CHARACTERIZATION OF PALMER DROUGHT INDEX AS A PRECURSOR FOR DROUGHT MITIGATION

by

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(ABSTRACT)

Coping with droughts involves two phases. In the first phase drought susceptibility of a region should be assessed for developing proper additional sources of supply which will be exploited during the course of a drought. The second phase focuses on the issuance of drought warnings and exercising mitigation measures during a drought . These kinds of information are extremely valuable to decision making authorities.

In this dissertation three broad schemes i) time series modeling, ii) Markov chain analysis, and iii) dynamical systems approach are put forward for computing the drought parameters necessary for understanding the scope of the drought. These parameters include drought occurrence probabilities, duration of various drought severity classes which describe a region's drought susceptibility, and first times of arrival for non drought classes which signify times of relief for a drought-affected region. These schemes also predict drought based on given current conditions.

In the time series analysis two classes of models; the fixed parameter and the time varying models are formulated. To overcome the bimodal behavior of the Palmer Drought Severity Index (PDSI), primarily due to the backtracking scheme to reset the temporary index values as the PDSI values, the models are fitted to the Z index in addition to the PDSI for the forecasting of the PDSI.

In the non-homogeneous Markov chain, monthly transition probabilities are utilized to evaluate steady state probabilities of occurrences of various drought severity classes,

expected class, duration, and times of first visit to different weather classes signifying entry into and exit out of drought classes. Also, a decision tree formulation is offered for operational decision making during the course of a dry spell.

In the dynamical systems approach, by considering the hydrological processes to be local phenomenon and the forcing function namely, the precipitation, to be triggered by a global phenomenon, a stochastic differential equation (SDE) formulation is obtained. From the SDE an equivalent partial differential equation, the Fokker Planck equation, is obtained to yield the probability density function of the drought index. As an example, the fluctuations in the global forcing is attributed to the El Nino Southern Oscillation (ENSO) phenomenon. Its effects are accounted for through the Southern Oscillation index (SOI) and the Sea Surface temperature (SST) index. The drift and diffusion functions employed in the Fokker Planck equation capture sudden changes in the index values. To handle nonlinear drift functions, a piecewise linearization scheme is suggested which, along with a constant diffusion function, leads to employing a segmentwise Gaussian distribution as the solution with considerable ease.

It is hoped that the proposed methodologies will help in the issuance of drought warnings in a timely manner without undue burden on the forecasters or the citizenry. The results should also help in planning for additional water resources for the drought prone regions.

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Chapter I

INTRODUCTION AND MOTIVATION

1.1 INTRODUCTION

Of all the natural environmental hazards on earth, drought is perhaps the most damaging. By its nature it develops slowly, frequently occupying vast areas and persisting over long periods. Drought is defined as an extended period of dry weather, specially one injurious to crops (Webster's Dictionary, 1986). In the U.S., in the 80s, drought occurred not only with increased frequency and over wide spread areas, but was experienced by the large population of the normally moist east. Also, the 1988 growing season was the driest since 1895 for many farming regions in the U.S. The 1988 drought clearly showed the extensive impact of water shortages on agriculture, livestock, urban and industrial water supplies, hydropower, navigation, forestry, wetlands, recreation, and the structural integrity of water facilities. As a result of the drought of 1988, the federal government spent $ 3.9 billion on drought relief programs and $ 2.5 billion on farm credit programs (Riebsame et al., 1990).

There are mainly meteorological, agricultural and hydrological droughts emphasizing conditions of precipitation, soil moisture, streamflow and water storage, respectively. From the view point of extent of water deficiency, droughts are classified as mild, moderate, severe and extreme types. It may also be worthwhile to distinguish between aridity and drought, both of which are characterized by lack of water. Aridity carries the connotation of a more or less permanent climatic condition, bringing about deserts as the companion land form. Drought, on the other hand, is a temporary condition, occurring in a climatic zone where precipitation is ordinarily adequate for vegetation or agriculture, river flow and water supplies. There are marginal areas on the globe, often called semi-arid, that are transition zones betwen truly arid areas and moist regions with more reliable precipitation. These zones are most frequented by droughts. However, keeping in view the past history of the occurrences of droughts, it is well understood that no region, no matter how well blessed with water from the sky, is free from drought (Landsberg, 1975). Even in today's age of high technology and instant communication, agricultural and livestock production in industralized as well as developing societies can be sharply reduced by drought related stresses (Glantz, 1994).

1.2 METHODS OF ANALYZING DROUGHTS

The methods of analyzing droughts can be broadly classified into three categories : i) index based approach, ii) time series analysis and iii) theory of runs. Studies related to these methods relevant to the scope of present work are reviewed in Chapter II. An index based approach typically computes the departure from normal. The normal may be simply the mean precipitation or as in the Palmer index (Palmer, 1965) could be an integrated measure of the climatically appropriate normal values. The index itself is set to represent the cumulative departures. There have been many attempts to forecast drought severity by using the time series models. In addition to the series of the meteorologic and hydrologic variables, Sun spot series, tree ring series and the index (like the Palmer index) series, just to cite a few, have been analyzed. In the case of the drought analysis, the time series models have had only limited success. This may be attributed to the use of the information in the data in a limited manner such as computing only the first two statistical moments. In the theory of runs a cutoff value, say X_0 is chosen for the series $X(t)$. The wet periods ($X(t) > X_0$) and dry periods ($X(t) < X_0$) are characterized in reference to the chosen threshold X_0. The probabilities of drought duration and severity are computed with the assumption that the realizations after exercising the threshold limit form a random sample (independent, identically distributed random variable). Such an assumption may not always be valid. An appropriate method of drought analysis should be able to describe various characteristics of droughts, for example, severity, duration, frequency and yield results which are useful for both the planning and operational aspects of drought related policies. The traditional methods of analysis seem to be deficient in one respect or the other.

1.3 MOTIVATION

A comprehensive criterion for assessment of droughts must consider both the supply and demand phenomena. In 1965, W.C. Palmer of the then U.S. Weather Bureau, now National Weather Service, developed an index called the Palmer Drought Severity Index (PDSI). The PDSI characterizes the weather conditions based on a physically based comprehensive water balance analysis. For operational purposes there is the modified version of the PDSI called the Palmer Hydrological Drought Index (PHDI) (Karl, 1986). The PHDI, unlike the PDSI, avoids the backtracking procedure. In this study, because of their distinct advantages, both the PDSI and PHDI are used to characterize the droughts. Particularly, the monthly Palmer index data of the Northwest division in Arizona (1895-1992), the San Joaquin Drainage division in California (1895-1992), and the Tidewater division in Virginia (1895-1990) have been used. Also, a methodology for complete characterization of drought should yield results which are helpful for the planning and operational aspects of drought management strategies. Typically, during droughts it is the responsibilty of a state level task force to recommend drought mitigation measures to the regulatory agencies to minimize drought impacts. Such recommendations are based on the impacts of drought on various resources.

A review of different state drought plans indicates a real need for information such as :
i) how frequently drought conditions can be expected in a region, ii) how long such conditions will last , iii) when the relief (normal) canditions can be expected, iv) can such drought conditions be predicted in advance ? , and v) are droughts in a region affected by some external factor ? Besides, the task force needs some "decision rules" with regard to the issuance of warnings. In this study, these aspects are grouped under the label of

"complete characterization of droughts". The probabilities of occurrences of certain drought severity classes, the durations, and times of return to a severer drought class define the drought proneness of a region. Such information also aids the decision makers to understand the drought behavior in quantitative terms. An enumerative decision tree encompassing all possible occurrences is offered for operational decision making during the course of a dry spell. Because the decision tree displays observed sequence of events upto the current period and offers all possible branchings from that point, the decision making process becomes more objective and less cumbersome. The specific objectives are given in the following section.

1.4 OBJECTIVES

The specific objectives of the present study are :

i) To fit fixed and time varying parameter time series models to the PDSI and the Z index for forecasting.

ii) To identify an appropriate probability model for the complete characterization of droughts using the physically based Palmer drought index.

iii) To analytically determine the probability density function of the Palmer index using the dynamical systems approach and to analyze the effects of external forcing on droughts in a region.

iv) To develop a "decision tree" based on the study results to be used as a tool for making operational decisions during droughts by drought monitoring agencies.

The sequence of steps followed in the development of these methodologies is described as follows. Chapter II contains a literature review. In chapter III, time series models are developed for the PDSI and the Z index. These models are used to forecast the PDSI. In chapter IV, a non-homogeneous Markov chain approach is used for determining the time of residence (duration), time of return, and the probability of occurrence of a particular drought severity class. Also, a procedure for predicting drought severity classes is offered. A drought is considered to be resulting from a combination of a global forcing action in the form of precipitation for which the source region can be far away and a local response defined by the region's hydrology in terms of the surface runoff and the moisture holding capacity within the soil. In chapter V, a dynamical systems approach which accounts for global forcing is considered. The method also explains the bi-modal distribution of the PDSI by pointing out two stable points on the wet and dry side respectively. The solution process is governed by the classical Fokker Planck equation. A procedure for its time variant solution is given, and its application for making adaptive forecasts is discussed. The analysis considers the ENSO phenomenon as the external force. Chapter VI describes the use of the study results in developing a decision tree for drought monitoring. For an illustrative example the Tidewater region, Virginia is chosen. Chapter VII summarizes the key results and offers recommendations for future research.

Chapter II

LITERATURE REVIEW

2.1 INTRODUCTION

The commonly used variables to study droughts include rainfall, temperature, evaporation, evapotranspiration, soil moisture, streamflow, reservoir tank levels and storages, and ground water levels. Studies have also been performed using crop parameters like critical growth stages, crop yields, fodder production, and land use classification.

A popular approach of drought characterization is through a drought index. Typically the approach consists of devising a criterion which indicates cumulative moisture deficiency or surplus in an area. Because of their simpler representation in the form of a numerical value or an alphabet, the indices serve as an effective communication tool in government agencies to take remedial actions. Moreover, the indices motivate for more detailed research in the area. In the following a brief description of drought indices is given.

2.2 METEOROLOGICAL DROUGHT INDICES

Rainfall has been mainly used to characterize the meteorologic drought condition. A commonly used procedure consists of comparing the depth of rainfall for a given period i.e. week, month, season or year with the long term mean of rainfall (normal) for a given duration. Based upon the review given in WMO (1975), the following indices are briefly annotated. Bates (1935) considered drought condition when annual precipitation is 75% of normal or when monthly precipitation is 60% of normal. Ramdas (1950) defined drought as a week with actual rainfall equal to a half of the normal rainfall or less. Gibbs and Mather (1967) used the decile (ten percent limits) of the rainfall frequency distribution. The first decile cutoff is that rainfall amount which will not be exceeded by only ten percent of the total indicating an abnormally dry condition.

Selyaninov (1930) suggested an index, k, based on the ratio of sum of rainfalls (mm) when mean temperature is above 10^{o} C and sum of rainfalls for the same period when the mean temperature was less than 10^{o} C. A dry spell is supposed to have occurred when $k < 1$ and a drought condition occurs $k < .5$. Koppen (1931) defined dry climate by $p < 2\,T$ for regions of winter rain and $p < 2T+14$ for regions of summer rain or no rainy season where p is annual precipitation in cm and T is mean temperature in ^{o}C. Palmer (1965) developed an index called the Palmer Drought Severity Index (PDSI) based on water balance computations in the root zone. The studies on the use of the PDSI are reviewed in section 2.5. Herbst et al. (1966) developed a technique for evaluating droughts using monthly rainfall data. The authors computed effective rainfall for month i as

$$E_i = P_i + W_i\,(P_{i-1} - M_{i-1}) \qquad (1)$$

where: E_i= effective rainfall for ith month; P_i= precipitation during ith month

W_i= weighting factor for ith month; P_{i-1} = precipitation for (i-1)th month

M_{i-1} = mean precipitation for (i-1)th month

The weighting factor (W_i) is calculated as

$$W_i = 0.1\,(1+\frac{Mi}{MAR/12}) \qquad (2)$$

where: Mi= mean precipitation for ith month computed based on long term precipitation record for ith month

MAR = mean annual precipitation for the area

Further, mean monthly deficit for ith month (MMD_i) is defined as

$$MMD_i = \frac{\sum_{j=1}^{n} (Eij - Mi)}{n} \quad ; \text{if } (Eij - Mi) > 0 \text{ then } (Eij-Mi) = 0 \qquad (3)$$

where : Eij = effective rainfall for ith month during jth year; n = number of years

The mean annual deficit (MAD) is then defined as

$$MAD = \sum_{i} MMD_i \; ; i = 1,..., 12 \qquad (4)$$

The authors proposed a method to identify beginning and ending of drought to compute the drought duration (D). The severity of drought was defined as

$$\text{Severity index} = Y \,.\, D \qquad (5)$$

Y = drought intensity ; D = drought duration

where:

$$Y = \frac{\sum_{i=1}^{D} [(Ei - Mi) - (MMDi)]}{\sum_{i=1}^{D} (MMDi)} \qquad (6)$$

Strommen et al. (1980) proposed an index based on weekly precipitation values. Eight weeks' precipitation values are accumulated and the deviation from corresponding eight weeks' mean precipitation data is computed. If the deviation falls below 60 % of the normal level , a drought warning is signalled. The choice of 8 weeks seems arbitrary. An objective method called precipitation anomaly classification (PAC) to identify and track significant global precipitation anomalies on time scales of a month and longer was developed at the Climate Analysis Center (CAC) of National Oceanic and Atmospheric Administration (NOAA) (Janowiak et al. 1986). The technique is a modified version of a method originated from the Australian Bureau of Meteorology (Lee, 1980) and requires only monthly accumulated precipitation values. The categories used in this method are determined by comparing the precipitation at a station to the Gamma distribution of the station's historical record. A severe precipitation deficiency classification is given when the rainfall for the past 3 months is among the lowest 5% of the site's historical record for that period. A serious deficiency exists when the rainfall for the 3 month period ranks within the lowest 10% but above the lowest 5% of the historical observations. When either the severe or serious categories are selected, the location is considered to be drought affected until either of the following two conditions are satisfied: i) the precipitation for the past month alone is sufficient to rank in the 30th percentile or greater of the historical record for the three month period starting with that month, or ii) the precipitation for the past three months ranks in the 70th percentile or greater of the historical record for the corresponding three month period. Meyer et al. (1992 a and b) described the development of a new crop specific drought index (CSDI) which is derived from the inputs of temperature, humidity, windspeed, solar radiation, and precipitation. The CSDI has been developed for corn and efforts are being continued for other crops. The CSDI values range from zero to one as the predicted production ranges from crop failure to bumper crop.

2.3 AGRICULTURAL DROUGHT INDICES

The agricultural drought indices have been mainly based on rainfall deficiency and occurrence of dry spells, evapotranspiration, soil moisture deficit and water balance computations. Van Bavel (1953) and Vellidis et al. (1985) have observed that agricultural drought should be defined on the basis of soil water status and resultant plant behavior. In the Van Bavel's study with tobacco crop, available water in soil was assumed to be 1.7 inch and a drought day was defined when available water reaches zero, and the incidence of drought was indicated by the number of drought days during the growing season. Van Bavel and Lillard (1957) defined probability of droughts in Virginia based on a method which consisted of making a daily balance of available soil moisture and noting the number of days (drought - days) on which supply of moisture is exhausted. The analysis was done for the growing season (April through September) and it was assumed that on April 1 the amount of water in root zone was at its maximum. Daily water balance was done by adding the rainfall and subtracting the estimated evapotranspiration data. The method gives probability of number of drought days based on amount of available water in the root zone in different parts of Virginia. Vellidis et al., (1985) updated the results of drought analysis by Van Bavel and Lillard (1957) by analyzing data upto 1983.

Thornthwaite and Mather (1955) developed a water balance approach and identified the extent and period of water deficit. The deficit was defined as actual evapotranspiration (AE) minus potential evapotranspiration (PE). Rodda (1965) carried out monthly water balance computations in Southeast England for 50 years assuming soil's field capacity as 5.5 inch. A drought month was indicated whenever the soil moisture was below field capacity. Federer (1980) used a daily water balance model to study agricultural drought in New Hampshire. A drought day was defined whenever soil moisture deficit exceeded 60 mm in growing season from June 1 through August 31.

The techniques of computing PE in water balance studies have varied in the past. Jensen et al. (1990) provide a good review of the ET methods. The approaches to compute amount of water extracted from soils whenever precipitation is less than PE have also varied in various studies. Thornthwaite and Mather (1955) assumed that the ratio of actual to potential ET decreases as a linear function of the amount of available water. Rodda (1965), however, assumed two soil layers in which soil moisture in top layer was assumed to get extracted completely when $P < PE$ and then moisture in lower layer is extracted proportional to available water. Such a layering scheme is also used in Palmer (1965). Aridity index (Ia) defined as the ratio of moisture deficit (PE-AE) and water need (PE), has also been used to characterize agricultural drought. Palmer (1968) further modified the PDSI to better reflect agricultural drought. He proposed a crop moisture index (CMI) which considers agricultural drought as ET deficit. The index is developed from moisture accounting procedures used in the PDSI and is the sum of an evapotranspiration anomaly and a moisture excess. Both terms are a function of the previous and current week. The evapotranspiration anomaly measures the evapotranspiration deficit and is weighted to be comparable for location and time of year. The moisture excess includes the runoff, soil recharge, and percent of field capacity during a week. If potential moisture demand exceeds available moisture supplies, the CMI is negative and if moisture exceeds demand, the index is positive. Negative values of CMI always mean that ET has been abnormally deficient. The values of the index range from

-4.0 to 3.0 and based on the values of CMI, various classes of agricultural drought have been defined. Motha and Heddinghaus (1986) reported the development of a moisture availability index (MAI) which is computed as the difference of weekly normalized precipitation and normalized potential evapotranspiration and provides a relative measure of moisture available for a given crop. In recent years, use of remote sensing technology has been explored to monitor drought conditions. Based on the data of radiation measurements, a normalized difference vegetation index (NDVI) is used to infer growth of vegetation. The maximum NDVI for a given season can serve as an indicator of drought severity by inferring deficiencies in the photosynthetic capacity in drought years as contrasted with other years (Tucker and Goward, 1987). Further refinements of NDVI have been reported to better assess impacts of weather on vegetation (Kogan, 1987 and 1990 b).

2.4 HYDROLOGICAL DROUGHT INDICES

Whipple (1966) defined a drought year as one in which the aggregate runoff is less than the long term average runoff. Yevjevich (1967) defined the term "hydrological drought" as deficiency in water supply or deficiency in precipitation, effective precipitation, runoff, or accumulated water in various storage capacities. Linsley et al. (1975) defined hydrologic drought as a period during which streamflows are inadequate to supply established uses under a given water management system. Hydrological droughts are related more to the manner in which precipitation shortfall affects the surface or subsurface water supply (i.e. streamflow, reservoir and lake levels, groundwater) than to precipitation shortfalls itself (Dracup et al., 1980; Klemes, 1987). Beran and Rodier (1985) discussed six types of hydrologic drought based upon variations in duration, season of year, and severity. A short duration (3 weeks to 3 months) runoff deficit experienced during the period of germination and plant growth could be catastrophic for farming if the sole source of irrigation water is river runoff. The second type occurs when the minimum discharge is significantly lower or more prolonged than the normal minimum but not necessarily advanced much in its position relative to the growing season and so less harmful to the agriculture. The third category occurs when there is significant deficit in the total amount of annual runoff which affects hydropower production and irrigation. The fourth category is defined in terms of below normal annual high water level of the river. This may introduce the need for pumping for irrigation. The fifth category refers to the deficiency in river discharge over several consecutive years. The sixth type refers to drought in which a significant natural depletion of aquifers occurs.

The variables used to characterize hydrologic drought include streamflow, reservoir levels, groundwater levels, and soil moisture level. Beran and Rodier (1985) discussed the use of pluviosity (ratio of actual rainfall during a year to the mean rainfall) and hydraulicity (ratio of actual river flow to mean river flow) to describe the drought conditions. The values of these variables classified on the basis of probability serve as an index for drought. One such scheme used in Europe is to divide the ranked pluviosities, and hydraulicities into the following five classes:

very wet : exceedence frequency between 0 and 15%

wet : exceedence frequency between 15 and 35%

normal: exceedence frequency between 35 and 65%

dry : exceedence frequency between 65 and 85%

very dry: exceedence frequency between 85 and 100%

Stockton (1984), while studying the long term streamflow (1932-80) in various regions in the U.S., considered hydrologic drought as any year among the driest 10 years in the time series of flows. Dezman et al. (1982) reported development of a surface water supply index (SWSI) in Colorado which integrates historical surface water supply data with current data of reservoir storage, streamflow and precipitation at high elevation into a single number. Cordery (1981,1983) used monthly water balance model to estimate soil water deficit based on which hydrological drought was defined.

Though the PDSI has been defined as a meteorologic index, hydrological parameters like runoff and recharge have been used in its development. Fieldhouse and Palmer (1965) noted that the PDSI should be related to water supplies in streams, lakes and reservoirs and hence be of interest to hydrologists. In view of the backcomputing procedure involved in computing PDSI (see Appendix I), Karl (1986) described an adjusted PDSI known as Palmer Hydrologic Drought Index (PHDI) which avoids the back computing problem and can be used in operational mode. Groundwater level fluctuations from long term levels have also been used to define drought intensity in conjunction with other variables (State Water Control Board, 1990).

A popular definition of drought in hydrology is based on the theory of runs (Mood, 1940; Yevjevich, 1967). Here a time series X(t) is truncated at a level X_O yielding distinct events of wet periods ($X(t) > X_O$) and dry periods ($X(t) < X_O$). Any uninterrupted sequence of dry period coincides with a drought of length equal to the number of dry periods within the same event. The selection of the threshold X_O is an important determinant of run properties. Yevjevich (1967) suggested that for site specific applications to streamflow X_O might be set at the average level of water use. Sen (1976) calculated drought duration probability by assuming that the joint probability between two successive random variables representing streamflow to be bi-variate normal. Dracup et al. (1980) investigated trade offs between the choice of X_O as the mean or as the median streamflow in studies of annual streamflows. Kottegoda (1980) descibed a procedure to find expected value of deficit run length and deficit run sum.

2.5 STUDIES ON THE PDSI

2.5.1 PDSI For Drought Analysis, Forest Fires and Crop Yield

PDSI has been used by various researchers to illustrate the areal extent and severity of drought in United States and elsewhere. Palmer (1967) studied abnormally dry weather of

1961-1966 in the northeastern United States using the index. Dickerson and Dethier (1970) used the PDSI for evaluating drought frequency in northeastern United States. Felch (1978) used the index to compare droughts of 1930s, 1950s and mid 1970s. Lawson et al. (1971) studied the spatial and temporal characteristics of droughts in Nebraska using the index. Klugman (1978) applied the PDSI to study droughts in 53 climatic divisions of upper midwest U.S. Skaggs (1975), Karl and Koscielny (1982) and Diaz (1983) studied the spatial and temporal characteristics of dry and wet episodes over the contiguous United States during 1895-1981. Puckett (1981) reconstructed a 230 year record of the PDSI for northern Virginia using a relationship with variations in the widths of tree ring. Haines et al. (1976 ,1978) reported the use of PDSI as a tool to monitor environmental conditions conducive to forest fire danger. Sakamoto (1978) described use of Palmer Z index in estimating yield of wheat crop in Australia. In applying the Palmer index to New South Wales, Australia, McDonald (1989) observed that the empirical constants which are developed and used in the U.S. to compute the index are also appropriate for New South Wales.

2.5.2 Time Series Modeling of the PDSI

Time series models have been fitted to the PDSI by Havens et al. (1968), Davis and Rappaport (1974) and Katz and Skaggs (1981). The latter examined the auto-regressive moving average (ARMA) models of various orders for 344 climatic divisions and found that an AR (1) was preferred for about 90% of the divisions. Rao and Padmanabhan (1984) studied the stochastic nature of the PDSI using monthly and annual PDSI series for Iowa (1930-1962) and Kansas (1887-1962). The authors used AR models of various orders to model the PDSI series and selected AR(3) and AR(5) models for monthly PDSI series in Kansas and Iowa, respectively. For annual PDSI series AR(5) and AR(4) models were selected for Kansas and Iowa data, respectively. The monthly models gave smaller mean square error to accurately forecast the drought indices one month ahead of time.

However, the annual models were not found to have better forecasting capabilities. The classical techniques of time series modeling are intended for making forecasts. They do not describe duration, intensity and long term probabilities. Moreover, the distribution of PDSI has been observed to be bi-modal (Alley, 1984, 1985; Eder et al. 1987; Heddinghaus and Sabol, 1991). Therefore, conventional time series models, which generally assume normality of data, are limited in their ability to capture the random variations in the index.

2.5.3 Sensitivity Studies and Limitations

Most of the studies during the 70s and early 80s as listed above used the PDSI for evaluating drought conditions and no efforts were directed towards evaluating the sensitivity of the PDSI to the prescribed and derived parameters within model calibration. Karl (1983) reported sensitivity of the spatial characteristics of drought duration indicated by the PDSI to values of the available water capacity (AWC) and weighting factor (k) (see Appendix I for details) as used in the computation of the index. To see the effects of AWC, drought durations in various states were computed using spatially varying values of the AWC as recommended by Palmer (1965). Then the AWC was kept constant (equal to 254 mm) for all states and again drought durations were computed. It was found that the fixed AWC value did not significantly change the drought durations as compared to the varying AWC. As per the computations of Palmer (1965), the k values change spatially and monthly. To see the effects of k, it was set equal to 1.0 for every state for every month. As a result it was found by Karl (1983) that the PDSI values got changed significantly but the duration of droughts were not altered. Therefore, Karl (1983) concluded that by altering the constants AWC and k, the drought duration was not significantly affected across the United States. The criterion of classifying weather conditions may affect the drought duration with varying AWC and k. Alley (1984) gave a detailed account of limitations and assumptions of the PDSI alongwith its computational procedure. Heddinghaus and Sabol (1991) discussed the PDSI in terms of the problems and solutions in its use in an operational mode and described results of a survey of PDSI users for its usefulness, accuracy and critical parameter involved and future changes desired. The authors cite the back tracking problem for PDSI's use in the operational mode and reported a modification to overcome this problem. Since June 1989, the sum of the wet and dry terms after they have been weighed by their probabilities is used as the operational PDSI. The survey results indicated that a majority of users used the index as an aid in observing the hydrologic conditions.

2.5.4 PDSI and Surface and Groundwater Resources

Alley (1985) reported a comparison of the PDSI with streamflow and groundwater indices for evaluating the hydrologic drought in New Jersey. The author reported the streamflow index to fluctuate in and out of subnormal conditions even during major dry episodes. The PDSI values showed more persistence of subnormal conditions than the streamflow and the groundwater. The groundwater indicated the occurrence of dry periods later than either the PDSI or the streamflow index. Bowles et al. (1980) applied the PDSI to evaluate the drought indices they developed for three municipal and three irrigation systems in Utah. Johnson and Kohne (1993) used the modified PDSI for operational mode called the Palmer Hydrologic Drought Index (PHDI) for evaluating the susceptibility of 516 reservoirs to droughts in the U.S. The authors used the PHDI values $\leq$ -3.0 to categorize severe and extreme drought for evaluating impacts on reservoirs. The study identified climatic divisions in the country with severe and extreme drought durations ranging from 12-48 months and it was observed that the interior portions of the country predominantly the Great Plains, are more susceptible to prolonged drought than the coastal areas. It was also reported that of 385 multiple purpose reservoirs, 52 were susceptible to severe or extreme drought longer than 36 consecutive months. Further, 80% of the reservoirs and 90% of the multiple purpose capacity are located in climate divisions with drought durations greater than 12 consecutive months and less than 36 months. In terms of the regional assessment, maximum number of multi-purpose reservoirs in the Missouri river and southwestern regions were affected by long duration (up to 48 months) droughts. The authors compared the index with the corresponding precipitation and annual runoff and observed that the index reflects a corresponding water deficiency, making it appropriate for measuring hydrologic drought. The study indicated that the midcontinent has the greatest potential for drought of long duration and noted that in the same region greatest reservoir storage capacities lie (Johnson and Kohne, 1993).

2.5.5 PDSI and State / Federal Drought Plans

The PDSI / PHDI is used in various forms in drought contingency plans (DCPs) developed by various states in recent years. Wilhite and Rhodes (1993) have discussed the progress made by the State governments in developing drought mitigation strategies. According to the authors during the widespread and severe drought of 1977-78, no state had prepared a formal strategy to handle the drought situation. In 1982 only three states had developed plans : South Dacota (1981), Colorado(1981), and New York (1982). During the past decade, an additional twenty-four states have developed and implemented formal DCPs. Therefore, now twenty seven states have DCPs including Arizona, California and Virginia, the states for which analyses have been carried out in the present study. The state DCPs have considerable diversity. Some are highly focused on municipal water use or conservation (such as New York) , agriculture (Nebraska), or multidimensional (Colorado)(Personal Communication, 1995a). Typically, in DCPs it is the responsibility of a Water Availability Task Force (WATF) to evaluate the status of water availability in the drought affected regions. For example, as per the Colorado Drought Response Plan (1981) one of the purposes of the WATF is to collect water availability data and evaluate them to assess the changes in water availability conditions and make long term projections. The Task Force collects various information including the Palmer index data for making such assessments.

Some DCPs have well defined criteria to activate the WATF. For example, in the Colorado Plan the WATF gets activated once the Palmer index or a state's Water Availability index assumes value between -1 and -2. In Virginia, the triggering factor for convening the Task force is a combination of the Palmer index, moisture deficits, surface and ground water levels, and other indices. No specific targets have been established for each triggering parameters but in the past the Task Force has convened when the Palmer index has fallen below -3.00 at the start of the summer ; when precipitation remains considerably below normal for several weeks ; or when there are widespread reports of water shortage caused by wells or streams drying up (Personal Communication, 1995 b).

Many states rely on the PDSI as a measure of drought severity and some supplement it with other indices. For example, Colorado, Oregon, and Montana use the Surface Water Supply Index (SWSI) for assessing water availability conditions in high altitudes along with the Palmer index. In New York, for determining drought status the state is divided into six regions in which four hydrologic indicators are monitored. Each indicator - precipitation, reservoir-lake storage, streamflow, and groundwater levels - is assigned a percent weight that reflects its importance as the source for priority water uses in each region. These weightings of the hydrologic indicators determine a region's drought stage when correlated with the point system of the state's drought index, which sets the criteria for normal conditions and the four drought stages of an alert, a warning, an energency, and a disaster. The Palmer index is considered along with the state index (Hrezo et al., 1986). In Virginia, a Drought Monitoring Task Force monitors the drought related conditions by compiling drought related data including the Palmer index. In Pennsylvania, the Department of Environmental Resources (PA DER) monitors five indicators of droughts which are : annual precipitation deficit, groundwater levels, streamflow levels, reservoir storage levels, and the PHDI. Kibler et al. (1987) reported refinement of critical threshold levels of the drought indicators used for drought triggering by PA DER. The study involved carrying out frequency analysis of 25 years of data of these indicators to establish monthly exceedence levels. The 25th, 10th and 5th quantiles of PHDI were suggested for drought watch, warning, and emergency, respectively. It was also suggested to include seasonal fluctuations of PHDI in triggering criterion and recommended an adjusted triggering criterion for droughts. Similar analysis was reported for other indicators. The State Water control Board (SWCB,1990) in Virginia carried out a study to analyze the droughts in the state during 1957-1987 using data of precipitation, streamflow, groundwater level and the PDSI. From PDSI view point, drought conditions were assumed if the index was below -2.00 for at least three consecutive months. The number of drought years ranged between 5-10 in various six climatic regions during the period of analysis. In addition, PDSI has been used by the Federal Government as one of the principal criteria for disaster designation (i.e. eligibility to receive federal drought relief ; Wilhite et al., 1986).

The studies reviewed above indicate the widespread use of the PDSI for a variety of purposes. There have been some attempts to forecast PDSI as a drought predictor. The studies in this direction have been mainly confined to applying time-series models. Tchaou et al. (1992) have demonstrated the use of a discrete Markov chain to analyse the stochastic bahavior of the index and reported good agreement between the observed values and the predicted values for 3-4 months lead time. However, the authors did not focus attention on the residence time, mean recurrence time, and the steady state probabilities for various drought classes which provide a comprehensive characterization of drought.

2.6 SUMMARY

The traditional methods of analyzing droughts are deficient in providing a characterization of droughts in terms of parameters necessary for drought planning as well as for drought warning in an operational setting. The Palmer index has been the most widely used drought index since its inception. Its determination from the water balance computations has the necessary physical support. A significant advantage of this index is its indication of the onset, the progression, and the amelioration of droughts. Of course, the index also points out the wet spell. Most of the literature fall under two broad areas namely the time series analysis and the theory of runs with the focus on predicting future index values and drought severity and duration, respectively. However, these methods suffer because of their use of the PDSI directly in the case of time series analysis and the estimation of the probabilities based on a threshold value and the assumption of independence in the case of run analysis.

The time series models in the present study employ both the normally transformed PDSI and the Z index. Moreover, both fixed parameter and time varying parameter models are suggested. The Markov chain analysis overcomes the difficulty of choosing a single cutoff value and the assumption of independence by employing a conditional transition probability structure. With the aid of the total probability theorem the necessary marginal and joint probabilities can be computed. This aspect provides for additional details such as the first passage times which are not possible with the run analysis. There is also a complete lack of methods for incorporating global forcing functions. In the present research, the dynamical systems approach is recommended which not only accommodates an external random forcing component but also provides for an analytical determination of the required probability density function of the drought process. An aspect of this literature survey is intended to take stock of current procedures on the issuance of drought warnings. It is found that almost all the schemes adopt a wait and see procedure (Wilhite ,1993). The proposed decision tree approach examines all possible occurrences to reach a decision. It is hoped that the methodologies proposed in this research would help to alleviate the situation.

Chapter III

TIME SERIES ANALYSIS

3.1 INTRODUCTION

In this chapter time series models are fitted to the PDSI data to forecast PDSI values. However, because of its bi-modal conditional distribution (Alley, 1984, 1985; Eder et al., 1987; Heddinghaus and Sabol, 1991) time series techniques have not been successful in modeling the PDSI. Havens et al. (1968), Davis and Rappoport (1974), Katz and Skaggs (1981) and Rao and Padmanabhnan (1984) applied time series analysis ragardless of the bi-modality of the index. Alley (1985) in his discussion pointed out while PDSI followed a bi-modal distribution, the Z index had a unimodal distribution. He suggested that the time series models could be fitted to the Z index (see Appendix I for details) which could then be used to develop forecasts for the PDSI. The goal of this chapter is to explore the potential of the time series techniques as a forecast tool in predicting the PDSI and Z index. The analysis is carried out with the monthly PDSI series (1895-1990) for the Tidewater region of Virginia. Two families of time series models with i) fixed parameters and ii) time varying parameters are developed. The fixed parameter models use the same parameters regardless of the month and the time varying models have month dependent parameters.

3.2 TIME SERIES MODELING USING TRANFORMED PDSI DATA

Because the Normal distribution provides good support for statistical inference and its advantages with the linear models, the time series variates are generally assumed to be normally distributed.

3.2.1 Fixed Parameter Model

The Box-Cox transformation for normality is applied to the monthly PDSI data as below (Box and Cox, 1964):

$$PDSI_T = [\ (PDSI + 6)^{LS} - 1\ ] / LS \qquad \text{if } LS \neq 0$$
$$= \ln [\ PDSI\ ] \qquad \text{if } LS = 0 \qquad (1)$$

where: $PDSI_T$ = transformed PDSI

LS = a parameter (lambda) which yields zero skew coefficient

For example, the LS parameter for the Tidewater PDSI data is 1.115. The statistics of the original PDSI data and Box-Cox transformed data are given in Table 3.1. The Box and Jenkins (1976) approach is adopted to select the appropriate time series model. Table 3.2 gives forms of the various models fitted to the data. Based on the minimum standard error of forecast and Akaike Information Criterion (AIC) (Akaike, 1974) an AR(1) model of the following type is selected :

$$PDSI_t = \phi\ PDSI_{t-1} + a_t \qquad (2)$$

where: a_t = Gaussian white noise i.e. E $[a_t]$ = 0.0; Cov $[a_t, a_{t+1}]$ = 0.0. and ϕ = .87

The Palmer equation (Palmer, 1965; Appendix I) has a coefficient of 0.897 for $PDSI_{t-1}$ which agrees quite well with the fitted model (Eq. 2). The model can now be used to develop the forecast function. We can write

$$PDSI_{t+l} = .87\ PDSI_{t-1+l} + a_{t+l} \qquad (3)$$

Taking conditional expectation, we get

$$PDSI_t(l) = .87\ [\ PDSI_{t-1+l}\ ] + [a_t] \qquad (4)$$

where: $PDSI_t(l)$ = l steps ahead forecast standing at origin t, and square brackets indicate observed or expected value.

Therefore, the n steps ahead forecast is given by

$$PDSI_t(n) = .87^n\ PDSI_t \qquad (5)$$

Eq. 5 is used to forecast the PDSI three months ahead of time for years 1931 and 1990. The results are presented in Table 3.6 along with the observed classes and Markov chain results stated as method # 1. The fixed parameter time series results are given under method # 2 in Table 3.6. It is seen from Eq. 5 that the use of conditional expectation as the forecast function for large n is unsuitable.

3.2.2 Time Varying Parameter Model

As the physical processes which affect the PDSI, i.e., precipitation and temperature, vary on a monthly basis, it is hypothesized that the parameter ϕ should also vary with months. Therefore, a time varying parameter model is fitted to the PDSI data after transforming the data on a monthly basis using the Box Cox transformation. The values of the LS parameter for each month which yield near normality of the transformed data are given in Table 3.3. The transformed monthly data series has zero mean after subtracting the monthly mean values. The variances and co-variances of the monthly data are given in Table 3.4. The general form of the time varying parameter model is

$$PDSI_t = \phi_t \, PDSI_{t-1} + a_t \qquad (6)$$

by multiplying both the sides by $PDSI_{t-1}$ and taking the expectation we obtain:

$$\phi_t = \{ Cov [PDSI_t \; PDSI_{t-1}] \} / \{ Var [PDSI_{t-1}] \} \qquad (7)$$

Also, taking the variance of both sides in Eq. 6 we get:

$$Var [a_t] = Var [PDSI_t] + \phi_t^2 \, Var [PDSI_{t-1}] - 2 \, \phi_t \, Cov [PDSI_t \; PDSI_{t-1}] \qquad (8)$$

Eqs. 7 and 8 are used to compute the parameters ϕ_t and $Var(a_t)$, respectively for each month The results are given in Table 3.3. It is observed from Table 3.3 that the values of the parameters ϕ_t and $Var[a_t]$ indeed vary significantly from month to month. The forecast function using Eq. 6 is now developed as

$$PDSI_{t+l} = \phi_{t+l} \, PDSI_{t+l-1} + a_{t+l} \qquad (9)$$

Taking the conditional expectation, we get

$$PDSI_t(l) = \phi_{t+1}\,[PDSI_{t+l-1}] + [a_{t+l}] \qquad (10)$$

for l=1 ; $PDSI_t(1) = \phi_{t+1}\,PDSI_t$

for l=2 ; $PDSI_t(2) = \phi_{t+2}\,.\,\phi_{t+1}\,PDSI_t$

for l=3 ; $PDSI_t(3) = \phi_{t+3}\,.\,\phi_{t+2}\,.\,\phi_{t+1}\,PDSI_t$ (11)

Equation 11 is used to forecast the weather classes three months ahead of time during 1931 and 1990 and the results are given in Table 3.6 as method #3. Following Alley's (1985) comments the Z index data are utilized to fit the time series models. In the following section the Z index based models are presented.

3.3 TIME SERIES MODELING of the Z INDEX

3.3.1 Fixed Parameter Model

Palmer (1965) defined the Z index as

$$Z = d\,.k \qquad (12)$$

where : d = difference between the observed precipitation P and the climatically appropriate precipitation P^*,

k = weighting factor

or; $d = P - P^*$ (13)

where : P = Actual precipitation

P^* = Climatically appropriate precipitation

A zero value of d (and hence of Z index) indicates climatically appropriate supply of moisture to meet the various demands which as per Palmer (1965) include evapotranspiration, recharge, runoff, and losses from the soil. The P^* is computed using:

$$P^* = ET^* + R^* + RO^* - L^* \qquad (14)$$

The climatically appropriate values of evapotranspiration ET^*, recharge R^*, runoff RO^*, and loss L^* are obtained by multiplying the potential values by the respective coefficients, which are obtained as the ratio of the average values to their potential values. These values are established through a hydrologic accounting procedure in the soil root zone. As a part of the index computation a particular calibration period is used. Karl (1986) observed that the Palmer Z index is much less sensitive to changes in the calibration periods and is also more responsive to short term moisture anomalies. Sakamoto (1978) illustrated the use of the Z index as a variable for estimating the yield of wheat crop in Australia.

In this study, two types of autoregressive models, namely, AR(1) and AR(2) are selected to represent the Z index series for the Tidewater region, Virginia. As shown in Table 3.2, the A.I.C. and the standard error of one step ahead forecast are minimal for the AR(2) model and is chosen to represent the Z-index as follows

$$Z_t = \phi_1 \; Z_{t-1} \; + \phi_2 \; Z_{t-2} \; + a_t \qquad (15)$$

where : estimates of ϕ_1 and ϕ_2 are found to be .153 and .082, respectively.

Now we can take the conditional expectation in Eq. 15 to forecast the values of the Z index.

For this purpose we write

$$Z_{t+l} = .153 \; Z_{t+l-1} \; + .082 \; Z_{t+l-2} + a_{t+l} \qquad (16)$$

Taking the conditional expectation, we get

$$Z_t(l) = .153 \; [Z_{t+l-1} \;] + .082[\; Z_{t+l-2} \;] + [a_{t+l}] \qquad (17)$$

Using Eq. 17, three step ahead forecasts of Z index are made from which the PDSI values are retrieved using the Palmer equation

$$PDSI_t = .897 \; PDSI_{t-1} + Z_t \,/\, 3 \qquad (17a)$$

The results obtained for years 1931 and 1990 are presented in Table 3.6 as method # 4. Method # 6 represents the PDSI values obtained from the Z index using the Markov chain approach.

3.3.2 Time Varying Parameter Model

As in the case of the PDSI data series, a time varying parameter model is fitted to the Z index. The estimation procedure for these parameters is as follows. A general form of the AR(2) model with time varying parameters can be written as

$$Z_t = \phi_{1t}\ Z_{t-1} + \phi_{2t}\ Z_{t-2} + a_t \tag{18}$$

multiplying both the sides by Z_{t-1} and Z_{t-2}, respectively ; we get

$$Z_t\ Z_{t-1} = \phi_{1t}\ Z_{t-1}^2 + \phi_{2t}\ Z_{t-1}\ Z_{t-2} + Z_{t-1}\ a_t \tag{19}$$

and ;

$$Z_t\ Z_{t-2} = \phi_{1t}\ Z_{t-2}\ Z_{t-1} + \phi_{2t}\ Z_{t-2}^2 + Z_{t-2}\ a_t \tag{20}$$

Taking the expectations of both sides in Eq. 19, we get

$$E[Z_t\ Z_{t-1}] = \phi_{1t}\ [\ Z_{t-1}^2\ ] + \phi_{2t}\ [\ Z_{t-1}\ Z_{t-2}\ ] + E[Z_{t-1}]\ E[a_t] \tag{21}$$

We know;

$$Cov\ [\ Z_{t-1}\ Z_t\ ] = E\ [\ Z_{t-1}\ .\ Z_t] - E\ [Z_{t-1}\ ]\ E\ [Z_t\ ] \tag{22}$$

combining Eqs. 21 and 22 , we get

$$Cov\ [Z_{t-1}\ Z_t\ ] = \phi_{1t}\ Var\ [Z_{t-1}] + \phi_{2t}\ Cov\ [Z_{t-1}\ Z_{t-2}] \tag{23}$$

Similarly from Eq. 20 we get

$$Cov\ [Z_{t-2}\ Z_t\ ] = \phi_{1t}\ Cov\ [Z_{t-1}\ Z_{t-2}] + \phi_{2t}\ Var\ [Z_{t-2}] \tag{24}$$

Solving Eqs. 23 and 24 for ϕ_{1t} and ϕ_{2t} we get

$$\phi_{1t} = \{\ Cov\ [Z_{t-1}\ Z_t\ ]\ /\ Var\ [Z_{t-1}]\} - \{\ Cov\ [Z_{t-1},\ Z_{t-2}]\ /\ Var\ [Z_{t-1}]\}.$$

$$\left\{(Cov\ (Z_{t-1},Z_t)Cov(Z_{t-1},\ Z_{t-2})-Cov(Z_{t-2}\ Z_t)Var[Z_{t-1}]\ )/\ ((Cov(Z_{t-1},Z_{t-2})^2 - Var[Z_{t-2}]\ Var[Z_{t-1}])\right\} \tag{25}$$

and;

$$\phi_{2t} = \left\{(Cov\ (Z_{t-1},Z_t)Cov(Z_{t-1},\ Z_{t-2})-Cov(Z_{t-2}\ Z_t)Var[Z_{t-1}]\ )/\ ((Cov(Z_{t-1},Z_{t-2})^2 - Var[Z_{t-2}]Var[Z_{t-1}])\right\} \tag{26}$$

From Eq. 18, $Var(a_t)$ is computed as

$$Var(a_t) = Var(Z_t) + \phi_{1t}^2\ Var\ (Z_{t-1}) + \phi_{2t}^2\ Var(Z_{t-2}) - 2\ \phi_{1t}\ Cov\ (Z_t\ Z_{t-1}) - 2\ \phi_{1t}\ \phi_{2t}\ Cov\ (Z_{t-2}\ Z_{t-1}) - 2\ \phi_{2t}\ Cov\ (Z_t\ Z_{t-2}\ ) \tag{27}$$

Eqs. 25, 26 ,and 27 are used to estimate the parameters ϕ_{1t} , ϕ_{2t} and Var(a_t) for each month, respectively which are given in Table 3.5. Using the values of ϕ_{1t} and ϕ_{2t} for different months, the one, two and three steps ahead forecasts are generated for the Z index which are then in turn converted into the PDSI values. The results are given in Table 3.6 as method # 5.

Towards the completion of the analysis, the non-homogeneous Markov chain approach as described in Chapter IV for the PDSI data is also used for modeling the Z index series. Karl (1986) gives the classification of weather into 7 classes based on the values of the Z index. Using this classification 22 classes are formed for the 12 monthly transition matrices. Using the prediction technique described in Chapter IV, one, two and three steps ahead forecasts of Z index are computed using the (22x22) transition matrices. These forecasts are used in turn to compute the PDSI values. The results are given in Table 3.6 as method # 6. In the following section, the use of the models is demonstrated with a numerical example.

3.4 NUMERICAL EXAMPLE

EXAMPLE : Monthly PDSI and Z index data (1895-1990) for Tidewater region in Virginia are given in Appendix III. Use the Box and Jenkins (1976) technique to fit the time series models to the PDSI and the Z index data. Use the fitted models to forecast the PDSI for August, September, and October, 1980 given that PDSI value for July 1980 is -2.02 and the Z index values for June and July 1980 are -3.52 and -2.73, respectively. Compare the results with the empirical data.

Solution : Fixed Parameter Model- PDSI

The following model as described in Eq. 3 is fitted to *transformed* PDSI data

$$PDSI_t = .87\ PDSI_{t-1} + a_t \qquad (20)$$

The 'n' steps ahead forecast using the model is given by Eq. 5 which is

$$PDSI_t(n) = .87^n \, PDSI_t \qquad (21)$$

A stepwise procedure to forecast PDSI values using the above equations for August, September, and October, 1980 with the known value of PDSI for July, 1980 is given below

<u>Step 1</u>: Transform the PDSI data using the Box Cox transformation (Eq. 1). An LS value of 1.115 transforms the PDSI data close to normality. Therefore, the transformed value of PDSI for July, 1980 will be

$$PDSI_t = ((-2.02 + 6)^{1.115} - 1) / 1.115 = 3.29$$

<u>Step 2</u> : Subtracting the mean of the series (Table 3.1 , mean = 5.76), we get -2.47.

<u>Step 3</u> : Compute one step ahead forecast by putting n=1 in Eq. 21.

$$PDSI_{July\text{ '}80}(1) = PDSI_{Aug.\ 80} = .87\ (-2.47) = -2.15$$

<u>Step 4</u>: Add the mean to get : -2.15 + 5.76 = 3.61

<u>Step 5</u> : Inverse-transform : $PDSI_{Aug.\ 80} = ((3.61 \times 1.115) + 1)^{1/1.115} -6 = -1.75$ or class 3

<u>Step 6</u> : For two steps ahead forecast, we put n=2 in Eq. 21.

$$PDSI_{July\text{ '}80}(2) = PDSI_{Sept.,\ 80} = .87^2\ (-2.47) = -1.87$$

<u>Step 7</u> : Add the mean value to get : -1.87 + 5.76 = 3.89

<u>Step 8</u> : Inverse-transform : $PDSI_{Sep.\ 80} = ((3.89 \times 1.115) + 1)^{1/1.115} - 6 = -1.51$ or class 3

<u>Step 9</u> : For three steps ahead forecast, we put n=3 in Eq. 21.

$$PDSI_{July'80}(3) = PDSI_{Oct.,80} = .87^3\ (-2.47) = -1.63$$

<u>Step 10</u>: Add the mean value : -1.63 + 5.76 = 4.13

<u>Step 11</u>: Inverse-transform : $PDSI_{Oct.'80} = ((4.13 \times 1.115) + 1)^{1/1.115} -6 = -1.31$ or class 2

Time Varying Parameter Model- PDSI

A general form of the time varying parameter model is given below

$$PDSI_t = \phi_t \, PDSI_{t-1} + a_t \qquad (22)$$

The value of ϕ_t for each month is given in Table 3.3 .Equation 11 yields the forecasts upto three steps ahead. A stepwise procedure using the time varying parameter model is as given below

Step 1 : Transform the PDSI data for each month using Box Cox transformation (Use Eq. 1 with LS=1.01). Therefore, the transformed value is

$$PDSI_t = ((-2.02 +6)^{1.01} - 1) / 1.01 = 3.01$$

Step 2 : Compute the mean of monthly transformed data and subtract the mean to get zero mean series. The mean for July month is computed as 4.93. Therefore, the mean subtracted value of the transformed PDSI for July 1980 is 3.01 - 4.93 = -1.92

Step 3 : Use Eq. 11 to compute one, two, and three steps ahead forecasts as

$$PDSI_{July\,'80}(1) = \phi_{Aug.} \, PDSI_{July\,'80} = .585 \text{ x } (-1.92) = -1.12$$

Step 4 : Inverse-transform $((-1.12 + 4.09) .83 + 1)^{1/.83} - 6 = -1.50$ or class 3

Step 5 : Two steps ahead forecast

$$PDSI_{July'80}(2) = \phi_{Aug.} \, \phi_{Sep.} \, PDSI_{July'80} = .585 \text{ x } .848 \text{ x } (-1.92) = -.952$$

Step 6 : Inverse-transform $((-.952 + 3.75) .77 + 1)^{1/.77} - 6 = -1.54$ or class 3

Step 7 : Three steps ahead forecast

$$PDSI_{July'80}(3) = \phi_{Aug.} \, \phi_{Sep.} \, \phi_{oct.} \, PDSI_{July'80} = .585 \text{ x } .848 \text{ x } 1.375 \, (-1.92) = -1.31$$

Step 8 : Inverse-transform : $((-1.31 + 5.08) 1.01 + 1)^{1/1.01} - 6 = -1.27$ or class 2

Fixed Parameter Model- Z Index

The following time series model is fitted to the Z index.

$$Z_t = .153 \, Z_{t-1} + .082 \, Z_{t-2} + a_t \qquad (23)$$

Given that the Z index values for June and July are -3.52 and -2.73 and the PDSI for July is -2.02, a stepwise procedure to predict PDSI for August, September and October months is given below

Step 1 : Z series has a mean of .0105 and subtract this mean from the given Z values. Hence, Z_{June} = -3.52 - .0105 = -3.5305 and Z_{July} = -2.73 - .0105 = -2.741

Step 2 : One step ahead forecast using Eq. 23 is

$$Z_{July'80}(1) = .153\ Z_{July'80} + .082\ Z_{June'80} = .153\ (-2.741) + .082\ (-3.5305)$$
$$= -.7089$$

Step 3 : Add the mean to get : -.7089 + .0105 = -.6984

Step 4 : Compute PDSI for August, 80 using

$$PDSI_{Aug.'80} = .897\ (\ PDSI_{July\ '80}) + Z_{Aug.'80}/\ 3.0 = .897\ (-2.02) + -.6984/3$$
$$= -2.053 \quad \text{or class 3}$$

Step 5 : Two steps ahead forecast is given as

$$Z_{July'80}(2) = .153\ Z_{July'80}(1) + .082\ Z_{July'80}$$

or;

$$Z_{July'80}(2) = .153\ (\ -.7089\ ) + .082\ (\ -2.741) = -.3333$$

Step 6: Add the mean to get -.3333 + .0105 = -.3228

Step 7 : Compute the PDSI value for September, 80 as

$$PDSI_{Sep.'80} = .897\ (\ PDSI_{Aug.\ '80}) + Z_{Sep.'80}/\ 3.0 = .897\ (-2.053) + -.3228/3$$

$$= -1.950 \text{ or class 3}$$

Step 8 : Three steps ahead forecast is given as

$$Z_{July'80}(3) = .153\ Z_{July'80}(2) + .082\ Z_{July'80}(1)$$

or ;

$$Z_{July'80}(2) = .153\ (-.3333) + .082\ (\ -.7089\ )$$
$$= -.1098$$

Step 9 : Add the mean to get -.1098 + .0105 = -.0993

Step 10 : Compute the PDSI value for October, 80 as

$$PDSI_{Oct.'80} = .897\ (\ PDSI_{Sep.\ '80}) + Z_{Oct..'80}/\ 3.0 = .897\ (-1.95) +(-.0993)/3$$
$$= -1.78 \text{ or class 3}$$

Time Varying Parameter Model- Z Index

The data are transformed using the Box Cox transformation on monthly basis. The general form of the model is

$$Z_t = \phi_{1t} Z_{t-1} + \phi_{2t} Z_{t-2} + a_t \qquad (24)$$

The stepwise procedure to predict the PDSI for August, September ,and October 1980 using the time varying Z index model is as follows

Step 1: For June and July 1980, the transformed values of the Z index are obtained as -3.52 and -2.73. From Eq. 24 we write

$$Z_{July'80}(1) = \phi_{1Aug.} Z_{July'80} + \phi_{2Aug.} Z_{June'80}$$

$$= .243\ (\ .86\ ) + .025\ (\ -2.02) = -.2595$$

Step 2 : Inverse-transform to get

$$Z_{July'80}(1) = (\ (\ -.2595 + 2.51\ )\ .385 + 1)^{1/.385} -6 = -.909$$

Step 3 : Therefore, for one step ahead forecast we write

$$PDSI_{Aug.'80} = .897\ (\ PDSI_{July\ '80}) + Z_{Aug.'80}/\ 3.0$$

$$PDSI_{Aug.'80} = .897\ (\ -2.02) + (-.909)/\ 3.0 = -2.12 \text{ or class } 3$$

Step 4 : For two steps ahead forecasts, we write

$$Z_{July'80}(2) = \phi_{1Sep.} Z_{July'80}(1) + \phi_{2Sep.} Z_{July'80}$$

$$Z_{July'80}(2) = .0806\ (\ -.2595\ ) + .1274\ (\ -.86) = -.1305$$

Step 5 : Inverse-transform to get

$$Z_{July'80}(2) = (\ (\ -.1305 + 1.75\ )\ .015 + 1)^{1/.015} -6 = -1.14$$

Step 6 : Therefore, for 2 steps ahead forecast we write

$$PDSI_{Sep.'80} = .897\ (\ PDSI_{Aug.\ '80}) + Z_{Sep.'80}/\ 3.0$$

$$PDSI_{Sep.'80} = .897\ (\ -2.12) + (-1.14)/\ 3.0 = -2.28 = \text{class } 3$$

Step 7 : For three steps ahead forecast, we write

$$Z_{July'80}(3) = \phi_{1Oct.} Z_{July'80}(2) + \phi_{2Oct.} Z_{july'80}(1)$$

$$Z_{July'80}(2) = .256\ (\ -.1305\ ) + .0589\ (\ -.2595\ ) = -.0487$$

Step 8 : Inverse-transform to get

$$Z_{July'80}(3) = (\ (\ -.0487 + 2.10\ )\ .195 + 1)^{1/.195} -6 = -.38$$

Step 9 : Therefore, for 3 steps ahead forecast we write

$PDSI_{Oct.'80} = .897\ (PDSI_{Sep.'80}) + Z_{Oct.'80}/\ 3.0$

$PDSI_{Sep.'80} = .897\ (-2.28) + (-.38)/\ 3.0 = -2.18 = \text{class } 3$

3.5 SUMMARY

The results given in Table 3.6 are the forecasted weather classes based on the PDSI values predicted 3 months ahead of time. For example, the January 1931 weather class is predicted by the various methods using the October 1930 observed weather class. The results are given for the years 1931 and 1990 as these represent relatively dry and wet years in the data record for the selected region. An examination of results obtained using the various methods reveals that the Markov chain model of PDSI gives the best results among all the methods. The time varying parameter models for both the PDSI and the Z index improve the results only marginally. The Z index models give relatively better results as compared with the PDSI time series models. From the present analysis it is seen that the time series models have only a limited potential to predict drought severity. The time series models use the information contained in the data only in the form of its covariances and the mean in contradistinction to the Markov chain analysis in which the data is grouped into classes dependent on the severity to compute the class to class transition probabilities. The latter analysis probes the data better and, with the aid of the transition matrix, a complete description of the drought characteristics is possible. The Markov chain analysis is given in the next chapter.

Table 3.1. Statistics of PDSI Data

Type	Mean	Median	Standard Deviation	Skew coeff.	Kurtosis
PDSI data	-0.0064	-0.0300	2.0549	-0.1093	2.575
Box-Cox Transformed data	5.7600	5.6800	2.5040	-0.0007	2.489

Table 3.2. Time Series Model Forms Fitted to PDSI and Z Index Data

Item	Model	Model form	AIC	S.E. of one step ahead Forecast
PDSI Box Cox Transformed Data	AR(1)	$PDSI_t = .87\ PDSI_{t-1} + a_t$	3764.1	1.23
	Seasonal	$(1-B)(1-B^{12})\ PDSI_t = (1-.9297\ B^{12})\ a_t$	3900.6	1.33
Z index Data	AR(2)	$Z_t = .153\ Z_{t-1} + .082\ Z_{t-2} + a_t$	4677.0	1.84
	AR(1)	$Z_t = .167\ Z_{t-1} + a_t$	4682.0	1.85

B = Backshift operator ; $(1-B)PDSI_t = PDSI_t - PDSI_{t-1}$; S.E.=Standard Error
AIC = Akaike Information Criterion; a_t = random term

Table 3.3. Values of Parameters LS, ϕ_t and Var [a_t] - Time Varying Parameter AR(1) model

Month	Lambda (LS)	ϕ_t	Var[a_t]
Jan.	1.405	0.922	3.956
Feb.	1.410	0.795	4.416
March	1.265	0.700	1.941
April	1.245	0.869	2.039
May	1.010	0.559	1.078
June	0.930	0.757	0.756
July	1.010	1.045	1.043
Aug.	0.830	0.585	0.753
Sept.	0.765	0.848	0.468
Oct.	1.005	1.375	0.938
Nov.	1.075	1.068	1.038
Dec.	1.355	1.257	5.167

Table 3.4. Variance and Covariances of Monthly PDSI Data

Month	Variance	Pair of Months	Co-variance
Jan.	17.2	Jan.-Feb.	13.7
Feb.	15.3	Feb.-Mar.	10.7
Mar.	9.4	Mar.-Apr.	8.2
Apr.	9.2	Apr.-May	5.1
May	3.9	May-Jun.	3.0
Jun.	3.0	Jun.-Jul.	3.1
Jul.	4.3	Jul.-Aug.	2.5
Aug.	2.2	Aug.-Sep.	1.9
Sep.	2.1	Sep.-Oct.	2.9
Oct.	4.9	Oct.-Nov.	5.2
Nov.	6.6	Nov.-Dec.	8.3
Dec.	15.6	Dec.-Jan.	14.4

Table 3.5. Values of Parameters Used in Time Varying AR(2) Model - Z index

Month	ϕ_{1t}	ϕ_{2t}	Var[a_t]
January	.039	.017	.057
February	.075	.157	1.57
March	.088	.388	.465
April	.301	-.025	1.11
May	.121	-.093	1.49
June	.098	.053	.847
July	.114	.053	.338
August	.243	.025	.502
September	.081	.127	.121
October	.256	.059	.176
November	.606	.515	.593
December	.327	.602	1.32

Table 3.6 Observed and Three Months Ahead Predicted Weather Classes, Tidewater Region, VA

Year	Observed	#1	#2	#3	#4	#5	#6
1931							
Jan.	7	7	6	6	6	7	6
Feb.	7	7	6	5	6	7	6
Mar.	7	7	6	5	6	7	6
Apr.	7	7	6	5	7	7	7
May	6	6	6	6	7	7	7
Jun.	5	6	6	5	6	7	7
Jul.	5	6	5	5	6	7	6
Aug.	5	6	5	5	5	5	5
Sep.	5	5	5	5	5	5	4
Oct.	5	5	5	5	5	5	4
Nov.	5	5	4	4	4	4	4
Dec.	6	5	4	4	4	5	4
1990							
Jan.	2	2	3	3	2	3	3
Feb.	3	3	3	3	2	1	2
Mar.	3	3	3	3	3	2	1
Apr.	3	3	3	3	3	2	1
May	2	3	3	3	3	3	3
Jun.	4	3	3	4	3	3	3
Jul.	4	3	4	4	4	3	2
Aug.	4	4	3	3	3	3	3
Sep.	4	4	4	4	4	4	4
Oct.	4	4	4	4	4	4	4
Nov.	4	4	4	4	4	4	4
Dec.	4	4	4	4	4	4	4

Note: Method #1: PDSI Markov chain; #2: AR(1) PDSI model with fixed parameter; #3: AR(1) PDSI time varying; #4: AR(2) Z index model with fixed parameters #5: AR(2) Z index model with time varying parameters ;; #6: Z index Markov chain.

CHAPTER IV

MONTHLY NON-HOMOGENEOUS MARKOV CHAIN ANALYSIS

4.1 INTRODUCTION

In chapter III time series forecast models have been developed for the PDSI and the Z indices. The time series models, however, do not provide information on drought duration, probabilities of transition between drought severity classes, steady state probabilities, and times of return to the various classes. In this chapter a Markov chain formulation capable of achieving these results is presented. Both Palmer (1965) and Karl (1986) present PDSI values into different classes ideally suited to a Markov chain formulation. Karl (1986) suggested another version called the Palmer Hydrologic Drought Index (PHDI) which avoids the backtracking to assign the correct state of drought dependent on wet or dry spell as done in the PDSI. In this chapter the various drought characteristics in terms of the probability of occurrence, the time of residence (duration), and the time of recurrence of various drought classes are presented. Also, formulations to predict the various drought classes are given.

4.2 MARKOV CHAIN MODEL

Consider Karl's (1986) seven class delineation of PDSI / PHDI (Table 4.1). Let the random variable X_n represent the drought (wet) class for month n. For example, $X_1 = X_{Jan} = 5$ represents the occurrence of class 5 in January. The underlying stochastic process is completely described by a Markov Chain if the transition probabilities denoted by $p_{i,j}^{n,n+1}$ for moving from class i in month n to class j in month n+1 and the initial class vector, $f^{(0)}$, describing the probabilities of the seven classes for the beginning month, are prescribed. A brief note on stochastic processes is given in Appendix II. Based on monthly Palmer index data for about 98 years, twelve monthly (non-homogeneous) transition probability matrices describing the class transfers from months January to February; February to March; ..., and December to January are formulated for the selected climatic divisions in Arizona, California, and Virginia. These matrices are assumed to be cyclic in the sense that there is no yearly variation; the transition probabilities depend only on the month and not on the year. The transition probability

$$p_{i,j}^{(n,n+1)} = P\,[X_{n+1} = j \mid X_n = i], \text{ for } i,j = 1,2,\ldots,7 \text{ and } n=1,2,\ldots, 12 \qquad (1)$$

is computed as :

$$p_{i,j}^{(n,n+1)} = N_{i,j}^{(n,n+1)} / N_i^{(n)} \qquad (2)$$

in which: $N_{i,j}^{(n,n+1)}$ = number of transitions from class i in month n to class j in month n+1; $N_i^{(n)}$ = number of occurrences of class i in month n. If $N_i^{(n)}$ is zero for some, i, we define $p_{i,j}^{(n,n+1)} = 1/7$ for all j = 1,2,...,7. Appendices III and IV give the Palmer index data and the transition probability matrices for Arizona, California and Virginia, respectively.

