THE APPLICATION OF MULTIUSER DETECTION
TO CELLULAR CDMA

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Abstract

This research investigates the application of multiuser detection to Code Division Multiple Access for cellular communications. This investigation focuses on the use of multiuser receivers at the base station of mobile radio systems. The first two chapters are dedicated to multiuser detection in general. An extensive literature survey is performed on the research concerning multiuser receivers to date. Six major receiver structures are chosen for extensive simulation studies. The bit error rate performance of these receivers is investigated in several system environments. Additionally, practical issues are considered such as computational complexity and robustness to code tracking errors. From this work, one receiver structure is identified for further study, namely multistage interference cancellation. The theoretical performance of this receiver is analyzed using a standard Gaussian Approximation and an Improved Gaussian Approximation for AWGN and fading environments. Additionally, the resistance of the receiver to interference energy levels is explored. Parameter estimation is an important issue for interference cancellation. Simple methods of improving parameter estimation are examined, as is the effect of parameter estimation error on system performance. A baseband hardware implementation is detailed and several design challenges are presented. Results are given for the performance of the implemented receiver and shown to match well with theory and computer simulation. Finally, the implications of this research are discussed.
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Chapter 1

Introduction

"It's dangerous to put limits on wireless."

These words spoken by Guglielmo Marconi nearly a century ago have never been more appropriate than they are today. While widespread mobile radio has been a long time coming (it was first introduced in the 1940's), in the past few years it has finally begun to realize its full potential. Jump started by the recent Federal Communications Commission (FCC) auctions of spectrum at 1.8 GHz for Personal Communications Services (PCS), commercial interest in wireless technologies has grown at a phenomenal rate. The interest from both the commercial side and the FCC are fueled by recent projections such as one provided by the Yankee Group in 1993 which predicted a growth in cellular, PCS and paging subscribers from approximately 20 million in 1991 to over 60 million in 1998 [112]. It would appear that wireless communications is finally being integrated into the everyday lives of people.

1.1 Cellular Radio

The first mobile telephone service offered to the public was introduced in 1946 in 25 major cities. These early systems had a single cell with a cell radius of approximately 50km and allowed only half-duplex communications [106]. Cellular radio theory and technology was subsequently developed by Bell Labs in the 50's and 60's. The concept mirrored the use of radio spectrum for commercial applications such as AM/FM radio and television. Frequencies could be reused by dividing the entire frequency band into n sections, where n is the number of cells per reuse pattern. The entire reuse pattern is repeated over a large geographical area, allowing multiple cells to utilize the same frequency band. Cells which use the same frequency are guaranteed some minimum spacing related to the reuse pattern.
While the ideas were in place by the early 60's, it wasn't until the late 70's when technology caught up with the cellular concept [80]. In 1983, the FCC allocated 40MHz of spectrum at 800MHz for cellular telephone and the Advanced Mobile Phone System (AMPS) was born. This spectrum was increased to 50MHz in 1989 and recently the FCC allocated spectrum in 1.8 GHz band to absorb the high and rising demand for wireless services.

It is against the backdrop of the burgeoning field of mobile communications which we discuss the need for better frequency utilization. Even with the recent allocation of frequency dedicated to commercial wireless, it is felt that the existing cellular technology (primarily still the same AMPS technology which was introduced 13 years ago) is simply inadequate to handle the predicted growth. Thus, the recent boom in commercial wireless has been accompanied by a surge in research aimed at improving wireless system design. These systems move from analog Frequency Modulation (FM) on which AMPS is based, to digital technology. With this move, more advanced multiple access schemes are possible as opposed to simple Frequency Division Multiple Access (FDMA). The primary technologies are Time Division Multiple Access (TDMA) and Code Division Multiple Access (CDMA).

To understand the difference between the three techniques consider Figure 1.1. In traditional multiple access, channels are allocated by frequency as shown in Figure 1.1(right) or what is called Frequency Division Multiple Access (FDMA). By using a different Radio Frequency (RF) carrier for each channel and having strict bandwidth requirements, users can simultaneously access the system without causing mutual interference. With the advent of digital technology, more sophisticated multiple access became possible. The first was TDMA, where multiple users transmit in the same frequency band but must co-ordinate transmissions to occur during different time slots. Allocating multiple channels to each RF carrier frequency improves the spectral utilization while reducing RF hardware costs. The second digital multiple access scheme, CDMA, is a relatively new technology which allows users to transmit simultaneously and in the same frequency band, but requires that high rate 'spreading' waveforms be used on top of the data waveform to further modulate the carrier. Ideally, the spreading waveforms of all users in the system are orthogonal, allowing perfect isolation between users.

It should be noted that while current technologies focus on CDMA and TDMA all inherently employ FDMA. That is, typical spectrum allocation will provide several MHz per system. Neither CDMA nor TDMA is being recommended over the entire band. Rather, the bandwidth is divided into frequency slots (e.g. 1.25MHz for IS-95 based CDMA and 200kHz for Groupe Spécial Mobile or GSM TDMA) and within specific frequency slots, TDMA or
Figure 1.1: Graphical Description of the Three Main Multiple Access Schemes

CDMA is employed. In recent years the benefits and drawbacks of TDMA and CDMA been debated exhaustively. However, the key point is that both will supply significant capacity improvements over traditional analog systems. Additionally, it is felt that neither technology in their current form will provide sufficient capacity for the wireless boom ahead. Thus, in this work we focus on a method to improve upon the current implementation of CDMA, namely multiuser detection.

1.2 Code Division Multiple Access

CDMA was born from spread spectrum technology which developed in the 1950's [100] for military applications. The general idea behind spread spectrum is to increase the utilized bandwidth in order to spread the energy of the signal over a large band. This spreading is performed in a pseudo-random fashion in order to prevent a receiver from capturing the energy without knowledge of a pre-defined pseudo-random code. This can be accomplished in
two main ways. The first, called Direct-Sequence Spread Spectrum (DS/SS), uses a symbol stream (or 'chip sequence') which has a rate $N$ times higher than the data rate, as shown in Figure 1.2 (left). This rise in the signaling rate results in an increase in the bandwidth utilization or 'spreading' of the data waveform by a factor of $N$, called the spreading factor, shown in Figure 1.2 (right). The spreading waveform which is used is a pseudo-random sequence which is known by both the receiver and transmitter. At the receiver, the sequence is regenerated and synchronously correlated with the incoming received signal. This correlation de-spreads the data waveform back to its original bandwidth by eliminating the high rate signal transitions, allowing for proper reception.

A second form of spread spectrum is called frequency hopping (FH/SS). In this scheme the signal bandwidth is kept constant, but the carrier frequency is changed in a pseudo-random fashion. This hopping of the carrier frequency among $N$ equally spaced frequencies spreads the actual used bandwidth by $N$. At the receiver, a locally generated hopping carrier must be created to properly demodulate the signal.

The purpose of spreading the bandwidth with either of these techniques is two-fold. First, by spreading the energy over a large bandwidth, the effective Signal-to-Noise Ratio (SNR) becomes extremely low. For example in a DS/CDMA system if a signal originally had a signal-to-noise ratio of 15dB, and a spreading waveform with a chip rate 31 times the data
rate is used, the resulting SNR at the receiver is \(15 - 10 \log(31) \approx 0dB\) due to the increased noise bandwidth. This low SNR prevents other users from receiving the transmission. The intended receiver however, can despread the signal by correlating the incoming signal with a locally generated version of the pseudo-random sequence. This brings the SNR back to 15dB by reducing the noise bandwidth. Thus, the spreading factor \(N\) is often termed the spreading gain or processing gain.

The second benefit to spreading the signal in this fashion is that the signal is resistant to interference. That is if an interferer or jammer transmits \(EdB\) of energy within some bandwidth \(B\), the correlation of the spreading sequence with the received signal (which contains an uncorrelated jamming signal) results in a spreading of the jamming signal over bandwidth \(NB\) while simultaneously increasing the desired user's energy in the desired bandwidth by \(N\). Thus, the interferer would require \(N\) times the power of a jammer in an unspread system in order to achieve the same jamming level. The key to this benefit is that the interfering signal is uncorrelated with the desired signal. This will be a significant issue in the discussion of CDMA.

As can be imagined, the preceding benefits of spread spectrum originally were most applicable to military communications. In these applications, security and anti-jamming are of prime importance. In 1979 Cooper et. al. proposed the use of spread spectrum in commercial applications amid considerable controversy[16]. A decade later Qualcomm, Inc. began championing CDMA and its IS-95 standard as the multiple-access technology of the future. While CDMA is based on spread spectrum, there are considerable differences between the military use of spread spectrum and commercial CDMA. Spread spectrum was originally designed for a few users trying to avoid interception and jamming. Bandwidth efficiency was of secondary importance. CDMA, on the other hand, is the application of spread spectrum to the multiple access challenge. Here we wish to support as many users as possible, so bandwidth efficiency is of primary concern. The anti-jamming and low probability of interception properties of spread spectrum can be used in several ways. First, by providing all users with ideally orthogonal spreading waveforms, we can support the same number of users as a TDMA or FDMA system in a given bandwidth assuming synchronous transmission. Second, by spreading the bandwidth of the signal beyond the coherence band of the channel we introduce frequency selective fading. This effect can be utilized as a form of diversity by employing a Rake receiver [103] to combine multipath. Third, if different spreading codes are overlayed on top of the waveforms of each cell, spreading codes and frequency bands can be reused in each cell. While the second point is an advantage over narrowband techniques in single link performance, it is the third factor which potentially
enables CDMA to provide significant capacity advantages. Current analog systems require a seven times frequency re-use pattern and TDMA proposals typically require a four cell re-use pattern. Thus, CDMA provides significant capacity advantages in a fundamental way.

Of course the argument is not so simple. As mentioned, the big caveat is that unlike TDMA or FDMA, orthogonality is maintained only when ideal codes (e.g. Walsh codes) are used in a perfectly synchronous channel. Unfortunately, synchronism can not be maintained on the cellular uplink from the mobile to the base station. This imperfection effects system performance in two ways. First, by introducing cross-correlation between spreading waveforms, we introduce in-cell interference which will limit the system capacity. The second effect is that the small cross-correlation introduces the need for strict power control. Even if there is a small amount of correlation between two spreading codes, if the received power of one user is significantly larger than a second user, the energy of the first will prevent the detection of the second. To avoid this problem, we require all users to transmit at levels that assure that the received signals of all users are approximately at the same power level.

These problems are the two major concerns about CDMA at this time: (1) the need for strict power control and the inability to ensure it, and (2) the limit in capacity due to interference. However, as we will discuss in the next section, these are not inherent problems of CDMA, rather they are the result of the conventional correlation receiver.

1.3 Multiuser Detection

As mentioned previously, if the spreading waveforms employed in CDMA are not received synchronously, there will be some cross-correlation between the waveforms resulting in mutual interference. While this interference is undesirable, abandoning the requirement of orthogonal waveforms provides significant implementation advantages. First, we need not employ cumbersome timing control among users, which may not be feasible in a mobile system. Second, we can now employ a larger set of spreading waveforms. That is, for a given spreading factor $N$, we are limited to $N$ orthogonal spreading codes. However, if non-orthogonal codes are used, significantly more codes are possible. The codes must be carefully designed, however, to assure some bound on maximum or average cross-correlation. This mutual interference is naturally mitigated by the fact that the voice activity during a given call is usually under 50% on a half-duplex link. Thus, the number of active users is approximately one half the number of actual users in the system. While TDMA and FDMA need sophisticated algorithms to take advantage of this inactivity, CDMA naturally takes advantage of it. By accepting some amount of interference, CDMA becomes a dynamic
channel sharing scheme. This flexibility is a strong advantage for CDMA.

However, we are still faced with the facts that the system is self-interference limited and that we require strict power control. These limitations are not inherent to CDMA, but result from the fact that we are using matched filter outputs (or simple correlators) to determine the most likely transmitted symbol. In an Additive White Gaussian Noise (AWGN) channel, this receiver structure is known to be optimal. However, in a multiple access environment, this is not the case. Since multiuser interference can often be adequately modeled as a Gaussian process [105], one might assume that the matched filter is essentially optimal for multiuser interference. However, this is not true [133]. Unfortunately, there are several cases (e.g. near-far situations) where the Gaussian Approximation for multiuser interference is grossly inaccurate. Additionally, the structured nature of multiple access interference allows for cancellation or rejection while AWGN has no inherent structure to exploit.

These facts led Poor in 1980 to improve the single user receiver (i.e. the conventional correlator) using non-Gaussian signal detection techniques [101]. In the early 1980's, the seminal work of Verdu [130] showed that the optimal receiver for multiple user channels provided significant gains over the conventional correlator. Additionally, Verdu showed that the optimal receiver was near-far resistant, i.e. its performance was invariant with respect to the received energy levels of all interferers. This receiver, termed a multiuser receiver¹, utilized all information available concerning the multiple users in the system. Essentially, Verdu recognized the analogy between multiuser interference and time-varying vectorized intersymbol interference (ISI). With this recognition it was shown that the optimal receiver in the presence of ISI (i.e. the Viterbi algorithm) could be adapted to provide optimal reception in multiuser environments.

Unfortunately, like many optimal solutions, the optimal receiver has a complexity which precludes its use in practical systems. However, the significant improvement in both performance and near-far resistance spurred research in sub-optimal solutions, and sub-optimal multiuser detection remains an extremely active research area.

1.4 Outline of Dissertation

The purpose of this work is to examine multiuser detection for cellular applications and focus on practical implementations of multiuser detection. This examination begins by an

¹There is some ambiguity in the literature as to the definition of a multiuser receiver. In this work, we define a multiuser receiver as any receiver which, working in a multiple user environment, is concerned with the detection of all of the users. This is contrasted to receivers that are designed to operate in a multiuser environment, but are concerned with only a single user's signal.
in-depth survey of the work performed to date in multiuser detection. This survey is presented in Chapter 2. The major receiver structures identified in Chapter 2 are then analyzed using a common simulation environment. The performance of these major structures are examined in a variety of situations such as AWGN, fading (both flat and frequency selective), near-far environments, and in the presence of practical imperfections such as time synchronization errors. The groundwork for this simulation study is presented in Chapter 3. Chapter 4 presents the simulation results created using a Multiuser Testbed as well as a brief discussion of other practical issues such as computational complexity and possible non-coherent implementations.

Based on the results of these simulations and discussions, one multiuser receiver is chosen as a focus for further analysis and hardware implementation. Multistage cancellation is found to have performance which is similar to other multiuser approaches while only requiring the same set of operations as the conventional receiver (e.g. correlations, additions). AWGN and fading analysis of multistage cancellation is presented in Chapter 5, and the performance in near-far situations is scrutinized in Chapter 6. A significant issue in interference cancellation (as opposed to interference rejection) is that we must accurately estimate the parameters of every user for cancellation to be successfully implemented. Parameter estimation (including amplitude, phase and timing estimation) is the focus of Chapters 7 and 8. Chapter 7 introduces the concept of parameter estimation and discusses simple methods of reducing the Mean Square Error of each estimate, while Chapter 8 examines the effect that estimation error has on system performance.

Chapter 9 discusses methods of improving interference cancellation such as reducing the bias in the decision statistic and employing adaptive antennas in co-ordination with multistage cancellation.

Hardware implementation is the subject of Chapter 10. Included in this chapter are the description and results of a baseband receiver design employing two stage cancellation. Multistage algorithms are developed for real time implementation, and methods for reducing the computational complexity are considered. Additionally, challenges are identified in taking the receiver to RF and possible solutions to these challenges are proposed. This discussion lays the groundwork for full scale implementation. Conclusions and future work are discussed in Chapter 11.

1.5 Contribution of this Work

The purpose of this thesis is the development of techniques for practically realizing the advantages of of multiuser detection. As noted in Section 1.3, this is an extremely active
area of research. The work presented here makes a number of novel contributions to the field. Significant contributions of this thesis include:

- A comprehensive comparison of alternative multiuser techniques on the basis of performance in multiple environments and complexity.

- The first closed form analytical result for the performance of multistage cancellation in fading environments.

- The first closed form analytical results for the performance of multistage interference cancellation in the presence of phase and timing errors.

- Discovery and analysis of a bias term which can degrade the performance of multistage interference cancellation and methods of mitigating it.

- The analysis of simple methods of improving parameter estimation in multiuser environments when using multistage interference cancellation.

- A baseband DSP hardware implementation of multistage interference cancellation, believed to be the first of its kind.
Chapter 2

A Survey of Multiuser Receivers for Cellular CDMA

2.1 Introduction

The tremendous increase in demand for wireless services has motivated a search for systems with significantly increased capacity as compared to today's standard analog cellular systems. One promising technique for better frequency utilization is Code Division Multiple Access (CDMA) [33]. However, in order to realize the full potential benefits of CDMA several advances must be made. For example, conventional receivers treat Multiple Access Interference (MAI), which is inherent in CDMA, as if it were additive noise. Unfortunately, this MAI is in general correlated with the desired user's signal and thus causes significant degradation. As a result, capacities for single cell CDMA systems can be substantially lower than those for orthogonal multiple access techniques such as Time Division Multiple Access (TDMA) or Frequency Division Multiple Access (FDMA). Also, if an interferer is significantly stronger than the desired user, the stronger interferer will dominate performance in a conventional receiver because of the near-far problem. Thus, CDMA performance can be greatly enhanced by receivers designed to compensate for MAI. Multiuser receivers represent one class of receivers that can improve CDMA performance.

This chapter presents a survey of multiuser receivers and their usefulness for CDMA, particularly at the base station. Cellular or Personal Communication Systems (PCS) design consists of two distinct problems, namely the design of the forward link from the base station to the mobile and the design of the reverse link from the mobile to the base station. The forward link can be designed so that users transmit with orthogonal spreading codes and all signals arrive at the mobile receiver with identical energy. However, the mobile receiver must
be inexpensive and have low power requirements. The reverse channel is more harsh, but can support a more sophisticated receiver. User signals arrive at the receiver asynchronously and can have significantly different energies, resulting in the near-far problem. In contrast to the mobile receiver, the base station receiver can be larger and more complex, have higher power consumption and use information available about the interfering signals. We focus on this latter situation because it is more feasible that the receiver can simultaneously detect signals from all users (i.e., implement multiuser reception). Even suboptimal multiuser receivers can show dramatic performance improvements when compared to conventional receivers, leading to the conclusion that many of the problems attributed to CDMA are not inherent, but instead result from the use of the conventional correlation receiver.

We will discuss both the optimal multiuser receiver as well as several suboptimal approaches. Section 2.2 presents the system model used throughout this thesis. In Section 2.3, the optimal multiuser receiver is reviewed. Section 2.4 examines several sub-optimal receivers, including linear and non-linear approaches. Finally, Section 2.5 suggests discusses the implications of the work surveyed here, motivating the work presented in the remainder of this thesis.

### 2.2 System Model

To facilitate the discussion of the multiuser receiver structures presented in this thesis, we provide a mathematical model of the CDMA system under consideration. The received signal can be represented as

\[ r(t) = \sum_{k=1}^{K} s_k(t - \tau_k) + n(t) \]  

(2.1)

where \( K \) users are independently transmitting bi-phase modulated signals in an AWGN channel, \( \tau_k \) is the delay of the \( k \)th user, \( n(t) \) is a Gaussian noise process with double-sided power spectral density \( \frac{N_0}{2} \), and

\[ s_k(t) = \sqrt{2P_k} b_k(t) a_k(t) \cos(\omega_c t + \theta_k) \]  

(2.2)

where \( P_k \) and \( \theta_k \) are the received power and phase of the \( k \)th user's signal respectively.

\[ b_k(t) = \sum_{i=0}^{N_b-1} b_{i,k} p_{T_b}(t - iT_b) \]  

(2.3)

is the data signal of the \( k \)th user with \( b_{i,k} \in \{-1, +1\} \), \( N_b \) is the length of the data sequence considered,

\[ p_{T_b}(t) = \begin{cases} 1 & 0 \leq t \leq T_b \\ 0 & \text{else} \end{cases} \]  

(2.4)
\( T_b \) is the duration of a data symbol, and the spreading waveform \( a_k(t) \) (also termed the pseudo-noise or PN code) is

\[
a_k(t) = \sum_{i=0}^{N_b-1} \sum_{j=0}^{N-1} a_{j,k} p_{T_c}(t - jT_c - iT_b)
\]

(2.5)

where \( N = \frac{T_b}{T_c} \) is the spreading gain as well (as the code length) \( a_{j,k} \epsilon \{-1, +1\} \), and \( T_c \) is the duration a rectangular chip.

Often it will be desirable to represent the system after despreading (i.e., after matched filtering). The output of a filter matched to the \( k \)th user's spreading waveform, after the \( ith \) bit interval (assuming perfect carrier and PN code phase tracking), can be represented as

\[
y_{i,k} = \int_{(i-1)T_b + \tau_k}^{iT_b + \tau_k} r(t)a_k(t - \tau_k - iT_b) \cos(\omega_c t + \theta_k)dt
\]

(2.6)

which leads to a vector representation of the matched filter outputs corresponding to the \( ith \) bit interval,

\[
y_i = R(-1)W_{i-1}b_{i-1} + R(0)W_i b_i + R(1)W_{i+1}b_{i+1} + n_i
\]

(2.7)

where \( y_i = [y_{i,1}, y_{i,2}, \ldots, y_{i,K}]^T \), \( R(i) \) is a \( K \times K \) matrix which represents the partial correlation between users over the \( ith \) relative bit interval (i.e. between the \( n \)th bit of user \( j \) and the \((n+i)\)th bit of user \( k \)). Thus, if \( \rho_{j,k}(i) \) are the elements of \( R(i) \),

\[
\rho_{j,k}(i) = \int_{\tau_j}^{iT_b + \tau_j} a_j(t - \tau_j)a_k(t - \tau_k - iT_b)dt,
\]

(2.8)

\( W_i \) is a diagonal matrix with vector \([\sqrt{w_{i,1}}, \sqrt{w_{i,2}}, \ldots, \sqrt{w_{i,K}}]^T \) along the diagonal, \( w_{i,k} = P_{i,k}T_b \), \((\cdot)^T \) represents the transpose operation, \( b_i = [b_{i,1}, b_{i,2}, \ldots, b_{i,K}]^T \), and \( n_i = [n_{i,1}, n_{i,2}, \ldots, n_{i,K}]^T \) is a vector of noise samples. It should be noted that by definition, \( R(i) = 0 \) for \(|i| > 1 \). In general we will drop the dependence of \( w_k \) on time. If we observe a sequence of \( N_b \) data symbols from each of \( K \) users, we can represent the observed sequence of matched filter outputs by

\[
y = RWb + n,
\]

(2.9)

where \( y = [y_1^T, y_2^T, \ldots, y_{N_b}^T]^T \), \( b \) and \( n \) are similarly defined, \( W \) is a \( KN_b \times KN_b \) diagonal matrix with \([W_1, W_2, \ldots, W_{N_b}]^T \) along the diagonal, and

\[
\mathcal{R} = \begin{pmatrix}
R(0) & R(-1) & 0 & \cdots & 0 \\
R(1) & R(0) & R(-1) & \vdots \\
0 & R(1) & R(0) & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & R(-1) \\
0 & \cdots & 0 & R(1) & R(0)
\end{pmatrix}.
\]

(2.10)
2.3 Optimal Multiuser Reception

2.3.1 AWGN Channels

The optimal receiver was first addressed in [111] for both the synchronous and asynchronous AWGN channels. Verdu expanded this work by more fully developing the mathematical model for the important case of the asynchronous channel and by determining the minimum receiver complexity [130]. Furthermore, Verdu developed probability of error bounds for the receiver.

It was noted in [130] that there is not a unique optimality criterion because transmitted signals are not independent conditioned on the received signal. One can optimize according to maximum likelihood sequence detection (global optimality) or one can optimize according to minimum probability of error (local optimality). The derivation in [130] pursues the former, due to implementation considerations. It should be noted that for low noise levels the two criteria will yield identical results [129]. For maximum likelihood sequence detection we desire to maximize the joint posteriori probability

\[ P[b | r(t)] \]  \hspace{1cm} (2.11)

where \( r(t) \) is defined in (2.1). If all input symbols are equally likely, this is equivalent to maximizing

\[ P[(r(t)|b)]. \] \hspace{1cm} (2.12)

For the AWGN channel this maximization results in

\[ b = \arg \max \left[ \exp \left( \frac{\Omega(b)}{2\sigma^2} \right) \right] \] \hspace{1cm} (2.13)

where \( \Omega(b) = 2 \int_{-\infty}^{\infty} S(b) r(t) dt - \int_{-\infty}^{\infty} S^2(b) dt \), \( \sigma^2 \) is the noise power, and

\[ S(b) = \sum_k s_k(t, b). \] \hspace{1cm} (2.14)

Verdu’s work showed that the minimal complexity receiver can be realized by equating multiple access interference to vectorized intersymbol interference (ISI). Treating MAI as vectorized ISI with memory equal to the number of interferers, it can be shown that \( \Omega(b) \) can be decomposed as

\[ \Omega(b) = \sum_{i=0}^{KN_b-1} \lambda_i(x_i, b_{\eta(i), \gamma(i)}) \] \hspace{1cm} (2.15)

13
where \( \lambda_i = b_{\gamma(i)} \left[ 2y_{\gamma(i)}, \gamma(i) - b_{\gamma(i)} \gamma(i) - \langle x_i, g_i \rangle \right] \), \( \gamma(i) = i - \left[ \frac{i-1}{K} \right] \). \( K \) represents the current user, \( \kappa(i) = \left[ \frac{i-1}{K} \right] + 1 \) represents the current bit, \( \left[ \cdot \right] \) represents the floor operation, \( x_i \) is the state vector at time \( i \), \( g_i \) is a \( (K - 1) \times 1 \) vector where the \( l \)th element is \( g_{i(l)} = \rho_{\gamma(i), \gamma(i-l)} \left( \text{rnd} \left[ \frac{2^{(i-l)} - \gamma(i)}{K} \right] \right) \), \( \text{rnd}[\cdot] \) is a rounding towards negative infinity, and \( \langle \cdot, \cdot \rangle \) is the dot product. The state vector \( x_i \) represents the previous \( K - 1 \) bit decisions, corresponding to the previous bit decisions of each of the other \( K - 1 \) users. With the decomposition in (2.15) we see that the maximum likelihood receiver can be implemented using a dynamic programming algorithm (i.e., the Viterbi algorithm). The algorithm has \( 2^{K-1} \) states, since each of the previous \( K - 1 \) decisions will impact the current decision, and \( K \) updates per bit decision due to the fact that, in this particular implementation, we treat the received bit of each user separately.\(^1\) Thus,

\[
\hat{b} = \arg \max \left( 2b^T y - y^T y \right) \tag{2.16}
\]

implying that the set of observations \( y \) is a sufficient set of statistics for determining \( \hat{b} \). Unlike the synchronous case, optimal asynchronous reception requires observation during more than a single symbol interval (i.e., Maximum Likelihood Sequence Estimation (MLSE)) as opposed to a simpler Maximum Likelihood Detector (MLD). Since each symbol will require complete observation of the overlapping interferers' symbols and since this can also be said of the interfering symbols, we cannot make any restrictions on the observation interval for optimal reception.

Thus, the optimal receiver consists of a bank of matched filters for each user followed by a decision algorithm which maximizes (2.16) and can be implemented using (2.15). As an

\(^1\)A somewhat more complex algorithm which requires only one update per bit decision is shown in [129] which does not fully take advantage of the decomposability of the log-likelihood function. This algorithm while requiring fewer updates, does have a larger state space, \( 2^K \).
example of optimal reception, consider a three-user asynchronous system and the resulting trellis as shown in Figure 2.2. We allow \( i = 3 \), that is we are concerned with the matched filter output of user \( \gamma(3) = 3 - [(3 - 1)/3]3 = 3 \) during bit interval \( \kappa(i) = [3/3] = 1 \). Notice that each node at epoch \( i = 3 \) has two branches entering it. To minimize calculations while determining the maximum likelihood path, we must eliminate one of the two entering paths. To determine the survivor branch, we compute the metric

\[
\lambda_3 = \hat{b}_{1,3} \left( 2y_{1,3} - w_{1,3}\hat{b}_{1,3} - \langle x, g \rangle \right)
\]

\[
= 2y_{1,3} - w_{1,3}\hat{b}_{1,3} - b_{1,3} \ast \rho_{1,3}(0) - b_{1,2} \ast \rho_{1,2}(0).
\]

where \( b_{1,1} \) and \( b_{1,2} \) are the previously decoded bits and make up the state vector and \( \hat{b}_{1,3} \in \{+1, -1\} \). As usual in Viterbi decoding, after approximately a decoding depth of 5-6 times the number of users, the most likely path will be the only path remaining and will denote the maximum likelihood estimate of the received bit sequence. It should be noted that for optimal reception, knowledge of the spreading codes, associated timing, and received energies of each user are required.

Verdu showed that the gains over conventional receivers were tremendous [130]. Moreover, Verdu developed a measure entitled asymptotic multiuser efficiency [130, 131] defined as

\[
\eta_k = \lim_{\sigma \to 0} \frac{e_k}{w_k}
\]

where \( \sigma^2 = \frac{N}{2} \) is the spectral level of the background AWGN, \( w_k \) is the energy required to obtain a given probability of error \( P_{e,k} \), and \( e_k \) is the effective energy of the \( k \)th user defined as the energy required to obtain the same probability of error in a single user.
environment. Thus the multiuser asymptotic efficiency varies between 0 and 1 and describes the degradation in performance due to MAI. The near-far resistance of a detector is defined to be the worst case multiuser asymptotic efficiency over all users' energies $\eta_k$. Formally [79],

$$\eta_k = \inf_{\eta_j \geq 0, j \neq k} \eta_k.$$  \hspace{1cm} (2.20)

This measure was created to measure the effectiveness of the receiver in an environment with loose or non-existent power control. As mentioned earlier, sensitivity to power control is a major concern in CDMA. The conventional receiver can be shown to have a near-far resistance of zero (which is intuitive), while the optimal receiver is guaranteed to have a near-far resistance greater than zero as long as all users' code sequences are linearly independent. Near-far resistance as defined is an asymptotic measure and may not accurately reflect a receiver's performance in practical situations. Thus, not all receivers with $\eta_k = 0$ are equal. For example, multistage cancellation is not in general near-far resistant. However, it shows significant near-far robustness when compared to the conventional receiver[11]. While near-far resistance gives a good measure of asymptotic behavior it is not a necessarily a good measure of robustness to strong interference. However, the excellent near-far resistance (as well as BER performance) of the optimal multiuser receiver has prompted a search for sub-optimal solutions which can approach this performance while maintaining a reasonable complexity.

2.3.2 Extensions to Fading Channels

While the research just discussed focused on optimal coherent reception in AWGN channels, subsequent work extended optimal reception to the case of fading channels. Specifically, optimal reception was investigated for Flat-Rician fading (assuming synchronous systems) [127], frequency selective Rician fading [128], Flat-Rayleigh fading [153], and general frequency selective multipath channels [28].

The optimal receiver for the flat-fading Rician channel case is derived by demonstrating that the Rician flat fading channel can be equated to an AWGN where a modified signal set is used [127]. It is found that the resulting receiver is also near-far resistant. Additionally, when the optimal receiver for the AWGN channel is used in the Rician fading channel, it is found to suffer performance degradation but still maintain near-far resistance. It should be noted that in addition to the knowledge of the spreading codes, received amplitudes, timing, and phases of each user which are required in the AWGN case, the optimal Rician fading receiver also requires knowledge (or estimation) of the fading variances of each user.
The previous result was generalized to the frequency selective Rician (asynchronous) channel in [128]. The frequency-selective Rician channel was equated to an AWGN channel with intersymbol interference and a modified signal set. Unlike the previous channel, the modified signal set results in a cross-correlation matrix which is not banded diagonal nor block Toeplitz (see equation 2.10), thus eliminating the possibility of a decomposition such as (2.15), resulting in a computational complexity which is not independent of the block length. The resulting optimal receiver is thus infeasible. The work, however, does provide bounds on both performance and near-far resistance for the frequency selective Rician fading channel. Again, it was found that the optimal receiver is near-far resistant and that the optimal receiver for the AWGN channel also results in near-far resistance when applied to this channel, although performance is less than optimal.

The optimal receiver was extended to single path Rayleigh fading channels in [153]. It was shown that when the knowledge of the fading coefficients are known, the optimal receiver is again a bank of matched filters followed by a dynamic programming algorithm (i.e. the Viterbi algorithm). The optimal receiver is shown to be near-far resistant and has performance rivaling the single-user case (i.e. isolated transmission in a flat Rayleigh fading channel). The more general case of frequency selective multipath channels was discussed in [27], where it was shown that the optimal detector is comprised of $KL$ matched filters (where $L$ is the number of paths), or equivalently $K$ Rake correlators.

2.3.3 Complexity

An extremely important consideration in multiuser detection algorithms is the computational complexity of the algorithm. As mentioned earlier, for AWGN channels the minimal complexity optimal multiuser receiver utilizes the Viterbi algorithm with $2^{K-1}$ states and $K$ updates per bit decision. Thus, while the algorithm is independent of the sequence length, it has a computational complexity which is exponential in the number of users [132]. This level of complexity is unacceptable for reasonable population sizes. However, due to the potential increase in performance and near-far resistance of the optimal multiuser receiver, we seek sub-optimal solutions which are less computationally intensive while maintaining significant performance advantages. Several such receiver structures are discussed in the next section.
2.4 Sub-optimal Multiuser Reception

The previous section mentioned that while significant performance gains could be achieved over the conventional matched filter receiver. However, the cost of this performance gain is exponential complexity in the number of users. In this section we investigate receivers which can approach the performance of the optimal receiver with significantly reduced computational complexity. These sub-optimal receivers can be broken down into two general categories, linear and non-linear as shown in Figure 2.3. Linear sub-optimal receivers create data estimates based upon linear transformations of the sufficient statistics (i.e., the matched filter outputs sampled at the symbol rate) while the non-linear implementations make decisions using non-linear transformations of the sufficient statistics.

2.4.1 Linear Receivers: The Decorrelator

In this section we consider suboptimal multiuser approaches which rely on a linear combination of the sufficient statistics for decisions. The first significant development of linear suboptimal reception was the decorrelating receiver reported in [78, 79, 68] for both synchronous and asynchronous AWGN channels. Focusing on the asynchronous case, we rewrite the set of sufficient statistics given in (2.9) as

\[ y = \mathcal{R}\psi b + n \]  

(2.21)
where \( y, R, W, b, \) and \( n \) were previously defined. A linear detector is one which makes decisions based on
\[
\hat{b} = \text{sgn}(Ty) = \text{sgn}(T(RWb + n))
\]
where \( T \) is a linear operator on \( y \).

Using near-far resistance to drive suboptimal receiver development, Lupas showed that a linear receiver can obtain optimal near-far resistance (that is the near-far resistance of the optimal receiver) \[79\]. Specifically, Lupas showed that the linear detector which accomplishes this is the decorrelating detector which is also maximum likelihood linear detector. If the multiple access channel (excluding noise) is viewed as a deterministic multi-input, multi-output linear filter with transfer function \( R \), then we can remove the interference in each of the matched filter outputs by applying the inverse transfer function. In other words, \( T = R^{-1} \) which leads to
\[
\hat{b} = \text{sgn}(R^{-1}y) = \text{sgn}(Wb + R^{-1}n).
\]
Since \( R \) as defined is simply the normalized cross-correlation between the users’ spreading codes, the decorrelator is independent of the received user energies. Because of this, the decorrelator is the optimal linear receiver for unknown signal energies \[79\]. This obviates the need for estimates of the received user energies. This is a significant advantage since energy estimates tend to be extremely noisy. Additionally, \( 2.24 \) shows that the data estimate is independent of the interfering powers thus providing near/far resistance.

There are, however, two main disadvantages of this receiver. The first is the need to calculate the inverse of the cross-correlation matrix in order to obtain the decorrelation coefficients. The second is that in high noise situations, the receiver performance can degrade due to the correlation introduced in the originally white noise. That is, similar to the ISI analog known as the zero-forcing equalizer, the application of the channel inverse creates correlated noise samples. This results in increased noise power which is relative to the cross-correlation between users. To show this, we define \( \Sigma_y = \text{var}[y] \) and
\[
\Sigma_y = E[yy^T] - E[y]E[y^T]
\]
\[
= E[(Wb + R^{-1}n)(Wb + R^{-1}n)^T] - (Wb)(Wb)^T
\]
\[
= E[(Wb + R^{-1}n)((Wb)^T + (R^{-1}n)^T)] - (Wb)(Wb)^T
\]
\[
= E[Wb(Wb)^T + Wb(R^{-1}n)^T + R^{-1}nb^TW^T + R^{-1}nn^T(R^{-1})^T] - (Wb)(Wb)^T
\]
\[
= WbE[n^T](R^{-1})^T + R^{-1}E[n]b^TW + R^{-1}E[n^T](R^{-1})^T
\]
where we have used \( E[n] = 0 \) and \( E[nn^T] = \sigma_n = \sigma^2 R \), and \( \sigma^2 \) is the power of the AWGN at the receiver. Thus the decorrelation process, while removing MAI, has enhanced the noise.

**Extensions**

The decorrelator as described does not require estimates of the users' received signal energies. However, it does require estimates of the timing and phase of each user along with knowledge of the spreading waveforms. The work presented in [123, 126] generalizes the decorrelator to the case of non-coherent demodulation (specifically differentially coherent PSK) where neither the received amplitudes nor the received phase need be estimated. This receiver, termed the bilinear detector, simply performs the decorrelating operation followed by differential detection. That is

\[
\hat{b}_{i,k} = \text{sgn} \left( \Re \left( \tilde{d}_{i-1,k} \tilde{d}_{i,k}^* \right) \right) 
\]

(2.25)

where \((\cdot)^*\) represents the complex conjugate, \(\Re(\cdot)\) takes the real part of the argument,

\[
\tilde{d}_i = R^{-1} y_i
\]

(2.26)

and \(y_i\) is the vector of matched filter outputs with the phase information still intact (i.e., no phase tracking or coherent combination was performed) during the ith bit interval, and where we have assumed a synchronous channel\(^2\). The performance is again invariant to the interfering powers thus obtaining near-far resistance and additionally does not require phase tracking, assuming that the phase is constant over two consecutive bit intervals.

Similar to the optimal receiver case, the decorrelator was generalized to fading channels in subsequent work. Specifically, the decorrelator was extended to flat and frequency selective Rician fading channels [127, 128] and flat and frequency selective Rayleigh fading channels (both coherent and differentially coherent) [153, 152, 154, 151, 150, 46]. The most notable difference between these detectors and the AWGN case described earlier is that the multipath versions treat resolvable multipath as individual users until a final decision is made. That is, the transform matrix \(R\) is now \(KL \times KL\) incorporating each of the \(L\) paths from each user. The outputs for each path are then combined according to the desired diversity combination scheme (e.g., maximal ratio combining).

\(^2\)The synchronous case was chosen for the sake of notational simplicity, but can be extended to the asynchronous case in a straightforward manner.
An alternative to this approach is to use a $K \times K$ correlation matrix which represents the correlation between $h_k(t)$ and $h_j(t)$ where $h_i(t)$ is the convolution of the impulse response of the $i$th user's channel with the $i$th user's spreading code [46]. Thus, the elements of $\mathbf{R}$ contain the multipath information directly. It was shown that this receiver outperforms the receiver which tracks the individual paths separately, albeit at the cost of higher complexity in a dynamic channel.

The preceding detectors, when considering ISI, incorporated only independent resolvable paths. That is, the effect of time dispersion on the individual symbols (due to the channel bandwidth limits) is not considered outside of resolvable multipath. The decorrelating detector of [65, 66] (referred to as a Joint Detection receiver) included the effects of ISI by incorporating this information into the correlation coefficients.

**Implementation Issues**

The discussion of the decorrelator up until this point has assumed that a finite block of data is received. This implies a level of co-ordination between users which, while not as complicated as attempting synchronism, certainly would be burdensome. Thus, we must assume that an infinite data stream is to be decoded. In this case, an infinite time delay is associated with the transform. An alternative to this is to operate on data on a block by block basis. By working on a block of data we ignore the bits immediately preceding and following the data block under consideration. Due to asynchronism between users, these edge bits will effect the performance of the desired user, leading to the conclusion that this suboptimal receiver is no-longer independent of interfering energy [143] (i.e., it is no longer near-far resistant). That is, if we operate on the $N_b$ bits from $M + 1$ to $M + N_b$, the $(M + 1)$th bit of user $k$ (assuming users are numbered such that $\tau_1 \leq \tau_2 \ldots \leq \tau_K$) will have a dependence on the $M$th bit of users $k + 1$ through $K$. Additionally, the $(M + N_b)$th bit of the $k$th user will be dependent on the $(M + N_b + 1)$th bit of users 1 through $k - 1$ which are not yet decoded.

A method to adjust the decisions to correct for edge effects was discussed in [138, 139]. The $M$th bit of each user will have been decoded previously and thus can be used to aid in the correction of the current data block. However, the $(M + N_b + 1)$th bit of each user will not have been detected. Making a hard decision on the correlator outputs is one option, but is not very reliable. It has been suggested that the use of convolutional coding could provide reliable estimates of the future bits required for edge correction [138, 139]. It was shown experimentally that this implementation, termed the 'sliding window decorrelator' is practically near-far resistant [138].
Another important implementation issue is the updating of the transform matrix. Traditional matrix inversion requires $O(K^3)$ operations (not taking into consideration the block-Toeplitz nature of the matrix). If this inversion has to be repeated each time a user enters or leaves the system or the channel changes, it could result in a significantly high computational complexity. Alternatively, if a single user enters or leaves the system, there exist computationally efficient methods for updating the inverse [57]. Specifically, it has been shown [57] that for a synchronous system, updating the matrix inverse due to an increase or decrease in the user population requires $O(K^2)$ operations. The work of [86] also discusses methods of adaptively updating the correlation matrix when a new user enters or leaves the system. This work assumes no knowledge of the new user’s spreading codes and demonstrates the performance degradation suffered when a new user is not properly removed.

Several researchers investigated reduced complexity versions of the decorrelator. One such detector is a pre-selection decorrelator [83] which decorrelates only the $N_{sel}$ interferers (with $N_{sel} < K$) which have the largest effect on the desired user and ignores the others. This relies on the fact that there will be relatively few interferers which have significant impact on the desired user. The smaller the value of $N_{sel}$, the lower the complexity, and the less the noise enhancement is. However, this also means that more interferers are not removed. Thus, a trade-off is made between the effect of noise and interference, as well as between performance and complexity.

A second complexity reduction technique is termed the one-shot decorrelator [56, 3]. The one-shot decorrelator reduces complexity by removing the memory of the decorrelator in an asynchronous system. This is accomplished by treating the $K$ user asynchronous channel as a $2K - 1$ user synchronous channel (i.e., each interfering bit is treated as a separate user). While this results in some complexity increase by effectively increasing the number of users from $K$ to $2K - 1$, the savings is realized in that the data block size which must be considered is only one bit (i.e., there is no memory). A disadvantage is that a different matrix is required for each desired user. A method to avoid this is to simply combine the data from a single one-shot receiver [3]. Another implementation of the one-shot decorrelator uses on-off keying (i.e., inserting an off period between symbols) to decouple adjacent bits of different users in the asynchronous channel [147].

Another variant of the decorrelator is the adaptive Least Squares decorrelator of [14] for synchronous CDMA systems. This receiver implements the decorrelating receiver in a particularly efficient way. Instead of computing the inverse matrix required to obtain the decorrelation coefficients, an RLS routine is run to adapt the filter coefficients to the proper
values. The received signal from equation (2.1) after sampling and downconversion in a synchronous system can be written as

\[ r(m) = \sum_{k=1}^{K} \sqrt{\frac{P_k}{2}} b_k(m)a_k(m) + n(m), \]  

(2.27)

where \( m = nT_s \), \( n \) is an integer and \( T_s \) is the sampling period. We can estimate the energies and bit sequences of the received signals using a \( K \)-tap filter. That is,

\[ \hat{r}(m) = \sum_{k=1}^{K} c_k(m-1)a_k(m) \]

(2.28)

\[ = C^T(m-1)A(m) \]  

(2.29)

where \( C(m) \) is the \( K \) dimensional vector of tap coefficients \( c_k \) at sample \( m \) and \( A(m) \) is a \( K \) dimensional vector of the \( m \)th samples of the \( K \) user waveforms (i.e. \( A(m) = [a_1(m), a_2(m), \ldots a_K(m)]^T \)). We can then update the coefficients \( c_k(m) \) using an RLS routine where the error is based on minimizing the Least Squares (LS) criterion:

\[ \sum_{k=1}^{K} |r(m) - \hat{r}(m)|^2. \]  

(2.30)

This leads to an estimate of the vector of received bits \( \hat{b} \) as

\[ \hat{b}_i = \text{sgn}[C(L)], \]  

(2.31)

where \( L \) is the number of samples per bit. In the limit, this detector converges to the decorrelator, that is

\[ \hat{b}_i = \text{sgn}[W_i b_i + R^{-1}\gamma], \]  

(2.32)

where \( \gamma \) is a \( K \) dimensional vector of Gaussian random variables and \( R \) is a synchronous, sampled version of (2.10) where there is no longer dependence on previous or successive bit intervals, that is

\[ R = \sum_{m=1}^{L} A(m)A^T(m), \]  

(2.33)

or \( R = R(0) \) and \( R(-1) = R(1) = 0 \). With this procedure, the matched filter operation is eliminated, as well as the matrix inversion. This increases the efficiency of the decorrelating receiver. However, this receiver is subject to the same pitfall as the decorrelator, poor performance for relatively weak users. Additionally, the tap weights must converge within \( L \) iterations. To improve receiver performance for weak users, the authors suggest the use of decision feedback which will be discussed in more detail later.
While the previous detectors utilize the inverse of the cross-correlation matrix to separate the signals of each user, it should be noted that this is not the only linear transformation which will perform the separation. In fact, any transform which diagonalizes $\mathcal{R}$ will separate the signals. One such example is the 'improved decorrelator' of [144]. This paper proposes a linear transformation $\mathbf{T} = \mathbf{I} - \mathbf{W}$ where

$$
\mathbf{W}^T = \begin{bmatrix}
0 & w_{12} & w_{13} & \cdots & w_{1K} \\
w_{21} & 0 & w_{23} & \cdots & w_{2K} \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
\vdots & & \ddots & 0 & w_{K-1,K} \\
w_{K1} & \cdots & w_{K,K-1} & 0
\end{bmatrix}.
$$

(2.34)

The output of the detector in the synchronous case is then $\mathbf{z} = \mathbf{T}\mathbf{y}$ or

$$z_{i,k} = y_{i,k} - \sum_{j \neq k} w_{jk}y_{i,j} + \eta_{i,k}
$$

(2.35)

where $\eta_{i,k} = n_{i,k} - \sum_{j \neq k} w_{jk}n_{i,j}$. It can be shown that if we choose $w_k = \hat{R}_k^{-1}\hat{\rho}_k$ where $\hat{R}_k$ is $\mathbf{R}$ with the $k$th column and row removed and $\hat{\rho}_k$ is the $k$th row of $\mathbf{R}$ with the $k$th element removed, the transformation will result in a diagonal matrix $\mathbf{T}\mathbf{R} = \text{diag}(1 - \sum_{j \neq 1} w_{j,1}\rho_{j,1}, \sum_{j \neq 1} w_{j,2}\rho_{j,2}, \cdots \sum_{j \neq 1} w_{j,K}\rho_{j,K})$. It can be shown that diagonal terms of $\mathbf{T}\mathbf{R}$ are upper bounded by the diagonal terms of $\mathbf{R}^{-1}$, and thus, the performance of the 'improved decorrelator' is upper bounded by the decorrelator which uses $\mathbf{T} = \mathbf{R}^{-1}$. The upper bound is reached in severe near-far situations [144].

The values of $\mathbf{W}$ can also be computed adaptively via the 'bootstrap algorithm' where [4]

$$w_k(i) = w_k(i - 1) + \mu y_k \text{sgn}[y_k]
$$

(2.36)

and $\mu$ is a forgetting factor. A similar idea is the adaptive detector of [29] which is termed the Orthogonalizing Matched Filter.

Other discussions of the decorrelator include a modified decorrelator for a quasi-synchronous, microcell environment [58], a linear decorrelating detector for narrowband interference suppression [108], a method of combating ISI via bit insertion [114], and an adaptive decorrelator implemented using a multidimensional lattice filter [53].

### 2.4.2 Linear Suboptimal Receivers: Minimum Mean Square Error

A key observation of [129] was the recognition of the analogous relationship between multiple access interference cancellation and the equalization of intersymbol interference. The
decorrelator mentioned previously is very similar to the zero-forcing equalizer for ISI channels. This observation lead to other transfers of ISI techniques. In [141], linear detectors based on minimum mean squared error (MMSE) were proposed. This detector attempts to minimize $E[(b - \hat{b})^T(b - \hat{b})]$ where $E[.]$ is the expected value and $\hat{b} = \text{sgn}(Ty)$ for some linear transform $T$. The linear transformation $T$ which attains the minimum value is

$$T = \left( R + \frac{N_0}{2W^2} \right)^{-1}. \tag{2.37}$$

This leads to the decision rule

$$\hat{b} = \text{sgn}\left[ (R + \frac{N_0}{2W^2})^{-1} y \right]. \tag{2.38}$$

It is shown in [141] that this receiver has performance which is upper-bounded by the decorrelator. Additionally, from (2.37) we can see that for $N_0 \approx 0$ (i.e. low noise situations) the two receivers are identical. In the opposite extreme ($N_0$ extremely large), the MMSE detector reduces to the conventional receiver. Like the decorrelating receiver, the MMSE receiver obtains optimal near-far resistance, however, unlike the decorrelator it requires knowledge of the received user energies. While slight performance improvement is found (esp. in the high noise case), the cost is high. Thus, this receiver does not offer significant advantages over the decorrelator. It is noted in [134] that this detector lends itself well to adaptive multiuser implementation. Specifically, if training sequences are employed, knowledge of all users’ spreading codes is not required. Instead, adaptive updates of the tap weights can converge to the necessary coefficients. The idea of MMSE and Weighted Least Squares (WLS) based detectors (which account for MAI as well as ISI channel effects) is developed in [23, 66, 67].

A constrained MMSE detector was proposed in [89] which approximates the MMSE detector by approximating the desired inverse matrix by polynomial expansion in $R$. It was shown that in a synchronous system, the MMSE detector is approximated extremely well with a 3rd order approximation in the low SNR case and a 10th order approximation in the high SNR case. Other extensions of the multiuser MMSE receiver include the addition of a power control algorithm when the MMSE receiver is used [73] and the addition of a multivariate MMSE employing decision feedback which will be presently discussed.

### 2.4.3 Non-Linear Suboptimal Receivers: Decision Feedback

The previous sections limited the discussion to linear sub-optimal detectors. As in the case of equalization, large performance improvements can often be realized if non-linear techniques are applied. We conveniently divide these techniques into three major categories:
decision feedback, multistage receivers, and other non-linear receivers. A classical example of non-linear equalization is decision feedback [104]. As with the zero-forcing equalizer and the MMSE equalizers, the concept of decision feedback from ISI equalization can be extended to multiuser channels.

