Failure Analysis of Notched Graphite-Epoxy Tubes

by

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Dissertation submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

in

Materials Engineering Science

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February 16, 1990

Blacksburg, Virginia
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(ABSTRACT)

Notched unidirectional graphite/epoxy tubes with fiber orientations of 2.5°, 15°, 45°, and 87.5° were failed in tension, compression, torsion, and combined compression-torsion loading. The stress field around the slot-like notches, aligned with the tube axis, was determined using an infinite flat plate elasticity solution with an elliptical hole. The normal stress ratio theory was used to predict crack location, crack direction, and failure stress. The experimental failure modes of the tubes were determined using scanning electron microscopy and related to the stress field in the vicinity of the notch.

The results showed that independent of loading the cracks usually initiated at the discontinuity of the notch where the semi-circular end intersects the straight sides and then grew along the fiber direction either at the fiber/matrix interface or within the matrix. The normal stress ratio theory correctly predicted the direction of crack growth but not the location of crack initiation since the model did not account for the notch discontinuity. The prediction of far-field failure stresses exhibited only limited agreement with the experimental results. When there was agreement, the predicted far-field failure stresses were dependent on the elliptical semi-axis ratio used to model the notch. The material principal stresses at the location of maximum normal stress ratio were correlated with the failure mode of the unidirectional tubes. Matrix failure tended to occur when the material principal shear stress and transverse stress (perpendicular to the fibers) were nearly equal in magnitude, while fiber/matrix interface failure was predominant when the stresses differed by a factor of 2.0 or more.

In addition, several notched angle-ply ± 87.5° tubes were failed in torsion. The normal stress ratio theory was applied to these tubes and correctly predicted that fiber breakage would occur. The predicted crack initiation stress agreed poorly with experimental results and
the location of the crack was influenced by the notch discontinuity not included in the model.
The direction of crack growth and the failure morphology of the angle-ply tubes were depen-
dent on the direction of torsion applied to the tubes.
Acknowledgements

This work was supported by Hercules, Inc., Magma, Utah, and the Center for Innovative Technology.

The author would like to thank Prof. C. T. Herakovich for his patience and guidance, also Prof. C. W. Smith, M. W. Hyer, R. E. Swanson, and Z. Gurdał for serving as committee members and for suggestions during the course of this study. The assistance of Garung Choksi, Jeff Hide, and especially Mara Knott was invaluable.

Special thanks goes to Mr. Don Pedersen and Mr. Bob Clark of Hercules, Inc., Magma, Utah, for the manufacture of the tubes. Mr. C. C. Poe, Jr., and Mr. George Trombly of NASA Langley deserve special acknowledgement for cutting the notches in the tubes. Also, Prof. Y. Renaldi must be recognized for insight into the difficulties of using complex numbers in computer programs.

This dissertation is dedicated to Ken, Arvonn, Kira, and Marel who provided support and encouragement to the author throughout this undertaking.
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1.0 Introduction

The increase in the use of fibrous composites for a wide variety of applications, particularly as critical structural members, has made it necessary to fully characterize not only the properties but also the failure modes of the material. An understanding of the failure modes is important not only to the theoreticians and experimentalists but also to the designer to permit optimum use of the unique properties of composite materials. Presently, material properties and response characteristics of composites are adequately predicted using mathematical models, but there remains a need for a mechanics based theory to characterize crack initiation and subsequent failure of composites. The economic use of composite materials may be lost through the need for overly conservative safety factors if the failure of a composite material cannot be accurately predicted.

The relationship between optimal structural design and failure mode is clearly demonstrated in experimental studies which have shown the effect of stacking sequence on failure mode. Figure 1 shows the failure modes for different angle-ply graphite-epoxy laminates. Clustered [(θ)_2/(−θ)_2], graphite-epoxy laminates failed parallel to the fibers by either fiber-matrix or intralaminar debonding. Alternating [(± θ)_2], laminates, however, exhibited fiber breakage in half the plies and either fiber-matrix debonding or matrix cracking in the remaining plies. As was shown by Herakovich, the strength of the laminate is related to the
Figure 1. Failure Modes of Angle-ply Graphite-Epoxy
failure mode. The 10° and 30° alternating laminates which exhibited a fiber breakage failure mode had failure strengths 30 and 59 percent greater, respectively, than the strengths of the clustered laminates. A fundamental understanding of this behavior will provide valuable assistance to the design process.

Before developing mechanics models and failure theories to predict laminate failure modes, a consistent method to characterize failure of a lamina is required. One of the fundamental failure problems is crack propagation in a unidirectional lamina. Fibrous graphite-epoxy composites have proven to be highly resistant to fatigue cracking and stable in many corrosive environments; however, the material is sensitive to defects that occur during both manufacture and service. Characterization of the crack growth from these flaws is necessary for the understanding and subsequent modeling and prediction of laminate failure modes.

The propagation of a crack in a unidirectional fibrous composite can occur either by growth across the fibers (fiber breakage) or between the fibers (matrix or fiber/matrix bond failures). Diverse criteria have been proposed for the prediction of crack growth initiation, direction, and stability. The conditions for crack growth by fiber breakage or even between the fibers are difficult to accurately predict, yet this is a necessary step toward a complete understanding of the fracture process.

Various theories have been evaluated on the basis of their ability to predict crack growth and failure loads. Of these, the best theories are based upon available analytical solutions that determine the stress distribution in the vicinity of idealized cracks in homogeneous, anisotropic materials for idealized loading and geometric configurations. In particular, the normal stress ratio theory has proven to be one of the more successful theories in its ability to predict crack growth and failure loads in notched unidirectional composites and hence has been chosen for use in this study.

The ultimate goal of current research efforts is to gain a better understanding of how fibrous composite laminates fail. Initially, crack propagation in unidirectional lamina was studied for tensile loading conditions. The next step was to study crack growth due to shear stresses in unidirectional lamina using off-axis tensile coupons and and losipescu tests. It is
the aim of this study to further evaluate the normal stress ratio theory by monitoring crack propagation in unidirectional tubular specimens subject to various loading conditions and consider the crack growth in one notched angle-ply laminate. This study has three main objectives. The first is to use the normal stress ratio theory to predict crack growth initiation and direction in the tension, compression, torsion, and combined torsion-compression loading of notched unidirectional graphite/epoxy tubes. By using tubular specimens, the effects of a finite width specimen are avoided and various combinations of axial and torsional loading can be considered. The second objective is to relate the observed failure modes to the predictions of the normal stress ratio theory and the stress field in the vicinity of the notch. The third objective is to apply the normal stress ratio theory to a shallow angle-ply laminate (± 87.5°).

This study is divided into the following five chapters. Chapter 2 reviews the published literature on fibrous, anisotropic tubes and the failure modes of fibrous composite materials. Chapter 3 describes the analytical model used to characterize a notched anisotropic tube. The experimental procedure is described in Chapter 4. The results and discussion are presented in Chapter 5 and conclusions are presented in Chapter 6.
2.0 Literature Review

2.1 Unnotched Anisotropic Cylinders

One of the first characterizations of the stress state in unidirectional composite tubes was presented by Sherr"er using an elasticity solution based on micromechanics models. He adapted a rectangular fiber model by Cutler and a hexagonal arrayed cylindrical fiber model by Hashin and Rosen to model thin and thick walled shells of various dimensions and fiber orientations. The technique based on the Hashin-Rosen model was shown to be better than the rectangular fiber model; however, it is very sensitive to the value chosen for the modulus of elasticity of the resin. Since the model is based on micromechanics, the solution is unwieldy and would become even more so in the presence of a notch.

Pagano and Whitney used a modified plane strain elasticity solution for a plate, combined with shell theory approximations, to illustrate the effect of anisotropy and geometry on the stress state of a tube subjected to simulated laboratory loads and end constraints. At a certain distance from the ends, the shell theory approximations were equivalent to the plate results.
Whitney and Halpin\textsuperscript{11} and Calcote\textsuperscript{12} applied Donnell's approximations to determine the response of composite tubes. They demonstrated that shell geometry is critical in determining the accuracy of the technique. In general, shell approximation techniques yield results similar to the plate solution for $R/t$ ratios greater than 20 where $R$ is the average radius of the tube and $t$ is the thickness.\textsuperscript{13}

Pagano\textsuperscript{14} developed an approximate elasticity solution for an anisotropic hollow circular cylinder subjected to surface tractions that are independent of the axial coordinate. This solution is expressed in the form of a Fourier series and can be readily applied to laminated cylinders.

Rousseau et al.\textsuperscript{15} developed an exact generalized plane strain elasticity solution for an anisotropic cylinder subject to internal pressure, external pressure, applied axial force, applied torque, and thermal loading. This technique has shown good agreement with experiment and is used in this study. This solution is described in Chapter 3 - Modeling.

### 2.2 Notched Anisotropic Cylinders

Several approximate solutions have been presented for a notched isotropic cylinder or shell.\textsuperscript{16, 17, 18} However, when attempting to derive the solutions for anisotropic materials, the problem becomes intractable and as yet no solution exists. For specially orthotropic materials, solutions utilizing higher order shell theories have been presented.\textsuperscript{19, 20, 21, 22} However, in these specially orthotropic solutions, it is assumed that the material properties can be modified to a pseudo-isotropic form; for example, the modulus, $E$, is approximated by $\sqrt{E_1E_2}$ where 1 refers to the longitudinal direction and 2 refers to the transverse direction, and the Poisson's ratio, $\nu$, is given by $\sqrt{\nu_{12}^2 \nu_{21}^2}$. These approximations may be acceptable for quasi-isotropic laminates,\textsuperscript{20, 21, 22} but are not generally acceptable for unidirectional tubes.
Carswell, et al\textsuperscript{13} utilized the finite element method to model the stress state of a notched composite tube. The purpose of the study was to determine the effect of hole or slit size on the stress concentration in (±55°) and (±75°) glass-polyester resin tubes subjected to tensile and hoop loading. In the model, it was assumed that the tubes had uniform thickness and quasi-isotropic properties. The quasi-isotropic assumption is not appropriate for either the unidirectional tubes or the angle-ply tubes studied.

Lakshminarayana et al\textsuperscript{14, 15} analyzed composite cylinders with holes using finite element analysis with an anisotropic triangular shell element. Their results show that coupled membrane-bending behaviour near the hole is affected by material properties, winding pattern, curvature, and loading. For axial tension loading of an orthotropic glass/epoxy tube, the calculated membrane stresses were almost identical to those calculated using a plate theory and the bending stresses were less than 10\% of the membrane stresses near the hole. The glass/epoxy tube had a 10\" radius, 30\" length, and 0.1\" thickness with a 1\" radius hole. The coupled membrane-bending behaviour at the hole for internal pressure loading is reduced for stiffer composite materials, i.e., higher the ratio of $\frac{E_1}{E_2}$ and is increased with a larger curvature factor: $\beta = \frac{r}{\sqrt{Rt}}$, where $r$ is the hole radius, $R$ is the tube radius, and $t$ is the wall thickness.

