is unitary because A is an orthogonal matrix, that is, $AA^{\mathrm{T}} = \mathrm{I}$. Thus the error term in equation (3.6) is still spherical. Since A is real-valued, so are Y^{*}, X^{*} and ε^{*}; consequently equation (3.6) can be estimated conveniently using ordinary regression packages.

3.1.2 Proof of Equivalence to Engle's Complex-Valued Transformation

This study shows that Harvey's real-valued transformation is equivalent to Engle's complex-valued transformation, and hence is equally valid for quantifying the frequency dependence of relationships between economic time series. The equivalence of these transformations can be proven by showing that the unitary transformation matrix A, given by Harvey, can be constructed by applying a series of row manipulations to Engle's complex-valued frequency domain model[10]:

Theorem 1:

Engle's complex-valued spectral regression (equation 3.3) can be transformed into Harvey's real-valued regression (equation 3.6) via row manipulations.

Proof: See Appendix D.

3.1.3 Implication

Since A (define in equation (3.7)) is a unitary transformation matrix, the error term in regression equation (3.6) is still spherical. Thus (3.6) can be considered as the real-valued frequency-domain representation of (3.1) and (3.2). The n "observations" of equation (3.6) now clearly correspond to the frequencies, as shown in Table 5[11].

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[10]These row manipulations are analogous to the manipulations used to "derive" the Prais-Winsten transformation to correct a regression equation for AR(1) serial correlation. {E.g., see Kmenta (1986, Pg 304,610)}. Here, of course, the objective is to bring together complex exponential terms to form real terms involving cosines and sines.

[11]Table 5 is given for even n; if n is odd, then "observation" number n - 1 and n correspond to frequency $2\pi * \{(n/2) - 1\} / n$.

Table 5
Real-Valued Observations in the Frequency Domain

Frequencies Corresponding to Each "Observation" in Equation (3.6)	
"Observation" Number	Frequency
1	0
2	$2\pi\{1 / n\}$
3	$2\pi\{1 / n\}$
4	$2\pi\{2 / n\}$
5	$2\pi\{2 / n\}$
6	$2\pi\{3 / n\}$
7	$2\pi\{3 / n\}$
. . .	
n - 2	$2\pi\{(n / 2) - 1\} / n$
n - 1	$2\pi\{(n / 2) - 1\} / n$
n	π

As a result, standard econometric methods can be sensibly applied to equation (3.6) to quantify, visualize and statistically test for variation in the coefficient vector, β, across frequencies.

3.2 Detecting Low-Frequency Versus High-Frequency Relationships Between Economic Time Series Via Regression Parameter Stability Tests

Having shown that a complex-valued regression model in the frequency domain can be conveniently transformed into a real-valued frequency-domain model, which is just an ordinary regression model, we can now apply standard regression parameter instability tests to it. This analysis can determine whether an individual parameter in the model is significantly unstable across frequencies. If it does, then we can conclude that the short-term relationship between the variables under study differs significantly from the long-term relationship in strength and/or sign. On the other hand, if it does not, then we can say that the short-term and the long-term relationships between the variables are invariant in term of strength and/or sign.

3.2.1 Literature Review on Regression Parameter Stability Tests

One of the classic problems in econometrics is testing whether the parameters of a regression model are stable in two or more separate subsamples. In the case of time series data, subsamples generally correspond to different economic environments, for example, different policy regimes. These econometric techniques are typically referred to as tests for structural change. Of course, these tests are also applicable to cross-section data, where the subsamples might be different groups of observations such as men and women, large firms and small firms, or industrialize countries and developing countries.

Various methods had been proposed to test for time-varying regression parameters.

One of the best-known of these tests was given by Chow (1960), which is commonly referred to as the Chow test. However, this test only considers two subperiods, and this is a limitation that can lead to a low power detection of the instability if the coefficients vary stochastically in every period. Cooley and Prescott (1973,1976) introduced the varying parameter regression model, in which the k dimensional coefficient vector β_t evolves in time according to a random walk:

$$\begin{aligned} \beta_t &= \beta_{t-1} + u_t, \qquad t = 1, \ldots, n, \\ u_t &\sim N(0, \sigma^2 P), \end{aligned} \tag{3.23}$$

where u_t is independent white noise series and P is an exogenously given (k x k) matrix. Since u_t has zero mean, the parameters do not change on average, but their variance grows linearly with time. Rosenberg (1973) proposed an alternative model, in which the time evolution of the coefficient vector is given by

$$\beta_t = (1 - \lambda)\beta^* + \lambda\beta_{t-1} + u_t, \qquad t = 1, \ldots, n, \tag{3.24}$$

where $0 < \lambda < 1$. In this model, β_t tends to converge to the fixed parameter vector β^* and ends up varying around β^* with a fixed steady state variance.

Several later diagnostic tests for unstable regression parameters at an unknown point of time have been constructed. These include the FH test provided by Farley, Hinich and McGuire (1975), the Cusum test and the Cusum of Square test provided by Brown, Durbin and Evans (1975), the VPR test provided by Garbade (1977), the LM test provided by

LaMotte and McWhorter (1978), and the Stabilogram provided by Ashley (1984).

I. The Farley-Hinich (FH)Test

This is a statistical test provided by Farley, Hinich and McGuire (1975) to test for variation in the regression coefficients across different time periods. The model they investigate is of the form

$$y_t = \beta_0 + (\beta_1 + \delta z_t) x_t + \varepsilon_t, \quad \text{with} \quad \varepsilon_t \sim \mathrm{NIID}(0, \sigma^2), \tag{3.25}$$

where z_t takes on values $\frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \ldots, 1$. This equation is estimated using OLS and the significance of δ is then tested by using ordinary methods.

II. The Brown, Durbin and Evans Tests

Brown, Durbin and Evans (1975) proposed two tests, the Cusum test and the Cusum Square test; both relied on a series of recursive residuals. These residuals are the scaled one-step-ahead post-sample forecasting errors generated by running repeated OLS regression, where at each repetition the sample size is increased by one period.

Consider the standard time domain regression model:

$$Y = X\beta + \varepsilon, \tag{3.26}$$

where $Y = [y_1, y_2, \ldots, y_n]^T$ is a $(n \times 1)$ vector, $X = [x_1^T, x_2^T, \ldots, x_n^T]^T$ is a $(n \times k)$ matrix, $x_i^T = [x_{i1}, x_{i2}, \ldots, x_{ik}]$ is the $(1 \times n)$ row vector of the i observation on k regressors,

$i \in [\,1, n\,]$, and $\varepsilon = [\,\varepsilon_1, \varepsilon_2, ..., \varepsilon_n\,]^{\mathrm{T}}$ is the $(n \times 1)$ independent disturbance vector. Let the column vector of parameters be β_r, where the subscript r indicates that it may vary over time; the hypothesis of parameter constancy across different time periods is therefore denoted by

$$\mathrm{H}_0 : \ \beta_1 = \beta_2 = \ \dots \ = \ \beta_n = \beta. \tag{3.27}$$

Applying OLS regression repeatedly on

$$Y_r \ = \ X_r \beta_r \ + \ \varepsilon_r , \qquad r \ = \ k{+}1, k{+}2, ..., n , \tag{3.28}$$

where $Y_r = [\,y_1, y_2, ..., y_r\,]^{\mathrm{T}}$, and $X_r = [\,x_1^{\mathrm{T}}, \, x_2^{\mathrm{T}}, \, ..., \, x_r^{\mathrm{T}}\,]^{\mathrm{T}}$, for $1 < r \leq n$, we can obtain the least-square estimate of β_r based on the first r observations. Under the null, this estimate is defined as

$$\hat{\beta}_r \ = \ (X_r^{\mathrm{T}} X_r)^{-1} X_r^{\mathrm{T}} Y_r , \tag{3.29}$$

where the matrix $(X_r^{\mathrm{T}} X_r)$ is assumed to be non-singular. The BDE recursive residual is computed as

$$w_r \ = \ \frac{y_r \ - \ x_r^{\mathrm{T}} \hat{\beta}_{r-1}}{\sqrt{[1 \ + \ x_r^{\mathrm{T}} (X_{r-1}^{\mathrm{T}} X_{r-1})^{-1} x_r]}} \ , \qquad r \ = \ k+1, \ k+2, \ ..., \ n, \tag{3.30}$$

and under the null hypothesis, they form a serially independent Gaussian process with mean zero and variance σ^2.

In the Cusum test, the test statistics is the cumulative sum series, defined as

$$W_r = \frac{1}{\hat{\sigma}} \sum_{i=k+1}^{r} w_i, \qquad r = k+1,\ k+2, \ldots,\ n, \tag{3.31}$$

where

$$\hat{\sigma}^2 = \frac{(Y - X\hat{\beta})^{\mathrm{T}}(Y - X\hat{\beta})}{n - k}. \tag{3.32}$$

Under the null of stability, W_r is approximately normally distributed with mean zero and variance $(n - k)$. The null hypothesis is rejected if

$$|W_r| > \left[a(n - k)^{\frac{1}{2}} + 2a(r - k)(n - k)^{-\frac{1}{2}}\right], \quad \text{for any } r \in [k+1, n]. \tag{3.33}$$

The scalar a is chosen to acquire the desired significance level α ; Brown, Durbin, and Evans suggested that

$$a = \begin{cases} 1.143, & \alpha = 0.01 \\ 0.948, & \alpha = 0.05 \\ 0.850, & \alpha = 0.10\,. \end{cases} \tag{3.34}$$

The test statistic in the "Cusum of Square" test is the cumulative sum of squares series, defined as

$$S_r = \frac{\sum_{i=k+1}^{r} w_i^2}{\sum_{i=k+1}^{n} w_i^2}, \qquad r = k+1,\ k+2,\ \ldots,\ n, \tag{3.35}$$

where S_r is a monotonically increasing sequence of positive real numbers with $S_n = 1$. Under the null hypothesis, S_r has a beta distribution with mean (r-k)/(n-k). Brown, Durbin, and Evans constructed a confidence interval for S_r as

$$\frac{(r - k)}{(n - k)} \pm c_0 , \tag{3.36}$$

where c_0 can be chosen from Table 1 of Durbin (1969). if

$$\left| S_r - \frac{(r - k)}{(n - k)} \right| > c_0, \qquad \text{for any} \quad r \in [k+1, n], \tag{3.37}$$

then the null hypothesis is rejected.

III. Stabilogram

This is a simple two-step test developed by Ashley (1984) to test for instability of regression parameters over different time periods. Consider the usual regression model of n observations:

$$Y = X\beta + \varepsilon , \tag{3.38}$$

where $\varepsilon \sim \text{NIID}\ (0, \sigma^2 \text{I})$. Without loss of generality, we suppose that the regressor under study is the k-th regressor. The first step of the test is to partition the sample period into r approximately equal subperiods of $m = \frac{n}{r}$ observations each, and then set up the r dummy variables $D^{(1)}$, $D^{(2)}$, ..., $D^{(r)}$, in such a way that $D^{(i)}$ is an n x 1 vector that consists of only the i-th subperiod observations of the k regressor , for $i = 1, 2, ..., r$. For example:

$$D^{(1)} = \begin{bmatrix} x_{1,k} \\ x_{2,k} \\ \cdot \\ \cdot \\ \cdot \\ x_{m,k} \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}, \quad D^{(2)} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ x_{m+1,k} \\ x_{m+2,k} \\ \cdot \\ \cdot \\ \cdot \\ x_{2m,k} \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}, \quad \ldots, \text{and } D^{(i)} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ x_{(i-1)m+1,k} \\ x_{(i-1)m+2),k} \\ \cdot \\ \cdot \\ \cdot \\ x_{im,k} \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}. \tag{3.39}$$

Define X_{k-1} as an n x (k-1) matrix which is equivalent to the X matrix in equation (3.38) excluding the k-th column, β_{k-1} as a k-1 vector which is equivalent to the vector β in equation (3.38) excluding the k-th element, $X_r = [D^{(1)}, D^{(2)},..., D^{(r)}]$ as an n x r matrix consists of the dummy variables, and $\delta = [\delta_1, \delta_2, \ldots, \delta_r]$ as the coefficient vector of X_r. Equation (3.38) is rewritten as

$$Y = X_{k-1}\beta_{k-1} + X_r\delta + \varepsilon. \tag{3.40}$$

The null hypothesis of stable coefficient corresponds to the r - 1 linear restrictions is

$$H_0 : \delta_1 = \delta_2 = \ldots = \delta_r . \tag{3.41}$$

The test statistic is

$$STAB = \frac{(RSS - URSS) \, / \, (r - 1)}{URSS \, / \, (n - k - r + 1)} , \tag{3.42}$$

which is distributed F(r-1, n-k-r+1) under the null. The RSS is the sum of squared residuals from equation (3.38), and $URSS$ is the sum of squared residuals from equation (3.40). Besides the F-test, this approach also proposed the "stabilogram," which is a plotting of confidence intervals of the estimated $\hat{\delta}_i$ versus the i subperiod , for i = 1, 2, ..., r. The instability of the parameter over different subperiod can also be observed from the "stabilogram."

3.2.2 Regression Parameter Stability Tests in the Frequency Domain

This is the second step of the BWFDR approach proposed by this study; the regression parameter stability tests discussed above are applied to the real-valued frequency-domain model given in equation (3.6), to quantify the strength and direction of the relationships across frequencies. By equating low frequency to long term and high frequency to short term, the characteristics of the short-term and the long-term relationships between the time series can be determined.

The conclusions that we can draw from the results of these tests are

(i) if the null hypothesis is rejected, the regression parameter of interest is unstable across frequencies; this means that there is a distinctive difference between the short-term and the long-term relationships in terms of strength and/or sign;

(ii) if the null hypothesis is not rejected then the relationship between the variables of interest is stable across frequencies; this means that the relationship is invariant in the short-term and in the long-term; it is stable across frequencies.

3.3 Back-Transforming a Frequency-Domain Regression Model to a Time-Domain Regression Model for Forecasting

The real-valued regression model in the frequency domain, given by equation (3.6), can be easily back transformed to a time-domain regression model by multiplying the equation with the transposed A matrix:

$$A^{\mathrm{T}}Y^{*} = A^{\mathrm{T}}X^{*}\beta + A^{\mathrm{T}}\varepsilon^{*} \tag{3.43}$$

or

$$A^{\mathrm{T}}AY = A^{\mathrm{T}}AX\beta + A^{\mathrm{T}}A\varepsilon. \tag{3.44}$$

Since A is an orthogonal matrix (so that $A^{\mathrm{T}}A = \mathrm{I}$), equation (3.44) is in fact identical to the model in the time domain in (3.1).

This back transforming process is important for the forecasting of time series. If the relationship between the time series in the model of interest is unstable across different frequency bands as indicated by the result of the BWFDR test described above, then the forecasting of y_t using the fixed-parameter model in the time domain can be improved by back transforming the varying- parameter model (in the frequency domain) into the time domain by multiplying it by the matrix A^{T}. The resulting time domain regression equation will have stable coefficients; it can be estimated and forecasted using ordinary methods.

More explicitly, consider the real-valued varying-parameter model in the frequency domain:

$$Y^* = X^{D*}\beta^D + \varepsilon^*, \tag{3.45}$$

where

$$Y^* = \left[y(0),\ \tilde{y}(1),\ \tilde{\tilde{y}}(1),\ \tilde{y}(2),\ \tilde{\tilde{y}}(2),\ \ldots \tilde{y}\left(\tfrac{n}{2}-1\right),\ \tilde{\tilde{y}}\left(\tfrac{n}{2}-1\right),\ y\left(\tfrac{n}{2}\right)\right]^{\mathrm{T}}, \tag{3.46}$$

$$X^{D*} = \left[x_1^*,\ x_2^*,\ \ldots,\ x_{k-1}^*,\ D_{(1)}^*,\ D_{(2)}^*,\ \ldots,\ D_{(r)}^*\right], \tag{3.47}$$

$$x_i^* = \left[x_i(0),\ \tilde{x}_i(1),\ \tilde{\tilde{x}}_i(1),\ \tilde{x}_i(2),\ \tilde{\tilde{x}}_i(2),\ \ldots \tilde{x}_i\left(\tfrac{n}{2}-1\right),\ \tilde{\tilde{x}}_i\left(\tfrac{n}{2}-1\right),\ x_i\left(\tfrac{n}{2}\right)\right]^{\mathrm{T}}, \tag{3.48}$$

and the dummy variables

$$D_{(1)}^* = \left[x_k(0),\ \tilde{x}_k(1),\ \tilde{\tilde{x}}_k(1),\ \ldots \tilde{x}_k\left(\tfrac{m}{2}\right),\ \tilde{\tilde{x}}_k\left(\tfrac{m}{2}\right),\ 0,\ 0,\ \ldots,\ 0\right]^{\mathrm{T}}, \tag{3.49}$$

$$\begin{aligned} D_{(2)}^* = \Big[&0,\ 0,\ \ldots,\ 0,\ \tilde{x}_k\left(\tfrac{m}{2}+1\right),\ \tilde{\tilde{x}}_k\left(\tfrac{m}{2}+1\right), \\ &\ldots,\ \tilde{x}_k\left(\tfrac{2m}{2}\right),\ \tilde{\tilde{x}}_k\left(\tfrac{2m}{2}\right),\ 0,\ 0,\ \ldots,\ 0\Big]^{\mathrm{T}}, \end{aligned} \tag{3.50}$$

$$D^*_{(i)} = \Big[0,\ 0,\ \ldots,\ 0,\ \tilde{x}_k\Big(\frac{(i-1)m}{2}+1\Big),\ \tilde{\tilde{x}}_k\Big(\frac{(i-1)m}{2}+1\Big),$$
$$\ldots,\ \tilde{x}_k\Big(\frac{im}{2}\Big),\ \tilde{\tilde{x}}_k\Big(\frac{im}{2}\Big),\ 0,\ 0,\ \ldots,\ 0\Big]^{\mathrm{T}}, \qquad (3.51)$$

$$D^*_{(r)} = \Big[0,\ 0,\ \ldots,\ 0,\ \tilde{x}_k\Big(\frac{(r-1)m}{2}+1\Big),\ \tilde{\tilde{x}}_k\Big(\frac{(r-1)m}{2}+1\Big),$$
$$\ldots,\tilde{x}_k\Big(\frac{rm}{2}\Big),\ \tilde{\tilde{x}}_k\Big(\frac{rm}{2}\Big),\ x_k\Big(\frac{n}{2}\Big)\Big]^{\mathrm{T}}. \qquad (3.52)$$

The ($\sim$) denotes a cosine-transformed "observation," and the ($\approx$) denotes a sine-transformed "observation", as discussed in Appendix D. In this case, the regressor of interest, the k-th regressor, is replaced by r dummy variables of $D^*_{(1)}, D^*_{(2)}, \ldots, D^*_{(r)}$, each of which consists of n observations[12]. As indicated by equation (3.49)- (3.52), each of the dummy variables consists of an equal number of non-zero observations, that is, $\left(\frac{n-2}{r}\right)$, except $D^*_{(1)}$ and $D^*_{(r)}$. These two dummy variables consist of $\left(\frac{n-2}{r}+1\right)$ non-zero observations because $D^*_{(1)}$ includes the observation corresponding to the zero frequency, and $D^*_{(r)}$, on the other hand, includes the observation corresponding to the folding frequency[13].