**Successive Cancellation**

The first discussion of decision feedback for multiuser reception is found in [135]. In this method, all users are ordered in decreasing received signal power. The strongest user is then detected first using the standard correlation receiver. The decision made on this user is then fed back to the detector of the second user to improve the estimate of that user. The previous decisions are utilized by using the symbol estimates, as well as an estimate of the user’s received energies and knowledge of the spreading codes, to remove the effects of the previous bits. Formally,

\[
\hat{b}_{i,k} = \text{sgn} \left[ y_{i,k} - \sum_{j=1}^{k-1} \left( \rho_{jk}(1) \sqrt{w_j} \hat{b}_{i-1,j} + \rho_{jk}(0) \sqrt{w_j} \hat{b}_{i,j} + \rho_{jk}(-1) \sqrt{w_j} \hat{b}_{i+1,j} \right) \right].
\]  

(2.39)

The signals which have estimates that are the most reliable (i.e., the strongest) are detected first. There are two reasons for this approach. First, since the strongest users have the most reliable estimates, we can be more confident about the quality of the remaining signal after cancellation. Additionally, the strongest users are more robust to interference, thus requiring less cancellation for detection.

The analysis of successive interference cancellation was extended in [42, 41, 98] by dealing with practical implementation issues as well as by analyzing the case of multipath fading. A non-trivial issue in this receiver structure is the estimation of the received energies of each user. The quality of these estimates can dominate the performance of the technique. In [98], outside energy estimates are avoided by using the output of conventional correlators as an estimate of the signal energy. This information is used to rank the signals for cancellation. Additionally, hard decisions are not made on individual correlator outputs. Rather the correlator output is used to estimate the received amplitude and data bit and is multiplied by the user spreading code to regenerate the interference. This soft-decision approach eliminates the need for separate energy estimates. The analysis of [98] was then extended to the case of multipath channels as well as to non-coherent demodulation. The latter analysis assumes no absolute phase information is available, thus showing that, like the previous detectors, successive cancellation can be extended to non-coherent reception.

Finally it was shown that performance can be improved by using the average of consecutive correlator outputs as an estimate rather than a single output. This improvement
increases with the number of bits over which the average is taken due to the reduction of the variance in the energy estimate. Of course, this is limited by the speed of any fading encountered. However, the authors offer that the correlation between symbols ten intervals apart at a frequency of 1.8 GHz is approximately 0.94 when the symbol rate is 30 ksym/s and the vehicle speed is 100 km/hr. This suggests that some amount of averaging would be beneficial even in fast fading environments.

In [64], several variations of successive cancellation were explored. It was shown that by using a more complicated reliability criterion for cancellation order (rather than simple energy estimates), performance could be improved. More explicitly, the work demonstrated that by cancelling in order of Signal-to-Interference Ratio (SIR) rather than received energy, some performance advantage could be gained. The cost for this increased performance is that SIR must be calculated for each user. Some simplifying modifications were also explored such as cancelling users in groups. Group cancellation was done to limit the number of estimation and cancellation steps. The drawback to such a scheme is that users in a particular group do not benefit from the cancellation of other users in the group. One such cancellation procedure divided users into reliability groups where reliability was determined by SIR and where the group size was determined by the number of users which fell into the particular reliability group. A similar procedure separates users into groups of fixed numbers where groups were determined by decreasing reliability while the reliability range of the group was variable. Both schemes, while providing some reduction in complexity, showed significant performance degradation. Groupwise cancellation was also examined in [122] where more promising results were obtained. Specifically, it was found that a group size of two was found to provide significant complexity reduction with a marginal sacrifice in performance.

Another drawback to successive cancellation is that the BER of users can vary significantly depending on when a user is cancelled. Specifically, if users have similar energy, that user which receives fewer cancellations will experience a worse BER since the small amount extra energy does not offset the penalty of receiving one less cancellation. The authors of [82] suggest a power control scheme which would alleviate this problem. By adjusting powers such that there is an optimal difference between users’ received powers, the performance of successive cancellation can be optimized. The cost, however, is that power control is required. Instead of trying to keep all users at equal power levels (which is necessary for the conventional correlation receiver), the goal is to ensure a specific power ranking which benefits successive cancellation. This theoretically ensures equal BER for all users with the average BER being the best possible for successive cancellation. This may not be attractive,
however, since power control can be burdensome and difficult to maintain.

The decision feedback schemes mentioned require the estimation of the amplitude, data, phase (for coherent demodulation), and timing of the interferer. If these estimations are not accurate, the performance of the technique degrades rapidly. The effect of error in the amplitude estimate for the successive cancellation scheme was discussed in [98]. Here it was shown that by reducing the amplitude estimation error through averaging performance could be enhanced. It was shown in [15] that errors in the timing of users can be tolerated for errors less than $0.2T_c$.

**Decorrelating Decision Feedback**

It was noted previously that a main drawback of the decorrelating receiver was the correlation it introduced to the originally white noise. In high noise situations, this can significantly degrade receiver performance. This problem can be avoided through a factorization of the cross-correlation matrix. This approach leads to a decorrelating decision feedback receiver as described in [20]. Specifically, considering a synchronous channel, the cross-correlation matrix $R$ (it is now a $K \times K$ positive definite matrix of synchronous cross-correlations) can be factored as

$$R = F^T F,$$  \hspace{1cm} (2.40)

where $F$ is a lower triangular matrix. If a filter with response $(F^T)^{-1}$ is then applied to the matched filter outputs, from (2.21) we obtain

$$\tilde{y} = FWb + n$$  \hspace{1cm} (2.41)

where $\tilde{y} = (F^T)^{-1}y$ and $n$ is a white Gaussian noise vector with autocorrelation matrix $\sigma^2 I$. In this transformation, we have retained the whiteness of the noise, but we have not totally decorrelated the matched filter outputs. This is analogous to applying a whitened matched filter as in ISI equalization. Since $F$ is lower triangular, we have eliminated all of the MAI from the first user's decision statistic. Additionally, we have eliminated all of the MAI from the second user's decision statistic, except interference from user 1. The third user's decision statistic contains correlated energy remaining from users one and two, and so on. If users are ranked according to their signal strengths, we can utilize our most confident estimate first. Additionally, if we then subtract out the estimate of the detected user from the received signal, detection of the second user can be improved. Assuming proper estimation, we can successively subtract users from the received signal until all users are detected. In this way, previous decisions are 'fed back' to assist in the current decision.
The detector for the strongest user is equivalent to the decorrelator. Assuming correct decisions, the weakest user's detector approaches a single user case.

This factorization can be viewed as a combination of a equalization feed-forward filter \(((F^T)^{-1})\) which eliminates multiuser interference due to future symbols and a feedback filter \(((F - \text{diag}(F_0)) \mathbf{W})\) which eliminates multiuser interference due to past symbols (i.e., decisions). This view results in a bit estimate

\[
\hat{b} = \text{sgn} [\text{diag}(F_0) \mathbf{Wb} + \mathbf{n} + \xi]
\]  

(2.42)

where \(\xi\) is the residual interference due to incorrect past decisions, i.e., \(\xi = (F - \text{diag}(F_0)) \mathbf{W}(\mathbf{b} - \hat{\mathbf{b}})\). It can be shown that the feedforward and feedback filters are optimal in the sense of SNR [24].

The decorrelating decision feedback detector was generalized in [137]. We can view the optimal receiver in terms of the whitening matched filter described earlier as the receiver which minimizes the metric

\[
||\hat{\mathbf{y}} - \mathbf{F}\mathbf{Wb}||.
\]  

(2.43)

This is contrasted to the decorrelating decision feedback receiver which seeks \(\hat{\mathbf{b}}_{i,k}\) to minimize

\[
\hat{\mathbf{y}}_{i,k} - \sqrt{2P_k F_{kk}} \hat{b}_{i,k} - \sum_{j=1}^{k-1} \sqrt{2P_j F_{jj}} \hat{b}_{i,j}.
\]  

(2.44)

Thus, while the MLD receiver must feed \(2^{k-1}\) vectors and metrics to user \(k\) and base a decision on all users, the decision feedback receiver requires that one vector of \(k - 1\) symbols be fed back to user \(k\) and that a decision be made based only on current and previous users. The generalized version of the decision feedback receiver feeds \(N_f\) partial symbol vectors and their corresponding metrics back to user \(k\). In the end, the vector with the smallest total metric is chosen as the decoded vector. If \(N_f\) is chosen as \(2^{k-1}\) the detector becomes the MLD receiver. When \(N_f = 1\), the receiver reduces to the decorrelating decision feedback receiver. As an example consider \(N_f = 2\). In this case, the strongest user is decoded first, and then a metric is associated with \(\hat{b}_{i,1} = \{+1, -1\}\). Both decisions and metrics are fed back to user 2. Four metrics associated with \([\hat{b}_{i,1}, \hat{b}_{i,2}] = \{[+1, +1], [+1, -1], [-1, +1], [-1, -1]\}\) are then calculated with the two having the largest metrics being fed to user 3. Again, four metrics are calculated (one using \(\hat{b}_{i,3} = +1\) and one using \(\hat{b}_{i,3} = -1\) with the two largest being passed on to user 4. This procedure continues until all users are detected and a final vector with the largest metric chosen. For the standard decorrelating decision feedback detector, only one bit per user is fed back, and no metrics are necessary.

This generalization allows better performance as \(N_f\) gets large. Unfortunately, the complexity grows significantly with \(N_f\). Thus, a large complexity increase is required for
marginally better performance. It was shown, however, that in the case of overloaded systems ($K > N$), this technique ($N_f = 4$) provided large performance gains when compared to $N_f = 1$.

As with the decorrelator, a drawback of the decorrelating decision feedback detector is the need to calculate the matrix coefficients, perform a matrix inversion, and in this case, perform a Cholesky decomposition prior to inversion. A method to avoid this calculation is to perform adaptive decorrelation [14] as discussed earlier for the decorrelator. To apply this idea to decision feedback, we adjust the detector as follows. The adaptive decorrelating algorithm is run for the received signal. Additionally, estimates of each user's received energy must be obtained. Using the bit estimate of the strongest user, as well as the energy estimate and spreading code, the strongest user is cancelled from the received signal. The resulting signal is then fed to a second adaptive algorithm which adapts to decorrelate the remaining signals. The second strongest user is then estimated and cancelled using the output of the adaptive detector as well as the energy estimate. This process continues until all users are decoded. Thus, the decorrelating decision feedback detector is implemented without the need for matrix calculation or inversion. However, $K$ adaptations are required, and $K - 1$ bit intervals of delay are experienced for each bit decision. Alternatively, the work of [57] discusses efficient methods of computing the Cholesky decomposition.

2.4.4 Non-Linear Receivers: Multistage Receivers

In the previous section, signal enhancement was accomplished through decision feedback. That is, previous bit decisions were used to cancel interference from the desired user's signal. This was accomplished both with and without prior equalization. In this section, instead of utilizing previous decisions to improve the signal quality, we use tentative decisions on each user to accomplish signal enhancement. This receiver structure is termed a multistage receiver because whenever decisions are made, they can be used to either make a final decision on the data or to enhance the signal through cancellation which leads to another stage of detection. As an example consider a conventional receiver. This receiver could be considered a one stage receiver since the initial data decisions are used directly to estimate the data. However, the conventional matched filter outputs could be used (along with knowledge of the spreading codes, timing and phases of each user) to regenerate the interference and cancel it. This cancellation can be done either prior to or after despreading.

This method is similar to the successive cancellation receiver discussed earlier. The differences are that (1) decisions made on the initial matched filter outputs are tentative for all users rather than final and (2) users are estimated in parallel rather than in succession.
The latter difference means that initial decisions on the weaker users do not benefit from the cancellation of the stronger users and consequently will not be very reliable. This drawback can be mitigated in subsequent stages of cancellation. The positive aspect of this type of cancellation is that all users benefit from cancellation.

We follow the development of the decision rule for the multistage receiver given in [124]. Using earlier definitions, we define decision statistics at bit interval \(i\) at stage 0 as

\[
y_i^{(0)} = R(-1)W_{(i-1)}b_{(i-1)} + R(0)W_{(i)}b_{(i)} + R(1)W_{(i+1)}b_{(i+1)} + n_i
\]  

(2.45)

which is equivalent to the matched filter outputs given in (2.7). Now for the multistage approach, the \(i\)th bit of user \(k\) is estimated at stage \(s + 1\) as

\[
\hat{b}_{i,k}^{(s+1)} = \arg \max_{\hat{b}_{i,k} \in \{ +1, -1 \}} \left[ L(b, r(t)) \right]
\]

(2.46)

where the maximization is performed by letting bit \(l\) of user \(j\) (where \(l \neq i\) if \(j = k\)) be equal to the respective estimates from stage \(s\) and

\[
L(b, r(t)) = \sum_i \left( b_i, 2y_i^{(0)} - R(0)b_i - 2R(1)b_{i-1} \right).
\]

(2.47)

If we retain only the terms in the summation affecting \(\hat{b}_{i,k}^{(s+1)}\), we arrive at

\[
\hat{b}_{i,k}^{(s+1)} = \text{sgn} \left[ y_{i,k}^{(0)} - \sum_{j=1}^{k-1} \rho_{j,k}(-1)\hat{b}_{i-1,j}^{(s)} - \sum_{j=k}^{K} \rho_{j,k}(0)\hat{b}_{i-1,j}^{(s)} - \sum_{j=K+1}^{K} \rho_{k,j}(1)\hat{b}_{i+1,j}^{(s)} \right].
\]

(2.48)

More explicitly, the decision statistic for the \((s + 1)\)th stage is formed by subtracting off the estimate of the interference based on the estimates from stage \(s\). Thus, like the optimal receiver, we require knowledge (or an estimate) of each user’s received energy level. Additionally, we require a method of obtaining bit estimates for the first stage. The simplest approach, and the one suggested in [124], is to utilize a conventional receiver for the first stage. Unlike the optimal detector, this approach has a complexity which is linear in the number of users rather than exponential.

Several important observations are made in [124]. First, it was shown that a two-stage receiver achieves a significant fraction of the performance improvement possible for the optimum receiver. Second, it was demonstrated that, in a two user system, performance of the desired user increases with increasing interference power, (i.e., near-far resistaace is
obtained). This is an intuitive result since the higher the interference power, the better we can estimate it and subtract it out. In fact it has been shown that if an interferer's power is sufficiently low, cancellation actually degrades performance [62]. It should be noted that this is not true in general (e.g., \( K > 2 \) and \( s < \infty \))[11]. Finally, it was shown that for high cross-correlations, the two stage receiver showed much less improvement over the conventional receiver. This suggests additional stages may be necessary in some situations. The main drawback of the receiver is the requirement that the energy of each user be estimated along with knowledge of each user's spreading waveform.

The work of [35] shows that a multistage receiver using matched filters at each stage could result in a theoretical maximum capacity per cell of 130% of the time-bandwidth product as the number of stages \( s \to \infty \). In other words, the total number of users supported is \( K = 1.3N \) where \( N \) is the processing gain of the system (although this does not take into consideration the out-of-cell interference which can be significant). While these results were for a synchronous system (which is the more difficult channel for random codes), they were tempered by simulation results which showed lower capacity gains.

A significant design issue for the multistage receiver is the quality of the data estimation in the first stage. If these estimates are poor, cancellation can actually degrade performance. It is well known that in a heavily loaded system, the performance of the conventional matched filter is unreliable. In fact, it is this which motivates the investigation of multiuser reception. Thus, in the multistage approach we may desire a more robust first stage. In [125, 21], it is suggested that the decorrelator discussed previously would be an appropriate choice for the first stage. This would provide reliable estimates in the presence of strong MAI and allow more accurate cancellation. This approach is found to be superior to the conventional first stage (for a two stage approach), particularly in high bandwidth utilization situations (i.e. high cross-correlation) [125]. This is to be expected, since the use of a conventional first stage ignores the MAI for first stage estimates resulting in poor estimation and cancellation. This receiver was modified to use a one-shot decorrelator in [56] to reduce complexity. The performance of a two stage receiver using a one-shot decorrelator in the first stage was shown to be lower bounded by the two-stage detector with a decorrelating first stage. However, the structure of the former is simpler with no memory requirement.

Another variation of the multistage receiver was proposed in [32]. This receiver combines the successive cancellation approach with the multistage approach. The first stage of this receiver is a successive cancellation receiver. Additionally, at subsequent stages the most reliable data estimates available are used. That is, in eliminating the interference from user \( k \) at stage \( s + 1 \), estimates from stage \( s + 1 \) are used for users 1 through \( (k - 1) \) (i.e., for
previously decoded users), while estimates from stage \( s \) are used for users \( (k + 1) \) through \( K \). More explicitly,

\[
\hat{b}_{k,i}^{(s+1)} = \text{sgn} \left[ y_{i,k}^{(0)} - \sum_{j=1}^{k-1} \rho_{k,j}^{(0)} \sqrt{\hat{w}_j} \hat{b}_{j,i}^{(s+1)} - \sum_{j=k+1}^{K} \rho_{k,j}^{(-1)} \sqrt{\hat{w}_j} \hat{b}_{k,j}^{(s+1)} \right. \\
\left. - \sum_{j=k+1}^{K} \rho_{k,j}^{(0)} \sqrt{\hat{w}_j} \hat{b}_{k,i}^{(s)} - \sum_{j=1}^{k-1} \rho_{k,j}^{(1)} \sqrt{\hat{w}_j} \hat{b}_{k,i+1}^{(s)} \right]
\]

(2.49)

where the first two summations represent the past MAI (which is estimated using results of the current stage) and the last two summations represent the future MAI (which is estimated using the previous stage results). In comparing this equation with equation (2.48), we see that the difference lies in the estimation of the users' bits which have been decoded in the current stage. While the approach of (2.48) does not use the updated bit estimates, the detector of (2.49) does.

Simulation results showed that this receiver structure outperformed the multistage receiver using a conventional first stage in a variety of situations. The largest performance advantage was found in near-far situations when there is significant cross-correlation. In this situation errors in the first stage (due to the use of the conventional receiver) are so severe that subsequent stages of cancellation cannot substantially improve performance. However, since successive cancellation performs well in a near-far situation, first stage estimates are much more reliable, allowing subsequent stages to perform significantly better.

An adaptive version of a multistage receiver was proposed in [116]. Here, a two stage receiver was discussed which utilized users' spreading codes but did not assume knowledge of users' signal energies. The scheme subtracts estimates of the transmitted symbol multiplied by an adaptive weight. This adaptive weight is adjusted iteratively until \( E[\hat{y}_{i,k}^2] \approx 0 \), where in a two user synchronous case, \( y_{i,k} = \sqrt{P_i} b_{i,k} + \rho_{k,j} \sqrt{P_j/2} - w_j \hat{b}_{i,j} + n_i \) and \( P_j \) is the \( j \)th user's received power, \( b_{i,j} \) is the \( j \)th user's symbol sent during the \( i \)th received interval, \( \hat{b}_{i,k} \) is the estimate of the \( k \)th interferer's symbol over the \( i \)th interval, \( \rho_{k,j} \) is the correlation between the spreading codes of user \( k \) and user \( j \), and \( w_j \) is the adaptive weight of user \( j \). It can be shown that in this two user example, if the users are of approximately equal strength and \( \rho_{k,j} \) is fairly low, \( w \) will adapt so as to eliminate the interfering signal. However, if \( \rho_{k,j}^2 P_i \gg P_j \), the weight will adapt to eliminate the desired signal. This problem is dealt with by imposing constraints on the weighting coefficients such that \( w_j = \min\{w_1, w_2\} \) or \( \max\{w_1, w_2\} = 0 \). The first constraint will force both \( y_1 \) and \( y_2 \) to eliminate the stronger signal weighted by \( \rho \). This will result in perfect cancellation for the weaker user, while the stronger user will suffer a reduction in wanted signal energy by a factor of \( \rho \) (while retaining the interfering signal). This technique would be successful if one user dominates.
second restriction would mean perfect cancellation for the weaker user, while detection of the second user would be identical to that of the conventional receiver. Performance curves showed that results were dependent on the cross-correlation between users, as well as on their relative energies. As the correlation between users grows, the first constraint results in poorer performance. The performance of the second constraint is highly dependent on relative energies, with better performance resulting from more disparate received energies.

Another adaptive scheme replaces the conventional first stage of the previous receiver with a decorrelating first stage [117]. Using the bit estimates from this first stage and adaptively estimating the powers of each user, MAI can be cancelled in stage 2. This receiver showed a large performance advantage over the single stage decorrelator and a slight advantage over the two stage receiver employing a decorrelating first stage and a second stage which performs cancellation using fixed weights (based on energy estimates). The latter advantage was seen primarily in the presence of weak interference. This is because, in this case, the reliability of the weak user’s estimate is poor, and the adaptive weighting reduces the effect of this user while the fixed scheme does not. By reducing the effect of a less reliable user, the noise added to the decision variable due to incorrect cancellation is mitigated. A thorough investigation of the convergence properties of this detector is given in [148].

The multistage concept was extended to multipath channels in [35, 146, 47]. Specifically, [146] presents a receiver which uses co-channel interference cancellers at each stage which cancel both multiple access interference as well as intersymbol interference due to multipath. One result presented here is that, if the mean square error in the estimate of the user energy is significantly greater than $10^{-3}$, the performance of the receiver degrades rapidly. This requires that the estimates of user power be accurate. The use of Rake receivers has also been suggested in combination with multistage receivers [63, 120].

The parallel cancellation receiver of [124] performs hard bit decisions at each stage, thus requiring separate information about signal energies. If a conventional receiver is used in the first stage, the correlator outputs can be used to estimate the received energies as well as bit estimates. While this introduces some amount of noise into the interference estimates, it alleviates the need for external energy estimates. This technique is suggested in [63, 120, 19]. The performance of such a receiver was analyzed extensively in [62, 61] and extended to fading channels in [9]. Following the notation used earlier for the multistage receiver of [124], we can define the $k$th user’s decision statistic at the $m$th stage of interference cancellation as

$$y_{i,k}^{(s+1)} = y_{i,k}^{(0)} - \sum_{l=k+1}^{K} \rho_{kl}(1)y_{i,l}^{*} - \sum_{l \neq k} \rho_{kl}(0)y_{i,l}^{(s)} - \sum_{l=1}^{k-1} \rho_{kl}(-1)y_{i+1,l}^{(s)}.$$  (2.50)
and \( \hat{b}_{i,k} = \text{sgn}(y_{i,k}) \). The main difference between (2.50) and (2.48) is that the energy estimate is included with the bit estimate in the decision statistic of the previous stage. This realization, while slightly less accurate than that obtained by using hard decisions, obviates the need for energy estimation. Furthermore, analysis of this receiver is possible for arbitrary stages as shown in [62, 61, 9].

In multistage interference cancellation, there are situations where cancellation of specific users does not improve performance, but can actually degrade performance. This is due to the poor quality of the estimate for the particular user. In this situation, the estimate is so poor that attempted cancellation actually adds noise power. This result is quantified in [62], where it is shown that for parallel, multistage interference cancellation, cancellation increases error probability if the following condition is not met:

\[
P_k > \frac{N_0^2}{2T} + \frac{1}{3N} \sum_{j=1 \atop j \neq k}^K P_j
\]  

(2.51)

where \( P_k \) is the received power of the \( k \)th user, \( N \) is the processing gain and \( T_h \) is the bit duration. With this information in hand, we can selectively cancel users according to received signal energy. This is a simple way to solve a major drawback of the parallel multistage approach.

An additional case for interference cancellation in a multistage design is a detection scheme which lies between the hard and soft decision cases. Similar to the use of erasures in coding theory, a decision is not made for some users if the bit decision is unreliable. This scheme is presented in [19]. At each stage a threshold \( \psi \) is set. The output of the correlator \( y_{i,k} \) then is interpreted as

\[
\hat{b}_{i,k} = \begin{cases} 
1 & y_{i,k} > \psi \\
0 & -\psi < y_{i,k} < \psi. \\
-1 & y_{i,k} < -\psi 
\end{cases}
\]  

(2.52)

Setting \( \psi = 0 \) is equivalent to hard decisions. The authors of [19] show that by setting \( \psi \) in the range 0.2 - 0.4, performance gains can be seen over hard decisions. This again shows that, when signal energies are sufficiently low, it does not pay to attempt to cancel them. This technique is similar to selective cancellation as suggested earlier.

It was also suggested earlier that correlator outputs might be used for energy estimates. Another method for estimating the energies of the users for multistage cancellation was proposed and analyzed in [28, 45]. In this scheme the channel gain parameter is estimated using an iterative algorithm called expectation maximization. This estimation showed very
low MSE after approximately 100 iterations. This resulted in excellent BER performance when compared to a receiver which uses simple correlator outputs for energy estimation.

Several researchers considered the addition of coding to multistage reception [110, 109, 27, 115]. As stated previously, the reliability of first stage estimates is of extreme importance. One method of increasing the reliability of an estimate is the use of coding. It is suggested in [110] that decoding be employed at each stage of a multistage receiver in order to improve estimation at each stage. Numerical results showed that decoding at each stage provided potential gains over a coded system which performed decoding only after all cancellation is complete. This results from the fact that the correction capability of the coding is not as powerful as the cancellation with improved estimates. This reiterates the importance of the estimates used for cancellation.

The discussion of multistage cancellation in this section has focused on coherent reception. One of the challenges of mobile communications is that coherent reception is often difficult due to the rapidity of phase fluctuations. One method of implementing coherent reception and multistage cancellation for the mobile channel is to employ pilot symbols [83]. By periodically inserting known symbols in the data stream, the carrier phase can be estimated and interpolated over the symbols between the pilot symbol occurrences. Alternatively, [8] shows that multistage cancellation can be performed non-coherently using either differential detection or M-ary signaling. While non-coherent cancellation does show significant improvement over conventional non-coherent reception, the advantage is not as large as with coherent reception. This is due to the fact that in the former amplitude estimation includes the effect of amplitude and phase with cancellation occurring in in-phase and quadrature channels independently. In the coherent case the phase information allowed in-phase and quadrature estimates to be combined eliminating the effect of phase on the amplitude estimate. Thus, the latter situation allows more accurate cancellation and larger performance gains.

Most of the discussion in this section considered only in-cell interference. Inter-cell interference was considered in [2, 18]. In particular, [2] showed that if information is available about out-of-cell users, significant improvement can be achieved by cancelling the most powerful of these users. If all users are cancelled, performance can degrade due to the unreliability of a majority of the out-of-cell users. However, by employing some selectivity in the cancellation process, we can show large gains over no cancellation of out-of-cell interference and cancellation of all out-of-cell interference.

Multistage cancellation depends heavily not just on the estimation of data bits, but also on the estimation of the energies, delays, and phases (assuming coherent reception)
of the interferers. Inaccurate estimation of any of these parameters (also called channel mismatch) will severely degrade the cancellation procedure. We have previously discussed the effect of energy estimation. It is found that using correlator outputs to estimate signal energy causes degradation when compared to perfect estimation [7]. Similarly, estimation of phase and timing also result in degradation from the ideal case. It was shown in [10] that imperfect knowledge of the code phase timing for a multistage receiver employing a conventional first stage can be a severe problem for errors greater than approximately $0.2T_c$. It was also shown that, for satisfactory performance, phase errors should be kept below $20^\circ$. The required accuracy of the phase estimate is confirmed in [72]. These are fairly strict requirements when compared to the conventional receiver. Additional work in [37, 36, 8] showed the effect of channel mismatch on the near-far resistance of the two stage receiver employing conventional receivers at each stage. Obviously, if the interference is not perfectly estimated, it cannot be removed entirely, thus causing problems as the interfering energy grows large. The simulation and analysis of [146] showed severe breakdown when the mean square error (MSE) of the estimates of all three channel parameters approached 0.1. This again suggests that very good estimates are required. One simple method of improving the channel gain estimate is to average over several bits to smooth out estimation error [7]. This averaging is, of course, limited by the coherence time of the channel.

In [99, 7], a comparison is made between cancellation performed using parallel cancellation (i.e., a multistage approach using matched filters at each stage) and successive cancellation (i.e., simple decision feedback). In [99], it is shown that the successive interference approach presented earlier outperforms this parallel cancellation scheme when received user powers vary greatly, while the parallel technique is superior when received powers are equal. This is due to the fact that in parallel cancellation if there are users who have extremely small signal powers, the estimates of these users is poor. Thus, subtracting the reconstructed signals of these users may add noise to the system rather than reduce it. As a result, it is shown in [7] that the use of selective cancellation can provide a performance gain for parallel cancellation when received powers vary greatly and thus provide superior performance to the successive scheme. It is also shown in [99, 7] that the successive scheme performs poorly in a perfect power control environment. This is intuitive since the users which are detected and cancelled first receive no benefit from cancellation while their signals are no more reliable than those estimated later.
2.4.5 Other Techniques

There are several other less popular sub-optimal approaches. We will consider a few of those here. In [143] the authors suggest replacing the Viterbi algorithm of the optimal receiver with a sequential decoding approach. Sequential decoding algorithms are sub-optimal approaches to decoding convolutional codes [77]. The sequential decoding algorithm is sub-optimal because it searches for the most likely path based on local metric values rather than evaluating all possible candidates. This algorithm is much less complex, but can perform badly in high noise situations. Consequently, applying sequential decoding to multiuser detection has similar features. While it reduces the complexity significantly when compared to the optimal receiver, it performs poorly (relative to the optimal receiver) in high interference situations.

A second modification of the Viterbi algorithm search used in the optimal receiver is suggested in [142]. In this scheme, the tree search of the Viterbi algorithm is reduced to a breadth-first algorithm which considers only $L$ paths at each stage, rather than all possible paths. Additionally, the tree-search algorithm which determines the transmitted sequence is augmented by a complex amplitude estimator. This estimate is recalculated at each stage but does not update previous metrics based on the new estimate. The algorithm shows near optimum performance with a complexity which is $O(LK^2)$, where $L$ is the number of paths kept at each stage and $K$ is the number of users. Other simplified search algorithms are presented in [27, 145, 140].

An interference cancellation technique proposed in [17, 121, 26, 25] suggests that multistage interference cancellation be performed in the frequency domain. The technique is based on the use of Walsh-Hadamard symbols as proposed in the IS-95 CDMA standard [33]. By accomplishing cancellation in the frequency domain, this technique avoids the regeneration of interference signals. The basic idea is very similar to the successive interference cancellation technique discussed earlier with the cancellation occurring in the Walsh domain rather than in the time domain. Synchronous transmission is assumed as is the use of Walsh-Hadamard codes for spreading. This type of spreading is referred to as 'intelligent spreading', since it is done not just to spread the signal bandwidth, but also to encode the sent data symbol. The $k$ data symbols can be encoded onto a $2^k$ chip length Walsh-Hadamard code word where all $2^k$ chip sets are completely orthogonal to each other.

Estimation of user energies is required, as is user ranking based on received energy. The receiver algorithm proceeds in the following manner. First the received signal is multiplied by the strongest user’s identification code (this code is overlaid on the Walsh-Hadamard,
code but does not contribute to spreading) to remove the effect of that code. A Walsh-Hadamard Transform (WHT) is then performed on this modified signal. The bin values of the transform show the correlation with each of the possible Walsh-Hadamard codes. The largest of these values signifies the code symbol of the desired user. This user can then be removed from the received signal by zeroing out the corresponding bin. By performing an inverse WHT and again multiplying by the strongest user's identification code, the received signal is regenerated minus the strongest user. This can be repeated for each of the users in decreasing order of their received energies.

Other techniques include the use of neural networks for demodulation [1, 91], as well as for channel estimation [44]. The work of [59] investigates the effect of monitoring users at several base stations and employing optimal diversity as well as multiuser reception. Results showed large gains can be achieved for a user which is received with moderate energy \(5dB < SNR < 8dB\) at two base stations. If the received energy at either base station is large, diversity provides no additional benefits, while if the received energy at both base stations is small, the diversity can not improve the already poor performance. Multiuser has also been investigated for use in optical systems [6, 94, 95].

### 2.4.6 Adaptive Antennas

Further studies have shown the benefits of combining adaptive antennas and multiuser receivers [70, 92, 55, 31][43, 54, 59, 71, 85, 93]. Specifically, optimal spatial filtering combined with multiuser reception is considered in [71, 85]. Joint detection (i.e., the decorrelator) is combined with spatial filtering in [92, 54, 93], while the work of [43] combines adaptive antennas with the adaptive decorrelator of [14]. Successive cancellation [31] and multistage reception [70] are also suggested for use in combination with adaptive antennas. These studies show that by separating users in space as well as by codes, even larger capacity gains are possible when compared to multiuser reception alone. Of course, there is a penalty to be paid in complexity. In Chapter 9, the combination of adaptive antennas with multistage reception is addressed in greater depth.

### 2.5 Summary

In this chapter, we have provided a description of the different classes of multiuser receivers proposed for CDMA systems. We have reviewed several designs, particularly those applicable for base station implementation. Some of the issues discussed include detection theory, complexity, performance, and near-far resistance.
It is clear that the multiuser receiver, if made economically feasible, can greatly increase the attractiveness of CDMA for cellular and PCS. In Chapters 3 and 4, we undertake a simulation study to compare the performance and complexity of alternative multiuser receiver implementations. Based on the results of this study, we select parallel multistage cancellation as providing an attractive combination of performance and complexity. In Chapters 5 through 9 we present results relating to the performance of multistage cancellation in practical environments. Then in Chapter 10, we present a DSP implementation of this technique.
Chapter 3

Simulation Procedure

3.1 Introduction

In this chapter we develop the basis for the simulations performed in this work. We present a simple development of the baseband equivalent of a bandpass BPSK spread spectrum system and show how this baseband equivalent is used to create a computer simulation. In Section 3.2 we describe the baseband equivalent of a bandpass BPSK system and show that the performance of the two systems is equal. In Section 3.3 we describe the performance of the sampled baseband equivalent, and Section 3.4 presents a step-by-step procedure for creating a bandpass equivalent simulation for a desired $\frac{E_b}{N_0}$. Section 3.5 discusses the generation of multipath fading channels using two separate methods, while Section 3.6 describes the overall simulation procedure. Section 3.7 presents an introduction to the Multiuser Receiver Testbed used to create the simulation results in this work and confidence intervals are the subject of Section 3.8.

3.2 System Model

Consider a bandpass BPSK spread spectrum system where the transmitted signal $s(t)$ is given as

$$s(t) = \sqrt{2P_{bp}}b(t)a(t)\cos(\omega_c t + \theta)$$  \hspace{1cm} (3.1)

where $P_{bp}$ is the power of the transmitted bandpass signal, $b(t) = \sum_{i=-\infty}^{\infty} b_i p_T(t)$ is the data signal, $b_i$ is a binary random variable representing the transmitted data which takes on $+1$ and $-1$ with equal probability, $p_T(t)$ is the unit pulse defined in equation (2.4), $T$ is the bit duration, $\omega_c$ is the carrier frequency, $\theta$ is a random phase, and $a(t)$ is the spreading waveform defined in (2.5). This model is analytically equivalent to a BPSK system with
the simple addition of a spreading waveform which modulates the BPSK data symbol.

At the receiver, the received signal $r(t)$ is simply the transmitted signal plus a bandpass Gaussian noise process

$$r(t) = s(t) + n(t)$$  \hspace{1cm} (3.2)$$

where $n(t)$ is a Gaussian random process with power spectral density

$$S_n(f) = \begin{cases} \frac{N_o}{2}, & |f - f_c| < \frac{B}{2} \\ 0, & \text{else} \end{cases}$$  \hspace{1cm} (3.3)$$

At the receiver the signal is mixed down to baseband, multiplied by the spreading waveform $a(t)$ to remove its effect, and integrated over the symbol duration to generate the decision statistic $Z^1$. That is,

$$Z = \int_0^T r(t)a(t)\cos(\omega_c t + \theta)dt.$$  \hspace{1cm} (3.4)$$

Substituting (3.2) for $r(t)$ results in

$$Z = \int_0^T \left(\sqrt{2P_{bp}}a^2(t)\cos^2(\omega_c t + \theta) + n(t)a(t)\cos(\omega_c t + \theta)\right)dt$$
$$= \int_0^T \sqrt{2P_{bp}}b(t)\cos(2\omega_c t + 2\theta) + \frac{1}{2}dt + \int_0^T n(t)a(t)\cos(\omega_c t + \theta)dt$$
$$= \sqrt{\frac{P_{bp}}{2}bT} + n$$  \hspace{1cm} (3.5)$$

$$= \sqrt{\frac{P_{bp}}{2}bT} + n$$  \hspace{1cm} (3.6)$$

where $Z$ is the decision statistic, $b$ is the current data bit, and $n = \int_0^T n(t)a(t)\cos(\omega_c t + \theta)dt$.

Now, for a BPSK system the bit-error-rate in an AWGN channel is determined by

$$P_{error} = Q\left(\sqrt{\frac{E[Z]^2}{\text{var}(Z)}}\right).$$  \hspace{1cm} (3.7)$$

From equation (3.6) the expected value of $Z$ is$^2$

$$E[Z] = E\left[\sqrt{\frac{P_{bp}}{2}bT} + n\right]$$
$$= \sqrt{\frac{P_{bp}}{2}bT},$$  \hspace{1cm} (3.8)$$

$^1$Note the use of notation $y$ and $Z$. The first represents the output of a correlator over a single bit interval. The second is the decision statistic. In the case of the conventional receiver, they are the same, but in general they are not.

$^2$Note that throughout this discussion all expectations are implicitly taken with respect to the data symbol $b$. 

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and the variance is

\[ \text{var}[Z] = E \left[ \left( \sqrt{\frac{P_{bp}}{2}} b T + n \right)^2 \right] - \left( \sqrt{\frac{P_{bp}}{2}} b T \right)^2 \]

\[ = E \left[ \frac{P_{bp}}{2} T^2 + 2 \sqrt{\frac{P_{bp}}{2}} b T n + n^2 \right] - \frac{P_{bp}}{2} T^2 \]

\[ = E[n^2] \]

\[ = E \left[ \left( \int_{0}^{T} n(t) a(t) \cos(\omega c t + \theta) dt \right)^2 \right] \]

\[ = E \left[ \int_{0}^{T} \int_{0}^{T} n(t)n(s)a^2(t) \cos(\omega c s + \theta) \cos(\omega c t + \theta) dsdt \right] \]

\[ = \int_{0}^{T} \int_{0}^{T} E[n(t)n(s)] \cos(\omega c s + \theta) \cos(\omega c t + \theta) dsdt \] (3.9)

where we have used the facts \( E[\delta_{k,i}^2] = 1 \) and \( E[n] = 0 \). Now, from the power spectral density given in (3.3), we know that if the system uses a sampling rate which an integer of the bandwidth, noise samples will be uncorrelated, i.e.

\[ E[n(t)n(s)] = \begin{cases} \sigma_n^2 \delta(t-s) & t = s \\ 0 & \text{else} \end{cases} \] (3.10)

where \( \sigma_n^2 \) is the total noise power. Thus,

\[ \text{var}[Z] = \frac{\sigma_n^2 T}{2} \] (3.11)

and

\[ P_{\text{error}} = Q \left( \sqrt{\frac{P_{bp} T}{\sigma_n^2}} \right) \] (3.12)

Now, any bandpass signal can be represented by

\[ x(t) = \text{Re} \left[ \tilde{x}(t)e^{j\omega c t} \right] \] (3.13)

where \( \tilde{x}(t) \) is the baseband equivalent of \( x(t) \). An alternate definition of (3.13) yields

\[ x(t) = \tilde{x}_I(t) \cos(\omega c t) - \tilde{x}_Q(t) \sin(\omega c t) \] (3.14)

where \( \tilde{x}_I(t) \) and \( \tilde{x}_Q(t) \) are the in-phase and quadrature parts of \( \tilde{x}(t) \) respectively. Using this definition we obtain the baseband equivalents of \( s(t) \) and \( n(t) \) as

\[ \tilde{s}(t) = \sqrt{2P_{bp}} b(t) a(t) \cos(\theta) + j\sqrt{2P_{bp}} b(t) a(t) \sin(\theta) \] (3.15)
and
\[ \tilde{n}(t) = n_I(t) + jn_Q(t) \]  
(3.16)

where \( n_I(t) \) and \( n_Q(t) \) are independent Gaussian processes each with power \( \sigma_n \). Using the complex baseband approach the decision statistic is now
\[ Z = \int_0^T \tilde{r}(t) (\cos(\theta) - j\sin(\theta)) \, dt \]  
(3.17)

where \( \tilde{r}(t) = \tilde{s}(t) + \tilde{n}(t) \). Simplifying we arrive at
\[ Z = \sqrt{2P_{bp}bT} + N_i + N_q \]  
(3.18)

where \( N_i = \int_0^T n_I(t)a(t) (\cos(\theta) - j\sin(\theta)) \, dt \) and \( N_q = \int_0^T n_Q(t)a(t) (\sin(\theta) + j\cos(\theta)) \, dt \). Again the performance of this system is determined by equation (3.7) where the mean and variance of \( Z \) are \( E[Z] = \sqrt{2P_{bp}bT} \) and \( \text{var}[Z] = E[N_i^2 + N_q^2] = 2\sigma_n^2T \). This gives a probability of error of
\[ P_{\text{error}} = Q \left( \sqrt{\frac{2P_{bp}T^2}{2T\sigma_n^2}} \right) \]  
\[ = Q \left( \sqrt{\frac{P_{bp}T}{\sigma_n^2}} \right) \]  
(3.19)

which is identical to the performance of the bandpass system as is expected.

### 3.3 Sampled Baseband System

In the previous section we showed that the baseband equivalent system provides identical performance to its bandpass counterpart. In this section we show how the baseband equivalent translates to a sampled waveform used in computer simulation. The sampled received signal can be represented as
\[ r_i = s_i + n_i \]  
\[ = \left( \sqrt{2P_{bp}\lfloor i/NN_s \rfloor a_{i/NN_s}} \cos(\theta) + n_{I_i} \right) \]  
\[ + j \left( \sqrt{2P_{bp}\lfloor i/NN_s \rfloor a_{i/NN_s}} \sin(\theta) + n_{Q_i} \right) \]  
(3.20)

where \( \lfloor \cdot \rfloor \) is the floor function, \( N_s \) is the number of samples per spreading symbol (or 'chip'), \( N \) is the spreading gain (i.e. the number of chips per bit) leading to \( NN_s \) samples per data symbol, \( n_{I_i} \) and \( n_{Q_i} \) are samples\(^3\) of independent Gaussian random processes, each with

---

\(^3\)As stated earlier we assume that the noise processes are sampled at an integer multiple of the noise bandwidth \( B \) resulting in uncorrelated (and independent since Gaussian) samples.
power $\sigma^2_n$, $a_j$ is the $j$th chip of the spreading sequence and $b_j$ is the $j$th received data symbol.

Analogous to the continuous case the decision statistic is a sum over one bit duration of the received samples multiplied by the spreading waveform and the appropriate phase term, i.e.

$$ Z = \sum_{i=0}^{NN_s-1} [\tau_i] a_{i/N_s} (\cos(\theta) - j \sin(\theta)) $$

$$ = \sqrt{2P_{bp}} b \cos^2(\theta) NN_s + \sqrt{2P_{bp}} b \sin^2(\theta) NN_s + \sum_{i=0}^{NN_s-1} n_{I_i} (\cos(\theta) - j \sin(\theta)) $$

$$ + \sum_{i=0}^{NN_s-1} n_{Q_i} (\sin(\theta) + j \cos(\theta)) $$

$$ = \sqrt{2P_{bp}} b NN_s + \sum_{i=0}^{NN_s-1} n_{I_i} (\cos(\theta) - j \sin(\theta)) + \sum_{i=0}^{NN_s-1} n_{Q_i} (\sin(\theta) + j \cos(\theta)) $$

(3.21)

The expected value of $Z$ is by inspection, $\sqrt{2P_{bp}} b NN_s$. The variance is

$$ \text{var}[Z] = E \left[ \left( \sum_{j=1}^{NN_s} n_{I_j} \right)^2 \right] + E \left[ \left( \sum_{j=1}^{NN_s} n_{Q_j} \right)^2 \right] $$

$$ = E \left[ \sum_{j=1}^{NN_s} n_{I_j}^2 \right] + E \left[ \sum_{j=1}^{NN_s} n_{Q_j}^2 \right] $$

$$ = 2NN_s \sigma^2_n $$

(3.22)

$$ = \sigma^2_n $$

(3.23)

where we assume that in-phase and quadrature noise processes are independent and of equal power. Again using equation (3.7) we obtain a measure for the probability of error,

$$ P_e = Q \left( \frac{P_{bp} NN_s}{\sigma^2_n} \right) $$

(3.24)

which is analogous to the continuous time case.

### 3.4 Generating a Desired $\frac{E_k}{N_o}$

Now that we have laid the groundwork for the simulation procedure, we present a step-by-step process for creating a desired $\frac{E_k}{N_o}$ in a simulation. First, the data waveform (in the absence of pulse-shaping) is a square pulse. In a simulation this reduces to creating a vector of $NN_s$ samples ($N_s$ samples per chip and $N$ chips per bit) each with amplitude $A = \sqrt{2P_{bp}}$

where the actual chosen value for $A$ or $P_{bp}$ is arbitrary as we will show. Since the expected value of the decision metric is simply the sum of these samples we achieve an expected value
of \( NN_s A \). Now, as shown previously, the variance of the decision metric is \( 2NN_s \sigma_n^2 \) where \( \sigma_n^2 \) is the power associated with each of the in-phase and quadrature components of the additive noise samples. Now, since we are running a simulation we require a value for \( \sigma_n^2 \) to generate a specific \( E_b/N_o \) (or SNR).

Sampled white Gaussian noise has a PSD defined by

\[
S_N(f) = \begin{cases} 
\frac{N_o}{2} & f \leq \frac{f_s}{2} \\
0 & \text{else} 
\end{cases}
\]  
\[ (3.25) \]

giving a noise power of \( \sigma_n^2 = \frac{N_o}{2} f_s \) where \( f_s \) is the sampling frequency. If the signal is sampled \( NN_s \) times per symbol (or bit), \( f_s = R_b NN_s \) where \( R_b \) is the bit rate. Since the bit rate is meaningless in a Monte Carlo simulation we arbitrarily set \( R_b = 1 \) giving \( f_s = NN_s \) and \( \sigma_n^2 = \frac{N_o NN_s}{2} \). Now, as stated we assume that we have specified \( \frac{E_b}{N_o} \). Thus,

\[
N_o = \frac{E_b}{\frac{E_b}{N_o}} 
\]  
\[ (3.26) \]

where at bandpass \( E_b = \frac{A^2 T}{2} \). Since we arbitrarily chose \( R_b = 1 \), we can simplify equation (3.26) as

\[
N_o = \frac{A^2}{2E_b/N_o}. 
\]  
\[ (3.27) \]

Combining (3.27) with the previous definition for noise power results in

\[
\sigma_n^2 = \frac{A^2 NN_s}{4E_b/N_o}. \]  
\[ (3.28) \]

Thus, given a desired \( E_b/N_o \) and a chosen value for \( A \), equation (3.28) determines the variance of the noise for each in-phase and quadrature received sample which is generated. Substituting this along with \( E[Z] = NN_s A \) into (3.12) provides a quick check on the simulated performance,

\[
P_e = Q \left( \sqrt{\frac{N^2 N_s^2 \frac{A^2}{2E_b/N_o} A^2}{4E_b/N_o}} \right) 
\]
\[
= Q \left( \sqrt{\frac{2E_b}{N_o}} \right) 
\]  
\[ (3.29) \]

which is the desired performance for uncoded BPSK. A somewhat simpler method is to create the decision statistic as

\[
Z = \sum_{j=0}^{NN_s-1} \text{Re}[r_j]a_{i/N_s} \cos(\theta) + \sum_{j=0}^{NN_s-1} \text{Im}[r_j]a_{i/N_s} \sin(\theta) \]  
\[ (3.30) \]
in which case the variance of each noise sample would be

$$\sigma_n^2 = \frac{A^2 NN_s}{2E_b/N_0}. \quad (3.31)$$

Thus, we have determined the amount of noise to add to a complex baseband system in order to generate a desired $E_b/N_0$. For a CDMA system, we must also generate interference. To do this, we simply add additional signal samples each with independent symbols, spreading waveforms, delays and phases, i.e.

$$r_i = \sum_{k=1}^{K} s_{k,i} + n_i \quad (3.32)$$

where

$$s_{k,i} = \left( \sqrt{2P_{e_k}} b_{k,[i/NN_s]} a_{k,[i/NN_s]} (\cos(\theta_k) + j \sin(\theta_k)) \right). \quad (3.33)$$

This obviously changes the performance of our system, but we are still guaranteed that the AWGN level is correct for the desired $E_b/N_0$. In order to generate different $E_b/N_0$ values for each user in the system, we simply create the noise with respect to an arbitrary user and then scale the amplitude of each of the other users to increase or decrease their $E_b/N_0$ as necessary.

### 3.5 Generating Rayleigh Fading Multipath Channels

In order to simulate realistic radio propagation channels, we must include the effects of multipath propagation. To do so we must understand the effect that multipath has on a received signal and how to emulate this at baseband. We provide two methods for doing this. The first is more straightforward but not as accurate, while the second more realistically models the channel but at the cost of higher complexity.

A received signal which is comprised of several time delayed scaled versions of the transmitted signal, but which has no line of sight component, will undergo small scale fading due to the fact that the signals will add together with different phases. It can be shown that the envelope of this fading follows a Rayleigh distribution [51]. Additionally, the rate at which this signal varies (known as the coherence time of the channel) is inversely related to the Doppler spread of the channel. A simple way to simulate this effect is to generate a Rayleigh random variable for each user which represents the magnitude of that user’s received signal over a specified time interval. This time interval (actually a block length) is equal to the coherence time of the channel. By varying this block length, we can simulate different coherence times and consequently different Doppler spreads. The variance of the associated Rayleigh variable is directly related to the energy in the user’s signal.
is, the variance $\sigma_R$ is related to the mean square of the distribution as $\sigma_R^2 = b(1 - \pi/4)$ where $b$ is the mean square or the energy of the desired signal.

Thus, to simulate a flat Rayleigh fading channel, we generate a random variable $r$ for each user which is Rayleigh distributed with variance $\sigma_R^2 = b(1 - \pi/4)$ and apply it to a block length inversely related to the desired Doppler spread. Since we are dealing with a complex baseband system, we may want to directly manipulate the in-phase and quadrature components rather than the magnitude. For a Rayleigh fading channel, the quadrature components vary with a Gaussian distribution which is zero mean and has a standard deviation

$$
\sigma_G = \frac{\sigma_R}{\sqrt{2 - \pi/2}} = \sqrt{b/2}.
$$

(3.34)

Thus, we can generate a Rayleigh fading signal by weighting the in-phase and quadrature channels by independent Gaussian random variables with standard deviations specified by (3.34).

To simulate frequency selective multipath channels, we can apply the above procedure to multiple paths each with independent delays and energies. Summing the paths together provides a simple frequency selective multipath channel. Additionally, the above flat fading model can be convolved with an empirically derived impulse response to generate a desired channel with a particular multipath delay profile.

The above describes a simple and easy method for generating a multipath fading channel. However, we may at times desire a more complex and accurate channel model. One relatively straightforward method is to exploit Gans’ model for the Doppler power spectrum [30]. Explicitly, Gans showed that for an omni-directional antenna with a gain of 1.5, the Doppler spectrum of the received signal can be modeled by

$$
S_{E_r}(f) = \frac{1.5}{\pi f_m \sqrt{1 - \left(\frac{f_c - f}{f_m}\right)}}
$$

(3.35)

where $f_c$ is the center frequency and $f_m$ is the maximum Doppler spread. Using this model for the Doppler spectrum, we can simulate a Rayleigh fading envelope by first generating independent complex Gaussian noise samples and filtering them using a Doppler shaped filter defined by $H(f) = \sqrt{S_{E_r}(f)}$ as shown in Figure 3.1. The filter outputs are then inverse Fast Fourier transformed. After taking the absolute value, each stream is squared and summed to provide the Rayleigh fading envelope. This envelope is then applied to the simulated signal. While this method is computationally more intensive than the previous, it more accurately reflects the channel and allows more precise control of the simulated
Doppler spread. If the exact Doppler spread is not a concern, the previous method may be more attractive.

