ON THE RELATIONSHIP OF DERIVATIVE ASSETS
TO THEIR UNDERLYING INSTRUMENTS

by

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(ABSTRACT)

The first essay, "Market Integration and Side by Side Trading of Derivative and Cash Instruments" inquires into the microstructure of integrated trading of derivative and cash instruments and proposes a spatial differentiation model as a framework for analysis. The model illustrates that when broker-dealers can execute cash and derivative transactions proximately they can increase their returns by serving a larger proportion of investors who hold diverse portfolios thereby helping investors to economize on transaction costs. The model predicts that transactions involving a cash and derivative will be effected through an integrated system.

The second essay, "Stock Index Futures Trading and Stock Market Volatility," reviews theoretical models and empirical evidence on the relationships between the level of futures trading and volatility. An empirical investigation is conducted by examining the relationship between the daily trading value of the S&P 500 stock index futures contract and the traded value of New York Stock Exchange stocks and considers whether there is higher price volatility in the stock markets when the level of trading in the futures markets is high relative to trading in the cash market. No evi-
idence, theoretical or empirical, is found to support the notion that futures trading leads to greater volatility in the underlying cash market.

The third essay, "Liquidation and Delivery Under Conditions of Manipulation," models how strategic traders would respond to manipulation given an option to liquidate or deliver on the contract. A perfect Bayesian equilibrium concept is used in which traders must decide whether to liquidate or deliver given the realization of the first period equilibrium futures price. If detected by floor brokers who competitively bid prices to their expected value, the manipulator will cause prices to move against him, raising the equilibrium price when he puts in orders to buy and lowering the price when he seeks to sell. Revelation of manipulation through prices also alters the behavior of other traders. An analysis of reactions in a simplified extensive form game indicates that detection of manipulation allows other market participants to strategically adjust their plans regarding liquidation and avoid incurring losses to the manipulator.
DEDICATION

This work is dedicated to my father, David Sylvester Brown.
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Chapter 1

Market Integration and Side by Side Trading
of Derivative and Cash Instruments

I. Introduction

Recently, the Chicago Board of Trade (CBOT) has sought to register its subsidiary, the Chicago Board Brokerage (CBB), with the Securities and Exchange Commission (SEC) as a broker-dealer of cash government securities. The CBOT has also sought registration of a government securities clearing agency, the Clearing Corporation for Options and Securities, and has signed an accord with the Bloomberg Financial News Service Corporation to list quotes generated from the trading of these securities. If the SEC approves the operation of this trading system, CBOT members and member firms will be able to trade government securities via a screen based trading and information system on the trading floor. Exchange officials have promoted the proposed entry into the cash government securities markets as a means to broaden market participation, increase price transparency, enhance credit, and improve surveillance and audit trails.¹ Notably, the addition of this instrument to the products already traded on the floor will initiate a market structure in which transactions in the cash instrument (Treasury bonds and notes) and the derivative (Treasury bond and note futures) could be traded "side by side" by the same individuals contemporaneously.²

¹Based on remarks by Fred Greée, CBOT Vice President of administration and planning, on Oct. 14, 1992.

²The term "derivative" here is used in the broad sense to refer to instruments whose payoffs are derived from the underlying value of a cash asset such as a commodity, bond, or stock price. Rather recently, it has come to refer more specifically to swaps and hybrid
Concerns have been raised that such integration of markets may affect market quality, create conflict of interest in principle-agent relationships, lead to trading abuses such as intermarket front-running, and may lead to the fragmentation of existing primary markets. This paper inquires into the microstructure of integrated trading and multimarket systems.

As evidence of the demand of investors and investment funds for immediacy and investment diversification, firms that act as intermediaries to multiple markets have prospered in the current marketplace. Concurrently, the physical location of the market place has become less relevant as telecommunications and computerized systems enable transactions in multiple markets for market participants who have never set foot on an exchange floor. Derivative exchanges, as developers and promoters of many of the new electronic systems, have been negotiating the listing of both cash and derivative products on their systems.³

The present paper proposes that market integration, physically or technologically, has the principal effect of altering the structure of transactions costs. Fixed costs, transactions fees, adverse selection and inventory holding costs are altered as a result of the integration of multiple markets. For example, to the extent that multimarket trading facilitates the simultaneity of multiple asset portfolio rebalancing,

³See, for example, "GLOBEX Looks to Add Equities and Options," Investors Business Daily, 9/15/92 and "CBOT Eying Leap into the Stock Market," by W. Crawford, Jr. in The Chicago Tribune, 10/22/92.
inventory holding risks are decreased. Also, by improving the linkages between the markets, integrated systems have the potential to lower intermarket frictions. Improved linkages imply faster convergence to market equilibrium, reducing the length of lags in price adjustment. The price discovery process is enhanced because traders themselves have access to the underlying market and thus prices become more informative and less noisy. As a result, an exchange that becomes more integrated will experience increased transaction volume and decreased market volatility.

To better understand the factors that lead to integrated markets, this paper proposes a spatial differentiation model in which the transactions costs associated with acquiring an investor's optimal portfolio determine his choice of broker-dealer. Broker-dealers compete for order flow on a basis of best execution for their customers. The model illustrates that when broker-dealers can execute cash and derivative transactions via a multimarket trading system, they can increase their profits by serving a larger proportion of investors who hold diverse portfolios. Since investors in both cash and derivatives will choose the broker-dealer who offers order execution at the lowest cost, ceteris paribus, the model suggests that all transactions involving a portfolio of cash and derivative assets will be affected through an integrated system or exchange.

The paper provides some historical background on side by side trading, the geographical expression of market integration, and outlines the regulatory and institutional considerations that have been raised with respect to integrated trading.
The microstructure of linked asset markets is then discussed, the characteristics of which suggest a spatial differentiation approach. The model is then presented, followed by the conclusion, and suggestions for future research.

II. Regulatory and Institutional Background

Shortly after the CBOT created the Chicago Board Options Exchange, an options-only exchange that began trading call options in 1973, preexisting stock exchanges began to introduce options markets: the American Stock Exchange (AMEX) in 1975, followed by the Philadelphia Stock Exchange (PHLX) and Pacific Stock Exchange (PSE) in 1976. In its approval of the PHLX options market, the first exchange to seek to trade an option on an underlying security also traded at the exchange, the SEC stated:

"the Exchange will physically separate the option trading floor from its regular floor for trading stocks and other securities to prevent visual and direct auditory communication between the two trading areas. ...primarily to bar the misuse in its options market of information obtained by floor members relating to activity in an underlying security where the information has not yet received public dissemination."4

The SEC view that options trading was particularly vulnerable to trading abuses and was not acceptably regulated lead to a moratorium on expansion of options markets beginning on July 15, 1977, until an exhaustive study could be completed. The Report of the Special Study of the Options Markets (Options Study), completed in 1979,

suggested that market integration, while it may improve market quality, could provide unfair informational advantages to market participants on the floor, enable manipulation and trade abuses, and complicate market surveillance. The Options Study further suggested that if markets were integrated to the point that marketmakers participated in both markets, the arrangement could lead to conflicts between the stock and options obligations of these marketmakers.\(^5\)

The specialist has an affirmative obligation to make a market in his specialty stock, maintain a market presence by continuous posting of quotes, and must trade in a stabilizing manner. It is a well documented tenet of market microstructure that these obligations leave the specialist exposed to exogenous inventory shocks and adverse selection bias.\(^6\) Rather than encourage specialists to hedge their substantial inventory risk, however, barriers to entry into risk shifting markets such as options and futures were erected. In the early years of options trading, anticipatory hedging of block transactions was interpreted by the SEC to violate exchange rules prohibiting transactions inconsistent with just and equitable principles of trade.\(^7\) Further, a market participant who had some knowledge of or was about to embark upon a large trade in

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\(^5\)Dual marketmaking is not likely to be incentive compatible. The lower risk associated with an integrated market would be unobtainable since affirmative obligations of a marketmaker to stabilize both markets would expose him to increased risk.

\(^6\)Hasbrouck and Sofianos (1992) find that the inventory holding risks are at least in part balanced by informational advantages embedded in the knowledge of the limit order book and computerized order flow.

\(^7\)For example, CBOE rule 4.1 was so interpreted.
stock (for example, because of their marketmaking activity) was prohibited from transacting in the option prior to the stock transaction. Even after regulatory barriers were relaxed, conditions were established to ensure that specialists do not acquire positions that exceed specified "hedge ratios".8

The potential for increased use or misuse of derivatives by market professionals under a regime of electronically integrated trading has contributed to further apprehension regarding the implementation of screen-based trading systems that facilitate multimarket trading. Of specific concern is the extent to which multimarket trading via automated trading systems may lead to improper trading practices. Domowitz (1992) notes that trading ahead of the customer "may be enabled by prior placement of personal orders in the different, but price-related commodity, from a single terminal location." There is also concern that the protocol for order entry would encourage front-running. For example, if orders are entered without recording of the time received, there would be no way to verify that which was initiated first: the customer order or that of the broker-terminal operator. Hence, execution priorities relating to price, order size, and time precedence are important considerations in the implementation of automated systems.9

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9See Harris (1990) for extensive discussion of the impact of trading procedures in automated systems.
Along with their concerns, practitioners and regulators have recognized that integrated trading may improve market quality by increasing the liquidity and depth of affected markets. The Options Study (1979) acknowledged that integrated trading would allow marketmakers like specialists the ability to hedge and reduce risk, encouraging them to commit capital and smooth order imbalances. Increased information flow to marketmakers regarding companion markets would lead to more rapid and accurate price adjustment.\textsuperscript{10} Further, because of improved price efficiency and lower risk associated with market making, "stock and options marketmakers may be able to narrow the spreads in their quotations, and by bidding and offering in greater size, to accumulate larger positions. This, in turn, may facilitate deeper and more liquid markets for both securities."\textsuperscript{11}

By combining inputs, such as using the same hardware or merging order routing and processing services, integrated markets could become more operationally efficient. Efficiency gains in the operation of integrated cash and derivatives markets may also result when clearing and settlement functions are consolidated. First, a futures contract could be marked-to-market based on the closing cash, which may be more easily accessed if the settlement price is based on contracts traded on the system. Further, by marking of cash positions to market on a daily basis, integrated clearing could improve risk management by lowering day to day risk, enhancing the integrity

\textsuperscript{10}Options Study, 879.

\textsuperscript{11}Ibid., Footnote 233, 878 - 879.
of the market. The implementation of a crossmargining system in which exchange
members are allowed to consolidate margining on their cash and derivative position
would decrease total margining of positions. Daigle (1991) notes that, in the absence
of a crossmargining system, intermarket spread positions, which extend across
regulatory jurisdictions (for example, when the positions are composed of cash and
futures, or, options and futures, etc.), are margined as separate positions. If integrated
trading is accompanied by a crossmargining system, then the benefits described in
Daigle, such as reduced overcollateralization and improvements in the synchronicity of
clearing and settlement, may result. Thus, unified clearing and margining will
improve the operational efficiency of the market by reducing cash flows between the
cash and derivative asset resulting in a more efficient use of capital.

A contiguous geographical location has also been found to have a positive
effect on the liquidity and the ultimate success (sustained volume) of companion
products. For example, a study by Silber (1981) found that the proximity of the
GNMA-CD (certificate delivery) contract to the GNMA-CDR (Collateralized
 Depository Receipt) on the Chicago Board of Trade (CBOT) encouraged spread
trading and cross hedging. A similar GNMA-CD contract introduced on the Amex
Commodity Exchange (ACE), which did not offer the proximity of the companion
product, failed to generate adequate volume to get off the ground. This evidence is

\footnote{See "CBOT Eyes Linking Members' Margin Data to New Trading System," Bondweek, May 6, 1992, p. 98.}

consistent with the notion that an arrangement that facilitates the trading of complementary assets by lowering the transactions costs associated with trading both assets increases the depth and liquidity of the assets.\textsuperscript{13}

The potential efficiency gains inherent in the integration of markets and the controversy surrounding that integration are in part explained by the unique microstructure of these markets. The market for financial assets, as a venue for the minimization of search costs and provision of liquidity, is characterized by economies of scale. Blume and Siegel [1989] note that, "Order flow attracts order flow." Market consolidation, consequent of the self-reinforcing demand for immediacy and liquidity, is not always viewed as desirable from an antitrust perspective. The institutional structure and regulation of securities markets has long been a matter of controversy in that, viewed through the economic paradigm of monopoly, the consolidation and integration of markets may be construed as anticompetitive.

Market frictions in asset markets, including fixed costs, discontinuities, and indivisibilities, constrain individuals from transacting in multiple markets costlessly and can result in fragmented markets. Among the frictions, fixed costs associated with trading such as exchange fees, membership or "seat" costs raise the cost of trading in multiple markets. A subset of investors and market professionals may find it too costly to maintain a membership in more than one market or they may face burdensome documentation and paperwork requirements whenever they effect trades in

\textsuperscript{13}For further discussion on the determinants of the fate of contracts, see Black (1986).
linked markets. Additional costs are entailed in the development and acquisition of the technology necessary to trade in multiple markets.

Transitory variation in the supply of liquidity creates discontinuities in the satisfaction of the demand for immediacy and impedes the efficient attainment of optimal positions. Further, assets are usually tradable in discrete quantities, presenting some economies and diseconomies of size.\textsuperscript{14} For a transaction to be consummated, there must be a buyer and a seller, at the same place, and the same time, willing to exchange the same asset. If there were constant costs per transaction, an abundant supply of liquidity, and assets were infinitely divisible, then investors would be able to fine tune their portfolio anytime they wanted utilizing as many markets as necessary. But frictions create lags in price adjustment that, in addition to introducing stale prices and distorting the price discovery process, impede the attainment of equilibrium. Further, the existence of these frictions leads investors to economize on the number and timing of adjustments made to their portfolios.

There is a specific role for derivative markets in models of investor decision making. In portfolio choice models, the introduction of derivatives into an incomplete market increases the expected utility of traders by allowing an investor to structure his portfolio according to his desired risk/return profile.\textsuperscript{15} Further, whereas securities and cash markets serve primarily as capital markets and facilitate the transfer of ownership

\textsuperscript{14}For example, market impact may give rise to diseconomies of size.

\textsuperscript{15}See, for e.g., Hakansson (1978).
rights, derivatives are a specialized vehicle for the transfer of price risk. By establishing standardized terms for their tradable assets, futures and options exchanges offer a lower order mapping that summarizes the preferences of highly diverse investors. If successful, these markets assemble the demands of investors in a way that fosters market depth and liquidity and allows users to minimize bargaining and search costs.

Transactions costs incorporate the cost of an exchange seat, transactions fees, inventory holding costs, and costs associated with adverse information. Existing research regarding the precise contribution of these components, particularly as they manifest in dealer markets, can be found in Roll (1984), who formalizes order processing costs and their influence on the bid-ask spread, Ho and Stoll (1981), who model inventory holding costs, and Glosten and Milgrom (1985), who give a prominent role to adverse information in determining the spread. Any change in market structure that changes these costs will also alter transactions costs faced by investors.

Taking the diverse microstructure and composition of the exchanges and the characteristics of cash and derivative assets, a differentiated products model provides a simple yet meaningful framework for analysis. Spatial differentiation models have provided insight into location choice, product differentiation, and non-price competition. In their review of the important literature on product differentiation, Eaton and Lipsey (1989) note that Hotelling’s (1929) work represented a breakthrough
in economic literature because of its contribution to our understanding of oligopolies and competition in differentiated goods. Tirole (1989) notes that these models provide insight and can help to explain the choices of utility maximizing individuals with heterogeneous tastes, the clustering of competitive firms at retail centers such as malls, and the selection process of voters with heterogenous beliefs.\textsuperscript{16} The model presented here is structurally similar to that of Eaton and Lipsey (1982). In their model, various indivisibilities in the activity of shopping motivate multipurpose shopping and the existence of central places. The natural application of such a model to the current problem stems from the structural requirements that the economy consist of at least two differentiated assets with equilibria derived from the optimizing behavior of individual agents with diverse demands.

\textit{III. Formulation of the Model}

The model consists of investors and broker-dealers. Investors maximize expected utility by acquiring at the least cost their individually optimal portfolios of cash and derivative assets. Broker-dealers execute the orders of investors in order to earn a return on brokerage activity, either through the market spread or commissions. Since broker-dealers earn a commission on each order they execute, they will seek to maximize the number of orders they handle subject to their costs. These costs include

\textsuperscript{16}Tirole (1989), in chapters 2 and 7, offers a good overview of spatial, horizontal, and vertical differentiation models, and suggests these and other applications.
transactions fees, inventory holding costs, and potential losses due to adverse information.

For convenience the cash asset is referred to as a stock and the derivative asset an option (on the stock) although they could just as easily be designated as treasury bonds and treasury bond futures. The precise relationship between the cash and the derivative need not be specified since it is assumed that the solution to an investors portfolio problem yields a particular demand, $x^k$, where $k$ indexes the type of portfolio. The type of portfolio is either one consisting of stocks, designated by superscript $S$, options, designated by superscript $O$, or both, designated by superscript 2. $S$ and $O$ are only sold in indivisible lots of size $1/\alpha$ and $1/\beta \geq 1$. Given this demand, investors must choose which broker-dealer(s) they wish to execute their transaction(s).

Each period, an investor evaluates her portfolio and decides whether it is necessary to rebalance. Given her evaluation, she chooses whether to notify her broker to implement a transaction in stocks, options, or both markets. Each investor $i$ solves for her individually optimal transaction, denoted $x^i_t$, and seeks to choose a broker-dealer who will minimize the transactions costs associated with executing her order and maintaining her account. Thus, $x_i$ denotes a particular portfolio demand, which includes stocks and/or options. It is assumed without loss of generality that all $x_i$, $i=0...I$, yield a particular distribution of demands that are disbursed uniformly over $i$.

Investors also seek to avoid risks associated with holding inventory in excess of their current portfolio demand. Since inventory holding costs are non-trivial, the
investor will never want put an order to trade more stocks than $1/\alpha$ and $1/\beta$. It is assumed that she rebalances her portfolio at a constant rate and for simplicity the rate is normalized to unity. The indivisibility of size in asset market transactions, $1/\alpha$, $1/\beta \geq 1$, means she cannot buy less than one lot of $S$ or $O$. Thus, the probability that an investor will need to rebalance her stock portfolio each period is given by $\alpha$ and similarly, the probability that an investor will adjust her options holdings in given by $\beta$. Investors are indifferent between brokers who offer execution of their desired portfolio position at the same cost and select randomly from these brokers.

