NUMERICAL STUDY ON THE VALIDITY OF THE QUASI-SPECULAR AND TWO-SCALE MODELS FOR ROUGH SURFACE PARAMETER ESTIMATION: ONE DIMENSIONAL SURFACES

by

Roger T. Marchand
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by

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(ABSTRACT)

This study examines the use of the quasi-specular and two-scale models in estimating rough surface parameters from the average radar cross section of randomly rough surfaces, with the goal of understanding what scattering mechanisms limit their applicability. The ranges of validity of these models are ascertained by comparing the average backscattered normalized radar cross section given by the models to results obtained using an exact numerical Monte Carlo approach. The advantage of using a numerical solution is that the exact surface parameters are known quantities. The surfaces studied here are rough in only one dimension (that is, they are grooved in one dimension or corduroy). The height of the surfaces are Gaussian distributed and have either a Gaussian or a Pierson-Moskowitz spectrum. For surfaces with Gaussian spectra, it is found that the quasi-specular model can be used to obtain good estimates of the surface parameters when diffraction and multiple scattering effects are not important. Approximate validity conditions are established for this model. For surfaces with Pierson-Moskowitz spectra, it is found that the quasi-specular model can be used to obtain good estimates of the surface parameters for backscattering angles of less than 20\(^\circ\), and a two-scale model can be used for backscattering angles of up to at least 60\(^\circ\). However, the quasi-specular model must be modified to use only a portion of the surface spectrum, and this modification shows problematic dependence on the surface roughness, the incident wavelength, and the incident polarization. Of particular importance in the estimation problem are the numerical fluctuations present in the Monte Carlo simulation and the angular region over which data is compared to the model. Both of these factors are explored.
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1.0 Introduction

1.1 Overview

From the oceans of Earth to the surface of Venus, the quasi-specular and two-scale models have seen extensive use in inversion algorithms to estimate surface roughness parameters from average backscattered radar cross sections [Tyler et al. 1992, Kim 1995, Jackson 1992, Brown 1979, etc.]. Despite wide usage of these models, no comprehensive study on their ranges of validity as an inversion tool is currently available. A literature review on surface parameter extraction for randomly rough surfaces using radar cross section data is presented in Chapter 2.

The propose of this study is to determine the accuracy of the quasi-specular and two-scale models in estimating rough surface parameters and to understand what scattering mechanisms limit their applicability. The validity of these models is determined by first comparing the average backscattered normalized radar cross section (NRCS) given by the models to results obtained using an exact numerical Monte Carlo approach. In the numerical Monte Carlo approach, a sample realization of a random rough surface with specific roughness parameters is generated. Then, using a Method of Moments (MOM) approach, the normalized radar cross section (NRCS) for the surface realization is calculated. This process is repeated many times for a number of different realizations to obtain a numerical average for the NRCS. Second, the quasi-specular and two-scale models are used to estimate the surface parameters from the numerically generated scattering data. Because the exact surface parameters are known, the accuracy of the models in estimating (or inverting) these surface parameters can be determined.
The following section gives a detailed description and summarizes the findings of this dissertation on a chapter by chapter basis. In short, the major tasks involved in this study are: (1) the development (chapters 3 and 4) and validation (chapters 5 and 6) of a numerical scattering code which can both generate randomly rough surfaces and compute their NRCS, and (2) evaluation of the quasi-specular and two-scale scattering models in terms of both the radar cross sections and their use in estimating surface parameters (Chapters 7 and 8).

1.2 Organization and Summary of Important Conclusions

Chapter 2 first reviews the literature on surface parameter extraction for randomly rough surfaces using average radar cross section data. In particular, the quasi-specular and two-scale models are introduced and the surface parameters which can be extracted using these models are discussed. Also, the results of earlier studies on their applicability are presented.

Chapter 3 then describes how the method of moments (MOM) can be used to obtain the surface currents on the boundary between two dielectric materials. Essentially, a pair of coupled integral equations which relate the incident and scattered fields to the fields on the boundary are converted into an approximately equivalent matrix problem. This matrix equation can then be solved using standard matrix techniques such as LU decomposition. In chapter 4, the normalized radar cross section (NRCS) is introduced and an expression is derived which determines the NRCS directly from the surface currents. The limitations of the MOM solutions are examined in chapters 5 and 6.

Chapter 5 examines the errors associated with the MOM solution for the currents and the normalized radar cross section in the limit where the boundary between the two dielectric materials is a plane. The solutions are examined in this limit because the exact solution for scattering of a plane wave is known for this simple geometry. It is shown in
this chapter that there is a basic tradeoff between the maximum value permitted in the real part of the dielectric constant and the number of Gauss-Legendre integration points which are required in numerically evaluating entries in the MOM matrices. It is also shown that because the scattering surface must be truncated in the MOM approach, a tapered incident field should be used to reduce the strength of the surface currents at the end of the truncated surface. If the incident field is not tapered sufficiently, spike-like discontinuities are generated in the MOM current solution at the ends of the scattering surface. These rapid transitions in the current can introduce significant errors in the scattered field calculations.

Chapter 6 introduces randomly rough surfaces, and the NRCS obtained using the MOM formulation (developed in chapters 3 and 4) is compared with results obtained by other researchers for rough surfaces with Gaussian and Pierson-Moskowitz spectra (these surfaces are defined in section 6.1). Also in chapter 6 the impact of surface truncation, very small scale surface roughness, and limitations of the Monte Carlo average are examined. To summarize these results; they are as follows.

(1) It is found that the length of the surface (compared to the size of the incident illumination) can affect the scattered field solution significantly for surfaces with Gaussian spectra. Generally, the smaller the surface, the greater the error and the smaller the backscattered angular region becomes where the scattering strength is sufficient to obtain accurate results.

(2) For the Pierson-Moskowitz spectrum, on the other hand, it is found that the surface length can be chosen much smaller than the correlation length for backscattering angles up to about 60°. This result is important because the correlation lengths of these surfaces are frequently in the range of a hundred of meters or more, and the scattering from surfaces this large is very difficult to solve using exact numerical approaches.
(3) Surfaces with Pierson-Moskowitz spectra contain notable levels of surface roughness which occur on spatial scales which are small compared to the wavelength of the incident electromagnetic wave. Fortunately, the average radar cross section is not sensitive to this very small scale portion of the surface spectrum. However, it should be noted that it is only the average which is not affected, and the radar cross section for each individual realization does have a small sensitivity to these features.

(4) Finally, it is difficult to obtain the NRCS using Monte Carlo techniques for surfaces with Gaussian spectra (whose scattering is dominated by specular scattering) at large backscattering angle. This is because infrequently occurring slopes, required to back-reflect rays at large backscattering angles, must be included to obtain a good average of the radar cross section. This observation was also made by Eric Thorsos [1988] for acoustic scattering from surfaces with Gaussian spectra.

Chapter 7 examines the use of the quasi-specular model to estimate the rms slope and the reflection coefficient from the average normalized radar cross section of randomly rough surfaces with Gaussian height distributions and with Gaussian spectra. (The quasi-specular model is introduced in chapter 2 and derived for one dimensional surfaces in Appendix B). Estimated values are determined by (1) examining the ratio of the average radar cross section at two backscattering angles and by (2) a numerical minimization approach which matches the model to the numerically generated backscattered average radar cross section data. Because of the finite number of surface realizations used to obtain the backscattering data, the data contains some small fluctuations. The impact of these fluctuations is studied, and it is found that confidence limits based on the assumption of Gaussian statistics can be used for the estimated parameters. In regards to the quasi-specular model's accuracy as a tool for estimating the rms slope and normal incidence flat surface reflection coefficient, the following results are found.
(1) The quasi-specular model does not accurately predict the scattering from Gaussian rough surfaces with Gaussian spectra at large backscattering angles \((45^\circ - 70^\circ)\), unless the surface has small rms (root mean squared) curvature. An approximate curvature condition under which the model is accurate is given by equation \((7.6-1)\). Further, the quasi-specular model becomes erroneous at these large backscattering angles in situations where the tangent plane approximation continues to give good results. This is because the stationary phase approximation used to obtain the quasi-specular model is more sensitive to the surface curvature than is the tangent plane approximation.

(2) At small backscattering angles (i.e. near normal incidence), the quasi-specular model yields very good estimates of the rms surface slope and normal incidence reflection coefficient when the rms slope is less than approximately \(18^\circ\) and rms height is greater than about half the incident wavelength. This rms height ensures that there is no significant coherent scattering and that the rms curvature remains sufficiently small (i.e., \(\sqrt{\langle \xi^2 \rangle} < 0.5/\lambda_o\)). It is possible to obtain good results with rms heights smaller than half a wavelength, but the rms curvature condition must still be satisfied (this requires the rms slope to be reduced below \(18^\circ\)). The above condition for the rms slope is less restrictive than the one given by Ulaby, Fung and Moore [1982 p. 949 and p.1815], who suggest a maximum rms slope of \(14.5^\circ\). On the other hand, these authors also suggest a radius of curvature limit which is very close to one wavelength. These values appear to have been obtained for the Kirchhoff approximation and applied to the quasi-specular model with the added constraint that the rms height be sufficiently large to eliminate any coherent scattering contribution.

(3) For surfaces with rms slopes greater than \(18^\circ\), the quasi-specular model's accuracy in estimating the root mean squared (rms) slope of the scattering surface is severely degraded by multiple scattering at small backscattering angles \((0^\circ - 45^\circ)\).
requirement that the rms surface slope be 18° or less (given above) ensures that there are no significant multiple scattering contributions to the radar cross section. The multiple scattering is a strong function of both the dielectric constant of the scattering material and the polarization of the incident field. When the rms curvature condition (equation 7.6-1) is not met and the dielectric constant of the scattering medium is large, multiple scattering is larger for TM (vertical) polarization than for TE (horizontal) polarization. When the curvature condition equation (7.6-1) is met and the scattering surface is a perfect electric conductor (PEC), the multiple scattering in the TM and TE polarizations are approximately the same. Regardless of the curvature and polarization, as the dielectric constant is reduced, the multiple scattering is reduced. However, the reduction in the multiple scattering (with respect to a decrease in the dielectric constant) is observed to be more rapid for TM polarization than for TE polarization. This effect appears to be a result of the Brewster angle. In principle, the quasi-specular model can be used for larger rms slopes than 18°, at least for TM polarization, but only when the dielectric constant of the surface is low. Finally, when the above curvature condition is met and the surface has large rms height (greater than 2 wavelengths), the surface scattering can be described in terms of ray scattering, and ray tracing solutions for the multiple scattering are possible.

Chapter 8 examines the accuracy of a modified quasi-specular model and a composite scattering model in estimating the wind speed from the average backscattered radar cross section of Gaussian surfaces with Pierson-Moskowitz spectra (i.e. surfaces with a Gaussian height distributions and with Pierson-Moskowitz spectra). This surface spectrum, defined in chapter 6.1, is only a function of the surface wind speed. Chapter 2 and chapter 8 will introduce and discuss the features of these two models in some detail. Both the models require choosing a separation wavenumber, k_o, (or a separation constant = k_o/k_o, where k_o is the electromagnetic wavenumber) which divides the surface spectrum into a large scale and a small scale region. Chapter 8 also examines the optimal value for this separation constant. Qualitatively, the separation constant should be chosen such
that the surface which would result from only the large scale portion of the spectrum
\((k<k_u)\) is gently undulating, while the surface resulting from only the small scale portion
of the spectrum has small height \((k>k_u)\). Important findings in this chapter include the
following results.

**For the quasi-specular model it is found that:**

1. At small backscattering angles (out to about 20 degrees), the quasi-specular
   model, modified in the sense that it depends only upon the large scale portion of the
   surface spectrum, is in fair agreement with the numerical calculations.

2. Good wind speed estimates were obtained using only the quasi-specular model
   in combinations with the optimal separation constant at wind speeds of 10, 15 and 20
   m/s, incident electromagnetic wavelengths of 3 and 30 centimeters, and both TE and TM
   polarization. However, the quasi-specular model's wind speed estimates were found to
   be very sensitivity to the separation constant. Optimal values of the separation constants
   are given in Table 8.3.1-1, and depend on polarization, electromagnetic wavelength, and
   wind speed.

**For a composite model it is found that:**

1. The composite model is observed to be in good agreement with numerical
   calculations at all backscattering angles up to 60 degrees.

2. The separation constant has little effect on the absolute difference between
   the numerically calculated average radar cross section and the composite scattering
   model, when the separation constant varies between roughly 2.5 and 6.0.

3. Finally, wind speed estimates based on the composite model tended to be
   slightly low, but in most cases the exact wind speed is within or very near the 95%
confidence limits of the estimated value. Wind speed estimates are also generally observed to be slightly better at an electromagnetic wavelength of 30 cm than at a wavelength of 3 cm. Perhaps most important, the composite model's wind speed estimates show little sensitivity to the separation constant.

1.3 Remarks on Notations

Before continuing with the dissertation, a few remarks on organization and notation may be helpful.

First, in regards to the organization of the equation and figure numbers, both figures and equations are labeled by the chapter and section number in which they appear followed by a dash ("--") and an additional number which specifies its location with respect to other equations or figures in that section. For example, "Figure 6.3.4-2" refers to the second figure used in section 6.3.4. Or for example, "equation (7.3.1-8)" refers to the eighth equation in section 7.3.1. Finally, figure numbers are always preceded by the word "figure" and equation numbers will be placed in parentheses.

Second, in the caption of many figures in chapters 5 through 7, the notation "10/1/1 evaluations" is used to specify the number of Gauss-Legendre integration points used in calculating the MOM matrix entries (the entries are defined in section 3.6). The first number, "10", is the number of points used to evaluate the self terms. The second number is the number of points used to evaluate the off diagonal matrix entries which are a function of $k_2$ (the wavenumber in the scattering material), and the third number is the number of points used to evaluate the off diagonal matrix entries which are a function of $k_1$. The importance of these three parameters are discussed in chapter 5.
2.0 Review of Surface Parameters Extraction from Randomly Rough Surfaces

2.1 Introduction

There is no closed form expression for the scattering from a randomly rough surface with a constant dielectric permittivity under all roughness conditions. As a result, attempts to estimate statistical rough surface parameters largely rely on the use of relatively simple analytical approximations or empirical models. In this chapter, the quasi-specular, Bragg, and two-scale models for the average radar cross section of randomly rough surfaces are introduced and reviewed. (Readers unfamiliar with the radar cross section or randomly rough surfaces may wish to consult sections 4.2 and 6.2, respectively.)

2.2 Quasi-Specular and Tangent Plane (Kirchhoff) Approximations

In the tangent plane approximation, the surface fields are approximated by the fields which would be present were a flat plane introduced at each point on the surface, perpendicular to the local surface normal, as is depicted in figure 2.2-1 below.

![Diagram of Tangent Plane](image)
The total field at each point on the surface is thus given by the sum of the incident field and the field reflected from an infinite flat plane using the appropriate Fresnel reflection coefficients. Expressions for the vector formulation of the Kirchhoff or tangent plane approximation are given by many authors [Ulaby, Moore, and Fung (1982), Sancer (1969), Ogilvy (1991)]. This approximation demands that the surface radius of curvature at every point on the surface be larger than the incident wavelength. Based on geometric arguments [Ogilvy 1991], the tangent plane approximation has been described as requiring,

\[ k_o r_c \cos^3(\theta_L) \gg 1, \text{ or } k_o r_c \cos(\theta_L) \gg 1, \text{ or } k_o r_c \gg 6\pi \]  \hspace{2cm} (2.2-1)

where, \( k_o \) is the electromagnetic wavenumber, 
\( \theta_L \) is the local incidence angle (as shown in figure 2.2-1), and 
\( r_c \) is the local radius of curvature.

The first condition is often found quoted in the literature. Numerical studies on the region of validity have added to the above criteria. Chen and Fung [1988] performed a numerical study on the region of validity. They compared one dimensional method of moments calculations of the average backscattered radar cross section for perfect electric conducting (PEC) surfaces having a Gaussian height spectra to the Kirchhoff approximation. They concluded that the conditions given by equation (2.2-1) are too restrictive. However, they examined the averaged backscattered radar cross section at only 0°, 10° and 20° degrees. Also, their averages are based on only 40 realizations (i.e. 40 random surfaces) and no analysis on the effect of using such a small number of realizations is presented.

Thorsos [1988] compared results from Monte Carlo calculations with the Kirchhoff approximation at large angles of incidence. He found that the Kirchhoff
approximation (with geometric shadow functions included) is valid when (1) the correlation length is greater than the electromagnetic wavelength and (2) the angle of incidence is less than or equal to twice the rms slope angle. Kim, Rodrigues, and Durden [1992] examined the Kirchhoff approximation without shadowing corrections for PEC surfaces with a power law height spectra. Although they provided no fixed criteria, they found that the region of validity is strongly dependent on the angle of incidence.

Simply approximating the surface fields with the tangent plane fields does not yield a closed form result for the surface's cross section. However by examining the high frequency limit, a stationary phase approximation can be made which simplifies the resulting integrals. In the high frequency limit, the average backscattered cross section per unit area is found as, [See Barrick 1968, Sancer 1969, Brown 1990]

$$\sigma^o(\theta) = \pi \sec^4 \gamma P_{\nu}\zeta(\zeta_x, \zeta_y)|\rho(0)|^2$$  \hspace{1cm} (2.2-2)

where, \( P_{\nu}\zeta \) is the probability density function (pdf) of the surface slopes,

\( \rho(0) \) is the normal incidence flat surface reflection coefficient,

\( \hat{k}_i = \sin(\theta)\hat{x} - \cos(\theta)\hat{z} \) is the incident wave direction,

\( \hat{k}_s = \sin(\theta)\hat{x} + \cos(\theta)\hat{z} \) is the backscattering direction,

\( \zeta_x = \frac{(k_x - \hat{k}_i) \cdot \hat{x}}{(k_z - \hat{k}_i) \cdot \hat{z}} \) is the specular slope in the x-direction,

\( \zeta_y = \frac{(k_y - \hat{k}_i) \cdot \hat{y}}{(k_z - \hat{k}_i) \cdot \hat{z}} \) is the specular slope in the y-direction,

\( \theta \) is the backscattering angle, and

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\[ \tan^2 \gamma = \zeta_x^2 + \zeta_y^2. \]

In the high frequency limit, the expression for the radar cross section, given as equation (2.2-2), is called the quasi-specular approximation. \( \zeta_x \) and \( \zeta_y \) are the particular values of the slopes which cause the incident ray traveling in the \( \hat{k}_i \) direction to be reflected into the \( \hat{k}_t \) direction. In effect, the reflection coefficient determines the magnitude of the scattered field and the slope pdf determines the angular distribution. Clearly then, the backscattered cross section can be used to extract the magnitude of the surface reflection coefficient as well as the distribution of surface slopes.

The quasi-specular model has been used to extract slope parameters by many investigators including Tyler et al [1992] for the planet Venus; Jackson [1992], Nickolaev, Yordanov, and Michalev [1992], Bufton et al [1983], Brown [1979], Barrick [1974], and Cox and Munk [1954] for scattering from the ocean; and Tsan et al [1988] for terrain. Often these investigators use the derived slope parameters to obtain additional information. For example, Brown [1979] and Jackson [1992] used the quasi-specular model to correlate the surface mean squared slope with the surface wind speed.

All of these studies used the quasi-specular results near nadir \((\theta_i, \approx 0^\circ)\). When near grazing angles \((\theta_i, \approx 90^\circ)\) the quasi-specular result can be modified by a shadowing function [Brown 1990, Thorsos 1988, Sancer 1969]. The shadowing function attempts to adjust the solution for the scattered field by incorporating the fact that some portions of the surface are not illuminated from (or do not scatter in) a given direction because they are obstructed by other portions of the surface.
Quantitative data on the quasi-specular model's range of validity, beyond that specified for the Kirchhoff approximation, is generally lacking. A numerical study by Kim, Rodrigues, and Durden [1992b] comparing the model to Monte Carlo simulations for one dimensional PEC surfaces with a power law spectrum \( W(k) = Nk^q \) with \( q = -2.5, -3.0 \) found that for horizontal polarization with \( k\sigma_h \cos \theta, > \pi \) (where \( \sigma_h \) is the root mean squared height of the surface) and an rms slope \(< 0.6\), the quasi-specular result is accurate to within 2 dB. However, they examined only a small number of cases and only at incident angles less than 40 degrees.

2.3 Bragg Scattering and First Order Boundary Perturbation

Using a well known boundary perturbation approach, the backscattered radar cross section per unit area to first order in height and slope for a dielectric surface is given by [Ulaby et al 1982, Valenzuela 1978, Barrick and Peak 1968] as,

\[
\sigma_{ij}^0(\Theta) = 4\pi k^4 \cos^4(\Theta) |g_{ij}(\Theta)|^2 W(2k \sin \Theta, 0) \tag{2.3-1}
\]

where, \( W(k_x, k_y) \) is the two-dimensional spectral density of the surface hieght (roughness)

\[
g_{hh}(\Theta) = \frac{(\varepsilon_r - 1)}{(\cos \Theta + \sqrt{(\varepsilon_r - \sin^2 \Theta)})^2}
\]

\[
g_{vv}(\Theta) = \frac{(\varepsilon_r - 1)[\varepsilon_r(1 + \sin^2 \Theta) - \sin^2 \Theta]}{[\varepsilon_r \cos \Theta + \sqrt{(\varepsilon_r - \sin^2 \Theta)}]^2}
\]

\[
g_{vh}(\Theta) = g_{hv}(\Theta) = 0.
\]

Depolarization is absent for first order, but is found when second order perturbation is examined [Valenzuela 1967]. However, it is important to realize that first order perturbation still includes some effects due to diffraction on the surface by the small
scale roughness. In reference to the region of validity of the perturbation approximation, the derivation of boundary perturbation requires that, [Ogilvy 1991]

\[ k|\zeta(x,y)| \ll 1 \quad \text{and} \quad |\nabla \zeta(x,y)| \ll 1. \]

Ulaby, Moore and Fung [1982] offer the following criteria, but stress that no precise validity conditions have been established,

\[ k\sigma_h < 0.3 \quad \text{and} \quad \frac{\sqrt{2} \sigma_h}{l} < 0.3 \quad (2.3-2) \]

where \( l \) is the correlation length, and \( \sigma_h \) is the root mean squared (rms) surface height.

Numerical evaluations have not completely confirmed these guidelines; such studies have found these conditions to be neither necessary nor sufficient. Thorsos and Jackson [1989] compared one dimensional Monte Carlo simulations for pressure release surfaces (equivalent to PEC surfaces in electromagnetic scattering) with a Gaussian spectra. They found the perturbation result was inaccurate for large correlation length. Similar work by Chen and Fung [1988] reached this same conclusion.

Kim, Rodrigues, and Durden [1992] studied one dimensional surfaces with power law spectra [\( W(k) = Nk^q \) with \( q = -3 \) or \(-2.5\)] and found that the height constraint suggested by Ulaby, Moore, and Fung [1982] is too stringent by at least a factor of two. Also, Thorsos [1990] studied the accuracy of the perturbation result for a Pierson-Moskowitz spectrum (a sea surface spectrum model which has power-law-like behavior at high frequencies) at large incident angles (70°-80°). He found that first order perturbation gave good results with \( k\sigma_h \) as high as 1.79 for these incidence angles.

From the form of the Bragg model, equation (2.3-1), it is obvious that the height spectrum (or at least a portion of it) can be extracted from backscatter measurements. A

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number of authors including Stover and Serati (1984), Barrick (1974), Larson and Wright (1975), Long and Trizna (1973) have used the Bragg model to extract information on the height spectrum. Barrick (1974) also showed that for moving surfaces, such as the ocean, the Doppler shift in the radar return from such surfaces corresponds to the radial velocity of the those waves at the Bragg frequency $(2k_z \sin \theta)$.

Besides the height spectrum, the Bragg return depends on the $g_{vw}$ and $g_{vh}$ coefficients. In principle, one should therefore be able to extract information on the dielectric constant (by using the ratio of the horizontal and vertical returns for example). However, no such attempts have been reported in the literature.

2.4 Composite Roughness or Two-Scale Models

Many surfaces cannot be described as having only small height (Bragg scattering model) or being slowly undulating (tangent plane model). Naturally, investigators have tried to combine these two models in describing surfaces with both kinds of roughness (i.e. composite or two-scale surfaces). The most straightforward, although certainly not rigorous, approach is to simply add the incoherent cross sections of the two surface types [Barrick and Peake 1968c, Fuks 1985], i.e.

$$\sigma^0(\theta) = \sigma_{\text{quasi-specular}}^0(\theta) + \sigma_{\text{Bragg}}^0(\theta) \quad (2.4-1)$$

Fuks [1985] argues that for two-scale surfaces, such as the ocean, there is a cut-off angle, $\theta_c$, separating a near nadir region where quasi-specular scattering is dominant from a region where Bragg scattering is dominant. In the quasi-specular region one can extract the magnitude of the reflection coefficient and find the mean square slope by fitting the shape of the average return to a model for the slope pdf. Far from nadir, the return would be dominated by Bragg scatter and one could obtain the roughness spectrum and the dielectric constant.
More elaborate models have been constructed by a superposition of both kinds of roughness [Wright 1968, Valenzuela 1978, Gorman and McDaniel 1983]. Wright [1968] introduced the "Tilted Bragg" model where the Bragg scattering from a tilted patch is averaged over all possible tilt angles (surface slopes). Using Wright's notation,

\[
\sigma^0_{\text{Tilted Bragg}}(\theta) = 16\pi k_0^4 \int |G(\theta, \alpha, \phi)|^2 W(2k_0 \cos(\theta + \alpha), 0) P(\alpha, \phi) d\alpha d\phi
\]

where, \(P(\alpha, \phi)\) is the probability density function of the slope angles,
\(\alpha\) is the angle between the patch normal and the mean surface normal in the plane of incidence,
\(\varphi\) is the angle between the patch normal and the mean surface normal perpendicular to the plane of incidence,
\(g_{\nu}\) and \(g_{hh}\) are the customary Bragg coefficients,

\[
G_{\nu} \equiv g_{\nu}(\theta + \alpha),
\]

\[
G_{\varphi} \equiv \frac{\tan \varphi}{\cos(\theta + \alpha)} (g_{\nu}(\theta + \alpha) - g_{hh}(\theta + \alpha)),
\]

\[
G_{hh} \equiv g_{hh}(\theta + \alpha) + \left[ \frac{\tan \varphi}{\cos(\theta + \alpha)} \right]^2 g_{\nu}(\theta + \alpha).
\]

Valenzuela [1978] points out that the tilted Bragg model still gives poor results near nadir, when compared to scattering from surfaces such as the ocean, and suggests adding the quasi-specular result to the tilted Bragg result. He further suggest that reflection coefficients modified for the small-scale roughness be used in connection with the quasi-specular model.
Besides these "physical" descriptions of composite surface scattering, more formal approaches have been pursued by splitting the spectrum into two portions. Brown [1978], developed a two-scale model for a PEC with a Gaussian pdf. He expressed the surface height spectrum as having two distinct regions with a dividing wavenumber \( k_d \) (i.e. the two dimensional surface spectrum, \( S(k_x,k_y) \), is given by \( S_d(k_x,k_y) + S_s(k_x,k_y) \) where \( S_d(k_x,k_y) = 0 \) for \( k_x \leq k_d \cap k_y \leq k_d \) and \( S_s(k_x,k_y) = 0 \) for \( k_x \geq k_d \cup k_y \geq k_d \)).

The surface which would result from only the large scale portion of the spectrum is considered gently undulating and is amenable to the Kirchhoff approximation, while a surface resulting from only the small scale portion of the spectrum satisfies perturbation requirements. In Brown's treatment, the small scale surface is treated as a slight disturbance of the large scale surface with the following results for the average backscattered radar cross section,

\[
\sigma_{pq}^0(\theta, \phi) \approx [\sigma_{pq}^0(\theta, \phi)]_0 + [\sigma_{pq}^2(\theta, \phi)]_1
\]

where, \([\sigma_{pq}^0(\theta, \phi)]_0\) is the zeroth order term (i.e. high frequency result) given by

\[
[\sigma_{pq}^0(\theta, \phi)]_0 = \frac{2}{\pi} \frac{\text{Sh}(\theta, \phi) \sec^4 \Theta}{2 \sqrt{\zeta^2_x > \zeta^2_y >}} \exp \left[ \frac{-\cos^2 \phi}{2 \left< \zeta^2_{\theta x} > \right>} + \frac{\sin^2 \phi}{2 \left< \zeta^2_{\theta y} > \right> \tan^2 \Theta} \right]
\]

\(< \zeta^2_{\theta x} >\) is the mean squared slope in x direction,

\(< \zeta^2_{\theta y} >\) is the mean squared slope in y direction,

\(\text{Sh}(\theta, \phi)\) is a shadowing function,

\([\sigma_{pq}^2(\theta, \phi)]_1\) is the first order term and is given by

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\[
[\sigma_{pq}^0(\theta, \phi)]_1 = \frac{k_0^2 \sec^2 \theta}{\pi \sqrt{\zeta_{lx}^2 + \zeta_{ly}^2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(k_x, k_y) \phi_0(u, v) \Gamma_{pq}^2(u, v) \exp \left[ \frac{-(k_{ax}-k_x)^2}{8k_0^2 \cos^2 \theta \zeta_{lx}^2} + \frac{(k_{oy}-k_y)^2}{8k_0^2 \cos^2 \theta \zeta_{ly}^2} \right] dk_x dk_y
\]

\[
- \int_{-k_d}^{k_d} \int_{-k_d}^{k_d} S(k_x, k_y) \phi_0(u, v) \Gamma_{pq}^2(u, v) \exp \left[ \frac{-(k_{ax}-k_x)^2}{8k_0^2 \cos^2 \theta \zeta_{lx}^2} + \frac{(k_{oy}-k_y)^2}{8k_0^2 \cos^2 \theta \zeta_{ly}^2} \right] dk_x dk_y,
\]

\[
\Gamma_{pq}(\zeta_{lx}, \zeta_{ly}) = \{2(\hat{n} \cdot \hat{e}_p)(\hat{n} \cdot \hat{e}_q) + (\hat{n} \cdot \hat{k}_l)^2(\hat{e}_q \cdot \hat{e}_p)\} \sqrt{1 + \zeta_{lx}^2 + \zeta_{ly}^2},
\]

\[
\hat{n}(\zeta_{lx}, \zeta_{ly}) = \frac{\hat{z} - \zeta_{lx} \hat{x} - \zeta_{ly} \hat{y}}{\sqrt{1 + \zeta_{lx}^2 + \zeta_{ly}^2}},
\]

\[
\hat{e}_{p/q} \text{ is the transmit / receive polarization,}
\]

\[
k_{ax} = -2k_0 \sin \theta \cos \phi, \quad k_{oy} = -2k_0 \sin \theta \sin \phi,
\]

\[
u = \frac{k_{ax} - k_x}{2k_0 \cos \theta} \quad \text{and} \quad \nu = \frac{k_{oy} - k_y}{2k_0 \cos \theta}.
\]

It is interesting to note that when \(8k_0^2 \cos^2 \theta < \zeta_{lx}^2\) and \(8k_0^2 \cos^2 \theta < \zeta_{ly}^2\) are small, the exponential terms in the above integrals are highly peaked and can be evaluated asymptotically. Doing so yields,

\[
[\sigma_{pq}^0(\theta, \phi)]_1 = 4\pi k_0^4 W(k_{ax}, k_{oy}) \Gamma_{pq}^2(0, 0).
\]

where \(S(k_{ax}, k_{oy}) = \frac{\pi}{2} W(k_{ax}, k_{oy})\),

\[
\Gamma_{hh}(0, 0) = \cos^4 \theta,
\]

\[
\Gamma_{vv}(0, 0) = (1 + \sin^2 \theta)^2,
\]

\[
\Gamma_{hv} = \Gamma_{vh} = 0.
\]

This is the classical perturbation solution for backscatter from a PEC.

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A number of authors have used models which incorporate this split spectrum idea, including Kim [1994], Brown [1992] and Durden and Vesecky [1989]. Kim [1994] used a composite model composed of adding the quasi-specular result to the tilted Bragg result in order to extract roughness parameters from Venus. Assuming that the large scale surface was isotropic and Gaussian, he modified the quasi-specular reflection coefficients by a factor of \( \exp[-4k^2\sigma_h^2] \) to account for the effect of the small scale roughness in the quasi-specular model. Other authors have developed similar effective reflection coefficients [Guissard A. and Sobieski P. 1987, Schiffer 1987]. Kim further assumed that the small scale surface spectrum took the form of a power law height spectrum (i.e. \( W(k)=Nk^q \)). In doing so, Kim was able to extract a large scale mean squared surface slope, a small scale surface rms height, a dielectric constant and a small scale spectral decay factor (q). Durden and Vesecky [1989], used a composite model similar to that used by Kim[1994]. They used the radar backscatter measurements of the ocean to determine parameters for a proposed empirical sea spectrum model. Brown [1992] used a simple model based on the work of Papa and Woodworth [1988], which is similar in form to equation (2.4-1). Using this model, Brown extracts the large scale mean squared slope, dielectric constant, small scale correlation length, and small scale height. Also, it should be noted that many authors using the quasi-specular model have long recognized the need to consider the extracted slope variances from surfaces such as the ocean to include only the ocean waves whose lengths are greater than the wavelength of the electromagnetic radiation [Valenzuela 1978, Brown 1979].

Of course, these two-scale models only bridge the gap between surfaces with small and large scale structure. In the more general case where there may be irregularities on the order of the incident wavelength, these models are expected to fail. However, there is a substantial lack of studies on the validity of the model under varying conditions. Fung and Chen [1985] compared a two-scale model to a surface which was obtained by superposition of two independent Gaussian profiles. They conclude from their study that
the two-scale approach works well when one Gaussian profile meets the Kirchhoff requirements and the other meets the Bragg scattering requirements.

Durden and Vesecky [1989] compared ocean measurements to a two-scale model (using a sea surface spectrum of their own design). They found good agreement but examined only a few situations, set no specific guidelines, and rendered no quantitative conclusion. Thorsos [1990] examined scattering from a Pierson Moskowitz (sea surface) spectrum at large incident angles (70°-80°). This spectrum (which is a function of wind speed) contains a large low frequency content and a high frequency region with a k⁻⁴ dependence (k⁻³ for one dimensional surfaces). Thorsos compared Monte Carlo simulations of one dimensional surface scattering to a very simple two-scale model by McDaniel and Gorman [1983]. Thorsos found that this composite model did not provide accurate results at these large incident angles.

2.5 Empirical Models

Because of the lack of an exact model for surface scattering under all conditions, there has been a great deal of effort given to the development of empirical models. The majority of the literature in this area has been concerned with two major approaches: (1) using an artificial neural network and (2) fitting to experimental data.

The use of an artificial neural network for parameter estimation has received much attention of late [Chuah and Chong 1993, Tzeng et al 1993, Hong et al 1992, Pierce, et al 1992] and such attempts include estimation of surface statistical parameters [Yoshitmoi et al 1993]. The transfer function of a neural network is determined by training the network using a sequence of known input and output data. A back propagating algorithm successively refines the transfer function based on the training sequence. Once the network has been trained, it can rapidly produce estimates. The major difficulty with neural networks is that they represent a true black box. There is no way to discern the
underlying physical processes that give rise to the network behavior which may be highly nonlinear. This means there is no way to determine the optimum set of input (or training) data or to detect poor connections with reality.

In the second approach, the scattering (usually backscattering) characteristics of a particular terrain, vegetation, etc. are measured, often at several frequencies and polarizations. The results are then fit to a fairly simple equation or set of equations that describe the scattering behavior as a function of the polarization, frequency, etc. [Ulaby, Moore, and Fung 1982/1986 and Polatin et al 1994 - virtually any current issue of IEEE Transactions on Geoscience and Remotes Sensing will have several articles using empirical models]. This approach has been used for many parameters ranging from soil moisture and corn growth to tree trunk thickness.

2.6 Summary

The estimation of rough surface statistical parameters largely relies on the use of closed form analytical approximations. Namely the quasi-specular model for high frequency measurements (which are usually conducted near nadir), the Bragg scattering model at low frequency (where measurements are usually made well off nadir), and the composite or two-scale models (which combine the above two approximations). From the quasi-specular model one can estimate the surface slope pdf and the magnitude of the reflection coefficient. From the Bragg scattering model, one can estimate the surface height spectrum (or at least a portion of it) and in principle the dielectric constant. The composite model which combines both of the above models allows estimation of all of the above surface parameters.

All of these analytical expressions are only valid under certain surface conditions. Although there are general qualitative expressions for these regions, little quantitative data is available. Only the Bragg model can be said to have a reasonably well defined
region of validity and then only for Gaussian and power law spectrums which exhibit noticeably differing results [Thorsos 1990].

Approaches based on empirical models are widely used. Although, empirical approaches are designed to be inverted over some specific region of validity, they rely strongly on the determination of "the ground truth." Further these approaches provide little physical insight into the scattering mechanisms (what is actually being monitored), so changes in these mechanisms (with respect to time of year for example) may not be adequately specified by the model parameters. Ultimately, the selection of frequencies and polarization (among other decisions) depends upon information provided by theoretical models.
3.0 Numerical Solution of the Scalar Extinction

Equations for One Dimensional Rough Surfaces

3.1 Introduction

In this chapter, a pair of coupled integral equations are cast into an approximately equivalent matrix equation which can be used to obtain the electric and magnetic fields on the boundary between two dielectric materials which are illuminated by an incident field. The technique, through which the matrix equation is obtained, is known as the Method of Moments (MOM). Once the surface fields (also known as surface currents) are found, the fields at any other point in space can be determined using the same integral equations used to obtain the surface fields. The average normalized radar cross section (NRCS) for a randomly rough surface can thus be obtained using a Monte Carlo approach. That is, (1) a sample realization of the randomly rough surface is generated, (2) the surface fields are then calculated using the method of moments, (3) using the surface fields the NRCS is computed (as is described in chapter 4), and (4) the first three steps are repeated many times and the results averaged.

The coupled integral equations used in this chapter are known as the extinction equations and have been derived by a number of authors [Nieto-Vesperinas 1991, Ishimaru 1989, Wolf 1973]. For completeness, a derivation is given in Appendix A. Actually, two slightly different pairs of equations are used in this study. One set is used for the case when the electric field is aligned with the one dimensional surface roughness, i.e. when the electric field is parallel to the grooved (or corduroy) surface. This situation is called transverse electric (TE) or horizontal (h) polarization. The other set is used when the magnetic field is aligned with the surface roughness. This situation is called transverse magnetic (TM) or vertical (v) polarization. Both sets of integral equations are
developed by application of Green's second scalar identity. Two equations are needed because the incident field creates both an electric and a magnetic surface current.

These equations are called the extinction equations because, "We know that under the influence of the incident electromagnetic field another field will be generated inside the dielectric material, which will have a different wave number and hence a different phase velocity. We may, therefore, say that inside the medium, the incident wave is somehow extinguished by the interaction with the medium and is replaced by a new wave propagated with the velocity $c/n$, where $n$ is the refractive index of the medium." [Wolf, 1973]. Mathematically, one obtains an integral expression of the form,

$$0 = \int_{S_s} E(\mathbf{r}_1) \frac{\partial G(\mathbf{r}, \mathbf{r}_1)}{\partial n_1} - G(\mathbf{r}, \mathbf{r}_1) \frac{\partial E(\mathbf{r}_1)}{\partial n_1} \, dS_1 + 4\pi E_s(\mathbf{r})$$

where, \(\mathbf{r}\) is the observation point (inside the dielectric material),
\(E\) is the total electric field = incident + scattered electric field,
\(E_i\) is the incident electromagnetic field, and
\(G\) is the Green's function for a unbounded homogenous region.

In this equation, the incident field is said to be canceled or extinguished by the contribution from the surface currents, because the sum of the incident field and the field scattered by the surface is zero at the observation point. This equation, derived in Appendix A, is included here only to help explain why these integral equations are called the extinction equations.

In the remaining sections of this chapter, the extinction equations are modified for numerical solution of the surface fields. Section 3.2 introduces the particular pair of integral equations which are used in this study for TE polarization. This section also
outlines the procedure by which the matrix formulation is obtained. Sections 3.3 and 3.4 then proceed to provide necessary modification to the discretized equation for implementation on a computer. Section 3.5 defines the form of the incident field used in this study, which must be explicitly specified. Finally, section 3.6 summaries the result for the TE case and presents the results for the TM case. The results for the TM case are obtained using precisely the same procedure used in sections 3.2-3.5 for the TE case.

3.2 Discretization of the Integral Equations

Using Green's second identity, a pair of coupled integral equations can be derived which give the scalar electric field off a surface in terms of the field and its normal derivative on that surface. The derivation of the integral extinction equations for a one dimensional surface is given is in appendix A. These equations for TE polarization are,

\[ E(\vec{r}) = E_i(\vec{r}) + \frac{1}{4\pi} \int_S \left[ E(\vec{r}_o) \frac{\partial G_1(\vec{r}, \vec{r}_o)}{\partial n_o} - G_1(\vec{r}, \vec{r}_o) \frac{\partial E(\vec{r}_o)}{\partial n_o} \right] dS_o \]  

and

\[ 0 = \frac{1}{4\pi} \int_S \left[ E(\vec{r}_o) \frac{\partial G_2(\vec{r}, \vec{r}_o)}{\partial n_o} - G_2(\vec{r}, \vec{r}_o) \frac{\partial E(\vec{r}_o)}{\partial n_o} \right] dS_o, \]  

where, \( E_i \) is the incident electric field,

\( E = \) the total electric field = incident + scattered fields,

\( G_i = -j\pi H_0^{(2)}(k_i |\vec{r} - \vec{r}_o|) \), which is the Green's function for a two dimensional unbounded homogeneous region with a dielectric constant \( \varepsilon_i \), a magnetic permeability \( \mu_o \), and a wave number \( k_i = \omega \sqrt{\mu_o \varepsilon_i} \) [Ishimaru 1991, p.153],

\( G_2 = -j\pi H_0^{(2)}(k_2 |\vec{r} - \vec{r}_o|) \), which is the Green's function for an unbounded homogeneous region with a wave number \( k_2 \) and a dielectric constant \( \varepsilon_o \),

\( \vec{r} \) is called the observation point, and can be either on or off the surface, and

\( H_0^{(2)}() \) is the zeroth order Hankel function of the second kind.
In this equation and in all other equations used throughout this study, both the incident and scattered fields are taken to have a \( \exp(j\omega t) \) time dependence where \( \omega = 2\pi f \) is known as the frequency, \( \lambda = \frac{c}{f} \) is known as the electromagnetic wavelength and \( c \) is the speed of light.

These integral equations can be solved approximately by taking the observation point to the surface and using a numerical technique known as the Method of Moments (MOM). In this approach, the surface is divided into \( N \) segments and on each segment the total field and its normal derivative are considered constant. That is, the surface currents are approximated by a sum of pulse functions. Using this pulse approximation for the current, i.e. the surface fields, equations (3.2-1) and (3.2-2) become,

\[
E(\vec{r}) = E_i(\vec{r}) + \frac{j}{4} \sum_{i=1}^{N} E(\vec{r}_i) \int_{S_i} \frac{\partial H_0^{(2)}(k_1 | \vec{r} - \vec{r}_o |)}{\partial n_o} dS_o
\]

\[
+\frac{j}{4} \sum_{i=1}^{N} \frac{\partial E(\vec{r}_i)}{\partial n} \int_{S_i} H_0^{(2)}(k_1 | \vec{r} - \vec{r}_o |) dS_o
\] (3.2-3)

and

\[
0 = -\frac{j}{4} \sum_{i=1}^{N} E(\vec{r}_i) \int_{S_i} \frac{\partial H_0^{(2)}(k_2 | \vec{r} - \vec{r}_o |)}{\partial n_o} dS_o
\]

\[
+\frac{j}{4} \sum_{i=1}^{N} \frac{\partial E(\vec{r}_i)}{\partial n} \int_{S_i} H_0^{(2)}(k_2 | \vec{r} - \vec{r}_o |) dS_o
\] (3.2-4)

where, \( i \) is the summation index = 1 .. \( N \),

\( N \) is number of segments,

\( S_i \) is the \( i \)-th segment on the surface, and

\( \vec{r}_i \) is the position vector to the center of the \( i \)-th segment.

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By choosing \( \mathbf{r} \) to be \( \mathbf{r}_i \) for each segment, a system of 2N equations with 2N unknowns is obtained. Therefore, one can solve for the approximate total electric field and its normal derivative at each point on the surface. However, great care must be taken when \( \mathbf{r} = \mathbf{r}_o \) (the so called self terms) because the Hankel function and its normal derivative are singular at this point. The following two sections provide the necessary modifications to the discretized extinction equations (i.e. equations 3.2-3 and 3.2-4) for implementation on a computer. Notice that an incident field term appears in equation (3.2-3). It is the incident field which creates the surface currents and this source term must be chosen. Section 3.5 defines the incident field used in this study.

### 3.3 Simple Terms

When \( \mathbf{r} \neq \mathbf{r}_o \) the Hankel function and its normal derivative are not singular, and therefore the integral portions of the equations (3.2-3) and (3.2-4), given below as equations (3.3-1) and (3.3-2), can be evaluated numerically.

\[
\int_{S_i} \frac{\partial \mathcal{H}_0^{(2)}(k|\mathbf{r} - \mathbf{r}_o|)}{\partial n_o} \, dS_o \quad (3.3-1)
\]

and

\[
\int_{S_i} \mathcal{H}_0^{(2)}(k|\mathbf{r} - \mathbf{r}_o|) \, dS_o , \quad (3.3-2)
\]

where, \( k \) is either \( k_1 \) or \( k_2 \).

It should be noted that the above integrations take place over a rough surface. However, this integration surface can be mapped onto one axis (for one dimensional surfaces).
Referring to Figure 3.3-1,

\[ ds = \sqrt{dx^2 + d\zeta^2} = dx \sqrt{1 + (\zeta_x)^2} \]  \hspace{1cm} (3.3-4)

and

\[ \hat{n} = \frac{\hat{z} - (\zeta_x)\hat{x}}{\sqrt{1 + (\zeta_x)^2}} \]  \hspace{1cm} (3.3-5)

where, \( \zeta(x) \) is the height of rough surface,
\( \hat{n} \) is the normal to the surface, and
\( \zeta_x \) is the derivative of the height with respect to \( x \) (i.e. \( \frac{d\zeta(x)}{dx} \)).