Utilizing the above mentioned finite element method, Sundaesan and Lakshminarayana\textsuperscript{16} considered notched crossply and angleply laminated glass/epoxy cylin-
ders subjected to axial tension. Their analysis showed that to minimize coupling which causes bending at the hole, the hole or notch must be small and the notch must be sufficiently far from the grips to avoid extraneous stresses. Based on this analysis, the length to radius ratio for the tubes in their study was 6.0 and the curvature factors, $\beta$, were 1.5 and 1.9. Their results showed fair agreement between theory and experiment at low load levels, i.e., in the region of the linear stress-strain response. Additional studies by Sundaesan\textsuperscript{27} using finite element analyses indicated that out-of-plane bending around noncircular notches is minimized if the longest dimension of the notch is aligned with the tube axis.
Since neither elasticity nor shell theory solutions for the notched anisotropic cylinder problem exist, plate theory solutions will be used to approximate the stresses at the notch in this study.

2.3 Anisotropic Fracture Theories

Due to the heterogeneous nature of fibrous composites, analysis of crack growth in these materials is very complicated. The use of isotropic fracture mechanics techniques for these materials is not appropriate because cracks in fibrous composites do not grow in a self-similar manner. There are two basic approaches for analysis of composites. One approach is to analyze the material on the micromechanical level. The second approach is to analyze the material on a macroscopic level.

**Micromechanics Approach**

Studies at the micromechanics level model the fiber, fiber/matrix interface, and the matrix properties as well as the geometric distribution of the distinct phases including thickness of the fiber-matrix interface. In order to model crack growth, assumptions must be made about factors such as the stress state within the different phases and the mode of failure. Because of these factors which increase the complexity of a model, the micromechanics approach is limited to simple laminates or unidirectional laminates. Furthermore, since so many assumptions are made, the applicability and accuracy of the technique is questionable.

**Macroscopic Approach**

Because of the difficulties associated with the micromechanics approach, a macroscopic level approach which incorporates the material inhomogeneity of composites into the analysis via equivalent material properties has been used by several investigators. The most popular approach is to evaluate the local stresses near a crack tip using a homogeneous anisotropic elasticity solution. The local stresses are then used in a crack growth theory to determine the
far-field stresses necessary to cause crack extension. Three crack growth theories (the tensor polynomial criterion, the strain energy density theory, and the normal stress ratio theory) are briefly described below.

The tensor polynomial criterion was originally developed for application to uncracked composites by Tsai and Wu.30 This criterion has been applied as a crack growth theory to the border of a region surrounding a crack tip by Lo et al.31 and others.3,3 Since the tensor polynomial was developed as a general failure criterion based on the existence of a failure surface in stress space, it does not have any hypotheses describing the parameters which may contribute to crack extension. The tensor polynomial criterion is written in the following form:

\[ f(\sigma_i) = F_i \sigma_i + F_{ij} \sigma_i \sigma_j \]  \hspace{1cm} (2.1)

where \( F_i \) and \( F_{ij} \) are strength tensors of the second and fourth order, respectively, and \( \sigma_i \) is the contracted form of the stress tensor. The strength tensors are defined such that failure corresponds to \( f(\sigma_i) = 1.0 \). For crack growth problems, it is assumed that the crack grows in the radial direction of maximum \( f(\sigma_i) \) when \( f(\sigma_i) \) equals 1.

The strain energy density theory has been applied to composites using the same form and approach as for isotropic materials. The theory hypothesizes that cracks will grow in the direction of minimum strain energy density factor.32 The strain energy density factor is a function of the local strain energy per unit volume around the crack tip. The theory does not account for the directional dependence of material strength.

The normal stress ratio theory was developed by Buczek and Herakovich3 as a crack growth criterion specifically for composites. It is a direct extension of the maximum normal stress theory based on the assumption that mode I crack opening controls crack growth. The directional dependence of material properties are included via a strength parameter. This theory also is used to predict the critical stress for crack extension which is defined as the far-field stresses causing the value of the normal stress ratio to equal 1 along the crack extension direction.
The normal stress ratio has been shown to predict the crack extension direction more accurately than either the strain energy density or the tensor polynomial techniques.\textsuperscript{\textregistered} In light of this correlation the normal stress ratio theory is being used exclusively in this study. A complete description of the normal stress ratio theory is included in the modeling section of this study (Chapter 3).

2.4 Composite Tube Testing

2.4.1 Specimen Design

The design of the gripping fixture for tubular specimens has received considerable attention from the standpoint of analyzing the stress concentration in the region of load introduction and maximizing biaxial load transfer. For testing graphite/epoxy laminates, the tubular specimens are either potted or cast into steel or aluminum end grips which are attached to the load frame. In order to reduce the stress concentration at the grips, the tubes have been reinforced at the ends with several different materials that taper to the gage section of the specimen. It should be noted that biaxial loading is often achieved by the application of internal pressure and axial load. A brief discussion of several end grips and their design follows.

Whitney et al.\textsuperscript{33, 34} provided a set of guidelines for determining the proper tube dimensions and discussed end attachments based on shell theory for unnotched tubes. They indicated that stress gradients through the thickness of unidirectional tubes are greater than those in laminates; hence greater care is required in designing the specimen holder. For combined axial, shear, and internal pressure loading, the stress gradients through the thickness of the tube decrease as the radius to thickness ratio increases. The degree to which
these gradients decrease also depends on the degree of anisotropy \( \frac{E_1}{E_2} \) and the fiber orientation. A conservative specimen length which insures minimum end effects is given as \((4R + L)\) where \(R\) is the radius of the tube and \(L\) is the gage length. Details on an appropriate test fixture are not provided.

Rizzo and Vicario\(^{25,26}\) used finite elements to characterize the stress distribution in gripped unnotched tubular specimens subject to axial, torsional, and internal pressure loading. They considered various radius to thickness ratios and length to diameter ratios of graphite/epoxy tubes. Their results showed good agreement with the shell theory approximations as developed by Whitney et al\(^{11}\) and provided further details of the stress state near the grips that are impossible to obtain with shell theory.

Using both theory and experiment, Whitney, Grimes, and Francis\(^{37}\) studied the effects of the end grips on the strength of symmetric, unidirectional, and bidirectional graphite/epoxy tubes. Collars consisting of three tapered steel sections bonded to each end of the tubes were used as grips for axial testing. Flat coupons were also tested for comparison. Analysis of the end tabs was accomplished using Flugge’s shell theory to determine the effect of end moments and load transfer on tube strength as predicted by the maximum strain failure criterion. For \(0^\circ\) unidirectional tubes, the experimental results indicated that tubes exhibit lower strength than flat coupons. It was concluded that the stress concentration in the tubes could be reduced if the end grips were designed to allow more flexibility or yielding at the adhesive bond.

Eckold et al\(^{38}\) used lamination theory and a semi-empirical criterion to predict failure of filament wound glass-epoxy tubes loaded axially and with internal pressure. These angle-ply tubes had fiber orientations of \(\pm 35^\circ, \pm 55^\circ,\) and \(\pm 72.5^\circ\). The tubes had a radius to thickness ratio of 35 and a radius to length ratio of 0.12. The threaded ends were cast onto the tubes and then mounted on the load frame. Origins of tube failure were not mentioned nor was the stress state near the grips.

Guess et al\(^{39,40,41}\) evaluated, both experimentally and analytically, two different end grip configurations for tubes subjected to uniaxial and biaxial loading. Graphite/epoxy and Kevlar/epoxy were made into either quasi-isotropic or unidirectional \(90^\circ\) tubes. Internal pres-
sure provided the biaxial stress state. The first grip consisted of an aluminum outer cap bonded to the outside of the tube and a threaded aluminum plug bonded to the inside of the tube. The threaded plug provided the connection to the load frame. This technique was not strong enough to test all the tubes considered and was difficult to assemble; therefore, an alternate threaded grip was developed. For the second grip, a glass cloth/epoxy overwrap was applied to the ends of the tubes, then cured. Threads were then machined into the wrap. A split collar aluminum cap was threaded onto the assembly and an aluminum plug with internal threads was fitted inside the tube. The split collar was mechanically clamped to the tube while the plug prevented collapse of the tube as well as a connection to the load frame.

Finite element analyses of both grips showed very little effect on the maximum stresses at the center of the gage length due to the different gripping techniques and predicted that failure would occur at the center of the gage section or at the grips. The uniaxial tests failed due to the tube pulling out from the grips for both composites tested. With the graphite specimens, the adhesive failed; for the Kevlar composites the failure of all but one tube occurred within the laminate in the region of the end grips. Specimens tested at high axial to hoop stress ratios tended to fail near the grips while those with low ratios failed in the gage sections. These fracture locations were in good agreement with the finite element predictions. There was also good correlation between the strains measured and predicted by the analysis. The authors concluded that both grips were acceptable for transferring load uniformly to the gage section of the tubes and that the threaded grip was easier to use.

In order to achieve a more uniform distribution of stress in a biaxially loaded tubular specimen, Duggan and Bailie proposed the concept of a constant compliance specimen. For this configuration, the tube specimen is tapered from the gage section to the tube ends while the tab material is thickest on both the inside and outside of the tube ends and tapered toward the gage section. The tube ends are then potted into steel grips which are attached to the load frame. Finite element analyses showed a drastic reduction in stress concentration with this configuration as compared to conventional tabbing. Experimental studies were conducted using a single graphite/epoxy cloth tube tabbed with glass/epoxy and loaded in axial tension,
torsion, and internal pressure. The specimen failed during the combined loading in the area of a manufacturing flaw. No detailed comparison was made with the finite element analyses.

Highten and Soden presented another method for gripping tubes for combined loading using tapered reinforcing tabs. The grip configuration was analyzed by modeling the tube, a reinforcing layer, and the grips as short parallel thin walled shells. The analysis was applied to 75° angle-ply glass/epoxy tubes which were tested for various axial to circumferential stress ratios. Utilizing the results of the analysis, they designed the taper, thickness, and length of a reinforcement layer which resulted in no experimental grip failures. The authors suggest that the details of the reinforcement shape where it blends into the gage length is critical in minimizing any stress concentrations.

Daniel et al used anisotropic finite elements to analyze the ends of tabbed quasi-isotropic cylinders. Various configurations were studied to minimize stress concentrations at the gage length-tab boundary when the cylinders were loaded biaxially. They found that internal pressure loading caused the maximum stress concentrations. Low stiffness tapered glass/epoxy tabs with tapered epoxy extensions reduce the stress concentration to less than 5% of the maximum applied stress. No experimental data were presented to validate the analysis.