A broker-dealer specializes in providing brokerage services to customers in stocks or options and his primary role is one of intermediary who allows investors to economize on the search and bargaining costs of transacting. Broker-dealers (henceforth referred to simply as brokers) in this model maximize revenue subject to operating costs. As noted above, brokers face operating costs, which include the cost of an exchange seat, transactions fees, inventory holding costs, and costs associated with executing orders of those with private information. A representative broker's profit can be expressed as:

$$\Pi^k_i(q_i,q_{-i}) = p^k_i(q_i,q_{-i})q_i - c_i^k [\kappa(q_i), \sigma^2 \tau] \quad k=S,O; \ i=0,...,J$$  \hspace{1cm} (1)

where $q_i$ is volume of customer orders for $k$ handled by a broker of type $k$, $p_i = p_i(P_b - P_a)$ in which $(P_b - P_a)$ is the bid-ask spread, $c$ gives the execution costs associated with buying or selling one lot of $k$, $\kappa$ represents exchange fees (costs associated with
registration, clearing and settlement, membership fees, etc.), and \( \sigma^2 \tau \) denotes inventory holding risk where \( \sigma^2 \) is the variance of asset prices and \( \tau \) denotes the length of time a naked position is held.

An exchange or a trading system primarily acts as a medium through which orders are matched and transactions efficiently processed. Dahlman (1979) notes that exchanges seek to establish trading rules and set their scope in such a way as to minimize search costs, contract enforcement costs, and other costs associated with information collection, bargaining and decision making. Maximizing the volume of transactions affected in exchanges increases their profit and helps maintain their viability as a market. Thus, an exchange of type \( k \) has a profit function that depends on volume:

\[
\Pi^j = V_j(q^j)P^k + \phi - K^j \quad j=S,O,2
\]

in which the volume \( V(q^j) \) is a function of quantity of stocks and options business conducted at the exchange, \( \phi \) denotes an arbitrarily small transaction fee collected and \( K \) is the fixed cost initiating a new exchange, or a different type asset, or system to the current exchange. \( P^k \) refers to the prices \( P^s \) and \( P^o \), which are taken as parametric.

The market is covered (all investors are served) and prices do not differ such that arbitrage using stocks and options is profitable.

**Equilibrium Conditions**

Necessary and sufficient conditions for an equilibrium can be described as
follows.

I. Brokers who are currently operating in stock or options markets cannot increase their business by routing order flow to another exchange.

In equilibrium, a broker distributes order flow optimally among exchanges. He knows the minimum cost portfolio offered at each exchange and knows that to maintain a customer’s business, he must route each customer’s order to the exchange that minimizes costs. In the model, a broker provides his customers with the lowest cost execution by routing each order to the exchange that minimizes the linear distance between the portfolio demand of each investor and the minimum cost transaction offered at the exchange. In equilibrium, brokers have no incentive to change the distribution of orders.

II. Existing exchanges must at least recoup the costs of operating through trading activity.

\[ R^j = \nu^j(q^k)P^k \leq K^j \quad j = S, O, 2 \]  

\[ R^{j+1} = \nu^{j+1}(q^k)P^k < K^{j+1} \quad j = S, O, 2 \]  

III. For any new exchange or new exchange system of type \( k \) that offers an additional asset \( J+1 \):

In equilibrium, the market is such that no new exchange would find it profitable to enter, nor would existing exchanges seek to introduce systems to accommodate other asset markets.
Description of Equilibria

An exchange or system in which stocks and options are traded side by side (referred to as a multiasset exchange) will be abbreviated by \( E^2 \); only options are traded, \( E^o \), and only stocks, \( E^s \). An exchange's type, stock or options, is superscripted while its reference point will be subscripted. The reference point corresponds to the portfolio of assets \( x_i \) offered at the exchange that minimizes the execution costs of the investor \( i \). Let \( a_i \) be the minimum cost portfolio of stocks available at exchange \( E_i^s \). Similarly, let \( b_i \) be the minimum cost portfolio of options available at exchange \( E_i^o \), \( b_2 \) be the minimum cost portfolio of options available at exchange \( E_2^o \), \( c_1 \) be the minimum cost portfolio of stocks and options available at \( E_1^o \), etc.

The investor whose demand for stocks, given by \( x_i \), just matches the minimum cost portfolio will find that \( a_i \) optimizes her objective precisely. She will obtain her optimal portfolio from broker \( F_i \). Other investors have differing demands, however, which lead them to seek the lower costs of a broker that offers a portfolio that provides a better match. The degree of market integration is inferred by the extent to which brokers bundle the portfolio demands of his customers and route them to exchanges offering stock, option, and stock and option portfolios. Increased order flow to integrated exchanges can be interpreted as 1) side by side trading in which stock and options trading occurs at the same location or, 2) the offering of stocks and options on the same automated trading system.
Proposition 1. At most one type of exchange, stocks-only or options-only, will serve the segment of the market served by an integrated exchange.

Formal proof of Proposition 1 is in the appendix. It is trivially true that a stock-only broker maximizing the volume of his business would, when faced with choosing between an E' or an E° with the same cost structure, not choose to operate at an E°. At an E°, only orders to sell or purchase options can be processed. However, choosing between an E' or an E°, ceteris paribus, a broker of stocks would prefer to operate at an E'. At an E' exchange, he can serve both stock-only customers and stock and option customers. He will be willing to allocate his business accordingly among exchanges to the extent that he does not lose more customers than he gains.

To derive Proposition 1, consider a representative broker F; who must choose an allocation of orders among exchanges referenced over i, but whose distribution is determined explicitly by the types of assets offered. The representative stock broker F; is able to specialize in trading assets with particular characteristics and can consolidate orders in such a way as to minimize his costs. His
movement in market space represents a change in the distribution of order flow among exchanges and thus, is an alteration of transactions costs faced by affected investors.

The choices of brokers like $E_i$ lead to substantial market integration even though single market exchanges maintain a localized market share of single market patrons. As the model is constructed (zero sum gains and losses), single markets suffer liquidity loss as other exchanges become more integrated and attract away business. For example, under Case 3 in the appendix, a broker handling stocks and options transactions must conduct options trading in an options-only exchange $E_o$, and his stocks business to a stock-only exchange. $E_i$ attains total order flow of:

$$V(q^{\theta}) = \alpha(1-\beta)(a_i - \frac{a_{i-1} - a_j}{2}) + \alpha \beta r \frac{a_i}{i}$$

(5)

The availability of both stock and options trading at $E_o$, however, would reduce volume at $E_i$ by the last term, $\alpha \beta t(a_i/i)$. As noted earlier, exchanges seek to maximize volume subject to fixed costs. The exchange faced with fixed costs that exceed the gain from increased volume resulting from introducing the companion asset or is constrained from introducing the companion assets experience a decrease in profits as a multiasset (offering options and stocks) market forms within its range.\(^\text{17}\) Thus, a single asset exchange subject to cost and/or regulatory constraints would be adverse to the entry of integrated exchanges that offer products additional to their product offerings.

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\(^\text{17}\)This real world assumption is one we have adopted in order to keep the model simple. If we wanted to calculate the cost of innovating we would also have to consider the reactions of other exchanges to the innovation.
For investors, however, transactions costs on average are either unchanged or lowered.

Proposition 1 also provides a theoretical explanation for the observation that exchanges place a high priority upon the offering of new products and other innovations. Unless an exchange provides an innovation that allows it to serve the diverse demands of the market, it has little chance of attracting order flow away from its competition.

A second proposition follows from the proof of proposition 1.

**Proposition 2.** Equilibrium is characterized by at least one E', an integrated exchange in which stocks and options are traded.

It has been shown that a broker-dealer who handles one type of order, operating in a market where his order flow can be assigned to any number of single asset exchanges, would nevertheless maximize his profits by directing his order flow to an integrated exchange. An exchange that is not constrained to operate as a single asset market has incentive (in the form of increased volume) to become an E'. This will be true as long as the gain from forming the exchange is greater than Kᵢ (equilibrium condition II). Thus, as long as equilibrium conditions I and II are satisfied, an exchange will have incentive to offer derivative and cash assets, and equilibrium is characterized by at least one such exchange.

Given a market in which single asset exchanges exist, there are pressures in the model world as constructed that propel the market toward integration. The market will integrate until equilibrium condition III is satisfied. Upon satisfaction of this
condition, the benefit to an exchange from market integration (increased volume) is
dissipated and an "optimal" number of multiasset exchanges will obtain.

The equilibria in this market have some interesting features. Most
straightforward among them is the following proposition.

Proposition 3. All orders involving both a stock and an option will be
executed in an exchange that offers both stocks and options.

Harris (1992) remarked that "diverse traders desire diverse markets."
Proposition 3 provides a theoretical construct for this notion and is derived from the optimizing
behavior of agents in a market characterized by reasonable simplifying assumptions.

While the market as constructed does not yield a unique equilibrium, conditions
can delineate the range of particular equilibrium configurations. In order to simplify
the expressions, we can define a ratio of parameters \( \lambda = \frac{\alpha P^K}{\beta^P K^S} \) and derive the
following equilibrium conditions as necessary for the existence of a stock, options, or
multiasset exchange:

\[
\begin{align*}
\text{If there exists an } & 1. E^s, \text{ then } \lambda > \frac{1}{1 - \beta}. \\
2. & E^o, \text{ then } \lambda < 1 - \alpha. \\
3. & E^2, \text{ then } \frac{1 - \alpha}{2} < \lambda < \frac{2}{1 - \beta}.
\end{align*}
\]

\(^{18}\)From his remarks in Screen-Based Trading and Beyond: The Future for Financial
Sufficient conditions for an equilibrium are as follows:

1. $\lambda \geq \frac{2}{1-\beta}$ is sufficient for an $E^s$,
2. $\lambda \leq \frac{1-\alpha}{2}$ is sufficient for an $E^o$, and
3. $1-\alpha \leq \lambda \leq 1-\beta$ is sufficient for an $E^2$.

(7)

Derivation and proof of these conditions are found in their entirety in the appendix.

In the process of deriving necessary conditions, an ancillary result emerges. In the market served by a multiasset exchange, an options-only exchange will not have incentive to enter when $\lambda > 1$. A stock-only exchange will not have incentive to enter when $\lambda < 1$. These results imply that when equilibrium contains multiasset exchanges, only one type of single asset exchange can also exist. Thus, one single asset market chooses to innovate by forming a multimarket exchange, it forces its competition to either do the same or suffer a loss of order flow to the multimarket exchange. An interpretation of this result of the model should not be overstated. The total diversion of volume to one exchange (market consolidation at its most extreme) is not inevitable but can be avoided by keeping pace with innovation and the integration of competing exchanges.

To complete the model, consider the equilibrium attained when all brokers compete for order flow under the assumption that the quantity of order flow they can attract does not affect the behavior of other brokers. Each broker $i$ of type $k$ would
like to operate as a monopolist over his portion of exchanges’ market share such that:

$$p_i(q_i^*, q_{-i}) + \frac{\partial p_i(q_i^*, q_{-i})}{\partial q_i} q_i^* = c_i' \left[ \kappa(q_i^*), \sigma^2 \tau \right] \quad i = 1, \ldots, I$$

(We drop the superscript for broker type.) The volume of customer orders handled by broker $i$, denoted below by $Q_i(q_i)$, is a function of the orders processed by other brokers who also are seeking to maximize their profit:

$$q_i^* = Q_i(q_1^*, \ldots, q_{i-1}^*)$$

$$\vdots$$

$$q_i^* = Q_i(q_1^*, \ldots, q_{i-1}^*)$$

which in equilibrium results in:

$$p_i(q_i^*, q_{-i}) + \frac{\partial p_i(q_i^*, q_{-i})}{\partial q_i} q_i^* = c_i' \left[ \kappa(q_i^*), \sigma^2 \tau \right] \quad i = 1, \ldots, I$$

Maintaining the same competitive assumptions with regard to brokers as those made for exchanges (new entry is not profitable and broker’s returns are adequate to maintain a market presence), the equilibrium spread is given by:

$$p_i^* = \frac{c_i \left[ \kappa(q_i^*), \sigma^2 \tau \right]}{q_i^*} \quad i = 1, \ldots, I$$

Since the structure of the market is one characterized by increasing returns, average
costs are decreasing in q. Thus, the spread for transactions decreases as the volume of orders increases.

One interesting characteristic of these type of equilibria is the result that even when competitive brokers earn zero profit, average costs may not be at their minimum. A classical interpretation of this result (referred to in the literature as Chamberlinian excess capacity) would be that, when free entry of broker-dealers into an exchange is allowed, equilibrium is characterized by more broker-dealers competing for a finite market share than is Pareto efficient.\textsuperscript{19} The model implies that increasing returns in the provision of a liquid market dictates that exchanges impose limitations on the number of broker-dealers in the market place. There are limitations on the number of members exchanges admit (there are a fixed number of tradable memberships or seats). Thus, this structural feature of exchange operation acts as a mechanism that ensures that the economic rent associated with seat ownership is not completely exhausted.

\textit{IV. Conclusion: Empirical Implications and Suggestions for Further Research}

The model shows that the preferences of investors and the optimization of brokers leads to substantial market integration even though single market exchanges maintain a specialized market share of single market investors. Non-integrated markets experience a decrease in trading volume as competing exchanges expand their

\textsuperscript{19}For discussion of the controversy surrounding the nature and efficiency of Chamberlin’s excess capacity theorem, see Eaton and Lipsey (1989), 763 - 764.
product line to include complementary assets, become more integrated, and attract away business. The exchange who finds the cost of integrating too high or is otherwise constrained from innovating (for example, by regulation providing exclusive rights to trading specific assets to particular exchanges) would experience a decrease in profits as an integrated market introduces products which satisfy similar portfolio demands. An interesting prediction of this analysis is that single asset exchanges subject to cost and/or regulatory constraints would be adverse to the introduction of integrated exchanges who offer products similar to their own. This is expected irrespective of investors, however, whose transactions costs on the whole are either unchanged or lowered.

In addition to the specific propositions set forth above, this paper suggests some discernable effects the implementation of an integrated trading system has on transactions costs. First, since integrated trading facilitates contemporaneous asset acquisition and hedging, inventory holding risk is reduced as market participants (including broker-dealers with multimarket access) can ensure themselves against price moves that may occur while holding a naked position. Adverse information costs are also mitigated because access to companion market information is improved and unprotected market positions can be hedged contemporaneously. Market integration may improve the operational efficiency of the market by unifying clearing and settlement, reducing cash flows between cash and derivative, and improving audit trail and surveillance capability. Thus, order processing costs per transaction are expected
to decrease with the introduction of integrated markets. A unified clearing and settlement system, with its potential to lower transactions costs and decrease default risk, contributes to the volume and liquidity of both markets.

For professional traders and the investing public, an integrated market allows market participants to economize on a variety of trading costs. For example, if the CBOT were granted recognition as an interdealer broker in government securities, exchange members who routinely trade in the futures markets would gain additional access to the cash market and information regarding the government securities markets. Market participants, particularly T-bond locals and broker-dealers on the trading floor, would experience reduced trading costs since additional access to the underlying market could decrease potential losses due to adverse information. To the extent that information transmission across the government securities futures and cash markets is improved, the price discovery function of the markets would be enhanced as intermarket frictions are reduced.

By improving the linkages between the markets, market integration lowers intermarket frictions. Chowdhry and Nanda (1991) show that when a security trades in multiple markets simultaneously, informed traders can profit in part due to informational disparities across markets. As noted, integrated markets will enhance information transmission between the cash and derivative. The ability of informed traders to capitalize on their information in multiple markets as described in Chowdhry and Nanda will be lessened in integrated markets since the frictions (e.g., separation in
the incorporation of information into prices), which detach the markets, will be decreased. Increased linkages imply faster convergence to market equilibrium, reducing the length of lags in price adjustment. This would enhance price discovery as prices become less noisy. Thus, integrated markets may decrease noise-based volatility in both the underlying cash and derivative.

The model lends itself to empirical analysis and suggests the course of future research. Studies have shown that transactions costs associated with trading an asset, in particular the bid-ask spread, decline following introduction of derivatives on that asset. Similarly, the model predicts that bid-ask spreads in both markets should decline following market integration. This prediction may be tested using an event-study methodology. For example, Damodaran and Lim (1991) found that bid-ask spreads associated with a particular stock decline following option listing. Another prediction of the model is that diverse traders such as those who are fairly well capitalized and diversified in their market activity, would be active users of integrated markets. Further, results of Glosten (1992) that predict that automated trading systems will come to dominate the financial markets are augmented here in the finding that such systems are likely to take the form of integrated, multiasset trading environments.
References


Appendix


Consider the market share of a stock-only exchange bounded by two integrated exchanges, referenced by minimum cost portfolios $c_1$ and $c_2$. The stock exchange's order flow would be given by $(c_2 - c_1)/2$. Equilibrium condition II suggests that:

$$\alpha (1 - \beta ) t \left[ \frac{c_2 - c_1}{2} \right] P s \geq K^s$$  \hspace{1cm} (1)

That is, the market share of this exchange must be large enough such that:

$$c_2 - c_1 \geq \frac{2K^s}{\alpha (1 - \beta ) t P^s}$$  \hspace{1cm} (2)

For this firm to remain a stock-only exchange, it must be the case that it has no incentive to introduce options (equilibrium condition III). Thus, for $E^*$ to exist in equilibrium, it must be the case that:

$$\beta t \left[ \frac{c_2 - c_1}{2} \right] K^o \leq K^o$$  \hspace{1cm} (3)

$$c_2 - c_1 \leq \frac{2K^o}{\beta t P^o}$$  \hspace{1cm} (4)

Thus, for a stock-only exchange to exist, a necessary condition is that both these equations are satisfied giving:
\[
\frac{2K^s}{\alpha (1-\beta) tP^s} \leq C_2 - C_1 < \frac{2K^o}{\beta tP^o}
\]

which yields:

\[
\frac{1}{1-\beta} < \frac{\alpha P^o K^o}{\beta P^o K^s} = \lambda
\]

Similarly, for \(E^o\) to exist in equilibrium, equilibrium condition II requires that:

\[
\beta (1-\alpha) t \left[ \frac{C_2 - C_1}{2} \right] P^o \geq K^o
\]

\[
C_2 - C_1 \geq \frac{2K^o}{\beta (1-\alpha) t P^o}
\]

\(E^o\) will not have incentive to introduce stocks as long as equilibrium condition III holds which yields:

\[
\alpha t \left( \frac{C_2 - C_1}{2} \right) P^o < K^s
\]

So, for an options-only exchange to exist in equilibrium, a necessary condition is:

\[
\frac{2K^o}{\beta (1-\alpha) t P^o} \leq C_2 - C_1 < \frac{2K^s}{\alpha t P^s}
\]

which yields:

\[
\lambda < 1-\alpha
\]

For at least one stock and one options exchange to cover costs at an \(E'\), the following can be shown:
\[
C_2 - C_1 \geq \frac{2(K^o + K^s)}{\alpha tP^s + \beta tP^o}
\]

(11)

For options-only exchange to not wish to enter equation 4 must be satisfied. The joint solution of equations 11 and 4 yield the condition that \( \lambda > 1 \). For a stock-only exchange to not have incentive to enter equation 2 must be satisfied. The solution of equations 11 and 2 leads to the condition that \( \lambda < 1 \). The contradiction of these results lead to the implication that when the equilibrium contains integrated exchanges, only one type of single asset exchange can also exist.

