Chapter 2

B Meson Physics

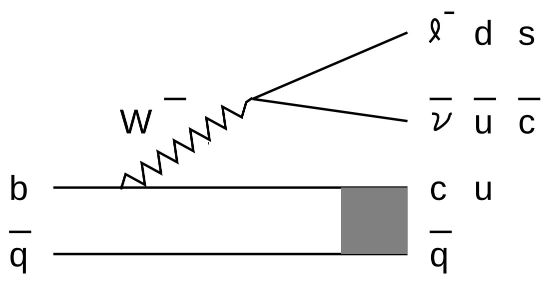
2.1 Introduction

The first impression of the B meson is that it has two distinct properties. (1) It has a longer lifetime compared with the other heavy flavor meson, D meson for example. (2) It has numerous amounts of decay channels with small branching fractions.

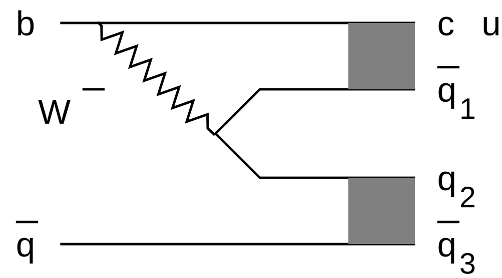
The long lifetime of the B meson is the result of the CKM matrix suppression. Because the t quark is much heavier than the b quark, b quark has to decay into a different generation, most likely to c quark. The corresponding V_{cb} value is a small number, about 0.04.

The heaviness of the b quark reserves a large amount of phase space for the b quark to decay. Unlike π, ρ, ω and strange flavored mesons which are saturated by a few decay modes, the B meson has enormous amounts of decay channels.

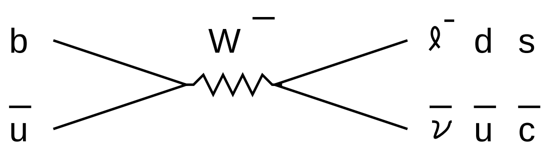
Figure 2.1 shows some Feynman Diagrams of B meson decays. We focus on the B semileptonic decay, which goes through the external spectator diagram where the virtual W boson materializes into a lepton and neutrino pair.



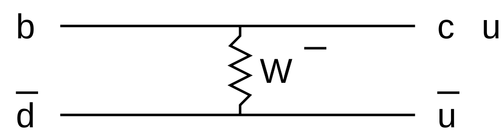
(a) Spectator



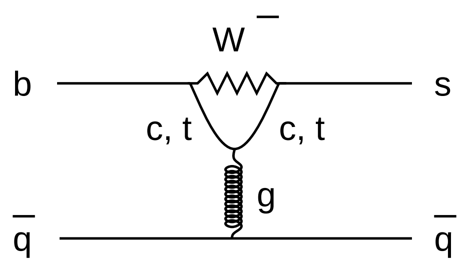
(b) Color Suppressed



(c) Annihilation



(d) Exchange



(e) Penguin

Figure 2.1: The Feynman Diagrams of B meson decay. Shown in the plot is the external spectator decay, internal spectator decay (color suppressed), annihilation of B meson, the exchange of W boson between the two B meson quarks and the penguin decay.

2.2 CKM Matrix and B Physics

In the Standard Model, B meson physics plays a key role to probe the CKM matrix, the matrix that couples the weak and the strong eigenstates. CKM matrix is also the source of CP violation in the Standard Model. As mentioned in chapter 1, the CKM matrix can be parameterized by the Wolfenstein parameterization [2]:

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

Among the four independent parameters, λ has been well measured from nuclear beta decay. The remaining A, ρ and η can be closely associated with B meson decay. There are four CKM elements that can be obtained from B physics measurement: V_{ub}, V_{cb}, V_{td} and V_{ts}. Knowing $V_{us} = \lambda = 0.22$ from the measurement of strange particle decay, we can obtain:

$$|V_{cb}/V_{us}^2| = A \quad and \quad |V_{ub}^*/V_{cd}V_{cb}| = \sqrt{\rho^2 + \eta^2} \tag{2.1}$$

The measurement of $|V_{cb}|$ essentially determines A. The constraint from $|V_{ub}|$ defines a circle in the (ρ, η) plane. $|V_{cb}|$ and $|V_{ub}|$ can be directly measured by the B meson semileptonic decay. $|V_{td}|$ and $|V_{ts}|$ can be indirectly inferred by the $B - \bar{B}$ mixing measurement.

Direct t quark decay measurement of CKM Matrix elements $|V_{tb}|, |V_{ts}|$ and $|V_{td}|$, is not yet feasible. For elements like $|V_{ud}|, |V_{cs}|, |V_{us}|, |V_{cd}|, |V_{cb}|$ and $|V_{ub}|$, we list their relative amplitudes and source of information in Table 2.1 [33].