3.6 Simulation Flow

The previous sections described the procedure for generating the channel model (i.e. AWGN level and Rayleigh envelope) used for the simulations in this work. We now present the flow of the overall simulation procedure. The flow diagram for the simulation procedure is presented in Figure 3.2. The first section of each simulation program is dedicated to defining the overall channel and system parameters, as well as defining the parameter which is to be varied. This variable can be the desired $E_b/N_0$, the number of users, the timing error variance, the interference power level, etc. and will be denoted by $v$.

After initialization, we determine the first value of $v$ and run a specified number of blocks for that value. That is, for each value of $v$ we desire to run a specified number of total bits
Figure 3.2: Flow Diagram for Simulation Procedure
$N_{\text{tot}}$. However, in order to ensure that the performance of a given user is not unduly influenced by parameters such as phase, delay, or spreading code, we will change these parameters for every group of symbols which we call blocks. This is particularly important if random spreading codes are assumed since a poor set of codes can severely degrade performance. Typical block sizes ranged from $N_b = 30$ to $N_b = 100$ bits. As described earlier, $N_b$ can also be chosen to match a desired coherence time if fading channels are being modeled. Thus, for each value of $v$, the phases, delays, spreading codes, and fading level if applicable, must be changed $N_{\text{tot}}/N_b$ times. If the Gans model is used for fading we create the fading envelopes of each user over $N_{\text{tot}}R_b$ seconds, where $R_b$ is the bit rate. These envelopes are created once for each value of $v$ and are then segmented and applied to individual blocks. Once the total number of blocks simulated for the current $v$ reaches $N_{\text{tot}}/N_b$, we calculate the overall probability of error for the current value of $v$ and increment $v$. The probability of error can also be averaged over all users with similar parameters in order to increase the number of observed errors. When $v$ reaches the defined limit, we end our simulation.

3.7 Multiuser Receiver Testbed

The majority of the simulation results$^4$ for multiuser reception presented in this work resulted from a MATLAB [81] Multiuser Receiver Testbed developed for the comparison of the six most common multiuser receivers. The testbed allows simulation in a variety of channel conditions and receiver impairments. Figure 3.3 shows a general diagram of the testbed. The testbed consists of several script files which simulate the performance of either the Multistage Rake, Decorrelator, MMSE, Successive Cancellation, or the Decision Feedback receivers. The conventional receiver is simulated within each program as a basis of comparison. These receivers will be discussed more in a later chapter. The program flow was discussed in the previous section. Each program is capable of varying the number of users, the $E_b/N_0$, the timing error variance as well as other various parameters which are specific to the receiver architecture. Additionally, each file has a secondary program which allows the variation of the power of a single dominant interferer to simulate near/far situations.

The program suite for multistage cancellation also includes individual programs which implement cancellation after despreading ($\text{MSRakeNarrow.m}$), amplitude estimate variation ($\text{MstageAmpEstError.m}$), and selective cancellation ($\text{MstageSelect.m}$).

Each script file program uses several functions from one of the three function libraries.

$^4$The phase and timing error results presented in Chapter 8 were the result of software developed in C.
Figure 3.3: Multiuser Testbed Developed for Simulation of Multiuser Receivers
The channel library contains functions which can implement AWGN, Rayleigh (flat and frequency selective), or Log-normal fading channels. The Receiver Library contains functions which perform basic receiver operations which are common to several receiver structures. The General Library contains general functions used to create data and spreading codes. The following provides a more detailed description of each of the library functions:

\[ \text{ cancelm } \] Cancels estimated signals from a given signal. Function inputs include the signal to be cancelled, amplitude and data estimates, phases (or estimates), delays (or estimates), and spreading codes of each user to be cancelled.

\[ \text{ channel.m } \] Creates the channel parameters including amplitudes, delays, and phases for each user. Inputs include channel type (AWGN, Rayleigh, or Log-Normal), the channel parameters if applicable, and whether the channel is synchronous or asynchronous.

\[ \text{ data.m } \] Creates random data for each user. Inputs include the number of users and the block length.

\[ \text{ PhaseEst.m } \] Estimates the phase of a signal using a short training sequence. Inputs include the received signal, the spreading codes, the delays (or estimates), and the training bits of each user.

\[ \text{ pncode.m } \] Creates the pseudo-noise spreading sequence of each user. Codes are random. Programs can also read in specific codes from a file.

\[ \text{ quantize.m } \] Quantizes an input signal using a mid-rise uniform quantizer. Inputs include the signal to be quantized and the number of bits used.

\[ \text{ rake.m } \] Creates the estimates of the amplitudes and data bits of each user using maximal ratio combining. Inputs include the received signal, the phases (or estimates) and delays (or estimates) of each path and spreading codes of each user.

\[ \text{ rakeIQ.m } \] Same as rake.m except that instead of amplitude estimates, the function returns the correlation of the spreading codes with the received in-phase (\( Z_i \)) and quadrature (\( Z_q \)) signals.

\[ \text{ rankpow.m } \] Sorts the received powers in order to rank them from strongest to weakest. Function returns the phases and delays of each user in the same order as the power ranking.

\[ \text{ regenerate.m } \] Regenerates a signal using estimates of the amplitude, data bit, delay, and phase, along with the knowledge of the spreading code. Inputs include
amplitude and data bit estimates, delays, phases, and the spreading codes.

select.m Performs threshold limiting on the amplitude estimates. The returned amplitude estimates are

\[
\hat{A}_{out} = \begin{cases} 
\hat{A}_{in} & \hat{A}_{in} > \text{Threshold} \\
0 & \hat{A}_{in} \leq \text{Threshold}
\end{cases}
\] (3.36)

Inputs include the amplitude estimates and the threshold.

smith.m Creates a Rayleigh fading envelope using the Smith model. See earlier section for detailed explanation.

spread.m Spreads input data by spreading codes. Inputs include the data and spreading codes of each user.

transmit.m Creates the received signal. Inputs include the delays, phases, spread data vectors, fading envelopes (if applicable) or amplitudes, and noise variance.

The preceding testbed was used to generate the majority of the results in this work and can be easily expanded by later students to include other receiver structures or channel models. The location of the files are described in Appendix G.

### 3.8 Confidence Intervals

Monte Carlo simulation is simply a sequence of Bernoulli trials where we count the number of successes (or errors in our context) and divide by the number of trials [52]. Thus, we estimate the true probability of error of a system \( p \) with the sample mean, i.e.

\[
\hat{p} = \frac{1}{n} \sum_{i=1}^{n} g(\hat{b}_i)
\] (3.37)

where \( n \) is the number of observed symbols and \( g(\hat{b}_i) \) is the error estimator

\[
g(\hat{b}_i) = \begin{cases} 
1 & \hat{b}_i \neq b_i \\
0 & \hat{b}_i = b_i
\end{cases}
\] (3.38)

Thus, assuming that all symbols will occur with equal probability, \( \hat{p} = m/n \) where \( m \) is the number of occurrences or errors. By the law of large numbers as \( n \to \infty \), \( \hat{p} \) converges to \( p \). However, to allow \( n \to \infty \) would require infinite processing time. Since we must run less than infinitely long processes, we would like to know how close our estimation is to the true bit error rate. This can be accomplished exactly by realizing that for a given \( n \), \( m\hat{p} \) is binomially distributed [52]. However, rather than obtaining an exact expression
for the confidence interval, we save computational power by using an approximation. One powerful and simple approximation is to assume that the $\hat{p}$ is Gaussian distributed, which is appropriate for large $n$. Using this approximation one can create a confidence interval of the form [52]

\[
P \left\{ \frac{n}{n + d_α^2} \left[ \hat{p} + \frac{d_α^2}{2n} - d_α \left( \frac{\hat{p}(1 - \hat{p})}{n} + \frac{d_α^2}{2n} \right)^{1/2} \right] \right. \\
\leq p \leq \left. \frac{n}{n + d_α^2} \left[ \hat{p} + \frac{d_α^2}{2n} + d_α \left( \frac{\hat{p}(1 - \hat{p})}{n} + \frac{d_α^2}{2n} \right)^{1/2} \right] \right\} = 1 - α
\]

(3.39)

where $d_α$ is chosen such that

\[
\frac{1}{\sqrt{2π}} \int_{-d_α}^{d_α} e^{-t^2/2} dt = 1 - α.
\]

(3.40)

Using this approximation it can be shown that a 95% confidence interval is equivalent to $(1.8\hat{p}, 0.55\hat{p})$ when $n = 10/\hat{p}$. Using this as a guide, our simulations required a minimum error count of $n = 10$. If we apply this to a typical simulation example, we can observe the maximum confidence interval for our simulations. In Figure 3.4, we plot the simulated BER for a multistage receiver along with the 90%, 95%, and 99% confidence intervals resulting from 10 observed errors. While this approximation may suffice as a maximum allowable confidence interval, in general we would like to narrow the interval considerably. Additionally, if only 10 errors were generated for a BER of $10^{-1}$ the Gaussian assumption may no longer hold. Thus, we typically run simulations such that approximately 10-50 errors would occur for the lowest BER desired. Since the same number of bits were run for all $E_b/N_0$ points in a given set, this resulted in order of magnitude increases in the number of observed errors at high ($10^{-1}$) BER’s. As an example we reproduce the same BER curve in Figure 3.5, but this time with the confidence intervals resulting from the actual number of errors observed. As can be seen the typical confidence intervals were significantly tighter than the maximum intervals. As noted, at higher BER’s the bounds are much tighter due to the larger number of observed errors.

3.9 Conclusion

In this chapter we have summarized the simulation procedure of a spread spectrum system which was used in this work. In subsequent chapters, we will apply this basic procedure to simulate more complex CDMA systems and receiver structures. Simulation of multiuser receiver structures will require modification of equation (3.4) where instead of simply basing
our decision on the correlator output $Z$, we will perform processing using each of the correlator outputs of each user to create the final decision metric.
Figure 3.4: Maximum Allowable Confidence Intervals for 90%, 95%, and 99% Using a Typical BER Simulation (Simulation shown: Multistage Cancellation in AWGN for processing gain = 31 and 20 users)
Figure 3.5: Typical Confidence Intervals for 90%, 95%, and 99% Using a Typical BER Simulation (Simulation shown: Multistage Cancellation in AWGN for processing gain = 31 and 20 users)
Chapter 4

A Simulation Comparison of Multiuser Receivers for Cellular CDMA

4.1 Introduction

In the previous chapter, we discussed the simulation methodology used throughout this thesis. This chapter presents a simulation comparison of five multiuser receivers and their usefulness for CDMA, particularly at the base station. The five structures of interest are the decorrelator, the MMSE receiver, the multistage parallel interference cancellation receiver, the successive interference cancellation receiver, and the decorrelating decision feedback receiver. We compare the receivers on the basis of performance in AWGN, Rayleigh fading and near-far situations, and complexity.

A brief review of multiuser receivers is presented in section 4.2. Section 4.3 describes the theoretical performance of the five multiuser receiver techniques in AWGN channels and compares the theoretical performance with simulation results. In section 4.4, simulation results are presented for near/far channels, flat Rayleigh fading, frequency selective fading, along with a presentation of the effects of code synchronization errors. Computational complexity of the proposed schemes is discussed in section 4.5. Section 4.6 discusses possible non-coherent reception for the a few of the structures. Conclusions are presented in section 4.7.
4.2 The Multiuser Receiver: A Brief History

One of the first investigations into multiuser reception of CDMA signals was presented in [111]. Here it was shown that optimal performance in synchronous situations requires estimates of all users for the bit period under consideration. It was also shown that optimal detection in the asynchronous case requires knowledge of the entire transmitted sequence for each user. The optimal detector was developed more rigorously in [130], where Verdu presents the optimal solution for the asynchronous case. It is shown that the complexity per binary decision is $O(2^{K-1})$ where $K$ is the number of simultaneous users. For a reasonable number of users, this level of complexity is impractical. However, the gains provided by the optimal receiver were significant enough to stimulate research on sub-optimal solutions. Some of the first sub-optimal solutions were the linear multiuser detectors of [78, 79] as well as those presented in [141]. The decorrelating detector of [78], while sub-optimal with respect to probability of error was still optimal with respect to near-far resistance, a measure developed in [79, 131]. Sub-optimal non-linear detectors using a multistage approach were proposed [69, 124, 125], while structures employing successive cancellation were presented in [135, 98] and similar non-linear receivers employing decision feedback were proposed in [20].

The previous papers commonly considered AWGN channels with coherent demodulation. Subsequent work adapted these ideas to channels which suffered from time-varying multipath fading and non-coherent demodulation. The optimal receiver of [130] was extended to Rayleigh fading channels in [153] as well as to asynchronous and synchronous Rician channels in [127, 128]. The decorrelator was also extended to non-coherent modulation, Rayleigh fading [152, 153], and Rician fading [127, 128]. Successive cancellation and multistage cancellation were also extended to fading channels in [98, 9]. A more thorough surveys can be found in [22].

4.3 Performance in AWGN Channels

In this section we derive the decision statistic (or metric) for each of the five receiver structures under consideration. Unlike Chapter 2, the decision statistics are presented here in complex baseband notation for ease of simulation. We present expressions for the probability of bit error for each in an AWGN channel with constant (although not necessarily equal) received user energies. These AWGN results are compared with simulation results.
4.3.1 System Model

For the development of the performance of different detectors, we require a common system model. The received baseband signal from user $k$ is defined as a complex baseband binary phase modulated waveform:

$$s_k(t) = \sqrt{P_k}a_k(t)b_k(t)e^{j\theta_k}$$  \hspace{1cm} (4.1)

where $P_k$ is the $k$th user’s received signal power, $a_k(t)$ and $b_k(t)$ are the spreading and data waveforms respectively (we assume rectangular pulses for both), and $\theta_k$ is the received phase of the $k$th user relative to some reference phase. Due to the asynchronous nature of the system uplink, the received signal is

$$r(t) = \sum_k s_k(t - \tau_k) + n(t)$$  \hspace{1cm} (4.2)

where we assume a single noise source from a common front end.

The set of sufficient statistics can be shown to be a set of matched filter outputs $y$ where the filters are matched to each user’s spreading code. In terms of the complex envelope, the vector of sufficient statistics is

$$y = y_I \cos(\Theta) + y_Q \sin(\Theta)$$  \hspace{1cm} (4.3)

where the $i$th matched filter output of the $k$th user is the $(i-1)K + k$th element of the vector $y$, $\Theta$ is a $KN_b \times KN_b$ diagonal matrix where the diagonal elements $\theta_{j,j}$ are the phases of the $i$th bit of the $k$th user and $j = (i-1)K + k$, $K$ is the number of users in the system, $N_b$ is the number of bits in the sequence under consideration and the in-phase and quadrature components are defined as

$$y_{I(i-1)K+k} = \int_{(i-1)T+\tau_k}^{iT+\tau_k} r_I(t)a_k(t - \tau_k)dt$$  \hspace{1cm} (4.4)

and

$$y_{Q(i-1)K+k} = \int_{(i-1)T+\tau_k}^{iT+\tau_k} r_Q(t)a_k(t - \tau_k)dt$$  \hspace{1cm} (4.5)

where $r_I(t) = Re[r(t)]$, $r_Q(t) = Im[r(t)]$ and $\tau_k$ is the relative delay of the $k$th user.

In matrix form we represent the set of matched filter outputs as

$$y_I = \mathcal{W}\cos(\Theta)b + n_I$$  \hspace{1cm} (4.6)

and

$$y_Q = \mathcal{W}\sin(\Theta)b + n_Q$$  \hspace{1cm} (4.7)
where $\mathcal{R}$ is a $KN_b \times KN_b$ matrix defined in 2.10 as

$$
\mathcal{R} = \begin{pmatrix}
R(0) & R(1) & 0 & \cdots & 0 \\
R(-1) & R(0) & R(1) & & \\
0 & R(-1) & R(0) & \ddots & 0 \\
& \vdots & \ddots & \ddots & R(1) \\
0 & \cdots & 0 & R(-1) & R(0)
\end{pmatrix}
$$

(4.8)

the $(k,l)$th element of the $K \times K$ matrix $\mathbf{R}(i)$ is defined by

$$
\rho_{k,l}(i) = \int_{-\infty}^{\infty} a_k(t - \tau_k) a_l(t + iT - \tau_l) dt,
$$

(4.9)

$\mathcal{W}$ is a $KN_b \times KN_b$ diagonal matrix of user received energies defined similar to $\Theta$, $\mathbf{b}$ is a $KN_b$ length vector with the $j = (i-1)K + k$th element equal to the $i$th data symbol of the $k$th user, and $\mathbf{n}_I$ and $\mathbf{n}_Q$ are vectors of colored noise samples at the matched filter outputs. If the users are numbered such that $\tau_1 < \tau_2 < \ldots < \tau_K$, then $\mathbf{R}(1)$ will be an upper triangular matrix with zeros along the diagonal, $\mathbf{R}(-1) = \mathbf{R}^T(1)$ and $\mathbf{R}(i) = 0 \ \forall |i| > 1$.

### 4.3.2 The Decorrelator

A linear detector is a detector where the decision metric is a linear transformation of the sufficient statistics, i.e.

$$
\hat{\mathbf{b}} = \text{sgn} \left[ T \mathbf{y}_I \cos(\Theta) + T \mathbf{y}_Q \sin(\Theta) \right].
$$

(4.10)

The decorrelator is a linear detector where $T = \mathcal{R}^{-1}$. As the name implies, the decorrelator removes the correlation between the elements of $\mathbf{y}$. More explicitly

$$
\hat{\mathbf{b}} = \text{sgn} \left[ \mathcal{R}^{-1} \mathbf{y}_I \cos(\Theta) + \mathcal{R}^{-1} \mathbf{y}_Q \sin(\Theta) \right] = \text{sgn} \left[ \mathcal{W} \mathbf{b} + \mathcal{R}^{-1} \mathbf{n} \right]
$$

(4.11)

This transformation is derived from the maximization of the likelihood function or equivalently the minimization of $(\mathbf{y} - \mathcal{R}\mathbf{b})^T \mathcal{R}^{-1} (\mathbf{y} - \mathcal{R}\mathbf{b})$ [141]. The probability of symbol error (equivalent to bit error in BPSK) of the $k$th user can be represented as [79]

$$
P_{k,i}(E) = Q \left( \sqrt{ \frac{E[y_{k,i}]}{\text{var}[y_{k,i}]} } \right),
$$

(4.12)

where $\mathbf{y}$ is the decision metric, $E[\mathbf{y}] = \mathcal{W} \mathbf{b}$ and $\text{var}[\mathbf{y}]$ needs to be determined and $Q(\cdot)$ is the standard $Q$-function. We define $\Sigma_y = \text{var}[\mathbf{y}]$ and as given by 2.24

$$
\Sigma_y = \sigma^2 (\mathcal{R}^{-1})^T
$$

(4.13)
where $\sigma^2$ is the power of the AWGN at the receiver. Using this result in equation (4.12) results in

$$P_{k,i}(E) = Q\left(\sqrt{\frac{w_{j,i}}{(R^{-1})_{j,j}N_o}}\right),$$

(4.14)

where $j = (i-1)K+k$, $N_o$ is the one-sided noise power spectral density, and we have assumed all data symbols are transmitted with equal probability. Thus, the performance of the decorrelator is identical to the single user case with the exception of the noise enhancement factor $(R^{-1})_{j,j}$. Since all of the elements of $R$ are less than or equal to one, we find that $(R^{-1})_{j,j} > 1$. Unfortunately, general statistics of $R^{-1}$ are not easily found, thus predicting error probabilities is best done using the actual correlation matrix of a known set of user codes and relative delays. In this work, we obtain an estimate of the performance by calculating the average of the elements along the diagonal of the inverse during the simulation runs. It should be noted that the decorrelator is a generalized version of the zero-forcing equalizer for ISI channels.

### 4.3.3 Minimum Mean Square Error Receiver

A similar receiver structure can be obtained if the transformation is sought which minimizes the mean square error of the bit estimate, $E[(b - \hat{b})^T(b - \hat{b})]$. In this case the linear transformation $T = R^{-1}$ used in (4.11) is replaced by $T = (R + N_o/2I)^{-1}$ where $I$ is a $KN \times KN$ identity matrix. The performance of the MMSE detector approaches the decorrelator as $N_o \to \infty$. As $N_o$ grows large, $T$ approaches an identity matrix scaled by $N_o/2$ and is thus reduced to the conventional receiver. Thus, the MMSE detector seeks to strike a balance between removing the interference and not enhancing the noise. At low $E_b/N_o$ the MMSE receiver should outperform the decorrelator, while the decorrelator should have superior BER performance at high $E_b/N_o$. Due to the residual interference this transformation results in an estimate which is biased, and its performance is dependent on the power levels of the interferers. This receiver is analogous to an equalizer which attempts to balance the removal of ISI and reduction of the noise enhancement.

### 4.3.4 Multistage Parallel Interference Cancellation

Multistage receivers are receivers which have multiple stages of interference estimation and cancellation. Although any receiver can be used at each stage (particularly the first) this approach is most often used in conjunction with parallel interference cancellation. In one version of this approach a conventional receiver is used in the first stage to estimate the channel gain and data symbol. These estimates along with independent delay estimates
are used to remove the interference from each of the desired users’ signals. The gain and data estimates can also be independent estimates derived from an outside source. This cancellation approach was first suggested in [68] and further developed in [69, 124]. In [124] it was suggested that each user’s signal could be iteratively estimated in parallel. The estimates for each user can then be used to reduce the interference of the other users by subtracting the estimate of each interferer from the desired user’s signal. Ideally, this would allow the elimination of all interfering signals from the desired user. However, due to the inaccuracy of the estimates, this interference will be subtracted imperfectly. Thus to overcome this, the entire process can be repeated for several stages. At each stage, better estimates of each user are produced, allowing more effective interference cancellation. In this paper we assume the use of matched filters at each stage for estimation. This allows a single estimate (the matched filter output) to be used for both the data symbol and the channel gain and alleviates the need for any outside estimates. While more robust receivers could be used in the first stage to improve performance, this approach is the most straightforward and allows reasonable complexity.

Mathematically we can represent the decision metric for an S-stage parallel cancellation scheme as

$$\hat{b} = \text{sgn} \left[ y_i^{(S)} \cos(\Theta) + y_Q^{(S)} \sin(\Theta) \right],$$

(4.15)

where

$$y_i^{(S)} = \frac{1}{T} \int_{(i-1)T+\tau_k}^{iT+\tau_k} \hat{r}_i^{(S)}(t) a_k(t - \tau_k) dt,$$

(4.16)

$$y_Q^{(S)} = \frac{1}{T} \int_{(i-1)T+\tau_k}^{iT+\tau_k} \hat{r}_Q^{(S)}(t) a_k(t - \tau_k) dt,$$

(4.17)

and

$$\hat{r}_i^{(S)}(t) = r_i(t) - \sum_{j \neq k} y_j^{(S)} a_j(t - \tau_j) \cos(\theta_j),$$

(4.18)

$$\hat{r}_Q^{(S)}(t) = r_Q(t) - \sum_{j \neq k} y_j^{(S)} a_j(t - \tau_j) \sin(\theta_j),$$

(4.19)

with \( j = (i-1)K + k \) for the \( i \)th bit of the \( k \)th user and \( \hat{r}_i^{(S)}(t) \) is the \( k \)th user’s signal after \( s-1 \) stage of cancellation. This implementation requires the estimation, regeneration, and cancellation of each interferer from each of the desired users. Since we must regenerate each of the wideband signals we refer to this as the wideband implementation. An alternate implementation which we call a narrowband implementation does not require regeneration. Instead, interference is cancelled from the narrowband outputs (the matched filter outputs) by using the estimates of the data symbol and channel gains as well as the known cross-correlations between users (derived using delay estimates). This narrowband implementation may require more memory to store the matrix \( \mathcal{R} \) but requires less computations.
because it avoids regeneration of the wideband signal. The computational trade-offs will be examined more in section 5. The theoretical performance of the two implementations is the same.

The analytical performance of this multistage parallel cancellation approach was derived in [61]. It was shown that in an AWGN channel the bit error rate of the receiver employing the standard Gaussian approximation for MAI at stage \( s \) is:

\[
P_k^s(E) = Q \left( \frac{1}{2E_b/N_0} \left( 1 - \frac{(K-1)^s}{3N} \right) + \frac{1}{(3N)^s} \left( \frac{(K-1)^s - (-1)^s \sum_j P_j}{P_k} + (-1)^s \right) \right)^{-1/2}
\]  

(4.20)

where \( K \) is the number of users and \( N \) is the processing gain.

The development of this equation assumes that \( Z_{k,i}^{(s)} \) is an unbiased estimate of \( \sqrt{P_k b} \) at each stage. Unfortunately, it is found that this is not the case. Rather, \( Z_{k,i}^{(s)} \) is biased particularly after the first stage of cancellation with the bias increasing with system loading. One method of alleviating this problem is to multiply the estimate by a back-off factor with a value in the range \([0, 1]\). This factor significantly reduces the bias at stage 2 and improves performance dramatically in heavily loaded systems. This back-off factor varies with both the stage of cancellation and the system loading. It is found that a factor of 0.5 in the first stage of cancellation is a good trade-off and results in performance improvements of an order of magnitude for loadings above 0.6N [7].

### 4.3.5 Successive Interference Cancellation

While the previous scheme suggests a cancellation of interference done in parallel and in multiple stages, a somewhat simpler approach is to estimate and cancel interference successively using feedback. In this approach users are first ranked according to their received powers, and then estimated and cancelled in order from strongest to weakest. This cancellation approach has two advantage. First, the strongest users cause the most interference. Thus it is most beneficial to eliminate these interferers first. Second, the strongest users provide the most reliable estimates and thus cause the least error in cancellation. The result is that each user is estimated and only cancelled once as opposed to \( s \) times in the parallel cancellation approach. This can provide a savings in computational complexity depending on the implementation. The performance relative to parallel cancellation is dependent on the spread of user powers. That is, for the equal power case the successive cancellation scheme performs significantly worse than the parallel approach. However, as the user powers get more widely distributed, the relative performance of the successive scheme improves.
The decision metric of the \( k \)-th user during the \( i \)-th bit interval is found to be

\[
y_{k,i} = y_{I_{k,i}} \cos(\theta_{k,i}) + y_{Q_{k,i}} \sin(\theta_{k,i})
\]  \( (4.21) \)

where

\[
y_{I_{k,i}} = \int_{(i-1)T+\tau_k}^{iT+\tau_k} \hat{r}_I^{(k)}(t) a_k(t-\tau_k) dt
\]  \( (4.22) \)

\[
y_{Q_{k,i}} = \int_{(i-1)T+\tau_k}^{iT+\tau_k} \hat{r}_Q^{(k)}(t) a_k(t-\tau_k) dt
\]  \( (4.23) \)

and

\[
\hat{r}_I^{(k)}(t) = r_I(t) - \sum_{i=1}^{N_b} \sum_{j=1}^{k-1} \frac{z_{j,i}}{T} a_j(t-\tau_k) \cos(\theta_j)
\]  \( (4.24) \)

\[
\hat{r}_Q^{(k)}(t) = r_Q(t) - \sum_{i=1}^{N_b} \sum_{j=1}^{k-1} \frac{z_{j,i}}{T} a_j(t-\tau_k) \sin(\theta_j)
\]  \( (4.25) \)

and \( \hat{r}^{(k)}(t) \) is the \( k \)-th user’s received signal after users 0 through \( k-1 \) have been estimated and cancelled. A theoretically equivalent approach is to perform cancellation without wide-band regeneration.

The performance of the successive cancellation detector in an AWGN channel can be predicted using equation (4.12) and \( \mathbb{E}[y_{k,i}] = w_{k,i} b_{k,i} \). The variance of the decision metric can be shown to be [7]:

\[
\text{var}[y_{k,i}] = \left[ \frac{N_o T}{4} + \frac{T^2}{6N} \sum_{j=2}^{K} P_j \right] \left( 1 + \frac{1}{3N} \right)^{k-1} - \frac{T^2}{6N} \sum_{j=2}^{k} \left( 1 + \frac{1}{3N} \right)^{k-1} P_j
\]  \( (4.26) \)

where \( N \) is the processing gain, \( K \) is the number of users and \( N_o \) is the one-sided noise power spectral density. This variance can be used in the Q-function to produce an analytical performance estimate.

### 4.3.6 Decorrelating Decision Feedback Receiver

Decision feedback multiuser detectors are non-linear receivers similar to decision feedback equalizers employed in single user channels with ISI [104]. In decision feedback multiuser detection, users are ranked in order of decreasing received power levels. Previous decisions are then used together with current statistics to estimate the current output bits. Decision feedback can be characterized by two matrix transformations: a whitening feedforward filter that operates on the matched filter outputs, and a feedback filter fed by the vector of previously made bit decisions.
The forward filter is obtained by first factoring the matrix of cross-correlations $\mathcal{R}$ from (8) using a Cholesky decomposition.

$$\mathcal{R} = \mathcal{F}^T \mathcal{F}$$  \hspace{1cm} (4.27)

Where $\mathcal{F}$ is a lower triangular matrix.

The optimal feedforward filter [24] is the noise whitening filter

$$\mathcal{G} = (\mathcal{F}^T)^{-1}$$  \hspace{1cm} (4.28)

which eliminates multiuser interference due to future inputs. When this filter is applied to the decision statistics $\mathbf{y}$ the resulting output vector is given by:

$$\hat{\mathbf{y}}_I = \mathcal{F}\mathcal{W}\cos(\Theta)\mathbf{b} + \mathbf{n}_I$$  \hspace{1cm} (4.29)

and

$$\hat{\mathbf{y}}_Q = \mathcal{F}\mathcal{W}\sin(\Theta)\mathbf{b} + \mathbf{n}_Q$$  \hspace{1cm} (4.30)

Where $\mathbf{n}_I$ and $\mathbf{n}_Q$ are white Gaussian noise vectors with autocorrelation $\mathcal{R}(\mathbf{n}) = \sigma^2 \mathcal{I}$.

The optimal feedback filter

$$\mathbf{B} = (\mathcal{F} - \text{diag}(\mathcal{F}))$$  \hspace{1cm} (4.31)

operates on previously made bit decisions ($\mathbf{B}$ is a lower triangular matrix with zeros along its diagonal). The feedback terms at the output of this filter eliminate all MAI provided that feedback data is correct. After feedback, the input to the decision devices is given by:

$$\mathbf{z} = \mathbf{z}_I \cos(\Theta) + \mathbf{z}_Q \sin(\Theta)$$  \hspace{1cm} (4.32)

where

$$\mathbf{z}_I = \hat{\mathbf{y}}_I - \mathcal{B}\mathcal{W}\cos(\Theta)\mathbf{b}$$  \hspace{1cm} (4.33)

$$\mathbf{z}_Q = \hat{\mathbf{y}}_Q - \mathcal{B}\mathcal{W}\sin(\Theta)\mathbf{b}$$  \hspace{1cm} (4.34)

and $\mathbf{b} = \text{sgn}(\mathbf{z})$.

The probability of error for the decision feedback, assuming that the feedback terms are correct was presented in [20]

$$P_{k,i}(E) = Q \left( \sqrt{\frac{2E_b F_{k,k}^2}{N_0}} \right)$$  \hspace{1cm} (4.35)

It can be shown that $F_{k,k}^2 \geq 1/(R_{k,k}^{-1})$ where equality is obtained for user 1. Thus, for the strongest user, the decorrelator and the decision feedback receivers present the same probability of error. For the other users the performance of the decision feedback strategy
is superior compared to that of the decorrelator provided feedback estimates are correct. For the weakest user, the ideal performance of decision feedback matches the single user bound since $F_{K_{N_b, KN_b}} = 1$.

An issue for this receiver structure is the estimation of the received energies. This estimation error will cause some degradation in the performance of the receiver. For the sake of simplicity we have assumed perfect estimation for this receiver, but in a realistic system estimation would be of concern.

### 4.3.7 Simulation Results

This section presents the results of simulations for AWGN channels. The first set of results are capacity curves for with $E_b/N_o = 8dB$, $N = 31$, and perfect power control. The simulation results are plotted along with the theoretical curves in Figure 4.1. The parallel scheme uses two stages of cancellation ($S = 3$) and a back-off factor of 0.5 in stage 2. The simulation results are shown to give excellent agreement with theoretical performance. We have not plotted the theoretical curves for the MMSE or decorrelating DF receivers to limit the number of plots. The MMSE performance is upper bounded by the decorrelator, while the decorrelating DF receiver will have a theoretical performance between the decorrelator and the single user bound.

For the perfect power control case we find that the decorrelator, MMSE, parallel canceller, and decorrelating DF detectors provide similar performance, although the latter two are slightly better. The successive canceller performs significantly worse than the other three receivers due to the lack of variance in the received powers. In fact the performance is not significantly better than the conventional receiver. The performance versus $E_b/N_o$ is given in Figure 4.2 for $K = 10$, $N = 31$, and perfect power control. Again we find significant improvement for the decorrelator, the parallel canceller, the MMSE and decorrelating DF receivers with each providing gains of over an order of magnitude at $10dB$, while the successive canceller provides a small improvement.

### 4.4 Realistic Channel Impairments

#### 4.4.1 Near-Far Channels

Previously we examined the improvement in capacity and BER performance possible with multiuser structures in perfect power control. However, as mentioned earlier, one of the drawbacks of the conventional receiver is that it is subject to the near/far problem. Thus we wish to examine the performance of each of the receiver structures in situations where
Figure 4.1: Bit Error Rate vs. Number of Users for Perfect Power Control ($E_b/N_0=8$ dB and processing gain = 31 )
Figure 4.2: BER vs. $E_b/N_0$ with Perfect Power Control (10 users and processing gain = 31)
a single interferer dominates the received power. Figure 4.3 presents the performance in the presence of two interferers, one with equal power to the desired user, and one with a power which varies from 10dB below the desired user to 30dB above the desired user. As expected the conventional receiver degrades quickly in the presence of strong interference. The successive canceller which benefits from diverse powers is found to be robust to strong interferers, as is the decorrelator which has a performance which is independent of user energies. The MMSE and decorrelating DF receivers show similar resistance. The parallel canceller is not as robust and shows slow degradation for high interference power. The parallel canceller suffers due to the fact that cancellation of the weak user is inaccurate in the first stage of cancelation due to the dominating interference. This poor cancellation serves to degrade the channel gain estimate of the strong user in the succeeding stage. Consequently, when the strong user is cancelled from the weak user's signal in second stage of cancellation it is done inaccurately causing problems for the weak user. This continues from stage to stage with slight improvement each time. Thus we find that as $s \to \infty$ the parallel scheme approaches the near/far resistance of the decorrelator and successive canceller. It can be shown that due to the channel estimation, the parallel approach is not near-far resistant in general [11]. Additionally, while the successive cancellation scheme is near-far resistant when a single interferer dominates, it is not near-far resistant in the more general case of two or more dominant interferers [11]. This is again due to the estimation required. However, both show significant robustness when compared to the conventional receiver. One way of improving the parallel receiver in such situations is to avoid cancelling the weak user since its channel gain is unreliable. This provides near/far resistance with only two stages of cancellation [7]. While the decision feedback will also require channel estimation (which we have ignored) its performance will not be as dependent on estimation error due to the feed-forward filtering.

### 4.4.2 Performance in Rayleigh Fading

To examine the performance of the receiver structures in a more realistic channel (rather than simple AWGN), simulations were performed for each detector in flat and frequency selective channels. While many researchers are now finding that the assumption of Rayleigh fading is particularly pessimistic for wideband channels, we use a Rayleigh fading model in order to generate results which are more easily compared with other research. Additionally, while we examine the effect of both flat and frequency selective fading, we assume in both

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1 As the bandwidth of a system increases, the number of resolvable multipaths increases. It has been noted that as more paths become resolvable, these paths are no longer Rayleigh distributed [106, 39]. Rather, the Rayleigh fading effect is mitigated, resulting in a reduced signal strength variance.
Figure 4.3: Performance degradation in Near/Far Channels ($E_b/N_0=5$dB for desired user and processing gain = 31)
cases that the coherence time of the channel is much greater than a single chip period (i.e. slow fading).

The performance results for each of the receiver structures in flat Rayleigh fading are presented in Figure 4.4. The channel is assumed flat with a single path experiencing Rayleigh fading with $\sigma = 0.93$ [107] and unit magnitude for the signals. It is assumed that the fading is slow (a coherence time of 50 bit intervals) and that the phase can be tracked with sufficient accuracy. Again, we find significant improvement over the conventional receiver with each of the receivers providing nearly equivalent performance. As in the AWGN case, the performance is extremely close to the single user bound. There does seem to be a slight disadvantage to the cancellation techniques likely due to the energy estimates.

One of the benefits of the use of spread spectrum is that it often has chip rates which allow the resolution of multipath, thus we wish to examine the performance in frequency selective channels. Results for two-ray frequency selective Rayleigh fading are presented in Figure 4.5. The receivers each use Rake receivers to combine the multipath using maximal ratio combining. The channel parameters are taken from measurement data presented in [107] and represent a strong main path and a fairly weak second path, i.e. $\sigma_1 = 0.93$ and $\sigma_2 = 0.28$. Again we assume that the fading is slow (coherence time = 50 bit intervals) and the delay of the second path relative to the first path is uniformly distributed on $[T_b/(NN_s), T_b/2]$ where $T_b$ is the bit duration, $N$ is the spreading gain and $N_s$ is the number of samples per chip. The results in Figure 4.5 show the difference between the decorrelating receivers and the cancellation techniques. Although the additional paths provide diversity, they also present additional MAI which degrades the estimates of the channel gains which affects the cancellation receivers more profoundly. Since the decorrelator does not require energy estimation, it is not affected as much as the two cancellation receiver structures. Additionally, since we have not modeled the estimation required in the decision feedback approach, we see that it too provides excellent performance. All show significant increases over the conventional receiver.

4.4.3 Delay Estimate Errors

We have assumed up to this point that the delays of each path for each user are known exactly. In a realistic system some delay estimation error will be present. Thus we examine the effect of delay estimation errors on the performance of each of the multiuser receivers. The estimation error is assumed to be Gaussian with some standard deviation in chips (or fraction of a chip). Since the number of samples per chip is finite, we can only simulate a finite number of possible delay errors. Thus while we generate a continuous delay estimation
Figure 4.4: BER vs. $E_b/N_o$ for Flat Rayleigh Fading (10 users, processing gain = 31, $\sigma = 0.93$, Coherence Time = 50 bit intervals)
Figure 4.5: BER vs. $E_b/N_o$ for Frequency Selective Rayleigh Fading (10 users, processing gain = 31, $\sigma_1 = 0.93$ and $\sigma_2 = 0.28$, Coherence Time = 50 bit intervals)
error, the actual error is taken as the generated error rounded up to the nearest sample. The effects of delay estimation error on the performance of each of the receivers studied for \( \frac{E_b}{N_0} = 8dB \), \( N = 31 \), \( K = 20 \) in a perfect power control AWGN channel are shown in Figure 4.6. As expected there comes a point where the attempted removal of interference becomes no longer useful and even harmful. We can also see that the performance degrades very rapidly for delay estimation errors. For an error standard deviation of only one-tenth of a chip, performance is degraded by more than an order of magnitude. These results show that timing errors are more critical for multiuser receivers than they are for matched filters since we not only lose correlation energy, but we also perform increasingly inaccurate cancellation. The combination of these two effects makes timing more critical. Thus, timing will be a significant design issue for multiuser implementation.

Figures 4.7 and 4.8 show the effect that timing errors have in flat Rayleigh fading and near-far channels. The degradation in Rayleigh fading channels is analogous to that seen in AWGN channels. Figure 4.8 shows that for even a small amount of timing error none of the receivers can maintain near-far resistance. This shows that for a realistic system loose power control will still be necessary.

4.5 Computational Complexity

The computational complexity of the detection scheme used in a system is extremely important for both implementation and simulation. Receiver structures with high computational complexity require extremely high speed processors for implementation and extremely long run times for simulation. Thus, we would like to examine the computational complexity of each of the detection schemes presented here. In the following examination of computational requirements we will define complexity as the number of real single floating point operations (with no parallelism) required per bit decision and will define this quantity in terms of the number of users \( K \), the frame length \( N_f \), the number of paths tracked (Rake fingers) \( L \), the spreading factor \( N \), the number of samples per chip \( N_s \), and the number of stages for the multistage receiver \( S \).

4.5.1 The Decorrelator

The decorrelator detection can be broken down into four distinct operations: (1) calculating the matched filter outputs; (2) multiplication of the matched filter outputs by \( R^{-1} \); (3) creating the decision metrics; and (4) creating the inverse of the correlation matrix. It can
Figure 4.6: Effect of Timing Errors (Delay Estimate Errors) on System Performance in AWGN Channel with Perfect Power Control ($\frac{E_b}{N_0} = 8 dB$, Processing gain = 31, $K = 20$)
Figure 4.7: Effect of Timing Errors (Delay Estimate Errors) on System Performance in Flat Rayleigh Fading AWGN Channel ($\frac{P_i}{N_o} = 30$dB, Processing Gain = 31, $K = 10$, Coherence Time = 50 bit intervals)
Figure 4.8: Effect of Timing Errors (Delay Estimate Errors) on System Performance in Near/Far Channels ($\frac{E_b}{N_0} = 30$dB, Processing Gain = 31, $K = 10$)
be shown that the total number of flops\(^2\) required per frame is

\[ C_{\text{decor}} = N_f L K (2 N N_s + N_f L K + 5) + 2 L K (K L - 1) N N_s + \frac{2}{3} (N_f L K)^3. \]  

(4.36)

This leads to computational complexity per bit decision of

\[ C_{\text{decor}}(b) = L K (2 N N_s + N_f L K + 5) + \frac{2 L K (K L - 1) N N_s}{N_f} + \frac{2}{3} N_f^2 (L K)^3. \]  

(4.37)

Significant reductions in the computational complexity of the decorrelator can be possible if the matrix inversion need not be done every frame. The rate of update will depend entirely on the rate at which users enter and leave the system, or timing information changes. It is mentioned in [22] that there exists an efficient method of updating the inverse coefficients rather than recalculating the entire matrix inversion each time the user configuration changes. This update requires approximately \(K L N_f\) computations. Using this algorithm provides large complexity reductions. This would provide a computational complexity per bit of

\[ C_{\text{decor}} = L K (2 N N_s + N_f L K + 5) + K L \]  

(4.38)

which is an extreme reduction in the computational complexity. The extent to which this reduction can be made will be directly related to the rate of change of the channels of each user.

### 4.5.2 Multistage Parallel Interference Cancellation

As mentioned previously the parallel cancellation approach can be implemented using either wideband or narrowband signals. While the theoretical performance of the two are identical, the computational and memory requirements of the two are significantly different. For the wideband approach, the spread signals of each user are regenerated and cancelled at each stage, whereas in the narrowband approach, the cancellation is performed using the channel gain and data estimates along with the cross-correlation information. This cancellation does not require regeneration of the wideband signals since cancellation is performed directly on the correlator outputs.

In the case of the wideband implementation, we must generate matched filter outputs for each user at each stage as well as regenerating and cancelling each path of each user in stages 2 through \(S\). In addition, stage \(S\) requires the maximal ratio combining of the individual paths to create the final decision statistic. It can be shown that the required

\(^2\)We define a floating point operation as any multiply or add. More complex operations such as division are considered multiple operations.
number of flops per frame is

$$C_{Parallel,\text{Wideband}} = N_f KL[(6NN_s + 7) - 4NN_s - 1]$$  \hspace{1cm} (4.39)

leading to a complexity per bit decision of

$$C_{Parallel,\text{Wideband}}(b) = KL[(6NN_s + 7) - 4NN_s - 1]$$  \hspace{1cm} (4.40)

While the narrowband approach does not require signal regeneration it does require that the correlation matrix of the users be generated and stored in memory. This leads to a complexity per frame of

$$C_{Parallel,\text{Narrowband}} = KLN_f[(4KL + 5) + 2NN_s - 4LK] + 2KL(KL - 1)NN_s$$  \hspace{1cm} (4.41)

and a complexity per bit decision of

$$C_{Parallel,\text{Narrowband}}(b) = KL[(4KL + 5) + 2NN_s - 4LK] + \frac{2KL(KL - 1)NN_s}{N_f}.$$  \hspace{1cm} (4.42)

### 4.5.3 Successive Interference Cancellation

Similar to the parallel cancellation scheme, the successive cancellation scheme incorporates estimation of the users, regeneration in the wideband case and cancellation. However, in this scheme each user is estimated and cancelled only once. However, users must be ranked each frame in order of received user powers. It can be shown that this leads to a computational complexity per frame in the wideband case of

$$N_fL[K(8NN_s + 12) - 6NN_s - 7] + K(2N_f + \log_2(K) + 1).$$  \hspace{1cm} (4.43)

This is equivalent to a computational complexity per bit decision of

$$L[K(8NN_s + 12) - 6NN_s - 7] + \frac{K}{N_f}(2N_f + \log_2(K) + 1).$$  \hspace{1cm} (4.44)

In the narrowband case we require

$$N_fL \left[ 2NN_sK + 5K + 8 \sum_{k=1}^{K-1} k + (K - 1)(5 + 2NN_s) \right] + 2KLNN_s(KL - 1) + 2KN_f + K + K\log_2(K)$$  \hspace{1cm} (4.45)

floating point operations per frame and

$$L \left[ 2NN_sK + 5K + 8 \sum_{k=1}^{K-1} k + (K - 1)(5 + 2NN_s) \right] + \frac{2KLNN_s(KL - 1) + K + K \log_2(K)}{N_f} + 2K$$  \hspace{1cm} (4.46)

flops per bit decision.
4.5.4 Decorrelating Decision Feedback

Implementation of a decision feedback detector involves estimation of the received energies, sorting of the users, computation of the matched filter bank outputs, generation of the forward filter using a Cholesky decomposition of the matrix $\mathcal{R}$, an inversion of the resulting triangular matrix, forward/feedback filtering, and computation of the decision statistics.

It can be shown that the number of flops required per frame is

$$
C_{DF} = 2N_f L K N_s + \frac{1}{3} (N_f L K)^3 + \frac{9}{2} (N_f L K)^2 - \frac{8}{3} N_f L K + K \log_2 (K) + C_{EngEst}. \quad (4.47)
$$

The complexity due to estimation of the received energies has been included as a separate term since there are different strategies that can be used for this operation. The complexity per bit decision is given by:

$$
C_{DF}(b) = 2L K N_s + \frac{1}{3} N_f^2 (L K)^3 + \frac{9}{2} N_f (L K)^2 - \frac{8}{3} L K + \frac{K \log_2 (K) + C_{EngEst}}{N_f}. \quad (4.48)
$$

We do not compute the computational complexity of the the MMSE because it will be certainly larger than the decorrelator since not only does it require computation of the cross-correlation matrix and a matrix inversion, but it also requires energy estimation.

One additional note is that the parallel cancellation scheme allows much of its computational burden to be split among separate processors while the successive scheme does not. This is particularly true of the regenerative scheme which requires only the actual cancellation to be performed successively. All other computations can be split among $KL$ processors where $K$ is the number of users and $L$ is the number of paths being tracked. The decorrelator and MMSE schemes may also exploit some amount of parallelism.

4.5.5 Results

The preceding equations were used to compare the computational complexity of the different receivers as well as the different possible implementations. Figure 4.9 shows the number of floating point operations required per bit decision for the multistage receiver implemented in both wideband (termed A) and narrowband (termed B), the successive cancellation receiver in both wideband (A) and narrowband (B) as well as the decorrelator. The results are shown for $N = 31$, $N_s = 4$, $L = 2$, $N_f = 100$, and $S = 3$. As can be seen if matrix recalculation and inversion is required (implementation A) the decorrelator is by far the most computationally expensive. However, if only one update is required per frame (implementation B) the decorrelator is the least expensive algorithm. This illustrates the importance of the rate of change of the channel parameters for each user. Additionally, the
narrowband implementations show computational advantages over the wideband approach for the cancellation receivers. The parallel cancellation scheme is more expensive than the successive scheme for both the narrowband and wideband approaches, although the advantage is not nearly as significant for the narrowband approach. The maximum computational requirements for a single processor in the parallel cancellation approach are also shown in Figure 4.9 (curves marked by '+'). It is seen that the maximum complexity required for a single processor is significantly less than the successive scheme which offers no complexity reduction due to parallelism. The computational complexity of the decorrelating decision feedback was not plotted since it will vary with the approach used to create the energy estimates. However, its computational complexity would be similar to the decorrelator for reasonable energy estimation algorithms, such as matched filtering.

4.6 Non-coherent Implementations

The receiver structures to this point have focused on coherent reception. While some researchers are investigating coherent reception using pilot symbols [83], typically the speed of phase fluctuation in the mobile channel discourages coherent reception. Thus we would like to investigate multiuser techniques which do not require a coherent phase reference. The most straightforward extension to non-coherent multiuser reception is to implement the preceding receivers using differential modulation. In this section we investigate the performance of the decorrelator, parallel cancellation, and successive cancellation employing differential detection. Additionally we show that the cancelation approaches can be extended to $M$-ary orthogonal signaling.

4.6.1 Differential Detection

The Decorrelator

Differential decoding requires the modulation symbols to be transmitted such that information is transmitted via changes in the phase of the signal rather than the absolute phase. It follows that the decision statistic of the conventional receiver is

$$Z_{i,k} = y_{i,k}^I * y_{i-1,k}^I + y_{i,k}^Q * y_{i-1,k}^Q.$$  \hspace{1cm} (4.49)

For a differential decorrelator [149] the transform $\mathcal{R}^{-1}$ would first be applied to the in-phase and quadrature arms prior to differential detection. This results in a straightforward extension to the coherent decorrelator. The simulated performance of the differential decorrelator in an AWGN channel with $K = 10$ users, spreading gain $N = 31$, and perfect power control
Figure 4.9: Computational Requirements for Multiuser Receiver Implementations (Processing Gain=31, Samples per Chip = 4, Paths = 2, Frame Size = 100 bits)
Figure 4.10: BER vs. $E_b/N_0$ with Perfect Power Control using Differential Detection (10 users and processing gain = 31)

is presented in Figure 4.10. The coherent conventional receiver is also shown for reference. We can see that there is the expected reduction in performance due to the non-coherent reception, but otherwise the decorrelator shows the types of gains over the conventional non-coherent receiver as is found in the coherent case. Performance in flat Rayleigh fading is shown in Figure 4.11 where $K = 10$, spreading gain $N = 31$, and $\sigma = 0.93$.