4.3 MONTHLY STEADY STATE PROBABILITIES

The probability of occurrence of a particular drought (wet) class will indicate proneness to drought/wet conditions. Let $f^{(k)}$ be the class probability row vector which lists $P[X_k = i]$ for i = 1, 2, . . ., 7 for the seven classes after k transitions given by

$$f^{(k)} = [\, f^{(0)}]\, [P_1]\, [P_2] \ldots [P_k] \qquad (3)$$

in which: $f^{(0)}$ is initial state probability row vector and P_1 = (7 x 7) monthly transition matrix associated with the starting month, say January to February, i.e., $P_1 = P^{(1,2)} = P^{(Jan., Feb.)}$. Of course, the starting month can be any one of the 12 months. Also, due to the cyclic nature of these matrices, the transition matrix for months 14 to 15 denoted by $P^{(14, 15)}$ is the same as $P^{(2,3)} = P^{(Feb., Mar.)}$, the February-March transition matrix.

For the long term, that is as $k \to \infty$ we would like to know whether $f^{(\infty)}$ has steady class probabilities independent of $f^{(0)}$. This will be true if the product of the transition matrices $[P_m]$ through $[P_k]$ denoted by $\phi^{(m,k)}$ called the composite matrix

$$\phi^{(m,k)} = [P_m]\,[P_{m+1}] \ldots [P_k] \tag{4}$$

is a constant stochastic matrix with identical rows (Isaacson and Madsen, 1976) for large k. For such a constant stochastic matrix it follows from Eq. (3) that $f_m^{(k)}$ will be independent of $f^{(0)}$; furthermore, each class has a steady state probability value corresponding to that class' (column) constant probability of $\phi^{(m,k)}$. However, because the beginning month, m, influences the value of $\phi^{(m,k)}$ the steady class probabilities of $f_m^{(k)}$ will depend on m. To interpret $f_m^{(k)}$ as $k \rightarrow \infty$, consider Eq. (4) as follows. The constant (identical rows) stochastic matrix for January is defined as the product of the sets of the consecutive 12 monthly matrices with the beginning matrix being that of January which is

$$\phi^{(1,\infty)} = [\text{ Jan }] = \{[P_1]\,[P_2] \ldots [P_{11}]\,[P_{12}]\}\;\{[P_1]\,[P_2] \ldots [P_{11}]\,[P_{12}]\} \ldots \tag{5}$$

Because [Jan] is a constant stochastic matrix it follows

$$\text{row [Jan]} = f_1^{(\infty)} \tag{6}$$

Now consider

$$\phi^{(2,\infty)} = [\text{ Feb }] = \{[P_2]\,[P_3] \ldots [P_{11}]\,[P_{12}]\,[P_1]\}\;\{[P_2]\,[P_3] \ldots [P_{11}]\,[P_{12}]\,[P_1]\} \ldots \tag{7}$$

and we obtain

$$[\text{Feb}] = [P_2]\,[P_3] \ldots [P_{11}]\,[P_{12}]\,[\text{Jan}]\,[P_1] \tag{8}$$

from which upon manipulation it follows

$$[\text{Feb}] = [\text{Jan}]\,[P_1] \tag{9}$$

and therefore

$$\text{row [Feb]} = f_2^{(\infty)} = \text{row [Jan]}\,[P_1]$$

similarly we can show

$$\text{row [Mar]} = f_3^{(\infty)} = \text{row [Feb]}\,[P_2]$$

$$\text{row [Apr]} = f_4^{(\infty)} = \text{row [Mar]}\,[P_3]$$

....

$$\text{row [Dec]} = f_{12}^{(\infty)} = \text{row [Nov]}\,[P_{11}]$$

$$\text{row [Jan]} = f_1^{(\infty)} = \text{row [Dec]}\,[P_{12}] \tag{10}$$

In general we obtain, *row* [Month n+1] = *row* [Month n] [P_n]. Eq. (10) provides a means to evaluate the monthly drought/wet steady class probabilities. It is a system of linear equations in terms of the monthly steady class probabilities.

Both the PDSI and PHDI data have been used to compute long term probabilities. The steady class probabilities for various weather classes for one climatic division each in Arizona, California and Virginia are given in Tables 4.2(a and b), 4.3(a and b) , and 4.4(a and b), respectively. In these tables , two values are reported for each month corresponding to each class. The top value represents the Markov analysis result and the bottom one is the result of empirically observed data. For instance, in Arizona the steady state probability of class 1 in January month obtained using Markov analysis of PDSI data is .0802 as against an empirical value of .0816. The results show that Markov analysis yields results which are in close agreement with the empirical observations. In the case of the PDSI data the sum of 12 months' average probabilities of drought classes (i.e. classes 5,6 and 7) is highest (37.5%) in Arizona followed by California (28.7%) and Virginia (24.6%).

Further, the 12 months' average probability using PDSI data for normal weather class (class 4) is 35.7% for Arizona as against 48.8% in California, and 48.9% in Virginia. These tables also contain Karl's (1986) estimates of the percent frequency distribution of PDSI computed across all months and climate divisions which compare well with our computations. Wallis (1993) and Guttman et al. (1992) give the probability of PDSI being less than -4.0 (class 7) in January for the northwest climatic division of Arizona between .01 and .05 which compares well with the value of 0.03 obtained in the present analysis. For the same region for the month of July, Guttman et al., (1992) give probability of PDSI being less than -4.0 to be between .01 and .05 and the present analysis gives this value as .07. It is of interest to note that for the Tidewater Region in Virginia for the months of August through October the probability of occurrence of a drought class (classes 5, 6, or 7) is close to 0.3 which agrees with the results of Van Bavel and Lillard (1957). Wallis (1993) reports the probability of PDSI being in class 7 for the month of July for Virginia as 0.00 - 0.05 and the present analysis yields 0.0208. For the same region, Guttman et al. (1992) reports the observed probability for class 7 in January as 0.01-.05 and the present analysis yields .0417. In Arizona, the months of April-July are found to have highest drought probabilities (40%). In California, the months of February - August (except May) are observed to have highest probabilities (≥ 30 %) of droughts. In Virginia for Tidewater region, the months of July through November are found most sensitive to droughts with about 27% or more long term probability. A comparison of results obtained using PDSI and PHDI data reveals that drought probabilities computed based on PHDI are higher in all 3 states as compared to probabilities computed based on PDSI. This is because PHDI responds slowly to changing weather conditions as compared to the PDSI.

4.4 EXPECTED UNINTERRUPTED RESIDENCE TIME

An important characteristic of drought is its duration, which is the expected uninterrupted residence time for a particular drought class. For instance, a process will stay in class i continuously for 'm' time periods when the following event occurs

$$\{ X_1 = i = X_2 = \ldots = X_{m-1} \mid X_0 = i\} \quad (11)$$

The probabilities of events specifying uninterrupted stay for different time periods in a particular class, i, can be computed as follows. For example, one month duration of stay denoted by m=1 starting with the month of January is given by

$$P [X_{Feb} \neq i \mid X_{Jan} = i] = P [m=1 \mid X_{Jan} = i] = 1 - p_{i,i}^{1,2} \quad (12)$$

where: $p_{i,i}^{1,2}$ = probability of moving from class i in January to the same class i, in February. Equation 12 says that the drought class 'i' is occupied for the thirty one days in January, i.e. 1 month and then on the first of February the drought class is no longer 'i' but some other j ≠ i. This interpretation also says that transition occurs on the last day of the given month. For m=2 starting from the month of January the probability can be computed by

$$P [m=2 \mid X_{Jan} = i] = P [X_{Mar} \neq i, X_{feb} = i \mid X_{jan} = i] \quad (13)$$

$$= P [X_{Feb} = i \mid X_{Jan} = i] P [X_{Mar} \neq i \mid X_{Feb} = i] \quad (14)$$

$$= p_{i,i}^{1,2} (1 - p_{i,i}^{2,3}) \quad (15)$$

Likewise, the probabilities of events defining consecutive stay for higher number of time steps can be computed. For example, for staying 12 time steps consecutively in class i, starting in January, the probability will be

$$P [m = 12 \mid X_{jan} = i] = p_{i,i}^{1,2} \, p_{i,i}^{2,3} \ldots p_{i,i}^{10,11} \, p_{i,i}^{11,12} (1 - p_{i,i}^{12,1}) \quad (16)$$

It is readily observed that the computation of probabilities for various events defining an uninterrupted stay in class i involves the multiplication of i^{th} row and i^{th} column entries (diagonal elements) of the consecutive transition matrices. If any one of these entries is zero, the computation stops at that point because all the remaining probabilities for higher durations of stay go to zero. Once the probabilities for uninterrupted stay for various time periods are computed, the expected uninterrupted residence time for class j, $E[R_{uj}]$, is given by

$$E [R_{uj} \mid \text{starting month}] = \sum_k k \, P [m = k \mid \text{starting month}] \quad (17)$$

where : R_{uj} = random variable describing uninterrupted stay in class j

Similarly, the variance of residence time i.e. $Var(R_{uj})$ can also be computed as below :

$$Var(R_{uj}) = E (R^2_{uj}) \mid \text{starting month}) - [E (R_{uj} \mid \text{starting month})]^2 \quad (18)$$

It is seen that the uninterrupted residence time and its variance is sensitive to the starting weather class. The expected uninterrupted residence times for various starting months and classes are given in Tables 4.5(a and b), 4.6(a and b), and 4.7(a and b) for the selected divisions in Arizona, California, and Virginia, respectively. In each of these tables three values are reported corresponding to each class and month. The top value represents the expected value of the uninterrupted residence time, the middle one gives the residence time as computed using observed data and the bottom figure represents the variance of the residence time.

For example, for Arizona it is seen that weather will stay uninterrupted in class 2 in January on an average for 1.7 months computed using PDSI data, which compares well with 1.3 months as given by observed data. The computed residence time is found to have variance of 1.6. It is seen that class 7 (extreme drought condition) has a relatively long residence time for Arizona as compared to California and Virginia. The computations of variance of residence times indicate large variability in uninterrupted residence time in Arizona and California as compared to Virginia. For Arizona and California, the average uninterrupted residence times for the drier classes (5, 6, and 7) are higher than the wet classes (1, 2, and 3) indicating that a drought spell once occurred would stay for a relatively longer period than a wet spell. For example, it is observed from Table 4.7(a) that by the end of April if one of the drought classes 5, 6, or 7 has occurred, then including the duration of 1 month for April, the drought would continue for another 1.5 months making the total duration to be about 2.5 months. A comparison of results obtained using PDSI and PHDI data indicate that both data sets give by and large similar residence time for each class for all the 3 states.

4.5 EXPECTED FIRST PASSAGE AND RECURRENCE TIMES

In characterizing droughts, it is also important to find how frequently the drought classes are visited in a region. For this purpose, the expected first passage time (m_{ij}) can be defined as the average time period taken for the process to go to a class, j, for the first time starting from some class, i. The starting month, n, is crucial in computing the expected first passage time in the case of the non-homogenous chain. Therefore, we let $m_{i,j}^{n}$ as the first passage time for a process to reach class, j, starting from class, i, in month, n. Mathematically,

$$m_{i,j}^{(n)} = (1)\, p_{i,j}^{(n,n+1)} + \sum_{k \neq j} p_{i,k}^{(n,n+1)} \left(m_{k,j}^{(n+1)} + 1\right) \qquad (19)$$

in which the first term says that class j, can be reached in one step in month (n+1) or the process can go to some class $k \neq j$ in one step as is indicated by 1 in summation term and it takes $m_{k,j}^{(n+1)}$ steps to reach j. This equation is simplified by combining the first term probability $p_{i,j}^{(n,n+1)}$ with the remaining sum of probabilities in the summation term to yield

$$m_{i,j}^{(n)} = 1 + \sum_{k \neq j} p_{i,k}^{(n,n+1)} m_{k,j}^{(n+1)} \tag{20}$$

For example, for n=1 we obtain

$$m_{i,j}^{(1)} = m_{ij}^{Jan} = 1 + \sum_{k \neq j} p_{i,k}^{(1,2)} m_{k,j}^{Feb} \tag{21}$$

in which : $p_{i,k}^{(1,2)}$ = January-February transition probability for reaching class, k, from class, i. The solution of system of linear equations in Eq. 20 yields the expected first passage times. The average time to return to the same class called the mean recurrence time (m_{ii}) can also be computed from Eq. 19 as

$$m_{i,i}^{(n)} = (1)\, p_{i,i}^{(n,n+1)} + \sum_{k \neq i} p_{i,k}^{(n,n+1)} (m_{k,i}^{(n+1)} + 1) \tag{22}$$

which simplifies to

$$m_{i,i}^{(n)} = 1 + \sum_{k \neq i} p_{i,k}^{(n,n+1)} m_{k,i}^{(n+1)} \tag{23}$$

The expected first passage times to class 4 for the selected climatic divisions of Arizona, California, and Virginia are given in Tables 4.8(a and b), 4.9(a and b), and 4.10(a and b), respectively. Each table gives two values corresponding to each class and month. The top value is computed using Markov analysis and bottom one is empirical. It can be seen that there is good agreement between computed and observed values. It is observed that the first passage times to class 4 from both drier (classes 5, 6 and 7) and wetter (1, 2, and 3) classes are the highest in the case of Arizona indicating higher possibility of non-normal weather in Arizona. If weather happens to be in class 7 ,the driest state, in March it is observed using PDSI data that it would need , on an average, 23.1 months to go to class 4 for the first time in Arizona, 10.8 months in California and 9.7 months in Virginia. In the case of transition from class 5 to class 4 in the same month the average time taken is 11.8, 5.6, and 5.7 months in Arizona, California, and Virginia, respectively.

The transition probability matrices show that, as far as class 5 is concerned, there is a strong tendency to stay in class 5 because of the high value of $p_{5,5}$. The relatively small values of the off diagonal elements determine the drift towards class 4. An intuitive substantiation of these first passage times may be gained as follows. From Table 4.12(a) it takes 21 months to reach class 4 from class 7 in Arizona. From Table 4.13(a) the average time spent in class 7 is 6.7 months. If we subtract this duration of stay in class 7, we obtain 14.3 months to reach class 4 which is close to the 14.9 months to reach class 4 from class 6. By the same argument using an average duration of stay in class 6 of 3.7 months, we obtain 10.6 months to reach class 4 from class 5 which agrees well with the value of 10.2 given in Table 4.12(a). A comparison of results obtained PDSI and PHDI data reveals that the expected first passage times computed using PHDI are slightly higher than PDSI based values. This agrees with the observation that once in non-normal weather conditions, PHDI returns to normal state slower as compared to PDSI.

4.6 MEAN MONTHLY HOMOGENEOUS MARKOV CHAIN ANALYSIS

Another approach to analyze weather transition patterns in a region is to use mean monthly homogeneous Markov chain. This can be done by computing transition probabilities giving emphasis on transitions among classes, irrespective of the months in which these take place during the year. Suppose N_T is the total number of data points used and N_{ij} is the number of times the process transits from class i to class j, regardless of the month, and N_i the total number of times the process is in class i, then

$$p_{i,j} = (N_{i,j} / N_T) / (N_i / N_T) = N_{i,j} / N_i \quad (24)$$

denotes the mean monthly probability of transition from class i to class j. In this manner the mean monthly transition matrix can be defined as $P = [\, p_{i,j} \,]$ for i,j = 1,2, ..., 7. These matrices are computed for the selected climatic division each in Arizona , California ,and Virginia and are given in Appendix IV. The mean monthly matrices represent a closed communicating class of all weather classes (1, 2, ..., 7). This is logical since in reality any class of weather is possible in a mean monthly transition. Further, the absence of any transient or absorbing classes indicates that neither there exists a weather class from which the system disappears forever nor there exists a permanent weather class in which the system is trapped.

4.6.1 Steady Class Probabilities

The mean monthly matrix represents an irreducible aperiodic Markov chain which has limiting probabilities which are independent of starting class. These steady class probabilities, denoted by the vector λ, are computed as the non-negative solution of (Ross, 1989).

$$\lambda_j = \sum_{i=1}^{7} \lambda_i \, p_{i,j} \quad \text{and} \quad \sum_{j=1}^{7} \lambda_j = 1.0 \qquad (25)$$

The steady class probabilities computed using mean monthly matrices are given in Tables 4.2(a and b), 4.3(a and b), and 4.4(a and b) for Arizona, California, and Virginia, respectively. These probabilities are quite close to the average values computed using the 12 different monthly matices and the empirical probabilities. Overall drought probabilities are found highest in Arizona and lowest in Virginia. The long term probabilities computed using PHDI data are found higher than the ones computed using PDSI data.

4.6.2 Mean Recurrence Time

The mean recurrence time, m_{jj}, for a class j is defined as the expected number of transitions until a Markov chain, starting in class j, returns to that class. Since, on the average, the chain will spend 1 unit of time for every m_{jj} units of time in class j, it follows that

$$m_{jj} = 1/\lambda_j \qquad (26)$$

Table 4.11(a and b) gives the mean first passage and mean recurrence times of class 4 computed using average of 12 monthly matrices computations, mean monthly matrices, and observed data. These results agree well. It can be observed that the mean recurrence time of class 4 (normal weather condition) is highest in Arizona. The results computed using PDSI and PHDI did not show significant difference.

4.6.3 Mean Passage Time

The mean monthly transition matrix can be analyzed to the find the mean passage time which gives an estimate of the number of transitions the process takes, on the average, to go from one particular class to another for the first time. The mean passage time is computed as

$$m_{ij} = 1 + \sum_{k \neq j} p_{ik}\, m_{kj} \qquad (27)$$

where: m_{ij} = mean passage time to go from class i to j; p_{ik} = one step transition probability to go from class i to k; m_{kj} = mean passage time to go from class k to j. Table 4.11(a and b) gives the comparison of mean first passage times to class 4 as computed using mean monthly matrix, 12 different monthly matrices, and empirical observations. The results computed by both techniques are quite close to empirical values. Tables 4.12(a and b) give comparison of average uninterrupted residence times computed using average of 12 monthly matrices and empirical observations. The results are in close agreement. It is obvious from the table that weather tends to stay in drought states for longer duration in Arizona followed by California and Virginia. The drought characteristics obtained using PDSI and PHDI data did not differ significantly.

4.7 MONTHLY DROUGHT CLASS PREDICTION

An important aspect of drought characterization is to predict its occurrence. The analysis using non-homogenous Markov chain can be used to forecast future classes of weather. Suppose $E[X_{feb} \mid X_{jan} = i]$ denote the expected weather class for February, given that during the month of January the observed weather class is, i. Then the conditional expected class for the month of February can be computed

$$E[X_{Feb} \mid X_{Jan} = i] = \sum_{j=1}^{7} j\, P(X_{Feb} = j \mid X_{Jan} = i) = \sum_{j=1}^{7} j\, p_{i,j}^{(1,2)} \qquad (28)$$

For the transition from February to March X_{feb} = NINT ($E[X_{feb} \mid X_{jan}]$,j), where : NINT = the nearest integer. Therefore,

$$PEXP[X_{Mar} \mid X_{Feb}] = \sum_{j=1}^{7} j\, p^{(2,3)}(NINT(E[X_{Feb} \mid X_{Jan}], j)) \qquad (29)$$

Also, the mode which has the greatest probability of occurrence may be used as the predicted value. Therefore,

$$PMODE[X_{mar} \mid X_{feb}] = k^*;\ k^* \text{ relates to } Max(p_{j,k}^{(2,3)}) \text{ for all k for given j} \qquad (30)$$

These approaches are satisfactory for making predictions for 4 months lead time, which is considered typical crop growing period. Some results of 4 months' ahead prediction for years 1931 and 1990 are given in Table 4.13(a and b) for Arizona, California, and Virginia, respectively. It is noted that for predicting the weather classes in Jan., 1931, the observed class in Sept., 1930 was used. The predicted values in general agreed well with the observed values.

In the present chapter the currently observed index values are used in finding the most probable drought characteristics in the future, i.e., given that a particular drought class has occurred, we can find information like : What is the most likely occurrence drought class?; What is the probability that a severer or a milder drought will occur over a chosen period ?; How long such a future drought can persist ?; What are the long term probabilities (steady state probabilities) for various drought classes defining the drought proneness of a region ?; How long will it take to get back to the normal class from a severer drought class ? These aspects are illustrated through a number of example problems in Appendix VI.

4.8 SUMMARY

The non-homogeneous Markov chain analysis provides useful drought characteristics such as the most likely state of drought severity at a location, drought duration, and period of return to various classes. The method is also successfully extended to predict drought classes, 4 months ahead of time, contingent on present month's observed state. A comparison of the results in Tables 4.2 through 4.4 indicates the highest long term drought probabilities in Arizona, followed by California and Virginia. The drought probabilities computed using PHDI are found higher than PDSI data which agrees with the observation that PHDI returns from non-normal weather condition to normal condition slower as compared to the PDSI. The characterization methodology for droughts proposed in this chapter should be useful in situations where short term predictions are needed, such as irrigation applications. While the mean monthly matrix provides simpler analysis, the monthly non-homogeneous matrices offer more flexibility by conditioning on the observed weather state in making the predictions.

Table 4.1. PDSI / PHDI Values and Corresponding Stochastic Classes

PDSI / PHDI value	Weather Spell	Stochastic Class
4.00 or more	extremely wet	1
3.00 to 3.99	severely wet	2
1.50 to 2.99	mild to mod. wetness	3
-1.49 to 1.49	near normal	4
-1.50 to -2.99	mild to mod.drought	5
-3.00 to -3.99	severe drought	6
-4.00 or less	extreme drought	7

Table 4.2(a). Monthly Steady Class Probabilities (Arizona), Computed and Empirical, PDSI Data

Classes→ Month ↓	1	2	3	4	5	6	7
Jan.	.0802	.0306	.1121	.4367	.2167	.0926	.0308
	.0816	.0306	.1224	.4286	.2143	.0918	.0306
Feb.	.0602	.0396	.1565	.3823	.2481	.0823	.0309
	.0612	.0408	.1633	.3776	.2449	.0816	.0306
March	.0600	.0689	.1588	.3404	.2560	.0824	.0309
	.0612	.0714	.1633	.3367	.2551	.0816	.0306
April	.0699	.0691	.2107	.2476	.2686	.0825	.0514
	.0714	.0714	.2142	.2449	.2653	.0816	.0510
May	.1094	.0798	.1505	.2164	.2582	.1341	.0514
	.1122	.0816	.1531	.2143	.2551	.1326	.0510
June	.1492	.0499	.1307	.2262	.2168	.1754	.0514
	.1530	.0510	.1327	.2245	.2143	.1735	.0502
July	.1095	.0899	.0608	.3475	.1342	.1858	.0721
	.1122	.0918	.0612	.3469	.1327	.1837	.0714
Aug.	.1096	.0199	.1320	.3980	.1855	.0929	.0618
	.1122	.0204	.1327	.3979	.1836	.0983	.0612
Sept.	.0997	.0299	.1017	.4080	.2057	.0928	.0618
	.1020	.0306	.1020	.4082	.2040	.0984	.0612
Oct.	.0798	.0201	.1420	.3977	.2261	.0722	.0618
	.0816	.0204	.1429	.3979	.2244	.0743	.0612
Nov.	.0599	.0400	.0914	.4793	.1952	.0721	.0618
	.0612	.0408	.0918	.4795	.1939	.0714	.0612
Dec.	.0699	.0503	.1221	.4079	.2361	.0618	.0515
	.0714	.0510	.1229	.4082	.2347	.0612	.0510
12 Months' Average	.0881	.0490	.1308	.3573	.2208	.1022	.0515
Mean Monthly Matrix	.0890	.0490	.1315	.3576	.2203	.1019	.0513
Karl (1986)	.05	.06	.17	.45	.17	.06	.04
Empirical 12 Months' Average	.0901	.0502	.1335	.3588	.2185	.1012	.0510

Table 4.2(b). Monthly Steady Class Probabilities (Arizona), Computed and Empirical, PHDI Data

Classes→ Month↓	1	2	3	4	5	6	7
Jan.	.0804	.0407	.1222	.3961	.2373	.0924	.0307
	.0816	.0408	.1326	.3878	.2347	.0918	.0306
Feb.	.0603	.0397	.1755	.3530	.2586	.0821	.0308
	.0612	.0408	.1837	.3469	.2551	.0816	.0306
March	.0602	.0689	.1788	.3101	.2689	.0823	.0308
	.0612	.0714	.1837	.3061	.2653	.0816	.0306
April	.0700	.0691	.2209	.2374	.2689	.0824	.0513
	.0714	.0714	.2245	.2347	.2653	.0816	.0510
May	.1095	.0798	.1606	.2061	.2584	.1342	.0513
	.1122	.0816	.1632	.2041	.2551	.1326	.0510
June	.1494	.0499	.1408	.1953	.2377	.1755	.0513
	.1531	.0510	.1429	.1939	.2347	.1735	.0510
July	.1195	.0900	.1311	.2466	.1549	.1859	.0720
	.1224	.0918	.1327	.2449	.1531	.1837	.0714
Aug.	.1097	.0598	.1414	.2704	.2477	.1032	.0618
	.1122	.0612	.1429	.2755	.2449	.1020	.0622
Sept.	.0997	.0399	.1923	.2249	.2885	.0929	.0618
	.1020	.0408	.1939	.2245	.2857	.0984	.0612
Oct.	.0798	.0301	.1920	.2964	.2677	.0722	.0618
	.0816	.0306	.1939	.2959	.2653	.0714	.0612
Nov.	.0599	.0501	.1519	.3777	.2265	.0722	.0618
	.0612	.0510	.1531	.3776	.2245	.0714	.0612
Dec.	.0699	.0503	.1421	.3774	.2469	.0618	.0515
	.0714	.0510	.1428	.3775	.2449	.0612	.0510
12 Months' Average	.0890	.0557	.1625	.2914	.2468	.1031	.0514
Mean Monthly Matrix	.0905	.0565	.1634	.2901	.2449	.1021	.0508
Karl (1986)	.0500	.0600	.1700	.4500	.1700	.0600	.0400
Empirical 12 Months' Average	.0910	.0570	.1658	.2857	.2441	.1020	.0510

Table 4.3(a) . Monthly Steady Class Probabilities (California), Computed and Empirical, PDSI Data

Classes→ Month ↓	1	2	3	4	5	6	7
Jan.	.0408	.0412	.1028	.5280	.2155	.0408	.0306
	.0408	.0408	.1020	.5306	.2143	.0408	.0306
Feb.	.0409	.0103	.2245	.4072	.2043	.0819	.0306
	.0408	.0102	.2245	.4081	.2041	.0816	.0306
March	.0205	.0715	.1326	.4383	.2551	.0716	.0120
	.0204	.0714	.1327	.4388	.2551	.0714	.0120
April	.0102	.0715	.1530	.4589	.1938	.0818	.0307
	.0102	.0714	.1531	.4592	.1939	.0816	.0306
May	.0306	.0408	.1530	.5099	.1632	.0613	.0409
	.0306	.0408	.1531	.5012	.1633	.0612	.0408
June	.0306	.0408	.1938	.4385	.1734	.0817	.0409
	.0306	.0408	.1939	.4388	.1735	.0816	.0408
July	.0306	.0510	.1938	.3977	.2041	.0715	.0511
	.0306	.0510	.1939	.3980	.2041	.0714	..0510
Aug.	.0306	.0204	.1734	.4590	.2143	.0613	.0409
	.0306	.0204	.1735	.4592	.2143	.0612	.0408
Sept.	.0408	.0306	.1326	.5100	.2041	.0510	.0307
	.0408	.0306	.1327	.5102	.2041	.0510	.0306
Oct.	.0204	.0408	.1224	.6121	.1326	.0510	.0204
	.0204	.0408	.1225	.6122	.1327	.0510	.0204
Nov.	.0204	.0102	.1632	.5815	.1837	.0204	.0204
	.0204	.0102	.1633	.5816	.1837	.0204	.0204
Dec.	.0306	0.0	.1836	.5101	.2245	.0408	.0102
	.0306	0.0	.1837	.5102	.2245	.0408	.0102
12 Months' Average	.0289	.0358	.1607	.4876	.1974	.0596	.0298
Mean Monthly Matrix	.0290	.0358	.1609	.4881	.1977	.0596	.0298
Karl (1986)	.0500	.0600	.1700	.4500	.1700	.0600	.0400
Empirical 12 Months' Average	.0289	.0357	.1607	.4881	.1973	.0595	.0298

Table 4.3(b). Monthly Steady Class Probabilities (California), Computed and Empirical, PHDI Data

Classes→ Month ↓	1	2	3	4	5	6	7
Jan.	.0508	.0408	.1838	.3887	.2424	.0625	.0309
	.0510	.0408	.1836	.3979	.2346	.0612	.0306
Feb.	.0610	.0102	.2623	.2830	.2582	.0944	.0309
	.0612	.0102	.2653	.2857	.2551	.0918	.0306
March	.0305	.0810	.1715	.3143	.3089	.0834	.0103
	.0306	.0816	.1734	.3163	.3061	.0816	.0102
April	.0203	.1013	.2024	.3051	.2568	.0830	.0312
	.0204	.1020	.2040	.3061	.2551	.0816	.0306
May	.0406	.0506	.2432	.3667	.1952	.0622	.0415
	.0408	.0510	.2448	.3673	.1938	.0612	.0408
June	.0406	.0506	.2637	.3362	.1847	.0826	.0415
	.0408	.0510	.2653	.3367	.1836	.0816	.0408
July	.0406	.0507	.2638	.3157	.2052	.0722	.0519
	.0408	.0510	.2653	.3163	.2040	.0714	.0510
Aug.	.0406	.0203	.2638	.3461	.2258	.0620	.0415
	.0409	.0204	.2653	.3469	.2244	.0612	.0408
Sept.	.0406	.0406	.2437	.3666	.2155	.0619	.0311
	.0408	.0408	.2448	.3673	.2042	.0612	.0306
Oct.	.0203	.0507	.2133	.4689	.1641	.0515	.0311
	.0204	.0510	.2142	.4693	.1632	.0510	.0306
Nov.	.0203	.0101	.2543	.4383	.2253	.0309	.0207
	.0204	.0102	.2551	.4387	.2244	.0306	.0204
Dec.	.0304	0.0	.2546	.3976	.2452	.0514	.0207
	.0306	0.0	.2551	.3979	.2448	.0510	.0204
12 Months' Average	.0364	.0422	.2350	.3606	.2273	.0665	.0320
Mean Monthly Matrix	.0364	.0423	.2353	.3601	.2270	.0662	.0318
Karl (1986)	.0500	.0600	.1700	.4500	.1700	.0600	.0400
Empirical 12 Months' Average	.0366	.0425	.2364	.3622	.2253	.0655	.0315

Table 4.4(a). Monthly Steady Class Probabilities (Virginia), Computed and Empirical, PDSI data

Classes→ Month ↓	1	2	3	4	5	6	7
Jan.	.0104 .0104	.0521 .0521	.2200 .2187	.5198 .5208	.1146 .1146	.0417 .0417	.0417 .0417
Feb.	.0104 .0104	.0314 .0313	.2299 .2292	.5411 .5417	.1249 .1250	.0312 .0313	.0312 .0313
March	0.0 0.0	.0627 .0625	.1775 .1771	.5414 .5416	.1666 .1667	.0208 .0208	.0312 .0313
April	0.0 0.0	.0731 .0729	.1669 .1667	.5415 .5417	.1458 .1458	.0417 .0417	.0312 .0313
May	.0418 .0417	.0313 .0313	.1669 .1667	.5624 .5625	.1354 .1354	.0521 .0508	.0104 .0104
June	.0104 .0104	.0417 .0417	.1668 .1667	.5625 .5625	.1458 .1458	.0625 .0625	.0104 .0104
July	0.0 0.0	.0626 .0625	.2085 .2083	.4584 .4583	.2083 .2083	.0416 .0416	.0208 .0208
Aug.	.0104 .0104	.0730 .0729	.1980 .1979	.4167 .4167	.2396 .2396	.0625 .0650	0.0 0.0
Sept.	.0104 .0104	.0938 .0938	.1355 .1354	.4584 .4583	.2187 .2188	.0625 .0625	.0208 .0208
Oct.	.0209 .0208	.0625 .0625	.2084 .2083	.3959 .3958	.2604 .2604	.0312 .0313	.0208 .0208
Nov.	.0209 .0208	.0521 .0521	.2605 .2604	.3959 .3958	.1979 .1979	.0417 .0417	.0312 .0313
Dec.	0.0 0.0	.0730 .0729	.2084 .2083	.4688 .4688	.1667 .1667	.0521 .0508	.0312 .0325
12 Months' Average	.0113	.0591	.1956	.4886	.1771	.0451	.0234
Mean Monthly Matrix	.0113	.0590	.1952	.4878	.1771	.0451	.0234
Karl (1986)	.05	.06	.17	.45	.17	.06	.04
Empirical 12 Months' Average	.0113	.0590	.1953	.4887	.1771	.0451	.0234

Table 4.4(b). Monthly Steady Class Probabilities (Virginia), Computed and Empirical, PHDI Data

Classes → Month ↓	1	2	3	4	5	6	7
Jan.	.0208	.0521	.2405	.4574	.1458	.0417	.0417
	.0208	.0508	.2396	.4583	.1458	.0417	.0417
Feb.	.0104	.0418	.2714	.4265	.1874	.0312	.0312
	.0104	.0417	.2708	.4270	.1875	.0313	.0313
March	.0104	.0626	.1982	.4893	.1873	.0208	.0312
	.0104	.0625	.1979	.4896	.1875	.0208	.0313
April	0.0	.0835	.1981	.4581	.1874	.0416	.0312
	0.0	.0833	.1979	.4583	.1875	.0417	.0313
May	.0417	.0417	.2085	.4790	.1666	.0521	.0104
	.0417	.0417	.2083	.4792	.1667	.0521	.0104
June	.0104	.0522	.2085	.4895	.1666	.0625	.0104
	.0104	.0521	.2083	.4896	.1667	.0625	.0104
July	0.0	.0730	.2188	.4479	.1978	.0416	.0208
	0.0	.0729	.2188	.4479	.1979	.0417	.0208
Aug.	.0104	.0730	.2501	.3229	.2707	.0729	0.0
	.0104	.0729	.2500	.3229	.2708	.0729	0.0
Sept.	.0104	.0938	.1563	.3750	.2812	.0625	.0208
	.0104	.0938	.1563	.3750	.2813	.0625	.0208
Oct.	.0208	.0625	.2188	.3437	.2916	.0417	.0208
	.0208	.0625	.2188	.3438	.2917	.0417	.0208
Nov.	.0208	.0521	.2709	.3437	.2395	.0417	.0312
	.0208	.0521	.2708	.3438	.2396	.0417	.0313
Dec.	.0104	.0729	.2604	.3646	.1979	.0625	.0312
	.0104	.0729	.2604	.3646	.1979	.0625	.0313
12 Months' Average	.0139	.0634	.2250	.4165	.2100	.0477	.0234
Mean Monthly Matrix	.0139	.0634	.2248	.4157	.2100	.0477	.0234
Karl (1986)	.0500	.0600	.1700	.4500	.1700	.0600	.0400
Empirical 12 Months' Average	.0139	.0634	.2248	.4168	.2102	.0478	.0235

Table 4.5(a). Expected Uninterrupted Residence Times (months), Computed and Empirical, Arizona, PDSI Data

Class→ Starting Month↓	1	2	3	4	5	6	7
Jan.	6.0 9.5 (24.2)	1.7 1.3 (1.6)	3.5 3.9 (5.7)	4.5 5.9 (20.5)	4.5 4.2 (11.2)	4.1 3.2 (9.0)	5.7 7.3 (35.0)
Feb.	6.6 11.3 (21.3)	2.1 1.8 (1.9)	3.3 3.6 (4.8)	4.8 6.2 (21.8)	4.2 3.8 (10.6)	3.9 4.3 (8.0)	7.1 6.7 (35.9)
Mar.	6.7 11.3 (18.1)	2.2 2.1 (1.4)	3.4 3.7 (3.5)	4.9 6.0 (22.7)	4.0 3.6 (10.0)	3.9 4.5 (6.7)	9.1 7.3 (26.1)
Apr.	5.7 11.3 (18.1)	1.7 1.6 (1.0)	2.7 2.8 (3.0)	5.8 7.0 (22.9)	3.6 3.1 (9.9)	3.9 4.4 (5.0)	8.1 8.8 (26.1)
May	4.7 7.7 (18.0)	1.7 1.6 (.7)	2.4 2.5 (2.6)	5.7 7.0 (22.1)	3.3 3.2 (10.3)	3.4 4.1 (4.3)	7.1 7.8 (26.0)
Jun.	3.7 5.3 (18.0)	1.4 1.6 (.3)	1.7 1.8 (2.6)	5.5 6.8 (21.4)	3.1 3.2 (11.4)	2.4 2.8 (4.3)	6.1 6.8 (26.1)
Jul.	4.1 5.9 (21.5)	1.1 1.1 (.1)	2.5 2.2 (4.0)	5.2 6.8 (21.0)	4.3 4.5 (14.1)	1.9 2.1 (5.0)	5.1 4.7 (26.1)
Aug.	4.9 6.8 (25.1)	1.0 1.0 (0)	2.3 1.8 (4.4)	5.3 6.4 (20.6)	4.3 4.0 (14.0)	2.9 2.8 (9.3)	5.8 7.3 (26.8)
Sep.	5.4 10.0 (26.7)	1.0 1.0 (0)	2.7 2.0 (5.5)	5.5 6.2 (19.9)	4.6 4.1 (13.5)	3.4 3.1 (11.7)	5.7 7.0 (26.8)
Oct.	5.5 11.3 (27.5)	1.0 1.0 (0)	2.5 2.4 (6.0)	5.3 5.6 (19.3)	4.3 4.5 (13.2)	4.3 3.9 (12.7)	5.7 6.5 (26.8)
Nov.	7.1 14.0 (25.1)	2.3 2.5 (1.5)	3.4 4.7 (7.7)	4.7 5.0 (19.2)	4.8 5.1 (12.2)	4.6 3.6 (11.7)	4.7 5.5 (26.8)
Dec.	6.1 11.6 (25.1)	1.6 1.6 (1.4)	3.6 4.2 (6.6)	4.6 4.9 (19.7)	3.9 4.1 (12.0)	5.1 3.5 (9.0)	4.4 5.4 (28.9)

Table 4.5(b). Expected Uninterrupted Residence Times (months), Computed and Empirical, Arizona, PHDI Data

Class→ Starting Month↓	1	2	3	4	5	6	7
Jan.	6.0	1.5	4.0	4.0	4.1	4.1	5.7
	9.5	1.3	4.2	4.8	4.1	3.2	7.3
	(24.5)	(1.3)	(7.6)	(13.2)	(10.7)	(9.2)	(35.1)
Feb.	6.7	2.1	3.5	4.0	4.0	4.0	7.1
	11.3	1.8	3.6	5.0	3.7	4.3	6.7
	(21.5)	(1.9)	(7.1)	(13.5)	(10.2)	(8.2)	(35.9)
Mar.	6.8	2.2	3.7	4.1	3.9	4.0	9.1
	11.3	2.1	3.8	5.2	3.6	4.6	7.3
	(18.0)	(1.4)	(6.1)	(13.8)	(9.7)	(7.0)	(26.1)
Apr.	5.8	1.7	3.3	4.7	3.6	4.0	8.1
	11.3	1.6	3.1	6.0	3.2	4.5	8.8
	(18.0)	(1.0)	(5.5)	(13.4)	(9.5)	(5.2)	(26.1)
May	4.8	1.7	3.2	4.5	3.4	3.5	7.1
	7.8	1.6	2.9	5.9	3.5	4.2	7.8
	(18.0)	(.7)	(4.9)	(12.7)	(9.7)	(4.5)	(26.1)
Jun.	3.8	1.5	2.6	4.1	2.9	2.5	6.1
	5.3	1.6	2.3	6.2	3.2	2.9	6.8
	(18.0)	(.4)	(4.7)	(12.4)	(10.2)	(4.5)	(26.1)
Jul.	3.9	1.1	2.6	3.7	3.9	2.1	5.1
	5.5	1.1	2.2	5.1	4.3	2.2	4.7
	(20.6)	(.2)	(5.0)	(12.6)	(13.5)	(5.2)	(26.1)
Aug.	4.9	1.2	2.9	3.6	4.4	2.7	5.8
	6.8	1.2	2.3	4.6	4.0	5.6	7.3
	(25.4)	(0.1)	(5.4)	(13.6)	(13.8)	(8.8)	(26.9)
Sep.	5.4	1.0	2.7	4.7	4.5	3.4	5.8
	10.0	1.0	2.1	5.4	3.8	3.1	7.0
	(27.1)	(0)	(5.6)	(14.7)	(13.4)	(11.8)	(26.8)
Oct.	5.5	1.7	2.6	4.8	4.5	4.3	5.7
	11.3	1.3	2.3	5.2	4.4	3.9	6.5
	(27.9)	(1.3)	(6.3)	(13.9)	(12.8)	(12.9)	(26.8)
Nov.	7.2	2.0	2.8	4.4	4.7	4.7	5.7
	14.0	2.2	3.3	5.1	4.6	3.6	5.5
	(25.4)	(1.3)	(7.7)	(13.5)	(11.5)	(11.9)	(26.8)
Dec.	6.2	1.6	3.8	4.3	4.1	5.1	4.4
	11.6	1.6	4.1	5.0	4.2	3.5	5.4
	(25.4)	(1.1)	(8.6)	(13.3)	(11.2)	(9.2)	(28.9)

Table 4.6(a). Expected Uninterrupted Residence Times (months), Computed and Empirical, California, PDSI data

Class→ Starting Month↓	1	2	3	4	5	6	7
Jan.	3.8	1.9	2.5	3.9	3.1	2.9	4.0
	4.5	1.5	2.0	4.1	3.0	2.5	5.0
	(17.9)	(3.3)	(4.9)	(18.1)	(8.3)	(4.8)	(14.0)
Feb.	3.7	3.6	2.5	5.0	3.6	2.5	3.0
	3.8	2.0	2.4	5.3	3.6	2.8	4.0
	(20.6)	(3.8)	(5.7)	(21)	(8.9)	(4.8)	(14.0)
Mar.	5.4	2.6	3.0	5.5	3.3	3.0	6.1
	5.5	2.1	2.7	5.7	3.0	2.7	9.0
	(26.7)	(3.8)	(6.7)	(20.8)	(9.0)	(5.2)	(17.2)
Apr.	8.7	2.7	3.7	6.4	4.1	2.8	5.1
	9.0	2.9	3.8	6.1	3.6	2.8	6.7
	(15.1)	(3.4)	(6.1)	(17.6)	(8.8)	(5.2)	(17.2)
May	7.8	3.0	4.1	5.9	4.5	3.5	6.2
	6.0	3.3	3.9	6.1	4.8	3.8	6.5
	(15.1)	(2.1)	(3.5)	(16.2)	(6.5)	(4.2)	(13.0)
Jun.	6.8	2.7	3.3	5.7	4.3	3.0	5.2
	5.0	2.8	3.4	6.0	4.5	3.1	5.5
	(15.1)	(1.0)	(3.0)	(14.3)	(4.6)	(3.5)	(13.0)
Jul.	5.8	1.7	2.8	5.5	3.5	2.7	4.2
	4.0	1.6	2.7	5.8	3.5	2.6	3.8
	(15.1)	(1.0)	(2.4)	(12.5)	(4.1)	(2.9)	(13.0)
Aug.	4.8	1.7	2.2	4.6	2.6	2.3	4.0
	3.0	1.5	2.2	4.7	2.8	2.0	3.5
	(15.1)	(.7)	(2.0)	(12.3)	(4.0)	(2.5)	(13.0)
Sept.	3.8	1.4	2.1	4.0	2.2	2.0	4.0
	5.5	1.3	1.9	4.1	2.5	1.6	3.3
	(15.1)	(.4)	(1.5)	(12.0)	(4.2)	(2.3)	(13.4)
Oct.	5.5	1.3	1.6	3.4	2.6	1.7	4.5
	9.0	1.3	1.4	3.7	2.6	1.4	3.5
	(15.1)	(.2)	(1.4)	(12.3)	(5.8)	(2.8)	(13.3)
Nov.	4.5	1.0	1.9	3.5	2.9	3.4	3.5
	8.0	1.0	1.8	3.8	3.1	4.5	2.5
	(15.1)	(0)	(2.0)	(13.9)	(7.0)	(4.4)	(13.3)
Dec.	3.5	1.3	1.7	3.9	3.1	2.4	5.0
	6.0	0	1.7	3.9	3.1	2.5	3.0
	(15.1)	(.9)	(2.6)	(16.4)	(7.7)	(4.4)	(14.0)

Table 4.6(b). Expected Uninterrupted Residence Times (months), Computed and Empirical, California, PHDI data

Class→ Starting Month ↓	1	2	3	4	5	6	7
Jan.	5.5 5.8 (30.9)	1.9 1.5 (3.2)	2.9 2.5 (7.9)	2.6 3.1 (8.6)	3.2 3.2 (8.2)	2.7 2.3 (4.7)	4.1 5.3 (15.3)
Feb.	4.5 4.2 (30.9)	3.5 2.0 (3.3)	3.2 2.9 (9.1)	3.6 4.8 (12.2)	3.4 3.6 (8.6)	2.5 2.7 (4.8)	3.1 4.3 (15.3)
Mar.	7.0 6.3 (37.8)	2.5 2.3 (3.3)	3.8 3.2 (9.8)	4.1 4.6 (13.0)	3.5 3.1 (8.9)	2.7 2.5 (5.2)	6.4 10.0 (18.7)
Apr.	8.9 8.0 (30.1)	2.5 2.5 (3.1)	4.3 3.9 (8.4)	5.0 5.2 (11.2)	3.6 3.3 (9.2)	2.8 2.8 (5.4)	5.4 7.0 (18.8)
May	7.9 6.0 (30.1)	2.9 3.0 (2.0)	4.4 2.3 (6.3)	4.9 5.2 (9.6)	4.4 4.5 (7.8)	3.6 4.0 (4.5)	6.6 7.0 (13.5)
Jun.	6.9 5.0 (30.1)	2.4 2.4 (1.4)	3.9 3.9 (5.3)	4.8 5.2 (7.4)	4.6 4.6 (5.1)	3.1 3.3 (3.7)	5.6 6.0 (13.5)
Jul.	5.9 4.0 (30.1)	1.7 1.6 (1.1)	3.5 3.4 (4.5)	4.5 4.8 (5.6)	3.8 3.8 (4.6)	2.8 2.7 (3.0)	4.6 4.2 (13.5)
Aug.	4.9 3.0 (30.1)	1.8 1.5 (.8)	2.9 2.8 (4.0)	3.6 3.8 (5.4)	2.8 2.9 (4.6)	2.6 2.2 (2.3)	4.5 4.0 (12.8)
Sept.	5.2 6.5 (33.4)	1.6 1.5 (.4)	2.5 2.2 (3.8)	3.0 3.3 (5.0)	2.3 2.8 (4.8)	1.9 1.5 (2.2)	4.7 4.0 (11.6)
Oct.	8.5 11.0 (30.9)	1.2 1.2 (.1)	2.2 2.0 (4.0)	2.5 2.8 (4.9)	2.7 3.0 (6.4)	1.7 1.4 (3.0)	3.7 3.0 (11.6)
Nov.	7.4 10.0 (30.9)	1.0 1.0 (0)	2.5 2.4 (5.3)	2.5 2.7 (5.9)	3.0 3.2 (7.5)	3.6 4.7 (4.5)	4.0 3.0 (11.9)
Dec.	6.5 9.0 (30.9)	1.3 0 (.8)	2.5 2.5 (6.3)	2.8 3.5 (7.4)	3.4 3.3 (8.0)	2.6 2.8 (4.5)	3.0 2.0 (11.9)

Table 4.7(a). Expected Uninterrupted Residence Times (months), Computed and Empirical, Virginia, PDSI Data

Class→ Starting Month ↓	1	2	3	4	5	6	7
Jan.	2	1.2	2.6	5.3	3.6	2.6	4.0
	2	1.2	2.8	6.3	2.9	2.9	4.0
	(0)	(.2)	(4.2)	(17.5)	(7.0)	(4.7)	(4.5)
Feb.	1.0	1.0	2.5	5.1	3.6	3.2	4.0
	1.0	1.0	2.2	6.7	2.8	3.3	4.0
	(0)	(0)	(4.3)	(16.7)	(7.1)	(4.3)	(2.0)
Mar.	1.2	2.1	2.9	5.0	3.1	3.3	3.0
	--	1.8	2.6	6.1	2.6	3.5	3.0
	(.2)	(.9)	(4.4)	(15.9)	(7.0)	(2.9)	(2)
Apr.	1.2	1.3	2.7	4.6	3.0	2.3	2.0
	--	1.1	2.4	6.0	2.4	2.5	2.0
	(.2)	(.8)	(4.1)	(15.5)	(7.3)	(2.9)	(2)
May	1.3	1.9	2.7	4.2	3.2	2.6	3.0
	1.3	1.3	2.7	5.2	2.5	2.6	3.0
	(.2)	(2.2)	(3.8)	(15.6)	(7.7)	(2.3)	(0)
Jun.	1.0	2.6	2.5	3.9	3.5	2.0	2.0
	1.0	3.0	2.6	4.7	3.2	1.7	2.0
	(0)	(2.1)	(3.6)	(16.4)	(7.8)	(2.1)	(0)
Jul.	1.1	2.2	2.4	4.3	3.5	2.0	1.0
	--	2.2	2.5	5.6	3.4	1.5	1.0
	(.1)	(1.6)	(3.6)	(18.3)	(7.3)	(2.1)	(0)
Aug.	1.0	1.7	2.1	4.7	3.4	2.0	1.5
	1.0	1.6	2.2	5.9	3.5	2.2	--
	(0)	(1.4)	(3.9)	(19.5)	(6.9)	(2.1)	(3.3)
Sept.	2.5	1.7	2.7	4.6	3.6	2.1	3.7
	3.0	1.7	2.7	5.9	3.6	2.2	4.5
	(.3)	(1.7)	(5.1)	(20.0)	(6.0)	(2.1)	(11.4)
Oct.	1.5	2.2	3.1	5.0	2.8	2.2	5.3
	1.5	2.0	2.7	6.1	2.8	2.3	8.5
	(.3)	(1.8)	(5.0)	(20.7)	(6.0)	(1.8)	(8.6)
Nov.	1.0	2.3	2.8	5.7	2.8	1.7	4.3
	1.0	2.0	2.8	6.8	2.8	1.5	5.3
	(0)	(.8)	(4.7)	(19.9)	(6.6)	(1.6)	(8.6)
Dec.	1.3	1.7	2.8	5.7	3.0	1.5	5.0
	--	1.7	3.0	6.6	2.6	1.4	6.0
	(.4)	(.4)	(4.4)	(18.4)	(7.5)	(2.0)	(4.5)

Note : -- denotes unavailability of data to compute empirical value

Table 4.7(b). Expected Uninterrupted Residence Times (months), Computed and Empirical, Virginia, PHDI Data

Class→ Starting Month ↓	1	2	3	4	5	6	7
Jan.	1.5	1.2	2.8	4.3	3.6	2.6	4.0
	1.5	1.2	2.9	6.0	3.2	2.0	4.0
	(.25)	(.2)	(4.1)	(11.2)	(7.6)	(4.6)	(4.5)
Feb.	1.0	1.0	2.4	4.3	3.3	3.2	4.0
	1.0	1.0	2.2	6.2	2.6	3.3	4.0
	(0)	(0)	(4.0)	(10.3)	(7.3)	(4.2)	(2.0)
Mar.	1.0	2.3	2.9	4.1	3.2	3.3	3.0
	1.0	1.8	2.7	5.4	2.5	3.5	3.0
	(0)	(1.5)	(4.0)	(9.5)	(7.2)	(2.6)	(2)
Apr.	1.2	1.5	2.7	3.8	3.0	2.3	2.0
	--	1.4	2.4	5.0	2.3	2.5	2.0
	(.2)	(1.4)	(3.6)	(8.9)	(7.5)	(2.6)	(2)
May	1.3	2.1	2.4	3.4	3.1	2.5	3.0
	1.3	1.5	2.7	4.5	2.3	2.6	3.0
	(.2)	(2.2)	(3.5)	(8.9)	(8.1)	(2.0)	(0)
Jun.	1.0	2.2	2.4	3.1	3.7	2.0	2.0
	1.0	2.6	2.5	4.6	3.2	1.7	2.0
	(0)	(1.9)	(3.6)	(9.3)	(8.6)	(1.7)	(0)
Jul.	1.1	2.0	2.5	3.2	3.8	1.9	1.0
	--	2.0	2.8	5.2	3.9	1.5	1.0
	(.1)	(1.6)	(3.7)	(10.7)	(7.8)	(1.6)	(0)
Aug.	1.0	1.7	2.1	3.8	3.6	1.8	1.5
	1.0	1.6	2.0	6.2	3.9	2.0	--
	(0)	(1.4)	(4.0)	(12.5)	(7.2)	(1.6)	(3.3)
Sept.	3.1	1.7	2.8	3.9	3.4	1.9	3.7
	5.0	1.7	2.7	5.9	3.6	2.2	4.5
	(2.1)	(1.7)	(5.7)	(13.3)	(6.7)	(1.6)	(11.4)
Oct.	2.1	2.1	3.4	4.2	2.9	1.9	5.3
	2.5	2.0	2.9	5.9	3.1	2.0	8.5
	(2.1)	(1.8)	(5.2)	(13.9)	(6.6)	(1.4)	(8.6)
Nov.	2.3	2.4	3.0	4.7	3.0	1.7	4.3
	2.0	2.0	3.0	5.9	3.2	1.5	5.3
	(1.7)	(.8)	(4.7)	(13.3)	(7.1)	(1.4)	(8.6)
Dec.	2.5	1.7	2.8	4.9	3.1	1.4	5.0
	2.0	1.7	2.8	6.3	2.9	1.3	6.0
	(.3)	(.4)	(4.3)	(11.7)	(7.5)	(1.7)	(4.5)

Table 4.8(a). Expected First Passage Time and Recurrence Time to Class 4, (months), Computed and Empirical, Arizona, PDSI Data

Class→ Starting Month ↓	1	2	3	4	5	6	7
Jan.	15.8 21.4	12.4 12.7	9.9 11.3	3.9 4.0	11.0 18.6	17.1 17.3	20.5 41.7
Feb.	15.3 23.3	13.4 8.8	10.4 13.9	3.2 2.6	11.7 17.5	17.8 22.3	20.3 30.3
Mar.	14.6 24.3	13.0 9.7	8.9 12.1	4.1 4.2	11.8 17.5	16.6 19.5	23.1 27.7
Apr.	13.6 23.9	12.7 5.6	7.2 10.2	2.6 2.6	11.5 13.5	13.4 26.0	22.1 28.6
May	12.6 16.7	10.6 7.8	5.2 9.3	2.1 1.6	9.9 9.8	12.7 24.6	21.1 27.6
Jun.	11.6 13.8	7.6 14.6	4.6 5.6	1.9 1.3	9.2 9.8	11.7 19.6	20.1 26.6
Jul.	11.6 14.5	10.6 16.2	3.8 2.7	2.4 1.5	11.1 9.4	10.2 17.1	19.1 29
Aug.	14.3 17.7	16.6 31.5	4.1 3.5	2.7 3.5	10.2 8.7	14.7 23.1	21.5 39.5
Sept.	15.6 21.0	7.3 11.7	6.8 5.1	2.1 1.4	9.4 10.8	14.7 22.7	21.7 38.2
Oct.	16.5 23.6	13.7 7.0	6.3 5.4	2.0 2.0	8.9 10.0	15.5 31.0	21.7 31.2
Nov.	15.6 25.2	15.3 10.0	9.8 8.8	2.7 3.5	9.5 15.6	16.5 19.1	20.8 30.2
Dec.	14.6 21.9	14.3 11.0	8.4 11.3	3.3 4.2	8.9 14.7	18.1 18.7	20.1 30.8

Table 4.8(b). Expected First Passage Time and Recurrence Time To Class 4 ,(months), Computed and Empirical, Arizona, PHDI Data

Class→ Starting Month ↓	1	2	3	4	5	6	7
Jan.	20.9 23.8	14.5 14.0	12.6 12.2	4.1 2.7	12.1 22.0	21.5 32.4	23.6 69.0
Feb.	20.7 25.3	17.6 12.3	12.2 13.0	4.3 4.2	13.5 19.8	21.1 42.5	23.3 47.0
Mar.	20.0 26.2	17.8 13.3	10.8 12.9	4.9 4.8	13.7 21.6	20.1 31.1	26.5 55.0
Apr.	19.0 26.0	17.8 8.9	9.4 11.6	3.0 2.9	13.7 16.7	17.0 37.6	25.5 53.8
May	18.0 19.2	15.1 11.3	6.8 10.2	2.3 1.6	11.6 12.8	16.6 32.6	24.5 52.8
Jun.	17.0 16.6	11.2 17.2	6.3 6.8	1.9 1.3	9.6 11.8	15.6 26.2	23.5 51.8
Jul.	17.1 16.3	13.1 18.4	3.9 2.8	3.0 1.7	10.1 9.3	14.4 25.3	22.5 48.6
Aug.	18.7 20.5	12.5 13.7	5.3 4.5	4.6 6.4	10.5 9.5	15.8 30.9	24.9 61.5
Sept.	21.5 23.7	7.6 11.3	6.1 4.4	2.9 1.7	10.0 13.0	16.5 29.9	25.4 58.8
Oct.	21.4 26.3	17.0 8.3	6.6 5.3	2.2 1.9	10.6 14.0	18.6 40.0	25.5 51.8
Nov.	22.0 27.3	17.8 12.6	8.0 6.8	3.2 4.8	11.0 17.5	22.5 32.6	23.7 50.8
Dec.	20.9 24.3	14.5 15.2	12.6 10.9	3.4 5.0	12.1 18.7	21.5 39.8	23.6 48.0

Table 4.9(a). Expected First Passage Time and Recurrence Time To Class 4 (months), Computed and Empirical, California, PDSI Data

Class→ Starting Month ↓	1	2	3	4	5	6	7
Jan.	5.1 6.5	6.0 2.0	3.6 2.5	3.2 2.9	6.2 11.2	5.6 7.5	10.9 10.0
Feb.	5.5 5.8	6.0 2.0	4.3 3.7	2.5 2.5	6.5 10.3	6.1 8.6	9.9 9.0
Mar.	6.5 5.5	5.0 5.3	4.9 5.5	2.8 2.3	5.6 7.0	10.2 15.4	10.8 9.0
Apr.	11.0 9.0	5.5 7.0	5.6 5.1	1.5 1.7	6.3 5.9	9.0 16.6	9.8 11.3
May	10.0 9.3	5.4 9.0	4.9 4.3	1.7 1.7	7.2 8.6	7.7 13.0	9.4 12.3
Jun.	9.0 8.3	4.6 7.5	3.9 3.7	1.6 1.3	6.0 6.9	6.9 11.3	8.4 11.3
Jul.	8.0 7.3	3.6 5.4	3.2 2.9	1.1 1.0	4.9 4.8	5.8 8.7	7.4 14.4
Aug.	7.0 6.3	5.1 9.5	2.8 2.4	1.3 1.3	4.1 5.5	5.1 7.5	6.7 9.3
Sept.	6.0 8.0	2.2 1.7	2.6 2.0	1.4 1.2	3.8 6.4	5.2 5.4	6.3 6.0
Oct.	6.4 9.0	3.7 3.3	1.9 1.4	2.2 2.0	5.0 7.7	5.3 6.8	7.9 7.5
Nov.	5.4 8.0	5.4 5.0	2.6 2.0	2.7 4.0	5.6 6.6	10.2 19.0	6.9 6.5
Dec.	4.4 6.0	6.3 0.0	3.2 2.6	2.3 2.1	5.9 10.2	9.2 12.5	11.9 11.0

Table 4.9(b). Expected First Passage Time and Recurrence Time To Class 4 (months), Computed and Empirical, California, PHDI Data

Class→ Starting Month ↓	1	2	3	4	5	6	7
Jan.	13.6 13.0	9.4 5.3	4.5 2.9	4.5 4.5	8.0 12.3	9.7 7.7	14.0 11.0
Feb.	12.6 11.7	9.6 4.0	5.7 4.8	3.4 3.6	6.8 9.9	11.4 11.2	13.0 10.0
Mar.	14.5 14.0	8.6 8.8	6.4 6.6	3.6 3.1	6.6 7.5	13.4 15.0	16.3 12.0
Apr.	16.3 13.5	8.0 10.1	6.8 6.1	2.0 2.2	6.5 5.7	12.4 18.5	15.3 12.3
May	15.3 17.5	8.4 11.6	5.7 4.9	2.2 2.0	8.1 8.3	10.8 14.2	16.0 15.5
Jun.	14.3 16.5	8.0 9.4	5.0 4.6	1.8 1.4	7.4 7.4	10.2 13.1	15.0 14.5
Jul.	13.3 15.5	7.6 8.2	4.4 3.9	1.1 1.0	6.2 5.6	8.9 9.9	14.0 17.0
Aug.	12.3 14.5	10.6 13.5	4.0 3.3	1.6 1.5	5.2 6.0	9.0 8.8	14.0 12.5
Sept.	13.0 17.8	6.2 5.3	3.3 2.3	1.7 1.3	4.8 7.1	8.7 5.8	15.7 9.3
Oct.	16.6 16.0	7.4 9.4	3.0 2.0	3.3 2.5	6.0 7.8	9.3 7.8	14.7 8.3
Nov.	15.6 15.0	15.6 16.0	4.1 3.2	4.1 5.8	7.0 6.6	13.5 14.7	13.9 8.0
Dec.	14.6 14.3	9.4 0.0	5.2 3.7	3.2 2.9	7.9 12.0	12.5 11.6	12.9 7.0

Table 4.10(a) Expected First Passage Time and Recurrence Time to Class 4, (months), Computed and Empirical, Virginia, PDSI data

Class→ Starting Month ↓	1	2	3	4	5	6	7
Jan.	8.1 4.0	4.7 3.8	4.6 5.2	1.8 1.4	5.2 4.3	6.8 8.0	11.2 15.5
Feb.	7.1 3.0	4.1 9.3	4.7 3.9	1.9 2.4	5.7 4.7	8.8 9.3	10.7 16
Mar.	6.5 --	6.1 5.2	4.2 4.9	1.6 1.8	5.7 6.3	8.9 9.0	9.7 15
Apr.	6.1 --	6.1 5.0	4.4 4.5	1.7 1.8	5.4 7.1	7.9 7.0	8.7 14
May	7.0 5.3	3.6 2.0	4.3 5.2	2.0 2.4	5.9 8.0	7.9 11.4	7.1 3
Jun.	8.4 7.0	7.7 6.3	4.5 6.0	2.9 2.5	6.2 9.3	7.1 8.0	6.1 2
Jul.	6.5 --	7.4 7.2	4.4 5.2	2.6 2.1	6.6 7.2	6.2 12.5	5.1 3
Aug.	7.2 5.0	6.7 6.7	4.9 4.6	2.1 2.7	5.8 5.1	8.2 14.0	6.4 --
Sept.	6.7 3.0	6.2 6.0	4.7 6.3	2.3 1.9	5.7 5.9	8.4 10.7	6.1 13
Oct.	5.7 2.5	6.0 6.2	4.5 4.6	2.3 2.5	4.9 5.6	9.8 11.3	10.1 17
Nov.	3.5 4.0	5.9 4.0	4.8 5.1	1.9 1.6	5.1 4.5	8.7 17	9.1 11
Dec.	6.8 --	5.0 3.6	4.6 5.8	1.5 1.5	5.2 4.0	6.7 9.4	12.2 18

Note : -- denotes unavailability of data to compute empirical result

Table 4.10(b). Expected First Passage Time and Recurrence Time To Class 4, (months), Computed and Empirical, Virginia, PHDI Data

Class→ Starting Month ↓	1	2	3	4	5	6	7
Jan.	9.1 6.0	6.3 4.8	5.2 5.9	2.2 1.6	5.2 4.6	8.5 9.0	14.2 15.8
Feb.	9.1 5.0	7.0 11.0	4.9 4.1	2.1 2.8	5.3 4.2	9.6 9.0	14.3 17.7
Mar.	8.6 4.0	8.1 7.3	5.1 5.3	2.0 2.2	6.0 6.4	10.0 7.0	13.3 16.7
Apr.	7.9 0.0	7.6 7.1	4.5 4.3	1.7 2.1	5.5 6.9	9.0 6.5	12.3 15.7
May	7.9 8.8	6.7 2.5	4.1 4.9	2.4 2.7	6.0 7.8	10.0 12.2	14.0 5.0
Jun.	9.2 11.0	6.9 6.8	4.5 5.6	3.1 2.5	6.7 9.4	9.6 8.3	13.0 4.0
Jul.	7.8 0.0	8.2 8.1	4.9 5.7	3.6 2.8	7.5 7.6	8.1 12.8	12.0 14.5
Aug.	8.8 9.0	8.2 8.4	4.4 4.1	2.7 3.6	6.5 5.5	11.0 13.7	8.7 0.0
Sept.	10.5 10.0	7.8 7.3	5.6 6.0	2.7 2.2	5.5 5.8	10.6 11.2	13.6 16.0
Oct.	9.5 8.0	8.0 6.7	5.5 5.1	2.8 2.8	5.3 6.0	10.7 9.3	14.6 14.5
Nov.	9.5 8.0	7.5 5.0	5.2 5.4	2.3 2.1	5.6 5.0	9.9 16.8	13.6 13.0
Dec.	10.1 7.0	6.9 4.9	5.0 5.5	1.5 1.7	5.6 4.3	7.4 9.0	15.2 19.7

Table 4.11(a). Mean First Passage Times and Mean Recurrence Times to Class 4, PDSI Data

Place ↓ Class→	1	2	3	4	5	6	7
Arizona							
12 Monthly Matrices' Average	14.3	12.3	7.1	2.7	10.3	14.9	21.0
Mean Monthly Matrix	13.7	12.1	7.4	2.8	10.7	13.9	19.5
Empirical	19.5	11.3	8.8	2.8	13.2	21.3	31.7
California							
12 Monthly Matrices' Average	7.0	4.9	3.6	2.0	5.6	7.2	8.9
Mean Monthly Matrix	6.9	4.8	3.8	2.1	5.8	7.6	9.9
Empirical	7.3	5.5	3.2	2.1	7.6	10.9	10.3
Virginia							
12 Monthly Matrices' Average	6.6	5.8	4.6	2.1	5.6	7.9	8.5
Mean Monthly Matrix	6.0	5.9	4.6	2.1	5.8	7.6	8.2
Empirical	4.3	5.4	5.0	2.1	5.9	10.7	13.2

Table 4.11(b) Mean First Passage Times and Mean Recurrence Times to Class 4, PHDI Data

Place ↓ Class→	1	2	3	4	5	6	7
Arizona							
12 Monthly Matrices' Average	19.8	14.7	8.4	3.3	11.5	18.4	24.4
Mean Monthly Matrix	22.0	16.9	8.6	3.4	11.7	17.5	22.9
Empirical	21.9	13.3	8.6	3.5	15.8	31.9	53.7
California							
12 Monthly Matrices' Average	14.3	9.1	4.8	2.7	6.7	10.8	14.6
Mean Monthly Matrix	14.4	8.3	5.0	2.8	7.0	11.7	15.6
Empirical	14.8	9.1	4.0	2.7	8.1	11.7	12.2
Virginia							
12 Monthly Matrices' Average	9.0	7.4	4.9	2.4	5.9	9.5	13.2
Mean Monthly Matrix	8.9	7.6	4.9	2.4	6.1	10.0	13.4
Empirical	7.8	6.8	5.1	2.4	6.0	10.7	14.8

Table 4.12(a) Average Uninterrupted Residence Times (months) and Empirical Average Residence Times, PDSI Data

Place ↓ Class →	1	2	3	4	5	6	7
Arizona							
12 Monthly Matrices' Average	5.5	1.6	2.8	5.2	4.1	3.7	6.7
Empirical Average	5.1	1.6	2.9	5.1	4.2	3.3	6.7
California							
12 Monthly Matrices' Average	5.3	2.1	2.6	4.5	3.3	2.7	5.0
Empirical Average	4.9	2.1	2.6	4.5	3.3	2.7	5.0
Virginia							
12 Monthly Matrices' Average	1.3	1.8	2.7	4.8	3.3	2.3	3.2
Empirical Average	1.4	1.9	2.7	4.9	3.3	2.2	3.9

Table 4.12(b) Average Uninterrupted Residence Times (months) and Empirical Average Residence Times, PHDI Data

Place ↓ Class →	1	2	3	4	5	6	7
Arizona							
12 Monthly Matrices' Average	5.6	1.6	3.1	4.2	4.0	3.7	6.3
Empirical Average	5.1	1.6	3.2	4.2	4.1	3.3	6.7
California							
12 Monthly Matrices' Average	6.6	2.0	3.2	3.7	3.4	2.7	4.6
Empirical Average	6.1	2.1	3.2	3.4	3.3	2.8	6.1
Virginia							
12 Monthly Matrices' Average	1.6	1.8	2.7	4.0	3.3	2.2	3.2
Empirical Average	1.6	1.9	2.7	3.9	3.4	2.0	3.9

Table 4.13(a). Prediction of Weather Classes, PDSI Data

1931 AZ	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
OBS.	4	3	4	3	3	3	2	1	1	1	1	1
PMODE	4	4	4	4	4	3	4	4	4	4	1	1
PEXP	4	4	4	4	4	3	4	3	4	4	1	1
1931 CA												
OBS.	4	5	5	5	5	5	5	5	5	6	4	3
PMODE	4	4	4	4	4	5	5	5	5	4	4	4
PEXP	4	4	4	4	4	5	5	5	5	5	5	5
1931 VA												
OBS.	7	7	7	7	6	5	5	5	5	5	5	6
PMODE	4	7	7	7	6	6	6	6	6	5	5	5
PEXP	5	5	5	7	6	6	6	5	5	5	5	5
1990 AZ												
OBS.	7	7	7	7	7	7	7	7	6	7	7	7
PMODE	7	7	7	7	7	7	7	7	7	7	7	7
PEXP	7	7	6	6	6	6	7	6	6	6	6	7
1990 CA												
OBS.	5	5	5	6	5	5	6	5	5	6	6	6
PMODE	4	4	4	5	5	5	5	6	5	4	4	4
PEXP	4	4	4	5	5	5	5	6	5	5	5	5
1990 VA												
OBS.	2	3	3	3	2	4	4	4	4	4	4	4
PMODE	3	3	3	3	3	3	3	3	4	4	4	4
PEXP	3	3	3	3	3	3	3	3	3	4	4	4

OBS.= OBSERVED Class ; PMODE = PREDICTED Class (MODE VALUE) ; PEXP = PREDICTED Class (EXPECTED VALUE)

Table 4.13(b). Prediction of Weather Classes, PHDI Data

1931 AZ	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
OBS.	4	3	4	3	3	3	2	1	1	1	1	1
PMODE	4	4	4	4	4	3	4	3	3	3	1	1
PEXP	4	4	4	4	4	3	4	4	3	3	3	1
1931 CA												
OBS.	4	5	5	5	5	5	5	5	5	6	5	3
PMODE	4	4	4	4	4	5	5	5	5	5	5	5
PEXP	4	4	4	4	4	5	5	5	5	5	5	5
1931 VA												
OBS.	7	7	7	7	6	5	5	5	5	5	5	6
PMODE	5	7	7	7	6	6	6	6	6	5	5	5
PEXP	7	7	7	7	6	6	6	5	5	5	5	5
1990 AZ												
OBS.	7	7	7	7	7	7	7	7	6	7	7	7
PMODE	7	7	7	7	7	7	7	7	7	7	7	7
PEXP	7	7	6	6	6	6	7	7	7	7	7	7
1990 CA												
OBS.	5	5	5	6	5	5	6	5	5	6	6	6
PMODE	4	4	4	5	5	5	5	6	5	5	5	5
PEXP	4	4	4	5	5	5	5	6	5	5	5	5
1990 VA												
OBS.	2	3	3	3	2	3	4	3	4	4	4	4
PMODE	3	3	2	3	3	3	3	3	2	4	4	4
PEXP	2	3	3	3	3	3	3	3	3	3	4	3

OBS.= OBSERVED Class ; PMODE = PREDICTED Class (MODE VALUE) ; PEXP = PREDICTED Class (EXPECTED VALUE)

Chapter V

DYNAMICAL BEHAVIOR OF DROUGHTS

5.1 INTRODUCTION

Chapter IV dealt with the characterization of droughts in terms of the steady state probabilities of occurrences of weather classes in various months, first passage times, and uninterrupted residence times and their variances ; also, a predictive technique for the weather classes has been suggested. The procedure offers forecasts with good accuracy up to 4 months in advance. The statistical measures of drought characteristics are derived as the expected values. To cope with the situations when sudden, extreme fluctuations in rainfall (and thereby in Palmer index) occur, two approaches are offered. One is the dynamical systems approach and the other to enumerate all possible occurrences no matter how small the probability of occurrence of an event is in terms of a decision tree. The current chapter deals with the dynamical systems approach while the decision tree approach is covered in chapter VI. In the dynamical systems approach it is stipulated that the time series of the variable characterizing the weather states represents a multistable dynamical system. This stipulation happens to agree very well with the empirically observed bi-modal / multi-modal distribution of the Palmer index (Alley, 1984, 1985; Eder et al., 1987; Heddinghaus and Sabol, 1991). Prolonged periods of droughts with abrupt transition to long periods of moisture abundance as often observed in several parts of the world also indicate the bi-modal behavior of the weather process.

