THE OCEAN TIDE AND WAVES BENEATH THE
ROSS ICE SHELF, ANTARCTICA,

by

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in
Geophysics

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This dissertation is dedicated to my wife, Lois, who quite literally traveled to one end of the earth to help me complete my formal education. On my behalf, she endured the rigors of life in Antarctica and spent long hours preparing this manuscript, and her efforts deserve special recognition.
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>THE OCEAN TIDE IN THE ROSS SEA</td>
<td>5</td>
</tr>
<tr>
<td>Introduction</td>
<td>5</td>
</tr>
<tr>
<td>Previous Work</td>
<td>7</td>
</tr>
<tr>
<td>Instrumentation</td>
<td>13</td>
</tr>
<tr>
<td>RISP Tidal Gravity Measurements</td>
<td>14</td>
</tr>
<tr>
<td>The Gravimetric Effect of the Ocean Tide</td>
<td>21</td>
</tr>
<tr>
<td>Tidal Harmonic Analysis</td>
<td>23</td>
</tr>
<tr>
<td>Uncertainties in the Harmonic Constants</td>
<td>31</td>
</tr>
<tr>
<td>Cotidal-Coamplitude Charts</td>
<td>36</td>
</tr>
<tr>
<td>Tide Prediction</td>
<td>58</td>
</tr>
<tr>
<td>Summary</td>
<td>63</td>
</tr>
<tr>
<td>TIDAL CURRENTS BENEATH THE ROSS ICE SHELF</td>
<td>64</td>
</tr>
<tr>
<td>Introduction</td>
<td>64</td>
</tr>
<tr>
<td>A Theoretical Basis for Tidal Current Calculations</td>
<td>66</td>
</tr>
<tr>
<td>Current Meter Data from J9</td>
<td>73</td>
</tr>
<tr>
<td>Theoretical J9 Tidal Current</td>
<td>83</td>
</tr>
<tr>
<td>Semidiurnal Current Resonance in the Ross Sea</td>
<td>94</td>
</tr>
<tr>
<td>Mass Transport by Tidal Currents at J9</td>
<td>97</td>
</tr>
<tr>
<td>Summary</td>
<td>98</td>
</tr>
<tr>
<td>FLEXURAL WAVES IN THE ROSS ICE SHELF</td>
<td>101</td>
</tr>
<tr>
<td>Introduction</td>
<td>101</td>
</tr>
<tr>
<td>Flexural Wave Theory</td>
<td>108</td>
</tr>
<tr>
<td>Topic</td>
<td>Page</td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>Digital Recorders for the Gravimeters</td>
<td>111</td>
</tr>
<tr>
<td>Gravimeter Frequency Response</td>
<td>114</td>
</tr>
<tr>
<td>Nontidal Wave Data</td>
<td>119</td>
</tr>
<tr>
<td>Speed and Direction of the Nontidal Waves at J9</td>
<td>125</td>
</tr>
<tr>
<td>Flexural Wave Power Spectrum</td>
<td>134</td>
</tr>
<tr>
<td>Summary</td>
<td>147</td>
</tr>
<tr>
<td>RESULTS</td>
<td>150</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>153</td>
</tr>
<tr>
<td>APPENDIX I</td>
<td>157</td>
</tr>
<tr>
<td>APPENDIX II</td>
<td>183</td>
</tr>
<tr>
<td>APPENDIX III</td>
<td>203</td>
</tr>
<tr>
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<td>204</td>
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INTRODUCTION

The quasi-periodic fluctuation in the elevation of the sea surface was one of the earliest observed physical phenomena. The ancient Greeks knew that the time of high tide was somehow related to the positions of the sun and moon, but it was not until the eighteenth century and Newton's formulation of the law of gravity that the tidal forces experienced by the waters on the surface of the earth could be expressed mathematically. The hydrodynamical equations for a column of water on a rigid, rotating earth were first formulated by Laplace in 1775. Since that time, rigorous solutions to these equations for the tide in various tesseral seas have been obtained. Unfortunately, these solutions shed little light on the nature of the ocean tide on the real earth. The irregular shapes of the ocean basins dictate a numerical solution to the problem, and such numerical methods for solving Laplace's tidal equations have become practical only within the past twenty years with the development of the electronic digital computer. In that time the equations have been used to infer the tide in a number of limited areas, and also for the tide in the world ocean. However, there is notable disagreement in the published maps of even the most important components of the global tide, made by different investigators. One area in which serious differences exist is in the southern Pacific Ocean. This uncertainty can be attributed largely to a lack of observational data in the region, particularly in the extreme south, adjacent to the Antarctic
continent. This dissertation represents the conclusion of an extended series of observations of the ocean tide in this region.

The Ross Sea is a marine embayment penetrating more than 1000 km into the Pacific sector of the Antarctic continent (Figure 1). The southern portion of this sea is covered by the Ross Ice Shelf, a tabular mass of floating ice which is almost everywhere between 300 m and 600 m thick. Situated for the most part between 78°S and 85°S latitude, and between 160°E and 210°E longitude, the Ross Ice Shelf compares in areal extent to France, covering about 560,000 sq km. A comprehensive research program, the Ross Ice Shelf Project (RISP), was begun in 1973 to investigate the interdependent glaciologic, oceanographic, geologic, and biologic phenomena pertaining to this region (Zumberge, 1971). A major effort in the project was to drill a hole through the ice shelf to provide direct access for current measurements, water temperature and salinity measurements, and sampling of the bottom sediments and bio-mass beneath the ice shelf.

The tide beneath the Ross Ice Shelf is of interest for several reasons. Knowledge of the variation of the tide in the Ross Sea region contributes to the understanding of the tide in the adjacent southern Pacific Ocean. Knowledge of the ocean currents beneath the ice shelf bears upon the problems of mass wastage by bottom melting of shelf ice, and the transport of marine organisms and sediment. Because of the ice cover, wind driven circulation is minimized, and the tidal current may be the largest component of water movement.
Figure 1. The Ross Sea area of Antarctica. Circles denote observation sites occupied in connection with RISP: triangles denote locations where observations of the ocean tide were made prior to RISP.
The interpretation of short-term current measurements made through boreholes in the ice shelf is facilitated if the tidal variation in the current is known.

The author has participated in research activities related to RISP since the summer of 1974, and traveled to Antarctica to participate in the field program during the 1974-75 and 1977-78 austral summers. Oceanographic data collected by the author and others, working in Antarctica in connection with RISP during four austral summers between 1973 and 1978, are the principal subject of this dissertation. The primary goal of this research program has been the determination of the spatial and temporal variation of the elevation of the ice shelf, due to the ocean tide in the underlying water-layer. Other topics investigated are the tidal currents in the water-layer, and some observed waves in the ice shelf with periods in the 1 min to 15 min range.
THE OCEAN TIDE IN THE ROSS SEA

Introduction

Measurements of the tide beneath the ice shelf cannot easily be made by conventional methods, which require access to the water. While instruments can be adapted for use in an access hole (Jacobs and others, 1978), the cost of drilling a series of access holes for tide measurements is prohibitive. In this study, the tidal elevation change at the surface of the ice due to the tide in the underlying water was inferred from measured tidal gravity variations. The increase in elevation of the floating ice shelf at high tide causes a change in the gravity at the surface which is detectable by a gravimeter. Gravity records were obtained by RISP personnel at eight locations (Figure 1, Table 1) distributed over the Ross Ice Shelf, using Geodynamics model TRG-1 tidal recording gravimeters.

RISP field operations usually commenced about 1 November and terminated before 1 February to take advantage of the relatively good weather of the austral summer. In the field, the gravimeters were installed in 5 m square Jamesway prefabricated buildings. For a stable foundation, the gravimeters were set on platforms mounted on 4 in square wooden posts extending approximately 3 m into the firn through a hole in the Jamesway floor. Electrical power was obtained from Onan 6 kw diesel generators. An attendant at each site was responsible for maintenance of the generator and heating stove, verifying that the gravimeter was operating normally, and placing calibration and hourly time signals on the record.
Table 1. Locations of stations cited in this study.

<table>
<thead>
<tr>
<th>Designation</th>
<th>Latitude</th>
<th>Longitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>82.53°S</td>
<td>166.00°W</td>
</tr>
<tr>
<td>Cape Adare</td>
<td>71.28°S</td>
<td>189.77°W</td>
</tr>
<tr>
<td>Cape Evans</td>
<td>77.63°S</td>
<td>193.60°W</td>
</tr>
<tr>
<td>Cape Royds</td>
<td>77.55°S</td>
<td>193.85°W</td>
</tr>
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<td>C13</td>
<td>79.25°S</td>
<td>189.67°W</td>
</tr>
<tr>
<td>C16</td>
<td>81.19°S</td>
<td>189.50°W</td>
</tr>
<tr>
<td>C36</td>
<td>79.75°S</td>
<td>169.05°W</td>
</tr>
<tr>
<td>F9</td>
<td>84.25°S</td>
<td>171.33°W</td>
</tr>
<tr>
<td>Hut Point</td>
<td>77.85°S</td>
<td>193.37°W</td>
</tr>
<tr>
<td>J9</td>
<td>82.37°S</td>
<td>168.63°W</td>
</tr>
<tr>
<td>Little America V (LAS)</td>
<td>78.20°S</td>
<td>162.27°W</td>
</tr>
<tr>
<td>McMurdo Base (McM)</td>
<td>77.85°S</td>
<td>193.34°W</td>
</tr>
<tr>
<td>McMurdo Sound</td>
<td>77.50°S</td>
<td>195.00°W</td>
</tr>
<tr>
<td>019</td>
<td>79.53°S</td>
<td>196.64°W</td>
</tr>
<tr>
<td>Roosevelt Island Camp (RI)</td>
<td>80.19°S</td>
<td>161.56°W</td>
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</tbody>
</table>
Previous Work

The early Antarctic expeditions traveled by ship, and naturally made observations of the ocean tide. Near the turn of the century, four British expeditions made tidal observations in the Ross Sea at (Table 1) Cape Adare (Bernacchi, 1901), Hut Point (Darwin, 1908), Cape Royds (Shackleton, 1909) and Cape Evans (Doodson, 1924). These measurements were not useful for the purpose of this study. The Cape Adare observations were made for one day only, and thus cannot be analyzed for tidal content. The remaining three locations are all on McMurdo Sound (near McM in Figure 1), near the site of major United States and New Zealand base camps and lengthy, higher quality records of the tide have been made there since 1957.

Beginning in the International Geophysical Year (IGY), 1957, ocean tide records have been made at several sites in the Ross Sea. In McMurdo Sound, near the northern extremity of the Ross Ice Shelf, sea level recordings have been reported for two locations near the site of McMurdo Station (McM, Figure 1) by MacDonald and Burrows (1959) and Heath (1971). The results of the analysis of these data have been summarized by Heath, and by Gilmour and others (1962). For the purpose of this dissertation, these two locations are so close together that they will be considered to be one station. At Little America V (LAS, Figure 1), on the eastern side of the ice front, gravity readings were made at one- or two-hour intervals from 11 June through 12 July, 1957, using a Frost gravimeter (Thiel and others, 1960). The observed change in gravity, shown in Figure 2, was shown to be
Figure 2. Gravimetric observations at Little America V, after Thiel and others (1960).
caused by the elevation of the station changing in response to the ocean tide beneath the ice shelf. McMurdo Sound and Little America V are the only places in the Ross Sea at which lengthy measurements of the height of the ocean tide were made prior to the present study.

The McMurdo and Little America records were made at the northern edge of the Ross Ice Shelf. There was also evidence developed for the existence of a tide extending beneath the southern extremity of the shelf, about 850 km distant from the ice front. At LAS current measurements made simultaneously with gravimetric surface elevation measurements demonstrated in a crude way that the volume of water flowing from beneath the shelf, per meter of section in a given time interval, was consistent with the flow required if the tide were the same everywhere beneath the shelf. More direct evidence came from gravimeter readings made during the Ross Ice Shelf Traverse (1957-58; Bennett, 1964). A Frost gravimeter was read at irregular intervals at several observation sites (Figure 3), which were occupied for periods of 12 to 48 hours. The gravimeter readings are reproduced in Figure 4, with dashed lines to illustrate that a diurnal fluctuation in gravity can account for the observed variation in the meter readings. A three-day-long study in November 1959 (Thiel, 1960) established the existence of a tide beneath the shelf at a point about 300 km south of the ice front. A graph of the meter readings (Figure 5) shows a diurnal gravity fluctuation, but the record was too short for meaningful analysis.
Figure 3. Sites of short term tidal gravity observations on the Ross Ice Shelf. Numbered stations were occupied during the 1957-58 Ross Ice Shelf Traverse (Bennett, 1964). Observations reported by Thiel (1960) were made at site labeled T.
Figure 4. Tidal fluctuations in the southern Ross Ice Shelf as indicated by observed gravity, after Bennett (1964).
Figure 5. Gravimetric recording of the ocean tide, reported by Thiel (1960), at a site on the Ross Ice Shelf.
Instrumentation

The gravity data obtained during RISP were recorded using three different Geodynamics model TRG-1 gravimeters. The basic unit is a North American exploration-type gravimeter that was modified by Geodynamics, Inc. for continuous and stable long-term recording of tidal gravity variations. The basic design of the gravimeter consists of a mass hinged on quartz wires, and supported by a zero-length spring (LaCoste, 1935). The gravimeter is mounted inside an air-tight container, which is in turn enclosed in a well-insulated wooden case. The manufacturer specifies that the temperature of the instrument, in the inner-most container, is controlled to within 0.005 C degrees of a manufacturer-determined optimum near 36°C. Changes in the gravitational force on the mass in the gravimeter are sensed by monitoring the position of the mass by means of a differential-capacitor transducer. A continuous record of the variations in gravity is obtained by recording a voltage, derived from the transducer, which is proportional to the mass position.

Instrument calibration is possible by means of an electrostatic force generated by applying a precise voltage to a fixed (capacitor) plate mounted over the gravimeter mass. The gravity-change equivalent of the electrostatic force is constant, and can be used to relate observed displacements of the mass to the gravity change. Calibration of the instrument to within about 3% was achieved by using earth-tide recordings made at Blacksburg, Virginia. The equivalent microgal shift in the mass position introduced by activation of the
calibration system was determined for each instrument by comparing the calibration pulse height with the amplitudes of the lunar diurnal (O$_1$) and lunar semidiurnal (M$_2$) constituents of the earth tide (Table 2). These amplitudes were assigned the assumed standard values of 34.7 \textmu gals and 56.5 \textmu gals, respectively. Each of these values is the product of the theoretical amplitude at 37.23° latitude on the surface of a rigid earth (Melchoir, 1966; Schureman, 1941), and a tidal gravimetric factor which accounts for elastic yielding of the earth. The following gravimetric factors for Blacksburg were used (Robinson, 1974; Jachens, 1971): $\delta_{O_1} = 1.16$, $\delta_{M_2} = 1.19$.

**RISP Tidal Gravity Measurements**

Eight tidal gravity stations (Figure 1, Table 1) were established on the Ross Ice Shelf during four austral summers between 1973 and 1978. The station positions were chosen, consistent with available logistical support, so as to cover the study area uniformly. In a given year, generally, two camps were established, one coordinated with other RISP programs and one self-sufficient. During the 1976-77 field season no RISP field program was conducted. At two stations, C13 and 019, two instruments were operated simultaneously. One station, C16, was reoccupied, so that two records from successive years were obtained there. A total of 11 records were obtained from eight RISP camps, and these are shown in Figure 6. The starting time and length of each record is shown in Table 3.
Table 2. Gravimeter calibration pulse gravity-change equivalents.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Calibration Pulse Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>826</td>
<td>82.2 μgals</td>
</tr>
<tr>
<td>735</td>
<td>133.0 μgals</td>
</tr>
<tr>
<td>VPI-17</td>
<td>126.0 μgals</td>
</tr>
</tbody>
</table>
Figure 6a. Tidal gravity records from RISP camps. Station positions are given in Figure 1 and Table 1. Record starting times are given in Table 3.
Figure 6b. Tidal gravity records from RISP camps. Station positions are given in Figure 1 and Table 1. Record starting times are given in Table 3.
Figure 6c. Tidal gravity records from RISP camps. Station positions are given in Figure 1 and Table 1. Record starting times are given in Table 3.
Figure 6d. Tidal gravity records from RISP camps. Station positions are given in Figure 1 and Table 1. Record starting times are given in Table 3.
Table 3. RISP tidal gravity record lengths and starting times.

<table>
<thead>
<tr>
<th>Station</th>
<th>Instrument</th>
<th>Length (Hours)</th>
<th>Starting Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>826</td>
<td>1056</td>
<td>20 Dec 73 0000 GMT</td>
</tr>
<tr>
<td>C13</td>
<td>735, 826</td>
<td>696</td>
<td>10 Nov 74 0000 GMT</td>
</tr>
<tr>
<td>C16</td>
<td>735</td>
<td>1068</td>
<td>9 Dec 76 1200 GMT</td>
</tr>
<tr>
<td></td>
<td>826</td>
<td>768</td>
<td>26 Dec 77 0000 GMT</td>
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<td>C36</td>
<td>826</td>
<td>816</td>
<td>29 Dec 74 0000 GMT</td>
</tr>
<tr>
<td>F9</td>
<td>826</td>
<td>1392</td>
<td>24 Nov 76 1200 GMT</td>
</tr>
<tr>
<td>J9</td>
<td>17</td>
<td>720</td>
<td>19 Nov 76 0000 GMT</td>
</tr>
<tr>
<td>O19</td>
<td>17, 735</td>
<td>936</td>
<td>6 Dec 77 0000 GMT</td>
</tr>
<tr>
<td>RI</td>
<td>17</td>
<td>858</td>
<td>23 Dec 74 0000 GMT</td>
</tr>
</tbody>
</table>
The Gravimetric Effect of the Ocean Tide

The diurnal variation of gravity was generally 300 µgals to 400 µgals at the time of tropic (diurnal spring) tide. This fluctuation is interpreted as resulting principally from changes in elevation and water-mass underlying the recording site associated with the ocean tide. Smaller contributions are due to the direct attraction of the moon and sun on the gravimeter, and from the vertical acceleration associated with elevation changes of the instrument. The ocean tide was discussed by Thiel and others (1960), and a portion of their development is presented here.

During high tide an additional mass of ocean water exists over a local region. The increase in elevation, $\Delta h$, of the floating shelf related to the introduction of this mass causes a decrease in the gravity field, $\Delta g_{FA}$, detected by a gravimeter on its surface according to the free air effect:

$$\Delta g_{FA} \text{ (mgals)} = -0.3086 \Delta h \text{ (meters)}$$

At the same time an increase in gravity can be expected from the attraction of the additional water mass, which can be adequately approximated by an infinite plate. Viewing this as similar to a Bouger correction, the gravity change, $\Delta g_{B}$, associated with an increased plate thickness, $\Delta h$, is given approximately by:

$$\Delta g_{B} \text{ (mgals)} = 0.04185 \rho \Delta h \text{ (meters)}$$

where $\rho$ is the plate (sea water) density in gm/cm$^3$. Combining these
two effects, and assuming a water density of 1.028 gm/cm$^3$, Thiel and others found the relationship between water-layer thickness change and gravity change to be:

$$\Delta h \text{ (meters)} = - 3.7653 \Delta g \text{ (mgals)}$$

A density of 1.030 gm/cm$^3$ to 1.032 gm/cm$^3$ would be more appropriate for the sea water beneath the Ross Ice Shelf (Arnold Gordon, personal communication). However these values would change the constant in equation (3) by less than 0.1%, and the difference is not important in this study.

The direct luni-solar gravity can be subtracted from the recorded gravity variations (Robinson and others, 1977). Well known formulas for the gravity tide on a rigid earth have been summarized by Longman (1959), and Broucke and others (1972). However, since the earth is not perfectly rigid, an allowance must be made for its elastic yielding. The effect of the elasticity is to increase the earth tide amplitude by about 16%, and to time-shift the maximum tide by a small amount, probably less than 20 min (Jachens, 1971). The amplitude of the earth tide is about one-fifth as large as the ocean tide effect at LAS, and decreases, due to increasing latitude, to about one-twentieth at F9. Because the earth tide is small compared to the ocean tide effect, the error in the tidal gravity, corrected for the luni-solar effect by assuming a 16% amplitude increase and no time-shift, is probably less than 2% for the amplitude and 5 min in the time-shift of the peaks near LAS. These
errors are smaller at more southern stations, and are acceptable in that they are smaller than the uncertainty in our instrument calibration.

The accelerations resulting directly from the vertical displace­ments are small, and need not be accounted for. An upper limit on these accelerations can be established based on a 1000 μgal tidal gravity change, which is larger than any observed. This gravity change corresponds to an elevation change (equation 3) of 377 cm. The gravity variation is diurnal, so the acceleration, \( a_D \), is approx­imatively:

\[
\begin{align*}
a_D &= 377 \times (2\pi/86400)^2 \times 10^6 \text{ μgal} \\
&= 2 \text{ μgal} \\
&= 0.2\% \text{ of the gravity change due to the elevation change. This is much less than the uncertainty in the instrument calibration.}
\end{align*}
\]

**Tidal Harmonic Analysis**

The tide generating force experienced by the water masses on the earth results primarily from the gravitational attraction of the moon and sun, and the earth's rotation. The height of the tide at a particular place may be represented harmonically by the formula (Schureman, 1941):

\[
h = \sum fH \cos (at + \phi - \kappa)
\]
where

\[ h = \text{height of the tide at time } t \]
\[ H = \text{mean amplitude of the tide constituent} \]
\[ f = \text{factor for correcting the mean amplitude } H \text{ to the year of } t, \text{ always near unity} \]
\[ a = \text{constituent frequency} \]
\[ \phi = \text{phase of the corresponding term in the tide generating force when } t = 0 \]
\[ \kappa = \text{phase lag of the constituent with respect to the generating force} \]

Although the tide-generating force contains a large number of constituents of the form indicated in equation (5), the tidal water level change at a given location can usually be represented accurately by a small number of constituents which have large amplitude. For each constituent in the sum, two quantities, the amplitude \( H \) and the phase \( \kappa \), must be determined from observational data. These quantities are time-independent and are called the harmonic constants.

