STOCHASTIC AVAILABILITY ANALYSIS AND MODELING
OF LONGWALL MINING OPERATIONS

by

Chunming Duan

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APPROVED:

E. Topuz, Chairman

M. Karmis

J. R. Lucas

G. H. Luttrell

S. Suboleski

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Blacksburg, Virginia
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Chunming Duan
Ertegrul Topuz, Chairman
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(ABSTRACT)

The objective of this research is to develop analytical approaches for assessment and prediction of the availability of longwall mining systems. After a functional analysis of longwall mining operations, the longwall production system is divided into four subsystems: coal-cutting, face-conveying, roof-support, and outby-haulage. The operating characteristics of the longwall system are then investigated based on the system configuration, component failure and repair processes, and rules of operation. Through use of the techniques of reliability assessment and stochastic systems analysis, five probability models are formulated and solved with respect to different longwall operating logic. The implementation of these models is demonstrated with a number of case studies. Furthermore, three important applications of the results have been identified for improvement of longwall performance: analysis of component importance, assessment and prediction of productivity, and optimization of system operational effectiveness.
This investigation provides a systematic methodology for evaluation of longwall operational effectiveness. A number of system effectiveness measures have been developed for longwall systems with various operating characteristics. Some of the measures include system availability, reliability, failure rate, mean time to failure, mean time to repair, the expected average of the number of system failures, and the limiting probabilities of system failure due to any subsystem. Explicit expressions of system availability are obtained for several practical cases. The methodology developed can be used as both an assessment tool and a design tool for improvement of the operational effectiveness of longwall mining systems.
DEDICATION

This dissertation is dedicated to my parents for their constant encouragement, support, and love during my education.
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I. INTRODUCTION

1.1 Research Background

Longwall mining provides the potential for greatly increased productivity, improved safety, better strata control, and a higher level of recovery. It is more amenable to automation and remote control. As a result, longwall mining has become an important mining method used in the U.S. coal mining industry. The share of longwall mining in the underground coal production of the United States has risen from 5% in the late 1970s to more than 25% today (Merrit, 1984; Sprouls, 1986). By the end of 1989, 95 longwall operations were functioning in the U.S. (Sprouls, 1990).

Despite the initial growth of the U.S. longwall industry since the 1970s, the number of longwalls has hovered around the 100 mark for the last nine years, whereas earlier predictions called for 200 units by now. Spectacular production rates have been achieved by certain longwalls, mainly because of utilization of larger and more powerful machines. But a large number of such operations have failed to meet the anticipated production targets. The range of longwall productivity is very large. In 1988, the reported productivity ranged from 640 to 6,000 clean tons per shift (Combs, 1990).
Various reasons were suggested to explain the slowness of the growth of the U.S. longwall industry, including large capital costs and uncertainty of demand forecasts. However, it is believed that some production figures are far below the expectation of mine operators, and therefore can be considered as a major contributing factor to the failure of some longwall operations. Historical figures for longwall system availability range from roughly 50 to 70%. The figures for face availability vary from 60 to 80% (Curry, et al., 1976; Pimentel, et al., 1981; Dunlap, 1990). Since longwall systems are capital intensive, high production rates must be achieved in order to make the installation economically viable. The large amount of downtime and high maintenance costs incurred in some of these operations have resulted in considerable financial loss to these capital-intensive systems. It can be concluded that low system availability contributes significantly to the marginal performance of many U.S. longwalls.

1.2 Statement of the Problem

Productivity problems can be solved through the development and acquisition of new equipment and through the application of sound engineering principles. Both approaches have merit and are a part of the evolutionary process of improving mining technology. However, over the years the art of systems engineering has not been emphasized in underground coal mines. As a result, many existing coal mining operations have not achieved their potential capabilities.

Coal mining requires intensive interaction between man, machine, and the
environment. Key factors for an efficient longwall operation are good system design, operational planning, and availability of equipment. All these factors are reflected by the availability and the life-cycle cost of the system in use. As longwall systems grow larger and more complex, a large number of closely interacting variables are involved in mine planning and operations. This complexity greatly increases difficulties in the control and optimization of the performance of longwall mining operations. Thus, conducting system effectiveness analysis becomes necessary to the successful operation of longwall mining. This kind of systems analysis has two important uses in helping coal mine operators compete in the coal industry: (1) to provide a qualitative and/or quantitative basis for evaluating the achievements during system acquisition and operation stages; (2) to provide a management discipline that will allow achievement of predicted optimum levels of system effectiveness.

System effectiveness is a measure of the ability of a system to accomplish its mission objectives. It represents the probability that the system can successfully meet an operational demand within a given period of time under specified conditions. For coal mining operations, the major mission objectives include meeting certain production, safety, and economic requirements. Among the most representative measures of effectiveness are reliability, maintainability, availability, productivity, and system life-cycle cost. Complementary to each other, these measures give a complete and overall picture of the operational effectiveness of a system. The actual measures selected for systems evaluation are dependent on the objective of the study and/or analytical techniques applied.

For longwall operations, system availability is one of the most important
measures of effectiveness. Availability is defined as the probability that the system is operating satisfactorily at any point in time under stated conditions. In practice, it is often expressed as the ratio of uptime to total time. The availability of a system is influenced by system design, working environment, management policies, and workers' skills. Relating both reliability and maintainability, availability has become probably the most valuable indicator of system efficiency and performance for maintained systems. The concept of availability is very helpful to systems analysis and evaluation of longwall mining operations in that it can quantify the productive capacity, the possibility of breakdown, and the maintenance requirements for a system. These quantifications assist in the comparison of the performance of systems, which are used under different geological/mining conditions or equipment supplied by different manufacturers and of different specifications.

Because of ever increasing concerns on the cost—effectiveness of longwall operations, many research projects have been undertaken. Most of these studies have been limited to the statistical analysis of historical delay data or computer simulation of longwall operations, rather than development of analytical models with optimization capabilities. To assess and predict the operational effectiveness of a longwall system, however, requires development of systematic approaches to enable the evaluation of the longwall performance in terms of equipment availability, reliability, maintainability, and the rules of operation. These approaches should include systems and industrial engineering guidelines, analytical models, and performance measurement techniques for use in the management of longwall operations.
1.3 Objectives of the Research

This research attempts to develop analytical approaches for assessment and prediction of the availability of longwall mining systems. The specific objectives include:

i) To understand the functional relationships between the various components in a longwall system and identify the system's operating logic.

ii) To develop analytical models for assessment of the availability, reliability, and maintainability of longwall systems with different failure-repair characteristics and rules of operation.

iii) To investigate both transient and steady-state availabilities of longwall operations by stochastically modeling the manner in which the longwall components interact with each other.

iv) To develop models, functions, and indices for analysis of component importance, assessment of productivity, and optimization of longwall operational effectiveness.

v) To present procedures and examples in application of the availability models and recommend the actions to be taken for improvement of longwall system performance.
vi) To provide recommendations for future research on development of decision support tools for longwall mining management.

1.4 Scope of the Research

Longwall mining is a complex and integrated system. The factors which influence system operation are closely interrelated and should be examined carefully. In order to achieve the above objectives, the concentration of this research is governed by the following guidelines.

i) For the mining industry, a reliability and maintainability concept should be broad in scope and allow a rather free interpretation of what constitutes a failure. In this study, a failure refers to any condition which causes a decrease in the production rate. Equipment breakdown and occurrence of geological/mining incidents are examples of these failure events, while the former is of primary interest in this research.

ii) Depending on the time interval considered, availability is classified into three categories: (1) instantaneous, (2) average, and (3) steady-state availabilities. The particular availability measure(s) chosen depends upon the mission requirements of the system. For a longwall system, which is expected to operate continuously for a certain period of time, the steady-state availability may be a more appropriate measure of system performance.

iii) A variety of factors are associated with longwall availabilities. In
addition to component reliability and maintainability, availability could include environmental factors, organizational policies, workers' skills, etc. However, since the mining environment, organizational policies, and human factors vary from mine to mine, they are not considered in this study.

iv) Besides equipment availability, the actual productivity of a longwall face also depends on the compatibility between the employed equipment components, and the interaction between the equipment and the geological conditions. Some of the major interactions include coal properties vs. cutting mechanism and shearer operation; shearer cutting speed vs. conveyor haulage speed or loading capacity; conveyor specifications vs. the power requirements; and roof support requirements vs. the behavior of the immediate roof. These problems were studied extensively in previous research on optimal design of longwall systems. Thus, this research will not address these topics and will assume that an optimal initial design was achieved for existing systems under study.

1.5 Organization of the Dissertation

In order to achieve the objectives given above, this dissertation is divided into six parts. Chapter 2 contains a review of the major work done in longwall systems analysis and modeling. In Chapter 3, after a functional description of longwall mining operations, the production system is divided into four subsystems. The system's operating logic is identified and then represented by functional diagrams. Chapter 4 develops five probability models for longwall system availability modeling. These models are formulated and solved with respect to a variety of
system operating logic. Chapter 5 contains three case studies for demonstrating the use of the availability models developed. The results from different models are compared with respect to real longwall operations. Chapter 6 presents three important applications of the modeling approaches. More models, functions, and indices are defined in this chapter. Finally, Chapter 7 summarizes the contribution of this research and presents suggestions for future research.

Five appendices are presented at the end of this dissertation. Appendix A contains a glossary of terms and symbols used in model formulation. To make the mathematical part of the dissertation easier to understand, Appendix B provides a brief introduction to the basic concepts and equations of stochastic reliability modeling. Appendix C lists the algorithm used for solving Markov models. Appendices D, E, and F contain the transition matrices and some intermediate results for Case studies 1, 2, and 3, respectively.
II. LITERATURE REVIEW

The studies on longwall systems analysis and design are often conducted by applying three types of techniques: (1) analytical methods, such as deterministic operations research models, (2) computer simulation, and (3) statistical analysis. Most of these studies can be divided into two groups based upon their objectives: optimization of longwall system design and planning, and analysis of longwall delays and availability. Some of the major work in these areas is summarized below.

2.1 Optimization of Longwall System Design and Planning

The wide variety of natural and regulatory conditions restricts the designer in his choice of the system parameters to achieve an optimal design, that is, matching engineering specifications to the mining conditions. The choice of equipment used, and the size and shape of panels, pillars, and entries is seriously affected by the overall strata condition. Reviewing the literature pertaining to the design of the layout of longwall systems indicates that the designers have used an iterative process based on practical experience to arrive at a few empirical formulas.

The research concerning longwall system design has concentrated on
determining the dependence of the coal-cutting process upon the characteristics of the coal and the cutting equipment, estimating the roof support requirements, and developing the most efficient design of longwall system. Two major fields in which systems engineering methods have been used are computer simulation of the longwall production process and the determination of optimal panel dimensions.

To simulate the design and development of the longwall system, many computer models have been developed. Manula, et al. (1979) developed a model consisting of three subassemblies — roof support, loader—shearer, and face conveyor. This model incorporated both deterministic and stochastic methods to calculate (1) the size and density of the roof supports; (2) the cutting force, power requirements, and haulage speed of the shearer; and (3) the conveyor size, speed, and power requirements.

Another simulation model of longwall operations was developed by Michalopoulos and Topuz (1984). This model has three functions: (1) estimating the production rate based on the characteristics of the equipment and the coal seam, (2) performing an event-oriented simulation of the longwall operations; and (3) fitting theoretical probability distributions to the input data. With this model, the productivity of a longwall face can be assessed based on its equipment design, panel dimensions, and operating methods.

In contrast to both of the models given above, which were established by incorporating both deterministic and stochastic methods, a completely deterministic, interactive computer program was constructed by Cole (1980). This model is capable of simulating the design and development of a fully retreating
longwall system or an alternating retreat system. A set of equations was formulated in the model to represent the relationships between design principles significant in constructing an effective longwall system. Major design parameters that were considered included the number of entries, pillar design, panel dimensions, the extraction sequence, and the number of shifts per day and days per week for the continuous miner units and the longwall units. Because of its interactive capability in the course of investigation, the model is flexible and can be used to assist the design engineer in evaluating the impact of managerial and engineering decisions in the overall productivity of a proposed longwall system at a low cost. If equipped with graphic capabilities and databases of seam characteristics and equipment specifications, this model could possibly be developed into a computer-aided design tool for longwall mining.

To explore the interactions between various influence factors to the longwall production, Kovach (1982) conducted a systems analysis of a longwall system. Besides such parameters as the panel dimensions, the number of entries, and production rates, other factors, including coal resource recovery, the number of continuous miner units required for development, and equipment reliability were also considered in this analysis. Also, a cost analysis of the longwall layout was made. By analyzing the relationship between the panel dimensions and the face equipment performance, a major conclusion was drawn: larger panel dimensions were not necessarily the correct approach for decreasing the non-productive times in longwall mining, because of the increased amount of equipment wear resulting from longer period of operation. It was indicated that, in general, production rates varied inversely with panel dimensions, and this occurred because the extraction of smaller panels was completed before the equipment began to wear out. However,
this conclusion probably took into consideration the pure extraction time and not the face move time in estimating the production rates.

For the last several years, a general trend, as shown in the latest longwall survey (Combs, 1990), has existed for longer panels and wider faces. The increased panel dimensions have partly resulted in increased productivity by reducing move times. However, some problems have been incurred from lengthening panels, such as deterioration in equipment and gate entries and incremental capital investment. Thus, optimization of panel dimensions has been a focus in some of the research regarding longwall system design.

As a part of the project on the design optimization of underground coal systems, an attempt was made at VPI & SU (1981) to optimize longwall panel dimensions. In order to find the optimal face length, an analytical model was developed to quantify the relationship between the face length and shift production. The optimum face lengths were then obtained with respect to each of these four different objectives: (1) the minimum average unit cost, (2) the maximum internal rate of return, (3) the maximum profitability index, and (4) the minimum present value of unit production cost.

Bessinger and Gentry (1985) provided another approach for optimization of longwall panel dimensions. Primarily as a design approach, their study incorporated equipment, geological, configurational, and productivity parameters. The total net present values were evaluated for a proposed longwall investment.

Grayson and Peng (1986) formulated the selection of the panel dimensions as a
linear programming problem. The objective function of this model is minimization of the total mining cost. This model was solved subject to constraints relative to the continuous mining section utilization requirements, the production shift availability, the demand for coal, and the longwall move requirements. The determination of optimal panel dimensions was illustrated for a proposed system by performing a sensitivity analysis on these factors.

A comprehensive analysis of longwall operations, which covered almost every aspect of longwall mining with an orientation towards optimal system design, was made by Peng and Chiang (1984). This work is considered as a classical textbook on longwall mining.

2.2 Analysis of Longwall Delays and Availability

Longwall mining is an integrated continuous production system. It is characterized by very close component interactions and is, therefore, prone to production delays. According to Duan, et al. (1988), these delays are attributed to these five sources: (1) system development and design, (2) equipment reliability and maintainability, (3) environmental factors, (4) organizational policies, and (5) human factors and safety. Thus, the design improvements in the individual components of a longwall system alone may not markedly increase the system's productivity.

By applying industrial and reliability engineering techniques, a number of studies have been conducted to analyze longwall system delays and availability.
Most of the studies were limited to statistical analysis of historical data, but the results are less than satisfactory. Usually a lack of data or inconsistent data reporting has caused too much variation to permit meaningful conclusions to be drawn from these studies.

The longwall system availability study made at Jet Propulsion Laboratory (Curry, et al., 1976, 1977) was probably the first among this kind of study. An effort was made to develop a computer program for data retrieval and analysis. Foreman reports were collected over an eleven-month period from one installation. Vital delay and production information was coded from these reports as input data. The system's availability was estimated from these data to be 76.5% of the available working time (total time minus the inherent delay time). If all types of delays (scheduled or unscheduled) were considered, the system availability would be below 60%. In order to identify the major delay sources, various delays were classified into two categories: inherent delays and face component delays. The inherent delays included external conveyor delays, travel time, and other external delays, while the face component delays were related to the shearer, the face conveyor, supports, pumps, and the stage loader. The shearer, face conveyor, and outby haulage were identified as the major contributors to system delays, accounting for about 85% of the total delays. This analysis cannot be generalized, however, since it is actually a case study of a single mine.

A more comprehensive data collection and delay analysis was made at KETRON, Inc. In an effort to assess the impact of differences in the face conveyor design versus the occurrences of specific delays, Herhal, et al. (1978) collected downtime data on longwall conveyors from a number of U.S. installations and
attempted to develop a database for medium seam conditions. Due to the limited number of sections studied, their effort could not conclusively isolate one or more factors or lead to a conclusion of the impact of the conveyor delays on production. But some valuable results were obtained regarding the effect of the conveyor design on its reliability, such as: multiple-strand chains exhibited a dramatic increase in chain breakage over single-strand chains; the chain reliability was inversely proportional to the number of chains; the flight reliability decreased with increased motor horsepower.

This study on the longwall conveyors was later expanded to include the whole longwall systems (Pimentel, et al., 1981). The database comprised time study data from six different longwall sections for a total of 22 shifts. In addition, 739 foreman reports were analyzed from six sections. The approach followed in this study was to start at the aggregate longwall system level and to stepwise analyze the system and components in more detail. At the system level, a new index, downtime per ton of coal produced, was used as a measure to compare the sensitivity of the shift production to changes in the downtime and changes in the production rate. The importance of the efforts to reduce the downtime in longwall operations was confirmed. In four out of six sections, the shift production was more sensitive to changes in downtime per ton than to changes in production rate. Across all of the mines surveyed, the shearer, the face conveyor, and the outby haulage were identified as the major delay sources. These three contributors accounted for 77% of all delay time on the foreman reports, and 73% in the time study data sets. The downtime behavior of these three equipment components was examined with respect to the frequency of occurrence and the duration of delays. Attempts were made to fit the exponential, the lognormal, the Weibull, and the
Gamma distributions to the observed data, but the results were unacceptable. Various sources of the input data and different methods of data collection were regarded as two major reasons for the poor fitting. The major information drawn from this analysis includes:

- The shearer exhibited the lowest availability of the three major components for system delays. The shearer–involved delays were characterized as being frequent in occurrence and short in duration. Bit change delays were found to be a major contributor to the shearer downtime.

- The face conveyors showed the highest availability of the three. The delays incurred were infrequent in occurrence, but long in duration. Conveyor hang-ups and chain, flight, and sprocket defects were all identified as major downtime sources of face conveyors.

- Most of the shearer and face conveyor delays were of a mechanical or operational type. Both the shearer and the face conveyor motors were identified as important downtime sources.

- Spalling of the face, which could cause rock/coal piling up under shearer and oversized lumps, was another major cause of system delay.

- The outby haulage system was medium in availability. The delays were low to medium in both frequency of occurrence and duration.

In general, two inhibiting factors encountered throughout this analysis made it
difficult to isolate specific problems and speak confidently on the importance of these conclusions to all longwall systems currently in use. They were the limited number of sections analyzed and a "dilution" effect that resulted from combining the data from various mines. Another problem was that the causes of the delays were not given in the established database. This information is essential to reducing longwall failures. Nevertheless, the methodology developed in this study is valuable for longwall production systems analysis, as addressed in Douglas (1980).

To illustrate the application of the reliability approach in analyzing underground coal mining operations, VPI & SU (1982) conducted a preliminary reliability study of a continuous mining system and a longwall system. Fourteen continuous mining sections and three longwall sections were analyzed for a total of about 1,200 continuous mining shifts and 500 longwall shifts. The longwall systems were found to operate only about 47% of the shift time. The failure time and downtime of a longwall system was successfully approximated by the exponential distributions. The plow and the face conveyor were identified as the weakest components in the longwall systems. Compared to the continuous mining systems, the longwall systems had more frequent failures but required less time to repair.

In constructing a simulation model for optimal longwall system design, Lee (1983) developed a computer code for the statistical analysis of longwall delays. The Weibull distribution was recommended to fit the delay duration times and the times to failure. As a result of advances in the computer software industry, however, the problem addressed in Lee's work can now be handled by a number of
commercial statistical packages on the market.

As an effort to apply the methodology of reliability engineering to analysis of longwall delays, Ramani et al. (1989) analyzed a number of cases based on time study data collected from a longwall face. An effort was made to separate the longwall face component delays from the geological and outby haulage delays. Five failure categories were generated. A Weibull distribution was then fitted to each data set. The results indicated that elimination of the geological delays alone could increase the system availability by 12%, and an increase of 23% in availability could be achieved by excluding all of the delays other than equipment failures associated with shearers, face conveyors, and the outby haulage. However, no failure data were given in this study for the roof support and other auxiliary longwall components. Thus, the impact of the shearer, face conveyor, and outby haulage related delays on the system’s availability can not be compared to that of other components. This comparison is required in order to identify the major delay sources and to confirm the results of the study.

The most comprehensive analysis of longwall delays was completed recently at Virginia Polytechnic Institute and State University by Dunlap, et al. (1989 and 1990). In this study, classification and statistical analyses were made of the delay data from a group of thirty-nine longwall sections located in the eastern and mideastern United States. Downtime data corresponding to over 14,000 shifts were classified according to equipment type, delay type, and specific delay event. A number of dBASE IV–based databases were constructed to allow flexible interrogation of the data. The downtime of the various equipment components and of the delay types was determined; machine availabilities and system
availabilities were calculated; and probability distributions were fit to the
time-to-failure and to the time-to-repair data sets, both for the principal
equipment types and for the longwall system as a whole.

In Dunlap's study, detailed availability and downtime analyses were made for
a group of twelve mines. The average face availability observed was 67.5%.
The average available production time per shift was about 420 minutes for these faces.
It was observed that 32.5% of the available production time was lost to equipment
delays; 23% of this lost time was due to the shearer breakdowns and servicing,
while 32% of the equipment delay time was due to the section belt and main
haulage. Reduction of equipment downtime should focus on these units. Inherent
delay was responsible for an average of 60 minutes per shift, or 28% of the total
production shift downtime. The main sources of inherent delay were travel time,
lunch time, safety talks, and face inspection.

Besides extensive research on statistical analyses of system delays, there were
a few theoretical studies conducted for systems analysis of mining operations. To
develop guidelines for estimating the availability of mining systems, and their
associated needs for spares and maintenance personnel, Lohman (1978) developed a
closed network queueing model of underground coal mining production, failure,
and repair. An underground mine was mathematically represented as a collection
of work sections that alternatively required servicing by one production crew and
one repair crew, each drawn from a respective pool of homogeneous crews. This
interaction was solved as a classical finite-state birth-and-death process. These
major conclusions were drawn from the sensitivity analysis of the model:
• The mean availability of a mining section had a theoretical limit of 
MTBF/(MTBF+MTTR), where MTBF is the mean time between failures 
and MTTR is the mean time to repair. This limit would be achieved only 
when there were so many production and repair crews that sections never 
needed to wait for each other. The groups of production crews should be 
0.85 to 1.0 times the number of sections, and the number of repair crews 
should exceed the quotient of the number of production crews divided by 
the maintainability ratio (MTBF/MTTR).

• Given a value for the maintainability ratio, representative of current 
operating experience, the section availability exhibited a steep improvement 
in response to a small amount of increase in MTBF and/or decrease in 
MTTR.

• The sensitivity of production at the mine portal to the availability of any 
link in the outby haulage system was exactly equal to the quantity of coal 
that the link was expected to receive from all sections and haulage links 
feeding into it, multiplying by the availability of all haulage links between 
it and the mine portal.

In two other theoretical studies, the techniques of stochastic systems analysis 
were applied to study serially-connected mining systems like longwall mining. 
Yegulalp (1978) analyzed the relationships between production cost and 
management policies regarding equipment selection, scheduled maintenance and 
repair procedures. The theory of Markov decision processes was employed to 
evaluate the production efficiency of a production and haulage system. The costs
associated with the transitions in a two-state Markov model was estimated. The results revealed certain functional relationships that lead to the formulation of optimum management policies. Pavlovic (1989) made an effort to adapt reliability theory to calculate the capacity of any mining system with continuous production process. Based on the assumption of series configuration, the reliability of a continuous mining system was evaluated. The capacity of the system was then determined according to the reliability parameters obtained. However, the methodology was developed mainly for analyzing open-cast coal mines.