With a change of variables, the two integral terms (equations 3.3-1 and 3.3-2) become,

\[ \frac{j}{4} \int_{S_0} H_0^{(2)}(k|\vec{r} - \vec{r}_o|) \, dS_o = \frac{j}{4} \int_{S_i} H_0^{(2)}(k|\vec{r} - \vec{r}_o|) \sqrt{1 + \zeta_o^2} \, dx_o , \]  \hspace{1cm} (3.3-6)

and

\[ \frac{-j}{4} \int_{S_o} \frac{\partial H_0^{(2)}(k|\vec{r} - \vec{r}_o|)}{\partial n_o} \, dS_o = \frac{-j}{4} \int_{S_i} \frac{\partial H_0^{(2)}(k|\vec{r} - \vec{r}_o|)}{\partial n_o} \sqrt{1 + \zeta_o^2} \, dx_o . \]  \hspace{1cm} (3.3-7)
Equation (3.3-6) is now in a sufficiently simple form that it can be easily integrated using standard numerical techniques, but the normal derivative in equation (3.3-7) must be further simplified.

Since \( \frac{\partial H_0^{(2)}(z)}{\partial z} = -H_1^o(z) \),

\[
\frac{\partial H_0^{(2)}(k|\vec{r} - \vec{r}_o|)}{\partial n_o} = \frac{\partial H_0^{(2)}(k|\vec{r} - \vec{r}_o|)}{\partial (k|\vec{r} - \vec{r}_o|)} \frac{\partial (k|\vec{r} - \vec{r}_o|)}{\partial n_o} = -H_1^o(k|\vec{r} - \vec{r}_o|) \frac{\partial (k|\vec{r} - \vec{r}_o|)}{\partial n_o} .
\]

(3.3-8)

Combing the definition of the normal derivative,

\[
\frac{\partial F}{\partial n} = \hat{n} \cdot \nabla F = \hat{n} \cdot \left( \frac{\partial F}{\partial x} \hat{x} + \frac{\partial F}{\partial z} \hat{z} \right),
\]

(3.3-9)

and the chain rule yields,

\[
\frac{\partial (k|\vec{r} - \vec{r}_o|)}{\partial x_o} = \frac{\partial (k\sqrt{(x-x_o)^2 + (z-\zeta(x_o))^2})}{\partial x_o},
\]

\[
= \frac{-k(x-x_o)}{\sqrt{(x-x_o)^2 + (z-\zeta(x_o))^2}}
\]

(3.3-10)

and,

\[
\frac{\partial (k|\vec{r} - \vec{r}_o|)}{\partial z_o} = \frac{\partial (k\sqrt{(x-x_o)^2 + (z-\zeta(x_o))^2})}{\partial z_o},
\]

\[
= \frac{-k(z-\zeta(x_o))}{\sqrt{(x-x_o)^2 + (z-\zeta(x_o))^2}}
\]

(3.3-11)

Inserting equations (3.3.5), (3.3-8), (3.3-9), (3.3-10) and (3.3-11) into equation (3.3-7), yields equation (3.3-12), which can also be integrated using standard techniques.

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\[
\int_{S_i} \frac{\partial H_0^{(2)}(k|\vec{r} - \vec{r}_o|)}{\partial n_o} \, dS_o
\]

\[
= \int_{S_i} \left( -\zeta_{x_o} \frac{k(x-x_o)}{\sqrt{(x-x_o)^2 + (z-\zeta(x_o))^2}} + \right)
\]

\[
\frac{k(z-\zeta(x_o))}{\sqrt{(x-x_o)^2 + (z-\zeta(x_o))^2}} \right) H_1^{(2)}(k|\vec{r} - \vec{r}_o|) dx_o
\]

\[
= \int_{S_i} \frac{-\zeta_{x_o} k(x-x_o) + k(z-\zeta(x_o))}{|\vec{r} - \vec{r}_o|} \right) H_1^{(2)}(k|\vec{r} - \vec{r}_o|) dx_o . \quad (3.3-12)
\]

Traditionally, method of moments codes for electromagnetic surface scattering from perfect electric conducting surfaces have used a single point evaluation of integral terms such as equation (3.3-6) and (3.3-12) in combination with a discretization length (i.e. pulse width) of approximately 1/10 of the incident wavelength [Thorsos 1989]. That is,

\[
\int_{S_i} H_0^{(2)}(k|\vec{r} - \vec{r}_o|) \sqrt{1 + \zeta^2_{x_o}} \, dx_o
\]

\[
\approx H_0^{(2)}(k|\vec{r} - \vec{r}_i|) \sqrt{1 + \zeta^2_{x_i}} \Delta x \quad (3.3-10)
\]

and

\[
\int_{S_i} \frac{-\zeta_{x_o} k(x-x_o) + k(z-\zeta(x_o))}{|\vec{r} - \vec{r}_o|} \right) H_1^{(2)}(k|\vec{r} - \vec{r}_o|) dx_o
\]

\[
\approx \frac{-\zeta_{x_i} k(x-x_i) + k(z-\zeta_i)}{|\vec{r} - \vec{r}_i|} \right) H_1^{(2)}(k|\vec{r} - \vec{r}_i|) \Delta x \quad (3.3-11)
\]

where, \( \vec{r}_i \rightarrow \vec{x}_i + \zeta(x_i) \hat{z} \) is the midpoint of segment \( S_i \), and

\( \Delta x \) is the length of the segment \( S_i \) mapped onto the \( x \) axis.

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In chapter 5, the validity of using a single point evaluation is examined in connection with dielectric surfaces. It is found that single point evaluations should be used only when the relative dielectric constant of the medium is small or has a large imaginary component.

3.4 Self Terms (\( \vec{r} = \vec{r}_o \))

Both the integrands in the discretized integral equations comprising (3.2-3) and (3.2-4) contain singularities when \( \vec{r} = \vec{r}_o \). The following two subsections provide approximations for these terms based on small argument expansion of the Hankel functions. The results obtained here are the same as those obtained by Thorsos [1989].

3.4.1 Zeroth Order Hankel Function Self Term

Abramowitz and Stegun [1972] give the small argument expansion of the zeroth order Hankel function of the second kind as,

\[
H_0^{(2)}(z) = J_0(z) - iY_0(z)
\]

where, \( J_0(z \to 0) \approx 1 \),

\[
Y_0(z \to 0) \approx \frac{2}{\pi} (\ln \left(\frac{z}{2}\right) + \gamma), \text{ and}
\]

\( \gamma \) is a constant.

The integration of the Hankel function in the small argument limit then becomes,

\[
\int_{S_i} \frac{H_0^{(2)}(k|\vec{r}_i - \vec{r}_o|)dS_o}{dS_o} \approx \int_{S_i} \left[ 1 - i\frac{2}{\pi} (\ln \left(\frac{k|\vec{r}_i - \vec{r}_o|}{2}\right) + \gamma) \right] dS_o
\]

\[
= \int_{S_i} \left[ 1 - \frac{2i}{\pi} (\ln \left(\frac{k|\vec{r}_i - \vec{r}_o|}{2}\right) + \gamma) \right] dS_o.
\]

where, \( \vec{r}_i \) is the midpoint of the segment \( S_o \), and \( \gamma = \ln(\gamma') \).
Similar to the simple terms, this integral can be mapped onto a flat surface. Substituting equation (3.3-4) into equation (3.4.1-1) yields,

\[
\int_{S_i} H_0^{(2)}(k|\vec{r}_i - \vec{r}_o|)dS_o \\
\approx \sqrt{1 + \zeta_{x_i}^2} \int_{S_i} \left[ 1 - \frac{2i}{\pi} \ln \left( \frac{k|x_i - x_o|}{2} \right) \right] d\zeta_{x_i} . \tag{3.4.1-2}
\]

Here, the derivative over each segment is assumed to be approximately constant. The integration in equation (3.4.1-2) can be further simplified by noting that,

\[
k|x_i - x_o| = k\sqrt{(x_i - x_o)^2 + (\zeta(x_i) - \zeta(x_o))^2} \\
= k|x_i - x_o| \sqrt{1 + \left( \frac{\zeta(x_i) - \zeta(x_o)}{x_i - x_o} \right)^2} = k|\Delta x| \gamma_m \tag{3.4.1-3}
\]

where, \( \gamma_m = \sqrt{1 + \left( \frac{\zeta(x_i) - \zeta(x_o)}{x_i - x_o} \right)^2} \), and \( \Delta x = x_o - x_i \).

Substituting equation (3.4.1-3) into equation (3.4.1-2) and performing a simple change of variables from \( x_o \) to \( \Delta x \) gives,

\[
\int_{S_i} H_0^{(2)}(k|\vec{r}_i - \vec{r}_o|)dS_o \\
\approx \sqrt{1 + \zeta_{x_i}^2} \int_{-\Delta x/2}^{\Delta x/2} \left[ 1 - \frac{2i}{\pi} \ln \left( \frac{k|\Delta x| \gamma_m}{2} \right) \right] d\Delta x \\
= \sqrt{1 + \zeta_{x_i}^2} \left\{ \Delta x - \frac{2i\Delta x}{\pi} \left[ \ln \left( \frac{k|\Delta x| \gamma_m}{2} \right) - 1 \right] \right\}
\]

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\[
= \sqrt{1 + \zeta_{ix}^2} \left\{ \Delta \varepsilon - \frac{2i\Delta \varepsilon}{\pi} \ln \left( \frac{k_{1/2}\Delta \varepsilon \gamma_m \gamma'}{2} \right) - \ln \varepsilon \right\} \\
= \sqrt{1 + \zeta_{ix}^2} \left\{ \Delta \varepsilon - \frac{2i\Delta \varepsilon}{\pi} \ln \left( \frac{k_{1/2}\Delta \varepsilon \gamma_m \gamma'}{2} \right) \right\} \\
= \sqrt{1 + \zeta_{ix}^2} \Delta \varepsilon \left\{ 1 - \frac{2i}{\pi} \ln \left( \frac{k_{1/2}\Delta \varepsilon \gamma_m \gamma'}{2} \right) \right\}
\]

(3.4.1-4)

where, \( \Delta \varepsilon \) is the length of the segment \( S_i \) projected onto the flat surface.

Again using the small argument approximation of the zeroth order Hankel function, the natural log in equation (3.4.1-4) can be rewritten as a zeroth order Hankel function,

\[
\int_{S_i} H_0^{(2)} (k | \vec{r} - \vec{r}_o |) dS_o \approx \Delta \varepsilon \sqrt{1 + \zeta_{ix}^2} H_0^{(2)} \left( \frac{k_{1/2}\Delta \varepsilon \gamma_m}{2} \right). 
\]

(3.4.1-5)

It should be noted that in equation (3.4.1-5), the argument of the Hankel function is proportional to \( k|\Delta \varepsilon| \). If the dielectric constant is large, \( k \) will also be large and the approximation will break down, unless the size of the discretization segment (and therefore \( \Delta \varepsilon \)) is made proportionally smaller. However, reducing the discretization size is impractical as it would dramatically increase the number of segments need to simulate the scattering surface. Instead, the integral can be broken into two portions, one which can be evaluated using the small argument approximation with \( k|\Delta \varepsilon| \) being small, and the other evaluated numerically, i.e.
\[
\int_{\Delta \varepsilon \mid r_i - r_o \mid > \Delta \varepsilon / 2} H_0^{(2)}(k \mid r_i - r_o \mid) dS_o \\
= \int_{-\Delta \varepsilon / 2}^{\Delta \varepsilon / 2} H_0^{(2)}(k \mid r_i - r_o \mid) dS_o + \int_{\Delta \varepsilon \mid r_i - r_o \mid > \Delta \varepsilon / 2} H_0^{(2)}(k \mid r_i - r_o \mid) dS_o \\
\approx \Delta \varepsilon' \sqrt{1 + \zeta_x^2} H_0^{(2)}(\frac{k |\Delta \varepsilon'| \gamma_m}{2e}) + \int_{\Delta \varepsilon \mid r_i - r_o \mid > \Delta \varepsilon / 2} H_0^{(2)}(k \mid r_i - r_o \mid) dS_o \\
(3.4.1-9)
\]

where, \( \Delta \varepsilon' \) is the part of the total integration length \( \Delta \varepsilon \) for which the small argument approximation is being used. However, \( \Delta \varepsilon' \) can be chosen arbitrarily small and in the limit as \( \Delta \varepsilon' \) goes to zero, the small argument approximation of the integral also goes to zero.

i.e.,

\[
\lim_{\Delta \varepsilon' \to 0} \Delta \varepsilon' \sqrt{1 + \zeta_x^2} H_0^{(2)}(\frac{k |\Delta \varepsilon'| \gamma_m}{2e}) \to 0
\]

This result is simple to prove using the small argument approximation and l'Hopital's rule (which proves that \( \lim_{x \to 0} [x \ln x] \to 0 \)). That is,

\[
\lim_{\Delta \varepsilon' \to 0} \Delta \varepsilon' \sqrt{1 + \zeta_x^2} H_0^{(2)}(\frac{k |\Delta \varepsilon'| \gamma_m}{2e}) \\
\approx \lim_{\Delta \varepsilon' \to 0} \sqrt{1 + \zeta_x^2} \Delta \varepsilon' \left[ 1 - \frac{2i}{\pi} \ln \left( \frac{k |\Delta \varepsilon'| \gamma_m \Phi'}{4e} \right) \right] \to 0
\]

Therefore, the Hankel function self term can also be calculated using numerical techniques by excluding the singular point. That is

\[
\int_{\Delta \varepsilon \mid r_i - r_o \mid > 0} H_0^{(2)}(k \mid r_i - r_o \mid) dS_o \approx \int_{\Delta \varepsilon \mid r_i - r_o \mid > 0} H_0^{(2)}(k \mid r_i - r_o \mid) dS_o
\]

(3.4.1-10)

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3.4.2 Normal Derivative Self Term

Both of the discretized integral equations (3.2-3) and (3.2-4) contain integral terms of the form \( \int_{S_i} \frac{\partial H^{(2)}(k|\vec{r} - \vec{r}_o|)}{\partial n_o} dS_o \). When \( \vec{r} = \vec{r}_o \), the integrand becomes singular. This situation is sometimes referred to as the normal derivative self term. In this section, it is shown that the normal derivative self term can be evaluated by choosing \( \vec{r} \) (known as the observation point) to be very close but just above surface, evaluating the integral, and then taking the limit as this point \( \vec{r} \) goes to the surface. Mathematically, this approach is expressed in equation (3.4.2-1).

\[
\frac{-j}{4} \int_{S_i} \frac{\partial H^{(2)}(k|\vec{r}_i - \vec{r}_o|)}{\partial n_o} dS_o = \lim_{\vec{r} \to \vec{r}_i} \frac{-j}{4} \int_{S_i} \frac{\partial H^{(2)}(k|\vec{r} - \vec{r}_o|)}{\partial n_o} dS_o \quad (3.4.2-1)
\]

Mapping the surface integration onto the \( x \) axis, and using the chain rule (as was done in section 3.2), equation (3.4.2-1) can be written in terms of a first order Hankel function as follows,

\[
\lim_{\vec{r} \to \vec{r}_i} \frac{-j}{4} \int_{S_i} \frac{\partial H^{(2)}(k|\vec{r} - \vec{r}_o|)}{\partial n_o} dS_o = \lim_{\vec{r} \to \vec{r}_i} \frac{-j}{4} \int_{S_i} \frac{\zeta_{x_o}k(x-x_o) + k(z - \zeta(x_o))}{|\vec{r} - \vec{r}_o|} H^{(2)}(k|\vec{r} - \vec{r}_o|) dx_o \quad . (3.4.2-2)
\]
Using the definition of the surface normal (equation 3.3-5), equation (3.4.2-2) can be rewritten as,

$$
\lim_{r \to r_i} \frac{-j}{4} \int_{S_i} \frac{\partial H_0^{(2)}(k|r - r_o|)}{\partial n_o} \, dS_o
$$

$$
= \lim_{r \to r_i} \frac{-j}{4} \int_{S_i} \hat{n} \cdot \frac{k(x - x_o)x + k(z - \zeta(x_o))z}{|r - r_o|} H_1^{(2)}(k|r - r_o|) \, dS_o
$$

$$
= \lim_{r \to r_i} \frac{-j}{4} \int_{S_i} \hat{n} \cdot \frac{k(r - r_o)}{|r - r_o|} \, dS_o. \quad (3.4.2-3)
$$

Recalling that the point \( r \) is chosen very close to the surface, the small kernel approximation for the first order Hankel function can be applied. Abramowitz and Stegun [1972] give the very small kernel approximation as,

$$
H_1^{(2)}(x) \approx \frac{2j}{\pi x}. \quad (3.4.2-4)
$$

Substituting (3.4.2-4) into (3.4.2-3) gives,

$$
\lim_{r \to r_i} \frac{-j}{4} \int_{S_i} \frac{\partial H_0^{(2)}(k|r - r_o|)}{\partial n_o} \, dS_o
$$

$$
\approx \lim_{r \to r_i} \frac{-j}{4} \int_{S_i} \hat{n} \cdot \frac{k(r - r_o)}{|r - r_o|} \frac{2j}{\pi |r - r_o|} \, dS_o
$$

$$
= \frac{1}{2\pi} \lim_{r \to r_i} \int_{S_i} \hat{n} \cdot \frac{r - r_o}{|r - r_o|^2} \, dS_o. \quad (3.4.2-5)
$$

Notice that the integral term now depends only on the geometry.

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The vector \((\vec{r} - \vec{r}_o)\) can be decomposed into the sum of two vectors, one in the direction of the surface normal and the other perpendicular to the normal (i.e. along the surface).

Referring to Figure 3.4.1-1,

\[ |\vec{r} - \vec{r}_o| = \sqrt{r^2 + r_n^2} \]

and so,

\[ \hat{n} \cdot \frac{\vec{r} - \vec{r}_o}{|\vec{r} - \vec{r}_o|^2} = |\hat{n}| \frac{\vec{r} - \vec{r}_o}{|\vec{r} - \vec{r}_o|^2} \cos \alpha = \frac{r_n}{r^2 + r_n^2}. \quad (3.4.2-6) \]

Thus equation (3.4.2-5) then becomes,

\[ \int_{S_i} \hat{n} \cdot \frac{\vec{r} - \vec{r}_o}{|\vec{r} - \vec{r}_o|^2} dS_o = \int \frac{r_n}{r^2 + r_n^2} dS_o = \int \frac{r_n}{r^2 \left( \frac{r_n^2}{r_n^2 + 1} \right)} dS_o. \quad (3.4.2-6) \]

Equation (3.2.4-6) can be further simplified by performing a change of variables,

\[ \tan \alpha = \frac{r_s}{r_n} \Rightarrow \sec^2 \alpha \, d\alpha \approx \frac{1}{r_n} \, dr_s = \frac{1}{r_n} \, dS, \]

which yields,

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\[
\int_{\varepsilon^2} \frac{r_n}{r_n^2 \left( \frac{r_n^2}{r_n^2 + 1} \right)} dS \approx \int_{\tan^{-1}(\varepsilon 2r_n)}^{\tan^{-1}(\varepsilon 2r_n)} \frac{r_n^2}{r_n^2 (r_n^2 + 1)} \sec^2 \alpha d\alpha
\]

\[= \int_{\tan^{-1}(\varepsilon 2r_n)}^{\tan^{-1}(\varepsilon 2r_n)} d\alpha = (\tan^{-1}\left(\frac{\varepsilon}{2r_n}\right) - \tan^{-1}\left(\frac{-\varepsilon}{2r_n}\right)) \quad (3.4.2-7)\]

where \(\left(\frac{\varepsilon}{2}, \frac{-\varepsilon}{2}\right)\) is the small segment of the surface, \(S_0\), over which the integration takes place. Therefore,

\[\frac{1}{2\pi} \lim_{r \to r_i} \int_{S_i} \hat{n} \cdot \frac{\vec{r} - \vec{r}_o}{|\vec{r} - \vec{r}_o|^2} dS_o = \frac{1}{2\pi} \lim_{r \to r_i} (\tan^{-1}\left(\frac{\varepsilon}{2r_n}\right) - \tan^{-1}\left(\frac{-\varepsilon}{2r_n}\right)), \quad (3.4.2-8)\]

and in the limit, as the observation point goes to the surface and \(r_n\) goes to zero with the final result,

\[-\frac{j}{4} \int_{S_i} \frac{\partial H_0^{(2)}(k|\vec{r} - \vec{r}_o|)}{\partial n_o} dS_i \]

\[\approx \frac{1}{2\pi} \lim_{r \to r_i} \int_{S_i} \hat{n} \cdot \frac{\vec{r} - \vec{r}_o}{|\vec{r} - \vec{r}_o|^2} dS \]

\[= \frac{1}{2\pi} (\tan^{-1}(\infty) - \tan^{-1}(\infty)) = \frac{1}{2\pi} \left(\frac{\pi}{2} - \frac{-\pi}{2}\right) = \frac{1}{2}. \quad (3.4.2-9)\]
3.5 Incident Field Tapering

It should be noted that in the extinction equations (3.2-1) and (3.2-2), every point on the surface may contribute to the field at any other point on the surface and that an incident field must be explicitly specified. The first observation means that the method of moments approximation of the equation must contain a full matrix - with memory requirements proportional to $N^2$. Since the computer can manipulate only a small number of surface segments, the surface must be truncated. The truncation of the surface results in surface current errors near the ends of the surface, where neighboring elements which may strongly affect the current have not been included. Both Chapters 5 and 6 will discuss the impact of the end effect errors. To reduce these errors, a tapered plane wave is chosen as the incident field. In this manner, the end effect errors are reduced because there is very little field strength near the truncation region. Equation (3.5-1) defines a tapered incident field, developed by Eric Thorsos [1988], which is used in this study.

\[
E_i(\vec{r}) = \exp \left[ -j(1 + w(\vec{r})) \hat{k} \cdot \vec{r} - \frac{(x-z \tan \theta)^2}{g^2} \right]
\]  

(3.5-1)

where, $g$ is the "spot size" on the surface, and

\[
w(\vec{r}) = \frac{2(x-z \tan \theta)^2/g^2 - 1}{(kg \sin \theta)^2}.
\]
3.6 Summary on the Numerical Implementation of the Scalar Extinction Equations for One Dimensional Rough Surfaces

To solve the extinction equations numerically, the scattering surface is divided into N segments on which the total field and its normal derivative are approximated as constants in each segment. By allowing the observation point to go to a point on each segment, a system of 2N equations with 2N unknowns is obtained. The problem of solving for the electric field and its normal derivative is then be cast into an approximately equivalent matrix equation of the form,

\[ Ax = b \]  \hspace{1cm} (3.6-1)

where, \( x[\ ] \) is a column matrix of the unknown surface fields,
\( b[\ ] \) is a column matrix which contains the incident field, and
\( A[\ , \] \) is a square matrix which approximates the interaction between each of the unknowns.

Using the results of sections 3.3 through 3.5, the matrix elements can be written as,

\[
x[n] = \begin{bmatrix} E(r_n^+) & 1 < n < N \\ \frac{\partial E(r_n^+)}{\partial n} & N + 1 < n < 2N \end{bmatrix}
\]

\[
b[n] = \begin{bmatrix} E_i(r_n) & 1 < n < N \\ 0 & N + 1 < n < 2N \end{bmatrix}
\]

\[
A[n,m] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}
\]
where, \( \mathbf{E} \) is the total electric field = incident + scattered electric field,
\( \mathbf{E}_i \) is the incident electric field,
\( N \) is the number of segments into which the surface has been divided,
n is the row index \( 1 \ldots 2N \),
m is the column index \( 1 \ldots 2N \),
\( \mathbf{r}_n \equiv x_n \mathbf{\hat{x}} + \zeta(x_n) \mathbf{\hat{z}} \) is the position vector to the middle of the \( n \)-th segment unless \( n>N \), when it is the position vector to the \( (n-N) \)-th segment,
\( \mathbf{r}_m \equiv x_m \mathbf{\hat{x}} + \zeta(x_m) \mathbf{\hat{z}} \) is the position vector to the middle of the \( m \)-th segment unless \( m>N \), when it is the position vector to the \( (m-N) \)-th segment,
\( \zeta(\mathbf{x}) \) is the height of the surface,
\( \zeta_{xm} = \frac{d\zeta(x_m)}{dx} \) is the derivative of the surface height evaluated at \( x_m \)
\( k_1, \varepsilon_1 \) is the wavenumber and dielectric constant on one side of the surface,
\( k_2, \varepsilon_2 \) is the wavenumber and dielectric constant on the other side of the surface,
n\( A_{11}, A_{12}, A_{21}, \) and \( A_{22} \) are each square \( \text{NxN} \) matrices,

\[
A_{11} = \begin{bmatrix}
\frac{i}{4} \frac{k_1 \left[ (\zeta(x_n) - \zeta(x_m) - \zeta_{xm}(x_m - x_n)) \right]}{|\mathbf{r}_n - \mathbf{r}_m|} H_1^{(2)}(k_1 |\mathbf{r}_m - \mathbf{r}_n|) \Delta x_m & \text{if } m \neq n \\
\frac{1}{2} & \text{if } m = n
\end{bmatrix}
\]

\[
A_{21} = \begin{bmatrix}
\frac{i}{4} \int_{s_m} \frac{k_2 \left[ (\zeta(x_n) - \zeta(x_m)) - \zeta_{xm}(x_m - x_n) \right]}{|\mathbf{r}_n - \mathbf{r}_m|} H_1^{(2)}(k_2 |\mathbf{r}_m - \mathbf{r}_n|) dx_m & \text{if } m \neq n \\
\frac{1}{2} & \text{if } m = n
\end{bmatrix}
\]

\[
A_{12} = \begin{bmatrix}
-\frac{i}{4} \Delta x H_0^{(3)}(k_1 |\mathbf{r}_m - \mathbf{r}_n|) \sqrt{1 + \varepsilon_{xm}^2} & \text{if } m \neq n \\
-\frac{i}{4} \Delta x H_0^{(3)} \left( \frac{k_1 |\Delta x| \sqrt{1 + \varepsilon_{xm}^2}}{2\varepsilon} \right) \sqrt{1 + \varepsilon_{xm}^2} & \text{if } m = n
\end{bmatrix}
\]

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\[ A_{22} = \begin{cases} \frac{-i}{4} \int_{S_m} H_0^{(2)}(k_2 | \vec{r}_m - \vec{r}_n |) \sqrt{1 + \frac{\zeta^2}{\omega^2}} \, dx_m & \text{if } m \neq n \\ \frac{-i}{4} \int_{\vec{r}_m = \vec{r}_n} H_0^{(2)}(k_2 | \vec{r}_m - \vec{r}_n |) \sqrt{1 + \frac{\zeta^2}{\omega^2}} \, dx_m & \text{if } m = n \end{cases} \]

The entries of matrices \( A_{11} \) and \( A_{12} \) use the single point evaluation approximation, defined in Section 3.3. In general, these entries can be evaluated more precisely using numerical integration. However, it will be shown in chapter 5 that such integration is not required when the dielectric constant \( \varepsilon_1 \) is close to that of free space.

Also, in the limit where the scattering material becomes a PEC the total electric field on the surface goes to zero. Therefore,

\[
\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} 0 \\ \frac{\partial E(\vec{r}_n)}{\partial n} \end{bmatrix} = \begin{bmatrix} E_i(\vec{r}_n) \\ 0 \end{bmatrix}
\]

which implies,

\[
\left[ \frac{\partial E(\vec{r}_n)}{\partial n} \right] [A_{12}] = \left[ E_i(\vec{r}_n) \right]. \quad (3.6-2)
\]

In effect, the integral extinction equations were developed using the continuity of the tangential electric and magnetic fields (see Appendix A). By taking into account the additional constraint that the tangential electric field is zero, the problem is reduced to solving for only the normal derivative of the electric field. This requires only a NxN matrix to be inverted. Finally, the development of the TM case follows the same approach as was used here for the TE case. On the following page, the linear system for the TM case is presented. The only difference between the two cases are (1) the unknowns become magnetic fields instead of electric fields and (2) the boundary conditions results in an \( \frac{\varepsilon_2}{\varepsilon_1} \) term in the \( A_{22} \) matrix.
In the TM case,

\[ x[n] = \begin{bmatrix} H(r_n) & 1 < n < N \\ \frac{\partial H(r_n)}{\partial n} & N+1 < n < 2N \end{bmatrix} \]

\[ b[n] = \begin{bmatrix} H_i(r_n) & 1 < n < N \\ 0 & N+1 < n < 2N \end{bmatrix} \]

\[ A[n,m] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \]

where, \( H \) is the total magnetic field = incident + scattered magnetic field,

\( H_i \) is the incident magnetic field,

\( N \) is the number of segments into which the surface has been divided,

\( n \) is the row index 1 ... 2N,

\( m \) is the column index 1 ... 2N,

\( \vec{r}_n \equiv x_n \hat{x} + z_n \hat{z} \) is the position vector to the middle of the \( n \)-th segment

unless \( n>N \), when it is the position vector to the \((n-N)\)-th segment,

\( \vec{r}_m \equiv x_m \hat{x} + z_m \hat{z} \) is the position vector to the middle of the \( m \)-th segment

unless \( m>N \), when it is the position vector to the \((m-N)\)-th segment,

\( k_1, \varepsilon_1 \) is the wavenumber and dielectric constant on one side of the surface,

\( k_2, \varepsilon_2 \) is the wavenumber and dielectric constant on the other side of the surface,

\( A_{11}, A_{12}, A_{21}, \) and \( A_{22} \) are each square NxN matrices,

\[ A_{11} = \begin{bmatrix} \frac{i}{4} k_1 [(\zeta(x_n) - \zeta(x_m)) - \frac{d\zeta(x_m)(x_m - x_n)}{|\vec{r}_m - \vec{r}_n|} \frac{H^{(2)}(k_1 |\vec{r}_m - \vec{r}_n|)\Delta x_m}{H_i}\frac{1}{2} & \text{if } \vec{r}_m \neq \vec{r}_n \\ \frac{1}{2} & \text{if } \vec{r}_m \neq \vec{r}_n \end{bmatrix} \]
\[ A_{21} = \left\{ \begin{array}{ll}
\frac{i}{4} \int_{S_m} k_2 \left[ \zeta(x_m) - \zeta(x_n) \right] \cdot \frac{d\zeta_{x_m}(x_m - x_n)}{d\zeta_{x_m}(x_m - x_n)} \frac{H^{(2)}_0(k_2 |r_m - r_n|)}{|r_m - r_n|} dx_m & \text{if } m \neq n \\
\frac{1}{2} & \text{if } m = n
\end{array} \right. \]

\[ A_{12} = \left\{ \begin{array}{ll}
-\frac{i}{4} \Delta x H^{(2)}_0(k_1 |r_m - r_n|) \sqrt{1 + \frac{d\zeta_{x_m}^2}{2e}} & \text{if } m \neq n \\
-\frac{i}{4} \Delta x H^{(2)}_0(k_1 |\Delta x| \sqrt{1 + \frac{d\zeta_{x_m}^2}{2e}}) \sqrt{1 + \frac{d\zeta_{x_m}^2}{2e}} & \text{if } m = n
\end{array} \right. \]

\[ A_{22} = \left\{ \begin{array}{ll}
-\frac{i}{4} \frac{\varepsilon_2}{\varepsilon_1} \int_{S_m} H^{(2)}_0(k_2 |r_m - r_n|) \sqrt{1 + \frac{d\zeta_{x_m}^2}{2e}} dx_m & \text{if } m \neq n \\
-\frac{i}{4} \frac{\varepsilon_2}{\varepsilon_1} \int_{S_m} H^{(2)}_0(k_2 |r_m - r_n|) \sqrt{1 + \frac{d\zeta_{x_m}^2}{2e}} dx_m & \text{if } m = n
\end{array} \right. \]

In the limit where the scattering material becomes a PEC, the total electric field on the surface goes to zero. The normal derivative of the magnetic field is proportional to the tangential electric field and therefore also goes to zero. This result is proven in Appendix B. Similar to TE case, the matrix problem reduces to a NxN problem, i.e.,

\[ \left[ H(r_n) \right] [A_{11}] = \left[ H_i(r_n) \right]. \]
4.0 Calculation of the Radar Cross Section (RCS) for One Dimensional Rough Surfaces Using the Discretized Extinction Equations

4.1 Introduction

In the previous chapter, the extinction equations were discretized using the method of moments approach. Once the resulting linear system has been solved, the field and its normal derivative are known on the scattering surface. One of the extinction equations can then be used in combination with the known surface currents to obtain the scattered field at any point in space. When combined with the definition of the normalized radar cross section (NRCS), the discretized equations can be altered into a form which does not require choosing an explicit distance from the scattering surface. That is, the NRCS is only a function of direction. In section 4.2, the radar cross section per unit length is defined for the tapered incident field introduced in section 3.5. Then in the following section, 4.3, the discretized extinction equation (obtained in chapter 3.0) is combined with this definition to find the radar cross section directly from the surface fields.

4.2 Defining the Radar Cross Section (RCS)

A primary tool in the design of radar systems is the radar equation. This equation is derived from a simple power balance approach, assuming that the scattering object is sufficiently far from the transmitter that the transmitted and return fields behave as spherical waves. That is, transmitted power is directed by an antenna, scatters off an object, and is then received by an antenna (not necessarily the same antenna used in transmitting). Although normally thought of in three dimensional terms, the radar equation can also be expressed in two dimensions as given below,
\[
\frac{P_r}{A_r} = \left(\frac{P_i G_i}{2\pi R_i}\right) \left(\frac{\sigma}{2\pi R_r}\right) \text{Loss}
\] (4.2-1)

where, \(P_r\) is the power received by the radar from the target,
\(P_i\) is the power transmitted from the radar,
\(G_i\) is the gain of the transmitting antenna,
\(A_r\) is the effective length of the receiving antenna,
\(\sigma\) is called the radar cross section (with units of length),
\(R_i\) is the distance from the transmitting antenna to the target,
\(R_r\) is the distance from the receiving antenna to the target, and

\(\text{Loss}\) contains all other sources of loss, i.e. atmospheric, etc.

The radar equations contains the following terms: (1) \(P_i G_i/2\pi R_i\) - the transmitted power density (directed by the transmitting antenna) which reaches the target, (2) \(\sigma/2\pi R_r\) - the ratio of the transmitted power density at the scattering object to the return power density at the receiving antenna, and (3) \(P_i A_r\) - the scattered power density gathered at the receiving antenna. The factor \(2\pi R\) appearing in the first and second terms is the "spreading" loss for cylindrical waves, and takes the place of the \(4\pi R^2\) which appears in the three dimensional formulation. The remaining term, \(\text{Loss}\), accounts for various loss mechanisms such as atmospheric attenuation, antenna blockage, etc. (Skolnik [1980] offers a good description of the various loss mechanisms). In the next two subsections, expressions are derived for the scattered power density and the incident power density for plane waves in two dimensional space. These results are then combined with the second term of the radar equation (given above) to obtain an expression for the normalized radar cross section (NRCS) in one dimension.
4.2.1 Scattered Power Density

The vector power density of an electromagnetic wave is given by,

\[ \vec{S} = \frac{1}{2} \text{Re} \left[ \vec{E} \times \vec{H}^* \right] \text{ [watts/m}^2\text{]} \]  

(4.2.1-1)

According to Maxwell's Equations, \(-j\omega \mu \vec{H} = \nabla \times \vec{E}\). Therefore, if the electric field is a plane wave, \(\vec{E} = E_o \exp \left[ -j \vec{k} \cdot \vec{r} \right] \hat{e}\), then the magnetic field is given by,

\[ j\omega \mu \vec{H} = -j\kappa_o \vec{k} \times \vec{E} \quad \Rightarrow \quad \vec{H} = -\frac{1}{\eta_o} \vec{k} \times \vec{E} \]  

(4.2.1-2)

where, \(\vec{E}\) is the electric field vector [volts/m],
\(\vec{H}\) is the magnetic field vector [amperes/m],
\(\hat{e}\) is the direction of the electric field vector (and defines the polarization),
\(\vec{k}\) \(= \kappa_o \hat{k} = \kappa_o (\cos \theta \hat{z} + \sin \theta \hat{x})\),
\(\eta_o\) is the impedance for free space[ohms],
\(\theta_i\) is the direction the plane wave is traveling with respect to the z axis (see figure 4.2.2-1), and
\(\kappa_o\) is the wavenumber [radians/s].

Substituting equation (4.2.1-2) into equation (4.2.1-11), gives:

\[ \vec{S} = \frac{1}{2} \text{Re} \left[ \vec{E} \times \vec{H}^* \right] = \frac{1}{2} \text{Re} \left[ \vec{E} \times (-\frac{1}{\eta_o} \vec{k} \times \vec{E}^*) \right] = \frac{1}{2\eta_o} \text{Re} \left[ \vec{E} \cdot \vec{E}^* \vec{k} - (\vec{E} \cdot \vec{k}) \vec{E}^* \right] \]

\[ = \frac{1}{2\eta_o} |\vec{E}|^2 \hat{k} \]  

(4.2.1-3)

If one is sufficiently far from the one dimensional target, the cylindrical wave front of the scattered field will not change significantly (with respect to the size of the receiving antenna) and can be treated as a plane wave. The cylindrical spreading loss, the reduction in power density as the wave front expands, is \(1/2\pi R\) and has already been included in the radar equation.

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4.2.2 Incident Power Density

Equation (4.2.1-3) gives the power density for a plane wave. However, as introduced in Chapter 3, a tapered plane wave will be required to obtain the MOM solution. To find the incident power density, the total power incident over the scattering surface can be divided by the area (length in the two-dimensional case) perpendicular to the incident beam. The total power incident on the surface is given by,

\[ P_i = \int S_i \cdot d\mathbf{S} = \int S_i \mathbf{k} \cdot d\mathbf{S} = \int \frac{1}{2\eta_o} |E_i|^2 \mathbf{k} \cdot \mathbf{n} \, dx. \]  

(4.2.2-1)

Substituting the electric field of the tapered plane wave, equation (4.2.2-2), into equation (4.2.2-1) yields equation (4.2.2-3).

\[ E_i(r) = \exp \left[ -j \mathbf{k} \cdot \mathbf{r} - (x - z \tan \theta)^2/g^2 \right]. \]  

(4.2.2-2)

\[ P_i = \int \frac{1}{2\eta_o} \exp \left[ -j \mathbf{k} \cdot \mathbf{r} - (x - z \tan \theta)^2/g^2 \right] |(\mathbf{k} \cdot \mathbf{n})| \, dx \]

\[ = \int \frac{1}{2\eta_o} \exp \left[ -2(x^2/g^2) \right] (\mathbf{k} \cdot \mathbf{n}) \, dx \]

\[ = \frac{1}{2\eta_o} \int \exp \left[ -2(x^2/g^2) \right] \cos(\theta_i) \, dx \]  

(4.2.2-3)

where, \( \theta_i \) is the angle formed by the direction of the incident ray and the normal to the mean surface. \( g \) is a tapering parameter which specifies the width of the incident field.

The integral in equation (4.2.2-3) can be evaluated using [Beyer 1991],

\[ \int_0^\infty x^n \exp \left[ -\alpha x \right] \, dx = \frac{\Gamma(n + 1)}{\alpha^{n+1}} \]  

(4.2.2-4)

where, \( \Gamma(\ ) \) is the gamma function. Selecting \( n = 0 \) and \( a = -2/g^2 \), it is obvious that equation (4.2.2-3) becomes,

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\[ P_i(\theta_i) = \frac{g \cos(\theta_i) \Gamma(1)}{2\eta_o} = \frac{g \cos(\theta_i)(\sqrt{\pi/2})}{2\eta_o} \]  

(4.2.2-3)

Taking \( L \) to be the length of the illuminated region on the surface, (as depicted below) the incident power density is given by,

\[ S_i(\theta_i) = \frac{P_i(\theta_i)}{L \cos \theta_i} = \frac{g (\sqrt{\pi/2})}{2L \eta_o} \]  

(4.2.2-4)

![Figure 4.2.2-1 - Depiction of Incident Beam Width](image)

4.2.3 Normalized Radar Cross Section (NRCS)

Using the radar equation (4.2-1), the radar cross section for a one dimensional surface is defined as,

\[ \sigma(\theta_i, \theta_s) = \lim_{r \to \infty} 2\pi r \left| \frac{\text{Scattered Power Density in } \theta_s \text{ direction}}{\text{Incident Power Density in } \theta_i \text{ direction}} \right| \]  

(4.2.3-1)

Taking the limit ensures cylindrical wave behavior of the scattered field. However, it is customary when discussing scattering from surfaces of large extent to use the RCS normalized by unit area (length in the 2D case). In this study, the RCS normalized by unit length will simply be referred to as the NRCS,

\[ \sigma'(\theta_i, \theta_s) = \frac{\sigma(\theta_i, \theta_s)}{L} \]  

(4.2.3-2)

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Using equations (4.2.3-1), (4.2.2-4) and (4.2.1-3) yields,

\[ \sigma^o(\theta_i, \theta_s) = \frac{1}{L} \lim_{r \to \infty} \frac{2\pi r}{L} \frac{\left| \text{Scattered Power Density in } \theta_z \text{ direction} \right|}{\left| \text{Incident Power Density in } \theta_i \text{ direction} \right|} \]

\[ \lim_{r \to \infty} 2\pi r \frac{|E_s(\vec{r})|^2}{g(\sqrt{\pi/2})} \text{ for the tapered plane wave.} \quad (4.2.3-3) \]

### 4.3 Calculating the Radar Cross Section

In this section, one of the discretized extinction equations (obtained in chapter 3) is modified to find the radar cross section directly from the surface fields. This is done by taking advantage of the observation point being very far from the surface. More specifically, the so-called far field transformation is achieved by applying a large argument approximation of the Hankel functions.

From equations (3.2-3), (3.3-6) and (3.3-12), the discretized integral equation for the scattered field can be written as,

\[ E(\vec{r}) = E_i(\vec{r}) \]

\[ \frac{i}{4} \sum_{m=1}^{N} \left\{ E(\vec{r}_m) \int_{S_i} \left[ \frac{k(x - x_o)}{(x - x_o)^2 + (z - \zeta(x_o))^2} H_1^0(k |\vec{r} - \vec{r}_o|) \right] \frac{\partial}{\partial n} \right\} \]

\[ \frac{i}{4} \sum_{m=1}^{N} \left\{ \frac{k(z - \zeta(x_o))}{\sqrt{(x - x_o)^2 + (z - \zeta(x_o))^2}} H_1^0(k |\vec{r} - \vec{r}_o|) \right\} \]

\[ + \frac{i}{4} \sum_{m=1}^{N} \left\{ \frac{\partial E(\vec{r}_m)}{\partial n} \int_{S_i} \left[ H_1^0(k |\vec{r} - \vec{r}_o|) \sqrt{1 + \zeta^2_{x_o}} \right] \right\} \quad (4.3-1) \]
where, \(i\) is the summation index = 1 .. \(N\),
\(N\) is the number of segments,
\(S_i\) is the \(i\)-th segment on the surface,
\(\vec{r}\) is the observation point,
\(\vec{r}_i\) is the position vector to the center of the \(i\)-th segment,
\(\vec{r}_o\) is the position vector to the integration point,
\(k\) is the wavenumber,
\(E_i\) is the incident field,
\(E_s\) is the scattered field,
\(E\) is the total field = incident + scattered fields,
\(H_0^{(2)}\) is the zeroth order Hankel function of the 2nd kind,
\(\zeta(x_o)\) is the height of the rough surface at \(x_o\), and
\(\zeta_{x_o} = \frac{d\zeta(x_o)}{dx}\) is the derivative of the height evaluated at \(x_o\).

Equation (4.3-1) can be further simplified by noticing that
\[
\frac{(z - \zeta(x_o))}{\sqrt{(x-x_o)^2 + (z-\zeta(x_o))^2}} = \sin(\theta_s) \quad (4.3-2)
\]
and
\[
\frac{(z - \zeta(x_o))}{\sqrt{(x-x_o)^2 + (z-\zeta(x_o))^2}} = \cos(\theta_s) \quad (4.3-3)
\]

where \(\theta_s\) is the scattering angle depicted in figure 4.3-1 below,

![Figure 4.3-1 - Depiction of Incident and Scattering Angles](image)

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Subtracting the incident field from both sides of equations (4.3-1) and substituting in equations (4.3-2) and (4.3-3) yields,

$$ E_z(\vec{r}) = -\frac{j}{4} \sum_{i=1}^{N} \left[ E(\vec{r}) \int_{S_i} \left[ k(-\zeta_{xo}\sin(\theta_s) + \cos(\theta_s)) \tilde{H}_1^2(k|\vec{r} - \vec{r}_o|) \right] d\chi_o \right] $$

$$ + \frac{j}{4} \sum_{i=1}^{N} \left\{ \frac{\partial E(\vec{r})}{\partial n} \int_{S_i} \left[ \tilde{H}_0^2(k|\vec{r} - \vec{r}_o|) \sqrt{1 + \frac{\zeta_{xo}^2}{\zeta_{xo}^2 + \zeta_{yo}^2}} \right] d\chi_o \right\} \quad (4.3-4) $$

Abramowitz and Stegun [1972] give the large argument expansion of the zeroth and first order Hankel functions of the second kind as,

$$ \lim_{z \to \infty} H_0^2(z) \approx \sqrt{\frac{2}{\pi z}} e^{-i(z - \pi/4)} \quad \text{and} \quad (4.3-5) $$

$$ \lim_{z \to \infty} H_1^2(z) \approx \sqrt{\frac{2}{\pi z}} e^{-i(z - 3\pi/4)} . \quad (4.3-6) $$

Substituting equations (4.3-5) and (4.3-6) into equation (4.3-4) yields,

$$ E_z(\vec{r}) \approx $$

$$ -\frac{j}{4} \sum_{i=1}^{N} \left[ E(\vec{r}) \int_{S_i} \left[ k(-\zeta_{xo}\sin(\theta_s) + \cos(\theta_s)) \frac{2}{\pi k |\vec{r} - \vec{r}_o|} e^{-i(k|\vec{r} - \vec{r}_o| - \pi/4)} \right] d\chi_o \right] $$

$$ + \frac{j}{4} \sum_{i=1}^{N} \left\{ \frac{\partial E(\vec{r})}{\partial n} \int_{S_i} \left[ \frac{2}{\pi k |\vec{r} - \vec{r}_o|} e^{-i(k|\vec{r} - \vec{r}_o| - \pi/4)} \sqrt{1 + \zeta_{xo}^2} \right] d\chi_o \right\} . \quad (4.3-7) $$

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Taking $| \vec{r} | \gg | \vec{r}_0 |$, then

$$\sqrt{| \vec{r} - \vec{r}_0 |} \approx \sqrt{| \vec{r} |}$$  \hspace{1cm} (4.3-8)$$

and

$$e^{-jkl(\vec{r} - \vec{r}_0)} \approx e^{-jkl| \vec{r} |} e^{j k_s \vec{r}_0}$$  \hspace{1cm} (4.3-9)$$

where $\vec{k}_s = k \cos(\theta_x) \hat{\imath} + k \sin(\theta_x) \hat{j}$.

