TIME DOMAIN SYNTHESIS APPLIED TO MODELING OF MICROWAVE
STRUCTURES AND MATERIAL CHARACTERIZATION

by

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(ABSTRACT)

In this dissertation a new time domain approach for the determination of material properties such as the complex permittivity and the complex permeability in a stripline geometry is presented. The new technique uses both Time Domain Reflectometry (TDR) and Time Domain Transmission (TDT) measurements for determining an optimum frequency dependent lossy transmission line model for the stripline under test. The optimization is done in the time domain by comparing the experimental TDR and TDT response waveforms with the simulated ones using a non-linear least squares fit. The conventional optimization algorithms have shown to be inefficient in this specific application. In this dissertation an efficient optimization algorithm which has been developed to suit this application is also presented. In general, the material properties in a stripline under test are related with the geometrical parameters of the line through complicated integral expressions. Using the proposed approach, the use of complicated integral expressions are avoided. The material properties such as the complex permittivity and the complex permeability are determined from the optimum lossy transmission line model. For this purpose, the frequency behavior of the line parameters have to be known beforehand in the form of causal mathematical models. The literature survey shows that, no causal model exists for the complex permittivity of thick film and polymer materials. The dissertation proposes a new causal model for this purpose.
In addition to the above, the dissertation also presents a new time domain synthesis technique which has several applications in microwave measurements and material characterization. The technique uses a general (lossy) transmission line synthesis approach to obtain an equivalent network model for a microwave device under test excited with a time domain step waveform. The response waveform acquired from a time domain network analyzer is divided into \( N \) equal time intervals. Each interval is synthesized by a lossy transmission line segment. The parameters of each line are determined by using an iterative least squares optimization technique to fit its simulated response to the measured waveform. The optimization is performed in the time domain by minimizing the error function due to the difference between the two waveforms. The validity of the proposed techniques are verified through both simulated and experimental results.
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CHAPTER 1

INTRODUCTION

The determination of material properties in a stripline geometry became very important since the growing dependence on stripline transmission media has increased in microwave integrated circuit applications. Due to its complicated geometrical structure, no exact formulation is available in the literature that relates the material properties of a stripline to its geometry. These difficulties have forced several researchers to make certain assumptions and come up with approximate formulations that are valid for certain dimensions and for characteristic impedances that are close to 50 Ω. In many applications, the limitations in technology prevents the user from constructing striplines with such characteristic impedances. With these limitations, it becomes very difficult to determine accurate properties of a wide class of materials such as thick film and polymers in a stripline geometry.

In this dissertation, a new technique for determining the material properties in a stripline geometry is proposed. The proposed technique uses time domain measurements and a frequency dependent lossy line approach for determining the complex permittivity and the complex permeability of a stripline. No assumptions are made on the geometrical parameters or the characteristic impedance. The technique assumes that a stripline under test can be accurately modeled by a lossy transmission line whose parameters are frequency dependent.

Using the proposed technique, the functional behavior of the lossy line parameters has to be known beforehand. In this regard, the dissertation presents existing models in the literature and discusses their applications to the determination of material properties. It is important to realize that any mathematical model that represents a physical system should be causal. The literature survey shows that no causal model is available for representing the complex
permittivity of thick film and polymer materials. Hence, in this dissertation a new, causal model is proposed for this purpose.

The material characterization process in a stripline geometry involves several steps. These can be summarized as follows: Fabrication, measurement and computer simulations. The fabrication process is a complete topic by itself, therefore only a summary is given in this dissertation. Measurements in the microwave frequency range are usually performed by either time or frequency domain techniques. Time domain techniques have some advantages over frequency domain techniques. Hence, in this dissertation time domain measurement techniques for the determination of material properties in a stripline geometry are presented. For the purpose of computer simulations, an equivalent network model is devised using the time domain measurement set up. The equivalent network model is then used in deriving the necessary mathematical formulation in terms of the scattering S-parameters that would simulate the measurement data.

Computer simulations involve the determination of the optimum values of the lossy line parameters through a non-linear least squares optimization technique. The optimization is done in the time domain by minimizing an objective function defined in terms of the lossy line parameters as the sum of squares of an error function. The error function is defined to be the difference between the experimental and simulated time domain responses of the stripline under test. The desired material properties such as the complex permittivity and the complex permeability are eventually determined from the optimal values of the lossy line parameters. The dissertation presents a modification to an existing algorithm to suit the above purpose.

Given an experimental time domain response waveform for a device under test, it is of practical interest to determine an equivalent network model for this device based on its time domain response. In the literature, this is known as the time domain network synthesis. Since the 1940's, several researchers have done a significant amount of work to solve this problem. The
majority of the research performed in this regard assumes that the impulse response of the device under is either known or can be easily determined from the input and output time domain responses. The next step involves the determination of the transfer function from the impulse response using Laplace transform techniques. This transfer function is then checked for realizability. If not realizable, it has to be approximated with one that is realizable. Once a realizable transfer function is obtained, conventional frequency domain techniques are utilized for obtaining an equivalent network model.

Several problems exist with the above network synthesis techniques. The first problem lies in the determination of the impulse response. In practice, this is not an easy task and sophisticated convolution techniques have to be used. Even after doing so, the exact determination of the impulse response might not be possible. Assuming that the impulse response is determined accurately, problems exist in the realizability of the transfer function. If the resulting transfer function is not realizable, then an approximate realizable transfer function will not necessarily give a time domain response that matches the experimental one.

To overcome the above problems a practical time domain synthesis technique is proposed in this dissertation. The proposed technique uses a general lossy transmission line synthesis approach to obtain an equivalent network model for a microwave device under test excited with a time domain step waveform. The response waveform acquired from a time domain network analyzer is divided into \( N \) equal time intervals. Each interval is synthesized by a lossy transmission line segment. The parameters of each line are determined by using an iterative least squares optimization technique to fit its simulated response to the measured waveform. The optimization is performed in the time domain by minimizing the error function due to the difference between the two waveforms.

The dissertation is divided into several chapters. Chapter 2 presents the literature survey and the proposed work. The determination of material properties of a stripline using a lossy
transmission line approach is discussed in chapter 3. In chapter 4, the time domain measurement and simulation techniques for the determination of material properties of a stripline is presented. A time domain optimization technique for the determination of material properties of a striplines is presented in chapter 5. In chapter 6, the experimental results on material characterization using a stripline geometry are discussed. Chapter 7 presents a practical time domain network synthesis technique. Finally, a summary and conclusions is provided in chapter 8.
CHAPTER 2

LITERATURE SURVEY AND THE PROPOSED WORK

2.1 INTRODUCTION

The growing dependence on stripline transmission media in RF, microwave, and millimeter-wave integrated circuit applications has attracted the attention of several researchers for the past three decades [1]-[3]. Stripline has some advantages over coaxial or waveguide circuitry especially whenever passive or active devices have to be included as part of the design. Its greatest use comes in circuits that require miniaturization or large bandwidth [4]. Furthermore, due to its shielded structure, a stripline has higher performance than other transmission lines in high frequency thick film and thin film circuit applications [5].

Striplines are formed by embedding a thin metal conductor of rectangular cross-section in a uniform dielectric material which in turn is sandwiched between two ground planes. A typical cross-section of a stripline geometry is shown in Figure 2.1.1. The dominant mode of propagation for this type of transmission line is TEM (transverse electromagnetic).

Cohn [2] is one of the pioneers which has contributed to the analysis of striplines. He has derived, using the conformal technique, an expression for the capacitance that allows the calculation of the characteristic impedance when the thickness "t" of the central strip is negligible, i.e. t = 0. For finite thickness, Wheeler’s formulae [1] are more accurate to within 0.5 percent for w/(b - t) < 10.

Due its complicated geometrical structure, the full field analysis of a stripline is very difficult. This makes it necessary to introduce certain assumptions on the geometrical parameters for analysis. These assumptions limit the user for the restrictions provided by the formulae
Figure 2.1.1  Stripline Configuration
derived by Cohn [2], Wheeler [1] and others [3]. In order to obtain the exact formulation for determining material properties in a stripline geometry, one has to use full-field analysis.

An important topic in time domain applications is device modeling. Based on a time domain reflectometry response waveform of a device under test, it is often important to come up with a network model that characterizes a device under test. Conventional time domain synthesis techniques require that the impulse response, \( h(t) \) is first determined from the excitation and response of the device under test. Later, this response is transformed into the frequency domain to obtain the transfer function, \( H(\omega) \). If this transfer function is in realizable form, then conventional frequency domain techniques are utilized for obtaining a network model. Otherwise, the transfer function has to be first approximated by one which is realizable. All this process leads to several numerical errors. Eventually, several iterations become necessary for obtaining an equivalent network model that has an impulse response very close to that of the device under test.

This chapter is divided into several sections. Section 2.2 presents the literature survey performed in the areas of material characterization using time domain techniques, transient analysis of transmission lines considering high frequency losses and time domain network synthesis. All this survey was necessary because it is directly related to the proposed research. In the light of the literature survey, the proposed work is presented in section 2.3. Finally a summary and conclusions are given in section 2.4.

2.2 LITERATURE SURVEY

2.2.1 Measurement of Intrinsic Properties of Materials by Time Domain Techniques

Recent improvements in the time domain reflectometry (TDR) and the time domain
transmission (TDT) instrumentation and measurement techniques have made it possible to make precise measurements of the complex permittivity and permeability over a wide frequency range from about 100 kHz to 25 GHz. Traditionally, such measurements have been made at fixed frequencies in the frequency domain using slotted line and impedance-bridge configuration [6]. Time domain methods have advantages over frequency domain methods. The most important advantage is the cost and the complexity of the measurement equipment. The instrumentation for the time domain measurement is much simpler and less expensive than that of a frequency domain method [7].

In order to obtain the intrinsic properties of materials, Nicholson and Ross [6] used an annular disk of material with permeability \( \mu = \mu_o \mu_r \), permittivity \( \epsilon = \epsilon_o \epsilon_r \) and thickness \( d \). They made time domain measurement on the material by installing it in a coaxial air filled line with characteristic impedance \( Z_o \) as shown in Figure 2.2.1(a). The characteristic impedance of the line in the range of \( 0 \leq z \leq d \) becomes

\[
Z_d = \sqrt{\frac{\mu_r}{\epsilon_r}} Z_o , \tag{2.2.1}
\]

where \( \mu_r \) and \( \epsilon_r \) can be complex. Assuming that \( d \) is infinite, then the reflection coefficients of a wave incident on the interface from the air-filled line would simply be given by

\[
\Gamma = \frac{Z_d - Z_o}{Z_d + Z_o} = \frac{\sqrt{\mu_r/\epsilon_r} - 1}{\sqrt{\mu_r/\epsilon_r} + 1} . \tag{2.2.2}
\]

For finite \( d \), the propagation coefficient between faces \( A \) and \( B \) of the slab is written as

\[
\tau = \exp\left[-j\omega\sqrt{\mu_e} d\right] = \exp\left[-j(\omega/c)\sqrt{\mu_r \epsilon_r} d\right] , \tag{2.2.3}
\]

where \( c \) is the speed of light in vacuum. Using the signal flow graph of Figure 2.2.1(b), the
scattering coefficients $S_{21}$ and $S_{11}$ of the slab are obtained to be as follows:

\[
S_{21}(\omega) = \frac{V_f}{V_{inc}} = \frac{(1 - \Gamma^2) \tau}{1 - \Gamma^2 \tau^2},
\]

(2.2.4)

and

\[
S_{11}(\omega) = \frac{V_r}{V_{inc}} = \frac{(1 - \tau^2) \Gamma}{1 - \Gamma^2 \tau^2}.
\]

(2.2.5)

Equations (1.2.4) and (1.2.5) can be simultaneously solved to yield

\[
\Gamma = X \pm \sqrt{X^2 - 1},
\]

(2.2.6)

where,

\[
X = \frac{1 - (S_{21}^2 - S_{11}^2)}{2 S_{11}}.
\]

(2.2.7)

In (2.2.6), the appropriate sign is chosen such that $|\Gamma| \leq 1$. Also,

\[
\tau = \frac{(S_{11} + S_{21}) - \Gamma}{1 - \Gamma (S_{11} + S_{21})}.
\]

(2.2.8)

From (2.2.1), we obtain

\[
\frac{\mu_r}{\varepsilon_r} = \left[ \frac{1 + \Gamma}{1 - \Gamma} \right]^2 = c_1.
\]

(2.2.9)

Furthermore, from (2.2.2), we obtain

\[
\mu_r \varepsilon_r = -\left[ \frac{c}{\omega d} \ln \left( \frac{1}{t} \right) \right]^2 = c_2.
\]

(2.2.10)
Figure 2.2.1  (a) A coaxial line with annular disk of material to be measured inserted;
(b) Signal flow graph for (a).
Using (2.2.9) and (2.2.10), Nicholson et al. [6] obtains the complex permeability and permittivity to be as follows:

\[ \mu_r = \sqrt{\epsilon_1 / \epsilon_2}, \quad (2.2.11) \]

and

\[ \epsilon_r = \sqrt{\epsilon_2 / \epsilon_1}. \quad (2.2.12) \]

Towards obtaining \( S_{11}(\omega) \) and \( S_{22}(\omega) \), Nicholson et al. [6] performed TDR and TDT measurements on the sample under test. They acquired the necessary response and reference waveforms and then used the discrete inverse Fourier transform.

Most of the methods for obtaining the intrinsic properties of materials using time domain measurements rely on frequency domain formulation for scattering coefficients. Cole [8] introduced a different approach based on relating the input admittance of the sample section, obtained from the observed incident and reflected pulses, to the dielectric properties and termination of the sample section. For a section of a coaxial line with free space geometric capacitance \( C_c \) per unit length subjected to a time dependent voltage \( v(t) \), the corresponding charge \( q(t) \) for a linear dielectric is given by

\[ q(t) = C_c \left[ \epsilon_{\infty} v(t) + \int_{-\infty}^{t} dt' \left( \frac{\partial}{\partial t} \xi(t-t') \right) v(t') \right]. \quad (2.2.13) \]

In (2.2.13), \( \epsilon_{\infty} \) represents polarization which responds instantaneously on the time scale of the measurements and \( \xi(t) \) corresponds to a dielectric response function in real time whose counterpart is the complex relative permittivity \( \epsilon^*(j\omega) \) as a function of frequency. From (2.2.13), the Fourier transforms of \( q(t) \) and \( v(t) \) are related by

\[ Q(j\omega) = \mathcal{F} \left[ q(t) \right] = C_c \left[ \epsilon_{\infty} + j\omega \Xi(j\omega) \right] V(j\omega) = C_c \epsilon^*(j\omega) V(j\omega), \quad (2.2.14) \]
where $\Xi(j\omega)$ is the Fourier Transform of $\xi(t)$, assumed to be zero at $t=0$, and $\epsilon^*(j\omega)$ is related to $\Xi(j\omega)$ and $\xi(t)$ by the second equality. The steady state current $I(j\omega)$ for voltage $V(j\omega)$ and sample admittance $Y(j\omega)$ are then

\[
I(j\omega) = \mathcal{F}[\partial q(t)/\partial t] = j\omega \epsilon^*(j\omega) C_c V(j\omega), \tag{2.2.15a}
\]

and

\[
Y(j\omega) = \frac{I(j\omega)}{V(j\omega)} = j\omega \epsilon^*(j\omega) C_c. \tag{2.2.15b}
\]

Let the incident and the reflected voltage pulses observed in a coaxial line between the pulse generator and sample be represented by $v_o(t)$ and $r(t)$. Then these two parameters are related to the voltage and current at sample input, $v(t)$ and $i(t)$, by: $v(t) = [v_o(t) + r(t)]$ and $i(t) = G_c[v_o(t) - r(t)]$. In the frequency domain, the input admittance $Y_{in}$ can then be obtained from the information by the relation

\[
Y_{in} = \frac{I(j\omega)}{V(j\omega)} = G_c \frac{[V_o(j\omega) - R(j\omega)]}{[V_o(j\omega) + R(j\omega)]}, \tag{2.2.16}
\]

where, $V_o(j\omega)$ and $R(j\omega)$ are the transforms of the observed pulses.

Now, suppose that a length of $d$ of the line past the input is filled with dielectric of relative permittivity $\epsilon^*(j\omega)$ and terminated by an admittance $Y_d$, then using transmission line analysis, Cole [8] obtained the following result,

\[
Y_{in} = \frac{Y_o + Y_d}{1 + Z_s Y_d}. \tag{2.2.17}
\]

For a non-magnetic sample with relative permeability $\mu^* = 1$, $Y_o$ and $Z_s$ are given by

\[
Y_o = j\omega \epsilon^*(j\omega) C_c d (\tan x/z), \tag{2.2.18a}
\]
and

$$Z_s = j \omega L_c d (\tan \frac{z}{z}) .$$  \hspace{1cm} (2.2.18b)$$

In (2.2.18b), $L_c$ is the geometric inductance per unit length and $G_c = (C_c/L_c)^{1/2}$. Furthermore, the factor $\tan \frac{z}{z}$ accounts for the propagation with finite speed in the sample, with the argument $z = \omega d \lambda / c$, where $c = \sqrt{1/L_c C_c} = 0.3 \text{ mm/ps}$. Combining (2.2.16), (2.2.17) and (2.2.18) the complex permittivity can be obtained as

$$
\varepsilon^*(j\omega) = \frac{G_c}{j\omega C_c d (\tan \frac{z}{z})} \left[ \frac{V_o(j\omega) - R(j\omega)}{V_o(j\omega) + R(j\omega)} \right] \left[ 1 + j\omega Y_d L_c d (\tan \frac{z}{z}) \right] - Y_d . \hspace{1cm} (2.2.19)
$$

Hence, using Cole's [8] technique, one needs to measure the incident and reflected voltage pulses in the time domain for a specific terminating admittance $Y_d$.

2.2.2 Transient Analysis of Transmission Lines Considering High-Frequency Losses

At very low frequencies ($f < 100 \text{ kHz}$), most transmission lines can be practically considered to be lossless. Typically, at frequencies in the range of 100 MHz region and above, transmission lines noticeably possess skin effect and dielectric losses. The skin effect losses are due to the imperfect conductors and the dielectric losses are due to imperfect dielectrics in the transmission lines. Skin effect losses could be due to cylindrical geometry which in turn can affect the transient behavior of transmission lines [9]-[11].

Wigington and Nahman [9] studied the transient behavior of coaxial cables by considering the skin effect of the conductors in coaxial cables as the distorting elements. In their paper, they presented generalized curves by which the response of any length of coaxial cable
could be predicted if one point on the attenuation versus frequency curve were known. For high frequencies, the skin effect impedance of a round wire is:

\[ Z_s = K \sqrt{\sigma}, \]  \hspace{1cm} (2.2.20)

and

\[ K = \frac{1}{2\pi r} \sqrt{\frac{\mu}{\sigma}}. \]  \hspace{1cm} (2.2.21)

where \( r \) is the wire radius, \( \mu \) is the permeability and \( \sigma \) is the conductivity of the wire. Neglecting dielectric losses \( (G = 0) \), the transfer function relating the output \( (V_2) \) to the input \( (V_1) \) voltages as functions of complex frequency is given by

\[ \frac{V_2}{V_1} = e^{-\gamma \ell} = e^{-\ell \sqrt{\frac{\ell^2 L C + s C K \sqrt{\sigma}}{}}}, \]  \hspace{1cm} (2.2.22)

where \( \ell \) is the line length, \( \gamma \) is the propagation constant, and \( L \) and \( C \) are inductance and capacitance per unit length of the transmission line, respectively. The above formulation is obtained by assuming that the transmission line is terminated to its characteristic impedance. Wigington and Nahman [9] expanded the square root in the exponent of (2.2.22) by the binomial expansion and retained only the first two terms. With this assumption, the transfer function becomes

\[ \frac{V_2}{V_1} = e^{-\ell [j\omega T + (K/2 R_o)j\omega^{1/2}]}, \]  \hspace{1cm} (2.2.23)

where \( R_o = \sqrt{L/C} \) and \( T = \sqrt{L/C} \). The impulse response of the line is the inverse Laplace transform of (2.2.23) and is given by
\[ g(t) = \alpha (t - T\ell)^{-3/2} e^{-\beta \sqrt{t}}, \quad (t - T\ell) \geq 0, \]
\[ = 0, \quad \quad (t - T\ell) < 0, \quad (2.2.24) \]

where,

\[ \alpha = \frac{\ell K}{4 R_o \sqrt{\pi}}, \]

and

\[ \beta = \left( \frac{\ell K}{4 R_o} \right)^2, \]

The above work neglected the low frequency effects. Later on, Nahman et al. [10] and Holt et al. [11] extended the skin effect loss model to below the high-frequency limit. In this case, the impedance per unit length of the transmission line becomes

\[ Z(j\omega) = j\omega L + A + B\sqrt{j\omega}. \quad (2.2.25) \]

For the high frequency limit, \( A = 0 \) and \( B = K \); thus,

\[ [Z(j\omega)]_{HF} = j\omega L + K\sqrt{j\omega}. \quad (2.2.26) \]

For low-frequency limit, \( A = R \) and \( B = 0 \); thus,

\[ [Z(j\omega)]_{LF} = j\omega L + R_{dc}. \quad (2.2.27) \]

In (2.2.27), \( R_{dc} \) is the dc resistance per unit length of the coaxial line. In the same paper, Nahman et al. showed that transition of \( R \) to \( K\sqrt{j\omega} \) with increasing \( |\omega| \) is described by a function comprised of modified Bessel functions (inner conductor) and modified Hankel functions (outer conductor).
After Wigington and Nahman [9], Nahman [12] showed that, for most coaxial cable transmission lines, with both conductor and dielectric losses present, the attenuation could be approximated by the real part of $K(j\omega)^m$, where $0 \leq m < 1$. He also presented step-response curves for the causal model $j\omega T + K(j\omega)^m$, and derived an integral expression for the step response containing the parameter $m$. With the above development, for a transmission line terminated with its characteristic impedance $Z_0(j\omega)$, the step response of the line becomes

$$e(t) = 1 - \frac{1}{\pi} \int_0^{\infty} \frac{e^{-rt} - a_1^m}{r} \sin(a_2 r^m) \, dr, \quad 0 \leq m < 1,$$

where,

$$a_1 = \frac{K^m \cos(m\pi)}{\cos(m\pi/2)}, \quad \text{and} \quad a_2 = \frac{K^m \sin(m\pi)}{\cos(m\pi/2)}.$$

Later, Jonscher [13] showed that many dielectric materials could be described by the following empirical law:

$$G(\omega) \propto |\omega|^m, \quad 0 < m < 1.$$

Curtins et al. [14] have used the above relation in their transmission line analysis and obtained very good agreement with experimental data.

The above methods utilize frequency domain techniques for developing dielectric loss and skin-effect models. The transient response is then obtained by using either the inverse Laplace or the inverse Fourier transforms. Another approach is to model the dielectric and skin-effect losses using time domain techniques. Some work done in this direction is reviewed below.

Brennan et al. [15] have modeled a section of transmission line exhibiting skin-effect
losses using four building blocks as shown in Figure 2.2.2. These blocks are repeated in cascade to model the overall skin-effect of a lossy transmission line. The number of quart blocks is dependent upon the amount of skin-effect loss in the line. The first building block is a lossless line having a delay of \( \ell_1 \) and characteristic impedance \( Z_{01} \). This section would be the desired model if no skin-effect losses existed. The second building block \( R_1 \) corresponds to the dc resistance per unit length of the line. The third and fourth building blocks correspond to the skin effect losses. Each consists of a very short lossless transmission line whose delay is much smaller than \( \ell_1 \), or \( (\ell_2, \ell_3 \ll \ell_1) \) and characteristic impedances are much larger than \( Z_{01} \), or \( (Z_{02}, Z_{03} \gg Z_{01}) \).

Computer implementation of lossy transmission lines using the above technique has been implemented by Sussman-Fort et al. [16]. In addition to this, Gruodis [17] has used the first three building blocks of Brennan et al. [15] to model uniform resistive transmission lines in a homogeneous medium.

Yen et al. [18] have performed a comprehensive study for the time domain transient analysis of lossy transmission lines suitable for desk-top computers. In their analysis, the line is divided into \( N \) sections. For each section, an equivalent circuit consisting of \( M \) resistors and \( M - 1 \) inductors is derived directly from the skin-effect differential equations to simulate the skin-effect loss. The skin-effect equivalent circuit and the transmission line equations are solved using a backward Euler integration method. They have derived a model which relates the new values of node voltages and line currents to their values at the previous step.

Yen et al. [18] started their analysis with Maxwell’s equations by neglecting the displacement current density. Hence,

\[
\nabla \times \vec{J} = -\mu_0 \sigma \frac{\partial \vec{H}}{\partial t},
\]

(2.2.30a)

and
\[ \nabla \times \vec{H} = \vec{J}, \quad (2.2.30b) \]

where, \( \mu_o = 4\pi \times 10^{-7} \) H/m is the permeability of free space, and \( \sigma \) is the conductance. Integration of (2.2.30b) yields

\[ I(r, t) = 2\pi r \ H_\phi(r, t). \quad (2.2.31) \]

Next substituting \( H_\phi(r, t) \) from (2.2.31) into (2.2.30a) and noting that \( \vec{J} \) is along the longitudinal direction, we obtain

\[ \frac{\partial J_\phi(r, t)}{\partial r} = \frac{\mu \sigma}{2\pi r} \frac{\partial I(r, t)}{\partial t}. \quad (2.2.32) \]

Next, dividing the center conductor into \( M \) concentric rings, and letting \( i = 1 \) be the outermost ring and \( i = M \) be the innermost ring as shown in Figure (2.2.3a), then (2.2.32) can be approximated by the following \( M - 1 \) equations;

\[ \frac{J_{i-1} - J_i}{r_{i-1} - r_i} = \frac{\mu \sigma}{2\pi r_i} \frac{\partial I_i(t)}{\partial t}, \quad i = 2, 3, \ldots, M. \quad (2.2.33) \]

In (2.2.33), \( J_i \) is the current density of the \( i \)-th ring, \( r_i \) is the outside radius of the \( i \)-th ring, and \( I_i \) is the current carried by all rings enclosed by \( r_i \) (\( I_{M+1} \) is zero). The current carried by the \( i \)-th ring with an area of \( A_i \) is \( I_i - I_{i+1} \). Hence, \( J_i \) can be expressed as

\[ J_i = \frac{I_i - I_{i+1}}{A_i}, \quad i = 1, 2, \ldots, M. \quad (2.2.34) \]

Substituting (2.2.34) into (2.2.33) and rearranging, yields the following set of \( M - 1 \) equations for the coaxial cable:
\[ R_{i-1}(I_{i-1} - I_i) - R_i(I_i - I_{i+1}) = L_i \frac{dI_i(t)}{dt}, \quad i = 2, 3, \ldots, M, \]  

where, \( R_i \) and \( L_i \) are the partial resistance and the partial inductances per unit length for the \( i \)-th ring given by

\[ R_i = \frac{1}{A_i \sigma}, \quad i = 1, 2, \ldots, M. \]  

and

\[ L_i = \frac{\mu (r_{i-1} - r_i)}{2 \pi r_i}, \quad i = 2, 3, \ldots, M. \]

In order to take the effect of the outer conductor of the coaxial cable, Yen et al. [18] multiplied the values of \( R_i \) and \( L_i \) of (2.236) by the ratio of the total dc resistance to that of the center conductor only. The equivalent circuit representation of (2.235) is shown in Figure 2.2.3(b).

### 2.2.3 Time Domain Network Synthesis

A time-domain synthesis problem is usually characterized by an excitation and a response, both of which are specified in terms of time [19]. The synthesis problem in the time domain consists of finding a function \( h(t) \), the unit-impulse response of a network, whose Laplace impulse transform \( H(s) \) (the network function) fulfills the necessary conditions of physical realizability [20]. Once \( H(s) \) is computed, the conventional network synthesis techniques can be used for obtaining an equivalent network.

Several researchers [22] have worked on this specific problem. A major problem with all these approaches is the assumption that \( h(t) \) is known analytically. This requires \( h(t) \) to be computed using other techniques such as deconvolution. If one uses deconvolution or other numerical techniques for the computation of the impulse response, then an extra effort has to be spent for finding a suitable analytical expression. Even after finding a suitable analytical
Figure 2.2.2  Brennan et al.'s time domain model for lossy line exhibiting skin-effect losses.
expression for \( h(t) \), it may or may not be suitable for ensuing these steps. If the analytical expression is not suitable for this purpose, one that is suitable has first to be found.

Kautz [21] solved the time domain synthesis problem by assuming that the impulse response \( h(t) \) of a given system can be expressed as follows:

\[
h(t) = \sum_{k=1}^{\infty} C_k \phi_k(t).
\]  

(2.2.37)

Since, (2.2.37) is not realizable as a finite network then it has to be approximated by a suitable impulse response. Kautz [21] has accomplished this by showing that the first \( n \) terms of (2.2.37) provides a good approximation. Let \( h^*(t) \) be the approximation to \( h(t) \); then \( h^*(t) \) can be written as

\[
h^*(t) = \sum_{k=1}^{n} C_k \phi_k(t).
\]  

(2.2.38)

In (2.2.37) and (2.2.38), the constants \( C_k \) are the coefficients of the expansions and the functions \( \phi_k(t) \) are chosen to form an orthonormal set, namely

\[
\int_0^{\infty} \phi_k(t) \phi_j(t) \, dt = \begin{cases} 
0 & \text{for } k \neq j \\
1 & \text{for } k = j 
\end{cases} \quad (k, j = 1, 2, \ldots)
\]  

(2.2.39)

It can be shown that the above choice of \( \phi_k(t) \) gives the minimum mean squared error between \( h(t) \) and \( h^*(t) \). Writing (2.2.38) as a sum of damped exponential and exponentially damped sinusoids, we obtain

\[
h^*(t) = \sum_{j=1}^{n} A_j e^{s_j t}.
\]  

(2.2.40)

The Laplace transform of (2.2.40) can be written as
Figure 2.2.3  (a) Center conductor ring definition and current distribution; (b) Equivalent circuit for skin-effect loss.
\[ H^*(s) = \frac{P(s)}{Q(s)} = \frac{A(s - \tilde{s}_1)(s - \tilde{s}_2) \cdots (s - \tilde{s}_m)}{(s - s_1)(s - s_2) \cdots (s - s_n)} = \sum_{j=1}^{n} \frac{A_j}{s - s_j}, \]  

(2.2.41)

where \( A \) is a real constant, \( n \) and \( m \) (\( m < n \)) are the number of poles and zeros, respectively. Furthermore, \( \tilde{s}_i \) and \( s_j \) are the zeros and poles of the system function, respectively.

For a given time domain excitation \( f_i(t) \) and response \( f_o(t) \), one way of determining \( h(t) \) is as follows:

\[ h(t) = \mathcal{L}^{-1}\{H(s)\} = \mathcal{L}^{-1}\left\{ \frac{F_o(s)}{F_i(s)} \right\}, \]  

(2.2.42)

where, \( F_o(s) \) and \( F_i(s) \) are the Laplace transforms of \( f_o(t) \) and \( f_i(t) \), respectively. By using a least squares fit between \( h(t) \) as given by (2.2.42) and \( h^*(t) \) as given by (2.2.38), the coefficients \( C_k \) and the functions \( \phi_k(t) \) can be determined. Hence, the residues and the poles of \( H^*(s) \) are computed and the network synthesis is carried out by using conventional frequency domain techniques [23].

Yengst [22] performed the approximation to a specified time domain response by starting with the same assumptions as Kautz [21] but approximated \( h(t) \) by an \( m \)-th order homogeneous difference equation of the form

\[ h^*_k(t) - A h^*_k(t - \Delta t) - B h^*_k(t - 2\Delta t) - \cdots - M h^*_k(t - m\Delta t) = 0. \]  

(2.2.43)

The coefficients of the above difference equation can be determined by a least-squares technique using \( h(t) \) and \( h^*(t) \). In this case, the order of the difference equation establishes the number of poles for the approximate function \( H^*(s) \) and the location of poles is computed directly from the difference equation. Once the poles are computed, the zeros of the transfer function can be determined from the original data and its derivative at \( t = 0 \) by successive application of the
initial value theorem.

A practical time domain network synthesis package was developed by Riad et al. [24] which uses both lumped and distributed network elements to synthesize the time domain reflectometry response (TDR) waveform of a microwave device under test (DUT). The modeling technique that utilizes the Modified Transient Circuit Analysis Package (MTCAP) is based on acquiring the TDR response waveform for the device under test at its interface port to the coaxial line. From the TDR response waveform, as well as any knowledge of the physical nature and structure detail of the DUT, a preliminary (rough) network model for the DUT is devised. In this simulation, a reference waveform is used to excite the network model and the reflection is computed as the simulated response of the DUT. Next, the simulated TDR waveform is compared to the experimental TDR waveform. The component values of the devised model are adjusted by the user in an iterative manner until the simulated response matches the measured TDR response. In this iterative process, each discontinuity is dealt with one at a time, in the order of their physical existence away from the launch end. Once a component value due to a discontinuity is optimized, any variation in the model component values of later discontinuities is not expected to affect the simulated response waveform of earlier times. When all the component values are adjusted such that the simulated response matches the experimental one, the resulting network provides an equivalent network for the DUT.

The MTCAP [24] simulation package, despite its practical use and advantages over the other theoretical methods, has two major disadvantages. The first disadvantage is the necessity of a user to interact with the package continuously during the whole iteration process. In this case, the user has to know intuitively how each network element behaves in a TDR environment and has to spend several hours in front of a personal computer until the iteration process is complete. The second disadvantage is the inability to model the losses. This results from the unavailability of a time domain lossy transmission line model to be used inside the package.
2.3 PROPOSED WORK

In this dissertation, our first objective is to develop a new technique for determining the intrinsic properties of a material in a stripline geometry without making any assumptions on the geometrical parameters. The proposed technique uses time domain measurements and a frequency dependent lossy transmission line approach to determine the intrinsic material properties such as the complex permittivity and complex the permeability of a stripline.

The second objective of this dissertation is to present a new and practical time domain network synthesis technique for microwave devices. In this case, the proposed technique uses a cascade of $N$-lossy transmission lines to model a time domain reflectometry (TDR) response waveform.

Towards achieving the first objective, a lossy transmission line representation is assumed for the stripline under test. The lossy line parameters are determined by time domain measurements and non-linear least squares optimization techniques. The approach is valid as long as the dominant mode of propagation in the stripline is approximately TEM. In the first part of this work, the relation between the material properties and lossy transmission line parameters are derived. Later, the functional behavior of the lossy line parameters are determined and a new model for the complex permittivity of microwave quality dielectric materials is developed. The integral expressions which relate the lossy line parameters such as the inductance, capacitance, resistance and conductance per unit length to the geometry of the lossy line are treated as constants to be determined through the optimization techniques. The intrinsic properties of the material are computed from the optimum values of the lossy line parameters. In the second part of this work, an existing non-linear optimization algorithm is modified to give an acceptable least squares solution for the above optimization problem.

Towards achieving the second objective, a new time domain synthesis technique which has several applications in microwave measurements and material characterization is proposed.
The technique uses a general lossy transmission line synthesis approach to obtain an equivalent network model for a microwave device under test excited with a time domain step waveform. The response waveform acquired from a time domain network analyzer is divided into \( N \) equal time intervals. Each interval is synthesized by a lossy transmission line segment. The parameters of each line are determined by using an iterative least squares optimization technique to fit its simulated response to the measured waveform. The optimization is performed in the time domain by minimizing the error function due to the difference between the two waveforms.

The dissertation provides both simulated and experimental results on the modeling of microwave devices with special application to the determination of the complex permittivity of materials in a stripline geometry.

### 2.4 SUMMARY AND CONCLUSIONS

In this chapter, the literature survey pertinent to the dissertation work was presented. In this process, the following topics were covered: Measurement of intrinsic properties of materials using time domain techniques, transient analysis of transmission lines considering high frequency losses and time domain network synthesis. The literature survey has shown that no exact formulation is available about the characteristic impedance and the propagation function of a stripline. This formulation is necessary if one wants to determine the exact relations between the material properties and the stripline geometry. The approximate formulation that is available in the literature makes certain assumptions on the geometry. This leads to erroneous results in the determination of material properties. The literature survey has also shown that, the measurement techniques available for material characterization using a stripline geometry require that, the characteristic impedance of the stripline under test is designed to be around 50 \( \Omega \). As a result of the above deficiencies, a new technique for material characterization using a stripline geometry is
proposed. The proposed technique uses a lossy transmission line approach to model a stripline under test. The approach requires that the functional behavior of the lossy line parameters are known beforehand.

Several researchers have proposed loss models for transmission lines. The majority of the models assume that the dielectric losses in the medium are negligible. As a result of this deficiency, a new model for the complex permittivity is proposed. The model is derived by realizing that the functional behavior of the admittance per unit length has to be dual to that of the impedance per unit length.