In the dynamical systems approach, it is argued that certain natural processes have a tendency to revolve around certain stable states and transitions from one state to another can be attributed to disturbances of external origin. For instance, in the case of the bi-modal distribution of the PHDI it can be assumed that the weather tends to stay around mainly two states; one representing dry conditions and the other wet conditions. An external force stochastic in nature can be considered responsible for aperiodic transitions of weather states between these two states. For example, the global phenomenon of El Nino- Southern Oscillation (ENSO) can be considered an external factor to account for greater fluctuations in precipitation in many regions of the world. Precipitation in turn affects the Palmer index. Hence ENSO phenomenon can be a relevant example of random force of external origin. Thus in regard to its temporal evolution the Palmer index will tend to lock itself around the value of one mode, but with a strong enough fluctuation in the climate it may shift to the other mode and will remain there until a large climatic fluctuation shifts it back to the previous mode.

In the dynamical systems approach the evolution of a process is represented by a stochastic differential equation and the solution of an equivalent Fokker Planck equation gives the probability density function of the process which is used to analyze the stable states of the system. The goal of the present chapter is, therefore, twofold : i) to explain the bi-modal behavior of Palmer index using dynamical systems approach and ii) to present a procedure to analyze the effects of the external forces on drought process in a region. Since PHDI is better suited for operational applications, we have chosen to use this index in this chapter.

5.2 FLUCTUATION INDUCED TRANSITIONS

5.2.1 Brownian Motion

In many circumstances the environment impinging on a dynamical system is a complex , noisy one. In such an environment spontaneous random variations from the means of the state variables called the fluctuations characterize the system. For example consider the Brownian motion of a particle. Let the mass of the particle be m. When this particle is immersed in a fluid, the fluid will exert a frictional force on the particle. The simplest expression for such a friction force (F) is given by the Stokes' law (Risken, 1984):

$$F = -\alpha \ V \tag{1}$$

where : $\alpha > 0$ is a constant which depends on the viscosity of the fluid and on the particle's mass and diameter and V = velocity of the particle.

Therefore, as per Newton's second law of motion we can write the equation of motion for the particle as :

$$m \frac{dV}{dt} = -\alpha \ V \tag{2}$$

or;

$$m \frac{dV}{dt} + \alpha V = 0 \tag{3}$$

Eq. 3 is a deterministic differential equation, and its solution gives how an initial velocity V(0) of the particle decreases to zero with time. Eq. 3 is valid only if the mass of the particle is large so that its velocity due to molecular bombardment in fluid is negligible. For small mass m the effects of molecular bombardment on velocity can not be neglected and so the solution of the deterministic Eq. 3 can not exactly describe the velocity of the particle. Instead we need to modify Eq. 2 by adding a fluctuating force F(t), due to the effect of molecular bombardment, which produces instantaneous random changes in the acceleration of the particle. Therefore, Eq. 2 is modified as :

$$m \frac{dV}{dt} = -\alpha \ V + F(t) \tag{4}$$

or;

$$\frac{dV}{dt} + \gamma \ V = \Gamma(t) \tag{5}$$

where: $\gamma = \frac{\alpha}{m}$ and $\Gamma(t) = \frac{F(t)}{m}$

The term $\Gamma(t)$ in Eq. 5 represents fluctuating force per unit mass (stochastic quantity) and is called Langevin force. Eq. 5 is called a stochastic differential equation because it contains the stochastic term $\Gamma(t)$. If a differential equation contains a stochastic term, like Eq. 5 as above, then the solution to the differential equation can only be described statistically. For this purpose some properties of the Langevin force $\Gamma(t)$ are required to be known. In many cases $\Gamma(t)$ is modeled as a Gaussian white noise. For white noise process it is assumed that the average of this force over the ensemble is zero. Also, the correlation function of the force $\Gamma(t)$ is assumed proportional to a δ function

or; $$E(\Gamma(t)) = 0 \text{ and } E(\Gamma(t) . \Gamma(t')) = q^2 \delta(t - t') \quad (6)$$

where: q^2 = variance of $\Gamma(t)$

The Gaussian white noise idealization of $\Gamma(t)$ can be explained as follows. The quantity giving rise to this fluctuating force can be assumed to arise primarily from the superposition of a large number of loosely coupled variables and, using the result of the central limit theorem, this force can be assumed to follow Gaussian distribution. Further, the fact that the variables are loosely coupled can be used to ensure statistical independence of the successive values of the force for different times.

$\Gamma(t)$ is a stochastic quantity in Eq. 5 which varies from system to system in the ensemble. As a result, the velocity V(t) will also become a stochastic quantity . Therefore, we may ask for the probability to find the velocity of the particle in the interval (V,V+dV); or in other words we may ask for number of the systems of the ensemble whose velocities are in the interval (V,V+dV) divided by the total number of the systems in the ensemble. For a continuous variable like velocity V, the answer to this question can be found by computing the probability density function p(V,t). A stochastic differential equation forced by the white noise, like Eq. 5 above, defines a class of random processes known as diffusion process, and the probability density function of the variable of interest, $p(V,t|v_0,t_0)$ in this case, obeys to an equation called the Fokker - Planck equation as below:

$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial v}F(v,t) + \frac{q}{2}\frac{\partial^2 p}{\partial v^2} \quad (7)$$

By solving Eq. 7 with proper initial condition, one obtains the probability density function $p(V,t|v_0,t_0)$ for all later times. The solution to the Fokker Planck equation is the probability density of the solution to the original differential equation. Mathematically, Eq. 7 is a linear second order partial differential equation of parabolic type. Roughly speaking, it is a diffusion equation with an additional first order derivative with respect to V. This equation is also called a forward Kolmogorov equation.

The general Fokker Planck equation for one variable x has the form :

$$\frac{\partial p(x,t|x_0,t_0)}{\partial t} = -\frac{\partial}{\partial x}A(x,t)p + \frac{1}{2}\frac{\partial^2}{\partial x^2}D(x,t)p \quad (8)$$

In Eq. 8, D(x,t) > 0 is called the diffusion function and A(x,t) the drift function. Nonstationary solutions of the Fokker Planck equation (Eq. 8) are generally difficult to obtain. A general expression for the nonstationary solution can be found only for special drift and diffusion functions. It is therefore important to discuss the drift and diffusion functions and their forms which yield analytical solution of the Fokker Planck equation.

5.2.2 Drift and Diffusion Functions of Continuous Markov Process

Appendix V gives a brief description on continuous Markov process representation. The propagator random variable for a process X_t denoted by K is defined as

$$K = X_{t+dt} - X_t \quad \text{given} \quad X_t = x \quad (9)$$

The mean function of K is A(x,t)dt and its variance is D(x,t)dt. The propagator density function is the same as the Makov state density function $p(x,t|x_0,t_0)$ except the range gets shifted. If a continuous Markov process is given to be in state x at time t, then its Markov state density function at time t is just a delta function spike at x. Now at infinitesimally later time t+dt , the infinite spike will have relaxed to a normal or Gaussian shaped peak, which is centered on x+A(x,t)dt and which has width of $2(D(x,t)\,dt)^{.5}$. Since the drifting of peak is controlled by A(x,t) function, it is called the drift function of the process. The diffusive spreading of peak is controlled by D(x,t) and so it is called the diffusion function of the process. The continuous Markov process X_t is temporally homogeneous if A(x,t) = A(x) and D(x,t) = D(x) ; and X_t is completely homogeneous if A(x,t) = A and D(x,t) = D. The functional forms of drift and diffusion functions dictate the possibility of obtaining closed form solutions of the Fokker Planck equation. For example, analytic solution of the Fokker Planck equation can be found for linear drift function and constant diffusion function. In this case one obtains Gaussian distribution for the stationary as well as for the non-stationary solutions (Risken, 1984). Also, based on the functional forms of the drift and diffusion functions of a continuous Markov process, moment evolution equations are derived for computing mean, variance and covariance of the process X_t. These equations can be solved analytically if A(x,t) is a polynomial in x of degree $\leq$ 1 and D(x,t) is a polynomial in x of degree $\leq$ 2. If these rather restrictive conditions on A(x,t) and D(x,t) are not satisfied, then either a numerical solution is hoped or suitable approximation of two functions A(x,t) and D(x,t) is done to obtain the solution (Gillespie, 1992).

Most continuous Markov processes of practical interest can be assumed to be temporally homogeneous which implies that : A(x,t) = A(x) ; and D(x,t) = D(x) (Gillespie, 1992). Kedam et al. (1994) described a probability distribution model for rain rate assuming it to follow a temporally homogeneous diffusion process. Temporal homogeneity essentially means that the first two moments of the propagator random variable (K) do not change over time. Temporal homogeneity does not restrict the variation of the moments of process X_t over time. For example, the Wiener process is characterized by : A(x,t) = A ;and D(x,t) = D > 0 ; which in fact is a completely homogeneous continuous Markov process. The Ornstein- Uhlenbeck process is characterized by A(x,t) = - k x ; D(x,t) = D (k > 0 ; D > 0) and is a temporally homogeneous process. Both the processes have time dependent moments. The following analysis assumes the process to be temporally homogeneous.

For some temporally homogeneous Markov processes the state density function $p(x,t|x_0,t_0)$ approaches, as $(t-t_0) \rightarrow \infty$, a stationary density function Ps(x). In such a circumstance we say that X(t) is a stable process (Gillespie, 1992). The stationary density function implies that a measurement of the process at any significantly large time t will be equivalent to sampling a random variable whose density function does not depend upon time. In the following computation of the steady state density function using the Fokker Planck equation (Eq. 8) is described.

5.2.3 Steady State Density Computation

There can be basically two approaches to find the steady state density of the variable of interest using the Fokker Planck equation. The first one is to explicitly solve the Fokker Planck equation (Eq. 8) for $p(x,t|x_0,t_0)$ using the proper drift and diffusion functions and then compute the limit $(t-t_0) \rightarrow \infty$; that is

$$Ps(x) = \lim_{(t-t0) \rightarrow \infty} p(x,t|x_0,t_0) \tag{10}$$

Another approach is to put the time derivative in the Fokker Planck equation (Eq. 8) equal to zero and then solve the resulting ordinary differential equation. This technique is discussed in the following. Putting time derivative equal to zero in Eq. 8, we get

$$-\frac{d}{dx}[A(x)Ps(x)] + \frac{1}{2}\frac{d^2}{dx^2}[D(x)Ps(x)] = 0 \tag{11}$$

Eq. 11 is an ordinary differential equation for Ps(x) and is relatively easier to solve than the partial differential Eq. 8. To solve Eq. 11 we write

$$\frac{d}{dx}[-A(x)Ps(x) + \frac{1}{2}\frac{d}{dx}(D(x)Ps(x))] = 0 \tag{12}$$

For Eq. 12 to be true, the quantity within the braces must be a constant with respect to x. Assuming as x approaches boundaries of the process, Ps(x) approaches zero and so the quantity in the braces in Eq. 12 should be zero at the steady state. That is

$$-A(x)Ps(x) + \frac{1}{2}\frac{d}{dx}(D(x)Ps(x)) = 0$$

$$\frac{d}{dx}(D(x)Ps(x)) = 2A(x)Ps(x)$$

$$\frac{d(D(x)Ps(x))}{D(x)Ps(x)} = \frac{2A(x)Ps(x)}{D(x)Ps(x)} \tag{13}$$

Integrating Eq. 13 we get

$$\ln (D(x)\, Ps(x)) = \int \frac{2A(x)}{D(x)} dx + \text{constant} \tag{14}$$

or; $$D(x)\,Ps(x) = C \exp\left(\int \frac{2A(x)}{D(x)} dx\right) \tag{15}$$

The equation for Ps(x) is then

$$Ps(x) = \frac{2K'}{D(x)} \exp(-\phi(x)) \tag{16}$$

where : $$\phi(x) = -\int_{\infty}^{x} \frac{2A(x)}{D(x)} dx \tag{17}$$

and : $$\frac{1}{K'} = \int_{-\infty}^{\infty} \frac{2}{D(x)} \exp(-\phi(x)) dx \tag{18}$$

It can be observed that if functional forms of A(x) and D(x) functions are known, the steady state density function for a temporally homogeneous process of interest can be evaluated using Eq. 16. In above relationships, the $\phi(x)$ is called the potential function and K' is the normalizing constant. It is observed from Eq. 16 that the Ps(x) will exist only if K' is defined as a finite, positive number.

5.3 DROUGHT MODELING USING PHDI

5.3.1 Distribution of the Palmer Drought Index

The computation procedure of the Palmer index is an involved one (see Appendix I for details). While commenting on the ARMA representation of the Palmer index, Alley (1984) pointed out that the switching among the temporary indices X_1, X_2, and X_3, one of which becomes the PDSI, (see Appendix I for definitions) during the computations might cause certain problems. In particular, for an established drought with $X(i)=X_3(i)$, the PDSI for the following month, $X(i+1)$, may be either $X_3(i+1)$ or $X_1(i+1)$. If set to $X_3(i+1)$, then $X(i+1)$ will be computed using $X_3(i)$ as per Palmer equation and will probably not deviate much from $X(i)$. On the other hand, if set to $X_1(i+1)$, then $X(i+1)$ will be positive and will be much different from $X(i)$. Similar results occur for established wet spells. In view of these Alley (1984, 1985) concluded that the conditional distribution of X(i+1) given X(i) tends to be bi-modal during periods of established droughts or wet spells and showed the same with the PDSI data of New Jersey and Iowa. Eder et al., (1987) reported unconditional bi-modal distribution of PDSI data over south-eastern United States. Similarly, Heddinghaus and Sabol (1991) reported unconditional bi-modal distribution of PDSI for 9 climatic divisions in Iowa. In view of the philosophy of the index computation procedure, the bi-modality in conditional distribution of the index seems more logical as compared to the marginal distribution. In order to verify this claim, conditional distribution of PHDI data for selected climatic divisions in AZ, CA and VA are examined. It is found that the conditional distribution of $PHDI_{i+1}$ is bi-modal only when $PHDI_i$ belonged to class 4. The dot plots of data are shown in Figures 5.1(a, b, and c) wherein bi-modality of $PHDI_{i+1}| PHDI_i \in$ class 4 is quite obvious. Figures 5.2(a, b, and c) show the relative frequency histograms of $PHDI_{i+1}| PHDI_i \in$ class 4 in respect of each state. Figures 5.3 and 5.4 show dot plots of $PHDI_{i+1}$ when $PHDI_i$ is in class 2 and of $PHDI_{i+1}$ when $PHDI_i$ is in class 6, respectively. It is observed that these plots do not exhibit distinct bi-modality and are rather close to unimodal type distribution. The relative frequency histograms are shown in Figures 5.5(a through c) and 5.6(a through c) which further support the claim made earlier. Based on these results it is concluded that the current month PHDI follows a bi-modal distribution only when the previous month it is in class 4. This seems to agree with the intuition that it is only when the weather is in normal state (class 4), it has a tendency to go to either a dry or a wet state in next time step.

5.3.2 Modeling of Droughts

Conventionally, the PHDI is computed on a monthly / weekly basis. However, to meaningfully define an intensity of dryness or wetness, we should be able to assess the changes (displacements) from a particular drought /wet value over infinitesimal periods. The Palmer index equation which is used for computing PHDI is

$$PHDI_{t+1} = .897\ PHDI_t + Z_{t+1}/3 \qquad (19)$$

where : $PHDI_{t+1}$= PHDI at time t+1

Z_{t+1} = Palmer's Z index which represents moisture anomaly conditions

For small time step dt, the Eq. 19 can be written as :

$$PHDI_{t+dt} - PHDI_t = -.103\ PHDI_t + Z_{t+dt}/3 \qquad (20)$$

or;

$$dX_t = -.103\ X_t + Z_{t+dt}/3 \qquad (21)$$

where: dX_t = total change in the PHDI value as dt goes to zero.

The total change in the PHDI value in the small time interval dt can be assumed to be composed of two components, namely , i) deterministic component, ii) stochastic component. The first term in the right hand side of Eq. 21 is a deterministic term at time (t+dt) and the second term which calls for the precipitation deficit between the current period t, and the future period , t+dt, is a random term . Therefore, an equivalent discrete form of Palmer equation (Eq. 19) can be written as :

$$X_t - a X_{t-1} = Y_t \ , t = 0,\ 1,\ 2,\ 3, \ldots \qquad (22)$$

where : X_t = Drought process as represented by the PHDI

Y_t = pure random process with variance σ_y^2 and represents term $Z_t/3$ term

a = correlation coefficient between X_t and X_{t-1}

Now we can obtain continuous analogue of Eq. 22 for representing the drought process as a continuos Markov process using PHDI. Eq. 22 can be written as :

$$X_t - a X_t + a X_t - a X_{t-1} = Y_t$$

or;

$$X_t (1 - a) + a (X_t - X_{t-1}) = Y_t$$

or;

$$(\frac{1-a}{a})\ X_t + \Delta X_t = \frac{Yt}{a} \qquad (23)$$

dividing by Δt both sides we get

$$\frac{\Delta Xt}{\Delta t} + \frac{(1-a)}{a}\frac{Xt}{\Delta t} = \frac{Yt}{a\Delta t} \qquad (24)$$

Taking $\Delta t \to 0$;

$$\frac{d\,Xt}{dt} + \beta Xt = \varepsilon(t) \qquad (25)$$

where: $$\beta = \frac{1-a}{a\Delta t}; \varepsilon(t) = \frac{Yt}{a\Delta t}$$

Eq. 24 is the continuous analogue to the discrete Palmer drought index equation and is similar to the stochastic differential Eq. 5. As we take limit $\Delta t \rightarrow 0$ in Eq. 24, the parameter β is assumed to remain finite. For this to be true , we require 'a' to approach unity as Δt approaches its limit zero. In other words the correlation between X_t and X_{t-1} is assumed to increase as dt tends to very small number. This is close to what happens in reality as we expect PHDI values to be correlated strongly if computed on shorter interval. As an illustration the variation in correlation coefficient with time interval of 1, 2, 3,...,12 months as computed using PHDI monthly data (1895-1992) for California is given in Table 5.1 which agrees with the stated assumption.

Another way to develop the stochastic differential equation representing the PHDI process is to relate the instantaneous rate of change of PHDI with the various factors which are responsible for such a change. For example, rate of change of PHDI can be related to hydro-meteorologic variables like precipitation, evapotranspiration, runoff, and infiltration through some parametric relationships. These relationships can be developed using empirical studies which relate PHDI to these variables.

Eq. 25 can also be written in general form as

$$dX_t = (f(x) + F) dt \tag{26}$$

where : f(x) is the deterministic component and F is the random component.

Because PHDI is a measure of the cumulative deficit/surplus of supply of moisture , the function f(x) will indicate average rate of change of cumulative deficit/surplus for instantaneous times. Therefore, f(x) during time interval dt will depend upon PHDI value as experienced at time t. This is quite obvious in Palmer's equation (Eq. 19) wherein PHDI at one time step later includes 89.7% of the PHDI as experienced during previous time step. However, this functional relationship (f(x) vs. x) has to be non-linear to account for the multimodal behavior of the PHDI distribution. The analysis of PHDI, as described by Eq. 25, requires the drift and diffusion functions. The following section contains the appropriate results.

5.3.3(a) Computation of the Drift and Diffusion Functions

The time evolution of the drought process Xt, which is assumed temporally homogeneous and is represented by PHDI, is completely determined by the forms of its two characterizing functions A(x) and D(x). The functional forms of these functions will dictate the steady state as well as the time varying density functions of the drought process. Therefore, determination of these functions plays a crucial role in analyzing the drought process Xt using the Fokker Planck equation. Ideally the functional forms of A(x) and D(x) functions should be determined based on physical considerations concerning the dynamics of the PHDI process. In the present case we have chosen to use the long term PHDI data for the purpose and adopted the following procedure to obtain the A(x) and D(x) functions for the drought process.

The propagator random variable for a continuous Markov process is described in Eq. 9 as

$$K = X_{t+dt} - X_t \quad \text{given } X_t = x$$

We know from Appendix V

$$E[K] = E[X_{t+dt} - X_t \mid X_t = x] = A(x)\,dt \qquad (27)$$

$$Var[K] = Var[X_{t+dt} - X_t \mid X_t = x] = D(x)\,dt \qquad (28)$$

Assuming dt=1.0, Eqs. 27 and 28 become:

$$E[K] = E[X_{t+1} - X_t \mid X_t = x] = A(x) \qquad (29)$$

$$Var[K] = Var[X_{t+1} - X_t \mid X_t = x] = D(x) \qquad (30)$$

We use Eqs. 29 and 30 to fit A(x) and D(x) functions using observed data of PHDI. As described in section 5.3.1, the distribution of $PHDI_{i+1} \mid PHDI_i \in$ class 4 is observed bi-modal while for $PHDI_{i+1} \mid PHDI_i \notin$ class 4 the distribution is observed unimodal. Since A(x) and D(x) functions affect the shape of the distribution, we will fit two separate sets of functions to model the bi-modal and unimodal distributions. We first discuss the procedure to fit A(x) and D(x) function for modeling $PHDI_{i+1} \mid PHDI_i \in$ class 4.

5.3.1.1 A(x) and D(x) Functions for $PHDI_{i+1} \mid PHDI_i \in$ class 4

Figures 5.1(b) and 5.2(b) show the dot plot and relative frequency histogram of $PHDI_{i+1} \mid PHDI_i \in$ class 4, respectively for the selected climatic division in California. In order to suitably represent the empirically observed bi-modal behavior as shown in these figures the drift function A(x) of the process should exihibit the seesaw behavior. In other words, A(x) should be non-linear function of x and should slow down and initiate process decay when PHDI reaches its extreme ends. The following form of A(x) function is adopted for the selected climatic division in California

$$A(x) = -x^3 - .2\,x^2 + 1.2\,x \tag{31}$$

which has the appropriate stable points as defined by the empirical conditional relative frequency histogram.

Having found the A(x) function we now use Eq. 30 to fit the D(x) function. For this purpose variance of $PHDI_{i+1} - PHDI_i \mid PHDI_i \in$ class 4 is computed using PHDI data from 1895-1992. The result turned out to be 1.4. Therefore, a constant function for D(x) is chosen :

$$D(x) = D = 1.4 \tag{32}$$

Eqs. 31 and 32 give the A(x) and D(x) functions for the PHDI process for (i+1)th month in California while PHDI is in class 4 range during ith month. Using Eq. 17, the potential function can now be computed as

$$\phi(x) = -\int^x \frac{2A(x)}{D(x)}\,dx \tag{33}$$

Putting A(x) and D(x) functional forms in Eq. 33, we get

$$\phi(x) = -\frac{2}{1.40}\int^x (-x^3 - .2x^2 + 1.2x)\,dx$$

or; $$\phi(x) = -\frac{2}{1.40}[\,-.25x^4 - .066x^3 + .6x^2] \tag{34}$$

Further, we can use Eq. 18 to compute normalizing constant K' as

$$\frac{1}{K'} = \int_{x1}^{x2} \frac{2}{1.40} \exp\,(-\phi(x))\; dx \tag{35}$$

The value of K' in Eq. 35 is evaluated by numerical integration between the limits of variation of PHDI which is taken to vary from -4.5 to 4.5. The result is

$$K' = .149 \tag{36}$$

The value of K' is finite and positive. Therefore, the steady state density of PHDI process should exist which is given by Eq. 16. The functional form of Ps(x) is then

$$Ps(x) = \frac{2x.149}{1.40} \exp\,((-\phi(x))$$

or; $$Ps(x) = .213 \exp\,(.36\,x^4 - .095\,x^3 + .86\,x^2\,) \tag{37}$$

Further, $\int_{a1}^{b1} Ps(x) = 1.0$ and $Ps(x) >= 0$ for $a1 <= x <= b1$; a1=-4.5, b1=4.5 which ensures that Ps(x) is a density function.

Figure 5.7 shows comparison of the density function as given by Eq. 37 with the empirical density. The empirical density function is developed using a conditional histogram as shown in Fig. 5.2(b). It is seen that the computed density function agrees well with the empirical density. Both density functions show bi-modality and have similar mode values.

The Ps(x) function as given by Eq. 37 can be analyzed for maximum and minimum points as below:

$$\frac{dP_s(x)}{dx} = .213\,(-1.44\,x^3 - .284\,x^2 + 1.72\,x) \exp\,(.36\,x^4 - .095\,x^3 + .86\,x^2\,) \tag{38}$$

It can be seen that $\frac{dP_s(x)}{dx} = 0$ for x = -1.2, 0, and 1.0. It can be further seen that

$$\frac{d^2Ps(x)}{dx^2} < 0 \text{ for } x = -1.2 \text{ and } 1.0 \text{ ; and } \frac{d^2Ps(x)}{dx^2} > 0 \text{ for } x = 0 \quad (39)$$

Therefore, x=-1.2 and 1.0 are two maxima of Ps(x) function and x=0 belongs to minima of the Ps(x) function. Since Ps(x) is a density function the maxima of the function are the modes of the distribution. Therefore, the values of PHDI=-1.2 and 1.0 can be said to be most likely states (stable points) of the weather. Having modeled the bi-modal distribution, we now apply the approach to model unimodal distribution.

5.3.1.2 A(x) and D(x) Functions for $PHDI_{i+1} \mid PHDI_i \notin$ class 4

It has been discussed in section 5.3.1 and shown in Figures 5.3 through 5.6 that if PHDI during ith month is not in class 4, then PHDI during the (i+1)th month follows a unimodal distribution. We now discuss a procedure to fit A(x) and D(x) functions for such a case. We have again selected the climatic division of California for demonstrating the approach to fit characterizing functions for $PHDI_{i+1} \mid PHDI_i \in$ class 6. As can be seen from Figures 5.4(b) and 5.6(b) that a PHDI value between -3.0 and -4.0 is the mode of the distribution. Therefore, we select the following form of function to represent the A(x) function

$$A(x) = -x - 3.31 \quad (40)$$

This function tries to bring the process close to -3.31 value while the process drifts far away from it. For D(x) function we computed variance of $PHDI_{i+1}\text{-}PHDI_i \mid PHDI_i \in$ class 6 using the PHDI data from 1895-1992. The result is

$$D(x) = D = 0.36 \quad (41)$$

Eqs. 40 and 41 give the A(x) and D(x) functions for the PHDI process during (i+1)th month while during ith month it is in class 6.

Using Eq. 17, the potential function $\phi(x)$ is computed as

$$\phi(x) = 2.78\, x^2 + 18.4\, x \quad (42)$$

Similarly, using Eq. 18 the normalizing constant K' is computed as

$$K' = .1027 \times 10^{-13} \quad (43)$$

It may be noted that the value of K' is computed by numerical integration between the observed limits of -5 to -1. The K' value is finite and positive and therefore, the steady stath function will exist and is given by Eq. 16 as

$$Ps(x) = (2x.1027x10^{-13})/.36\ [\exp(-2.78\ x^2 - 18.4\ x)] \quad (44)$$

Figure 5.8 gives comparison of empirical and computed density functions. A good match between empirical and computed density functions is observed in Figure 5.8.

The Ps(x) function as given by Eq. 44 can be analyzed for maximum and minimum points as below

$$Ps(x) = .571 \times 10^{-13} \exp[-2.78\ x^2 - 18.4\ x]$$

It is checked that $\int_{a1}^{b1} Ps(x) = 1.0$ and $Ps(x) >= 0$ for $a1 <= x <= b1$; a1=-1.0, b1=-5.0 which ensures that Ps(x) is a density function.

$$\frac{dP_s(x)}{dx} = .571\ (-5.56\ x - 18.4) \exp(-2.78\ x^2 - 18.4\ x) \quad (45)$$

It can be seen that $\frac{dP_s(x)}{dx}|_{x=-3.31} = 0$. Further, $\frac{d^2P_s(x)}{dx^2}|_{x=-3.31} < 0$.

The analysis fits bi-modal / unimodal densities of PHDI and results agree well with the empirical data. Using these density functions the probability of the current month's PHDI being in a certain range can be computed given the PHDI of the previous month. An example problem showing such an application follows.

EXAMPLE : During a particular month the PHDI value of -3.5 is recorded in the San Jaoquin division, California. Using the classification of weather given in Table 4.1 of Chapter IV, compute the following :

a) Probability that PHDI will be in between -3.0 and -1.5 during the following month.

b) Can we use Markov chain approach for solving this problem ?

c) What will be the probability of PHDI being less than -1.5 during (i+1)th month if its value during ith month is -1.1.

Solution : We will use the density functions developed in above sections to solve the problem. The stepwise solution procedure follows :

a) Step 1 : Referring to Table 4.1 in Chapter IV, we classify PHDI value of -3.5 to fall in class 6. In section 5.3.1.2 we found that if PHDI is in class 6 in month 'i', then the steady state density function for 'i+1' month will be given by

$$Ps(x) = .571x\ 10^{-13} \exp [-2.78\ x^2 - 18.4\ x]\ ;\quad -5 \leq x \leq -1$$

Step 2 : Use Ps(x) function to compute probability of PHDI being in between -3.0 and -1.5 as

$$P [-3.0 \leq x \leq -1.5] = \int_{-3}^{-1.5} Ps(x)\, dx$$

$$= \int_{-3}^{-1.5} .571x\ 10^{-13} \exp [-2.78\ x^2 - 18.4\ x] = .231 \text{ or } 23.1\ \%$$

b) Step 1 : We can use the mean monthly matrix as described in chapter IV. We know from step 1 of part (a) that PHDI is in class 6 during ith month. The range of PHDI specified for i+1th month is for class 5 as given in Table 4.1 of Chapter IV. Therefore, the required probability will be one step transition probability from class 6 to class 5 in the mean monthly matrix for California given in Appendix IV. The result is

$$p_{65}^{i,i+1} = .2597 \text{ or } 25.97\ \%$$

It may, however, be noted that we can not compute probability of PHDI being in any range in month (i+1) using Markov chain approach unless that range is used as one of the classes in the Markov chain analysis.

c) Step 1 : Given that PHDI during ith month is -1.1 which falls in class 4. We will use the bi-modal density function in section 5.3.1.1 to compute the probability

$$Ps(x) = .213 \exp (.36\ x^4 - .095\ x^3 + .86\ x^2)\ ; -4.5 \leq x \leq 4.5$$

Step 2 : Compute probabilty that PHDIduring (i+1)th month will be less than -1.49

$$P\,[\,PHDI_{i+1} < -1.49\,] = \int_{-4.5}^{-1.49} Ps(x)\,dx$$

$$= \int_{-4.5}^{-1.49} .213 \exp(.36\,x^4 - .095\,x^3 + .86\,x^2) = .098 \text{ or } 9.8$$

This example demonstated that using the current month's PHDI we can compute the probability of PHDI falling in any range in the following month.

5.3.3(b) Effects of Fluctuating Force on Steady State Distribution

In this section the significance of the forcing term F is analyzed. Consider Eq. 26 given by

$$dX_t = (\,f(x) + F\,)\,dt$$

Taking conditional variance

$$Var\,(\,dX_t \mid X_t{=}x\,) = dt^2\,Var\,(\,F\,) \tag{46}$$

or;

$$D(x)\,dt = dt^2\,Var\,(\,F\,)$$

or;

$$D(x) = dt\,Var\,(\,F\,) \tag{47}$$

Eq. 47 indicates that the diffusion function D(x) of the process is dictated by the variance of the fluctuating force F. Now since D(x) affects the time varying and steady state density functions of the process, variance of the fluctuating force should also be responsible for determining the shape of the steady state and time varying density function. In order to evaluate such effects, the values of D(x) function are varied in computations of steady state density. The varying values of D(x) simulate varying strength of the fluctuating force. The effects of D(x) function on bi-modal and unimodal density functions are discussed separately.

5.3.3.1 Effects of D(x) on Steady State Distribution of $PHDI_{i+1} \mid PHDI_i \in$ class 4

Four different values of D(x)= D (i.e. .05, 1.4, 10, and 100) are used and the resulting steady state density functions are shown in Figure 5.9. It is seen that with low values of D(x) function, meaning less variability in external force, unimodal density function is obtained. Once D(x) function assumes a certain value then bi-modality of distribution is observed. For very high values of the D(x) a rather flat steady state density function is observed. This indicates that with small strength of external force the process has a tendency to stay around one of the stable modes of the distribution. With increase in strength of fluctuation force the process has a tendency to visit two or more stable states. The transition between the states can therefore be attributed to fluctuating force strength.

5.3.3.2 Effects of D(x) on Steady State Distribution of $PHDI_{i+1} \mid PHDI_i \notin$ class 4

Four different values of D(x) = D (i.e. .1, .36, 1.0, and 100.0) are used to compute the steady state density function. The results are shown in Figure 5.10. It is seen that with low values of D(x) the process stays close to the mode value. The density function spreads out with increasing value of D(x). However, unlike earlier case here the density function remains unimodal. With D(x) approaching very high value a flat densty function is observed.

5.3.4 Computation of Time Varying Density

A procedure to obtain A(x) and D(x) functions for temporally homogeneous PHDI process (X_t) has been described in section 5.3.3. Using these characterizing functions, the steady state density functions of the PHDI process have been computed for the following two cases : i) $PHDI_{i+1} \mid PHDI_i \in$ class 4, and ii) $PHDI_{i+1} \mid PHDI_i \in$ class 6. The A(x) and D(x) functions obtained for these two cases are

$A(x) = -x^3 - .2x^2 + 1.2x$; $D(x) = 1.4$ for $PHDI_{i+1} \mid PHDI_i \in$ class 4

and ; $A(x) = -x - 3.31$; $D(x) = .36$ for $PHDI_{i+1} \mid PHDI_i \in$ class 6

Using the Fokker Planck equation (Eq. 8) we can write the time derivative of the probability density function of the process $PHDI_{i+1} \mid PHDI_i \in$ class 6 as

$$\frac{\partial p(x,t|x_0,t_0)}{\partial t} = -\frac{\partial}{\partial x}(-x-3.31)p + \frac{1}{2}\frac{\partial^2}{\partial x^2}(.36)p \tag{48}$$

Similarly, for the case $PHDI_{i+1} \mid PHDI_i \in$ class 4 the time derivative of the probability density function is written as

$$\frac{\partial p(x,t|x_0,t_0)}{\partial t} = -\frac{\partial}{\partial x}(-x^3 - .2x^2 + 1.2x)p + \frac{1}{2}\frac{\partial^2}{\partial x^2}(1.4)p \tag{49}$$

The time varying density of the PHDI process $p(x,t|x_0,t_0)$ can be obtained by solving Eqs. 48 and 49. As discussed in section 5.2.2, the closed form solution of the Fokker Planck equations can be obtained for linear A(x) and constant D(x) functions. While the latter condition is met in both the cases the former condition is met only in the case of $PHDI_{i+1} \mid PHDI_i \in$ class 6. Therefore, analytical solution can be obtained for Eq. 48 only. In case of Eq. 49 A(x) function can be described using combination of linear segments which can then be used to obtain a solution close to its analytical solution.

5.3.4.1 Time Varying Density of $PHDI_{i+1} \mid PHDI_i \in$ class 6 process

The characterizing functions for the process $PHDI_{i+1} \mid PHDI_i \in$ class 6 are given as

$A(x) = -x - 3.31$; $D(x) = D = .36$

The corresponding Fokker Planck equation is given by

$$\frac{\partial p(x,t|x_0,t_0)}{\partial t} = -\frac{\partial}{\partial x}(-x-3.31)p + \frac{1}{2}\frac{\partial^2}{\partial x^2}(.36)p \tag{51}$$

The analytical solution of Eq. 51 is given as

$$p(x,t|x_0,t_0) = [\pi D(1-e^{-2(t-t_0)})]^{-0.5} e^{\left[-\frac{(x-(-3.31+x_0e^{-(t-t_0)}))^2}{D(1-e^{-2(t-t_0)})}\right]} \tag{52}$$

Putting D = 0.36 we get

$$p(x,t|x_0,t_0) = [1.13(1-e^{-2(t-t_0)})]^{-0.5} e^{[-\frac{(x-(-3.31+x_0e^{-(t-t_0)}))^2}{.36(1-e^{-2(t-t_0)})}]} \quad (53)$$

Using Eq. 53 with known values of x_0 at time t_0, the probability density function of the process X_t at time t can be computed.

For the situation when (t-t_0) → ∝, the time varying probability density function given by Eq. 53 should yield the steady state density function given by Eq. 44. In order to examine this take limit (t-t_0) → ∝ in Eq. 53 and we get

$$Ps(x) = _{(t-t0)\rightarrow\propto} p(x,t|x_0,t_0) = [1.13]^{-.5} \exp [(-(x+3.31)^2)/.36]$$

or; $$Ps(x) = .57 \times 10^{-13} \exp [-2.78x^2 - 18.4 x] \quad (54)$$

It is seen that the steady state density function (Ps(x)) computed using the time varying density function ($P(x,t|x_0,t_0)$ is the same as in Eq. 44.

We will now examine how the time varying density can be used in adaptive forecast. A careful examination of Eq. 53 indicates that $PHDI_{i+1}|PHDI_i \in$ class 6 is the normal random variable with mean -3.31 + x_0 exp(-(t-t_0)) and variance .18(1-exp(-2(t-t_0)).

Assuming the interval t-t_0 to be unity we get

$$p(x,t \mid x_0,t_0) = \text{Density function of } N [-3.31 + .37 x_0 , .16] \quad (55)$$

It is seen that the time dependent density function changes with x_0. Suppose x_0 = -3.0 (a value within class 6) at time t=t_0 , then the probability density function of PHDI process at t=t+1 will be

$$p(x,t+1 \mid x_0 = -3, t_0) = \text{Density function of } N [-4.42, .16] \quad (56)$$

Having the pdf of the process known the probabilities of events involving varying values of PHDI can be obtained contingent on already realized value of the index. In this way every time a new value is realized the probability density function gets updated accordingly, a mechanism which can not be introduced in the Markov chain procedure.

5.3.4.2 Time Varying Density of $PHDI_{i+1}$ | $PHDI_i \in$ class 4

The time varying probability density function for $PHDI_{i+1}$ | $PHDI_i \in$ class 4 process can not be computed analytically due to the non-linear nature of the A(x) function. One possible way to find the solution is to linearize the function within its range of variation. For each linearized section the analytical solution can be obtained using the procedure descibed in the above section. For example if we represent the third order polynomial (A(x)) by two linear functions we can use the analysis given in earlier section to fit normal density for each of these function and when combined together two normal distributions can yield bi-modal density function similar to the empirical results.

We have discussed in section 5.3.3 how an external force can affect the density function of a drought characterizing variable like the Palmer index. In the following section a brief description of a global phenomenon, called the ENSO, which is known to affect drought events in various parts of the world is discussed alongwith its relationship to the Palmer index values in the study region.

5.4 EL NINO SOUTHERN OSCILLATION (ENSO)

ENSO is an acronym which combines two phenomena: the El Nino (oceanic) and the Southern Oscillation (atmospheric). The term El Nino (the child Jesus) referred originally to a relatively weak and warm southward oceanic current that develops almost annually along the coast of southern Ecuador and northern Peru around Christmas (Philander, 1990) The term Southern Oscillation (SO) was proposed by Walker and Bliss (1932) to identify a global scale phenomenon characterized by, among other features, a seesaw in the atmospheric pressure field difference between the eastern and western tropical Pacific. A measure of the state of the SO is defined by Southern Oscillation Index (SOI), which is based on standardized sea level pressure difference between Tahiti (17^0S, 150^0 W) and Darwin (12^0S 131^0E). Figure 5.11 shows nine month running mean of monthly sea level pressure anomalies at Darwin and at Tahiti during period of Jan. 1935-May 1991. The seesaw behavior is quite obvious in the figure. The data shows a correlation of -.75. While the pressure oscillation associated with the SO has been known for about a century, it was not until late 1950s and the 1960s that the connection between the SO and the sea surface temperature (SST) in the western tropical Pacific was demonstrated (Berlage, 1957; Bjerknes, 1966, 1969). Generally, SST along the western coast of south America and along the equator in the central and eastern Pacific is anomalously cold for its latitude. These cool SSTs stabilize the lower atmosphere, inhibiting precipitation and giving rise to the hyperarid climate of coastal Peru. This aridity extends well into the central Pacific, occupying a wedge shaped region that extends as far west as the dateline and southeastward from this general vicinity to about the latitude of Santiago, Chile, along the South American coast. This region is referred to as the Pacific 'dry zone'. Every few years this aridity is broken by periodic heavy rainfall episodes lasting several months associated with a dramatic increase of equatorial Pacific SST. This phenomenon, which occurs every 2-7 years, is called El Nino.

During an El Nino event Ecuador and northern Peru are particularly susceptible to flooding. The effects of an El Nino event are actually felt worldwide ranging from flooding to drought. Table 5.2 gives description of such effects as experienced during the major El Nino event of 1982-83. The El Nino is sometimes also called the warm event referring to relatively warm sea surface temperature along equatorial Pacific. The non-El Nino period is then characterized as the cold event which is called La Nina (Philander, 1990). Various studies have been reported evaluating the effects of a La Nina event. The drought of 1988 in the U.S. is attributed to a La Nina event (Trenberth and Brantator,1992). A number of researchers have tried to identify the occurrences of warm and cold events in the past several years (Quinn, 1992; Kiladis and Diaz, 1989). The phenomenon of ENSO is considered to describe both the warm (El Nino) and the cold (La Nina) events. During an El Nino event the southern oscillation (SO) has negative phase while during a LaNina event SO has positive phase. In other words, during an El

Nino event the SOI is low negative as a result of reduced westward pressure gradient over the equatorial Pacific. On the other hand the La Nina event is characterized by a high positive SOI caused due to increased westward pressure gradient over the equatorial Pacific. In general ENSO cycle describes the evolution of coupled ocean-atmosphere system in which El Nino is one phase and La Nina is the other phase.

Various researchers have evaluated tropical and extratropical response to the ENSO events. Ropelewski and Halpert (1986, 1987, 1989) and Kiladis and Diaz (1989) have identified areas of the world where the ENSO events are related to the mean temperature and the precipitation anomalies. Strong increases in precipitation are experienced during an El Nino event over the central Pacific , in the narrow coastal zone of Ecuador and Peru, southeastern USA, over a region south of India and over eastern equatorial Africa. During the same El Nino event, rainfall gets reduced in tropics over the western tropical pacific ocean, Indonesia, Australia, India, southeastern Africa and northeastern South America (Peixoto and Oort, 1992). Woolhiser et al., (1993) reported a three month lag between SOI and precipitation in the western USA and suggested the SOI as a precursor for daily rainfall prediction. The ENSO events are also related to streamflow responses in different parts of the world (Simpson et al., 1993; Dracup and Kahya, 1993, Cayan and Webb, 1992). A number of indices have been developed to characterize an ENSO event which are based on number of variables like pressure, rainfall and sea surface temperature in tropical Pacific region. The following section gives a brief description of such indices.

5.4.1 Indices of ENSO Phenomenon

5.4.1.1 Pressure Indices

A popular way to characterize the southern oscillation phenomenon, the atmospheric component of ENSO, is to use the atmospheric pressure at Tahiti and Darwin. The Tahiti-Darwin SO index has been computed in several different ways by different investigators (Chen, 1982; Trenberth, 1984). Wright (1984) has noted that the mean pressure anomaly at Darwin only has also been used as an index of SO. In all such indices, the annual cycle of pressure at each station is removed by forming anomalies , or differences , from the long term monthly averages. These monthly anomalies are then normalized by the appropriate monthly standard deviations to produce standardized values; zero mean and unit variance. The 1951-80 period is used to as the base period for computations of mean and standard deviations. The three forms of Tahiti-Darwin indices as reported by Ropelewski and Jones (1987) are as given below:

A) CAC (Climatic Analysis Center) Version of Tahiti-Drawin Index : In this form of SO index , the difference of the standardized values (standardized Tahiti-standardized Drawin) is itself standardized.

B) Trenberth (1976) Tahiti-Darwin Index : In this form of index, the Tahiti and Darwin pressure anomalies are separately normalized by the mean of the 12 month standard deviations. This index does not have standard deviation of 1.0.

C) Troup's Tahiti-Darwin Index : In this form the anomalies of monthly pressure differences, Tahiti minus Darwin, are standardized by the standard deviation of Tahiti minus Drawin series.

Roplewski and Jones (1987) report availability of Tahiti-Drawin monthly S.O index for a period ranging from 1882-1986. Wright (1989) has given Tahiti-Darwin index, averaged over 3 months, for a period of 1851-1985 which is reproduced in Appendix VII for the period 1895- 1984. Pressure indices are generally criticized for high month to month variability and are only useful when smoothed in time or averaged over seasons. Pressure series also suffer from changes in station location, instrument or time of observation, or inhomogeneities due to having to combine from different stations over different periods (Wright, 1975).

5.4.1.2 Sea Surface Temperature Index

In order to define an index of SO based on SST, a 'core region' in the central and eastern equatorial Pacific is defined based on studies of Weare (1986). The data used to compute the index were monthly anomalies in SST (relative to 1949-68) in 4^0 latitude by 10^0 latitude boxes. A provisional SST index as described by Wright (1984) is defined as the mean SST anomaly over all available 4^0x10^0 boxes in the region 6^0N - 6^0S , 180-90^0W. The time series of provisional index was correlated with SST time series in each 4^0x10^0 box and a new core region was defined based on areas showing high correlation values. Based on these new chosen boxes, SST index was defined as the anomaly over all chosen boxes. Wright (1989) has given monthly values of SST index from 1880-1986. Appendix VII contains monthly values of SST index for the period of 1895- 1984. As can be seen in the Appendix, for January 1898, the index value is -26 which means the SST in this region is -.26^0 C below average. For computing annual value of index, the author has taken average of monthly values from April through March. The SST index does not exhibit high monthly variability like the pressure indices and are therefore ralatively more useful. SST indices are, however, subject to inhomogeneities due to changes in method of measurement.

In order to evaluate the consistency in pressure and SST indices of ENSO, long term monthly values of these indices are correlated and the results are given in Table 5.3. It is observed that generally the indices are well correlated except in the month of December.

5.4.1.3 Rainfall Index

A core region (160^0 E - 150^0 W) close to the equator is used to compute mean of rainfall recorded over stations within the area, to be used as an index of the SO. Wright (1989) has reported monthly rainfall index values from 1893-1983 with some missing values. The author has studied the correlation among various indices (pressure, rainfall, and SST) and used the relationship to fill the missing values. The correlation between the rainfall and SST index is reported as 0.81 and the correlation between SST index and Tahiti-Darwin index is reported as .67. Wright (1989) also observed good correlation between SST and air temperature in many areas of the Pacific and described an air temperature index of SO defined in the same way as the SST index. However, values of this index are not reported.

5.5 PALMER DROUGHT INDEX AND ENSO INDICES

To assess the effects of ENSO events as reflected by the SST and SOI indices on Palmer's drought index, long term data are correlated. The SST and SO index values, as given by Wright (1989), are correlated with the PHDI values for the selected climatic divisions in Arizona, California and Virginia. For this purpose monthly values of all indices are chosen for the period of 1895-1984.

Table 5.4 gives the correlation between Palmer's index (PHDI) and the ENSO indices (SST and SOI) for three different regions in the USA. It is observed that there is no significant correlation between Palmer index and the ENSO indices. For example, the correlation between Palmer index in January month in Arizona and SST and SO indices is found to be .063 and .076, respectively which is insignificant. In general, the correlation coefficients between Palmer index and ENSO indices are found low. The correlation between consecutive differences ($PHDI_{Feb}$ - $PHDI_{Jan}$) and SST and SO indices was also performed but no case the correlation coefficient exceeded .3. The Z index data were also correlated with the SST and SO indices and once again no significant correlation was found. Based on these results no conclusive observations of ENSO effects as reflected by PHDI in the chosen areas of study can be made.

5.6 SUMMARY

The dynamical systems approach is used to explain the bi-modal conditional distribution of the Palmer index. The conditional time dependent and steady state density functions are developed for the Palmer index. The functional form of the drift and diffusion functions affect the shape of distribution. The time dependent density can be used to make the adaptive forecasts. The effects of external disturbances on the probability density function of Palmer index are analyzed. A brief description on the ENSO phenomenon along with the indices which are used to characterize the process is given. A correlation study between the long term data of ENSO indices and the Palmer index did not indicate significant correlation in the study region. However, the methodology is general and could be applied to other regions with significant correlation between the PDSI and ENSO indices.