Swanson et al completed several studies on biaxial testing of composite tubes. The gripping technique used to support the tube for axial, torsional, and internal pressure loading consisted of overwrapping the tube with glass/epoxy cloth, a tapered low modulus epoxy reinforcement, and aluminum rings. The fiberglass cloth was cast against a grooved clamp that attaches to the load frame. The quasi-isotropic laminates tested were made of AS4/3501-6 graphite/epoxy. Preliminary analysis of the gripping configuration was accomplished using linear elastic finite element analysis. The chosen design yielded stress concentrations of less than 1% the maximum applied stress either by internal pressure or axial loading. No cases of grip failure were reported in their experiments.

Based on the literature, tapered external reinforcement of the tube is necessary to minimize stress concentrations at the grips, especially for biaxially loaded specimens. The ta-
pering of internal tube plugs appears to slightly decrease the stress concentration at the grips. Therefore, the tube gripping design utilized in this work includes the reinforcement of the tubes with an extra layer of epoxy on the outside of the tubes and the tapering of the aluminum plugs that are bonded to the inside of the tubes.

2.4.2 Test Results

Several experimental studies on composite tubes have been completed by Swanson et al.\textsuperscript{45, 46, 47, 48} and Cole and Pipes.\textsuperscript{49} The tubes were either quasi-isotropic or 90\textdegree\ unidirectional graphite/epoxy loaded in tension, torsion, or biaxially by internal pressure. The unidirectional tube studies compared the shear properties determined by torsional loading of tubes versus the losipescu specimen.\textsuperscript{50} The tension and biaxial tests on quasi-isotropic material were conducted to determine the strength and elastic properties. Their work did not include the effect of flaws or notches on the failure of tubes.

Liber, Daniel, et al.\textsuperscript{51, 52} evaluated mechanical properties of quasi-isotropic laminated tubes loaded axially and biaxially at high strain rates. They did not examine the effects of flaws on tube failure.

Catastrophic failure of notched, quasi-isotropic, graphite/epoxy cylinders under internal pressure were studied by Chang and Mar.\textsuperscript{22} The notches in these large diameter tubes were slots cut at various angles to the tube axis. The slots were covered with aluminum strips to maintain the internal pressure. A method was presented that successfully predicted the failure pressure of the cylinders based on the failure of notched flat coupons.

In the work by Sundaresan et al.,\textsuperscript{26} the strain distribution around circular holes in cross-ply and angle-ply glass/epoxy tubes loaded axially was evaluated using finite element analyses. Fair agreement was reported between the experimental results and predictions.
The above works do not directly apply to the notched unidirectional tubes studied in this investigation; however, they provide insight into some of the considerations necessary in testing tubular specimens.

2.5 Phenomenological Aspects of Crack Growth

Fractography is necessary to determine the mode of failure at the tip of a crack in a composite, particularly the degree of different modes of failure. This section summarizes the phenomena associated with the fracture processes and crack growth in fibrous composites as described in the literature.

Several fracture mechanisms have been observed in cracked fibrous composite materials. These mechanisms are sensitive to the type of loading and the orientation of the loading axis with respect to the fiber direction, as well as the properties of the fiber and matrix. The mechanisms are best described in terms of unidirectional composites that are loaded axially or in shear. Descriptions of the failure modes applicable to fibrous epoxy-matrix materials follow.

2.5.1 Axial Failure Modes

In a unidirectional composite with tensile stresses applied parallel to the fiber direction, cracks extend by several different modes (Figure 2): 1) fiber pull out, 2) fiber debonding, 3) fiber fracture, and 4) matrix failure.53 54 55

The fiber pull out failure mode can occur when fibers break at flaws or weak points in the fiber. If the bond strength between the matrix and fiber is low, cracks may develop along the fiber/matrix interface in the region of this fiber break and grow allowing the fiber to be
Figure 2. Mechanisms of Axial Failure in Unidirectional Composites
pulled out of the matrix. If the bond strength is higher, a fracture may occur within the matrix leaving a layer of resin attached to the fiber which is then separated from the bulk of the matrix.

Fiber debonding or fiber-matrix interfacial failure is characterized by crack growth along the interface between fibers and matrix. In glass reinforced plastics, fiber debonding is indicated by the formation of splits or cracks at the fiber matrix interface before a matrix crack reaches and passes the fiber as seen in Figure 2. The splits are caused by the high stress field preceding the crack. In most cases, the debonding occurs after a matrix crack passes the fiber due to the mismatch in strain between the fiber and matrix. Subsequent to debonding, fiber pull out may be observed since the fibers will not necessarily fail in the same plane as the crack faces. Between the fibers, brittle fracture of the matrix is commonly seen.

Fiber failure in brittle composites with strong interfacial bonding is characterized by a tendency for cracks to jump from fiber break to fiber break rather than directly through the matrix independent of broken fibers. Thus, small zones within a fracture surface are observed where groups of fibers are related by a common origin of failure with no fiber pull out (Figure 3). These zones appear to break independently of each other.

The appearance of matrix failures depend on the characteristics of the resin. If the failure mode is ductile, the epoxy will exhibit large shear lips or ductile tearing. For epoxy matrices that fail in a brittle manner, the degree of epoxy phase separation has been shown to be important. If there is a phase separation, the failure will occur at the phase boundaries producing a cellular or rough appearing fracture surface, which can be observed at high magnification (appx. 5000X). The shape of these regions may be distorted by the presence of shear forces which will elongate the depressions left by removed material. With no phase separation, the matrix will show a fracture surface typical of brittle failure with a river pattern forming and pointing out the direction of crack growth.

For compressive loading, the fracture surface is more difficult to resolve due to the obliteration of the details during loading. Purslow has identified the primary modes of fiber failure are shearing, buckling, and microbuckling. Shear failure is considered the ideal form
Figure 3. Fiber Failure in Brittle Unidirectional Composites
of compressive failure. It occurs when there is a high interfacial bond energy between the matrix and fibers causing the fibers to shear and the matrix to fail concurrently. Fibers that have failed due to gross buckling of the specimen appear bent in uniform rows on the fracture surface or some distance away. Microbuckling failure is characterized by individual fibers having regions of both compressive and tensile failure in their cross section. As microbuckling progresses, the movement of areas already broken may cause these areas to contact each other and abrade, removing material from the fracture surface.

2.5.2 Shear Failure Modes

In off-axis tension tests and rail shear tests of unidirectional composites, a shearing failure mode is observed.\textsuperscript{55 56 57} A series of micro-fractures form within the matrix between the fibers. These micro-fractures originate within a shear band and form normal to the resolved tensile component of stress between fibers in the matrix. With increasing stress, these fractures elongate and curve as they reach the limits of the shear stress zone. They then coalesce at ultimate failure. This process is schematically illustrated in Figure 4. This type of failure leaves characteristic cusps or hackles on the fracture surface. If there are additional tensile stresses, the microcracks will not be aligned at 45° as depicted in Figure 4, but at some other angle normal to the tensile component of stress within the shear band. The ductility of the matrix material will determine the manner in which the hackles coalesce.

Several studies detailing the morphology of fracture surfaces associated with a notch in a graphite/epoxy composite were conducted in association with fracture mechanics experiments where notches were aligned parallel to the fibers\textsuperscript{57 58 59} or with interlaminar imbedded notches for analyzing the delamination process.\textsuperscript{60} The failure modes observed in these studies were primarily the shearing and matrix failure modes described previously.
Figure 4. Mechanisms of Shear Failure in Unidirectional Composites
3.0 Modeling

3.1 Notch Models

In order to provide a location for crack initiation, notches were machined into the tubes. The notches used in this study are not ellipses or cracks (which have been analyzed using infinite notched plate elasticity), rather the notches are slots with straight sides and semicircular ends as shown in Figure 5. This section describes the models used to approximate the notch geometry.

One technique is to model the notch as a sharp crack. This method has been shown to yield limited agreement with experimental results. However, this approach is limited in three ways. First, it can only be used for comparison with experiments where the actual notch is truly sharp. This can only be accomplished in unidirectional composites with the notch aligned parallel to the fibers and then with difficulty. The notches in this study could not be made sharp. Another limitation of the sharp crack solution is that the stresses are infinite at the crack tip necessitating the selection of a critical distance from the crack tip for failure analysis. The third difficulty is that the solution is applicable only to loading configurations where the crack does not close. In this study, the notches are aligned parallel to the tube axis.
Figure 5. Notch Models
<table>
<thead>
<tr>
<th>Notch Length 2b</th>
<th>Semi-Axis Model a/b</th>
<th>Radius Model a/b</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2&quot;</td>
<td>0.035</td>
<td>0.265</td>
</tr>
<tr>
<td>0.5&quot;</td>
<td>0.014</td>
<td>0.187</td>
</tr>
</tbody>
</table>

which results in crack closure for some loading configurations. Accordingly, the sharp crack model is not the choice for this study.

Another approach is to model the slot with an ellipse. When modeling a slot with an ellipse, there are two extremes that envelop the slot geometry. First, the ellipse can be of the same length and have the same minimum radius of the notch tip. By matching the radius and length, the width of the ellipse is greater than that of the notch. This model (Fig. 5b) will be referred to as the radius model in this study. The other extreme is to match the semi-axes of the ellipse with the length and width of the slot. This approximation (Fig. 5c) is called the semi-axis model in this study. For the notches used in the experimental study, the two notch models have the a/b ratios listed on Table 1.

As a third approximation, the notch can be modeled as a circular hole (a/b = 1.0) since the ends of the notch are semi-circular. This model will be referred to as the circle model and is included in this study for completeness.

### 3.2 Infinite Notched Plate Analysis
Figure 6. Elliptical Notch in Unidirectional Lamina under Biaxial Loading
3.2.1 Unidirectional Lamina

Since there is no tractable analytical technique for evaluating the stresses in a notched anisotropic cylinder, a notched infinite plate theory approach was used in this study. Lekhnitskii\textsuperscript{12} and Savin\textsuperscript{63} developed a solution for an elliptical hole (semi-axis ratio of $a/b$) at the center of an infinite homogeneous anisotropic plate (Figure 6) utilizing a complex potential technique. This method of solution is described below.