The time domain network synthesis problem has attracted the attention of several researchers since 1940's. The bulk of the work done in this regard assumes that the impulse response is known beforehand. The transfer function of the DUT is computed by using inverse Laplace transform techniques. Once this is done, conventional frequency domain techniques are utilized to solve the synthesis problem. In practice, the impulse response is not available. Hence, it has to be determined using the input and output responses of the device under test. However, this is not an easy task. In this dissertation, a practical time domain network synthesis technique is proposed that does not require the direct knowledge of the impulse response.
CHAPTER 3

DETERMINATION OF MATERIAL PROPERTIES OF A STRIPLINE USING A LOSSY TRANSMISSION LINE APPROACH

3.1 INTRODUCTION

The determination of material properties in different geometries has been an active research topic for several years. In general, exact formulation between material properties and the measurement geometry is obtained by full field analysis. For a stripline geometry, no full field analysis is available in the literature and the formulation that is available is only valid for certain dimensions. Furthermore, the available formulation is valid only for characteristic impedances that are close to 50 Ω (a standard used in most microwave measurements). In certain applications, where the stripline is constructed from thick film or polymer dielectric materials, it is very difficult to obtain a 50 Ω characteristic impedance. Due to these limitations, it was necessary to come up with new techniques that are independent of the dimensions and the characteristic impedance of the stripline under test.

In this chapter a new technique for the determination of material properties such as the complex permittivity and the complex permeability in a stripline geometry is presented. It is proposed that a stripline under test can be modeled by a lossy transmission line. The technique requires a priori knowledge of the functional behavior of the lossy line parameters. This functional behavior has to be in the form of a mathematical model that satisfies the causality relations. The chapter presents several models that are available in literature. The literature survey has shown that, no causal model is available for the complex permittivity that represents
the thick film and polymer dielectric materials. Hence, a causal model for the complex permittivity which is derived from both analytical and empirical expressions is proposed.

This chapter is divided into several sections. In section 3.2, the relation between material properties and lossy transmission line parameters is presented. The existing and the proposed mathematical models for the lossy transmission line parameters are presented in section 3.3. The applications of these models to the determination of material properties in a stripline geometry is also discussed in this section. The causality relation of the propagation function that uses the proposed model is discussed in section 3.4. Finally, a brief summary and conclusions is given in section 3.5.

3.2 RELATION BETWEEN MATERIAL PROPERTIES AND LOSSY TRANSMISSION LINE PARAMETERS

In this section, the relation between the transmission line equations and material properties will be derived. In the first part, transmission line equations are derived using an electromagnetic fields approach. In the second part, these equations are used in deriving the material properties.

3.2.1 An Electromagnetic Fields Approach to the Transmission Line Equations

In order to derive the transmission line parameters using an electromagnetic fields approach, first the Maxwell’s equations are introduced. Maxwell’s equations in complex phasor notation which takes into account the material properties can be written as follows [25]:

\[
\nabla \times \vec{E} = - j \omega \vec{B} = - j \omega \mu^{*} (\omega) \vec{H},
\]

\[
\nabla \times \vec{H} = j \omega \vec{D} + \vec{J} = (j \omega \varepsilon^{*} (\omega) \sigma_{c}) \vec{E},
\]
\[ \vec{\nabla} \cdot \vec{D} = \vec{\nabla} \cdot \epsilon^*(\omega) \vec{E} = \rho^*(\omega), \quad (3.2.3) \]
\[ \vec{\nabla} \cdot \vec{B} = 0, \quad (3.2.4) \]

where,

\[ \vec{E} \quad = \text{electric field intensity (V/m)}; \]
\[ \vec{D} \quad = \text{magnetic flux density (As/m}^2\text{)}; \]
\[ \vec{H} \quad = \text{magnetic field intensity (A/m)}; \]
\[ \vec{B} \quad = \text{magnetic flux density (Vs/m}^2\text{)}; \]
\[ \vec{J} \quad = \text{electric current density (A/m}^2\text{)}; \]
\[ \rho^*(\omega) \quad = \text{electric charge density (As/m}^3\text{)}; \]
\[ \mu^*(\omega) \quad = \text{complex permeability (H/m)}; \]
\[ \epsilon^*(\omega) \quad = \text{complex permittivity (F/m)}; \]
\[ \sigma_c \quad = \text{conductivity (S/m)}. \]

The complex permittivity \( \epsilon^*(\omega) \) can be written as

\[ \epsilon^*(\omega) = \epsilon_o \epsilon'_r(\omega) (1 - j \tan \delta_e) = \epsilon_o \left( \epsilon'_r(\omega) - j \epsilon''_r(\omega) \right), \quad (3.2.5) \]

where,

\[ \epsilon_o \quad = \text{permittivity of free space} = 8.854185338 \times 10^{-12} \text{ (F/m)}; \]
\[ \epsilon'_r(\omega) \quad = \text{real part of relative permittivity of the material}; \]
\[ \epsilon''_r(\omega) \quad = \text{imaginary part of relative permittivity of the material}; \]
\[ \tan \delta_e \quad = \text{dielectric dissipation factor (loss tangent)}. \]

Similarly, the complex permeability \( \mu^*(\omega) \) can be written as
\[ \mu^*(\omega) = \mu_0 \mu'_r(\omega) (1 - j \tan \delta_{\mu}) = \mu_0 \left( \mu'_r(\omega) - j \mu''_r(\omega) \right), \tag{3.2.6} \]

where,

\[ \mu_0 = \text{permeability of free space} = 4 \pi \times 10^{-7} \text{ (H/m)}; \]
\[ \mu'_r(\omega) = \text{real part of relative permeability of the material}; \]
\[ \mu''_r(\omega) = \text{imaginary part of relative permeability of the material}; \]
\[ \tan \delta_{\mu} = \text{magnetic dissipation factor}. \]

Let \( z \) be the direction of propagation. Then, the differential operator \( \nabla \) or \( \nabla \) which appears in Maxwell's equations (3.2.1)-(3.2.4), can be separated into its transverse and longitudinal parts as follows:

\[ \nabla = \nabla_t + \nabla_z \frac{\partial}{\partial z}. \tag{3.2.7} \]

Similarly, the vector \( \vec{r} \) which defines the position can be separated into longitudinal and transverse components as follows:

\[ \vec{r} = \vec{r}_t + \vec{r}_z z. \tag{3.2.8} \]

The same relation holds for each of the fields defined in (3.2.1)-(3.2.4),

\[ \vec{E} = \vec{E}_t + \vec{E}_z E_z. \tag{3.2.9a} \]

and

\[ \vec{H} = \vec{H}_t + \vec{H}_z H_z. \tag{3.2.9b} \]

By the TEM assumption, the electric and magnetic field intensities are transverse to the direction
of propagation, i.e. $H_z = 0$ and $E_z = 0$. Hence, Maxwell’s curl equations become, after introducing the relations given by (3.2.7) through (3.2.9),

$$\mathbf{\varepsilon}_z \times \frac{\partial \mathbf{E}}{\partial z} = -j \omega \mu^*(\omega) \mathbf{H}, \quad (3.2.10)$$

and

$$\mathbf{\varepsilon}_z \times \frac{\partial \mathbf{H}}{\partial z} = -j \omega \epsilon^*(\omega) \mathbf{E}. \quad (3.2.11)$$

Within any cylindrical coordinate system, Maxwell’s equations are partially separable. This implies that the electrical and magnetic fields are products of a transverse function and a longitudinal function. Hence, they can be written as follows:

$$\mathbf{E}(r_l, z) = \mathbf{F}(r_l) \ V(z), \quad (3.2.12)$$

and

$$\mathbf{H}(r_l, z) = \mathbf{G}(r_l) \ I(z). \quad (3.2.13)$$

In (3.2.12) and (3.2.13), $V(z)$ and $I(z)$ are line voltage and line current, respectively and the functions $\mathbf{F}(r_l)$ and $\mathbf{G}(r_l)$ are the transverse electric and magnetic field distributions having dimensions m$^{-1}$.

For a two conductor transmission line, the line voltage is given by,

$$\int_A^B \mathbf{E} \cdot d\mathbf{l} = V(z). \quad (3.2.14)$$

In (3.2.14), $A$ and $B$ are points on the boundaries of each conductor shown in Figure 3.2.1. After substituting, (3.2.12) into (3.2.14) and simplifying, we obtain,
\[ \int_A^B \vec{F} \cdot d\vec{l} = 1. \quad (3.2.15) \]

Equation (3.2.15) provides a condition of normalization for the transverse electric distribution \( \vec{F} \).

Similarly, for a two conductor transmission line, the line current \( I(z) \) is given by

\[ \oint_C \vec{H} \cdot d\vec{l} = I(z). \quad (3.2.16) \]

where, \( C \) is a contour which surrounds one of the conductors of the transmission line as shown in Figure 3.2.2. Next, substituting (3.2.13) into (3.2.16) and simplifying, we obtain,

\[ \oint_C \vec{G} \cdot d\vec{l} = 1. \quad (3.2.17) \]

Equation (3.2.17) provides a condition of normalization for the transverse magnetic distribution function \( \vec{G} \).

### 3.2.2 Derivation of Material Properties

Having introduced the relations in the previous sub-section, we are now in a position to derive the relation between material properties and transmission line parameters. In this regard, the following assumptions are made:

1. \( \epsilon^*(\omega) = \epsilon'_r(\omega) \epsilon_0 ; \)
2. \( \mu^*(\omega) = \mu'_r(\omega) \mu_0 ; \)
3. \( \sigma_c = \infty . \)

The first two assumptions imply that the dielectric and the magnetic material inside the
Figure 3.2.1 Integration contours for calculating the capacitance per unit length of a transmission line.
transmission line are lossless and the third assumption imply that the conductors of the transmission line are perfect. With the above assumptions, the electric field vanishes within the conductors. Hence, the surface charge density is given by

\[
\rho_s(\omega) = \mathbf{n} \cdot \mathbf{D} = \varepsilon'_s(\omega) \varepsilon_0 \mathbf{n} \cdot \mathbf{E} = \varepsilon'_r(\omega) \varepsilon_0 \mathbf{n} \cdot \mathbf{F} V(z) .
\]  

(3.2.18)

In (3.2.18), \( \mathbf{n} \) corresponds to a unit vector normal to the surface. The total charge \( Q(\omega) \) on a section conductor of length \( \Delta z \) (\( \ll \lambda \)) is obtained by integrating the surface charge density over the surface of the conductor. Hence, the total charge is given by

\[
Q(\omega) = \int_{S_A} \rho_s(\omega) \, dA \simeq \varepsilon'_r(\omega) \varepsilon_0 \Delta z \int_{C_A} \mathbf{n} \cdot \mathbf{E} \, dl .
\]  

(3.2.19)

The integration surfaces \( SA \) and \( SA' \) are defined in Figure 3.2.1. Surface \( SA \) is located directly on the metal boundary, whereas surface \( SA \) lies infinitely close to the boundary but on the side of the propagation medium. Furthermore, the contour \( CA \) is located within the surface \( SA' \). The capacitance per unit length is obtained by dividing \( Q(\omega) \) by the line voltage \( V(z) \) and the length \( \Delta z \) and making use of the normalization condition given by (3.2.15). Hence,

\[
C(\omega) = \frac{Q(\omega)}{V \Delta z} = \varepsilon'_r(\omega) \varepsilon_0 \int_{C_A} \mathbf{n} \cdot \mathbf{F} \, dl .
\]  

(3.2.20)

To obtain the inductance per unit length we first calculate the magnetic flux \( \phi(\omega) \) through a small length of line \( \Delta z \). Hence \( \phi(\omega) \) is given by

\[
\phi(\omega) = \int_{S} \mathbf{B} \cdot \mathbf{n} \, dA = \mu'_r(\omega) \mu_0 I(z) \Delta z \int_{A} \left( \mathbf{\vec{c}}_x \times \mathbf{G} \right) \cdot d\mathbf{l} .
\]  

(3.2.21)

The integration contours for the magnetic flux are shown in Figure 3.2.2. The inductance per
unit length is obtained by dividing $\phi(\omega)$ by the line current $I(z)$ and the length $\Delta z$ and making use of the normalization condition given by (3.2.17). Hence,

$$L_e(\omega) = \frac{\phi(\omega)}{I_{\Delta z}} = \mu_r(\omega) \mu_0 \int_A^B (\vec{e}_z \times \vec{G}) \cdot d\vec{l}.$$  \hfill (3.2.22)

To derive the conductance per unit length $G(\omega)$ of the line, we remove the assumption that the line is free of dielectric losses [26]. The shunt admittance per unit length $Y(\omega)$ of a lossless transmission line is given by

$$Y(\omega) = j \omega C(\omega) = j \omega \epsilon'_r(\omega) \epsilon_o \int_{CA} \vec{n} \cdot \vec{F} \ dl.$$  \hfill (3.2.23)

For a transmission line that contains lossy dielectric material, $\epsilon'_r(\omega) \epsilon_o$ is replaced by the complex permittivity, $\epsilon^*(\omega) = \epsilon'_r(\omega) \epsilon_o - j \epsilon''_r(\omega) \epsilon_o$. Hence, (3.2.24) becomes

$$Y(\omega) = j \omega \epsilon'_r(\omega) \epsilon_o \int_{CA} \vec{n} \cdot \vec{F} \ dl + \omega \epsilon''_r(\omega) \epsilon_o \int_{CA} \vec{n} \cdot \vec{F} \ dl.$$  \hfill (3.2.24)

In terms of network representation, the admittance per unit length $Y(\omega)$ of a transmission line is given by

$$Y(\omega) = G(\omega) + j \omega C(\omega).$$  \hfill (3.2.25)

Comparing (3.2.24) with (3.2.25) yields

$$G(\omega) = \omega \epsilon''_r(\omega) \epsilon_o \int_{CA} \vec{n} \cdot \vec{F} \ dl.$$  \hfill (3.2.26)

Next, to derive the resistance per unit length $R(\omega)$ of the line, we remove the assumption that the
Figure 5.2.2  Integration contours for calculating the inductance per unit length of a transmission line.
line is free of magnetic losses. The series impedance per unit length \( Z(\omega) \) of a lossless transmission line is given by

\[
Z(\omega) = j \omega L_e(\omega) = j \omega \mu'_r(\omega) \mu_o \int_A^B (\vec{e}_z \times \vec{G}) \cdot \, \, d\vec{l} .
\] (3.2.27)

For a transmission line that contains lossy magnetic material, \( \mu'_r(\omega)\mu_o \) is replaced by the complex permittivity, \( \mu^*(\omega) = \mu'_r(\omega)\mu_o - j \mu''_r(\omega)\mu_o \). Hence, (3.2.27) becomes

\[
Z(\omega) = j \omega \mu'_r(\omega) \mu_o \int_A^B (\vec{e}_z \times \vec{G}) \cdot \, \, d\vec{l} + \omega \mu''_r(\omega) \mu_o \int_A^B (\vec{e}_z \times \vec{G}) \cdot \, \, d\vec{l} .
\] (3.2.28)

In terms of network representation, the impedance per unit length \( Z(\omega) \) of a transmission line is given by

\[
Z(\omega) = R(\omega) + j \omega L_e(\omega) .
\] (3.2.29)

Comparing (3.2.28) with (3.2.29) yields

\[
R(\omega) = \omega \mu''_r(\omega) \mu_o \int_A^B (\vec{e}_z \times \vec{G}) \cdot \, \, d\vec{l} .
\] (3.2.30)

Next, to derive the dc resistance per unit length \( R_{dc} \) and the skin effect impedance per unit length \( Z_{sk}(\omega) \), we remove the assumption that the line has conductors with infinite conductivity. When the transmission line has conductors with finite conductivity \( \sigma_c \neq \infty \), then the modes of propagation can not be transverse electromagnetic. In such a case, longitudinal components of either the electric or the magnetic field or both, are necessary to satisfy the boundary conditions at the dielectric-metal interface. This results to the existence of transverse electric (TE) and/or transverse magnetic (TM) modes. If these modes are strong, then, the
dominant mode of propagation can not be anymore TEM. For certain geometries, the longitudinal components of the fields are much smaller than the transverse ones and the dominant mode of propagation keeps most of the characteristics of the lossless TEM mode. The transmission lines that posses these properties are called quasi-TEM lines. A stripline whose cross-section is shown in Figure 1.1.1 falls into this category.

At low frequencies and at the zero frequency limit, the current is evenly distributed over the whole cross-section of the conductors. The dc resistance per unit length of a section of uniform conductor having a conductivity $\sigma_c$ and cross section $S$ is given by

$$R_{dc} = \frac{1}{S \sigma_c}.$$  \hspace{1cm} (3.2.31)

In a conductor having finite conductivity, both the electric and magnetic field penetrate into the conductors. The internal magnetic energy contributes to the inductance per unit length of the line and produces an additional term called the internal inductance per unit length $L_i(\omega)$. The internal inductance per unit length is obtained by integrating the magnetic energy over the conductors cross-section, which gives

$$L_i(\omega) = \frac{\mu_r(\omega) \mu_o \int |\vec{H}|^2 \, dA}{S \int |I(z)|^2 \, dz}.$$ \hspace{1cm} (3.2.32)

Due to a finite conductivity, a skin effect resistance per unit length is also introduced [27]. The skin effect resistance per unit length $R_{sk}(\omega)$ is related to $L_i(\omega)$ by

$$R_{sk}(\omega) = j \omega L_i(\omega).$$ \hspace{1cm} (3.2.33)

Hence, the skin effect impedance per unit length is defined by
\[ Z_{sk}(\omega) = R_{sk}(\omega) + j \omega L_{c}(\omega). \] (3.2.34)

Using the above derivations, the total series impedance per unit length of the lossy transmission line is defined as

\[ Z(\omega) = R_{dc} + R(\omega) + j \omega L_{c}(\omega) + Z_{sk}(\omega), \] (3.2.35)

and the total shunt admittance per unit length is defined as

\[ Y(\omega) = G(\omega) + j \omega C(\omega). \] (3.2.36)

Having defined the impedance and the admittance per meter of a lossy transmission line, the propagation function can defined as [25]

\[ \gamma(\omega) = \sqrt{Z(\omega)Y(\omega)}, \] (3.2.37)

and, the characteristic impedance as

\[ Z_0(\omega) = \sqrt{\frac{Z(\omega)}{Y(\omega)}}. \] (3.2.38)
3.3 MODELS FOR LOSSY TRANSMISSION LINE PARAMETERS AND THEIR APPLICATION TO THE DETERMINATION OF MATERIAL PROPERTIES OF A STRIPLINE

3.3.1 Discussion

In practice, the accurate determination of both the complex permittivity and the complex permeability of a material in a striplike geometry requires the knowledge of the propagation function as well as the characteristic impedance. The propagation function and the characteristic impedance can be determined from the measurement data through numerical computation by using two approaches. The first approach involves the use of analytical formulation that relates the measured data to the propagation function and the characteristic impedance [28]. Once the propagation function and the characteristic impedance are obtained, the complex permittivity and the complex permeability are determined from these parameters directly. The second approach which is discussed in this dissertation requires the determination of the lossy transmission line parameters such as $C(\omega), L_c(\omega), Z_{sk}(\omega), R(\omega), R_{dc}$ and $G(\omega)$. In this case, the measured data are indirectly related to the lossy line parameters through the characteristic impedance and the propagation function by (3.2.37) and (3.2.38), respectively. The real parts of the permittivity and the permeability are obtained using (3.2.20) and (3.2.22) and the corresponding imaginary parts are obtained using (3.2.26) and (3.2.30), respectively.

In general, the lossy transmission line parameters such as $C(\omega), L_c(\omega), Z_{sk}(\omega), R(\omega), R_{dc}$ and $G(\omega)$ are functions of frequency and depend on the type of material used. In order to make use of the second approach for determining the intrinsic properties of materials, a model has to be assumed for each of the lossy line parameters.

The purpose of this section is to present the concept of using lossy transmission line formulation in the determination of material properties of a stripline. Towards this goal, it is
proposed that a stripline can be represented (modeled) by a single lossy transmission line. In the previous section, it was shown that the transmission line parameters are related to the line geometry by integral expressions which are independent of frequency. These integral expressions become quite complicated for a stripline geometry. In certain cases, it is extremely difficult to solve them exactly without making certain assumptions on the line geometry. However, when a solution is obtained for a fixed geometry the result is simply a constant number. In the light of this discussion, the integral expressions presented in the previous section will be treated as constants and determined using a non-linear least squares optimization technique. The concept and approach of this optimization technique will be presented in chapter 5.

3.3.2 Existing models and their applications

Several researchers have developed models for the skin effect impedance per unit length \( Z_{sk}(\omega) \) and the conductivity per unit length \( G(\omega) \) [12]-[14], [29]-[30]. A model for the skin effect impedance per unit length was originally developed by Nahman in 1962 for the sub-nanosecond to nanosecond domain transient analysis of coaxial conductor cables [12] is given below. The model specifies \( Z_{sk} \) as

\[
Z_{sk} = K_1 (j\omega)^p ; \quad 0 < p < 1,
\]  

(3.3.1)

where,

\[
K_1 = \text{high frequency loss coefficient per unit length (}\Omega/[(\text{radians/sec})^p]]; \\
p = \text{high frequency loss exponent.}
\]

The skin effect impedance as defined by (3.3.1) is a generalization of the conventional skin effect surface impedance and is the result of an electric field penetrating into a metal of a given dc conductivity \( \sigma_c \). However, this relation is not exact and is valid only in an approximate sense. For an infinite plane conductor, the skin effect surface impedance varies in direct
proportion to $\sqrt{j\omega}$. On the other hand, for a cylindrical conductor, the surface impedance is expressed in terms of Bessel functions. However, if the electromagnetic field depth of penetration is very small compared to the radius of curvature of the cylinder surface, the surface appears to be plane and the $\sqrt{j\omega}$ predominates [31].

As mentioned earlier, the skin effect impedance per unit length $Z_{sk}(\omega)$ can be written as a combination of the skin effect resistance per unit length $R_{sk}(\omega)$ and the internal inductance per unit length, $L_i(\omega)$. In order to write (3.3.1) in terms of $R_{sk}(\omega)$ and $L_i(\omega)$, Euler’s formula [32] which is expressed as

$$e^{j\theta} = \cos \theta + j \sin \theta$$  \hspace{1cm} (3.3.2)

is used. By making use of the Euler’s formula, the complex number $j$ can be written as $j = e^{j\pi/2}$. Using this relation and (3.3.2), $j^p$ can be written as

$$j^p = \left(e^{j\pi/2}\right)^p = \cos \left(\frac{\pi p}{2}\right) + j \sin \left(\frac{\pi p}{2}\right).$$  \hspace{1cm} (3.3.3)

Substituting (3.3.3) into (3.3.1) and rearranging in the form of (3.2.34), we obtain

$$Z_{sk}(\omega) = K_1 \omega^p \cos \left(\frac{\pi p}{2}\right) + j \omega K_1 \omega^{p-1} \sin \left(\frac{\pi p}{2}\right).$$  \hspace{1cm} (3.3.4)

Comparing (3.2.34) with (3.3.4) gives

$$R_{sk}(\omega) = K_1 \omega^p \cos \left(\frac{\pi p}{2}\right),$$  \hspace{1cm} (3.3.5a)

and,

$$L_i(\omega) = K_1 \omega^{p-1} \sin \left(\frac{\pi p}{2}\right).$$  \hspace{1cm} (3.3.5b)
The dc resistance per unit length is a function of the dc conductivity and the cross-sectional area of the line. Hence, it can be modeled as a constant as follows:

\[ R_{dc} = K_2, \quad (3.3.6) \]

where, \( K_2 \) is a constant having the dimensions of \( \Omega/m \).

The permeability of most dielectric materials, including diamagnetic, paramagnetic and antiferromagnetic materials is approximately the same as that of free space \( \mu_0 \) (i.e. \( \mu'_e = 1 \)). Ferromagnetic and ferrimagnetic materials exhibit much higher permeability than free space and are usually lossy. The complex permeability of a ferromagnetic material is usually a tensor due to its inhomogeneous and anisotropic structure. This requires the treatment of Maxwell's equations in a tensor form which can become quite involved and complicated. Hence, in this dissertation, it will be assumed that the complex permeability of the material used in constructing the stripline under test is a scalar quantity. An expression for \( \mu^*(\omega) \) using the modified Bloch-Bloembergen damping form [33] is given by

\[ \mu^*(\omega) = \mu_0 + \frac{\mu_0 \omega_m \omega_0}{\omega_0^2 - \omega^2 + \frac{1}{\tau^2} + 2 j \frac{\omega}{\tau}}, \quad (3.3.7) \]

where,

\[ \omega_m = \frac{\chi_o \omega_0}{\mu_0}; \]

\( \chi_o = \) static susceptibility (F/m);

\( \omega_0 = \) ferromagnetic resonant frequency (rad/sec);

\( \tau = \) relaxation time (sec).

From (3.3.7), \( \mu'_e(\omega) \) and \( \mu'_e(\omega) \) are obtained as follows:
\[ \mu'_\omega(\omega) = \frac{\text{Re}[\mu^*(\omega)]}{\mu_0} = 1 + \frac{\omega_m \omega_0 \left[ \omega_o^2 - \omega^2 + \frac{1}{\tau^2} \right]}{\left[ \omega_o^2 - \omega^2 + \frac{1}{\tau^2} \right]^2 + 4 \frac{\omega^2}{\tau^2}} , \]  

(3.3.8)

and

\[ \mu''_\omega(\omega) = -\frac{\text{Im}[\mu^*(\omega)]}{\mu_0} = -\frac{2 \frac{\omega}{\tau} \omega_m \omega_0}{\left[ \omega_o^2 - \omega^2 + \frac{1}{\tau^2} \right]^2 + 4 \frac{\omega^2}{\tau^2}} . \]  

(3.3.9)

where, \(\text{Re}[]\) and \(\text{Im}[]\) are the real and imaginary parts of \(\[]\).

Using (3.3.8) and (3.2.22), a model for the external inductance per unit length \(L_e(\omega)\) is obtained as

\[ L_e(\omega) = \mu_0 \left[ 1 + \frac{\omega_m \omega_0 \left[ \omega_o^2 - \omega^2 + \frac{1}{\tau^2} \right]}{\left[ \omega_o^2 - \omega^2 + \frac{1}{\tau^2} \right]^2 + 4 \frac{\omega^2}{\tau^2}} \right] A_1 , \]

(3.3.10)

where, \(A_1\) is a constant which is given by the following integral expression

\[ A_1 = \int_A^B (\vec{e}_z \times \vec{G}) \cdot d\vec{l} . \]

(3.3.11)

On the other hand, the resistance per unit length \(R(\omega)\) that is responsible for the magnetic losses is obtained by substituting (3.3.9) into (3.2.30), which yields

\[ R(\omega) = \omega \mu_0 \left[ \frac{2 \frac{\omega}{\tau} \omega_m \omega_0}{\left[ \omega_o^2 - \omega^2 + \frac{1}{\tau^2} \right]^2 + 4 \frac{\omega^2}{\tau^2}} \right] A_1 . \]

(3.3.12)
The unknown parameters in (3.3.11) and (3.3.12) are $\omega_m$, $\omega_o$, $\tau$ and $A_1$. These parameters can be determined through several measurement techniques. Once, they are determined $\mu^*_1(\omega)$ and $\mu^*_2(\omega)$ can be calculated using (3.3.8) and (3.3.9), respectively.

Having presented a model for the complex permeability, it is now the time to present a model for the complex permittivity. This model is derived from the well known Debye equation and is given by [34]

$$
\varepsilon^*(\omega) = \varepsilon_o \left[ \varepsilon'_{\infty} + \frac{(\varepsilon'_s - \varepsilon'_\infty)}{1 + j \omega \tau_e} \right], 
$$

(3.3.13)

where, $\varepsilon'_s$ and $\varepsilon'_\infty$ are the relative permittivities at zero and infinite frequencies, respectively. Furthermore, $\tau_e$ is new relaxation time constant related to the original relaxation time $\tau$ by

$$
\tau_e = \tau \frac{\varepsilon'_s + 2}{\varepsilon'_\infty + 2} .
$$

(3.3.14)

From (3.3.13), $\varepsilon'_s(\omega)$ and $\varepsilon''(\omega)$ are obtained as follows:

$$
\varepsilon'_s(\omega) = \frac{\text{Re} [\varepsilon^*(\omega)]}{\varepsilon_o} = \varepsilon'_\infty + \frac{(\varepsilon'_s - \varepsilon'_\infty)}{1 + \omega^2 \tau_e^2} ,
$$

(3.3.15)

and

$$
\varepsilon''(\omega) = \frac{-\text{Im} [\varepsilon^*(\omega)]}{\varepsilon_o} = \frac{(\varepsilon'_s - \varepsilon'_\infty) \omega \tau_e}{1 + \omega^2 \tau_e^2} .
$$

(3.3.16)

Using (3.3.15) and (3.2.20), a model for $C(\omega)$ can be written as

$$
C(\omega) = \varepsilon_o \left[ \varepsilon'_{\infty} + \frac{(\varepsilon'_s - \varepsilon'_\infty)}{1 + \omega^2 \tau_e^2} \right] A_2 ,
$$

(3.3.17)
where, $A_2$ is a constant which is given by the integral expression

$$
A_2 = \int_{CA} \vec{r} \cdot \vec{F} \ dl .
$$

(3.3.18)

Furthermore, using (3.3.16) and (3.2.26), the conductance per unit length that is responsible for the dielectric losses is given by

$$
G(\omega) = \omega \varepsilon_o \left[ \frac{(\varepsilon'_r - \varepsilon'_\infty) \omega \tau_e}{1 + \omega^2 \tau_e^2} \right] A_2 .
$$

(3.3.19)

Reference [25] shows that the constants $A_1$ and $A_2$ are related to each other by the following relation,

$$
A_1 A_2 = 1 .
$$

(3.3.20)

It has been shown that, the Debye model provides a very good approximation for determining the dielectric properties of certain class of liquids [35]-[36]. The unknown parameters in (3.3.17) and (3.3.19) are $\varepsilon'_r$, $\varepsilon'_\infty$ and $\tau$. These parameters can be determined through several measurement techniques. Once, they are determined $\varepsilon'_r(\omega)$ and $\varepsilon''_r(\omega)$ can be calculated using (3.3.15) and (3.3.16), respectively.

### 3.3.3 A new model for the complex permittivity

In the previous section, a model for the complex permeability as well as for the complex permittivity was presented. These models can be used for determining the intrinsic properties of some materials. Typically, the Debye model is only applicable for representing the dielectric properties of certain class of liquids. On the other hand, the Bloch-Bloembergen model is
applicable for determining the magnetic properties of materials in general. The validity of the Debye model has been verified experimentally [35]-[36].

The real and imaginary parts of any physical imittance function has to be related by a pair of Hilbert transforms. This assures that the resulting network is causal. Both the Debye and the Bloch-Bloembergen models satisfy the Hilbert transform pairs. The literature survey has shown that no causal model was available for representing the dielectric properties of solid materials such as polymers and thick-film pastes. This has made it necessary to come up with a new model for representing the dielectric properties of solid materials of interest. Hence, in this section a new model for the complex permittivity is presented. For simplicity, it will be assumed that the dielectric material used in constructing the stripline is non-magnetic (i.e. \( \mu_r(\omega) = 1 \)). Using this assumption and the formulation presented in the previous section, the external inductance per meter \( L_e(\omega) \) is given by

\[
L_e(\omega) = \mu_0 A_1 = L_o = \text{constant.} \tag{3.3.21}
\]

Before introducing the new model for the complex permittivity, let us show that the real and imaginary parts of \( \varepsilon^*(\omega) \) are interrelated with each other by a pair of Hilbert transforms. These are known as the Kramers-Kronig relations [25] and are given by

\[
\varepsilon'_r(\omega) = \varepsilon'_{r\infty} + \frac{2}{\pi} \mathcal{P} \int_0^\infty \frac{\xi \varepsilon'_r(\xi)}{\xi^2 - \omega^2} \, d\xi, \tag{3.3.22}
\]

and

\[
\varepsilon''_r(\omega) = \frac{2}{\pi} \frac{\omega}{\varepsilon'_{r\infty}} \mathcal{P} \int_0^\infty \frac{\varepsilon'_r(\xi) - \varepsilon'_{r\infty}}{\xi^2 - \omega^2} \, d\xi. \tag{3.3.23}
\]

where, \( \varepsilon'_{r\infty} \) is the value of the relative permittivity at infinite frequency, while \( \mathcal{P} \) denotes the
Cauchy principal value of the integral [37]. The inspiration for the new model resulted from Jonscher’s work [13], [29]-[30] who has shown that many dielectric materials may be described by the following empirical law:

\[ G(\omega) \propto |\omega|^q ; \quad 0 < q < 1. \]  

(3.3.24)

where, q is a high frequency loss exponent. Curtins et al. [14] have used this model in their analysis for the pulse behavior of transmission lines with dielectric losses. In their analysis, they have used the following approximate model for the capacitance per unit length of the line:

\[ C(\omega) = C_0 = \text{constant}. \]  

(3.3.25)

Based on Curtins et al.’s definition of \( C(\omega) \), \( \epsilon'_r(\omega) \) is obtained using (3.2.20) to be as follows:

\[ \epsilon'_r(\omega) = \frac{C_0}{\epsilon_0 A_0} = \epsilon'_{r\infty}. \]  

(3.3.26)

In order to obtain \( \epsilon''_r(\omega) \), we substitute (3.3.26) into (3.3.23) which yields

\[ \epsilon''_r(\omega) = \frac{2}{\pi} \omega \int_0^\infty \frac{\epsilon'_{r\infty} - \epsilon'_{r\infty}}{\xi^2 - \omega^2} d\xi = 0. \]  

(3.3.27)

Using the above result, the actual value for \( G(\omega) \) is obtained by substituting (3.3.27) into (3.2.26), which gives

\[ G(\omega) = 0. \]  

(3.3.28)

Obviously, (3.3.28) is not the same as (3.3.24). The above analysis shows that if one specifies
$G(\omega)$ to be a function of frequency, then $C(\omega)$ has to be a function of frequency too, such that the Kramers-Kronig relations are satisfied. Hence, Curtins et al. attempt to use Jonscher’s result for $G(\omega)$ with a constant value for $C(\omega)$ was erroneous. The proper approach would be to obtain $C(\omega)$ from $G(\omega)$ according to the Kramers-Kronig relations. The derivation of the new model uses this approach as explained in the following.

For a non-magnetic material, the models for $L_e(\omega)$, $Z_{sk}(\omega)$ and $R_{dc}$ were previously discussed. Using these models, the impedance per unit length of the line $Z(\omega)$ becomes

$$Z(\omega) = R_{dc} + j \omega L_e(\omega) + Z_{sk}(\omega) = R_{dc} + j \omega \left(L_e(\omega) + L_i(\omega)\right) + R_{sk}(\omega)$$

$$= K_2 + j \omega \left(L_o + K_1 \omega^{p-1} \sin \left(\frac{\pi \rho}{2}\right) + K_1 \omega^p \cos \left(\frac{\pi \rho}{2}\right)\right). \quad (3.3.29)$$

Towards deriving the new model for the complex permittivity, we claim that the admittance per unit length $Y(\omega)$ and the impedance per unit length $Z(\omega)$ should be the dual of each other. The duality in this context implies that the two functions have the same functional behavior. Based on this conjecture, the proposed model for $Y(\omega)$ is shown below

$$Y(\omega) = K_4 + j \omega \left(C_o + K_3 \omega^{q-1} \sin \left(\frac{\pi \rho}{2}\right) + K_3 \omega^q \cos \left(\frac{\pi \rho}{2}\right)\right). \quad (3.3.30)$$

where,

$$K_4 = G_{dc} = \text{dc conductance per unit length (S/m)};$$

$$C_o = \text{high frequency limit capacitance per unit length (F/m)};$$

$$K_3 = \text{high frequency admittance coefficient per unit length (S/[m(radians/sec)]^q)};$$

$$q = \text{high frequency admittance exponent}.$$

The dc conductance per unit length is defined as
\[ G_{dc} = \sigma_d \int_{C_A} \vec{n} \cdot \vec{F} \ dl = \sigma_d A_2. \tag{3.3.31} \]

In (3.3.31), \( \sigma_d \) is the conductivity (S/m) of the dielectric material (ideally \( \sigma_d = 0 \)). On the other hand, the high frequency limit capacitance per unit length \([36]\) is defined by

\[ C_o = \lim_{\omega \to \infty} \left[ \frac{Y(\omega)}{j\omega} \right]. \tag{3.3.32} \]

Comparing (3.3.30) with (3.2.25), the new expressions for \( C(\omega) \) and \( G(\omega) \) are obtained as follows:

\[ C(\omega) = C_o + K_3 \omega^q - 1 \sin\left(\frac{\pi q}{2}\right), \tag{3.3.33} \]

and

\[ G(\omega) = G_{dc} + K_3 \omega^q \cos\left(\frac{\pi q}{2}\right). \tag{3.3.34} \]

Using (3.3.33), the following expression is obtained for \( \epsilon'_r(\omega) \):

\[ \epsilon'_r(\omega) = \frac{C_o + K_3 \omega^q - 1 \sin\left(\frac{\pi q}{2}\right)}{\epsilon_o A_2}. \tag{3.3.35} \]

Furthermore, using (3.3.34), the following expressions are obtained for \( \sigma_d \) and \( \epsilon''_r(\omega) \):

\[ \sigma_d = \frac{G_{dc}}{A_2}, \tag{3.3.36} \]

and

\[ \epsilon''_r(\omega) = \frac{K_3 \omega^q - 1 \cos\left(\frac{\pi q}{2}\right)}{\epsilon_o A_2}. \tag{3.3.37} \]

Next, we will show that the new model for \( \epsilon'_r(\omega) \) and \( \epsilon''_r(\omega) \) as defined by (3.3.35) and
(3.3.37), respectively, satisfy the Kramers-Kronig causality relations.