Table 5.1 Variation in Correlation Coefficient between PHDI for Jan. and all other months

Mon.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Jan.	.899	.794	.695	.640	.582	.553	.536	.473	.422	.397	.423

Table 5.2 Major ENSO 1982-83 Effects (Source: NOAA, 1993)

Location	Phenomenon	Victims	Damage
United States			
1. Mountain and Pacific States	Storms	45 dead	$1.1 billion
2. Gulf States	Flooding	50 dead	$1.1 billion
3. Hawaii	Hurricane	1 dead	$230 million
4. Northeaster US	Storms	66 dead	---
5. Cuba	Flooding	15 dead	$170 million
6. Mexico- Central America	Drought	---	$600mmillion
7. Ecuador and Norhtern Peru	Flooding	600 dead	#650 million
8. Sothern Peru-Western Bolivia	Drought	---	$ 240 million
9. Southern Brazil, Northern Argentina, Eastern Paraguay	Flooding	170 dead 600,000 evacuated	$ 3 billion
10.Bolivia	Flooding	50 dead, 26000 homeless	$ 300 million
11. Tahiti	Hurricane	1 dead	$50 million
12. Australia	Drought, fires	71 dead	$2.5 billion
13. Indonesia	Drought	340 dead	$500 million
14. Philippines	Drought	---	$450 million
15. Southern China	Wet weather	600 dead	$600 milion
16. Southern India	Drought	---	$150 million
17. Middle East, chiefly Lebanon	Cold, snow	65 dead	$50 million
18. Southern Africa	Drought	Disease, Starvation	$1 billion
19.Norhtern Africa,Iber-ian Peninsula	Drought	---	$200 million
20. Western Europe	Flooding	25 dead	$200 milion

Table 5.3 Correlation Between Monthly SOI and SST Index of ENSO

Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
.822	.810	.708	.667	.671	.614	.684	.722	.761	.806	.838	-.013

Table 5.4 Correlation Between Monthly SOI and SST Indices And Palmer Index

	Sea Surface Temperature Index			Southern Oscillation Index		
	AZ	CA	VA	AZ	CA	VA
Jan.	.063	.141	-.057	.076	.131	.02
Feb	.112	.166	.064	.148	.172	.023
Mar	.190	.101	.058	.128	.068	-.039
Apr.	.216	.058	.135	.178	.077	.018
May	.191	.269	.028	.202	.136	-.028
Jun.	.102	.190	-.062	-.063	-.073	-.040
Jul.	-.052	.142	-.162	-.10	-.068	-.159
Aug.	-.136	.063	-.216	-.104	-.041	-.210
Sep.	.075	.167	-.226	-.005	.094	-.217
Oct.	.011	.078	-.200	.038	.124	-.229
Nov.	.112	.103	-.080	.101	.102	-.133
Dec.	.111	.062	-.077	.178	.272	-.090

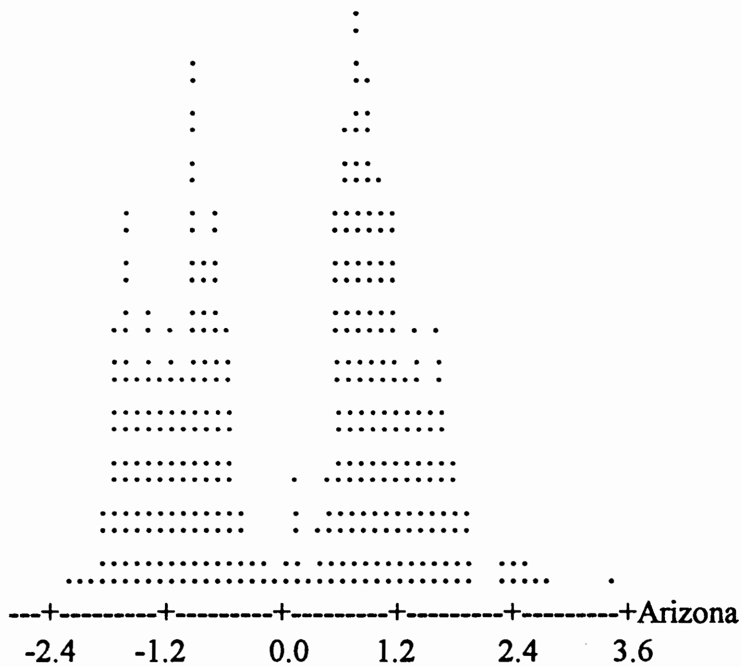


Figure 5.1(a) : Dot Plot of $PHDI_{i+1}$| $PHDI_i$ is in class 4

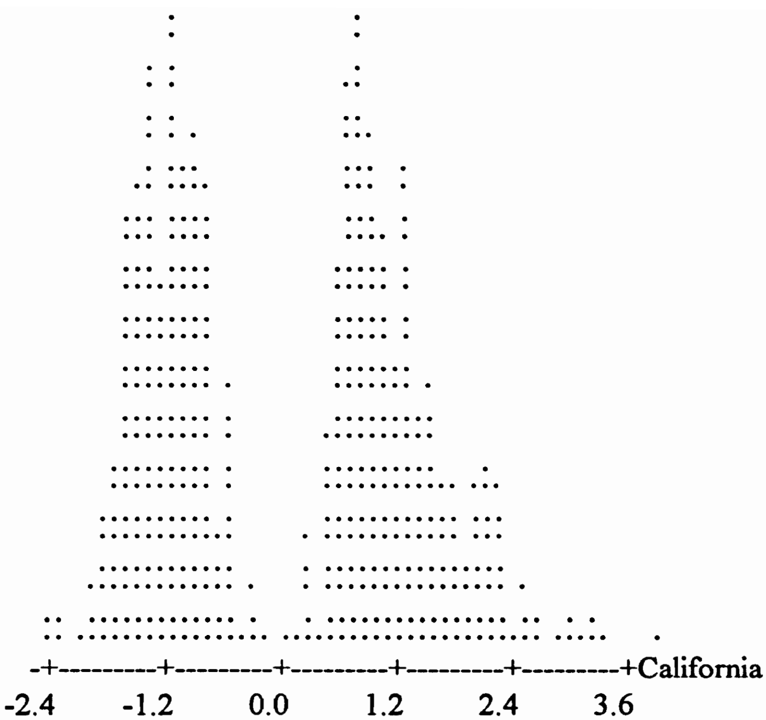


Figure 5.1(b) : Dot Plot of $PHDI_{i+1}$| $PHDI_i$ is in class 4

Each dot represents 2 points

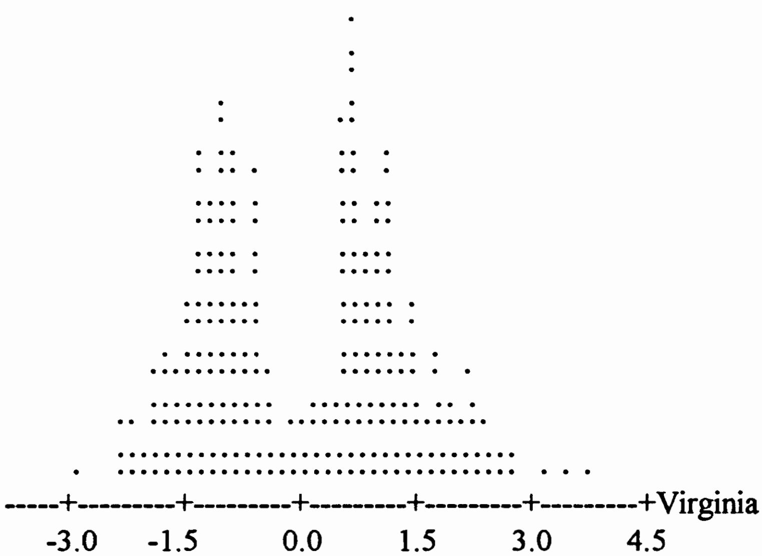


Figure 5.1(c) : Dot Plot of $PHDI_{i+1}$| $PHDI_i$ is in class 4

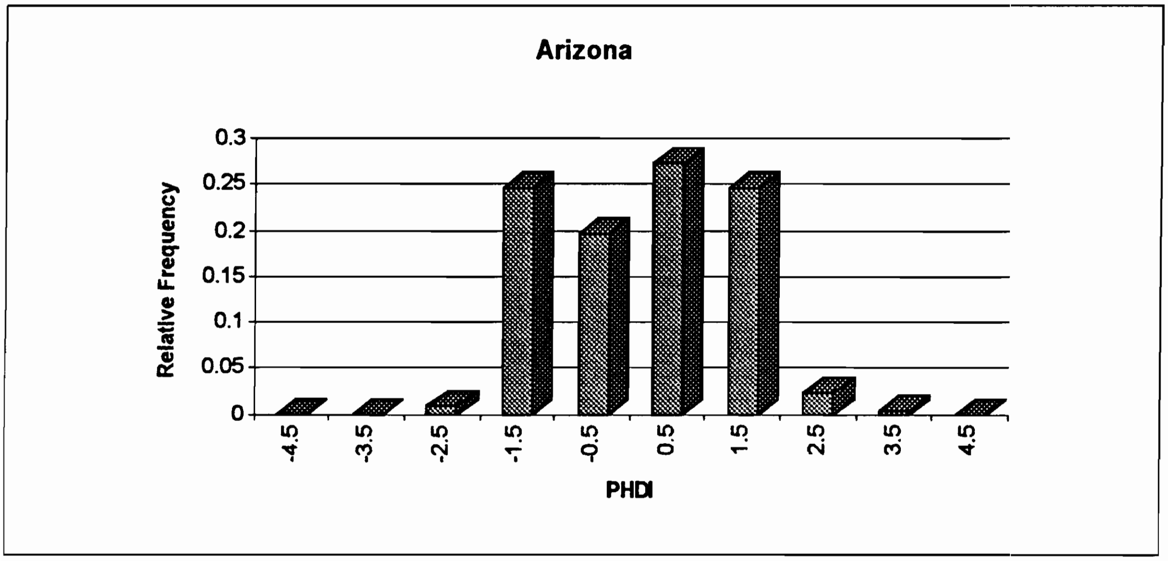


Figure 5.2(a): Relative Frequency Histogram, $PHDI_{i+1} \mid PHDI_i \in 4$

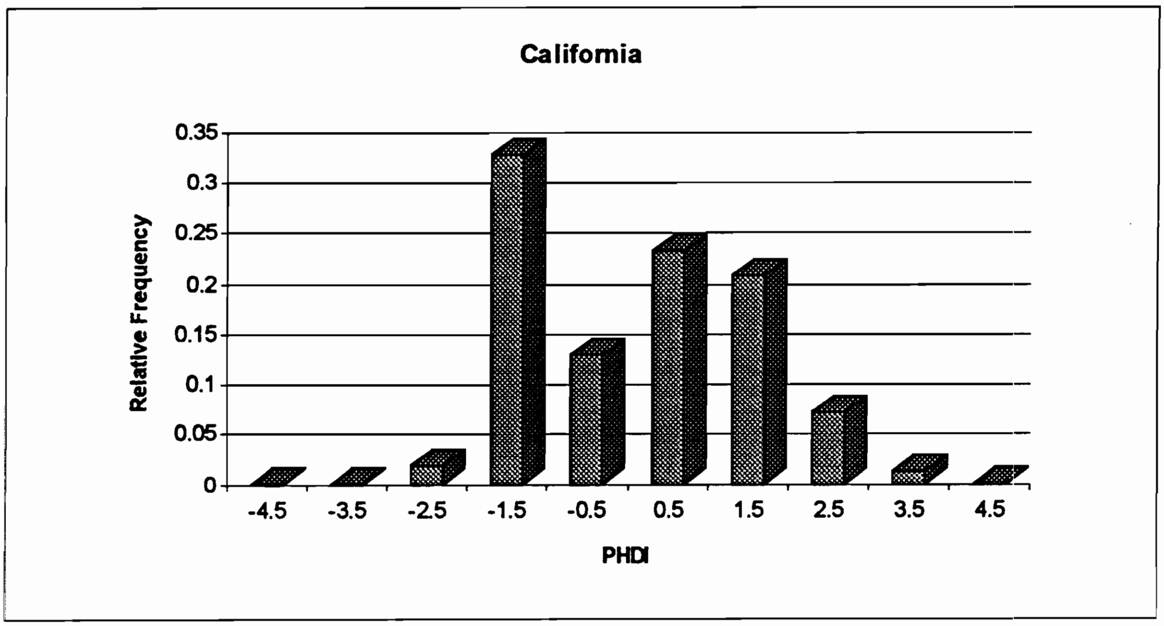


Figure 5.2(b): Relative Frequency Histogram, $PHDI_{i+1} \mid PHDI_i \in 4$

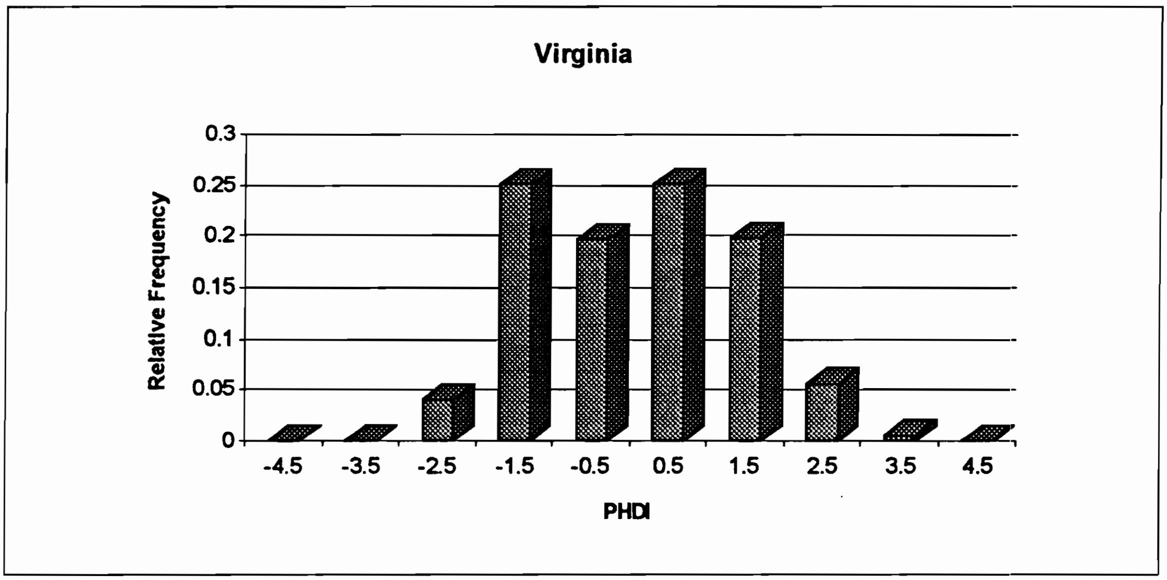


Figure 5.2(c): Relative Frequency Histogram, $PHDI_{i+1}$ | $PHDI_i \in 4$

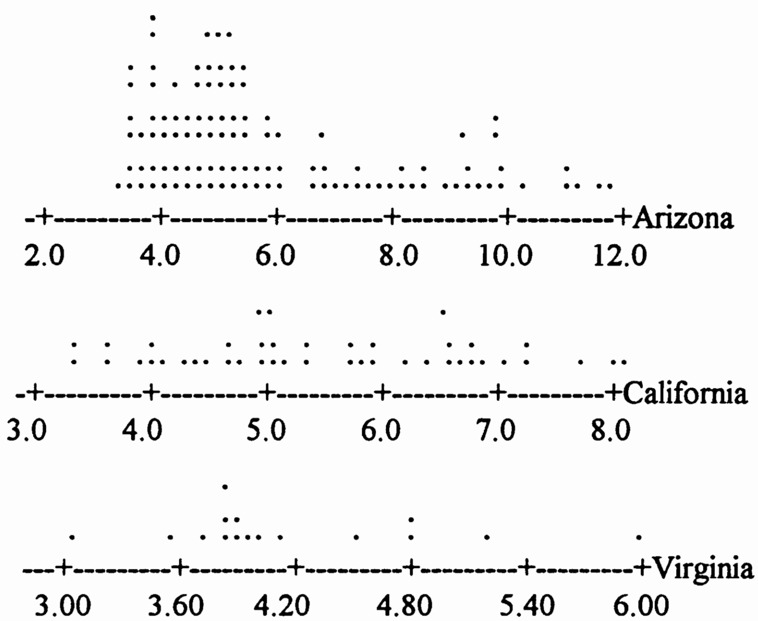


Figure 5.3 : Dot Plot of $PHDI_{i+1}$| $PHDI_i$ is in class 2

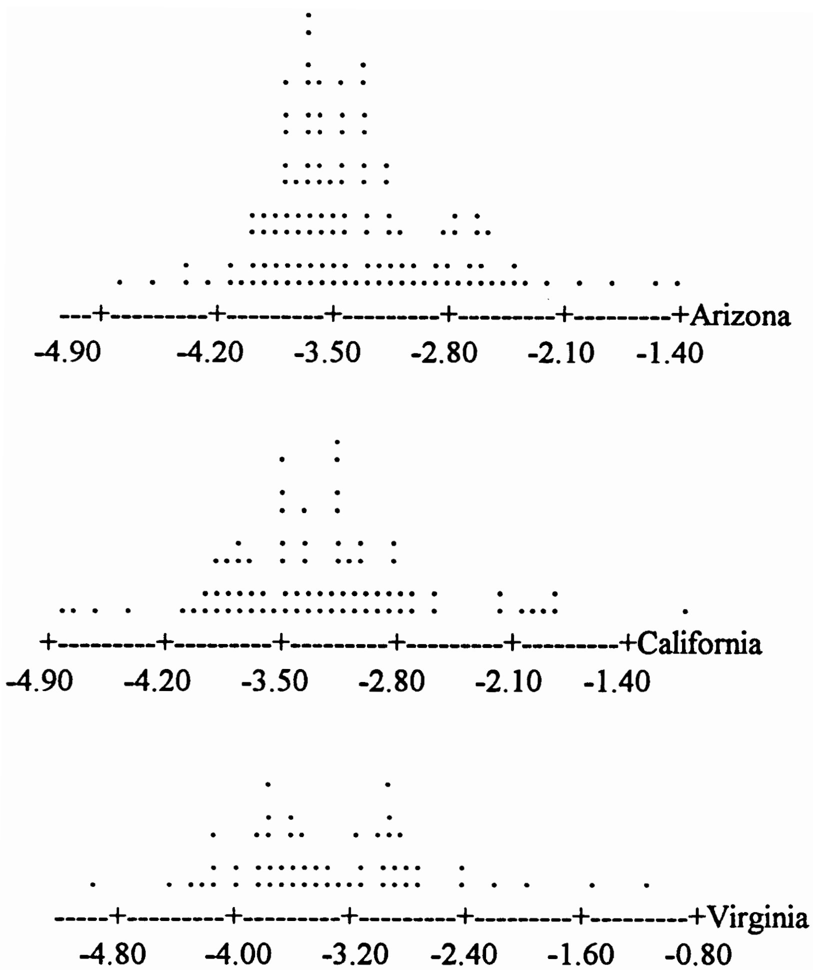


Figure 5.4 : Dot Plot of $PHDI_{i+1}$| $PHDI_i$ is in class 6

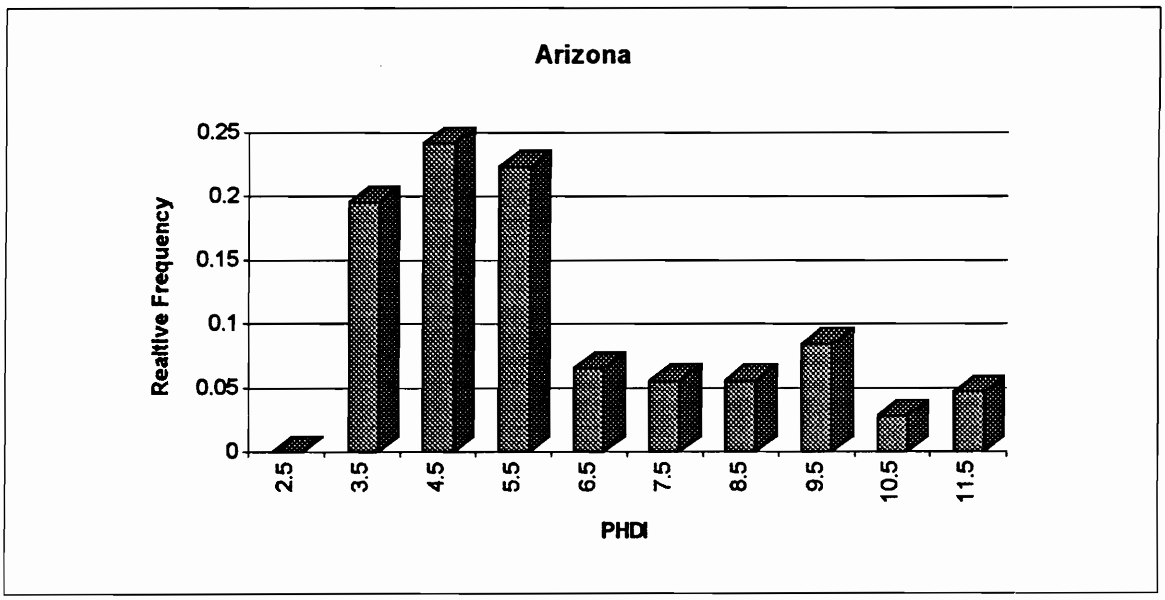


Figure 5.5(a): RELATIVE FREQUENCY HISTOGRAM, $PHDI_{i+1}$ | $PHDI_i \in$ class 2

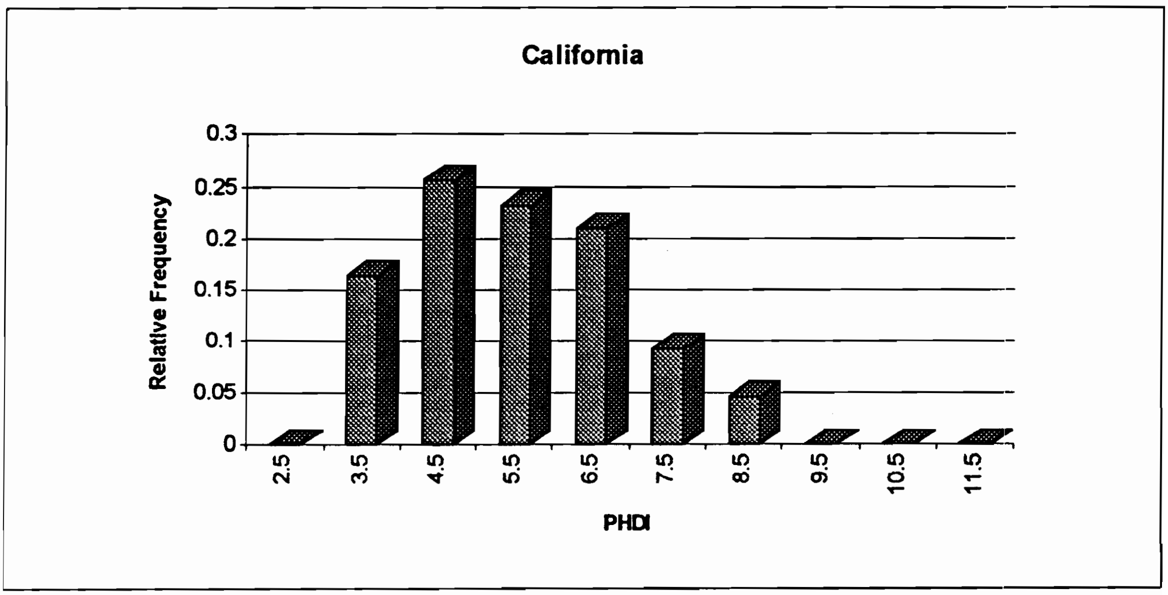


Figure 5.5(b): Relative Frequency Histogram, $PHDI_{i+1} \mid PHDI_i \in$ class 2

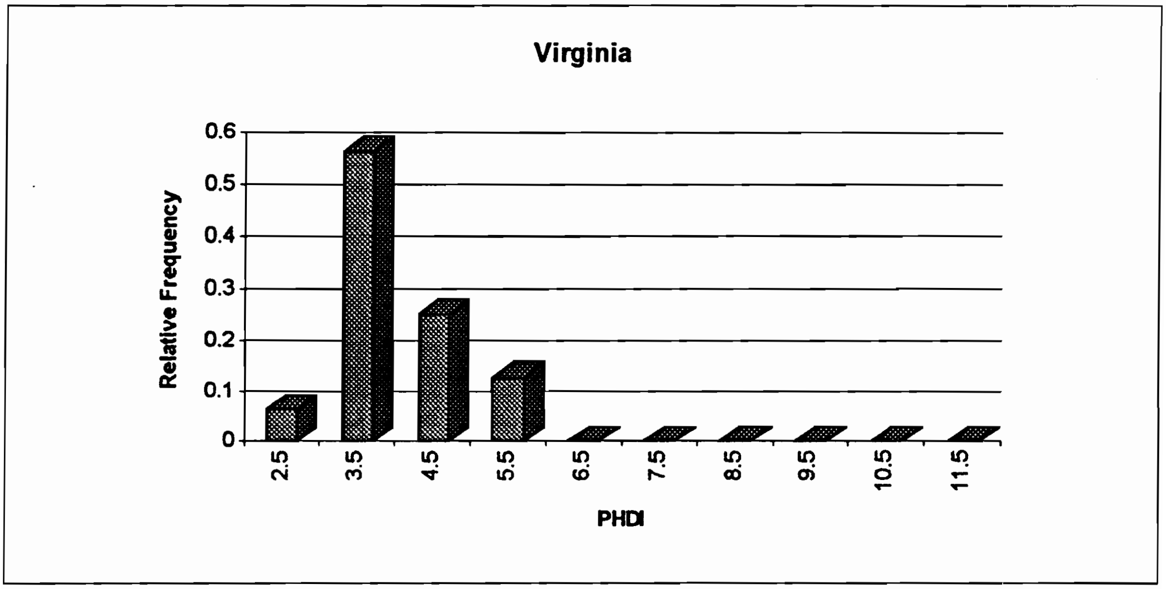


Figure 5.5(c): Relative Frequency Histogram, $PHDI_{i+1}$ | $PHDI_i \in$ class 2

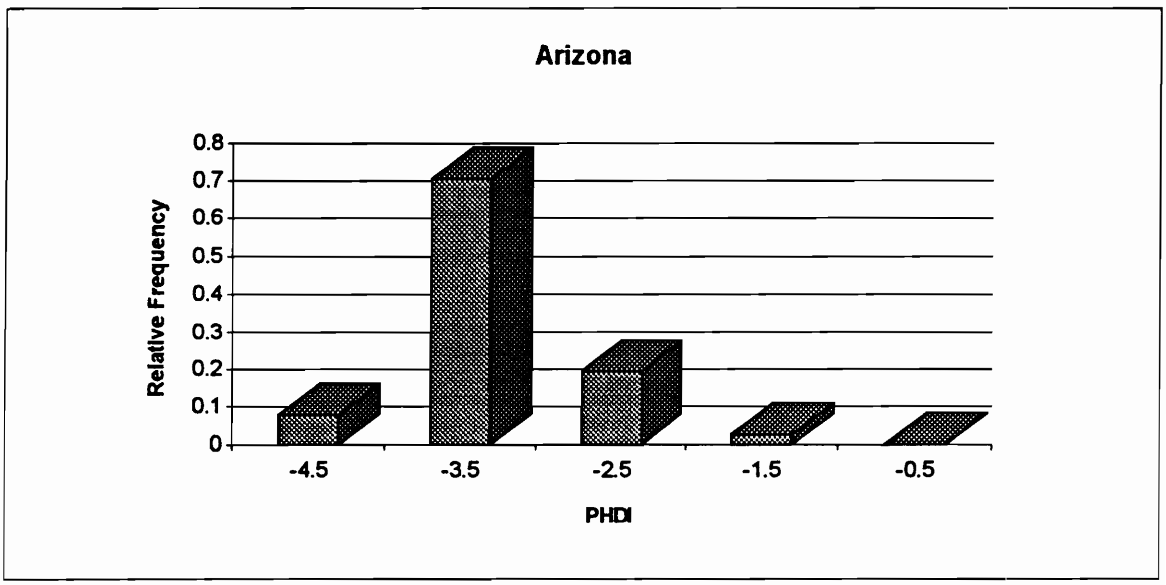


Figure 5.6(a): Relative Frequency Histogram, $PHDI_{i+1} \mid PHDI_i \in$ class 6

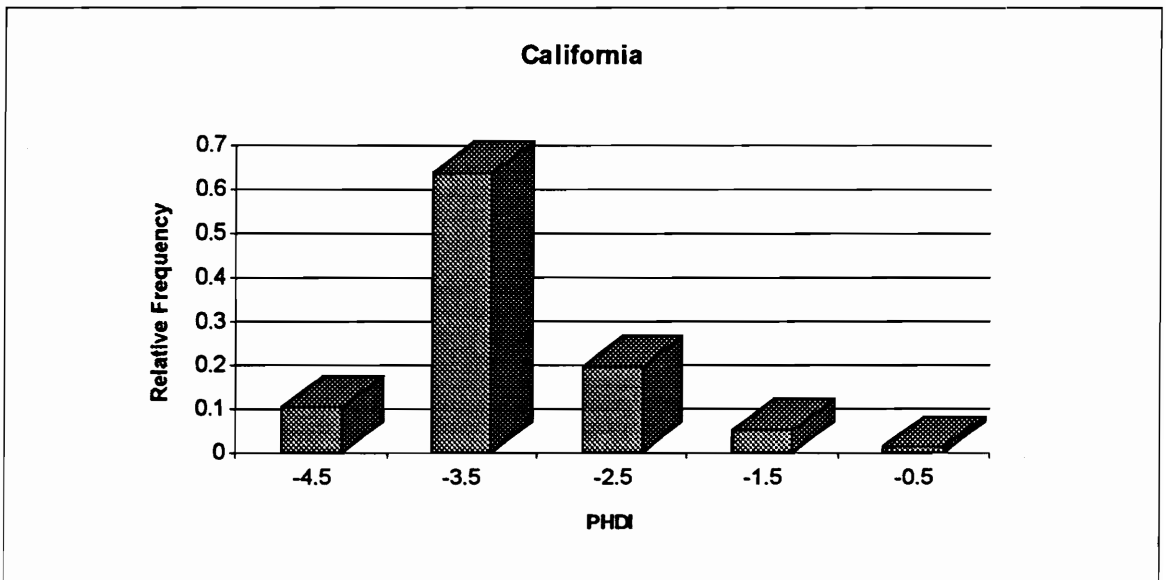


Figure 5.6(b): Relative Frequency Histogram, $PHDI_{i+1} \mid PHDI_i \in$ class 6

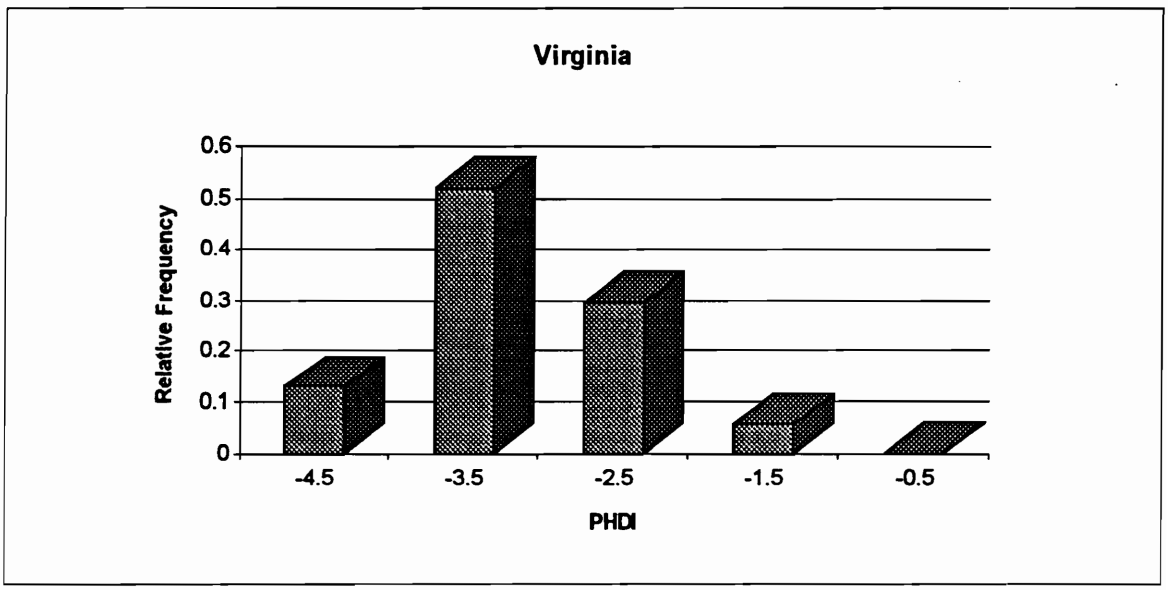


Figure 5.6(c): Relative Frequency Histogram, $PHDI_{i+1} \mid PHDI_i \in$ class 6

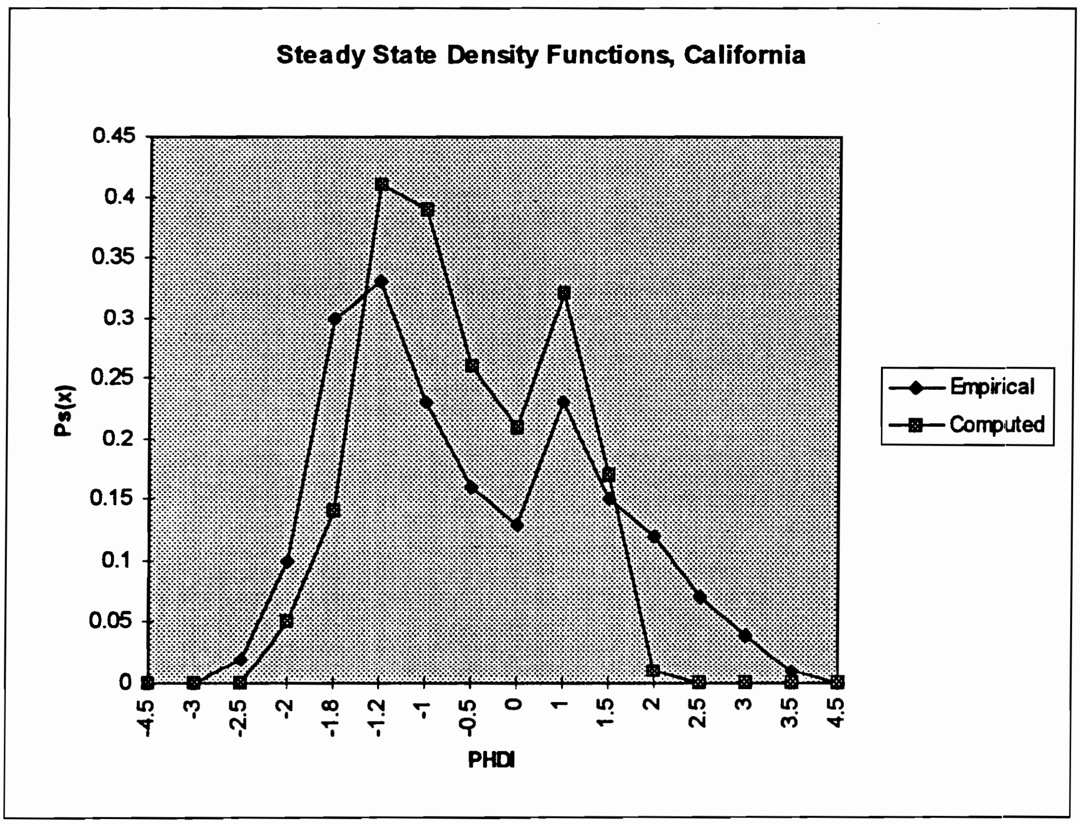


Figure 5.7 Empirical and Computed Steady State Density, $PHDI_{i+1}|PHDI_i \in$ class 4

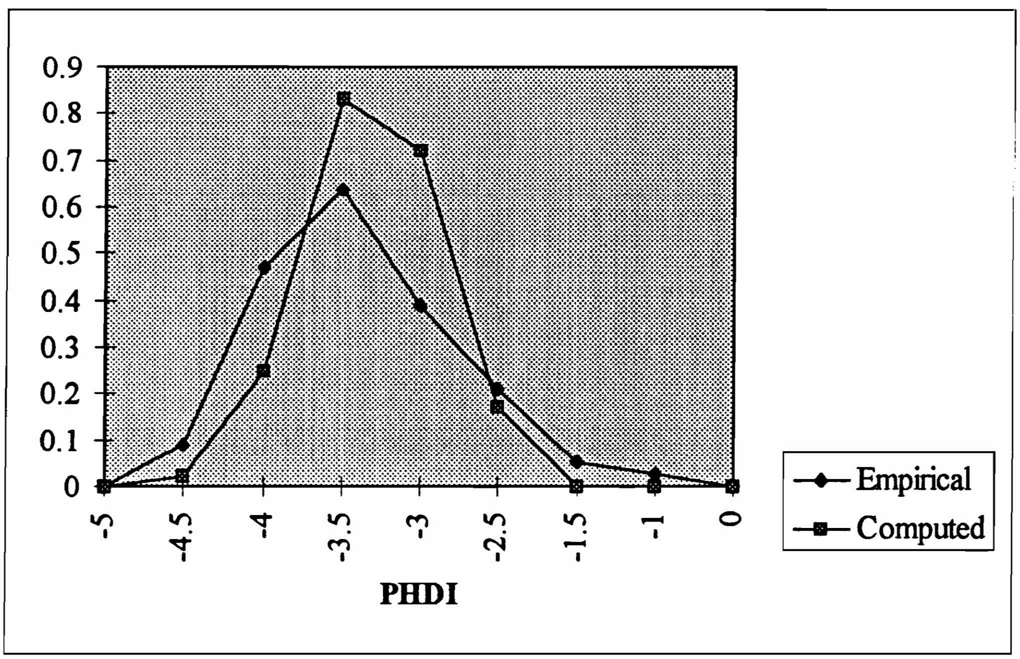


Figure 5.8 Empirical and Computed Steady State Density, $PHDI_{i+1}|PHDI_i \in$ class 6

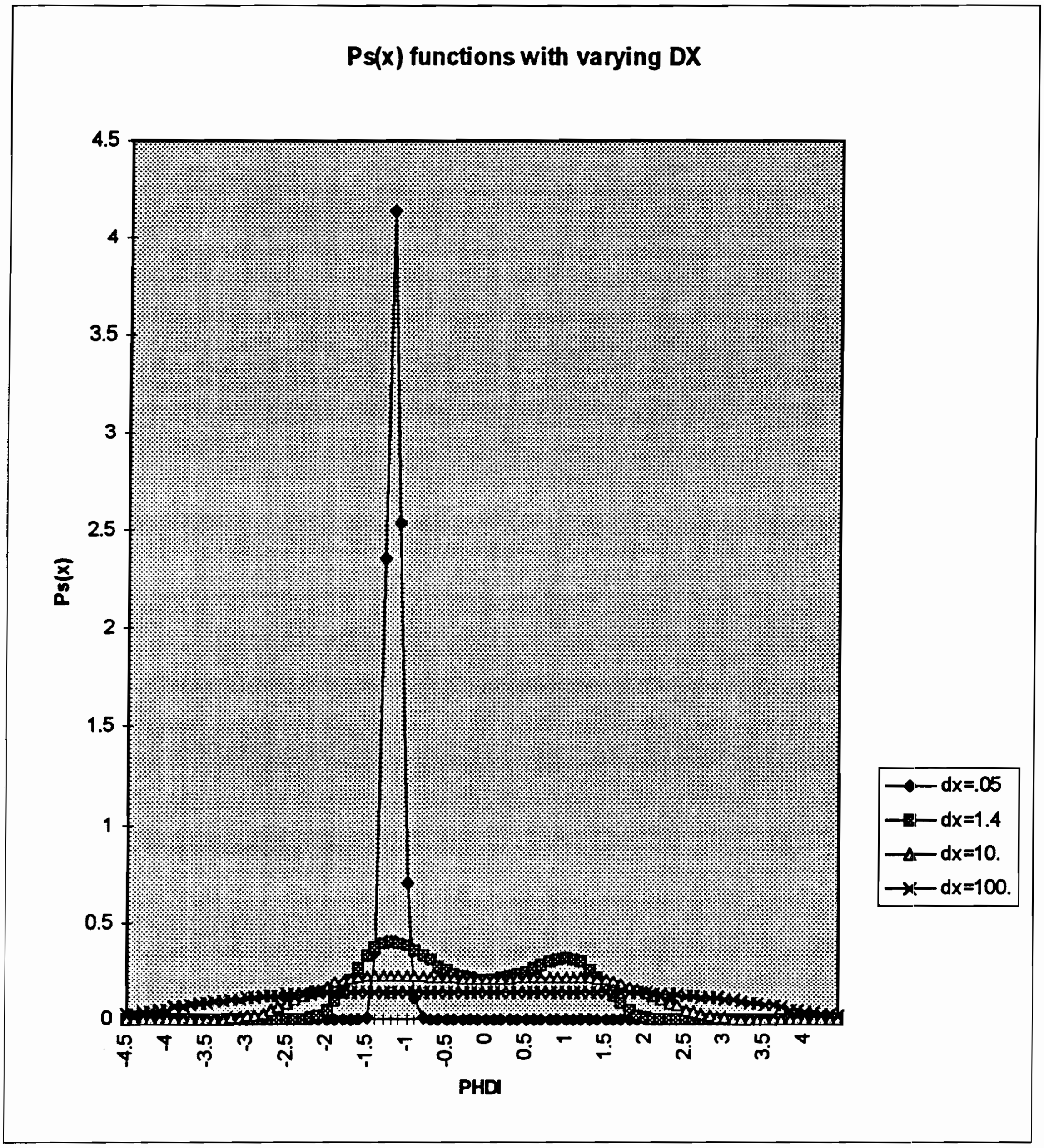


Figure 5.9 Steady State Density Function of $PHDI_{i+1}|PHDI_i \in$ class 4 with Varying D(x)

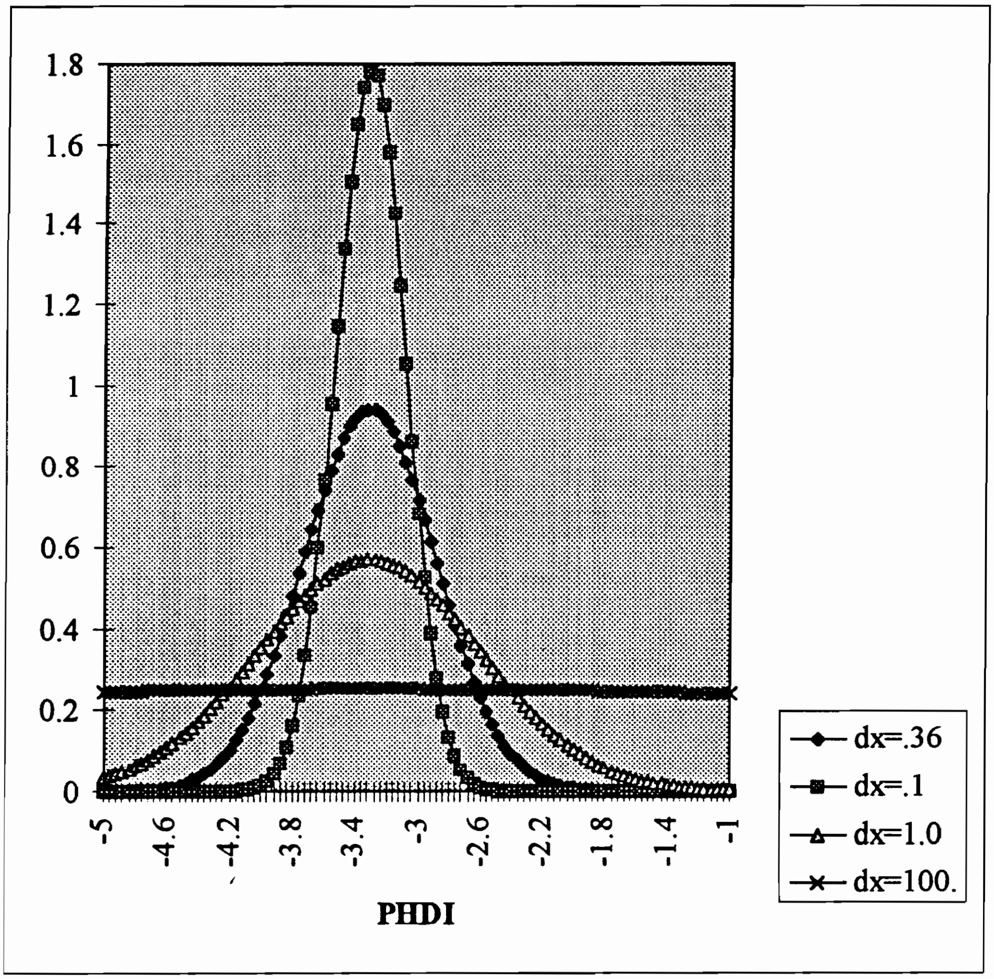


Figure 5.10 Steady State Density Function of $PHDI_{i+1}|PHDI_i \in$ class 6 with Varying D(x)

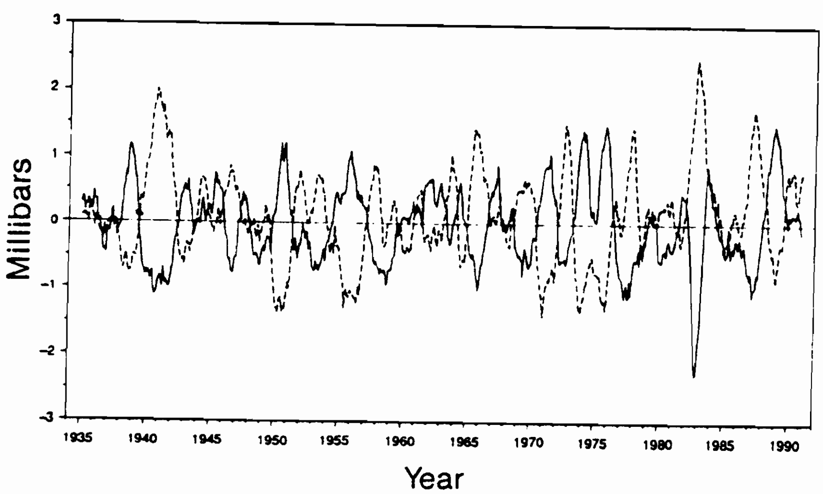


Figure 5.11 Nine-month Running Mean of Monthly Sea Level Pressure Anomalies at Darwin, Australia (dashed line) and at Tahiti (solid line)

Chapter VI

PALMER INDEX AS A PRECURSOR FOR DROUGHT WARNING

6.1 INTRODUCTION

The objective of the present chapter is to show the practical usefulness of the results obtained in the previous chapters. The results should be useful in a decision making context of drought mitigation practices adopted by various agencies. The management strategies to mitigate the impacts of drought have been formulated in almost every state of the Union. These strategies generally fall into four categories : i) curtailment of water usage; ii) supply augmentation by development of wells for groundwater extraction and storage of surface water in reservoirs; iii) procurement of water either by sale or through transfer of water rights in the form of a lease or by ownership from the riparian land owners; and iv) additional procurement by water transfer from neighboring basins under inter-basin water transfer agreements. While the first two strategies are implemented widely towards an impending drought, the third and the fourth pose both legal and political complications in addition to the technical aspects. Except in the case of California, a concerted effort towards acquiring water usage rights from the owners is not a major issue. Interbasin transfer has led to special difficulties when the consented parties face widespread drought in unison. New Jersey, Pennsylvania, and New York have an agreement which considers such an eventuality (Wilhite, 1990). The aforementioned matters are fully addressed under the institutional aspects of water allocation(Cox, 1995). Except for a few cases typically invloving large cities, exercising drought mitigation efforts is considered a local problem. Currently, there is no federal organization in the U.S. responsible for monitoring drought conditions. However, in view of the widespread droughts of the past two decades various state governments have come up with drought contigency plans (DCPs) including AZ, CA, and VA. All states use the PDSI as a measure of drought severity and typically supplement it with other indices. In addition, PDSI has been used by the Federal Government as one of the principal criteria for disaster designation, that is, to assess the eligibility to receive federal drought relief (Wilhite et al., 1986). In the operational mode in a drought emergency, it is the responsibility of a water availability task force (WATF) to evaluate the status of the supply of water in various regions. The WATF recommends mitigating measures based on trigger threshold values for defining droughts. For example, in the Colorado Plan the trigger values correspond to PDSI values between -1 and -2. In Virginia, drought is indicated when : i) precipitation is less than 85 percent of the 30 year mean for at least three consecutive months; ii) PDSI is below -2.00 for at least three consecutive months; iii) streamflow is within the lowest 25 percent of mean monthly flow for at least three consecutive months ; and iv) groundwater level is within the lowest 25 percent of the average monthly level for three consecutive months. Based on the above criteria, Table 6.1 (State Water Control Board, 1990) shows the drought years for the period 1957-1987 for the Tidewater climatic division in Virginia. The concerned statistics are reported in Tables 6.2, 6.3, and 6.4. These tables contain the Blackwater river at Luni streamflow data, groundwater level for well # 58B13 data and PDSI data, respectively. The correlations between the streamflow - PDSI and groundwater - PDSI are given in Tables 6.5 and 6.6, respectively. Also, the 25th percentile streamflow values and mean groundwater levels are given in Tables 6.7 and 6.8. It is clear that level of the threshold plays a crucial role in the Virginia definition of drought occurrence because of its "wait and see" nature with the waiting period being 3 months. For a "here and now" decision, while the curtailment of water usage at an initial stage might help to err on the conservative side, it may also impose an undue burden at an early stage. Therefore, a good predictive model is essential for making responsible decisions.

In this chapter a decision tree analysis is put forward for making operational decisions with regard to a progressing drought. The decision tree shows all possible drought state occurrences in terms of the PDSI. A secondary drought measure, the precipitation deficit is also shown in the decision tree (Figs. 1 and 2). The decision tree displays the pathway to reach the current state of drought along with all possible ways of leaving that state with the associated probabilities. The decision tree extends over a critical period starting from a drought trigger month to the end of the water year. The drought trigger month is chosen to be the month of May. In the ensuing sections a precipitation deficit / surplus analysis is carried out based on: i) calendar year, and ii) water year which constitutes the secondary drought measure to accompany the PDSI values. It also serves well to select the drought trigger month.

6.2 PRECIPITATION DEFICIT / SURPLUS ANALYSIS- CALENDAR YEAR

6.2.1 Analysis of Drought Years

Table 6.9 lists the drought years (based on calendar years) as identified by the SCWB (1990) analysis. The table also contains the rainfall received and its deficit from long term average. In order to ascertain the patterns of cumulative rainfall shortage, the drought years are analyzed for rainfall deficit on a monthly basis. Table 6.10 contains the results for each drought year along with the observed Z index and PDSI values for the period. Based on long term average monthly rainfall values, deficit / surplus in rainfall for each month is computed and is accumulated starting in January for each drought year. For example, it is seen that during 1965 drought year a deficit of 1.13 inch is experienced during the month of January. The accumulated deficit upto May 1965 went upto 5.75 inch. Based on the analysis of rainfall deficit for each drought year, Table 6.11 gives accumulated rainfall deficit upto the months of April, May ,and June and the corresponding means.

6.2.2 Analysis of Non-Drought Years

For comparison purposes deficit / surplus characteristics of those years which were not classified as drought years in SWCB, 1990 are studied. These non-drought years are listed along with the rainfall received in Table 6.12. The deficit / surplus characteristics of these non- drought years are given in Table 6.13. It is seen that all non-drought years had a surplus accumulation of rainfall from the normal. These cumulative surpluses and the corresponding means upto April, May and June months for each year are given in Table 6.14. It is seen that for non-drought years there has been on an average 5.7 inch surplus rainfall from January through May.

6.2.3 Analysis of Low Rainfall Years

It is observed that, during the 31 year period, a few years received rainfall less than the normal, but are not classified as drought years under the criteria described by the SWCB (1990). Such years are referred to as the low rainfall years here and are given in Table 6.15 along with the rainfall received. The deficit / surplus characteristics of low rainfall years are analyzed in Table 6.16.

Table 6.17 gives a summary of the accumulated average deficit / surplus rainfall upto April, May and June months for : i) drought years , ii) non-drought years, and iii) low rainfall years. It is only during the drought years that the cumulative deficit increases consistently upto June. For example, upto May end the cumulative deficit is of the order of 3.10 inch in drought years which rose to 3.6 inch by the end of June. During non-drought years the cumulative surplus consistently increases. The unique pattern observed in the accumulation of surplus during non-drought years separates them from the other drought and low rainfall years.

6.2.4 Identification of Starting Month For Droughts

To establish the most critical month for the commencement of a drought, the PDSI data of the region from 1895-1990 are analyzed. A drought event is considered to have occurred when the PDSI value falls in class 5. For the entire period of record such events were counted and the results are given in Table 6.18. It is seen that 19 percent of the total historical drought events began in the month of July and 55 % of drought events began in the months of June through September. The analysis of steady state probabilities as given in Chapter IV also indicates July through October as the months having the highest probability of droughts. Therefore, from the view point of drought monitoring and for making recommendations for appropriate mitigation measures it is important to watch the deficit or surplus as it builds from January through May. In other words for the Tidewater region the Virginia Drought Monitoring Task Force (VDMTF) should examine the deficit situation at the end of May every year. If the cumulative deficit at this time is greater than or equal to 3.1 inch (average deficit upto May month experienced during past drought years), there is a good possibility of drought extending through the coming months. Of course, the PDSI values do indicate this behavior. Referring to Table 6.10 it is seen that during the drought years for the month of May the PDSI is consistently below -1.5 indicating either drought state 5 or 6. Therefore, it can be assumed that during a year when the cumulative deficit upto May has been of the order of 3.1 inch, the most probable state during May month would be either 5 or 6.

In the following section a decision tree analysis is presented. It not only shows the progression of drought upto the current month (period) but also yields further branchings and the associated probabilities into the future.

6.2.5 Decision Tree Analysis

Based on section 6.2.4 results, the month of May can be considered the trigger month for droughts whenever the PDSI values fall in classes 5 or 6. Considering the termination of crop growing season and the onset of winter, the month of September is chosen to define a critical period for the issuance of drought warnings. It should be noted that the technique proposed is flexible and can be used with any other starting month. The monthly transition matices using PDSI data (1957-87), given in SCWB, 1990, are given in Table 6.19. It is seen that being in state 5 in May , the weather will transit to either state 5 with .8 probability or to state 6 with .2 probability in June. A complete description of weather state transitions from May through September along with the associated probabilities is shown in Figures 6.1 and 6.2 for the starting states of 5 and 6 in May, respectively.

While the PDSI is a holistic index for interacting moisture related mechanisms, in a decision making context it is useful to display the other measures of deficits as well. As pointed out before in this study the rainfall deficit / surplus is chosen to be the secondary indicator. It is seen in Figure 6.1 that to go from state 5 in May to state 5 in June, there will be a deficit of .47 inch in June from the normal rainfall. To compute this amount, all state 5 in May to state 5 in June transitions for the period are considered as given in Table 6.20. It is seen that first such event took place in May 1967 and the computed deficit for June 1967 is 1.62 inch. Likewise for the other events occurred in 1969, 1976, and 1981 the deficits are shown. For each month corresponding to the various drought states the deficit / surplus amounts are computed (Table 6.20) and are shown in Figure 6.1. The various possible paths of weather transition are also shown. Further, at each level of transition the expected duration of drought is indicated within curly brackets. Using this decision tree we can work out total deficit / surplus for each possible path of transition from May to September and its associated probability level. For example, the event when May = 5; June = 5; July = 5; August = 5; and September = 6 (we refer to this event as

55556) will have a probability of

$$P\,[\text{ June} = 5\text{ ; July} = 5\text{; Aug.} = 5\text{; Sep.} = 6 \mid \text{May} = 5\ ]$$

$$= P_{55}^{MayJun}\, P_{55}^{Jun\,Jul}\, P_{55}^{Jul\,Aug}\, P_{56}^{Aug\,Sep} = .8\ .67\ .72\ .3 = .1158$$

The event 55556 involves a total deficit of 4.53 inch (.47 inch June; .35 inch July; .56 inch August; and 3.15 inch September) and has a probability of 11.58%. Likewise probability of all possible events and associated deficits / surplus are computed and are given in Table 6.21. It must be noted that the precipitation deficits shown on Figs. 1 and 2 provide additional detail and are not the CAFEC (Climatically Appropriate For Existing Conditions) deficits used to compute the Z and PDSI values. From Table 6.21 it is seen that there is 62 % probability that there will be a deficit in September when the drought state is 5 in May. It should also be remembered when a particular branch is followed in the decision tree the deficits at each node should be added to the May cumulative deficit to obtain the total deficit at that node (for that month). For example, traversing along the branch 5-6-6 starting in May in Figure 6.1 yields a total deficit of -3.10 -2.07 +0.26 = -4.91 inches by the end of July. Considering a typical drought year is declared when the rainfall shortage is around 8 inches, about 5 inches shortage at the end of July may require serious consideration.

Figure 6.2 gives the decision tree with the starting state of 6 during May. It is seen that the information given in Figures 6.1 and 6.2 provides a decision making body like the VDMTF (Virginia Drought Monitoring Task Force) an in depth analysis of all possible scenarios should conditions for an impending drought develop. It is obvious that the analysis can be done starting any month and for any region. The values of deficit or surplus amounts can be updated by using a longer period of data and can be periodically updated with the new data. In the following section a second analysis based on a water year basis is suggested. Also, as a secondary indicator a three month running sum of rainfall deficits is adopted. With the calendar year and water year based analyses the observed drought year patterns are well captured.

6.3 PRECIPITATION DEFICIT / SURPLUS ANALYSIS - WATER YEAR

In this analysis rainfall data are analyzed on water year basis. For example, water year 1981 is considered from October, 1980 to September, 1981. The SCWB (1990) gives rainfall data for a period from 1958-1987 (water years) for the climatic divisions in Virginia. This analysis uses these data for the Tidewater division. Based on the 30 year data, the annual average rainfall is computed as 44.13 inches. The 30 year period is classified into four category of years, namely, Drought years, Low rainfall years, Normal years, and Non-drought years. The details are given in Table 6.22.

6.3.1 Analysis of Drought Years

Table 6.23 gives the drought years along with deficit / surplus rainfall accumulated for selected 3 month periods. The 3 month periods end in October, January, March, and May. These periods are consistent with the period chosen by the SCWB (1990) for defining droughts. For example, for water year 1965, which is classified as a drought year, a cumulative surplus rainfall of 4.66 inch is observed upto October month which includes deficit / surplus during August, September, and October, 1964. In 1965 water year a cumulative deficit of 2.45 inch is observed up to January which includes deficit / surplus observed over November, December, and January months of the water year. In the similar way, cumulative deficit / surplus figures for all drought years are computed and are given in Table 6.23.

6.3.2 Analysis of Low Rainfall Years

Table 6.24 gives the surplus / deficit analysis of the years which received rainfall above 40 inches but less than the normal rainfall of 44.13 inches. The deficit / surplus rainfall amounts accumulated over the 3 month periods are given in Table 6.24.

6.3.3 Analysis of Normal Years

Table 6.25 gives the surplus / deficit analysis of years which received rainfall close to the normal rainfall of 44.13 inches.

6.3.4 Analysis of Non- Drought Years

Table 6.26 gives the surplus / deficit analysis of years which received rainfall about 3 inches or more than the normal rainfall.

6.3.5 Summary of Water Year Analysis

Based on the analysis described in Tables 6.23 through 6.26, the average cumulative deficit/ surplus amounts up to the months of January, March, and May for each category of years are given in Table 6.27. The values within parentheses in this table give the range of variation which is chosen to represent most of the analyzed years. It is seen from Table 6.27 that during the drought years the 3 month cumulative deficit consistently increase from January through May months. On the other hand a consistent pattern of increasing cumulative rainfall surplus is observed for the non- drought years. Such contrasting patterns can be used to differentiate a drought year from the other years and hence is useful in drought monitoring operations.

The water year based analysis has the advantage of incorporating information about surplus / deficit amounts experienced over the previous water year. For example, it is seen in Table 6.23 during water year 1981, the cumulative deficit up to October 1980, the first month of water year 1981, is of the order of 2.58 inches which includes the carried over deficits from the previous water year's August and September months. It is further seen that the cumulative deficit increased to 5.27 inches by January. The year 1981 is reported to be a worse drought year (Birch and Ulrich, 1982). Similar drought situation occurred in year 1966. Therefore, based on this analysis the following two deterministic rules can be developed for issuing drought warnings.

Rule # 1 : Observe 3 months' cumulative deficit / surplus rainfall amounts up to October month. If it is more than 2.5 inch, then observe the 3 months' deficit up to January. If this deficit is more than 4.5 inch, issue warning of a possible drought during the current water year.

Rule # 2 : If 3 month cumulative rainfall deficit upto October is less than 2.5 inch, then observe 3 month cumulative deficits up to the months of January, March, and May. If these deficits fall in the ranges of 0.5-2.5 inch, 1-3 inch, and 1-5 inch at the end of January, March, and May months, respectively then there is a good possibility of drought extending to the coming months in the current water year.

If the Rule # 1 is satisfied, then the decision maker may like to issue warning right in February month. For Rule # 2, the warning will be issued in June similar to the case of calendar year analysis.

Once drought warnings are issued the decision makers would like to know the extent of shortages the forthcoming months would face. In a way similar to the calendar year analysis we can use the Markov matrices, developed in Chapter IV, to assign the likely states of weather for the forthcoming months. In the case of the calendar year analysis the average amount of rainfall shortage / surplus corresponding to a specific monthly weather transition is indicated on the decision tree as a secondary indicator of drought progress. In the water year analysis the Palmer equation (see Appendix I for details) itself is used to compute the average CAFEC deficit / surplus to accompany the transition from one class to another. An example of such computation is given below

Suppose during the month of May, weather is in state 5 and then it transits to state 6 during June. Using Palmer equation we can write

$$PDSI_{June} = .897\ PDSI_{May} + Z_{june}/3 \qquad (1)$$

$PDSI_{June}$= weather class 6 = -3.5 (middle value of class 6; Table 4.1)

$PDSI_{May}$ = weather class 5 = -2.25 (middle value of class 5; Table 4.1)

Using Eq. 1 we can calculate Z_{june} as

$$Z_{june} = 3\ (\ -3.5\ \ + .897\ (2.25)\ ) = -4.45$$

We know from Appendix I (Eq. 8) that

$$Z = d.\ k \qquad (2)$$

where : k = a weighting factor for a month and a region ; d = CAFEC deficit of moisture from normal demand

Therefore, $d_{June} = Z_{june}\ /\ k_{june}$

The values of k are given for some locations in Palmer (1965). The values given for Scranton, PA are chosen as the representative values for the Tidewater region, Virginia. The actual values of k for the Tidewater region should be available but could not be obtained at the time of the analysis. Based on the adopted values of k, the deficit of moisture for transition from class 5 in May to class 6 in June is then computed as

$$d_{June} = -4.45\ /\ 1.17 = -3.80 \text{ inch}$$

Table 6.28 gives the deficit / surplus values of moisture, computed using above procedure, for various possible monthly transitions during the period of drought. For any other transition not covered in Table 6.28, the computations can be made in the manner illustrated above. Using the deficit / surplus amounts and the associated probabilities, the decision maker can make appropriate decisions about the mitigation measures once drought warnings are issued. An example application for the drought year 1981 is given in the following section.

6.3.6 Application of Drought Warning Technique to 1981 Drought Year

Birch and Ulrich (1982) reported serious drought conditions in the Tidewater region in Virginia during 1980-81. As seen in Table 6.23 the accumulated 3 month deficit during 1981 water year up to October month is 2.58 inches which rose to 5.27 inches by January. Therefore, as per rule # 1 drought warning should be issued in February 1981. Figure 6.4 shows progression of wet / dry states during 1981 water year starting October

1980. The observed weather state during October 1980 is 5 (see corresponding PDSI value in Table 6.4). The weather state progression along with associated probabilities, as shown in Figure 6.4, give amount of expected deficit / surplus (underline values) and compare these with the actual observed values (square brackets). The actual deficit / surplus amounts as computed using values given in Table 6.27 agree reasonably well with the observed values. The 3 month cumulative deficits are shown within parentheses. From October upto December the mode probabilities serve as very good predictors. For Dec. - Jan. transition a deviation from the mode occurs, that is instead of $5 \rightarrow 5$ transition $5 \rightarrow 6$ occurs. This could be argued that the high deficit value would warrant the decision maker to consider this case even beforehand. Figure 6.4 illustrates these scenarios. The actual states of weather indicated within curly brackets are compared with the mode weather states (encircled ones). It is seen that out of 12 months, the mode weather states agree with the actual observed states for 75% of time. From Figure 6.4 for the water year decision rules 1 and 2, one would have generated a warning in February. From the calendar year analysis (Figs. 6.1 and 6.2) a warning is not needed in June 1981 because the prevailing drought severity is at state 5 and not at 6. For typical crop growing periods of 3 month duration 1980 June or 1981 June as starting months do not indicate severe drought. This example shows short term recoveries within a long spell of droughts. Such droughts may matter only in filling surface water supply reservoirs. From irrigation consideration the 1981 drought should not have had great impact. It can, therefore, be concluded that the technique described here describes the past drought events very well and can therefore be useful for drought monitoring operations.

6.4 SUMMARY

A typical decision making analysis recommends associating a utility function for each branch of the tree along with the probability, as shown in Figure 6.3. The reward or penalty is incurred dependent on whether the action of issuing the warning coincides with the realization of drought or not. Of course, determining the reward / penalty coefficients C1, C2, C3 and C4 as shown in Figure 6.3 is a difficult task. Whenever, 0.2 C1 + 0.8 C2 > 0.2 C3 + 0.8 C4 a warning should be issued.

In this chapter a decision tree framework has been put forward for use in issuing drought warnings. The key advantage is the enumeration of all possible sequences of occurrences. With such a procedure, a decision maker can observe how the progression of drought has taken place up to certain stage indicated by a particular month. For any such stage, one is provided with all possible future scenarios with their associated probabilities. As recommended before, certain secondary measures of drought can also be associated with each drought state (node) for a particular month (stage) of the decision tree which should provide an intuitive feel in deciding the future courses of action.