Like the failures and delays during longwall panel extraction, the problem of moving the face equipment from one panel to the next has always been a critical issue to any longwall operation. It is noted that the impact of reducing move time on system production is as important as that of increasing the availability of the shearer. This is especially true when one considers that an already low face availability is, in effect, further reduced by face-to-face moves. The magnitude of this problem has increased with the use of heavier shield supports and larger face lengths.

To alleviate the burden and to increase the overall availability of the longwall systems in production, Pimentel, et al. (1982) made a study of face-to-face moves for the longwall equipment. This study attempted to improve equipment moves in terms of time, safety, and costs by designing systems, methods, and techniques for efficiently disassembling, transporting, and reassembling an installation. After reviewing current U.S. and foreign practices and strategies of face-to-face moves, an industrial engineering study was conducted to analyze four U.S. longwall moves using existing records and on-site time and method studies. The positive and
negative aspects of these moves with respect to time, manpower use, cost, and safety were identified by use of the CPM, PERT, and cost–effective analyses. As a result, a handbook for efficient face–to–face moves was developed (Adam, et al., 1982). This handbook covered various aspects related to face moves, such as mine planning and development, equipment selection, pre–move preparation, dismantling and withdrawal operations, etc.
III. SYSTEM FUNCTIONAL ANALYSIS AND REPRESENTATION

3.1 Introduction

An essential part of system availability analysis is the employment of a functional approach as a basis for the identification and representation of the system's operating logic. The operating logic of a system is characterized by: the component-failure process, the repair or maintenance process, the system configuration, and the state in which the system is defined as failed. In this chapter, after a functional description of longwall mining systems, the production system is decomposed into four functional entities: the coal-cutting subsystem, the face-conveying subsystem, the roof-support subsystem, and the outby-haulage subsystem. The functional relationships among these subsystems are then analyzed to understand how they are functionally connected and how they interact to keep production continuous. After identification of the functions of these subsystems, the system's operating logic is represented by reliability-block diagrams and state-transition diagrams. These results provide a basis for development of availability models in the next chapters.
3.2 System Description

The operation of underground coal mining consists of two kinds of activities: production and logistics. The emphasis of this study will be on the production activities. In longwall mining, production takes place along a long face, blocked out between two sets of entries, or gates (see Fig. 1). Depending on whether the entries are driven simultaneously with or prior to mining the longwall face, advance and retreat mining can be differentiated. The retreat system is used almost exclusively in the U.S., mainly because of safety regulations.

The basic concept of modern longwall mining includes keeping open only a narrow strip along the immediate mining area. Along this narrow strip — longwall face, a coal-winning machine pulls itself back and forth to cut slices of coal, or webs, from the face, and then to dump the mined coal to a continuous transportation system on the face. The roof above this mining area is protected by a system of self-advancing hydraulic supports. As the coal-winning machine mines a strip, the whole system is moved forward, and the roof is allowed to cave behind the face. The weight of the main roof strata is distributed in the solid coal ahead of the face, along the ribs and possibly in the caved gob area.

The production activities of a longwall mining system are divided into two categories: panel development and panel extraction. The activities associated with panel development are almost the same as in continuous mining. Since longwall mining consists essentially of panel extractions, this study is concentrated on the production activities within the panel.
Figure 1. Panel layout and development of a retreat longwall system
(Source: Katen, 1981)
Figure 2. Basic equipment components on a longwall face

(Source: Stefanko, 1984)
The equipment used in longwall mining includes the production equipment on the face and the outby-haulage equipment (see Fig. 2). The face-production equipment consists of three main components: a coal-winning machine, a haulage system, and a roof-support system. These components are integrated to achieve three major functions of longwall mining: coal cutting, coal haulage, and roof support. In addition to the major face equipment, there are several smaller but equally important machines associated with a longwall system. Usually a stage loader is used as an intermediate conveyor between the face conveyor and the panel belt. If necessary, a crusher is used to reduce the size of the mined coal. In some cases, the gate entries may need additional supports. As a result, it is common to find single hydraulic props and cribbing, or timbering, as necessary. Equipment in the gate roads includes hydraulic pumps to maintain the pressure required by roof supports, and electrical devices to drive and control the operations on the longwall face.

3.3 System Decomposition and Functional Analysis

In analyzing the major mining activities at a longwall mining section, the production system can be decomposed into four entities: the coal-cutting subsystem, the face-conveying subsystem, the roof-support subsystem, and the outby-haulage subsystem. These four subsystems are of primary interest in this study. Their composition, functions, and operational characteristics will be examined below. Figure 3 illustrates the functional flows for these subsystems. Other auxiliary subsystems in the longwall system will not be emphasized for either of the following reasons: (1) their effect on the production process of the
Figure 3. Functional flow of longwall mining systems
overall system is indirect; or (2) their share in system downtime is almost negligible with respect to the four subsystems given above. The ventilation subsystem and the electric—distribution subsystem are two examples, each standing for either case, respectively. With the electric—distribution subsystem, as indicated by Hassan (1981), the total downtime for underground coal mining faces averages only 12.29 hours per year.

3.3.1 The coal—cutting subsystem

The coal—cutting subsystem is mainly composed of a coal—winning machine, either a shearer or a plough. The shearer will be of primary consideration in this study because of its popularity. The basic shearing machine consists of an electric motor, cutting drums and bits, a hydraulic haulage gearcase, and a cutter drum—gearcase, all mounted on a steel bedplate. Shearers can be classified as single or double drum, according to the number of cutting drums, or as ranging arm or gearhead, depending on whether the drum is fixed.

Riding upon an armored face conveyor, the shearer moves back and forth along the longwall face to cut and then load coal. For its propulsion, either chain haulage or various chainless haulage systems are available. The chainless haulage systems in the U.S. are currently of either the chain—and—track or wheel—and—rack type. The shearer unit operations, however, are independent of the type of haulage. The cyclic activities of the shearer include sumping, coal cutting, tramming without cutting, and reversing direction.
3.3.2 The face-conveying subsystem

The major component of this subsystem is an armored face conveyor (AFC). The components of the AFC include line pans, chains, flight bars, drivehead, gearboxes, and accessories. The conveyor serves to transport the mined coal to the headgate, as a track for the shearer or plow, and as an anchoring point for advancement of the roof supports. Hence, the face-conveying subsystem provides not only the transportation of coal from the face to the headgate, but also a linkage between all of the independent face equipment to form a cohesive mechanical system. The activities of a face-conveying subsystem fall into three states:

- **Operating** — the conveying subsystem is up and running
- **Down** — the conveying equipment itself is down
- **Ready** — the conveying subsystem is capable of operating, but is off due to some other system delays

3.3.3 The outby-haulage subsystem

The outby-haulage subsystem includes a stage loader/breaker and a section—belt conveyor or a railway. It serves to transfer the mined coal from the longwall face to a main haulageway. The stage loader, which is a short chain conveyor similar to the AFC in structure, receives the coal from the delivery end of the AFC and transfers it to the section—belt conveyor. The mined coal is then transferred to the main haulageway.
3.3.4 The roof-support subsystem

The roof-support subsystem consists of a number of powered supports, and a power pack (a pump unit and a sealed tank), which supplies the high- or low-pressure fluid to the hydraulic jacks. Generally, a powered support has four major elements: canopy, caving shield, hydraulic legs, and base plate. The functions of the roof-support subsystem include: holding up the roof, pushing the face-chain conveyor, and providing a safe environment for all associated mining activities.

The basic unit operations of the roof supports are: supporting the roof, supporting the roof and snaking the conveyor, lowering and advancing the support, and resetting the support. The actual sequence of these operations depends on the types of supports and the operating system used. In the Immediate Forward Support (IFS) or One Web system, the supports are advanced before the conveyor is snaked. Delays can be caused in the unit operations by equipment failures, such as failures of the many hydraulic parts, and by adverse environmental conditions. According to their effect on the performance of the overall production process, the delays occurring with roof supports can be classified into two groups:

- **Interruptive delays** — the delays resulting in shutdown of the production process of the overall system;
- **Non-interruptive delays** — the delays which either have no immediate effect on the production flow of the system, or affect system performance to a certain extent, but do not shut down production.
3.3.5 Discussions

Longwall mining is an integrated production system. The major mining activities in the system are cyclic and the system is repairable. Under given geological conditions and equal management and work force capabilities, the productivity of a longwall face is mainly determined by: system availability, the capacity of coal cutting and conveying machines, and the panel dimensions, particularly the face length. Compared to other coal-mining systems underground, mechanized longwall mining has two unique features in system dynamics:

- The self-advancing mechanism
- Continuous production/hauling

In order to implement these two functions, the major equipment components of the coal-cutting, the face-conveying, the roof-support, and the outby- haulage subsystems are closely interconnected. The cutting equipment is usually supported by the haulage equipment. Both the cutting machine and the haulage equipment are advanced by the roof-support equipment, while the roof-support equipment uses the haulage equipment to advance itself.

From both a mining and an industrial engineering standpoint, these features of longwall systems provide important advantages in system operation and management. The production process is completely continuous and can be fully mechanized, and even automated. Since longwall operations are highly centralized, the communication, the supervision, and the system measurement and analysis in longwall operations are facilitated. As a result, longwall mining
provides the potential for high productivity. However, the close interdependence and the lack of redundancy among major face equipment bring about a significant weak point in longwall operations. As a result of this interdependence, the system downtime can result from any failure not only in the production equipment, but also in the haulage equipment, and in some cases, even from failures in the roof-support equipment. The output of longwall systems, therefore, depends both on the individual performance of the equipment components and on their operational interactions. To a great extent, these functional relationships among major face equipment explain the reason for the vulnerability of longwall operations to various delays in production.

3.4 Representation of the System Operating Logic

As specified in the last section, the basic system under this study is composed of four subsystems: coal-cutting, face-conveying, roof-support, and outby-haulage. It is initially operable, and operates continuously for some period of time. At any time the system is either operating or failed. Repaired components in the system are assumed to function like new components. According to the functional analyses given above, this system's operating logic can be further represented by reliability-block diagrams or state-transition diagrams. These representations provide a basis for development of system availability models and make it possible to evaluate the system's availability quantitatively.
3.4.1 Critical components vs. noncritical components

Depending on their effect on system performance, the components in the system can be differentiated as critical or noncritical. The critical components are those for which a single failure renders the system inoperative. As indicated by the component functional analyses above, the coal-cutting subsystem, the face-conveying subsystem, and the outby-haulage subsystem belong to this category. Whether these subsystems function successfully has a direct effect on system operation. The failure of any equipment in these subsystems can shut down the production process of the overall system. During the mission of the longwall system, these critical components are either operating or failed. Thus, the operating status of critical components can be represented by two states: operating and failed. The overall system fails if any of the critical components is in the failed state.

In contrast to those critical components, the operating status of individual roof supports does not necessarily have a direct and immediate effect on the system's production sequence. Some support failures can force production to halt, while other failures may let the system function normally while repair actions take place simultaneously. Still other cases may occur between these two extremes. For example, when some of the supports are not functioning, the shearer may have to reduce its tramming speed in order to meet ground-control requirements. As a result, delays are incurred in the system production process, and the overall system is operating at a degraded level of performance. Thus, the roof-support subsystem is critical only when supports fail in certain numbers, or under certain situations.
The effect of the roof-support subsystem as a noncritical component on the performance of the overall system can be modeled in two ways. One is to use the concept of three-state components; the other is to consider the subsystem as a k-out-of-n structure.

**Three-state component:** Unlike the case of critical components, the operating status of the roof-support subsystem may not be represented simply as either operating or failed. For realistic modeling of the functions of the roof supports in the context of the overall system, this subsystem can be assumed to stay in one of the three states below:

- *Operating* — All supports are functioning.
- *Partially operating* — The failure of the roof supports makes the overall system operate at a degraded level of performance.
- *Failed* — The failure of the roof supports shuts down the overall production process completely.

Given these definitions, it is possible to evaluate the performance of the roof-support subsystem by using any techniques applied to critical components. This method of representation is preferred and will be used in this study.

**k-out-of-n structure:** Another way to assess the effect of the roof-support subsystem on the performance of the overall system is to represent the subsystem as a k-out-of-n structure. That is, composed of n identical supports (statistically), the subsystem is regarded as operable if and only if at least k out of n units function. Thus, the failure of a single support, or even a few supports in
this structure, may not hinder the successful operation of the longwall system. But when more than \((n-k)\) units are not functioning, the subsystem is considered as failed. In this case, the overall system is regarded as failed due to the failure of the roof–support subsystem.

### 3.4.2 Reliability-block diagram representation

The functional relationships of critical components in the system can be represented by a series configuration. Namely, the functioning of the system depends on the proper operation of all critical components, and the failure of any critical component can result in system failure. But for the roof–support subsystem, it is neither in series nor in parallel redundancy with any of these three critical components. Nevertheless, with either representation for its operating and failure characteristics as given in Section 3.4.1, the roof–support subsystem can still be regarded as the fourth component in this series structure. The only difference between the roof–support subsystem and those critical components is in how they are defined as failed. As an example, the operating logic of the system can be represented by Fig. 4, where components 1, 2, 3, and 4 denote the coal-cutting, the face–conveying, the outby–haulage, and the roof–support subsystems, respectively.

### 3.4.3 State-transition-diagram representation

If the components of a system are interdependent and closely interactive, the state–transition diagram is more appropriate in representing the system's operating logic. A state–transition diagram depicts topologically the
Figure 4. Reliability—block diagram of the longwall system
state-to-state transitions of the system. The nodes in this diagram represent various system states, and the rays between the nodes correspond to the interstate transitions. Each ray is valued by the transition rate from one state to another. When the transition rates are constant, the system is Markovian, and the state-transition diagram is often called a Markov diagram. The actual construction of state-transition diagrams for a system depends on: the system configuration, the component failure and repair process, and the definition of system states. In the next chapter, state-transition diagrams will be used as a major tool for formulation of availability models.
IV. MODEL DEFINITION, FORMULATION AND SOLUTION

4.1 Introduction

As defined in the last chapter, the basic system under this study is composed of four subsystems: coal-winning, face-conveying, outby-haulage, and roof-support. It is initially operable and operates continuously for some period of time. At any given time, the system is either operating or failed. Repaired components in the system are assumed to function like new components. In order to study the availability of this system under different operating logic, five probability models are defined by use of the system representations given above (see Table 1). These models are formulated and then solved by either of these approaches: direct availability and reliability assessment or Markov modeling.

4.2 Methodologies

With respect to different ways of system abstraction and representation, two techniques are used for development of availability models: direct availability and reliability assessment, and Markov modeling. The rationale behind selecting these approaches stems from two aspects:
Table 1. Model Definition

1. Series—Configuration Systems:
   - Model A  Independent component failure—repair processes
   - Model B  Suspended animation

2. Systems subject to Partial Failures of the Roof—Supports:
   - Model C  Multiple repair policy
   - Model D  First-Come—First-Serve repair policy
   - Model E  Priority repair policy
• The system's operating logic, including the system structure, the failure-repair characteristics of the equipment in the system, and the functional relationships between the equipment components, can be well described by such analytical approaches.

• The models developed by use of these techniques are, in general, mathematically tractable enough to represent either transient or steady-state behavior of the system.

4.2.1 Direct availability and reliability assessment

The direct assessment techniques are based on a series of mathematical theories of reliability analysis. This approach often assumes a coherent system with statistically independent components and a well-defined structure. It begins with construction of a reliability-block diagram to represent the operating logic of the system. The reliability-block diagram is a circuitless diagram, with an input and output, whose blocks represent the components of the system. The connections between the blocks describe the relationships between the various components. The system operates if there exists a successful path between the components. Thus, by applying the theories of reliability assessment, the availability of the system can be evaluated directly based on the system's structure and its component failure and repair characteristics.

4.2.2 Markov modeling

In practice a longwall mining system could have a very complex operating
logic and dynamic characteristics. For example, some of the components in the system may be greatly interdependent. The characteristic parameters of the system, particularly the occurrence of failures and the duration of downtimes, vary randomly. In order to consider the dependencies between the various components in such a longwall system, Markov modeling can be used along with the direct-assessment techniques.

Among various methods of stochastic systems analysis, Markov modeling is probably one of the most important approaches. The Markov modeling method assumes a sequential process with no memory of prior events (known as Markovian property). In applying the Markovian approach for system availability analysis, exponential distributions are often assumed for failure times and repair times.

A Markov model is constructed by identifying the modes of component operations in a system with the states in a Markov chain. After all of the mutually exclusive states of the system are defined, the transitions between these system states are depicted topologically with a state-transition diagram. Markovian state equations are then developed to describe the probabilistic transitions from initial states to final states. Thus, the probabilities for the system in various states are obtained by solving these state equations. The system's availability can then be derived from the probabilities of the system in those operating states.
4.3 Series—Configuration Systems

The functioning of the longwall system depends on the proper operation of each component. As noted in Section 3.4.1, the subsystems in a longwall system may have quite different effects on system availability. If these subsystems are either operating or failed in any given time, and the failure of any of them can result in system failure, the system can then be described by a pure series structure. Two models are developed below to evaluate the availability of such a system. For convenience in model formulation, the functions and parameters for the system are denoted by subscript $s$, while subscript $i$ is used for components.

4.3.1 Model 1 — independent component failure-repair processes

This model represents the system as a series structure. The failures and repairs of the components in the system are assumed mutually statistically independent ($s$—independent). While a failed component is being repaired, other components in the system remain in operation. Thus, the failure rates of the nonfailed components are the same as when the system is operating.

The direct assessment techniques are used to develop this model. The operating logic of this system is illustrated by Fig. 4 on page 37. Since the failures and repairs of the components in the system are $s$—independent, the system's reliability $R_s(t)$ and availability $A_s(t)$ must be the product of the component reliabilities and the product of the component availabilities. That is,
\[ R_s(t) = \prod_{i=1}^{4} R_i(t), \] (4.1)

\[ A_s(t) = \prod_{i=1}^{4} A_i(t). \] (4.2)

The system failure rate is the sum of the individual failure rates, given by

\[ \lambda_s(t) = \sum_{i=1}^{4} \lambda_i(t). \] (4.3)

The reliability and failure rate for any component have the following relationships with the probability distribution function of its times to failure, \( F_i(t) \) (see Appendix B):

\[ R_i(t) = 1 - F_i(t), \]

\[ \lambda_i(t) = \frac{dF_i(t)}{dt} \cdot \frac{1}{1 - F_i(t)}. \]

If the system's reliability function is known, the mean time to failure of the system can be obtained by

\[ MTTF_s = \int_0^\infty R_s(t) \, dt. \] (4.4)

For any component \( i \), let the probability density functions of its times to failure (TTF) and its times to repair (TTR) be \( f_i(t) \) and \( g_i(t) \), respectively. If the
expected values of these times are finite,

$$\nu_i = \int_0^\infty t \times f_i(t) dt,$$  
(4.5)

$$\tau_i = \int_0^\infty t \times g_i(t) dt,$$  
(4.6)

then the system's availability and MTTF$_s$ have the following asymptotic behaviors:

$$A_s = \lim_{t \to \infty} A_s(t) = \prod_{i=1}^4 \left[ \frac{\nu_i}{\tau_i + \nu_i} \right],$$  
(4.7)

$$\text{MTTF}_s = \left[ \frac{\prod_{i=1}^4 \frac{1}{\nu_i}}{\sum_{i=1}^4 \frac{1}{\nu_i}} \right]^{-1}.$$  
(4.8)

When $\tau_i/\nu_i \ll 1$, the system's availability can be approximated by

$$A_s = 1 - \sum_{i=1}^4 \frac{\tau_i}{\nu_i}.$$  
(4.9)

For any system, since the following relationship exists between its steady-state availability $A_s$, mean time to failure MTTF$_s$, and mean time to repair MTTR$_s$,

$$A_s = \frac{\text{MTTF}_s}{\text{MTTF}_s + \text{MTTR}_s},$$  
(4.10)
the system's mean time to repair is obtained as

\[ \text{MTTR}_s = \text{MTTF}_s \left[ \frac{1}{A_s} - 1 \right] \]

\[ = \left[ \prod_{i=1}^{4} \left( \frac{\tau_i}{\nu_i} + 1 \right) - 1 \right] \left( \sum_{i=1}^{4} \frac{1}{\nu_i} \right)^{-1}. \tag{4.11} \]

According to Barlow and Proschan (1973), the long-run expected average of the number of system failures is given by

\[ N_t = A_s / \text{MTTF}_s \]

\[ = \left[ \prod_{i=1}^{4} \left( \frac{1}{\nu_i} \right) \right] \left[ \prod_{i=1}^{4} \frac{\nu_i}{\tau_i + \nu_i} \right]. \tag{4.12} \]

As an example, consider that all of the components in the longwall system have exponentially-distributed TTF's and TTR's, that is, the failure rates and repair rates of the components are constant. If the failure rate and the repair rate of component \( i \) are \( \lambda_i \) and \( \mu_i \), respectively, the following results can be obtained:

\[ R_s(t) = \exp \left[ -\sum_{i=1}^{4} (\lambda_i)t \right], \tag{4.13} \]

\[ A_s(t) = \prod_{i=1}^{4} \left[ \frac{\mu_i}{\lambda_i + \mu_i} + \frac{\lambda_i}{\lambda_i + \mu_i} e^{- (\lambda_i + \mu_i)t} \right]. \tag{4.14} \]
\[ A_s = \prod_{i=1}^{4} \left[ \frac{\mu_i}{\lambda_i + \mu_i} \right], \quad (4.15) \]

\[ \lambda_s = \sum_{i=1}^{4} \lambda_i, \quad (4.16) \]

\[ \text{MTTF}_s = \left( \sum_{i=1}^{4} \lambda_i \right)^{-1}, \quad (4.17) \]

\[ \text{MTTR}_s = \left( \prod_{i=1}^{4} \left[ \frac{\lambda_i}{\mu_i} + 1 \right] - 1 \right) \left( \sum_{i=1}^{4} \lambda_i \right)^{-1}, \quad (4.18) \]

\[ N_f = \left( \sum_{i=1}^{4} \lambda_i \right)^{-1} \prod_{i=1}^{4} \left[ \frac{\mu_i}{\lambda_i + \mu_i} \right]. \quad (4.19) \]

When a system is failed, the limiting probability that the system breakdown is due to the failure of component \( i \) can be estimated by

\[ P_i = (1 - A_s) \left[ \frac{\lambda_i}{\lambda_s} \right]. \quad (4.20) \]

### 4.3.2 Model B — Suspended animation

This model represents the same type of system as in Model A, except that no components fail while the system is de-energized. Thus, other units in the system are suspended during repair of a failed component. Such suspension is often called suspended animation. This model is developed by a combination of the direct assessment techniques and Markov modeling.
The system's operating logic can be illustrated by a state-transition diagram (see Fig. 5). In this diagram, the state in which the system is operating is defined as State 0, and the state in which the system is down due to the failure of component \( i \) is defined as State \( i \). Because of the characteristics of series-configuration systems, the only transitions possible in the interval \((t, t+\Delta t)\) are \( i \to 0 \), or \( 0 \to i \) for \( i = 1, \ldots, 4 \).

