Chapter 5

Measurement of $B^0 - \bar{B}^0$ Mixing

5.1 Introduction

In this chapter, we measure the $B - \bar{B}$ mixing parameter χ_d, the probability that a B^0 meson decays as its anti-particle, $\bar{B}^0$, or vice versa:

$$\chi_d = \frac{N(B^0 \to \bar{B}^0)}{N(B^0 \to B^0) + N(B^0 \to \bar{B}^0)} \tag{5.1}$$

The measurement of the mixing parameter χ_d is an important input to calculate the mass difference between the two CP eigenstates of the neutral B meson. The relation of χ_d to the mass difference between the two neutral B mesons, ΔM, and their averaged lifetime, Γ, is shown as follows:

$$\chi_d = \frac{(\Delta M/\Gamma)^2}{2((\Delta M/\Gamma)^2 + 1)} \tag{5.2}$$

ARGUS [55] and CLEO-II [49] have independently measured the mixing parameter χ_d by using di-lepton events. One hard lepton with momentum greater than 1.4 GeV/c is used to tag the flavor of a B meson. The charge of the second lepton, if it is a primary lepton from the other B meson, should carry the opposite sign with respect to the tag lepton, or else carry the same sign if the other B mixes. Lepton pairs of the same sign divided by all lepton pairs gives χ_d, as follows:

$$\chi_d = \frac{1}{(1-\Lambda)} \frac{N(\ell^\pm \ell^\pm)}{N(\ell^\pm \ell^\pm) + N(\ell^\pm \ell^\mp)} \tag{5.3}$$

The Λ accounts for the component of charged B in the data:

$$\Lambda = \frac{f_{+-} b_+^2}{f_{+-} b_+^2 + f_{00} b_0^2} \tag{5.4}$$

where f_{+-} and f_{00} refer to the fractions of charged and neutral B mesons in $\Upsilon(4S)$ events, and the b_+ and b_0 correspond to the semileptonic branching fractions of B^- and $\bar{B}^0$, respectively.

The method does not distinguish B^- from $\bar{B}^0$, making the uncertainty of the composition of B^- the biggest systematic error. The systematic error on Λ dominates the measurement.

Later CLEO II [48] published another measurement based on a method similar to our analysis. A hard lepton and a soft pion were used to obtain the *partial reconstruction* of $\bar{B}^0 \to D^{*+} \ell^- \nu$, $D^{*+} \to D^0 \pi^+$ to identify B^0 meson. Then a search is made for additional leptons in the tagged event. With the charge correlation of this additional lepton and the tag lepton, they are able to obtain the mixed signal when the additional lepton has the same charge as the tag, or an unmixed event when the additional lepton and tag lepton have opposite charges.

5.2 Motivation

The previous CLEO measurement of χ_d using the partial reconstruction method relies on a very complicated method to extract the mixing parameter. With the amount of B meson events we now have, we can implement this measurement in a way that is less complicated and still reduce the systematic error. Since it is already five years after the most recent measurement of χ_d has been reported, and much progress in B physics has been made since then, we felt that revisiting this measurement was worth while.

5.3 Analysis Overview

This analysis is the extension of the B^0 semileptonic branching fraction measurement in the previous chapter. We use **partial reconstruction** to tag $\bar{B}^0$ and search for any additional leptons in the same event, then construct the additional lepton momentum spectrum accordingly. The method of *partial reconstruction* and additional lepton momentum spectrum reconstruction have been described in detail in chapter 4.

One major modification needs to be made here. To measure χ_d, we need to separate the additional lepton according to the sign of its charge with respect to the charge of the tag lepton. We construct four additional lepton momentum spectra in our χ_d measurement: for each species of additional lepton (electron or muon), we also separate the additional leptons which carry the same charge as the tagged lepton, *like sign lepton spectra*, from the additional leptons carrying opposite charges, *unlike sign lepton spectra*. After reconstructing the four additional lepton spectra, a theoretical function was used to estimate the primary portions to obtain the *like sign pair* entries $N(\ell^{\pm}_{add}, \ell^{\pm}_{tag})$, and the *unlike sign pair* entries $N(\ell^{\pm}_{add}, \ell^{\mp}_{tag})$. After the necessary corrections for $N(\ell^{\pm}_{add}, \ell^{\pm}_{tag})$ and $N(\ell^{\pm}_{add}, \ell^{\mp}_{tag})$, we can obtain the mixing parameter χ_d.

5.4 Data and Event Cuts

We use $3.1 fb^{-1}$ $\Upsilon(4S)$ resonance and $1.6 fb^{-1}$ off-resonance data, as well as 19 million generic $B\bar{B}$ Monte Carlo events.

The event cuts are the same as for the semileptonic analysis except that we adopt an improved ghost track killer TRKMNG [35], "TRKMAN New Generation", in place of the older ghost track eliminator, TRKMAN.

5.5 Second Lepton Spectra Recreation

The procedure to construct the second lepton spectrum is very similar to that employed in the B^0 semileptonic branching fraction analysis. In addition to separating the additional leptons according to the charge relative to the tag leptons, we also subtract the leptons from π^0 Dalitz decay and photon conversion before the correction of lepton detection efficiency was made on the raw additional lepton spectra. Unlike in our semileptonic branching fraction analysis, we do not subtract the $B \to J/\Psi \to \ell^+\ell^-$ in our spectrum, but rather use its lepton shape from generic $B\bar{B}$ Monte Carlo along with lepton shapes from primary and secondary decays and other sources to fit the spectra.

The collections of $\tilde{M}_\nu^2$ distribution for like sign lepton pairs can be found in Figure 5.1, Figure 5.2 and Figure 5.3. The unlike sign lepton pairs can be found in Figure 5.4, Figure 5.5 and Figure 5.6. The table of like sign and unlike sign lepton spectra before we subtract the π^0 Dalitz decay and photon conversion leptons are tabulated in Table 5.1 for like sign pairs, and in Table 5.2 for unlike sign pairs.

After we reconstruct our additional lepton spectra, we fit the spectra with lepton shapes from primary, secondary, J/ψ, D_s, Λ_c, τ and $b \to \bar{c} \to \ell^-$ all together to estimate the primary

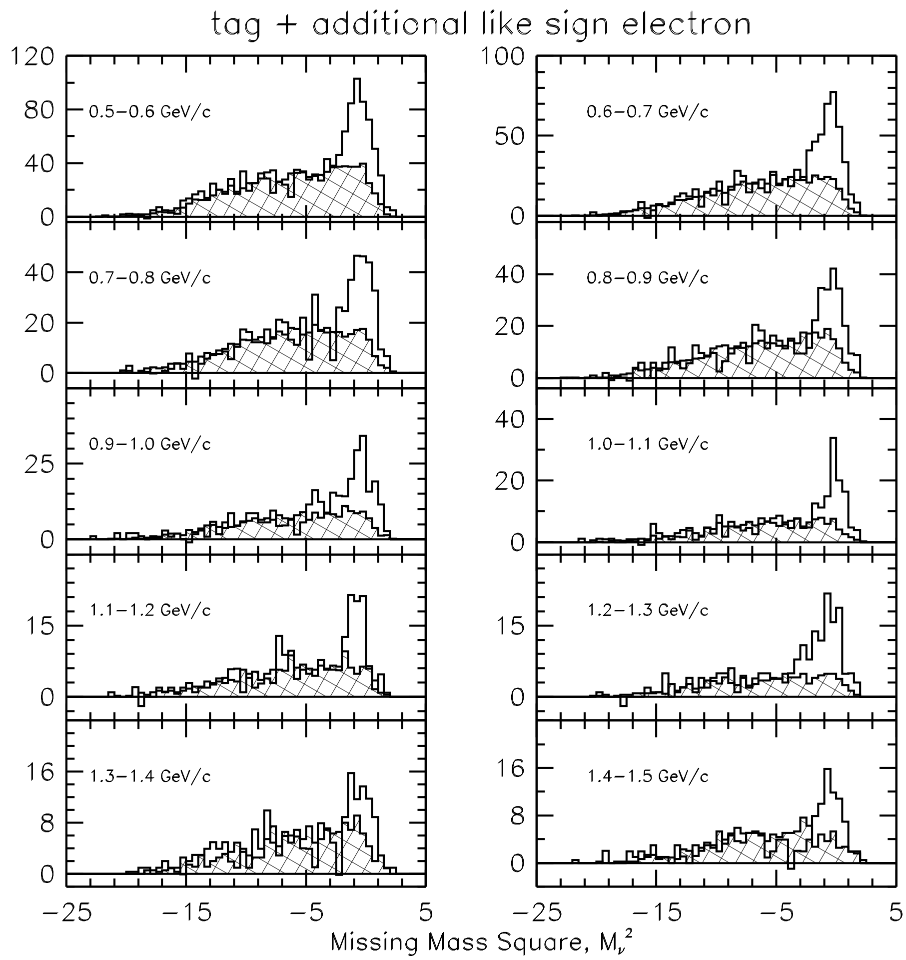

Figure 5.1: The bin-by-bin $\tilde{M}^2$ distribution of additional "like sign" electrons from 0.5-1.5 GeV/c. Continuum and fake candidates have been subtracted and the dotted line is the combinatoric background simulated by Monte Carlo.