**Parallel Cancellation**

For non-coherent parallel cancellation we follow the same approach as coherent cancellation with the exception that cancellation is done in both in-phase and quadrature channels.
Figure 4.11: BER vs. $E_b/N_o$ for Flat Rayleigh Fading (10 users, processing gain = 31, $\sigma = 0.93$, Coherence Time = 50 bit intervals)
separately. The estimate used for cancellation is simply the in-phase or quadrature channels correlated with a synchronous copy of the spreading code. These estimates will contain inherent phase information for the cancellation process. That is the expected value of \( \int r_i(t)a(t-\tau_k)dt = \sqrt{P_k}b \cos(\theta_k) \). A similar result is true for the quadrature channel. Once these estimates of the I&Q interferers are removed, the I&Q channels are again passed through a matched filter for re-estimation. Once cancellation is complete (i.e. all s stages), the resulting in-phase and quadrature estimates are combined according to (4.49) to form the final decision statistic. The performance of the differential version of parallel cancellation is shown in Figure 4.10 for an AWGN channel with \( K = 10 \) users, spreading gain \( N = 31 \), and perfect power control. Performance improvements over the conventional receiver are similar to the coherent case. Results for flat Rayleigh fading are presented in Figure 4.11. Here we see the degradation due to the additional noise involved in the cancellation process. In the coherent case we know the phase explicitly while in the current case we are implicitly cancelling using an estimate of the phase. This additional noise degrades the accuracy of the cancellation. Thus the performance is still as good as the non-coherent decorrelator which has no such degradation.

**Successive Cancellation**

Successive cancellation using differential encoding is similar to the parallel case with the exception that users are cancelled only once and in descending order of received power levels. The performance is for AWGN and flat Rayleigh fading are given in Figures 4.10 and 4.11. The results are consistent with coherent reception.

### 4.6.2 M-ary Orthogonal Signaling

Another method of non-coherent reception which is in use today in M-ary orthogonal signaling [118]. The performance of successive cancellation using this modulation format was discussed in [99]. We present simulations for successive and parallel cancellation. M-ary Walsh codes are used to represent \( \log_2(M) \) symbols to provide a spreading of \( M/\log_2(M) \). Additionally, the signal is transmitted over both in-phase and quadrature channels to provide a means of non-coherent reception. The two channels are spread with separate I&Q short spreading codes before being spread with a user-defined long code. By applying the two spreading codes an additional spreading gain is achieved.

At the receiver, the incoming signal is mixed down with both an in-phase and a quadrature channel without a phase reference and despread with the appropriate long user code as well as the associated I or Q short code. This provides four sets of decision statistics at
the receiver after despreading and taking the Walsh transform [5]:

\[
Z_{I,I}^m = \int r(t) \cos(\omega_c t) W_m(t) a_k(t - \tau_k) a_I(t - \tau_k) dt
\]  
\[
Z_{I,Q}^m = \int r(t) \cos(\omega_c t) W_m(t) a_k(t - \tau_k) a_Q(t - \tau_k) dt
\]  
\[
Z_{Q,I}^m = \int r(t) \sin(\omega_c t) W_m(t) a_k(t - \tau_k) a_I(t - \tau_k) dt
\]  
\[
Z_{Q,Q}^m = \int r(t) \sin(\omega_c t) W_m(t) a_k(t - \tau_k) a_Q(t - \tau_k) dt
\]  

where \( r(t) \) is the received signal, \( a_k(t) \) is the individual user's long spreading code, \( a_I(t) \) and \( a_Q(t) \) are the I&Q short codes and \( W_m(t) \) is the \( m \)th Walsh code. By combining these using \( Z = (Z_{I,I} + Z_{Q,Q})^2 + (Z_{I,Q} - Z_{Q,I})^2 \) we can determine the most likely transmitted Walsh function. Using the estimate of the Walsh function, the known timing information and the four estimates given above, we can estimate the signals in the in-phase and quadrature arms and perform cancellation, i.e.

\[
r_I^j(t) = r_I(t) - \sum_{j \neq k} Z_{I,I}^m W_m(t - \tau_k) a_k(t - \tau_k) a_I(t - \tau_k) - \sum_{j \neq k} Z_{I,Q}^m W_m(t - \tau_k) a_k(t - \tau_k) a_Q(t - \tau_k)
\]

and

\[
r_Q^k(t) = r_Q(t) - \sum_{j \neq k} Z_{Q,I}^m W_m(t - \tau_k) a_k(t - \tau_k) a_I(t - \tau_k) - \sum_{j \neq k} Z_{Q,Q}^m W_m(t - \tau_k) a_k(t - \tau_k) a_Q(t - \tau_k).
\]

Once cancellation has been performed, re-estimation can be performed as described above using the new I&Q signals for each user. The parallel and successive schemes will be similar with the difference being that users will be cancelled once and in order of descending powers in the successive scheme, whereas in the parallel scheme, estimation will occur in parallel and the cancellation process will occur \( S \) times.

The performance of 64-ary signaling in AWGN is shown for 10 users in perfect power control in Figure 4.12 for both successive and parallel cancellation. The long and short codes have a chip rate which is 4 times that of the Walsh chip rate providing a spreading gain of \( \frac{64}{\log_2(64)} = 42.7 \). While the successive cancellation provides some gain, parallel cancellation \((s = 3)\) shows well over a magnitude of improvement for \( \frac{E_b}{N_0} \geq 6 \text{dB} \).
Figure 4.12: BER vs. $E_b/N_0$ Using Multistage Cancellation, Successive Cancellation, or Conventional Reception in AWGN for $M$-ary Orthogonal Signaling (10 users, $M = 64$, Spreading Gain = 42.7)
4.7 Illustrations

In this section we represent the performance results presented earlier in several forms other than bit error rate curves to better demonstrate multiuser detection. We focus on multistage cancellation and divide this section into two parts. The first half of the section demonstrates the improvement in the decision statistic as well as the constellation diagram (assuming coherent reception). These results are simply meant to illustrate the performance improvement from a different direction than simple BER curves and perhaps provide some insight into the performance.

4.7.1 Improvements to the Decision Statistic

We first examine the advantage of interference cancellation by showing the improvement in the decision statistic. Figure 4.13 presents a histogram of the decision statistic with 5 users when using the conventional receiver, two-stage and three-stage cancellation. The large improvement with only two stages is evident. Results for 25 users are presented in Figure 4.14. Again, we can see a significant improvement with two stages of cancellation. One thing to note in this plot is the bias towards zero in the distributions. This bias can result in significant performance degradation as will be discussed in Chapter 5. Methods of mitigating this effect are discussed in Chapter 9.

A second illustration of the improvement possible with multistage detection is the tightening of the constellation plot. That is, we can see that the complex error is reduced with each stage of cancellation. Figure 4.15 shows the variation in the received complex amplitude for a zero phase signal using conventional and multistage receivers. The variation for 25 users is presented in Figure 4.16. The increase in system loading has a large impact on the conventional receiver, whereas the multistage receiver shows significant robustness.

4.7.2 Applications

In this section we show results for common applications such as speech and image transmission. These results put the BER curves displayed previously into perspective and allow us to get a feel for the gains achievable when using multistage cancellation.

Figure 4.17 presents a transmitted voice signal and Figure 4.18(top) shows the output of a heavily loaded system (10 users with a spreading gain of 15) when using conventional reception and 8-bit quantized PCM voice coding. The signal is extremely noisy, but can be improved somewhat by low-pass filtering as shown in Figure 4.18(bottom). PCM is a
Figure 4.13: Histogram of the Decision Statistic for 5 Users with Perfect Power Control ($E_b/N_0 = \infty$)
Figure 4.14: Histogram of the Decision Statistic for 25 Users with Perfect Power Control \((E_b/N_0 = \infty)\)
Figure 4.15: Constellation Plot for 5 Users with Perfect Power Control ($E_b/N_0 = 20$dB)
Figure 4.16: Constellation Plot for 5 Users with Perfect Power Control ($E_b/N_0 = 20$dB)
Figure 4.17: Original Voice Waveform Used for Demonstration
fairly robust scheme, and even in this heavily loaded system the message can be understood, although the signal is far from toll quality. The output of a three stage cancellation receiver with and without low-pass filtering is shown in Figure 4.19. The signal quality is improved tremendously and toll-quality speech is achieved. In this heavily loaded system, the conventional receiver would require significant coding (if the BER is low enough to allow coding gains) while the multistage receiver would require little to no coding. In a system employing more sophisticated speech coding techniques, the errors would be even more costly. It is likely that even with coding the conventional receiver would provide unsatisfactory performance. Thus, the interference cancellation allows a bandwidth savings by significantly reducing the coding requirements.

Unlike speech, data transmission is not as amenable to bit errors. As an illustration consider image transmission using JPEG video compression of the image in Figure 4.20. In a relatively lightly loaded system (5 users with a spreading gain of 15) and $E_b/N_0 = 10\text{dB}$, the conventional receiver fails completely as shown in Figure 4.20(top). Data compression is not as forgiving as voice and thus requires significantly lower bit-error rates. The transmission using a three stage cancellation receiver is shown in Figure 4.20(bottom). The cancellation is able to remove all data errors. Again, this cancellation directly relates to a savings in bandwidth since coding can be reduced or in the case of Automatic Repeat Request (ARQ) systems, we require fewer transmissions leading to a higher throughput. A second example is shown in Figure 4.21.

4.8 Conclusions and Future Research

In this chapter we have provided a comparison of five multiuser receivers proposed for CDMA systems. The issues discussed include detection theory, complexity, capacity, and near-far resistance. Additionally, non-coherent versions of these receivers were also presented.

We have shown that in an AWGN channel with perfect or imperfect power control multiuser receivers provide significant gains over the conventional matched filter. It was found that for perfect power control, the multistage parallel cancellation receiver, decorrelator, MMSE and decorrelating decision feedback receivers outperform the successive cancelation approach. However, as the power control error is increased, the performance of successive cancellation rivals that of the decorrelator, while the parallel cancellation receiver shows a slow degradation. All are significantly more robust to energy disparity than the conventional receiver.

It was also shown that the each of the multiuser receivers provide significant gains over
Figure 4.18: Voice Waveforms for Conventional Receiver in Heavy Loading (Users = 10, Spreading Gain = 15, $E_b/N_o = 10dB$)
Unfiltered Output of Stage 3

Filtered Output of Stage 3

Figure 4.19: Voice Waveforms for a Three Stage Cancellation Receiver in Heavy Loading (Users = 10, Spreading Gain = 15, $E_b/N_o = 10dB$)
Figure 4.20: JPEG Image 'Bird' Received Using Conventional and Three Stage Cancellation Receivers (Users = 5, Spreading Gain = 15, $E_b/N_0 = 10dB$)
Figure 4.21: JPEG Image 'Balloons' Received Using Conventional and Three Stage Cancellation Receivers (Users = 5, Spreading Gain = 15, $E_b/N_o = 10dB$)
the conventional receiver in the presence of flat and frequency selective Rayleigh fading. Due to the degradation in the accuracy of the energy estimates, particularly in frequency selective channels, we found that the two cancellation approaches are not as robust as the other three structures.

In terms of complexity it was shown that the decorrelator (and thus the MMSE and decorrelating DF structures) was extremely sensitive to the amount of update required for the matrix inverse coefficients. If a new matrix inversion must be performed at regular intervals, the complexity becomes a major impediment to implementation. However, if simple coefficient updates will provide sufficient dynamics, the scheme is the least computationally intensive. Between the two cancellation schemes, successive is overall less computationally intensive, but the parallel scheme is more flexible allowing slower processors in parallel to perform the computation. The successive scheme provides no such flexibility. Also, cancellation approaches which avoid regeneration of the wideband signal can provide significant computational savings at the cost of memory storage requirements. The exact amount of savings achieved will depend on the rate of update required for the correlation matrix.

Timing misalignment was shown to be crucial but not fatal to each of the five receivers. The decorrelator, MMSE, and DF receivers appear to be more sensitive to timing misalignment, although none of the structures can withstand timing errors above 0.3Tc where Tc is the chip period. Non-coherent versions of each of the receivers are possible with performance being analogous to the coherent case.

Based on the simulation results, we have determined that parallel multistage cancellation provides a good combination of performance, near-far resistance, robustness to timing errors, and complexity. The remainder of this thesis focuses exclusively on the parallel multistage cancellation technique.
Chapter 5

Analysis of Multistage Interference Cancellation

The previous chapter showed the potential performance benefits that can be achieved by applying multiuser detection to cellular CDMA systems. Additionally, it was shown that the computational complexity of techniques such as the decorrelator are extremely dependent on the channel coherence time. For moderate channel dynamics, these receivers can require an impractical amount of computation. The cancellation detectors, however, have complexities which are relatively independent of the fading rate. The overall computational requirements of the successive cancellation scheme may be slightly lower than the parallel scheme, but will require faster processors due to its lack of parallel operations. Additionally, the successive scheme performs poorly in situations where the received energies of the users are similar. Thus, the remainder of this work focuses on multistage cancellation. In this section we study the analytical performance of multistage interference cancellation much more closely.

The first section of this chapter details a common method used to approximate the performance of CDMA systems, namely the Gaussian Approximation [105]. This approximation has been shown to provide accurate results for systems with large loading and relatively equal powers. However, it has been shown give optimistic results in certain situations, motivating the development of an Improved Gaussian Approximation [75]. We present the performance approximation for multistage cancellation receiver using the Gaussian Approximation in Section 2, as derived in [60] and extend it by applying the Improved Gaussian Approximation in section 3. Section 4 presents numerical examples along with limited simulation results. The analysis assumes that the decision statistic used to estimate the data symbols is unbiased. In section 5 we show that in general this is not true and explain the adjustment required to account for this bias. It is found that in lightly loaded
systems this bias is not significant, but as loading increases, the bias term becomes more significant and must be considered in the analysis.

5.1 The Gaussian Approximation

The Gaussian Approximation (GA) is a method to approximate the bit error rate of a direct sequence CDMA system by modeling the decision statistic used for symbol estimation as a Gaussian random variable. In a conventional system, the decision statistic, denoted by $Z$, is simply the output of a filter matched to the spreading code of the desired user. From equation (2.6) this becomes

$$Z_{k,i}^{(s)} = \int_{iT+\tau_k}^{(i+1)T+\tau_k} r(t) a_k(t - \tau_k) \cos(\omega_c t + \phi_k) dt$$

where $Z_{k,i}^{(s)}$ is the decision statistic of the $k$th user during bit interval $i$ at stage $s$. The decision statistic can then be modeled as the sum three main components:

$$Z_{k,i}^{(1)} = \eta + \sqrt{\frac{P_c}{2}} T b_{k,i} + \sum_{j=1,j\neq k}^{K} I_{j,k}^{(1)}$$

where $\eta$ represents the correlation of the desired user's spreading code with AWGN, the second component is the constant desired term, and the third term represents the correlation of the desired spreading code with each of the interfering signals. The GA says that since the desired term is constant and $\eta$ is Gaussian, if we can model the interference term as a Gaussian random variable, the decision statistic will be Gaussian. If the decision statistic is Gaussian, the performance is evaluated as

$$P_e^{(1)} = Q \left( \sqrt{\frac{E[Z_{k,i}|b_{k,i}]^2}{\text{var}[Z_{k,i}|b_{k,i}]}} \right).$$

The expected value is evaluated as

$$E[Z_{k,i}|b_{k,i}] = \sqrt{\frac{P_c}{2}} T b_{k,i} + E[\eta] + E \left[ \sum_{j=1,j\neq k}^{K} I_{j,k}^{(1)} \right].$$

The second term is evaluated as zero since $\int_{iT+\tau_k}^{(i+1)T+\tau_k} n(t) a_k(t - \tau_k) \cos(\omega_c t + \phi_k) dt = 0$. Additionally, the variance of $\eta$ is

$$\sigma_{\eta}^2 = \frac{N_o T_b}{4}$$

provided that $\omega_c >> 2/T_b$. We now require the statistics of $\sum_{j=1,j\neq k}^{K} I_{j,k}^{(1)}$ in order to complete the analysis. The Central Limit Theorem states that a sum of $n$ independent
random variables \( y = \sum_j x_j \) will have a probability distribution which tends toward a Gaussian distribution as \( n \) grows large if the variance of \( y \) is not dominated by any one term in the summation. That is, for independent \( x_j, \sigma^2_y = \sum_j \sigma^2_{x_j} \). The Central Limit Theorem requires independence between \( x_j \) and \( \sigma^2_{x_j} << \sigma^2_y \) \( \forall j \). Thus, if these conditions hold, we can model \( \sum_{j=1, j\neq k}^{K} I_{j,k}^{(1)} \) as a Gaussian random variable.

It is shown in \([75, 74]\) that we can express \( I_{j,k}^{(1)} \) in a form which allows for a more detailed evaluation. The delays of each interferer relative to the desired user, \( \tau_j \) may be expressed as \( \tau_j = \gamma_j T_c + \Delta_j \), where \( \gamma_j = \lfloor \frac{T_D}{j} \rfloor \) is the integer number of chips of delay, and \( \Delta_j \in (0, T_c) \) is the residual chip delay. Using this notation we write

\[
I_{j,k}^{(1)} = \sqrt{\frac{P_j}{2}} \cos(\phi_j) \left[ \Delta_j \sum_{l=0}^{N-1} \tilde{a}_{j,l-1-\gamma_j} a_{k,l} + (T_c - \Delta_j) \sum_{l=0}^{N-1} \tilde{a}_{j,l-\gamma_j} a_{k,l} \right],
\]

(5.6)

where

\[
\tilde{a}_{j,l} = \begin{cases} b_{j,-1} a_{j,l}, & l < 0 \\ b_{j,0} a_{j,l}, & l \geq 0. \end{cases}
\]

(5.7)

Further manipulation yields

\[
I_{j,k}^{(1)} = T_c \sqrt{\frac{P_j}{2}} \cos(\phi_j) W_{j,k},
\]

(5.8)

where \([74]\)

\[
W_{j,k} = X_j + \left( 1 - \frac{2\Delta_j}{T_c} \right) Y_j + \left( 1 - \frac{\Delta_j}{T_c} \right) U_j + \left( \frac{\Delta_j}{T_c} \right) V_j,
\]

(5.9)

\( X_j \) and \( Y_j \) have binomial probability mass functions which are given by

\[
p_{X_j}(l) = \binom{A}{\frac{l+A}{2}} 2^{-A}, \quad l = -A, -A+2, \ldots, A-2, A,
\]

(5.10)

\[
p_{Y_j}(l) = \binom{B}{\frac{l+B}{2}} 2^{-B}, \quad l = -B, -B+2, \ldots, B-2, B,
\]

(5.11)

and the random variables \( U_j \) and \( V_j \) have the binary probability mass functions

\[
p_{U_j}(l) = \frac{1}{2}, \quad l \in \{\pm 1\}
\]

(5.12)

\[
p_{V_j}(l) = \frac{1}{2}, \quad l \in \{\pm 1\}.
\]

(5.13)

The dependence of \( W_{j,k} \) on \( k \) comes through the terms \( A \) and \( B \). The parameter \( A \) is the number of times consecutive chips are equal in the sequence \( \{a_{k,0}, \ldots, a_{k,N-1}\} \), and the parameter \( B \) is the number of times consecutive chips are unequal in the sequence \( \{a_{k,0}, \ldots, a_{k,N-1}\} \). Since there are exactly \( N \) bits in this sequence, \( A \) and \( B \) are related by
\( A + B = N - 1 \). Assuming that the spreading sequence of user \( k \) is generated randomly, then \( B \) is also a binomial random variable with probability mass function

\[
p_B(j) = \binom{N - 1}{j} 2^{1-N}, \quad j = 0, \ldots, N - 1.
\]

(5.14)

The standard Gaussian Approximation assumes that the variables \( X_j, Y_j, U_j, \) and \( V_j \) are all independent random variables. However, as we can see, \( X_j \) and \( Y_j \) are both dependent on the random variable \( B \). Thus, they are not independent, nor are the individual \( I_{j,k}^{(1)} \) terms in the summation, since \( B \) is a property of user \( k \) and thus the same for all \( I_{j,k}^{(1)} \). The terms are independent conditioned on \( B \), however. This fact will be explored more later.

Continuing with the standard Gaussian Approximation, we can easily see that \( E[W_j] = 0 \) since each term in (5.9) is zero mean, leading to the conclusion that \( E[Z_{k,i}b_{k,i}] = \sqrt{\frac{P_k}{2} T} b_{k,i} \). Due to the assumed independence of \( I_{j,k}^{(1)} \), the variance of the summation can be reduced to the summation of variances. That is

\[
E \left[ \left( \sum_{j=1, j \neq k}^{K} I_{j,k}^{(s)} \right)^2 \right] = E \left[ \sum_{j=1, j \neq k}^{K} (I_{j,k}^{(s)})^2 \right].
\]

(5.15)

Further, assuming the independence of terms in (5.9), we can show that

\[
E[W_j^2] = E\left[ X_j^2 + \left( 1 - \frac{2\Delta_j}{T_c} \right)^2 Y_j^2 + \left( 1 - \frac{\Delta_j}{T_c} \right)^2 U_j^2 + \left( \frac{\Delta_j}{T_c} \right)^2 V_j^2 \right].
\]

(5.16)

Since \( U_j \) and \( V_j \) are binomial random variables, \( E[U_j^2] = E[V_j^2] = 1 \). Additionally, it can be shown that given \( B \), \( E[X_j^2] = N - B - 1 \) and \( E[Y_j^2] = B \). Finally, since \( \Delta_k \) is uniform on \([0, T_c]\), \( E[\Delta_k] = T_c/2 \) and \( E[\Delta_k^2] = T_c^2/3 \). Using these values in (5.16) results in

\[
E[W_j^2|B] = (N - B + 1) + \frac{1}{3} B + \frac{2}{3}.
\]

(5.17)

Strictly, we should average over the distribution of \( B \). However, the standard Gaussian Approximation assumes that \( B \) takes on its expected value, \( E[B] = \frac{N-1}{2} \). Substituting this value into (5.17) and subsequently into (5.8) leads to

\[
E \left[ (I_{j,k}^{(1)})^2 \right] = \frac{T_c^2 P_j}{2} E[\cos^2(\phi_j)] E[W_j^2]
\]

\[= \frac{T_c^2 P_j}{4} \left( 3N - 2((N-1)/2) - 1 \right) \]

\[= \frac{T_c^2 P_j N}{6}. \]

(5.18)

The variance of the interference term is then

\[
E \left[ \left( \sum_{j=1, j \neq k}^{K} I_{j,k}^{(s)} \right)^2 \right] = \frac{T_c^2 N}{6} \sum_{j \neq k} P_j.
\]

(5.19)
giving an overall variance of the decision statistic as

\[
\text{var}[Z_{k,i}|b_{k,i}] = \frac{N_o T_b}{4} + \frac{T_b^2}{6N} \sum_{j \neq k} P_j
\]

(5.20)

where we have used the fact that \( T_b = NT_c \). If this expression is used in (5.3) we arrive at the standard Gaussian Approximation for \( P_e \),

\[
P_e = Q\left(\sqrt{\left(\frac{N_o}{2T_b P_k} + \frac{1}{3N} \sum_{j \neq k} P_j P_k\right)^{-1}}\right).
\]

(5.21)

### 5.2 Analysis Using the Gaussian Approximation

The preceding summarizes the Gaussian Approximation for the conventional receiver. We now wish to analyze multistage interference cancellation as presented in Figure 5.1. Because of complexity of analysis of systems employing multistage interference cancellation, most work has relied heavily on simulation techniques to supplement analysis as explained in [99, 19]. One exception to this is a closed form expression for the probability of bit error after an arbitrary number of stages of interference cancellation which was presented in [60]. This expression is an extension of the Gaussian approximation to Multistage Cancellation and is given by the formula:

\[
P_{b_k}^{(s)} = Q\left\{ \left[ \frac{1}{2} \frac{E_{b_k}}{N_o} \left( 1 - \left( \frac{K-1}{3N} \right)^s \right) \right] + \frac{1}{(3N)^s} \left( (K - 1)^s - (-1)^s \left( \sum_{j=1}^{K} \frac{P_j}{P_k} \right) + (-1)^s \right) \right\}^{-1/2}
\]

(5.22)

where \( K \) is the number of simultaneous users sharing the channel, \( N \) is the number of chips per bit (processing gain), \( P_{b_k}^{(s)} \) is the probability of bit error at stage \( s \) for the \( k \)th user, \( E_{b_k} \) is the energy per bit for the \( k \)th user, \( N_o \) is the one-sided power spectral density of the Gaussian noise, \( S \) the number of stages in the receiver, and \( P_k \) is the power of the \( k \)th user. The key result for this formula is presented in [60], and a complete derivation of this result with several extensions is presented in [62]. A recursive expression based on a simple Gaussian approximation was also presented in [90].
Figure 5.1: Multistage Interference Cancellation
Furthermore, it is shown in [62] that interference cancellation on the kth user with power $P_k$ is only effective if the condition

$$P_k > \frac{N_0}{2T} + \frac{1}{3N} \left( \sum_{j=1, j \neq k}^{K} P_j \right)$$

(5.23)

is satisfied, confirming that interference cancellation will be ineffective on sufficiently weak interfering signals.

Simulations show equation (5.22) to be accurate for a small number stages, for bit error rates above $10^{-4}$ and light loading. However, for low bit error rates, simulations show this result to be optimistic. The assumption that interference terms from different users are independent is only an approximation. As a result, it is well known that the Gaussian approximation becomes inaccurate at low bit error rates when there are a small number of users or when a few interferers dominate due to a near/far problem [75]. Note that after several stages of cancellation, even if interferers initially had equal power, that the power of the uncancelled signals may be unevenly distributed, because for any particular instance, interference cancellation will be more effective for some users than for others. Furthermore, equation (5.22) is applicable only to the case of fixed signal powers $\{P_k\}$. In mobile environments of practical interest, we may want to consider signal powers which are randomly distributed. Thus we would like to include the second order statistics of MAI.

There exist analytical techniques for overcoming some of the inaccuracies of the standard Gaussian approximation. The authors of [75] and [74] suggest a method for numerical integration; however, this technique can become quite computationally intensive. It is shown in [88] that an improved Gaussian approximation can be developed by conditioning the result on the variance of the total multiple access interference. Holtzman [40] showed that a quite accurate result can be obtained using only the mean and variance of the variance of multiple access interference. Liberti [76] has generalized Holtzman's result to the case of unequal and randomly distributed interference powers.

In this work, we extend the analysis of adaptive multistage interference cancellation of [60] by applying the analytic approach of [40] and [76]. This allows us to develop more accurate performance results particularly at low bit error rates and relatively lightly loaded systems, and it also allows us to analyze more general cases of randomly fading signals.

### 5.3 Analysis Using An Improved Gaussian Approximation

We now derive the probability of error for a multistage interference cancellation system using the Improved Gaussian Approximation. At stage $s$ of the receiver, the decision statistic $Z_{k,s}^{(s)}$
can be used to form an unbiased estimate of the product $\sqrt{P_k}b_{k,i}$, where $\tilde{b}_{k,i} = \frac{Z_{k,i}^{(s)}}{|Z_{k,i}^{(s)}|}$ is an estimate of the $k$th user's data bit, and $\hat{P}_k = 2 \left( \frac{Z_{k,i}^{(s)}}{|Z_{k,i}^{(s)}|} \right)^2$ is an estimate of the $k$th user's power. Using these estimates, we form an estimate $\hat{s}_k^{(s)}(t)$ of user $k$'s signal after stage $s$:

$$\hat{s}_k^{(s)}(t) = \frac{2}{T}a_k(t)\cos(\omega_c t + \phi_k) \sum_{i=-\infty}^{\infty} Z_{k,i}^{(s)} p_{T}(t - iT). \quad (5.24)$$

Although we could form the estimates $\tilde{b}_{k,i}$ and $\hat{P}_k$ separately, we are most interested in the product $\sqrt{P_k}b_{k,i}$, so $Z_{k,i}^{(s)}$ appears directly in equation (5.24) and $\hat{s}_k^{(s)}(t)$ is an unbiased estimate of $s_k(t)$. Thus the correlator output can be used to create the combined estimate of the data bit and the signal level. The signal level estimate can be improved if the absolute value of correlator outputs are averaged over several bit intervals reducing the variability of the estimate as suggested in [99, 90]. The amount of averaging which can be performed is highly dependent on the rate of fading experienced by the channel and complicates the analysis. We assume no averaging is performed.

Interference cancellation is performed by regenerating and subtracting the estimated signals of the interfering users from the received signal $r(t)$ to form a new received signal $r_k^{(s)}(t)$ for the $k$th user after stage $s$, given by

$$r_k^{(s)}(t) = r(t) - \sum_{j=1,j\neq k}^{K} \hat{s}_k^{(s)}(t-\tau_k)$$

$$= n(t) + \sqrt{2P_k}b_k(t-\tau_k)a_k(t-\tau_k)\cos(\omega_c t + \phi_k)$$

$$+ \sum_{j=1,j\neq k}^{K} \left[ s_j(t-\tau_j) - \hat{s}_j^{(s)}(t-\tau_j) \right]. \quad (5.25)$$

The decision statistic $Z_{k,i}^{(s+1)}$ for the $i$th bit of the $k$th user at stage $s+1$ (i.e., after $s$ stages of interference cancellation) is formed by correlating $r_k^{(s)}(t)$ with the $k$th user's signature signal:

$$Z_{k,i}^{(s+1)} = \int_{T+\tau_k}^{(T+1)T+\tau_k} r_k^{(s)}(t)a_k(t-\tau_k)\cos(\omega_c t + \phi_k) dt. \quad (5.26)$$

Using this procedure, an arbitrary number of stages of interference cancellation may be performed to form successive estimates of the data transmitted by each user.

The BER is evaluated through examination of the decision statistic given in equation (5.26). By substituting the expression for $r_k^{(s)}(t)$ from equation (5.25) into (5.26), we can write the decision statistic at stage $s+1$ analogous to the (5.4) for the conventional receiver, as the sum of noise, desired signal and multiple access interference components:

$$Z_{k,i}^{(s+1)} = \eta + \sqrt{P_k/T}b_{k,i} + \sum_{j=1,j\neq k}^{K} j_{j,k}^{(s)} \quad (5.27)$$
where \( \eta \) is a Gaussian random variable representing thermal noise with zero mean and variance \( \frac{N_0 T}{4} \), the desired signal component is given by \( \sqrt{\frac{P_k}{2}} T b_{k,i} \), and \( I_{j,k}^{(s)} \) is the residual interference caused by user \( j \) to user \( k \) due to imperfect cancellation at receiver stage \( s \).

First, consider an evaluation of the multiple access interference at stage 1. As shown previously, if the standard Gaussian approximation is used, we model each interference term as an independent Gaussian random variable with mean zero and variance which may be computed to be:

\[
\mathbb{E}_{\phi_j, \tau_j, \{b_{k,i}\}, \{a_{k,i}\}} \left[ \left( I_{j,k}^{(1)} \right)^2 \right] = \frac{P_k T_c^2 N}{6}
\]

(5.28)

where \( \mathbb{E}_{\phi_j, \tau_j, \{b_{k,i}\}, \{a_{k,i}\}} [\cdot] \) is the expectation taken over \( \phi_j, \tau_j, \{b_{k,i}\}, \{a_{k,i}\} \).

As can be inferred from equation (5.26), a flaw in the standard Gaussian approximation is that each multiple access interference term \( I_{j,k}^{(1)} \) is a function of the \( k \)th user’s spreading signal \( a_k(t) \). As a result, the terms \( I_{j,k}^{(1)} \) are not actually independent, but rather are conditionally independent given \( a_k(t) \) through the term \( B \). It is shown in [88] that, in fact, these terms are conditionally independent given the number of times the spreading sequence of user \( k \) changes signs during the transitions between adjacent chips (\( B \)), the delay of user \( k \), \( \tau_k \), and the relative phase of user \( k \), \( \phi_k \).

If we let the random variable \( \Psi \) be the conditional variance of the total multiple access interference, given by

\[
\Psi = \mathbb{E} \left[ \left( \sum_{j=1, j \neq k}^{K} I_{j,k} \right)^2 \right] \{\phi_j\}, \{\tau_j\}, \{P_j\}, B,
\]

(5.29)

then it is shown in [88] that the BER of user \( k \) at the first stage is given by:

\[
P_{k}^{(1)} = \mathbb{E}_{\Psi} \left[ Q \left( \frac{P_k T_c^2}{2 \left( \Psi + \frac{N_0 T}{4} \right)} \right) \right]
\]

\[
= \int_{0}^{\infty} Q \left( \frac{P_k T_c^2}{2 \left( \psi + \frac{N_0 T}{4} \right)} \right) p_{\Psi}(\psi) d\psi,
\]

(5.30)

where \( p_{\Psi}(\psi) \) is the probability density function of \( \Psi \). In [75, 74], a method is given to find arbitrarily tight upper and lower bounds on the integral in (5.30), but the method of evaluation proposed in [88] is much simpler.

An even simpler approximation is developed in [40], based on the fact that any continuous function \( f(x) \) can be expanded using a Taylor series. Expressed in terms of differences rather than derivatives, this Taylor series expansion takes the form

\[
f(x) = f(\mu) + (x-\mu) \left( \frac{f(\mu + h) - f(\mu - h)}{2h} \right) + \frac{1}{2} (x-\mu)^2 \left( \frac{f(\mu + h) - 2f(\mu) + f(\mu - h)}{h^2} \right) + \ldots
\]

(5.31)
If we let $\mu = \mathbb{E}[x]$ and $\sigma^2 = \mathbb{E}[(x - \mu)^2]$, then
\[
\mathbb{E}[f(x)] \approx f(\mu) + \frac{\sigma^2}{2} \left( \frac{f(\mu + h) - 2f(\mu) + f(\mu - h)}{h^2} \right).
\] (5.32)

Holtzman [40] argues that $h = \sqrt{3}\sigma$ is an appropriate choice, yielding
\[
\mathbb{E}[f(x)] \approx \frac{2}{3} f(\mu) + \frac{1}{6} f(\mu + \sqrt{3}\sigma) + \frac{1}{6} f(\mu - \sqrt{3}\sigma).
\] (5.33)

If we now let $\mu_\Psi$ and $\sigma^2_\Psi$ be the mean and variance respectively of the random variable $\Psi$, and use the approximation of (5.33) to evaluate the expectation in (5.30), a new approximation for the probability of error is obtained:
\[
P_{b_k}^{(i)} \approx \frac{2}{3} Q\left( \frac{P_k T^2}{2 (\mu_\Psi + \frac{N_c T^2}{4})} \right) + \frac{1}{6} Q\left( \frac{P_k T^2}{2 (\mu_\Psi + \sqrt{3}\sigma_\Psi + \frac{N_c T^2}{4})} \right)
+ \frac{1}{6} Q\left( \frac{P_k T^2}{2 (\mu_\Psi - \sqrt{3}\sigma_\Psi + \frac{N_c T^2}{4})} \right).
\] (5.34)

The parameters $\mu_\Psi$ and $\sigma^2_\Psi$ are evaluated in [40] for the case of fixed equal signal powers. Liberti [76] has generalized this result to the case of randomly distributed signal powers with an arbitrary distribution. Suppose that the received signal power $P_j$ from the interfering user $j$ is randomly distributed with mean $\mu_{P_j}$ and variance $\sigma^2_{P_j}$. Then [76] shows that the mean and variance of $\Psi$ are given by:
\[
\mu_\Psi = \frac{T_c^2 N}{6} \sum_{j=1, j \neq k}^K \mu_{P_j}
\] (5.35)

and
\[
\sigma^2_\Psi = \left( \frac{T_c^4}{4} \right) \left[ \frac{23N^2 + 18N - 18}{360} \sum_{j=1, j \neq k}^K \mu^2_{P_j} + \frac{7N^2 + 2N - 2}{40} \sum_{j=1, j \neq k}^K \sigma^2_{P_j} + \frac{N - 1}{36} \sum_{j=1, j \neq k}^K \sum_{j=1, j \neq k}^K \mu_{P_j} \mu_{P_j} \right].
\] (5.36)

The special case of fixed signal powers may be obtained by substituting $\mu_{P_j} = P_j$ and $\sigma^2_{P_j} = 0$.

Equation (5.34), when combined with equations (5.35) and (5.36) provides a framework for analyzing the performance of a multistage receiver using the improved Gaussian approximation. Equation (5.34) also provides a method to compute the probability of error for a CDMA system with randomly distributed signal powers, provided that the statistics of
those signal power distributions are available. We can view the case of adaptive interference cancellation within this framework. At each stage, the interference cancellation will imperfectly cancel the interfering signals, so that the remaining signals will have randomly distributed amplitudes. By recursively computing the statistics of the signal powers at each stage of interference cancellation, we can accurately compute the performance of the multistage receiver.

Equation (5.27) provides an expression for the decision statistic $Z^{(s)}_{k,i}$ for the $i$th bit of the $k$th user at receiver stage $s$. Using equations (5.25) and (5.26), we can express $I^{(s+1)}_{j,k}$, the multiple access interference caused by user $j$ to user $k$ at stage $s + 1$ in terms of the decision statistic at the previous stage $s$:

$$I^{(s+1)}_{j,k} = \cos(\phi_j) \left[ \left( \frac{P_j}{2} b_{j,i} - \frac{Z_{j,i}^{(s)}}{T} \right) \int_{iT}^{iT+\tau_j} a_j(t - \tau_j) a_k(t) dt + \left( \frac{P_j}{2} b_{j,i} - \frac{Z_{j,i}^{(s)}}{T} \right) \int_{iT+\tau_j}^{(i+1)T} a_j(t - \tau_j) a_k(t) dt \right].$$

(5.37)

The multiple access interference in equation (5.37) is divided into two terms, reflecting the fact that in general two bits from user $j$ will overlap with one bit from user $k$. We wish to manipulate the decision statistic into a form in which equation (5.34) is applicable. We define the variable $\nu^{(s+1)}_{j,i}$ as

$$\nu^{(s+1)}_{j,i} = P_j - \sqrt{8P_j b_{j,i-1}} \frac{Z_{j,i}^{(s)}}{T} + 2 \left( \frac{Z_{j,i}^{(s)}}{T} \right)^2.$$

(5.38)

We may interpret $\nu^{(s+1)}_{j,i}$ as the effective power remaining in the interfering signal from user $j$ after $s$ stages of interference cancellation, and rewrite equation (5.37) as

$$I^{(s+1)}_{j,k} = \cos(\phi_j) \left[ \frac{\nu^{(s+1)}_{j,i-1}}{2} b_{j,i-1} \int_{iT}^{iT+\tau_j} a_j(t - \tau_j) a_k(t) dt + \sqrt{\frac{\nu^{(s+1)}_{j,i}}{2} b_{j,i} \int_{iT+\tau_j}^{(i+1)T} a_j(t - \tau_j) a_k(t) dt} \right].$$

(5.39)

If the signal power statistics for each user are constant throughout the time interval considered by the receiver, then $\nu^{(s+1)}_{j,i-1}$ and $\nu^{(s+1)}_{j,i}$ are identically distributed. We can now rewrite equation (5.39) in the same form as (5.6) so that

$$I^{(s+1)}_{j,k} = \sqrt{\frac{\nu^{(s+1)}_{j,i}}{2}} \cos(\phi_j) \left[ \Delta_j \sum_{l=0}^{N-1} \tilde{a}_{j,l-1-\gamma_l} a_{k,l} + (T_c - \Delta_j) \sum_{l=0}^{N-1} \tilde{a}_{j,l-\gamma_l} a_{k,l} \right].$$

(5.40)
Note that the equation (5.40) is identical to equation (5.6) except that the random variable $\nu_{j,i}^{(s+1)}$ representing the power remaining in user $j$'s signal after $s$ stages of interference cancellation is substituted for the random variable $P_j$ representing the received power from user $j$. With the interference expressed in this form, we can now apply equation (5.34) to evaluate error probability. All that remains is to compute the mean $\mu_{P_j}^{(s+1)}$ and variance $(\sigma_{P_j}^{(s+1)})^2$ of the effective signal power for user $j$ at stage $s + 1$.

We compute the mean $\mu_{P_j}^{(s+1)}$ by taking the expectation of equation (5.38)

$$\mu_{P_j}^{(s+1)} = E[\nu_{j,i}^{(s+1)}] = E \left[ P_j - \sqrt{8 P_j b_{j,i-1}} \frac{Z_{j,i}^{(s)}}{T} + 2 \left( \frac{Z_{j,i}^{(s)}}{T} \right)^2 \right].$$  \hspace{1cm} (5.41)

Substituting the expression for the decision statistic in equation (5.27), we have

$$\mu_{P_j}^{(s+1)} = E \left[ P_j - \frac{\sqrt{8 P_j b_{j,i-1}}}{T} \left( \eta + \sqrt{\frac{P_k}{2T} T b_{k,i} + \sum_{j=1,j \neq k}^K I_{j,k}^{(s)}} \right) + \frac{2}{T^2} \left( \eta + \sqrt{\frac{P_k}{2T} T b_{k,i} + \sum_{j=1,j \neq k}^K I_{j,k}^{(s)}} \right)^2 \right]$$

$$= 2E \left[ \left( \frac{\eta}{T} \right)^2 \right] + \frac{2}{T^2} E \left[ \left( \sum_{j=1,j \neq k}^K I_{j,k}^{(s)} \right)^2 \right].$$  \hspace{1cm} (5.42)

where we have made use of the fact that $E[\eta] = 0$ and $E[I_{j,k}] = 0$. It can be shown that $E \left[ \left( \frac{\eta}{T} \right)^2 \right] = \frac{N}{4T^2}$ and $E \left[ \left( \sum_{j=1,j \neq k}^K I_{j,k}^{(s)} \right)^2 \right] = E \left[ \sum_{j=1,j \neq k}^K \left( I_{j,k}^{(s)} \right)^2 \right]$, where we have made use of the fact that the terms $I_{j,k}^{(s)}$ are uncorrelated (although not independent). We now express the variance of the multiple access interference in terms the mean signal power $\mu_{P_j}^{(s)}$ at the previous stage $s$:

$$\frac{1}{T^2} E \left[ \sum_{j=1,j \neq k}^K \left( I_{j,k}^{(s)} \right)^2 \right] = \frac{1}{T^2} E \left[ \sum_{j=1,j \neq k}^K \left( \cos(\phi_j) \sqrt{\frac{\nu_{j,i}^{(s)}}{2}} \int_{(i+1)T}^{iT} \tilde{a}_j(t - \tau_j) a_k(t) dt \right)^2 \right]$$

$$= \frac{1}{T^2} E \left[ \sum_{j=1,j \neq k}^K T^2 \cos^2(\phi_j) \frac{\nu_{j,i}^{(s)}}{2} W_j^2 \right]$$

$$= \frac{1}{6N} \sum_{j=1,j \neq k}^K \mu_{P_j}^{(s)},$$  \hspace{1cm} (5.43)

where we have made use of the facts that $\frac{N}{2T} = N$, $E[W_j^2] = \frac{2N}{3}$, $E[\cos^2(\phi_j)] = \frac{1}{2}$, and $E[\nu_{j,i}^{(s)}] = \mu_{P_j}^{(s)}$. Combining equations (5.42) and (5.43), we have a recursive formula for the
mean:

\[ \mu_{P_j}^{(s+1)} = \frac{N_o}{2T} + \frac{1}{3N} \sum_{j=1, j \neq k}^{K} \mu_{P_j}^{(s)} \]  \hspace{1cm} (5.44)

It is shown by mathematical induction in Appendix A that the recursion (5.44) leads to the result

\[ \mu_{P_j}^{(s+1)} = \frac{N_o}{2T} \left[ \frac{1}{1 - \frac{K-1}{3N}} \right] + \frac{1}{(3N)^s} \left[ \frac{(K-1)^s - (-1)^s}{K} \left( \sum_{j=1}^{K} \mu_{P_j}^{(1)} \right) + (-1)^s \mu_{P_k}^{(1)} \right] \]  \hspace{1cm} (5.45)

where \( \mu_{P_j}^{(1)} = E[P_j] \). If we assume \( \sigma_{\phi}^2 = 0 \), then equation (5.45) can be used in conjunction with (5.34) to compute a standard Gaussian approximation. This is equivalent to the approach taken in [60].

In order to include the second order effects introduced by the multiple access interference, we must develop an expression for the variance of the remaining interference power after \( s \) stages of interference cancellation for use in equation (5.34). From (5.38) we can find the variance as

\[ (\sigma_{P_k}^{(s+1)})^2 = \frac{1}{E} \left[ (\nu_k^{(s+1)})^2 \right] - (\mu_{P_k}^{(s+1)})^2. \]  \hspace{1cm} (5.46)

In Appendix B we show that this can be solved as a recursive formula for stage \( s + 1 \):

\[ (\sigma_{P_k}^{(s+1)})^2 = \frac{N_o^2}{2T^2} + \frac{4N_o}{T^3} \mu_{\phi(s)} - \frac{4}{T^4} (\mu_{\phi(s)})^2 + \frac{9(4N^2 - 3N)}{40N^4} \sum_{j \neq k} \left( (\sigma_{P_j}^{(s)})^2 + (\mu_{P_j}^{(s)})^2 \right) \]

\[ + \frac{4N^2 - 9N + 13}{12N^4} \sum_{j \neq k, l \neq k, l \neq j} \mu_{P_j}^{(s)} \mu_{P_l}^{(s)} \]  \hspace{1cm} (5.47)

where \( \mu_{\phi(s)} \) is defined in (5.35).

It is interesting to view the asymptotic values of (5.45) and (5.47). First from (5.45), letting \( s \to \infty \) and assuming that \( 3N < K - 1 \):

\[ \mu_{P_j}^{(\infty)} = \frac{N_o}{2T} \left[ \frac{1}{1 - \frac{K-1}{3N}} \right]. \]  \hspace{1cm} (5.48)

This result shows that the multiple access interference cannot be totally eliminated. Additionally, while an expression for \( (\sigma_{P_k}^{(\infty)})^2 \) is not possible without a closed form expression for (5.47), we can see that as \( s \to \infty \) the variance will converge to a small but non-zero value due to the first term of (5.47) as well as its dependence on the mean. The reason for these asymptotic values is the noise term present in (5.45) and (5.47). The thermal noise prevents perfect estimation and thus perfect cancellation is impossible regardless of the number of stages.
5.4 Numerical Results

We can now apply the improved Gaussian approximation to determine the second order effects of multiple access interference on the multistage receiver. Figure 5.2 shows the capacity in terms of the number of simultaneous users for a given bit error rate for the case of perfect power control for the standard Gaussian approximation (GA), the improved Gaussian approximation (IGA) given here, and simulation results. All users are assumed to have a constant power with $\frac{E_b}{N_0} = 17$ dB and processing gain $N = 31$. In this instance the standard Gaussian approximation is optimistic. Additionally, with the exception of the result for the second stage at 30 users, the improved formulation more accurately predicts user performance. The improvement is most apparent at low bit error rates and relatively light loading. The match is less good for heavily loaded systems. At heavy loading we see a dependence on the presence of the desired bit in the cancellation process. That is, as the number of users increase, some bias in the decision statistic is introduced which makes the approximation less valid at stages beyond stage 1.

Figure 5.3 shows the results for $\frac{E_b}{N_0} = 12$ dB and $N = 31$ for both the standard Gaussian approximation and the improved Gaussian approximation. While the difference does not appear as pronounced, the standard approximation is again optimistic. Another important result to be observed from Figure 3 is the cross-over at approximately 90 users. We can see that when a system is sufficiently loaded interference cancellation not only fails to be effective, but actually degrades performance. This can be understood by realizing that when the overall interference level grows sufficiently high, estimates of the interferers' signals will be poor and thus when cancellation is attempted, average interference will actually be increased. This phenomenon was also observed in [90].

While the Improved Gaussian Approximation derived here can provide more accurate results for AWGN channels at low BER's and loadings, the more important use of the approach is the modeling of the second order effects of the MAI. To show the effect that $\sigma_\Psi$ has on the performance of multistage interference cancellation, we first examine the results for perfect power control with $\frac{E_b}{N_0} = 10$ dB, and $N = 63$. ¹ The probability of error is plotted in Figure 5.4 versus the number of users in the system for up to five stages (i.e. four stages of interference cancellation.) Although the shape of the curves in Figure 5.4 appears to be different than that of Figures 5.2 and 5.3, this is due to the portion of the curves pictured. The curves actually have the same fundamental shape, but we picture different

¹It should be noted that we have increased the value of the spreading gain from $N = 31$ to $N = 63$ for this analysis. For the previous analysis a low value for $N$ is desirable for simulation purposes. However, for the current analysis where $\sigma_\Psi > 0$, equation (5.34) is valid only for $\mu_\Psi + \frac{E_b}{4} > \sqrt{3}\sigma_\Psi$ [40]. Increasing $N$ increases the range of $K$ for which this relation holds.
Figure 5.2: Capacity Predicted by Simulation ('o' and '×'), Standard (dotted lines), and Improved Gaussian (solid lines) Approximations for Constant User Powers ($N = 31$, $\frac{E_b}{N_0} = 17$dB)

portions of each. Figure 5.4 shows the lower left half of the curves while Figures 5.2 and 5.3 show the right portion of the curves. If Figure 5.4 were extended to the right we would find that the curves would approach a limiting value similar to Figures 5.2 and 5.3.

Also plotted in Figure 5.4 is the performance for $s \to \infty$ and a single user channel. Several points can be made from this figure. First, it is apparent that multistage interference cancellation can provide a significant capacity increase over the conventional receiver. For a required BER of $10^{-4}$, a conventional receiver ($s = 1$) provides a capacity of approximately 5 users. Three stages of interference cancellation ($s = 4$) increases this capacity 10 times. If a lower BER ($10^{-2}$) is required as is the case in a coded system, three stages of interference cancellation provides a capacity increase of approximately 4 times. Thus, as noted previously, tremendous gains are possible with this receiver structure under ideal conditions (i.e. no multipath, no timing errors, etc.). A second point to be seen from Figure
Figure 5.3: Capacity Predicted by Standard (dotted lines) and Improved (solid lines) Gaussian Approximations for Constant User Powers \((N = 31, \frac{E_k}{N_0} = 12dB)\)

5.4 is the performance of the receiver in the limit as the number of stages of interference cancellation goes to infinity. For a required system BER of \(10^{-4}\), the maximum increase in capacity achievable (i.e. for \(s \to \infty\)) is approximately 11.6 times. Thus approximately 86% of the possible capacity gain is achieved with three stages of interference cancellation \((s = 4)\). Also, the first stage of cancellation \((s = 2)\) achieves the largest increase and the relative increase in capacity is smaller for each additional stage. Since complexity (i.e. cost) and delay are both linearly related to the number of stages, it is not beneficial to increase use more than a few stages, which also agrees with the results of \([90, 62]\).

Figure 5.4 also shows that for \(\sigma_{P_k} = 0, N = 63,\) and \(\frac{E_k}{N_0} = 10dB\) the standard Gaussian approximation provides nearly equivalent results to the IGA. This shows that for high \(N\), moderate \(\frac{E_k}{N_0}\), and \(\sigma_{P_k} = 0\), the standard approximation is sufficient. Finally, the performance of a single user system is plotted in Figure 5.4 for reference. As \(K\) increases for constant \(\frac{E_k}{N_0}\), the gap between the best cancellation system and the single user increases.
Figure 5.4: Standard (dotted line) and Improved Gaussian (solid line) Approximations at Each Stage of Interference Cancellation for Constant Desired User Interferers ($N = 63, \frac{E_k}{N_0} = 10\text{dB}, \sigma_{P_k}^{(1)} = 0$)

Figure 5.5 shows the results for $\frac{E_k}{N_0} = 10\text{dB}, N = 63, \mu_{P_k} = 2\text{dB}, \sigma_{P_k} = 2\text{dB}$ and a constant desired user power. This represents the case of imperfect power control where it has been found that the received power even under power control varies with a log-normal distribution with a standard deviation of 1-2dB [136]. The curves appear very similar to those in Figure 5.4. Again the standard approximation provides sufficient accuracy compared to the IGA. Thus, for $\sigma_{P_k} = 2\text{dB}$ we can effectively ignore the effects of MAI power variance. Figure 5.6 shows the results for $\sigma_{P_k}$ increased to 6dB. This case is typical for a situation where power control is not used, where slow fading within a building or microcell results in a log-normal distribution with larger variance [113, 97]. For this situation we find that the there is significant degradation from the case of perfect power control for the first stage. For stages 2-5 we find less degradation. The standard Gaussian approximation does not show this degradation since it does not take into account $\sigma_{P_k}$. The reason that the
Figure 5.5: Standard (dotted line) and Improved Gaussian (solid line) Approximations at Each Stage of Interference Cancellation for Constant Desired User and Randomly Distributed Interferers ($N = 63$, $\frac{E_k}{N_0} = 10$dB, $\sigma_{P_k}^{(1)} = 2$dB)

degradation is reduced as $s$ increase is that the receiver works to reduce the variation in the MAI power and thus as $s$ increases the effect of $\sigma_{P_k}$ decreases. This will be addressed more later.