2.3 Semileptonic B Decay

Semileptonic B decays proceeds when a b quark decays to a c or u quark and a virtual W^- boson, then this virtual W^- boson subsequently decays to a $(\ell^-, \bar{\nu}_\ell)$ pair. The accompanying

Table 2.1: Source and Relative Strength of Weak Transition

Relative amplitude	$\lvert V_{ij}\rvert$	Source of information	example
≈ 1	$\lvert V_{ud}\rvert$	Nuclear β decay	$n \to pe^-\bar{\nu}_e$
≈ 1	$\lvert V_{cs}\rvert$	Charmed particle decay	$D^0 \to K^- e^+ \nu_e$
≈ 0.22	$\lvert V_{us}\rvert$	Strange particle decay	$K \to \pi\ell\nu$
≈ 0.22	$\lvert V_{cd}\rvert$	neutrino production of charm	$\nu_\mu d \to \mu^- c$
≈ 0.04	$\lvert V_{cb}\rvert$	B decay to Charm	$B^0 \to D^{*+}\ell^-\bar{\nu}_\ell$
≈ 0.003	$\lvert V_{ub}\rvert$	Charmless B decay	$B \to \pi(\rho)\ell\bar{\nu}_\ell$

light u or d quark that is not involved in this process is known as the spectator quark. The Feynman Diagram of B semileptonic decay is shown in Figure 2.1.a.

The partial width for the weak decay of the free quark Q to fermions of non-zero masses was calculated by Corthes [22]. The calculation for $Q \to qW^*$, $W^* \to f_1\bar{f}_2$, where f_1, f_2 are fermions with $m_Q \ll m_W$, is

$$\Gamma(Q \to q f_1 \bar{f}_2) = \Gamma_0 N_c \mid V_{qQ} \mid^2 \mid V_{f_2 f_1} \mid^2 I(\frac{m_q}{m_Q}, \frac{m_{f_1}}{m_Q}, \frac{m_{f_2}}{m_Q}) \tag{2.2}$$

where,

$$\Gamma_0 = G_F^2 m_Q^5 / 192\pi^3 \tag{2.3}$$

$$\begin{aligned} I(x,y,z) &= 12\int_{(x+y)^2}^{(1-z)^2} \frac{ds}{s}(s - x^2 - y^2)(1 + z^2 - s) \\ &\times([s-(x-y)^2][s-(x+y)^2][(1+z)^2 - s][(1-z)^2 - s])^{1/2} \end{aligned}$$

and,

- N_c, 3 for hadronic decay, 1 for semileptonic decay.
- $V_{f_2 f_1}$ is the CKM element for $W \to \bar{f}_2 f_1$.

The function I has a familiar form as μ decay when only one final state particle has non-zero mass:

$$I(x,0,0) = I(0,x,0) = I(0,0,x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 lnx$$

The B semileptonic branching fraction is given by:

$$Br(B \to X\ell\bar{\nu}_\ell) = \frac{\Gamma(B \to X\ell\bar{\nu}_\ell)}{\Gamma(B \to All)}$$

where,

$$\begin{aligned}
\Gamma(b \to X\ell\bar{\nu}_\ell) &\approx \Gamma(b \to ce^-\bar{\nu}) = \Gamma(b \to c\mu^-\bar{\nu}) \\
\Gamma(B \to All) &\approx \Gamma(b \to ce^-\nu) + \Gamma(b \to c\mu^-\nu) + \Gamma(b \to c\tau^-\nu) + \\
&\quad \Gamma(b \to c\bar{u}d) + \Gamma(b \to c\bar{c}s) + \Gamma(b \to c\bar{u}s) + \\
&\quad \Gamma(b \to c\bar{c}d)
\end{aligned}$$

We have ignored the small contribution from $b \to u$, the rare decay and the penguin process in our B meson total decay width approximation. This will lead us to a crude estimation of the B semileptonic branching fraction as:

$$Br(B \to X\ell\bar{\nu}_\ell) \approx \frac{r_\ell}{r_\ell + r_\tau + 3\eta_{\bar{u}d}r_{\bar{u}d} + 3\eta_{\bar{c}s}r_{\bar{c}s} + 3\eta_{\bar{u}s}r_{\bar{u}s} + 3\eta_{\bar{c}d}r_{\bar{c}d}} \qquad (2.4)$$

where,

- η accounts for the QCD factor for each channel.
- r_ℓ,r_τ, $r_{\bar{u}d}$, $r_{\bar{c}s}$, $r_{\bar{u}s}$ and $r_{\bar{c}d}$ accounts for the available phase space.
- the number 3 accounts for the 3 colors of quark.

Ignoring the small contributions from $b \rightarrow c\bar{u}s$ and $b \rightarrow c\bar{c}d$ and the QCD corrections for each channel, we can calculate the branching fraction with the assigned values of the masses for leptons and quarks. Since the masses of lepton are well measured, and the masses of neutrino are very small, we have $r_\ell = 0.45$ and $r_\tau = 0.12$. The quark masses bear some uncertainties. We assign the values of our best knowledge, $m_u \approx m_d \approx 0$, $m_s = 0.5$ GeV/c^2 and $m_c = 1.4$ GeV/c^2 to obtain $r_{\bar{u}d} \approx r_\ell \approx 0.45$ and $r_{c\bar{s}} \approx r_\tau \approx 0.12$ [14]. We obtain $Br(\bar{B} \rightarrow X\ell\bar{\nu}_\ell)$:

$$Br(\bar{B} \rightarrow X\ell\bar{\nu}_\ell) = \frac{\Gamma_\ell}{\Gamma_B} = \frac{r_\ell}{2r_\ell + r_\tau + 3r_{\bar{u}d} + 3r_{c\bar{s}}} = 16.5\% \tag{2.5}$$

