Control Charts Applying A Sequential Test at Fixed Sampling Intervals With Optional Sampling at Fixed Times

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(ABSTRACT)

In recent years, variable sampling interval (VSI) control charts have been intensively investigated. In contrast to traditional fixed sampling interval (FSI) control charts, VSI charts vary the sampling interval as a function of the data. VSI charts detect many process changes faster than their FSI counterparts. A disadvantage, however, of VSI charts as recently formulated is that the advance prediction of sampling times is impossible for more than the next sample. A control chart is proposed which applies a sequential probability ratio test (SPRT) at fixed sampling intervals, the *SPRT* chart, to monitor the mean of a process with a normal distribution. A natural modification of the SPRT chart, the SPRT chart with *sampling at fixed times* (SFT), is also proposed in which samples are always taken at pre-specified, equally spaced fixed times, with additional samples taken between these times as indicated by the data. A third control chart is introduced as a generalization of the VSI cumulative sum (CUSUM) chart that uses two sampling intervals, called the *universal* CUSUM (UC) chart, in order to address the need for a general framework for the study of control charts that are equivalent to a sequence of SPRT's. The UC chart can also be viewed as a generalization of the SPRT chart. The integral equation approach is adapted for the evaluation of properties of both the unmodified and modified with SFT versions of the SPRT chart, such as average time to signal (ATS), steady state ATS (SSATS), and average number of observations to signal (ANOS). After comparisons are performed within the general framework of the UC chart, the unmodified SPRT chart is found to be more efficient than both the FSI and VSI $\bar{X}$ charts and the FSI CUSUM chart, though very similar in efficiency to the VSI CUSUM chart. The modified SPRT chart with SFT is found to be more efficient than all five of the other control charts, including its unmodified version and the VSI CUSUM chart. General guidelines are provided for the design of both versions of the SPRT chart.

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Chapter 1

Introduction

Statistical process control (SPC) refers to the statistical procedures employed to maintain or even improve the quality of various processes which are stochastic in nature. SPC is principally concerned with *control charts* and *acceptance sampling plans.* Acceptance sampling plans provide inspection procedures with known risks of passing nonconforming output (Duncan (1982)), but will not be considered here. Statistical experimental design may be used in the study and establishment of SPC procedures, as well as in the design of the process itself. However, this aspect will not be considered here either. Here the concentration will be on control charts.

Control charts are graphical devices widely used to monitor a process of interest for the purpose of determining whether the process is in control, for bringing an out-of-control process into control, and for maintaining statistical control. A control chart is maintained by taking samples from the process and plotting in time order on the chart a relevant statistic, called the *control statistic*, computed from the samples. A region, called the *signal region*, is designated on the chart, such that if the process is in control, a value of the control statistic will only rarely fall within the signal region. A value of the control statistic within the signal region is taken as an indication that an *assignable cause* of variation, as it is called, has changed the process. Such an indication is called a *signal.* When a signal is given, then the process should be investigated so that the assignable cause can be found, addressed, and (hopefully) eliminated. Control charts can be used to monitor a wide variety of univariate and multivariate measurable process characteristics. However, here the focus will be on univariate control charts for the mean μ of a single measurable process characteristic X, and μ_0 will denote the target value for μ. The most common control charts for μ are the $\overline{X}$ chart introduced by Shewhart (1931), the *cumulative sum* (CUSUM) chart proposed by Page (1954), and the *exponentially weighted moving average* (EWMA) chart introduced by Roberts (1959).

Two general types of control charts are the *fixed sampling interval* (FSI) and the *variable sampling interval* (VSI) charts. Traditional control charts are FSI charts. FSI charts use a fixed-length sampling interval, whereas VSI charts vary the sampling interval as a function of the data. That is, after a sample is taken, if there is an indication of a process change, then the time interval until the next sample should be short, but if there is no indication of a process change, then it should be long. In recent years, VSI charts have received great attention and their statistical properties have been intensively investigated (for examples, see Reynolds et al. (1988), Cui and Reynolds (1988), Reynolds (1989), Reynolds and Arnold (1989), Chengular-Smith et al. (1989), Saccucci et al. (1990), Rendtel (1990), Reynolds et al. (1990a, b), Reynolds (1991), Amin and Hemasinha (1991), Shamma et al. (1991), Amin and Letsinger (1991), Runger and Pignatello (1991), Runger and Montgomery (1993), and Vaughn (1993)). Using the VSI feature in control charts substantially reduces the detection time of process changes.

Two other general types of control charts are the *fixed sample size* (FSS) and the *variable sample size* (VSS) charts. This distinction is independent of whether a control chart is an FSI or a VSI chart. That is, in general, control charts can be simultaneously FSI-FSS, VSI-FSS, FSI-VSS, VSI-VSS, or even of other types. Traditionally, control charts have been FSI and FSS charts, simultaneously. FSS charts use a fixed sample size, whereas VSS charts vary the sample size as a function of the data. That is, after a sample is taken, if there is an indication of a process change, then the size of the next sample should be large, but if there is no indication of a process change, then it should be small. VSS charts have been investigated by Sawalapurkar-Powers et al. (1990) Rendtel (1990), and Park and Choi (1993). Using the VSS feature in FSI charts substantially reduces the detection time of process changes. However, using the VSS feature in VSI charts only moderately reduces the detection time of process changes.

Both the VSI and VSS approaches to sampling are forms of *dynamic sampling*, as it is called. Dynamic sampling uses the following idea:

> In many situations, the variance of the control statistic σ^2_{CS} is due to sampling error or other sources of random variation. In such situations, σ^2_{CS} may be controlled by changing the sampling rate. Rather than sampling at a fixed rate, in the dynamic version, the sampling rate may be varied as desired, as long as some constraint on the average sampling rate is satisfied (Assaf et al. (1992)).

The introduction of dynamic sampling features to traditional control charts offers a substantial improvement to their statistical efficiency, without increased sampling. Alternatively, this introduction can be used to reduce the amount of sampling without reducing their statistical efficiency.

The SPC problem of detecting a change in the drift coefficient of a Brownian motion process has received much attention, because diffusion theory yields certain explicit results which seem impossible to compute in discrete time (see Shiryayev (1963), Roberts (1966), Bather (1967), Bather (1976), Assaf

(1988), Assaf and Ritov (1989), and Assaf et al. (1992)). A dynamic sampling procedure for the problem was suggested by Assaf and Ritov (1989), which called for performing a sequence of extremely short sequential probability ratio tests (SPRT's), each of them done in zero time with infinitesimal time between consecutive SPRT's, subject to a constraint on the average sampling rate. The dynamic sampling procedure suggested was shown to be vastly superior to fixed-rate counterparts, such as the FSI CUSUM chart, in the continuous-time context of a Brownian motion (Assaf and Ritov (1989)). This procedure, however, has two major disadvantages. The first disadvantage is that it can be described only as an extreme limit of procedures that can reasonably be implemented in practice, since it requires the sampling of miniscule "fractions of observations", in order to satisfy the average sampling rate constraint, which can only be done in theory. The other disadvantage is that diffusion theory provides poor (asymptotic) approximations to results for discrete-time processes (Pollak and Siegmund (1985)), which are far more in number, and therefore of greater interest. Assaf et al. (1992) stressed that Assaf and Ritov's (1989) procedure can be described only as a remote limit of practical procedures even in the continuous-time context of Brownian motion. Thus, as suggested by Assaf and Ritov (1989), the procedure is of very limited practical interest. In view of these strong criticisms, and since the research presented here is strongly motivated by practical application, all developments and evaluations will be made exclusively in the context of discrete time. In fact, a discrete-time treatment of Assaf and Ritov's (1989) dynamic sampling procedure will be introduced and evaluated. This treatment will provide the foundation for all the developments presented, as components of the broad spectrum of this dynamic sampling control chart research. It should be noted, however, that the proposed discrete-time treatment of Assaf and Ritov's (1989) dynamic sampling procedure, and even the procedure itself, were conceived completely independently from the continuous time research of Assaf and Ritov (1989). The research presented in this dissertation was initiated in order to address the need for control chart procedures that are highly efficient, that accurately utilize and reflect information about the process, and that are convenient for practical applications.

The **first objective** of this research is to introduce a dynamic sampling control chart in the context of discrete time, defined as a sequence of similar SPRT's, each for testing the hypothesis that the process is in control ($\mu = \mu_0$) and without any sampling between consecutive SPRT's. In addition, definitions will be provided and expressions in terms of integral equations will be developed for the properties of this control chart, together with numerical approximations to these properties.

The **second objective** of this research is to introduce a second general dynamic sampling control chart (also in the context of discrete time) that will be defined as a generalization of the VSI CUSUM chart and that can also be viewed as a generalization of the control chart from the first objective. The motivation for the definition of this general control chart came from the need to develop a general framework for the study of control charts that are equivalent to a sequence of SPRT's and for the quantitative comparison of control charts that use different sample sizes and/or different sampling methods (for example, VSS sequential sampling versus FSS sampling). In addition, definitions will be provided and expressions in terms of integral equations will be developed for the properties of this general control chart, together with numerical approximations to these properties. The framework of the general control chart will then be used to compare the proposed dynamic sampling control chart from the first objective with the $\overline{X}$ and CUSUM charts, both with and without the VSI feature.

A critical disadvantage in the practical application of VSI control charts as recently formulated is that their sampling intervals are irregular, with the advance prediction of sampling times being impossible for more than the next sample. That is, the sampling times may not correspond to the natural periods in the process, such as work shifts for plant personnel (Reynolds et al. (1988) and Reynolds et al. (1990a)). A modification of significant practical importance to the VSI feature exists in which samples are always taken at pre-specified, equally spaced fixed times, with additional samples taken between these times as a function of the process data. This modification was briefly discussed by Reynolds et al. (1988) and Reynolds et al. (1990a) for the VSI $\overline{X}$ and VSI CUSUM charts, respectively, but without actually evaluating the modification for either chart. In agreement with Reynolds et al. (1990a), Lucas and Saccucci (1990) noted that, although the mathematics required to evaluate the properties of the VSI CUSUM chart are much more cumbersome, the implementation of this modification in the VSI CUSUM chart is much more practical than the VSI feature alone. In fact, the **third objective** of this research is to implement the modification in the proposed dynamic sampling control chart from the first objective, evaluate the chart's properties under the modification, and compare the modified chart with its unmodified version and with the $\overline{X}$ and CUSUM charts, both with and without the VSI feature.

Finally, the **fourth objective** of this research is to provide guidelines for the choice of parameters in the design of the proposed dynamic sampling control chart from the first objective and in the design of its modification from the third objective. It should be noted that Stoumbos and Reynolds (1992b) presented a modular FORTRAN 77 software for the evaluation of statistical properties and the constrained determination of parameters for the proposed control chart from the first objective and for its modification from the third objective. A modular FORTRAN 77 software was also presented by Stoumbos and Reynolds (1992a) for the general dynamic sampling control chart from the second objective.

The special case of the proposed dynamic sampling control chart from the first objective where each SPRT is conducted in negligible time will receive some special attention. In this case it will be shown that the chart reduces to an FSI control chart that satisfies the modification from the third objective, though without being an FSS chart or even a VSS chart in the usual sense. This model is similar to that of Assaf and Ritov (1989), with the important difference, however, of the much more practical context of discrete time. For an example, consider the case of a battery production line in an electronics industry, where the voltage of each battery produced can be tested in negligible time using a voltmeter. Additional examples from Assaf and Ritov (1989), Wilson et al. (1979), and Weatherall and Haskey (1976) from electronics and biomedical applications are referenced in Section 5.5.

It is important to note that the parameters that characterize all of the control charts considered here are independent of the sample size used at each sampling point. This allows for meaningful direct comparisons between control charts that use different sample sizes and/or different sampling methods. Also note that for the remaining part of this dissertation, the references to the discrete-time context of this research will be suppressed, and the discrete-time context is to be assumed unless stated otherwise. Finally, note that the expression "dynamic sampling" for the control charts considered will also be suppressed for the most part in the remaining chapters. These chapters are arranged in an order similar to the presentation of the above objectives, with the exception of Chapter

2 which provides the literature review and background terminology.

Chapter 2

Literature Review and Terminology

2.1 Control Charts

Control charts were first developed by W. A. Shewhart (Shewhart (1931)). The simplest and most widely used control chart is the $\overline{X}$ chart, introduced by Shewhart (1931), and is also known as the *Shewhart* $\overline{X}$ chart (see Figure 2.1.1). The $\overline{X}$ chart is designed to detect changes in the mean μ of a measurable characteristic X of a process of interest from a target value μ_0. Random samples of size n are taken at fixed regular time intervals, and the sample mean $\overline{X}$ is plotted in time order on a chart which has two control limits with a centerline between the two limits. The centerline is positioned at μ_0, and the control limits are placed at $\mu_0 \pm L\sigma_{\overline{X}}$, where $\sigma_{\overline{X}}$ is the standard deviation of $\overline{X}$, and L is a specified constant. When a value of $\overline{X}$ falls outside the control limits, a signal is given that an assignable cause of variation has shifted μ away from μ_0, and rectifying action is taken. The constant L is chosen so that the chart has a small probability of a false signal, called a *false alarm*, when the process is in control, and is usually taken to be equal to 3 (3-sigma limits). In general, the expected number of false alarms produced by a control chart per unit of time is called the *false alarm rate* (FAR).

The $\overline{X}$ chart is easy to construct and implement, and has the ability to detect large changes in μ quickly, but is slow to signal small or moderate ones. Various modifications of the chart have been proposed to improve this weak point. They have been in the form of added supplementary rules, called *runs rules*. Page (1955) incorporated warning lines within the control limits of the $\overline{X}$ chart,

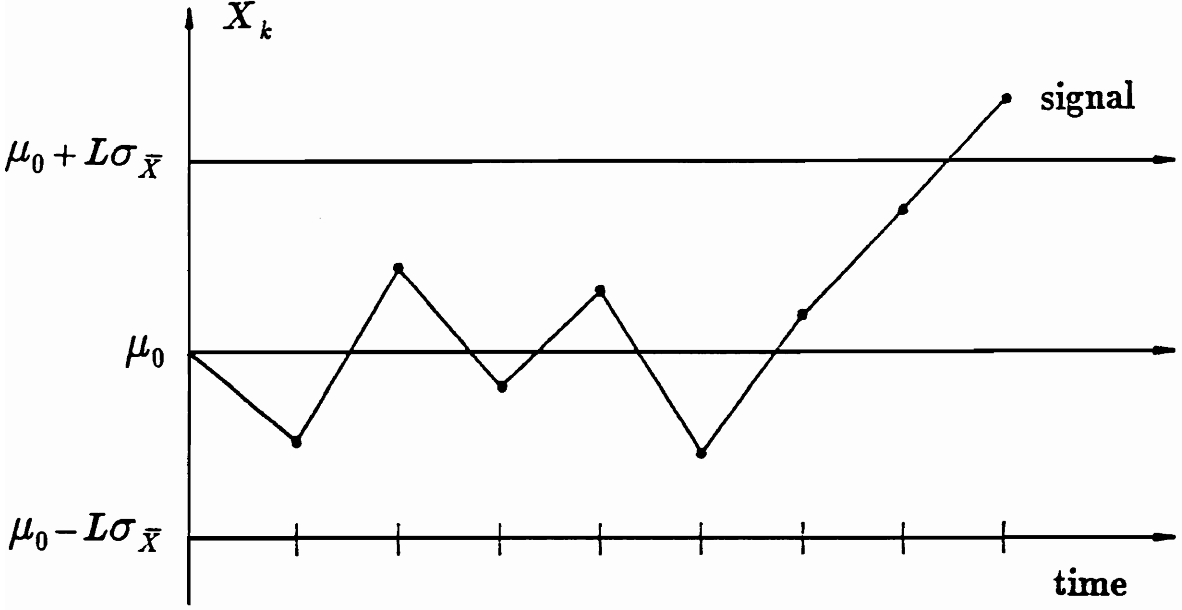

Figure 2.1.1. $\overline{X}$ (Shewhart) Chart.

suggesting that a signal be given when r out of the last N sample means fall between the warning lines and the control limits. The addition of warning lines enabled the $\overline{X}$ chart to use the information from the last r samples instead of using only the last sample, thus improving its efficiency in detecting small or moderate shifts. To make the $\overline{X}$ chart more sensitive to such shifts while still using a small sample size, Weiler (1953) and Moore (1958) proposed that an additional signal be given if a specific number of consecutive means fell on the same side of the centerline. In fact, Weiler (1953) went even further to recommend that the (production) process be stopped if a given number of consecutive means went outside the control limits. Even though the introduction of runs rules to the $\overline{X}$ chart improves its efficiency, the simultaneous use of several different runs rules can increase the frequency of false alarms, which is undesirable.

Page (1954) proposed the widely used CUSUM chart for detecting shifts in μ (see Figure 2.1.2). As indicated by its name (cumulative sum chart), the CUSUM chart is based on sums of values of statistics computed from multiple samples rather than individual values of statistics calculated from single samples, which makes it more sensitive to small and moderate shifts. The CUSUM chart is maintained by taking random samples of size n at fixed time intervals and plotting the control statistic in time order on a chart. The CUSUM statistic for detecting a positive shift in μ can be written as

$$Y_k = \max(0,\ Y_{k-1}) + Z_k - \gamma \qquad \text{for } k = 1, 2, \ldots, \tag{2.1.1}$$

where $Y_0 = y_0$ is the specified starting value of the control statistic, $Z_k = (\overline{X}_k - \mu_0)/\sigma_{\overline{X}}$, and $\gamma > 0$ is a specified chart parameter, called the *reference*

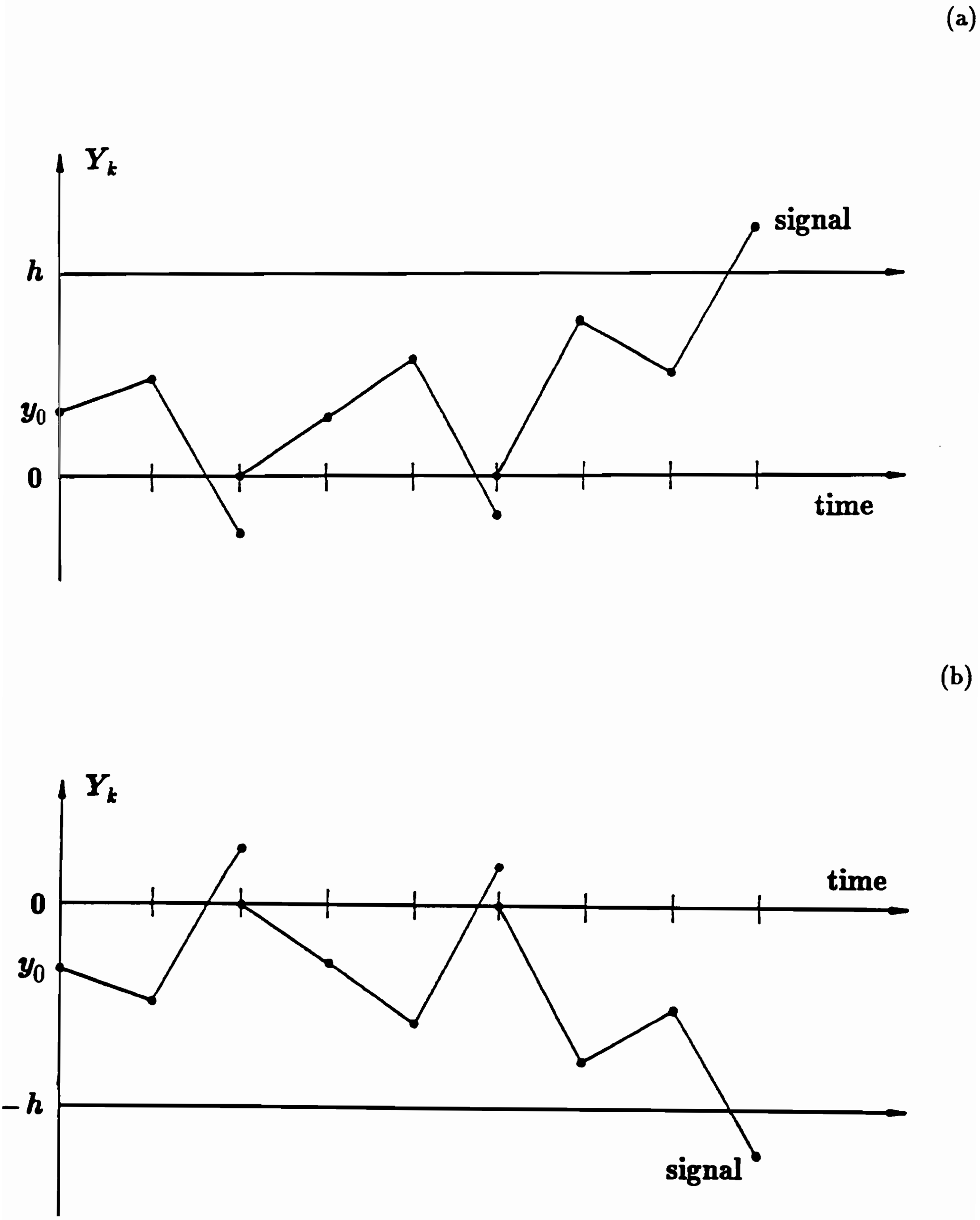

Figure 2.1.2. CUSUM Chart for Detecting (**a**) Positive and (**b**) Negative Shifts.

value (Page (1961)). The starting value y_0 is usually taken to be 0. The CUSUM procedure signals and rectifying action is taken when the control statistic exceeds the control limit h, where $h \geq 0$. The limit h is chosen to attain a certain desired FAR. Similarly, the CUSUM statistic for detecting a negative shift in μ can be written as

$$Y_k = \min(0,\ Y_{k-1}) + Z_k + \gamma \qquad \text{for } k = 1, 2, \ldots, \tag{2.1.2}$$

and the chart signals when the statistic is less than the preset control limit $-h$. The CUSUM scheme can be derived from the sequential probability ratio test (SPRT) for testing the null hypothesis H_0: $\mu = \mu_0$ against the alternative hypothesis H_1: $\mu = \mu_1$, where $\mu_1 = \mu_0 + 2\gamma\sigma_{\overline{X}}$. Optimality theory of SPRT's indicates that γ should be chosen to be $(\mu_1 - \mu_0)/2\sigma_{\overline{X}}$, where μ_1 is the shift in μ which should be detected the quickest (Gosh (1970), pp. 93 – 98). In practice, γ is usually chosen this way.

Originally, for the detection of two-sided deviations in μ, Page (1954) suggested the application of two one-sided CUSUM procedures simultaneously, one to detect an increase and the other a decrease. However, the difficulty of deriving properties of this scheme caused the search for alternative procedures. Barnard (1959) re-presented the scheme using a masking device, called the *V-mask* (see Figure 2.1.3). He proposed that cumulative sums

$$S_k = \sum_{i=1}^{k} (X_i - \mu_0) \qquad \text{for } k = 1, 2, \ldots, \tag{2.1.3}$$

be plotted, and then a V-shaped mask with an angle 2ϕ, a horizontal axis, and a vertex of distance d from the current point be placed on the chart. The procedure

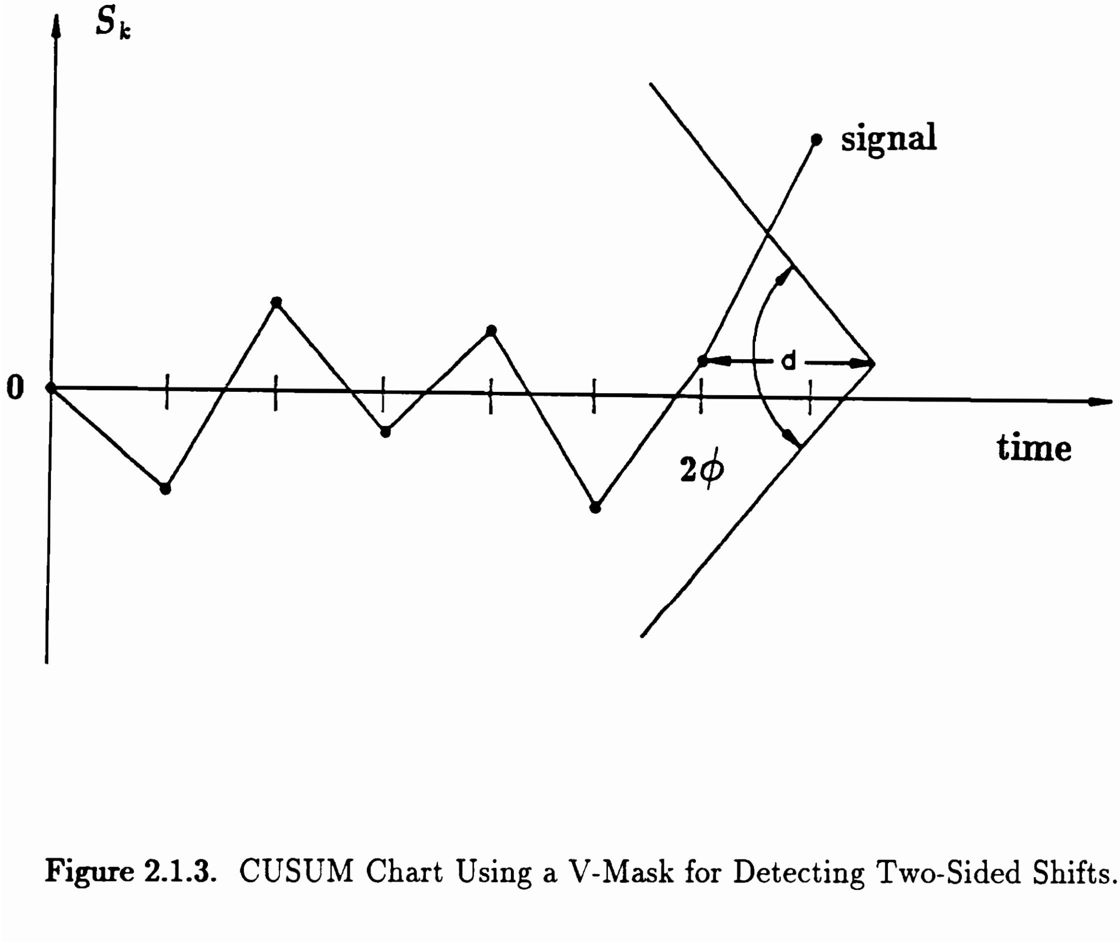

Figure 2.1.3. CUSUM Chart Using a V-Mask for Detecting Two-Sided Shifts.

continues as long as all of the previous path of sums is visible within the V-mask. If part of the path disappears under the lower arm of the V-mask, then this is taken as a signal that μ has decreased below μ_0, but if part of the path disappears over the upper arm of the V-mask, then it is taken as a signal that μ has increased above μ_0. The two quantities giving the shape and position of the V-mask, ϕ and d, are chosen to give the scheme desired FAR characteristics. Barnard's (1959) V-mask scheme is equivalent to Page's (1954) scheme which applies two one-sided CUSUM procedures simultaneously. However, Page's (1954) scheme is much more widely used in practice and has received much greater attention in CUSUM chart research.

Crosier (1986) developed a new two-sided CUSUM scheme that uses the control statistic

$$S_k = \begin{cases} 0 \quad , \quad \text{if } C_k \le \gamma\sigma_{\overline{X}} \\ \\ (S_{k-1} + \overline{X}_k - \mu_0) \cdot (1 - \gamma\sigma_{\overline{X}} \,/\, C_k), \quad \text{if} \quad C_k > \gamma\sigma_{\overline{X}} \end{cases} \tag{2.1.4}$$

$$\text{for } k = 1, 2, \ldots,$$

where $S_0 = 0$ and $C_k = |\,S_{k-1} + \overline{X}_k - \mu_0\,|$. Crosier's procedure signals at the k^{th} sample if $|\,S_k\,| > h\sigma_{\overline{X}}$, where $h \ge 0$ is preset according to a certain FAR, and has efficiency equivalent to that of the above two conventional two-sided CUSUM procedures. Champ and Woodall (1987) showed that CUSUM schemes are much more sensitive to small and moderate shifts than are Shewhart schemes (even with runs rules).

Some processes may be more likely to be off-target at the beginning of a control procedure, due to starting problems, or immediately after a signal, because of ineffective control action. To account for such problems, a *first initial response*

(FIR) feature may be added to a control chart. Put in general terms, a control chart is said to have an FIR feature if the starting value of its control statistic, when standardized with respect to μ_0, exceeds 0 in absolute value. Lucas and Crosier (1982a) added an FIR feature to the CUSUM chart, increasing its sensitivity at start-up or after a restart, following an out-of-control signal.

Lucas (1982) created the *combined* Shewhart-CUSUM chart, by adding Shewhart control limits to the CUSUM chart. This scheme signals at a given sampling point, if either the CUSUM chart or the superimposed Shewhart chart signals. The combined chart not only detects small and moderate shifts quickly, but also large ones. It is, however, sensitive to outliers in the data. Lucas and Crosier (1982b) developed a more robust CUSUM procedure, providing moderate protection against outliers that might otherwise cause an out-of-control signal.

Roberts (1959) introduced the EWMA chart for monitoring μ, in which the most recent sample mean is assigned a specified weight λ, where $0 < \lambda \leq 1$, and every previous point's sample mean is assigned a fraction $(1-\lambda)$ of the weight of its immediate successor (see Figure 2.1.4). The EWMA control statistic can be written as

$$Y_k = (1-\lambda) Y_{k-1} + \lambda \overline{X}_k \qquad \text{for } k = 1, 2, \ldots, \tag{2.1.5}$$

where $Y_0 = y_0$ is the specified starting value of the control statistic, usually taken to be 0. The parameter λ is called the *smoothing* constant of the EWMA chart. Successive random samples of size n taken at given regular time intervals are used to obtain values of the control statistic which are plotted on a chart that has two control limits with a centerline between the two limits. The centerline is positioned at μ_0 and the control limits are usually placed at $\mu_0 \pm L\sigma_Y$, where σ_Y is

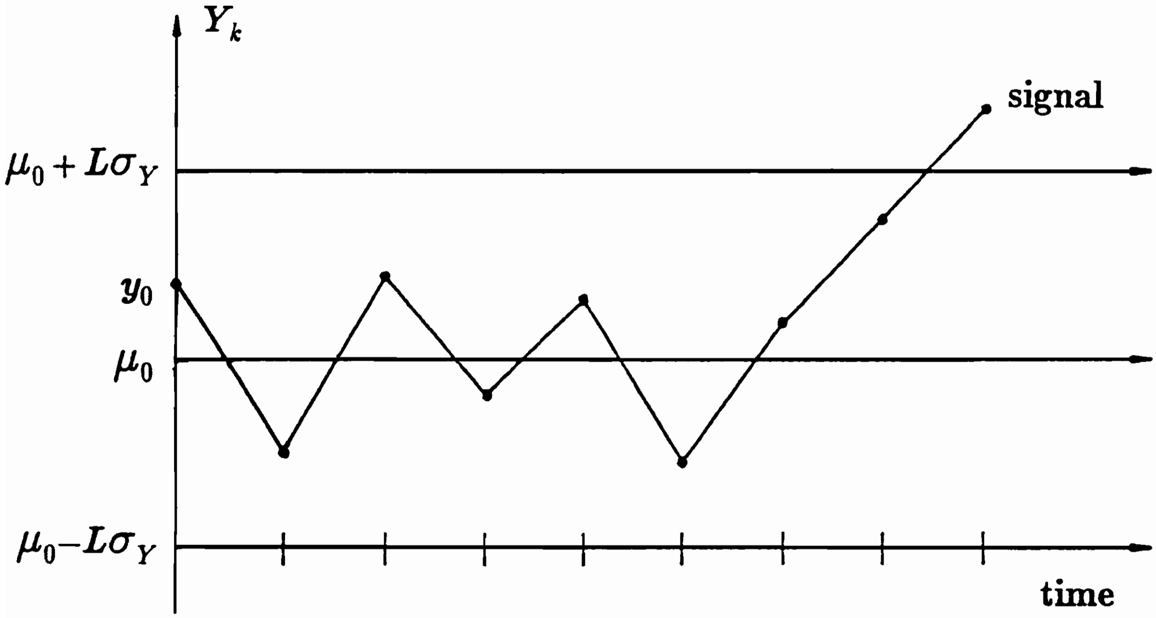

Figure 2.1.4. EWMA Chart.

the asymptotic standard deviation of the EWMA control statistic, given by $\sigma_{\overline{X}}(\lambda/(2-\lambda))^{\frac{1}{2}}$, and L is a specified constant. The constant L is chosen according to the desired FAR and is frequently taken to be equal to 3 (3-sigma limits). The EWMA procedure signals and rectifying action is taken when the control statistic falls outside the control limits.

The EWMA chart can be thought of as a control scheme between the $\overline{X}$ and the CUSUM schemes (Hunter (1986)). When λ is close to 0, the last observations receive less weight, and the EWMA is similar to the CUSUM. On the other hand, when $\lambda = 1$, the EWMA chart puts all of the weight on the last sample mean and is equivalent to the $\overline{X}$ chart. Lucas and Saccucci (1990) showed that the properties of EWMA schemes are very close to those of CUSUM schemes. They also considered several enhancements to EWMA schemes, which included the FIR feature, a combined Shewhart-EWMA procedure, and a robust EWMA chart that provides protection against occasional outliers in the data. Recently, the EWMA chart has received great attention, because it is highly efficient, easy to implement and interpret, and naturally two-sided.

Yashchin (1989) introduced the *geometric* CUSUM (GC) chart for detecting changes in μ. To detect an increase in μ, the GC chart uses the control statistic

$$Y_k = \max(0, (1-\lambda)Y_{k-1}) + Z_k - \gamma \qquad \text{for } k = 1, 2, \ldots, \tag{2.1.6}$$

where Y_0, Z_k, and γ are defined as in the CUSUM chart, and where λ is defined as in the EWMA chart. The GC signals the k^{th} sample if $Y_k > h$, where $h \geq 0$ is chosen to satisfy the desired FAR. On the other hand, to detect a decrease in μ, the GC chart uses the control statistic

$$Y_k = \min(0, (1-\lambda)Y_{k-1}) + Z_k - \gamma \qquad (2.1.7)$$
$$\text{for } k = 1, 2, \ldots,$$

and signals at the k^{th} sample if $Y_k < -h$. A two-sided GC chart is obtained by using two one-sided GC charts simultaneously. The one-sided CUSUM and the one-sided EWMA with a reflecting barrier at zero are special cases of the one-sided GC chart. However, when $\gamma = 0$, the two-sided GC is equivalent to two one-sided EWMA's with reflecting barriers at zero, but is not equivalent to the regular two-sided EWMA chart. The GC chart was investigated in a more general form by Champ et al. (1991).

2.2 VSI and VSS Control Charts

The concept of varying the sampling interval as a function of the observed data has been used extensively in SPC in the development of sampling plans for environmental monitoring (see Arnold (1970), Crigler (1973), Smeach and Jernigan (1977), and Crigler and Arnold (1979, 1986)), but not until recently in that of control charts. The development of VSI control charts was motivated by the following intuitive idea: The traditional practice in maintaining a control chart is to take samples from the process at fixed-length sampling intervals. However, if the control statistic falls close to the signal region (but not actually within it), then naturally, a change in the process is suspected. In this situation, since it is important to quickly detect such changes, it would be reasonable to take another sample shortly, rather than wait the usual fixed-length sampling interval. Additional information about the process can thus be obtained quickly for fast confirmation or denial of the above suspicion. On the other hand, if the control statistic falls far from the signal region, so that there is no indication of a process change, it would be appropriate to wait longer than the usual fixed-length interval for the next sample, decreasing the amount of unnecessary sampling.

Reynolds et al. (1988) and Runger and Pignatiello (1991) introduced the VSI feature to the $\overline{X}$ chart, substantially reducing its reaction time to small and moderate shifts, and Cui and Reynolds (1988) investigated the properties of the VSI version of the $\overline{X}$ chart with runs rules. Runger and Pignatiello (1991), however, used the new term *adaptive sampling* to refer to the VSI feature. The VSI feature was also introduced to the CUSUM chart by Reynolds et al. (1990a), substantially improving its efficiency, and showing that the VSI CUSUM chart is by far superior to the VSI $\overline{X}$ chart. Saccucci et al. (1990) investigated the properties of the VSI EWMA chart and showed that properties of VSI EWMA schemes are very similar to those of VSI CUSUM schemes. Reynolds and Arnold (1989) showed that for one-sided VSI $\overline{X}$ charts, the use of exactly two sampling intervals, the shortest possible and the longest possible, is optimal for detecting a specific shift in μ, in the sense of minimizing the expected length of time from the beginning of the process to the time point the chart produces a signal. Reynolds (1989) showed that Reynolds and Arnold's (1989) result is much more general in that it is also true for two-sided VSI $\overline{X}$ charts, under a very general condition discussed in Section 2.8, and for all one-sided VSI charts in the class of charts that can be modeled as a Markov process. This class includes the Shewhart, CUSUM and EWMA charts.

The concept of varying the sample size as a function of the observed data has received much less attention than that of varying the sampling interval in the development of control charts. Similarly to VSI control charts, the development of VSS control charts was motivated by the following intuitive idea: The usual practice in maintaining a control chart is to use samples of fixed size at each sampling point. However, if the control statistic falls close to the signal region (but not actually within it), then naturally, a change in the process is suspected. In this situation, it would be reasonable to use a sample size larger than the usual fixed one at the next sampling point, obtaining extra information about the process for confirmation or denial of the above suspicion. On the other hand, if the control statistic falls far from the signal region, so that there is no indication of a process change, it would be appropriate to use a sample size smaller than the usual fixed one at the next sampling point, decreasing the amount of unnecessary sampling.

Sawalapurkar-Powers et al. (1990) introduced the VSS feature to the FSI $\bar{X}$ chart, reducing its reaction time to small and moderate shifts, and to the VSI $\bar{X}$ chart, also reducing its reaction to similar shifts, though relatively less. They also introduced the VSS feature to the FSI CUSUM chart, substantially improving its efficiency. However, when Sawalapurkar-Powers et al. (1990) and Rendtel (1990) added the VSS feature to the VSI CUSUM chart the improvement was only moderate. Sawalapurkar-Powers et al. (1990) also concluded that the pure VSI versions of the $\bar{X}$ and CUSUM charts, respectively, however the $\bar{X}$ chart's superiority is less substantial than the CUSUM chart's superiority.

2.3 Other Control Charts

Very recently, Daudin (1992) proposed a new procedure for detecting shifts in μ, the *double sampling* (DS) $\bar{X}$ chart, which is the control chart counterpart to DS plans in acceptance sampling. The procedure assumes that two successive samples can be taken without any intervening time, and therefore, coming from the same probability distribution. Specifically, the following procedure was proposed:

At given time intervals, first a sample of size n_1 is taken and its mean $\overline{X}_1$ is computed. There are three possibilities at this stage

- if $\overline{X}_1$ falls inside the limits $\mu_0 \pm L_1\sigma_{\overline{X}_1}$, conclude the process is in control, where $\sigma_{\overline{X}_1}$ is the standard deviation of $\overline{X}_1$;
- if $\overline{X}_1$ falls outside the limits $\mu_0 \pm L\sigma_{\overline{X}_1}$, conclude the process is out of control;
- if $\overline{X}_1$ falls outside the limits $\mu_0 \pm L_1\sigma_{\overline{X}_1}$, but within $\mu_0 \pm L\sigma_{\overline{X}_1}$, take a second sample of size n_2, compute its mean $\overline{X}_2$, and go to the next stage.

At the second stage, decisions are based on the combined sample mean $\overline{Y} = (n_1\overline{X}_1 + n_2\overline{X}_2) / (n_1 + n_2)$. At this stage there are only two possibilities:

- if $\overline{Y}$ falls inside the limits $\mu_0 \pm L_2\sigma_{\overline{Y}}$, conclude that the process is in control, where $\sigma_{\overline{Y}}$ is the standard deviation of $\overline{Y}$;
- if $\overline{Y}$ falls outside the limits $\mu_0 \pm L_2\sigma_{\overline{Y}}$, conclude that the process is out of control.

The DS $\overline{X}$ chart is an FSI dynamic sampling scheme that uses a complex procedure for deciding if the process is under control. However, closer examination reveals its relation to VSI and VSS procedures. It is reasonable to identify a trivial interval between the (possibly) two consecutive samples taken at a given sampling point, which suggests a VSI-type structure in the DS $\overline{X}$ chart.

On the other hand, it is reasonable to say that the sample size is varied between n_1 and $n_1 + n_2$ (or even n_1 and n_2) observations at each sampling point, which in turn suggests a VSS-type structure in the DS $\overline{X}$ chart. Daudin (1992) showed that the DS $\overline{X}$ chart offers better efficiency than the $\overline{X}$ chart (even with the VSI feature). He also showed that it is better than the FSI versions of the EWMA, CUSUM, and combined Shewhart-CUSUM charts for detecting large shifts, but is much worse for detecting small ones.

If the objective is to control the process variance σ^2, then the R chart or the S^2 chart can be used. The former uses the sample range R as the control statistic and is easy to use and interpret. The latter uses the sample variance S^2 as the control statistic and is more efficient. Both of these charts are usually designed under the assumption that the process quality characteristic has a normal distribution.

Control charts are not only used to monitor quantitative characteristics but also qualitative ones. Examples of the latter are the p chart and the c chart. The p chart monitors the variation in the proportion of defectives of the process' output. The VSS feature may be added to the p chart, appropriately varying the control limits at each sampling point. The c chart monitors the variation in the number of defects per unit of process output. The number of defects per unit are assumed to follow a Poisson distribution and the control limits are set up accordingly.

2.4 Multivariate Control Charts

In many situations, the simultaneous monitoring of two or more (related) process characteristics is required. For the simultaneous monitoring of multiple means under normality assumptions, the χ^2 chart can be used, if the dispersion matrix Σ is known, or the Hotelling's T^2 chart, if Σ is unknown. Hotelling (1947) pioneered in multicharacteristic SPC by introducing the T^2 chart, whose control limits are based on the chi-squared distribution, being the asymptotic distribution of its control statistic. Jackson (1956) and Ghare and Torgerson (1968) presented *control ellipses* for the bivariate case of simultaneously monitoring two means. For the same case, Jackson and Morris (1957) recommended that the T^2 chart be used together with principal component analysis, if the dispersion matrix is near singular. Jackson (1959) extended their bivariate results to the multivariate case, by developing a method using the residuals associated with the principal components to monitor the unexplained variation. Alternative residual analysis procedures such as these were investigated by Jackson and Mudholkar (1979). Jackson (1981a, b, c) provided a detailed summary of principal components methods and discussed their relationship to SPC.

Reynolds and Ghosh (1981) suggested several Shewhart type schemes for the simultaneous monitoring of the mean and variance of a univariate process, and discussed multivariate extensions. The problem of simultaneously controlling multiple qualitative process characteristics was investigated by Marcucci (1982). Alt (1984) discussed multivariate control charts for the mean vector and dispersion matrix of a process. A Shewart type procedure was proposed by Tuprah and Woodall (1986) for the simultaneous monitoring of the two standard deviations of a bivariate normal process.

Woodall and Ncube (1985) discussed the simultaneous use of a set of univariate CUSUM procedures for controlling the mean vector of a multivariate normal process. A method based on SPRT's for constructing multivariate CUSUM charts was presented by Alwan (1986). Healy (1987) developed a sequential CUSUM-type scheme in terms of sums of Hotelling's T^2 statistics for the detection of shifts in the process dispersion parameters. Two multivariate generalizations of the CUSUM procedure were heuristically derived by Crosier (1988). Lowry et al. (1992) presented a multivariate extension of the EWMA chart and used simulation to show that it is more efficient than several other multivariate control charts. Two CUSUM charts for controlling the mean vector of a multivariate normal process were proposed by Pignatiello and Runger (1990).

A very recent development in multicharacteristic SPC has been the investigation of VSI multivariate control charts. Chengular-Smith et al. (1989) considered VSI Shewhart control schemes for the simultaneous monitoring of the mean and variance of a univariate normal process. The use of VSI multivariate Shewhart schemes for controlling the mean vector of a multivariate normal process was discussed by Chengular-Smith et al. (1990). Cho (1991) investigated VSI multivariate Shewhart, CUSUM, and EWMA procedures for monitoring the mean vector and dispersion matrix of a multivariate normal process, both separately and simultaneously.

2.5 Control Charts in the Context of Brownian Motion

The continuous-time SPC problem of monitoring the drift coefficient of a Brownian motion process has received much attention both in the Bayesian and non-Bayesian frameworks, because diffusion theory yields certain results which seem impossible to compute in discrete time (Pollak and Siegmund (1985)). Shiryayev (1963) and Roberts (1966) independently presented a Bayesian control procedure for this problem, called the *Shiryayev-Roberts* procedure. The Shiryayev-Roberts procedure assumes that the point in time that the drift coefficient changes has a (prior) exponential distribution and signals such a change when a certain posterior probability exceeds a threshold b, chosen to satisfy a certain FAR. For a particular loss structure, Shiryayev (1963) showed that the procedure is a limit of Bayes rules. Optimality results for the Shiryayev-Roberts procedure were established by Bather (1967), first in the discrete-time context and then extended to the continuous-time version of Brownian motion. However, explicit calculations were only provided for the latter.

Bather (1976) designed a non-Bayesian control scheme for monitoring the drift of a Brownian motion, investigating the following question: When information relevant to a sequence of decisions is collected by statistical sampling, does controlling the local sampling rate provide more accurate information at certain times? The determination of an optimal scheme depends on the solution of a complicated free boundary problem, which remains unsolved in the general case. However, some limited optimality results were obtained by examining special cases. Pollak and Siegmund (1985) systematically compared the FSI CUSUM and Shiryayev-Roberts procedures in the context of controlling the drift of a Brownian motion, concluding that neither of the two procedures is dramatically better than the other. Examples of more complex diffusion theory problems, however, were also considered for which the Shiryayev-Roberts procedure seemed more easily adapted than the CUSUM procedure.

Assaf (1988) proposed a dynamic sampling procedure for the model of detecting a change in the drift of a Brownian motion in the Bayesian framework with the time of change having a (prior) exponential distribution. The procedure varies the sampling rate between zero and infinity, depending on the information at hand, and uses an interpretable, though complicated, stopping rule to signal a change. The procedure was shown to be significantly superior to constant sampling rate procedures, such as the Shirayev-Roberts, in the continuous-time context of Brownian motion, with the comparison being most favorable when the expected time until change tends to infinity.

A dynamic sampling procedure for the non-Bayesian version of Assaf's (1988) model was suggested by Assaf and Ritov (1989), which called for conducting a sequence of extremely short SPRT's, each of them done in zero time with infinitesimal time between consecutive SPRT's, subject to a constraint on the average sampling rate. The dynamic sampling procedure suggested was shown to be five to six times more efficient than fixed-rate counterparts, such as the FSI CUSUM chart, in the continuous-time context of Brownian motion. The superiority of this procedure stems from the fact that it is actually equivalent to a Bayes procedure (with a generalized prior), which can be described as a limit of procedures in which the maximal permissible sampling rate converges to infinity (Assaf and Ritov (1989)).

A major criticism of control procedures developed in the continuous-time context of Brownian motion is that the machinery of diffusion processes provides poor (asymptotic) approximations to the corresponding results in discrete time (Pollak and Siegmund (1985)). Thus, since SPC applications are predominantly discrete-time, continuous-time results should be used cautiously in obtaining comparative information on which to base selection of a control procedure.

The two dynamic sampling procedures proposed by Assaf (1988) and Assaf and Ritov (1989), respectively, are subject to a second major criticism, which lies in the difficulty of their practical application. These procedures can be described only as an extreme limit of practical procedures, since both require the sampling of miniscule "fractions of observations," in order to satisfy the sampling rate constraint, which can only be done in theory. Assaf et al. (1992) stressed that

Assaf's (1988) and Assaf and Ritov's (1989) procedures can be described only as a remote limit of practical procedures, even in the continuous-time context of Brownian motion. Thus, as proposed by Assaf (1988) and Assaf and Ritov (1989), respectively, the two procedures are of very limited practical interest.

Assaf et al. (1992) presented a non-Bayesian procedure for monitoring the drift of a Brownian motion as a combination of the FSI CUSUM and Assaf and Ritov's (1989) procedures. The combined procedure with parameters $0 \leq l \leq \mathrm{w} \leq r$ is defined as follows: Begin with the CUSUM statistic, until its value hits the warning line w. Once w is hit, perform (in zero time) an SPRT with starting value w, rejection limit r, and lower limit l. A shift is declared upon hitting r. If l is hit before r, an FSI CUSUM procedure with starting value l is initiated until hitting w again, and so on. This procedure attempts to combine the advantages of fixed-rate and dynamic sampling and can reasonably be carried out in practical situations, where the information can be obtained continuously (or at least very quickly). Important criticisms of the combined procedure, however, are that it is complicated and only developed in the continuous-time context of Brownian motion.

2.6 Properties of Control Charts

Due to background noise in the process data, a control chart often fails to immediately detect a process change or could even falsely signal such a change when the process is in control. The efficiency of a control chart in detecting process changes is determined by the distribution of the length of time it takes the chart to produce a signal. Detecting process changes quickly and with few false alarms are performance objectives of a control chart.

In the SPC literature, the *run length* is defined to be the number of samples required by the chart to produce a signal, and the *average run length* (ARL) is the expected value of the run length. The ARL is the most commonly used criterion for the quantitative evaluation and comparison of FSI control charts, because the time required for an FSI chart to signal is essentially the product of the (fixed) sampling interval length and the run length. When the process is in control, then the ARL should be large so that the frequency of false alarms is low, avoiding unnecessary investigations that can be costly and decrease productivity. When there is a change in the process, however, then the ARL should be small so that the change is quickly detected, reducing the cost of producing nonconforming product.

With VSI control charts, the time required for a chart to signal is not (essentially) a constant multiple of the run length, because the sampling interval varies randomly with the data. Hence, the ARL is not a suitable measure of VSI chart performance, and to assess such performance, the time to signal must be directly evaluated, separately from the run length. The *time to signal* is defined to be the length of time from the start of the process (time zero) to the point that the chart signals, and the *average time to signal* (ATS) is the expected value of the time to signal (see Yashchin (1987) and Reynolds et al. (1988)). The ATS is computed under the assumption that the shift occurs at time zero.