From equation (5.34) we see that the mean and variance of $\Psi$ (the MAI power) determine system performance for a constant $\frac{E_k}{N_0}$. Thus we would like to see the effect that the multistage receiver has on these statistics. Figure 5.7 shows the mean of the MAI variance for up to five stages of multiple access interference cancellation for the case where all users have a mean $\frac{E_k}{N_0}=10$ dB, the desired user has constant signal power, the interferers have a signal power standard deviation of 6dB, $N = 63$, and $K = 60$. From equation (5.48) we can determine the asymptotic limit, $\mu_{(\Psi)}^{(\infty)} = 0.0465$. From Figure 5.7 we can see that $\mu_{(\Psi)}^{(s)}$ approaches its asymptotic limit very quickly. It can be shown that that the asymptotic limit of $(\sigma_{(\Psi)}^{(\infty)})^2$ is approximately $1.5^{-4}$ in this case. In Figure 5.8 we find that $(\sigma_{(\Psi)}^{(s)})^2$ also

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approaches its limit quickly. This shows that few stages are required to achieve the desired performance, which agrees with previous analysis. Additionally, this result shows that the interference cannot be totally removed. Thus, the receiver cannot achieve the performance of the single user receiver which would require total interference elimination. This also agrees with previous analytic results.

Another case of interest is the case where the desired user as well as the interferers undergoes log-normal fading. Figure 5.9 presents the case where the desired user has a log-normal distribution with $\mu_{P_k} = 2dB$ and $\sigma_{P_k} = 2dB$. This figure can be interpreted as the degradation in the receiver performance when imperfect power control is used. Comparing Figure 5.9 with Figure 5.5 it can be seen that the effect of fading on signal power is much more significant than the effect of fading on the power levels of the interferers. In this case if a BER of $10^{-2}$ is required for adequate system performance, three stages of interference
Figure 5.7: Mean of MAI Variance (Power) at Each Stage of Interference Cancellation for Constant Desired User and Randomly Distributed Interferers \(N = 63, K = 60, \frac{E_b}{N_0} = 10\text{dB}, \mu_{P_k}^{(1)} = 2\text{dB}, \sigma_{P_k}^{(1)} = 6\text{dB}\)

cancellation provide an eight times increase in capacity. Thus, even in fading environments the multistage cancellation approach can provide significant gains.

Figure 5.10 shows the case where the log-normal fading has \(\mu_{P_k} = 2\text{dB}\) and \(\sigma_{P_k} = 6\text{dB}\). Comparing this figure to Figure 5.9 we see that the higher variance causes a larger degradation as expected. However, the cancellation still provides significant gain over the conventional receiver \(s = 1\) although it is relative to the desired system performance. While this is true of all the figures shown here, it is especially true in this instance due to the small spread of probability of error. Again comparing this figure to Figure 5.6 we see that the variance of the desired user has a significantly larger effect on the system performance than does the variance of the interferers.

Finally, we would like to compare our receiver performance with the optimal receiver. The work of [143] gives results for the optimal receiver when all users have equal and constant
powers, $K = 6$, and $N = 63$. These results are reproduced in Figure 5.11 along with the results for the conventional receiver (stage 1) and a three stage interference cancellation receiver under the same operating conditions. The results show that with three stages of interference cancellation, the multistage receiver has performance very near optimum.

What wasn’t analyzed here is the effect of heavy loading on the analytical assumptions. There is some indication that there are dependencies between interference estimates and the desired user’s data which may cause problems in heavily loaded systems. This dependence was not modeled here and may cause some inaccuracies in heavily loaded systems. This issue is addressed next.
Figure 5.9: Error Rates vs. Capacity for Lognormally Distributed Desired User and Interferers ($N = 63, \frac{E_b}{N_0} = 10\text{dB}, \mu_{P_k}^{(1)} = 2\text{dB}, (\sigma_{P_k}^{(1)}) = 2\text{dB}$)
Figure 5.10: Error Rates vs. Capacity for Lognormally Distributed Desired User and Interferers ($N = 63, \frac{E_b}{N_o} = 10\, \text{dB}, \mu_{P_k}^{(1)} = 2\, \text{dB}, (\sigma_{P_k}^{(1)}) = 6\, \text{dB}$)
Figure 5.11: Comparison of Multistage Receiver and Optimal Receiver for Constant and Equivalent User Powers \((N = 63, K = 6)\)
5.5 Analysis of Adaptive Multistage Interference Cancellation for CDMA in Heavily Loaded Systems

In the previous section we showed that in lightly loaded systems the Gaussian Approximation can be somewhat inaccurate. This inaccuracy is most noticed in fading channels and when there is a disparity in the received user powers. We also noted that in heavily loaded systems there exists some biasing in the decision statistic which we have not yet modeled. We examine that phenomenon in more detail in this section. We show that this bias can have significant impact on system performance and must be taken into account as the system becomes heavily loaded.

Revisiting the decision statistic at stage 1 given in equation (5.1), we can rewrite the decision statistic as

\[ Z^{(1)}_{k,i} = \sqrt{\frac{P_k}{2}} T_b b_{k,i} + \eta + \sum_{j \neq k} \sqrt{2P_j} \cos(\phi_j - \phi_k) T_c W_{j,k} \]  

(5.49)

where

\[ W_{j,k} = \int_{(i-1)T_b + \tau_k}^{iT_b + \tau_k} b_j(t - \tau_j)a_j(t - \tau_j)a_k(t - \tau_k)dt. \]  

(5.50)

At stage two, the decision statistic given in equation (5.27) can be rewritten as

\[ Z^{(2)}_{k,i} = \sqrt{\frac{P_k}{2}} T_b b_{k,i} + \eta + \sum_{j \neq k} \int_{(i-1)T_b + \tau_k}^{iT_b + \tau_k} \left( \sqrt{2P_j} b_j(t - \tau_j) \cos(\phi_j) a_j(t - \tau_j) - \frac{2}{T_b} \sum_{n=\infty}^{\infty} Z^{(1)}_{j,n} P_{T_b}(t - nT_b) a_j(t - \tau_j) \cos(\phi_j) a_k(t - \tau_k) \cos(\phi_k)dt \right) \]  

\[ \]  

(5.51)

Substituting (5.49) into (5.51) and dropping the noise term \( \eta \) (we are most interested in the heavily loaded case where we are interference limited), we obtain

\[ Z^{(2)}_{k,i} = \sqrt{\frac{P_k}{2}} T_b b_{k,i} + \left\{ \sum_{j \neq k} \int_{(i-1)T_b + \tau_k}^{iT_b + \tau_k} \left[ \left( \sqrt{2P_j} b_j(t - \tau_j) - \frac{2}{T_b} \sum_{n=\infty}^{\infty} \left( \sqrt{\frac{P_j}{2}} T_b b_{j,i} + \sum_{m \neq j} \sqrt{2P_m} \cos(\phi_m - \phi_j) T_c W_{m,j,i} \right) \right) a_j(t - \tau_j) \cos(\phi_j) a_k(t - \tau_k) \cos(\phi_k)dt \right] \right\} \]  

\[ \]  

(5.52)

Simplifying,

\[ Z^{(2)}_{k,i} = \sqrt{\frac{P_k}{2}} T_b b_{k,i} + \sum_{j \neq k} \int_{(i-1)T_b + \tau_k}^{iT_b + \tau_k} \left[ \left( -\frac{2}{T_b} \sum_{m \neq j} \sqrt{2P_m} \cos(\phi_m - \phi_j) T_c W_{m,j,i} \right) a_j(t - \tau_j) \cos(\phi_j) a_k(t - \tau_k) \cos(\phi_k)dt \right]. \]  

(5.53)
Since the terms in the innermost summation will not vary with time, we can move them outside the integration:

\[
Z_{k,i}^{(2)} = \sqrt{\frac{P_k}{2}} T_b b_{k,i} + \sum_{j \neq k} \left[ \left( -\frac{2}{T_b} \sum_{m \neq j} \sqrt{2P_m} \cos(\phi_m - \phi_j) T_c W_{m,j,i} \right) \cdot \int_{(i-1)T_b+\tau_k}^{iT_b+\tau_k} a_j(t-\tau_j) a_k(t-\tau_k) \cos(\phi_j - \phi_k) dt \right]
\]

\[
= \sqrt{\frac{P_k}{2}} T_b b_{k,i} + \sum_{j \neq k} \left[ \left( -\frac{2}{T_b} \sum_{m \neq j} \sqrt{2P_m} \cos(\phi_m - \phi_j) T_c W_{m,j,i} \right) \cos(\phi_j - \phi_k) W_{k,j,i} \right] \quad (5.54)
\]

In the previous section, we assumed that the residual interference terms were uncorrelated. Thus we modeled the decision statistic as

\[
Z_{k,i}^{(2)} = \sqrt{\frac{P_k}{2}} T_b b_{k,i} + \sum_{j \neq k} \hat{I}_{j,k}^{(2)} \quad (5.55)
\]

where again, we have ignored the thermal noise and \( \hat{I}_{j,k}^{(s)} \) is the residual cross-correlation between users \( j \) and \( k \) at stage \( s \) as defined in equation (5.40). Previously, we assumed that \( \mathbb{E} [\hat{I}_{j,k}^{(s)}] = 0 \). However, as we can see from equation (5.54), there will be a small negative mean. To see this more clearly, let us examine the residual cross-correlation term in more detail. The expected value of the term (conditioned on user \( j \)) is

\[
\mathbb{E} [\hat{I}_{j,k}^{(s)}] = \mathbb{E} \left[ \left( -\frac{2}{T_b} \sum_{m \neq j} \sqrt{2P_m} \cos(\phi_m - \phi_j) T_c W_{m,j,i} \right) \cos(\phi_j - \phi_k) W_{k,j,i} \right]
\]

\[
= \left( -\frac{2}{T_b} \sum_{m \neq j, m \neq k} \sqrt{2P_m} T_c \mathbb{E} [\cos(\phi_m - \phi_j) W_{m,j,i}] \right) \mathbb{E} [\cos(\phi_j - \phi_k) W_{k,j,i}]
\]

\[
- \frac{2}{T_b} \sqrt{2P_k} T_c \mathbb{E} [\cos^2(\phi_k - \phi_j) W_{k,j,i}^2]
\]

\[
= - \frac{2}{T_b} \sqrt{2P_k} T_c \left[ \frac{12N}{3} \right]
\]

\[
= - \frac{2T_b \sqrt{2P_k}}{3N} \quad (5.56)
\]

Thus, we have a small bias term associated with each \( \hat{I}_{j,k}^{(s)} \). The resulting expected value of the decision statistic at stage 2 is then (assuming equal powers)

\[
\mathbb{E} [Z_{k,i}^{(2)}] = \sqrt{\frac{P_k}{2}} T_b b_{k,i} - \frac{2(K-1)T_b}{3N} \sqrt{P_k} b_{k,i}
\]

(5.57)
Figure 5.12: Reduction in the Mean of the Decision Statistic in the Second Stage

At small values of $K$, this bias will not significantly affect performance, and can thus be ignored. However, as the number of users grows, the bias will become more significant.

To test this analysis, several simulations were run for a perfect power control AWGN channel. Adjusting the previous result for a complex baseband simulation to allow comparison with simulation, results in a bias of $-\frac{T_k \sqrt{P_k}}{3N}$ for each interferer. This bias is plotted versus the number of users for a perfect power control case along with the results of simulations in Figure 5.12. As we can see, the simulation results agree exceptionally well with the analysis.

It is also obvious from Figure 5.12 that as the number of users increases, the bias is significant and must be taken into consideration when predicting system performance. To
Figure 5.13: Predicted and Simulated Performance of Heavily Loaded System with Perfect Power Control (20 users, spreading gain = 31)

see this more directly, consider a system with perfect power control, spreading gain of 31, and a heavy system loading (20 users). The predicted system performance is plotted in Figure 5.13 for the Gaussian approximation with and without the biasing term. The simulated system performance is also plotted. As we can see, ignoring the bias term at $E_b/N_0 = 10dB$ would result in predicted performance which is approximately an order of magnitude greater than the actual performance. Thus, we must include this biasing effect to obtain reliable results.

As further confirmation of the analysis of the bias in the decision statistic, we return to the histogram of the decision statistic presented in Chapter 4. Figure 5.14 presents the histogram of a system with no noise and 25 users. Two sets of plots are presented. The
inside plots are the decision statistic without manipulation while the outside plots have the bias removed. The second set of curves were the result of simulations aimed at verifying the bias' cause. This was generated as follows. User 1 was denoted as the desired user and removed exactly from the received signal using perfect knowledge of the received signal. This new received signal was then used to create the decision statistic of the other users. These estimates were used to cancel the interferers from the original received signal. This residual signal now contains only the desired user, and the estimates used to remove the interferers are truly independent of the desired signal. Thus, no bias is introduced as seen in Figure 5.14. This procedure is by no means a practical solution to the bias problem. Rather, it is a simple way to verify the origination of the bias effect.

The results presented in the previous section assumed that the decision statistic is unbiased. In low to moderately loaded systems, these equations can be used directly to approximate system performance. However, we have just seen that in lightly loaded systems, the zero bias assumption will result in significantly optimistic results. Thus, if we are to apply these equations, (as well as significantly improve system performance) we must find a method of reducing this bias. A means for mitigating the bias is discussed in Chapter 9.

5.6 Discussion

In this chapter we have developed an improved Gaussian approximation for the performance of a parallel multistage interference cancellation receiver. This new formulation allows second order effects of multiple access interference to be considered in performance calculations. This allows better performance prediction in cases where interference power has a random distribution. Also, we have shown that in some cases first order statistics are sufficient for accurate performance prediction. Specifically, when processing gain, $N$, is large, $\frac{P_k}{\sigma^2}$ is low, $\sigma_{P_k}$ is low, or the number of users relative to the processing gain is large, MAI power variance has little effect on receiver performance.

Capacity curves for the important case of log-normally distributed powers were presented for the multistage receiver which showed significant performance improvement over the conventional receiver. Additionally, performance comparable to the optimal receiver was demonstrated. It was also shown that the multistage receiver effectively reduced MAI by demonstrating that the mean and variance of the MAI variance is reduced close to asymptotic values with only a few stages of cancellation.

Additionally, we showed that the assumption of an unbiased decision statistic is not justified in heavily loaded systems and can result in significantly optimistic performance prediction. In order to apply the equations discussed in this chapter in heavily loaded
Figure 5.14: Histogram of the Decision Statistic in Heavily Loaded System With and Without Bias Removal (25 users, spreading gain = 31)
systems (with the exception of the last section) some method of bias reduction is required. This will be addressed more in Chapter 9.
Chapter 6

The Near-Far Resistance of Interference Cancellation Receivers

6.1 Introduction

Conventional receivers treat Multiple Access Interference (MAI) which is inherent in CDMA, as if it were additive noise. Unfortunately, this MAI is in general correlated with the desired user's signal and thus causes significant degradation. If an interferer is significantly stronger than the desired user, it will dominate performance in a conventional receiver due to the near-far problem. This problem requires that strict power control be employed in current CDMA system proposals. However, strict power control can be a significant complexity burden and can be difficult to maintain. Thus, receivers which are resistant to large power disparities are desired.

In section 6.2 the concept of optimum multiuser asymptotic efficiency is reviewed along with previous applications. Section 6.3 presents results for the general interference cancellation case. This result is then applied to multistage and successive cancellation in section 6.4. Simulation results for near-far environments are also presented. Section 6.5 defines a measure termed near-far robustness and presents results for multistage cancellation. The implications of this research are discussed in section 6.6.
6.2 Optimum Multiuser Asymptotic Efficiency

Optimum multiuser asymptotic efficiency is a measure developed by Verdu [131] to quantify the degradation in performance that multiple access interference causes a given receiver structure. Formally the asymptotic efficiency of the $k$th user is defined as

$$\eta_k = \sup \left\{ 0 \leq r \leq 1; \lim_{N_o \to 0} P_k(N_o)/Q\left(\sqrt{\frac{2rw_k}{N_o}}\right) < +\infty \right\} \tag{6.1}$$

where $w_k$ is the received energy of the $k$th user, $P_k(N_o)$ is the probability of error of the receiver under consideration for a specified noise level and $Q\left(\sqrt{\frac{2w_k}{N_o}}\right)$ is the performance of the matched filter receiver in the absence of multiple access interference. In the limit as $N_o \to 0$, $\eta_k$ is equal to the ratio of the energy required by the receiver under consideration to achieve a specified BER to the energy required in the isolated transmission case (often termed the effective energy). Formally,

$$\eta_k = \lim_{N_o \to 0} \frac{e_k(N_o)}{w_k} \tag{6.2}$$

where $e_k$ is the effective energy. Thus, in essence $\eta_k$ provides a measure of the loss in performance due to the presence of interference. The near-far resistance is defined as the worst case optimum multiuser asymptotic efficiency over all energies.

The conventional matched filter receiver when used in a multiple access environment will have a performance given by

$$P_k(N_o) = E\left[ Q\left(\sqrt{\frac{\sqrt{w_k} + \sum_{j=1}^{k-1} \sqrt{w_j} |\rho_{j,i-k+1}| + \sum_{j\neq k} \sqrt{w_j} |\rho_{j,i-k+1}| + \sum_{j=k+1}^{K} \sqrt{w_j} |\rho_{j,i-k+1}|}{\sqrt{N_o/2}}\right)^2\right] \tag{6.3}$$

where the expectation is taken over all possible bit sequences and $\rho_{j,i}(i)$ is defined in (2.8). For low noise situations, the expectation will be dominated by the $Q$-function with the smallest argument. Assuming constant energies, this will occur when $b_{j,i-1} = -sgn[\rho_{j,k}(1)]$, for $j < k$, $b_{j,i} = -sgn[\rho_{j,k}(0)]$, and $b_{j,i+1} = -sgn[\rho_{j,k}(-1)]$, for $j > k$. In this case the effective energy is

$$e_k = \left(\sqrt{w_k} - \sum_{j=1}^{k-1} \sqrt{w_j} |\rho_{j,k}(1)| + \sum_{j\neq k} \sqrt{w_j} |\rho_{j,k}(0)| + \sum_{j=k+1}^{K} \sqrt{w_j} |\rho_{j,k}(-1)|\right)^2$$

$$= \left(\sqrt{w_k} - \sum_{j\neq k} \sqrt{w_j} |\rho_{j,k}|\right)^2, \tag{6.4}$$
where
\[
\rho_{j,k} = \begin{cases} 
(|\rho_{j,k}(1)| + |\rho_{j,k}(0)|) & j < k \\
(|\rho_{j,k}(0)| + |\rho_{j,k}(-1)|) & j > k, \\
1 & j = k
\end{cases}
\]

Thus the multiuser asymptotic efficiency of the conventional receiver can be expressed as [130]
\[
\eta_k^C = \left[ \max \left\{ 0, 1 - \sum_{j \neq k} \sqrt{w_j/w_k \rho_{j,k}} \right\} \right]^2.
\] (6.5)

Thus the conventional receiver has an asymptotic multiuser efficiency which is dependent on \( w_j \) (the energy of the interferers) and consequently has a near-far resistance of zero provided \( \rho_{j,k} > 0 \). That is as \( w_j \to \infty \) for any \( j \neq k \), \( \eta_k^C = 0 \) if the waveforms are not orthogonal.

In contrast, the optimal multiuser receiver has a near-far resistance which is guaranteed to be greater than zero [131]. While in general the expression for \( \eta_k \) in the case of the optimal receiver is non-trivial, the two-user case can be expressed as
\[
\eta_k^O = 1 - \max \left\{ 0, \quad 2\rho_{1,2}(0)\sqrt{w_j/w_k} - w_j/w_k, \quad 2\rho_{1,2}(1)\sqrt{w_j/w_k} - w_j/w_k, \quad 2\rho_{1,2}(0)\sqrt{w_j/w_k} + 2\rho_{1,2}(1)\sqrt{w_j/w_k} - 2w_j/w_k \right\}.
\] (6.6)

As can be seen, \( \eta_k^O \) achieves its minimum value at \( \sqrt{w_j/w_k} = 2 \cdot \min \{|\rho_{1,2}(0)|, |\rho_{1,2}(1)|\} \) which is \( \eta_{\text{min}}^O = \min \left\{ 1 - \rho_{1,2}(0), 1 - \rho_{1,2}(1), 1 - \rho_{1,2}(0) - \rho_{1,2}(1), 1 - \rho_{1,2}(0) - \rho_{1,2}(1) + \frac{(|\rho_{1,2}(0)| - |\rho_{1,2}(1)|)^2}{2} \right\} \). Thus, \( \eta_{\text{min}}^O \) is independent of the interfering energy and is near-far resistant provided that \( |\rho_{j,k}(i)| < 1 \).

A third example of optimum multiuser asymptotic efficiency is for the decorrelator. The performance of the decorrelator for the \( k \)th user over the \( i \)th bit interval can be shown to be
\[
P_e(N_o) = Q \left( \sqrt{\frac{2w_k}{N_o T_{m,m}}} \right).
\] (6.7)

where \( T = \mathcal{R}^{-1} \) and \( m = (i - 1)K + k \). Thus \( \eta_k^D \) can be found to be
\[
\eta_m^D = \frac{1}{T_{m,m}}.
\] (6.8)

For the two user case (assuming constant energies) this reduces to [78]
\[
\eta_k^D = \sqrt{1 - (\rho_{1,2}(0) + \rho_{1,2}(1))^2(1 - (\rho_{1,2}(0) - \rho_{1,2}(1))^2)}.
\] (6.9)

As in the case of the optimal receiver, the decorrelator is near-far resistant, i.e. the minimum value of \( \eta_k^D \) is independent of the interfering energy and is greater than zero provided that
\( \rho_{1,2}^2(0) + \rho_{1,2}^2(1) < 1 \). As an example we provide numerical results for the two user case when \( \rho_{1,2}(0) = 0.3 \) and \( \rho_{1,2}(1) = 0.5 \) [79] in Figure 6.1. Here it is explicitly shown that the conventional receiver, unlike the optimal and the decorrelator, has an optimal multiuser efficiency which goes to zero for sufficiently strong interference.

### 6.3 Interference cancellation

For any interference cancellation receiver, performance enhancement is attempted by estimating the interference and removing it through regeneration and cancellation. Specifically, assuming that all users in the system are being tracked (i.e. we have code and phase lock on all users) the receiver can regenerate the interference for removal by estimating the received energy and data symbols of each interferer. Formally the decision metric for an interference cancellation receiver is,

\[
b = \text{sgn} \left[ \mathcal{R}\mathcal{W} b + n - \hat{\mathcal{R}}\hat{\mathcal{W}} \hat{b} \right]
\]

where \( \hat{\mathcal{R}} = \mathcal{R} - \mathcal{I} \), \( \mathcal{I} \) is a \( KN_b \times KN_b \) identity matrix, \( \hat{\mathcal{W}} \) is an estimate of the received energy levels \( \mathcal{W} \), and \( \hat{b} \) is the estimated data of the interferers. The probability of bit error can be expressed as

\[
P_e|b = E \left[ Q \left( \frac{\sqrt{(\mathcal{R}\mathcal{W} b - \hat{\mathcal{R}}\hat{\mathcal{W}} \hat{b})^2}}{N_0/2} \right) \right].
\]

Assuming that all data sequences are equally likely and arbitrarily assuming that \( b_{k,i} = 1 \) the probability of error is

\[
P_e = \sum_{b \in \{+1,-1\}^{N_{\mathcal{K}}} \atop b_{k,i} = 1} \frac{1}{2^{K(N-1)}} Q \left( \frac{\sqrt{(\mathcal{R}\mathcal{W} b - \hat{\mathcal{R}}\hat{\mathcal{W}} \hat{b})^2}}{N_0/2} \right).
\]

Simplifying the numerator, we define \( \hat{\mathcal{W}} = \mathcal{W} + \mathcal{W}^e \) and \( \hat{b} = b + b^e \) where \( \mathcal{W}^e \) and \( b^e \) are the errors involved in the energy and data estimates respectively. Using these definitions,

\[
\mathcal{R}\mathcal{W} b - \hat{\mathcal{R}}\hat{\mathcal{W}} \hat{b} = \mathcal{R}\mathcal{W} b - (\mathcal{R} - \mathcal{I})(\mathcal{W} + \mathcal{W}^e)(b + b^e)
\]

\[
= \mathcal{R}\mathcal{W} b - \mathcal{R}\mathcal{W} b + \mathcal{W} b - \hat{\mathcal{R}}(\mathcal{W}^e b + \mathcal{W}^e b^e)
\]

\[
= \mathcal{W} b - \hat{\mathcal{R}}(\mathcal{W}^e b + \mathcal{W}^e b^e)
\]

\[
= \mathcal{W} b - \hat{\mathcal{R}} \mathcal{E}
\]

where \( \mathcal{E} = (\mathcal{W}^e b + \mathcal{W}^e b^e) \) is the residual interference. As before, the probability of error in (6.12) will be dominated by the element in the summation with smallest Q-function.
Figure 6.1: Comparison of Asymptotic Efficiencies for Conventional, Optimal, and Decorrelating Receivers as a Function of Interfering Energy
argument. Thus we can approximate the probability of error as

$$P_e = Q \left( \sqrt{\frac{\sqrt{w_k} - \sum_{j=1}^{k-1} |\rho_{j,k}(1)| |\mathcal{E}_{j,i-1}| - \sum_{j \neq k} |\rho_{j,k}(0)| |\mathcal{E}_{j,i}| - \sum_{j=k+1}^{K} |\rho_{j,k}(-1)| |\mathcal{E}_{j,i+1}|}{N_c/2}} \right)$$

(6.14)

where \( \mathcal{E}_{j,i} = \sqrt{w_j^e} b_{j,i} + \sqrt{w_j^e} b_{j,i}^e + \sqrt{w_j^e} b_{j,i}^e \). Thus using this result in (6.1) results in an expression for the optimum asymptotic multiuser efficiency for an interference cancellation receiver:

$$\eta^I_k = \max^2 \left\{ 0, 1 - \sum_{j=1}^{k-1} |\rho_{j,k}(1)| \frac{|\mathcal{E}_{j,i-1}|}{\sqrt{w_k}} - \sum_{j \neq k} |\rho_{j,k}(0)| \frac{|\mathcal{E}_{j,i}|}{\sqrt{w_k}} - \sum_{j=k+1}^{K} |\rho_{j,k}(-1)| \frac{|\mathcal{E}_{j,i+1}|}{\sqrt{w_k}} \right\}.$$  

(6.15)

For the two-user case, (6.15) is simplified to

$$\eta^I_k = \left[ \max \left\{ 0, 1 - |\rho_{1,2}(1)| \frac{|\mathcal{E}_{1,i-1}|}{\sqrt{w_k}} - |\rho_{1,2}(0)| \frac{|\mathcal{E}_{1,i}|}{\sqrt{w_k}} \right\} \right]^2.$$  

(6.16)

### 6.4 Numerical and Simulation Results

With the asymptotic efficiency of a cancellation receiver in hand, we first examine a multistage parallel cancellation approach. If conventional correlator outputs are used to estimate the data and received energy of each user, we can represent the energy estimation error (for the case of low thermal noise) as

$$\sqrt{w_k^e} = \sum_{j \neq k} |\rho_{j,k}| \sqrt{w_j^e}$$  

(6.17)

and \( b_{k,i} = 2 \cdot P_{e_k} \). Again neglecting noise,

$$P_{e_j} = \begin{cases} 0 & \sqrt{w_k} < \sum_{j \neq k} \sqrt{w_j} |\rho_{j,k}| \\ 1 & \sqrt{w_k} \geq \sum_{j \neq k} \sqrt{w_j} |\rho_{j,k}| \end{cases}.$$  

(6.18)

Using these values for the two user case in (6.16) and assuming that \( \rho_{j,k} |\mathcal{E}_{j,i}| = \rho_{j,k} |\mathcal{E}_{j,i+1}| = \rho_{j,k} 2P_{e_j} (\sqrt{w_j^e} + \sqrt{w_j^e} - \sqrt{w_k^e}) \) (which is the worst case) results in:

$$\eta^I_k = \max^2 \left\{ 0, 1 - \rho_{1,2} \left[ |\rho_{1,2}(-1 + 2 \cdot P_{e_j}) + \sqrt{\frac{w_j}{w_k}} 2P_{e_j}| \right] \right\}.$$  

(6.19)

The important result in the previous equation is that the near-far resistance of user \( k \) depends on the received energy of user \( j \) in two ways. First the last term increases with increasing \( \sqrt{w_j} \) due to \( \sqrt{\frac{w_j}{w_k}} \). Second, the terms including \( P_e \) will decrease with increasing
\( \sqrt{\omega_j} \). The second effect will dominate the first since the Q-function will decrease much faster than its argument increases. Additionally, for the low noise case, \( P_{ej} = 0 \) for sufficiently large \( \sqrt{\omega_j} \). Specifically, when \( \sqrt{\omega_j} > \sqrt{\omega_k \rho_{j,k}} \), \( \eta_k = 1 - \rho_{1,2}^2 \), i.e. \( \eta_k \) is independent of interfering energy. For the case where \( \sqrt{\omega_j} < \sqrt{\omega_k \rho_{j,k}} \), we find that \( \eta_k = 1 - \rho_{j,k} \left[ \rho_{j,k} + 2 \sqrt{\omega_j \omega_k} \right] \) and \( \eta_k \) will decrease as \( \omega_j \) increases. The worst case arises when \( \sqrt{\omega_j} = \sqrt{\omega_k \rho_{j,k}} \) and \( \eta_k = 1 - 3 \rho_{j,k}^2 \). Thus, \( \eta_k > 0 \) provided \( \rho_{1,2} < \sqrt{1/3} = 0.5774 \).

As an example consider the two user system discussed previously with \( \rho_{1,2}(0) = 0.3 \) and \( \rho_{1,2}(1) = 0.5 \) (i.e. \( \rho_{1,2} = 0.8 \)). Figure 6.2 shows the asymptotic multiuser efficiency as the energy of user 2 grows large. As expected the performance is not degraded by strong interference. In fact the performance is enhanced. Degradation occurs when interference is fairly weak. Also, while the efficiency is invariant as the interference grows strong, it is significantly less than the optimal and decorrelating receivers. This is due to the fact that we now require energy estimation in order to remove interference. Using correlator outputs to estimate the energy results in a limitation which is directly related to the correlation between the two signals. Since this is a strongly correlated case, the efficiency is low. The poor performance at low interfering powers may seem counterintuitive. However, it can be explained as follows. When interferers are weak, they will be estimated poorly and thus cause a degradation in the performance of the stronger user due to imperfect cancellation. If perfect energy estimates are obtained as in the case of [124], the performance is enhanced as shown in Figure 6.2. Unfortunately, perfect amplitude estimates are not realistic. Also shown in Figure 6.2 is a case where \( \rho_{1,2} = 0.5 \). In this situation the two stage receiver is near-far resistant as predicted. The conventional receiver is replotted for comparison.

The previous example showed that for a two-user system, a multistage cancellation receiver can provide near-far resistance for a range of cross-correlations. Unfortunately this is not true in the general case of \( K > 2 \). This can be best seen by example. Consider a lightly loaded system with \( K = 3 \) and a set of relative delays such that \( \rho_{k,j}(i) = 1/10 \) for \( k \neq j \). This obviously represents a very amenable system with light loading and small cross-correlation. The asymptotic efficiency is plotted in Figure 6.3. As we can see, even in this very pleasant environment the receiver is not near far resistant. The reason for this is straightforward. While in the two user case increasing interference power is beneficial since it affords more accurate cancellation, in the case where \( K > 2 \) this is no longer the case. While increasing interference power does lead to more accurate cancellation of the strong interferer, it hinders cancellation of all other interferers (which are not present in the two user case). From equation (6.17) we see that estimation error of the weaker interferers is directly related to the power of the strong interferer. Thus as its power increases, the
Figure 6.2. Asymptotic Efficiencies of Two Stage Cancellation Receiver for 2 Users as a Function of Interfering Energy
estimation of the weaker interferers degrades leading to poor cancellation of these interferers and thus degradation in the desired user's performance. Also shown in Figure 6.3 is the asymptotic efficiencies for the same example when an additional stage of cancellation is used. The third stage (i.e. second cancellation stage) increases the efficiency greatly although it is still heading toward zero, albeit at a much slower rate. Since the efficiency is still decreasing the near-far resistance of the multistage receiver with $K > 2$ is zero.

To emphasize the importance of the energy estimates in the asymptotic efficiency of the multistage receiver, consider the near ideal cross-correlation considered previously. The asymptotic efficiency of a two stage receiver is plotted in Figure 6.3 for perfect energy estimates. We now see that near-far resistance is obtained. It is obvious that the energy estimate error dominates the performance of the multistage receiver.

A second type of interference cancellation is successive cancellation where previously detected bits are fed back along with energy estimates to cancel interference. The main differences between this technique and the previous is that all users are cancelled only once, and cancellation is done in descending order of received energies. This is done to allow the more reliable estimates to be used first and allow the weaker user to benefit from the most cancellation. In this case the energy estimate error for user $k$ can be expressed by (again assuming the use of correlator outputs for energy estimates)

\[
\sqrt{w_k^e} = \sum_{j=1}^{k-1} \rho_{j,k} \sqrt{w_j^e} + \sum_{k+1}^{K} \rho_{j,k} \sqrt{w_j}.
\]  

One thing to notice about the previous equation is the lack of dependence on $\sqrt{w_j}$ which is the energy of the strongest user. Since the estimation error is a function of the estimation error of previously detected users and the energies of subsequently detected users, the energy of the strongest user will not effect the estimation of any other user. Thus we would expect the receiver to be resistant to a single dominant interferer. We can see this in Figure 6.4 where the previous examples of $K = 2$ and $K = 3$ are shown. As expected, both examples illustrate the independence of the receiver to a dominating interferer regardless of the number of users. This contrasts with the multistage receiver which in independent of the strong user only in a two-user system.

However, while the receiver performance is invariant to the power of the strongest user, it is susceptible to the power of the second strongest user. Let us examine the case for $K = 4$, $\rho_{i,j} = 1/10 \forall j \neq k$, and allow two users to grow strong. Figure 6.5 shows that independence is no longer maintained and $\eta_k$ goes to zero for sufficiently strong interference. The case where only a single interferer dominates is also plotted. This case does show independence to the interfering energy. Thus the receiver structure is not near-far resistant in the true
Figure 6.3: Asymptotic Efficiencies of Two Stage Cancellation Receiver for 3 Users as a Function of Interfering Energy
Figure 6.4: Asymptotic Efficiencies of Successive Cancellation Receiver for 2 or 3 Users as a Function of Interfering Energy
sense because there exist combinations of interfering energy which cause $\eta_k^C = 0$. The explanation for the lack of near-far resistance is straightforward. The cancellation of the strongest user is dependent on the energy of the second user which is now very strong. The estimation error of user one’s energy will be directly related to the strength of the other users including the second user which is large. This estimation error is propagated to all users, since each will benefit or suffer from the cancellation of the strongest user. The performance of all other users in the system while independent of the strongest user’s energy will not then be independent of the second strongest user’s received energy. Thus a second strong interferer will degrade performance even though a single dominant interferer will not. In Figure 6.5 we also plot the asymptotic efficiency for the case of two strong interferers when perfect estimation is obtained. Again we find that perfect estimation allows near-far resistance.

To reinforce these near-far resistance calculations we ran simulations to examine the performance of a conventional receiver, a multistage cancellation receiver, and a successive cancellation receiver in an AWGN near-far environment. Figure 6.6 shows the results for the conventional and two-stage receivers for $E_b/N_o = 5dB$ and a spreading gain $N = 15$ for 2 or 3 users. The conventional receivers performance degrades quickly as expected. Also, as predicted the two-stage receiver shows approximately single user performance when there are only two users in the system regardless of interfering energy. However, when there are three users, the performance shows susceptibility to interfering energy. Both situations show dramatic improvement over the conventional receiver.

Figure 6.7 shows the performance of the conventional and successive cancellation receivers for $E_b/N_o = 5dB$ and a spreading gain $N = 15$ for 2, 3 or 4 users. For the cases where there are 2 or 3 users, only a single interferer dominates. However, when $K = 4$, we allow two interferers to grow strong. In the latter case we see near-far robustness but not resistance. Again, the successive cancellation receiver shows large benefits when compared to the conventional receiver. Thus, our simulation results support our near-far resistance analysis.

6.5 Near/Far Robustness

In the previous sections we showed that cancellation receivers are not formally near-far resistant. However, it was shown that cancellation receivers provide a greater robustness to interfering signals than the conventional receiver. The asymptotic efficiency is a measure which provides insight into the performance in the worst possible case. While this certainly provides some insight into the receiver’s performance, it does not truly characterize the
Figure 6.5: Asymptotic Efficiencies of Successive Cancellation Receiver for 4 Users as a Function of Interfering Energy
Figure 6.6: Simulation Results for Multistage Cancellation as a Function of the Strongest Interferer's Energy ($E_b/N_0 = 5dB$, $N = 15$)
Figure 6.7: Simulation Results for Successive Cancellation as a Function of the Strongest Interferer's Energy ($E_b/N_o = 5dB$, $N = 15$)
performance in the non-limiting case. In this section we define a measure of near-far robustness which can be applied to more practical situations, and provides some additional insight into the receiver performance.

To measure the level of immunity to strong interferers achieved by a receiver we introduce a performance measure termed Near/Far Robustness. Near/far robustness, \( \Upsilon \), is defined as the increase in interference power of a single user from the equal power case which results in an increase in the BER of the desired user by an order of magnitude over the BER achievable in the isolated transmission case. The term gives a measure of the increase allowable by a rogue interferer before significant degradation. Formally,

\[
\Upsilon = \frac{w_i(10P_e)}{wd} \bigg|_{P_e = 10Q(\sqrt{2wd/N_0})} \quad (6.21)
\]

where \( w_i(x) \) is the energy of a single interferer which results in \( P_e = x \), \( w_d \) is the energy of the desired user, and \( N_0 \) is the one-sided power spectral density of the noise.

As an example consider Figure 6.8 where we present simulation results for the conventional and three stage cancellation receivers for \( N = 31 \), \( E_s/N_0 = 8\,\text{dB} \), and \( K = 4, 8, 12, 16 \) as the power of a single interferer grows large. Also shown on the graph is a line marking the point at which the BER is an order of magnitude greater than the single user case. Using the above definition, the near-far robustness of the conventional receiver when there are four users in the system is approximately 1dB, while the three stage cancellation receiver shows a near-far robustness of approximately 25dB. Thus, the increase in a single interferer's power by a very small amount pushes the performance in a conventional receiver to an unacceptable level, while the cancellation receiver can withstand an increase in any given interferer's power of 25dB. This is partially due to the fact that the conventional receiver performs so poorly for four users in perfect power control. However, from the plot we can see that the conventional receiver begins degradation as soon as the interferer's power level increases beyond the perfect power control case. The performance of the multistage receiver, however, remains constant until the interferer's power is 10dB greater than the other users in all 4 cases.

As the number of users increases to \( 8 \rightarrow 16 \) we see that the conventional receiver has a near-far robustness of \( \Upsilon^C = 0 \), that is the performance is above the maximum allowable BER in perfect power control. The cancellation receiver on the other hand provides a near-far robustness of \( \Upsilon^{IC} = 21\,\text{dB} \), \( \Upsilon^{IC} = 17\,\text{dB} \), and \( \Upsilon^{IC} = 15\,\text{dB} \) for loadings of 8, 12, and 16 users respectively. Thus, when the system is loaded at over 50% of the spreading gain, we can still withstand an increase in a single interferer of 15dB before performance exceeds our threshold. Additionally, when the interferer does not exceed 10dB above the other users, we
Figure 6.8: Simulation Results for Conventional and Multistage Cancellation for 4, 8, 12, and 16 users as a Single Interferer Grows Strong ($E_b/N_o = 8dB$, $N = 31$)
see no performance degradation when compared to perfect power control (although there is of course a degradation compared to the single users case due to the interference, but not the lack of power control). Thus, while both the conventional receiver and the cancellation receivers have a near-far resistance of zero, they provide significantly different performance in near-far situations. This is a performance advantage of the multistage receiver which cannot be seen in either asymptotic multiuser efficiency nor near-far resistance. However, the measure defined here, near-far robustness, does illustrate the advantage of multistage cancellation in near-far situations.

6.6 Summary

Near-far resistance is an important feature in CDMA receiver design due to the complexity burden associated with power control and the high penalty paid for imperfect power control. We have examined interference cancellation receivers and investigated their near-far resistance. It was shown that while the two-stage receiver is near-far resistant for the degenerative two user case, it is in general not near-far resistant due to the inaccuracy of the energy estimates. Additionally, it was shown that if energy estimates were improved near-far resistance is possible. Successive cancellation receivers were shown to be near-far resistant when a single interferer dominates. However, if more than one interferer’s energy grows large, this receiver is no longer near-far resistant. Again, near-far resistance can be obtained by using perfect energy estimates.
Chapter 7

Parameter Estimation

7.1 Introduction

In this chapter we investigate an important aspect of detection theory in general and multistage cancellation in particular, namely parameter estimation. In order to decode the transmitted information embedded in an RF carrier, we must estimate some parameter of that carrier. Specifically we must either estimate the phase, the amplitude or the frequency, since these are the main characteristics of a carrier wave which we can modulate. Typically, however, these parameters are of varying interest to the receiver. For example in classical AM modulation there is no need to estimate the phase of the received signal, since it is only the amplitude which carries information. Additionally, in PSK we are not interested in the amplitude of the signal other than whether it is positive or negative. While the frequency does not explicitly carry any information in PSK, we are interested in its value, as we need to track it in order to get proper phase estimation. In this case we need not estimate the frequency, but we must be able to adjust our local replica of the carrier to match the incoming wave's frequency (i.e. track the frequency using a closed loop circuit) to facilitate demodulation. Thus, classical receivers have varying needs when it comes to parameter estimation.

Interference cancellation receivers are somewhat different. While classical receivers need to estimate parameters only well enough to obtain the desired information, receivers which employ interference cancellation must estimate these parameters more accurately. Not only is the desired information sought, but we also require parameter estimation in order to regenerate the interfering signals and remove their effect from the desired user's signal. Thus, estimation errors will not only result in increased possibility of symbol error in the desired signal, but will also hinder cancellation of the interfering signals.
Timing is an additional parameter that is not used to convey information, but is essential in signal demodulation and detection. We must accurately know the symbol timing of the desired signal in order to know the appropriate time interval over which to detect the data symbol. In CDMA, proper timing must also be obtained for the PN sequence, making the timing requirements more strict. Thus, timing errors are just as crucial, if not more so, than other parameter errors.

In this chapter we will examine the estimation of signal amplitude, phase/frequency, and timing for multistage interference cancellation. The first section will focus on amplitude estimation and show how amplitude estimation can be improved by using simple averaging techniques. The primary means of amplitude estimation to be explored is through the use of matched filter outputs. The second section investigates the estimation of phase and frequency through the use of small training sequences. Again some improvement in this estimation can be achieved through averaging techniques. The third section investigates the estimation of code and bit timing i.e., synchronization and tracking and shows how simple feedback filters can improve synchronization. This is very important since we cannot realize the benefits of interference cancellation, if we exceed the receiver's ability to acquire a signal.

7.2 Previous Work

As mentioned in an earlier chapter, the optimal multiuser receiver requires knowledge of the amplitudes and phases of the users. Thus, early work suggested means of obtaining these parameters using test sequences (i.e. known bits) [129]. Later work on parameter estimation for DS/CDMA includes joint maximum likelihood sequence and amplitude estimation which does not assume the use of training sequences [102]. Unfortunately, this method has a complexity that is exponential in the number of bits transmitted. Methods which which may be implemented with more practical levels of complexity include a joint detection and parameter estimation scheme which utilizes a reduced tree search algorithm for sequence estimation and an adaptive complex amplitude estimator which has a complexity $O(L K^2)$ where $L$ is the number of nodes or paths retained in the reduced search and $K$ is the number of users. Other work in the area of parameter estimation includes the sequential amplitude estimators of [119], methods not requiring timing information [87], and near-far resistant estimators using training sequences [96]. Other work assuming known training sequences is described in [84].

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7.3 Amplitude Estimation

Perhaps one of the simplest but most crucial aspects of interference cancellation is the estimation of the channel gain (i.e., the received signal amplitude). Initial work on multistage cancellation showed the tremendous potential of the technique, but assumed that perfect estimation of the received energy. This is impossible to obtain and in practice amplitude estimates are noisy. For example, assume that we use the absolute value of the correlator outputs to estimate the amplitude of a desired signal. Formally,

\[ \hat{A}_{k,i} \left| \frac{1}{T_b} \int_{(i-1)T_b+\tau_k}^{iT_b+\tau_k} r(t)a_k(t) \cos(\omega_c t + \theta_k) dt \right| \]  \hspace{1cm} (7.1)

where \( r(t), a_k(t), \tau_k, T_b, \omega_c, \) and \( \theta_k \) are defined in Chapter 2 and \( A_k = \sqrt{2P_k} \) is the amplitude of the received signal. Using previous definitions,

\[ \hat{A}_{k,i} = \left| \frac{2Z_{k,i}}{T_b} \right| \hspace{1cm} (7.2) \]

Figure 7.1 shows a typical example of the amplitude estimate in a lightly loaded (5 users, spreading gain of 31, and \( E_b/N_0 = 8 \text{dB} \)) CDMA system. As we can see, there is significant variance in the amplitude estimate even in light loading.

Of course, as the loading increases this variance increases. To illustrate this, Figure 7.2 displays the mean square error (MSE) of the amplitude estimate as the the capacity increases. This estimation error will cause performance degradation for the interference cancellation receiver as will be shown in the next section. Thus, we would like to reduce this variance as much as possible. As mentioned previously, there are more robust techniques to improve upon this estimation rather than using simple matched filter outputs (e.g. \([87]\)). However, these techniques are complex and thus require significant computational power. Since multistage cancellation is already increasing complexity considerably over the conventional receiver, we would like to keep the estimation as simple as possible.

One simple method of reducing the amplitude variance when the channel gain is constant is to average the estimate over time. For a constant channel gain in AWGN, we should be able to reduce the variance to an arbitrarily small number. To show this we know that the amplitude estimation \( \hat{A} \), in the absence of multiple access interference, is a random variable with a folded Gaussian pdf. The estimation when using bit averaging is simply the classical sample mean

\[ \hat{A}_{b_{ak}} = \frac{1}{N_a} \sum_{i=1}^{N_a} \hat{A}_{k,i} \]  \hspace{1cm} (7.3)

where \( N_a \) is the number of bits over which the average is taken. For independent random variables, the expected value of this estimate is \( \frac{N_a\mu_A}{N_b} = \mu_A \) while the variance is \( \frac{N_a\sigma^2_A}{N_b} = \frac{\sigma^2_A}{N_b} \).
Figure 7.1: Typical One Bit Amplitude Estimate, Actual Amplitude and Averaged Estimate in CDMA System (5 users, spreading gain = 31, and $E_b/N_0 = 8$dB)
Figure 7.2: Mean Squared Error of Amplitude Estimate for AWGN Channel (spreading gain = 31, and $E_b/N_0 = 8\,$dB)
In the absence of interference, \( Z_{K,l} \) is a Gaussian random variable with mean \( \mu = A T_b \) and variance \( \sigma^2 \), and \( \hat{A} \) is a folded Gaussian with mean \( \mu_A = \frac{1}{T_b} \lambda \sqrt{\frac{2\sigma^2}{\pi}} e^{-\mu^2/2\sigma^2} + \frac{A}{T_b} \). Thus, the amplitude estimate \( \hat{A}_{ba} \) will be a biased estimate with mean \( A + \frac{1}{T_b} \lambda \sqrt{\frac{2\sigma^2}{\pi}} e^{-\mu^2/2\sigma^2} \). Additionally, the estimator is not consistent, although \( \lim_{N_b \to \infty} \sigma_A = 0 \). However, the estimation is certainly improved over the single point estimation \( \hat{A} = \left| \frac{Z}{T_b} \right| \).

In CDMA we of course are not faced with a simple AWGN channel. However, as we have shown in previous chapters, the multiple access interference can be modeled as a Gaussian random variable. Thus, it is very likely that the amplitude estimate can be considered to have a folded Gaussian pdf and that averaging over consecutive bit periods will smooth out the effect of MAI. This is indeed shown to be the case in Figure 7.1 for the case of 5 users, spreading gain of 31, and \( E_b/N_0 = 8\text{dB} \). Here we show typical example of the variance in the amplitude estimate as well as the improvement possible by a simple averaging. The averaged estimator is a cumulative average over the entire bit sequence. That is the value at 5 bits represents the value of averaging over 5 bits, while the value at 15 bits is the value after averaging over 15 bits. The predicted bias at \( N_p = 50 \) is evident.

Another factor in the improvement of the amplitude estimation is that the estimator will improve at each stage of cancellation assuming that interference is reduced. That is, provided that the first stage of cancellation does not worsen the situation, the estimates at stage two will be improved over the initial estimates. This can be seen in Figure 7.2 where improved estimates are created by passing the signal after cancellation through a matched filter. The reduction in the mean square error (MSE) of the amplitude estimate is significant. Thus, at subsequent stages the cancellation will lead to improved estimation which will in turn improve the cancellation. Additionally, these estimates can also be averaged over time to reduce the variance. To examine both of these effects we ran simulations to determine an empirical measure of the accuracy of the amplitude estimate using averaging and after cancellation. Figure 7.3 shows the MSE of the amplitude estimate for both a lightly loaded system (\( K = 5 \)) and a heavily loaded system (\( K = 20 \)) as the number of bits over which the average is taken is increased. We can see that the majority of the gain is in the first few bits suggesting that perhaps averaging over a long time interval is an unnecessary complication. We can also see that the estimate after cancellation also is improved by the averaging process. The two effects together provide dramatic improvement of the amplitude estimate, even in a heavily loaded system.