Proof:

We will first show that (3.3.22) is satisfied. Hence, substituting (3.3.37) into (3.3.22) and using table of integrals [38], we obtain

\[
\epsilon'_r(\omega) = \epsilon'_\infty + \frac{2 K_3 \cos \left(\frac{\pi q}{2}\right)}{\pi \epsilon \frac{\omega}{A_2}} P \int_0^\infty \frac{\xi q}{\xi^2 - \omega^2} d\xi
\]

\[
= \epsilon'_\infty + \frac{K_3 \cos \left(\frac{\pi q}{2}\right)}{\pi \epsilon \frac{\omega}{A_2}} \int_0^\infty \left[ \frac{\xi q - 1}{\xi + \omega} + \frac{\xi q - 1}{\xi - \omega} \right] d\xi
\]

\[
= \epsilon'_\infty + \frac{K_3 \omega q^{-1} \cos \left(\frac{\pi q}{2}\right)}{\epsilon \frac{\omega}{A_2}} \left[ \frac{1 - \cos (\pi q)}{\sin (\pi q)} \right].
\]

(3.3.38)

In order to simplify (3.3.38) further, we use the following half angle trigonometric relation:

\[
\tan \left(\frac{q}{2}\right) = \pm \sqrt{\frac{1 - \cos q}{1 + \cos q}} = \frac{1 - \cos q}{\sin q} = \frac{\sin q}{1 + \cos q}.
\]

(3.3.39)

Substituting (3.3.39) into (3.3.38) and simplifying yields

\[
\epsilon'_r(\omega) = \epsilon'_\infty + \frac{K_3 \omega q^{-1} \sin (\pi q)}{\epsilon \frac{\omega}{A_2}}.
\]

(3.3.40)

By taking \(\epsilon'_\infty\) to be \(\frac{C_0}{\epsilon \frac{\omega}{A_2}}\), (3.3.40) becomes equivalent to (3.3.35). This proves the first part of the Kramers-Kronig relation. To complete the proof, we substitute (3.3.37) into (3.3.23) and use table of integrals [38]. Hence, we have
\[ \epsilon_r''(\omega) = \frac{2 \omega}{\pi} \frac{K_3 \sin \left( \frac{\pi q}{2} \right)}{\epsilon_0 A_2} p \int_0^\infty \frac{\xi^{q-1}}{\xi^2 + \omega^2} d\xi \]

\[ = \frac{2 \omega}{\pi} \frac{K_3 \sin \left( \frac{\pi q}{2} \right)}{\epsilon_0 A_2} \left[ \frac{\pi}{2} \left( \omega^{q-1} \csc (\pi q) + \omega^{q-1} \cot (\pi q) \right) \right] \]

\[ = \frac{K_3 \omega^{q-1} \sin \left( \frac{\pi q}{2} \right)}{\epsilon_0 A_2} \left[ 1 + \cos \left( \frac{\pi q}{2} \right) \right]. \quad (3.3.41) \]

Next, substituting (3.3.39) into (3.3.41) and simplifying further yields

\[ \epsilon_r''(\omega) = \frac{K_3 \omega^{q-1} \cos \left( \frac{\pi q}{2} \right)}{\epsilon_0 A_2}. \quad (3.3.42) \]

Comparing (3.3.42) with (3.3.37) shows that the two equations are in full agreement. Hence, the new model for the complex permittivity satisfies the Kramers-Kronig causality relations.

Q. E. D.

In the analysis of material properties, it is typical to discuss and often specify the ratio of \( \epsilon_r''(\omega) \) to \( \epsilon_r'(\omega) \). This ratio was defined in the section 3.2.1 as the dielectric dissipation factor or the loss tangent (\( \tan \delta_c \)). Using the above definitions for \( \epsilon_r'(\omega) \) and \( \epsilon_r''(\omega) \), the loss tangent becomes

\[ \tan \delta_c = \frac{\epsilon_r''(\omega)}{\epsilon_r'(\omega)} = \frac{1}{\frac{K_3 \omega^{q-1} \cos \left( \frac{\pi q}{2} \right)}{C_0 + \tan \left( \frac{\pi q}{2} \right)}.} \quad (3.3.43) \]

It is also of practical interest to define a modified form of the loss tangent as

\[ \tan \delta'_c = \frac{\epsilon_r''(\omega)}{\epsilon_r'(\omega) - \epsilon_r'(\omega_\infty)} = \cot \left( \frac{\pi q}{2} \right). \quad (3.3.44) \]

It is observed from (3.3.44) that, the modified loss tangent is independent of frequency. This is in
complete contrast with the Debye behavior for which this ratio is equal to \( \omega \tau_e \). Johnscher [29]-[30] has arrived to the same conclusion using an empirical analysis and has defined this as the "universal" dielectric response. He has shown that the validity of this response extends for a wide range of dielectrics from \( \omega_p = \frac{1}{\tau_e} \) to the onset of phonon and quantum phenomena at around 1 - 10 GHz.

The formulation presented in this section can be applied for the determination of the dielectric properties of a material in a stripline geometry. The unknown parameters are \( K_1, K_2, K_3, K_4, p, q, L_o \) and \( C_o \). Once these parameters are determined using the optimization technique of chapter 5, the material properties can be directly computed.

### 3.4 CAUSALITY RELATIONS FOR THE PROPAGATION FUNCTION

In this section, the causality properties of the propagation function will be presented. The propagation function as defined in the previous section by (3.2.37) can be written as

\[
\gamma(j\omega) = \alpha(\omega) + j \beta(\omega),
\]

(3.4.1)

where, \( \alpha(\omega) \) is the attenuation function (Np/m), while \( \beta(\omega) \) is the phase function (rad/m). These two functions are interrelated with each other through a pair of Hilbert transforms as follows [39]-[40]:

\[
\beta(\omega) = - \frac{2}{\pi} \omega P \int_0^\infty \frac{\alpha(\xi)}{\omega^2 - \xi^2} d\xi,
\]

(3.4.2)

and,

\[
\alpha(\omega) = \frac{2}{\pi} P \int_0^\infty \frac{\xi \beta(\xi)}{\omega^2 - \xi^2} d\xi.
\]

(3.4.3)
Using the relations for $Z(\omega)$ and $Y(\omega)$ defined in section 3.3.3, $\gamma(j\omega)$ can be written as follows:

$$
\gamma(j\omega) = \sqrt{Z(\omega) Y(\omega)} = \left(\left[K_2 + j\omega L_o + K_1 (j\omega)^p \right] \left[K_4 + j\omega C_o + K_3 (j\omega)^q \right]^{\frac{1}{2}} \right.
$$

$$
= \left(\left[K_2 + j\omega \left(L_o + K_1 \omega^{p-1} \sin \left(\frac{\pi}{2}\right) + K_1 \omega^p \cos \left(\frac{\pi}{2}\right) \right) \right] \cdot \left[K_4 + j\omega \left(C_o + K_3 \omega^{q-1} \sin \left(\frac{\pi}{2}\right) + K_3 \omega^q \cos \left(\frac{\pi}{2}\right) \right) \right]\right)^{\frac{1}{2}}. \tag{3.4.4}
$$

Squaring both sides of (3.4.4) and separating into real and imaginary parts yields

$$
\gamma^2(j\omega) = A(\omega) + jB(\omega), \tag{3.4.5}
$$

where,

$$
A(\omega) = \left(\left[K_2 + K_1 \omega^p \cos \left(\frac{\pi}{2}\right) \right] \left[K_4 + K_3 \omega^q \cos \left(\frac{\pi}{2}\right) \right] \right.
$$

$$
- \left[\omega L_o + K_1 \omega^p \sin \left(\frac{\pi}{2}\right) \right] \left[\omega C_o + K_3 \omega^q \sin \left(\frac{\pi}{2}\right) \right], \tag{3.4.6}
$$

$$
B(\omega) = \left(\left[K_2 + K_1 \omega^p \cos \left(\frac{\pi}{2}\right) \right] \left[\omega C_o + K_3 \omega^q \sin \left(\frac{\pi}{2}\right) \right] \right.
$$

$$
+ \left[\omega L_o + K_1 \omega^p \sin \left(\frac{\pi}{2}\right) \right] \left[K_4 + K_3 \omega^q \cos \left(\frac{\pi}{2}\right) \right]. \tag{3.4.7}
$$

Next, squaring both sides of (3.4.1), we obtain

$$
\gamma^2(j\omega) = \alpha^2(\omega) - \beta^2(\omega) + 2 j \alpha(\omega) \beta(\omega). \tag{3.4.8}
$$
Comparing (3.4.8) with (3.4.5) yields

\[ A(\omega) = \alpha^2(\omega) - \beta^2(\omega), \quad (3.4.9) \]

and

\[ B(\omega) = 2 \alpha(\omega) \beta(\omega). \quad (3.4.10) \]

From (3.4.10), we obtain

\[ \alpha(\omega) = \frac{B(\omega)}{2 \beta(\omega)}. \quad (3.4.11) \]

Substituting (3.4.11) into (3.4.9) gives

\[ \beta^4(\omega) + A(\omega) \beta^2(\omega) - \frac{1}{4} B(\omega) = 0. \quad (3.4.12) \]

To solve (3.4.12), a biquadratic formula [41] is used. By retaining only the positive roots, the following expression is obtained for \( \beta(\omega) \):

\[ \beta(\omega) = \sqrt[4]{\frac{1}{2} \left( -A(\omega) + \sqrt{A^2(\omega) + B^2(\omega)} \right)}. \quad (3.4.13) \]

Substituting (3.4.13) into (3.4.9) and simplifying yields an expression for \( \alpha(\omega) \) which is given by

\[ \alpha(\omega) = \sqrt[4]{\frac{1}{2} \left( A(\omega) + \sqrt{A^2(\omega) + B^2(\omega)} \right)}. \quad (3.4.14) \]

The analytical evaluation of (3.4.2) and (3.4.3) based on the above phase and attenuation characteristics is extremely difficult. Hence, the causality properties of these functions will not be analyzed through the Hilbert transform pairs. Transfer functions that can be specified by either
their attenuation or phase characteristics are minimum phase transfer functions. Nahman [12], presented a set of conditions for the attenuation and phase of a minimum phase transfer function. These conditions are called the Bode conditions and are given by

\[
\lim_{\omega \to \infty} \left[ \frac{\gamma(j\omega)}{j\omega} \right] = 0.
\]  

(3.4.15)

Separating into phase and attenuation components, we obtain

\[
\lim_{\omega \to \infty} \left[ \frac{\beta(\omega)}{\omega} \right] = 0, \tag{3.4.16a}
\]

\[
\lim_{\omega \to \infty} \left[ \frac{\alpha(\omega)}{\omega} \right] = 0. \tag{3.4.16b}
\]

Again, the application of (3.4.16a) to (3.4.13) and (3.4.16b) to (3.4.14) is very cumbersome. Hence, we will apply (3.4.15) to (3.4.4) directly. Rearranging (3.4.4) such that the limiting term \((j\omega)\) is outside, we have

\[
\gamma(j\omega) = j\omega \sqrt{\frac{1}{C_o}} \left\{ \left[ \frac{K_2}{j\omega L_o} + 1 + \frac{K_1}{j\omega L_o}^p \right] \left[ \frac{K_4}{j\omega C_o} + 1 + \frac{K_3}{j\omega C_o}^q \right] \right\}^{1/2}. \tag{3.4.17}
\]

Next, taking the limit of (3.4.17) as \(\omega\) tends to \(\infty\), we obtain

\[
\lim_{\omega \to \infty} \left[ \frac{\gamma(j\omega)}{j\omega} \right] = \sqrt{\frac{1}{L_o C_o}} \neq 0. \tag{3.4.18}
\]

The above result shows that, the limiting term does not vanish at infinite frequency. Hence, the transfer function which uses the propagation function of (3.4.4) is not of minimum phase type. The term \(\sqrt{\frac{1}{L_o C_o}}\) constitutes a pure phase shift represented by the ideal lossless line. However, this phase shift does not introduce any corresponding attenuation. From this result, we can
conclude that (3.4.15) should be applied only to the modified propagation function. Namely,

\[ \gamma'(j\omega) = \alpha(\omega) + j \beta'(\omega), \quad (3.4.19) \]

where,

\[ \beta'(j\omega) = \beta(\omega) - \omega \sqrt{L_o C_o}. \quad (3.4.20) \]

With the above definition, \( \beta'(j\omega) \) becomes the phase shift without transmission delay component. The propagation function as defined by (3.4.19) yields a transfer function of the minimum phase type. We can now state an important property of minimum phase transfer functions: “Any minimum phase transfer function is causal” [39]-[40]. Therefore, the proposed model for the propagation function leads to a causal transfer function.

3.5 SUMMARY AND CONCLUSIONS

In this chapter, a new technique for the determination of material properties such as the complex permittivity and the complex permeability in a stripline geometry was presented. To overcome the limitations in stripline formulation available in literature, it was proposed to model a stripline under test with a lossy transmission line. Towards this goal, the chapter has provided derivations for the relations between material properties and lossy transmission line parameters. The technique requires a priori knowledge of the functional behavior of the lossy line parameters.

The chapter presented several mathematical models for both the complex permittivity and the permeability that are available in literature. These models are only capable for representing the properties of certain class of materials. Since, no causal model was available in the literature for the complex permittivity that would closely represent the thick film and polymer dielectric materials, it was necessary to introduce a new, causal model. The new model was shown to
satisfy the Kramers-Kronig causality relations. Based on the new model for the complex permittivity, it was also shown that $\epsilon'_*(\omega)$ and $\tan \delta'_*(\omega)$ had the same frequency behavior. In the last section, the chapter presented the causality properties of the propagation function based on the new model.
CHAPTER 4

TIME DOMAIN MEASUREMENT AND SIMULATION
TECHNIQUES FOR THE DETERMINATION OF MATERIAL
PROPERTIES OF A STRIPLINE

4.1 INTRODUCTION

In the previous chapter, a lossy transmission line approach was presented for the
determination of material properties such as the complex permittivity and the complex
permeability in a stripline geometry. The success of this approach, similar to other approaches,
depends on the accuracy and reliability of both the measurement techniques and the
instrumentation. One of the objectives in using a stripline geometry is to have the ability to
determine material properties at microwave frequencies. Measurements at microwave frequencies
are accomplished by either frequency or time domain techniques. Frequency domain
measurements are performed by using frequency domain network analyzers. Similarly, time
domain measurements are performed by using time domain network analyzers.

A frequency domain network analyzer uses a sine wave as the stimulus signal which is
applied to the device under test (DUT). By sweeping the source over desired frequency range, the
frequency response of the device can be measured. On the other hand, a time domain network
analyzer uses a step like signal as the stimulus. An ideal step waveform contains all the
harmonics in the frequency domain. Hence, a single time domain measurement can provide the
frequency response of a DUT over a wide bandwidth. The range of this bandwidth is dependent
upon the transition duration of the step stimulus. Time domain measurements have some
advantages over frequency domain measurements. The most important advantage is the fact that
a time domain response can reveal information about the discontinuities in the DUT. The type of each discontinuity is directly determined from the shape of the time domain response waveform. Furthermore, the location of each discontinuity in the time domain response waveform can be directly translated to the physical dimensions along the DUT. A second advantage is that the time domain network analyzer has much lower cost than its frequency domain counterpart.

In this chapter, time domain measurement and simulation techniques for the determination of material properties in a stripline geometry are presented. Using the time domain measurement set up used in the experiments, an equivalent network model is devised. A microwave measurement system usually uses coaxial reference transmission lines for transmitting and receiving the signal. A stripline is a planar device and special adapters that can adapt the coaxial geometry of the reference transmission lines to the planar geometry of the stripline are necessary for making measurements. In this chapter, it is proposed that each adapter and the stripline under test can be modeled by a lossy transmission line. The propagation function and the characteristic impedance of the lossy lines are based on the models presented in the previous chapter. In order to understand the performance and behavior of the equivalent network model, several computer simulations are presented.

This chapter is divided into several sections. Section 4.2 explains the concept and approach of the proposed time domain measurements and simulation techniques for determining the material properties in a stripline geometry. The measurement set up and its corresponding equivalent network model is discussed in section 4.3. Section 4.4 presents the derivation of the S-parameters based on the equivalent network. The sensitivity analysis on the lossy line parameters and the complex permittivity is provided in section 4.5. Finally, a summary and conclusions is presented for this chapter in section 4.6.
4.2 CONCEPT AND APPROACH OF TIME DOMAIN MEASUREMENTS AND SIMULATION TECHNIQUES

The determination of material properties in a stripline geometry using the proposed technique involves several steps. Given a dielectric material whose properties need to be determined, the first step starts with constructing the stripline. This step, itself involves several stages and the details are explained in chapter 6. The second step involves the measurements which are performed using a time domain network analyzer. For each stripline under test, two types of measurements are performed. In the first measurement, a step like signal is sent down a transmission line to the stripline under test (SUT) and then, the reflected signal is acquired through a data acquisition system. In the second measurement, a step like signal is sent through the SUT and the transmitted signal is acquired. The third step involves computer simulations. As mentioned in the previous chapter, the stripline under test is modeled by a lossy transmission line. The objective is to use the measurement data together with an optimization algorithm for determining the optimum values of the lossy transmission line parameters. The material properties of the stripline under test are determined from the the optimum values of the lossy transmission line parameters.

The measurement set up consists of a signal generator, a digitizing oscilloscope, a pair of coaxial reference transmission lines, a pair of coaxial to planar adapters and other accessories. For proper computer simulation, it is necessary to have an equivalent network model for the measurement set up including the SUT. The signal generator is modeled by a signal source and a series impedance having a value of 50 Ω. The reference transmission lines are usually modeled by lossless transmission lines having a characteristic impedance of 50 Ω. Since the reference waveforms are acquired at the end of the reference transmission lines, they are usually omitted from being part of the equivalent network model. Similar to the SUT, each coaxial to planar adapter is also modeled by a lossy transmission line. The idea behind having an equivalent
network model is to have the ability of deriving the desired mathematical formulation for computer simulation. The experimental time domain responses are related to scattering S-parameters through Fourier transform pairs. Since the lossy transmission line parameters are functions of frequency, it is more appropriate to derive the desired mathematical formulation for computer simulation in terms of the scattering S-parameters.

The optimization process for determining the optimal values of lossy transmission line parameters can be performed either in the frequency domain or the time domain. However, since the experimental waveforms are acquired in the time domain, it is much more efficient from numerical point of view to perform the optimization process in the time domain. The simulated time domain response is obtained by convolving the inverse Fourier transform of the appropriate S-parameter with the incident step waveform acquired from the time domain network analyzer. The analytical expressions for the S-parameters derived using the equivalent network model is extremely complicated. Hence, both the inverse Fourier transformation and the convolution has to be carried out numerically.

The ultimate goal of the optimization process is to minimize an objective function which is defined to be the least squares sum of the difference between the simulated and experimental time domain responses. The optimization process starts up with a set of initial values for the lossy line parameters. It then computes the objective function by going through the calculation of the S-parameters, inverse Fourier transformation and convolution steps. Based on a given tolerance value, the algorithm starts iterating through the line parameters. At each iteration the objective function is recalculated. If the absolute value of the objective function is reduced, the iteration becomes a success, otherwise it becomes a failure. The process continues until the tolerance value is satisfied. Once the optimal solution is obtained, the dielectric properties of the stripline under test are calculated using the optimal values of the lossy line parameters.
4.3 THE MEASUREMENT SET UP AND THE CORRESPONDING EQUIVALENT NETWORK MODEL

4.3.1 The Measurement Set Up

The measurement set up used in the experiments is shown in Figure 4.3.1. In this set up, the Hp 54121A test set contains the TDR pulse generator and the sampling heads (detectors). The Hp 54120A digitizing oscilloscope mainframe is a menu driven device with a CRT display and a microprocessor unit for monitoring the Hp 54121A test set, performing certain functions on the waveforms and communicating with the personal computer (PC). The Hp 54120A digitizing oscilloscope mainframe together with the Hp 54121A test set forms the Hp 54120T time domain network analyzer (TDNA) [42]. Time domain network analysis includes both time domain reflectometry (TDR) and time domain transmission (TDT) measurements.

Time domain reflectometry (TDR) sends a step like signal down a transmission line to a device under test (DUT) and then measures the reflections from the DUT. In Figure 4.3.1, channel 1 of the Hp 54121A test set is utilized for accomplishing this task. The TDR signature reveals information about the characteristic impedance of the DUT as well the nature and the location of each discontinuity along the DUT. By choosing a suitable time window, the TDR technique allows the user to concentrate on a designated region of the DUT. This can be thought of being similar to real time filtering.

Time domain transmission (TDT) measurements are performed by sending a step like signal through the DUT. In Figure 4.3.2, channel 1 of the Hp 54121A test set is utilized for sending down the voltage step waveform and channel 4 is used for receiving it from the DUT. The TDT signature reveals information about the gain, the propagation delay and the dispersion properties of the DUT.

In this specific application the device under test is a stripline. In order to use the TDNA
for TDR and TDT measurements, it is necessary to have special adapters for adapting the coaxial geometry of the reference transmission lines to the planar geometry of the stripline under test. These adapters are called Eisenhart Launchers [43] and are manufactured by Cascade Microtech™. These launchers are designed such that an abrupt discontinuity is avoided by matching the field patterns between the coaxial cable and the stripline with a gradual taper. Meanwhile, the reference transmission lines used in the experimental set up are precision 3.5 mm coaxial lines having a bandwidth of 26.5 GHz and a characteristic impedance of 50 Ω.

The proposed technique requires a response and a reference waveform for each measurement. The measurement starts up by first choosing a time epoch that would show both the beginning and end of the first reflection due to the stripline under test. The time delay is also adjusted such that the reference waveform is displayed on the digitizing oscilloscope at least one division beyond the starting point. The TDR and the TDT response waveforms are then acquired through the PC by using a software program written in BASIC and an Hp-Guru card. In order to acquire the desired reference waveforms, both of the reference transmission lines are disconnected from the adapters. The TDR reference waveform is acquired by connecting a coaxial short circuit termination (calibration standard) to the end of the reference transmission line of channel 1. Furthermore, the TDT reference waveform is acquired by mating the two reference transmission lines together and performing the data acquisition from channel 4.

4.3.2 The Equivalent Network Model

The purpose of devising an equivalent network model for the experimental set up of Figure 4.3.1 is to have the opportunity of simulating this set up on a digital computer. The computer simulation can provide reliable results whenever the equivalent network representation closely approximates the experimental set up. The equivalent network model shown in Figure 4.3.2 is proposed to accomplish this task. This model is chosen to be a frequency domain
Figure 4.3.1  Time domain measurement set up used in the experiments.
Figure 4.3.2  Equivalent network model for the measurement set up of Figure 4.3.1.
representation of an actual time domain model, hence all the variables are functions of frequency. The calibration process as described above, takes care of all the imperfections in the reference transmission lines, hence, they are not shown as part of the model.

The signal generator is represented by a voltage source, $V_g(\omega)$ and an internal impedance, $Z_g(\omega)$. The load, which corresponds to the impedance of the reference transmission line connected to channel 4, is represented by an impedance, $Z_L(\omega)$. In this specific application, both the generator and the load impedances have a value of 50 $\Omega$. The coaxial to planar adapters A and B are represented by lossy transmission lines having characteristic impedances of $Z_1(\omega)$ and $Z_3(\omega)$, and propagation functions of $\gamma_1(\omega)$ and $\gamma_3(\omega)$, respectively. Furthermore, the stripline under test is represented by a lossy transmission line having a characteristic impedance of $Z_2(\omega)$ and a propagation function of $\gamma_2(\omega)$.

4.4 DERIVATION OF S-PARAMETERS FOR COMPUTER SIMULATION USING THE EQUIVALENT NETWORK MODEL

4.4.1 Derivation of the Overall S-Parameters

The derivation of the overall S-parameters for computer simulation will be based on the equivalent network model of Figure 4.3.2. All variables with upper case letters will be assumed to be functions of frequency and all variables with lower case letters will be assumed to be functions of time, unless otherwise stated. For a major part of the formulation to follow, the functional dependence of the variables will not be shown. Since the TDR and TDT waveforms are directly related to the scattering S-parameters by Fourier transformations, the derivation will involve the use of S-parameters whose definitions and applications are presented in Appendix A.

Referring to Figure 4.3.2, the terminal voltages $V_1$ and $V_L$ can be written as a sum of
incident and reflected waveforms as follows:

\[ V_1 = V_{i1} + V_{r1}, \]  
(4.4.1)

and

\[ V_L = V_{iL} + V_{rL}. \]  
(4.4.2)

The input reflection coefficient is defined as

\[ S_{11} = \frac{b_1}{a_1} \bigg|_{a_2 = 0} = \frac{V_{r1}}{V_{i1}}. \]  
(4.4.3)

On the other hand, the forward transmission gain is defined by

\[ S_{21} = \frac{b_2}{a_1} \bigg|_{a_2 = 0} = \frac{V_{rL}}{V_{iL}} \frac{Z_g}{Z_L}. \]  
(4.4.4)

In order to obtain \( S_{11} \) and \( S_{21} \) of the network shown in Figure 4.3.2, we shall be utilizing the signal flow graph approach for the scattering S-parameters and verify our results using the T-parameters approach. Probably, the easiest technique for obtaining the desired S-parameters is to assume that each transmission line is doubly matched from both sides and then treat the discontinuities at the plane boundaries by discontinuity matrices [44]. With this assumptions in mind, the S-parameters of each transmission line is given by

\[ [S]_i = \begin{bmatrix} 0 & z_i \\ z_i & 0 \end{bmatrix}, \quad i = 1, 2, 3. \]  
(4.4.5)

where,

\[ z_i = e^{-\gamma_i l_i}. \]
On the other hand, the S-parameter discontinuity matrix for each plane boundary is defined as

\[
[S_D]_{ij} = \begin{bmatrix} \rho_{ij} & z_{ij} \\ y_{ij} & -\rho_{ij} \end{bmatrix}, \quad ij = 1g, 21, 32, L3
\]  

(4.4.6)

where,

\[
\rho_{ij} = \frac{Z_i - Z_j}{Z_i + Z_j},
\]

\[
y_{ij} = (1 + \rho_{ij}) \sqrt{\frac{Z_j}{Z_i}},
\]

\[
z_{ij} = (1 - \rho_{ij}) \sqrt{\frac{Z_i}{Z_j}}.
\]

It should be noted that the lower case variables \( \rho_{ij}, y_{ij}, z_{ij} \) are frequency dependent variables.

Using the above definitions, the signal flow graph representation of the equivalent network model becomes as shown in Figure 4.4.1. Applying the Mason’s rule as stated in Appendix A to Figure 4.4.1, we obtain

\[
S_{11} = \frac{A}{D},
\]  

(4.4.7)

where,

\[
D = 1 + \rho_{1g}\rho_{21}z_{12}^2 + \rho_{21}\rho_{32}z_{22}^2 + \rho_{32}\rho_{L3}\rho_{32}z_{32}^2 + \rho_{1g}\rho_{32}\rho_{L3}z_{23}^2 + \rho_{1g}\rho_{L3}\rho_{L3}z_{32}^2 + \rho_{1g}\rho_{L3}\rho_{L3}z_{32}^2 + \rho_{1g}\rho_{L3}\rho_{L3}z_{32}^2 + \rho_{1g}\rho_{L3}\rho_{L3}z_{32}^2
\]

\[
+ \rho_{21}\rho_{L3}\rho_{L3}z_{32}^2 + \rho_{1g}\rho_{21}\rho_{L3}\rho_{L3}z_{32}^2 + \rho_{1g}\rho_{21}\rho_{L3}\rho_{L3}z_{32}^2 + \rho_{1g}\rho_{21}\rho_{L3}\rho_{L3}z_{32}^2 + \rho_{21}\rho_{L3}\rho_{L3}z_{32}^2 + \rho_{21}\rho_{L3}\rho_{L3}z_{32}^2
\]

and
\[ A = \rho_{1g} z_1 + \rho_{32} \bar{z}_1 \bar{z}_2 y_{1g} z_{21} (1 + \rho_{32} L_3 z_3^2) + \rho_{1g} z_1 \bar{z}_2 \bar{z}_3 y_{1g} z_{21} \bar{z}_{21} y_{3} \bar{z}_{32} z_{32} \]
\[ + \rho_{21} \bar{z}_1 \bar{z}_2 y_{1g} \bar{z}_1 \bar{z}_3 (1 + \rho_{21} \rho_{32} \bar{z}_2 + \rho_{32} \rho_{1g} \bar{z}_2 + \rho_{21} \rho_{32} \rho_{1g} \bar{z}_2 \bar{z}_3 y_{32} \bar{z}_{32} + \rho_{21} \rho_{32} \rho_{1g} \bar{z}_2 \bar{z}_3) . \]

Noting that \( y_{ij} = (1 - \rho_{ij}^2) \) and simplifying, we obtain

\[ D = 1 + \rho_{1g} \rho_{32} \bar{z}_1 + \rho_{32} \rho_{1g} \bar{z}_2 + \rho_{32} \rho_{1g} \bar{z}_3 + \rho_{1g} \rho_{32} \bar{z}_2 \bar{z}_2 + \rho_{1g} \rho_{32} \rho_{1g} \bar{z}_2 \bar{z}_3 + \rho_{21} \rho_{32} \rho_{1g} \bar{z}_2 \bar{z}_3, \tag{4.4.8} \]

and

\[ A = \rho_{1g} + \rho_{21} \bar{z}_1 + \rho_{1g} \rho_{21} \rho_{32} \bar{z}_2 + \rho_{32} \rho_{1g} \bar{z}_2 + \rho_{1g} \rho_{32} \rho_{1g} \bar{z}_2 \bar{z}_3 + \rho_{21} \rho_{32} \rho_{1g} \bar{z}_2 \bar{z}_3 + \rho_{1g} \rho_{21} \rho_{32} \bar{z}_2 \bar{z}_3 + \rho_{1g} \rho_{21} \rho_{32} \rho_{1g} \bar{z}_2 \bar{z}_3. \tag{4.4.9} \]

The forward transmission gain is given by

\[ S_{21} = \frac{z_1 z_2 z_3 (1 + \rho_{1g})(1 + \rho_{21})(1 + \rho_{32})(1 + \rho_{L3}) \sqrt{\frac{Z_g}{Z_L}}}{D}. \tag{4.4.10} \]

From (4.4.3) and (4.4.7), the ratio of the reflected voltage at the sending end (generator) to the incident voltage is given by

\[ \frac{V_{r1}}{V_{i1}} = A. \tag{4.4.11} \]

Furthermore, from (4.4.4) and (4.4.10), the ratio of the reflected voltage at the receiving end (transmitted voltage) to the incident voltage at the sending end is given by

\[ \frac{V_{rL}}{V_{i1}} = \frac{z_1 z_2 z_3 (1 + \rho_{1g})(1 + \rho_{21})(1 + \rho_{32})(1 + \rho_{L3})}{D}. \tag{4.4.12} \]
To verify the above results, we use the T-parameter approach. Using the scattering S-parameter to T-parameter conversion formula from Appendix A, the T-matrix for each lossy transmission line becomes

$$
\begin{bmatrix}
T^1_i
\end{bmatrix} = \begin{bmatrix}
x_i^{-1} & 0 \\
0 & x_i
\end{bmatrix}, \quad i = 1, 2, 3. 
$$

(4.4.13)

On the other hand, the T-parameter discontinuity matrix for each plane boundary becomes

$$
\begin{bmatrix}
TD_{ij}
\end{bmatrix} = \begin{bmatrix}
y_{ij}^{-1} & \rho_{ij}y_{ij}^{-1} \\
\rho_{ij}y_{ij}^{-1} & y_{ij}^{-1}
\end{bmatrix}, \quad ij = 1g, 21, 32, L3. 
$$

(4.4.14)

Using the properties of T-parameters, the overall (total) T-parameter matrix of the equivalent network is obtained to be as follows:

$$
\begin{bmatrix}
T
\end{bmatrix}_T = \begin{bmatrix}
TD_{1g} & T_1 & TD_{21} & T_2 & TD_{32} & T_3 & TD_{L3} & T_4
\end{bmatrix} = \begin{bmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{bmatrix},
$$

(4.4.15)

where,

$$
T_{11} = y_{1g}^{-1}x_1^{-1} - y_{21}^{-1}x_2^{-1} - y_{32}^{-1}x_3^{-1} - y_{L3}^{-1}(1 + \rho_{1g}x_1^2 + \rho_{21}x_2^2 + \rho_{32}x_3^2 + \rho_{1g}x_1^2x_2^2 + \rho_{32}x_3^2x_3^2 + \rho_{1g}x_1^2x_2^2x_3^2),
$$

$$
T_{12} = y_{1g}^{-1}x_1^{-1} - y_{21}^{-1}x_2^{-1} - y_{32}^{-1}x_3^{-1} - y_{L3}^{-1}(\rho_{L3} + \rho_{1g}\rho_{21}\rho_{L3}x_1^2 + \rho_{21}\rho_{32}\rho_{L3}x_2^2 + \rho_{1g}\rho_{32}\rho_{L3}x_3^2 + \rho_{1g}\rho_{21}\rho_{32}\rho_{L3}x_1^2x_2^2 + \rho_{1g}\rho_{21}\rho_{32}\rho_{L3}x_1^2x_3^2 + \rho_{1g}\rho_{32}\rho_{L3}x_2^2x_3^2),
$$

$$
T_{21} = y_{1g}^{-1}x_1^{-1} - y_{21}^{-1}x_2^{-1} - y_{32}^{-1}x_3^{-1} - y_{L3}^{-1}(\rho_{1g} + \rho_{21}x_1^2 + \rho_{1g}\rho_{21}\rho_{32}x_2^2 + \rho_{32}x_3^2x_3^2 + \rho_{1g}\rho_{32}\rho_{L3}x_1^2x_2^2 + \rho_{1g}\rho_{21}\rho_{L3}x_2^2x_3^2 + \rho_{1g}\rho_{32}\rho_{L3}x_1^2x_3^2 + \rho_{1g}\rho_{21}\rho_{L3}x_2^2x_3^2 + \rho_{L3}x_1^2x_2^2x_3^2),
$$
Figure 4.4.1  Signal flow graph representation of the equivalent network model of Figure 4.3.2.
\[ T_{22} = y_{19}^{-1} x_{1}^{-1} y_{21}^{-1} x_{2}^{-1} y_{32}^{-1} x_{3}^{-1} y_{L3}^{-1} \left( \rho_{19} \rho_{L3} + \rho_{21} \rho_{L3} x_{2}^2 + \rho_{19} \rho_{21} \rho_{32} \rho_{L3} x_{3}^2 + \rho_{32} \rho_{L3} x_{1}^2 x_{2}^2 \right) + \rho_{19} \rho_{32} x_{3}^2 + \rho_{21} \rho_{32} x_{3}^2 + \rho_{19} \rho_{21} x_{2}^2 x_{3}^2 + x_{1}^2 x_{2}^2 x_{3}^2 \right). \]

Next, using the conversion formula from T- to S-parameters as stated in Appendix A, we obtain the desired S-parameters. Hence,

\[ S_{11} = \frac{T_{21}}{T_{11}} = \frac{A}{D}, \quad (4.4.16) \]

\[ S_{21} = \frac{1}{T_{11}} = \frac{y_{19} x_{1} y_{21} x_{2} y_{32} x_{3} y_{L3}}{D} \]

\[ = \frac{z_{1} x_{2} x_{3} (1 + \rho_{19})(1 + \rho_{21})(1 + \rho_{32})(1 + \rho_{L3}) \sqrt{\frac{Z_{g}}{Z_{L}}}}{D}. \quad (4.4.17) \]

From the above analysis it can be observed that the two approaches give identical results.

### 4.4.2 Derivation of the S-parameters for the Adapter Regions

As mentioned in section 4.4.2, each coaxial to planar adapter will be modeled by a lossy transmission line. By doing so, the reference plane can be chosen to be at the end of the coaxial reference transmission line. This allows the user to utilize the well established coaxial standards. On the other hand, if the reference plane is chosen to be at the end of the coaxial to planar adapter, then it becomes necessary to use planar calibration standards. A major disadvantage of planar standards is the fact that they are not yet well developed.