Because of the series configuration, the reliability, failure rate, and \( \text{MTTF}_s \) of the system are the same as those derived in Model A; they are:

\[
R_s(t) = \prod_{i=1}^{4} R_i(t), \quad (4.21)
\]

\[
\lambda_s(t) = \sum_{i=1}^{4} \lambda_i(t), \quad (4.22)
\]

\[
\text{MTTF}_s = \left[ \sum_{i=1}^{4} \frac{1}{\lambda_i} \right]^{-1}. \quad (4.23)
\]

For any arbitrary component failure-time distributions with mean \( \nu_i \), and any component repair-time distributions with mean \( \tau_i \), the following limiting results can be obtained for the system:

\[
A_s = \left[ 1 + \sum_{i=1}^{4} \frac{\tau_i}{\nu_i} \right]^{-1}, \quad (4.24)
\]
Figure 5. State-transition diagram of the system in Model B
\[ MTTR_s = \left[ \sum_{i=1}^{4} \frac{\tau_i}{\nu_i} \right] \left[ \sum_{i=1}^{4} \frac{1}{\nu_i} \right]^{-1}, \]  

(4.25)

\[ N_t = \left[ \sum_{i=1}^{4} \frac{1}{\nu_i} \right] \left[ 1 + \sum_{i=1}^{4} \frac{\tau_i}{\nu_i} \right]^{-1}. \]  

(4.26)

When a system is failed, the probability that the system breakdown is due to the failure of component \( i \) is \( P_i(t) \), which has a limiting value

\[ P_i = \left[ \frac{\tau_i}{\nu_i} \right] \left[ 1 + \sum_{i=1}^{4} \frac{\tau_i}{\nu_i} \right]^{-1}. \]  

(4.27)

As given in Equation (4.25), the mean time to repair of the system is actually the weighted mean of the component repair times \( \tau_i \), with the weighted factors being the ratio of component failure rates to the total failure rate. Thus, if each component’s MTTR is multiplied by its expected fraction of occurrence and the resulting values summed, then the expected system repair time is obtained.

As a special case, if the failure times and repair times of all the components are exponentially distributed with the failure rates and the repair rates given by \( \lambda_i \) and \( \mu_i \), respectively, the system’s availability function can be developed through a Markovian model below.

Let \( P_i(t) \) denote the probability that the system is in state \( i \) at time \( t \), for \( i = 0, \ldots, 4 \), then
\[ P_0(0) = 1, \quad P_i(0) = 0, \quad \text{and} \quad \sum_{i=0}^{4} P_i(t) = 1. \] (4.28)

During a small interval of time \( \Delta t \), the probability of system failure is given by
\[ \sum_{i=1}^{4} \lambda_i \Delta t, \quad \text{and the probability that a repair is completed is} \quad \sum_{i=1}^{4} P_i(t) \mu_i \Delta t, \quad \text{therefore}, \]

\[ P_0(t+\Delta t) = P_0(t) \left[ 1 - \sum_{i=1}^{4} \lambda_i \Delta t \right] + \sum_{i=1}^{4} P_i(t) \mu_i \Delta t + o(\Delta t), \]

and so the derivative of \( P_0(t) \), \( P_0'(t) \) can be expressed as:
\[ P_0'(t) = -\sum_{i=1}^{4} \lambda_i P_0(t) + \sum_{i=1}^{4} \mu_i P_i(t). \] (4.29)

Similarly, for \( i = 1, \ldots, 4 \),

\[ P_i(t+\Delta t) = P_0(t) \lambda_i \Delta t + P_i(t)(1-\mu_i \Delta t) + o(\Delta t) \]

\[ P_i'(t) = -\mu_i P_i(t) + \lambda_i P_0(t). \] (4.30)

Equations (4.29) and (4.30) can be expressed in a matrix form,
\[ \frac{d}{dt}(P) = A \times P \] (4.31)

where \( P \) is the transpose of vector \([P_0(t), P_1(t), P_2(t), P_3(t), P_4(t)]\), and \( A \) is a transition matrix, given by
\[
A = \begin{bmatrix}
\lambda_1 & \mu_1 & \mu_2 & \mu_3 & \mu_4 \\
-\Sigma \lambda_i & & & & \\
\lambda_1 & -\mu_1 & 0 & 0 & 0 \\
\lambda_2 & 0 & -\mu_2 & 0 & 0 \\
\lambda_3 & 0 & 0 & -\mu_3 & 0 \\
\lambda_4 & 0 & 0 & 0 & -\mu_4
\end{bmatrix}
\]

By implementing a Laplace transform of both sides of (4.29), we obtain

\[
s P_0'(s) - 1 = -\Sigma \lambda_i P_0'(s) + \Sigma \mu_i P_i'(s), \quad (4.32)
\]

and from (4.30),

\[
s P_i'(s) = -\mu_i P_i'(s) + \lambda_i P_0'(s) \quad (4.33)
\]

and equivalently, for \( i = 1, \ldots, 4, \)

\[
P_i'(s) = \left[ \frac{\lambda_i}{s + \mu_i} \right] P_0'(s). \quad (4.34)
\]

Substituting (4.34) into (4.32) we obtain

\[
s P_0'(s) - 1 = -\Sigma \lambda_i P_0'(s) + \Sigma \left[ \frac{\mu_i \lambda_i}{s + \mu_i} \right] P_0'(s)
\]
\[ P'_0(s) = s \left[ 1 + \sum_{i=1}^{4} \frac{\lambda_i}{s + \mu_i} \right]^{-1}. \]  \hspace{1cm} (4.35)

Expanding equation (4.35) by the method of partial fractions, one can then compute the inverse transform of \( P'_0(s) \), that is, \( P_0(t) \). The instantaneous system availability, \( A_s(t) \), is clearly given by \( P_0(t) \), since State \( 0 \) is the only system operating state. The steady-state system availability is given by

\[ A_s = \lim_{t \to \infty} P_0(t) = \lim_{s \to 0} s P'_0(s) \]

\[ = \left[ 1 + \sum_{i=1}^{4} \frac{\lambda_i}{\mu_i} \right]^{-1} \hspace{1cm} (4.36) \]

The limiting probability that the system is down due to the failure of component \( i \) is given by

\[ P_i = \lim_{t \to \infty} P_i(t) = \lim_{s \to 0} s P'_i(s) \]

\[ = \frac{\lambda_i}{\mu_i} \left[ 1 + \sum_{i=1}^{4} \frac{\lambda_i}{\mu_i} \right]^{-1} \hspace{1cm} (4.37) \]

The long-run expected average number of system failures, \( N_f \), is obtained as
\[ N_f = \left[ \sum_{i=1}^{4} \frac{\lambda_i}{\lambda_i} \right] \left[ 1 + \sum_{i=1}^{4} \frac{\lambda_i}{\mu_i} \right]^{-1}. \] (4.38)

### 4.4 Systems Subject to Partial Failures of the Roof Supports

In Models A and B all components in the system are treated equally. Each of them is considered to be critical to the operation of the system, and a single failure of any component renders the system inoperative. In practice, however, these components can have quite different effects on system performance. For realistic modeling of the functions of the roof supports in the context of the overall system, the roof-support subsystem can be regarded as a three-state component. During its mission, this subsystem stays in one of the three states: normally operating, partially operating, and failed.

In the following, three models will be developed by applying the concept of three-state components to the roof-support subsystem. The method of representation given in Section 3.4.1 will be used for the roof-support subsystem, while the specifications given in Model B apply to the other three subsystems. Markov modeling is employed to formulate these models by assuming constant component failure rates and repair rates.

For the system defined, all of the possible states in which the system may stay are given in Table 2. According to this definition, the system operates fully in
Table 2. System Transition States in Models C, D and E

<table>
<thead>
<tr>
<th>State</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>All subsystems are operating.</td>
</tr>
<tr>
<td>1</td>
<td>The coal—cutting subsystem fails.</td>
</tr>
<tr>
<td>2</td>
<td>The face—conveying subsystem fails.</td>
</tr>
<tr>
<td>3</td>
<td>The outby haulage subsystem fails.</td>
</tr>
<tr>
<td>4</td>
<td>The roof—support subsystem is partially operating.</td>
</tr>
<tr>
<td>5</td>
<td>The roof—support subsystem is completely down.</td>
</tr>
<tr>
<td>6</td>
<td>When the roof—support subsystem is partially operating, the coal—cutting subsystem fails.</td>
</tr>
<tr>
<td>7</td>
<td>When the roof—support subsystem is partially operating, the face—conveying subsystem fails.</td>
</tr>
<tr>
<td>8</td>
<td>When the roof—support subsystem is partially operating, the outby—haulage subsystem fails.</td>
</tr>
</tbody>
</table>
State 0, at a degraded level of performance in State 4, and is down in other states. The availability of the system can then be expressed as

\[ A_d(t) = P_0(t) + \delta P_4(t), \]  

where \( 0 < \delta < 1 \). A factor \( \delta \) is introduced in Equation (4.39) in order to realistically represent the system performance in State 4. \( \delta \) is actually a factor by which the system performance is derated if the system is in State 4, compared to that in State 0. The initial condition for these models is

\[ P_0(0) = 1, \quad \text{and} \quad P_i(0) = 0 \quad \text{for } i \neq 0. \]

\[ (4.40) \]

### 4.4.1 The availability and reliability of the roof-support subsystem

As discussed in Section 3.4.1, the effect of the roof-support subsystem on system availability can be modeled by a three-state Markov model. This model represents a situation where when some of the supports fail, the overall system operates partially, and if a catastrophic failure occurs the system fails. When the roof-support subsystem is operating partially, a repair process is initiated to put back the system to its fully operational state. Once the system is failed, it is repaired back to its normal operating state.

According to the model definitions above, the state-transition diagram for the roof-support subsystem is shown in Fig. 6. Given the interstate transition rates \( \lambda_i \)'s and \( \mu_i \)'s, the availability of the subsystem can be obtained by Markov
Figure 6. State-transition diagram of the roof-support subsystem
modeling. The Markov state equations are given below:

\[ P_0'(t) = -(\lambda_4 + \lambda_5)P_0(t) + \mu_4P_4(t) + \mu_5P_5(t), \]

\[ P_4'(t) = \lambda_4P_0(t) - (\mu_4 + \lambda_6)P_4(t), \]

\[ P_5'(t) = \lambda_5P_0(t) + \lambda_6P_4(t) - \mu_5P_5(t). \]

Let \( P_0(0) = 1, P_4(0) = 0, \) and \( P_5(0) = 0, \) the Laplace transforms of these state probabilities are

\[ P_0(s) = \left[ (s + \lambda_4 + \lambda_5) - \left[ \mu_4 + \frac{\mu_5 \lambda_6}{s + \mu_5} \right] \left[ \frac{\lambda_4}{s + \mu_4 + \lambda_6} \right] \right]^{-1}, \quad (4.41) \]

\[ P_4(s) = \left[ \frac{\lambda_4}{s + \mu_4 + \lambda_6} \right] P_0(s), \quad (4.42) \]

\[ P_5(s) = \left[ \frac{\lambda_5}{s + \mu_5} \right] P_0(s) + \left[ \frac{\lambda_6}{s + \mu_5} \right] P_4(s). \quad (4.43) \]

Expanding Equations (4.41), (4.42), and (4.43) will give state probabilities \( P_0(t), \)

\( P_4(t), \) and \( P_5(t). \) As a function of \( P_0(t) \) and \( P_4(t), \) the availability of the roof-support subsystem can be determined accordingly.

In Figure 6, by letting \( \mu_5 = 0, \) and \( P_1(0) = 1, P_4(0) = P_5(0) = 0, \) the reliability of the component can be found in a similar way. The Laplace transforms of \( P_0(t) \) and \( P_4(t) \) are given by
\[ P_0(s) = \left( s + \lambda_4 + \lambda_5 + \frac{\lambda_4 \mu_4}{s + \mu_4 + \lambda_6} \right)^{-1}, \]  \hspace{1cm} (4.44)

\[ P_4(s) = \left( \frac{\lambda_4}{s + \mu_4 + \lambda_6} \right) P_0(s). \]  \hspace{1cm} (4.45)

If \( R(t) = P_0(t) + P_4(t) \), then

\[ \text{MTTF} = \lim_{s \to 0} R(s) = \lim_{s \to 0} \left[ P_0(s) + P_4(s) \right] \]

\[ = \frac{\lambda_4 + \lambda_6 + \mu_4}{(\lambda_4 + \lambda_5) \lambda_6 + \lambda_5 \mu_4}. \]  \hspace{1cm} (4.46)

If \( R(t) = P_0(t) + \delta P_4(t) \), then

\[ \text{MTTF} = \lim_{s \to 0} \left[ P_0(s) + \delta P_4(s) \right] \]

\[ = \left[ 1 + \frac{\delta \lambda_4}{\mu_4 + \lambda_6} \right] \left[ \frac{\mu_4 + \lambda_6}{\lambda_4 \lambda_6 + \lambda_5 \mu_4 + \lambda_5 \lambda_6} \right]. \]  \hspace{1cm} (4.47)

In these equations, \( \lambda_4 \) and \( \lambda_5 \) represent the failure rate from the normally operating state to the partially operating state or to the failed state, respectively; and \( \lambda_6 \) is the transition rate from the partially operating state to the failed state. In practice, the data for these transition rates are not reported separately in many data collections, such as foreman production records. In such cases, however, the ratios between these parameters can be estimated according to the operator's experience. For example, say \( \lambda_5/\lambda_4 = a \), and \( \lambda_6/\lambda_4 = \beta \). Since MTTF, \( \mu_4 \), and \( \mu_5 \)
are easy to collect, the values of $\lambda_4$, $\lambda_5$, and $\lambda_6$ can be estimated based on the relationships given in Equation (4.46) or (4.47).

### 4.4.2 Model C — the multiple repair policy

This model assumes that the longwall system is served by the multiple repair policy. This implies that multiple repair facilities and groups of crew are available for equipment repairs. Thus, each component failure can be handled independently.

According to the state definitions given in Table 2 on page 55, the state–transition diagram of the system can be constructed (Fig. 7). Given the interstate transition rates, $\lambda_j$'s and $\mu_j$'s, the corresponding Markovian state equations are

\[
P'_0(t) = \left[ \sum_{i=1}^{5} \lambda_i \right] P_0(t) + \sum_{i=1}^{5} \mu_i P_i(t),
\]

\[
P'_1(t) = \lambda_1 P_0(t) - \mu_1 P_1(t) + \mu_4 P_6(t),
\]

\[
P'_2(t) = \lambda_2 P_0(t) - \mu_2 P_2(t) + \mu_4 P_7(t),
\]

\[
P'_3(t) = \lambda_3 P_0(t) - \mu_3 P_3(t) + \mu_4 P_8(t),
\]

\[
P'_4(t) = \lambda_4 P_0(t) - (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_6 + \mu_4) P_1(t) + \mu_1 P_6(t) + \mu_2 P_7(t) + \mu_3 P_8(t),
\]
Figure 7. State-transition diagram of the System in Model C
\[ P_0'(t) = \lambda_0 P_0(t) + \lambda_6 P_4(t) - \mu_5 P_5(t), \]
\[ P_6'(t) = \lambda_1 P_4(t) - (\mu_1 + \mu_4) P_6(t), \]
\[ P_7'(t) = \lambda_2 P_4(t) - (\mu_2 + \mu_4) P_7(t), \]
\[ P_8'(t) = \lambda_3 P_4(t) - (\mu_3 + \mu_4) P_8(t). \]

This system of equations can be represented in a matrix form below:

\[
\frac{d}{dt}(P) = A \times P, \tag{4.48}
\]

where \( P = [P_0(t), P_1(t), P_2(t), \cdots, P_8(t)]^T \), and \( A \) is a transition matrix, given by

\[
\begin{bmatrix}
-\Sigma_1 & \mu_1 & \mu_2 & \mu_3 & \mu_4 & \mu_5 & 0 & 0 & 0 \\
\lambda_1 & -\mu_1 & 0 & 0 & 0 & 0 & \mu_4 & 0 & 0 \\
\lambda_2 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & \mu_4 & 0 \\
\lambda_3 & 0 & 0 & -\mu_3 & 0 & 0 & 0 & 0 & \mu_4 \\
\lambda_4 & 0 & 0 & 0 & -\Sigma_2 & 0 & \mu_1 & \mu_2 & \mu_3 \\
\lambda_5 & 0 & 0 & 0 & \lambda_6 & -\mu_5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_1 & 0 & -(\mu_4 + \mu_1) & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & -(\mu_4 + \mu_2) & 0 \\
0 & 0 & 0 & 0 & \lambda_3 & 0 & 0 & 0 & -(\mu_4 + \mu_3)
\end{bmatrix},
\]

where \( \Sigma_1 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 \), and \( \Sigma_2 = \mu_4 + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_6 \).
The state probabilities $P_i(t)$, for $i = 0, ..., 8$, can be obtained by solving Equation (4.48) under initial condition (4.40). The system availability is then given by Equation (4.39), that is:

$$A_s(t) = P_0(t) + \delta P_4(t).$$

### 4.4.3 Model D — the First-Come-First-Serve repair policy

Independent repair actions are assumed in Model C for all components in the longwall system. This maintenance policy requires that in case of multiple failures occurring during system operation, each equipment component should be served independently. Models D and E study the availability of the system if there is only one set of repair crew and facilities available. Thus, only one failed component can be served each time. The First-Come-First-Serve repair policy (FCFS) is investigated in Model D.

Based upon the system state definitions in Table 2 on page 55, the possible transitions between the states for this model are illustrated in Fig. 8. In States 6, 7 and 8, the coal-cutting, the face-conveying, or the outby-haulage subsystem fails while the roof-support subsystem is partially failed. Since the roof-support subsystem fails before the other subsystems in these cases, it is repaired first according to the FCFS repair policy.

The Markov equations for this problem have the same form as Equation (4.48), but with the transition matrix $A$ given below (page 65):
Figure 8. State-transition diagram of the System in Model D
\[
\begin{bmatrix}
-\Sigma_1 & \mu_1 & \mu_2 & \mu_3 & \mu_4 & \mu_5 & 0 & 0 & 0 \\
\lambda_1 & -\mu_1 & 0 & 0 & 0 & 0 & \mu_4 & 0 & 0 \\
\lambda_2 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & \mu_4 & 0 \\
\lambda_3 & 0 & 0 & -\mu_3 & 0 & 0 & 0 & 0 & \mu_4 \\
\lambda_4 & 0 & 0 & 0 & -\Sigma_2 & 0 & 0 & 0 & 0 \\
\lambda_5 & 0 & 0 & 0 & \lambda_6 & -\mu_5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_1 & 0 & -(\mu_4 + \mu_1) & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & -(\mu_4 + \mu_2) & 0 \\
0 & 0 & 0 & 0 & \lambda_3 & 0 & 0 & 0 & -(\mu_4 + \mu_3)
\end{bmatrix},
\]

where $\Sigma_1 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5$, and $\Sigma_2 = \mu_4 + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_6$.

Given this transition matrix, the probabilities $P_i(t)$'s can be determined by solving the corresponding system of Markov equations. The system's availability function is then obtained through Equation (4.39).

### 4.4.4 Model F — the priority repair policy

This model examines longwall system availability under the priority repair policy. According to this repair policy, if more than one component fails during system operation, the component which has the greatest effect on system performance is served first. After the priority repair is completed, the repair actions for other failed components will start from the beginning.

For this model, the possible transitions between the system states are illustrated in Fig. 9. As indicated by the functional analysis of longwall operations, the coal-cutting, the face-conveying, and the outby-haulage subsystems are three critical components for the availability of longwall systems.
Figure 9. State-transition diagram of the System in Model E
Thus, when the system is in State 6, 7 or 8, according to the priority policy, any of the critical components is served before the roof-support subsystem.

The Markov equations have the same form as Equation (4.48), but with the transition matrix $A$ given below:

$$
\begin{bmatrix}
-\Sigma_1 & \mu_1 & \mu_2 & \mu_3 & \mu_4 & \mu_5 & 0 & 0 & 0 \\
\lambda_1 & -\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_2 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_3 & 0 & 0 & -\mu_3 & 0 & 0 & 0 & 0 & 0 \\
\lambda_4 & 0 & 0 & 0 & -\Sigma_2 & 0 & \mu_1 & \mu_2 & \mu_3 \\
\lambda_5 & 0 & 0 & 0 & \lambda_6 & -\mu_5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_1 & 0 & -(\mu_4+\mu_1) & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & -(\mu_4+\mu_2) & 0 \\
0 & 0 & 0 & 0 & \lambda_3 & 0 & 0 & 0 & -(\mu_4+\mu_3) \\
\end{bmatrix}
$$

where $\Sigma_1 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5$, and $\Sigma_2 = \mu_4 + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_6$.

Given this transition matrix, the state probabilities $P_i(t)$'s can be derived by solving the corresponding system of Markov equations. The system's availability function is then obtained by using Equation (4.39).

### 4.5 Extensions of the Modeling Approaches

In practice, a longwall system may have more complicated operating characteristics than those represented by the models developed above. For example, the component failure-repair processes in the system could be
interdependent; for some of the components, their TTF's or TTR's may be arbitrarily distributed; the system's availability could be affected by corrective maintenance (CM) as well as preventive maintenance (PM), etc. However, some of these problems can be solved by extending the methodologies developed in the previous chapters. In this section, the possible extensions of the modeling approaches to two non-Markov problems are discussed first. Then an approach is introduced for evaluating the effect of PM on system availability.

4.5.1 Common-cause failures

So far, all of the probability models have been formulated by assuming s-independent component failures in the system. In practice, though dependent failures may occur among some components in a system, it is difficult to incorporate these types of failure quantitatively in system modeling. As an example to deal with such problems, the common-cause failures, a special type of dependent failures which could occur in the operation of longwall systems, are studied below by employing a Markov modeling approach.

**Definition:** Common-cause failures denote any instance where multiple units or components fail due to a single cause. In longwall operations, this type of failure includes environmental and external impacts which can shut off system production accidentally. Some of the examples are:

- external catastrophe, such as roof falls, drainage, and gas problems
- operation and maintenance errors
- external power failure
As an example, Model C is extended below to consider the common-cause failures in a longwall system. In developing this model, the common-cause failures are assumed to be statistically independent of individual component failures. The distribution of the times to such failures and the corresponding repair times are generally considered to be exponentially distributed. These considerations are justified by the work of Astashkin (1970):

Analysis of statistical data on accidents and traverse of geological disturbances in the Donets coalfield reveals that the holdups of normal functioning of the face can be represented by Poisson's law. With certain assumptions, the distribution of the times of series accidents and encounters with geological disturbance also obeys an exponential law.

The state definitions of the system are similar to those given in Table 2 on page 55. However, two more states need to be added for the common-cause failures (see Table 3). The corresponding state-transition diagram of the system is given in Fig. 10. The resulting Markovian state equations are the same as Equation (4.48), with the exception of transition matrix A, which is given below:

\[
\begin{bmatrix}
-\Sigma_1 & \mu_1 & \mu_2 & \mu_3 & \mu_4 & \mu_5 & 0 & 0 & 0 & \lambda_m & 0 \\
\lambda_1 & -\mu_1 & 0 & 0 & 0 & 0 & \mu_4 & 0 & 0 & 0 & 0 \\
\lambda_2 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & \mu_4 & 0 & 0 & 0 \\
\lambda_3 & 0 & 0 & -\mu_3 & 0 & 0 & 0 & \mu_4 & 0 & 0 & 0 \\
\lambda_4 & 0 & 0 & 0 & -\Sigma_2 & 0 & \mu_1 & \mu_2 & \mu_3 & 0 & \lambda_m \\
\lambda_5 & 0 & 0 & 0 & \lambda_6 & -\mu_5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_1 & 0 & -(\mu_4+\mu_1) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & -(\mu_4+\mu_2) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_3 & 0 & 0 & 0 & -(\mu_4+\mu_3) & 0 & 0 \\
\mu_m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_m & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_m & 0
\end{bmatrix}
\]

where \( \Sigma_1 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 \), \( \Sigma_2 = \mu_4 + \lambda_6 + \lambda_1 + \lambda_2 + \lambda_3 \), and \( \lambda_m \) and \( \mu_m \)
Table 3. System State Definitions Considering Common—Cause Failures

<table>
<thead>
<tr>
<th>State</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>All subsystems are operating.</td>
</tr>
<tr>
<td>1</td>
<td>The coal—cutting subsystem fails.</td>
</tr>
<tr>
<td>2</td>
<td>The face—conveying subsystem fails.</td>
</tr>
<tr>
<td>3</td>
<td>The outby haulage subsystem fails.</td>
</tr>
<tr>
<td>4</td>
<td>The roof—support subsystem is partially operating.</td>
</tr>
<tr>
<td>5</td>
<td>The roof—support subsystem is completely down.</td>
</tr>
<tr>
<td>6</td>
<td>When the roof—support subsystem is partially operating, the coal—cutting subsystem fails.</td>
</tr>
<tr>
<td>7</td>
<td>When the roof—support subsystem is partially operating, the face—conveying subsystem fails.</td>
</tr>
<tr>
<td>8</td>
<td>When the roof—support subsystem is partially operating, the outby—haulage subsystem fails.</td>
</tr>
<tr>
<td>9</td>
<td>With all components operating, common—cause failures shut down the system.</td>
</tr>
<tr>
<td>10</td>
<td>With the coal—cutting, the face—conveying, and the outby—haulage subsystems operating and the roof—support subsystem partially operating, common—cause failures shut down the system.</td>
</tr>
</tbody>
</table>
Figure 10. State-transition diagram of the system with common-cause failures.
are the failure rates and the repair rates for the common—cause failures, respectively.