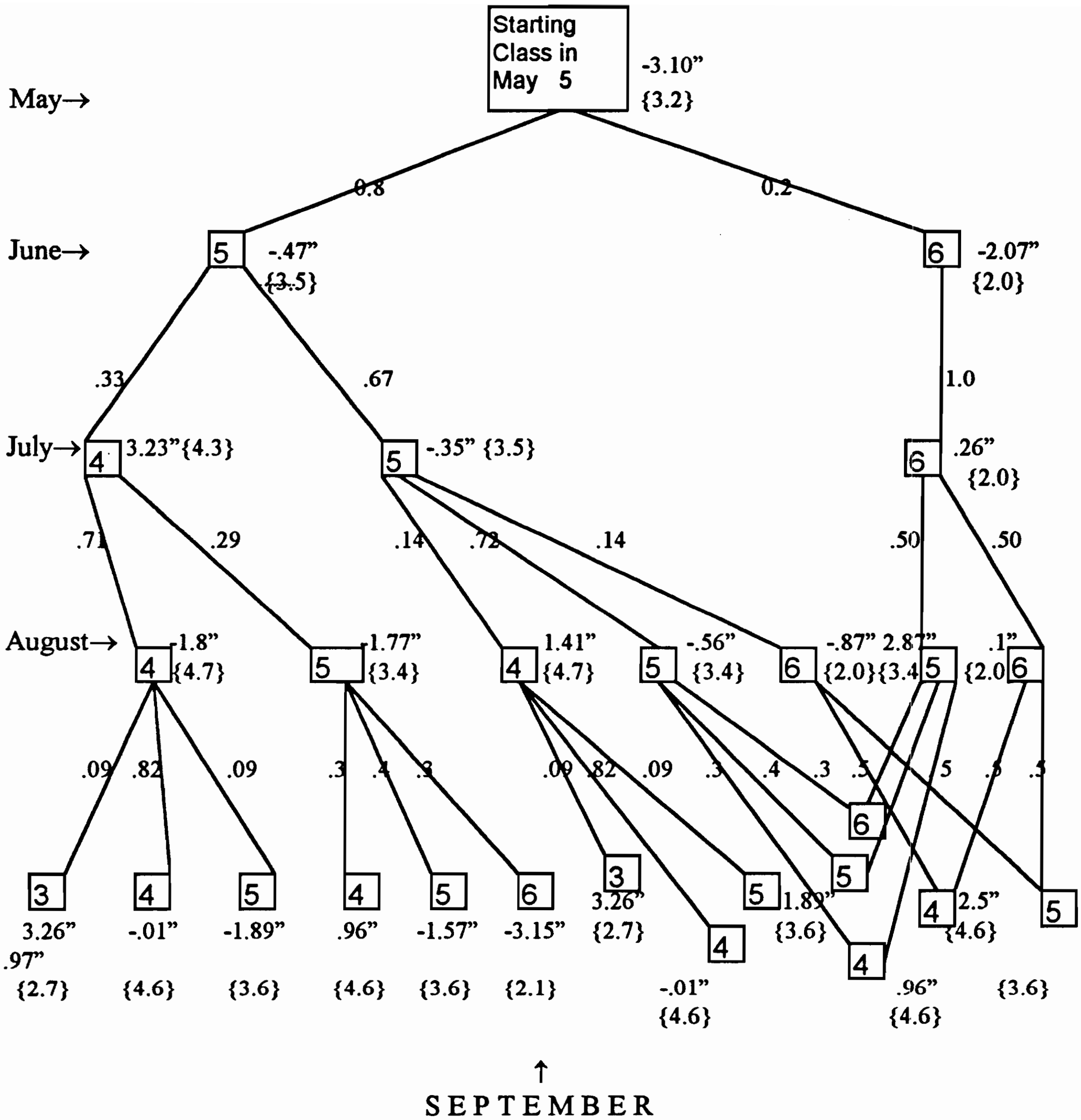


Figure 6.1 Decision Tree of Possible Actions, May State 5

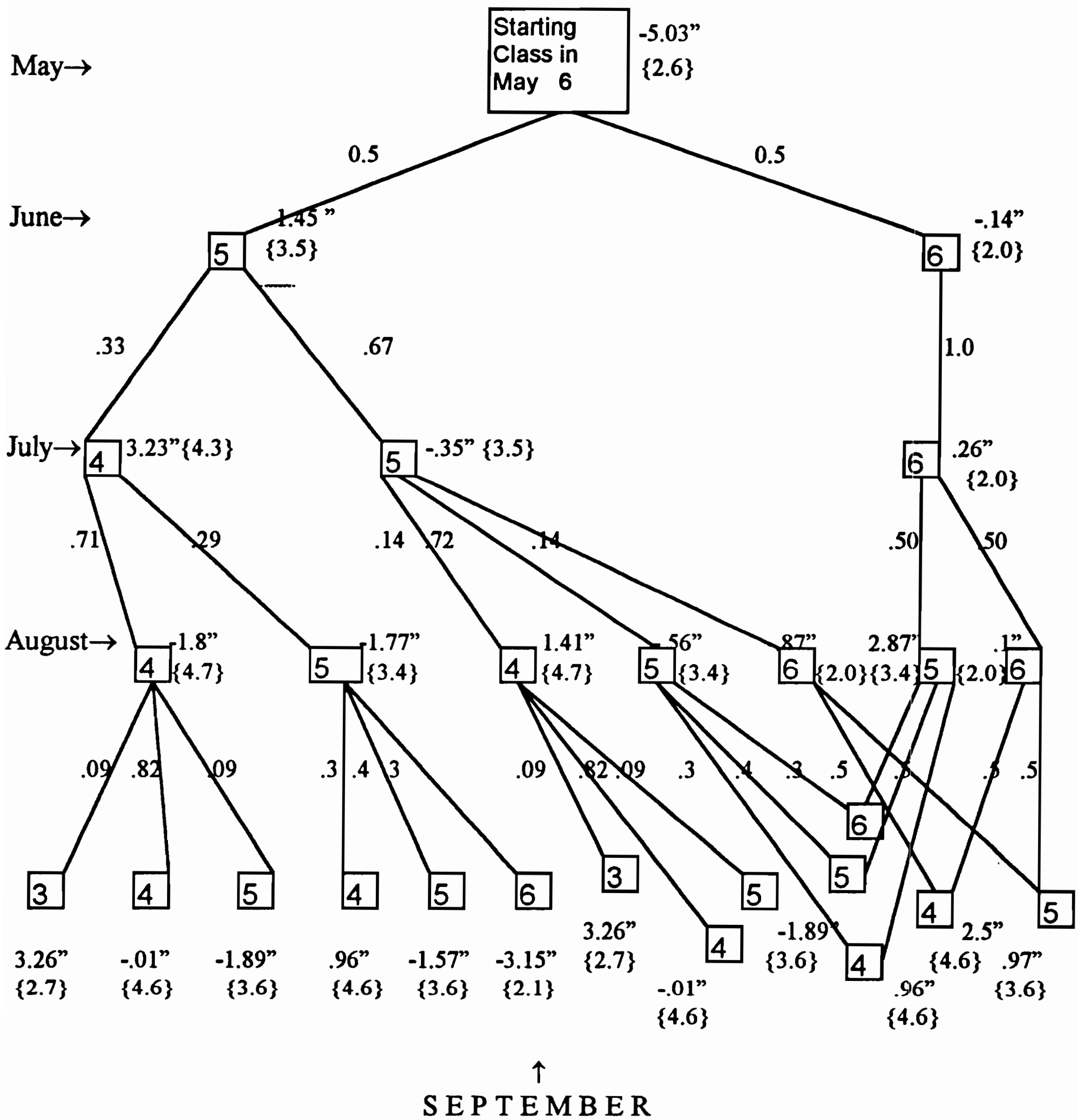


Figure 6.2 Decision Tree of Possible Actions, May State 6

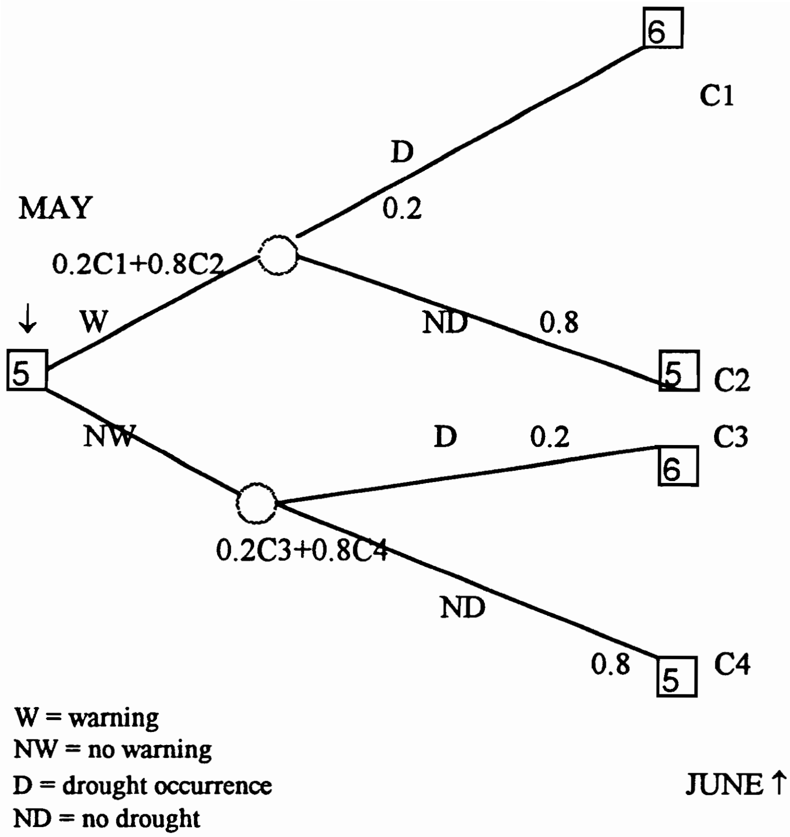
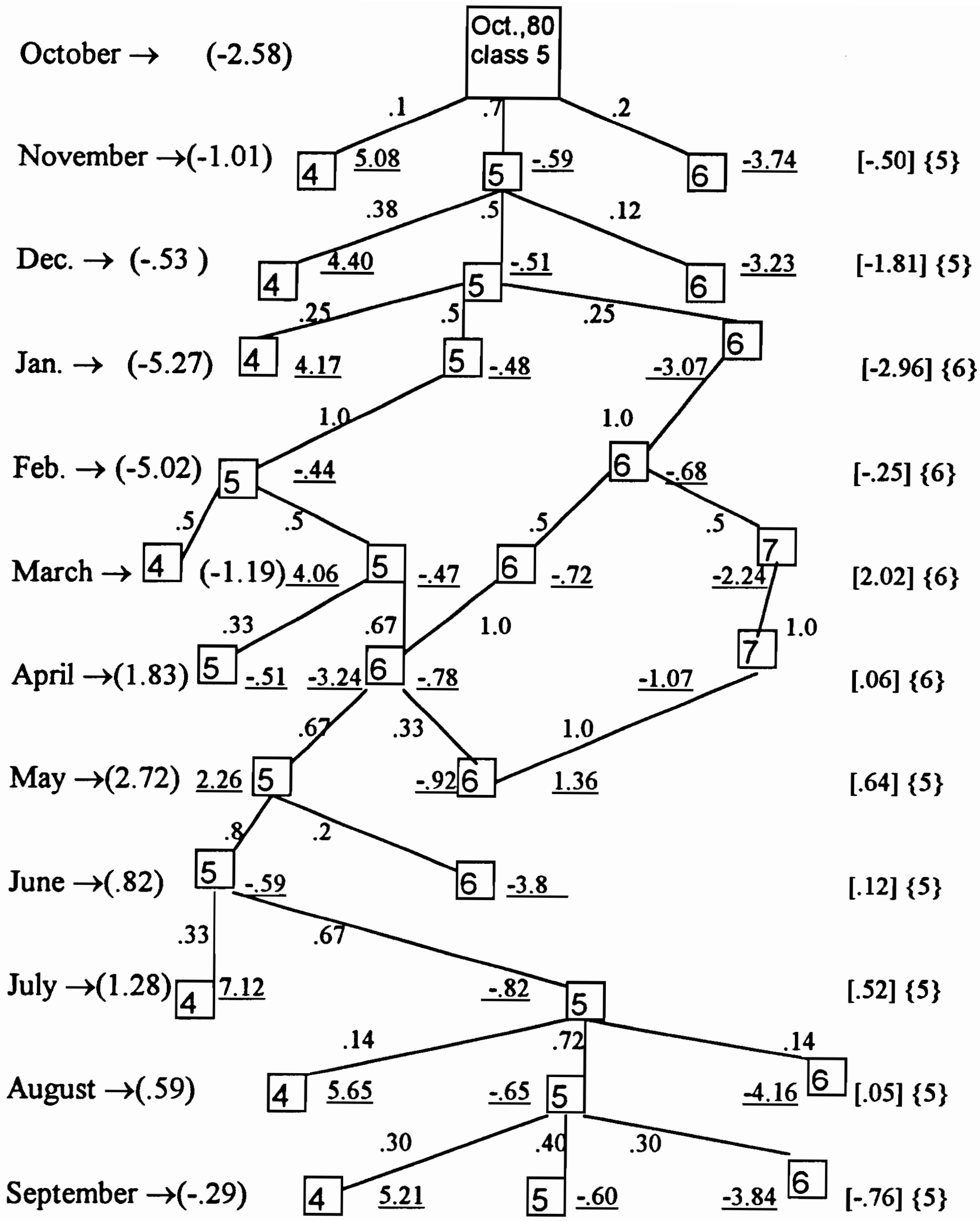


Figure 6.3. Utility Based Decision Tree



Underline values = expected monthly surplus / deficit amounts (inches) as per Table 6.28
[.] = observed monthly surplus / deficit (inches)
(.) = 3 month cumulative surplus / deficit (inches)
{.} = observed weather class; minus sign indicates deficit

Figure 6.4 Decision Tree Analysis- 1981 Drought Year

Table 6.1. Summary of Drought Occurrences, 1957 -1987, Tidewater Region, VA

Year	PR	PI	SF	GW	Year	PR	PI	SF	GW
1957					1973				
1958					1974				
1959					1975				
1960					1976		x	x	
1961					1977				x
1962					1978				
1963	x				1979				
1964					1980	x	x	x	x
1965	x	x	x		1981		x	x	x
1966		x	x		1982				
1967		x			1983			x	
1968	x	x			1984				
1969					1985		x	x	x
1970	x				1986	x	x	x	x
1971					1987			x	
1972									

x indicates drought condition; PR=precipitation; PI=PDSI; SF=streamflow;
GW= groundwater level

Table 6.2. Monthly Streamflow (cfs), Blackwater River at Luni, Tidewater Region, VA

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
1957	695	1759	1499	737	114	297	13.2	13.5	33.3	265	475	1598
1958	1022	91	1386	1368	1382	827	265	920	263	100	182	246
1959	1042	487	358	1131	235	58.9	446	207	39.6	330	415	584
1960	597	1392	1171	832	383	98.9	201	967	1507	279	287	326
1961	690	1404	1075	810	1015	478	216	39.3	17.8	910	424	1130
1962	1720	1271	1460	1126	279	231	795	183	29.2	25.4	395	400
1963	802	890	1326	276	180	1463	178	10.6	38.4	7	147	396
1964	922	1149	850	576	543	30	12	109	319	738	226	694
1965	643	828	954	747	247	87.8	120	43	10	2.7	2.8	7.66
1966	40.2	406	517	146	410	253	16.8	146	8	34.7	65.7	137
1967	659	807	656	262	257	41.7	56.9	500	153	63.9	59.2	758
1968	976	376	919	471	160	388	400	21.3	1.9	2.9	72.7	140
1969	508	959	1370	736	216	270	254	1248	104	134	152	526
1970	696	1092	742	1262	597	65.6	518	115	13.2	4.9	54.3	114
1971	254	875	746	882	373	307	89.8	131	50.2	1097	700	620
1972	517	1020	703	723	1115	460	517	355	141	1528	1179	1450
1973	865	1173	900	1168	532	416	300	141	16.8	18.8	34.8	562
1974	894	1039	698	630	159	116	32.9	130	419	25.4	41.4	301
1975	1092	1364	2373	1121	425	143	1434	325	1678	1041	721	719
1976	1252	1407	518	303	101	102	4.5	0.4	20.8	218	256	610
1977	793	473	1298	434	409	128	109	3	11.3	72	1233	863
1978	2155	782	1575	902	1376	248	107	820	48.2	35.1	117	832
1979	1693	1200	2145	1664	1346	1345	112	185	1893	537	1652	702
1980	1687	727	1154	792	346	31.2	2.6	0.1	0.1	0	0.4	6.19
1981	35.6	145	141	166	83.1	796	23.8	18.8	60.8	12.8	34.9	263
1982	779	1075	1340	329	348	232	204	807	24	70.7	252	1031
1983	630	1594	1421	1855	427	328	13.8	0.7	0.1	0.7	96.8	847
1984	1141	1259	1694	1905	442	403	314	328	23.4	2.3	13	183
1985	541	1014	471	155	62.3	126	96.5	300	625	983	1274	1192
1986	395	992	616	243	68.5	8.6	6.4	200	56	1.7	10.5	359
1987	2235	1539	1202	1731	615	849	58.6	3	3.6	1.2	19.2	288

Table 6.3. Groundwater Level Data 1977-1987, Well # 58B13, Tidewater Region, VA

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
77	9.6	9.7	7.9	9.1	9.2	9.4	10.6	11.6	12.2	12.4	10.1	7.7
78	5.9	7.4	5.4	7.2	6.9	7.9	9.9	10.6	11.4	12.2	12.6	12.1
79	8.1	5.4	5.9	6.5	5.1	7.5	8.2	8.1	8.6	9.3	7.9	8.7
80	6.9	7.1	6.4	7.9	8.5	9.8	10.9	11.7	12.5	12.9	13.3	13.3
81	13.4	12.6	12.3	12.2	11.8	11.2	11.6	12.1	12.4	12.8	13.0	12.3
82	8.6	5.8	6.5	8.0	8.9	9.3	9.9	9.4	9.4	8.3	8.0	6.9
83	7.2	5.7	6.3	5.9	7.7	8.2	9.7	10.7	10.7	10.6	8.7	6.9
84	6.6	5.8	5.3	5.4	7.9	8.6	8.2	9.5	10.4	10.9	11.3	11.0
85	9.5	7.4	7.4	8.9	10.2	10.3	10.8	11.3	11.5	10.9	7.6	7.3
86	8.0	6.9	8.2	9.0	10.1	11.0	11.4	11.8	11.9	12.7	13.1	13.3
87	8.3	5.9	6.6	6.5	7.9	9.3	10.6	11.1	8.9	10.2	10.5	10.1

Data represent groundwater level below ground in feet

Table 6.4 PDSI Data, Tidewater Region, 1957-87, Source: SWCB (1990)

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1957	1.53	2.04	2.24	-0.65	-1.17	-1.19	-2.2	0.01	0.1	0.8	1.66	2.62
1958	2.64	2.18	3.71	3.79	4.25	4.7	3.96	4.84	3.62	3.83	3.31	3.65
1959	-0.54	0.95	-0.88	-0.24	-0.98	-1.6	0.89	-0.87	-1.14	0.96	1.28	1.12
1960	0.91	1.35	0.04	-0.61	0.8	0.59	1.07	1.3	2.38	2.61	1.93	1.79
1961	1.79	2.55	2.47	2.14	3.05	3.81	2.79	2.2	1.23	1.89	1.3	2.11
1962	2.71	2.63	2.77	2.92	2.17	2.77	2.84	2.09	1.95	1.34	2.18	2.32
1963	2.03	1.98	2.16	-1.14	-1.28	1.76	0.94	-1.75	-1.25	-2.1	0.47	0.55
1964	0.88	1.68	-0.48	-0.29	-1	-1.23	-1.46	-1.45	0.89	1.72	-0.24	0.33
1965	-0.27	-0.58	-0.35	-0.5	-1.48	0.38	0.71	-0.77	-1.47	-2.13	-3.28	-4.47
1966	-4.21	-3.57	-4.06	-4.01	-3.14	-2.61	-2.88	-3.07	-2.49	-2.42	-3	-2.83
1967	-2.76	-2.14	-2.52	-3.19	-2.68	-2.8	-2.63	-1.66	-1.75	-2.11	-2.28	-1.01
1968	-0.94	-1.58	-1.15	-1.34	-1.24	-0.98	-1.12	-1.79	-2.35	-2.49	-2.45	-2.42
1969	-2.31	-1.97	-1.29	-1.51	-1.79	-1.7	0.39	0.93	0.83	0.47	0.27	1.34
1970	-0.88	0.95	-1.23	1.29	-0.55	-0.67	-0.46	-1.32	-2.07	-2.73	-2.73	-2.77
1971	0.03	0.49	0.44	0.08	1.14	-0.48	-0.96	-0.9	-1.16	1.49	1.45	-0.7
1972	-0.79	0.76	0.42	0.39	2.2	3.58	3.14	2.2	2.45	2.88	3.8	3.71
1973	3.17	3.1	2.79	2.88	2.7	2.9	-0.62	-0.45	-1.1	-1.46	-2.21	0.86
1974	0.93	0.81	1.06	0.55	-0.48	-0.08	-0.42	0.09	0.34	-0.43	-1.14	0.17
1975	0.88	1.09	2.86	2.79	2.3	1.79	2.58	1.65	3.43	3.53	3.13	3.23
1976	3.29	-0.79	-1.41	-2.39	-2.09	-1.94	-2.22	-2.67	0.48	1.7	1.57	1.73
1977	1.62	-0.28	-0.39	-0.94	-0.78	-1.13	-1.88	-2.59	-3.11	0.86	1.38	2
1978	3.47	2.45	3.58	3.62	4.21	4.2	3.64	3.1	2.01	1.11	1.14	1.22
1979	2.41	3.2	3.07	3.24	4.02	4.11	3.87	3.44	5.5	5.14	5.79	4.71
1980	4.76	3.97	4.27	0.05	-0.05	-1.06	-1.84	-2.98	-3.84	-2.9	-2.77	-2.97
1981	-3.56	-3.19	-3.45	-3.44	-2.66	-2.36	-2.21	-2.16	-2.12	-1.84	-2.35	0.77
1982	1.19	1.95	1.86	1.65	1.2	1.66	1.78	2.08	1.73	1.68	1.91	2.13
1983	1.48	2	2.58	3.81	3.58	3.39	-1.3	-2.39	-0.03	0.12	0.61	1.81
1984	1.96	2.05	3.47	3.94	4.17	-0.56	-0.09	-0.8	-1.41	-2.38	-2.56	-3.14
1985	-2.84	-2.16	-2.62	-3.77	-3.67	-3.33	-3.21	-3.09	0.77	1.3	2.39	-0.58
1986	-0.64	-0.73	-1.7	-2.23	-2.56	-3.3	-3.74	-2.94	-3.67	-3.92	-4.17	-3.12
1987	2.38	2.08	1.52	2.08	-0.71	-0.25	-1.27	-2.16	-1.91	-2.11	-2.09	-1.93

Table 6.5 Correlation Between PDSI and Streamflow Data, Tidewater Region, VA

Mon	J-J	F-F	MM	A-A	MM	J-J	J-J	A-A	S-S	O-O	N-N	D-D
CC	.72	.45	.74	.84	.73	.45	.50	.58	.64	.60	.75	.61

CC = correlation coefficient; J-J = January PDSI & January streamflows

Table 6.6 Correlation Between PDSI and Groundwater Level Data, Tidewater Region, VA

Mon	J-J	F-F	MM	A-A	MM	J-J	J-J	A-A	S-S	O-O	N-N	D-D
CC	-.78	-.72	-.83	-.87	-.84	-.90	-.62	-.79	-.64	-.59	-.65	-.60

CC = correlation coefficient; J-J = January PDSI & January groundwater levels

Table 6.7 Lowest Quartile Streamflows (25th percentile) , cfs, Blackwater River at Luni, Tidewater Region, VA

Mon	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug	Sep.	Oct.	Nov	Dec.
Q	496	695	642	530	175	83	29	19	11	9	51	127

Q = lower quartile streamflows (cfs)

Table 6.8 Long Term Average Groundwater Level in Well # 58B13, Tidewater Region, VA

Mon	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug	Sep.	Oct.	Nov	Dec.
GW	8.2	7.3	7.37	8.06	8.67	9.41	10.1	10.7	10.9	11.3	10.8	10.1

GW = groundwater level (feet below ground surface)

Table 6.9. Drought Years and Associated Rainfall(inch), Tidewater Area, VA, 1957-1987

Year	1965	1966	1968	1976	1980	1981	1985	1986
Rainfall	29.26	38.10	35.65	38.96	35.73	42.71	45.20	34.59
D. / S.	-14.61	-5.77	-8.22	-4.91	-8.14	-1.16	1.33	-9.28

D. / S. = deficit / surplus [minus sign indicates deficit], Long term average annual rainfall = 43.87 inch

Table 6.10. Deficit Characteristics of Drought Years, PDSI and Z Indices

Mon	Jan	Feb	Mar	Apr	May	Jun	Jul
Avg.	3.51	3.23	3.93	2.92	3.7	4.52	4.52
<u>1965</u>	2.38	2.07	3.55	2.43	1.11	4.51	6.40
D. / S.	-1.13	-1.16	-.38	-.49	-2.59	.81	1.88
Σ D/S	-1.13	-2.29	-2.67	-3.16	-5.75	-4.94	-3.06
Z	-1.22	-1.45	.35	-.47	-3.48	.73	1.48
K	1.08	1.25	-.92	.96	1.34	.90	.79
PDSI	-.27	-.58	-.35	-.50	-1.48	.38	.71
<u>1966</u>	4.21	3.64	1.61	2.04	4.58	3.53	3.31
D. / S.	.70	.41	-2.32	-.88	.88	-.17	-1.21
Σ D/S	.70	1.11	-1.21	-2.09	-1.21	-1.38	-2.59
Z	-1.24	.58	-2.55	-.98	1.52	.08	-1.70
K	1.77	1.41	1.10	1.11	1.73	-.47	1.40
PDSI	-4.21	-3.57	-4.06	-4.01	-3.14	-2.16	-2.88
<u>1968</u>	2.85	.97	4.32	2.40	3.15	3.72	4.4
D. / S.	-.66	-2.26	.39	-.52	-.55	.02	-.12
Σ D/S	-.66	-2.92	-2.53	-3.05	-3.60	-3.58	-3.7
Z	-.45	-2.72	.69	-.83	-.15	-.15	-.59
K	.68	1.20	1.77	1.60	.27	-7.5	4.9
PDSI	-.94	-1.58	-1.15	-1.34	-1.24	-0.98	-1.12
<u>1976</u>	4.01	1.66	2.50	0.92	3.73	3.35	3.58
D. / S.	.50	-1.57	-1.43	-2.0	.03	-.35	-.94
Σ D/S	.50	-1.07	-2.50	-4.50	-4.47	-4.82	-5.76
Z	.96	-2.84	-1.99	-3.25	.23	-.68	-1.45
K	1.92	1.81	1.39	1.63	7.67	1.94	1.54
PDSI	3.29	-.79	-1.41	-2.39	-2.09	-1.94	-2.22

Table 6.10 continue

1980	4.41	2.02	4.92	3.48	3.54	0.85	3.33
D. / S.	0.90	-1.21	.99	.56	-.16	-2.85	-1.19
Σ D/S	0.90	-.31	.68	1.24	1.08	-1.77	-2.96
Z	1.42	-1.31	1.92	.23	-.20	-3.52	-2.73
K	1.58	1.08	1.94	0.41	1.25	1.24	2.29
PDSI	4.76	3.97	4.27	.05	-.05	-1.06	-1.84
1981	.64	3.05	6.0	3.07	4.45	3.85	5.15
D. / S.	-2.87	-.18	2.07	.15	.75	.15	.63
Σ D/S	-2.87	-3.05	-.98	-.83	-.08	.07	.70
Z	-3.27	-.36	-1.81	-.74	1.37	-.57	.01
K	1.14	2.0	-.87	-4.9	1.83	-3.8	.02
PDSI	-3.56	-3.19	-3.45	-3.44	-2.66	-2.36	-2.21
1985	3.78	3.78	2.25	0.62	3.38	3.56	4.78
D. / S.	.27	.55	-1.68	-2.30	-.32	-.14	.26
Σ D/S	.27	.82	-.86	-3.16	-3.48	-3.62	-3.36
Z	-.09	.8	-2.02	-4.1	-.87	-.36	-.43
K	-.33	1.46	1.20	1.78	2.70	2.57	-1.65
PDSI	-2.84	-2.16	-2.62	-3.77	-3.67	-3.33	-3.21
1986	2.74	2.59	1.28	1.63	2.20	1.63	4.25
D. / S.	-.77	-.64	-2.65	-1.29	-1.50	-2.07	-.27
Σ D/S	-.77	-1.41	-4.06	-5.35	-6.85	-8.92	-9.19
Z	-.75	-.91	-3.14	-2.13	-1.93	-3.54	-2.41
K	.97	1.42	1.19	1.65	1.29	1.71	8.90
PDSI	-.64	-.73	-1.70	-2.23	-2.56	-3.30	-3.74

D. / S. = deficit / surplus (inch); Z = Z index; K = Z / (D. / S.); Σ D/S =Accumulated deficit / surplus

Table 6.11. Accumulated Deficit (inch) Upto Selected Months during Drought Years

Year	1965	1966	1968	1976	1980	1981	1985	1986	Avg.
April	-3.16	-2.09	-3.05	-4.50	1.24	-.83	-3.16	-5.35	-2.61
May	-5.75	-1.21	-3.60	-4.47	1.08	-.08	-3.48	-6.85	-3.10
June	-4.94	-1.38	-3.58	-4.82	-1.77	.07	-3.62	-8.92	-3.60

Avg. = average value of accumulated deficits

Table 6.12. Non- Drought Years and Associated Rainfall (inch), Tidewater Area, VA

Year	1961	1962	1975	1979	1984
Rainfall	49.30	48.27	55.56	60.91	43.52

Long term annual rainfall = 43.87 inch

Table 6.13. Analysis of Accumulated Deficits (inch), Non-Drought Years

Mon	Jan	Feb	Mar	Apr	May	Jun	Jul
Avg.	3.51	3.23	3.93	2.92	3.7	4.52	4.52
1961	3.39	5.21	4.24	2.63	6.29	5.66	2.93
D. / S.	-.12	1.98	.31	-.29	2.59	1.96	-1.59
Σ D/S	-.12	1.86	2.17	1.88	4.47	6.43	4.84
1962	5.16	3.34	4.18	4.16	2.65	5.37	5.39
D. / S.	1.65	.11	.25	1.24	-1.05	1.67	0.87
Σ D/S	1.65	1.76	2.01	3.25	2.20	3.87	4.74
1975	5.12	3.81	7.49	3.15	3.12	2.86	8.29
D. / S.	1.61	0.58	3.56	0.23	-.58	-.84	3.77
Σ D/S	1.61	2.19	5.75	5.98	5.40	4.56	8.33
1979	6.44	5.04	4.01	4.36	6.75	4.04	5.20
D. / S.	2.93	1.81	.08	1.44	3.05	0.34	0.68
Σ D/S	2.93	4.74	4.82	6.26	9.31	9.65	10.33
1984	3.77	3.92	6.81	4.76	5.31	2.23	6.44
D. / S.	0.26	0.69	2.88	1.84	1.61	-1.47	1.92
Σ D/S	0.26	0.95	3.83	5.67	7.28	5.81	7.73

D./ S. = deficit / surplus in inch; Σ D/S = accumulated deficit / surplus ; Avg. = long term average rainfall

Table 6.14. Accumulated Surplus (inch) Upto Selected Months During Non- Drought Years

Year	1961	1962	1975	1979	1984	Avg.
April	1.88	3.25	5.98	6.26	5.67	4.60
May	4.47	2.20	5.40	9.30	7.28	5.70
June	6.43	3.87	4.56	9.65	5.81	6.10

Avg. = average accumulated surplus value

Table 6.15. Low Rainfall Years and Rainfall (inch), Tidewater Area, VA, 1957-87

Year	1963	1967	1970	1974	1977
Rainfall	36.16	38.2	36.63	41.37	41.74

Long term annual average rainfall = 43.87 inch

Table 6.16. Analysis of Accumulated Deficits (inch), Low Rainfall Years

Mon	Jan	Feb	Mar	Apr	May	Jun	Jul
Avg.	3.51	3.23	3.93	2.92	3.70	3.70	4.52
1963	2.83	3.03	4.74	0.83	2.81	7.62	1.69
D. / S.	-.68	-.20	.81	-2.09	-.89	3.92	-2.83
Σ D/S	-.68	-.88	-.07	-2.16	-3.05	.87	-1.96
1967	2.75	3.53	2.14	1.29	3.52	2.08	4.66
D. / S.	-.76	.30	-1.79	-1.63	-.18	-1.62	0.14
Σ D/S	-.76	-.46	-2.25	-3.88	-4.06	-5.68	-5.54
1970	2.09	3.24	4.01	3.58	2.29	3.14	5.83
D. / S.	-1.42	.01	.08	0.66	-1.41	-.56	1.31
Σ D/S	-1.42	-1.41	-1.33	-.67	-2.08	-2.64	-1.33
1974	3.96	2.91	4.58	2.27	3.62	3.87	3.82
D. / S.	.45	-.32	.65	-.65	-.08	.17	-0.7
Σ D/S	.45	.13	.78	.13	.05	.22	-0.48
1977	3.10	2.16	3.57	2.35	3.88	2.15	3.27
D. / S.	-.41	-1.07	-.36	-.57	.18	-1.55	-1.25
Σ D/S	-.41	-1.48	-1.84	-2.41	-2.23	-3.78	-5.03

D./ S. = deficit / surplus in inch; Σ D/S = accumulated deficit / surplus ; Avg. = long term average rainfall

Table 6.17. Accumulated Average Deficit / Surplus Rainfall (inch) Starting January Month

Deficit / Surplus ↓	Drought Years	Non -Drought Years	Low Rainfall Years
April	-2.61	4.60	-1.79
May	-3.10	5.70	-2.27
June	-3.60	6.10	-2.20

Table 6.18. Percentage of Drought Events Starting Various Months, Tidewater Area, VA, Data Period 1895-1990

Mon	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
%	0	6	11	4	4	8	19	17	11	8	6	6

Drought event = weather state entering class 5

Table 6.19. Monthly Transition Matrices, Tidewater Region, VA, PDSI data, 1957-1987 (Data Source : SWCB, 1990)

January - February

0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.3333	0.3333	0.3333	0.0000	0.0000	0.0000
0.0000	0.1111	0.7778	0.1111	0.0000	0.0000	0.0000
0.0000	0.0000	0.2308	0.6923	0.0769	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000

February - March

0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429
0.3333	0.3333	0.3333	0.0000	0.0000	0.0000	0.0000
0.0000	0.2727	0.6364	0.0909	0.0000	0.0000	0.0000
0.0000	0.0000	0.0909	0.8182	0.0909	0.0000	0.0000
0.0000	0.0000	0.0000	0.5000	0.5000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.5000	0.5000
0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429

March - April

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.1111	0.6667	0.2222	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.8333	0.1667	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.3333	0.6667	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

April - May

0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429
0.8000	0.2000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.1667	0.5000	0.3333	0.0000	0.0000	0.0000
0.0000	0.0000	0.0769	0.9231	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.6667	0.3333	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000

May - June

0.7500	0.0000	0.0000	0.2500	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.2500	0.7500	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.1429	0.7857	0.0714	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.8000	0.2000	0.0000
0.0000	0.0000	0.0000	0.0000	0.5000	0.5000	0.0000
0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429

June - July

0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.3333	0.3333	0.3333	0.0000	0.0000	0.0000
0.0000	0.0000	0.6000	0.4000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.7500	0.2500	0.0000	0.0000
0.0000	0.0000	0.0000	0.3333	0.6667	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429

July - August

0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429
0.2500	0.5000	0.2500	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.7143	0.2857	0.0000	0.0000
0.0000	0.0000	0.0000	0.1429	0.7143	0.1429	0.0000
0.0000	0.0000	0.0000	0.0000	0.5000	0.5000	0.0000
0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429

August - September

0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.5000	0.0000	0.5000	0.0000	0.0000	0.0000	0.0000
0.0000	0.2000	0.6000	0.2000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0909	0.8182	0.0909	0.0000	0.0000
0.0000	0.0000	0.0000	0.3000	0.4000	0.3000	0.0000
0.0000	0.0000	0.0000	0.5000	0.5000	0.0000	0.0000
0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429

September - October

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.6000	0.4000	0.0000	0.0000	0.0000
0.0000	0.0000	0.2143	0.5714	0.2143	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.3333	0.3333	0.3333	0.0000
0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429

October - November

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.1667	0.5000	0.3333	0.0000	0.0000	0.0000
0.0000	0.0000	0.2727	0.6364	0.0909	0.0000	0.0000
0.0000	0.0000	0.0000	0.1000	0.7000	0.2000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429

November - December

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.8333	0.1667	0.0000	0.0000	0.0000
0.0000	0.0000	0.3000	0.7000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.3750	0.5000	0.1250	0.0000
0.0000	0.0000	0.0000	0.0000	0.5000	0.0000	0.5000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000

December - January

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.6667	0.0000	0.3333	0.0000	0.0000	0.0000
0.0000	0.1250	0.7500	0.1250	0.0000	0.0000	0.0000
0.0000	0.0000	0.0909	0.9091	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.2500	0.5000	0.2500	0.0000
0.0000	0.0000	0.5000	0.0000	0.5000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

Table 6.20. Computation of Rainfall Deficit / Surplus for Weather Class Transitions

Month								
M.-J.	5→5	5→6	6→5	6→6				
1967	-1.62							
1969	-.07							
1976	-.35							
1981	.15							
1986		-2.07						
1966			-.17					
1985				-.14				
Avg.	-.47	-2.07	-.17	-.14				
J.-Ju.	5→4	5→5	6→6					
1959	4.17							
1969	2.29							
1966		-1.21						
1967		.14						
1976		-.94						
1981		.63						
1985			.26					
Avg.	3.23	-.35	.26					
Ju.- A.	4→4	4→5	5→4	5→5	5→6	6→5	6→6	
1959	-2.40							
1960	1.39							
1964	.38							
1965	-2.26							
1969	2.17							
1970	-2.25							
1971	.87							
1973	1.52							
1974	.88							
1984	-2.09							
1963		-2.26						
1968		-.87						
1983		-2.19						
1987		-1.75						
1957			1.41					
1967				2.91				
1976				-1.61				

Table 6.20 cont.

1977				-1.25				
1980				-2.43				
1981				-.41				
1966					-.87			
1986						2.87		
1985							.1	
Avg.	-.18	-1.77	1.41	-.56	-.87	2.87	.1	
A.-S.	4→3	4→4	4→5	5→4	5→5	5→6	6→4	6→5
1960	3.26							
1957		.82						
1959		-.64						
1964		2.68						
1965		1.86						
1969		-.68						
1971		-.67						
1973		-1.77						
1974		.52						
1984		-2.19						
1970			-1.89					
1963				.81				
1976				1.52				
1983				.56				
1967					-1.8			
1968					-2.83			
1981					-1.63			
1987					-.01			
1977						-2.30		
1980						-3.16		
1986						-3.98		
1985							2.5	
1966								.97
Avg.	3.26	-.01	-1.89	.96	-1.57	-3.15	2.5	.97

Table 6.21. Weather Transition Events from May through September, Deficits / Surplus And Associated Probabilities

Events	Probability	Deficit / Surplus (inch)
55443	.0169	4.22
55444	.1537	0.95
55445	.0169	-0.93
55454	.0230	1.95
55455	.0306	-0.58
55456	.0230	-2.16
55543	.0067	3.85
55544	.0615	0.58
55545	.0067	-1.30
55554	.1158	-0.42
55555	.1544	-2.95
55556	.1158	-4.53
55564	.0375	0.81
55565	.0375	-.72
56656	.0300	-1.87
56655	.0400	-.51
56654	.0300	2.02
56664	.0500	0.79
55565	.0500	-0.74

minus sign indicates deficit

Table 6.22 Analysis of Rainfall Data (Water Year Basis)

Non - Drought Years		Drought Years		Average Years		Low Rainfall Years	
Year	Rainfall	Year	Rainfall	Year	Rainfall	Year	Rainfall
1958	58.76	1965	36.77	1961	44.15	1959	40.92
1960	54.00	1966	33.95	1969	44.25	1963	41.72
1962	49.55	1967	36.16	1974	45.08	1964	40.77
1972	50.46	1968	35.20	1983	44.19	1971	41.79
1973	47.33	1970	37.93	1987	45.03		
1975	52.41	1976	37.75				
1978	54.23	1977	37.04				
1979	60.56	1980	36.16				
1982	47.09	1981	42.94				
1984	51.87	1985	39.31				
		1986	36.47				

Annual Average Rainfall = 44.13 inch (water year basis)

Table 6.23 Three Months' Cumulative Deficit / Surplus (inches) Up To Various Months; Drought Years

Year	October	January	March	May
1965	4.66	-2.45	-2.88	-3.71
1966	-5.93	-4.72	-1.42	-2.57
1967	-.53	-3.21	-2.46	-3.85
1968	-.78	.19	-2.74	-.93
1970	.31	-.51	-1.54	-.92
1976	4.27	-.21	-2.71	-3.65
1977	2.64	-2.00	-2.05	-1.00
1980	6.34	1.47	.47	1.14
1981	-2.58	-5.27	-1.19	2.72
1985	-6.12	-1.49	-1.07	-4.55
1986	4.28	-.16	-4.27	-5.69

minus sign indicates deficit

Table 6.24 Three Months' Cumulative Deficit / Surplus (inches) Up To Various Months; Low Rainfall Years

Year	October	January	March	May
1959	3.33	-2.31	-3.7	-.67
1963	-2.53	1.71	-.28	-2.42
1964	-3.96	-.98	.73	-3.36
1971	-5.48	-.49	-.29	1.07

Minus sign indicates deficit

Table 6.25 Three Months' Cumulative Deficit / Surplus (inches) Up To Various Months; Normal Years

Year	October	January	March	May
1961	5.24	-3.12	1.96	2.36
1969	-3.90	-1.27	-.92	-1.77
1974	-.96	1.65	.57	-.33
1983	.02	-.49	.96	4.68
1987	-1.16	5.34	2.02	-.89

Minus sign indicates deficit

Table 6.26 Three Months' Cumulative Deficit / Surplus (inches) Up To Various Months; Non- Drought Years

Year	October	January	March	May
1959	3.33	-2.31	-3.7	-.67
1960	.57	-.91	-.88	.24
1962	.77	1.67	1.80	.19
1972	5.69	-3.15	-.46	.52
1973	.81	1.75	-.80	.77
1975	-.39	.18	5.54	2.96
1978	.19	6.12	3.21	5.30
1979	-3.82	4.43	4.61	4.32
1982	-1.10	.08	1.94	-1.25
1984	-.56	3.94	3.62	6.08

Minus sign indicates deficit

Table 6.27 Three Months' Cumulative Deficit / Surplus (inches) Up To Various Months

Category	January	March	May
Drought Years	-1.7 (-0.5 to -2.5)	-2 (-1 to - 3)	-2.15 (-1 to - 5)
Low RainfallYrs.	-.52	-.90	-1.35
Normal Years	.42	.92	.81
Non-Drought Yrs.	1.2	1.5	1.85

Minus sign indicates deficit; (.) = range of variation

Table 6.28 Deficit / Surplus Amounts (inches) Associated with Weather State Transitions

Transition→ Month ↓	4→5	5→6	6→7	5→5	6→6	7→7	7→6	6→5	5→4
J.-F.	-4.27	-2.82	-2.11	-.44	-.68	-.88	1.02	1.69	3.83
F.-M.	-4.53	-2.99	-2.24	-.47	-.72	-.93	1.08	1.79	4.06
M.-A.	-4.91	-3.24	-2.43	-.51	-.78	-1.07	1.17	1.94	4.40
A.-M.	-5.72	-3.77	-2.82	-.59	-.92	-1.18	1.36	2.26	5.10
M.-J.	-5.77	-3.80	-2.84	-.59	-.92	-1.18	1.38	2.28	5.17
J.-J.	-7.94	-5.24	-3.92	-.82	-1.27	-1.63	1.89	3.14	7.12
J.-A.	-6.31	-4.16	-3.11	-.65	-1.01	-1.30	1.51	2.50	5.65
A.-S.	-5.82	-3.84	-2.87	-.60	-.93	-1.20	1.39	2.30	5.21
S.-O.	-5.67	-3.74	-2.80	-.59	-.91	-1.17	1.35	2.24	5.08
O.-N.	-5.67	-3.74	-2.80	-.59	-.91	-1.17	1.35	2.24	5.08
N.-D.	-4.90	-3.23	-2.43	-.51	-.78	-1.07	1.17	1.94	4.40
D.-J.	-4.66	-3.07	-2.30	-.48	-.74	-.96	1.11	1.84	4.17

minus sign indicates deficit; J.-F. = January to February

Chapter VII

SUMMARY and RECOMMENDATIONS

In this dissertation, time series models, a Markov chain approach, and a dynamical systems approach have been proposed to predict droughts. The time series models reproduce the statistical moments of the historical data and lend an analytical framework for forecasting with designated confidence levels. Because they preserve the historical moments, they may be run for a long period called a generation scheme to mark the worst possibilities. In the specific application to the PDSI to overcome its bi-modal distribution nature, the Z index is considered. It is explained that the bi-modality is primarily due to the backtracking computation procedure. The Z index depends on the monthly deficit and a region and month dependent multiplying factor. It is shown to have a unimodal distribution and is recommended for time series analysis.

In the auto-regressive schemes, certain fractions of the past values are summed and a perturbation solely dependent on the variance of the random shock term is added to yield the predicted index value. This approach indicates that, no matter how many of the past realized values one can incorporate, still there is a variability in making the prediction for the next period. The variance of the random term and the coefficients are determined by the second order (covariance) moments of the data series.

In comparison, in the Markov chain approach data is stratified into classes, and class to class transition probabilities are estimated. These transition probabilities are then manipulated to obtain the parameters helpful in drought planning such as the occurrence probability, expected duration, and first times of return. The method also predicts the most likely and expected drought classes. Within a decision making framework, the method lays out all possible occurrences with the associated probabilities. This decision tree enables a decision maker to follow the realized path exactly and from that point to follow a future branch based on the drought progressive pattern, associated probabilities, and some secondary measures of drought. If a value system can be imposed, the decision on whether to issue a drought warning or not can be decided at each node of the tree. Such a value assignment should be possible because the government agencies routinely assess the damage in terms of dollars after certain disasters.

Based on the idea that runoff processes are local in nature but the precipitation process is determined on a global basis, a dynamical systems formulation is also considered. The global forcing is brought in through the Southern Oscillation index (SOI) and the Sea Surface Temperature (SST) index. An important benefit is the determination of the probability density function of the drought process by analytical means through the Fokker Planck equation. This should be compared with the time series analysis in which a normal distribution is assumed and therefore proper transformations to data are necessary; in the Markov chain a distribution free, frequency interpretation of probability is employed. However, in terms of the end applications the Markov chain approach and the dynamical systems approach serve the same purpose because both of them exploit the conditional probability.

It is also anticipated that the drift and the diffusion functions employed in the Fokker Planck equation would respond more quickly to sudden changes in the weather as opposed to obtaining the probabilities from long term data. The dynamical systems approach and the time series analysis deal with the index values themselves while the Markov chain deals with the index classes. Note that there is a limit to shortening the class interval in the Markov chain in order to obtain stable transition probabilities. However, in the dynamical analysis such restrictions are not encountered, except for the difficulties faced in synthesizing the drift and diffusion functions. In this dissertation, for an empirically obtained drift function, a piecewise linear approximation scheme is suggested. When the linearization interval gets shorter, a segment dependent but constant diffusion value is acceptable. This scheme leads to the result that by employing suitably determined Gaussian distribution for each segment, any true distribution (without the linearization) can be approximated well.

The practical values of the results are in providing the parameter values for assessing the drought proneness of a region; for developing buffer resources to cope with the droughts based on the expected duration and times of return; for deciding then and there what mitigation measures to be taken during the course of a drought based on the decision tree analysis.

For future the research in this area the following recommendations are made :

i) In the time series analysis relate the residual term to the physical processes and obtain a derived distribution. The Palmer CAFEC (Climatically Appropriate For Existing Conditions) deficit equation has some difficulties in determining the covariance structure of certain variables such as recharge and zonal soil moisture storage.

ii) In the Markov chain analysis one may explore weekly chains with larger number of states. Evaluate reward / penalty functions for the decision tree based on governmental disaster damage assessment procedures.

iii) a) In the dynamical systems approach develop assessment methods for the drift and diffusion functions.

b) develop numerical procedure for the general forms of the Fokker - Planck equation.

c) explore various choices of global forcing from appropriate source regions. Note in this context results from the general circulation models can be combined.

iv) The waterbalance models can be improved to yield more accurate representation of the index values.

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PALMER DROUGHT SEVERITY INDEX (PDSI)

The PDSI was developed by W.C. Palmer in year 1965 . It was published as a research paper (#45) entitled 'Meteorologic Drought' by the U.S. Weather Bureau , U.S. Dept. of Commerce (Palmer, 1965). A brief description of the index computation procedure is as below:

PROCEDURE

The computation procedure of Palmer index begins with water balance calculations (usually done on a monthly or weekly basis) in the area under consideration using historic record of precipitation and temperature. The soil is divided into two layers, the upper layer, which is assumed to retain 25 mm of available water, and the lower layer, for which the local soil conditions determine the water holding characteristics. It is assumed that moisture cannot be removed from (or recharged to) the lower layer until all the available moisture has been removed from (replenished in) the upper layer. Calculations of the potential evapotranspiration (PE) are done using Thornthwaite's method (Thornthwaite, 1948) . Whenever PE is greater than precipitation (P) for a month, evapotranspiration losses occur from the soil. The loss of water from upper and lower layers is calculated using separate methods as follows. If PE > P , then :

$$L_S = \text{Min} \{ S_S , (PE-P) \} \quad (1)$$

$$L_u = \{ (PE - P) - L_S \} S_u / AWC \ , L_u \leq S_u \quad (2)$$

where : L_S = loss of water from upper layer

L_u = loss of water from lower layer

S_u = water stored in the lower layer in the beginning of the month

S_S = water stored in the upper layer in the beginning of the month

Three other terms are computed as a part of water balance , namely , potential recharge (PR), potential loss (PL) and potential runoff (PRO). Potential recharge is defined as the amount of moisture required to bring the soil to field capacity or,

$$PR = AWC - (S_s + Su) \quad (3)$$

where : AWC = combined available capacity of both soil layers

Potential loss is defined as the amount of moisture that could be lost from the soil to evapotranspiration provided precipitation during the period was zero or,

$$PL = PL_s + PL_u \quad (4)$$

where : $PL_s = \min(PE, S_s)$ = potential loss from upper layer

$PL_u = (PE - PL_s) S_u / AWC ; PL_u \leq S_u$

= potential loss from lower layer

Potential runoff is defined as potential precipitation minus potential recharge. Potential precipitation is taken equal to AWC. Therefore,

$$PRO = AWC - PR = S_s + S_u \quad (5)$$

Based on long term available records of precipitation and temperature , water balance on monthly or weekly basis is done and all components as described above are computed for each month. Having completed the water balance , the long term averages of these components are computed and four coefficients are defined as given below:

$$\alpha_j = \overline{ET} / \overline{PE}$$

$$\beta_j = \overline{R} / \overline{PR}$$

$$\gamma_j = \overline{RO} / \overline{PRO}$$

$$\delta_j = \overline{L} / \overline{PL}; j = 1,2,\ldots,12 \quad (6)$$

The overbar indicates long term average values of respective components. A separate set of coefficients is determined for each of 12 months which vary from place to place. These coefficients represent ratio of long term averages of actual and potential values of a particular water balance component. For example, if $\alpha = .7$ for some region for January month, it means on an average in that region actual ET is 70% of the PE in the month of

January. These coefficients are then used to compute cilmatically appropriate for existing conditions (CAFEC) precipitation (P^*). P^* ,therefore, indicates the amount of precipitation that should actually occur in an area to meet normal (average) demands and is given by ;

$$P^* = \alpha_j \; PE + \beta_j \; PR + \gamma_j \; PRO - \delta_j \; PL \qquad 6(a)$$

Now based on values of actual and CAFEC precipitation values, water deficiency or surplus is computed as :

$$d = P - P^* = P - (\alpha_j \; PE + \beta_j \; PR + \gamma_j \; PRO - \delta_j \; PL) \qquad (7)$$

where: P = actual precipitation during month

A moisture anamoly index , Z, is then defined as :

$$Z = d \, . \, k \qquad (8)$$

where : k is a weighting factor which varies depending upon the month and the location

The purpose of the weighting factor is to adjust the departures from normal precipitation such that they are comparable among different areas and different months. k tends to be large in arid regions than in humid regions. Therefore, same value of d will give higher value of Z index in arid region than in humid region which is logical in the sense that a deficit may not be considered that serious in a humid area as in an arid area.

The Z index, therefore, gives relative departure of weather of a particular month and location from the average moisture condition of that month in the area under consideration. Palmer evaluated the accumulation of the Z index for the 13 driest intervals in his two original study areas (central Iowa and western Kansas) and found a linear relationship betwen accumulated Z and length of dry period. The accumulated Z values were considered to represent extreme drought condition and were, therefore, assigned a numerical drought severity of -4.0. A horizontal line indicated normal condition and the body of graph between normal and -4.0 intensity lines was arbitrarily divided to represent mild, moderate and severe drought conditions. Palmer fitted the following equation for severe drought (index value -4) :

$$-4 = \sum_{n=1}^{i} Z_n /(.309\, i + 2.691) \qquad (9)$$

where : i = duration over which deficit Z_n is to be accumulated. Palmer proposed the drought index for "i months duration" as

$$X_i = \sum_{n=1}^{i} Z_n /(.309\, i + 2.691) \qquad (10)$$

From which we obtain :

$$dX_i = X_i - X_{i-1} = [\sum_{n=1}^{i} Z_n - \sum_{n=1}^{i-1} Z_n \frac{.309i+2.691}{.309(i-1)+2.691}] /(.309\, i+2.691)$$

(11)

If we can approximate the ratio in the second term within the brackets to be unity we obtain :

$$dX_i = Z_i /(.309\, i + 2.691) \qquad (12)$$

Specially for unit time period increments , i.e. duration i=1 we obtain :

$$dX_i = Z_i / 3 \qquad (13)$$

which was somewhat arbitrarily generalised by Palmer as :

$$dX_i = (Z_i/3) + C\, X_{i-1} \qquad (14)$$

If we use Eq. 10 , we have :

$$\sum_{n=1}^{i} Z_n = X_i \; (.309\, i + 2.691) \qquad (15)$$

and;

$$\sum_{n=1}^{i-1} Z_n = X_{i-1} \,(.309\, i -.309 +2.691) \qquad (16)$$

From Eqs. 15 and 16 we obtain :

$$Z_i = (.309\, i +2.691)\, dX_i + .309\, X_{i-1} \qquad (17)$$

Using Eq. 17 in Eq.14 yields the coefficient C as :

$$C = .103 \; \{ (1-i) [\frac{X_i}{X_{i-1}} - 1] - 1 \} \qquad (18)$$

When $X_i = X_{i-1}$, Eq. 18 yields $C = -.103$. Palmer chose to use this value as the general coefficient in Eq. 14. Therefore :

$$dX_i = (Z_i/3) - .103\, X_{i-1} \tag{19}$$

$$X_i = .897\, X_{i-1} + Z_i/3 \tag{20}$$

Note the dependence of dX_i on X_{i-1} and the random perturbation in Z_i

The weighting factor (k) was earlier computed as the ratio of avarage water demand (PE +R) and average water supply (P+L) which did not give satisfactory results. It was then argued that to maintain a severity of -4.0 for 12 months , Σ Z should be -25.60 which is calculated as below:

$$X_i = -4.0 \;;\; i = 12\;;$$

So; $$Z = -4.0\ [\,2.691 + (.309 \times 12)] = -25.60$$

Now this Σ Z of 25.60 when divided by sum of 12 months' departure 'd' should give avarage k value or;

$$k = -25.60 \Big/ \sum_{i=1}^{12} d_i \tag{21}$$

Based on information available from 9 different places, a semi-log plot was developed between k and [(PE+R+RO)/(P+L) + 2.8] / D and the following equation was fitted:

$$k = 1.5 \log \{ [(PE+R+RO)/(P+L) + 2.8\,] \,/\, D\} + .50 \tag{22}$$

Here, it was assumed that the weighting factor (k) should depend upon average water supply and water demand but the water demand should also include the avarage runoff (R). Also, it was assumed that k varies inversely with D, which is the mean of the absolute values of d . The monthly weighting factor k_j was then finally expressed as :

$$k_j = \frac{17.67\, K_j}{\sum_{i=1}^{12} D_i \cdot K_i} \quad , \quad j = 1,2,\ldots, 12 \tag{23}$$

Having established the above equations, Palmer(1965) devised a method to establish the beginning and end of drought or wet spell. A brief description of various steps involved in computation of the index is given as below :

Step 1 : Definitions of X_1, X_2, and X_3

In order to establish the beginning and end of drought or wet spell , Palmer (1965) suggested to compute three indices as below:

X_1 = severity index for a wet spell that is becoming established

X_2 = severity index for a drought that is becoming established

X_3 = severity index for any wet spell or any drought that has become established

Step 2 : Established Spells

Drought spell established means X_2 or $X \leq -1.00$

Wet spell established means X_1 or $X \geq 1.00$ (24)

Step 3 : End of Established Spells

An established drought or wet spell is considered to definitely end when the index reaches the near normal category which lies between -.50 and .50

Step 4 : Criteria for Ending Established Drought Spell in ith Month

This happens when $Z_i \geq Z_e(i)$

where: Z_i = Z index for ith month

$Z_e(i)$ = moisture needed to reduce the severity of an established drought spell to -.5 in a single month. This is computed as below:

$$Z_e(i) = -2.691\ X_3(i-1) - 1.5 \quad (25)$$

This equation is obtained by replacing X_i by X_{3i} and putting its value as -.5 in Eq. 20.

Step 5 :. Criteria for Ending Established Wet Spell in ith Month

This happens when $Z_i \leq Z_e(i)$

where: Z_i = Z index for ith month

$Z_e(i)$ = moisture to be lost to increase the severity of an established wet spell to .5 in a single month. This is computed as below:

$$Z_e(i) = -2.691\ X_3(i-1) + 1.5 \tag{26}$$

This equation is obtained by replacing X_i by X_{3i} and putting its value as .5 in Eq. 20.

Step 6 : 'Z' Value for Ending Drought spell

The Z value needed to maintain index of -.50 month to month will be:

$$-.5 = .897(-.5) + Z/3 \quad \text{or;} \quad Z = -.15 \tag{27}$$

So, Z index value > -.15 will tend to end a drought spell.

Therefore, a term effective wetness (U(i)) is defined as:

$$U(i) = Z_i + .15 \tag{28}$$

Effective wetness is used when termination of dryness is probed.

Step 7. 'Z' Value for Ending Wet spell

The Z value needed to maintain index of .50 month to month will be:

$$.5 = .897(.5) + Z/3 \quad \text{or;} \quad Z = .15 \tag{29}$$

So; Z index value <.15 will tend to end a wet spell.

Therefore, another term effective dryness (U(i)) is defined as:

$$U(i) = Z_i - .15 \tag{30}$$

Effective dryness is used when termination of wet spell is probed.

Step 8 : Probability of Ending Drought during i^{th} Month

Palmer (1965) used the following equation for computing probability of ending of an established spell:

$$P_e(i) = \sum_{j=0}^{j^*} U(i-j) \;/\; (Z_{ei} + \sum_{j=1}^{j^*} U(i-j)) \qquad (31)$$

In Eq. 31, j^* corresponds to number of successive values of U(i) computed immediately prior to the current month. For example, suppose in a particular year established drought conditions continued till December month. This means the index (X) remained ≤ -1.0 until December. Suppose in January the Z index is ≥-.15 , so it will tend to end the drought. Therefore, we will calculate the probability of ending the drought in January as follows:

$$P_{eJan} = U_{Jan} \,/\, Z_{eJan} \quad (\text{put i=jan. and } j^* = o \text{ in Eq. 31}) \qquad (32)$$

= (Effective wetness in Jan.) / (Moisture needed to bring index to -.5)

or; $$P_{eJan} = (Z_{Jan} + .15) / (-2.691\, X_{3Dec} - 1.5) \qquad (33)$$

Here all values are known so the probability can be calculated. If this value comes out to be 100% then the drought spell will be considered to have ended in the month of January. If not, then the probability will be computed for February month as below:

P_{eFeb} = (moisture accumulated till Feb.) / (Moisture accumulated in Jan. + Moisture needed to end drought in the month of Feb.) or;

$$P_{eFeb} = (U_{Jan} + U_{Feb}) \,/\, (U_{Jan} + Z_{eFeb}) \quad (\text{put i=feb and } j^*=1 \text{ in Eq. 31}) \qquad (34)$$

Similarly; $$P_{eMar} = (U_{Jan} + U_{Feb} + U_{Mar}) \,/\, (U_{Jan} + U_{Feb} + Z_{eMar}) \qquad (35)$$

This way we keep computing P_e for all months until we get either zero or 100 value. If anytime the numerator is less than zero, it is taken zero. In the same way probability of ending wet spell is computed . Here the effective wetness is replaced by the effective dryness and Z_e as mentioned for the wet spell in step 5 above is used.

<u>Step 9 : Index Assignment Procedure</u>

Suppose, at any particular time, a dry spell is established, which means the index value X is ≤ -1.0 . Now, observe the subsequent month's Z index value. Whenever, Z ≥ -.15, it means the ongoing drought may end. So, start calculating the P_e for the following months. Suppose during these computations, the P_e becomes zero before attaining value equal to 100. Then for all months the index value is assigned equal to X_3 value. If, however, the P_e becomes 100, then we start assigning index value backwards. We assign index value equal to X_1 till it has positive value. Once X_1 becomes zero, we assign X_2 value to the index till X_2 becomes zero. X_1 and X_2 can not be zero at the same time only except the case when a spell is established. When a spell is established, the index value is equal to X_3.

In view of the back tracking procedure involved in computation of PDSI, it is not used as a real time index for making operational decsions. The necessity for an index that could be used operationally led to an adjusted PDSI known as Palmer Hydrological Drought Index (PHDI) (Karl, 1986). The main differences between PDSI and PHDI are listed in Table I.1.

In order to illustrate the procedure of assignment of both indices i.e. PDSI and PHDI, sample computations have been done in respect of a climatic division in California and the values of indices agree with the data provided by N.C.D.C., Asheville, N.C. These computations are given in Table I.2.

It can be observed from Table I.2 that the Z index during January, 1895 is 4.08. Therefore, using Eq. 20 the PDSI and PHDI values are computed as 1.36 assuming values of these indices during previous month as zero. Since index values are > 1.00, it is a situation of established wet spell. Therefore, only X_3 computations are made and we watch values of Z index in subsequent months to explore possibilities of ending wet spell. During March 1895 the Z index falls to -1.06 which is less than .15 and as per step 7 above, we will now explore possibility of ending wet spell by computing the probabilty term P_e. The P_e for March is calculated as :

$$P_e(\text{March}) = U_{Mar} / Z_{eMar} = (Z_{mar} - .15) / (-2.691\, X_{3Feb} + 1.5)$$
$$= (-1.06 - .15) / (-2.691(1.33) + 1.5) = 58\ \%$$

Now since $0 < P_e < 100$, we start calculating X_1, X_2, and X_3 using Eq. 20. We get $X_1 = -.35$; $X_2 = -.35$ and $X_3 = .84$. Since X_1 can not be negative, we put it to be zero.

For March we assign PHDI = .84 but we can not assign PDSI at this stage since $0 < P_e < 100$. We keep computing P_e for subsequent months. For example :

$$P_e(\text{April}) = (U_{mar} + U_{apr}) / (U_{mar} + Z_{eApr})$$

$$= \{(-1.06 - .15) + (0 - .15)\} / \{(-1.06 - .15) + (-2.691 X_{3Mar} + 1.5)\}$$

$$= 69\ \%$$

The P_e becomes zero in the month of June 1895. Therefore, the wet spell did not end and we assign values of X_3 to PDSI. In July of 1896 again the possibility of ending wet spell arose and once again P_e values are calculated. The P_e value became 100% in January 1897 which indicated that the wet spell got ended. The PDSI values are assigned backwards from January 1897 to July 1896. The PHDI value changed sign first time since P_e became 100 %. It can be observed from that Table that most of time the PDSI and PHDI values are similar. They only differ when we explore possibilty of ending of an established spell.

Finally, some important points are listed as below:

i) During established spell, $P_e = 0.0$, and we compute X_3 only

ii) For established drought spell, if $Z \geq -.15$, P_e calculations start. For established wet spell, if $Z \leq .15$, P_e calculations start. If $0 < P_e < 100$, we compute all X_1, X_2 and X_3. If $X1$ comes negative, we put $X1=0.0$, if $X2$ comes positive, we put $X2= 0.0$

iii) For P_e calculations, if denominator becomes negative, we put it equal to zero.

iv) If P_e becomes 100, then from next month onwards we compute X_1 and X_2 only and do not compute P_e. We keep computing X_1 and X_2 and when either $X_1 \geq 1.00$ or $X_2 \leq -1.00$ we assume the corresponding spell has been established, and compute then X_3 only by transferring the calculations to X_3. Now we monitor the Z index value for the coming months and as per condition specified in ii) above, P_e calculations are started and the process goes on.

Note: Heddinghaus and Sabol (1991) have reported a procedure that has been introduced since June, 1989 at N.C.D.C., Asheville to overcome back tracking difficulty of PDSI.

This method takes the sum of the wet and dry terms after they are weighed by their probabilities during the period $0 < Pe < 100$. For established spells, procedure remains unaffected.

Table I.1 Differences between PDSI and PHDI

PDSI	PHDI
i) It is the original version of Palmer (1965) index. It can not be used in operational applications due to its backtracking procedure.	It is the operational version of the Palmer index and can be used in operational purposes as it aviods backtracking procedure.
ii) PDSI abruptly returns to near-normal levels during first month in a sequence of months with sufficient moisture to end a drought.	ii) PHDI usually more gradual in its return to near normal levels.
iii) The PDSI and PHDI are identical during an established spell. They, however, differ during the onset and ending of a spell. The PDSI values are assigned following the P_e values during these periods.	iii) The PHDI does not change sign until P_e equals 100%. The values of PHDI are regularly assigned while $0 < P_e < 100$ but PDSI values are assigned only when P_e is either 0 or 100%.
iv) PDSI is generally classified as meteorological index.	iv) PHDI is generally classified as a hydrological index.

Table I.2 Index Assignment Calculations, California, San Jaoquin Division, Jan. 1895- Jan. 1897

Month	Z	P_e (%)	X_1	X_2	X_3	PDSI	PHDI
Jan.1895	4.08	0.0	0.0	0.0	1.36	1.36	1.36
Feb.	0.33	0.0	0.0	0.0	1.33	1.33	1.33
Mar.	-1.06	58.0	0.0	-.35	.84	.84	.84
Apr.	0.0	69.0	0.0	-.31	.75	.75	.75
May	.96	29.0	.32	0.0	1.00	1.00	1.00
Jun.	2.94	0.0	0.0	0.0	1.88	1.88	1.88
Jul.	2.14	0.0	0.0	0.0	2.40	2.40	2.40
Aug.	1.20	0.0	0.0	0.0	2.55	2.55	2.55
Sep.	.64	0.0	0.0	0.0	2.50	2.50	2.50
Oct.	-1.03	22.5	0.0	-.34	1.90	1.90	1.90
Nov.	-.32	34.0	0.0	-.42	1.60	1.60	1.60
Dec.	-1.59	76.0	0.0	-.91	.90	.90	.90
Jan.1896	2.78	17.0	.93	0.0	1.73	1.73	1.73
Feb.	-2.63	90.0	0.0	-.88	.68	.68	.68
Mar.	.76	75.0	.25	-.54	.86	.86	.86
Apr.	3.28	0.0	0.0	0.0	1.87	1.87	1.87
May	1.1	0.0	0.0	0.0	2.04	2.04	2.04
Jun.	.94	0.0	0.0	0.0	2.48	2.48	2.48
Jul.	-.79	18.0	0.0	-.26	1.96	-.26	1.96
Aug.	-.72	38.0	0.0	-.47	1.52	-.47	1.52
Sep.	.01	28.0	0.0	-.42	1.36	-.42	1.36
Oct.	-.81	71.0	0.0	-.65	.96	-.65	.96
Nov.	1.2	47.0	0.0	-.18	1.26	-.18	1.26
Dec.	-.62	70.0	0.0	-.37	.92	-.37	.92
Jan.1897	-1.76	100.0	0.0	-.92	0.0	-.92	-.92

APPENDIX II

STOCHASTIC PROCESSES

A stochastic process is a system which evolves in time while undergoing chance fluctuations. Such a system can be described by defining a family of random variables, $\{X_t, t \in T\}$, where X_t measures, at time t, the aspect of the system which is of interest. The family $\{X_t, t \in T\}$ may be thought of as the path of a particle moving randomly in space S, its position at time t being X_t. A record of one of these paths is called a realisation of the process. As an example, X_t might represent number of cars passing a particular intersection at time t. As time passes, cars will arrive and leave, so the value of the random variable X_t will change with time. At any time, X_t takes, let's say, one of the values 0,1,2,.... which are called states of the process and these possible values of X_t are referred to as the state space. The state space is discrete if it contains a finite number of points ; otherwise it is continuous. If the parameter set T is a set of integers or a subset thereof, the process is called a discrete parameter process. If T is a subset of real line, then the process is called a continuous parameter process. The process $\{X_t, t \in T\}$ may also be multi-dimensional. For example, $X_t = (X_{t1}, X_{t2})$ represents a two-dimensional stochastic process where for example, X_{t1} represents the maximum and X_{t2} represents the minimum temperature at a place at time t. Conventionally, in case of discrete time , the parameter generally used with random variable X is 'n' i.e. the family of random variables is represented by $\{X_n, n=0,1,2,...\}$. In case of continuous time both the symbols $\{X_t, t \in T\}$ and $\{X_{(t)}, t \in T\}$ are used. The parameter t is usually interpreted as time, though it may represent such characters as distance, length , thickness and so on. Some authors call the discrete parameter family as the stochastic sequence, and the continuous parameter family as a stochastic process. It is of interest to know the relations between the random variables (X_ts) for different fixed values of t which is studied using the theory of probability. In some cases, the random variables X_t, i.e. members of the family $\{X_t, t \in T\}$ are mutually independent but most generally we come across processes whose members are mutually dependent. It is this relationship among the random variables which is important in studying stochastic processes.

MARKOV PROCESS

The Markov assumption is formulated in terms of the conditional probabilities. If the ordering of times is : $n \geq n-1 \geq \dots \geq 2 \geq 1$; the conditional probability is determined entirely by the knowledge of the most recent condition, i.e.