The state under which no stock-only exchange will enter is observed when equation 1 is not satisfied:

\[
C_2 - C_1 < \frac{2K^s}{\alpha (1-\beta) tP^s}
\]

(12)

For an options broker to wish to operate in an \( E' \), the market must be large enough to support an additional broker:

\[
C_2 - C_1 \geq \frac{K^o}{\beta tP^o}
\]

(13)

Equations 12 and 13 provide the sufficient conditions for equilibrium with integrated and stock-only exchanges:
\begin{equation}
\lambda < \frac{2}{1 - \beta}
\end{equation}

Similarly, for a stock broker to enter an \( E' \) requires:

\begin{align}
C_2 - C_1 & < \frac{2K^o}{\beta (1 - \alpha) tP^o} \\
C_2 - C_1 & \geq \frac{K^o}{\alpha tP^o}
\end{align}

which solve for the sufficient condition for an options-only exchange in equilibrium:

\begin{equation}
\lambda > \frac{1 - \alpha}{2}
\end{equation}

A necessary condition for an \( E' \) in equilibrium is one in which 14 and 16 are satisfied:

\begin{equation}
\frac{1 - \alpha}{2} < \lambda < \frac{2}{1 - \beta}
\end{equation}

With the sufficient condition for an equilibrium in which only \( E' \) exist is one in which necessary conditions for stock-only and options-only exchanges are not present. This occurs when equations 6 and 10 are not satisfied, thus giving:

\begin{equation}
1 - \alpha \leq \lambda \leq \frac{1}{1 - \beta}
\end{equation}

\textit{Proof of Proposition 1:}

\textbf{CASE 1.} Consider the broker \( F^o_i \) who apportions his orders among exchanges \( E^o_i \) and \( E^o_{i+1} \) and who wishes to offer a minimum cost stock portfolio, which has characteristics that are demanded by investors over the interval \([0,b_i]\). By construction, broker \( F^o_i \)
would like to choose an allocation that maximizes the number of investors he serves. This allocation corresponds to the selection of minimum cost portfolio, which we shall denote \( a_i \). Suppose \( F_i \) chooses to offer portfolio \( a_i \), the maximum number of customer orders he handles is given by:

\[
q_i^s = \alpha (1 - \beta) t \left( a_i + \frac{a_i + a_{i+1}}{2} \right)
\]

\[
q_i^o = \alpha \beta t \left( a_i + a_{i+1} \right) - b_i
\]

Note that \( q_i^s \) denotes the volume of customer orders who seek stock-only transactions and \( q_i^o \) denotes the volume of customer orders who seek stock with options transactions. Broker \( F_i \) could increase the number of customers, both stock customers and stock and options customers if he offers a portfolio with characteristics of \( a_{i+1} \).

This scenario violates equilibrium condition I, thus is not an equilibrium.

**CASE 2.** Consider the broker \( F_i \), choosing a minimum cost portfolio at \( a_i \) who seeks to maximize the customers he serves over the interval \([0, a_{i+1}]\). If the minimum cost portfolio he offers is given by \( a_i \), and he must compete with multiple stock-only exchanges, his order flow would be:

\[
q_i^s = \left( \frac{a_i - a_{i-1} + a_{i+1} - a_i}{2} \right) \alpha (1 - \beta) t
\]

\[
= \alpha (1 - \beta) t \frac{(a_{i+1} - a_{i-1})}{2}
\]

---

1This order flow would be divided randomly among all brokers offering \( a_i \). His order flow is at maximum when he is the only broker offering \( a_i \).
\[ q_i^2 = \alpha \beta t \left[ \frac{a_i}{i} + \frac{a_2 - a_1}{i-1} + \ldots + \frac{a_{i-1} - a_{i-2}}{2} + (a_i - a_{i-1}) + (\xi_i - a_i) \right] \] (21)

where \( \xi_i \) identifies the customer who is indifferent to broker \( F_i \) and broker \( F_{i+1} \). Also note that \( \xi_i - a_i \) is identical to \( a_{i+1} - b_i \). From equation 20, as broker \( F_i \) alters his offering of stocks toward that of \( a_{i+1} \), he lowers the transactions costs of some customers while concomitantly raising that of others. Similarly, as he moves toward portfolio \( a_{i+1} \), his gain for stock-only customers is exactly offset by his losses. In equation 21, however, for stock with options customers, as broker \( F_i \) diversifies toward \( b_i \) (increasing his \( E_{i+1}^b \) business or, in essence, making it less costly for stock with options customers) his stock with options business increases. This scenario also violates equilibrium condition 1 and is not an equilibrium.

**CASE 3.** If broker \( F_i \) should take all his business to \( E_{i+1}^b \) then his order flow would be given by:

\[ q_{i-1}^{\delta} = \alpha (1 - \beta) t \left( \frac{a_{i+1} - a_{i-2}}{4} \right) \] (22)

\[ q_{i-1}^2 = \alpha \beta t \left[ \frac{a_i}{i} + \frac{a_2 - a_1}{i-1} + \ldots + \frac{a_{i-1} - a_{i-2}}{2} + \frac{a_{i+1} - b_i}{2} \right] \] (23)

Such an equilibrium would represent a high degree of market consolidation (multiple brokers of the same type offering the same minimum cost portfolio). But is the market in equilibrium? If \( F_i \) adjusts his portfolio by \( \delta \), from \( a_{i+1} \) to \( a_{i+1}^\delta \), he would

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lower transactions costs and capture a larger share of the stock-only business from a_i-2 to a_i-1 to have a market share of:

$$q_{i-1-6}^S = \alpha (1-\beta) t \left( \frac{a_{i-1} - a_{i-2}}{2} \right)$$

$$q_{i-2-6}^S = \alpha \beta t \left[ \frac{a_i}{i} + \frac{a_{i-1} - a_i}{i-1} + \ldots + \frac{a_{i-2} - a_{i-1}}{2} \right]$$

In this case, F_i loses some stock with options business because the last term is lost to brokers at E_i. The change in stock-only business is:

$$q_{i-1-6}^S - q_{i-1}^S = \alpha (1-\beta) t \left\{ \frac{1}{2} a_{i-1} - \frac{1}{4} \left[ a_{i+1} + a_{i-2} \right] \right\}$$

If F_i could take his stock and options business to E_i, his gain (or loss) in stock-only customers would be the same. Thus, to show that a_i is not an equilibrium it is only necessary to show that broker F_i's stock with options business would be improved by routing order flow to E_i.

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\(^2\)We ignore the incremental terms of \(\phi \cdot b_i\) and \(\frac{a_{i-1} - a_{i-1-6}}{2}\).
CASE 4. Suppose broker $F_i^t$ can conduct all his investors stock business in $E_i^3$.

Customer demand would then be given by:

\[ q_i^s = \alpha (1 - \beta) t \left( \frac{b_1 - a_{i-1} + \frac{a_{i+1} - b_1}{2}}{2} \right) \]
\[ = \alpha (1 - \beta) t \left( \frac{a_{i+1} - a_{i-1}}{2} \right) \quad (27) \]

\[ q_i^2 = \alpha \beta t (b_1) + \alpha \beta t (\xi_2 - b_1) \quad (28) \]

Note that because of $\phi$, brokers whose market is given from $(0, a_i)$ direct order flow strictly to $E_i^1$. In addition, broker $F_i^t$ may also attract the business of customers from $\xi_2 - a_i - \phi$ where $\xi_2$ is the investor whose costs are the same regardless of whether her order goes to $E_i^1$ or to other exchanges. Since $\phi > 0$ it must also be the case, given our assumptions, that $\xi_2 > b_1 = a_i$ (the locus of $E_i^1$). Does $F_i^t$ have incentive to deviate from his current assignment of customer orders?

If he reallocates $\delta > 0$ to other exchanges he will lose stock with options business to those who were otherwise indifferent to him and other $a_i$'s given by:

\[ q_i^2 - q_{i-\delta}^2 = \alpha \beta t \left( \frac{a_i}{1} + \frac{a_2 - a_1}{2} + \ldots + \frac{a_{i-1} - a_{i-2}}{2} \right) \]
\[ = \alpha \beta t \left[ D \right] > 0 \quad (29) \]

The difference in stock with options business given by an assignment of $a_{i-1}$ and $b_1$ is:

\[ \alpha \beta t (\xi_1) - \alpha \beta t [D] - 1/2 (a_{i+1} - b_1) \quad (30) \]

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Adding zero conveniently and substituting:

\[ \alpha \beta t(a_{i,1}) + \alpha \beta t(\xi_1 - a_{i-1}) - \alpha \beta t[D] - \alpha \beta t \frac{1}{2} (\xi_2 - a_{i-1}) > 0 \]

since \( a_{i-1} > D, \quad \xi_1 > \xi_2 \) \hspace{1cm} (31)

Thus, broker \( F_i^r \)'s assignment of stock orders to stock-only exchanges when a multimarket exchange is within his range violates equilibrium condition 1. \( F_i^r \) maximizes the volume of his business by directing all his orders to \( E_i^3 \).
Chapter 2

Stock Index Futures Trading and Stock Market Volatility

I. Introduction

Futures and cash markets work together in our economy to facilitate the transfer of ownership and to control price risks associated with ownership. Occasionally, however, commentators raise concerns about the observed level of futures trading relative to the volume of trading in cash markets. Motivating these concerns is the notion that a "balance" between futures trading volume and trading volume in the underlying cash market promotes stability in the cash market. For example, participants at the International Organization of Securities Commissioners (IOSCO) meeting proposed a project that "would focus upon the need to maintain balance between the stock and derivative markets in order to avoid adverse effects on the stability of the stock market."\(^1\) Others have suggested that the availability of futures on stock indexes, such as the S&P 500 stock index futures contract, triggers investment migration from the cash market to the futures market, adversely affecting cash market liquidity.\(^2\)

Bhattacharya and Spiegel (1991) advance a model in which the trading of a basket of securities, such as the S&P 500 stock index futures contract, in the presence of asym-

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\(^1\) IOSCO Report (1990), Santiago, Chile, p. 25.

\(^2\) See, e.g., Gammill and Perold (1989).
metric information, decreases the stability of the underlying cash market and may even cause the cash market to "break down".

Despite increased concern over the trading of stock index futures, no empirical evidence has conclusively supported the view that high levels of futures trading, either alone or relative to trading in the cash market, result in elevated volatility. To the contrary, several studies show a negative relationship between stock price volatility and trading volume in futures markets. The primary purpose of this paper is to examine the relationship between trading in the cash and futures markets. To this end, two questions are asked. First, is there an optimal relationship between the level of trading in the two markets? Second, does high trading volume in the futures market relative to the cash market lead to higher price volatility in the cash market? Following a review of theoretical and empirical research on trading volume in the futures and cash markets, an empirical investigation is conducted that focuses on the Chicago Mercantile Exchange's S&P 500 futures contract and stocks traded on the New York Stock Exchange (NYSE). The stock market is investigated for two reasons. First, recent concern over trading levels in the futures market has concentrated on stock index futures. Second, the cash market for stocks is moderately centralized, making cash market trading volume easier to ascertain. The decentralized structure of cash trading in agricultural or natural resource commodities renders measurement of trading volume in these markets difficult.

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With respect to the relationship between the volume of trading in futures and cash markets, the empirical analysis indicates that the ratio of the daily trading value of S&P 500 futures⁴ to the traded value of New York Stock Exchange (NYSE) stocks generally hovers around 1 and varies by day of the week. In addition, the ratio of futures trading to cash market trading has no explanatory power with respect to volatility in the cash equity market. Furthermore, higher levels of contemporaneous cash trading are associated with higher cash price volatility, while higher levels of contemporaneous futures trading are associated with lower levels of cash price volatility.

The remainder of this paper is structured as follows. Section II explores the theoretical explanations for trading and market volume that appear in the existing academic literature. Section III reviews the existing empirical evidence on the relationships between volume and price volatility within a market and between futures volume and cash price volatility. Section IV outlines the empirical methods and data used to estimate the relationship between trading volume, relative trading levels, and cash price volatility. Section V presents evidence showing that price volatility is related to the outright levels of trading in the cash and futures markets, but not to the

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⁴ We define this value as the daily volume of trading times the daily settlement price times 500. While we refer to this measure as the value of futures trading, it should be noted that the value of a futures contract at any point in time tends toward zero. However, in expressing concerns over the impact of futures trading, commentators often cite this measure (see, e.g., SEC (1988) Chapter 3).
balance of trading between futures and cash equities. Section VI concludes with a summary of results and consideration of public policy implications.

II. Theoretical Considerations

Ross [1989] notes that

"...the exact mechanism by which prices incorporate information is still a mystery and an attendant theory of volume is simply missing."

Although a unified theory of volume has not been articulated in financial economics, two general classes of models, portfolio choice models and asymmetric information models, provide insight about trading.

In standard portfolio choice models, investors are symmetrically informed about the future values of tradable assets suggesting that trading is related to differences in investors' risk and return profiles.\(^5\) Futures trading will have the potential to increase investors' utility when markets are "incomplete." Since an incomplete market is one in which investors may experience constraints on their ability to hedge portfolio risks, the introduction of futures into an incomplete market will increase the scope of alternatives

\(^5\)A number of non-standard approaches have appeared in the literature. E.g., Telser [1986] provides another unique discussion of futures volume and its relation to cash volume, likening the relationship to that between fractional reserves and the money supply. See also Karpoff [1986, 1987] and Carlton [1983]. In addition to asymmetric information, our current discussion also abstracts from the effect of market imperfections on trading volume. Constantinides [1986] has shown that the inclusion of transactions costs in a continuous trading environment causes the volume of trade to diminish, as one would expect. Also, to the extent that futures represent a low-cost trading vehicle, the introduction of futures will increase trading volume.
for tailoring a portfolio's risk/return profile to the investor. An increase in expected utility is not related directly to the volume of trade, but is instead achieved by the reallocation of portfolio risk and return accompanying the exchange of financial claims. Thus, changes in the volume of trading are largely irrelevant to the "production" of utility by investors.

In portfolio choice models, investors use futures contracts for their general effect on return distributions. For example, by cross-hedging, an investor can use a stock index futures contract to hedge the risk inherent in any equity portfolio with which the price of the stock index futures contract is correlated. Thus, among other things, the initial portfolios of investors and the set of securities available for trading have a direct effect on the volume of futures trading. Because the price of a particular futures contract will be correlated with a number of assets, trading volume in a futures contract over an interval of time need not be less than the available supply of the cash commodity or asset underlying it.

Asymmetric information models employ a rational expectations framework in which traders formulate their demands using all available information, including what they are able to deduce about the information possessed by other traders. Many models posit a class of traders, referred to as noise or liquidity traders, whose need for

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6See, e.g., Hakansson [1982].

immediacy in transacting is such that they must trade irrespective of the price. Kyle
[1985] shows that trading volume of informed traders responds to changes in the
volume of trade by liquidity traders. In the words of Blume and Siegel [1989], "Order
flow attracts order flow." Since the actions of liquidity traders are taken to be
exogenous, however, these models cannot explain the total volume of trading.

Asymmetric information models also emphasize the importance of the trading
mechanism on the volume of trade. Futures markets are generally set up as continuous
auction markets, whereas cash markets vary widely from dealer markets (bid and ask
prices posted by a specialist who takes the opposing side of all orders) to batch
markets (prices set at specific times during the day, with all trades executed at the
posted price). To the extent that different trading mechanisms cause differences in the
way prices incorporate information, such mechanisms will affect the actions of
informed traders and, hence, trading volume. For example, Easley and O'Hara [1988]
show that markets with a limit book (such as in equities markets) differ from markets
without a limit book (such as in futures markets) in their price responsiveness to
trading volume. Trading volume may be sensitive to the trading mechanism because
the futures market may enhance price discovery over that which was available with
only a cash market, causing trading volume in the cash market to rise or fall depend-
ing on the how the rational expectations equilibrium sorts itself out. Trading volume
may migrate from one market to the other depending on how market conditions affect
the rewards to information trading under two different market trading mechanisms.
Further, differing trading mechanisms in futures and cash markets can result in different equilibrium bid-ask spreads. Because trading volume is thought to be inversely related to spreads, the relation between trading volume in cash and futures markets will be affected by the spreads in the two markets.

Thus, the two major classes of models have predictions for futures and cash volume both reach similar conclusions. Both portfolio choice models and asymmetric information models suggest that futures trading may result in either an increase or decrease in cash volume. In addition, both types of models imply that any change in the relationship between futures market trading volume and cash market trading volume accompanies an increase in investors' expected utility. Finally, neither model suggests that any definable level of balance should exist between the markets.

III. Empirical Considerations: Sources of Futures Volume

Complications such as measurement problems, diversity of cash activity, and the absence of a unified theoretical treatment of volume, may explain why no empirical studies have addressed the question of balance between markets. The discussion of portfolio choice models and asymmetric information models suggests that no predictable empirical relationship exists between futures volume and volume in the cash market or between the relative levels of trading in the two markets and price volatility in either market.
Several empirical studies have considered the relationship between trading volume and volatility within a market. Karpoff (1987) summarizes the theories and empirical evidence of a number of studies that deal with intramarket volume and volatility, and identifies two general results. First, volume is positively related to the absolute value of price changes in a market. Second, volume and the price change per se are positively related. While these findings appear at odds, Karpoff argues that the models used to estimate the relationship between volume and volatility are misspecified. In actuality, an asymmetric price-volume relationship may exist, resulting in limited support for both theories. Nonetheless, his review of studies demonstrates the existence of a positive empirical relationship between volume and volatility within a market.

The relationship between volume in one market and price volatility in a related market is another extension in the literature. A number of authors have considered whether futures trading is related to higher or lower cash price volatility. No consensus on the relationship, however, has been forthcoming. Figlewski (1981) and others maintain that conflicting arguments exist as to why futures trading may increase or decrease volatility in the cash market. For example, the ability of cash market participants to hedge with futures improves the functioning of the cash market resulting in lower cash price volatility. Similarly, futures trading may increase the informational efficiency of the cash market and decrease its price volatility. Alternatively, if futures prices are distorted by manipulation or technical factors, this may lead to increased

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volatility. Figlewski also contends that if increased hedging demand is not offset by enough speculation, or if futures traders are less informed than cash market participants, volatility in the cash markets could also be increased.