The S-parameters for the adapter regions can also be derived by using the equivalent network model of Figure 4.3.2. The objective is to utilize the same TDR data acquired for the stripline under test to model the adapter regions. In the simulation process, this can be accomplished by choosing a time window that shows only the adapter region. The most effective
approach for this modeling is to use only TDR data. Hence, we are mainly interested in $S_{11}$ for the adapter region. To derive $S_{11}$ for the coaxial to planar adapter A, we will assume that the lossy transmission line used for modeling the stripline region and the one used for modeling the coaxial to planar adapter B is infinitely long, i.e. $\ell_2, \ell_3 = \infty$. Based on this assumption, (4.4.8) and (4.4.9) become,

$$D^a = 1 + \rho_{1g}\rho_{21}x_1^2,$$  \hspace{1cm} (4.4.18)

and

$$A^a = \rho_{1g} + \rho_{21}x_1^2.$$  \hspace{1cm} (4.4.19)

Hence,

$$S_{11}^a = \frac{A^a}{D^a} = \frac{\rho_{1g} + \rho_{21}x_1^2}{1 + \rho_{1g}\rho_{21}x_1^2}.$$  \hspace{1cm} (4.4.20)

In order to model the coaxial to planar adapter B, the coaxial to planar adapters A and B in Figure 4.3.1 has to be interchanged and a new TDR data has to be acquired. This is basically due to a disadvantage of the TDNA shown in Figure 4.3.1 having only one signal generator at channel 1. To derive $S_{11}^b$, we assume that the lossy transmission lines 1 and 3 in Figure 4.3.2 are interchanged and $\ell_1, \ell_2 = \infty$. Hence,

$$S_{11}^b = \frac{A^b}{D^b} = \frac{\rho_{3g} + \rho_{23}x_3^2}{1 + \rho_{3g}\rho_{23}x_3^2}.$$  \hspace{1cm} (4.4.21)

where,

$$\rho_{3g} = \frac{Z_3 - Z_g}{Z_3 + Z_g},$$

and
4.5 COMPUTER SIMULATIONS

4.5.1 Sensitivity Analysis on the Lossy Line Parameters

In order to have a proper understanding on how the lossy line parameter affect the TDR and TDT responses several sensitivity analysis have been performed. These analyses were based on the S-parameters derived in the previous section. In the simulation process, an actual time domain step waveform acquired from an Hp 54120T TDNA was used. The S-parameters derived in the previous section are functions of frequency. The simulated time domain responses are obtained by convolving the inverse Fourier Transform [45] of the S-parameters with the actual step waveform.

The simulated TDR and TDT response waveforms are calculated by making use of the Fourier and the convolution integrals. Let \( x(t) \) be a time domain signal and \( X(\omega) \) be its frequency domain representation. The two variables are then related by the following Fourier transform pairs [45]:

\[
x(t) = \mathcal{F}^{-1}\{X(\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} \, d\omega,
\]

and

\[
X(\omega) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \, dt.
\]

Using the above representation, the time domain input reflection coefficient and the forward transmission gain are defined respectively, as
\[ s_{11}(t) = \mathcal{F}^{-1}\{S_{11}(\omega)\} , \quad (4.5.3) \]
and
\[ s_{21}(t) = \mathcal{F}^{-1}\{S_{21}(\omega)\} . \quad (4.5.4) \]

Next, suppose that \( v_{i1}(t) \) is the time domain incident (step) waveform acquired from the Hp 54120T TDNA. Then, the simulated TDR response can be obtained by using the following convolution relation:

\[ v_{r1}(t) = v_{i1}(t) * s_{11}(t) = \int_{0}^{\infty} v_{i1}(\tau) s_{11}(t - \tau) \, d\tau , \quad (4.5.5) \]

On the other hand, the simulated TDT response is obtained by a similar relation given by

\[ v_{t}(t) = v_{i1}(t) * s_{21}(t) = \int_{0}^{\infty} v_{i1}(\tau) s_{21}(t - \tau) \, d\tau . \quad (4.5.6) \]

It should be noted that, in (4.5.6), \( v_{t}(t) \equiv v_{rL}(t) = \mathcal{F}^{-1}\{V_{rL}(\omega)\} \).

In this simulation process, the adapters A and B were assumed to be identical and were modeled with lossless transmission lines having a time delay, \( \tau = 1.75 \times 10^{-10} \) seconds and a characteristic impedance of \( R = 50 \) Ω. Hence, the propagation functions and the characteristic impedances of the transmission lines used for modeling the two adapters become

\[ \gamma_1 = \gamma_3 = j\omega \tau , \]

and

\[ Z_1 = Z_3 = R . \]

The stripline under test was modeled by a lossy transmission line having a propagation function and a characteristic impedance given by
\[
\gamma_2(\omega) = \left[ K_2 + j\omega \left( L_e + K_1\omega^{p-1}\sin\left(\frac{\pi p}{2}\right) + K_1\omega^p\cos\left(\frac{\pi p}{2}\right)\right) \right] \\
\times \left[ K_4 + j\omega \left( C_o + K_3\omega^{q-1}\sin\left(\frac{\pi q}{2}\right) + K_3\omega^q\cos\left(\frac{\pi q}{2}\right)\right) \right]^\frac{1}{2}, 
\]
(4.5.7)

\[
Z_2(\omega) = \frac{K_2 + j\omega \left( L_e + K_1\omega^{p-1}\sin\left(\frac{\pi p}{2}\right) + K_1\omega^p\cos\left(\frac{\pi p}{2}\right)\right) + K_1\omega^p\cos\left(\frac{\pi p}{2}\right)}{K_4 + j\omega \left( C_o + K_3\omega^{q-1}\sin\left(\frac{\pi q}{2}\right) + K_3\omega^q\cos\left(\frac{\pi q}{2}\right)\right) + K_3\omega^q\cos\left(\frac{\pi q}{2}\right)}.
\]
(4.5.8)

Note that, in (4.5.7) and (4.5.8), \( K_2 = R_{dc}, \) \( C = C_o + K_3\omega^{q-1}\sin\left(\frac{\pi q}{2}\right), \) \( Z_{sk} = K_1(j\omega)^p, \)
\( G = K_3\omega^q\cos\left(\frac{\pi q}{2}\right), \) and \( K_4 = G_{dc}. \)

In order to see the effect of each parameter on the TDR and TDT responses, the parameters in (4.5.7) and (4.5.8) were varied one at a time while keeping the rest of them constant. The stripline under test was assumed to have a length of 3" in all the computer simulations. The sensitivities of the lossy line parameters are directly related to the characteristic impedance \( Z_2(\omega). \) Since 50 \( \Omega \) is a standard in microwave measurements, it was chosen as a reference impedance for computer simulations. Hence, to observe this variation, the simulation was performed on two cases: one with \( |Z_2(\omega)| > 50 \Omega \) and the other with \( |Z_2(\omega)| < 50 \Omega. \) The simulated TDR responses have been plotted to show the local reflection coefficient as a function of time rather than the reflected voltage \( v_{r1}(t). \) This gives a better intuitive understanding about the sensitivities of the different parameters. The local reflection coefficient is defined as the ratio of the reflected voltage \( v_{r1}(t) \) to the maximum value of the incident waveform \( v_{i1}(t). \) In other words,

\[
\text{local reflection coefficient} = \frac{v_{1r}(t)}{\max(v_{i1}(t))}.
\]
(4.5.9)

The incident TDR waveform \( v_{i1}(t) \) acquired from Hp 54120T TDNA is shown in Figure 4.5.1.
Its maximum value is approximately 0.2 Volts. This waveform was acquired by terminating the reference transmission line with a short circuit termination (calibration standard) and then negating the end result. Furthermore, the incident TDT reference waveform was acquired by performing a through measurement with the two reference transmission lines. This waveform is shown in Figure 4.5.2.

The effects of varying the external inductance per unit length $L_e$ are shown in Figures 4.5.3 through 4.5.6. These responses were obtained by setting the capacitance per unit length $C$ to a constant and the loss parameters $K_1$, $p$, $R_{dc}$, $K_3$, $q$, and $G_{dc}$ to zero. From these figures, it can be observed that as $L_e$ increases, both the characteristic impedance and the time delay of the lossy line increases. In Figures 4.5.3 and 4.5.4, where $|Z_2(\omega)| > 50 \, \Omega$, an increase in the characteristic impedance translates to an increase in the magnitude of the TDR response and to a decrease in the magnitude of the TDT response, respectively. On the other hand, in Figures 4.5.5 and 4.5.6, where $|Z_2(\omega)| < 50 \, \Omega$, an increase in the characteristic impedance translates to a decrease in the absolute magnitude of the TDR response and increase in the magnitude of the TDT response, respectively.

The effects of varying the capacitance per unit length $C$ are shown in Figures 4.5.7 through 4.5.10. These responses were obtained by setting the external inductance per unit length $L_e$ to a constant and the loss parameters $K_1$, $p$, $R_{dc}$, $K_3$, $q$, and $G_{dc}$ to zero. From these figures, it can be observed that as $C$ increases, the characteristic impedance of the lossy line decreases and the time delay increases. In Figures 4.5.7 and 4.5.8, where $|Z_2(\omega)| > 50 \, \Omega$, a decrease in the characteristic impedance translates to a decrease in the magnitude of the TDR response and to an increase in the magnitude of the TDT response, respectively. On the other hand, in Figures 4.5.9 and 4.5.10, where $|Z_2(\omega)| < 50 \, \Omega$, a decrease in the characteristic impedance translates to an increase in the absolute magnitude of the TDR response and to a decrease in the magnitude of the TDT response, respectively.
Figure 4.5.1  TDR reference waveform acquired from the Hp 54129T TDNA and used in the TDR computer simulations.
Figure 4.5.2  TDT reference waveform acquired from the Hp 54120T TDNA and used in the TDT computer simulations.
Figure 4.5.3  Simulated TDR response waveform: Effect of varying the external inductance per unit length $L_e$ ($C = 4\varepsilon_0 \text{ F/m}, R_{dc} = 0 \text{ \Omega/m}, Z_{sk} = 0 \text{ \Omega/m}, G = 0 \text{ S/m}, G_{dc} = 0 \text{ S/m},$

$R_L = 50 \Omega$).
Figure 4.5.4  Simulated TDT response waveform: Effect of varying the external inductance per unit length $L_e$ ($C = 4\varepsilon_0 F/m$, $R_{dc} = 0 \ \Omega/m$, $Z_{sk} = 0 \ \Omega/m$, $G = 0 \ S/m$, $G_{dc} = 0 \ S/m$).

$R_L = 50 \ \Omega$.
Figure 4.5.5  Simulated TDR response waveform: Effect of varying the external inductance per unit length $L_e$ ($C = 25\varepsilon e$ F/m, $R_{dc} = 0$ Ω/m, $Z_{zk} = 0$ Ω/m, $G = 0$ S/m, $G_{dc} = 0$ S/m, $R_L = 50$ Ω).
Figure 4.5.6  Simulated TDT response waveform: Effect of varying the external inductance per unit length $L_e$ ($C = 25\epsilon_o \ F/m$, $R_{dc} = 0 \ \Omega/m$, $Z_{sk} = 0 \ \Omega/m$, $G = 0 \ S/m$, $G_{dc} = 0 \ S/m$, $R_L = 50 \ \Omega$).
Figure 4.5.7  Simulated TDR response waveform: Effect of varying the capacitance per unit length $C$ ($L_e = 0.4 \mu H/m$, $R_{dc} = 0 \Omega/m$, $Z_{sk} = 0 \Omega/m$, $G = 0 S/m$, $G_{dc} = 0 S/m$, $R_L = 50 \Omega$).
Figure 4.5.8  Simulated TDT response waveform: Effect of varying the capacitance per unit length $C$ ($L_e = 0.4 \mu_\Omega$ H/m, $R_{dc} = 0$ $\Omega$/m, $Z_{lk} = 0$ $\Omega$/m, $G = 6$ S/m, $G_{dc} = 0$ S/m, $R_L = 50$ $\Omega$).
Figure 4.5.9  Simulated TDR response waveform: Effect of varying the capacitance per unit length $C$ ($L_e = 0.07 \mu \text{H/m}$, $R_{dc} = 0 \Omega / \text{m}$, $z_{sh} = 0 \Omega / \text{m}$, $G = 0 \text{ S/m}$, $G_{dc} = 0 \text{ S/m}$, $R_L = 50 \Omega$).
Figure 4.5.16  Simulated TDT response waveform: Effect of varying the capacitance per unit length $C$ ($L_c = 0.87 \mu \text{H/m}$, $R_{dc} = 0 \Omega/\text{m}$, $z_{sh} = 0 \Omega/\text{m}$, $G = 0 \text{S/m}$, $G_{dc} = 0 \text{S/m}$, $R_L = 50 \Omega$).
The effects of varying the dc resistance per unit length $R_{dc}$ are shown in Figures 4.5.11 through 4.5.14. These responses were obtained by setting the capacitance per unit length $C$ and the external inductance per unit length $L_e$ to constant values and the loss parameters $K_1$, $p$, $K_3$, $q$, and $G_{dc}$ to zero. From these figures, it can be observed that as $R_{dc}$ increases, the characteristic impedance of the lossy line increases but the time delay stays unchanged. Referring to Figure 4.5.11, the TDR signature from 0.8 to 1.4 ns corresponds to the lossy transmission line ( stripline) region. This region determines the characteristic impedance and the time delay of the line under investigation. On the other hand, the TDR signature from 1.4 to 2.0 ns corresponds to the third transmission line (adapter B) and the load region. This figure shows that, an increase in $R_{dc}$ affects the magnitude of the TDR response in the lossy line ( stripline) region in a different manner than the one in the adapter and the load region. The same thing is true for the TDR response waveform of Figure 4.5.13. In Figures 4.5.11 and 4.5.12, where $|Z_2(\omega)| > 50 \ \Omega$, an increase in $R_{dc}$ causes an increase in the magnitude of the TDR response and a decrease in the magnitude of the TDT response, respectively. On the other hand, in Figures 4.5.13 and 4.5.14, where $|Z_2(\omega)| < 50 \ \Omega$, an increase in $R_{dc}$ causes a decrease in both the absolute magnitudes of the TDR and the TDT responses. This analysis shows that, a variation in $R_{dc}$ makes a greater effect on both the TDR and the TDT responses for the case with $|Z_2(\omega)| < 50 \ \Omega$ than the one with $|Z_2(\omega)| > 50 \ \Omega$.

The effects of varying the skin effect impedance per unit length $Z_{sk}$ are shown in Figures 4.5.15 through 4.5.18. This sensitivity analysis was performed by varying only the high frequency loss term $K_1$, while keeping the high frequency loss exponent $p$ constant. These responses were obtained by setting the capacitance per unit length $C$ and the external inductance per unit length $L_e$ to constant values and the loss parameters $R_{dc}$, $K_2$, $q$, and $G_{dc}$ to zero. From these figures, it can be observed that as $Z_{sk}$ increases, both the characteristic impedance and the time delay of the lossy line increases. However, the increase in the time delay is almost negligible for low values of $K_1$. Also, with the increase in $Z_{sk}$, the magnitude of the TDR responses in the lossy
Figure 4.5.11  Simulated TDR response waveform: Effect of varying the dc resistance per unit length \( R_{dc} \) \( (L_e = 0.4 \mu \text{H/m}, C = 4 \varepsilon_o \text{ F/m}, Z_{sk} = 0 \Omega/\text{m}, G = 0 \text{ S/m}, G_{dc} = 0 \text{ S/m}, \)

\[ R_L = 50 \Omega \).
Figure 4.5.12  Simulated TDT response waveform: Effect of varying the dc resistance per unit length $R_{dc}$ ($L_c = 0.4\mu_\text{o} \text{ H/m}, C = 4\epsilon_\text{o} \text{ F/m}, Z_{sk} = 0 \Omega/m, G = 0 \text{ S/m}, G_{dc} = 0 \text{ S/m},$

\[ R_L = 50 \Omega \].
Figure 4.5.13  Simulated TDR response waveform: Effect of varying the dc resistance per unit length $R_{dc}$ ($L_c = 0.07 \mu_0$ H/m, $C = 25 \varepsilon_0$ F/m, $Z_{zh} = 0$ $\Omega$/m, $G = 0$ S/m, $G_{dc} = 0$ S/m, $R_L = 50$ $\Omega$).
Figure 4.5.14  Simulated TDT response waveform: Effect of varying the dc resistance per unit length $R_{dc}$ ($L_e = 0.07 \mu \text{H/m, } C = 25\varepsilon_0 \text{ F/m, } Z_{sk} = 0 \Omega/m, G = 0 \text{ S/m, } G_{dc} = 0 \text{ S/m,}$

$$R_L = 50 \Omega$$).
line (stripline) regions get affected in a different manner than the ones in the adapter and the load regions. In Figures 4.5.15 and 4.5.16, where $|Z_2(\omega)| > 50 \, \Omega$, an increase in $Z_{sk}$ causes an increase in the magnitude of the TDR response and a decrease in the magnitude of the TDT response, respectively. On the other hand, in Figures 4.5.17 and 4.5.18, where $|Z_2(\omega)| < 50 \, \Omega$, an increase in $Z_{sk}$ causes a decrease in both the absolute magnitudes of the TDR and the TDT responses. This analysis shows that, a variation in $Z_{sk}$ makes a greater effect on both the TDR and the TDT responses for the case with $|Z_2(\omega)| < 50 \, \Omega$ than the one with $|Z_2(\omega)| > 50 \, \Omega$.

Similar to the above case, a sensitivity analysis was also performed by varying only the high frequency loss exponent $p$, while keeping the high frequency loss term $K_1$ constant. These results are shown in Figures 4.5.19 through 4.5.22. The argument presented in the previous paragraph applies to this analysis too. However, it can be observed that both the TDR and TDT responses are more sensitive to the high frequency loss exponent $p$, than the high frequency loss term $K_1$.

The effects of varying the conductance per unit length $G$ are shown in Figures 4.5.23 through 4.5.26. This sensitivity analysis was performed by varying only the high frequency loss term $K_3$, while keeping the high frequency loss exponent $q$ constant. These responses were obtained by setting the capacitance constant per unit length $C_o$ and the external inductance per unit length $L_e$ to constant values and the loss parameters $R_{dc}$, $K_1$, $p$, and $G_{dc}$ to zero. It should be noted that, $G$ and $C$ are related to each other by the Kramers-Kronig causality relations. Hence, when $G$ is varied $C$ varies too. From these figures, it can be observed that as $G$ increases, the characteristic impedance of the lossy line decreases, but the time delay line increases. However, the increase in the time delay is again almost negligible for low values of $K_3$. Also, with the increase in $G$, the magnitude of the TDR responses in the lossy line (stripline) regions get affected in a different manner than the ones in the adapter and the load regions. In Figures 4.5.23 and 4.5.24, where $|Z_2(\omega)| > 50 \, \Omega$, an increase in $G$ causes a decrease in the magnitude of
Figure 4.5.15  Simulated TDR response waveform: Effect of varying the skin effect impedance per unit length $Z_{sk}$ by changing $K_1 (L_e = 0.4 \mu \text{H/m}, C = 4\varepsilon_o \text{F/m}, R_{dc} = 0 \Omega/m, G = 0 \text{S/m}, G_{dc} = 0 \text{S/m}, R_L = 50 \Omega)$. 

- $Z_{sk}=0.1 (j\omega)^{0.1} \Omega/m$
- $Z_{sk}=10 (j\omega)^{0.1} \Omega/m$
- $Z_{sk}=20 (j\omega)^{0.1} \Omega/m$
Figure 4.5.16 Simulated TDT response waveform: Effect of varying the skin effect impedance per unit length $Z_{sk}$ by changing $K_1 \left( L_c = 0.4\mu_0 \, \text{H/m}, C = 4\varepsilon_0 \, \text{F/m}, R_{dc} = 0 \, \Omega/m, G = 0 \, \text{S/m}, G_{dc} = 0 \, \text{S/m, } R_L = 59 \, \Omega \right)$. 
Figure 4.5.17  Simulated TDR response waveform: Effect of varying the skin effect impedance per unit length $Z_{sk}$ by changing $K$ ($L_c = 0.07 \mu \text{H/m}, C = 25\varepsilon_0 \text{F/m}, R_{dc} = 0 \Omega/\text{m}, G = 0 \text{S/m}, G_{dc} = 0 \text{S/m}, R_L = 50 \Omega$).
Figure 4.5.18  Simulated TDT response waveform: Effect of varying the skin effect impedance per unit length $Z_{sk}$ by changing $K_1$ ($L_e = 0.07 \mu H/m, C = 25 \varepsilon_o \text{ F/m, } R_{dc} = 0 \Omega/m, G = 0$ $S/m, G_{dc} = 0 S/m, R_L = 50 \Omega$).
Figure 4.5.19  Simulated TDR response waveform: Effect of varying the skin effect impedance per unit length $Z_{sk}$ by changing $p$ ($L_o = 0.4 \mu \text{H/m}, C = 4\varepsilon_o \text{ F/m}, R_{dc} = 6 \Omega/m, G = 0 \text{ S/m},$

$$G_{dc} = 0 \text{ S/m}, R_L = 50 \Omega).$$
Figure 4.5.20 Simulated TDT response waveform: Effect of varying the skin effect impedance per unit length $Z_{sk}$ by changing $p$ ($L_e = 0.4\mu_0$ H/m, $C = 4\varepsilon_0$ F/m, $R_{dc} = 0$ $\Omega$/m, $G = 0$ S/m, $G_{dc} = 0$ S/m, $R_L = 50$ $\Omega$).
Figure 4.5.21  Simulated TDR response waveform: Effect of varying the skin effect impedance per unit length $Z_{sk}$ by changing $p$ ($L_c = 0.07 \mu H/m$, $C = 25 \epsilon_0 F/m$, $R_{dc} = 0 \Omega/m$, $G = 0 S/m$, $G_{dc} = 0 S/m$, $R_L = 50 \Omega$).
Figure 4.5.22  Simulated TDT response waveform: Effect of varying the skin effect impedance per unit length $Z_{sk}$ by changing $p$ ($J_c = 0.07\mu_0$ H/m, $C = 25\varepsilon_o$ F/m, $R_{dc} = 0$ Ω/m, $G = 0$ S/m, $G_{dc} = 0$ S/m, $R_L = 50$ Ω).
both the TDR and the TDT responses. On the other hand, in Figures 4.5.25 and 4.5.26, where $|Z_2(\omega)| < 50 \ \Omega$, an increase in $G$ causes an increase in the absolute magnitude of the TDR response and a decrease in the magnitude of the TDT response, respectively. This analysis shows that, a variation in $G$ makes a greater effect on both the TDR and the TDT responses for the case with $|Z_2(\omega)| > 50 \ \Omega$ than the one with $|Z_2(\omega)| < 50 \ \Omega$.

Similar to the above case, a sensitivity analysis was also performed by varying only the high frequency loss exponent $q$, while keeping the high frequency loss term $K_3$ constant. These results are shown in Figures 4.5.27 through 4.5.30. The argument presented in the previous paragraph applies to this analysis too. However, it can be observed that both the TDR and TDT response are more sensitive to the high frequency loss exponent $q$, than the high frequency loss term $K_3$.

The effects of varying the dc conductance per unit length $G_{dc}$ are shown in Figures 4.5.31 through 4.5.34. These responses were obtained by setting the capacitance per unit length $C$ and the external inductance per unit length $L_e$ to constant values and the loss parameters $R_{dc}$, $K_1$, $p$, $K_3$ and $q$ to zero. From these figures, it can be observed that as $G_{dc}$ increases, the characteristic impedance of the lossy line decreases, but the time delay stays unchanged. Also, with the increase in $G_{dc}$, the magnitude of the TDR responses in the lossy line (stripline) regions get affected in a different manner than the ones in the adapter and the load regions. In Figures 4.5.31 and 4.5.32, where $|Z_2(\omega)| > 50 \ \Omega$, an increase in $G_{dc}$ causes a decrease in the magnitude of both the TDR and the TDT responses. On the other hand, in Figures 4.5.33 and 4.5.34, where $|Z_2(\omega)| < 50 \ \Omega$, an increase in $G_{dc}$ causes an increase in the absolute magnitude of the TDR response and a decrease in the magnitude of the TDT response, respectively. This analysis shows that, a variation in $G_{dc}$ makes a greater effect on both the TDR and the TDT responses for the case with $|Z_2(\omega)| > 50 \ \Omega$ than the one with $|Z_2(\omega)| < 50 \ \Omega$. 
Figure 4.5.23  Simulated TDR response waveform: Effect of varying the conductance per unit length $G$ by changing $K_3$ ($L_e = 0.4\mu_0$ H/m, $C_o = 4\epsilon_0$ F/m, $R_{dc} = 0$ $\Omega$/m, $Z_{sk} = 0$ $\Omega$/m, $G_{dc} = 0$ S/m, $R_L = 50$ $\Omega$).
Figure 4.5.24  Simulated TDT response waveform: Effect of varying the conductance per unit length $G$ by changing $K_3$ ($L_e = 0.4 \mu \text{H/m}, C_o = 4 \epsilon_0 \text{F/m}, R_{dc} = 0 \Omega/\text{m}, Z_{sk} = 0 \Omega/\text{m}, G_{dc} = 0 \text{S/m}, R_L = 50 \Omega$).
Figure 4.5.25  Simulated TDR response waveform: Effect of varying the conductance per unit length $G$ by changing $K_3$ ($L_e = 0.07\mu_\Omega \text{ H/m, } C_o = 25\epsilon_o \text{ F/m, } R_{dc} = 0 \Omega/m, Z_{ek} = 0 \Omega/m,$

\[ G_{dc} = 0 \text{ S/m, } R_L = 50 \Omega). \]
Figure 4.5.26  Simulated TDT response waveform: Effect of varying the conductance per unit length $G$ by changing $K_3$ ($L_e = 0.07\mu\text{H/m}, C_o = 25\epsilon_o \text{ F/m}, R_{dc} = 0 \Omega/\text{m}, Z_{sk} = 0 \Omega/\text{m},$

$$G_{dc} = 0 \text{ S/m}, R_L = 50 \Omega).$$
Figure 4.5.27  Simulated TDR response waveform: Effect of varying the conductance per unit length $G$ by changing $q$ ($L_e = 0.4 \mu \text{o} \text{H/m}, C_o = 4 \epsilon_o \text{F/m}, R_{dc} = 0 \Omega/\text{m}, Z_{sk} = 0 \Omega/\text{m},$

$$G_{dc} = 0 \text{S/m}, R_L = 50 \Omega).$$
Figure 4.5.28  Simulated TDT response waveform: Effect of varying the conductance per unit length $G$ by changing $q$ ($L_c = 0.4 \mu \text{H/m}$, $C_o = 4 \epsilon_o \text{ F/m}$, $R_{dc} = 0 \Omega/m$, $Z_{sk} = 0 \Omega/m$, $G_{dc} = 0 \text{ S/m}$, $R_L = 50 \Omega$).
Figure 4.5.29  Simulated TDR response waveform: Effect of varying the conductance per unit length $G$ by changing $q$ ($L_e = 0.07 \mu \text{H/m}, C_o = 25 \varepsilon_o \text{F/m}, R_{dc} = 0 \Omega/\text{m}, Z_{sk} = 0 \Omega/\text{m},$

$G_{dc} = 0 \text{ S/m}, R_L = 50 \Omega$).
Figure 4.5.30  Simulated TDT response waveform: Effect of varying the conductance per unit length $G$ by changing $q (L_c = 0.07 \mu H/m, C_o = 25 \epsilon_o F/m, R_{dc} = 0 \Omega/m, Z_{sk} = 0 \Omega/m, G_{dc} = 0 \ S/m, R_L = 50 \ \Omega)$. 

\[ G = 1 \times 10^{-5} \ \omega^{0.20} \cos(0.10\pi) \ \text{S/m} \]
\[ G = 1 \times 10^{-5} \ \omega^{0.30} \cos(0.19\pi) \ \text{S/m} \]
\[ G = 1 \times 10^{-5} \ \omega^{0.42} \cos(0.21\pi) \ \text{S/m} \]
Figure 4.5.31  Simulated TDR response waveform: Effect of varying the dc conductance per unit length $G_{dc}$ ($L_e = 0.4 \mu \text{H/m}, C = 4 \epsilon_0 \text{F/m, } R_{dc} = 0 \Omega/m, Z_{sk} = 0 \Omega/m, G = 0 \text{S/m, } R_L = 50 \Omega$).
Figure 4.5.32  Simulated TDT response waveform: Effect of varying the dc conductance per unit length \( G_{dc} \) \((L_e = 0.4 \mu \text{H/m}, C = 4 \varepsilon \text{ F/m}, R_{dc} = 0 \Omega/\text{m}, Z_{sk} = 0 \Omega/\text{m}, G = 0 \text{ S/m}, R_L = 50 \Omega)\).
Figure 4.5.33  Simulated TDR response waveform: Effect of varying the dc conductance per unit length $G_{dc}$ ($L_e = 0.07\mu_h$ H/m, $C = 25\epsilon_o$ F/m, $R_{dc} = 0$ $\Omega$/m, $Z_{sk} = 0$ $\Omega$/m, $G = 0$ S/m, $R_L = 50$ $\Omega$).
Figure 4.5.34  Simulated TDT response waveform: Effect of varying the dc conductance per unit length $G_{dc}$ ($L_e = 0.07 \mu m$, $C = 25 \varepsilon_o F/m$, $R_{dc} = 0 \Omega/m$, $Z_{sk} = 0 \Omega/m$, $G = 0 S/m$, $R_L = 50 \Omega$).
The effects of varying the load impedance $R_L$ are shown in Figures 4.5.35 through 4.5.38. These responses were obtained by setting the capacitance per unit length $C$ and the external inductance per unit length $L_e$ to constant values and the loss parameters $R_{dc}$, $K_1$, $p$, $K_3$, $q$ and $G_{dc}$ to zero. From Figures 4.5.35 and 4.5.37, it is clear that the effect of the load impedance is observable on the TDR signature beyond 1.7 ns. In Figure 4.5.35, where $|Z_2(\omega)| > 50 \Omega$, as $R_L$ increases (decreases) the magnitude of the TDR response in the load region increases (decreases) too. On the other hand, in Figure 4.5.37, where $|Z_2(\omega)| < 50 \Omega$, as $R_L$ increases (decreases) the absolute magnitude of the TDR response in the load region decreases (increases). Furthermore, for both cases, as $R_L$ increases (decreases), the magnitude of the TDT responses decreases (increases).

4.5.2 Sensitivity Analysis on $\varepsilon'(\omega)$ and $\tan \delta(\omega)$

The purpose of this analysis is to have an understanding about the effect of varying $\varepsilon'(\omega)$, and $\tan \delta(\omega)$ on the time domain responses. This analysis will reveal the capability of both the TDR and TDT responses for determining the real part of the permittivity and the loss tangent. The first sensitivity analysis was done on $\varepsilon'(\omega)$. For this purpose, a reference value for $\varepsilon'_{r_{\infty}}$ was chosen to be 1.6. Later, by keeping the rest of the line parameters fixed, the value of $\varepsilon'_{r_{\infty}}$ was increased by 2.5% and 5%. For each case, the simulated TDR and TDT responses were computed. Using the TDR response waveform for $\varepsilon'_{r_{\infty}}$=1.6 as a reference data, the relative error in each TDR response was computed by subtracting it from the reference data accordingly. From the relative error, it becomes easier to have an understanding about how accurately $\varepsilon'(\omega)$ can be determined from the TDR response. The simulated TDR waveforms and their relative error with respect to the reference data are shown in Figures 4.5.39 and 4.5.40, respectively.

Similarly, using the TDT response waveform for $\varepsilon'_{r_{\infty}}$=1.6 as a reference data, the relative error in each TDT response was computed by subtracting it from the reference data accordingly. The simulated TDT waveforms and their relative error with respect to the reference data are
Figure 4.5.35  Simulated TDR response waveform: Effect of varying the load impedance $R_L$

\[ L_e = 0.4\mu_0 \text{ H/m}, \quad C = 4\varepsilon_0 \text{ F/m}, \quad R_{dc} = 0 \Omega/\text{m}, \quad Z_{sk} = 0 \Omega/\text{m}, \quad G = 0 \text{ S/m}, \]

\[ G_{dc} = 0 \Omega \].
Figure 4.5.36  Simulated TDT response waveform: Effect of varying the load impedance $R_L$

$$I_e = 0.4 \mu \text{H/m, } C = 4\epsilon_0 \text{ F/m, } R_{dc} = 0 \Omega/\text{m, } Z_{sk} = 0 \Omega/\text{m, } G = 0 \text{ S/m, }$$

$$G_{dc} = 0 \Omega.$$
Figure 4.5.37  Simulated TDR response waveform: Effect of varying the load impedance $R_L$

$$L_c = 0.07 \mu \text{H/m, } C = 25\epsilon_0 \text{ F/m, } R_{dc} = 0 \Omega/\text{m, } Z_{sk} = 0 \Omega/\text{m, } G = 0 \text{ S/m, }$$

$$G_{dc} = 0 \Omega).$$
Figure 4.5.38  Simulated TDT response waveform: Effect of varying the load impedance $R_L$

$\left( L_e = 0.07 \mu_\text{H/m}, C = 25 \epsilon_\text{F/m}, R_{dc} = 0 \Omega/m, Z_{sh} = 0 \Omega/m, G = 0 \text{S/m}, \right.$

$G_{dc} = 0 \Omega).$
shown in Figures 4.5.41 and 4.5.42, respectively. Comparing the results of Figure 4.5.40 with those of Figure 4.5.42, it can be observed that the TDR response is more sensitive to the changes in $\varepsilon'_r(\omega)$ than the corresponding TDT response. Hence, TDR techniques should yield much better accuracy in determining $\varepsilon'_r(\omega)$.

The above analysis was repeated for the loss tangent, $\tan \delta_L(\omega)$. A reference value was chosen for the loss tangent to be $1.074 \times 10^{-3}$ at 5 GHz. Then, by varying the high frequency admittance exponent $q$, the loss tangent was increased by 5% and 10% and the corresponding TDR and TDT responses were computed. When doing this, the rest of the line parameters were kept constant. The relative errors of the TDR and the TDT responses with respect to the reference data are shown in Figures 4.5.43 and 4.5.44, respectively. From these figures, it can be observed that as $\tan \delta_L(\omega)$ increases, the absolute magnitude of the relative error in the TDR response increases too. However, the magnitude of the relative error in the TDT response stays constant but the time delay changes slightly. Hence, an accurate determination of $\tan \delta_L(\omega)$ requires a proper detection of the time shift in the TDT response.

In order to observe how the conductance per unit length $G$ affects the frequency behavior of $\tan \delta_L(\omega)$ and $\varepsilon'_r(\omega)$, the high frequency admittance constant $K_3$ was varied while keeping the rest of the line parameters constant. The plot of $\varepsilon'_r(\omega)$ versus frequency is shown in Figure 4.5.45. From this figure, it can be observed that as $G$ increases $\varepsilon'_r(\omega)$ increases too. This increase is much more drastic in the low frequencies than in the high frequencies. As the frequency increases, $\varepsilon'_r(\omega)$ reaches its limiting value of $\varepsilon'_{\text{r\infty}}=1.6$. On the other hand, the plot of $\tan \delta_L(\omega)$ versus frequency is shown in Figure 4.5.46. The conductance per unit length has a similar effect on $\tan \delta_L(\omega)$ as on $\varepsilon'_r(\omega)$.
Figure 4.5.39  Simulated TDR response waveform: Effect of varying the relative permittivity

\[ \varepsilon_{\infty} \left( L_e = 0.4 \mu_e \text{ H/m, } C = [4\varepsilon_o + 1 \times 10^{-5} \omega^{-0.8} \sin(0.1\pi)] \frac{\text{F/m}}{\text{F/m}}, Z_{sk} = 0.1(j\omega)^{0.1} \Omega/\text{m}, \right. \]

\[ R_{dc} = 5 \Omega/\text{m, } G = [1 \times 10^{-5} \omega^{0.2} \cos(0.1\pi)] \frac{\text{S/m}}{\text{S/m}}, G_{dc} = 0.01 \text{ S/m, } R_L = 50 \Omega). \]
Figure 4.5.40  Relative error in the reflection coefficient to show the effect of varying $\varepsilon'_{\infty}$

$\left( I_c = 0.4 \mu_A \text{ H/m}, \ C = \left[ 4\varepsilon_{\infty} + 1 \times 10^{-5} \omega^{-0.8} \sin (0.1\pi) \right] \text{ F/m}, \ Z_{sk} = 0.1(j\omega)^{0.1} \Omega/m, \right.$

$R_{dc} = 5 \Omega/m, \ G = \left[ 1 \times 10^{-5} \omega^{0.2} \cos (0.1\pi) \right] \text{ S/m}, \ G_{dc} = 0.01 \text{ S/m}, \ R_L = 50 \Omega). \right)$
Figure 4.5.41 Simulated TDT response waveform: Effect of varying the relative permittivity

\[
\epsilon'_\infty (L_e = 0.4\mu_\circ \text{H/m}, C = [4\epsilon_\circ + 1 \times 10^{-5}\omega^{-0.8}\sin (0.1\pi)] \text{F/m}, Z_{sk} = 0.1(j\omega)^{0.1} \Omega/\text{m},
\]

\[
R_{dc} = 5 \Omega/\text{m}, G = [1 \times 10^{-5}\omega^{0.2}\cos (0.1\pi)] \text{S/m}, G_{dc} = 0.01 \text{S/m}, R_L = 50 \Omega).
\]
Figure 4.5.42  Relative error in the TDT response to show the effect of varying $\varepsilon_{r\infty}$

$$(L_c = 0.4 \mu_0 \text{ H/m}, \quad C = [4 \varepsilon_0 + 1 \times 10^{-5} \omega^{-0.8} \sin (0.1\pi)] \text{ F/m}, \quad Z_{sk} = 0.1(j\omega)^{0.1} \Omega/\text{m},)$$

$$(R_{dc} = 5 \Omega/\text{m}, \quad G = [1 \times 10^{-5} \omega^{0.2} \cos (0.1\pi)] \text{ S/m}, \quad G_{dc} = 0.01 \text{ S/m}, \quad R_L = 50 \Omega).$$
Figure 4.5.43  Relative error in the reflection coefficient to show the effect of varying \( \tan \delta(\omega) \).