Reynolds et al. (1988) defined the *number of samples to signal* to be the number of samples required by the chart to signal, and the *average number of samples to signal* (ANSS) to be the expected value of the number of samples to signal. The definition of the number of samples to signal is identical to that of the run length. Reynolds et al. (1988), however, suggested that the former term be adopted for clarity because it is more descriptive, and because in FSI control chart terminology the run length relates to both number of samples and time.

The ATS is defined to be the expected detection time of a shift under the simplifying assumption that the shift occurs at time zero. Although there may be control chart applications where the process starts being out of control at time zero, in most practical situations, it is more reasonable to assume that the process starts being in control at time zero and then changes at some random time in the future. In this situation the appropriate measure of detection time is the length of time from the process change to the chart signal, called the *adjusted* time to signal. The expected value of the adjusted time to signal is called the adjusted ATS. The determination of the distribution of the adjusted time to signal is complicated by the fact that the time point of the shift may fall within a sampling interval. Reynolds et al. (1990a) noted that for charts such as the CUSUM and EWMA, this determination is further complicated by the fact that the control statistic at the time of the shift may not have its starting value. Thus, to determine the distribution of the adjusted time to signal, it is necessary to determine the distribution of the time point of the shift within the sampling interval and (possibly) that of the control statistic at the time of the shift.

Reynolds et al. (1990a) discussed two approaches to determining the distribution of the control statistic at the time of the shift. One approach involves assuming that the time point that the shift occurs has a particular distribution, such as the exponential distribution. Reynolds et al. (1990a) remarked that this approach appears to be quite difficult. The other approach involves assuming that the process has been running for a long enough time that the control statistic has reached a steady-state or stationary distribution at the sampling point just before the shift. Lucas and Saccucci (1990) defined a control statistic to be in steady-state if the process has been in control long enough for the effect of its starting value to be negligible. Under the assumption that the control statistic has reached a stationary distribution by the time the shift occurs, the adjusted ATS is usually called the *steady-state* ATS (Reynolds (1991)). Analogously, steady-state versions can also be defined for other control chart properties, such as the ARL.

For the meaningful study of the steady-state properties of a control chart, it is necessary to make some assumption about the false alarms that occur before the shift actually occurs (Reynolds et al. (1990a)). In the context of FSI CUSUM schemes, Roberts (1966), Taylor (1968), Yashchin (1985), and Crosier (1986) have investigated various approaches to studying steady-state properties. Based on Taylor's (1968) simulation results, Crosier (1986) recommended two approaches. The first approach was to use the stationary distribution of the control statistic, conditional on no false alarm occurring before the shift. The second approach, called *cyclical*, assumes that the control statistic is immediately reset to its starting value after a false alarm. The cyclical approach, however, has the disadvantage of yielding steady-state results that depend on the particular starting value used to reset the control statistic. Because of this, Yashchin (1985) and Reynolds et al. (1990a) adopted the first approach for the investigation of the steady-state behavior of combined Shewhart-CUSUM and VSI CUSUM schemes, respectively. Reynolds (1991) also adopted the first approach to provide a unified framework for the evaluation of properties of control charts such as the CUSUM and EWMA, which have control statistics that can be modeled as Markov processes.

For control charts that can be modeled as a Markov chain, the stationary distribution of the control statistic, conditional on no false alarms, is determined by the normalized left eigenvector $\underline{\pi}$ of the transition distribution matrix corresponding to the dominant eigenvalue (Darroch and Seneta (1965)). In the context of VSI control charts, when the control statistic has reached a stationary distribution, the frequency with which sampling intervals of different lengths are used depends on $\underline{\pi}$. Also, the probability of the shift falling in an interval of a particular length depends on this length. For these reasons, Reynolds et al. (1990a) and Reynolds (1991) assumed that the probability of the shift falling in an interval of a particular length is proportional to the product of the interval's length and the frequency of its use when the control statistic is in steady state. For FSI control charts this frequency is simply one hundred percent.

For the same sampling effort, VSI control charts will detect most process changes faster than their FSI counterparts, both in the zero-time and steady-state senses (for examples, see Reynolds et al. (1988), Reynolds et al. (1990a), and Saccucci et al. (1990)). Reynolds et al. (1990a) noted that unless the process change is very large, the time to signal has a distribution with a right tail much like that of an exponential distribution, and its standard deviation is approximately equal to its mean. When the process is in control, the VSI feature has a negligible effect on the standard deviation, and when the process is out of control, the VSI feature reduces the mean reaction time enough that the standard deviation is actually lower than in the corresponding FSI chart (Reynolds et al. (1990a)).

A possible disadvantage of VSI control charts relative to their FSI counterparts is the lack of regular sampling intervals. VSI charts switch back and forth between sampling intervals of different lengths in an unpredictable manner, potentially increasing administrative costs and human efforts. A measure for the quantitative evaluation of the switching behavior of a VSI control chart is the *average switching rate* (ASWR), defined to be the expected number of switches before the signal divided by the ARL. From an administrative point of view, a low ASWR value is desirable. An ASWR value close to zero, however, is usually not achievable in a chart that is responsive to process changes (Reynolds (1991)). The switching behavior of VSI charts has been investigated by Amin and Letsinger (1991), Shamma et al. (1991), and Amin and Hemasinha (1991).

Other useful measures for the quantitative evaluation and/or comparison of control charts are the *average number of observations to signal* (ANOS), the *average sampling interval* (ASI), the *average observation rate* (AOR), and the FAR. The ANOS is defined to be the expected number of single observations required by the chart to produce a signal. For FSS charts, the ANOS is simply the product of the (fixed) sample size and the ARL. For VSS charts, however, the sample size is random, and thus the ANOS is not simply a constant multiple of the ARL. Hence, to evaluate the sampling costs of a VSS control chart, it is necessary to evaluate the ANOS, in addition to the ARL. The ASI is defined to be the expected length of time between two samples of the chart and is meaningful only in the context of VSI control charts. The AOR is defined to be the expected number of observations sampled by the chart per unit time, and as defined in Section 2.1, the FAR is the expected number of false alarms produced by the chart per unit of time.

2.7 Evaluation of Properties of Control Charts

For the $\overline{X}$ chart, the distribution of the run length is geometric with parameter p, where p is the probability of the chart signaling at a given sampling point. Thus, the mean and variance of the $\overline{X}$ chart's run length are

$$\text{ARL} = \frac{1}{p} \quad , \tag{2.7.1}$$

and

$$\mathrm{Var}(\text{run length}) = \frac{1-p}{p^2} \quad , \tag{2.7.2}$$

respectively. Ghosh et al. (1981) noted, however, that when using an $\overline{X}$ chart to monitor the process mean, if the process variance is estimated, then the t-statistics based on the estimate will no longer be independent. Further, they showed that using a small sample size to estimate the variance would inflate both the mean and variance of the chart's run length. The properties of the $\overline{X}$ chart are relatively easy to evaluate, compared to evaluating those of charts such as the CUSUM and EWMA. The properties of the CUSUM and EWMA charts are difficult to evaluate because no explicit expressions are available for the distributions of the charts' respective run length and time to signal. Page (1954) and Ewan and Kemp (1960), however, showed that the run length distribution of the CUSUM is approximately geometric when the ARL is large.

Two widely used methods for the accurate evaluation of properties of control charts whose control statistics can be modeled as Markov processes are the *integral equation* (IE) method and the *Markov chain* (MC) method. The former is used for charts with continuous control statistics, whereas the latter can be used both for charts with discrete and charts with continuous control statistics. The IE method uses an integral equation involving the property of interest and then applies numerical integration to approximate the integral. The approximation converts the integral equation to a system of linear equations which is solved for the property of interest. Originally, Page (1954) developed the IE method to compute the ARL of the CUSUM chart, and later van Dobben de Bruyn (1968) used it to evaluate the properties of the same chart. Goel and Wu (1971) used the IE method to construct a nomogram based on ARL contours for the design of CUSUM charts, and Crowder (1987) used it to compute the ARL of the EWMA chart.

When the control statistic is discrete (for example, when the observations are count data) and can be modeled as a Markov process with the chart's signal region corresponding to an absorbing state, then the machinery of Markov chains can be used, yielding exact calculations of the chart's properties. When the control statistic is continuous, however, it may be possible to replace the statistic by a discretized version which is a Markov chain, and obtain very accurate approximations to the chart's properties. The MC method was originally developed by Brooks and Evans (1972) to evaluate the ARL of the CUSUM chart. Lucas and Crosier (1982b) and Woodall (1984) developed similar Markov chain models to compute the ARL of two-sided CUSUM schemes. Reynolds et al. (1990a) used the MC method to evaluate the properties of the VSI CUSUM chart and to show that the VSI feature substantially improves the ATS and SSATS performances of the CUSUM chart. Lucas and Saccucci (1990) used the MC method to compute the properties of several enhanced EWMA-type schemes. Rendtel (1990) employed the MC method to assess the properties of VSI and VSS CUSUM schemes for controlling the proportion of defectives coming from a continuous production process. A framework was provided by Reynolds (1991) for the unified treatment of the IE and MC methods, which have traditionally been viewed as separate methods. The MC method is more flexible for evaluating certain properties that are difficult to compute with the IE method, but the IE method offers higher precision for the same computational effort.

In traditional SPRT terminology, the *sample number* is defined to be the number of samples required by an SPRT to either reject or "accept", and the *average sample number* (ASN) is the expected value of the sample number. Page (1954) showed that the CUSUM procedure is equivalent to a sequence of independent SPRT's, each having the CUSUM's control limit as its rejection limit and 0 as its "acceptance" limit. To quantify the number of SPRT's in a CUSUM chart, define the *number of tests to signal* to be the number of SPRT's required by the CUSUM chart to produce a signal, and the *average number of tests to signal* (ANTS) to be the expected value of the number of tests to signal. The distribution of the number of tests to signal is geometric with parameter p_s, where p_s is the rejection probability of a given SPRT. Thus, the ANTS and the ARL of the CUSUM chart are

$$\text{ANTS} = \frac{1}{p_s} \quad , \tag{2.7.3}$$

and

$$\text{ARL} = \text{ANTS} \cdot \text{ASN} \quad , \tag{2.7.4}$$

respectively.

Several approximations to the ARL of the CUSUM chart have been derived using the analogy between this chart and the sequence of SPRT's. Reynolds (1975) used a limiting argument based on Wald's (1947) diffusion theory approximations to the ASN and to p_s to obtain an approximation formula to the CUSUM's ARL for a normal process. He also used a continuous-time Brownian motion process as an approximation to the CUSUM's discrete-time control statistic to obtain the same approximation formula, though without assuming a normal process. Reynold's (1975) Brownian motion approximation uniformly underestimates the value of the CUSUM's ARL. For small and moderate values of the CUSUM's signal limit, the discrepancy is very serious. Reynolds (1975), however, discovered empirically that using an appropriately shifted signal limit in the approximation formula yields reasonable approximations to the CUSUM's ARL for most shifts. Khan (1978) also used the analogy between the CUSUM chart and the sequence of SPRT's to provide Wald-type diffusion theory approximations to the ARL of CUSUM procedures for an exponential family of densities. As Reynolds' (1975) Brownian motion approximation, Kahn's (1978) Wald-type approximations yield poor results.

Siegmund (1975, 1979, 1985a, 1985b), Pollak and Siegmund (1975, 1985), and Park (1992) stressed that Wald's (1947) asymptotic diffusion approximations to the SPRT's ASN and p_s are inaccurate when applied to discrete-time practical situations and should be avoided. Another drawback of Wald's (1947) approximations is that they are not valid when the SPRT's "acceptance" limit is 0, as in CUSUM procedures. Consequently, a number of discrete-time methods have been developed as improvements on Wald's (1947) approximations. These methods are called *corrected diffusion approximations* (Siegmund (1985a)) or *corrected Browninan motion approximations* (Siegmund (1985b)) because they introduce correction terms to the SPRT's limits in Wald's (1947) expressions, in order to improve the accuracy. The correction terms estimate the "excess over the limits" of the SPRT's statistic in discrete time.

Siegmund (1975) proposed corrected diffusion approximations suggested by results in renewal theory. These approximations offer substantially improved accuracy when compared to those of Wald (1947), though only for normally distributed observations. For the special case where the "acceptance" limit is 0, Siegmund (1975) obtained complete asymptotic expansions of the ASN and of p_s in terms of reciprocal powers of the rejection limit, but under restrictive assumptions. A major drawback of Siegmund's (1975) corrected approximations is the great difficulty involved in their computation, being functions of the three first moments of ladder distributions (Siegmund (1975)). Siegmund (1979) derived "usable" corrected diffusion approximations to random walk problems in both finite and infinite time. As Siegmund's (1975) corrected approximations, those of Siegmund (1979) are based on moments of ladder distributions and are of similar accuracy. Siegmund (1979), however, succeeded in developing an algorithm for the numerical evaluation of his 1979 approximations. Siegmund (1985a) provided a survey of the development of corrected diffusion approximations up to that time and discussed advantages and disadvantages of these as he compared them.

Goal and Wu (1971) used the IE method to obtain highly accurate approximations to the SPRT's ASN and p_s. Then, they used the approximations in equations **(2.7.3)** and **(2.7.4)** to accurately evaluate the ARL of the CUSUM chart. Recently, corrected diffusion approximations to these were proposed by Park (1992). Park's (1992) corrections involve estimating the expected value of the "excess over the limits" of the SPRT's statistic in discrete time, using the *condition of before-stopping time* (CBST) method. The CBST method, developed by Park (1992), is a numerical method which conditions on the value of the SPRT's statistic immediately before the stopping time. Park's (1992) approximations were much less accurate than those obtained using the IE method, but of similar accuracy to the ones proposed by Siegmund (1975, 1979).

The computational effort required in approximating the ASN and p_s, using corrected diffusion approximations, is similar to that required by the IE and MC methods. Corrected diffusion approximations, however, are less accurate than those obtained using the IE or MC methods (see Goel and Wu (1971), Siegmund (1979, 1985a) and Park (1992)). When the SPRT's "acceptance" limit is close to 0, as in CUSUM procedures, the discrepancy is serious (see Siegmund (1975, 1979, 1985a)). Although corrected diffusion approximations are not especially good approximations to the corresponding quantities in discrete time, they can provide useful information for the quantitative assessment of the discrete-time behavior of

SPC procedures developed in the context of continuous time (Pollak and Siegmund (1985)). Pollak (1987) provided diffusion theory corrections for the discrete-time evaluation of the Shiryayev-Roberts procedure. Pollak's (1987) corrections are related, though not completely analogous to Siegmunds' (1979) corrections and are also difficult to compute (Pollak and Siegmund (1985)).

Earlier, Nadler and Robbins (1971) studied the two-sided CUSUM procedure as proposed by Page (1954) to approximate the null distribution of the control statistic and the ARL of the procedure. In order to obtain these approximations, they derived the distribution of a Brownian motion process and the Laplace transformation of the time at which the run length exceeds a given value. Nadler and Robbins (1971) used Donsker's (1951) theorem to show that the asymptotic distribution of the control statistic does not depend on the particular distribution of the observations, which gives the procedure a nonparametric character. They concluded, however, that the quality of their approximation to the ARL of Page's (1954) two-sided CUSUM is poor and depends on the underlying distribution.

In the continuous-time context of monitoring the drift coefficient of a Brownian motion process, Shiryayev (1963) and Roberts (1966) independently used asymptotic diffusion theory results to evaluate the ATS of the Shiryayev-Roberts procedure. Asymptotic diffusion theory was also used to evaluate the ATS of Bather's (1976) control scheme for monitoring the drift of a Brownian motion. Pollak and Siegmund (1985) used the machinery of diffusion processes to make a quantitative comparison of the Shiryayev-Roberts and CUSUM procedures on the basis of SSATS performance for several different Brownian motion SPC problems. Pollak and Siegmund (1985) noted that the comparison was made in the context of continuous time in order to use the asymptotic results of diffusion theory to perform accurate SSATS evaluations, which seemed impossible in discrete time. The asymptotic machinery of diffusion processes was also used by Assaf (1988), Assaf and Ritov (1989), and Assaf et al. (1992) to compute the SSATS of their respective procedures, proposed for the model of detecting a change in the drift of a Brownian motion.

2.8 Optimality Results for Control Charts

Lorden (1971) formulated a problem on the optimal stopping time for detecting a change in distribution and proposed simple rules which are asymptotically optimal in a well defined, although complex, sense. The problem is of central importance in SPC and has applications in other areas, such as reliability theory (Lorden (1971)). Moustakides (1989) showed that the CUSUM's stopping rule is minimax in the sense of Lorden (1971). Ritov (1990) gave an alternative proof to Moustakides' (1986) result, showing that the CUSUM procedure defines the optimal (Bayes) stopping time in a specific stochastic game between nature and the statistician. In this game, nature chooses the change point, and the statistician chooses the stopping time. Both make their decision after considering all of the observed data.

In the context of continuous time, Shiryayev (1963) adopted the Bayesian framework in order to solve the problem of optimally detecting a change in the drift coefficient of a Brownian motion process, subject to a limitation on the number of false alarms. He showed that the Shiryayev-Roberts procedure minimizes the SSATS under the assumption that tbe time at which the drift coefficient changes has a (prior) exponential distribution. Shiryayev (1963) attempted the same problem in the context of discrete time, though without any definitive results. Limited results, however, for the discrete-time version of

Shiryayev's (1963) (Bayesian) model were established by Bather (1967). He showed that the optimal policy for the period following any false alarm must be identical to the procedure applied initially, before the first inspection. In addition, he proved the existence of a critical level *l*, which is a function of the expected number of false alarms, such that sampling should continue as long as the Shiryayev-Roberts' (discrete-time) posterior probability is less than *l*, with immediate inspection otherwise. Bather (1967), however, did not determine the critical level for the optimal policy.

Pollak (1985) proved an optimality property for the Shiryayev-Roberts procedure in the continuous-time setting, which is considerably stronger than Lorden's (1971) asymptotic optimality. A brief discussion of this result was provided by Pollak and Siegmund (1985). Assaf (1988) derived the optimal policy for the dynamic sampling version of Shiryayev's (1963) continuous-time Bayesian model, where the amount of sampling may vary as desired, subject to a constraint on the average amount sampled. Assaf et al. (1992) noted that the CUSUM scheme is a special case of the combined procedure that Assaf et al. (1992) introduced, and that the CUSUM scheme is the best combined procedure when the switching cost is too high. Further, they concluded that the combined procedure can be optimal only in a sense that coincides with an optimality property of the CUSUM scheme, when the CUSUM is the best combined procedure. Assaf et al. (1992) also extended Ritov's (1990) proof to establish a minimax optimality property of the combined procedure in the context of continuous time.

Consider a control chart that is used to monitor a process of interest in order to detect changes in a (random) measurable quality characteristic whose distribution depends on a parameter θ. Let θ_0 represent the target value for θ, and assume that if an assignable cause occurs, then any resulting change in the process will be reflected by a change in θ. The *sampling interval function* of a control chart is a function of the control statistic which specifies the length of the sampling interval to be used next, based on the currently observed value of the control statistic. Following the development in Reynolds (1989) for the $\overline{X}$ chart and in Reynolds (1991) for general one- and two-sided control charts, the sampling interval function of a control chart is said to be *unbiased* if

$$\mathrm{ARL}(\theta) \leq \mathrm{ARL}(\theta_0) \quad \text{for all } \theta \neq \theta_0\,, \tag{2.8.1}$$

and

$$\mathrm{ATS}(\theta) \leq \mathrm{ATS}(\theta_0) \quad \text{for all } \theta \neq \theta_0\,. \tag{2.8.2}$$

As noted in Section 2.2, Reynolds and Arnold (1989) showed that for one-sided VSI $\overline{X}$ charts, the use of exactly two sampling intervals, the shortest possible and the longest possible, is optimal for detecting a specific shift in the process mean, in the sense of minimizing the ATS. Reynolds (1989) showed that Reynolds and Arnold's (1989) result is much more general in that it is also true for two-sided VSI $\overline{X}$ charts with unbiased sampling interval functions, and for all one-sided VSI charts in the class of charts that can be modeled as a Markov process. Reynolds and Arnold's (1989) and Reynolds' (1989) results were also independently established by Runger and Pignatiello (1991). Reynolds (1991) derived some general optimality results for the choice of the sampling interval function of a control chart, in the sense of minimizing the ATS.

The SSATS performance of the VSI $\overline{X}$ chart was optimized by Runger and Montgomery (1993). By discretizing the possible values of the (standardized) $\overline{X}$ statistic, he transformed the problem of minimizing the SSATS to a nonlinear optimization problem in a form that applies to the Kuhn-Tucker Theorem. After obtaining a discrete solution to the optimization problem, Runger and

Montgomery (1993) used the continuous analogue to translate the solution and obtain the (continuous) multi-interval SSATS-optimal sampling interval function. Expressions for the SSATS-optimal sampling interval function were obtained for both one- and two-sided VSI $\overline{X}$ charts. These expressions hold for any $\overline{X}$ chart in which the control statistic has the monotone likelihood ratio property with respect to the process parameter under control (Runger and Montgomery (1993)). Runger and Montgomery (1993), however, recommended using VSI $\overline{X}$ charts with only two sampling intervals, because such charts can be constructed that will perform comparably to much more complicated SSATS-optimal counterparts. The very small advantage of an SSATS-optimal VSI $\overline{X}$ chart for only a specific shift does not justify its extra complexity (Runger and Montgomery (1993)).

2.9 Control Charts in the Presence of Serial Correlation

In most SPC applications it is assumed that the observations from the process output are realizations of independent and identically distributed (i.i.d.) random variables. For some processes, however, this assumption is not realistic because the observations are serially correlated. One of the early investigations on the effect of serial correlation was conducted by Goldsmith and Whitfield (1961) for the CUSUM chart. They used a Monte Carlo simulation study for a process operating at its target value to conclude that positive serial correlation decreases the ARL of a CUSUM chart, while negative serial correlation increases the ARL. Johnson and Bagshaw (1974) and Bagshaw and Johnson (1975) used respective Brownian motion approximations to the CUSUM's control statistic in order to study the effects of first order autoregressive (AR(1)) and first order moving average (MA(1)) processes on CUSUM charts. For the AR(1) processes, they reached the same conclusions as Goldsmith and Whitfield (1961). For the MA(1) processes, however, they found that the CUSUM's ARL is increased both by the presence of positive and of negative serial correlation. Kartha and Abraham (1979) conducted simulation studies of ARL behavior of the CUSUM chart for AR(1), MA(1), and first order mixed autoregressive-moving average (ARMA(1, 1)) processes. Their results for the AR(1) and MA(1) processes agreed with the respective results of Goldsmith and Whitfield (1961), of Johnson and Bagshaw (1974), and of Bagshaw and Johnson (1975). Kartha and Abraham (1979) also found that the effect of the ARMA(1, 1) process on the CUSUM's ARL is similar to that of the MA(1) process. VanBrackle (1991) investigated the effect of an AR(1) process on both the CUSUM and EWMA control procedures. He concluded that for both procedures, positive serial correlation decreases the ARL when the process is in control, or when it has undergone a small shift in the mean. For a process, however, that has undergone a larger shift in the mean, the ARL of both EWMA and CUSUM procedures increases with positive and negative serial correlation. VanBrackle (1991) also concluded that for the same FAR, the ARL of both the CUSUM and EWMA procedures is larger for an AR(1) process than for an i.i.d. process.

Recently, various modifications to control charts have been investigated for the purpose of controlling serially correlated processes. Athanasios and Stamboulis (1978) modified the control limits of the Shewhart chart to account for the dependence in low-order autoregressive processes. Berthouex et al. (1978) used stochastic model fitting to study serially correlated data from sewage treatment plants. Their results suggested fitting a time series model to the data and plotting the residuals from the model on Shewhart and CUSUM charts. Abraham and Kartha (1979) called any structural change in a time series-modeled process a *forecast instability* and suggested using the residuals from the model in four different control charts, to detect forecast instability. The four charts are the Shewhart and CUSUM charts, a modified CUSUM chart, and a Shewhart chart for the first lag autocorrelation among the (one step ahead forecast) residuals. The modified CUSUM chart accumulates products of the residuals and their corresponding derivatives (with respect to the model parameter), to detect forecast instability. Ermer (1980) proposed using a Shewhart chart for the sum of squares of residuals to monitor a time series-modeled process. Dooley et al. (1986) suggested using Shewhart and CUSUM charts based on the residuals or on their autocorrelations to detect changes in a time series-modeled process.

Alwan and Roberts (1988) proposed a two-chart control procedure for monitoring a serially correlated process. They noted that such a process drifts about its mean value with time and proposed using a *common-cause* control chart as a guide in seeking a better understanding of the process' drifting behavior. In addition, they proposed using a *special-cause* control chart to detect process changes in a traditional manner. The common-cause chart is a Shewhart-type chart, though without control limits, based on the fitted values from a time series model of the process. Instead of control limits, the common-cause chart uses *action* limits determined by economic considerations of the costs of operating out of control. The action limits are not to be used to signal the presence of an assignable cause of variation, but rather to indicate that the process has drifted far from its target value, and rectifying action must be taken. The special-cause chart is simply a Shewhart chart based on the residuals of the fitted time series model.

Alwan and Roberts (1988) did not evaluate any of the properties of the common- and special-cause charts they proposed. Wardell et al. (1990a), however, developed a method for deriving the run length distribution of the special-cause chart for mixed autoregressive-moving average models of any finite order and for two types of shifts, Wardell et al. (1990b) investigated four charts in the presence of serial correlation. These were the traditional Shewhart and EWMA charts, the common-cause chart with added control limits, and the special-cause chart. Response surface analysis was used to perform quantitative comparisons of the four charts on the basis of ARL. The comparisons were performed in the context of an ARMA(1, 1) process, and two methodologies, the *dynamic step response function* and the *impulse response function*, were introduced for the interpretation of the comparisons' results. The (traditional) EWMA chart was found to be very efficient in detecting small shifts and to perform well for large shifts when the autoregressive parameter is negative, but the moving average parameter is positive. The (traditional) Shewhart, common-cause, and special-cause charts, however, were found to perform well for most ARMA(1, 1) models, though only for large shifts.

Yashchin (1993) presented a method for assessing the run length distribution of a general CUSUM control chart when the underlying data exhibit the presence of moderate serial correlation. The method replaces a sequence of correlated observations by an i.i.d. sequence for which the run length distribution is approximately the same. Formulas for several special cases involving ARMA models and variability monitoring were developed. Yashchin's (1993) method is relatively simple to apply, provides exact results when no serial correlation is present, and leads to approximations that can be considered acceptable in many practical situations when the magnitude of the serial correlation is not too large, especially for Gaussian processes.

Chapter 3

The SPRT Chart

3.1 Introduction

Suppose that X denotes a measurable characteristic of a process of interest whose distribution is normal with mean μ and standard deviation σ. Let μ_0 denote the target value for μ, and assume that if an assignable cause occurs, then any resulting change in the process will be reflected by a change in μ. Even though a process change can generate many types of deviations from μ_0, the simple case will be considered here where μ shifts from μ_0 to $\mu_1 \neq \mu_0$ and stays at μ_1 until the shift is detected and corrective action is taken. In practice it may be necessary to estimate μ_0 and/or σ from past data, but for simplicity it will be assumed here that μ_0 and σ are known.

The proposed *SPRT* chart is a control chart defined as a sequence of similar SPRT's, each for testing the null hypothesis H_0: $\mu = \mu_0$ against the alternative hypothesis H_1: $\mu = \mu_1$ and without any sampling between consecutive SPRT's. At each sampling point, an SPRT is conducted by taking sequentially random samples of size $n \geq 1$, with a time interval $t_w \geq 0$ between consecutive samples *within* the SPRT. The time interval $t_b > 0$ *between* consecutive SPRT's is assumed to be at least as large as the time interval t_w. When $n > 1$, the SPRT's are applied to (the averages of) groups of n observations rather than to individual observations as in the usual case.

To fix an example, consider the model $(n, t_w, t_b) = (1, 0, 1)$, for the case of a computer chip production line in an electronics industry where the speed of each chip produced can be measured in negligible time using an electronic device, and where the time unit is one hour. In this case, every hour on the hour, for example, individual computer chips could be sampled sequentially in negligible time and their speed measured one chip at a time, applying an SPRT for testing the hypothesis that the process is in control. To fix a second example, consider the model $(n,\ t_w,\ t_b) = (5,\ 0.25,\ 2)$, for the same case of the computer chip production line where the time unit is one hour, but where the temperature that each chip reaches in 15 minutes of operating time can be measured. In this case, starting at time 8:45 a.m., for example, computer chips could be sampled sequentially every 15 minutes in groups of five and their individual temperatures measured five chips at a time, applying an SPRT for testing the hypothesis that the process is in control. That is, the SPRT's are applied to (the averages of) groups of 5 measurements rather than to individual measurements as in the usual case of the first example. The next SPRT starts being applied in the same manner two hours after the last observation for the 8:45 a.m. SPRT is taken, provided that this SPRT does not reject the hypothesis that the process is in control. Additional SPRT's are applied similarly until the hypothesis that the process is in control is rejected and a signal is given.

3.2 Description of the SPRT Chart

Specifically, the SPRT chart can be described as follows:

Consecutive sequences S_1, S_2, ... of random samples of size $n \geq 1$ are taken from the process at times T_1, T_2, ..., respectively, to detect any deviation of μ from μ_0. Let T_i be the time that sequence S_i is started, and let N_i denote the number of samples in sequence S_i, for $i = 1,\ 2, \ldots$ Assume that observations between samples within and across sequences are independent, and that the process starts at time $T_0 = 0$, but no sample is taken before time $T_1 = t_{b_0}$. Let

$\underline{X}_{ij} = (X_{ij1}, X_{ij2}, \ldots, X_{ijn})$ represent the j^{th} sample in sequence S_i, and $\overline{X}_{ij}$ its sample mean, for $i = 1, 2, \ldots$ and $j = 1, \ldots, N_i$. Let $t_w \geq 0$ be the sampling interval between samples $\underline{X}_{ij}$ and $\underline{X}_{i,j+1}$, for $i = 1, 2, \ldots$ and $j = 1, \ldots, N_i - 1$, and let $t_b > 0$ be the sampling interval between samples $\underline{X}_{iN_i}$ and $\underline{X}_{i+1,1}$, for $i = 1, 2, \ldots$, with $0 \leq t_w \leq t_b$. That is, t_w is the time interval between samples *within* sequences, and t_b is the one *between* sequences. Then, if I_i denotes the time interval between times T_i and T_{i+1}, I_i satisfies

$$I_i = \begin{cases} t_{b_0} \quad , & \text{if } i = 0 \\ (N_i - 1)\, t_w + t_b \quad , & \text{if } i = 1, 2, \ldots \end{cases} \tag{3.2.1}$$

Standardizing the deviation of $\overline{X}_{ij}$ from μ_0 in terms of units of σ, let

$$Z_{ij} = (\overline{X}_{ij} - \mu_0) \,/\, \sigma \quad . \tag{3.2.2}$$

Define the control statistic of the *upper* one-sided SPRT chart for detecting an increase in μ to be

$$U_{ij} = \begin{cases} u_0 + \sum_{j'=1}^{j} W_{ij'} \quad , & \text{if } i = 1 \\ \sum_{j'=1}^{j} W_{ij'} \quad , & \text{if } i = 2, 3, \ldots \end{cases} \tag{3.2.3}$$

$$\text{for } j = 1, \ldots, N_i \quad ,$$

where u_0 is the specified starting value of the control statistic, $W_{ij'} = \sqrt{n}\,(Z_{ij'} - \gamma)$, and $\gamma > 0$ is a specified chart parameter. Define the upper one-sided SPRT chart as a sequence of similar SPRT's for testing the null hypothesis H_0: $\mu = \mu_0$ against the alternative hypothesis H_1: $\mu = \mu_1$, where $\mu_1 = \mu_0 + 2\gamma\sigma$, with a time interval t_b

and no sampling between consecutive SPRT's. Also, define the chart's i^{th} SPRT to be conducted at time T_i and using the samples from sequence S_i to compute the control statistic in **(3.2.3)** as its test statistic. That is, when $n > 1$, the SPRT's are applied to (the averages of) groups of n observations rather than to individual observations, as in the usual case. At the i^{th} SPRT, define the upper one-sided SPRT chart to stop sampling and signal if $U_{ij} > h$, to continue sampling as long as $g \leq U_{ij} \leq h$, and to stop sampling and conclude that the process is in control if $U_{ij} < g$. If $U_{ij} < g$, then proceed to time T_{i+1} and conduct the next SPRT. Note that N_i represents the sample number of the i^{th} SPRT of the upper one-sided SPRT chart. For an example of the upper one-sided SPRT chart, see Figure 3.2.1. It can easily be seen that the control statistic in **(3.2.3)** is the loglikelihood ratio (apart from a constant multiple) in the SPRT for testing H_0: $\mu = \mu_0$ against H_1: $\mu = \mu_1$, where $\mu_1 = \mu_0 + 2\gamma\sigma$.

Similarly, define the control statistic of the *lower* one-sided SPRT chart for detecting a decrease in μ to be

$$U_{ij} = \begin{cases} u_0 + \sum_{j'=1}^{j} V_{ij'} \quad, & \text{if } i = 1 \\ \\ \sum_{j'=1}^{j} V_{ij'} \quad, & \text{if } i = 2, 3, \ldots \end{cases} \qquad \textbf{(3.2.4)}$$

$$\text{for } j = 1, \ldots, N_i \quad ,$$

where u_0 is the specified starting value of the control statistic, $V_{ij'} = \sqrt{n}\ (Z_{ij'} + \gamma)$, and $\gamma > 0$ is a specified chart parameter. Define the lower one-sided SPRT chart as a sequence of similar SPRT's for testing the null hypothesis H_0: $\mu = \mu_0$ against the alternative hypothesis H_1: $\mu = \mu_1$, where $\mu_1 = \mu_0 - 2\gamma\sigma$, with a time interval t_b

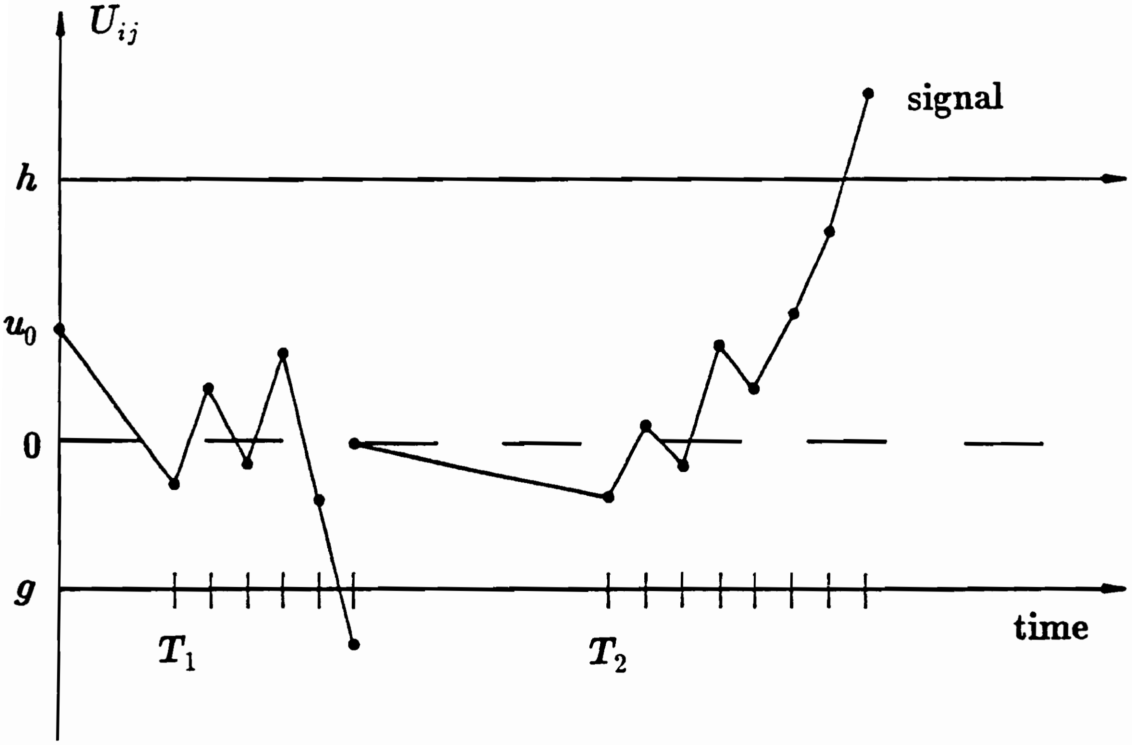

Figure 3.2.1. Upper One-Sided SPRT Chart.

and no sampling between consecutive SPRT's. Also, define the chart's i^{th} SPRT to be conducted starting at time T_i and using the samples from sequence S_i to compute the control statistic in **(3.2.4)** as its test statistic. As in the upper one-sided SPRT chart, in the lower one, when $n > 1$ the SPRT's are applied to (the averages of) groups of n observations rather than to individual observations, as in the usual case. At the i^{th} SPRT, define the lower one-sided SPRT chart to stop sampling and signal if $U_{ij} < -h$, to continue sampling as long as $-h \leq U_{ij} \leq -g$, and to stop sampling and conclude that the process is in control if $U_{ij} > -g$. If $U_{ij} > -g$, then proceed to time T_{i+1} and conduct the next SPRT. Note that, as in the upper one-sided SPRT chart, N_i represents the sample number of the i^{th} SPRT of the lower one-sided SPRT chart. For an example of the lower one-sided SPRT chart, see Figure 3.2.2. It can easily be seen that the control statistic in **(3.2.4)** is the loglikelihood ratio (apart from a constant multiple) in the SPRT for testing H_0: $\mu = \mu_0$ against H_1: $\mu = \mu_1$, where $\mu_1 = \mu_0 - 2\gamma\sigma$.

A two-sided control procedure for the detection of two-sided deviations in μ will not be considered here. However, two one-sided SPRT charts may be applied simultaneously to detect such deviations, one to detect an increase in μ and the other a decrease. The two-sided extension of the SPRT chart is a nontrivial task, and for further discussion the reader is refered to Section 7.2. Without loss of generality, the upper one-sided SPRT chart will be considered here, and for the remaining part of this paper "one-sided SPRT chart" or simply "SPRT chart" is understood to mean "upper one-sided SPRT chart".

Since the control statistic of the SPRT chart is expressed in terms of the standardized Z_{ij} in **(3.2.2)**, it is convenient to standardize μ also and take the process parameter to be

$$\delta = (\mu - \mu_0) / \sigma \quad , \qquad \textbf{(3.2.5)}$$

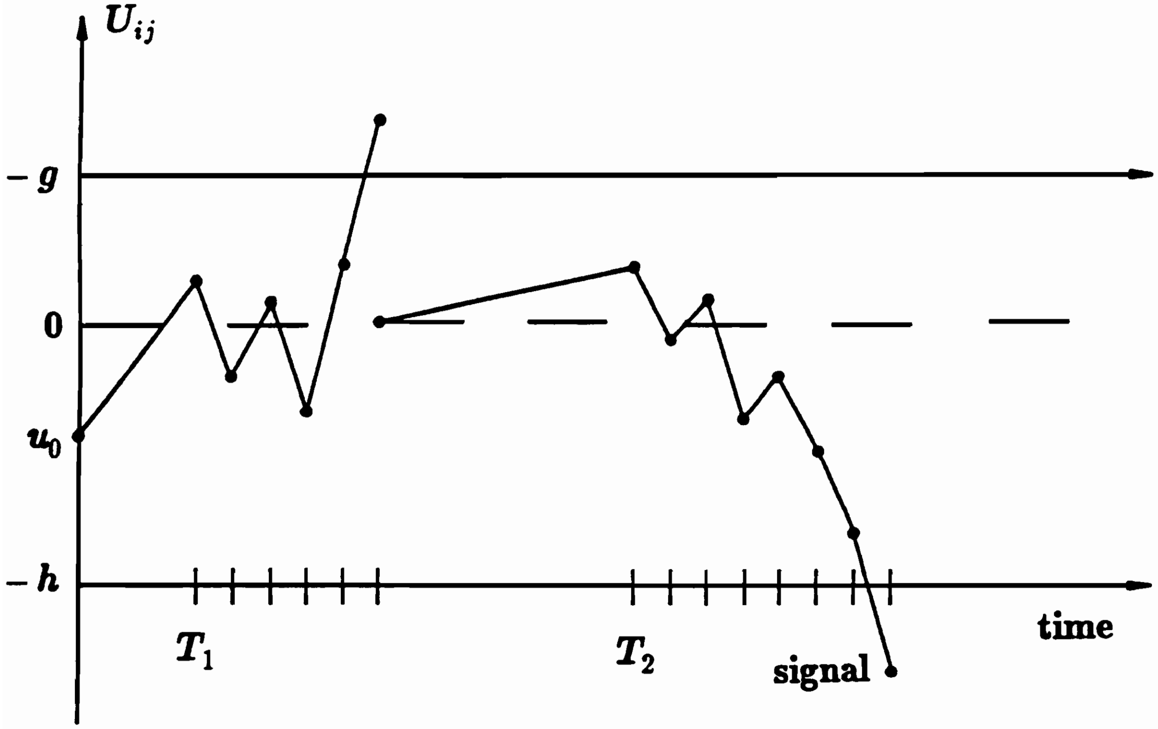

Figure 3.2.2. Lower One-Sided SPRT Chart.

and the target value to be $\delta_0 = 0$. In this case, a standardized shift in the process mean is given by δ.

The parameters g, h, and γ are chosen so that the SPRT chart will have certain properties. For example, if the SPRT chart is designed to detect a shift from $\delta_0 = 0$ to a specified value δ_1, then optimality theory of SPRT's indicates that γ should be chosen to be $\delta_1/2$ (Ghosh (1970), pp. 93 – 98). Once γ is determined, g and h could be chosen to attain a desired FAR. Following the CUSUM chart terminology, γ is called the reference value of the SPRT chart.

3.3 Definitions of Properties of the SPRT Chart

The properties of the SPRT chart considered here are specifically defined as follows:

(i) The *average sample number* (ASN) is the expected value of the number of samples in a given SPRT of the chart required by the control statistic to either produce a signal or drop below the in-control limit g.

(ii) The *average number of tests to signal* (ANTS) is the expected value of the number of SPRT's required by the chart to produce a signal.

(iii) The *average run length* (ARL) is the expected value of the number of samples (over all the SPRT's) required by the chart to produce a signal.

(iv) The *average number of observations to signal* (ANOS) is the expected value of the total number of single observations (over all samples and SPRT's) required by the chart to produce a signal.

(v) The *average time to signal* (ATS) is the expected value of the length of time from the start of the process (time $T_0 = 0$) to the point that the chart produces a signal.

(vi) The *steady-state average time to signal* (SSATS) is the expected value of the length of time from the point that a shift occurs to the point that the chart produces a signal, when the control statistic has reached a stationary distribution at the sampling point just before the shift.

(vii) The *average test interval* (ATI) is the expected value of the length of time between the beginnings of two consecutive SPRT's of the chart.

(viii) The *average sampling interval* (ASI) is the expected value of the length of time between any two consecutive samples of the chart (ignoring the SPRT's).

(ix) The *average observation rate* (AOR) is the expected value of the number of single observations sampled by the chart per unit of time.

(x) The *false alarm rate* (FAR) is the expected value of the number of signals produced by the chart per unit of time, when the process is in control ($\delta = 0$).

3.4 Expressions for Properties of the SPRT Chart

Expressions will now be developed for the properties of the SPRT chart defined in Section 3.3. The dependence of these expressions on the process parameter δ will be suppressed, and the expressions will be approximated numerically in Section 3.5. It will be assumed that when δ is constant, the joint distribution of the control statistics U_{11}, U_{12}, U_{13}, ..., U_{21}, U_{22}, U_{23}, ... does not depend on the sampling times T_1, $T_1 + t_w$, $T_1 + 2t_w$, ..., T_2, $T_2 + t_w$, $T_2 + 2t_w$, ... That is, when δ is constant, the length of the time interval between consecutive samples within or between SPRT's (t_w or t_b) does not affect the distribution of the control statistic. Since the observations of the SPRT chart are assumed to be i.i.d., the SPRT chart satisfies this assumption. Let f_W denote the density of the increments in equation **(3.2.3)**. That is, f_W is the normal density with mean $\sqrt{n}\,(\delta-\gamma)$ and variance 1.

The ASN of the SPRT chart, conditional on the starting value u_0 of the control statistic, satisfies the integral equation

$$\mathrm{ASN}(u_0) \;=\; 1 \;+\; \int_g^h \mathrm{ASN}(w)\, f_W(w-u_0)\,\mathrm{d}w \quad . \tag{3.4.1}$$

Note that suppressing the starting value u_0 of the control statistic, $\mathrm{ASN} = \mathrm{E}(N_i)$, for all i. Similarly to the ASN in **(3.4.1)**, the ARL of the SPRT chart, conditional on the starting value u_0 of the control statistic, satisfies the integral equation

$$\mathrm{ARL}(u_0) \;=\; 1 \;+\; \int_{-\infty}^h \mathrm{ARL}(w)\, f_W(w-u_0)\,\mathrm{d}w \quad . \tag{3.4.2}$$

As defined in Section 3.3, the ANTS of the SPRT chart, conditional on the starting value u_0 of the control statistic, is given by

$$\mathrm{ANTS}(u_0) \;=\; \frac{\mathrm{ARL}(u_0)}{\mathrm{ASN}(u_0)} \quad . \tag{3.4.3}$$

The ATS of the SPRT chart, conditional on the starting value u_0 of the control statistic, satisfies the integral equation

$$\mathrm{ATS}(u_0) = t_{b_0} + \int_{-\infty}^h \mathrm{ATS}(w) f_W(w-u_0)\,\mathrm{d}w \quad . \tag{3.4.4}$$

An indirect expression for $\mathrm{ATS}(u_0)$ for the SPRT chart is given by

$$\begin{aligned}\mathrm{ATS}(u_0) &= \mathrm{E}\left(\sum_{i=0}^{R-1} I_i + (N_R - 1)\, t_w \mid u_0\right) \\ &= t_{b_0} + \mathrm{ANTS}(u_0)\,(\mathrm{ASN}(u_0) - 1)\, t_w + (\mathrm{ANTS}(u_0) - 1)\, t_b \quad , \qquad (3.4.5)\end{aligned}$$

by Wald's identity, where R denotes the number of tests to signal for the SPRT chart.

As defined in Section 3.3, the ANOS, ATI, ASI, and AOR of the SPRT chart, conditional on the starting value u_0 of the control statistic, are given by

$$\mathrm{ANOS}(u_0) = n \cdot \mathrm{ARL}(u_0) \quad , \qquad \textbf{(3.4.6)}$$

$$\mathrm{ATI}(u_0) = \frac{\mathrm{ATS}(u_0)}{\mathrm{ANTS}(u_0)} \quad , \qquad \textbf{(3.4.7)}$$

$$\mathrm{ASI}(u_0) = \frac{\mathrm{ATS}(u_0)}{\mathrm{ARL}(u_0)} \quad , \qquad \textbf{(3.4.8)}$$

and

$$\mathrm{AOR}(u_0) = \frac{\mathrm{ANOS}(u_0)}{\mathrm{ATS}(u_0)} \quad , \qquad \textbf{(3.4.9)}$$

respectively. Note that suppressing the starting value u_0 of the control statistic, $\mathrm{ATI} = \mathrm{E}(I_i)$, for all i. Also, as defined in Section 3.3, the FAR of the SPRT chart, conditional on the starting value u_0 of the control statistic, is given by

$$\mathrm{FAR}(u_0) = \frac{1}{\mathrm{ATS}_0(u_0)} \quad , \qquad \textbf{(3.4.10)}$$

where $\mathrm{ATS}_0(u_0)$ denotes ATS (u_0) when $\delta = 0$.

Recall from Section 2.8 that the sampling interval function $l(u)$ of a control chart is a function of the control statistic which specifies the length of the sampling interval to be used next, based on the currently observed value u of the control statistic. Ignoring the SPRT's in the SPRT chart, the sampling interval function of the *upper* one-sided SPRT chart is

$$l_U(u) = \begin{cases} t_w , & \text{if } u \epsilon [g, h] \\ t_b , & \text{if } u \epsilon (-\infty, g) \end{cases} , \tag{3.4.11}$$

and the sampling interval function of the *lower* one-sided SPRT chart is

$$l_L(u) = \begin{cases} t_w , & \text{if } u \epsilon [-h, -g] \\ t_b , & \text{if } u \epsilon (-g, +\infty) \end{cases} . \tag{3.4.12}$$

That is, the in-control limit g is the boundary between the regions specifying the use of t_w and t_b. It should be noted that in practice, the initial sampling interval t_{b_0} may be taken to be different than $l_U(u_0)$ for an upper one-sided SPRT chart, or $l_L(u_0)$ for a lower one.

By definition, the computation of the SSATS of the SPRT chart requires the determination of the distribution of the control statistic at the last sample before the shift in δ. In agreement with Reynolds et al. (1990a), this distribution will be assumed to be proportional to the product of a stationary distribution of the control statistic and the sampling interval function. Various stationary distributions have been used for the evaluation of steady-state properties of control charts (see, for examples, Crosier (1986), Yashchin (1985), and Reynolds et al. (1990a)). Darroch and Seneta (1965) provide a good discussion of stationary distributions in absorbing Markov chains. Following Yashchin (1985) and Reynolds et al. (1990a), the stationary distribution conditional on no false alarms will be used here. The integral equation satisfied by the SSATS of the SPRT chart involves complicated mathematical expressions that may be derived using the framework provided by Reynolds (1991), but are beyond the scope of this paper. Hence, this integral equation will not be developed here. A numerical approximation, however, to the SSATS of the SPRT chart will be developed in Section 3.5, together with a numerical approximation to the variance of the steady-state time to signal of this chart.

3.5 Numerical Evaluations of Properties of the SPRT Chart

As noted in Section 2.7, two widely used methods for accurately evaluating properties of control charts whose control statistics can be modeled as Markov processes are the IE method and the MC method. The MC method offers flexibility for evaluating certain properties that are difficult to compute with the IE method, but the IE method offers higher precision for the same computational effort. Since expressions in terms of integral equations are available from Section 3.4 for the properties of the SPRT chart, flexibility is not needed, and thus the IE method will be used here within the unified framework developed by Reynolds (1991).