Empirical evidence on the futures volume and cash price volatility relationship reflects the ambivalence of the theoretical arguments. A review of several recent studies reveals inconsistent conclusions regarding the relationship between the stock and stock index futures markets. Early studies by Santoni (1987) and Edwards (1988) found that cash price volatility either decreased marginally or not at all following the introduction of stock index futures trading in 1982. McGartland and Wang (1989), however, note that these studies are not conclusive because the authors did not control for the possibility that changes in volatility may have been due to changes in fundamental economic volatility. McGartland and Wang controlled for this by using an over-the-counter index, assumed to be free of the influence of futures trading, as a proxy to capture general economic conditions. Although they found that cash price volatility increased by a significant, but small, amount when a daily measure of futures trading was employed, this effect disappeared when a bi-weekly measure of volatility was used. Similarly, Harris (1989), using non-S&P 500 stocks as a control, found that stock price volatility increased a statistically significant but economically small amount following the advent of futures trading. Stoll and Whaley (1986,1987) also found short-term cash price volatility increases during the "triple-witching" hour, which they attributed to index arbitrage.
In a more recent study, Bessembinder and Seguin (1991) consider the interrelationship between spot trading volume, futures trading activity and cash price volatility. They found that unexpected cash market trading volume had a much larger effect on volatility than expected cash market trading volume. In addition, they found a significant decline in the coefficients relating cash price volatility to spot trading volume when futures trading began. Furthermore, they found that stock market volatility was negatively associated with the level of open interest in stock index futures. Their study, however, did not control for other changes which may have occurred in the economy subsequent to the beginning of futures trading.

General conclusions can be drawn from the empirical studies of futures trading and cash price volatility. First, within the cash market, a definitive link exists between price volatility and the volume of trading. This is clear from the articles surveyed by Karpoff (1987). In addition, although to a significantly lesser degree, there is evidence that short-run stock price volatility may be positively related to futures trading. This relationship is less clear and possibly negative when one considers contemporaneous trading in the markets à la Bessembinder and Seguin (1991). With respect to balance, empirical studies to date have not dealt directly with the question of whether an imbalance in trading volumes is related to volatility in cash markets. And while theoretical models of trading do not address the issue directly, the existing empirical work on volume and price volatility can be extended to investigate the relationship of balance to cash market volatility.
IV. Empirical Methods and Data

The goal of this study is to identify the relationship, if one exists, between stock price volatility and the balance of trading between the cash and futures markets for stocks. We proceed in three general steps. First, the characteristics of trading volume in futures and cash markets are investigated. Then, a series is constructed in order to examine the relative trading volume in the two markets and control for nominal differences in the values of the instruments traded in order to investigate the notion of market balance. To this end, the balance series is defined as the ratio of the daily volume of the S&P 500 futures times the daily settlement price times 500 to the daily dollar value NYSE share volume. Two unconditional volatility series, the daily standard deviation and price range, are visually and statistically inspected to see how they have changed over time, whether obvious shifts have occurred and whether recurring patterns exist. For example, a number of researchers have identified day-of-the-week patterns in the equities markets with respect to volume and returns.\(^8\) The results of this exercise will help in properly modeling price volatility later.

The nature of the relationship during what might be termed "crisis times" in the market are then considered. Days of high relative price volatility, identified as days with either large percentage price ranges, high intraday price volatility or large absolute returns are examined to determine whether a disproportionate number of

these days also exhibit balance ratios that differ substantially from normal. Days exhibiting abnormally high (or low) balance ratios are examined for a concurrence of elevated volatilities as measured by the indices identified above.

Finally, using a methodology introduced by Schwert (1990) that allows for unbiased estimation of daily standard deviations conditional on observable variables, the relationship between balance and price volatility in the markets is tested. Controlling for other factors that may influence cash price volatility, this method involves iterating between a return and a standard deviation equation of the following forms:

\[ R_t = \alpha_1 + \sum_{j=1}^{n} \gamma_j R_{t-j} + \sum_{j=1}^{4} \rho_j d_{j} + \sum_{j=1}^{n} \psi_j \delta_{t-j} + U_t \]  \hspace{1cm} (1)

\[ \delta_t = \alpha_2 + \sum_{t=1}^{4} \eta_t d_{t} + \sum_{j=1}^{n} \beta_j \delta_{t-j} + \sum_{j=1}^{n} \omega_j \hat{d}_{t-j} + \varepsilon_t \]  \hspace{1cm} (2)

where \( R_t = \ln(P_t/P_{t-1}) \) is the return on day \( t \), \( P_t \) is the closing price on day \( t \), \( d_t \) is a day of the week dummy variable where Friday serves as the control, \( U_t \) is the residual from equation 1, and \( \hat{\delta}_t = \sqrt{\hat{U}_t / \pi} \) is the estimated conditional return standard deviation on day \( t \). The procedure for estimating these equations is to first estimate equation 1, without the conditional return standard deviations, using ordinary least squares (OLS). The residuals from equation 1 are then used to estimate the condi-
tional return standard deviation as noted above, and these estimates are used to estimate equation 2 with OLS. The predicted values of equation 2 are then used in a weighted least squares regression of equation 1 where the weight equals

\[ 1/(\hat{\sigma}_t/n/2)^2 \]

This procedure is iterated three times, as suggested by Davidian and Carroll (1987).

Of primary interest in this study is equation 2, which models the conditional standard deviation of prices. To evaluate the relationship between volatility, individual trading volumes, and the balance ratio, equation 2 is modified to include these variables as follows:

\[
\hat{\sigma}_t = \alpha_2 + \sum_{i=1}^{4} \gamma_i d_t + \sum_{j=1}^{n} \beta_j \hat{\sigma}_{t-j} + \sum_{j=1}^{n} \omega_j \hat{U}_{t-j} + \phi_1 CV_t + \phi_2 FV_t + \phi_3 \frac{FV_t}{CV_t} + \varepsilon_t \quad (2a)
\]

where CV, and FV, are the dollar values of NYSE stocks and S&P 500 futures, respectively, traded on day t. The value of futures trading is measured as the settlement price of the nearby futures contract times 500 times the number of contracts traded.

Bessembinder and Seguin (1991) also estimate a model that allows the relation between volumes and volatility to change with the number of active traders and the capital they bring to the market. This modification is based on models developed by Kyle (1985) and Admati and Pfleiderer (1988). These models show that in markets containing discretionary liquidity traders that can allocate their demand over different periods, market makers can use past order flows to forecast liquidity demand in the
current period. This implies that the relationship between volume and price volatility is not constant. To control for this, the procedure of Bessembinder and Seguin (1991) uses a standardized volume series, defined as the ratio of daily trading volume to the average trading volume over the prior 100 days.

Daily data from the NYSE proxies for the value of stocks traded during the period April 4, 1982 through March 30, 1990. The daily volume and settlement prices of the S&P 500 index futures were obtained from the Commodity Futures Trading Commission’s (CFTC) "Permanent Records" database. Intraday cash values of the S&P 500 index were obtained from the Chicago Mercantile Exchange’s (CME) time and sales data.

V. Relationships Between Volume and Volatility

Before proceeding to the estimation of equations 1 and 2a, it is useful to inspect the summary statistics on the data series. Figure 1 graphs the NYSE share volume, the S&P 500 index futures volume, and the ratio of these two series in panels a, b and c, respectively. Panels a and b show a distinct growth in cash and futures trading during the sample period. In addition, panel b shows substantial growth in S&P 500 futures trading during the early life of the contract. Panel c of the graph shows that by mid-1984, the growth of futures trading relative to the cash market leveled off, as indicated by the leveling off of the balance ratio. Following the market crash of October 1987, the volume of trading in both the cash and futures markets drops off sharply and then

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begins to rise again. The graph of volume ratios, however, confirms that the futures trading volume dropped off more than volume in the stock market. One should also note that the futures series also appears more volatile following the crash.

In addition to the growth of trading in the cash and futures markets, several authors have identified daily patterns in trading. To investigate whether there are daily patterns in volume, further details of the volume series are presented in Table I. An F-test confirms that the daily volume of trading in both the cash and futures markets, as well as the ratio of futures to cash market trading, differ by the day of the week, thus justifying the inclusion of the daily dummies in equations 1 and 2. Table I also shows a clear trading pattern throughout the week. For both the cash and futures series, volume rises through Wednesday and then falls through Friday. The ratio of futures trading to cash market trading, however, falls throughout the week.

Statistics for the unconditional standard deviation and percentage price range are also shown in Table I. The standard deviation measure is the daily standard deviation of five minute returns on the S&P 500 index:

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9 See French (1980) and Jain and Joh (1988).
\[
SDEV_t = \sum_{i=1}^{N} \frac{(R_{i,t} - \bar{R}_t)^2}{N-1}
\]

where \( R_{i,t} = \ln(P_{i,t}/P_{i,t-1}) \) is the return for interval \( i \) on day \( t \), \( \bar{R}_t \) is the average return on day \( t \), and \( P_{i,t} \) is the last price in interval \( i \) of day \( t \). The range is the daily range of the index divided by the average value of the index on that day:

\[
\%\text{RANGE}_t = \frac{P_{\text{high}} - P_{\text{low}}}{\bar{P}_t}
\]

where \( P_{\text{high}} \) is the highest value of \( P_{i,t} \), and \( P_{\text{low}} \) is the lowest value of \( P_{i,t} \) on day \( t \). The F-test indicates that neither of these series exhibits a consistent daily effect.

The standard deviation and range series are graphed in Figures 2 and 3. The graph of standard deviations, Figure 2, shows a fairly stable level of price volatility, although with some spikes, through 1985, followed by a period of increased volatility, and more frequent and extreme spikes, from 1986 through 1989. The price range series, Figure 3, does not show a similar increase during this latter period. This rough observation suggests that prices during the latter period tended to move up and down
more during the day (exhibiting higher intraday volatility) instead of in a smooth fash-
ion.\textsuperscript{10}

Finally, Figures 4 and 5 plot the daily unconditional standard deviation and
percentage range series against the ratio of futures to cash volumes. No obvious
pattern can be identified that indicates a relationship between cash price volatility and
the balance ratio. The estimation of equations 1 and 2 will formally test this assertion.
The ratios of futures to cash volume below approximately 0.85 reflect the early
history of trading in futures. Spikes in volatility towards the center of the distribution
reflect the market crash periods of 1987 and 1989. These spikes, however, are not
generally associated with the greater imbalance in volumes.

To explore the relationship between cash market price volatility and the ratio of
futures to cash trading and to ascertain whether the market's most volatile days
coincide with days of unbalanced trading, Table II reports the ten highest uncondition-

\textsuperscript{10} Consider, for example, the two series graphed below. The range in each series is
identical. However, the series which fluctuates as it moves from 10 to 1 has a higher
standard deviation of returns than the smooth series.
al standard deviation, percentage range, and balance days. Observations from pre-June 1984 have been dropped to control for the early growth in the S&P stock index futures volume. For the ten highest observations of the standard deviations and ranges, the concurrent balance measure is reported. As a benchmark, the averages of the series are presented in the bottom row of the table. With respect to the unconditional standard deviation of prices, in eight of ten cases the measure of balance is below the mean level of balance for the period. For the percentage range series, the level of balance on seven of the ten days was below the average. It should be noted that most of the days identified here were during the October 1987 market crash period. The ten highest balance days are shown in the far right columns of Table II along with the sample period averages. Of these ten days, four experience below-average standard deviations, but seven are associated with below-average price ranges. Thus, at least at the upper extreme, days experiencing a high ratio of futures trading to cash market trading are not associated with high levels of price volatility. Furthermore, highly volatile days appear to experience lower ratios of futures to cash market activity.

We also consider the relationship between returns and balance. Table III shows the highest and lowest twenty average return days, as measured by the average of five-minute returns during a day, and the associated balance ratios. For the twenty highest return days, eleven show a balance level above the average, while balance levels on

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11 As with most successful futures contracts, volume during the early stages subsequent to introduction picks up gradually as the contract becomes more liquid.
the remaining nine days were below the average. On fifteen of the twenty lowest return days, however, the ratio is above the average, indicating that futures trading tends to be higher relative to the cash market when prices fall. Three of the five days when futures trading was low relative to the cash market were associated with the crash in October 1987. This result is, in general, contrary to an assertion made by the SEC Staff Report (1988), which noted, "During a market break, liquidity [in the futures markets] disappears at a rate geometrically larger than does liquidity in the lower-leveraged stock market." Thus, possibly due to short-sale restrictions in the stock market, liquidity falls off faster in the stock market than in the futures markets when prices plummet. These results are also consistent with Karpoff's (1988) observation that a positive relationship between volume and price changes per se in the stock market is likely caused by short-sale restrictions, based on the absence of a similar relationship in the futures market.

Table IV reports the results of regression equation 2a. Four sets of estimates are presented in this table to account for shifts in structure of the model. Columns 1 and 2 contain the estimates of the equation using raw volumes. The estimates in column 1 do not control for the initial period of futures trading growth, while those in column 2 add intercept and slope dummy variables to serve this purpose. These variables are equal to 0 for the period prior to July 1, 1984 and 1 otherwise. The choice of date was made based on the flattening of the balance series previously referred to with respect to Figure 1 Panel c. The results of the standardized volume
series are presented in columns 3 and 4 of Table II. Again, the equation in column 4 has dummy variables to control for the initial growth in futures trading.

The results of the regression 2a are consistent with the results reported by Bessembinder and Seguin (1991). In column 1, the coefficient on cash volume is positive and significant, indicating a positive relationship between cash trading volume and cash price volatility. The coefficient on futures volume is negative and significant, indicating that higher futures trading activity is associated with lower cash price volatility. The coefficient on the ratio of futures trading volume to cash trading volume indicates no significant association between balance and cash price volatility.

The estimated coefficients for the equation which controls for the growth of futures trading, column 2, are similar to those obtained for the model which does not control for this period with the exception of the balance coefficients. Cash price volatility remains positively associated with cash market volume, but negatively associated with futures market volume. The coefficients for balance indicate that during the early portion of the series, balance is positively associated with volatility. The relationship, however, reverses in the later period, as indicated by the significant negative coefficient for the latter part of the sample period. It is also important to note that while the balance coefficient indicates a positive relationship during the earlier period, cash price volatility did not necessarily rise during this period. A rising balance ratio indicates either falling cash volume or rising futures volume. Either case implies a lower level of cash price volatility.
The results of the detrended volume regressions, columns 3 and 4 of Table IV, are similar to those of the raw volume regressions. As with those results, the coefficients on the Monday dummy are significant and positive. The coefficients on cash market volume are also significant and positive, indicating that when volume is high relative to average volume over the previous 100 days, price volatility tends to also be high. The coefficients on futures volume and balance, however, are not significant. Thus, activity in the futures market, whether considered alone or relative to that in the cash market, has little influence on cash prices over the sample period. Instead, the primary relationship between volume and volatility exists within the cash market.

VI. Conclusions

This study examines the relationship of the balance between futures and cash trading volume, to cash price volatility. Considering the roles of cash and futures markets in an economy and noting that while the two markets are interrelated, there is no a priori reason to believe that the levels of trading in the markets are inextricably linked. Theoretical models establish a basis for trading in cash and futures markets but do not support the assertion that trading volumes should necessarily be in some sort of balance between the two markets.

Finally, an empirical study of balance between trading in the stock market and stock index futures market looks at whether there is a relationship between trading volumes in the two markets. The results indicate that the value of trading in S&P 500
futures contracts rapidly increased to a level at or marginally above that found in the cash market. The ratio of these activities has remained fairly stable since approximately mid-1984.

The daily unconditional standard deviations and price ranges during the period exhibit some spikes and some elevation in the level of daily standard deviation of prices during the period 1986 through 1989. These spikes in the standard deviation and ranges were examined to see whether they were associated with a relatively high level of futures trading to cash market activity. On a majority of days, the level of futures trading relative to cash market trading was below average. Hence, with respect to highly volatile days, there is no evidence that cash prices are unduly affected by volume imbalances.

A more general analysis of the relationship between balance and volatility was performed by modelling the conditional standard deviation of prices as a function of futures and cash market volumes and their relative value to each other. This analysis confirmed the findings of other studies showing a positive relationship between cash market volume and cash price volatility. The analysis, however, failed to turn up a significant relationship between balance and price volatility.

While Congress has not recently sought to ban futures trading in any particular market, congressional and regulatory efforts to increase futures margins, impose transactions taxes, and increase trading restrictions are not uncommon. Implementation of these proposals would have the economic result of curtailing futures trading by
making it either more costly to trade and more difficult to trade, particularly during times of market stress. Although several comprehensive studies, including that of the Federal Reserve, found that margins are not an effective way to control volatility, proposals for both higher and "coordinated" margins are frequently advanced. Some have argued that transactions taxes and other measures to reduce futures trading volume are justified because they "have the beneficial effects of curbing instability introduced by speculation."12 While these initiatives may be pursued for other purposes, this investigation provides no theoretical or empirical support for regulatory policy that seeks to curtail trading in pursuit of market stability. Regulatory initiatives designed to limit futures trading premised on the assumption that futures trading has a destabilizing effect on the cash market are unsupported by the findings of this paper. The premise that excessive futures volume or an imbalance between futures and cash volume leads to cash market volatility remains unsubstantiated.

References


### TABLE 1

Descriptive Statistics and F-Test Results

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'Significance level of 1%.'
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## TABLES
Ten Highest and Lowest Average Return Days and the Ratio of Futures to Cash Market Volume.

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<td>1-21-85</td>
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<td>4-3-87</td>
<td>.00031</td>
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<tr>
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*Indicates balance level above the mean balance level of 1.0133.
### TABLE 6
Regression of Estimated S&P 500 Return Standard Deviations on Cash, Futures and Relative Futures to Cash Volume.