The tidal gravity records obtained during RISP were analyzed for the harmonic constants of the tide by the standard method of the National Ocean Survey (Schureman, 1941). In addition, the LAS record in Figure 2 was digitized at one-hour intervals and subjected to the same analysis. The results of the analysis are presented in Table 4. Also in the table are the harmonic constants of the tide at McMurdo (Gilmour, 1963). The analysis indicated that the height of the ocean
Table 4. Tidal harmonic constituents in the southern Ross Sea.

<table>
<thead>
<tr>
<th>Site</th>
<th>K1</th>
<th>P1</th>
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<th>M2</th>
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<td>Base</td>
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<td>McM</td>
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<td>186</td>
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<td>O19</td>
<td>31</td>
<td>208</td>
<td>10</td>
<td>208</td>
<td>29</td>
<td>196</td>
</tr>
<tr>
<td>RI</td>
<td>44</td>
<td>160</td>
<td>15</td>
<td>160</td>
<td>38</td>
<td>140</td>
</tr>
</tbody>
</table>

*a Amplitude in cm

b Phase angle in degrees relative to the Greenwich Meridian
tide in the southern Ross Sea can be represented by six harmonic constituents. The three largest of these are diurnal, and account for the diurnal character of the observed gravity tide (Figure 6). These seven constituents comprise the four largest diurnal constituents, and the three largest semidiurnal constituents in the tide-producing force. Table 4 shows the frequency, maximum amplitude, and amplitude at 78°S latitude of the luni-solar gravity for each of the 9 greatest constituents of the tide-producing force. The results (Table 4) for LAS are about 20% larger than reported by Thiel and others (1960). The difference is due to the luni-solar effect, for which they made no correction.

Calculations required in the analysis were performed using a FORTRAN computer program (Dennis and Long, 1971) developed by the National Oceanic and Atmospheric Administration (NOAA). This particular program was used because it was designed for 29-day data series. The 29-day length is approximately a multiple of the synodic periods of the more important constituents of the tide-producing force. It is considered the minimum length necessary for a satisfactory determination of the most important constituents (Schureman, 1941, p. 51, 52), and it is appropriate for the lengths of the RISP tidal gravity records.

The use of 29-day data windows limits the frequency domain resolution of the analysis. The 29-day boxcar (rectangular) window used adequately separates all the important constituents (Table 5) except P₁ from K₁. Figure 7 shows the amplitude of the boxcar window in
<table>
<thead>
<tr>
<th>Constituent</th>
<th>Frequency (°/hr)</th>
<th>Maximum At 78°S Latitude (µgals)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q₁</td>
<td>13.3986609</td>
<td>6</td>
</tr>
<tr>
<td>Q₁</td>
<td>13.9430356</td>
<td>31</td>
</tr>
<tr>
<td>P₁</td>
<td>14.9589314</td>
<td>14</td>
</tr>
<tr>
<td>K₁</td>
<td>15.0410686</td>
<td>44</td>
</tr>
<tr>
<td>N₂</td>
<td>28.4397295</td>
<td>14</td>
</tr>
<tr>
<td>v₂</td>
<td>28.5125831</td>
<td>3</td>
</tr>
<tr>
<td>M₂</td>
<td>28.9841042</td>
<td>75</td>
</tr>
<tr>
<td>S₂</td>
<td>30.0</td>
<td>35</td>
</tr>
<tr>
<td>K₂</td>
<td>30.0821373</td>
<td>9</td>
</tr>
</tbody>
</table>
Figure 7a. Relative positions of the diurnal tide constituents in the frequency-domain window associated with a 29-day rectangular time-domain window. Constituent frequencies are given in Table 5.
Figure 7b. Relative positions of the semidiurnal tide constituents in the frequency-domain window associated with a 29-day rectangular time-domain window. Constituent frequencies are given in Table 5.
the frequency domain, and the relative positions of the more important constituents. The NOAA analysis separates $P_1$ and $K_1$ by assuming that the relationship between the constituents in the observations is the same as in the tide-generating force. The validity of this assumption on a world-wide basis is indicated by a test reported by Schureman (1941, p. 79). In that test 60 stations in different parts of the world where the harmonic constants had been determined from observations were selected, and a comparison was made between the constants obtained by inference and from observations. The following results were obtained: maximum difference 8.2 cm, 49°; average difference 0.9 cm, 8°; differences less than 1.5 cm, 85%; differences less than 3.0 cm, 92%; differences less than 10°, 76%; differences less than 20°, 92%. In most parts of the world, then, $P_1$ can be adequately separated from $K_1$ by inference. In the Ross Sea the validity of the inference technique can be tested using the data from two RISP camps, F9 and C16, and from McMurdo. Gilmour and others (1962) reported the analysis of records obtained at McMurdo Sound during the 29-day periods whose central days were 17 December 1957, 16 January 1958, 16 June 1958, and 16 July 1958. It is seen that the records occur in two pairs separated by one-half year. For the constituent $P_1$, the amplitude average from the four sets was 8.8 cm with the largest deviation from the mean being 0.3 cm or 3.4%. The largest deviation from the mean phase was 5°. At RISP camp F9, 58-days of data were obtained during the 1976-77 field season, which were analyzed as independent 29-day segments. The means of the
PI harmonic constants were 13.8 cm and 205°, with deviations of 0.3 cm (2.2%) and 2°. At C16, records obtained during the 1976-77 and 1977-78 field seasons provide independent 29-day sets for analysis. The means of the PI constants in this case were 10.6 cm and 199° with deviations of 0.4 cm (3.8%) and 0°. The evidence that PI and Kl can be separated reliably by inference based on the relationship of their driving forces is compelling.

Uncertainties in the Harmonic Constants

The uncertainties in the harmonic constants of the Ross Sea tide determined in this study (Table 4) can be estimated in two ways: (1) examination of Fourier amplitude spectra, and (2) comparison of the values obtained for independent 29-day records at C16 and F9.

Fourier amplitude spectra calculated from a representative 29-day data segment from each of the eight RISP camps are presented in Figure 8. Each of these spectra clearly shows peaks at the diurnal tidal frequencies, the largest peak being due to the combined effect of constituents PI and K1. All of the spectra except C13 also contain peaks at the semidiurnal tidal frequencies. The noise level on the spectra is generally near 5 μgals, except on the C13 record, which corresponds to a 2 cm uncertainty in the amplitudes in Table 3. Viewing the pair of harmonic constants as a vector, the uncertainty in the phase can be estimated by taking the noise to be a second vector of 2 cm amplitude oriented normal to the first. For the diurnal constituents the amplitudes are typically greater than
Figure 8a. Fourier amplitude spectra of the last 29 days of the RISP gravity records (Figure 6). The lines near frequency $15^\circ$/hr are due to the diurnal constituents. The semidiurnal constituents are near $30^\circ$/hr.
Figure 8b. Fourier amplitude spectra of the last 29 days of the RISP gravity records (Figure 6). The lines near frequency 15°/hr are due to the diurnal constituents. The semidiurnal constituents are near 30°/hr.
34 cm ($P_1$ and $K_1$ may be taken together). The maximum effect on the diurnal phase by the 2 cm noise level is approximately $4^\circ$, or 16 min uncertainty in the time of the constituent extrema. For the semidiurnal constituents the southeastern stations, Base, J9, and F9, have amplitudes near 8 cm, indicating a phase uncertainty of approximately $14^\circ$, or 30 min in the time domain. For the remaining stations, except C13, the semidiurnal amplitudes are near 4 cm, indicating a phase uncertainty of approximately $27^\circ$, or about 1 hour in the time domain. No estimates are made for the uncertainties in the harmonic constants at C13, although the results of the analysis are given in Table 4. It is noted that in spite of the apparently high noise level there, the values are consistent with those from other locations on the western side of the shelf (McM, O19, and C16).

Inferences of the uncertainties at F9 and C16 based on independent record segments are summarized in Table 6. The deviation from the mean values for constituents $P_1$ and $K_1$ have already been discussed in connection with the validity of the theoretical separation of these two constituents. In general, the deviation of the amplitudes from the mean of the two segments is less than 1 cm, the exception being the F9 $O_1$ amplitudes where the deviation is 2 cm. The deviations of the diurnal phases are $5^\circ$ or less. The semidiurnal phase deviations are $4^\circ$ or less at F9 where the semidiurnal amplitudes are near 10 cm, and are $25^\circ$ or less at C16 where the amplitudes are near 3 cm. These values are consistent with the error estimates based on the amplitude spectra.
Table 6. Differences in the harmonic constants determined from independent record segments from C16 and F9.

<table>
<thead>
<tr>
<th>Constituent</th>
<th></th>
<th>F9</th>
<th></th>
<th>C16</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Deviation</td>
<td>Mean</td>
<td>Deviation</td>
</tr>
<tr>
<td>K₁</td>
<td>41 cm</td>
<td>0.8 cm</td>
<td>31 cm</td>
<td>0.6 cm</td>
</tr>
<tr>
<td></td>
<td>206°</td>
<td>2°</td>
<td>200°</td>
<td>2°</td>
</tr>
<tr>
<td>O₁</td>
<td>40 cm</td>
<td>2.1 cm</td>
<td>27 cm</td>
<td>1.0 cm</td>
</tr>
<tr>
<td></td>
<td>190°</td>
<td>2°</td>
<td>190°</td>
<td>5°</td>
</tr>
<tr>
<td>M₂</td>
<td>8 cm</td>
<td>0.5 cm</td>
<td>3 cm</td>
<td>0.5 cm</td>
</tr>
<tr>
<td></td>
<td>258°</td>
<td>3°</td>
<td>310°</td>
<td>16°</td>
</tr>
<tr>
<td>S₂</td>
<td>11 cm</td>
<td>0.1 cm</td>
<td>2 cm</td>
<td>0.2 cm</td>
</tr>
<tr>
<td></td>
<td>142°</td>
<td>4°</td>
<td>160°</td>
<td>25°</td>
</tr>
<tr>
<td>N₂</td>
<td>8 cm</td>
<td>0.6 cm</td>
<td>4 cm</td>
<td>0.5 cm</td>
</tr>
<tr>
<td></td>
<td>138°</td>
<td>4°</td>
<td>147°</td>
<td>14°</td>
</tr>
</tbody>
</table>
Cotidal-Coamplitude Charts

The primary goal of this effort was to determine the spatial variation of the height of the ocean tide beneath the Ross Ice Shelf. Within the previously discussed uncertainties this has been successfully accomplished, and maps showing the spatial variation of the amplitude and phase of the principal constituents of the tide have been prepared. Contours of constituent amplitude (coamplitude lines), and of constituent phase (cotidal lines) for $K_1$, $O_1$, $M_2$, $N_2$, and $S_2$ are presented in Figures 9-13 respectively. No separate map was drawn for $P_1$. The $P_1$ map is the same as that for $K_1$, with the amplitudes scaled by one-third. The cotidal lines can be viewed as the position at different times of the crest of a fictitious wave that would exist if only that particular constituent were disturbing the sea surface. The height of the wave crest above mean sea-level is not necessarily constant along the crest. If the cotidal lines are nearly parallel, then the tide propagates as a progressive wave, and the speed of the tidal wave is approximately that of a shallow-water gravity wave, given by

$$c_w = \sqrt{gh}$$

where $g$ is the gravitational acceleration (9.8 m/sec$^2$), and $h$ is the water depth.

The diurnal cotidal lines (Figures 9, 10) display a simple, nearly parallel pattern. The spacing between 10° lines is in the 100 km to 150 km range, which is appropriate for the 150 m to 400 m...
Figure 9. Cotidal (dashed) and coamplitude (solid) lines for the constituent $K_1$. Cotidal lines are labeled in degrees relative to Greenwich: coamplitude labels are in centimeters. The lines are the same for the constituent $P_1$ except that the amplitudes are scaled by one-third.
Figure 10. Cotidal (dashed) and coamplitude (solid) lines for the constituent $O_1$. Cotidal lines are labeled in degrees relative to Greenwich: coamplitude labels are in centimeters.
Figure 11. Cotidal lines for $M_2$, showing an amphidromic region near RISP camp C13. Lines are labeled in degrees relative to Greenwich.
Figure 12. Cotidal lines for N$_2$ showing an amphidromic region near RISP camp C13. Lines are labeled in degrees relative to Greenwich.
Figure 13. Cotidal lines for $S_2$ showing an amphidromic region near RISP camp Cl3. Lines are labeled in degrees relative to Greenwich.
thickness of the water-layer beneath the ice shelf. The diurnal coamplitude lines shown are more complex than could have been drawn by simple interpolation between points where measurements were made. The location of these contours was based on an observed relationship between the constituent amplitude and the thickness of the water-layer beneath the station, together with a map of the thickness of the water-layer beneath the ice (Greishar and Bentley, 1978; Figure 14). This relationship is illustrated in Figure 15, where curves have been fitted to the data on the assumption that the relationship is similar to that between the wave height and ocean depth for a wave in a shallow canal of slowly varying depth. In such a canal the wave amplitude is inversely proportional to the fourth root of the depth (Lamb, 1941). Curves of this form are intuitively attractive since the wave trajectories are nearly constant and parallel. For the constituents \( K_1 \) and \( O_1 \), proportionality constants of 144 and 129, respectively, for amplitude in centimeters and depth in meters, were found by minimizing the root-mean-square (RMS) error with respect to the constant. The minimum RMS errors were found to be 1.0 cm (\( K_1 \)) and 2.4 cm (\( O_1 \)). These values are about the same as the estimated uncertainty in the constituent amplitudes. Only data from the RISP camps were used to fit the curves in Figure 15, and the C13 data were excluded because of the greater uncertainty in those values. However, the C13 data are not inconsistent with the relationship, although the deviations are larger, as can be seen in Figure 15. The LAS data were not used because the station was at the ice front,
Figure 14. Contours of water-layer thickness beneath the floating ice shelf in meters (Greischar and Bentley, 1978).
Figure 15. $K_1$ and $O_1$ constituent amplitudes plotted versus water-layer thickness for the RISP camps. Curves of the form $A = Ch^{-1/4}$ were fitted to the data on the assumption that the relationship is analogous to that for a wave in a shallow canal. The proportionality constants are 144 and 129 (amplitude in centimeters and water thickness in meters) for $K_1$ and $O_1$, respectively.
and the water depth there changes rapidly from 580 m to 360 m. This rapid change is inconsistent with the assumption that the water-layer thickness is slowly changing. Similarly, the data from McMurdo Sound were recorded near shore, and the complex topography of the sound is inconsistent with the required assumption.

The ocean tide in the Ross Sea must merge with the tide in the southern Pacific Ocean along the common boundary. The cotidal and coamplitude contours for the diurnal constituents shown in Figures 9 and 10 are reasonably continuous with corresponding contours for the same constituents shown on global ocean tide maps. For reference, global ocean tide maps by Estes (1977) and Tiron and others (1967) for the constituent $K_1$ are shown in Figures 16 and 17, respectively. A $K_1$ map for the Pacific Ocean only by Bogdanov (1973) is shown in Figure 18. Maps by the same authors for the constituent $O_1$ are shown in Figures 19, 20, and 21. For both $K_1$ and $O_1$, the map by Estes shows the phase to be about $30^\circ$, or 2 hr, behind the observed phase; Tiron's map shows the phase to be about $15^\circ$, or 1 hr, ahead of the observation. Bogdanov's maps appear to agree with the observed phases within about $15^\circ$, although his maps only extend to 66°S latitude, about 1300 km north of the Ross Ice Shelf. This agreement is quite good considering the dearth of observational values in the Ross Sea available at the time the global maps were made.

The spatial variation of the semidiurnal constituents is more complex. Because of the relatively greater uncertainties no co-amplitude contours have been drawn: it is expected that the more
Figure 16. Global $K_1$ cotidal (dashed) and coamplitude (solid) lines from Estes (1977). Cotidal lines are labeled in solar hours (15°/hr) relative to Greenwich. Amplitudes are shown in decimeters.
Figure 17. Global $K_1$ cotidal (Roman numerals) and coamplitude (Arabic numerals) lines from Tiron and others (1967). Cotidal lines are labeled in constituent hours (Table 5) relative to Greenwich. Amplitudes are in centimeters.
Figure 18. Pacific Ocean $K_1$ cotidal (solid) and coamplitude (dashed) lines from Bogdanov (1973). Cotidal lines are labeled in constituent hours (Table 5) relative to Greenwich. Amplitudes are in centimeters.
Figure 19. Global $O_1$ cotidal (dashed) and coamplitude (solid) lines from Estes (1977). Cotidal lines are labeled in solar hours (15°/hr) relative to Greenwich. Amplitudes are shown in decimeters.
Figure 20. Global $O_1$ cotidal (Roman numerals) and coamplitude (Arabic numerals) lines from Tiron and others (1967). Cotidal lines are labeled in constituent hours (Table 5) relative to Greenwich. Amplitudes are in centimeters.
Figure 21. Pacific Ocean $O_1$ cotidal (solid) and coamplitude (dashed) lines from Bogdanov (1973). Cotidal lines are labeled in constituent hours (Table 5) relative to Greenwich. Amplitudes are in centimeters.
complex configuration of the cotidal contours would preclude a
canal-type amplitude-depth dependence, such as that observed for the
diurnal constituents. There is a similar phase variation in each of
the three semidiurnal constituents having amplitude sufficiently
great as to be measurable in this study (Figures 11-13). The small
amplitudes in the northwestern portion of the region, and the
regular increase in the constituent phase clockwise around the
shelf indicate the existence of an amphidromic region (an area
surrounding a notide point from which the radiating cotidal lines
progress through all hours of the tidal cycle (Schureman, 1949))
near C13.

The semidiurnal constituent maps should merge with corresponding
maps for the global ocean tide in the southern Pacific Ocean region.
Recent maps of $M_2$ have been made by Zahel (1977), Estes (1977),
Bogdanov (1973), Henderschott and Munk (1970), and Tiron and others
(1967). These are shown in Figures 22-26, respectively. These maps
can be divided into two classes according to the direction of in­
creasing phase at the northern extremity of the Ross Ice Shelf, i.e.,
from LAS to McMurdo. This study indicates that the $M_2$ phase is about
35° at LAS and approximately 0° at McMurdo, and the cotidal-coampli-
tude map (Figure 11) reflects these values. In Figure 11 the phase
is shown decreasing progressively from LAS to McMurdo, from 35° to
0°. The global maps by Estes, Bogdanov, and Tiron are consistent with
this direction of decreasing phase. Estes' map indicates a decrease
from about 130° at LAS to about 90° at McMurdo. Tiron's map shows
Figure 22. Global $M_2$ cotidal (solid) and coamplitude (dashed) lines from Zahel (1977). Cotidal lines are labeled in constituent hours (Table 5) relative to Greenwich. Amplitudes are in centimeters.
Figure 23. Global $M_2$ cotidal (dashed) and coamplitude (solid) lines from Estes (1977). Cotidal lines are labeled in solar hours ($15^\circ$/hr) relative to Greenwich. Amplitudes are shown in decimeters.
Figure 24. Pacific Ocean M_2 cotidal (solid) and coamplitude (dashed) lines from Bogdanov (1973). Cotidal lines are labeled in constituent hours (Table 5) relative to Greenwich. Amplitudes are in centimeters.
Figure 25. Global M\textsubscript{2} cotidal and coamplitude lines from Henderschott and Munk (1970). Cotidal lines are labeled in degrees relative to Greenwich. Amplitudes are in centimeters.
Figure 26. Global $M_2$ cotidal (Roman numerals) and coamplitude (Arabic numerals) lines from Tiron and others (1967). Cotidal lines are labeled in constituent hours (Table 5) relative to Greenwich. Amplitudes are in centimeters.
a decrease from 120° at LAS to 60° at McMurdo. Bogdanov's map is consistent with these two, but does not extend sufficiently far south for detailed comparison. The global maps by Zahel, and Henderschott and Munk show increasing constituent phase from LAS to McMurdo, and are thus inconsistent with the map presented in Figure 11. Zahel shows the phase to increase from 237° at 160°W longitude (near LAS) to 265° at 190°W longitude (near McMurdo). The map by Henderschott and Munk shows the phase increasing from about 0° at LAS to about 30° at McMurdo. An interpretation of the data which is consistent with Zahel's global M_2 map requires a complex variation in the LAS-C36-Roosevelt Island area. Such an interpretation is presented in Figure 27.

**Tide Prediction**

The maps of Figures 9-11 describe the spatial and temporal variation of the six most significant harmonic constituents of the ocean tide in the southern Ross Sea, within the uncertainties previously considered. These maps can be used to predict the tide at any point on the ice shelf, for any time interval. To do this, values of amplitude and phase read from the maps are used to solve equation (5). To illustrate the accuracy with which these six harmonic constituents represent the elevation changes of the ice shelf, the record for Base has been reconstructed from the harmonic constants determined from the record. For convenient comparison, the observed and reconstructed records are presented together in Figure 28. To illustrate the time-invariant nature of the harmonic constants, and the accuracy to which
Figure 27. Ross Sea $M_2$ cotidal lines consistent with both the RISP data and Zahel's global $M_2$ in the southern Pacific Ocean (Figure 22).
Figure 28. Comparison of observed Base tidal gravity record with the Base tidal gravity synthesized from the six largest constituents of the tide, and the record for nearby (Figure 1) J9 hindcast from data obtained three years later.
the tide can be predicted for past or future times, the J9 tide has been hindcast for the time interval of the Base record. This record is also presented in Figure 28. The quantities required in equation (5) for the prediction are summarized in Table 7.