Reference data parameters: \( L_o = 0.4 \mu H/m, \ C = [4\epsilon_o + 1 \times 10^{-5} \omega^{-0.8} \sin (0.1\pi)] \ F/m, \)
\( Z_{sk} = 0.1(j\omega)^{0.1} \Omega/m, \ R_{dc} = 5 \Omega/m, \ G = [1 \times 10^{-5} \omega^{0.2} \cos (0.1\pi)] \ S/m, \ G_{dc} = 0.01 \ S/m, \)
\( R_L = 50 \Omega, \ (\tan \delta(\omega) = 1.074 \times 10^{-3} \) at 5 GHz).
Figure 4.5.44 Relative error in the TDT response to show the effect of varying tan δ(ω).

Reference data parameters: $L_e = 0.4 \mu_H/m$, $C = [4\varepsilon_0 + 1 \times 10^{-5} \omega^{-0.8}\sin(0.1\pi)] F/m$,

$Z_{sk} = 0.1(j\omega)^{0.1} \Omega/m$, $R_{dc} = 5 \Omega/m$, $G = [1 \times 10^{-5} \omega^{0.2}\cos(0.1\pi)] S/m$, $G_{dc} = 0.01 S/m$,

$R_L = 50 \Omega$, ($tan \delta_e(\omega) = 1.074 \times 10^{-3}$ at 5 GHz).
Figure 4.5.45  Effect of the conductance per unit length $G$ on the relative permittivity $\varepsilon'(\omega)$

($L_e = 0.4 \mu_0$ H/m, $C_o = 4 \varepsilon_o \text{ F/m, } Z_{sk} = 0 \Omega/m, R_{dc} = 0 \Omega/m, G_{dc} = 0 S/m, R_L = 50 \Omega$).
Figure 4.5.46  Effect of the conductance per unit length $G$ on the loss tangent $\tan \delta_c(\omega)$

$(L_e = 0.4\mu \text{H/m}, C_o = 4\epsilon \text{ F/m}, Z_{sk} = 0 \Omega/m, R_{dc} = 0 \Omega/m, G_{dc} = 0 \text{ S/m}, R_L = 50 \Omega)$. 
4.6 SUMMARY AND CONCLUSIONS

In this chapter, time domain measurement and simulation techniques for the
determination of material properties of a stripline was presented. The chapter first presented the
theory of time domain measurement techniques. Later, the measurement set up used in the
experiments was discussed. An equivalent network model that closely simulates this set up was
then proposed. Using the equivalent network model, the S-parameters were derived. These S-
parameters were then used in computer simulation. The sensitivity analysis on the lossy line
parameters were performed by varying each parameter of the line one at a time while keeping the
others constant. The simulated time domain response for each case was obtained by convolving
the inverse Fourier Transform of the related S-parameter with an actual step waveform acquired
from an Hp 54120T TDNA.

The sensitivity analysis on the lossy line parameters has shown that the effect of each line
parameter was dependent upon the magnitude of the characteristic impedance of the line under
test, i.e. $Z_2(\omega)$. Each parameter had a different effect on the time domain response. On the
other hand, the sensitivity analysis on $\varepsilon'_e(\omega)$ has shown that the TDR data is more sensitive to
the changes in $\varepsilon'_e(\omega)$ than the TDT data. For the loss tangent, $\tan \delta_e(\omega)$, the reverse was true.
CHAPTER 5

A TIME DOMAIN OPTIMIZATION TECHNIQUE FOR THE DETERMINATION OF MATERIAL PROPERTIES OF A STRIPLINE

5.1 INTRODUCTION

In the previous chapter, the measurement set up and the equivalent network model pertinent to material characterization using a stripline geometry was discussed. In addition to this, the derivation of the desired S-parameters for computer simulation was also performed. The S-parameters are functions of the characteristic impedances and propagation functions of the lossy transmission lines used in the equivalent network model. In order to determine the lossy line parameters for each line, it is necessary to utilize optimization techniques. The optimization can be done either in the time domain or in the frequency domain. Since, all the measurements are done in the time domain, it was found out to be more practical to perform the optimization in the time domain. This is basically due to the numerical error involved in transforming the time domain data to the frequency domain using the Fast Fourier Transformation (FFT) [45].

Conventional optimization algorithms utilize the derivatives of the error function for minimization which has to be continuously differentiable. In the current application, since the simulated time domain response is not available analytically, it is not possible to obtain an analytical expression for the error function. Furthermore, the error function does not have any continuous derivatives. In certain cases, finite difference techniques can be used for computing the derivative of the error function and then utilize conventional optimization techniques. However, it has been found out that, in many applications, such an attempt leads to many
numerical errors, particularly in the vicinity of the minimum. Hence, the time domain optimization problem has to be solved by an optimization algorithm that does not require the derivative of the error function. Several optimization algorithms of this nature exist in literature. Since, writing the computer code of an optimization algorithm is an extremely tedious task, we have attempted to utilize one whose code was available in literature.

In this chapter, an optimization technique for the determination of material properties of a stripline is presented. The technique uses a modification of Powell's [46] algorithm to obtain the best non-linear least squares fit between the simulated and experimental time domain responses. The purpose of the optimization is to determine the set of values for the lossy transmission line parameters that give the best fit. This set is said to be an optimal solution for the optimization process. From the optimal solution, the material properties of the stripline under test are obtained. The optimization algorithm has been furnished in a software package called Time Domain Material Characterization (TDMC).

This chapter is divided into several sections. Section 5.2 presents the concept and approach of the non-linear least squares optimization technique for time domain applications. The details of the Time Domain Material Characterization (TDMC) package is described in section 5.3. Finally, a summary and conclusions is presented in section 5.4.

5.2 A NON-LINEAR LEAST SQUARES OPTIMIZATION TECHNIQUE FOR
TIME DOMAIN APPLICATIONS

5.2.1 Discussion

Before discussing the optimization algorithm, it is necessary to define an objective function [47]-[49] which describes the return or benefit from any proposed solution [48].
\[ E_2(\tilde{X}) = \int_0^\infty w(t) e^2(t) \, dt , \quad (5.2.1) \]

where, \( w(t) \) is a weighting function and \( \tilde{X} \) is the argument of the objective function defined in terms of the parameters of the lossy line as follows:

\[ \tilde{X} = (R_{dc}, L_o, K_1, p, C_o, K_3, q, G_{dc}) . \quad (5.2.2) \]

Furthermore, the error function \( e(t) \) is defined by

\[ e(t) = v_s(t) - v_e(t) , \quad (5.2.3) \]

where \( v_s(t) \) and \( v_e(t) \) are the simulated and experimental time domain responses, respectively.

In practice, the data acquired from the measurement equipment is discrete. Hence, the integral defined in (5.2.1) is replaced by a summation and the time dependent functions by sequences as follows:

\[ E_2(\tilde{X}) = \sum_{m=1}^{M} w(t_m) e^2(t_m) , \quad (5.2.4) \]

where, \( M \) is the total number of points used in data acquisition. In the specific application being considered, the weighting function is taken as unity (i.e. \( w(t_m) = 1 \)), because it gives fairly acceptable results. There are three main reasons for using a least squares error criterion for optimization [50]: (1) the minimization of a sum of squared error criterion usually leads to acceptably good data models, (2) this criterion has been thoroughly studied and many insightful theorems are available for its analysis, and, (3) in applications where ideal data is corrupted by additive Gaussian noise, the least squares error model often corresponds to the maximum likelihood estimate.
The simulated time domain response is obtained by convolving the time domain step waveform as acquired from the Hp 54120 TDNA with the corresponding S-parameter which has to be a function of time too. The S-parameters as derived in the previous chapter are functions of frequency. Due to their complicated structure, it is extremely difficult to transform them to the time domain by using analytical formulation. Hence, in order to obtain the S-parameters as functions of time, Inverse Fourier Transform (IFFT) techniques have to be utilized. The simulated TDR response waveform is given by

\[ v_{rd}(t) = v_{i1}(t) * s_{11}(t) = \int_{0}^{\infty} v_{i1}(\tau) s_{11}(t-\tau) \, d\tau , \] (5.2.5)

and the simulated TDT response by

\[ v_{td}(t) = v_{i1}(t) * s_{21}(t) = \int_{0}^{\infty} v_{i1}(\tau) s_{21}(t-\tau) \, d\tau . \] (5.2.6)

From (5.2.5) and (5.2.6), it can be observed that the computation of the simulated time domain responses involves two stages: (1) computation of the inverse Fourier Transform of the S-parameters, (2) convolution of the step response \( v_{i1}(t) \) with the time dependent S-parameters.

### 5.2.2 A Modification of Powell's Algorithm

We start this section by first presenting Powell's original algorithm and then present the modifications. Powell’s algorithm [46] is a modification of a quadratically convergent method proposed by Smith [51]. It locates the minimum of a positive definite quadratic function \( f(\mathbf{X}) \) by making use of some properties of conjugate directions. In order to understand the algorithm, we need to state some definitions and theorems [52].
Definition 5.2.1

Two vectors \( \tilde{u} \) and \( \tilde{v} \) are said to be conjugate with respect to the positive definite symmetric matrix \( \tilde{A} \) if

\[
\tilde{u}^T \tilde{A} \tilde{v} = 0 . \tag{5.2.7}
\]

Theorem 5.2.1

If \( \tilde{A} \) is positive definite symmetric, \( \tilde{A} \tilde{X} = \tilde{b} \), and \( \{ \tilde{u}_1, \ldots, \tilde{u}_m \} \) is a set of nonzero conjugate directions, then

\[
\tilde{W} = \tilde{X} - \sum_{i=1}^{m} \left( \frac{\tilde{u}_i^T \tilde{b}}{\tilde{u}_i^T \tilde{A} \tilde{u}_i} \right) \tilde{u}_i . \tag{5.2.8}
\]

is conjugate to each of \( \tilde{u}_1, \ldots, \tilde{u}_m \).

Corollary 5.2.1

If \( m = n \) in Theorem 5.2.1, then \( \tilde{W} = 0 \), so

\[
\tilde{X} = \sum_{i=1}^{m} \left( \frac{\tilde{u}_i^T \tilde{b}}{\tilde{u}_i^T \tilde{A} \tilde{u}_i} \right) \tilde{u}_i . \tag{5.2.9}
\]

Theorem 5.2.2

If \( \tilde{A} \) is positive definite symmetric,

\[
f(\tilde{X}) = \tilde{X}^T \tilde{A} \tilde{X} - 2 \tilde{b}^T \tilde{X} + \tilde{c} , \tag{5.2.10}
\]
for some $\bar{b} \in \bar{R}^n$ and $\bar{c} \in \bar{R}$, and $\bar{u}_1, \ldots, \bar{u}_m$ is a set of nonzero conjugate directions, then the minimum of $f(\bar{X})$ in the space spanned by $\bar{u}_1, \ldots, \bar{u}_m$ occurs at the point $\sum_{i=1}^{m} \xi_i \bar{u}_i$, where

$$\xi_i = \frac{\bar{u}_i^T \bar{b}}{\bar{u}_i^T \bar{A} \bar{u}_i}. \quad (5.2.11)$$

**Theorem 5.2.3**

Using the notation of Theorem 5.2.2, a fixed $j$ satisfying $1 \leq j \leq m$, and fixed $\bar{\zeta}_1, \ldots, \bar{\zeta}_{j-1}, \bar{\zeta}_{j+1}, \ldots, \bar{\zeta}_m$, the minimum of

$$\bar{\phi}_j(\bar{\zeta}_j) = f \left( \sum_{i=1}^{m} \bar{\zeta}_i \bar{u}_i \right) \quad (5.2.12)$$

occurs at $\bar{\zeta}_j = \bar{\xi}_j$.

From Theorem 5.2.2 and 5.2.3, it can be observed that the minimum of the quadratic function $f(\bar{X})$ can be found by $n$ one-dimensional minimizations along nonzero conjugate directions $\bar{u}_1, \ldots, \bar{u}_n$. Brent [52] claims that the order in which the one-dimensional minimizations are performed is irrelevant. This result can be used, provided that we have the capability of generating sets of conjugate directions. Powell [46] accomplishes this task by making use of the following theorem:

**Theorem 5.2.4**

If the minimum of $f(\bar{X})$ as defined by (5.2.10) in the direction $\bar{u}$ from the point $\bar{X}_i^*$ is at $\bar{X}_i$, for $i = 0, 1$, then $\bar{X}_i - \bar{X}_o$ is conjugate to $\bar{u}$.

The proofs of the above theorems is beyond the scope of this dissertation. The interested reader may refer to [52].
Let \( \tilde{X}_o \) be the initial approximation to the minimum, and let \( \tilde{u}_1, \ldots, \tilde{u}_n \) be the columns of the identity matrix. Then, using Powell’s original algorithm as presented in [46], an iteration of the basic procedure is given as follows:

1. For \( i = 1, \ldots, n \), compute \( \xi_i \) to minimize \( f(\tilde{X}_{i-1} + \xi_i \tilde{u}_i) \), and define
   \[
   \tilde{X}_i = \tilde{X}_{i-1} + \xi_i \tilde{u}_i.
   \]

2. For \( i = 1, \ldots, n-1 \), replace \( \tilde{u}_i \) by \( \tilde{u}_{i+1} \).

3. Replace \( \tilde{u}_n \) by \( \tilde{X}_n - \tilde{X}_o \).

4. Compute \( \xi \) to minimize \( f(\tilde{X}_o + \xi \tilde{u}_n) \), and replace \( \tilde{X}_o \) by \( \tilde{X}_o + \xi \tilde{u}_n \).

If \( f(\tilde{X}) \) is a non-quadratic function, the above iteration is repeated until some stopping criterion is satisfied. On the other hand, if \( f(\tilde{X}) \) is quadratic, let us consider what happens after the \( k \)-th iteration, where \( 1 \leq k \leq n \). By theorem 5.2.4, \( \tilde{u}_{n-k+1}, \ldots, \tilde{u}_n \) become conjugate and the choice of \( \tilde{u}_n \) at step 3 becomes applicable. After \( n \) iterations the minimization along \( n \) conjugate directions has been accomplished and the minimum has been reached if the \( \tilde{u}_i \) are all nonzero. This statement is true only if \( \xi_1 \neq 0 \) at each iteration.

Zangwill [53] has observed that, even if \( f(\tilde{X}) \) is a quadratic function, one of the iterations may have \( \xi_1 = 0 \). This drawback is due to the procedure of replacing, at each stage, \( \tilde{u}_i \) by \( \tilde{X}_n - \tilde{X}_o \). Once this happens, the minimum of the function \( f \) is only found over a subspace of \( \mathbb{R}^n \) and the algorithm gives the wrong answer. Although it is very unlikely that \( \xi_1 \) will become 0, Powell has discovered that the directions \( \tilde{u}_1, \ldots, \tilde{u}_n \) often become nearly linearly dependent. Hence, in the same paper [46], Powell proposed the following procedure for ensuring a reasonable convergence:

1. For \( i = 1, \ldots, n \), compute \( \xi_i \) to minimize \( f(\tilde{X}_{i-1} + \xi_i \tilde{u}_i) \), and define
   \[
   \tilde{X}_i = \tilde{X}_{i-1} + \xi_i \tilde{u}_i.
   \]

2. Find the integer \( m, 1 \leq m \leq n \), so that \( \{f(\tilde{X}_{m-1}) - f(\tilde{X}_m)\} \) is a maximum, and define
   \[
   \Delta = f(\tilde{X}_{m-1}) - f(\tilde{X}_m).
   \]
3. Calculate $f_3 = f(2\bar{X}_n - \bar{X}_o)$, and define $f_1 = f(\bar{X}_o)$ and $f_2 = f(\bar{X}_n)$.

4. If either $f_3 \geq f_1$ and/or $(f_1 - 2f_2 + f_3) \cdot (f_1 - f_2 - \Delta)^2 \geq \frac{1}{2} \Delta(f_1 - f_3)^2$ use the old directions $\bar{u}_1, \ldots, \bar{u}_n$ for the next iteration and use $\bar{X}_n$ for the next $\bar{X}_o$, otherwise,

5. defining $\bar{u} = (\bar{X}_n - \bar{X}_o)$, calculate $\xi$ so that $(\bar{X}_n + \xi \bar{u})$ is a minimum, use $\bar{u}_1, \ldots, \bar{u}_{m-1}, \bar{u}_{m+1}, \ldots, \bar{u}_n, \bar{u}$ as the directions and $\bar{u}_n + \xi \bar{u}$ as the starting point for the next iteration.

With the above modification, one of the mutually conjugate directions may be thrown away, so that more than $n$ iterations become required to find an exact minimum of a quadratic. This in turn implies that the property of quadratic convergence is given up.

Powell’s modified algorithm was further modified and written in FORTRAN code by Press et al. [54]. Their modification is as follows:

Define,

$$
\begin{align*}
  f_o &\equiv f(\bar{X}_o), \\
  f_n &\equiv f(\bar{X}_n) \quad \text{and} \quad f_e \equiv f(2\bar{X}_n - \bar{X}_o).
\end{align*}
$$

(5.2.13)

Note that $f_e$ is the function value at an extrapolated point somewhat further along the proposed new direction. Also define $\Delta f$ to be the magnitude of the largest decrease along one particular direction of the present basic procedure iteration. Then,

1. If $f_e \geq f_o$, then keep the old set of directions for the next basic procedure, because the average direction $\bar{X}_n - \bar{X}_o$ is all finished.

2. If $2(f_o - 2f_n + f_e) \cdot [(f_o - f_n) - \Delta f]^2 \geq (f_o - f_e)^2 \Delta f$, then retain the old set of directions for the next basic procedure, because (i) either the decrease along the average direction was not primarily due to any single direction’s decrease, or (ii) there is a substantial second derivative along the average direction and the minimum is in the vicinity.
The termination criterion used in the POWELL subroutine written by Press et al. [54] is as follows: Let $ftol$ be the fractional tolerance in the function value such that failure to decrease by more than this amount on one iteration implies that the minimum is reached. Then the termination criterion is given by

$$2|f_o - f_n| \leq ftol \cdot (|f_o| + |f_n|).$$

(5.2.14)

### 5.2.3 Solution of the Time Domain Optimization Problem

In order to solve the time domain optimization problem, Press et al.'s [54] POWELL subroutine was utilized with slight modifications. Powell's [46] algorithm as discussed in the previous section assumes that the function $f(\bar{X})$ to be minimized does not have any constraints. The optimization problem discussed in this chapter is one with constraints. The constraints are due to the physical properties of the lossy line parameters and are given as follows:

$$R_{dc} \geq 0,$$

(5.2.15a)

$$L_o > 0,$$

(5.2.15b)

$$K_1 \geq 0,$$

(5.2.15c)

$$0 < p < 1,$$

(5.2.15d)

$$C_o > 0, \quad \epsilon'_\infty \geq 1,$$

(5.2.15e)

$$K_3 \geq 0,$$

(5.2.15f)

$$0 < q < 1,$$

(5.2.15g)

and

$$G_{dc} \geq 0,$$

(5.2.15h)

The above constraints have been implemented inside the POWELL subroutine [54]. The non-negativeness constraint is enforced by checking the sign of every element of $\bar{X}$ before every
function evaluation. If a certain element is less than zero, then that element is replaced by its absolute value. After enforcing non-negativeness, the constraints given by (5.2.15d) and (5.2.15g) are implemented by replacing the appropriate element of $\vec{X}$, say $\vec{X}_i$, with $a \cdot \vec{X}_i$, where $a$ is an empirical scaling factor. In the current application $a$ was chosen to be $1 \times 10^{-2}$.

In the previous chapter, we have discussed the sensitivity of the lossy line parameters. It has been shown that each parameter affects different regions in a different fashion. The proposed time domain optimization technique should take this into consideration. In its current form, Powell's algorithm [46] is supposed to minimize any specified function $f(\vec{X})$, whether quadratic or non-quadratic. In the current application, $f(\vec{X}) = E_2(\vec{X})$. It has been shown in the previous chapter that the simulated time domain responses are most sensitive to $L_o$ and $C_o$. Hence it is extremely important that the initial conditions for these parameters are determined with a reasonable accuracy before the POWELL subroutine is called.

The time domain optimization problem involves the determination of the optimum lossy transmission line parameters for both the adapters and the stripline under test so that $E_2(\vec{X})$ is minimum. The most efficient approach to determine the optimum values for these parameters is to perform the optimization for each region separately. This approach is explained in the following sections.

### 5.2.3.1 Determination of The Optimum Line Parameters for the Adapters

Before discussing the details on how to determine the optimum line parameters for the adapters shown in Figure 4.3.1, we will make some assumptions. The adapters used in the measurement set up are designed to have a smooth transmission from the coaxial, reference transmission line to the planar, stripline under test with a characteristic impedance of approximately 50 Ω. Due to this property, it will be assumed that each adapter can be modeled with a real characteristic impedance, $R$, and time delay, $\tau$. 
In order to describe the optimization process, let \( v^a_{re}(t) \) and \( v^b_{re}(t) \) be the experimental TDR response waveforms acquired for adapters A and B, respectively. Furthermore, let \( v_{i1}(t) \) be the experimental TDR reference (step) waveform. Then, the initial conditions for each adapter of the adapter parameters are obtained as follows: First obtain the first derivatives of both the experimental response and reference waveforms using a backward difference formula defined by

\[
f'(t_k) \approx \frac{f(t_k) - f(t_k - \Delta t)}{\Delta t}, \quad k = 1, 2, \ldots, M. \tag{5.2.16}
\]

In (5.2.16), \( t_k \) corresponds to discrete time points, \( \Delta t \) is the time interval and \( M \) is the total number of points used in the data acquisition from the Hp 54120T TDNA. The backward difference formula can not provide the derivative at \( t_1 = 0 \), hence \( f'(0) \) is taken as zero. Typical graphs of \( v^a_{re}(t) \), \( v^b_{re}(t) \) and \( v_{i1}(t) \) and their derivatives computed using the backward difference formula are shown in Figures 5.2.1, 5.2.2 and 5.2.3, respectively. Let \( t^a_{re}, t^b_{re} \) and \( t_{i1} \) be the time instants which correspond to the locations of the spikes having the maximum absolute magnitude in the derivatives of \( v^a_{re}(t) \), \( v^b_{re}(t) \), and \( v_{i1}(t) \), respectively. A good initial value for the time delay of adapter A, \( \tau^a_o \), and the time delay of adapter B, \( \tau^b_o \) can be obtained by using the following relations:

\[
\tau^a_o = \frac{t^a_{re} - t_{i1}}{2}. \tag{5.2.17}
\]

\[
\tau^b_o = \frac{t^b_{re} - t_{i1}}{2}. \tag{5.2.18}
\]

On the other hand, initial values for the characteristic impedance of adapter A, \( R^a_o \), and the characteristic impedance of adapter B, \( R^b_o \) can be obtained as follows: Define the reflection coefficient due to the discontinuities in adapter A as
Figure 5.2.1  A typical graph showing $v_{re}^a(t)$ and its derivative.
Figure 5.2.2  A typical graph showing $v_{re}^b(t)$ and its derivative.
Figure 5.2.3  A typical graph showing $v_{i1}(t)$ and its derivative.
\[ p^a_o = \frac{v^a_{re}(t_{1i} + r^a_o)}{v_{i1}(t_M)} , \]  
\[ (5.2.19) \]

and the reflection coefficient due to the discontinuities in adapter B as

\[ p^b_o = \frac{v^b_{re}(t_{1i} + r^b_o)}{v_{i1}(t_M)} . \]  
\[ (5.2.20) \]

Then, initial values for \( R^a \) and \( R^b \) can be taken, respectively, as

\[ R^a_o = \frac{Z_g (1 + \rho^a_o)}{(1 - \rho^a_o)} , \]  
\[ (5.2.21) \]

and

\[ R^b_o = \frac{Z_g (1 + \rho^b_o)}{(1 - \rho^b_o)} . \]  
\[ (5.2.22) \]

Next define a new objective function for computing the optimum parameters for adapter A as follows:

\[ E^a_2 (\tau^a, R^a) = \sum_{k = M_1}^{M_2} [e^a(t_k)]^2 , \]  
\[ (5.2.23) \]

where, the error function \( e^a(t_k) \) is defined by

\[ e^a(t_k) = v^a_{rs}(t_k) - v^a_{re}(t_k) , \quad k = 1, \ldots, M. \]  
\[ (5.2.24) \]

Furthermore, \( M_1 \) and \( M_2 \) are defined as follows:

\[ M_1 = t_{1i}/\Delta t , \]
and

\[ M_2 = t_{re}^a / \Delta t , \]

The optimal values for \( \tau^a \) and \( R^a \) are then obtained by using the above initial conditions in the POWELL subroutine and the objective function defined in (5.2.23).

Once \( \tau^a \) and \( R^a \) are computed, we can proceed in computing the optimal values for \( \tau^b \) and \( R^b \). For this purpose, define the following objective function:

\[ E_2^b (\tau^b, R^b) = \sum_{k = M_1}^{M_3} [e^b(t_k)]^2 , \]  

(5.2.25)

where, the error function \( e^b(t_k) \) is defined by

\[ e^b(t_k) = v_{rd}^b(t_k) - v_{re}^b(t_k) , \quad k = 1, \ldots, M . \]  

(5.2.26)

Furthermore, \( M_1 \) is as defined above and \( M_3 \) as follows:

\[ M_3 = t_{re}^b / \Delta t . \]

Similarly, the optimal values for \( \tau^b \) and \( R^b \) are obtained by using the above initial conditions in the POWELL [54] subroutine and the objective function defined in (5.2.25).

5.2.3.2 Determination of The Optimum Lossy Line Parameters for the Stripline

Having explained the determination of the line parameters for the adapters, it is now time describe how the optimal values for the lossy transmission line representing the stripline is determined. We start again with the initial conditions. Let \( v_{re}^a(t) \) be the experimental TDR
response waveform acquired for the overall network that includes the adapters A and B, and the stripline under test. Then, compute the derivative of \( v^*_{re}(t) \) using the backward difference formula as defined by (5.2.16). A typical graph of \( v^*_{re}(t) \) and its derivative is shown in Figures 5.2.4. Let \((t^*_{re})_1\) and \((t^*_{re})_2\) be the time instants which correspond to the locations of the spikes having the first and the second highest absolute magnitudes in the derivative of \( v^*_{re}(t) \). An approximate value for the time delay due to the stripline under test alone is then given by

\[
\tau^* = \frac{[(t^*_{re})_1 - (t^*_{re})_2]}{2},
\]

(5.2.27)

Next, an approximate value for the characteristic impedance of the stripline under test can be obtained as follows: Define the reflection coefficient due to the mismatch in the stripline as

\[
\rho^* = \frac{[v^*_{re}(t_s + \tau^*)](1 - (\rho^2))}{v_{in}(t_M)},
\]

(5.2.28)

where, \( t_s = \min \{ [(t^*_{re})_1, (t^*_{re})_2] \}\), and \( \rho^2 \) is the reflection coefficient due to the discontinuities in adapter A obtained by using the optimal value of the time delay, \( \tau^o \). Then, an approximate value for the characteristic impedance of the stripline under test is given by

\[
R^* = \frac{R^o (1 + \rho^*)}{(1 - \rho^*)}.
\]

(5.2.29)

Using (5.2.27) and (5.2.29), and assuming that the physical length of the stripline under test is \( \ell \) meters, we can obtain the following initial values for \( L_o \) and \( C_o \):

\[
(L_o)_o = \frac{\tau^* R^*}{\ell^o},
\]

(5.2.30)

and
Figure 5.2.4 A typical graph showing $v_{re}(t)$ and its derivative.
\[(C_o)_o = \frac{\tau_o^*}{R_o^* \ell}.\]  

(5.2.31)

In chapter 3, we have assumed that the stripline under test is non-magnetic. To elaborate further, let us recall the following formulation:

\[L_o(\omega) = \mu_o A_1 = L_o = \text{constant},\]  

(5.2.32)

\[C_o = \lim_{\omega \to \infty} \left[ \frac{Y(\omega)}{j\omega} \right] = \epsilon'_\infty \epsilon_o A_2,\]  

(5.2.33)

and

\[A_1 \cdot A_2 = 1.\]  

(5.2.34)

From the above formulation, it can be observed that it is more efficient from the optimization point of view to optimize for \(A_1\) instead of \(L_o\). The reason for this is that, once \(A_1\) is fixed at a certain value, then \(A_2\) is not anymore arbitrary, but has to be obtained using (5.2.34). Furthermore, from (5.2.33) it can be observed that, it is more intuitive to optimize \(\epsilon'_\infty\) directly instead of \(C_o\). Using \(A_1\) and \(\epsilon'_\infty\) as the new variables for optimization, the initial conditions become

\[(A_1)_o = \frac{\tau_o^* R_o^*}{\mu_o \ell},\]  

(5.2.35)

and

\[(\epsilon'_\infty)_o = \frac{(\tau_o^*)^2}{\mu_o \epsilon_o \ell^2}.\]  

(5.2.36)

Assuming that the optimal parameters for both adapters are computed beforehand and taking into consideration the results of sensitivity analysis of chapter 4, the objective function for
the stripline under test can be written as

\[ E_2^s (\vec{X}) = \sum_{k = M_4}^{M} [e^s (t_k)]^2, \]

(5.2.37)

where, \( \vec{X} \) is the new argument of the objective function defined in terms of the parameters of the lossy line as follows:

\[ \vec{X} = (R_{dc}, A_1, K_1, p, \epsilon'_{\infty}, K_3, \eta, G_{dc}) \]

(5.2.38)

Furthermore, the error function \( e^s(t_k) \) is defined by

\[ e^s(t_k) = v^s_{rs}(t_k) - v^s_{re}(t_k), \quad k = 1, \ldots, M, \]

(5.2.39)

and \( M_4 \) by

\[ M_4 = \frac{t_s}{\Delta t}. \]

The starting and the ending points for the evaluation of the least squares summation was chosen based on the sensitivity analysis explained in chapter 4. For example, consider the summation in (5.2.37). Recall that the optimization for the stripline is performed after the optimal values for both adapter parameters are determined. This implies that the simulated and experimental responses in the time interval \( t = [0, t_s] \) is already matched in a least squares sense. Hence, there is no point in wasting computer time in computing the summation for this interval.
5.3 THE TIME DOMAIN MATERIAL CHARACTERIZATION (TDMC) PACKAGE FOR DETERMINING THE MATERIAL PROPERTIES OF A STRIPLINE

5.3.1 Description of the Software Package

The non-linear least squares optimization technique discussed in the previous section has been implemented in a software package using FORTRAN language. The package, which is named as the Time Domain Material Characterization (TDMC) is based on the theory and measurement set up presented in the current and the previous chapters. A general information flowchart which would help in describing its features is shown in Figure 5.3.1.

The input data file of TDMC should be named as filename0.DAT, where filename0 is an arbitrary user-defined file name but its extension is fixed as DAT. The format of the input data file is shown in Figure 5.3.2. The package requires at least one set of experimental time domain response and reference data for operation. This could be either TDR or TDT data. However, it is recommended that a set of TDR data is always included, since it gives more accurate results on the dielectric constant. The text in Figure 5.3.2 which appears in upper case letters has to be exactly the same, because the package compares them to a preset format for identifying the information. However, the order of this information is arbitrary. The lossy line parameters provided in this data file are used as initial conditions in the optimization algorithm, except for $A_1$ and $\epsilon_{\infty}$, where the package provides the user with the option of calculating them internally. In this specific application, the load resistance $R_L$ is set to 50 $\Omega$.

All of the data beyond the equality sign which is shown in italics should be real numbers, and provided in mks units, except for the reference and response file names, where it has to be in text format. The response and reference data file names, referred as filename1, filename2, filename3 and filename4 are user-defined and should be different from each other. On the other
hand, the response and reference data file extensions, referred as filetype are also user defined and can be the same for all four files. The TDMC package assumes that these files are in the Hp 54120T TDNA format. This format assumes that all the data is listed in a single column. The first data in the column should correspond to the number of data points and the last data should correspond to the time per division used in the acquisition process. The data points corresponding to the waveform should be listed in between. The units of the data corresponding to the waveform should be in Volts.

The TDMC software package is executed as follows: TDMC filename0. Assuming that the adapters are identical, then the TDR reference and response waveforms for the stripline under test are used for computing the optimal adapter parameters. The initial conditions for the adapter parameters are computed internally. If the assumption that the two adapters are identical is not desirable, then the TDR response and reference waveforms are used for computing the optimal parameters for adapter A only. For computing the optimal parameters of adapter B, separate TDR response and reference data has to be provided. The computation is then done in a similar manner as in the case of adapter A. Once the optimization process for the two adapters are completed, then the software package asks the user if he/she desires \((A_1)\) and \((c'_{\text{wave}})\) to be calculated internally. After this question is properly answered, the program calls the POWELL subroutine and starts the iterative process for optimization. The status of each iteration is displayed on the computer terminal. When the iteration is complete, the package plots the experimental and simulated TDR responses on the terminal and asks the user if he/she desires to use TDT data for optimization. If the user answers "YES", then the program uses the optimal solution for the stripline parameters as initial conditions for the new iteration. Once this iteration is also complete, the package asks the user if he/she wants to reiterate the optimization process.

When the optimization process is complete, the package prints the following output data into a floppy diskette or hard disk, depending on from where the data file filename0.DAT was
Figure 5.3.1 Information flowchart for the Time Domain Material Characterization (TDMC) software package.
REFLECTION REFERENCE FILE = filename1.filetype
REFLECTION RESPONSE FILE = filename2.filetype
TRANSMISSION REFERENCE FILE = filename3.filetype
TRANSMISSION RESPONSE FILE = filename4.filetype
LENGTH OF STRIPLINE (m) = \( \ell \)
DC RESISTANCE (Rdc) = \( (R_{dc})_o \)
INDUCTANCE CONSTANT (A1) = \( (A_1)_o \)
CAPACITANCE CONSTANT (Epsr infinity) = \( (\varepsilon'_\infty)_o \)
CONDUCTANCE CONSTANT (K3) = \( (K_3)_o \)
CONDUCTANCE EXPONENT (q) = \( (q)_o \)
DC CONDUCTANCE CONSTANT (sigma) = \( (\sigma_{dc})_o \)
SKIN EFFECT CONSTANT (K1) = \( (K_1)_o \)
SKIN EFFECT EXPONENT (p) = \( (p)_o \)
SHUNT LOAD RESISTANCE (RL) = \( (R_L)_o \)

Figure 5.3.2 Input data file format for TDMC software package.
read: \texttt{filename0.OUT}, \texttt{filename0.TDR}, \texttt{filename0.TDT} and \texttt{filename0.EPS}. The file having the extension \texttt{OUT} contains the optimal values for the adapters and the stripline under test. It also provides the number of iterations it took the program to reach the optimal solution and the sum of least squares of the error function at the optimal solution. The file having the extension \texttt{EPS} contains the listing of $\epsilon'_s(\omega)$ and $\tan \delta_e(\omega)$ as a function of frequency. On the other hand, the files having the extensions \texttt{TDR} and \texttt{TDT} contain the listings of the experimental and simulated TDR and TDT data as a function of time at the optimal solution, respectively.

\textbf{Example 5.3.1}

In order to describe the input and output files of the TDMC software package further, suppose that the input data file is named as \texttt{PYR251.DAT}. Assume that the stripline under test has a physical length of 3 inches. Let the TDR reference and response files be named as \texttt{PYR25RF1.HP} and \texttt{PYR25RS1.HP}, respectively. Furthermore, let the TDT reference and response files be named as \texttt{PYR25TF1.HP} and \texttt{PYR25TS1.HP}, respectively. Then, a typical input data file would look like one as shown in Figure 5.3.3. When executed, the TDMC software package would generate output data files with the file name \texttt{PYR25R1} and extensions as discussed above. Typical outputs of \texttt{PYR251.OUT}, \texttt{PYR251.EPS}, \texttt{PYR251.TDR}, \texttt{PYR251.TDT} are shown in Figures 5.3.4 through 5.3.7, respectively.