<table>
<thead>
<tr>
<th></th>
<th>Raw Volumes</th>
<th>Standardized Volumes</th>
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<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Intercept</td>
<td>-1.4879</td>
<td>-3.5927</td>
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<tr>
<td></td>
<td>(-0.665)</td>
<td>(-9.017)</td>
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<tr>
<td>DUMMY</td>
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<tr>
<td></td>
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<tr>
<td>Monday</td>
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<td>2.8425</td>
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<tr>
<td></td>
<td>(3.522)</td>
<td>(2.742)</td>
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<tr>
<td>Tuesday</td>
<td>18.710</td>
<td>6.962</td>
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<td></td>
<td>(2.007)</td>
<td>(0.686)</td>
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<td>1.751</td>
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<td></td>
<td>(0.189)</td>
<td>(-0.414)</td>
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<td>Thursday</td>
<td>4.728</td>
<td>-5.251</td>
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<tr>
<td></td>
<td>(0.509)</td>
<td>(-0.520)</td>
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<td>10 Lagged Volatility Estimates</td>
<td>0.3950</td>
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<td>10 Lagged Unexpected Returns</td>
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<td></td>
<td>(24.602)</td>
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<td>Cash Volume</td>
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<td>Cash Volume*DUMMY</td>
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<td>Futures Volume</td>
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<td>Futures Volume*DUMMY</td>
<td>31.70</td>
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<td>Futures/Cash Volume</td>
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<td>(-9.622)</td>
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<td>Futures/Cash Volume*DUMMY</td>
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<tr>
<td>Adjusted R²</td>
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<td>0.31</td>
</tr>
</tbody>
</table>
FIGURE 1

Panel a

DAILY DOLLAR VALUE OF NYSE SHARE VOLUME

Panel b

DAILY DOLLAR VALUE OF S&P 500 FUTURES VOLUME

Panel c

DAILY RATIO OF THE DOLLAR VALUE OF S&P 500 FUTURES VOLUME TO THE DOLLAR VALUE NYSE SHARE VOLUME

71
FIGURE 4

DAILY STANDARD DEVIATION OF RETURNS
BY RATIO OF THE DOLLAR VALUE OF S&P 500 FUTURES VOLUME
TO THE DOLLAR VALUE OF NYSE SHARE VOLUME

RATIO OF FUTURES VOLUME TO CASH VOLUME
FIGURE 5

DAILY PERCENTAGE RANGE OF RETURNS
BY RATIO OF THE DOLLAR VALUE OF S&P 500 FUTURES VOLUME
TO THE DOLLAR VALUE OF NYSE SHARE VOLUME

RATIO OF FUTURES VOLUME TO CASH VOLUME
Chapter 3

Liquidation and Delivery Under Conditions of Manipulation

1. Introduction

It has been noted that futures markets are perhaps the closest approximation of the perfect competition paradigm since many buyers and sellers trade a homogeneous good in a competitive auction setting.\(^1\) In the case of futures markets, the homogeneous good is a contract to buy or sell a physical commodity or a financial instrument at a specified future date (the settlement date). The competitive auction is one in which traders call out bids (offers to buy) and asks (offers to sell) to each other in a futures pit.\(^2\) In addition to providing a market in which producers and consumers can hedge current or anticipated cash market positions, the futures markets provide price quotes that reflect the anticipated value of the underlying instrument in the future.

Given the importance of futures markets as a risk sharing and price discovery mechanism, it is of importance to market practitioners and economists to know under what conditions futures prices may become distorted or manipulated.

We define manipulation as strategic trading by a dominant trader that results in prices that differ from their expected value absent manipulation. The Commodity Exchange Act (CEA), which defines the regulatory and legal environment in which

\(^{1}\) Anderson and Sundaresan (1984) note that "The fact that anyone may buy or sell futures contracts for homogeneous goods in centralized markets at publicly quoted prices suggests that these markets come close to fulfilling the standard criteria for perfect markets."

\(^{2}\) A pit is a designated area on an exchange floor where futures contracts are traded.
futures markets operate, notes that futures markets "are susceptible to excessive speculation and can be manipulated, controlled, cornered or squeezed." Prevention of manipulation is a consistent theme in the CEA, but, oddly, there is no definition of manipulation there. One must rely on administrative and case law as it has been applied to the manipulation cases to find the pieces to a definition for manipulation. A review of the cases brought by the CFTC reveals that for the purpose of proving manipulation in court, three elements must be established: 1) a dominant position, 2) an artificial price, and 3) an intent to manipulate.

Manipulation of futures prices results from an exercise of some strategic advantage. The strategic advantage may be the result of private information, some specialized knowledge of market structure, or comparative size in relation to the rest of the market. A number of authors have suggested that anonymity, or the ability to acquire open positions without disclosing them to others, is a feature that makes futures markets especially conducive to manipulation. Kyle (1984) notes that "if the rest of the market knew what the squeezer was up to, prices would adjust quickly to levels such that the squeezer could not engineer a squeeze in which he expected to make a profit." Easterbrooke (1986) elaborated this point noting "if the adverse parties had been aware of the large position, they could have stayed out of the market, liquidated their positions earlier, or prepared to take or make delivery." While these conjectures are intuitively appealing, they raise a number of questions.

---

3 *Commodity Exchange Act*, [Necessity for Regulation], Section 3.
If futures are priced via a competitive auction, how does a large, potentially monopolistic, trader acquire his dominant position without alerting others in the market? Without the standard assumption that the large trader has private information on the moves of others, can the large trader still engineer a manipulation? If, as is implicit in the above assertions, the large trader is able to acquire his position without the knowledge of others in the market, is he assured of a monopoly profit? How would revelation of his presence alter the behavior of other traders? Could they, as suggested by Easterbrook, adopt a strategy in which early liquidation or delivery provisions protect them from losses? Or, does commitment to the price specified in the contract lock them into a no-win situation? No one has investigated how strategic traders could best respond to manipulation given an option to liquidate or deliver on the contract. The present paper seeks to do so.

In order to investigate these questions, a perfect Bayesian equilibrium concept is used in which 1) traders move simultaneously submitting market orders for futures contracts and 2) given the realization of the first period equilibrium futures price, they decide whether to liquidate or deliver on the contract. Imperfect information about the manipulator's effect on prices, the sequential nature of the problem, and the importance of beliefs to the outcome suggest a step-by-step approach. In order to see what affects the decision making process in equilibrium, futures trading is modeled such that the activity of hedging and speculation dictates the nature of a player's utility maximization problem. Longs and shorts have utility functions of the negative exponential type and maximize their final wealth under risk aversion. This
specification is a particularly insightful one, providing a general equilibrium framework that is consistent with classic interpretations of risk premia, basis risk, and market efficiency.

Under conditions of manipulation players must operate under a regime of imperfect information: a manipulator may be able to obtain a substantial proportion of short (long) positions without the knowledge of other traders in the market. We consider the reactions of traders to revelation of manipulation through equilibrium prices. With a decision variable that embodies the offset or deliver option, the model is made tractable by imposing restrictions on the market participants, their beliefs, and their strategy space.

An analysis of reactions in a simplified extensive form game indicates that detection of manipulation by other market participants is costly to the manipulator. If detected by floor brokers who competitively bid prices to their expected value, the manipulator’s trading will cause prices to move against him, raising the equilibrium price when he puts in orders to buy and lowering the price when he seeks to sell. In addition, strategic traders in the market who have the option to offset their position prior to delivery or to hold and make delivery will revise their expectations if they detect the manipulator. Under simplifying assumptions, the model indicates traders strategically adjust their plans regarding liquidation and avoid incurring losses to the manipulator. Under certain circumstances in which the manipulator is detected early enough, the manipulator loses money and the other traders in the market profit fortuitously.
Such a result has not been achieved in any other model of manipulation, primarily because the traders opposite the manipulator have been either competitive (earning zero profits) or atomistic (usually following a random trading process). Among the papers that do not provide the manipulator with private information on the moves of others, Pirrong (1993) shows that a large long trader earns positive expected profits in equilibrium when his trade is obscured by noise traders and the marginal cost of delivery is increasing. We find that by allowing one strategic move to imperfectly informed traders, manipulative activity does not yield the certain bounty to large traders that other models predict.

The model does not explicitly consider the arrival of any outside information and does not allow for inside information. The purpose here is not to model insider trading (for which there are already numerous successful efforts), but rather to model manipulation by a large trader whose advantage lies in anonymity and, therefore, whose actions are unanticipated by holders of open futures positions.

We begin with a conceptual literature review and then develop a model of futures trading. We solve for equilibrium prices when manipulation is unanticipated and solve for prices when manipulation is anticipated. We then construct a simplified extensive form game in order to analyze the effects of detection on trader strategies. The results are presented, followed by suggestions for further research, and concluding comments.
II. Review of the Literature

The various models of manipulation posed in the literature can be regarded as studies on the optimizing behavior of agents when the assumptions of perfect competition are violated. The perfect competition model rests upon three broad assumptions, 1) perfect information, 2) prices clear markets, and 3) many small buyers and sellers. Following from the violation of these assumptions, models of manipulation can be classified into three categories: 1) asymmetric information models, 2) frictional models, and 3) monopoly models. In the asymmetric information models, an agent has access to, or can create the appearance of, some piece of news which is not available to other agents. In the frictional models, some structural feature of the market affords strategic players the opportunity to trade profitably. Frictional models include those that suggest that transportation, transformation, and delivery costs give rise to marginal cost relationships that can be exploited. The third type of manipulation model, which we refer to as a monopoly model, considers the effects on equilibrium of the presence of a trader in the market who is not a price taker.

In the asymmetric information models, market power is derived from the possession of information unavailable to the public or other traders. Actually possessing information may not even be necessary. Merely the perception thereof may signal to other traders 1) not to trade, or 2) that the potential for manipulation should be impounded into their quoted prices.

Easterbrook (1986) refers to manipulation as a "misrepresentation of the size
of one’s position. The release of spurious information may provide an indication of intent. But one might question to whom the manipulator misrepresents his position. It is certainly not the holder of the opposite side of the position. A holder of a contract in the futures market will not generally know the specific identity of the holder of the opposite side of his position. The clearing house is formally the holder of the other side of all contracts. As an intermediary, the clearing house ensures the performance of both holders. Futures markets are unique in that a trader does not necessarily know the composition of trading in the market. If there is a dominant trader in the market, it may be difficult to distinguish his activity from the other traders in the market. While he may have to report his positions to regulatory authorities, there is no legal requirement that he reveal his positions to other traders.

In Kyle’s (1984) paper, the manipulator or "squeezer" has private information about the amount of (exogenous) hedging, which allows him to formulate his order to buy or sell futures contracts. There are two qualities of good in this model, a cheapest to deliver grade and a more expensive grade. Risk neutral speculators observe aggregate order flow, surmise whether hedging is active or inactive, then bid futures prices to reflect the expected value of the commodity delivered. For each contract on which the squeezer takes delivery beyond the supply of the cheapest to deliver grade, he can receive a spot price that includes a quality differential, and thus, he profits.

David Newbery (1984) models a market in which participants operate under

\* Easterbrook (1986), 106.
full public information but must predict what output and futures positions the dominant producer will take. The market consists of a dominant producer and a fringe of small risk averse competitive producers. Newbery shows that, regardless of his risk aversion, the dominant producer can profit from manipulation. Further, if the competitive fringe producers are more risk averse, he can profit by varying his production stochastically. The activity of the dominant trader also reduces bias in the futures price, thereby increasing market efficiency.

In general, manipulation can only occur when some aspect of market efficiency is violated. Virtually all the models below require an implicit assumption of an inefficient marketplace in which informationless trading affects prices. In an early paper on the subject, Grossman and Stiglitz (1975) suggest that markets cannot be efficient when information is not costlessly obtained by all market participants. In a rational expectations framework, the assumption that prices reflect informationless trade can be considered a violation of the Muth rational expectations hypothesis or the efficient markets hypothesis. Generically, these hypotheses state that in an efficient market, current and expected prices embody all available information. The extent to which expected prices and realized prices differ is stochastic, and therefore cannot be the basis for a profitable strategy like those described below.

Pirrong (1993), and Pirrong and Kormendi (1990), in their evaluation of delivery points, specify marginal relationships that might give rise to frictional manipulation. In a long manipulation, they show that the marginal cost of making delivery can be increasing over some quantity intervals in the delivery market due to
economic frictions, such as transportation costs. If the shorts were compelled to make
delivery over these intervals, they would be forced into an increasing costs range since
they would have to go outside the delivery market to secure more distant supplies.
Therefore, they would be willing to offset at a higher futures price rather than make
delivery. The long affects this manipulation by standing for delivery on some of his
position, and liquidating when he can receive the higher price. The eventual profit he
can earn is a function of the elasticity of demand he faces in the delivery market. The
more elastic demand for the commodity in the delivery market, the less the
introduction of these additional supplies will affect the spot price the long can receive.

In Jarrow's (1990a and 1990b) papers, the market power of the manipulator
stems from his size and an assumption of intertemporal differences in the price impact
of the manipulator's trades. The importance of size is self evident in the corner
strategy, the manipulator must control more than the supply of the asset - he owns
some and contracts to buy more from those who do not (they short sell). The short
sellers will be unable to acquire the asset and will have to pay a premium to get out of
the contract. The plausibility of the intertemporal differences argument rests on the
assumption that the manipulator is able to establish a trend with his initial trading
(have a herd effect), but not with his reversing trade. There is also a tacit
informational asymmetry here. The large trader is assumed to know when market
conditions are ripe for herding, for example, by taking advantage of portfolio insurance
programs, or be able to mimic trading patterns which turn on technical analysis
programs.
Kyle (1984), Jarrow (1992), Kumar and Seppi (1992), and Pirrong (1993) present a stylized representation of a market in which there is a persistent inefficiency built in. In general, these models include one strategic player, non-strategic competitive market makers (a Bayesian price updating mechanism), and atomistic liquidity or "noise" traders. Noise traders, who are so called because the distribution describing their behavior is randomly and independently distributed, stand ready to lose money repeatedly because their demand for liquidity is such that they must trade. Results suggesting a positive expected profit are obtained in these models, but the question remains whether such profits would still obtain in a model with strategic or profit maximizing traders.

Since anonymity is an important feature of futures market, Chichilnisky (1984) proposes an imperfect information model in which strategic traders are not fully informed about the characteristics of other traders in the market. Traders are able, however, to observe the actions of other players in the game. Since prices may not reflect all information available to the traders, manipulation is seen as a problem of market efficiency. Chichilnisky notes that disclosure of the manipulator's activity may lead to less manipulation, but it imposes costs on the market by restricting entry. Chichilnisky's model attempts to illuminate issues related to manipulation, but assumptions about a player's ability to observe other trader's actions in a repeated game setting are not consistent with the concept of anonymity in futures markets.

Game theoretic models, like that of Chichilnisky, may be best suited to dispensing with a fundamental element of manipulation, that of intent. Simply put, in
a game theoretic model, agents are assumed to have the intent of maximizing profit or utility, and their decision about what actions they take are derived directly by plugging their set of actions into their payoff function. Players seek to maximize their payoff given the rules of the game. In such an environment, manipulation may be a player’s Nash equilibrium strategy. In many bargaining games with asymmetric or imperfect information, the naive truth telling behavior is not a Nash equilibrium message. Players following their own best strategies, however, may result in non-Pareto optimal outcomes. A Nash equilibrium, although individually optimal, may result in a social loss of efficiency.

From the outset, it is important to note that manipulation, considered a negative and destabilizing activity of market participants, is a very broad and nebulous concept. As noted, a manipulator must have or appear to have some strategic advantage such as private information, some specialized skill, or size in order to profit from his trade. Dishonesty, avarice, or deceit are not necessary conditions. In fact, the successful manipulation employs the familiar strategy of buying low and selling high. In many circumstances, it is individually rational for traders to act upon any strategic advantage they possess.

As noted, the limitation of the models of manipulation described above is the absence of any strategic players opposite the manipulator in the markets modeled.

---

3 Since the negative connotations associated with manipulation are difficult to wrest, one might wish to employ another terminology. For example, Gastineau and Jarrow (1990) refer to the agent in their paper as a "yama", the Japanese word for mountain.
This approach seems necessary since the degree of complexity increases as strategic agents are introduced into economic models. Since the strategic variables of position size, order form, and time of execution define such a large universe, solving for a meaningful equilibrium or set of equilibria is a difficult task, if not impossible. Hence, the model developed below restricts strategic variables to the bare minimum necessary to capture the essence of manipulation. Since the primary aim of this paper is to gain intuition on the decision of liquidate or deliver under conditions of manipulation, the strategic variable introduced describes the proportion of positions that would optimally be liquidated when manipulation is probable.

III. Model of Futures Trading

The economy is delineated by three discrete periods: \( t = \{1, T - 1, T\} \) where \( T \) denotes the time of contract settlement. Traders can submit orders to trade futures contracts in period 1. They must then decide whether to liquidate at period \( T-1 \), or hold until delivery at the settlement date, period \( T \). A short's costs (long's revenues) are composed of the costs (revenues) of taking the position, which is given by the spot price, \( S_t \) and the futures price, \( F_t \). Payoffs to the shorts (longs) are the difference between those costs and revenues. If we assume parties to the futures contract post margin on \( F_t \), for example, \( mF_t \), where \( m \) is a percentage of \( F_t \), then in the profit calculation of this game, payoffs remain unchanged. Hence, we abstract from the issue of margin by assuming that players post (or collect) 100% margin and thus,
simplify calculations without any loss of generality.⁶

There are \( i = 1, \ldots, I \) sellers and \( j = 1, \ldots, J \) buyers. The buyers and sellers in this model can only submit market orders. Since limit orders are most likely to be executed when they are close to the market range, the complication of modeling demand and supply schedules is avoided without detracting from the fundamental issues addressed here.⁷

Consider a model in which long and short traders, whether hedger or speculator, have utility functions of the following negative exponential type:

\[
U_i (\bar{\pi}) = -e^{-a_i \bar{\pi}} = -\exp (-a_i \bar{\pi})
\]  

(1)

where traders maximize their final wealth and \( a_i \) is the Arrow-Pratt coefficient of risk aversion. Thus, with the assumption:

\[
\bar{\pi} \sim N (E\bar{\pi}, \sigma^2_{\bar{\pi}})
\]

(2)

the position which maximizes expected utility is given by:

\[
E [-\exp (-a_i \bar{\pi})] = -\exp [-a_i (E\bar{\pi} - \frac{1}{2} a_i \sigma^2_{\bar{\pi}})]
\]

A function of an individual or firm \( j \) who maximizes the expected utility of profit over \( t \) given that he acquires the spot commodity in period 1 and holds no futures is given

---

⁶ Unlike securities margins, futures margins are good faith deposits -- performance bonds -- which insure that future financial obligations of both parties will be met.

⁷ Easley and O’Hara (1991) find that market prices are influenced by order type when the type of order is correlated with information about the value of the asset.
by:

$$\bar{\pi}_j = -S_j q_j + \bar{S}_j q_j - C(q_j)$$

(4)

where $S_T$ is the final period spot price, and $C(q_j)$ is the cost of carrying the position, for example, the cost of storing $q_j$ until the final period. Alternatively, $C(q_j)$ may be thought of as the cost of producing output $q_j$. In either case, $C(q_j)$ is convex in $q_j$ since storage costs rise in quantity stored and similarly, interest income is decreased as more initial wealth is used to finance production. Note that whether this trader is risk neutral or risk averse, a long cash position, defined as $q_j > 0$, indicates that this individual expects the final spot price of the commodity to at least compensate his initial investment and marginal costs.

The spot holder faces risks in the price he receives for his holdings. The existence of the futures market allows the cash market participant to hedge the risks he faces in the spot market by allowing him to transfer the uncertainty associated with spot price changes to the futures market speculator. Consider the objective of such an individual who maximizes his expected wealth via transactions in the futures market:

$$\bar{\pi}^U = -S_j q_j + \bar{S}_j (q_j - x_j) - C(q_j) + \bar{F}_1 x_i + \delta x_i - C(x_i)$$

(5)

where $\bar{F}_1$ represents the revenue from (cost of) taking the futures position, $x_i$, $\delta$ represents any premium (or discount) that may apply upon delivery of the commodity to satisfy the futures position, and $C(x_i)$ is the cost associated with making delivery of $x_i$, where $C'(x_i) > 0$.