The reconstructed Base record correctly reproduces the principal features of the observed Base record, including the days of tropic and equatorial tide, and the time of day of high and low water. However, there are differences between the two records which can be attributed to several factors. The most notable difference is caused by instrument drift, which the reconstructed record does not reproduce. Differences in the tidal variations result from the uncertainties in the harmonic constants, and from the failure to include more than six of the tide-constituents in the reconstructed record. There are certainly more than six constituents with nonzero amplitude present in the Ross Sea: all of the constituents in Table 5, and others, are likely to be present. This is indicated by the number of spikes (more than 5) in the diurnal and semidiurnal frequency ranges of the Fourier spectra of the records (Figure 8). Although the amplitudes of these lesser constituents are too small to permit the accurate determination of their harmonic constants from the records obtained in this study, their aggregate effect can be observable.

The success of the prediction technique is evidenced by the striking similarity of the reconstructed Base and hindcast J9 records. Since these two stations are near to each other (Figure 1), it may be
Table 7. Parameters used to hindcast the J9 record to 20 Dec 1973 starting time.\textsuperscript{a}

<table>
<thead>
<tr>
<th>Constituent</th>
<th>$f$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>1.0</td>
<td>1.6°</td>
</tr>
<tr>
<td>$K_1$</td>
<td>1.010</td>
<td>6.9°</td>
</tr>
<tr>
<td>$O_1$</td>
<td>1.016</td>
<td>107.1°</td>
</tr>
<tr>
<td>$M_2$</td>
<td>1.002</td>
<td>117.8°</td>
</tr>
<tr>
<td>$N_2$</td>
<td>1.002</td>
<td>11.4°</td>
</tr>
<tr>
<td>$S_2$</td>
<td>1.0</td>
<td>0.0°</td>
</tr>
</tbody>
</table>

\textsuperscript{a} Values from Schureman (1941).
anticipated that the tide will be similar, but not identical, at the two locations.

Summary

The elevation of the Ross Ice Shelf varies in response to the ocean tide in the subjacent water-layer. The motion can be represented accurately by a sum of six harmonic constituents. Maps of the amplitude and phase of each of these were produced from measurements of tidal variations of gravity. The tide is principally diurnal; the diurnal constituent amplitudes are approximately three times larger than the semidiurnal amplitudes. At tropic (diurnal spring) tide the diurnal elevation fluctuation is in the 1 m to 2 m range.

The diurnal tide constituents (Figures 9 and 10) enter the Ross Ice Shelf region as a wave progressing towards the southwest. The height of the wave generally increases from near 10 cm in the southern Pacific Ocean to approximately 30 cm in the southern Ross Sea. This increase can be attributed to constructive interference between the wave and its reflection from the southwestern coast of the Ross Sea. Within the Ross Ice Shelf region, the height of the tide is closely related to the local water-layer thickness. The semidiurnal tidal waves rotate clockwise about an amphidromic point near C13.
TIDAL CURRENTS BENEATH THE ROSS ICE SHELF

Introduction

One of the measurable physical manifestations of the ocean tide is the tidal current. It is essentially the particle motion associated with the tidal wave that propagates in the sea. Like the surface motion, the currents can be decomposed into a quasi-harmonic series, as given in equation (5). For the current associated with a particular tide constituent, the periodic flow does not result in any net transport of the water mass: the water particles move around an ellipse or back and forth along a line.

Currents are important because they transport energy and mass, including heat, solutes, sediments, living organisms and nutrients. In most bodies of water, the tidal currents are only one aspect of the horizontal water movement. Other currents are caused by wind-stress at the surface, and temperature and density gradients in the water mass. Because of the ice cover, wind-driven circulation in the southern Ross Sea is minimized, but other currents are likely to exist. Knowledge of these currents bears upon the interpretation of observational data related to the physical and biological processes taking place in the water-layer beneath the ice shelf. One of the principal objectives of RISP is to determine whether the bottom surface of the Ross Ice Shelf is melting, or accreting mass by freezing sea water. In the larger perspective, this question is related to whether the Antarctic ice cap is growing or melting. Knowledge of the
ocean current beneath the ice shelf is important in determining the source of heat and solutes indicated by water temperature and salinity data from beneath the shelf. Biologically, knowledge of the current is important in determining the range over which nutrients may be transported, and from which biological specimens may be attracted by bait placed in the water.

In addition to the importance of the currents in relationship to other disciplines, the physical environment beneath the Ross Ice Shelf is unique, and the current is interesting in itself. Beneath the Ross Ice Shelf there are two surfaces which resist the flow of the water, and the effect of the upper surface on the current may shed additional light on the mechanism of friction in the global ocean. Also, because of the high latitude, or more accurately, the proximity of the rotational pole, there is a resonance effect which may greatly increase the amplitudes of the semidiurnal currents. The effect of this resonance on the tide in the Ross Sea is interesting, particularly because the tidal elevation changes are principally diurnal.

The objective of this portion of the dissertation is to develop a technique for calculating the currents associated with the ocean tide beneath the ice shelf. The results are then evaluated by comparing the calculated currents to those which have been measured with a current meter that was lowered through a borehole into the sea beneath the J9 camp. In addition, the area over which a particular mass of water moves in the course of a tidal cycle is estimated.
A Theoretical Basis for Tidal Current Calculations

The relationship between the height of the tide and the associated tidal current was first expressed mathematically by Laplace in 1775. These equations (the Laplace Tidal Equations, LTE) can be written (Doodson, 1958) as:

\[
\begin{align*}
\frac{\partial u}{\partial t} - 2 \Omega \cos \theta \, v &= -\frac{g}{a} \frac{\partial}{\partial \theta} \left( \zeta - \bar{\zeta} \right) - G \\
\frac{\partial v}{\partial t} + 2 \Omega \cos \theta \, u &= -\frac{g}{a} \frac{1}{\sin \theta} \frac{\partial}{\partial \chi} \left( \zeta - \bar{\zeta} \right) - F \\
\frac{1}{a \sin \theta} \left[ \frac{\partial}{\partial \theta} (hu \sin \theta) + \frac{\partial}{\partial \chi} (hv) \right] &= - \frac{\partial \zeta}{\partial t}
\end{align*}
\]  

(6a) (6b) (6c)

where:

- \( \theta \) is the colatitude,
- \( \chi \) is the east longitude,
- \( \Omega \) is the angular frequency of rotation of the earth,
- \( \omega = 2 \Omega \cos \theta \)
- \( a \) is the mean spherical earth radius,
- \( g \) is the mean gravitational acceleration,
- \( \zeta \) is the height of the tide,
- \( \bar{\zeta} \) is the height of the fictitious equilibrium tide,
- \( h \) is the (undisturbed) thickness of the water layer,
- \( u,v \) are the current speeds south and east, respectively, and
- \( G,F \) represent the south and east components of the frictional forces (per unit mass)
For the equations in this form, it has been assumed that the current speeds, \( u \) and \( v \), are independent of depth, and that the vertical current speed is small compared to the horizontal. Essentially this is the shallow-water wave assumption, which is true for the tide. A criterion that a wave be a shallow-water wave is (Kinsman, 1965, p. 126), for wave period \( T \):

\[
T^2 \geq 200 \ h \ \frac{2\pi}{g}
\]

Taking 700 meters as representative of the greatest thickness of the water-layer beneath the Ross Ice Shelf, waves having periods longer than 5 minutes can be considered shallow. Tidal periods are far greater than this.

The first two (6a,b) of the LTE relate the currents, and their time derivatives, to the surface elevations. Inasmuch as the most significant constituents of the tidal elevations have been determined, these two equations can be solved simultaneously for the horizontal current components. Setting \( \zeta^* = \zeta - \overline{\zeta} \), and rewriting 6a and 6b in vector notation:

\[
\frac{\partial}{\partial t} \left( \begin{array}{c} u \\ v \end{array} \right) = - \frac{g}{a} \left( \begin{array}{c} \frac{\partial \zeta^*}{\partial \theta} \\ \frac{1}{\sin \theta} \frac{\partial \zeta^*}{\partial \chi} \end{array} \right) - \omega \left( \begin{array}{c} -v \\ u \end{array} \right) - \left( \frac{C}{F} \right)
\]

In this form, the equations may be solved step-wise in time for given initial values of \( u \) and \( v \). A FORTRAN program to solve this equation was written, which implements the International Mathematics and
Statistics Library (IMSL, 1977) subroutine DREBS. Subroutine Drebs is based on an algorithm by Bulirsch and Stoer (1966) that is said to have computational advantages over the Runge-Kutta method. The computation time required to solve this equation is long, depending on the number of time-steps desired in the solution. This program, presented in Appendix I, provides the option of treating the friction terms $F$ and $G$ in the different ways that are described in the following discussion.

Thus far the mathematical form of the retarding forces represented by $F$ and $G$ has not been specified. At the time of this writing, the phenomenon of energy dissipation in marine circulation is not well understood. However, important insights have been gained in recent years with expanded research efforts motivated by the need for extensive exploitation of off-shore resources (Nihoul, 1977, p. v). For the water layer beneath the Ross Ice Shelf, there are insufficient observational data upon which to base a detailed model of the retarding effects of the sea floor and lower surface of the ice shelf. For that reason, and consistent with the shallow water nature of the tidal wave, the retarding effects will be treated here as a body force, acting on the entire water column, and increasing as the speed of the current increases. Classically, two forms of the frictional forces have been used when bottom friction has been treated in this fashion, one nonlinear and one linear. The nonlinear form can be traced to Young (1813), and is expressed by the equation:
The value of the proportionality constant, $k_n$, is determined empirically. Retarding effects of this form have been included as part of the description of friction by a number of investigators, including Estes (1977) and Zahel (1977), in their recent attempts to solve the LTE for the global ocean tide. The linear treatment of the frictional forces is attractive because of its mathematical simplicity, and its application can be traced to Guldberg and Mohn (1876). For $G$ and $F$ linearly dependent on the horizontal current components, the frictional terms can be expressed as:

$$ \begin{bmatrix} G \\ F \end{bmatrix} = k_1 \begin{bmatrix} u \\ v \end{bmatrix} $$

(10)

As in the nonlinear case (equation 9) the proportionality constant, $k_1$, is determined empirically.

If it is assumed that the frictional forces are linear, at least to a first approximation, a simpler method can be used to solve equation (8) for the current components. Substituting from equation (10) equation (8) may be written:

$$ \frac{\partial}{\partial t} \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} k_1 & -\omega \\ \omega & k_1 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \frac{g}{a} \sin \theta \begin{bmatrix} \frac{\partial \xi^*}{\partial \theta} \\ \frac{\partial \xi^*}{\partial x} \end{bmatrix} $$

(11)
Since the equation is linear, the solution for each constituent of the tide will be independent of all the others. So for the constituent having frequency \( \sigma \), let

\[
\begin{align*}
    u &= A_u \cos (\sigma t - \phi_u) \\
    v &= A_v \cos (\sigma t - \phi_v) \\
    \zeta &= A_\zeta \cos (\sigma t - \phi_\zeta)
\end{align*}
\]

where \( A \) and \( \phi \) are the amplitude and phase, respectively, and apply:

\[
A \cos (\sigma t - \phi) = A \cos \phi \cos \sigma t + A \sin \sigma \sin \sigma t
\]

Also let

\[
\begin{align*}
    c_i &= A_i \cos \phi_i \\
    s_i &= A_i \sin \phi_i
\end{align*}
\]

Then differentiating and substituting into equation (11) yields:

\[
\begin{bmatrix}
    -\sigma \sin \sigma t & \sigma \cos \sigma t & 0 & 0 \\
    0 & 0 & -\sigma \sin \sigma t & \sigma \cos \sigma t
\end{bmatrix}
\begin{bmatrix}
    c_u \\
    s_u \\
    c_v \\
    s_v
\end{bmatrix}
\]
\[
\begin{align*}
\begin{bmatrix}
k_1 - \omega \\
\omega & k_1
\end{bmatrix}
\begin{bmatrix}
\cos \sigma t & \sin \sigma t & 0 & 0 \\
0 & 0 & \cos \sigma t & \sin \sigma t
\end{bmatrix}
\begin{bmatrix}
c_u \\
s_u \\
c_v \\
v
\end{bmatrix}
\end{align*}
\]
\[
= - \frac{g}{a}
\begin{bmatrix}
\cos \sigma t & \sin \sigma t & 0 & 0 \\
0 & 0 & \cos \sigma t & \sin \sigma t
\end{bmatrix}
\begin{bmatrix}
\frac{\partial c_\zeta}{\partial \theta} \\
\frac{\partial s_\zeta}{\partial \theta} \\
\frac{1}{\sin \theta} \frac{\partial c_\zeta}{\partial \chi} \\
\frac{1}{\sin \theta} \frac{\partial s_\zeta}{\partial \chi}
\end{bmatrix}
\]

Since this equation is true for all times, \( t \), it must be true for \( \sigma t = 0^\circ \) and \( 90^\circ \) in particular. Thus

\[
\begin{align*}
\begin{bmatrix}
\sigma & -k_1 & 0 & \omega \\
-k_1 & \sigma & \omega & 0 \\
0 & -\omega & \sigma & -k_1 \\
-\omega & 0 & -k_1 & -\sigma
\end{bmatrix}
\begin{bmatrix}
c_u \\
s_u \\
c_v \\
v
\end{bmatrix}
= \frac{g}{a}
\begin{bmatrix}
\frac{\partial s_\zeta}{\partial \theta} \\
\frac{\partial c_\zeta}{\partial \theta} \\
\frac{1}{\sin \theta} \frac{\partial s_\zeta}{\partial \chi} \\
\frac{1}{\sin \theta} \frac{\partial c_\zeta}{\partial \chi}
\end{bmatrix}
\end{align*}
\]
The harmonic constants of the currents may be obtained directly from the spatial derivatives of the harmonic constants of the elevations using this equation, provided that the frictional forces are specified linearly (equation 10). Two FORTRAN computer programs were written, one of which solves equation (12), while the other reconstructs the current variation with time, for a specific time interval, by substituting the resultant harmonic constants into equation (5). These programs are presented in Appendix II.

In the preceding discussion, two methods for calculating the current associated with a given tidal elevation variation are described. These methods are fundamentally different. The Bulirsch-Stoer method (equation 8) includes both the transient and steady-state parts of the solution, while the other (equation 12) yields only the steady-state part. The transient part of the solution depends on the initial values selected for u and v to start the solution, and should decay with time, so that the two methods produce the same result in the steady-state limit. Thus a comparison of the solutions obtained using both methods to solve the same problem gives a direct measure of the effect of the choice of the initial current on the solution obtained using the Bulirsch-Stoer method.

To make an exact comparison, the nonlinear friction calculation in the program using the Bulirsch-Stoer routine (Appendix I) must be replaced by the linear calculation, but this change involves only one statement in the program. It was found that agreement between the two methods was better than 0.1 cm/sec and 1° after the first 6 days.
of record, for any initial values of \( u \) and \( v \) on the order of \( \pm 10 \) cm/sec or smaller. This is consistent with Estes (1977) statement that for a time-stepped solution to the complete LTE (equations 6 a, b, c) the steady-state solution is assured in 8 tidal periods.

Theoretical calculations of the tidal current are useful only insofar as they correctly predict the current actually present in the sea. Therefore it is necessary to compare the theoretical results with actual measurements of the current beneath the Ross Ice Shelf. In addition, the theoretical calculations are sensitive to the values of the friction coefficients, \( k_1 \) and \( k_n \). The nature of this sensitivity will be explicitly derived in a later section. Because of this sensitivity, it is necessary to use current measurements to guide the selection of the values of the friction coefficient used in the theoretical calculations.

Current Meter Data from J9

Ocean current measurements have been made in the Ross Sea at several locations in open water north of the Ross Ice Shelf (Jacobs and others, 1974), and in McMurdo Sound (Gilmour, 1963; Gilmour and others, 1962). However these locations lie essentially outside the domain of this study. The maps presented here (Figures 9-13, 27), and particularly the semidiurnal ones, cannot be extended north of the ice front. Further, the tide in McMurdo Sound will be greatly influenced by the shape of the sound, and it is not the purpose of this study to produce detailed maps of the tide there. Measurements of the current beneath the ice shelf have been made at one location,
J9 camp. During the austral summer of 1977-78 a hole was bored through the 420 m thick ice shelf to the underlying water. Physical and biological data obtained using that hole were the subject of a RISP symposium at the 1978 Spring Meeting of the American Geophysical Union (AGU). Papers presented at the symposium were abstracted in the AGU bulletin, Eos (June, 1978).

Preliminary results of the current measurements made using the J9 access hole have been published by Jacobs and others (1978). The current records obtained by Jacobs (personal communication) are presented in Table 8 and Figure 29. These records are revisions of the preliminary published data. Briefly, a Geodyne savonius rotor current meter was suspended beneath the ice for periods up to several hours, in the interval from 20 to 27 December 1977. The longest continuous segment of current record obtained spans only six hours, much less than a single cycle of any tide constituent. At the time of this writing, the 1978-79 RISP field program is underway, and additional access holes have been made at J9. However, current records obtained through these holes are not yet available.

Inasmuch as Jacobs' 1977-78 data have been discussed only in a brief and preliminary fashion in the literature, some elaboration on the quality of the data is in order here. The current meter records three parameters: the speed of the current as indicated by the rotation of a horizontal rotor, the direction of the current with respect to the meter case as indicated by a moving vane, and the orientation of the case with respect to magnetic north as indicated by a compass.
Table 8. Current meter data obtained at J9 during December 1977 by Jacobs and others (1978; Stan Jacobs, personal communication).

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</table>

\(^a\)Greenwich Mean Time.

\(^b\)Current speed (cm/sec).

\(^c\)Direction of the current (away from the meter). Measured in degrees north through east.
Figure 29. J9 current meter data from Table 8 in vector form at one hour intervals. The current direction is specified away from the observer. For this display, speeds below the instrument threshold, and not given in the table, were taken to be 1 cm/sec.
In operation at J9 the meter was lowered through the access hole on a cable, and suspended in the water column on a cable having length equal to the 420 m thickness of the ice shelf at J9, plus the distance from the meter to the bottom of the ice shelf. The position of the meter in the water column for each of the 5 records (Figure 29) is shown in Figure 30.

There is some indication that the vane measuring the current direction was not working properly on at least one of the records. On record CM5 the vane position remained nearly constant, and the change in current direction on that record is due to rotation of the meter case on the suspending cable. It is possible that on this record, and on others, that the vane was entirely or partially fouled by slushy ice as the meter was lowered through the access hole. The temperature of the water in the column below J9 is shown in Figure 30 by a graph of measurements made by Jacobs. At all points the temperature is below the freezing point of fresh water, and it is slightly colder, 0.3°C degree, at the top of the column where CM5 was obtained. If the vane was fouled by ice containing enough salt, it could have been freed by melting while the meter was suspended in the water. Such melting, if it occurred, would have been somewhat slower near the bottom of the ice shelf, where CM5 was recorded.

Although the current directions measured by Jacobs must be accepted with reservation due to the possibility of instrument problems, his are the first measurements of the ocean current beneath the Ross Ice Shelf, and the data presented in Table 8 and Figure 29
Figure 30. Generalized graph of temperature measurements made by Jacobs in the water column at J9. Also shown are the depths at which current measurements were made.
appear reasonable in the absence of conclusive evidence to the contrary. These measurements appear to confirm the supposition that the current is principally a tidal current. Firstly, the largest current speed observed was near 18 cm/sec, but neither the speed nor the direction of the current was constant. Additionally, the direction changes appear to be tidal, in the 15° to 30° per hour range. On records CM2 and CM4 the azimuth change is clockwise, while CM5 shows a counterclockwise azimuth change. The azimuthal variations of the remaining record, CM3, are irregular. On each record, the current vectors form part of an ellipse having the major axis oriented generally northwest-southeast.

**Theoretical J9 Tidal Current**

The tidal current at J9 was calculated for the month of December 1977, a time interval that spans the measurements made by Jacobs at that location. The calculations were based on the cotidal-coamplitude maps of Figures 9-13. Both the linear and nonlinear forms of the frictional force, as previously described, were used. The friction coefficients $k_1$ and $k_n$ were chosen so that the greatest speed of the theoretical current was about the same as the greatest current speed measured by Jacobs (18 cm/sec, Table 8). For the semidiurnal amplitudes, which were not mapped, the values observed at J9 were used. The calculated current is presented in Figure 31. Shown are the southward and eastward components for both treatments of friction. The linear friction calculation yields harmonic constants for the
Figure 31. Current components south and east at J9, calculated the month of December 1977. The upper curves (a) are based on the non-linear friction assumption (equation 9) with $k_n$ equal to $3 \times 10^{-6}$. The lower curves (b) are based on the linear friction assumption (equation 10) with $k_l$ equal to $5 \times 10^{-5}$ sec$^{-1}$. 
tidal current, and these are presented in Table 9 for three values of the friction coefficient $k_1$.

The values of the friction coefficients required to reduce the theoretical current speeds to the maximum observed during Jacobs' measurements are 5 to 10 times larger than corresponding values which have been deduced or assumed for other parts of the global ocean. For the linear term (equation 10), the value $5 \times 10^{-5} \text{ sec}^{-1}$ yielded a maximum current speed of 17 cm/sec. For the nonlinear term (equation 9), $k_n$ equal to $3 \times 10^{-6}$ yielded a maximum current speed of 19 cm/sec during the month of December, 1977. These values may be compared to corresponding values from the literature. Neumann (1968, pp. 166, 167, 300) summarizes several estimates of the linear coefficient, and concludes that values of $k_1$ in the range $10^{-6}$ to $10^{-5} \text{ sec}^{-1}$ are appropriate in different areas, the value being higher in shallow seas. For the nonlinear coefficient the value 0.003 cm h$^{-1}$ has been widely used (Estes, 1977; Zahel, 1977; Grijalva, 1964). At J9 where the water layer is 238 m thick, this becomes $1.3 \times 10^{-7}$.

Since these values of $k_1$ and $k_n$ have been applied to areas where the seabed is the only surface resisting the current, they may be doubled for comparison with the Ross Sea values because the flow is also opposed by friction due to the bottom of the floating ice shelf.