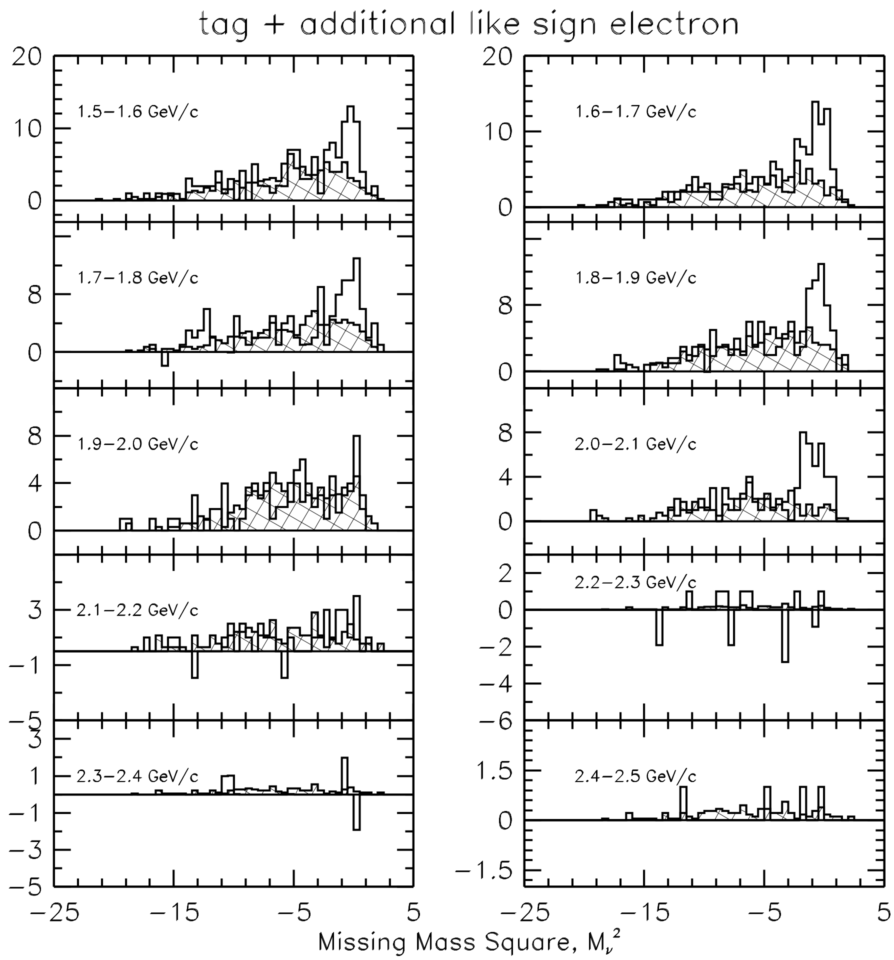

Figure 5.2: The bin-by-bin $\tilde{M}^2$ distribution of second "like sign" electrons from 1.5-2.5 GeV/c. Continuum and fake candidates have been subtracted and the dotted line is the combinatoric background simulated by Monte Carlo.

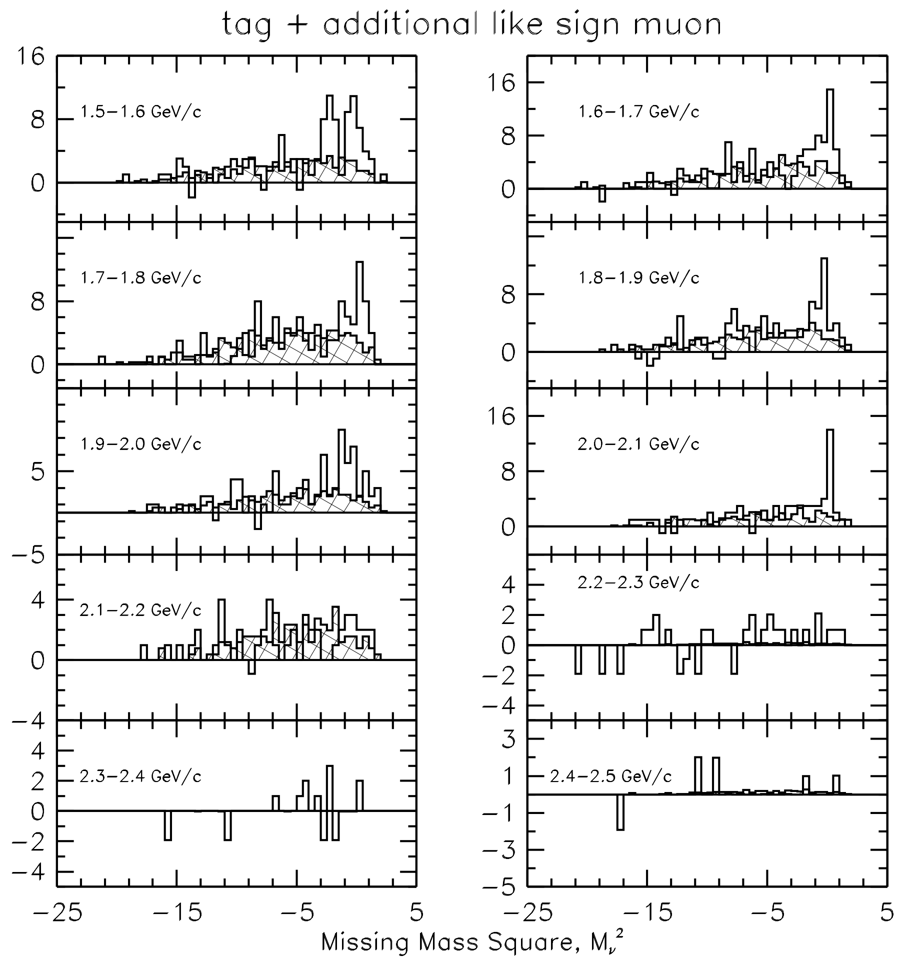

Figure 5.3: The bin-by-bin $\tilde{M}^2$ of second "like sign" muons from 1.5-2.5 GeV/c. Continuum and fake candidates have been subtracted and the dotted line is the combinatoric background simulated by Monte Carlo.