Approximations to the ASN, ARL, and ATS of the SPRT chart may be obtained by replacing integral equations **(3.4.1)**, **(3.4.2)**, and **(3.4.4)** with three systems of algebraic equations, respectively, and then solving for the unknown variables. Since **(3.4.1)**, **(3.4.2)**, and **(3.4.4)** are Fredholm equations of the second kind, the integrals in these will be approximated using Gaussian quadrature.

Before proceeding further, recall that U_{ij} denotes the control statistic in **(3.2.3)**, and f_W the density of the increments in **(3.2.3)**. That is f_W is the normal density with mean $\sqrt{n}\,(\delta-\gamma)$ and variance 1.

Let u_1, u_2, ..., u_m be the m quadrature roots for integrating over the interval $[g,\ h]$, and let a_1, a_2, ..., a_m be the corresponding weights. Define the $(m+1)\times(m+1)$ matrix

$$\mathbf{Q} = [q_{ij}] \tag{3.5.1}$$

as

$$q_{11} = P\Big(U_{i'j'} < g \mid U_{i',\,j'-1} = 0\Big) = \Phi(g - \sqrt{n}\,(\delta - \gamma)) \quad ,$$

$$q_{1j} = a_{j-1} f(u_{j-1} \,|\, 0) = a_{j-1} f_W(u_{j-1})$$

$$\text{for } j = 2, \ldots, m+1,$$

$$q_{i1} = P\Big(U_{i'j'} < g \mid U_{i',\,j'-1} = u_{i-1}\Big) = \Phi(g - u_{i-1} - \sqrt{n}\,(\delta - \gamma))$$

$$\text{for } i = 2, \ldots, m+1,$$

and

$$q_{ij} = a_{j-1} f(u_{j-1} \,|\, u_{i-1}) = a_{j-1} f_W(u_{j-1} - u_{i-1})$$

$$\text{for } i,\, j = 2, \ldots, m+1,$$

and the $1\times(m+1)$ vector

$$\underline{q}\,(u_0) = [q_i(u_0)] \tag{3.5.2}$$

as

$$q_1(u_0) = \mathrm{P}\Big(U_{i'j'} < g \mid U_{i',\,j'-1} = u_0\Big) = \Phi(g - u_0 - \sqrt{n}\,(\delta - \gamma)) \quad ,$$

and

$$q_i(u_0) = a_{i-1} f(u_{i-1} \mid u_0) = a_{i-1} f_W(u_{i-1} - u_0)$$

$$\text{for } i = 2, \ldots, m+1,$$

where Φ is the standard normal cumulative distribution function (c.d.f.), and $f(\,\cdot\,|\,\cdot\,)$ is the conditional density of $U_{i'j'}$ given a value of $U_{i',\,j'-1}$.

Let $\mathbf{Q}^*$ be the $m\times m$ submatrix of $\mathbf{Q}$, obtained by deleting the first row and first column of $\mathbf{Q}$, and let $\underline{q}^*(u_0)$ be the $1\times m$ subvector of $\underline{q}\,(u_0)$, obtained by deleting the first element of $\underline{q}\,(u_0)$. Then, if $\widetilde{\text{ASN}}(u_0)$ and $\widetilde{\text{ARL}}(y_0)$ represent approximations to $\text{ASN}(u_0)$ and $\text{ARL}(u_0)$ for the SPRT chart, respectively, as shown by Reynolds (1991),

$$\widetilde{\text{ASN}}(u_0) \;=\; 1 \,+\, \underline{q}^{*}(u_0)\,(\mathbf{I}^{*}-\mathbf{Q}^{*})^{-1}\,\underline{1}^{*} \quad , \tag{3.5.3}$$

where $\mathbf{I}^{*}$ is the $m \times m$ identity matrix, and $\underline{1}^{*}$ is the $m \times 1$ vector of 1's, and

$$\widetilde{\text{ARL}}(u_0) \;=\; 1 \,+\, \underline{q}\,(u_0)\,(\mathbf{I}-\mathbf{Q})^{-1}\,\underline{1} \quad , \tag{3.5.4}$$

where I is the $(m+1) \times (m+1)$ identity matrix, and $\underline{1}$ is the $(m+1) \times 1$ vector of 1's. Appropriately, using the approximations $\widetilde{\text{ASN}}(u_0)$ and $\widetilde{\text{ARL}}(u_0)$ in equation **(3.4.3)**, an approximation $\widetilde{\text{ANTS}}(u_0)$ to $\text{ANTS}(u_0)$ for the SPRT chart is given by

$$\widetilde{\text{ANTS}}(u_0) \;=\; \frac{\widetilde{\text{ARL}}(u_0)}{\widetilde{\text{ASN}}(u_0)} \quad . \tag{3.5.5}$$

Define the $(m+1) \times 1$ vector

$$\underline{l} \;=\; [l_j] \tag{3.5.6}$$

as

$$l_1 \;=\; t_b \quad ,$$

and

$$l_j \;=\; t_w$$

$$\text{for } j \;=\; 2, \ldots, m \quad .$$

Then, as shown by Reynolds (1991), an approximation $\widetilde{\text{ATS}}(u_0)$ to $\text{ATS}(u_0)$ for the SPRT chart is given directly by

$$\widetilde{\text{ATS}}(u_0) \;=\; t_{b_0} \,+\, \underline{q}\,(u_0)\,(\mathbf{I}-\mathbf{Q})^{-1}\,\underline{l} \quad . \tag{3.5.7}$$

The same approximation is given indirectly by

$$\widetilde{\mathrm{ATS}}(u_0) = t_{b_0} + \widetilde{\mathrm{ANTS}}(u_0)(\widetilde{\mathrm{ASN}}(u_0)-1)\,t_w + (\widetilde{\mathrm{ANTS}}(u_0)-1)\,t_b \quad , \tag{3.5.8}$$

substituting the approximations $\widetilde{\mathrm{ASN}}(u_0)$ and $\widetilde{\mathrm{ANTS}}(u_0)$ in **(3.4.5)**.

Appropriately using the approximations $\widetilde{\mathrm{ASN}}(u_0)$, $\widetilde{\mathrm{ARL}}(u_0)$, $\widetilde{\mathrm{ANTS}}(u_0)$, and $\widetilde{\mathrm{ATS}}(u_0)$ in equations **(3.4.6)**, **(3.4.7)**, and **(3.4.8)**, approximations $\widetilde{\mathrm{ANOS}}(u_0)$, $\widetilde{\mathrm{ATI}}(u_0)$, and $\widetilde{\mathrm{ASI}}(u_0)$ to $\mathrm{ANOS}(u_0)$, $\mathrm{ATI}(u_0)$, and $\mathrm{ASI}(u_0)$ for the SPRT chart, respectively, are given by

$$\widetilde{\mathrm{ANOS}}(u_0) = n \cdot \widetilde{\mathrm{ARL}}(u_0) \quad , \tag{3.5.9}$$

$$\widetilde{\mathrm{ATI}}(u_0) = \frac{\widetilde{\mathrm{ATS}}(u_0)}{\widetilde{\mathrm{ANTS}}(u_0)} \quad , \tag{3.5.10}$$

and

$$\widetilde{\mathrm{ASI}}(u_0) = \frac{\widetilde{\mathrm{ATS}}(u_0)}{\widetilde{\mathrm{ARL}}(u_0)} \quad . \tag{3.5.11}$$

In turn, appropriately using the approximation $\widetilde{\mathrm{ANOS}}(u_0)$ together with the approximation $\widetilde{\mathrm{ATS}}(u_0)$ in equation **(3.4.9)**, an approximation $\widetilde{\mathrm{AOR}}(u_0)$ to $\mathrm{AOR}(u_0)$ for the SPRT chart is given by

$$\widetilde{\mathrm{AOR}}(u_0) = \frac{\widetilde{\mathrm{ANOS}}(u_0)}{\widetilde{\mathrm{ATS}}(u_0)} \quad . \tag{3.5.12}$$

Let $\widetilde{\mathrm{ATS}}_0(u_0)$ denote the approximation to $\mathrm{ATS}_0(u_0)$ for the SPRT chart when $\delta = 0$. Then, using this approximation in equation **(3.4.10)**, an approximation $\widetilde{\mathrm{FAR}}(u_0)$ to $\mathrm{FAR}(u_0)$ for the SPRT chart is given by

$$\widetilde{\mathrm{FAR}}(u_0) = \frac{1}{\widetilde{\mathrm{ATS}}_0(u_0)} \quad . \tag{3.5.13}$$

Concluding, the *steady-state* time to signal T_{ss} of the SPRT chart is defined to be the length of time from the point that a shift occurs to the point that the chart produces a signal, when the control statistic has reached a stationary distribution at the sampling just before the shift. Let $\underline{\pi}$ denote the normalized left eigenvector of $\mathbf{Q}$ corresponding to the dominant eigenvalue, and let

$$\underline{\alpha} = \underline{\pi}\, \mathbf{L} \,/\, \underline{\pi}\, \underline{l} \quad , \tag{3.5.14}$$

where $\mathbf{L}$ is the $(m+1)\times(m+1)$ diagonal matrix that has $\underline{l}$ on the main diagonal. Then as shown by Reynolds (1991), approximations $\widetilde{\mathrm{SSATS}}$ $\left(= \widetilde{\mathrm{E}}\,(T_{ss})\right)$ and $\widetilde{\mathrm{Var}}(T_{ss})$ to the mean and variance of T_{ss}, respectively, are given by

$$\widetilde{\mathrm{SSATS}} = \widetilde{\mathrm{E}}\,(T_{ss}) = \underline{\alpha}\,[(\mathbf{I}-\mathbf{Q})^{-1} - \mathbf{I}/2]\,\underline{l} \quad , \tag{3.5.15}$$

and

$$\widetilde{\mathrm{Var}}(T_{ss}) = \underline{\alpha}\,[(2(\mathbf{I}-\mathbf{Q})^{-1}-\mathbf{I})\cdot(\mathbf{L}/2-(\underline{l}\,\underline{\alpha}\,/4))\cdot(2(\mathbf{I}-\mathbf{Q})^{-1}-\mathbf{I})-\mathbf{L}/6]\,\underline{l} \ . \tag{3.5.16}$$

Note that an accurate numerical approximation $\underline{\tilde{\pi}}$ to the eigenvector $\underline{\pi}$ in **(3.5.14)** may be obtained using the *power method* (see Burden and Faires (1989), pp. 492 – 508).

Chapter 4

The Universal CUSUM Chart

4.1 Introduction

The general assumptions made in the first paragraph of Section 3.1 for the SPRT chart are likewise made here for the proposed *universal* CUSUM (UC) chart. For the convenience of the reader, this paragraph is repeated below:

> Suppose that X denotes a measurable characteristic of a process of interest whose distribution is normal with mean μ and standard deviation σ. Let μ_0 denote the target value for μ, and assume that if an assignable cause occurs, then any resulting change in the process will be reflected by a change in μ. Even though a process change can generate many types of deviations from μ_0, the simple case will be considered here where μ shifts from μ_0 to $\mu_1 \neq \mu_0$ and stays at μ_1 until the shift is detected and corrective action is taken. In practice it may be necessary to estimate μ_0 and/or σ from past data, but for simplicity it will be assumed here that μ_0 and σ are known.

The UC chart is defined as a generalization of the VSI CUSUM chart that uses two sampling intervals and can also be viewed as a generalization of the SPRT chart. The motivation for the definition of the UC chart came from the need to develop a general framework for the study of control charts that are equivalent to a sequence of SPRT's and for the quantitative comparison of control charts that use different sample sizes and/or different sampling methods (for example, VSS sequential sampling versus FSS sampling).

4.2 Description of the UC Chart

Suppose that random samples of size $n \geq 1$ are taken from the process at times T_1, T_2, ..., to detect any deviation of μ from μ_0. Assume that observations between samples are independent, and that the process starts at time $T_0 = 0$, but no sample is taken before time $T_1 = d_0$. Let $\underline{X}_i = (X_{i1}, X_{i2}, \ldots, X_{in})$ represent the sample taken at time T_i, and $\overline{X}_i$ its sample mean, for $i = 1, 2, \ldots$. Standardizing the deviation of $\overline{X}_i$ from μ_0 in terms of units of σ, let

$$Z_i = (\overline{X}_i - \mu_0) / \sigma \quad . \qquad \textbf{(4.2.1)}$$

Define the control statistic of the *upper* one-sided UC chart for detecting an increase in μ to be

$$Y_i = MAX_{i-1} + Z_i - \gamma \quad , \qquad \textbf{(4.2.2)}$$

where

$$MAX_{i-1} = \begin{cases} Y_{i-1} \quad , & \text{if } Y_{i-1} \geq g \\ 0 \quad , & \text{if } Y_{i-1} < g \quad , \end{cases}$$

and where $Y_0 = y_0$ is the specified starting value of the control statistic, and $\gamma > 0$ and g are specified chart parameters. Define the upper one-sided UC chart to signal at the i^{th} sample if $Y_i > h$, where $h \geq 0$ and $h \geq g$. The control statistic in (4.2.2) begins accumulation from 0 (*resets* to 0) on the next sample whenever its current value drops below g.

Similarly, define the control statistic of the *lower* one-sided UC chart for detecting a decrease in μ to be

$$Y_i = MIN_{i-1} + Z_i + \gamma \quad , \tag{4.2.3}$$

where

$$MIN_{i-1} = \begin{cases} Y_{i-1} \quad , \text{ if } Y_{i-1} \leq -g \\ \\ 0 \quad , \text{ if } Y_{i-1} > -g \quad , \end{cases}$$

and where $Y_0 = y_0$ is the specified starting value of the control statistic, and $\gamma > 0$ and g are specified chart parameters. Define the lower one-sided UC chart to signal at the i^{th} sample if $Y_i < -h$, where $h \geq 0$ and $h \geq g$. The control statistic in **(4.2.3)** resets to 0 on the next sample whenever its current value exceeds $-g$.

As noted in Sections 2.2 and 2.8, Reynolds (1989) showed that for one-sided VSI charts in the class of charts that can be modeled as a Markov process, the use of exactly two sampling intervals, the shortest possible and the longest possible, is optimal for detecting a specific shift in μ in the sense of minimizing the ATS. Hence, since the one-sided UC chart belongs to this class, define the sampling interval function of the *upper* one-sided UC chart to be

$$l_U(y) = \begin{cases} d_1 \ , \text{ if } y \epsilon [c, h] \\ \\ d_2 \ , \text{ if } y \epsilon (-\infty, c) \quad , \end{cases} \tag{4.2.4}$$

and the sampling interval function of the *lower* one-sided UC chart to be

$$l_L(y) = \begin{cases} d_1 \ , \text{ if } y \epsilon [-h, -c] \\ \\ d_2 \ , \text{ if } y \epsilon (-c, +\infty) \quad , \end{cases} \tag{4.2.5}$$

where $0 \leq d_1 \leq d_2$, $d_2 > 0$, and $-\infty < c \leq h$. That is, d_1 and d_2 represent the two sampling intervals used by the one-sided UC chart, and c represents the boundary between the region specifying the use of d_1 and d_2. Given that the upper one-sided UC does not signal at time T_i and the value y of the control statistic in **(4.2.2)** is observed, then the next sample is taken at time $T_{i+1} = T_i + l_U(y)$. Similarly, given that the lower one-sided UC does not signal at time T_i and the value y of the control statistic in **(4.2.3)** is observed, then the next sample is taken at time $T_{i+1} = T_i + l_L(y)$. For an example of an upper one-sided UC chart, see Figure 4.2.1, and for an example of a lower one, see Figure 4.2.2. When $d_1 = d_2$, the UC chart is an FSI control chart, but when $d_1 < d_2$, it is a VSI control chart. It should be noted that in practice, it may be desirable to take the initial sampling interval d_0 to be different than $l_U(y_0)$ for an upper one-sided UC chart, or $l_L(y_0)$ for a lower one.

A two-sided control procedure for the detection of two-sided deviations in μ was not considered in the development of the SPRT chart. Likewise, a two-sided control procedure for the detection of such deviations will not be considered here, in the development of the UC chart. However, two one-sided UC charts may be applied simultaneously to detect such deviations, one to detect an increase in μ and the other a decrease. Without loss of generality, the upper one sided UC will be considered here, and for the remaining part of this paper "one-sided UC chart" or simply "UC chart" is understood to mean "upper one-sided UC chart".

As in the development of the SPRT chart in Chapter 3, since the control statistic of the UC chart is expressed in terms of the standardized Z_i in **(4.2.1)**, it is convenient to standardize μ also and take the process parameter to be δ, which is defined in **(3.2.5)**. In this case, the target value is given by $\delta_0 = 0$, and a

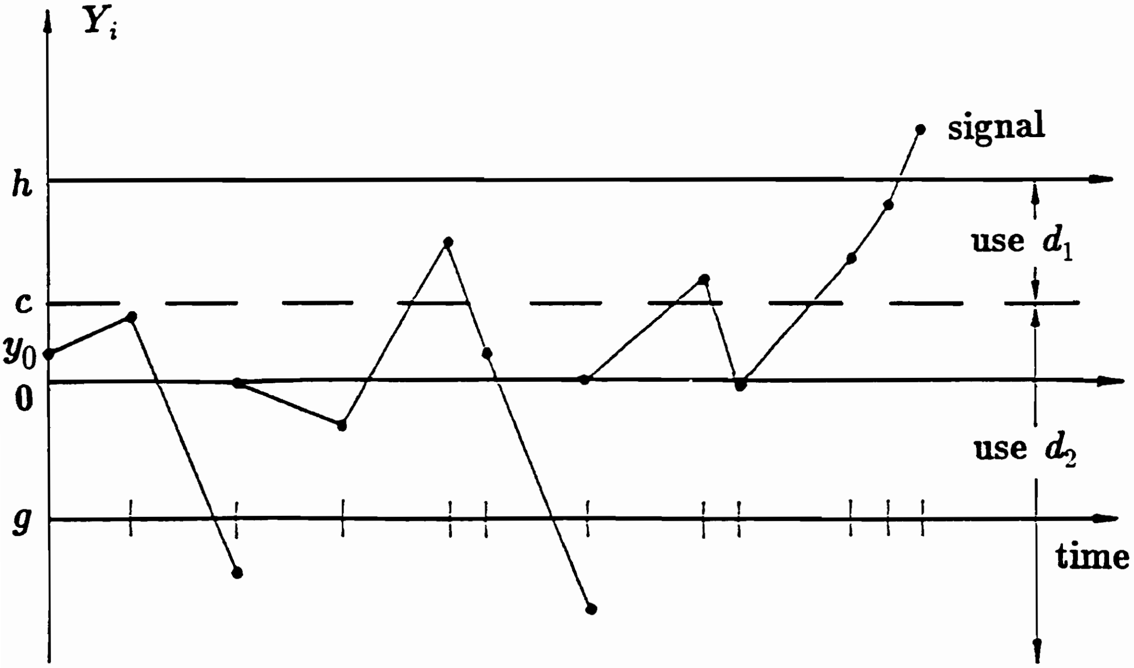

Figure 4.2.1. Upper One-Sided UC Chart.

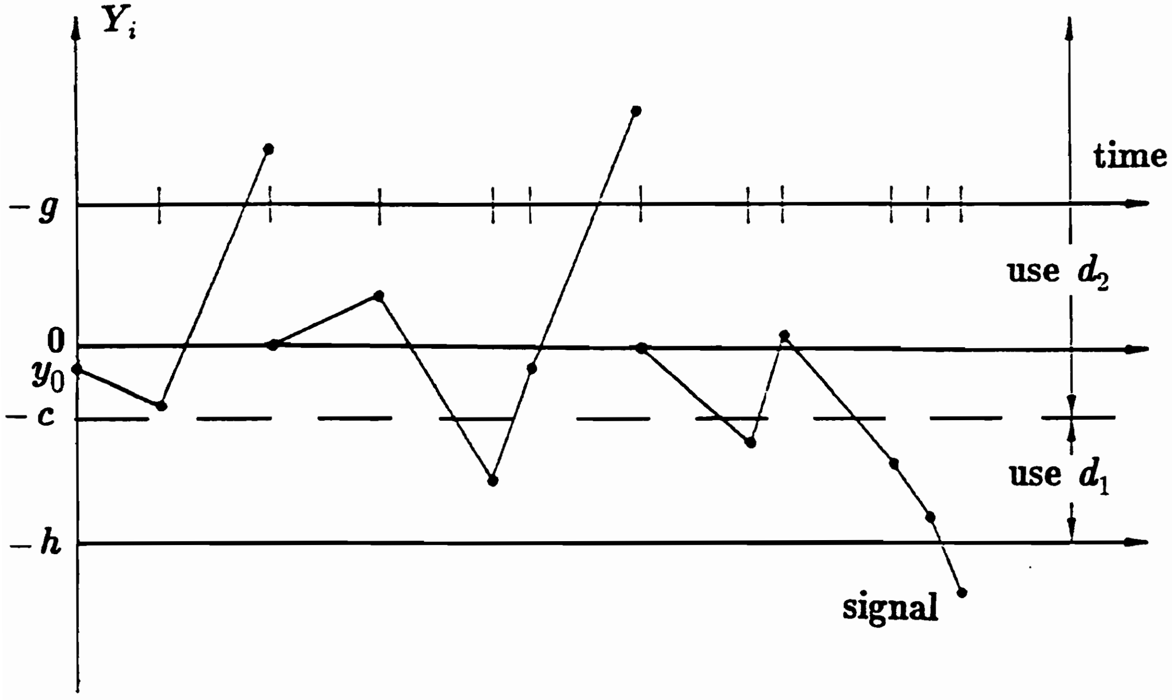

Figure 4.2.2. Lower One-Sided UC Chart.

standardized shift in the process mean is given by δ.

The control statistic of the UC chart is very similar to that of the CUSUM chart. Page (1954) showed that the (upper one-sided) CUSUM chart is equivalent to a sequence of SPRT's, with the control statistic corresponding to the j^{th} observation of the i^{th} SPRT given by equation **(3.2.3)**, and the in-control (or reset) limit g equal to 0. Similarly, the UC chart is equivalent to a sequence of SPRT's with the same control statistic, but with g not necessarily equal to 0. That is, apart from the sampling interval function, the UC chart is equivalent to the SPRT chart. In fact, the SPRT chart could have been introduced indirectly as a special case of the UC chart, using the notation developed in this section, with the risk, however, of failing to convey the conception of the SPRT chart purely as a sequence of SPRT's. On the other hand, the UC chart could have been introduced as a generalization of the SPRT chart, using the notation developed in Chapter 3, just by generalizing the sampling interval function of the SPRT chart to that of the UC chart. This, however, could mask the FSS-VSI structure of the SPRT chart discussed in the following chapters.

In addition to the SPRT chart, special cases of the UC chart are the one-sided FSI $\bar{X}$ chart and CUSUM charts and the one-sided VSI $\bar{X}$ and CUSUM charts that use two sampling intervals. Specifically, the UC chart reduces to the one-sided

(i) FSI $\bar{X}$ chart with $\gamma\sigma$ limit, when $g = h = 0$ and $d_1 = d_2$ (clearly);

(ii) VSI $\bar{X}$ chart with $\gamma\sigma$ limit, when $g = h = 0$;

(iii) FSI CUSUM chart, when $g = 0$ and $d_1 = d_2$;

(iv) VSI CUSUM chart, when $g = 0$;

(v) SPRT chart, when $g = c$.

For examples of the upper one-sided FSI $\bar{X}$, VSI $\bar{X}$ chart, and VSI CUSUM charts, see Figures 4.2.3, 4.2.4, and 4.2.5, respectively.

The parameters g, h, c, and γ are chosen so that the UC chart will have certain properties. For example, if the UC chart is to be designed to detect a shift from $\delta_0 = 0$ to a specified value δ_1, then optimality theory of SPRT's indicates that γ should be chosen to be $\delta_1/2$ (Ghosh (1970), pp. 93 – 98). Once γ is determined, g and h could be chosen to attain a specified ARL when $\delta = 0$. Finally, once g, h, and γ are determined, c could be chosen to attain a specified ASI when $\delta = 0$, thus providing the UC chart with desired FAR characteristics. As for the SPRT chart, following the CUSUM chart terminology, γ is called the reference value of the UC chart.

4.3 Definitions of Properties of the UC Chart

Since, apart from the sampling interval function, the UC chart is equivalent to the SPRT chart, definitions specific to the properties of the UC chart are the same as those specific to the properties of the SPRT chart. The properties considered in Chapter 3 for the SPRT chart will also be considered in this chapter for the more general UC chart. For the convenience of the reader, the definitions from Section 3.3 are repeated below:

(i) The *average sample number* (ASN) is the expected value of the number of samples in a given SPRT of the chart required by the control statistic to either produce a signal or drop

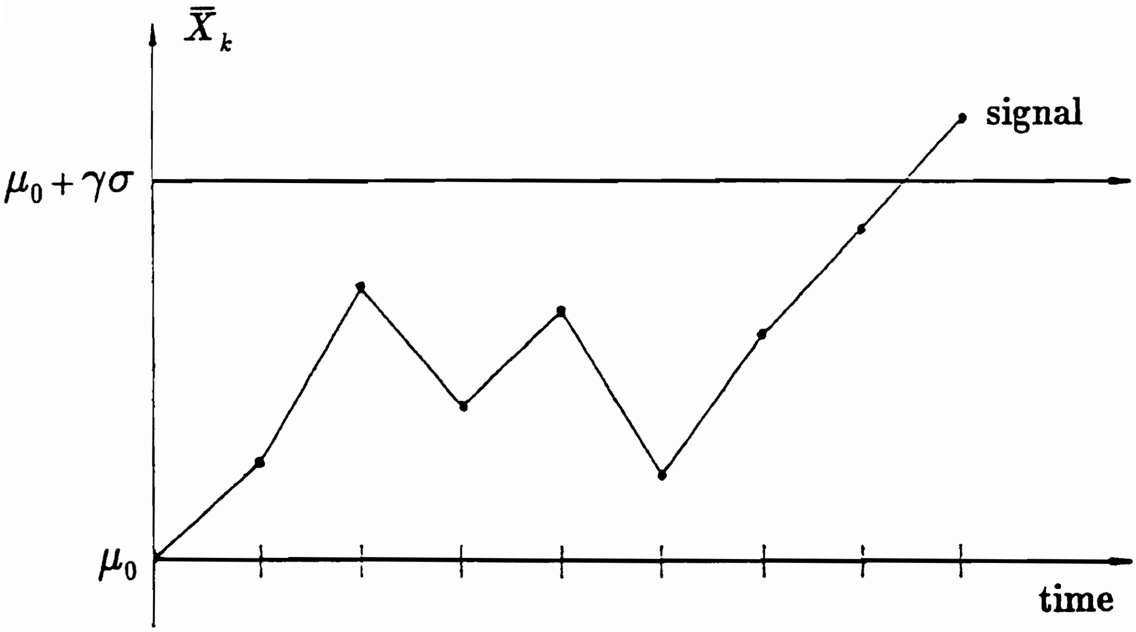

Figure 4.2.3. Upper One-Sided FSI $\bar{X}$ Chart.

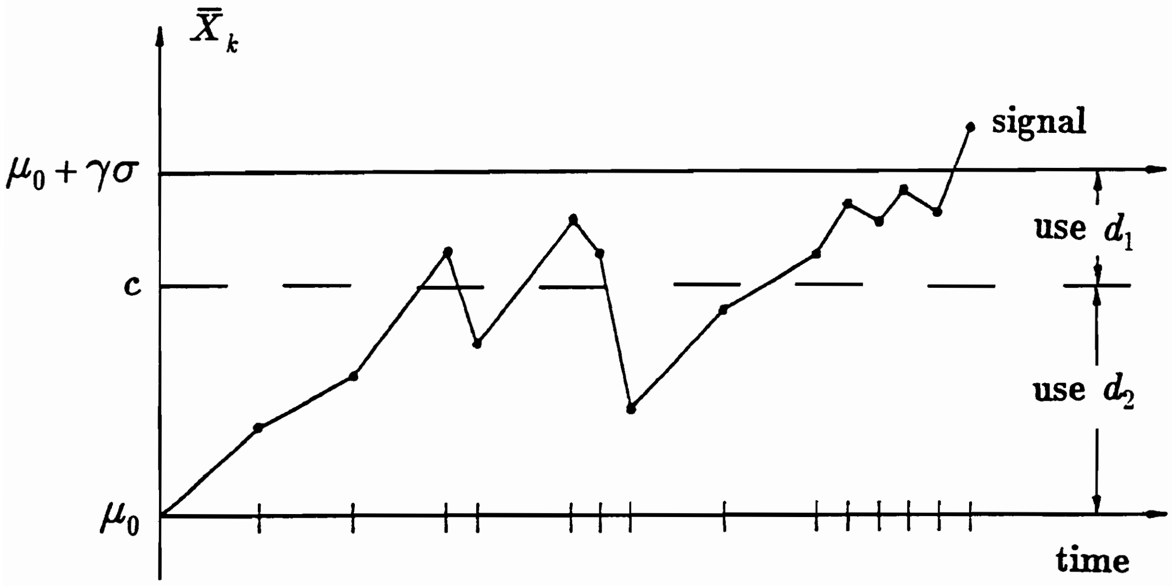

Figure 4.2.4. Upper One-Sided VSI $\overline{X}$ Chart.

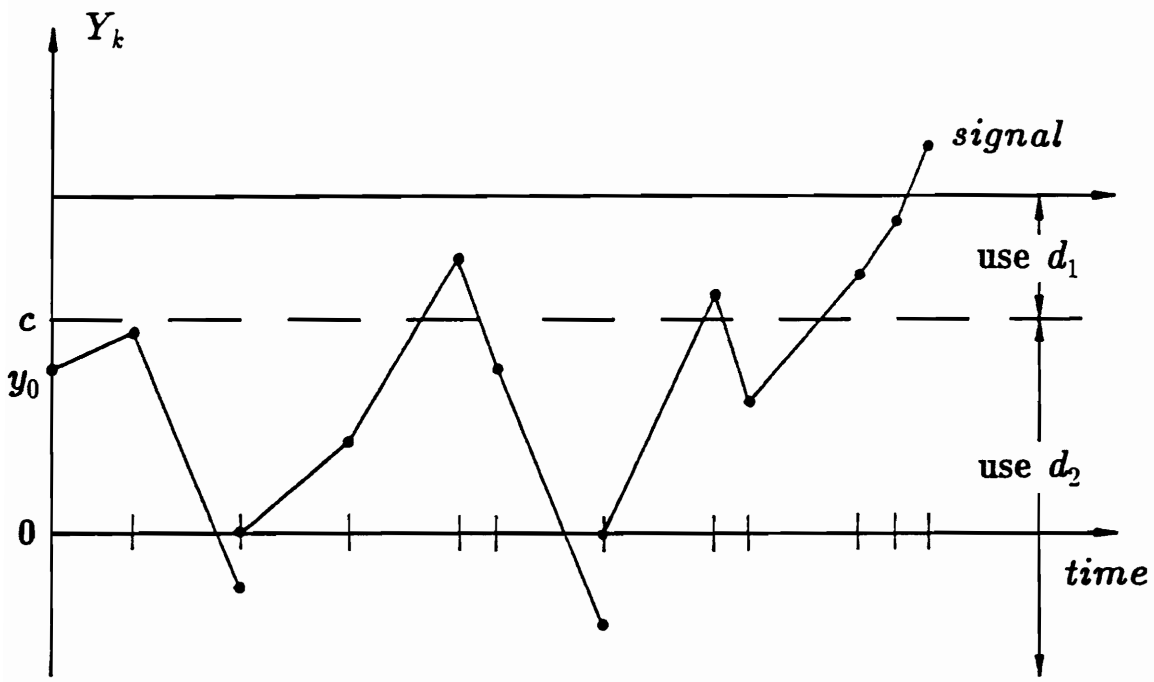

Figure 4.2.5. Upper One-Sided VSI CUSUM Chart.

below the in-control (or reset) limit g.

(ii) The *average number of tests to signal* (ANTS) is the expected value of the number of SPRT's required by the chart to produce a signal.

(iii) The *average run length* (ARL) is the expected value of the number of samples (over all the SPRT's) required by the chart to produce a signal.

(iv) The *average number of observations to signal* (ANOS) is the expected value of the total number of single observations (over all samples and SPRT's) required by the chart to produce a signal.

(v) The *average time to signal* (ATS) is the expected value of the length of time from the start of the process (time $T_0 = 0$) to the point that the chart produces a signal.

(vi) The *steady-state average time to signal* (SSATS) is the expected value of the length of time from the point that a shift occurs to the point that the chart produces a signal, when the control statistic has reached a stationary distribution at the sampling point just before the shift.

(vii) The *average test interval* (ATI) is the expected value of the length of time between the beginnings of two consecutive SPRT's of the chart.

(viii) The *average sampling interval* (ASI) is the expected value of the length of time between any two consecutive samples of the chart (ignoring the SPRT's).

(ix) The *average observation rate* (AOR) is the expected value of the number of single observations sampled by the chart per unit of time.

(x) The *false alarm rate* (FAR) is the expected value of the number of signals produced by the chart per unit of time, when the process is in control ($\delta = 0$).

4.4 Expressions for Properties of the UC Chart

The SPRT chart-type structure of the UC chart will now be used to develop the properties of the UC chart defined in Section 4.3. The dependence of these expressions on the process parameter δ will be suppressed, and the expressions will be approximated numerically in Section 4.5. As for the SPRT chart, it will be assumed that when δ is constant, the value of the sampling interval function does not affect the distribution of the control statistic. Since the observations of the UC chart are assumed to be i.i.d., the UC chart satisfies this assumption. Recall from Section 3.4 that f_W denotes the density of the increments in equation **(3.2.3)**. That is, f_W is the normal density with mean $\sqrt{n}\,(\delta-\gamma)$ and variance 1.

The ASN of the UC chart, conditional on the starting value y_0 of the control statistic, satisfies the integral equation

$$\mathrm{ASN}(y_0) \;=\; 1 \;+\; \int_{g}^{h} \mathrm{ASN}(w)\, f_W\,(w-y_0)\,\mathrm{d}w \quad . \tag{4.4.1}$$

Similarly, the ARL of the UC chart, conditional on the starting value y_0 of the control statistic, satisfies the integral equation

$$\mathrm{ARL}(y_0) \;=\; 1 \;+\; \int_{-\infty}^{h} \mathrm{ARL}(w)\, f_W\,(w-y_0)\,\mathrm{d}w \quad . \tag{4.4.2}$$

The ATS of the UC chart, conditional on the starting value y_0 of the control statistic, satisfies the integral equation

$$\text{ATS}(y_0) = d_0 + \int_{-\infty}^{h} \text{ATS}(w)\, f_W(w-y_0)\, dw \quad . \tag{4.4.3}$$

This expression is a simple generalization of expression **(3.4.4)** for the ATS of the SPRT chart.

As defined in Section 4.3, the ANTS, ANOS, ATI, ASI, and AOR of the UC chart, conditional on the starting value y_0 of the control statistic, are given by

$$\text{ANTS}(y_0) = \frac{\text{ARL}(y_0)}{\text{ASN}(y_0)} \quad , \tag{4.4.4}$$

$$\text{ANOS}(y_0) = n \cdot \text{ARL}(y_0) \quad , \tag{4.4.5}$$

$$\text{ATI}(y_0) = \frac{\text{ATS}(y_0)}{\text{ANTS}(y_0)} \quad , \tag{4.4.6}$$

$$\text{ASI}(y_0) = \frac{\text{ATS}(y_0)}{\text{ARL}(y_0)} \quad , \tag{4.4.7}$$

and

$$\text{AOR}(y_0) = \frac{\text{ANOS}(y_0)}{\text{ATS}(y_0)} \quad , \tag{4.4.8}$$

respectively. Also, as defined in Seciton 4.3, the FAR of the UC chart, conditional on the starting value y_0 of the control statistic, is given by

$$\text{FAR}(y_0) = \frac{1}{\text{ATS}_0(y_0)} \quad , \tag{4.4.9}$$

where $\text{ATS}_0(y_0)$ denotes $\text{ATS}(y_0)$ when $\delta = 0$. Note that, apart from the ATS, the above integral equations and expressions are the same as their respective SPRT chart counterparts.

By definition, the computation of the SSATS of the UC chart requires the determination of the distribution of the control statistic at the last sample before the shift in δ. As for the SPRT chart, this distribution will be assumed to be proportional to the product of a stationary distribution of the control statistic and the sampling interval function, with the stationary distribution conditional on no false alarms used here. The integral equation satisfied by the SSATS of the UC chart involves complicated mathematical expressions that may be derived using the framework provided by Reynolds (1991), but are beyond the scope of this paper. Hence, as for the SPRT chart, this integral equation will not be developed here. A numerical approximation, however, to the SSATS of the UC chart will be developed in Section 4.5, together with a numerical approximation to the variance of the steady-state time to signal of this chart.

4.5 Numerical Evaluations of Properties of the UC Chart

Since expressions in terms of integral equations are available from Section 4.4 for the properties of the UC chart, the very accurate IE method will be used here for their evaluation, within the unified framework developed by Reynolds (1991). The SPRT chart-type structure of the UC chart will be used to develop approximations to its properties by generalizing the approximations developed in Section 3.5 for the properties of the SPRT chart.

Approximations to the ASN, ARL, and ATS of the UC chart may be obtained by replacing integral equations **(4.4.1)**, **(4.4.2)**, and **(4.4.3)** with three systems of algebraic equations, respectively, and then solving for the unknown variables. Since **(4.4.1)**, **(4.4.2)**, and **(4.4.3)** are Fredholm equations of the second kind, the integrals in these will be approximated using Gaussian quadrature. When $g < c \leq h$, however, the Gaussian quadrature formulae for equation **(4.4.3)** cannot be applied directly on the interval $[g, h]$, because the sampling interval function of the UC chart is discontinuous at the point c, and c may not fall on a boundary between subintervals of the quadrature on $[g, h]$. The same problem arises with the numerical approximation of the SSATS of the UC chart when $g < c \leq h$. This problem is easily overcome by partitioning $[g, h]$ into the two subintervals $[g, c)$ and $[c, h]$ and then applying Gaussian quadrature separately on each of the two subintervals, since the sampling interval function of the UC chart is continuous on the individual subintervals. Hence, in obtaining approximations to the properties of the UC chart, the following three cases will be considered:

$$\text{Case (i):} \quad g < c \leq h \quad ,$$

$$\text{Case (ii):} \quad c \leq g \leq h \quad ,$$

and

$$\text{Case (iii):} \quad g \leq h \text{ and } c \leq h \quad .$$

Note that Cases (i) and (ii) form a partition of Case (iii), which is the general case.

Before proceeding with Case (i), recall that U_{ij} denotes the control statistic in **(3.2.3)**, and f_W denotes the density of the increments in **(3.2.3)**. That is, f_W is the normal density with mean $\sqrt{n}\,(\delta-\gamma)$ and variance 1.

Case (i): $g < c \leq h$

Let $y_{11}, y_{12}, \ldots, y_{1m_1}$ be the m_1 quadrature roots for integrating over the interval $[g, c)$, and let $a_{11}, a_{12}, \ldots, a_{1m_1}$ be the corresponding weights. Let $y_{21}, y_{22}, \ldots, y_{2m_2}$ be the m_2 quadrature roots for integrating over the interval $[c, h]$, and let $a_{21}, a_{22}, \ldots, a_{2m_2}$ be the corresponding weights. Let $y_i = y_{1i}$, for

$i=1,\ 2,\ \ldots,\ m_1$, and let $y_{m_1+i}=y_{2i}$, for $i=1,\ 2,\ \ldots,\ m_2$. Similarly, let $a_i=a_{1i}$, for $i=1,\ 2,\ \ldots,\ m_1$, and let $a_{m_1+i}=\ a_{2i}$, for $i=1,\ 2,\ \ldots,\ m_2$. Define the $(m_1+m_2+1)\times(m_1+m_2+1)$ matrix

$$\mathbf{Q}\ =\ [q_{ij}] \tag{4.5.1}$$

as

$$q_{11}\ =\ \mathrm{P}\,(\mathrm{U}_{i'j'}<g\,|\,\mathrm{U}_{i',\,j'-1}=0)\ =\ \Phi(g-\sqrt{n}\,(\delta-\gamma))\quad ,$$

$$q_{1j}\ =\ a_{j-1}\ f(y_{j-1}\,|\,0)\ =\ a_{j-1}\ f_W(y_{j-1})$$

$$\text{for } j\ =\ 2,\ldots,\ m_1+m_2+1\quad ,$$

$$q_{i1}\ =\ \mathrm{P}\,(U_{i'j'}<g\,|\,U_{i',\,j'-1}=y_{i-1})\ =\ \Phi(g-y_{i-1}-\sqrt{n}\,(\delta-\gamma))$$

$$\text{for } i\ =\ 2,\ldots,\ m_1+m_2+1\quad ,$$

and

$$q_{ij}\ =\ a_{j-1}\ f(y_{j-1}\,|\,y_{i-1})\ =\ a_{j-1}\ f_W(y_{j-1}-y_{i-1})$$

$$\text{for } i,\ j\ =\ 2,\ldots,\ m_1+m_2+1\quad ,$$

and the $1\times(m_1+m_2+1)$ vector

$$\underline{q}\,(y_0)\ =\ [q_i(y_0)] \tag{4.5.2}$$

as

$$q_1(y_0)\ =\ \mathrm{P}\Big(U_{i'j'}<g\,|\,U_{i',\,j'-1}=y_0\Big)\ =\ \Phi(g-y_0-\sqrt{n}(\delta-\gamma))\quad ,$$

and

$$q_i(y_0)\ =\ a_{i-1}\ f(y_{i-1}\,|\,y_0)\ =\ a_{i-1}\ f_W(y_{i-1}-y_0)$$

$$\text{for } i\ =\ 2,\ldots,\ m_1+m_2+1\quad ,$$

where Φ is the standard normal c.d.f., and $f(\,\cdot\,|\,\cdot\,)$ is the conditional density of $U_{i'j'}$ given a value of $U_{i',\ j'-1}$.

Let $\mathbf{Q}^*$ be the $(m_1+m_2)\times(m_1+m_2)$ submatrix of $\mathbf{Q}$, obtained by deleting the first row and first column of $\mathbf{Q}$, and let $\underline{q}^*(y_0)$ be $1\times(m_1+m_2)$ subvector of $\underline{q}\,(y_0)$, obtained by deleting the first element of $\underline{q}\,(y_0)$. Then, if $\widetilde{\mathrm{ASN}}(y_0)$ and $\widetilde{\mathrm{ARL}}(y_0)$ represent approximations to $\mathrm{ASN}(y_0)$ and $\mathrm{ARL}(y_0)$ for the UC chart, respectively, as shown by Reynolds (1991),

$$\widetilde{\mathrm{ASN}}(y_0) \;=\; 1 \;+\; \underline{q}^*(y_0)(\mathbf{I}^*-\mathbf{Q}^*)^{-1}\,\underline{1}^* \quad , \tag{4.5.3}$$

where $\mathbf{I}^*$ is the $(m_1+m_2)\times(m_1+m_2)$ identity matrix, and $\underline{1}^*$ is the $(m_1+m_2)\times 1$ vector of 1's, and

$$\widetilde{\mathrm{ARL}}(y_0) \;=\; 1 \;+\; \underline{q}\,(y_0)(\mathbf{I}-\mathbf{Q})^{-1}\,\underline{1} \quad , \tag{4.5.4}$$

where $\mathbf{I}$ is the $(m_1+m_2+1)\times(m_1+m_2+1)$ identity matrix, and $\underline{1}$ is the $(m_1+m_2+1)\times 1$ vector of 1's. Note that approximations to $\mathrm{ASN}(y_0)$ and $\mathrm{ARL}(y_0)$ for the UC chart are also given by equations **(3.5.3)** and **(3.5.4)**, respectively, which use a single quadrature on the whole interval $[g,\ h]$.

Define the $(m_1+m_2+1)\times 1$ vector

$$\underline{l} \;=\; [l_j] \tag{4.5.5}$$

as

$$l_j \;=\; d_2$$

$$\text{for } j \;=\; 1, \ldots, m_1+1 \quad ,$$

and

$$l_j \;=\; d_1$$

$$\text{for } j \;=\; m_1+2, \ldots, m_1+m_2+1 \quad .$$

Then as shown by Reynolds (1991), an approximation $\widetilde{\mathrm{ATS}}(y_0)$ to $\mathrm{ATS}(y_0)$ for the

UC chart is given by

$$\widetilde{\mathrm{ATS}}(y_0) = d_0 + \underline{q}\,(y_0)(\mathbf{I}-\mathbf{Q})^{-1}\,\underline{l} \quad . \qquad \textbf{(4.5.6)}$$

Concluding Case (i), as for the SPRT chart, the steady-state time to signal T_{ss} of the UC chart is defined to be the length of time from the point that a shift occurs to the point that the chart produces a signal, when the control statistic has reached a stationary distribution at the sampling point just before the shift. Let $\underline{\pi}$ denote the normalized left eigenvector of $\mathbf{Q}$ corresponding to the dominant eigenvalue, and let

$$\underline{\alpha} = \underline{\pi}\ \mathbf{L}/\underline{\pi}\,\underline{l} \quad , \qquad \textbf{(4.5.7)}$$

where $\mathbf{L}$ is the $(m_1+m_2+1)\times(m_1+m_2+1)$ diagonal matrix that has $\underline{l}$ on the main diagonal. Then, as shown by Reynolds (1991), approximations $\mathrm{SS}\widetilde{\mathrm{A}}\mathrm{TS}(=\widetilde{\mathrm{E}}\ (T_{ss}))$ and $\mathrm{V}\widetilde{\mathrm{A}}\mathrm{R}(T_{ss})$ to the mean and variance of T_{ss}, respectively, are given by

$$\mathrm{SS}\widetilde{\mathrm{A}}\mathrm{TS} = \widetilde{\mathrm{E}}\ (T_{ss}) = \underline{\alpha}\,[(\mathbf{I}-\mathbf{Q})^{-1} - \mathbf{I}/2]\,\underline{l} \quad , \qquad \textbf{(4.5.8)}$$

and

$$\widetilde{\mathrm{Var}}(T_{ss}) = \underline{\alpha}\,[(2(\mathbf{I}-\mathbf{Q})^{-1}-\mathbf{I})\cdot(\mathbf{L}/2-(\underline{l}\,\underline{\alpha}/4))\cdot(2(\mathbf{I}-\mathbf{Q})^{-1}-\mathbf{I})-\mathbf{L}/6]\,\underline{l} \ . \qquad \textbf{(4.5.9)}$$

Note that an accurate numerical approximation $\widetilde{\underline{\pi}}$ to the eigenvector $\underline{\pi}$ in **(4.5.7)** may be obtained using the power method.

Next Case (ii) will be considered. Unless redefined in Case (ii), the notation used in Case (i) will also hold in Case (ii).

Case (ii): $c \le g \le h$

Let $y_1, \ldots, y_m$ be the m quadrature roots for integrating over the interval $[g,\ h]$, and let $a_1, \ldots, a_m$ be the corresponding weights. Define the $(m+2)\times(m+2)$ matrix

$$\mathbf{Q} = [q_{ij}] \tag{4.5.10}$$

as

$$q_{11} = q_{21} = \mathrm{P}\,(U_{i'j'} < c \mid U_{i',\,j'-1} = 0) = \Phi(c - \sqrt{n}\,(\delta-\gamma)) \quad ,$$

$$q_{12} = q_{22} = \mathrm{P}\,(c \le U_{i'j'} \le g \mid U_{i',\,j'-1} = 0)$$

$$= \Phi(g - \sqrt{n}\,(\delta-\gamma)) - \Phi(c - \sqrt{n}\,(\delta-\gamma)) \quad ,$$

$$q_{1j} = q_{2j} = a_{j-2}\, f(y_{j-2} \mid 0) = f_W(y_{j-2})$$

$$\text{for } j = 3, \ldots, m+2 \quad ,$$

$$q_{i1} = \mathrm{P}\,(U_{i'j'} < c \mid U_{i',\,j'-1} = y_{i-2}) = \Phi(c - y_{i-2} - \sqrt{n}\,(\delta-\gamma))$$

$$\text{for } i = 3, \ldots, m+2 \quad ,$$

$$q_{i2} = \mathrm{P}\,(c < U_{i'j'} \le g \mid U_{i',\,j'-1} = y_{i-2})$$

$$= \Phi(g - y_{i-2} - \sqrt{n}(\delta-\gamma)) - \Phi(c - y_{i-2} - \sqrt{n}\,(\delta-\gamma))$$

$$\text{for } i = 3, \ldots, m+2 \quad ,$$

and

$$q_{ij} = a_{j-2}\, f(y_{j-2} \mid y_{i-2}) = a_{j-2}\, f_W(y_{j-2} - y_{i-2})$$

$$\text{for } i,\ j = 3, \ldots, m+2 \quad ,$$

and the $1 \times (m+2)$ vector

$$\underline{q}\,(y_0) \;=\; [q_i(y_0)] \tag{4.5.11}$$

as

$$q_1(y_0) \;=\; \mathrm{P}\,(U_{i'j'} < c \mid U_{i',\,j'-1} = y_0) \;=\; \Phi(c - y_0 - \sqrt{n}\,(\delta - \gamma)) \quad ,$$

$$\begin{aligned} q_2(y_0) \;&=\; \mathrm{P}\,(c \leq U_{i'j'} \leq g \mid U_{i',\,j'-1} = y_0) \\ &=\; \Phi(g - y_0 - \sqrt{n}\,(\delta - \gamma)) \;-\; \Phi(c - y_0 - \sqrt{n}\,(\delta - \gamma))\;, \end{aligned}$$

and

$$q_i(y_0) \;=\; a_{i-2}\; f(y_{i-2} \mid y_0) \;=\; a_{i-2}\; f_W(y_{i-2} - y_0)$$

$$\text{for } i,\, j \;=\; 3, \ldots, m+2 \quad .$$

Let $\mathbf{Q}^*$ be the $m \times m$ submatrix of $\mathbf{Q}$, obtained by deleting the first two rows and first two columns of $\mathbf{Q}$, and let $\underline{q}^*(y_0)$ be the $1 \times m$ subvector of $\underline{q}\,(y_0)$, obtained by deleting the first two elements of $\underline{q}\,(y_0)$. Then as shown by Reynolds (1991),

$$\widetilde{\mathrm{ASN}}(y_0) \;=\; 1 \;+\; \underline{q}^*(y_0)(\mathbf{I}^* - \mathbf{Q}^*)^{-1}\, \underline{1}^* \quad , \tag{4.5.12}$$

where $\mathbf{I}^*$ is the $m \times m$ identity matrix, and $\underline{1}^*$ is the $m \times 1$ vector of 1's, and

$$\widetilde{\mathrm{ARL}}(y_0) \;=\; 1 \;+\; \underline{q}(y_0)(\mathbf{I} - \mathbf{Q})^{-1}\, \underline{1} \quad , \tag{4.5.13}$$

where $\mathbf{I}$ is the $(m+2) \times (m+2)$ identity matrix, and $\underline{1}$ is the $(m+2) \times 1$ vector of 1's.