Substituting equations (4.3-8) and (4.3-9) into (4.3-7) gives,

$$E_s(\vec{r}) \approx$$

$$- \frac{j}{4} \sum_{i=1}^{N} \left\{ \frac{2}{\pi k l | \vec{r} |} e^{-j(kl| \vec{r} | - \pi/4)} \int_{S_i} \left[ k(-\zeta_{x_0} \sin(\theta_x) + \cos(\theta_x)) e^{j k_s \vec{r}_0} \right] d\vec{x}_0 \right\}$$

$$+ \frac{j}{4} \sum_{i=1}^{N} \left\{ \frac{2}{\pi k l | \vec{r} |} e^{-j(kl| \vec{r} | - \pi/4)} \int_{S_i} \left[ e^{j k_s \vec{r}_0} \sqrt{1 + \zeta_{x_0}^2} \right] d\vec{x}_0 \right\}.$$  \hspace{1cm} (4.3-10)$$

Evaluating the integrals in equation (4.3-10) at the midpoint of each segment,

$$E_s(\vec{r}) \approx$$

$$- \frac{j}{4} \sum_{i=1}^{N} \left\{ \frac{2}{\pi k l | \vec{r}_i |} e^{-j(kl| \vec{r}_i | - \pi/4)} \left[ k(-\zeta_{x_i} \sin(\theta_x) + \cos(\theta_x)) e^{j k_s \vec{r}_0} \right] \Delta x \right\}$$

$$+ \frac{j}{4} \sum_{i=1}^{N} \left\{ \frac{2}{\pi k l | \vec{r}_i |} e^{-j(kl| \vec{r}_i | - \pi/4)} \left[ e^{j k_s \vec{r}_0} \sqrt{1 + \zeta_{x_i}^2} \right] \Delta x \right\}$$  \hspace{1cm} (4.3-11)$$

where $\Delta x$ is the length of the segment $S_i$ projected onto the $x$ axis.

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Finally, substituting equation (4.4-11) into the definition of the NRCS, equation (4.2.3-1), yields,

$$
\sigma^o(\theta_i, \theta_s) \approx \frac{2\pi}{g(\sqrt{\pi/2})^2} \frac{1}{2\pi k} \Delta x
\left| \sum_{i=1}^{N} \left[ E(\bar{r}_i) k \left( -\zeta_{x_i} \sin(\theta_s) + \cos(\theta_i) \right) + j \sqrt{1 + \zeta_{x_i}^2} \frac{\partial E(\bar{r}_i)}{\partial n} \right] e^{-j\bar{r}_i \cdot \bar{r}_i} \right|^2,
$$

which is the desired result.
5.0 The Flat Surface Limit

5.1 Introduction

In chapter three, a pair of coupled integral equations was converted into an approximately equivalent matrix problem, which can be used to obtain an approximate solution for the surface fields (i.e. the surface currents) on the boundary between two homogenous materials. In this chapter, a portion of the error produced using the approximate matrix formulation is examined by solving the problem of scattering from a flat interface. This flat surface limit is examined because the exact solution for plane wave illumination is known in this case.

The organization of this chapter is as follows: Section 5.2 first demonstrates the error in the method of moments (MOM) solution. Section 5.3 then examines how the errors in the MOM solution for the surface currents are related to the discretization of the integral equations and the truncation of the scattering surface. It is shown that there is a basic tradeoff between the maximum value permitted the real part of the dielectric constant of the scattering medium and the number of Gauss-Legendre integration points used in evaluating the entries in the MOM matrices. It is also shown that if the incident field is not tapered sufficiently, spikes are generated in the MOM solution for the current at the ends of the scattering surface. These current spikes introduce small but potentially significant errors in the scattered field calculations in all scattering directions. Section 5.4 then reexamines the approximate matrix equation for the surface current. It is shown that the error in the numerical solution for the normal derivative of the electric field on a flat perfect electric conductor (PEC) is exactly twice the error in the case of a fictitious boundary, i.e., a boundary where the material on both sides of the flat interface is the same. Although, this relationship is not exact for a rough interface, it is expected to be approximately true for surfaces with small heights and small slopes.
5.2 Method of Moment (MOM) Solution Errors in the Flat Surface Limit

In this section, errors produced in the method of moments (MOM) solution of the scattered field are examined by comparing the numerical results with a nearly exact solution in the flat surface limit. When the scattering surface is flat, the exact electric field and its normal derivative on the surface (that is the surface currents) can be expressed using Fresnel reflection coefficients. For plane wave illumination [Balanis 1989],

\[ E(r_s) = (1 + \rho(\theta_i))E_i(r_s), \]  
(5.2-1)

and

\[ \frac{\partial E(r_s)}{\partial n} = (1 - \rho(\theta_i))(ik)E_i(r_s). \]  
(5.2-2)

where \( r_s \) is a point on the flat surface,

\[ k_i = k_1(-\cos \theta, \hat{z} + \sin \theta, \hat{x}) = k_{i\hat{z}} + k_{i\hat{x}} \hat{x} \] is the incident wavevector,

\[ k_1 = \frac{2\pi}{\lambda_o} \sqrt{\varepsilon_{r1}} \] is the wavenumber (in the upper material),

\( \lambda_o \) is the free space wavelength,

\( \varepsilon_{r1} \) is the relative dielectric constant (in the upper material),

\( \theta_i \) defines the direction of propagation of the plane wave relative to the flat surface normal (see figure 4.3-1),

\( E_i \) is the incident field,

\( E \) is the total electric field = incident + scattered electric fields,

\( \rho(\theta_i) \) is the Fresnel reflection coefficient.

It should be noted that the incident field used in this study is not a plane wave, but is tapered (or smoothly windowed) to reduce the effects of truncating the scattering surface. (The exact form of the Gaussian tapered incident field used in this study is given in section 5.3.3 and the so called end effect errors are examined in more detail in this section and again in chapter six). However to a very good approximation, the tapered field can be used with the Fresnel reflection coefficient at the incident angle. This is
because the tapered field is actually a spectrum of plane waves traveling in slightly
different directions. This spectrum, however, has a narrow angular width (typically, the 3
dB width is less than 2°).

As an example of the MOM solution, figure 5.2-1 plots the bistatic normalized
radar cross section (NRCS) for an incidence angle of 0° (i.e. normal incidence) for a flat
surface between two media with different dielectric constants. (The NRCS is defined in
chapter four). In this example, and in all the other cases examined throughout this study,
the dielectric constant of the upper region, that is the region in which the incident field is
generated, has been chosen to be equal to that of free space, i.e. ε_{r1}=1.0. The dielectric
constant of the lower region, that is the region containing the material off of which the
incident field is being scattered, has been chosen to have an relative dielectric constant
which is either infinite, i.e. a PEC, for the solid line in figure 5.2-1 or ten, i.e. ε_{r2} = 10.0,
for the dashed line. Here the subscript "r1" or "r2" specify region 1 and region 2
respectively. The Gaussian like shape of the scattered field depicted in the figure is a
result of the angular spread of the tapered incident field discussed in the proceeding
paragraph. If the scattering surface were infinite and the incident field a plane wave, the
NRCS would be a delta function. The width of the angular spread is controlled by a
tapering parameter "g" which will be discussed in section 5.3.3. Also in regards to figure
5.2-1, the rapidly oscillating NRCS at scattering angles greater than 14° and less than -14°
is not the correct solution but a result of numerical error in the solution process.

It should be noted that in this example, the length of the current pulses, i.e. the
length of the segments into which the surface is subdivided, was chosen as one tenth the
free space wavelength (λ_0). Also, in evaluating the MOM matrix entries (defined in
Section 3.6), a ten point Gauss-Legendre integration was used to evaluate the self terms
which contain a zeroth order Hankel function, i.e. the diagonal entries of the matrices A_{12}.  

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Figure 5.2-1 Example of Flat Surface Normalized Radar Cross Section (NRCS) for Flat Surfaces, 80 $\lambda_o$ Long with $\Delta x = 0.1 \lambda_o$, $\theta_i = 0^\circ$, TE Polarization, and $g = 5 \lambda_o$. 

PEC

$\varepsilon_{r2} = 10.0$
and $A_{22}$, and one point was used to evaluate the off diagonal entries of all of the MOM matrices, i.e. $A_{11}$, $A_{12}$, $A_{21}$ and $A_{22}$. The impact of these parameters will be discussed in the following sections.

Using equation (5.2-1), the error in the MOM result can be examined either on the surface or in the far field by using the same far field transform that is used for the MOM currents (see chapter four). Figure 5.2-2, for example, plots the absolute error in the NRCS, defined as $|Exact - Calculated|$, for flat surfaces with three different dielectric constants. Referring to figure 5.2-2, it is interesting to note that the absolute error is greater for a surface with a relative dielectric constant of ten than for the perfect electric conducting (PEC) surface. Also, when the relative dielectric constant of the lower region is equal to the relative dielectric constant of the upper region, $\varepsilon_{r1} = \varepsilon_{r2} = 1.0$, the error is less than that for the PEC, but does not go to zero. Said another way, even when the materials on both side of the boundary are identical so there is no physical boundary the MOM solution still produces a scattered field!

Alternatively, one can plot the relative error in the calculated NRCS, i.e. $|Exact - Calculated|/|Calculated|$. This is done in figure 5.2-3. This definition for the relative error has been chosen over the common definition $|Exact - Calculated|/|Exact|$ because the exact solution is zero in the no contrast limit and in other situations the exact solution can be sufficient small to cause some computational difficulties. Examining figure 5.2-3, shows that the relative error in the MOM solution for the PEC is quite small (less than 0.5%) in the region the where the NRCS is greater than 90 dB. For the $\varepsilon_{r2} = 10$ case, however, the relative error is unacceptably high at roughly 11%. Only the scattering directions -$14^\circ$ to $+14^\circ$ have been displayed because outside this region the exact solution is much less than the computed solution, and so the relative error is essentially 100%.
Figure 5.2-2: Examples of Absolute Error in the NRCS, i.e. abs(Exact - Calculated), for a Flat Surface, 80 $\lambda_0$ Long with $\Delta x = 0.1 \lambda_0$, $\theta_i = 0^\circ$, TE Polarization, and $g = 5 \lambda_0$. 
Figure 5.2-3 Relative Error in NRCS, i.e. abs(Exact - Calculated)/abs(Calculated), for a Flat Surface, 80 $\lambda_o$ Long with $\Delta x = 0.1 \lambda_o$, $\theta_i = 0^\circ$, TE Polarization, and $g = 5 \lambda_o$.
The obvious questions raised by these two figures are (1) what is the source of these errors, (2) why does the $\varepsilon_{12} = 10.0$ surface produce greater absolute and relative error than the PEC surface, and (3) can the no contrast solution provide either quantitative or qualitative information about these errors? All of these questions are addressed in the following sections.

Also of interest, is the fact that the absolute error in the NRCS (figure 5.2-2) has the same shape as the NRCS itself and correspondingly the relative error is nearly constant as a function of the scattering angle. This occurs because the errors in the MOM current solutions have almost the same Gaussian shape as the tapered incident field. Figure 5.2-4, for example, plots the magnitude of the error in the electric surface current relative to the current at $x = 0$ (the center of the incident illumination) for both the PEC and $\varepsilon_{12} = 10$ cases examined above. Also plotted in figure 5.2-4 is the absolute error in the electric surface current for a flat PEC, but with an incident angle of 20°. This later curve was added to demonstrate that this same effect occurs away from normal incidence. There is however a phase difference between the exact current and the current error.

Figure 5.2-5, plots the phase difference between the current solution and the current errors, for the same cases displayed in figure 5.2-4. This plot shows that the phase shift between the current solution and current error is roughly constant in the strongly illuminated region of the surface.

Figures 5.2-4 and 5.2-5 raise the question, if the relative error in the surface current is 3.3% for the PEC, why is the relative error in the NRCS less than 0.5%. After all, the scattered field far from the surface is essentially a Fourier transform of the surface current (that is a linear transform). If the error in the current has the same shape as the exact current ($J_{exact}$) and a constant phase shift, then the relative error in the scattered field will have the same phase and magnitude as the relative error in the current, i.e.
Figure 5.2-4 Magnitude of Error in Electric Surface Current Relative to the Maximum Current, i.e. \( \frac{\text{abs(Exact-Calculated)}}{\text{abs(Exact at } x=0)} \) for a PEC and \( \varepsilon_r = 10.0 \) at \( \theta_i = 0^\circ \). 

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Figure 5.2-5 Phase Difference between the Exact Electric Current and the Current Error, i.e. phase(Exact)-phase(Exact - Calculated), for a PEC and $\varepsilon_r = 10$ Flat Surfaces $80\lambda_o$ Long with $\Delta x = 0.1\lambda_o$. TE Polarization, and $g = 5\lambda_o$.
\[ E_s = \text{Far\_Field\_Transform}[(1 + \varepsilon e^{-j\Delta})J_{\text{exact}}] = (1 + \varepsilon e^{-j\Delta})\text{Far\_Field\_Transform}[J_{\text{exact}}]. \]

The answer to this question lies in the relative phases of the surface current and the surface current error. Suppose that the scattered electric field is given by the sum of the exact (or correct) solution \((X e^{j\Phi})\) and some error \((\varepsilon X e^{j(\Phi + \Delta)})\) which has a magnitude equal to some small value, \(\varepsilon\), times the magnitude of the exact solution and a phase difference, \(\Delta\). That is,

\[ E_s = (X e^{-j\Phi} + (\varepsilon X)e^{-j(\Phi + \Delta)}) \quad (5.2-1) \]

The NRCS is proportional to the magnitude of the electric field squared,

\[ |E_s|^2 = E_s E_s^* = (X e^{-j\Phi} + (\varepsilon X)e^{-j(\Phi + \Delta)})(X e^{-j\Phi} + (\varepsilon X)e^{-j(\Phi + \Delta)})^* \]

\[ = XX + \varepsilon XX e^{-j\Delta} + \varepsilon XX e^{j\Delta} + (\varepsilon X)(\varepsilon X) \]

\[ = X^2 + 2\varepsilon XX \cos(\Delta) + \varepsilon^2 X^2. \quad (5.2-2) \]

Therefore, the relative error in the NRCS would be given by,

\[ \frac{|E_s|^2 - |X e^{-j\Phi}|^2}{|E_s|^2} = \frac{2\varepsilon \cos(\Delta) + \varepsilon^2}{1 + 2\varepsilon \cos(\Delta) + \varepsilon^2}. \quad (5.2-3) \]

which can be larger or smaller than the magnitude of the relative error in the scattered field, \(\frac{|E_s - X e^{-j\Phi}|}{|E_s|} = \frac{\varepsilon}{1 + \varepsilon}\), depending on the phase difference between the error and the exact solution.
For the PEC surface, the error is approximately 92.2° out of phase with the exact solution, so the error in the NRCS is much smaller than the 3.3% error in the magnitude of the current and the scattered field. In the $\varepsilon_{r_2} = 10$ case, the current error and scattered field errors are about 107.5° out of phase, so the error in the current contributes more to the NRCS. It should be stressed that the above analysis explains why the relative error in the electric surface current is larger than the relative error in the NRCS for the PEC. It does not explain why the relative error in the current is larger for the $\varepsilon_{r_2} = 10$ case than for the PEC case!

Finally, it should be noted that in the cases where $\varepsilon_{r_2} = 10$ and $\varepsilon_{r_2} = 1.0$, there is both an electric and a magnetic surface current. The error in magnetic surface current, not displayed here, also has the same Gaussian profile in amplitude and a nearly constant phase shift.

5.3 Sources of Error

In the following three subsections, sources of error in MOM solutions are evaluated (in the flat surface limit) by examining the impact of varying (1) the number of integration points used to evaluate entries in the MOM matrices, (2) the computer precision (single or double), and (3) the length of the scattering surface which is longer than the illuminated surface.

5.3.1 Discretization Error

The terminology, "discretization error", actually embraces two primary sources of error which interact closely. First, in the MOM approach the surface was discretized into small segments over which the current was assumed to be constant. Only to the degree that the current can be considered constant will the MOM solution be accurate. Second, there are numerical errors in the calculation of the interaction terms between the surface
current elements, i.e. the MOM matrix entries. Calculating the entries in MOM interaction matrix (which is defined in section 3.6 using four square submatrices $A_{11}$, $A_{12}$, $A_{21}$, and $A_{22}$) involves numerical integrations of Hankel functions. For a flat surface at normal incidence, the exact current distribution is smooth such that discretization intervals of less than half a wavelength can be expected to produce good results. In this situation, the accurate calculation of the MOM matrix elements is generally the limiting factor on solution accuracy. In the following two subsections, the impact of varying the number of numerical integration points used in calculated the MOM matrix entries is examined. In the first subsection, the off-diagonal matrix entries are examined (referred to in chapter three as simple terms) and in the second subsection the diagonal (or self terms) are examined. The diagonal terms are examined separately because a special asymptotic form for these integral terms was developed in chapter three based on the small argument approximation of the Hankel function.

5.3.1.1 The Off-Diagonal Matrix Entries

In this first subsection, the effect of varying the number of Gauss-Legendre integration points, used to calculate the off-diagonal entries in the MOM matrices, is examined with respect to the MOM current solution. It will be shown that there is a basic tradeoff between the maximum value permitted the real part of the relative dielectric constant and the number of integration points that must be used in evaluating the off-diagonal entries. For a flat surface, if the number of integration points is chosen too small, the errors in the matrix terms (which are complex numbers) can add in a coherent like fashion, i.e. all the errors have the same phase. This results in large current errors in the MOM solution. Before examining this error, however, it will prove helpful to first examine the behavior of the zeroth order Hankel function. In figure 5.3.1.1-1, the real and imaginary parts of the both the zeroth and first order Hankel functions of the second kind, $H^{(2)}_{0,1}(kr)$, are plotted for $0.1\lambda_o \leq r \leq 1.0\lambda_o$ and $k = 2\pi\sqrt{\varepsilon_r/\varepsilon_o}$ with $\varepsilon_r = 1, 10$ and 100.
Figure 5.3.1.1-1 Plot of First and Second Order Hankel Functions for Lossless Dielectrics
These three plots show that both the real and imaginary parts of the Hankel functions have a decaying oscillatory behavior (away from the origin) and that the period of the oscillations is proportional to $1/\sqrt{\varepsilon_r}$. This figure suggests, that using a $1/10$ of a wavelength discretization length ($\Delta x = 0.1\lambda_o$) with a single point evaluation of the Hankel function will be an acceptable approximation when $\varepsilon_r = 1.0$, because the Hankel function is varying slowly on this scale. However when $\varepsilon_r$ is large (and the Hankel functions vary rapidly) either the discretization length will have to be greatly reduced (e.g. $\Delta x = \lambda_o/100$ for $\varepsilon_r = 100.0$) or more than one integration point will be required to evaluate the integrals.

Turning to the error in the MOM calculated surface currents, figure 5.3.1.1-2 plots the error in the electric surface current evaluated at $x = 0$ (the location of the peak error) as a function of the relative dielectric constant of the scattering surface for varying numbers of integration points. In this example, the incident field is TE polarized with an incident angle of $0^\circ$ and a spot size of $5\lambda_o$. In this figure (and other figures throughout this dissertation) the notation "x/y/z" is used to specify the number of Gauss-Legendre integration points used in evaluating the MOM matrix entries. For example, "10/3/1" indicates that 10 integration points are used to evaluate the self terms (i.e. the diagonal entries), 3 integration points are used to evaluate the off-diagonal matrix entries which are a function $\varepsilon_2$ (the dielectric constant of the scattering material), and 1 integration point is used to evaluate the off-diagonal matrix entries which are a function of $\varepsilon_1$. The use of 10 integration points for evaluating the self term will be addressed in the following section, but for the moment assume that this is a sufficient number of points for a very accurate evaluation.
Figure 5.3.1.1-2 shows that when one integration point is used to evaluate the off-diagonal terms of the matrices $A_{21}$ and $A_{12}$ (which are a function of $\varepsilon_{r_2}$) then the error in the surface current peaks at roughly $\varepsilon_{r_2} = 100, 400, \text{and} 900$. In light of the previous discussion about the Hankel functions, this is not surprising. These "error resonances" occur because the integration errors in each segment tend to cancel each other out except when the discretization size is an integer multiple of the period in the Hankel function oscillations. In this situation, the error in each segment (a complex number) has the same phase. Thus when $\varepsilon_{r_2} = 100$, the period of the Hankel function is $0.1 \lambda_o$ which is equal to the discretization size. Similarly when $\varepsilon_{r_2} = 400 \text{ and } 900$, the period of the Hankel functions oscillations are $0.05 \text{ and } 0.0333 \lambda_o$, respectively. Therefore, the discretization length is 2 and 3 times the period of the Hankel functions oscillations. If the plot were continued for larger dielectric constants, there would be peaks at $\varepsilon_{r_2} = 1600, 2500, \text{etc.}$

By using more than one integration point to evaluate the off-diagonal matrix elements, the errors resulting from the oscillatory behavior is reduced. However, the larger $\varepsilon_{r_2}$, the faster the function oscillates, and the greater the number of integration points required to correctly perform the integrations. Thus there is a basic tradeoff between the number of points required in evaluating the integrals and the maximum permitted real dielectric constant. For example, figure 5.3.1.1-2 also plots the electric surface current error using 3, 5 and 10 integration points for each matrix entry. Using 3 points strongly suppresses the first error peak, but not the second. Similarly, using 5 points strongly suppresses the first two error peaks, but not the third at $\varepsilon_r = 900$.

What happens when loss (i.e. an imaginary part of the dielectric constant) is included? Figure 5.3.1.1-3 plots the zeroth and first order Hankel functions for $\varepsilon_r = 10.0, 10.0 - 1.0j, \text{and } 10 - 2.0j$. These plots demonstrate that the imaginary component causes the Hankel functions to decay more rapidly (away from the origin); the larger the
Figure 5.3.1.1-3 Plot of First and Second Order Hankel Functions for Lossy Dielectrics
imaginary component, the more rapid the decay. This suggests that the imaginary component of the dielectric constant does not place any additional constraints on the number of integration points. Figure 5.3.1.1-4 confirms this conclusion by plotting the relative error in the surface current where the real part of the relative dielectric constant is fixed at 1.0 and the imaginary component ranges from -1j to -000j. In this figure, the relative error in the electric surface current remains small when using either 10 or single point evaluations of the off-diagonal terms which are a function of ε_{r2} (that is 10/10/1 or 10/1/1 integration points). It should be noted that the roughly 3.3% error observed in the electric current is largely a result of using one point evaluation of the integrations associated with ε_{r1} = 1.0. The reader may recall that this is the same error that was found in the PEC example in section 5.2 (see figure 5.2-4). As indicated in figure 5.3.1.1-4, the 3.3% error can be reduced by increasing the number of integration points used for all the off-diagonal terms (i.e. 10/3/3 or 10/10/3 integration points).

Figures 5.3.1.1-2 and 5.3.1.1-4 plotted the electric current errors obtained for a TE (horizontally) polarized incident field. When a TM (vertically) polarized incident field is used, somewhat different error behavior is observed, as is shown in figure 5.3.1.1-5. As in the TE case, the surface is a flat interface, the incident angle is 0° and g = 5 λ₀. Also, 10 points are used to evaluated the self term, and 1 point is used to evaluate the Hankel functions related to the ε_{r1}. Again, the use of 10 points for the self term will be addressed in the following section, but for the moment assume this is sufficient for a highly accurate evaluation. Unlike the TE case, the TM "error resonances" do not appear to coincide with the periodicity in the Hankel function oscillations. As will be shown presently, this result occurs because, although the scattering from a flat surface for either TE and TM polarization should be identical at normal incidence, the coupled scalar extinction equations (used to obtain the surface currents) are NOT identical. (A derivation of the extinction equations is provided appendix A).
Figure 5.3.1.1-4 Relative Error in Electric Surface Current evaluated at $x = 0$ versus Loss in Relative Dielectric Constant with $\text{Real}(\varepsilon_r) = 1.0$ for a Flat Surface, $80 \lambda_o$ Long with $\Delta x = 0.1 \lambda_o$, $\theta_i = 0^\circ$, TE Polarization, and $g = 5 \lambda_o$
Number of Integration Points = Self Term/Lower Region/Upper Region

abs(Exact-Calculated)/abs(Calculated)

0.30

0.25

0.20

0.15

0.10

0.05

0.00

0

100

200

300

400

500

600

700

800

900

1000

Relative Dielectric Constant

10/1/1

10/10/1

TM polarization

Figure 5.3.1.1-5 Relative Error in Electric Surface Current evaluated at $x = 0$
versus Real Relative Dielectric Constant and Number of Integration Points
for a Flat Surface, $80 \lambda_o$ Long with $\Delta x = 0.1 \lambda_o$, $\theta_i = 0^\circ$, TM Polarization, and $g = 5 \lambda_o$
In the next several pages, an expression for the electric surface current (which does not depend on knowing the magnetic surface current) is obtained for plane wave illumination of a flat interface. That is, the coupled integral equations for the electric and magnetic surface currents can be decoupled in this situation. Using these expressions (one for TE and one for TM polarization), the error resonances displayed in figures 5.3.1.1-2 and 5.3.1.1-5 can be better understood.

In chapter three, the two discretized integral equations for the TE case were given as equations (3.2-3) and (3.2-4), which are reproduced below as equations (5.3.1.1-1) and (5.3.1.1-2).

\[
E(\vec{r}) = E_i(\vec{r}) + \frac{j}{4} \sum_{m=1}^{N} E(r_i) \int_{S_i} \frac{\partial H^{(2)}_0(k_1 |\vec{r} - \vec{r}_o|)}{\partial n_o} dS_o \tag{5.3.1.1-1}
\]

\[
+ \frac{j}{4} \sum_{m=1}^{N} \frac{\partial E(r_i)}{\partial n} \int_{S_i} H^{(2)}_0(k_1 |\vec{r} - \vec{r}_o|) dS_o
\]

and

\[
0 = \frac{-j}{4} \sum_{m=1}^{N} E(r_i) \int_{S_i} \frac{\partial H^{(2)}_0(k_2 |\vec{r} - \vec{r}_o|)}{\partial n_o} dS_o \tag{5.3.1.1-2}
\]

\[
+ \frac{j}{4} \sum_{m=1}^{N} \frac{\partial E(r_i)}{\partial n} \int_{S_i} H^{(2)}_0(k_2 |\vec{r} - \vec{r}_o|) dS_o
\]

where \(i\) is the summation index = 1 .. N,

\(N\) is the number of segments into which the surface has been divided,

\(S_i\) is the \(i\)-th segment on the surface,

\(\vec{r}_i\) is the position vector to the center of the \(i\)-th segment,

\(H^{(2)}_0\) is the zeroth order Hankel function of the 2nd kind,

\(E\) is the total electric field,
\( E_i \) is the incident field,
\( k_2 \) is the wavenumber for the lower region,
\( k_1 \) is the wavenumber for the upper region, and
\( \vec{r} \) is chosen to be a point on the surface.

For a perfectly flat surface illuminated by a plane wave at normal incidence, the surface fields will be the same everywhere on the surface. So, the field quantities may be removed from the summation, such that,

\[
E = E_i + \frac{\mathbf{j}}{4} \sum_{\mu=1}^{N} \int_{S_i} \frac{\partial \mathbf{H}_0^{(2)}(k_1 | \vec{r} - \vec{r}_{\circ} |)}{\partial \mathbf{n}_{\circ}} dS_{\circ} + \frac{\partial E}{\partial \mathbf{n}} \sum_{\mu=1}^{N} \int_{S_i} \mathbf{H}_0^{(2)}(k_1 | \vec{r} - \vec{r}_{\circ} |) dS_{\circ},
\]

(5.3.1.1-3)

and

\[
0 = \frac{\mathbf{j}}{4} \sum_{\mu=1}^{N} \int_{S_i} \frac{\partial \mathbf{H}_0^{(2)}(k_2 | \vec{r} - \vec{r}_{\circ} |)}{\partial \mathbf{n}_{\circ}} dS_{\circ} + \frac{\partial E}{\partial \mathbf{n}} \sum_{\mu=1}^{N} \int_{S_i} \mathbf{H}_0^{(2)}(k_2 | \vec{r} - \vec{r}_{\circ} |) dS_{\circ}
\]

(5.3.1.1-4)

In the case of a flat surface, the integration of the normal derivative of the zeroth order Hankel function is zero everywhere except at the point where \( \vec{r} = \vec{r}_{\circ} \). It was shown in chapter three, that when \( \vec{r} = \vec{r}_{\circ} \), this integration yields a value of 1/2. Therefore, equations (5.3.1.1-3) and (5.3.1.1-4) can be reduced to,

\[
E = E_i + \frac{1}{2} E + \frac{\mathbf{j}}{4} \sum_{\mu=1}^{N} \int_{S_i} \mathbf{H}_0^{(2)}(k_1 | \vec{r} - \vec{r}_{\circ} |) dS_{\circ},
\]

(5.3.1.1-5)

and

\[
0 = \frac{1}{2} E + \frac{\mathbf{j}}{4} \sum_{\mu=1}^{N} \int_{S_i} \mathbf{H}_0^{(2)}(k_2 | \vec{r} - \vec{r}_{\circ} |) dS_{\circ}.
\]

(5.3.1.1-6)

Combining the equation (5.3.1.1-5) and (5.3.1.1-6), one obtains the following expression for the normal derivative of the electric field, which is proportional to the electric surface current (a proof of this is given in appendix B.7)

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\[
\frac{\partial E}{\partial n} = E_i \sum_{i=1}^{N} \frac{1}{4} \int_{S_i} H_0^2(k_2 | \vec{r} - \vec{r}_o |) \, dS_o + \frac{j}{4} \int_{S_i} H_0^2(k_1 | \vec{r} - \vec{r}_o |) \, dS_o
\]

(5.3.1.1-7)

In this equation, one can readily see how the addition of errors in the numerical evaluation of the Hankel function can cause dramatic errors in the solution. When the error in the terms \[\int_{S_i} H_0^2(k_2 | \vec{r} - \vec{r}_o |) \, dS_o\] becomes large (because it add in phase), the normal derivative of the electric field moves towards zero and away from the correct solution, \[2j\pi(1 - \rho(0))E_i\]. (A similar expression can be obtain for the magnetic surface current, which is proportion to the tangential electric field).

The discretized integral extinction equations in the TM case, are given below as equations (5.3.1.1-8) and (5.3.1.1-9).

\[
H(\vec{r}) = H_i(\vec{r}) + \frac{j}{4} \sum_{i=1}^{N} H(\vec{r}_i) \int_{S_i} \frac{\partial H_0^{(2)}(k_1 | \vec{r} - \vec{r}_o |)}{\partial n_o} \, dS_o
\]

\[+ \frac{j}{4} \sum_{i=1}^{N} \frac{\partial H(\vec{r}_i)}{\partial n} \int_{S_i} H_0^{(1)}(k_1 | \vec{r} - \vec{r}_o |) \, dS_o \quad (5.3.1.1-8)
\]

and

\[0 = \frac{j}{4} \sum_{i=1}^{N} H(\vec{r}_i) \int_{S_i} \frac{\partial H_0^{(2)}(k_2 | \vec{r} - \vec{r}_o |)}{\partial n_o} \, dS_o
\]

\[+ \frac{j}{4} \sum_{i=1}^{N} \frac{\varepsilon r_2}{\varepsilon r_1} \frac{\partial H(\vec{r}_i)}{\partial n} \int_{S_i} H_0^{(2)}(k_2 | \vec{r} - \vec{r}_o |) \, dS_o \quad (5.3.1.1-9)
\]

The TM equations are the same as the TE equations (5.3.1.1-1) and (5.3.1.1-2) except, the magnetic field becomes the unknown quantity and the second integral term of the equation (5.3.1.1-9) is multiplied by a factor \[\frac{\varepsilon r_2}{\varepsilon r_1}\].

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Simplifying the equations for a flat surface with a normally incident plane wave (as was done for the TE case above) yields the following expression for the tangential magnetic field,

\[
H = 2H_i \left[ 1 - \frac{\frac{1}{4} \sum_{m=1}^{N} \int_{S_i} H_0^{(2)}(k_1 \lvert \vec{r} - \vec{r}_o \rvert) \, dS_o}{\frac{\varepsilon_{r2}}{\varepsilon_{r1}} \frac{1}{4} \sum_{m=1}^{N} \int_{S_i} H_0^{(2)}(k_2 \lvert \vec{r} - \vec{r}_o \rvert) \, dS_o + \frac{1}{4} \sum_{m=1}^{N} \int_{S_i} H_0^{(2)}(k_1 \lvert \vec{r} - \vec{r}_o \rvert) \, dS_o} \right].
\]  

(5.3.1.1-10)

In this equation, the behavior of the error in the current is more complex than in equation (5.3.1.1-7). Although the error in the integral terms \( \frac{1}{4} \sum_{m=1}^{N} \int_{S_i} H_0^{(2)}(k_2 \lvert \vec{r} - \vec{r}_o \rvert) \) increases rapidly as the relative dielectric constant \( \varepsilon_{r2} \) approaches the resonance values, that portion of equation (5.3.1.1-10) which contains this term is already becoming increasing unimportant because of the influence of factor \( \frac{\varepsilon_{r2}}{\varepsilon_{r1}} \). As a result, the error resonances shift towards lower values of the dielectric constant \( \varepsilon_{r1} \).

In summary, section 5.3.1.1 showed that there is a basic tradeoff between the maximum value permitted the real part of the relative dielectric constant and the number of integration points required in evaluating the off-diagonal entries in the MOM matrices. In principle, the size of the MOM discretizations can be used to reduce the errors in lieu of increasing the number of integration points. However, this is not a very practical solution because it greatly increases the size of the MOM matrices. When the number of integration points is too small, errors in the matrix terms can add in phase (for a flat surface) and may result in a large error in the surface current. Also, these discretization errors where shown to affect the solution of TE and TM problem is a slightly different manner. In either case, the number of evaluations used over each integration segment should be chosen sufficiently large to keep the discretization errors small.

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Finally, when the scattering from a perfect electric conductor is desired, the
difficulty with the large dielectric constants can be avoided because the electric field goes
to zero on the surface. This leaves only the tangential magnetic field as an unknown, and
so the MOM matrix formulation can be modified such that the matrices \( A_{21} \) and \( A_{22} \)
(whose off diagonal matrix elements are proportional to \( \varepsilon_\rho \)) do not have to be calculated.
Both the PEC and dielectric formulations are given in section 3.6.

5.3.1.2 The Diagonal Matrix Entries

In the previous section, the effect of the number of Gauss-Legendre integration
points used to calculate the off-diagonal entries in the MOM matrices on the MOM
current solution was examined. In this section, the integration of the diagonal entries of
the MOM matrix \( A_{12} \) and \( A_{22} \) are examined. These diagonal terms which involve
integrating the zeroth order Hankel function over its square root singularity can be
obtained numerically or by using an analytical approximation based on a small argument
expansion of the Hankel function (see section 3.4 for further details). However, the
analytic approximation becomes invalid as the dielectric constant of the scattering
material becomes large. It should also be mentioned that the remaining MOM matrices
\( A_{11} \) and \( A_{21} \) contain diagonal elements which involve integrating the normal derivative of
the zeroth order Hankel function over its square root singularity. These integrations yield
a principal value of 1/2 which is exact in the flat surface limit. Before examining the error
in the MOM surface currents, it should also be noted that because the integrals are
symmetric, the numerical integrations are performed over only half the segment length.

Figure 5.3.1.2-1 shows the relative error in the electric surface current (evaluated
at \( x = 0 \)) as a function of the relative dielectric constant of the scattering material using
both the small argument approximation and numerical integration. In this comparison,
the incident field is TE polarized and the dielectric constant \( \varepsilon_\rho \) has been chosen to be purely
real. Figure 5.3.2-1 shows that, as anticipated, the small argument approximation begins
Figure 5.3.1.2-1 Relative Error in Electric Surface Current Evaluated at $x = 0$ Due to the Small Argument Approximation of the Self Term versus Real Dielectric Constant for a Flat Surface, $80 \lambda_0$ Long with $\Delta x = 0.1 \lambda_0$, $\theta_i = 0^\circ$, TE Polarization, and $g = 5 \lambda_0$.
Figure 5.3.1.2-2 Relative Error in Electric Surface Current Evaluated at $x = 0$ Due to the Small Argument Approximation of the Self Term versus Loss in the Dielectric Constant for a Flat Surface, 80 $\lambda_o$ Long with $\Delta x = 0.1 \lambda_o$, $\theta_i = 0^\circ$, TE, $g = 5 \lambda_o$, $\text{Re}(\varepsilon_{12}) = 1$
to fail as the relative dielectric constant of the scattering material is increased. The figure also shows that using as few as 5 integration points results in current errors of less than 1% percent. Further, there is only a small improvement between using 10 integration points (with errors of about 0.25%) and 20 points (with errors of about 0.12%). Good results are also obtained using numerical integration when the scattering material is lossy, as is shown in figure 5.3.1.2-2.

The reader may recall from section 3.4 that the analytical approximation for the diagonal terms of the $A_{12}$ and $A_{22}$ matrices was obtained by (1) expanding the Hankel function in a small argument expansion, (2) evaluating the integration, and (3) recasting the integration result back into a Hankel function (again based on the small argument expansion). This third step is not strictly required, but is valid when the small argument approximation can be applied. In both of the proceeding figures, the relative error in the current initially increases as the relative dielectric constant of the scattering surface increases, but the error eventually begins to decrease. This decrease occurs because of the recasting of the analytical expression in step (3) forces the analytical approximation to have the correct result in the limit as the dielectric constant approach infinity. If the result from step (2) is used directly then the error in surface current will continue to increase as the dielectric constant is increased.

5.3.2 Numerical Precision Error

Because of the large memory required to store the MOM matrices, it is desirable to use single precision arithmetic. However, since large matrices must be inverted this may result in precision related errors. To investigate this possibility, the same calculations can performed using both single and double precision arithmetic. Figure 5.3.2-1, for example, plots the relative error in the electric surface current (evaluated at $x = 0$) versus the relative dielectric constant of the lower region, using single and double precision storage. As is readily apparent, there is no discernible difference in the currents.
Figure 5.3.2-1 Effect of Computer Precision on Relative Error in Electric Surface Current Evaluated at $x = 0$ versus Real Relative Dielectric Constant for a Flat Surface, $80 \lambda_o$ long with $\Delta x = 0.1 \lambda_o$, $\theta_i = 0^\circ$, TM Polarization, and $g \approx 5 \lambda_o$.
5.3.3 Surface Truncation Error

The computer can store and process only a relatively small number of unknowns, both because of the large memory requirements (proportional to $4N^2$, where $N$ in the number of surface segments) and because of the large number of floating point operations required to solve the matrix equation (proportional to $8N^3$ using LU decomposition). As a result, it is desirable to approximate the scattering from the (infinitely) long surface by as short a (truncated) surface as possible. However, the truncation of the surface can results in surface current errors near the ends of the surface, where neighboring segments which may strongly effect the current have not been included.

To avoid these end effect errors, the incident field is tapered to reduce the current near the ends of the surface. Figure 5.3.3-1, for example, plots the magnitude of the electric surface current for a flat PEC 80 $\lambda_o$ long. The figure shows, that the solution obtained using plane wave illumination contains large spikes in the current at the ends of the surface. A tapered incident field, defined by equation (5.3.3-1), with a tapering parameter of $g = 40 \lambda_o$ produces similar, but smaller current spikes, while a spot size of $10 \lambda_o$ produces no noticeable spikes.

Because the far field transformation (essentially a Fourier transform) of a delta function is a constant, the end current spikes can be expected to contribute to the scattered field in all scattering directions. Also, it should be noted that the Fourier transform of a constant current over a finite length (of the surface) is the well known sinc function. As a result, plane wave illumination of the truncated surface results in a radar cross section which is a sinc function plus a wide angle error component due to the erroneous end currents. This is not to say that the sinc function is not the correct solution for the scattering from the truncated surface. The essential point is that we are interested in the effects of the surface roughness on the scattered field for a surface which

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Figure 5.3.3-1 Example of End Effect Currents in MOM Solution Using 10/-1 Evaluations for a Flat PEC Surface, 80 \( \lambda_o \) Long with \( \Delta x = 0.1 \lambda_o \), \( \theta_i = 0^\circ \), TE Polarization, and \( g = 5 \lambda \).
is not truncated. The truncation is a limitation of the numerical solution technique and not a part of the problem of interest so we desire to eliminate the effects of the truncation. For a flat surface, elimination of the truncation effects is easily achieved by tapering the incident beam and this has the added bonus or removing the erroneous current spikes at the truncation points. However, tapering the incident field is not effective for some rough surfaces. The impact of surface truncation for rough surfaces will be discussed in chapter 6.

One possible incident field tapering, derived by Eric Thorsos [1989], is

\[ E_i(r) = \exp \left[ -j(1 + w(r))\hat{k} \cdot r - (x - z \tan \theta)^2 / g^2 \right] \]  
(5.3.3-1)

where \( w(x, z) = \frac{2(x - z \tan \theta)^2 / g^2 - 1}{(kg \cos \theta)^2} \).

The parameter "\( g \)" is called the (half) spot size and determines the length on the surface over which the magnitude of incident field drops by 1/e from its maximum value. The development of this equation starts with an angular spectrum of plane waves representation for the incident field,

\[ E_i(r) = \frac{2}{\sqrt{\pi} (kg \cos \theta_i)} \int_{-\pi/2}^{\pi/2} \exp \left( \frac{-(\theta' - \theta_i)^2}{\frac{2}{kg \cos \theta_i}} \right) \exp \left( -jk_i \cdot r \right) d\theta'. \]  
(5.3.3-2)

Equation (5.3.3-1) is then obtained from equation (5.3.3-2) by: (a) performing a change of variable, \( \delta = \theta' - \theta_i \), (b) extending the limits of integration to \( \pm \infty \) and (c) expanding the term \( \hat{k}_i \cdot r = k(\cos(\theta' + \delta)x + \sin x(\theta' + \delta)z) \) in \( \delta \), and retaining terms up to order \( \delta^2 \).

Thorsos states that the resulting equation, (5.3.3-1), is accurate to order \( \frac{1}{(kg \cos \theta)^2} \).

However in extending the limits beyond \( \pi/2 \), Thorsos has allowed for contributions to the
incident field representing plane wave with incident angle of greater than 90 degrees. Therefore, an additional constraint must be placed on $g$ to prevent any "significant" incident field contribution from these unrealistic angles. One possible form for this condition is given as follows,

$$g > \frac{A \sqrt{2}}{k(\frac{\pi}{2} - |\theta_i|)\cos \theta_i}$$

(5.3.3-3)

where "A" is a constant which specifies precisely how much of the incident field spectrum must be within the integral (equation 5.3.3-2) before the limits are extended to infinity. "A" is essentially twice the square root of the number of 1/e reductions in the magnitude of spectrum. For example if "A" equals 4, then $\exp \left\{ \frac{-(\theta' - \theta_i)^2}{2kg \cos \theta_i} \right\} \theta_{\frac{\pi}{2}} = \exp(-8)$. In this study, a value for "A" of 3 to 4 was observed to be an effective threshold. At large incidence angles, this additional constraint dominates the condition given by Thorsos, $kg \cos \theta >> 1$. Figure 5.3.3-2 compares the two conditions as a function of incidence angle, where "A" has been selected at 3.

In summary, because the scattering surface must be truncated for numerical purposes, a tapered incident field should be used to minimize the effects of the surface truncation on the scattered field solution. The price to be paid for tapering the incident field is that we are no longer solving the problem of a plane wave scattered from a rough surface but, in effect, the scattering of a narrow angular spectrum of plane waves. One advantage of the latter is its closeness to a real antenna pattern.
Figure 5.3.3-2 Plot of Minimum Required Spot Size (g) vs. Incident Field Angle
5.4 The No Contrast Solution as an Error Estimate in the TE case

In this section, the impact of errors in the MOM matrix entries are examined from the perspective of a matrix inversion problem. It is shown that the discretization error in the numerical solution of the normal derivative of the electric field for a flat perfect electric conducting (PEC) surface is exactly twice the error in the case of a fictitious boundary, where the dielectric constant on both sides of the interface is the same. Although, this relationship is not exact for a rough interface, it is expected to be approximately true for surfaces with small heights and small slopes. Consequently, we hypothesize that the no contrast solution can be used as an estimate of the error in the PEC solution.

In Chapter 3, the method of moments approach was used to convert the coupled scalar extinction equations into an approximately equivalent linear system of the form,

\[
\begin{bmatrix}
E_i \\
0
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix} \begin{bmatrix}
E \\
\frac{\partial E}{\partial n}
\end{bmatrix}
\]

(5.4-1)

where \(A_{11}, A_{12}, A_{21}, \text{ and } A_{22}\) are each \(N \times N\) matrices defined in section 3.6,

\(E_i\) is a column matrix of length \(N\), which contains the incident field on evaluated on the \(N\) discretized segments of the surface,

\(E\) is a column matrix of length \(N\) for the total electric field on the \(N\) discretized segments,

\(\frac{\partial E}{\partial n}\) is a column matrix of length \(N\) for the unknown normal derivative of total electric field on the \(N\) surface segments, and

\(N\) is the number of segments into which the surface has been divided.

The solution for the surface fields is given by,
\[
\begin{bmatrix}
  \frac{E}{\partial n} \\
  \frac{\partial E}{\partial n}
\end{bmatrix}
= \begin{bmatrix}
  A_{11} & A_{12} \\
  A_{21} & A_{22}
\end{bmatrix}^{-1}
\begin{bmatrix}
  E_i \\
  0
\end{bmatrix}.
\]  

(5.4-2)

Equation (5.4-2) can also be written as [Beyer 1991],

\[
E = (A_{11} - A_{12} A_{22}^{-1} A_{21})^{-1} E_i ,
\]  

(5.4-3a)

and

\[
\frac{\partial E}{\partial n} = (A_{12} - A_{11} A_{21}^{-1} A_{22})^{-1} E_i .
\]  

(5.4-3b)

When the surface is flat, the matrices \( A_{11} \) and \( A_{21} \) become diagonal,

\[
A_{11} = -A_{21} = \frac{1}{2} I
\]  

(5.4-4)

where \( I \) is the identity matrix. Substituting equation (5.4-4) into equations (5.4-3a) and (5.4-3b) yields,

\[
E_{\text{flat}} = (\frac{1}{2} I + \frac{1}{2} A_{12} A_{22}^{-1})^{-1} E_i
\]  

(5.4-5a)

and

\[
\frac{\partial E}{\partial n_{\text{flat}}} = (A_{12} + A_{22})^{-1} E_i.
\]  

(5.4-5b)

where the subscript "flat" indicates the flat surface limit has been applied.

In the case when the material on both sides of the interface is the same, i.e. \( k = k_1 = k_2 \) or \( \varepsilon = \varepsilon_1 = \varepsilon_2 \), examining the MOM matrices shows that,

\[
A_{12} = A_{22}
\]  

(5.4-6a)

and

\[
A_{11} = A_{21} - I.
\]  

(5.4-6b)
It should be noted that since the same numerical procedure will be used, all of the entries in $A_{12}$ will be exactly the same as $A_{22}$ (including any errors in the calculations of these entries). The same holds true of the matrices $A_{11}$ and $A_{21}$ except along the diagonal.