For comparison of the calculated current with the measurements, hourly values of the theoretical current, at times corresponding to Jacobs' current meter runs, are presented in Table 10, for both linear and nonlinear treatments of the friction. The same data are
Table 9. Harmonic constants of the J9 tidal current, based on a linear friction model, for different coefficients of friction.

<table>
<thead>
<tr>
<th>Friction Coefficient</th>
<th>$10^{-6} \text{ sec}^{-1}$</th>
<th>$10^{-5} \text{ sec}^{-1}$</th>
<th>$5 \times 10^{-5} \text{ sec}^{-1}$</th>
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<td></td>
<td>A^a</td>
<td>A</td>
<td>A</td>
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<tr>
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<td>101</td>
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</tr>
<tr>
<td>east</td>
<td>85.5</td>
<td>191</td>
<td>1.9</td>
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<td>south</td>
<td>29.5</td>
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^a Amplitude in cm/sec

^b Phase angle in degrees relative to the Greenwich Meridian
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<th>Case II^c</th>
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a Greenwich Mean Time  
b Nonlinear friction (equation 9), $k_1 = 3 \times 10^{-6}$  
c Linear friction (equation 10), $k_n = 5 \times 10^{-5}$ sec$^{-1}$  
d Measured in cm/sec  
e Measured degrees north through east
presented vectorally in Figure 32. Although the calculated and measured currents do not compare favorably, there are important similarities between the two. Both indicate that the azimuthal variation of the current is sometimes clockwise and sometimes counterclockwise. The theoretical calculations show that this results from the fact that the azimuthal change of the diurnal current constituents is clockwise, while the semidiurnal is counterclockwise. Figure 33 shows the current, in vector form, due to each of the constituents separately. In the figure, the theoretical current constituents given in Table 9, for \( k_2 \) equal to \( 5 \times 10^{-5} \) sec\(^{-1}\), have been plotted at intervals of one (solar) hour. It is seen that the constituents each form an ellipse having the major axis oriented northwest-southeast, the sense of rotation being clockwise for all the diurnal constituents, and counterclockwise for all the semidiurnal constituents. These characteristics are similar to the observations made earlier about Jacobs' data.

Current calculations based on cotidal charts consistent with Zahel's global map of \( M_2 \) (Figure 27) have also been considered. It is emphasized that the author prefers the relatively simple amphidromic \( M_2 \) variation of Figure 11, and regards Figure 27 as an unnecessarily complex interpretation of the data. Current calculations based on the \( M_2 \) chart in Figure 27, using the linear friction model with \( k_2 \) equal to \( 5 \times 10^{-5} \) sec\(^{-1}\), yield the following harmonic constants for the \( M_2 \) current constituent: southward, 4.2 cm/sec and 39°; eastward, 5.6 cm/sec and 139°. This current is presented
Figure 32a. Theoretical currents from Table 10 in vector form. Numbers are hours Greenwich Mean Time. These are the Case I data, calculated using a nonlinear friction model. These vectors may be compared to the measured current shown in Figure 29.
Figure 32b. Same as Figure 32a, except that the Case II data, calculated using a linear friction model, are displayed.
Figure 33. Current due to each constituent of the tide displayed separately in vector form, at hourly intervals. Time 0 is taken, for each constituent, at the passage of the equilibrium constituent over the Greenwich Meridian. Also shown is the current inferred from the M₂ cotidal chart consistent with Zahel's global M₂ (Figure 27).
vectorally in Figure 27 for comparison with the current associated with the amphidromic $M_2$ chart. These current speeds are somewhat greater than the speeds implied by the amphidromic map (Table 9). For comparison with the observations, the theoretical calculation must include the semidiurnal constituents $S_2$ and $N_2$, in addition to $M_2$. Since it is generally true that the response of the world ocean to the tide generating force is similar for all the semidiurnal constituents, it may be anticipated that if the $M_2$ variation shown in Figure 27 is correct, then cotidal charts for $S_2$ and $N_2$ will be similar to Figure 27, though not identical. Calculations of the J9 tidal current, based on semidiurnal cotidal charts similar to Figure 27 for all three semidiurnal constituents, failed to produce improved agreement with the measured current. However, one significant observation was made: the value of the friction coefficient required to reduce the maximum theoretical current to the 18 cm/sec level was about 50% larger for these charts than for the amphidromic charts of Figures 11-13.

**Semidiurnal Current Resonance in the Ross Sea**

Calculations of the tidal current at J9 indicate that the semidiurnal components of the current are larger than the diurnal components. This contrasts with the spectrum of the water level fluctuation which is dominated by the diurnal constituents. In addition, it was observed that while larger values of the friction coefficient reduce the amplitude of the semidiurnal current components,
the diurnal components are insensitive to changes in the value of
the coefficient (Table 9). The relationship between the linear
coefficient of friction and the current can be more clearly seen
in the explicit solution to equation (12):

\[
\begin{pmatrix}
\frac{\partial c_v}{\partial \theta} \\
\frac{\partial c_v}{\partial x}
\end{pmatrix} = \frac{g}{\alpha D} \begin{pmatrix}
A_1 & A_2 & A_3 & A_4 \\
A_2 & -A_1 & A_4 & -A_3 \\
A_3 & -A_4 & A_1 & A_2 \\
A_4 & A_3 & A_2 & -A_1
\end{pmatrix} \begin{pmatrix}
\frac{\partial s_v}{\partial \theta} \\
\frac{\partial c_v}{\partial x}
\end{pmatrix}
\]

(13)

where

\[
\begin{align*}
A_1 &= h^2 \sigma [h^2 (\sigma^2 - \omega^2) + k_1^2] \\
A_2 &= -h k_1 [h^2 (\sigma^2 + \omega^2) + k_1^2] \\
A_3 &= 2 \, h^3 \sigma \omega k_1 \\
A_4 &= h^2 \omega [h^2 (\sigma^2 - \omega^2) - k_1^2] \\
D &= [h^2 (\sigma^2 - \omega^2) - k_1^2]^2 + 4h^2 \sigma^2 k_1^2
\end{align*}
\]

At the high latitude of the Ross Sea, \(\omega\) is semidiurnal, near
29.7°/hr. For the diurnal constituents the quantity \((\sigma^2 - \omega^2)\) in
equation (13) is relatively large, and the terms, particularly the
divisor \(D\), are not greatly affected by changes in \(k_1\). However, for
the semidiurnal constituents \((\sigma^2 - \omega^2)\) is near zero, and the theoretical currents are sensitive to the value of \(k_1\).

The close dependence of the semidiurnal current amplitudes on the magnitude of the frictional force can be viewed as the result of a resonance between the semidiurnal and inertia currents. Inertia currents are a well-known and important component of the current in the global ocean. They arise when there is an imbalance between the inertial, Coriolis, and external forces acting on the water, as for example with the rapid passage of a meteorological front. Assuming that for some unspecified reason the external forces in the LTE (equations 6a,b) go to zero, and neglecting the frictional terms, the currents must satisfy the relationship (Neumann, 1968, p. 150):

\[
\frac{\partial u}{\partial t} = \omega v
\]

\[
\frac{\partial v}{\partial t} = -\omega u
\]

which has the solution

\[
u = \sin \omega t
\]

\[
v = \cos \omega t
\]

Thus, inertia currents have frequency \(\omega\), which in general is intermediate to the diurnal and semidiurnal constituents of the tide, but at the high latitude of the Ross Sea is quite close to the semidiurnal frequencies. An inertial current, once established, may persist for many hours, subject to the frictional effects which reduce its speed.
Mass Transport by Tidal Currents at J9

A tidal current is cyclic in nature, and the net flow averaged over one cycle is zero. However, within a cycle the water mass is transported over a closed path, and as mentioned in the Introduction, the area covered by this motion is of interest. The trajectory of a particular mass of water can be obtained by integrating the speed, as a function of time, of that mass of water. Essentially this requires knowledge of the material description of the motion. In this study the ocean current has been determined at a single point in space, i.e., the spatial description of the motion is known. For the purpose of obtaining an estimate of the area over which water-borne material may be transported by the tide, it is assumed that the tidal current is approximately the same in some region around J9. Under this assumption, the desired trajectory of the water mass is the integral of the current as previously determined.

The precise area traversed by the tidal current will change as the phase relationships between various constituents of the current change. However, what is desired here is an estimate of the greatest size of the area which water-borne material may cover following the tidal current. For the estimate, a diurnal current having a constant speed of 20 cm/sec will be used. For such a current:

\[
\begin{align*}
    u &= 20 \text{ cm/sec} \cos \frac{15^\circ}{\text{hr}} t \\
    v &= 20 \text{ cm/sec} \sin \frac{15^\circ}{\text{hr}} t
\end{align*}
\]
The displacements are

\[ X_p = 3 \text{ km} \sin \frac{15^\circ}{\text{hr}} t \]

\[ Y_p = -3 \text{ km} \cos \frac{15^\circ}{\text{hr}} t \]

which is the equation of a circle having 3 km radius. The area enclosed by this circle is 30 square kilometers. A similar semi-diurnal current would enclose an area about one-fourth this size.

It is concluded that the tidal current at J9 is capable of transporting water-borne material over an area within a 3 km radius of J9. This same estimate applies to transport by inertia currents, provided that the speed of the current during the inertial event does not exceed 40 cm/sec.

**Summary**

The objective of this part of the dissertation has been to learn what current is associated with the tide beneath the Ross Ice Shelf. In this effort, two FORTRAN computer programs have been developed to calculate the current from a set of cotidal-coamplitude charts, and one which calculates the current for a specified time interval from the harmonic constants of the current. Both of these methods are based on the assumption that the tide can be treated as a shallow-water wave. Within this assumption, one method (equation 8; Appendix I) allows for a general, i.e. nonlinear, treatment of the retarding forces experienced by the tide. The second method
 Presupposes that each constituent of the tide can be considered independently of the others by using a linear representation of the retarding forces, but is computationally about 100 times faster than the more general treatment.

A number of important results have been obtained from calculations of the tidal current based on the cotidal-coamplitude maps in Figures 9-13 and 27. It has been shown that the semidiurnal current components are quite important, even though the tidal elevation changes are predominately diurnal. It has been demonstrated that the relatively greater semidiurnal currents may be viewed as the result of a near resonance between the semidiurnal tidal components and the inertia currents in the Ross Sea. Because of this resonance effect, the semidiurnal current amplitudes are sensitive to the treatment of the retarding forces which act on the tidal wave, resulting in an opportunity to study the retarding forces more closely than is generally possible. The calculations of this section indicate that to treat friction as it has been elsewhere in the global ocean results in tidal currents which are unreasonably large. To reduce the theoretical current to the level of the observations requires that the frictional force be increased by a factor of 5 to 10 over previous treatments. This is not necessarily unreasonable, but it does point out the need to investigate other possible mathematical formulations of the retarding forces. It is possible that alternate treatments of friction would require that the shallow-water wave
assumption be abandoned, a step that would literally add a new dimension to the complexity of solving the Laplace Tidal Equations.

The characteristics of the current observed at J9 during December 1977 are consistent with the theoretical calculations of this section. However, agreement between the particular current observed and the calculations was not obtained. It is not clear at this time whether the differences are due to the theoretical calculations, particularly the treatment of friction, or to the previously discussed problems of ice jamming the current meter at J9. This question will be answered in the next year or two when the results of more recent current measurements at J9 become available. Other related questions involve the effect of the lesser semidiurnal constituents (Table 5). The aggregate effect of these constituents on the current may be important, especially since they too will be amplified by near resonance with the inertial currents.

The estimate of tidal transport of material beneath the ice shelf indicates that over several tidal cycles a given mass of water may traverse an area of some 30 square kilometers. This implies that in the absence of other strong currents, waterborne material would stay within about 3 km of J9 over a period of several days. Given tidal currents of similar amplitude elsewhere, this estimate may be applied to the entire Ross Ice Shelf region.
Introduction

The principal subject of the research program described in this dissertation has been the ocean tide in the Ross Ice Shelf region of Antarctica. However, the signals recorded during the field program by the gravimeters were of two types. In addition to the tidal gravity changes associated with variations in the elevation of the ice shelf, the RISP gravimeters recorded a continuous motion containing periods shorter than 20 minutes. The nature of these waves is illustrated in Figure 34, which shows segments of the record widely separated points on the ice shelf. Referring to the figure, and to the station positions in Figure 1, it is seen that these oscillations appear to have smaller amplitudes and longer periods at stations farther from the ice front. In addition these oscillations have greater amplitude during bad weather. This is illustrated in Figure 35, which shows record segments from C13. Camp C13 was occupied early in the 1974-75 field season, mostly during the month of November, and the weather at that time was stormy. The segments shown were obtained during a storm on 20 November 1974, and during calmer weather on 6 December. For the study of the tide, these waves were removed from the record by manually drawing a smooth curve on the field records, corresponding to the apparent beam positions in the absence of the oscillations. Thus the oscillations do not appear on RISP tidal gravity records presented in Figure 6.
Figure 34a. Twelve-hour long segments of the field record from seven of the RISP tidal gravity camps (Figure 1) show the character of the short period motion of the ice shelf superposed on the tidal cycle. The time scale is slightly different on each record due to differing chart recorder speeds. The record segments from J9, C16 and F9 are simultaneous, beginning at 1200 GMT 12 Dec 1976. The Base segment begins at 1200 GMT 14 Jan 1974. The C36 and Roosevelt Island camp records are simultaneous, and begin at 1200 GMT 9 Jan 1975. The 019 segment begins at 1200 GMT 3 Jan 1978.
Figure 34b. Refer to Figure 34a for description.
Figure 34c. Refer to Figure 34a for description.
Figure 34d. Refer to Figure 34a for description.
Figure 35. Short period vibrations observed at C13 during and after a storm are illustrated by (a) and (b), respectively.
The waves present on the RISP gravity records are similar to the short period motions of the gravimeter beam at Little America V, except for the period range of the motion. Thiel and others (1960) report observations of waves having periods shorter than 1 minute, and having much larger amplitudes than are present on the RISP records. According to their report, the gravimeter at LAS was unreadable, swinging from stop to stop, until midwinter when the Ross Sea north of the ice shelf was covered by sea ice. The observations at LAS are quite consistent with the RISP data. LAS was located essentially at the ice front, and, again, the motion appears to decrease in amplitude and increase in period as distance from the ice front, or open water, increases.

During the first years of the project, these nontidal waves were observed on all the RISP tidal gravity records, but no special effort was made to study them. However, some consideration was given to their nature and source, and it was hypothesized that they are elastic flexural waves, generated at the ice front by the ocean swell, and propagating in the ice and subjacent water layers. A field experiment was conducted in the last field season of the project to test this hypothesis, by measuring the speed and direction of propagation of these waves at J9. In the experiment, three gravimeters were operated simultaneously on the apices of an equilateral triangle having 5 km long sides. The data were recorded digitally at a 4 sec digitizing interval on recorders designed and assembled for that purpose. The new recorders were necessary because the
nontidal waves were poorly recorded on the strip-chart recorders used for the tidal data (Figure 34). A short sample of the digitally recorded data from J9 is presented in Figure 36. Also shown are the positions of the recording sites relative to J9 camp. The broadening of the trace from satellite camp Beta is caused by a slightly higher level of electrical noise generated by that instrument.

Flexural Wave Theory

It is well known that the ocean swell penetrates the ice covered regions of the oceans, but that in doing so its spectrum is altered, the shorter periods being rapidly attenuated (Wadham, 1973). Studies of this phenomenon have been done, for the most part, on floating sea ice a few meters thick. To the author's knowledge, it has not been previously studied in a body of ice as thick as the Ross Ice Shelf overlying water column of similar thickness.

The theory of elastic waves in floating ice has been well developed, although it is, perhaps, not widely known. The theory was first investigated by Greenhill (1887), whose work was extended by Ewing and Crary (1934). This result has been verified by Wilson (1958) and by Wadham (1973). An experimental study was reported by Clements and others (1958). Flexural waves in a floating ice sheet are analogous to Rayleigh waves on a solid, and gravity waves on water. They are inversely dispersed, with the phase speed given by the following:
Figure 36. Waves at J9. The record segments shown were recorded beginning at 1058:52 GMT 26 Nov 1977. Also shown is a plan of the tripartite array used to record the wave motion. The arrow and shading superposed on the array indicate the direction of propagation of the waves across the array.
\[ c^2 = \frac{\frac{\alpha}{K} + \frac{k^3 D}{\rho_w}}{\frac{\rho_i}{\rho_w} H_k + \frac{1}{\alpha} \coth (\alpha k)} \]  \hspace{1cm} (14) 

where \( D \) = flexural rigidity of the ice sheet,

\[ D = \frac{E H^3}{12 \left(1 - \nu^2\right)} \approx c_i^2 \rho_i \frac{H^3}{12} \]

\[ \alpha = 1 - \frac{c^2}{c_w^2} \frac{1}{2} \]

\( c \) = flexural wave phase speed

\( k \) = wave number = \( 2\pi / \) wave length

\( c_w \) = speed of sound in water

\( c_i \) = speed of sound in ice

\( \rho_w \) = density of the water

\( \rho_i \) = density of the ice

\( h \) = thickness of the water layer

\( H \) = thickness of the ice

\( g \) = acceleration of gravity

\( \nu \) = Poisson's ratio

\( E \) = Young's modulus

According to equation (14), the speed of the flexural wave at \( J9 \) is period dependent. A theoretical dispersion curve, based on the theory by Ewing and Crary (1934; equation 14) is presented in
Figure 37. In calculating this curve, the following physical parameters were used: $c_w = 3700 \text{ m/sec}$ and $c_i = 1450 \text{ m/sec}$ (Robertson, 1975); $\rho_w = 1.03 \text{ gm/cm}^3$ and $\rho_i = 0.86 \text{ gm/cm}^3$; $h = 240 \text{ m}$ and $H = 240 \text{ m}$; $g = 9.83 \text{ m/sec}^2$. From the figure, it is seen that in the period range of the J9 waves (apparently longer than 1 min), the flexural wave speed decreases monotonically to the shallow-water wave speed of 48 m/sec. For practical purposes, waves longer than 10 min period could be considered to travel at the shallow-water speed.

Digital Recorders for the Gravimeters

A new method of recording the gravimeter output signal was required for quantitative studies of the nontidal motion of the ice shelf. The tidal data were obtained by continuously recording the gravimeter output signal on a strip-chart recorder (Figure 34). Time was kept by an independent crystal-controlled clock that placed hourly tic marks on the record. The paper speed on the chart recorders was nominally 1 in/hr, driven by a motor synchronized to the 60 Hz power line. However, in Antarctica the line frequency was poorly controlled, and subject to short term fluctuations as great as 10%. In addition, the amplitude resolution possible with the chart recorders was poor. The sensitivity of the chart recorder was set so that the tidal cycle occupied the full width of the chart. At this setting, the nontidal waves usually had a 1/2 in to 1 in range. This was felt to be inadequate.

New microprocessor-based digital recorders were designed and constructed to supplement the strip-chart machines. These recorders
Figure 37. Flexural wave dispersion curve for J9.
are comprised of 8 basic modules. The central module is an analog-to-digital (AD) converter that samples the analog signal from the gravimeter and outputs a corresponding 12-bit binary number. A crystal-controlled clock module generates the signals that trigger the AD converter at specified intervals. There is a memory module for short-term data storage, and a cassette tape module for permanent data storage. Two modules, a typewriter-like keyboard and television monitor, allow an operator to pass instructions to the digitizer and check for proper operation. A serial interface module allows for communication between the digitizer and another machine. After the data has been recorded, it can be retrieved and transmitted to a larger computer for analysis through this interface.

The last module contains the microprocessor which controls all the other modules according to a program which is stored in the memory. The manufacturers and model numbers of the component parts of this system are given in Appendix III. The controlling program, developed by the author, is also discussed.

The program under which the digitizer operates samples and records the gravimeter signal at selected intervals of 1 sec to 256 sec. The program includes a clock which, in conjunction with the clock module, keeps time in terms of the year, day of the year, and time of day. The clock drift rate is better than 1 sec per day. The data acquired by the digitizer were stored on tape cassettes, which were ultimately transmitted to the IBM machine in the Virginia Polytechnic Institute (VPI) Computing Center. A
program which causes the digitizer to read the data tapes and transmit them to the VPI main computer via the serial interface was also developed by the author.

**Gravimeter Frequency Response**

The tripartite measurement of the wave speed and direction requires that the time lag of the arrival of the wave along the various sides of the array be accurately measured. Since the flexural wave hypothesized here is dispersive, the time lags may be period dependent. That is, for a dispersive wave the phase changes of the Fourier component waves, at two stations relative to the third, are likely to be nonlinearly dependent on frequency. Although the three gravimeters used in this study are similarly constructed, it is certain that they are not identical (Table 2). Therefore it was necessary to measure the frequency response of each of the instruments. For measurement of the wave speed, the data must be corrected for differences in the phase response between the three gravimeters. Calculation of the amplitude spectrum of the nontidal wave requires a correction for the amplitude response of the gravimeter.

Determination of the frequency response characteristics of the gravimeters used in this study was based on the assumption that they behave linearly. Similar instruments have been modeled as underdamped harmonic oscillators, using a 17 sec natural period and 0.7 critical damping (Hunkins, 1962). Observations of the response of the instruments to an impulsive disturbance indicate that these values are approximately correct, but a more accurate determination
was desired. The frequency response was determined by comparing the known frequency content of a 10 sec long calibration pulse to the response of the instrument to the pulse. The frequency content of a 10 sec pulse is illustrated in Figure 38. As a check on this procedure, one instrument (number 735) was calibrated on the long-period shake-table at Carnegie Institute, Department of Terrestrial Magnetism, in Washington, D. C.