Define the $(m+2) \times 1$ vector

$$\underline{l} \;=\; [l_j] \tag{4.5.14}$$

as

$$l_1 \;=\; l_2 \;=\; d_2 \quad ,$$

and

$$l_j = d_1$$

$$\text{for } j = 3, \ldots, m+2 \quad .$$

Then as shown by Reynolds (1991),

$$\widetilde{\mathrm{ATS}}(y_0) = d_0 + \underline{q}\,(y_0)(\mathbf{I}-\mathbf{Q})^{-1}\,\underline{l} \quad . \tag{4.5.15}$$

Concluding Case (ii), let $\underline{\pi}$ denote the normalized left eigenvector of $\mathbf{Q}$ corresponding to the dominant eigenvalue, and let

$$\underline{\alpha} = \underline{\pi}\ \mathbf{L}/\underline{\pi}\,\underline{l} \quad , \tag{4.5.16}$$

where $\mathbf{L}$ is the $(m+2)\times(m+2)$ diagonal matrix that has $\underline{l}$ on the main diagonal. Then as shown by Reynolds (1991),

$$\mathrm{SS}\widetilde{\mathrm{ATS}} = \underline{\alpha}\,[(\mathbf{I}-\mathbf{Q})^{-1} - \mathbf{I}/2]\,\underline{l} \quad , \tag{4.5.17}$$

and

$$\widetilde{\mathrm{Var}}(T_{ss}) = \underline{\alpha}\,[(2(\mathbf{I}-\mathbf{Q})^{-1}-\mathbf{I})\cdot(\mathbf{L}/2-(\underline{l}\,\underline{\alpha}/4))\cdot(2(\mathbf{I}-\mathbf{Q})^{-1}-\mathbf{I})-\mathbf{L}/6]\,\underline{l} \ . \tag{4.5.18}$$

Note that an accurate numerical approximation $\tilde{\underline{\pi}}$ to the eigenvector $\underline{\pi}$ in **(4.5.16)** may be obtained using the power method.

Finally, in order to obtain approximations to the remaining properties of the UC chart defined in Section 4.3, the general case, Case (iii), will be considered. In Case (iii), when referring to the approximations given above, no distinction will be made between Cases (i) and (ii).

Case (iii): $g \le h$ and $c \le h$

Appropriately using the approximations $\widetilde{\mathrm{ASN}}(y_0)$, $\widetilde{\mathrm{ARL}}(y_0)$, $\widetilde{\mathrm{ATS}}(y_0)$ in equations **(4.4.4)**, **(4.4.5)**, and **(4.4.7)**, approximations $\widetilde{\mathrm{ANTS}}(y_0)$, $\widetilde{\mathrm{ANOS}}(y_0)$, and $\widetilde{\mathrm{ASI}}(y_0)$ to $\mathrm{ANTS}(y_0)$, $\mathrm{ANOS}(y_0)$, and $\mathrm{ASI}(y_0)$ for the UC chart, respectively, are given by

$$\widetilde{\mathrm{ANTS}}(y_0) = \frac{\widetilde{\mathrm{ARL}}(y_0)}{\widetilde{\mathrm{ASN}}(y_0)} , \tag{4.5.19}$$

$$\widetilde{\mathrm{ANOS}}(y_0) = n \cdot \widetilde{\mathrm{ARL}}(y_0) , \tag{4.5.20}$$

and

$$\widetilde{\mathrm{ASI}}(y_0) = \frac{\widetilde{\mathrm{ATS}}(y_0)}{\widetilde{\mathrm{ARL}}(y_0)} . \tag{4.5.21}$$

In turn, appropriately using approximations $\widetilde{\mathrm{ANTS}}(y_0)$ and $\widetilde{\mathrm{ANOS}}(y_0)$ together with the approximation $\widetilde{\mathrm{ATS}}(y_0)$ in equations **(4.4.6)** and **(4.4.8)**, approximations $\widetilde{\mathrm{ATI}}(y_0)$ and $\widetilde{\mathrm{AOR}}(y_0)$ to $\mathrm{ATI}(y_0)$ and $\mathrm{AOR}(y_0)$ for the UC chart, respectively, are given by

$$\widetilde{\mathrm{ATI}}(y_0) = \frac{\widetilde{\mathrm{ATS}}(y_0)}{\widetilde{\mathrm{ANTS}}(y_0)} , \tag{4.5.22}$$

and

$$\widetilde{\mathrm{AOR}}(y_0) = \frac{\widetilde{\mathrm{ANOS}}(y_0)}{\widetilde{\mathrm{ATS}}(y_0)} . \tag{4.5.23}$$

Concluding this section, and hence this case, let $\widetilde{\mathrm{ATS}}_0(y_0)$ denote the approximation to $\mathrm{ATS}(y_0)$ for the UC chart when $\delta = 0$. Then, using this approximation in equation **(4.4.9)**, an approximation $\widetilde{\mathrm{FAR}}(y_0)$ to $\mathrm{FAR}(y_0)$ for the UC chart is given by

$$\widetilde{\mathrm{FAR}}(y_0) = \frac{1}{\widetilde{\mathrm{ATS}}_0(y_0)} . \tag{4.5.24}$$

Chapter 5

Comparison of the SPRT Chart With the $\overline{X}$ and CUSUM Charts

5.1 Introduction

In evaluating the SPRT chart, it will be compared with the $\overline{X}$ and CUSUM charts, both with and without the VSI feature, within the general framework of the UC chart. That is, the notation used to represent the parameters of the general UC chart will be used here to represent the respective parameters of the SPRT, $\overline{X}$, and CUSUM charts, as special cases of the UC chart. The quantitative comparisons will be based on SSATS, ATS, and ANOS performances, without considering any FIR features. That is, the starting values of the control statistics were chosen to be 0 for all charts. For purposes of fair comparison and evaluation of the SPRT, $\overline{X}$, and CUSUM charts, all charts were matched up so that the two equations in the system

$$\begin{aligned} \mathrm{ATS}(0) &= 740.8 \quad \text{when } \delta = 0 \\ \mathrm{AOR}(0) &= 5 \qquad\;\, \text{when } \delta = 0 \end{aligned} \tag{5.1.1}$$

were satisfied simultaneously. Note that 740.8 corresponds to the in-control ATS of the one-sided FSI $\overline{X}$ chart with the "standard" $3\sigma/\sqrt{n}$ limit, and obviously, when $\delta = 0$, the SSATS of a control chart is defined to be simply its ATS. When the control charts are matched according to system **(5.1.1)**, the performance of the charts can be evaluated by computing the ATS (both steady-state and zero-time) and ANOS values of the charts at various values of δ to determine which chart is the most efficient in detecting changes in δ. The chart with the smallest ATS at a particular δ is the most efficient chart at that δ. When any two charts have the same ATS efficiency at a particular δ, then the chart with the smaller ANOS at δ has the advantage of sampling less at the δ, thus reducing sampling costs.

The properties of the SPRT, $\overline{X}$, and CUSUM charts, as special cases of the UC chart, are determined by the choice of the parameters n, γ, g, h, c, d_0, d_1, and d_2. For the FSI charts, the unit time was chosen as the sampling interval so that $d_1 = d_2 = 1$. For the VSI charts, the 12 pairs of sampling intervals (d_1, d_2), for $d_1 = 0$, 0.2, 0.5 and $d_2 = 2$, 1.75, 1.5, 1.25, were chosen. The choices for (d_1, d_2) were made so that a broad range of sampling interval combinations were considered, while maintaining reasonable respective efficiencies for the charts. For all of the charts, the starting sampling interval $d_0 = 1$ was used, and the sample size $n = 5$ was considered. The sample sizes $n = 1$ and 3 were also considered, though only for the VSI charts. Clearly, the sample sizes $n = 1$ and 3 could not have been considered for the FSI charts without violating the second equation in system (5.1.1). Likewise, the sample size $n = 1$ could not have been considered for the VSI charts that use $d_1 = 0.2$ or 0.5 without violating the second equation in system (5.1.1). Thus, by system (5.1.1), when $\delta = 0$, for all charts that use sample size

(i) $n = 1$, $\text{ASI} = 0.2$;

(ii) $n = 3$, $\text{ASI} = 0.6$;

(iii) $n = 5$, $\text{ASI} = 1$.

For the SPRT and CUSUM charts, the reference values $\gamma = 0.1$, 0.15, 0.2, 0.25, and 0.3 were used covering a large number of choices, while maintaining reasonable efficiencies for the broad range of shifts considered. The shifts considered were $\delta = 0.25$, 0.5, 0.75, 1, 1.5, 2, and 3. The remaining parameters were determined (appropriately for each chart) to satisfy system **(5.1.1)**. Specifically, for the

(i) SPRT chart, given values for n, γ, d_0, d_1, and d_2, g and h were determined as solutions to system **(5.1.1)**;

(ii) $\overline{X}$ chart, given values for n, d_0, d_1, and d_2, γ and c were determined as solutions to system **(5.1.1)**;

(iii) CUSUM chart, given values for n, γ, d_0, d_1, and d_2, h and c were determined as solutions to system **(5.1.1)**.

Recall from Section 4.2, that by definition, for the

(i) SPRT chart, $g = c$;

(ii) $\overline{X}$ chart, $g = h = 0$;

(iii) CUSUM chart, $g = 0$.

The constrained determination of the parameters and the evaluation of the SSATS, ATS, and ANOS of the $\overline{X}$ and CUSUM charts was performed using the modular FORTRAN 77 computer software developed by Stoumbos and Reynolds (1992a), for $m_1 = m_2 = m = 96$ Gaussian quadrature points. The constrained determination of the parameters and the evaluation of the SSATS, ATS, and ANOS of the SPRT chart, however, was performed using the modular FORTRAN 77 computer software developed by Stoumbos and Reynolds (1992b), for $m = 96$ Gaussian quadrature points. Unlike the software developed by Stoumbos and Reynolds (1992a) for the general UC chart, that developed by Stoumbos and Reynolds (1992b) was designed specifically for the SPRT chart, thus exploiting the particular structure of the SPRT chart for increased computational efficiency.

It should be noted that both of these softwares exclusively employ double precision throughout all computations, and that even 24 quadrature points were enough to maintain complete agreement (up to the four decimals considered) with the same computations performed using 96 quadrature points, for a large subset of parameter combinations and both softwares. For the convenience of the reader, all tables provided in this chapter are placed at the end of the chapter. The results provided in these tables hold for both upper and lower charts. Note that values for the parameters g, h, and c are provided only in Tables 5.2.1 – 5.2.60, which also provide numerical SSATS values for matched one-sided SPRT, $\bar{X}$ and CUSUM charts.

5.2 Comparisons Based on SSATS Performance

As noted in Section 5.1, Tables 5.2.1 – 5.2.60 provide numerical SSATS values for matched one-sided SPRT, $\bar{X}$, and CUSUM charts, along with respective values for the parameters g, h, and c. In particular, Tables 5.2.1 – 5.2.20 provide SSATS values for the negligible sampling interval $d_1 = 0$, Tables 5.2.21 – 5.2.40 provide SSATS values for the medium sampling interval $d_1 = 0.2$, and Tables 5.2.41 – 5.2.60 provide SSATS values for the larger sampling interval $d_1 = 0.5$. Throughout Tables 5.2.1 – 5.2.60, SSATS values for the FSI and VSI $\bar{X}$ charts and the FSI CUSUM charts are appropriately repeated to aid the reader's comparisons.

In comparing the charts using Tables 5.2.1 – 5.2.60, it is clear that the SPRT chart is substantially more efficient than both the FSI and VSI $\bar{X}$ charts and the FSI CUSUM chart in terms of SSATS performance. Although still substantially more efficient, the improvement that the SPRT chart offers is not as large for the FSI CUSUM chart as for the FSI and VSI $\bar{X}$ charts. In fact, the efficiency of the SPRT chart is very similar to that of the VSI CUSUM chart, particularly when comparing these two charts for the sample size that offers the "best" overall SSATS performance for the set of shifts considered. Specifically, for the negligible sampling interval $d_1 = 0$, using Tables 5.2.1 – 5.2.20, it can be seen that the sample size $n = 1$ offers higher efficiency to the SPRT and VSI CUSUM charts for small shifts, but the sample sizes $n = 3$ and 5 offer higher efficiency for moderate and larger shifts. The improvement, however, attained by using $n = 1$ is more substantial than that attained by using $n = 3$ or 5, thus rendering $n = 1$ the better choice for the SPRT and VSI CUSUM charts when $d_1 = 0$. To the VSI $\overline{X}$ chart, however, $n = 5$ offers uniformly much higher SSATS efficiency than $n = 1$ or 3 when $d_1 = 0$. For the positive sampling intervals $d_1 = 0.2$ and 0.5, using Tables 5.2.21 – 5.2.60, it can be seen that the sample size $n = 5$ offers almost uniformly higher SSATS efficiency to the SPRT and VSI CUSUM charts, thus rendering $n = 5$ clearly the better choice. In fact, for the larger sampling interval $d_1 = 0.5$, the benefits provided by using $n = 5$ are indeed substantial. Likewise, to the VSI $\overline{X}$ chart, $n = 5$ offers substantially higher SSATS efficiency when $d_1 = 0.2$ or 0.5, though not uniformly. For very large shifts, $n = 3$ offers higher efficiency to the $\overline{X}$ chart when $d_1 = 0.2$ or 0.5.

The SSATS results in Tables 5.2.1 – 5.2.60 show that the sampling interval d_1 should be as small as possible for all charts, with $d_1 = 0$ being the best choice. These results also show that for the SPRT and CUSUM charts, the reference values $\gamma = 0.1$ and 0.3 provide rather poor choices for the set of shifts considered, compared to $\gamma = 0.15$, 0.2, and 0.25. In fact, for the SPRT chart, $\gamma = 0.1$ provides uniformly worse SSATS efficiency than $\gamma = 0.15$ when $d_1 = 0$. A closer examination of the SSATS results in Tables 5.2.1 – 5.2.60 reveals that the VSI CUSUM chart is somewhat more efficient than the SPRT chart in detecting small shifts, but the SPRT chart is somewhat more efficient in detecting moderate and large shifts. If for given values of γ and the other chart parameters, the VSI CUSUM chart is more efficient in detecting small shifts than its matched SPRT chart counterpart, then, since the SPRT chart is more efficient in detecting moderate and large shifts, the SPRT chart can match the efficiency of the VSI CUSUM chart in detecting the small shifts. This can be achieved by appropriately reducing the γ-value of the SPRT chart, thus attaining overall SSATS efficiency similar to that of the VSI CUSUM chart's, for the set of shifts considered.

5.3 Comparisons Based on ATS Performance

Tables 5.3.1 – 5.3.60 provide numerical ATS values for matched one-sided SPRT, $\bar{X}$, and CUSUM charts. In particular, Tables 5.3.1 – 5.3.20 provide ATS values for the negligible sampling interval $d_1 = 0$, Tables 5.3.21 – 5.3.40 provide ATS values for the medium sampling interval $d_1 = 0.2$, and Tables 5.3.41 – 5.3.60 provide ATS values for the larger sampling interval $d_1 = 0.5$. Throughout Tables 5.3.1 – 5.3.60, ATS values for the FSI and VSI $\bar{X}$ charts and the FSI CUSUM charts are appropriately repeated to aid the reader's comparisons.

The comparisons and discussion from the previous section for the SSATS performance apply in a completely analogous manner to the (zero-time) ATS performance. Thus, to avoid redundancy, such analogous comparisons and discussion will not be repeated here. Note, however, that Reynolds' (1989) theoretical results, which show that d_1 should be as short as possible but d_2 should be as long as possible if the ATS is used as the criterion of performance, are illustrated for the SPRT and the VSI $\bar{X}$ and CUSUM charts by the numerical results provided in Tables 5.3.1 – 5.3.60, with only a few exceptions concerning

SPRT charts. These exemptions are due to the fact that Reynolds' (1989) results hold for a more restricted control chart model which assumes that the limits g and h do not change for different values of d_1 and/or d_2, unlike an SPRT chart's limits.

5.4 Comparisons Based on ANOS Performance

Tables 5.4.1 – 5.4.60 provide numerical ANOS values for matched one-sided SPRT, $\overline{X}$, and CUSUM charts. In particular, Tables 5.4.1 – 5.4.20 provide ANOS values for the negligible sampling interval $d_1 = 0$, Tables 5.4.21 – 5.4.40 provide ANOS values for the medium sampling interval $d_1 = 0.2$, and Tables 5.4.41 – 5.4.60 provide ANOS values for the larger sampling interval $d_1 = 0.5$. Although ANOS values for the SPRT chart do depend on the sampling interval function, those for the VSI $\overline{X}$ and CUSUM charts do not. Throughout Tables 5.4.1 – 5.4.60, however, ANOS values for the VSI as well as the FSI $\overline{X}$ and CUSUM charts are appropriately repeated to aid the reader's comparisons.

In comparing the charts using Tables 5.4.1 – 5.4.60, it is clear that the SPRT chart is substantially more efficient than the $\overline{X}$ chart in terms of ANOS performance. The ANOS efficiency of the SPRT chart, however, is very similar to that of the CUSUM chart. As noted in the previous paragraph, the ANOS performances of the FSI $\overline{X}$ and CUSUM charts are identical to those of their VSI counterparts, for the same sample size.

For the negligible sampling interval $d_1 = 0$, using Tables 5.4.1 – 5.4.60, it can be seen that the improvement that the sample size $n = 1$ offers to the ANOS efficiency of the SPRT chart is quite small, compared to the ANOS efficiency offered by the sample size $n = 3$ or 5. A similar improvement to the ANOS efficiency of the CUSUM chart is offered by $n = 1$. For the positive sampling intervals $d_1 = 0.2$ and 0.5, using Tables 5.4.21 – 5.4.60, it can be seen that the sample size $n = 5$ offers ANOS efficiency to the SPRT chart that is somewhat lower than that offered by the sample size $n = 3$. This also holds for the CUSUM chart. The gains, however, in SSATS and ATS efficiencies by using $n = 5$ for the SPRT and CUSUM charts, respectively, are substantially greater than the gains in ANOS efficiency by using $n = 3$. Finally, to the $\overline{X}$ chart, $n = 5$ offers much higher overall ANOS efficiency than $n = 1$ or 3, even though the efficiency is lower for very large shifts.

5.5 Concluding Remarks

From the comparisons made in Sections 5.2 – 5.4, it is clear that the SPRT chart is substantially more efficient than both the FSI and VSI $\overline{X}$ charts and the FSI CUSUM chart, though very similar in efficiency to the VSI CUSUM chart. In fact, often the SPRT chart is practically identical to the VSI CUSUM chart (for examples, see Tables 5.2.2 – 5.2.4, 5.2.7, 5.2.8, for $n = 1$, Tables 5.2.25, 5.2.26, 5.2.29, 5.2.30, 5.2.49 – 5.2.51, 5.2.60, for $n = 5$, and the respective tables of ATS and ANOS values). An important difference, however, between the SPRT and the VSI CUSUM charts lies in their sampling interval functions. The VSI CUSUM chart randomly varies the sampling intervals between the short interval d_1 and the long interval d_2, with the advance prediction of sampling times being impossible for more than the next sample. The SPRT chart, however, can be thought of as a more convenient and predictable "rearrangement" of the VSI CUSUM chart, which groups together within each SPRT the short d_1's, separating these by a single long d_2 only between consecutive SPRT's. The structure of the SPRT chart, as reflected by its sampling interval function, thus provides a very intuitive regularity to the process as a sequence of SPRT's (for testing the hypothesis that the process is in control) which are conducted as fast as possible, with the time interval d_2 between consecutive SPRT's typically determined by efficiency requirements and/or administrative considerations.

A modification of significant practical importance to the SPRT chart exists in which samples are always taken at pre-specified, equally spaced fixed times, with additional samples taken between these times as a function of what is observed from the process. This modification was briefly discussed by Reynolds et al. (1988) and Reynolds et al. (1990), though for the VSI $\bar{X}$ and VSI CUSUM charts, respectively, and without actually evaluating the modification for either chart. The modification for the SPRT chart is developed in the next chapter, and even though it is naturally suitable and simple for the SPRT chart, such a development would be quite complicated for the VSI CUSUM chart because of the cumbersome mathematics required to evaluate the mean and standard deviation of its time to signal, particularly for the steady-state case (Lucas and Saccucci (1990)). Lucas and Saccucci (1990), however, noted that the implementation of this modification in the VSI CUSUM chart is much more practical than the VSI feature alone.

One of Shewhart's (1931) most significant contributions to SPC was the concept of the *rational subgroup.* "Ideally such a subgroup is a sample in which all of the items are produced under conditions in which only random effects are responsible for the observed variation. Potential causes of extraneous variability such as differing raw materials, personnel spindles, test conditions and so on will then occur between subgroups rather than within them. If there is an art to setting up control charts, it comes down to the art of rational subgrouping." (Nelson (1988)). It is important to recognize that observations taken too infrequently or irregularly do not reflect the rational subgroup concept, and thus do not provide sufficient information about the process (Woodall (1990)). Hence, the VSI CUSUM chart will not necessarily reflect the rational subgroup concept in its entirety (in its SPRT's), especially without taking great care (Woodall (1990)). The SPRT chart, however, reflects the rational subgroup concept much better, because its sampling interval function groups together the observed information in each SPRT in as short a time period as possible. This is particularly true for a relatively short sampling interval d_1.

Before concluding this section, and hence this chapter, the model of the negligible sampling interval $d_1 = 0$ with unit sample size $n = 1$ deserves special attention. As discussed in Section 2.5, this model has received much attention in the last 30 years, though only in the continuous-time context of monitoring the drift of a Brownian motion process. To fix an example, consider the case of a transistor production line in an electronics industry, where each transistor produced can be tested in negligible time using an electronic device (Assaf and Ritov (1989)). Other interesting applications are discussed by Wilson et al. (1979) and Weatherall and Haskey (1976). The former are concerned with monitoring the quality of repeated radioimmunoassays, the latter with surveillance of birth records for a possible increase in frequency of genetic malformations.

From the (discrete-time) quantitative comparisons made in Sections 5.2 – 5.4, it can be seen that the model $(d_1,\ n) = (0,\ 1)$ offers the "best" overall efficiency to both the SPRT and VSI CUSUM charts, with the efficiency of the SPRT chart being very similar to that of the VSI CUSUM chart. For this model, however, unlike the VSI-FSS CUSUM chart, the SPRT chart can be considered dually as a VSI-FSS control chart, and also as an FSI control chart which is not an FSS control chart. As an FSI control Chart, the SPRT chart uses the fixed sampling interval d_2 between consecutive sampling points/SPRT's, with the sample size at a given sampling point being the sample number of the corresponding SPRT. That is, the sample size is not fixed but is rather determined by what is being observed at the current sampling point, one observation at a time, with the advance prediction of the sample size being impossible even for the next sampling point. Hence as an FSI chart, the SPRT chart is not a VSS chart in the usual sense, and the more descriptive expression *sequential sample size* (SSS) control chart is introduced for the SPRT chart. Even though the FSI-SSS SPRT chart could present administrative problems, it should be more suitable for most applications than a VSI-FSS control chart, such as the VSI-FSS CUSUM chart, since the latter presents similar administrative problems in addition to the unwelcomed VSI irregularity.

Table 5.2.1

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.10			1.34	3.46	1.82	1.34	0.10			
(d_1, d_2)	(0.00, 2.00)			FSI	(0.00, 2.00)			FSI	(0.00, 2.00)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	6.50	7.05	8.14	136.04	190.08	104.26	79.51	21.24	6.52	6.99	7.70
0.50	2.53	2.54	2.66	32.92	49.58	15.75	9.78	8.39	2.57	2.53	2.62
0.75	1.67	1.59	1.56	10.26	13.52	3.13	2.00	5.14	1.71	1.59	1.58
1.00	1.34	1.24	1.19	4.00	4.23	1.31	1.11	3.69	1.36	1.24	1.21
1.50	1.09	1.03	1.01	1.07	1.22	1.01	1.00	2.35	1.10	1.03	1.02
2.00	1.02	1.00	1.00	0.58	1.01	1.00	1.00	1.71	1.03	1.00	1.00
3.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.20	1.00	1.00	1.00
h	20.65	11.43	8.39	0.00	0.00	0.00	0.00	8.62	20.71	11.47	8.62
g	-0.40	0.28	0.63	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.40	0.28	0.63	0.00	-4.74	-3.68	-3.00	8.62	-0.33	0.31	0.84

Table 5.2.2

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.10			1.34	3.46	1.82	1.34	0.10			
(d_1, d_2)	(0.00, 1.75)			FSI	(0.00, 1.75)			FSI	(0.00, 1.75)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	6.51	7.25	8.87	136.04	193.06	108.27	84.89	21.24	6.52	7.11	8.05
0.50	2.49	2.53	2.75	32.92	51.08	16.88	11.03	8.39	2.51	2.50	2.69
0.75	1.61	1.53	1.53	10.26	14.04	3.29	2.14	5.14	1.62	1.53	1.58
1.00	1.26	1.15	1.10	4.00	4.33	1.24	1.03	3.69	1.26	1.15	1.16
1.50	0.99	0.92	0.89	1.07	1.11	0.88	0.88	2.35	0.99	0.92	0.90
2.00	0.91	0.88	0.8	0.58	0.89	0.88	0.88	1.71	0.91	0.88	0.88
3.00	0.88	0.88	0.88	0.50	0.88	0.88	0.88	1.20	0.88	0.88	0.88
h	20.69	11.37	8.21	0.00	0.00	0.00	0.00	8.62	20.71	11.47	8.62
g	-0.23	0.42	0.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.23	0.42	0.80	0.00	-4.66	-3.56	-2.82	8.62	-0.21	0.51	1.17

Table 5.2.3

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.10			1.34	3.46	1.82	1.34	0.10			
(d_1, d_2)	(0.00, 1.50)			FSI	(0.00, 1.50)			FSI	(0.00, 1.50)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	6.55	7.60	10.31	136.04	196.69	113.47	92.50	21.24	6.55	7.28	8.67
0.50	2.46	2.56	2.99	32.92	52.96	18.45	13.04	8.39	2.46	2.50	2.86
0.75	1.55	1.48	1.55	10.26	14.72	3.57	2.46	5.14	1.55	1.48	1.66
1.00	1.18	1.07	1.04	4.00	4.50	1.20	1.00	3.69	1.18	1.08	1.17
1.50	0.88	0.80	0.78	1.07	1.02	0.76	0.75	2.35	0.88	0.81	0.82
2.00	0.79	0.76	0.75	0.58	0.77	0.75	0.75	1.71	0.79	0.76	0.75
3.00	0.75	0.75	0.75	0.50	0.75	0.75	0.75	1.20	0.75	0.75	0.75
h	20.71	11.25	7.89	0.00	0.00	0.00	0.00	8.62	20.71	11.47	8.62
g	-0.06	0.58	1.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.06	0.58	1.02	0.00	-4.57	-3.41	-2.57	8.62	-0.06	0.78	1.67

Table 5.2.4

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.10			1.34	3.46	1.82	1.34	0.10			
(d_1, d_2)	(0.00, 1.25)			FSI	(0.00, 1.25)			FSI	(0.00, 1.25)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	6.62	8.25	14.23	136.04	201.29	120.58	104.70	21.24	6.61	7.58	10.10
0.50	2.43	2.65	3.71	32.92	55.43	20.79	16.90	8.39	2.43	2.54	3.37
0.75	1.50	1.46	1.73	10.26	15.66	4.09	3.32	5.14	1.50	1.47	1.96
1.00	1.10	1.00	1.05	4.00	4.77	1.22	1.08	3.69	1.10	1.04	1.38
1.50	0.79	0.69	0.67	1.07	0.94	0.64	0.64	2.35	0.79	0.72	0.85
2.00	0.68	0.63	0.63	0.58	0.65	0.63	0.63	1.71	0.68	0.64	0.66
3.00	0.63	0.63	0.63	0.50	0.63	0.63	0.63	1.20	0.63	0.63	0.63
h	20.70	11.04	7.18	0.00	0.00	0.00	0.00	8.62	20.71	11.47	8.62
g	0.13	0.79	1.34	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	0.13	0.79	1.34	0.00	-4.46	-3.20	-2.16	8.62	0.14	1.17	2.65

Table 5.2.5

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.15			1.34	3.46	1.82	1.34	0.15			
(d_1, d_2)	(0.00, 2.00)			FSI	(0.00, 2.00)			FSI	(0.00, 2.00)		
n δ	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	6.47	7.02	8.12	136.04	190.08	104.26	79.51	22.16	6.50	7.02	8.00
0.50	2.07	2.14	2.29	32.92	49.58	15.75	9.78	7.57	2.18	2.14	2.33
0.75	1.39	1.38	1.39	10.26	13.52	3.13	2.00	4.43	1.48	1.39	1.39
1.00	1.17	1.14	1.12	4.00	4.23	1.31	1.11	3.10	1.23	1.14	1.14
1.50	1.03	1.02	1.01	1.07	1.22	1.01	1.00	1.92	1.06	1.02	1.02
2.00	1.01	1.00	1.00	0.58	1.01	1.00	1.00	1.41	1.01	1.00	1.01
3.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	0.84	1.00	1.00	1.00
h	15.78	8.75	6.48	0.00	0.00	0.00	0.00	6.52	16.01	8.75	6.52
g	-0.92	-0.10	0.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.92	-0.10	0.30	0.00	-4.74	-3.68	-3.00	6.52	-0.64	-0.10	0.33

Table 5.2.6

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.15			1.34	3.46	1.82	1.34	0.15			
(d_1, d_2)	(0.00, 1.75)			FSI	(0.00, 1.75)			FSI	(0.00, 1.75)		
n δ	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	6.49	7.23	8.84	136.04	193.06	108.27	84.89	22.16	6.49	7.22	8.47
0.50	2.03	2.13	2.37	32.92	51.08	16.88	11.03	7.57	2.10	2.13	2.38
0.75	1.32	1.32	1.35	10.26	14.04	3.29	2.14	4.43	1.38	1.32	1.35
1.00	1.08	1.05	1.03	4.00	4.33	1.24	1.03	3.10	1.12	1.05	1.06
1.50	0.92	0.90	0.89	1.07	1.11	0.88	0.88	1.92	0.94	0.90	0.90
2.00	0.89	0.88	0.88	0.58	0.89	0.88	0.88	1.41	0.89	0.88	0.88
3.00	0.88	0.88	0.88	0.50	0.88	0.88	0.88	0.84	0.88	0.88	0.88
h	15.86	8.75	6.41	0.00	0.00	0.00	0.00	6.52	16.01	8.75	6.52
g	-0.71	0.07	0.49	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.71	0.07	0.49	0.00	-4.66	-3.56	-2.82	6.52	-0.53	0.67	0.59

Table 5.2.7

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.15			1.34	3.46	1.82	1.34	0.15			
(d_1, d_2)	(0.00, 1.50)			FSI	(0.00, 1.50)			FSI	(0.00, 1.50)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	6.52	7.57	10.28	136.04	196.69	113.47	92.50	22.16	6.52	7.50	9.23
0.50	2.00	2.15	2.58	32.92	52.96	18.45	13.04	7.57	2.04	2.14	2.51
0.75	1.26	1.27	1.36	10.26	14.72	3.57	2.46	4.43	1.30	1.26	1.37
1.00	1.00	0.96	0.96	4.00	4.50	1.20	1.00	3.10	1.02	0.96	1.01
1.50	0.81	0.78	0.77	1.07	1.02	0.76	0.75	1.92	0.82	0.78	0.79
2.00	0.77	0.75	0.75	0.58	0.77	0.75	0.75	1.41	0.77	0.75	0.76
3.00	0.75	0.75	0.75	0.50	0.75	0.75	0.75	0.84	0.75	0.75	0.75
h	15.93	8.72	6.25	0.00	0.00	0.00	0.00	6.52	16.01	8.75	6.52
g	-0.49	0.26	0.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.49	0.26	0.73	0.00	-4.57	-3.41	-2.57	6.52	-0.40	0.28	0.97

Table 5.2.8

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.15			1.34	3.46	1.82	1.34	0.15			
(d_1, d_2)	(0.00, 1.25)			FSI	(0.00, 1.25)			FSI	(0.00, 1.25)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	6.59	8.22	14.20	136.04	201.29	120.58	104.70	22.16	6.59	7.93	10.83
0.50	2.00	2.23	3.24	32.92	55.43	20.79	16.90	7.57	1.99	2.18	2.92
0.75	1.21	1.24	1.53	10.26	15.66	4.09	3.32	4.43	1.22	1.23	1.57
1.00	0.92	0.89	0.96	4.00	4.77	1.22	1.08	3.10	0.93	0.89	1.09
1.50	0.71	0.67	0.66	1.07	0.94	0.64	0.64	1.92	0.71	0.67	0.71
2.00	0.65	0.63	0.63	0.58	0.65	0.63	0.63	1.4	0.65	0.63	0.64
3.00	0.63	0.63	0.63	0.50	0.63	0.63	0.63	0.84	0.63	0.63	0.63
h	15.98	8.63	5.84	0.00	0.00	0.00	0.00	6.52	16.01	8.75	6.52
g	-0.26	0.48	1.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.26	0.48	1.09	0.00	-4.46	-3.20	-2.16	6.52	-0.23	0.59	1.68

Table 5.2.9

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.20			1.34	3.46	1.82	1.34	0.20			
(d_1, d_2)	(0.00, 2.00)			FSI	(0.00, 2.00)			FSI	(0.00, 2.00)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	7.69	8.24	9.36	136.04	190.08	104.26	79.51	24.77	7.55	8.15	9.36
0.50	1.87	1.96	2.12	32.92	49.58	15.75	9.78	7.26	2.03	1.98	2.12
0.75	1.25	1.27	1.30	10.26	13.52	3.13	2.00	4.02	1.38	1.29	1.30
1.00	1.09	1.09	1.08	4.00	4.23	1.31	1.11	2.74	1.17	1.10	1.08
1.50	1.01	1.01	1.00	1.07	1.22	1.01	1.00	1.66	1.04	1.01	1.00
2.00	1.00	1.00	1.00	0.58	1.01	1.00	1.00	1.23	1.01	1.00	1.00
3.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	0.60	1.00	1.00	1.00
h	12.71	7.03	5.23	0.00	0.00	0.00	0.00	5.23	13.12	7.09	5.23
g	-1.41	-0.43	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	-1.41	-0.43	0.01	0.00	-4.74	-3.68	-3.00	5.23	-0.85	-0.37	0.01

Table 5.2.10

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.20			1.34	3.46	1.82	1.34	0.20			
(d_1, d_2)	(0.00, 1.75)			FSI	(0.00, 1.75)			FSI	(0.00, 1.75)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	7.70	8.45	10.11	136.04	193.06	108.27	84.89	24.77	7.57	8.41	10.02
0.50	1.83	1.95	2.19	32.92	51.08	16.88	11.03	7.26	1.94	1.95	2.18
0.75	1.18	1.20	1.26	10.26	14.04	3.29	2.14	4.02	1.28	1.21	1.25
1.00	0.99	0.99	0.99	4.00	4.33	1.24	1.03	2.74	1.06	0.99	0.99
1.50	0.89	0.89	0.88	1.07	1.11	0.88	0.88	1.66	0.92	0.89	0.88
2.00	0.88	0.88	0.88	0.58	0.89	0.88	0.88	1.23	0.89	0.88	0.88
3.00	0.88	0.88	0.88	0.50	0.89	0.88	0.88	0.60	0.88	0.88	0.88
h	12.81	7.07	5.21	0.00	0.00	0.00	0.00	5.23	13.12	7.09	5.23
g	-1.15	-0.24	0.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	-1.15	-0.24	0.22	0.00	-4.66	-3.56	-2.82	5.23	-0.75	-0.22	0.24

Table 5.2.11

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.20			1.34	3.46	1.82	1.34	0.20			
(d_1, d_2)	(0.00, 1.50)			FSI	(0.00, 1.50)			FSI	(0.00, 1.50)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	7.74	8.81	11.60	136.04	196.69	113.47	92.50	24.77	7.62	8.81	11.03
0.50	1.80	1.96	2.39	32.92	52.96	18.45	13.04	7.26	1.87	1.96	2.32
0.75	1.12	1.15	1.25	10.26	14.72	3.57	2.46	4.02	1.18	1.15	1.25
1.00	0.90	0.90	0.92	4.00	4.50	1.20	1.00	2.74	0.95	0.90	0.92
1.50	0.78	0.77	0.76	1.07	1.02	0.76	0.75	1.66	0.80	0.77	0.76
2.00	0.76	0.75	0.75	0.58	0.77	0.75	0.75	1.23	0.76	0.75	0.75
3.00	0.75	0.75	0.75	0.50	0.75	0.75	0.75	0.60	0.75	0.75	0.75
h	12.91	7.09	5.14	0.00	0.00	0.00	0.00	5.23	13.12	7.09	5.23
g	-0.88	-0.03	0.48	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.88	-0.03	0.48	0.00	-4.57	-3.41	-2.57	5.23	-0.63	-0.03	0.56

Table 5.2.12

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.20			1.34	3.46	1.82	1.34	0.20			
(d_1, d_2)	(0.00, 1.25)			FSI	(0.00, 1.25)			FSI	(0.00, 1.25)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	7.81	9.49	15.65	136.04	201.29	120.58	104.70	24.77	7.73	9.41	12.98
0.50	1.78	2.04	3.02	32.92	55.43	20.79	16.90	7.26	1.81	2.02	4.22
0.75	1.06	1.12	1.40	10.26	15.66	4.09	3.32	4.02	1.09	1.12	1.38
1.00	0.82	0.82	0.90	4.00	4.77	1.22	1.08	2.74	0.85	0.82	0.94
1.50	0.67	0.65	0.65	1.07	0.94	0.64	0.64	1.66	0.69	0.65	0.66
2.00	0.64	0.63	0.63	0.58	0.65	0.63	0.63	1.23	0.64	0.63	0.63
3.00	0.63	0.63	0.63	0.50	0.63	0.63	0.63	0.60	0.63	0.63	0.63
h	13.01	7.07	4.90	0.00	0.00	0.00	0.00	5.23	13.12	7.09	5.23
g	-0.61	0.22	0.86	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.61	0.22	0.86	0.00	-4.46	-3.20	-2.16	5.23	-0.48	0.24	1.15

Table 5.2.13

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.25			1.34	3.46	1.82	1.34	0.25			
(d_1, d_2)	(0.00, 2.00)			FSI	(0.00, 2.00)			FSI	(0.00, 2.00)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	10.03	10.59	11.72	136.04	190.08	104.26	79.51	28.73	9.48	10.18	11.62
0.50	1.82	1.91	2.07	32.92	49.58	15.75	9.78	7.26	1.99	1.94	2.07
0.75	1.19	1.21	1.25	10.26	13.52	3.13	2.00	3.78	1.33	1.25	1.25
1.00	1.05	1.06	1.06	4.00	4.23	1.31	1.11	2.51	1.14	1.08	1.06
1.50	1.00	1.00	1.00	1.07	1.22	1.01	1.00	1.49	1.03	1.01	1.00
2.00	1.00	1.00	1.00	0.58	1.01	1.00	1.00	1.04	1.01	1.00	1.00
3.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	0.53	1.00	1.00	1.00
h	10.56	5.82	4.33	0.00	0.00	0.00	0.00	4.35	11.14	5.94	4.35
g	-1.89	-0.73	-0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	-1.89	-0.73	-0.25	0.00	-4.74	-3.68	-3.00	4.35	-1.01	-0.58	-0.23

Table 5.2.14

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.25			1.34	3.46	1.82	1.34	0.25			
(d_1, d_2)	(0.00, 1.75)			FSI	(0.00, 1.75)			FSI	(0.00, 1.75)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	10.03	10.79	12.49	136.04	193.06	108.27	84.89	28.73	9.54	10.54	12.49
0.50	1.78	1.89	2.14	32.92	51.08	16.88	11.03	7.26	1.90	1.91	2.14
0.75	1.11	1.14	1.20	10.26	14.04	3.29	2.14	3.78	1.22	1.16	1.20
1.00	0.95	0.96	0.96	4.00	4.33	1.24	1.03	2.51	1.03	0.97	0.96
1.50	0.88	0.88	0.88	1.07	1.11	0.88	0.88	1.49	0.91	0.88	0.88
2.00	0.88	0.88	0.88	0.58	0.89	0.88	0.88	1.04	0.88	0.88	0.88
3.00	0.88	0.88	0.88	0.50	0.89	0.88	0.88	0.53	0.88	0.88	0.88
h	10.68	5.88	4.35	0.00	0.00	0.00	0.00	4.35	11.14	5.94	4.35
g	-1.57	-0.52	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	-1.57	-0.52	-0.02	0.00	-4.66	-3.56	-2.82	4.35	-0.92	-0.44	-0.02

Table 5.2.15

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.25			1.34	3.46	1.82	1.34	0.25			
(d_1, d_2)	(0.00, 1.50)			FSI	(0.00, 1.50)			FSI	(0.00, 1.50)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	10.06	11.15	14.04	136.04	196.69	113.47	92.50	28.73	9.64	11.05	13.83
0.50	1.74	1.91	2.33	32.92	52.96	18.45	13.04	7.26	1.82	1.91	2.31
0.75	1.04	1.08	1.19	10.26	14.72	3.57	2.46	3.78	1.12	1.09	1.19
1.00	0.85	0.86	0.89	4.00	4.50	1.20	1.00	2.51	0.91	0.87	0.89
1.50	0.77	0.76	0.76	1.07	1.02	0.76	0.75	1.49	0.79	0.76	0.76
2.00	0.75	0.75	0.75	0.58	0.77	0.75	0.75	1.04	0.76	0.75	0.75
3.00	0.75	0.75	0.75	0.50	0.75	0.75	0.75	0.53	0.75	0.75	0.75
h	10.81	5.92	4.33	0.00	0.00	0.00	0.00	4.35	11.14	5.94	4.35
g	-1.25	-0.28	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	-1.25	-0.28	0.25	0.00	-4.57	-3.41	-2.57	4.35	-0.81	-0.26	0.28

Table 5.2.16

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.25			1.34	3.46	1.82	1.34	0.25			
(d_1, d_2)	(0.00, 1.25)			FSI	(0.00, 1.25)			FSI	(0.00, 1.25)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	10.11	11.85	18.22	136.04	201.29	120.58	104.70	28.73	9.80	11.85	16.27
0.50	1.72	1.98	2.95	32.92	55.43	20.79	16.90	7.26	1.76	1.98	2.73
0.75	0.97	1.05	1.32	10.26	15.66	4.09	3.32	3.78	1.03	1.05	1.30
1.00	0.77	0.78	0.87	4.00	4.77	1.22	1.08	2.51	0.81	0.78	0.87
1.50	0.65	0.64	0.65	1.07	0.94	0.64	0.64	1.49	0.67	0.64	0.65
2.00	0.63	0.63	0.63	0.58	0.65	0.63	0.63	1.04	0.64	0.63	0.63
3.00	0.63	0.63	0.63	0.50	0.63	0.63	0.63	0.53	0.63	0.63	0.63
h	10.93	5.94	4.19	0.00	0.00	0.00	0.00	4.35	11.14	5.94	4.35
g	-0.93	-0.01	0.66	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.93	-0.01	0.66	0.00	-4.46	-3.20	-2.16	4.35	-0.66	-0.01	0.80

Table 5.2.17

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.30			1.34	3.46	1.82	1.34	0.30			
(d_1, d_2)	(0.00, 2.00)			FSI	(0.00, 2.00)			FSI	(0.00, 2.00)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	13.51	14.11	15.23	136.04	190.08	104.26	79.51	33.80	12.18	12.97	14.69
0.50	1.87	1.96	2.12	32.92	49.58	15.75	9.78	7.52	2.02	1.98	2.11
0.75	1.15	1.18	1.21	10.26	13.52	3.13	2.00	3.67	1.30	1.23	1.23
1.00	1.03	1.04	1.05	4.00	4.23	1.31	1.11	2.35	1.12	1.07	1.05
1.50	1.00	1.00	1.00	1.07	1.22	1.01	1.00	1.35	1.02	1.00	1.00
2.00	1.00	1.00	1.00	0.58	1.01	1.00	1.00	0.88	1.00	1.00	1.00
3.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	0.51	1.00	1.00	1.00
h	8.97	4.91	3.65	0.00	0.00	0.00	0.00	3.70	9.69	5.11	3.70
g	-2.37	-1.02	-0.49	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	-2.37	-1.02	-0.49	0.00	-4.74	-3.68	-3.00	3.70	-1.15	-0.75	-0.43

Table 5.2.18

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.30			1.34	3.46	1.82	1.34	0.30			
(d_1, d_2)	(0.00, 1.75)			FSI	(0.00, 1.75)			FSI	(0.00, 1.75)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	13.48	14.25	15.96	136.04	193.06	108.27	84.89	33.80	12.29	13.45	15.79
0.50	1.83	1.95	2.19	32.92	51.08	16.88	11.03	7.52	1.94	1.95	2.18
0.75	1.07	1.10	1.16	10.26	14.04	3.29	2.14	3.67	1.19	1.13	1.16
1.00	0.92	0.94	0.95	4.00	4.33	1.24	1.03	2.35	1.01	0.95	0.95
1.50	0.88	0.88	0.88	1.07	1.11	0.88	0.88	1.35	0.90	0.88	0.88
2.00	0.88	0.88	0.88	0.58	0.89	0.88	0.88	0.88	0.88	0.88	0.88
3.00	0.88	0.88	0.88	0.50	0.89	0.88	0.88	0.51	0.88	0.88	0.88
h	9.10	4.98	3.69	0.00	0.00	0.00	0.00	3.70	9.69	5.11	3.70
g	-1.99	-0.78	-0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	-1.99	-0.78	-0.25	0.00	-4.66	-3.56	-2.82	3.70	-1.06	-0.62	-0.23

Table 5.2.19

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.30			1.34	3.46	1.82	1.34	0.30			
(d_1, d_2)	(0.00, 1.50)			FSI	(0.00, 1.50)			FSI	(0.00, 1.50)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	13.46	14.57	17.50	136.04	196.69	113.47	92.50	33.80	12.46	14.13	17.49
0.50	1.80	1.96	2.39	32.92	52.96	18.45	13.04	7.52	1.86	1.95	2.38
0.75	0.99	1.04	1.15	10.26	14.72	3.57	2.46	3.67	1.09	1.05	1.15
1.00	0.82	0.84	0.86	4.00	4.50	1.20	1.00	2.35	0.89	0.85	0.86
1.50	0.76	0.76	0.76	1.07	1.02	0.76	0.75	1.35	0.78	0.76	0.76
2.00	0.75	0.75	0.75	0.58	0.77	0.75	0.75	0.88	0.76	0.75	0.75
3.00	0.75	0.75	0.75	0.50	0.75	0.75	0.75	0.51	0.75	0.75	0.75
h	9.24	5.05	3.70	0.00	0.00	0.00	0.00	3.70	9.69	5.11	3.70
g	-1.61	-0.52	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	-1.61	-0.52	0.05	0.00	-4.57	-3.41	-2.57	3.70	-0.95	-0.44	0.05

Table 5.2.20

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.30			1.34	3.46	1.82	1.34	0.30			
(d_1, d_2)	(0.00, 1.25)			FSI	(0.00, 1.25)			FSI	(0.00, 1.25)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	13.48	15.25	21.75	136.04	201.29	120.58	104.70	33.80	12.71	15.15	20.51
0.50	1.77	2.03	3.01	32.92	55.43	20.79	16.90	7.52	1.81	2.03	2.88
0.75	0.93	1.00	1.28	10.26	15.66	4.09	3.32	3.67	1.00	1.01	1.26
1.00	0.73	0.75	0.84	4.00	4.77	1.22	1.08	2.35	0.78	0.76	0.84
1.50	0.64	0.64	0.64	1.07	0.94	0.64	0.64	1.35	0.66	0.64	0.64
2.00	0.63	0.63	0.63	0.58	0.65	0.63	0.63	0.88	0.63	0.63	0.63
3.00	0.63	0.63	0.63	0.50	0.63	0.63	0.63	0.51	0.63	0.63	0.63
h	9.38	5.10	3.64	0.00	0.00	0.00	0.00	3.70	9.69	5.11	3.70
g	-1.23	-0.22	0.48	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	-1.23	-0.22	0.48	0.00	-4.46	-3.20	-2.16	3.70	-0.81	-0.21	0.53

Table 5.2.21

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.20, 2.00)		FSI	(0.20, 2.00)		FSI	(0.20, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	11.55	10.43	136.04	125.14	87.46	21.24	11.55	10.22
0.50	4.53	3.80	32.92	26.87	13.64	8.39	4.53	3.78
0.75	2.87	2.33	10.26	7.93	3.52	5.14	2.87	2.33
1.00	2.17	1.75	4.00	3.28	1.67	3.69	2.17	1.76
1.50	1.58	1.34	1.07	1.26	1.01	2.35	1.58	1.34
2.00	1.33	1.19	0.58	0.89	0.92	1.71	1.33	1.19
3.00	1.11	1.08	0.50	0.77	0.90	1.20	1.11	1.08
h	11.46	8.49	0.00	0.00	0.00	8.62	11.47	8.62
g	-0.06	0.49	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.06	0.49	0.00	-3.92	-3.14	8.62	-0.06	0.61