The equilibrium equations for the case where there are no body forces have the form:

$$\sigma_{ii,j} = 0$$  \hspace{1cm} (3.1)

These equations are identically satisfied for a plane stress state using the stress function, $U$, defined by

$$\sigma_x = \frac{\partial^2 U}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 U}{\partial x^2}, \quad \tau_{xy} = -\frac{\partial^2 U}{\partial x \partial y}$$  \hspace{1cm} (3.2)

A second requirement of an elasticity solution is that the strain compatibility equations be satisfied. The important compatibility equation for this problem is

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = 0$$  \hspace{1cm} (3.3)

The strain-stress relations for a composite modeled as a homogeneous anisotropic material are:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$  \hspace{1cm} (3.4)
where \( A_{ij} \) are the compliance coefficients which are expressed in terms of the elastic constants and the fiber orientation, \( \theta \), (Figure 6) as

\[
A_{11} = S_{11} \cos^3 \theta \sin \theta + (2S_{12} + S_{66}) \cos^2 \theta \sin^2 \theta + S_{22} \sin^3 \theta \cos \theta
\]

\[
A_{12} = S_{12} (\sin^3 \theta \cos \theta + \cos^3 \theta \sin \theta) + (S_{11} + S_{22} - S_{66}) \cos^2 \theta \sin^2 \theta
\]

\[
A_{22} = S_{11} \sin^3 \theta \cos \theta + (2S_{12} + S_{66}) \cos^2 \theta \sin^2 \theta + S_{22} \cos^3 \theta \sin \theta
\]

\[
A_{16} = (2S_{11} + 2S_{12} - S_{66}) \cos^3 \theta \sin \theta - (2S_{22} + 2S_{12} - S_{66}) \sin^3 \theta \cos \theta
\]

\[
A_{26} = (2S_{11} + 2S_{12} - S_{66}) \sin^3 \theta \cos \theta - (2S_{22} + 2S_{12} - S_{66}) \cos^3 \theta \sin \theta
\]

\[
A_{66} = 2(2S_{11} + 2S_{12} - 4S_{66}) \cos^2 \theta \sin^2 \theta + S_{66}(\cos^3 \theta \sin \theta + \sin^3 \theta \cos \theta)
\]

where

\[
S_{11} = \frac{1}{E_1}, \quad S_{22} = \frac{1}{E_2},
\]

\[
S_{12} = -\frac{v_{12}}{E_1} = -\frac{v_{21}}{E_2}
\]

\[
S_{66} = \frac{1}{G_{12}}
\]

and \( E_1 \) is the Young’s modulus in the fiber direction, \( E_2 \) is the Young’s modulus in the transverse direction, \( v_{21} \) and \( v_{12} \) are the Poisson’s ratios, and \( G_{12} \) is the shear modulus.

The strain-stress relations (Eqn. 3.4) are substituted into the compatibility equation (Eqn. 3.3) which then are written in terms of the stress function, \( U \) (Eqn. 3.2), to obtain the governing partial differential equation:

\[
\frac{\partial^4 U}{\partial x^4} - \frac{2A_{26}}{A_{22}} \frac{\partial^4 U}{\partial x^3 \partial y} + \frac{(2A_{12} + A_{66})}{A_{22}} \frac{\partial^2 U}{\partial x \partial y} - \frac{2A_{16}}{A_{22}} \frac{\partial^4 U}{\partial x \partial y^3} + \frac{A_{11}}{A_{22}} \frac{\partial^4 U}{\partial y^4} = 0
\]
Assuming that the stress function has the form:

\[ U = e^{x+\mu y} \]  \hspace{1cm} (3.8)

where \( \mu \) are the roots of the characteristic equation:

\[ A_{11}\mu^3 - 2A_{16}\mu^2 + (2A_{12} + A_{66})\mu - 2A_{26}\mu + A_{22} = 0 \]  \hspace{1cm} (3.9)

The roots of this equation, \( \mu_1 \) and \( \mu_2 \), are complex and not equal. These roots are functions of the material properties via the \( A_{ij} \).

The solution to the governing equation (Eqn. 3.7) is obtained in terms of two complex potential functions, \( \phi_1(Z_1) \) and \( \phi_2(Z_2) \), which have the following form:

\[ U = 2\text{Re}[F_1(Z_1) + F_2(Z_2)] \]  \hspace{1cm} (3.10)

where

\[ \phi_1 = \frac{dF_1(Z_1)}{dZ_1}, \quad \phi_2 = \frac{dF_2(Z_2)}{dZ_2} \]  \hspace{1cm} (3.11)

The complex variables, \( Z_1 \) and \( Z_2 \), are defined as:

\[ Z_1 = x + \mu_1 y, \quad Z_2 = x + \mu_2 y \]  \hspace{1cm} (3.12)

Savin\textsuperscript{63} develops the stresses for the infinite anisotropic plate with an elliptical hole in terms of the complex potentials as:

\[ \sigma_x = \bar{\sigma}_x + 2\text{Re}\{\mu_1^2\phi_1'(Z_1) + \mu_2^2\phi_2'(Z_2)\} \]  \hspace{1cm} (3.13)

\[ \sigma_y = \bar{\sigma}_y + 2\text{Re}\{\phi_1'(Z_1) + \phi_2'(Z_2)\} \]  \hspace{1cm} (3.14)

\[ \tau_{xy} = \bar{\tau}_{xy} - 2\text{Re}\{\mu_1\phi_1'(Z_1) + \mu_2\phi_2'(Z_2)\} \]  \hspace{1cm} (3.15)

Modeling
Figure 7. Conformal Mapping of an Ellipse into a Unit Circle

\[ z = c\left(\zeta + \frac{m}{\zeta}\right) \]
The $\phi_i$ functions are obtained by applying the boundary conditions (there are no tractions on the hole boundary) and using the conformal mapping of an ellipse in the $Z$ plane on to a unit circle in the $\zeta$ plane as illustrated in Figure 7. The mapping functions are of the form:

$$Z_k = \frac{a - i \mu_k b}{2} \zeta + \frac{a + i \mu_k b}{2 \zeta} \quad (k = 1, 2) \quad (3.16)$$

After mapping, the resulting derivatives of the complex potentials, $\phi'_1$ and $\phi'_2$, are expressed as:

$$\phi'_1 = K_1 \left\{ 1 - \sqrt{\frac{Z_1^2}{Z_1^2 - (a^2 + \mu_1^2 b^2)}} \right\} \quad (3.17)$$

$$\phi'_2 = K_2 \left\{ 1 - \sqrt{\frac{Z_2^2}{Z_2^2 - (a^2 + \mu_2^2 b^2)}} \right\} \quad (3.18)$$

where

$$K_1 = \frac{i(a - i \mu_1 b)}{2(\mu_1 - \mu_2)(a^2 + \mu_1^2 b^2)} \left[ \bar{\sigma}_x b + \bar{\sigma}_y j a \mu_2 + \bar{\tau}_{xy} (b \mu_2 + ja) \right] \quad (3.19)$$

$$K_2 = \frac{i(a - i \mu_2 b)}{2(\mu_1 - \mu_2)(a^2 + \mu_2^2 b^2)} \left[ \bar{\sigma}_x b + \bar{\sigma}_y j a \mu_1 + \bar{\tau}_{xy} (b \mu_1 + ja) \right] \quad (3.20)$$

This analytical solution for an infinite plate with an elliptical hole yields real stresses on the edge of the notch.

If the $a/b$ ratio equals 1, then the notch is a circular hole. Furthermore, it has been shown that if the $a/b$ ratio is 0.005 or less (for a notch aligned with the y axis, or 200 and greater for a notch aligned with the x axis), the stresses approach those calculated by the crack-like notch model.

When using computers to calculate the stresses utilizing the complex potential solution presented above, the stresses may become discontinuous due to digital computations. Techniques to avoid this problem are described in Appendix A.
Figure 6. Tangential Stress as a Function of Position for Tension Loading
Figure 9. Tangential Stress as a Function of Position for Torsional Loading
Elliptical holes with different a/b ratios can significantly affect the stresses in the vicinity of the notch. This effect is shown for tension loading of an 87.5° lamina in Figure 8 and for torsional loading in Figure 9. In these figures the stress, \( \sigma_\alpha \), tangent to the ellipse boundary at the edge of the hole, is plotted with respect to position around the notch. The tangential stress is concentrated along the sides of the notch in the case of tensile loading with the circle notch model having the largest stresses (Figure 8). It is noted that as the a/b ratio approaches a crack-like notch (a/b = 0.005 approximately), \( \sigma_\alpha \) starts becoming unstable as indicated by its increase near the notch tip. For torsional loading, the tangential stress approaches a singularity near notch tip of the semi-axis notch model. The radius and circle notch models have a maximum tangential stress less than half that of the semi-axis notch model (Figure 9). As indicated, the different a/b ratios have a considerable effect on the magnitude of stress concentration around the notch.

### 3.2.2 Laminate Analysis

The plate solution detailed above assumes effective properties, i.e. homogeneous anisotropic linear elastic material. For laminates, it is necessary to consider the stresses within each ply. Although there are several techniques for determining the ply stresses from a Lekhnitskii type solution,\(^{31}\) the simplest is to apply classical lamination theory.\(^{31}\) First, the characteristic equation (Eqn. 3.9) is determined using the laminate compliance matrix which has different values for \( A_{ij} \) than those given in Eqn. 3.4. Specifically, \( A_{ij} \) are determined using

\[
A_{ij} = \left[ \frac{1}{2H} \sum_{k=1}^{n} Q_{ij}^{(k)}(t_k) \right]^{-1}
\]

(3.21)

Here, \( Q_{ij}^{(k)} \) is the stiffness coefficients of the \( k^{th} \) ply with thickness \( (t_k) \) and 2H is the total thickness of the laminate. The \( Q_{ij}^{(k)} \) terms are
\[
\begin{align*}
Q_{11}^{(k)} & = U_1 + U_2 \cos 2\theta + U_3 \cos 4\theta \\
Q_{22}^{(k)} & = U_1 - U_2 \cos 2\theta + U_3 \cos 4\theta \\
Q_{21}^{(k)} & = Q_{12} = U_3 - U_3 \cos 4\theta \\
Q_{68}^{(k)} & = U_2 - U_3 \cos 2\theta \\
Q_{61}^{(k)} & = Q_{16} = \frac{1}{2U_2} \sin 2\theta + U_3 \sin 4\theta \\
Q_{62}^{(k)} & = Q_{26} = \frac{1}{2U_2} \sin 2\theta - U_3 \sin 4\theta
\end{align*}
\]

(3.22)

where \( U_i \) are the invariants

\[
\begin{align*}
U_1 & = \frac{1}{8} (3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}) \\
U_2 & = \frac{1}{2} (Q_{11} - Q_{22}) \\
U_3 & = \frac{1}{8} (Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}) \\
U_5 & = \frac{1}{2} (Q_{11} - U_3)
\end{align*}
\]

(3.23)

and the \( Q_{ij} \) are given by:

\[
Q_{11} = \frac{\varepsilon_1}{1 - \nu_{12}\nu_{21}}
\]
\[ Q_{22} = \frac{E_2}{1 - v_{12}^2} \]  
(3.24)

\[ Q_{12} = Q_{11}v_{21}, \quad Q_{66} = G_{12} \]

The notched plate solution (Eqns. 3.14, 3.15, and 3.16) is used to determine the stresses in the vicinity of the notch. Given these stresses, the laminate midplane strains, \( \varepsilon^T_j \), are determined using Eqn. 3.8 with the \( A_{ij} \) of Eqn. 3.21. Assuming that the strains are the same in every lamina, the inplane stress components are given by:

\[ \sigma^{(k)}_{ij} = \frac{Q^{(k)}_{ij} \varepsilon^T_j}{Q_{12}} \]  
(3.25)

With these stresses, the normal stress ratio theory can then be used to determine the ply where crack initiation occurs and the direction of crack growth with respect to the fiber orientation within each ply.