Substituting equations (5.4-6a) and (5.4-6b) into (5.5-3a) and (5.4-3b) yields,

$$E_{nc} = E_i \quad (5.4-7a)$$

and

$$\frac{\partial E}{\partial n_{nc}} = -(A_{21}^{-1}A_{22})^{-1}E_i = -A_{21}(A_{22}^{-1})E_i = -A_{21}(A_{12}^{-1})E_i \quad (5.4-7b)$$

where the subscript "nc" indicates the resultant matrix equation in the no contrast limit.

Equation (5.4-7a) indicates that the numerical solution of the electric field should be obtained with almost no error in the flat surface limit. Effectively, the errors in the calculations of the matrix entries of $A_{12}$ and $A_{22}$ are the same and will cancel out except for small precision related error. After obtaining this result, the errors in the MOM solution for the electric surface field were examined, and were found to be very small ($< 10^{-4}$). (Note: the magnitude of the incident field can be specified arbitrarily and was chosen to be 1 in the computer code). In the limit where the materials are the same on both sides of the interface (i.e. the no contrast limit) and the surface is flat then the normal derivative of the electric field can be further reduced to, (i.e. combining 5.4-5b and 5.4-6a)

$$\frac{\partial E}{\partial n_{flat,nc}} = \frac{1}{2} A_{12}^{-1}E_i \quad (5.4-8)$$

where the subscript "flat nc" indicates the flat surface and no contrast limits.
In the case of a PEC, the total tangential electric field on the surfaces zero,

\[ E = 0. \quad (5.4-9) \]

Substituting (5.4-9) into the matrix expression (5.4-1) yields the following equation for the normal derivative on the surface,

\[ \frac{\partial E}{\partial n_{PEC}} = A_{12}^{-1} E_i. \quad (5.4-10) \]

where the subscript "PEC" indicates the scattering surface has been restricted to a PEC.

Comparing equations (5.4-8) and (5.4-10) shows that the matrix equation for the electric surface current of the PEC case is exactly twice the equation for the no contrast case in the flat surface limit. The entries of the matrix \( A_{12} \) only depend on the dielectric constant of the upper region (\( \varepsilon_1 = 1.0 \)) and not on the scattering material. Therefore, with the exception of numerical precision relate errors, the errors in the solution for the electric current on a flat PEC will be exactly twice the error found in the flat no contrast solution. Further, since the scattered far field is a linear transformation of the surface currents, the error in the scattered field for a flat PEC will be twice the solution for the scattered field calculated in no contrast limit (remember any scattered field in the no contrast case is an error). This result is demonstrated in figure 5.4-1.

When the surface is not flat, the simple relationship between the no contrast solution, equation (5.4-7b), and the PEC, equation (5.4-10) is lost. However, examination of the off-diagonal terms in the matrix \( A_{21} \) (defined in section 3.6) shows that these terms are small when the surface has small slopes and small heights. Therefore, the no contrast solution can be expected to provide a good measure of the PEC error with such small roughness. (Essentially, the diagonal of matrix \( A_{21} \) represents the Kirchhoff (or single scattering) current. In the limit of a flat surface, the Kirchhoff result is the
Figure 5.4-1 Magnitude of Electric Field Error for a PEC and the No Contrast Solution for Flat Surfaces, 80 $\lambda_0$ Long with $\Delta x=0.1 \lambda_0$, $\theta_i=0^\circ$, TE Polarization, and $g=5 \lambda_0$
exact result. As long the surface has small heights and small slopes there is very little multiple scattering, the Kirchhoff current will be the dominant current, and the matrix $A_{12}$ will be diagonally dominant. Also, examining equation (5.4-5) shows that there no simple relationship between the current errors in the no contrast case and any other dielectric configuration, and finally, it should be emphasized that the above relationship takes advantage of the symmetry in the MOM matrices for the TE case. In the TM case, the presence of the addition term $e_2$ in $A_{22}$ disrupts this symmetry.

5.5 Conclusions

In this chapter, the errors produced by numerical solution of the extinction equations from a perfectly flat interface were examined. It was found that the solution suffered from a number of errors which can broadly be classified as discretization errors and finite surface length errors. Of these, the discretization errors were the most significant.

In regards to discretization errors, section 5.3.1.1 found that there is a basic tradeoff between the maximum value permitted for the real part of the dielectric constant and the number of Gauss-Legendre integration points used in evaluating the off-diagonal entries in the MOM matrices. In principle, the size of the MOM discretizations can be reduced in lieu of increasing the number of integration points. However, this is not a very practical solution because it would greatly increase both the computer memory and time requirements. For a flat surface, when the number of integration points is too small, errors in the matrix terms can add in phase resulting in large errors in the surface currents. In section 5.3.1.2 it was shown that, the diagonal terms of the MOM matrix $A_{21}$ (defined in Section 3.6) should also evaluated using numerical integration, because the small argument approximation fails as the relative dielectric constant of the scattering material is increased. It was found that as few as five Gauss-Legendre integration points can be used to evaluate these self-terms.
When the scattering from a perfect electric conductor is desired, the difficulty with the large dielectric constants can be avoided because the electric field goes to zero on a PEC. This leaves only the magnetic field as an unknown, so the MOM matrix equation can modified such that the matrices $A_{21}$ and $A_{22}$ need not be calculated.

In regards to finite length of the calculation surface, section 5.3.3 found that because the scattering surface must be truncated for a numerical solution, a tapered incident field should be used to minimize the effect of the surface truncation on the scattered field. If the incident field is not tapered sufficiently, spikes are generated in the current solution at the ends of the truncated surface.

In section 5.4 it was shown that surface current errors in the solution for the normal derivative of the electric field for a flat PEC is exactly twice the error found for a flat no contrast surface (for TE polarization). Although, this relationship is not exactly correct for a rough interface, it is expected to be approximately true for surfaces with small heights and small slopes.

Finally, it should be noted that because the scattering surface examined in this chapter is flat, the matrix terms which contain integrations of the normal derivative of the Green's function contribute only a principle value to the scattering problem. That is, the off-diagonal terms in the $A_{11}$ and $A_{21}$ matrices are zero in the flat surface limit. Therefore, this analysis does not evaluate errors in the calculation of these off-diagonal terms. However, the derivative of the Green's function, away from its singularity, is proportional to a first order Hankel function (see section 3.4) and so we anticipate that the same integration conditions developed for the zeroth order Hankel function apply for the first order Hankel function as well.
6.0 Randomly Rough Surfaces

6.1 Introduction

This chapter examines the average normalized radar cross section (NRCS) obtained by Monte Carlo solution of the extinction equations for randomly rough surfaces with Gaussian distributed heights and both Gaussian and Pierson-Moskowitz spectra. In this approach a sample realization of a rough surface with specific statistical roughness parameters is randomly generated. Then using a Method of Moments (MOM) solution of the integral extinction equations (see chapters 3 and 4), the NRCS for the randomly generated surface is calculated. This process is repeated many times to obtain a numerical average for NRCS.

In the second section of this chapter the idea behind treating the rough surface as a random process and a technique to generate a random realization of such a surface is introduced. In the third section, the average NRCS obtained by a Method of Moments (MOM) solution of the extinction equations, referred to in this chapter as the "extinction code", is compared with the results of other investigators. The extinction code results are shown to be in good agreement with other scattering codes (for surfaces with Gaussian and Pierson-Moskowitz spectra) and with measurements (for surfaces with Gaussian spectra).

Finally, in the fourth section, the impact of surface truncation, very small scale surface features, and the limitations of the Monte Carlo average are examined. It is shown that the length of the surface can significantly affect the NRCS calculations for surfaces with Gaussian spectra at large backscattering angles. For Pierson-Moskowitz spectra, on the other hand, it is found that the surface length can be chosen much smaller than the surface correlation length for backscattering angles up to about 60° with little
resulting error. This result is important because the correlation length of these surfaces may be in the range of hundreds of meters. It is also shown that the average radar cross section is not sensitive to the very small scale portion of the Pierson-Moskowitz spectrum (that is those feature of the rough surface which vary on a spatial scale which is less than half the incident electromagnetic wavelength). Lastly, in regards to the Monte Carlo approach, it is shown that this technique is difficult to apply at large backscattering angles for surfaces with Gaussian spectra when the surface scattering is dominated by specular events. This is because infrequently occurring slopes must be included to obtain a good average of the radar cross section.

6.2 Describing a Random Surface

It is not possible to exactly predict the profile of many surfaces such as wind driven seas or landscapes nor for most radar applications is it desirable to do so. Such rough surfaces can be described in terms of their deviation from a smooth reference surface using statistical techniques [Ogilvy 1991]. Essentially, the roughness is characterized by a spread of heights about a reference surface and the variation of these heights along the surface and in time. For this study, the reference surface is taken to be a flat plane and time is fixed.

Because of its mathematical simplicity, the two point (or joint) height probability density function (pdf) of the surface is assumed to be Gaussian, i.e.,

\[
pdf_{\zeta_1,\zeta_2}(\zeta_1,\zeta_2) = \frac{1}{2\pi\sigma_h^2\sqrt{1-c^2}} \exp \left( \frac{1}{2\sigma_h^2} \left[ \frac{-\zeta_1^2}{2\sigma_h^2} + \frac{-\zeta_2^2}{2\sigma_h^2} \right] \right) \quad (6.2-1)
\]

where \(\zeta_{1,2}\) is the height at point 1 or 2, respectively, on the surface, and is a random variable with zero mean,
\[ c \] is the correlation coefficient, defined as \( c = \frac{\langle \zeta_1 \zeta_2 \rangle}{\sigma_h^2} \),

\[ \sigma_h^2 = \langle \zeta^2 \rangle \] is the variance of the height (which will taken to be the same at every point on the surface), and

\[ \langle > \] denotes averaging.

Equation (6.2-1) specifies the probability that at point 1, the surface will have a height between \( \zeta_1 \) and \( \zeta_1 + d\zeta \) and at point 2 will have a height between \( \zeta_2 \) and \( \zeta_2 + d\zeta \). Also, the single point or marginal height probability density function (obtained by integrating equation (6.2-1) over all possible values of \( \zeta_2 \)) is also Gaussian, i.e.,

\[
\text{pdf}_\zeta (\zeta_1) = \frac{1}{\sigma_h \sqrt{2\pi}} \exp \left(-\frac{\zeta_1^2}{2\sigma_h^2}\right),
\]

where \( \sigma_h \) is called the root mean squared (rms) height of the surface.

Essentially, equation (6.2-2) gives the probability that a point on the surface will have a height between \( \zeta_1 \) and \( \zeta_1 + d\zeta_1 \). It should be noted that because any linear transformation of a Gaussian pdf is also Gaussian [Shanmugan and Breipohl 1988], the slopes and all higher order derivatives of a surface with a Gaussian pdf will also have a Gaussian distribution. For example,

\[
\text{pdf}_{\zeta_s} (\zeta_s) = \frac{1}{\sigma_s \sqrt{2\pi}} \exp \left(-\frac{\zeta_s^2}{2\sigma_s^2}\right)
\]

is the single point slope pdf, where \( \zeta_s \) is the slope of the surface and \( \sigma_s \) is the root mean squared (rms) slope.
6.2.1 The Correlation Function and The Power Spectral Density

Specifying the marginal height distribution does not fully characterize the surface, because the statistical variation of the heights along the surface must also be specified. The statistical variation of the heights for a second order process such as the Gaussian can be specified by the correlation coefficient or by the correlation function (sometimes called the autocorrelation function),

\[ C(x_1, x_2) \equiv \sigma_h^2 c = \langle \zeta(x_1) \zeta(x_2) \rangle. \]  \hspace{1cm} (6.2.1-1)

where \( \zeta = \zeta(x) \) is the height of the surface, written as an explicit function of \( x \)

\[ c \] is the correlation coefficient, and

\[ C \] is correlation function.

Many surfaces can be treated as statistically homogeneous, that is the statistics are independent of location on the surface. In such a situation, the marginal height pdf is the same everywhere on the surface and the correlation function can be written as \( C(x_1, x_2) = C(|x_1 - x_2|) = C(\Delta x) \). Similar to the height pdf, the correlation function is also frequently assumed to be Gaussian in form,

\[ C(\Delta x) = \sigma_h^2 \exp \left( -\frac{(\Delta x)^2}{l^2} \right) \]  \hspace{1cm} (6.2.1-2)

where \( l = \sqrt{\frac{\sigma_h}{\sigma_s}} \) is the correlation length.

The Fourier transform of the correlation function is the power spectral density (psd) or, as it is sometimes called, the spectral density or simply the spectrum, i.e.,

\[ S(K) = \int_{-\infty}^{\infty} C(\Delta x) e^{-jK\Delta x} d\Delta x \]  \hspace{1cm} (6.2.1-3)

where \( K \) is the spatial frequency.
Some authors use a spectrum which differs from the above definition by a factor of $1/2\pi$ - for which the symbol $W(K)$ will be used. That is, $W(K)$ is the spectrum $S(K)$ normalized such that

$$\int_{-\infty}^{\infty} W(K) dK = \sigma_h^2,$$  \hspace{1cm} (6.2.1-4)

For the Gaussian correlation function the spectrum is given by,

$$W(K) = \frac{i\sigma_h^2}{2\sqrt{\pi}} \exp\left[\frac{-K^2l^2}{4}\right].$$ \hspace{1cm} (6.2.1-5)

Many real surfaces have correlation functions which depart rather significantly from a Gaussian model. In this study, surfaces with Pierson-Moskowitz spectra will also be examined. This spectrum of the ocean surface, proposed by Willard J. Pierson Jr. and Lionel Moskowitz in 1964, is based on a combination of buoy measurements and hydrodynamic models. The use of buoy measurements means that the shape of the proposed spectrum was not measured in the centimeter range so its behavior is essentially imposed by hydrodynamic models. According to Moskowitz, this spectrum is only valid for "fully developed seas in the absence of swell and tidal forces." That is, the model is valid only when: (1) the wind has been blowing at a constant rate in one direction for a substantial distance (estimated at anywhere from 100 to 710 miles [Moskowitz 1964]), (2) the dominant forcing function is the local wind, i.e. there is no wave energy from nearby regions (introducing swell) or strong ocean current-wave interactions, and (3) in deep waters where coastal and tidal effects are not significant. In addition to the above limitations, the measurements were made in two dimensions and a one dimensional modification of this spectrum, developed by Thorsos [1990], will be used here.
However, since the goal of this study is to examine the accuracy of the specular point and two scale scattering theories in extracting surface parameters such as the rms slope; the Pierson-Moskowitz spectrum does represent a reasonable two scale surface. The one-dimensional Pierson-Moskowitz spectrum is given by Thorsos [1990],

\[ W(K) = \frac{\alpha}{4|K|^3} \exp \left( -\frac{\beta g^2}{K^2 U^4} \right) \]  

(6.2.1-6)

where \( K \) is the spatial frequency [rad/m],
\( \alpha \) is a constant, 0.0081,
\( g \) is the gravitational acceleration constant, 9.81 [m/s²],
\( U \) is the wind velocity 19.5 meters above sea level, and
\( \beta \) is a constant 0.74.

6.2.2 Generating a Random Surface

One technique that can be used to generate realizations of spatially homogeneous random processes is a spectral technique discussed by Ogilvy [1991], Thorsos [1988], and Shiozuka [1970]. Using this technique, a set of \( N \) points with spacing, \( \Delta x \), over a length \( L = N \Delta x \) is generated as follows:

\[ h(x_n) = \frac{1}{L} \sum_{i=-N/2}^{N-1} F_i(K_i) \exp \left( jK_i x_n \right) \]  

(6.2.2-1)

where \( x_n = n \Delta x \) \( n = 1, 2, \ldots N \),
\( L = N \Delta x \),
\( K_i = \frac{2\pi i}{L} \).
\[
F(K) = \sqrt{2\pi L W(K)} \left\{ \begin{array}{ll}
N_i(0,1) + jM_i(0,1) & i \neq 0 \text{ and } i \neq \frac{N}{2} \\
\sqrt{2} & i = 0 \text{ or } i = \frac{N}{2}
\end{array} \right\} \quad \text{with } i \geq 0,
\]

\[
F(K_i) = F(K_{-i})^* \quad \text{with } i \leq 0.
\]

and the arrays, \(N_i(0,1)\) and \(M_i(0,1)\), are filled with independently generated random numbers which are Gaussian distributed with zero mean and unit variance and the * denotes the complex conjugate.

Note that two independent random numbers are generated for each point in the sampled spectrum, \(i = 1, 2, \ldots \frac{N}{2} - 1\) and one number is generated for the points \(i = 0\) and \(i = \frac{N}{2}\). Thus a total of \(N\) independent random numbers must be generated to obtain a height profile with the desired spectrum.

That the \(h(x_s)\) sequence will have a Gaussian pdf follows from the well known property of Gaussian random variables, that a sum of independent Gaussian distributed random variables will itself be Gaussian distributed [Shanmugan and Breipohl 1988]. To see that this sequence has the desired correlation function, one can apply the definition for the correlation function, i.e.

\[
C(x_1, x_2) = \langle h(x_1)h(x_2) \rangle
\]

\[
= \left\langle \left[ \frac{1}{L} \sum_{i = \frac{N}{2}}^{N-1} F(K_{i1}) \exp(jK_{i1}x_1) \right] \left[ \frac{1}{L} \sum_{i = -\frac{N}{2}}^{-1} F(K_{i2}) \exp(jK_{i2}x_2) \right] \right\rangle
\]

\[
= \left\langle \frac{1}{L^2} \sum_{i = \frac{N}{2}}^{N-1} \sum_{i = -\frac{N}{2}}^{-1} F(K_{i1})F(K_{i2}) \exp(jK_{i2}x_2) \exp(jK_{i1}x_1) \right\rangle
\]

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\[ i_1 = N - 1, i_2 = N - 1 \]
\[ \frac{1}{L^2} \sum_{i_1 = -N/2}^{N/2} \sum_{i_2 = -N/2}^{N/2} \langle F(K_{i_1})F(K_{i_2}) \rangle \exp(jK_{i_2}x_2) \exp(jK_{i_1}x_1) \]  

(6.2.2-2)

where \( \langle F(K_{i_1})F(K_{i_2}) \rangle = 2\pi L \sqrt{W(K_{i_1})} \sqrt{W(K_{i_2})} \langle A(i_1)B(i_2) \rangle \),

\[
A(i_1) = \begin{cases} 
\left( \frac{N_1(0,1) + jM_1(0,1)}{\sqrt{2}} \right) & i_1 > 0 \\
\left( \frac{N_1(0,1) - jM_1(0,1)}{\sqrt{2}} \right) & i_1 < 0 \text{ and } i_1 \neq -\frac{N}{2} \\
N_1(0,1) & i_1 = 0 \text{ or } i_1 = \frac{N}{2}
\end{cases}
\]

\[
B(i_2) = \begin{cases} 
\left( \frac{N_2(0,1) + jM_2(0,1)}{\sqrt{2}} \right) & i_2 > 0 \\
\left( \frac{N_2(0,1) - jM_2(0,1)}{\sqrt{2}} \right) & i_2 < 0 \text{ and } i_2 \neq -\frac{N}{2} \\
N_2(0,1) & i_2 = 0 \text{ or } i_2 = \frac{N}{2}
\end{cases}
\]

\( \langle \rangle \) denotes averaging.

Notice that except when \( i_1 = -i_2 \) or \( i_1 = i_2 = \frac{N}{2} \) that the average \( \langle A(i_1)B(i_2) \rangle \) is zero. For example when \( i_1 \neq i_2, i_1 > 0 \) and \( i_2 > 0 \) then,

\[
\left\langle \left( \frac{N_1(0,1) + jM_1(0,1)}{\sqrt{2}} \right) \left( \frac{N_2(0,1) + jM_2(0,1)}{\sqrt{2}} \right) \right\rangle
\]

\[ = \left\langle \frac{N_1(0,1)jM_2(0,1)}{2} \right\rangle + \left\langle \frac{N_2(0,1)jM_1(0,1)}{2} \right\rangle + \left\langle \frac{N_1(0,1)N_2(0,1)}{2} \right\rangle + \left\langle \frac{jM_1(0,1)jM_2(0,1)}{2} \right\rangle
\]

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But since $N_t(0,1)$ and $M_t(0,1)$ are independent random numbers with zero mean,

$$\left\langle \left(\frac{N_{i_1}(0,1) + jM_{i_1}(0,1)}{\sqrt{2}}\right)\left(\frac{N_{i_2}(0,1) + jM_{i_2}(0,1)}{\sqrt{2}}\right) \right\rangle = \frac{<N_{i_1}(0,1)><jM_{i_2}(0,1)>}{2} + \frac{<N_{i_2}(0,1)><jM_{i_2}(0,1)>}{2} + \frac{<N_{i_1}(0,1)> <N_{i_2}(0,1)>}{2} + \frac{<jM_{i_1}(0,1)> <jM_{i_2}(0,1)>}{2} = 0.$$ 

However when, $i = i_1 = -i_2$, then,

$$\left\langle \left(\frac{N_i(0,1) + jM_i(0,1)}{\sqrt{2}}\right)\left(\frac{N_i(0,1) - jM_i(0,1)}{\sqrt{2}}\right) \right\rangle = \left\langle \frac{N_i(0,1)^2 + M_i(0,1)^2}{2} \right\rangle = 1.$$ 

Also when $i = i_1 = i_2 = 0$

$$\langle N_i(0,1)N_i(0,1) \rangle = \langle N_i(0,1)^2 \rangle = 1.$$ 

Therefore, equation (6.2.2-2) reduces to,

$$C(x_1, x_2) = \frac{2\pi}{L} \sum_{i=-\frac{N}{2}}^{\frac{N}{2}-1} W(K_i) \exp(jK_i(x_2 - x_1)). \quad (6.2.2-3)$$

The above equation is easily recognized as the discrete inverse Fourier transform of the spectrum. As an example, figure 6.2.2-1 plots the numerically estimated spectrum, height pdf and slope pdf for a small number of surface realizations with Gaussian spectra generated using this technique.
Estimated Spectrum, Height PDF, and Slope PDF

Gaussian Spectrum $\sigma_h = 0.7 \lambda_o$, $\sigma_s = 17.5^\circ$
25 Surfaces of length $40 \lambda_o$, $\Delta x = 0.1 \lambda_o$
(10,000 sample points)

Estimated Spectrum

Figure 6.2.2-1 Example of Estimated Spectrum, Height PDF, and Slope PDF
6.3 Comparison of Randomly Rough Surface Scattering Results

In this section, the validity of the average NRCS obtained from the extinction code is demonstrated through comparisons to both measurements and other scattering codes for surfaces with Gaussian and Pierson-Moskowitz spectra.

6.3.1 Statistical Variation in the Numerical Calculations

Before comparing the average NRCS results, it should be stressed that the average NRCS obtained by a Monte Carlo approach will have some fluctuations due to the finite number of samples used to construct the average. As an example, figure 6.3.1-1 plots two realizations of a Gaussian surfaces, that is a surface with both a Gaussian height pdf and a Gaussian correlation function. Figure 6.3.1-2 shows the resulting bistatic NRCS for one of the example surfaces. (The bistatic NRCS is the NRCS evaluated when the incidence angle is fixed and the observation or scattering angle is allowed to vary.) Notice that the NRCS shows considerable variations with respect to the scattering angle. To obtain a mean scattering profile, the cross sections from many randomly generated surfaces (also called realizations) are averaged. Figure 6.3.1-3 shows the resulting average bistatic NRCS for the same type of surface used in Figure 6.3.1-2, but with the cross section from 100, 400 and 800 realizations having been averaged. In this figure, the NRCS has been plotted in both a linear scale and in dB, a logarithmic scale (10 \log_{10}). In both scales, one can observe that as the number of realizations is increased, the fluctuations in the NRCS are reduced and in principle the closer the computed average will be to the true mean. In chapter 7, a derivation is provided which shows that the fluctuations in the NRCS can be expected to reduce as one over the square root of the number of surfaces used to construct the average. Also, as can be observed in the figure, the variations in the NRCS are largest where the NRCS is itself largest.
Figure 6.3.1-1 Examples of Gaussian Random Rough Surfaces
Figure 6.3.1-2 Example of Bistatic NRCS from One Surface Realization of a PEC Surface with a Gaussian Spectrum, $\sigma_h = 0.7 \lambda_o$, $\sigma_s = 17.55^\circ$, $80 \lambda_o$ Long with $\Delta x = 0.1 \lambda_o$ Using $10^{-}/1$ Evaluations, TE Polarization, $\theta_i = 0^\circ$, and $g = 5 \lambda_o$.
Figure 6.3.1-3 Example of Fluctuations in the Average NRCS for PEC Surfaces with Gaussian Spectra, \( \sigma_h = 0.71 \lambda_o \) and \( \sigma_s = 17.55^\circ \), \( 80 \lambda_o \) Long with \( \Delta x = 0.1 \lambda_o \) Using 10/-1 Evaluations, TE Polarization, \( \theta_i = 0^\circ \), and \( g = 5 \lambda_o \)
6.3.2 Comparison of Rough Surface Scattering Codes for Perfect Electric Conductors with Gaussian Spectra

Dr. David Kapp [1996] has written a similar methods of moments code for scattering from perfect electric conducting (PEC) surfaces following the work of Dr. Eric Thorsos at the University of Washington [1989]. This code has been verified through comparison to analytical models such as perturbation theory and Kirchhoff theory, as well as, smoothing and the method of ordered multiple interactions (MOMI) where those approaches are valid.

Figures 6.3.2-1 through 5 compare Kapp's PEC code to the extinction code for surfaces with Gaussian spectra. In the first four comparisons, the correlation length, \( L \), is fixed at 1.8 \( \lambda_o \) and the rms height is 0.2, 0.5, 1.0, and 1.5 \( \lambda_o \). In each case, the surface is 80 \( \lambda_o \) long with a discretization spacing of 0.1 \( \lambda_o \). A 10 point Gauss-Legendre integration is used to evaluate the Green's function self terms and 1 point is used for the off diagonal terms. The figures show that good agreement is found in every case, considering that different realizations of the surfaces are used by each code so there is statistical variability in the averages. These four comparisons were chosen because (1) much of the work presented here will involve surfaces which are sufficiently rough so as not to produce a coherent return and (2) Nieto-Vesperians [1991] studied these same cases. Nieto-Vesperians results (not shown here) agree very well with Kapp's and the extinction code results. Figure 6.3.2-5 compares Kapp's PEC code and the extinction code for a Gaussian surface with small heights and small slopes. The appearance of a specular component at 0° results from the small height of the surface roughness. This case was examined by Thorsos [1989] whose results also agree very well with those of Kapp and the extinction code.
Figure 6.3.2-1 Comparison of Kapp's PEC Code and the Extinction Code for 100 Realizations of PEC Surfaces with Gaussian Spectra, $\sigma_h = 0.2 \lambda_o$ and $L = 1.8 \lambda_o$, 80 $\lambda_o$ Long with $\Delta x = 0.1 \lambda_o$ Using 10/-/1 Evaluations, Both Polarizations, $\theta_i = 0^\circ$, and $g = 5 \lambda_o$
Figure 6.3.2-2 Comparison of Kapp's PEC Code and the Extinction Code for 100 Realizations of PEC Surfaces with Gaussian Spectra, \( \sigma_h = 0.5 \lambda_0 \) and \( L = 1.8 \lambda_0 \), 80 \( \lambda_0 \) Long with \( \Delta x = 0.1 \lambda_0 \). Using 10/1 Evaluations, Both Polarizations, \( \theta_i = 0^\circ \), and \( g = 5 \lambda_0 \).
Figure 6.3.2-3 Comparison of Kapp's PEC Code and the Extinction Code for 100 Realizations of PEC Surfaces with Gaussian Spectra, $\sigma_h = 1.0 \lambda_o$ and $L = 1.8 \lambda_o$, $80 \lambda_o$ Long with $\Delta x = 0.1 \lambda_o$ Using 10/-/1 Evaluations, Both Polarizations, $\theta_i = 0^\circ$, and $g = 5 \lambda_o$. 
Figure 6.3.2-4 Comparison of Kapp's PEC Code and the Extinction Code for 100 Realizations of PEC Surfaces with Gaussian Spectra, $\sigma_h = 1.5 \lambda_o$ and $L = 1.8 \lambda_o$, $80 \lambda_o$ Long with $\Delta x = 0.1 \lambda_o$ Using 10/-/1 Evaluations, Both Polarizations, $\theta_i = 0^\circ$, and $g = 5 \lambda_o$
Figure 6.3.2-5 Comparison of Kapp's PEC Code and the Extinction Code
for 100 PEC Surfaces with Gaussian Spectra, $\sigma_h = 0.053 \lambda_o$, $\sigma_s = 9.5^\circ$, and $L = 0.45 \lambda_o$
80 $\lambda_o$ Long with $\Delta x = 0.1 \lambda_o$ Using 10/-/1 Evaluations, Both Polarizations, $\theta_i = 0^\circ$, and $g = 15 \lambda_o$
6.3.3 Comparison of Rough Surface Scattering Codes for non Perfect Electric Conductors with Gaussian Spectra

In this section, the extinction code results are compared with those of A.J. Sant [1991] and Maradudin [1989] for scattering from randomly rough surfaces which are not perfectly conducting. Figures 6.3.3-1 and 6.3.3-2 compare Maradudin's calculations of average scattering cross section from a silver surface at optical frequencies to the extinction code results for the same roughness and dielectric parameters. There are slight differences between the two calculations resulting from slight difference in the sampling of the surface and evaluation of the MOM matrix entries, but these can be expected to have a small effect on the average NRCS calculations. (More specifically, Maradudin used 1000 realizations, the small argument approximation for the Hankel function self terms, 1 point evaluation of all off-diagonal matrix elements, and a surface length of 41.8 \( \lambda_o \) with a discretization length of \( \Delta x = 0.161 \lambda_o \) - which are not unreasonable choices for this surface. The values used by the extinction code are listed in the figure caption.) Maradudin plotted what he called the differential reflection coefficient, which is essentially the same as the normalized radar cross section except for a constant factor. Clearly the two scattering codes produce results which are in reasonable agreement.

A.J. Sant constructed grooved, i.e. one dimensional or corduroy, randomly rough surfaces with Gaussian spectra from silicone elastomer and epoxy resins. He then measured the scattering from these surfaces at optical frequencies and compared the measurements with his own numerical calculations. With the exception of the off-nadir TE case, Sant's measurements and computations are in fair agreement. (Sant's TE measurements and computations do not appear to be in close agreement. Sant however, describes the comparison as "good" in all of these cases. Construction, characterization and measurement of the diffusers was a complex task leaving many possible questions.)

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Figure 6.3.3-1 Comparison of Extinction Code and Maradudin's Scattering Code for 500 Surfaces with Gaussian Spectra: $\varepsilon_r = 17.2 - 0.498j$, $\sigma_h = 1.91 \lambda_o$ and $L = 3.183 \lambda_o$ $90 \lambda_o$ Long with $\Delta x = 0.1 \lambda_o$ Using 10/2/3 Evaluations, TM Polarization, $\theta_i = 0^\circ$ and $g = 10.446 \lambda_o$. 
Figure 6.3.3-2 Comparison of Extinction Code and Maradudin's Scattering Code
for 500 Surfaces with Gaussian Spectra: $\varepsilon_r = 17.2 - 0.498j$, $\sigma_h = 1.91 \lambda_o$ and $L = 3.183 \lambda_o$
90 $\lambda_o$ Long with $\Delta x = 0.1 \lambda_o$ Using 10/2/3 Evaluations, TM Polarization, $\theta_i = 20^\circ$ and $g = 10.446 \lambda_o$
Figures 6.3.3-3 through 6.3.3-8 compare Sant's results with extinction code computations using the same surface roughness and dielectric parameters. The only noteworthy differences between the Sant's problem setup and that of the extinction code is that Sant program used 1 point evaluation of all off-diagonal matrix elements, a surface length of 40 $\lambda_0$ and a plane wave incident field. Sant's Scattering Cross Section (SCS) is essentially the same as our NRCS except for a constant factor. The figures show that the extinction code results are in reasonable agreement with Sant's calculations, taking into account the expected statistical variation in the average NRCS, except at very large scattering angles. In this region the extinction code results more closely agree with Sant's measurements, this is especially noticeable in figure 6.3.3-8. Errors of this type are typical when using plane wave illumination, as was used by Sant in his computations.

6.3.4 Comparison of Rough Surface Scattering Codes for Pierson-Moskowitz Spectra

Monte Carlo calculations of scattering from two-scale surfaces, such as Pierson-Moskowitz surfaces, have not received as much attention in the literature as Gaussian surfaces (see also section 2.3). Figures 6.3.4-1 and 6.3.4-2 compare the extinction code results with those obtained by Thorsos [1990] for the scattering of acoustic waves from a rough pressure release (or Dirichlet) boundary. In these two cases, all of the parameters (wind speed, surface length, discretization length, etc) are the same. The additional parameter $k_c$ appearing in the caption of both these figures is the cutoff wavenumber used in generating the random surfaces. The importance of this parameter will be discussed in section 6.4.3. The "Scattering Strength" used by Thorsos is the same as the NRCS except for a constant factor. Also, Thorsos defined the scattering angle as the compliment of the incidence angle. Consequently, Thorsos' 90° scattering angle is equivalent to 0° incidence for the extinction code results. As in previous examples, good agreement is observed between Thorsos' results and those obtained with the extinction code.
Figure 6.3.3.3-3 Comparison of Extinction Code and A.J. Sant's Code and Measurements for 800 Surfaces with Gaussian Spectra: $\varepsilon_r = 1.991$, $\sigma_h = 1.93 \lambda_o$ and $L = 5.02 \lambda_o$ for $80 \lambda_o$ Long with $\Delta x = 0.1 \lambda_o$. Using 10/3/3 Evaluations, TE Polarization, $\theta_i = 0^\circ$ and $g = 15 \lambda_o$. 
Figure 6.3.3-4 Comparison of Extinction Code and A.J. Sant's Code and Measurements for 800 Surfaces with Gaussian Spectra: $\varepsilon_i = 1.991$, $\sigma_h = 1.93 \lambda_o$ and $L = 5.02 \lambda_o$

80 $\lambda_o$ Long with $\Delta x = 0.1 \lambda_o$ Using 10/3/3 Evaluations, TE Polarization, $\theta_i = 20^\circ$ and $g = 15 \lambda_o$
Figure 6.3.3-5 Comparison of Extinction Code and A.J. Sant's Code and Measurements for 800 Surfaces with Gaussian Spectra: $\varepsilon_r = 1.991$, $\sigma_h = 1.93 \lambda_o$ and $L = 5.02 \lambda_o$ 80 $\lambda_o$ Long with $\Delta x = 0.1 \lambda_o$ Using 10/3/3 Evaluations, TE Polarization, $\theta_i = 40^\circ$ and $g = 15 \lambda_o$
Figure 6.3.3-6 Comparison of Extinction Code and A.J. Sant's Code and Measurements for 400 Surfaces with Gaussian Spectra: \( \varepsilon_r = 1.991, \sigma_h = 1.93 \ \lambda_o \) and \( L = 5.02 \ \lambda_o \)
80 \( \lambda_o \) Long with \( \Delta x = 0.1 \ \lambda_o \) Using 10/3/3 Evaluations, TM Polarization, \( \theta_i = 0^\circ \) and \( g = 10 \ \lambda_o \)
Figure 6.3.3-7 Comparison of Extinction Code and A.J. Sant's Code and Measurements for 400 Surfaces with Gaussian Spectra: $\varepsilon_r = 1.991$, $\sigma_h = 1.93 \lambda_o$ and $L = 5.02 \lambda_o$

80 $\lambda_o$ Long with $\Delta x = 0.1 \lambda_o$ Using 10/3/3 Evaluations, TM Polarization, $\theta_i = 20^\circ$ and $g = 10 \lambda_o$
Figure 6.3.3-8 Comparison of Extinction Code and A.J. Sant's Code and Measurements for 400 Surfaces with Gaussian Spectra: $\varepsilon_r = 1.991$, $\sigma_h = 1.93 \lambda_o$ and $L = 5.02 \lambda_o$.

80 $\lambda_o$ Long with $\Delta x = 0.1 \lambda_o$ Using 10/3/3 Evaluations, TM Polarization, $\theta_i = 40^\circ$ and $g = 10 \lambda_o$. 
Figure 6.3.4-1 Comparison of Extinction Code and Thorsos' Code
for 50 PEC Surfaces with Pierson-Moskowitz Spectra, $U_{19.5} = 20$ m/s, $\lambda_o = 7.5$ m, $k_e = 2k_o$
180 $\lambda_o$ Long with $\Delta x = 0.2 \lambda_o$, TE Polariation, $\theta_i = 70^\circ$, $g = 40 \lambda_o$ and 10/-1 Evaluations
Figure 6.3.4-2 Comparison of Extinction Code and Thorsos' Code for 50 PEC Surfaces with Pierson-Moskowitz Spectra, $U_{19.5} = 10 \text{ m/s}$, $\lambda_o = 7.5 \text{ m}$, $k_c = 2k_o$, $180 \lambda_o$ Long with $\Delta x = 0.2 \lambda_o$, TE Polarization, $\theta_i = 70^\circ$, $g = 40 \lambda_o$ and 10/1/1 Evaluations
6.4 Accuracy of the Rough Surface Scattering Code

The previous section showed that the extinction code produces results which are in good qualitative agreement with the results obtained by other researchers. This is not to say any of these numerical calculations are "exact". As with the flat surface, the solution for randomly rough surfaces will suffer from discretization and finite domain errors. However unlike the flat surface case, there is no "exact" solution for comparison and the accuracy of the solution is not known except to say that these sources of error are believed to be small because (1) the errors were found to be small in the flat surface limit and (2) when one decreases the discretization size or increases the number of integration points (for example) there is no significant change in the average NRCS - with one important exception. The NRCS at large backscattering angles is sensitive to the truncation of the scattering surface. This problem is addressed in sections 6.4.1 and 6.4.2 for surfaces with Gaussian and Pierson-Moskowitz spectra respectively. In addition to the surface truncation, the NRCS for individual surfaces with Pierson-Moskowitz spectra is slightly affected by the spectral cutoff used in generating the randomly rough surface. However as in shown in section 6.4.3, the average cross section for is not affected by the spectral cutoff. Finally, section 6.4.4 points out an important limitation in the Monte Carlo average for the scattering from very slowly undulating random surfaces. Specifically, some important events are so unlikely to occur that they are difficult to include in the average cross section computation.

6.4.1 Effect of Surface Truncation on Scattering from Surfaces with a Gaussian Spectra

In section 5.3.3, the effects of surface truncation on the surface currents and scattered fields for a flat surface were examined. It was noted that when the incident field was not sufficiently tapered, the MOM current solution contained spikes in the currents which would then appear as nearly angle independent errors in the scattered field. Also, a step change in the current produces a far zone scattered field in all directions well away.
from the specular scattering direction. By tapering the incident field, one can reduce the
currents near the domain edges which in turn greatly reduces the impact of the surface
truncation.

For rough surfaces however, the incident field may be scattered by the surface
roughness along the surface so that it reaches the truncation points. As a result, tapering
the incident field does not ensure that the end currents are small. Figures 6.4.1-1 and
6.4.1-2, for example, plot the magnitude of the electric surface current and the NRCS for
one realization of a rough PEC which is truncated at 40 and at 400 wavelengths.
Examining the surface current reveals that when the incident angle is 0° (figure 6.4-1)
there is very little current outside the illumination region and correspondingly this is very
little difference in the bistatic radar cross sections for the two surface lengths. However
when the incident angle is 70° (figure 6.4.1-2), the scattered field will propagate along the
surface much more than 40 wavelengths and consequently there are very significant
changes in the bistatic NRCS for the two surface lengths, especially where the NRCS is
small. This end effect is usually easy to identify when examining the backscattered radar
cross section. The backscattered NRCS is the NRCS evaluated with the observation (or
scattering) angle is in the direction from which the incident field originated (i.e. in the
antispecular direction) or to use the notation of chapter 4 (see also figure 4.3-1) \( \theta_s = - \theta_i \).
Figure 6.4.1-3, for example, plots the backscattered NRCS for the same surfaces used in
figures 6.4.1-1 and 6.4.1-2. The backscattered cross sections for both surface lengths are
in good agreement up to about 25°. In this angular region (\( \theta_o = 0° \) to 25°), the
backscattered NRCS is accurate, because the computed backscattered power is much
greater than the error introduced due to truncation of the surface. (This case was one of
the worst observed in regards to the angular width of the valid backscattering region.
Also, it was frequently observed that the angular width of the valid backscattering region
was smaller for TM (v) polarization than for TE (h) polarization.)
Figure 6.4.1-1 Example of Surface Truncation Effects for $\theta_i = 0^\circ$.

1 Realization of a PEC Surface with Gaussian Spectrum, $\sigma_h = 0.707 \lambda_0$ and $L = 4.5 \lambda_0$,
with $\Delta x = 0.1 \lambda_0$ Using -/-/1 Evaluations, TM Polarization, and $g = 5 \lambda_0$. 
Figure 6.4.1-2 Example of Surface Truncation Effects for $\theta = 70^\circ$,
1 Realization of a PEC Surface with Gaussian Spectrum, $\sigma_h = 0.707 \lambda$ and $L = 4.5 \lambda$, 
with $\Delta x = 0.1 \lambda$ Using -1/-1 Evaluations, TM Polarization, and $g = 5 \lambda$.
Figure 6.4.1-3 Effect of Surface Truncation on Backscattered NRCS for 1 Realization of a PEC Surface with Gaussian Spectrum, $\sigma_h = 0.707 \lambda_o$ and $L = 4.5 \lambda_o$, with $\Delta x = 0.1 \lambda_o$ Using -/-/1 Evaluations, TM Polarization, and $g = 5 \lambda_o$. 
6.4.2 Effect of Surface Truncation on Scattering from Surfaces with Pierson-Moskowitz Spectra

When calculating the scattering from rough surfaces, one would like to choose the illuminated zone sufficiently large to contain several surface correlation lengths, so that one can be confident that all important scattering mechanisms have been included in the calculation. This is easily accomplished for surfaces with Gaussian spectra which have correlation lengths for the largest height surfaces examined in this study of only 20 times the length of the incident wavelength. However, Pierson-Moskowitz surfaces have correlation lengths that are quite large with respect to microwave wavelengths. At a wind speed of 15 m/s, for example, the period of the dominant wave in the spectrum is 205.17 meters long.

Recently, Kapp and Brown [1996] have developed a new technique to numerically calculate the scattering from PECs. This new technique, called the Method of Ordered Multiple Interactions (MOMI), is an exact solution to the same degree that a MOM solution is exact, but does not require the storage of a large matrix such as is used by the MOM. Using this new technique it is now possible, although still time consuming, to calculate the scattering from these very long surfaces.

Figure 6.4.2-1, for example, compares the average backscattered NRCS computed using MOMI from 10 surfaces with Pierson-Moskowitz spectra, but where the surfaces have been truncated at two different lengths - 820 meters and 12 meters. The figure shows that there is very good agreement between the solutions down to incident angles of about 60 degrees. This shows that there are no significant large scale multiple scattering mechanism for these surfaces at backscattering angles less than 60°. (Note: Based on the discussion in section 5.3.3, a spot size of 15 wavelengths will begin to give erroneous results at incident angles of approximately 72° or greater).
Figure 6.4.2-1 Effect of Surface Truncation on Average Backscattered NRCS for 10 Realizations of PEC Surfaces with Pierson-Moskowitz Spectra, $U_{1/5} = 15$ m/s, $\lambda_o = 0.10$ m, and $k_c = 2k_o$

Using MOMI with $\Delta x = 0.15 \lambda_o$, 10/-/1 Evaluations, TE Polarization, and $g = 15 \lambda_o$
Figure 6.4.2-2a compares the average backscattered NRCS obtained using the 820 meter long surfaces but for two different illumination spot sizes. One illumination spot size is 205 meters (equal to the period of the dominate water wave) and the other is 1.5 meters. The plots show that there is agreement in the scattering results, considering that the fluctuations in the cross sections are still quite large since the NRCS from only 10 surfaces have been averaged (95% confidence limits are about 4.3 dB for each cross section). By increasing the number of cross sections averaged, the fluctuations can be decreased. However, for ONE 820 meters long surface, 10 days were required to calculate the NRCS. In addition, the fluctuations in the average can be expected to drop off as only the square root of the number of surfaces averaged. Both of the above points make it impractical to attempt to decrease the fluctuations any further by this means. However, in figure 6.4.2-1 it was observed that the backscattered NRCS for a short surfaces (12 meters) gave the same results as a long surface (820 meters) for backscattering angles of less than about 60 degrees, when the same spot size is used. Therefore, the an average NRCS with smaller fluctuations can be obtained for the illumination spot size of 1.5 meters by using many short surfaces instead of 10 long surfaces. Figure 6.4.2-2b, therefore, compares the average NRCS obtained from the 10 long surfaces with the 205 meter spot size and 200 short surfaces with a 1.5 spot size. Figure 6.4.2-2b is an improvement over figure 6.4.2-2a because although the fluctuations in the results for the 820 meter spot size (dot-dot line) remain large (about 4.3 dB) the fluctuations for the 1.5 meter spot size (solid line) have been reduced to about 0.7 dB. Figure 6.4.2-2b shows that the average cross section for the small spot size is in good agreement with the large spot size, because it is well within the expected confidence limits for the fluctuations in the 10 surface average. Similar results to those of figures 6.4.2-1 and 6.4.2-2 where also obtain for wind speeds of 10 and 20 m/s and for both polarizations.
Surface length = 820 meters (54,710 unknowns)

- **g = 1.5 meters**
- **g = 205 meters**

**Figure 6.4.2-2a** Effect of the Incident Beam Spot Size on the Average Backscattered NRCS Using 10 PEC Surfaces with Pierson-Moskowitz Spectra, $U_{19.5} = 15 \text{ m/s}$, $\lambda_o = 0.10 \text{ m}$, $k_o = 2k_o$. MOMI with $\Delta x = 0.15 \lambda_o$, 10/-1 Evaluations and TE Polarization
Figure 6.4.2-2b Effect of the Incident Beam Spot Size on the Average Backscattered NRCS Using PEC Surfaces with Pierson-Moskowitz Spectra, $U_{19.5} = 15 \text{ m/s}, \lambda_o = 0.10 \text{ m}, k_c = 2k_o$, MOMI with $\Delta x = 0.15 \lambda_o$, 10/-/1 Evaluations and TE Polarization
In summary, the previous two paragraphs suggest that the scattering from Pierson-Moskowitz surfaces can be determined using illuminated surfaces which are much smaller than the correlation length of the surface heights. This result seems sensible because: (1) Pierson-Moskowitz surfaces possess large amounts of small scale surface structure, i.e. strong Bragg scattering mechanisms which dominate the backscattered field away from nadir and (2) the large scale surface features, which cause the correlation length to be so large, are slowly undulating and do not have large slopes. For example, at a wind speed of 20 m/s, the dominant period wave is 364.75 meters long but the surface has an rms height only of 2.13 meters. Therefore, the very long surface waves really are just adding a small tilt to the surface.