$P [X_n = a_n \mid X_{n-1} = a_{n-1}, X_{n-2} = a_{n-2}, \dots, X_1 = a_1]$ will be same as :

$$P [X_n = a_n \mid X_{n-1} = a_{n-1}] \qquad (1)$$

In other words, the Markov process has the property that, given the present state of the process, the future state becomes conditionally independent of the past. The Markov assumption is extremely powerful . For it means that we can define everything in terms of the simple conditional probability $P [X_n = a_n \mid X_{n-1} = a_{n-1}]$. For example, the joint density $P [X_n = a_n, X_{n-1} = a_{n-1}, \dots, X_1 = a_1]$ can be expressed simply as:

$$P (X_n = a_n \mid X_{n-1} = a_{n-1}) . P (X_{n-1} = a_{n-1} \mid X_{n-2} = a_{n-2}) \dots P (X_2 = a2 \mid X_1 = a_1) . P (X_1 = a_1) \qquad (2)$$

A discrete Markov parameter process is called a Markov chain. A Markov chain is said to be stationary or homogeneous in time if the probability of going from one state to another is independent of the time at which the step is being made i.e. for all states i and j the following holds good:

$$P [X_n = j \mid X_{n-1} = i] = P [X_{n+k} = j \mid X_{n+k-1} = i]$$

$$\text{for } k = -(n-1), -(n-2), \dots, -1, 0, 1, 2, \dots \qquad (3)$$

The Markov chain is said to be non-stationary or non-homogeneous if the condition stated in Eq. 3 fails.

GAUSSIAN PROCESS

If the joint distribution of (X_{t1}, X_{t2}, ..., X_{tn}) is multivariate normal for all t1,t2,..., tn, the process [X_t] is said to be a Gaussian process. Two important entities associated with any Gaussian process [X_t] are its mean μ_t and auto-covariance C_{ts} which are defined as:

$$\mu_t = E[X_t] \quad \text{and} \quad C_{ts} = E[(X_t - \mu_t)(X_s - \mu_s)] \qquad (4)$$

Both of above quantities are defined for all t ,s in T. The knowledge of these two entities is sufficient to characterise a Gaussian process completely. However, if we know the mean and auto-covariance of some process [X_t] , it will not necessarily be a Gaussian process.

STATIONARITY

Many important stochastic processes menifest the same statistical properties at one time as another, and the ensemble of processes distinguishes no particular time over any other time. The first order probability density function $f_x(x,t)$ of the process i.e. the density function of a sample X=X(t) taken at an arbitrary time t, then does not depend on t or;

$$f_x(x,t) = f_x(x,) \quad , \text{for } \forall \ t \qquad (5)$$

The second order joint density function depends only on the interval between sampling times , or;

$$f_x(x1,t1 ; x2,t2) = f_{x1x2}(x1,x2;t2-t1) , \ \forall \ (t1,t2) \qquad (6)$$

and not on t1 and t2 individually. What matters is the interval between the sampling times. In general, the joint probability function of n samples x1,x2,...,xn depends only on the differences between the sampling times or;

$$f_x(x1,t1;x2,t2;...,xn,tn) = f_x(x1,x2,...,xn; t2-t1,t3-t1,..., tn-t1) \qquad (7)$$

Such a stochastic process is said to be stationary process. In other words ; if for arbitrary t1,t2,...,tn , the joint distribution of the vector random variables $(X_{t1}, X_{t2},...,X_{tn})$, and $(X_{t1+h}, X_{t2+h},...,X_{tn+h})$ are the same for all h>0 , then the process $\{X_t , t \in T\}$ is said to be stationary of order n. In simple terms , stationarity of a process implies that the probabilistic structure of the process is invariant under translation of the time axis. In general terms, a stochastic process is stationary only if all of its moments behave with regard to time shift.

When the random proces is stationary, its expected value E $[X_t]$ and variance must be independent of the time t. The mean value of a stochastic process can be expressed as:

$$E [X_t] = \mu_t = \int_{-\infty}^{\infty} x f_x (x,t) dx \qquad (8)$$

Often μ_t represents a signal and the difference $X_t - \mu_t$ represents a random noise which is ordinarily taken to have expected value zero. The range of values over which the random variable X_t is dispersed is measured by the standard deviation $\sigma(t)$,which is the square root of the variance or;

$$\sigma^2(t) = Var (X_t) = E [(X_t - \mu_t)^2] = \int_{-\infty}^{\infty} x^2 f_x (x,t) dx - (\mu_t)^2 \qquad (9)$$

When the stochastic process X_t is thought of as the sum of signal and noise, Var (X_t) represents, in suitable units, the average power of the noise.

The degree of linear auto-dependence of the random variables in a stochastic process is measured by the auto-covariance function. The auto-covariance C_k between two random variables X_t and X_{t+k} may be determined by :

$$C_k = \frac{1}{N-k} \sum_{t=1}^{N-k} (X_t - X^*) (X_{t+k} - X^*) \quad ; 0 \leq k < N \qquad (10)$$

where: C_k = auto-covariance at lag k

k represents the time lag (or distance) between the correlated pairs

(X_t, X_{t+k})

X^{*} = sample mean

N = # of observations

C_k in Eq. 10 is the unbiased estimator of population auto-covariance function . A dimensionless measure of linear dependence is obtained by dividing C_k in Eq. 10 by estimate of variance C_o which gives :

$$r_k = C_k / C_o = \left[\frac{1}{N-k} \sum_{t=1}^{N-k} (X_t - X^{*})(X_{t+k} - X^{*})\right] / \left[(X_t - X^{*})^2\right] \qquad (11)$$

where: r_k = lag k autocorrelation coefficient or the auto-correlation function (ACF). The plot of r_k vs. k is called correlogram.

When the random process is stationary , its expected value, E (X_t) and variance must be independent of time t. In such case the auto-covariance function C_k will depend on only the interval k because the joint probability density function $f_x(X_t, X_{t+k})$ depends on the sampling times t and t+k only through interval k and the mean values μ_t and μ_{t+k} appearing there are independent of t and t+k. Sometimes a process may have a constant expected value with auto-covariance function depending upon only the time interval . This process is called weakly stationary process. However, in such cases the joint probability density function of n variables may also depend on the sampling times besides the differences between the sampling times. For most weakly stationary processes the auto- covariance function drops to zero as the lag time tends to infinity which infers that samples of most processes taken at times separated by a very long interval tend to be uncorrelated.

ENSEMBLE MEAN AND VARIANCE

In a stochastic process, when we talk of the statistical properties like mean and standard deviation of the process, it is the ensemble mean and standard deviation. To clarify the point , let's take an example of unit white noise Gaussian process. This process is a collection of random variables with zero mean and unit standard deviation. We can get several realisations of the process using a computer program. Suppose we generate 1000 realisations of the process with each having 100 numbers. Let us index each realisation by w ; so we have w ranging from 1 - 1000. Let the realisations be denoted as w_1 , w_2 ,...,w_{1000}. Let the numbers of first realisation be noted as $w_1(1),w_1(2),...,w_1(n)$. Here we have taken n to be 100 as each realisation has 100 numbers or in other words the time parameter is a set of 100 integers 1,2,..., 100. Therefore, at a fixed time 'n' general representation of a reaslisation will be $w_1(n),w_2(n),...,w_{1000}(n)$ and specifically $w_1(1),w_2(1),...,w_{1000}(1)$ will be members of the first realisation. Now we can verify the statistical properties of computer generated random variables using the ensemble of realisations. It is to be noted that the expectation operator (E) always means averaging over the ensemble and never over a single sequence (which in this case will be averaging over 100 numbers of a single realisation). Therefore, to verify that :

$E[w_2] = 0$ and $Var[w_2] = 1.0$, we would compute :

$$m_2 = \frac{1}{1000} \sum_{h=1}^{1000} w_2(h) \tag{12}$$

In other words to estimate mean m_2, we will get average over values of all 1000 realisations at time t=2.0. m_2 can now be compared with zero for verifying the mean of the white noise process. For variance we can compute :

$$Var(w_2) = \frac{1}{999} \sum_{h=1}^{1000} (w_2(h) - m_2) \tag{13}$$

The variance computed as above can be compared with 1.0 for testing the white noise process. It is therefore emphasized that averaging over an ensemble of many different realisations of the same random sequence is the only way to measure statistical properties such as mean and variance. Ensembles have statistical properties , whereas one individual realisation of a random sequence does not. The need to have ensemble of realisation is much more when we want to verify independence. Suppose we want to see whether random variables w2 and w6 are independent. If we had just only one realisation , we would have one specific value of w2 and one specific value of w6. In this situatuion it is meaningless to talk whether these two numbers are independent or not. However, if we had ensemble of 1000 realisations and if we could believe in the Gaussianess of the distribution , it would be suffice to verify that w2 and w6 are uncorrelated. For this purpose we could compute the sample covariance as:

$$C_{26} = \frac{1}{999} \sum_{w=1}^{1000} [\, W_w(2) - m_2 \,] [W_w(6) \; - m6 \,] \qquad (14)$$

Provided that C_{26} is suitably near zero , we could reliably conclude that w2 and w6 are uncorrelated.

ERGODIC PROCESSES

In the earlier example , we had 1000 strings of 100 numbers each. Here the time index was denoted to vary from 1-100 and the ensemble variable was denoted by w, which was a discrete variable from 1-1000. Such kind of experiments are possible in computer simulation when it is easy to genarate many versions of a sequence with the same statistical characteristics. However, in real world one may not have the luxury of having more than one realisation of a physical process and therefore the possibility of ensemble averaging does not exist. In such a situation if statistical procedures are to be involved with any degree of rigor, it is necessary to assume that the stochastic process under investigation is stationary and a long time record of the process is available. Now the basic problem to consider is to find statistical properties of such a process for which only one realisation is available. Let the process be denoted as { X(t) } . By definition,

$$\mu_X(t) = E[\, X_t \,] \qquad (15)$$

If the mean $\mu_X(t)$ really varies with time , and if we have only one realisation X(.) to work with, there is no way to estimate mean , because we can not take ensemble average over only one realisation. The only option ,however, we have is to take average over values of single realisation at different times. The only one assumption that workers in this field have found useful here is that the process { X_t } is stationary and that the mean is some constant value, μ_X. Now if the data record of a continuous parameter process extends over an interval $T1 \leq t \leq T2$, one possibility of estimating μ_X is to compute the time average , or;

$$X(T1,T2) = \frac{1}{T2-T1} \int_{T1}^{T2} X(t)\, dt \qquad (16)$$

There is still no mathematical property of stationary Gaussian process that allows one to conclude , in general , that X has any relation to μ_X . One, therefore, has to specifically postulate that the process in question possesses an additional property that permits one to estimate or approximate μ_X by X. This additional property is called ergodicity. In simple terms , a process first of all has to be stationary in order that the ensemble average be constant. If it is ergodic, then it is legitimate to estimate parameters that are defined as ensemble averages by computing time averages.

PALMER INDEX DATA

(a) Arizona, PDSI, 1895-1992 (Monthly)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
1895	1.5	1.4	0.8	0.8	1.4	1.5	-0.8	0.0	.56	0.1	0.6	-0.2
1896	-0.62	-1.2	-1.89	-2.31	-2.72	-3.34	-2.59	-2.64	-2.16	-2.22	-1.97	-2.37
1897	-1.37	-1.22	-1.18	-1.82	-2.28	-2.67	-3.03	0.04	1.46	1.73	-0.33	-0.54
1898	-0.11	-0.71	-1.01	-1.63	-1.71	-2	-2.15	-1.48	-2.27	-2.5	-2.31	-2.15
1899	-2.04	-2.3	-2.87	-3.42	-3.76	-3.76	-3.99	-4.24	-4.84	-4.38	-4.39	-4.45
1900	-4.73	-5.01	-5.43	-4.65	-5.12	-5.5	-5.85	-6.24	-5.64	-5.44	-4.65	-4.85
1901	-4.45	-3.22	-3.27	-3.62	-3.57	-3.78	-3.55	-3.17	-3.42	-3.19	-3.53	-3.7
1902	-3.71	-3.91	-3.92	-4.65	-4.91	-5.2	-5.51	-5.21	-5.46	-5.58	-4.11	-3.75
1903	-3.55	-3.3	-2.86	-2.72	-2.74	-2.89	-3.44	-3.65	-2.78	-2.85	-3.25	-3.41
1904	-3.62	-3.55	-4.1	-4.76	-5.06	-5.39	-4.95	1.5	1.33	1.02	0.38	0.13
1905	0.64	1.91	3.88	5.47	6.79	8.48	7.18	6.02	6.55	5.46	6.57	6.07
1906	5.81	5.09	5.3	5.12	5.35	5.32	4.83	4.97	3.74	2.95	2.92	3.42
1907	4.25	3.53	3.25	2.96	3.15	3.1	-0.08	0.18	-0.7	-0.08	-0.39	-0.7
1908	-0.97	-0.58	-0.7	-0.9	-1.03	-1.33	0.26	0.21	0.56	0.68	0.09	0.46
1909	0.91	0.67	1.16	0.99	0.93	0.9	0.73	1.22	-0.26	-0.58	0.36	0.68
1910	0.79	-0.45	-1	-1.42	-1.85	-2.02	-1.82	-2.01	-2.18	-2.2	0.27	-0.34
1911	0.73	0.79	0.76	0.49	0.18	0.21	0.74	0.13	1.35	1.47	-0.42	-0.61
1912	-1.17	-1.8	1.05	1.65	1.63	1.33	1.79	1.49	1.66	2.45	-0.37	-0.61
1913	-0.82	-0.37	-0.57	-0.89	-1.19	-1.52	-1	-1.26	-1.75	-1.7	-1.11	-1.45
1914	-0.96	-0.97	-1.17	-1.73	-2.08	-0.04	0.51	0.49	-0.55	-0.16	-0.55	0.4
1915	0.94	0.95	1.03	1.77	2.31	2.7	3.33	2.39	1.58	0.98	0.98	1.21
1916	3.53	2.94	2.96	2.88	3.26	3.76	4.32	3.95	4.18	4.66	3.83	3.37
1917	3.5	3.04	2.42	3.51	4.18	4.11	4.79	-0.33	-0.3	-0.7	-1.1	-1.62
1918	0.12	0.07	0.61	0.46	0.24	0.44	0.52	0.58	1.06	0.91	0.94	0.73
1919	0.2	0.43	0.67	0.64	-0.17	-0.6	1.64	1.41	2.78	2.87	3.7	3.14
1920	2.98	2.97	3.17	3.01	3.07	3.26	-0.29	-0.25	-0.31	0.41	-0.3	-0.57
1921	-0.63	-1.04	-1.51	-1.68	-1.62	-2.11	1.02	1.83	1.47	1.26	0.81	1.04
1922	1.38	1.19	1.54	1.73	2.03	2.47	2.37	2.24	-0.28	-0.6	-0.33	-0.43
1923	-0.61	-0.88	-0.6	-0.44	-0.97	-1.35	0.08	0.83	1.75	1.59	1.73	1.9
1924	-0.48	-1.09	0.4	0.89	-0.39	-0.89	-0.79	-1.6	-1.74	-1.42	-1.69	-1.25
1925	-1.63	-2.09	-2.28	-1.82	-2.18	0.22	0.24	0.27	0.99	1.58	0.03	-0.44
1926	-0.77	-1.4	0.34	2.54	2.86	2.72	-0.13	-0.99	1.15	-0.51	-0.7	-0.22
1927	-0.77	0.29	0.51	0.76	0.47	0.64	0.17	0.56	1.82	1.8	-0.25	-0.09
1928	-0.71	-0.82	-1.38	-1.88	-2.4	-2.81	-3.14	-2.89	-3.37	-3.03	-3.04	-2.87
1929	-2.72	-2.73	-2.6	-2.16	-2.65	-2.87	-2.65	-2.13	-1.23	-1.51	-1.77	-2.12

1930	0.44	-0.44	0.62	0.63	0.96	0.89	1.01	-0.34	0.33	0.05	0.58	0.16
1931	0.04	1.83	1.38	1.9	2.23	2.66	3.59	6.35	5.2	4.33	4.7	5.05
1932	4.29	5.84	5.02	4.6	4.91	5.69	5.33	5.15	4.97	4.66	3.69	4.39
1933	4.82	4.12	3.35	3.78	3.92	4.43	4.32	-0.43	-0.87	-1.08	-1.11	-1.47
1934	-1.86	-2.1	-3.05	-3.41	-3.81	-3.57	-3.64	1.08	-0.68	-0.94	-0.93	0.75
1935	1.2	2.47	2.69	2.64	3.1	4	3.85	4.41	4.7	-0.4	-0.74	-0.97
1936	-1.54	-0.56	-0.51	-0.77	-1.09	-1.08	-0.22	-0.61	-0.99	1.48	0.89	2.59
1937	2.67	2.91	3.6	3.54	4.39	5.29	5.98	5.13	4.87	3.84	2.96	2.87
1938	2.22	2.3	2.81	2.55	2.64	2.65	2.84	2.33	1.49	1.16	0.76	1.73
1939	2.13	2.54	2.29	2.54	2.96	3.26	3.69	3.38	9.99	10.04	9.33	8.2
1940	7.8	8.13	7	7.34	8	8.61	7.3	6.72	9.93	9.94	9.11	9.36
1941	8.68	9.58	10.42	11.13	11.33	11.85	11.97	11.16	9.86	10.05	9.48	9.03
1942	7.67	6.73	5.81	5.42	4.65	4.01	3.43	4.43	3.37	2.78	2.26	2.05
1943	2.39	2.14	2.23	2.63	2.3	1.97	1.79	1.77	1.91	1.83	1.21	1.29
1944	1.28	2.44	2.44	2.62	3.01	3.53	3.09	2.57	1.63	0.87	1.45	1.56
1945	1.16	1.39	2.27	2.25	2.54	2.94	3.99	5.01	4.22	4.56	3.83	3.98
1946	3.45	2.97	2.73	2.76	2.39	1.97	2.44	2.91	2.73	3.79	4.36	4.44
1947	-0.25	-0.63	-0.92	-1.06	-0.86	-0.82	-1.26	-0.72	-1.4	0.23	0.39	1.06
1948	-0.52	0.43	0.74	0.63	0.51	0.71	0.45	0.61	-0.56	-0.27	-0.68	0.33
1949	1.7	1.6	1.5	1.68	2.27	2.89	-0.25	-0.59	-1.01	-1.13	-1.45	-1.14
1950	-1.48	-1.66	-1.96	-2.77	-3.16	-3.53	-2.37	-2.79	-2.78	-3.2	-3.4	-3.82
1951	-3.62	-3.61	-3.74	-3.27	-2.83	-3.17	-3.45	1.55	1.09	1.32	1.32	1.95
1952	2.12	1.55	2.47	3.22	3.62	4.08	3.75	-0.38	-0.62	-1.24	-0.71	-0.26
1953	-0.83	-1.19	-1.63	-1.7	-1.93	-2.09	-1.2	-0.92	-1.62	-1.67	-1.91	-2.11
1954	0.38	0.24	1.23	0.9	0.89	1.08	-0.28	-0.9	-1.35	-1.78	-2.08	-2.11
1955	-1.57	-1.64	-1.87	-2.09	-2.31	-2.36	0.88	1.67	-0.77	-1.28	-1.13	-1.5
1956	-1.71	-1.87	-2.46	-2.83	-3.25	-3.55	-3.29	-3.97	-4.42	-4.3	-4.33	-4.53
1957	-3.68	-3.68	-3.77	-3.77	-3.32	-3.28	-3.36	-2.8	-3.29	1.53	1.72	1.28
1958	0.71	0.73	1.7	2.13	2.32	-0.09	-0.66	-1.01	-0.63	-0.54	-0.5	-1.21
1959	-1.61	-1.2	-1.66	-2.39	-2.88	-3.26	-3.32	0.64	0.25	0.6	0.65	1.08
1960	1.02	1.05	-0.49	-0.72	-0.84	-1.06	-1.73	-2.33	-1.83	-1.52	-1.02	-1.43
1961	-1.66	-2.22	-2.31	-2.63	-3.01	-3.53	-3.65	-2.41	-2.64	-2.67	-2.46	-2.34
1962	-2.28	-1.43	-1.31	-1.71	-1.89	-1.84	-2.44	-3.11	-2.67	-2.64	-2.99	-2.89
1963	-3.06	-3.05	-3.13	-3.13	-3.62	-3.79	-4.41	-3.86	-3.06	-2.61	-1.93	-2.28
1964	-2.55	-2.85	-2.59	-2.47	-2.62	-2.8	-2.91	-2.22	-2.51	-2.97	-2.85	-2.83
1965	-2.96	-2.74	0.13	2.05	2.6	2.52	-0.07	-0.62	-0.92	-1.19	1.58	2.94
1966	2.41	2.5	2.07	1.9	2.39	2.63	-0.27	-0.87	-1.11	-0.92	-0.92	0.91
1967	-0.11	-0.6	-0.97	-0.8	-0.81	-0.74	0.39	0.64	1.39	1.18	1.25	1.58
1968	-0.44	-0.7	-0.99	-1.3	-1.72	-1.57	-1.19	-1.74	-2.21	-2.19	-2.27	-2.3
1969	0.33	1.06	1.06	0.81	1.05	1.08	1.14	-0.11	-0.06	-0.24	-0.31	-0.9
1970	-1.37	-1.8	-1.42	-1.41	-2.04	-2.41	-2.49	-2.01	-2.26	-2.44	-2.15	-2.07
1971	-2.44	-2.25	-2.65	-2.96	-2.46	-2.87	-3.4	-2.51	-2.91	-2.07	-2.04	-1.5

1972	-1.9	-2.33	-3.25	-3.43	-3.75	-3.08	-3.65	-0.03	0.16	1.27	1.93	1.74
1973	1.78	1.91	3.38	3.27	4.2	5.26	-0.16	-0.76	-1.38	-1.72	-1.42	-1.81
1974	-1.26	-1.55	-1.78	-2.1	-2.54	-2.97	-2.7	-3.3	-3.51	-2.68	-2.37	-2.27
1975	-2.43	-2.41	-1.8	-1.11	-1.14	-1.56	-1.55	-2.17	-2.28	-2.37	-2.13	-2.33
1976	-2.86	-2.03	-2.01	-1.3	-1.52	-1.89	-1.72	-2.5	1.49	1.57	-0.4	-0.78
1977	-1	-1.62	-1.86	-2.42	-1.81	-2.07	-2.29	-2.21	-2.42	-2.49	-2.6	-2.66
1978	1.01	1.54	2.44	2.95	3.6	4.15	3.45	2.49	1.8	2.08	2.95	3.39
1979	4.07	3.94	4.42	4.19	5.2	5.83	5.51	5.01	3.62	2.82	2.25	1.78
1980	2.63	3.54	3.76	3.66	4.22	4.76	4.33	-0.74	-1.17	-1.22	-1.63	-2.19
1981	-2.76	-2.95	-2.37	-2.73	-2.71	-3.17	-3.28	-3.21	-2.71	-2.05	-1.9	-2.41
1982	-2.32	-2.22	-1.89	-2.02	-2.32	-2.71	-3.05	0.67	0.52	0.23	0.77	1.06
1983	0.72	0.69	1.44	1.63	1.47	1.29	0.83	1.67	1.62	1.77	-0.06	-0.23
1984	-0.93	-1.54	-2.22	-2.69	-3.43	-3.67	1.44	1.71	0.91	0.53	0.73	1.89
1985	1.97	-0.17	-0.31	-0.41	-0.41	-0.46	-0.6	-1.77	-1.72	-1.58	-0.56	-0.96
1986	-1.62	-1.82	-1.91	-2.47	-3.04	-3.52	-3.66	-3.77	-3.4	-3.16	-2.94	-2.73
1987	-2.6	-2.67	-2.81	-3.47	-3.73	-3.86	-4.05	-4.3	-4.36	-3.38	-2.48	-2.36
1988	-2.17	-2.2	-2.73	-2.07	-2.56	-2.87	-3.29	-2.59	-3.11	-3.6	-3.56	-3.55
1989	-3.13	-3.29	-3.91	-4.83	-5.03	-5.33	-5.48	-5.49	-5.75	-5.57	-5.66	-5.7
1990	-5.26	-4.85	-5.1	-5.28	-5.16	-5.21	-4.44	-4.42	-3.96	-4.11	-4.04	-4.09
1991	-3.84	-4.02	-2.85	-2.85	-3.08	-3.15	-3.43	-3.96	-3.87	-3.94	-3.85	-3.66
1992	-3.45	0.46	2.31	2.05	2.75	2.59	-0.71	-1.17	-1.8	0.57	0.05	0.57

(b) Arizona, PHDI, 1895-1992 (Monthly)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
1895	1.52	1.42	0.84	0.87	1.46	1.57	0.59	0.55	1.03	0.12	0.69	-0.29
1896	-0.62	-1.2	-1.89	-2.31	-2.72	-3.34	-2.59	-2.64	-2.16	-2.22	-1.97	-2.37
1897	-1.37	-1.22	-1.18	-1.82	-2.28	-2.67	-3.03	-2.68	-0.98	1.73	1.22	0.85
1898	1.14	-0.71	-1.01	-1.63	-1.71	-2	-2.15	-1.48	-2.27	-2.5	-2.31	-2.15
1899	-2.04	-2.3	-2.87	-3.42	-3.76	-3.76	-3.99	-4.24	-4.84	-4.38	-4.39	-4.45
1900	-4.73	-5.01	-5.43	-4.65	-5.12	-5.5	-5.85	-6.24	-5.64	-5.44	-4.65	-4.85
1901	-4.45	-3.22	-3.27	-3.62	-3.57	-3.78	-3.55	-3.17	-3.42	-3.19	-3.53	-3.7
1902	-3.71	-3.91	-3.92	-4.65	-4.91	-5.2	-5.51	-5.21	-5.46	-5.58	-4.11	-3.75
1903	-3.55	-3.3	-2.86	-2.72	-2.74	-2.89	-3.44	-3.65	-2.78	-2.85	-3.25	-3.41
1904	-3.62	-3.55	-4.1	-4.76	-5.06	-5.39	-4.95	-2.94	-2.65	-2.55	-2.83	-2.75
1905	-1.94	1.91	3.88	5.47	6.79	8.48	7.18	6.02	6.55	5.46	6.57	6.07
1906	5.81	5.09	5.3	5.12	5.35	5.32	4.83	4.97	3.74	2.95	2.92	3.42
1907	4.25	3.53	3.25	2.96	3.15	3.1	2.7	2.61	1.64	2.02	1.49	0.99
1908	-0.97	-0.58	-0.7	-0.9	-1.03	-1.33	-0.93	-0.86	0.56	0.68	0.09	0.46
1909	0.91	0.67	1.16	0.99	0.93	0.9	0.73	1.22	0.84	-0.58	0.36	0.68
1910	0.79	-0.45	-1	-1.42	-1.85	-2.02	-1.82	-2.01	-2.18	-2.2	-1.7	-1.87
1911	-0.94	-0.72	-0.58	-0.72	-0.91	-0.76	0.74	0.13	1.35	1.47	0.9	0.57
1912	-1.17	-1.8	-0.57	1.65	1.63	1.33	1.79	1.49	1.66	2.45	1.83	1.37
1913	0.96	1.22	0.85	-0.89	-1.19	-1.52	-1	-1.26	-1.75	-1.7	-1.11	-1.45
1914	-0.96	-0.97	-1.17	-1.73	-2.08	-1.9	-1.2	-1.04	-1.49	-1	-1.3	-0.77
1915	0.94	0.95	1.03	1.77	2.31	2.7	3.33	2.39	1.58	0.98	0.98	1.21
1916	3.53	2.94	2.96	2.88	3.26	3.76	4.32	3.95	4.18	4.66	3.83	3.37
1917	3.5	3.04	2.42	3.51	4.18	4.11	4.79	3.97	3.56	2.76	2	1.17
1918	1.17	1.01	1.46	1.21	0.92	1.05	1.07	1.07	1.5	1.31	1.29	1.05
1919	0.2	0.43	0.67	0.64	-0.17	-0.6	1.64	1.41	2.78	2.87	3.7	3.14
1920	2.98	2.97	3.17	3.01	3.07	3.26	2.63	2.37	2.04	2.24	1.71	1.24
1921	0.99	-1.04	-1.51	-1.68	-1.62	-2.11	-0.88	1.83	1.47	1.26	0.81	1.04
1922	1.38	1.19	1.54	1.73	2.03	2.47	2.37	2.24	1.73	1.21	1.28	1.03
1923	0.69	-0.88	-0.6	-0.44	-0.97	-1.35	-1.12	0.83	1.75	1.59	1.73	1.9
1924	1.22	-1.09	-0.58	0.89	-0.39	-0.89	-0.79	-1.6	-1.74	-1.42	-1.69	-1.25
1925	-1.63	-2.09	-2.28	-1.82	-2.18	-1.74	-1.52	-1.3	0.99	1.58	1.44	0.86
1926	-0.77	-1.4	-0.92	2.54	2.86	2.72	2.31	1.19	2.22	1.48	1.09	1.38
1927	0.67	0.89	1.04	1.24	0.9	1.02	0.17	0.56	1.82	1.8	1.37	1.36
1928	0.59	-0.82	-1.38	-1.88	-2.4	-2.81	-3.14	-2.89	-3.37	-3.03	-3.04	-2.87
1929	-2.72	-2.73	-2.6	-2.16	-2.65	-2.87	-2.65	-2.13	-1.23	-1.51	-1.77	-2.12
1930	-1.46	-1.75	-0.95	-0.78	0.96	0.89	1.01	0.56	0.83	0.05	0.58	0.16
1931	0.04	1.83	1.38	1.9	2.23	2.66	3.59	6.35	5.2	4.33	4.7	5.05
1932	4.29	5.84	5.02	4.6	4.91	5.69	5.33	5.15	4.97	4.66	3.69	4.39
1933	4.82	4.12	3.35	3.78	3.92	4.43	4.32	3.45	2.61	2.04	1.69	1.04

1934	-1.86	-2.1	-3.05	-3.41	-3.81	-3.57	-3.64	-2.19	-2.64	-2.7	-2.5	-1.49
1935	-0.81	2.47	2.69	2.64	3.1	4	3.85	4.41	4.7	3.82	3.05	2.42
1936	1.51	2.17	1.94	1.43	0.89	0.69	1.37	0.81	-0.99	1.48	0.89	2.59
1937	2.67	2.91	3.6	3.54	4.39	5.29	5.98	5.13	4.87	3.84	2.96	2.87
1938	2.22	2.3	2.81	2.55	2.64	2.65	2.84	2.33	1.49	1.16	0.76	1.73
1939	2.13	2.54	2.29	2.54	2.96	3.26	3.69	3.38	9.99	10.04	9.33	8.2
1940	7.8	8.13	7	7.34	8	8.61	7.3	6.72	9.93	9.94	9.11	9.36
1941	8.68	9.58	10.42	11.13	11.33	11.85	11.97	11.16	9.86	10.05	9.48	9.03
1942	7.67	6.73	5.81	5.42	4.65	4.01	3.43	4.43	3.37	2.78	2.26	2.05
1943	2.39	2.14	2.23	2.63	2.3	1.97	1.79	1.77	1.91	1.83	1.21	1.29
1944	1.28	2.44	2.44	2.62	3.01	3.53	3.09	2.57	1.63	0.87	1.45	1.56
1945	1.16	1.39	2.27	2.25	2.54	2.94	3.99	5.01	4.22	4.56	3.83	3.98
1946	3.45	2.97	2.73	2.76	2.39	1.97	2.44	2.91	2.73	3.79	4.36	4.44
1947	3.73	2.94	2.29	1.81	1.71	1.49	0.82	1.13	-1.4	-1.02	-0.74	1.06
1948	-0.52	0.43	0.74	0.63	0.51	0.71	0.45	0.61	-0.56	-0.27	-0.68	0.33
1949	1.7	1.6	1.5	1.68	2.27	2.89	2.34	1.74	1.08	0.74	-1.45	-1.14
1950	-1.48	-1.66	-1.96	-2.77	-3.16	-3.53	-2.37	-2.79	-2.78	-3.2	-3.4	-3.82
1951	-3.62	-3.61	-3.74	-3.27	-2.83	-3.17	-3.45	-1.55	-1.68	-1.16	-0.91	1.95
1952	2.12	1.55	2.47	3.22	3.62	4.08	3.75	2.98	2.39	1.46	1.72	1.92
1953	1.12	0.56	-1.63	-1.7	-1.93	-2.09	-1.2	-0.92	-1.62	-1.67	-1.91	-2.11
1954	-1.51	-1.45	1.23	0.9	0.89	1.08	0.68	-0.9	-1.35	-1.78	-2.08	-2.11
1955	-1.57	-1.64	-1.87	-2.09	-2.31	-2.36	-1.24	1.67	0.74	-1.28	-1.13	-1.5
1956	-1.71	-1.87	-2.46	-2.83	-3.25	-3.55	-3.29	-3.97	-4.42	-4.3	-4.33	-4.53
1957	-3.68	-3.68	-3.77	-3.77	-3.32	-3.28	-3.36	-2.8	-3.29	-1.42	-0.92	-1.09
1958	-1.42	-1.18	1.7	2.13	2.32	1.99	1.21	0.66	0.87	0.81	0.7	-1.21
1959	-1.61	-1.2	-1.66	-2.39	-2.88	-3.26	-3.32	-2.34	-2.42	-1.79	-1.5	-0.85
1960	-0.71	1.05	-0.49	-0.72	-0.84	-1.06	-1.73	-2.33	-1.83	-1.52	-1.02	-1.43
1961	-1.66	-2.22	-2.31	-2.63	-3.01	-3.53	-3.65	-2.41	-2.64	-2.67	-2.46	-2.34
1962	-2.28	-1.43	-1.31	-1.71	-1.89	-1.84	-2.44	-3.11	-2.67	-2.64	-2.99	-2.89
1963	-3.06	-3.05	-3.13	-3.13	-3.62	-3.79	-4.41	-3.86	-3.06	-2.61	-1.93	-2.28
1964	-2.55	-2.85	-2.59	-2.47	-2.62	-2.8	-2.91	-2.22	-2.51	-2.97	-2.85	-2.83
1965	-2.96	-2.74	-2.33	2.05	2.6	2.52	2.18	1.41	0.9	-1.19	1.58	2.94
1966	2.41	2.5	2.07	1.9	2.39	2.63	2.08	1.24	0.78	0.78	0.61	1.46
1967	1.19	0.57	-0.97	-0.8	-0.81	-0.74	0.39	0.64	1.39	1.18	1.25	1.58
1968	0.98	0.57	-0.99	-1.3	-1.72	-1.57	-1.19	-1.74	-2.21	-2.19	-2.27	-2.3
1969	-1.73	-0.8	-0.6	-0.68	1.05	1.08	1.14	0.91	0.85	0.58	-0.31	-0.9
1970	-1.37	-1.8	-1.42	-1.41	-2.04	-2.41	-2.49	-2.01	-2.26	-2.44	-2.15	-2.07
1971	-2.44	-2.25	-2.65	-2.96	-2.46	-2.87	-3.4	-2.51	-2.91	-2.07	-2.04	-1.5
1972	-1.9	-2.33	-3.25	-3.43	-3.75	-3.08	-3.65	-3.3	-2.8	-1.39	1.93	1.74
1973	1.78	1.91	3.38	3.27	4.2	5.26	4.56	3.47	2.42	1.68	1.63	0.93
1974	1.19	0.66	-1.78	-2.1	-2.54	-2.97	-2.7	-3.3	-3.51	-2.68	-2.37	-2.27
1975	-2.43	-2.41	-1.8	-1.11	-1.14	-1.56	-1.55	-2.17	-2.28	-2.37	-2.13	-2.33

1976	-2.86	-2.03	-2.01	-1.3	-1.52	-1.89	-1.72	-2.5	-0.75	1.57	1.01	-0.78
1977	-1	-1.62	-1.86	-2.42	-1.81	-2.07	-2.29	-2.21	-2.42	-2.49	-2.6	-2.66
1978	-1.37	-0.6	2.44	2.95	3.6	4.15	3.45	2.49	1.8	2.08	2.95	3.39
1979	4.07	3.94	4.42	4.19	5.2	5.83	5.51	5.01	3.62	2.82	2.25	1.78
1980	2.63	3.54	3.76	3.66	4.22	4.76	4.33	3.15	2.31	1.91	1.18	-2.19
1981	-2.76	-2.95	-2.37	-2.73	-2.71	-3.17	-3.28	-3.21	-2.71	-2.05	-1.9	-2.41
1982	-2.32	-2.22	-1.89	-2.02	-2.32	-2.71	-3.05	-2.06	-1.94	-1.97	-1.21	-0.71
1983	-0.86	-0.74	1.44	1.63	1.47	1.29	0.83	1.67	1.62	1.77	1.53	1.19
1984	-0.93	-1.54	-2.22	-2.69	-3.43	-3.67	-1.85	-1.24	-1.73	-1.85	-1.4	1.89
1985	1.97	1.6	1.28	1.01	0.87	0.69	-0.6	-1.77	-1.72	-1.58	-0.56	-0.96
1986	-1.62	-1.82	-1.91	-2.47	-3.04	-3.52	-3.66	-3.77	-3.4	-3.16	-2.94	-2.73
1987	-2.6	-2.67	-2.81	-3.47	-3.73	-3.86	-4.05	-4.3	-4.36	-3.38	-2.48	-2.36
1988	-2.17	-2.2	-2.73	-2.07	-2.56	-2.87	-3.29	-2.59	-3.11	-3.6	-3.56	-3.55
1989	-3.13	-3.29	-3.91	-4.83	-5.03	-5.33	-5.48	-5.49	-5.75	-5.57	-5.66	-5.7
1990	-5.26	-4.85	-5.1	-5.28	-5.16	-5.21	-4.44	-4.42	-3.96	-4.11	-4.04	-4.09
1991	-3.84	-4.02	-2.85	-2.85	-3.08	-3.15	-3.43	-3.96	-3.87	-3.94	-3.85	-3.66
1992	-3.45	-2.64	2.31	2.05	2.75	2.59	1.61	0.92	-1.8	-1.04	-1.4	-0.73

c) California, PDSI, 1895-1992 (Monthly)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
1895	1.36	1.33	0.84	0.76	1	1.88	2.4	2.55	2.5	1.9	1.6	0.9
1896	1.74	0.68	0.86	1.87	2.04	2.48	-0.26	-0.48	-0.42	-0.65	-0.18	-0.37
1897	-0.92	0.52	0.83	-0.69	-1.3	-1.73	-2.08	-2.14	-2.3	-1.09	-1.35	-1.7
1898	-2.45	-3.1	-3.82	-4.5	-4.03	-4.8	-5.2	-5.09	-4.54	-4.25	-4.19	-4.6
1899	-4.3	-5.08	-3.78	-4.04	-3.67	-3.23	-3.18	-3.03	-3.12	-1.85	-1.3	-1.13
1900	-1.39	-2.09	-2.41	0.43	0.91	1.26	1.1	0.87	0.58	1.09	2.25	1.41
1901	1.76	2.4	1.38	1.52	1.74	1.9	1.73	1.54	1.72	1.92	0.01	-0.67
1902	-1.43	0.37	0.58	0.73	0.79	1.06	1.04	0.99	-0.36	0.31	0.79	0.23
1903	0.73	0.2	1.12	-0.5	-0.8	-0.99	-1.23	-1.2	-1.44	-1.66	-0.83	-1.51
1904	-2.16	0.73	2.2	2.3	1.79	1.52	1.11	0.95	2.47	3.14	-0.57	-1
1905	-1.38	-1.64	-1.1	-1.66	1.14	1.3	1.42	-0.07	-0.34	-0.88	-0.79	-1.37
1906	1.44	1.26	3.1	2.99	4.69	5.32	6.39	5.67	4.91	3.78	3.19	4.46
1907	5.31	4.6	6.9	-0.43	-0.56	0.26	0.38	0.42	-0.15	-0.15	-1.09	-0.5
1908	-0.28	-0.25	-0.96	-1.52	0.68	0.75	0.94	0.65	0.78	0.73	-0.35	-0.97
1909	3.22	3.54	3.33	-0.73	-1.15	-1.38	-1.5	-1.41	0.14	0.43	1.33	2.58
1910	2.75	-0.61	-0.84	-1.43	-1.93	-2.35	-2.55	-2.57	-1.98	-1.99	-2.38	-2.58
1911	2.78	2.47	3.24	3.21	2.95	2.96	3.01	0	-0.19	-0.65	-1.21	-1.7
1912	-2.02	-3.17	0.26	1.01	1.84	2.39	2.58	2.62	3.22	3.11	2.94	-0.82
1913	-0.7	-1.29	-1.74	-0.05	0.29	0.47	0.88	1.1	-0.32	-0.91	1.12	2.48
1914	5.27	4.92	3.5	3.49	3.15	3.15	3.09	2.82	2.42	1.99	0.96	1.11
1915	1.45	2.62	1.91	1.86	4.4	4.61	5.01	4.86	4.21	3.18	2.47	2.64
1916	5.95	-0.22	-0.45	-1.06	-1.14	-1.14	0.21	0.41	0.36	1.17	0.88	1.49
1917	0.84	1.24	0.63	0.43	0.56	0.77	0.74	-0.23	-0.61	-1.19	-1.48	-2.51
1918	-3.37	-3.11	0.95	-0.31	-0.4	-0.49	-0.89	-0.96	1.66	1.51	1.64	-0.37
1919	-1.12	0.55	0.57	-0.56	-0.92	-1.3	-1.49	-1.55	-1.14	-1.22	-1.84	-1.51
1920	-2.61	-3.03	0.7	1.19	0.81	0.85	0.83	0.92	0.63	1.18	2.18	2.73
1921	3.19	2.24	1.66	1	2.09	2.39	2.35	-0.08	-0.18	-0.61	-1.27	1.62
1922	1.43	2.16	2.1	1.6	1.84	1.8	1.63	1.44	0.87	0.86	1.27	2.31
1923	-0.2	-0.92	-1.88	1.33	0.99	1.18	1.52	1.56	2.03	-0.19	-0.98	-1.81
1924	-2.41	-3.34	-3.51	-3.9	-4.88	-5.67	-5.96	-5.64	-5.47	0.82	0.54	0.71
1925	-0.83	-0.72	-0.83	0.65	1.44	2.02	2.24	2.11	2.08	1.97	-0.19	-0.85
1926	-1.16	-1.23	-2.36	-1.34	-1.47	-1.59	-1.93	-1.93	-2.08	-2.12	1.93	1.55
1927	1.13	1.81	1.45	1.71	1.5	1.56	1.52	1.3	0.87	1.41	1.59	-0.09
1928	-0.84	-1.47	-0.93	-0.98	-0.98	-1.07	-1.12	-1.11	-1.37	-1.69	-1.29	-1.26
1929	-1.68	-1.92	-1.9	0.32	-0.36	0.62	0.94	0.92	-0.26	-0.82	-1.7	-1.61
1930	0.38	-0.43	-0.43	0.04	0.81	1.3	1.51	1.47	1.63	1.33	1.44	-1.01
1931	-1.17	-1.57	-2.36	-2.66	-2.78	-2.7	-2.88	-2.85	-2.85	-2.99	0.37	2.02
1932	1.77	2.11	-0.84	-0.94	-0.54	-0.44	-0.43	-0.39	-0.6	-1.1	-1.89	-2.04
1933	-1.15	-1.67	-1.85	-2.18	0.59	0.97	1.36	-0.18	-0.48	-0.56	-1.5	-1

1934	-1.63	-1.54	-2.68	-3.52	-3.74	-3.82	-3.91	-3.77	-3.51	0.28	0.65	0.51
1935	0.66	0.31	0.73	2.33	-0.24	-0.17	-0.3	-0.21	-0.56	-0.27	-0.71	-1.22
1936	-1.38	2.5	-0.45	-0.54	-0.67	-0.42	-0.31	-0.44	-0.74	-0.31	-1.23	0.74
1937	0.69	2.01	2.49	-0.08	-0.49	-0.61	-0.68	-0.81	-1.12	-1.48	-1.85	0.74
1938	0.53	2.2	3.82	3.94	-0.15	0.14	0.14	-0.11	-0.23	0.51	-0.56	-1.2
1939	-1.51	-1.67	-1.66	-2.06	-1.9	-1.89	-1.9	-1.84	0.69	0.93	-0.91	-1.8
1940	1.23	2.41	0.05	-0.29	-0.7	-1.2	-1.62	-1.7	-1.84	-1.21	-1.79	1.34
1941	1.22	2.27	2.12	2.98	2.53	2.34	2.34	2.19	1.71	1.79	1.18	2.33
1942	2.01	1.74	1.22	2.06	2.48	2.76	2.98	2.74	2.15	1.45	1.77	1.58
1943	2.13	1.82	2.78	2.55	-0.23	0.04	0.31	0.37	-0.33	-0.41	-1.04	-1.38
1944	-1.59	0.71	0.13	0.54	0.52	0.79	1.21	1.37	1	0.97	2.02	1.69
1945	0.62	1.5	2.1	1.44	1.39	1.63	1.7	1.51	1.13	2.11	2.05	2.64
1946	-0.74	-0.81	-0.43	-1.1	-0.93	-1.03	-0.87	-0.8	-0.98	0.44	1.36	1.42
1947	-0.81	-1.2	-1.44	-1.78	-1.88	-2.03	-2.06	-1.86	-1.91	-1.02	-1.37	-2.01
1948	-3	-3.34	0.29	1.69	2.04	2.43	2.7	2.7	-0.23	-0.48	-1.15	-1.03
1949	-1.4	-1.35	-0.57	-1.32	-1.09	-1.24	-1.39	-1.28	-1.49	-1.86	-2	-2.21
1950	0.31	-0.16	-0.31	-0.35	-0.42	-0.46	-0.36	-0.45	-0.43	0.73	2.2	2.61
1951	-0.13	-0.36	-0.91	-0.76	-0.74	-0.6	-0.53	-0.51	-0.83	0.16	0.21	1.61
1952	2.4	1.93	2.74	2.6	-0.35	-0.33	0.38	0.38	-0.06	-0.8	-0.74	0.84
1953	-0.36	-1.25	-1.57	0.29	0.65	1.34	2.29	2.25	-0.32	-0.56	-0.71	-1.39
1954	-1.45	-1.54	-0.76	-0.98	-1.24	-1.1	-1.04	-1.04	-1.25	-1.72	-1.62	-1.38
1955	-1.01	-1.05	-1.83	0.4	0.64	0.84	1.03	1.19	-0.28	-0.9	-0.89	2.86
1956	3.03	2.34	1.08	1.37	1.88	2.06	2.14	2.01	1.69	1.97	-0.94	-1.73
1957	-1.86	-2.02	-2.21	0.01	1.26	1.73	1.92	1.83	1.51	1.69	1.12	0.95
1958	0.74	1.53	2.89	3.9	3.63	3.58	3.56	3.43	3.46	-0.77	-1.43	-2.4
1959	-2.61	-2	-2.88	-3.28	-3.45	-3.78	-4.02	-3.89	-1.99	-2.5	-3.25	-3.81
1960	-3.84	-3.17	-3.39	-3.04	-3.02	-2.81	-2.96	-2.95	-3.01	-3.12	-2.18	-2.54
1961	-2.95	-3.54	-3.62	-3.76	-3.49	-3.5	-3.63	-3.28	-3.07	-3.27	-2.87	-3.05
1962	-3.37	-0.99	-1	-1.65	-1.75	-1.84	-1.82	-1.7	-1.79	-1.19	-1.91	-2.51
1963	-2.4	0.06	0.19	1.63	1.66	1.93	2.14	2.29	2.21	2.38	2.79	-0.78
1964	-1.23	-2.19	-2.48	-2.55	0.02	0.05	0.04	0.15	-0.14	0.21	0.81	2.06
1965	-0.2	-0.87	-1.23	0.88	0.49	0.52	0.84	1.56	1.29	0.59	1.85	2.01
1966	-0.7	-0.98	-1.88	-2.41	-2.84	-3.24	-3.34	-3.17	-3.14	-3.47	0.28	1.38
1967	1.5	0.54	1	3.3	2.92	3	3.12	-0.02	0.07	-0.54	-0.6	-0.82
1968	-1.22	-1.73	-2.05	-2.36	-2.49	-2.67	-2.82	-2.47	-2.6	0.34	0.55	1
1969	3.21	4.75	-0.39	-0.09	-0.44	0.19	0.41	0.37	0.16	0.49	-0.44	-0.7
1970	-0.02	-0.44	-0.69	-0.81	-1.18	-1.09	-1.16	-1.25	-1.49	-1.71	1.23	1.8
1971	-0.69	-1.31	-1.6	-1.63	0.66	1.05	1.4	1.36	-0.12	-0.45	-0.49	0.7
1972	-0.84	-1.51	-2.67	-2.79	-3.26	-3.53	-3.74	-3.49	0.06	0.11	1.08	1.03
1973	1.45	2.13	2.36	-0.58	-1.02	-1.52	-1.89	-1.86	-1.99	0.55	1.09	1.28
1974	-0.04	-0.74	0.43	0.67	0.27	0.22	0.5	0.47	0.02	0.51	-0.54	-0.8
1975	-1.57	0.25	0.86	1.26	0.84	0.76	0.67	1.23	0.89	1.92	-0.47	-1.41

1976	-2.54	-2.85	-3.53	-3.4	-4.14	-4.94	-5.29	-4.24	-2.34	-2.36	-2.79	-3.49
1977	-4	-4.82	-5.43	-6.17	-5.49	-6.51	-6.97	-6.49	-6.1	-6.07	-5.77	0.72
1978	1.26	2.31	3.1	3.98	3.23	2.87	2.56	2.26	3.73	-0.78	-0.82	-1.13
1979	0.25	0.96	1.16	-0.49	-0.72	-1.07	-1.23	-1.24	-1.4	0.38	-0.15	-0.48
1980	1.21	2.28	1.99	1.64	1.67	1.67	2.07	2	-0.28	-0.77	-1.56	-2.1
1981	-1.8	-2.27	-1.66	-1.7	-1.64	-1.72	-2.03	-2.05	-2.18	0.9	1.54	1.32
1982	1.65	1.49	2.95	3.8	3.09	3.21	3.23	3.2	4.87	5.7	6.63	6.59
1983	6.76	7.14	8.2	8.09	7.27	6.71	6.19	6.58	6.82	5.94	7.31	7.8
1984	-1.08	-1.23	-1.77	-1.85	-2.11	-2.19	-2.09	-2	-2.19	-1.44	-0.19	-0.32
1985	-1.05	-1.32	-0.87	-1.57	-1.99	-2.21	-2.42	-2.39	0.44	0.47	1.51	1.22
1986	0.67	2.97	3.29	-0.25	-0.47	-0.7	-0.82	-0.82	0.76	-0.58	-1.34	-2.05
1987	-2.36	-2.29	-2.02	-2.58	-2.78	-3.04	-3.14	-2.92	-2.96	-2.55	-2.48	-2.25
1988	-2.28	-2.96	-3.68	-3.04	-2.79	-2.54	-2.3	-2.21	-2.33	-2.84	-2.28	-1.93
1989	-2.62	-2.72	-2.32	-2.8	-2.73	-2.8	-2.85	-2.65	1.38	1.71	-0.49	-1.57
1990	-1.83	-2.01	-2.69	-3.1	-2.56	-2.91	-3.05	-2.97	-2.89	-3.26	-3.58	-3.94
1991	-4.67	-5.35	-2.76	-2.93	-2.64	-2.23	-1.82	-1.62	-1.82	-1.06	-1.59	-1.95
1992	-2.48	-1.82	-1.74	-2.4	-2.96	-3.31	-2.83	-2.68	-2.82	-1.89	-2.57	0.78

(d) California, PHDI, 1895-1992 (Monthly)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
1895	1.36	1.33	0.84	0.76	1	1.88	2.4	2.55	2.5	1.9	1.6	0.9
1896	1.74	0.68	0.86	1.87	2.04	2.48	1.96	1.52	1.37	0.96	1.26	0.92
1897	-0.92	0.52	0.83	-0.69	-1.3	-1.73	-2.08	-2.14	-2.3	-1.09	-1.35	-1.7
1898	-2.45	-3.1	-3.82	-4.5	-4.03	-4.8	-5.2	-5.09	-4.54	-4.25	-4.19	-4.6
1899	-4.3	-5.08	-3.78	-4.04	-3.67	-3.23	-3.18	-3.03	-3.12	-1.85	-1.3	-1.13
1900	-1.39	-2.09	-2.41	-1.72	-1.02	1.26	1.1	0.87	0.58	1.09	2.25	1.41
1901	1.76	2.4	1.38	1.52	1.74	1.9	1.73	1.54	1.72	1.92	1.72	0.87
1902	-1.43	-0.92	-0.57	0.73	0.79	1.06	1.04	0.99	-0.36	0.31	0.79	0.23
1903	0.73	0.2	1.12	0.92	-0.8	-0.99	-1.23	-1.2	-1.44	-1.66	-0.83	-1.51
1904	-2.16	-1.21	2.2	2.3	1.79	1.52	1.11	0.95	2.47	3.14	2.24	1.53
1905	0.89	-1.64	-1.1	-1.66	1.14	1.3	1.42	1.21	0.8	-0.88	-0.79	-1.37
1906	1.44	1.26	3.1	2.99	4.69	5.32	6.39	5.67	4.91	3.78	3.19	4.46
1907	5.31	4.6	6.9	5.76	4.99	4.74	4.39	4.02	3.45	3.08	1.81	2.11
1908	2.06	1.84	0.92	-1.52	-0.69	0.75	0.94	0.65	0.78	0.73	-0.35	-0.97
1909	3.22	3.54	3.33	2.26	1.53	1.03	0.66	-1.41	-1.13	-0.7	1.33	2.58
1910	2.75	1.86	1.37	0.56	-1.93	-2.35	-2.55	-2.57	-1.98	-1.99	-2.38	-2.58
1911	2.78	2.47	3.24	3.21	2.95	2.96	3.01	2.71	2.24	1.53	0.74	-1.7
1912	-2.02	-3.17	-2.58	-1.53	1.84	2.39	2.58	2.62	3.22	3.11	2.94	1.82
1913	1.67	0.83	-1.74	-1.61	-1.15	-0.82	0.88	1.1	0.67	-0.91	1.12	2.48
1914	5.27	4.92	3.5	3.49	3.15	3.15	3.09	2.82	2.42	1.99	0.96	1.11
1915	1.45	2.62	1.91	1.86	4.4	4.61	5.01	4.86	4.21	3.18	2.47	2.64
1916	5.95	5.12	4.34	3.23	2.71	2.32	2.29	2.27	2.03	2.67	2.22	2.7
1917	1.92	2.21	1.5	1.21	1.26	1.4	1.3	0.94	-0.61	-1.19	-1.48	-2.51
1918	-3.37	-3.11	-1.84	-1.96	-1.88	-1.82	-2.08	-2.03	1.66	1.51	1.64	1.1
1919	-1.12	0.55	0.57	-0.56	-0.92	-1.3	-1.49	-1.55	-1.14	-1.22	-1.84	-1.51
1920	-2.61	-3.03	-2.02	-1.24	-1.38	-1.11	-0.93	-0.65	-0.79	1.18	2.18	2.73
1921	3.19	2.24	1.66	1	2.09	2.39	2.35	2.03	1.71	1.08	-1.27	1.62
1922	1.43	2.16	2.1	1.6	1.84	1.8	1.63	1.44	0.87	0.86	1.27	2.31
1923	1.87	0.94	-1.88	1.33	0.99	1.18	1.52	1.56	2.03	1.64	0.65	-1.81
1924	-2.41	-3.34	-3.51	-3.9	-4.88	-5.67	-5.96	-5.64	-5.47	-4.09	-3.86	-3.24
1925	-3.74	-3.33	-3.17	-2.19	-1.11	2.02	2.24	2.11	2.08	1.97	1.57	0.73
1926	-1.16	-1.23	-2.36	-1.34	-1.47	-1.59	-1.93	-1.93	-2.08	-2.12	1.93	1.55
1927	1.13	1.81	1.45	1.71	1.5	1.56	1.52	1.3	0.87	1.41	1.59	1.34
1928	-0.84	-1.47	-0.93	-0.98	-0.98	-1.07	-1.12	-1.11	-1.37	-1.69	-1.29	-1.26
1929	-1.68	-1.92	-1.9	-1.39	-1.6	-0.81	0.94	0.92	-0.26	-0.82	-1.7	-1.61
1930	-1.06	-1.38	-1.28	-1.11	0.81	1.3	1.51	1.47	1.63	1.33	1.44	-1.01
1931	-1.17	-1.57	-2.36	-2.66	-2.78	-2.7	-2.88	-2.85	-2.85	-2.99	-2.32	2.02
1932	1.77	2.11	1.06	0.76	0.99	0.93	0.8	0.71	-0.6	-1.1	-1.89	-2.04
1933	-1.15	-1.67	-1.85	-2.18	-1.36	-0.78	1.36	1.04	0.61	-0.56	-1.5	-1

1934	-1.63	-1.54	-2.68	-3.52	-3.74	-3.82	-3.91	-3.77	-3.51	-2.88	-2.18	-2.03
1935	-1.62	-1.73	-1.1	2.33	1.85	1.7	1.38	1.3	0.8	0.94	-0.71	-1.22
1936	-1.38	2.5	1.79	1.47	1.14	1.2	1.14	0.86	-0.74	-0.31	-1.23	0.74
1937	0.69	2.01	2.49	2.15	1.51	1.19	0.93	0.64	-1.12	-1.48	-1.85	-0.92
1938	-0.96	2.2	3.82	3.94	3.38	3.17	2.86	2.46	2.07	2.36	1.56	0.71
1939	-1.51	-1.67	-1.66	-2.06	-1.9	-1.89	-1.9	-1.84	-0.96	0.93	-0.91	-1.8
1940	1.23	2.41	2.21	1.69	1.08	-1.2	-1.62	-1.7	-1.84	-1.21	-1.79	1.34
1941	1.22	2.27	2.12	2.98	2.53	2.34	2.34	2.19	1.71	1.79	1.18	2.33
1942	2.01	1.74	1.22	2.06	2.48	2.76	2.98	2.74	2.15	1.45	1.77	1.58
1943	2.13	1.82	2.78	2.55	2.06	1.88	1.96	1.85	1.33	1.08	-1.04	-1.38
1944	-1.59	-0.72	-1.15	-0.61	0.52	0.79	1.21	1.37	1	0.97	2.02	1.69
1945	0.62	1.5	2.1	1.44	1.39	1.63	1.7	1.51	1.13	2.11	2.05	2.64
1946	1.63	1.31	1.47	0.61	0.6	-1.03	-0.87	-0.8	-0.98	0.44	1.36	1.42
1947	-0.81	-1.2	-1.44	-1.78	-1.88	-2.03	-2.06	-1.86	-1.91	-1.02	-1.37	-2.01
1948	-3	-3.34	-2.7	-1	2.04	2.43	2.7	2.7	2.19	1.7	0.8	0.72
1949	-1.4	-1.35	-0.57	-1.32	-1.09	-1.24	-1.39	-1.28	-1.49	-1.86	-2	-2.21
1950	-1.67	-1.66	-1.65	-1.55	-1.5	-1.43	-1.23	-1.23	-1.13	0.73	2.2	2.61
1951	2.21	1.74	0.97	0.93	0.77	0.76	0.69	0.59	-0.83	0.16	0.21	1.61
1952	2.4	1.93	2.74	2.6	1.99	1.76	1.96	1.79	1.55	0.65	0.56	1.35
1953	0.84	-1.25	-1.57	-1.12	-0.62	1.34	2.29	2.25	1.7	1.25	0.91	-1.39
1954	-1.45	-1.54	-0.76	-0.98	-1.24	-1.1	-1.04	-1.04	-1.25	-1.72	-1.62	-1.38
1955	-1.01	-1.05	-1.83	-1.25	-0.84	0.84	1.03	1.19	0.78	-0.9	-0.89	2.86
1956	3.03	2.34	1.08	1.37	1.88	2.06	2.14	2.01	1.69	1.97	0.82	-1.73
1957	-1.86	-2.02	-2.21	-1.97	1.26	1.73	1.92	1.83	1.51	1.69	1.12	0.95
1958	0.74	1.53	2.89	3.9	3.63	3.58	3.56	3.43	3.46	2.34	1.35	-2.4
1959	-2.61	-2	-2.88	-3.28	-3.45	-3.78	-4.02	-3.89	-1.99	-2.5	-3.25	-3.81
1960	-3.84	-3.17	-3.39	-3.04	-3.02	-2.81	-2.96	-2.95	-3.01	-3.12	-2.18	-2.54
1961	-2.95	-3.54	-3.62	-3.76	-3.49	-3.5	-3.63	-3.28	-3.07	-3.27	-2.87	-3.05
1962	-3.37	-0.99	-1	-1.65	-1.75	-1.84	-1.82	-1.7	-1.79	-1.19	-1.91	-2.51
1963	-2.4	-2.09	-1.74	1.63	1.66	1.93	2.14	2.29	2.21	2.38	2.79	1.73
1964	1.02	-2.19	-2.48	-2.55	-2.27	-2	-1.8	-1.5	-1.49	-1.12	0.81	2.06
1965	1.65	0.79	-1.23	0.88	0.49	0.52	0.84	1.56	1.29	0.59	1.85	2.01
1966	1.11	0.64	-1.88	-2.41	-2.84	-3.24	-3.34	-3.17	-3.14	-3.47	-2.83	-1.42
1967	-1	-1.71	-1.02	3.3	2.92	3	3.12	2.78	2.56	1.76	1.46	1.03
1968	-1.22	-1.73	-2.05	-2.36	-2.49	-2.67	-2.82	-2.47	-2.6	-1.99	-1.55	-0.87
1969	3.21	4.75	3.87	3.73	2.98	2.86	2.81	2.52	2.09	2.22	1.55	1.09
1970	1.59	1	0.6	-0.81	-1.18	-1.09	-1.16	-1.25	-1.49	-1.71	1.23	1.8
1971	0.92	-1.31	-1.6	-1.63	-0.8	1.05	1.4	1.36	1.1	0.65	-0.49	0.7
1972	-0.84	-1.51	-2.67	-2.79	-3.26	-3.53	-3.74	-3.49	-3.07	-2.7	-1.44	-1.23
1973	-0.57	2.13	2.36	1.54	0.88	-1.52	-1.89	-1.86	-1.99	-1.24	1.09	1.28
1974	1.11	-0.74	0.43	0.67	0.27	0.22	0.5	0.47	0.02	0.51	-0.54	-0.8
1975	-1.57	-1.16	0.86	1.26	0.84	0.76	0.67	1.23	0.89	1.92	1.25	-1.41

1976	-2.54	-2.85	-3.53	-3.4	-4.14	-4.94	-5.29	-4.24	-2.34	-2.36	-2.79	-3.49
1977	-4	-4.82	-5.43	-6.17	-5.49	-6.51	-6.97	-6.49	-6.1	-6.07	-5.77	-4.45
1978	-3.38	-1.86	-0.64	3.98	3.23	2.87	2.56	2.26	3.73	2.56	2.18	1.56
1979	1.64	2.21	2.28	1.55	1.12	0.58	-1.23	-1.24	-1.4	-0.87	-0.94	-1.18
1980	1.21	2.28	1.99	1.64	1.67	1.67	2.07	2	1.51	0.84	-1.56	-2.1
1981	-1.8	-2.27	-1.66	-1.7	-1.64	-1.72	-2.03	-2.05	-2.18	-1.06	1.54	1.32
1982	1.65	1.49	2.95	3.8	3.09	3.21	3.23	3.2	4.87	5.7	6.63	6.59
1983	6.76	7.14	8.2	8.09	7.27	6.71	6.19	6.58	6.82	5.94	7.31	7.8
1984	5.91	5.04	3.86	3.19	2.42	1.87	1.55	1.27	0.74	1.19	2.17	1.8
1985	0.84	-1.32	-0.87	-1.57	-1.99	-2.21	-2.42	-2.39	-1.7	-1.45	1.51	1.22
1986	0.67	2.97	3.29	2.7	2.18	1.68	1.31	1.09	1.74	0.98	-1.34	-2.05
1987	-2.36	-2.29	-2.02	-2.58	-2.78	-3.04	-3.14	-2.92	-2.96	-2.55	-2.48	-2.25
1988	-2.28	-2.96	-3.68	-3.04	-2.79	-2.54	-2.3	-2.21	-2.33	-2.84	-2.28	-1.93
1989	-2.62	-2.72	-2.32	-2.8	-2.73	-2.8	-2.85	-2.65	-1	1.71	1.04	-1.57
1990	-1.83	-2.01	-2.69	-3.1	-2.56	-2.91	-3.05	-2.97	-2.89	-3.26	-3.58	-3.94
1991	-4.67	-5.35	-2.76	-2.93	-2.64	-2.23	-1.82	-1.62	-1.82	-1.06	-1.59	-1.95
1992	-2.48	-1.82	-1.74	-2.4	-2.96	-3.31	-2.83	-2.68	-2.82	-1.89	-2.57	-1.53