### 3.3 Unnotched Tube Analysis

Rousseau et al. 15 developed a generalized plane strain elasticity solution for an anisotropic cylinder subjected to internal pressure, external pressure, applied axial force and torque, and thermal loading. The solution utilizes polar coordinates and the nomenclature illustrated in Figure 10. The solution applied to a single layer is described as follows. Since the temperature of the tube is uniform and the applied pressures are independent of \( \theta \), none of the components of displacement depend on the circumferential coordinate and the derivatives of all stress, strain, and displacement components with respect to \( \theta \) are zero. Assuming generalized plane deformation, the stresses and strains are independent of \( x \) away from the
Figure 10. Geometry of Unnotched Anisotropic Tube
ends of the tube. The final assumption is that the radial displacement \( w \), is a function of the radial coordinate, \( r \), only. Based on the above assumptions the displacements are given by:

\[
U = u(x,r) \tag{3.26a}
\]

\[
v = v(x,r) \tag{3.26b}
\]

\[
w = w(r) \tag{3.26c}
\]

where \( u \) is the axial displacement and \( v \) is the hoop \((\theta)\) displacement. The strain-displacement relations in reduced form are given by:

\[
\varepsilon_x = \frac{\partial u}{\partial x} \tag{3.27a}
\]

\[
\varepsilon_\theta = \frac{w}{r} \tag{3.27b}
\]

\[
\varepsilon_r = \frac{\partial w}{\partial r} \tag{3.27c}
\]

\[
\gamma_{\theta r} = \frac{\partial v}{\partial r} - \frac{v}{r} \tag{3.27d}
\]

\[
\gamma_{xr} = \frac{\partial u}{\partial r} \tag{3.27e}
\]

\[
\gamma_{x\theta} = \frac{\partial r}{\partial x} \tag{3.27f}
\]

The thermo-elastic constitutive relation for a monoclinic material in cylindrical coordinates is given by:
\[
\begin{bmatrix}
\sigma_x \\
\sigma_\theta \\
\sigma_r \\
\tau_{\theta r} \\
\tau_{x r} \\
\tau_{x \theta}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\
C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\
C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36} \\
0 & 0 & 0 & C_{44} & C_{45} & 0 \\
0 & 0 & 0 & C_{45} & C_{55} & 0 \\
C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x - \varepsilon_T \\
\varepsilon_\theta - \varepsilon_T \\
\varepsilon_r - \varepsilon_T \\
\gamma_{\theta r} \\
\gamma_{x r} \\
\gamma_{x \theta} - \gamma_{x T}
\end{bmatrix}
\]

(3.28)

Of the six compatibility equations, three are automatically satisfied and three remain in simplified form as:

\[
\frac{\partial^2 \varepsilon_x}{\partial r^2} = 0
\]

(3.29a)

\[
\frac{1}{r} \frac{\partial \varepsilon_x}{\partial r} = 0
\]

(3.29b)

\[
\frac{1}{2} \frac{\partial}{\partial r} \left[ \frac{1}{r} \frac{\partial}{\partial r} (\gamma_{x \theta}) \right] = 0
\]

(3.29c)

The three equilibrium equations (neglecting the body forces) can be simplified based on the assumption given above and written:

\[
\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0
\]

(3.30a)

\[
\frac{\partial \tau_{\theta r}}{\partial r} + \frac{2\tau_{\theta r}}{r} = 0
\]

(3.30b)

\[
\frac{\partial \tau_{x r}}{\partial r} + \frac{\tau_{x r}}{r} = 0
\]

(3.30c)

The equations given above are sufficient to provide a complete solution for a single layer. When applied to a laminate, the analysis must account for the variation in properties from
layer to layer and the required continuity between layers. Boundary conditions, continuity of tractions and displacements between layers, zero rigid body motion, and constant applied load and torque are required to determine the constants that satisfy the equations describing the problem.

The boundary conditions for the inner and outer radii are: at the inner radius, $\sigma_r$ must equal the internal pressure and the surface shear stresses $\tau_\theta$, and $\tau_x$ must be zero and at the outer surface, $\sigma_r$ must equal the external pressure and the shear stresses, $\tau_\theta$ and $\tau_x$, must equal 0. Continuity requires that the tractions and the displacements at the interface between the $k^{th}$ and the $(k+1)$ layers be continuous.

The applied axial force, $F_x$, and applied torque, $T_x$, acting on the cross section of the tube are specified quantities. The applied forces are related to the ply stresses via the following equilibrium equations:

$$2 \pi \sum_{k=1}^{N} \int_{r_{(k-1)}}^{r_k} \sigma_x^{(k)} r dr = F_x$$

(3.31a)

$$2 \pi \sum_{k=1}^{N} \int_{r_{(k-1)}}^{r_k} \tau_{x \theta}^{(k)} r^2 dr = T_x$$

(3.31b)

The displacements are assumed to have the form:

$$u^{(k)}(x,r) = \epsilon_x^0 x$$

(3.32a)

$$v^{(k)}(x,r) = \gamma_x^0 xr$$

(3.32b)
\[ w^{(k)}(r) = A^{(k)} r^{\lambda^{(k)}} + B^{(k)} r^{-\lambda^{(k)}} + \left[ \frac{C_{12}^{(k)} - C_{13}^{(k)}}{C_{33}^{(k)} - C_{22}^{(k)}} \right] \varepsilon^o r + \left[ \frac{C_{26}^{(k)} - 2C_{36}^{(k)}}{4C_{33}^{(k)} - C_{22}^{(k)}} \right] \gamma^o r^2 + \left[ \frac{\Sigma^{(k)}}{C_{33}^{(k)} - C_{22}^{(k)}} \right] r \] (3.32c)

where:

\[ \lambda^{(k)} = \sqrt{\frac{C_{13}^{(k)}}{C_{33}^{(k)}}} \] (3.33a)

\[ \Sigma^{(k)} = (C_{13}^{(k)} - C_{12}^{(k)}) \varepsilon^{T(k)}_x + (C_{23}^{(k)} - C_{22}^{(k)}) \varepsilon^{T(k)}_\theta + (C_{33}^{(k)} - C_{23}^{(k)}) \varepsilon_r^{T(k)} + (C_{36}^{(k)} - C_{26}^{(k)}) \gamma^{T(k)}_{x\theta} \] (3.33b)

and \( \varepsilon^{T(k)}_x, \varepsilon^{T(k)}_\theta, \varepsilon_r^{T(k)}, \gamma^{T(k)}_{x\theta} \) are free thermal strains given by \( \varepsilon^{T(k)}_x = \alpha^o \Delta T \) The unknowns are \( \varepsilon^o, \gamma^o, A^{(k)}, \) and \( B^{(k)}. \) These are determined by solving simultaneously the equations for equilibrium (Eqn. 3.30) with the boundary conditions given above and the applied force/torque equations (Eqn. 3.31).

The resulting stresses are given by the following equations:

\[ \sigma^{(k)}_x = C_{11}^{(k)} \varepsilon_x^{(k)} + C_{12}^{(k)} \varepsilon_y^{(k)} + C_{13}^{(k)} \varepsilon_r^{(k)} + C_{16}^{(k)} \gamma_{x\theta}^{(k)} \] (3.34a)

\[ \sigma^{(k)}_\theta = C_{12}^{(k)} \varepsilon_x^{(k)} + C_{22}^{(k)} \varepsilon_\theta^{(k)} + C_{23}^{(k)} \varepsilon_r^{(k)} + C_{26}^{(k)} \gamma_{x\theta}^{(k)} \] (3.34b)

\[ \sigma^{(k)}_r = C_{13}^{(k)} \varepsilon_x^{(k)} + C_{23}^{(k)} \varepsilon_\theta^{(k)} + C_{33}^{(k)} \varepsilon_r^{(k)} + C_{36}^{(k)} \gamma_{x\theta}^{(k)} \] (3.34c)

\[ \tau^{(k)}_{x\theta} = C_{16}^{(k)} \varepsilon_x^{(k)} + C_{26}^{(k)} \varepsilon_\theta^{(k)} + C_{36}^{(k)} \varepsilon_r^{(k)} + C_{66}^{(k)} \gamma_{x\theta}^{(k)} \] (3.34d)

where:

\[ \varepsilon^{(k)}_x = \varepsilon_x^o + \alpha^{(k)} \Delta T \] (3.35a)

\[ \gamma^{(k)}_{x\theta} = \gamma_{x\theta}^o + \gamma^{(k)}_{x\theta} \Delta T \] (3.35b)
\[ \varepsilon_\theta^{(k)} = \left[ \frac{C_{12}^{(k)} - C_{13}^{(k)}}{C_{33}^{(k)} - C_{22}^{(k)}} \right] \varepsilon_\theta + \left[ \frac{C_{26}^{(k)} - 2C_{36}^{(k)}}{4C_{33}^{(k)} - C_{22}^{(k)}} \right] \gamma_{xf}r \\
+ \left[ \frac{\Sigma^{(k)}}{C_{33}^{(k)} - C_{22}^{(k)}} \right] + A^{(k)}r^{-\lambda^{(k)} - 1} + B^{(k)}r^{-\lambda^{(k)} - 1} + a_r^{(k)} \Delta T \tag{3.35c} \]

\[ \varepsilon_r^{(k)} = \left[ \frac{C_{12}^{(k)} - C_{13}^{(k)}}{C_{33}^{(k)} - C_{22}^{(k)}} \right] \varepsilon_r + 2 \left[ \frac{C_{26}^{(k)} - 2C_{36}^{(k)}}{4C_{33}^{(k)} - C_{22}^{(k)}} \right] \gamma_{xf}r \\
+ \left[ \frac{\Sigma^{(k)}}{C_{33}^{(k)} - C_{22}^{(k)}} \right] + \lambda^{(k)}A^{(k)}r^{-\lambda^{(k)} - 1} - \lambda^{(k)}B^{(k)}r^{-\lambda^{(k)} - 1} + a_r^{(k)} \Delta T \tag{3.35d} \]