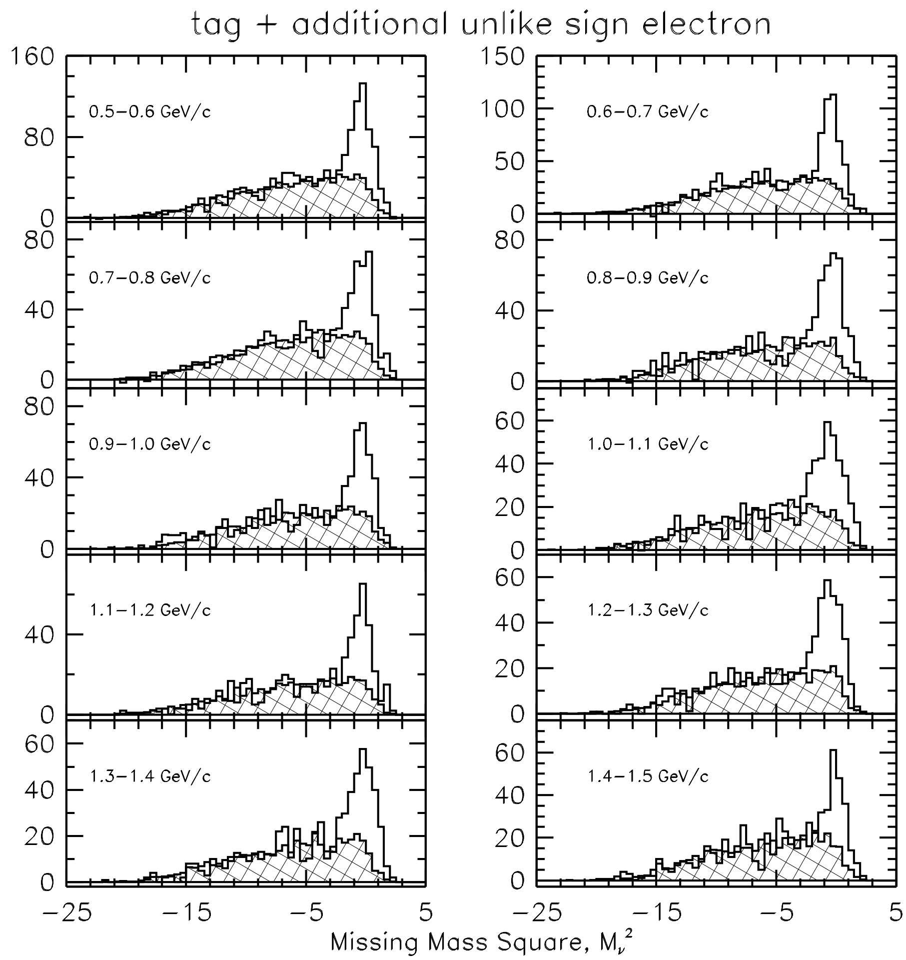

Figure 5.4: The bin-by-bin $\tilde{M}^2$ distribution of second "unlike sign" electrons from 0.5-1.5 GeV/c. Continuum and fake candidates have been subtracted and the dotted line is the combinatoric background simulated by Monte Carlo.

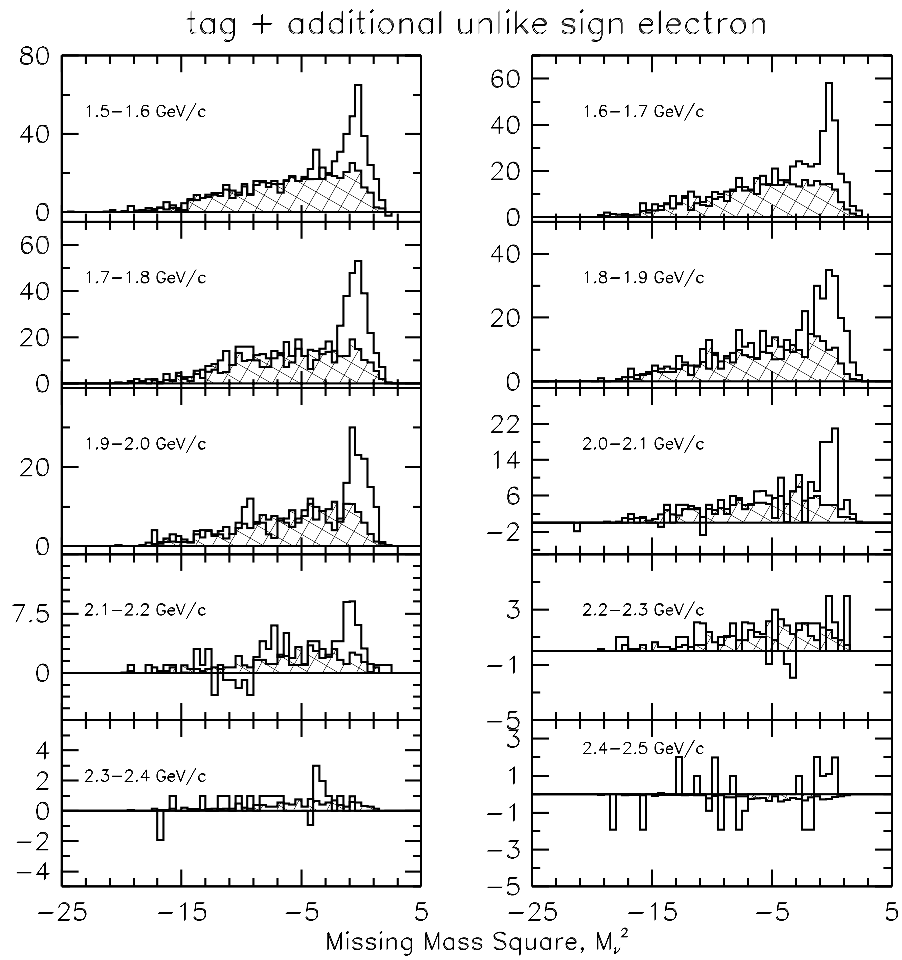

Figure 5.5: The bin-by-bin $\tilde{M}^2$ distribution of second "unlike sign" electrons from 1.5-2.5 GeV/c. Continuum and fake candidates have been subtracted, dotted line is the combinatoric background simulated by Monte Carlo.

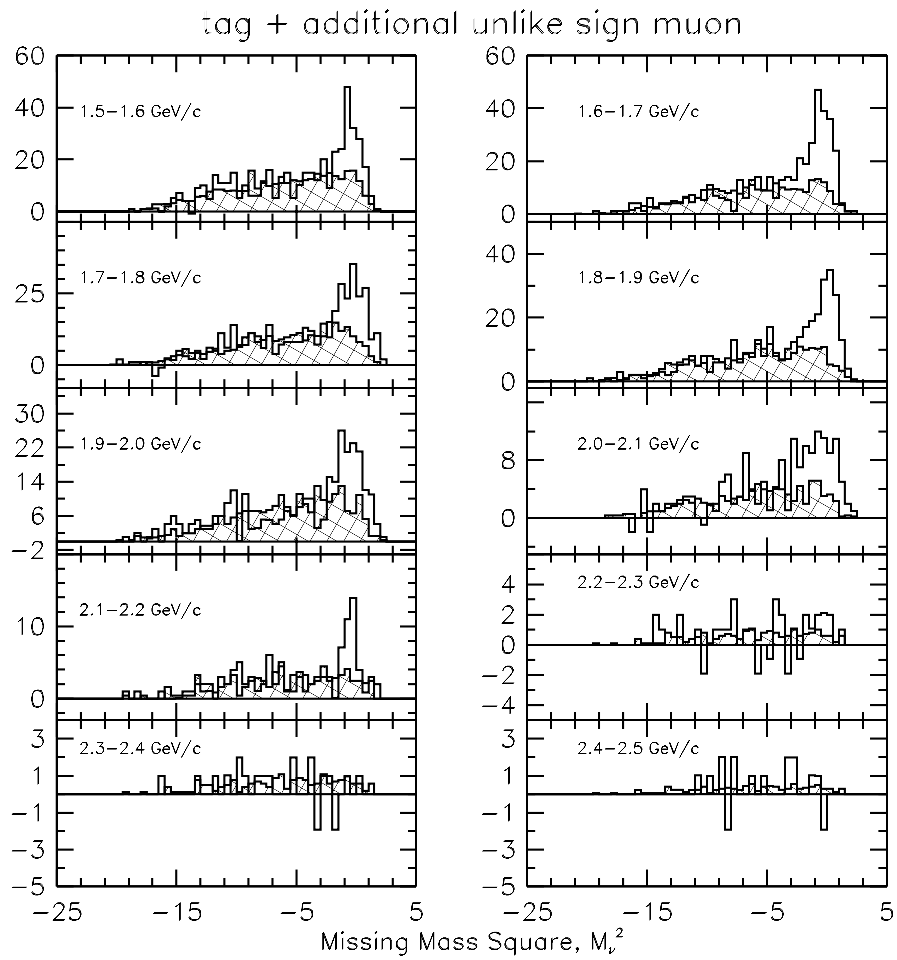

Figure 5.6: The bin-by-bin $\tilde{M}^2$ distribution of second "unlike sign" muons from 1.5-2.5 GeV/c. Continuum and fake candidates have been subtracted and the dotted line is the combinatoric background simulated by Monte Carlo.

entries. The primary and secondary lepton normalizations are allowed to float freely, with J/ψ, D_s, Λ_c, τ and $b \to \bar{c} \to \ell^-$ leptons normalized to the primary lepton portions. The normalization factor of leptons from J/ψ, D_s, Λ_c, τ and $b \to \bar{c} \to \ell^-$ was derived from the PDG-98 value. Their shape was taken with the most updated generic CLEO $B\bar{B}$ Monte Carlo.

The fit to our additional lepton spectra is shown in Figure 5.7 and Figure 5.8 for like sign and unlike sign lepton momentum distributions, respectively. We have like sign pair = 562.37 ± 55.68, unlike sign pair = 2763.96 ± 105.30.

5.6 Calculation of χ_d

The ratio of like sign to all sign lepton pairs, denoted as χ_{tag}, needs to be adjusted to obtain χ_d. As we noted in a previous chapter, the charged B meson pair B^+B^- could enter our signal tag through $B \to (D^{**}/D^*\pi)X\ell\bar{\nu}_\ell$ decay to contaminate our mixing measurement. We need to estimate the portion of B^+B^- in the tag with care.