The results of the amplitude and phase response measurements are presented in Figures 39 and 40, respectively. It was found that the frequency response of the instruments is more complex than allowed by a simple harmonic oscillator model. In particular, meter 735, whose amplitude response is shown in Figure 39 appears to exhibit at least two resonances, one near 0.022 Hz (45 sec period) and one near 0.059 Hz (17 sec period). The principal amplitude resonance being near 0.059 Hz. The phase response of meter 735 (Figure 40) also indicates a complex behavior. Firstly, the frequency at which 90° phase lag occurs is 0.022 Hz. For an underdamped harmonic oscillator would be the natural frequency, 0.059 Hz. Secondly, the phase response at frequencies higher than 0.1 Hz is unlike that of a simple harmonic oscillator.

In order to calculate the nontidal wave speed it was necessary to know the relative phase responses of the three instruments at periods longer than 1 min. Also, knowledge of the amplitude response of only one instrument was needed to calculate the wave amplitude spectrum. This information is given in Figures 39 and 40. It is
Figure 38. Amplitude spectrum of a 10 sec-long rectangular pulse.
Figure 39. Meter 735 amplitude response, measured on the shake-table at Carnegie Institute.
Figure 40. Gravimeter phase response curves. Solid curves were determined from the instrument response to a 10 sec-long calibration pulse. Crosses denote meter 735 phase measured on the shake-table at Carnegie Institute.
noted that for the periods of interest in this study, the phase response of meter 826, used at J9 base camp, relative to meter 735, used at Alpha, corresponds approximately to a time lag of 2 sec. Similarly, the response of 826 relative to meter 17, used at satellite camp Beta, corresponds approximately to a time lag of 3 sec.

**Nontidal Wave Data**

The tripartite array at J9 was operated during the period 22 to 26 November 1977. Approximately 33 hours of simultaneous data from all three instruments were obtained. Noisy record segments, due to earthquakes, camp noise, and instrument problems, were rejected. The time intervals spanned by these records are given in Table 11; the records are presented in Figure 41. The long period variation, particularly evident on the longer records, is the tidal gravity variation.

A detailed plan of the J9 tripartite array is shown in Figure 42. The dimensions of the array were determined using a surveyor's transit and chain. The angles may be regarded as accurate to 0.01 degree: The distances are accurate to ± 1 meter. From the figure it is seen that the array forms a nearly perfect equilateral triangle.

The length of a side of the array was limited by field considerations. The 5 km dimension was roughly the greatest distance at which the satellite camps could be seen from J9 base camp on a clear day.
Table 11. Times of the record segments from the J9 tripartite array.

<table>
<thead>
<tr>
<th>Beginning time (GMT)</th>
<th>Length</th>
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</thead>
<tbody>
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</tr>
<tr>
<td>22:38:04 23 Nov 77</td>
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<td>14 hr 26 min 40 sec</td>
</tr>
<tr>
<td>14:48:24 25 Nov 77</td>
<td>2 hr 6 min 40 sec</td>
</tr>
<tr>
<td>8:18:52 26 Nov 77</td>
<td>5 hr 59 min 56 sec</td>
</tr>
</tbody>
</table>
Figure 41a. Simultaneous record segments from the J9 tripartite array. Long period variations are due to the ocean tide and to instrument drift. Spikes on the records are caused by vehicle movement nearby. Small amplitude earthquake signals are also visible. The upper record is from J9 Base Camp, the center record is from satellite camp Alpha and the lower record is from satellite camp Beta. The starting time and length of each segment is given in Table 11. A short segment is shown more clearly, and with a time scale in Figure 36.
Figure 41b. Refer to Figure 41a for description.
Figure 4lc. Refer to Figure 4la for description.
Figure 42. Dimensions and orientation of the J9 tripartite array. Distances are accurate to ± 1 m, interior angles of the array are accurate to 0.01°.
Speed and Direction of the Nontidal Waves at J9

Examination of the short record segments shown in Figure 36 reveals that the wave motion is similar, though not identical, at the three stations in the array. Further, the motion at sites Alpha and Beta occurs more or less synchronously, or with Beta slightly in the lead, while the motion at J9 clearly precedes both Alpha and Beta.

The relationship between the relative arrival times of the wave across the array, and the wave velocity is now explicitly developed. Referring to Figure 43, let \( s \) be the wave speed and \( \phi \) be the direction of propagation, and let time be measured from the arrival of the wave front at J9. Denote the arrival times at satellite camps Alpha and Beta by \( \Delta_\alpha \) and \( \Delta_\beta \), respectively. Then

\[
\Delta_\alpha = \frac{D_\alpha}{s}
\]

\[
\Delta_\beta = \frac{D_\beta}{s}
\]

Referring to Figures 42 and 43, it can be seen that:

\[
D_\alpha = 5007 \, \text{m} \sin \left(201^\circ - \phi\right)
\]

\[
D_\beta = 5037 \, \text{m} \sin \left(\phi - 81^\circ\right)
\]

or

\[
D_\alpha = -1794 \, \text{m} \cos \phi + 4674 \, \text{m} \sin \phi
\]

\[
D_\beta = -4974 \, \text{m} \cos \phi + 786.4 \, \text{m} \sin \phi
\]
Figure 43. Schematic representation of the nontidal wave crossing the J9 tripartite array. Array dimensions are given in Figure 36.
Divide by $\Delta_\alpha$ and $\Delta_\beta$ to get $s$, equate, and multiply by $\Delta_\alpha \Delta_\beta$ to get:

$$\sin \phi \ (\Delta_\beta \ 4674 \text{ m} - \Delta_\alpha \ 786.4 \text{ m}) = \cos \phi \ (\Delta_\beta \ 1794 \text{ m} - \Delta_\alpha \ 4974 \text{ m})$$

So that

$$\phi = \tan^{-1} \left( \frac{\Delta_\beta \ 1794 - \Delta_\alpha \ 4974}{\Delta_\beta \ 4674 - \Delta_\alpha \ 786.4} \right)$$

Having $\phi$, then $D_\alpha$ or $D_\beta$ can be calculated, from which the wave speed $s$ can be found from:

$$s = \frac{D_\alpha}{\Delta_\alpha} = \frac{D_\beta}{\Delta_\beta}$$

Calculations of the speed of the J9 wave from the observational data, for comparison with Figure 37, were done in two steps. Firstly, a single representative speed was calculated, neglecting the (suspected) dispersive quality of the wave. Secondly, it was attempted to measure the dispersion by bandpass filtering the data, and calculating a speed representative of each band. Calculation of the wave velocity across the J9 array requires that the time lags $\Delta_\alpha$ and $\Delta_\beta$ be determined from the field observations. These values were obtained by pair-wise crosscorrelation of the data from J9, Alpha, and Beta. For the required calculations 16 nonoverlapping segments of the wave record (Table 11, Figure 41) were selected. Portions of the record containing camp noise or seismic waves were avoided, but some low-level noise on the segments was unavoidable. Most of the windows were either 120 min or 113 min 20 sec in length, depending on the available record. Results of the crosscorrelation are presented in
Table 12. It can be seen from the table that the lags $\Delta_\alpha$ and $\Delta_\beta$ are roughly equal, while the crosscorrelation of the records from Alpha and Beta, $\Delta_{\alpha-\beta}$, is greatest near zero lag. Crosscorrelations of the records from Alpha and Beta were used only as a qualitative check on the values of $\Delta_\alpha$ and $\Delta_\beta$, because there is a greater possibility that this value, being near zero, will be disturbed by signals that travel across the array at high speed (a few kilometers per second). The crosscorrelation process is sensitive to coherent energy between the two time series. Because of the short dimensions of the array, energy crossing it at high speeds will result in an increased correlation near zero lag. In this study signals due to earthquakes, microseisms, and timing pulses would appear to arrive simultaneously at all stations, and thus interfere with the correlation process.

Wave velocities calculated from pairs of $\Delta_\alpha$ and $\Delta_\beta$ are given in Table 12, along with the lags. There is a considerable scatter in the velocities, reflecting the scatter in the lags. To extract a single representative value for the velocity from the scatter, the concept of order statistics (Bennett and Franklin, 1954, p. 157) was applied. The application of order statistics allows a representative velocity to be determined, within error limits, without requiring an assumption that the scatter in the observations has a specific form, Gaussian for example. While this calculation implicitly assumes that the wave is not dispersive, it is recognized that dispersion could effect the scatter in the observations of the wave
Table 12. Time lags and wave velocities from the unfiltered J9 data.

<table>
<thead>
<tr>
<th>Start (GMT)</th>
<th>$\Delta_\alpha$ (sec)</th>
<th>$\Delta_\beta$ (sec)</th>
<th>$s$ (m/sec)</th>
<th>$\phi$ (degrees)</th>
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Table 12 (continued)

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<td>78</td>
<td>79</td>
<td>141</td>
</tr>
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</table>
speed. A plot of the wave speed-direction pairs from Table 12 is presented in Figure 44. From the figure, it appears that while the distribution of the wave directions may be Gaussian, the wave speeds are clustered between 50 m/sec and 58 m/sec, and only one value falls below the shallow water speed of 48 m/sec.

The median value of the wave speeds observed at J9 (Table 12) is 54.5 m/sec and the median direction is N140°E. Referring to the tables provided by Bennett and Franklin, at the 95% confidence level the median of a very large number of such observations would fall between 50 m/sec and 65 m/sec for the speed, and between 131° and 145° for the direction of propagation. Wave speeds in this range are appropriate for flexural waves having periods from 2 min to 6 min (Figure 43), and these periods are evident in the wave recordings shown in Figure 36. The range of wave directions is indicated in Figure 36 by the shaded zone surrounding an arrow superposed on the figure.

The preceding calculation shows that the nontidal waves at J9 are progressive, propagating towards the southeast at a speed in the range predicted by flexural wave theory. However, the speed calculated was averaged, in some sense, over all periods. Recognizing that the flexural wave is dispersive, an effort was made to detect dispersion in the observed waves. In this effort, the wave records were bandpass filtered, before the wave speed was determined. The frequency windows used are illustrated in Figure 45, wherein the windows are shown relative to the dispersion curve for the flexural
Figure 44. Nontidal wave speeds and directions determined from the unfiltered J9 data (Table 12).
Figure 45. Flexural wave dispersion curve for J9. Also shown are cosine-tapered frequency windows used to filter the data in an attempt to measure the dispersion.
wave. In this figure the frequency scale is linear, rather than logarithmic as in Figure 43. Four cosine-tapered windows, each 0.002 Hz wide, were used, centered at 0.001 Hz, 0.002 Hz, 0.0033 Hz, and 0.005 Hz. The results of the speed determinations using the filtered data are presented in Table 13. Not shown are the results for the window centered at 0.005 Hz: in that case the maximum cross-correlation occurred at zero lag, for almost all record segments. For the remaining windows, the median speed and the range of the median at the 95% confidence level are as follows: for the window centered at 0.001 Hz, 52 m/sec and 44 m/sec to 68 m/sec; 0.002 Hz, 51 m/sec and 31 m/sec to 56 m/sec; 0.0033 Hz, 50 m/sec and 36 m/sec to 58 m/sec. Although the uncertainties are large, these values are quite close to the speeds predicted by flexural wave theory. Referring to Figure 45, the theory predicts that the speed increases from 48 m/sec for a 0.001 Hz wave to 52 m/sec for a 0.0033 Hz wave.

**Flexural Wave Power Spectrum**

In the preceding discussion it has been shown that the nontidal waves observed at J9 propagate at the speed and direction expected for flexural waves generated at the ice front. It will now be shown that a mechanism exists at the ice front for generating flexural waves in the observed period range. Central to this discussion is the power spectrum of the nontidal waves at J9. It is shown that the spectrum of the ocean swell at the ice front, as determined by extrapolation of the J9 spectrum, is consistent with the spectrum of the ocean swell in the Pacific Ocean.
Table 13. Time lags and wave velocities from the J9 data, bandpass filtered using a cosine tapered frequency window 0.002 Hz wide. Record starting times are the same as in Table 12.

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<th>Center Frequency 0.002 Hz</th>
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</table>
The power spectrum of the nontidal waves at J9 has been calculated from the wave records taken using meter 735 (Figure 40). The spectrum, corrected for the instrument response according to the measured response curve (Figure 38), is presented in Figure 46. It is seen that the wave spectrum decreases with increasing frequency from about $5 \times 10^{-2} \text{ cm}^2/\text{Hz}$ at 0.001 Hz to about $2 \times 10^{-7} \text{ cm}^2/\text{Hz}$ at 0.01 Hz.

The interchange of energy between the wave in the ocean and the wave in the ice shelf is a complex phenomenon. However, from physical arguments it can be seen that the transmission of energy into the ice covered region is biased towards the long period waves. The geometry of the ice front is illustrated in Figure 47, based on ice and water thicknesses, and elevation measurements at Little America V (Thiel and others, 1960). The phase speeds of flexural waves in the ice shelf region and gravity waves in the open sea, for this geometry, are presented in Figure 48. The flexural wave speeds were obtained from equation (14) using $h$ equal to 400 m and $H$ equal to 230 m. The gravity wave speeds were calculated according to the relationship (Kinsman, 1965, p. 126):

$$c^2 = \frac{g}{k} \tanh (kh)$$

(15)

using $h$ equal to 600 m. It is seen from the figure that at periods shorter than 100 sec there is an increasing contrast between the speeds of the gravity and flexural waves.

A quantitative estimate of the amplitude of the flexural wave, relative to the amplitude of the ocean swell, is based on the
Figure 46. Power spectrum of the flexural waves observed at J9.
Figure 47. The geometry of the ice front based on ice and water thicknesses at Little America V (Thiel and others, 1960).
Figure 48. Speeds of flexural and gravity waves near the ice front, based on the geometry shown in Figure 47.
continuity of pressure across the ice front (the $x = 0$ plane in Figure 47). On either side of the ice front, the pressure in the water layer is given by the formula (Kinsman, 1965, p. 144; Ewing and Crary, 1934):

$$p = -w(z) + \frac{\rho \sigma^2 \cosh k(z + h) \eta}{k \sinh kh}$$

where $\eta = \cos (kx - \sigma t)$ represents the wave,

$P$ = the total pressure,

$z$ = the vertical coordinate, positive upward,

$t$ = the weight per unit area of the mass above depth $z$,

$h$ = thickness of the water layer

In the open sea, the motion is described by linear gravity wave theory (Kinsman, 1965, Chapter 3), while in the ice covered region the motion is described by the flexural wave theory of Ewing and Crary (1934). On either side of the ice front equation (16) is interpreted in a slightly different way. The quantities $h$, $k$, and $\eta$ differ in the two regions, and in each case $z = 0$ is taken at the top of the undisturbed water column, which changes across the ice front.

The two theories do not merge across the ice front. Each assumes lateral homogeneity, and neither correctly describes the motion of the water at the ice front. To estimate the flexural wave height, it is assumed that the pressure, averaged over the thickness of the water column beneath the ice, can be calculated from either theory. Integrating equation (16) for the gravity wave side of the boundary, the average pressure is given by:
\[
\bar{P} = \frac{1}{h_w} \int_{-h_w}^{h_w} \left[ -\rho_w g (\eta_w - z) + \rho_w \sigma^2 \cosh k_w (z + h_w) \frac{\eta_w}{k_w \sinh k_w h_w} \right] dz
\]

where \( \bar{P} \) = the average pressure

\( h_w \) = the water thickness in the open sea

\( k_w \) = the wavenumber of the wave having frequency \( \sigma \) according to gravity wave theory

\( \eta_w \) = the surface disturbance by the gravity wave

Performing the integration, and applying the small amplitude assumption by neglecting terms in \( \eta \) compared to \( h \), yields:

\[
\bar{P}_{h_i} = -\rho_w g \left( h_i h_i - h_i^2 \right) + \rho_w \sigma^2 \frac{\sinh k_i h_i}{k_i^2 \sinh k_i h_i} \quad (17)
\]

Equation (16) is applied to the flexural wave side of the boundary in a similar way, with \( h_w \) replaced by \( h_i \), the water thickness beneath the ice, \( k_w \) replaced by \( k_i \) as predicted by flexural wave theory, and \( \eta_w \) replaced by \( \eta_i \), the surface disturbance due to the flexural wave. The limits of the integral on the ice shelf side are \(-h_i\) to \( \eta_i\).

Integrating and applying the small amplitude assumption as before yields:

\[
\bar{P}_{h_i} = -\rho_w g \left( h_i h_i - h_i^2 \right) + \rho_w \sigma^2 \frac{\eta_i}{k_i^2} \quad (18)
\]

Equating (17) and (18) and solving for \( \eta_i \) in terms of \( \eta_w \) yields:
The amplitude ratio of the flexural wave to the ocean swell calculated according to this relationship is presented in Figure 49.

The wave numbers necessary for the calculation were obtained from the phase speeds given in Figure 48.

In the preceding discussion it is shown that long period (Figure 49) waves in the ocean can generate flexural waves in the ice shelf, but that at shorter periods the transfer of energy into the ice covered region is inefficient. After the flexural wave is generated, the spectrum may change as the wave propagates away from the ice front. The attenuation of flexural waves by plastic behavior (creep) in the ice layer has been discussed by Wadhams (1973). The derivation of Wadhams' theory is lengthy, and his paper is readily available, so only his result is given here. He found that the flexural wave amplitude diminishes with distance traveled according to the relationship:

\[ a^2(x) = \frac{1}{2Sx + a^2(x = 0)} \]  

(20)

where \( x \) is the distance traveled, and \( S \) is a function of wavelength, ice and water thicknesses and densities, and the speed of sound in ice and water. Critical assumptions made in the development of this theory are that the ice creep can be modeled by the flow law of Glen (1955), and that the tensile stress in the ice due to bending
Figure 49. The ratio of the flexural wave amplitude to the ocean swell amplitude at the ice front, based on equation (9).
can be calculated using linear elastic theory. Wadhams finds that
the former assumption fits his observations and those of Robin (1963)
well. However, the tensile stress in the ice will be relaxed by the
creep, and the value calculated from linear theory may be too large.
This over estimate will ultimately cause the estimated energy loss
due to creep to be too great.

Calculations of Wadhams' factor $S$ were made using the ice
parameters previously used to calculate the flexural wave speed, and
the value $1.8 \times 10^{8}$ (mks units) for the flow law parameter for the
Ross Ice Shelf (Thomas, 1971 quoted by Wadhams, 1973). The variation
of $S$ with period for three combinations of ice and water thickness
representative of the Ross Ice Shelf region is shown in Figure 50.
The amplitude reduction due to creep is not linear, depending on the
amplitude (equation 20). However, an upper limit on the effect can
be made by taking $x$ equal to 1000 km and $a(x = 0)$ to be 1 cm. The
attenuation thus predicted reduces the amplitude of the wave by less
than 1% at all periods.

The estimates of the wave attenuation made by applying Wadhams' 
theory indicate that creep in the ice is not important for the 
waves observed at J9. Inspection of the field records (Figures 34 
and 35) neither confirm nor contradict this conclusion. On the 
records from C36, Base, C16, and F9 it appears that the shorter 
periods are attenuated with distance from the ice front. However,
the C13 record does not fit this pattern, having greater short period 
motion than any other record, but being intermediate to C36 and
Figure 50. The factor $S$ in equation (20) for ice and water thicknesses typical in the Ross Ice Shelf region.
Base in distance to open water. The Roosevelt Island camp and 019 records indicate that topography greatly influences the spectrum. Both stations are shielded from open water by land to the north (Figure 1), and the records contain less short period motion than the distance from open water would indicate.

The J9 spectrum can be compared to the spectrum of the ocean swell in the Pacific Ocean. Wave spectra measured by Munk (1962) in the western Pacific and at Hawaii are reproduced in Figure 51. The J9 spectrum, corrected for the effect of the ice front according to Figure 49 (equation 19) is superposed on Munk's figure. From the figure it is evident that the Pacific Ocean swell does include the waves necessary to generate the flexural waves observed at J9. It is especially noteworthy that the peak in the Pacific Ocean spectrum at frequencies higher than 40 cycles per kilosecond (periods shorter than 25 sec) is not observed on the ice shelf. The absence of this peak is consistent with the predicted response of the ice shelf to the swell (Figure 49).

Summary

The nontidal waves observed throughout the Ross Ice Shelf have been shown to be flexural waves. It has been shown that the waves are progressive, and propagate at the speed predicted by flexural wave theory. Further, the direction of propagation is away from the ice front, and it has been shown that the Pacific Ocean swell is
Fig. 3. A typical spectrum for Camp Pendleton, California. The two curves designate the spectra of surface elevation at distances of 8000 and 13,000 ft from the beach, respectively, and corresponding water depths of 20 and 100 ft.

Fig. 4. Background spectra at Mar del Plata, Argentina; Acapulco, Mexico; Camp Pendleton, California; and at Lahaina Wharf, Maui, Hawaii.

Figure 51. Illustrations from Munk (1960) showing the spectrum of the Pacific Ocean swell. Circles superposed on Munk's figures show the J9 spectrum (Figure 46) corrected for the effect of the ice front (Figure 49).
capable of generating flexural waves in the ice shelf having the observed spectrum.
RESULTS

The elevation at the surface of the Ross Ice Shelf fluctuates in response to the ocean tide beneath the shelf. The motion can be described in terms of six harmonic constituents (Table 4), three diurnal and three semidiurnal. The tide is principally diurnal, the diurnal constituent amplitudes being roughly three times as large as the semidiurnal amplitudes. The range of the tropic tide is about 1 m near the ice front, and can be as great as 2 m in the southeastern part of the sea where the water is shallow. The spatial variation of the tide has been described by cotidal-coamplitude maps for these constituents. The diurnal constituents can be viewed as a relatively simple progressive wave that propagates southeastward across the region (Figures 9, 10). The diurnal tidal wave amplitudes are closely related to the thickness of the water column (Figure 15). Within the Ross Sea the diurnal constituents are larger than in the adjacent Pacific Ocean because of constructive interference between the tidal wave and its reflection from the southeastern coast of the sea. The semidiurnal constituents exhibit a more complex variation. The semidiurnal tidal waves rotate clockwise around an amphidromic region near Cl3 (Figures 11-13). For the diurnal constituents the cotidal contours mapped in this study are consistent with available global ocean tide maps in the southern Pacific Ocean (Figures 16-21). For the semidiurnal the situation is more complicated. Available maps showing the constituent M_2 in
the southern Pacific (Figures 22-26) are of two types: those that show the phase decreasing eastward from McMurdo, and those that show increasing phase eastward. The discovery in this study that there is an amphidromic region within the Ross Sea is an important step in reconciling the different maps of $M_2$ in this region.