The solution presented above is valid as long as \( C_{33}^{(k)} \) does not equal \( C_{22}^{(k)} \) which occurs for transversely isotropic and isotropic materials (see Eqn. 3.35c&d). Modifications to the Rousseau solution required to account for isotropic and transversely isotropic materials affect only the hoop displacement, \( w \). The differential equation used to define the displacement \( w \) is derived by writing the stresses in terms of displacements, then substituting them into the equilibrium equation, Eqns. 3.30, namely

\[ \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} - \frac{w}{r^2} = \frac{\Sigma^{(k)}}{C_{33}^{(k)} r} \tag{3.36} \]

Solving this equation for \( w \) yields:

\[ w^{(k)}(r) = A_r^{(k)}r + \frac{B_r^{(k)}}{r} + \frac{\Sigma^{(k)}}{2C_{22}^{(k)}} r \ln r \tag{3.37} \]

where \( A_r^{(k)} \) and \( B_r^{(k)} \) are the unknown constants analogous to \( A^{(k)} \) and \( B^{(k)} \) in Eqns. 3.32 above. The displacements, \( u(x,r) \) and \( v(x,r) \), are the same as presented in Eqns. 3.32a and 3.32b. The equations for the stresses (Eqns. 3.34) are the same; however, the strains \( \varepsilon_\theta \) and \( \varepsilon_r \) are now:

\[ \varepsilon_\theta^{(k)} = A_r^{(k)} + \frac{B_r^{(k)}}{r^2} + \frac{C_{26}^{(k)} - 2C_{36}^{(k)}}{3C_{22}^{(k)}} \gamma_{xf}r \\
+ \frac{\Sigma^{(k)}}{2C_{22}^{(k)}} ln r + a_\theta^{(k)} \Delta T \tag{3.38a} \]
\[
e_i^{(k)} = A_i^{(k)} - \frac{B_i^{(k)}}{r^2} + 2 \frac{C_i^{(k)} - 2D_i^{(k)}}{3C_{22}^{(k)}} \gamma_{x_i}^0 r + \frac{\eta_i^{(k)}}{2C_{22}^{(k)}} (\ln r + 1) + \alpha_i^{(k)} \Delta T \tag{3.38b}
\]

There is no significant difference between this solution and the completely anisotropic solution with \( E_2 \) and \( E_3 \) differing by between 5 and 10%.

### 3.4 Effects of Tube Curvature

It has been shown that the stress state in a unidirectional tube with a large \( R/t \) ratio is similar to that predicted for a flat coupon of infinite width subjected to equivalent membrane.\textsuperscript{10, 38, 65} Figure 11 shows the effect of pure shear for various \( R/t \) ratios, calculated using the generalized plain strain elasticity solution for an unnotched tube developed by Rousseau et al\textsuperscript{65} (detailed in 3.3 above) normalized with respect to the laminated plate theory solution. The figure indicates that for torsional loading there is less than 10% difference between the shear stress in a plate and tube for the \( R/t \) ratio in this study (\( R/t = 12.5 \)). Similar differences arise for the other loading. Accordingly, a notched flat plate solution should provide a reasonable estimate of the notch tip stresses for the tube of this study. However, to insure that curvature does not significantly affect the stress state, the following scheme is used to account for curvature effects.

In order to include curvature effects into the notched plate solution, the stresses that occur in a tube, (\( \sigma_x \), \( \sigma_y \), and \( \tau_{xy} \)), subjected to similar loading are used as the plate far-field stresses (\( \bar{\sigma}_x \), \( \bar{\sigma}_y \), and \( \bar{\tau}_{xy} \)). The tube stress along the circumference, \( \sigma_{th} \), and the shear stress, \( \tau_{th} \), are non-zero on the inner and outer radius and therefore, are used as limiting far-field stresses. The effect of utilizing this technique to account for curvature is shown in Figure 12. For this figure, a 45° plate with a vertical notch (\( a/b = 0.035 \)) is loaded in tension (along the \( Y \)-axis). Using the tube stresses in the infinite notched plate analysis alters the stress state.
Figure 11. Comparison of Plate and Tube Stresses for Shear Loading
Figure 12. Effect of Curvature on Stress Tangent to the Notch
at the notch, as shown for the stress tangent to the ellipse boundary at the edge of the notch, $\sigma_\phi$, plotted with respect to position around the notch. On the inside radius, the tangential stress reaches a maximum negative value, and on the outside radius, a maximum positive value is attained, while the plate solution yields a maximum $\sigma_\phi$ at $0^\circ$ and $180^\circ$. To determine the effect of curvature, if any, the stress state of notches at both the inner and outer radius of the tube is evaluated and presented in the results.

### 3.5 Normal Stress Ratio Theory

The normal stress ratio (NSR) is a phenomenological criterion originally developed by Buczek and Herakovich\textsuperscript{244} to predict the direction of crack growth in composites. The theory assumes that cracks grow due to tensile stresses only. The direction of crack extension corresponds to the maximum value of the ratio, $R(\phi)$, which is defined as the ratio of the normal stress acting on a radial plane defined by the angle, $\phi$, to the tensile strength normal to that plane:

$$R(\phi) = \frac{\sigma_{\phi\phi}}{T_{\phi\phi}}$$  \hspace{1cm} (3.1)

The tensile strength normal to the $\phi$ plane is assumed to have the form:

$$T_{\phi\phi} = X_t \sin^2 \beta + Y_t \cos^2 \beta$$  \hspace{1cm} (3.2)

where $\beta$ is the angle between the radial plane, $\phi$, and the fiber direction, $\theta$, as shown in Figure 13. The selection of this form of $T_{\phi\phi}$ is based on the following requirements for the directional dependence of $T_{\phi\phi}$:
Figure 13. Geometry of the Normal Stress Ratio
1. For isotropic materials, $T_{\phi\phi}$ must be independent of $\phi$.

2. In a plane perpendicular to the fibers, $T_{\phi\phi}$ must equal the fiber direction tensile strength $X_r$.

3. In a plane parallel to the fibers, $T_{\phi\phi}$ must equal the transverse tensile strength of the composite, $Y_r$.

The normal stress ratio theory has been applied by Beuth and Herakovich² to predict the critical applied far-field stresses for crack growth. The critical stresses are defined as the smallest far field stresses causing $R(\phi) = 1$ at some point near an existing flaw. It is obvious that the normal stress ratio theory is a variation of the maximum stress theory for isotropic materials modified to account for anisotropic strength. In particular, $T_{\phi\phi}$ provides the directional strength dependence expected for anisotropic materials.

In this study, the normal stress ratio is evaluated along the elliptical boundary itself,³ where the stresses are not singular as in the sharp crack model. Figure 14 illustrates the procedure used to apply the normal stress ratio to an elliptical notch as developed by Gurdal and Herakovich.³ The NSR is evaluated over a range of radial directions at each point along the elliptical boundary. The radial directions, $\Omega$, are varied over 180° bounded by the tangent to the ellipse at each position defined by the angle $\phi$. Predicted crack extension direction $\Omega_c$ occurs in the direction of maximum normal stress ratio at a point on the ellipse ($\phi_c$). Thus both the site of origin of crack extension and the direction of crack growth are obtained.
Figure 14. Normal Stress Ratio Applied to an Elliptical Hole
3.6 Influence of Notch Geometry and Curvature on NSR

Predictions

The normal stress ratio theory predictions are influenced by the notch geometry and tube curvature. In particular, the effect of the a/b ratio, distance from the notch boundary, and the inclusion of curvature effects should be considered.

When determining the far field stress for crack initiation using the normal stress ratio with a notched infinite plate solution, the predicted critical stress can be adjusted to account for the finite geometry of the test specimen via factors such as a specific distance from the notch, tube length, and/or notch length. As the distance from the notch increases the critical far-field stress for a 2.5° tube with an elliptical hole (a/b = 0.265) loaded in negative torsion increases as shown in Figure 15. As the distance from the notch increases, the effects of tube curvature become more significant. At approximately 0.04" from the notch, the critical stress predicted for the inner and outer tube radius differ from the plate stress by 10%.

The effect of the a/b ratio on the critical stresses predicted by the normal stress ratio theory follow the trends shown in Figure 16 and Figure 17. For axial loading (both tension and compression), as the a/b ratio increases, the critical stress decreases. For both positive and negative torsional loading (positive torsion is in the counter clockwise direction from the top of the tube), the critical stress reaches a maximum at an a/b ratio near, but not exactly, 1.0. These trends are the same for all fiber orientations considered in this study. The crack solution can be approximated by an a/b ratio of 0.005 for a vertical notch (or 200.0 for a horizontal notch).

It is interesting to note that the NSR theory critical stress predictions for axial loading decrease with increasing a/b ratio similar to the fracture toughness for mode I loading of notched isotropic plates (Kc). For torsional loading, the critical stress reaches a maximum near a/b = 1 similar to Kii in notched isotropic materials.70 71
Figure 15. Increase in the Critical Stress with Distance from the Notch
Figure 16. Critical Failure Stress vs a/b Ratio for a 15° Tube in Tension
Figure 17. Critical Failure Stress vs a/b Ratio for a 15° Tube in Torsion
Figure 18. Normal Stress Ratio for Different a/b Ratios
Figure 19. Effect of Curvature on Normal Stress Ratio
The position of crack initiation, $\phi_c$, where the normal stress ratio is a maximum is affected by the $a/b$ ratio as well as the fiber orientation and loading. Figure 18 demonstrates variation of the normal stress ratio with $a/b$ ratio around an elliptical hole in an $87.5^\circ$ tube loaded in positive torsion. As shown, the location of maximum normal stress ratio, $\phi_c$, varies from $144^\circ$ to $93^\circ$ using the different notch models.

With the effects of tube curvature introduced via alteration of the far-field stresses, $\phi_c$ changes by less than $5^\circ$ for the cases examined in this study. As an example, Figure 19 shows the normal stress ratio calculated using the inside radius, plate, and outside radius conditions on an $87.5^\circ$ tube in positive torsion for the radius notch model. Here the maximum normal stress ratio occurs at $111^\circ$ at the inside radius, at $110^\circ$ for the plate, and at $109^\circ$ for the outside radius. Since the location of crack initiation is not affected significantly by the curvature correction scheme, $\phi_c$ calculated using plate conditions are presented in the tabulated results in Chapter 5.

The predicted direction of crack extension, $\Omega_c$, is parallel to the fibers (within $3^\circ$) for almost all $a/b$ ratios, fiber orientations, and loading of unidirectional lamina. Similarly, introduction of the curvature corrections does not affect the crack growth angle. Thus, in the tabulated results only the critical crack growth direction determined using plate conditions are listed. An exception occurs with compression loading which can be explained by the dependance of the normal stress ratio theory upon tensile stresses for crack growth. With compression, the tensile stresses at the notch are very small or zero.
4.0 Experimental Program

4.1 Specimen Configuration

Tubular specimens provide a unique opportunity for investigating the properties of unidirectional composite materials. The major advantage in using tubes is the ability to arbitrarily vary axial and torsional loadings. In addition, any effects that may arise due to the finite width of the specimen and free edge effects are avoided. The main disadvantages in using tubular specimens are the difficulties associated with gripping the specimen and the effects of curvature on the stress state in the specimen.