The real-valued regression model in the frequency domain (3.45) is then back transformed to a time domain model by multiplying it with A^{T}:

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[12]This real-valued-varying-parameter model in the frequency domain is constructed based on the varying-parameter model in the time domain given by Ashley (1984).

[13]In this case, the number of observations in the sample data set, n, is even; if n is odd, there is no observation corresponding to the folding frequency. If $(n\text{-}2)/r$ is not an integer, the "subperiods" are made as equal in length as possible, subject to the restriction that the cosine and sine terms for frequency $2\pi m/n$ are always kept together in the same subperiod.

$$A^{\mathrm{T}} Y^{*} \;=\; A^{\mathrm{T}} X^{D*} \beta^{D} \;+\; A^{\mathrm{T}} \varepsilon^{*} \tag{3.53}$$

or

$$Y \;=\; X^{D} \beta^{D} \;+\; \varepsilon . \tag{3.54}$$

Let *msfe* be the mean squared forecast error computed from the one-step-ahead forecast errors of the fixed-parameter model in (3.1), and $msfe^{*}$ be the mean squared forecast error computed from the one-step-ahead forecast errors of the back transformed varying-parameter model in (3.54):

$$msfe \;=\; \frac{1}{n-1} \sum_{t=1}^{n-1} (fe_{t+1})^{2}, \qquad \text{where } fe_{t+1} \;=\; y_{t+1} \;-\; \hat{\beta}_{t} x_{t+1}, \tag{3.55}$$

$$msfe^{*} \;=\; \frac{1}{n-1} \sum_{t=1}^{n-1} (fe_{t+1}^{*})^{2}, \qquad \text{where } fe_{t+1}^{*} \;=\; y_{t+1} \;-\; \hat{\beta}_{t}^{D} x_{t+1}, \tag{3.56}$$

we conjecture that

$$\frac{msfe^{*}}{msfe} \;<\; 1, \tag{3.57}$$

that is, the correctly specified varying-parameter model should forecast better than the misspecified fixed-parameter model.

CHAPTER 4

AN EMPIRICAL EXAMPLE USING MACROECONOMIC DATA

4.1 The Jaditz-Sayers Paradox

Jaditz and Sayers (1994) re-examine the relationship between the consumer price index (CPI) and the producer price index (PPI) to investigate the mechanism of how inflation is transmitted through the economy; they explored the extent to which the consumer price index can be predicted from its own history and on its relationship with the producer price index and other financial variables. They used the criterion of post-sample forecast performance to assess the degree to which the growth rate of PPI Granger-causes fluctuations in the growth rate of CPI.

Their finding, however, is paradoxical: while in-sample hypothesis tests strongly reject the hypothesis that the CPI and PPI are unrelated (as do similar tests reported in the literature), including the history of PPI does not seem to improve the prediction of the changes of CPI. How is it, they ask, that if two variables are so closely related in-sample, models estimated based on this relationship perform so poorly in post-sample forecasting?

A variety of studies have investigated the relationship between the CPI and PPI. In general, the results obtained by these studies are consistent with the in-sample inference of Jaditz and Sayers; they have reported that the fluctuations in producer prices precede fluctuations in consumer prices. The theoretical reasoning is very straight forward: when input costs increase, the price of the finished good several months later will have to be raised. This is referred to as cost-push inflation by Phelps (1961).

This theory has extensive empirical support. Engle (1978), for example, examines the relationship between the wholesale price of raw food products, the wage rate in the food industry, and the food component of the consumer price index for sixteen cities, and finds that there is a lag between the producer price index and the consumer price index. Silver and Wallace (1980), and Guthrie (1981) also present evidence for a unidirectional causal relationship from changes in producer prices to future changes in consumer prices; Colclough and Lange (1982), and Cushing and McGarvey (1990) report evidence for bidirectional causality between changes in producer prices and changes in consumer prices.

Jaditz and Sayers use monthly data of 160 observations for the period of March, 1981 to June, 1994 to examine the one-step-ahead forecasts of the growth rate of CPI and the growth rate of PPI [14]. They specify and estimate the following two models for CPI using 60 in-sample observations :

$$\underset{}{CPI_t} = \underset{(5.31)}{0.0016} + \underset{(5.34)}{0.4614}\, CPI_{t-1} + \varepsilon_t , \qquad R^2 = 0.218 ; \tag{4.1}$$

$$CPI_t = \underset{(6.32)}{0.002} + \underset{(2.62)}{0.272}\, CPI_{t-1} + \underset{(2.32)}{0.104}\, PPI_{t-1} + \epsilon_t , \qquad R^2 = 0.250. \tag{4.2}$$

The higher R^2 for the VAR model provides in-sample evidence that lagged PPI granger-

[14]This time period is selected to avoid the potential empirical complication attributable to the monetary policy regime shift undertaken at the Federal Reserve Bank in the early 1980's.

causes CPI fluctuations. However, the post-sample one-step-ahead MSFE over the remaining 100 periods (updating the coefficient estimates as each new observation is added to the sample) is actually 2% higher for the VAR model than for the AR(1) model.

4.2 Data and Methodology

This empirical study is performed with the purpose of resolving the Jaditz-Sayers paradox, so the same data set is used. The series are

(1) the Consumer Price Index (CPI): " U.S. City Average, All Items, All Urban Consumers" Index. Source: Bureau of Labor Statistics,

(2) the Producer Price Index (PPI): "All Commodities" Producer Price Index. Source: Bureau of Labor Statistics,

for the time period of March 1981 to June, 1994. These series are transformed into the growth rates by taking log first differences. The time plots of the annualized growth rates of the CPI and of the PPI are shown in Figure 2 and Figure 3. Looking at the plot of CPI growth rates, it seems likely that the first 16 observations (March, 1981 - June, 1982) and the last 36 observations (July, 1991 - June, 1994) of CPI are generated from a different system, so these observations are dropped. The rest of the data set is then divided into a 72 observation initial sample (July, 1982 - June, 1988) and a 36 observation post-sample forecasting period (July, 1988 - June 1991).

The relationship between these two price indices is re-examined using the BWFDR approach proposed by this dissertation. The regression model in the time domain:

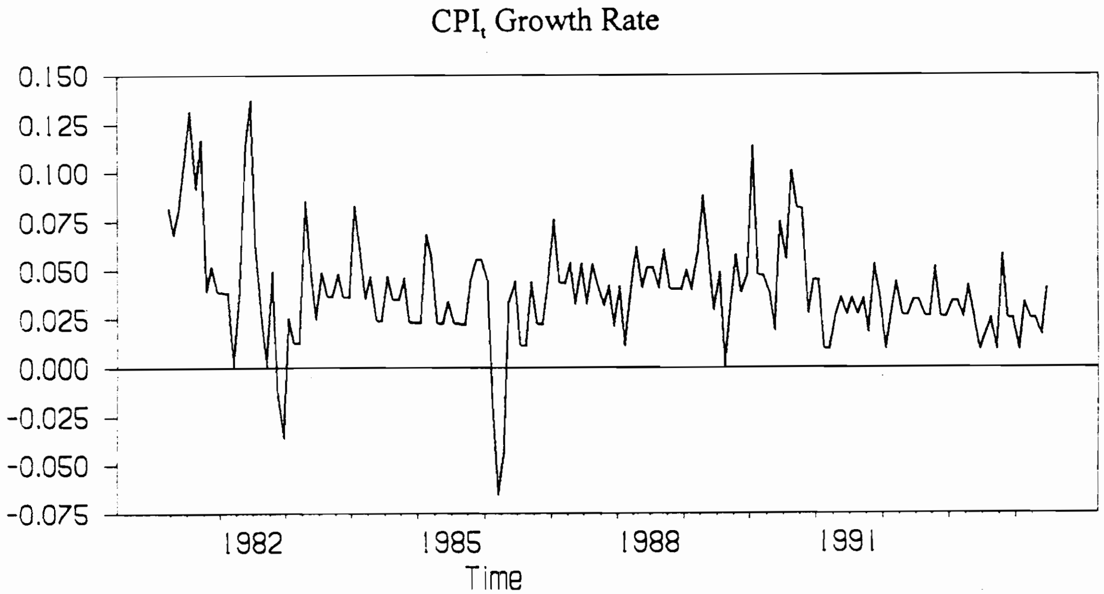

Figure 2
Time Plot of CPI$_t$ Growth Rate

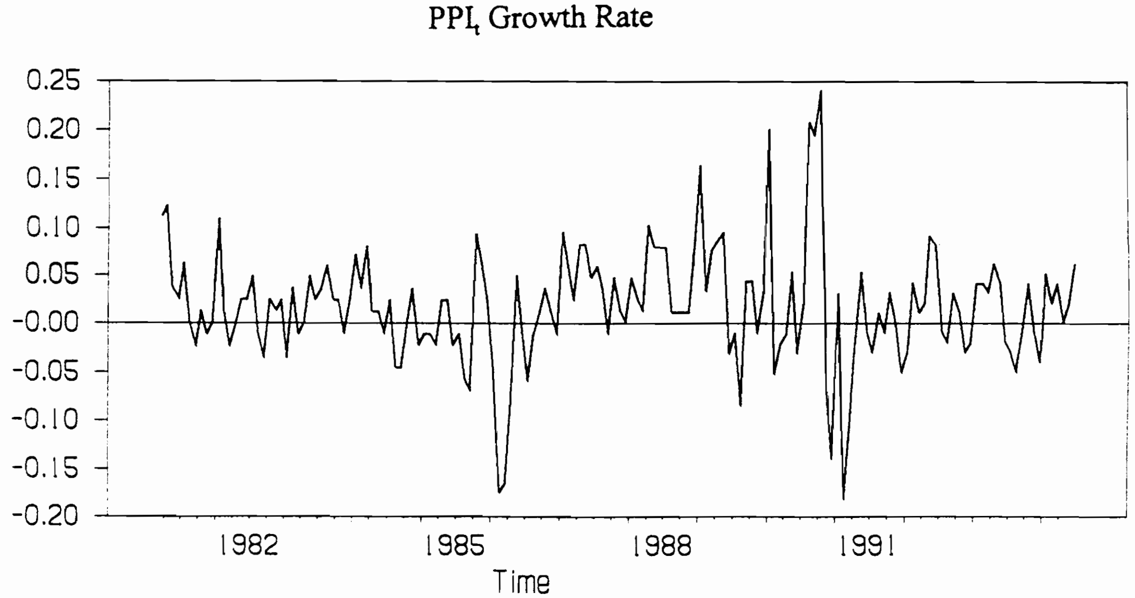

Figure 3
Time Plot of PPI$_t$ Growth Rate

$$Y = X\beta + \varepsilon, \tag{4.3}$$

where $Y = \begin{bmatrix} \mathrm{CPI}_t \\ \mathrm{CPI}_{t+1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{CPI}_{t+n-1} \end{bmatrix}$ is a $(n \times 1)$ vector, $X = \begin{bmatrix} 1 & \mathrm{CPI}_{t-1} & \mathrm{PPI}_{t-1} \\ 1 & \mathrm{CPI}_t & \mathrm{PPI}_t \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 1 & \mathrm{CPI}_{t+n-2} & \mathrm{PPI}_{t+n-2} \end{bmatrix}$ is a

$(n \times 3)$ matrix, β is a (3×1) parameter vector, and ε is a $(n \times 1)$ disturbance vector, is transformed into a real-valued frequency-domain regression model by premultiplying it with the unitary transformation matrix A given in subsection (3.1.1):

$$AY = AX\beta + A\varepsilon, \tag{4.4}$$

or

$$Y^* = X^*\beta + \varepsilon^*. \tag{4.5}$$

The "observations" of this model are now correspond to the frequencies j, $j \in \left[0, \frac{1}{2}\right]$, as the example demonstrated in Table 5 in subsection (3.1.3).

4.3 Empirical Results

Plots of $\left\{\mathrm{PPI}_{t-1} - \overline{\mathrm{PPI}}\right\}$ in the time domain and transformed $\left\{\mathrm{PPI}_{t-1} - \overline{\mathrm{PPI}}\right\}$ in the frequency domain using this 72 observation sample period (July, 1982 - June, 1988) are given in Figure 4 and Figure 5; similar plots are given for CPI_t in Figure 6 and Figure 7.