The tidal current beneath the Ross Ice Shelf at J9 has been calculated from the cotidal-coamplitude charts (Figures 32, 33), and compared to the measured current (Figure 29). A number of important insights have been gained from these calculations. Firstly, the semidiurnal current components are relatively more important than the semidiurnal components of the tidal height, because of a resonance between the semidiurnal and inertia currents. The semidiurnal current amplitudes appear to be roughly the same as the diurnal current amplitudes (Tables 4, 9). Secondly, the calculations of this dissertation show that the tidal current is rotary, and that the sense of rotation is clockwise for the diurnal constituents and counterclockwise for the semidiurnal constituents (Figure 33). The differences between the calculated and measured currents may arise from the presence of significant nontidal currents beneath the ice shelf, and from the simplifying assumptions made in the calculations. In the theory it is assumed that the motion of the water is nearly uniform throughout the water column, and that the retarding effect of the ice and sea floor can be treated as a body force that increases with the speed of the current. Because of the resonance condition, calculations of the semidiurnal
current components are sensitive to these assumptions. In future studies it may be necessary to extend the theory to allow for variation in the current with position in the water column. More current data from beneath the shelf, and especially simultaneous measurements at different levels in the water column, would be helpful in the development of a more general method for calculating the tidal current.

The observed motion of the ice shelf at periods shorter than 20 min (Figure 34) has been identified as elastic flexural waves which are generated at the ice front by the action of the ocean swell on the ice shelf, and propagate southward. The wave speed was predicted within the uncertainty of the measurement by the classical flexural wave theory. The theory has previously been shown to apply in ice from less than 1 m to about 50 m thick. In this study that range has been extended to about 500 m.
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APPENDIX I

The FORTRAN computer program developed by the author to calculate the tidal current step-wise in time from a given cotidal-coamplitude chart is presented in this appendix. This program solves equation (8) using the IMSL subroutine DREBS. The equation is specified in subroutine DFN of the program, and changes in the equation, including different treatments of the dissipation terms (equations 9 and 10) are made by changing the FORTRAN code within that subroutine.

Subroutine DREBS and UERTST are proprietary products belonging to International Mathematical and Statistical Libraries, Inc. (IMSL), in Houston, Texas. These subroutines are reproduced in this dissertation with the permission of IMSL, in order that the results can be duplicatable by others. These subroutines may not be extracted for other purposes or used as the basis for any software development.
IMPLICIT REAL*8 (A-H,O-Z)
REAL*4 NU(3000),NV(3000)
DIMENSION CURR(21),MK(59),CNES(2),ERRORS(7),M(2,9)
COMMON /STAT/ ERRCONS,TIME,COLAT,DEPTH,W,SVRA,PI,DELT,DELX,IPRINT
COMMON /MARK/ U,V
COMMON /MKEY/ TIME,VI,VI,ERROR,MAX,ISTEP,STPRINT,YORDER
COMMON /TABLE/ TOLD(58),DTOLD(58),DTXOLD(58),THER
EQUIVALENCE (CURR(1),U)
EXTERNAL OPN,FCN

C
C IN THIS PROGRM, DISTANCES ARE IN C4, TIMES ARE IN SECONDS
C
CNES(1)=1.0
CNES(2)=1.0
ER=0
INC=0

C
C READ ALL INPUT
C
CALL DATAIN
TIME=TIME1
U=UI
V=VI
1001 CONTINUE
C
C SET THE TIME STEP
C
TIME=ISTEP
INC=INC+1
C
C SET THE POINTER FOR A NEW TABLE
C
IEND=0

GET THE NEXT ESTIMATE OF THE CURRENTS USING THE IMLS SUBROUTINE

IF(IPRIN.EQ.1 .OR. IPRINT.EQ.2)
  WRITE(6,1002) CURNOW,TIME,INC,STPMN,CSRTOL,ERRORS,ONES,ICORDER
1002 FORMAT(10612.5,216)
  CALL DRESST0P,CURNOW,TIME,2,ICORDER,3,0,INC,STPMN,CSRTOL,ERRORS
  ,ONES,WR,IEP)
IF(IPRIN.EQ.1 .OR. IPRINT.EQ.2)
  WRITE(6,1003) CURNOW,TIME,INC,ERRORS,IER,IEND
1003 FORMAT(10612.5,216)

CHECK THE ERROR INDICATOR

IF(IER.NE.0) STOP

SAVE VALUES OF CURRENT IN CM/SEC TO OUTPUT LATER

NV(INC)=U
NV(INC)=V

CHECK FOR END OF SERIES

IF (TIME.GE.TMAX) GO TO 1000

CHECK RECOMMENDED TIME STEP

IF(STEP.LE.INC) GO TO 1001
WRITE(6,1004) STEP,INC,TIME
1004 FORMAT('WARNING - THE SPECIFIED TIME STEP ',12.5,
            , ' MAY BE TOO LARGE. ' ,12.5, ' IS RECOMMENDED. TIME=',12.5,
            )
:612.9
GO TO 1001
1000 CONTINUE
   WRITE (2,1002) TEND, TINC, TSTEP, TIME
1005 FORMAT (4G14.6)
   DTOR=3.141592654/180.0
   AMP=1d0*srt(U1*U1+V1*V1)
   AZ1I=DATAN2(V1,-U1)/DTOR
   IF (AZ1I.LT.0.) AZ1I=AZ1I+360.0
   DO 2001 I=1,TINC
      TAMP=SQRT(U1(I)**2+V1(I)**2)
      TAZI=ATAN2(V1(I),-U1(I))/DTOR
      IF (TAZ1.LT.0.) TAZ1=TAZ1+360.0
      U1(I)=TAMP
      V1(I)=TAZ1
   2001 CONTINUE
   WRITE (2,2002) AMP,AZ1I,(U1(I),V1(I),I=1,TINC)
2002 FORMAT (12F6.1/12F6.1/12F6.1/12F6.1/12F6.1/)
STOP
END
SUBROUTINE OFG(CURRNO,TIME,TINC,SLOPES)
IMPLICIT REAL (A-H,O-Z)
DIMENSION CURRNO(100),SLOPES(100)
COMMON /STAY/ FRCSNS,MLENG,CELL,DEPHE,WDIVER,PI,D_Y,EDEL,DELX,IPRINT
ENTRY FCN(REC,TIME,CURRNO,SLOPES)
U=CURRNO(1)
V=CURRNO(2)
C
C  CALCULATE FRICTIONAL TERMS
C
TEMP=FRCSNS*SQRT(U*U+V*V)
FRICTO=-TEMP*U
EFICTV = -TEMP*V

CALL CALCULATE CORRECTED TERMS

CALL GRADS(TIME, DZSDT, DZSDX)
FORADV = -GVRA*DZSDT
FORADV = -GVRA*DZSDX/DSTK(UCLAT)

CALL THE TERMS TO GET THE TIME DERIVATIVE

SLOPES(1) = EFICTV + COPH + FORADV
SLOPES(2) = EFICTV + COPH + FORADV
IF (PRINT.EQ.1 .OR. PRINT.EQ.3)
    WRITE(6,1) TEMP, EFICTV, EFICTV, COPH, COPH, FORADV, FORADV, SLOPES
1 FORMAT(6HDFH,4HLOG12.5)
RETURN
END

SUBROUTINE GRADS(TIME, DZSDT, DZSDX)
IMPLIED REAL*8 (A-H,O-Z,_)-
COMMON /STAT/ TOLST, TOLST, TOLST, TOLST, TOLST, TOLST, TOLST, TOLST,
COMMON /TOL/ TOLST(B), TOLST(B), TOLST(B), TOLST(B), TOLST(B), TOLST(B)

KEEP A TABLE OF UP TO 5E VALUES

IF (END.EQ.6) GO TO 10
D2 1 = 1, END
IF (TIME, NE, TOLC(1)) GO TO 2
GO TO 11
2 CONTINUE
10 IEND = IEND + 1
C C DELT IS THE THETA GRID SPACING IN RADIANS, DELX IS THE CHI
C DIRECTION SPACING.
C C GET THE HEIGHT OF THE DRIVING FORCE NEARBY
C CALL HEIGHT(NORTH, NSEAST, NEAST, NWEST, TIME)
C C DIVIDE BY THE GRID SPACING TO ESTIMATE THE DERIVATIVE
C
DZSOT = (NORTH - NMARK) / (2. * DELT)
DZSOX = (NEAST - NWEST) / (2. * DELX)
TOLU(IEND) = TOLU(IEND) + 1
DTOLU(IEND) = DZSOT
DXOLU(IEND) = DZSOX
IF (IPRINT.EQ.1, OR, IPRINT.EQ.3) THEN
WRITE (6, 1) TIME, HEIGHT, NSEAST, NEAST, NWEST, DZSOT, DZSOX
1 FORMAT ('GRADS     , 10D12.5')
RETURN
11 DZSOT = TOLU(1)
DZSOX = DXOLU(1)
IF (IPRINT.EQ.1, OR, IPRINT.EQ.3) THEN
WRITE (6, 1) TIME, DZSOT, DZSOX
RETURN
END
SUBROUTINE HEIGHT(NMARK, NSEAST, NEAST, NWEST, TIME)
C C CALCULATE THE HEIGHT OF THE TIDE - EQUILIBRIUM HEIGHT.
IMPLICIT REAL*8 (A-H,O-Z)
COMMON /STAT/ FREQN,MLONG,CLAT,DEPTH,GOVRA,P1,DELT,DELX,IPRIN
COMMON /CONS/ YN(7),HR(7),HS(7),HE(7),HR(7),SP(7),PO(7),PNK(7),PSK:
(7),PKN(7),PNK(7),ON(7),DS(7),DE(7),CL(17)

THE TIDE IS ASSUMED TO BE THE SUM OF SEVEN CONSTITUENTS
P1, A1, O1, Q1, Q2, N2, S2

HOUR=0.00
HS00=0.00
HEAS=0.00
HRES=0.00

FR IS THE MODE FACTOR; FN IS THE AMPLITUDE
SP IS CONSTITUENT SPEED IN DEGREES PER HOUR
TIME IS IN SECONDS
PO IS THE GREENWICH EQUILIBRIUM ARGUMENT AT TIME ZERO
PNK IS THE CONSTITUENT PHASE LAG
ON IS THE EQUILIBRIUM TIDE AMPLITUDE

DO 1 I=1,7
   TEMP=SP(I)*TIME+PO(I)
   HOUR=HOUR+FR(I)*TAN(I)*DGDCS(TEMP-PNK(I))-ON(I)*DGDCS(TEMP))
   HS00=HS00+FR(I)*HS(I)*DGCOS(TEMP-PSK(I))-DS(I)*DGDCS(TEMP))
   HEAS=HEAS+FR(I)*HE(I)*DGDCS(TEMP-PNK(I))-OC(I)*DGDCS(TEMP))
   HRES=HRES+FR(I)*HR(I)*DGDCS(TEMP-PNK(I))-OW(I)*DGDCS(TEMP)
1 RETURN

END

SUBROUTINE DATAIN

READ ALL INPUT AND INITIALIZE SURF ARRAYS
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION ID(7),SPDEG(7),OMAX(7)
COMMON /STAT/ PCONS,DLONG,DEPH,VI,STEP,IPRINT
COMMON /CONS/ IN(7),IM(7),HE(7),RI(7),SP(7),PI(7),PNK(7),PSK
: (7),PEK(7),PEK(7),O(7),O(7),O(7),O(7),O(7),O(7)
COMMON /INIT/ T1M,UL,VI,CRF,TOL,MAX,TSR,STPMIN,IC,ORDER
DATA ID/14,23,14,23,14,23,14,23,14,23/
DATA SPDEG/14,9,5,6,4,0,1,5,9,4,6,0,1,3,9,4,3,6,0,0,
:13,5,9,6,0,5,6,2,8,9,6,4,1,0,2,0,2,8,9,7,2,9,6,6,3,0,0,0,0/
DATA OMAX/4,68,14,16,10,06,1,73,24,24,1,09,11,23/
DELX=DELX*10
DEPT=DEPT*10.

CONVERT TIMES TO SECONDS

TMAX=TMAX*3600.
TSTEP=TSTEP*3600.
STPMIN=STPMIN*3600.
TMAX=TMAX+3600.

GRAVITY DIVIDED BY EARTH RADIUS

GVEK=980.76.374E9
ESPEED=2.*PI/(2.*3600.)
N=2.*ESPEED*DCOS(COLAT)

GET SPEEDS, EQUILIBRIUM AMPLITUDES, AND INITIALIZE NODE FACTORS

DO 2 I=1,7
SP(I)=SPDEG(I)*10/3600.
PP(I)=0.
HP(I)=0.
HS(I)=0.
HE(I)=0.
HR(I)=0.
PNK(I)=0.
PSK(I)=0.
PRE(I)=0.
PSK(I)=0.
PF(I)=0.

FIRST FOUR CONSTITUENTS ARE INTERNAL

"
C
IF(1,56,5) GO TO 3
DN(1)=DAX(I)@DFACT(CULAT-DELT)
DS(1)=DAX(I)@DFACT(CULAT+DELT)
DE(1)=DAX(I)@DFACT(CULAT)
DL(1)=DL(1)
GO TO 2
3 DL(1)=DAX(I)@SFAC(CULAT-DELT)
DS(1)=DAX(I)@SFAC(CULAT+DELT)
DE(1)=DAX(I)@SFAC(CULAT)
DS(1)=DS(1)
2 CONTINUE
C
READ CONSTITUENT LABEL, MEAN NODAL FACTOR FOR THE SERIES,
GREENWICH EQUILIBRIUM ARGUMENT AT TIME=0, AND AMPLITUDE
AND GREENWICH PHASE OF THE CONSTITUENT AT POINTS NORTH,
SOUTH, EAST, AND WEST, RESPECTIVELY.
C
998 READ(1,4,END=999) LBL, A1, A2, A3, A4, A5, A6, A7, A8, A9, A10
4 FORMAT(A2,5X,16F5.0)
5 WRITE(2,7) LBL, A1, A2, A3, A4, A5, A6, A7, A8, A9, A10
7 FORMAT(1X,A2,5X,F7.3,9X,F7.1)
1=0
30 5 J=1,7
5 IF(LBL.EQ.'01(D1)) I=J
IF(1.EQ.0) CALL EXIT
FM(I)=A1
PO(I)=A2@TOR
HM(I)=A3
PSK(I)=A4@TOR
BS(I)=A5
PSK(I)=A6@TOR
HE(1) = A7
PEK(1) = A8
HT(1) = A9
PWK(1) = A10
GOTO 998
979 CONTINUE
IF(IPK1=1 .AND. IPK1=4) RETURN
WRITE(6,997) U1,V1,T1,MAX,STEP,STEPH,
DRCOS,ERRTOL,ORDER,IPK1
DO 996 I=1,7
996 WRITE(6,997) I,SP(1),FN(1),P0(1),DI(1),DS(1),DE(1),DA(1),
HE(1),PSK(1),III(1),PEK(1),HT(1),PWK(1)
997 FORMAT(* DATASETS , 10F12.5/10X,6G12.5/) RETURN
END
C
SUBROUTINE DRES (DEN,Y,T,N,JM,RO,JSTART,H,imin,eps,r,s,tk,ier) DTRB0010
C
C DRES-------------S/0------LIBRARY-------------------------------------------------DTRB0030
C
C FUNCTION
C
* FIRST ORDER DIFFERENTIAL EQUATION SOLVER
C, THE METHOD OF BULIRSCH - STECK FOR
C
C USAGE
C, CALL DRES(DEN,Y,1,N,JM,RO,JSTART,H,imin,
C, eps,r,s,tk,ier)
C
C PARAMETERS
C, DEN - USER SUPPLIED EXTERNAL SUBROUTINE,
C, DEN(Y,T,N,DY). DEN MUST CALCULATE DY(I) =
C, F(Y(1),Y(2),...,Y(N),T) FOR I=1,2,...,N.
C, Y - ON INPUT Y(1),Y(2),...,Y(N) ARE INITIAL VALUES.
C, ON OUTPUT Y(1),Y(2),...,Y(N) CONTAIN AN
C, APPROXIMATE SOLUTION TO Y AT T (AS SET ON
C, OUTPUT).
T - \( t \) IS THE INDEPENDENT VARIABLE. ON INPUT, \( t \) SHOULD CONTAIN THE INITIAL VALUE OF THE INDEPENDENT VARIABLE. ON OUTPUT, \( t \) CONTAINS THE UPDATED VALUE OF THE INDEPENDENT VARIABLE.

N - THE NUMBER OF EQUATIONS IN THE SYSTEM

JH - THE MAXIMUM ORDER OF THE RATIONAL APPROXIMATION. \( J \) MUST BE LESS THAN \( T \).

IND - CONVERGENCE CRITERION (INPUT)

IND = 1 SPECIFIES THE STANDARD ERROR TEST
IND = 2 SPECIFIES THE RELATIVE ERROR TEST
IND = 3 SPECIFIES THE ABSOLUTE ERROR TEST

JSTART - AN INPUT INDICATOR WITH THE FOLLOWING MEANINGS:

0 - PERFORM THE FIRST STEP

-1 - REPEAT THE LAST STEP WITH A NEW VALUE OF \( t \) OR \( J \). THE INITIAL VALUES OF \( y \), \( s \), AND \( t \) ARE SET TO THE INITIAL VALUES OF \( y \), \( s \), AND \( t \) FROM THE MOST RECENT CALL TO THE ROUTINE WITH JSTART = 0.

H - ON INPUT, \( h \) IS AN INITIAL GUESS FOR THE STEP SIZE.

ON OUTPUT, \( h \) CONTAINS A SUGGESTED STEP SIZE FOR THE NEXT STEP. THE SUGGESTED VALUE MAY BE LARGER OR SMALLER THAN THE ORIGINAL STEP SIZE.

MAX - MAX IS THE SMALLEST PERMISSIBLE STEP SIZE. ORDERS WILL DECREASE THE STEP SIZE UNTIL CONVERGENCE CAN BE OBTAINED.

EPS - ERROR TOLERANCE

P - ON OUTPUT, THE \( n \)-VECTOR \( p \) CONTAINS THE
ABSOLUTE ERRORS IN EACH COMPONENT FOR THE CURRENT STEP.

J

S - A VECTOR CONTAINING EITHER (1) THE LARGEST VALUE OF EACH Y COMPUTED SINCE THE START OF THE INTEGRATION FOR THE STANDARD ERROR TEST, DTR0530

(2) THE LARGEST VALUE OF EACH Y COMPUTED DURING THE CURRENT STEP BEING TAKEN FOR RELATIVE ERROR TEST, OR (3) THE VALUE 1.0 FOR THE ABSOLUTE ERROR TEST.

S MUST BE INITIALIZED IN ACCORDANCE WITH THE ERROR TEST SELECTED BEFORE THE FIRST CALL TO DREBS. IF IND = 1 OR 2, S(I) SHOULD BE EQUAL Y(I) ON THE FIRST CALL. IF IND = 3, S(I) SHOULD EQUAL I.

WK - WORKING STORAGE OF DIMENSION 29*N

IER - ERROR INDICATOR

TERMINAL ERROR = 128 + N

N = 1 INDICATES CONVERGENCE COULD NOT BE OBTAINED WITH CURRENT VALUES OF H AND HN, Y, Y', AND U HAVE BEEN UPDATED.

N = 2 INDICATES J4 IS LESS THAN 1 OR GREATER THAN 6. SET J4 = 6.

PRECISION - SINGLE/DIABLE

REQUIRES IML ROUTINES - DRTST

LANGUAGE - FORTRAN

LATEST REVISION - APRIL 5, 1977

SUBROUTINE DREBS(DY,Y,T,H,J4,IND,ISTART,HMIN,EP5,R,S,WKIER)

DIMENSION Y(J4),S(I),R(N),W(7),WK(11)
C DOUBLE PRECISION Y,1,H,X1N, EPS, R, S, K, D, Z0TOP, A, G,
C 1
C DATA
C DATA
C LOGICAL
C IER=0
C N2=N+1
C N3=N2+1
C N4=N3+1
C N5=N4+1
C N6=N5+N5+N2
C N7=N6+N5+N3
C N8=N7+N5+N3
C N2P1=N2+1
C N3P1=N3+1
C N6P1=N6+1
C IF (JMAX. GE. 10 .AND. JM. LE. 6) GO TO 5
C IER = 66
C JM=0
C C FOR AN EXTRAPOLATION OF ORDER JM,
C JM+1 APPROXIMATIONS ARE REQUIRED.
C THREE MORE ARE ALLOWED IN ATTEMPTING
C TO ACHIEVE CONVERGENCE.
C C JMAX = JM+4
C IF (JSTART. GE. 10) GO TO 135
C C INITIALIZATION
C SAVE THE INITIAL VALUES FOR THE DEPEN-
C DENT VARIABLES AND THE ERROR TEST
C C VECTOR FOR THE STEP
C C TOLJ = 1
C JK+4=JK
C DO 10 I = 1, N
WK(1) = Y(1)
4 = JK4+1
WK(JK4) = S(1)
10 CONTINUE

CALL DEF(Y1, T, H, WK(3P1))

15 BH = .FALSE.

KONVF = .TRUE.

20 K = H+I

BG = .FALSE.