4.5.2 Non-exponential event times

So far, the Markovian modeling of longwall systems has been carried out by assuming exponentially—distributed component failure times and repair times. The resulting Markov models are time—homogeneous. In more realistic modeling of longwall operations, however, it is not always possible to describe all components' failure and repair times by exponential distributions. As a result, the system availability model may not be Markovian. Because of the analytical and computational difficulties, not much has been done when failure and repair times are other than exponential. Two methods are recommended in this section for dealing with these types of problems: the steady state approach and the dummy—state method.

Steady—state approach:

Models A and B developed above were formulated without specifying any particular probability distributions. The results derived for these models are, thus, valid for systems or components with any failure or repair time distributions. One of the important results is that no matter what the failure and repair time distributions are, the steady—state availability of the system is given by

\[
A = \frac{MTTF_s}{MTTF_s + MTTR_s},
\]

(4.49)
given that the PDF's of the system's times to failure and times to repair, \( f_s(t) \) and \( g_s(t) \), are known, and the following quantities are finite:

\[
\text{MTTF}_s = \nu = \int_0^\infty t \times f_s(t) \, dt,
\]

\[
\sigma^2_{\nu} = \int_0^\infty (t-\nu)^2 f_s(t) \, dt,
\]

\[
\text{MTTR}_s = \tau = \int_0^\infty t \times g_s(t) \, dt,
\]

\[
\sigma^2_{\tau} = \int_0^\infty (t-\tau)^2 g(t) \, dt.
\]

These formulae assume a steady-state condition based upon expected values.

**Dummy-state method:**

After studying the behavior of multi-component systems, Doyon (1975) indicated that most reliability models, particularly those for repairable equipment where either or both the times to failure and times to repair are not Poisson or exponentially distributed, or where failures can occur while the equipment is being repaired, fall into the category of imbedded Markov chains. Such imbedded Markov chains can be treated as Markov chains at the instant of transition. Thus, resolving methods can be used to reduce a semi-Markov stochastic model with non-exponentially distributed times between events to a simpler form. The Markovian approach can still be used after such resolution.
One example is that the PDF of the time to events is a certain type of the gamma distribution, called Erlang distribution, which has a PDF as below:

$$f(t) = \frac{\lambda^k t^{k-1} e^{-\lambda t}}{(k-1)!}, \quad \lambda > 0, \text{ and } k \text{ is a positive integer.}$$

An Erlangian distribution with an integer-valued parameter $k$ may be regarded as the convolution of $k$ exponential distributions. Thus, the state transition of a system (failure or repair events in this study) corresponding to an Erlangian distribution can be decomposed into $k$ transitions corresponding to the same exponential law. This convolution can be illustrated by Fig. 11. $1_1, 1_2, ..., 1_{k-1}$ in this figure are dummy states, which enable the process to be converted into a Markovian one. Using these pseudo states as additional states in the model, the Markovian approach can be applied with a constant rate of transfer between these states. As defined by introduction of the additional states, the system may not be the exact physical equivalent of the system under consideration, but mathematically both would be the same.

### 4.5.3 Preventive maintenance

So far, the availability models have been developed by considering only one type of repair actions — corrective maintenance. System performance, however, could be improved by preventive maintenance (PM) as well. PM is an activity undertaken regularly at preselected intervals, while the system is satisfactorily operating, to reduce or eliminate the accumulated deterioration. The actions taken may include such functions as periodic inspection, servicing, scheduled
Figure 11. Convolution of an Erlang distribution
replacement of critical parts, calibration, overhaul, and so on.

PM is designed to optimize the related concepts of reliability effectiveness and the costs that accrue when a system needs to be repaired. As a result, PM actions may be able either to increase a system's availability or to reduce the total maintenance costs. Therefore, PM should be considered under the following situations:

- The failure rate of the system and/or units is increasing with time.
- The preventive maintenance actions cost less than the corresponding corrective maintenance actions.

In this section, only the effect of PM on system availability is discussed, while the effect of PM on the operating costs of a system is not investigated.

Generally speaking, for a system with increasing failure rate, the more frequent performance of the PM, the higher value of the system's MTTF. In longwall operations, mechanical failures due to part wearout of face equipment are one of the most common problems. This type of failure increases with time, and thus, it may be served more efficiently by PM. More frequent PM actions, however, may not necessarily increase the system’s availability. For a system intended for continuous service, such as longwall mining systems, both CM and PM actions must be taken during the duty time and consequently. The time available for system production is reduced because of PM actions. Thus, there must be a tradeoff in determining PM policies, depending on the availability and cost requirements for the system.

In order to quantitatively assess the effect of PM on system availability, an
achieved availability function is developed based on the following considerations. In this study, CM policy is such that repair or replacement begins only when the system fails due to failure of any subsystem. By contrast, PM is scheduled at age $T$ of the system, which starts at $t = 0$, and is actually performed only if the system has not failed before age $T$. In performing PM, every unit is checked, and any failed part is replaced by a new and statistically identical unit. The system is, therefore, restored to a like-new condition after each PM action. The time to perform PM is assumed to be a constant for the system and its components, since PM is more likely than CM to be nearly constant in duration because of its scheduled nature (Cho, 1963).

Considering both CM and PM, the system availability function is termed as the achieved availability ($A_s(T)$), given by

$$A_s(T) = \frac{MTTF_T}{MTTF_T + MT_T}, \quad (4.50)$$

where $MTTF_T$ and $MT_T$ are the mean uptime and the mean downtime of the system, respectively, with consideration of both CM and PM actions, and subscript $T$ denotes the PM interval.

Given system reliability $R_s(t)$, the mean uptime of the system under PM schedule $T$ can be derived from

$$MTTF_T = \int_0^T R_s(t) \, dt. \quad (4.51)$$
The mean system downtime $\bar{M}_T$ due to both CM and PM is given by

$$\bar{M}_T = \bar{M}_{ct}[1 - R_s(T)] + \bar{M}_{pt}[R_s(T)],$$  \hspace{1cm} (4.52)

where $\bar{M}_{ct}$ is the mean corrective maintenance time, which is equal to MTTR$_s$, and $\bar{M}_{pt}$ represents the mean time of preventive maintenance actions.

Substituting equations (4.51) and (4.52) into Equation (4.50) gives a formula for computing the achieved system availability:

$$A_{sT} = \frac{\int_0^T R_s(T)dt}{\int_0^T R_s(t)dt + \text{MTTR}_s [1 - R_s(T)] + \bar{M}_{pt}[R_s(T)]}.$$  \hspace{1cm} (4.53)

Now let us compare $A_{sT}$ and $A_s$. Under PM schedule $T$, whether the availability of a longwall system can be increased by PM depends on the length of mean PM time $\bar{M}_{pt}$. If $\bar{M}_{pt}$ is shorter than $\bar{M}_{ct}$ (or MTTR$_s$) to some extent, the achieved system availability can be increased by performance of PM. Clearly, when $T \rightarrow \infty$, we have

$$A_{sT} = A_s = \frac{\text{MTTF}_s}{\text{MTTF}_s + \text{MTTR}_s}.$$  \hspace{1cm} (4.54)
V. CASE STUDIES

In this chapter three case studies are presented to demonstrate the implementation of the availability models developed. The solutions of these models will be compared with respect to real longwall mining systems.

5.1 Introduction

The data for these case studies were collected from three longwall faces in Virginia and West Virginia. The data sources are either foreman production records and maintenance records, or time studies. The information collected includes mining characteristics, equipment type, scheduled delay time, the exact time of occurrence for all longwall delays, and the duration of each delay.

A number of dBASE-based databases were constructed to facilitate the analysis and classification of various delays occurred. The times to failure and the times to repair for each equipment component were grouped and calculated according to four categories: coal-cutting, face-conveying, roof-support, and outby-haulage. The TTF's were measured between the ending of one delay and the start of the next delay. The TTR includes all active portions of downtime, i.e., isolation of the failed component, removal, repair and/or replacement, and
checkout. A VAX-based statistical software package, UNIFIT, was then used for fitting probability distributions to the TTF and TTR data sets of each subsystem. Four families of distributions were hypothesized for the observed data sets: exponential, Gamma, lognormal, and Weibull. The parameters of these hypothesized distributions were estimated by the method of maximum likelihood. According to the Chi-square goodness-of-fit test, exponential distributions provide acceptable fits to the TTF and TTR data from each of these three case studies. Thus, the failure rates and the repair rates of each subsystem can be obtained from the parameters of the corresponding exponential distributions.

However, the data collected are not adequate to fully use Models C, D and E. For example, for the roof-support subsystem, the data for the transition rates between the operating, partially operating, and failed states are not available for all of the three cases. Instead, these parameters are estimated by assuming these relative ratios: \( \lambda_5/\lambda_4 = 0.2, \lambda_6/\lambda_4 = 0.4 \). The average time to perform a PM action for the roof-support subsystem, \( T \), is assumed to be 20 minutes. Since the actual \( \delta \) value is unknown for these cases, a \( \delta \) value of 0.5 is used to make the results from different models comparable. To a certain extent, the corresponding model solutions will be affected due to these assumptions. But the relative behaviors of the system represented by the results of those models should be the same, and thus, can reflect the true pictures of the real system. Besides, since the primary objective of these case studies is to demonstrate the use of the modeling approaches, absolute accuracy is not pursued for the specific results.

In the following, Models A through E are solved respectively for each case study. Both transient and steady-state solutions are derived. The algorithm used
for solving the corresponding Markov equations is given in Appendix C. For Models A and B, the functions and indices related to both system availability and reliability are obtained. However, because of the complexity of the problems, only system availability is evaluated for Models C, D, and E. Finally, the solutions from different models are compared.

5.2 Case Study 1

5.2.1 Data Inputs

The data for this case study were collected from a longwall mine in Virginia, operating in the Pocahontas #3 seam. The average thickness of the seam is 60 inches. The average depth of the overburden is 2,000 feet. The roof consists of massive shaley sandstone, and the floor is sandy shale. The mining face has no water and spalls well.

A retreating longwall system with four-entry gate roads is employed at this mine. The panel is about 5,560 feet long and 600 feet wide, with an average production of 1,300 tons per shift. An Anderson Mavor 500 DERS of 500 hp is used for coal winning, along with Eicotak haulage. The face conveyor is a double inboard 22 mm chain conveyor made by American Longwall. Powered by three 175 hp motors, the conveyor operates at a speed of 265 feet per minute and provides side discharge to the strogelader-crusher. Westfalia Lunen four-legged shields are used, with a yielding capacity of 580 tons. The shields are controlled by "Piano Key" manual controls. All face equipment operates at 950 volts.
### Table 4. MTTF and MTTR Data of Case Study 1

<table>
<thead>
<tr>
<th>Category</th>
<th>MTTF (min.)</th>
<th>MTTR (min.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal cutting</td>
<td>214.1740</td>
<td>59.9863</td>
</tr>
<tr>
<td>Face conveying</td>
<td>814.2580</td>
<td>49.2414</td>
</tr>
<tr>
<td>Outby haulage</td>
<td>639.9880</td>
<td>34.8936</td>
</tr>
<tr>
<td>Roof support</td>
<td>342.7300</td>
<td>34.0000</td>
</tr>
<tr>
<td>System</td>
<td>89.9952</td>
<td>47.4125</td>
</tr>
</tbody>
</table>

### Table 5. Component Failure and Repair Rates of Case Study 1

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Failure rate</th>
<th>Repair rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal cutting</td>
<td>0.00466910</td>
<td>0.01667047</td>
</tr>
<tr>
<td>Face conveying</td>
<td>0.00122811</td>
<td>0.02030811</td>
</tr>
<tr>
<td>Outby haulage</td>
<td>0.00156253</td>
<td>0.02865855</td>
</tr>
<tr>
<td>Roof support</td>
<td>0.00291775</td>
<td>0.02941176</td>
</tr>
</tbody>
</table>

Note: The failure and repair rates are in number of occurrences per minute.
The data from 200 continuous production shifts were collected at this panel. In Table 4, the MTTF's and MTTR's are listed for the coal—cutting, face—conveying, roof—support and outby—haulage subsystems. As the basic inputs to those availability models, the failure rate and the repair rate of each component were calculated in Table 5. For the roof—support subsystem, given that its MTTF = 342.73 minutes, MTTR = 34.00 minutes, \( \lambda_5/\lambda_4 = 0.2 \), \( \lambda_6/\lambda_4 = 0.4 \), and \( T = 20 \) minutes, the transition rates between its operating, partially operating, and failed states were estimated as follows: \( \lambda_4 = 0.01232886, \lambda_5 = 0.00246577, \lambda_6 = 0.00493154, \mu_4 = 0.05000000, \mu_5 = 0.02941176 \).

The achieved availability of the system is given by \( \frac{MTTF}{(MTTF + MTTR)} \), which is equal to 65.50%. This figure can be used as a benchmark for verifying the results derived from the mathematical models.

### 5.2.2 Solutions of Models 1 and 2

**Model A:**

According to Model A, the reliability and availability functions for this system were obtained as follows:

\[
R_a(t) = \exp(-0.01037749t),
\]

\[
A_s(t) = 0.63551870 \left[ 1 + 0.28008208 \times \exp(-0.02133957t) \right]
\]
\[ \times \left( 1 + 0.06047395 \times \exp(-0.02153623t) \right) \]
\[ \times \left( 1 + 0.05452227 \times \exp(-0.03022108t) \right) \]
\[ \times \left( 1 + 0.09920345 \times \exp(-0.03232951t) \right). \]

The variation of the system availability with time is illustrated in Fig. 12 on page 86. As shown in this figure, the system availability decreases with time. After about four hours of system operation, it approaches a steady-state value of 63.55%. Some important steady-state results were also obtained for this system: system failure rate \( (\lambda_s) \), MTTFs, MTTRs, system availability \( (A_s) \), the expected average of the number of system failures \( (N_f) \), and the limiting probability that the system is down due to the failure of any particular subsystem \( (P_i)'s \). These results are listed in Table 6 on page 87. According to the system-state probabilities given by this model, the largest contributor to the unavailability of this system is the coal-cutting subsystem, followed by the roof-support and the outby-haulage subsystems, with the face-conveying subsystem being the smallest.

**Model B:**

The system reliability function has the same form as that represented by Model A, that is:

\[ R_a(t) = \exp(-0.01037749t). \]
In order to find the availability function of the system, the corresponding problem of Markov processes was formulated. The transition matrix for this problem, \( A_b \) is given in Appendix D. Through solving this system of differential equations, \( P_i(t) \)'s, the probabilities that the system is in State \( i \) at time \( t \), for \( i = 0, 1, 2, 3, \) and \( 4 \), were obtained below:

\[
\begin{bmatrix}
P_0(t) \\
P_1(t) \\
P_2(t) \\
P_3(t) \\
P_4(t)
\end{bmatrix} = K_1 + K_2 \exp(-0.03579074t) + K_1 \exp(-0.02196192t) \\
+ K_4 \exp(-0.01876582t) + K_5 \exp(-0.02890790t),
\]

where \( K_1 \) represents the \( i^{th} \) column of matrix \( K_b \), which is given in Appendix D. Actually, \( K_1 \) describes the steady-state values of \( P_0(t) \), ..., \( P_5(t) \), which are listed separately in Table 6.

Since State 0 is the only operating state for the system, \( A_5(t) = P_0(t) \). An explicit expression of the system availability function is then given by

\[
A_5(t) = 0.23222252 \times \exp(-0.03579074t) + 0.06475212 \times \exp(-0.02196192t) \\
+ 0.03286423 \times \exp(-0.01876582t) + 0.00094330 \times \exp(-0.02890790t) \\
+ 0.66921783.
\]

In Figure 12, the variation of the system availability is shown as a function of
Figure 12. System Availability Functions of Models A & B (Case Study 1)
Table 6. Steady-State Results of Models A and B (Case Study 1)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model A</th>
<th>Model B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_s$</td>
<td>0.63551870</td>
<td>0.66921783</td>
</tr>
<tr>
<td>$\lambda_s$ (1/min.)</td>
<td>0.01037749</td>
<td>0.01037749</td>
</tr>
<tr>
<td>MTTFs (min.)</td>
<td>96.36240812</td>
<td>96.36240812</td>
</tr>
<tr>
<td>MTTRs (min.)</td>
<td>55.26555845</td>
<td>47.63018094</td>
</tr>
<tr>
<td>$N_f$ (1/min.)</td>
<td>0.00659509</td>
<td>0.00694480</td>
</tr>
<tr>
<td>$P_1$</td>
<td>0.16398954</td>
<td>0.18743592</td>
</tr>
<tr>
<td>$P_2$</td>
<td>0.04313411</td>
<td>0.04047025</td>
</tr>
<tr>
<td>$P_3$</td>
<td>0.05487962</td>
<td>0.03648728</td>
</tr>
<tr>
<td>$P_4$</td>
<td>0.10247803</td>
<td>0.06638872</td>
</tr>
</tbody>
</table>
time. The system approaches a steady state after about four hours of continuous operation. The steady-state availability is about 66.92%, which is more than 3% higher than that given by Model A. Other steady-state values for this system are listed in Table 6, along with those of Model A. According to the system-state probabilities given by this model, the largest contributor to system unavailability is the coal-cutting subsystem, followed by the roof-support and the face-conveying subsystems. The outby-haulage subsystem is the smallest contributor.

5.2.3 Solutions of Models C, D and E

Model C:

The transition matrix for the corresponding Markov problem, \( A_c \), is given in Appendix D. The state probabilities of the system were obtained by solving a system of linear differential equations under this initial condition: \( I = [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^{T} \). They are listed as below:

\[
\begin{bmatrix}
P_0(t) & P_1(t) & P_2(t) & P_3(t) & P_4(t) & P_5(t) & P_6(t) & P_7(t) & P_8(t)
\end{bmatrix}^T
= K_1 + K_2 \exp(-0.08590595t) + K_3 \exp(-0.07519666t)
+ K_4 \exp(-0.06941336t) + K_5 \exp(-0.05886968t)
+ K_6 \exp(-0.01875885t) + K_7 \exp(-0.02193203t)
+ K_8 \exp(-0.03636090t) + K_9 \exp(-0.02889424t),
\]

where \( K_i \) represents the \( i \)th column of matrix \( K_c \), which is given in Appendix D. Actually, \( K_1 \) describes the steady-state values of \( P_0(t), ..., P_8(t) \). These limiting
results are listed separately in Table 7 on page 95.

Since the availability of this system is a function of $P_0(t)$ and $P_4(t)$, the system's availability function can be determined accordingly. If $A_s(t) = P_0(t) + \delta P_4(t)$, for $0 \leq \delta \leq 1$, the availability function becomes

$$A_s(t) = (0.55603944 + \delta \times 0.11368751) + (0.07501444 - \delta \times 0.07911091) \exp(-0.08590595t) + (0.02765569 - \delta \times 0.02962436) \exp(-0.07519666t) + (0.00484113 - \delta \times 0.00525283) \exp(-0.06941336t) + (0.10176131 - \delta \times 0.11510440) \exp(-0.05889681t) + (0.02517327 + \delta \times 0.00761120) \exp(-0.01875885t) + (0.04726143 + \delta \times 0.01557363) \exp(-0.02193203t) + (0.16164205 + \delta \times 0.09196841) \exp(-0.03636090t) + (0.00061124 + \delta \times 0.00025175) \exp(-0.02889424t).$$

For $\delta = 0.5$, Figure 13 was drawn to illustrate the availability of the system as a function of time. Since the system is fully operating initially, $A(0) = 100\%$. With the passage of time, the availability of the system decreases continuously and approaches a steady-state value (61.29%) after about three hours.

**Model D:**

For transition matrix $A_d$ given in Appendix D, the state probabilities of the system were obtained as below:
Figure 13: The system availability function of Model C (Case study 1)
\[
\begin{bmatrix}
P_0(t) & P_1(t) & P_2(t) & P_3(t) & P_4(t) & P_5(t) & P_6(t) & P_7(t) & P_8(t) & P_9(t)
\end{bmatrix}^T
\]

\[=
K_1 + K_2 \exp(-0.07553183t) + K_3 \exp(-0.04344926t)
+ K_4 \exp(-0.04073192t) + K_5 \exp(-0.01882515t)
+ K_6 \exp(-0.02220159t) + K_7 \exp(-0.02895479t),
\]

where \(K_i\) represents the \(i\)th column of matrix \(K_d\), which is given in Appendix D. The steady-state values of \(P_0(t), ..., P_9(t)\) are listed in Table 7 on page 95, along with those of Model C. Given that \(A_5(t) = P_0(t) + \delta P_1(t), 0 \leq \delta \leq 1\), the availability function is

\[
A_5(t) = (0.55029587 + \delta \times 0.10874147)
+ (0.19847674 - \delta \times 0.18621700) \exp(-0.07553183t)
- (0.55577836 + \delta \times 0.36174134) \exp(-0.04344926t)
+ (0.73217886 + \delta \times 0.41676801) \exp(-0.04073192t)
+ (0.02293832 + \delta \times 0.00649136) \exp(-0.01882515t)
+ (0.05124608 + \delta \times 0.01572059) \exp(-0.02220159t)
+ (0.00064249 + \delta \times 0.00023690) \exp(-0.02895479t).
\]

For \(\delta = 0.5\), Figure 14 illustrates the variation of system availability as a function of time. The system's availability decreases continuously from 100% and approaches a steady-state value of 60.47% after about three hours.