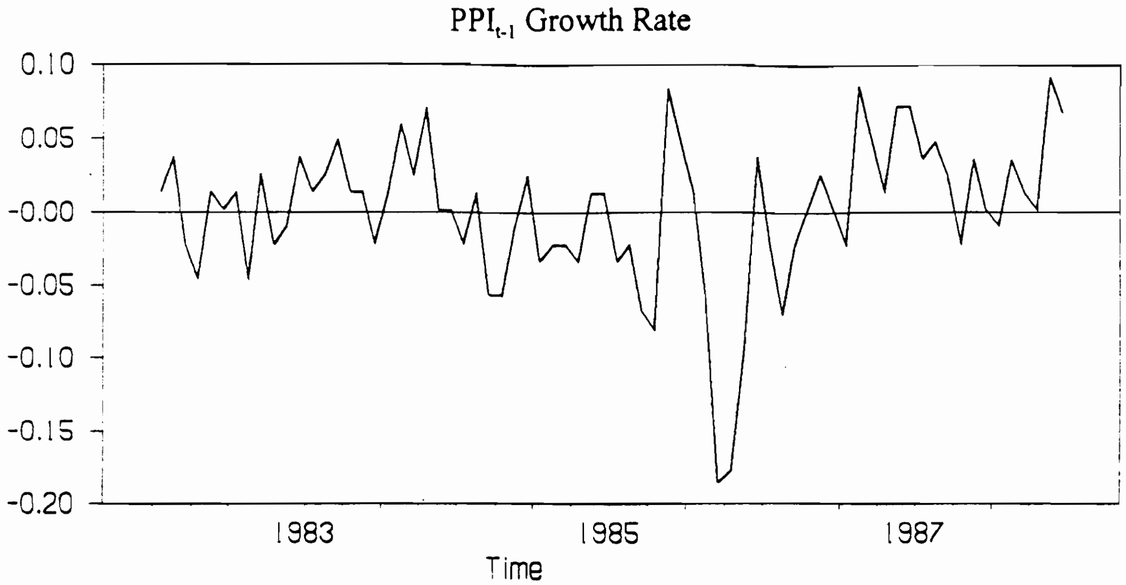

Figure 4
PPI$_{t-1}$ Growth Rate in the Time Domain

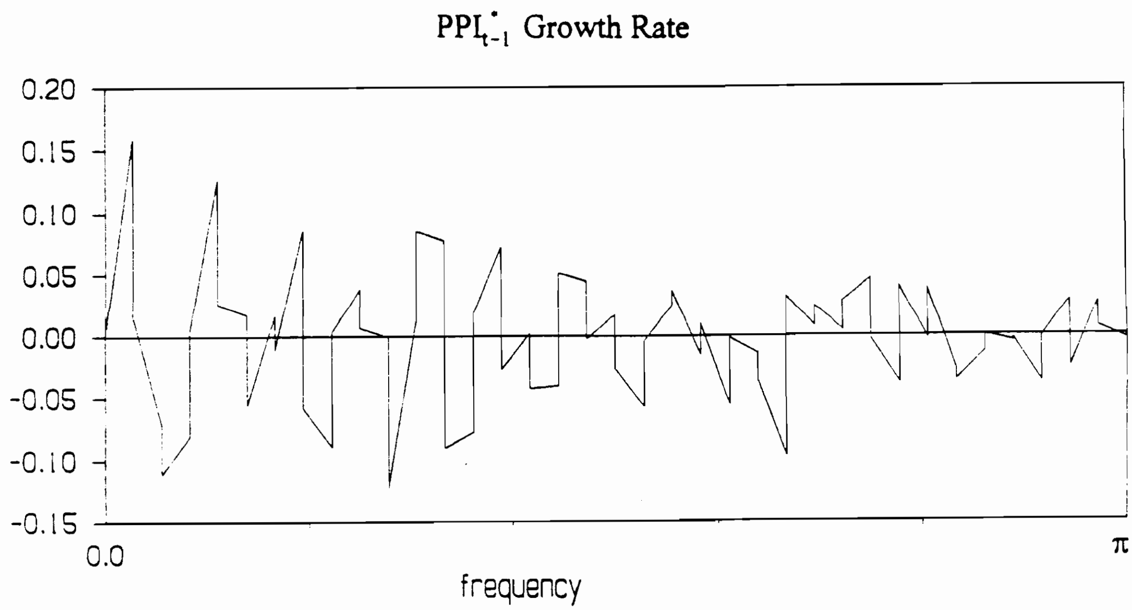

Figure 5
PPI$^{*}_{t-1}$ Growth Rate in the Frequency Domain

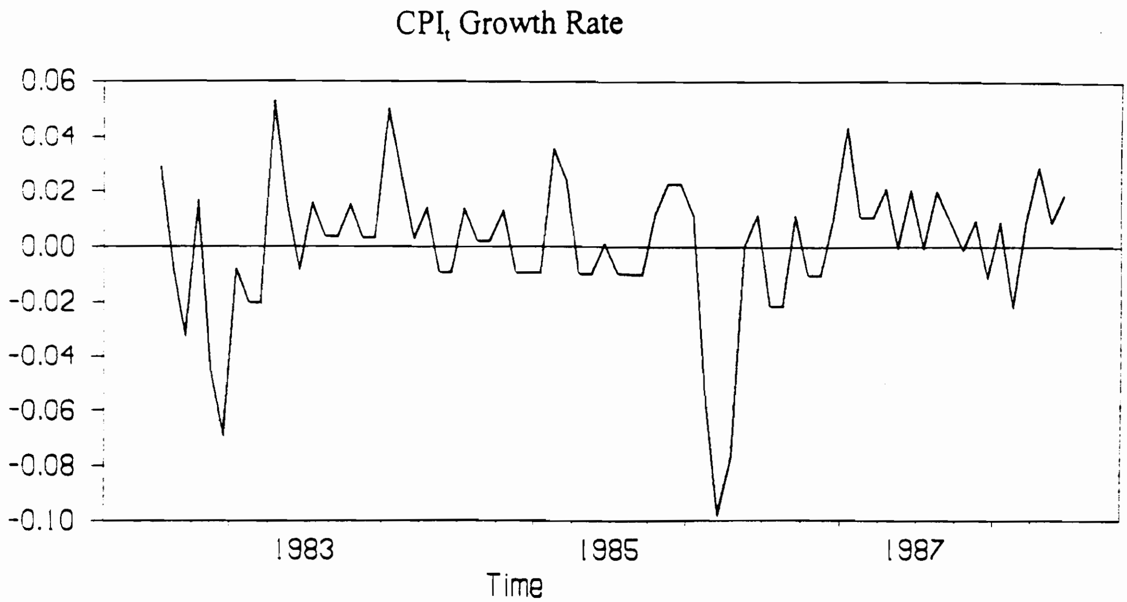

Figure 6
CPI$_t$ Growth Rate in the Time Domain

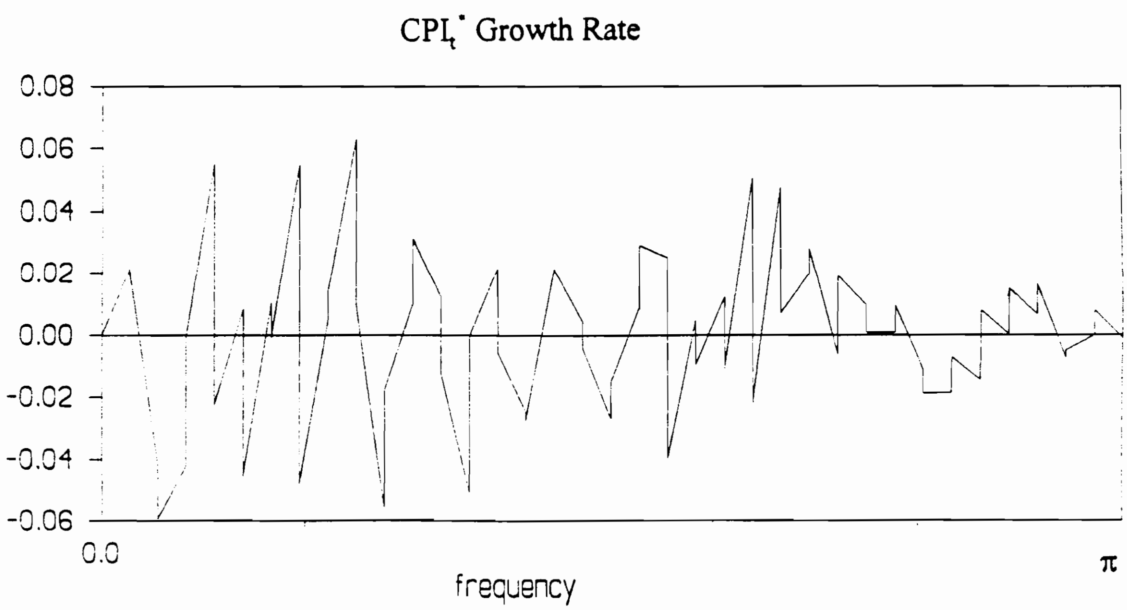

Figure 7
CPI$_t^*$ Growth Rate in the Frequency Domain

None of these plots indicates the presence of any serious non-stationarity problems in either the time domain or frequency domain regressions.

Table 6 gives the results from estimating equation (4.3) using OLS over this sample period; OLS estimation of equation (4.5) gives identical results. Consistent with Jaditz and Sayers' results over somewhat different sample period (and with many other results in the literature), these results indicate a significant positive relationship between the growth rate in the CPI and the lagged growth rate in the PPI; indeed, the coefficient on PPI_{t-1} is significant at the 0.5% level.

Table 6
The Estimated Time Domain Model

ESTIMATION BY LEAST SQUARES - IN THE TIME DOMAIN

THE DEPENDENT VARIABLE = CPIG
THE # OF OBSERVATIONS = 72

VARIABLE	COEFFICIENT	T-RATIO	STD ERROR	P-LEVEL
CONSTANT	0.001981	5.476832	0.000362	0.000001
CPIGLAG1	0.208007	1.871071	0.111170	0.065576
PPIGLAG1	0.180933	2.914003	0.062091	0.004806

4.3.1 Varying Parameter Test Results in the Time Domain

Farley-Hinich Test

Table 7 gives the results of a Farley-Hinich test applied to the coefficient on lagged PPI in this regression. The "trend" variable corresponds to z_t in equation (3.25) above. The coefficient on this regressor is not significant (its estimated t-ratio is only 0.99), so this test provides no evidence of parameter instability in the time domain regression.

Table 7
FH Test in the Time Domain

THE DEPENDENT VARIABLE = CPIG
THE # OF OBSERVATIONS = 72

VARIABLE	COEFFICIENT	T-RATIO	STD ERROR	P-LEVEL
CONSTANT	0.001962	5.416770	0.000362	0.000001
CPIGLAG1	0.213370	1.916821	0.111315	0.059464
PPIGLAG1	0.043824	0.289125	0.151574	0.773365
TREND	0.218679	0.991604	0.220531	0.324905

BDE (Cusum Square) Test

Figure 8 shows that the plot of Cusum Square statistic Sr lies within the range of $\left(\frac{r-k}{n-k} - C_0(\tilde{\alpha}, \tilde{n}), \frac{r-k}{n-k} + C_0(\tilde{\alpha}, \tilde{n}) \right)$, or $\left| Sr - \frac{(r-k)}{(n-k)} \right| \ngtr C_0(\tilde{\alpha}, \tilde{n})$, for $r = k+1, k+2, \ldots, n$; where $\tilde{\alpha} = \frac{\alpha}{2} = 0.005$, $\tilde{n} = \frac{1}{2}(n - k) - 1 = 34$, and $C_0 = 0.253$; which implies that the null hypothesis of parameter stability is not rejected at the 1% significant level. This result is consistent with that of the Farley-Hinich test, that is, the relationship between CPI and PPI is stable in the time domain.

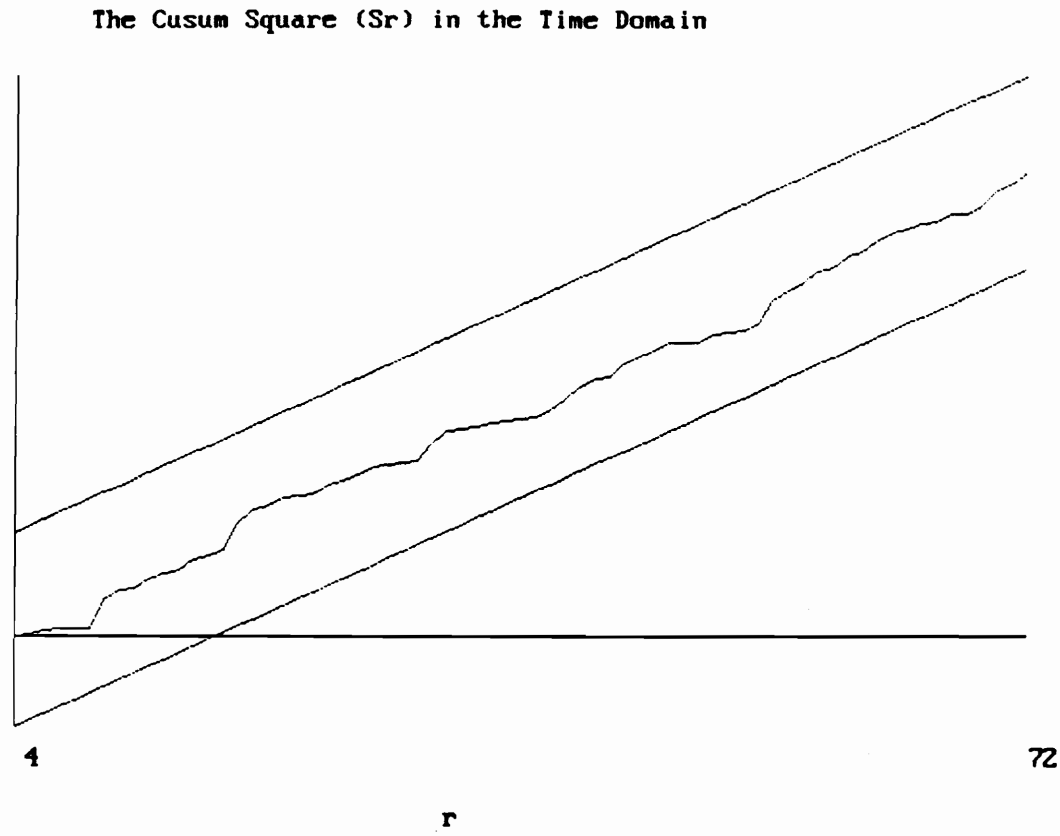

Figure 8
Cusum Square Test (5% Level)
in the Time Domain

Stabilogram Test

Partitioning the 72 sample observations into 3 subperiods and estimating the stabilogram regression, we see that the coefficient on lagged PPI, in Table 8, does appear to be larger in the middle subperiod, but the F test shows that this apparent parameter variation is not significant at the 5% level.

Table 8
Stabilogram Test in the Time Domain

THE DEPENDENT VARIABLE = CPIG
THE # OF SAMPLE BLOCKS = 3
THE # OF OBSERVATIONS = 72

VARIABLE	COEFF	T-RATIO	STD ERR	P-LEVEL	BLOCK LENGTH	OBS INTERVAL
CONSTANT	0.002308	6.038087	0.000382	0.000000	--	--
CPIGLAG1	0.192424	1.761618	0.109232	0.082696	--	--
PPIGLAG1	-0.020682	-0.150171	0.137724	0.881082	25	1 - 25
	0.295741	3.720603	0.079487	0.000409	24	26 - 49
	0.065241	0.660650	0.098753	0.511103	23	50 - 72

THE STAB STATISTIC IS F(2, 67) = 2.701259 WITH P LEVEL 0.07443163

As did Jaditz and Sayers, we conclude that the relationship between CPI and lagged PPI is significant, positive and (apparently) reasonably stable in the time domain.

4.3.2 Varying Parameter Test Results in the Frequency Domain

The same tests are repeated for the regression model in the frequency domain (4.3), to examine the "low-frequency" versus the "high-frequency" relationships between CPI and lagged PPI.

Farley-Hinich Test

Note that the estimated coefficient on the "trend" variable (analogous to the "δ" coefficient on z_t in equation (3.25)) has an estimated t-ratio of -2.74 now that the regression equation has been transformed into the frequency domain. The Farley-Hinich test thus rejects the null hypothesis that the coefficient on transformed PPI_{t-1} is stable across frequencies at the 1% level.

Table 9
FH Test in the Frequency Domain

THE DEPENDENT VARIABLE = CPIG
THE # OF OBSERVATIONS = 72

VARIABLE	COEFFICIENT	T-RATIO	STD ERROR	P-LEVEL
CONSTANT	0.002044	5.897324	0.000347	0.000000
CPIGLAG1	0.125024	1.131462	0.110498	0.261834
PPIGLAG1	0.380335	4.050648	0.093895	0.000133
TREND	-0.585120	-2.740702	0.213493	0.007825

BDE (Cusum Square) Test

Figure 9 - 10 indicate that the null hypothesis of parameter stability is rejected at the 1% significant level. These results also shows that the relationship between CPI and lagged PPI is unstable in the frequency domain.

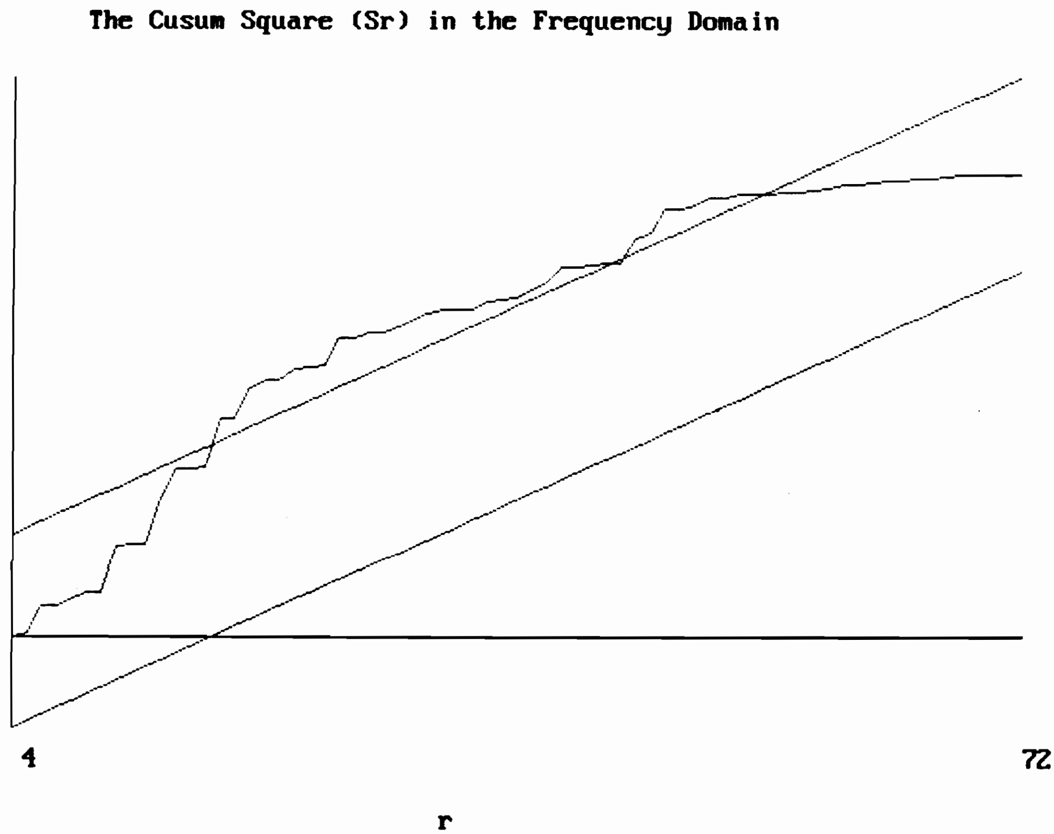

Figure 9
Cusum Square Test (5% Level)
in the Frequency Domain

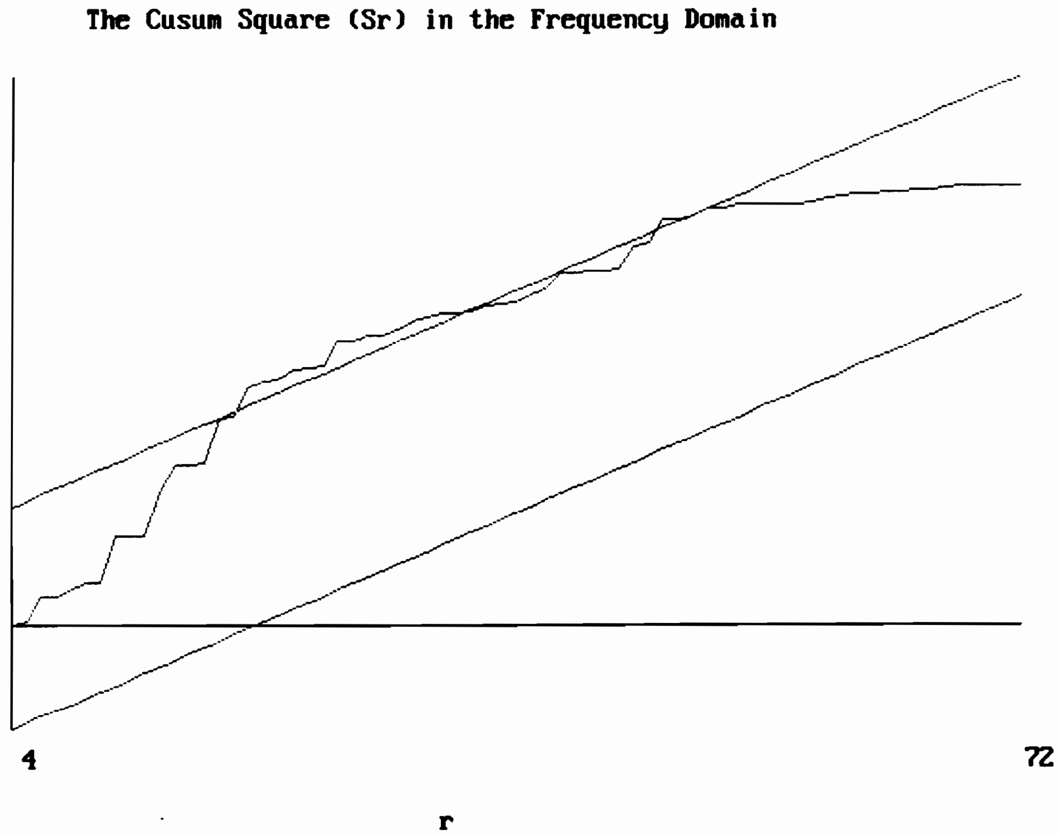

Figure 10
Cusum Square Test (1% Level)
in the Frequency Domain

Stabilogram Test

As with the time-domain testing, we partition the sample "period" into 3 "subperiods." The first subperiod consists of the observation at frequency zero plus twelve pairs of observations corresponding to the cosine and sine transformation at frequencies $\frac{1}{72}, \frac{2}{72}, \ldots, \frac{12}{72}$. The second subperiod consists of twelve pairs of observations corresponding to frequencies $\frac{13}{72}, \frac{14}{72}, \ldots, \frac{24}{72}$, and the final subperiod consists of 11 pairs of observations corresponding to frequencies $\frac{25}{72}, \frac{26}{72}, \ldots, \frac{35}{72}$ plus a single observation at the folding frequency of $\frac{36}{72}$ [15] .

Table 10 summarizes the estimation results obtained by replacing the (transformed) lagged growth rate of PPI by three dummy variables, partitioned into these three "subperiod" as in equation (3.49) - (3.52).

Looking at these results we see that the stabilogram F test allows one to reject the null hypothesis that all three coefficients are the same at the 5% significant level. Instead, the stabilogram results indicate that the relationship is positive and highly significant ($p < .05\%$) at low frequencies but not significant at all at medium or high frequencies.