M = 1

JG = 2

JS = 3

JJ = 0

USE THE FUNCTION ROUTINE TO OBTAIN
THE INITIAL SLOPES
DZ = DY/DX

THE LOGICAL VARIABLE, BH, DETERMINES
WHETHER THE STEPSIZE HAS BEEN
HALVED, INITIALLY FALSE.
LATER BH IS FALSE IF THE STEPSIZE IS
CUT BY A FACTOR NOT 2

PRESET THE CONVERGENCE SUCCESS FLAG
TRUE

ADVANCE THE INDEPENDENT VARIABLE BY
THE STEPSIZE, K

SET THE SWITCH ON FOR THE FIRST SET
OF COEFFICIENTS, D

INITIALIZE THE M SEQUENCE...
H/H, H/H, H/H/J

JJ IS THE INDEX FOR THE ARRAY OF
VALUES SAVED IN CASE THE INTERVAL
MUST BE HALVED
C
C
1 Je = N - N
1 J f = N - N
1 c = 1 J 0
1 7 = 1 J 7
DO 125 J = 1, J MAX
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
INTRODUCTION STEP
KIDPLN + EXTRAPOLATION
C
DTRB1450
DTRB1460
DTRB1470
DTRB1480
DTRB1490
DTRB1500
DTRB1510
DTRB1520
DTRB1530
DTRB1540
DTRB1550
DTRB1560
DTRB1570
DTRB1580
DTRB1590
DTRB1600
DTRB1610
DTRB1620
DTRB1630
DTRB1640
DTRB1650
DTRB1660
DTRB1670
DTRB1680
DTRB1690
DTRB1700
DTRB1710
DTRB1720
DTRB1730
DTRB1740
DTRB1750
DTRB1760

SET THE VALUES OF THE EXTRAPOLATION COEFFICIENTS TO THEIR CORRECT VALUES FOR THIS EXTRAPOLATION STEP

DO 125 J = 1, J MAX

D(2) = 1.77777777777777777
D(4) = 7.11111111111111111
D(6) = 25.44444444444444444
D(2) = 1.777777
D(4) = 7.111111
D(6) = 25.44444
DTRB1450
DTRB1460
DTRB1470
DTRB1480
DTRB1490
DTRB1500
DTRB1510
DTRB1520
DTRB1530
DTRB1540
DTRB1550
DTRB1560
DTRB1570
DTRB1580
DTRB1590
DTRB1600
DTRB1610
DTRB1620
DTRB1630
DTRB1640
DTRB1650
DTRB1660
DTRB1670
DTRB1680
DTRB1690
DTRB1700
DTRB1710
DTRB1720
DTRB1730
DTRB1740
DTRB1750
DTRB1760

IF THE ORDER OF THE EXTRAPOLATION STEP BEING COMPUTED IS LESS THAN JM/2
DTRB1720

SET KONV = FALSE
DTRB1730

IF ( J . LE. ( JM/2 ) ) KONV = .TRUE.
DTRB1740

IF ( J . LE. ( JMAX+1 ) ) GO TO 35
DTRB1750

DTRB1760
RESTRICT THE ORDER OF THE EXTRAPOLATION TO JP

ADJUST THE EXTRAPOLATION COEFFICIENT

DISCOURAGE THE STEP-INCREASING FACTOR BY A FACTOR OF SQRT(J) SINCE CONVERGENCE HAS NOT OBTAINED IN JM EXTRAPULATIONS

THE NUMBER, J, OF EXTRAPULATIONS HAS NOT EXCEEDED JM

FIND D(J) = ((H DIVIDED BY H/M) x 2

ADJUST THE FACTOR, FC, USED TO ADJUST THE STEPSIZE FOR THE NEXT STEP TO BE TAKEN

MODIFIED MIDPOINT RULE USED TO FIND FIRST VALUE FOR THIS EXTRAPULATION STEP

IF THE STEPSIZE HAS NOT BEEN HALVED OR IF THE ORDER OF THE EXTRAPULATION STEP EXCEEDS THAT FOR WHICH PREVIOUSLY COMPUTED VALUES WERE SAVED, THEY...
C
IF (R.XH1,<H1) OR (J.XE,(JMAX-1)) GO TO 50
C
OTHERWISE THE VALUES HAVE BEEN SAVED
AND CAN BE RESTORED
C
IJK1=N
IJK2=N2
IJK6=1J6
IJK7=1J7
DO 45 I=1,N
   IJK2=IJK2+1
   IJK7=IJK7+1
   WK(IJK2)=WK(IJK7)
   IJK1=IJK1+1
   IJK6=IJK6+1
   WK(IJK1)=WK(IJK6)
45 CONTINUE
GO TO 75
C
COMPUTE STARTING VALUES FOR THE MODIFIED MIDPOINT RULE
C
50 IJK1=N
IJK2=N2
IJK3=N3
DO 55 I=1,N
   IJK1=IJK1+1
   WK(IJK1)=WK(I)
   IJK2=IJK2+1
   IJK3=IJK3+1
   WK(IJK2)=WK(I)+6*WK(IJK3)
55 CONTINUE
K1=4/2
XU=T
C
THE MEMBER OF THE H SEQUENCE
C
DTRB2090
DTRB2100
DTRB2110
DTRB2120
DTRB2130
DTRB2140
DTRB2150
DTRB2160
DTRB2170
DTRB2180
DTRB2190
DTRB2200
DTRB2210
DTRB2220
DTRB2230
DTRB2240
DTRB2250
DTRB2260
DTRB2270
DTRB2280
DTRB2290
DTRB2300
DTRB2310
DTRB2320
DTRB2330
DTRB2340
DTRB2350
DTRB2360
DTRB2370
DTRB2380
DTRB2390
DTRB2400
C    BEING USED BY THE MIDPOINT INTEGRATION
C    THIS RULE IS H/M. COMPUTE THE END  OF THE STEP FOR EACH DEPENDENT VAR1-DTRB2430
C
DO 70 K = 2,N
XU = XU + G
CALL OFH(WK(N2P1),XU,N,WK(N8P1))  
IIK1=N
IIK2=N2
IIK8=N8
DO 60 I = 1,N
IIK1=IIK1+1
IIK8=IIK8+1
U = WK(IJK1) + 5*WK(IJK8)
IIK2=IIK2+1
WK(IJK1) = WK(IJK2)
WK(IJK2) = U
60 CONTINUE

C IN CASE THE INTERVAL MUST BE HALVED
C NEXT TIME, SAVE THE VALUES AT HALFWAY DTRB2600
C ALLING (KH=4/2) THE STEP UNLESS K=3 DTRB2610
IF ((K .NE. KH) .OR. (K .EQ. 3)) GO TO 70
IIJ = IIJ+II
II6=II6+II
II7=II7+II
IIK1=II
IIK2=II
IIK3=II
IIK7=II
DO 65 I = 1,N
IIK2=IIK2+1
IIK7=IIK7+1
65
\[\text{WK}(\text{ijk}7) = \text{WK}(\text{ijk}2)\]
\[\text{ijk}6 = \text{ijk}6 + 1\]
\[\text{ijk}1 = \text{ijk}1 + 1\]
\[\text{WK}(\text{ijk}6) = \text{WK}(\text{ijk}1)\]

65 \text{ CONTINUE} \\
70 \text{ CONTINUE} \\
75 \text{ CALL \texttt{DFN} (WK(N2PL),A,N,WK(N8PL))} \\
1 \text{ijk}1 = \text{N} \\
1 \text{ijk}2 = \text{N}2 \\
1 \text{ijk}5 = \text{N}5 \\
1 \text{k}5 = \text{N}5 \\
1 \text{ijk}5 = \text{N}8 \\
\text{DO 115} \text{ I} = 1, \text{N} \\

C \text{ V IS USED TO SAVE THE VALUE OBTAINED BY THE MIDPOINT RULE USING THE PREVIOUS VALUES.} \\
C \text{ THE FIRST TIME THROUGH THIS VALUE IS DTRB2900} \\
C \text{ L IS LESS THAN 2} \\
C

1 \text{ijk}1 = \text{ijk}1 + 1 \\
1 \text{ijk}2 = \text{ijk}2 + 1 \\
1 \text{ijk}5 = \text{ijk}5 + 1 \\
1 \text{k}5 = \text{k}5 + 1 \\
\text{IF (L} * \text{BE} . 2) \text{ V} = \text{dcl}(\text{ijk}5) \\

C \text{ COMPUTE THE FINAL VALUE OBTAINED FOR THIS MEMBER OF THE A SEQUENCE BY THE MODIFIED MIDPOINT RULE} \\
C

1 \text{ijk}6 = \text{ijk}6 + 1 \\
\text{WK}(\text{ijk}5) = (\text{WK}(\text{ijk}2) + \text{WK}(\text{ijk}1) + \text{G} * \text{WK}(\text{ijk}8)) * \text{HALF} \\
\text{C} = \text{WK}(\text{ijk}5) \\
\text{TA} = \text{C} \\

C \text{ AT LEAST TWO VALUES ARE NEEDED TO} \\

176
C IF (L .LT. 2) GO TO 90
C IF THE VALUE JUST COMPUTED BY THE
C MIDPOINT RULE SHOWS A LARGE JUMP FROM
C THE PREVIOUS, HALVE THE INTERVAL
C
C1 IF ((ABS(V) .LT. CUBITUP .LT. ABS(C)) .AND. (H .NE. HMIN)) .AND.
C (J.GT.JM/2+1)) GO TO 130
C
C1 PERFORM THE L STEPS FOR THE CURRENT
C LTH GROUP EXTRAPOLATION STEP. IF THE
C DENOMINATOR OF THE RATIONAL FUNCTION
C GOES TO ZERO AT ANY STEP, SET DT AT
C THAT STEP TO ITS VALUE JUST BEFORE
C
C IP5=1K5
DO 85 K = 2,L
IP5=IP5+1
AL = L(K) = V
B = BL-C
U = V
IF (B .EQ. 0.) GO TO 80
B = (C-V)/B
U = C*B
C = B1*B
80 IF (K.LT.L) V=WR(IP5)
WR(IP5) = U
TA = U + TA
85 CONTINUE
C USE THE ERROR ROUTINE FOR EACH
C DEPENDENT VARIABLE TO CHECK WHETHER
C CONVERGENCE HAS BEEN ACHIEVED
C
C1 90 GO TO (95,100,105),IND
C1 95 UST=ABS(TA)
95 \textbf{UST} = \textbf{ABS} (\textbf{TA})
\textbf{100} \textbf{IF} \textbf{UST} \cdot \textbf{GT} \cdot \textbf{S} (\textbf{1}) \textbf{S} (\textbf{1}) = \textbf{UST}
\textbf{GO TO} \textbf{100}
\textbf{CL100} \textbf{S} (\textbf{1}) = \textbf{DABS} (\textbf{Y} (\textbf{1}))
\textbf{110} \textbf{S} (\textbf{1}) = \textbf{ABS} (\textbf{Y} (\textbf{1}))
\textbf{GO TO} \textbf{110}
\textbf{120} \textbf{S} (\textbf{1}) = 1.
\textbf{CL110} \textbf{P} (\textbf{1}) = \textbf{DABS} (\textbf{Y} (\textbf{1}) - \textbf{TA})
\textbf{110} \textbf{R} (\textbf{1}) = \textbf{ABS} (\textbf{Y} (\textbf{1}) - \textbf{TA})
\textbf{Y} (\textbf{1}) = \textbf{TA}
\textbf{IF} (\textbf{S} (\textbf{1}) \cdot \textbf{LT} \cdot \textbf{EPS}) \textbf{S} (\textbf{1}) = \textbf{EPS}
\textbf{IF} (\textbf{R} (\textbf{1}) \cdot \textbf{LT} \cdot \textbf{EPS} \cdot \textbf{S} (\textbf{1})) \textbf{KONV} = \textbf{.FALSE.}
\textbf{115} \textbf{CONTINUE}
\textbf{IF} (\textbf{KONV}) \textbf{GO TO} \textbf{155}
\textbf{C}
\textbf{D} (\textbf{3}) = 4.
\textbf{D} (\textbf{5}) = 16.
\textbf{C}
\textbf{S} (\textbf{1}) = 1.
\textbf{C}
\textbf{IJK4} = \textbf{N4}
\textbf{DO} 120 \textbf{I} = 1, \textbf{N4}
\textbf{IJK4} = \textbf{IJK4} + 1
\textbf{S} (\textbf{1}) = \textbf{KREAT} (\textbf{IJK4})
\textbf{120} \textbf{CONTINUE}
\textbf{C}
\textbf{TAKE THE NEXT MEMBER}
\textbf{OF THE M SEQUENCE}
\textbf{AND GO BACK FOR THE}
\textbf{RESET THE EXTRAPOLATION COEFFICIENTS}
\textbf{RESET THE EXTRAPOLATION COEFFICIENTS}
\textbf{FLIP THE DO SWITCH FOR THE NEXT SET}
\textbf{OF COEFFICIENTS}
\textbf{RESET S}
\textbf{RESET THE EXTRAPOLATION COEFFICIENTS
C
125 CONTINUE

NEXT Extrapolation
DTRB3690
DTRB3700
C
IF, AFTER ALL THE EXTRAPOLATIONS
DTRB3710
C
ALLOWED, CONVERGENCE HAS NOT BEEN
DTRB3720
C
ACHIEVED, ATTEMPT TO HALVE H SO THAT
DTRB3730
C
THE SAVED VALUES CAN BE USED (SET BH
DTRB3740
C
TRUE FOR THIS PURPOSE)
DTRB3750
C
IF HALVING H MAKES IT LESS THAN HMIN,
DTRB3760
C
SET H = HMIN.
DTRB3770
C
IN THIS CASE THE SAVED VALUES CANNOT
DTRB3780
C
BE USED.
DTRB3790
C
IF H HAD ALREADY BEEN AT HMIN,
DTRB3800
C
CONVERGENCE CANNOT BE ACHIEVED FOR
DTRB3810
C
THIS HMIN AND THIS EPS CRITERION.
DTRB3820
C
SET KHIV FALSE.
DTRB3830

BH = (.NOT. BH)

C1130 IF (DABS(H) .LE. DABS(HMIN)) GO TO 150
C130 IF (ABS(H) .LE. ABS(HMIN)) GO TO 150
C1
H = H - HALF
C1
IF (DABS(H) .GE. DABS(HMIN)) GO TO 20
C1
IF (ABS(H) .GE. ABS(HMIN)) GO TO 20
C1
H = SIGN(HMIN,H)
C1
H = SIGN(HMIN,H)
C1
GO TO 15
C135 DC: 140 I = 1, N
C140 CONTINUE
C145 CONTINUE

Y(I) = AK(I)

IJK4 = N4

D1 145 I = 1, N
IJK4 = IJK4 + 1
S(I) = S(I)(IJK4)

145 CONTINUE
T = TOLD
GO TO 15
150 KONVF = .FALSE.
C WHETHER OR NOT CONVERGENCE HAS BEEN
C ACHIEVED SET A NEW SUGGESTED STEP SIZE
C FOR THE NEXT STEP
C ASSIGN THE END OF STEP VALUE TO THE
C INDEPENDENT VARIABLE

155 H = FC*H
   F = A
   IF (KONVF) GO TO 160
   IER = 129
160 IF (IER .EQ. 0) GO TO 9005
9000 CONTINUE
   CALL UERST1 (IER, CHDREBS )
9005 RETURN
   END
C SUBROUTINE UERST1 (IER, NAME)
C
C UERST1 ----------------- LIBRARY 1 -------------------------------
C
C FUNCTION
C USAGE
C PARAMETERS
C LANGUAGE

DIRB4010
DTK4020
DTP4030
DIRB4040
DIRB4050
DIRB4060
DIRB4070
DIRB4080
DIRB4090
DIRB4100
DIRB4110
DIRB4120
DIRB4130
DIRB4140
DIRB4150
DIRB4160
DIRB4170
UERT0010
UERT0020
UERT0030
UERT0040
UERT0050
UERT0060
UERT0070
UERT0080
UERT0090
UERT0100
UERT0110
UERT0120
UERT0130
UERT0140
UERT0150
C                          --------- UERT0160
C LATEST REVISION - JANUARY 18, 1974  

C
C SUBROUTINE UERTST(IER,NAME)
C
C DIMENSION ITYP(5,4),IBIT(4)
C INTEGER*2 NAME(3)
C INTEGER WARN,WARF,TERM,PRINTER
C EQUIVALENCE (IBIT(1),WARN),(IBIT(2),WARF),(IBIT(3),TERM)
C DATA ITYP /'WARN','ING',' ',' ',' ',' ',
C * 'WARN','ING(','WITH',' FIX'),' ','
C * 'TERM','INAL',' ',' ','
C * 'NON-','DEFI','NED ',' ',' 
C * INIT / 32,64,128,0/
C DATA PRINTK / 6/
C IER2=IER
C IF (IER2 .GE. WARN) GO TO 5
C
C IER1=4
C GO TO 20
C 5 IF (IER2 .LT. TERM) GO TO 10
C
C IER1=3
C GO TO 20
C 10 IF (IER2 .LT. WARF) GO TO 15
C
C IER1=2
C GO TO 20
C
C 15 IER1=1
C
C 20 IER2=IER2-128(IER1)
C

PR 1 RT ERROR MESSAGE

WRITE (PRINTF,25) (IYPAI,1,1E1),I,=1,2),NAME,IER2,ICR

25 FORMAT(* **I H S L(UERTST) ***,5A4,4X,3A2,4X,12, 
* (IFR = ',13,')*)

RETURN
END
APPENDIX II

Two FORTRAN computer programs developed by the author to calculate the harmonic constants for the tidal current directly from the harmonic constants of the tidal elevation (equation 12), and to generate a time series from the harmonic constants (equation 5), are presented in this appendix. If it is assumed that each harmonic constituent of the tide can be treated separately, these programs can be used to provide the same results as the program of Appendix I, but at significant savings in computation time.
PROGRAM TO INFER TIDAL CURRENTS FROM GIVEN SURFACE
ELEVATIONS. COS UNITS ARE USED UNLESS OTHERWISE NOTED. ALL INPUT IS THROUGH SUBROUTINE DATAIN.

IMPLICIT REAL*8 (A-H, O-Z)
REAL*8 IFIRST, JFIRST, KFIRST, IFST, JFST, KFST
DIMENSION A(110), R(4)
COMMON /BLOCK1/UI(200), U2(200), VI(200), V2(200)
COMMON /BLOCK2/ZS1(200), ZS2(200)
COMMON /BLOCK3/MAPLAT(200), ELONG(200), DEPTH(200), SIGMA
COMMON /BLOCK4/Z1(200), Z2(200), ZB1(200), ZB2(200)
COMMON /BLOCK5/IFCUNS, CSYS, MPTS, MAXITK, LORDER
COMMON /BLOCK6/NUMPNT(20, 20), ELONG, 4CCLAT
COMMON /BLOCK12/TIDE, AMPMAX

IEL=0
PL=3.141592654
ESPLED=2.*PI/(24.*3600.)
ACCEL=980.
ERAD=6.3748
DTOR=2.*PI/360.

READ ALL INPUT AND IDENTIFY TIDAL CONSTITUENT
call datain(1er)
call phase(1er)
if(ier.eq.-1) stop

SUBTRACT EQUILIBRIUM ELEVATIONS FROM GIVEN SURFACE
call zobar
do 1 i=1, mpts
\[ ZS1(I) = Z1(I) - ZB1(I) \]
\[ ZS2(I) = Z2(I) - ZB2(I) \]

C
SOLUTION IS PERFORMED 'MATIR' TIMES TO ALLOW
SUCCESSIVE CORRECTION FOR THE EFFECT OF FRICTION.
INITIALIZE CURRENTS. MAGNITUDE OF THE GUESS DETER-
MINES THE MAGNITUDE OF THE FRICTIONAL TERM ON THE
FIRST ITERATION.

DO 2 I=1,MPTS
   U1(I)=1.
   U2(I)=0.
   V1(I)=1.
   V2(I)=0.
2
DO 3 IMX=1,MAXITR
DO 4 I=1,MPTS
SET UP COEFFICIENTS MATRIX

FD=FRCONS*DSQR(T(U1(I)**2+U2(I)**2+V1(I)**2+V2(I)**2)/DEPTH(I)

FD=FRCONS
GB=FD
W=2.*ESPEED*DG/S(COLAT(I))
A(1) =SIGMA
A(5) =-GB
A(9) =0.
A(13) = w
A(2) = A(5)
A(6) = -SIGMA
A(10) = W
A(14) = 0.
A(3) = 0.
A(7) = -W
A(11) = SIGMA
A(15) = -F3
A(4) = -W
A(8) = 0.
A(12) = A(15)
A(16) = -SIGMA

C
SET UP RIGHT-HAND-SIDE VECTOR
APPROXIMATE DERIVATIVES
C
CALL PARTAL(I, DZ1DT, DZ2DT, DZ1DX, DZ2DX, (ER)
IF (ER(R) < 1) STOP
R(1) = DZ2DT*ACCEL*ERAD
R(2) = DZ1DT*ACCEL*ERAD
R(3) = DZ2DX*ACCEL*ERAD*DSIN(COLAT(1))
R(4) = DZ1DX*ACCEL*ERAD*DSIN(COLAT(1))

C
SOLVE FOR THE CURRENTS
C
CALL SOLVER(4, A, R, GDET)
IF (GDET EQ 0.00) STOP
U1(1) = R(1)
U2(1) = R(2)
V1(1) = R(3)
V2(1) = R(4)
CONTINUE
CONTINUE

PRINT THE RESULTS OF THE CURRENT CALCULATIONS
IN POLAR COORDINATES

DO 9 I=1, MPTS
AMP=DSQRT(U1(I)**2+U2(I)**2)
PHSU=DATAN2(U2(I),U1(I))/DTRK
IF(PHSU.LT.0.) PHSU=PHSU+360.
AMPV=DSQRT(V1(I)**2+V2(I)**2)
PHSV=DATAN2(V2(I),V1(I))/DTRK
IF(PHSV.LT.0.) PHSV=PHSV+360.
WRITE(2,5) I, AMPU, PHSU, AmpV, PHSV
9 FORMAT(15,F10.2,F10.2,F10.2,F10.2)
STOP
END

SUBROUTINE AROUND(INDEX, L9NORTH, LSOUTH, LEAST, LWEST, INDEX)

FIND THE NUMBERS OF THE POINTS AROUND POINT NUMBER
INDEX THAT WILL BE INVOLVED IN THE APPROXIMATION
OF THE DERIVATIVES. AT ANY POINT OF THE GRID THERE
MUST BE AT LEAST THREE CONSECUTIVE POINTS ALONG A LINE
THROUGH THAT POINT, INCLUDING THAT POINT. L9NORTH AND
LSOUTH ARE THE CLOSEST POINTS ALONG A LONGITUDE LINE.