The previous discussion focused on an AWGN channel. Extending this analysis to a flat Rayleigh fading channel, we find that the same approach can be used provided that the coherence time of the channel is considered. For instance let us consider a very slow
Figure 7.3: Mean Squared Error of Amplitude Estimate for AWGN Channel with 5 and 20 Users (spreading gain = 31, and $E_b/N_0 = 8$dB, Doppler Spread = 10Hz)
fading channel with a mobile unit speed of 10km/hr, a data rate of 10kpbs, and a transmit frequency of 2.2GHz with a corresponding Doppler spread of approximately 20Hz. This represents a slow fading channel with a correspondingly large coherence time. Thus we would expect to find that a long averaging time would be very useful. Figure 7.4 (top) shows a typical amplitude variation over 5ms (equal to 50 bits) for the given channel along with typical amplitude estimates both before (left) and after (right) cancellation. In this example there are 5 users and each has an average $E_b/N_o = 15dB$. As we can see, there is significant estimation error even after cancellation. It stands to reason that averaging would be beneficial for such a channel. Figure 7.4 (bottom) show typical results for the same channel when averaging is used over 10 bit periods. As anticipated, the variation is reduced considerably in both cases. In order to quantify this gain we ran several simulations to determine the MSE resulting from averaging over 1 to 20 bit periods. This is shown in Figure 7.5 for both the conventional correlator output and a stage of cancellation. From this figure, we can make two observations. First, as in the AWGN case, the majority of the reduction in MSE is obtained in the first few bits of averaging. This is coherent with theory since the variance decreases by $\frac{1}{N}$. The second observation is that, particularly in the 20 users case, the cancellation does significantly reduce the estimate variance for a given averaging interval. The first observation leads us to the conclusion that, again, a small number of bits would be the best choice for reduction in the variance while keeping the memory requirements low. $^1$

We found that averaging is beneficial for both AWGN and very slow, flat Rayleigh fading channels. We would like to examine the effects in fading channels with much shorter coherence times as well. Let us consider a faster$^2$ fading channel a mobile unit speed of 150km/hr, a data rate of 10kpbs and a transmit frequency of 2.2GHz with a corresponding Doppler spread of approximately 305Hz. This is an extremely large Doppler spread. A typical envelope is shown in Figure 7.6 (top) along with estimates corresponding to matched filter outputs when 5 users are in the system with a spreading gain of 31 and average $E_b/N_o = 15dB$. We also show the estimates after two stages of cancellation. Obviously, the envelope is varying much more rapidly than in the previous case. Also, the estimates result in significant error although the estimates after cancellation are less noisy. Typical envelopes and estimates when averaging is performed over two bit periods are shown in

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$^1$We need not keep all bits in memory. Instead the use of a feedback structure with a single feedback tap weighted by the desired memory factor (i.e. values closer to unity result in longer memory while values closer to zero will result in a shorter memory).

$^2$It should be noted that while we use the term 'faster' we are not implying that we are simulating a 'fast' fading channel in the normal use of the term. That is, the fading is still slow relative to a single chip interval, and is thus not technically 'fast'.
Figure 7.4: Typical Amplitude Estimate and Actual Amplitude when Using Single Bit Estimates and Averaging over 10 Bits in Slower Rayleigh Fading (5 users, spreading gain = 31, $E_b/N_0 = 15$dB, Doppler spread = 20Hz)
Figure 7.5: Mean Squared Error of Amplitude Estimate for Slower Fading Rayleigh Channel with 5 and 20 Users (spreading gain = 31, and $E_b/N_0 = 8$dB, Doppler Spread = 10Hz)
Figure 7.6 (middle). The averaging of only two bit periods shows significant improvement in the estimate, particularly after a stage of cancellation. However, due to the extremely fast variation in the channel we would not expect longer term averaging to be beneficial. Figure 7.6 (bottom) shows the result of averaging over 10 bit intervals. Obviously, this provides a low pass filtering effect on the amplitude estimate which prevents the estimate from adequately tracking the channel. A good approximation is to keep the averaging interval within the coherence time of the channel. While the coherence time is not a strict definition, we can approximate the coherence time by [106]

\[ T_C \approx \frac{9}{16\pi f_m} \]  \hspace{1cm} (7.4)

where \( f_m \) is the maximum Doppler frequency. Using this equation for the two channels under consideration, we arrive at coherence times of 9ms and 0.6ms for the slower and faster fading channels respectively. This corresponds to approximately 6 and 90 bit intervals. Thus, we would expect long term averaging (e.g. 10 bits) to be beneficial in the slower channel, but not in the faster fading channel. In the latter channel, 2-5 bits would probably be beneficial\(^3\). Since this is where the majority of the gain is anyways, we conclude that averaging is beneficial even in faster fading environments\(^4\).

Figure 7.7 shows the MSE resulting from averaging performed in the faster fading environment (\( f_m = 305Hz \)) in both lightly loaded and heavily loaded systems. As can be seen, the MSE is decreased by averaging over a small number of bits, but increases as the number of bits increases, as expected. We conclude that the optimal averaging time will be related to the coherence time of the channel. Again, we see that averaging combined with the cancellation can provide significant reduction in amplitude variance with a small increase in complexity.

As a final examination of amplitude estimation, we examine the performance of this simple estimator with the performance of the maximum likelihood linear estimator. If a training sequence is used (i.e. the bits are known) the maximum likelihood linear estimator is [87]

\[ \hat{A}_{ML} = (B'R^{-1}B)^{-1}(B'y) \]  \hspace{1cm} (7.5)

where \( B \) is a \( KN_b \) by \( KN_b \) matrix with the training sequence \( b \) along the diagonal, and \( R, b, \) and \( y \) are defined in Chapter 2. Like the ML linear symbol estimates, these estimates will be independent of the received energies of each user, but will suffer from enhanced noise due to the decorrelating effect.

\(^3\) Obviously this corresponds only to the data rate examined (10kbps). For slower data rates we may not be able to perform any averaging in a faster channel, while for higher data rates longer averaging may be possible.

\(^4\) Similar results were also noted in [98]
Figure 7.6: Typical Amplitude Estimate and Actual Amplitude when Using Single Bit Estimates and Averaging over 2 and 10 Bits in Faster Rayleigh Fading (5 users, spreading gain = 31, $E_b/N_0 = 15$dB, Doppler spread = 305Hz)
Figure 7.7: Mean Squared Error of Amplitude Estimate for Fast Fading Rayleigh Channel with 5 and 20 Users (spreading gain = 31, and $E_b/N_0 = 8$dB, Doppler Spread = 305Hz)
Figure 7.8 shows the performance of both estimators as the number of observations (i.e. averaging intervals) increases for a near-far situation. The desired user and a single interferer are at $E_b/N_0 = 8dB$ while a dominant interferer is at $E_b/N_0 = 18dB$. As we can see, the ML estimator is significantly more accurate than the simple correlation estimate for no averaging ($N_{bo} = 1$) and before cancellation. However, after two stages of cancellation, the correlation estimate shows performance which rivals the ML estimate. Also, as the number of intervals or observations increases, the difference between the estimates gets small. Thus, the simple correlator estimate performs well in the near-far environment after cancellation.

7.4 Phase Estimation

Traditional receivers, including CDMA receivers, must have knowledge of the phase and frequency of the signal which is being received for coherent reception. While the mobile channel does not usually lend itself to coherent demodulation due to the rapid phase variation, some researchers are suggesting coherent demodulation by using pilot symbols [83]. We wish to examine the error involved in estimating the phase/frequency of a desired signal. As stated previously, we do not simply wish to track the phase/frequency, rather we need to estimate it. This is because tracking typically involves closed loop solutions which match the incoming phase/frequency by eliminating the error between a locally generated version of the carrier and the incoming signal. However, if we are to remove the effects of one signal from another, we cannot track them independently. That is if signal A is tracked by its tracking loop and signal B by its tracking loop, the effect of signal A on B after downconversion can not be predicted based on signal A without knowing the relative phase/frequency between the two tracking loops. Since the loops are closed, this information will be difficult to obtain. Additionally, we wish to track several users and would thus require the relative phase and frequency between each possible combination. A more amenable solution is to mix the entire received signal down by tracking a single strong user and then estimating all users' phase and frequency relative to the arbitrary reference. We may then use these estimates to remove the effect of all interfering signals on the desired signal. This discussion will be expanded in Chapter 10. For now we wish to examine the accuracy of possible phase estimation procedures.

The output of the in-phase channel correlator for user $k$ after symbol epoch $i$ can be represented as

$$Z_{ik} = \int_{(i-1)T_b + \tau_k}^{iT_b + \tau_k} r_i(t)a_k(t)dt$$  \hspace{1cm} (7.6)
Figure 7.8: Mean Squared Error of Amplitude Estimate for Correlation Estimate and Maximum Likelihood Estimate in Near/Far Environment (3 users, spreading gain = 31, and $E_b/N_0 = [8\mathrm{dB}, 8\mathrm{dB}, 18\mathrm{dB}]$)
where $r_i(t)$ is the received signal in the in-phase channel, and we have not used any phase information. From the definition of $r(t)$ we observe that $\mathbb{E}[Z_{I_{k,i}}] = \sqrt{2P_kT}b_{k,i}\cos(\theta_k)$ where $\theta_k$ is the phase offset of the desired signal from the locally generated mixer.  \footnote{It should be noted that equation 7.1 assumes the use of a tracking loop such that the signal is mixed down with the correct phase offset included. What is being suggested here is that the mixing process instead will use a regenerated carrier which is intentionally of a different phase and possibly frequency. This will be taken care of by correlating in both in-phase and quadrature arms and combining the two outputs using phase estimates.} Defining $Z_{Q_{k,i}}$ similarly we can then estimate the phase by

$$\hat{\theta}_k = \arctan\left(\frac{Z_{Q_{k,i}}}{Z_{I_{k,i}}}\right). \quad (7.7)$$

and the expected value of $\hat{\theta}_k$ is

$$\mathbb{E}\left[\hat{\theta}\right] = \arctan\left(\frac{\sqrt{2P_kT}b_{k,i}\sin(\theta_k)}{\sqrt{2P_kT}b_{k,i}\sin(\theta_k)}\right) \quad (7.8)$$

$$= \arctan\left(\frac{\sin(\theta_k)}{\cos(\theta_k)}\right) \quad (7.9)$$

which would seem to provide us with our desired result. However, in the result there exists an ambiguity of $\pi$. That is, if $Z_I < 0$ we do not know if that corresponds to $\cos(\theta_k) < 0$ and $b_{k,i} = -1$ or $\cos(\theta) > 0$ $b_{k,i} = 1$. The same can be said for $Z_Q$. Thus, in order to properly estimate the phase, we require additional information. The most straightforward method of obtaining additional information is to periodically send pilot symbols. These symbols would provide bit (or symbol) information which would allow the quadrant ambiguity to be removed. Unfortunately, we have seen that the correlator outputs are extremely noisy especially in heavily loaded systems. These noisy outputs will result in a phase estimate which is noisier still due to the inherent division which must be performed followed by the arctan operation. Thus, even more than for the case of amplitude estimates, we require several consecutive estimates to average out the effects of noise and interference. As an example consider the constellation plots of Figure 7.9. The top plot shows the phase estimate relative to the actual phase for a single bit estimate. That is, the location away from the point [-1,0] or [1,0] represents the error in the phase estimate. Obviously, the phase estimates using a single bit estimate are extremely noisy even after cancellation. The bottom figure shows phase estimation error when estimation is averaged over 10 bits. That is the in-phase and quadrature correlations are each averaged after first eliminating the bit information. Formally,

$$\hat{\theta}_{b_{k,i}} = \arctan\left(\frac{\sum_{i=1}^{N_b}Z_{Q_{k,i}}b_{k,i}}{\sum_{i=1}^{N_b}Z_{I_{k,i}}b_{k,i}}\right) \quad (7.11)$$
We can see that this averaging reduces the noise in the estimates significantly, particularly for the second stage estimates. Of course, the pilot symbols add overhead to the system and reduce throughput, especially when the channel is changing rapidly. Thus, in some instances this may not be practical and non-coherent demodulation will be required.

To quantify the improvement we ran simulations to calculate the MSE in the phase estimate. Figure 7.10 shows the MSE in the phase estimate as the system capacity increases for an AWGN channel, spreading gain of 31 and estimation occurring over 10 bits. The phase is regenerated after each stage of cancellation to obtain improved estimates as was done in the case of amplitude estimation. Figure 7.11 shows the variation in MSE as the number of training bits is varied. As we have found in the case of amplitude estimation, the combination of averaging and cancellation allows for significant reduction in the estimate MSE. Figure 7.11 suggests that using more than 10 bits would not provide further reduction in MSE. In an environment where the channel is changing rapidly, this overhead may prove to be impractical. For example, consider a Doppler spread of 300 Hz with a corresponding coherence time of 0.6 ms. This would correspond to approximately 6 bits for a 10 kbps data rate, which is obviously too short to allow phase estimation. For a more amenable delay spread of 20 Hz or corresponding coherence time of 9 ms, we would require an overhead of 11% to continuously update the phase estimate. This too may be unreasonable. If so, differentially coherent demodulation may be desirable. The trade-offs between the two approaches will be more thoroughly discussed in chapter 10.

7.5 Timing Estimation

The third estimation required for interference cancellation (and any receiver structure) is timing. While traditional systems require accurate knowledge of the timing of the incoming symbol stream for proper demodulation, CDMA receivers have the additional requirement of chip timing. While the two are very similar, the latter requires significantly more accuracy. Additionally, interference cancellation receivers require even more accuracy as will be shown in the next chapter. Since multiuser receivers propose to increase the capacity of CDMA systems, they must allow for improved signal acquisition capabilities as well. Ultimately, a receiver is limited in its capacity by the amount of interference which can be suffered and still allow proper acquisition and tracking of the signal of interest [13]. We must show then that multistage receivers can provide significant improvements to traditional synchronization techniques. In this section we examine ways of improving synchronization.

To keep the discussion general we use simplistic analysis with performance metrics being probability of false alarm (the probability that we lock onto the wrong code phase) and the
One Bit Estimation

Ten Bit Estimation

Figure 7.9: Typical Phase Estimate Relative to Actual Phase in Stage 1 and Stage 2 Using Single Bit and 10 Bit Estimates (10 users, spreading gain = 31, and $E_b/N_o = 8$dB)
Figure 7.10: Mean Squared Error of Phase Estimate in AWGN Channel (spreading gain = 31, 10 Training Bits, and $E_b/N_0 = 8$dB)
Figure 7.11: Mean Squared Error of Phase Estimate in AWGN Channel vs. the Number of Training Bits Used (spreading gain = 31, 10 users and $E_b/N_0 = 8$dB)
mean time to acquisition which is measured in units of dwell times. We use a simple matched filter acquisition technique. That is we pass the incoming signal through a matched filter and wait for an output which crosses a specified threshold. Once this occurs, we assume synchronization has been obtained. A typical matched filter output for an AWGN channel with 10 users and $E_b/N_o = 8$ dB is shown in Figure 7.12 (top, left) where we have normalized to $N \times N_s$, the number of samples per bit. Additionally, we have currently assumed that phase tracking is performed separately and prior to code acquisition.

One method of improving the tracking in a multistage cancellation routine is to synchronize using the residual signal. That is, assuming the system is in a state of equilibrium, a user will come onto the system where all other users are being tracked and canceled from the received signal. If this is the case, the signal which remains after all users have been cancelled will contain noise, residual interference and the user which has not been acquired. This signal will likely be much more amenable to synchronization. A matched filter output after cancellation of interferers is shown in Figure 7.12 (top, right). While the signal is not tremendously improved, there is a discernible difference.

A second simple method is similar to the ideas discussed previously. If we employ a single tap IIR filter with a delay of $N$ samples, we can sum consecutive peaks resulting in a higher peak-to-sidelobe ratio provided that (1) the peak doesn’t shift over the time considered, and (2) the sidelobes tend to have large values at different places each bit interval. The filter described is presented in Figure 7.13 (top) where $\frac{1}{1-b_0}$ represents the memory of the filter. A typical output for 10 users, a spreading gain of 31, and $E_b/N_o$ is shown in Figure 7.12 (bottom, left) where $b_0 = 0.9$ and the output is normalized be $\frac{N_s}{1-b_0}$. We can see that the filter output enhances the peaks significantly. This filter can also be used with the residual signal as described above. Doing so results in the output shown in Figure 7.12 (bottom, right) where we have used the same system parameters as in the previous case. Comparing the four filter outputs in Figure 7.12 we can see the large benefit of using such a simple scheme by observing the distinctness of the correlation peaks. The use of such a filter, however, does assume that the channel remains relatively static over the integration time which is $\frac{1}{1-b_0}$ bit intervals. For the filter considered previously, this would correspond to 10 bits. For a data rate of 10kbps this corresponds to a coherence time of 1ms. Using equation (7.4) we would be limited to a Doppler spread of below 180Hz. Thus for all but extremely fast fading channels, such a technique would provide a simple and effective means of improving synchronization.

As in the previous analysis, we would like to quantify the improvement possible from this filter structure. Here we undertake a simple analysis with the main objective being to
Figure 7.12: Typical Correlator Outputs for Matched Filter and Enhanced IIR Filter in Stage 1 and Stage 2 (10 users, spreading gain = 31, and $E_b/N_o = 8$dB)
Figure 7.13: Synchronization Loops Assumed for Timing Analysis For Coherent Acquisition (top) and Non-coherent Acquisition (bottom)
show the relative gains when compared to matched filter acquisition. Strategies which can be applied to the matched filter technique to improve its performance, should also apply to the IIR structure as well. The two metrics investigated for synchronization improvement are probability of false alarm and mean time to acquisition. The first measure simply presents the probability that the incorrect code phase is chosen. More explicitly we choose a threshold which when crossed, indicates that the correct code phase has been found. Since the delays are known, we can compare this estimate with the actual delay and determine if an error has occurred. By repeating the process we can approximate the probability of false alarm. Additionally, by keeping track of the number of code phases which must be cycled through before acquisition is achieved, we can approximate the mean acquisition time in units of dwell times. Footnote Dwell times are an unspecified unit of time which corresponds to the integration over one code phase. The probability of false alarm for both the matched filter and the IIR structure are given in Figure 7.14 when the signal used to acquire is either the received signal (stage 1) or the residual signal (stage 2). The system capacity is increased from 5 to 35 users in an AWGN channel with spreading gain of 31 and $E_b/N_0 = 8$ dB. The threshold used is 0.85 assuming that the matched filter output is normalized to $N \cdot N_s$ and the IIR filter output is normalized to $NN_s/(1 - b_0)$. The probability of false alarm is reduced dramatically when considering both the use of the IIR filter structure as well as the residual interference signal. We see a similar reduction in the mean acquisition time shown in Figure 7.15 where we have assumed identical system conditions. Additionally, we assume a penalty time of 5 dwell times. The penalty time is used to represent the additional time required to synchronize when a false peak is detected. This means that on average it will take 5 dwell times to realize we have made a mistake in acquisition.

Since the threshold considered can make a large difference on our statistics, we would like to see the effect which the threshold has on the probability of false alarm and the mean acquisition time. Obviously, as the threshold increases, the probability of false alarm decreases. However, the mean acquisition time will be long for both small and large thresholds. This is because if the threshold is too low, the false alarms and consequent penalty times will drive the acquisition up. However, for large thresholds the acquisition time will also grow large, thus resulting in an optimum threshold level when considering acquisition time. Figures 7.16 and 7.17 show the variance with threshold for the probability of false alarm and acquisition time respectively for the case of 15 users, $E_b/N_0 = 8$ dB, spreading gain of 31 and penalty time of 5 dwell times. From these graphs we can see that for 15 users, a threshold of 0.85 may be satisfactory for the IIR filter structure, but the matched filter would work better with a higher threshold of say 1.2. However, the IIR structure still
Figure 7.14: Probability of False Alarm vs. System Capacity for Conventional Matched Filter and FIR Filter (spreading gain = 31, $E_b/N_0 = 8$ dB, threshold = 0.85)
Figure 7.15: Mean Acquisition Time vs. System Capacity for Conventional Matched Filter and IIR Filter (spreading gain = 31, $E_b/N_0 = 8$dB, Threshold = 0.85, Penalty=5 dwell times)
provides sufficient gains over the matched filter.

Finally, we would like to consider the effect of the coherent assumption. That is, we have until this point assumed that code acquisition would take place after carrier phase acquisition, allowing coherent reception while synchronizing. However, in a high interference situation this may not be possible, thus we would like to examine the effect of non-coherent synchronization using both the matched filter and the IIR structure as shown in Figure 7.13 (bottom). The resulting probability of false alarm and mean acquisition time are shown in Figures 7.18 and 7.19 respectively. From these figures we find that the matched filter is not affected by the change, however the IIR structure is. When using the received signal as input to the IIR structure, the probability of false alarm and mean acquisition time
Figure 7.17: Mean Acquisition Time vs. Threshold for Conventional Matched Filter and IIR Filter (spreading gain = 31, $E_b/N_0 = 8$dB, 15 users, Penalty=5 dwell times)
are increased dramatically. This is not the case when the residual signal is used. Thus, removing the coherent assumption does affect the gains achieved by the IIR filter without using cancellation. However, the gains are still valid when using the residual signal for synchronization.

### 7.6 Conclusion

In this section we have examined means of estimating the amplitude, phase, and timing of a signal of interest as well show the improvement possible by using simple averaging techniques. We also show the improvement gained from one stage to the next by re-estimating...
Figure 7.19: Mean Acquisition Time vs. System Capacity for Non-coherent Conventional Matched Filter and IIR Filter (spreading gain = 31, $E_b/N_0 = 8$dB, Threshold = 1.0, Penalty=5 dwell times)
after interference has been removed. We found that significant reductions in parameter estimation error were possible. In the next chapter we examine the effects that these errors have on the receiver performance.
Chapter 8

Effect of Parameter Estimation Error on System Performance

8.1 Introduction

In the previous chapter we discussed the importance of parameter estimation in interference cancellation. Particularly, we examined the difficulty associated with the estimation of the signal amplitude, phase/frequency and timing in a multiuser environment. In this chapter we will focus on the effect that estimation errors have on the receiver performance, specifically, the bit error rate of the receiver. In the first section, we concentrate on performance degradation due to errors in amplitude estimation. Theoretical results are compared with simulation results. Of more importance to receiver performance is the effect of phase and timing errors. These effects are studied in more detail in sections 2 and 4. Section 3 examines the comparison of differentially demodulated PSK with phase estimation. Finally we present conclusions in section 5.

8.2 Amplitude Errors

In the previous chapter we examined the the accuracy of simple amplitude estimation methods and found that we can significantly reduce the amplitude estimation error through the use of simple averaging techniques along with cancellation. Now we would like to examine the effect which these errors have on the receiver performance.

To examine the effect of amplitude estimate variation consider equation 5.38 where we define the residual interference power $\nu$. That is we define

$$\sqrt{2\nu_k,i} = \sqrt{2P_kb_{k,i}} - \frac{2}{T} \frac{\hat{A}_k Z_{k,i}}{|Z_{k,i}|}$$

(8.1)
where $\hat{A}$ is the estimate of the received amplitude which was defined previously. If we assume that $\hat{A}$ is an unbiased estimate of the amplitude (we will address this assumption more later), we can we can find the variance of the residual interference (i.e. the expected interference power) as

$$
E \left[ (\sqrt{2\nu_{k,i}})^2 \middle| b_{k,i} \right] = E \left[ \left( \sqrt{2P_k} b_{k,i} - \hat{A}_{k,i} \frac{Z_{k,i}}{|Z_{k,i}|} \right)^2 \middle| b_{k,i} \right] \\
= E \left[ 2P_k - 2\sqrt{2P_k} b_{k,i} \hat{A}_{k,i} \frac{Z_{k,i}}{|Z_{k,i}|} + \hat{A}_{k,i}^2 \middle| b_{k,i} \right] \\
= 2P_k - 2\sqrt{2P_k} b_{k,i} E \left[ \hat{A}_{k,i} \frac{Z_{k,i}}{|Z_{k,i}|} \middle| b_{k,i} \right] + E \left[ \hat{A}_{k,i}^2 \middle| b_{k,i} \right] 
$$

(8.2)

If we can assume an unbiased estimator, then $E \left[ \hat{A}_{k,i} \middle| b_{k,i} \right] = \sqrt{2P_k}$ and $E \left[ \hat{A}_{k,i}^2 \middle| b_{k,i} \right] = \text{var} \left( \hat{A}_{k,i} \right) + 2P_k$. Additionally assuming independence between the amplitude estimate and the data estimate, with $E \left[ \frac{Z_{k,i}}{\hat{A}_{k,i}^2} \middle| b_{k,i} \right] = b_{k,i}(1 - P_e)$ where $P_e$ is the probability of symbol error, it can be shown that

$$
E [2\nu_{k,i} \middle| b_{k,i}] = \text{var} \left( \hat{A}_{k,i} \right) + 8P_k P_e. 
$$

(8.3)

Using this equation along with equations 5.27 and 5.39 results in an expression for the variance of the decision statistic at stage $s$ as

$$
\text{var} \left[ Z_{k,i}^{(s)} \right] = \frac{N_0 T}{T} + \frac{(K - 1)T^2}{12N} \left( \text{var} \left( \hat{A}_{k,i}^{(s-1)} \right) + 8P_k P_e^{(s-1)} \right) 
$$

(8.4)

where $P_e^{(s-1)}$ and $\hat{A}_{k,i}^{(s-1)}$ are the probability of symbol error and amplitude estimate at stage $s$ respectively, and we have assume that we have perfect power control. We must now address the assumptions used to arrive at equation (8.4). First, we assume that the amplitude estimate is unbiased. However, we showed in the last chapter that using the absolute value of the decision statistic does introduce a bias of $\sqrt{2\alpha^2 \pi e^{-\mu^2/2\alpha^2}}$ where $\mu$ is the expected value of $Z$ or $\mu = \sqrt{2P_k T b_{k,i}}$ and $\sigma$ is the variance of $Z$. For large values of SINR, the bias is negligible. If bit averaging is assumed, we must also include this bias, since each term in the averaging will contain roughly the same bias.

The second assumption was that the amplitude and bit estimate are independent which lead to $E \left[ \hat{A}_{k,i} \frac{Z_{k,i}}{|Z_{k,i}|} \middle| b_{k,i} \right] = \sqrt{2P_k} b_{k,i}(1 - 2P_e)$. However, this assumption is not true since we use the correlator outputs to estimate each. It is shown in Appendix E that if we use equation (7.3) as the amplitude estimate

$$
E \left[ \hat{A}_{k,i} \frac{Z_{k,i}}{|Z_{k,i}|} \middle| b_{k,i} \right] = \sqrt{2P_k} b_{k,i} \left( \frac{1}{N_a} + (1 - 2P_e) \frac{N_a - 1}{N_a} \right) + \frac{N_a - 1}{N_a} (1 - 2P_e) b_{k,i} \sqrt{\frac{2\alpha^2}{\pi} e^{-\frac{\mu^2}{2\alpha^2}}}. 
$$

(8.5)
For $N_o >> 1$ and $SINR >> 1$, the assumption of independence does not significantly affect the results.

Figure 8.1 shows system performance in a heavily loaded system with an AWGN channel and $E_b/N_o = 8$dB and spreading gain = 31. We can see that the simulation results match favorably with the analytical results and that there is certainly degradation as the variance increases. Figure 8.2 presents the performance when perfect amplitude estimates are used as opposed to single bit time correlation estimates. We find that there is a small gain for perfect amplitude estimates at stage 2 and that the gain is reduced at stage 3. In this situation, the amplitude estimates are relatively good due to the moderate loading and perfect power control. Additionally, after a stage of cancellation, the improvement in the amplitude estimate results in performance that is nearly equal to the case of perfect amplitude estimation.

### 8.3 Performance Analysis with Phase Errors

In any practical system, there also exists the possibility of phase tracking errors. We examine coherent reception in order to get rough estimates of the importance of phase tracking. Restating equation (3.4) to account for phase errors results in the decision statistic for stage 1

$$
Z^{(1)}_{k,i} = \int_{iT+\tau_k}^{(i+1)T+\tau_k} r(t) u_k(t - \tau_k) \cos(\omega_c t + \phi_k + \psi) dt, \quad (8.6)
$$

where $\psi$ is the phase tracking error. As in the timing error case, we assume that all users undergo the same phase error. Simplifying this as before results in

$$
Z^{(1)}_{k,i} = \eta + \sqrt{\frac{P_k}{2}} b_{k,i} T \cos(\psi) + \sum_{j=1 \atop j \neq k}^K I_{j,k}. \quad (8.7)
$$

Again we can represent this value with a Gaussian random variable with mean $E \left[ Z^{(1)}_{k,i} \mid \{ P_k \} \right] = \sqrt{\frac{P_k}{2}} b_{k,i} T \cos(\psi)$ and variance given by (8.31). Extending this procedure to stage $s$, we obtain

$$
Z^{(s)}_{k,i} = \int_{iT+\tau_k}^{(i+1)T+\tau_k} \hat{r}^{(s)}_k(t - \tau_k) \cos(\omega_c t + \phi_k + \psi) dt, \quad (8.8)
$$

where

$$
\hat{r}^{(s)}_k(t) = r(t) - \sum_{j=1 \atop j \neq k}^K \frac{2Z^{(s-1)}_{j,i}}{T} a_j(t - \tau_j) \cos(\omega_c t + \phi_j + \psi). \quad (8.9)
$$
Figure 8.1: Effect of Amplitude Error Variance on System Performance in Heavily loaded System (spreading gain = 31, $E_b/N_0 = 8$dB, 20 users)
Figure 8.2: Performance vs. Signal Power for Perfect and Imperfect Amplitude Estimation in Moderately Loaded System (spreading gain = 31, 10 users)
This can be simplified as
\[
Z_{k,i}^{(s)} = \eta + \sqrt{\frac{P_k}{2}} b_{k,i} T \cos(\psi) + \sum_{j=1 \atop j \neq k}^{K} I_{j,k},
\] (8.10)

where
\[
I_{j,k} = \int_{i \tau_k}^{(i+1)T+\tau_k} a_k(t - \tau_k) \cos(\omega_c t + \phi_k + \psi) (s_j(t - \tau_j) - \hat{s}_j(t - \tau_j)) dt.
\] (8.11)

Examining the third term, we can expand the estimation error as
\[
s_j(t) - \hat{s}_j(t) = \sqrt{2P} a_j(t - \tau_j) b_j(t - \tau_j) \cos(\omega_c t + \phi_j) - \frac{2Z_j^{(s-1)}}{T} a_j(t - \tau_j) \cos(\omega_c t + \phi_j + \psi).
\] (8.12)

This expression can then be broken into in-phase and quadrature components with respect to the desired user. Assuming all phases are relative to the desired user ($\phi_k = 0$), we obtain
\[
s_j(t) - \hat{s}_j(t) = \sqrt{2P} a_j(t - \tau_j) b_j(t - \tau_j) [\cos(\phi_j) \cos(\omega_c t) - \sin(\phi_j) \sin(\omega_c t)]
\]
\[
- \frac{2Z_j^{(s-1)}}{T} a_j(t - \tau_j) [\cos(\phi_j + \psi) \cos(\omega_c t) - \sin(\phi_j + \psi) \sin(\omega_c t)].
\] (8.13)

Rearranging yields
\[
s_j(t) - \hat{s}_j(t) = \left[ \sqrt{2P} b_j(t - \tau_j) \cos(\phi_j) - \frac{2Z_j^{(s-1)}}{T} \cos(\phi_j + \psi) \right] a_k(t - \tau_j) \cos(\omega_c t)
\]
\[
- \left[ \sqrt{2P} b_j(t - \tau_j) \sin(\phi_j) - \frac{2Z_j^{(s-1)}}{T} \sin(\phi_j + \psi) \right] a_k(t - \tau_j) \sin(\omega_c t).
\] (8.14)

Only the first term will be in-phase with the desired signal, allowing (8.11) to be simplified to
\[
\hat{I}_{j,k}^{(s)} = \frac{\sqrt{2P} \cos(\phi_j)}{2} \int_{i \tau_k}^{(i+1)T+\tau_k} b_j(t - \tau_j) a_j(t - \tau_j) a_k(t - \tau_k) dt
\]
\[
- \frac{2Z_j^{(s-1)}}{T} \cos(\phi_j + \psi) \int_{i \tau_k}^{(i+1)T+\tau_k} a_j(t - \tau_j) a_k(t - \tau_k) dt
\]
\[
= \frac{\sqrt{2P} \cos(\phi_j)}{2} T \cdot \hat{W}_j - \frac{Z_j^{(s-1)}}{T} \cos(\phi_j + \psi) T \cdot \hat{V}_j,
\] (8.15)

where $\hat{V}_j$ is defined similarly to $\hat{W}_j$ with $E[\hat{V}_j] = 0$ and $E[\hat{V}_j^2] = \frac{2N}{3}$. Returning to (8.10), we can see that
\[
E \left[ Z_{k,i}^{(s)} | \{ P_k \} \right] = \sqrt{\frac{P_k}{2} T b_{k,i} \cos(\psi)}.
\] (8.16)
Since \( \text{E}[\hat{I}_{j,k}^{(s)}] = 0 \),

\[
\text{var} \left( \hat{I}_{j,k}^{(s)} \right| \{ P_k \} \right) = \text{E} \left[ \left( \hat{I}_{j,k}^{(s)} \right)^2 \right] \\
= \text{E} \left[ \frac{P_j}{2} \cos^2(\phi_j) \cos(\phi_j + \psi) W_j + 2 \sqrt{\frac{P_j}{2}} \cos(\phi_j) \frac{Z_{j,i}^{(s-1)}}{T} \cos(\phi_j + \psi) W_j V_j \right] \\
+ \frac{(Z_{j,i}^{(s-1)})^2}{T^2} \cos^2(\phi_j + \psi) V_j^2 ,
\]

(8.17)

which simplifies to

\[
\text{var} \left( \hat{I}_{j,k}^{(s)} \right| \{ P_k \} \right) = \frac{NT^2 P_j}{6}(1 - \cos^2(\psi)) + \frac{1}{3N} \text{var} \left( Z_{j,i}^{(s-1)} \right) .
\]

(8.18)

Substituting this into (8.38) results in

\[
\text{var} \left( Z_{k,i}^{(s)} \right) = \frac{N_0 T}{4} + \sum_{j=1, j \neq k}^K \left[ \frac{NT^2 P_j}{6}(1 - \cos^2(\psi)) + \frac{1}{3N} \text{var} \left( Z_{j,i}^{(s-1)} \right) \right] .
\]

(8.19)

It is shown in Appendix B that solving this iterative equation results in

\[
\text{var} \left( Z_{k,i}^{(s)} \right) = \frac{N_0 T}{4} \left[ 1 - \left( \frac{K-1}{3N} \right)^s \right] + \frac{T^2(1 - \cos^2(\psi))}{2K} \left[ \frac{1 - \left( \frac{K-1}{3N} \right)^s}{1 - \left( \frac{1}{3N} \right)^s} \sum_{j=1}^K P_j - \frac{1 - \left( \frac{1}{3N} \right)^s}{1 + \left( \frac{1}{3N} \right)^s} \right] \\
\frac{T^2(1 - \cos^2(\psi))}{2} P_k \left[ \frac{3N + \left( \frac{1}{3N} \right)^s}{1 + \left( \frac{1}{3N} \right)^s} \right] + \frac{T^2}{2(3N)^s} \left[ \frac{(K-1)^s - (-1)^s}{K} \sum_{j=1}^K P_j + (-1)^s P_k \right] .
\]

(8.20)

An expression for probability of bit error may be obtained by substituting (8.16) and (8.20) into (8.43). Furthermore, the limit of BER as \( s \to \infty \) is found to be

\[
P_k^{(\infty)} = Q \left\{ \frac{1}{2 \frac{K}{N_o} \cos^2(\psi)} \left[ \frac{1}{1 - \left( \frac{K-1}{3N} \right)^s} \right] \\
+ \frac{1 - \cos^2(\psi)}{\cos^2(\psi)} \left[ \frac{1}{1 - \left( \frac{K-1}{3N} \right)^s} - \frac{1}{1 + \left( \frac{1}{3N} \right)^s} \right] \frac{\sum_{j=1}^K P_j}{KP_k} - \frac{1 - \cos^2(\psi)}{\cos^2(\psi)} \left[ \frac{1}{1 + \left( \frac{1}{3N} \right)^s} \right] \right\}^{-\frac{1}{2}}
\]

(8.21)

where it has been assumed that \( K - 1 < 3N \).

The results of phase errors are shown in Figures 8.3 through 8.5. A comparison of analytic curves based on the results of section 4 with simulation results is given in Figure 8.3. Simulation results are again very consistent with the analytic curves. Figure 8.4 shows the effect on capacity for \( \frac{K}{N_o} = 8 \)dB and \( N = 31 \). As in the case of timing errors, we find that while phase errors degrade performance as well as reduce performance gains, the multistage
approach still outperforms the conventional receiver. In fact, phase errors seem to be less critical than timing errors. Figure 8.5 shows the effect phase errors have on required $\frac{E_b}{N_0}$ for $K = 10$ and $N = 31$. Again, we find that phase errors do not eliminate performance gains. It should be noted that in many systems non-coherent reception would be employed, while we have assumed coherent reception.
8.4 Phase Estimation vs. Differential Demodulation

As discussed in Chapter 9, the use of pilot symbols for phase estimation may be cumbersome in a fast fading channel. Thus, we would like to examine the performance of non-coherent methods of cancellation to determine if phase estimation can be avoided entirely. In Chapter 4 we showed that non-coherent multistage cancellation was possible and resulted in significant improvement over the conventional differential receiver. However, it was also explained that due to the fact that the cancellation variables include both the effects of amplitude and phase, they would be more noisy than those used in the coherent situation. Thus, the improvement over the conventional receiver was not nearly as great as in the coherent case. However, we would like to determine if the performance is similar coherent reception with phase estimation. If the performance of the two situations are comparable, it would be advisable to utilize the non-coherent scheme and avoid the necessity of phase estimates entirely.

Figure 8.6 shows the performance of each technique in an AWGN channel as the capacity in the system increases for the case of $E_b/N_a = 8$ dB, spreading gain =31, and 10 pilot symbols are used. We find that not only does the phase estimation receiver outperform the non-coherent receiver, but it performs as well as the perfect phase estimation receiver. To verify this result we examine the phase estimation error for $N_a = 10$ in Figure 7.11. We see that for 10 users and $N_a = 10$, the mean square estimation error is approximately $0.016 \text{rad}^2$. This corresponds approximately to a phase error of less than 8 degrees. Referring to Figure 8.3 we see that for this amount of phase error there is virtually no degradation in receiver performance. However, the differentially coherent receiver, while providing gains over the conventional differentially coherent conventional receiver, obtains a BER which is significantly worse than the coherent receiver. This suggests that phase estimation is preferable assuming that the coherence time is such that it allows phase estimation.

8.5 Performance Analysis with Timing Errors

In a practical system perfect chip timing may not be possible. In this section we will examine the effects of chip misalignment. Timing misalignment affects interference cancellation in two ways. First, correlation with the desired user's spreading code is imperfect, resulting in reduced received signal power. Second, the cancellation will be imperfect due to the time offset between the actual interfering signal and the estimated interfering signal. To show the effect that these will have on the receiver performance we will begin by examining the decision statistic from the first stage. Adjusting the decision statistic of (3.4) to account
Figure 8.3: Degradation Caused by Phase Errors ($E_b/N_o = 8$dB, Users=10, Spreading Gain=31) ['x' = simulated].
Figure 8.4: Capacity Curves for Perfect Phase Synchronization and Phase Error = 20 deg ($E_b/N_o = 8$dB, Spreading Gain=31).
Figure 8.5: Bit Error Rate vs. $\frac{E_b}{N_0}$ for Perfect Phase Synchronization and Phase Error = 20 deg ($E_b/N_0 = 8$ dB, Spreading Gain = 31).
Figure 8.6: Performance Comparison Between Differentially Coherent Reception and Coherent Reception with Phase Estimation in AWGN ($E_b/N_0 = 8$dB, Spreading Gain=31, Pilot Symbols = 10).
for timing misalignment results in
\[ Z_{k,i}^{(1)} = \int_{(i+1)T + \tau_k - \epsilon}^{iT + \tau_k - \epsilon} r(t) a_k(t - \tau_k + \epsilon) \cos(\omega_c t + \phi_k) dt \] (8.22)
where \( 0 \leq \epsilon < T_c \) represents the timing error associated with the locally generated version of the desired user's spreading code. In order to ease the analysis it is assumed that each user is subject to the same timing error. Although modeling timing errors as independently distributed random variables [15] may more accurately reflect the underlying situation, fixed timing errors lead to a tractable analysis which accurately reflects the qualitative effects of the timing errors. Simplifying (8.22) using the notation of [105] leads to the expression
\[ Z_{k,i}^{(1)} = \eta + \frac{P_k}{2} b_{k,i} (NT_c - (2B + 1 - Q_k)\epsilon) + \sum_{i=1, j \neq k}^{K} I_{j,k} \] (8.23)
where \( \eta \) is a Gaussian random variable with zero mean and variance \( \frac{N_0 T}{4} \), \( B \) is a binomial random variable with mean \( \frac{N-1}{2} \) representing the number of chip alternations in the desired user's spreading code, \( Q_k \) is a binary random variable with values \{\pm 1\} occurring with equal probability, \( I_{j,k} = \frac{P_k}{2} T_c \cos(\phi_j - \phi_k) W_j \), and \( W_j \) is a random variable with zero mean and variance \( \frac{2N}{3} \). The first term represents the contribution to the decision statistic from thermal noise, the second term represents the imperfect correlation between the desired user's received signal and the desired user's spreading code, and the third term represents the effect of multiple access interference due to the non-orthogonality of the simultaneous users. Using the standard Gaussian approximation, we can model this decision statistic as a Gaussian random variable with mean
\[ \mathbb{E}[Z_{k,i}^{(1)} | P_k] = \frac{P_k}{2} b_{k,i} N(T_c - \epsilon) \] (8.24)
and variance
\[ \text{var} \left( Z_{k,i}^{(1)} | \{ P_k \} \right) = \frac{N_0 T}{4} + \mathbb{E} \left[ \left( \sum_{i=1, j \neq k}^{K} I_{j,k} \right)^2 | \{ P_k \} \right] \] (8.25)
where we have assumed that \( B \) and \( Q_k \) take on their expected values of \( \frac{N-1}{2} \) and 0 respectively. If we make the assumption that the terms \( I_{j,k} \) are independent \(^1\) then
\[ \mathbb{E} \left[ \left( \sum_{i=1, j \neq k}^{K} I_{j,k} \right)^2 \right] = \sum_{i=1, j \neq k}^{K} \mathbb{E}[I_{j,k}]^2. \] (8.26)
\(^1\)It is shown in [88] that in fact \( \{ I_{j,k} \} \) are not independent, but for a large number of users \( K \) the effect on the results is insignificant.
From (8.22) we can write \( I_{j,k} \) as
\[
I_{j,k} = \int_{iT + \tau_k - \epsilon}^{(i+1)T + \tau_k - \epsilon} \sqrt{2P_j a_j(t - \tau_j)b_j(t - \tau_j) \cos(\omega_c t + \phi_j)a_k(t - \tau_k + \epsilon) \cos(\omega_c t + \phi_k)} dt. \quad (8.27)
\]
Simplifying this expression, we find each term in (8.26) has conditional variance
\[
E \left[ I_{j,k}^2 | P_j \right] = E \left[ \frac{P_j}{2} T_c^2 \cos^2(\phi_j - \phi_k) W_j^2 | P_j \right], \quad (8.28)
\]
where from [74]
\[
W_j = X_j + \left(1 - \frac{2\delta_j}{T_c}\right) Y_j + \left(1 - \frac{\delta_j}{T_c}\right) U_j + \frac{\delta_j}{T_c} V_j, \quad (8.29)
\]
\(X_j\) and \(Y_j\) are binomial random variables with mean \(\frac{N-1}{2}\), \(U_j\) and \(V_j\) are binary random variables equally distributed on \(\{\pm 1\}\), and \(\delta_j\) is a random variable representing the time offset of the \(j\)th user with respect to the desired user's locally generated spreading code and is uniformly distributed on \([0, T_c]\). It can be shown that \(X_j, Y_j, U_j,\) and \(V_j\) are uncorrelated leading to \(E[W_j^2] = \frac{2N}{3}\) [88]. Using this we can show that
\[
E \left[ I_{j,k}^2 | P_j \right] = \frac{P_j N T_c^2}{6}. \quad (8.30)
\]
Thus, we determine the variance of the decision statistic at stage 1 to be
\[
\text{var} \left( Z_{k,i}^{(1)} | \{ P_k \} \right) = \frac{N_0 T}{4} + \frac{N T_c^2}{6} \sum_{j=1}^{K} P_j. \quad (8.31)
\]
Extending this to stage \(s\), from (5.26) the decision statistic can be represented as
\[
Z_{k,i}^{(s)} = \int_{jT + \tau_k - \epsilon}^{(i+1)T + \tau_k - \epsilon} \hat{e}_k^{(s)}(t)a_k(t - \tau_k + \epsilon) \cos(\omega_c t + \phi_k) dt. \quad (8.32)
\]
As before, we can simplify this equation as
\[
Z_{k,i}^{(s)} = \eta + \sqrt{\frac{P_k}{2} b_{k,i}(N T_c - (2B + 1 - Q_k) \epsilon)} + \sum_{j=1, j \neq k}^{K} I_{j,k}^{(s)}. \quad (8.33)
\]
Using the definition of \(\hat{s}_k(t)\), \(I_{j,k}^{(s)}\) is found to be
\[
I_{j,k}^{(s)} = \int_{jT + \tau_k}^{(i+1)T + \tau_k} (s_j(t - \tau_j) - \hat{s}_j(t - \tau_j + \epsilon))a_k(t - \tau_k + \epsilon) \cos(\omega_c t + \phi_k) dt. \quad (8.34)
\]
Equation (8.34) simplifies to
\[
I_{j,k}^{(s)} = T_c \cos(\phi_j - \phi_k) \left[ \sqrt{\frac{P_k}{2} b_{j,i} W_j - \frac{Z_{j,i}^{(s-1)}}{T} S_j} \right]. \quad (8.35)
\]
where similarly to \( W_j, S_j \) can be defined as
\[
S_j = X_j + \left(1 - \frac{2(\delta_j + \epsilon)}{T_c}\right) Y_j + \left(1 - \frac{\delta_j + \epsilon}{T_c}\right) U_j + \left(\frac{\delta_j + \epsilon}{T_c}\right) V_j. \tag{8.36}
\]
The expected value of \( Z_{k,i}^{(s)} \) given the received powers \( \{P_k\} \) is
\[
E \left[ Z_{k,i}^{(s)} \mid \{P_k\} \right] = \sqrt{\frac{P_k}{2}} b_{k,i} N(T_c - \epsilon) \tag{8.37}
\]
where we have allowed \( B \) and \( Q_j \) to take on their expected values. Again assuming no correlation between terms \( I_{j,i}^{(s)} \) the variance of \( Z_{k,i}^{(s)} \) is
\[
\text{var} \left( Z_{k,i}^{(s)} \mid \{P_k\} \right) = \frac{N_0 T}{4} + \sum_{j=1}^{K} \sum_{j \neq k} \text{E} \left[ \left( I_{j,k}^{(s)} \right)^2 \mid \{P_k\} \right]. \tag{8.38}
\]
We must now find an expression for \( \text{E} \left[ \left( I_{j,k}^{(s)} \right)^2 \mid \{P_k\} \right] \). Expanding and simplifying (8.35) results in the expression
\[
\text{E} \left[ \left( I_{j,k}^{(s)} \right)^2 \mid \{P_k\} \right] = \frac{T_c^2}{2} \text{E} \left[ \frac{P_j}{2} W_j^2 - 2 \sqrt{\frac{P_k}{2}} W_j Z_{j,i}^{(s-1)} \frac{T_c}{S_j} + \frac{(Z_{j,i}^{(s-1)})^2}{T^2} S_j^2 \right] \mid \{P_k\} \right]. \tag{8.39}
\]
It can be shown that \( \text{E}[W_k S_k] = \text{E}[W_k^2] = \frac{2N}{3} \) and that \( \text{E}[S_k^2] = \frac{2N}{3} + \frac{2N_0^2}{T_c^2} \). Using this along with (8.37) and the fact \( \text{E}[x^2] = \text{var}(x) + (\text{E}[x])^2 \) allows the simplification of (8.39) to
\[
\text{E} \left[ \left( I_{j,k}^{(s)} \right)^2 \mid \{P_k\} \right] = \frac{T_c^2 N}{3} \left[ \frac{P_j}{2} \left( -\frac{1}{2} + \frac{\epsilon}{T_c} + \frac{1}{2} \left(1 - \frac{\epsilon}{T_c}\right)^2 \left(1 + \frac{3\epsilon^2}{T_c^2}\right) \right) + \frac{1 + \frac{3\epsilon^2}{T_c^2}}{T^2} \text{var} \left( Z_{j,i}^{(s-1)} \right) \right]. \tag{8.40}
\]
Now substitution of this result into (8.38) results in
\[
\text{var} \left( Z_{k}^{(s)} \right) = \frac{N_0 T}{4} + \frac{N T^2}{3} \left[ \alpha \sum_{j=1}^{K} \sum_{j \neq k} \text{var} \left( Z_{j,i}^{(s-1)} \right) \right] \tag{8.41}
\]
where \( \alpha = -\frac{1}{2} + \frac{\gamma}{T_c} + \frac{1}{2} \left(1 - \frac{\epsilon}{T_c}\right)^2 \left(1 + \frac{3\epsilon^2}{T_c^2}\right) \) and \( \gamma = \frac{1 + \frac{3\epsilon^2}{T_c^2}}{T^2} \) and the subscript \( i \) is now dropped. It is shown in Appendix A by mathematical induction that solving this iterative equation gives
\[
\text{var} \left( Z_{k}^{(s)} \right) = \frac{N_0 T}{4} \left[ \frac{1 - \left(\frac{\gamma T^2(K-1)}{3N}\right)^s}{1 - \left(\frac{T^2(K-1)}{3N}\right)^s} \right] + \alpha \frac{K}{\gamma} \left[ \frac{1 - \left(\frac{T^2(K-1)}{3N}\right)^s}{1 - \left(\frac{T^2(K-1)}{3N}\right)^s} \right] \sum_{j=1}^{K} P_j + \frac{\alpha}{\gamma} \frac{P_k}{\gamma} \left[ \frac{-\left(\frac{T^2(K-1)}{3N}\right)^s}{1 + \left(\frac{T^2(K-1)}{3N}\right)^s} \right] \left[ \frac{\gamma T^2(K-1)}{3N} \right] \left[ \frac{K}{2} \left(1 - \frac{1}{N} \sum_{j=1}^{K} \left(-\frac{\gamma T^2(K-1)}{3N}\right)^s P_j \right) \right] \tag{8.42}
\]
Since we can model $Z_k$ as a Gaussian variable, the probability of error of user $k$ at stage $s$ can be calculated as

$$P_k^{(s)} = Q\left\{ \sqrt{\frac{\mathbb{E}[Z_k^{(s)}]}{\text{var}(Z_k^{(s)})}} \right\}$$

(8.43)

where $Q(\cdot)$ is the well-known Q-function. Strictly speaking, since we are assuming random signature sequences, the value of $\mathbb{E}[Z_k^{(s)}]$ should be averaged over $B$ which has a binomial distribution. However, it was found that there was little difference between averaging over the binomial distribution and simply using the expected value for $B$. If specific codes are used, specific values for $B$ can be used in the expression. Using $\mathbb{E}[B] = \frac{N-1}{2}$ results in

$$P_k^{(s)} = Q\left\{ \left(1 - \frac{1}{2E_b/N_0\epsilon_o} \right)^s \left[ 1 - \left(\frac{\gamma_o(K-1)}{3N} \right)^s \right] + \frac{2\alpha}{K\gamma_0\epsilon_o} \left[ 1 - \left(\frac{\gamma_o(K-1)}{3N} \right)^s - \frac{1}{1 + \left(\frac{\gamma_o}{3N} \right)^s} \right] \sum_{j=1}^{K} P_j + \frac{2\alpha}{\gamma_0\epsilon_o} \left[ \frac{\gamma_o}{3N} \right]^s \left[ \frac{K - 1}{K} - \frac{(-1)^s \sum_{j=1}^{K} P_j}{P_k} + (-1)^s \right] \right\}^{-1/2},$$

(8.44)

where $\epsilon_o = 1 - \frac{1}{2\epsilon_c}$ and $\gamma_o = \gamma T^2$.