This simplified picture needs some corrections. We need to add the gluon radiation and gluon exchange process [15, 16] to the calculation, as illustrated in the Feynman Diagram in Figure 2.2. The new result is to enhance the hadronic width in spectator decay processes. Gluon exchange can modify the hadronic event exemplified as follows. From Equation 2.2, the decay width of $b \rightarrow c\bar{u}d$ can be written as:

$$\Gamma(b \rightarrow c\bar{u}d) = 3\Gamma_0 I(\frac{m_c}{m_b}, 0, 0)\eta J \tag{2.6}$$

where:

- $\eta = \frac{c_-^2 + 2c_+^2}{3}$ for hard gluon exchange
  - $c_\pm$ is the Wilson Coefficient. $c_+ = [\frac{\alpha_s(\mu)}{\alpha_s(M_W)}]^{-6/23}$. $c_- = [\frac{\alpha_s(\mu)}{\alpha_s(M_W)}]^{12/23}$.
- V_{ud} (from Equ 2.2) is approximated as 1.
- μ is the renormalization scale. This is a non-physical artifact of the calculation, which only includes the effects of virtual gluons in the momentum range from μ to M_W.
- J: for softer gluon momentum below the cutoff μ.

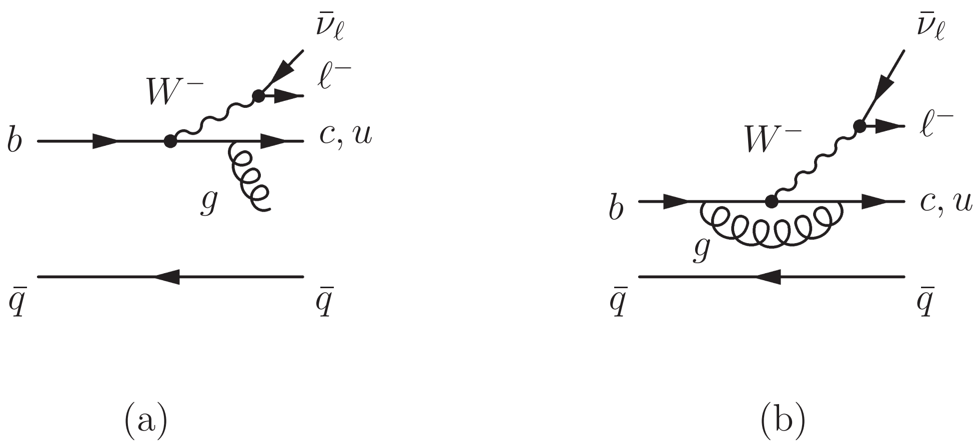


Figure 2.2: First order gluon correction in B semileptonic decay. (a) shows gluon radiation and (b) illustrates gluon exchange between initial and final quarks

- $\Gamma_0 = \frac{G_F^2 M_Q^5}{192\pi^3}$.

For μ taken to be m_b, we have the value $\eta \approx 1.$ and J $\approx$ 1.15. This results in the enhancement of the hadronic width by a factor of 1.27 in the calculation. If we calculate the decay width using $m_b = 4.6$ (5.0), $m_c = 1.2$ (1.7), $m_s = 0.15$ (0.3) and $m_u = m_d = 0$ (0.16), we have the following result:

$$B_{sl} = (12.2 \pm 0.45(scale) \pm 0.8(\alpha_s))\% \tag{2.7}$$

for light masses calculation and

$$B_{sl} = (14.4 \pm 0.45(scale) \pm 0.8(\alpha_s))\% \tag{2.8}$$

for heavy masses calculation. The error on scale comes from the uncertainty of μ when we varied μ between $1/2m_b$ and m_b.

One of the mysteries in B meson semileptonic decay is the large discrepancy between the model prediction and the experimentally measured value. The experimental value of the inclusive semileptonic branching fraction is well below 12.5%, the lower bound of the theoretical model prediction [57]. This discrepancy is about at least 4 σ. This needs a 10-11% reduction of the theoretical prediction in the semileptonic decay width to match with the experimental value, which requires 20% enhancement of the hadronic decay width to make it work.

Our lack of knowledge of the non-perturbative QCD process prohibits us from calculating hadronic currents in B meson semileptonic decay width. Various phenomenological models have emerged to simulate the dynamic system for calculation. These models can be classified into two categories: inclusive and exclusive models. The inclusive model works on the quark level. The exclusive model assumes B semileptonic decays are saturated by a few resonant final states. Both models provide lepton spectra consistent with measurement [46]. In our measurement, we don't intend to examine the validity of these models, but rather through the lepton spectra these models provided to obtain the B meson semileptonic branching fraction, the $\Upsilon(4S)$ $B^0 - \bar{B}^0$ mixing parameter, χ_d, and the lifetime ratio of charged to neutral B mesons, τ_+/τ_0.

2.3.1 Inclusive Model (ACCMM)

In the pioneering work by Altarelli, Cabibbo, Corbo Maiani and Martinelli (ACCMM) [17], the momentum of the spectator quark in B meson semileptonic decay is described by a Gaussian distribution $f(p)$:

$$f(p) = \frac{4}{\sqrt{\pi} p_f^3} exp(-p^2/p_f^2) \tag{2.9}$$

The mass of the b quark is assigned by an off-shell mass squared:

$$m_b^2 = m_B^2 + m_{sp}^2 - 2m_B\sqrt{p^2 + m_{sp}^2} \tag{2.10}$$

where: p_f is the Fermi momentum parameter determining the width of $f(p)$. m_B is the mass of the B meson and m_{sp} is the mass of spectator quark.