Table 5.2.22

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.20, 1.75)		FSI	(0.20, 1.75)		FSI	(0.20, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	11.64	10.93	136.04	127.62	91.70	21.24	11.63	10.47
0.50	4.51	3.85	32.92	27.49	14.55	8.39	4.51	3.80
0.75	2.82	2.28	10.26	7.99	3.60	5.14	2.82	2.30
1.00	2.11	1.67	4.00	3.23	1.60	3.69	2.11	1.70
1.50	1.50	1.24	1.07	1.18	0.91	2.35	1.50	1.25
2.00	1.25	1.08	0.58	0.80	0.81	1.71	1.25	1.09
3.00	1.02	0.98	0.50	0.69	0.80	1.20	1.02	0.98
h	11.46	8.35	0.00	0.00	0.00	8.62	11.47	8.62
g	0.11	0.67	0.00	0.00	0.00	0.00	0.00	0.00
c	0.11	0.67	0.00	-3.80	-2.96	8.62	0.12	0.91

Table 5.2.23

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.20, 1.50)		FSI	(0.20, 1.50)		FSI	(0.20, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	11.79	11.95	136.04	130.86	97.82	21.24	11.74	10.91
0.50	4.51	4.00	32.92	28.37	16.06	8.39	4.50	3.91
0.75	2.78	2.27	10.26	8.12	3.81	5.14	2.78	2.33
1.00	2.04	1.60	4.00	3.18	1.55	3.69	2.04	1.67
1.50	1.42	1.13	1.07	1.09	0.82	2.35	1.42	1.16
2.00	1.16	0.97	0.58	0.72	0.72	1.71	1.16	0.98
3.00	0.94	0.88	0.50	0.60	0.70	1.20	0.94	0.88
h	11.42	8.07	0.00	0.00	0.00	8.62	11.47	8.62
g	0.30	0.90	0.00	0.00	0.00	0.00	0.00	0.00
c	0.30	0.90	0.00	-3.66	-2.71	8.62	0.35	1.39

Table 5.2.24

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.20, 1.25)		FSI	(0.20, 1.25)		FSI	(0.20, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	12.10	14.90	136.04	135.41	107.95	21.24	11.91	11.94
0.50	4.54	4.50	32.92	29.72	19.06	8.39	4.51	4.25
0.75	2.75	2.36	10.26	8.37	4.43	5.14	2.75	2.52
1.00	1.99	1.55	4.00	3.17	1.60	3.69	1.99	1.78
1.50	1.34	1.01	1.07	1.01	0.72	2.35	1.34	1.13
2.00	1.08	0.84	0.58	0.64	0.62	1.71	1.08	0.88
3.00	0.85	0.76	0.50	0.52	0.60	1.20	0.85	0.77
h	11.30	7.42	0.00	0.00	0.00	8.62	11.47	8.62
g	0.53	1.24	0.00	0.00	0.00	0.00	0.00	0.00
c	0.53	1.24	0.00	-3.46	-2.29	8.62	0.69	2.32

Table 5.2.25

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.20, 2.00)		FSI	(0.20, 2.00)		FSI	(0.20, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	11.86	10.57	136.04	125.14	87.46	22.16	11.82	10.55
0.50	3.69	3.31	32.92	26.87	13.64	7.57	3.99	3.31
0.75	2.46	2.03	10.26	7.93	3.52	4.43	2.48	2.03
1.00	1.87	1.56	4.00	3.28	1.67	3.10	1.89	1.56
1.50	1.40	1.23	1.07	1.26	1.01	1.92	1.40	1.23
2.00	1.19	1.11	0.58	0.89	0.92	1.41	1.20	1.11
3.00	0.99	1.00	0.50	0.77	0.90	0.84	0.99	1.00
h	8.67	6.51	0.00	0.00	0.00	6.52	8.75	6.52
g	-0.50	0.14	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.50	0.14	0.00	-3.92	-3.14	6.52	-0.42	0.14

Table 5.2.26

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.20, 1.75)		FSI	(0.20, 1.75)		FSI	(0.20, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	11.93	11.05	136.04	127.62	91.70	22.16	11.92	10.90
0.50	3.95	3.36	32.92	27.49	14.55	7.57	3.95	3.34
0.75	2.41	1.98	10.26	7.99	3.60	4.43	2.42	1.98
1.00	1.81	1.48	4.00	3.23	1.60	3.10	1.81	1.48
1.50	1.32	1.13	1.07	1.18	0.91	1.92	1.32	1.13
2.00	1.11	1.01	0.58	0.80	0.81	1.41	1.11	1.01
3.00	0.91	0.90	0.50	0.69	0.80	0.84	0.91	0.90
h	8.72	6.47	0.00	0.00	0.00	6.52	8.75	6.52
g	-0.30	0.34	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.30	0.34	0.00	-3.80	-2.96	6.52	-0.27	0.39

Table 5.2.27

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.20, 1.50)		FSI	(0.20, 1.50)		FSI	(0.20, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	12.07	12.05	136.04	130.86	97.82	22.16	12.07	11.48
0.50	3.94	3.50	32.92	28.37	16.06	7.57	3.94	3.43
0.75	2.37	1.97	10.26	8.12	3.81	4.43	2.37	1.97
1.00	1.75	1.41	4.00	3.18	1.55	3.10	1.75	1.43
1.50	1.24	1.03	1.07	1.09	0.82	1.92	1.24	1.03
2.00	1.03	0.91	0.58	0.72	0.72	1.41	1.03	0.91
3.00	0.83	0.79	0.50	0.60	0.70	0.84	0.83	0.79
h	8.75	6.34	0.00	0.00	0.00	6.52	8.75	6.52
g	-0.07	0.60	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.07	0.60	0.00	-3.66	-2.71	6.52	-0.07	0.76

Table 5.2.28

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.20, 1.25)		FSI	(0.20, 1.25)		FSI	(0.20, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	12.36	14.96	136.04	135.41	107.95	22.16	12.33	12.67
0.50	3.97	3.96	32.92	29.72	19.06	7.57	3.97	3.71
0.75	2.34	2.06	10.26	8.37	4.43	4.43	2.34	2.08
1.00	1.69	1.38	4.00	3.17	1.60	3.10	1.69	1.45
1.50	1.16	0.92	1.07	1.01	0.72	1.92	1.16	0.95
2.00	0.95	0.80	0.58	0.64	0.62	1.41	0.95	0.81
3.00	0.74	0.67	0.50	0.52	0.60	0.84	0.74	0.68
h	8.74	5.98	0.00	0.00	0.00	6.52	8.75	6.52
g	0.20	0.98	0.00	0.00	0.00	0.00	0.00	0.00
c	0.20	0.98	0.00	-3.46	-2.29	6.52	0.21	1.44

Table 5.2.29

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.20, 2.00)		FSI	(0.20, 2.00)		FSI	(0.20, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	13.61	12.06	136.04	125.14	87.46	24.77	13.36	12.04
0.50	3.73	3.10	32.92	26.87	13.64	7.26	3.77	3.10
0.75	2.24	1.86	10.26	7.93	3.52	4.02	2.27	1.86
1.00	1.70	1.44	4.00	3.28	1.67	2.74	1.72	1.45
1.50	1.28	1.16	1.07	1.26	1.01	1.66	1.29	1.16
2.00	1.10	1.07	0.58	0.89	0.92	1.23	1.11	1.07
3.00	0.96	0.93	0.50	0.77	0.90	0.60	0.96	0.93
h	6.90	5.22	0.00	0.00	0.00	5.23	7.09	5.23
g	-0.90	-0.17	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.90	-0.17	0.00	-3.92	-3.14	5.23	-0.66	-0.16

Table 5.2.30

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.20, 1.75)		FSI	(0.20, 1.75)		FSI	(0.20, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	13.66	12.54	136.04	127.62	91.70	24.77	13.48	12.54
0.50	3.71	3.14	32.92	27.49	14.55	7.26	3.73	3.14
0.75	2.19	1.81	10.26	7.99	3.60	4.02	2.21	1.81
1.00	1.63	1.37	4.00	3.23	1.60	2.74	1.65	1.37
1.50	1.20	1.07	1.07	1.18	0.91	1.66	1.21	1.07
2.00	1.02	0.97	0.58	0.80	0.81	1.23	1.03	0.97
3.00	0.87	0.83	0.50	0.69	0.80	0.60	0.88	0.83
h	6.97	5.23	0.00	0.00	0.00	5.23	7.09	5.23
g	-0.66	0.06	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.66	0.06	0.00	-3.80	-2.96	5.23	-0.52	0.06

Table 5.2.31

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.20, 1.50)		FSI	(0.20, 1.50)		FSI	(0.20, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	13.77	13.56	136.04	130.86	97.82	24.77	13.69	13.32
0.50	3.71	3.27	32.92	28.37	16.06	7.26	3.71	3.24
0.75	2.14	1.80	10.26	8.12	3.81	4.02	2.15	1.79
1.00	1.57	1.30	4.00	3.18	1.55	2.74	1.58	1.30
1.50	1.12	0.97	1.07	1.09	0.82	1.66	1.13	0.97
2.00	0.94	0.87	0.58	0.72	0.72	1.23	0.95	0.87
3.00	0.79	0.73	0.50	0.60	0.70	0.60	0.79	0.73
h	7.04	5.19	0.00	0.00	0.00	5.23	7.09	5.23
g	-0.39	0.34	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.39	0.34	0.00	-3.66	-2.71	5.23	-0.34	0.38

Table 5.2.32

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.20, 1.25)		FSI	(0.20, 1.25)		FSI	(0.20, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	14.04	16.53	136.04	135.41	107.95	24.77	14.03	14.82
0.50	3.73	3.71	32.92	29.72	19.06	7.26	3.73	3.51
0.75	2.11	1.88	10.26	8.37	4.43	4.02	2.11	1.87
1.00	1.51	1.27	4.00	3.17	1.60	2.74	1.51	1.28
1.50	1.05	0.87	1.07	1.01	0.72	1.66	1.05	0.88
2.00	0.86	0.76	0.58	0.64	0.62	1.23	0.86	0.77
3.00	0.71	0.62	0.50	0.52	0.60	0.60	0.71	0.62
h	7.08	4.99	0.00	0.00	0.00	5.23	7.09	5.23
g	-0.09	0.75	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.09	0.75	0.00	-3.46	-2.29	5.23	-0.09	0.96

Table 5.2.33

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.20, 2.00)		FSI	(0.20, 2.00)		FSI	(0.20, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	16.70	14.79	136.04	125.14	87.46	28.73	15.88	14.53
0.50	3.71	3.06	32.92	26.87	13.64	7.26	3.73	3.06
0.75	2.11	1.76	10.26	7.93	3.52	3.78	2.16	1.77
1.00	1.59	1.37	4.00	3.28	1.67	2.51	1.62	1.38
1.50	1.20	1.12	1.07	1.26	1.01	1.49	1.22	1.12
2.00	1.04	1.03	0.58	0.89	0.92	1.04	1.05	1.03
3.00	0.92	0.91	0.50	0.77	0.90	0.52	0.93	0.91
h	5.65	4.30	0.00	0.00	0.00	4.35	5.94	4.35
g	-1.28	-0.45	0.00	0.00	0.00	0.00	0.00	0.00
c	-1.28	-0.45	0.00	-3.92	-3.14	4.35	-0.86	-0.39

Table 5.2.34

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.20, 1.75)		FSI	(0.20, 1.75)		FSI	(0.20, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	16.68	15.23	136.04	127.62	91.70	28.73	16.07	15.17
0.50	3.68	3.09	32.92	27.49	14.55	7.26	3.69	3.09
0.75	2.06	1.71	10.26	7.99	3.60	3.78	2.09	1.71
1.00	1.52	1.29	4.00	3.23	1.60	2.51	1.54	1.29
1.50	1.12	1.02	1.07	1.18	0.91	1.49	1.14	1.02
2.00	0.96	0.93	0.58	0.8[illegible]	0.81	1.04	0.97	0.93
3.00	0.84	0.80	0.50	0.69	0.80	0.52	0.85	0.81
h	5.74	4.34	0.00	0.00	0.00	4.35	5.94	4.35
g	-0.99	-0.20	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.99	-0.20	0.00	-3.80	-2.96	4.35	-0.72	-0.19

Table 5.2.35

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.20, 1.50)		FSI	(0.20, 1.50)		FSI	(0.20, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	16.73	16.23	136.04	130.86	97.82	28.73	16.35	16.20
0.50	3.67	3.21	32.92	28.37	16.06	7.26	3.67	3.21
0.75	2.01	1.69	10.26	8.12	3.81	3.78	2.03	1.69
1.00	1.46	1.22	4.00	3.18	1.55	2.51	1.47	1.22
1.50	1.05	0.93	1.07	1.09	0.82	1.49	1.05	0.93
2.00	0.88	0.83	0.58	0.72	0.72	1.04	0.89	0.83
3.00	0.76	0.71	0.50	0.60	0.70	0.52	0.76	0.71
h	5.83	4.34	0.00	0.00	0.00	4.35	5.94	4.35
g	-0.69	0.11	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.69	0.11	0.00	-3.66	-2.71	4.35	-0.55	0.11

Table 5.2.36

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.20, 1.25)		FSI	(0.20, 1.25)		FSI	(0.20, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	16.93	19.25	136.04	135.41	107.95	28.73	16.80	18.12
0.50	3.69	3.64	32.92	29.72	19.06	7.26	3.69	3.52
0.75	1.98	1.77	10.26	8.37	4.43	3.78	1.98	1.75
1.00	1.40	1.19	4.00	3.17	1.60	2.51	1.40	1.19
1.50	0.97	0.83	1.07	1.01	0.72	1.49	0.97	0.83
2.00	0.80	0.72	0.58	0.64	0.62	1.04	0.80	0.73
3.00	0.68	0.61	0.50	0.52	0.60	0.52	0.68	0.61
h	5.91	4.25	0.00	0.00	0.00	4.35	5.94	4.35
g	-0.36	0.54	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.36	0.54	0.00	-3.46	-2.29	4.35	-0.32	0.63

Table 5.2.37

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.20, 2.00)		FSI	(0.20, 2.00)		FSI	(0.20, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	21.23	18.79	136.04	125.14	87.46	33.80	19.23	17.84
0.50	3.86	3.69	32.92	26.87	13.64	7.52	3.82	3.13
0.75	2.05	1.70	10.26	7.93	3.52	3.67	2.09	1.72
1.00	1.52	1.45	4.00	3.28	1.67	2.35	1.56	1.33
1.50	1.15	1.12	1.07	1.26	1.01	1.35	1.17	1.09
2.00	1.00	0.97	0.58	0.89	0.92	0.88	1.01	0.99
3.00	0.87	0.85	0.50	0.77	0.90	0.51	0.89	0.90
h	4.72	3.61	0.00	0.00	0.00	3.70	5.11	3.70
g	-1.64	-0.71	0.00	0.00	0.00	0.00	0.00	0.00
c	-1.64	-0.71	0.00	-3.92	-3.14	3.70	-1.02	-0.59

Table 5.2.38

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.20, 1.75)		FSI	(0.20, 1.75)		FSI	(0.20, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	21.04	19.11	136.04	127.62	91.70	33.80	19.50	18.67
0.50	3.82	3.70	32.92	27.49	14.55	7.52	3.78	3.17
0.75	1.99	1.65	10.26	7.99	3.60	3.67	2.02	1.66
1.00	1.45	1.38	4.00	3.23	1.60	2.35	1.48	1.24
1.50	1.07	1.04	1.07	1.18	0.91	1.35	1.08	0.99
2.00	0.92	0.89	0.58	0.80	0.81	0.88	0.93	0.89
3.00	0.79	0.76	0.50	0.69	0.80	0.51	0.81	0.80
h	4.82	3.66	0.00	0.00	0.00	3.70	5.11	3.70
g	-1.31	-0.43	0.00	0.00	0.00	0.00	0.00	0.00
c	-1.31	-0.43	0.00	-3.80	-2.96	3.70	-0.89	-0.39

Table 5.2.39

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.20, 1.50)		FSI	(0.20, 1.50)		FSI	(0.20, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	20.93	20.01	136.04	130.86	97.82	33.80	19.88	19.97
0.50	3.81	3.78	32.92	28.37	16.06	7.52	3.77	3.30
0.75	1.94	1.63	10.26	8.12	3.81	3.67	1.96	1.63
1.00	1.38	1.32	4.00	3.18	1.55	2.35	1.40	1.17
1.50	0.99	0.96	1.07	1.09	0.82	1.35	1.00	0.89
2.00	0.84	0.81	0.58	0.72	0.72	0.88	0.85	0.79
3.00	0.72	0.68	0.50	0.60	0.70	0.51	0.72	0.70
h	4.93	3.70	0.00	0.00	0.00	3.70	5.11	3.70
g	-0.97	-0.11	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.97	-0.11	0.00	-3.66	-2.71	3.70	-0.73	-0.11

Table 5.2.40

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.20, 1.25)		FSI	(0.20, 1.25)		FSI	(0.20, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	21.00	22.97	136.04	135.41	107.95	33.80	20.48	22.39
0.50	3.81	3.73	32.92	29.72	19.06	7.52	3.79	3.67
0.75	1.91	1.71	10.26	8.37	4.43	3.67	1.91	1.70
1.00	1.32	1.28	4.00	3.17	1.60	2.35	1.33	1.13
1.50	0.91	0.87	1.07	1.01	0.72	1.35	0.92	0.80
2.00	0.76	0.72	0.58	0.64	0.62	0.88	0.76	0.69
3.00	0.64	0.59	0.50	0.52	0.60	0.51	0.64	0.60
h	5.03	3.70	0.00	0.00	0.00	3.70	5.11	3.70
g	-0.60	0.35	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.60	0.35	0.00	-3.46	-2.29	3.70	-0.50	0.38

Table 5.2.41

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$ *, and CUSUM Charts*

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.50, 2.00)		FSI	(0.50, 2.00)		FSI	(0.50, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	18.92	14.13	136.04	164.89	101.47	21.24	18.76	14.12
0.50	7.51	5.51	32.92	46.09	20.00	8.39	7.50	5.50
0.75	4.66	3.44	10.26	15.70	5.91	5.14	4.67	3.44
1.00	3.39	2.55	4.00	6.36	2.52	3.69	3.40	2.55
1.50	2.22	1.79	1.07	1.64	1.03	2.35	2.23	1.79
2.00	1.67	1.43	0.58	0.72	0.79	1.71	1.68	1.43
3.00	1.15	1.17	0.50	0.43	0.75	1.20	1.16	1.17
h	10.76	8.61	0.00	0.00	0.00	8.62	11.47	8.62
g	-2.12	0.15	0.00	0.00	0.00	0.00	0.00	0.00
c	-2.12	0.15	0.00	-4.65	-3.43	8.62	-1.05	0.16

Table 5.2.42

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$ *, and CUSUM Charts*

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.50, 1.75)		FSI	(0.50, 1.75)		FSI	(0.50, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	18.87	14.34	136.04	165.31	103.94	21.24	18.76	14.25
0.50	7.49	5.52	32.92	46.16	20.45	8.39	7.48	5.51
0.75	4.64	3.40	10.26	15.69	5.93	5.14	4.65	3.40
1.00	3.37	2.50	4.00	6.34	2.47	3.69	3.38	2.50
1.50	2.20	1.72	1.07	1.62	0.97	2.35	2.21	1.72
2.00	1.65	1.37	0.58	0.70	0.73	1.71	1.66	1.37
3.00	1.13	1.11	0.50	0.41	0.69	1.20	1.14	1.11
h	10.94	8.55	0.00	0.00	0.00	8.62	11.47	8.62
g	-1.67	0.37	0.00	0.00	0.00	0.00	0.00	0.00
c	-1.67	0.37	0.00	-4.56	-3.25	8.62	-0.93	0.43

Table 5.2.43

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.50, 1.50)		FSI	(0.50, 1.50)		FSI	(0.50, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	18.84	14.80	136.04	165.87	107.65	21.24	18.76	14.48
0.50	7.47	5.57	32.92	46.26	21.23	8.39	7.47	5.53
0.75	4.63	3.37	10.26	15.69	6.01	5.14	4.63	3.38
1.00	3.35	2.44	4.00	6.32	2.43	3.69	3.36	2.45
1.50	2.18	1.65	1.07	1.60	0.91	2.35	2.19	1.65
2.00	1.63	1.29	0.58	0.68	0.66	1.71	1.64	1.30
3.00	1.12	1.04	0.50	0.39	0.63	1.20	1.12	1.04
h	11.13	8.39	0.00	0.00	0.00	8.62	11.47	8.62
g	-1.21	0.63	0.00	0.00	0.00	0.00	0.00	0.00
c	-1.21	0.63	0.00	-4.44	-3.00	8.62	-0.77	0.84

Table 5.2.44

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.50, 1.25)		FSI	(0.50, 1.25)		FSI	(0.50, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	18.82	16.30	136.04	166.70	114.21	21.24	18.78	14.99
0.50	7.46	5.79	32.92	46.43	22.92	8.39	7.46	5.67
0.75	4.61	3.37	10.26	15.70	6.30	5.14	4.62	3.43
1.00	3.34	2.36	4.00	6.30	2.43	3.69	3.34	2.45
1.50	2.16	1.54	1.07	1.58	0.85	2.35	2.17	1.59
2.00	1.61	1.18	0.58	0.66	0.60	1.71	1.62	1.22
3.00	1.10	0.96	0.50	0.36	0.56	1.20	1.10	0.97
h	11.31	7.89	0.00	0.00	0.00	8.62	11.47	8.62
g	-0.73	1.02	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.73	1.02	0.00	-4.27	-2.57	8.62	-0.55	1.67

Table 5.2.45

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.50, 2.00)		FSI	(0.50, 2.00)		FSI	(0.50, 2.00)	
n δ	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	20.30	14.57	136.04	164.89	101.47	22.16	19.49	14.55
0.50	6.88	4.87	32.92	46.09	20.00	7.57	6.75	4.88
0.75	4.08	2.95	10.26	15.70	5.91	4.43	4.03	2.95
1.00	2.91	2.18	4.00	6.36	2.52	3.10	2.88	2.18
1.50	1.87	1.53	1.07	1.64	1.03	1.92	1.86	1.53
2.00	1.41	1.25	0.58	0.72	0.79	1.41	1.39	1.25
3.00	0.96	0.97	0.50	0.43	0.75	0.84	0.94	0.97
h	7.66	6.50	0.00	0.00	0.00	6.52	8.75	6.52
g	-3.36	-0.26	0.00	0.00	0.00	0.00	0.00	0.00
c	-3.36	-0.26	0.00	-4.65	-3.43	6.52	-1.31	-0.23

Table 5.2.46

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.50, 1.75)		FSI	(0.50, 1.75)		FSI	(0.50, 1.75)	
n δ	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	20.08	14.74	136.04	165.31	103.94	22.16	19.49	14.74
0.50	6.82	4.88	32.92	46.16	20.45	7.57	6.73	4.88
0.75	4.04	2.92	10.26	15.69	5.93	4.43	4.01	2.92
1.00	2.87	2.13	4.00	6.34	2.47	3.10	2.86	2.13
1.50	1.84	1.47	1.07	1.62	0.97	1.92	1.84	1.47
2.00	1.38	1.19	0.58	0.70	0.73	1.41	1.37	1.19
3.00	0.93	0.91	0.50	0.41	0.69	0.84	0.92	0.91
h	-7.89	6.52	0.00	0.00	0.00	6.52	8.75	6.52
g	-2.70	-0.01	0.00	0.00	0.00	0.00	0.00	0.00
c	-2.70	-0.01	0.00	-4.56	-3.25	6.52	-1.20	-0.01

Table 5.2.47

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.50, 1.50)		FSI	(0.50, 1.50)		FSI	(0.50, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	19.89	15.15	136.04	165.87	107.65	22.16	19.50	15.06
0.50	6.77	4.93	32.92	46.26	21.23	7.57	6.72	4.92
0.75	4.00	2.90	10.26	15.69	6.01	4.43	3.99	2.89
1.00	2.84	2.08	4.00	6.32	2.43	3.10	2.84	2.08
1.50	1.82	1.41	1.07	1.60	0.91	1.92	1.82	1.41
2.00	1.35	1.12	0.58	0.68	0.66	1.41	1.35	1.13
3.00	0.90	0.85	0.50	0.39	0.63	0.84	0.90	0.85
h	8.13	6.48	0.00	0.00	0.00	6.52	8.75	6.52
g	-2.03	0.30	0.00	0.00	0.00	0.00	0.00	0.00
c	-2.03	0.30	0.00	-4.44	-3.00	6.52	-1.06	0.33

Table 5.2.48

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.50, 1.25)		FSI	(0.50, 1.25)		FSI	(0.50, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	19.75	16.58	136.04	166.70	114.21	22.16	19.52	15.71
0.50	6.73	5.14	32.92	46.43	22.92	7.57	6.71	5.04
0.75	3.98	2.91	10.26	15.70	6.30	4.43	3.97	2.91
1.00	2.82	2.03	4.00	6.30	2.43	3.10	2.82	2.05
1.50	1.79	1.32	1.07	1.58	0.85	1.92	1.80	1.34
2.00	1.33	1.05	0.58	0.66	0.60	1.41	1.33	1.06
3.00	0.88	0.75	0.50	0.36	0.56	0.84	0.88	0.77
h	8.38	6.25	0.00	0.00	0.00	6.52	8.75	6.52
g	-1.36	0.73	0.00	0.00	0.00	0.00	0.00	0.00
c	-1.36	0.73	0.00	-4.27	-2.57	6.52	-0.85	0.97

Table 5.2.49

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.50, 2.00)		FSI	(0.50, 2.00)		FSI	(0.50, 2.00)	
n δ	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	24.62	16.58	136.04	164.89	101.47	24.77	21.76	16.35
0.50	6.90	4.62	32.92	46.09	20.00	7.26	6.45	4.62
0.75	3.88	2.67	10.26	15.70	5.91	4.02	3.67	2.69
1.00	2.70	1.95	4.00	6.36	2.52	2.74	2.57	1.96
1.50	1.71	1.37	1.07	1.64	1.03	1.66	1.63	1.38
2.00	1.27	1.15	0.58	0.72	0.79	1.23	1.21	1.16
3.00	0.88	0.81	0.50	0.43	0.75	0.60	0.85	0.81
h	5.76	5.14	0.00	0.00	0.00	5.23	7.09	5.23
g	-4.62	-0.62	0.00	0.00	0.00	0.00	0.00	0.00
c	-4.62	-0.62	0.00	-4.65	-3.43	5.23	-1.51	-0.51

Table 5.2.50

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.50, 1.75)		FSI	(0.50, 1.75)		FSI	(0.50, 1.75)	
n δ	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	23.83	16.68	136.04	165.31	103.94	24.77	21.76	16.60
0.50	6.74	4.62	32.92	46.16	20.45	7.26	6.43	4.62
0.75	3.78	2.64	10.26	15.69	5.93	4.02	3.65	2.64
1.00	2.63	1.90	4.00	6.34	2.47	2.74	2.55	1.91
1.50	1.65	1.31	1.07	1.62	0.97	1.66	1.61	1.32
2.00	1.22	1.09	0.58	0.70	0.73	1.23	1.19	1.10
3.00	0.85	0.75	0.50	0.41	0.69	0.60	0.83	0.75
h	5.98	5.20	0.00	0.00	0.00	5.23	7.09	5.23
g	-3.74	-0.33	0.00	0.00	0.00	0.00	0.00	0.00
c	-3.74	-0.33	0.00	-4.56	-3.25	5.23	-1.40	-0.30

Table 5.2.51

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.50, 1.50)		FSI	(0.50, 1.50)		FSI	(0.50, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	23.16	17.03	136.04	165.87	107.65	24.77	21.78	17.03
0.50	6.62	4.66	32.92	46.26	21.23	7.26	6.42	4.66
0.75	3.71	2.62	10.26	15.69	6.01	4.02	3.63	2.62
1.00	2.57	1.86	4.00	6.32	2.43	2.74	2.53	1.86
1.50	1.61	1.25	1.07	1.60	0.91	1.66	1.59	1.25
2.00	1.19	1.03	0.58	0.68	0.66	1.23	1.17	1.03
3.00	0.82	0.69	0.50	0.39	0.63	0.60	0.81	0.69
h	6.24	5.23	0.00	0.00	0.00	5.23	7.09	5.23
g	-2.86	0.01	0.00	0.00	0.00	0.00	0.00	0.00
c	-2.86	0.01	0.00	-4.44	-3.00	5.23	-1.26	0.01

Table 5.2.52

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.50, 1.25)		FSI	(0.50, 1.25)		FSI	(0.50, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	22.63	18.39	136.04	166.70	114.21	24.77	21.82	17.90
0.50	6.52	4.86	32.92	46.43	22.92	7.26	6.41	4.80
0.75	3.65	2.64	10.26	15.70	6.30	4.02	3.61	2.64
1.00	2.52	1.82	4.00	6.30	2.43	2.74	2.51	1.83
1.50	1.57	1.19	1.07	1.58	0.85	1.66	1.56	1.19
2.00	1.15	0.97	0.58	0.66	0.60	1.23	1.15	0.97
3.00	0.79	0.62	0.50	0.36	0.56	0.60	0.79	0.62
h	6.52	5.14	0.00	0.00	0.00	5.23	7.09	5.23
g	-1.98	0.48	0.00	0.00	0.00	0.00	0.00	0.00
c	-1.98	0.48	0.00	-4.27	-2.57	5.23	-1.07	0.56

Table 5.2.53

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.50, 2.00)		FSI	(0.50, 2.00)		FSI	(0.50, 2.00)	
n δ	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	33.19	20.06	136.04	164.89	101.47	28.73	25.24	19.24
0.50	7.50	4.61	32.92	46.09	20.00	7.26	6.44	4.58
0.75	3.94	2.52	10.26	15.70	5.91	3.78	3.46	2.53
1.00	2.67	1.81	4.00	6.36	2.52	2.51	2.37	1.82
1.50	1.65	1.27	1.07	1.64	1.03	1.49	1.47	1.28
2.00	1.22	1.04	0.58	0.72	0.79	1.04	1.08	1.05
3.00	0.82	0.76	0.50	0.43	0.75	0.53	0.78	0.76
h	4.52	4.18	0.00	0.00	0.00	4.35	5.94	4.35
g	-5.88	-0.95	0.00	0.00	0.00	0.00	0.00	0.00
c	-5.88	-0.95	0.00	-4.65	-3.43	4.35	-1.67	-0.72

Table 5.2.54

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.50, 1.75)		FSI	(0.50, 1.75)		FSI	(0.50, 1.75)	
n δ	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	31.03	20.03	136.04	165.31	103.94	28.73	25.25	19.57
0.50	7.18	4.60	32.92	46.16	20.45	7.26	6.42	4.58
0.75	3.77	2.48	10.26	15.69	5.93	3.78	3.44	2.49
1.00	2.55	1.76	4.00	6.34	2.47	2.51	2.34	1.77
1.50	1.57	1.21	1.07	1.62	0.97	1.49	1.45	1.22
2.00	1.15	0.98	0.58	0.70	0.73	1.04	1.06	0.99
3.00	0.78	0.70	0.50	0.41	0.69	0.53	0.76	0.70
h	4.72	4.26	0.00	0.00	0.00	4.35	5.94	4.35
g	-4.79	-0.63	0.00	0.00	0.00	0.00	0.00	0.00
c	-4.79	-0.63	0.00	-4.56	-3.25	4.35	-1.57	-0.52

Table 5.2.55

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.50, 1.50)		FSI	(0.50, 1.50)		FSI	(0.50, 1.50)	
n δ	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	29.16	20.23	136.04	165.87	107.65	28.73	25.29	20.13
0.50	6.91	4.62	32.92	46.26	21.23	7.26	6.41	4.62
0.75	3.62	2.46	10.26	15.69	6.01	3.78	3.42	2.46
1.00	2.45	1.71	4.00	6.32	2.43	2.51	2.33	1.71
1.50	1.50	1.16	1.07	1.60	0.91	1.49	1.43	1.16
2.00	1.10	0.93	0.58	0.68	0.66	1.04	1.04	0.93
3.00	0.74	0.64	0.50	0.39	0.63	0.53	0.74	0.64
h	4.96	4.33	0.00	0.00	0.00	4.35	5.94	4.35
g	-3.69	-0.25	0.00	0.00	0.00	0.00	0.00	0.00
c	-3.69	-0.25	0.00	-4.44	-3.00	4.35	-1.43	-0.23

Table 5.2.56

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.50, 1.25)		FSI	(0.50, 1.25)		FSI	(0.50, 1.25)	
n δ	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	27.63	21.45	136.04	166.70	114.21	28.73	25.34	21.27
0.50	6.68	4.80	32.92	46.43	22.92	7.26	6.40	4.78
0.75	3.51	2.48	10.26	15.70	6.30	3.78	3.40	2.48
1.00	2.36	1.68	4.00	6.30	2.43	2.51	2.30	1.68
1.50	1.44	1.10	1.07	1.58	0.85	1.49	1.41	1.10
2.00	1.05	0.86	0.58	0.66	0.60	1.04	1.02	0.86
3.00	0.72	0.58	0.50	0.36	0.56	0.53	0.72	0.58
h	5.24	4.33	0.00	0.00	0.00	4.35	5.94	4.35
g	-2.60	0.26	0.00	0.00	0.00	0.00	0.00	0.00
c	-2.60	0.26	0.00	-4.27	-2.57	4.35	-1.25	0.28

Table 5.2.57

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.50, 2.00)		FSI	(0.50, 2.00)		FSI	(0.50, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	47.65	25.16	136.04	164.89	101.47	33.80	29.72	23.03
0.50	8.76	4.82	32.92	46.09	20.00	7.52	6.65	4.71
0.75	4.22	2.44	10.26	15.70	5.91	3.67	3.35	2.45
1.00	2.77	1.72	4.00	6.36	2.52	2.35	2.23	1.73
1.50	1.67	1.19	1.07	1.64	1.03	1.35	1.36	1.21
2.00	1.22	0.95	0.58	0.72	0.79	0.88	1.00	0.96
3.00	0.80	0.76	0.50	0.43	0.75	0.51	0.69	0.75
h	3.67	3.47	0.00	0.00	0.00	3.70	5.11	3.70
g	-7.15	-1.26	0.00	0.00	0.00	0.00	0.00	0.00
c	-7.15	-1.26	0.00	-4.65	-3.43	3.70	-1.81	-0.90

Table 5.2.58

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.50, 1.75)		FSI	(0.50, 1.75)		FSI	(0.50, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	42.99	24.82	136.04	165.31	103.94	33.80	29.75	23.47
0.50	8.20	4.79	32.92	46.16	20.45	7.52	6.63	4.71
0.75	3.96	2.40	10.26	15.69	5.93	3.67	3.33	2.41
1.00	2.59	1.66	4.00	6.34	2.47	2.35	2.21	1.67
1.50	1.55	1.13	1.07	1.62	0.97	1.35	1.33	1.14
2.00	1.15	0.89	0.58	0.70	0.73	0.88	0.98	0.90
3.00	0.74	0.69	0.50	0.41	0.69	0.51	0.67	0.69
h	3.85	3.56	0.00	0.00	0.00	3.70	5.11	3.70
g	-5.84	-0.90	0.00	0.00	0.00	0.00	0.00	0.00
c	-5.84	-0.90	0.00	-4.56	-3.25	3.70	-1.71	-0.71

Table 5.2.59

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.50, 1.50)		FSI	(0.50, 1.50)		FSI	(0.50, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	38.82	24.72	136.04	165.87	107.65	33.80	29.80	24.19
0.50	7.69	4.79	32.92	46.26	21.23	7.52	6.62	4.76
0.75	3.72	2.37	10.26	15.69	6.01	3.67	3.31	2.37
1.00	2.44	1.61	4.00	6.32	2.43	2.35	2.19	1.62
1.50	1.45	1.08	1.07	1.60	0.91	1.35	1.31	1.08
2.00	1.06	0.83	0.58	0.68	0.66	0.88	0.96	0.84
3.00	0.69	0.63	0.50	0.39	0.63	0.51	0.65	0.63
h	4.05	3.65	0.00	0.00	0.00	3.70	5.11	3.70
g	-4.53	-0.49	0.00	0.00	0.00	0.00	0.00	0.00
c	-4.53	-0.49	0.00	-4.44	-3.00	3.70	-1.58	-0.43

Table 5.2.60

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.50, 1.25)		FSI	(0.50, 1.25)		FSI	(0.50, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	35.28	25.64	136.04	166.70	114.21	33.80	29.89	25.63
0.50	7.24	4.94	32.92	46.43	22.92	7.52	6.61	4.94
0.75	3.52	2.39	10.26	15.70	6.30	3.67	3.30	2.39
1.00	2.30	1.58	4.00	6.30	2.43	2.35	2.17	1.59
1.50	1.36	1.02	1.07	1.58	0.85	1.35	1.29	1.02
2.00	0.99	0.77	0.58	0.66	0.60	0.88	0.94	0.77
3.00	0.64	0.57	0.50	0.36	0.56	0.51	0.63	0.57
h	4.32	3.70	0.00	0.00	0.00	3.70	5.11	3.70
g	-3.22	0.05	0.00	0.00	0.00	0.00	0.00	0.00
c	-3.22	0.05	0.00	-4.27	-2.57	3.70	-1.40	0.05

Table 5.3.1

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.10			1.34	3.46	1.82	1.34	0.10			
(d_1, d_2)	(0.00, 2.00)			FSI	(0.00, 2.00)			FSI	(0.00, 2.00)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	6.50	7.05	8.14	136.54	190.08	104.26	79.51	24.76	6.52	7.06	8.11
0.50	2.53	2.54	2.66	33.42	49.58	15.75	9.78	10.39	2.57	2.57	2.85
0.75	1.67	1.59	1.56	10.76	13.52	3.13	2.00	6.61	1.71	1.61	1.70
1.00	1.34	1.24	1.19	4.50	4.24	1.31	1.11	4.90	1.36	1.25	1.26
1.50	1.09	1.03	1.01	1.57	1.22	1.01	1.00	3.31	1.10	1.04	1.02
2.00	1.02	1.00	1.00	1.08	1.01	1.00	1.00	2.54	1.03	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.98	1.00	1.00	1.00

Table 5.3.2

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.10			1.34	3.46	1.82	1.34	0.10			
(d_1, d_2)	(0.00, 1.75)			FSI	(0.00, 1.75)			FSI	(0.00, 1.75)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	6.64	7.38	8.99	136.54	193.18	108.40	85.02	24.76	6.65	7.39	8.82
0.50	2.62	2.66	2.88	33.42	51.20	17.01	11.15	10.39	2.63	2.73	3.18
0.75	1.74	1.65	1.65	10.76	14.17	3.42	2.26	6.61	1.75	1.71	1.91
1.00	1.38	1.27	1.23	4.50	4.46	1.37	1.16	4.90	1.39	1.31	1.38
1.50	1.11	1.04	1.02	1.57	1.24	1.01	1.00	3.31	1.12	1.05	1.04
2.00	1.03	1.00	1.00	1.08	1.02	1.00	1.00	2.54	1.03	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.98	1.00	1.00	1.00

Table 5.3.3

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.10			1.34	3.46	1.82	1.34	0.10			
(d_1, d_2)	(0.00, 1.50)			FSI	(0.00, 1.50)			FSI	(0.00, 1.50)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	6.80	7.85	10.56	136.54	196.94	113.72	92.75	24.76	6.80	7.84	9.92
0.50	2.71	2.81	3.24	33.42	53.21	18.70	13.29	10.39	2.71	2.94	3.69
0.75	1.80	1.73	1.80	10.76	14.97	3.82	2.71	6.61	1.80	1.85	2.26
1.00	1.43	1.32	1.29	4.50	4.75	1.45	1.24	4.90	1.43	1.40	1.62
1.50	1.13	1.05	1.03	1.57	1.27	1.01	1.00	3.31	1.13	1.08	1.11
2.00	1.04	1.01	1.00	1.08	1.02	1.00	1.00	2.54	1.04	1.01	1.01
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.98	1.00	1.00	1.00

Table 5.3.4

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.10			1.34	3.46	1.82	1.34	0.10			
(d_1, d_2)	(0.00, 1.25)			FSI	(0.00, 1.25)			FSI	(0.00, 1.25)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	6.99	8.62	14.60	136.54	201.67	120.96	105.08	24.76	7.00	8.49	12.05
0.50	2.81	3.02	4.09	33.42	55.80	21.16	17.28	10.39	2.81	3.26	4.62
0.75	1.87	1.83	2.11	10.76	16.04	4.46	3.70	6.61	1.88	2.07	2.91
1.00	1.48	1.37	1.43	4.50	5.14	1.60	1.46	4.90	1.48	1.55	2.16
1.50	1.16	1.07	1.05	1.57	1.31	1.02	1.01	3.31	1.16	1.14	1.40
2.00	1.05	1.01	1.00	1.08	1.02	1.00	1.00	2.54	1.05	1.02	1.07
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.98	1.00	1.00	1.00

Table 5.3.5

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.15			1.34	3.46	1.82	1.34	0.15			
(d_1, d_2)	(0.00, 2.00)			FSI	(0.00, 2.00)			FSI	(0.00, 2.00)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	6.47	7.02	8.12	136.54	190.08	104.26	79.51	24.70	6.50	7.02	8.07
0.50	2.07	2.14	2.29	33.42	49.58	15.75	9.78	9.07	2.18	2.14	2.31
0.75	1.39	1.38	1.39	10.76	13.52	3.13	2.00	5.56	1.48	1.39	1.41
1.00	1.17	1.14	1.12	4.50	4.24	1.31	1.11	4.06	1.23	1.14	1.13
1.50	1.03	1.02	1.01	1.57	1.22	1.01	1.00	2.71	1.06	1.02	1.01
2.00	1.01	1.00	1.00	1.08	1.01	1.00	1.00	2.10	1.01	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.56	1.00	1.00	1.00

Table 5.3.6

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.15			1.34	3.46	1.82	1.34	0.15			
(d_1, d_2)	(0.00, 1.75)			FSI	(0.00, 1.75)			FSI	(0.00, 1.75)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	6.61	7.35	8.97	136.54	193.18	108.40	85.02	24.70	6.62	7.35	8.77
0.50	2.15	2.25	2.50	33.42	51.20	17.01	11.15	9.07	2.23	2.26	2.55
0.75	1.45	1.44	1.48	10.76	14.17	3.42	2.26	5.56	1.51	1.44	1.53
1.00	1.20	1.17	1.16	4.50	4.46	1.37	1.16	4.06	1.25	1.17	1.18
1.50	1.05	1.02	1.01	1.57	1.24	1.01	1.00	2.71	1.06	1.02	1.01
2.00	1.01	1.00	1.00	1.08	1.02	1.00	1.00	2.10	1.02	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.56	1.00	1.00	1.00

Table 5.3.7

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.15			1.34	3.46	1.82	1.34	0.15			
(d_1, d_2)	(0.00, 1.50)			FSI	(0.00, 1.50)			FSI	(0.00, 1.50)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	6.77	7.82	10.53	136.54	196.94	113.72	92.75	24.70	6.77	7.80	9.87
0.50	2.25	2.40	2.83	33.42	53.21	18.70	13.29	9.07	2.29	2.41	2.93
0.75	1.51	1.52	1.61	10.76	14.97	3.82	2.71	5.66	1.55	1.53	1.74
1.00	1.25	1.21	1.21	4.50	4.75	1.45	1.24	4.06	1.27	1.22	1.30
1.50	1.06	1.03	1.02	1.57	1.27	1.01	1.00	2.71	1.07	1.03	1.03
2.00	1.02	1.00	1.00	1.08	1.02	1.00	1.00	2.10	1.02	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.56	1.00	1.00	1.00

Table 5.3.8

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.15			1.34	3.46	1.82	1.34	0.15			
(d_1, d_2)	(0.00, 1.25)			FSI	(0.00, 1.25)			FSI	(0.00, 1.25)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	6.97	8.60	14.57	136.54	201.67	120.96	105.08	24.70	6.97	8.45	12.01
0.50	2.35	2.60	3.62	33.42	55.80	21.16	17.28	9.07	2.36	2.65	3.70
0.75	1.58	1.62	1.90	10.76	16.04	4.46	3.70	5.56	1.59	1.67	2.21
1.00	1.30	1.26	1.34	4.50	5.14	1.60	1.46	4.06	1.30	1.30	1.61
1.50	1.09	1.04	1.03	1.57	1.31	1.02	1.01	2.71	1.09	1.05	1.11
2.00	1.02	1.00	1.00	1.08	1.02	1.00	1.00	2.10	1.03	1.01	1.01
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.56	1.00	1.00	1.00

Table 5.3.9

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.20			1.34	3.46	1.82	1.34	0.20			
(d_1, d_2)	(0.00, 2.00)			FSI	(0.00, 2.00)			FSI	(0.00, 2.00)		
n δ	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	7.69	8.24	9.36	136.54	190.08	104.26	79.51	26.77	7.55	8.15	9.36
0.50	1.79	1.96	2.12	33.42	49.58	15.75	9.78	8.48	2.03	1.98	2.12
0.75	1.25	1.27	1.30	10.76	13.52	3.13	2.00	4.97	1.38	1.29	1.30
1.00	1.07	1.09	1.08	4.50	4.24	1.31	1.11	3.56	1.17	1.10	1.08
1.50	1.01	1.01	1.00	1.57	1.22	1.01	1.00	2.35	1.04	1.01	1.03
2.00	1.00	1.00	1.00	1.08	1.01	1.00	1.00	1.91	1.01	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.15	1.00	1.00	1.00

Table 5.3.10

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.20			1.34	3.46	1.82	1.34	0.20			
(d_1, d_2)	(0.00, 1.75)			FSI	(0.00, 1.75)			FSI	(0.00, 1.75)		
n δ	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	7.83	8.57	10.24	136.54	193.18	108.40	85.02	26.77	7.69	8.54	10.17
0.50	1.96	2.07	2.32	33.42	51.20	17.01	11.15	8.48	2.07	2.08	2.32
0.75	1.31	1.33	1.38	10.76	14.17	3.42	2.26	4.97	1.40	1.34	1.39
1.00	1.12	1.11	1.11	4.50	4.46	1.37	1.16	3.56	1.18	1.12	1.12
1.50	1.02	1.01	1.01	1.57	1.24	1.01	1.00	2.35	1.04	1.01	1.01
2.00	1.00	1.00	1.00	1.08	1.02	1.00	1.00	1.91	1.01	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.15	1.00	1.00	1.00

Table 5.3.11

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.20			1.34	3.46	1.82	1.34	0.20			
(d_1, d_2)	(0.00, 1.50)			FSI	(0.00, 1.50)			FSI	(0.00, 1.50)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	7.99	9.06	11.85	136.54	196.94	113.72	92.75	26.77	7.87	9.06	11.42
0.50	2.05	2.21	2.64	33.42	53.21	18.70	13.29	8.48	2.12	2.21	2.65
0.75	1.37	1.40	1.50	10.76	14.97	3.82	2.71	4.97	1.43	1.40	1.54
1.00	1.15	1.15	1.17	4.50	4.75	1.45	1.24	3.56	1.20	1.15	1.19
1.50	1.03	1.02	1.01	1.57	1.27	1.01	1.00	2.35	1.05	1.02	1.01
2.00	1.01	1.00	1.00	1.08	1.02	1.00	1.00	1.91	1.01	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.15	1.00	1.00	1.00

Table 5.3.12

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.20			1.34	3.46	1.82	1.34	0.20			
(d_1, d_2)	(0.00, 1.25)			FSI	(0.00, 1.25)			FSI	(0.00, 1.25)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	8.18	9.86	16.02	136.54	201.67	120.96	105.08	26.77	8.10	9.81	13.77
0.50	2.15	2.41	2.39	33.42	55.80	21.16	17.28	8.48	2.19	2.41	3.32
0.75	1.43	1.49	1.78	10.76	16.04	4.46	3.70	4.97	1.47	1.50	1.89
1.00	1.20	1.20	1.28	4.50	5.14	1.60	1.46	3.56	1.23	1.20	1.38
1.50	1.05	1.03	1.03	1.57	1.31	1.02	1.01	2.35	1.06	1.03	1.05
2.00	1.01	1.00	1.00	1.08	1.02	1.00	1.00	1.91	1.02	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.15	1.00	1.00	1.00

Table 5.3.13

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.25			1.34	3.46	1.82	1.34	0.25			
(d_1, d_2)	(0.00, 2.00)			FSI	(0.00, 2.00)			FSI	(0.00, 2.00)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	10.03	10.59	11.72	136.54	190.08	104.26	79.51	30.38	9.48	10.18	11.62
0.50	1.82	1.91	2.07	33.42	49.58	15.75	9.78	8.31	1.99	1.94	2.07
0.75	1.19	1.21	1.25	10.76	13.52	3.13	2.00	4.62	1.33	1.25	1.25
1.00	1.05	1.06	1.06	4.50	4.24	1.31	1.11	3.24	1.14	1.08	1.06
1.50	1.00	1.00	1.00	1.57	1.22	1.01	1.00	2.14	1.03	1.01	1.00
2.00	1.00	1.00	1.00	1.08	1.01	1.00	1.00	1.67	1.01	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.04	1.00	1.00	1.00

Table 5.3.14

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.25			1.34	3.46	1.82	1.34	0.25			
(d_1, d_2)	(0.00, 1.75)			FSI	(0.00, 1.75)			FSI	(0.00, 1.75)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	10.16	10.91	12.62	136.54	193.18	108.40	85.02	30.38	9.66	10.66	12.62
0.50	1.90	2.02	2.26	33.42	51.20	17.01	11.15	8.31	2.03	2.03	2.26
0.75	1.23	1.26	1.32	10.76	14.17	3.42	2.26	4.62	1.35	1.25	1.32
1.00	1.07	1.08	1.09	4.50	4.46	1.37	1.16	3.24	1.15	1.09	1.09
1.50	1.01	1.01	1.00	1.57	1.24	1.01	1.00	2.14	1.03	1.01	1.00
2.00	1.00	1.00	1.00	1.08	1.02	1.00	1.00	1.67	1.01	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.04	1.00	1.00	1.00