The inclusion of $\delta x_i - C(x_i)$ allows us to capture the heterogeneity of the
underlying instrument and the role it plays in a classic squeeze. While standardization remains an important feature of futures contracts, many contracts provide for the delivery of different grades, coupons, maturities, etc., of the underlying instrument. In addition, at the option of the seller, certain agricultural commodities are deliverable at various locations. Because of the heterogeneity these options introduce into the futures instrument, contracts specify premiums and discounts (similar conceptually to conversion factors in financial futures) that apply when the instrument delivered differs from par. \( C(x_i) \) represents costs to making delivery through the futures market. An example of these costs may include shipping the commodity from the field to the delivery market, storage costs at the delivery point, or costs associated with transforming the commodity into a state that meets the specifications of the contract.

It may be required that delivery be made to specific warehouses that require a fee for certification or handling. Ideally, exchanges would like to set \( \delta_i = C'(x_i) \). Uncertainty, heterogeneity, and discontinuities in the marginal cost, however, make it difficult to set premiums or discounts that perfectly normalize the underlying asset to a par value.

Thus, equilibrium prices will reflect unaccounted for options associated with delivery.\(^4\)

Given a particular \( q_i \), the futures position that maximizes expected utility is:

\[
    x_i^{D*} = \frac{\bar{p}_i - E_i S_x + \delta - C'(x_i)}{a_i \sigma_{SS}} + q_i
\]

Implicit in the above formulation is the requirement that the futures position be

settled entirely by delivery. In this sense, the specification described above is
indistinguishable from a forward market. As noted above, the costs of delivering the
commodity, expressed in the model above as $C(x_i)$, are non-trivial. Suppose that an
individual hedger in this model would like to obtain risk shifting advantages of the
futures market without directly incurring the costs of making delivery. Consider the
formulation in which the trader could completely offset his futures position in a period
just prior to delivery, which we denote $T-1$:

$$\pi_i^L = S_1 q_i + S_T q_i - C(q_i) + F_{T-1} x_i - F_{T-1} x_i$$

such that $F_{T-1}$ represents the price associated with liquidating that position prior to
delivery.

The resulting position is given by:

$$x_i^{L*} = \frac{E_i F_{T-1} - F_{T-1}}{\alpha_i \sigma_F} - \frac{\sigma_F}{\sigma_F} q_i$$

This expression describes the optimal futures position of a hedger who offsets his
entire futures position. Since he does not deliver, he does not directly incur any costs
associated with delivery.

In order to specify a model in which traders have an option to liquidate or
deliver, it is necessary to construct utility functions that incorporate a decision variable
whose value represents such a choice. Consider such a variable $\alpha^h$, where $\alpha \in (0,1)$.
is the proportion of positions that are liquidated by the hedger via a reversing
transaction prior to period $T$. In the final period, positions still held by traders, i.e.
\((1 - \alpha^h)x^h\) are settled by purchase or sale in the delivery market. We can drop the subscripts i and j by denoting \(x^h < 0\) as a short futures position and \(x^h > 0\), a long futures position. Incorporating this into the hedgers problem:

\[
\pi^h = -S_1q + \bar{S}_Tq - C(q) - \bar{F}_1x^h + \alpha^h x^h \bar{F}_{T-1}^h + (1 - \alpha^h) x^h [\bar{S}_T - \delta + C'(x)]
\]  

(9)

The optimizing position is given by:

\[
x^h = \frac{(1 - \alpha^h) [E_1S_T - \delta + C'(x)] + \alpha^h E_1F_{T-1} - \bar{F}_1^h + \alpha^h q [\sigma_{ss} + \alpha^h \sigma_{sf}]}{\alpha^h [\sigma_{ss} + \alpha^h \sigma_{sf} + 2 (1 - \alpha^h) \alpha^h \sigma_{sf}]}(10)
\]

Given that a long cash position, \(q > 0\), is hedged by a short futures, \(x^h < 0\), optimal \(q\) is also influenced by the futures position:

\[
q = \frac{-S_1 + E_1S_T - C'(q) + \alpha^h \sigma_{ss} + \alpha^h \sigma_{sf} x^h}{\alpha^h \sigma_{ss}}(11)
\]

To incorporate the behavior of speculators (defined as traders who do not carry cash positions) into the model consider a speculator who does not hold \(q\):

\[
\pi^s = -\bar{F}_1x^s + \alpha^s x^s \bar{F}_{T-1} + (1 - \alpha^s) x^s [\bar{S}_T - \delta + C'(x)]
\]  

(12)

The position of a speculator with risk aversion coefficient \(\alpha'\) is given by:

\[
x^s = \frac{(1 - \alpha^s) [E_1S_T - \delta + C'(x)] + \alpha^s E_1F_{T-1} - \bar{F}_1^s}{\alpha^s [\sigma_{ss} + \alpha^s \sigma_{sf} + 2 (1 - \alpha^s) \alpha^s \sigma_{sf}]}(13)
\]

Previous literature shows that the existence of incomplete information, either as a result of asymmetrically informed traders or due to the presence of a large trader or "price maker", is sufficient to motivate a price response to signals that partially reveal this information. Typically, market participants infer information directly from prices.
or from any information made public regarding trading activity. In market microstructure models, information is extracted from current quotes, past prices, the level of volume, and open interest, that are generally available to all market participants. In models based on the Kyle (1985) and Glosten and Milgrom (1985), risk neutral market makers competitively set quotes or prices to their expected value, earning zero profit.

Unlike market makers or "specialists" in the securities markets, futures market makers do not post quotes and are under no affirmative obligation to make a market. Market makers in futures pits include scalpers, floor traders (who trade for their own account) and floor brokers, who also process customer orders. In general, these market makers do not know the composition of order flow or the precise information content of market orders. The prices they quote in the process of open outcry can be thought of as a momentary and complete summarization of a subjective probability assessment given their available information. These market makers rely on signals including the volume of orders entering the pit, the types of orders, whether they are buy or sell, limit order, market order, the size of order, etc. While none of these signals are strategy proof manipulable, market makers must be successful at least enough of the time to post an economic profit, otherwise they would exit the market.

Utilizing a methodology similar to that developed in Glosten and Milgrom (1985) and Easley and O'Hara (1987), assume that floor brokers in the pit match orders to buy with orders to sell and absorb order imbalance such that the expected utility of their profits derived from their determination of market clearing futures prices is the same whether they assume an open position, $z_t$, or merely match orders:
Equilibrium in the futures market requires in each period net supply of futures contracts to sum to zero. In order to examine the properties of equilibrium without the influence of manipulation, noise, or market frictions, we first solve for the equilibrium prices when hedgers and speculators who submit orders to the futures market are matched by risk neutral floor brokers. While it is perhaps more unlikely for a speculator to make or take delivery than a hedger, if a priori hedgers have no greater propensity to carry a position to delivery than speculators, then at time $1$, $\alpha^h = \alpha^s = \alpha$.

As noted, floor brokers, in addition to trading for their own account, can process customer orders and can implement "cross trades" in which orders to sell are paired with orders to buy. When net hedging equals the net speculative positions, the following is obtained:

$$F_1^* = (1 - \alpha) \left[ E_1 S_T - \delta + C' (x) \right] + \alpha \left[ E_1 F_{T-1} \right] - A \left[ (1 - \alpha) \sigma_{ss} + \alpha \sigma_{sf} \right] q$$ \hspace{1cm} (15)

in which $A = \alpha^h \alpha^s / (\alpha^h + \alpha^s)$. Since total long positions equal total short positions (net supply is zero) $x$ without superscripts now refers to total long or total short positions taken.

Evaluating the hedgers optimal position given $q$, the equilibrium expected spot price, $E_i(S)$, is given by:

$$E_i S_T = S_1 + C' (q) + \alpha^h \sigma_{ss} q + (A - \alpha^h) \rho^2 q$$ \hspace{1cm} (16)

where:
\[
\rho^2 = \frac{\left[ (1-\alpha) \sigma_{ss} + \alpha \sigma_{sf} \right]^2}{(1-\alpha)^2 \sigma_{ss} + \alpha^2 \sigma_{ff} + 2 (1-\alpha) \alpha \sigma_{sf}}
\]

(17)

Note that \( \rho^2 \) is the familiar measure of hedging effectiveness, although its components are weighted by the decision variable \( \alpha \). Thus, the expected spot price is a function of the current spot price, \( S_t \), the cost of carry \( C'(q) \), and the final collection of terms, which shall henceforth be referred to as the expected risk premium, \( E\phi \).

At this juncture, we define terms that will allow us to identify the role of realized and expected basis in the liquidation versus deliver decision. We refer to the delivery basis as the difference between delivery premia or discounts and the marginal cost of delivery: \( DB = \delta - C'(\alpha) \). The liquidation basis is the difference between the expected spot price at delivery and the expected futures price just prior to delivery: \( LB = E_sS_T - E_fT,T \).

Substituting these expected basis terms and the expected spot price into the forecast of the first period futures price we obtain the following:

\[
F_1^* = S_t + C'(q) + E\phi - \alpha LB - (1-\alpha) DB - A \left[ (1-\alpha) \sigma_{ss} + \alpha \sigma_{sf} \right] q
\]

(18)

Thus, the risk premium inherent in the expected spot price, \( E\phi \), is transferred from the spot market to the futures market via the futures price. In a simpler framework, Kamara (1990) proposes that \( E\phi \) combined with \( A[.]q \) offsets the basis terms under a pure expectations hypothesis, i.e. \( DB + LB = E\phi + A[.]q \). Hence, the futures markets are efficient when \( F_t = E_rS_T = S_t + C'(q) \). Kamara suggests that \( A[.]q \) indicates that hedgers also speculate on futures prices. If \( A[.]q > 0 \), then the speculative position is
long.\(^9\) Note also that \(A[Jq\) moves opposite the risk premium above, suggesting that via these terms, risk is transferred from hedger to speculator. As in Kamara and without loss of generality, \(A[Jq\) is subsumed in the risk premium, \(E\delta\), embedded in the first period futures price.

Combining the basis terms, we define the difference between the expected futures price and the expected spot price at time \(t\) to be given by \(E_tB\) such that:

\[
E_tB = E_tF_{T-1} - [E_tS_T - \delta + C'(x)]
\]  

(19)

The value of this difference, which we refer to as the expected basis following the convention of Kamara, is crucial to the liquidation versus deliver decision. If this expression is expected to be positive, short position holders \((x^k < 0)\) would have a predisposition to liquidate while long position \((x^k > 0)\) holders would prefer to hold until delivery. If \(\alpha^h = \alpha^l\), we can also rule out a non-zero expected basis at time \(0\). Thus, if \(E_tB = 0\) ex ante, then the forecast of first period futures price would be:

\[
\hat{P}_1 = E_1(S_T|Z_1 = 0) = S_1 + C'(q^*) + E\delta
\]  

(20)

**IV. Pricing Strategies Under Conditions of Manipulation**

In order to calculate the equilibrium futures price under conditions of manipulation, we must consider the prices that floor brokers face when they assume the other side of order imbalance. Floor brokers will absorb order imbalance at a

\(^9\) This becomes clear when one realizes that \(A[Jq\) is also a component of the hedge ratio.
price that is incentive compatible. The incentive compatible price corresponds to the
cost of making or accepting delivery in the spot market. The market is one in
which the spot price at settlement is determined in localized delivery markets (outlined
by the contract). Thomas Kilcollin (1984) discusses the economics of a localized
market (consisting of the spot markets in the delivery point locations) in his comment
on Kyle's (1984) paper that is created for the physical commodity during the delivery
month. The equilibrium derived above is one in which supply of the commodity in
the delivery market equals the demand at the equilibrium quantity of cash commodity,
$q^*$. Thus, the profitability of a manipulation will depend on the demand and supply
schedules in the delivery market.

Pirrong (1993) shows specifically that a long manipulation is profitable because
the marginal cost of making delivery of quantities above $q^*$ is increasing. Similarly,
the decreasing marginal revenue of accepting deliveries above $q^*$ makes manipulation
profitable in a short manipulation. Consistent with that formulation, the supply curve
in the delivery market faced by shorts is kinked, with bid prices for the delivered
commodity constant for quantities below $q^*$. Thereafter, the spot price for acquiring
stocks of the commodity is increasing in $x$. As the quantity demanded increases, the
cost of supplying the delivery market increases. This implies that a representative
hedger i faces a marginal cost of delivery that depends on whether the commodity he

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10 One can think of floor brokers in the model acting as agents of spot market participants
who absorb order imbalance in order to arbitrage expected differences in spot and futures
prices.
delivers is in a deliverable condition, in which case, $C'(x_i) = 0$, or not, in which case, he must pay $C'(x_i)$.

Encompassing the paradigms of manipulation, the price on which competitive risk neutral floor brokers would be willing to assume order imbalance can be expressed in the following manner:

$$F_1^* = E_1 \{ S_T | z_1 \neq 0 \} = S_1 + C'(q^*) + E \phi + g(\Omega, x^m, C'(x)) \quad (21)$$

where $\Omega_i$ denotes (positive) private information held by the manipulator and $x^m$ denotes his position. If the manipulator knows that the final spot price will be high as a result of his spot market activities, this would constitute private information, $\Omega_i$. Although we have not directly specified $g(.)$, our earlier description of the paradigms and our derivations indicate that $g(.)$ is nondecreasing in all its arguments. Further, note that the unconditional expectation of futures prices given no private information, friction, or noncompetitive dominant trader, is the expected value of the commodity, in effect, the expected spot price derived in equation 20.

Since the floor brokers do not know $g(.)$, they must calculate $E_i S_T$ based on their assessment that the dominant trader is manipulating the market long or short, using any and all market information that may signal potential manipulation. Consistent with Kyle (1984) and similar microstructure models thereafter, we adopt a framework in which floor brokers condition the deviation of prices from the unmanipulated value on net order imbalance. Thus, $g(\cdot | z_i)$ depends on the signal $z_i$, which summarizes first period order imbalance in which $z_i > 0$ signifies an order
imbalance in which buy orders exceed sell orders.

\[
F_1^* = E_1(S_T | z_1 = x^m + x^h + x^g) = \sum_{x^m} S_T(x^m) \Pr \{ x^m | z_1 \} 
\]

(22)

where using Bayes’ theorem:

\[
\frac{\Pr \{ x^m | z_1 \}}{\Pr \{ x^m = \theta \} \Pr \{ z_1 | x^m = \theta \}} = \frac{\Pr \{ x^m = \theta \} \Pr \{ z_1 | x^m = \theta \}}{\Pr \{ x^m = \theta \} \Pr \{ z_1 | x^m = \theta \} + \sum_{x^m} \Pr \{ x^m \neq \theta \} \Pr \{ z_1 | x^m \neq \theta \}} 
\]

(23)

where \( \theta \) is the strategy space of the manipulator, i.e. the set from which the manipulator chooses his position, and \( S_T(x^m) \) is the spot price of the commodity in the delivery market expected to prevail given the manipulator’s position.

The manipulator maximizes his expected profit over \( x^m \in \theta \):

\[
\mathbb{E} \Pi = \sum_{x^m} \left[ R(x^m) - (E_1(S_T | z_1) x^m) \Pr \{ x^m \} \right] 
\]

(24)

where:

\[
R(x^m) = \alpha^m x^m E_1 F_{T-1} + (1 - \alpha^m) x^m E_1 S_T 
\]

(25)

Thus, this general equation describing the manipulator’s objective function allows for an \( \alpha^m \), an option to offset or deliver after establishing an open position \( x^m \).

In order to gain some intuition on prices that would prevail in an equilibrium with manipulation, it is useful to limit the strategy space of the manipulator such that the manipulator’s optimal strategy is given by either a long position denoted by L, a short position denoted by N, or no position, \( x^m \in \{ L, N, \theta \} \). Define \( \lambda \) as the
probability of a long manipulation that results in an elevated spot price, $S^+_t$. If the manipulator is detected in period 1, floor brokers act as Bayesian competitors and bid prices to $S^+_t = S^* - C'(x)$ with probability $\lambda$, and $S^+_t = S^* - \delta'(x)$ with probability $1 - \lambda$. We make a notational change, explicitly $\delta'(x)$, and define it as the marginal discount emanating from the downward sloping demand in the delivery market for quantities exceeding $q^*$. Given that the manipulator has entered the market, the floor brokers wish to bid prices according to the probability that the manipulator will attempt a short or long manipulation.

$$S^* = E[\xi_s^+ | x^m = L] \cdot Pr \{x^m = L\} + E[\xi_s^+ | x^m = N] \cdot Pr \{x^m = N\}$$

$$= S^+_1 \lambda + S^+_1(1 - \lambda) \tag{26}$$

If long and short manipulations are equally as likely, the pregame (prior) strategy is to set $\lambda = \frac{1}{2}$ when the dominant trader is present and when he is not. However, when information is complete and the floor brokers know with certainty that the dominant trader is manipulating, they would set $\lambda = 1$ when he takes a long position in period 1 ($x^m = L$), and $\lambda = 0$ when he takes a short position ($x^m = N$). This suggests the one period updating formula of $\lambda_t$ for a given signal, $z_1$, is:

$$\lambda_t(z_1) = Pr \{S_t = S^+_t | z_1\} = 1 \cdot Pr \{x^m = L | z_1\} + 0 \cdot Pr \{x^m = N | z_1\} + \lambda \cdot Pr \{x^m = 0 | z_1\} \tag{27}$$

Floor broker’s use their subjective probability beliefs regarding the actions of traders in the market when they competitively bid to the equilibrium $F^*_1$. Denote $v$ as the probability that the manipulator has entered. Using an approach similar to that of Easley and O’Hara (1987), the subjective probability beliefs about the fraction of order
flow that floor brokers believe to be that of the various position takers in the market influence the equilibrium price. Thus, we denote \( \mu \) as the fraction of order flow that the floor brokers believe to be that of the manipulator given he has entered, \( 0 < \mu < 1 \).

To help retain our intuition about the positions of hedgers and speculators, denote \( \chi^s \) as the fraction of total order flow believed to be long in t=1 and \( \chi^h \) as the fraction of total order flow believed to be short. Using Bayes' rule above, we can then solve for the conditional probability that a long manipulation is eminent given net buy pressure in the first period:

\[
\lambda (z_1 > 0) = \frac{\mu v + \chi^S}{\lambda \mu v + \chi^S} > \lambda \tag{28}
\]

This implies that the probability of an elevated spot price due to a long manipulation is increased when order imbalance is net buy.