6.4.3 Very Small Scale Surface Structure

A potential source of difficulty with the Pierson Moskowitz spectrum is that it contains spatial components which are smaller in wavelength than the incident wavelength at microwave frequencies. Due to viscous damping effects on the ocean surface, the Pierson-Moskowitz spectrum could justifiably be truncated somewhere near 1 mm [Apel 1994]. However, due to the relatively small number of unknowns that can be easily manipulated by the computer, it is desirable to sample the surface as infrequently as possible because each sample point requires finding two unknowns (i.e. the electric field and the magnetic field). For incident electromagnetic wavelengths greater than a few centimeters sampling the surface near 1 mm becomes problematic. For example, if the incident wavelength is 10 cm then sampling the surface at 1 mm means that there would be 100 sample points per electromagnetic wavelength, and since a total surface length of at least 40 wavelengths is needed, a minimum of 8000 unknowns would be required.

Fortunately, these very small scale spatial components of the surface spectrum do not appear to be very important to the average radar cross section [Thorsos 1990]. Figure 6.4.3-1, for example, shows the average NRCS obtained from 200 surfaces, where the
cutoff in the Pierson Moskowitz surface used to generate these surfaces was \( \lambda_c = \lambda_o/2 \) and \( \lambda_c = \lambda_o/10 \) (\( \lambda_o \) is the electromagnetic wavelength and \( k = 2\pi/\lambda \)). To highlight potentially small changes, the difference in the two radar cross sections is also plotted. As the figure clearly demonstrates, removing the surface components between \( \lambda_c = \lambda_o/2 \) and \( \lambda_c = \lambda_o/10 \) has no significant impact on the average cross section for backscattering angles less than 60 degrees. It should be stressed that the cutoff should not be chosen less than \( k_c = 2k_o \), because these components of the surface structure can (and do) cause Bragg scattering.

It is important to note that for any particular realization of the surface, these very small scale components of the surface spectrum do in fact cause small changes in the surface current and in the radar cross section. Figure 6.4.3-2, for example, shows the difference in the NRCS obtained from 1 surface realization, where the cutoff in the Pierson Moskowitz surface used to generate these surfaces was selected at \( \lambda_c = \lambda_o/2 \) and \( \lambda_c = \lambda_o/10 \). Clearly the difference in the radar cross sections due to these very small scale features is small (compared with the fluctuations in the cross section itself) and appear to fluctuate about a mean value of zero. The net result being that these fluctuations do not contribute to the average cross section, and would only slightly increase higher order moments of the cross section. The results in figure 6.4.3-2 were obtained using a very dense sampling of the surface (\( \Delta x = 0.01 \lambda_o \)). This dense sampling was used because the scattering from a surface with a Pierson-Moskowitz spectrum has a slight sensitivity to the sampling interval. Figure 6.4.3-3, for example, shows the difference in the NRCS obtained from 1 surface realization where the cutoff in the Pierson Moskowitz spectrum used to generate this surface was selected at \( \lambda_c = \lambda_o/2 \) and the same surface was sampled at both \( \Delta x = 0.01 \lambda_o \) and \( \Delta x = 0.1 \lambda_o \). With this cutoff, decreasing the sampling interval beyond \( \Delta x = 0.01 \lambda_o \) does not show any further change in the radar cross section. What this results means is that there is a small error in the radar cross section when the surface
Figure 6.4.3-1 Effect of the Surface Spectral Cutoff on the Average Backscattered NRCS
Using MOMI and 200 PEC Gaussian Surfaces with Pierson-Moskowitz Spectra, $U_{19.5} = 20$ m/s,
$\lambda_o = 3$ cm, 80 $\lambda_o$ Long with $\Delta x = 0.1 \lambda_o$, 10/-1 Evaluations, TE Polarization, and $g = 15 \lambda_o$.
Figure 6.4.3-2 Effect of the Surface Spectral Cutoff on the Backscattered NRCS
Using MOMI and 1 PEC Gaussian Surface with a Pierson-Moskowitz Spectrum, $U_{19.5} = 20$ m/s, $\lambda_o = 3$ cm, 80 $\lambda_o$ Long with $\Delta x = 0.01$ $\lambda_o$, 10/1/1 Evaluations, TE Polarization, and $g = 15$ $\lambda_o$
Figure 6.4.3-3 Effect of the Discretization Size on the Backscattered NRCS
Using MOMI and 1 PEC Gaussian Surfaces with a Pierson-Moskowitz Spectrum, $U_{19.5} = 20$ m/s, $\lambda_o = 3$ cm, $k_c = 2 k_o$, 80 $\lambda_o$ Long, 10^-1 Evaluations, TE Polarization, and $g = 15$ $\lambda_o$
is sampled at \( \lambda_c/10 \) even when the surface contains spatial variations only up to \( \lambda_c/2 \). Much like the difference in the cross section due to the very small scale features, this error is small and appears to fluctuate about a mean value of zero and does not significantly influence the average radar cross section. It should be stressed that this sampling related error is strongly tied to the small scale structure and does not appear when using either a Gaussian spectrum (with an rms height greater than one quarter of a wavelength) or a Pierson Moskowitz spectrum when the spectral cut off is chosen equal to the incident wavelength. Again, the minimum spectral cutoff should not be chosen less than \( \lambda_c/2 \) in order to include all Bragg scattering elements. Also included in figure 6.4.3-2 is a difference plot, where the NRCS was obtained using a small scale curvature correction factor for the self terms of the matrix equation. This correction factor, discussed by Kapp [1996], is numerically inexpensive to calculate, and as the figure indicates, significantly reduces the sampling related error.

In summary of section 6.4.3, the average radar cross section for surfaces with Pierson-Moskowitz spectra is not sensitive to the very small scale portion of the surface spectrum for backscattering angles less than 60 degrees. However, the radar cross section obtained for one surface with a Pierson-Moskowitz spectrum does have a slight sensitivity to this portion of the surface spectrum. Also, to obtain highly accurate results for each individual cross section, the sampling frequency must be chosen much higher than the cutoff frequency - although this restriction can be relaxed by using a self-term curvature correction. Finally, although the example used in this section was for TE polarization, at a wind speed of 20 m/s and an electromagnetic wavelength of 3 cm, very similar results were observed for TM polarization, at electromagnetic wavelengths of 3 and 30 cm, and wind speeds from 5 to 20 m/s.
6.4.4 The Monte Carlo Solution and Low Probability Events

A serious limitation to any Monte Carlo simulation is the possibility that some important events may be sufficiently unlikely to occur that they will not be properly included in the simulation. Such a situation is encountered when scattering from surfaces with large rms height, small rms slope, and Gaussian spectra - that is surfaces which do not contain any significant spatial variation even 10 times the length of the incident wavelength. The scattering from such surfaces is dominated by specular (i.e. ray-like) scattering, as will be shown in chapter 7. However, these surfaces contain important specular facets (i.e. slopes) which occur very infrequently. For example, figure 6.4.4-1 compares the theoretical probability density function (pdf) of the surface slopes for a Gaussian surface with a rms height of 2.24 \( \lambda_n \) and rms slope 24.1\(^\circ\), and two estimated slope pdfs (essentially histograms of the slope). The slope given along the y-axis of figure 6.4.4-1 is the rate of change in the height of the rough surface with respect to the mean scattering surface (plane). This value can be converted into the angle formed between a line with this slope and the mean scattering plane by take its tangent, i.e. \( \theta = \tan \zeta x \). Figure 6.4.4-1 shows that when 200 surfaces which were 800 times longer than the incident wavelength were examined, no slope greater than about 1.7 was found. The slope was determined for the surface height profile using a finite difference formula, \( \zeta x = \frac{1}{\Delta \sigma} [\zeta(x + \Delta x) - \zeta(x)] \). This means that none of the 200 surfaces contained a slope which could specularly backscatter a ray incident at 60 degrees. When the number of surfaces examined was increased to 2000, a few slopes of up to 2.1 were found. These slopes can back reflect rays up to about 65 degrees. Note, this is not an error in the surface generation program, the likelihood that a point on the surface will have a slope of 2 or greater is 3.385E-4 \%. When the large slopes are not included in the Monte Carlo sample population, the average backscattered cross section is significantly underestimated at large backscattering angles, as is depicted in figure 6.4.4-2.
Figure 6.4.4-1 Effect of Finite Sample Population on Surface Slope Distribution for Surfaces with Gaussian Spectra (estimates obtained via difference formula with $\Delta x = 0.1 \lambda_o$).

Estimated from 2000 surface of $80 \lambda_o$ (1,600,000 Samples)

Estimated from 200 surfaces of $80 \lambda_o$ (160,000 Samples)
Figure 6.4.4-2 Influence of Low Slope Probability Event on Monte Carlo Averages for Surfaces with Gaussian Spectra, $\sigma_h = 2.24 \lambda_o$, $\sigma_s = 24.1^\circ$, and $\epsilon_r = 2.0$, $80 \lambda_o$ Long with $\Delta x = 0.1 \lambda_o$ Using 10/3/2 Evaluations, TM Polarization, and $g = 15 \lambda_e$
6.5 Conclusions

In this chapter, the average normalized radar cross section (NRCS) produced by Monte Carlo simulation of the scattering from randomly rough surfaces with Gaussian and Pierson-Moskowitz spectra was examined. The extinction code results were found to be in good agreement with other scattering codes (for Gaussian and Pierson Moskowitz Spectra) and with measurements (for Gaussian Spectra).

Also, the impact of surface truncation, very small scale surface features, and limitations of the Monte Carlo average were examined. It was found that the length of the surface can significantly affect the solution for surfaces with Gaussian spectra. Generally, the smaller the surface, the greater the influence of the surface truncation, and the smaller the angular region becomes where the scattering strength is sufficient to obtain accurate results. For Pierson-Moskowitz spectra, on the other hand, it was found that the surface length can be chosen much smaller than the surface correlation length for backscattering angles up to about 60°.

It was also found that the average radar cross section is not sensitive to the very small scale surface features of surfaces with Pierson-Moskowitz spectra (that is the surface spectral components at spatial wavelengths shorter than $\lambda_c/2$) for backscattering angle less than 60 degrees. However, the radar cross section obtained for a single surface with a Pierson-Moskowitz spectrum does have a slight sensitivity to these subwavelength features, and in order to obtain highly accurate results the sampling frequency must be chosen much higher than the cutoff frequency. This restriction can be somewhat relaxed by using a self-term curvature correction discussed by Kapp [1996]. Finally, in regards to the Monte Carlo approach, it was shown that this technique is difficult to apply at large backscattering angles for surfaces with Gaussian spectra whose scattering is dominated by specular scattering. This is because infrequently occurring slopes must be included to obtain a good average for the radar cross section.

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7.0 Estimation of Surface Parameters using the Quasi-Specular Model for Surfaces with Gaussian Spectra

7.1 Introduction

This chapter examines the use of the quasi-specular model to estimate the rms slope and the reflection coefficient magnitude of randomly rough surfaces with Gaussian height distributions and Gaussian spectra. The limitations of the model are first examined by comparing the average backscattered normalized radar cross section (NRCS) given by the model with results obtained using a numerically "exact" method of moments (MOM) Monte Carlo solution. Second, the quasi-specular model is used to estimate the surface parameters from the numerically generated NRCS data. Because the exact surface parameters used to generate the numerical data are known, the accuracy of the model-derived estimates can be determined.

The organization of this chapter is as follows. In section two, the numerically "exact" average backscattered NRCS is compared with results predicted by the quasi-specular model to determine where the model and the MOM solutions are in good qualitative agreement. It will be shown that near nadir (looking directly down at the surface) the quasi-specular model compares favorably with the MOM calculations until multiple scattering effects become significant. Well away from nadir, however, the quasi-specular model under predicts the scattering due to the diffraction contribution of intermediate scale surface structure, i.e., surface spectral components which are 3 to 10 times the size of the incident wavelength.

In the third section of this chapter, techniques are introduced to determine the surface root mean squared (rms) slope and the (normal incidence flat surface) reflection coefficient magnitude from the MOM-generated average NRCS. These techniques...
involve either direct manipulation of the model or a numerical minimization procedure. In the fourth section, the impact of the finite sample size used to obtain average NRCS is examined. In essence, variations in the average radar cross section cause the parameters estimated from these data to also vary. Finally in the fifth section, the inversion approaches are applied to the numerical data to obtain a quantitative description of the model’s accuracy.

Before continuing, a brief review of the quasi-specular model for one dimensional surface scattering is required. The quasi-specular model is obtained by making two important approximations to the surface integral formulation of the scattering problem; (1) the Kirchhoff or tangent plane approximation and (2) a stationary phase or high frequency approximation. The full derivation is provided in Appendix B, and the results for the average backscattered NRCS are given below in equations (7.1-1). In the backscattered case (θs = −θi = θ or \( \hat{k}_s = -\hat{k}_i \)).

\[
\sigma^0_{bs}(\theta) = \frac{\pi}{\cos^3(\theta)} |\rho(0)|^2 \left[ \text{pdf}_{\zeta_x} (\tan \theta) \right].
\] (7.1-1)

where \( \text{pdf}_{\zeta_x} (\zeta_x) \) is the probability density function (pdf) of the surface slopes, \( \zeta_x \), \( \zeta_x = \tan \theta_i \) is the surface slope for specular reflection, i.e.,

\[
\zeta_x = \frac{(\hat{k}_s - \hat{k}_i) \cdot \hat{z}}{(\hat{k}_s - \hat{k}_i) \cdot \hat{\zeta}}
\]

and the incident (\( \hat{k}_i \)) and scattered (\( \hat{k}_s \)) unit vector directions are,

\[
\hat{k}_i = \sin(\theta_i)\hat{x} - \cos(\theta_i)\hat{z} \quad \text{and} \quad \hat{k}_s = \sin(\theta_s)\hat{x} + \cos(\theta_s)\hat{z}.
\]

In should be noted that as a result of the high frequency approximation, only those points on the surface whose slopes (i.e. tangent planes) specularly reflect the incident ray into the scattering direction contribute to the cross section. This is why the slope in the above equation, \( \zeta_x \), is only a function of the incident and scattering vectors.

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\[ \rho_H(\theta_L) = \frac{\mu_2 k_1 \cos(\theta_L) - \mu_1 \sqrt{k_2^2 - k_1^2 \sin^2(\theta_L)}}{\mu_2 k_1 \cos(\theta_L) + \mu_1 \sqrt{k_2^2 - k_1^2 \sin^2(\theta_L)}} \]

is the TE Fresnel reflection coefficient, and

\[ \rho_V(\theta_L) = \frac{\mu_1 k_1 \cos(\theta_L) - \mu_2 \sqrt{k_2^2 - k_1^2 \sin^2(\theta_L)}}{\mu_1 k_1 \cos(\theta_L) + \mu_2 \sqrt{k_2^2 - k_1^2 \sin^2(\theta_L)}} \]

is the TM Fresnel reflection coefficient. The subscript H or V specifies the polarization. H indicates horizontal (TE) polarization, which occurs when the electric field is parallel to the one dimensional (i.e. grooved or corduroy) surface. V indicates vertical (TM) polarization in which the magnetic field is parallel to the one dimensional surface. \( \theta_L = \frac{\theta_i + \theta_s}{2} \) is the local angle of incidence at the tangent plane. In the backscattering case \( \theta_L = \theta_i \) and \( |\rho_H(\theta)| = |\rho_V(\theta)| = \rho(0) \). \( k_{1,2} = \frac{2\pi}{\lambda_o} \sqrt{\mu_{r_{1,2}} \varepsilon_{r_{1,2}}} \) is the wavenumber in upper medium (#1) or lower medium (#2) respectively, \( \mu_{1,2} = \mu_{r_{1,2}} \mu_o \) is the magnetic permeability, \( \mu_o \) is the free space magnetic permeability, \( \varepsilon_{1,2} = \varepsilon_{r_{1,2}} \varepsilon_o \) is the dielectric permittivity, \( \varepsilon_o \) is the free space dielectric permittivity, and \( \lambda_o \) is the free space wavelength of the incident field. In this chapter, the material above the scattering surface is taken to be that of free space (\( \varepsilon_{r_1} = 1.0 \)) and the scattering material will be restricted to real values (\( \varepsilon_{r_2} = \varepsilon_r = \) a real number).

It should be noted the reflection coefficient introduces no angular dependence in the average backscattered NRCS (equation 7.1-1). Therefore, according to the quasi-specular model, if the NRCS is divided by the magnitude of the reflection coefficient squared then the resulting ratio will only be a function of the surface roughness. This effect will be used both in comparing the model with MOM calculations (to determine where the model is valid) and in extracting a rms slope from the numerically generated data.

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7.2 Comparison of the Simulations and the Model

In this section numerical simulations, i.e. the MOM-based calculations, of the average NRCS are compared to the quasi-specular model for surfaces which have both a Gaussian height probability density function and a Gaussian correlation function. Such surfaces can be characterized by two values: (1) the rms slope, $\sigma_s$, and (2) the rms height, $\sigma_h$. Chapter 6 provides more details on the surface characterization.

Consider first figure 7.2-1, which shows the TE (horizontal) polarized average backscattered NRCS divided by the square of the magnitude in the reflection coefficient for a rms height of 0.71 $\lambda_0$ and a rms slope of 17.55°. The rms height is specified with respect to the wavelength because the scattering results scale with respect to the incident wavelength. The NRCS has been divided by the reflection coefficient because the quasi-specular model (equation 7.1-2) suggests that this ratio will be independent of the surface material, i.e., the scattering will only be a function of the surface roughness. The figure shows that for this case, there is only a very slight difference in the near nadir region between the normalized solution for a perfect electric conducting (PEC) material and the normalized solution for an $\varepsilon_r = 2.0$ material. This result is predicted by the model. The figure also shows that both of the normalized MOM calculated results are in close agreement with the normalized quasi-specular model near nadir, but that they begin to diverge at large backscattering angles. One concludes that for these roughness parameters, the quasi-specular model is in good agreement with the method of moments solution.
Figure 7.2-1 Comparison between the Quasi-Specular Model and MOM Calculations for 5200 Surface Realizations with Gaussian Spectra, $\sigma_h = 0.71 \lambda_o$ and $\sigma_s = 17.55^\circ$, 80 $\lambda_o$ Long with $\Delta x = 0.1 \lambda_o$ Using 10/3/2 Evaluations ($\varepsilon_r = 2$) and 10/-1 (PEC), TE Polarization, and $g = 15 \lambda_o$
Consider now figure 7.2-2, which shows the TE backscattered average NRCS divided by the square of the magnitude in the reflection coefficient for the same rms height as in figure 7.2-1 but a rms slope of $24.1^\circ$. Unlike the previous example (rms slope $17.55^\circ$), there is now a very noticeable difference in the scattering between the MOM calculated results for the PEC and the $\varepsilon_r = 2.0$ material. This is not predicted by the model. The figure also shows that near nadir, the scattering from the $\varepsilon_r = 2.0$ material remains much closer to the quasi-specular model than does the result for the PEC material. One concludes from this second example, that the quasi-specular model has lost accuracy for these particular roughness parameters and that the validity of the model will also be a function of the surface material. Although not readily apparent in these figures, a careful examination of the data reveals that as the rms slope increased, the difference between the quasi-specular model and the MOM calculations also increased at the largest backscattering angles. In the following two subsections, the reasons for the models departure from the MOM results at large and small backscattering angles is investigated.

### 7.2.1 The Quasi-Specular Model at Large Backscattering Angles

Readers familiar with the Kirchhoff approximation might suspect that this approximation is responsible for the model's poor behavior at large backscattering angles, because the validity requirements for the Kirchhoff approximation are strongly dependent on the angle of incidence and because Thorsos' [1988] study found the Kirchhoff approximation breaks down at large incidence angles (although he did not address the particular roughness parameters used here). Alternatively, readers familiar with the two scale model might suspect that Bragg scattering, which is dominant at these angles for so called two-scale surfaces, might be the cause of the problem. However, neither of these explanations is correct, as will be shown using figure 7.2.1-1.
Figure 7.2-2 Comparison between the Quasi-Specular Model and MOM Calculations for 8000 Surface Realizations with Gaussian Spectra, $\sigma_h = 0.71 \lambda_0$ and $\sigma_s = 24.1^\circ$, 80 $\lambda_0$ Long with $\Delta x = 0.1 \lambda_0$ Using 10/3/2 Evaluations ($\varepsilon_r = 2$) and 10/-/1 (PEC), TE Polarization, and $g = 15 \lambda_0$. 
Figure 7.2.1-1 Comparison between the Kirchhoff, Quasi-Specular and MOM Solution for 1600 Surface Realizations with Gaussian Spectra, $\sigma_h = 0.71 \lambda_o$ and $\sigma_s = 24.10^\circ$, 80 $\lambda_o$ Long with $\Delta x = 0.1 \lambda_o$ Using 10/~/1 Evaluations, TE Polarization, and $g = 15 \lambda_o$. 

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This figure compares: (1) a method of moments result using the entire surface spectrum, (2) the quasi-specular model using the known rms slope, (3) a method of moments result for the same set of surfaces but with surface spectral components of less than three times the incident wavelength removed, and (4) a numerical result for the average NRCS obtained by using the Kirchhoff approximation for the surface currents but without using the high frequency approximation. (An expression for the scattering using only the Kirchhoff result for Gaussian surfaces with Gaussian spectra is given by Ulaby, Fung and Moore [1982] chapter 12). This graph shows two important effects. First, there is virtually no small scale structure or components of the surface spectrum which could contribute to Bragg scattering, i.e. the surface contains no spatial components whose wavelength is even 3 times longer than the wavelength of the incident electromagnetic field. This is evidenced by the fact that the estimated rms slope for the filtered surfaces is 23.98° compared to a unfiltered value of 24.10° and that there is no significant change in the method of moments solutions when the surface spectrum is filtered to remove all of the small scale structure. Second, the tangent plane approximation is in good agreement with the method of moments solution whereas the quasi-specular model is not. Therefore, it is clear that the difference between the method of moments solution and the quasi-specular model is not due to a breakdown in the Kirchhoff approximation or the presence of small scale structure, i.e. Bragg scattering mechanisms. This leaves the stationary phase approximation as the likely source of the difficulty. In the stationary phase approximation, surface height derivatives of order two or greater are discarded such that (see appendix B),

\[
\lim_{k \to \infty} \int \exp \left[ -j \Delta k_x (\Delta x) - j \Delta k_z \left( \zeta_x (\Delta x) + \zeta_{xx} (\Delta x)^2 \right) + \text{HOT} \right] d\Delta x,
\]

\approx \lim_{k \to \infty} \int \exp \left[ -j \Delta k_x (\Delta x) - j \Delta k_z \zeta_x (\Delta x) \right] d\Delta x

= \lim_{k \to \infty} \int \exp \left[ -j \Delta x (\Delta k_x + \Delta k_z \zeta_x) \right] d\Delta x

(7.2.1-1)

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where $\Delta k_x = (k_{xx} - k_{ix}) = k(\sin \theta_s - \sin \theta_i)$, and

$$\Delta k_z = (k_{zz} - k_{iz}) = k(\cos \theta_s + \cos \theta_i)$$

Therefore when $k$ (the wavenumber) is very large, $-\frac{\Delta k_x}{\Delta k_z} = \zeta_x(x)$ and $\frac{1}{2\Delta k_z} \zeta_{xx}(x) +$ the higher order terms (HOT) are small there is a contribution to the integration. For Gaussian surfaces the root mean squared (rms) curvature is given by,

$$\sqrt{\left<\zeta_{xx}\right>^2} = \sqrt{\frac{3\sigma_s^4}{\sigma_h^2}}.$$  \hspace{1cm} (7.2.1-2)

where $\sigma_h$ is the rms surface height and $\sigma_s$ is the rms slope. This suggests that decreasing the rms slope or increasing the rms height should improve the stationary phase approximation. Naturally, the question arises as to how either decreasing the slope variance or increasing the height variance effects the surface spectrum. As depicted in figure 7.2.1-2, either increasing the rms height or decreasing the rms slope causes the Gaussian spectrum to become more peaked, with greater spectral power at "low" spatial frequencies and less spectral power at higher spatial frequencies. A more rigorous proof of this behavior is provided in Appendix C.

Not surprisingly therefore, agreement between the quasi-specular model and the MOM calculations in the previous example (figure 7.2-2) is improved when either a larger rms height is used or a smaller rms slope is used or when the surface spectrum is filtered to remove the components of the surface spectrum which are 3 to 10 times the length of the incident wavelength ($10 \lambda_o > \lambda_s > 3 \lambda_o$), i.e., the intermediate scale components. Figure 7.2.1-3, for example, demonstrates this improved agreement at large backscattering angles when increasing the rms height or filtering the surface as described above.
Figure 7.2.1-2 Comparison of Three Gaussian Spectra
Unfiltered Spectrum, $\sigma_h = 1.58 \lambda_o$ and $\sigma_s = 24.1^\circ$  

$\lambda_s < 10 \lambda_o$ removed and $\sigma_h = 0.71 \lambda_o$

Figure 7.2.1-3 Comparison of the NRCS in the Absence of Intermediate Frequency Surface Components for 2000 Surface Realizations with Gaussian Spectra, $80\lambda_o$ Long with $\Delta x = 0.1 \lambda_o$ 
Using 10/-/1 Evaluations, TE Polarization, and $g = 15 \lambda_o$
This example combined with a number of numerical calculations for other values of the rms height and slope (not shown here) and the result to be presented in section 7.5 suggests that for Gaussian surfaces, the quasi-specular model compares favorably with MOM calculations at large backscattering angle (but not necessarily greater than about 68°, which is the limit of the calculations) for both PEC and dielectric materials when,

$$\sqrt{\langle \zeta_{xx}^2 \rangle} = \sqrt{\frac{3 \sigma_h^4}{\sigma_s^2}} \leq 0.15/\lambda_o. \quad (7.2.1-7)$$

This condition is significantly more restrictive than the curvature limit frequently associated with the Kirchhoff approximation [Ulaby, Fung, and Moore p. 949, 1815, and Appendix 12F],

$$\sqrt{\langle \zeta_{xx}^2 \rangle} \leq 1/\lambda_o. \quad (7.2.1-8)$$

7.2.2 The Quasi-Specular Model at Small Backscattering Angles

In section 7.2, it was observed that there is a noticeable difference in the scattering between a PEC and an \( \varepsilon_r = 2.0 \) material at small backscattering angles when the rms slope is 24.1° and the rms height is 0.71 \( \lambda_o \). Also, the results from the \( \varepsilon_r = 2.0 \) material remained much closer to the quasi-specular model than those from the PEC material. Because the surface roughness is exactly the same for both materials (in fact identical surface profiles were used), this suggests that multiple scattering is contributing to the scattered field for both materials, as is depicted below in figure 7.2.2-1.
A Single Scattered Ray will be proportional to \(|R(0)|^2\)

A Double Scattered Ray will be proportional to \(|R(a)|^2|R(b)|^2\)

Figure 7.2.2-1 Ray Image of Multiple Scattering

This is because the scattering strength of a ray which is backscattered from a surface only one time, can be expected to be proportional to the magnitude of the reflection coefficient (evaluated at normal incidence) squared - as is found in the quasi-specular result. Whereas, a ray which reflects from a surface two times will be proportional to the product of two reflection coefficients (each evaluated at its local angle of incidence) squared. Therefore twice scattered rays, which are not included in the quasi-specular model, can be expected to increase the radar cross section for both PECs and dielectric materials, but the increase would be larger for PECs because the reflection coefficient for PECs is larger than that for dielectric materials.
This multiple scattering view is further confirmed by examining the NRCS obtained from the Kirchhoff current ($J^{KA}$) and the first iteration current ($J^1$) for a PEC. The first iteration current is obtained from the 2nd kind Magnetic Field Integral Equation (MFIE), which can be written as [Kapp 1996],

$$J = J^{KA} + LJ.$$  \hspace{1cm} (7.2.2-1)

where $J$ is the total electric current (density) [Amperes/m] = $\hat{n} \times \vec{H} = \vec{H}_{\tan}$, $J^{KA}$ is the Kirchhoff current $= 2\hat{n} \times \vec{H}_i$, $L$ is an integral operator, $\vec{H}_{\tan}$ is the tangential magnetic field, $\vec{H}$ is the total magnetic field = incident + scattered magnetic fields, and $\vec{H}_i$ is the incident magnetic field.

Because of the symmetry of the two dimensional problem (i.e. a one dimensional or grooved surface) the magnetic field is confined to be either parallel or in the plane of the surface grooves (depending on the polarization). As a result, the MFIE can be written in one of two scalar forms with the direction of the currents suppressed. Equation (7.2.2-1) can be solved through a well known iterative process, whereby the total current is first assumed to be given by the Kirchhoff result (sometimes called the zeroth order iterate). A corrected current is then determined by applying the operator L to the Kirchhoff result and adding the result to the Kirchhoff current. This process is then repeated until the current converges, i.e.,

$$J^0 = J^{KA}$$
$$J^1 = J^{KA} + LJ^{KA}$$
$$J^2 = J^{KA} + LJ^1$$
$$\ldots$$
$$J^N = J^{KA} + LJ^{N-1}.$$  \hspace{1cm} (7.2.2-2)

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The zeroth order current is essentially the single scattering result and the factor, $L_{M}$ in the first order term is a multiple scattering correction. Unfortunately, this procedure is not always numerically stable [Wingham and Devayya 1992]. However, we have found that for surfaces with very large rms heights, in excess of roughly 2 wavelengths, and moderate rms slopes, e.g. 24.1°, the iterations converge.

Figure 7.2.2-2, compares the Kirchhoff result, the 1st iteration result, and the full MOM solution for a surface with a rms slope of 24.1° and a rms height of $7.1 \lambda_0$. The figure shows that the first iteration term is in very close agreement with the full method of moments result, but differs from the (Monte Carlo) Kirchhoff result in the near nadir region. The fact that the first iteration and the MOM calculations agree closely, clearly indicates that multiple scattering is largely responsible for the difference in the average cross section at these small backscattering angles.

7.2.3 Remarks on the NRCS for TM (vertical) polarization and Ray Bouncing Models

In this section, the scattering of a TM (vertically) polarized field is compared to the quasi-specular model and to the scattering for TE (horizontal) polarization. Differences in the TE and TM results are used to draw some additional conclusions about the nature of the multiple scattering.

Average NRCS results for the same roughness conditions used in figure 7.2-2 (TE polarization) are given in figure 7.2.3-1 for TM polarization. As in the TE case, the quasi-specular model under predicts the scattering at large backscattering angles for TM polarization. However, unlike the TE case, at small backscattering angles the results for the low dielectric material remain in very good agreement with the quasi-specular model. That is, multiple scattering appears to significantly affect the PEC results but does not significantly influence the low dielectric material results for TM polarization.
Figure 7.2.2-2 Comparison of Kirchhoff, 1st Iteration and MOM Results for 800 Surface Realizations with Gaussian Spectra, $\sigma_h = 7.1 \lambda_0$ and $\sigma_s = 24.10^\circ$, 240 $\lambda_0$ Long with $\Delta x = 0.1 \lambda_0$ Using 10/1 Evaluations, TE Polarization, and $g = 40 \lambda_0$. 

- Monte Carlo MOM Calculations
- Monte Carlo 1st Order Iteration Results
- Monte Carlo Kirchhoff Results
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Figure 7.2.3-1 Comparison between the Quasi-Specular Model and MOM Calculations for 2600 Surface Realizations with Gaussian Spectra, $\sigma_h = 0.71 \lambda_o$ and $\sigma_s = 24.1^\circ$, 80 $\lambda_o$ Long with $\Delta x = 0.1 \lambda_o$ Using 10/3/2 Evaluations ($\epsilon_r = 2$) and -/-/1 (PEC), TM Polarization, and $g = 15 \lambda_o$.
This effect can be predicted using a double ray bounce model of multiple scattering. In the backscattered direction, any ray which is multiply scattered by two bounces will have to undergo a 180° change in direction, as illustrated in figure 7.2.3-2. Therefore, one can plot the product of the reflection coefficients for every possible pair of angles. This is done in figure 7.2.3-3 for both TE and TM polarization, where the normalized double bounce reflection coefficient is defined as,

\[ R_{\text{double bounce}}(\theta = a) = \frac{|\rho_{HV}(\theta)|^2 |\rho_{HV}(90 - \theta)|^2}{|\rho_{HV}(0)|^2}. \]  

(7.2.3-1)

In the TM case, it is the Brewster angle which causes the double bounce reflection coefficient to be substantially lower than that for the TE case for dielectric materials. In fact, as is suggested by this figure the multiple scattering in the TE case with \( \varepsilon_r = 2.0 \) is stronger than for the TM case with \( \varepsilon_r = 10 \). This will be observed later in section 7.5.

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Figure 7.2.3-3 Examples of the Double Bounce Reflection Coefficient versus the Double Bounce Angle for Both Polarizations
It should be noted that according to the double bounce ray model, there would be no difference in the multiple scattering from a PEC under TE or TM polarization. However as is shown in figure 7.2.3-4, this is not true when the rms height is 0.71 $\lambda_o$ and the rms slope is 24.1°. On the other hand, it does appear to be true when the rms height is increased to 2.25 $\lambda_o$. Also recall from section 7.2.1, that for this same rms slope when the rms height was choose as 0.71 $\lambda_o$, a noticeable difference was observed between the quasi-specular model and the MOM result at large backscattering angles, but this difference disappeared when the rms height was increased.

The above two observations suggests that a ray bounce model for multiple scattering should not be employed for surfaces with significant spectral components three to ten times the wavelength of the incident field. This is because the intermediate scale components of the surface spectrum can add sufficient curvature to the surface such that the incident field is being diffracted, and not strictly ray scattered. In the case of the Gaussian surfaces with rms heights of 0.71 $\lambda_o$, scattering due to diffraction is greater than the specular scattering at large backscattering angles and so it is not surprising to discover that some of the multiple scattering is diffraction induced. Although it is not easily observed in these examples, the backscattering at large angles is in fact larger for TM polarization than for TE polarization. This suggest that the diffraction is greater for TM polarization and consequently one should expect any diffraction induced multiple scattering to also be larger for TM than for TE polarization. In section 7.5, this trend of greater diffraction using TM polarization is also observed. In the case of the surfaces with the rms heights of 2.25 $\lambda_o$ (and consequently much smaller rms curvature), the backscattering does not reveal the presence of diffraction and in this situation the multiple scattering can be described using a ray bouncing or ray tracing model.
Figure 7.2.3-4 Comparison between the Quasi-Specular Model and MOM Calculations for PEC Surfaces with Gaussian Spectra, $\sigma_h = 0.71 \lambda_o$ and $\sigma_h = 2.25 \lambda_o$, $\sigma_s = 24.1^\circ$, 80 $\lambda_o$, Long with $\Delta x = 0.1 \lambda_o$ Using 10/3/2 Evaluations ($\varepsilon_r = 2$) and $-/-/1$ (PEC), TM Polarization, and $g=15 \lambda_o$. 

$\sigma_h = 0.71 \lambda_o$ 

$\sigma_h = 2.25 \lambda_o$
7.3 Inversion Techniques

In this section, two techniques are introduced for determining the rms slope and reflection coefficient magnitude from the backscattered NRCS "data". The first technique, which will be called the two angle approach, involves direct manipulation of the quasi-specular model to obtain an analytical expression for the rms slope using the NRCS at two backscattering angles. The second approach involves using a numerical minimization procedure. In the minimization approach a function, called the merit function, is used to describe the difference between the model and the "data" for a given set of model parameters. The set of model parameters which give the smallest value for the merit function, i.e. its minimum, are the values which are said to best fit the model to the data. In section 7.3.2, a maximum likelihood estimator is derived for use in the minimization approach.

7.3.1 The Two Angle Approach for Estimating the Slope Variance

The quasi-specular model predicts that the dielectric constant of the scattering surface introduces no angular dependence in the backscattered field (see equation 7.1-2). Thus by taking the ratio of the backscattered NRCS at any two angles, the dependence of the NRCS on the dielectric constant can be removed leaving only a function of the surface slopes, i.e.

\[
\frac{\sigma_{bs}(\theta_1)}{\sigma_{bs}(\theta_2)} = \frac{\pi \sec^3(\theta_1) P \nu z'(\tan \theta_1)}{\pi \sec^3(\theta_2) P \nu z'(\tan \theta_2)}. \quad (7.3.1-1)
\]

The rough surfaces considered in this analysis are Gaussian distributed in height. Because the linear transformation of a Gaussian pdf is also Gaussian [Shanmugan and Breipohl 1988], the slopes will also be Gaussian distributed. For a Gaussian pdf the two angle equation (7.3.1-1) can be easily manipulated to determine the variance of the surface slopes.
\[ \sigma_s^2 = \frac{1}{2} \tan^2 \theta_2 \tan^2 \theta_1 \ln \left[ \frac{\sigma_s^2(\theta_1)}{\sigma_s^2(\theta_2)} \right] - 3 \ln \left[ \frac{\cos(\theta_2)}{\cos(\theta_1)} \right] \] (7.3.1-2)

where \( P_{\nu_\zeta}(\tan \theta) \equiv \frac{1}{\sigma_s \sqrt{2\pi}} \exp \left[ -\frac{\tan^2 \theta}{2\sigma_s^2} \right] \).

Once the slope variance in determined, the magnitude of the reflection coefficient can then be found using the NRCS at either of the two backscattering angles as,

\[ |\rho(0)| = \sqrt{\frac{\sigma_{bs}^2(\theta)}{\pi \sec^3 \theta P_{\nu_\zeta}(\tan(\theta))}}. \] (7.3.1-3)

7.3.2 Minimization using the Maximum Likelihood Estimator (MLE)

Alternatively, the data can be fit to the model using a minimization approach. In the minimization approach a function, called the merit function, is defined which describes the difference between the model and the data for a given set of model parameters. The set of model parameters which give the smallest value for the merit function, i.e. its minimum, are the values which best fit the model to the data. For example, a commonly used merit function is the least squares function,

\[ f_{\text{least square}}(a_1, a_2, \ldots, a_m) = \sum_{i=1}^{N} \left( \text{data}(x_i) - \text{model}(x_i, a_1, a_2, \ldots, a_m) \right)^2 \] (7.3.2-1)

where the "a's" are the model parameters. The least squares merit function is the Maximum Likelihood Estimator (MLE) for data that is independently Gaussian distributed with equal variance [Shanmugan and Breipohl 1988]. That is, if each data point \( \{ \text{data}(x_i); i = 1, 2, \ldots, N \} \) has some (random) error which is independent (of the other data points) and Gaussian distributed around the true value with the same variance. Then, the likelihood that a particular data set will occur is given by the product of the probabilities for each data point, or

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\[ L(a_1, a_2, \ldots, a_m) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi \sigma}} \exp\left( -\frac{(\text{data}(x_i) - \text{model}(x_i, a_1, a_2, \ldots, a_m))^2}{2 \sigma^2} \right) \]

(7.3.2-2)

where \( L \) is called the likelihood function.

Maximizing this likelihood function is the same as maximizing its logarithm.

\[ \ln(L(a_1, a_2, \ldots, a_m)) = -N \ln(\sqrt{2\pi \sigma}) - \sum_{i=1}^{N} \frac{(\text{data}(x_i) - \text{model}(x_i, a_1, a_2, \ldots, a_m))^2}{2 \sigma^2}. \]

(7.3.2-3)

Ignoring constant terms and multiplying equation (7.3.2-3) by \(-2\sigma^2\) yields the merit function given by equation (7.3.2-1). It should be noted that by multiplying equation (7.3.2-3) by a negative value converts the problem of maximizing the likelihood to a problem of finding the minimum.

In a similar fashion, a MLE for the average NRCS (obtained by averaging the NRCS from a finite number of surfaces realizations) can be determined if one can obtain the pdf for the NRCS at each backscattering angle. In the following two subsections, an approximate pdf for the average NRCS is derived, and used to construct a MLE merit function.

7.3.2.1 An Approximate Probability Density Function for the NRCS

Assume that the scattering by a rough surface can be described by a collection of \( N \) scattering centers randomly distributed about the surface. The scattered field in a given direction can then be represented as the complex sum of the fields scattered from each these points [Beckman 1963, p.119],

\[ \text{Scattered Field}(\theta_s) = \sum_{N} A_i e^{j\phi_i} = \sum_{N} (X_i + jY_i) \]

(7.3.2.1-1)

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If the number of scatters is sufficiently large and the scattering centers are randomly located at distances large compared to the wavelength, then the real and imaginary components of the scattered field will have Gaussian probability density functions (pdf). Therefore the magnitude of the field (given by the square root of the sum of the squares of the real and imaginary parts of the field) will have a Rayleigh pdf and the normalized radar cross section (proportional to the square of the magnitude) will have an exponential pdf [Starks and Wood 1986, p.93]. Interestingly, numerical studies have shown that the radar cross section is nearly exponentially distributed for as few as 6 point scatters [Daba and Bell 1995].

From statistics, the probability density function (pdf) of a sum of $n$ independent random variables is given by the convolution of their pdfs [Shanmugan and Breipohl 1988, p.91], i.e.,

$$ y = \sum_{i=1}^{n} x_i \Rightarrow pdf_y = pdf_{x1} * pdf_{x2} * pdf_{x2}...pdf_{xn} \quad (7.3.2.1-2) $$

Since the NRCS from each surface realization is independent and exponentially distributed, a Laplace transform can be used to rewrite the above expression as,

$$ L_t[\text{pdf}_y] = L_t[\text{pdf}_{x1}]L_t[\text{pdf}_{x2}]L_t[\text{pdf}_{x2}]...L_t[\text{pdf}_{xn}] = L_t[\text{pdf}_x]^n \quad (7.3.2.1-3) $$

where $L_t[f]$ is the Laplace transform of the function $f$,

$$ pdf_x = \frac{1}{\sigma} e^{-\frac{x}{\sigma}} \quad (x > 0) $$

is the pdf of the NRCS for one surface realization,

$pdf_y$ is the pdf for a sum of NRCSs, and

$\sigma$ is the standard deviation, which is also equal to the mean for the exponential distribution!
From the CRC’s Laplace transform table [Beyer 1989], one finds

\[
L_t[pdf_z] = L_t\left[\frac{1}{\sigma} e^{-\frac{x}{\sigma}}\right] = \frac{1}{s + \frac{1}{\sigma}}
\]

(7.3.2.1-4)

Raising equation (7.3.2.1-4) the to power \(n\) (the number of RCS being averaged) and taking the inverse Laplace transform yields,

\[
pdf_y = L_t^{-1}\left[\left(\frac{1}{s + \frac{1}{\sigma}}\right)^n\right] = \frac{1}{(n-1)!} \frac{y^{n-1}}{\sigma^n} \exp\left[-\frac{y}{\sigma}\right] \text{ for } y > 0
\]

(7.3.2.1-5)

Finally to get the pdf for the sample average NRCS, one simply divides \(y\) by number of surfaces averages, such that

\[
z = \frac{1}{n} y = \frac{1}{n} \sum_{i=1}^{n} x_i
\]

(7.3.2.1-6)

where, \(z\) is the population average for the NRCS in one scattering direction.

Therefore, after a simply linear transformation [Shanmugan and Breipohl 1988, p.67], one arrives at an approximate pdf for the average NRCS

\[
pdf_z(z) = \frac{n}{(n-1)!} \frac{(nz)^{n-1}}{\sigma^n} \exp\left[-\frac{nz}{\sigma}\right].
\]

(7.3.2.1-7)

From the above pdf it is straight forward to determine the mean and standard deviation of the average NRCS,

\[
\text{mean} = E[z] = \int_{0}^{\infty} z \frac{n}{(n-1)!} \frac{(nz)^{n-1}}{\sigma^n} \exp\left[-\frac{nz}{\sigma}\right] dz
\]

\[
= \int_{0}^{\infty} \frac{1}{(n-1)!} \frac{(nz)^{n}}{\sigma^n} e^{-\frac{nz}{\sigma}} dz
\]
which after performing a change of variable \( x = \frac{-nx}{\sigma} \) yields,

\[
= \frac{\sigma}{n(n-1)!} \left[ \int_0^\infty x^n e^{-x} \, dx \right] = \frac{\sigma}{n!} [n!] = \sigma. \tag{7.3.2.1-8}
\]

Therefore, the mean of the average NRCS is (not surprisingly) the mean value of the NRCS. Furthermore, the variance is given by,

\[
\text{variance} = E[(x - \text{mean})^2] = E[x^2] - \text{mean}^2
\]

\[
= \int_0^\infty x^2 \frac{n}{(n-1)!} \left( \frac{nx}{\sigma} \right)^n \exp\left[ -\frac{nx}{\sigma} \right] dx - \sigma^2
\]

\[
= \int_0^\infty \frac{\sigma}{n(n-1)!} \left( \frac{nx}{\sigma} \right)^{n+1} \exp\left[ -\frac{nx}{\sigma} \right] dx - \sigma^2
\]

\[
= \frac{\sigma^2}{n^2(n-1)!} \left[ \int_0^\infty x^{n+1} e^{-x} \, dx \right] - \sigma^2 = \frac{\sigma^2}{n^2(n-1)!} [(n+1)!] - \sigma^2
\]

\[
= \frac{(n+1)\sigma^2}{n} - \sigma^2 = \frac{\sigma^2}{n} \tag{7.3.2.1-9}
\]

Therefore, the standard deviation of the fluctuations in the average NRCS can only be expected to (1) drop as one over the square root of the number of realizations used to obtain the average and (2) be proportional to the average value.
7.3.2.2 A Maximum Likelihood Estimator for the NRCS

In this section, the approximate pdf for the NRCS derived in previous section is used to obtain a MLE for the NRCS. Under the assumption that the scattering centers are randomly positioned and distance larger than a wavelength (that is, there is no phase correlation between the scattered fields produced at each scattering center) and that the average NRCS is an independent random variable at each scattering angle, the likelihood function for the averaged NRCS is given by the product of the probabilities at each backscattering angle, i.e.,

\[
L_{\text{Average}_\text{NRCS}}(\sigma_s, \rho(0^\circ)) = \prod_{j=1}^{N} \frac{n}{(n-1)!} \left( \frac{n \cdot \text{data}(\theta_j)}{\sigma_j^n} \right) \exp \left[ -\frac{n \cdot \text{data}(\theta_j)}{\sigma_j} \right]
\]

(7.3.2.2-1)

where \( N \) is the number of backscattering angles,

\( n \) is the number of single surface radar cross sections used to obtain the average,

\( \theta_j \) are the backscattering angles at which the model and data are compared, and

\( \sigma_j \) is the value of the average NRCS given by the model, which in this case is the quasi-specular model, i.e.

\[
\sigma_j = \sigma(\theta_i, \sigma_s, \rho(0^\circ)) = \text{quasi}_\text{specular}(\theta_i, \sigma_s, \rho(0^\circ)).
\]

(7.3.2.2-2)

In this study, the "data" will be the MOM generated average NRCS.