4.2 Tube Fabrication

Filament wound tubes, nominally 2.0 in. OD x 0.08 in thick x 10.0 in. long, consisting of sixteen plies of AS4/3501-6 graphite-epoxy were manufactured by Hercules, Inc., Magna, Utah.
Special handling was required to insure that the tubes were truly unidirectional, that is the tubes were wound in one direction only. Tubes with fiber orientations of 2.5°, 15°, 45°, and 87.5° (measured from the axial direction) were fabricated. The 2.5° and the 87.5° specimens were studied because they were the closest approximation to 0° and 90° fiber orientations which could be obtained due to the limitations of the winding machines. One set of tubes was conventionally wound having a nominal fiber orientation of alternating plies of +87.5° and -87.5°. These tubes are referred to as having a ± 87.5° fiber orientation or ± 87.5 angle-ply.

Manufacture of the unidirectional tubes was difficult, especially for the 2.5° tubes. The major problem was removal of the tubes from the mandrel after curing. Because of the relative thermal contraction, the graphite-epoxy tube is in a shrink fit condition on the aluminum mandrel after curing. To overcome this problem, the aluminum mandrel was sleeved with teflon which has a much greater coefficient of thermal expansion than either the tube or the mandrel and shrinks readily away from the tube upon cooling.72

The manufacturing procedure used is described as follows. The tubes were wound on a dual material mandrel with an inner mandrel made of a 1.12" diameter aluminum rod 42" long covered by a 36" long, 0.25" thick teflon tube. After each ply was wound, it was cellowrap compacted (wrapped in heat shrink tape then heated to avoid using an autoclave), except after the fifth and tenth plies, at which time the tube was vacuum bagged and heat compacted at 200°F under vacuum for 10 minutes. Following the sixteenth ply, the tube was vacuum bagged and cured. The curing process began with a vacuum drawn to 22" of mercury and 100 psi pressure. When the autoclave pressure reached 25 psi, the bag was vented. Next it was heated to 275°F and held for 30 minutes, then heated to 350°F and held for 120 minutes. Finally, the pressure was released and the assembly cooled.
Figure 20. Micrographs of Filament Wound Tube of AS4/3501-6
4.3 Material

The tubes were made of tows of AS4 fibers consisting of approximately 12,000 filaments. Each filament has an average diameter of 0.0003 inches. The tubes have an average fiber volume fraction of 74% as measured from 10 photomicrographs of 2.5° tube sections at a magnification of approximately 200X. Six straight lines were drawn through each photo and the amount of matrix material along each line measured. These linear fractions, which are the same as the volume fraction, were averaged to yield the overall fiber volume fraction. A photomicrograph of a typical 2.5° transverse section is shown in Figure 20. Resin-rich regions are present around each tow. This is common for filament wound materials. There were very few voids evident in the material at this magnification.

Because the material used in this study had an unusually high fiber volume fraction (74%), it was necessary to experimentally determine the material properties given in Table 2. These properties were determined as described in Appendix B.

To insure that the properties were reasonable, they were compared to the same graphite/epoxy material (AS4/3501-6) with a lower fiber volume fraction. Using the rule of mixtures, $E_i$ and $X_i$ correspond to the values for the material with a fiber volume fraction of 58%$^9$ (specifically, $E_i = 18.3$ Msi, $X_i = 210$ ksi for the 58% volume fraction material). The values of $Y$ and $S$ do not differ from the 58 vol% material since they are influenced by the matrix material properties ($Y = 7.75$ ksi and $S = 14.4$ ksi). The other properties are reasonable when compared with calculations using the Aboudi's constitutive model$^{13}$ for determining material properties ($E_2 = 1.88$ Msi, $v_{12} = 0.235$, and $G_{12} = 1.10$ Msi).

4.4 Specimen Preparation
Table 2. Material Properties of Tubes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$</td>
<td>19.75 Msi</td>
</tr>
<tr>
<td>$E_2$</td>
<td>1.44 Msi</td>
</tr>
<tr>
<td>$G_{12}$</td>
<td>0.94 Msi</td>
</tr>
<tr>
<td>$v_{12}$</td>
<td>0.276</td>
</tr>
<tr>
<td>$X_r$</td>
<td>259 ksi</td>
</tr>
<tr>
<td>$Y_r$</td>
<td>7.2 ksi</td>
</tr>
<tr>
<td>$S$</td>
<td>12.0 ksi</td>
</tr>
</tbody>
</table>

After manufacture at Hercules, the tubes were shipped to Virginia Polytechnic Institute and State University where they were ultrasonically c-scanned using an Automation Industries, Inc., Sperry Products Div. Reflectoscope S80 type 500800 Model 50D907 and a US 450 Series Laboratory Scanner. This c-scan unit is designed for flat specimens; thus, a fixture was developed to roll the tubes beneath the transducer. The fixture (Figure 21) is similar to one constructed by Rose and coworkers. A 5 MHz ultrasonic transducer with a 1.25" focal length was used. The signal was reflected off a steel rod positioned approximately 0.25" from the inside of the tube.

After scanning, notches were machined in the tubes at NASA-Langley Research Center using a Branson Ultrasonic Impact Grinder Model UAM-20 with 6 mil steel shim stock used as the cutting tool. The corners of the shim stock were rounded so that sharp corners were avoided; the notch was then a slot with semi-circular ends (Figure 22). In order to insure a clean notch with no fraying of the fibers on the inside diameter, the tube was supported with an aluminum plug during machining.

The notches were measured after machining and found to have nominal dimensions of 0.007" wide by either 0.2" or 0.5" long. The notch length of 0.2" was selected so that the entire notch could be viewed using the video equipment described below. The 0.5" notches were made to overcome an unsatisfactory attempt to make notches using a 5 mil carbide slot cutter. The slot cutter left the notch ends with sharp corners and a curve through the thickness of the tube with the same radius as the cutter. An attempt was made to clean up these ends with a
Figure 21. Apparatus for C-scanning Tubes
Figure 22. Example of a 0.2" notch in a graphite/epoxy tube
5 mil diamond impregnated wire but this proved unsatisfactory because the wire was actually wider than the slot cutter (approx. 7 mils to 5 mils) making it difficult to thread the wire through the notch and it was difficult to keep the wire perpendicular to the tube while cutting. Therefore, these tubes were notched again in the same location using the ultrasonic impact grinder with half inch long shim stock.

The tubes were c-scanned a second time after the notches were introduced. This was done to insure that no damage was incurred by the notching process. Sample c-scans before and after notching are shown in Figure 23. This figure shows that no damage was introduced as a result of the cutting process.

Before the tubes were attached to the gripping assembly, they were dried in a National Appliance Co. Model 5851 vacuum oven at a temperature of nominally 100°F drawn to a vacuum of 28” of mercury for at least 24 hours. The average weight loss after this time in the oven was 0.06%. Thus, to determine if this was a reasonable drying time, several tubes were dried for up to 2 weeks. Their average total weight loss during this time was 0.07%. Accordingly, the 24 hr. drying period was deemed sufficient to remove moisture. The tubes were weighed using a Mettler Instrument Corp. Balance Model H80.

### 4.5 Load Introduction

The tube gripping assembly used in this study consisted of aluminum plugs that were bonded to the tube, steel sleeves to transfer the load from the plugs to an adaptor, and a hardened steel adaptor that fit the testing machine grips. This assembly is shown in Figure 24.

After notching, c-scanning, and drying, the tubes were prepared for bonding to the aluminum plugs. If the inside surface of the tube was smooth, it was roughened using a coarse rat tail file, then cleaned with a spray degreaser (Micro-Measurements M-line accessories
Figure 23. Ultrasonic C-scans of a 2.5° Tube With and Without a Notch
Figure 24. Gripping Assembly for Loading the Tubes
FTF-1 mild degreaser). Care was taken to insure that the filing did not break any fibers or penetrate deeply into the tube. All 87.5° tubes had a rough inside surface after manufacture because the inside of the tubes was machined slightly to remove any teflon or excess epoxy matrix.

Aluminum plugs that fit the inside diameter were first grit blasted using a Pangborn Grit Blaster in the Industrial Engineering and Operations Research Dept., Virginia Polytechnic Institute and State University. Next, the plugs were vapor degreased over boiling propanol for approximately 30 minutes. After degreasing, rubber gloves were worn by the investigator and all areas that contacted the plugs or tubes were cleaned with spray degreaser to insure that no oils, grease or other foreign products contaminated the surfaces until the bonding process was complete. The adhesive used to bond the tube to the plugs was Hysol 934, a high strength epoxy. To insure that the gap between the tube and the aluminum plug was maintained between 3-10 mils, 1% by weight of 0.003-0.005 in. diameter glass beads were added to the adhesive before bonding. Using a stainless steel spatula, the adhesive-bead mix was vigorously rubbed onto the plugs and the inside of the tube. The rubbing insured that all the fissures on the surfaces were coated and there was good contact between the adhesive and the surfaces.

After preparation, one end of the tube was bonded to the plugs and then the other end immediately following. With both end plugs bonded in place, the assembly was made concentric by keeping the plugs aligned with the outside radius of the tubes using the set up shown in Figure 25. Hysol 934 Epoxy was used to coat the outside of the tube in the region of the grip to provide added strength in the grip region. The tubes were left at room temperature and not moved for at least 12 hrs to allow proper setting of the adhesive. Finally, the tube/plug assembly was cured in a Blue M Electric Co., Stabl-therm Powermatic 70 Oven for a minimum of 24 hrs at 100°F.

After curing the adhesive, the tube assemblies were strain gaged with Texas Measurement type FRA-2 rosettes (0-45-90 degree with 2 mm gages). The notched tubes had two
Figure 25. Alignment Fixture to Insure Concentricity of Tubes
gages mounted on the outside: one on the side opposite the notch and another approximately one inch below the notch.

The gaged tubes were placed into the steel collar assembly shown in Figure 26. The collar transferred the load from the grip insert to the tubes using 3/8 in steel drill stock. Two different collars were used for testing. One was Schedule 40 Black pipe and the other was machined from thick walled steel pipe. The thick walled steel pipe was used for loading tubes in the direction of their maximum strength, specifically: 2.5° and 15° tubes loaded axially, 45° tubes loaded in positive torsion, and the 87.5° tubes loaded biaxially.