The χ_{tag} can be written as follows:

$$\chi_{tag} = \frac{N(\ell^\pm_{add}, \ell^\pm_{tag})}{N(\ell^\pm_{add}, \ell^\pm_{tag}) + N(\ell^\pm_{add}, \ell^\mp_{tag})} = \frac{N(\ell^\pm_{add}, \ell^\pm_{tag})/N(tag)}{N(\ell^\pm\ell^\pm + \ell^\pm\ell^\mp)/N(tag)} \tag{5.5}$$

Since the $N(\ell^\pm_{add}, \ell^\pm_{tag})$ is the like sign pair where the additional leptons are from primary decay, it can only come from the mixed B^0. The ratio of $N(\ell^\pm_{add}, \ell^\pm_{tag})/N(tag)$ thus can be written as:

$$N(\ell^\pm_{add}, \ell^\pm_{tag})/N(tag) = \chi_d b_0 f_{0,tag} \tag{5.6}$$

Where $f_{0,tag}$ is the fraction of tag that are from B^0 decay. The b_0 corresponds to the B^0

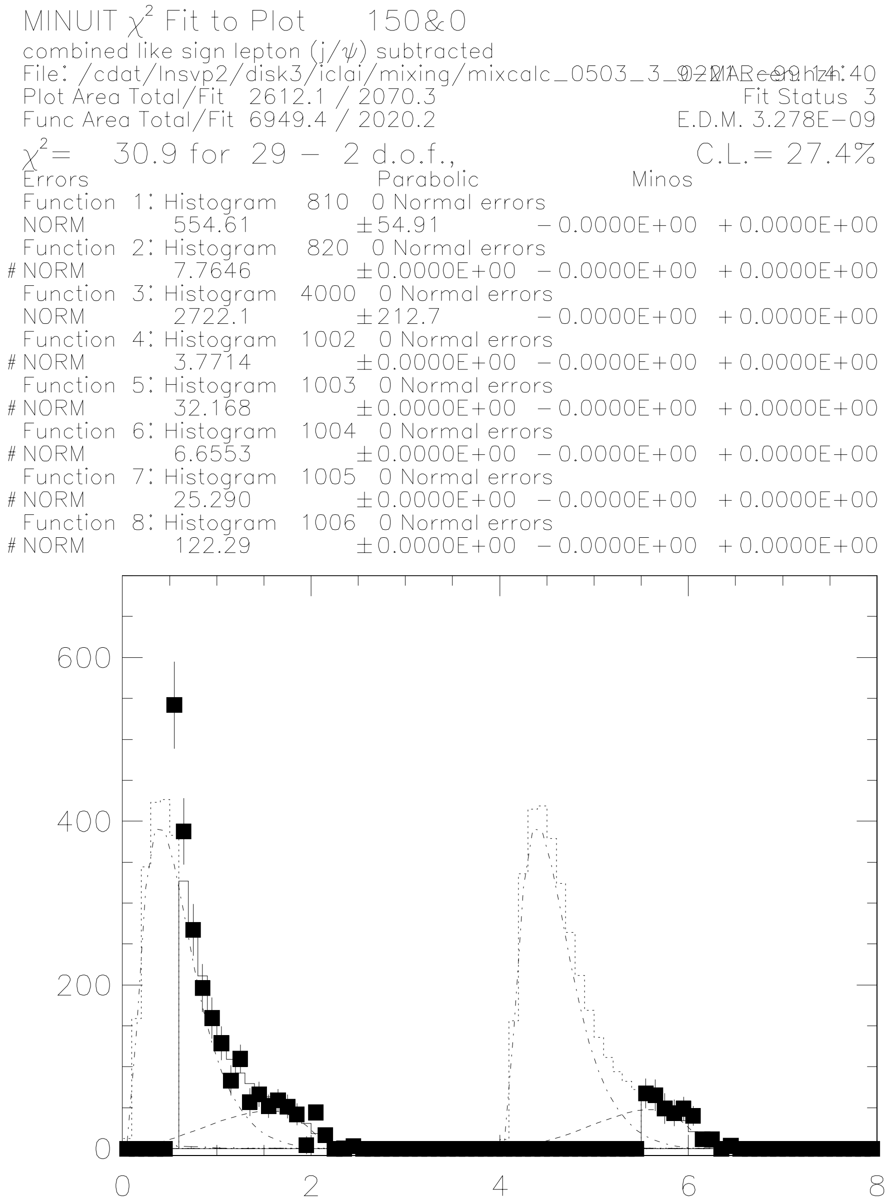

Figure 5.7: The second lepton spectrum for second leptons which have the same charge sign as the tag lepton. The ACCMM model is used for the simultaneous fit of electrons (0-4 GeV/c) and muons (4-8 GeV/c)

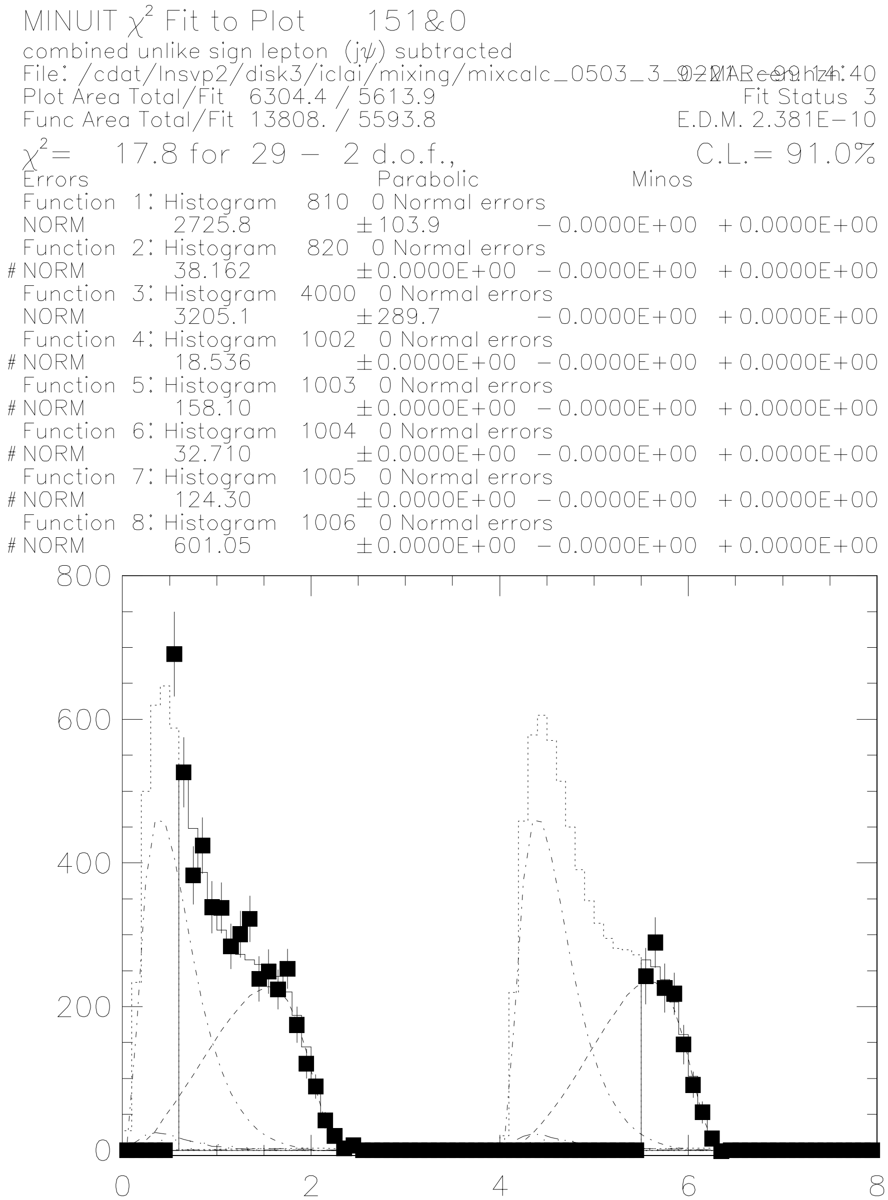

Figure 5.8: The second lepton spectrum for second leptons which have the opposite charge sign as the tag lepton. The ACCMM model is used for the simultaneous fit of electrons (0-4 GeV/c) and muons (4-8 GeV/c)

semileptonic branching fraction.