We then obtain the following semileptonic partial width:

$$\frac{d\Gamma_{sl}}{dx} = \frac{G_F^2 V_{cb}^2 m_b^5}{96\pi^3}\Phi(x,\epsilon)[1 - \frac{2\alpha_s}{3\pi}G(x,\epsilon)] \quad (2.11)$$

The free quark distribution term, $\Phi(x,\epsilon)$ [19], and the gluon radiation term, $G(x,\epsilon)$, can be described as:

$$\Phi(x,\epsilon) = \frac{x^2(1-\epsilon^2-x)^2}{(1-x)^3}[(1-x)(3-2x)+(3-x)\epsilon^2] \quad (2.12)$$

$$\begin{aligned} G(x,\epsilon) &\approx \frac{1}{x_M}2ln(x_M - x)[2x_M + (2-x_M)ln(1-x_M)] \\ x &\rightarrow x_M \end{aligned}$$

where $x = 2E_\ell/m_b$, $\epsilon = m_c/m_b$ and $x_M = 1-(m_c/m_b)^2$.

The ACCMM model prediction for the $b \rightarrow c\ell^-\bar{\nu}_\ell$ inclusive lepton spectrum has given a nice fit to the experimental measurement [46]. Its reliability for the $b \rightarrow u\ell^-\bar{\nu}_\ell$ lepton spectrum at endpoint region is controversial. For our measurement, since $b \rightarrow u\ell^-\bar{\nu}_\ell$ only makes a very small contribution, the uncertainty of $b \rightarrow u\ell^-\bar{\nu}_\ell$ is managed by estimating the systematic error.

2.3.2 Exclusive Model Prediction

General Form

The hadronic current can be constructed from the available four-vectors that are momentum and spin polarization vectors: $H^\mu = V^\mu - A^\mu$, where V^μ and A^μ correspond to vector and axial vector current, respectively. The hadronic current in B semileptonic decay can be categorized as the following:

- $P(Q\bar{q}) \rightarrow P^{'}(q^{'}\bar{q})\ell\nu$: pseudo-scalar to pseudo-scalar decay, such as $B^0 \rightarrow D^- \ell^+ \nu_\ell$.
- $P(Q\bar{q}) \rightarrow V(q^{'}\bar{q})\ell\nu$: pseudo-scalar to vector decay, like $\bar{B}^0 \rightarrow D^{*+}\ell^-\bar{\nu_\ell}$ for example.

$P \rightarrow P^{'}\ell\nu$**:**

Since the axial vector current disappears in this type of decay [24, 60]:

$$< P^{'}(p^{'})|V^\mu - A^\mu|P(p) >=< P^{'}(p^{'})|V^\mu|P(p) > \tag{2.13}$$

we can have:

$$< P^{'}(p^{'})|V^\mu|P(p) >= f_+(q^2)(p+p^{'})^\mu + f_-(q^2)(p-p^{'})^\mu \tag{2.14}$$

If we only refer to leptons as e or μ in B meson semileptonic decay, then in the limit of $m_\ell \rightarrow 0$, terms proportional to $(p-p^{'})$ are negligible because $(p-p^{'})^\mu L_\mu = 0$ at small lepton mass. Thus for $\ell = e$ or μ, the hadronic current can be written in terms of a single form factor $f_+(q^2)$:

$$< P^{'}(p^{'})|V^\mu|P(p) >= f_+(q^2)(p+p^{'})^\mu \tag{2.15}$$

$P \rightarrow V\ell\nu$**:**

For $P(Q\bar{q}) \rightarrow V(q^{'}\bar{q})\ell\nu$ type of decay, the hadronic current is written as:

$$
\begin{aligned}
< V^{'}(p^{'},\varepsilon)|V^{\mu}-A^{\mu}|P(p)> &= \frac{2i\epsilon^{\mu\nu\alpha\beta}}{M+m_V}\varepsilon^{*}_{\nu}p^{'}_{\alpha}p_{\beta}V(q^2) \\
&- (M+m_V)\varepsilon^{*\mu}A_1(q^2) \\
&+ \frac{\varepsilon^{*}q}{M+m_V}(p+p^{'})^{\mu}A_2(q^2) \\
&+ 2m_V\frac{\varepsilon^{*}q}{q^2}q^{\mu}A_3(q^2) \\
&- 2m_V\frac{\varepsilon^{*}q}{q^2}q^{\mu}A_0(q^2)
\end{aligned}
$$

where,

- $V^{\mu}=\bar{q}^{'}\gamma^{\mu}Q$.
- $A^{\mu}=\bar{q}^{'}\gamma^{\mu}\gamma_5 Q$.
- $A_3(q^2)=\frac{M+m_V}{2m_V}A_1(q^2)-\frac{M-m_V}{2m_V}A_2(q^2)$.
- $A_0(0)=A_3(0)$.

If we only refer e and μ as the lepton in B meson semileptonic decay, we can obtain the following simplified form:

$$
\begin{aligned}
< V^{'}(p^{'},\varepsilon)|V^{\mu}-A^{\mu}|P(p)> &= \frac{2i\epsilon^{\mu\nu\alpha\beta}}{M+m_V}\varepsilon^{*}_{\nu}p^{'}_{\alpha}p_{\beta}V(q^2) \\
&- (M+m_V)\varepsilon^{*\mu}A_1(q^2) \\
&+ \frac{\varepsilon^{*}q}{M+m_V}(p+p^{'})^{\mu}A_2(q^2)
\end{aligned}
$$

There are several models devised to predict the exclusive decay. We discuss the one used in this analysis, the ISGW Model.