**Model E:**

The transition matrix for this Markov problem, \(A_e\), is given in Appendix D. Solving a system of differential equations, we obtained the state probabilities of
Figure 14. The system availability function of Model D (Case study 1)
the system as below:

\[
\begin{bmatrix}
P_0(t) & P_1(t) & P_2(t) & P_3(t) & P_4(t) & P_5(t) & P_6(t) & P_7(t) & P_8(t) & P_9(t)
\end{bmatrix}^T
= K_1 + K_2 \exp(-0.07660583t) + K_3 \exp(-0.03598599t) \\
+ K_4 \exp(-0.01516897t) + K_5 \exp(-0.01876767t) \\
+ K_6 \exp(-0.02197161t) + K_7 \exp(-0.02892625t) \\
+ K_8 \exp(-0.02793405t) + K_9 \exp(-0.01997131t),
\]

where \(K_i\) stands for the \(i\)th column of matrix \(K_e\) (see Appendix D). The steady-state values of \(P_0(t), \ldots, P_9(t)\) are listed in Table 7, along with those of Models C and D. Given that \(A_s(t) = P_0(t) + \delta P_4(t)\), for \(0 \leq \delta \leq 1\), the availability function is then obtained as

\[
A_s(t) = (0.54654996 + \delta \times 0.12266791) \\
+ (0.17952681 - \delta \times 0.19181697) \exp(-0.07660583t) \\
+ (0.18468092 + \delta \times 0.05970319) \exp(-0.03598599t) \\
+ (0.00492754 - \delta \times 0.00475201) \exp(-0.01516897t) \\
+ (0.02700658 + \delta \times 0.00554748) \exp(-0.01876767t) \\
+ (0.05393186 + \delta \times 0.01048788) \exp(-0.02197161t) \\
+ (0.00096935 + \delta \times 0.00005691) \exp(-0.02892625t) \\
+ (0.00162836 - \delta \times 0.00116926) \exp(-0.02793405t) \\
+ (0.00077863 - \delta \times 0.00072693) \exp(-0.01997131t).
\]

For \(\delta = 0.5\), Figure 15 illustrates the availability of the system as a function of time. The availability of this system decreases continuously with time and approaches a steady-state value of 60.79%.
Figure 15. The system availability function of Model B (Case study 1)
Table 7. Limiting State Probabilities of Models C, D, & E (Case Study 1)

<table>
<thead>
<tr>
<th>Probability</th>
<th>Model C</th>
<th>Model D</th>
<th>Model E</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₀</td>
<td>0.55603944</td>
<td>0.55029587</td>
<td>0.54654996</td>
</tr>
<tr>
<td>P₁</td>
<td>0.17961671</td>
<td>0.18458457</td>
<td>0.15307887</td>
</tr>
<tr>
<td>P₂</td>
<td>0.03851514</td>
<td>0.03985454</td>
<td>0.03305199</td>
</tr>
<tr>
<td>P₃</td>
<td>0.03425668</td>
<td>0.03593223</td>
<td>0.02979916</td>
</tr>
<tr>
<td>P₄</td>
<td>0.11368751</td>
<td>0.10874147</td>
<td>0.12266791</td>
</tr>
<tr>
<td>P₅</td>
<td>0.06567849</td>
<td>0.06436766</td>
<td>0.06638869</td>
</tr>
<tr>
<td>P₆</td>
<td>0.00796182</td>
<td>0.01015450</td>
<td>0.03435709</td>
</tr>
<tr>
<td>P₇</td>
<td>0.00198584</td>
<td>0.00267093</td>
<td>0.00741820</td>
</tr>
<tr>
<td>P₈</td>
<td>0.00225837</td>
<td>0.00339824</td>
<td>0.00668814</td>
</tr>
</tbody>
</table>
Figure 16. Comparison of system availabilities for different $\delta$ values (Case 1)
With respect to different $\delta$ values, the system availabilities from Models C, D, and E are compared in Fig. 16. The system under the multiple repair policy (Model C) demonstrates the highest availability for any $\delta$ values. When $\delta > 0.3$, the system served by the priority repair policy (Model E) has a higher availability than the system under FCFS (Model D), but the opposite is true if $\delta < 0.3$. By using the actual availability of the system (65.5%) as a benchmark, the $\delta$ value for this system is estimated to be between 0.85 and 0.95.

5.3 Case Study 2

5.3.1 Data inputs

The data for this case study were collected from a longwall mine in West Virginia. The seam mined is a part of the Eagle seam, with an average thickness of 54 inches. The average depth of the overburden is 550 feet. A four-entry longwall retreating system was used, with mining height equal to 50 inches. The longwall panel is about 5,300 feet long and 520 feet wide.

A Westfalia Lunen Gleithobel plow is used at the face, driven by two motors of 200 hp. The face conveyor is a single inboard 30 mm chain conveyor also made by Westfalia Lunen. The conveyor is powered by three 125 hp motors. Westfalia Lunen 2-legged shields are used for roof support, each with a yielding capacity of 560 tons. Manual controls are used for moving the supports.

Data from 171 production shifts were collected from this panel. As the basic
Table 8. MTTF and MTTR Data of Case Study 2

<table>
<thead>
<tr>
<th>Category</th>
<th>MTTF (min.)</th>
<th>MTTR (min.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal cutting</td>
<td>779.5650</td>
<td>96.7969</td>
</tr>
<tr>
<td>Face conveying</td>
<td>1825.0500</td>
<td>57.4783</td>
</tr>
<tr>
<td>Outby haulage</td>
<td>494.4270</td>
<td>28.2019</td>
</tr>
<tr>
<td>Roof support</td>
<td>1194.6200</td>
<td>24.5385</td>
</tr>
<tr>
<td>System</td>
<td>159.4330</td>
<td>51.0968</td>
</tr>
</tbody>
</table>

Table 9. Component Failure and Repair Rates of Case Study 2

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Failure rate</th>
<th>Repair rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal cutting</td>
<td>0.00128277</td>
<td>0.01033091</td>
</tr>
<tr>
<td>Face conveying</td>
<td>0.00054793</td>
<td>0.01739787</td>
</tr>
<tr>
<td>Outby haulage</td>
<td>0.00202254</td>
<td>0.03545860</td>
</tr>
<tr>
<td>Roof support</td>
<td>0.00083709</td>
<td>0.04075229</td>
</tr>
</tbody>
</table>

Note: The failure and repair rates are in number of occurrences per minute.
inputs to those availability models, the MTTF and MTTR data, and the failure and repair rates for each subsystem are listed in Tables 8 and 9. For the roof—support subsystem, given its MTTF = 1194.62 minutes, MTTR = 24.54 minutes, $\lambda_5/\lambda_4 = 0.2$, $\lambda_6/\lambda_4 = 0.4$, and $T = 20$ minutes, the transition rates between its operating, partially operating, and failed states were estimated as below: $\lambda_4 = 0.00390986$, $\lambda_5 = 0.00078197$, $\lambda_6 = 0.00156394$, $\mu_4 = 0.05000000$, $\mu_5 = 0.04075229$. The achieved availability of the system is given by $\text{MTTF}/(\text{MTTF} + \text{MTTR})$, which is equal to 75.73%.

5.3.2 Solutions of Models 1 and 2

Model A:

According to Model A, the reliability and availability functions for this system are obtained as follows:

$$R_s(t) = \exp(-0.00469033t),$$

$$A_s(t) = 0.79943004 \left[ 1 + 0.12416784 \times \exp(-0.01161368t) \right]$$

$$\times \left[ 1 + 0.03149410 \times \exp(-0.01794580t) \right]$$

$$\times \left[ 1 + 0.05703956 \times \exp(-0.03748115t) \right]$$

$$\times \left[ 1 + 0.02054084 \times \exp(-0.04158937t) \right].$$
The variation of the system availability with time is illustrated in Fig. 17. A steady-state availability of 79.94% is approached after about five hours of system operation. Other steady-state results of this model are given in Table 10. As shown in this table, from high to low, the order of component importance to system unavailability is: the outby-haulage, the coal-cutting, the roof-support, and the face-conveying subsystems.

**Model B:**

The system reliability function has the same form as that represented by Model A, that is:

\[
R_s(t) = \exp(-0.00469033t).
\]

For the corresponding problem of Markov processes, whose transition matrix is given in Appendix E, the probability that the system is in State \( i \) at time \( t \), for \( i = 0, 1, 2, 3, \) and \( 4 \), were obtained as below:

\[
\begin{bmatrix}
P_0(t) \\
P_1(t) \\
P_2(t) \\
P_3(t) \\
P_4(t)
\end{bmatrix}
= K_1 + K_2 \exp(-0.01139645t) + K_3 \exp(-0.04207720t)
+ K_4 \exp(-0.01795471t) + K_5 \exp(-0.03720164t),
\]

where \( K_i \) stands for the \( i \)th column of matrix \( K_b \), which is given in Appendix E.
Figure 17. System Availability Functions of Models A & B (Case Study 2)
Table 10. Steady-State Results of Models A and B (Case Study 2)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model A</th>
<th>Model B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_s$</td>
<td>0.79943004</td>
<td>0.81087064</td>
</tr>
<tr>
<td>$\lambda_s$ (1/min.)</td>
<td>0.00469033</td>
<td>0.00469033</td>
</tr>
<tr>
<td>MTTF$_s$ (min.)</td>
<td>213.20477814</td>
<td>213.20477814</td>
</tr>
<tr>
<td>MTTR$_s$ (min.)</td>
<td>53.49120087</td>
<td>49.72838085</td>
</tr>
<tr>
<td>$N_f$ (1/min.)</td>
<td>0.00374959</td>
<td>0.00380325</td>
</tr>
<tr>
<td>$P_1$</td>
<td>0.05485428</td>
<td>0.10068405</td>
</tr>
<tr>
<td>$P_2$</td>
<td>0.02343085</td>
<td>0.02553764</td>
</tr>
<tr>
<td>$P_3$</td>
<td>0.08648895</td>
<td>0.04625171</td>
</tr>
<tr>
<td>$P_4$</td>
<td>0.03579588</td>
<td>0.01665596</td>
</tr>
</tbody>
</table>
The limiting values of these state probabilities are provided in Table 10.

The system availability function is given by \( P_0(t) \), that is:

\[
A_s(t) = 0.07633455 \times \exp(-0.01139645t) + 0.04525022 \times \exp(-0.04207720t) \\
+ 0.03098637 \times \exp(-0.01795471t) + 0.03655842 \times \exp(-0.03720164t) \\
+ 0.81087064.
\]

The variation of the system availability with time is illustrated in Fig. 17. A steady-state availability of 81.09\% is approached after about four hours of system operation. This figure is slightly higher than that given in Model A. Other steady-state results of this model are given in Table 10, along with those of Model A. According to the state probabilities, from high to low, the order of component importance to system unavailability is: the coal-cutting, the outby-haulage, the face-conveying, and the roof-support subsystems.

5.3.3 Solutions of Models C, D and E

Model C:

For transition matrix \( A_c \) as given in Appendix E, the state probabilities of the system were obtained as below:

\[
\begin{bmatrix}
P_0(t) & P_1(t) & P_2(t) & P_3(t) & P_4(t) & P_5(t) & P_7(t) & P_8(t) & P_9(t) \\
\end{bmatrix}^T \\
= K_1 + K_2 \exp(-0.08803483t) + K_3 \exp(-0.01139613t)
\]
\[ + K_4 \exp(-0.01795418t) + K_5 \exp(-0.06822615t) \\
+ K_6 \exp(-0.05396464t) + K_7 \exp(-0.04230362t) \\
+ K_8 \exp(-0.03716466t) + K_9 \exp(-0.06204508t), \]

where \( K_i \) represents the \( i \)th column of matrix \( K_c \) (see Appendix E). The steady-state values of \( P_0(t), ..., P_8(t) \) are listed in Table 11 on page 110.

The system availability function is

\[
A_s(t) = (0.75636568 + \delta \times 0.05454544) \\
+ (0.00399961 - \delta \times 0.00407276) \exp(-0.08803483t) \\
+ (0.06987688 + \delta \times 0.00641159) \exp(-0.01139613t) \\
+ (0.02788936 + \delta \times 0.00303784) \exp(-0.01795418t) \\
+ (0.00412090 - \delta \times 0.00425915) \exp(-0.06822615t) \\
+ (0.05971201 - \delta \times 0.06456675) \exp(-0.05396464t) \\
+ (0.03821765 + \delta \times 0.01445579) \exp(-0.04230362t) \\
+ (0.02792690 + \delta \times 0.00687589) \exp(-0.03716466t) \\
+ (0.01189102 - \delta \times 0.01242789) \exp(-0.06204509t). \]

For \( \delta = 0.5 \), Figure 18 illustrates the availability of the system as a function of time. The system availability decreases continuously with time and approaches a steady-state value (78.36%) after about five hours.
Figure 18. The system availability function of Model C (Case study 2)
Model D:

The state probabilities of the system were obtained as below:

\[
\begin{bmatrix}
P_0(t)
\ P_1(t)
\ P_2(t)
\ P_3(t)
\ P_4(t)
\ P_5(t)
\ P_6(t)
\ P_7(t)
\ P_8(t)
\ P_9(t)
\end{bmatrix}^T
= \mathbf{K}_1 + \mathbf{K}_2 \exp(-0.01141054t) + \mathbf{K}_3 \exp(-0.01797828t)
+ \mathbf{K}_4 \exp(-0.06060496t) + \mathbf{K}_5 \exp(-0.04383036t)
+ \mathbf{K}_6 \exp(-0.04611703t) + \mathbf{K}_7 \exp(-0.03796075t),
\]

where \( \mathbf{K}_i \) represents the \( i \)th column of matrix \( \mathbf{K}_d \) (see Appendix E). The steady-state probabilities are listed in Table 11 on page 110, along with those of Model C.

The system availability function is given by

\[
A_s(t) = (0.75437831 + \delta \times 0.05322381)
+ (0.07032870 + \delta \times 0.00624850) \exp(-0.01141054t)
+ (0.02895384 + \delta \times 0.00302374) \exp(-0.01797828t)
+ (0.08530266 - \delta \times 0.06428979) \exp(-0.06060496t)
+ (0.19721917 + \delta \times 0.06654971) \exp(-0.04383036t)
- (0.17438267 + \delta \times 0.07331193) \exp(-0.04611703t)
+ (0.03820001 + \delta \times 0.00855597) \exp(-0.03796075t).
\]

For \( \delta = 0.5 \), Figure 19 illustrates the availability of the system as a function of time. The system availability decreases continuously with time and gradually approaches a steady-state value of 78.10\%.
Figure 19. The system availability function of Model D (Case study 2)
Model E:

The system's state probabilities under this model are given by

\[
\begin{bmatrix}
P_0(t) & P_1(t) & P_2(t) & P_3(t) & P_4(t) & P_5(t) & P_6(t) & P_7(t) & P_8(t) & P_9(t)
\end{bmatrix}^T
\]

\[
= K_1 + K_2 \exp(-0.06229960t) + K_3 \exp(-0.04215081t)
+ K_4 \exp(-0.03720451t) + K_5 \exp(-0.03288464t)
+ K_6 \exp(-0.01139646t) + K_7 \exp(-0.01795472t),
+ K_8 \exp(-0.01003766t) + K_9 \exp(-0.01716089t),
\]

where \( K_i \) represents the \( i \)th column of matrix \( K_e \) (see Appendix E). The steady-state values of these probabilities are also listed in Table 11, along with those of Models C and D.

The availability function is

\[
A_s(t) = (0.75371940 + \delta \times 0.05715113)
+ (0.06412556 - \delta \times 0.06698036) \exp(-0.06229960t)
+ (0.04142885 + \delta \times 0.00653561) \exp(-0.04215081t)
+ (0.03421362 + \delta \times 0.00221975) \exp(-0.03720451t)
+ (0.00588066 - \delta \times 0.00557178) \exp(-0.03288464t)
+ (0.07095998 + \delta \times 0.00534689) \exp(-0.01139646t)
+ (0.02880769 + \delta \times 0.00215680) \exp(-0.01795472t)
+ (0.00045328 - \delta \times 0.00045121) \exp(-0.01003766t)
+ (0.00041096 - \delta \times 0.00040683) \exp(-0.01716089t).
\]
Figure 20. The system availability function of Model B (Case study 2)
Table 11. Limiting State Probabilities of Models C, D, & E (Case Study 2)

<table>
<thead>
<tr>
<th>Probability</th>
<th>Model C</th>
<th>Model D</th>
<th>Model E</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0</td>
<td>0.75636568</td>
<td>0.75437831</td>
<td>0.75371940</td>
</tr>
<tr>
<td>P1</td>
<td>0.09952958</td>
<td>0.10027846</td>
<td>0.09358794</td>
</tr>
<tr>
<td>P2</td>
<td>0.02509546</td>
<td>0.02543469</td>
<td>0.02373770</td>
</tr>
<tr>
<td>P3</td>
<td>0.04496302</td>
<td>0.04606520</td>
<td>0.04299176</td>
</tr>
<tr>
<td>P4</td>
<td>0.05454544</td>
<td>0.05322381</td>
<td>0.05715113</td>
</tr>
<tr>
<td>P5</td>
<td>0.01660670</td>
<td>0.01651785</td>
<td>0.01665592</td>
</tr>
<tr>
<td>P6</td>
<td>0.00115976</td>
<td>0.00136548</td>
<td>0.00709635</td>
</tr>
<tr>
<td>P7</td>
<td>0.00044344</td>
<td>0.00058326</td>
<td>0.00179992</td>
</tr>
<tr>
<td>P8</td>
<td>0.00129092</td>
<td>0.00215295</td>
<td>0.00325987</td>
</tr>
</tbody>
</table>
Figure 21. Comparison of system availabilities for different $\delta$ values (Case 2)
For $\delta = 0.5$, Figure 20 on page 109 shows the variations of the system availability as a function of time. The availability of the system decreases continuously with time and approaches a steady-state value (78.23%) after about five hours.

With respect to different $\delta$ values, the system availabilities from Models C, D and E can be given in Fig. 21. The system under the multiple repair policy (Model C) demonstrates the highest availability for any $\delta$ values. When $\delta > 0.19$, the system served by the priority repair policy (Model E) has a higher availability than the system under FCFS (Model D), but the opposite is true if $\delta < 0.19$. The difference between the estimated availabilities and the actual availability can be explained by the selection of $\delta$ value. For the actual availability of 75.73%, this system's $\delta$ value is estimated between 0 and 0.1.

5.4 Case Study 3

5.4.1 Data inputs

The data for this case study were collected from a longwall mine in Virginia. The seam mined is a part of the Pocahontas #3 seam. The average thickness of the seam is 72 inches. The average depth of the overburden is 1,500 feet. The roof is shale with 0 - 3 inches draw slate, and the floor is firm shale. Horizontal cleats exist at the face, but water was not found. The mining face spalls well.

This four-entry retreatting system has a panel length of 5,200 feet and a face
Table 12. MTTF and MTTR Data of Case Study 3

<table>
<thead>
<tr>
<th>Category</th>
<th>MTTF (min.)</th>
<th>MTTR (min.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal cutting</td>
<td>947.7180</td>
<td>44.2857</td>
</tr>
<tr>
<td>Face conveying</td>
<td>3324.8700</td>
<td>51.1852</td>
</tr>
<tr>
<td>Outby haulage</td>
<td>634.7520</td>
<td>30.8125</td>
</tr>
<tr>
<td>Roof support</td>
<td>1418.4200</td>
<td>47.2979</td>
</tr>
<tr>
<td>System</td>
<td>172.5800</td>
<td>39.1835</td>
</tr>
</tbody>
</table>

Table 13. Component Failure and Repair Rates of Case Study 3

<table>
<thead>
<tr>
<th>Coal cutting</th>
<th>Failure rate</th>
<th>Repair rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal cutting</td>
<td>0.00105517</td>
<td>0.02258065</td>
</tr>
<tr>
<td>Face conveying</td>
<td>0.00030076</td>
<td>0.01953690</td>
</tr>
<tr>
<td>Outby haulage</td>
<td>0.00157542</td>
<td>0.03245436</td>
</tr>
<tr>
<td>Roof support</td>
<td>0.00070501</td>
<td>0.02114259</td>
</tr>
</tbody>
</table>

Note: The failure and repair rates are in number of occurrences per minute.
length of 620 feet. The average production at this face is about 1,500 tons per shift. The shearer used is a JOY 1LS–6 DERS of 500 hp, with Eicotrak haulage. The face conveyor is a double inboard 30-mm chain conveyor made by American Longwall, powered by two 300 hp motors. Westfalia Lunen four-legged shields are used for roof support, each with a yielding capacity of 580 tons. Manual controls are used for moving the supports. All face equipment is powered by 950 volts.

Data of 245 production shifts were collected at this panel. As the basic inputs to those availability models, the MTTF's and MTTR's, and the failure and repair rates for each subsystem are listed respectively in Tables 12 and 13. For the roof-support subsystem, given its MTTF = 1418.42 minutes, MTTR = 47.30 minutes, $\lambda_5/\lambda_4 = 0.2$, $\lambda_6/\lambda_4 = 0.4$, and $T = 20$ minutes, the transition rates between its operating, partially operating, and failed states were estimated as follows: $\lambda_4 = 0.00332300$, $\lambda_5 = 0.00066460$, $\lambda_6 = 0.00132920$, $\mu_4 = 0.05000000$, $\mu_5 = 0.02114259$. The achieved availability of the system is estimated to be 81.50%.

5.4.2 Solutions of Models A and B

Model A:

According to Model A, the reliability and availability functions for this system are given by

\[ R_s(t) = \exp(-0.00363636t), \]

\[ A_s(t) = 0.86835908\left[1 + 0.04672877 \times \exp(-0.02363582t)\right] \]
\[ \times \left[ 1 + 0.01539465 \times \exp(-0.01983766t) \right] \times \left[ 1 + 0.04854258 \times \exp(-0.03402978t) \right] \times \left[ 1 + 0.03334548 \times \exp(-0.02184760t) \right]. \]

The variation of the system availability with time is illustrated in Fig. 22. The steady-state availability is 86.84%. Other steady-state results of this model are given in Table 14. As shown in this table, from high to low, the order of component importance to system unavailability is: the outby–haulage, the coal–cutting, the roof–support, and the face–conveying subsystems.

**Model B:**

The system reliability function has the same form as that represented by Model A, that is:

\[ R_B(t) = \exp(-0.00363636t). \]

For transition matrix: \( A_B \) given in Appendix F, the state probabilities of the system were obtained as follows (on page 117):
Figure 22. System Availability Functions of Models A & B (Case Study 3)
Table 14. Steady-State Results of Models A and B (Case Study 3)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model A</th>
<th>Model B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_s$</td>
<td>0.86835908</td>
<td>0.87411710</td>
</tr>
<tr>
<td>$\lambda_s$ (1/min.)</td>
<td>0.00363636</td>
<td>0.00363636</td>
</tr>
<tr>
<td>MTTF$_s$ (min.)</td>
<td>275.00042104</td>
<td>275.00042104</td>
</tr>
<tr>
<td>MTTR$_s$ (min.)</td>
<td>41.68932950</td>
<td>39.60321878</td>
</tr>
<tr>
<td>$N_f$ (1/min.)</td>
<td>0.00315766</td>
<td>0.00317860</td>
</tr>
<tr>
<td>$P_1$</td>
<td>0.03819840</td>
<td>0.04084642</td>
</tr>
<tr>
<td>$P_2$</td>
<td>0.01088804</td>
<td>0.01345672</td>
</tr>
<tr>
<td>$P_3$</td>
<td>0.05703221</td>
<td>0.04243190</td>
</tr>
<tr>
<td>$P_4$</td>
<td>0.02552228</td>
<td>0.02914786</td>
</tr>
</tbody>
</table>
\[ \begin{bmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \end{bmatrix} = K_1 + K_2 \exp(-0.03433770t) + K_3 \exp(-0.02382301t) \\
+ K_4 \exp(-0.01968930t) + K_5 \exp(-0.02150085t), \]

where \( K_i \) stands for the \( i \)th column of matrix \( K_b \) (see Appendix F). The limiting values of these state probabilities are presented in Table 14. The system's availability function is given by:

\[ A_s(t) = 0.06369560 \times \exp(-0.03433770t) + 0.05123491 \times \exp(-0.02382301t) \\
+ 0.00378490 \times \exp(-0.01968930t) + 0.00716752 \times \exp(-0.02150085t) \\
+ 0.87411710. \]

The variation of the system availability with time is illustrated in Fig. 22, with a steady-state value of 87.41%. Other steady-state results of this model are given in Table 14, along with those of Model A. As shown in Table 14, the order of component importance to system unavailability is the same as given by Model A.