---

[15] There is an observation at the folding frequency because the sample length is even. There is only one observation at the folding frequency because the Fourier transformation at the folding frequency is real-valued; the sine-transform part is thus precisely zero.

Table 10
Stabilogram Test in the Frequency Domain

THE DEPENDENT VARIABLE = CPIG
THE # OF FREQUENCY BANDS = 3
THE # OF OBSERVATIONS = 72

VARIABLE	COEFF	T-RATIO	STD ERR	P-LEVEL	BAND LENGTH (OBS)	BAND LENGTH (FREQ)	FREQ INTERVAL
CONSTANT	0.002041	5.723174	0.000357	0.000000	--	--	-- --
CPIGLAG1	0.156859	1.388685	0.112955	0.169532	--	--	-- --
PPIGLAG1	0.275214	3.680198	0.074782	0.000466	25	13	.000 - .167
	-0.022507	-0.209077	0.107651	0.835023	24	12	.181 - .333
	0.148688	0.827429	0.179699	0.410934	23	12	.347 - .500

THE STAB STATISTIC IS F(2, 67) = 3.2921583 WITH P LEVEL 0.04264103

Figures 11-14 plot the stabilograms for partitioning the frequency domain observations into 2 through 5 bands. In each case, the middle horizontal line corresponds to the estimated coefficient for this band, bracketed by lines ± 2 estimated standard deviations above and below it.

These plots consistently show that there is a positive, significant relationship between CPI and lagged PPI at the low frequencies, but no significant relationship at the high frequencies. In other words, there is a significant "long-term" relationship between CPI and lagged PPI, but there is no significant "short-term" relationship between these two variables.

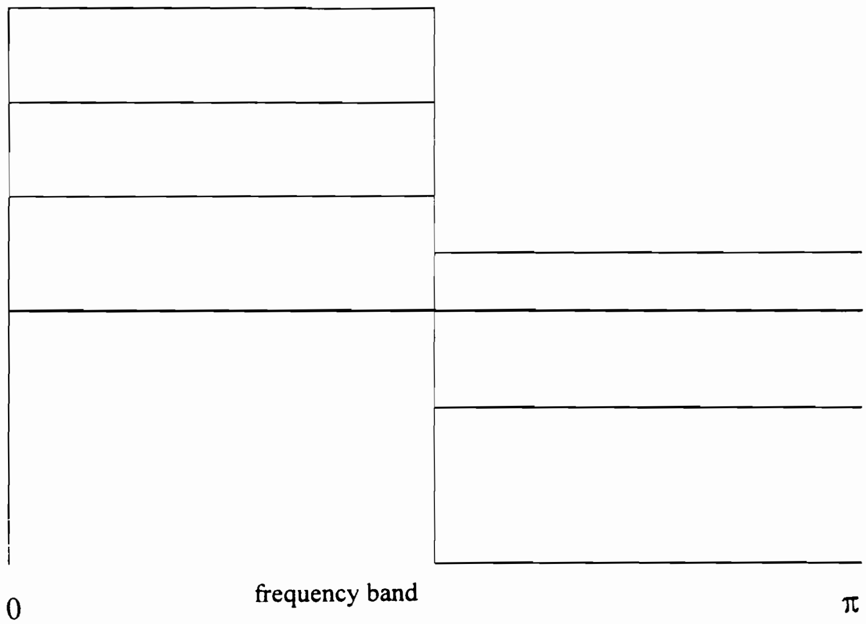

Figure 11
The 2-Band Stabilogram

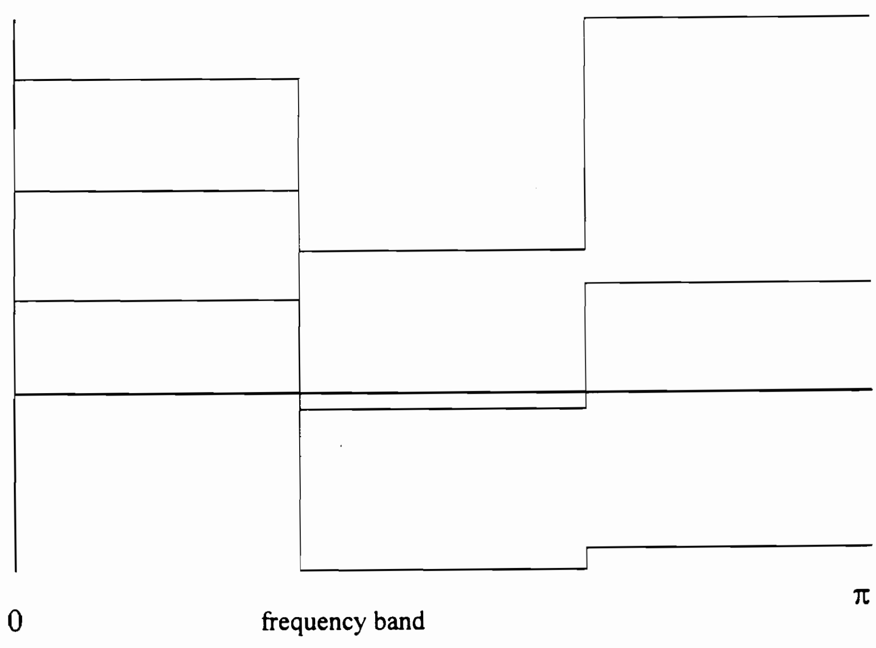

Figure 12
The 3-Band Stabilogram

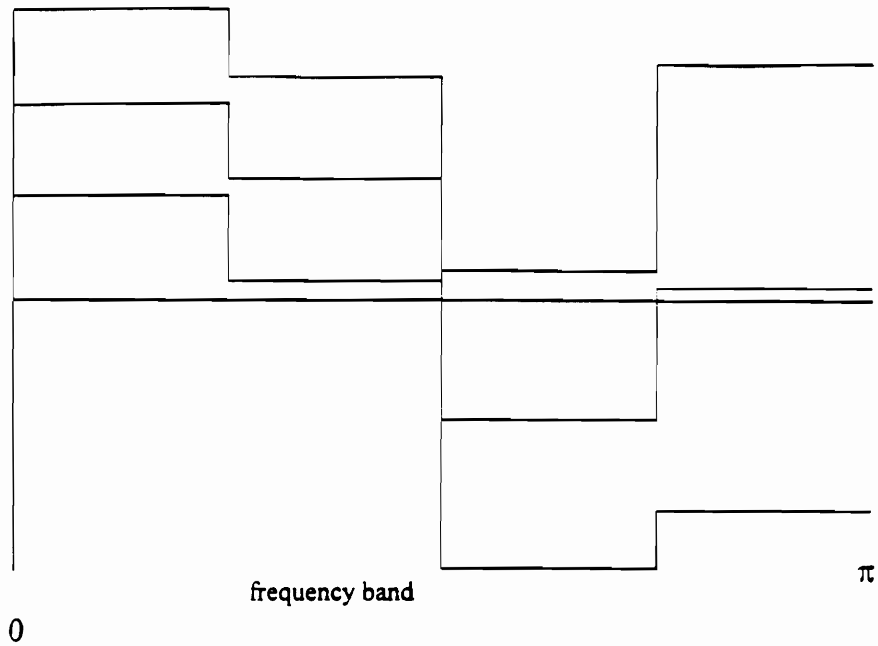

Figure 13
The 4-Band Stabilogram

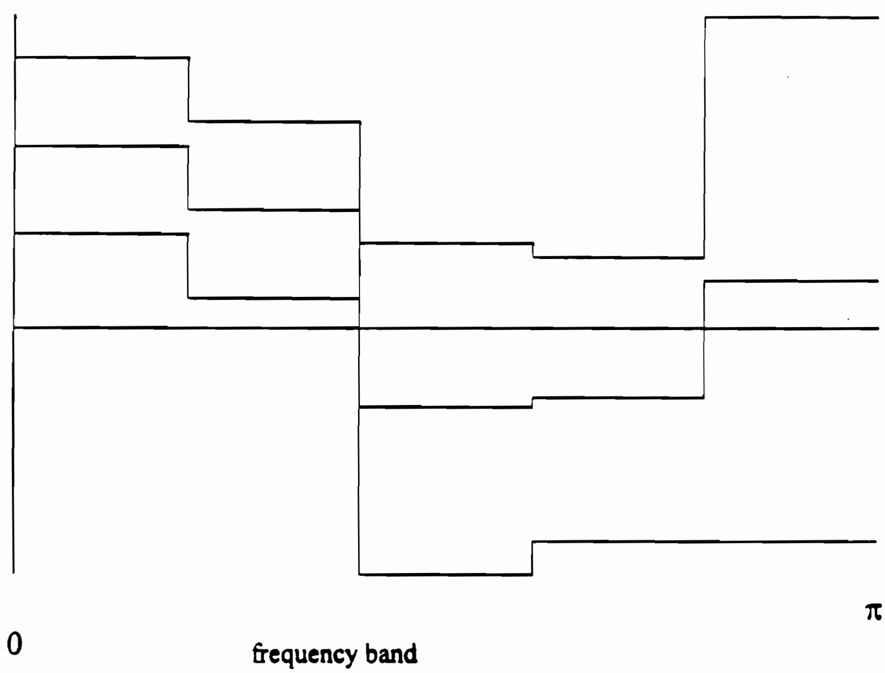

Figure 14
The 5-Band Stabilogram

The results from these regression parameter stability test in the frequency domain indicate that Jaditz and Sayers' fixed-parameter time domain model is seriously misspecified. Most likely this is why it failed to forecast better than an AR(1) model outside of its fitting period.

4.3.3 Forecasting

Having shown that the relationship between the fluctuations of CPI and PPI is unstable across frequencies, we back-transform the (more) correctly specified frequency-domain varying-parameter model into the time domain for forecasting. Figure 15 shows the back-transformations of the three dummy variables $D^*_{(1)}$, $D^*_{(2)}$, and $D^*_{(3)}$ (at different frequency bands) for the lagged fluctuations of PPI from the frequency domain to the time domain. We can see from these plots that the back-transformation of the low-frequency dummy variable is relatively smooth; the smoothness of these back-transformed variables decreases for the middle and high frequency bands.

If $\hat{\beta}^D$ is the updated estimate of the parameter vector from the back-transformed model given by (3.54), then the one-step-ahead forecast error for the growth rate of CPI is

$$fe_t = y_t - X_t^D \hat{\beta}^D_{t-1}. \tag{4.6}$$

The MSFE (Mean Squared Forecast Error) given by models of various numbers of the dummy variables are summarized in Table 11.

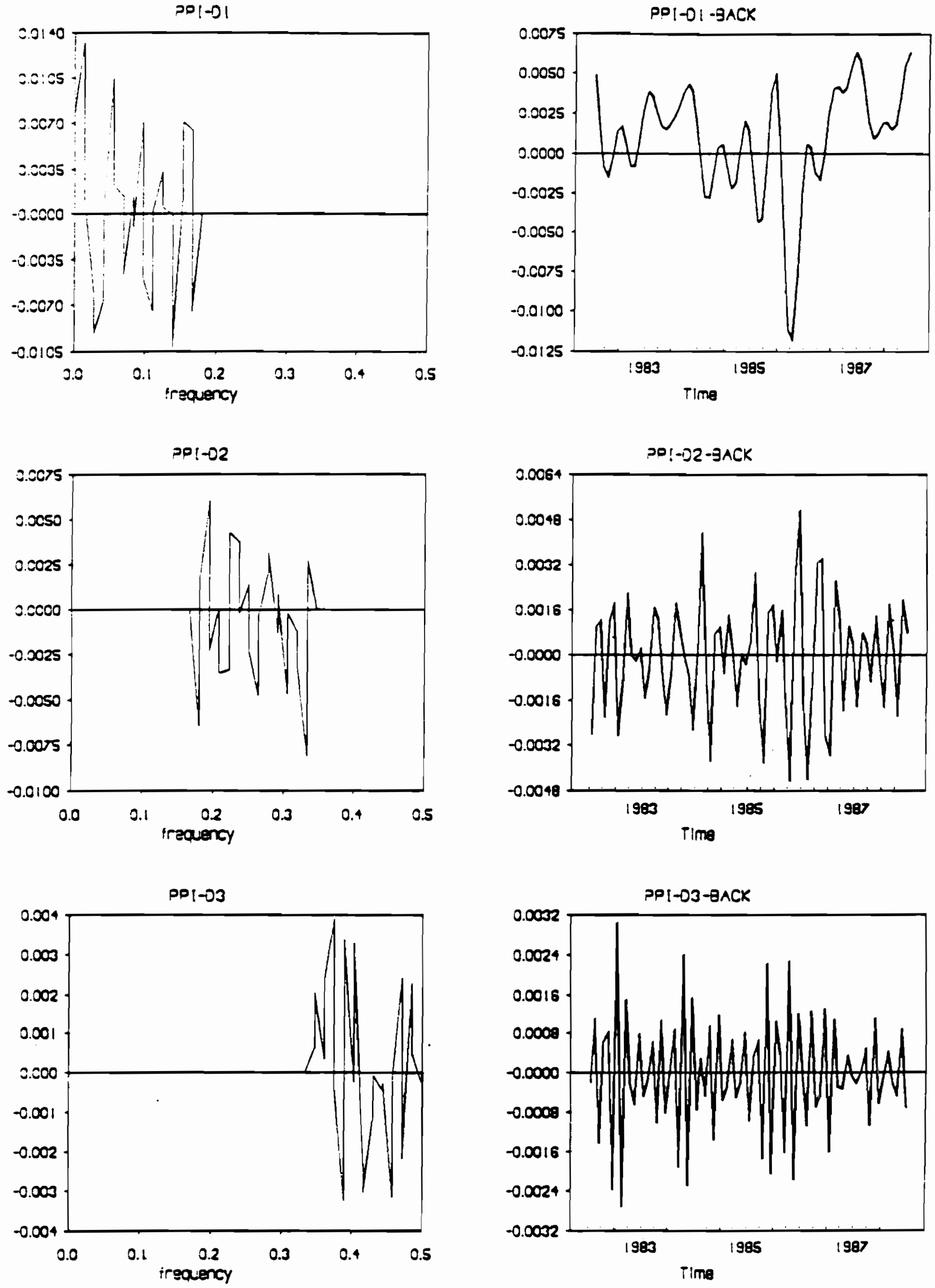

Figure 15
Back-Transformation of the 3-Band Model

Table 11
Forecasting Results (MSFE)[16]

	Sample	Post-Sample
Time Domain (AR)	0.03707	0.04147
Time Domain (VAR)	0.03301	0.04637
Stabilogram (2 bands)	0.02818	0.05193
Stabilogram (3 bands)	0.03036	0.03709
Stabilogram (4 bands)	0.02716	0.04996
Stabilogram (5 bands)	0.02682	0.04466
Stabilogram (6 bands)	0.02587	0.04546
Stabilogram (7 bands)	0.02703	0.04299
Stabilogram (8 bands)	0.02487	0.04580

The "sample" column of Table 11 gives the in-sample (fitting results) for each model. As in Jaditz and Sayers results, the bivariate {VAR} model for the growth rate in CPI fits better than the univariate {AR(1)} model but does not forecast as well over the post-sample period.

[16]Sample period is July 1982 to June 1988 and post-sample period is July 1988 to June 1991.

The three-band stabilogram model fits better than either of these models and also forecasts better over the post-sample period, providing a post-sample MSFE reduction of 11% relative to the AR(1) model and of 20% relative to the VAR model[17]. Results of 4 - 8 band models indicate that more complicated stabilogram models usually fit better but forecast worse. Since it is so easy to over-fit with this technique, it seems evident that one should routinely choose the number of stabilogram bands based on post-sample forecasting rather than on in-sample fitting.

A number of variations on these models were explored. One variation was to omit individually insignificant frequency-band terms from the regression specification. For example, in the three-band case, this would imply keeping only the (back-transformed) low-frequency term. Another set of variations involved various forms of smoothing the transformed PPI_{t-1} series prior to partitioning it into dummy variables. None of these variations yielded models with improved post-sample forecasting performance, so they are not described here in further detail.

4.4 Conclusions

This empirical example demonstrates that it is not sufficient to examine the stability of a relationship across time -- there is a very real possibility that a relationship can be unstable across frequencies. This example resolves the Jaditz-Sayers paradox: their fixed-

[17]These MSFE reductions are significant at the 12 to 28% and 1 to 7% levels respectively, using the bootstrap-based test given in Ashley (1994). A range of significant levels is given to account for the fact that the bootstrap is only asymptotically justified. The outputs of these tests are given in Appendix E and F.

parameter VAR model did not forecast better than the AR(1) model for the growth rate of the CPI because the coefficient on lagged PPI is unstable across frequencies. Once this instability was correctly modeled, the model including information on lagged PPI does indeed forecast (at least somewhat) better than the AR(1) model.

CHAPTER 5

DISCUSSIONS AND CONCLUSIONS

5.1 Discussions

This study emphasizes the importance of the distinction between high-frequency, or "short-term" and low-frequency, or "long-term" relationships; and the existing techniques related to this distinction are discussed in detail. The Geweke approach, which at first glance appears to be an elegant way to quantify the strength of relationships across frequencies, turns out to be inadequate. Two counter examples are presented demonstrating the inadequacy of this approach. Theorem 1 in the first example demonstrates that the Geweke Measure at various frequencies is not a measure of the frequency dependence of the relationship between time series, but rather a measure of the degree to which serially uncorrelated shocks in one time series are converted into more (or less) smoothly persistent fluctuations in the other series. Theorem 2 shows how to generate data in which the relationship between two time series is constructed so as to be frequency dependent. In the second example, such data is generated showing explicitly that the Geweke measure can easily fail to detect the frequency dependence in the relationship.

A new method of detecting and disentangling low-frequency versus high-frequency relationships is proposed in this study. It is referred to as the Block-Wise-Frequency-Dependent-Regression Approach (BWFDR Approach). This approach is a three-step procedure. The first step transforms the time domain model of interest into a real-valued model in the frequency domain using the unitary transformation matrix given by Harvey (1978). This study has shown that this model can be derived from Engle's (1974) complex-valued finite Fourier-transform model, leading to an equivalent but vastly more convenient model. Since the resulting regression model is functionally identical to an ordinary regression model, the second step of this approach reduces the problem of testing for a frequency dependent relationship to the already-solved problem of testing the constancy of a regression coefficient across sample observations. Standard regression parameter stability tests such as Farley, Hinich and McGuire (1975), Brown, Durbin and Evans (1975) and Ashley (1984) can then be applied. The third step back-transforms the frequency domain model to the time domain for forecasting.