Table 5.3.15

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.25			1.34	3.46	1.82	1.34	0.25			
(d_1, d_2)	(0.00, 1.50)			FSI	(0.00, 1.50)			FSI	(0.00, 1.50)		
n / δ	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	10.31	11.40	14.29	136.54	196.94	113.72	92.75	30.38	9.89	11.30	14.11
0.50	1.99	2.16	2.58	33.42	53.21	18.70	13.29	8.31	2.07	2.16	2.58
0.75	1.29	1.33	1.44	10.76	14.97	3.82	2.71	4.62	1.37	1.34	1.44
1.00	1.10	1.11	1.14	4.50	4.75	1.45	1.24	3.24	1.16	1.12	1.14
1.50	1.02	1.01	1.01	1.57	1.27	1.01	1.00	2.14	1.04	1.01	1.01
2.00	1.00	1.00	1.00	1.08	1.02	1.00	1.00	1.67	1.01	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.04	1.00	1.00	1.00

Table 5.3.16

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.25			1.34	3.46	1.82	1.34	0.25			
(d_1, d_2)	(0.00, 1.25)			FSI	(0.00, 1.25)			FSI	(0.00, 1.25)		
n / δ	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	10.49	12.23	18.60	136.54	201.67	120.96	105.08	30.38	10.18	12.23	16.85
0.50	2.10	2.35	3.33	33.42	55.80	21.16	17.28	8.31	2.14	2.35	3.22
0.75	1.35	1.42	1.70	10.76	16.04	4.46	3.70	4.62	1.40	1.42	1.73
1.00	1.14	1.16	1.24	4.50	5.14	1.60	1.46	3.24	1.18	1.16	1.28
1.50	1.03	1.02	1.02	1.57	1.31	1.02	1.01	2.14	1.05	1.02	1.03
2.00	1.01	1.00	1.00	1.08	1.02	1.00	1.00	1.67	1.01	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.04	1.00	1.00	1.00

Table 5.3.17

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.30			1.34	3.46	1.82	1.34	0.30			
(d_1, d_2)	(0.00, 2.00)			FSI	(0.00, 2.00)			FSI	(0.00, 2.00)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	13.51	14.11	15.23	136.54	190.08	104.26	79.51	35.22	12.18	12.97	14.69
0.50	1.87	1.96	2.12	33.42	49.58	15.75	9.78	8.45	2.02	1.98	2.11
0.75	1.15	1.18	1.21	10.76	13.52	3.13	2.00	4.43	1.30	1.23	1.23
1.00	1.03	1.04	1.05	4.50	4.24	1.31	1.11	3.03	1.12	1.07	1.05
1.50	1.00	1.00	1.00	1.57	1.22	1.01	1.00	1.97	1.02	1.00	1.00
2.00	1.00	1.00	1.00	1.08	1.01	1.00	1.00	1.46	1.00	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.00

Table 5.3.18

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.30			1.34	3.46	1.82	1.34	0.30			
(d_1, d_2)	(0.00, 1.75)			FSI	(0.00, 1.75)			FSI	(0.00, 1.75)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	13.60	14.38	16.08	136.54	193.18	108.40	85.02	35.22	12.42	13.57	15.92
0.50	1.95	2.07	2.31	33.42	51.20	17.01	11.15	8.45	2.06	2.07	2.31
0.75	1.19	1.23	1.29	10.76	14.17	3.42	2.26	4.43	1.32	1.26	1.29
1.00	1.05	1.06	1.07	4.50	4.46	1.37	1.16	3.03	1.13	1.08	1.07
1.50	1.00	1.00	1.00	1.57	1.24	1.01	1.00	1.97	1.03	1.01	1.00
2.00	1.00	1.00	1.00	1.08	1.02	1.00	1.00	1.46	1.01	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.00

Table 5.3.19

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.30			1.34	3.46	1.82	1.34	0.30			
(d_1, d_2)	(0.00, 1.50)			FSI	(0.00, 1.50)			FSI	(0.00, 1.50)		
n / δ	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	13.71	14.82	17.75	136.54	196.94	113.72	92.75	35.22	12.71	14.38	17.74
0.50	2.05	2.21	2.64	33.42	53.21	18.70	13.29	8.45	2.11	2.20	2.63
0.75	1.24	1.29	1.40	10.76	14.97	3.82	2.71	4.43	1.34	1.30	1.40
1.00	1.07	1.09	1.11	4.50	4.75	1.45	1.24	3.03	1.14	1.10	1.11
1.50	1.01	1.01	1.01	1.57	1.27	1.01	1.00	1.97	1.03	1.01	1.01
2.00	1.00	1.00	1.00	1.08	1.02	1.00	1.00	1.46	1.01	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.00

Table 5.3.20

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.30			1.34	3.46	1.82	1.34	0.30			
(d_1, d_2)	(0.00, 1.25)			FSI	(0.00, 1.25)			FSI	(0.00, 1.25)		
n / δ	1	3	5	5	1	3	5	5	1	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	13.85	15.63	22.12	136.54	201.67	120.96	105.08	35.22	13.08	15.52	20.98
0.50	2.15	2.41	3.39	33.42	55.80	21.16	17.28	8.45	2.18	2.40	3.30
0.75	1.30	1.38	1.65	10.76	16.04	4.46	3.70	4.43	1.37	1.38	1.66
1.00	1.11	1.13	1.22	4.50	5.14	1.60	1.46	3.03	1.16	1.13	1.23
1.50	1.02	1.01	1.02	1.57	1.31	1.02	1.01	1.97	1.04	1.01	1.02
2.00	1.00	1.00	1.00	1.08	1.02	1.00	1.00	1.46	1.01	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.00

Table 5.3.21

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.20, 2.00)		FSI	(0.20, 2.00)		FSI	(0.20, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	12.68	11.02	136.54	125.38	87.56	24.76	12.68	11.03
0.50	5.11	4.07	33.42	27.10	13.74	10.39	5.11	4.17
0.75	3.29	2.51	10.76	8.16	3.62	6.61	3.29	2.58
1.00	2.52	1.89	4.50	3.52	1.77	4.90	2.52	1.94
1.50	1.87	1.46	1.57	1.49	1.11	3.31	1.87	1.47
2.00	1.61	1.30	1.08	1.12	1.02	2.54	1.61	1.31
3.00	1.37	1.20	1.00	1.00	1.00	1.98	1.37	1.20

Table 5.3.22

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.20, 1.75)		FSI	(0.20, 1.75)		FSI	(0.20, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	12.87	11.61	136.54	127.93	91.90	24.76	12.87	11.56
0.50	5.19	4.22	33.42	27.80	14.75	10.39	5.20	4.41
0.75	3.33	2.56	10.76	8.31	3.80	6.61	3.33	2.72
1.00	2.54	1.91	4.50	3.54	1.80	4.90	2.54	2.01
1.50	1.88	1.46	1.57	1.49	1.11	3.31	1.88	1.48
2.00	1.61	1.29	1.08	1.12	1.02	2.54	1.61	1.31
3.00	1.37	1.19	1.00	1.00	1.00	1.98	1.37	1.20

Table 5.3.23

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.20, 1.50)		FSI	(0.20, 1.50)		FSI	(0.20, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	13.11	12.71	136.54	131.26	98.12	24.76	13.12	12.39
0.50	5.29	4.46	33.42	28.77	16.36	10.39	5.32	4.80
0.75	3.38	2.64	10.76	8.52	4.11	6.61	3.40	2.97
1.00	2.56	1.94	4.50	3.58	1.85	4.90	2.58	2.17
1.50	1.88	1.44	1.57	1.49	1.12	3.31	1.89	1.51
2.00	1.61	1.28	1.08	1.12	1.02	2.54	1.61	1.31
3.00	1.37	1.19	1.00	1.00	1.00	1.98	1.37	1.20

Table 5.3.24

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.20, 1.25)		FSI	(0.20, 1.25)		FSI	(0.20, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	13.50	15.69	136.54	135.89	108.35	24.76	13.50	14.02
0.50	5.40	5.03	33.42	30.20	19.46	10.39	5.50	5.53
0.75	3.43	2.81	10.76	8.85	4.83	6.61	3.52	3.48
1.00	2.59	1.98	4.50	3.65	2.00	4.90	2.65	2.56
1.50	1.88	1.42	1.57	1.50	1.12	3.31	1.91	1.68
2.00	1.60	1.25	1.08	1.12	1.02	2.54	1.61	1.34
3.00	1.37	1.17	1.00	1.00	1.00	1.98	1.37	1.20

Table 5.3.25

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.20, 2.00)		FSI	(0.20, 2.00)		FSI	(0.20, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	12.64	11.00	136.54	125.38	87.56	24.70	12.64	10.99
0.50	4.40	3.53	33.42	27.10	13.74	9.07	4.44	3.53
0.75	2.80	2.18	10.76	8.16	3.62	5.56	2.83	2.18
1.00	2.17	1.69	4.50	3.52	1.77	4.06	2.19	1.69
1.50	1.66	1.34	1.57	1.49	1.11	2.71	1.67	1.35
2.00	1.45	1.22	1.08	1.12	1.02	2.10	1.46	1.22
3.00	1.24	1.11	1.00	1.00	1.00	1.56	1.24	1.11

Table 5.3.26

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.20, 1.75)		FSI	(0.20, 1.75)		FSI	(0.20, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	12.83	11.58	136.54	127.93	91.90	24.70	12.83	11.52
0.50	4.48	3.68	33.42	27.80	14.75	9.07	4.50	3.70
0.75	2.84	2.24	10.76	8.31	3.80	5.56	2.86	2.26
1.00	2.19	1.71	4.50	3.54	1.80	4.06	2.20	1.72
1.50	1.67	1.34	1.57	1.49	1.11	2.71	1.67	1.35
2.00	1.46	1.22	1.08	1.12	1.02	2.10	1.46	1.22
3.00	1.24	1.11	1.00	1.00	1.00	1.56	1.24	1.11

Table 5.3.27

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.20, 1.50)		FSI	(0.20, 1.50)		FSI	(0.20, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	13.08	12.68	136.54	131.26	98.12	24.70	13.08	12.34
0.50	4.58	3.91	33.42	28.77	16.36	9.07	4.58	3.99
0.75	2.90	2.32	10.76	8.52	4.11	5.56	2.90	2.41
1.00	2.22	1.74	4.50	3.58	1.85	4.06	2.22	1.79
1.50	1.68	1.34	1.57	1.49	1.12	2.71	1.68	1.36
2.00	1.46	1.21	1.08	1.12	1.02	2.10	1.46	1.22
3.00	1.24	1.10	1.00	1.00	1.00	1.56	1.24	1.11

Table 5.3.28

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.20, 1.25)		FSI	(0.20, 1.25)		FSI	(0.20, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	13.47	15.66	136.54	135.89	108.35	24.70	13.46	13.97
0.50	4.71	4.46	33.42	30.20	19.46	9.07	4.72	4.57
0.75	2.96	2.51	10.76	8.85	4.83	5.56	2.97	2.75
1.00	2.25	1.80	4.50	3.65	2.00	4.06	2.26	2.00
1.50	1.68	1.33	1.57	1.50	1.12	2.71	1.68	1.40
2.00	1.46	1.21	1.08	1.12	1.02	2.10	1.46	1.22
3.00	1.24	1.07	1.00	1.00	1.00	1.56	1.24	1.11

Table 5.3.29

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.20, 2.00)		FSI	(0.20, 2.00)		FSI	(0.20, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	14.19	12.39	136.54	125.38	87.56	26.77	14.01	12.37
0.50	4.07	3.28	33.42	27.10	13.74	8.48	4.16	3.28
0.75	2.52	1.99	10.76	8.16	3.62	4.97	2.59	1.99
1.00	1.97	1.56	4.50	3.52	1.77	3.56	2.01	1.56
1.50	1.53	1.27	1.57	1.49	1.11	2.35	1.55	1.27
2.00	1.35	1.18	1.08	1.12	1.02	1.91	1.37	1.18
3.00	1.20	1.03	1.00	1.00	1.00	1.15	1.20	1.03

Table 5.3.30

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.20, 1.75)		FSI	(0.20, 1.75)		FSI	(0.20, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	14.36	12.99	136.54	127.93	91.90	26.77	14.23	12.98
0.50	4.16	3.42	33.42	27.80	14.75	8.48	4.21	3.42
0.75	2.57	2.05	10.76	8.31	3.80	4.97	2.61	2.05
1.00	1.99	1.58	4.50	3.54	1.80	3.56	2.01	1.58
1.50	1.54	1.27	1.57	1.49	1.11	2.35	1.55	1.27
2.00	1.36	1.18	1.08	1.12	1.02	1.91	1.37	1.18
3.00	1.20	1.03	1.00	1.00	1.00	1.15	1.20	1.03

Table 5.3.31

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.20, 1.50)		FSI	(0.20, 1.50)		FSI	(0.20, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	14.59	14.10	136.54	131.26	98.12	26.77	14.53	13.93
0.50	4.26	3.66	33.42	28.77	16.36	8.48	4.28	3.66
0.75	2.62	2.14	10.76	8.52	4.11	4.97	2.64	2.15
1.00	2.02	1.62	4.50	3.58	1.85	3.56	2.03	1.63
1.50	1.55	1.27	1.57	1.49	1.12	2.35	1.55	1.28
2.00	1.36	1.18	1.08	1.12	1.02	1.91	1.37	1.18
3.00	1.20	1.03	1.00	1.00	1.00	1.15	1.20	1.03

Table 5.3.32

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.20, 1.25)		FSI	(0.20, 1.25)		FSI	(0.20, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	14.98	17.16	136.54	135.89	108.35	26.77	14.97	15.74
0.50	4.39	4.19	33.42	30.20	19.46	8.48	4.39	4.17
0.75	2.69	2.32	10.76	8.85	4.83	4.97	2.69	2.41
1.00	2.05	1.68	4.50	3.65	2.00	3.56	2.05	1.76
1.50	1.56	1.27	1.57	1.50	1.12	2.35	1.56	1.30
2.00	1.37	1.17	1.08	1.12	1.02	1.91	1.37	1.18
3.00	1.20	1.02	1.00	1.00	1.00	1.15	1.20	1.03

Table 5.3.33

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.20, 2.00)		FSI	(0.20, 2.00)		FSI	(0.20, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	17.13	15.05	136.54	125.38	87.56	30.38	16.43	14.80
0.50	3.98	3.20	33.42	27.10	13.74	8.31	4.08	3.21
0.75	2.36	1.87	10.76	8.16	3.62	4.62	2.45	1.89
1.00	1.83	1.48	4.50	3.52	1.77	3.24	1.89	1.49
1.50	1.44	1.23	1.57	1.49	1.11	2.14	1.47	1.23
2.00	1.28	1.13	1.08	1.12	1.02	1.67	1.30	1.14
3.00	1.16	1.01	1.00	1.00	1.00	1.04	1.18	1.01

Table 5.3.34

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.20, 1.75)		FSI	(0.20, 1.75)		FSI	(0.20, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	17.24	15.61	136.54	127.93	91.90	30.38	16.71	15.55
0.50	4.07	3.35	33.42	27.80	14.75	8.31	4.13	3.35
0.75	2.40	1.93	10.76	8.31	3.80	4.62	2.47	1.94
1.00	1.86	1.50	4.50	3.54	1.80	3.24	1.90	1.50
1.50	1.45	1.23	1.57	1.49	1.11	2.14	1.47	1.23
2.00	1.29	1.13	1.08	1.12	1.02	1.67	1.30	1.14
3.00	1.17	1.01	1.00	1.00	1.00	1.04	1.18	1.01

Table 5.3.35

ATS Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.20, 1.50)		FSI	(0.20, 1.50)		FSI	(0.20, 1.50)	
δ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	17.42	16.72	136.54	131.26	98.12	30.38	17.09	16.70
0.50	4.17	3.58	33.42	28.77	16.36	8.31	4.20	3.58
0.75	2.46	2.02	10.76	8.52	4.11	4.62	2.49	2.02
1.00	1.88	1.54	4.50	3.58	1.85	3.24	1.91	1.54
1.50	1.46	1.23	1.57	1.49	1.12	2.14	1.47	1.23
2.00	1.29	1.13	1.08	1.12	1.02	1.67	1.30	1.14
3.00	1.17	1.01	1.00	1.00	1.00	1.04	1.18	1.01

Table 5.3.36

ATS Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.20, 1.25)		FSI	(0.20, 1.25)		FSI	(0.20, 1.25)	
δ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	17.76	19.84	136.54	135.89	108.35	30.38	17.63	18.84
0.50	4.30	4.11	33.42	30.20	19.46	8.31	4.30	4.06
0.75	2.52	2.20	10.76	8.85	4.83	4.62	2.53	2.23
1.00	1.92	1.60	4.50	3.65	2.00	3.24	1.93	1.63
1.50	1.47	1.23	1.57	1.50	1.12	2.14	1.48	1.24
2.00	1.30	1.13	1.08	1.12	1.02	1.67	1.30	1.14
3.00	1.18	1.01	1.00	1.00	1.00	1.04	1.18	1.01

Table 5.3.37

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.20, 2.00)		FSI	(0.20, 2.00)		FSI	(0.20, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	21.54	18.99	136.54	125.38	87.56	35.22	19.72	18.08
0.50	4.07	3.27	33.42	27.10	13.74	8.45	4.14	3.27
0.75	2.26	1.81	10.76	8.16	3.62	4.43	2.37	1.83
1.00	1.74	1.42	4.50	3.52	1.77	3.03	1.82	1.44
1.50	1.38	1.19	1.57	1.49	1.11	1.97	1.41	1.20
2.00	1.24	1.09	1.08	1.12	1.02	1.46	1.26	1.09
3.00	1.10	1.00	1.00	1.00	1.00	1.01	1.13	1.00

Table 5.3.38

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.20, 1.75)		FSI	(0.20, 1.75)		FSI	(0.20, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	21.49	19.44	136.54	127.93	91.90	35.22	20.08	19.01
0.50	4.16	3.42	33.42	27.80	14.75	8.45	4.20	3.41
0.75	2.31	1.86	10.76	8.31	3.80	4.43	2.39	1.88
1.00	1.77	1.44	4.50	3.54	1.80	3.03	1.82	1.45
1.50	1.39	1.19	1.57	1.49	1.11	1.97	1.42	1.20
2.00	1.24	1.09	1.08	1.12	1.02	1.46	1.26	1.09
3.00	1.11	1.00	1.00	1.00	1.00	1.01	1.13	1.00

Table 5.3.39

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.20, 1.50)		FSI	(0.20, 1.50)		FSI	(0.20, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	21.53	20.46	136.54	131.26	98.12	35.22	20.55	20.42
0.50	4.26	3.65	33.42	28.77	16.36	8.45	4.27	3.65
0.75	2.36	1.95	10.76	8.52	4.11	4.43	2.41	1.95
1.00	1.79	1.48	4.50	3.58	1.85	3.03	1.83	1.48
1.50	1.40	1.20	1.57	1.49	1.12	1.97	1.42	1.20
2.00	1.25	1.09	1.08	1.12	1.02	1.46	1.26	1.09
3.00	1.12	1.00	1.00	1.00	1.00	1.01	1.13	1.00

Table 5.3.40

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.20, 1.25)		FSI	(0.20, 1.25)		FSI	(0.20, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	21.73	23.52	136.54	135.89	108.35	35.22	21.24	22.99
0.50	4.38	4.19	33.42	30.20	19.46	8.45	4.38	4.15
0.75	2.43	2.13	10.76	8.85	4.83	4.43	2.45	2.14
1.00	1.83	1.55	4.50	3.65	2.00	3.03	1.85	1.55
1.50	1.41	1.20	1.57	1.50	1.12	1.97	1.42	1.21
2.00	1.25	1.09	1.08	1.12	1.02	1.46	1.26	1.09
3.00	1.13	1.00	1.00	1.00	1.00	1.01	1.13	1.00

Table 5.3.41

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.50, 2.00)		FSI	(0.50, 2.00)		FSI	(0.50, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	21.29	15.70	136.54	165.47	101.72	24.76	21.78	15.71
0.50	8.82	6.30	33.42	46.67	20.24	10.39	9.24	6.30
0.75	5.69	3.98	10.76	16.28	6.16	6.61	5.99	3.99
1.00	4.29	3.00	4.50	6.94	2.77	4.90	4.52	3.00
1.50	3.01	2.15	1.57	2.23	1.28	3.31	3.16	2.16
2.00	2.41	1.77	1.08	1.30	1.04	2.54	2.52	1.77
3.00	1.85	1.49	1.00	1.01	1.00	1.98	1.93	1.49

Table 5.3.42

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.50, 1.75)		FSI	(0.50, 1.75)		FSI	(0.50, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	21.48	15.98	136.54	165.91	104.25	24.76	21.80	15.99
0.50	8.94	6.37	33.42	46.76	20.76	10.39	9.25	6.42
0.75	5.77	4.01	10.76	16.30	6.24	6.61	5.99	4.05
1.00	4.35	3.00	4.50	6.94	2.78	4.90	4.52	3.03
1.50	3.05	2.15	1.57	2.23	1.28	3.31	3.16	2.16
2.00	2.44	1.76	1.08	1.30	1.04	2.54	2.52	1.77
3.00	1.87	1.49	1.00	1.01	1.00	1.98	1.93	1.49

Table 5.3.43

ATS Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.50, 1.50)		FSI	(0.50, 1.50)		FSI	(0.50, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	21.66	16.46	136.54	166.50	108.03	24.76	21.84	16.44
0.50	9.08	6.47	33.42	46.89	21.60	10.39	9.26	6.62
0.75	5.86	4.03	10.76	16.32	6.38	6.61	6.00	4.16
1.00	4.42	2.99	4.50	6.95	2.80	4.90	4.52	3.08
1.50	3.09	2.12	1.57	2.23	1.28	3.31	3.16	2.16
2.00	2.46	1.74	1.08	1.30	1.04	2.54	2.52	1.77
3.00	1.89	1.49	1.00	1.01	1.00	1.98	1.93	1.49

Table 5.3.44

ATS Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.50, 1.25)		FSI	(0.50, 1.25)		FSI	(0.50, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	21.83	17.90	136.54	167.34	114.65	24.76	21.90	17.34
0.50	9.20	6.67	33.42	47.08	23.36	10.39	9.28	7.04
0.75	5.94	4.03	10.76	16.35	6.74	6.61	6.01	4.43
1.00	4.48	2.94	4.50	6.95	2.87	4.90	4.53	3.26
1.50	3.12	2.05	1.57	2.23	1.29	3.31	3.16	2.21
2.00	2.49	1.67	1.08	1.30	1.04	2.54	2.52	1.78
3.00	1.91	1.46	1.00	1.01	1.00	1.98	1.93	1.49

Table 5.3.45

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.50, 2.00)		FSI	(0.50, 2.00)		FSI	(0.50, 2.00)	
n δ	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	21.27	15.67	136.54	165.47	101.72	24.70	21.72	15.66
0.50	7.42	5.45	33.42	46.67	20.24	9.07	8.11	5.47
0.75	4.61	3.38	10.76	16.28	6.16	5.56	5.10	3.39
1.00	3.45	2.55	4.50	6.94	2.77	4.06	3.82	2.55
1.50	2.43	1.85	1.57	2.23	1.28	2.71	2.67	1.85
2.00	1.97	1.55	1.08	1.30	1.04	2.10	2.14	1.55
3.00	1.53	1.27	1.00	1.01	1.00	1.56	1.61	1.28

Table 5.3.46

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.50, 1.75)		FSI	(0.50, 1.75)		FSI	(0.50, 1.75)	
n δ	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	21.45	15.94	136.54	165.91	104.25	24.70	21.75	15.94
0.50	7.60	5.55	33.42	46.76	20.76	9.07	8.12	5.55
0.75	4.72	3.42	10.76	16.30	6.24	5.56	5.10	3.42
1.00	3.53	2.57	4.50	6.94	2.78	4.06	3.82	2.57
1.50	2.48	1.86	1.57	2.23	1.28	2.71	2.67	1.86
2.00	2.01	1.55	1.08	1.30	1.04	2.10	2.14	1.55
3.00	1.54	1.28	1.00	1.01	1.00	1.56	1.61	1.28

Table 5.3.47

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.50, 1.50)		FSI	(0.50, 1.50)		FSI	(0.50, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	21.62	16.43	136.54	166.50	108.03	24.70	21.79	16.39
0.50	7.77	5.67	33.42	46.89	21.60	9.07	8.13	5.69
0.75	4.84	3.47	10.76	16.32	6.38	5.56	5.11	3.48
1.00	3.61	2.58	4.50	6.95	2.80	4.06	3.82	2.59
1.50	2.53	1.85	1.57	2.23	1.28	2.71	2.67	1.86
2.00	2.05	1.55	1.08	1.30	1.04	2.10	2.14	1.55
3.00	1.55	1.27	1.00	1.01	1.00	1.56	1.61	1.28

Table 5.3.48

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.50, 1.25)		FSI	(0.50, 1.25)		FSI	(0.50, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	21.78	17.86	136.54	167.34	114.65	24.70	21.84	17.29
0.50	7.95	5.91	33.42	47.08	23.36	9.07	8.14	6.00
0.75	4.96	3.52	10.76	16.35	6.74	5.56	5.11	3.65
1.00	3.70	2.57	4.50	6.95	2.87	4.06	3.82	2.68
1.50	2.59	1.82	1.57	2.23	1.29	2.71	2.67	1.87
2.00	2.08	1.53	1.08	1.30	1.04	2.10	2.14	1.55
3.00	1.57	1.22	1.00	1.01	1.00	1.56	1.61	1.28

Table 5.3.49

ATS Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.50, 2.00)		FSI	(0.50, 2.00)		FSI	(0.50, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	24.28	17.37	136.54	165.47	101.72	26.77	23.57	17.22
0.50	6.72	5.06	33.42	46.67	20.24	8.48	7.61	5.11
0.75	3.97	3.02	10.76	16.28	6.16	4.97	4.61	3.07
1.00	2.94	2.27	4.50	6.94	2.77	3.56	3.41	2.30
1.50	2.08	1.66	1.57	2.23	1.28	2.35	2.37	1.68
2.00	1.69	1.44	1.08	1.30	1.04	1.91	1.91	1.45
3.00	1.41	1.07	1.00	1.01	1.00	1.15	1.51	1.08

Table 5.3.50

ATS Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.50, 1.75)		FSI	(0.20, 1.75)		FSI	(0.20, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	24.16	17.61	136.54	165.91	104.25	26.77	23.60	17.55
0.50	6.92	5.16	33.42	46.76	20.76	8.48	7.62	5.18
0.75	4.09	3.08	10.76	16.30	6.24	4.97	4.61	3.09
1.00	3.02	2.30	4.50	6.94	2.78	3.56	3.41	2.31
1.50	2.13	1.67	1.57	2.23	1.28	2.35	2.37	1.68
2.00	1.73	1.45	1.08	1.30	1.04	1.91	1.91	1.45
3.00	1.44	1.07	1.00	1.01	1.00	1.15	1.51	1.08

Table 5.3.51

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.50, 1.50)		FSI	(0.50, 1.50)		FSI	(0.50, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	24.06	18.07	136.54	166.50	108.03	26.77	23.64	18.07
0.50	7.14	5.30	33.42	46.89	21.60	8.48	7.62	5.30
0.75	4.22	3.14	10.76	16.32	6.38	4.97	4.61	3.14
1.00	3.11	2.32	4.50	6.95	2.80	3.56	3.41	2.32
1.50	2.18	1.68	1.57	2.23	1.28	2.35	2.37	1.68
2.00	1.77	1.45	1.08	1.30	1.04	1.91	1.91	1.45
3.00	1.46	1.08	1.00	1.01	1.00	1.15	1.51	1.08

Table 5.3.52

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.50, 1.25)		FSI	(0.50, 1.25)		FSI	(0.50, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	24.00	19.48	136.54	167.34	114.65	26.77	23.71	19.10
0.50	7.35	5.55	33.42	47.08	23.36	8.48	7.64	5.56
0.75	4.37	3.22	10.76	16.35	6.74	4.97	4.62	3.25
1.00	3.22	2.34	4.50	6.95	2.87	3.56	3.41	2.37
1.50	2.25	1.67	1.57	2.23	1.29	2.35	2.37	1.68
2.00	1.82	1.44	1.08	1.30	1.04	1.91	1.91	1.45
3.00	1.48	1.07	1.00	1.01	1.00	1.15	1.51	1.08

Table 5.3.53

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.50, 2.00)		FSI	(0.50, 2.00)		FSI	(0.50, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	31.41	20.64	136.54	165.47	101.72	30.38	26.78	19.96
0.50	6.50	4.94	33.42	46.67	20.24	8.31	7.47	5.01
0.75	3.59	2.81	10.76	16.28	6.16	4.62	4.32	2.88
1.00	2.61	2.08	4.50	6.94	2.77	3.24	3.14	2.14
1.50	1.84	1.54	1.57	2.23	1.28	2.14	2.17	1.57
2.00	1.54	1.30	1.08	1.30	1.04	1.67	1.75	1.34
3.00	1.20	1.01	1.00	1.01	1.00	1.04	1.44	1.02

Table 5.3.54

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.50, 1.75)		FSI	(0.50, 1.75)		FSI	(0.50, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	30.28	20.75	136.54	165.91	104.25	30.38	26.82	20.37
0.50	6.71	5.05	33.42	46.76	20.76	8.31	7.47	5.08
0.75	3.70	2.86	10.76	16.30	6.24	4.62	4.32	2.90
1.00	2.69	2.11	4.50	6.94	2.78	3.24	3.14	2.14
1.50	1.89	1.56	1.57	2.23	1.28	2.14	2.17	1.57
2.00	1.57	1.32	1.08	1.30	1.04	1.67	1.75	1.34
3.00	1.24	1.01	1.00	1.01	1.00	1.04	1.44	1.02

Table 5.3.55

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.50, 1.50)		FSI	(0.50, 1.50)		FSI	(0.50, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	29.27	21.09	136.54	166.50	108.03	30.38	26.88	21.00
0.50	6.93	5.19	33.42	46.89	21.60	8.31	7.48	5.19
0.75	3.84	2.93	10.76	16.32	6.38	4.62	4.32	2.94
1.00	2.78	2.15	4.50	6.95	2.80	3.24	3.15	2.15
1.50	1.95	1.57	1.57	2.23	1.28	2.14	2.17	1.57
2.00	1.60	1.33	1.08	1.30	1.04	1.67	1.75	1.34
3.00	1.29	1.02	1.00	1.01	1.00	1.04	1.44	1.02

Table 5.3.56

ATS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.50, 1.25)		FSI	(0.50, 1.25)		FSI	(0.50, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	28.43	22.41	136.54	167.34	114.65	30.38	26.96	22.25
0.50	7.17	5.45	33.42	47.08	23.36	8.31	7.50	5.44
0.75	4.00	3.03	10.76	16.35	6.74	4.62	4.33	3.03
1.00	2.89	2.18	4.50	6.95	2.87	3.24	3.15	2.19
1.50	2.01	1.57	1.57	2.23	1.29	2.14	2.17	1.57
2.00	1.64	1.33	1.08	1.30	1.04	1.67	1.75	1.34
3.00	1.34	1.02	1.00	1.01	1.00	1.04	1.44	1.02

Table 5.3.57

ATS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.50, 2.00)		FSI	(0.50, 2.00)		FSI	(0.50, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	44.39	25.55	136.54	165.47	101.72	35.22	31.08	23.65
0.50	6.73	5.06	33.42	46.67	20.24	8.45	7.59	5.09
0.75	3.37	2.68	10.76	16.28	6.16	4.43	4.16	2.77
1.00	2.40	1.95	4.50	6.94	2.77	3.03	2.97	2.03
1.50	1.69	1.44	1.57	2.23	1.28	1.97	2.03	1.49
2.00	1.41	1.19	1.08	1.30	1.04	1.46	1.65	1.23
3.00	1.08	1.00	1.00	1.01	1.00	1.01	1.33	1.00

Table 5.3.58

ATS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.50, 1.75)		FSI	(0.50, 1.75)		FSI	(0.50, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	41.13	25.38	136.54	165.91	104.25	35.22	31.14	24.16
0.50	6.93	5.16	33.42	46.76	20.76	8.45	7.60	5.16
0.75	3.48	2.74	10.76	16.30	6.24	4.43	4.16	2.79
1.00	2.48	1.99	4.50	6.94	2.78	3.03	2.97	2.03
1.50	1.73	1.46	1.57	2.23	1.28	1.97	2.03	1.49
2.00	1.45	1.20	1.08	1.30	1.04	1.46	1.65	1.23
3.00	1.10	1.00	1.00	1.01	1.00	1.01	1.33	1.00

Table 5.3.59

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.50, 1.50)		FSI	(0.50, 1.50)		FSI	(0.50, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	38.14	25.45	136.54	166.50	108.03	35.22	31.21	24.96
0.50	7.15	5.29	33.42	46.89	21.60	8.45	7.60	5.28
0.75	3.61	2.81	10.76	16.32	6.38	4.43	4.16	2.83
1.00	2.56	2.03	4.50	6.95	2.80	3.03	2.97	2.04
1.50	1.78	1.48	1.57	2.23	1.28	1.97	2.03	1.49
2.00	1.48	1.22	1.08	1.30	1.04	1.46	1.65	1.23
3.00	1.13	1.00	1.00	1.01	1.00	1.01	1.33	1.00

Table 5.3.60

ATS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.50, 1.25)		FSI	(0.50, 1.25)		FSI	(0.50, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	740.80	740.80	740.80	740.80	740.80	740.80	740.80	740.80
0.25	35.55	26.49	136.54	167.34	114.65	35.22	31.32	26.48
0.50	7.36	5.54	33.42	47.08	23.36	8.45	7.62	5.54
0.75	3.78	2.91	10.76	16.35	6.74	4.43	4.17	2.91
1.00	2.67	2.07	4.50	6.95	2.87	3.03	2.97	2.07
1.50	1.84	1.49	1.57	2.23	1.29	1.97	2.03	1.49
2.00	1.53	1.23	1.08	1.30	1.04	1.46	1.65	1.23
3.00	1.18	1.00	1.00	1.01	1.00	1.01	1.33	1.00

Table 5.4.1

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.10			1.34	3.46	1.82	1.34	0.10			
(d_1, d_2)	(0.00, 2.00)			FSI	(0.00, 2.00)			FSI	(0.00, 2.00)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	123.64	123.70	123.93	682.72	1507.33	917.13	682.72	123.81	123.66	123.72	123.81
0.50	51.56	51.67	51.35	167.11	650.34	269.81	167.11	51.94	51.60	51.74	51.94
0.75	32.48	32.68	32.47	53.81	297.35	93.98	53.81	33.07	32.53	32.75	33.07
1.00	23.72	24.01	23.97	22.48	143.99	38.58	22.48	24.49	23.76	24.07	24.49
1.50	15.45	15.89	16.17	7.83	40.01	10.36	7.83	16.53	15.48	15.93	16.53
2.00	11.50	12.06	12.37	5.38	13.86	4.82	5.38	12.72	11.53	12.10	12.72
3.00	7.68	8.56	9.86	5.00	3.10	3.10	5.00	9.92	7.70	8.58	9.92

Table 5.4.2

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.10			1.34	3.46	1.82	1.34	0.10			
(d_1, d_2)	(0.00, 1.75)			FSI	(0.00, 1.75)			FSI	(0.00, 1.75)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	123.65	123.70	124.42	682.72	1507.33	917.13	682.72	123.81	123.66	123.72	123.81
0.50	51.58	51.57	50.99	167.11	650.34	269.81	167.11	51.94	51.60	51.74	51.94
0.75	32.51	32.58	32.06	53.81	297.35	93.98	53.81	33.07	32.53	32.75	33.07
1.00	23.75	23.92	23.60	22.48	143.99	38.58	22.48	24.49	23.76	24.07	24.49
1.50	15.47	15.82	15.90	7.83	40.01	10.36	7.83	16.53	15.48	15.93	16.53
2.00	11.52	12.01	12.12	5.38	1 .86	4.82	5.38	12.72	11.53	12.10	12.72
3.00	7.69	8.52	9.79	5.00	3.10	3.10	5.00	9.92	7.70	8.58	9.92

Table 5.4.3

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.10			1.34	3.46	1.82	1.34	0.10			
(d_1, d_2)	(0.00, 1.50)			FSI	(0.00, 1.50)			FSI	(0.00, 1.50)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	123.66	123.77	126.18	682.72	1507.33	917.13	682.72	123.81	123.66	123.72	123.81
0.50	51.60	51.41	50.49	167.11	650.34	269.81	167.11	51.94	51.60	51.74	51.94
0.75	32.52	32.40	31.36	53.81	297.35	93.98	53.81	33.07	32.53	32.75	33.07
1.00	23.76	23.75	22.93	22.48	143.99	38.58	22.48	24.49	23.76	24.07	24.49
1.50	15.48	15.68	15.41	7.83	40.01	10.36	7.83	16.53	15.48	15.93	16.53
2.00	11.53	11.90	11.68	5.38	13.86	4.82	5.38	12.72	11.53	12.10	12.72
3.00	7.70	8.45	9.60	5.00	3.10	3.10	5.00	9.92	7.70	8.58	9.92

Table 5.4.4

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.10			1.34	3.46	1.82	1.34	0.10			
(d_1, d_2)	(0.00, 1.25)			FSI	(0.00, 1.25)			FSI	(0.00, 1.25)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	123.66	124.11	134.45	682.72	1507.33	917.13	682.72	123.81	123.66	123.72	123.81
0.50	51.59	51.13	50.15	167.11	650.34	269.81	167.11	51.94	51.60	51.74	51.94
0.75	32.52	32.07	30.05	53.81	297.35	93.98	53.81	33.07	32.53	32.75	33.07
1.00	23.76	23.43	21.56	22.48	143.99	38.58	22.48	24.49	23.76	24.07	24.49
1.50	15.48	15.43	14.29	7.83	40.01	10.36	7.83	16.53	15.48	15.93	16.53
2.00	11.52	11.07	10.87	5.38	13.86	4.82	5.38	12.72	11.53	12.10	12.72
3.00	7.70	8.31	8.78	5.00	3.10	3.10	5.00	9.92	7.70	8.58	9.92

Table 5.4.5

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.15			1.34	3.46	1.82	1.34	0.15			
(d_1, d_2)	(0.00, 2.00)			FSI	(0.00, 2.00)			FSI	(0.00, 2.00)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	123.36	123.44	123.68	682.72	1507.33	917.13	682.72	123.52	123.37	123.43	123.52
0.50	44.85	45.14	45.25	167.11	650.34	269.81	167.11	45.33	45.02	45.15	45.33
0.75	27.10	27.49	27.69	53.81	297.35	93.98	53.81	27.79	27.29	27.50	27.79
1.00	19.41	19.87	20.19	22.48	143.99	38.58	22.48	20.28	19.58	19.87	20.28
1.50	12.41	12.99	13.47	7.83	40.01	10.36	7.83	13.53	12.56	12.99	13.53
2.00	9.17	9.84	10.47	5.38	13.86	4.82	5.38	10.50	9.30	9.85	10.50
3.00	6.10	6.64	7.71	5.00	3.10	3.10	5.00	7.79	6.18	6.64	7.79

Table 5.4.6

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.15			1.34	3.46	1.82	1.34	0.15			
(d_1, d_2)	(0.00, 1.75)			FSI	(0.00, 1.75)			FSI	(0.00, 1.75)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	123.37	123.43	124.18	682.72	1507.33	917.13	682.72	123.52	123.37	123.43	123.52
0.50	44.91	45.15	45.12	167.11	650.34	269.81	167.11	45.33	45.02	45.15	45.33
0.75	27.17	27.49	27.52	53.81	297.35	93.98	53.81	27.79	27.29	27.50	27.79
1.00	19.47	19.87	20.03	22.48	143.99	38.58	22.48	20.28	19.58	19.87	20.28
1.50	12.46	12.99	13.43	7.83	40.01	10.36	7.83	13.53	12.56	12.99	13.53
2.00	9.21	9.85	10.41	5.38	13.86	4.82	5.38	10.50	9.30	9.85	10.50
3.00	6.13	6.64	7.57	5.00	3.10	3.10	5.00	7.79	6.18	6.64	7.79

Table 5.4.7

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.15			1.34	3.46	1.82	1.34	0.15			
(d_1, d_2)	(0.00, 1.50)			FSI	(0.00, 1.50)			FSI	(0.00, 1.50)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	123.38	123.50	125.94	682.72	1507.33	917.13	682.72	123.52	123.37	123.43	123.52
0.50	44.96	45.11	44.96	167.11	650.34	269.81	167.11	45.33	45.02	45.15	45.33
0.75	27.23	27.45	27.17	53.81	297.35	93.98	53.81	27.79	27.29	27.50	27.79
1.00	19.52	19.83	19.68	22.48	143.99	38.58	22.48	20.28	19.58	19.87	20.28
1.50	12.51	12.96	13.07	7.83	40.01	10.36	7.83	13.53	12.56	12.99	13.53
2.00	9.24	9.82	10.29	5.38	13.86	4.82	5.38	10.50	9.30	9.85	10.50
3.00	6.15	6.62	7.25	5.00	3.10	3.10	5.00	7.79	6.18	6.64	7.79

Table 5.4.8

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.15			1.34	3.46	1.82	1.34	0.15			
(d_1, d_2)	(0.00, 1.25)			FSI	(0.00, 1.25)			FSI	(0.00, 1.25)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	123.38	123.85	134.22	682.72	1507.33	917.13	682.72	123.52	123.37	123.43	123.52
0.50	45.00	45.01	45.17	167.11	650.34	269.81	167.11	45.33	45.02	45.15	45.33
0.75	27.27	27.31	26.49	53.81	297.35	93.98	53.81	27.79	27.29	27.50	27.79
1.00	19.57	19.69	18.89	22.48	143.99	38.58	22.48	20.28	19.58	19.87	20.28
1.50	12.54	12.85	12.40	7.83	40.01	10.36	7.83	13.53	12.56	12.99	13.53
2.00	9.27	9.74	9.99	5.38	13.86	4.82	5.38	10.50	9.30	9.85	10.50
3.00	6.17	6.57	6.48	5.00	3.10	3.10	5.00	7.79	6.18	6.64	7.79

Table 5.4.9

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.20			1.34	3.46	1.82	1.34	0.20			
(d_1, d_2)	(0.00, 2.00)			FSI	(0.00, 2.00)			FSI	(0.00, 2.00)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	134.00	133.98	133.87	682.72	1507.33	917.13	682.72	133.87	133.65	133.74	133.87
0.50	41.79	42.13	42.38	167.11	650.34	269.81	167.11	42.38	42.09	42.21	42.38
0.75	24.01	24.46	24.84	53.81	297.35	93.98	53.81	24.84	24.37	24.56	24.84
1.00	16.80	17.32	17.80	22.48	143.99	38.58	22.48	17.80	17.14	17.42	17.80
1.50	10.52	11.16	11.76	7.83	40.01	10.36	7.83	11.76	10.80	11.23	11.76
2.00	7.71	8.42	9.54	5.38	13.86	4.82	5.38	9.54	7.93	8.47	9.54
3.00	5.10	6.05	5.75	5.00	3.10	3.10	5.00	5.75	5.25	6.06	5.75

Table 5.4.10

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.20			1.34	3.46	1.82	1.34	0.20			
(d_1, d_2)	(0.00, 1.75)			FSI	(0.00, 1.75)			FSI	(0.00, 1.75)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	133.93	133.83	134.11	682.72	1507.33	917.13	682.72	133.87	133.65	133.74	133.87
0.50	41.88	42.19	42.36	167.11	650.34	269.81	167.11	42.38	42.09	42.21	42.38
0.75	24.12	24.53	24.80	53.81	297.35	93.98	53.81	24.84	24.37	24.56	24.84
1.00	16.90	17.39	17.76	22.48	143.99	38.58	22.48	17.80	17.14	17.42	17.80
1.50	10.59	11.20	11.73	7.83	40.01	10.36	7.83	11.76	10.80	11.23	11.76
2.00	7.76	8.46	9.52	5.38	13.86	4.82	5.38	9.54	7.93	8.47	9.54
3.00	5.14	6.05	5.73	5.00	3.10	3.10	5.00	5.75	5.25	6.06	5.75

Table 5.4.11

ANOS Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.20			1.34	3.46	1.82	1.34	0.20			
(d_1, d_2)	(0.00, 1.50)			FSI	(0.00, 1.50)			FSI	(0.00, 1.50)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	133.86	133.74	135.55	682.72	1507.33	917.13	682.72	133.87	133.65	133.74	133.87
0.50	41.96	42.21	42.34	167.11	650.34	269.81	167.11	42.38	42.09	42.21	42.38
0.75	24.21	24.56	24.64	53.81	297.35	93.98	53.81	24.84	24.37	24.56	24.84
1.00	16.98	17.42	17.60	22.48	143.99	38.58	22.48	17.80	17.14	17.42	17.80
1.50	10.66	11.23	11.62	7.83	40.01	10.36	7.83	11.76	10.80	11.23	11.76
2.00	7.82	8.47	9.43	5.38	13.86	4.82	5.38	9.54	7.93	8.47	9.54
3.00	5.18	6.06	5.65	5.00	3.10	3.10	5.00	5.75	5.25	6.06	5.75

Table 5.4.12

ANOS Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.20			1.34	3.46	1.82	1.34	0.20			
(d_1, d_2)	(0.00, 1.25)			FSI	(0.00, 1.25)			FSI	(0.00, 1.25)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	133.78	133.90	143.43	682.72	1507.33	917.13	682.72	133.87	133.65	133.74	133.87
0.50	42.03	42.19	42.77	167.11	650.34	269.81	167.11	42.38	42.09	42.21	42.38
0.75	24.29	24.53	24.30	53.81	297.35	93.98	53.81	24.84	24.37	24.56	24.84
1.00	17.06	17.39	17.13	22.48	143.99	38.58	22.48	17.80	17.14	17.42	17.80
1.50	10.73	11.20	11.26	7.83	40.01	10.36	7.83	12.76	10.80	11.23	11.76
2.00	7.87	8.45	9.11	5.38	13.86	4.82	5.38	9.54	7.93	8.47	9.54
3.00	5.21	6.05	5.43	5.00	3.10	3.10	5.00	5.75	5.25	6.06	5.75

Table 5.4.13

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.25			1.34	3.46	1.82	1.34	0.25			
(d_1, d_2)	(0.00, 2.00)			FSI	(0.00, 2.00)			FSI	(0.00, 2.00)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	153.14	152.98	152.30	682.72	1507.33	917.13	682.72	151.92	151.48	151.66	151.92
0.50	40.90	41.25	41.54	167.11	650.34	269.81	167.11	41.55	41.27	41.39	41.55
0.75	22.13	22.60	23.06	53.81	297.35	93.98	53.81	23.11	22.66	22.84	23.11
1.00	15.06	15.62	16.17	22.48	143.99	38.58	22.48	16.21	15.58	15.85	16.21
1.50	9.29	9.87	10.67	7.83	40.01	10.36	7.83	10.70	9.63	10.04	10.70
2.00	6.69	7.38	8.34	5.38	13.86	4.82	5.38	8.37	7.01	7.50	8.37
3.00	4.41	5.58	5.17	5.00	3.10	3.10	5.00	5.18	4.62	5.66	5.18

Table 5.4.14

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.25			1.34	3.46	1.82	1.34	0.25			
(d_1, d_2)	(0.00, 1.75)			FSI	(0.00, 1.75)			FSI	(0.00, 1.75)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	152.76	152.46	151.92	682.72	1507.33	917.13	682.72	151.92	151.48	151.66	151.92
0.50	41.01	41.33	41.55	167.11	650.34	269.81	167.11	41.55	41.27	41.39	41.55
0.75	22.27	22.72	23.11	53.81	297.35	93.98	53.81	23.11	22.66	22.84	23.11
1.00	15.19	15.73	16.21	22.48	143.99	38.58	22.48	16.21	15.58	15.85	16.21
1.50	9.31	9.95	10.69	7.83	40.01	10.36	7.83	10.70	9.63	10.04	10.70
2.00	6.76	7.43	8.37	5.38	13.86	4.82	5.38	8.37	7.01	7.50	8.37
3.00	4.45	5.62	5.18	5.00	3.10	3.10	5.00	5.18	4.62	5.66	5.18

Table 5.4.15

ANOS Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.25			1.34	3.46	1.82	1.34	0.25			
(d_1, d_2)	(0.00, 1.50)			FSI	(0.00, 1.50)			FSI	(0.00, 1.50)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	152.42	151.96	152.63	682.72	1507.33	917.13	682.72	151.92	151.48	151.66	151.92
0.50	41.01	41.37	41.57	167.11	650.34	269.81	167.11	41.55	41.27	41.39	41.55
0.75	22.39	22.81	23.07	53.81	297.35	93.98	53.81	23.11	22.66	22.84	23.11
1.00	15.19	15.81	16.17	22.48	143.99	38.58	22.48	16.21	15.58	15.85	16.21
1.50	9.31	10.01	10.66	7.83	40.01	10.36	7.83	10.70	9.63	10.04	10.70
2.00	6.76	7.48	8.33	5.38	13.86	4.82	5.38	8.37	7.01	7.50	8.37
3.00	4.45	5.65	5.17	5.00	3.10	3.10	5.00	5.18	4.62	5.66	5.18

Table 5.4.16

ANOS Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart			$\overline{X}$ Chart				CUSUM Chart			
γ	0.25			1.34	3.46	1.82	1.34	0.25			
(d_1, d_2)	(0.00, 1.25)			FSI	(0.00, 1.25)			FSI	(0.00, 1.25)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	152.10	151.66	159.51	682.72	1507.33	917.13	682.72	151.92	151.48	151.66	151.92
0.50	41.18	41.39	42.07	167.11	650.34	269.81	167.11	41.55	41.27	41.39	41.55
0.75	22.50	22.84	22.91	53.81	297.35	93.98	53.81	23.11	22.66	22.84	23.11
1.00	15.42	15.85	15.91	22.48	143.99	38.58	22.48	16.21	15.58	15.85	16.21
1.50	9.49	10.04	10.45	7.83	40.01	10.36	7.83	10.70	9.63	10.04	10.70
2.00	6.89	7.50	8.08	5.38	13.86	4.82	5.38	8.37	7.01	7.50	8.37
3.00	4.54	5.66	5.13	5.00	3.10	3.10	5.00	5.18	4.62	5.66	5.18