### 5.4.3 Solutions of Models C, D and E

**Model C:**

The transition matrix for this Markov problem, \( A_c \), is given in Appendix F.
The state probabilities of the system were obtained as follows:

\[
\begin{bmatrix}
P_0(t) & P_1(t) & P_2(t) & P_3(t) & P_4(t) & P_5(t) & P_6(t) & P_7(t) & P_8(t) & P_9(t) 
\end{bmatrix}^T
= K_1 + K_2 \exp(-3.08453270t) + K_3 \exp(-0.07373397t)
+ K_4 \exp(-0.06977419t) + K_5 \exp(-0.05404193t)
+ K_6 \exp(-0.03434817t) + K_7 \exp(-0.02383584t)
+ K_8 \exp(-0.01968828t) + K_9 \exp(-0.02151083t),
\]

where \(K_i\) represents the \(i\)th column of matrix \(K_C\) (see Appendix F). The steady-state probabilities are listed in Table 15 on page 125.

The availability function of the system is given by

\[
A_\delta(t) = (0.82279835 + \delta \times 0.05136727)
+ (0.00318190 - \delta \times 0.00321789) \exp(-0.08453270t)
+ (0.00241782 - \delta \times 0.00245138) \exp(-0.07373397t)
+ (0.00047131 - \delta \times 0.00047845) \exp(-0.06977419t)
+ (0.06017488 - \delta \times 0.06161453) \exp(-0.05404193t)
+ (0.05436784 + \delta \times 0.01000392) \exp(-0.03434817t)
+ (0.04660437 + \delta \times 0.00535219) \exp(-0.02383584t)
+ (0.00339369 + \delta \times 0.00033978) \exp(-0.01968828t)
+ (0.00658935 + \delta \times 0.00069908) \exp(-0.02151083t).
\]

For \(\delta = 0.5\), Figure 23 illustrates the availability of the system as a function of time. The system availability decreases continuously with time and gradually approaches a steady-state value (84.85%) after about three hours.
Figure 23. The system availability function of Model C (Case study 3)
Model D:

The state probabilities of the system were given by

\[
\begin{bmatrix}
P_1(t) & P_2(t) & P_3(t) & P_4(t) & P_5(t) & P_6(t) & P_7(t) & P_8(t) \\
\end{bmatrix}^T
= K_1 + K_2 \exp(-0.05870580t) + K_3 \exp(-0.03499314t)
+ K_4 \exp(-0.02390012t) + K_5 \exp(-0.01969394t)
+ K_6 \exp(-0.02149809t) + K_7 \exp(-0.04810292t),
\]

where \(K_i\) represents the \(i\)th column of matrix \(K_d\) (see Appendix F). The steady-state probabilities are given in Table 15 on page 125. The availability function of this system is

\[
A_s(t) = (0.82131497 + \delta \times 0.05029860)
+ (0.07241132 - \delta \times 0.05413037) \exp(-0.05870580t)
+ (0.07829177 + \delta \times 0.01350278) \exp(-0.03499314t)
+ (0.04768938 + \delta \times 0.00521968) \exp(-0.02390012t)
+ (0.00346427 + \delta \times 0.00033303) \exp(-0.01969394t)
+ (0.00620447 + \delta \times 0.00062930) \exp(-0.02149809t)
- (0.02937618 + \delta \times 0.01585302) \exp(-0.04810292t).
\]

For \(\delta = 0.5\), Figure 2c illustrates the availability of the system as a function of time. After about three hours of operation, the system gradually approaches its steady-state availability (84.65%).
Figure 24. The system availability function of Model D (Case study 3)
Model E:

The state probabilities of this system were obtained by

\[
\begin{bmatrix}
P_0(t) & P_1(t) & P_2(t) & P_3(t) & P_4(t) & P_5(t) & P_6(t) & P_7(t) & P_8(t) & P_9(t)
\end{bmatrix}^T
\]

\[= K_1 + K_2 \exp(-0.06014536t) + K_3 \exp(-0.03433794t) + K_4 \exp(-0.03075735t) + K_5 \exp(-0.02382541t) + K_6 \exp(-0.01968961t) + K_7 \exp(-0.02157003t) + K_8 \exp(-0.02181752t) + K_9 \exp(-0.01932270t),
\]

where \( K_i \) represents the \( i \)th column of matrix \( K \) (see Appendix F). The steady-state probabilities are provided in Table 15, along with those of Models C and D. The system availability function is given by

\[
A_s(t) = (0.82096843 + \delta \times 0.05314866) + (0.05607058 - \delta \times 0.05716475) \exp(-0.06014536t) + (0.05982348 + \delta \times 0.00401677) \exp(-0.03433794t) + (0.00318637 - \delta \times 0.00332114) \exp(-0.03075735t) + (0.04819129 + \delta \times 0.00355697) \exp(-0.02382541t) + (0.00357803 + \delta \times 0.00019555) \exp(-0.01968961t) + (0.00678514 + \delta \times 0.00308564) \exp(-0.02157003t) + (0.00094543 - \delta \times 0.00311285) \exp(-0.02181752t) + (0.00045085 - \delta \times 0.00040484) \exp(-0.01932270t).
\]

For \( \delta = 0.5 \), the availability of the system is shown in Fig. 25 as a function of time. The steady-state availability is 84.75%.
Figure 25. The system availability function of Model E (Case study 3).
Table 15. Limiting State Probabilities of Models C, D & E (Case Study 3)

<table>
<thead>
<tr>
<th>Probability</th>
<th>Model C</th>
<th>Model D</th>
<th>Model E</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₀</td>
<td>0.82279885</td>
<td>0.82131497</td>
<td>0.82096843</td>
</tr>
<tr>
<td>P₁</td>
<td>0.04010209</td>
<td>0.04072953</td>
<td>0.03836299</td>
</tr>
<tr>
<td>P₂</td>
<td>0.01323514</td>
<td>0.01341802</td>
<td>0.01263836</td>
</tr>
<tr>
<td>P₃</td>
<td>0.04145287</td>
<td>0.04231042</td>
<td>0.03985197</td>
</tr>
<tr>
<td>P₄</td>
<td>0.05136727</td>
<td>0.05029860</td>
<td>0.05314866</td>
</tr>
<tr>
<td>P₅</td>
<td>0.02909338</td>
<td>0.02897955</td>
<td>0.02914784</td>
</tr>
<tr>
<td>P₆</td>
<td>0.00074677</td>
<td>0.00106147</td>
<td>0.00248358</td>
</tr>
<tr>
<td>P₇</td>
<td>0.00022217</td>
<td>0.00030256</td>
<td>0.00081819</td>
</tr>
<tr>
<td>P₈</td>
<td>0.00098145</td>
<td>0.00158483</td>
<td>0.00257998</td>
</tr>
</tbody>
</table>
Figure 26. Comparison of system availabilities for different $\delta$ values (Case 3)
With respect to different $\delta$ values, the system availabilities from Models C, D and E are compared in Fig. 26. The system under the multiple repair policy (Model C) demonstrates the highest availability for any $\delta$ values. When $\delta > 0.05$, the system served by the priority repair policy (Model E) has a higher availability than the system under FCFS (Model D), but the opposite is true if $\delta < 0.05$. The difference between the availabilities given in Figures 23, 24 and 25 and the system's actual availability can be explained by the selection of the $\delta$ value. Given an actual availability of 31.5%, the system's $\delta$ value must be close to 0.

5.5 Discussions and Comparisons of the Results

For each of the case studies, the system availability is assessed and predicted under different rules of system operation. Compared with the figures for the achieved system availability, the ranges given by the availability models are quite close. According to these case studies, the results from different models are compared in two groups.

5.5.1 Comparisons of Model A and Model B

i) Both models give identical values for system reliability, failure rate, and MTTF’s. For a series-structure system whose components fail independently, its reliability is the product of the reliabilities of its components, and its failure rate is the sum of the failure rates of individual components.

ii) If a system is initially operable, the system's availability will decrease
gradually with time and approaches a steady-state value after several hours of continuous operation. However, Model A shows a lower availability, both transient and steady-state, than Model B. The difference between the steady-state availabilities from both models varies from 1 to 4%. This implies that when one of the components in a longwall system is under repair, this system would have a higher availability if other operating components are suspended.

iii) The MTTR of the system represented by Model A is longer. This is due to the fact that for the system represented by Model A, more than one failure could contribute to a single renewal of the system. Suppose that component 1 fails at time \( t_1 \), and its TTR is \( r_1 \). During the repair of this component, component 2 fails at time \( t_2 \), with a TTR equal to \( r_2 \). Then two cases could occur as below:

1. If \( t_1 + r_1 < t_2 + r_2 \), the TTR of the system (\( TTR_s \)) is given by \( (t_2 - t_1) + r_2 \). Since \( t_2 \geq t_1 \), this \( TTR_s \) must be greater than both \( r_1 \) and \( r_2 \). Thus, in this case, the TTR of the system is longer than the TTR of each individual component failure.

2. If \( t_1 + r_1 \geq t_2 + r_2 \), \( TTR_s = r_1 \), which is equal to the TTR of the component which failed first.

Thus, the MTTR of the system modeled by Model A must be greater than or at least equal to that under Model B.

iv) The long-run expected average of the number of system failures is slightly smaller for Model A. Under the assumption of independent component failures, more than one failure could contribute to a single system failure. As a result, the number of system failures is less than the number of component failures. In contrast, according to Model B, if other components are suspended during the
repair of a failed component, the number of system failures should be nearly equal to the number of component failures.

v) These two models give different pictures of the limiting probabilities of system failure due to any particular component. Not only are the exact values different, but also the order of the relative importance between individual components may vary. For example, in Case study 1, Model A gives the order of component contribution to system unavailability as the coal-cutting, the roof-support, the outby-haulage, and the face-conveying subsystems. In contrast, the order given by Model B is the coal-cutting, the roof-support, the face-conveying, and the outby-haulage subsystems. The cause for such a variation is due to the theories employed in determination of the system-state probabilities. According to Model A, the contribution of a component to system unavailability is mainly dependent on the ratio between its failure rate and the system failure rate, i.e. $\lambda_i/\lambda_s$. However, the ratio of the failure rate and the repair rate of a component is used in Model B to evaluate this contribution. Since both the failure rate and the repair rate are considered, Model B provides a more accurate estimate for component importance.

5.5.2 Comparisons of Models C, D, and E

i) As shown by the results from all these three models, system availabilities decrease continuously with time and approaches a steady-state value after three to five hours of operation.

ii) For each case study, the systems under the multiple repair policy
demonstrate a higher availability than under the FCFS and the priority policies.

iii) Compared with those systems served by the multiple and the FCFS repair policies, the systems under the priority repair policy are more likely to stay in those states in which the roof-support subsystem is partially operating, i.e. States 4, 6, 7, and 8. This is due to the fact that during the period when the roof-support subsystem is partially operating, any failures occurring to the other components will be served first under the priority repair policy.

iv) These three models give an identical order of component importance to system unavailability. This order of component importance for each case study agrees with that given by Model B.

v) When a system is subject to partial failures of the roof supports, the system's availability is also a function of $\delta$ — the derating factor of system performance when the roof-support subsystem is partially operating. The availability is the highest when $\delta = 1$, and lowest when $\delta = 0$. In both cases the roof-support subsystem is reduced to a two-state component. If $\delta$ is close to 1, the system under the priority repair policy will have a higher availability than that under the FCFS repair policy. However, when $\delta$ is close to 0, the reverse will be true.

vi) If the roof-support subsystem is quite reliable (MTTF is large), compared to other subsystems, there is no significant difference in system availabilities due to the selection of the repair policies. Thus, the policy which has the lowest maintenance cost should be employed in this case.
VI. APPLICATIONS OF THE RESULTS

6.1 Introduction

Computing the availability or reliability of a system is not generally the final stage in systems analysis. The methodology of system availability analysis developed above is very useful for planning and evaluation of longwall operations. Three important applications for this methodology include: component importance and parametric analysis, assessment and prediction of system productivity, and optimization of system operational effectiveness. In the following, only methodologies to illustrate these applications are developed. Specific examples are not given since the task is beyond the scope of this research.

6.2 Component Importance and Parametric Analysis

At system development and operation stages, the management of a longwall mining operation often encounters the following questions:

- Which parameters of the system have the greatest effect on system availability and production?
- Which subsystems or components must be improved and with what priority
in order to increase system availability and reliability?

- Which component must receive priority repair when the system is down?

Providing answers to these questions is important for successful management of longwall mining operations.

In order to answer these questions, it is necessary to take account of the importance of a particular component for system availability. In systems engineering, a component's contribution to system failure is termed as its importance. The importance factors are thus a function of component failure and repair characteristics, system configuration, and time. Knowing the relative importance of components in a longwall system is a valuable tool for upgrading system reliability and safety. Inspection, maintenance, and failure detection may be carried out according to their order of importance for the performance of the overall system. The composition of the repair crew and facilities can be determined accordingly. Systems can be upgraded by improving components with relatively large importance. If cost optimization is the objective, the importance indices provide valid weighting factors for optimization of this improvement.

In the following, a set of importance factors are given for evaluation of the importance of each subsystem in a longwall system. These factors can be formulated explicitly based upon the availability functions developed in Chapter 4. Though transient availabilities are considered in some of the functions, they can be replaced by their corresponding steady-state availability functions, if necessary. No economic considerations will be introduced at this stage.
6.2.1 Parameter importance factor

The parameter importance factor, \( PI_j \), can be used to find the parameters which have the greatest effect on system availability. It is defined as

\[
PI_j = \frac{\partial \bar{A}_s}{\partial x_j},
\]

where \( x_j \) can be any parameters of the system or its subsystems, such as the failure rate and repair rate. \( \bar{A}_s \) is system unavailability, which equals \( 1 - A_s \).

6.2.2 Marginal importance factor

The Marginal importance factor, \( B_{i}(t) \), is defined as the partial derivative of system unavailability \( \bar{A}_s \) with respect to the unavailability of its components (Birnbaum, 1969); that is:

\[
B_{i}(t) = \frac{\partial \bar{A}_s}{\partial \bar{A}_i}(t),
\]

where \( \bar{A}_i(t) \) is the unavailability function of component \( i \), given by \( 1 - A_i(t) \). This factor enables the variations of system unavailability to be measured as a function of the unavailability of any component in the system. Thus, the sensitivity of the system's availability with respect to the availabilities of individual subsystems can be assessed.
6.2.3 Critical importance factor

The critical importance factor is the probability of a component failing, given that the system has failed (Lambert, 1975). For the longwall system, it can be expressed as

\[ C_i(t) = \frac{\overline{A}_i}{\overline{A}_s} \times \frac{\partial \overline{A}_s}{\partial A_i}(t). \]  

(6.3)

This importance factor may provide satisfactory answers to the first two questions mentioned at the beginning of this section when system availability is of interest.

6.3 Productivity Assessment and Prediction

The availability functions are useful in assessment and prediction of system productivity. By incorporating the availability function of a longwall mining system into its potential production rate, simple analytical models can be formulated to represent the system's productivity measures as functions of system availability, as well as reliability and maintainability parameters of major equipment components. The system's productivity can then be assessed and predicted based on different management policies, design principles, and types of equipment used. As examples, three indices are defined below in order to quantify the effect of the unavailability of longwall systems on its production.
6.3.1 Lost production per shift

\[ LQ = (1 - A_s) \times Q_r \times T_a, \]  

where \( A_s \) is system availability, \( Q_r \) is production rate, or potential production capacity of the system, and \( T_a \) denotes the time available for producing coal, which is the total shift time minus necessary or scheduled delays.

6.3.2 Lost production shifts per year

\[ LPS = (1 - A_s)N_s, \]  

where \( N_s \) is the average number of working shifts per year. Actually, LPS reflects the total downtime of the system in a year.

6.3.3 Shift production

\[ Q_s = Q_c \times N_c, \]  

where \( Q_c \) denotes average production per cycle and \( N_c \) is the number of cycles completed per shift, which can be determined as below.

Production per cycle:

\[ Q_c = L \times m \times w \times s \times r, \]  

(6.7)
where \( L \), \( m \), and \( w \) are the face length, the seam thickness, and the web depth in feet, respectively; \( s \) is the specific weight of coal in tons/ft\(^3\); and \( r \) is the recovery rate within the face.

**Number of cycles per shift:**

\[
N_c = \frac{T_a}{T_c},
\]

(6.8)

where \( T_c \) stands for the time required for completing a cutting cycle, which depends on the mechanism of the coal-cutting machine. For bidirectional cutting,

\[
T_c = \frac{L}{V \times A_s} + T_t,
\]

(6.9)

where \( V \) is the haulage speed of the shearer, and \( T_t \) denotes the turning around time of the shearer at the top and bottom ends of the face.

### 6.4 Optimization of System Operational Effectiveness

After evaluating the performance of a longwall system, one is faced with choices which can lead to improvements in its operational effectiveness. Approaches to improvement requiring no capital investment should, of course, be made first. Most changes, however, will require a cash outlay. Management must, then, be convinced of the cost effectiveness of any proposed plan. The measures for achieving a certain level of system availability have to be economically feasible.
The sensitivity analysis of various parameters of longwall equipment should be supported by cost–benefit analysis in order to obtain an economically optimal longwall operation. Consequently, the operations of longwall mining must be optimized with respect to both production and cost control.

In this section, based on an analysis of life–cycle costs for longwall operations, the major costs incurred in achieving a certain level of system availability are examined. Two optimization models are then formulated by incorporating cost factors into system availability functions.

6.4.1 Cost categories and functions

For a longwall mining system, the total system cost includes the cost of purchasing and installation, and operating cost. In performing a systems analysis, consideration should be made to the costs incurred not only in system operation stage, but also in the system acquisition stage. Since a modern longwall system is highly expensive to develop, a systematic analysis of its life–cycle costs is particularly important to successfully operating a longwall face. The costs incurred in achieving a certain level of system availability are analyzed below.

The concept of availability includes reliability as well as maintenance. The cost incurred in designing reliability and maintainability into a system is a major part of the capital cost. Besides corrective maintenance (CM), preventive maintenance (PM) may be another measure to increase system availability or reduce the total maintenance costs. An optimal PM schedule should be decided in order to minimize the cost of maintenance while maintaining the highest possible
availability under given conditions. Overall, in achieving a certain level of system availability, the cost incurred for each subsystem consists of four components:

**Reliability and maintainability apportioning:**

This is the cost of designing for a failure rate $\lambda$ and repair rate $\mu$ of individual units into the subsystem. It has the following structure:

$$C_d = f_i(\lambda) + g_i(\mu).$$  \hspace{1cm} (6.10)

**Unit cost of corrective maintenance:**

$$C_{cm} = h_i(\mu).$$  \hspace{1cm} (6.11)

**Unit cost of preventive maintenance:**

$$C_{pa} = l_i(\mu).$$  \hspace{1cm} (6.12)

**Opportunity cost:**

This function represents the cost incurred due to lost production when the system is idled. It is given by

$$C_{op} = m_i(\mu) + n_i(M_{pt}),$$  \hspace{1cm} (6.13)

where $M_{pt}$ is the average duration of preventive maintenance actions.
If the improvement efforts for system availability are nonduplicative, that is, the changing of the failure rate of a component does not affect the repair time of that component and vice versa, the cost functions for each subsystem can be summed up to obtain the total cost for the system. For the mission time of $t$, the total system cost is then given by

$$C(t) = \sum_{i=1}^{n} [f_i(\lambda_i) + g_i(\mu_i)] + \sum_{i=1}^{n} t \times \lambda_i h_i(\mu_i) + m_i(\mu_i)] + \frac{t}{I} \left[ \sum_{i=1}^{n} 1_{i}(\mu_i) + n(M_{pl}) \right],$$

(6.14)

where $I$ is the time interval for preventive maintenance.

In Equation (6.14) each subsystem is given the weighting factor ($t \times \lambda_i$) for its corrective maintenance cost, since the subsystem with a smaller failure rate will need less corrective maintenance. Most of the theoretical studies on optimal maintenance scheduling find an optimal $I$ for an infinite time period of operation, but many practical cases may not be so. Thus, a finite mission time $t$ is considered in this equation in order to make the model more flexible.

6.4.2 Model formulation

If system availability is examined with various costs incurred, the system's operational effectiveness can be optimized. Assuming that each subsystem has a minimum requirement for reliability and maintainability, the optimization models
can be formulated in two ways. This type of nonlinear optimization problem can be solved by the Lagrange multipliers, dynamic programming, or sequential unconstrained minimization techniques.

Maximization of system availability subject to budget $C_0$:

Maximize $A_s$

subject to:

$C(t) \leq C_0$

$l_i \leq \lambda_i \leq m_i$, for $i = 1,...,n$

$f_i \leq \mu_i \leq g_i$, for $i = 1,...,n$

$v \leq I \leq w$

determine $I$, and $\lambda_i, \mu_i$, for $i = 1,...,n$,

where $l_i, m_i, f_i, g_i, v, w$ are constants. Since each subsystem may have a minimum requirement for reliability and maintainability, these parameters are assumed in order to generate reasonable solutions.

This model is useful during preliminary system design analysis. Since the cost of purchasing and installation is related to the design cost of the equipment, such an optimization model may be used in the selection of equipment from different manufacturers.
Minimization of the total cost while availability requirement $A_0$ is met:

Minimize $C(t)$

subject to:

$A_s \geq A_0$

$l_i \leq \lambda_i \leq m_i$, for $i = 1, \ldots, n$

$f_i \leq \mu_i \leq g_i$, for $i = 1, \ldots, n$

$v \leq I \leq w$

determine $I$, and $\lambda_i$, $\mu_i$, for $i = 1, \ldots, n$.

where $l_i$, $m_i$, $f_i$, $g_i$, $v$, and $w$ are constants.

This model is useful during final design or redesign stage of a longwall system when availability requirements have been set.
VII. SUMMARY AND CONCLUSIONS

7.1 Summary

In this dissertation, the techniques of reliability engineering and stochastic systems analysis have been applied to develop a methodology for longwall availability assessment and prediction. After a functional analysis of longwall mining operations, the longwall production system under study was decomposed into four subsystems: coal-cutting, face-conveying, outby-haulage, and roof-support. The operating characteristics of the system were analyzed based on the system configuration, component failure and repair processes, and rules of operation. The system's operating logic was then represented by reliability-block diagrams and state-transition diagrams.

Based on these system representations, five probability models were developed for availability analysis of longwall systems. Models A and B study the longwall production process as a series-configuration system. Independent component failure-repair processes and suspended animation were investigated respectively in either of the models. Models C, D and E assess the availability of longwall systems under partial failure of roof supports. These models were developed by incorporating the concept of three-state components into a Markov model. Three maintenance policies, multiple repair, First-Come-First-Serve (FCFS), and
priority repair, were examined respectively under these three models. Some measures of system effectiveness were obtained by solving these models, including system availability, reliability, the mean time to failure (MTTF), the mean time to repair (MTTR), and the long-run expected number of system failures.

Three case studies were presented to illustrate the implementation of the availability models. Each model was solved explicitly with respect to each case study. Both the transient and steady-state system availabilities were obtained. Explicit functions were given for system availability and reliability. The variation of system availability was examined as a function of time.

Three important applications of these modeling approaches have been identified: component importance and parametric analysis, assessment and prediction of system productivity, and optimization of system operational effectiveness. Models, functions, and indices were defined for improvement of longwall system performance.