The denominator, $N(\ell^{\pm}\ell^{\pm}+\ell^{\pm}\ell^{\mp})/N(tag)$, can be written as:

$$N(\ell^{\pm}\ell^{\pm}+\ell^{\pm}\ell^{\mp})/N(tag) = f_{0,tag}b_0 + f_{+,tag}b_+ \tag{5.7}$$

The denominator is essentially the semileptonic branching fraction of our tagged event, b_{tag}, the quantity we measured in the previous chapter. The $f_{0,tag}$ and $f_{+,tag}$ are the fractions of neutral and charged B in our tag, and b_+ and b_0 are the charged and neutral B semileptonic branching ratios respectively.

From this information, we can write χ_{tag} as follows:

$$\chi_{tag} = \frac{\chi_d b_0 f_{0,tag}}{f_{0,tag}b_0 + f_{+,tag}b_+} \tag{5.8}$$

We define $\alpha \equiv \frac{f_{+-}b_+}{f_{00}b_0}$, where f_{+-} and f_{00} correspond to the production fraction of B^+B^- and $B^0\bar{B}^0$ in $\Upsilon(4S)$. From the discussion in chapter 4 on $f_{+,tag}$, we can write $f_{+,tag} = 2\alpha f^{**}/(1+2\alpha)$ and $f_{0,tag} = 1 - f_{+,tag}$. We thus obtain χ_d in terms of α, f^{**} and b_+/b_0:

$$\chi_d = \chi_{d,tag}(1 + (\frac{f^{**}(\frac{2\alpha}{1+2\alpha})}{1 - f^{**}(\frac{2\alpha}{1+2\alpha})})(\frac{\tau_+}{\tau_0})) \tag{5.9}$$

where f^{**} is the fraction of $B \to (D^{**}/D^*\pi)\ell\nu$ in our tagged $\tilde{M}_\nu^2$ signal region, and $\tau_+/\tau_0 = b_+/b_0$ through the isospin symmetry of B semileptonic decay.

We estimate f^{**} as follows: From Monte Carlo we found that the ratio of the entries of $(B^-, B^0) \to (D^{**}/D^*\pi)\ell\nu$ to $B^0 \to D^*\ell\nu$ in our tag signal region is about 15.9%. We scale this number by taking the ratio of $Br(B \to D^{**}\ell\nu)$ [66] vs. its respective number in Monte Carlo, and divide by the ratio of $Br(B \to D^*\ell\nu)$ (from PDG) with its respective number in Monte Carlo:

$$N(D^{**}/D^*)_{new} = N(D^{**}/D^*)_{old} * (\frac{Br(B \to D^{**})}{Br(B \to D^*)})_{new}/(\frac{Br(B \to D^{**})}{Br(B \to D^*)})_{old}$$
$$f^{**}_{new} = N(D^{**}/D^*)_{new}/[1 + N(D^{**}/D^*)_{new}]$$

The inputs to rescale the f^{**} are:

- $(Br(B \to D^{**}\ell\nu) * Br(D^{**} \to D^{*\pm}))_{old} = 2.357\%$. This is estimated from prelib Monte Carlo.
- $Br(B \to D^*\ell\nu)_{old} = 5.44\%$ obtained from prelib Monte Carlo.
- $(Br(B \to D^{**}\ell\nu) * Br(D^{**} \to D^{*\pm}))_{new} = (1.88 \pm 0.30 \pm 0.23)\%$. Obtained from 1997 LEP measurement [66].
- $Br(B \to D^*\ell\nu)_{new} = (4.60 \pm 0.27)\%$. obtained from PDG 98.
- $(D^{**}/D^*)_{old} = 0.159$ obtained from analyzing the $\widetilde{M^2_\nu}$ in prelib Monte Carlo sample.

We found the value of $f^{**} = 0.150 \pm 0.025$, This number is consistent with $f^{**} = 0.14 \pm 0.08$ [48] which was derived by a CLEO-I.5 direct measurement [51]. The small value of f^{**} has constrained the dependence on α and τ_+/τ_0 in this analysis, as can be seen from Equation 5.9.

The dependence on α and τ_+/τ_0 in previous $\Upsilon(4S)$ $B - \bar{B}$ mixing analyses is expressed through the quantity Λ, which is the biggest systematic error on previous CLEO dilepton analyses on mixing as seen in equation 5.4:

$$\Lambda = \frac{f_{+-}b_+^2}{f_{+-}b_+^2 + f_{00}b_0^2} = \frac{\alpha\tau_+/\tau_0}{1 + \alpha\tau_+/\tau_0}$$

Comparing with previous CLEO measurements, this analysis has less dependence on those two quantities, α and τ_+/τ_0, for a small f^{**}. We will discuss this more.

5.7 Result

Our fit with the primary lepton shape extracted by the ACCMM model gives us 562.37 ± 55.68 for the like sign entries and 2763.96 ± 105.30 for the unlike sign pair sample. The χ_{tag}, ratio of like sign pairs divided by like sign plus unlike sign pairs, is 0.169 ± 0.017.

We take $\alpha = 1.15 \pm 0.13$, cited from the weighted error average of the two CLEO measurements on α as derived in the previous chapter. The $\tau_+/\tau_0 = 1.04 \pm 0.04$, is the value taken from PDG-98. f^{**} is taken to be 0.150 ± 0.025. We then calculate χ_d using Equation 5.9. We obtain:

$$\boxed{\chi_d = 0.189 \pm 0.019}$$

where the error is the statistical error only.

5.8 Systematic Error

The χ_d systematic error associated with the lepton detection efficiency canceled when we took the ratio of entries of like sign to all sign leptons. The sources of systematic error come from: the uncertainties on primary and secondary lepton shapes in our fit, the uncertainties of leptons from π^0 Dalitz decay and photon conversions, the fake leptons and leptons from other sources like J/ψ, D_s, Λ_c, τ, upper vertex charm $b \to \bar{c} \to \ell^-$. We also have dependence on f^{**}, α and τ_+/τ_0.

5.8.1 Primary Lepton Spectrum

Because different models predict different primary lepton shapes, the result of the fit will be different. We replace the primary lepton shape predicted by the ACCMM model in our fit by using the shape predicted by ISGW** model. We find the deviation on χ_d due to this model difference to be 2.2%.

5.8.2 Secondary Lepton Spectrum

We take the lepton spectrum of charm semileptonic decay from the 1985 MARK-III analysis [65], and boost this spectrum into the lab frame with the most recent CLEO measured inclusive D meson momentum distribution [74], to obtain the function for our secondary lepton spectrum in our fit. The uncertainty of the secondary spectrum comes from the uncertainty of the D inclusive momentum spectrum. The systematic error on χ_d from the uncertainty of this secondary spectrum, as taken from the biggest variation in our fit, is evaluated as 1.3%.

5.8.3 Fake Lepton

Fake "additional leptons" in the like sign and unlike sign pairs must be subtracted. Like our semileptonic branching fraction analysis, we varied the fake rate for electrons and muons up and down 50% and 25%, respectively, to evaluate the difference caused from the uncertainty of fake rate. We found out the effect from the fake leptons has been as small as 0.4%(0.1%) for fake electrons (muons).

5.8.4 Leptons from $\pi^0 \rightarrow e^+e^-\gamma$ and Photon Conversion

Electrons are created from π^0 Dalitz decay and photon conversions, when photons interact with detector material to produce lepton pairs. Electrons generated from conversion are soft, mostly below 500 MeV/c. The electron positron pairs created from π^0 Dalitz decay travel almost parallel to each other in the lab frame. The combination of the di-lepton opening angle cut and the momentum requirement throw out 99.8% of electrons from $\pi^0 \rightarrow e^+e^-\gamma$ process.

We use Monte Carlo to estimate the portion of π^0 Dalitz decay and photon conversion electrons in the lepton spectrum. We normalize their contributions to the primary leptons portion, then subtract them from our raw lepton spectrum. Since our cut for additional leptons is identical to the previous CLEO study on the inclusive lepton analysis [46], the normalization factor of π^0 Dalitz and photon conversion was taken directly from that analysis. We varied their contributions according to the uncertainty of the normalization and found our fit is not sensitive to this uncertainty. This is not too surprising, because we fit our electrons and muons simultaneously. Because we only accept high momentum muons (from 1.5-2.5 GeV/c), the fit has severely constrained the impact of the variation of leptons below 800 Mev/c. The χ_d systematic error due to the uncertainty of electrons from $\pi^0 \rightarrow e^+e^-\gamma$ is about 0.1%, and 0.1% for electrons from photon conversion.