Letting $s \to \infty$ gives the asymptotic result

$$P_k^{(s)} = Q\left\{ \left(1 - \frac{1}{2E_b/N_0\epsilon_o} \right)^s \left[ 1 - \left(\frac{\gamma_o(K-1)}{3N} \right)^s \right] + \frac{2\alpha}{K\gamma_0\epsilon_o} \left[ 1 - \left(\frac{\gamma_o(K-1)}{3N} \right)^s - \frac{1}{1 + \left(\frac{\gamma_o}{3N} \right)^s} \right] \sum_{j=1}^{K} P_j \right\}^{-1/2},$$

(8.45)

where both (8.44) and (8.45) assume that $K - 1 < 3N$.

Using the results of the previous section we are able to create performance curves to explore the effect of timing errors on the multistage receiver. First, we validate the analytic results with simulations. Figure 8.7 shows analytic curves and simulation results for $\frac{E_b}{N_0} = 8dB, K = 10,$ and $N = 31$ for timing errors varying from 0 to $0.4T_c$. As can be seen, extremely consistent results are obtained. Additionally, these curves show that as the timing error increases, the gain due to interference cancellation is reduced. However, for errors up to $0.4T_c$, the multistage approach still outperforms a conventional receiver. This demonstrates that timing errors, while important in determining receiver performance, do not eliminate the benefits of this multistage approach.

To examine the effect timing errors have on capacity, we set all received powers equal, $\frac{E_b}{N_0} = 8dB$ with $N = 31$ and vary $K$. Figure 8.8 shows the results for ideal timing and for timing error $\epsilon = 0.2T_c$. While timing error does significantly affect performance, it
does not fully diminish the gains realized by the multistage receiver. For example, when considering perfect timing and a required error probability of error of $10^{-2}$, a three stage interference cancellation receiver can theoretically accommodate approximately 37 users, as opposed to 10 users for the conventional receiver (stage 1). However, when considering non-ideal timing, specifically $\epsilon = 0.2T_c$, the three stage receiver is reduced to a capacity of approximately 13 users, while the conventional receiver is reduced to approximately 5 users. Thus, considering only timing imperfections, we can see that the gain due to multiple access interference (MAI) cancellation is reduced from 370% to 260%. It should be emphasized that this takes into consideration only timing errors, while other departures from the ideal case are not considered.

Another way of analyzing the effect of timing errors is to show the effect that timing errors have on required $\frac{E_b}{N_0}$. To show this, we set all received powers equal, $K = 10$, $N = 31$ and vary $\frac{E_b}{N_0}$. The results are shown in Figure 8.9 for no timing error and for a timing error of $0.2T_c$. Again we can see that while the gains are reduced in the presence of timing errors, they are still significant.

Next, we investigate the effects of timing errors when pulse shaping is used. Specifically we chose square root raised cosine pulse shaping where the square of the pulse frequency response is

$$P^2(f) = \begin{cases} \frac{1}{2W} & 0 \leq |f| < f_1 \\ \frac{1}{4W} \left( 1 - \sin \left[ \frac{\pi (f - W)}{2W - 2f_1} \right] \right) & f_1 \leq |f| < 2W - f_1 \\ 0 & \text{else} \end{cases} \tag{8.46}$$

where $f_1$ is a frequency definition related to the bandwidth $W$ through the roll-off factor $\alpha = 1 - \frac{f_1}{W}$. The roll-off factor $\alpha$ varies between 0 and 1 with $\alpha = 0$ corresponding to a rectangular 'brick wall' filter response and $\alpha = 1$ corresponding to a very gradual roll-off in frequency and high damping in the time domain.

In examining the effect of timing when employing pulse shaping, analysis proves to be intractable. Thus, the results presented here rely on simulation. Two extremes of the square root raised cosine pulse were examined, namely $\alpha = 0$ and $\alpha = 1$. In these simulations 10 samples were used per chip and the filter impulse response was extended over 5 chip intervals including the desired chip interval. The results for $\frac{E_b}{N_0} = 8dB$, $N = 31$, and $K = 10$ are shown in Figure 8.10. As can be seen, for $\alpha = 1$ the results are very close to those for rectangular pulse shaping given in Figure 8.7. Thus, for $\alpha = 1$, pulse shaping does not make the design more sensitive to timing errors.

The second case examined is $\alpha = 0$. While $\alpha = 0$ is not practical, it provides a feel for the extreme case. This pulse, while resulting in better spectral characteristics, has considerably less energy in the desired bit interval and causes more degradation to
adjacent bits if timing is not perfect. This is seen in Figure 8.10. The figure shows that indeed this extreme case causes much more rapid degradation in the presence of timing errors. However, this degradation is found in a conventional receiver (stage 1) as well. The multistage interference cancellation receiver degrades slightly faster than the conventional receiver but still outperforms the conventional receiver until the error goes beyond one half of a chip period which is extremely large.

While the AWGN channel allows us to analytically examine the effects of tracking errors, it is desirable to obtain performance results for a more practical channel model. Towards this end, simulations were performed for a frequency selective Rayleigh fading channel. This channel was chosen to reflect the multipath resolution capability of CDMA systems. In particular we assume a three ray channel where each component has a Rayleigh distributed amplitude with standard deviations of 0.93, 0.73 and 0.28. This reflects a channel with two fairly strong components and a third weak component and is derived from measurement data presented in [107]. Figure 8.11 shows the effect which timing errors have on receiver performance in such a channel. Clearly, timing errors affect the performance of a three-stage receiver more significantly than a conventional receiver (stage 1). However, as in the AWGN case, even at an error of one-half of a chip, the interference cancellation scheme performs better than the conventional receiver.

Figure 8.12 shows the reduction in capacity for a conventional receiver, a receiver employing successive cancellation, and a receiver employing parallel cancellation in an AWGN channel with $N = 31$ and $\frac{E_b}{N_0} = 10dB$ and perfect power control. Results for successive cancellation are taken from [15] and reflect a Gaussian distributed timing error with a standard deviation of $\sigma_c = 0, 0.2T_c$. In the absence of timing errors we can see the large performance advantage of a three stage parallel interference cancellation receiver. However, with a timing error of $0.2 \times T_c$, the advantage is reduced significantly, although not eliminated.
8.6 Conclusions

In this chapter we have derived equations which approximate bit-error rate for multistage interference cancellation in the presence of amplitude, timing and phase estimation errors. These expressions were used along with simulation results to show that the multistage interference cancellation receiver is fairly robust to estimation errors. Analytic and simulation results proved to be extremely consistent. Further it was shown that pulse shaping can make timing errors more significant, but does not eliminate gains due to interference cancellation unless error is extreme. While this analysis does not take into consideration several other practical degradations which affect mobile receiver performance, we have shown that minor tracking or phase errors alone will not severely degrade the gains achieved through MAI cancellation. Finally, it was shown that coherent reception with phase estimation provides significantly better performance than a differentially demodulated multistage cancellation scheme.
Figure 8.7: Degradation Caused by Timing Errors ($E_b/N_o = 8$dB, Users=10, Spreading Gain=31) ['x' = simulated].
Figure 8.8: Capacity Curves for Perfect Timing and Timing Error=$0.2T_c$ ($E_b/N_0 = 8$ dB, Spreading Gain=31).
Figure 8.9: Bit Error Rate vs. $\frac{E_b}{N_0}$ for Perfect Timing and Timing Error $= 0.2T_c$ (Users $= 10$, Spreading Gain $= 31$).
Figure 8.10: Degradation Caused by Timing Errors When Pulse Shaping is Employed ($\frac{E_b}{N_0} = 8\text{dB, Users}=10, \text{Spreading Gain}=31$).
Figure 8.11: Degradation Caused by Timing Errors for Three Ray Multipath Fading ($\frac{E_b}{N_0} = 8\text{dB}$, Users=10, Spreading Gain=31, Path Gains:$\sigma_1 = 0.93$, $\sigma_2 = 0.73$, $\sigma_3 = 0.28$).
Figure 8.12: Comparison of Conventional, Successive IC and Parallel IC Receivers ($\frac{E_b}{N_0} = 10$, Users=10, Spreading Gain=31, perfect power control)
Chapter 9

Improvements to Multistage Interference Cancellation

In this chapter we discuss two methods of improving multistage interference cancellation, bias reduction and adaptive antennas. The first technique refers to the fact reported in Chapter 5 that multistage interference cancellation introduces a bias in the decision statistic at stage 2. This bias is especially harmful in a heavily loaded system since (a) the bias is directly proportional to the system loading and is in the direction of the decision boundary, and (b) the variance is also proportional to the system loading. The combination of these two facts reduces the effectiveness interference cancellation in this case. The second improvement discussed here is the use of adaptive antennas along with interference cancellation. Many researchers have suggested the combination of multiuser detection and adaptive antennas [70, 92, 55, 31, 43, 54, 59, 71, 85, 93]. We show that there exist several cases where the combination of the multistage cancellation and adaptive antennas would be beneficial. Additionally, we suggest a method for combining the two.

9.1 Bias Reduction as a Means to Improving Interference Cancellation

As discussed in Chapter 5, multistage interference cancellation introduces a bias in the decision statistic at stage two which is directly proportional to the system loading. This bias is created due to the fact that the amplitude estimates of each interferer from stage 1, will be correlated with the desired user’s amplitude and bit values. By removing the interference, we actually remove a small part of the desired energy. At low loadings, this is tolerable and does not significantly affect system performance. However, as the system grows heavily
loaded, the performance degrades quickly due to the loss of energy in the decision statistic. Additionally, as the system loading increases, the noise in the decision statistic grows. This exacerbates the bias problem causing significant performance degradation. As an example, consider the bit-error-rate performance displayed in Figure 9.1. This figure presents the simulated performance for 20 users in an AWGN channel when no effort is taken to reduce the bias at stage 2. Additionally, we have plotted the theoretical performance assuming no bias is present. There is a significant difference between the predicted performance and the actual performance.

One simple method to reduce the bias at stage 2 is to not attempt to remove the interference entirely. That is if we multiply the amplitude estimate of each interferer by some factor $c$ where $0 < c \leq 1$, we do not attempt to remove all of a given interference signal. This benefits us because we are now removing less of the desired signal's energy, thus
reducing the bias. Additionally, in a heavily loaded system, the estimates of the interference will not be as accurate as in a lightly loaded system. Thus, cancelling conservatively would seem to be prudent.

It can be shown that the bias is negatively correlated with the transmitted bit only in even stages of cancellation. Additionally, the bias introduced by each interferer is reduced by a factor of $1/N$ at each stage. Thus, stage three will have a positive bias weighted by $1/N^2$. This small positive bias is marginally helpful. However, by stage 4 the bias while again being negative, is weighted by $1/N^3$ and is thus no longer a problem. Thus, it is only in stage two that this bias needs to be considered. This is fortunate since we do not wish to continue weighting the interference estimates by some number less than one since it will limit the improvement seen by interference cancellation.

To demonstrate the performance improvement seen when applying a bias reduction factor, consider the performance shown in Figure 9.2. Here we again have 20 users in an AWGN channel. However, we now do not attempt full interference cancellation in stage two. Instead, we weight the amplitude estimate by a factor of $c = 0.5$. This is only done in stage two. The performance improvement when compared to Figure 9.1 is evident. Not only is performance more in line with the theoretical prediction shown in Figure 9.1, but the performance is approaching the single user bound after three stages.

Now that we have demonstrated the benefit of weighting the estimate in stage two, we would like to see how this weighting effects performance as it is varied. Figure 9.3 shows the performance of a 20 user system in AWGN as the weighting factor is varied between 0 and 1. Obviously, there exists an optimal value. A weighting which is too small eliminates the effect of stage two entirely. Notice that the performance of stage two for $b = 0$ is identical to stage one and stage three’s performance mirrors that of stage two when $b = 1$ (i.e. it has effectively become stage two). However, as $b$ is increased from zero, the performance is improved until an optimum value is reached. Above this value the cancellation at stage two introduces a bias which begins to negate the benefits of interference removal.

Based on these results, we conclude that in a heavily loaded system, a weighting factor of one-half should provide significant gains. This factor will increase as loading goes down. Although not explicitly stated, all simulations in this work use a weighting factor of $b = 1/2$ in heavily loaded systems. It should be emphasized that this weighting is used only in stage two. Again, this is because of two separate factors. First, the bias in stage 3 (and all odd numbered stages) is positively correlated with the desired bit. Thus, it can only help performance. Second, the bias goes down by $(1/N)^{(s-1)}$ where $N$ is the spreading factor and $s$ is the stage. Thus by stage 4, the bias is insignificant.
Figure 9.2: Performance of Parallel Cancellation in a Heavily Loaded System with Back-off Factor
Figure 9.3: Effect of Bias Reduction Factor on Performance of Parallel Cancellation in a Heavily Loaded AWGN Channel (Note: The bias reduction factor is applied only in stage 2 or the 1st stage of cancellation)
9.2 Adaptive Antennas

In recent years several researchers have suggested that multiuser reception and adaptive antennas are synergistic and could be used together beneficially [70, 92, 55, 31, 43, 54, 59, 71, 85, 93]. We show here that in several situations multistage cancellation and adaptive antenna algorithms can be combined to provide performance gains. Switched beam diversity is also currently being considered as method of increasing capacity. However, multistage cancellation can be readily used along with switched beams providing a multiplicative gain. We consider the use adaptive antenna arrays in this section, but the discussion also applies to switched beam antennas.

Intuitively, multistage cancellation and adaptive antennas are complimentary technologies. The former performs cancellation in the time domain (or perhaps the code domain) whereas the latter performs spatial filtering. If two signals have significantly different angles of arrival, an adaptive antenna can perform spatial filtering and remove mutual interference. However, if two signals arrive with similar angles of arrival, an adaptive antenna cannot provide any gain. In CDMA, the spreading codes themselves will provide some isolation. However, in a near-far scenario, this gain may be insufficient. It is in such a case when interference cancellation provides additional benefit. Additionally, in the case of overloading, the received signals of some users may not be incident on the main lobe of any beam. In this case, the antenna itself creates a near-far situation, which would result in significant performance degradation. Some form of cancellation would be beneficial in this situation.

However, adding multistage cancellation to a pure spatial diversity system would not be beneficial. That is, in a system where a beam is formed for each user, adding multistage cancellation is not worthwhile for two reasons: (1) the complexity of putting a multistage receiver on K array ports would be prohibitive, and (2) information from one beam could not be used to cancel interference in another due to the different RF front ends (and thus different phase and frequency information) used on each.

Adding interference cancellation would be more beneficial in a system which has a fixed number of ports which is less than the number of users. Consider the system presented in Figure 9.4. This system has M ports running a multi-target Constant Modulus Algorithm (CMA) [34]. At the output of each port is a multistage cancellation receiver which performs cancellation on the array output from that port. By using a selectivity level, the cancellation receiver could cancel only users with significant power in that array output. The final estimates from each port could then be combined using maximal ratio combining, assuming that multiple ports have estimates of a given user. In such a system, multiple users would be present in each beam and it is possible that a given user may be present in more than

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Figure 9.4: Proposed Use of Multistage Cancellation with Adaptive Antenna Employing a Multi-target Constant Modulus Algorithm

a single port output. Thus, providing multistage cancellation on each port would provide additional interference reduction and resistance to near-far situations when the two users in question have similar angles of arrival.

As evidence that the two techniques are synergistic consider the performance of an eight element array in an AWGN channel with $E_b/N_0 = 8dB$. The simulated performance of such a system is presented in Figure 9.5 along with a conventional correlation receiver and a three stage cancellation receiver. As we can see, at low to moderate loading the array performs extremely well. In fact due to the antenna gain, the system performs better than a single user system at $E_b/N_0 = 8dB$ for low loading (with an omni-directional antenna). However, as the loading grows large, the array begins to have problems resolving all of the users as shown in Figure 9.6. In this figure we display the antenna gain from each of the eight ports. As we can see, in general the array performs well. However, for some users, the array does not capture the signal properly. This results in performance degradation as shown in Figure 9.5. The performance of the Three Stage cancellation receiver degrades much more gracefully. This suggests that in overloading cases, multistage cancellation would be beneficial.

Another situation where cancellation can be useful is the case of crowding, i.e., when several signals have angles of arrival which are similar. In such a case, an array's resolution is limited by the number of elements used, particularly in the case of a linear array with an endfire beam. Consider the case shown in Figure 9.2. This is a case of crowded users with AOA's of 0.10,20,25,30,45,50,and 60 degrees. Here we see that in one case the array
Figure 9.5: Comparison of the BER Performance of Three Stage Cancellation, an Eight Element Adaptive Array and the Conventional Receiver ($E_b/N_0 = 8$ dB, spreading gain = 15)
Figure 9.6: Antenna Gain Patterns vs. Angle of Arrival for Each of 8 ports for Uniform Angle Spread ($E_b/N_0 = 8$dB, spreading gain = 15, and 8 users)
Figure 9.7: Antenna Gain Patterns vs. Angle of Arrival for Each of 8 ports for Crowded Case ($E_b/N_o = 8$dB, spreading gain = 15, and 8 users)
was able to resolve the user at 60° with one port. However, the crowding near zero (i.e. at end-fire) caused the array significant problems and it was unable to resolve a single user in the range of 0-20 degrees. This causes the array’s performance to degrade by over an order of magnitude as shown in Table 9.1. In such a case, interference cancellation would be extremely valuable. The performance of the cancellers at each array output would be similar to the case of 2-4 users for an omni directional antenna. That is the performance should be approximately 3e – 4 as suggested in Figure 9.5 regardless of the angles of arrival. (Excepting of course the case where all signals arrive at the receiver with identical angles of arrival. In such a case, the performance would be similar to an 8 users multistage receiver.) Thus in cases of crowding or overloading, the array suffers significant degradation. However, by supplying some isolation between users, the array can reduce the interference seen by the individual multistage receivers. Additionally, in cases of low loading the adaptive array would serve to significantly reduce the noise level seen by the cancellation receiver. Thus, the array can improve the performance of the multistage cancellation receiver, while the cancellation receiver improves the performance of the adaptive array in cases of overloading and crowding.

9.3 Summary

In this chapter, we have considered two techniques for improving the results of multistage interference cancellation. One of these techniques is a method for reducing the the effects of the bias introduced by interference cancellation at heavy loading. The second is a discussion of how interference cancellation may be combined with adaptive antenna technology to yield further performance improvements. In the next chapter, we present a description of a DSP-based implementation of interference cancellation.
Chapter 10

Hardware Development of a Two Stage Interference Cancellation Receiver

To date all of the research presented in communications literature has focused on theoretical or simulation analysis of the multistage receiver. To the best of our knowledge implementation has not been addressed. In this chapter we attempt to fill this void by detailing the development of a baseband implementation of a two-stage interference cancellation receiver using commercial floating-point DSP processors. We present the algorithm development as well as results. The extension of this receiver architecture to RF is also explored and specific challenges identified. The groundwork is laid for meeting these challenges and steps for future development are outlined.

Section 10.1 explores the implementation of the multistage interference cancellation algorithm. Simplifications are discussed as well as a real-time algorithm. Section 10.2 briefly presents the system parameters chosen for this development. The development phases are presented in Section 10.3 along with results from each phase. Section 10.4 details the extension to RF including preliminary design work, specific challenges associated with multistage cancellation, and possible solutions to these challenges. Conclusions are presented in Section 10.5.
10.1 Algorithm Development

10.1.1 Simplifying the Decision Metric

As stated in Chapter 5, the decision metric for multistage interference cancellation can be written as

$$
\hat{b}_{i,k} = \text{sgn} \left[ \int_{(i-1)T+\tau_k}^{iT+\tau_k} \left( r(t) - \sum_{m \neq k} \hat{\delta}_m(t - \tau_m) a_k(t - \tau_k) \cos(\omega_c t + \theta_k) dt \right) \right] \quad (10.1)
$$

where

$$
\hat{\delta}_m(t) = \hat{A}_m \hat{b}_m(t) a_m(t) \cos(\omega_c t + \theta_m)
$$

is the estimated signal for user $k$ and $\hat{A}_m$ and $\hat{b}_m(t)$ are the estimates of the $m$th user's amplitude and data symbol respectively. Using a soft-decision approach, we can estimate the combination $A_m b_n(t)$ by the correlator output over the appropriate interval divided by the integration time, i.e., $\hat{A}_k \hat{b}_{i,k} = Z_{i,k}/T$. Now since our processing will be performed at baseband, we would like to represent the above equations at baseband. Simply,

$$
\hat{b}_{i,k} = \text{sgn} \left[ \int_{(i-1)T+\tau_k}^{iT+\tau_k} \left( r_b(t) - \sum_{m \neq k} \hat{A}_m \hat{b}_m(t) a_m(t) e^{j\theta_m} \right) a_k(t - \tau_k) e^{-j\theta_k} dt \right]. \quad (10.3)
$$

Using a direct approach we could create $K$ replicas of the received signal and define the received signal for user $k$ as

$$
r_b^{(k)} = r_b(t) - \sum_{m \neq k} \hat{A}_m \hat{b}_m(t) a_m(t)
$$

and estimate the data symbol directly using equation (10.3). However, this would require $K(K-1)NN_s$ operations where $K$ is the number of users, $N$ is the spreading gain and $N_s$ is the number of samples per chip. In other words, the computational complexity is $O(K^2)$. Fortunately, there exists a computationally less intensive approach. Specifically, we can cancel each user from the received signal to create a residual signal. We can then use this residual signal to improve the original amplitude estimate. If we define $\tilde{r}_b(t)$ as the residual signal at baseband, i.e.

$$
\tilde{r}_b(t) = r_b(t) - \sum_{k=1}^{K} \hat{A}_k \hat{b}_k(t - \tau_k) a_k(t - \tau_k) e^{j\theta_k}, \quad (10.5)
$$

then our decision becomes

$$
\hat{b}_{i,k} = \text{sgn} \left[ \int_{(i-1)T+\tau_k}^{iT+\tau_k} \left( \tilde{r}_b(t) + \hat{A}_k \hat{b}_k(t) a_k(t) e^{j\theta_k} \right) a_k(t - \tau_k) e^{-j\theta_k} dt \right]. \quad (10.6)
$$
Now if we recognize that
\[ \int_{-(i-1)T+\tau_k}^{iT+\tau_k} \hat{A}_k \hat{b}_k(t - \tau_k) a_k(t - \tau_k) e^{j\theta_k} a_k(t - \tau_k) e^{-j\theta_k} = T \hat{A}_k \hat{b}_{i,k} \]
\[ = Z_{i,k}, \] (10.7)
then we can simplify the decision variable as
\[ \hat{b}_{i,k} = \text{sgn} \left[ Z_{i,k} + \int_{-(i-1)T+\tau_k}^{iT+\tau_k} \hat{r}_b(t) a_k(t - \tau_k) e^{-j\theta_k} dt \right]. \] (10.9)

That is, we can use the correlator output \( Z_{k,i}^{(1)} \) of the first stage directly and simply add it to the correlation of the desired signal’s spreading code with the residual signal. Thus, we require only the subtraction of every signal once and a single correlation and addition per user to complete stage 2. Thus, the complexity is approximately \( 3KNN_s \) floating point operations or \( O(K) \) complexity.

To extend this to an arbitrary number of stages we would simply repeat the above procedure for each stage. That is, for an \( s \) stage receiver, the estimate at stage \( s \) would be
\[ \hat{b}_{i,k} = \text{sgn} \left[ Z_{i,k}^{(s-1)} + \int_{-(i-1)T+\tau_k}^{iT+\tau_k} \hat{r}_b(t)^{(s-1)} a_k(t - \tau_k) e^{-j\theta_k} dt \right], \] (10.10)
where
\[ Z_{i,k}^{(s-1)} = Z_{i,k}^{(s-2)} + \int_{-(i-1)T+\tau_k}^{iT+\tau_k} \hat{r}_b(t)^{(s-2)} a_k(t - \tau_k) e^{-j\theta_k} dt \] (10.11)
and
\[ \hat{r}_b(t)^{(s-1)} = r_b(t) - \sum_{m=1}^{K} \hat{A}_m^{(s-1)} \hat{b}_m(t)^{(s-1)} a_m(t). \] (10.12)

Thus in general we have a complexity which is \( O(SKNN_s) \), i.e. linear in users, stages \( (S) \), and samples per symbol. Although we have not shown it here, the extension to multipath is straightforward. We simply add an arm for each tracked multipath and perform maximal ratio combining for each data symbol estimate. The complexity in the multipath case will additionally have an additional complexity which is linear in the number of paths tracked. Also, in the baseband processing, the complex operations are replaced by two real operations (in-phase and quadrature channels).

### 10.1.2 Real-Time Algorithm

The above decision metric can be applied either in continuous time processing or by block processing. In a synchronous system there would be no real difference between the two. However, for an asynchronous system, care must be taken in deriving the real-time processing flow. The reason for this can be seen in Figure 10.1. Since there is overlap between the
bits of different users, we cannot process a bit in its entirety without performing processing on overlapping bits. A block processing scheme would be satisfactory but it does incur a delay which is directly proportional to the block size. Another approach is to process the bits as they come in real time. To understand the approach taken here refer to Figures 10.1 and 10.2. The first step is to perform a correlation between the spreading code associated with the signal under consideration and the received signal at baseband. (It is assumed that we have already locked onto the correct code-phase as well as carrier phase.) This is done for the current bit of each user (assume arbitrarily that it is denoted as 'bit 1'). In order to assure that enough samples have arrived for each user, we can simply allow two data bits of any specific user to arrive which would assure that all users have at least one bit available. Thus, the first step is to create the first stage decision statistic for 'bit 1' of each user (Figure 10.1 (a)). This is stored and used as an estimate of each user's data bit and received power in the cancellation. Since there is overlap between 'bit 1' of user $k$ and 'bit 2' for any user $j$ where $j < k$ (assuming users are arbitrarily numbered with increasing delays), we need to create the first stage decision statistics of each user during bit interval 2 (Figure 10.1 (b)) before proceeding with cancellation. Once correlation is completed for each user over bits one and two, cancellation over bit interval one may occur (Figure 10.1 (c)). This is done by multiplying the correlation score (i.e. $Z_{i,k} = \hat{A}_k \hat{b}_{i,k}$) by the spreading code of each user and using the resulting signal estimate to subtract out each estimated signal from the received signal. This residual signal is then stored and saved for processing in stage 2.

Due to the overlap of bit two and bit one as described in the preceding paragraph, we cannot proceed with stage 2 processing until interference has been removed from all samples effecting bit one of each user. Thus, we need to cancel interference during bit intervals one and two before stage 2 processing can begin on bit number one. However, as explained for bit one, we cannot cancel interference over bit interval two before we perform initial correlations over bit interval three. Thus, the next step is to perform correlation over bit interval three (Figure 10.1 (d)) followed by stage 1 cancellation on bit 2 of each user (Figure 10.1 (e)). Once this is done stage two processing can be performed on bit 1 (Figure 10.1 (f)). In stage 2, we correlate the residual signal with the spreading code of each user to obtain a correction factor which is added to the correlation result of the first stage as described in Section 1. The result is then used as the decision statistic. (If additional stages were going to be used, we would need to recreate the estimated signals based on these new correlation results and create a new residual signal using the original received samples and the new signal estimates.) After stage two is completed, we must then

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perform correlation on bit interval four. This is then followed by stage one cancellation on bit three before performing stage two processing on bit two. This loop will then continue with initial correlation performed on bit interval \( n \), followed by stage one processing of bit \( n - 1 \). and subsequent stage two processing of bit \( n - 2 \) before starting again with bit \( n + 1 \). This procedure is shown in Figure 10.2. The specific processing for stage one correlation, stage one cancellation, and stage two processing is depicted in Figures 10.3-10.5.

10.2 Choice of Parameters

This section details the choice of system parameters such as spreading code length, the sampling rate, the codes used, the phases and delays chosen for each user.

The parameters were largely driven by two factors: (1) the specifications for the GloMo project of which this development is a part, and (2) the processing capabilities. The GloMo project specifies a low spreading rate of 15, but a high bit rate of 128kbps. These specifications allow slow motion video to be transmitted while keeping the processing as low as possible. The spreading codes were created as pseudo-Gold codes of length 15. The data rate specification, however, was not possible under the constraints of a single DSP processor.

The number of users in the system is arbitrary if a low data rate is acceptable. Tests were run for 2-10 users and typical data rates were 4kbps with 4 users and 1kbps with 10 users. The sampling rate was chosen as \( N_s = 4 \) samples per chip\(^1\). This specification was determined by using the information in Chapter 4 where we showed that for reasonable performance, timing errors should be kept under \( 0.2T_c \). The timing error is directly related to the number of samples per chip which determines our resolution. With \( N_s = 4 \), we have a worst case error of \( T_c/8 \) (assuming proper tracking) which will provide adequate performance.

The phases and delays were chosen randomly for each user, but assumed constant throughout a single test. The specific delays can have a large effect as we will see in a later section. The spreading codes were also kept constant throughout testing.

10.3 Development Phases

The hardware development was performed in distinct phases which are detailed in this section. These development phases lead naturally to a full scale RF implementation which will be discussed in the next section. Phase 1 focused on algorithm development and was

\(^1\)Several of the tests were also run at \( N_s = 2 \). This was allowable when the timing jitter was not great.
Figure 10.1: Bit Timing Associated with Two Stage Interference Cancellation
Figure 10.2: Flow Diagram
Figure 10.3: Flow Diagram for Correlate Routine

Get Pointer
User \(k, \text{ Bit } i\)

\[ Z_i = \text{Correlate (PN Code } k, r_i) \]
\[ Z_q = \text{Correlate (PN Code } k, r_q) \]

\[ A = Z_i \cos(\theta) + Z_q \sin(\theta) \]
\[ A_i = A \cos(\theta) \]
\[ A_q = A \sin(\theta) \]

Store \(A_i \& A_q\)

\[ k = k + 1 \]
\[ k > K ? \]

No \hspace{1cm} Yes, exit.
Figure 10.4: Flow Diagram for Stage 1
Figure 10.5: Flow Diagram for Stage 2
done using the software simulator. Phase 2 moved the algorithms onto the DSP chip, but did not perform real-time processing of the received data. Instead, the DSP read data from a stored file and displayed the output. Phase 3 moved to real time processing. In this stage a multiuser signal is received in real time, synchronization is achieved (albeit assisted synchronization), and cancellation is performed in real time. This phase allowed data streams to be monitored and BER's to be calculated.

10.3.1 Development Phase 1

The first phase in the development concentrated on the development of the algorithms for baseband processing. The baseband processing algorithms discussed in the previous section were coded using Analog Devices 21000 Family assembly language for use on the ADSP-21020 processor. These algorithms were tested in Phase 1 to insure that processing was taking place as expected. The Phase 1 testing is described in Figure 10.6. The received data was created using Matlab software [81] for both in-phase and quadrature channels as shown in Figure 10.6. This data was then included in the DSP code at compile time and placed in the simulated data memory.\(^2\) The ADSP-21020 simulator [48] was used to test the algorithms extensively by saving output data such as initial soft estimates of the data, as well as soft and hard estimates after stage 2. These data were compared with the input symbols as well as simulation results from Matlab to verify proper operation. An example of the soft outputs from stages 1 and 2 (where again stage 1 corresponds to the conventional receiver) are shown in Figure 10.7 for the case of 10 users with random (but known a priori) phases and delays and perfect power control. The original input symbols were a repeating pattern of seven bits \(0100111\) to allow easy verification. This input data along with the first stage soft estimates are presented in Figure 10.7 (top) where we have presented two samples of each point for clarity of presentation. Note the data estimate error marked by the bold circle. The corresponding stage 2 soft estimates along with the original data symbols are presented in Figure 10.7(bottom). Note that the data estimation error which occurred in stage 1 is corrected in stage 2.

10.3.2 Development Phase 2

The second stage of development in the hardware implementation was carried out as described in Figure 10.8. In this phase we extend the procedure of Phase 1 to now include interaction with external hardware and real data, although not in real time. To demonstrate

\(^2\)Due to the existence of dual memory memory busses on the 21020 (i.e. data and program memory), we actually were able to store data in both program and data memory allowing many computations for I&Q channels to occur in parallel.
Figure 10.6: Implementation Phase 1
Figure 10.7: Example Output of Phase 1 Receiver Test
the effect of cancellation visually, we fed a single tone signal from a signal generator to a Harris HC-55564 codec as shown in Figure 10.8. This data was downloaded to memory through the receiver code which was running on chip using the ADSP EZ-Ice Emulator. The emulator is similar to the simulator mentioned previously with the exception that the code runs on the actual chip rather than being simulated. The emulator allows access to all memory as well as to all of the ALU and DAG registers. This allowed testing of the code on the chip while still retaining easy access to memory and registers, which is important since the chip behaves slightly different than the simulator. The received data was then ported to Matlab \[81\] software to add multiuser interference. The resulting signal was subsequently embedded into the assembly code at assemble time and stored in data memory. The algorithm codes were then run on chip with the output being fed via output flags, which are accessible via the EZ-lab evaluation boards, to the Harris codec. The Harris codec output was then fed to an oscilloscope for visual display. Example outputs are shown in Figures 10.9 and 10.10.

The purpose of the second phase was two-fold. First, running the algorithm on chip is different from running the code in the simulator. As good as the simulator models the chip's function, there are distinct differences in their performance. Thus, it is important to run the code on chip with a controlled input (i.e. through a file) to allow higher level debugging. Second, by using a sinusoid as the input, we can visually verify the gains achieved by interference cancellation. Figure 10.9 shows the original digital bit stream without interference as well as the resulting sinusoid output from the Harris 55564 Codec. 3

Note that since the codec uses adaptive delta modulation, a string of ones corresponds to the output increasing, and a string of zeros corresponds to the output decreasing. The toggling between one and zero at peaks and valleys represents where the codec's step size is too large and cannot adapt to the drop in the rate of change. Also note that due to the slow data rate ($\approx 15\text{ksps}$ or $500\text{bps}$) a string of ones or zeros results in a slow decay in the bit stream. The output corresponding to the conventional and two-stage cancellation receivers are shown in Figure 10.10 (top and bottom, respectively). The interference contains three interferers of four users total with one interferer having a signal power level which is 14dB higher than the desired user. The BER for the conventional receiver was approximately 10\%, while the two stage cancellation receiver obtained a BER on the order of 1e-3. There is a noticeable difference between the two outputs. The top figure shows the response of the conventional receiver. We see that there are considerable errors in the bit stream as marked by the lack of upswing in the sinusoid in the right part of the graph. However, in

3These plots were obtained by performing a screen dump from an HP 55420A digital oscilloscope.
the bottom graph we do not see such degradation for the two stage cancellation receiver output. This phase showed proper functioning of the DSP algorithms on chip as opposed to simple simulator tests.

10.3.3 Phase 3

The final phase of implementation accomplished as part of this work is described in Figures 10.11-10.13. The first extension necessary to the previous code was to include a synchronization stage. Since the input signal will have an arbitrary delay associated with it, we must synchronize to the chip timing of the incoming signal. Since the development is initially concerned with testing the algorithms we simplified the synchronization problem in
Figure 10.9: Implementation Phase 2: Output for Original Data Stream and Resulting Analog Output Using Harris Codec
Figure 10.10: Implementation Phase 2: Output for 4 User system with Near Far Problem (top) Conventional Receiver, (bottom) Two Stage Cancellation Receiver
the following ways. First, the transmitted signal initially contained only a single user. The receiver was given 50 symbols to achieve synchronization lock before the interference was introduced. Second, since we are controlling the interference we know the relative delays between the different users. Thus, once we synchronize to the first user, we know the timing of the other users in the system. While this simplifies the problem for the time being, eventually this problem will need to be addressed. This will be investigated more later.

In order to test the synchronization algorithms, as well as provide sanity checks on the interaction between the chip and external devices, we initially used the set-up given in Figure 10.11. A single user input was generated by a data generator and spreader board which is outlined in Figure 10.12. This board uses a 4 stage shift register to generate a length 15 maximal sequence which is used to spread the data. The data is generated by a variable length PN sequence generator which can be configured to provide data sequences which range from 7 bits to over a billion bits in length. This board will also serve as the basis of a transmitter board in the RF implementation stage. Diagrams of the individual elements of this board are presented in Appendix F. Again, this set-up provided initial verification of the algorithms in real time as well as verification of the synchronization procedure.

Once this segment of Phase 3 was completed, testing with multiuser signals commenced. The procedure is detailed in Figure 10.13. Multiuser signals were generated using a separate ADSP-21020 and converted to an analog signal using the AD7769 DAC. In this phase the generated signal was changed to a zero phase signal. This was done for two reasons. First, the complex case is a simple extension of the zero phase case and thus provides no extra insight from an experimental point of view. Second, by removing the phase, we can run the system at a higher rate and can achieve twice the interference for a given number of users. Thus we can test the receiver's performance more adequately with fewer users. The resulting analog signal was directly transferred to a second AD7769 working as an ADC. The ADC delivers samples directly to the BUS of the 21020 which processed the input samples and created data estimates for each user.

Example outputs from a HP54520A digital oscilloscope are presented in Figures 10.14 through 10.16 for the case of 4 users. The input data stream for the user shown is again 0100111 to allow easy identification. Figures 10.14 and 10.16 show a close view of the multiuser interference signal. Five distinct levels are clearly identifiable which is what is expected in the zero phase case.

The final testing of in Phase 3 involved the determination of the Bit Error Rate for varying interference levels. BER curves were generated by creating data using a software

\footnote{In the random phase case, half of the interference will be on average out of phase with the desired user.}
Figure 10.11: Implementation Phase 3 for a Single User Input
Figure 10.12: Direct Sequence Data Generator and Spreader Board
Figure 10.13: Implementation Phase 3 for Multiuser Input
PN sequence generator at the transmitter. This generator was also employed at the receiver to provide a reference bit stream to test for bit errors. In order to perform this testing, bit level synchronization was required within the transmitted stream. To accomplish this, a known bit stream was transmitted after sufficient bits were sent to allow synchronization. The receiver searched for this known bit sequence after synchronization and started the PN generator immediately following the recognition of the known sequence. The results of several BER tests for various interference levels are given in Figure 10.17 along with theoretical and computer simulation results for the case of high noise ($E_b/N_0 = 30$dB) and zero phase. The most noticeable thing about these results are the wide variance in the BER values obtained. Points were generated for various combinations of delays. It was found that at low to moderate loading levels, the BER was extremely dependent on the relative delays between users. For some combinations the interference was extremely bad, while for others it was fairly low. On average however, the results provide good agreement with theoretical and simulation results. We should note that at these low numbers of users and spreading, we expect that even the Improved Gaussian Approximation would be inaccurate. It should be noted that these results were obtained by using a factor of $1/2$ in stage two as described in Chapter 9 to reduce the bias in the decision statistic. To provide further evidence that this factor is crucial, the BER obtained with 10 users when not using any bias reduction was 0.27 or an increase in over 1500%!

10.4 Next Phase Implementation

The next step in the development of this receiver is to move the receiver to RF. This development involves several challenges. In this section we present logical steps towards the design and identify the challenges involved. Additionally, we present some initial design work and present possible solutions to the specific challenges. This section should serve as guide to the further development which will continue after the completion of the current work.

10.4.1 Future Design Challenges

The next step in the development can be broken down into several parts including: (1) the addition of a third stage of processing; (2) the addition of multipath processing; (3) moving the data rate up to the specified data rate$^5$; (4) the addition of RF transmission and reception; (5) extending the synchronization to acquire all users; and (6) phase estimation.

$^5$As mentioned previously, this development is part of a larger project which was a targeted data rate of 128kbps at a spreading rate of 15.
Figure 10.14: Four Level MultiUser Signal and Demodulated Data

Figure 10.15: Four Level MultiUser Signal and Demodulated Data
Figure 10.16: MultiUser Received Signal

The first two steps are simple extensions and can be done rather seamlessly with the current code. The addition of stages to the algorithm can be accomplished as described in Section 1. Adding paths is akin to adding users with additional instructions to perform maximal ratio combining. The only difficulty with these steps is the addition of step (3). Specifically, the implementation of additional stages or paths is straightforward however, a single 21020 processor is incapable of supporting a data rate of 128kbps, even at a spreading gain of 15 (and a sampling rate of 4 times per chip.) Thus a more sophisticated processor will be required.

The addition of an RF front end in and of itself should be rather straightforward. The RF front end design is shown in Figure 10.18.\textsuperscript{6} The RF front itself does not present any distinct design challenges due to multiuser detection. However, it can be noted that the RF front end is common to all users. The reason for this is straightforward. Assume that two signals are received using distinct front ends. The first signal with carrier $\cos(\omega_c + \theta_1)$ is mixed down using oscillator A, $\cos(\omega_c t + \omega_A t + \theta_A)$, while the second signal with carrier $\cos(\omega_c t + \theta_2)$ is mixed down with oscillator B, $\cos(\omega_c t + \omega_B t + \theta_B)$ where $\omega_A$ and $\theta_A$ are

\textsuperscript{6}While the original design was done by the author as part of this work, the design as shown also represents some modifications performed by Ni\text{ij}er Correal.
Figure 10.17: Comparison of BER Test Results for Baseband Receiver with Computer Simulation and Theoretical Results Using a Gaussian Approximation ($E_b/N_0 \approx 30$dB, Spreading Gain = 31, Gold Codes, Zero Phase)
the carrier and phase difference between the incoming carrier and oscillator A. The signal coming from RF chain A after low-pass filtering and assuming perfect closed loop carrier and phase tracking is \( m_1(t - \tau_1) + m_2(t - \tau_2) \cos(\omega_A + \theta_A) \) while the signal coming from RF chain B is \( m_2(t - \tau_2) + m_1(t - \tau_1) \cos(\omega_B + \theta_B) \) where \( m_k(t) \) is simply the entire message signal of user \( k \) including the data stream and spreading waveform. As we can see, we immediately have a problem. Due to the closed loop tracking used, we have eliminated phase and frequency offset of signal 1 in chain A and signal 2 in chain B. However, if we are to estimate the signal of user 2 and remove it from user 1’s signal, we can not use the information from chain B. This is because the signal in chain B has a completely different phase and frequency offset than signal in chain A. This is true for all signals in the system if separate front ends are used. Thus, in order to allow straightforward cancellation, we must mix all users down with a single front end.

This requirement introduces a large problem. Namely, all but one signal in the system will have a phase and likely a frequency offset. In order to perform cancellation we must estimate these offsets. The coherence time of the channel as well as the resulting frequency offset will determine the rate at which the estimation will be necessary. This will be discussed more in the next section.

The fifth challenge will also be significant. As mentioned in Chapter 7, synchronization is an important issue. If acquisition is not possible, interference cancellation becomes useless. There are solutions to this problem however, as will be discussed in the following section.

The last challenge mentioned (phase estimation) is directly related to the previous problem of RF transmission. Since a common front end is required, we must find a method to eliminate the phase and frequency offset of each user. This too will be discussed in the next section.

### 10.4.2 Possible Solutions

In this section we discuss possible solutions to each of the challenges discussed in the previous section. These discussions should be helpful in the final phase of development. First, we discuss the issue of DSP processors. As mentioned the most significant barrier to implementation of additional stages and multipath tracking is not technical but practical. The current implementation simply does not have the processor speed to implement these additions. Several processors were examined and we show that the ADSP-2106x SHARC is a good choice to fill the need. The significant challenges of phase tracking and full signal acquisition are also explored and possible solutions are detailed.
Figure 10.18: Design for the RF Front End
Table 10.1: Performance Comparison of 32-bit Floating Point DSP Processors

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Chip</th>
<th>Clock Speed</th>
<th>Clock Cycles / instruction</th>
<th>Rating</th>
<th>1024pt FFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analog Devices</td>
<td>ADSP-2106x</td>
<td>25ns</td>
<td>1</td>
<td>120MFLOPS</td>
<td>0.46ms</td>
</tr>
<tr>
<td>AT&amp;T</td>
<td>ADSP32C/3210</td>
<td>20ns</td>
<td>4</td>
<td>25MFLOPS</td>
<td>3.2ms</td>
</tr>
<tr>
<td>Motorola</td>
<td>DSP96002</td>
<td>25ns</td>
<td>2</td>
<td>60MFLOPS</td>
<td>1.04ms</td>
</tr>
<tr>
<td>TI</td>
<td>TMS320C40</td>
<td>20ns</td>
<td>2</td>
<td>50MFLOPS</td>
<td>1.93ms</td>
</tr>
</tbody>
</table>

DSP Challenges

We have mentioned that the current processor is not capable of handling the processing requirements of the next phase of development. Thus, we must now decide upon the hardware and software which will best suit the remainder of the project. The main piece of hardware required is a DSP processor. Due to the dynamic range and ease of programming provided by floating point processing, we do not consider fixed point DSPs. In fact, the discussion will be limited to 32-bit floating point processors. When comparing DSP processors, there are several issues to consider. Among these are processor speed, on-chip memory, ease of programming, development tools available, support, external interface ease, and multiprocessing ability. A further consideration in this case is experience. It is important, when time is a consideration, to consider the experience of the personnel in the lab with different processors. The more experience available, the more favorable the DSP becomes since it require will less time to traverse the chip’s learning curve. We will look at each of these categories in turn.

Several speed measures for the four considered DSP processors are given in Table 10.1. (The four chips considered were the best chips available from each of the major DSP manufacturers.) While MFLOPS is the often quoted measure of processor speed, it can be deceiving. This is because a MFLOP rate is normally a peak rate based on instructions which perform the largest number of operations. However, if typically these are not the most common instructions used, the sustained number of operations per second can be significantly less than this. Thus, we also look at the instruction cycle of each of the chips as well as the 1024-point complex FFT time. The instruction cycle is determined from the clock speed and the number of clock cycles required to perform an instruction. From Table 10.1, we can see that the ADSP2106x is the fastest processor, with a 25 ns cycle time, the highest FLOP rating and the best FFT time. With the large number of operations required by multistage cancellation, speed becomes an extremely important consideration.
Table 10.2: Memory and I/O Comparison of 32-bit Floating Point DSP Processors

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Chip</th>
<th>I/O</th>
<th>Memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analog Devices</td>
<td>ADSP-2106x</td>
<td>1, 240Mbytes/s</td>
<td>120Kword data</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6, 20Mbytes/s</td>
<td>80Kword program</td>
</tr>
<tr>
<td>AT&amp;T</td>
<td>ADSP32C/3210</td>
<td>1 16bit serial</td>
<td>1.5Kwords</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 32 bit parallel</td>
<td></td>
</tr>
<tr>
<td>Motorola</td>
<td>DSP96002</td>
<td>non dedicated</td>
<td>1 Kword data</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2 Kword program</td>
</tr>
<tr>
<td>TI</td>
<td>TMS320C40</td>
<td>6, 20Mbytes/s</td>
<td>2 Kwords</td>
</tr>
</tbody>
</table>

As mentioned, the receiver structure presented here is extremely computationally intensive. While a fast processor will be helpful, the computational demand of 10-15 users at nearly 8Mmps indicates that a single processor will simply be inadequate. Thus, multiprocessing is a significant consideration. Table 10.2 shows the I/O ports for interprocessor communications for each chip under examination. It is seen that only the ADSP2106x and the TMS320C40 can truly be considered multi-processor oriented.

On-chip memory is another important consideration. If data must be retrieved from off-chip, either time must be sacrificed (slow external memory) or money must be sacrificed (fast external memory). It is thus desirable to have a large amount of on-chip program and data memory. The memory available for each of the chips considered is displayed in Table 4. The ADSP2106x has a significantly larger amount of on-chip memory available with as much as 4Mbits of SRAM.

Development tools are also a significant consideration. Of high importance are hardware emulators, software simulators, and C language compilers. These tools will make a big impact on the speed of implementation. Fortunately, each of the chips considered thus far have these development tools available. Another development tool to consider is a block diagram oriented software tool. One such tool is available by converting SPW [49] simulation code to assembly language. Software is currently available to convert SPW code to DSP96002 assembly language. Further, software is being developed to do the same for Analog Devices chips. A similar development tool is Hypersignal for Windows by Hyperception, Inc [50]. This software allows block diagram oriented development for Analog Devices chips including the 2106x. Also of great importance is the availability of a development board for the chip. Each of the chips considered is available on a DSP development board.

Some considerations are somewhat subjective, including ease of programming, support,
and experience. From the comments made by several involved in DSP development, it was found that Analog Devices has the most straightforward assembly language. Finally, experience is a key factor. Several researchers within the group currently have experience using Analog Devices processors. Staying with Analog Devices certainly has the advantage of shorter development time.

Based on the previous discussion, it has become apparent that the Analog Devices 2106x (SHARC) is the best choice for this project. It is the fastest chip, has by far the most on-chip memory at 4Mbits, and is most readily used on multiprocessing with 240Mbytes of I/O bandwidth. At present Bittware and Analog Devices both offer PC plug-in boards with a single SHARC processor. Further, both boards allow expansion to multiple processor configurations through SHARCPAC expansion modules available from Bittware. LSI is also in the process of creating development boards based on the SHARC. We have currently purchased two SHARC EZ-Lab development boards and will require additional SHARC processors for final implementation.

10.4.3 Phase and Frequency Offset

We have shown that due to the unique nature of interference cancellation, a single RF front end is desired for all users, since closed loop tracking of the phase and frequency is not amenable to interference cancellation. There are two possible solutions to this problem. The first is to perform phase estimation as outlined in Chapters 7 and 8. It was shown that by using a combination of averaging and cancellation, we can obtain reasonable phase estimation in a multiuser environment. The downside to this approach is that pilot symbols are required, which can be cumbersome. In a fast fading environment, this may simply require too much bandwidth.

The second solution is differentially coherent demodulation and cancellation as described in Chapters 4 and 8. While the results shown previously (see Figures 4.10 and 8.6) suggest that gains are achievable over conventional differential demodulation, there is a significant performance degradation suffered when going from coherent to differentially coherent demodulation. This trade-off will need to be studied experimentally.

10.4.4 Adding Full Synchronization

The final challenge associated with the next phase of implementation is the addition of synchronization for each user. Currently synchronization is obtained for a single user and the other users' delays with respect to the desired user are known a priori. The synchronization of all users will need to be obtained explicitly. In a steady state system, it can be assumed
that during a given time interval, only a single new user is entering the system. In such a situation, synchronization can be eased by using the residual signal for acquisition rather than the received signal. That is, the residual signal as defined in equation (10.5) will have interference of each of the synchronized users due only to cancellation error, as well as the signal of the unsynchronized user. This signal will have significantly less interference than the original received signal and thus will allow acquisition in high interference environments. The actual algorithm used for synchronization needs to be studied further (see [12]), but the IIR filter approach presented in Chapter 7 is one possible solution. If this approach is taken, the non-coherent version presented in Figure 7.13 should be used since phase information will not be available until after synchronization. After acquisition, phase estimation must be performed which can be followed by coherent processing for finer code tracking.