ISGW Model

The model developed by Isgur, Scora, Grinstein and Wise [18], called the ISGW Model, assumes the B semileptonic decays are saturated by a few resonant final states: $D(1^1S_0)$, $D^*(1^3S_1)$, and the higher resonance states called D^{**}. The hadronic currents of those resonant final states are expressed by form factors. The ISGW model input of the hadronic current of decay mode of $B \to D\ell\nu$ and $B \to D^*\ell\nu$ is expressed below:

$$
\begin{aligned}
< D|A_\mu|\bar{B} > &= 0 \\
< D|V_\mu|\bar{B} > &= f_+(p_B + p_D)_\mu + f_-(p_B - p_D)_\mu \\
< D^*|A_\mu|\bar{B} > &= f\epsilon^*_\mu + a_+(\epsilon^* * p_B)(p_B + p_{D^*})_\mu + a_-(\epsilon^* * p_B)(p_B - p_{D^*})_\mu \\
< D^*|V_\mu|\bar{B} > &= ig\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}(p_B + p_{D^*})^\rho(p_B + p_{D^*})^\sigma
\end{aligned}
$$

The f, a and g are the form factors. These form factors are functions of q^2, the mass of the *virtual* W boson.

ISGW [18] argues the heaviness of the b quark makes it possible to use non-relativistic approximations to model the B meson decay. The form factors are obtained by solving the Schrödinger equation with a coulomb plus linear potential for the bound state of the B meson:

$$V(r) = -\frac{4\alpha_s}{3r} + c + br \tag{2.16}$$

where $\alpha_s = 0.5$, c = -0.84 GeV and b = 0.18 GeV/c^2. We can have the q^2 dependence on the form factor $F(q^2)$ as:

$$F(q^2) \propto F(q^2_{max})exp(\frac{q^2 - q^2_{max}}{\kappa Q^2}) \tag{2.17}$$

where κ is a parameter introduced to account for the relativistic effect.

The ISGW Model calculates the form factor value of the minimum recoil of the final state meson, where the q^2 is maximum, then extrapolate to other q^2 regions to obtain the lepton momentum spectrum distribution.

2.3.3 The Lepton Spectrum of B Semileptonic Decay

In our analysis, the lepton spectrum of B semileptonic decay plays a key role to access the measurements of the semileptonic branching fraction of neutral B, the $B^0 - \bar{B}^0$ mixing parameter χ_d and the lifetime ratio of B^+ vs. B^0. Here we draw a more qualitative picture of inclusive lepton spectrum of B semileptonic decay.

The lepton spectrum is strongly affected by the following three factors: (1) the V-A coupling, (2) the quantum number of the daughter hadron from B semileptonic decay and (3) the distribution of q^2, which is the square of the four momentum of the W boson.

V-A coupling

If we first neglect the quantum number of the daughter meson and focus on the quark level, then the $b \to c\ell^-\bar{\nu}$ and $b \to u\ell^-\bar{\nu}$ produces the c- and u- quarks that are predominately helicity $\lambda = -1/2$ in association with a charged lepton that is almost purely helicity $\lambda = -1/2$. This allows the collinear configuration where the charged lepton recoils against the neutrino and the daughter quark, leading to a higher possible lepton energy.

For semileptonic charm decay, $c \to s\ell^+\nu$ produces a helicity $\lambda = -1/2$ s-quark and a helicity $\lambda = +1/2$ lepton in the decay. The collinear configuration of a lepton recoiling against the daughter quark and the neutrino is forbidden, thus leading to a softer lepton spectrum.

Angular Momentum of the Daughter Meson

The above discussion ignores the daughter meson's quantum number. The quantum number of the daughter meson affects the lepton spectrum through the cosine angle distribution, $cos\theta_\ell$, where the θ_ℓ is the angle between the lepton and the daughter meson in the rest frame of the B meson. If a spin-0 daughter meson (like $\bar{B} \rightarrow D\ell^-\bar{\nu_\ell}$ or $\bar{B} \rightarrow \pi\ell^-\bar{\nu_\ell}$) is produced from a B semileptonic decay, then the virtual W^- must have helicity 0 in order to conserve both momentum and spin in the B meson rest frame. The $cos\theta_\ell$ distribution of the lepton will be proportional to $sin^2\theta_\ell$, a situation not really favoring higher lepton momentum.

If this B decays semileptonically to a spin-1 daughter meson, like $\bar{B} \rightarrow D^*\ell^-\bar{\nu_\ell}$ or $\bar{B} \rightarrow \rho\ell^-\bar{\nu_\ell}$, then the helicity information of daughter c- or u-quarks is preserved since this quark needs to combine with the spectator quark to form a spin-1 meson, which tends to have helicity $\lambda = -1$ rather than $\lambda = +1$. This forces the predominance of $\lambda_{W^-} = -1$ over $\lambda_{W^-} = +1$, and consequently, the $cos\theta_\ell$ distribution of the lepton will be dominated by $(1 + cos\theta_\ell)^2$.