5.8.5 Other Leptons

The "other leptons" are leptons from sources like J/ψ, D_s, Λ_c, τ decays and from upper vertex charm mesons, $b \rightarrow \bar{c} \rightarrow \ell$. Except for the J/ψ and τ leptons, leptons from D_s, Λ_c and upper vextex charm ($b \rightarrow \bar{c} \rightarrow \ell^-$, W boson decayed to charm in external spectator decay), all have momentum spectra similar to the secondary leptons. We use generic Monte Carlo to obtain the respective lepton momentum shapes for each source and fix their normalization to the primary lepton portion in our fit to obtain the central χ_d value. The systematic error for the "other leptons" comes from the uncertainty of their normalization scale. We vary it up and down according to the statistical error of this normalization to see how our χ_d has changed. The biggest deviation is taken to be our systematic error.

In Figure 5.9 (Figure 5.10) are the momentum spectra of electrons (muons) from: J/ψ, D_s, Λ_c, τ and upper vertex charm, $b \to \bar{c} \to \ell^-$, in comparison with the leptons from secondary decay, $b \to c \to \ell^+$.

J/ψ leptons

Leptons from $B \to J/\psi \to \ell^+\ell^-$ need to be subtracted. The product branching fraction $Br(B \to J/\psi X)$*$Br(J/\psi \to \ell^+\ell^-)$ is $(0.068 \pm 0.004)\%$, or $(0.68 \pm 0.04)\%$ with respect to the primary lepton abundance.

The lepton spectrum of J/ψ was taken from a newly updated generator level $B\bar{B}$ Monte Carlo. The uncertainty on the absolute branching fraction, $Br(B \to XJ/\psi)$* $Br(J/\Psi \to \ell^+\ell^-)$ is the source of the systematic error. We normalized the contribution of J/ψ leptons to the primary leptons in our fit, and varied the portion of this absolute normalization up and down accordingly. The biggest deviation from our central value is about 0.1%.

τ Leptons

Leptons from $B \to X\tau\bar{\nu_\tau}, \tau \to \nu_\tau\ell\bar{\nu_\ell}$ have a slightly different momentum spectrum from D_s and Λ_c leptons. Its relative abundance is also very small. From PDG 98, we found $Br(\bar{B} \to X\tau\bar{\nu_\tau}) = (2.75 \pm 0.3 \pm 0.37)\%$ and $Br(\tau \to \nu_\tau\ell\bar{\nu_\ell}) = (17.37 \pm 0.09)\%$. The product branching fraction of $Br(\bar{B} \to X\tau\bar{\nu_\tau})$ * $Br(\tau \to \nu_\tau\ell\bar{\nu_\ell})$ is $(0.48 \pm 0.05)\%$. We estimated

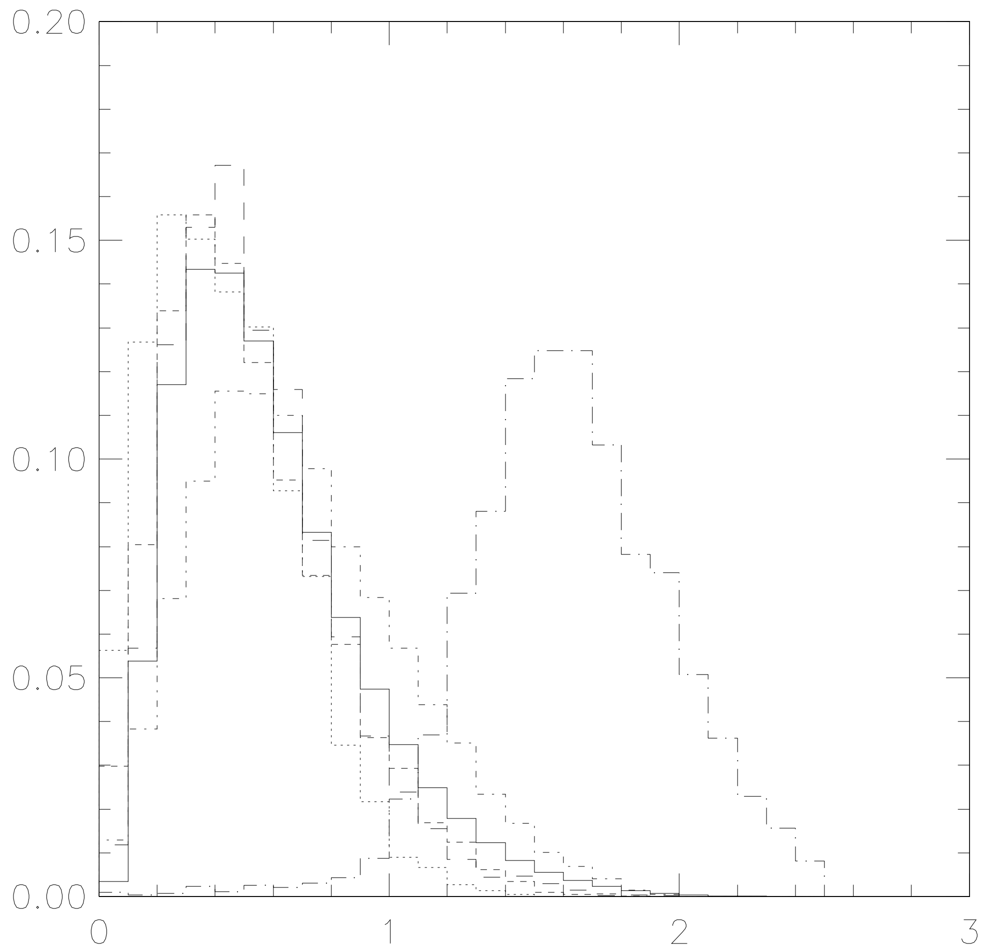

Figure 5.9: The electron spectrum shape from J/ψ (long-dash dot), D_s(short dash), Λ_c(dot), τ(short-dash dot), and $b \to \bar{c} \to \ell^-$ (long-dash) in comparison with the secondary shape (histogram).

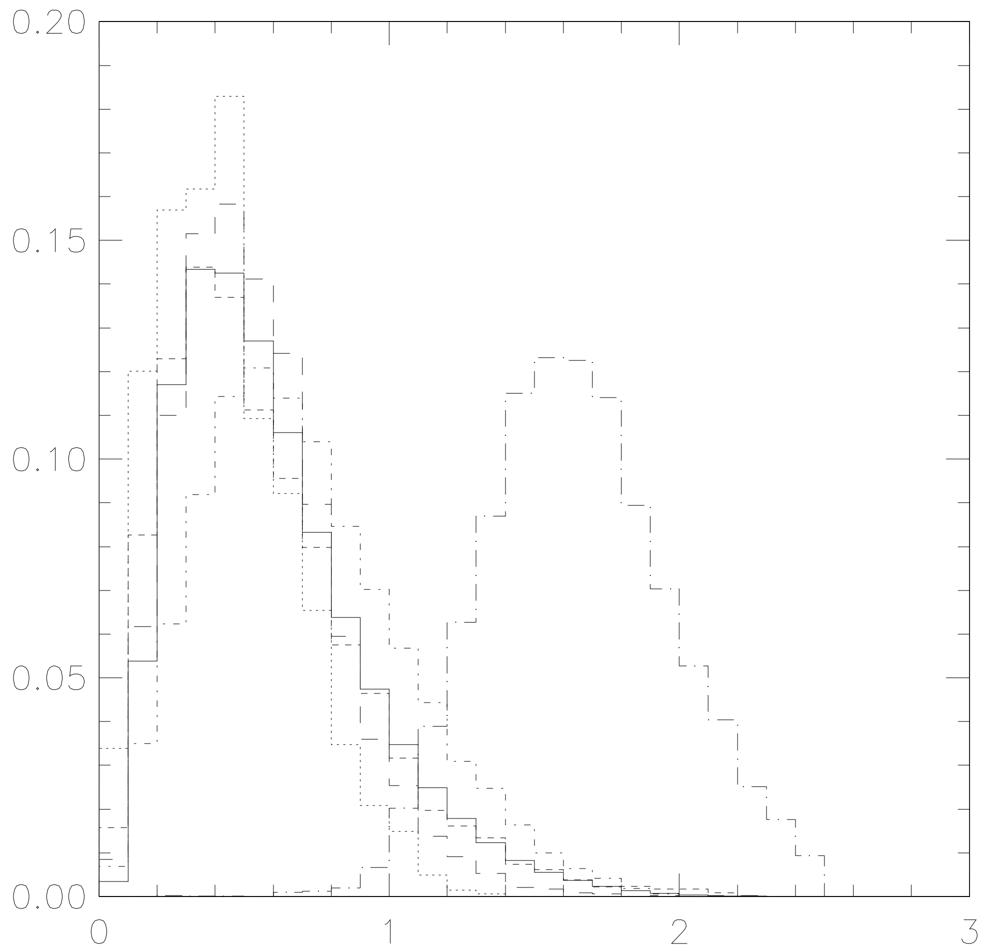

Figure 5.10: The muon spectrum shape from J/ψ (long-dash dot), D_s(short dash), Λ_c(dot), τ(short-dash dot), and $b \rightarrow \bar{c} \rightarrow \ell^-$ (long-dash) in comparison with the secondary shape (histogram).

its effect on mixing by employing the same procedure as above and found its impact is also small, about 0.1% deviation from central χ_d value.