Table 5.4.17

ANOS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.30			1.34	3.46	1.82	1.34	0.30			
(d_1, d_2)	(0.00, 2.00)			FSI	(0.00, 2.00)			FSI	(0.00, 2.00)		
n δ	1	3	5	5	1	3	5	5	1	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	180.09	179.72	178.35	682.72	1507.33	917.13	682.72	176.11	175.21	175.59	176.11
0.50	41.72	42.06	42.32	167.11	650.34	269.81	167.11	42.26	41.97	42.09	42.26
0.75	21.01	21.50	22.01	53.81	297.35	93.98	53.81	22.13	21.70	21.88	22.13
1.00	13.86	14.43	15.04	22.48	143.99	38.58	22.48	15.17	14.55	14.80	15.17
1.50	8.26	8.92	9.76	7.83	40.01	10.36	7.83	9.85	8.80	9.19	9.85
2.00	5.94	6.70	7.22	5.38	13.86	4.82	5.38	7.32	6.35	6.87	7.32
3.00	3.89	4.78	5.04	5.00	3.10	3.10	5.00	5.05	4.16	5.01	5.05

Table 5.4.18

ANOS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.30			1.34	3.46	1.82	1.34	0.30			
(d_1, d_2)	(0.00, 1.75)			FSI	(0.00, 1.75)			FSI	(0.00, 1.75)		
n δ	1	3	5	5	1	3	5	5	1	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	178.94	178.40	176.81	682.72	1507.33	917.13	682.72	176.11	175.21	175.59	176.11
0.50	41.80	42.11	42.29	167.11	650.34	269.81	167.11	42.26	41.97	42.09	42.26
0.75	21.18	21.65	22.10	53.81	297.35	93.98	53.81	22.13	21.70	21.88	22.13
1.00	14.01	14.57	15.13	22.48	143.99	38.58	22.48	15.17	14.55	14.80	15.17
1.50	8.36	9.01	9.83	7.83	40.01	10.36	7.83	9.85	8.80	9.19	9.85
2.00	6.01	6.76	7.29	5.38	13.86	4.82	5.38	7.32	6.35	6.87	7.32
3.00	3.94	4.86	5.05	5.00	3.10	3.10	5.00	5.05	4.16	5.01	5.05

Table 5.4.19

ANOS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.30			1.34	3.46	1.82	1.34	0.30			
(d_1, d_2)	(0.00, 1.50)			FSI	(0.00, 1.50)			FSI	(0.00, 1.50)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	177.94	177.10	176.15	682.72	1507.33	917.13	682.72	176.11	175.21	175.59	176.11
0.50	41.87	42.13	42.27	167.11	650.34	269.81	167.11	42.26	41.97	42.09	42.26
0.75	21.34	21.78	22.13	53.81	297.35	93.98	53.81	22.13	21.70	21.88	22.13
1.00	14.16	14.69	15.17	22.48	143.99	38.58	22.48	15.17	14.55	14.80	15.17
1.50	8.47	9.11	9.85	7.83	40.01	10.36	7.83	9.85	8.80	9.19	9.85
2.00	6.09	6.82	7.32	5.38	13.86	4.82	5.38	7.32	6.35	6.87	7.32
3.00	3.99	4.94	5.05	5.00	3.10	3.10	5.00	5.05	4.16	5.01	5.05

Table 5.4.20

ANOS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart			$\bar{X}$ Chart				CUSUM Chart			
γ	0.30			1.34	3.46	1.82	1.34	0.30			
(d_1, d_2)	(0.00, 1.25)			FSI	(0.00, 1.25)			FSI	(0.00, 1.25)		
δ \ n	1	3	5	5	1	3	5	5	1	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	177.07	175.95	181.21	682.72	1507.33	917.13	682.72	176.11	175.21	175.59	176.11
0.50	41.93	42.11	42.70	167.11	650.34	269.81	167.11	42.26	41.97	42.09	42.26
0.75	21.48	21.86	22.09	53.81	297.35	93.98	53.81	22.13	21.70	21.88	22.13
1.00	14.30	14.78	15.05	22.48	143.99	38.58	22.48	15.17	14.55	14.80	15.17
1.50	8.58	9.18	9.73	7.83	40.01	10.36	7.83	9.85	8.80	9.19	9.85
2.00	6.18	6.86	7.19	5.38	13.86	4.82	5.38	7.32	6.35	6.87	7.32
3.00	4.04	4.99	5.04	5.00	3.10	3.10	5.00	5.05	4.16	5.01	5.05

Table 5.4.21

ANOS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.20, 2.00)		FSI	(0.20, 2.00)		FSI	(0.20, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	123.72	123.79	682.72	917.13	682.72	123.81	123.72	123.81
0.50	51.74	51.59	167.11	269.81	167.11	51.94	51.74	51.94
0.75	32.75	32.73	53.81	93.98	53.81	33.07	32.75	33.07
1.00	24.07	24.20	22.48	38.58	22.48	24.49	24.07	24.49
1.50	15.93	16.33	7.83	10.36	7.83	16.53	15.93	16.53
2.00	12.09	12.52	5.38	4.82	5.38	12.72	12.10	12.72
3.00	8.58	9.89	5.00	3.10	5.00	9.92	8.58	9.92

Table 5.4.22

ANOS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.20, 1.75)		FSI	(0.20, 1.75)		FSI	(0.20, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	123.72	124.00	682.72	917.13	682.72	123.81	123.72	123.81
0.50	51.73	51.28	167.11	269.81	167.11	51.94	51.74	51.94
0.75	32.74	32.39	53.81	93.98	53.81	33.07	32.75	33.07
1.00	24.06	23.89	22.48	38.58	22.48	24.49	24.07	24.49
1.50	15.93	16.12	7.83	10.36	7.83	16.53	15.93	16.53
2.00	12.09	12.32	5.38	4.82	5.38	12.72	12.10	12.72
3.00	8.58	9.85	5.00	3.10	5.00	9.92	8.58	9.92

Table 5.4.23

ANOS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.20, 1.50)		FSI	(0.20, 1.50)		FSI	(0.20, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	123.70	125.02	682.72	917.13	682.72	123.81	123.72	123.81
0.50	51.66	50.76	167.11	269.81	167.11	51.94	51.74	51.94
0.75	32.67	31.76	53.81	93.98	53.81	33.07	32.75	33.07
1.00	23.99	23.31	22.48	38.58	22.48	24.49	24.07	24.49
1.50	15.88	15.70	7.83	10.36	7.83	16.53	15.93	16.53
2.00	12.05	11.93	5.38	4.82	5.38	12.72	12.10	12.72
3.00	8.55	9.72	5.00	3.10	5.00	9.92	8.58	9.92

Table 5.4.24

ANOS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.20, 1.25)		FSI	(0.20, 1.25)		FSI	(0.20, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	123.73	130.82	682.72	917.13	682.72	123.81	123.72	123.81
0.50	51.47	50.14	167.11	269.81	167.11	51.94	51.74	51.94
0.75	32.47	30.46	53.81	93.98	53.81	33.07	32.75	33.07
1.00	23.81	22.02	22.48	38.58	22.48	24.49	24.07	24.49
1.50	15.73	14.68	7.83	10.36	7.83	16.53	15.93	16.53
2.00	11.94	11.12	5.38	4.82	5.38	12.72	12.10	12.72
3.00	8.48	9.13	5.00	3.10	5.00	9.92	8.58	9.92

Table 5.4.25

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.20, 2.00)		FSI	(0.20, 2.00)		FSI	(0.20, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	123.46	123.54	682.72	917.13	682.72	123.52	123.43	123.52
0.50	45.00	45.31	167.11	269.81	167.11	45.33	45.15	45.33
0.75	27.34	27.77	53.81	93.98	53.81	27.79	27.50	27.79
1.00	19.74	20.26	22.48	38.58	22.48	20.28	19.87	20.28
1.50	12.90	13.52	7.83	10.36	7.83	13.53	12.99	13.53
2.00	9.78	10.49	5.38	4.82	5.38	10.50	9.85	10.50
3.00	6.60	7.78	5.00	3.10	5.00	7.79	6.64	7.79

Table 5.4.26

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.20, 1.75)		FSI	(0.20, 1.75)		FSI	(0.20, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	123.45	123.75	682.72	917.13	682.72	123.52	123.43	123.52
0.50	45.09	45.22	167.11	269.81	167.11	45.33	45.15	45.33
0.75	27.44	27.66	53.81	93.98	53.81	27.79	27.50	27.79
1.00	19.82	20.16	22.48	38.58	22.48	20.28	19.87	20.28
1.50	12.96	13.45	7.83	10.36	7.83	13.53	12.99	13.53
2.00	9.82	10.46	5.38	4.82	5.38	10.50	9.85	10.50
3.00	6.62	7.69	5.00	3.10	5.00	7.79	6.64	7.79

Table 5.4.27

ANOS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.20, 1.50)		FSI	(0.20, 1.50)		FSI	(0.20, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	123.43	124.78	682.72	917.13	682.72	123.52	123.43	123.52
0.50	45.14	45.04	167.11	269.81	167.11	45.33	45.15	45.33
0.75	27.49	27.38	53.81	93.98	53.81	27.79	27.50	27.79
1.00	19.87	19.89	22.48	38.58	22.48	20.28	19.87	20.28
1.50	12.99	13.23	7.83	10.36	7.83	13.53	12.99	13.53
2.00	9.85	10.36	5.38	4.82	5.38	10.50	9.85	10.50
3.00	6.64	7.44	5.00	3.10	5.00	7.79	6.64	7.79

Table 5.4.28

ANOS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.20, 1.25)		FSI	(0.20, 1.25)		FSI	(0.20, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	123.47	130.59	682.72	917.13	682.72	123.52	123.43	123.52
0.50	45.12	44.99	167.11	269.81	167.11	45.33	45.15	45.33
0.75	27.47	26.71	53.81	93.98	53.81	27.79	27.50	27.79
1.00	19.85	19.17	22.48	38.58	22.48	20.28	19.87	20.28
1.50	12.97	12.63	7.83	10.36	7.83	13.53	12.99	13.53
2.00	9.83	10.10	5.38	4.82	5.38	10.50	9.85	10.50
3.00	6.63	6.74	5.00	3.10	5.00	7.79	6.64	7.79

Table 5.4.29

ANOS Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.20, 2.00)		FSI	(0.20, 2.00)		FSI	(0.20, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	134.37	133.95	682.72	917.13	682.72	133.87	133.74	133.87
0.50	41.86	42.36	167.11	269.81	167.11	42.38	42.21	42.38
0.75	24.17	24.82	53.81	93.98	53.81	24.84	24.56	24.84
1.00	17.07	17.78	22.48	38.58	22.48	17.80	17.42	17.80
1.50	10.98	11.74	7.83	10.36	7.83	11.76	11.23	11.76
2.00	8.28	9.53	5.38	4.82	5.38	9.54	8.47	9.54
3.00	6.01	5.74	5.00	3.10	5.00	5.75	6.06	5.75

Table 5.4.30

ANOS Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.20, 1.75)		FSI	(0.20, 1.75)		FSI	(0.20, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	134.17	133.88	682.72	917.13	682.72	133.87	133.74	133.87
0.50	42.02	42.38	167.11	269.81	167.11	42.38	42.21	42.38
0.75	24.34	24.84	53.81	93.98	53.81	24.84	24.56	24.84
1.00	17.21	17.80	22.48	38.58	22.48	17.80	17.42	17.80
1.50	11.08	11.76	7.83	10.36	7.83	11.76	11.23	11.76
2.00	8.36	9.54	5.38	4.82	5.38	9.54	8.47	9.54
3.00	6.03	5.75	5.00	3.10	5.00	5.75	6.06	5.75

Table 5.4.31

ANOS Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.20, 1.50)		FSI	(0.20, 1.50)		FSI	(0.20, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	133.95	134.56	682.72	917.13	682.72	133.87	133.74	133.87
0.50	42.14	42.34	167.11	269.81	167.11	42.38	42.21	42.38
0.75	24.48	24.74	53.81	93.98	53.81	24.84	24.56	24.84
1.00	17.34	17.70	22.48	38.58	22.48	17.80	17.42	17.80
1.50	11.17	11.69	7.83	10.36	7.83	11.76	11.23	11.76
2.00	8.43	9.49	5.38	4.82	5.38	9.54	8.47	9.54
3.00	6.05	5.70	5.00	3.10	5.00	5.75	6.06	5.75

Table 5.4.32

ANOS Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.20, 1.25)		FSI	(0.20, 1.25)		FSI	(0.20, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	133.76	139.92	682.72	917.13	682.72	133.87	133.74	133.87
0.50	42.21	42.52	167.11	269.81	167.11	42.38	42.21	42.38
0.75	24.56	24.40	53.81	93.98	53.81	24.84	24.56	24.84
1.00	17.41	17.29	22.48	38.58	22.48	17.80	17.42	17.80
1.50	11.22	11.39	7.83	10.36	7.83	11.76	11.23	11.76
2.00	8.47	9.24	5.38	4.82	5.38	9.54	8.47	9.54
3.00	6.06	5.51	5.00	3.10	5.00	5.75	6.06	5.75

Table 5.4.33

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.20, 2.00)		FSI	(0.20, 2.00)		FSI	(0.20, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	154.48	152.94	682.72	917.13	682.72	151.92	151.66	151.92
0.50	40.93	41.49	167.11	269.81	167.11	41.55	41.39	41.55
0.75	22.20	22.97	53.81	93.98	53.81	23.11	22.84	23.11
1.00	15.27	16.09	22.48	38.58	22.48	16.21	15.85	16.21
1.50	9.63	10.62	7.83	10.36	7.83	10.70	10.04	10.70
2.00	7.21	8.28	5.38	4.82	5.38	8.37	7.50	8.37
3.00	5.45	5.16	5.00	3.10	5.00	5.18	5.66	5.18

Table 5.4.34

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.20, 1.75)		FSI	(0.20, 1.75)		FSI	(0.20, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	153.67	152.17	682.72	917.13	682.72	151.92	151.66	151.92
0.50	41.12	41.54	167.11	269.81	167.11	41.55	41.39	41.55
0.75	22.42	23.08	53.81	93.98	53.81	23.11	22.84	23.11
1.00	15.46	16.19	22.48	38.58	22.48	16.21	15.85	16.21
1.50	9.76	10.68	7.83	10.36	7.83	10.70	10.04	10.70
2.00	7.30	8.35	5.38	4.82	5.38	8.37	7.50	8.37
3.00	5.52	5.17	5.00	3.10	5.00	5.18	5.66	5.18

Table 5.4.35

ANOS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.20, 1.50)		FSI	(0.20, 1.50)		FSI	(0.20, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	152.88	152.02	682.72	917.13	682.72	151.92	151.66	151.92
0.50	41.27	41.55	167.11	269.81	167.11	41.55	41.39	41.55
0.75	22.63	23.10	53.81	93.98	53.81	23.11	22.84	23.11
1.00	15.64	16.21	22.48	38.58	22.48	16.21	15.85	16.21
1.50	9.89	10.69	7.83	10.36	7.83	10.70	10.04	10.70
2.00	7.39	8.37	5.38	4.82	5.38	8.37	7.50	8.37
3.00	5.58	5.18	5.00	3.10	5.00	5.18	5.66	5.18

Table 5.4.36

ANOS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.20, 1.25)		FSI	(0.20, 1.25)		FSI	(0.20, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	152.10	156.29	682.72	917.13	682.72	151.92	151.66	151.92
0.50	41.36	41.80	167.11	269.81	167.11	41.55	41.39	41.55
0.75	22.78	22.95	53.81	93.98	53.81	23.11	22.84	23.11
1.00	15.79	16.01	22.48	38.58	22.48	16.21	15.85	16.21
1.50	9.99	10.54	7.83	10.36	7.83	10.70	10.04	10.70
2.00	7.47	8.18	5.38	4.82	5.38	8.37	7.50	8.37
3.00	5.64	5.14	5.00	3.10	5.00	5.18	5.66	5.18

Table 5.4.37

ANOS Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.20, 2.00)		FSI	(0.20, 2.00)		FSI	(0.20, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	183.71	180.05	682.72	917.13	682.72	176.11	175.59	176.11
0.50	41.81	42.31	167.11	269.81	167.11	42.26	42.09	42.26
0.75	21.01	21.87	53.81	93.98	53.81	22.13	21.88	22.13
1.00	14.01	14.92	22.48	38.58	22.48	15.17	14.80	15.17
1.50	8.65	9.67	7.83	10.36	7.83	9.85	9.19	9.85
2.00	6.53	7.13	5.38	4.82	5.38	7.32	6.87	7.32
3.00	4.56	5.04	5.00	3.10	5.00	5.05	5.01	5.05

Table 5.4.38

ANOS Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.20, 1.75)		FSI	(0.20, 1.75)		FSI	(0.20, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	181.50	177.94	682.72	917.13	682.72	176.11	175.59	176.11
0.50	41.96	42.31	167.11	269.81	167.11	42.26	42.09	42.26
0.75	21.29	22.03	53.81	93.98	53.81	22.13	21.88	22.13
1.00	14.23	15.07	22.48	38.58	22.48	15.17	14.80	15.17
1.50	8.79	9.78	7.83	10.36	7.83	9.85	9.19	9.85
2.00	6.62	7.24	5.38	4.82	5.38	7.32	6.87	7.32
3.00	4.68	5.04	5.00	3.10	5.00	5.05	5.01	5.05

Table 5.4.39

ANOS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.20, 1.50)		FSI	(0.20, 1.50)		FSI	(0.20, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	179.46	176.26	682.72	917.13	682.72	176.11	175.59	176.11
0.50	42.07	42.27	167.11	269.81	167.11	42.26	42.09	42.26
0.75	21.54	22.13	53.81	93.98	53.81	22.13	21.88	22.13
1.00	14.46	15.16	22.48	38.58	22.48	15.17	14.80	15.17
1.50	8.94	9.85	7.83	10.36	7.83	9.85	9.19	9.85
2.00	6.71	7.31	5.38	4.82	5.38	7.32	6.87	7.32
3.00	4.80	5.05	5.00	3.10	5.00	5.05	5.01	5.05

Table 5.4.40

ANOS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.20, 1.25)		FSI	(0.20, 1.25)		FSI	(0.20, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	177.49	178.52	682.72	917.13	682.72	176.11	175.59	176.11
0.50	42.12	42.45	167.11	269.81	167.11	42.26	42.09	42.26
0.75	21.74	22.10	53.81	93.98	53.81	22.13	21.88	22.13
1.00	14.66	15.10	22.48	38.58	22.48	15.17	14.80	15.17
1.50	9.08	9.79	7.83	10.36	7.83	9.85	9.19	9.85
2.00	6.80	7.25	5.38	4.82	5.38	7.32	6.87	7.32
3.00	4.91	5.04	5.00	3.10	5.00	5.05	5.01	5.05

Table 5.4.41

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.50, 2.00)		FSI	(0.50, 2.00)		FSI	(0.50, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	121.83	123.80	682.72	917.13	682.72	123.81	123.72	123.81
0.50	49.67	51.94	167.11	269.81	167.11	51.94	51.74	51.94
0.75	31.09	33.04	53.81	93.98	53.81	33.07	32.75	33.07
1.00	22.76	24.49	22.48	38.58	22.48	24.49	24.07	24.49
1.50	15.06	16.53	7.83	10.36	7.83	16.53	15.93	16.53
2.00	11.44	12.72	5.38	4.82	5.38	12.72	12.10	12.72
3.00	8.10	9.92	5.00	3.10	5.00	9.92	8.58	9.92

Table 5.4.42

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.50, 1.75)		FSI	(0.50, 1.75)		FSI	(0.50, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	122.49	123.77	682.72	917.13	682.72	123.81	123.72	123.81
0.50	50.32	51.75	167.11	269.81	167.11	51.94	51.74	51.94
0.75	31.56	32.89	53.81	93.98	53.81	33.07	32.75	33.07
1.00	23.12	24.34	22.48	38.58	22.48	24.49	24.07	24.49
1.50	15.29	16.43	7.83	10.36	7.83	16.53	15.93	16.53
2.00	11.61	12.62	5.38	4.82	5.38	12.72	12.10	12.72
3.00	8.24	9.91	5.00	3.10	5.00	9.92	8.58	9.92

Table 5.4.43

ANOS Values for Matched One-Sides SPRT, $\bar{X}$ *, and CUSUM Charts*

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.50, 1.50)		FSI	(0.50, 1.50)		FSI	(0.50, 1.50)	
n / δ	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	123.04	123.93	682.72	917.13	682.72	123.81	123.72	123.81
0.50	50.90	51.35	167.11	269.81	167.11	51.94	51.74	51.94
0.75	32.02	32.47	53.81	93.98	53.81	33.07	32.75	33.07
1.00	23.47	23.97	22.48	38.58	22.48	24.49	24.07	24.49
1.50	15.52	16.17	7.83	10.36	7.83	16.53	15.93	16.53
2.00	11.79	12.37	5.38	4.82	5.38	12.72	12.10	12.72
3.00	8.37	9.86	5.00	3.10	5.00	9.92	8.58	9.92

Table 5.4.44

ANOS Values for Matched One-Sided SPRT, $\bar{X}$ *, and CUSUM Charts*

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.10		1.34	1.82	1.34	0.10		
(d_1, d_2)	(0.50, 1.25)		FSI	(0.50, 1.25)		FSI	(0.50, 1.25)	
n / δ	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	123.46	126.18	682.72	917.13	682.72	123.81	123.72	123.81
0.50	51.39	50.49	167.11	269.81	167.11	51.94	51.74	51.94
0.75	32.43	31.36	53.81	93.98	53.81	33.07	32.75	33.07
1.00	23.80	22.93	22.48	38.58	22.48	24.49	24.07	24.49
1.50	15.74	15.41	7.83	10.36	7.83	16.53	15.93	16.53
2.00	11.95	11.68	5.38	4.82	5.38	12.72	12.10	12.72
3.00	8.49	9.60	5.00	3.10	5.00	9.92	8.58	9.92

Table 5.4.45

ANOS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.50, 2.00)		FSI	(0.50, 2.00)		FSI	(0.50, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	121.73	123.55	682.72	917.13	682.72	123.52	123.43	123.52
0.50	41.46	45.27	167.11	269.81	167.11	45.33	45.15	45.33
0.75	24.67	27.73	53.81	93.98	53.81	27.79	27.50	27.79
1.00	17.72	20.23	22.48	38.58	22.48	20.28	19.87	20.28
1.50	11.60	13.50	7.83	10.36	7.83	13.53	12.99	13.53
2.00	8.84	10.48	5.38	4.82	5.38	10.50	9.85	10.50
3.00	6.17	7.75	5.00	3.10	5.00	7.79	6.64	7.79

Table 5.4.46

ANOS Values for Matched One-Sided SPRT,
$\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.50, 1.75)		FSI	(0.50, 1.75)		FSI	(0.50, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	122.34	123.52	682.72	917.13	682.72	123.52	123.43	123.52
0.50	42.44	45.33	167.11	269.81	167.11	45.33	45.15	45.33
0.75	25.32	27.79	53.81	93.98	53.81	27.79	27.50	27.79
1.00	18.29	20.28	22.48	38.58	22.48	20.28	19.87	20.28
1.50	11.89	13.53	7.83	10.36	7.83	13.53	12.99	13.53
2.00	9.05	10.50	5.38	4.82	5.38	10.50	9.85	10.50
3.00	6.24	7.79	5.00	3.10	5.00	7.79	6.64	7.79

Table 5.4.47

ANOS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.50, 1.50)		FSI	(0.50, 1.50)		FSI	(0.50, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	122.84	123.68	682.72	917.13	682.72	123.52	123.43	123.52
0.50	43.38	45.25	167.11	269.81	167.11	45.33	45.15	45.33
0.75	25.99	27.69	53.81	93.98	53.81	27.79	27.50	27.79
1.00	18.67	20.19	22.48	38.58	22.48	20.28	19.87	20.28
1.50	12.20	13.47	7.83	10.36	7.83	13.53	12.99	13.53
2.00	9.28	10.47	5.38	4.82	5.38	10.50	9.85	10.50
3.00	6.32	7.41	5.00	3.10	5.00	7.79	6.64	7.79

Table 5.4.48

ANOS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.15		1.34	1.82	1.34	0.15		
(d_1, d_2)	(0.50, 1.25)		FSI	(0.50, 1.25)		FSI	(0.50, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	123.23	125.94	682.72	917.13	682.72	123.52	123.43	123.52
0.50	44.23	44.96	167.11	269.81	167.11	45.33	45.15	45.33
0.75	26.65	27.17	53.81	93.98	53.81	27.79	27.50	27.79
1.00	19.18	19.68	22.48	38.58	22.48	20.28	19.87	20.28
1.50	12.52	13.07	7.83	10.36	7.83	13.53	12.99	13.53
2.00	9.51	10.29	5.38	4.82	5.38	10.50	9.85	10.50
3.00	6.44	7.25	5.00	3.10	5.00	7.79	6.64	7.79

Table 5.4.49

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.50, 2.00)		FSI	(0.50, 2.00)		FSI	(0.50, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	138.57	134.51	682.72	917.13	682.72	133.87	133.74	133.87
0.50	37.29	42.13	167.11	269.81	167.11	42.38	42.21	42.38
0.75	20.83	24.56	53.81	93.98	53.81	24.84	24.56	24.84
1.00	14.65	17.56	22.48	38.58	22.48	17.80	17.42	17.80
1.50	9.46	11.61	7.83	10.36	7.83	11.76	11.23	11.76
2.00	7.15	9.43	5.38	4.82	5.38	9.54	8.47	9.54
3.00	5.46	5.65	5.00	3.10	5.00	5.75	6.06	5.75

Table 5.4.50

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.50, 1.75)		FSI	(0.50, 1.75)		FSI	(0.50, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	137.37	134.12	682.72	917.13	682.72	133.87	133.74	133.87
0.50	38.45	42.31	167.11	269.81	167.11	42.38	42.21	42.38
0.75	21.54	24.76	53.81	93.98	53.81	24.84	24.56	24.84
1.00	15.14	17.73	22.48	38.58	22.48	17.80	17.42	17.80
1.50	9.76	11.71	7.83	10.36	7.83	11.76	11.23	11.76
2.00	7.36	9.51	5.38	4.82	5.38	9.54	8.47	9.54
3.00	5.63	5.72	5.00	3.10	5.00	5.75	6.06	5.75

Table 5.4.51

ANOS Values for Matched One-Sided SPRT, $\bar{X}$*, and CUSUM Charts*

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.50, 1.50)		FSI	(0.50, 1.50)		FSI	(0.50, 1.50)	
δ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	136.26	133.87	682.72	917.13	682.72	133.87	133.74	133.87
0.50	39.64	42.38	167.11	269.81	167.11	42.38	42.21	42.38
0.75	22.33	24.84	53.81	93.98	53.81	24.84	24.56	24.84
1.00	15.68	17.80	22.48	38.58	22.48	17.80	17.42	17.80
1.50	10.10	11.76	7.83	10.36	7.83	11.76	11.23	11.76
2.00	7.61	9.54	5.38	4.82	5.38	9.54	8.47	9.54
3.00	5.77	5.75	5.00	3.10	5.00	5.75	6.06	5.75

Table 5.4.52

ANOS Values for Matched One-Sided SPRT, $\bar{X}$*, and CUSUM Charts*

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.20		1.34	1.82	1.34	0.20		
(d_1, d_2)	(0.50, 1.25)		FSI	(0.50, 1.25)		FSI	(0.50, 1.25)	
δ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	135.30	135.55	682.72	917.13	682.72	133.87	133.74	133.87
0.50	40.77	42.34	167.11	269.81	167.11	42.38	42.21	42.38
0.75	23.18	24.64	53.81	93.98	53.81	24.84	24.56	24.84
1.00	16.30	17.60	22.48	38.58	22.48	17.80	17.42	17.80
1.50	10.48	11.62	7.83	10.36	7.83	11.76	11.23	11.76
2.00	7.90	9.43	5.38	4.82	5.38	9.54	8.47	9.54
3.00	5.90	5.65	5.00	3.10	5.00	5.75	6.06	5.75

Table 5.4.53

ANOS Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.50, 2.00)		FSI	(0.50, 2.00)		FSI	(0.50, 2.00)	
n δ	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	178.33	154.99	682.72	917.13	682.72	151.92	151.66	151.92
0.50	35.98	41.19	167.11	269.81	167.11	41.55	41.39	41.55
0.75	18.52	22.57	53.81	93.98	53.81	23.11	22.84	23.11
1.00	12.69	15.75	22.48	38.58	22.48	16.21	15.85	16.21
1.50	8.07	10.41	7.83	10.36	7.83	10.70	10.04	10.70
2.00	6.22	8.04	5.38	4.82	5.38	8.37	7.50	8.37
3.00	4.21	5.12	5.00	3.10	5.00	5.18	5.66	5.18

Table 5.4.54

ANOS Values for Matched One-Sided SPRT, $\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.50, 1.75)		FSI	(0.50, 1.75)		FSI	(0.50, 1.75)	
n δ	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	171.26	153.62	682.72	917.13	682.72	151.92	151.66	151.92
0.50	37.19	41.42	167.11	269.81	167.11	41.55	41.39	41.55
0.75	19.22	22.86	53.81	93.98	53.81	23.11	22.84	23.11
1.00	13.15	15.99	22.48	38.58	22.48	16.21	15.85	16.21
1.50	8.34	10.55	7.83	10.36	7.83	10.70	10.04	10.70
2.00	6.39	8.21	5.38	4.82	5.38	8.37	7.50	8.37
3.00	4.45	5.15	5.00	3.10	5.00	5.18	5.66	5.18

Table 5.4.55

ANOS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.50, 1.50)		FSI	(0.50, 1.50)		FSI	(0.50, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	164.84	152.30	682.72	917.13	682.72	151.92	151.66	151.92
0.50	38.46	41.54	167.11	269.81	167.11	41.55	41.39	41.55
0.75	20.04	23.06	53.81	93.98	53.81	23.11	22.84	23.11
1.00	13.70	16.17	22.48	38.58	22.48	16.21	15.85	16.21
1.50	8.67	10.67	7.83	10.36	7.83	10.70	10.04	10.70
2.00	6.60	8.34	5.38	4.82	5.38	8.37	7.50	8.37
3.00	4.73	5.17	5.00	3.10	5.00	5.18	5.66	5.18

Table 5.4.56

ANOS Values for Matched One-Sided SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart		$\bar{X}$ Chart			CUSUM Chart		
γ	0.25		1.34	1.82	1.34	0.25		
(d_1, d_2)	(0.50, 1.25)		FSI	(0.50, 1.25)		FSI	(0.50, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	159.36	152.63	682.72	917.13	682.72	151.92	151.66	151.92
0.50	39.70	41.57	167.11	269.81	167.11	41.55	41.39	41.55
0.75	20.98	23.07	53.81	93.98	53.81	23.11	22.84	23.11
1.00	14.35	16.17	22.48	38.58	22.48	16.21	15.85	16.21
1.50	9.06	10.66	7.83	10.36	7.83	10.70	10.04	10.70
2.00	6.84	8.33	5.38	4.82	5.38	8.37	7.50	8.37
3.00	5.05	5.17	5.00	3.10	5.00	5.18	5.66	5.18

Table 5.4.57

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.50, 2.00)		FSI	(0.50, 2.00)		FSI	(0.50, 2.00)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	250.05	185.21	682.72	917.13	682.72	176.11	175.59	176.11
0.50	37.35	42.10	167.11	269.81	167.11	42.26	42.09	42.26
0.75	17.22	21.37	53.81	93.98	53.81	22.13	21.88	22.13
1.00	11.43	14.50	22.48	38.58	22.48	15.17	14.80	15.17
1.50	7.16	9.38	7.83	10.36	7.83	9.85	9.19	9.85
2.00	5.48	6.85	5.38	4.82	5.38	7.32	6.87	7.32
3.00	3.47	5.03	5.00	3.10	5.00	5.05	5.01	5.05

Table 5.4.58

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.50, 1.75)		FSI	(0.50, 1.75)		FSI	(0.50, 1.75)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	230.90	181.76	682.72	917.13	682.72	176.11	175.59	176.11
0.50	38.52	42.26	167.11	269.81	167.11	42.26	42.09	42.26
0.75	17.89	21.72	53.81	93.98	53.81	22.13	21.88	22.13
1.00	11.85	14.78	22.48	38.58	22.48	15.17	14.80	15.17
1.50	7.41	9.58	7.83	10.36	7.83	9.85	9.19	9.85
2.00	5.68	7.03	5.38	4.82	5.38	7.32	6.87	7.32
3.00	3.61	5.03	5.00	3.10	5.00	5.05	5.01	5.05

Table 5.4.59

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.50, 1.50)		FSI	(0.50, 1.50)		FSI	(0.50, 1.50)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	213.27	178.35	682.72	917.13	682.72	176.11	175.59	176.11
0.50	39.70	42.32	167.11	269.81	167.11	42.26	42.09	42.26
0.75	18.69	21.01	53.81	93.98	53.81	22.13	21.88	22.13
1.00	12.37	15.04	22.48	38.58	22.48	15.17	14.80	15.17
1.50	7.70	9.76	7.83	10.36	7.83	9.85	9.19	9.85
2.00	5.90	7.22	5.38	4.82	5.38	7.32	6.87	7.32
3.00	3.80	5.04	5.00	3.10	5.00	5.05	5.01	5.05

Table 5.4.60

ANOS Values for Matched One-Sided SPRT,
$\overline{X}$, and CUSUM Charts

	SPRT Chart		$\overline{X}$ Chart			CUSUM Chart		
γ	0.30		1.34	1.82	1.34	0.30		
(d_1, d_2)	(0.50, 1.25)		FSI	(0.50, 1.25)		FSI	(0.50, 1.25)	
δ \ n	3	5	5	3	5	5	3	5
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.25	197.85	176.15	682.72	917.13	682.72	176.11	175.59	176.11
0.50	40.80	42.27	167.11	269.81	167.11	42.26	42.09	42.26
0.75	19.66	22.13	53.81	93.98	53.81	22.13	21.88	22.13
1.00	13.02	15.17	22.48	38.58	22.48	15.17	14.80	15.17
1.50	8.07	9.85	7.83	10.36	7.83	9.85	9.19	9.85
2.00	6.16	7.32	5.38	4.82	5.38	7.32	6.87	7.32
3.00	4.08	5.05	5.00	3.10	5.00	5.05	5.01	5.05

Chapter 6

The SPRT Chart With Sampling at Fixed Times

6.1 Introduction

As previously noted, a critical disadvantage in the practical application of VSI control charts as recently formulated is that their sampling intervals are irregular, with the advance prediction of sampling times being impossible for more than the next sample. That is, the sampling times may not correspond to the natural periods in the process, such as work shifts for plant personnel or production cycles. A modification, however, of significant practical importance to the VSI feature, and hence to the SPRT chart, was discussed in Section 5.5. In this modification, samples are always taken at pre-specified, equally spaced fixed times, with additional samples taken between these times as a function of the process data. The phrase *sampling at fixed times* (SFT) is introduced to describe this modification, and under the modification, the SPRT chart will be called an SPRT chart with SFT. Since for the special case $t_w = 0$, the SPRT chart is an FSI control chart, the practical importance of the SFT modification to the SPRT chart manifests itself in the case which is more often encountered where $t_w > 0$.

The SFT modification is naturally suitable and simple for the SPRT chart and affects only the expressions for the properties of the SPRT chart which are associated with the time to signal, together with the corresponding numerical approximations to these properties. The expressions for all other properties of the SPRT chart with SFT and their corresponding approximations are the same as those provided in Sections 3.4 and 3.5 for the unmodified SPRT chart. Before proceeding further in this chapter, the reader should review the notation introduced in Chapter 3 for the unmodified SPRT chart, with particular attention to that introduced in Section 3.2.

6.2 Description of the SPRT Chart With SFT

Recall from Section 3.2 that for the unmodified SPRT chart, T_i denotes the starting time for the chart's i^{th} SPRT, N_i denotes the sample number for the chart's i^{th} SPRT, and I_i denotes the length of time between times T_i and T_{i+1}, for $i = 1, 2, \ldots$. These and all other notations for the unmodified SPRT chart will also be used for the modified SPRT chart with SFT, unless otherwise specified. Specifically, the SPRT chart with SFT can be described as follows:

For the SPRT chart, as defined in Chapter 3, suppose that samples of size $n \geq 1$ are always taken at pre-specified, equally spaced fixed times t_1, t_2, ..., with the time interval d between times t_i and t_{i+1} satisfying

$$d = t_{i+1} - t_i = k \cdot t_w \tag{6.2.1}$$

$$\text{for all } i = 1, 2, \ldots,$$

where $k > 0$ is an integer-valued specified chart parameter. Define the chart's first SPRT to be conducted starting at time $T_1 = t_1$. Define the chart's second SPRT to be conducted starting at time $T_2 = T_1 + I_1 = t_1 + I_1$, where

$$\begin{aligned} I_1 &= \left(\left\lfloor \frac{(N_1 - 1)\, t_w}{k\, t_w} \right\rfloor + 1 \right) k\, t_w \\ &= \left(\left\lfloor \frac{(N_1 - 1)}{k} \right\rfloor + 1 \right) k\, t_w \quad , \end{aligned}$$

and where $\lfloor \cdot \rfloor$ is the floor function, which is defined as the greatest integer no larger than the argument of the function. Define the chart's third SPRT to be conducted starting at time $T_3 = T_2 + I_2 = t_1 + I_1 + I_2$, where

$$I_2 = \left(\left\lfloor \frac{(N_2 - 1)}{k} \right\rfloor + 1 \right) k \, t_w \quad .$$

$$\begin{matrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{matrix}$$

Define the chart's i^{th} SPRT to be conducted starting at time $T_i = T_{i-1} + I_{i-1} = t_1 + \sum_{i'=1}^{i-1} I_{i'}$, where

$$I_{i-1} = \left(\left\lfloor \frac{(N_{i-1} - 1)}{k} \right\rfloor + 1 \right) k \, \mathrm{t}_w \quad .$$

$$\begin{matrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{matrix}$$

That is, in general, the i^{th} SPRT of the SPRT chart with SFT is defined to be conducted starting at time

$$T_i = \begin{cases} t_1, & \text{if } \ i = 1 \\ t_1 + \sum_{i'=1}^{i-1} I_{i'} \ , & \text{if } i = 2, 3, \ldots \ , \end{cases} \tag{6.2.2}$$

where

$$I_{i'} = \left(\left\lfloor \frac{(N_{i'} - 1)}{k} \right\rfloor + 1 \right) k \, t_w \quad . \tag{6.2.3}$$

As defined for the SPRT chart with SFT, samples are always taken at pre-specified, equally spaced fixed times t_1, t_2, …, with up to k additional samples taken between any pair of consecutive times $(t_i,\ t_{i+1})$. Given any pair of consecutive times $(t_i,\ t_{i+1})$, the additional samples between these times are taken sequentially, t_w time units apart, as a function of the process data, with the possibility of even no samples being taken between times t_i and t_{i+1}. The fixed sampling times t_1, t_2, … and the parameter k (and hence, the sampling interval t_w) would typically be determined by administrative considerations and efficiency requirements of the process. For an example of an upper one-sided SPRT chart with SFT, see Figure 6.2.1, and for an example of a lower one, see Figure 6.2.2.

For the special case $(t_w,\ n)(=(d_1,\ n))=(0,\ 1)$, the SPRT chart with SFT reduces to the unmodified FSI-SSS SPRT chart with $d=t_b(=d_2)$, which received special attention in Section 5.5. The practical significance of the SFT modification to the SPRT chart manifests itself in the more often encountered cases where $t_w>0$. Clearly, the assumption that $t_w=0$, and hence the model $(t_w,\ n)=(0,\ 1)$, is an oversimplification of what takes place in practice, in many processes.

6.3 Expressions and Numerical Evaluations of Properties of the SPRT Chart With SFT

Expressions will now be developed for the properties of the SPRT chart with SFT, together with numerical approximations to these properties. These expressions and approximations are simple modifications of the corresponding expressions and approximations of the unmodified SPRT chart. Clearly, the SFT modification only affects those properties of the SPRT chart which are associated with the time to signal, together with the corresponding approximations to these

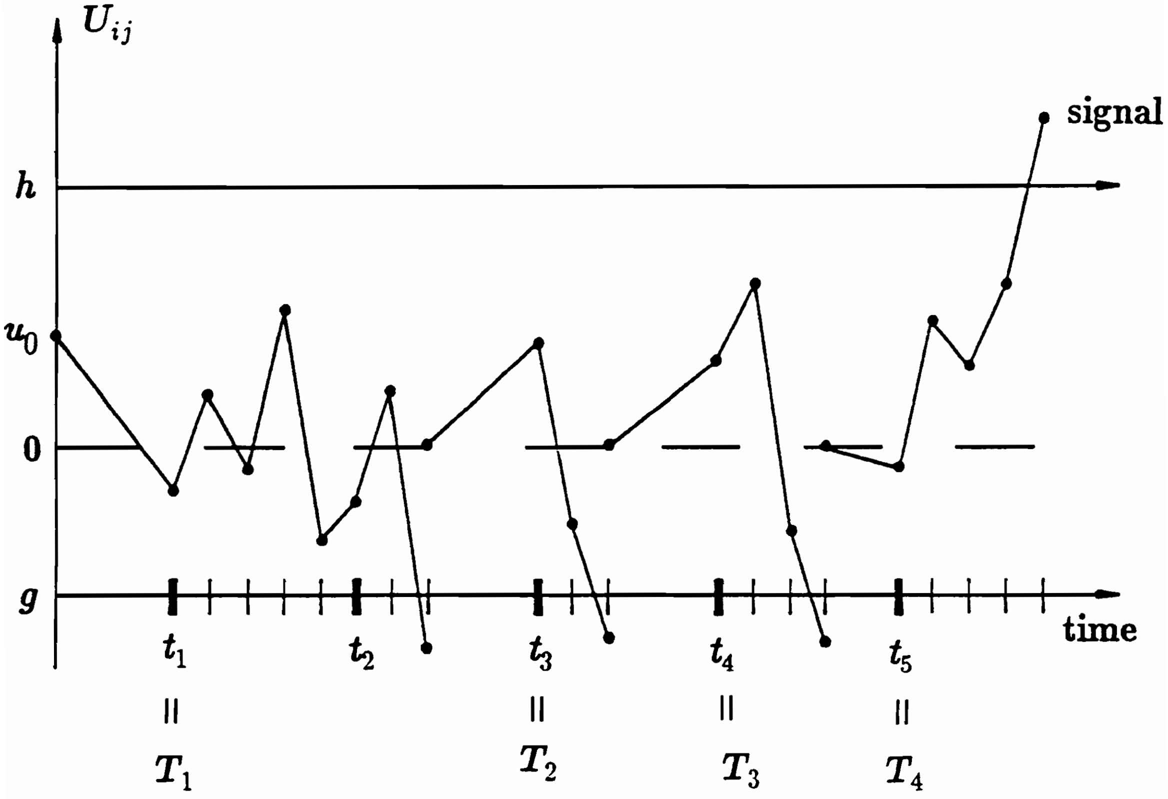

Figure 6.2.1. Upper One-Sided SPRT Chart With SFT for $k = 5$.

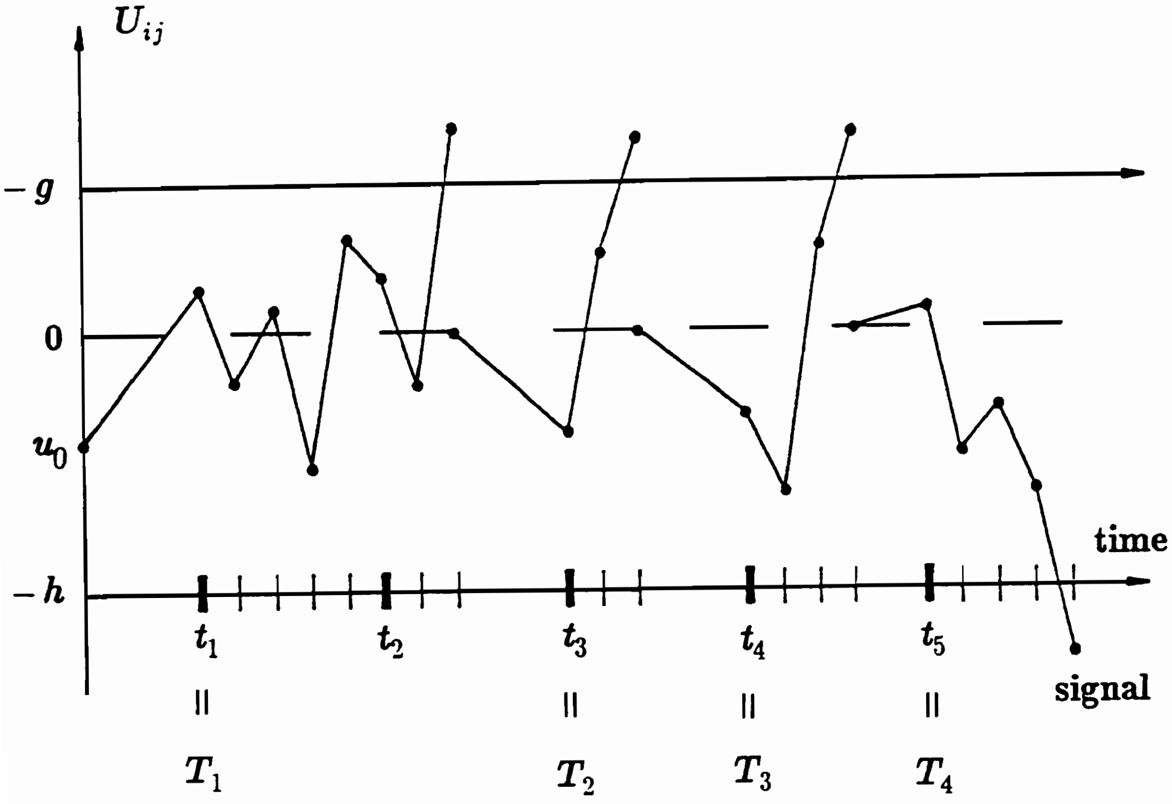

Figure 6.2.2. Lower One-Sided SPRT Chart With SFT for $k=5$.

properties. That is, the SFT modification affects neither the ASN, ANTS, ARL, or ANOS of the SPRT chart, nor their corresponding approximations. Note that in this section, the dependence of the expressions and approximations on the process parameter δ will be suppressed.

Under the SFT modification of the SPRT chart, the length of the sampling interval t_b between consecutive SPRT's is no longer fixed, but rather varies as a function of the process data. To make this distinction, for the SPRT chart with SFT, let the sampling interval between the i^{th} and $(i+1)^{st}$ consecutive SPRT's be denoted by T_{b_i}, for $i = 1, 2, \ldots$. Then,

$$T_{b_i} = I_i - (N_i - 1)\, t_w \quad , \tag{6.3.1}$$

where I_i is given in equation **(6.2.3)**. The possible values for the sampling interval T_{b_i} are t_w, $2t_w$, $\ldots$, $k\, t_w$, for all i. The ATI of the SPRT chart with SFT, conditional on the starting value u_0 of the control statistic, is given by

$$\mathrm{ATI}(u_0) = \left(\left\lfloor \frac{(\mathrm{ASN}(u_0) - 1)}{k} \right\rfloor + 1 \right) k\, t_w \quad , \tag{6.3.2}$$

where $\mathrm{ASN}(u_0)$ is given in equation **(3.4.1)**. Note that suppressing the starting value u_0 of the control statistic, $\mathrm{ATI} = \mathrm{E}(I_i)$, for all i. Then, conditional on the starting value u_0 of the control statistic,

$$\mathrm{E}(T_{b_i} \mid u_0) = \mathrm{ATI}(u_0) - (\mathrm{ASN}(u_0) - 1)\, t_w \quad . \tag{6.3.3}$$

Appropriately using the approximation $\widetilde{\mathrm{ASN}}(u_0)$ from equation **(3.5.3)** in equation **(6.3.2)**, an approximation $\widetilde{\mathrm{ATI}}(u_0)$ to the $\mathrm{ATI}(u_0)$ of the SPRT chart is given by

$$\widetilde{\mathrm{ATI}}(u_0) = \left(\left\lfloor \frac{(\widetilde{\mathrm{ASN}}(u_0) - 1)}{k} \right\rfloor + 1 \right) k\, t_w \quad . \tag{6.3.4}$$

In turn, appropriately using the approximation $\widetilde{\mathrm{ATI}}(u_0)$ together with the approximation $\widetilde{\mathrm{ASN}}(u_0)$ in equation **(6.3.3)**, an approximation $\widetilde{\mathrm{E}}\,(T_{b_i} \mid u_0)$ to $\mathrm{E}(T_{b_i} \mid u_0)$ is given by

$$\widetilde{\mathrm{E}}\,(T_{b_i} \mid u_0) = \widetilde{\mathrm{ATI}}(u_0) - (\widetilde{\mathrm{ASN}}(u_0) - 1)\,t_w \quad . \tag{6.3.5}$$

Expressions for the properties of the modified SPRT chart with SFT are simply obtained by appropriately replacing the sampling interval t_b with the mean sampling interval $\mathrm{E}(T_{b_i} \mid u_0)$ in the expressions from Section 3.4 for the properties of the unmodified SPRT chart. Similarly, approximations to the properties of the modified SPRT chart with SFT are simply obtained by appropriately replacing the sampling interval t_b with the approximation $\widetilde{\mathrm{E}}\,(T_{b_i} \mid u_0)$ to the mean of the sampling interval T_{b_i} in the approximations from Section 3.5 for the properties of the unmodified SPRT chart.