This quantum number effect gives us the softer lepton spectrum for pseudoscalar to pseudoscalar decay, $P \rightarrow P^{'}\ell^-\bar{\nu_\ell}$, than the pseudoscalar to vector decay, $P \rightarrow V\ell^-\bar{\nu_\ell}$, in the B meson sector.

For charm meson decay, because of the predominance of helicity $\lambda = +1/2$ in charm semileptonic decays, it is just the reverse: the $P \rightarrow V\ell^+\nu_\ell$ has a harder lepton spectrum than the $P \rightarrow P^{'}\ell^+\nu_\ell$.

q^2 distribution

Finally, if we are able to have higher q^2 for B semileptonic decay, then the phase space tends to allow higher lepton energy. However, the q^2 distribution in B decay also depends on

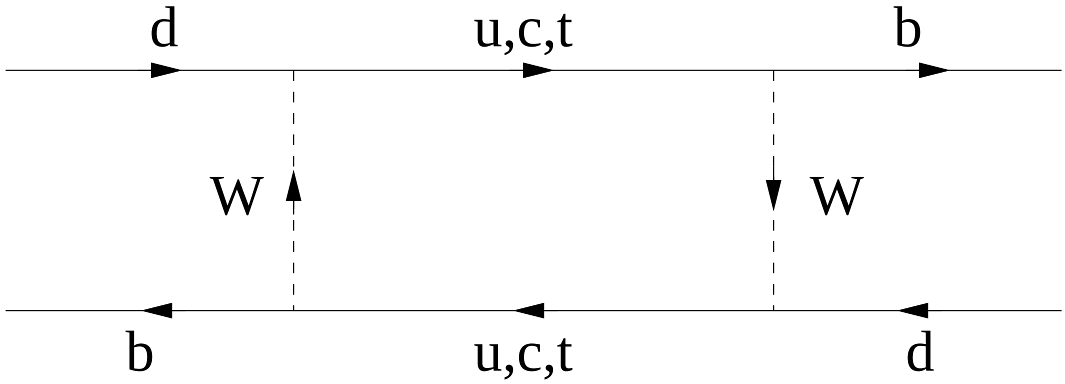


Figure 2.3: Feynman "Box" diagram for $B^0 - \bar{B}^0$ mixing

the quantum number of the daughter meson. We can take a look at the $P \rightarrow P' \ell^- \bar{\nu}_\ell$ decay rate:

$$\frac{d\Gamma}{dq^2} = \frac{G^2}{24\pi^3} |V_{pp'}| P_{p'}^3 f_+^2(q^2) \tag{2.18}$$

We immediately see that the high q^2 is suppressed by the low momentum value of $P_{p'}^3$ for that q^2.

For $P \rightarrow V \ell^- \bar{\nu}_\ell$, the maximum q^2 corresponds to the total overlap of the initial and final meson wave function, the form factor will have biggest value there.

2.4 $B - \bar{B}$ mixing

Oscillation between particle and anti-particle was predicted in 1955 [26], and was observed in 1956 in the $K^0 - \bar{K}^0$ [27], and again in 1987 in the $B^0 - \bar{B}^0$ system [28]. These oscillations are described in the Standard Model by second order weak transitions, as demonstrated in the known *Feynman Box Diagram* in Fig 2.3

An arbitrary neutral B meson state $\psi(t) = a(t)|B^0> + b(t)|\bar{B}^0>$, is governed by the time dependent Schröndinger equation:

$$H\begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix} = \begin{pmatrix} M - i\frac{1}{2}\Gamma & M_{12} - \frac{1}{2}i\Gamma_{12} \\ M_{12}^* - \frac{1}{2}i\Gamma_{12}^* & M - i\frac{1}{2}\Gamma \end{pmatrix}\begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix}$$

Diagonalization of the hamiltonian matrix leads to CP eigenstates B_L and B_H which are linear combination of flavor eigenstates. we obtain:

$$|B_L >= p|B^0 > +q|\bar{B}^0 >) \tag{2.19}$$

$$|B_H >= p|B^0 > -q|\bar{B}^0 >) \tag{2.20}$$

The eigenvalues of the two new eigenstates B_H, B_L are:

$$m_{H,L} = M \pm Re[(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)] \tag{2.21}$$

$$\Gamma_{H,L} = \Gamma \mp 2Im[(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)] \tag{2.22}$$

The proper time evolution of an initially ($t = 0$) pure B^0 or $\bar{B^0}$ is given by:

$$\begin{aligned} |B^0_{phys}(t) > &= e^{(-\Gamma t/2)}e^{(-iMt)} \\ &\times (cos(\Delta Mt/2)|B^0 > +i(q/p)sin(\Delta Mt/2)|\bar{B^0} >) \end{aligned}$$

$$\begin{aligned} |\bar{B^0}_{phys}(t) > &= e^{(-\Gamma t/2})e^{(-iMt)} \\ &\times (i(p/q)sin(\Delta Mt/2)|B^0 > +cos(\Delta Mt/2)|\bar{B^0} >) \end{aligned}$$

where $\Delta M = M_H - M_L$ and $\Gamma \approx \Gamma_L = \Gamma_H$. The quantities of M_H, M_L and $\Gamma_{L,H}$ are the masses and decay widths of the two heavy and light B CP eigenstates. p, q are related to the CP violation parameter ϵ, as we saw in theoretical calculation [29]:

$$|\frac{q}{p}| = \frac{1-\epsilon}{1+\epsilon} = [\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}]^{1/2} \tag{2.23}$$