Leptons from D_s Decay

The leptons from D_s decay have a spectrum that is similar to that of our secondary leptons. Because D_s can be produced from upper or lower vertex, its decaying lepton will contaminate our lepton charge sign correlation, since upper and lower vertex leptons have opposite charge signs but a similar momentum spectrum. We have to estimate it with care.

We calculate the product branching fraction of $Br(\bar{B} \to \bar{D}_s \to \ell^-)$ to be $(0.63 \pm 0.13)\%$ from PDG 98, about 5.8% of primary leptons. Our lepton momentum threshold cut suppresses 88.5% and 99.6% of electrons and muons from D_s decay, respectively. We estimate the effect of the remainder on mixing by taking the lepton spectrum shape from Monte Carlo and varying it according to the uncertainty in its absolute normalization to our primary leptons in the fit. It is found that its effect is mostly absorbed by our secondary function and its impact has no sensitivity to our fit. We found its deviation is as small as 0.002%. Being conservative, we assign it to be 0.1% here.

Λ_c Leptons

Leptons from Λ_c decay also have a similar spectrum compared to that of secondary leptons. Its contribution is also far less than D_s leptons in the additional lepton population. From PDG 98, we compute the product branching fraction of $Br(\bar{B} \to \Lambda_c \to X\ell\nu)$ to be $(0.13 \pm 0.04)\%$. Using an identical method to the D_s leptons contribution, we found it generated a small effect, about 0.1% deviation on our χ_d measurement.

Leptons from Upper Vertex Charm $b \to \bar{c} \to \ell^-$

Leptons from $B \to \bar{D} \to \ell^-$ have the same charge as the primary leptons, but with much smaller momentum because the small phase space for upper vertex charm production does not boost its daughter lepton as much. The branching fraction for the upper vertex charm production in B decay, $B \to \bar{D^0} + D^- X$, is found to be 0.079 ± 0.022 [72]. This number is combined with the semileptonic branching fraction of D^0 and D^+ respectively to arrive at the branching fraction of $B \to \bar{D} \to \ell$. We take an equal mixture of D^0 to D^- in upper vertex charm production.

Since there is no measurement of $B \to \bar{D^*} X$, we use $Br(B \to \bar{D^*})$ from Monte Carlo and $Br(D^* \to D\pi)$ from 1998 PDG value to arrive at the value of $Br(B \to \bar{D^*} \to \bar{D})$. We multiply the $D^0(D^+)$ semileptonic branching fraction with this number to obtain the branching fraction of $B \to \bar{D} \to \ell$ when $\bar{D}$ meson comes from the decay of upper vertex vector D^*.

We combine the two results to obtain the branching fraction of leptons from upper vertex charm in B decay, $(2.39 \pm 0.40)\%$. We used the identical procedure for leptons from D_s mentioned earlier and found our χ_d value is insensitive to the uncertainty associated with upper vertex charm leptons. The systematic error associated with this uncertainty is found to be about 0.1%. This is not surprising, because the lepton spectrum of upper vertex charm has a shape similar to that of our secondary lepton spectrum. Our simultaneous fit procedure also has constrained the impact of the uncertainty of upper vertex charm.

5.8.6 Uncertainty on f^{**}

We have estimated f^{**} to be 0.150 ± 0.025. We varied f^{**} up and down 1 σ to estimate the systematic error on χ_d, and found it to be 2.0%.

5.8.7 Systematic Error from α and τ_+/τ_0

The error on α and τ_+/τ_0 was used to vary the values of α and τ_+/τ_0 up and down, respectively, to estimate the associated systematic error on χ_d. We found that the systematic error due to α to be 0.5% and to τ_+/τ_0 to be 0.4%. As explained earlier, the small χ_d dependence on α and τ_+/τ_0 can be traced to the small value of f^{**} in our conversion from $\chi_{d,tag}$ to χ_d in Equation 5.9.

5.8.8 Summary of Systematic Errors

We summarize the systematic errors in Table 5.8.8.

5.9 Summary

We measure the $\bar{B}^0 - B^0$ mixing parameter χ_d through partial reconstruction of $\bar{B}^0 \rightarrow D^{*+}\ell^-\bar{\nu}$ and the inclusive lepton momentum spectrum which is enriched in $\bar{B}^0$. This measurement makes no assumption of the fraction of B^+B^- pair in $\Upsilon(4S)$ decays. We measure χ_d:

$$\chi_d = 0.189 \pm 0.019(stat) \pm 0.005(sys) \pm 0.001(\alpha, \frac{\tau_+}{\tau_0})$$

The last error is the error on α and τ_+/τ_0.

In our previous analysis, we have obtained the ratio of charged to neutral B meson semileptonic branching fraction, shown in equation 4.19:

$$\frac{b_+}{b_0} = 0.952^{+0.117+0.060}_{-0.081-0.051}$$

If we replace the $\tau_+/\tau_0 = 1.04 \pm 0.04$ with the b_+/b_0 value shown above, the χ_d value is $0.187 \pm 0.019 \pm 0.005^{+0.003}_{-0.002}$. The last error is from the uncertainty of the b_+/b_0 where we take the square root of the sum of squared statistical and systematic errors.

We list the measured χ_d between the two CLEO analyses with similar methods of *partial reconstruction* on the same dataset in Table 5.4. We find the two agree well with each other.

We also compare this result with the previous CLEO measurement and scale the difference of α and τ_+/τ_0. Our result has the lowest dependence on the different value of α and τ_+/τ_0, as shown in Table 5.5.

From the averaged LEP measurement of $x_d = 0.734 \pm 0.035$ [67], χ_d can be calculated as follows:

$$\begin{aligned}\chi_d &= \frac{(\frac{\Delta M}{\Gamma})^2}{(2 * (1 + (\frac{\Delta M}{\Gamma})^2))} = \frac{x_d^2}{2 * (1 + x_d^2)} \\ &= 0.175 \pm 0.011(\sigma(x_d))\end{aligned}$$

This is consistent with our result of χ_d.