7.2 Conclusions

The major purpose of a systems model is not the duplication of the real world, but the reduction of a real world situation into a manageable size and solvable problem. The model result is then a prediction of the long-run performance of a system of interest. Thus, the main emphasis of this study is on the techniques rather than the results since the operating characteristics and environment vary widely from mine to mine. On the basis of this study, the following conclusions
can be drawn:

i) The methodology presented in this dissertation will facilitate quantitative evaluation of the availability, reliability and productivity of longwall operations. The potential of its application to improvement of longwall operational effectiveness is evident. As an assessment tool, the modeling approaches can be used for system availability evaluation, optimization of repair policies, identification of major contributors to system delays, and selection of the best modes of system operation. Also, these methods can be used as a design tool for system design improvement and optimization of operational effectiveness for the system’s life cycle.

ii) The functional analysis of longwall systems indicates that the coal-winning machine, the face conveyor, and the outby haulage are three critical components for system availability. The roof supports, however, do not have such a direct influence on the performance of the overall system.

iii) As a subsystem, the roof supports’ effect on system performance can be modeled either as a three-state component or as a k-out-of-n structure, while the former is used in this study. According to the concept of three-state components, the states in which the roof-support subsystem stays are defined as operating, partially operating, and failed. When this subsystem is in the partially operating state, the overall system operates at a degraded level of performance. As indicated in the case studies, this definition can reflect the operating and failure characteristics of roof supports in the production process of longwall mining.
iv) The three case studies indicate that if a longwall system is initially operating normally, the system's availability will decrease gradually with time, and approach a steady-state value after several hours of continuous operation.

v) According to the results of these case studies, when one of the critical components in a longwall system is under repair, this system would have a higher availability if other operating components are suspended. The increase in steady-state availabilities ranges from 1 to 4%.

vi) The contribution of any subsystem to system unavailability is largely dependent on the ratio between that subsystem's failure rate and its repair rate, i.e. \( \lambda/\mu \). Thus, more attention should be given to items with high failure rates and/or long repair times.

vii) In general, the system served by the multiple repair policy demonstrates a higher availability than under the FCFS and the priority policies. However, if the roof-support subsystem is quite reliable (MTTF is large), compared to other subsystems, there is no significant difference in system availabilities due to the selection of the repair policies. In this case, the policy which leads to the lowest maintenance cost should be employed.

viii) Compared with those systems served by the multiple and the FCFS repair policies, the systems under the priority repair policy are more likely to stay in those states in which the roof-support subsystem is partially failed.

ix) If a longwall system is subject to the influence of the partial failures of the
roof supports, the system's availability is also a function of $\delta$ — the derating factor of system performance when the roof-support subsystem is partially operating. The availability is the highest when $\delta = 1$, and lowest when $\delta = 0$. In both cases the roof-support subsystem is reduced to a two-state component. If $\delta$ is close to 1, the system under the priority repair policy will have a higher availability than that under the FCFS repair policy. However, when $\delta$ is close to 1, the reverse will be true.

x) The productivity of a longwall operation depends on the system's availability, production rate, and the time available for production. The longwall production and downtime data collected indicate that a significant portion of shift time is lost to so-called inherent delays, such as travel time, lunch time, face checks, safety and production talks, etc. The amount of these delays ranges from 30 minutes to more than 70 minutes per shift (Dunlap, et al., 1989). These delays have to be reduced in order to increase the productivity of any longwall operation.

xi) In order to use the availability models developed, the times to failure and the repair times should be given for each of the subsystems. Currently, the only sources of operational level data are often foreman reports. Indeed, these reports can provide management with valuable information, but these data have to be implemented with time study information if a comprehensive and well-defined set of performance measures, such as availability, are to be developed. The data should be used with careful scrutiny to verify that the results of the analysis are reliable and valid measures for the operation under study. The period covered by the reports being analyzed must be large enough to render reliable statistics. On the other hand, it must be ascertained that there have been no significant changes
in methods, conditions, or procedures during this period. Another consideration should be that the foreman and crews may have an interest in overstating production and equipment downtime. Even if the data are accurate, the analyst should consider that underground mining systems are extremely difficult to supervise. The foreman cannot be expected to report all downtime, including the frequent delays of short duration which, as many studies indicated, typically account for as much as is reported on foreman reports. Thus, a methodology should be developed for systematic analysis of foreman reports. The work by Dunlap, et al. (1989) is a good starting point for this task.

7.3 Recommendations for Further Research

There are many other areas related to the availability and reliability of longwall mining systems. Much work can be done by extending the approaches developed in this dissertation. The following areas are identified for additional study.

i) This longwall availability study has been conducted at the system level. Down to the subsystem level, however, two questions should be asked: (1) Why and how can equipment failure happen? (2) How can equipment reliability be increased? In order to answer these questions, two types of systems analysis needs to be carried out for longwall components: fault tree analysis, and failure mode and effect analysis. These analyses should cover the following aspects:

- Examining the structural configuration of components in the system
- Constructing basic event trees to derive the possible accident chains for
component failures

- Estimating the probability of component failure based on the probability of the occurrence of the basic events
- Identifying component failure modes
- Detecting the weak points in system design
- Proposing possible design modifications

ii) The actual availability of a longwall system is affected by more factors than those covered in this study. Besides the availability of the system itself, environmental factors, organizational policies, human factors, and safety should be examined in order to increase the system's efficiency. Approaches need to be developed to study these intangible factors.

iii) Sensitivity analysis of the availability models needs to be conducted to estimate the effect of variations in the failure and repair rates to differences in mine maintenance programs and operating procedures. The uncertainty and wide variation of the failure rate, and the repair rate of longwall equipment suggest that such an analysis on the availability functions is a worthwhile study.

iv) In Chapter 6, importance functions, production models, and optimization models were formulated. After these models and indices are defined explicitly, it is possible to develop a complete decision-support system for longwall system availability management.
REFERENCES


Stefanko, R., 1983, Coal Mining Technology: Theory and Practice, SME of AIME, New York, N.Y.


APPENDICES

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APPENDIX A. GLOSSARY

A.1 Definitions

System partitioning

Three levels of system partition are used in this study:

- **System**: A group of subsystems especially integrated to perform a specific operational function or functions. In this study, the system includes the equipment, operating environment, personnel, and activities involved in coal production in a longwall operation.

- **Subsystem, Component, Unit**: A collection of parts which represent a self-contained element of a complete operating system and perform a function necessary to the operation of that system, e.g., the shearer, and the roof supports in longwall systems.

- **Part, Element**: The least subdivision of a system. Examples include the chains in AFC, hydraulic valves in roof supports, and drums in a shearer.

Failure events

A failure refers to any event which disables production equipment from mining and transporting coal. Equipment breakdown and environmental accidents are
examples of the failure events. But if the state of a longwall system is defined in
terms of the conditions affecting its production rate, the term "failure" can be used
to refer to any condition causing a decrease in the production rate.

**Failure rate**

The rate at which failures occur in a specified time interval is called the failure
rate for that period of time. In probabilistic terms, the instantaneous failure rate
at time \( t \), \( \lambda(t) \), can be defined as follows:

\[
\lambda(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \cdot \text{Prob}\{\text{System will fail during } [t, t+\Delta t], \text{ given that }
\text{no failure has occurred during } [0,t]\}.
\]

**Repair rate**

The repair rate of a component represents the number of failures corrected in a
unit period of time. The instantaneous repair rate at time \( t \), \( \mu(t) \), can be defined as

\[
\mu(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \cdot \text{Prob}\{\text{System will be repaired during } [t, t+\Delta t], \text{ given}
\text{that it has not failed during } [0,t]\}.
\]

**Downtime**

The downtime of a system refers to the total time from a system failure until
return to service. In this study this downtime will be considered as a random variable and referred to, for convenience, as *repair time*.

**Reliability**

The reliability of a system represents the probability that this system performs a given function for a definite period of time under specified conditions. The reliability function, $R(t)$, can be defined as

$$R(t) = \text{Prob \{System is operable during } [0,t] \}. $$

**Maintainability**

Maintainability denotes the probability that a failed system or unit will be restored to satisfactory condition in a specified period of time. The maintainability function, $M(t)$, is defined as

$$M(t) = \text{Prob \{System is repaired during } [0,t] | S \text{ is down at } t=0 \}. $$

This concept applies only to repairable systems or units.

**Availability**

The availability of a system is defined as the probability that the system is operating satisfactorily at any point in time under stated conditions. Depending on the time interval considered, availability can be classified into three functions:
- Instantaneous availability:

\[ A(t) = \text{Prob (System is operable at time } t) \].

- Average availability:

\[ A(T) = \frac{1}{T} \int_{0}^{T} A(t) \, dt. \]

- Steady-state availability:

\[ A = \lim_{t \to \infty} A(t). \]

Availability and reliability are equivalent in the case of non-repairable systems.

Mean time to failure (MTTF)

The mean time to failure of a system represents the expected value of its times to failure. This index can be estimated by

\[ \text{MTTF} = \frac{\text{total operating hours}}{\text{number of failures}}. \]
Mean time to repair (MTTR)

The mean time to repair of a system is the expected value of its times to repair. This measure can be calculated by

\[ MTTR = \frac{\sum (M_{ct})_i}{f_c} \]

where \((M_{ct})_i\) is active repair time per corrective maintenance action, and \(f_c\) is the total number of corrective maintenance actions. When preventive maintenance is performed in the system, MTTR is often termed as mean corrective maintenance time \((M_{ct})\).

System effectiveness

System effectiveness is a measure of the ability of a system to accomplish its objective. It is often defined as the probability that the system can successfully meet an operational demand within a given time when operated under specified conditions. Reliability, maintainability, availability, and efficiency are some of the commonly used measures of system effectiveness.
A.2 Notations

A \quad \text{Steady-state availability.}

A(t) \quad \text{Availability function.}

A(s) \quad \text{Laplace transform of } A(t).

A(T) \quad \text{Average availability during period } [0,T].

CM \quad \text{Corrective maintenance.}

F(t) \quad \text{Probability distribution function of failure times.}

F(s) \quad \text{Laplace transform of } F(t).

f(t) \quad \text{Probability density function of failure times.}

f(s) \quad \text{Laplace transform of } f(t).

G(t) \quad \text{Probability distribution function of repair times.}

G(s) \quad \text{Laplace transform of } G(t).

g(t) \quad \text{Probability density function of repair times.}

g(s) \quad \text{Laplace transform of } g(t).

M(t) \quad \text{Maintainability function.}

M(s) \quad \text{Laplace transform of } M(t).

MTBF \quad \text{Mean time between failures.}

MTTF \quad \text{Mean time to failure.}

MTTR \quad \text{Mean time to repair.}

P_i(t) \quad \text{The probability that the system is state } i \text{ at time } t.

P_i(s) \quad \text{Laplace transform of } P_i(t).

P_i'(t) \quad \text{The derivative of } P_i(t).

P_i'(s) \quad \text{Laplace transform of } P_i'(t).

PM \quad \text{Preventive maintenance.}
R(t)  Reliability function.
R(s)  Laplace transform of R(t).
TTF   Time to failure.
TTR   Time to repair.
λ(t)  Failure rate at time t.
μ(t)  Repair rate at time t.
A     Transition matrix.
K     Constant matrix for system state probabilities.
P     Probability vector [P_1(t), P_2(t), ⋯, P_n(t)].

For those functions and parameters which apply to both a system and its subsystems, subscript s is used to denote the system, while subscript i (i = 1, ⋯, n) is used for subsystem i.
APPENDIX B. BASIC CONCEPTS AND EQUATIONS OF
STOCHASTIC RELIABILITY MODELING

The fundamental concepts and basic relationships of reliability engineering pertinent to this dissertation are introduced below. Some equations, which are taken from named references, are presented here without including the details of their derivations. Those who are interested in a more thorough exposition of the subject are urged to peruse the given references (Pages and Gondran, 1986; Cox, 1963).

B.1 Failure Time Distribution, Reliability Function, and Failure Rate

Let X denote the time to failure of a unit, and assume that

\[ F(t) = P(X < t), \]

and

\[ f(t) = \frac{dF(t)}{dt}. \]

The reliability of the unit is given by
\[ R(t) = 1 - F(t). \]  \hfill (B.1)

Clearly, \( R(t) \) is decreasing with time, and \( \lim_{t \to \infty} R(t) = 0. \)

The failure rate of the unit is given by

\[ \lambda(t) = \frac{f(t)}{R(t)} \]  \hfill (B.2)

\[ = \frac{\frac{dF(t)}{dt}}{1 - F(t)} \]  \hfill (B.3)

\[ = -\frac{\frac{dR(t)}{dt}}{R(t)}. \]  \hfill (B.4)

Consequently, reliability function \( R(t) \) and probability density function \( f(t) \) can be expressed as the functions of failure rate \( \lambda(t) \):

\[ R(t) = \exp \left[ -\int_0^t \lambda(u)du \right], \]  \hfill (B.5)

\[ f(t) = -\frac{dR(t)}{dt} \]  \hfill (B.6)

\[ = \lambda(t)R(t) \]

\[ = \lambda(t)\exp \left[ -\int_0^t \lambda(u)du \right]. \]  \hfill (B.7)
As shown above, functions $f(t)$, $F(t)$, $R(t)$, and $\lambda(t)$ are closely related. If any
one of them is known, the other three can be determined by Equations (B.1 –
B.7).

The mean time to failure of the unit is given by

$$MTTF = \int_0^\infty t \cdot f(t) \, dt$$

(B.8)

$$=: -\int_0^\infty \left[ \frac{dR(t)}{dt} \right] dt = -t \cdot R(t) \bigg|_{t=0}^{t=\infty} + \int_0^\infty R(t) \, dt$$

$$=: \int_0^\infty R(t) \, dt.$$

(B.9)

For a physically realizable system, it must fail after a finite amount of operating
time, i.e. $MTTF < \infty$, so we must have $tR(t) \to 0$ as $t \to \infty$.

Since the Laplace transform of $\int_0^t R(u) \, du$ is

$$\left\{ \int_0^t R(u) \, du, \, s \right\} = \frac{R(s)}{s},$$

then we must have

$$MTTF = \lim_{t \to \infty} \int_0^t R(u) \, du$$
\[
= \lim_{s \to 0} R(s). \tag{B.10}
\]

### B.2 Repair Time Distribution, Maintainability Function, and Repair Rate

Let \( Y \) denote the time to repair of a unit, and assume that

\[
G(t) = P(Y < t),
\]

and

\[
g(t) = \frac{dG(t)}{dt}.
\]

The maintainability of the unit is then given by

\[
M(t) = G(t). \tag{B.11}
\]

Thus \( M(t) \) is an increasing function, and \( \lim_{t \to \infty} M(t) = 1. \)

The repair rate of the unit is given by

\[
\mu(t) = \frac{\frac{dM(t)}{dt}}{1 - M(t)}, \tag{B.12}
\]

and the following relationships exist:
\[ M(t) = 1 - \exp\left[ -\int_0^t \mu(u) \, du \right], \quad (B.13) \]

\[ g(t) = \mu(t)[1-M(t)] \]
\[ = \mu(t)\exp\left[ -\int_0^t \mu(u) \, du \right]. \quad (B.14) \]

Thus, if any of \( g(t) \), \( G(t) \), \( M(t) \), or \( \mu(t) \) is given, the other three are determined by Equations (B.11 – B.14).

The mean time to repair can be obtained as

\[ MTTR = \int_0^\infty [1-M(t)] \, dt. \quad (B.15) \]

But since the Laplace transform of \( MTTR \) is given by

\[ L \left\{ \int_0^t [1-M(t)] , s \right\} = \frac{1}{s} - M(s) / s, \]

then we obtain

\[ MTTR = \lim_{s \to 0} \left[ \frac{1}{s} - M(s) \right]. \quad (B.16) \]
B.3 Reliability and Availability Functions of One–unit Systems

Depending on the characteristics of the unit's failure and repair process, three cases are discussed separately, beginning with the simplest.

Constant failure rate $\lambda$ and repair rate $\mu$:

The reliability function, $R(t)$, and the maintainability function, $M(t)$, are given by

\[ R(t) = e^{-\lambda t}, \quad (B.17) \]

\[ M(t) = 1 - e^{-\mu t}. \quad (B.18) \]

From equations (B.8) and (B.15), it can be shown that

\[ MTTF = \frac{1}{\lambda}, \quad (B.19) \]

\[ MTTR = \frac{1}{\mu}. \quad (B.20) \]

If $A(0) = 1$, the availability function is given by

\[ A(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t}. \quad (B.21) \]

Consequently, the steady–state availability ($A$) is obtained as
\[ A = \lim_{t \to \infty} A(t) = \frac{\mu}{\lambda + \mu}. \quad (B.22) \]

Constant failure rate \( \lambda \), but general PDF \( g(t) \) for repair times:

The Laplace transform of availability function is given by (Pages and Gondran, 1986)

\[ A(s) = \frac{A_0 + (1-A_0)g(s)}{s + \lambda - \lambda \cdot g(s)}, \quad (B.23) \]

where \( A_0 = A(0) \), and \( g(s) \) is the Laplace transform of \( g(t) \). If the system is operating initially, that is, \( A(0) = 1 \), we have

\[ A(s) = \frac{1}{s + \lambda - \lambda \cdot g(s)}. \quad (B.24) \]

This equation, when inverted, will yield the time dependent pointwise availability availability \( A(t) \). Suppose

\[ \tau = \int_0^\infty t \cdot g(t) dt < \infty, \]

the steady-state availability of the system is given by

\[ A = A(\infty) = \frac{1}{1 + \lambda \tau}. \quad (B.25) \]
General failure and repair time distributions:

Let the PDF's of times to failure and repair times be \( f(t) \) and \( g(t) \), respectively. When \( A(\infty) = 1 \), the Laplace transform of \( A(t) \) is given by \( (\text{Pages and Gondran, 1986}) \)

\[
A(s) = \frac{1 - f(s)}{s[1 - f(s)g(s)]}, \quad (B.26)
\]

where \( f(s) \) and \( g(s) \) are Laplace transforms of \( f(t) \) and \( g(t) \), respectively. The inverse of this equation will give availability function \( A(t) \). However, for all density functions except for the simplest ones, the equation is very difficult to solve mathematically. Only asymptotic results are given here.

Assume that the following quantities are finite:

\[
\nu = \text{MTTF} = \int_0^\infty t \times f(t) \, dt, \quad (B.27)
\]

\[
\sigma_\nu^2 = \int_0^\infty (t-\nu)^2 f(t) \, dt, \quad (B.28)
\]

\[
\tau = \text{MTTR} = \int_0^\infty t \times g(t) \, dt \quad (B.29)
\]

\[
\sigma_\tau^2 = \int_0^\infty (t-\tau)^2 g(t) \, dt. \quad (B.30)
\]

The steady-state availability of the system is then obtained below by
manipulating Equation (B.26):

\[ A = \lim_{t \to \infty} A(t) = \frac{1}{1 + \rho} \quad \text{(B.31)} \]

\[ = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}}. \quad \text{(B.32)} \]

where \( \rho = \tau/\nu \).

If MTTF and A are known, MTTR can be derived from equation (B.32), given by

\[ \text{MTTR} = \frac{\text{MTTF}(1 - A)}{A}. \quad \text{(B.33)} \]

Since the mean time between failures (MTBF) of the unit is equal to MTTF + MTTR, the long-run expected number of failures \( (N_f) \) can be computed by

\[ N_f = 1/\text{MTBF} = A/\text{MTTF}. \quad \text{(B.34)} \]
APPENDIX C. ALGORITHM FOR SOLVING
THE MARKOVIAN PROBLEMS

In Chapter 5, the Markovian problems given in Models B, C, D, and E were solved numerically to find the transient state probabilities. The major part of this task is to solve a homogeneous linear system of differential equations, that is, \( dP/dt = A \times P \). Since many computer software packages are available for this purpose, only a simple procedure is given here. The inputs which must be given are the transition matrix \( A \) and the initial condition for this problem, which is denoted by a column vector \( I \).

1. Compute the eigenvalues of matrix \( A \), \( r_1, r_2, \ldots, r_n \), and their corresponding eigenvectors \( V_1, V_2, \ldots, V_n \).

2. Find the general solutions for the problem.

   Depending on the nature of these eigenvalues, three cases exist:

   (i) All eigenvalues are real and distinct. The general solution is given by

\[
P = c_1 \, V_1 \exp(r_1 t) + \ldots + c_n \, V_n \exp(r_n t), \tag{C.1}
\]

   where \( c_1, c_2, \ldots, c_n \) are \( n \) constants.
(ii) Some eigenvalues occur in complex conjugate pairs. For example, suppose that \( r_1 = \lambda + i\mu \), and \( r_2 = \lambda - i\mu \), and that \( r_3, ..., r_n \) are all real and distinct. Let the corresponding eigenvectors be \( \mathbf{V}_1 = a + ib \), \( \mathbf{V}_2 = a - ib \); the general solution to this problem is then given by

\[
P = c_1 \mathbf{u}(t) + c_2 \mathbf{v}(t) + c_3 \mathbf{V}_3 \exp(r_3t) + ... + c_n \mathbf{V}_n \exp(r_nt),
\]

where \( \mathbf{u}(t) = \exp(\lambda t)(a \cos \mu t - b \sin \mu t) \), and \( \mathbf{v}(t) = \exp(\lambda t)(a \sin \mu t + b \cos \mu t) \).

(iii) Some eigenvalues are repeated. If the eigenvectors associated with the repeated eigenvalues are independent, the solution is the same as given in Equation (C.1). However, if these eigenvectors are not independent, the situation becomes quite complicated and can only be dealt with individually. Consult Ross (1984) for this case.