The decay width and mass difference are:

$$\Delta\Gamma = \Gamma_H - \Gamma_L = -4Im[(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)]^{1/2} \tag{2.24}$$

$$\Delta M = M_H - M_L = 2Re[(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)]^{1/2} \tag{2.25}$$

For a B^0 created at time $t = 0$, the probability to find a $B^0(\bar{B}^0)$ at a time t is given by $P_{B^0}(t)(P_{\bar{B}^0}(t))$ as:

$$P_{B^0}(t) = \frac{1}{2}e^{-\Gamma t}(1 + cos(\Delta M t)) \tag{2.26}$$

$$P_{\bar{B}^0}(t) = \frac{1}{2}e^{-\Gamma t}|\frac{q}{p}|^2(1 - cos(\Delta M t)) \tag{2.27}$$

In the B meson system, $|\Delta\Gamma| \ll \Delta M$, we have $\epsilon \ll 1$, then $q \approx p$ [30], we have the following:

$$\Delta\Gamma \approx 2Re\Gamma_{12}, \quad \Delta M \approx 2ReM_{12} \tag{2.28}$$

The frequency of $B^0 - \bar{B}^0$ oscillations is driven by the mass difference between the two CP eigenstates of neutral B meson, ΔM:

$$\Delta M = 2Re\sqrt{(M_{12} - \frac{i\Gamma_{12}}{2})(M_{12}^* - \frac{i\Gamma_{12}^*}{2})} \tag{2.29}$$

This ΔM is evaluated from box diagram calculations. With the above approximation, we can calculate ΔM through the matrix element, M_{12} [30], as:

$$\Delta M \approx 2ReM_{12} = \frac{G_F^2 M_W^2}{8\pi^2} < \bar{B}^0|j_\mu^{V-A} j^{V-A,\mu}|B^0 > \sum_{u,c,t} \lambda_i \lambda_j A_{ij} \eta_{QCD} \tag{2.30}$$

where λ_i is the CKM matrix element, $V_{ib}^* V_{id}$, and η_{QCD} is the QCD correction factor, which is calculated to be in the range 0.78-0.85 [31].

The unitarity of the CKM matrix requires:

$$\sum_{u,c,t} \lambda_i = 0 \tag{2.31}$$

Thus we can remove the u quark dependency:

$$\sum_{u,c,t} \lambda_i \lambda_j A_{ij} = (\lambda_c^2 U_{cc} + \lambda_t^2 U_{tt} + 2\lambda_c \lambda_t U_{ct}) \tag{2.32}$$

where

$$U_{ij} = A_{uu} - A_{ij} - A_{ui} - A_{uj} \tag{2.33}$$

A_{ij} is evaluated by loop integral and it is found that $U_{cc}, U_{ct} \ll U_{tt}$. On the other hand, $|\lambda_c| \approx |\lambda_t|$, so the contribution from the t quark dominates $\bar{b}d - b\bar{d}$ processes.

$$U_{tt} \approx \frac{m_t^2}{M_W^2} F(\frac{m_t^2}{M_W^2}) \tag{2.34}$$

with:

$$F(z) = \frac{1}{4} + \frac{9}{4(1-z)} - \frac{3}{2(1-z)^2} - \frac{3z^2 lnz}{2(1-z)^3} \tag{2.35}$$

The matrix element of $< \bar{B}^0 | j_\mu^{V-A} j^{V-A,\mu} | B^0 >$ can be written as:

$$\begin{aligned} < \bar{B}^0 | j_\mu^{V-A} j^{V-A,\mu} | B^0 > &= < \bar{B}^0 | [\bar{b}\gamma_\mu (1-\gamma_5) d][\bar{b}\gamma_\mu (1-\gamma_5) d | B^0 > \\ &= \boxed{B_B \tfrac{4}{3} f_B^2 M_B} \end{aligned}$$

where,

- B_B is the bag parameter.
- f_B is the decay constant.

- M_B is the mass of the B meson.

Putting all things together, we have the expression for ΔM:

$$\Delta M = \frac{G_F^2}{6\pi^2} B_B f_B^2 M_b |V_{tb}^* V_{td}|^2 m_t^2 F(\frac{m_t^2}{M_W^2}) \eta_{QCD} \tag{2.36}$$

From time integrated rates $N(B^0)$ and $N(\bar{B}^0)$, with the approximation $|q| \approx |p|$:

$$N(B^0) = \int_0^\infty P_{B^0}(t)dt = \frac{1}{2}[\frac{1}{\Gamma} + \frac{\Gamma}{\Gamma^2 + (\Delta M)^2}] \tag{2.37}$$

$$N(\bar{B}^0) = \int_0^\infty P_{\bar{B}^0}(t)dt = \frac{1}{2}[\frac{1}{\Gamma} - \frac{\Gamma}{\Gamma^2 + (\Delta M)^2}] \tag{2.38}$$

one can determine the parameter $r \equiv \frac{N(\bar{B}^0)}{N(B^0)}$:

$$r = \frac{(\Delta M/\Gamma)^2}{2 + (\Delta M/\Gamma)^2} \tag{2.39}$$

The value x_d is defined as:

$$x_d \equiv \frac{\Delta M}{\Gamma} \tag{2.40}$$

Through this expression and the evaluation of the loop diagram mentioned earlier, we can relate x_d through our information of ΔM:

$$x_d = \tau_{B^0} \frac{G_F^2}{6\pi^2} B_B f_B^2 m_b |V_{tb}^* V_{td}|^2 m_t^2 F(\frac{m_t^2}{M_W^2}) \eta_{QCD} \tag{2.41}$$

$F(\frac{m_t^2}{M_W^2})$ is a function that decreases slowly with the t-quark mass. The dominant uncertainty is the decay constant f_B, which has not been measured with statistical significance through B^- leptonic decay.