Finally, taking the natural log of equation (7.3.2.2-1), omitting constant terms, and multiplying by -\( n \) yields a simple merit function which can be minimized to obtain the model parameters (i.e. the rms slope and the reflection coefficient) which best fit the quasi-specular model to the data.

\[
f_{\text{NRCS}}(\sigma_s, \rho(0^\circ)) = \sum_{i=1}^{N} \left[ \ln[\sigma(\theta_i, \sigma_s, \rho(0^\circ))] + \frac{\text{data}(\theta_i)}{\sigma(\theta_i, \sigma_s, \rho(0^\circ))} \right]
\]

(7.3.2.2-3)

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7.3.3 Minimization using the Least Absolute Deviate

In addition to the MLE estimator derived above, a more traditional minimization based on the least absolute difference, equation (7.3.3-1), also known as the L1 norm will be examined in sections 7.4 and 7.5. This minimization function is well known for its stability and is more robust with respect to outlying data points (that is the effect of data points which are unlikely to occur) than the least squares merit function based on Gaussian statistics [Shanmugan and Breipohl 1988].

\[ f_{L1}(a_1, a_2, \ldots, a_m) = \sum_{i=1}^{N} \left[ \text{data}(x_i) - \text{model}(x_i, a_1, a_2, \ldots, a_m) \right]. \]  

(7.3.3-1)

Also, the data will be analyzed using this same merit function, but with both the data and the model first converted into dB, i.e.

\[ f_{L1\_dB}(a_1, a_2, \ldots, a_m) = \sum_{i=1}^{N} \left| 10 \log \left[ \text{data}(x_i) \right] - 10 \log \left[ \text{model}(x_i, a_1, a_2, \ldots, a_m) \right] \right|. \]  

(7.3.3-2)

This second case, equation (7.3.3-2), will be referred to as "minimization in dB" when used in sections 7.4 and 7.5. It is useful to examine the radar cross section data in a log scale, i.e. in dB, because this data may have values which differ (depending on the scattering angle) by several orders of magnitude.

Lastly, there are a large number of numerical techniques for finding the minimum of a function [Press et al 1989]. In this study, a simple downhill simplex method was employed because this technique only requires function evaluations (no derivatives) and proved to be very robust.
7.4 Inversion using Monte Carlo Simulated Data

In this section, the effect of fluctuations in the numerically computed average NRCS (due to the finite number of samples used to construct the average) on the inversion procedures is examined. Essentially, these fluctuations cause the estimated parameters to themselves be random variables. Therefore this analysis will require specifying both the accuracy of the model (i.e. the error in the estimates) and the precision with which these values have been determined. From a statistical point of view, we want to know the so called "goodness of fit" for the estimated parameters. As an example of how these fluctuations effect the estimation of the surface parameters, figure 7.4-1 plots two averaged normalized radar cross sections obtained by averaging the MOM calculated NRCS from two different sets of 200 surfaces. After minimization using the least absolute merit function, equation (7.3.3-1), the first data set is found to have a relative error in the estimated rms slope of 0.1 % while the second data set is found to have an relative error of 4.6 %. The relative error is defined by,

\[
\text{relative error} = \frac{\text{Known Value} - \text{Estimated Value}}{\text{Known Value}}. \tag{7.4-1}
\]

It should also be noted that in this example, the estimates of the magnitude in the reflection coefficient are greater than one - a situation which is physically impossible. This effect occurs because: (1) as was explored in section 7.2.2, the average NRCS is under predicted by the quasi-specular model due to the effects of multiple scattering. As a result, when the model is fit to the data, a reflection coefficient greater than one in needed to compensate for this additional scattering. (2) In the minimization process, no constraint was placed on the maximum value allowed by the reflection coefficient. Although such a constraint can easily be imposed in the minimization approach, the same is not true for the two angle approach which yields similarly high values for the reflection coefficient. Also, constraining the reflection coefficient to be one or less is only effective.
Inversion Technique: L1 minimization of backscattered NRCS at $0^\circ$, $0.5^\circ$ ... $52^\circ$

![Graph showing backscattered NRCS against backscattering angle with various fits and data sets.]

Figure 7.4-1 Example of Parameter Estimation in the Presence of Finite Sample Averaged NRCS 200 PEC Surface Realizations with Gaussian Spectra, $\sigma_b = 0.71 \lambda_o$ and $\sigma_s = 24.10^\circ$, $80 \lambda_o$ Long with $\Delta x = 0.1 \lambda_o$ Using 10/1 Evaluations, TE Polarization, and $g = 15 \lambda_o$. 
in preventing overestimation of the reflection coefficients for PECs (or at the very best highly reflective materials). The multiple scattering occurs for dielectric materials, as will be shown in section 7.5, and also results in overestimation of their reflection coefficients. Finally, constraining the reflection coefficient can be effective when large angular regions of the backscattered data are being used - in effect causing the minimization routine to ignore the regions where the NRCS is too large to be matched. However, if the region of interest is restricted to a small angular region, constraining the reflection coefficient in the minimization routines often results in completely erroneous values for the rms slope.

7.4.1 Fluctuations in the NRCS and the Two Angle Approach

A significant difficulty with the two angle approach is that an estimate based on the NRCS at only two backscattering angles is very sensitive to fluctuations in the average NRCS data. If the backscattered NRCS is known at several angles, then results of the two angle approach can be averaged over all possible pairs of angles to reduce the impact of the fluctuations. This approach was used by Y. Kim [1995] for data obtained from the planet Venus. Figure 7.4.1-1, for example, plots a case where the estimated slopes, obtained using the two angle approach, are particularly bad. When the two angles, used to obtain the slope estimate are close together, the fluctuations in the NRCS can cause the NRCS to appear to increase as the backscattering angle is increasing, a situation which produces extreme outliers (including negative values) for the slope variance. As a result, some kind of windowing (which prevents the use of angles which are "close" together) or a more robust estimator of the average must be used. In this study both a square windowing function and a partial averaging procedure (which ignores a fixed percentage of the highest and lowest values) are used. Both techniques are found to produce reasonable results.
Inversion Technique: Two Angle Approach with $\theta_1 = 10^\circ$ and $\theta_2 = 0^\circ, 0.5^\circ \ldots 52^\circ$

Figure 7.4.1-1 Example of Two Angle Estimate of Slope Variance from Backscattered NRCS
200 PEC Surface Realizations with Gaussian Spectra, $\sigma_h = 0.71 \lambda_o$ and $\sigma_s = 24.10^\circ$, $80 \lambda_o$ Long with $\Delta x = 0.1 \lambda_o$ Using $10/-/1$ Evaluations, TE Polarization, and $g = 15 \lambda_o$. 
7.4.2 Fluctuations in the NRCS and Confidence Limits

Because the fluctuations in the MOM generated average normalized radar cross section (due to the finite number of samples used to construct the average) cause the estimated parameters to fluctuate, some measure of the precision with which the estimates have been obtained is needed. In this section, it is argued that confidence limits based on the assumption of Gaussian statistics are an appropriate measure. This is accomplished by subdividing the total number of cross sections available to be averaged into sets of 200 to 1600 cross sections. The inversion techniques developed in section 7.3 are then used to estimate the surface parameters for each set. The resulting estimates (or more precisely their errors relative to the exact values) are then calculated and compared to a Gaussian distribution using the Kolmogorov-Smirnov test. It is found that the distribution of the errors compares favorably with a Gaussian distribution. Also, it is shown that the Gaussian confidence limits remain essentially unchanged regardless of the number of sets into which the total number of cross sections are divided.

For example, figure 7.4.2-1 plots a histogram of the relative error in the estimated rms slope and the estimated magnitude of the reflection coefficient obtained from the average NRCS grouped into 96 sets of 200 cross sections each. Each cross section is obtained by a MOM solution for a surface with $\varepsilon_r = 50.0$ and a Gaussian spectrum ($\sigma_n = 0.71 \lambda$, and $\sigma_r = 17.55\%$) using TM polarization and a spot size of 5 wavelengths. In this figure, the two angle approach with partial averaging (introduced in the previous section) is used to obtain the estimated parameters. When this data is compared to a Gaussian pdf using the Kolmogorov-Smirnov Test [Press et al 1989], it returns a significance level (i.e. a pdf match) of 80.5% for the rms slope and 41.9% for the reflection coefficient. Therefore, if it is hypothesized that "this random variable is Gaussian distributed" then the Kolomogorov-Smirnov tests indicates that we can not reject this hypothesis with any
Figure 7.4.2-1 Histogram of the Relative Error in the Estimated RMS Slope and Reflection Coefficient Using the Two Angle Approach Averaged (10/90) Over All Pairs of Angles 0°, 0.5° ... 65° for 96 sets of 200 Surface Realizations with Gaussian Spectra, \( \sigma_h = 0.71 \lambda_o \) and \( \sigma_s = 17.55^\circ \), \( \varepsilon_r = 50.0, 40 \lambda_o \) Long with \( \Delta x = 0.1 \lambda_o \) Using 10/3/2 Evaluations, TM Polarization, and \( g = 5 \lambda_o \).
confidence. It does NOT mean that the random variable is Gaussian distributed. However, it does suggest that computing a confidence limit based on the assumption of Gaussian statistics is a reasonable measure for the "goodness of fit."

As another example, figure 7.4.2-2 plots a histogram of the relative error in the estimated slope variance and the estimated magnitude of the reflection coefficient obtained from 48 sets of 200 surfaces each with a larger rms slope than the previous example. In this second example the scattering surface is a PEC with $\sigma_h = 0.71 \lambda_o$ and $\sigma_s = 24.1^\circ$, and minimization with the L1 norm is used to obtain the estimated parameters. When this histogram is compared to a Gaussian pdf using the Kolmogorov-Smirnov Test, it returns a significance level (pdf match) of 93.0% for the rms slope and 98.2% for the reflection coefficient. It should be stressed that these two examples are not "best case" results but appear to be quite typical. (These are the only cases for which more than 5000 realizations were generated). In fact, a significance level of only 41.9% for the reflection coefficient in the first example is unusually low. Significance levels of greater than 65% are the norm and values above 90% are not rare.

Interestingly, in a few cases the two angle approach was found to give pdf match of less than 30%, with both the windowing and partial averaging approaches. When the two angle results were reexamined, it was discovered that a few erroneous and very large estimates, i.e. severe outliers had still managed to influence the results. When the size of the window or the reject percentage of the partial averaging technique was then increased, both techniques again yielded Gaussian-like distributions.
Figure 7.4.2-2 Histogram of the Relative Error in the Estimated RMS Slope and Reflection Coefficient Using L1 Minimization Over All Angles 0°, 0.5° ... 30° for 48 sets of 200 Surface Realizations with Gaussian Spectra, $\sigma_h = 0.71 \lambda_o$ and $\sigma_s = 24.1°$, PEC, 80 $\lambda_o$ Long with $\Delta x = 0.1 \lambda_o$ Using 10/-/1 Evaluations, TE Polarization, and $g = 15 \lambda_o$.
Naturally the question arises as to whether it is better to estimate the surface parameters from 96 sets of 200 cross sections each or 48 sets of 400 each. The average relative error, standard deviation, 95% confidence limit and Kolmogorov-Smirnov test results for several possible configurations are summarized in the following tables for the two previous examples.

Table 7.4.2-1a Relative Error in Estimated RMS Slope
(for a Gaussian spectrum with \( \sigma_h = 0.71 \lambda_0 \) and \( \sigma_z = 17.55^\circ \))

<table>
<thead>
<tr>
<th></th>
<th>Average Relative Error</th>
<th>Standard Deviation</th>
<th>95% Confidence Limit</th>
<th>Kolmogorov Smirnov Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>96 sets of 200</td>
<td>-0.0383</td>
<td>0.0349</td>
<td>0.0071</td>
<td>80.5 %</td>
</tr>
<tr>
<td>48 sets of 400</td>
<td>-0.0410</td>
<td>0.0270</td>
<td>0.0078</td>
<td>50.9 %</td>
</tr>
<tr>
<td>24 sets of 800</td>
<td>-0.0415</td>
<td>0.0205</td>
<td>0.0086</td>
<td>81.7 %</td>
</tr>
<tr>
<td>12 sets of 1600</td>
<td>-0.0421</td>
<td>0.0134</td>
<td>0.0085</td>
<td>93.2 %</td>
</tr>
<tr>
<td>1 set of 19200</td>
<td>-0.0406</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 7.4.2-1b Relative Error in Estimated Reflection Coefficient
(for a Gaussian spectrum with \( \sigma_h = 0.71 \lambda_0 \) and \( \sigma_z = 17.55^\circ \))

<table>
<thead>
<tr>
<th></th>
<th>Average Relative Error</th>
<th>Standard Deviation</th>
<th>95% Confidence Limit</th>
<th>Kolmogorov Smirnov Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>96 sets of 200</td>
<td>0.0311</td>
<td>0.0237</td>
<td>0.0048</td>
<td>41.9 %</td>
</tr>
<tr>
<td>48 sets of 400</td>
<td>0.0349</td>
<td>0.0170</td>
<td>0.0050</td>
<td>86.0 %</td>
</tr>
<tr>
<td>24 sets of 800</td>
<td>0.0364</td>
<td>0.0127</td>
<td>0.0053</td>
<td>71.1 %</td>
</tr>
<tr>
<td>12 sets of 1600</td>
<td>0.0381</td>
<td>0.0072</td>
<td>0.0046</td>
<td>95.5 %</td>
</tr>
<tr>
<td>1 set of 19200</td>
<td>0.0376</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 7.4.2-2a Relative Error in Estimated RMS Slope
(for a Gaussian spectrum with $\sigma_h = 0.71 \lambda_s$ and $\sigma_s = 24.1\%$)

<table>
<thead>
<tr>
<th></th>
<th>Average Relative Error</th>
<th>Standard Deviation</th>
<th>95% Confidence Limit</th>
<th>Kolmogorov Smirnov Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>48 sets of 200</td>
<td>0.0674</td>
<td>0.0373</td>
<td>0.0108</td>
<td>93.0 %</td>
</tr>
<tr>
<td>24 sets of 400</td>
<td>0.0691</td>
<td>0.0245</td>
<td>0.0103</td>
<td>96.4 %</td>
</tr>
<tr>
<td>12 sets of 800</td>
<td>0.0728</td>
<td>0.0171</td>
<td>0.0109</td>
<td>64.0 %</td>
</tr>
<tr>
<td>1 set of 9600</td>
<td>0.0719</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 7.4.2-2b Relative Error in Estimated Reflection Coefficient
(for a Gaussian spectrum with $\sigma_h = 0.71 \lambda_s$ and $\sigma_s = 24.1\%$)

<table>
<thead>
<tr>
<th></th>
<th>Average Relative Error</th>
<th>Standard Deviation</th>
<th>95% Confidence Limit</th>
<th>Kolmogorov Smirnov Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>48 sets of 200</td>
<td>-0.0229</td>
<td>0.0104</td>
<td>0.0030</td>
<td>98.2 %</td>
</tr>
<tr>
<td>24 sets of 400</td>
<td>-0.0220</td>
<td>0.0087</td>
<td>0.0037</td>
<td>85.7 %</td>
</tr>
<tr>
<td>12 sets of 800</td>
<td>-0.0219</td>
<td>0.0053</td>
<td>0.0033</td>
<td>84.0 %</td>
</tr>
<tr>
<td>1 set of 9600</td>
<td>-0.0232</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Interestingly, the confidence limits remain roughly the same regardless of the number of cross sections in each set. This occurs because the Gaussian confidence limits are proportional to standard deviation and inversely proportional to the square root of the number of data points (i.e. the number of sets). One can observe in the above tables that the standard deviation of the error in the estimated parameters is decreasing at each step by roughly the square root of 2. This is not completely surprising since the fluctuations in the NRCS are expected to reduce as approximately one over the square root of the number of cross sections averaged (see section 7.3.2). Thus in regards to confidence limits, one can conclude that the manner in which the radar cross sections are combined does not matter significantly so long as the number of radar cross sections
averaged in each set is sufficiently large to obtain a good average and the number of sets is sufficiently large to obtain an accurate assessment of the spread in the estimates.

7.5 Inversion results

In this section, the inversion techniques developed in section 7.3 are applied to the MOM-generated average normalized radar cross sections. Because the exact surface parameters used in computing these cross sections are known, the accuracy of the model's estimates can be obtained.

Figures 7.5-1a and 7.5-1b illustrate the type of results which will be examined in this section. Figure 7.5-1a plots the relative error in the estimated rms slope and figure 7.5-1b plots the relative error in the estimated reflection coefficient. As in section 7.4, the relative error is defined as,

$$\text{relative error} = \frac{\text{Known Value} - \text{Estimated Value}}{\text{Known Value}}$$

In both these plots, the rms surface height has as been fixed at 0.71 $\lambda_o$ while the rms slope and the (normal incidence flat surface) reflection coefficient of the scattering surface have been allowed to vary. The exact reflection coefficient is plotted along the horizontal axis. Also, both the TE (h) and TM (v) polarization results are given, where the reflection coefficient has been defined such that it is positive in the TM case and negative in the TE case. The relative error for several rms slopes are plotted as piecewise continuous line segments with a label specifying the exact rms slope. In figure 7.5-1a, for example, the lines (one TE and one TM) which are closest to the top of the graph give the relative error in the estimated rms slope when the actual rms slope is 32.31°. Finally, the error bars are the approximate Gaussian confidence limits (see section 7.4.2 for more information on the Gaussian confidence limits). The size of the confidence intervals

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Figure 7.5-1a Effect of the Exact Slope and Reflection Coefficient on the Estimated Slope
For Surfaces with Gaussian Spectra, $\sigma_h = 0.71 \lambda_0$ and $\sigma_s = 17.55^\circ$, 24.1$^\circ$, 28.71$^\circ$ and 32.31$^\circ$
Using L1 Minimization of the Quasi-Specular Model to the MOM Computed Average
Backscattered NRCS 0$^\circ$, 0.5$^\circ$ to 45$^\circ$
Figure 7.5-1b Effect of the Exact Slope and Exact Reflection Coefficient on the Estimated Reflection Coefficient for Surfaces with Gaussian Spectra, $\sigma_h = 0.71 \lambda_o$ and $\sigma_s = 17.55^\circ, 24.1^\circ, 28.71^\circ$ and $32.31^\circ$. Using L1 Minimization of the Quasi-Specular Model to the MOM Computed Average Backscattered NRCS $0^\circ, 0.5^\circ$ to $45^\circ$
depend on a number of factors such as the total number of surface realizations used to obtain the average cross section and the number of backscattering angles used to obtain estimated parameters.

Figure 7.5-1a shows the impact of several of the differences identified in section 7.2 between the quasi-specular model and the MOM calculations. In section 7.2 it was observed that (1) as the rms slope was increased from 17.55° to 24.1° there was an increase in the multiple scattering and (2) that the multiple scattering was stronger for TM than TE polarization for PEC surfaces but weaker for TM than TE polarization with low dielectric constants. The differences between the two polarizations are a result of differences in the multiple scattering (due to the effects of curvature induced diffraction and Brewster angle effects). The inversion results shows these same trends. (1) As the rms slope is increased above 17.55°, the (magnitude of the) error in both the estimated rms slope and estimated reflection coefficient also increases, and (2) the error is a strong function of the dielectric constant of the material (the error being larger for larger dielectric constants) and polarization (the error being larger for TM than TE polarization with PEC surfaces and smaller for TM than TE polarization with weak dielectric surfaces). In this example, minimization based on the least absolute merit function (i.e. the L1 norm) was used over the backscattered angular region from 0 to 45 degrees. The following subsections will examine how these results are affected by the inversion technique, the angular region examined, and the rms surface height.

7.5.1 Comparison of Inversion Techniques

In this section, all of the inversion techniques developed in section 7.3 are applied to a typical case in order to demonstrate the effect of the inversion approach on the estimated parameters. In particular, the scattering surfaces used here will have rms heights of 0.71 \( \lambda_o \), rms slopes of 24.1°, and real dielectric constants which for a flat surface at normal incidence would give reflection coefficients of 1.0, 0.75, 0.5, and 0.2. Chapter 7, Estimation of Surface Parameters using the Quasi-Specular Model - page 192
It is shown that if the backscattered NRCS changes significantly (roughly more than about 1 order of magnitude) in the angular region which is used to estimate the surface parameters, then the inversion techniques of section 7.3 can produce widely varying results. When however the angular region of interest is sufficiently narrow, all of the estimation procedures yield equivalent results.

Figures 7.5.1-1a and 7.5.1-1b compare the relative error in the estimated parameters using the backscattered NRCS from 0 to 65 degrees obtained via: (1) the two angle approach using a 10/90 average, i.e. the largest and smallest 10% of the values obtained using the two angle approach over all pairs of angles are ignored, (2) the two angle approach using a 10° window, i.e. using the two angle approach over all pairs of angles more than 10° apart, (3) L1 minimization, i.e. minimization of the difference between the quasi-specular model and the MOM data based on equation (7.3.3-1), (4) L1 minimization in dB, i.e. minimization based on equation (7.3.3-2), and (5) minimization using the maximum likelihood estimator (MLE), derived in section 7.4.2.

Figure 7.5.1-1a shows that the error in the estimated rms slope is significantly influenced by the inversion approach. The relative error in the rms slope is several percent larger using L1 minimization (in a linear scale) than for any of the other inversion techniques. On the other hand, the "minimization in dB" and minimization based on the maximum likelihood estimator produced the smallest error. It should also be noted that the relative error using the L1 minimization (linear scale) is positive (except at εr = 2.0) which means that the estimated rms slope is too small, while for the minimization in dB and the maximum likelihood estimator approaches, the relative error is negative which means that the estimated rms slope is too large (see also section 7.4-1). Finally, as the dielectric constant of the material is reduced, the relative error obtained using the maximum likelihood estimator approach remains roughly the same, while the relative error obtained for the other approaches decreases.
Figure 7.5.1-1a Comparison of Inversion Techniques for Estimating the RMS Slope Using the Average Backscattered NRCS from 0°, 0.5° to 65° for Surfaces with Gaussian Spectra, $\sigma_h = 0.71 \lambda_o$ and $\sigma_s = 24.10^\circ$
Figure 7.5.1-1b Comparison of Inversion Techniques for Estimating the (Normal Incidence Flat Surface) Reflection Coefficient using the Average Backscattered NRCS from 0°, 0.5° to 65° for Surfaces with Gaussian Spectra, $\sigma_h = 0.71 \lambda_0$ and $\sigma_s = 24.10°$
Why are there such significant differences between the inversion approaches? Recall from the discussion of section 7.2 that the MOM calculated NRCS departs from the quasi-specular model in two important ways. First, at large backscattering angles, surface curvature (tied to the presence of intermediate scale roughness) causes the average NRCS to be larger than is predicted by the quasi-specular model and more importantly to decay much more slowly (with respect to the backscattering angle) than is predicted by the quasi-specular model. As a result of the slow drop off in the NRCS, one would expect that the estimate of the rms slope will be too large. Also, because increasing the rms slope greatly increases the magnitude of the quasi-specular model at large backscattering angles, the estimated reflection coefficient is reduced to compensate for this additional scattering. Conversely at small backscattering angles, multiple scattering causes the NRCS to be larger and most of the time to decay more rapidly than is predicted by the quasi-specular model. As a result of the more rapid reduction in the NRCS, the estimated rms slope will be too small. To compensate for both the additional scattering cross section (due to the multiple scattering) and the small estimate in the rms slope, the estimated reflection coefficient generally becomes too large.

When however the entire backscattered NRCS is used to estimate the rms slope, the two sources of error tend to cancel each other out, and because the different inversion approach are influenced to a different degree by each type of error, they produce different results. In the case of the least absolute merit function, (i.e. the L1 norm) each data point (i.e. the average NRCS at each backscattering angle) is weighted equally on a linear scale. Thus the L1 minimization places most emphasis on differences in the nadir region because the NRCS is largest in this region. Since the error due to multiple scattering is largest in the near nadir region, this minimization approach is the most sensitive to the multiple scattering. Accordingly, the L1 minimization estimates show more sensitivity to the dielectric constant, and for the larger dielectric constants yield rms slopes which are too small and reflection coefficients which are too large. On the other hand, the
"minimization in dB", which uses a 10 base logarithm, and the MLE, which uses a natural log, emphasizes the region far from normal incidence where small values are amplified. Therefore, the rms slope estimates are too large, and the estimated reflection coefficients tend towards smaller values. Finally the two angle approach produces results which generally lie somewhere between the MLE and L1 minimization approaches. Essentially, the two angle approach treats the estimate from every pair of angles equally, and so the errors in the two regions are more evenly balanced.

When the angular region of interest is sufficiently narrow all of the estimation procedures will yield equivalent result. This is demonstrated in figures 7.5.1-2 and 7.5.1-3, where the angular region has been reduced such that the backscattered cross section changes by roughly 1 order of magnitude. It should be noted that in both of these figures, not only do the two angle, MLE and L1 approaches produce equivalent results, but the (magnitude of the) relative error in the estimated rms slope has increased in both cases from that of figure 7.5.1-1a.

Not surprisingly, in figure 7.5.1-2 where the backscattering angles are relatively small, the rms slope estimates show even more sensitivity to the dielectric constant. In figure 7.5.1-3, where the error in the backscattering cross section is dominated by curvature induced errors, no significant sensitivity to the dielectric constant is observed. Finally, in both figures the confidence limits have become noticeably larger. This is also not surprising since as the number of angles used to obtain the estimated is decreased, one would expected a greater sensitivity to random fluctuations in the numerically approximated average of the radar cross section.
Figure 7.5.1-2 Comparison of Inversion Techniques Using the Average Backscattered NRCS from 0°, 0.5° to 30° for Surfaces with Gaussian Spectra, $\sigma_h = 0.71 \lambda_o$ and $\sigma_s = 24.10^\circ$
Figure 7.5.1-3 Comparison of Inversion Techniques Using the Average Backscattered NRCS from 60°, 60.5° to 65° for Surfaces with Gaussian Spectra, $\sigma_n = 0.71 \lambda_0$ and $\sigma_s = 24.10°$
In general, it should be realized that the difference between the MOM generated average cross section and the quasi-specular model varies significantly with respect to the backscattering angle. By using a range of angles to obtain an estimate, these errors are in some sense smoothed. In the following two sections, the effect of varying the dielectric constant, rms slope, and rms height of the surface on the estimated (normal incidence flat surface) reflection coefficient and the rms slope are investigated by using the average NRCS from 0 to 45 and from 45 to 68 degrees.

7.5.2 Parameter Estimation with the RMS Height Fixed

This section examines the impact of changing the rms slope and dielectric constant of the surface upon the relative error in the estimated reflection coefficient and the estimated rms slope. Figures 7.5.2-1a and 7.5.2-1b plot the relative error in these estimated surface parameters, when the rms height ($\sigma_h$) is fixed at 0.71 $\lambda_0$. The solid line in these figures is the same result which was discussed in the introductory portion of section 7.5. What has been added to figures here are the inversion results based on the Maximum Likelihood Estimator (MLE). One observes that in both figures, the MLE results show very much the same trends as the L1 minimization results. Following the discussion of the previous section, the L1 minimization results emphasize the smaller backscattering angles while the MLE minimization results emphasize the larger backscattering angles. In figure 7.5.2-1a, one observes that error in estimates obtained using L1 minimization are larger than the errors obtained using MLE minimization. Therefore, the errors in the estimated parameters (and by inference the difference between the exact NRCS and quasi-specular result) increases as the rms slope increases. This observation is in agreement with the comparisons of section 7.2. Figure 7.5.2-2b, shows that both of the minimization approaches yield estimated reflection coefficients which are too large. This is largely due to the additional scattering added by multiple scattering mechanisms. As with the error in the estimated rms slope, the magnitude of the error in the estimated reflection coefficient increases as the rms slope increases.

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Figure 7.5.2-1a Effect of the Exact Slope and Reflection Coefficient on the Estimated Slope for Surfaces with Gaussian Spectra, $\sigma_h = 0.71 \lambda$, and $\sigma_s = 17.55^\circ, 24.1^\circ, 28.71^\circ$ and $32.31^\circ$ (Estimated values were obtained using L1 and MLE minimization at the backscattering angles $0^\circ, 0.5^\circ \ldots 45^\circ$)
Figure 7.5.2-1b Effect of the Exact Slope and Reflection Coefficient on the Estimated Reflection Coefficient for Surfaces with Gaussian Spectra, $\sigma_n = 0.71 \lambda_s$ and $\sigma_s = 17.55^\circ, 24.1^\circ, 28.71^\circ$ and $32.31^\circ$, (Estimated values were obtained using L1 and MLE minimization at the backscattering angles $0^\circ, 0.5^\circ..45^\circ$)
The results of figures 7.5.2-1a and 1b are obtained with the rms height fixed at 0.71 \( \lambda_o \). However, these same trends are observed at all rms heights examined in this study, that is rms heights ranging from 2.25 to 0.25 wavelengths. Section 7.5.3 will discuss the impact of the rms height in more detail.

Turning to the range of backscattering angles from 45 to 68 degrees, figures 7.5.2-2a and 7.5.2-2b plot the relative error in the estimated reflection coefficient and the estimated rms slope. In these figures, both the two angle and MLE approaches are used to obtain the estimated parameters. Because the radar cross section changes by several orders of magnitude in the backscattering angular region from 45 to 68 degrees, it is not surprising that the two approaches yield somewhat different results. None the less, both techniques show that as the rms slope is increased the magnitude of the relative error in both the estimated rms slope and estimated reflection coefficient increases.

At first, these results may seem contrary to section 7.2.1 where it was stated that increasing the rms slope should improve agreement between the MOM calculations and the quasi-specular model. The resolution of these apparently contradictory results lies in the angular region used to obtain the estimated parameters. In section 7.2, the expression "large backscattering angles" was essentially being used qualitatively to describe the angular region where the NRCS rapidly decreases and difference between the MOM results and the model are large. For surfaces with a Gaussian spectra and a rms slope of 24.1°, the NRCS begins this rapidly decrease at backscattering angle greater than roughly 50 degrees. However, for rms slopes of 28.7° or 32.3°, the NRCS does not begin this rapid decrease until much larger backscattering angles (roughly 70 degrees). As a result, figure 7.5.2 is somewhat misleading, because increasing the rms slope has not so much reduced the error as shifted it towards larger backscattering angles. This effect is shown in demonstrated figure 7.5.2-3.
Figure 7.5.2-2a Effect of the Exact Slope and Reflection Coefficient on the Estimated Slope For Surfaces with Gaussian Spectra, $\sigma_h = 0.71 \lambda_o$ and $\sigma_s = 24.1^\circ, 28.71^\circ$ and $32.31^\circ$ Evaluated by Applying MLE Minimization of the Quasi-Specular Model and the Two Angle Equation (with 05/95 Averaging) to the Average Backscattered NRCS from $45^\circ$, $45.5^\circ$ to $68^\circ$
Figure 7.5.2-2b Effect of the Exact Slope and Reflection Coefficient on the Estimated Reflection Coefficient for Surfaces with Gaussian Spectra, $\sigma_h = 0.71 \lambda_o$ and $\sigma_s = 24.1^\circ, 28.71^\circ$ and $32.31^\circ$

Evaluated by applying MLE Minimization of the Quasi-Specular Model and the Two Angle Equation (with 05/95 Averaging) to the Average Backscattered NRCS from $45^\circ, 45.5^\circ$ to $68^\circ$
Figure 7.5.2-3 Comparison between the Quasi-Specular Model and MOM Calculations for 2000 Surfaces with Gaussian Spectra, \( \sigma_h = 0.71 \lambda_0 \), \( \sigma_s = 24.1^\circ \) and 32.3\(^\circ \), 80 \( \lambda_0 \) Long with \( \Delta x = 0.1 \lambda_0 \), Using 10/3/2 Evaluations (\( \varepsilon_r = 2 \)) and 10/-1/1 (PEC), and \( g = 15 \lambda_0 \)
In summary, section 7.5.2 (Parameter Estimation with the RMS Height Fixed) shows that for small backscattering angles: (1) as the rms slope is increased above 17.55°, the error in both the estimated rms slope and estimated reflection coefficient increases, and (2) this error, which is due to multiple scattering, is a function of the dielectric constant of the material (the error being larger for larger dielectric constants) and polarization (the error being larger for TM than TE polarization with PEC surfaces and smaller for TM than TE polarization with weak dielectric surfaces).

7.5.3 Parameter Estimation with the RMS Slope Fixed

In this section, the impact of changing the rms height and dielectric constant of the surface is examined. As in the previous section, the impacts are investigated by fitting the MOM computed average NRCS from 0 to 45 and from 45 to 68 degrees to the quasi-specular model.

For Gaussian surfaces with Gaussian spectra, as the rms surface height is increased the intermediate scale components of the surface spectrum are reduced in magnitude. As a result, the rms surface curvature is also reduced and better agreement is observed between the quasi-specular model and the MOM calculations at large backscattering angles. This result is clearly observed in figures 7.5.3-1a and 7.5.3-1b. Figure 7.5.3-1a shows that the magnitude of the relative error in the estimated rms slope increases as the rms height is decreased. (The relative error, defined by equation 7.4-1, is negative because estimated value is larger than the actual value). Although in this example, the rms slope is fixed at 24.1°, the exact same trends is observed at a rms slope of 17.5° with rms heights of 2.25, 0.71, 0.5 and 0.25 wavelengths and at a rms slope of 32.3° with rms heights of 2.25 and 0.71 wavelengths.
Figure 7.5.3-1a Error in Estimated RMS Slope (45°-68°) verses the Exact RMS Height and Reflection Coefficient for Surfaces with Gaussian Spectra, $\sigma_h = 2.25, 0.71, 0.50, 0.36$, $0.25 \lambda_o$ and $\sigma_s = 24.1^\circ$, Using the Two Angle Formula with 05/95 Averaging over all pairs of Backscattering Angles 45°, 45.5° to 68°.
Figure 7.5.3-1b Error in Estimated Reflection Coefficient (45°-68°) verses the Exact RMS Height and Reflection Coefficient for Surfaces with Gaussian Spectra, $\sigma_h = 2.25, 0.71, 0.50, 0.36, 0.25 \lambda_o$ and $\sigma_s = 24.1°$, Using the Two Angle Formula with 05/95 Averaging over all pairs of Backscattering Angles 45°, 45.5° to 68°
It is also interesting to note that in figures 7.5.3-1a and 7.5.3-1b the confidence limits become larger as the rms height is increased. This effect appears to have its origin in a deficiency of large slopes required to specularly backscatter. As was discussed in section 6.4.4, the probability of occurrences for large slopes is very low for surfaces with Gaussian spectra and large rms heights. The larger the rms height, the less likely one is to encounter a point on the surface with a slope sufficiently large to back reflect an incident ray into the backscattering direction. Thus as the rms height increases, not only does the quasi-specular scattering become more important, but it also becomes increasingly unlikely to occur. As a result, several "scattering centers" are not illuminated on each surface realization and the fluctuations in the NRCS do not appear to drop as one over the square root of the number of surfaces averaged (as predicted in section 7.3.2.1).

Now turning attention to the cross section at small backscattering angles, figures 7.5.3-2a and 7.5.3-2b plot the relative error in the estimated rms slope and the estimated reflection coefficient magnitude when the actual rms slope is fixed at 17.55° and the rms height varies between 0.25 \( \lambda_0 \) and 2.25 \( \lambda_0 \). Figure 7.5.3-2a shows that the error in the estimated rms slope is not very large at any of these heights, and with respect to the confidence limits is essentially zero when the rms height is less than 0.5 \( \lambda_0 \). The small but measurable error which is observed for rms heights of 0.25 \( \lambda_0 \) and 0.36 \( \lambda_0 \) is partially due to multiple scattering and partially due to surface curvature. In section 7.2.1 it was observed that for a rms height of 0.71 \( \lambda_0 \), when the rms surface curvature became too large the quasi-specular model was in poor agreement with both the Kirchhoff and the MOM scattering results at large backscattering angles. An approximate rms curvature limit required to obtain a good comparison at large backscattering angles up to about 68° was found as,

\[
\sqrt{\langle r^{2}_{xx} \rangle} = \sqrt{\frac{3\sigma^4_x}{\sigma^2_h}} \leq 0.15/\lambda_0.
\]

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Figure 7.5.3-2a Effect of the Exact Height and Reflection Coefficient on the Estimated Slope.

For Surfaces with Gaussian Spectra, $\sigma_h = 2.25, 0.71, 0.50, 0.36, 0.25 \lambda_o$ and $\sigma_s = 17.55^\circ$.

Using L1 Minimization over all Backscattering Angles $0^\circ, 0.5^\circ$ to $45^\circ$. 
Figure 7.5.3-2b Effect of the Exact Height and Reflection Coefficient on the Estimated Reflection Coefficient for Surfaces with Gaussian Spectra, $\sigma_h = 2.25, 0.71, 0.50, 0.36, 0.25 \lambda_o$ and $\sigma_t = 24.1^\circ$ Using L1 Minimization over all Backscattering Angles $0^\circ, 0.5^\circ$ to $45^\circ$
As the rms height is decreased, the rms curvature continues to increase until the quasi-specular and Kirchhoff models do not agree even at small backscattering angles. This effect is shown in figure 7.5.3-3. One observes that in this case (rms height of 0.25 \( \lambda_o \) and rms slope of 17.55\(^o\)) the MOM and Kirchhoff results but not the quasi-specular result are in good agreement for backscattering angles greater than about 15 degrees. As was discussed in section 7.2.1, this suggest that the surface curvature has become too large for the stationary phase approximation used in the quasi-specular model. At backscattering angles of less than about 15 degrees, on the other hand, the quasi-specular result is smaller than both the MOM and Kirchhoff results. However, the MOM result is also greater than the Kirchhoff result, which indicates the presence of multiple scattering (as was discussed in section 7.2.2). These two observations suggest that the error in the estimated rms slope is due to a complex interplay of both surface curvature and multiple scattering. Further, the fact that no multiple scattering or curvature related error is observed with the rms height greater than 0.71 \( \lambda_o \), indicates that the multiple scattering is not due to multiple bounce rays but tied to diffraction by the surface curvature.

As the rms slope is increased, one expects that both the curvature related error and multiple scattering related error would increases because (1) the likelihood of a double bounced backscattered ray increases and (2) the rms curvature will increase, and correspondingly any curvature induced multiple scattering so should also increase. Figures 7.5.3-4a and 7.5.3-4b plot the relative error in the estimated rms slope and reflection coefficient when the actual surface rms slope is fixed at a higher value of 24.1\(^o\). Both these figures show that the error in the estimated parameters does increase.
Figure 7.5.3-3 Comparison between the Quasi-Specular Model, Kirchoff Model and MOM Calculations at Small Backscattering Angles for Surfaces with Gaussian Spectra, \( \sigma_h = 0.25 \lambda_o \) and \( \sigma_s = 17.55^\circ \), 80 \( \lambda_o \) Long with \( \Delta x = 0.1 \lambda_o \) Using 10/3/2 Evaluations (\( \varepsilon_r = 2 \)) and 10/-/-1 (PEC), and \( g = 15 \lambda_o \).
Figure 7.5.3-4a Effect of the Exact Height and Reflection Coefficient on the Estimated Slope For Surfaces with Gaussian Spectra, $\sigma_h = 2.25, 0.71, 0.50, 0.36, 0.25 \lambda_o$ and $\sigma_s = 24.1^\circ$, Using the L1 minimization over all Backscattering Angles $0^\circ, 0.5^\circ$ to $45^\circ$
Figure 7.5.3-4b Effect of the Exact Height and Reflection Coefficient on the Estimated Reflection Coefficient for Surfaces with Gaussian Spectra, $\sigma_h = 2.25, 0.71, 0.50, 0.36, 0.25 \lambda_o$ and $\sigma_s = 24.1^\circ$

Using L1 Minimization over all Backscattering Angles $0^\circ, 0.5^\circ$ to $45^\circ$
In summary section 7.5.3 Parameter Estimation with the RMS Slope Fixed, shows that for a fixed rms slope, as the rms height is decreased the relative error in the estimated rms slope and reflection coefficient obtained using the NRCS from 45-68 degrees increases and by inference the difference between the MOM results and the quasi-specular result is also increasing. This results is in agreement with earlier observations that reducing the rms surface curvature produces better agreement between the quasi-specular model and MOM calculations at "large backscattering angles." At small backscattering angles (0-45 degrees), multiple scattering significantly degrades the quasi-specular model's accuracy when the rms slope ($\sigma_s$) is greater than approximately 18°. In addition, the numerical results suggest that the rms height must also greater than roughly 0.5 $\lambda_o$. This rms height ensures that there is no significant coherent scattering and that the rms curvature remains sufficiently small, ($\sqrt{\langle \xi^2 \rangle} < 0.5 \lambda_o$) so that there is very little curvature related error.

7.6 Conclusions

In this chapter, the use of the quasi-specular model to estimate the rms slope and the reflection coefficient from the average normalized radar cross section of randomly rough surfaces with Gaussian height distributions and Gaussian spectra was examined. The estimated values were determined by examining the ratio of the cross section at two angles and by using a numerical minimization approach. Minimization was performed using both a traditional least absolute difference merit function and an approximate maximum likelihood estimator for the average radar cross section.

In regards to the quasi-specular model's accuracy as a tool for estimating the rms slope and normal incidence flat surface reflection coefficient magnitude, the following findings were made:
(1) The quasi-specular model cannot be used to obtain good estimates of the rms surface slope or the magnitude of the surface reflection coefficient from Gaussian rough surfaces with Gaussian spectra at large backscattering angles (45° - 70°), unless the surface has small rms (root mean squared) curvature. An approximate curvature condition under which the model is accurate is given below in equation (7.6-1). Further, the quasi-specular model becomes erroneous at these large incidence angles in situations where the tangent plane approximation continues to give good results. A similar observation was made by Thorsos [1988] for surfaces with smaller rms heights than were examined here. This is because the stationary phase approximation used to obtain the quasi-specular model is more sensitive to the surface curvature than is the tangent plane approximation.

\[ \sqrt{\left\langle \zeta_{xx}^2 \right\rangle} = \sqrt{\frac{3\sigma_z^4}{\sigma_h^2}} < 0.15/\lambda_0. \]  

(7.6-1)

(2) At small backscattering angles or near normal incidence, the quasi-specular model yields very good estimates of the rms surface slope and normal incidence reflection coefficient when the rms slope is less than approximately 18° and the rms height is greater than about half the incident wavelength. This rms height ensures that there is no significant coherent scattering and that the rms curvature remains sufficiently small.

\[ \sqrt{\left\langle \zeta_{xx}^2 \right\rangle} < 0.5/\lambda_0. \]  

(7.6-2)

It is possible to obtain good results with rms heights smaller than half a wavelength, but the curvature condition (equation 7.6-2) must still be satisfied; that is, the rms slope will have to be decreased below 18°. This rms slope condition is less restrictive than the condition given by Ulaby, Fung and Moore [1982 p. 949 and p.1815], who suggest a maximum rms slopes of 14.5°. These authors also suggest a radius of curvature

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limit which is very close to one wavelength. These values appear to have been obtained for the Kirchhoff approximation, and applied to the quasi-specular model with the added constraint that the rms height be sufficiently large to eliminate any coherent scattering contributions.

(3) For surfaces with rms slopes greater than 18°, the quasi-specular model's accuracy in estimating the rms slope of the scattering surface is severely degraded by multiple scattering at small backscattering angles. The requirement that the rms surface slope be 18° or less given above ensures that there is no significant multiple scattering contributions to the radar cross section at small backscattering angles. The multiple scattering is also a function of both the dielectric constant of the scattering surface and the polarization of the incident field.

When the curvature condition (equation 7.6-1) is not met, multiple scattering is larger for TM (vertical) polarization than for TE (horizontal) polarization when the dielectric constant of the scattering medium is large. When the curvature condition equation (7.6-1) is met and the scattering surface is a perfect electric conductor (PEC), the multiple scattering in the TM and TE polarizations is the same. Regardless of the curvature and polarization, as the dielectric constant is reduced, the multiple scattering is reduced. This reduction in multiple scattering is more rapid for TM polarization as a result of the Brewster angle. In principle, the quasi-specular model can be used for rms slopes larger than 18° (at least for TM polarization), but only when the dielectric constant of the surface is very low.

Finally, when the curvature condition (equation 7.6-1) is met and the surfaces have large rms height (greater than 2 wavelengths), the surface scattering can be described in terms of ray scattering and ray tracing solution should be possible. In addition, numerical iteration was found to converge under these conditions.
Table 7.6-1 Summary of Validity Conditions for the Quasi-Specular Model
(for Surfaces with Gaussian Height Distributions and Gaussian Spectra)

Small Backscattering Angles: \[ \sqrt{\langle \zeta_{xx}^2 \rangle} < 0.5/\lambda_0 \text{ and } \sigma_z \leq 18^\circ \]

Large Backscattering Angles: \[ \sqrt{\langle (\zeta_{xx})^2 \rangle} = \sqrt{\frac{4}{3} \frac{\sigma_z}{\sigma_n^2}} < 0.15/\lambda_0 \]
8.0 Estimating the Wind Speed for Surfaces with Pierson-Moskowitz Spectra

8.1 Introduction

Many surfaces cannot be described as having only small heights (Bragg scattering) or being slowly undulating (Kirchhoff scattering). Surfaces which contain both types of roughness are sometimes called two-scale or composite surfaces. A surface with a Pierson-Moskowitz spectrum belongs to the two-scale category, and as will be shown in section 8.2, the scattering from such surfaces is not well described by either Bragg or quasi-specular scattering models alone. The Pierson-Moskowitz spectrum was first introduced in chapter 6 as a model for the ocean surface and requires only one parameter, the wind speed at 19.5 meters above the mean surface. Figure 8.1-1 shows the Pierson-Moskowitz spectrum for wind speeds of 10 and 20 m/s. As this plot illustrates, much of the surface wave spectrum energy is located in the neighborhood of wavelengths longer than about 31 meters ($K = 2\pi/\lambda < 0.2$ [rads/m]). As the wind speed increases, the low frequency portion of the spectrum increases significantly with the peak shifting towards smaller wavenumber or longer spatial wavelengths. Those surface waves which are shorter than about 6 meters ($K > 1.0$) are essential unchanged by the wind speed.

In this chapter, both the quasi-specular model and a composite model will be used to estimate the wind speed from the average normalized radar cross section of these surfaces. First, however, section 8.2 compares the numerically evaluated average normalized radar cross section (NRCS) to the cross section predicted by the Bragg model, the quasi-specular model, and the tilted Bragg model. It is found that for small backscattering angles (out to about 20 degrees) the quasi-specular model with a filtered or effective rms slope is in fair agreement with the numerical calculations.
The tilted Bragg model, on the other hand, is in poor agreement at these small scattering angles but agrees well at backscattering greater than about 35 degrees. A composite model constructed by addition of the tilted Bragg and the quasi-specular model (with a modified reflection coefficient), is found to be in reasonable agreement at all backscattering angles up to 60 degrees. Unfortunately, because of limitations in the size of the scattering surface which can be efficiently solved numerically, the calculated average cross sections are only believed to be valid for backscattering angles of less than 60 degrees. This problem is discussed in section 6.4. As a result, this chapter presents no backscattering cross section data beyond 60 degrees.