(e) Virginia, PDSI, 1895-1990 (Monthly)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
1895	0.98	0.56	0.54	1.51	1.82	-0.17	-0.05	-0.67	-1.73	-1.93	-2.07	-2.3
1896	-2.4	0.47	0.89	-0.84	-0.82	0.3	0.9	0.4	0.82	-0.4	-0.17	-0.7
1897	-1.01	1.09	-0.2	-0.51	-0.14	-0.47	-0.15	-0.51	-1.2	0.52	0.69	0.77
1898	-0.44	-1.22	-1.31	0.22	0.55	-0.57	-0.45	-0.32	-0.47	0.74	1.06	1.16
1899	1.21	2.31	3.03	-0.37	-0.53	-0.96	-0.98	-0.95	-0.88	-0.64	-1.07	-1.52
1900	-1.42	0.61	0.77	0.57	0.09	0.64	-0.68	-1.68	-1.95	-2.45	-2.4	-2.52
1901	-2.53	-2.97	0.11	1.34	2.03	1.83	2.21	3.14	3.21	2.79	2.47	3.68
1902	3.16	3.69	-0.38	-0.61	-1.11	-0.99	-1.54	-1.93	0.53	1.31	1.57	1.97
1903	2.01	2.52	3.04	3.16	2.17	2.85	2.54	2.73	2.31	2.54	-0.14	-.29
1904	-0.49	-0.43	-0.7	-0.96	-1.25	0.42	0.87	0.75	0.86	0.7	0.69	1.01
1905	0.8	1.15	-0.55	-0.62	0.15	0.19	1.54	1.91	-0.05	-0.47	-0.98	0.85
1906	1	0.63	1.23	-0.76	-1.02	0.57	1.49	3.16	2.53	3.41	3	2.82
1907	1.64	1.33	0.63	1.1	1.36	2.33	1.76	1.64	2.02	1.6	2.49	2.78
1908	2.84	3.02	2.14	1.6	1.87	1.8	2.21	3.02	2.75	3.06	2.53	2.64
1909	1.95	1.68	1.62	1.52	1.65	2.26	-0.44	-0.33	-0.49	-0.83	-1.66	-1.85
1910	-1.73	-1.54	-2.04	0.32	0.37	1.47	-0.15	0.16	-0.33	0.31	-0.17	-.06
1911	-0.16	-0.58	0	0.49	-1.17	-1.49	-2.31	-1.58	-2.09	-2.11	0.4	0.5
1912	0.24	0.36	1.84	1.55	1.72	1.8	-0.27	-0.85	-0.24	-0.71	-0.92	-.95
1913	-1.07	-1.36	0.38	0.22	0.46	0.32	0.02	0.03	0.17	0.36	-0.36	-.52
1914	-0.54	-0.21	-0.21	-0.31	-1.18	-1.53	-1.75	-2.14	-2.32	-2.39	-2.38	.59
1915	1.48	0.02	-0.59	-1.42	0.24	0.79	0.45	1.15	-0.59	-0.41	-0.76	-.62
1916	-1.36	-1	-1.35	-1.49	-0.02	1.1	2.17	-0.35	-0.36	-0.57	-0.98	-.99
1917	-1.01	-1.08	0.66	0.47	0.2	0.53	1.43	1.05	1.48	2.19	1.59	1.20
1918	1.84	-0.95	-1.01	1.23	-0.32	-0.26	-0.27	-0.33	0.32	-0.64	-1.16	.09
1919	0.34	0.14	0.17	0.08	0.64	0.67	2.21	-0.09	-0.7	-1.46	-1.73	-2.03
1920	-2.06	0.68	0.61	0.99	0.41	0.94	1.27	2.25	2.15	1.38	2.07	2.26
1921	2.1	-0.11	-1.16	-1.34	0.69	-0.63	-0.8	-1.48	-2.02	-2.57	-2.56	-3.0
1922	0.28	0.78	1.17	0.6	0.63	1.09	2.03	2.09	-0.45	-0.38	-1.09	.09
1923	0.18	0.15	0.75	0.92	-0.43	-1.06	-0.73	-0.83	-0.73	-0.95	-1.05	-1.55
1924	0.28	0.19	0.62	0.95	1.88	2.1	1.82	1.69	3.19	2.61	2.17	2.08
1925	2.69	-0.79	-1.45	-1.74	-1.97	-2.52	-2.98	-3.22	-3.86	-3.78	-3.63	-3.68
1926	-2.91	-2.37	-2.39	-2.56	-3	-3.1	-2.77	-2.49	-2.69	-2.81	-2.16	-1.60
1927	-2.26	-2.11	-2.81	0.67	0.28	0.4	0.54	0.8	0.2	0.73	0.67	1.35
1928	-0.49	-0.59	-0.99	0.83	0.67	1.05	0.78	2.2	3.8	2.84	2.04	1.26
1929	0.96	1.5	1.15	1.14	1.26	1.98	1.59	1.1	1.26	2.25	2.21	-.02
1930	-0.17	-1.07	-1.66	-1.86	-2.36	-2.27	-3.26	-3.96	-4.51	-4.87	-5.14	-5.19
1931	-5.66	-5.81	-5.06	-4.34	-3.18	-2.88	-2.67	-1.64	-1.6	-1.95	-2.94	-3.66
1932	-3.17	-3.54	-2.68	-2.54	-2.28	-2.36	-2.69	-3.7	-4.14	0.7	0.92	1.31
1933	1.2	-0.25	-0.86	-0.32	-0.15	-0.95	0.12	1.94	-1.21	-1.74	-2.13	-2.62
1934	-3.16	0.71	1.41	0.86	1.89	1.3	1.82	1.6	2.93	2.09	2.12	1.61

1935	1.77	1.33	0.84	1.63	1.62	1.4	1.83	2.04	3.43	2.76	2.97	2.67
1936	4.1	4.15	3.89	3.81	-1.09	-1.05	-1.39	-2.05	-2.27	-2.35	-2.95	0.36
1937	2.46	1.96	1.33	2.4	2.12	2.22	2.36	2.68	2.55	3.23	3.93	3.04
1938	2.54	1.64	1.28	1.18	1.38	2.73	3.76	2.47	3.44	3.02	2.88	2.36
1939	2.05	2.55	2.36	2.41	1.24	1.13	1.86	2.59	-0.9	0.64	0.86	-.55
1940	-0.81	-0.91	-1.18	0.85	-0.04	-0.83	-1.01	0.72	0.36	0.15	0.98	-.55
1941	-0.84	-1.26	-1.5	-1.45	-2.29	-2.24	-1.95	-2.43	-3.38	-4.23	-5.07	-5.48
1942	-5.63	-5.69	-4.34	-5.05	-5.5	-5.45	-5.34	1.13	1.37	2.6	-0.45	-.08
1943	-0.43	-0.74	-0.69	-0.7	-0.46	-0.85	-0.98	-1.99	-1.84	-1.25	-1.48	-1.89
1944	0.17	0.9	1.9	1.96	-1.21	-2.29	-2.58	-2.68	-1.88	-1.77	-1.19	-1.19
1945	-1.42	-0.93	-2.3	-2.5	0.18	0.68	3.68	3	3.19	2.66	2.63	3.88
1946	3.06	2.55	1.44	1.27	2.48	2.43	2.24	2.38	-0.02	-0.2	-0.45	-1.01
1947	-0.61	-1.27	-1.41	-1.59	-1.8	-1.36	-1.47	-2.23	-1.75	-2.17	0.76	0.38
1948	0.68	0.47	0.14	0.75	1.84	1.54	1.01	1.51	1.22	1.17	2.4	2.92
1949	-0.6	-0.5	-0.99	-1.33	0.53	0.29	0.18	1.17	1.09	0.81	0.96	-.4
1950	-1.19	-1.84	-1.73	-1.98	-1.72	-2.23	0.99	0.93	1.24	-0.3	-0.54	-.6
1951	-1.38	-1.85	-1.82	-1.78	-1.92	-1.1	-1.32	-1.22	-1.89	-2.03	0.89	1.02
1952	1.68	1.81	2.1	1.95	-0.2	-1.05	-1.8	-1.71	-2.08	-2.24	-1.38	-1.07
1953	-1.4	-1.4	-1.27	-0.9	-1.11	-1.13	-2.14	-1.59	-1.35	-1.93	-2.17	-2.38
1954	-1.54	-2.17	-1.92	-2.2	-1.25	-2.26	-2.49	-2.63	-3.19	-3.42	-3.69	-3.8
1955	-4.3	-3.85	-3.57	-3.63	-3.86	-3.12	-3.5	1.7	2.66	2.43	2.23	-.47
1956	-0.86	-0.35	-0.49	-0.31	-0.43	-0.87	0.8	0.56	0.71	1.84	1.85	1.64
1957	1.61	2	2.15	-0.59	-1.18	-1.36	-2.4	0.28	0.41	1.14	2.17	2.96
1958	2.87	2.89	3.68	3.79	4.32	4.58	3.86	5.05	3.81	4.02	3.53	3.74
1959	-0.71	-1.24	-1.17	-0.47	-1.28	-2.02	1.09	-0.87	-1.07	1.02	1.5	-.12
1960	-0.32	0.43	0	-0.56	0.88	0.48	1.1	1.46	2.62	2.82	2.1	1.84
1961	1.74	2.41	2.31	2.02	3.05	3.61	2.58	2.13	1.23	1.94	1.42	2.19
1962	2.75	2.54	2.64	2.82	2.02	2.43	2.54	1.93	1.88	1.32	2.37	2.39
1963	1.98	1.79	1.94	-1.08	-1.23	1.56	-1	-1.74	-1.17	-2	0.66	0.69
1964	0.92	1.6	-0.49	-0.27	-1.06	-1.44	-1.61	0.06	1.03	1.89	-0.25	0.23
1965	-0.41	-0.85	-0.64	-0.73	-1.82	-1.39	-0.75	-1.41	-1.99	-2.56	-3.64	-5.01
1966	-4.91	-4.21	-4.63	-4.48	-3.51	-3.12	-3.36	-3.38	-2.69	-2.57	-3.05	-2.98
1967	-2.99	-2.48	-2.82	-3.41	-2.85	-3.1	-2.82	-1.54	-1.59	-1.92	-2	-.79
1968	-0.86	-1.67	-1.27	-1.42	-1.32	-1.24	-1.31	-1.9	-2.4	-2.49	-2.32	-2.39
1969	-2.41	-2.19	-1.55	-1.7	-2.02	-2.08	0.52	1.29	1.21	0.85	0.73	1.68
1970	-0.46	0.03	0.35	0.53	-0.62	-0.92	-0.59	-1.38	-2.05	-2.69	-2.57	-2.76
1971	0	0.35	-0.03	-0.32	1.17	-0.94	-1.37	-1.06	-1.24	1.57	1.55	-.78
1972	-0.98	0.65	0.3	0.31	1.13	2.42	2.21	1.36	1.78	2.38	3.5	3.4
1973	2.74	2.6	2.36	2.53	2.34	2.35	-0.63	-0.21	-0.83	-1.2	-1.88	.75
1974	0.74	0.51	0.74	0.17	0.17	0.33	0.01	0.29	0.6	-0.4	-1.01	0.1
1975	0.77	0.9	2.51	2.48	2	1.35	2.39	1.47	3.43	3.48	3.1	3.05
1976	3.05	-0.95	-1.51	-2.44	-2.11	-2.12	-2.39	-2.71	0.54	1.7	1.6	1.66

1977	-0.03	-0.5	-0.66	-1.13	-0.95	-1.44	-2.13	-2.66	-3.13	0.91	1.57	2.32
1978	3.61	2.41	3.41	3.48	4.16	3.96	3.45	3.04	1.95	1.09	1.29	1.27
1979	2.48	3.16	2.94	3.17	4.08	3.99	3.84	3.47	5.62	5.23	5.98	-.57
1980	-0.04	-0.47	0.64	0.65	-0.07	-1.24	-2.02	-3.05	-3.87	-2.9	-2.7	-2.97
1981	-3.76	-3.49	-3.73	-3.6	-2.77	-2.67	-2.39	-2.2	-2.12	-1.79	-2.24	0.68
1982	1.04	1.71	1.6	1.46	0.99	1.27	1.54	2.03	1.72	1.7	2.05	2.16
1983	1.36	1.78	2.3	3.55	3.37	-0.01	-1.41	-2.34	0.05	0.22	0.89	1.94
1984	2	2.02	3.33	3.81	4.12	-0.72	-0.1	-0.85	-1.42	-2.36	-2.45	-3.17
1985	-2.88	-2.31	-2.75	-3.83	-3.73	-3.46	-3.25	-0.03	0.87	1.45	2.65	-.67
1986	-0.85	-1.07	-2	-2.51	-2.89	-3.77	-4.19	-3.03	-3.74	-3.96	-4.14	0.43
1987	2.25	1.84	1.27	1.86	-0.89	-0.63	-1.66	-2.39	-2.04	-2.2	-2.03	-1.96
1988	-1.88	-1.54	-1.85	-1.65	-0.8	-0.92	-1.24	-1.57	-1.82	-1.64	-1.03	-1.74
1989	-2.15	0.74	2.21	2.91	2.97	3.26	3.22	3.69	3.87	3.96	4.25	3.90
1990	3.46	2.78	2.33	2.01	3.06	-0.97	-1.34	-0.36	-0.77	-0.6	-1	-1.01

f) Virginia, PHDI, 1895-1990 (Monthly)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
1895	0.98	0.56	0.54	1.51	1.82	1.46	1.42	0.65	-1.73	-1.93	-2.07	-2.3
1896	-2.4	-1.68	-1.04	-1.77	-1.65	-1.19	0.9	0.4	0.82	-0.4	-0.17	-0.7
1897	-1.01	1.09	0.78	-0.51	-0.14	-0.47	-0.15	-0.51	-1.2	-0.55	0.69	0.77
1898	-0.44	-1.22	-1.31	-0.95	0.55	-0.57	-0.45	-0.32	-0.47	0.74	1.06	1.16
1899	1.21	2.31	3.03	2.35	1.91	1.22	0.98	0.81	0.7	0.78	-1.07	-1.52
1900	-1.42	-0.66	0.77	0.57	0.09	0.64	-0.68	-1.68	-1.95	-2.45	-2.4	-2.52
1901	-2.53	-2.97	-2.55	-1.05	2.03	1.83	2.21	3.14	3.21	2.79	2.47	3.68
1902	3.16	3.69	2.93	2.36	1.55	1.4	0.61	-1.93	-1.21	1.31	1.57	1.97
1903	2.01	2.52	3.04	3.16	2.17	2.85	2.54	2.73	2.31	2.54	2.14	1.75
1904	1.34	1.21	0.77	-0.96	-1.25	-0.7	0.87	0.75	0.86	0.7	0.69	1.01
1905	0.8	1.15	-0.55	-0.62	0.15	0.19	1.54	1.91	1.66	1.06	-0.98	0.85
1906	1	0.63	1.23	0.59	-1.02	0.57	1.49	3.16	2.53	3.41	3	2.82
1907	1.64	1.33	0.63	1.1	1.36	2.33	1.76	1.64	2.02	1.6	2.49	2.78
1908	2.84	3.02	2.14	1.6	1.87	1.8	2.21	3.02	2.75	3.06	2.53	2.64
1909	1.95	1.68	1.62	1.52	1.65	2.26	1.58	1.49	1.15	0.63	-1.66	-1.85
1910	-1.73	-1.54	-2.04	-1.51	-1.27	1.47	1.17	1.21	0.75	0.98	0.7	0.72
1911	-0.16	-0.58	0	0.49	-1.17	-1.49	-2.31	-1.58	-2.09	-2.11	-1.49	-1.19
1912	-1.28	-1	1.84	1.55	1.72	1.8	1.34	0.6	1.05	-0.71	-0.92	-0.95
1913	-1.07	-1.36	-0.84	-0.87	0.46	0.32	0.02	0.03	0.17	0.36	-0.36	-0.52
1914	-0.54	-0.21	-0.21	-0.31	-1.18	-1.53	-1.75	-2.14	-2.32	-2.39	-2.38	-1.55
1915	1.48	1.35	0.62	-1.42	-1.03	0.79	0.45	1.15	0.66	0.72	-0.76	-0.62
1916	-1.36	-1	-1.35	-1.49	-1.36	1.1	2.17	1.59	1.38	0.99	-0.98	-0.99
1917	-1.01	-1.08	0.66	0.47	0.2	0.53	1.43	1.05	1.48	2.19	1.59	1.2
1918	1.84	0.7	-1.01	1.23	0.79	0.73	0.62	-0.33	0.32	-0.64	-1.16	-0.95
1919	-0.6	-0.7	-0.58	-0.59	0.64	0.67	2.21	1.9	1.08	-1.46	-1.73	-2.03
1920	-2.06	-1.17	-1.05	0.99	0.41	0.94	1.27	2.25	2.15	1.38	2.07	2.26
1921	2.1	1.77	-1.16	-1.34	0.69	-0.63	-0.8	-1.48	-2.02	-2.57	-2.56	-3
1922	-2.41	-1.64	-1	-1.34	-1.12	1.09	2.03	2.09	1.42	1.3	-1.09	-0.89
1923	-0.7	-0.63	0.75	0.92	-0.43	-1.06	-0.73	-0.83	-0.73	-0.95	-1.05	-1.55
1924	-1.11	-1.05	0.62	0.95	1.88	2.1	1.82	1.69	3.19	2.61	2.17	2.08
1925	2.69	1.62	0.72	-1.74	-1.97	-2.52	-2.98	-3.22	-3.86	-3.78	-3.63	-3.68
1926	-2.91	-2.37	-2.39	-2.56	-3	-3.1	-2.77	-2.49	-2.69	-2.81	-2.16	-1.6
1927	-2.26	-2.11	-2.81	-1.86	-1.98	-1.63	-1.28	-0.83	-1.26	-0.58	0.67	1.35
1928	0.73	-0.59	-0.99	0.83	0.67	1.05	0.78	2.2	3.8	2.84	2.04	1.26
1929	0.96	1.5	1.15	1.14	1.26	1.98	1.59	1.1	1.26	2.25	2.21	1.78
1930	1.6	-1.07	-1.66	-1.86	-2.36	-2.27	-3.26	-3.96	-4.51	-4.87	-5.14	-5.19
1931	-5.66	-5.81	-5.06	-4.34	-3.18	-2.88	-2.67	-1.64	-1.6	-1.95	-2.94	-3.66
1932	-3.17	-3.54	-2.68	-2.54	-2.28	-2.36	-2.69	-3.7	-4.14	-3.02	-2.41	-1.68
1933	-1.48	-1.58	-2.05	-1.39	-1.11	-1.81	-1.51	1.94	1.18	-1.74	-2.13	-2.62

1934	-3.16	-2.13	-1.14	-1.42	1.89	1.3	1.82	1.6	2.93	2.09	2.12	1.61
1935	1.77	1.33	0.84	1.63	1.62	1.4	1.83	2.04	3.43	2.76	2.97	2.67
1936	4.1	4.15	3.89	3.81	2.33	2.01	1.36	-2.05	-2.27	-2.35	-2.95	-2.29
1937	2.46	1.96	1.33	2.4	2.12	2.22	2.36	2.68	2.55	3.23	3.93	3.04
1938	2.54	1.64	1.28	1.18	1.38	2.73	3.76	2.47	3.44	3.02	2.88	2.36
1939	2.05	2.55	2.36	2.41	1.24	1.13	1.86	2.59	1.43	1.92	2.01	1.25
1940	0.81	-0.91	-1.18	0.85	-0.04	-0.83	-1.01	0.72	0.36	0.15	0.98	-0.55
1941	-0.84	-1.26	-1.5	-1.45	-2.29	-2.24	-1.95	-2.43	-3.38	-4.23	-5.07	-5.48
1942	-5.63	-5.69	-4.34	-5.05	-5.5	-5.45	-5.34	-3.66	-2.93	-1.26	-1.58	-1.09
1943	-1.33	-1.55	-1.42	-1.36	-1.05	-1.37	-1.46	-2.41	-2.22	-1.59	-1.79	-2.16
1944	-1.77	-0.84	1.9	1.96	-1.21	-2.29	-2.58	-2.68	-1.88	-1.77	-1.19	-1.19
1945	-1.42	-0.93	-2.3	-2.5	-2.06	-1.33	3.68	3	3.19	2.66	2.63	3.88
1946	3.06	2.55	1.44	1.27	2.48	2.43	2.24	2.38	2.12	1.72	1.27	-1.01
1947	-0.61	-1.27	-1.41	-1.59	-1.8	-1.36	-1.47	-2.23	-1.75	-2.17	-1.19	-1.37
1948	-0.89	-0.94	-1.12	0.75	1.84	1.54	1.01	1.51	1.22	1.17	2.4	2.92
1949	2.03	1.85	1.12	0.56	1.04	0.74	0.59	1.53	1.41	1.11	1.22	0.69
1950	-1.19	-1.84	-1.73	-1.98	-1.72	-2.23	-1	-0.86	1.24	0.82	-0.54	-0.6
1951	-1.38	-1.85	-1.82	-1.78	-1.92	-1.1	-1.32	-1.22	-1.89	-2.03	-0.93	-0.62
1952	1.68	1.81	2.1	1.95	1.55	-1.05	-1.8	-1.71	-2.08	-2.24	-1.38	-1.07
1953	-1.4	-1.4	-1.27	-0.9	-1.11	-1.13	-2.14	-1.59	-1.35	-1.93	-2.17	-2.38
1954	-1.54	-2.17	-1.92	-2.2	-1.25	-2.26	-2.49	-2.63	-3.19	-3.42	-3.69	-3.8
1955	-4.3	-3.85	-3.57	-3.63	-3.86	-3.12	-3.5	-1.43	2.66	2.43	2.23	1.53
1956	0.93	1.26	0.96	0.99	0.73	-0.87	0.8	0.56	0.71	1.84	1.85	1.64
1957	1.61	2	2.15	1.34	0.55	-1.36	-2.4	-1.88	-1.52	-0.59	2.17	2.96
1958	2.87	2.89	3.68	3.79	4.32	4.58	3.86	5.05	3.81	4.02	3.53	3.74
1959	2.65	1.77	1.53	1.96	0.89	-2.02	-0.72	-1.52	-1.65	1.02	1.5	1.22
1960	0.89	1.22	1.09	-0.56	0.88	0.48	1.1	1.46	2.62	2.82	2.1	1.84
1961	1.74	2.41	2.31	2.02	3.05	3.61	2.58	2.13	1.23	1.94	1.42	2.19
1962	2.75	2.54	2.64	2.82	2.02	2.43	2.54	1.93	1.88	1.32	2.37	2.39
1963	1.98	1.79	1.94	0.66	-1.23	1.56	-1	-1.74	-1.17	-2	-1.13	-0.92
1964	0.92	1.6	0.95	1.02	-1.06	-1.44	-1.61	-1.39	1.03	1.89	1.45	1.53
1965	0.96	-0.85	-0.64	-0.73	-1.82	-1.39	-0.75	-1.41	-1.99	-2.56	-3.64	-5.01
1966	-4.91	-4.21	-4.63	-4.48	-3.51	-3.12	-3.36	-3.38	-2.69	-2.57	-3.05	-2.98
1967	-2.99	-2.48	-2.82	-3.41	-2.85	-3.1	-2.82	-1.54	-1.59	-1.92	-2	-0.79
1968	-0.86	-1.67	-1.27	-1.42	-1.32	-1.24	-1.31	-1.9	-2.4	-2.49	-2.32	-2.39
1969	-2.41	-2.19	-1.55	-1.7	-2.02	-2.08	-1.35	1.29	1.21	0.85	0.73	1.68
1970	1.05	0.97	1.19	1.28	-0.62	-0.92	-0.59	-1.38	-2.05	-2.69	-2.57	-2.76
1971	-2.48	-1.87	-1.71	-1.83	1.17	-0.94	-1.37	-1.06	-1.24	1.57	1.55	0.61
1972	-0.98	0.65	0.3	0.31	1.13	2.42	2.21	1.36	1.78	2.38	3.5	3.4
1973	2.74	2.6	2.36	2.53	2.34	2.35	1.48	1.68	0.87	-1.2	-1.88	-0.94
1974	-0.77	-0.84	0.74	0.17	0.17	0.33	0.01	0.29	0.6	-0.4	-1.01	-0.8
1975	0.77	0.9	2.51	2.48	2	1.35	2.39	1.47	3.43	3.48	3.1	3.05

1976	3.05	1.79	0.95	-2.44	-2.11	-2.12	-2.39	-2.71	-1.88	1.7	1.6	1.66
1977	1.46	0.83	-0.66	-1.13	-0.95	-1.44	-2.13	-2.66	-3.13	-1.9	-0.94	2.32
1978	3.61	2.41	3.41	3.48	4.16	3.96	3.45	3.04	1.95	1.09	1.29	1.27
1979	2.48	3.16	2.94	3.17	4.08	3.99	3.84	3.47	5.62	5.23	5.98	4.79
1980	4.77	3.84	4.09	3.74	3.29	1.78	0.68	-3.05	-3.87	-2.9	-2.7	-2.97
1981	-3.76	-3.49	-3.73	-3.6	-2.77	-2.67	-2.39	-2.2	-2.12	-1.79	-2.24	-1.33
1982	-0.76	1.71	1.6	1.46	0.99	1.27	1.54	2.03	1.72	1.7	2.05	2.16
1983	1.36	1.78	2.3	3.55	3.37	3.02	1.3	-2.34	-2.04	-1.66	-0.79	1.94
1984	2	2.02	3.33	3.81	4.12	2.97	3.21	2.12	1.25	-2.36	-2.45	-3.17
1985	-2.88	-2.31	-2.75	-3.83	-3.73	-3.46	-3.25	-2.95	-1.77	-0.92	2.65	1.71
1986	1.29	0.85	-2	-2.51	-2.89	-3.77	-4.19	-3.03	-3.74	-3.96	-4.14	-3.28
1987	-1.07	-1.15	-1.41	1.86	1.26	1.3	-1.66	-2.39	-2.04	-2.2	-2.03	-1.96
1988	-1.88	-1.54	-1.85	-1.65	-0.8	-0.92	-1.24	-1.57	-1.82	-1.64	-1.03	-1.74
1989	-2.15	-1.18	2.21	2.91	2.97	3.26	3.22	3.69	3.87	3.96	4.25	3.9
1990	3.46	2.78	2.33	2.01	3.06	1.78	1.13	1.85	1.22	1.18	0.59	-1.01

(g) Z Index Data, Tidewater Region, Virginia, 1895-1990 (Monthly)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
1895	2.93	-0.95	0.13	3.06	1.41	-0.51	0.31	-1.87	-3.4	-1.12	-1.02	-1.34
1896	-0.99	1.42	1.4	-2.53	-0.19	0.89	1.9	-1.24	1.39	-1.21	0.58	-1.65
1897	-1.13	3.28	-0.61	-0.98	0.95	-1.05	0.83	-1.13	-2.23	1.56	0.67	0.45
1898	-1.32	-2.47	-0.64	0.66	1.07	-1.72	0.2	0.23	-0.55	2.23	1.18	0.62
1899	0.52	3.65	2.89	-1.11	-0.59	-1.47	-0.35	-0.21	-0.07	0.44	-1.49	-1.68
1900	-0.18	1.83	0.66	-0.36	-1.24	1.68	-2.04	-3.21	-1.33	-2.1	-0.62	-1.1
1901	-0.8	-2.11	0.34	3.72	2.47	0.04	1.7	3.46	1.18	-0.26	-0.11	4.41
1902	-0.42	2.56	-1.14	-0.8	-1.71	0.01	-1.94	-1.65	1.58	2.52	1.19	1.68
1903	0.73	2.13	2.34	1.3	-2	2.71	-0.03	1.36	-0.41	1.38	-0.41	-0.49
1904	-0.71	0.04	-0.95	-0.98	-1.17	1.26	1.47	-0.08	0.57	-0.22	0.19	1.17
1905	-0.32	1.29	-1.65	-0.38	0.46	0.17	4.1	1.57	-0.14	-1.28	-1.66	2.55
1906	0.71	-0.81	2.01	-1.56	-1.01	1.71	2.93	5.49	-0.93	3.41	-0.17	0.41
1907	-2.68	-0.42	-1.69	1.61	1.12	3.33	-0.99	0.18	1.63	-0.63	3.16	1.64
1908	1.05	1.42	-1.73	-0.96	1.31	0.37	1.78	3.12	0.15	1.77	-0.66	1.11
1909	-1.25	-0.19	0.34	0.2	0.86	2.34	-1.33	0.21	-0.57	-1.19	-2.75	-1.09
1910	-0.18	0.02	-1.96	0.97	0.23	3.42	-0.44	0.47	-1	0.92	-0.52	0.28
1911	-0.31	-1.3	0.01	1.45	-3.51	-1.31	-2.92	1.46	-2.01	-0.71	1.21	0.43
1912	-0.64	0.45	4.54	-0.31	0.99	0.76	-0.81	-1.81	1.55	-1.49	-0.86	-0.35
1913	-0.67	-1.21	1.14	-0.35	0.77	-0.26	-0.82	0.03	0.42	0.64	-1.07	-0.61
1914	-0.2	0.81	-0.08	-0.35	-2.7	-1.42	-1.12	-1.73	-1.19	-0.93	-0.71	1.77
1915	2.86	0.05	-1.77	-2.67	0.73	1.72	-0.77	2.22	-1.09	0.37	-1.16	0.16
1916	-2.41	0.65	-1.33	-0.85	-0.07	3.29	3.55	-1.05	-0.15	-0.73	-1.4	-0.35
1917	-0.35	-0.54	1.99	-0.38	-0.67	1.06	2.86	-0.69	1.61	2.59	-1.13	-0.68
1918	2.3	-2.85	-0.48	3.7	-0.95	0.07	-0.12	-0.24	0.97	-1.93	-1.74	0.27
1919	0.76	-0.49	0.14	-0.21	1.69	0.3	4.83	-0.26	-1.87	-2.51	-1.25	-1.43
1920	-0.73	2.03	0	1.33	-1.42	1.71	1.28	3.33	0.4	-1.66	2.51	1.21
1921	0.2	-0.33	-3.17	-0.91	2.07	-1.89	-0.7	-2.27	-2.07	-2.28	-0.77	-2.13
1922	0.85	1.58	1.4	-1.33	0.26	1.58	3.15	0.81	-1.35	0.07	-2.24	0.26
1923	0.3	-0.02	1.85	0.73	-1.28	-2.05	0.69	-0.55	0.05	-0.89	-0.58	-1.83
1924	0.84	-0.17	1.33	1.17	3.09	1.26	-0.21	0.17	5.03	-0.75	-0.52	0.41
1925	2.46	-2.36	-2.22	-1.32	-1.23	-2.26	-2.14	-1.64	-2.92	-0.95	-0.71	-1.29
1926	1.18	0.72	-0.78	-1.27	-2.09	-1.24	0.03	-0.02	-1.37	-1.2	1.1	1.02
1927	-2.47	-0.27	-2.75	2	-0.95	0.44	0.54	0.96	-1.56	1.65	0.04	2.27
1928	-1.47	-0.46	-1.37	2.49	-0.22	1.36	-0.5	4.51	5.48	-1.7	-1.52	-1.71
1929	-0.52	1.93	-0.57	0.31	0.72	2.53	-0.54	-0.99	0.82	3.38	0.55	-0.59

1930	0.01	-2.75	-2.1	-1.11	-2.08	-0.46	-3.65	-3.11	-2.88	-2.47	-2.33	-1.72
1931	-3.02	-2.21	0.47	0.59	2.13	-0.06	-0.27	2.26	-0.38	-1.53	-3.58	-3.05
1932	0.33	-2.11	1.5	-0.4	-0.03	-0.92	-1.74	-3.85	-2.48	2.09	0.9	1.44
1933	0.07	-0.74	-1.9	1.35	0.4	-2.45	0.36	5.49	-1.66	-1.95	-1.72	-2.12
1934	-2.45	2.13	2.31	-1.21	3.34	-1.17	1.95	-0.1	4.5	-1.63	0.74	-0.86
1935	0.97	-0.77	-1.07	2.63	0.49	-0.18	1.73	1.19	4.8	-0.94	1.49	0
1936	5.11	1.44	0.48	0.98	-3.26	-0.24	-1.34	-2.39	-1.31	-0.95	-2.53	1.09
1937	6.41	-0.76	-1.28	3.63	-0.1	0.96	1.11	1.68	0.45	2.82	3.08	-1.45
1938	-0.57	-1.89	-0.59	0.1	0.96	4.49	3.91	-2.7	3.66	-0.18	0.51	-0.67
1939	-0.2	2.14	0.2	0.88	-2.76	0.07	2.55	2.77	-2.7	1.92	0.87	-1.65
1940	-0.95	-0.55	-1.07	2.56	-0.13	-2.38	-0.8	2.17	-0.87	-0.52	2.53	-1.64
1941	-1.05	-1.52	-1.12	-0.3	-2.98	-0.54	0.15	-2.02	-3.61	-3.59	-3.82	-2.79
1942	-2.14	-1.94	2.3	-3.49	-2.91	-1.55	-1.36	3.39	1.07	4.12	-1.35	0.97
1943	-1.07	-1.07	-0.08	-0.25	0.49	-1.29	-0.68	-3.32	-0.17	1.2	-1.09	-1.67
1944	0.51	2.25	3.28	0.75	-3.63	-3.62	-1.58	-1.09	1.57	-0.23	1.17	-0.37
1945	-1.05	1.03	-4.41	-1.31	0.54	1.57	9.21	-0.9	1.49	-0.62	0.73	4.56
1946	-1.25	-0.58	-2.55	-0.07	4.04	0.59	0.19	1.13	-0.05	-0.55	-0.8	-1.83
1947	0.88	-2.16	-0.82	-0.98	-1.12	0.79	-0.77	-2.71	0.73	-1.79	2.27	-0.9
1948	1.03	-0.43	-0.83	1.86	3.5	-0.32	-1.12	1.83	-0.4	0.22	4.05	2.31
1949	-1.79	0.1	-1.63	-1.32	1.59	-0.57	-0.24	3.02	0.12	-0.48	0.68	-1.2
1950	-2.48	-2.34	-0.24	-1.28	0.17	-2.06	2.98	0.12	1.22	-0.89	-0.82	-0.34
1951	-2.54	-1.85	-0.47	-0.46	-0.94	1.84	-0.98	-0.12	-2.37	-1.02	2.66	0.66
1952	2.32	0.88	1.45	0.21	-0.6	-2.6	-2.58	-0.3	-1.64	-1.11	1.87	0.5
1953	-1.32	-0.43	-0.03	0.72	-0.92	-0.41	-3.37	0.99	0.23	-2.17	-1.32	-1.28
1954	1.79	-2.39	0.08	-1.43	2.16	-3.4	-1.41	-1.17	-2.51	-1.67	-1.88	-1.47
1955	-2.67	0.03	-0.34	-1.3	-1.82	1.03	-2.09	5.11	3.39	0.14	0.16	-1.42
1956	-1.32	1.27	-0.52	0.4	-0.46	-1.45	2.41	-0.49	0.64	3.6	0.59	-0.05
1957	0.4	1.68	1.08	-1.78	-1.95	-0.89	-3.55	0.83	0.5	2.3	3.43	3.06
1958	0.63	0.96	3.26	1.46	2.77	2.1	-0.72	4.75	-2.17	1.82	-0.22	1.72
1959	-2.12	-1.82	-0.18	1.75	-2.6	-2.59	3.26	-2.62	-0.87	3.06	1.75	-0.37
1960	-0.63	1.28	-0.01	-1.66	2.65	-0.93	2.01	1.4	3.93	1.41	-1.29	-0.11
1961	0.27	2.53	0.43	-0.15	3.72	2.63	-1.97	-0.55	-2.06	2.53	-0.97	2.74
1962	2.38	0.22	1.07	1.38	-1.55	1.88	1.06	-1.02	0.43	-1.1	3.57	0.78
1963	-0.51	0.07	1	-3.25	-0.78	4.68	-3	-2.52	1.18	-2.86	1.98	0.29
1964	0.9	2.34	-1.46	0.5	-2.47	-1.45	-0.95	0.17	2.95	2.89	-0.75	0.69
1965	-1.22	-1.45	0.35	-0.47	-3.48	0.73	1.48	-2.2	-2.18	-2.33	-4.01	-5.26
1966	-1.24	0.58	-2.55	-0.98	1.52	0.08	-1.7	-1.09	1.04	-0.46	-2.25	-0.72
1967	-0.97	0.62	-1.8	-2.62	0.63	-1.63	-0.14	2.98	-0.63	-1.49	-0.81	3.01

1968	-0.45	-2.72	0.69	-0.83	-0.15	-0.15	-0.59	-2.19	-2.08	-1.01	-0.25	-0.95
1969	-0.8	-0.08	1.25	-0.93	-1.48	-0.82	1.57	2.45	0.16	-0.71	-0.09	3.09
1970	-1.39	0.09	0.97	0.65	-1.85	-1.1	0.72	-2.58	-2.43	-2.54	-0.48	-1.36
1971	-0.01	1.05	-0.09	-0.88	3.51	-1.96	-1.58	0.51	-0.86	4.7	0.45	-2.34
1972	-0.83	1.94	-0.84	0.13	2.54	4.22	0.13	-1.88	1.7	2.35	4.09	0.79
1973	-0.94	0.42	0.09	1.23	0.21	0.76	-1.89	1.06	-1.91	-1.37	-2.41	2.24
1974	0.2	-0.45	0.85	-1.5	0.08	0.53	-0.86	0.84	1.01	-1.21	-1.94	0.31
1975	2.04	0.64	5.11	0.67	-0.68	-1.32	3.52	-2.01	6.32	1.22	-0.08	0.81
1976	0.96	-2.84	-1.99	-3.25	0.23	-0.68	-1.45	-1.7	1.63	3.64	0.21	0.68
1977	-0.09	-1.42	-0.64	-1.62	0.2	-1.77	-2.5	-2.27	-2.2	2.72	2.28	2.72
1978	4.57	-2.49	3.76	1.27	3.1	0.69	-0.31	-0.16	-2.32	-1.99	0.94	0.34
1979	4.04	2.8	0.32	1.59	3.72	0.98	0.79	0.06	7.52	0.58	3.86	-1.71
1980	1.42	-1.31	1.92	0.23	-0.2	-3.52	-2.73	-3.73	-3.4	1.71	-0.29	-1.65
1981	-3.27	-0.36	-1.81	-0.74	1.37	-0.57	0.01	-0.15	-0.44	0.32	-1.9	2.05
1982	1.29	2.34	0.2	0.07	-0.97	1.16	1.2	1.95	-0.3	0.47	1.58	0.95
1983	-1.71	1.68	2.12	4.46	0.55	-0.02	-4.2	-3.23	0.16	0.53	2.08	3.42
1984	0.77	0.69	4.54	2.48	2.1	-2.17	1.63	-2.29	-1.95	-3.25	-1	-2.94
1985	-0.09	0.8	-2.02	-4.1	-0.87	-0.36	-0.43	-0.1	2.62	1.99	4.07	-2
1986	-0.75	-0.91	-3.14	-2.13	-1.93	-3.54	-2.41	2.18	-3.07	-1.81	-1.76	1.3
1987	5.6	-0.55	-1.13	2.16	-1.22	0.51	-3.3	-2.69	0.29	-1.11	-0.18	-0.41
1988	-0.35	0.45	-1.41	0.02	2.05	-0.62	-1.25	-1.38	-1.24	0	1.3	-2.44
1989	-1.75	2.22	4.63	2.78	1.08	1.8	0.89	2.4	1.67	1.48	2.09	0.27
1990	-0.12	-0.99	-0.47	-0.26	3.8	-2.91	-1.4	2.52	-1.33	0.25	-1.38	-0.32

APPENDIX IV

MONTHLY TRANSITION MATRICES, ARIZONA, PDSI DATA

January - February

0.7500	0.2500	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.3333	0.6667	0.0000	0.0000	0.0000	0.0000
0.0000	0.0833	0.7500	0.1667	0.0000	0.0000	0.0000
0.0000	0.0000	0.1190	0.7381	0.1429	0.0000	0.0000
0.0000	0.0000	0.0000	0.1429	0.8571	0.0000	0.0000
0.0000	0.0000	0.0000	0.1111	0.0000	0.7778	0.1111
0.0000	0.0000	0.0000	0.0000	0.0000	0.3333	0.6667

February - March

0.8333	0.1667	0.0000	0.0000	0.0000	0.0000	0.0000
0.2500	0.5000	0.2500	0.0000	0.0000	0.0000	0.0000
0.0000	0.2500	0.6875	0.0625	0.0000	0.0000	0.0000
0.0000	0.0000	0.1081	0.7838	0.1081	0.0000	0.0000
0.0000	0.0000	0.0000	0.1250	0.7917	0.0833	0.0000
0.0000	0.0000	0.0000	0.0000	0.1250	0.7500	0.1250
0.0000	0.0000	0.0000	0.0000	0.3333	0.0000	0.6667

March - April

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.1429	0.7143	0.1429	0.0000	0.0000	0.0000	0.0000
0.0000	0.1250	0.8750	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.1818	0.6667	0.1515	0.0000	0.0000
0.0000	0.0000	0.0000	0.0800	0.8400	0.0800	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.7500	0.2500
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

April - May

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.5714	0.4286	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.2381	0.7143	0.0476	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.8333	0.1667	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.7692	0.2308	0.0000
0.0000	0.0000	0.0000	0.0000	0.1250	0.8750	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

May - June

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.5000	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0667	0.8000	0.1333	0.0000	0.0000	0.0000
0.0000	0.0000	0.0476	0.8571	0.0952	0.0000	0.0000
0.0000	0.0000	0.0000	0.0800	0.7600	0.1600	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

June - July

0.6667	0.2667	0.0000	0.0667	0.0000	0.0000	0.0000
0.2000	0.4000	0.0000	0.4000	0.0000	0.0000	0.0000
0.0000	0.2308	0.3077	0.4615	0.0000	0.0000	0.0000
0.0000	0.0000	0.0909	0.8636	0.0455	0.0000	0.0000
0.0000	0.0000	0.0000	0.2381	0.4762	0.2857	0.0000
0.0000	0.0000	0.0000	0.0588	0.1176	0.7059	0.1176
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

July - August

0.6364	0.0909	0.0000	0.2727	0.0000	0.0000	0.0000
0.4444	0.1111	0.3333	0.1111	0.0000	0.0000	0.0000
0.0000	0.0000	0.6667	0.3333	0.0000	0.0000	0.0000
0.0000	0.0000	0.1176	0.7941	0.0882	0.0000	0.0000
0.0000	0.0000	0.0000	0.0769	0.7692	0.1538	0.0000
0.0000	0.0000	0.0556	0.2778	0.2778	0.3333	0.0556
0.0000	0.0000	0.1429	0.0000	0.0000	0.1429	0.7143

August - September

0.7273	0.2727	0.0000	0.0000	0.0000	0.0000	0.0000
1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.4615	0.5385	0.0000	0.0000	0.0000
0.0000	0.0000	0.1026	0.7949	0.1026	0.0000	0.0000
0.0000	0.0000	0.0000	0.1111	0.7222	0.1667	0.0000
0.0000	0.0000	0.0000	0.0000	0.3333	0.5556	0.1111
0.0000	0.0000	0.0000	0.0000	0.0000	0.1667	0.8333

September - October

0.8000	0.1000	0.0000	0.1000	0.0000	0.0000	0.0000
0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.1000	0.7000	0.2000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0750	0.8500	0.0750	0.0000	0.0000
0.0000	0.0000	0.0000	0.1000	0.8500	0.0500	0.0000
0.0000	0.0000	0.1111	0.0000	0.2222	0.5556	0.1111
0.0000	0.0000	0.0000	0.0000	0.0000	0.1667	0.8333

October - November

0.6250	0.3750	0.0000	0.0000	0.0000	0.0000	0.0000
0.5000	0.0000	0.5000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0714	0.4286	0.5000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0513	0.8974	0.0513	0.0000	0.0000
0.0000	0.0000	0.0000	0.2273	0.6818	0.0909	0.0000
0.0000	0.0000	0.0000	0.0000	0.2857	0.7143	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

November - December

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2500	0.7500	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.2222	0.6667	0.1111	0.0000	0.0000	0.0000
0.0000	0.0000	0.1277	0.8085	0.0638	0.0000	0.0000
0.0000	0.0000	0.0000	0.0526	0.9474	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.2857	0.7143	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.1667	0.8333

December - January

0.8571	0.0000	0.0000	0.1429	0.0000	0.0000	0.0000
0.4000	0.4000	0.2000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.7500	0.2500	0.0000	0.0000	0.0000
0.0000	0.0256	0.0256	0.7949	0.1538	0.0000	0.0000
0.0000	0.0000	0.0000	0.3043	0.6522	0.0435	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.4000	0.6000

MONTHLY TRANSITION MATRICES, ARIZONA, PHDI DATA

January - February

0.7500	0.2500	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.2500	0.7500	0.0000	0.0000	0.0000	0.0000
0.0000	0.0769	0.8462	0.0769	0.0000	0.0000	0.0000
0.0000	0.0000	0.0789	0.7632	0.1579	0.0000	0.0000
0.0000	0.0000	0.0435	0.1739	0.7826	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.1111	0.7778	0.1111
0.0000	0.0000	0.0000	0.0000	0.0000	0.3333	0.6667

February - March

0.8333	0.1667	0.0000	0.0000	0.0000	0.0000	0.0000
0.2500	0.5000	0.2500	0.0000	0.0000	0.0000	0.0000
0.0000	0.2222	0.6667	0.1111	0.0000	0.0000	0.0000
0.0000	0.0000	0.1176	0.7353	0.1471	0.0000	0.0000
0.0000	0.0000	0.0400	0.1200	0.7600	0.0800	0.0000
0.0000	0.0000	0.0000	0.0000	0.1250	0.7500	0.1250
0.0000	0.0000	0.0000	0.0000	0.3333	0.0000	0.6667

March - April

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.1429	0.7143	0.1429	0.0000	0.0000	0.0000	0.0000
0.0000	0.1111	0.8333	0.0556	0.0000	0.0000	0.0000
0.0000	0.0000	0.1667	0.6667	0.1667	0.0000	0.0000
0.0000	0.0000	0.0385	0.0769	0.8077	0.0769	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.7500	0.2500
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

April - May

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.5714	0.4286	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.2273	0.7273	0.0455	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.8261	0.1739	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.7692	0.2308	0.0000
0.0000	0.0000	0.0000	0.0000	0.1250	0.8750	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

May - June

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.5000	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0625	0.8125	0.1250	0.0000	0.0000	0.0000
0.0000	0.0000	0.0500	0.8500	0.1000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.8400	0.1600	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

June - July

0.7333	0.2667	0.0000	0.0000	0.0000	0.0000	0.0000
0.2000	0.4000	0.4000	0.0000	0.0000	0.0000	0.0000
0.0000	0.2143	0.6429	0.1429	0.0000	0.0000	0.0000
0.0000	0.0000	0.1053	0.8421	0.0526	0.0000	0.0000
0.0000	0.0000	0.0000	0.2609	0.4783	0.2609	0.0000
0.0000	0.0000	0.0000	0.0000	0.1765	0.7059	0.1176
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

July - August

0.5833	0.4167	0.0000	0.0000	0.0000	0.0000	0.0000
0.4444	0.1111	0.4444	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.5385	0.4615	0.0000	0.0000	0.0000
0.0000	0.0000	0.1250	0.7500	0.1250	0.0000	0.0000
0.0000	0.0000	0.0000	0.2000	0.6667	0.1333	0.0000
0.0000	0.0000	0.0000	0.0000	0.5556	0.3889	0.0556
0.0000	0.0000	0.0000	0.0000	0.1429	0.1429	0.7143

August - September

0.7273	0.2727	0.0000	0.0000	0.0000	0.0000	0.0000
0.3333	0.1667	0.5000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.7143	0.2857	0.0000	0.0000	0.0000
0.0000	0.0000	0.2222	0.5556	0.2222	0.0000	0.0000
0.0000	0.0000	0.0000	0.1250	0.7500	0.1250	0.0000
0.0000	0.0000	0.0000	0.0000	0.4000	0.5000	0.1000
0.0000	0.0000	0.0000	0.0000	0.0000	0.1667	0.8333

September - October

0.8000	0.2000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0526	0.6316	0.3158	0.0000	0.0000	0.0000
0.0000	0.0000	0.1364	0.7727	0.0909	0.0000	0.0000
0.0000	0.0000	0.0000	0.1786	0.7857	0.0357	0.0000
0.0000	0.0000	0.0000	0.1111	0.2222	0.5556	0.1111
0.0000	0.0000	0.0000	0.0000	0.0000	0.1667	0.8333

October - November

0.6250	0.3750	0.0000	0.0000	0.0000	0.0000	0.0000
0.3333	0.3333	0.3333	0.0000	0.0000	0.0000	0.0000
0.0000	0.0526	0.5789	0.3684	0.0000	0.0000	0.0000
0.0000	0.0000	0.1034	0.8621	0.0345	0.0000	0.0000
0.0000	0.0000	0.0000	0.1923	0.7308	0.0769	0.0000
0.0000	0.0000	0.0000	0.0000	0.2857	0.7143	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

November - December

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2000	0.6000	0.2000	0.0000	0.0000	0.0000	0.0000
0.0000	0.1333	0.4667	0.4000	0.0000	0.0000	0.0000
0.0000	0.0000	0.1622	0.7838	0.0541	0.0000	0.0000
0.0000	0.0000	0.0000	0.0909	0.9091	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.2857	0.7143	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.1667	0.8333

December - January

0.8571	0.1429	0.0000	0.0000	0.0000	0.0000	0.0000
0.4000	0.4000	0.2000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.7143	0.2857	0.0000	0.0000	0.0000
0.0000	0.0278	0.0278	0.8056	0.1389	0.0000	0.0000
0.0000	0.0000	0.0000	0.2083	0.7500	0.0417	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.4000	0.6000

MONTHLY TRANSITION MATRICES, CALIFORNIA, PDSI DATA

January - February

0.7500	0.0000	0.0000	0.2500	0.0000	0.0000	0.0000
0.2500	0.2500	0.5000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.6000	0.4000	0.0000	0.0000	0.0000
0.0000	0.0000	0.2692	0.5769	0.1538	0.0000	0.0000
0.0000	0.0000	0.0000	0.1905	0.5714	0.2381	0.0000
0.0000	0.0000	0.0000	0.2500	0.0000	0.7500	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

February - March

0.5000	0.2500	0.0000	0.2500	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.1818	0.5000	0.3182	0.0000	0.0000	0.0000
0.0000	0.0250	0.0500	0.7250	0.2000	0.0000	0.0000
0.0000	0.0000	0.0000	0.1000	0.8000	0.1000	0.0000
0.0000	0.0000	0.0000	0.5000	0.0000	0.5000	0.0000
0.0000	0.0000	0.0000	0.0000	0.3333	0.3333	0.3333

March - April

0.5000	0.0000	0.0000	0.5000	0.0000	0.0000	0.0000
0.0000	0.5714	0.1429	0.2857	0.0000	0.0000	0.0000
0.0000	0.1538	0.5385	0.3077	0.0000	0.0000	0.0000
0.0000	0.0233	0.1628	0.6977	0.1163	0.0000	0.0000
0.0000	0.0000	0.0000	0.3200	0.5600	0.1200	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.7143	0.2857
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

April - May

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.5714	0.2857	0.1429	0.0000	0.0000	0.0000
0.1333	0.0000	0.6667	0.2000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0667	0.9111	0.0222	0.0000	0.0000
0.0000	0.0000	0.0000	0.2632	0.6842	0.0526	0.0000
0.0000	0.0000	0.0000	0.0000	0.2500	0.5000	0.2500
0.0000	0.0000	0.0000	0.0000	0.0000	0.3333	0.6667

May - June

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.7500	0.2500	0.0000	0.0000	0.0000	0.0000
0.0000	0.0667	0.9333	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0800	0.8600	0.0600	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.8125	0.1875	0.0000
0.0000	0.0000	0.0000	0.0000	0.1667	0.8333	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

June - July

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0526	0.8421	0.1053	0.0000	0.0000	0.0000
0.0000	0.0000	0.0698	0.8605	0.0698	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.9412	0.0588	0.0000
0.0000	0.0000	0.0000	0.0000	0.1250	0.7500	0.1250
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

July - August

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.4000	0.2000	0.4000	0.0000	0.0000	0.0000
0.0000	0.0000	0.7895	0.2105	0.0000	0.0000	0.0000
0.0000	0.0000	0.0256	0.9744	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0500	0.9500	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.2857	0.7143	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.2000	0.8000

August - September

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.5000	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.1176	0.5882	0.2941	0.0000	0.0000	0.0000
0.0000	0.0000	0.0667	0.8889	0.0444	0.0000	0.0000
0.0000	0.0000	0.0000	0.1905	0.7619	0.0476	0.0000
0.0000	0.0000	0.0000	0.1667	0.1667	0.6667	0.0000
0.0000	0.0000	0.0000	0.0000	0.2500	0.0000	0.7500

September - October

0.5000	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.3333	0.0000	0.6667	0.0000	0.0000	0.0000
0.0000	0.0769	0.6923	0.2308	0.0000	0.0000	0.0000
0.0000	0.0000	0.0600	0.8800	0.0600	0.0000	0.0000
0.0000	0.0000	0.0000	0.4500	0.4500	0.1000	0.0000
0.0000	0.0000	0.0000	0.2000	0.2000	0.6000	0.0000
0.0000	0.0000	0.0000	0.3333	0.0000	0.0000	0.6667

October - November

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.2500	0.5000	0.2500	0.0000	0.0000	0.0000
0.0000	0.0000	0.3333	0.6667	0.0000	0.0000	0.0000
0.0000	0.0000	0.1500	0.7000	0.1500	0.0000	0.0000
0.0000	0.0000	0.0769	0.3077	0.5385	0.0769	0.0000
0.0000	0.0000	0.0000	0.4000	0.4000	0.2000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

November - December

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.5000	0.5000	0.0000	0.0000	0.0000
0.0000	0.0000	0.1754	0.6316	0.1930	0.0000	0.0000
0.0000	0.0000	0.0000	0.2778	0.6111	0.1111	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
0.0000	0.0000	0.0000	0.5000	0.0000	0.0000	0.5000

December - January

0.6667	0.0000	0.0000	0.3333	0.0000	0.0000	0.0000
0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429
0.1111	0.1111	0.2778	0.5000	0.0000	0.0000	0.0000
0.0000	0.0408	0.0816	0.7551	0.1224	0.0000	0.0000
0.0000	0.0000	0.0455	0.1818	0.6818	0.0909	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.5000	0.5000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

MONTHLY TRANSITION MATRICES, CALIFORNIA, PHDI DATA

January - February

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2500	0.2500	0.5000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.6111	0.3889	0.0000	0.0000	0.0000
0.0000	0.0000	0.3333	0.4359	0.2308	0.0000	0.0000
0.0000	0.0000	0.0000	0.1304	0.6522	0.2174	0.0000
0.0000	0.0000	0.0000	0.1667	0.1667	0.6667	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

February - March

0.5000	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.1154	0.5769	0.3077	0.0000	0.0000	0.0000
0.0000	0.0357	0.0714	0.6429	0.2500	0.0000	0.0000
0.0000	0.0000	0.0000	0.2000	0.7200	0.0800	0.0000
0.0000	0.0000	0.0000	0.0000	0.4444	0.5556	0.0000
0.0000	0.0000	0.0000	0.0000	0.3333	0.3333	0.3333

March - April

0.6667	0.3333	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.6250	0.3750	0.0000	0.0000	0.0000	0.0000
0.0000	0.1176	0.6471	0.2353	0.0000	0.0000	0.0000
0.0000	0.0645	0.1613	0.6129	0.1613	0.0000	0.0000
0.0000	0.0000	0.0333	0.2333	0.6333	0.1000	0.0000
0.0000	0.0000	0.0000	0.0000	0.1250	0.6250	0.2500
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

April - May

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.5000	0.5000	0.0000	0.0000	0.0000	0.0000
0.1000	0.0000	0.7500	0.1500	0.0000	0.0000	0.0000
0.0000	0.0000	0.1000	0.8333	0.0667	0.0000	0.0000
0.0000	0.0000	0.0400	0.3200	0.6000	0.0400	0.0000
0.0000	0.0000	0.0000	0.0000	0.2500	0.5000	0.2500
0.0000	0.0000	0.0000	0.0000	0.0000	0.3333	0.6667

May - June

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.8000	0.2000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0417	0.8750	0.0833	0.0000	0.0000	0.0000
0.0000	0.0000	0.1111	0.8056	0.0833	0.0000	0.0000
0.0000	0.0000	0.0000	0.1053	0.7368	0.1579	0.0000
0.0000	0.0000	0.0000	0.0000	0.1667	0.8333	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

June - July

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.8000	0.2000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0385	0.8462	0.1154	0.0000	0.0000	0.0000
0.0000	0.0000	0.0909	0.8485	0.0606	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.9444	0.0556	0.0000
0.0000	0.0000	0.0000	0.0000	0.1250	0.7500	0.1250
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

July - August

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.4000	0.6000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.8462	0.1538	0.0000	0.0000	0.0000
0.0000	0.0000	0.0323	0.9677	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.2857	0.7143	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.2000	0.8000

August - September

0.7500	0.2500	0.0000	0.0000	0.0000	0.0000	0.0000
0.5000	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0769	0.7692	0.1538	0.0000	0.0000	0.0000
0.0000	0.0000	0.0882	0.8529	0.0588	0.0000	0.0000
0.0000	0.0000	0.0455	0.1364	0.7727	0.0455	0.0000
0.0000	0.0000	0.0000	0.0000	0.1667	0.8333	0.0000
0.0000	0.0000	0.0000	0.0000	0.2500	0.0000	0.7500

September - October

0.5000	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.5000	0.5000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0417	0.6667	0.2917	0.0000	0.0000	0.0000
0.0000	0.0000	0.0833	0.8333	0.0833	0.0000	0.0000
0.0000	0.0000	0.0000	0.4286	0.4762	0.0952	0.0000
0.0000	0.0000	0.0000	0.0000	0.5000	0.5000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

October - November

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.2000	0.8000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.4762	0.5238	0.0000	0.0000	0.0000
0.0000	0.0000	0.2174	0.5870	0.1957	0.0000	0.0000
0.0000	0.0000	0.0625	0.3125	0.5625	0.0625	0.0000
0.0000	0.0000	0.0000	0.0000	0.8000	0.2000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.3333	0.6667

November - December

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.6000	0.4000	0.0000	0.0000	0.0000
0.0000	0.0000	0.2093	0.5349	0.2558	0.0000	0.0000
0.0000	0.0000	0.0455	0.2727	0.5909	0.0909	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

December - January

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429
0.0800	0.0800	0.5200	0.3200	0.0000	0.0000	0.0000
0.0000	0.0513	0.1026	0.6923	0.1538	0.0000	0.0000
0.0000	0.0000	0.0435	0.1304	0.7391	0.0870	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.6000	0.4000
0.0000	0.0000	0.0000	0.0000	0.0000	0.5000	0.5000

MONTHLY TRANSITION MATRICES, VIRGINIA, PDSI DATA

January - February

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.2000	0.6000	0.2000	0.0000	0.0000	0.0000
0.0000	0.0952	0.6667	0.2381	0.0000	0.0000	0.0000
0.0000	0.0000	0.1000	0.8400	0.0600	0.0000	0.0000
0.0000	0.0000	0.0000	0.2727	0.7273	0.0000	0.0000
0.0000	0.0000	0.0000	0.2500	0.2500	0.5000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.2500	0.7500

February - March

0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.6667	0.3333	0.0000	0.0000	0.0000
0.0000	0.2273	0.5000	0.2727	0.0000	0.0000	0.0000
0.0000	0.0000	0.0769	0.8269	0.0962	0.0000	0.0000
0.0000	0.0000	0.0000	0.1667	0.8333	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.3333	0.6667	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

March - April

0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429
0.0000	0.8333	0.0000	0.1667	0.0000	0.0000	0.0000
0.0000	0.1176	0.7059	0.1765	0.0000	0.0000	0.0000
0.0000	0.0000	0.0769	0.8654	0.0577	0.0000	0.0000
0.0000	0.0000	0.0000	0.1875	0.6875	0.1250	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

April - May

0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429
0.5714	0.1429	0.1429	0.1429	0.0000	0.0000	0.0000
0.0000	0.1250	0.6250	0.2500	0.0000	0.0000	0.0000
0.0000	0.0000	0.0962	0.8654	0.0385	0.0000	0.0000
0.0000	0.0000	0.0000	0.2857	0.6429	0.0714	0.0000
0.0000	0.0000	0.0000	0.0000	0.5000	0.5000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.6667	0.3333

May - June

0.2500	0.5000	0.0000	0.2500	0.0000	0.0000	0.0000
0.0000	0.3333	0.0000	0.6667	0.0000	0.0000	0.0000
0.0000	0.0625	0.6875	0.2500	0.0000	0.0000	0.0000
0.0000	0.0000	0.0926	0.8148	0.0926	0.0000	0.0000
0.0000	0.0000	0.0000	0.2308	0.6154	0.1538	0.0000
0.0000	0.0000	0.0000	0.0000	0.2000	0.8000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

June - July

0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.7500	0.2500	0.0000	0.0000	0.0000	0.0000
0.0000	0.0625	0.6250	0.3125	0.0000	0.0000	0.0000
0.0000	0.0185	0.1667	0.6667	0.1481	0.0000	0.0000
0.0000	0.0000	0.0000	0.2143	0.7143	0.0714	0.0000
0.0000	0.0000	0.0000	0.0000	0.3333	0.5000	0.1667
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

July - August

0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429
0.1667	0.6667	0.1667	0.0000	0.0000	0.0000	0.0000
0.0000	0.1000	0.6500	0.2500	0.0000	0.0000	0.0000
0.0000	0.0227	0.0909	0.7045	0.1818	0.0000	0.0000
0.0000	0.0000	0.0000	0.1000	0.7500	0.1500	0.0000
0.0000	0.0000	0.2500	0.2500	0.0000	0.5000	0.0000
0.0000	0.0000	0.0000	0.5000	0.0000	0.5000	0.0000

August - September

0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.1429	0.4286	0.4286	0.0000	0.0000	0.0000	0.0000
0.0000	0.2105	0.4211	0.3684	0.0000	0.0000	0.0000
0.0000	0.0250	0.0500	0.8000	0.1250	0.0000	0.0000
0.0000	0.0000	0.0000	0.2174	0.6522	0.1304	0.0000
0.0000	0.0000	0.0000	0.0000	0.1667	0.5000	0.3333
0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429

September - October

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.1111	0.3333	0.5556	0.0000	0.0000	0.0000	0.0000
0.0000	0.2308	0.5385	0.2308	0.0000	0.0000	0.0000
0.0000	0.0000	0.1818	0.7273	0.0909	0.0000	0.0000
0.0000	0.0000	0.0000	0.0476	0.9524	0.0000	0.0000
0.0000	0.0000	0.0000	0.1667	0.1667	0.5000	0.1667
0.0000	0.0000	0.0000	0.5000	0.0000	0.0000	0.5000

October - November

0.5000	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000
0.1667	0.5000	0.3333	0.0000	0.0000	0.0000	0.0000
0.0000	0.0500	0.7500	0.2000	0.0000	0.0000	0.0000
0.0000	0.0000	0.2105	0.7105	0.0789	0.0000	0.0000
0.0000	0.0000	0.0000	0.2800	0.6400	0.0800	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.6667	0.3333
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

November - December

0.0000	0.5000	0.0000	0.5000	0.0000	0.0000	0.0000
0.0000	0.8000	0.2000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0800	0.6400	0.2800	0.0000	0.0000	0.0000
0.0000	0.0000	0.0789	0.8158	0.1053	0.0000	0.0000
0.0000	0.0000	0.0000	0.2632	0.5789	0.1579	0.0000
0.0000	0.0000	0.0000	0.0000	0.2500	0.5000	0.2500
0.0000	0.0000	0.0000	0.3333	0.0000	0.0000	0.6667

December - January

0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429
0.0000	0.5714	0.2857	0.1429	0.0000	0.0000	0.0000
0.0500	0.0500	0.7000	0.2000	0.0000	0.0000	0.0000
0.0000	0.0000	0.1136	0.8864	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.2500	0.5625	0.1875	0.0000
0.0000	0.0000	0.0000	0.2000	0.4000	0.2000	0.2000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

MONTHLY TRANSITION MATRICES, VIRGINIA, PHDI DATA

January - February

0.5000	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.2000	0.8000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0870	0.7391	0.1739	0.0000	0.0000	0.0000
0.0000	0.0000	0.1136	0.7727	0.1136	0.0000	0.0000
0.0000	0.0000	0.0000	0.2143	0.7857	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.5000	0.5000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.2500	0.7500

February - March

0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2500	0.0000	0.7500	0.0000	0.0000	0.0000	0.0000
0.0000	0.1923	0.4615	0.3462	0.0000	0.0000	0.0000
0.0000	0.0000	0.0976	0.8049	0.0976	0.0000	0.0000
0.0000	0.0000	0.0000	0.2778	0.7222	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.3333	0.6667	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