The empirical example presented in this study shows that a model (back transformed from a frequency domain model) that allows varying parameters across frequencies can significantly improve the forecasting performance over a misspecified fixed-parameter model. This empirical study is motivated by the Jaditz-Sayers Paradox (1994): if the growth rate in the PPI leads the growth rate in the CPI, why is the post-sample forecasting performance of the bivariate model based on this relationship so poor? Jaditz and Sayers' empirical results show the Granger causality running from the producer price index to the consumer price index in-sample, but then the fixed-parameter time-domain model fails to forecast post-sample. With the purpose of resolving Jaditz and Sayers' problem, we re-examine the lag relationship between the two price indices using the BWFDR approach. We find that the coefficient of the lagged PPI is significantly positive and apparently reasonably stable over time in-sample, but it is unstable across frequency in-sample. This indicates that the Jaditz and Sayers' fixed-parameter model in the frequency domain is misspecified. Their misspecification is an important source for the bivariate model's poor post-sample forecasting performance.

The result of the post-sample forecasts, using the BWFDR approach, shows that the model that allows the variation of parameters across frequency bands does significantly improve the forecasting performance over the fix-parameter model. It also shows that adding the lagged PPI into the information set provides a modest gain in the forecasts of CPI, and thus resolves the Jaditz-Sayers paradox.

5.2 Conclusions

The distinction between high-frequency, or "short-term" and low-frequency, or "long-term" relationships between time series is important to both statistical inference and forecasting. Ignoring it can lead to false conclusions in both theory and practice. The BWFDR approach proposed by this study is a simple and effective way of assessing linear frequency-dependent relationships between time series. This approach also provides a varying-parameter model which can significantly improve forecasting performance where a relationship is different at different frequencies.

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Appendix A
Proof of Lemma 1

Rewrite the simple Geweke model (2.40) in the matrix form:

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} 1+\phi B & \rho B \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_t \\ v_t \end{pmatrix} = \begin{pmatrix} E & F \\ G & H \end{pmatrix} \begin{pmatrix} u_t \\ v_t \end{pmatrix}. \tag{A.1}$$

For this special case of the Geweke model, $E = 1 + \phi B$ and its Fourier transformation is

$$E_\omega = 1 + \phi e^{-i\omega}, \tag{A.2}$$

where $i = \sqrt{-1}$ and ω is the angular frequency. Similarly, $F = \rho B$ and its Fourier transformation is

$$F_\omega = \rho e^{-i\omega}. \tag{A.3}$$

The Geweke measure of frequency dependent relationship for this model is defined as

$$\begin{aligned} f_{y-x}(\omega) &= \ln\left\{\frac{|S_x(\omega)|}{|E_\omega \sigma_u^2 E_\omega'|}\right\} \\ &= \frac{1}{2}\ln\left\{\frac{S_x(\omega)^2}{[E_\omega \sigma_u^2 E_\omega']^2}\right\}, \end{aligned} \tag{A.4}$$

where E_ω' and F_ω' are respectively the conjugate and transposition of E_ω and F_ω, and $S_x(\omega)$ is the spectral density given by

$$S_x(\omega) = E_\omega \sigma_u^2 E_\omega' + F_\omega \sigma_v^2 F_\omega'. \tag{A.5}$$

The first component on the right hand side of (A.5) is contributed by the innovation of the

x_t series, whereas the second component is contributed by the innovation of the y_t series. By substituting E_ω and F_ω in equation (A.5) by (A.2) and (A.3) yield

$$\begin{aligned} S_x(\omega) &= (1 + \phi e^{-i\omega})\sigma_u^2(1 + \phi e^{i\omega}) + \rho e^{-i\omega}\sigma_v^2 \rho e^{i\omega} \\ &= \sigma_u^2\left[1 + \phi e^{i\omega} + \phi e^{-i\omega} + \phi^2\right] + \rho^2\sigma_v^2 \\ &= \sigma_u^2\left[1 + \phi^2 + \phi(e^{i\omega} + e^{-i\omega})\right] + \rho^2\sigma_v^2 \\ &= \sigma_u^2\left[1 + \phi^2 + 2\phi\cos(\omega)\right] + \rho^2\sigma_v^2. \end{aligned} \tag{A.6}$$

Hence, the Geweke measure of frequency dependent relationship between y_t and x_t can be computed as

$$f_{y\to x}(\omega) = \frac{1}{2}\ln\left\{\frac{[\sigma_u^2(1 + \phi^2 + 2\phi\cos(\omega)) + \rho^2\sigma_v^2]^2}{[\sigma_u^2(1 + \phi^2 + 2\phi\cos(\omega))]^2}\right\}. \tag{A.7}$$

For all $\rho \neq 0$, when $\phi \to 1$, then

$$f_{y\to x}(0) \to \ln\left(1 + \frac{\rho^2\sigma_v^2}{4\sigma_u^2}\right) \approx 0\text{, since } \frac{\rho^2\sigma_v^2}{4\sigma_u^2} \text{ is very small,} \tag{A.8}$$

and

$$f_{y\to x}(\pi) \to \ln\left(\frac{\rho^2\sigma_v^2}{0}\right) \approx \infty. \tag{A.9}$$

On the other hand, when $\phi \to -1$, then

$$f_{y\to x}(0) \to \ln\left(\frac{\rho^2\sigma_v^2}{0}\right) \approx \infty, \tag{A.10}$$

and

$$f_{y \to x}(\pi) \quad \to \quad \ln\left(1 \ + \ \frac{\rho^2 \sigma_v^2}{4\,\sigma_u^2} \right) \quad \approx \quad 0 \,. \tag{A.11}$$

Appendix B
Derivation of the Real-Valued Component $Q_{j,t-1}$

The term ρv_{t-1} in model (2.43) can be represented by its finite Fourier transformation over a frequency block of size m :

$$\frac{1}{m}\sum_{j=0}^{m-1} r_j e^{i2\pi(m-2)\frac{j}{m}} v_{t-1}(j), \tag{B.1}$$

where $r_j = \rho$, for $j \in [\,0, \mathrm{m}\text{-}1]$, $v_{t-1}(j)$ is a complex-valued variable indexed by '(j)' and j is the frequency. Note that the parenthesis '( )' is used to symbolize complex-valued variables. The subscript t of $v_t\,(j)$ indicates that $v_t\,(j)$ is the j component of the finite Fourier transformation of v_t , which is based on a frequency block that starts from the observation { t } and ends at the observation { t - (m-1) }. For simplicity, let

$$\widetilde{Q}_t(j) \;=\; \frac{1}{m} e^{i2\pi(m-1)\frac{j}{m}} v_t(j), \text{ for } j \,\in\, [0, m-1], \tag{B.2}$$

where the inverse finite Fourier transformation of $v_t\,(j)$ is given by

$$v_t(j) \;=\; \sum_{l=0}^{m-1} e^{-i2\pi l\frac{j}{m}} v_{l+t-(m-1)}. \tag{B.3}$$

We can avoid the complexity of complex-valued variables in running regressions by combining the conjugate pairs in equation (B.2) to obtain real-valued components. Since $j \;=\; \frac{m}{2}$ is the frequen the fold, it is clear that $\widetilde{Q}_{t-1}(j)$ at $j \;=\; \frac{m}{2} - k$ is the conjugate of $\widetilde{Q}_{t-1}(j')$ at $j' \;=\; \frac{m}{2} \,+\, k,$ for $k \in [\,1, \frac{m}{2} - 1]$ and $r_{\frac{m}{2}-k} \;=\; r_{\frac{m}{2}+k}$.

The combination of the conjugate pairs of $\widetilde{Q}_{t-1}(\frac{m}{2} - k)$ and $\widetilde{Q}_{t-1}(\frac{m}{2} + k)$ is organized as

$$\sum_{j=0}^{\frac{m}{2}} r_j Q_{j,t-1} = r_0\widetilde{Q}_{t-1}(0) + \sum_{k=1}^{\frac{m}{2}-1} r_{\frac{m}{2}-k}\Big\{\widetilde{Q}_{t-1}(\tfrac{m}{2}-k) + \widetilde{Q}_{t-1}(\tfrac{m}{2}+k)\Big\} + r_{\frac{m}{2}}\widetilde{Q}_{t-1}(\tfrac{m}{2}), \tag{B.4}$$

where

$$
\begin{aligned}
Q_{0,t-1} &= \widetilde{Q}_{t-1}(0), \\
Q_{1,t-1} &= \widetilde{Q}_{t-1}(1) + \widetilde{Q}_{t-1}(m-1), \\
Q_{2,t-1} &= \widetilde{Q}_{t-1}(2) + \widetilde{Q}_{t-1}(m-2), \\
&\cdot \\
&\cdot \\
&\cdot \\
Q_{\frac{m}{2}-2,t-1} &= \widetilde{Q}_{t-1}(\tfrac{m}{2}-2) + \widetilde{Q}_{t-1}(\tfrac{m}{2}+2), \\
Q_{\frac{m}{2}-1,t-1} &= \widetilde{Q}_{t-1}(\tfrac{m}{2}-1) + \widetilde{Q}_{t-1}(\tfrac{m}{2}+1), \text{ and} \\
Q_{\frac{m}{2},t-1} &= \widetilde{Q}_{t-1}(\tfrac{m}{2}).
\end{aligned}
\tag{B.5}
$$

Given that

$$
\widetilde{Q}_{t-1}(j) = \frac{1}{m} e^{i2\pi(m-2)\frac{j}{m}} \sum_{l=1}^{m-2} e^{-i2\pi l\frac{j}{m}} v_{l+t-(m-1)} , \tag{B.6}
$$

the term at the zero frequency ,

$$
\widetilde{Q}_{t-1}(0) = \frac{1}{m} \sum_{l=0}^{m-1} v_{l+t-(m-1)}, \tag{B.7}
$$

and the term at the folding frequency ,

$$
\begin{aligned}
\widetilde{Q}_{t-1}(\tfrac{m}{2}) &= \frac{1}{m} e^{i2\pi(m-2)(\frac{m}{2})(\frac{1}{m})} \sum_{l=0}^{m-1} e^{-i2\pi l(\frac{m}{2})(\frac{1}{m})} v_{l+t-(m-1)} \\
&= \frac{1}{m} e^{i\pi(m-2)} \sum_{l=0}^{m-1} e^{-i\pi l} v_{l+t-(m-1)} \\
&= \frac{1}{m} (-1)^{m-2} \sum_{l=0}^{m-1} (-1)^{l} v_{l+t-(m-1)}
\end{aligned}
\tag{B.8}
$$

are real valued terms.

Combining the conjugate pair of $\widetilde{Q}_{t-1}(\frac{m}{2}-k)$ and $\widetilde{Q}_{t-1}(\frac{m}{2}+k)$ for $k \in [1, \frac{m}{2}-1]$:

$$\widetilde{Q}_{t-1}(\tfrac{m}{2}-k) \quad + \quad \widetilde{Q}_{t-1}(\tfrac{m}{2}+k)$$

$$= \frac{1}{m} e^{i2\pi(m-2)(\frac{m}{2}-k)(\frac{1}{m})} \sum_{l=0}^{m-1} e^{-i2\pi l(\frac{m}{2}-k)(\frac{1}{m})} v_{l+t-(m-1)}$$

$$+ \quad \frac{1}{m} e^{i\frac{2\pi}{m}(m-2)(\frac{m}{2}+k)} \sum_{l=0}^{m-1} e^{-i\frac{2\pi}{m}l(\frac{m}{2}+k)} v_{l+t-(m-1)}$$

$$= \frac{1}{m} e^{i\pi(m-2)} e^{-i\frac{2\pi}{m}k(m-2)} \sum_{l=0}^{m-1} e^{-i\pi l} e^{i\frac{2\pi}{m}kl} v_{l+t-(m-1)}$$

$$+ \quad \frac{1}{m} e^{i\pi(m-2)} e^{i\frac{2\pi}{m}k(m-2)} \sum_{l=0}^{m-1} e^{-i\pi l} e^{-i\frac{2\pi}{m}kl} v_{l+t-(m-1)}$$

$$= \frac{(-1)^{m-2}}{m} \left(\sum_{l=0}^{m-1} (-1)^l \exp[-i\frac{2\pi}{m}k(m-2-l)]\, v_{l+t-(m-1)} \right.$$

$$\left. + \sum_{l=0}^{m-1} (-1)^l \exp[i\frac{2\pi}{m}k(m-2-l)]\, v_{l+t-(m-1)} \right) \tag{B.9}$$

$$= \frac{(-1)^{m-2}}{m} \sum_{l=0}^{m-1} (-1)^l\, 2\cos[\frac{2\pi}{m}k(m-2-l)]\, v_{l+t-(m-1)}$$

$$= \frac{2}{m}(-1)^{m-2} \sum_{l=0}^{m-1} (-1)^l \cos\left[\frac{2\pi}{m}(\frac{m}{2}-j)(2+l)\right] v_{l+t-(m-1)} \,,$$

since $j = \frac{m}{2} - k$ or $k = \frac{m}{2} - j$, and

$$
\begin{aligned}
\cos\left[\frac{2\pi}{m}k(m-2-l)\right] &= \cos\left[2\pi k - \frac{2\pi}{m}k(2+l)\right] \\
&= \cos[2\pi k]\cos\left[\frac{2\pi}{m}k(2+l)\right] + \sin[2\pi k]\sin\left[\frac{2\pi}{m}k(2+l)\right] \\
&= \cos\left[\frac{2\pi}{m}k(2+l)\right] \\
&= \cos\left[\frac{2\pi}{m}(\frac{m}{2}-j)(2+l)\right].
\end{aligned}
\tag{B.10}
$$

As a result, the real-valued component $Q_{j,\,t\text{-}1}$ can be written as

$$
Q_{j,t-1} = \begin{cases} \frac{1}{m}\sum_{l=0}^{m-1} v_{l+t-(m-1)}, & \text{for } j = 0, \\ \frac{2}{m}(-1)^{m-2}\sum_{l=0}^{m-1}(-1)^l\cos\left[\frac{2\pi}{m}(\frac{m}{2}-j)(2+l)\right]v_{l+t-(m-1)}, & \text{for } j = 1,2,\ldots,(\frac{m}{2}-1), \\ \frac{1}{m}(-1)^{m-2}\sum_{l=0}^{m-1}(-1)^l v_{l+t-(m-1)}, & \text{for } j = \frac{m}{2}. \end{cases}
\tag{B.11}
$$

Appendix C
Proof of Lemma 2

To prove that

$$\sum_{j=0}^{m-1} r_j Q_{j,t-1} = \rho v_{t-1}, \text{ for all } r_j = \rho,\ j \in [0, m-1]. \tag{C.1}$$

As defined in Appendix B,

$$\sum_{j=0}^{\frac{m}{2}} r_j Q_{j,t-1} = \sum_{j=0}^{m-1} r_j \widetilde{Q}_{t-1}(j), \tag{C.2}$$

where

$$\widetilde{Q}_{t-1}(j) = \frac{1}{m} e^{i2\pi(m-2)\frac{j}{m}} \sum_{l=1}^{m-2} e^{-i2\pi l \frac{j}{m}} v_{l+t-(m-1)}\ , \tag{C.3}$$

then

$$\sum_{j=0}^{\frac{m}{2}} r_j Q_{j,t-1} = \sum_{j=0}^{m-1} r_j \widetilde{Q}_{t-1}(j) = \sum_{j=0}^{m-1} r_j \left\{ \frac{1}{m} e^{i2\pi(m-2)\frac{j}{m}} \sum_{l=1}^{m-2} e^{-2\pi l \frac{j}{m}} v_{l+t-(m-1)} \right\}. \tag{C.4}$$

Substituting r_j by ρ, for all $j \in$ [0, m-1], yield

$$\begin{aligned}\sum_{j=0}^{\frac{m}{2}} r_j Q_{j,t-1} &= \frac{1}{m} \rho \sum_{j=0}^{m-1} e^{i2\pi(m-2)\frac{j}{m}} \sum_{l=1}^{m-2} e^{-i2\pi l \frac{j}{m}} v_{l+t-(m-1)} \\ &= \frac{1}{m} \rho \sum_{l=1}^{m-2} v_{l+t-(m-1)} \sum_{j=0}^{m-1} e^{i2\pi(m-2)\frac{j}{m}} e^{-i2\pi l \frac{j}{m}}.\end{aligned} \tag{C.5}$$

Note that $\sum_{j=0}^{m-1} e^{i2\pi(m-2)\frac{j}{m}} e^{-i2\pi l \frac{j}{m}}$ is a delta function. According to the identity of the Transposed Orthogonality Relations:

$$\sum_{j=0}^{m-1} \exp(i w_j t)\ \exp(-i w_j s) = \begin{cases} m, & \text{if } t = s \\ 0, & \text{otherwise;} \end{cases} \tag{C.6}$$

$$w_j = 2\pi \frac{j}{m} \ ;$$

it is clear that

$$\sum_{j=0}^{m-1} e^{i2\pi(m-2)\frac{j}{m}}\ e^{-i2\pi l \frac{j}{m}} = \begin{cases} m, & \text{if } l = m-2 \\ 0, & \text{otherwise.} \end{cases} \tag{C.7}$$

Hence,

$$\sum_{j=0}^{\frac{m}{2}} r_j Q_{j,t-1} = \frac{1}{m}\ \rho\, v_{(m-2)+t-(m-1)}\ m, \tag{C.8}$$

because all the terms with $l \neq m - 2$ vanished, and finally,

$$\sum_{j=0}^{\frac{m}{2}} r_j Q_{j,t-1} = \rho\, v_{t-1}. \tag{C.9}$$

As a result, model (2.45) is equivalent to model (2.43), and is referred to as the Frequency Block model.