10.5 Discussion

This chapter has outlined the implementation of a baseband two stage interference cancellation receiver as well as laying the ground work for future development of an RF version. We have shown that the mathematical algorithms can be implemented in hardware and do achieve gains which are similar to those shown theoretically. This development lays the groundwork for a full-scale multistage cancellation receiver implementation.
Chapter 11

Summary and Future Work

This dissertation examined the use of multiuser detection at the base station of a cellular CDMA system. We summarize the results from this work in section 1. Several of the areas studied here are ripe for additional research. These future research areas are discussed in Section 2. Conclusions are presented in Section 3.

11.1 Summary of Results

This work has studied the use of multiuser detection in CDMA cellular systems. The very active area of multiuser detection research was thoroughly discussed in Chapter 1. From this investigation, six major receiver architectures were identified for simulation study including the conventional correlation receiver, multistage cancellation, successive cancellation, the decorrelating detector, the MMSE detector, and decision feedback employing a decorrelating feed-forward filter. These receivers were simulated using a Multiuser Testbed developed for this project. While each of these receiver structures have been previously discussed in the literature, they had yet to be directly compared using a single simulation procedure. Comparisons were made for AWGN, flat and frequency selective fading, and near-far situations. Additionally, results for timing error for each of the receiver structures were presented which were previously existent only for successive cancellation and the decorrelator in the literature. The effects which time synchronization errors have on system performance were studied for both perfect power control and near-far situations. Other issues studied were computational complexity and differentially coherent modulation. Multistage cancellation results for differentially coherent detection were presented for the first time. It is found that among these comparisons, multistage cancellation provides a good trade-off between performance and computational complexity. This receiver structure becomes the focus of
the rest of this work.

Chapter 5 presents theoretical analysis for multistage cancellation. This work augments previous analysis on this receiver structure and presents an analysis using an improved Gaussian Approximation for both AWGN and fading cases. A significant result of this analysis was that interference cancellation introduces a bias into the decision statistic which increases with the number of users. This bias can dominate performance in heavy loading situations.

The analysis of multistage cancellation as well as successive cancellation in near-far situations is presented in Chapter 6. It is found that interference cancellation in general cannot provide infinite resistance to large interference powers. It was shown however, that in some degenerate cases (e.g. two users for the multistage cancellation receiver and a single dominant interferer for the successive cancellation receiver) the receivers do provide near-far resistance. Additionally, while not satisfying the mathematical definition of near-far resistance in general, both structures provide significant near-far performance improvements when compared to the conventional receiver. A measure termed near-far robustness is developed and used to quantify this improvement.

A significant issue in interference cancellation (as opposed to interference rejection) is parameter estimation. For cancellation to perform properly, reliable estimates are required of the interferer's amplitude, phase, and timing. In Chapter 7 we present some simple but effective ways to significantly improve the estimation of these parameters in a multiuser environment using interference cancellation and averaging. The reduction in MSE is shown to be significant when employing these techniques together. These reductions in MSE translate directly into performance improvement, as shown in Chapter 8. In this chapter we present a detailed analysis of amplitude, phase, and timing errors for multistage cancellation. It is found that in the case of phase and timing errors, multistage cancellation is less robust to estimation error than conventional reception, but still significantly outperforms conventional reception with reasonable phase and timing errors.

A method of mitigating the bias effect discussed in Chapter 5 is presented in Chapter 9. It is found that by weighting the amplitude estimate in stage 2 of a multistage cancellation receiver, we can reduce the bias in the decision statistic. This method greatly improves the performance of the multistage receiver and is necessary in heavily loaded systems.

Chapter 10 presents the steps followed in the design of a baseband two stage cancellation receiver. Algorithms were created for real-time processing and minimizing required computations. The development phases are discussed performance results are presented. While
there was found to be significant variance in the performance of the receiver\textsuperscript{1}, particularly for low loading, it is found that the two stage receiver performs significantly better than the conventional receiver. Additionally, challenges are identified and solutions proposed for the implementation of the receiver at RF. The most significant challenges will be the implementation of full synchronization and the estimation of carrier phase and frequency offset. The latter challenge will require robust and frequent phase estimation as discussed in Chapter 7, or differentially coherent cancellation may be necessary as discussed in Chapters 4 and 8.

11.2 Future Work

The research described in this work leads naturally to several extensions. Here we present a few of these.

1. The Multiuser Testbed can be expanded in several ways. First, new receiver architectures can be added to the existing libraries. Of prime importance would be single users receivers such as Time Dependent Adaptive Filters [38]. These receivers have primarily been proposed for mobile unit implementation due to the fact that they do not require knowledge of the other users in the system. A direct comparison of these receiver structures in a multiuser uplink with multiuser receivers would be extremely interesting. It is possible that while multiuser receivers utilize knowledge of the interference, the cost in applying this knowledge may outweigh the gain achieved by using it. Additionally, while the current software allows several channel models to be simulated, the addition of actual measurement data would be an excellent augmentation.

2. Multiuser detection in general, and multistage cancellation specifically, may be compatible with adaptive antenna algorithms such as CMA as discussed in Chapter 9. The cursory work in Chapter 9 could be expanded to study specifically the case of random angles of arrival where interference cancellation can be beneficial even in the underloaded case since the separation between users may not be large enough for resolution. There are several practical issues related to combining these technologies and are ready for further research.

3. Extending the baseband receiver to additional stages and multipath tracking will require very little extra development outside of increasing the processing

\textsuperscript{1}As discussed in Chapter 10, performance in low loading was extremely dependent on the relative delays of the users.
power through a multiprocessor architecture. In the upcoming year, this receiver architecture will be moved to a reconfigurable computing platform as part on an ongoing DARPA sponsored project. While moving to a multiprocessor architecture will provide its own challenges, the modification to the codes and algorithms should be straightforward.

4. Extending the baseband receiver to include synchronization and Radio Frequency (RF) reception. These will provide larger challenges, but will help answer the remaining questions concerning the practicality of multistage reception. The current receiver performs synchronization on a single user only. Extending this to other users would require a more robust synchronization technique. Employing the Infinite Impulse Response (IIR) filter approach discussed in Chapter 7 should provide some robustness to the interference as will the use of the residual signal for tracking as opposed to using the original received signal. The move to RF will require phase estimation as outlined in Chapter 10. Chapter 7 provides methods of improving phase estimation for the multiuser environment. The implementation of these techniques should help the phase estimation problem. This problem could be a research in itself itself. If there exists a significant frequency offset, phase estimation may not be possible. In such a case, differentially coherent cancellation may be desirable as discussed in Chapters 4 and 10.

5. Other extensions include studying the current receiver in multipath fading (either by introducing fading in software or hardware), near-far environments and using fixed-point processing.

11.3 Conclusions

This research has shown that multiuser reception can be a practical and very beneficial augmentation of cellular CDMA systems. Several practical issues were analyzed such as parameter estimation and the effect of estimation errors. Additionally, multiuser detection was implemented at baseband and shown to provide significant performance improvement. This work has also thoroughly investigated the performance of multistage detection analytically and through extensive simulation. Of particular interest among these results was this fact that this type of interference cancellation introduces a bias in the decision statistic which can devastate performance. Means of mitigating this effect were presented and analyzed via simulation.
This work has laid the groundwork for the next step in multistage cancellation: full RF implementation. By identifying and investigating the key issues in multistage cancellation, this work has advanced the knowledge of multistage cancellation and has helped to show that such an implementation is both practical and beneficial. The push for higher and higher spectral efficiency will likely result in interference cancellation techniques being used in cellular systems. By investigating issues crucial to multistage cancellation implementation, it is hoped that we have taken a step towards showing that multiuser detection (specifically multistage interference cancellation) can help cellular CDMA provide the high spectral efficiency that will be required.
Appendix A

Proof of Average Interference Power

From equation (5.44) the mean of the interference power at stage \( s + 1 \) must satisfy the recursive formula:

\[
\mu^{(s+1)}_{P_j} = \frac{N_o}{2T} + \frac{1}{3N} \sum_{j=1, j \neq k}^{K} \mu^{(s)}_{P_j}
\]  

(A.1)

where the initial condition is given for \( s = 1 \) by \( \mu^{(1)}_{P_k} = \mathbb{E}[P_k] \). We would like to show that the mean at stage \( s + 1 \) has the form of equation (5.45). We can show this to be true by mathematical induction. First we use the initial condition \( s = 1 \) to find the mean at stage 2.

\[
\mu^{(2)}_{P_j} = \frac{N_o}{2T} \left[ 1 - \frac{(K-1)^{1}}{1 - \frac{1}{3N}} \right] + \frac{1}{3N} \left[ \frac{(K-1)^{1} - (-1)^{1}}{K} \left( \sum_{j=1}^{K} \mu^{(1)}_{P_j} \right) + (-1)^{1} \mu^{(1)}_{P_k} \right]
\]

(A.2)

which agrees with the result implicit in (5.44). Now we assume that (5.45) is true for stage \( s \) and would like to find the mean at stage \( s + 1 \) using (5.44):

\[
\mu^{(s+1)}_{P_{i,k}} = \frac{N_o}{2T} + \frac{1}{3N} \left( \sum_{j=1, j \neq k}^{K} \left\{ \frac{N_o}{2T} \left[ 1 - \frac{(K-1)^{s-1}}{1 - \frac{1}{3N}} \right] \right. \right.
\]

\[
+ \frac{1}{(3N)^{s-1}} \left[ \frac{(K-1)^{s-1} - (-1)^{s-1}}{K} \sum_{j=1}^{K} \mu^{(1)}_{P_j} + (-1)^{s-1} \mu^{(1)}_{P_k} \right] \left( \right) \right) \right) (A.3)
\]

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\[
\begin{align*}
&= \frac{N_o}{2T} + \frac{1}{3N} \left\{ \frac{N_o(K - 1)}{2T} \left[ 1 - \left( \frac{K - 1}{3N} \right)^{s-1} \right] \right\} \\
&\quad + \frac{K - 1}{(3N)^s} \left[ \frac{(K - 1)^{s-1} - (-1)^{s-1}}{K} \sum_{j=1}^{K} \mu_{P_j}^{(1)} + (-1)^{s-1} \sum_{j=1, j \neq k}^{K} \mu_{P_k}^{(1)} \right] \tag{A.4}
\end{align*}
\]

\[
\begin{align*}
&= \frac{N_o}{2T} \left\{ \frac{1 - \frac{K - 1}{3N} + \frac{K - 1}{3N} - \left( \frac{K - 1}{3N} \right)^{s}}{1 - \frac{K - 1}{3N}} \right\} \\
&\quad + \frac{1}{(3N)^s} \left[ \frac{(K - 1)^{s} - (-1)^{s}}{K} \sum_{j=1}^{K} \mu_{P_j}^{(1)} - (-1)^{s-1} \sum_{j=1}^{K} \mu_{P_k}^{(1)} + (-1)^{s-1} \sum_{j=1, j \neq k}^{K} \mu_{P_k}^{(1)} \right]. \tag{A.5}
\end{align*}
\]

After some final manipulation we obtain:

\[
\mu_{P; k}^{(s+1)} = \frac{N_o}{2T} \left\{ \frac{1 - \left( \frac{K - 1}{3N} \right)^{s}}{1 - \frac{K - 1}{3N}} \right\} + \frac{1}{(3N)^s} \left[ \frac{(K - 1)^{s} - (-1)^{s}}{K} \sum_{j=1}^{K} \mu_{P_j}^{(1)} + (-1)^{s} \mu_{P_k}^{(1)} \right]. \tag{A.6}
\]

which completes the proof of equation (5.45).
Appendix B

Derivation of Expression for Variance

We wish to evaluate the variance of the residual interference power, after \( s \) stages of interference cancellation given by:

\[
(s_{P_k}^{(s+1)})^2 = \mathbb{E} \left[ (\nu_k^{(s+1)})^2 \right] - (\mu_{P_k}^{(s+1)})^2. \tag{B.1}
\]

From (5.38) we have the expression for the residual power after stage \( s \):

\[
\nu_k^{(s+1)} = P_j - \sqrt{8P_j b_{j,i-1}} \frac{Z_{j,i}^{(s)}}{T} + 2 \left( \frac{Z_{j,i}^{(s)}}{T} \right)^2. \tag{B.2}
\]

Substituting (B.1) into (B.2) results in:

\[
(s_{P_k}^{(s+1)})^2 = \mathbb{E} \left[ P_k^2 - 2\sqrt{8P_k b_k} \left( \frac{Z_{k}^{(s)}}{T} \right)^2 - 4\sqrt{8P_k} \left( \frac{Z_{k}^{(s)}}{T} \right)^3 b_k + 4 \left( \frac{Z_{k}^{(s)}}{T} \right)^4 \right] - (\mu_{P_k}^{(s+1)})^2. \tag{B.3}
\]

First, taking the expectation of the foregoing expression conditioned on \( P_k \) we have the following terms:

\[
\mathbb{E} \left[ \frac{Z_{k}^{(s)}}{T} \left| P_k \right. \right] = \frac{\sqrt{P_k}}{2} b_k \tag{B.4}
\]

\[
\mathbb{E} \left[ \left( \frac{Z_{k}^{(s)}}{T} \right)^2 \left| P_k \right. \right] = \frac{N_o}{4T} + \frac{P_k}{2} + \frac{\mathbb{E} \left[ \left( \sum_{j \neq k} I_{j,k}^{(s)} \right)^2 \left| P_k \right. \right]}{T^2} \tag{B.5}
\]
\[
E \left[ \left( \frac{Z_{k}^{(s)}}{T} \right)^{3} \mid P_{k} \right] = \frac{3N_{o}}{4T} \sqrt{\frac{P_{k}}{2} b_{k}} + \frac{3E \left[ \left( \sum_{j \neq k} I_{j,k}^{(s)} \right)^{2} \mid P_{k} \right]}{T^{2}} \sqrt{\frac{P_{k}}{2} b_{k}} + \left( \sqrt{\frac{P_{k}}{2} b_{k}} \right)^{3}
\]

(B.6)

\[
E \left[ \left( \frac{Z_{k}^{(s)}}{T} \right)^{4} \mid P_{k} \right] = \frac{P_{k}^{2}}{4} + \frac{3N_{o}P_{k}}{4T} + \frac{3E \left[ \left( \sum_{j \neq k} I_{j,k}^{(s)} \right)^{2} \mid P_{k} \right]}{T^{2}}
+ \frac{3P_{k}N_{o}E \left[ \left( \sum_{j \neq k} I_{j,k}^{(s)} \right)^{2} \mid P_{k} \right]}{T^{3}} + \frac{E \left[ \left( \sum_{j \neq k} I_{j,k}^{(s)} \right)^{4} \mid P_{k} \right]}{T^{4}} + \frac{3N_{o}}{16T^{2}}
\]

(B.7)

Substituting these values into (B.3) and simplifying allows cancellation of all terms with strict dependence on \( P_{k} \) resulting in:

\[
(\sigma_{P_{k}}^{(s+1)})^{2} = E_{P_{k}} \left[ \frac{3N_{o}}{T^{2}} \right] + \frac{6N_{o}E \left[ \left( \sum_{j \neq k} I_{j,k}^{(s)} \right)^{2} \mid P_{k} \right]}{T^{3}} + \frac{4E \left[ \left( \sum_{j \neq k} I_{j,k}^{(s)} \right)^{4} \mid P_{k} \right]}{T^{4}} \right] - (\mu_{P_{k}}^{(s+1)})^{2}
\]

(B.8)

where \( E_{P_{k}}[\cdot] \) is the expectation taken over \( P_{k} \). Since the first term is constant, we must evaluate only the second and third terms. Thus,

\[
E_{P_{k}} \left[ E \left[ \sum_{j \neq k} I_{j,k}^{(s)} \mid P_{k} \right] \right] = E \left[ \text{var} \left( \sum_{j \neq k} I_{j,k}^{(s)} \right) \right] = E \left[ \Psi^{(s)} \right] = \mu_{\Psi^{(s)}}
\]

(B.9)

and

\[
E_{P_{k}} \left[ E \left[ \left( \sum_{j \neq k} I_{j,k}^{(s)} \right)^{4} \mid P_{k} \right] \right] = E \left[ \sum_{j \neq k} I_{j,k}^{(s)} \right]^{4} + \frac{3}{2} \sum_{j \neq k} \sum_{l \neq j} E \left[ I_{j,k}^{(s)} \right]^{2} \left( I_{l,k}^{(s)} \right)^{2}
\]

(B.10)

Evaluating the first term of (B.10):

\[
E \left[ \sum_{j \neq k} I_{j,k}^{(s)} \right]^{4} = E \left[ \sum_{j \neq k} \left( T_{c} \sqrt{\frac{\nu_{j}^{(s)}}{2} \cos(\phi_{j}) W_{j}} \right)^{4} \right]
\]

(B.11)

\[
= \frac{374}{32} \sum_{j \neq k} E \left[ (\nu_{j}^{(s)})^{2} \right] E \left[ (W_{j})^{4} \right]
\]

where from (5.46)

\[
E \left[ (\nu_{j}^{(s)})^{2} \right] = (\sigma_{\nu_{j}^{(s)}})^{2} + (\mu_{\nu_{j}^{(s)}})^{2}
\]

(B.12)

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We evaluate $E[(W_j)^4]$ by expanding (5.9)

\[
E[(W_j)^4] = E \left[ X_j^4 + (1 - \frac{2\Delta_j}{T_c})^4 Y_j^4 + (1 - \frac{\Delta_j}{T_c})^4 U_j^4 + \left( \frac{\Delta_j}{T_c} \right)^4 V_j^4 \right] \\
+ 6X_j^3 \left( 1 - \frac{2\Delta_j}{T_c} \right)^2 Y_j^2 + 6X_j^2 \left( 1 - \frac{\Delta_j}{T_c} \right)^2 U_j^2 + 6X_j \left( \frac{\Delta_j}{T_c} \right)^2 V_j^2 \\
+ 6 \left( 1 - \frac{2\Delta_j}{T_c} \right)^2 Y_j^2 \left( \frac{\Delta_j}{T_c} \right)^2 V_j^2 + 6 \left( 1 - \frac{\Delta_j}{T_c} \right)^2 U_j^2 \left( \frac{\Delta_j}{T_c} \right)^2 V_j^2 \right] \quad (B.13)
\]

Since $\Delta_j$ is uniformly distributed on $[0, T_c]$, it is straightforward to compute the moments:

\[
E \left[ (1 - \frac{2\Delta_j}{T_c})^4 \right] = E \left[ (1 - \frac{\Delta_j}{T_c})^4 \right] = E \left[ \left( \frac{\Delta_j}{T_c} \right)^4 \right] = \frac{1}{5} \quad (B.14)
\]

\[
E \left[ (1 - \frac{2\Delta_j}{T_c})^2 \right] = E \left[ (1 - \frac{\Delta_j}{T_c})^2 \right] = E \left[ \left( \frac{\Delta_j}{T_c} \right)^2 \right] = \frac{1}{3} \quad (B.15)
\]

\[
E \left[ \left( 1 - \frac{2\Delta_j}{T_c} \right)^2 \left( 1 - \frac{\Delta_j}{T_c} \right)^2 \right] = E \left[ (1 - \frac{2\Delta_j}{T_c})^2 \left( \frac{\Delta_j}{T_c} \right)^2 \right] = \frac{2}{15} \quad (B.16)
\]

\[
E \left[ \left( 1 - \frac{\Delta_j}{T_c} \right)^2 \left( \frac{\Delta_j}{T_c} \right)^2 \right] = \frac{1}{10} \quad (B.17)
\]

Furthermore, individual terms in equation (B.13) may be evaluated using $E[(W_j)^4] = E_B \left[ E[W_j^4 | B] \right]$ by taking the conditional expectation given $B$ and then taking the expectation over $B$, yielding:

\[
E[X_j^2] = E[Y_j^2] = E[X_j^2 U_j^2] = E[Y_j^2 U_j^2] = E[Y_j^2 V_j^2] = \frac{N - 1}{2} \quad (B.18)
\]

\[
\]

\[
E[X_j^4] = E[Y_j^4] = \frac{3N^2 - 7N + 4}{4} \quad (B.20)
\]

\[
E[X_j^2 Y_j^2] = \frac{N^2 - 5N + 2}{4} \quad (B.21)
\]

Inserting the previous equations into (B.13), we find that

\[
E[W_j^4] = \frac{12N^2 - 9N}{5} \quad (B.22)
\]

The second term in (B.10) can be evaluated as:

\[
E \left[ 3 \sum_{j \neq k} \sum_{l \neq j} \left( T_c \left( \frac{\nu(s)}{2} \cos(\phi_j) W_j \right)^2 \left( T_c \sqrt{\frac{\nu(s)}{2} \cos(\phi_l) W_l} \right) \right) \right] = \frac{T_c^4(4N^2 - 9N + 13)}{48} \sum_{j \neq k} \sum_{l \neq j, l \neq k} \mu_{P_j} \mu_{P_l} \quad (B.23)
\]

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Finally, by substituting the previous results along with

$$\mu_{P_k}^{(s+1)} = \frac{N_o}{2T} + \frac{2\mu_{\psi(s)}}{T^2}$$

(B.24)

into equation (B.3), we obtain after some manipulation:

$$\begin{align*}
(\sigma_{P_k}^{(s+1)})^2 &= \frac{N_o^2}{2T^2} + \frac{4N_o}{T^3 \mu_{\psi(s)}} - \frac{4}{T^4} (\mu_{\psi(s)})^2 + \frac{9(4N^2 - 3N)}{40N^4} \sum_{j \neq k} \left( (\sigma_{P_j}^{(s)})^2 + (\mu_{P_j}^{(s)})^2 \right) \\
&\quad + \frac{4N^2 - 9N + 13}{12N^4} \sum_{j \neq k} \sum_{l \neq k, l \neq j} \mu_{P_j}^{(s)} \mu_{P_l}^{(s)}
\end{align*}$$

(B.25)

which is the desired result.
Appendix C

Proof of Variance in the Presence of Timing Errors

From equation (8.41) the variance of the decision statistic of the multistage receiver in the presence of timing errors must satisfy the recursive relation

\[
\text{var} \left( Z_{k,i}^{(s)} \right) = \frac{N_o T}{4} + \frac{T^2 N}{3} \left[ \alpha \sum_{\kappa=1}^{K} P_\kappa + \gamma \sum_{\kappa=1}^{K} \text{var} \left( Z_{k,i}^{(s-1)} \right) \right].
\]  \hspace{1cm} (C.1)

We postulate that

\[
\text{var} \left( Z_{k,i}^{(s)} \right) = \frac{N_o T}{4} \left[ 1 - \left( \frac{T^2 (K-1)}{3N} \right)^s \right] + \frac{\alpha}{K \gamma} \left[ 1 - \left( \frac{T^2 (K-1)}{3N} \right)^s \right] \left[ 1 - \left( \frac{T^2 (K-1)}{3N} \right)^s \right] - \frac{1}{1 + \left( \frac{T^2}{3N} \right)} \sum_{\kappa=1}^{K} P_\kappa \\
+ \frac{\alpha}{\gamma} \left[ \left( \frac{-T^2}{3N} \right) - \left( \frac{-T^2}{3N} \right)^s \right] P_k + \frac{1}{2 \gamma} \left( \frac{T^2 \gamma}{3N} \right)^s \left[ \frac{(K-1)^s}{K} - \frac{(-1)^s}{K} \sum_{\kappa=1}^{K} P_\kappa + (1-s) P_k \right].
\]  \hspace{1cm} (C.2)

First, we check (C.2) for \( s = 1 \):

\[
\text{var} \left( Z_{k,i}^{(1)} \right) = \frac{N_o T}{4} \left[ 1 - \left( \frac{T^2 (K-1)}{3N} \right) \right] + \frac{\alpha}{K \gamma} \left[ 1 - \left( \frac{T^2 (K-1)}{3N} \right) \right] - \frac{1}{1 + \left( \frac{T^2}{3N} \right)} \sum_{\kappa=1}^{K} P_\kappa \\
+ \frac{\alpha}{\gamma} \left[ \left( \frac{-T^2}{3N} \right) - \left( \frac{-T^2}{3N} \right)^s \right] P_k + \frac{1}{2 \gamma} \left( \frac{T^2 \gamma}{3N} \right) \left[ \frac{(K-1)}{K} - \frac{(-1)^s}{K} \sum_{\kappa=1}^{K} P_\kappa + (-1) P_k \right]
\]

\[
= \frac{N_o T}{4} + \frac{\alpha}{K \gamma} \left( 1 - \frac{1}{K} \right) \sum_{\kappa=1}^{K} P_\kappa + \frac{\alpha}{\gamma} (0) P_k + \frac{T^2}{6N} \left( \sum_{\kappa=1}^{K} P_\kappa - P_k \right)
\]

\[
= \frac{N_o T}{4} + \frac{T^2 N}{6} \sum_{\kappa=1}^{K} P_\kappa - \frac{T^2 N}{6} P_k
\]  \hspace{1cm} (C.3)
where we have used the fact that $T = T_c N$. We can see that this result checks with that of (8.31). Now we assume that (C.2) holds for stage $s$ and would like to find the variance at stage $s + 1$. From (C.1) we have

\[
\text{var} (Z_{k,t}^{(s+1)}) = \frac{N_0 T}{4} + \frac{T_c^2 N}{3} \left\{ \alpha \sum_{\kappa=1}^{K} P_\kappa + \gamma \sum_{\kappa=1}^{K} \left[ \frac{N_0 T}{4} \left( \frac{1 - \left( \frac{\gamma T^2 (K-1)}{3N} \right)^s}{1 - \left( \frac{\gamma T^2 (K-1)}{3N} \right)^s} \right) + \frac{\alpha}{K \gamma} \left( \frac{1 - \left( \frac{\gamma T^2 (K-1)}{3N} \right)^s}{1 - \left( \frac{\gamma T^2 (K-1)}{3N} \right)^s} \right) \sum_{\kappa=1}^{K} P_\kappa + \frac{\alpha}{\gamma} \left( \frac{1 - \left( \frac{-\gamma T^2}{3N} \right)^s}{1 + \left( \frac{-\gamma T^2}{3N} \right)^s} \right) P_k + \frac{1}{2\gamma} \left( \frac{T^2 \gamma}{3N} \right)^s \left( \frac{(K-1)^s - (-1)^s}{K} \sum_{\kappa=1}^{K} P_\kappa + (-1)^s P_k \right) \right] \right\}.
\]

(C.4)

Rearranging results in

\[
= \frac{N_0 T}{4} \left[ 1 + \frac{(K-1)\gamma T_c^2 N}{3} \left[ \frac{1 - \left( \frac{\gamma T^2 (K-1)}{3N} \right)^s}{1 - \left( \frac{\gamma T^2 (K-1)}{3N} \right)^s} \right] \right] + \frac{T_c^2 N}{3} \alpha \sum_{\kappa=1}^{K} P_\kappa

+ \frac{T_c^2 N}{3} \gamma \frac{\alpha}{\gamma} \left[ 1 - \left( \frac{\gamma T^2 (K-1)}{3N} \right)^s \right] \frac{1 - \left( \frac{-\gamma T^2}{3N} \right)^s}{1 + \left( \frac{-\gamma T^2}{3N} \right)^s} \sum_{\kappa=1}^{K} P_\kappa

+ \frac{T_c^2 N}{3} \gamma \frac{1}{2\gamma} \left( \frac{T^2 \gamma}{3N} \right)^s \sum_{\kappa=1}^{K} (-1)^s P_\kappa.
\]

(C.5)

Simplifying the double summations provides

\[
= \frac{N_0 T}{4} \left[ 1 + \frac{(K-1)\gamma T_c^2 N}{3} \left[ \frac{1 - \left( \frac{\gamma T^2 (K-1)}{3N} \right)^s}{1 - \left( \frac{\gamma T^2 (K-1)}{3N} \right)^s} \right] \right] + \frac{T_c^2 N}{3} \alpha \sum_{k=1}^{K} P_k - P_k

+ \frac{T_c^2 N}{3} \frac{\alpha}{K} \left[ 1 - \left( \frac{\gamma T^2 (K-1)}{3N} \right)^s \right] \frac{1 - \left( \frac{-\gamma T^2}{3N} \right)^s}{1 + \left( \frac{-\gamma T^2}{3N} \right)^s} \left( K - 1 \right) \sum_{\kappa=1}^{K} P_\kappa

+ \frac{T_c^2 N}{3} \alpha \left[ 1 - \left( \frac{\gamma T^2}{3N} \right)^s \right] \left( \sum_{\kappa=1}^{K} P_\kappa - P_k \right) + \frac{1}{2\gamma} \left( \frac{T^2 \gamma}{3N} \right)^{s+1} \left( \frac{(K-1)^s - (-1)^s}{K} \right) (K-1) \sum_{\kappa=1}^{K} P_\kappa

+ \frac{1}{2\gamma} \left( \frac{T^2 \gamma}{3N} \right)^{s+1} \sum_{\kappa=1}^{K} (-1)^s P_\kappa.
\]

(C.6)
Multiplying through and collecting like terms gives

\[
\frac{N_c T}{4} \left[ 1 - \frac{(k-1)\gamma T^2 N}{3} + \frac{(k-1)\gamma T^2 N}{3} - \frac{\gamma T^2 (K-1)}{3N} \right] + \frac{\alpha}{K \gamma} \sum_{\kappa=1}^{K} P_\kappa \left\{ \frac{T^2 \gamma}{3N} K + \frac{T^2 (K-1)}{3N} \left[ 1 - \frac{\gamma T^2 (K-1)}{3N} \right] \right\} + \frac{1}{\gamma} \sum_{\kappa=1}^{K} P_\kappa \left\{ \frac{T^2 \gamma}{3N} K + \frac{T^2 (K-1)}{3N} \left[ 1 - \frac{\gamma T^2 (K-1)}{3N} \right] \right\} + \frac{1}{2\gamma} \left( \frac{T^2 \gamma}{3N} \right)^{s+1} \frac{(K-1)^{s+1} - K(-1)^s - (-1)^{s+1}}{K} \sum_{\kappa=1}^{K} P_\kappa \\
+ \frac{1}{2\gamma} \left( \frac{T^2 \gamma}{3N} \right)^{s+1} \sum_{\kappa=1, \kappa \neq k}^{K} (-1)^s P_\kappa. \quad (C.7)
\]

Collecting common denominators and simplifying results in

\[
\frac{N_c T}{4} \left[ 1 - \frac{(\gamma T^2 (K-1))^{s+1}}{3N} \right] + \frac{\alpha}{K \gamma} \sum_{\kappa=1}^{K} P_\kappa \left\{ \frac{\gamma T^2 (K-1)}{3N} - \frac{\gamma T^2 (K-1)}{3N} \right\} - \frac{(-\gamma T^2)^s}{3N} K \\
- \left( \frac{\gamma T^2 (K-1)}{3N} \right) \left[ 1 - \frac{(-\gamma T^2)^s}{3N} \right] - \left( \frac{\gamma T^2}{3N} \right) K \left( \frac{-\gamma T^2}{3N} \right)^{s+1} \right\} \right\} + \frac{1}{\gamma} \left( \frac{T^2 \gamma}{3N} \right)^{s+1} \frac{(K-1)^{s+1} - (-1)^{s+1}}{K} \sum_{\kappa=1}^{K} P_\kappa - (-1)^s \sum_{\kappa=1}^{K} P_\kappa + (-1)^s \sum_{\kappa=1}^{K} P_\kappa \right\} \right\} \right\} \right\}. \quad (C.8)
\]

Adding and subtracting 1 from the second group of terms and continuing simplification of the other groups gives

\[
\frac{N_c T}{4} \left[ 1 - \frac{(\gamma T^2 (K-1))^{s+1}}{3N} \right] + \frac{\alpha}{K \gamma} \sum_{\kappa=1}^{K} P_\kappa \left\{ 1 - \frac{(\gamma T^2 (K-1))^{s+1}}{3N} \right\} - \frac{(-\gamma T^2)^s}{3N} K \left( \frac{-\gamma T^2}{3N} \right)^{s+1} \right\} \right\} + \frac{1}{\gamma} \left( \frac{T^2 \gamma}{3N} \right)^{s+1} \frac{(K-1)^{s+1} - (-1)^{s+1}}{K} \sum_{\kappa=1}^{K} P_\kappa - (-1)^s \sum_{\kappa=1}^{K} P_\kappa + (-1)^s \sum_{\kappa=1}^{K} P_\kappa \right\} \right\} \right\}. \quad (C.8)
\]

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\[
\frac{\alpha}{\gamma} P_k \left[ \frac{\left(\frac{-T^2}{3N}\right) - \left(\frac{-T^2}{3N}\right)^{s+1}}{1 - \left(\frac{-T^2}{3N}\right)} \right] \\
+ \frac{1}{2\gamma} \left(\frac{T^2\gamma}{3N}\right)^{s+1} \left[ \frac{(K - 1)^{s+1} - (-1)^{s+1}}{K} \sum_{k=1}^{K} P_k - (-1)^s P_k \right].
\]

Placing all terms above common denominators results in

\[
= \frac{N_0 T}{4} \left[ 1 - \left(\frac{-T^2(K-1)}{3N}\right)^{s+1} \right] + \frac{\alpha}{K\gamma} \sum_{k=1}^{K} P_k \left\{ 1 - \left(\frac{-T^2(K-1)}{3N}\right)^{s+1} \right\} \\
- \frac{1}{2\gamma} \left(\frac{T^2\gamma}{3N}\right)^{s+1} \left[ \frac{(K - 1)^{s+1} - (-1)^{s+1}}{K} \sum_{k=1}^{K} P_k + (-1)^s P_k \right].
\]

Finally, simplifying the middle term provides

\[
\text{var} (\varphi_{k,i}^{(s+1)}) = \frac{N_0 T}{4} \left[ 1 - \left(\frac{-T^2(K-1)}{3N}\right)^{s+1} \right] \\
+ \frac{\alpha}{K\gamma} \sum_{k=1}^{K} P_k \left[ 1 - \left(\frac{-T^2(K-1)}{3N}\right)^{s+1} \right] \\
- \frac{1}{2\gamma} \left(\frac{T^2\gamma}{3N}\right)^{s+1} \left[ \frac{(K - 1)^{s+1} - (-1)^{s+1}}{K} \sum_{k=1}^{K} P_k + (-1)^s P_k \right].
\]

which is the desired result.
Appendix D

Proof of Variance with Phase Errors

From (8.20) we know that the variance of the decision statistic in the presence of phase errors satisfies the recursive relation

$$\text{var} \left( Z_{k,i}^{(s)} \right) = \frac{N_0 T}{4} + \sum_{\kappa=1}^{K} \left[ \frac{T_c^2 N P_\kappa}{3} \left( 1 - \cos^2(\psi) \right) + \frac{1}{3N} \text{var} \left( Z_{k,i}^{(s-1)} \right) \right]. \quad (D.1)$$

We postulate that the recursive equation in (D.1) can be solved to give

$$\text{var} \left( Z_{k,i}^{(s)} \right) = \frac{N_0 T}{4} \left[ 1 - \left( \frac{K-1}{3N} \right)^s \right] + \frac{T_c^2 (1 - \cos^2(\psi))}{2} \left[ \frac{1}{K} \sum_{k=1}^{K} P_k \frac{1 - \left( \frac{K-1}{3N} \right)^s}{1 - \left( \frac{K-1}{3N} \right)} - \frac{1 - \left( \frac{1}{3N} \right)^s}{1 + \left( \frac{1}{3N} \right)} \right]$$

$$- P_k \left[ \frac{1}{3N} - \left( \frac{1}{3N} \right)^s \right] + \frac{T_c^2}{2(3N)^s} \left[ \frac{(K-1)^s - (-1)^s}{K} \sum_{k=1}^{K} + (-1)^s P_k \right]. \quad (D.2)$$

Letting $s = 1$ results in

$$\text{var} \left( Z_{k,i}^{(1)} \right) = \frac{N_0 T}{4} \left[ 1 - \left( \frac{K-1}{3N} \right) \right] + \frac{T_c^2 (1 - \cos^2(\psi))}{2} \left[ \frac{1}{K} \sum_{k=1}^{K} P_k \frac{1 - \left( \frac{K-1}{3N} \right)}{1 - \left( \frac{K-1}{3N} \right)} - \frac{1 - \left( \frac{1}{3N} \right)}{1 + \left( \frac{1}{3N} \right)} \right]$$

$$- P_k \left[ \frac{1}{3N} - \left( \frac{1}{3N} \right)^s \right] + \frac{T_c^2}{2(3N)} \left[ \frac{(K-1) - (-1)}{K} \sum_{k=1}^{K} + (-1) P_k \right]$$

$$= \frac{N_0 T}{4} + \frac{T_c^2 N}{6} \sum_{\kappa=1}^{K} P_\kappa \quad (D.3)$$
which agrees with the result from (8.31). Now assuming that (D.2) holds for stage \( s \), we can solve for the variance at stage \( s + 1 \) as

\[
\text{var} \left( Z_{k,s}^{(s+1)} \right) = \frac{N_c T}{4} + \sum_{\kappa = 1}^{K} \left[ \frac{T_c^2 N P_k}{3} (1 - \cos^2(\psi)) + \frac{1}{3N} \left\{ \frac{N_c T}{4} \left[ \frac{1 - \left( \frac{(K-1)}{3N} \right)^s}{1 - \left( \frac{(K-1)}{3N} \right)^s} \right] \right. \right.
\]

\[
+ T^2 (1 - \cos^2(\psi)) \left[ \frac{1}{K} \sum_{k=1}^{K} P_k \frac{1 - \left( \frac{(K-1)}{3N} \right)^s}{1 - \left( \frac{(K-1)}{3N} \right)^s} - \frac{1 - \left( \frac{-1}{3N} \right)^s}{1 + \left( \frac{1}{3N} \right)^s} \right] - \frac{1}{3N} \sum_{k=1}^{K} P_k \frac{1}{3N} - \left( \frac{-1}{3N} \right)^s \right]
\]

\[
+ \frac{T^2}{2(3N)^s} \left[ \frac{(K - 1)^s}{K} \sum_{k=1}^{K} P_k + (-1)^s P_k \right] \right) \right]. \tag{D.4}
\]

Applying the summation throughout the second term and rearranging gives

\[
= \frac{N_c T}{4} \left[ 1 + \frac{K - 1}{3N} \left( \frac{1 - \left( \frac{(K-1)}{3N} \right)^s}{1 - \left( \frac{(K-1)}{3N} \right)^s} \right) \right] + \frac{NT_c^2}{6} (1 - \cos^2(\psi)) \sum_{\kappa = 1}^{K} P_k
\]

\[
+ \frac{T^2 (1 - \cos^2(\psi))}{2} \frac{1}{3N} \left[ \frac{1}{K} \sum_{k=1}^{K} P_j \frac{1 - \left( \frac{(K-1)}{3N} \right)^s}{1 - \left( \frac{(K-1)}{3N} \right)^s} - \frac{1 - \left( \frac{-1}{3N} \right)^s}{1 + \left( \frac{1}{3N} \right)^s} \right] - \sum_{k=1}^{K} P_k \frac{1}{3N} - \left( \frac{-1}{3N} \right)^s \right]
\]

\[
+ \frac{1}{3N} \frac{T^2}{2(3N)^s} \left[ \frac{(K - 1)^s}{K} \sum_{k=1}^{K} P_j + (-1)^s \sum_{k=1}^{K} P_k \right]. \tag{D.5}
\]

Placing the first term above a common denominator and simplifying summations gives

\[
= \frac{N_c T}{4} \left[ 1 - \left( \frac{(K-1)}{3N} \right)^s + \left( \frac{(K-1)}{3N} \right)^{s+1} \right] + \frac{NT_c^2}{6} (1 - \cos^2(\psi)) \left[ \sum_{\kappa = 1}^{K} P_\kappa - P_k \right]
\]

\[
+ \frac{T^2 (1 - \cos^2(\psi))}{6NK} \left[ 1 - \left( \frac{(K-1)}{3N} \right)^s + \frac{1 - \left( \frac{-1}{3N} \right)^s}{1 + \left( \frac{1}{3N} \right)^s} \right] (K - 1) \sum_{k=1}^{K} P_k
\]

\[
+ \frac{T^2 (1 - \cos^2(\psi))}{6N} \left( \sum_{k=1}^{K} P_k - P_k \right) \left[ \frac{1}{3N} - \left( \frac{-1}{3N} \right)^s \right]
\]

\[
+ \frac{T^2}{2(3N)^s+1} \left[ \frac{(K - 1)^s}{K} \sum_{k=1}^{K} P_k + (-1)^s \sum_{k=1}^{K} P_k \right]. \tag{D.6}
\]

Collecting like terms results in

\[
= \frac{N_c T}{4} \left[ 1 - \left( \frac{(K-1)}{3N} \right)^{s+1} \right] + \]
\[
\frac{NT^2}{6} (1 - \cos^2(\psi)) \left[ \sum_{\kappa=1}^{K} P_\kappa \left( \frac{1}{3N} + \frac{K-1}{3NK} \left[ 1 - \left( \frac{(K-1)}{3N} \right)^s \right] \right) \right] \\
- \frac{1}{3N} \left[ \frac{1}{3N} - \left( \frac{1}{3N} \right)^s \right] - P_k \left( \frac{1}{3N} - \frac{1}{3N} \left[ 1 + \left( \frac{1}{3N} \right)^s \right] \right) \\
+ \frac{T^2}{2(3N)^{s+1}} \left[ \frac{(K-1)^{s+1} - K(-1)^s - (-1)^{s+1}}{K} \sum_{\kappa=1}^{K} P_\kappa + (-1)^s \sum_{\kappa=1}^{K} P_\kappa \right].
\]

Further rearrangement yields
\[
= \frac{N_o T}{4} \left[ \frac{1 - \left( \frac{(K-1)}{3N} \right)^{s+1}}{1 - \left( \frac{(K-1)}{3N} \right)} \right] + \frac{NT^2}{6} (1 - \cos^2(\psi)) \left\{ \sum_{\kappa=1}^{K} P_\kappa \left( \frac{1}{3N} + \frac{1}{K} \left[ \frac{\left( \frac{(K-1)}{3N} \right) - \left( \frac{(K-1)}{3N} \right)^{s+1}}{1 - \left( \frac{(K-1)}{3N} \right)} \right) \right) \\
- \frac{\left( \frac{(K-1)}{3N} \right)^s}{1 + \left( \frac{1}{3N} \right)^s} \right\} - P_k \left( \frac{1}{3N} + \frac{1}{3N} \left[ 1 + \left( \frac{1}{3N} \right)^s \right] \right) \\
+ \frac{T^2}{2(3N)^{s+1}} \left[ \frac{(K-1)^{s+1} - (-1)^s + 1}{K} \sum_{\kappa=1}^{K} P_\kappa + (-1)^s \sum_{\kappa=1}^{K} P_\kappa \right].
\]

Collecting common denominators in the second group of terms gives
\[
= \frac{N_o T}{4} \left[ \frac{1 - \left( \frac{(K-1)}{3N} \right)^{s+1}}{1 - \left( \frac{(K-1)}{3N} \right)} \right] + \frac{NT^2}{6} (1 - \cos^2(\psi)) \left\{ \sum_{\kappa=1}^{K} P_\kappa \left( \frac{K}{3N} + \frac{1 - \left( \frac{(K-1)}{3N} \right) + \left( \frac{(K-1)}{3N} \right)^{s+1}}{1 - \left( \frac{(K-1)}{3N} \right)} \right) \\
- \left( \frac{(K-1)}{3N} \right)^s + \left( \frac{(K-1)}{3N} \right)^s - \left( \frac{1}{3N} \right)^s \right\} - K \left[ \frac{- \left( \frac{1}{3N} \right)^s - \left( \frac{1}{3N} \right)^{s+1}}{1 + \left( \frac{1}{3N} \right)^s} \right] \\
- P_k \left( \frac{\left( \frac{1}{3N} - \frac{1}{3N} \right)^{s+1}}{1 + \left( \frac{1}{3N} \right)^s} \right) \right\} + \frac{T^2}{2(3N)^{s+1}} \left[ \frac{(K-1)^{s+1} - (-1)^s + 1}{K} \sum_{\kappa=1}^{K} P_\kappa + (-1)^s \sum_{\kappa=1}^{K} P_\kappa \right].
\]

Simplifying the first term of the second group of terms and collecting the rest over a common denominator results in
\[
= \frac{N_o T}{4} \left[ \frac{1 - \left( \frac{(K-1)}{3N} \right)^{s+1}}{1 - \left( \frac{(K-1)}{3N} \right)} \right] + \frac{NT^2}{6} (1 - \cos^2(\psi)) \left\{ \sum_{\kappa=1}^{K} P_\kappa \left[ \frac{\left( \frac{1}{3N} \right)^s - \left( \frac{1}{3N} \right)^{s+1}}{1 - \left( \frac{(K-1)}{3N} \right)} \right) \right) \\
+ \frac{1 + \frac{1}{3N} + \left( \frac{1}{3N} \right)^s - \frac{K-1}{3N} - K \left( \frac{1}{3N} \right)^s + K \left( \frac{1}{3N} \right)^s - K \left( \frac{1}{3N} \right)^{s+1}}{1 + \left( \frac{1}{3N} \right)^s} \\
- \left( \frac{(K-1)}{3N} \right)^s \right\} - P_k \left( \frac{\left( \frac{1}{3N} - \frac{1}{3N} \right)^{s+1}}{1 + \left( \frac{1}{3N} \right)^s} \right) \right\} + \frac{T^2}{2(3N)^{s+1}} \left[ \frac{(K-1)^{s+1} - (-1)^s + 1}{K} \sum_{\kappa=1}^{K} P_\kappa + (-1)^s \sum_{\kappa=1}^{K} P_\kappa \right].
\]

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\[ + \frac{T^2}{2 (3N)^{s+1}} \left[ \frac{(K - 1)^{s+1} - (-1)^{s+1}}{K} \sum_{k=1}^{K} P_k + (-1)^{s+1} P_k \right] \]  

Finally, simplification yields

\[ \text{var} \left( Z^{(s)}_{k,t} \right) = \frac{N_s T}{4} \left[ \frac{1 - \left( \frac{(K-1)}{3N} \right)^{s+1}}{1 - \left( \frac{(K-1)}{3N} \right)^{s+1}} \right] \]

\[ + \frac{NT_c^2}{6} (1 - \cos^2(\psi)) \left[ \sum_{k=1}^{K} P_k \frac{i}{K} \left( \left[ \frac{1 - \left( \frac{(K-1)}{3N} \right)^{s+1}}{1 - \left( \frac{(K-1)}{3N} \right)^{s+1}} - \frac{1 - \left( \frac{1}{3N} \right)^{s+1}}{1 + \left( \frac{1}{3N} \right)^{s+1}} \right) \right] \]

\[ - P_k \left( \frac{\frac{1}{3N} - \left( \frac{-1}{3N} \right)^{s+1}}{1 + \left( \frac{1}{3N} \right)^{s+1}} \right) \] + \[ \frac{T^2}{2 (3N)^{s+1}} \left[ \frac{(K - 1)^{s+1} - (-1)^{s+1}}{K} \sum_{k=1}^{K} P_k + (-1)^{s+1} P_k \right] \]

which is the desired result.
Appendix E

Derivation of Expected Value of Combined Amplitude and Bit Estimate When Using Averaged Amplitude Estimate

We wish to find $E[\hat{A}_{k,i}\hat{b}_{k,i}]$ where $\hat{A}_{k,i}$ is the amplitude estimate defined here as

$$\hat{A}_{k,i} = \frac{1}{N_a} (|Z_{k,i}| + |Z_{k,i-1}| + |Z_{k,i-2}| \ldots + |Z_{k,i-N_a+1}|)$$  \hspace{1cm} (E.1)

$N_a$ is the number of bit intervals over which an average is taken, $Z_{k,i}$ is the correlator output for user $k$ over bit interval $i$, and $\hat{b}_{k,i}$ is the bit estimate for user $k$ during bit interval $i$ defined as

$$\hat{b}_{k,i} = \frac{Z_{k,i}}{|Z_{k,i}|}.$$  \hspace{1cm} (E.2)

The combined expected value is then

$$E[\hat{A}_{k,i}\hat{b}_{k,i}] = E\left[\frac{Z_{k,i}}{|Z_{k,i}|} \frac{1}{N_a} (|Z_{k,i}| + |Z_{k,i-1}| + |Z_{k,i-2}| \ldots + |Z_{k,i-N_a+1}|)\right].$$  \hspace{1cm} (E.3)

Simplifying,

$$E[\hat{A}_{k,i}\hat{b}_{k,i}] = E\left[\frac{1}{N_a} Z_{k,i} + \frac{1}{N_a} \frac{Z_{k,i}}{|Z_{k,i}|} |Z_{k,i-1}| + \ldots + \frac{1}{N_a} \frac{Z_{k,i}}{|Z_{k,i}|} |Z_{k,i-N_a+1}|\right]$$

$$= \frac{1}{N_a} \sqrt{\frac{P_k}{2} T_b b_{k,i}} + \frac{1}{N_a} E\left[\frac{Z_{k,i}}{|Z_{k,i}|}\right] E[|Z_{k,i-1}|] + \ldots$$

$$+ \frac{1}{N_a} E\left[\frac{Z_{k,i}}{|Z_{k,i}|}\right] E[|Z_{k,i-N_a+1}|]$$  \hspace{1cm} (E.4)
Assuming independence between consecutive bits (which is a simplification since this is not true in an asynchronous case) and identical distributions for each correlator output considered,

\[ E[\hat{A}_{k,i} b_{k,i}] = 1N_a \sqrt{\frac{P_k}{2}} T b_{k,i} + \frac{N_a - 1}{N_a} \mathbb{E}\left[ \frac{Z_{k,i}}{|Z_{k,i}|} \right] \mathbb{E}[|Z_{k,i-j}|]. \]  

(E.5)

Now the expected value of the bit estimate is \( E[|Z_{k,i}||Z_{k,i}|] = b_{k,i}(1 - P_e) \) where \( P_e \) is the probability of error. Now since \( Z_{k,i} \) is Gaussian (using a Gaussian Approximation), \( |Z_{k,i}| \) is a folded Gaussian. Thus

\[ \mathbb{E}[|Z_{k,i-j}|] = \sqrt{\frac{P_k}{2}} T + \sqrt{\frac{2\sigma_Z^2}{\pi}} e^{-\frac{P_e T^2}{4\sigma_Z^2}}. \]  

(E.6)

Thus, after minimal manipulation,

\[ E[\hat{A}_{k,i} b_{k,i}] = \sqrt{\frac{P_k}{2}} T b_{k,i} \left[ \frac{1}{N_a} + (1 - 2P_e) \frac{N_a - 1}{N_a} \right] + \frac{N_a - 1}{N_a} (1 - 2P_e) b_{k,i} \sqrt{\frac{2\sigma_Z^2}{\pi}} e^{-\frac{P_e T^2}{4\sigma_Z^2}}. \]  

(E.7)

which is the result we wished to show.
Appendix F

Description of PN Sequence Generator

This appendix presents diagrams detailing the development of a simple PN sequence generator board which creates random data bits using a variable length PN sequence generator. The data is spread by a length 15 code-on-pulse spreading waveform which is a maximal sequence (taps at 1 and 4). This is generated using a four stage PN sequence generator. The data length is variable between 7 and 1 billion (4 to 30 stages).
Figure F.1: Direct Sequence Spreader Board
Figure F.2: PN Sequence Generator (logic diagram)
Figure F.3: PN Sequence Generator
Figure F.4: Data Reset/Clear Board
Figure F.5: Data Generator Board
Appendix G

Directory Structure for Multiuser Testbed

This appendix presents a description of the directory structure of the multiuser testbed.
Figure G.1: Directory Structure for Multiuser Testbed
Bibliography


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