The mixing parameter χ_d is the probability that a $B^0/(\bar{B}^0)$ decays into its anti-particle $\bar{B}^0/(B^0)$. It is equal to:

$$
\begin{aligned}
\chi_d &= \frac{N(\bar{B}^0)}{N(\bar{B}^0) + N(B^0)} \\
&= \frac{r}{1+r} \\
&= \frac{x_d^2}{2(1+x_d^2)}
\end{aligned}
$$

By looking at the formula for x_d in equation 2.41, we immediately expect a bigger $B-\bar{B}$ mixing rate than $D-\bar{D}$ mixing. The $D-\bar{D}$ mixing diagram is similar to B-mixing by replacing t and b quark with b and c quark and substituting the CKM element $|V_{tb}^* V_{td}|^2$ by $|V_{cb}^* V_{ub}|^2$. The discovery of the t quark by Fermilab has the t quark mass 38 times heavier than b-quark mass. The CKM element in $D-\bar{D}$ mixing, compared with the rate of $B-\bar{B}$ mixing, is also suppressed by the factor of $(|V_{cb}|/|V_{tb}|)^2$. B^0 also has a longer lifetime than D^0: $\tau(B^0) = 1.56$ ps and $\tau(D^0) = 0.42$ ps. All of these factors collectively assert that $D-\bar{D}$ mixing has a much smaller oscillation rate than $B-\bar{B}$ mixing.

The $\Upsilon(4S)$ resonance also provides a special environment for $B-\bar{B}$ mixing measurement. The B meson pair produced at the $\Upsilon(4S)$ is in a coherent state $J^P = 1^-$, which forbids the co-existence of $(B-B)$ or $(\bar{B}-\bar{B})$ meson pair at the same time. For B to decay into $\bar{B}$ or vice versa, it can happen only when the other $\bar{B}$ (or B) has decayed away. If we are able to tag the flavor of one b quark, and found out the other b quark carries the same flavor as the tagged b quark, then we know this is a mixed event.

The $B-\bar{B}$ mixing measurement in the 80's has helped to set the limit of the t quark mass before its discovery in 1994. The big mixing rate measured at that time surprised a lot of people, and the indication of a large t quark mass was recognized.

$B-\bar{B}$ mixing also provides another channel to measure the CP violation in B meson decay. *Indirect CP violation* is caused by the interference between B and $\bar{B}$ to a CP eigenstate to which both states can decay. Since this analysis does not focus on the discovery of CP violation in B meson decay, it will not be elaborated here.

2.5 Lifetime Ratio of B^+/B^0

The measurement of the B meson semileptonic branching fraction combined with the lifetime measurement of the B meson can lead us to the information of the B meson total decay width because:

$$B_{sl} \equiv \frac{\Gamma(B \to X\ell\nu)}{\Gamma(B \to all)} = \Gamma(B \to X\ell\nu)\tau_B \tag{2.42}$$

In the $\Upsilon(4S)$ experiment, we can not directly measure the lifetime of the B meson because B meson pair is produced nearly at rest and will decay immediately without traveling further before CLEO detector can catch its trace. What we can do in $\Upsilon(4S)$ experiment is to measure the ratio of the semileptonic branching fraction of charged and neutral B meson, then through isospin symmetry of B^- and $\bar{B}^0$ in the B semileptonic decay, we can obtain the lifetime ratio of charged vs. neutral B mesons, $\tau_{B^-}/\tau_{\bar{B}^0}$, as follows:

$$\begin{aligned} \frac{Br(B^+ \to X\ell\bar{\nu}_\ell)}{Br(B^0 \to X\ell\bar{\nu}_\ell)} &= (\frac{\Gamma(B^+ \to X\ell\bar{\nu}_\ell)}{\Gamma(B^+)})/(\frac{\Gamma(B^0 \to X\ell\bar{\nu}_\ell)}{\Gamma(B^0)}) \\ &= \frac{\tau_{B^-}}{\tau_{\bar{B}^0}} \end{aligned}$$

Since the B meson lifetime is inversely proportional to the B decay width and we know the heaviness of b-quark makes the spectator diagram the pre-dominant mechanism of B meson decay, we can argue that $\bar{B}^0$ and B^- have similar lifetime via $\bar{B}^0, B^-$ isospin symmetry.

The theoretical predictions of the B meson lifetime ratio vary, but all of them are not too far from unity. For example, Neubert [70] gives the estimate of lifetime ratio:

$$\frac{\tau_{B^-}}{\tau_{\bar{B^0}}} = 1 + O(1/m_b^3) \tag{2.43}$$

where O is the $1/m_b^3$ correction. In another literature, Bigi concludes [34]:

$$\frac{\tau_{B^-}}{\tau_{B^0}} = 1 + 0.05[\frac{f_B^2}{(200MeV)^2}] \tag{2.44}$$