Similarly, we can derive the conditional probability the manipulator is attempting a short manipulation given negative order imbalance:

\[
\lambda (z_1 < 0) = \frac{\chi^h}{(1 - \lambda) \mu v + \chi^h} < \lambda \tag{29}
\]

Thus, based upon the observed order imbalance and the subjective beliefs floor brokers have regarding \( v, \mu, \) and \( \chi \), floor brokers calculate their conditional expectations and use these to set the futures price that they bid or offer to assume order imbalance. The floor brokers bid competitively to the equilibrium price, which is a linear combination of the price they would be willing to pay (receive) given no manipulation and the price they would be willing to pay (receive) given manipulation. Given a positive
order imbalance, floor brokers will bid prices to the following first period equilibrium futures price:

$$F_1^*(z_1 > 0) = S^* + \frac{\sigma^2_s}{S^* - S^-} \left[ \frac{\nu \mu}{\lambda \nu \mu + \chi^s} \right]$$  \hspace{1cm} (30)

where $\sigma^2_s$ is the variance of $S^*$. Thus, if floor brokers are faced with positive order imbalance, i.e. net buying pressure, they will only sell for a price that is higher than the no manipulation price, $S^*$. Similarly, a negative order imbalance signals potential short manipulation, and floor brokers bid to the following equilibrium futures price:

$$F_1^*(z_1 < 0) = S^* - \frac{\sigma^2_s}{S^* - S^-} \left[ \frac{\nu \mu}{(1-\lambda) \nu \mu + \chi^b} \right]$$  \hspace{1cm} (31)

Differentiating 28 indicates that $\lambda(z_1 > 0)$ is increasing in $\mu \nu$, suggesting an important result about detection and manipulation. Floor brokers formulate their subjective probability beliefs using all available information about the likelihood and magnitude of manipulation. Any information about the make up of order flow and what it means for expected prices is used by floor brokers to set their bids and asks. As the probability that the manipulator has entered increases, equilibrium bids and asks move farther away from the equilibrium price under conditions of no manipulation. As the fraction of order flow they believe is coming from the manipulator increases, the more they competitively bid prices against the manipulator. Hence, detection by floor brokers imposes a direct cost on the manipulator via the equilibrium futures price. If he is detected, even to a small degree, the cost of attaining his initial futures position increases.
Since hedgers and speculators do not know the probability assessments of the floor brokers, the first period futures price provides a signal that they can use to optimally choose $\alpha$, the decision variable for liquidation or deliver. Recall that $\alpha$ is a component of both equilibrium futures and settlement price (through the correlation coefficient). However, an individual hedger or speculator cannot assess how his choice of $\alpha$ will influence prices. This follows since traders do not know the composition of order flow. By inspection of the expected profit functions in equations 9 and 12, one can see that the short futures holder at T-1 will rationally choose to offset in T-1 when the expected basis they face is negative and deliver when it is positive.

V. The End Game in Liquidation Strategies

The preceding sections have laid out the optimizing strategies of each market participant given their expectations about future prices. We now wish to consider the reactions of traders to revelation of those prices given their beliefs about the manipulators activities. Imperfect information about the manipulator's effect on prices, the sequential nature of the problem, and the importance of beliefs to the outcome suggest a perfect Bayesian equilibrium concept. In this section we impose further restrictions on the market participants, their beliefs, and their strategy space so that the model satisfies the requirements of a finite game, thus ensuring an equilibrium exists.
and is meaningful.¹

By definition, a perfect Bayesian equilibrium is one in which:

1. Given the realization of the first period futures price, players form beliefs about whether the market is being manipulated. Since hedgers and speculators operate under imperfect information, these beliefs are represented by probabilities. Players use Bayes' rule and their equilibrium strategies to determine their beliefs.

2. Given their beliefs about the actions of the manipulator, the strategies of the players are sequentially rational. This means that their choice of \( \alpha^k \) is Nash given the beliefs and strategies of the other players.

The object of this end game for each player including hedgers, speculators, and the manipulator, is to select the value of \( \alpha^k \) that maximizes their expected payoff given what they expect other players in the game to do. Players in this game are sequentially rational in the sense that, given their initial positions, they choose their \( \alpha^k \) using their beliefs about what the manipulator will do given that he has or has not been detected. Formally, we can now define an equilibrium as follows:

**Def.** This manipulation game, consisting of strategies \( \alpha^k_i \in [0,1] \) and beliefs, \( B(x^m, p(z, \cdot)) \), attains a perfect Bayesian equilibrium when, for any player \( i \) of type \( k \) and any strategy \( \alpha^k \):

¹Kreps and Wilson (1982) prove that there exists a sequential equilibrium in any finite game. Since sequential equilibria encompass perfect Bayesian equilibria, constructing a finite game will enable us to answer the questions we have posed regarding detection of manipulation without a detailed proof of the existence of equilibrium.
\[ E[\Pi_t^k (\alpha_i^k, \Sigma_{-i}) \mid B(x^m, p(z_t)) ] \approx E[\Pi_t^k (\alpha_i^k, \Sigma_{-i}) \mid B(x^m, p(z_t)) ] \] (32)

where \( \Sigma_{-i} \) represents the strategies of other players in the game, and \( p(z_t) \) denotes the probability of detecting the manipulator such that \( \Sigma p(z_t) = 1 \).

The Signalling Model

As noted, hedgers and speculators estimate the first period futures price and this allows them to formulate demands for futures contracts. The extent to which their forecast differs from the realized futures price, \( F_1^* \), acts as a signal \( \gamma \) that provides information on expected prices in the liquidation and delivery period. Thus, traders update expected prices according to

\[ E_{T-1} F_{T-1} = E_1 F_{T-1} + f(\gamma) \] (33)

such that \( f(\gamma) > 0 \) and \( f(\gamma) = 0 \) when \( \gamma = 0 \).

Assuming normality of \( F_1 \) and \( S_1 \), traders employ Bayesian updating to form their expectations:

\[ E_{T-1} F_{T-1} = \frac{\left[ E_1 S_T + C'(x) - \delta'(x) \right] \left( \frac{1}{\sigma_{SS}} \right) + F_1^* \left( \frac{1}{\sigma_{FF}} \right)}{\frac{1}{\sigma_{SS}^2} + \frac{1}{\sigma_{FF}^2}} \] (34)

\[ E_{T-1} F_{T-1} = \left[ E_1 S_T + C'(x) - \delta'(x) \right] \tau_S + F_1^* \tau_F \] (35)

When constructing an estimate of liquidation prices, both hedgers and speculators will be highly responsive to price signals originating in the futures market, assigning greater precision to the observation of the realized futures price than to the prior
estimate constructed from cash prices. This condition leads directly to the following
precision relation:

$$\tau_s < \tau_F, \quad \tau_s + \tau_F = 1 \quad \Rightarrow \quad \tau_F \geq \frac{1}{2}$$  \hspace{1cm} (36)

Using equations 33, 35 and 36 above:

$$f(\gamma) = \tau_F \left[ g(\cdot | z_1) - C'(x) + \delta'(x) \right]$$  \hspace{1cm} (37)

substituting \(\tau_F\) from equation 36 above leads to the result that \(\tau_F(\gamma) \geq \gamma/2\) that, when
combined with the fact that \(f(\gamma)\) is increasing in \(\gamma\), yields the following inferences:

$$f(\gamma) = \begin{cases} 
>0 & \text{for } \gamma > 0 \\
<0 & \text{for } \gamma < 0 \\
=0 & \text{for } \gamma = 0 
\end{cases}$$  \hspace{1cm} (38)

We can now describe the optimal liquidation and delivery strategies for hedgers and
speculators given a positive, negative, or flat signal derived from the realized value of
the \(t = 1\) equilibrium futures price.

We have shown that hedgers’ positions, \(x^h\), and speculators’ positions, \(x^s\),
depend on prices that are expected to prevail in the spot market at settlement. This is
true regardless of the risk aversion of traders. We have also shown that deviation of
futures prices from the expected spot price will cause traders to adjust their liquidation
strategies. In order to simplify the analysis, we assume that all traders who hold open
positions in the futures markets after period 1 seek to maximize expected payoff by
choosing an optimal \(\alpha\). In effect, the traders hold open positions and must now decide
what is the best liquidation strategy. The model can now be viewed as is a one shot
decision game in which traders choose to liquidate or deliver given their open positions.

To identify the equilibrium reactions of hedgers and speculators to signals indicating manipulation, backward induction dictates that we take their positions as exogenous and consider their best response given their initial positions. As before, we consider the results when hedgers hold short positions and the speculators hold long positions. We also concentrate our discussion on long manipulation, keeping in mind that the model is symmetric. Some results for a short manipulation are also presented.

In order to evaluate candidates for equilibria, we impose the following restriction on the behavior of hedgers and speculators:

1. Hedgers and speculators, after having observed the first period futures price, will maintain their current \( \alpha \) when they do not detect a manipulation via price. Conversely, they will revise their \( \alpha \) when prices signal that a manipulation is occurring.

The \( t=1 \) equilibrium in which \( F_t = E_t (S_T | z_t = 0) \), leads hedgers and speculators to believe that their prior forecast of futures prices was correct. They will therefore decline to revise expected prices. Recall that arbitrage by floor brokers leads to zero expected basis, so if there is no reason to expect a manipulation is occurring, hedgers and speculators will maintain their pregame strategy, which is assumed to be one in which \( \alpha^h = \alpha^s = \alpha \).

Note that hedgers and speculators, however, do not generally use the futures market to facilitate spot market transactions. Under conditions of no manipulation, it
is a simple exercise to show that when hedgers and speculators offset their entire position in $T-1$, the futures price in $T-1$ is still that given in equation 20. Completely offsetting their position in $T-1$ has no effect on futures prices. Floor brokers match offsetting orders and bid prices to $F^*_{r,1} = E_1 (S_T \mid z_1 = 0) = S^*$. Initially assume that this is the case, in fact, $\alpha^h = \alpha^s = 1$, giving another simplifying assumption:

2. **Ceteris paribus**, hedgers and speculators prefer to offset their contracts in $T-1$.

   We will relax assumption 2 later in this section and show that the liquidate strategy dominates the hold strategy for hedgers and speculators in most subgames considered.

   Under conditions of an unanticipated manipulation, however, orders to offset open positions before delivery will not match. This is because the manipulator, by adopting a hold strategy, seeks to create a liquidity shortage in the period prior to delivery and profit from it. The following rules of the game simplify our analysis:

3. Floor brokers will absorb order imbalance in T-1 at a price that is incentive compatible, in this model that is the price that offsets the cost of making or accepting delivery.

4. If the manipulator is undetected in period 1, $F^*_{1} = E_1 (S_T \mid z_1 = 0) = S^*$. If the manipulator is detected in period 1, floor brokers act as Bayesian competitors and bid prices to $S^*_r = S^* + C'(\chi)$ with probability $\lambda$, and $S^*_r = S^* - \delta'(\chi)$ with probability $1 - \lambda$.

5. When floor brokers detect manipulation, they set $\lambda = 1$ if they believe a long manipulation is occurring, and $\lambda = 0$ if they believe a short manipulation is in
progress.

Under conditions of manipulation, traders making or taking delivery who do not have preexisting agreements or arrangements may find they cannot receive $S^*$ for the commodity they accept. If the demand for the commodity in the delivery market is downward sloping, quantities brought into the delivery market in excess of $q^*$ will depress the price below $S^*$. The manipulator accepting delivery may also face this depressed price, which would eat away at his expected profits. Since this would cut into the profits gained from manipulation, it is referred to as the cost of "burying the body." The existence of the cost of burying the body imposes a constraint on the size of position on which the manipulator will make or take delivery. The manipulator will optimize accordingly. He may make arrangements such that the position he holds to delivery, $(1-\alpha)x^n$, can be bought or sold for $S^*$. This is not true for other traders in the market, however, who, by assumption 2, do not seek to conduct cash market transactions via the futures markets. At most, traders can secure willing buyers and sellers for the price of $S^*$ up to the quantity of $q^*$. Floor brokers who acquire open positions face prices in the spot market given by the incentive compatible prices described in assumptions 3 and 4, above.

These simplifying assumptions allow us to calculate payoffs to choosing liquidate and hold strategies under a number of possible combinations of strategies. These payoffs are presented in the appendix and provide the basis for the following propositions.
Proposition 1:

If the manipulator is undetected via order imbalance, hedgers and speculators will decline to revise $\alpha^h = \alpha^s$ and the manipulator will adopt a hold strategy. In the case of a long manipulation, the manipulator will stand for delivery of $(1-\alpha^m)x^m$ such that $\alpha^m < 1 - q^*/x^m$.

The amount for which the long manipulator stands for delivery must exceed $q^*$, the equilibrium quantity of commodity in the delivery market. This implies the $x^m$ must also be greater than $q^*$. At quantities above $q^*$, shorts face an increasing marginal cost of delivery, $C'(x)$, when they have not made preparation for delivery.

When the long manipulator stands for delivery of a quantity in excess of $q^*$, the floor brokers will bid equilibrium prices to $S^* + C'(x)$ with probability 1 in period T-1. The manipulator receives this for the proportion of his position that he offsets in T-1, $\alpha^mx^m$.

Proof of proposition 1 follows from the fact that if the manipulator stands for delivery of less than $q^*$, floor brokers will bid to the equilibrium price of $S^*$, earning the manipulator zero payoff. If the manipulator stands for delivery of $(1-\alpha^m)x^m > q^*$, his payoff is given by $C'(x)\alpha^mx^m > 0$. Thus, the manipulator’s hold strategy includes standing for delivery of some optimal proportion of his open positions that exceeds $q^*$. A similar result is obtained in the calculation of payoffs in a short manipulation.

Proposition 2:

When the manipulator is undetected, he earns a payoff on the proportion of his position he offsets in T-1 corresponding to the marginal cost of delivery when
he manipulates long. When he manipulates short, he earns a payoff on the proportion of his position he offsets in T-1 corresponding to \( \delta'(x) \), the marginal discount emanating from the downward sloping demand in the delivery market for quantities exceeding \( q^* \).

In a short manipulation, the manipulator declares his intention to make delivery of an amount that exceeds the equilibrium quantity of commodity in the delivery market. His strategy is to swamp the delivery market such that floor brokers will bid prices in the liquidation period to the incentive compatible price of \( S^* - \delta'(x) \). Unsuspecting speculators who are holding contracts calling for delivery of commodity they had no intention of accepting delivery on, will be forced to accept the discounted price in order to liquidate their positions. This brings up the next proposition.

**Proposition 3:**

When the manipulator is undetected, the profits and losses of hedgers and speculators depend on whether their initial positions are the same as the manipulator, long or short, or opposing. If their open position is the same sign as the manipulator, they will profit fortuitously on their position. Again, their payoff will correspond to the marginal values described above. If their position is on the opposing side, the loss on the position will likewise be levied on their offsetting trade in T-1.

Proposition 3 relies on the highly plausible assumptions regarding the incentive compatibility of prices and the predisposition of traders to liquidate early for its derivation. The payoff calculations on which this proposition is derived are given in
the appendix. One can see that in the case of a long manipulation, short hedgers pay
\( C'(x) \) to offset their position. Speculators on the long side offsetting their position find
themselves the recipients of an unanticipated manipulation premium courtesy of the
unwitting hedgers. In the case of a short manipulation, the reverse occurs, with
hedgers profiting fortuitously while speculators pick up the tab. As we see in the next
proposition, however, detection of the manipulator allows hedgers and speculators to
strategically dodge the prospect of undesirable losses due to manipulation.

**Proposition 4:**

When the long manipulator is detected, the best response of short hedgers who
cannot secure the commodity at a price that is less than the incentive
compatible price is to choose \( \alpha^h = 1 \) and earn \( \Pi^h = 0 \). Short hedgers who can
secure the commodity will choose \( \alpha^h = 0 \) and earn \( \Pi^h = C'(x)x^h \). The best
response of speculators is to choose \( \alpha^s = 1 \) and earn \( \Pi^s = 0 \).

The signaling model indicates that traders on the opposing side of the
manipulator who do not have supplies in a deliverable condition or are not prepared to
take delivery, having detected the manipulator's activity, would prefer to liquidate their
futures positions by purchasing or selling offsetting contracts to net out their futures
position in the period prior to settlement, \( T-J \). Because of the incentive compatibility
of the prices bid on the floor, traders who hold open positions opposite the
manipulator optimal strategy choose \( \alpha = 1 \) and liquidate their entire futures position
since they know they can pay/receive no more/less than they would if they performed
or accepted delivery.
Short traders opposite the long manipulator who have supplies in a deliverable condition or can secure them at reasonable prices will optimally choose \( \alpha^h = 0 \) and will make delivery since, having detected the manipulator, they will not wish to liquidate their position at potentially biased prices during the liquidation period. They have no incentive to offset their position in \( T-1 \). Their best response is to deliver the commodity to the manipulator, receiving the first period futures price biased by detection. This results in the next proposition:

**Proposition 5:**

Even without collusion or precommitment on the part of strategic traders opposite the manipulator, detection leads to an \( \alpha^k \) for the population of opposing traders between 0 and 1 that denies the manipulator a positive payoff. Specifically, in a long manipulation, the best response to detection is \( \alpha^h = 1 + q^*/x^h \). In a short manipulation, the speculator’s best response to detection is \( \alpha^s = 1 - q^*/x^s \).

In a long manipulation, \( \alpha^h \) is derived by considering the supply schedule faced by hedgers who, if they choose to deliver, must secure commodity in the delivery market. In the short manipulation, the demand schedule gives the decreasing marginal revenue expected when the manipulator makes delivery of quantities beyond \( q^* \). Thus, shorts can secure buyers willing to pay \( S^* \) for the commodity up to \( q^* \). Since prices in the liquidation period are incentive compatible, long position holder opposite a short manipulation will offset all contracts for which they can receive less than \( S^* \).

Thus, detection of the manipulator allows traders who hold positions opposite
him to strategically choose the $\alpha$ that minimizes the price they pay to liquidate their position. Detection of the manipulator also influences those who hold positions on the same side as the manipulator. Detection of a long manipulation stems from positive net order imbalance, i.e. $x^t + x^m > -x^h$. Given knowledge of the manipulation, holders of long positions given by $x^t$ have no incentive to take delivery.