Table 5.1: Like Sign Lepton Spectrum

(GeV/c)	Data	MC	Data-MC	ϵ_{ID}	net event
electrons					
0.5-0.6	502.5 ± 26.1	205.3 ± 13.0	297.2 ± 29.1	0.548	542.4 ± 53.2
0.6-0.7	354.7 ± 21.5	128.8 ± 9.7	225.9 ± 23.6	0.582	388.1 ± 40.5
0.7-0.8	247.4 ± 17.0	90.7 ± 7.4	156.6 ± 18.5	0.585	267.6 ± 31.7
0.8-0.9	205.6 ± 15.2	89.1 ± 8.2	116.5 ± 17.3	0.593	196.3 ± 29.1
0.9-1.0	147.2 ± 13.5	51.7 ± 6.5	95.5 ± 15.0	0.598	159.7 ± 25.1
1.0-1.1	117.5 ± 11.6	38.2 ± 5.0	79.3 ± 12.6	0.615	128.9 ± 20.5
1.1-1.2	89.3 ± 10.2	37.5 ± 5.3	51.8 ± 11.5	0.621	83.4 ± 18.5
1.2-1.3	94.5 ± 9.8	26.3 ± 4.4	68.2 ± 10.7	0.622	109.7 ± 17.2
1.3-1.4	75.5 ± 8.8	40.3 ± 6.0	35.2 ± 10.6	0.618	57.0 ± 17.2
1.4-1.5	67.3 ± 8.6	25.8 ± 4.4	41.5 ± 9.7	0.625	66.4 ± 15.4
1.5-1.6	55.6 ± 7.9	22.5 ± 4.6	33.1 ± 9.1	0.635	52.1 ± 14.3
1.6-1.7	59.7 ± 7.7	22.0 ± 4.1	37.7 ± 8.8	0.635	59.3 ± 13.8
1.7-1.8	58.8 ± 7.7	26.1 ± 5.0	32.7 ± 9.2	0.636	51.5 ± 14.5
1.8-1.9	51.8 ± 7.2	25.1 ± 4.6	26.7 ± 8.5	0.638	41.9 ± 13.4
1.9-2.0	24.0 ± 5.4	21.3 ± 4.4	2.7 ± 7.0	0.635	4.3 ± 11.1
2.0-2.1	34.9 ± 5.9	6.7 ± 1.9	28.2 ± 6.2	0.637	44.3 ± 9.7
2.1-2.2	13.0 ± 3.6	2.1 ± 1.1	10.8 ± 3.8	0.648	16.7 ± 5.8
2.2-2.3	0.1 ± 2.4	0.1 ± 0.4	0.0 ± 2.4	0.630	-0.1 ± 3.8
2.3-2.4	0.1 ± 2.4	0.0 ± 0.0	0.0 ± 2.4	0.619	0.0 ± 3.8
2.4-2.5	2.0 ± 1.4	0.0 ± 0.0	2.0 ± 1.4	0.615	3.2 ± 2.3
muons					
1.5-1.6	41.9 ± 6.6	14.2 ± 3.5	27.7 ± 7.5	0.409	67.6 ± 18.4
1.6-1.7	49.2 ± 7.9	18.9 ± 4.4	30.3 ± 9.0	0.463	65.4 ± 19.5
1.7-1.8	45.9 ± 8.0	22.1 ± 4.4	23.7 ± 9.1	0.483	49.2 ± 18.9
1.8-1.9	37.6 ± 7.1	15.7 ± 4.0	21.9 ± 8.1	0.503	43.5 ± 16.2
1.9-2.0	38.4 ± 6.7	13.4 ± 3.5	25.1 ± 7.5	0.513	48.8 ± 14.7
2.0-2.1	30.6 ± 5.6	9.4 ± 3.2	21.2 ± 6.4	0.528	40.2 ± 12.1
2.1-2.2	14.7 ± 3.9	8.4 ± 2.5	6.3 ± 4.6	0.540	11.7 ± 8.6
2.2-2.3	6.1 ± 4.2	0.1 ± 0.4	6.0 ± 4.2	0.529	11.3 ± 7.9
2.3-2.4	-0.1 ± 2.4	0.0 ± 0.1	-0.1 ± 2.4	0.540	-0.2 ± 4.4
2.4-2.5	1.8 ± 1.4	0.0 ± 0.0	1.8 ± 1.4	0.536	3.4 ± 2.6

Table 5.2: Unlike Sign Lepton Spectrum

(GeV/c) electrons	Data	MC	Data-MC	ϵ_{ID}	net event
0.5-0.6	598.9 ± 29.3	220.2 ± 13.7	378.7 ± 32.4	0.548	691.2 ± 59.1
0.6-0.7	483.6 ± 25.8	177.1 ± 11.7	306.5 ± 28.3	0.582	526.6 ± 48.6
0.7-0.8	364.7 ± 21.2	140.6 ± 10.6	224.2 ± 23.7	0.585	383.0 ± 40.5
0.8-0.9	368.8 ± 21.4	117.1 ± 9.3	251.7 ± 23.3	0.593	424.1 ± 39.3
0.9-1.0	325.7 ± 19.6	123.3 ± 9.8	202.4 ± 21.9	0.598	338.4 ± 36.6
1.0-1.1	309.0 ± 19.9	101.5 ± 8.7	207.5 ± 21.8	0.615	337.3 ± 35.4
1.1-1.2	271.7 ± 17.4	95.5 ± 8.9	176.3 ± 19.5	0.621	283.9 ± 31.5
1.2-1.3	292.9 ± 17.8	105.8 ± 9.2	187.1 ± 20.1	0.622	300.8 ± 32.2
1.3-1.4	295.9 ± 18.0	96.7 ± 8.5	199.2 ± 20.0	0.618	322.2 ± 32.3
1.4-1.5	256.9 ± 17.5	107.8 ± 9.1	149.2 ± 19.7	0.625	238.6 ± 31.6
1.5-1.6	268.9 ± 17.1	110.9 ± 9.4	158.0 ± 19.5	0.635	248.8 ± 30.7
1.6-1.7	224.2 ± 16.1	81.9 ± 7.7	142.2 ± 17.8	0.635	223.8 ± 28.0
1.7-1.8	244.8 ± 15.7	84.3 ± 8.0	160.5 ± 17.6	0.636	252.5 ± 27.7
1.8-1.9	183.0 ± 14.1	71.8 ± 7.7	111.3 ± 16.1	0.638	174.5 ± 25.2
1.9-2.0	124.0 ± 11.6	47.5 ± 6.1	76.5 ± 13.2	0.635	120.6 ± 20.7
2.0-2.1	83.1 ± 9.7	26.7 ± 4.8	56.4 ± 10.8	0.637	88.5 ± 17.0
2.1-2.2	34.2 ± 7.1	7.3 ± 2.3	26.9 ± 7.5	0.648	41.6 ± 11.6
2.2-2.3	14.0 ± 4.4	1.7 ± 0.6	12.3 ± 4.5	0.630	19.6 ± 7.1
2.3-2.4	1.9 ± 1.4	0.3 ± 0.2	1.7 ± 1.4	0.619	2.7 ± 2.3
2.4-2.5	4.2 ± 3.9	0.0 ± 0.0	4.1 ± 3.9	0.615	6.7 ± 6.4
muons					
1.5-1.6	177.3 ± 14.0	78.1 ± 7.9	99.2 ± 16.1	0.409	242.4 ± 39.3
1.6-1.7	196.5 ± 14.9	62.4 ± 6.9	134.1 ± 16.4	0.463	289.2 ± 35.4
1.7-1.8	177.1 ± 14.6	68.2 ± 7.5	108.9 ± 16.4	0.483	225.6 ± 34.0
1.8-1.9	166.4 ± 13.4	56.9 ± 6.4	109.6 ± 14.9	0.503	217.9 ± 29.6
1.9-2.0	131.9 ± 12.0	56.3 ± 7.3	75.5 ± 14.1	0.513	147.2 ± 27.4
2.0-2.1	69.3 ± 8.7	21.3 ± 4.0	48.0 ± 9.6	0.528	90.9 ± 18.2
2.1-2.2	42.8 ± 7.7	14.3 ± 3.0	28.5 ± 8.3	0.540	52.7 ± 15.4
2.2-2.3	9.8 ± 4.6	1.2 ± 0.6	8.6 ± 4.7	0.529	16.2 ± 8.8
2.3-2.4	-0.4 ± 2.4	0.4 ± 0.2	-0.7 ± 2.4	0.540	-1.4 ± 4.4
2.4-2.5	0.0 ± 2.4	0.0 ± 0.0	0.0 ± 2.4	0.536	0.0 ± 4.4

Table 5.3: Systematic Errors on χ_d.

Source	$\sigma_{sys}/value(\%)$	$\delta(x_d)/x_d(\%)$
Fake electron	50%	0.4%
Fake muon	25%	0.1%
primary spectrum	isgw**	2.2%
secondary spectrum		1.3%
γ conversion		0.1%
$\pi^0 \to e^+e^-$		0.1%
D_s		0.1%
Λ_c		0.1%
τ		0.1%
Ψ contribution		0.1%
$\bar{B} \to \bar{D} \to \ell^-$		0.1%
f^{**}	17%	2.0%
α	12.2%	0.5%
τ_+/τ_0	4%	0.4%
Net systematic error		3.3%

Table 5.4: Comparison with previous CLEO χ_d measurement using partial reconstruction technique in the same dataset.

Method	$f_{+-}/f_{00} = 1$ $\tau_+/\tau_0 = 1.00 \pm 0.14$	$f_{+-}/f_{00} = 1.13 \pm 0.14$ $\tau_+/\tau_0 = 1.04 \pm 0.04$
previous analysis	0.149 ± 0.023	0.155 ± 0.024
this analysis	0.151 ± 0.034	0.152 ± 0.034

Table 5.5: Comparison with Previous CLEO χ_d Value.

Method	$\alpha = 1.0 \pm 0.15$ $\tau_+/\tau_0 = 1.0 \pm 0.14$	$\alpha = 1.15 \pm 0.13$ $\tau_+/\tau_0 = 1.04 \pm 0.04$
CLEO-93 d-lepton	$0.157 \pm 0.016 \pm 0.018^{+0.028}_{-0.021}$	$0.172 \pm 0.018 \pm 0.018^{+0.010}_{-0.011}$
CLEO-93	$0.149 \pm 0.023 \pm 0.019 \pm 0.010$	$0.155 \pm 0.024 \pm 0.019 \pm 0.004$
This analysis	$0.188 \pm 0.019 \pm 0.005 \pm 0.003$	$0.189 \pm 0.019 \pm 0.005 \pm 0.001$