In section 8.3, a modified quasi-specular model and the composite model described above are used to estimate the wind speed from the numerically calculated data. Both of these models require choosing a separation wavenumber, $k_{sb}$ (or a separation constant defined as $k_{s}/k_{sb}$ where $k_{s}$ is the electromagnetic wavenumber) which divides the surface spectrum into large scale and small scale regions. This separation constant is discussed in section 8.2; in section 8.3.1, a minimization technique is used to determine the optimal choice for this parameter.

8.2 Description and Comparison of Two-scale Scattering Models

In this section, the numerically calculated average normalized radar cross section (NRCS) is compared to the cross section predicted by the Bragg, quasi-specular, tilted Bragg, and a composite model. It is shown that the composite model which adds the tilted Bragg result to the quasi-specular result with a modified reflection coefficient compares favorably with the numerically determined result.

Recently, Kapp and Brown [1996] developed a new technique to numerically calculate the scattering from perfect electric conducting surfaces. This new technique, called the Method of Ordered Multiple Interactions (MOMI), is an exact solution to the...
same degree that a method of moments solution is exact but does not require the storage or inversion of the large interaction matrix required by the traditional method of moments. This technique is used to calculate the NRCS throughout this chapter because it is faster than the MOM approach. The details of this technique are given by Kapp and Brown [1996] and Kapp [1996].

The average TE polarized NRCS, obtained using the Method of Ordered Multiple Interactions (MOMI), for a Gaussian surface with a Pierson-Moskowitz spectrum at a wind speed of 15 m/s and an incident electromagnetic wavelength of 3 cm is compared in figure 8.2-1 with the results obtained using the quasi-specular model, 1st order perturbation (here referred to as the Bragg model), and the tilted Bragg model. All three of these models are introduced in Chapter 2. This figure shows that the quasi-specular model is in good agreement with the nearly exact MOMI solution for backscattering angles up to about 20 degrees. The Bragg model, on the other hand, is in poor agreement with the MOMI solution at all backscattering angles, although at backscattering angle greater than about 35 degrees this model does have a similar shape to that of the MOMI result. Finally, the tilted Bragg model is in good agreement with the MOMI calculation for backscattering angles greater than about 35 degrees.

The reader may recall from chapter 2 that the tilted Bragg model is itself a two-scale scattering model. In the tilted Bragg approach the surface spectrum is separated into a large scale and small scale region separated by a wavenumber, \( k_d \). For convenience, the normalized quantity \( \frac{k_o}{k_d} \), referred to as the separation constant, will be used to specify the value of the separation wavenumber \( (k_d) \) in this report. The tilted Bragg result is then obtained by averaging the traditional Bragg scattering result over the large scale slopes formed from the part of the spectrum having \( k < k_d \), i.e.,
Figure 8.2-1 Comparison of the Bragg, Tilted Bragg and Quasi-Specular Models to a Numerical Monte Carlo Solution for the Average Backscattered NRCS using $k_o/k_d = 6.05$, $\rho = -1.0$
2000 PEC Surfaces with Pierson-Moskowitz spectra, $80 \lambda_o$ Long with $\Delta x = 0.1 \lambda_o$
Wind Speed = 15 m/s, $\lambda_o = 3$ cm, TE polarization, and $g = 15$
\[ \sigma_{\text{Tilted Bragg}}^2(\theta) = \left\langle \sigma_{\text{Bragg,1D}}^2(\theta + \tan^{-1}(\nabla \zeta)) \right\rangle \sigma_{s,ls} \]  

(8.2-2)

where \( \sigma_{\text{Bragg,1D}}^2(\theta) = 8\pi k^3 \cos^4(\theta) |g(\theta)|^2 W_{\text{filtered}}(2k \sin \theta, k_d) \).

\[ W_{\text{filtered}}(k, k_d) = \begin{cases} 1 & \text{if } k \geq k_d \\ 0 & \text{if } k < k_d \end{cases} \]

is the small scale portion of the spectrum,

\[ \sigma_{s,ls} = \sqrt{k_d} \int_{-k_d}^{k_d} W(k) dk \]

is the rms slope of the large scale portion of the spectrum,

\[ \left\langle \cdot \right\rangle_{\sigma_{s,ls}} \]

denotes averaging over the large scale slopes, and

\[ |g(\theta)|^2 \]

is a polarization dependent constant which is defined in section 2.3.

It should be noted that both the Bragg and tilted Bragg models plotted in figure 8.2-1 use the high pass filtered spectrum, \( W_{\text{filtered}} \). This is why the Bragg model peaks at about 4.5 degrees and then goes to zero for smaller angles of incidence. Also, the quasi-specular model uses only the large scale rms slope defined above. For a Pierson-Moskowitz surface, the total rms slope is in fact infinite because \( W(k) \) is proportional to \( 1/k^3 \) for large \( k \).

Clearly, figure 8.2-1 shows that neither the tilted Bragg nor the quasi-specular results is sufficient to accurately model the entire backscattered radar cross section. This result was noticed by Valenzuela [1978], among others, who suggested adding the quasi-specular result to the tilted Bragg result. However, as can be observed in figure 8.2-2, simple addition of the two models, produces a result which is too large at small backscattering angles. As discussed in chapter 2, a number of authors have suggested reducing the quasi-specular contribution with an effective reflection coefficient, the argument being that the small scale roughness effectively reduces the specular scattering area reflectivity.

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Figure 8.2-2 Comparison of Composite (Tilted Bragg + Quasi-Specular) Model to a Numerical Monte Carlo Solution for the Average Backscattered NRCS using $k_\alpha/k_d = 6.05$, $\rho = -1.0$

2000 PEC Surfaces with Pierson-Moskowitz spectra, $80 \lambda_\alpha$ Long with $\Delta x = 0.1 \lambda_\alpha$

Wind Speed = 15 m/s, $\lambda_\alpha = 3$ cm, TE polarization, and $g = 15$
One such model which has been used with some success [Kim 1994, Rodriguez and Kim 1992, Schiffer 1987] is given below as equation (8.2-3). As is observed in figure 8.2-2, this model appears to be in good agreement with the MOMI result. Similarly good agreement is also observed with this two-scale model at winds speed of 10, 15 and 20 m/s, at both polarizations and at incident wavelengths of 3 and 30 centimeters.

\[
\sigma_{\text{composite}}^0 (\theta) = \sigma_{\text{Tilted_Bragg}}^0 (\theta) + \sigma_{\text{Quasi_Specular}}^0 (\theta) e^{-4k^2 \sigma_{h,ss}^2} \quad (8.2-3)
\]

where \( \sigma_{h,ss} = \sqrt{\frac{2}{\pi} \int_{k_d}^{\infty} W(k) dk} \) is the rms height of the small scale portion of the spectrum.

This composite model will be used to estimate the wind speed from the numerically calculated average radar cross sections later in this chapter. In the remainder of this section and in following sections, this model will be referred to simply as "the composite model."

An important and obvious question about all of these models is how to choose the separation constant. One approach is to find the value for the separation constant which minimizes the difference between the numerical calculations and the model (using the correct wind speed). This approach will be used in section 8.3.1. In a similar (but more limited) study to that being reported here, Durden and Vesecky [1990] used a minimization approach to determine the optimal separation constant. In their study, Durden and Vesecky used Gaussian surfaces with a power law spectra, TE polarization, and the composite model given above in equation (8.2-3). They found that the optimal separation constant varied from about 1 to 5. This result is essentially in agreement with comparisons of two-scale models to measured sea surface data [see summary and discussion given by Jackson et al 1992].
Qualitatively, the separation constant should be chosen such that the surface which would result from only the large scale portion of the spectrum is gently undulating, while the surface resulting from only the small scale portion of the spectrum satisfies perturbation requirements. Based on this premise, Brown [1978] has argued for a separation constant of approximately 3.

What effect does the separation constant have on the tilted Bragg and modified quasi-specular models? As the separation constant decreases, the large scale rms slope will increase. Therefore, the quasi-specular model will broaden out (with respect to the backscattering angle), but will also decrease very slightly in magnitude at nadir. This effect is demonstrated in figure 8.2-3, which compares the quasi-specular model using separation constants of 2.5 and 6.05. For the tilted Bragg model, a larger rms slope means that the traditional Bragg model will be averaged over a larger range of slopes. This has the effect of increasing the cross section at large backscattering angles. At small backscattering angles however, the cross section drops, as is shown in figure 8.2-4. This drop occurs because as the separation constant is decreased, the spectrum used in the Bragg model is cutoff at larger spatial frequencies. This means that the peak in the Bragg result of figure 8.2-1 moves from about 4.5 degrees out to about 11.5 degrees. Interestingly, as the separation constant is decreased the increase in the radar cross section of the quasi-specular model is very nearly matched by the decrease in the cross section of the tilted Bragg model in the backscattering directions from about 15 to 35 degrees.
Figure 8.2-3 Effect of the Separation Constant on the Quasi-Specular Model
Wind Speed = 15 m/s, $\lambda_o = 3$ cm, TE polarization

Figure 8.2.4 Effect of the Separation Constant on the Tilted Bragg Model

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For TM polarization, the Bragg model's radar cross section does not change as rapidly at large backscattering angles as it does for TE polarization. As a result, when the Bragg result is averaged in the tilted Bragg model, one would expect the impact of the averaging will not be as significant for TM polarization as for TE polarization. Figures 8.2-5 and 8.2-6 compare the composite model using separation constants of 6.05 and 2.5 to the numerically determined result for TE and TM polarizations respectively. Because it is difficult to see the impact of changing the separation constant in these figures, figure 8.2-7 plots the difference (in dB) between the composite model's cross section for the two values of the separation constant. As can be clearly observed in figure 8.2-7, the TE result is more sensitive to the separation constant at large backscattering angles than is the TM result. Also, in either case, increasing the separation constant reduces the average cross section at backscattering angles of less than about 35 degrees by less than half a dB, with the peak effect occurring at about 30 degrees. Very similar results to those shown in figure 8.2-7 are obtained for wind speeds of 10, 15 and 20 m/s and at electromagnetic wavelengths of 3 and 30 centimeters.

In summary, this section demonstrated that at small backscattering angles (out to about 20 degrees) the quasi-specular model with a filtered or effective rms slope is in fair agreement with the numerical calculations. The tilted Bragg model, on the other hand, is in poor agreement at these small scattering angle, but agrees well at backscattering greater than about 35 degrees. Finally, a composite model constructed by addition of the tilted Bragg and the quasi-specular model (with a modified reflection coefficient) is found to be in reasonable agreement at all backscattering angles up to 60 degrees. All three of these models require choosing a separation wavenumber, $k_d$ (or a separation constant $= \frac{k_0}{k_d}$), which divides the spectrum into a large scale and a small scale region. It was observed that varying the separation constant over the range from 2 to 6 resulted in
Wind Speed = 15 m/s, $\lambda_o = 3$ cm, TE polarization

Figure 8.2-5 Effect of the Separation Constant on the Composite Model, TE Polarization
Wind Speed = 20 m/s, $\lambda_0 = 3$ cm, TM polarization

Figure 8.2-6  Effect of the Separation Constant on the Composite Model, TM polarization
Figure 8.2.7 Difference in the Composite Model NRCS for $k_y/k_d = 6.05$ and $k_y/k_d = 2.50$.
relatively small (0.5 dB) changes in the composite model's cross section (see figure 8.2-7). These changes are a function of polarization and backscattering angle. In particular, the changes at large backscattering angles (greater than roughly 40 degrees) were observed to be larger for TE polarization than for TM polarization. The physical reasoning behind this observation is that surface tilting is more important for TE polarization than for TM polarization.

8.3 Estimating the Wind Speed and the Separation Constant

In the previous section, the normalized radar cross sections of a quasi-specular and a composite models were compared to numerical results for scattering from Gaussian surfaces with Pierson-Moskowitz spectra. In this section, these two models are used to estimate the wind speed from the numerical results using a minimization approach. Since the composite model was observed to be in good agreement with the numerical results (e.g. figure 8.2-5), one might naturally believe that the wind speed estimates will be quite good. On the other hand, although the height of the surface is a strong function of the wind speed, neither the large scale rms slope nor the small scale region of the spectrum is strongly dependent on the wind speed. As a result, the radar cross section does not change very much as a function of the wind speed. For example, figure 8.3-1 compares the composite model for wind speeds of 15 m/s and 12.3 m/s. As the plot indicates, although the wind speed changes by about 18% there is only a small change of about 3% in the large scale rms slope. Further, the radar cross section at both these wind speeds can be described as being in good agreement with the numerical result obtained using a wind speed of 15 m/s.

In subsection 8.3.2, a minimization approach will be used to estimated the wind speed from the numerical data. Before proceeding with this effort, selection of the separation constant (used in both the composite and the quasi-specular models) is first investigated in section 8.3.1, also using numerical minimization.
Figure 8.3-1 Comparison of the Two-Scale Model using Wind Speeds of 15.0 and 12.3 m/s, $k_c/k_d = 6.05$, and $\rho = -1.0$ to the MOMI Result using a Wind Speed of 15 m/s, 2000 PEC Pierson-Moskowitz surfaces, 80 $\lambda_o$ long with $\Delta x = 0.1 \lambda_o$, TE Polarization, and $g = 15 \lambda_o$.
It is of course also possible to extract the normal incidence reflection coefficient (or the dielectric constant) using either the quasi-specular or the composite model. However, it turns out that the estimated reflection coefficient is obtained with no significant error at all wind speeds and electromagnetic wavelengths examined in this chapter. Figure 8.3-2, for example, plots the relative error in the estimated reflection coefficient versus the wind speed for an incident wavelength of 3 centimeters. As in chapter 7, the total number of available cross sections where subdivided into 10 sets of 200 cross sections in order to obtain approximate confidence limits on the estimated parameter. As the figure clearly demonstrates, there is essentially no discernible error in the estimated reflection coefficient. Finally, only perfect electric conduction surfaces (PECs) have been examined in the following subsections.

8.3.1 Finding the Optimal Separation Constant

In this section, the effect of the separation constant on the absolute difference between the numerically calculated average cross section and a scattering model is examined. The absolute difference in dB, defined below in equation 8.3.1-1, is calculated by summing the absolute value of the difference between the numerical result and the scattering model for a given wind speed, electromagnetic wavelength, and set of backscattering angles. The optimal value for the separation constant is defined by the value which minimizes this difference.

\[
\text{Absolute Difference}(U_{19.5}, k_d, k_o) = \sum_\theta |10 \log(\sigma_{\text{MOMI}}^\theta(U_{19.5}, k_o)) - 10 \log(\sigma_{\text{model}}^\theta(U_{19.5}, k_d))|
\]  

where \( \theta \) is the backscattering angle,
\( U_{19.5} \) is the wind speed at 19.5 above the mean surface,
\( k_d \) is the separation wave number, and
\( k_o \) is the wavenumber of the incident electromagnetic wave, \( \frac{2\pi}{\lambda_o} \).

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Figure 8.3-2 Error in Estimated Reflection Coefficient Obtained by L1 Minimization in dB
Using the Average Backscattered NRCS at Angles of 0°, 0.5° ... 20°, the Optimal Separation Constant, 10x200 PEC Pierson-Moskowitz surfaces, 80 λ_0 long with Δx=0.1 λ_0, TE polarization, and g = 15 λ_0.
Figure 8.3.1-1, for example, plots the absolute difference between the MOMI result and the composite model (which is described in section 8.2). In this figure the absolute difference is plotted as a function of the separation constant using precisely the same wind speed (i.e. the same surface spectrum) in both the model and the MOMI calculations. The figure shows that there is a broad minimum in the difference plot at wind speeds of 10, 15 and 20 m/s. This suggests that changing the separation constant over the range from about 2 to 6 does not significantly change the composite model over the backscattering angles from 0 to 20 degrees with respect to the numerical calculations for TE polarization and an incident wavelength of 3 cm. A similar broad minimum to that observed in figure 8.3.1-1 was found by Durden and Vesecky [1990], who examined the rms difference (as opposed to the absolute difference) between their scattering code results and a similar composite model, using only TE polarization and Gaussian surfaces with a power law spectrum.

Figure 8.3.1-2, shows that when the set of backscattering angles is increased to include all the angles from 0 to 60 degrees, the absolute difference curve remains fairly broad with respect to the separation constant, but the minimum in the curve occurs at higher values of the separation constant. For a wind speed of 15 m/s the minimum value occurs at 6.05, which is why this value was used in examples of section 8.2. This tendency towards larger separation constants is not consistent with the study by Durden and Vesecky [1990], who found that the optimal value for the separation constant decreased at larger backscattering angles. However, when the electromagnetic wavelength is increased to 30 centimeters, as shown in figure 8.3.1-3, the optimal separation constant does not shift towards larger separation constants, although the minimum is not as broad.
Figure 8.3.1-1 Absolute Difference in dB between MOMI and the Composite Model at $\lambda_o = 3$ cm Using the Backscattering Angles $0^\circ, 0.5^\circ \ldots 20^\circ$ versus the Separation Constant, 2000 PEC Pierson-Moskowitz surfaces $80 \lambda_o$ Long with $\Delta x=0.1 \lambda_o$, $g=15 \lambda_o$, and TE pol.
Figure 8.3.1-2 Absolute Difference in dB between MOMI and the Composite Model at $\lambda_o = 3$ cm versus the Separation Constant using Backscattering Angles of 0°, 0.5° ... 60°, 2000 PEC Pierson-Moskowitz surfaces $80 \lambda_o$ Long with $\Delta_x=0.1 \lambda_o$, $g = 15 \lambda_o$ and TE pol.
Figure 8.3.1-3 Absolute Difference in dB between MOMI and the Composite Model at $U_{19.5} = 15$ m/s, $\lambda_o = 3$ and $30$ cm versus the Separation Constant, using 2000 PEC Pierson-Moskowitz surfaces $80\lambda_o$ Long with $\Delta x = 0.1\lambda_o$, $g = 15\lambda_o$, and TE pol.
It should also be noted that the minimum in the absolute difference is much smaller for 30 centimeters than it is for 3 centimeters. This means that the composite model more closely matches the numerical result at 30 centimeters than at 3 centimeters. Very similar results are observed at winds speeds of 10 and 20 m/s, to those presented here for 15 m/s.

The composite model results for TM polarization are not unlike those obtained for TE polarization, in the sense that the absolute difference plots show a broad minimum for separation constants between roughly 2.5 and 6. Figure 8.3.1-4, for example, plots the absolute difference curves for TM polarization at a wind speed of 20 m/s with electromagnetic wavelengths of both 3 and 30 centimeters and using the backscattering angles from both 0 to 20 degrees and 0 to 60 degrees. It should be noted that in the TM case, the absolute difference curves using the backscattering angles from 0 to 60 degrees indicate less sensitivity to the separation constant than is observed for TE polarization (see figure 8.3.1-3). This is not surprising since tilting effects are less important for TM polarization at large backscattering angle (see section 8.2).

Now turning our attention to the quasi-specular model, figure 8.3.1-5 compares the absolute difference curves obtained using only the quasi-specular model to those obtained using the composite model. Recall that for two-scale surfaces, the quasi-specular model is applied by using only the large scale portion of the surface spectrum which is why the quasi-specular model is a function of the separation constant. The figure shows that the minimum in the absolute difference curves are significantly narrower for the quasi-specular model than for the composite model. The two models are essentially identical for small values of the separation constant; as the separation constant becomes small the tilted Bragg portion of the composite model becomes negligible, leaving only the quasi-specular portion.
Figure 8.3.1-3 Absolute Difference in dB between MOMI and the Composite Model versus the Separation Constant for TM Polarization and $U_{19.5} = 20$ m/s, 2000 PEC Pierson-Moskowitz Surfaces $80 \lambda_o$ Long with $\Delta x=0.1 \lambda_o$, and $g = 15 \lambda_o$.
Figure 8.3.1-5 Comparison of Absolute Difference Curves between the Quasi-Specular and Composite Models using $U_{19.5} = 15$ m/s, 2000 PEC Pierson-Moskowitz Surfaces

$80 \lambda_o$ Long with $\Delta x = 0.1 \lambda_o$, and $g = 15 \lambda_o$
Table 8.3.1-1 gives the optimal value for the separation constant (that is the minimum in the absolute difference curve) for wind speeds of 10, 15, and 20 m/s, electromagnetic wavelengths of 3 and 30 centimeters, and the quasi-specular model. This table is included because it will be shown in the following section that wind speed estimates obtained using only the quasi-specular model are sensitive to the separation constant.

Table 8.3.1-1 Optimal Separation Constant for the quasi-specular Model

<table>
<thead>
<tr>
<th>Wind Speed</th>
<th>10 m/s</th>
<th>15 m/s</th>
<th>20 m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_o = 3$ cm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TE Polarization</td>
<td>3.07</td>
<td>2.62</td>
<td>3.63</td>
</tr>
<tr>
<td>TM Polarization</td>
<td>2.66</td>
<td>2.00</td>
<td>3.33</td>
</tr>
<tr>
<td>$\lambda_o = 30$ cm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TE Polarization</td>
<td>2.24</td>
<td>2.21</td>
<td>2.71</td>
</tr>
<tr>
<td>TM Polarization</td>
<td>1.91</td>
<td>1.78</td>
<td>2.24</td>
</tr>
</tbody>
</table>

As the table indicates, the optimal separation constant is slightly larger for TE polarization than for TM polarization and is also larger at $\lambda_o = 3$ cm than at $\lambda_o = 30$ cm. Although there are noticeable changes with respect to wind speed, no clear pattern is evident. It should be mentioned that the above values are obtained using a numerically calculated average cross section, which contains small fluctuations due to the finite number of cross section available to be averaged. The above values represent essentially a best estimate for the actual optimum values. In regards to the polarization, however, identical surfaces profiles were used for both polarizations at a wind speed of 10 m/s. It seems reasonable to assume that any shift in these numbers (due to small fluctuations in radar cross section caused by a finite sample population) would affect the results at both polarizations equally and therefore the separation constant is almost certainly slightly larger for TE polarization.

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In summary, this section examined the effect of the separation constant on the absolute difference between the numerically calculated average radar cross section and both the quasi-specular and composite scattering models as a function of polarization, wind speed, and electromagnetic wavelength. It was generally found that the separation constant had little effect on the absolute difference between the composite model and the numerically calculated average cross section, when the separation constant varied between roughly 2.5 and 6.0. The effect of the separation constant was especially small when the angular backscattering region of interest was taken as 0 to 20 degrees. When the angular backscattering region was changed to include all angles from 0 to 60 degrees, the separation constant was observed to have an increased effect for both polarizations, but especially for TE polarization. This is understandable because: (1) tilting effects are important at backscattering angles greater than roughly 25 degrees (especially for TE polarization) and (2) unlike the situation at small backscattering angles (0 to 20 degrees), at backscattering angles greater than roughly 35 degrees the quasi-specular model essentially does not contribute to the radar cross section and so the optimal choice for the separation constant is the value which best matches the tilted Bragg contribution to the cross section. For the quasi-specular model, on the other hand, the separation constant was found to have significant influence on the absolute difference between the model and the numerical calculations. Optimal values of the separation constants for the quasi-specular model appear to be a function of polarization, electromagnetic wavelength, and wind speed.
8.3.2 Estimating the Wind Speed

In this section, minimization based on the least absolute difference will be used to estimated the wind speed from the MOMI generated average normalized radar cross sections at wind speeds of 10, 15, and 20 m/s and incident wavelengths of 3 and 30 centimeters. The "optimal" value of the separation constant described in the previous section is frequently used. It should be noted that the word "optimal" used here does not mean that this is the value of the separation constant which necessarily yields wind speed estimates which are as close as possible to the actual wind speeds - although these values may do this. Here, the word "optimal" simply means that this is the value of the separation constant which gives the minimum value for the absolute difference when the exact wind speed is used in the model.

To illustrate the impact of the wind speed on the absolute difference curves examined in the previous section, figure 8.3.2-1 plots the absolute difference between the MOMI generated average cross section and both the composite and quasi-specular models, where MOMI uses a wind speed of 15 m/s and the models use wind speeds of 13.4 and 15 m/s.

This figure shows that the absolute difference between the quasi-specular model and the MOMI results is affected by the wind speed. At a separation constant of 3, for example, the absolute difference for the quasi-specular model is lower using a wind speed of 15 m/s than using a wind speed of 13.4 m/s. Therefore, the minimization of the wind speed is going to yield a wind speed estimate which is closer to 15.0 m/s than to 13.4 m/s. However, if a separation constant of 2 is used, then the minimization approach is going to yield a wind speed estimate which is closer to 13.4 m/s than to 15.0 m/s. In this cases, the "optimal" value of the separation constant (3.06) does lead to a good value for the wind speed estimated.
Figure 8.3.2-1 Wind Speed Effect on the Absolute Difference Curves at $\lambda_o = 3$ cm versus the Separation Constant using the Backscattering Angles $0^\circ$, $0.5^\circ$...$20^\circ$, 2000 PEC Pierson-Moskowitz surfaces $80 \lambda_o$ Long with $\Delta x = 0.1 \lambda_o$, TE pol. and $g = 15 \lambda_o$
For the composite model, on the other hand, the figure shows that the absolute difference between the composite model and the MOMI results is actually slightly lower using a wind speed of 13.4 m/s rather than 15 m/s for all values of the separation constant between 2 and 6. Therefore, the minimization approach is going to yield a wind speed estimate which is closer to 13.4 m/s than to 15 m/s regardless of the separation constant. As will be shown presently, the trends demonstrated in this example are also observed at (1) wind speed of 10, 15 and 20 m/s, (2) incident wavelengths of 3 and 30 centimeters and (3) for both polarizations.

First, examining the effect of the actual wind speed on the estimated value for the wind speed, figure 8.3.2-2 plots the estimated wind speed versus the actual wind speed using an incident wavelength of 3 centimeters, TE polarization, and the optimal separation constant. The estimated values are determined by finding the value of the wind speed in the composite model which minimizes the absolute difference (in dB) between the MOMI calculated cross section and the composite model. As was done in the previous chapter, the total number of cross sections available to be averaged are grouped into 10 sets of 200 cross sections each in order to obtain approximate confidence limits based on the assumption of Gaussian statistics. Section 7.4 provides additional details regarding this point. Figure 8.3.2-2 shows that the estimated wind speed tends to be somewhat on the low side, although the exact value (represented by the dash-dot line) is in or near the edge of the confidence limits. When the electromagnetic wavelength is reduced from 3 to 30 cm (figure 8.3.2-3), similar results are obtained to those at 3 cm except that the estimated wind speeds at 30 cm are closer to the exact value and the confidence limits are somewhat smaller.
Figure 8.3.2-2 Estimated Wind Speed from Absolute Difference Minimization at $\lambda_o = 3$ cm Using the Optimal Separation Constant at each Wind Speed, 10x200 PEC Pierson-Moskowitz Surfaces $80 \lambda_o$ long with $\Delta x = 0.1 \lambda_o$, TE pol. and $g = 15 \lambda_o$. 

- Backscattering Angles 0, 0.5 - 20° - Composite Model
- Backscattering Angles 0, 0.5 - 60° - Composite Model

$\lambda_o = 3$ cm - TE polarization
Figure 8.3.2-3 Estimated Wind Speed from Absolute Difference Minimization at $\lambda_o = 30$ cm Using the Optimal Separation Constant at each Wind Speed, 10x200 PEC Pierson-Moskowitz Surfaces 80 $\lambda_o$ long with $\Delta x = 0.1 \lambda_o$, TE polarization and $g = 15 \lambda_o$
For the composite model, the wind speed estimates are not very sensitive to the separation constant. Figures 8.3.2-4 through 8.3.2-7 plot the estimated wind speed versus the actual wind speed (as in the previous two figures) only in these figures the separation constant is fixed at either 2.5 or 6.0. All four figures show that the estimated wind speed is not significantly affected by the change in the separation constant.

It should also be noted that at either electromagnetic wavelength, the confidence limits increase as the wind speed increases. This effect can be attributed to the fact that the spot size used to calculate the radar cross section is significantly smaller than the period of the peak components in the Pierson-Moskowitz spectrum. Further, as the wind speed increases, the peak in the surface spectrum moves to longer and longer wavelengths with the result that more of the surface must be illuminated to correctly account for all of the large scale tilting effects. This effect would be most important for the largest backscattering angles with TE polarization and in the intermediate angles (25° - 35°) for both polarizations - i.e. the backscattering angles where the tilting effect would be expected to be the most important. Finally, regarding the composite model, there does not appear to be much polarization sensitivity in the wind speed estimates. Figures 8.3.2-8 and 8.3.2-9, for example, compare the estimated wind speeds for both polarizations.
Figure 8.3.2-4 Estimated Wind Speed from Absolute Difference Minimization at $\lambda_0 = 3$ cm

Using a Fixed Separation Constant, Backscattering Angles $0^\circ, 0.5^\circ, 20^\circ$.
10x200 PCD-Person-Moskovitz surfaces $80 \lambda_0$ long with $\Delta \lambda = 0.1 \lambda_0$, TE pol. and $g = 15 \lambda_0$.

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Figure 8.3.2-5 Estimated Wind Speed from Absolute Difference Minimization at $\lambda_o = 3$ cm Using a Fixed Separation Constant, Backscattering Angles $0^\circ, 0.5^\circ ... 60^\circ$

10x200 PEC Pierson-Moskowitz Surfaces $80 \lambda_o$ long with $\Delta x = 0.1 \lambda_o$, TE pol. and $g = 15 \lambda_o$
Figure 8.3.2-6 Estimated Wind Speed from Absolute Difference Minimization at $\lambda_o = 30$ cm
Using a Fixed Separation Constant, Backscattering Angles $0^\circ$, $0.5^\circ$ ... $20^\circ$,
10x200 PEC Pierson-Moskowitz surfaces $80 \lambda_o$ long with $\Delta x = 0.1 \lambda_o$, TE pol. and $g = 15 \lambda_o$
Figure 8.3.2-7 Estimated Wind Speed from Absolute Difference Minimization at $\lambda_o = 30$ cm
Using a Fixed Separation Constant, Backscattering Angles $0^\circ, 0.5^\circ, ..., 60^\circ$,
10x200 PEC Pierson-Moskowitz Surfaces 80 $\lambda_o$ long with $\Delta x = 0.1 \lambda_o$, TE pol. and $g = 15 \lambda_o$.
Figure 8.3.2-8 Estimated Wind Speed from Absolute Difference Minimization at $\lambda_o = 3$ cm Using the Optimal Separation Constant, Backscattering Angles $0^\circ, 0.5^\circ \ldots 20^\circ$, 10x200 PEC Pierson-Moskowitz Surfaces $80 \lambda_o$ long with $\Delta x = 0.1 \lambda_o$, and $g = 15 \lambda_o$.
Figure 8.3.2-9 Estimated Wind Speed from Absolute Difference Minimization at $\lambda_0 = 30$ cm Using the Optimal Separation Constant, Backscattering Angles $0^\circ$, $0.5^\circ$ ... $60^\circ$, $10 \times 200$ PEC Pierson-Moskowitz Surfaces $80 \lambda_0$ long with $\Delta x = 0.1 \lambda_0$, and $g = 15 \lambda_0$. 
Now turning our attention to the quasi-specular model, figure 8.3.2-10 plots the wind speed estimated using only the quasi-specular model along with the optimal separation constant, which is different at each wind speed, polarization, and electromagnetic wavelength (see section 8.3.1 for further details on the optimal separation constant for this model). The figure shows that using the quasi-specular model and the optimal separation constant produces wind speed estimates which are as good, if not better, than those obtained using the composite model. However, these good estimates are only obtained by using the optimal separation constant. Figures 8.3.2-11 and 8.3.2-12, for example, show that the estimate in the quasi-specular model is strongly influenced by the separation constant. This occurs because the quasi-specular model is actually being used to estimate the rms slope of the scattering surface, and the only impact of the separation constant is to define the portion of the surface spectrum which will be used to relate the rms slope to the wind speed. In finding the optimal separation constant for the quasi-specular model, one is effectively matching the estimated rms slope to the desired wind speed.

In summary, section 8.3.2 shows that wind speed estimates based on the composite model tended to be slightly low but in most cases the exact wind speed was within or very near the 95% confidence limits. Wind speed estimates were also generally observed to be slightly better at an electromagnetic wavelength of 30 cm than at 3 cm. Also, the composite model's wind speed estimates showed little sensitivity to the separation constant, which was varied between 2.5 and 6.0. Good wind speed estimates can also be obtained using only the quasi-specular model. However, the quasi-specular model's wind speed estimates are sensitive to the separation constant, i.e. the cutoff value used to filter the surface spectrum used in the model.
Figure 8.3.2-10 Estimated Wind Speed using the Quasi-Specular Model and the Optimal Separation Constant at Each Wind Speed and $\lambda_o$. Backscattering Angles $0^\circ, 0.5^\circ \ldots 20^\circ$, 10x200 PEC Pierson-Moskowitz Surfaces $80 \lambda_o$ long with $\Delta x=0.1 \lambda_o$, and $g=15 \lambda_o$. 

Estimates obtained using the Quasi-Specular Model and the Optimal $K_d$. 

- TE Polarization - $\lambda_o = 3 \text{ cm}$ 
- TE Polarization - $\lambda_o = 30 \text{ cm}$ 
- TM Polarization - $\lambda_o = 3 \text{ cm}$ 
- TM Polarization - $\lambda_o = 30 \text{ cm}$
Figure 8.3.2-11 Estimated Wind Speed using the Quasi-Specular Model and a Fixed Separation Constant at $\lambda_o = 3$ cm, Backscattering Angles $0^\circ, 0.5^\circ ... 20^\circ$, 10x200 PEC Pierson-Moskowitz Surfaces $80 \lambda_o$ long with $\Delta x = 0.1 \lambda_o$, TE pol. and $g = 15 \lambda_o$
Figure 8.3.2-12 Estimated Wind Speed using the Quasi-Specular Model and a Fixed Separation Constant at $\lambda_o = 30$ cm, Backscattering Angles 0°, 0.5° ... 20°, 10x200 PEC Pierson-Moskowitz Surfaces 80 $\lambda_o$ long with $\Delta x = 0.1 \lambda_o$, TM pol. and $g = 15 \lambda_o$. 

$\lambda_o = 30$ cm - $k_o/k_d = 2.0$

$\lambda_o = 30$ cm - $k_o/k_d = 3.0$
8.4 Conclusions

Many surfaces cannot be described as having only small heights (Bragg Scattering) or being slowly undulating (Kirchhoff Scattering). Such surfaces which contain both types of roughness are sometimes called two-scale or composite surfaces. In this chapter, both a modified quasi-specular and a composite model were used to estimate the wind speed from the average normalized radar cross section of Gaussian surfaces with Pierson-Moskowitz Spectra. Both of these models require choosing a separation wavenumber, \( k_d \), (or a separation constant = \( \frac{k_o}{k_d} \), where \( k_o \) is the electromagnetic wavenumber) which divides the spectrum into a large scale and small scale regions. Qualitatively, the separation constant should be chosen such that the surface which would result from only the large scale portion of the spectrum is gently undulating, while the surface resulting from only the small scale portion of the spectrum satisfies perturbation requirements.

For the quasi-specular model it was found that:

(1) At small backscattering angles (out to about 20 degrees), the quasi-specular model based only on the large scale portion of the surface spectrum was in fair agreement with the numerical calculations.

(2) Good wind speed estimates were obtained using only the quasi-specular model in combinations with the optimal separation constant at wind speeds of 10, 15, and 20 m/s; incident electromagnetic wavelengths of 3 and 30 centimeters, and both TE and TM polarization. However, the quasi-specular model's wind speed estimates were found to be sensitive to the separation constant. Optimal values of the separation constants are given in Table 8.3.1-1, and depend on polarization, electromagnetic wavelength, and wind speed.
For a composite model constructed by addition of the tilted Bragg model and the quasi-specular model (with a modified reflection coefficient), it was found that:

(1) The composite was observed to be in good agreement with numerical calculations at all backscattering angles up to 60 degrees.

(2) It was generally found that the separation constant had little effect on the absolute difference between numerical calculations and the composite model, when the separation constant varied between roughly 2.5 and 6.0. The effect of the separation constant was especially small when the angular backscattering region of interest was taken as 0 to 20 degrees. When the angular backscattering region was changed to include all angles from 0 to 60 degrees, slightly greater dependence on the separation constant was observed for both polarizations, but especially for TE polarization. This occurs because: (1) tilting effects are important at backscattering angles greater than roughly 25 degrees (especially for TE polarization) and (2) unlike the situation at small backscattering angles (0 to 20 degrees) the quasi-specular model essentially does not contribute to the radar cross section and so the optimal choice for the separation constant is the value which best matches the tilted Bragg model to the numerical result.

(3) Finally, wind speed estimates based on the composite model tended to be slightly low, but in most cases the exact wind speed was within or very near the 95% confidence limits of the estimated value. Wind speed estimates were also generally observed to be slightly better at an electromagnetic wavelength of 30 cm than at 3 cm. Perhaps most important, the composite model's wind speed estimates showed little sensitivity to the separation constant.


9.0 **Executive Summary**

The propose of this study was to determine the accuracy of the quasi-specular and two-scale models in estimating rough surface parameters and to understand what scattering mechanism limit their applicability. The validity of these models was examined by first comparing the average backscattered normalized radar cross section (NRCS) given by the models to results obtained using a numerical Monte Carlo approach. In this approach, a sample realization of a rough surface with specific roughness parameters is randomly generated. Then, using a Method of Moments (MOM) technique, the normalized radar cross section for the randomly generated surface is calculated. This process is repeated many times to obtain a numerical average for the NRCS. Second, the quasi-specular and two-scale models were used to estimate the surface parameters from the numerically generated NRCS data. Because the exact surface parameters are known, the accuracy of the models in estimating (or inverting) these surface parameters could be determined. In short, the major tasks involved in this study were: (1) the development (chapters 3 and 4) and validation (chapters 5 and 6) of a numerical scattering code which can both generate randomly rough surfaces and compute their radar cross sections, and (2) the evaluation of the quasi-specular and two-scale scattering models in terms of both their radar cross sections and their use in estimating surface parameters (Chapters 7 and 8).

Important limitations regarding the numerical solution of the radar cross section using method of moments techniques are discussed in chapters 5 and 6. The most important of these restrictions are:

1. The length of the surface can significantly effect the scattered field solution for surfaces with Gaussian spectra. Generally, the smaller the surface, the greater the error and the smaller the backscattered angular region becomes where the scattering strength is sufficient to obtain accurate results.
(2) For Pierson-Moskowitz spectra, on the other hand, it is found that the surface length can be chosen much smaller than the correlation length for backscattering angles up to about 60°. This result is important because the correlation length of these surfaces may exceed a hundred meters, and the average scattering from surfaces this large can not currently be solved using exact numerical approaches.

(3) Surfaces with Pierson-Moskowitz spectra, contain notable levels of surface roughness which occur on spatial scales which are comparable to or smaller than the wavelength of the incident electromagnetic wave. Fortunately, the average radar cross section is not sensitive to very small scale portion of the surface spectrum, i.e., the surface spectral components smaller than half the incident wavelength. However, it should be noted that it is only the average which is not effected, and the radar cross section for each individual surface does have a slight sensitivity to the these features.

(4) It is difficult to obtain the NRCS using Monte Carlo techniques for surfaces with Gaussian spectra whose scattering is dominated by specular scattering events at large backscattering angle. This is because infrequently occurring slopes, required to back-reflect rays at large backscattering angles, must be included to obtain a good average of the radar cross section. This observation was also made by Eric Thorsos [1988] for acoustic scatting from surface with Gaussian spectra.

Chapter 7 examines the use of the quasi-specular model for surfaces with Gaussian height distribution and Gaussian spectra. In regards to the quasi-specular model's accuracy as a tool for estimating the rms slope and normal incidence flat surface reflection coefficient (the two surface parameters which can be obtained using this model) from the average radar cross section of these surfaces, the following findings were made:
(1) The quasi-specular model does not accurately predict the scattering from Gaussian rough surfaces with Gaussian spectra at large backscattering angles (45° - 70°), unless the surface has small rms (root mean squared) curvature. An approximate curvature condition under which the model is accurate is given by equation (7.6-1). Further, the quasi-specular model can become erroneous at these large incidence angles although the tangent plane approximation continues to give good results. This is because the stationary phase approximation used to obtain the quasi specular model is more sensitive to the surface curvature than is the tangent plane approximation.

(2) For near normal incidence or small backscattering angles, the quasi specular model yields very good estimates of the rms surface slope and normal incidence reflection coefficient when the rms slope is less than approximately 18° and the rms height is greater than about half the incident wavelength. This rms height ensures that there is no significant coherent scattering and that the rms curvature remains sufficiently small \( \left( \sqrt{\frac{\zeta^2_{xx}}{\zeta_{xx}}} < 0.5/\lambda_o \right) \). It is possible to obtain good results with an rms height smaller than half a wavelength, but the rms curvature condition must still be satisfied, this requires the rms slope to be reduced below 18°. This upper bound is somewhat less restrictive then the condition suggested by Ulaby, Fung and Moore [1982 p. 949 and p.1815], who suggest a maximum rms slope of 14.5°. These authors also suggest a radius of curvature limit which is very close to one wavelength. These values appear to have been obtained for the Kirchhoff approximation and applied to the quasi-specular model with the added constraint that the rms height be sufficiently large to eliminate any coherent scattering contribution.

(3) For surfaces with rms slopes greater than 18°, the quasi specular model's accuracy in estimating the root mean squared (rms) slope of the scattering surface is severely degraded by multiple scattering at small backscattering angles (0° - 45°). The

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requirement that the rms surface slope be 18° or less ensures that there is no significant multiple scattering contributions to the radar cross section. The multiple scattering is a strong function of both the dielectric constant of the scattering material and the polarization of the incident field. When the rms curvature condition (equation 7.6-1) is not met, multiple scattering is larger for TM (vertical) polarization than for TE (horizontal) polarization when, in addition, the dielectric constant of the scattering medium is large. When the curvature condition equation (7.6-1) is met and the scattering surface is a perfect electric conductor (PEC), the multiple scattering for TM and TE polarizations is approximately the same. Regardless of the curvature and polarization, as the dielectric constant is reduced, the multiple scattering is also reduced. This reduction in multiple scattering is more rapid for TM polarization as a result of the Brewster angle. In principle, the quasi specular model can be used for larger rms slopes than 18° (at least for TM polarization) but only when the dielectric constant of the surface is low. Finally, when the above curvature condition is met and the surface has large rms height (greater than 2 wavelengths), the surface scattering can be described in terms of ray scattering and ray tracing solutions for the multiple scattering are possible.

Chapter 8 examined the accuracy of a modified quasi-specular model and a composite scattering model in estimating the wind speed from the average backscattered radar cross section of Gaussian surfaces with Pierson-Moskowitz spectra. (This spectrum, defined in chapter 6.1 is only a function of the wind speed.) This type of surface is called a two-scale surface, because it contains (1) variation in height which are large (compared to the electromagnetic wavelength), but vary slowly along the surface and (2) variations which are small in height, but vary rapidly along the surface. Both the modified quasi-specular model and the composite scattering model require choosing a separation wavenumber, \( k_a \) (or a separation constant = \( k_a/k_o \), where \( k_o \) is the electromagnetic wavenumber) which divides the surface spectrum into a large scale and a
small scale region. Chapter 8 also examines the optimal value for this separation constant.

For the quasi-specular model it was found that:

(1) At small backscattering angles (out to about 20 degrees), the quasi-specular model, modified in the sense that is depends only the large scale portion of the surface spectrum, is in fair agreement with the numerical calculations.

(2) Good wind speed estimates were obtained using only the quasi specular model in combinations with the optimal separation constant at wind speeds of 10, 15 and 20 m/s, incident electromagnetic wavelengths of 3 and 30 centimeters, and both TE and TM polarization. However, the quasi specular model's wind speed estimates were found to be very sensitivity to the separation constant. Optimal values of the separation constants are given in Table 8.3.1-1, and depend on polarization, electromagnetic wavelength, and wind speed.

For a composite model constructed by addition of the tilted Bragg model and the quasi-specular model with a modified reflection coefficient it was found that:

(1) The composite model was observed to be in good agreement with numerical calculations at all backscattering angles up to 60 degrees.

(2) The separation constant has little effect on the difference between the numerically calculated average radar cross section and the composite scattering model, when the separation constant varies between roughly 2.5 and 6.0.

(3) Wind speed estimates based on the composite model tended to be slightly low, but in most cases the exact wind speed was within or very near the 95% confidence limits of the estimated value. Wind speed estimates were also generally observed to be slightly
better at an electromagnetic wavelength of 30 cm than at a wavelength of 3 cm. Perhaps most important, the composite model's wind speed estimates showed little sensitivity to the separation constant.
References


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Appendix A  Derivation of the Scalar Extinction Equations

A.1  Introduction

In this appendix, a set of coupled integral equations are derived which give the electric field above a one dimensional rough surface given the electric field and its normal derivative on the surface. These integral equations are developed by application Green's second scalar identity. Two equations are desired because the incident field creates both a electric and magnetic surface current, which must be found to solve the scattering problem. These equations are called the extinction equations because, "We know that under the influence of the incident electromagnetic field another field will be generated inside the dielectric material, which will have a different wave number and hence a different phase velocity. We may, therefore, say the inside the medium, the incident wave is somehow extinguished by the interaction with the medium and is replaced by a new wave propagated with the velocity c/n, where n is the refractive index of the medium." [Wolf, 1973]. Mathematically, one obtains an integral expression of the form,

\[ 0 = \int_{S_s} E(r_1) \frac{\partial G(r, r_1)}{\partial n_1} - G(r, r_1) \frac{\partial E(r_1)}{\partial n_1} \, ds_1 + 4\pi E_i(r) \]

where, \( r \) is a point inside the space occupied by the scattering material.
\( E \) is the total electric field = incident + scattered
\( E_i \) is the incident electromagnetic field
\( G \) is the Green's function for a unbounded homogenous region.

In this equation, incident field is canceled or extinguished by the contribution from the surface currents. This equation is carefully derived in Section A.3 and is included here only to help explain why these integral equations are called the extinction equations.
In the following four sections, the coupled extinction equations are derived for the case when the electric field is parallel to the one dimensional (i.e. grooved) surface. This situation is called TE or horizontal polarization. In Section A.2, the scattering geometry and notation are explained. Section A.3 then derives one of the two integral equations by applying Green's identity to the space which is not occupied by the scattering material (i.e. the space in which the incident field propagates to the scattering surface). This integral equation gives the electric field parallel to a one dimensional rough surface given the electric field and its normal derivative on the surface. Section A.4 then derives an additional integral equation by applying Green's Identity to the space which is occupied by the scattering material. The two integral equation are then coupled by application of boundary conditions, to yield two equations with two unknowns. Finally, the derivation for the case when the magnetic field is parallel to the one dimensional surface (i.e. TM or vertical polarization) follows exactly this same procedure, the results of which are summarized in section A.6.