We can now summarize what we have discovered regarding the reactions of traders in our model to the events of detection and when the manipulator’s activity is not detected in a normal form game representation, given in figure 6.
\[ x^m > 0, \ x^h < 0, \ x^s > 0 \]

I. Detected: \( \lambda = 1 \)

**Hedger, Speculator Strategies**

<table>
<thead>
<tr>
<th>Manipulator Strategies</th>
<th>Liquidate</th>
<th>Hold</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha^h = \alpha^s = 1 )</td>
<td>(-C'(x)(1-\alpha^m)x^m,0,0)</td>
<td>(-C'(x)(1-\alpha^m)x^m,C'(x)q^*,0)</td>
</tr>
<tr>
<td>( \alpha^m = \alpha^s )</td>
<td>(-C'(x)x^m,-C'(x)x^s,-C'(x)x^t)</td>
<td>(-C'(x)x^m,-C'(x)x^s,-C'(x)x^t)</td>
</tr>
<tr>
<td>Liquidate</td>
<td>(\alpha^m = 1)</td>
<td></td>
</tr>
<tr>
<td>( \alpha^m = 1 )</td>
<td>(0,0,0))</td>
<td>(0,0,0))</td>
</tr>
</tbody>
</table>

II. Not Detected: \( \lambda = 1/2 \)

**Hedger, Speculator Strategies**

<table>
<thead>
<tr>
<th>Manipulator Strategies</th>
<th>Liquidate</th>
<th>Hold</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha^h = \alpha^s = 1 )</td>
<td>(-C'(x)x^m,C'(x)x^s,C'(x)x^t)</td>
<td>(\alpha^h = 1+q^*/x^h, \ \alpha^s = 1)</td>
</tr>
<tr>
<td>( \alpha^m = \alpha^s )</td>
<td>(C'(x)x^m,C'(x)(x^h+q^*),)</td>
<td>(C'(x)x^t)</td>
</tr>
<tr>
<td>Liquidate</td>
<td>(\alpha^m = 1)</td>
<td></td>
</tr>
<tr>
<td>( \alpha^m = 1 )</td>
<td>(0,0,0))</td>
<td>(0,0,0))</td>
</tr>
</tbody>
</table>

**THE NORMAL FORM OF THE LONG MANIPULATION SUBGAME**

Figure 6
We can now describe the equilibrium in our liquidation strategy game.

**Proposition 6:**

There exists a perfect Bayesian equilibrium in liquidation strategies in which hedgers and speculators offset their open positions when a manipulation is undetected. The manipulator’s best strategy given the events that he is detected and not detected is to respond with a hold strategy in either case. When the manipulator is detected, traders who hold open positions opposite the manipulator who can make arrangements in the delivery market will do so and choose to hold and perform on the contract at delivery. Traders holding open positions on the opposing side of the manipulator who do not have supplies in a deliverable condition or are not prepared to take delivery, having detected the manipulator’s activity, will liquidate their futures positions by purchasing or selling offsetting contracts to net out their futures position in the period prior to settlement, T-1. Further, when the manipulator is detected, traders holding open positions on the same side as the manipulator will choose to offset their position in the period prior to delivery.

We begin the proof of this proposition by considering the optimal strategies of hedgers and speculators. Above, we initially assumed that hedgers and speculators would play the liquidation strategy when the manipulator is undetected via first period futures prices. We found that if they both respond with liquidate and the manipulator is present, players on the same side as the manipulator profit fortuitously while players
opposite the manipulator suffer a loss. We now discuss possible outcomes when the hedgers and speculators respond with a hold strategy when the manipulator is not detected.

If adopting a hold strategy protects traders from losses when the manipulator is detected, what about when the manipulator is not detected? Can hedgers and speculators protect themselves from losses to a manipulator by adopting a hold strategy all the time? To answer this question, we must look at payoff calculations when the traders adopt a hold strategy when the manipulator is present and undetected, and when he is not present.

If the manipulator does not enter and both hedgers and speculators who can secure buyers and sellers for the commodity at $S^*$ hold, standing for and making delivery of $q^*$, their payoff would be zero. As before, those who cannot secure arrangements in the delivery market would liquidate. Since the open interest offset would equal, prices would be given by $S^*$ in all three periods. Payoffs again would be zero. As show earlier, regardless of what level $\alpha$ is, if hedgers and speculators in the market have the same liquidation strategies, i.e. $\alpha^s = \alpha^h$, and the manipulator is not active the equilibrium price is $S^*$.

If the manipulator did enter, is not detected, and hedgers and speculators chose hold, the manipulator still profits. The best that the hedgers and speculators who hold can do is break even. This is because they took their initial positions for $S^*$, made delivery arrangements, then performed on $q^*$ of their initial positions. As in the case
when manipulation is detected, traders on the opposing side who cannot make delivery provision will lose \( C'(x) \). Summing over all traders who have open positions opposite the manipulator, payoffs are negative. Similarly, the best traders on the same side as the manipulator who had adopted the defensive hold strategy can do is break even. Those who could or did not choose to deliver will be glad they didn’t; they will profit fortuitously on every contract they liquidate in \( T-1 \).

These results indicate that the liquidation strategy will dominate the hold strategy for both hedgers and speculators. First, traders break even when they liquidate and there is no manipulation. Second, if there is an equal probability that the manipulator will go long or short, that is, \( \lambda = \frac{1}{2} \), then there is an equal probability that a trader will find himself in a position to profit fortuitously from manipulation. The only way a trader can realize this potential payoff is through liquidation. If he adopts a hold strategy without even knowing whether a manipulation is on or not, the best he can do is break even. Adopting a liquidation strategy when the manipulator is undetected does open up the possibility of a loss, but the probability of a loss due to manipulation is the same as when traders liquidate in a detected manipulation.

The last thing that remains for us to establish is that the manipulator’s best response to both the event that his presence is not evident and the event of his detection is to maintain the hold strategy. This is clear when one considers that payoffs when the manipulator holds are greater than or equal to payoffs from early offset in both the event of detection and not. Figure 7 provides a summary of the
decision tree of the manipulator and the payoffs he can expect from choosing the 
liquidate or hold strategies.

Ultimately, the payoffs to the manipulator depend on whether he has been 
detected by other traders or not, and if he is detected in time for traders to adopt a 
defensive liquidation strategy. As discussed in the previous section, the probability 
that a manipulation is detected depends on the subjective probability beliefs of the 
floor brokers since the signals provide incomplete information about the manipulator’s 
activity.

Denote the probability of detection of the long manipulation as \( p(x^m | z) \) such 
that \( \sum_d p(x^m | z) = 1 \). The expected profit of the manipulator, given that he is attempting 
a long manipulation, is:

\[
E \pi^m = [ -C'(x)(1 - \alpha^m)x^m] p(x^m | z) + [C'(x) \alpha^m x^m](1 - p(x^m | z)) \\
= C'(x)x^m (\alpha^m - p(x^m | z))
\]

This implies that long manipulation is profitable if the proportion of contracts he 
liquidates at the incentive compatible prices exceeds the probability of his orders being 
detected. His profits are increasing in the marginal cost of delivery, his position size, 
and in the amount he can liquidate at the incentive compatible price. His profits are 
decreasing in the probability of detection, which increases with order imbalance. This 
suggests that a manipulator must acquire his open position \( x^m \) as quietly as possible, 
being careful not to create order imbalance and alert the floor brokers to his presence. 
In a more dynamic setting, the manipulator would strategically "break up" his order in
its size and over time to minimize the probability he will create order imbalance and have an effect on prices.

VI. Conclusions and Future Research

The introduction to this paper noted that previous research on futures market manipulation suggested a number of unanswered questions. A model was constructed to answer, at least partially, these questions. First, possible utility functions of hedgers and speculators in the markets were examined to see what factors influence their decision making. The model was solved for equilibrium prices that would prevail absent manipulation, i.e. when floor brokers matched the orders of hedgers and speculators. Futures prices were found to reflect the current spot price, the cost of carry, and a risk premium. Floor brokers, acting as Bayesian competitors, would adjust their bids given a possibility of manipulation. Examination of the Bayesian mechanism yielded a relationship between the beliefs of the floor brokers and their bidding behavior: as the probability that the manipulator has entered increases, equilibrium bids and asks move farther away from the equilibrium price under conditions of no manipulation. Finally, we boiled what we learned from those exercises down to a strategic game in order to investigate how traders would respond to those prices. In conclusion, we want to look at what answers we were able to find.

The first question involved the pricing of futures contracts in a market where manipulation occurs. If futures are priced via a competitive auction, how does a large,
potentially monopolistic, trader acquire his dominant position without alerting others in the market? In general, we proposed that a manipulator can acquire his dominant position without alerting other traders in the market because they have imperfect information about his activity. If traders knew the manipulator’s orders for futures contracts they could detect his intentions perfectly. Prices would reflect his effect on the market perfectly (The price would equal the expected value of the spot commodity at T under conditions of manipulation with probability 1). The market imperfection arises from the fact that traders cannot distinguish the manipulator’s orders from others.

Without the standard assumption that the large trader has private information on the moves of others, can the large trader still engineer a manipulation? Our examination of literature on manipulation, particularly that of Pirrong (1993), revealed an affirmative answer. Pirrong’s research indicates that the increasing marginal cost of delivery makes a long manipulation profitable because shorts are forced to make delivery of commodity that must be brought in from outside the delivery market, entailing higher costs of making delivery on their open contracts. The manipulator squeezes the shorts by standing for delivery of more than the deliverable supply, making delivery of the more costly commodity necessary. Shorts, who prefer not to make delivery of the expensive commodity will be willing to liquidate at the higher price they would have paid, i.e. the marginal cost. This concept was captured in the model by requiring that equilibrium prices in the liquidation market be incentive
compatible. The results indicate that private information on the moves of others is not a necessary condition for a profitable manipulation. If the large trader is able to acquire his position without the knowledge of others in the market, incentive compatibility results in his earning a payoff on the contracts he offsets corresponding to the marginal cost and revenue associated with making or taking delivery.

In the model, given that they are aware of the possibility of manipulation, floor brokers bid competitively to an equilibrium price, a probability weighted average of the price they would be willing to pay (receive) given no manipulation and the price they would be willing to pay (receive) given manipulation. By limiting the strategy space of the manipulator such that the manipulator’s optimal strategy was given by either a long position, a short position, or no position, we were able to uncover some results regarding equilibrium prices under conditions of manipulation. As the fraction of order flow floor brokers believe is coming from the manipulator increases, the more they competitively bid prices against the manipulator. Hence, detection by floor brokers imposes a direct cost on the manipulator via the equilibrium futures price. If he is detected by the floor, the cost of attaining his initial futures position increases.

In addition, other traders may observe the deviation of futures prices from their expected value and use it as a signal to help them adjust their liquidation and delivery strategies.

Our final line of questions concerned how revelation of a manipulator’s presence would alter the behavior of other traders. We found that Easterbrook’s
proposition that traders could adopt a strategy in which early liquidation or delivery provisions protect them from losses was true when prices accurately reflect the manipulator's activity. Although we did not allow mixed strategies in the liquidation game, it is clear that allowing for the prospect that prices only partially reflect the manipulator's activity would result in losses for those traders who are unable to make delivery provisions. Thus, precommitment to the price specified in the contract does not lock traders into a no-win situation. Allowing floor brokers mixed pricing strategies would change this result since the first period price when the manipulator is partially detected would be different than the perfectly revealing price bid to in the liquidation period.

While we were able to answer the questions using a step-by-step approach, we have said very little about equilibrium and its qualities. Thus, the path for future research is to uncover the conditions under which equilibrium exists in a more generalized framework. Such a model would yield some interesting comparative static results. In a market microstructure analysis, the model has implications relating to the effect of manipulation on traders' willingness to use futures markets, the depth and liquidity of futures markets in general and over the life of a contract, the cost of using futures markets or the bid-ask spread, and the volatility of prices. Further, superimposed in a more dynamic setting, it would of interest whether or not the game described is strategy proof.
Decision Tree of Dominant Trader

Figure 7
References

(see also comments by Robert Rosenthal and James Hayes)

see also comments by Richard Kihlstrom.


see also comments by Steve Salant and Thomas Kicicollin.


see also comments by Louis Phlips and Lawrence White.

Peck, Anne and Williams, Jeffrey, "Market Concentration and Deliveries on Commodity Futures Contracts," mimeo, Food Research Institute, 1990.


Appendix

The equilibrium prices bid and offered by the floor brokers indicate the extent to which a manipulator's activity is disguised by the trading activity of hedgers and speculators. In the following payoff calculations, we assume that when the manipulator is undetected, his activity is just disguised, yielding no order imbalance and, thus, an equilibrium price of $S^*$. If the manipulator is detected, futures prices will be bid to their incentive compatible levels depending on the direction of the order imbalance that must be absorbed by the floor.

1. Long manipulation, undetected, $\alpha^h = \alpha^s = \alpha^m = 1$ [x^h < 0, x^s > 0, x^m > 0]

$$\Pi^m = -S^*x^m + S^*x^m = 0$$
$$\Pi^h = -S^*x^h + S^*x^h = 0$$
$$\Pi^g = -S^*x^g + S^*x^g = 0$$

2. Long manipulation, undetected, $\alpha^h = \alpha^s = 1$, $\alpha^m < 1 - q^*/x^m$

$$\Pi^m = -S^*x^m + (S^* + C'(x)) \alpha^mx^m + S^*(1 - \alpha^m) x^m = C'(x) x^m > 0$$
$$\Pi^h = -S^*x^h + (S^* + C'(x)) \alpha^hx^h = C'(x) x^h < 0$$
$$\Pi^g = -S^*x^g + (S^* + C'(x)) \alpha^gx^g = C'(x) x^g > 0$$

3. Long manipulation, undetected, $\alpha^h = 1 + q^*/x^h$, $\alpha^s = \alpha^m = 1$

$$\Pi^m = -S^*x^m + S^*x^m = 0$$
$$\Pi^h = -S^*x^h + S^*\alpha^hx^h + S^*(1 - \alpha^h) x^h = 0$$
$$\Pi^g = -S^*x^g + S^*x^g = 0$$

4. Long manipulation, undetected, $\alpha^h = 1 + q^*/x^h$, $\alpha^s = 1$, $\alpha^m < 1 - q^*/x^m$

$$\Pi^m = -S^*x^m + (S^* + C'(x)) \alpha^mx^m + S^*(1 - \alpha^m) x^m = C'(x) x^m > 0$$
$$\Pi^h = -S^*x^h + (S^* + C'(x)) (x^h + q^*) + S^* (-q^*) = C'(x) (x^h + q^*) < 0$$
$$\Pi^g = -S^*x^g + (S^* + C'(x)) x^g = C'(x) x^g > 0$$

5. Long manipulation, undetected, $\alpha^h = 1 + q^*/x^h$, $\alpha^s = 1 - q^*/x^s$, $\alpha^m < 1 - q^*/x^m$

$$\Pi^m = -S^*x^m + (S^* + C'(x)) \alpha^mx^m + S^*(1 - \alpha^m) x^m = C'(x) x^m > 0$$
$$\Pi^h = -S^*x^h + (S^* + C'(x)) (x^h + q^*) + S^* (-q^*) = C'(x) (x^h + q^*) < 0$$
$$\Pi^g = -S^*x^g + (S^* + C'(x)) (x^g - q^*) + S^* (q^*) = C'(x) (x^g - q^*)$$
6. Long manipulation, detected, $\alpha^h = 1 + q^*/x^h$, $\alpha^x = 1 - q^*/x^x$, $\alpha^m < 1 - q^*/x^m$

$$\Pi^m = -( S^* + C^l(x) ) x^m + S^* x^m = -C^l(x) x^m < 0$$

$$\Pi^h = -( S^* + C^l(x) ) x^h + S^* x^h = -C^l(x) x^h > 0$$

$$\Pi^s = -( S^* + C^l(x) ) x^s + S^* x^s = -C^l(x) x^s < 0$$

7. Long manipulation, detected, $\alpha^h = 1 + q^*/x^h$, $\alpha^x = 1$, $\alpha^m < 1 - q^*/x^m$

$$\Pi^m = -( S^* + C^l(x) ) x^m + ( S^* + C^l(x) ) \alpha^m x^m + S^* (1 - \alpha^m) x^m = -C^l(x) (1 - \alpha^m) x^m < 0$$

$$\Pi^h = -( S^* + C^l(x) ) x^h + S^* x^h = -C^l(x) x^h > 0$$

$$\Pi^s = -( S^* + C^l(x) ) x^s + S^* x^s = -C^l(x) x^s < 0$$

8. Long manipulation, detected, $\alpha^h = \alpha^x = \alpha^m = 1$

$$\Pi^m = -( S^* + C^l(x) ) x^m + S^* x^m = -C^l(x) x^m < 0$$

$$\Pi^h = -( S^* + C^l(x) ) x^h + S^* x^h = -C^l(x) x^h > 0$$

$$\Pi^s = -( S^* + C^l(x) ) x^s + S^* x^s = -C^l(x) x^s < 0$$

9. Long manipulation, detected, $\alpha^h = \alpha^x = 1$, $\alpha^m < 1 - q^*/x^m$

$$\Pi^m = -( S^* + C^l(x) ) x^m + ( S^* + C^l(x) ) \alpha^m x^m + S^* (1 - \alpha^m) x^m = C^l(x) (1 - \alpha^m) x^m < 0$$

$$\Pi^h = -( S^* + C^l(x) ) x^h + S^* x^h = C^l(x) x^h = 0$$

$$\Pi^s = -( S^* + C^l(x) ) x^s + S^* x^s = C^l(x) x^s = 0$$

10. Long manipulation, detected, $\alpha^h = 1 + q^*/x^h$, $\alpha^x = \alpha^m = 1$

$$\Pi^m = -( S^* + C^l(x) ) x^m + S^* x^m = -C^l(x) x^m < 0$$

$$\Pi^h = -( S^* + C^l(x) ) x^h + S^* (x^h + q^*) + S^* (-q^*) = -C^l(x) x^h > 0$$

$$\Pi^s = -( S^* + C^l(x) ) x^s + S^* x^s = -C^l(x) x^s < 0$$

To illustrate the symmetry of the model, we present some results from the short manipulation:

11. Short manipulation, detected, $\alpha^h = 1$, $\alpha^x = 1 - q^*/x^x \ [x^h < 0, x^x > 0, x^m < 0]$

$$\Pi^m = -( S^* - \delta'(x) ) x^m + ( S^* - \delta'(x) ) \alpha^m x^m + S^* (1 - \alpha^m) x^m = \delta'(x) (1 - \alpha^m) x^m < 0$$

$$\Pi^h = -( S^* - \delta'(x) ) x^h + S^* (x^h + q^*) + S^* (-q^*) = -C^l(x) x^h > 0$$

$$\Pi^s = -( S^* - \delta'(x) ) x^s + S^* x^s = -C^l(x) x^s < 0$$
12. Short manipulation, undetected, $\alpha^{h} = \alpha^{s} = 1, \alpha^{m} < 1 - q^{*}/x^{m}$

\[
\Pi^{m} = -S^{*}x^{m} + (S^{*}-\delta'(x)) \alpha^{m}x^{m} + S^{*}(1-\alpha^{m})x^{m} = -\delta'(x)\alpha^{m}x^{m} > 0 \\
\Pi^{h} = -S^{*}x^{h} + (S^{*}-\delta'(x)) \alpha^{h}x^{h} = -\delta'(x)x^{h} > 0 \\
\Pi^{s} = -S^{*}x^{s} + (S^{*}-\delta'(x)) \alpha^{s}x^{s} = -\delta'(x)x^{s} < 0
\]
Vita


[Signature]

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