\textbf{5.3.2 Verification of the TDMC Software Package and the Optimization Algorithm}

In order verify the TDMC software package and the time domain optimization algorithm, some computer simulations have been performed. The simulations were performed by first generating both TDR and TDT data using the formulation presented in the previous chapter. Later, this data was given to the TDMC package as if it were an experimental data and was asked to give the optimal parameters for the adapters and the stripline under test. The results for
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>REFLECTION REFERENCE FILE</td>
<td>PYR25RF1.HP</td>
</tr>
<tr>
<td>REFLECTION RESPONSE FILE</td>
<td>PYR25RS1.HP</td>
</tr>
<tr>
<td>TRANSMISSION REFERENCE FILE</td>
<td>PYR25TF1.HP</td>
</tr>
<tr>
<td>TRANSMISSION RESPONSE FILE</td>
<td>PYR25TS1.HP</td>
</tr>
<tr>
<td>LENGTH OF STRIPLINE (m)</td>
<td>2.72E-2</td>
</tr>
<tr>
<td>DC RESISTANCE (Rdc)</td>
<td>0.0</td>
</tr>
<tr>
<td>INDUCTANCE CONSTANT (A1)</td>
<td>2.0E-2</td>
</tr>
<tr>
<td>CAPACITANCE CONSTANT (Epsr infinity)</td>
<td>3.0</td>
</tr>
<tr>
<td>CONDUCTANCE CONSTANT (K3)</td>
<td>1.0E-2</td>
</tr>
<tr>
<td>CONDUCTANCE EXPONENT (a)</td>
<td>0.2</td>
</tr>
<tr>
<td>DC CONDUCTANCE CONSTANT (sigma)</td>
<td>0.0</td>
</tr>
<tr>
<td>SKIN EFFECT CONSTANT (K1)</td>
<td>1.0E-5</td>
</tr>
<tr>
<td>SKIN EFFECT EXPONENT (p)</td>
<td>0.4</td>
</tr>
<tr>
<td>SHUNT LOAD RESISTANCE (RL)</td>
<td>50.0</td>
</tr>
</tbody>
</table>

Figure 5.3.3  A typical input data file for the TDMC package: PYR25RJ1.DAT.
THE CONSTRAINED OPTIMAL SOLUTION FOR ADAPTER

\[ \text{ Tau1 } = 1.260000 \times 10^{-10} \text{ (Sec)} \]

\[ \text{ R1 } = 4.890000 \times 10^1 \text{ (Ohms)} \]

\[ \text{ Tau3 } = 1.260000 \times 10^{-10} \text{ (Sec)} \]

\[ \text{ R3 } = 4.890000 \times 10^1 \text{ (Ohms)} \]

THE CONSTRAINED OPTIMAL SOLUTION FOR STRIPLINE USING TDT DATA

\[ \text{ L0 } = 5.367196 \times 10^{-2} \times \text{ Mu0 (H/m)} \]

\[ \text{ Epsrinf } = 2.266680 \times 10^0, \text{ C0 = Epsrinf \times Eps0 \times Mu0/L0 (F/m)} \]

\[ \text{ Rdc } = 2.040418 \times 10^{-4} \text{ (Ohms/m)} \]

\[ \text{ Sigmad } = 6.389046 \times 10^{-10}, \text{ Gdc = Sigmad \times Mu0/L0 (S/m)} \]

\[ \text{ K1 } = 9.059438 \times 10^{-3} \text{ (Ohms/[m/(radians/sec)]**p)} \]

\[ \text{ p } = 3.086624 \times 10^{-1} \]

\[ \text{ K3 } = 1.442651 \times 10^{-4} \text{ (S/[m/(radians/sec)]**q)} \]

\[ \text{ q } = 3.121070 \times 10^{-1} \]

\[ \text{ RL } = 5.000000 \times 10^1 \text{ (Ohms)} \]

NUMBER OF ITERATIONS TAKEN IN THE FINAL OPTIMIZATION = 47

SUM OF LEAST SQUARES OF FUNCTION AT OPTIMAL SOLUTION = 1.5726E - 03

Figure 5.3.4 A typical output data from the TDMC package: PYR25R1.OUT.
<table>
<thead>
<tr>
<th>FREQ(HZ)</th>
<th>EPSR</th>
<th>TAN DELTA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.250E + 08</td>
<td>2.580E + 00</td>
<td>2.274E − 01</td>
</tr>
<tr>
<td>2.500E + 08</td>
<td>2.461E + 00</td>
<td>1.480E − 01</td>
</tr>
<tr>
<td>3.750E + 08</td>
<td>2.414E + 00</td>
<td>1.142E − 01</td>
</tr>
<tr>
<td>5.000E + 08</td>
<td>2.387E + 00</td>
<td>9.471E − 02</td>
</tr>
<tr>
<td>6.250E + 08</td>
<td>2.370E + 00</td>
<td>8.182E − 02</td>
</tr>
<tr>
<td>7.500E + 08</td>
<td>2.358E + 00</td>
<td>7.255E − 02</td>
</tr>
<tr>
<td>8.750E + 08</td>
<td>2.349E + 00</td>
<td>6.551E − 02</td>
</tr>
<tr>
<td>1.000E + 09</td>
<td>2.342E + 00</td>
<td>5.994E − 02</td>
</tr>
<tr>
<td>1.125E + 09</td>
<td>2.336E + 00</td>
<td>5.541E − 02</td>
</tr>
<tr>
<td>1.250E + 09</td>
<td>2.331E + 00</td>
<td>5.165E − 02</td>
</tr>
<tr>
<td>1.375E + 09</td>
<td>2.327E + 00</td>
<td>4.845E − 02</td>
</tr>
<tr>
<td>1.500E + 09</td>
<td>2.323E + 00</td>
<td>4.571E − 02</td>
</tr>
<tr>
<td>1.625E + 09</td>
<td>2.320E + 00</td>
<td>4.332E − 02</td>
</tr>
<tr>
<td>1.750E + 09</td>
<td>2.318E + 00</td>
<td>4.121E − 02</td>
</tr>
<tr>
<td>1.875E + 09</td>
<td>2.315E + 00</td>
<td>3.934E − 02</td>
</tr>
<tr>
<td>2.000E + 09</td>
<td>2.313E + 00</td>
<td>3.767E − 02</td>
</tr>
<tr>
<td>2.125E + 09</td>
<td>2.311E + 00</td>
<td>3.616E − 02</td>
</tr>
<tr>
<td>2.250E + 09</td>
<td>2.310E + 00</td>
<td>3.479E − 02</td>
</tr>
<tr>
<td>2.375E + 09</td>
<td>2.308E + 00</td>
<td>3.354E − 02</td>
</tr>
<tr>
<td>2.500E + 09</td>
<td>2.307E + 00</td>
<td>3.240E − 02</td>
</tr>
<tr>
<td>2.625E + 09</td>
<td>2.305E + 00</td>
<td>3.135E − 02</td>
</tr>
</tbody>
</table>

Figure 5.3.5  A typical output data from the TDMC package: PYR25R1.EPS.
<table>
<thead>
<tr>
<th>TIME(S)</th>
<th>VRE (VOLTS)</th>
<th>VRT (VOLTS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000E+00</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>1.563E-11</td>
<td>-6.250E-04</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>3.125E-11</td>
<td>-6.880E-04</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>4.688E-11</td>
<td>-6.250E-04</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>6.250E-11</td>
<td>-5.940E-04</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>7.813E-11</td>
<td>-4.070E-04</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>9.375E-11</td>
<td>-2.820E-04</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>1.094E-10</td>
<td>-1.570E-04</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>1.250E-10</td>
<td>2.500E-04</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>1.406E-10</td>
<td>3.430E-04</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>1.563E-10</td>
<td>5.000E-04</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>1.719E-10</td>
<td>8.120E-04</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>1.875E-10</td>
<td>8.120E-04</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>2.031E-10</td>
<td>8.120E-04</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>2.188E-10</td>
<td>7.180E-04</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>2.344E-10</td>
<td>7.810E-04</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>2.500E-10</td>
<td>9.370E-04</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>2.656E-10</td>
<td>1.156E-03</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>2.813E-10</td>
<td>1.500E-03</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>2.969E-10</td>
<td>1.500E-03</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>3.125E-10</td>
<td>1.406E-03</td>
<td>0.000E+00</td>
</tr>
</tbody>
</table>

Figure 5.3.6 A typical output data from the TDMC package: PYR25R1.TDR.
<table>
<thead>
<tr>
<th>TIME(S)</th>
<th>VTE (VOLTS)</th>
<th>VTT (VOLTS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000E+00</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>3.125E-11</td>
<td>-3.125E-05</td>
<td>7.001E-07</td>
</tr>
<tr>
<td>4.688E-11</td>
<td>-3.125E-05</td>
<td>-8.641E-08</td>
</tr>
<tr>
<td>6.250E-11</td>
<td>9.375E-05</td>
<td>3.750E-07</td>
</tr>
<tr>
<td>7.813E-11</td>
<td>-3.125E-05</td>
<td>-7.814E-08</td>
</tr>
<tr>
<td>9.375E-11</td>
<td>0.000E+00</td>
<td>5.150E-07</td>
</tr>
<tr>
<td>1.094E-10</td>
<td>-3.125E-05</td>
<td>1.440E-07</td>
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<tr>
<td>1.250E-10</td>
<td>9.375E-05</td>
<td>8.946E-07</td>
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<tr>
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<td>5.201E-07</td>
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<tr>
<td>1.563E-10</td>
<td>3.125E-05</td>
<td>1.032E-06</td>
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<tr>
<td>1.719E-10</td>
<td>-6.250E-05</td>
<td>8.308E-07</td>
</tr>
<tr>
<td>1.875E-10</td>
<td>-6.250E-05</td>
<td>1.499E-06</td>
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<tr>
<td>2.031E-10</td>
<td>1.875E-04</td>
<td>1.769E-06</td>
</tr>
<tr>
<td>2.188E-10</td>
<td>0.000E+00</td>
<td>2.586E-06</td>
</tr>
<tr>
<td>2.344E-10</td>
<td>6.250E-05</td>
<td>3.783E-06</td>
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<tr>
<td>2.500E-10</td>
<td>0.000E+00</td>
<td>5.067E-06</td>
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<tr>
<td>2.656E-10</td>
<td>6.250E-05</td>
<td>9.768E-06</td>
</tr>
<tr>
<td>2.813E-10</td>
<td>9.375E-05</td>
<td>1.880E-05</td>
</tr>
<tr>
<td>2.969E-10</td>
<td>6.250E-05</td>
<td>4.689E-05</td>
</tr>
<tr>
<td>3.125E-10</td>
<td>1.562E-04</td>
<td>1.201E-04</td>
</tr>
</tbody>
</table>

Figure 5.3.7  A typical output data from the TDMC package: PYR25R1.TDT.
two different cases are shown below.

**Verification Case 1**

The simulated data for this case was generated by assuming that the two adapters are identical. Each adapter was modeled with a lossless transmission line having the following specifications:

\[ \tau = \tau_1 = \tau_3 = 1.75 \times 10^{-10} \text{ seconds} \]

and

\[ R = R_1 = R_3 = 50 \ \Omega \]

On the other hand, the stripline under test was modeled with a transmission line having the following specifications:

\[ L_e = 0.4 \mu_0 \ \text{H/m} \]

\[ C = 4 \epsilon_0 \ \text{F/m} \]

\[ Z_{sk} = 0 \ \Omega/\text{m} \]

\[ R_{dc} = 0 \ \Omega/\text{m} \]

\[ G = 0 \ \text{S/m} \]

\[ G_{dc} = 0 \ \text{S/m} \]

\[ R_L = 50 \ \Omega \]

\[ \ell = 3'' \]

The above data was used for generating simulated TDR and TDT responses. These responses were then provided to the TDMC software package as experimental data and was asked to give the optimum parameters for the transmission lines. The tolerance value given to the software
package was $1 \times 10^{-7}$. The results of the optimization are shown below.

\[
\begin{align*}
\tau^{\text{op}} & = 1.748061 \times 10^{-10} \text{ seconds}, \\
R^{\text{op}} & = 49.97881 \ \Omega, \\
I_e^{\text{op}} & = 0.3984285 \mu \text{ H/m}, \\
C^{\text{op}} & = 4.0155461 \epsilon_o + 1.775979 \times 10^{-7} \omega^{-0.99895} \sin (5.25435 \times 10^{-4} \pi) \text{ F/m}, \\
Z_{st}^{\text{op}} & = 3.894693 \times 10^{-10} (j\omega)^{0.528} \Omega/\text{m}, \\
R_{dc}^{\text{op}} & = 4.627330 \times 10^{-7} \Omega/\text{m}, \\
C^{\text{op}} & = 1.775979 \times 10^{-7} \omega^{1.05087 \times 10^{-3}} \cos (5.25435 \times 10^{-4} \pi) \text{ S/m}, \\
C_{dc}^{\text{op}} & = 2.727421 \times 10^{-4} \text{ S/m},
\end{align*}
\]

where, the superscript \text{"op"} refers to the optimum solution. Comparing the above optimal solution with the actual parameters, it can be observed that some discrepancy exists, especially in the loss parameters. Theoretically, the optimization algorithm was supposed to find the exact solution, since we have used the same formulation to generate the simulated data. However, in each function evaluation, the algorithm has to go through an inverse FFT computation. This results in some loss of numerical accuracy. It is interesting to look at the computation error in some of the parameters that have high sensitivity on the TDR and TDT responses. Hence,

\[
\begin{align*}
\% \text{ error in } \tau & = \left| \frac{1.75 \times 10^{-10} - 1.748061 \times 10^{-10}}{1.75 \times 10^{-10}} \right| = 0.11, \\
\% \text{ error in } R & = \frac{|50.0 - 49.97881|}{50.0} = 0.042, \\
\% \text{ error in } L_e & = \frac{|0.4 - 0.3984285|}{0.4} = 0.39, \\
\% \text{ error in } C_o & = \frac{|4.0 - 4.0155461|}{4.0} = 0.39.
\end{align*}
\]
From the above results, it can be observed that the error in the line parameters which are directly related to the time delay and the characteristic impedance is less 0.4%. On the other hand, the line parameters which are related to the dielectric and conductor losses are in the order of \(1 \times 10^{-7}\), except for \(G_{dc}\), which is on the order of \(1 \times 10^{-4}\). In the current application, this error is considered to be negligible.

The comparison between the simulated and verified TDR and TDT responses are shown in Figures 5.3.8 and 5.3.9, respectively. These figures display a perfect match between the simulated and verified responses. This also proves that the small discrepancy between the simulated and verified line parameters is also negligible when it comes to the time domain responses.

**Verification Case 2**

The simulated data for this case was also generated by assuming that the two adapters are identical. Each adapter was again modeled with a lossless transmission line having the following specifications:

\[
\tau = \tau_1 = \tau_3 = 1.75 \times 10^{-10} \text{ seconds},
\]

and

\[
R = R_1 = R_3 = 50 \, \Omega.
\]

On the other hand, the stripline under test was modeled with a transmission line having the following specifications:

\[
L_e = 0.07 \mu_0 \, \text{H/m},
\]

\[
C = 25 \epsilon_0 + 1.0 \times 10^{-5} \, \omega^{-0.8} \sin (0.1 \pi) \, \text{F/m},
\]
Figure 5.3.8  Comparison between the simulated and verified TDR responses for verification case 1.
Figure 5.3.9  Comparison between the simulated and verified TDT responses for verification case 1.
\[ Z_{sk} = 1.0 \times 10^{-2} (j\omega)^{0.25} \, \text{\Omega/m}, \]
\[ R_{dc} = 5 \, \text{\Omega/m}, \]
\[ G = 1.0 \times 10^{-5} \omega^{0.2} \cos (0.1\pi) \, \text{S/m}, \]
\[ G_{dc} = 1 \times 10^{-2} \, \text{S/m}, \]
\[ R_L = 50 \, \Omega, \]
\[ \ell = 3''. \]

The above data was used for generating simulated TDR and TDT responses. These responses were provided to the TDMC software package as experimental data and was asked to give the optimum parameters for the transmission lines with a tolerance value of \(1 \times 10^{-7}\). The results of the optimization are shown below.

\[ \tau^{op} = 1.748061 \times 10^{-10} \text{ seconds}, \]
\[ R^{op} = 49.97881 \, \Omega, \]
\[ L_e^{op} = 0.069999936 \mu_\text{H/m}, \]
\[ C^{op} = 25.00000857\sigma + 1.004516 \times 10^{-5} \omega^{-0.8000004} \sin (0.0999998\pi) \, \text{F/m}, \]
\[ Z_{sk}^{op} = 1.001562 \times 10^{-2} (j\omega)^{0.25} \, \text{\Omega/m}, \]
\[ R_{dc}^{op} = 5.002978 \, \text{\Omega/m}, \]
\[ G^{op} = 1.004516 \times 10^{-5} \omega^{0.199996} \cos (0.0999998\pi) \, \text{S/m}, \]
\[ G_{dc}^{op} = 1.000009 \times 10^{-2} \, \text{S/m}. \]

For above case, the discrepancy between the simulated and verified lossy line parameters seems to be less than those of case 1. The computation error for each line parameter is as follows:

\[
\text{% error in } \tau = \frac{|1.75 \times 10^{-10} - 1.748061 \times 10^{-10}|}{1.75 \times 10^{-10}} = 0.11, \]
\[
\% \text{ error in } R = \frac{|50.0 - 49.97881|}{50.0} = 0.042, \\

\% \text{ error in } L_e = \frac{|0.07 - 0.069999936|}{0.07} = 9.14 \times 10^{-5}, \\

\% \text{ error in } C_o = \frac{|25.0 - 25.00000857|}{25.0} = 3.43 \times 10^{-5}, \\

\% \text{ error in } K_1 = \frac{|1.0 \times 10^{-2} - 1.001562 \times 10^{-2}|}{1.0 \times 10^{-2}} = 0.152, \\

\% \text{ error in } p = \frac{|0.25 - 0.25|}{0.25} = 0.0, \\

\% \text{ error in } K_3 = \frac{|1.0 \times 10^{-5} - 1.004516 \times 10^{-5}|}{1.0 \times 10^{-5}} = 0.452, \\

\% \text{ error in } q = \frac{|0.2 - 0.1999996|}{0.2} = 2.0 \times 10^{-4}, \\

\% \text{ error in } R_{dc} = \frac{|5.0 - 5.002978|}{5.0} = 0.05956, \\

\% \text{ error in } G_{dc} = \frac{|1.0 \times 10^{-2} - 1.000009 \times 10^{-2}|}{1.0 \times 10^{-2}} = 9.0 \times 10^{-4}.
\]

From the above results, it can be observed that the error in the line parameters is less than 0.5%. This error is again due to the computation of the numerical inverse FFT in each function evaluation and is negligible for the current application.

The comparison between the simulated and verified TDR and TDT responses are shown in Figures 5.3.10 and 5.3.11, respectively. These figures also display a perfect match between the simulated and verified responses.
Figure 5.3.10  Comparison between the simulated and verified TDR responses for verification case 2.
Figure 5.3.11  Comparison between the simulated and verified TDT responses for verification case 2.
5.4 SUMMARY AND CONCLUSIONS

In this chapter, a non-linear optimization technique for time domain applications was presented. The technique used a modification of Powell's [46] algorithm. The chapter presented Powell's original algorithm, the problems associated with it and the modifications done to overcome these problems. The modified Powell's algorithm was applied to the determination of the optimum lossy line parameters. A constraint, non-linear, least squares objective function was defined to suit the application. The proposed optimization technique has been implemented in a software package called the Time Domain Material Characterization (TDMC). The chapter provided an information flow chart about the package and presented examples about typical input and output file structures. Finally, two verification cases have been presented about the validity of the software package and the error bounds of the optimization technique.
CHAPTER 6

EXPERIMENTAL RESULTS ON MATERIAL CHARACTERIZATION USING A STRIPLINE GEOMETRY

6.1 INTRODUCTION

This chapter presents experimental results on material characterization using a stripline geometry. In order to verify the validity of both the simulation technique and the TDMC software package, some experiments were performed on well characterized samples constructed from Rogers RT/Duroid®. The results on this verification are presented in section 6.2 [35]. The remaining part of the chapter is devoted to obtain experimental data (measurements) on Du Pont polymers and conventional thick film pastes.

Section 6.3 presents results on Pyralux and WA-Adhesive. These materials are Du Pont polymer products which are used in the construction of multilayer electronic circuits. The results on Du Pont thick film dielectric and conductor pastes are presented in section 6.4. Finally, a summary and conclusions is presented in section 6.5.

6.2 EXPERIMENTAL VERIFICATION OF THE TDMC SOFTWARE PACKAGE ON KNOWN SAMPLES

6.2.1 Fabrication of Samples Using RT/Duroid® Composite Laminates

The Rogers RT/Duroid composite laminates are commercially available with copper cladding on both sides. In order to use these laminates in stripline construction, certain
procedures have to be followed. Each stripline is constructed using the following steps [56]:

1. Cut two laminates in 3"x1" dimensions.

2. Design the center conductor pattern using a computer aided design (CAD) package for hybrid microcircuit applications.

3. Using a 10:1 expansion ratio, scribe the pattern of step 2 on a master artwork using a diamond-tip plotter.

4. Following common photographic process, reduce the patterns to their final, desired size on negative photoplates.

5. Coat one side of each laminate with positive photoresist by spinning at 2500 rpm for 30 seconds.

6. Prebake at 90 °C for 15 minutes.

7. Expose the side of one of the laminates having the photoresist with Ultra-Violet (UV) through the negative photoplate for 8 seconds.

8. Develop and wash the exposed regions.

9. Post bake for 15 minutes at 110 °C.

10. Etch the copper in the exposed regions of the photoresist by using ferric chloride based etching solution.

Using the above procedure, one laminate was fully etched on one side to form the top conductor and the first dielectric layer and the second laminate was partially etched on one side to form the center conductor, the second dielectric layer and the bottom conductor of the stripline. The stripline was then formed by putting the two laminates together and clamping them with a vice to avoid air gap formation.
6.2.2 Results on Stripline Constructed from Rogers RT/Duroid 5880®

Before discussing the experimental results on a stripline constructed from Rogers RT/Duroid 5880® composite laminates, we shall present the manufacturer specifications for the dielectric constant and the loss tangent. These specifications are listed at a measurement frequency of 10 GHz and are given as follows:

\[ \epsilon'_r = 2.20 \pm 0.02 \]
\[ \tan \delta_e = 9.0 \times 10^{-4} \]

Using the procedure explained in section 6.2.1, a stripline was constructed from Rogers RT/Duroid 5880® with the following dimensions:

\[ w = 25 \text{ mils}, \ b = 125 \text{ mils}, \ t = 2.8 \text{ mils}, \ \ell = 3'' \]

Using the experimental set up of Figure 4.3.1, the necessary TDR and TDT response and reference waveforms were obtained. These waveforms were then given to the TDMC software package and the optimal line parameters for the adapters and the stripline under test were obtained.

The optimal line parameters for adapters A and B are given by

\[ \tau_1 = \tau_3 = 37.59793 \text{ ps} \]

and

\[ R_1 = R_3 = 50.46455 \Omega \]

Furthermore, the optimal line parameters for the stripline under test were obtained to be as follows:
\( L_o = 0.3907551 \mu_o \text{ H/m}, \)

\( C(\omega) = \left[ 5.600663928 \epsilon_o + 6.079899 \times 10^{-7} \omega^{-0.6} \sin (0.19991\pi) \right] \text{ F/m}, \)

\( R_{dc} = 1.464661 \times 10^{-5} \text{ } \Omega/\text{m}, \)

\( Z_{ak}(\omega) = 1.557315 \times 10^{-5} (j\omega)^{0.6} \text{ } \Omega/\text{m}, \)

\( G(\omega) = \left[ 6.079899 \times 10^{-7} \omega^{0.4} \cos (0.19991\pi) \right] \text{ S/m}, \)

and

\( G_{dc} = 1.05882 \times 10^{-8} \text{ S/m}. \)

The comparison between the experimental and the simulated TDR response waveforms is shown in Figure 6.2.1. This figure shows that the stripline region, extending approximately from 0.65 ns to 1.45 ns, is matched only in an average sense. The inductive discontinuities at the beginning as well as at the end of the stripline response waveform are due to the solder used in connecting the End Launch Jack (ELJ) coaxial to planar adapters (connectors). The ELJ connectors were used instead of the Cascade Microtech adapters, because the sample thickness was too thick. On the other hand, the comparison between the experimental and the simulated TDT response waveforms are shown in Figure 6.2.2. In this figure, a perfect match can be observed between the two responses.

As an outcome of the TDMC software package, the plot of the real part of the complex permittivity, \( \epsilon'_r(\omega) \) versus frequency is shown in Figure 6.2.3. From this figure, it can be observed that, the value of \( \epsilon'_r(\omega) \) at 10 GHz is approximately 2.193 and its limiting value is approximately 2.19. Comparing these results with the manufacturer specifications, the percent error is given by

\[
\% \text{ error in } \epsilon'_r(\omega) \text{ at 10 GHz} = \frac{|2.20 - 2.193|}{2.20} = 0.318.
\]

The above result shows that the proposed technique and the TDMC software package are very
Figure 6.2.1 Comparison between the experimental and the simulated TDR response waveforms for the stripline constructed from RT/Duroid 5880®.
Figure 6.2.2  Comparison between the experimental and the simulated TDT response waveforms for the stripline constructed from RT/Duroid 5880®.
efficient and reliable tools in determining \( \varepsilon'_r(\omega) \). On the other hand, the plot of the loss tangent, \( \tan \delta'_c(\omega) \) versus frequency is shown in Figure 6.2.4. From this figure, the value of \( \tan \delta'_c(\omega) \) at 10 GHz is obtained to be approximately \( 2.93 \times 10^{-3} \). Comparing this result with the manufacturer specifications, it can be observed that the proposed technique has overestimated the value of the loss tangent. This is mainly due to the imperfect samples used in the measurements. Hence, it is more logical to name the simulated loss tangent as the apparent loss tangent.

6.2.3 Results on Stripline Constructed from Rogers RT/Duroid 5870®

The manufacturer specifications for the dielectric constant and the loss tangent of RT/Duroid 5870® at a measurement frequency of 10 GHz are given as follows:

\[
\begin{align*}
\varepsilon'_r & = 2.33 \pm 0.02 \\
\tan \delta'_c & = 1.2 \times 10^{-3}
\end{align*}
\]

Using the procedure explained in section 6.2.1, a stripline was constructed from Rogers RT/Duroid 5870® with the following dimensions:

\( w = 50 \text{ mils}, \quad b = 257 \text{ mils}, \quad t = 0.7 \text{ mils}, \quad \ell = 3'' \).

Using the experimental set up of Figure 4.3.1, the necessary TDR and TDT response and reference waveforms were obtained. Again, due to large sample thickness, the ELJ connectors were used instead of the Cascade Microtech adapters. These waveforms were then given to the TDMC software package and the optimal line parameters for the adapters and the stripline under test were obtained.

The optimal line parameters for adapters A and B are given by
Figure 6.2.3  Plot of $\varepsilon'_r(\omega)$ versus frequency for the stripline constructed from RT/Duroid 5880®.
Figure 6.2.4  Plot of $\tan \delta_c(\omega)$ versus frequency for the stripline constructed from RT/Duroid 5880®.
\[ \tau_1 = \tau_3 = 37.95384 \text{ ps}, \]

and

\[ R_1 = R_3 = 51.13024 \Omega. \]

Furthermore, the optimal line parameters for the stripline under test were obtained to be as follows:

\[ L_o = 0.4061525 \mu \Omega \text{ H/m}, \]

\[ C(\omega) = \left[ 5.808343 \varepsilon_0 + 8.9605998 \times 10^{-8} \omega^{-0.592} \sin (0.20389\pi) \right] \text{ F/m}, \]

\[ R_{dc} = 4.055514 \times 10^{-9} \Omega/m, \]

\[ Z_{se}(\omega) = 1.700333 \times 10^{-11} (j\omega)^{0.389} \Omega/m, \]

\[ G(\omega) = \left[ 8.960598 \times 10^{-8} \omega^{0.408} \cos (0.20389\pi) \right] \text{ S/m}, \]

and

\[ G_{dc} = 1.61284419 \times 10^{-9} \text{ S/m}. \]

The comparison between the experimental and the simulated TDR and TDT responses are shown in Figures 6.2.5 and 6.2.6, respectively. In both figures, a very good match is observed between the experimental and the simulated responses.

As an outcome of the TDMC software package, the plot of the real part of the complex permittivity, \( \varepsilon'_r(\omega) \) versus frequency is shown in Figure 6.2.7. From this figure, it can be observed that, the value of \( \varepsilon'_r(\omega) \) at 10 GHz is approximately 2.36. The limiting value of \( \varepsilon'_r(\omega) \) is approximately 2.36 too. Comparing these results with the manufacturer specifications, the percent error is given by

\[ \% \text{ error in } \varepsilon'_r(\omega) \text{ at 10 GHz} = \left| \frac{2.33 - 2.36}{2.33} \right| = 1.288 \]
Figure 6.2.5  Comparison between the experimental and the simulated TDR response waveforms for the stripline constructed from RT/Duroid 5870®.
Figure 6.2.6  Comparison between the experimental and the simulated TDT response waveforms for the stripline constructed from RT/Duroid 5870®.
On the other hand, the plot of the loss tangent, \(\tan \delta_e(\omega)\) versus frequency is shown in Figure 6.2.8. From this figure, the value of \(\tan \delta_e(\omega)\) at 10 GHz is obtained to be approximately 5.625x10\(^{-4}\). Comparing this result with the manufacturer specifications, it can be observed that the proposed technique has underestimated the value of the loss tangent at this frequency. Again, this was due to the imperfect samples used in the experiment. The above experimental results also show that a loss tangent of 1.2x10\(^{-3}\) is obtained at 2.75 GHz.

6.3 EXPERIMENTAL RESULTS ON DU PONT POLYMERS\(^\circledR\)

6.3.1 Fabrication of Samples Using Du Pont Pyralux\(^\circledR\)

The fabrication of striplines using Du Pont Pyralux\(^\circledR\) is somehow different from those using RT/Duroid\(^\circledR\) composite laminates. Du Pont Pyralux\(^\circledR\) is a polymer material that is obtained from successive lamination of Du Pont Kapton\(^\circledR\) and Du Pont WA-Adhesive\(^\circledR\) polymer materials. The lamination is performed by using a soft-pad system. Du Pont manufactures Pyralux\(^\circledR\) in several forms; some with copper cladding and some without. The structural details of the stripline constructed from Du Pont Pyralux\(^\circledR\) is shown in Figure 6.3.1. The etching of the center conductor is performed following the same steps discussed in section 6.2.1 except the samples are cut in 3"x3" dimensions. The lamination of the samples are performed using the following steps [57]:

1. Punch out any window regions in the appropriate layers.
2. Clean all parts with a lint-free (chamois) cloth.
3. Dry all cleaned parts in the oven for approximately one hour.
4. Turn on the press equipment, and set the platen temperature to 350 °F.
5. If necessary, blank out the soft-pad system.
Figure 6.2.7  Plot of $\varepsilon_r'(\omega)$ versus frequency for the stripline constructed from RT/Duroid 5870®.
Figure 6.2.8  Plot of $\tan \delta_\varepsilon(\omega)$ versus frequency for the stripline constructed from RT/Duroid 5870°.
6. Remove parts from oven and store under laminar flow hood until cooled to room temperature.

7. At room temperature, stack the parts in the appropriate order in the laminating die.

8. Place the die in heated press and increase the pressure to 335 psi.

9. Maintain both heat and pressure for approximately one hour duration.

10. Turn off the heat, maintaining the exerted pressure in order to prevent warping of the sample during the cooling process.

11. Increase the flow of cooling water through the platens in a gradual manner.

12. Release the pressure as soon as the platen temperature approaches room temperature.

13. Remove the die from the press.

14. Remove laminated sample from the die.

In step 7, the stacking up of parts is done in the following order:

   a. Laminating die (top).
   b. 200A FEP film.
   c. Bond paper.
   d. Clear vinyl sheet.
   e. Bond paper.
   f. 200A FEP film.
   g. Sample to be laminated stacked up in the same order as depicted in Figure 6.3.1.
   h. Laminating die (bottom).

The soft-pad system is composed of the materials shown in steps (b) through (f). The window areas of the sample to be laminated is put toward the soft-pad layers. If the sample to be laminated has windows at the bottom side, then the soft-pad system has to be repeated for the bottom part too.
6.3.2 Results on Stripline Constructed from Du Pont Pyralux®

Using the procedure explained in section 6.3.1, a stripline was constructed from Du Pont Pyralux® with the following dimensions:

\[ w = 50 \text{ mils}, \quad b = 8.8 \text{ mils}, \quad t = 2.8 \text{ mils}, \quad \ell = 2.835". \]

Using the experimental set up of Figure 4.3.1, the necessary TDR and TDT response and reference waveforms were obtained at a room temperature of 70°F and a humidity of 70%. These waveforms were then given to the TDMC software package and the optimal line parameters for the adapters and the stripline under test were obtained.

The optimal line parameters for adapters A and B are given by

\[ \tau_1 = \tau_3 = 0.1233966 \text{ ns}, \]

and

\[ R_1 = R_3 = 48.53422 \Omega. \]

Furthermore, the optimal line parameters for the stripline under test were obtained to be as follows:

\[ L_o = 3.439252 \times 10^{-2} \mu_0 \text{ H/m}, \]

\[ C(\omega) = \left[ 84.8227463 \varepsilon_\infty + 1.658418 \times 10^{-4} \omega^{0.692} \sin (0.15414 \pi) \right] \text{ F/m}, \]

\[ R_{dc} = 4.083377 \times 10^{-6} \Omega/\text{m}, \]

\[ Z_{sk}(\omega) = 2.51723 \times 10^{-8} (j\omega)^{0.876} \Omega/\text{m}, \]

\[ G(\omega) = \left[ 1.658418 \times 10^{-4} \omega^{0.308} \cos (0.15414 \pi) \right] \text{ S/m}, \]

and
Figure 6.3.1 Structural details for the stripline constructed from Du Pont Pyralux®.
\[ G_{dc} = 1.2474 \times 10^{-6} \text{ S/m}. \]

The comparison between the experimental and the simulated TDR and TDT responses are shown in Figures 6.3.2 and 6.3.3, respectively. Both figures display a perfect match between the simulated and experimental responses. The plot of the real part of the complex permittivity, \( \varepsilon'_r(\omega) \) versus frequency is shown in Figure 6.3.4. From this figure, it can be observed that, the limiting value of \( \varepsilon'_r(\omega) \) is approximately 2.92. On the other hand, the plot of the loss tangent, \( \tan \delta_c(\omega) \) is shown in Figure 6.3.5. This figure shows that the value of \( \tan \delta_c(\omega) \) at 2 GHz is approximately 0.02.

### 6.3.3 Fabrication of Samples Using Du Pont WA-Adhesive®

The fabrication of striplines using Du Pont WA-Adhesive® is exactly the same as the those using Du Pont Pyralux®. Du Pont WA-Adhesive® is a polymer material used mainly as an adhesive in fabricating polymers like Pyralux®. Du Pont manufactures WA-Adhesive® in the form of thin film layers having a thickness of 1 mil and a protective layer (or film). The structural details of the stripline constructed from Du Pont WA-Adhesive® is shown in Figure 6.3.6.

### 6.3.4 Results on Stripline Constructed from Du Pont WA-Adhesive®

Using the procedure explained in section 6.3.1, a stripline was constructed from Du Pont WA-Adhesive® with the following dimensions:

\[ w = 25 \text{ mils}, \ b = 3.4 \text{ mils}, \ t = 1.4 \text{ mils}, \ \ell = 3.0'' \]

The necessary TDR and TDT response and reference waveforms were obtained at a room temperature of 66 °F and a humidity of 52% using the experimental set up of Figure 4.3.1. These
Figure 6.3.2  Comparison between the experimental and the simulated TDR response waveforms for the stripline constructed from Du Pont Pyralux®.
Figure 6.3.3 Comparison between the experimental and the simulated TDT response waveforms for the stripline constructed from Du Pont Pyralux®.
Figure 6.3.4 Plot of $\varepsilon_r'(\omega)$ versus frequency for the stripline constructed from Du Pont Pyralux®.
Figure 6.3.5  Plot of $\tan \delta_c(\omega)$ versus frequency for the stripline constructed from Du Pont Pyralux®.
Figure 6.3.6  Structural details for the stripline constructed from Du Pont WA-Adhesive®.
waveforms were then used in the optimization process and the optimal line parameters for the adapters and the stripline under test were obtained.

The optimal line parameters for adapters A and B are given by

$$ \tau_1 = \tau_3 = 0.1173966 \text{ ns}, $$

and

$$ R_1 = R_3 = 48.82979 \text{ } \Omega. $$

Furthermore, the optimal line parameters for the stripline under test were obtained to be as follows:

$$ L_o = 1.898656 \times 10^{-2} \text{ } \mu_0 \text{ H/m}, $$

$$ C(\omega) = \left[ 145.4553115 \varepsilon_o + 1.532571 \times 10^{-4} \omega^{-0.7} \sin (0.1542135\pi) \right] \text{ F/m}, $$

$$ R_{dc} = 5.361177 \times 10^{-5} \text{ } \Omega/m, $$

$$ Z_{sk}(\omega) = 5.370947 \times 10^{-6} (j\omega)^{0.683} \text{ } \Omega/m, $$

$$ G(\omega) = \left[ 1.532571 \times 10^{-4} \omega^{0.3} \cos (0.1542135\pi) \right] \text{ S/m}, $$

and

$$ G_{dc} = 0.1949 \text{ } \text{ S/m}. $$

The comparison between the experimental and the simulated TDR and TDT response waveforms are shown in Figures 6.3.7 and 6.3.8, respectively. In both figures, a very good match can be observed between the experimental and simulated responses.

The plot of the real part of the complex permittivity, $\varepsilon'_r(\omega)$ versus frequency is shown in Figure 6.3.9. From this figure, it can be observed that, the limiting value of $\varepsilon'_r(\omega)$ is approximately 2.76. On the other hand, the plot of the loss tangent, $\tan \delta'_r(\omega)$ is shown in Figure 6.3.10. This figure shows that the value of $\tan \delta'_r(\omega)$ at 2 GHz is 0.01085.
Figure 6.3.7  Comparison between the experimental and the simulated TDR response waveforms for the stripline constructed from Du Pont WA-Adhesive®.
Figure 6.3.8  Comparison between the experimental and the simulated TDT response waveforms for the stripline constructed from Du Pont WA-Adhesive®.
Figure 6.3.9  Plot of $\varepsilon'_f(\omega)$ versus frequency for the stripline constructed from Du Pont WA-Adhesive®.
Figure 6.3.10  Plot of $\tan \delta_c(\omega)$ versus frequency for the stripline constructed from Du Pont WA-Adhesive®.
6.4 EXPERIMENTAL RESULTS ON DU PONT THICK FILM PASTES®

6.4.1 Fabrication of Samples Using Du Pont Thick Film Pastes®

The structural details for fabricating striplines using Du Pont thick film pastes® is shown in Figure 6.4.1. The striplines are fabricated on a 96% Alumina substrate having a thickness of 25 mils. The following steps explain how the processing is accomplished.