A.2 Problem Setup

Green's second identity in scalar form is given by [Balanis 1989],

\[
\int_v \left[ u \nabla^2 v - v \nabla^2 u \right] dv = \int_s \left[ u \frac{\partial v}{\partial n} - v \frac{\partial u}{\partial n} \right] ds \tag{A.2-1}
\]

where, \( u \) and \( v \) are any scalar functions with continuous first and second derivatives in the volume of integration. The surface integration occurs over the surface which encloses the volume and the surface normal points into the volume. In the next several subsections, Green's second identity will be applied to the one dimensional surface problem (i.e. two dimensional space) depicted below in Figure A.2-1.
The upper volume, \( V_1 \), is bounded by the surfaces \( S_4 \) and \( S_1 \). The lower volume, \( V_2 \), is bounded by the surfaces \( S_2 \) and \( S_1 \). Each volume has a dielectric constant, \( \varepsilon \), which is homogeneous but not necessarily the same, and a magnetic permeability equal to that of free space. The upper volume has a scalar field electric field which is given by the sum of and incident and scattered field (i.e. \( E_i = E_s + E_c \)). The incident field, which is generated by some electric current source in the upper region (i.e. \( f_{\text{source}} \)), is chosen as the field that would exist if the current sources were radiating in an unbounded space (i.e. there was no scattering surface). The lower volume has a scalar field electric field, \( E_2 \). In both domains, the electric field vector points along the \( y \) axis (i.e. in or out of the page and parallel to the one-dimensional or grooved surface). This situations is called the transverse electric (TE) case. In the alternative case (electric field vector in the \( x-z \) plane) is known as the transverse magnetic (TM) case because the magnetic field, \( H \), is then parallel to the grooved surface. The derivation in the TM case is similar to that of the TE case except that the \( H \) field is used instead of the \( E \) field and a slightly different set of boundary conditions are required. The results for the TM case are summarized in section A.6.

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A.3 Green's Identity applied to the Upper Volume

In this section, one of the two desired integral equations is derived by applying Green's second scalar identity to the upper volume. This is accomplished by choosing one of the arbitrary scalar functions \((u)\) to be the total electric field, which satisfies the scalar wave equation. The other scalar function \((v)\) is chosen to be the well known Green's function for an unbounded homogenous region. Combining the scalar wave equation and the definition of the Green's function allows the left hand side of Green's Identity (i.e. the volume integration) to be reduced to the value of the field at one point in space, which is called the observation point.

Let \(\vec{r}\) (called the observation point) and \(\vec{r}_1\) (called the integration or source point) be in the upper volume, \(V_1\). Also let,

\[
u = u_{1}(\vec{r}_1) \quad \text{and} \quad \nu = G_{1}(\vec{r}, \vec{r}_1)
\]

where, \(E_{1}(\vec{r}_1)\) is the total electric field = incident + scattered electric field, \(G_{1}(\vec{r}, \vec{r}_1)\) is the Green's function for an unbounded homogeneous region.

Green's function is defined as the solution to the following differential equation in a unbounded homogeneous region with a dielectric constant \(\varepsilon_{i}\),

\[
\nabla_1^2G_{1}(\vec{r}, \vec{r}_1) + k_1^2G_{1}(\vec{r}, \vec{r}_1) = 4\pi\delta(\vec{r} - \vec{r}_1) \quad (A.3.1)
\]

where \(\lim_{|\vec{r}| \to \infty} \left( \frac{\partial}{\partial r} + jk_1 \right)G_{1}(\vec{r}, \vec{r}_1) \to 0\), \(\nabla_1^2 = (\nabla_1 \cdot \nabla_1)\) is the Laplacian operator with respect to the \(r_1\) coordinates, \(k_1 = \frac{2\pi}{\lambda_1}\) is the wave number in \(V_1\), \(\lambda_1\) is the wavelength, which is a function of frequency and material constants, and \(\delta(\cdot)\) is the delta function.

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Essentially, the Green's function is the impulse response (to borrow a term from linear system theory) for the electric field, such that the field produced by an arbitrary current source can be represented by a superposition of Green's functions. Equation (A.3-2) is known as the radiation condition (or Sommerfeld condition). This condition is required because solution to the differential equation (A.3-1) for the Green's function results in two solutions. One represents and outgoing wave and the other an incoming wave. This radiation condition forces the Green's function to contain only the physically meaningful outgoing wave [Ishimaru p.124, 1991].

Substituting for the functions \( u \) and \( v \) in equation (A.2-1) yields equation (A.3-3), where the volume integration takes place over the entire upper volume and the surface integrations take place over surfaces \( S_s \) and \( S_1 \). Note, \( \vec{r}_1 \) is the integration point and \( \hat{n}_1 \) is the normal, which points into the volume \( V_1 \).

\[
\int_{V_1} E_1(\vec{r}_1) \nabla^2 G_1(\vec{r}, \vec{r}_1) - G_1(\vec{r}, \vec{r}_1) \nabla^2 E_1(\vec{r}_1) \, dV_1 = \\
\int_{S_1+S_s} E_1(\vec{r}_1) \frac{\partial G(\vec{r}, \vec{r}_1)}{\partial n_1} - G(\vec{r}, \vec{r}_1) \frac{\partial E_1(\vec{r}_1)}{\partial n_1} \, dS_1.
\]  

(A.3-3)

The above expression can be further simplified by application of the Helmholtz's equation. Helmholtz's (scalar) wave equation (sometimes call the scalar wave equation) is obtained by manipulating of Maxwell's equations, and can be found in any graduate electromagnetics book [Balanis 1989],

\[
\nabla^2 E_1(\vec{r}_1) + k_1^2 E_1(\vec{r}_1) = f_{\text{source}}(\vec{r}_1) \quad (A.3-4a)
\]

and

\[
\nabla^2 E_2(\vec{r}_1) + k_1^2 E_2(\vec{r}_1) = 0 \quad (A.3-4b)
\]
where $f_{\text{source}}(\vec{r}_1)$ is a current source in $V_1$ which generates the incident field. Using $E_i = E_i + E_s$ and substituting equations (A.3-4a) and (A.3-4b) into the left hand side of equation (A.3-3) yields,

\[
\int_{V_1} E_i(\vec{r}_1) \nabla_1^2 G(\vec{r}, \vec{r}_1) - G(\vec{r}, \vec{r}_1) \nabla_1^2 E_i(\vec{r}_1) \, dV_1 = \\
\int_{V_1} E_i(\vec{r}_1) \nabla_1^2 G(\vec{r}, \vec{r}_1) - G(\vec{r}, \vec{r}_1) \nabla_1^2 E_i(\vec{r}_1) \, dV_1 + \\
\int_{V_1} E_s(\vec{r}_1) \nabla_1^2 G(\vec{r}, \vec{r}_1) - G(\vec{r}, \vec{r}_1) \nabla_1^2 E_s(\vec{r}_1) \, dV_1 = \\
\int_{V_1} E_i(\vec{r}_1) \nabla_1^2 G(\vec{r}, \vec{r}_1) - G(\vec{r}, \vec{r}_1) \left( -k_1^2 E_i(\vec{r}_1) + f_{\text{source}}(\vec{r}_1) \right) \, dV_1 + \\
\int_{V_1} E_s(\vec{r}_1) \nabla_1^2 G(\vec{r}, \vec{r}_1) - G(\vec{r}, \vec{r}_1) \left( k_1^2 E_s(\vec{r}_1) \right) \, dV_1 = \\
\int_{V_1} E_i(\vec{r}_1) \left( \nabla_1^2 G(\vec{r}, \vec{r}_1) + k_1^2 G(\vec{r}, \vec{r}_1) \right) \, dV_1 + \\
-\int_{V_1} G(\vec{r}, \vec{r}_1) f_{\text{source}}(\vec{r}_1) \, dV_1 + \\
\int_{V_1} E_s(\vec{r}_1) \left( \nabla_1^2 G(\vec{r}, \vec{r}_1) + k_1^2 G(\vec{r}, \vec{r}_1) \right) \, dV_1 = \\
\int_{S_1 + S_s} E_i(\vec{r}_1) \frac{\partial G(\vec{r}, \vec{r}_1)}{\partial n_1} - G(\vec{r}, \vec{r}_1) \frac{\partial E_i(\vec{r}_1)}{\partial n_1} \, dS_1. \quad (A.3-5)
\]

It is straigthforward to show that the middle integral term on the left hand side of equation (A.3-5), is simply equal to $4\pi$ times incident field (by substituting it to equation A.3-4a) i.e.,

\[
-\int_{V_1} G(\vec{r}, \vec{r}_1) f_{\text{source}}(\vec{r}_1) \, dV_1 = -4\pi E_i(\vec{r}). \quad (A.3-6)
\]

This is because, this integral is simply a representation of the incident field in terms of the "system" impulse response. Using equation (A.3-6) and the definition of the Green's function (equation A.3-1), equation (A.3-5) becomes,

\begin{equation*}
\text{Appendix A, Derivation of Scalar Extinction Equations - page 286}
\end{equation*}
4\pi \int_{V_1} E_i(\vec{r}_1) \delta(\vec{r} - \vec{r}_1) dV_1 - 4\pi E_i(\vec{r}) + 4\pi \int_{V_1} E_s(\vec{r}_1) \delta(\vec{r} - \vec{r}_1) dV_1 =

\int_{S_1 + S_s} \left[ E_1(\vec{r}_1) \frac{\partial G(\vec{r}, \vec{r}_1)}{\partial n_1} - G(\vec{r}, \vec{r}_1) \frac{\partial E_1(\vec{r}_1)}{\partial n_1} \right] dS_1. \quad (A.3-6)

Notice that part of the surface integration occurs over \( S_1 \). However if \( S_1 \) is taken to be infinitely far away and the radiation condition (equation A.3-2) ensures that the Green's function and its derivative will be zero at infinity. Also, recalling that the observation point \( \vec{r} \) was specifically chosen to be inside the upper volume yields,

\[ 4\pi E_s(\vec{r}) = \int_{S_s} \left[ E_1(\vec{r}_1) \frac{\partial G(\vec{r}, \vec{r}_1)}{\partial n_1} - G(\vec{r}, \vec{r}_1) \frac{\partial E_1(\vec{r}_1)}{\partial n_1} \right] dS_1. \quad (A.3-7a) \]

By adding the incident field to both sides of the equation,

\[ 4\pi E_1(\vec{r}) = 4\pi E_i(\vec{r}) +. \]

\[ \int_{S_s} \left[ E_1(\vec{r}_1) \frac{\partial G(\vec{r}, \vec{r}_1)}{\partial n_1} - G(\vec{r}, \vec{r}_1) \frac{\partial E_1(\vec{r}_1)}{\partial n_1} \right] dS_1 \quad (A.3-7b) \]

Note, if the observation points had been chosen to be outside of the upper volume, then equations (A.3-6) would have reduced to,

\[ 0 = \int_{S_s} \left[ E_1(\vec{r}_1) \frac{\partial G(\vec{r}, \vec{r}_1)}{\partial n_1} - G(\vec{r}, \vec{r}_1) \frac{\partial E_1(\vec{r}_1)}{\partial n_1} \right] dS_1 + 4\pi E_i(\vec{r}). \quad (A.3-8) \]
In equation (A.3-8), the contribution from the incident field is exactly canceled or 
_exinguished_ by the contribution of the surface currents. This is why these integral 
equations are called the extinction equations.

### A.4  Green's Identity Applied to the Lower Volume

In this section, the second desired integral equations is derived by applying
Green's second scalar identity to the lower volume. This is accomplished in the same 
manner as the equation in the previous section, only there is no incident field.

Let \( \vec{r} \) (defined as the observation point) be in the upper volume, \( V_1 \), and let \( \vec{r}_2 \) (defined as the integration or source point) be in the lower volume, \( V_2 \). Also let,

\[
\begin{align*}
    u & = E_2(\vec{r}_2) \\
    v & = G_2(\vec{r},\vec{r}_2)
\end{align*}
\]

where, \( E_2(\vec{r}_2) \) is the scalar electric field in the lower region.
\( G_2(\vec{r},\vec{r}_2) \) is the Green's function in the lower medium material.

The Green's function is defined as the solution to the following differential equation in a 
unbounded homogeneous region with a dielectric constant \( \varepsilon_2 \),

\[
\nabla^2 G_2(\vec{r},\vec{r}_2) + \kappa_2^2 G_2(\vec{r},\vec{r}_2) = 4\pi\delta(\vec{r} - \vec{r}_2) \quad (A.4-1)
\]

where, \( \lim_{|\vec{r}| \to \infty} \left( \frac{\partial}{\partial r} + jk_2 \right) G_2(\vec{r},\vec{r}_2) \to 0 \quad (A.4-2) \)

\( \nabla_2^2 \) is the Laplacian operator taken with respect to the \( r_2 \) coordinates.
\( \kappa_2 \equiv \frac{2\pi}{\lambda_2} \) is the wavenumber in \( V_2 \).
\( \lambda_2 \) is the wavelength in the lower medium.

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Substituting the functions $u$ and $v$ into equation (A.2-1) yields equation (A.4-3), where the volume integration takes place over the entire lower volume and the surface integrations take place over surfaces $S_1$ and $S_2$. Note, $\vec{r}_2$ is the integration point and the normal, $\hat{n}_2$, points into the volume $V_2$.

$$
\int_{V_2} \left[ E_2(\vec{r}_2) \nabla^2 G_2(\vec{r}, \vec{r}_2) - G_2(\vec{r}, \vec{r}_2) \nabla^2 E_2(\vec{r}_2) \right] dV_2 = 
$$

$$
\int_{S_2 + S_1} \left[ E_2(\vec{r}_2) \frac{\partial G_2(\vec{r}, \vec{r}_2)}{\partial n_2} - G_2(\vec{r}, \vec{r}_2) \frac{\partial E_2(\vec{r}_2)}{\partial n_2} \right] dS_2. \quad (A.4-3)
$$

Using the scalar wave equation for the total field in the lower region,

$$
\nabla^2 E_2(\vec{r}) + k^2 E_2(\vec{r}) = 0 \quad (A.4-3)
$$

and the definition of the Green's function (A.4-1) yields,

$$
4\pi \int_{V_2} E_2(\vec{r}_2) \delta(\vec{r} - \vec{r}_2) dV_2 = 
$$

$$
\int_{S_2 + S_1} \left[ E_2(\vec{r}_2) \frac{\partial G_2(\vec{r}, \vec{r}_2)}{\partial n_2} - G_2(\vec{r}, \vec{r}_2) \frac{\partial E_2(\vec{r}_2)}{\partial n_2} \right] dS_2. \quad (A.4-4)
$$

Recalling that the observation point $\vec{r}$ was specifically chosen to be inside the upper volume yields the desired result,

$$
0 = \int_{S_s} E_2(\vec{r}_2) \frac{\partial G_2(\vec{r}, \vec{r}_2)}{\partial n_2} - G_2(\vec{r}, \vec{r}_2) \frac{\partial E_2(\vec{r}_2)}{\partial n_2} dS_2. \quad (A.4-5)
$$

This is an extinction equation similar to equation (A.3-8), only there is no incident field generated in the lower medium. Essentially this equation indicates that the surface fields extinguish themselves in the upper region (with respect to the Green's function for the lower region).
A.5 Boundary Conditions for TE Case

Because there are two unknowns, two boundary conditions are required to couple the integral equations derived in sections A.3 and A.4. In particular, the continuity of the tangential electric and magnetic fields are used. However in equation (A.3-7) and (A.4-5), the unknowns are the scalar (tangential) electric field and the scalar (tangential) normal derivative of the electric field. It can be shown that for a TE polarized electric field, \( \frac{\partial E}{\partial n} = j\omega \mu H_{\parallel} \), the proof of which is given in appendix B. The boundary conditions can then be written as,

\[
E = E_1 = E_2 \tag{A.5-1}
\]

and

\[
\frac{\partial E(\vec{r}_o)}{\partial n_o} = \frac{\partial E(\vec{r}_1)}{\partial n_1} = -\frac{\partial E(\vec{r}_2)}{\partial n_2} \tag{A.5-2}
\]

where \( \hat{n}_o = \hat{n}_1 = -\hat{n}_2 \). \( \text{(A.5-3)} \)

Substituting equations (A.5-1), (A.5-2) and (A.5-3) into the integral equations (A.3-7) and (A.4-5) yields the final result,

\[
4\pi E(\vec{r}) = 4\pi E_1(\vec{r}) + \int_{S_o} \left[ E(\vec{r}_o) \frac{\partial G_1(\vec{r},\vec{r}_o)}{\partial n_o} - G_1(\vec{r},\vec{r}_o) \frac{\partial E(\vec{r}_o)}{\partial n_o} \right] dS_o \tag{A.5-4}
\]

and

\[
0 = \int_{S_o} E(\vec{r}_o) \frac{\partial G_2(\vec{r},\vec{r}_o)}{\partial n_o} - G_2(\vec{r},\vec{r}_o) \frac{\partial E(\vec{r}_o)}{\partial n_o} dS_o. \tag{A.5-5}
\]

These two equations form a pair of coupled integral equations which can be used to solve for the scalar electric field above the rough surface. Note that both of these equation were obtained by choosing the observation point in the upper volume. A similar set can be obtained by choosing the observation point in the lower volume.
A.6 The Integral Equations in the TM Case.

In the TM case, the development of the integral equations is precisely the same as the TE case, except that the scalar magnetic field, $H$, is used instead of the scalar electric field. Also, the boundary conditions become,

$$ H = H_1 = H_2 $$

(A.6-1)

and,

$$ \frac{\partial H(r_0)}{\partial n_o} = \frac{e_2}{e_1} \frac{\partial H_1(r_1)}{\partial n_1} = -\frac{\partial H_2(r_2)}{\partial n_2} $$

(A.6-2)

where $\hat{n}_o = \hat{n}_1 = -\hat{n}_2$.

(A.6-3)

Equation (A.6-3) results for the continuity of the tangential electric field. It contains the term $\frac{e_2}{e_1}$ because the normal derivative of the magnetic field is given by,

$$ \hat{n}_o \times j_0 e E = j_0 e E_{\tan} = -\frac{\partial H}{\partial n_o} $$

(A.6-4)

Therefore the normal derivative is proportional to the product of the dielectric constant and the tangential electric field. This result is proven in appendix B. The resulting coupled integral equations for TM polarization are,

$$ 4\pi H(r) = 4\pi H_1(r) + $$

$$ \int_{S_y} H(r_0) \frac{\partial G_1(r, r_0)}{\partial n_o} - G_1(r, r_0) \frac{\partial H(r)}{\partial n_o} dS_o, $$

(A.6-5)

and

$$ 0 = \int_{S_y} H(r) \frac{\partial G_2(r, r_0)}{\partial n_o} - G_2(r, r_0) \frac{e_2}{e_1} \frac{\partial H(r)}{\partial n_o} dS_o. $$

(A.6-6)

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Appendix B  Derivation of the Quasi-Specular Model for
Random One Dimensional Surfaces

B.1  Introduction

In this appendix, the quasi-specular model for the average normalized radar cross
section is derived for randomly rough one dimensional surfaces. Although this result has
been derived by several authors for scattering from two dimensional surfaces [Kodis 1966
& Barrick 1968a, Sancer 1969], the result for one-dimensional surfaces has not been
published. The derivation presented here is similar to that used by Sancer. In Sancer's
approach, an approximate integral equation which gives the magnitude of the electric field
above the rough surface given the electric and magnetic currents on the surface is
averaged before spatial integration is applied. The integral equation is approximate in the
sense that the surface currents are obtained using the well known tangent plane or
Kirchhoff approximation and that a stationary phase approximation is employed. The
approach used by Kodis and Barrick differs from Sancer's in applying spatial integration
before averaging. Both approaches yield the same result, but each gives somewhat
different insight into the problem and each contains different levels of mathematical
difficulty. A good review of the techniques used by Kodis and Barrick, as well as those
used by Sancer is given by Brown [1990].

The appendix is organized as follows. Section B.2 first defines the normalized
radar cross section (NRCS) used in this derivation. The third section then presents an
integral equation which gives the scattered field far from the scattering surface, given the
surface fields. This approximate equations is obtained by simplifying one of the integral
extinction equations (derived in Appendix A) by assuming that the observation point (i.e.
the point in space where one wants to know the scattered field) is very far from the
surface. Afterwards the fourth section modifies the approximate integral equation using
the tangent plane approximation and the fifth section then applies a stationary phase approximation to obtain a closed form expression for the average NRCS. This closed form expression is called the quasi-specular model. The derivation presented in section B.2 through B.5 is for TE polarization. The derivation in the TM case follows exactly the same procedure as that used for the TE case, and the results are given in section B.6.

Finally in the section B.7, it is shown that the normal derivative of the magnetic field is proportional to the tangential electric field. This is done because the integral formulations used throughout this dissertation make use of the normal derivative of the surface field. Other formulations use the electric and magnetic fields (or currents) directly. This proof has been included to show (in part) that regardless of the formulation, one still requires both electric and magnetic surface fields to obtain the scattered field (from a dielectric surface). Also, understanding the relation between the normal derivatives and the surface fields is required to determine the boundary conditions used in Appendix A.

B.2 Definition of the Normalized Radar Cross Section

From the radar equation, the radar cross section (RCS) for a one dimensional surface is defined as,

$$\sigma(\theta_i, \theta_s) = \lim_{r \to \infty} \frac{2\pi r}{|\text{Scattered Power Density in } \theta_s \text{ direction}|} |\text{Incident Power Density in } \theta_i \text{ direction}|. \quad (B.2-1)$$

It is customary when dealing with scattering from surfaces of large extent to use the RCS normalized by area (length in the 1D surface case). The RCS normalized by unit length is simply be referred to as the normalized radar cross section (NRCS), given below as equation (B.2-2),

Appendix B, Derivation of the Quasi-Specular Model - page 293
\[
\sigma^0(\theta_i, \theta_s) = \frac{\sigma(\theta_i, \theta_s)}{L}.
\] (B.2-2)

For an incident plane wave, \(E_i(\vec{r}) = E_0 \exp\left[-jk_i \cdot \vec{r}\right]\), which illuminates the surface over a region \((0 < x < L)\), equation (B.2-2) yields,

\[
\sigma^0(\theta_i, \theta_s) = \lim_{r \to \infty} 2\pi r \frac{|E_s(\vec{r})|^2}{|E_o|^2 L}.
\] (B.2-3)

### B.3 The Far Field Form of the Integral Equation

Using Green's second scalar identity, it can be shown that the following integral equation (B.3-1) gives the electric field scattered by a one dimensional surface, given the total electric field and its normal derivative on the surface [Appendix A and Nieto-Vesperinas 1991]. This equation assumes that the electric field is parallel to grooved one-dimensional surface.

\[
E_s(\vec{r}) = \frac{j}{4} \int_{S_o} \left[ E(\vec{r}_o) \frac{\partial H_0^{(2)}(k|\vec{r} - \vec{r}_o|)}{\partial n_o} - \frac{\partial E(\vec{r}_o)}{\partial n_o} H_0^{(2)}(k|\vec{r} - \vec{r}_o|) \right] dS_o
\] (B.3-1)

where \(S_o\) is the scattering surface,

- \(k\) is the wavenumber,
- \(E_s\) is the scattered electric field,
- \(E_i\) is the incident electric field,
- \(E\) is the total electric field \(= E_i + E_s\), and
- \(H_0^{(2)}\) is a zeroth order Hankel function of the 2nd kind.

This equation can be simplified by taking into account that the scattered field will be examined only very far from the surface. This so-called far field transformation is achieved by applying a large argument approximation of the Hankel functions in equation Appendix B, Derivation of the Quasi-Specular Model - page 294
(B.3-1), in combination with a few additional approximations stemming from the observation point being very far from the surface. This procedure is used in Chapter 4 with the result,

\[
E_s(r) = \frac{i}{4} \left( \frac{2}{\pi k |r|^4} \right) e^{-jk|r|^2 - \pi/4} \int_{S_o} \left[ \frac{\partial E(r_o)}{\partial n_o} - \left( k(k_s \cdot \hat{n}) \right) E(r_o) e^{i\pi/2} \right] e^{jk_s \cdot r_o} \sqrt{1 + \frac{r_o^2}{\lambda_n}} \, dx_o. \quad \text{(B.3-2)}
\]

Equation (B.3-2) is the same as equation (4.3-12) except that in equation (4.3-12) the surface was assumed to be divided into a finite number of segments on which the surface fields were constant.

**B.4 The Tangent Plane Approximation**

In this section, the surface fields in the integral equation (B.3-2) are approximated by the fields which would be present were a flat plane introduced at each point on the surface perpendicular to the local surface normal, as is depicted in figure B.4-1. This approximation is known as the tangent plane or Kirchhoff approximation.

![Diagram](image)

Figure B.4-1 - Example of a Local Tangent Plane
Taking the incident field to be a TE polarized plane wave and using the definition of the normal derivative, equation (3.3-6), gives

\[ E_i(\vec{r}_o) = E_o e^{-j \vec{k}_i \cdot \vec{r}_o} \]  

(B.4-1a)

and

\[ \frac{\partial E_i(\vec{r}_o)}{\partial n_o} = -j(\hat{n} \cdot \vec{k}_i) E_o e^{-j \vec{k}_i \cdot \vec{r}_o}. \]  

(B.4-1b)

where \( \vec{k}_i = k_{ix} \hat{x} + k_{iz} \hat{z} = \sin(\theta_i) \hat{x} - \cos(\theta_i) \hat{z} \) is the direction in which the incident field is propagating, and

\( \theta_i \)

is the incidence angle (defined in figure 4.3-1).

Therefore using the tangent plane approximation, the total fields on the surface can be written using the well known Fresnel flat surface reflection coefficients,

\[ E^{TP}(\vec{r}_o) = [1 + \rho_H(\theta_L)] E_i(\vec{r}_o) = [1 + \rho_H(\theta_L)] E_o e^{-j \vec{k}_i \cdot \vec{r}_o} \]  

(B.4-2a)

and

\[ \frac{\partial E^{TP}(\vec{r}_o)}{\partial n_o} = [1 - \rho_H(\theta_L)] \frac{\partial E_i(\vec{r}_o)}{\partial n_o} \]

\[ = -j(\hat{n} \cdot \vec{k}_i) [1 - \rho_H(\theta_L)] E_o e^{-j \vec{k}_i \cdot \vec{r}_o} \]  

(B.4-2b)

where \( \rho_H(\theta_L) \) is the Fresnel reflection coefficient for horizontal (TE) polarization evaluated at the local angle of incidence. Note: This derivation is for the TE case. In the TM case, the vertical Fresnel reflection coefficient is used,

\[ \rho_H(\theta_L) = \frac{\mu_2 k_1 \cos(\theta_L) - \mu_1 \sqrt{k_2^2 - k_1^2 \sin^2(\theta_L)}}{\mu_2 k_1 \cos(\theta_L) + \mu_1 \sqrt{k_2^2 - k_1^2 \sin^2(\theta_L)}}. \]  

(B.4-3)

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Also, $\mu_1$ and $k_1$ are the magnetic permeability and wavenumber of the "upper" region where the incident field is generated, while $\mu_2$ and $k_2$ are the magnetic permeability and wavenumber of the scattering surface. $\theta_i$ is the local angle of incidence (depicted in figure B.4-1) and is a function of the surface slope at the source point and the angle of incidence ($\theta_s$) of the incident plane wave; "TP" denotes the tangent plane approximation.

Substituting the approximate surface fields into the approximate far field integral (equation B.3-14) yields,

$$E_s^{TP}(\vec{r}) \approx \frac{i}{4} \frac{2}{\pi k|\vec{r}|} e^{-j(\vec{k}_i \cdot \vec{r})} - \frac{\pi}{4} \int_{S_o} kR(\frac{d\zeta(x_o)}{dx})E_o e^{-j\vec{k}_i \cdot \vec{r}_o} e^{j(k_2 \cdot \vec{r}_o - \omega t)} \, dx_o.$$  (B.4-4)

where $R_H(\frac{d\zeta(x_o)}{dx}) = [-j(\hat{n} \cdot \hat{k}_i) \left\{ 1 - \rho_H(\theta_L(\frac{d\zeta(x_o)}{dx})) \right\} ]$

$$= -(\hat{k}_s \cdot \hat{n}) \left\{ 1 + \rho_H(\theta_L(\frac{d\zeta(x_o)}{dx})) \right\} e^{j\pi/2} \sqrt{1 + \left( \frac{d\zeta(x_o)}{dx} \right)^2},$$  (B.4-5)

and

$$\hat{k}_s = k_{xx} \hat{x} + k_{xz} \hat{z} = \sin(\theta_s) \hat{x} + \cos(\theta_s) \hat{z}.$$  

It should be noted that the variable $R$ (given by equation B.4-5) is a function of the incident and scattering direction. In order to simplify the notation slightly, the angular dependence will not be expressed directly.
B.5 High Frequency Limit

Finally, to obtain the closed form quasi-specular result, the average radar cross section is evaluated in the high frequency limit. Using the definition of the NRCS (equation B.2.3),

$$\sigma^0(\theta_i, \theta_s) = \lim_{r \to \infty} 2\pi r \frac{|E_{\text{s}}(r)|^2}{|E_o|^2 L},$$

and $E_{\text{s}}^{TP}(r)$ obtained from the far field tangent plane approximation (equation B.4.4) yields,

$$E_{\text{s}}^{TP}(r)E_{\text{s}}^{TP*}(r) \approx \frac{kE_0^2}{8\pi |r|^2} \int_{S_1} \int_{S_2} R_H^{-1}\left(\frac{dz(x_1)}{dx}\right)R_H^*\left(\frac{dz(x_2)}{dx}\right)e^{i(k_{sx} - k_{sx})_x'(x_1 - x_2) + j(k_{sx} - k_{sx})_z'(z(x_1) - z(x_2))} \, dx_1 \, dx_2$$

$$\approx \frac{kE_0^2}{8\pi |r|^2} \int_{S_1} \int_{S_2} \left\{ R_H^{-1}\left(\frac{dz(x_1)}{dx}\right)R_H^*\left(\frac{dz(x_2)}{dx}\right) e^{i(k_{sx} - k_{sx})_x'(x_1 - x_2) + j(k_{sx} - k_{sx})_z'(z(x_1) - z(x_2))} \right\} \, dx_1 \, dx_2. \quad \text{(B.5-1)}$$

and

$$\sigma^0_{TP}(\theta_i, \theta_s) \approx \frac{k}{4L} \int_{S_1} \int_{S_2} \left\{ R_H^{-1}\left(\frac{dz(x_1)}{dx}\right)R_H^*\left(\frac{dz(x_2)}{dx}\right) e^{i(k_{sx} - k_{sx})_x'(x_1 - x_2) + j(k_{sx} - k_{sx})_z'(z(x_1) - z(x_2))} \right\} \, dx_1 \, dx_2. \quad \text{(B.5-3)}$$

where $\theta_i$ and $\theta_s$ are given in figure B.4.1, and "*" denotes the complex conjugate.

Expanding $z(x_2)$ in a Taylor series about $x_1$,

$$z(x_2) = z(x_1) + \frac{dz(x_1)}{dx}(x_2 - x_1) + \frac{d^2z(x_1)}{dx^2} \frac{(x_2 - x_1)^2}{2} + \frac{d^3z(x_1)}{dx^3} \frac{(x_2 - x_1)^3}{6} + \ldots \quad \text{(B.5-4)}$$

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and substituting it in equation (B 6-3) yields,

\[
\sigma_{TP}(\theta_1, \theta_3) \approx \frac{k}{4L} \int_{S_1} \int_{S_2} \{R(H) \frac{dz(x_1)}{dx} R^*(H) \frac{dz(x_2)}{dx} \} dx_1 dx_2,
\]

where \( \Delta k_x = (k_{sx} - k_{tx}) = k(\sin \theta_3 - \sin \theta_1) \), and

\( \Delta k_z = (k_{sz} - k_{tz}) = k(\cos \theta_3 + \cos \theta_1) \).

We next make a change of variables and take the high frequency limit, as follows:

Let \( u = \Delta k_z (x_2 - x_1) \), which implies \( du = \Delta k_z dx_2 \) and

\[
\lim_{k_0 \to \infty} \sigma_{TP}(\theta_1, \theta_3) \approx \lim_{k_0 \to \infty} \frac{k}{4L \Delta k_z} \int_{S_1} \int_{S_2} \{R(H) \frac{dz(x_1)}{dx} R^*(\frac{dz(x_1) - \frac{u}{\Delta k_z}}{dx}) \} dx_1 du.
\]

Interchanging the order of the integrations and the limit yields,

\[
\lim_{k_0 \to \infty} \sigma_{TP}(\theta_1, \theta_3) \approx \frac{1}{4L(\cos(\theta_3) + \cos(\theta_1))} \int_{S_1} \int_{S_2} \lim_{k_0 \to \infty} \{R(H) \frac{dz(x_1)}{dx} R^*(\frac{dz(x_1) - \frac{u}{\Delta k_z}}{dx}) \} \frac{dx_1 du}{dx_1 dx_2}
\]

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Evaluating the limit gives,

\[
\lim_{k_0 \to \infty} \sigma_{TP}^o(\theta, \theta_s) \approx \frac{1}{4L(\cos(\theta_s) + \cos(\theta))} \int_{S_1} \int_{S_2} R_H\left(\frac{dx(x_1)}{dx}\right) R_{H'}^*(\frac{dx(x_1)}{dx}) e^{-j\mu} \left[ \frac{\Delta k_x + dx(x_1)}{\Delta k_x} \right] dx_1 du. \quad (B.5-8)
\]

Since \( \int_{\infty}^{\infty} e^{j\alpha v} dv = 2\pi \delta(\alpha) \), the integration with respect to \( u \) can be performed, and

\[
\lim_{k_0 \to \infty} \sigma_{TP}^o(\theta, \theta_s) \approx \frac{1}{4L(\cos(\theta_s) + \cos(\theta))} \int_{S_1} R_H\left(\frac{dx(x_1)}{dx}\right) R_{H'}^*(\frac{dx(x_1)}{dx}) 2\pi \delta\left[ \frac{\Delta k_x + dx(x_1)}{\Delta k_x} \right] dx_1. \quad (B.5-9)
\]

In the above expression, the only random variable which remains is the slope. Therefore, the average NRCS can be obtained by averaging over the surface slopes.

\[
< \lim_{k_0 \to \infty} \sigma_{TP}^o(\theta, \theta_s) > \approx \frac{1}{4L(\cos(\theta_s) + \cos(\theta))} \int_{S_1} R_H\left(\frac{dx(x_1)}{dx}\right) R_{H'}^*(\frac{dx(x_1)}{dx}) 2\pi \delta\left[ \frac{\Delta k_x + dx(x_1)}{\Delta k_x} \right] dx_1 >
\]

\[
\approx \frac{1}{4L(\cos(\theta_s) + \cos(\theta))} \int_{S_1} \int_{-\infty}^{\infty} R_H(\alpha) R_{H'}^*(\alpha) 2\pi \delta\left[ \frac{\Delta k_x + dx(x_1)}{\Delta k_x} + \alpha \right] \left[ pdf_{\zeta}(\alpha) \right] d\alpha dx_1. \quad (B.5-10)
\]

where \( pdf_{\zeta}(\alpha) \) is the probability density function of the surface slope.

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Evaluating the $\alpha$ integral yields,

\[
< \lim_{k_0 \to \infty} \sigma^0_{TP}(\theta_i, \theta_s) > \approx \frac{\pi}{2L(\cos(\theta_s) + \cos(\theta_i))} |R_H(\frac{\Delta k_x}{\Delta k_z})|^2 \int_{S_1} \left[ pdf_{\nu_\varphi}(\frac{\Delta k_x}{\Delta k_z}) \right] \, d\chi_1
\]

\[
\approx \frac{\pi}{2(\cos(\theta_s) + \cos(\theta_i))} |R_H(\frac{\Delta k_x}{\Delta k_z})|^2 \left[ pdf_{\nu_\varphi}(\frac{\Delta k_x}{\Delta k_z}) \right]
\]

(B.5-11)

where \( |R_H(\frac{\Delta k_x}{\Delta k_z})|^2 = |(\hat{N} \cdot \hat{k}_i)(1 - \rho_H(\theta_L)) + (\hat{k}_s \cdot \hat{N})(1 + \rho_H(\theta_L))|^2 \),

\[
\frac{\Delta k_x}{\Delta k_z} = \frac{\sin(\theta_s) - \sin(\theta_i)}{\cos(\theta_s) + \cos(\theta_i)},
\]

\[
\theta_L = \frac{\theta_i + \theta_s}{2}, \text{ and}
\]

\[
\hat{N}(\frac{\Delta k_x}{\Delta k_z}) = \hat{z} - \left( \frac{\Delta k_x}{\Delta k_z} \right) \hat{x}.
\]

In the backscattering case ($-\theta_s = \theta_i = \theta$), the above expression simply reduces to

\[
< \lim_{k_0 \to \infty} \sigma^0_{TP}(\theta, -\theta) > \approx \frac{\pi}{\cos^3(\theta)} |r_H(\theta)|^2 \left[ pdf_{\nu_\varphi}(\tan(\theta)) \right]
\]

(B.5-12)

which is the desired result.
B.6 TM Polarization

Similar to that of the TE case, Green's second scalar identity can be used to obtain the following integral equation for TM polarized fields (i.e. the magnetic field parallel to the grooved surface). This equation, given below as equation (B.6-1), gives the magnetic field scattered by a one dimensional surface given the magnetic field and its normal derivative on the surface [Nieto-Vesperinas 1991].

\[
H_s(\vec{r}) = \frac{j}{4} \int_{S_o} \left[ H(\vec{r}_o) \frac{\partial H_0^2(k|\vec{r} - \vec{r}_o|)}{\partial n_o} - \frac{\partial H(\vec{r}_o)}{\partial n_o} H_0^2(k|\vec{r} - \vec{r}_o|) \right] dS_o
\]  
\hspace{3cm} (B.6-1)

where \( S_o \) is the scattering surface,
\( k \) is the wavenumber,
\( H_s \) is the scattered magnetic field,
\( H_i \) is the incident field,
\( H \) is the total magnetic field = \( H_i + H_s \) and
\( H_0^{(0)} \) is the zeroth order Hankel function of the 2nd kind.

The TM derivation follows previously the same steps used in the TE case, with the result,

\[
< \lim_{k_o \to \infty} \sigma_{\text{RP}}^0(\theta_i, \theta_s) > = \frac{\pi}{2(\cos(\theta_s) + \cos(\theta_i))} |R_{\chi}(-\frac{\Delta k_x}{\Delta k_z})|^2 \left[ \rho \nu_{\chi}(-\frac{\Delta k_x}{\Delta k_z}) \right]
\]  
\hspace{3cm} (B.6-2)

where \( |R_{\chi}(-\frac{\Delta k_x}{\Delta k_z})|^2 = |(\vec{N} \cdot \hat{k})[1 - \rho \nu(\theta_s)] + (\hat{k}_s \cdot \vec{N})[1 + \rho \nu(\theta_s)]|^2 \), and

\[
\rho \nu(\theta) = \frac{\mu_1 k_1 \cos(\theta) - \mu_2 \sqrt{k_2^2 - k_1^2 \sin^2(\theta)}}{\mu_1 k_1 \cos(\theta) + \mu_2 \sqrt{k_2^2 - k_1^2 \sin^2(\theta)}}.
\]
The only difference between the TM results and TE results lies in the use of the TM (vertical) reflection coefficients verses the TE (horizontal) reflection coefficients. In the backscattering case ($\theta_s = -\theta_i$) the expressions become identical since $\rho_H(0) = -\rho_E(0)$.

### B.7 The One Dimension Normal Derivative and the Tangential Field

In this section, it is shown that the normal derivative of the magnetic field is proportional to the tangential electric field. The integral formulations used throughout this dissertation make use of the normal derivative of the surface field, either electric or magnetic depending on the polarization. An understanding the relation between the normal derivatives and the fields is required to formulated the boundary conditions used in Appendix A.

From Maxwell's equations, $\nabla \times \vec{H} = j\omega \varepsilon \vec{E}$. \hspace{1cm} (B.7-1)

where $\omega$ is the frequency of electromagnetic wave in radians per second and $\varepsilon$ is the dielectric constant of the material in which the wave is propagating.

In the one dimension TM case $H(\vec{r}) = H_y(x, z)\hat{y}$. \hspace{1cm} (B.7-2)

Therefore, using equation (B.7-1) and (B.7-2) yields

\[
\hat{n} \times j\omega \varepsilon \vec{E} = j\omega \varepsilon E_{\text{tan}} = \hat{n} \times (\nabla \times \vec{H})
\]

\[
= \hat{n} \times \left[ -\frac{\partial H_y}{\partial z} \hat{z} + \frac{\partial H_y}{\partial x} \hat{x} \right]
\]
\[
\begin{align*}
\hat{z} - \frac{dz(x)}{dx} \hat{x} & = \left[ \frac{\partial H_y}{\partial z} \hat{x} + \frac{\partial H_y}{\partial x} \hat{z} \right] \\
& \times \left[ \frac{\hat{y} - \left( \frac{\partial H_y}{\partial z} \hat{x} + \frac{\partial H_y}{\partial x} \hat{z} \right) \hat{y}}{\sqrt{1 + \left( \frac{dz(x)}{dx} \right)^2}} \right] \\
& = \frac{\partial H_y}{\partial z} + \frac{dz(x)}{dx} \frac{\partial H_y}{\partial x} \hat{y} \\
& \times \left[ \frac{\hat{y} - \left( \frac{\partial H_y}{\partial z} \hat{x} + \frac{\partial H_y}{\partial x} \hat{z} \right) \hat{y}}{\sqrt{1 + \left( \frac{dz(x)}{dx} \right)^2}} \right] \\
& = \frac{\hat{y} - \frac{dz(x)}{dx} \hat{x}}{\sqrt{1 + \left( \frac{dz(x)}{dx} \right)^2}} \left[ \frac{\frac{\partial H_y}{\partial z} \hat{x} + \frac{\partial H_y}{\partial x} \hat{z}}{\sqrt{1 + \left( \frac{dz(x)}{dx} \right)^2}} \right] \\
\end{align*}
\]

\[j \omega E_{\text{tan}} = -(\hat{n} \cdot \nabla) \hat{y} = -\frac{\partial H_y}{\partial n} \hat{y} \quad (B.7-3)\]

Equation (B.7-3) shows that the normal derivative of the magnetic field is proportional to the dielectric constant and the tangential electric field. Similarly, one can show that,

\[\nabla \times \vec{E} = -j \omega \mu \vec{H} \quad (B.7-4)\]

yields,

\[\frac{\partial E_y}{\partial n} = j \omega \mu H_{\text{tan}} \quad (B.7-5)\]
Appendix C  The Gaussian Spectrum

In chapter seven, the question arises as to how either decreasing the slope variance or increasing the height variance affects the Gaussian surface spectrum. From section 6.1, the Gaussian spectrum is given as,

\[ W(K) = \frac{l \sigma_h^2}{2 \sqrt{\pi} e} \left( \frac{k^2 \sigma_h^2}{4} \right) = \frac{\sqrt{2} \sigma_h^3}{2 \sqrt{\pi} \sigma_s} e \frac{k^2 \sigma_h^2}{2 \sigma_s^2}. \] (C-1)

where \( l \) is the correlation length, \( \sigma_h \) is the rms height, \( \sigma_s \) is the rms slope, \( K = \frac{2 \pi}{\lambda_s} \) is the spatial wavenumber of the surface spectrum and \( \lambda_s \) is the spatial wavelength. To determine how the surface spectrum changes with respect to small changes in the rms height, one can examine the derivative of equation (C-1) with respect to the rms height. Doing so yields,

\[ \frac{dW(K)}{d\sigma_h} = \frac{3}{\sigma_h} \left( \frac{\sqrt{2} \sigma_h^3}{2 \sqrt{\pi} \sigma_s} e \frac{k^2 \sigma_h^2}{2 \sigma_s^2} \right) - \frac{2K^2 \sigma_h}{2 \sigma_s^2} \left( \frac{\sqrt{2} \sigma_h^3}{2 \sqrt{\pi} \sigma_s} e \frac{k^2 \sigma_h^2}{2 \sigma_s^2} \right) \] (C-2)

When right hand side of equation (C-2) is less than zero the spectra decreases as the rms height is increased. Therefore,

\[ \frac{3}{\sigma_h} \left( \frac{\sqrt{2} \sigma_h^3}{2 \sqrt{\pi} \sigma_s} e \frac{k^2 \sigma_h^2}{2 \sigma_s^2} \right) - \frac{2K^2 \sigma_h}{2 \sigma_s^2} \left( \frac{\sqrt{2} \sigma_h^3}{2 \sqrt{\pi} \sigma_s} e \frac{k^2 \sigma_h^2}{2 \sigma_s^2} \right) < 0, \]

implies,
\[ 3 - \frac{2K^2 \sigma_h}{2\sigma_s^2} = 3 - \frac{K^2 l^2}{2} < 0 \]

or

\[ K > \frac{\sqrt{6}}{l}. \]  

(C-3)

Equation (C-3) indicates that components of surface spectrum with wavenumbers greater than the square root of 6 divided by the correlation length decrease as the rms height is increased. On the other hand, components of the surface spectrum with a wave numbers less than this value will increase as the rms height is increased. For the example of section 7.2-2, the rms slope was 24.1° and the rms height was 0.71 \( \lambda_0 \). Therefore, components of the surface spectrum with a wavelength 5.8 times that of the incident wavelength (i.e. \( \lambda_s > 5.8 \lambda_0 \)) or less will begin to decrease as the rms height is increased.

Similarly, taking the derivative of the Gaussian spectrum, equation (C-1), with respect to the rms slope and finding the conditions for which this expression is negative yields,

\[ K < \frac{\sqrt{7}}{l}. \]  

(C-4)

Therefore, the components of the surface spectrum with a spatial wavenumbers greater than the square root of two divided by the correlation length will decrease as the rms slope is decreased. For the example of section 7.2-2, the components of the surface spectrum with a wavelength 10 times or less than that of the incident wavelength (i.e. \( \lambda_s > 10 \lambda_0 \)) will begin to decrease as the rms slope is decreased.

In summary, as either the rms height is increased or the rms slope is decreased, the Gaussian spectrum becomes more peaked, with greater spectral power at "low" spatial frequencies and less spectral power at higher spatial frequencies.

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Vita

Roger T. Marchand received both his B.S.E.E. in 1990 and his M.S.E.E. in 1993 from Virginia Tech. His masters thesis work involved simulation and measurement of parasitic inductance and capacitance in microelectronic circuitry. Since 1993, he has been working on his Ph.D. with the ElectroMagnetics Interaction Laboratory (EMIL) in the area of electromagnetic scattering and inverse problems. Dr. Marchand is a member of the IEEE and also has a B.A. in History.