Appendix D
Proof of Theorem 1

To show that Engle's complex-valued spectral regression model (3.3) can be transformed into Harvey's real-valued regression model (3.6) via row manipulations. Consider the t row of the simplified version of model (3.1):

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t, \quad t = 1, 2, ..., n, \tag{D.1}$$

where y and x are the time series under study, β_0 is the intercept, and $\varepsilon \sim \text{NIID}(0, \sigma^2)$ is the disturbance. This is referred to as a simplified version because only one explanatory variable is considered. The finite, discrete Fourier transformation of (D.1), given by Engle is

$$\sum_{t=0}^{n-1} y_t e^{-i2\pi t \frac{j}{n}} = \beta_0 + \beta_1 \sum_{t=0}^{n-1} x_t e^{-i2\pi t \frac{j}{n}} + \sum_{t=0}^{n-1} \varepsilon_t e^{-i2\pi t \frac{j}{n}} \tag{D.2}$$

or

$$y(j) = \beta_0 + \beta_1 x(j) + \varepsilon(j), \quad j = 0, 1, 2, ..., n\text{-}1, \tag{D.3}$$

where $i = \sqrt{-1}$ and j is the frequency. Note that the observations above the folding frequency of $j = \frac{n}{2}$ or the folding angular frequency of $\omega = 2\pi(j/n) = \pi$, are the mirror images or the complex conjugates of the observations below the folding frequency. The observations on the row corresponding to the frequency $j = \frac{n}{2} + k$ is the complex conjugate of the observations on the row corresponding to the frequency $j' = \frac{n}{2} - k$, for $k \in [1, \frac{n}{2} - 1]$, so they contain identical information and thus can be combined to yield a useful real-valued term. These row manipulations are done in two ways:

$$\frac{y(j) + y(j')}{2} = \beta_0 + \beta_1 \left[\frac{x(j) + x(j')}{2}\right] + \frac{\varepsilon(j) + \varepsilon(j')}{2} \tag{D.4}$$

and

$$\frac{y(j) - y(j')}{2i} = 0 + \beta_1\left[\frac{x(j) - x(j')}{2i}\right] + \frac{\varepsilon(j) - \varepsilon(j')}{2i}. \tag{D.5}$$

The first transformation given by equation (D.4) contains information about the real part of the complex-valued observations, whereas the second transformation given by (D.5) contains information about the imaginary part. The first transformation is known as the cosine transformation:

$$\begin{aligned}\frac{y(j) + y(j')}{2} &= \frac{1}{2}\left[\sum_{t=0}^{n-1} y_t e^{i2\pi t\frac{j}{n}} + \sum_{t=0}^{n-1} y_t e^{-i2\pi t\frac{j}{n}}\right]\\ &= \frac{1}{2}\sum_{t=0}^{n-1} y_t\left[e^{i2\pi t\frac{j}{n}} + e^{-i2\pi t\frac{j}{n}}\right]\\ &= \frac{1}{2}\sum_{t=0}^{n-1} y_t\left[\cos\left(2\pi t\frac{j}{n}\right) + i\sin\left(2\pi t\frac{j}{n}\right) + \cos\left(2\pi t\frac{j}{n}\right) - i\sin\left(2\pi t\frac{j}{n}\right)\right]\\ &= \sum_{t=0}^{n-1} \cos\left(2\pi t\frac{j}{n}\right) y_t,\end{aligned} \tag{D.6}$$

and the second transformation is known as the sine transformation:

$$\begin{aligned}\frac{y(j) - y(j')}{2i} &= \frac{1}{2i}\left[\sum_{t=0}^{n-1} y_t e^{i2\pi t\frac{j}{n}} - \sum_{t=0}^{n-1} y_t e^{-i2\pi t\frac{j}{n}}\right]\\ &= \frac{1}{2i}\sum_{t=0}^{n-1} y_t\left[e^{i2\pi t\frac{j}{n}} - e^{-i2\pi t\frac{j}{n}}\right]\\ &= \frac{1}{2i}\sum_{t=0}^{n-1} y_t\left[\cos\left(2\pi t\frac{j}{n}\right) + i\sin\left(2\pi t\frac{j}{n}\right) - \cos\left(2\pi t\frac{j}{n}\right) + i\sin\left(2\pi t\frac{j}{n}\right)\right]\\ &= \sum_{t=0}^{n-1} \sin\left(2\pi t\frac{j}{n}\right) y_t.\end{aligned} \tag{D.7}$$

Similarly,

$$\frac{x(j) + x(j')}{2} = \sum_{t=0}^{n-1} \cos\left(2\pi t \frac{j}{n}\right) x_t, \tag{D.8}$$

$$\frac{x(j) - x(j')}{2i} = \sum_{t=0}^{n-1} \sin\left(2\pi t \frac{j}{n}\right) x_t, \tag{D.9}$$

$$\frac{\varepsilon(j) + \varepsilon(j')}{2} = \sum_{t=0}^{n-1} \cos\left(2\pi t \frac{j}{n}\right) \varepsilon_t, \quad \text{and} \tag{D.10}$$

$$\frac{\varepsilon(j) - \varepsilon(j')}{2i} = \sum_{t=0}^{n-1} \sin\left(2\pi t \frac{j}{n}\right) \varepsilon_t. \tag{D.11}$$

When these manipulations are applied simultaneously on a set of complex-valued observations, it is referred to as the cosine-sine transformation.

If the complex-valued regression model given by Engle (equation 3.3) is cosine-sine transformed, the resulting model can be written as

$$Y' = X'\beta + \varepsilon', \tag{D.12}$$

where the typical s observation of Y' is

$$y_s = \begin{cases} \sum_{t=0}^{n-1} y_t, & \text{for } s = 1, \\ \sum_{t=0}^{n-1} \cos\left[\frac{\pi t(s-1)}{n}\right] y_t, & \text{for } s = 2, 4, 6, \ldots, (n-2) \text{ or } (n-1), \\ \sum_{t=0}^{n-1} \sin\left[\frac{\pi (t-1)(s-1)}{n}\right] y_t, & \text{for } s = 3, 5, 7, \ldots, (n-1) \text{ or } n, \\ \sum_{t=0}^{n-1} (-1)^t y_t, & \text{for } s = n \text{ and } n \text{ is even}, \end{cases} \tag{D.13}$$

and similarly for x_s and ε_s. Equation (D.12) can be rewritten as

$$CY = CX\beta + C\varepsilon, \tag{D.14}$$

where the typical element of matrix C, c_{ts}, is

$$c_{t,s} = \begin{cases} 1, & \text{for } t = 1; \\ \cos\left[\dfrac{\pi t(s-1)}{n}\right], & \text{for } t = 2, 4, 6, \ldots, (n-2) \text{ or } (n-1); \\ \sin\left[\dfrac{\pi (t-1)(s-1)}{n}\right], & \text{for } t = 3, 5, 7, \ldots, (n-1) \text{ or } n; \\ (-1)^{s+1}, & \text{for } t = n \text{ and } n \text{ is even}, \quad s = 1, \ldots, n. \end{cases} \tag{D.15}$$

Note that matrix C is not an orthogonal matrix because $CC^T \neq \mathrm{I}$, then equation (D.14) is not a unitary transformation of equation (3.1). To preserve the information in the data, matrix C is manipulated in such a way that the resulting matrix, $\widetilde{C}$, is a unitary transformation (orthogonal) matrix, which has the property of $\widetilde{C}\widetilde{C}^{\mathrm{T}} = \mathrm{I}$.

Constructing an Orthogonal Matrix from Matrix *C*

Given that

$$C = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \cos\left[\frac{2\pi}{n}(1)(0)\right] & \cos\left[\frac{2\pi}{n}(1)(1)\right] & \cdots & \cos\left[\frac{2\pi}{n}(1)(n-1)\right] \\ \sin\left[\frac{2\pi}{n}(1)(0)\right] & \sin\left[\frac{2\pi}{n}(1)(1)\right] & \cdots & \sin\left[\frac{2\pi}{n}(1)(n-1)\right] \\ \cos\left[\frac{2\pi}{n}(2)(0)\right] & \cos\left[\frac{2\pi}{n}(2)(1)\right] & \cdots & \cos\left[\frac{2\pi}{n}(2)(n-1)\right] \\ \sin\left[\frac{2\pi}{n}(2)(0)\right] & \sin\left[\frac{2\pi}{n}(2)(1)\right] & \cdots & \sin\left[\frac{2\pi}{n}(2)(n-1)\right] \\ \vdots & & & \\ \cos\left[\frac{2\pi}{n}(\frac{n}{2}-2)(0)\right] & \cos\left[\frac{2\pi}{n}(\frac{n}{2}-2)(1)\right] & \cdots & \cos\left[\frac{2\pi}{n}(\frac{n}{2}-2)(n-1)\right] \\ \sin\left[\frac{2\pi}{n}(\frac{n}{2}-2)(0)\right] & \sin\left[\frac{2\pi}{n}(\frac{n}{2}-2)(1)\right] & \cdots & \sin\left[\frac{2\pi}{n}(\frac{n}{2}-2)(n-1)\right] \\ \cos\left[\frac{2\pi}{n}(\frac{n}{2}-1)(0)\right] & \cos\left[\frac{2\pi}{n}(\frac{n}{2}-1)(1)\right] & \cdots & \cos\left[\frac{2\pi}{n}(\frac{n}{2}-1)(n-1)\right] \\ \sin\left[\frac{2\pi}{n}(\frac{n}{2}-1)(0)\right] & \sin\left[\frac{2\pi}{n}(\frac{n}{2}-1)(1)\right] & \cdots & \sin\left[\frac{2\pi}{n}(\frac{n}{2}-1)(n-1)\right] \\ 1 & -1 & \cdots & -1 \end{bmatrix} \tag{D.16}$$

According to the orthogonality of sinusoids:

$$\sum_{t=0}^{n-1} \cos\left(\frac{2\pi}{n} jt\right) \cos\left(\frac{2\pi}{n} kt\right) = \begin{cases} \frac{n}{2}, & j = k \neq 0 \text{ or } \frac{n}{2} \\ n, & j = k = 0 \text{ or } \frac{n}{2} \\ 0, & j \neq k \end{cases} \tag{D.17}$$

$$\sum_{t=0}^{n-1} \sin\left(\frac{2\pi}{n} jt\right) \sin\left(\frac{2\pi}{n} kt\right) = \begin{cases} \frac{n}{2}, & j = k \neq 0 \text{ or } \frac{n}{2} \\ 0, & \text{otherwise} \end{cases} \tag{D.18}$$

$$\sum_{t=0}^{n-1} \cos\left(\frac{2\pi}{n} jt\right) \sin\left(\frac{2\pi}{n} kt\right) = 0, \tag{D.19}$$

it is clear that

$$CC^T = \begin{bmatrix} n & 0 & 0 & \dots & 0 \\ 0 & \frac{n}{2} & 0 & \dots & 0 \\ 0 & 0 & \frac{n}{2} & \dots & 0 \\ & & & \ddots & \\ 0 & 0 & 0 & \dots & n \end{bmatrix}. \tag{D.20}$$

An orthogonal matrix $\widetilde{C}$ can be constructed from C such that $\widetilde{C}\widetilde{C}^T = I$:

$$\widetilde{C} = \begin{bmatrix} \frac{1}{\sqrt{n}} & \frac{1}{\sqrt{n}} & \cdots & \frac{1}{\sqrt{n}} \\ \sqrt{\frac{2}{n}}\cos\left[\frac{2\pi}{n}(1)(0)\right] & \sqrt{\frac{2}{n}}\cos\left[\frac{2\pi}{n}(1)(1)\right] & \cdots & \sqrt{\frac{2}{n}}\cos\left[\frac{2\pi}{n}(1)(n-1)\right] \\ \sqrt{\frac{2}{n}}\sin\left[\frac{2\pi}{n}(1)(0)\right] & \sqrt{\frac{2}{n}}\sin\left[\frac{2\pi}{n}(1)(1)\right] & \cdots & \sqrt{\frac{2}{n}}\sin\left[\frac{2\pi}{n}(1)(n-1)\right] \\ \sqrt{\frac{2}{n}}\cos\left[\frac{2\pi}{n}(2)(0)\right] & \sqrt{\frac{2}{n}}\cos\left[\frac{2\pi}{n}(2)(1)\right] & \cdots & \sqrt{\frac{2}{n}}\cos\left[\frac{2\pi}{n}(2)(n-1)\right] \\ \sqrt{\frac{2}{n}}\sin\left[\frac{2\pi}{n}(2)(0)\right] & \sqrt{\frac{2}{n}}\sin\left[\frac{2\pi}{n}(2)(1)\right] & \cdots & \sqrt{\frac{2}{n}}\sin\left[\frac{2\pi}{n}(2)(n-1)\right] \\ \vdots & & & \\ \sqrt{\frac{2}{n}}\cos\left[\frac{2\pi}{n}(\frac{n}{2}-2)(0)\right] & \sqrt{\frac{2}{n}}\cos\left[\frac{2\pi}{n}(\frac{n}{2}-2)(1)\right] & \cdots & \sqrt{\frac{2}{n}}\cos\left[\frac{2\pi}{n}(\frac{n}{2}-2)(n-1)\right] \\ \sqrt{\frac{2}{n}}\sin\left[\frac{2\pi}{n}(\frac{n}{2}-2)(0)\right] & \sqrt{\frac{2}{n}}\sin\left[\frac{2\pi}{n}(\frac{n}{2}-2)(1)\right] & \cdots & \sqrt{\frac{2}{n}}\sin\left[\frac{2\pi}{n}(\frac{n}{2}-2)(n-1)\right] \\ \sqrt{\frac{2}{n}}\cos\left[\frac{2\pi}{n}(\frac{n}{2}-1)(0)\right] & \sqrt{\frac{2}{n}}\cos\left[\frac{2\pi}{n}(\frac{n}{2}-1)(1)\right] & \cdots & \sqrt{\frac{2}{n}}\cos\left[\frac{2\pi}{n}(\frac{n}{2}-1)(n-1)\right] \\ \sqrt{\frac{2}{n}}\sin\left[\frac{2\pi}{n}(\frac{n}{2}-1)(0)\right] & \sqrt{\frac{2}{n}}\sin\left[\frac{2\pi}{n}(\frac{n}{2}-1)(1)\right] & \cdots & \sqrt{\frac{2}{n}}\sin\left[\frac{2\pi}{n}(\frac{n}{2}-1)(n-1)\right] \\ \frac{1}{\sqrt{n}} & -\frac{1}{\sqrt{n}} & \cdots & -\frac{1}{\sqrt{n}} \end{bmatrix}. \tag{D.21}$$

This matrix is identical to the matrix A given by Harvey, therefore the transformed Engle's model:

$$\widetilde{C}Y = \widetilde{C}X\beta + \widetilde{C}\varepsilon, \tag{D.22}$$

is equivalent to Harvey's real-valued model (3.6).

Appendix E
Output of the Bootstrap Estimates
(3-Band vs AR)

```
PARAMETER STATEMENT USED IN SOURCE CODE WAS:

(NPEQN=2, NPDATA=200, NPPARM=20, NPLAM=20, NPDIST=1000,
 NPSIM=401, NPPOLY=50, NPFRAK=25, NPDECK=97, NPLAG=18,
 NPGAP=50, NPMXLG=30, NPMAXIT=2,NPBIAS=80)

CALCULATION BEGINS: 11:47:51.52    11/09/95

*************************************************************
*                                                           *
*        A STATISTICAL INFERENCE ENGINE                     *
*         FOR SMALL, DEPENDENT SAMPLES                      *
*                                                           *
*               R. ASHLEY                                   *
*                                                           *
*             VERSION  2.1025                               *
*                                                           *
*                                                           *
*                                                           *
* S WORK WAS SUPPORTED BY NSF GRANT SES8922394              *
*                                                           *
*       COPYRIGHT (C) 1992 RICHARD ASHLEY                   *
*                                                           *
*************************************************************

CPI PPI AR STAB 3-BAND

NDIM    =     2 IS DIMENSIONALITY OF VAR SYSTEM

NOBSIN  =     36 IS NUMBER OF SAMPLE OBSERVATIONS
              IN ORIGINAL DATA SET READ IN

NOBSAMP =     36 IS NUMBER OF OBSERVATIONS IN EACH
              DATA SET GENERATED FOR USE IN TEST.
              CAN BE SET NOT EQUAL TO NOBSIN IF
              NUSAMP=1 & (IBIAS = 0 OR IBIAS > 1)
              & ISIMDAT=0 & IMPORTON=0 & ISAMP=0
              & IBALON=0.

MAXLG   =     1 IS MAXIMUM LAG IN VAR

NBIAS   =     20 SAMPLE SIZE FOR COMPUTING BIAS
              (TYPICAL VALUE: 20)

NBIAS0  =     0 SAMPLE SIZE FOR COMPUTING BIAS
              USING INITIAL SAMPLE DATA
              (SET TO 0 WHEN IBIAS > 1 OR < 0)
```

NENSM =	2000 # OF POINTS FOR ESTIMATED DIST FCN (TYPICAL VALUE: 2000)
NUMSIM =	101 # OF REPEATS (TYPICAL VALUE: 101)
ISPLAY =	500 # OF REPETITIONS BEFORE ITERATION COUNT DISPLAYED ON SCREEN (TYPICAL VALUE: 500)
NUSAMP =	0 --> USE ACTUAL SAMPLE DATA TO INITIATE ALL NUMSIM SIMULATIONS 1 --> USE PSEUDO DATA INSTEAD TO INITIATE ALL AFTER THE FIRST (DEFAULT)
IMPRNT =	0 SWITCH FOR PRINTING IMPORTANCE SAMPLING NONCONVERGENCE MESSAGES 0 --> NO PRINT 1 --> PRINT
LAMCALC =	0 SWITCH FOR LAMBDA CALCULATION: 0 -> EXACT; NN -> SIMULATE WITH NN*NEST DATA POINTS
ISEED =	3463 INITIAL SEED FOR RANDOM NUMBER GENERATOR
ISEDIN =	0 {NOT USED IN DISTRIBUTION VERSIONS}
INFLAG =	50 ACTUAL LENGTH OF INFINITE ORDER LAG STRUCTURE FOR INVERSION IN CALCULATING LAMBDA. (TYPICAL VALUE: 50)
NUMLAM =	1 DIMENSIONALITY OF LAMBDA (IGNORED IF IMPORTON = 1)
ISTASHE =	95 NONSTATIONARITY TEST PARAMETER: (ONLY RELEVANT IF ICARE = 1) ISTASHE > 0 --> TEST WHETHER ALL ROOTS OF VAR DETERMINANT EXCEED (100./FLOAT(ISTASHE)). (TYPICAL VALUE: ISTASHE = 95)
ISAVE =	0 SWITCH FOR SAVING ESTIMATED DFS TO FILE. ISAVE=1 -> SAVE THEM
ISAVP =	0 {NOT IMPLEMENTED IN DISTRIBUTION VERSION}
ICARE =	1 SWITCH FOR ELIMINATING NONSTATIONARY MODELS FROM CONSIDERATION. ICARE=1 -> SKIP THEM
IOLSTEK =	1 SWITCH FOR SELECTING REGRESSION TECHNIQUE. IOLSTEK = 1 -> USE SVD

ALGORITHM. OTHERWISE, USE FASTER
NON-SVD ALGORITHM

IBALON = 0 SWITCH FOR SELECTING BALANCED
RESAMPLING IN THE BIAS CORRECTION
ROUTINE.
IBALON=0 -> BALANCE OFF (DEFAULT)
IBALON=1 -> BALANCE ON

ISAMP = 0 SWITCH FOR RESAMPLING (WHERE FEASIBLE)
WITHOUT REPLACEMENT
ISAMP=0 -> REPLACE (DEFAULT)
ISAMP=1 -> DO NOT REPLACE
{NOT IMPLEMENTED IN DISTRIBUTION VERSION}

INFKIND = 0 SWITCH FOR INFERENCE KIND
INFKIND=0 --> MULTIPLICATIVE
(DEFAULT FOR RATIOS)
INFKIND=1 --> ADDITIVE
INFKIND=2 --> RAW VALUES

MIDNITE = 0 SWITCH FOR RESTARTING ANY SIMULATION
THAT STRADDLES MIDNIGHT, LOUSING
UP CALC. OF # OF TICKS USED.
(ONLY USED WITH IMPORTANCE SAMPLING)
MIDNITE = 1 --> RESTART
MIDNITE = 0 --> IGNORE

ISIMDAT = 0 SWITCH FOR SELECTING TAYLOR/THOMPSON
NONPARAMETRIC DENSITY ESTIMATION
ALGORITHM (INCOMPATIBLE WITH
IMPORTANCE SAMPLING)

ISIMDAT > 0 --> USE IT
ISIMDAT = 0 --> USE DIRAC COMB -- THE BOOTSTRAP
ISIMDAT < 0 --> USE IT WITH -ISIMDAT-1
NEAREST NEIGHBORS

IMPORTON = 0 SWITCH FOR SELECTING IMPORTANCE
RESAMPLING. IMPORTON = 1 -->
IMPORTANCE SAMPLING IS ENABLED.