1. Design the stripline pattern using a computer aided package for hybrid microcircuit applications.

2. Using a 10:1 expansion ratio, print the pattern of step 1 on a rubylith (master artwork).

3. Following common photographic process, reduce the patterns to their original size on negative photoplates.

4. Clean and wet the screens having a screen mesh size of 325 count.

5. Apply hybrid screen emulsion to the screens and dry them.

6. Place the negative photoplates on the screens with the mate side towards the emulsion and expose for 6 minutes under Ultra Violet (UV) light.

7. Remove the negative photoplates from the screens and wash the screen in warm water until all the emulsion from the unexposed areas is removed.

8. Air dry the screens.

9. Print-Dry-Fire first metallization layer to form the bottom ground plane.

10. Print-Dry-Fire first dielectric layer.

11. Print-Dry-Fire second dielectric layer.

12. Print-Dry-Fire second (inner) metallization layer to form the center conductor.

13. Print-Dry-Fire third dielectric layer.

14. Print-Dry-Fire fourth dielectric layer.

15. Print-Dry-Fire top metallization layer to form the top ground plane.
Figure 6.4.1  Structural details for the stripline constructed from Du Pont Thick-Film Pastes®.
The printing process is accomplished using a conventional thick film printer. The drying process is done in a Blue M® oven at 150 °C for 10 minutes, and firing takes place in a multizone firing furnace with 10 minutes at a peak firing temperature of 850 °C for one hour total duration.

6.4.2 Results on Stripline Constructed from Du Pont 5704® Dielectric and Du Pont 6134® Conductor Pastes

Using the procedure explained in section 6.4.1, a stripline was constructed from Du Pont 5704® dielectric and Du Pont 6134® conductor pastes with the following dimensions:

\[ w = 25 \text{ mils}, \ b = 4.86 \text{ mils}, \ t = 0.453 \text{ mils}, \ \ell = 3.0''. \]

The necessary TDR and TDT response and reference waveforms were obtained using the experimental set up of Figure 4.3.1. These waveforms were then given to the TDMC software package and the optimal line parameters for the adapters and the stripline under test were obtained.

The optimal line parameters for adapters A and B are given by

\[ \tau_1 = \tau_3 = 45.21131 \text{ ps}, \]

and

\[ R_1 = R_3 = 49.80339 \Omega. \]

Furthermore, the optimal line parameters for the stripline under test were obtained to be as follows:

\[ L_o = 3.256824 \times 10^{-2} \mu_0 \text{ H/m}, \]
\[ C(\omega) = \left[ 298.9666 \varepsilon_0 + 9.197860 \times 10^{-3} \omega^{-0.718} \sin(0.14075\pi) \right] \text{ F/m}, \]
\[ R_{dc} = 1.307512 \times 10^{-5} \text{ \Omega/m}, \]
\[ Z_{sk}(\omega) = 8.329005 \times 10^{-9} (j\omega)^{0.138} \text{ \Omega/m}, \]
\[ G(\omega) = \left[ 9.197860 \times 10^{-3} \omega^{0.282} \cos(0.14075\pi) \right] \text{ S/m}, \]

and
\[ G_{dc} = 6.5147057 \times 10^{-2} \text{ S/m}. \]

The comparison between the experimental and the simulated TDR responses is shown in Figure 6.4.2. The discontinuities that can be observed in this figure between 0.9 ns to 1.2 ns are due to the microstrip region. Since, these discontinuities were not modeled separately, a proper match could not be obtained. A more efficient approach requires the use of additional lossy lines for modeling this region.

The plot of the real part of the complex permittivity, \( \varepsilon'_r(\omega) \), versus frequency is shown in Figure 6.4.3. From this figure, it can be observed that, the limiting value of \( \varepsilon'_r(\omega) \) is approximately 9.737. On the other hand, the plot of the loss tangent, \( \tan \delta_\varepsilon(\omega) \) is shown in Figure 6.4.4. This figure shows that the value of \( \tan \delta_\varepsilon(\omega) \) at 2 GHz is 0.1609. The high loss is mainly due to the lossy conductor used in the fabrication process. The Du Pont 6134® conductor paste is manufactured using Silver Palladium (AgPd) conductor, which is known to be a very lossy material.

### 6.4.3 Results on Stripline Constructed from Du Pont 5704® Dielectric and Du Pont 6160® Conductor Pastes

Using the procedure explained in section 6.4.1, a stripline was constructed from Du Pont 5704® dielectric and Du Pont 6160® conductor pastes with the following dimensions:
Figure 6.4.2  Comparison between the experimental and the simulated TDR response waveforms for the stripline constructed from Du Pont 5704® dielectric and Du Pont 6134® conductor pastes.
Figure 6.4.3  Plot of $\varepsilon'_e(\omega)$ versus frequency for the stripline constructed from Du Pont 5704\textsuperscript{®} dielectric and Du Pont 6134\textsuperscript{®} conductor pastes.
Figure 6.4.4  Plot of $\tan \delta_e(\omega)$ versus frequency for the stripline constructed from Du Pont 5704® dielectric and Du Pont 6134® conductor pastes.
\( w = 10 \text{ mils}, \ b = 4.88 \text{ mils}, \ t = 0.472 \text{ mils}, \ \ell = 3.0'' \).

Using the experimental set up of Figure 4.3.1, the necessary TDR and TDT response and reference waveforms were obtained. These waveforms were then given to the TLEM software package and the optimal line parameters for the adapters and the stripline under test were obtained.

The optimal line parameters for adapters A and B are given by

\[
\tau_1 = \tau_3 = 37.87401 \text{ ps},
\]

and

\[
R_1 = R_3 = 49.18436 \Omega.
\]

Furthermore, the optimal line parameters for the stripline under test were obtained as follows:

\[
L_o = 5.588595 \times 10^{-2} \mu_\text{o} \text{ H/m},
\]

\[
C(\omega) = \left[164.9413135 \epsilon_\text{o} + 5.460635 \times 10^{-5} \omega^{-0.594} \sin (0.202819\pi)\right] \text{ F/m},
\]

\[
R_{dc} = 0.1368968 \Omega/\text{m},
\]

\[
Z_{sk}(\omega) = 3.706049 \times 10^{-4} (j\omega)^{0.4965} \Omega/\text{m},
\]

\[
G(\omega) = \left[5.460635 \times 10^{-8} \omega^{0.4056} \cos (0.202819\pi)\right] \text{ S/m},
\]

and

\[
G_{dc} = 8.9779542 \times 10^{-8} \text{ S/m}.
\]

The comparison between the experimental and the simulated TDR response waveforms is shown in Figure 6.4.5. Again, the discontinuities between 0.9 ns to 1.2 ns are due to the microstrip region and are not properly matched.
The plot of the real part of the complex permittivity, $\varepsilon'_r(\omega)$ versus frequency is shown in Figure 6.4.6. From this figure, it can be observed that, the limiting value of $\varepsilon'_r(\omega)$ is approximately 9.22. On the other hand, the plot of the loss tangent, $\tan \delta_r(\omega)$ is shown in Figure 6.4.7. This figure shows that the value of $\tan \delta_r(\omega)$ at 2 GHz is 0.0292. The Du Pont 6160\textsuperscript{®} conductor paste is manufactured using Silver (Ag) conductor. The low loss tangent shows that, this material is not as lossy as the Silver Palladium conductor paste.

### 6.4.4 Results on Stripline Constructed from Du Pont 5704\textsuperscript{®} Dielectric and Du Pont 5715\textsuperscript{®} Conductor Pastes

Again, using the procedure explained in section 6.4.1, a stripline was constructed from Du Pont 5704\textsuperscript{®} dielectric and Du Pont 5715\textsuperscript{®} conductor pastes with the following dimensions:

$$w = 25 \text{ mils}, \quad b = 4.65 \text{ mils}, \quad t = 0.31 \text{ mils}, \quad \ell = 3.0''.$$  

Using the experimental set up of Figure 4.3.1, the necessary TDR and TDT response and reference waveforms were obtained. These waveforms were then given to the TDMC software package and the optimal line parameters for the adapters and the stripline under test were obtained.

The optimal line parameters for adapters A and B are given by

$$\tau_1 = \tau_3 = 53.94639 \text{ ps},$$

and

$$R_1 = R_3 = 52.14774 \Omega.$$  

Furthermore, the optimal line parameters for the stripline under test were obtained to be as
Figure 6.4.5  Comparison between the experimental and the simulated TDR response waveforms for the striplia constructed from Du Pont 5704® dielectric and Du Pont 6160® conductor pastes.
Figure 6.4.6  Plot of $\varepsilon_r'(\omega)$ versus frequency for the stripline constructed from Du Pont 5704® dielectric and Du Pont 6134® conductor pastes.
Figure 6.4.7  Plot of $\tan \delta_e(\omega)$ versus frequency for the stripline constructed from Du Pont 5704® dielectric and Du Pont 6134® conductor pastes.
follows:
\[ I_o = 4.766912 \times 10^{-2} \mu_0 \text{ H/m,} \]
\[ C(\omega) = \left[ 171.825722 \varepsilon_o + 1.122615 \times 10^{-3} \omega^{-0.6} \sin(0.14424\pi) \right] \text{ F/m,} \]
\[ R_{dc} = 3.834475 \times 10^{-7} \text{ \Omega/m,} \]
\[ Z_{sk}(\omega) = 1.412880 \times 10^{-7} (j\omega)^{0.783} \text{ \Omega/m,} \]
\[ G(\omega) = \left[ 1.122615 \times 10^{-3} \omega^{0.2884} \cos(0.14424\pi) \right] \text{ S/m,} \]
and
\[ G_{dc} = 7.687596 \times 10^{-2} \text{ S/m.} \]

The comparison between the experimental and the simulated TDR response waveforms is shown in Figure 6.4.8. From this figure, several discontinuities can be observed along the line. This is due to the improper fabrication and processing that might have occurred during the manufacturing process.

The plot of the real part of the complex permittivity, \( \epsilon'_r(\omega) \) versus frequency is shown in Figure 6.4.9. From this figure, it can be observed that, the limiting value of \( \epsilon'_r(\omega) \) is approximately 8.19. On the other hand, the plot of the loss tangent, \( \tan \delta(\omega) \) is shown in Figure 6.4.10. This figure shows that the value of \( \tan \delta(\omega) \) at 2 GHz is 0.0426. The Du Pont 5715\textsuperscript{©} conductor paste is manufactured using gold (Au) conductor. The loss tangent shows that, this material is more lossy than Silver, but less lossy than the Silver Palladium conductor pastes.
Figure 6.4.8  Comparison between the experimental and the simulated TDB response waveforms for the stripline constructed from Du Pont 5704® dielectric and Du Pont 5715® conductor pastes.
Figure 6.4.9  Plot of $\varepsilon_r(\omega)$ versus frequency for the stripline constructed from Du Pont 5704© dielectric and Du Pont 5715© conductor pastes.
Figure 6.4.10  Plot of $\tan \delta_e(\omega)$ versus frequency for the stripline constructed from Du Pont 5704® dielectric and Du Pont 5715® conductor pastes.
6.5 SUMMARY AND CONCLUSIONS

The first part of this chapter presented experimental results on commercially available materials. Comparison of these results with the manufacturer's specifications indicates that, the error in the real part of permittivity, $\epsilon'_r(\omega)$, was found to be less than 1.3% for both samples. On the other hand, there was some discrepancy in the determination of the loss tangent. This discrepancy was mainly due to imperfect samples used in the experiments.

In the second part of this chapter, results on Du Pont Polymer® materials were presented. For Du Pont Pyralux®, the limiting value of $\epsilon'_r(\omega)$ and the value of $\tan \delta'_e(\omega)$ at a frequency of 2 GHz were found to be 2.92 and 0.02, respectively. Furthermore, for Du Pont WA-Adhesive® the limiting value of $\epsilon'_r(\omega)$ and the value of $\tan \delta'_e(\omega)$ at a frequency of 2 GHz were found to be 2.76 and 0.01, respectively.

In the last part of this chapter, results on Du Pont thick-film® materials were presented. Theoretically, the dielectric constant and the loss tangent should be the same for all the thick-film samples, since the same dielectric paste was used in their construction. However, the results show that both the dielectric constant and the loss tangent change as a consequence of both the conductor and interface properties. For the stripline constructed using Du Pont 6134® conductor paste, the limiting value of $\epsilon'_r(\omega)$ and the value of $\tan \delta'_e(\omega)$ at a frequency of 2 GHz were found to be 9.737 and 0.1609, respectively. On the other hand, for the stripline constructed using Du Pont 6160® conductor paste, the limiting value of $\epsilon'_r(\omega)$ and the value of $\tan \delta'_e(\omega)$ at a frequency of 2 GHz were found to be 9.22 and 0.0292, respectively. Finally, for the stripline constructed using Du Pont 5715® conductor paste, the limiting value of $\epsilon'_r(\omega)$ and the value of $\tan \delta'_e(\omega)$ at a frequency of 2 GHz were found to be 8.19 and 0.04926, respectively. The above results show that, the highest loss and surface roughness occur with Du Pont Silver-Palladium 6134® conductor paste and the least with Du Pont Silver 6160® conductor paste.
CHAPTER 7

A PRACTICAL TIME DOMAIN NETWORK SYNTHESIS
TECHNIQUE

7.1 INTRODUCTION

In chapter 4, we have presented a new technique for material characterization using a stripline geometry. Using the proposed technique, the coaxial to planar adapters and the stripline under test were modeled by lossy transmission lines. In certain applications, it might not be possible to model the stripline under test accurately with a single transmission line. Also, in other applications it is desirable to obtain an equivalent network model for a device under test based on its time domain reflectometry (TDR) response waveform. The technique proposed in this chapter is geared towards solving both problems.

Time domain synthesis of electrical networks has attracted the attention of several researchers for almost five decades. In fact, the word "time domain synthesis" is a misnomer, because, the actual synthesis is usually performed in the frequency domain. However, the calculation of the error function in the optimization process is mainly accomplished in the time domain. In time domain synthesis, one is usually given an excitation and a response as a function of time and is asked to determine the electrical network whose response is closest to the given one.

Most of the research done in this regard was based on the assumption that the impulse response $h(t)$ of the given system can be easily determined from the given input and output relations. In practice, this is not always true. In fact, in many practical cases the exact determination of the impulse response is impossible. Hence, one desires to determine the

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equivalent network for a given time domain response without the need of calculating the impulse response. This is possible either by assuming that the functional behavior of the impulse response or the type of the network is known beforehand.

In this chapter, a practical time domain network synthesis technique is presented. The technique uses a lossy transmission line synthesis approach to obtain an equivalent network model for a microwave device under test excited with a time domain step waveform. The response waveform acquired from a time domain network analyzer is divided into \( N \) equal time intervals. Each interval is synthesized by a lossy transmission line segment. The parameters of each line are then determined by using the optimization algorithm presented in chapter 5.

The chapter is divided into several sections. Section 7.2 describes the time domain synthesis problem and discusses the proposed concept and approach of a practical solution. The derivation of the S-parameters and the optimization techniques for the proposed time domain synthesis problem is presented in section 7.3. The description and verification of a software package written for this purpose is presented in section 7.4. Finally, a summary and conclusions is presented in section 7.5.

7.2 THE TIME DOMAIN SYNTHESIS PROBLEM AND THE PROPOSED SOLUTION

7.2.1 Description of the Time Domain Synthesis Problem

A time-domain synthesis problem is usually characterized by an excitation and a response, both of which are specified in terms of time [19]. The synthesis problem in the time domain consists of finding a function \( h(t) \), the unit-impulse response of a network, whose Laplace transform \( H(s) \) (the network function) fulfills the necessary conditions of physical realizability
[20]. Once $H(s)$ is computed, the conventional network synthesis techniques can be used for obtaining an equivalent network.

A major problem with all the approaches presented in the literature is the assumption that $h(t)$ is known analytically [20]-[22]. This requires $h(t)$ to be computed using other techniques such as deconvolution. If one uses deconvolution or other numerical techniques for the computation of the impulse response, then an extra effort has to be spent for finding a suitable analytical expression. Even after finding a suitable analytical expression for $h(t)$, it may or may not be suitable for proceeding with the synthesis steps. If the analytical expression is not suitable for this purpose, one that is suitable has first to be found.

To avoid all these problems, a novel time domain synthesis approach has been developed. The synthesis technique uses a cascade of lossy transmission lines for synthesizing the time domain reflectometry response waveform of a microwave network due to an arbitrary step excitation. The concept and approach of this technique is described in the following section.

### 7.2.2 Concept and Approach of the Proposed Time Domain Synthesis Technique

The concept of generalized time domain synthesis has evolved from experience with time domain modeling using the Modified Transient Circuit Analysis Package (MTCAP) [24]. The modeling technique that utilizes the MTCAP simulation package is based on acquiring the TDR response waveform for the device under test (DUT) at its interface port to the coaxial line. A reference waveform is acquired by replacing the DUT by a coaxial (standard) short network termination. All waveforms are to be acquired using a computer interfaced wideband sampling oscilloscope. From the TDR response waveform, as well as any knowledge of the physical nature and structure detail of the DUT, a preliminary (rough) network model for the DUT is devised. The reference waveform, which is obtained by negating the reflection from the standard short together with the preliminary model are used to simulate the experimental TDR set up of the
DUT. In this simulation, the reference waveform is used to excite the network model and the reflection is computed as the simulated response of the DUT. Next, the simulated TDR waveform is compared to the experimental TDR waveform. The component values of the devised model are adjusted in an iterative manner until the simulated response matches the measured TDR response. In this iterative process, each discontinuity is dealt with one at a time, in the order of their physical existence away from the launch end. Once a component value due to a discontinuity is optimized, any variation in the model component values of later discontinuities is not expected to affect the simulated response waveform of earlier times. When all the component values are adjusted such that the simulated response matches the experimental one, the resulting network provides an equivalent network for the DUT.

Experience with MTCAP has also shown that, an arbitrary time domain discontinuity can be modeled using a cascade of lossless transmission lines provided that the loss mechanism involved in the DUT is not too large. This experience has led the author to the conclusion that an arbitrary TDR response waveform can be modeled by using a cascade of lossy transmission lines. Since the idea is to have an automated approach rather than one like MTCAP, the technique has to use a suitable optimization technique. The optimization technique proposed in chapter 5 is directly applicable to solve this problem. For this optimization technique a priori model has to be assumed for the characteristic impedance and the propagation function for each transmission line. This a priori model was explicitly defined in chapter 5 and is restated below.

\[
\gamma(\omega) = \left\{ K_2 + j\omega \left( L_e + K_1 \omega^{-1} \sin \left(\frac{\pi P}{2}\right) + K_1 \omega \cos \left(\frac{\pi P}{2}\right) \right) \right\} \cdot \left[ K_4 + j\omega \left( C_o + K_3 \omega^{-1} \sin \left(\frac{\pi Q}{2}\right) + K_3 \omega \cos \left(\frac{\pi Q}{2}\right) \right) \right]^\frac{1}{2},
\]  

(7.2.1)

\[
Z(\omega) = \frac{K_2 + j\omega \left( L_e + K_1 \omega^{-1} \sin \left(\frac{\pi P}{2}\right) + K_1 \omega \cos \left(\frac{\pi P}{2}\right) \right)}{\sqrt{K_4 + j\omega \left( C_o + K_3 \omega^{-1} \sin \left(\frac{\pi Q}{2}\right) + K_3 \omega \cos \left(\frac{\pi Q}{2}\right) \right)}}.
\]

(7.2.2)
In (7.2.1) and (7.2.2), the parameters of the lossy line are given by

\[ L_c = A_1 \mu_0, \quad (7.2.3a) \]
\[ R_{dc} = K_2, \quad (7.2.3b) \]
\[ C(\omega) = C_o + K_3 \omega^{q-1} \sin \left( \frac{\pi q}{2} \right) = \frac{e^{\sigma T_{\infty}}}{A_1} + K_3 \omega^{q-1} \sin \left( \frac{\pi q}{2} \right), \quad (7.2.3c) \]
\[ Z_{sk}(\omega) = K_1 (j\omega)^p, \quad (7.2.3d) \]
\[ G(\omega) = K_3 \omega^{q} \cos \left( \frac{\pi q}{2} \right), \quad (7.2.3e) \]
\[ G_{dc} = K_4. \quad (7.2.3f) \]

The proposed time domain synthesis starts with dividing the time axis of the TDR response waveform of the DUT into \( N \) equal time intervals of length \( \Delta T \) as shown in Figure 7.2.1. Let \( M \) be the number of points acquired from a TDNA for the TDR response waveform. Then, the first time interval, i.e. \( 0 \leq t_k \leq T_1 \) \((k = 1, \ldots, M)\) as shown in Figure 7.2.2 is synthesized by a lossy transmission line having a propagation constant of \( \gamma_1 \) and a characteristic impedance of \( Z_1 \). The parameters of the first transmission line are then determined by using the non-linear least squares optimization technique explained in chapter 5. Further details on how the optimization technique is used is explained in the next section. Once, \( \gamma_1 \) and \( Z_1 \) are optimized, their values become fixed for all \( t > T_1 \). Later, the optimum parameters for the second transmission line, i.e. \( \gamma_2 \) and \( Z_2 \) are computed by concentrating in the time interval \( T_1 \leq t_k \leq T_2 \) \((k = 1, \ldots, M)\). Once, \( \gamma_2 \) and \( Z_2 \) are optimized, their values become fixed for all \( t > T_2 \). The process is repeated in this manner until the time interval \( T_{N-1} \leq t_k \leq T_N \) \((k = 1, \ldots, M)\) is synthesized and the optimum values for \( \gamma_N \) and \( Z_N \) are determined. Once this is done, the network of \( N \) cascaded lossy transmission line becomes an equivalent network representation for the DUT.
Figure 7.2.1 A typical time domain response waveform divided into $N$ intervals for the purpose of time domain network synthesis.
Figure 7.2.2  An explanation of different stages in the proposed time domain synthesis technique.
7.3 FURTHER DETAILS ON THE PROPOSED TIME DOMAIN SYNTHESIS TECHNIQUE AND THE DERIVATION OF THE S-PARAMETERS FOR N-CASCaded LOSSY TRANSMISSION LINES

7.3.1 Further Details on the Proposed Time Domain Synthesis Technique

As mentioned in the previous section, for each time interval, the parameters of the lossy line are determined by the optimization technique presented in chapter 5. This section presents further details about the use of this optimization technique for accomplishing the synthesis task. For the first time interval, i.e. $0 \leq t_k \leq T_1$ ($k = 1, \ldots, M$), the objective function is defined as follows:

$$E_2^{(1)}(\bar{X}^{(1)}) = \sum_{k=0}^{M_1} [c^{(1)}(t_k)]^2,$$  \hspace{1cm} (7.3.1)

where, $\bar{X}^{(1)}$ is the argument of the objective function defined in terms of the parameters of the lossy line and is given by

$$\bar{X}^{(1)} = (R_{dc}^{(1)}, A_1^{(1)}, K_1^{(1)}, R^{(1)}, r_{\infty}^{(1)}, K_3^{(1)}, q^{(1)}, G_{dc}^{(1)}).$$  \hspace{1cm} (7.3.2)

In (7.3.1), the subscript “2” refers to the least squares definition of the error function and the superscript “(1)” corresponds to the transmission line number whose parameters are to be optimized. Define $\Delta t$ to be the time interval used in acquiring the experimental TDR response waveform from a time domain network analyzer (TDNA). Then, the upper limit of the summation is defined as follows:

$$M_1 = \frac{T_1}{\Delta t} = \frac{\Delta T}{\Delta t}.$$  \hspace{1cm} (7.3.3)
Furthermore, the error function \( e^{(1)}(t) \) is defined by

\[
e^{(1)}(t_k) = \left[ v_{r1}^{(1)}(t_k) - v_{re}(t_k) \right] u(t_k - T_1); \quad k = 1, \ldots, M. \tag{7.3.4}
\]

In (7.3.4), \( v_{re}(t) \) corresponds to the experimental TDR response for the DUT acquired from a TDNA. On the other hand, \( v_{r1}^{(1)}(t) \) corresponds to the simulated TDR response for a single transmission line terminated by a load impedance. The load impedance for this case corresponds to an approximate value for the characteristic impedance of the second transmission line. Let \( v_{i1}(t) \) be the incident step-like signal used for exciting the DUT. Then, the simulated TDR response is obtained by convolving the incident step waveform acquired from a TDNA with the inverse Fourier Transform of the appropriate input reflection coefficient as follows:

\[
v_{r1}^{(1)}(t_k) = s_{i1}^{(1)}(t_k) \ast v_{r1}^{(1)}(t_k); \quad k = 1, \ldots, 4M, \tag{7.3.5}
\]

where, \( \ast \) denotes convolution in the time domain and is carried out over \( 4M \) points by zero padding.

For the second time interval, i.e. \( T_1 \leq t_k \leq T_2 \) (\( k = 1, \ldots, M \)), the objective function is defined as follows:

\[
\mathcal{E}_2^{(2)}(\bar{X}^{(2)}) = \sum_{k=M_1}^{M_2} [e^{(2)}(t_k)]^2, \tag{7.3.6}
\]

where, \( \bar{X}^{(2)} \) is the argument of the objective function defined in terms of the parameters of the lossy line and is given by

\[
\bar{X}^{(2)} = (R_{dc}^{(2)}, A_1^{(2)}, K_1^{(2)}, p^{(2)}, \varepsilon_r^{(2)}, K_3^{(2)}, q^{(2)}, C_{dc}^{(2)}). \tag{7.3.7}
\]
For this case, the upper limit of the summation is defined as follows:

\[
M_2 = \frac{T_2}{\Delta t} = 2 \frac{\Delta T}{\Delta t} .
\]  
(7.3.8)

Furthermore, the error function \( e^{(2)}(t) \) is defined by

\[
e^{(2)}(t_k) = \left[ v^{(2)}_{r1}(t_k) - v_{rc}(t_k) \right] u(t_k - T_2) ; \quad k = 1, \ldots, M.
\]  
(7.3.9)

In (7.3.9), \( v^{(2)}_{r1}(t) \) corresponds to the simulated TDR response for first two cascaded transmission lines terminated by a load impedance. The load impedance for this case corresponds to an approximate value for the characteristic impedance of the third transmission line. The simulated TDR response is obtained by convolving the incident step waveform acquired from a TDNA with the inverse Fourier Transform of the appropriate input reflection coefficient as follows:

\[
v^{(2)}_{r1}(t_k) = s^{(2)}_{11}(t_k) * v_{11}(t_k) ; \quad k = 1, \ldots, 4M.
\]  
(7.3.10)

For the last time interval, i.e. \( T_{N-1} \leq t_k \leq T_N \) \((k = 1, \ldots, M)\), the objective function is defined as follows:

\[
E_2^{(N)}(\vec{X}^{(N)}) = \sum_{k=M_{N-1}}^{M} [e^{(N)}(t_k)]^2 ,
\]  
(7.3.11)

where, \( \vec{X}^{(N)} \) is the argument of the objective function defined in terms of the parameters of the lossy line and is given by

\[
\vec{X}^{(N)} = (R^{(N)}_{dc}, A_1^{(N)}, K_1^{(N)}, p^{(N)}, c^{(N)}_{rco}, K_3^{(N)}, q^{(N)}, G^{(N)}_{dc}) .
\]  
(7.3.12)
For this case, the lower limit of the summation is defined by

\[ M_{N-1} = \frac{T_{N-1}}{\Delta t} = \frac{(N-1) \Delta T}{\Delta t}, \quad (7.3.13) \]

and the error function \( e^{(N)}(t) \) is given by

\[ e^{(N)}(t_k) = \left[ v_{r1}^{(N)}(t_k) - v_{re}(t_k) \right]; \quad k = 1, \ldots, M. \quad (7.3.14) \]

In (7.3.9), \( v_{r1}^{(N)}(t) \) corresponds to the simulated TDR response for the \( N \) cascaded transmission lines terminated by a load impedance. The load impedance for this case corresponds to an approximate value for the characteristic impedance of the load terminating the DUT. The simulated TDR response is obtained by convolving the incident step waveform acquired from a TDNA with the inverse Fourier Transform of the overall input reflection coefficient as follows:

\[ v_{r1}^{(N)}(t_k) = s_{11}^{(N)}(t_k) * v_{11}(t_k); \quad k = 1, \ldots, 4M. \quad (7.3.15) \]

7.3.2 Derivation of the S-Parameters for N-Cascaded Lossy Transmission Lines

In this section, the derivation of the scattering S-parameters used in calculating the simulated time domain response waveforms are discussed. Although, an attempt was made to calculate a closed form expression for the generalized S-parameters of \( N \) cascaded lossy transmission lines, no success was obtained. Since the simulated time domain response is calculated in an iterative manner using a computer, the S-parameters for each line can be stored numerically in an array format. Hence, all we need is a closed form expression for the S-parameters of two general cascaded networks. Let, \([S]^{(1)}\) and \([S]^{(2)}\) be the S-parameter matrices of networks one and two, respectively. Also, let \([S]\) be the overall S-parameter matrix of the two cascaded networks. Then, using the theory explained in Appendix A, the overall S-parameters of
the two cascaded networks become as follows:

\[ S_{11} = S_{11}^{(1)} + \frac{S_{21}^{(1)} S_{11}^{(2)} S_{12}^{(1)} S_{11}^{(2)}}{1 - S_{11}^{(2)} S_{22}^{(1)}}, \]  

(7.3.16a)

\[ S_{12} = \frac{S_{12}^{(1)} S_{12}^{(2)}}{1 - S_{11}^{(2)} S_{22}^{(1)}}, \]  

(7.3.16b)

\[ S_{21} = \frac{S_{21}^{(1)} S_{21}^{(2)}}{1 - S_{11}^{(2)} S_{22}^{(1)}}, \]  

(7.3.16c)

\[ S_{22} = S_{22}^{(2)} + \frac{S_{21}^{(2)} S_{11}^{(2)} S_{12}^{(2)}}{1 - S_{11}^{(2)} S_{22}^{(1)}}. \]  

(7.3.16d)

Consider the network shown in Figure 7.2.2a. In order to calculate the overall S-parameters at node A, we need to use the formulation given in (7.3.16) twice. In this process, we shall be utilizing the discontinuity matrix approach of chapter 4. For this case, we have two discontinuity matrices, one due to the mismatch from the generator impedance and the other one due to the mismatch from the load impedance. Substituting the discontinuity matrix due the mismatch from the generator impedance and the S-parameters for the lossy transmission line (as defined in chapter 4) into (7.3.16), we obtain

\[ S_{11} = \rho_{1g}, \]  

(7.3.17a)

\[ S_{12} = x_1 (1 - \rho_{1g}), \]  

(7.3.17b)

\[ S_{21} = x_1 (1 + \rho_{1g}), \]  

(7.3.17c)

\[ S_{22} = -\rho_{1g} x_1^2. \]  

(7.3.17d)
Now, letting the S-parameters given by (7.3.17) be $S_{ij}^{(1)}$ ($i, j = 1, 2$) and the discontinuity matrix due to the mismatch from the load impedance be $S_{ij}^{(2)}$ ($i, j = 1, 2$). Then, the overall S-parameters of the network shown in Figure (7.2.2a) are given by

\[
S_{11}^{(1)} = \frac{\rho_{1g} + \rho_{21} \rho_{1g}^{2} z_{1}^{2}}{1 + \rho_{21} \rho_{1g}^{2} z_{1}^{2}},
\]

\[
S_{12}^{(1)} = \frac{z_{1} (1 - \rho_{1g}) (1 - \rho_{21})}{1 + \rho_{21} \rho_{1g}^{2} z_{1}^{2}},
\]

\[
S_{21}^{(1)} = \frac{z_{1} (1 + \rho_{1g}) (1 + \rho_{21})}{1 + \rho_{21} \rho_{1g}^{2} z_{1}^{2}},
\]

\[
S_{22}^{(1)} = -\frac{\rho_{21} - \rho_{1g} \rho_{1g}^{2} z_{1}^{2}}{1 + \rho_{21} \rho_{1g}^{2} z_{1}^{2}}.
\]

\[\text{(7.3.18a)}\]
\[\text{(7.3.18b)}\]
\[\text{(7.3.18c)}\]
\[\text{(7.3.18d)}\]

In (7.3.17) and (7.3.18) the frequency dependent variables $\rho_{1g}$, $\rho_{21}$, and $z_{1}$ are defined as follows:

\[
\rho_{1g} = \frac{Z_{1} - Z_{g}}{Z_{1} + Z_{g}},
\]

\[
\rho_{21} = \frac{Z_{2} - Z_{1}}{Z_{2} + Z_{1}},
\]

\[
z_{1} = e^{-\gamma_{1} \ell_{i}},
\]

\[\text{(7.3.19a)}\]
\[\text{(7.3.19b)}\]
\[\text{(7.3.19c)}\]

The physical lengths of the lossy transmission lines at each iteration are calculated by using the following relation:

\[
\ell_{i} = \frac{\Delta T}{2} \frac{c}{\sqrt{\epsilon_{r_{\infty}}}}, \quad i = 1, 2, \ldots, N,
\]

\[\text{(7.3.20)}\]

where, $c$ is the speed of light in vacuum.
During the synthesis process, once the optimum parameters of the first transmission line are calculated, then the S-parameters given by (7.3.17) are stored in an array format. These S-parameters correspond to the cascade of the discontinuity matrix due to the generator impedance with the S-parameters of the first transmission line.

Next, consider the network shown in Figure 7.2.2b. The overall S-parameters at node A are calculated using the same approach presented above. Again, we have two discontinuity matrices, one due to the mismatch from transmission line one and the other one due to the mismatch from the load impedance. In the first place, the S-parameters for the cascade of transmission line two together with the discontinuity matrix due to the mismatch from the load impedance are calculated by using the formulation given by (7.3.16). Later, using these S-parameters and the discontinuity matrix due to the mismatch from transmission line 1, the S-parameters of the last section are calculated. The overall S-parameters at node A are finally calculated by using the S-parameters of the last section together with those which are stored in an array format. After the synthesis process for the second transmission line is complete, the S-parameters are re-calculated by excluding the effect of the load impedance and the final result is stored in the same array. This process is repeated in the same manner until the synthesis process is complete.

7.4 THE TIME DOMAIN NETWORK SYNTHESIS (TDNS) PACKAGE FOR MICROWAVE DEVICE MODELING

7.4.1 Description of the Software Package

The theory presented in this chapter and the optimization algorithm presented in chapter 5 have been implemented in a software package named as the Time Domain Network Synthesis
(TDNS). A general information flowchart which would help in describing its features is shown in Figure 7.4.1.

Similar to the TDMC software package, the input data file of TDNS should be named as \texttt{filename0.DAT}, where \texttt{filename0} is an arbitrary user defined file name but its extension is fixed as \texttt{DAT}. The format of the input data file is shown in Figure 7.4.2. The package requires a set of experimental time domain response and reference data for operation. In this specific application the response and reference data should be of TDR type. The text in Figure 7.4.2 which appears in upper case letters has to be exactly the same, because the package compares them to a preset format for identifying the information. However, the order of this information is arbitrary. The lossy line parameters provided in this data file are used as initial conditions in the optimization algorithm for the parameters of the first transmission line. The initial conditions of each consecutive transmission line is taken to be the optimal values of the previous transmission line.

All of the data beyond the equality sign which is shown in italics should be real numbers, and provided in mks units, except for the reference and response file names, where it has to be in text format. The response and reference data file names, referred as \texttt{filename1}, and \texttt{filename2} are user defined and should be different from each other. On the other hand, the response and reference data file extensions, referred as \texttt{filetype} are also user defined and can be the same for both files. The TDNS package assumes that these files are in the Hp 54120T TDNA format. This format assumes that all the data is listed in a single column. The first data in the column should correspond to the number of data points and the last data should correspond to the time per division used in the acquisition process. The data points corresponding to the waveform should be listed in between. The units of the data corresponding to the waveform should be in Volts.

The TDNS software package is executed as follows: \texttt{TDNS filename0}. After the
execution, the package asks the user to enter the number of transmission lines for synthesis. After this question is properly answered, the program calls the POWELL subroutine and starts the iterative process for optimization. The status of each iteration is displayed on the computer terminal. When the iteration for the first transmission line is complete, the package plots the experimental and simulated TDR responses. When this is done, the program proceeds to synthesize the second time interval and the process continues in this manner until all the intervals are synthesized.

When the optimization process is complete, the package prints the following output data into a floppy diskette or hard disk, depending on from where the data file filename0.DAT is read: filename0.OUT, and filename0.TDR. The file having the extension OUT contains the optimal values for the parameters of each transmission line. It also provides the number of iterations it took the program to reach the optimal solution and the sum of least squares of the error function at the optimal solution for each transmission line. On the other hand, the file having the extensions TDR contains the listings of the experimental and simulated TDR data as a function of time at the optimal solution of \( N \) cascaded lossy transmission lines. The format of the above two files is similar to those given by the TDMC software package.