Chapter 7

Comparison of the SPRT Chart With SFT With the Regular SPRT, $\overline{X}$, and CUSUM Charts

7.1 Introduction

For the remainder of this dissertation, the unmodified SPRT chart will be referred to as the *regular* SPRT chart, and the modified SPRT chart with SFT simply as the SPRT chart with SFT. In evaluating the SPRT chart with SFT, it will be compared with the regular SPRT chart and with the $\overline{X}$ and CUSUM charts, both with and without the VSI feature, within the general framework of the UC chart. As in Chapter 5, the quantitative comparisons will be based on SSATS, ATS, and ANOS performances, without considering any FIR features. That is, the starting values of the control statistics were chosen to be 0 for all charts. Also as in Chapter 5, for purposes of fair comparison and evaluation of the SPRT chart with SFT and of the regular SPRT, $\overline{X}$, and CUSUM charts, all charts were matched up so that the two equations in the system

$$\begin{aligned} \mathrm{ATS}(0) &= 740.8 \quad \text{when } \delta = 0 \\ \mathrm{AOR}(0) &= 5 \qquad\ \ \text{when } \delta = 0 \end{aligned} \tag{7.1.1}$$

were satisfied simultaneously. Note that system (7.1.1) is the same as system (5.1.1), and recall that 740.8 corresponds to the in-control ATS of the one-sided FSI $\overline{X}$ chart with the "standard" $3\sigma/\sqrt{n}$ limit, and that obviously, when $\delta = 0$, the SSATS of a control chart is defined to be simply its ATS. When the control charts are matched according to system (7.1.1), the performance of the charts can be evaluated by computing the ATS (both steady-state and zero-time) and ANOS values of the charts at various values of δ to determine which chart is the most efficient in detecting changes in δ. As noted in Chapter 5, the chart with the smaller ATS at a particular δ is the most efficient chart at that δ. When any two charts have the same ATS efficiency at a particular δ, then the chart with the smaller ANOS at δ has the advantage of sampling less at δ, thus reducing sampling costs.

The properties of the SPRT chart with SFT are determined by the choice of the parameters n, γ, g, h, d_0, d_1 ($= t_w$), and d (or equivalently, k). Similarly, the properties of the regular SPRT, $\overline{X}$, and CUSUM charts, as special cases of the UC chart, are determined by the choice of parameters n, γ, g, h, d_0, d_1, and d_2. As in Chapter 5 for the FSI charts, the unit time was chosen as the sampling interval so that $d_1 = d_2 = 1$. For the SPRT chart with SFT, the two pairs of sampling intervals $(d_1, d) = (0.5, 2.0)$ and $(0.5, 1.5)$ were chosen. Likewise, for the other VSI charts, the two pairs of sampling intervals $(d_1, d_2) = (0.5, 2.0)$ and $(0.5, 1.5)$ were chosen. For all of the charts, the starting sampling interval $d_0 = 1$ was used. It was shown in Chapter 5 that the sample size $n = 5$ offers substantially higher SSATS and ATS efficiencies than sample sizes smaller than $n = 5$ for the sampling interval $d_1 = 0.5$. Therefore, no sample sizes other than $n = 5$ were considered. Thus, by system (7.1.1), for all charts, ASI = 1 when $\delta = 0$. For the SPRT chart with SFT and the regular SPRT and CUSUM charts, the reference values $\delta = 0.15$, 0.25, 0.35, and 0.4 were used in order to cover a large number of choices while maintaining reasonable efficiencies for the broad range of shifts considered. The shifts considered were $\delta = 0.1$, 0.15, 0.2, 0.25, 0.5, 0.75, 1, 1.5, 2 and 3. The remaining parameters were determined (appropriately for each chart) to satisfy system (7.1.1). Specifically, for the SPRT chart with SFT, given values for n, γ, d_0, d_1, and d, g and h were determined as solutions to system (7.1.1). For the regular SPRT, $\overline{X}$, and CUSUM charts, the parameters were determined in the same way as in Section 5.1. That is, for the

(i) regular SPRT chart, given values for n, γ, d_0, d_1, and d_2, g and h were determined as solutions to system (7.1.1);

(ii) $\overline{X}$ chart, given values for n, d_0, d_1, and d_2, γ and c were determined as solutions to system (7.1.1);

(iii) CUSUM chart, given values for n, γ, d_0, d_1, and d_2, h and c were determined as solutions to system (7.1.1).

Recall from Section 4.2, that by definition, for the

(i) SPRT chart (both, regular and with SFT), $g = c$;

(ii) $\overline{X}$ chart, $g = h = 0$;

(iii) CUSUM chart, $g = 0$.

As in Chapter 5, the constrained determination of parameters and the evaluation of the SSATS, ATS and ANOS of the $\overline{X}$ and CUSUM charts was performed using the modular FORTRAN 77 computer software developed by Stoumbos and Reynolds (1992a), for $m_1 = m_2 = m = 96$ Gaussian quadrature points. The constrained determination of the parameters and the evaluation of the SSATS, ATS, and ANOS of the regular SPRT chart and of the SPRT chart with SFT, however, was performed using the modular FORTRAN 77 computer software developed by Stoumbos and Reynolds (1992b), for $m = 96$ Gaussian quadrature points. As noted in Section 5.1, unlike the software developed by Stoumbos and Reynolds (1992a) for the general UC chart, that developed by

Stoumbos and Reynolds (1992b) was designed specifically for the SPRT chart (both regular and with SFT), thus exploiting the particular structure of the SPRT chart for increased computational efficiency. Recall that both of these softwares exclusively employ double precision throughout all computations. For the convenience of the reader, all tables provided in this chapter are placed at the end of the chapter. The results provided in these tables hold for both upper and lower charts.

7.2 Comparisons Based on SSATS, ATS, and ANOS Performance

Tables 7.2.1 – 7.2.24 provide numerical SSATS, ATS, and ANOS values for matched one-sided SPRT with SFT, regular SPRT, $\bar{X}$, and CUSUM charts. In particular, Tables 7.2.1 – 7.2.8 provide SSATS values, along with values for the parameters g, h, and c, Tables 7.2.9 – 7.2.16 provide ATS values, and Tables 7.2.17 – 7.2.24 provide ANOS values. To avoid redundancy, values for the parameters g, h, and c are only included in Tables 7.2.1 – 7.2.8. Throughout Tables 7.2.1 – 7.2.24, however, SSATS, ATS, and ANOS values for the FSI as well as the VSI $\bar{X}$ and CUSUM charts are appropriately repeated to aid the reader's comparisons.

In comparing the charts using Tables 7.2.1 – 7.2.8, it is clear that the SPRT chart with SFT is substantially more efficient than both the FSI and VSI $\bar{X}$ charts and the FSI CUSUM chart in terms of SSATS performance. Although still substantially more efficient in terms of SSATS performance, the improvement that the SPRT chart with SFT offers is not as large for the regular SPRT and VSI CUSUM charts as for the FSI and VSI $\bar{X}$ charts and the FSI CUSUM chart. Specifically, the SPRT chart with SFT is uniformly more efficient for all shifts considered, except for the shifts $\delta = 0.5$ and 3. For the very large shift $\delta = 3$, the SSATS efficiency of the SPRT chart with SFT is very similar to that of the regular SPRT and VSI CUSUM charts, while for the shift $\delta = 0.5$, the SPRT chart with SFT appears to be somewhat less efficient than the regular SPRT and VSI CUSUM charts. A closer examination of Tables 7.2.1 – 7.2.8, however, reveals that the SSATS performance which is uniformly best in detecting a shift of $\delta = 0.5$ is that of the SPRT chart with SFT from Table 7.2.8. This SSATS performance is at least 28.1 percent faster than all others from all charts in Tables 7.2.1 – 7.2.8.

The comparison and discussion from the previous paragraph for the SSATS performance apply in a completely analogous manner to the (zero-time) ATS performance in Tables 7.2.9 – 7.2.16. Thus, to avoid redundancy, such analogous comparisons and discussion will not be repeated here.

Recall that although ANOS values for the regular SPRT chart and the SPRT chart with SFT do depend on the sampling interval function, those for the VSI $\overline{X}$ and VSI CUSUM charts do not. That is, the ANOS values for any VSI $\overline{X}$ and any VSI CUSUM chart are exactly the same as those for their respective FSI counterparts, and no distinction need be made between FSI and VSI $\overline{X}$ and CUSUM charts when considering ANOS performance. In comparing the charts using Tables 7.2.17 – 7.2.24, it is clear that the SPRT chart with SFT is substantially more efficient than the $\overline{X}$ chart in terms of ANOS performance. The ANOS efficiency of the SPRT chart with SFT, however, is very similar to that of the regular SPRT and CUSUM charts.

From the above comparisons, it is clear that the SFT modification to the SPRT chart, as developed in Chapter 6, not only has minimal negative effects on the SPRT chart's high efficiency, but also provides a substantial improvement to this high efficiency for many shifts, rendering the SPRT chart with SFT the best choice among the charts compared. This is especially true, since unlike the VSI $\overline{X}$ and CUSUM charts, the SPRT chart with SFT, by definition, incorporates the desirable feature of sampling at pre-specified, equally spaced fixed times. Concluding, recall that for the special case of $d_1(= t_w) = 0$, the SPRT chart with SFT reduces to the FSI regular SPRT chart with $d = d_2$ $(= t_b)$, which received great attention in Chapter 5. However, as discussed in Chapter 6, the practical significance of the SPRT chart with SFT manifests itself in the more often encountered cases where $d_1 > 0$.

Table 7.2.1

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.15$		$\gamma = 1.34$		$\gamma = 0.15$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 2.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$
δ	$n = 5$					
0.00	740.80	740.80	740.80	740.80	740.80	740.80
0.10	69.54	86.60	363.36	321.93	109.33	86.35
0.15	28.07	39.91	258.96	216.16	55.30	39.80
0.20	14.38	22.35	162.76	147.07	32.89	22.30
0.25	16.16	14.57	136.04	101.47	22.16	14.55
0.50	5.15	4.87	32.92	20.00	7.57	4.88
0.75	2.20	2.95	10.26	5.91	4.43	2.95
1.00	1.70	2.18	4.00	2.52	3.10	2.18
1.50	1.28	1.53	1.07	1.03	1.92	1.53
2.00	1.11	1.25	0.58	0.79	1.41	1.25
3.00	0.93	0.97	0.50	0.75	0.84	0.97
h	6.48	6.50	0.00	0.00	6.52	6.52
g	0.30	-2.56	0.00	0.00	0.00	0.00
c	0.30	-2.56	0.00	-3.43	6.52	-0.23

Table 7.2.2

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.15$		$\gamma = 1.34$		$\gamma = 0.15$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 1.50$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$
δ	$n = 5$					
0.00	740.80	740.80	740.80	740.80	740.80	740.80
0.10	81.60	90.07	363.36	331.56	109.33	89.47
0.15	34.23	41.75	258.96	225.35	55.30	41.45
0.20	17.39	23.35	162.76	154.85	32.89	23.19
0.25	10.55	15.15	136.04	107.65	22.16	15.06
0.50	5.47	4.93	32.92	21.23	7.57	4.92
0.75	2.01	2.90	10.26	6.01	4.43	2.89
1.00	1.51	2.08	4.00	2.43	3.10	2.08
1.50	1.09	1.41	1.07	0.91	1.92	1.41
2.00	0.94	1.12	0.58	0.66	1.41	1.13
3.00	0.76	0.85	0.50	0.63	0.84	0.85
h	6.25	6.48	0.00	0.00	6.52	6.52
g	0.73	0.30	0.00	0.00	0.00	0.00
c	0.73	0.30	0.00	-3.00	6.52	0.33

Table 7.2.3

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.25$		$\gamma = 1.34$		$\gamma = 0.25$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 2.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$
δ	$n = 5$					
0.00	740.80	740.80	740.80	740.80	740.80	740.80
0.10	112.53	134.65	363.36	321.93	155.20	129.29
0.15	48.31	64.51	258.96	216.16	80.60	61.49
0.20	22.98	34.13	162.76	147.07	45.95	32.54
0.25	12.51	20.06	136.04	101.47	28.73	19.24
0.50	4.87	4.61	32.92	20.00	7.26	4.58
0.75	1.95	2.52	10.26	5.91	3.78	2.53
1.00	1.48	1.81	4.00	2.52	2.51	1.82
1.50	1.13	1.27	1.07	1.03	1.49	1.28
2.00	0.99	1.04	0.58	0.79	1.04	1.05
3.00	0.82	0.76	0.50	0.75	0.52	0.76
h	4.33	4.18	0.00	0.00	4.35	4.35
g	-0.25	-0.95	0.00	0.00	0.00	0.00
c	-0.25	-0.95	0.00	-3.43	4.35	-0.72

Table 7.2.4

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.25$		$\gamma = 1.34$		$\gamma = 0.25$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 1.50$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$
δ	$n = 5$					
0.00	740.80	740.80	740.80	740.80	740.80	740.80
0.10	121.81	134.65	363.36	331.56	155.20	133.75
0.15	54.17	64.56	258.96	225.35	80.60	64.22
0.20	26.32	34.29	162.76	154.85	45.95	34.11
0.25	14.29	20.23	136.04	107.65	28.73	20.13
0.50	5.11	4.62	32.92	21.23	7.26	4.62
0.75	1.78	2.46	10.26	6.01	3.78	2.46
1.00	1.32	1.71	4.00	2.43	2.51	1.71
1.50	0.97	1.16	1.07	0.91	1.49	1.16
2.00	0.83	0.93	0.58	0.66	1.04	0.93
3.00	0.67	0.64	0.50	0.63	0.52	0.64
h	4.33	4.33	0.00	0.00	4.35	4.35
g	0.26	-0.25	0.00	0.00	0.00	0.00
c	0.26	-0.25	0.00	-3.00	4.35	-0.23

Table 7.2.5

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.35$		$\gamma = 1.34$		$\gamma = 0.35$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 2.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$
δ	$n = 5$					
0.00	740.80	740.80	740.80	740.80	740.80	740.80
0.10	159.70	189.44	363.36	321.93	199.17	170.45
0.15	76.94	100.20	258.96	216.16	110.38	87.39
0.20	38.60	55.28	162.76	147.07	64.44	47.48
0.25	20.56	32.13	136.04	101.47	39.83	27.58
0.50	5.43	5.28	32.92	20.00	8.01	4.98
0.75	1.89	2.44	10.26	5.91	3.63	2.42
1.00	1.39	1.66	4.00	2.52	2.25	1.67
1.50	1.05	1.13	1.07	1.03	1.24	1.14
2.00	0.90	0.89	0.58	0.79	0.77	0.90
3.00	0.81	0.75	0.50	0.75	0.50	0.75
h	3.12	2.92	0.00	0.00	3.21	3.21
g	-0.72	-1.57	0.00	0.00	0.00	0.00
c	-0.72	-1.57	0.00	-3.43	3.21	-1.06

Table 7.2.6

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.35$		$\gamma = 1.34$		$\gamma = 0.35$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 1.50$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$
δ	$n = 5$					
0.00	740.80	740.80	740.80	740.80	740.80	740.80
0.10	164.10	182.10	363.36	331.56	199.17	176.08
0.15	80.49	95.40	258.96	225.35	110.38	91.29
0.20	41.10	52.48	162.76	154.85	64.44	49.93
0.25	22.16	30.57	136.04	107.65	39.83	29.06
0.50	5.64	5.16	32.92	21.23	8.01	5.05
0.75	1.72	2.35	10.26	6.01	3.63	2.34
1.00	1.23	1.55	4.00	2.43	2.25	1.56
1.50	0.89	1.01	1.07	0.91	1.24	1.02
2.00	0.75	0.77	0.58	0.66	0.77	0.77
3.00	0.67	0.63	0.50	0.63	0.50	0.63
h	3.20	3.12	0.00	0.00	3.21	3.21
g	-0.14	-0.72	0.00	0.00	0.00	0.00
c	-0.14	-0.72	0.00	-3.00	3.21	-0.61

Table 7.2.7

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.40$		$\gamma = 1.34$		$\gamma = 0.40$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 2.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$
δ	$n = 5$					
0.00	740.80	740.80	740.80	740.80	740.80	740.80
0.10	184.22	218.41	363.36	321.93	219.75	189.61
0.15	93.87	121.81	258.96	216.16	125.82	100.82
0.20	49.02	69.72	162.76	147.07	74.95	56.02
0.25	26.59	41.25	136.04	101.47	46.68	32.78
0.50	6.09	6.04	32.92	20.00	8.73	5.39
0.75	1.92	2.50	10.26	5.91	3.67	2.43
1.00	1.38	1.64	4.00	2.52	2.19	1.63
1.50	1.02	1.08	1.07	1.03	1.15	1.09
2.00	0.88	0.86	0.58	0.79	0.70	0.86
3.00	0.81	0.75	0.50	0.75	0.50	0.75
h	2.70	2.50	0.00	0.00	2.82	2.82
g	-0.94	-1.87	0.00	0.00	0.00	0.00
c	-0.94	-1.87	0.00	-3.43	2.82	-1.21

Table 7.2.8

SSATS Values With g, h, and c Boundary Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.40$		$\gamma = 1.34$		$\gamma = 0.40$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 1.50$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$
δ	$n = 5$					
0.00	740.80	740.80	740.80	740.80	740.80	740.80
0.10	185.54	206.81	363.36	331.56	219.75	195.76
0.15	95.43	113.36	258.96	225.35	125.82	105.31
0.20	50.43	64.24	162.76	154.85	74.95	58.96
0.25	27.66	37.91	136.04	107.65	46.68	34.62
0.50	3.60	5.78	32.92	21.23	8.73	5.49
0.75	1.73	2.38	10.26	6.01	3.67	2.35
1.00	1.21	1.52	4.00	2.43	2.19	1.52
1.50	0.86	0.96	1.07	0.91	1.15	0.96
2.00	0.73	0.73	0.58	0.66	0.70	0.73
3.00	0.67	0.63	0.50	0.63	0.50	0.63
h	2.80	2.70	0.00	0.00	2.82	2.82
g	-0.33	-0.94	0.00	0.00	0.00	0.00
c	-0.33	-0.94	0.00	-3.00	2.82	-0.76

Table 7.2.9

ATS Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.15$		$\gamma = 1.34$		$\gamma = 0.15$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 2.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$
δ	$n = 5$					
0.00	740.80	740.80	740.80	740.80	740.80	740.80
0.10	71.66	88.97	363.86	322.18	114.22	88.74
0.15	29.61	41.65	259.46	216.41	59.04	41.55
0.20	15.69	23.70	187.15	147.32	35.91	23.67
0.25	17.43	15.67	136.54	101.72	24.70	15.66
0.50	5.83	5.45	33.42	20.25	9.07	5.47
0.75	3.21	3.38	10.76	6.16	5.56	3.39
1.00	2.53	2.55	4.50	2.77	4.06	2.55
1.50	1.85	1.85	1.57	1.28	2.71	1.85
2.00	1.55	1.55	1.08	1.04	2.10	1.55
3.00	1.27	1.27	1.00	1.00	1.56	1.28

Table 7.2.10

ATS Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.15$		$\gamma = 1.34$		$\gamma = 0.15$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 1.50$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$
δ	$n = 5$					
0.00	740.80	740.80	740.80	740.80	740.80	740.80
0.10	83.80	92.62	363.86	331.93	114.22	92.08
0.15	35.84	43.66	259.46	225.72	59.04	43.42
0.20	18.71	24.88	187.15	155.23	35.91	24.77
0.25	11.73	16.43	136.54	108.03	24.70	16.39
0.50	6.18	5.67	33.42	21.60	9.07	5.69
0.75	3.04	3.47	10.76	6.38	5.56	3.48
1.00	2.44	2.58	4.50	2.80	4.06	2.59
1.50	1.81	1.85	1.57	1.28	2.71	1.86
2.00	1.53	1.55	1.08	1.04	2.10	1.55
3.00	1.22	1.27	1.00	1.00	1.56	1.28

Table 7.2.11

ATS Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.25$		$\gamma = 1.34$		$\gamma = 0.25$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 2.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$
δ	$n = 5$					
0.00	740.80	740.80	740.80	740.80	740.80	740.80
0.10	113.97	135.84	363.86	322.18	158.07	130.67
0.15	49.51	65.40	259.46	216.41	82.90	62.55
0.20	24.08	34.83	187.15	147.32	47.88	33.39
0.25	13.61	20.64	136.54	101.72	30.38	19.96
0.50	5.34	4.94	33.42	20.25	8.31	5.01
0.75	2.80	2.81	10.76	6.16	4.62	2.88
1.00	2.13	2.08	4.50	2.77	3.24	2.14
1.50	1.57	1.54	1.57	1.28	2.14	1.57
2.00	1.33	1.30	1.08	1.04	1.67	1.34
3.00	1.02	1.01	1.00	1.00	1.04	1.02

Table 7.2.12

ATS Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.25$		$\gamma = 1.34$		$\gamma = 0.25$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 1.50$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$
δ	$n = 5$					
0.00	740.80	740.80	740.80	740.80	740.80	740.80
0.10	123.34	135.84	363.86	331.93	158.07	135.28
0.15	55.42	65.77	259.46	225.72	82.90	65.44
0.20	27.42	35.29	187.15	155.23	47.88	35.12
0.25	15.33	21.09	136.54	108.03	30.38	21.00
0.50	5.68	5.19	33.42	21.60	8.31	5.19
0.75	2.72	2.93	10.76	6.38	4.62	2.94
1.00	2.12	2.15	4.50	2.80	3.24	2.15
1.50	1.57	1.57	1.57	1.28	2.14	1.57
2.00	1.33	1.33	1.08	1.04	1.67	1.34
3.00	1.02	1.02	1.00	1.00	1.04	1.02

Table 7.2.13

ATS Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.35$		$\gamma = 1.34$		$\gamma = 0.35$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 2.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$
δ	$n = 5$					
0.00	740.80	740.80	740.80	740.80	740.80	740.80
0.10	160.70	189.97	363.86	322.18	201.11	171.37
0.15	77.84	100.59	259.46	216.41	112.00	88.14
0.20	39.47	55.57	187.15	147.32	65.85	48.11
0.25	21.47	32.35	136.54	101.72	41.07	28.12
0.50	5.78	5.41	33.42	20.25	8.86	5.34
0.75	2.65	2.61	10.76	6.16	4.34	2.73
1.00	1.93	1.86	4.50	2.77	2.89	1.96
1.50	1.39	1.35	1.57	1.28	1.83	1.41
2.00	1.14	1.11	1.08	1.04	1.32	1.16
3.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 7.2.14

ATS Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.35$		$\gamma = 1.34$		$\gamma = 0.35$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 1.50$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$
δ	$n = 5$					
0.00	740.80	740.80	740.80	740.80	740.80	740.80
0.10	165.24	183.07	363.86	331.93	201.11	177.14
0.15	81.49	96.21	259.46	225.72	112.00	92.19
0.20	42.02	53.17	187.15	155.23	65.85	50.70
0.25	23.06	31.19	136.54	108.03	41.07	29.75
0.50	6.12	5.61	33.42	21.60	8.86	5.54
0.75	2.61	2.75	10.76	6.38	4.34	2.78
1.00	1.96	1.94	4.50	2.80	2.89	1.97
1.50	1.41	1.39	1.57	1.28	1.83	1.41
2.00	1.16	1.14	1.08	1.04	1.32	1.16
3.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 7.2.15

ATS Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.40$		$\gamma = 1.34$		$\gamma = 0.40$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 2.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$
δ	$n = 5$					
0.00	740.80	740.80	740.80	740.80	740.80	740.80
0.10	185.04	218.69	363.86	322.18	221.39	190.39
0.15	94.63	121.99	259.46	216.41	127.22	101.47
0.20	49.79	69.83	187.15	147.32	76.19	56.57
0.25	27.39	41.32	136.54	101.72	47.78	33.27
0.50	6.37	6.08	33.42	20.25	9.51	5.72
0.75	2.65	2.61	10.76	6.16	4.34	2.73
1.00	1.88	1.80	4.50	2.77	2.80	1.91
1.50	1.33	1.28	1.57	1.28	1.71	1.36
2.00	1.10	1.07	1.08	1.04	1.22	1.11
3.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 7.2.16

ATS Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.40$		$\gamma = 1.34$		$\gamma = 0.40$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 1.50$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$
δ	$n = 5$					
0.00	740.80	740.80	740.80	740.80	740.80	740.80
0.10	186.52	207.59	363.86	331.93	221.39	196.68
0.15	96.32	114.02	259.46	225.72	127.22	106.10
0.20	51.27	64.81	187.15	155.23	76.19	59.65
0.25	28.49	38.42	136.54	108.03	47.78	35.24
0.50	4.60	6.16	33.42	21.60	9.51	5.96
0.75	2.61	2.75	10.76	6.38	4.34	2.78
1.00	1.91	1.88	4.50	2.80	2.80	1.92
1.50	1.35	1.33	1.57	1.28	1.71	1.36
2.00	1.11	1.10	1.08	1.04	1.22	1.11
3.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 7.2.17

ANOS Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.15$		$\gamma = 1.34$		$\gamma = 0.15$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 2.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$
δ	$n = 5$					
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.10	573.03	571.84	1819.32	1819.32	571.08	571.08
0.15	296.11	295.53	1297.32	1297.32	295.20	295.20
0.20	179.96	179.68	935.76	935.76	179.56	179.56
0.25	123.68	123.55	682.72	682.72	123.52	123.52
0.50	45.25	45.27	167.11	167.11	45.33	45.33
0.75	27.70	27.73	53.81	53.81	27.79	27.79
1.00	20.19	20.23	22.48	22.48	20.28	20.28
1.50	13.47	13.50	7.83	7.83	13.53	13.53
2.00	10.47	10.48	5.38	5.38	10.50	10.50
3.00	7.72	7.75	5.00	5.00	7.79	7.79

Table 7.2.18

ANOS Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.15$		$\gamma = 1.34$		$\gamma = 0.15$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 1.50$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$
δ	$n = 5$					
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.10	591.56	573.04	1819.32	1819.32	571.08	571.08
0.15	305.42	296.11	1297.32	1297.32	295.20	295.20
0.20	184.52	179.96	935.76	935.76	179.56	179.56
0.25	125.93	123.68	682.72	682.72	123.52	123.52
0.50	44.96	45.25	167.11	167.11	45.33	45.33
0.75	27.17	27.69	53.81	53.81	27.79	27.79
1.00	19.69	20.19	22.48	22.48	20.28	20.28
1.50	13.07	13.47	7.83	7.83	13.53	13.53
2.00	10.29	10.47	5.38	5.38	10.50	10.50
3.00	7.25	7.71	5.00	5.00	7.79	7.79

Table 7.2.19

ANOS Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.25$		$\gamma = 1.34$		$\gamma = 0.25$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 2.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$
δ	$n = 5$					
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.10	792.60	810.21	1819.32	1819.32	790.33	790.33
0.15	415.87	426.07	1297.32	1297.32	414.52	414.52
0.20	240.11	245.50	935.76	935.76	239.39	239.39
0.25	152.31	154.99	682.72	682.72	151.92	151.92
0.50	41.54	41.19	167.11	167.11	41.55	41.55
0.75	23.06	22.57	53.81	53.81	23.11	23.11
1.00	16.17	15.75	22.48	22.48	16.21	16.21
1.50	10.67	10.41	7.83	7.83	10.70	10.70
2.00	8.34	8.04	5.38	5.38	8.37	8.37
3.00	5.17	5.12	5.00	5.00	5.18	5.18

Table 7.2.20

ANOS Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.25$		$\gamma = 1.34$		$\gamma = 0.25$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 1.50$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$
δ	$n = 5$					
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.10	793.98	792.58	1819.32	1819.32	790.33	790.33
0.15	416.75	415.86	1297.32	1297.32	414.52	414.52
0.20	240.65	240.11	935.76	935.76	239.39	239.39
0.25	152.62	152.30	682.72	682.72	151.92	151.92
0.50	41.57	41.54	167.11	167.11	41.55	41.55
0.75	23.07	23.06	53.81	53.81	23.11	23.11
1.00	16.17	16.17	22.48	22.48	16.21	16.21
1.50	10.66	10.67	7.83	7.83	10.70	10.70
2.00	8.33	8.34	5.38	5.38	8.37	8.37
3.00	5.17	5.17	5.00	5.00	5.18	5.18

Table 7.2.21

ANOS Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.35$		$\gamma = 1.34$		$\gamma = 0.35$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 2.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$
δ	$n = 5$					
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.10	1030.67	1082.89	1819.32	1819.32	1005.54	1005.54
0.15	577.62	614.74	1297.32	1297.32	560.02	560.02
0.20	340.39	364.01	935.76	935.76	329.25	329.25
0.25	212.10	226.27	682.72	682.72	205.35	205.35
0.50	44.67	44.93	167.11	167.11	44.29	44.29
0.75	21.50	20.80	53.81	53.81	21.70	21.70
1.00	14.23	13.61	22.48	22.48	14.47	14.47
1.50	8.94	8.49	7.83	7.83	9.13	9.13
2.00	6.43	6.11	5.38	5.38	6.58	6.58
3.00	5.01	5.01	5.00	5.00	5.02	5.02

Table 7.2.22

ANOS Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.35$		$\gamma = 1.34$		$\gamma = 0.35$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 1.50$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$
δ	$n = 5$					
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.10	1007.00	1030.60	1819.32	1819.32	1005.54	1005.54
0.15	561.06	577.57	1297.32	1297.32	560.02	560.02
0.20	329.91	340.36	935.76	935.76	329.25	329.25
0.25	205.76	212.08	682.72	682.72	205.35	205.35
0.50	44.32	44.67	167.11	167.11	44.29	44.29
0.75	21.69	21.51	53.81	53.81	21.70	21.70
1.00	14.46	14.23	22.48	22.48	14.47	14.47
1.50	9.12	8.94	7.83	7.83	9.13	9.13
2.00	6.58	6.43	5.38	5.38	6.58	6.58
3.00	5.02	5.01	5.00	5.00	5.02	5.02

Table 7.2.23

ANOS Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.40$		$\gamma = 1.34$		$\gamma = 0.40$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 2.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$	$d_2 = 1.00$	$d_2 = 2.00$
δ	$n = 5$					
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.10	1153.91	1227.00	1819.32	1819.32	1106.93	1106.93
0.15	671.36	728.11	1297.32	1297.32	636.11	636.11
0.20	404.70	443.92	935.76	935.76	380.93	380.93
0.25	254.13	279.55	682.72	682.72	238.91	238.91
0.50	48.79	50.14	167.11	167.11	47.57	47.57
0.75	21.51	20.80	53.81	53.81	21.69	21.69
1.00	13.68	13.00	22.48	22.48	14.01	14.01
1.50	8.27	7.81	7.83	7.83	8.55	8.55
2.00	5.95	5.71	5.38	5.38	6.12	6.12
3.00	5.00	5.00	5.00	5.00	5.01	5.01

Table 7.2.24

ANOS Values for Matched One-Sided SPRT With SFT, Regular SPRT, $\bar{X}$, and CUSUM Charts

	SPRT Chart					
	SFT	Regular	$\bar{X}$ Chart		CUSUM Chart	
	$\gamma = 0.40$		$\gamma = 1.34$		$\gamma = 0.40$	
	$d_1 = 0.50$		$d_1 = 1.00$	$d_1 = 0.50$	$d_1 = 1.00$	$d_1 = 0.50$
	$d = 1.50$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$	$d_2 = 1.00$	$d_2 = 1.50$
δ	$n = 5$					
0.00	3704.00	3704.00	3704.00	3704.00	3704.00	3704.00
0.10	1115.14	1153.81	1819.32	1819.32	1106.93	1106.93
0.15	642.29	671.28	1297.32	1297.32	636.11	636.11
0.20	385.12	404.65	935.76	935.76	380.93	380.93
0.25	241.62	254.10	682.72	682.72	238.91	238.91
0.50	47.83	48.78	167.11	167.11	47.57	47.57
0.75	21.69	21.51	53.81	53.81	21.69	21.69
1.00	13.97	13.68	22.48	22.48	14.01	14.01
1.50	8.51	8.27	7.83	7.83	8.55	8.55
2.00	6.09	5.95	5.38	5.38	6.12	6.12
3.00	5.01	5.00	5.00	5.00	5.01	5.01

Chapter 8

Guidelines for the Design of Regular SPRT Charts and of SPRT Charts With SFT, Conclusions, and Future Research

8.1 Guidelines for the Design of Regular SPRT Charts and of SPRT Charts With SFT

As in Chapters 5 and 7, the general framework of the UC chart will also be adopted for the discussions made in this last chapter. The design of a regular SPRT chart, as a special case of the UC chart, requires the specification of the sample size n, reference value γ, sampling intervals d_0, d_1 and d_2, and the chart's limits g and h. Similarly, the design of an SPRT chart with SFT requires the specification of the chart parameters n, γ, d_0, d_1 ($= t_w$), d (or equivalently, k), g, and h.

First, general guidelines will be provided for the design of a regular SPRT chart, and then they will be naturally extended for the design of an SPRT chart with SFT. Both the numerical SSATS results in Tables 5.2.1 – 5.2.60 and the numerical ATS results in Tables 5.3.1 – 5.3.60 showed that d_1 should be as small as possible, with $d_1 = 0$ offering the best efficiency. In particular, the ATS results in Tables 5.3.1 – 5.3.60 also showed that d_2 should be as large as possible if the ATS is used as the criterion of performance. These numerical ATS results are in general agreement with the theoretical results of Reynolds (1989), with only a few exceptions. These exceptions are due to the fact that Reynolds' (1989) results hold for a more restricted control chart model which assumes that the limits g and h do not change for different values of d_1 and/or d_2, unlike an SPRT chart's limits. If the SSATS is used as the criterion of performance, however, the SSATS results in Tables 5.2.1 – 5.2.50 show that d_2 should not be excessively large. Taken as a whole, the results in Tables 5.2.1 – 5.2.60 and Tables 5.3.1 – 5.3.60 show that a relatively small value of d_2 is preferable for large shifts in δ, and a relatively large value of d_2 is preferable for small shifts in δ. In most cases, a medium to small value of d_2, say $1.25 \leq d_2 \leq 1.75$, produces a chart that gives good performance over a large number of shifts. Given that a value for d_1 is chosen, then the choice of a value for d_2 depends somewhat on d_1. Specifically,

(i) if $0 \leq d_1 < 0.1$, then a value of $d_2 = 1.5$ or 1.25 produces a regular SPRT chart that gives good performance over a large number of shifts;

(ii) if $d_1 > 0.1$, then a value of $d_2 = 1.75$, 1.5, or even 1.25 (if $d_1 > 0.3$), produces a regular SPRT chart that gives good performance over a large number of shifts.

It should be noted that in practice, it would probably be preferable to use "convenient" sampling intervals, as indicated by administrative considerations, so this should also be considered in choosing d_1 and d_2.

As noted in Section 3.2, the reference value γ is usually chosen according to the size of the shift δ that can be expected. For example, if a regular SPRT chart is designed to detect a shift from $\delta_0 = 0$ to a specified value δ_1, then optimality theory of SPRT's indicates that γ should be chosen to be $\delta_1/2$ (Ghosh (1970), pp. 93 – 98). In choosing the value for γ, however, care should be taken so that the corresponding regular SPRT chart gives good performance over a large number of shifts.

An important question in the design of regular SPRT charts is what sample size n to use. The choice of n depends on d_1 as well as γ. If $d_1 = 0$, then $n = 1$ produces a regular SPRT chart that gives the best performance uniformly for all values of γ. The results from Chapter 5, together with other very extensive results not explicitly reported in this dissertation, show that if AOR_0 denotes the desired AOR when $\delta = 0$, then for $2 \leq \text{AOR}_0 \leq 5$, the sample sizes that produce a regular SPRT chart giving good performance over a large number of shifts are:

(i) for $0 \leq d_1 < 0.01$, $n = 1$ or 2, with $n = 1$ recommended for $\gamma < 0.25$, and $n = 2$ recommended for $\gamma \geq 0.25$;

(ii) for $0.01 < d_1 \leq 0.03$, $n = 2$ or $\min(3, \lfloor \text{AOR}_0 \rfloor)$ with $n = 2$ recommended for $\gamma < 0.25$, and $n = \min(3, \lfloor \text{AOR}_0 \rfloor)$, recommended for $\gamma \geq 0.25$;

(iii) for $0.03 < d_1 \leq 0.07$, $n = \min(3, \lfloor \text{AOR}_0 \rfloor)$ or $\min(4, \lfloor \text{AOR}_0 \rfloor)$, with $n = \min(3, \lfloor \text{AOR}_0 \rfloor)$ recommended for $\gamma < 0.25$, and $n = \min(4, \lfloor \text{AOR}_0 \rfloor)$ recommended for $\gamma \geq 0.25$

(iv) for $0.07 < d_1 \leq 0.15$, $n = \min(4, \lfloor \text{AOR}_0 \rfloor)$ or $\min(5, \lfloor \text{AOR}_0 \rfloor)$, with $n = \min(4, \lfloor \text{AOR}_0 \rfloor)$ recommended for $\gamma < 0.25$, and $n = \min(5, \lfloor \text{AOR}_0 \rfloor)$ recommended for $\gamma \geq 0.25$;

(v) for $0.15 < d_1$, $n = \min(5, \lfloor \text{AOR}_0 \rfloor)$ for all γ values.

Note that

$$\min(n_1, n_2) = \begin{cases} n_1 & , \text{ if } n_1 \leq n_2 \\ n_2 & , \text{ if } n_1 \leq n_2 \end{cases} \tag{8.1.1}$$

and recall from Section 6.2 that $\lfloor . \rfloor$ is the floor function, which is defined as the greatest integer no larger than the argument of the function.

Additional questions in the design of regular SPRT charts are what sampling interval d_0 to use before the first sample when the chart is first put into operation and what starting value of the control statistic to use. These questions will not be addressed here. However, for a good discussion of these two questions, the reader is referred to the discussion by Reynolds et al. (1990a), though for the VSI CUSUM chart, and to Lucas and Crosier's (1982a) paper on the FIR feature, though for the FSI CUSUM chart. Reynolds' et al. (1990a) discussion and Lucas and Crosier's (1982a) investigation also apply to the regular SPRT chart.

Finally, as discussed in Section 5.1, given values for n, γ, d_0, d_1, and d_2, the limits g and h are determined as solutions to the system of equations

$$\begin{aligned} \text{ATS} &= \text{ATS}_0 \\ \text{AOR} &= \text{AOR}_0 \quad , \end{aligned} \tag{8.1.2}$$

where ATS_0 and AOR_0 denote the ATS and AOR, respectively, of the regular SPRT chart, when $\delta = 0$. The extensive tables in Chapter 5, Tables 5.2.1 – 5.2.60, 5.3.1 – 5.3.60, and 5.4.1 – 5.4.60 may be used as aids in the design of regular SPRT charts. An additional, much more extensive collection of detailed tables for the properties of the regular SPRT chart is available from the author of this dissertation. These tables cover a broad range of parameter combinations in detail, including respective values for the limits g and h, and contain values for all of the properties of the regular SPRT chart defined in Section 3.3 (including SSATS, ATS, and ANOS) for a broad range of shifts. These tables also may be used as aids in the design of regular SPRT charts.

The above general guidelines for the design of regular SPRT charts apply in a completely analogous manner to the design of SPRT charts with SFT. The recommended guidelines for choosing a value of d_2 for the regular SPRT chart are the same as those for choosing the value of the sampling interval d, between the "pre-specified, equally spaced fixed times" in the SPRT chart with SFT, with the additional restriction that d be a constant multiple of $d_1(=\mathrm{t}_w)$. In addition to efficiency requirements, the choice of d would typically be determined by administrative considerations, as noted in Section 6.3.

In conclusion, the modular FORTRAN 77 software developed by Stoumbos and Reynolds (1992b) may be used for the evaluation of statistical properties and constrained determination of parameters (according to system **(8.1.1)**) in the design of a regular SPRT chart or an SPRT chart with SFT. A machine-readable copy of this software may be obtained from the principal developer, who is also the author of this dissertation. The software is very user-friendly, efficient (quick), and accurate. For details about the software, see Stoumbos and Reynolds' (1992b) paper which includes an example of the use of this software, along with its complete listings.

8.2 Conclusions

Extensive comparisons of the regular SPRT chart with the $\overline{X}$ and CUSUM charts, both with and without the VSI feature, were made in Chapter 5. In Chapter 7, the SPRT chart with SFT was compared with the regular SPRT and with the $\overline{X}$ and CUSUM charts, both with and without the VSI feature.

The results in Chapter 5 demonstrated that the regular SPRT chart is substantially more efficient than both the FSI and VSI $\bar{X}$ charts and the FSI CUSUM chart, though very similar in efficiency to the VSI CUSUM chart. Unlike the VSI CUSUM chart, however, the structure of the regular SPRT chart, as reflected by its sampling interval function, provides a very intuitive and rather convenient regularity to the process as a sequence of SPRT's (for testing the hypothesis that the process is in control) which are conducted as fast as possible and without any sampling between consecutive SPRT's. In fact, for the negligible sampling interval $d_1 = 0$, the regular SPRT chart is an FSI control chart.

In particular, the results in Chapter 5 demonstrated that the model $(d_1, n) = (0, 1)$ offers the "best" overall efficiency to both the regular SPRT and VSI CUSUM charts, with the efficiency of the regular SPRT chart being very similar to that of the VSI CUSUM chart. For this model, however, unlike the VSI-FSS CUSUM chart, the regular SPRT chart can be considered dually as a VSI-FSS control chart, and also as an FSI-SSS control chart. Although the FSI-SSS regular SPRT chart could present administrative problems, it should be more suitable for most applications than a VSI-FSS control chart, such as the VSI-FSS CUSUM chart, since the latter presents similar administrative problems in addition to the unwelcomed VSI irregularity.

The SFT modification of the regular SPRT chart, which was developed in Chapter 6, is naturally suitable and simple for the regular SPRT chart because of the structure of its sampling interval function. The implementation of this modification in VSI control charts, and hence in the regular SPRT chart, is much more practical than the VSI feature alone. Since for the special case $d_1 = 0$, the regular SPRT chart is an FSI control chart, the practical importance of the SFT modification to the SPRT chart manifests itself in the more often encountered cases where $d_1 > 0$. It is important to note that the implementation of the SFT modification in the VSI CUSUM chart would be quite complicated because of the cumbersome mathematics required to evaluate the VSI CUSUM chart's properties.

The results in Chapter 7 demonstrated that the SPRT chart with SFT is substantially more efficient than both the FSI and VSI $\bar{X}$ charts and the FSI CUSUM chart. Although still substantially more efficient, the improvement that the SPRT chart with SFT offers is not as large for the regular SPRT and VSI CUSUM charts as for the FSI and VSI $\bar{X}$ charts and the FSI CUSUM chart. Thus, it is clear that the SFT modification to the SPRT chart, as developed in Chapter 6, not only has minimal negative effects on the SPRT chart's high efficiency, but also provides a substantial improvement to this high efficiency for many shifts, rendering the SPRT chart with SFT the best choice among the control charts compared in this dissertation. This is especially true, since unlike the VSI $\bar{X}$ and CUSUM charts, the SPRT chart with SFT, by definition, incorporates the desirable feature of sampling at pre-specified, equally spaced fixed times.

Finally, the regular SPRT chart and, hence, the SPRT chart with SFT reflect the rational subgroup concept much better than the VSI CUSUM chart, because their sampling interval functions group together the observed information in each SPRT in as short a time period as possible. This is particularly true for a relatively short sampling interval d_1. Observations taken too infrequently or irregularly do not reflect the rational subgroup concept, and thus do not provide sufficient information about the process. Hence, the VSI CUSUM chart will not necessarily reflect the rational subgroup concept in its entirety (in its SPRT's), especially without taking great care.

8.3 Future Research

In this last section, topics for future research beyond this dissertation will be discussed. Some of these topics have been and/or are currently under investigation by Stoumbos and Reynolds.

Two-sided control procedures for the detection of two-sided deviations in μ were not considered here for either the regular SPRT chart, the SPRT chart with SFT, or the UC chart. For any one of these charts, however, two one-sided such charts may be applied simultaneously to detect two-sided deviations in μ, one to detect an increase and the other a decrease. When two such one-sided VSI control charts are applied simultaneously to form a two-sided control procedure, the two one-sided charts may specify different sampling intervals. For example, the upper one-sided control chart could specify d_1, while the lower one-sided control chart could specify d_2, or vice-versa. For a two-sided VSI control procedure, a reasonable rule for specifying the length of the sampling interval is to use the short d_1 if either one-sided chart specifies d_1 and use the long d_2 if both charts specify d_2 (Reynolds et al. (1990a)). Clearly, two-sided VSI control procedures constitute non-trivial extensions of their one-sided counterparts. Since in practice, the majority of control chart applications are two-sided, it is important that two-sided counterparts to the regular SPRT chart, the SPRT chart with SFT, and the UC chart be developed and evaluated. This is particularly important for the very efficient and practical SPRT chart with SFT.

In this research, the sample number of the SPRT's was not restricted by any upper bounds. It may be, however, that for some processes, there are practical and economical limitations on the sample number requiring it not to exceed a specified fixed positive integer n_s. This is particularly true for the cases where the sampling interval d_1 is negligible or small. Hence, further research must be conducted for the development and evaluation of the *truncated* SPRT chart, a modification of the regular SPRT chart which applies a truncated SPRT at fixed sampling intervals. For the cases where the sampling interval d_1 is medium or large, truncation is much less important, and the benefits of truncation should not outweigh the loss in efficiency.

A problem which arises for most all control charts is that in practice, it may be necessary to estimate the target value μ_0 and/or the process standard deviation σ from past data, but for simplicity it was assumed here that μ_0 and σ are known. Hence, further investigation needs to be directed at developing methods to obtain "good" estimates of μ_0 and/or σ, when μ_0 and/or σ are unknown, for the regular SPRT chart, the SPRT chart with SFT, and the UC chart. Once these estimates are obtained, efforts should be made to evaluate the properties of these control charts and study the effects of the estimates of μ_0 and σ on the respective efficiencies of the three control charts introduced.

In this research, the focus was on the univariate SPC problem of monitoring the mean μ of a single measurable characteristic X whose distribution is normal. There are many univariate SPC applications, however, in which the monitoring of different parameters from various distributions is required, besides the mean of a univariate normal distribution. In fact, there are univariate SPC applications in which the monitoring of two or more parameters in a univariate process is required. There are also many multivariate SPC applications in which the simultaneous monitoring of two or more related process characteristics is required. The monitoring of multiple quality characteristics can be done using multiple control charts or a single control chart which uses an appropriate univariate control statistic for monitoring multivariate data. Since the common control statistic of the regular SPRT chart, the SPRT chart with SFT, and the UC chart is the loglikelihood ratio, these three control charts naturally lend themselves to the univariate and multivariate SPC applications discussed in this paragraph, and extensions of these control charts to these applications deserve investigation.

The properties of the regular SPRT chart, the SPRT chart with SFT, and the UC chart were developed in this research under the assumption that the observations from the process are independent. For some processes, however, this assumption may not be realistic because there is positive serial correlation between closely spaced observations. When considering the numerical evaluations of the properties from this research for a serially correlated process, the small sampling interval d_1 should be large enough so that successive samples are approximately independent. In certain applications where serial correlation is present, even this solution may be artificial or impractical. Approaches such as the ones discussed in Section 2.9 can be used to address the complications associated with serially correlated data. Of these approaches, in agreement with Woodall (1990), the approach by Alwan and Roberts (1988) seems preferable to attempts to use a single modified control chart to monitor serially correlated data. The SPRT chart (both regular and with SFT) and the UC chart could be used with serially correlated observations, but in this case the properties must be developed and evaluated under a model which allows for serial correlation. Clearly, the practical importance of investigating the properties of the SPRT chart under such a model is quite substantial, particularly for the SFT modification.

In this research, the properties of the regular SPRT chart, the SPRT chart with SFT, and the UC chart have been developed and evaluated for the case in which the shift in process mean remains constant until the shift is detected and the assignable cause is eliminated. This model, however, is a simplification of what may occur in certain applications where the process mean gradually drifts away from the process target. Hence, further research must be conducted for the development and evaluation of the properties of the three charts introduced under a model which allows for a drift in the process mean.

Recall that the motivation for the development of the UC chart came from the need to develop a general framework for the study of control charts that are equivalent to a sequence of SPRT's and for the quantitative comparison of control charts that use different sample sizes and/or different sampling methods. The general framework of the UC chart can also be used together with the modular FORTRAN 77 software developed by Stoumbos and Reynolds (1992a) for the chart, in order to investigate the important control chart design question of what the length of the sampling interval d_2 should be (or what the length of the sampling interval d should be, for the SFT model) for the SPRT, VSI CUSUM, and VSI $\overline{X}$ charts, in order to "best" detect certain shifts while maintaining a desired level of overall efficiency. This investigation is well underway by Stoumbos and Reynolds, and recommendations are becoming available, as briefly presented in Section 8.1. For example, for the regular SPRT and VSI CUSUM charts, if good protection from very large shifts is desirable while also maintaining a high level of overall efficiency, then a shorter sampling interval d_2 is preferable when possible, together with a small reference value γ, particularly for a small sampling interval d_1. This recommendation is different from the existing notion of using large reference values for good protection from large shifts.

Efforts should be made to investigate the theoretical optimality properties of the SPRT chart (both regular and with SFT). Intuitively, it seems that just as the CUSUM chart's stopping rule, that of the SPRT chart should also be minimax in the sense of Lorden (1971). The two different approaches that Moustakides (1986) and Ritov (1990) used to establish that the CUSUM's stopping rule is minimax in the sense of Lorden (1971) should also be successful in establishing minimax optimality in the sense of Lorden (1971) for the SPRT chart. Such a theoretical investigation for the SPRT chart is underway and promising results appear to have been reached.

For a more complete treatment of this research, the VSS feature should be introduced to both the regular SPRT chart and the SPRT chart with SFT, and the properties of the VSS versions of these charts should be evaluated. Then, in order to assess the performances of the regular VSS SPRT chart and the VSS SPRT chart with SFT, quantitative comparisons would need to be made of these two VSS charts with their FSS counterparts, and also with the FSS and VSS versions of the FSI and VSI $\bar{X}$ and CUSUM charts. The VSS version of the general UC chart would provide an appropriate framework for such comparisons.

In conclusion, the SPRT chart's model (both regular and with SFT) naturally lends itself to automatic process control (APC) applications, because it is maintained by continuously/sequentially obtaining samples as fast as possible at sampling points in order to conduct SPRT's, in turn, as fast as possible. That is, the SPRT chart's model can be used to develop and investigate the properties of a control scheme which employs APC theory and even an integrated control scheme which combines automatic and statistical process control, while maintaining high efficiency.

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