IMPSTART = 1000 NUMBER OF REPETITIONS TO BE USED
FOR CALCULATING IMPORTANCE PROBS
(SET TO NENSM/2 WHEN IMPORTANCE
SAMPLING TURNED OFF.)

IBAKUP = 0 PROGRAM WILL BACKUP INFO NEEDED
TO RESTART THE CALCULATIONS AT THE
END OF EACH COMPLETED SIMULATION
IF IBAKUP SET TO ONE

IRESTRT = 0 SET TO ONE IF THIS RUN IS A RESTART
(NORMALLY SET TO ZERO)

IBUSE = 0 IBUSE = 1 --> READ IN BETA
IBUSE = 0 --> ESTIMATE OPTIMAL BETA

BETA = 0.400 WEIGHT TO PLACE ON RESULT FROM
FIRST IMPSTART REPETITIONS. (MUST BE
A FRACTION IN [0,1] (THIS INPUT
NOT USED UNLESS IBUSE = 1)

BETAMIN = 0.100 MINIMUM WEIGHT TO PLACE ON RESULT
FROM FIRST IMPSTART REPETITIONS.
MUST BE FRACTION IN [0,1]. (THIS
INPUT NOT USED UNLESS IBUSE = 0)

VMAX = 80.000 IF AN IMPORTANCE PROB DIST. IS
GENERATED THAT YIELDS AN ESTIMATED
VARIANCE REDUCTION RATIO > VMAX
THEN IT IS DISCARDED AND NEW
ESTIMATE IS MADE. (THIS INPUT NOT
USED UNLESS IBUSE = 0)

IBIAS = 0 IBIAS = 0 --> NO BIAS CORRECTION)
= 1 --> BIAS CORRECTION
> 1 --> ESTIMATE BIAS ONCE-
AND-FOR-ALL IN FIRST SIMULATION
WITH IBIAS REPETITIONS.
< 0 --> ESTIMATE BIAS (ONCE)
FOR EACH SIMULATION USING -IBIAS
SEPARATE REPETITIONS.

JSPARE = 0 IF JSPARE > 1 AND ICARE = 1 THEN AT
MOST JSPARE NONSTATIONARY MODELS ARE
REJECTED IN A ROW DURING THE BIAS
CORRECTION ITERATIONS.

SAMPLE DATA READ FROM FILE FCUPAR.DAT :

DATA FOR SERIES NUMBER 1 AR TIME-DOMAIN

0.09477061	0.00793602	0.20740090	-0.02963139
0.03591376	0.03476131	0.11611040	-0.00113170
0.19432560	0.36927910	0.01477930	-0.12779630
0.13401610	-0.33463310	0.07041211	0.21241130
-0.04434340	0.09714258	0.61421610	-0.16772380
0.05664780	-0.02245578	-0.14778390	0.37534060
0.04237246	0.47761740	0.16748600	0.21600030
-0.24126320	0.09578415	0.03410300	-0.26314140
-0.14259620	0.00824319	0.02177149	-0.08229384

DATA FOR SERIES NUMBER 2 STAB 3-BAND

-0.07928773	-0.10471860	0.15125860	-0.01993551
0.07147363	0.01709951	0.00681679	-0.18478060
0.06743756	0.25758500	0.03820030	-0.21395760
0.04354060	-0.32052060	0.08980758	0.22863530
-0.01427093	0.05915373	0.54152060	-0.18314520
-0.06109499	-0.03007639	-0.08367777	0.31590030
0.08193654	0.48641250	0.12862030	0.02697197

-0.44366980	-0.11784320	0.09515977	-0.09501950
-0.08903138	0.07243574	0.09283744	-0.13784640

TEST STATISTICS USED:

1: MEAN SQUARE OF STAB 3-BAND / MEAN SQUARE OF AR TIME-DOMAI

VAR SPECIFICATION:

FOR EQUATION 1 THE LAG STRUCTURE ON VARIABLE 1 CONTAINS LAGS:

FOR EQUATION 1 THE LAG STRUCTURE ON VARIABLE 2 CONTAINS LAGS:

FOR EQUATION 2 THE LAG STRUCTURE ON VARIABLE 1 CONTAINS LAGS:

FOR EQUATION 2 THE LAG STRUCTURE ON VARIABLE 2 CONTAINS LAGS:

INDGAP = GAP SUFFICIENT FOR INDEPENDENCE = 50
WEIGHT = PARAMETER FOR BIAS ROUTINE = 1.00
IPRNTF = {NOT USED} = 0
NUMITER = MAX # ITERATIONS FOR BIAS ROUTINE = 1

ESTIMATED VAR USING SAMPLE DATA
(NOT BIAS-CORRECTED)

EQN VAR LAG	ESTIMATE	T-STAT	PROB	ADJ. R-SQD	S-SQRD
1 CONST.	0.571222E-01	1.68	0.101	0.000	0.402858E-01

EQN VAR LAG	ESTIMATE	T-STAT	PROB	ADJ. R-SQD	S-SQRD
2 CONST.	0.220919E-01	0.67	0.510	0.000	0.385872E-01

FOR TEST STATISTIC #1:

MEAN SQUARE OF STAB 3-BAND / MEAN SQUARE OF AR TIME-DOMAI

SAMPLE VALUE = 0.894456E+00

1ST PART OF EACH SIMULATION USES IMPSTART = 1000 REPETITIONS

2ND PART OF EACH SIMULATION USES NENSM - IMPSTART = 1000 REPETITIONS

FOR MEAN SQUARE OF STAB 3-BAND / MEAN SQUARE OF AR TIME-DOMAI

PROBABILITY THAT TRUE > 1.000

	PROBABILITY	PROB (1ST)	PROB (2ND)
SMPL	0.2295	0.2340	0.2250
1	0.1035	0.1140	0.0930

2	0.2615	0.2700	0.2530
3	0.2035	0.1990	0.2080
4	0.3130	0.3120	0.3140
5	0.2440	0.2400	0.2480
6	0.2115	0.2000	0.2230
7	0.2945	0.2920	0.2970
8	0.2600	0.2460	0.2740
9	0.3105	0.3170	0.3040
10	0.0575	0.0660	0.0490
11	0.2860	0.2820	0.2900
12	0.0230	0.0250	0.0210
13	0.1345	0.1280	0.1410
14	0.1520	0.1500	0.1540
15	0.2740	0.2630	0.2850
16	0.1775	0.1740	0.1810
17	0.1100	0.1080	0.1120
18	0.0800	0.0710	0.0890
19	0.2615	0.2620	0.2610
20	0.2950	0.2940	0.2960
21	0.1840	0.1760	0.1920
22	0.2450	0.2500	0.2400
23	0.3005	0.3140	0.2870
24	0.1085	0.0980	0.1190
25	0.0835	0.0860	0.0810
26	0.2875	0.2970	0.2780
27	0.3215	0.3280	0.3150
28	0.1990	0.2090	0.1890
29	0.0210	0.0190	0.0230
30	0.0495	0.0510	0.0480
31	0.2325	0.2360	0.2290
32	0.1855	0.1730	0.1980
33	0.1000	0.1040	0.0960
34	0.2055	0.1920	0.2190
35	0.0535	0.0520	0.0550
36	0.1680	0.1690	0.1670
37	0.0825	0.0800	0.0850
38	0.2150	0.2060	0.2240
39	0.1365	0.1410	0.1320
40	0.3065	0.3010	0.3120
41	0.1165	0.1130	0.1200
42	0.2795	0.2750	0.2840
43	0.3385	0.3570	0.3200
44	0.2195	0.2180	0.2210
45	0.2620	0.2600	0.2640
46	0.0635	0.0630	0.0640
47	0.1650	0.1700	0.1600
48	0.1420	0.1320	0.1520
49	0.2450	0.2570	0.2330
50	0.0470	0.0440	0.0500
51	0.1720	0.1750	0.1690
52	0.2965	0.3100	0.2830
53	0.2095	0.2190	0.2000
54	0.2540	0.2390	0.2690
55	0.0955	0.0980	0.0930
56	0.2780	0.2610	0.2950

57	0.3680	0.3770	0.3590
58	0.3145	0.3060	0.3230
59	0.3220	0.3170	0.3270
60	0.2580	0.2490	0.2670
61	0.1760	0.1770	0.1750
62	0.2415	0.2280	0.2550
63	0.0835	0.0910	0.0760
64	0.2155	0.2190	0.2120
65	0.3050	0.2900	0.3200
66	0.3510	0.3520	0.3500
67	0.2215	0.2320	0.2110
68	0.2250	0.2320	0.2180
69	0.2070	0.2110	0.2030
70	0.1430	0.1460	0.1400
71	0.0890	0.0940	0.0840
72	0.2615	0.2520	0.2710
73	0.1240	0.1150	0.1330
74	0.1780	0.1760	0.1800
75	0.1025	0.1050	0.1000
76	0.2110	0.2170	0.2050
77	0.2145	0.2170	0.2120
78	0.3140	0.3080	0.3200
79	0.3280	0.3260	0.3300
80	0.3755	0.3810	0.3700
81	0.2360	0.2390	0.2330
82	0.2020	0.1870	0.2170
83	0.2995	0.3180	0.2810
84	0.3315	0.3370	0.3260
85	0.2440	0.2370	0.2510
86	0.1840	0.1860	0.1820
87	0.0695	0.0830	0.0560
88	0.1615	0.1550	0.1680
89	0.2735	0.2880	0.2590
90	0.2815	0.2740	0.2890
91	0.0840	0.0930	0.0750
92	0.0995	0.0990	0.1000
93	0.2120	0.2130	0.2110
94	0.2845	0.2820	0.2870
95	0.1930	0.1890	0.1970
96	0.3570	0.3690	0.3450
97	0.1215	0.1170	0.1260
98	0.3645	0.3820	0.3470
99	0.2685	0.2800	0.2570
100	0.0585	0.0620	0.0550

MEAN	0.2068	0.2069	0.2066
ST.DEV.	0.0913	0.0923	0.0911
MEDIAN	0.2132	0.2170	0.2145
RANGE	0.3545	0.3630	0.3490
IQR	0.1513	0.1585	0.1510

DECILES AND HALF-DECILES:

MINIMUM	0.0210	0.0190	0.0210
0.5	0.0535	0.0520	0.0500
1.0	0.0800	0.0800	0.0750
1.5	0.0890	0.0940	0.0890
2.0	0.1035	0.1050	0.1000
2.5	0.1240	0.1170	0.1320
3.0	0.1520	0.1500	0.1540
3.5	0.1760	0.1740	0.1750
4.0	0.1855	0.1860	0.1920
4.5	0.2055	0.2000	0.2050
5.0	0.2132	0.2170	0.2145
5.5	0.2215	0.2280	0.2230
6.0	0.2440	0.2390	0.2400
6.5	0.2580	0.2500	0.2570
7.0	0.2620	0.2620	0.2690
7.5	0.2795	0.2800	0.2830
8.0	0.2945	0.2920	0.2890
8.5	0.3050	0.3080	0.3040
9.0	0.3145	0.3170	0.3200
9.5	0.3385	0.3520	0.3300
MAXIMUM	0.3755	0.3820	0.3700

INTERDECILE RANGE:

IDR	0.2345	0.2370	0.2450

CALCULATION ENDS: 11:53:30.79 11/09/95

****

ESTIMATED VAR USING SAMPLE DATA
(NOT BIAS-CORRECTED)

EQN VAR LAG	ESTIMATE	T-STAT	PROB	ADJ. R-SQD	S-SQRD
1 CONST.	0.632597E-01	1.76	0.088	-0.020	0.411078E-01
1 1	-0.987037E-01	-0.57	0.575		

EQN VAR LAG	ESTIMATE	T-STAT	PROB	ADJ. R-SQD	S-SQRD
2 CONST.	0.219011E-01	0.64	0.523	-0.030	0.397540E-01
2 1	0.802707E-02	0.05	0.964		

****

ESTIMATED VAR USING SAMPLE DATA
(NOT BIAS-CORRECTED)

EQN VAR LAG	ESTIMATE	T-STAT	PROB	ADJ. R-SQD	S-SQRD
1 CONST.	0.610871E-01	1.55	0.132	-0.052	0.435839E-01
1 1	-0.924504E-01	-0.51	0.612		
1 2	0.495796E-01	0.27	0.786		

EQN VAR LAG	ESTIMATE	T-STAT	PROB	ADJ. R-SQD	S-SQRD
2 CONST.	0.277412E-01	0.78	0.440	-0.056	0.414703E-01
2 1	-0.890665E-03	0.00	0.996		
2 2	-0.872272E-01	-0.49	0.630		

****

ESTIMATED VAR USING SAMPLE DATA
(NOT BIAS-CORRECTED)

EQN VAR LAG	ESTIMATE	T-STAT	PROB	ADJ. R-SQD	S-SQRD
1 CONST.	0.489153E-01	1.14	0.262	-0.078	0.452922E-01
1 1	-0.917612E-01	-0.50	0.623		
1 2	0.579876E-01	0.31	0.756		
1 3	0.112086E+00	0.61	0.549		

EQN VAR LAG	ESTIMATE	T-STAT	PROB	ADJ. R-SQD	S-SQRD
2 CONST.	0.255331E-01	0.69	0.499	-0.088	0.435063E-01
2 1	0.437928E-02	0.02	0.981		
2 2	-0.765560E-01	-0.42	0.681		
2 3	-0.866080E-01	-0.47	0.642		

****

ESTIMATED VAR USING SAMPLE DATA
(NOT BIAS-CORRECTED)

EQN VAR LAG	ESTIMATE	T-STAT	PROB	ADJ. R-SQD	S-SQRD
1 CONST.	0.663376E-01	1.46	0.157	-0.070	0.461639E-01
1 1	-0.640789E-01	-0.34	0.736		
1 2	0.655195E-01	0.35	0.729		
1 3	0.880184E-01	0.47	0.643		
1 4	-0.220478E+00	-1.16	0.257		

EQN VAR LAG	ESTIMATE	T-STAT	PROB	ADJ. R-SQD	S-SQRD
2 CONST.	0.336753E-01	0.87	0.391	-0.078	0.444174E-01
2 1	-0.119229E-01	-0.06	0.950		
2 2	-0.992681E-01	-0.53	0.602		
2 3	-0.885855E-01	-0.47	0.639		
2 4	-0.215752E+00	-1.15	0.261		

****

Appendix F
Output of the Bootstrap Estimates
(3-Band vs VAR)

PARAMETER STATEMENT USED IN SOURCE CODE WAS:

```
(NPEQN=2, NPDATA=200, NPPARM=20, NPLAM=20, NPDIST=1000,
 NPSIM=401, NPPOLY=50, NPFRAK=25, NPDECK=97, NPLAG=18,
 NPGAP=50, NPMXLG=30, NPMAXIT=2,NPBIAS=80)

CALCULATION BEGINS: 01:27:31.92     11/07/95

*************************************************************
*                                                           *
*        A STATISTICAL INFERENCE ENGINE                     *
*         FOR SMALL, DEPENDENT SAMPLES                      *
*                                                           *
*                 R. ASHLEY                                 *
*                                                           *
*               VERSION  2.1025                             *
*                                                           *
*                                                           *
*                                                           *
* THIS WORK WAS SUPPORTED BY NSF GRANT SES8922394 *
*                                                           *
*      COPYRIGHT (C) 1992 RICHARD ASHLEY                    *
*                                                           *
*************************************************************

CPI PPI VAR STAB 3-BAND

NDIM    =     2 IS DIMENSIONALITY OF VAR SYSTEM

NOBSIN  =     36 IS NUMBER OF SAMPLE OBSERVATIONS
              IN ORIGINAL DATA SET READ IN

NOBSAMP =     36 IS NUMBER OF OBSERVATIONS IN EACH
              DATA SET GENERATED FOR USE IN TEST.
              CAN BE SET NOT EQUAL TO NOBSIN IF
              NUSAMP=1 & (IBIAS = 0 OR IBIAS > 1)
              & ISIMDAT=0 & IMPORTON=0 & ISAMP=0
              & IBALON=0.

MAXLG   =     4 IS MAXIMUM LAG IN VAR

NBIAS   =     20 SAMPLE SIZE FOR COMPUTING BIAS
               (TYPICAL VALUE: 20)

NBIAS0  =     0 SAMPLE SIZE FOR COMPUTING BIAS
              USING INITIAL SAMPLE DATA
              (SET TO 0 WHEN IBIAS > 1 OR < 0)
```