### 7.4.2 Verification of the TDNS Software Package

In order to verify the TDNS software package and the time domain optimization algorithm, some computer simulations have been performed. The simulations were performed by first generating the TDR data using a set of transmission lines with known parameters. Later, this data was given to the TDNS package as if it were an experimental data and was asked to give the optimal parameters for each transmission line. The results for two different cases are shown below.
Figure 7.4.1 Information flowchart for the Time Domain Material Characterization (TDMS) software package.
REFLECTION REFERENCE FILE  =  filename1.filetype

REFLECTION RESPONSE FILE  =  filename2.filetype

DC RESISTANCE  (Rdc)  =  \( (R_{dc})_o \)

INDUCTANCE CONSTANT  (A1)  =  \( (A_1)_o \)

CAPACITANCE CONSTANT  (Epsr infinity)  =  \( (\varepsilon'_{\infty})_o \)

CONDUCTANCE CONSTANT  (K3)  =  \( (K_3)_o \)

CONDUCTANCE EXPONENT  (q)  =  \( (q)_o \)

DC CONDUCTANCE CONSTANT  (sigma)  =  \( (\sigma_{dc})_o \)

SKIN EFFECT CONSTANT  (K1)  =  \( (K_1)_o \)

SKIN EFFECT EXPONENT  (p)  =  \( (p)_o \)

SHUNT LOAD RESISTANCE  (RL)  =  \( (R_{L'})_o \)

Figure 7.4.2  Input data file format for TDNS software package.
Verification Case 1

The simulated data for this case was generated by using a cascade of two lossless transmission lines having the following specifications:

**Actual parameters for the first transmission line:**

\[ L_e^{(1)} = \mu_0 \text{ H/m,} \]
\[ C^{(1)} = 10 \epsilon_0 \text{ F/m,} \]
\[ Z_{sk}^{(1)} = 0 \Omega/\text{m,} \]
\[ R_{dc}^{(1)} = 0 \Omega/\text{m,} \]
\[ G^{(1)} = 0 \text{ S/m,} \]
\[ G_{dc}^{(1)} = 0 \text{ S/m,} \]
\[ R_L^{(1)} = 168.50 \Omega. \]

**Actual parameters for the second transmission line:**

\[ L_e^{(2)} = 2 \mu_0 \text{ H/m,} \]
\[ C^{(2)} = 10 \epsilon_0 \text{ F/m,} \]
\[ Z_{sk}^{(2)} = 0 \Omega/\text{m,} \]
\[ R_{dc}^{(2)} = 0 \Omega/\text{m,} \]
\[ G^{(2)} = 0 \text{ S/m,} \]
\[ G_{dc}^{(2)} = 0 \text{ S/m,} \]
\[ R_L^{(2)} = 168.50 \Omega. \]

The above data was used for generating simulated TDR response. This response was then provided to the TDNS software package as experimental data and was asked to give the optimum parameters for the transmission lines. The tolerance value given to the software package was \(1 \times 10^{-7}\). The results of the optimization are shown below.
Optimum parameters for the first transmission line:

\[ L_e^{(1)\text{op}} = 1.000015 \mu_0 \, \text{H/m}, \]
\[ C^{(1)\text{op}} = 9.99985 \epsilon_0 + 2.478672 \times 10^{-6} \omega^{-0.968} \sin (0.0157841 \pi) \, \text{F/m}, \]
\[ Z_{sk}^{(1)\text{op}} = 4.55167 \times 10^{-2} (j\omega)^{0.0037} \, \Omega/m, \]
\[ R_{dc}^{(1)\text{op}} = 1.69067 \times 10^{-2} \, \Omega/m, \]
\[ G^{(1)\text{op}} = 2.478672 \times 10^{-6} \omega^{0.031568} \cos (0.0157841 \pi) \, \text{S/m}, \]
\[ G_{dc}^{(1)\text{op}} = 1.07317 \times 10^{-5} \, \text{S/m}. \]

Optimum parameters for the second transmission line:

\[ L_e^{(2)\text{op}} = 2.000345 \mu_0 \, \text{H/m}, \]
\[ C^{(2)\text{op}} = 9.9982703 \epsilon_0 + 3.068664 \times 10^{-6} \omega^{-0.958} \sin (0.02108 \pi) \, \text{F/m}, \]
\[ Z_{sk}^{(2)\text{op}} = 1.102597 \times 10^{-5} (j\omega)^{0.3868} \, \Omega/m, \]
\[ R_{dc}^{(2)\text{op}} = 5.745915 \times 10^{-5} \, \Omega/m, \]
\[ G^{(2)\text{op}} = 3.06866 \times 10^{-6} \omega^{0.04217} \cos (0.02108 \pi) \, \text{S/m}, \]
\[ G_{dc}^{(2)\text{op}} = 2.5211796 \times 10^{-5} \, \text{S/m}. \]

where, the superscript "\text{op}" refers to the optimum solution. Comparing the above optimal solution with the actual parameters, it can be observed that some discrepancy exists, especially in the loss parameters. Theoretically, the optimization algorithm was supposed to find the exact solution, since we have used the same formulation to generate the simulated data. However, in each function evaluation, the algorithm has to go through an inverse FFT computation. This results in some loss of numerical accuracy. The computation error in some of the parameters that have high sensitivity on the TDR response is given below.

\[
\text{\% error in } L_e^{(1)} = \left| \frac{1.0 - 1.000015}{1.0} \right| = 0.0015 ,
\]
\[ \% \text{ error in } C_o^{(1)} = \left| \frac{10.0 - 9.99985}{10.0} \right| = 0.0015, \]

\[ \% \text{ error in } L_e^{(2)} = \left| \frac{2.0 - 2.000345}{2.0} \right| = 0.01725, \]

\[ \% \text{ error in } C_o^{(2)} = \left| \frac{10.0 - 9.99827}{10.0} \right| = 0.01729. \]

From the above results, it can be observed that the error in the line parameters which have high sensitivity on the TDR response is less than 0.02%. In the current application, this error is considered to be negligible. The comparison between the simulated and verified TDR responses is shown in Figure 7.4.3. This figure displays a perfect match between the simulated and verified responses.

**Verification Case 2**

The simulated data for this case was generated by using a cascade of three lossy transmission lines having the following specifications:

**Actual parameters for the first transmission line:**

\[ L_e^{(1)} = 0.1 \mu \text{H/m,} \]

\[ C^{(1)} = 50 \epsilon_o + 1.0 \times 10^{-5} \omega^{-0.92} \sin (0.04 \pi) \text{ F/m,} \]

\[ Z_{sk}^{(1)} = 1.0 \times 10^{-6} (j\omega)^{0.05} \Omega/\text{m,} \]

\[ R_{dc}^{(1)} = 1.0 \times 10^{-5} \Omega/\text{m,} \]

\[ G^{(1)} = 1.0 \times 10^{-5} \omega^{0.08} \cos (0.04 \pi) \text{ S/m,} \]

\[ G_{dc}^{(1)} = 1.0 \times 10^{-7} \text{ S/m.} \]
Figure 7.4.3  Comparison between the simulated and verified TDR responses for verification case 1.
Actual parameters for the second transmission line:

\[ L_e^{(2)} = 0.2 \mu_\text{o} \text{ H/m}, \]
\[ C^{(2)} = 50 \epsilon_\text{o} + 2.0 \times 10^{-5} \omega^{-0.84} \sin (0.68 \pi) \text{ F/m}, \]
\[ Z_{sk}^{(2)} = 2.0 \times 10^{-6} (j\omega)^{0.1} \Omega/\text{m}, \]
\[ R_{dc}^{(2)} = 2.0 \times 10^{-5} \Omega/\text{m}, \]
\[ G^{(2)} = 2.0 \times 10^{-5} \omega^{0.16} \cos (0.08 \pi) \text{ S/m}, \]
\[ G_{dc}^{(2)} = 1.0 \times 10^{-7} \text{ S/m}. \]

Actual parameters for the third transmission line:

\[ L_e^{(3)} = 0.4 \mu_\text{o} \text{ H/m}, \]
\[ C^{(3)} = 50 \epsilon_\text{o} + 4.0 \times 10^{-5} \omega^{-0.68} \sin (0.16 \pi) \text{ F/m}, \]
\[ Z_{sk}^{(3)} = 4.0 \times 10^{-6} (j\omega)^{0.2} \Omega/\text{m}, \]
\[ R_{dc}^{(3)} = 4.0 \times 10^{-5} \Omega/\text{m}, \]
\[ G^{(3)} = 4.0 \times 10^{-5} \omega^{0.32} \cos (0.16 \pi) \text{ S/m}, \]
\[ G_{dc}^{(3)} = 1.0 \times 10^{-7} \text{ S/m}. \]

The above data was used for generating the simulated TDR response. This response was then provided to the TDNS software package as experimental data and was asked to give the optimum parameters for the transmission lines. The tolerance value given to the software package was again $1 \times 10^{-7}$. The results of the optimization are shown below.

Optimum parameters for the first transmission line:

\[ L_e^{(1)\text{op}} = 0.1000333 \mu_\text{o} \text{ H/m}, \]
\[ C^{(1)\text{op}} = 49.983465 \epsilon_\text{o} + 1.454358 \times 10^{-10} \omega^{-0.77} \sin (0.11423 \pi) \text{ F/m}, \]
\[ Z_{sk}^{(1)\text{op}} = 9.67103 \times 10^{-5} (j\omega)^{0.04998} \Omega/\text{m}, \]
\[ R_{dc}^{(1)\text{op}} = 2.615977 \Omega/\text{m}, \]
\[ G^{(1)op} = 1.454358 \times 10^{-10} \omega^{0.228} \cos (0.11423 \pi) \text{ S/m,} \]
\[ G^{(1)dc}_{dc} = 6.14763 \times 10^{-8} \text{ S/m.} \]

**Optimum parameters for the second transmission line:**

\[ L^{(2)op}_c = 0.1989767 \mu \text{H/m,} \]
\[ C^{(2)op} = 50.30629 \epsilon_o + 1.777565 \times 10^{-5} \omega^{-0.801} \sin (0.09922 \pi) \text{ F/m,} \]
\[ Z^{(2)op}_{sk} = 1.969491 (j\omega)^{0.656} \Omega/\text{m,} \]
\[ R^{(2)op}_{dc} = 8.001656 \times 10^{-6} \Omega/\text{m,} \]
\[ G^{(2)op} = 1.777565 \times 10^{-5} \omega^{0.198} \cos (0.09922 \pi) \text{ S/m,} \]
\[ G^{(2)op}_{dc} = 9.999492 \times 10^{-3} \text{ S/m.} \]

**Optimum parameters for the third transmission line:**

\[ L^{(3)op}_c = 1.989767 \mu \text{H/m,} \]
\[ C^{(3)op} = 50.30629 \epsilon_o + 1.777565 \times 10^{-5} \omega^{-0.801} \sin (0.09922 \pi) \text{ F/m,} \]
\[ Z^{(3)op}_{sk} = 1.969491 (j\omega)^{0.656} \Omega/\text{m,} \]
\[ R^{(3)op}_{dc} = 8.001656 \times 10^{-6} \Omega/\text{m,} \]
\[ G^{(3)op} = 1.777565 \times 10^{-5} \omega^{0.198} \cos (0.09922 \pi) \text{ S/m,} \]
\[ G^{(3)op}_{dc} = 9.999492 \times 10^{-3} \text{ S/m.} \]

Comparing the above optimal solution with the actual parameters, it can be observed that some discrepancy exists, especially in the loss parameters. This discrepancy is again due to the errors accumulated from the calculation of the inverse FFT at each iteration. The computation error in some of the parameters that have high sensitivity on the TDR response is given below.

\[
\% \text{ error in } L^{(1)}_c = \left| \frac{0.1 - 0.10100333}{0.1} \right| = 0.033 ,
\]
\[
\% \text{ error in } C_o^{(1)} = \frac{|50.0 - 49.98346|}{50.0} = 0.033, \\
\% \text{ error in } L_e^{(2)} = \frac{|0.2 - 0.1989767|}{0.2} = 0.511, \\
\% \text{ error in } C_o^{(2)} = \frac{|50.0 - 50.30629|}{50.0} = 0.612, \\
\% \text{ error in } L_e^{(3)} = \frac{|0.4 - 0.4002135|}{0.4} = 0.053, \\
\% \text{ error in } C_o^{(3)} = \frac{|50.0 - 49.974226|}{50.0} = 0.052.
\]

From the above results, it can be observed that the error in the line parameters which have high sensitivity on the TDR response is less than 0.65%. In the current application, this error can still be considered to be negligible. The comparison between the simulated and verified TDR responses is shown in Figure 7.4.4. This figure displays a perfect match between the simulated and verified responses.
Figure 7.4.4  Comparison between the simulated and verified TDR responses for verification case 2.
7.5 SUMMARY AND CONCLUSIONS

In this chapter, a practical time domain network synthesis technique has been presented. The technique used a cascade of \( N \) lossy transmission lines to synthesize a given time domain response. This was accomplished by dividing the time axis of the given time domain response waveform into \( N \) equal time intervals of length \( \Delta T \). Each time interval was then synthesized by a lossy transmission line. The optimum parameters of each line was determined by using a non-linear least squares optimization algorithm. The proposed synthesis technique has been implemented in a software package named as the Time Domain Network Synthesis (TDNS) package. The chapter provided an information flow chart about the package and presented the format of the input file structure. Finally, two verification cases have been presented about the validity of the software package and the error bounds of the optimization technique.
CHAPTER 8

SUMMARY AND CONCLUSIONS

In this dissertation, a new technique for the determination of material properties such as the complex permittivity and the complex permeability in a stripline geometry was presented. The stripline under test were modeled by a lossy transmission line. The functional behavior of the lossy transmission line parameters was defined as a priori model beforehand. With this approach, no assumptions were made about the geometrical parameters or the characteristic impedance of the stripline under test. The proposed technique used time domain measurements and a non-linear least squares optimization technique for determining the optimal values of the lossy transmission line parameters. Finally, the material properties such as the complex permittivity and the complex permeability were determined from the optimal values of the lossy transmission line parameters.

The literature survey has shown that the models available for solid dielectric materials are non-causal. In this dissertation, it has been shown that any physical model has to be causal and should satisfy a pair of Hilbert transforms. Specifically, the real and imaginary parts of the complex permittivity are related by a pair of Hilbert transform known as the Kramers-Kronig relations. The dissertation has presented a new causal model for the complex permittivity. A detailed proof showing the causality of this model was also presented. Its validity was verified through several experiments on well characterized dielectric materials such as RT/Duroid 5880© and RT/Duroid 5870©.

Based on the proposed experimental set up for the stripline under test, an equivalent network model was devised. The coaxial to planar adapters used for adapting the coaxial geometry of the reference transmission lines to the planar geometry of the stripline were also
modeled by lossy transmission lines. Using the equivalent network model, the necessary scattering S-parameters that would simulate the measurement data were derived. The simulated time domain response waveform was obtained by convolving the inverse Fourier transform of the appropriate S-parameter with the incident step waveform. The dissertation has presented several computer simulations showing how different lossy line parameters affect the time domain response waveform. These simulations have also shown that, the real part of the complex permittivity, \( \varepsilon'_\infty(\omega) \) is more sensitive to the time domain reflectometry data than the time domain transmission one. The inverse was true for the loss tangent, \( \tan \delta(\omega) \).

The lossy transmission line parameters for both the coaxial to planar adapters and the stripline under test were determined by using a non-linear least squares optimization technique. For this purpose an objective function was defined to be the sum of squares of an error function. The error function was in turn defined to be the difference between the simulated and experimental time domain responses. Due to the complicated mathematical formulation involved, it was not possible to obtain an analytical formula for the error function. In addition to this, computer simulations has shown that the error function was not continuously differentiable. Due to these limitations it was necessary to find an optimization algorithm that did not require the calculation of derivatives of the error function. Literature survey has shown that the best algorithm for this purpose was Powell’s algorithm [46]. Powell’s algorithm in its original form had some problems. Hence, some modifications were necessary. These modifications were also found to be available in literature. The modified Powell’s algorithm is an optimization algorithm that does not use any constraints. Since the lossy line parameters have to be non-negative then it was necessary to make further modifications in Powell’s algorithm to take care of the necessary constraints.

Using the proposed time domain technique and the modified Powell’s algorithm a user friendly software package was written for material characterization. This software package which
was named as the Time Domain Material Characterization (TDMC) package was also presented in this dissertation. The dissertation has provided detailed explanation about the input and output file formats. It has also presented two cases for verifying and showing the validity of the proposed technique. Several experimental results on the complex permittivity of RT/Duroid®, Du Pont thick-film pastes® and Du Pont polymers® were also presented as an outcome of the TDMC software package.

The last chapter of this dissertation has presented a practical time domain synthesis technique. Using the proposed time domain synthesis technique, a time domain reflectometry response waveform obtained by exciting a device under test by a step like signal was first divided into \( N \) equal time intervals. Later, each interval was synthesized by a lossy transmission line. The parameters of each line were determined by using an iterative non-linear least squares optimization technique to fit the simulated time domain response to the experimental one. The optimization was performed in the time domain by minimizing the error function due to the difference between the two waveforms. For this case, the Powell’s modified optimization algorithm was found to give very good results.

The proposed time domain optimization technique was also implemented in a software package called the Time Domain Network Synthesis (TDNS) package. The dissertation has presented an information flow chart about the software package and it has provided detailed explanation about the input and output file formats. In order to show the validity of both the technique and the software package, two verification cases were also presented.

Based on our findings, we are proposing the following topics as a possible direction for future research:

Using Powell’s modified optimization algorithm, it takes several iterations to reach an optimal solution. This sometimes takes several hours on a conventional IBM 386™
personal computer. Hence, some research can be done for improving the convergence time.

The proposed model was only verified for thick film and polymer dielectric materials. It would be interesting to test its validity for general dielectric materials.

Using the proposed time domain material characterization technique, some problems were encountered in the effect of conductor and interface properties. Future research can be in the direction of solving this problem.

The proposed time domain network synthesis technique was not verified using experimental data. Some work should be done in this direction.

In the time domain synthesis problem, the S-parameters of the cascaded lossy transmission lines are calculated in the frequency domain and then converted to the time domain using an inverse Fast Fourier Transform (FFT) algorithm. It was found out that the calculation of the inverse FFT at each iteration causes some numerical leakage problems. A future research direction would be the solution of this problem.
APPENDIX A

SCATTERING PARAMETERS AND THEIR PROPERTIES

A.1 INTRODUCTION

Linear or non-linear networks operating with signals sufficiently small to cause the network to respond in a linear manner, can be completely characterized by parameters measured at the network terminals (ports). Once the parameters of a network have been determined, its behavior in any external environment can be easily predicted. Scattering S-parameters are used in microwave design and testing because they are easier to measure and work with at high frequencies than other kinds of parameters. Measuring most other parameters calls for the input and output of the device to be successfully opened and short circuited. This is difficult to achieve even at RF frequencies where lead inductance and capacitance make short and open circuits difficult to obtain. At higher frequencies these measurements require tuning stubs, separately adjusted at each measurement frequency, to reflect short or open circuit conditions to the device terminals. However, this is both tedious and inconvenient. Furthermore, in active devices this can cause the device to oscillate, making the measurement difficult and invalid. S-parameters, on the other hand, are usually measured with the device embedded between a 50 Ω load and source, and there is little chance for oscillations to occur.

Section A.2 presents the definition of scattering S-parameters. The scattering transfer parameters or T-parameters and their applications are presented in section A.3. Finally the use of signal flow graphs in S-parameter design are given in section A.4.
A.2 DEFINITION OF SCATTERING S-PARAMETERS

In order to define the scattering S-parameters, consider the general two-port network shown in Figure A.2.1. The variables $a_i$ and $b_i$ are normalized complex voltage waves incident on and reflected from the $i$-th port of the network. They are defined in terms of the terminal voltage $V_i$, the terminal current $I_i$, and an arbitrary reference impedance $Z_i$, as follows:

$$a_i = \frac{V_i + Z_i I_i}{2 \sqrt{\text{Re} [Z_i]}},$$  \hspace{1cm} (A.2.1)

and

$$b_i = \frac{V_i - Z_i^* I_i}{2 \sqrt{\text{Re} [Z_i]}},$$ \hspace{1cm} (A.2.2)

where the asterisk denotes the complex conjugate, and $\text{Re} [\cdot]$ corresponds to the real part of $\cdot$.

For most practical measurements and calculations, it is more convenient to assume that the reference impedance $Z_i$ is positive and real. Therefore, in the rest of this appendix, all variables and parameters will be referenced to a single positive real impedance $Z_o$. The independent variables $a_1$ and $a_2$ are normalized incident voltages and are defined as follows:

$$a_1 = \frac{V_1 + Z_o I_1}{2 \sqrt{Z_o}} = \frac{\text{voltage wave incident on port 1}}{\sqrt{Z_o}} = \frac{V_{i1}}{\sqrt{Z_o}},$$ \hspace{1cm} (A.2.3)

$$a_2 = \frac{V_2 + Z_o I_2}{2 \sqrt{Z_o}} = \frac{\text{voltage wave incident on port 2}}{\sqrt{Z_o}} = \frac{V_{i2}}{\sqrt{Z_o}}.$$ \hspace{1cm} (A.2.4)

Furthermore, the dependent variables $b_1$ and $b_2$ are normalized reflected voltages and are defined by
Figure A.2.1  Two port network showing incident \((a_1, a_2)\) and reflected \((b_1, b_2)\) waves used in S-parameter definitions.
\[ b_1 = \frac{V_1 - Z_o I_1}{2 \sqrt{Z_o}} = \frac{\text{voltage wave reflected from port 1}}{\sqrt{Z_o}} = \frac{V_{r1}}{\sqrt{Z_o}}, \quad (A.2.5) \]

\[ b_2 = \frac{V_2 - Z_o I_2}{2 \sqrt{Z_o}} = \frac{\text{voltage wave reflected from port 2}}{\sqrt{Z_o}} = \frac{V_{r2}}{\sqrt{Z_o}}. \quad (A.2.6) \]

The linear equations describing the two-port network are then given by:

\[ b_1 = S_{11} a_1 + S_{12} a_2, \quad (A.2.7) \]

and

\[ b_2 = S_{21} a_1 + S_{22} a_2, \quad (A.2.8) \]

or in matrix form,

\[
\begin{bmatrix}
  b_1 \\
  b_2 \\
\end{bmatrix} =
\begin{bmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22} \\
\end{bmatrix}
\begin{bmatrix}
  a_1 \\
  a_2 \\
\end{bmatrix}. \quad (A.2.9)
\]

Using the above representation, the S-parameter matrix is defined as

\[
\begin{bmatrix}
  S \\
\end{bmatrix} =
\begin{bmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22} \\
\end{bmatrix}.
\]

Conceptually, the S-parameters are defined as follows:

\[ S_{11} = \frac{b_1}{a_1} \bigg|_{a_2 = 0} = \text{Input reflection coefficient with the output port terminated by a} \]
matched load \((Z_L = Z_o\) sets \(a_2 = 0\)),

\[ S_{22} = \frac{b_2}{a_2} \bigg|_{a_1 = 0} = \text{Output reflection coefficient with the input port terminated by a matched load \((Z_L = Z_o\) and \(V_L = 0\))}, \]

\[ S_{21} = \frac{b_2}{a_1} \bigg|_{a_2 = 0} = \text{Forward transmission (insertion) gain with the output port terminated in a matched load}, \]

\[ S_{12} = \frac{b_1}{a_2} \bigg|_{a_1 = 0} = \text{Reverse transmission (insertion) gain with the input port terminated in a matched load}. \]

Referring to Figure A.2.1, it can be observed that the terminal voltages at each port can be written in terms of the sum of the incident and reflected waves, namely

\[ V_1 = V_{i1} + V_{r1}, \quad \text{(A.2.10)} \]

and

\[ V_2 = V_{i2} + V_{r2}. \quad \text{(A.2.11)} \]

Next, define the input impedance at port 1 as

\[ Z_1 = \frac{V_1}{I_1}, \quad \text{(A.2.12)} \]

then, the following relation is obtained for \(S_{11}\):

\[ S_{11} = \frac{b_1}{a_1} = \frac{V_1}{I_1} \frac{1 - Z_o}{V_1 + Z_o} = \frac{Z_1 - Z_o}{Z_1 + Z_o}. \quad \text{(A.2.13)} \]
A.3 DEFINITION OF SCATTERING T-PARAMETERS AND THE DERIVATION OF THEIR RELATION TO THE SCATTERING S-PARAMETERS

The chain scattering parameters, also called the scattering transfer parameters or T-parameters are defined in such a way that the incident waves $a_1$ and $b_1$ in Figure A.2.1 are the dependent variables and the reflected waves $a_2$ and $b_2$ are the independent variables. Hence, the $T$-parameters are defined by

$$
\begin{bmatrix}
    a_1 \\
    b_1
\end{bmatrix} =
\begin{bmatrix}
    T_{11} & T_{12} \\
    T_{21} & T_{22}
\end{bmatrix}
\begin{bmatrix}
    b_2 \\
    a_2
\end{bmatrix}.
$$

(A.3.1)

Using the above representation, the $T$-parameter matrix is defined as

$$
[T] =
\begin{bmatrix}
    T_{11} & T_{12} \\
    T_{21} & T_{22}
\end{bmatrix}.
$$

In order to derive the relationship between the $S$- and $T$-parameters, we proceed as follows:

First we will obtain $[S]$ in terms of $[T]$. Writing (A.3.1) in the form of algebraic equations gives

$$
a_1 = T_{11}b_2 + T_{12}a_2,
$$

(A.3.2)

$$
b_1 = T_{21}b_2 + T_{22}a_2.
$$

(A.3.3)
Figure A.3.1 Cascade connection of two-port networks.
From (A.3.2), we have

\[ b_2 = \frac{1}{T_{11}} a_1 - \frac{T_{12}}{T_{11}} a_2. \]  
(A.3.4)

Substituting (A.3.4) into (A.3.3) and rearranging gives

\[ b_1 = \frac{T_{21}}{T_{11}} a_1 + \left( T_{22} - \frac{T_{21} T_{12}}{T_{11}} \right) a_2. \]  
(A.3.5)

Combining (A.3.4) and (A.3.5) in matrix form yields

\[
\begin{bmatrix}
  b_1 \\
  b_2
\end{bmatrix}
= \begin{bmatrix}
  \frac{T_{21}}{T_{11}} & T_{22} - \frac{T_{21} T_{12}}{T_{11}} \\
  \frac{1}{T_{11}} & -\frac{T_{12}}{T_{11}}
\end{bmatrix}
\begin{bmatrix}
  a_1 \\
  a_2
\end{bmatrix}. 
\]  
(A.3.6)

Comparing (A.3.6) with (A.2.9) gives

\[
\begin{bmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{bmatrix}
= \begin{bmatrix}
  \frac{T_{21}}{T_{11}} & T_{22} - \frac{T_{21} T_{12}}{T_{11}} \\
  \frac{1}{T_{11}} & -\frac{T_{12}}{T_{11}}
\end{bmatrix}. 
\]  
(A.3.7)

Next, we want to write \([T]\) in terms of \([S]\). From (A.2.8), we have

\[ a_1 = \frac{1}{S_{21}} b_2 - \frac{S_{22}}{S_{21}} a_2. \]  
(A.3.8)

Substituting (A.3.8) into (A.2.7) and rearranging gives
\[ b_1 = \frac{S_{11}}{S_{21}} b_2 + \left( S_{12} - \frac{S_{11} S_{22}}{S_{21}} \right) a_2. \] (A.3.9)

Combining (A.3.8) and (A.3.9) in matrix form yields

\[
\begin{bmatrix}
  a_1 \\
  b_1
\end{bmatrix} = \begin{bmatrix}
  \frac{1}{S_{21}} & -\frac{S_{22}}{S_{21}} \\
  \frac{S_{11}}{S_{21}} & S_{12} - \frac{S_{11} S_{22}}{S_{21}}
\end{bmatrix} \begin{bmatrix}
  b_2 \\
  a_2
\end{bmatrix}. \quad (A.3.10)
\]

Comparing (A.3.10) with (A.3.1) gives

\[
\begin{bmatrix}
  T_{11} & T_{12} \\
  T_{21} & T_{22}
\end{bmatrix} = \begin{bmatrix}
  \frac{1}{S_{21}} & -\frac{S_{22}}{S_{21}} \\
  \frac{S_{11}}{S_{21}} & S_{12} - \frac{S_{11} S_{22}}{S_{21}}
\end{bmatrix}. \quad (A.3.11)
\]

Having derived the relationship between S- and T-parameters, we can now proceed to the derivation of an elegant property of T-parameters as applied to cascaded networks. Towards this goal, consider the cascade connection of two-port networks as shown in Figure A.3.1 [58]. By definition, the T-parameters of each network can be written as

\[
\begin{bmatrix}
  a_1 \\
  b_1
\end{bmatrix} = \begin{bmatrix}
  T_{11}^a & T_{12}^a \\
  T_{21}^a & T_{22}^a
\end{bmatrix} \begin{bmatrix}
  b_2 \\
  a_2
\end{bmatrix}, \quad (A.3.12)
\]

and

\[
\begin{bmatrix}
  a'_1 \\
  b'_1
\end{bmatrix} = \begin{bmatrix}
  T_{11}^b & T_{12}^b \\
  T_{21}^b & T_{22}^b
\end{bmatrix} \begin{bmatrix}
  b'_2 \\
  a'_2
\end{bmatrix}. \quad (A.3.13)
\]
Referring to Figure A.3.1, we can observe that

\[
\begin{bmatrix}
  b_2 \\
  a_2
\end{bmatrix}
= \begin{bmatrix}
  a'_1 \\
  b'_1
\end{bmatrix}
\]  

(A.3.14)

By substituting (A.3.13) into (A.3.12) we obtain the following relation:

\[
\begin{bmatrix}
  a_1 \\
  b_1
\end{bmatrix}
= \begin{bmatrix}
  T_{11}^a & T_{12}^a \\
  T_{21}^a & T_{22}^a
\end{bmatrix}
\begin{bmatrix}
  T_{11}^b & T_{12}^b \\
  T_{21}^b & T_{22}^b
\end{bmatrix}
\begin{bmatrix}
  b'_2 \\
  a'_2
\end{bmatrix}
\]  

(A.3.15)

Equation (A.3.15) shows that the T-parameters of a cascaded network is equivalent to the product of the T-parameters of the individual networks. This property is extremely useful in the microwave analysis of cascaded networks.

A.4 USE OF SIGNAL FLOW GRAPHS IN S-PARAMETER APPLICATIONS

Microwave measurement techniques can be analyzed more simply by using signal flow graphs instead of the customary scattering matrices to describe the microwave networks used in the measuring system. This is basically due to the fact that, the flow graphs of individual networks are simply joined together when the networks are cascaded and the solution for the system can be written down by inspection of the over-all flow graph by application of the non-touching loop rule [59]-[61].

The signal flow graph is a method of writing a set of equations, whereby the variables are
represented by points and the interrelations by directed lines giving a direct picture of signal flow. The algebra of flow graphs leading to solutions by direct inspection has been developed by S. J. Mason [60]-[61]. The following rules will be used for building up a network of flow graph [62]-[63]:

1. Each variable, \( a_1, a_2, b_1, \) and \( b_2 \) will be designated as a node.
2. Each of the S-parameters will be a branch.
3. Branches enter dependent variable nodes, and emanate from the independent variable nodes.
4. Each node is equal to the sum of the branches entering it.

The signal flow graph of the S-parameters of a two-port network is shown in Figure A.4.1(a). Observe that \( b_1 \) and \( b_2 \) are the dependent nodes and \( a_1 \) and \( a_2 \) the independent nodes. The complete signal graph of the two-port network is shown in Figure A.4.1(b). This signal flow graph shows the relationship between the traveling waves. The incident wave \( a_1 \) at port 1 gets partly transmitted (i.e., \( S_{21}a_1 \)) to become part of \( b_2 \), and partly reflected (i.e., \( S_{11}a_1 \)) to become part of \( b_1 \). Similarly, the incident wave \( a_2 \) at port 2 gets partly transmitted (i.e., \( S_{12}a_2 \)) to become part of \( b_1 \), and partly reflected (i.e., \( S_{22}a_2 \)) to become part of \( b_2 \).

In order to obtain the signal flow graph of a typical microwave network we need examine some typical cases such as a network with a signal generator having a certain internal impedance, and the signal graph of a load impedance. Figure A.4.2(a) shows a voltage source generator with internal impedance \( Z_g \). The terminal voltage \( V_s \) can be written as

\[
V_s = V_g + I_s Z_g .
\] (A.4.1)

Equation (A.4.1) can be written in terms of traveling waves as follows:
Figure A.4.1  (a) Signal flow graph for the scattering parameter equations; (b) complete signal flow graph of a two port network.
\[ V_{is} + V_{rs} = V_g + \left( \frac{V_{is}}{Z_o} - \frac{V_{rs}}{Z_o} \right) Z_g. \] (A.4.2)

Solving for the reflected wave \( V_{rs} \), we obtain

\[ V_{rs} = \frac{1}{Z_g + Z_o} \left[ V_g Z_o + V_{is} (Z_g - Z_o) \right]. \] (A.4.3)

Define,

\[ b_s = \frac{V_{rs}}{\sqrt{Z_o}}, \]

\[ a_s = \frac{V_{is}}{\sqrt{Z_o}}, \]

\[ b_g = \frac{V_g \sqrt{Z_o}}{Z_g + Z_o}, \]

and

\[ \Gamma_g = \frac{Z_g - Z_o}{Z_g + Z_o}. \]

Next, dividing both sides of (A.4.3) by \( \sqrt{Z_o} \) and using the above definitions we obtain

\[ b_s = b_g + a_s \Gamma_g. \] (A.4.4)

The signal flow graph of Figure A.4.2(a) which is shown in Figure A.4.2(b) is obtained using (A.4.4) and the signal flow graph rules defined previously.

Next, we are interested in obtaining the signal flow graph of a load impedance as shown in Figure A.4.3. The terminal voltage \( V_L \) can be written as

\[ V_L = Z_L I_L. \] (A.4.5)
Figure A.4.2  Signal flow graph of a voltage-source generator.
In terms of traveling waves, (A.4.5) becomes

\[ V_{iL} + V_{rL} = Z_L \left( \frac{V_{iL}}{Z_o} - \frac{V_{rL}}{Z_o} \right). \]  

(S.4.6)

Solving for \( V_{iL} \), we obtain

\[ V_{rL} = \frac{V_{iL} (Z_L - Z_o)}{Z_L + Z_o}. \]  

(S.4.7)

Define,

\[ b_L = \frac{V_{rL}}{\sqrt{Z_o}}, \]

\[ a_L = \frac{V_{iL}}{\sqrt{Z_o}}, \]

and

\[ \Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}. \]

Next, dividing both sides of (A.4.3) by \( \sqrt{Z_o} \) and using the above definitions we obtain

\[ b_L = a_L \Gamma_L. \]  

(S.4.8)

The signal flow graph follows from (S.4.8) and is shown in Figure A.4.3b. To demonstrate the utility of flow graphs, a two-port network is embedded between a signal-generator and a load. Combining the signal flow graphs of Figures A.4.1 through A.4.3 accordingly, a signal flow graph can be drawn for the system which is shown in Figure A.4.4.

To determine the ratio or transfer function \( T \) of a dependent to an independent variable, the Mason’s rule, or the non-touching loop rule which is defined below is applied [58].
Figure A.4.3  Signal flow graph of a load impedance.
Figure A.4.4  Signal flow graph of a two port network inserted between a signal-generator and a load impedance.
\[ T = \frac{P_1 \left[ 1 - \sum L(1) + \sum L(2) - \cdots \right] + P_2 \left[ 1 - \sum L(1) - \sum L(2) + \cdots \right] + \cdots}{1 - \sum L(1) + \sum L(2) - \sum L(3) + \cdots}. \] (A.4.9)

The different terms in (A.4.9) are defined as follows:

The terms \( P_1, P_2, \) and so on, are the different paths connecting the dependent and independent variables whose transfer function \( T \) is to be determined. A path is defined as a set of consecutive, codirectional branches along which no node is encountered more than once as we move in the graph from the independent to the dependent node. The value of the path is the product of all branch coefficients along the path.

The term \( \sum L(1) \) is the sum of all first-order loops. A first-order loop is defined as the product of the branches encountered in a round trip as we move from a node in the direction of the arrows back to that original node. The term \( \sum L(2) \) is the sum of all second-order loops. A second-order loop is defined as the product of any two non-touching first-order loops. Hence, in general, \( \sum L(i) \) corresponds to the sum of all \( i \)-th order loops, where, an \( i \)-th order loop is defined as the product of any \( i \) non-touching first-order loops.

The term \( \sum L(1)^P \) is the sum of all first-order loops that do not touch the path \( P \) between the independent and dependent variables. The term \( \sum L(2)^P \) is the sum of all second-order loops that do not touch the path \( P \) between the independent and dependent variables. Hence, in general, \( \sum L(i)^P \) corresponds to the sum of all \( i \)-th order loops that do not touch the path \( P \) between the independent and dependent variables.
BIBLIOGRAPHY


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