March - April

0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.8333	0.1667	0.0000	0.0000	0.0000	0.0000
0.0000	0.1053	0.7368	0.1579	0.0000	0.0000	0.0000
0.0000	0.0000	0.0851	0.8085	0.1064	0.0000	0.0000
0.0000	0.0000	0.0000	0.1667	0.7222	0.1111	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

April - May

0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429
0.5000	0.2500	0.2500	0.0000	0.0000	0.0000	0.0000
0.0000	0.1053	0.6842	0.2105	0.0000	0.0000	0.0000
0.0000	0.0000	0.1136	0.8409	0.0455	0.0000	0.0000
0.0000	0.0000	0.0000	0.2778	0.6667	0.0556	0.0000
0.0000	0.0000	0.0000	0.0000	0.5000	0.5000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.6667	0.3333

May - June

0.2500	0.5000	0.2500	0.0000	0.0000	0.0000	0.0000
0.0000	0.5000	0.5000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0500	0.6000	0.3500	0.0000	0.0000	0.0000
0.0000	0.0000	0.1087	0.7609	0.1304	0.0000	0.0000
0.0000	0.0000	0.0000	0.3125	0.5625	0.1250	0.0000
0.0000	0.0000	0.0000	0.0000	0.2000	0.8000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

June- July

0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.6000	0.2000	0.2000	0.0000	0.0000	0.0000
0.0000	0.1000	0.5500	0.3500	0.0000	0.0000	0.0000
0.0000	0.0213	0.1915	0.6596	0.1277	0.0000	0.0000
0.0000	0.0000	0.0000	0.2500	0.6875	0.0625	0.0000
0.0000	0.0000	0.0000	0.0000	0.3333	0.5000	0.1667
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

July - August

0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429
0.1429	0.5714	0.2857	0.0000	0.0000	0.0000	0.0000
0.0000	0.0952	0.7143	0.1905	0.0000	0.0000	0.0000
0.0000	0.0233	0.1395	0.5814	0.2326	0.0233	0.0000
0.0000	0.0000	0.0526	0.0526	0.7895	0.1053	0.0000
0.0000	0.0000	0.0000	0.2500	0.2500	0.5000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000

August - September

0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.1429	0.4286	0.4286	0.0000	0.0000	0.0000	0.0000
0.0000	0.1667	0.3750	0.4583	0.0000	0.0000	0.0000
0.0000	0.0323	0.0968	0.7097	0.1613	0.0000	0.0000
0.0000	0.0000	0.0000	0.1154	0.7692	0.1154	0.0000
0.0000	0.0000	0.0000	0.0000	0.2857	0.4286	0.2857
0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429

September - October

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.1111	0.3333	0.5556	0.0000	0.0000	0.0000	0.0000
0.0000	0.2000	0.5333	0.2667	0.0000	0.0000	0.0000
0.0000	0.0000	0.1944	0.6944	0.1111	0.0000	0.0000
0.0000	0.0000	0.0370	0.1481	0.8148	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.3333	0.5000	0.1667
0.0000	0.0000	0.0000	0.0000	0.0000	0.5000	0.5000

October - November

0.5000	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000
0.1667	0.5000	0.3333	0.0000	0.0000	0.0000	0.0000
0.0000	0.0476	0.8095	0.1429	0.0000	0.0000	0.0000
0.0000	0.0000	0.2121	0.6667	0.1212	0.0000	0.0000
0.0000	0.0000	0.0000	0.2857	0.6429	0.0714	0.0000
0.0000	0.0000	0.0000	0.0000	0.2500	0.5000	0.2500
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

November - December

0.5000	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.8000	0.2000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0769	0.7308	0.1923	0.0000	0.0000	0.0000
0.0000	0.0000	0.1515	0.7576	0.0909	0.0000	0.0000
0.0000	0.0000	0.0000	0.2174	0.6522	0.1304	0.0000
0.0000	0.0000	0.0000	0.0000	0.2500	0.5000	0.2500
0.0000	0.0000	0.0000	0.0000	0.0000	0.3333	0.6667

December - January

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.5714	0.4286	0.0000	0.0000	0.0000	0.0000
0.0400	0.0400	0.6400	0.2800	0.0000	0.0000	0.0000
0.0000	0.0000	0.0882	0.9118	0.0000	0.0000	0.0000
0.0000	0.0000	0.0526	0.2105	0.5789	0.1579	0.0000
0.0000	0.0000	0.0000	0.1667	0.5000	0.1667	0.1667
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

MEAN MONTHLY TRANSITION MATRICES

AZ, PDSI

0.8019	0.1415	0.0000	0.0566	0.0000	0.0000	0.0000
0.3559	0.3898	0.2034	0.0508	0.0000	0.0000	0.0000
0.0000	0.1274	0.6561	0.2166	0.0000	0.0000	0.0000
0.0000	0.0024	0.0911	0.8034	0.1031	0.0000	0.0000
0.0000	0.0000	0.0000	0.1284	0.7588	0.1128	0.0000
0.0000	0.0000	0.0168	0.0588	0.1513	0.6975	0.0756
0.0000	0.0000	0.0167	0.0000	0.0167	0.1167	0.8500

AZ, PHDI

0.8037	0.1963	0.0000	0.0000	0.0000	0.0000	0.0000
0.3134	0.3731	0.3134	0.0000	0.0000	0.0000	0.0000
0.0000	0.1026	0.6821	0.2154	0.0000	0.0000	0.0000
0.0000	0.0029	0.1091	0.7640	0.1239	0.0000	0.0000
0.0000	0.0000	0.0105	0.1324	0.7561	0.1010	0.0000
0.0000	0.0000	0.0000	0.0083	0.2167	0.7000	0.0750
0.0000	0.0000	0.0000	0.0000	0.0333	0.1167	0.8500

CA, PDSI

0.7941	0.0882	0.0000	0.1176	0.0000	0.0000	0.0000
0.0714	0.5238	0.2143	0.1905	0.0000	0.0000	0.0000
0.0212	0.0688	0.6085	0.3016	0.0000	0.0000	0.0000
0.0000	0.0070	0.1099	0.7801	0.1030	0.0000	0.0000
0.0000	0.0000	0.0086	0.1983	0.6940	0.0991	0.0000
0.0000	0.0000	0.0000	0.1286	0.1429	0.6286	0.1000
0.0000	0.0000	0.0000	0.0571	0.0571	0.0857	0.8000

CA, PHDI

0.8372	0.1628	0.0000	0.0000	0.0000	0.0000	0.0000
0.0600	0.5200	0.4200	0.0000	0.0000	0.0000	0.0000
0.0144	0.0432	0.6871	0.2554	0.0000	0.0000	0.0000
0.0000	0.0117	0.1408	0.7089	0.1385	0.0000	0.0000
0.0000	0.0000	0.0227	0.1932	0.6970	0.0871	0.0000
0.0000	0.0000	0.0000	0.0130	0.2597	0.6364	0.0909
0.0000	0.0000	0.0000	0.0000	0.0541	0.1351	0.8108

VA, PDSI

0.3077	0.5385	0.0000	0.1538	0.0000	0.0000	0.0000
0.1176	0.4706	0.3088	0.1029	0.0000	0.0000	0.0000
0.0044	0.1156	0.6267	0.2533	0.0000	0.0000	0.0000
0.0000	0.0053	0.1103	0.7954	0.0890	0.0000	0.0000
0.0000	0.0000	0.0000	0.2059	0.6961	0.0980	0.0000
0.0000	0.0000	0.0192	0.0769	0.2308	0.5385	0.1346
0.0000	0.0000	0.0000	0.1111	0.0000	0.1481	0.7407

VA, PHDI

0.3750	0.5625	0.0625	0.0000	0.0000	0.0000	0.0000
0.1233	0.4658	0.3973	0.0137	0.0000	0.0000	0.0000
0.0039	0.1042	0.6293	0.2625	0.0000	0.0000	0.0000
0.0000	0.0063	0.1315	0.7474	0.1127	0.0021	0.0000
0.0000	0.0000	0.0124	0.2066	0.7025	0.0785	0.0000
0.0000	0.0000	0.0000	0.0364	0.3273	0.5091	0.1273
0.0000	0.0000	0.0000	0.0000	0.0000	0.2593	0.7407

APPENDIX V

CONTINUOUS MARKOV PROCESS REPRESENTATION

The representation of the PDSI process as a continuous Markov process is motivated in Chapter V. Here it is assumed that changes in state are occurring continually all the time and the state space is continuous. A continuous stochastic process can be visualised as family of random variables $\{X(t), t \in T\}$ such that the state of the system is characterised at every instant over a finite or infinite interval. The system is then defined for a continuous range of times and we say that we have a family of random variables in continuous time. A continuous Markov process can be thought of as systems whose memory time is so small that, on the time scale on which we carry out observations, it is fair to regard them as being well approximated by a Markov process. Gardiner (1985) observes that for a Markov process , it can be shown that with probability one, the sample paths are continuous function of t, if for any e > 0 we have

$$\lim_{\Delta t \rightarrow} 1/dt \int_{|x-z|>\varepsilon} dx\, p(x,t+dt \ |z,t) = 0 \qquad (1)$$

uniformly in z, t and dt . This means that the probability for the final position x to be finitely different from z goes to zero faster than dt, as dt goes to zero. Gillespie (1992) has described a propagator random variable to describe continuous Markov process as follows. Suppose a process is in state x at time t or, X(t)= x . Then by the infinitesimally later time t+dt , the system will have evolved to some new state that is displaced from x by an amount ,say K , where K is given by :

$$K(dt\,;x,t) \;=\; X(t+dt) - X(t), \quad \text{given } X(t) = x \qquad (2)$$

Here K, which is a random variable for a fixed x and t, is called propagator of the process X(t). The propagator tells us where the process will be at the infinitesimally later time t+dt given that it starts from state x at time t, which in this case will be x+ K(dt;x,t). Here dt is a real variable whose allowed range is the open interval $(0,\varepsilon)$, where ε is positive but 'arbitrarily close to zero'. Let the density of the propagator random variable (K) be expressed as R(s|dt; x,t). The continuous Markov process is defined using the density R(s|dt:x,t) which has to follow the following two conditions : i) R(s|dt;x,t) varies smoothly with each of its three parameters dt, x and t, and ii) R(s|dt;x,t) is practically zero everywhere outside an infinisimally small neighborhood of s=0

Gillespie (1992) has shown that K(dt;x,t) is a normal random variable with mean and variance given by functions A(x,t)dt and D(x,t)dt, respectively. Therefore :

$$R(s|\ dt;\ x,t) = \frac{1}{(2\pi\ D\ (x,t)\ dt\ )^{.5}}\ \exp\{-(s-A(x,t)dt)^2/(2\ D(x,t)dt)\} \quad (3)$$

It can be noticed in Eq. 3 that the functions A(x,t)dt and D(x,t)dt completely characterize the continuous Markov process X(t). This is because these two functions completely determine the propagator density function R(s|dt; x,t) which in turn determines the Markov state density function $P(x,t|x_o,t_o)$. The state density function will determine the density of state variable X at time t given that at time t_o the state variable has value x_o. If A(x,t) and D(x,t) functions are independent of x and t, then such a process is called completely homogeneous Markov process. In particular, if A(x,t) = A; and D(x,t) = D ≥ 0; such a process is commonly called a Wiener process.

The time evolution equation for $P(x,t|x_o,t_o)$, for fixed x_o and t_o , for a continuous Markov process with characterising functions A(x,t) and D(x,t) is given by:

$$\frac{\partial}{\partial t}\ p(x,t|x_o,t_o) = -\ \frac{\partial}{\partial x}[A(x,t)\ p(x,t|\ x_o,t_o)]\ +$$

$$1/2\frac{\partial^2}{\partial x^2}\ [D(x,t)\ p(x,t|x_o,t_o)] \quad (4)$$

Eq. 4 is called the Fokker Planck equation. With the known initial conditions, solution of Eq. 4 gives a time evolution for $p(x,t|x_o,t_o)$, for fixed x_o and t_o. Therefore, this equation describes a process in which X(t) has continuous sample paths. The origin of the name 'Fokker Planck Equation' is from the work of Fokker (1914) and Planck (1917) where the former investigated Brownian motion in a radiation field and the latter attempted to build a complete theory of fluctuations.

Gillespie (1992) observes that most continuous Markov processes of practical interest are temporally homogeneous which implies :

$$A(x,t) = A(x) \quad ; \text{and} \quad D(x,t) = D(x) \tag{5}$$

Therefore, for a temporary homogeneous process Eq. 4 becomes :

$$\frac{\partial}{\partial t} p(x,t|x_o,t_o) = - \frac{\partial}{\partial x}[A(x,t)\, p(x,t|\, x_o,t_o)] + 1/2 \frac{\partial^2}{\partial x^2} [D(x,t)p(x,t|x_o,t_o)] \tag{6}$$

The temporally homogeneous Markov Processes sometimes approach steady state density as (t-to) goes to infinity (Gillespie, 1992). The specific conditions for existence of the steady state density can be described as below:

Steady density function infers :

$$\lim_{t \to 0} p(x,t|x_o,t_o) = P_s(x) \tag{7}$$

One way to compute the steady state density will be to solve Eq. 6 explicitly for $p(x,t|x_o,t_o)$ and then compute the limit as given by Eq. 7. Another way is to calculate $P_s(x)$ such that :

$$\lim_{t-t0 \to \infty} \frac{\partial}{\partial t} P(x,t|x_o,t_o) = 0 \tag{8}$$

Substituting the condition as stated in Eq. 8 in Eq. 6 , we get :

$$0 = - \frac{d}{dx} [\, A(x)\, P_s(x)] + \frac{1}{2} \frac{d^2}{dx^2} [D(x)\, P_s(x)\,]$$

or;

$$\frac{d}{dx} \{- [\, A(x)\, P_s(x)] + \frac{1}{2} \frac{d}{dx} [D(x)\, P_s(x)\,]\, \} = 0 \tag{9}$$

Eq. 9 implies that the quantity under braces must be constant with respect to x and taking this constant to be equal to zero, we get :

$$\frac{d}{dx} [D(x)\, P_s(x)\,] = 2\, A(x)\, P_s(x) \tag{10}$$

Dividing by $D(x)P_S(x)$ both sides , we get ;

$$\frac{d(D(x)P_S(x))}{D(x)P_S(x)} = 2 \frac{A(x)}{D(x)} \quad (11)$$

Integration gives :

$$\ln (D(x)\, P_S(x)) = 2 \frac{A(x)}{D(x)} \, dx + \text{constant}$$

or;

$$D(x)\, P_S(x) = \text{constant} \, . \, \exp \left(\int_x 2 \frac{A(x)}{D(x)} \, dx \right)$$

Solving for $P_S(x)$, we get :

$$P_S(x) = \frac{2K'}{D(x)} \exp\left(-\phi(x)\right) \quad (12)$$

where: $\phi(x)$ = potential function = $- \int_x 2 \frac{A(x)}{D(x)} \, dx$ (13)

and K' = normalising constant which is equal to :

$$\frac{1}{K'} = \int_x \frac{2}{D(x)} \exp\left(-\phi(x)\right) dx \quad (14)$$

Eq. 12 indicates that existence of $P_S(x)$ mainly depends on K'. Therefore, prime requirement of existence of $P_S(x)$ can then be expressed in terms of the values of K' which must exist as a finite non-zero number or;

$$0 < K' < \infty \quad (15)$$

It can be observed that as long as K' is finite, $P_S(x)$ will not only exist but will also be everywhere non-zero. Here $\phi(x)$ is called the potential function. Differentiating $\phi(x)$ in Eq. 13 with respect to x, we get :

$$\phi'(x) = \frac{-2A(x)}{D(x)} \quad (16)$$

The slope of the $\phi(x)$ curve, therefore, depends upon A(x) and D(x) . Since D(x) is always positive as it denotes the variance, the slope will mainly depend upon A(x).

Therefore, if $A(x)$ is positive , the $\phi(x)$ will be sloping downward and the process moves to the right and if $A(x)$ is negative, the process moves to the left and $\phi(x)$ slopes upward.

The slope of $\phi(x)$ curve will depend upon magnitude of $|A(x)|$. So the process in state x at time t has a probabilistic bias to move in the next dt in direction that decreases the function $\phi(x)$ and with the increase of local slope of $\phi(x)$, this probabilistic bias will also increase. Therefore, $\phi(x)$ can be taken as analogous to the potential energy function in classical mechanics.

Now as the process keeps advancing the value of $\phi(x)$ also changes. The process has stochastic tendency to move toward local minimum of the potential function $\phi(x)$. The regions where $\phi(x)$ is small corresponds to higher value of $P_S(x)$. In terms of potential energy and probability density it can be interpreted that the state corresponding to minimum values of the potential energy correspond to maximum probability of occurrence which can then be taken as the 'Stable States' of the process. These stable states can be related to the multi-modal behavior of the distribution of some random variable which in our case happens to be the PDSI. It can also be noted from Eq. 12 that the values of $P_S(x)$ will also depend on $D(x)$. However, for a constant value of $D(x)$, the peaks of $P_S(x)$ will coincide with the minima of the $\phi(x)$.

APPENDIX VI

NUMERICAL EXAMPLES

1.0 INTRODUCTION

In chapter IV non-homogeneous Markov chain analysis of long term Palmer index data is presented. In this approach currently observed index values are used in finding the most probable drought characteristics in the future, i.e. given that a particular drought class has occurred, we can find information like : What is the most likely occurrence drought class?; What is the probability that a severer or a milder drought will occur over a chosen period ?; How long such a future drought can persist ?; What are the long term probabilities (steady state probabilities) for various drought classes defining the drought proneness of a region ?; How long will it take to get back to the normal class from a severer drought class ? These aspects are illustrated through a number of example problems in the following.

2.0 APPLICATION EXAMPLES

EXAMPLE 1 : The monthly PDSI data from 1895-1990 for Tidewater region in Virginia are given in Appendix III. Compute the following :

a) Steady state probabilities of wet, dry, and normal weather classes for different months using the three PDSI classes given in Table VI.1 with the aid of the Markov chain.

b) State the empirical steady state probabilites computed from the data directly.

c) Compare analytical and empirical probabilites and identify months that have largest and smallest probability of droughts on long run as indicated by PDSI data.

Solution : The stepwise solution procedure using the Markov chain technique, described in detail in section 4.3 of chapter IV, is given below :

a) Step 1 : Allocate PDSI values in respect of Tidewater division in Virginia in Appendix III into their appropriate classes according to Table VI.1. The results are in Table VI.2.

Step 2 : Compute the twelve monthly transition probability matrices with the transition probabilities given by

$$P_{i,j}^{(n,n+1)} = N_{i,j}^{(n,n+1)} / N_i^{(n)} \qquad (1)$$

where: $N_{i,j}^{(n,n+1)}$ = number of transitions from class i in month n to class j in month n+1;

$N_i^{(n)}$ = number of occurrences of class i in month n.

The transition probabilty matrices are given in Table VI.3.

Step 3 : Compute monthwise steady state probabilities of each class using

$$\phi^{(m,k)} = [P_m]\,[P_{m+1}] \ldots [P_k] \qquad (1a)$$

where : $\phi^{(m,k)}$ is a constant stochastic matrix with identical rows for large k, and

$[P_m]$ is transition matrix starting month m.

The results are shown in Table VI.4.

b) Step 1 : Compute the empirical probabilities by

$$EMPROB_{ij} = N_{ij} / N_j \;\; ; i=1,2,\text{and } 3 : j=1,2,3,\ldots, 12. \qquad (2)$$

where : $EMPROB_{ij}$ = empirical probability of class i in month j

N_{ij} = number of occurrences of class i in month j

N_j = number of ocurrences of month j in data period

For example to compute empirical probability for class 1 in January ($EMPROB_{1Jan}$) count the occurrences of class 1 in January for the ninety six years of data using Table VI.2. The counts are N_{1Jan}=27 and N_{Jan} = 96. Therefore,

$$EMPROB_{1Jan} = 27/\,96 = .2813$$

Likewise empirical probabilities for each class can be computed. Results are given within brackets in Table VI.4. They are quite close to the analytical probabilities.

c) Step 1 : For July through November there is more than 27 % chance of occurrence of a drought. During January - February there is less than 20 % chance of a drought.

This example shows that the non-homogeneous Markov chain approach can describe the long term behavior of weather well. The key advantage is that a formal analytical framework is provided by the Markov chain formulation. The agreement with the empirical probabilities demonstrate that a laborious empirical analysis is not needed. Besides, the Markov analysis provides additional results merely by the use of transition martices as illustrated in the following examples such as drought duration and time of returning to the same drought class after exiting and more importantly forecasting droughts contingent on the observed severity status of a drought.

EXAMPLE 2: Using the long term PDSI data of Virginia given in Appendix III and the corresponding weather classes given in Table VI.2, compute the following :

a) Probability that the weather will stay in class 3 (drought class) for five months continuously given that drought class 3 begins in the month of July.

b) Monthwise expected duration (residence time) for each weather class.

c) Variance of residence time for each month and class and coefficient of variation to compare relative variability.

d) Compare results with empirical observations and draw appropriate conclusions.

Solution : We use the Markov chain procedure to solve the problem in following steps :

a) Step 1 : Compute one step transition matrices for each month following the procedure as described in step 2 in part (a) of Example 1.

Step 2 : Define the following event in order for weather to stay in class 3 (drought class) for 1 month given that it begins in the month of July

$$[\ X_{Aug.} \neq 3 \mid X_{Jul.} = 3\ ]$$

The above event will correspond to 1 month duration of drought beginning in July. The probability of above event can be computed using the July- August transition matrix in Table VI.3 as follows

$$P\,[\,X_{Aug.} \neq 3 \mid X_{Jul.} = 3\,] = 1 - p_{33}^{Jul\text{-}Aug} = 1 - .8077 = .1923 \qquad (4)$$

Similarly, the event defining continuous stay of weather in class 3 for 5 months beginning in the month of July can be written as

$$[\,X_{Aug.} = 3,\ X_{Sep.} = 3,\ X_{Oct.} = 3,\ X_{Nov.} = 3,\ X_{Dec.} \neq 3 \mid X_{Jul.} = 3\,]$$

Using the property of Markov chain, the probability of above event can be computed as

$$[\,X_{Aug.} = 3,\ X_{Sep.} = 3,\ X_{Oct.} = 3,\ X_{Nov.} = 3,\ X_{Dec.} \neq 3 \mid X_{Jul.} = 3\,]$$

$$(1 - p_{33}^{Nov.\text{-}Dec.})\ p_{33}^{Oct.\text{-}Nov.}\ p_{33}^{Sep.\text{-}Oct.}\ p_{33}^{Aug.\text{-}Sep.}\ p_{33}^{Jul.\text{-}Aug.} \qquad (5)$$

Using the transition probabilities matrices given in Table VI.3, put the appropriate probabilities in Eq. 5 and we get

$$P\,[\,X_{Aug.} = 3,\ X_{Sep.} = 3,\ X_{Oct.} = 3,\ X_{Nov.} = 3,\ X_{Dec.} \neq 3 \mid X_{Jul.} = 3\,] =$$

$$(1 - .7692)\,(.7667)\,(.8966)\,(.8276)\,(.8077) = .1061 \qquad (6)$$

Therefore, the probability of having continuous occurrence of class 3 drought starting in the month of July is 10.61 %.

b) Step 1 : Following the procedure in step 2 of part (a) we can calculate probability of continuous stay or duration of "n" months in each drought class. As an illustration, the probabilities for varying length of uninterrupted residence times in class 3 beginning in July are given in Table VI.5. Then using Eq. 17 of chapter IV the expected value of uninterrupted residence time can be computed. The results are given in Table VI.6.

c) Step 1 : Using the probabilities computed in step 3, we can compute the variance of uninterrupted residence time using Eq. 18 as given in chapter IV. The results are given within parentheses(.) in Table VI.6. Divide the standard deviation by the computed mean value to compute the coefficient of variation. The results are given within square brackets [.] in Table VI.6.

d) Step 1 : In order to compute average (duration) empirical uninterrupted residence time, use the following equation

$$EMPUNRE_{ij} = (1/N_{ij}) \sum_{n=1}^{Nij} (UNRE_{ij})_n \qquad (7)$$

where : $EMPUNRE_{ij}$ = empirical uninterrupted residence time in class j in month i

$UNRE_{ij}$ = length of uninterrupted stay in class i beginning in month j

N_{ij} = total number of occurrences of class i in month j.

For demonstation, we compute empirical average empirical uninterrupted residence time of weather class 1 given that class 1 begins in the month of February.

From Table VI.2, count occurrences of class 1 in the month of February. It is found as 26. Hence, $N_{1Feb.}$=26 . Now for each occurrence of class 1 in February, find the length of continuous stay in class 1. For example, the first ocuurrence of class 1 in data record is in February 1899 (see Table VI.2). At this occurrence weather stayed in class 1 continuously for 2 months beginning February 1899. Therefore, for the first event the length of continuous stay is 2 months. Likewise compute length of continuous stay for each of 26 events. Now use Eq. 7 to compute empirical average uninterrupted residence time as follows

$EMPUNRE_{1Feb.}$=

(2+1+9+17+5+1+3+1+1+3+1+3+2+11+7+8+2+1+5+8+10+2+4+4+1+4)/26 = 4.46 mon.

It can be observed from Table VI.6 that analytical value of uninterrupted residence time for class 1 in February month is 4.4 months which compares very well with the empirical calculation of 4.5 months.

There are 4 values reported for each month and class in Table VI.6. The first value gives the expected value of the uninterrupted residence time (duration), the value within the parentheses gives its variance, the value within square brackets gives the coefficient of variation and the last value gives the empirical results. For example, it can be observed that once drought conditions are set in the month of January, these will prevail on an average for 6.1 months. Based on coefficient of variation it is seen that all the months have about the same variability for drought durations.

This example demonstrated application of non-homogeneous Markov chain approach to assess the duration of drought once it begins. Once again the analytical results agree well with the observed values. Having computed the persistance time in a drought class, one is interested to know, how long will it take to the relief situation in terms of normal class or wet class. Intuitively, by the definition of duration of drought we also imply its termination. The termination has to be in terms of the normal or the wet class. Because drought progression is continuous, that is to reach the wet state the process must go through the normal state. Therefore, the time to reach the normal state must be the same as the drought class duration. The last columns of Tables VI.6 and VI.7 support this conclusion. By the same argument first columns of the Tables VI.6 and VI.7 agree for the wet state. To interpret the time of return to the normal class (class 4), first of all note from the transition matrices in Table VI.3 that the staying power of probability of returning to class 2 or the probability of having unit period (1 month) time of return is about .8. For a recurrence time period of more than 1 month, the weather class has to necessarily migrate to the wet class 1 or the relatively dry class 3 which have about .1 probability and the weather lingers in those states about 4 and 6 months, respectively before returning to the normal class. Therefore, the time of return to the normal class having left from the normal class is 0.1 (4+6) + 0.8 (1) which is 1.8 months. These results are formally derived in Example 3.

EXAMPLE 3 : Appendix III gives monthly PDSI data for Tidewater region in Virginia from 1895-1990. Using the weather classification as given in Table VI.1, compute the following :

a) Average time of transition from drought class 3 to normal class 2 and from wet class 1 to normal class 2.

b) Average recurrence time for class 2 in each month.

c) Verify the results of parts (a) and (b) empirically and draw relevant conclusions.

Solution : Markov chain procedure is adopted to solve this problem. The stepwise procedure is as follows

a) Step 1 : Compute one step transition probability matrices for each month following the procedure as described in step 2 in part (a) of Example 1.

Step 2 : Say, M_{ij}^{n} is the average length of time it takes the weather to transit to class j, starting from class i, in month n. The equation for M_{ij}^{n} is given by

$$M_{ij}^{n} = (1)\, P_{ij}^{n,n+1} + \sum_{k \neq j} P_{ik}^{n,n+1} \,(1 + M_{kj}^{n+1}) \tag{9}$$

Suppose we need to compute $M_{32}^{Feb.}$, for this we write

$$M_{32}^{Feb.} = (1)\, P_{32}^{Feb.\text{-}Mar.} + P_{31}^{Feb.\text{-}Mar.}(1 + M_{12}^{Mar.}) + P_{33}^{Feb.\text{-}Mar.}(1 + M_{32}^{Mar.}) \tag{10}$$

Substituting values of transition probabilities from Table VI.3 in Eq. 10 we get

$$M_{32}^{Feb.} = 1.0 + .8889\, M_{32}^{Mar.} \tag{11}$$

Likewise we can set up similar equation to compute $M_{32}^{Mar.}$ as

$$M_{32}^{Mar.} = (1)\, P_{32}^{Mar.\text{-}Apr.} + P_{31}^{Mar.\text{-}Apr.}(1 + M_{12}^{Apr.}) + P_{33}^{Mar.\text{-}Apr.}(1 + M_{32}^{Apr.}) \tag{12}$$

Substituting the values of the transition probabilities from Table VI.3 we get

$$M_{32}^{Mar.} = 1.0 + .8571\, M_{32}^{Apr.} \tag{13}$$

In a similar way we can develop equations for $M_{32}^{Apr.}$, M_{32}^{May}, ..., $M_{32}^{Jan.}$ and these simultaneous linear equations can be solved for the unknowns M_{32}^{n} and in general M_{ij}^{n}. The results are given in Table VI.7.

b) Step 1 : The average recurrence time of class 2 can be computed using similar procedure as described in step 2 of part (a) above. Equation 23 in chapter IV is written by

$$M_{ii}^{n} = 1 + \sum_{k \neq i} P_{ik}^{n,n+1}\, M_{ki}^{n+1} \tag{14}$$

Substituting i = 2 ; n=Jan. , we get

$$M_{22}^{Jan.} = 1 + P_{21}^{Jan.\text{-}Feb.}\, M_{12}^{Feb.} + P_{23}^{Jan.\text{-}Feb.}\, M_{32}^{Feb.} \tag{15}$$

Substituting values of transition probabilities using Table VI.3 we get

$$M_{22}^{Jan.} = 1 + .1\, M_{12}^{Feb.} + .06\, M_{32}^{Feb.} \tag{16}$$

Likewise for i=2 ; and n=Feb. , we get

$$M_{22}^{Feb.} = 1 + P_{21}^{Feb.\text{-}Mar.}\, M_{12}^{Mar.} + P_{23}^{Feb.\text{-}Mar.}\, M_{32}^{Mar.} \tag{17}$$

Substituting values of transition probabilities we get

$$M_{22}^{Feb.} = 1 + .0769\, M_{12}^{Mar.} + .0962\, M_{32}^{Mar.} \tag{18}$$

Similar equations can be developed for $M_{22}^{Mar.}$, $M_{22}^{Apr.}$, ..., $M_{22}^{Dec.}$ and using the values of M_{12} and M_{32} computed in step 2 of part (a) for various months we can compute the values of M_{22} for each month. The results are given in Table VI.7.

c) Step 1 : For empirical computations of M_{12}, M_{32}, and M_{22} for various months, we refer to Table VI.2 and compute the first passage time or recurrence time using the following equation :

$$M_{ij}^{n} = (1/N_i^{n})\left(\sum_{k=1}^{Ni^{n}} (\text{PASSAGE}_{ij}^{n})_k\right) \tag{19}$$

where : M_{ij}^{n} = Average first passage time to go from class i to j in month n.

$(\text{PASSAGE}_{ij}^{n})_k$ = Number of months to go from class i in month n to j for the first time for event k

N_i^{n} = number of events in which class i begins in month n.

For illustration, we compute $M_{32}^{Feb.}$ empirically. For this purpose count number of occurrences of class 3 in February month as observed in data given in Table VI.2. This is counted as 18. So, $N_3^{Feb.} = 18$. As can be seen from Table VI.2, class 3 during February was first realized in year 1901. It can also be noticed that in March 1901 weather class realized is 2. Therefore, for this event :

$$(\text{PASSAGE}_{32}^{Feb.})_1 = 1.0$$

Likewise length of period for each of 18 events is obtained from Table VI.2 and then $M_{32}^{Feb.}$ is computed as :

$$M_{32}^{Feb.} = (1+2+14+2+20+8+6+5+4+3+10+22+10+1+5+10+6+3)\,/\,18 = 7.3$$

In the same way, empirical values of M_{22} can be calculated for each month. The results are given in Table VI.7 as values within parentheses. It is observed that analytical results obtained using methodology described in the present study agree quite well with the empirical observations.

The kind of analysis given in Example 3 should be useful at planning stages. The analysis provides information on the duration with which a drought state or any other weather class will repeat itself (recurrence time). Of course, the expected time to recover, that is, the first passage time from a drought state to the normal state can also be obtained. From Table VI.7 it is noted that the relatively small value of about 2 months for the recurrence time for the normal class is due to the 80 % probability of staying in the normal class month after month. However, a drought once occurred may last for about six months. For a water rich state such a duration may not be an issue if buffer sources are properly developed as would be required in a planning study.

Example 4 provides a holistic application of Markov chain results. The seventeen month drought starting from July 1980 occurred in the Tidewater region, VA is analyzed. The results agree quite well with the observed data.

EXAMPLE 4 : A 17 month long drought occurred in the Tidewater region of Virginia during 1980-81 . Birch and Ulrich (1982) reported that in the Spring of 1980 drought conditions emerged in the Tidewater region in Virginia which ultimately turned out to be the most severe drought in many parts of Virginia in this century. Drought conditions started in July 1980 and by mid- August, reservoirs serving South-east Virginia dropped to 65 % of capacity, and mandatory conservation measures were enacted in Virginia Beach and other southeastern localities. By October, the drought had dropped the reservoirs to almost half their capacity, and water rationing plans, the first of their kind in Virginia, took effect for more than 690,000 customers in Norfolk and Virginia Beach. At the request of Virginia Beach and Norfolk, Gov. John N. Dalton declared an emergency under Emergency Services and Disaster Act of 1973. By November 1980, 72 localities had been declared drought disaster areas. The mandatory conservation measures initiated in Virginia Beach in August 1980 were not lifted until August 1981. Drought conditions ended in December 1981. Table VI.8 gives PDSI based weather classes classified as per the classification given in Table VI.1 during 1980-81 in the area.

Using the analysis presented in Examples 1 through 3, explain / compute the following

a) Beginning of drought event of 1980-81 in the month of July.

b) Expected duration of drought predicted in July 1980 and its comparison with the actual duration.

c) Predict one to four months ahead weather classes using Markov chain procedure.

Solution : In order to solve this problem, the results obtained in Examples 1 through 3 will be used. A stepwise procedure follows

a) <u>Step 1</u> : For the commencement of drought in the month of July, consider the transition matrices given in Table VI.3. As given in Table VI.8, the weather class was normal

(class 2) in January 1980. Now referring to January- February transition matrix in Table VI.3, there is 84% probability that weather class will stay normal in the month of February if the class during Janaury is normal. Likewise the probabilities of staying in class 2 in subsequent months (upto July) are given in Table VI.9. It can be seen that it is only in the transition from June to July the probability of staying in normal class reduces drastically from mid eighty to mid sixty percent. We again refer to transition matrices and observe that with weather being in class 2 in January 1980, there is only 6% probability of having drought class in February as per the Janaury-February transition matrix. The probability of transition to drought class next month having been in normal class in previous month is given in Table VI.10 for various pair of months. It is seen from Table VI.10 that the probability of transition to drought class from normal class in previous month suddenly increases in June to July transition. Therefore, combining the inferences of results given in Tables VI.9 and VI.10, we can explain high probability of drought commmencement in the month of July. This can also be observed in Table VI.4 wherein July is among the months which have higher long term probability of experiencing droughts. Incidently, the 1980-81 drought did begin in July 1980.

b) Step 1 : The uninterrupted residence times for various weather classes in different months are computed in Example 2 and the results are given in Table VI.6. It is seen that if drought state begins in July month, then on an average it will last for 5.6 months. The probabilities of having one to several months of uninterrupted stay in drought class beginning July have also been computed in Example 2. The results are given in Table VI.5.

It is observed that there is 2% chance that there will be a drought event of 20 months or more starting the month of July in Tidewater region in Virginia. The drought event of 1980-81 lasted for 17 months which has 1% probability as per present analysis. If we were to make prediction of drought duration when it started in July 1980, an expected value forecast will be of 5.6 months. This is not close to the actual duration of 17 months. However, by referring to part (c) proper forecasts can be made.

c) Step 1 : A procedure to predict PDSI given the current state of weather is given in section 4.7 in chapter IV. This technique is employed to predict PDSI classes ahead of time. Table VI.11 gives weather states predicted using Markov chain model 1,2, 3 and 4 months ahead of time from July 1980 to December 1981. It is noted that in this case the seven weather classes have been based on the classification given in Table 4.1 of chapter IV. It is seen that the Markov model gives satisfactory forecasts of weather states for 13 months out of 17 months with a lead time of one month and for 10 months with lead time of 2 months.

Table VI.1 PDSI Based Weather Classification

Class	PDSI	Category
1	> 1.49	Wet
2	> = -1.49 ; <= 1.49	Normal
3	< -1.49	Drought

Table VI.2 Weather Classes, Tidewater Region, Virginia

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1895	2	2	2	1	1	2	2	2	3	3	3	3
1896	3	2	2	2	2	2	2	2	2	2	2	2
1897	2	2	2	2	2	2	2	2	2	2	2	2
1898	2	2	2	2	2	2	2	2	2	2	2	2
1899	2	1	1	2	2	2	2	2	2	2	2	3
1900	2	2	2	2	2	2	2	3	3	3	3	3
1901	3	3	2	2	1	1	1	1	1	1	1	1
1902	1	1	2	2	2	2	3	3	2	2	1	1
1903	1	1	1	1	1	1	1	1	1	1	2	2
1904	2	2	2	2	2	2	2	2	2	2	2	2
1905	2	2	2	2	2	2	1	1	2	2	2	2
1906	2	2	2	2	2	2	2	1	1	1	1	1
1907	1	2	2	2	2	1	1	1	1	1	1	1
1908	1	1	1	1	1	1	1	1	1	1	1	1
1909	1	1	1	1	1	1	2	2	2	2	3	3
1910	3	3	3	2	2	2	2	2	2	2	2	2
1911	2	2	2	2	2	2	3	3	3	3	2	2
1912	2	2	1	1	1	1	2	2	2	2	2	2
1913	2	2	2	2	2	2	2	2	2	2	2	2
1914	2	2	2	2	2	3	3	3	3	3	3	2
1915	2	2	2	2	2	2	2	2	2	2	2	2
1916	2	2	2	2	2	2	1	2	2	2	2	2
1917	2	2	2	2	2	2	2	2	2	1	1	2
1918	1	2	2	2	2	2	2	2	2	2	2	2
1919	2	2	2	2	2	2	1	2	2	2	3	3
1920	3	2	2	2	2	2	2	1	1	2	1	1
1921	1	2	2	2	2	2	2	2	3	3	3	3
1922	2	2	2	2	2	2	1	1	2	2	2	2
1923	2	2	2	2	2	2	2	2	2	2	2	3
1924	2	2	2	2	1	1	1	1	1	1	1	1
1925	1	2	2	3	3	3	3	3	3	3	3	3
1926	3	3	3	3	3	3	3	3	3	3	3	3
1927	3	3	3	2	2	2	2	2	2	2	2	2
1928	2	2	2	2	2	2	2	1	1	1	1	2

1929	2	1	2	2	2	1	1	2	2	1	1	2
1930	2	2	3	3	3	3	3	3	3	3	3	3
1931	3	3	3	3	3	3	3	3	3	3	3	3
1932	3	3	3	3	3	3	3	3	3	2	2	2
1933	2	2	2	2	2	2	2	1	2	3	3	3
1934	3	2	2	2	1	2	1	1	1	1	1	1
1935	1	2	2	1	1	2	1	1	1	1	1	1
1936	1	1	1	1	2	2	2	3	3	3	3	2
1937	1	1	2	1	1	1	1	1	1	1	1	1
1938	1	1	2	2	2	1	1	1	1	1	1	1
1939	1	1	1	1	2	2	1	1	2	2	2	2
1940	2	2	2	2	2	2	2	2	2	2	2	2
1941	2	2	3	2	3	3	3	3	3	3	3	3
1942	3	3	3	3	3	3	3	2	2	1	2	2
1943	2	2	2	2	2	2	2	3	3	2	2	3
1944	2	2	1	1	2	3	3	3	3	3	2	2
1945	2	2	3	3	2	2	1	1	1	1	1	1
1946	1	1	2	2	1	1	1	1	2	2	2	2
1947	2	2	2	3	3	2	2	3	3	3	2	2
1948	2	2	2	2	1	1	2	1	2	2	1	1
1949	2	2	2	2	2	2	2	2	2	2	2	2
1950	2	3	3	3	3	3	2	2	2	2	2	2
1951	2	3	3	3	3	2	2	2	3	3	2	2
1952	1	1	1	1	2	2	3	3	3	3	2	2
1953	2	2	2	2	2	2	3	3	2	3	3	3
1954	3	3	3	3	2	3	3	3	3	3	3	3
1955	3	3	3	3	3	3	3	1	1	1	1	2
1956	2	2	2	2	2	2	2	2	2	1	1	1
1957	1	1	1	2	2	2	3	2	2	2	1	1
1958	1	1	1	1	1	1	1	1	1	1	1	1
1959	2	2	2	2	2	3	2	2	2	2	1	2
1960	2	2	2	2	2	2	2	2	1	1	1	1
1961	1	1	1	1	1	1	1	1	2	1	2	1
1962	1	1	1	1	1	1	1	1	1	2	1	1
1963	1	1	1	2	2	1	2	3	2	3	2	2
1964	2	1	2	2	2	2	3	2	2	1	2	2
1965	2	2	2	2	3	2	2	2	3	3	3	3
1966	3	3	3	3	3	3	3	3	3	3	3	3

1967	3	3	3	3	3	3	3	3	3	3	3	2
1968	2	3	2	2	2	2	2	3	3	3	3	3
1969	3	3	3	3	3	3	2	2	2	2	2	1
1970	2	2	2	2	2	2	2	2	3	3	3	3
1971	2	2	2	2	2	2	2	2	2	1	1	2
1972	2	2	2	2	2	1	1	2	1	1	1	1
1973	1	1	1	1	1	1	2	2	2	2	3	2
1974	2	2	2	2	2	2	2	2	2	2	2	2
1975	2	2	1	1	1	2	1	2	1	1	1	1
1976	1	2	3	3	3	3	3	3	2	1	1	1
1977	2	2	2	2	2	2	3	3	3	2	1	1
1978	1	1	1	1	1	1	1	1	1	2	2	2
1979	1	1	1	1	1	1	1	1	1	1	1	2
1980	2	2	2	2	2	2	3	3	3	3	3	3
1981	3	3	3	3	3	3	3	3	3	3	3	2
1982	2	1	1	2	2	2	1	1	1	1	1	1
1983	2	1	1	1	1	2	2	3	2	2	2	1
1984	1	1	1	1	1	2	2	2	2	3	3	3
1985	3	3	3	3	3	3	3	2	2	2	1	2
1986	2	2	3	3	3	3	3	3	3	3	3	2
1987	1	1	2	1	2	2	3	3	3	3	3	3
1988	3	3	3	3	2	2	2	3	3	3	2	3
1989	3	2	1	1	1	1	1	1	1	1	1	1
1990	1	1	1	1	1	2	2	2	2	2	2	2

Table VI.3 Monthly Transition Matrices, 3 Classes, Tidewater Region, Virginia

Class↓→	1	2	3
	Jan. →	Feb.	
1	.7778	.2222	.0000
2	.1000	.8400	.0600
3	.0000	.2105	.7895
	Feb. →	Mar.	
1	.7308	.2692	.0000
2	.0769	.8269	.0962
3	.0000	.1111	.8889
	Mar. →	Apr.	
1	.8261	.1739	.0000
2	.0769	.8846	.0385
3	.0000	.1429	.8571
	Apr. →	May	
1	.7826	.2174	.0000
2	.0943	.8679	.0377
3	.0000	.1500	.8500
	May →	Jun.	
1	.6957	.3043	.0000
2	.0926	.8333	.0741
3	.0000	.1579	.8421
	Jun. →	Jul.	
1	.7619	.2381	.0000
2	.1818	.6545	.1636
3	.0000	.1500	.8500

	Jul. →	Aug.	
1	.0877	.1923	.0000
2	.1136	.7045	.1818
3	.0385	.1538	.8077
	Aug. →	Sep.	
1	.7407	.2593	.0000
2	.0750	.8000	.1250
3	.0000	.1724	.8276
	Sep. →	Oct.	
1	.8696	.1304	.0000
2	.1818	.7273	.0909
3	.0000	.1034	.8966
	Oct. →	Nov.	
1	.8571	.1429	.0000
2	.2105	.7105	.0789
3	.0000	.2333	.7667
	Nov. →	Dec.	
1	.7500	.2500	.0000
2	.0789	.8158	.1053
3	.0000	.2308	.7692
	Dec. →	Jan.	
1	.8148	.1852	.0000
2	.1136	.8864	.0000
3	.0000	.2083	.7917

Table VI.4 Steady State Probabilities, Virginia, Three Classes

Month ↓ Class →	1	2	3
Jan.	.2820 (.2813)	.5188 (.5208)	.1976 (.1979)
Feb.	.2712 (.2708)	.5401 (.5417)	.1871 (.1875)
Mar.	.2397 (.2396)	.5404 (.5417)	.2183 (.2188)
Apr.	.2396 (.2396)	.5509 (.5521)	.2079 (.2083)
May	.2395 (.2396)	.5614 (.5625)	.1975 (.1979)
Jun.	.2186 (.2188)	.5719 (.5729)	.2079 (.2083)
Jul.	.2705 (.2708)	.4575 (.4583)	.2703 (.2708)
Aug.	.2809 (.2813)	.4159 (.4167)	.3015 (.3021)
Sep.	.2392 (.2396)	.4575 (.4583)	.3015 (.3021)
Oct.	.2912 (.2917)	.3951 (.3958)	.3119 (.3125)
Nov.	.3328 (.3333)	.3951 (.3958)	.2703 (.2708)
Dec.	.2808 (.2813)	.4679 (.4688)	.2495 (.2500)

Table VI.5 Probability of Various Residence Times for Drought Class Beginning July, Tidewater Region, Virginia.

Residence Time (months)	Probability	Residence Time (months)	Probability
1	.19	11	.02
2	.14	12	.02
3	.07	13	.02
4	.14	14	.01
5	.11	15	.01
6	.07	16	.01
7	.06	17	.01
8	.02	18	.01
9	.03	19	.01
10	.03	>=20	.02

Expected Residence time = 5.6 months ; Variance = 26.6, Coefficient of variation = .92

Table VI.6 Uninterrupted Residence Times, Tidewater Region, Virginia, (months)

Starting State→ Month ↓	1	2	3
Jan.	4.4(15.8)[.90]{4.9}	5.4(18.0)[.79]{6.5}	6.1(29.2)[.89]{7.2}
Feb.	4.4(16.1)[.91]{4.5}	5.2(17.1)[.80]{6.8}	6.4(28.2)[.83]{7.1}
Mar.	4.6(16.4)[.88]{5.0}	5.1(16.1)[.79]{6.3}	6.1(27.6)[.86]{7.6}
Apr.	4.4(16.5)[.92]{4.7}	4.7(15.7)[.84]{6.1}	6.0(27.1)[.87]{8.2}
May	4.3(17.1)[.96]{4.8}	4.2(15.8)[.95]{5.3}	5.8(26.8)[.90]{8.4}
June	4.7(17.8)[.90]{6.1}	3.8(16.5)[1.07]4.7}	5.8(26.6)[.89]{8.4}
July	4.9(17.6)[.86]{5.6}	4.3(18.7)[1.01]5.7}	5.6(26.6)[.92]{7.4}
Aug.	4.8(17.3)[.87]{5.2}	4.7(19.9)[.95]{6.0}	5.7(26.8)[.91]{6.8}
Sep.	5.2(16.5)[.78]{6.0}	4.7(20.5)[.96]{6.0}	5.7(26.8)[.91]{7.2}
Oct	4.8(15.9)[.83]{4.8}	5.1(21.2)[.90]{6.2}	5.2(27.1)[1.0]{6.8}
Nov	4.4(15.8)[.90]{4.8}	5.7(20.4)[.79]{6.9}	5.5(28.4)[.97]{7.0}
Dec	4.6(15.8)[.86]{5.2}	5.8(18.9)[.75]{6.8}	5.8(29.2)[.93]{6.7}

(.) = variance , [.] = Coefficient of variation , {.}= empirical

Table VI.7 First Passage/ Recurrence Times to/of Class 2, (months)

Month	M_{12}	M_{22}	M_{32}
Jan.	4.4(4.9)	1.8(1.4)	6.5(7.4)
Feb.	4.4(4.5)	2.0(2.4)	6.5(7.3)
Mar.	4.6(5.0)	1.6(1.8)	6.2(7.8)
Apr.	4.4(4.7)	1.6(1.8)	6.1(8.4)
May	4.3(4.8)	1.9(2.3)	6.0(8.6)
Jun.	4.7(6.1)	2.8(2.5)	5.9(8.8)
Jul.	4.9(5.6)	2.6(2.1)	5.8(7.7)
Aug.	4.8(5.2)	2.1(2.7)	5.7(6.9)
Sep.	5.2(6.0)	2.4(1.9)	5.7(7.4)
Oct.	4.8(4.8)	2.4(2.5)	5.2(6.9)
Nov.	4.4(4.8)	2.0(1.6)	5.5(7.2)
Dec.	4.6(5.2)	1.5(1.5)	5.9(6.9)

M_{12} = Average length of time to go to from class 1 to class 2; (.) = empirical

Table VI.8 PDSI Based Weather Classes, Tidewater Region, Virginia, 1980-1981

Mon →	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
'80	2	2	2	2	2	2	3	3	3	3	3	3
'81	3	3	3	3	3	3	3	3	3	3	3	2

Table VI.9 Probability of Transition from Normal to Normal Class

Months	J-F	F-M	M-A	A-M	M-J	J-J
Probability	.84	.83	.89	.87	.83	.66

J-F = January-February

Table VI.10 Probability of Transition to Drought Class from Normal Class

Months	J-F	F-M	M-A	A-M	M-J	J-J
Probability	.06	.10	.04	.04	.07	.16

J-F = January-February

Table VI.11 Observed and Predicted PDSI Classes, July 1980 - December 1981

1980	July	Aug.	Sep.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.
Obs.	5	6	6	5	5	5	6	6	6
1 step	4	5	6	6	5	5	5	6	6
2 steps	4	4	5	6	6	5	5	5	6
3 steps	4	4	4	5	6	6	5	5	5
4 steps	4	4	4	4	5	6	5	5	5
1981	**Apr.**	**May**	**Jun.**	**Jul.**	**Aug.**	**Sep.**	**Oct.**	**Nov.**	**Dec.**
Obs.	6	5	5	5	5	5	5	5	4
1 step	6	5	5	5	5	5	5	5	5
2 steps	6	5	5	5	5	5	5	5	5
3 steps	6	5	5	5	5	5	5	5	5
4 steps	5	5	5	5	5	5	5	5	5

Note : Weather classification as per Table 4.1, Chapter IV

SOUTHERN OSCILLATION INDEX (SOI) (1895-1984)

Source : Wright (1989)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
1895	0	0	-7	-7	-7	0	0	0	-2	-2	-2	0
96	-1	-1	-4	-4	-4	7	7	7	18	18	18	-1
97	9	9	-1	-1	-1	-3	-3	-3	-11	-11	-11	9
98	-6	-6	-14	-14	-14	-3	-3	-3	-10	-10	-10	-6
99	-20	-20	-1	-1	-1	15	15	15	19	19	19	-20
1900	19	19	7	7	7	6	6	6	1	1	1	19
1	8	8	1	1	1	5	5	5	3	3	3	8
2	-2	-2	7	7	7	19	19	19	15	15	15	-2
3	24	24	6	6	6	-8	-8	-8	-12	-12	-12	24
4	-14	-14	-8	-8	-8	5	5	5	10	10	10	-14
5	35	35	14	14	14	9	9	9	6	6	6	35
6	22	22	2	2	2	-9	-9	-9	-6	-6	-6	22
7	-7	-7	-1	-1	-1	2	2	2	3	3	3	-7
8	1	1	-7	-7	-7	-1	-1	-1	-10	-10	-10	1
9	-8	-8	-9	-9	-9	-18	-18	-18	-14	-14	-14	-8
10	-16	-16	-17	-17	-17	-14	-14	-14	-15	-15	-15	-16
11	-12	-12	7	7	7	10	10	10	14	14	14	-12
12	15	15	7	7	7	-3	-3	-3	1	1	1	15
13	4	4	-4	-4	-4	9	9	9	14	14	14	4
14	31	31	10	10	10	17	17	17	26	26	26	31
15	18	18	8	8	8	-13	-13	-13	-14	-14	-14	18
16	-11	-11	-11	-11	-11	-28	-28	-28	-30	-30	-30	-11
17	-36	-36	-16	-16	-16	-26	-26	-26	-31	-31	-31	-36
18	-15	-15	4	4	4	10	10	10	14	14	14	-15
19	17	17	8	8	8	4	4	4	4	4	4	17
20	10	10	-2	-2	-2	-9	-9	-9	-2	-2	-2	10
21	-20	-20	-11	-11	-11	-12	-12	-12	-2	-2	-2	-20
22	-5	-5	-7	-7	-7	-5	-5	-5	-7	-7	-7	-5
23	-11	-11	-4	-4	-4	-6	-6	-6	10	10	10	-11
24	3	3	-6	-6	-6	-6	-6	-6	14	14	14	3
25	-8	-8	-8	-8	-8	7	7	7	18	18	18	-8
26	31	31	13	13	13	-1	-1	-1	-8	-8	-8	31
27	-10	-10	-8	-8	-8	0	0	0	4	4	4	-10
28	12	12	-4	-4	-4	0	0	0	-6	-6	-6	12
29	-2	-2	-5	-5	-5	7	7	7	5	5	5	-2
30	-3	-3	7	7	7	14	14	14	14	14	14	-3
31	13	13	6	6	6	-3	-3	-3	-7	-7	-7	13

32	6	6	2	2	2	-4	-4	-4	-7	-7	-7	6
33	-7	-7	-5	-5	-5	-8	-8	-8	-16	-16	-16	-7
34	-23	-23	-5	-5	-5	2	2	2	0	0	0	-23
35	0	0	-8	-8	-8	-1	-1	-1	-11	-11	-11	0
36	3	3	-14	-14	-14	2	2	2	4	4	4	3
37	-4	-4	-6	-6	-6	-1	-1	-1	0	0	0	-4
38	-13	-13	-7	-7	-7	-25	-25	-25	-14	-14	-14	-13
39	-27	-27	-12	-12	-12	-5	-5	-5	15	15	15	-27
40	8	8	15	15	15	24	24	24	22	22	22	8
41	36	36	12	12	12	25	25	25	18	18	18	36
42	17	17	2	2	2	-6	-6	-6	-9	-9	-9	17
43	-24	-24	-11	-11	-11	-3	-3	-3	-12	-12	-12	-24
44	8	8	-3	-3	-3	3	3	3	5	5	5	8
45	-11	-11	-7	-7	-7	-13	-13	-13	-6	-6	-6	-11
46	-6	-6	8	8	8	10	10	10	14	14	14	-6
47	9	9	-1	-1	-1	-11	-11	-11	-12	-12	-12	9
48	2	2	1	1	1	6	6	6	0	0	0	2
49	10	10	0	0	0	10	10	10	0	0	0	10
50	-18	-18	-19	-19	-19	-27	-27	-27	-18	-18	-18	-18
51	-26	-26	13	13	13	12	12	12	19	19	19	-26
52	18	18	0	0	0	-3	-3	-3	0	0	0	18
53	13	13	17	17	17	12	12	12	10	10	10	13
54	4	4	-2	-2	-2	-4	-4	-4	-2	-2	-2	4
55	-13	-13	-3	-3	-3	-21	-21	-21	-24	-24	-24	-13
56	-20	-20	-16	-16	-16	-15	-15	-15	-11	-11	-11	-20
57	-6	-6	8	8	8	6	6	6	13	13	13	-6
58	20	20	6	6	6	-4	-4	-4	6	6	6	20
59	22	22	-8	-8	-8	9	9	9	-7	-7	-7	22
60	-3	-3	-8	-8	-8	-3	-3	-3	-7	-7	-7	-3
61	-5	-5	10	10	10	3	3	3	0	0	0	-5
62	-21	-21	4	4	4	3	3	3	-9	-9	-9	-21
63	0	0	-6	-6	-6	9	9	9	17	17	17	0
64	13	13	-12	-12	-12	-12	-12	-12	-15	-15	-15	13
65	5	5	4	4	4	24	24	24	24	24	24	5
66	12	12	16	16	16	-1	-1	-1	3	3	3	12
67	-15	-15	-1	-1	-1	-4	-4	-4	1	1	1	-15
68	-4	-4	-2	-2	-2	-7	-7	-7	5	5	5	-4
69	15	15	7	7	7	8	8	8	13	13	13	15
70	15	15	1	1	1	-2	-2	-2	-22	-22	-22	15
71	-22	-22	-24	-24	-24	-8	-8	-8	-21	-21	-21	-22
72	-7	-7	8	8	8	19	19	19	17	17	17	-7
73	22	22	0	0	0	-13	-13	-13	-28	-28	-28	22
74	-35	-35	-20	-20	-20	-9	-9	-9	-10	-10	-10	-35

75	1	1	-14	-14	-14	-26	-26	-26	-28	-28	-28	1
76	-27	-27	-8	-8	-8	14	14	14	2	2	2	-27
77	1	1	15	15	15	23	23	23	21	21	21	1
78	29	29	1	1	1	-5	-5	-5	5	5	5	29
79	1	1	4	4	4	-5	-5	-5	4	4	4	1
80	4	4	13	13	13	4	4	4	6	6	6	4
81	3	3	10	10	10	-11	-11	-11	-1	-1	-1	3
82	-8	-8	4	4	4	32	32	32	39	39	39	-8
83	61	61	23	23	23	7	7	7	-6	-6	-6	61
84	-3	-3	2	2	2	3	3	3	1	1	1	-3

SEA SURFACE TEMPERATURE (SST) INDEX (1895-1984)

Source : Wright (1989)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
1895	-50	-85	-31	-11	53	33	-21	39	77	37	87	47
96	53	10	37	6	11	33	84	153	114	110	165	176
97	169	165	65	-79	5	-1	28	-17	11	-33	-31	-30
98	-26	-134	-52	-48	-33	-43	-58	-67	-24	-50	-71	-63
99	-67	-41	-64	-15	-26	-14	25	58	104	232	164	168
1900	163	137	91	98	108	92	78	83	98	80	19	68
1	56	33	-15	25	-1	12	-34	-11	-1	1	10	7
2	52	51	41	70	61	70	146	110	176	177	172	169
3	170	117	101	14	20	-17	-66	-70	-47	-46	-51	-85
4	-52	-5	0	-21	-16	19	80	87	68	118	100	119
5	97	98	92	77	155	151	135	82	164	127	112	122
6	142	95	48	110	23	2	15	-67	-38	-67	-90	-20
7	-45	-55	-65	-42	16	20	-19	0	91	-57	28	46
8	-58	42	-23	-32	-43	-95	-84	-47	-117	-71	-59	-77
9	-83	-129	-46	-49	-86	-81	-54	-78	-77	-108	-132	-117
10	-117	-82	-107	-123	-78	-82	-48	-52	-83	-36	-67	-55
11	-69	-72	-63	-80	-14	1	53	88	106	111	138	215
12	122	87	58	91	37	-24	19	-58	26	-7	29	-18
13	-38	15	3	-11	-49	26	34	21	43	42	52	106
14	139	106	168	93	141	93	67	53	86	207	161	148
15	133	93	148	111	104	3	-28	-87	-15	-175	-125	-70
16	9	-101	-57	-109	-107	-79	-85	-94	-96	-96	-127	-125
17	-149	-103	-114	-24	-76	1	-46	-59	9	-103	-85	-43
18	-125	-96	25	-8	16	16	58	139	82	133	82	106
19	111	120	141	148	113	136	32	17	27	60	-31	40
20	76	111	52	41	57	-29	-91	-13	27	-14	13	-13
21	64	-17	-87	-11	-12	13	12	23	7	3	-75	36
22	-53	-18	-9	31	31	-97	33	-25	-52	-84	-94	-17
23	-38	-84	6	23	44	10	24	51	83	90	107	68
24	36	8	26	1	-64	-58	-97	-69	-63	-98	-99	-51
25	-117	-84	-36	-79	8	5	47	44	70	68	103	144
26	120	89	82	88	45	8	42	22	-4	-47	-55	-50
27	48	30	-16	-44	8	12	-25	10	16	46	22	24
28	15	5	-10	0	8	-29	-23	-2	12	-8	-10	1
29	-61	-39	0	4	-23	25	11	19	27	37	52	27
30	25	20	18	28	29	20	70	60	102	129	164	153
31	147	102	89	93	55	36	31	18	-22	-66	-29	-33
32	-22	1	54	21	67	41	9	17	18	27	-5	-8

33	-7	0	-28	15	-25	-40	-56	-49	-50	-93	-85	-97
34	-48	-90	-48	-16	-9	-5	-12	14	-18	-12	-23	-48
35	-53	-26	-29	-35	-21	-22	-44	22	-1	21	-8	-39
36	7	16	1	-7	8	-28	-5	-18	-4	50	-6	6
37	-47	-25	4	-39	-18	-9	16	-3	17	-3	5	5
38	-56	-39	-40	-13	-39	-60	-58	-69	-78	-112	-46	-87
39	-63	-92	-79	-21	31	5	47	38	40	69	93	133
40	71	75	102	106	71	95	71	67	93	227	155	190
41	166	181	104	84	80	89	58	60	159	82	146	126
42	67	45	27	45	-1	-7	-46	-71	-98	-141	-150	-135
43	-126	-102	-110	-68	-19	-35	5	1	-8	-61	-66	-39
44	-63	-1	1	25	22	33	45	14	2	-53	-45	-32
45	-33	-19	-95	-135	4	7	-51	-68	-32	-60	5	-32
46	3	5	10	32	-40	-127	-38	-54	37	-8	-36	-26
47	-6	5	9	7	15	14	-26	-35	-26	-40	-21	-32
48	-2	21	39	34	24	4	-81	-36	-66	-70	-5	-1
49	-42	17	-23	99	14	-44	-40	-35	-70	-67	-141	-83
50	-104	-148	-25	-54	-115	-71	-85	-59	-77	-45	-110	-79
51	-45	-21	-21	15	32	33	98	101	74	89	132	88
52	69	53	12	56	-6	-20	-10	-10	19	23	-3	-30
53	54	32	37	101	56	51	60	16	67	9	54	8
54	4	5	12	-95	-73	-101	-110	-92	-86	-106	-87	-101
55	-65	-27	-67	-59	-98	-104	-90	-64	-132	-160	-185	-132
56	-139	-64	-67	-62	-57	-53	-60	-72	-73	-52	-84	-63
57	-58	0	24	60	86	86	98	123	54	84	117	129
58	141	132	98	69	50	54	42	33	-2	20	22	54
59	48	40	27	42	29	-3	-25	-12	-28	30	3	-7
60	-9	-7	7	23	23	-10	-1	22	18	-17	-23	-20
61	-17	29	0	21	1	43	-38	-35	-71	-82	-27	-32
62	-22	-16	-37	-44	-35	-21	-7	0	-26	-30	-33	-63
63	-43	-36	1	18	6	27	81	74	86	86	104	102
64	66	26	-17	-37	-83	-92	-46	-82	-75	-58	-94	-95
65	-52	-10	-4	13	66	84	92	118	126	146	142	147
66	110	82	62	39	-15	24	22	10	-2	-7	-27	-33
67	-42	-20	-36	-45	-4	23	-16	-37	-66	-59	-50	-52
68	-76	-87	-72	-34	-46	1	38	43	28	33	65	75
69	98	67	60	53	103	72	38	57	73	90	89	109
70	89	48	17	41	1	-45	-90	-86	-87	-86	-116	-133
71	-117	-111	-88	-55	-47	-53	-54	-59	-66	-65	-84	-80
72	-38	-6	4	48	82	84	113	154	143	174	204	207
73	159	117	57	-10	-30	-70	-75	-86	-92	-97	-129	-129
74	-144	-98	-69	-54	-51	-27	-32	-26	-25	-60	-64	-63
75	-44	-35	-49	-30	-48	-91	-76	-79	-96	-118	-107	-135

76	-137	-86	-49	-28	-9	29	48	70	70	96	86	66
77	78	45	46	1	34	27	23	15	29	57	62	59
78	67	23	6	-23	-12	-49	-27	-42	-18	-8	-5	35
79	3	11	40	35	42	53	12	18	100	42	45	59
80	47	40	24	28	39	58	24	-14	18	7	35	53
81	-23	-44	-20	-35	-14	1	-30	-48	7	24	-19	25
82	31	29	22	37	76	98	70	97	136	184	192	250
83	221	187	167	141	164	155	81	84	60	-16	-45	-9
84	-24	31	-14	-5	-5	-33	-2	2	23	-35	-31	-42

VITA

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