NENSM =	2000 # OF POINTS FOR ESTIMATED DIST FCN (TYPICAL VALUE: 2000)
NUMSIM =	101 # OF REPEATS (TYPICAL VALUE: 101)
ISPLAY =	500 # OF REPETITIONS BEFORE ITERATION COUNT DISPLAYED ON SCREEN (TYPICAL VALUE: 500)
NUSAMP =	0 --> USE ACTUAL SAMPLE DATA TO INITIATE ALL NUMSIM SIMULATIONS 1 --> USE PSEUDO DATA INSTEAD TO INITIATE ALL AFTER THE FIRST (DEFAULT)
IMPRNT =	0 SWITCH FOR PRINTING IMPORTANCE SAMPLING NONCONVERGENCE MESSAGES 0 --> NO PRINT 1 --> PRINT
LAMCALC =	0 SWITCH FOR LAMBDA CALCULATION: 0 -> EXACT; NN -> SIMULATE WITH NN*NEST DATA POINTS
ISEED =	3463 INITIAL SEED FOR RANDOM NUMBER GENERATOR
ISEDIN =	0 {NOT USED IN DISTRIBUTION VERSIONS}
INFLAG =	50 ACTUAL LENGTH OF INFINITE ORDER LAG STRUCTURE FOR INVERSION IN CALCULATING LAMBDA. (TYPICAL VALUE: 50)
NUMLAM =	1 DIMENSIONALITY OF LAMBDA (IGNORED IF IMPORTON = 1)
ISTASHE =	95 NONSTATIONARITY TEST PARAMETER: (ONLY RELEVANT IF ICARE = 1) ISTASHE > 0 --> TEST WHETHER ALL ROOTS OF VAR DETERMINANT EXCEED (100./FLOAT(ISTASHE)). (TYPICAL VALUE: ISTASHE = 95)
ISAVE =	0 SWITCH FOR SAVING ESTIMATED DFS TO FILE. ISAVE=1 -> SAVE THEM
ISAVP =	0 {NOT IMPLEMENTED IN DISTRIBUTION VERSION}
ICARE =	1 SWITCH FOR ELIMINATING NONSTATIONARY MODELS FROM CONSIDERATION. ICARE=1 -> SKIP THEM
IOLSTEK =	1 SWITCH FOR SELECTING REGRESSION TECHNIQUE. IOLSTEK = 1 -> USE SVD

ALGORITHM. OTHERWISE, USE FASTER
NON-SVD ALGORITHM

IBALON = 0 SWITCH FOR SELECTING BALANCED
RESAMPLING IN THE BIAS CORRECTION
ROUTINE.
IBALON=0 -> BALANCE OFF (DEFAULT)
IBALON=1 -> BALANCE ON

ISAMP = 0 SWITCH FOR RESAMPLING (WHERE FEASIBLE)
WITHOUT REPLACEMENT
ISAMP=0 -> REPLACE (DEFAULT)
ISAMP=1 -> DO NOT REPLACE
{NOT IMPLEMENTED IN DISTRIBUTION VERSION}

INFKIND = 0 SWITCH FOR INFERENCE KIND
INFKIND=0 --> MULTIPLICATIVE
(DEFAULT FOR RATIOS)
INFKIND=1 --> ADDITIVE
INFKIND=2 --> RAW VALUES

MIDNITE = 0 SWITCH FOR RESTARTING ANY SIMULATION
THAT STRADDLES MIDNIGHT, LOUSING
UP CALC. OF # OF TICKS USED.
(ONLY USED WITH IMPORTANCE SAMPLING)
MIDNITE = 1 --> RESTART
MIDNITE = 0 --> IGNORE

ISIMDAT = 0 SWITCH FOR SELECTING TAYLOR/THOMPSON
NONPARAMETRIC DENSITY ESTIMATION
ALGORITHM (INCOMPATIBLE WITH
IMPORTANCE SAMPLING)

ISIMDAT > 0 --> USE IT
ISIMDAT = 0 --> USE DIRAC COMB -- THE BOOTSTRAP
ISIMDAT < 0 --> USE IT WITH -ISIMDAT-1
NEAREST NEIGHBORS

IMPORTON = 0 SWITCH FOR SELECTING IMPORTANCE
RESAMPLING. IMPORTON = 1 -->
IMPORTANCE SAMPLING IS ENABLED.

IMPSTART = 1000 NUMBER OF REPETITIONS TO BE USED
FOR CALCULATING IMPORTANCE PROBS
(SET TO NENSM/2 WHEN IMPORTANCE
SAMPLING TURNED OFF.)

IBAKUP = 0 PROGRAM WILL BACKUP INFO NEEDED
TO RESTART THE CALCULATIONS AT THE
END OF EACH COMPLETED SIMULATION
IF IBAKUP SET TO ONE

IRESTRT = 0 SET TO ONE IF THIS RUN IS A RESTART
(NORMALLY SET TO ZERO)

IBUSE =	0 IBUSE = 1 --> READ IN BETA IBUSE = 0 --> ESTIMATE OPTIMAL BETA
BETA =	0.400 WEIGHT TO PLACE ON RESULT FROM FIRST IMPSTART REPETITIONS. (MUST BE A FRACTION IN [0,1] (THIS INPUT NOT USED UNLESS IBUSE = 1)
BETAMIN =	0.100 MINIMUM WEIGHT TO PLACE ON RESULT FROM FIRST IMPSTART REPETITIONS. MUST BE FRACTION IN [0,1]. (THIS INPUT NOT USED UNLESS IBUSE = 0)
VMAX =	80.000 IF AN IMPORTANCE PROB DIST. IS GENERATED THAT YIELDS AN ESTIMATED VARIANCE REDUCTION RATIO > VMAX THEN IT IS DISCARDED AND NEW ESTIMATE IS MADE. (THIS INPUT NOT USED UNLESS IBUSE = 0)
IBIAS =	0 IBIAS = 0 --> NO BIAS CORRECTION) = 1 --> BIAS CORRECTION > 1 --> ESTIMATE BIAS ONCE-AND-FOR-ALL IN FIRST SIMULATION WITH IBIAS REPETITIONS. < 0 --> ESTIMATE BIAS (ONCE) FOR EACH SIMULATION USING -IBIAS SEPARATE REPETITIONS.
JSPARE =	0 IF JSPARE > 1 AND ICARE = 1 THEN AT MOST JSPARE NONSTATIONARY MODELS ARE REJECTED IN A ROW DURING THE BIAS CORRECTION ITERATIONS.

SAMPLE DATA READ FROM FILE FCUPVAR.DAT :

DATA FOR SERIES NUMBER 1		VAR TIME-DOMAIN	
0.01758820	-0.06770422	0.21864850	0.01112860
0.04579812	0.04431736	0.02791780	-0.20295160
0.17750360	0.32225150	-0.00558770	-0.20487530
0.18941060	-0.28238980	0.15539760	0.16733880
-0.04777102	0.13588460	0.61006950	-0.29661010
0.15261590	0.03229529	-0.11786700	0.31107200
0.15291220	0.49549450	0.03678969	0.08037531
-0.43185600	0.16212100	0.17615930	-0.26510150
-0.02503212	0.06595015	0.03639887	-0.11083440
DATA FOR SERIES NUMBER 2		STAB 3-BAND	
-0.07928773	-0.10471860	0.15125860	-0.01993551
0.07147363	0.01709951	0.00681679	-0.18478060
0.06743756	0.25758500	0.03820030	-0.21395760
0.04354060	-0.32052060	0.08980758	0.22863530
-0.01427093	0.05915373	0.54152060	-0.18314520
-0.06109499	-0.03007639	-0.08367777	0.31590030

0.08193654	0.48641250	0.12862030	0.02697197
-0.44366980	-0.11784320	0.09515977	-0.09501950
-0.08903138	0.07243574	0.09283744	-0.13784640

TEST STATISTICS USED:

1: MEAN SQUARE OF STAB 3-BAND / MEAN SQUARE OF VAR TIME-DOMA

VAR SPECIFICATION:

FOR EQUATION 1 THE LAG STRUCTURE ON VARIABLE 1 CONTAINS LAGS:

FOR EQUATION 1 THE LAG STRUCTURE ON VARIABLE 2 CONTAINS LAGS:

FOR EQUATION 2 THE LAG STRUCTURE ON VARIABLE 1 CONTAINS LAGS:

FOR EQUATION 2 THE LAG STRUCTURE ON VARIABLE 2 CONTAINS LAGS:

INDGAP = GAP SUFFICIENT FOR INDEPENDENCE = 50
WEIGHT = PARAMETER FOR BIAS ROUTINE = 1.00
IPRNTF = {NOT USED} = 0
NUMITER = MAX # ITERATIONS FOR BIAS ROUTINE = 1

ESTIMATED VAR USING SAMPLE DATA
(NOT BIAS-CORRECTED)

EQN VAR LAG	ESTIMATE	T-STAT	PROB	ADJ. R-SQD	S-SQRD
1 CONST.	0.495999E-01	1.26	0.217	0.000	0.496041E-01

EQN VAR LAG	ESTIMATE	T-STAT	PROB	ADJ. R-SQD	S-SQRD
2 CONST.	0.233316E-01	0.65	0.520	0.000	0.412060E-01

FOR TEST STATISTIC #1:

MEAN SQUARE OF STAB 3-BAND / MEAN SQUARE OF VAR TIME-DOMA

SAMPLE VALUE = 0.799949E+00

1ST PART OF EACH SIMULATION USES IMPSTART = 1000 REPETITIONS
2ND PART OF EACH SIMULATION USES NENSM - IMPSTART = 1000 REPETITIONS

FOR MEAN SQUARE OF STAB 3-BAND / MEAN SQUARE OF VAR TIME-DOMA

PROBABILITY THAT TRUE > 1.000

	PROBABILITY	PROB (1ST)	PROB (2ND)
SMPL	0.0230	0.0190	0.0270

1	0.0235	0.0250	0.0220
2	0.0235	0.0250	0.0220
3	0.1410	0.1400	0.1420
4	0.0150	0.0100	0.0200
5	0.0150	0.0180	0.0120
6	0.0525	0.0520	0.0530
7	0.0215	0.0190	0.0240
8	0.0175	0.0180	0.0170
9	0.0045	0.0060	0.0030
10	0.0055	0.0060	0.0050
11	0.0100	0.0090	0.0110
12	0.0150	0.0170	0.0130
13	0.0220	0.0260	0.0180
14	0.1040	0.0940	0.1140
15	0.0090	0.0090	0.0090
16	0.0085	0.0080	0.0090
17	0.0465	0.0410	0.0520
18	0.0395	0.0400	0.0390
19	0.0310	0.0300	0.0320
20	0.0695	0.0770	0.0620
21	0.0405	0.0320	0.0490
22	0.1180	0.1140	0.1220
23	0.0200	0.0180	0.0220
24	0.0080	0.0080	0.0080
25	0.0935	0.0850	0.1020
26	0.1030	0.0930	0.1130
27	0.0285	0.0300	0.0270
28	0.1150	0.1110	0.1190
29	0.0200	0.0190	0.0210
30	0.0815	0.0780	0.0850
31	0.0110	0.0100	0.0120
32	0.0380	0.0400	0.0360
33	0.0050	0.0060	0.0040
34	0.0000	0.0000	0.0000
35	0.0030	0.0040	0.0020
36	0.0065	0.0040	0.0090
37	0.0060	0.0050	0.0070
38	0.0255	0.0250	0.0260
39	0.0550	0.0550	0.0550
40	0.0285	0.0270	0.0300
41	0.1015	0.1080	0.0950
42	0.0465	0.0410	0.0520
43	0.0505	0.0490	0.0520
44	0.0515	0.0470	0.0560
45	0.1310	0.1400	0.1220
46	0.0365	0.0340	0.0390
47	0.0005	0.0010	0.0000
48	0.0020	0.0020	0.0020
49	0.0760	0.0770	0.0750
50	0.0125	0.0120	0.0130
51	0.0290	0.0260	0.0320
52	0.0850	0.0820	0.0880
53	0.0275	0.0300	0.0250
54	0.0095	0.0110	0.0080
55	0.0180	0.0190	0.0170

56	0.0335	0.0320	0.0350
57	0.0775	0.0840	0.0710
58	0.0365	0.0300	0.0430
59	0.0125	0.0090	0.0160
60	0.0025	0.0010	0.0040
61	0.1530	0.1570	0.1490
62	0.0340	0.0350	0.0330
63	0.0055	0.0060	0.0050
64	0.0365	0.0370	0.0360
65	0.0055	0.0060	0.0050
66	0.0025	0.0020	0.0030
67	0.0775	0.0710	0.0840
68	0.0285	0.0340	0.0230
69	0.0635	0.0690	0.0580
70	0.0015	0.0010	0.0020
71	0.1085	0.1110	0.1060
72	0.0045	0.0040	0.0050
73	0.0065	0.0050	0.0080
74	0.0195	0.0240	0.0150
75	0.0445	0.0520	0.0370
76	0.0155	0.0170	0.0140
77	0.0675	0.0650	0.0700
78	0.0060	0.0050	0.0070
79	0.0150	0.0150	0.0150
80	0.1075	0.1080	0.1070
81	0.0010	0.0020	0.0000
82	0.0915	0.1000	0.0830
83	0.1270	0.1390	0.1150
84	0.0835	0.0840	0.0830
85	0.0820	0.0690	0.0950
86	0.0700	0.0610	0.0790
87	0.0135	0.0110	0.0160
88	0.1080	0.0990	0.1170
89	0.0210	0.0250	0.0170
90	0.0930	0.0870	0.0990
91	0.0960	0.0980	0.0940
92	0.0050	0.0070	0.0030
93	0.0305	0.0350	0.0260
94	0.0035	0.0030	0.0040
95	0.0265	0.0210	0.0320
96	0.0375	0.0340	0.0410
97	0.2535	0.2810	0.2260
98	0.0035	0.0060	0.0010
99	0.0020	0.0030	0.0010
100	0.0000	0.0000	0.0000

MEAN	0.0427	0.0426	0.0429
ST.DEV.	0.0443	0.0457	0.0435
MEDIAN	0.0280	0.0265	0.0260
RANGE	0.2535	0.2810	0.2260
IQR	0.0610	0.0605	0.0615

DECILES AND HALF-DECILES:

MINIMUM	0.0000	0.0000	0.0000

0.5	0.0015	0.0010	0.0010
1.0	0.0030	0.0030	0.0030
1.5	0.0050	0.0050	0.0040
2.0	0.0060	0.0060	0.0070
2.5	0.0085	0.0080	0.0090
3.0	0.0125	0.0100	0.0120
3.5	0.0150	0.0170	0.0150
4.0	0.0195	0.0190	0.0170
4.5	0.0220	0.0250	0.0220
5.0	0.0280	0.0265	0.0260
5.5	0.0305	0.0300	0.0320
6.0	0.0365	0.0340	0.0360
6.5	0.0405	0.0400	0.0430
7.0	0.0515	0.0520	0.0530
7.5	0.0695	0.0690	0.0700
8.0	0.0815	0.0780	0.0830
8.5	0.0930	0.0870	0.0950
9.0	0.1040	0.1000	0.1070
9.5	0.1180	0.1140	0.1190
MAXIMUM	0.2535	0.2810	0.2260

INTERDECILE RANGE:

IDR	0.1010	0.0970	0.1040

CALCULATION ENDS: 01:33:04.32 11/07/95

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ESTIMATED VAR USING SAMPLE DATA
(NOT BIAS-CORRECTED)

EQN VAR LAG	ESTIMATE	T-STAT	PROB	ADJ. R-SQD	S-SQRD
1 CONST.	0.617668E-01	1.54	0.134	0.019	0.486405E-01
1 1	-0.227797E+00	-1.27	0.214		

EQN VAR LAG	ESTIMATE	T-STAT	PROB	ADJ. R-SQD	S-SQRD
2 CONST.	0.229183E-01	0.62	0.538	-0.033	0.425698E-01
2 1	0.152968E-01	0.08	0.934		

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ESTIMATED VAR USING SAMPLE DATA
(NOT BIAS-CORRECTED)

EQN VAR LAG	ESTIMATE	T-STAT	PROB	ADJ. R-SQD	S-SQRD
1 CONST.	0.720603E-01	1.70	0.101	0.007	0.492807E-01
1 1	-0.261467E+00	-1.41	0.169		
1 2	-0.143726E+00	-0.78	0.441		

EQN VAR LAG	ESTIMATE	T-STAT	PROB	ADJ. R-SQD	S-SQRD
2 CONST.	0.252856E-01	0.67	0.508	-0.061	0.437333E-01
2 1	0.169102E-01	0.09	0.929		
2 2	-0.835893E-01	-0.45	0.657		

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ESTIMATED VAR USING SAMPLE DATA
(NOT BIAS-CORRECTED)

EQN VAR LAG	ESTIMATE	T-STAT	PROB	ADJ. R-SQD	S-SQRD
1 CONST.	0.701781E-01	1.53	0.136	-0.028	0.510117E-01
1 1	-0.258003E+00	-1.35	0.187		
1 2	-0.137135E+00	-0.71	0.486		
1 3	0.238049E-01	0.13	0.900		

EQN VAR LAG	ESTIMATE	T-STAT	PROB	ADJ. R-SQD	S-SQRD
2 CONST.	0.275741E-01	0.72	0.480	-0.090	0.449185E-01
2 1	0.102303E-01	0.05	0.957		
2 2	-0.830981E-01	-0.44	0.663		
2 3	-0.910573E-01	-0.48	0.632		

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ESTIMATED VAR USING SAMPLE DATA
(NOT BIAS-CORRECTED)

EQN VAR LAG	ESTIMATE	T-STAT	PROB	ADJ. R-SQD	S-SQRD
1 CONST.	0.784783E-01	1.61	0.119	-0.055	0.523178E-01
1 1	-0.256402E+00	-1.33	0.196		
1 2	-0.152563E+00	-0.77	0.450		
1 3	-0.471422E-02	-0.02	0.981		
1 4	-0.104996E+00	-0.55	0.588		

EQN VAR LAG	ESTIMATE	T-STAT	PROB	ADJ. R-SQD	S-SQRD
2 CONST.	0.336753E-01	0.87	0.391	-0.078	0.444174E-01
2 1	-0.119229E-01	-0.06	0.950		
2 2	-0.992681E-01	-0.53	0.602		
2 3	-0.885855E-01	-0.47	0.639		
2 4	-0.215752E+00	-1.15	0.261		

****

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Member of the Honor Society of PHI KAPPA PHI
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