3. Determine the constants \( c_1, c_2, ..., c_n \).

These constants can be determined based on initial condition \( \mathbf{I} \). For example, for the solution given by Equation (C.1), the following result can be obtained:

\[
\mathbf{C} = \mathbf{V}^{-1} \cdot \mathbf{I},
\]

where \( \mathbf{C} = [c_1 \ c_2 \ ... \ c_n]^T \), and \( \mathbf{V} \) is a square matrix whose columns are composed of \( \mathbf{V}_1, \mathbf{V}_2, ..., \mathbf{V}_n \).
APPENDIX D. TRANSITION MATRICES AND CONSTANTS FOR
DETERMINING STATE PROBABILITIES (CASE STUDY 1)
Transition matrix of Model B (Case study 1)

\[
A_b = \begin{bmatrix}
-0.01037749 & 0.01667047 & 0.02030811 & 0.02865855 & 0.02941176 \\
0.00466910 & -0.01667047 & 0 & 0 & 0 \\
0.00122811 & 0 & -0.02030811 & 0 & 0 \\
0.00156253 & 0 & 0 & -0.02865855 & 0 \\
0.00291775 & 0 & 0 & 0 & -0.02941176
\end{bmatrix}
\]

Constants for the state probabilities in Model B (Case study 1)

\[
K_b = \begin{bmatrix}
0.66921783 & 0.23222252 & 0.06475212 & 0.03286423 & 0.00094330 \\
0.18743592 & -0.05670788 & -0.05713640 & -0.07323173 & -0.00035991 \\
0.04047025 & -0.01842030 & -0.04808470 & 0.02616952 & -0.00013471 \\
0.03648728 & -0.05087561 & 0.01510865 & 0.00519082 & -0.00591115 \\
0.06638872 & -0.10621871 & 0.02536033 & 0.00900716 & 0.00546247
\end{bmatrix}
\]
Transition matrix of Model C (Case study 1)

\[ A_c = \begin{bmatrix}
-0.02225437 & 0.01667047 & 0.02030811 & 0.02865855 & 0.05000000 & 0.02941176 & 0 & 0 & 0 \\
0.00466910 & -0.01667047 & 0 & 0 & 0 & 0.05000000 & 0 & 0 & 0 \\
0.00122811 & 0 & -0.02030811 & 0 & 0 & 0 & 0.05000000 & 0 & 0 \\
0.00156253 & 0 & 0 & -0.02865855 & 0 & 0 & 0 & 0.05000000 & 0 \\
0.01232886 & 0 & 0 & 0 & -0.06239129 & 0.01667047 & 0.02030811 & 0.02865855 & 0 \\
0.00246577 & 0 & 0 & 0 & 0.00493154 & -0.02941176 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.00466910 & 0 & -0.06667047 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.00122811 & 0 & -0.07030811 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.00156253 & 0 & 0 & 0 & -0.0786585
\end{bmatrix} \]
Constants for the expression of state probabilities in Model C (Case study 1)

\[
K_c = 
\begin{pmatrix}
0.55603944 & 0.07501444 & 0.02765569 & 0.00484113 & 0.10176131 & 0.02517327 & 0.04726143 & 0.16164205 & 0.00061124 \\
0.17961671 & -0.01892663 & -0.01606579 & -0.00890521 & 0.07037119 & -0.07403952 & -0.05738506 & -0.07430494 & -0.00036075 \\
0.03851514 & -0.00615218 & -0.00739824 & 0.00722017 & 0.01278337 & 0.02580725 & -0.04791514 & -0.02272948 & -0.00013090 \\
0.03425668 & -0.01694440 & 0.01343710 & 0.00690356 & 0.00977876 & 0.00497603 & 0.01416723 & -0.05484591 & -0.00572907 \\
0.11368751 & -0.07911091 & -0.02962436 & -0.00525283 & -0.11510440 & 0.00761120 & 0.01557363 & 0.09196841 & 0.00025175 \\
0.06567849 & 0.00363171 & 0.00170148 & 0.00034917 & 0.01075167 & 0.00935017 & 0.02584824 & -0.12262215 & 0.00531125 \\
0.00796182 & 0.01920289 & 0.01622285 & 0.00894166 & -0.06889485 & 0.00074173 & 0.0162533 & 0.01416746 & 0.00003112 \\
0.00198584 & 0.00622887 & 0.00744228 & -0.00720988 & -0.01235842 & 0.00018133 & 0.00039536 & 0.00332715 & 0.00000747 \\
0.00225837 & 0.01705621 & -0.01337100 & -0.00088778 & -0.00908865 & 0.00019854 & 0.00042898 & 0.00339743 & 0.00000790
\end{pmatrix}
\]
Transition matrix of Model D (Case study 1)

\[ A_d = \begin{pmatrix}
-0.02225437 & 0.01667047 & 0.02030811 & 0.02865855 & 0.05000000 & 0.02941176 & 0 & 0 & 0 \\
0.00466910 & -0.01667047 & 0 & 0 & 0 & 0.05000000 & 0 & 0 & 0 \\
0.00122811 & 0 & -0.02030811 & 0 & 0 & 0 & 0.05000000 & 0 & 0 \\
0.00156253 & 0 & 0 & -0.02865855 & 0 & 0 & 0 & 0.05000000 & 0 \\
0.01232886 & 0 & 0 & 0 & -0.06239129 & 0 & 0 & 0 & 0 \\
0.00246577 & 0 & 0 & 0 & 0.00493154 & -0.02941176 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.00466910 & 0 & -0.05000000 & 0 \\
0 & 0 & 0 & 0 & 0.00122811 & 0 & 0 & -0.05000000 & 0 \\
0 & 0 & 0 & 0 & 0.00156253 & 0 & 0 & 0 & -0.05000000
\end{pmatrix} \]
### Constants for the expression of state probabilities in Model D (Case study 1)

\[
K_d = \begin{pmatrix}
0.55029587 & 0.19847674 & -0.55577836 & 0.73217886 & 0.02293832 & 0.05124608 & 0.00064249 \\
0.18458457 & -0.04467136 & 0.57832015 & -0.57837945 & -0.07226706 & -0.06712870 & -0.00045813 \\
0.03985454 & -0.01252385 & 0.17602665 & -0.17922639 & 0.02761832 & -0.05157806 & -0.00017120 \\
0.03593223 & -0.01877281 & 0.35040103 & -0.38574584 & 0.00529925 & 0.01924368 & -0.00635754 \\
0.10874147 & -0.18621700 & -0.36174134 & 0.41676801 & 0.00649136 & 0.01572059 & 0.00023690 \\
0.06436766 & 0.00930048 & 0.22470970 & -0.34104564 & 0.00836651 & 0.02827779 & 0.00602350 \\
0.01015450 & 0.03405419 & -0.25783463 & 0.20996069 & 0.00097222 & 0.00254047 & 0.00005256 \\
0.00267093 & 0.00895725 & -0.06781806 & 0.05522541 & 0.00025572 & 0.00069452 & 0.00001382 \\
0.00339824 & 0.01139635 & -0.08628523 & 0.07026405 & 0.00032536 & 0.00088364 & 0.00001759
\end{pmatrix}
\]
Transition matrix of Model E (Case study 1)

\[
A_e = \\
\begin{pmatrix}
-0.02225437 & 0.01667047 & 0.02030811 & 0.02865855 & 0.05000000 & 0.02941176 & 0 & 0 & 0 \\
0.00466910 & -0.01667047 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.00122811 & 0 & -0.02030811 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.00156253 & 0 & 0 & -0.02865855 & 0 & 0 & 0 & 0 & 0 \\
0.01232886 & 0 & 0 & 0 & -0.06239129 & 0.01667047 & 0.02030811 & 0.02865855 & 0 \\
0.00246577 & 0 & 0 & 0 & 0.00493154 & -0.02941176 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.00466910 & 0 & -0.01667047 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.00122811 & 0 & 0 & -0.02030811 & 0 \\
0 & 0 & 0 & 0 & 0.00156253 & 0 & 0 & 0 & -0.02865850 \\
\end{pmatrix}
\]
Constants for the expression of state probabilities in Model E (Case study 1)

\[ K_e = \]

\[
\begin{bmatrix}
0.54654996 & 0.17952681 & 0.18468092 & 0.00492754 & 0.02700658 & 0.05393186 & 0.00090935 & 0.00162836 & 0.00077863 \\
0.15307887 & -0.01398554 & -0.04464252 & 0.01532275 & -0.06012518 & -0.04750168 & -0.00036929 & -0.00067501 & -0.00110138 \\
0.03305199 & -0.00391630 & -0.01446678 & 0.00117754 & 0.02153084 & -0.03981612 & -0.00013813 & -0.00026224 & 0.00283919 \\
0.02979916 & -0.00585051 & -0.03938202 & 0.00057077 & 0.00426641 & 0.01260221 & -0.00565796 & 0.00351188 & 0.00014005 \\
0.12266791 & -0.19181697 & 0.05970319 & -0.00475021 & 0.00554748 & 0.01048788 & 0.00005691 & -0.00116926 & -0.00072693 \\
0.06638869 & 0.01066408 & -0.11405274 & -0.00079168 & 0.00882646 & 0.02482545 & 0.00550111 & -0.00118500 & -0.00017637 \\
0.03435709 & 0.01494298 & -0.01443192 & -0.01477132 & -0.01235066 & -0.00923743 & -0.00020168 & 0.00048469 & 0.00102826 \\
0.00741820 & 0.00418440 & -0.00467679 & -0.00113517 & 0.00442270 & -0.00774285 & -0.00000811 & 0.00018830 & -0.00265069 \\
0.00668814 & 0.00625103 & -0.01273132 & -0.00055023 & 0.00087637 & 0.00245069 & -0.00033213 & -0.00252174 & -0.00013075
\end{bmatrix}
\]
APPENDIX E. TRANSITION MATRICES AND CONSTANTS FOR
DETERMINING STATE PROBABILITIES (CASE STUDY 2)
Transition matrix of Model B (Case study 2)

\[
A_b = \begin{pmatrix}
-0.00469033 & 0.01033091 & 0.01739787 & 0.03545860 & 0.04075229 \\
0.00128277 & -0.01033091 & 0 & 0 & 0 \\
0.00054793 & 0 & -0.01739787 & 0 & 0 \\
0.00202254 & 0 & 0 & -0.03545860 & 0 \\
0.00083709 & 0 & 0 & 0 & -0.04075229
\end{pmatrix}
\]

Constants for the state probabilities in Model B (Case study 2)

\[
B_b = \begin{pmatrix}
0.81087064 & 0.07633455 & 0.04525022 & 0.03098637 & 0.03655842 \\
0.10068405 & -0.09189689 & -0.00182842 & -0.00521373 & -0.00174525 \\
0.02553764 & 0.00696935 & -0.00100464 & -0.03049083 & -0.00101150 \\
0.04625171 & 0.00641629 & -0.01388276 & 0.00358041 & -0.04242058 \\
0.01665596 & 0.00217670 & -0.02858941 & 0.00113777 & 0.00861890
\end{pmatrix}
\]
Transition matrix of Model C (Case study 2)

\[
A_c = 
\begin{pmatrix}
-0.00854507 & 0.01033091 & 0.01739787 & 0.03545860 & 0.05000000 & 0.04075229 & 0 & 0 & 0 \\
0.00128277 & -0.01033091 & 0 & 0 & 0 & 0 & 0.05000000 & 0 & 0 \\
0.00054793 & 0 & -0.01739787 & 0 & 0 & 0 & 0 & 0.05000000 & 0 \\
0.00202254 & 0 & 0 & -0.03545860 & 0 & 0 & 0 & 0 & 0.05000000 \\
0.00390986 & 0 & 0 & 0 & -0.05541718 & 0 & 0.01033091 & 0.01739787 & 0.03545860 \\
0.00078197 & 0 & 0 & 0 & 0.00156394 & -0.04075229 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.00128277 & 0 & -0.06033091 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.00054793 & 0 & 0 & -0.06739787 & 0 \\
0 & 0 & 0 & 0 & 0.00202254 & 0 & 0 & 0 & -0.08545860 
\end{pmatrix}
\]
### Constants for the expression of state probabilities in Model C (Case study 2)

\[ K_C = \]

\[
\begin{bmatrix}
0.75636568 & 0.00399961 & 0.06987688 & 0.02788936 & 0.00412090 & 0.05971201 & 0.03821765 & 0.02792690 & 0.01189102 \\
0.09952958 & -0.00018737 & -0.09203656 & -0.00529609 & -0.00063894 & 0.01315259 & -0.00314193 & -0.00204446 & -0.00928681 \\
0.02509546 & -0.00010757 & 0.00690204 & -0.03049519 & -0.00281607 & 0.00270637 & -0.00147446 & -0.00108934 & 0.00127875 \\
0.04496302 & -0.00319462 & 0.00623724 & 0.00348245 & 0.00050842 & 0.00467705 & -0.01624126 & -0.04154672 & 0.00111441 \\
0.05454544 & -0.00407276 & 0.00641159 & 0.00303784 & -0.00425915 & -0.06456675 & 0.01445579 & 0.00687589 & -0.01242789 \\
0.01660670 & 0.00006857 & 0.00220291 & 0.00116499 & 0.00012516 & 0.00410870 & -0.03383756 & 0.00908441 & 0.00047613 \\
0.00115976 & 0.00018858 & 0.00016807 & 0.00009196 & 0.00069200 & -0.01300987 & 0.00102863 & 0.00038073 & 0.00930013 \\
0.00044344 & 0.00010814 & 0.00006273 & 0.00003367 & 0.00281756 & -0.00263362 & 0.00031564 & 0.00012462 & -0.00127217 \\
0.00129092 & 0.000319743 & 0.00017509 & 0.00009102 & -0.00049989 & -0.00149447 & 0.00677750 & 0.00028796 & -0.00107356
\end{bmatrix}
\]
**Transition matrix of Model D (Case study 2)**

\[ \text{Ad} = \]

\[
\begin{bmatrix}
-0.00854507 & 0.01033091 & 0.01739787 & 0.03545860 & 0.05000000 & 0.04075229 & 0 & 0 & 0 \\
0.00128277 & -0.01033091 & 0 & 0 & 0 & 0.05000000 & 0 & 0 & 0 \\
0.00054793 & 0 & -0.01739787 & 0 & 0 & 0 & 0.05000000 & 0 & 0 \\
0.00202254 & 0 & 0 & -0.03545860 & 0 & 0 & 0 & 0.05000000 & 0 \\
0.00390986 & 0 & 0 & 0 & -0.05541718 & 0 & 0 & 0 & 0 \\
0.00078197 & 0 & 0 & 0 & 0.00156394 & -0.04075229 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.00128277 & 0 & -0.05000000 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.00054793 & 0 & 0 & -0.05000000 & 0 \\
0 & 0 & 0 & 0 & 0.00202254 & 0 & 0 & 0 & -0.05000000 \\
\end{bmatrix}
\]
Constants for the expression of state probabilities in Model D (Case study 2)

\[ K_d = \begin{bmatrix}
0.75437831 & 0.07032870 & 0.02895384 & 0.08530266 & 0.19721917 & -0.17438267 & 0.03820001 \\
0.10027846 & -0.09318134 & -0.00564868 & -0.00991061 & -0.02820423 & 0.04008963 & -0.00342323 \\
0.02543469 & 0.00717704 & -0.03179090 & -0.00492567 & -0.01526828 & 0.02133788 & -0.00196475 \\
0.04606520 & 0.006593585 & 0.00389635 & -0.03124046 & -0.17794402 & 0.21222752 & -0.05960045 \\
0.05322381 & 0.00624850 & 0.00302374 & -0.06428979 & 0.06654971 & -0.07331193 & 0.00855597 \\
0.01651785 & 0.00220734 & 0.00120181 & 0.00170462 & -0.08391590 & 0.04679023 & 0.01549406 \\
0.00136548 & 0.00020771 & 0.00012113 & 0.00777646 & 0.01383679 & -0.02421919 & 0.00091163 \\
0.00058326 & 0.00008872 & 0.00005174 & 0.00332168 & 0.00591033 & -0.01034513 & 0.00038940 \\
0.00215295 & 0.00032749 & 0.00019098 & 0.01225112 & 0.02181643 & -0.03818633 & 0.00143736
\end{bmatrix} \]
Transition matrix of Model E (Case study 2)

\[ A_e = \]

\[
\begin{pmatrix}
-0.00854507 & 0.01033091 & 0.01739787 & 0.03545860 & 0.05000000 & 0.04075229 & 0 & 0 & 0 \\
0.00128277 & -0.01033091 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.00054793 & 0 & -0.01739787 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.00202254 & 0 & 0 & -0.03545860 & 0 & 0 & 0 & 0 & 0 \\
0.00390986 & 0 & 0 & 0 & -0.05541718 & 0 & 0.01033091 & 0.01739787 & 0.0354586 \\
0.00078197 & 0 & 0 & 0 & 0.00156394 & -0.04075229 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.00128277 & 0 & -0.01033091 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.00054793 & 0 & 0 & -0.01739787 & 0 \\
0 & 0 & 0 & 0 & 0.00202254 & 0 & 0 & 0 & -0.0354586 \\
\end{pmatrix}
\]

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## Constants for the expression of state probabilities in Model E (Case study 2)

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<td>0.00183313</td>
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<td>0.00854253</td>
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<td>-0.00990707</td>
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<td>0.00854253</td>
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<td>0.09253577</td>
<td>0.00156824</td>
<td>0.00169014</td>
<td>0.00183313</td>
<td>0.00039347</td>
<td>0.00854253</td>
</tr>
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<td>-0.00990707</td>
<td>-0.00990707</td>
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<td>-0.00462904</td>
</tr>
</tbody>
</table>
APPENDIX F. TRANSITION MATRICES AND CONSTANTS FOR
DETERMINING STATE PROBABILITIES (CASE STUDY 3)
Transition matrix of Model B (Case study 3)

\[
A_b = \begin{bmatrix}
-0.00363636 & 0.02258065 & 0.01953690 & 0.03245436 & 0.02114259 \\
0.00105517 & -0.02258065 & 0 & 0 & 0 \\
0.00030076 & 0 & -0.01953690 & 0 & 0 \\
0.00157542 & 0 & 0 & -0.03245436 & 0 \\
0.00070501 & 0 & 0 & 0 & -0.02114259
\end{bmatrix}
\]

Constants for the state probabilities in Model B (Case study 3)

\[
K_b = \begin{bmatrix}
0.87411710 & 0.06369560 & 0.05123491 & 0.00378490 & 0.00716752 \\
0.04084642 & -0.00571654 & -0.04351532 & 0.00138126 & 0.00700403 \\
0.01345672 & -0.00129423 & -0.00359520 & -0.00746939 & -0.00109764 \\
0.04243190 & -0.05328150 & 0.00935155 & 0.00046712 & 0.00103089 \\
0.02914786 & -0.00340323 & -0.01347594 & 0.00183611 & -0.01410480
\end{bmatrix}
\]
Transition matrix of Model C (Case study 3)

\[ A_c = \begin{pmatrix}
-0.06891895 & 0.02250695 & 0.00030076 & 0.00066460 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 \\
0.02250695 & -0.06891895 & 0.00030076 & 0.00066460 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 \\
0.00030076 & 0.00066460 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 \\
0.00066460 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 \\
0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 \\
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0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 \\
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0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230 & 0.00033230
\end{pmatrix} \]
### Constants for the expression of state probabilities in Model C (Case study 3)

\( K_c = \)

\[
\begin{pmatrix}
0.62279885 & 0.00318190 & 0.00241782 & 0.00047131 & 0.06017488 & 0.05436784 & 0.04660437 & 0.00339369 & 0.00658935 \\
0.04010209 & -0.00028347 & -0.00224208 & 0.00018005 & 0.00355200 & -0.00604818 & -0.04379303 & 0.00135524 & 0.00717419 \\
0.01323514 & -0.00006437 & -0.0017548 & -0.00060637 & 0.00120850 & -0.00139265 & -0.00367018 & -0.00741970 & -0.00111489 \\
0.04145287 & -0.00243813 & 0.00044145 & 0.00005975 & 0.00352148 & -0.05387703 & 0.00935354 & 0.00045221 & 0.00103116 \\
0.05136727 & -0.00321789 & -0.00245138 & -0.00047845 & -0.06161453 & 0.01000392 & 0.00535219 & 0.00033978 & 0.00069908 \\
0.02909338 & 0.0003411 & 0.00003140 & 0.00000664 & 0.00127376 & -0.00374312 & -0.01414182 & 0.00186142 & -0.01441578 \\
0.00074677 & 0.00028409 & 0.00224277 & -0.00017989 & -0.00350692 & 0.00027610 & 0.00011586 & 0.00000667 & 0.00001444 \\
0.00022217 & 0.00006454 & 0.00017566 & 0.00060642 & -0.00119595 & 0.00008550 & 0.00003522 & 0.00000205 & 0.00000438 \\
0.00098145 & 0.00243921 & -0.00044286 & -0.00005944 & -0.00341642 & 0.00032762 & 0.00014384 & 0.00000853 & 0.00001807
\end{pmatrix}
\]
Transition matrix of Model D (Case study 3)

\[
A_d = \\
\begin{pmatrix}
-0.00691895 & 0.02258065 & 0.0195369 & 0.03245436 & 0.05000000 & 0.02114259 & 0 & 0 & 0 \\
0.00105517 & -0.02258065 & 0 & 0 & 0 & 0.05000000 & 0 & 0 & 0 \\
0.00030076 & 0 & -0.0195369 & 0 & 0 & 0 & 0.05000000 & 0 & 0 \\
0.00157542 & 0 & 0 & -0.03245436 & 0 & 0 & 0 & 0.05000000 & 0 \\
0.00332300 & 0 & 0 & 0 & -0.05426055 & 0 & 0 & 0 & 0 \\
0.00066460 & 0 & 0 & 0 & 0.00132920 & -0.02114259 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.00105517 & 0 & -0.05000000 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.00030076 & 0 & 0 & -0.05000000 & 0 \\
0 & 0 & 0 & 0 & 0.00157542 & 0 & 0 & 0 & -0.05000000
\end{pmatrix}
\]
### Constants for the expression of state probabilities in Model D (Case study 3)

\[
K_d = \begin{bmatrix}
0.82131497 & 0.07241132 & 0.07829177 & 0.04768938 & 0.00346427 & 0.00620447 & -0.02937618 \\
0.04072958 & -0.01119566 & -0.01047992 & -0.04613342 & 0.00146712 & 0.00712352 & 0.01848877 \\
0.01341802 & -0.00294317 & -0.00239889 & -0.00397654 & -0.00768705 & -0.00112079 & 0.00470842 \\
0.04231042 & -0.02300277 & -0.07650074 & 0.01062445 & 0.00049554 & 0.00105089 & 0.04502222 \\
0.05029860 & -0.05413037 & 0.01350278 & 0.00521968 & 0.00033303 & 0.00062930 & -0.01585302 \\
0.02897955 & 0.00063428 & -0.00505255 & -0.01400979 & 0.00189488 & -0.01395211 & 0.00150574 \\
0.00106147 & 0.00656077 & 0.00094941 & 0.00021102 & 0.00001160 & 0.00002330 & -0.0081757 \\
0.00030256 & 0.00187005 & 0.00027062 & 0.00006015 & 0.00000331 & 0.00000664 & -0.00251331 \\
0.00158483 & 0.00979555 & 0.00141752 & 0.00031507 & 0.00001731 & 0.00003478 & -0.01316506
\end{bmatrix}
\]
Transition matrix of Model $E$ (Case study 3)

$$A_e = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.0001695 & 0.02256065 & 0.01953690 & 0.00105517 & 0.000157542 & 0.00006460 & 0 & 0 & 0 & 0 \\
0.00000000 & 0.05000000 & 0.02111239 & 0.00000000 & 0.00000000 & 0.00000000 & 0 & 0 & 0 & 0 \\
0.00000000 & 0.03245436 & 0.03245436 & 0.00000000 & 0.00000000 & 0.00000000 & 0 & 0 & 0 & 0 \\
0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0 & 0 & 0 & 0 \\
0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0 & 0 & 0 & 0 \\
0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0 & 0 & 0 & 0 \\
0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0 & 0 & 0 & 0 \\
0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0 & 0 & 0 & 0 \\
0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0 & 0 & 0 & 0 \\
0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0.00000000 & 0 & 0 & 0 & 0 \\
\end{bmatrix}$$
Constants for the expression of state probabilities in Model E (Case study 3)

\[ K_e = \]

\[
\begin{bmatrix}
0.82096843 & 0.0560708 & 0.05982348 & 0.00318637 & 0.04819129 & 0.00357803 & 0.00678514 & 0.00094543 & 0.00045086 \\
0.03836299 & -0.00157500 & -0.00536892 & -0.00041119 & -0.04085128 & 0.00130591 & 0.00706423 & 0.00130724 & 0.00014602 \\
0.01263836 & -0.00041528 & -0.00121562 & -0.00008541 & -0.00337973 & -0.00704696 & -0.00100372 & -0.00012468 & 0.00063304 \\
0.03985197 & -0.00319004 & -0.05003627 & 0.00295806 & 0.00879846 & 0.0004160 & 0.00098299 & 0.00014003 & 0.00005409 \\
0.05314866 & -0.05716475 & 0.00401677 & -0.00332114 & 0.00355697 & 0.00019555 & 0.00308564 & -0.0031285 & -0.00040484 \\
0.02914784 & 0.00099271 & -0.00341770 & 0.00023888 & -0.01370046 & 0.00181550 & -0.02014519 & 0.00519945 & -0.00013104 \\
0.00248358 & 0.00160572 & -0.00036049 & 0.00042858 & -0.00301521 & 0.00007137 & 0.0032165 & -0.00430409 & -0.00013112 \\
0.00081819 & 0.00042338 & -0.00008162 & 0.00008902 & -0.00024946 & -0.00038514 & -0.00045646 & 0.00041051 & -0.00056844 \\
0.00257998 & 0.00325227 & -0.00335962 & -0.00308318 & 0.00064941 & 0.0002413 & 0.00044662 & -0.00048104 & -0.00004857
\end{bmatrix}
\]
VITA

Chunming Duan was born in April 7, 1962 in Li County, Hunan, China. He received his B.S. and M.S. degrees in Mining Engineering from Central-South University of Technology (CSUT), Changsha, China in 1982 and 1985. Then he worked briefly as a research associate in the Laboratory of Continuous Mining Technology at CSUT. In 1986, under a scholarship from the Chinese government, Mr. Duan came to Virginia Tech to study for his Ph.D. in Mining Engineering.

Chunming Duan