COMMON / BLOCKS/NUMPT(20,20), ALONG, NUMLAT
DO 1 J=LALONG
DO 1 I=1, NUMLAT
IF(NUMPT(I,J).NE.INDEX) GO TO 1
1 CONTINUE

LD=1-1
IF(LN.LT.1) LN=I+2
LS=I+1
IF(LS.LT.REGRID) LS=I+2
LE=J+1
IF(LE.LT.MILNG) LE=J+2
LW=J-1
IF(LW.LT.1) LW=J+2
LNORTH=NUMPNT(LN,J)
IF(LNORTH.GT.0) GO TO 2
LN=I+2
LNORTH=NUMPNT(LN,J)
2 LSOUTH=NUMPNT(LS,J)
IF(LSOUTH.GT.0) GO TO 3
LS=I-2
LSOUTH=NUMPNT(LS,J)
3 LEAST=NUMPNT(I,LE)
IF(LEAST.GT.0) GO TO 4
LE=J-2
LEAST=NUMPNT(I,LE)
4 LWEST=NUMPNT(I,LW)
IF(LWEST.GT.0) GO TO 5
LW=J+2
LWEST=NUMPNT(I,LW)
5 CONTINUE
RETURN

1 CONTINUE
IFK=-1
RETURN
END

SUBROUTINE DATAFIN(IER)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION UBT(30),FMT(16),NCHECK(200)
COMMON /BLOCK1/ U1(200), U2(200), V1(200), V2(200)
COMMON /BLOCK2/ COLAT(200), ELONG(200), R(200), SIGMA
COMMON /BLOCK3/ Z1(200), Z2(200), ZR1(200), ZR2(200)
COMMON /BLOCK4/ R(200), R(200), R(200), R(200)
COMMON /BLOCK5/ R(200), R(200), R(200), R(200)
COMMON /BLOCK6/ R(200), R(200), R(200), R(200)
COMMON /BLOCK7/ R(200), R(200)
COMMON /BLOCK12/ TIDE, AMPMAX
DATA SPHET/SPHET/ , PI=3.141592654,
, DTOK=2.*PI/360. ,
, ERAD=6.374EB8,
, ATUL=2.*PI*EFAU/360.
WHITE(6,46)
40 FORMAT(' CONTROL INPUT IS ON UNIT 1, GRID INPUT IS'
, ' ON UNIT 1, RESULTS ARE OUTPUT ON UNIT 2')
C
C TWO TITLE CARDS ARE REQUIRED
C
WRITE(6,32)
32 FORMAT(' INPUT TWO LINES AS COMMENT')
READ(1,30) GMT
30 FORMAT(15A4)
WRITE(2,31) GMT
31 FORMAT(1X,15A4)
C
'TIDE' IS THE TIDAL CONSTITUENT. SEE SUBROUTINE PHASE.
'MAXITR' IS THE NUMBER OF PASSES MADE TO CORRECT
FOR FRICTION.
C
WRITE(6,47)
47 FORMAT(' ENTER TIDE CONSTITUENT')
READ(1,30) TIDE
WRITE(2,31) TIDE
WRITE(2,48)
48 FORMAT(' ENTER NO. OF PASSES TO CORRECT FOR FRICTION, 2 DIGITS')
READ(1,5) MAXITR
5 FORMAT(12)
WRITE(2,1) MAXITR
WRITE(6,481)
481 FORMAT(' ENTER COEFFICIENT OF FRICTION TERM')
READ(1,*1) FGONS
WRITE(2,482) FGONS
+32 FORMAT(E20.5)
C EITHER A POLAR STEREOGRAPHIC, OR SPHERICAL COORDINATE SYSTEM
C CAN BE USED.
C WRITE(6,49)
49 FORMAT(' SPHERICAL COORDINATES OR NAVIGATION GRID')
READ(1,30) CSYS
WRITE(2,311) CSYS
IF(CSYS.EQ.'SPHE') GOTO 50
WRITE(6,51)
51 FORMAT(' ON THE NAVIGATION GRID, ONE DEGREE IS EQUIVALENT TO 111.3
 :17 KM*/' GRID DIRECTIONS NORTH AND EAST ARE NEGATIVE VALUED')
50 CONTINUE
C 'MCOLAT' IS THE NUMBER OF LATITUDE LINES IN THE GRID.
C 'MOLONG' IS THE NUMBER OF LONGITUDE LINES IN THE GRID.
C 'MCPTS' IS THE NUMBER OF POINTS ON THE SEA.
C READ(1,1) MCOLAT,MOLONG
1 FORMAT(515)
C  READ IN GRID POINT NUMBERS BY ROWS.  GRID INTERSECTIONS
C  OUTSIDE THE AREA OF INTEREST MUST BE NUMBERED 0.
C
C  READ(1,2) ((NUMPNT(1,J),J=1,MLONG),I=1,MCOLAT)
2  FORMAT(1615)
   NMAX=0
   MPTS=0
   DO 37  I=1, MCOLAT
   DO 37  J=1, MLONG
   NMAX=MAX(NMAX, NUMPNT(1,J))
   IF(NUMPNT(1,J).GT.0) MPTS=MPTS+1
37  CONTINUE
   WRITE(2,14)
14  FORMAT(' THE FOLLOWING GRID WAS GIVEN (NORTH IS AT THE TOP)')
   DO 15  K=1, MCOLAT
15  WRITE(2,16) (NUMPNT(K,J),J=1,MLONG)
16  FORMAT(25J5)
C
C  READ IN PHYSICAL PARAMETERS AT EACH GRID POINT BY FORMAT
C  *FMT'
C  'FMT' IS THE COLATITUDE OF THE POINT.  'FLONG' IS
C  THE EAST LONGITUDE OF THE POINT.  'H' IS THE WATER DEPTH
C  IN METERS.
C
C  WRITE(2,10)
10  FORMAT(' THE FOLLOWING PHYSICAL PARAMETERS WERE READ',
     :'/1X, 'GRID POINT',/7X, 'LATITUDE',/6X, 'LONGITUDE',/9X, 
     :'DEPTH',/11X, 'ID',/9X, 'PHASE',/3X, 'NUMBER',/7X, 
     :'(CO- OR GRID)',/3X, 'WEST',/8X, 'METERS',/6X, 'CM',/ 
     :'X', 'DEGREES')'
   READ(1,4) FMT
4  FORMAT(20A4)
DO 34 I=1, NPTS
34 NCHECK(1)=0
3 READ(1, FMT, END=999) I, A, B, C, D, E
1 IF (I.EQ.0 .OR. I.GT. NPTS) GO TO 7
1 IF (NCHECK(I).EQ.1) GO TO 7
NCHECK(I)=1
WPITE (2, 11) I, A, B, C, D, E
11 FORMAT (10, 2F15.5, F15.0, 4F15.2)

CHANGE FROM POLAR TO RECTANGULAR FORM
L1(I) = D*DCS(I*6.28)  
L2(I) = D*DSS(I*6.28)

CHANGE DEPTH TO CENTIMETERS
H(I) = C*100.

THE NAVIGATION GRID
1 IF (CSYS.EQ. 'SINE') GO TO 52
Y(I) = A*ATCL
X(I) = B*ATCL
GO TO 3

SPHERICAL WAS RIGHT, RADIANS AND EAST LONGITUDE ARE REQUIRED

52 COLAT(I) = A*DTOR
ELONG(I) = 2. *PI-B*DTOR
GO TO 3
999 GO 36 I=1, NPTS
1 IF (NCHECK(I).NE.1) GO TO 7
36 CONTINUE
C IN CASE WE ARE USING THE NAVIGATION GRID, CALCULATE THE
C LONGITUDES AND EAST LONGITUDES
C IF(CSYS.EQ.SPHERE) RETURN
50 53 I=1,NPTS
C CFACT=4.*ERAD**2/(X(I)**2+Y(I)**2)
C CFACT2=1.00-CFACT
C COLAT(I)=DAPCOS(CFACT2/(2.00-CFACT2))
C ELONG(I)=DMCOS((ELONG(I)+2.*P1,2.*P1)
53 CONTINUE
RETURN
7 WRITE(2,9) 1
9 FORMAT('EXECUTION TERMINATED BY SUBROUTINE DATA1!', 'ERROR IN INPUT DATA', 'SOMETHING STRANGE ABOUT GRID POINT', 'NUMBERED', '15', 'HAS BEEN FOUND.' )
1ER=-1
RETURN
END
SUBROUTINE PHASE(1ER)
C FIND THE SPEED 'SIGMA' AND THE AMPLITUDE OF THE RIGID
C EARTH GRAVITY TIDE FROM THE VARIABLE 'TIDE'
C RIGID EARTH TIDE AMPLITUDES IN MICROGALS (MELCHORF)
C O1=-31, P1-15, K1-44, H2-14, S2-34, A2-15
C
C IMPLICIT REAL*(A-H,L-Z)
C COMMON/CLOCK,CLAT,ELONG,DEPTH,SIGMA
C COMMON/CLOCK,TIDE,AMPMAX
C DATA C1,P1,A1,K1,H2,S2,A2/01,'P1','15','44','14','34','15','
: 'N2' /
ACCEL=980.
BTR=2.*3.14159/360.
ERAD=6.*374
IF(TIME.EQ.01) GO TO 1
IF(TIME.EQ.PI) GO TO 21
IF(TIME.EQ.AK1) GO TO 22
IF(TIME.EQ.AM2) GO TO 3
IF(TIME.EQ.SZ) GO TO 4
IF(TIME.EQ.AM2) GO TO 5
WRITE(6,10) TIME
10 FORMAT(' SPECIFIED TIME ('',A4,'') WAS NOT FOUND.')
JER=-1
RETURN
1 SIGMA=13.9430350
AMPMA=10.06
GO TO 6
21 SIGMA=14.9589314
AMPMA=4.98
GO TO 6
22 SIGMA=15.0410680
AMPMA=14.16
GO TO 6
3 SIGMA=26.9841042
AMPMA=24.24
GO TO 6
4 SIGMA=30.
AMPMA=11.28
GO TO 6
5 SIGMA=28.4397299
AMPMA=4.69
6 CONTINUE
CONVERT SIGMA TO RADIANS PER SECOND

SIGMA=SIGMA*DTOR/3600.
RETURN
END

SUBROUTINE ZBAR

COMPUTE AMPLITUDE AND GREENWICH PHASE OF EQUILIBRIUM TIDE

IMPLICIT REAL*8 (A-H,O-Z)
COMMON/BLCK3/COLAT(200),ECLS(200),DEPTH(200),SIGMA
COMMON /BLOCK4/Z1(200),Z2(200),Z61(200),Z32(200)
COMMON /BLOCK5/FRCNS,CSYS,MPTS,MAXIT,ICORDER
COMMON/BLCK12/TIDE,AMPMAX
PI=3.141593
DTOR=2.*PI/360.
DO 1 K=1,MPTS

FIND THE LATITUDE

LAT=DABS(COLAT(K)-PI/2.)

COMPUTE THE AMPLITUDE AND PHASE OF THE EQUILIBRIUM TIDE

MUST CHOOSE BETWEEN DIURNAL AND SEMIDIURNAL CONSTITUENT

XX=22.5*DTOR/3600.
IF(SIGMA.GT.XX) GO TO 2

DIURNAL
A.MPEQ=AMP.MAX*DSIN(2.*TLAT)
PSEQ=0.00
GO TO 3
C
C SEMIDIURNAL
C
2 A.MPEQ=AMP.MAX*(COS(TLAT))*2
PSEQ=0.00
3 CONTINUE
C
C CONVERT FROM POLAR TO RECTANGULAR REPRESENTATION
C
ZB1(K)=AMPEQ*DCOS(PHSEQ)
ZB2(K)=AMPEQ*DSIN(PHSEQ)
1 CONTINUE
RETURN
END
SUBROUTINE PARTIAL (1, A, B, C, D, IEK)
C
C COMPUTE DERIVATIVES AT POINT '1'
C
IMPLICIT REAL*8 (A-H, O-Z)
REAL*8 I.FIRST, J.FIRST, K.FIRST, IFIRST, JFIRST, KFIRST
COMMON /BLOCK1/U1(200),U2(200),V1(200),V2(200)
COMMON /BLOCK2/ZS1(200),ZS2(200)
COMMON /BLOCK3/CCLAT(200),DELTIN(200),LEPTIN(200),SIGMA
COMMON /BLOCKS/FCSIN,SYS1,NPTS,MAXITR,OCRER
COMMON /BLOCK7/X(200),Y(200)
DATA SPINE/'SPINE'/
EPA0=6.374E8
C
C FIND POINTS TO BE USED IN DERIVATIVES
CALL ARGUND(I,J,K,KK, JJ, IER)

IF (CSYS.EQ.SPHE) GO TO 3

NAVIGATION GRID

AA = Y(K) - Y(I)
BB = Y(J) - Y(I)
CC = X(KK) - X(I)
DD = X(JJ) - X(I)

GO TO 2

SPHERICAL GRID

3 AA = COLAT(K) - COLAT(I)
BB = COLAT(J) - COLAT(I)
CC = ELONG(KK) - ELONG(I)
DD = ELONG(JJ) - ELONG(I)

CONTINUE

FIRST = (AA*AA-DB*DB)/(AA*BB+BB-AA*AA)
JFIRST = AA/(AA*BB+BB-AA*AA)
KFIRST = BB/(AA*BB+BB-AA*AA)
IFIRST = (CC*CC-DD*DD)/(CC*DD-DD-CC*CC)
JFIRST = CC/(CC*DD-DD-CC*CC)
KFIRST = DD/(CC*DD-DD-CC*CC)

A = IFIRST*ZS1(I) + JFIRST*ZS1(J) + KFIRST*ZS1(K)
B = IFIRST*ZS2(I) + JFIRST*ZS2(J) + KFIRST*ZS2(K)
C = IFIRST*ZS1(I) + JFIRST*ZS1(JJ) + KFIRST*ZS1(KK)
D = IFIRST*ZS2(I) + JFIRST*ZS2(JJ) + KFIRST*ZS2(KK)

IF (CSYS.EQ.SPHE) RETURN

CONVERT DERIVATIVES FROM STEREGRAPHIC SYSTEM TO SPHERICAL SYSTEM
SUBROUTINE SOLVER(N,A,R,G)
C
C SUBROUTINE TO SOLVE SIMULTANEOUS EQUATIONS BY METHOD
C OF ELIMINATION (GAUSS-JORDAN). 'N' IS THE NUMBER OF
C EQUATIONS. 'G' IS THE DETERMINANT OF THE COEFFICIENTS
C MATRIX 'A'. 'R' IS THE RIGHT-HAND-SIDE VECTOR.
C
C IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION A(N,N),R(N)
G=1.0D0
DO 1 I=1,N
G=G*A(I,I)
1 IF(G.EQ.0.0D0) RETURN
TEMP=1.0D0/A(I,I)
DO 2 J=1,N
2 A(I,J)=A(I,J)*TEMP
K(I)=R(I)*TEMP
DO 1 K=1,N
IF(K.EQ.1) GO TO 1
U=A(K,I)
DO 3 J=1,N
3 A(K,J)=A(K,J)-U*A(I,J)
1 CONTINUE
RETURN
END
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION AMP(744),AZI(744)

PROGRAM TO CALCULATE CURRENT AMPLITUDES AND DIRECTIONS
DURING THE MONTH OF DECEMBER, 1977 FROM THE STEADY-
STATE SOLUTIONS TO THE LTE.

U=CURRENT SLOTH
V=CURRENT EAST
AZI=AZIMUTH MEASURED NORTH THRU EAST

U(T) = FP1*AP1U*COS((SP1*T+PH1P1-PHSP1U)*DTOR)
   + FK1*AK1U*COS((SK1*T+PH1K1-PHSK1U)*DTOR)
   + FL1*AL1U*COS((SL1*T+PH1L1-PHSL1U)*DTOR)
   + FN2*AN2U*COS((SN2*T+PH1N2-PHSN2U)*DTOR)
   + FS2*AS2U*COS((SS2*T+PH1S2-PHSS2U)*DTOR)

V(T) = FP1*AP1V*COS((SP1*T+PH1P1-PHSP1V)*DTOR)
   + FK1*AK1V*COS((SK1*T+PH1K1-PHSK1V)*DTOR)
   + FL1*AL1V*COS((SL1*T+PH1L1-PHSL1V)*DTOR)
   + FN2*AN2V*COS((SN2*T+PH1N2-PHSN2V)*DTOR)
   + FS2*AS2V*COS((SS2*T+PH1S2-PHSS2V)*DTOR)

J9 CURRENT AMPLITUDES

AP1U=1.14
AP1V=0.64
AK1U=3.39
AK1V=1.92
AL1U=3.01
AL1V=2.15
AN2U = 4.43
AN2V = 2.68
AN2U = 4.83
AN2V = 2.54
AS2U = 3.91
AS2V = 1.86

J9 CURRENT PHASES

PHSP1U = 149.
PHSP1V = 82.
PHSK1U = 148.
PHSK1V = 83.
PHSK2U = 126.
PHSK2V = 65.
PHSM2U = 148.
PHSM2V = 220.
PHSN2U = 52.
PHSN2V = 146.
PHSS2U = 56.
PHSS2V = 126.

MODE FACTORS

FP1 = 1.0
FP1 = 0.887
FP1 = 0.614
FM2 = 1.036
FM2 = 1.036
FS2 = 1.0

EQUILIBRIUM ARGUMENTS
C

PH1=20.3
PH2=343.3
PH3=253.1
PH4=238.4
PH5=12.2
PH6=0.0

C

CONSTITUENT SPEEDS

C

SP1=14.958931400
SR1=15.041068600
S01=13.543075600
S14=28.984104200
S22=28.435729500
S52=30.000000000

C

DTP=3.14159265410/180.00
DC1=1.744
T=DFLG(A1(I-1)
CV=U(1)
CV=V(T)
AMP(I)=DSQRT(CU+CV+CU+CV)
AZ(I)=D3TAN(2(CV-CU)/D1UR
IF(AZ(I)=7.773, AZ(I)=AZ(I)+360.
WRITE(6,2) (AMP(I), AZ(I), I=1,744)
FORMAT(1X,12E3.1/1X,12E3.1/1X,12E3.1/1X,12E3.1/1X)
STOP
END
APPENDIX III

The digital recorders used on the gravimeters to record the flexural waves in the ice shelf are the subject of this appendix. The recorders were based on the Sol-20 computer manufactured by Processor Technology Corporation of Pleasanton, California, with 16K memory, audio cassette tape recorder, and video monitor. This micro-computer was adapted for use as a digitizer by the installation of the following modules, which plug into the card cage of the computer: 88-ADC analog-to-digital converter by MITS Corporation of Albuquerque, New Mexico; 88-SPM clock module by International Data Systems, Incorporated of Falls Church, Virginia; and PIC-8 priority interrupt controller by IMSAI Manufacturing Corporation of San Leandro, California. The digitizing functions were controlled on an interrupt basis, hence the need for the PIC-8 module.

System software was based on machine language programs provided by the manufacturer with each of the system components although extensive modifications were made to the software provided. This software is distributed on a proprietary basis to the purchaser of the hardware. Because the system is both modular and programmable it is readily adaptable to a variety of functions, including processing of the acquired data. In service in Antarctica, the system proved to be rugged and reliable.
VITA

Richard Turl Williams II was born December 23, 1950 in Richmond, Kentucky. He obtained his elementary and high school education in the Madison County, Kentucky, school system. He graduated from high school in the spring of 1968 and entered Eastern Kentucky University, in the fall of that year. He majored in mathematics and physics and received his Bachelor of Science degree in June 1973. The following September he enrolled in the Graduate School at Virginia Polytechnic Institute and State University, in Blacksburg, Virginia as a geophysics major in the Department of Geological Sciences, and completed his Master of Science degree in July 1976. He is a member of the Society of Exploration Geophysicists.

Richard T. Williams II
THE OCEAN TIDE AND WAVES BENEATH THE
ROSS ICE SHELF, ANTARCTICA

by

Richard T. Williams II

(ABSTRACT)

Widely spaced tidal gravity records have been used to determine
the spatial and temporal variation of the ocean tide beneath the
Ross Ice Shelf. Cotidal-coamplitude maps have been drawn for the
six greatest harmonic constituents of the tide. These are \( K_1, P_1, O_1, M_2, S_2, \) and \( N_2 \). The tide is principally diurnal, the diurnal
amplitudes being roughly 3 times longer than the semidiurnals. The
range of the tropic tide is about 1 m at the northern extremity of
the ice shelf, and can be as great as 2 m in the southeastern part
of the region. The diurnal constituents can each be viewed as a
wave that propagates towards the southwest across the sea, having
an amplitude that is closely related to the thickness of the water-
layer beneath the ice. For each of the semidiurnal constituents
there is an amphidromic region located within the Ross Sea near
80° S latitude, 190° W longitude, and having a clockwise sense of
rotation.

Theoretical calculations of the tidal current indicate that
the semidiurnal and diurnal current constituents have roughly the
same amplitude. The semidiurnal current is magnified by near resonance with the inertia current due to the high latitude of the sea. Because of the resonance, calculations of the semidiurnal components of the tidal current are sensitive to the treatment of the retarding effects of the ice shelf and sea floor.

Waves having periods shorter than 20 min were observed in the ice shelf. These have been identified as flexural waves that are generated by the action of the ocean swell on the northern edge of the shelf. The observed speed of these waves was predicted within the uncertainty of the measurement by the classical flexural wave theory.