4.6 Test Procedure

The tube assemblies were tested using an Instron Servo-Hydraulic Biaxial Testing Machine Model 1350. The tubes were tested in tension, compression, negative and positive torsion, and combined compression/negative torsion. The axial tests were run in stroke control with a strain rate of approximately 0.1%/min. During the axial loading, the torque on the tube was maintained at zero load permitting free rotation of the tube. The torsion tests were conducted in rotary control with a shear strain rate of 0.1%/min and the load feedback was kept at zero load allowing free axial displacement. The biaxial tests were conducted in load control with both the axial load and torque being applied at a rate that corresponded to 1 ksi/min. (This corresponds to an axial strain rate of approximately 0.5%/min and a shear strain rate of about 0.1%/min.)

During the tests, the formation and growth of cracks from the notch were monitored using a video camera, monitor, and video tape recorder. Also, a microphone was positioned near the specimen to record any sounds emitted during the test. The video recording system used included the following items: Sony Videocassette Recorder Model VO-5800H, Panasonic WV-5470 Video Monitor, Realistic Stereo Electret Microphones Model 33-1065, Anchor Broad-
Figure 26. Complete Tube Assembly and Load Train
Figure 27. Testing Machine, Data Acquisition System, and Video System Used for the Experiments
cast Monitor Model 1400, Panasonic TV camera model WV1800, and Questar QM1 telescope. The telescope magnified the notch making it easier to see the crack formation. During the tests, data were acquired using a Solatron Orion Data Logger Model 3530F attached to an IBM AT Personal Computer. Strains, loads, and stroke/rotary feedback were recorded for all the tests. These data were recorded and analyzed using a version of MATPAC™ modified to work with the Solatron Data Logger. The stress-strain curves from all tests are shown in Appendix C. The testing machine, data acquisition, and video system used for the experiments are shown in Figure 27. All notched tests were videotaped using Sony KCA-60BRK tape.

4.7 Fractography

After each tube failed, great care was taken in removing the specimens from the testing machine to preserve the fracture surfaces for analysis using a scanning electron microscope (SEM). It was especially difficult to avoid further failure or damage of the fracture region after compression tests. One technique to minimize this damage was to keep the failed surfaces apart by carefully unloading the tubes and placing several layers of soft paper between the surfaces before removing the tube from the testing machine.

The use of an SEM requires a small cubic specimen with sides of approximately 0.5” in length. To remove the fracture surface near the notch, a Dremel Moto-tool Model 280-5 with a cut off wheel #409 was utilized. It was found that if the wheel was used for very short periods of time in the vicinity of the notch then the pieces did not significantly heat up. The pieces were carefully covered in soft tissue paper to avoid further damage to the fracture surface and stored until SEM analysis.

A Phillips Model SEM-550 scanning electron microscope in the Department of Plant Pathology, Virginia Polytechnic Institute and State University, was used for analysis of the fracture surfaces. The specimens were mounted on aluminum stubs using colloidal graphite.
paste and silver paint was put on the edges of the samples to insure a good electrical contact with the stub. In the SEM, samples were observed at both low and high magnifications to detail the nature of fracture.

4.8 Test Matrix

Tubes of the four fiber orientations were tested in tension, torsion, and biaxially. All except the 15° tubes were tested in compression. The biaxial loading tests were conducted in combined compression/negative torsion to test the normal stress ratio theory under the most severe conditions. The angle-ply tubes were tested in torsion only. The test matrix is listed on Table 3. Details of the tests are located in the next chapter - Results and Discussion.
<table>
<thead>
<tr>
<th>Loading</th>
<th>Fiber Orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension</td>
<td>2.5°, 15°, 45°, 87.5°</td>
</tr>
<tr>
<td>Compression</td>
<td>2.5°, 45°, 87.5°</td>
</tr>
<tr>
<td>Positive Torsion</td>
<td>2.5°, 15°, 45°, 87.5°, ± 87.5°</td>
</tr>
<tr>
<td>Negative Torsion</td>
<td>2.5°, 15°, 45°, 87.5°, ± 87.5°</td>
</tr>
<tr>
<td>Combined</td>
<td>2.5°, 15°, 45°, 87.5°</td>
</tr>
</tbody>
</table>
5.0 Results and Discussion

5.1 Crack Growth Location and Direction

Before describing the experimental results, it is necessary to define the crack initiation location and crack growth direction. The position of crack initiation, $\phi_c$, is measured clockwise positive from the y-axis as shown in Figure 28. The direction of crack growth, $\Omega_c$, is measured from the y-axis, clockwise positive.

Since the notch is not an ellipse, there are two special cases for the location of crack initiation. These are illustrated in Figure 29. One occurs when the crack begins where the semi-circular end intersects the straight side of the slot. It is called an “intersection failure” in the tabulated results. The location of these points is at $4.5^\circ$, $-4.5^\circ$, $175.5^\circ$, and $-175.5^\circ$. When a crack originates along the straight side of a notch, the failure is referred to as “side failure”.

The cracks emanating from the notch were symmetric for most of the tests regardless of the loading. For example, a $2.5^\circ$ tube loaded in negative torsion has cracks located at $\phi_c = 4.5^\circ$ and $-175.5^\circ$ (intersection failure) with growth angles, $\Omega_c$ of $2.5^\circ$ and $182.5^\circ$. This is shown in Figure 30.
Figure 28. Notation Used to Define Crack Growth Geometry
Figure 29. Illustration of Crack Initiation Location
Figure 30. Failure of 2.5° Tube in Negative Torsion
<table>
<thead>
<tr>
<th>Fiber Orientation (degrees)</th>
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<th>Ultimate Failure Stress (ksi)</th>
<th>Ultimate Failure Strain (%)</th>
<th>Crack Growth</th>
</tr>
</thead>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>Position $\phi_e$</td>
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</tr>
<tr>
<td></td>
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<td>4.5°i</td>
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<td>45</td>
<td>TE45F3</td>
<td>6.1</td>
<td>0.26</td>
<td>-4.5°i</td>
</tr>
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<td>TE45D3</td>
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<td>0.32</td>
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<td>-1.87</td>
<td>4.5°i</td>
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</table>

* - Intersection Failure
* - Tube Did Not Fail
$\ddagger$ - Side Failure
1 - Not Applicable
2 - Tube Failed at Grips
5.2 Unidirectional Tubes with 0.1" Notches

5.2.1 Axial Loading

The results of unidirectional tubes with 0.1" notches tested axially (both tension and compression) are summarized in Table 4. There were no tubes that failed by slow, stable crack growth. Audible cracking sounds heard during the tests could not be associated with crack growth.

The far-field stress-strain curves for the tests are presented in Appendix C.

5.2.1.1 Tensile Loading

Tubes with fiber orientations of 2.5°, 15°, 45°, and 87.5° were tested in tension. The 2.5° tubes did not fail when loaded to the testing machine capacity of 20 kips and the 87.5° tube failed at the grips. The 15° tubes failed at an average stress of 26.4 ksi with the cracks initiating at \( \phi_c = 4.5° \) and growing parallel to the fibers at \( \Omega_c = 15° \) (Table 4). One 45° tube, TE45D3, failed along the straight sides of the notch, while the other, TE45F3, failed at \( \phi_c = -4.5° \). The average failure stress for the 45° tubes was 6.7 ksi. Both the 15° and 45° tubes exhibited crack growth parallel to the fibers.

The critical far-field stresses as predicted using the normal stress ratio for the 2.5°, the 15°, and the 45° tubes loaded in tension are listed in Table 5. For the 2.5° tubes, the predicted critical failure stress ranges from 22.3 to 27.0 ksi for the different notch geometry and curvature effects. These stress levels are well below the maximum experimental far-field stress of approximately 42 ksi.

The 15° tests show good agreement with the radius model which predicted critical stresses of 28.4 ksi for the plate conditions and 25.5 ksi at the outer radius as compared to the
<table>
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<tr>
<th>Fiber Orientation</th>
<th>Experimental Stress (ksi)</th>
<th>Notch Model</th>
<th>Predicted Critical Stress</th>
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</thead>
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<td>Plate (ksi)</td>
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<td></td>
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<td>23.0</td>
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<td>Semi-Axis Radius</td>
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<td>-26.5</td>
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<td>45° Compression</td>
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<td>Semi-Axis Radius</td>
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<td>Circle</td>
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</table>

* Tube did not fail
Figure 31. Schematic of Crack Growth in 15° Tubes in Tension
<table>
<thead>
<tr>
<th>Fiber Orientation</th>
<th>Tube</th>
<th>Experimental Crack Growth</th>
<th>Notch Model</th>
<th>Predicted Crack Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Position $\phi_c$</td>
<td>Direction $\Omega_c$</td>
<td></td>
</tr>
<tr>
<td><strong>Tension</strong></td>
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<td>4.5° i</td>
<td>15°</td>
<td>Circle</td>
</tr>
<tr>
<td>15°</td>
<td>TE15C1B</td>
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<td></td>
<td>Semi-Axis Radius</td>
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<td></td>
<td>TE15A2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45°</td>
<td>TE45F3</td>
<td>-4.5° i</td>
<td>225°</td>
<td>Circle</td>
</tr>
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<td></td>
<td>TE45D3</td>
<td></td>
<td>45°</td>
<td>Semi-Axis Radius</td>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>Compression</strong></td>
<td></td>
<td>4.5° i</td>
<td>45°</td>
<td>Circle</td>
</tr>
<tr>
<td>45°</td>
<td>C45E3</td>
<td></td>
<td></td>
<td>Semi-Axis Radius</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>87.5°</td>
<td>C90L1A</td>
<td>4.5° i</td>
<td>87.5°</td>
<td>Circle</td>
</tr>
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<td></td>
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<td></td>
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</tbody>
</table>

s - Side Failure  
i - Intersection Failure
experimental value of 26.4 ksi (Table 5). The crack growth initiation site for the 15° tubes is predicted to be \( \phi_c = 65°, 32°, \) or 5° depending on the notch model with \( \Omega_c \) of 16° or 17° as shown on schematically on Figure 31 and listed in Table 5. Experimentally, the crack formed parallel to the fibers at a position of 4.5° yielding good agreement with the semi-axis model. However, it is noted that the crack is located at the intersection of the straight sides with the semi-circular notch ends.

The failure stresses of 45° tube tests agree best with the radius model predictions of critical far-field stress (5.7 ksi experimental vs. 7.7-9.2 ksi predicted) as indicated in Table 5. The predicted crack growth sites for the 45° tube in tension are 69°, 5°, and 37° for the different notch models and the crack growth direction angle is 44° for all models. The experimental results show cracks growing from the notch side or at the intersection opposite the predicted side (at -4.5°) (Figure 32 and Table 6). The actual direction of crack growth is parallel to the fibers; however, only specimen TE45D3 (which failed along the notch sides) has the cracks growing at 45°. Therefore, there is only limited agreement with the predictions.

The above results indicate that for tension loading of a unidirectional tube, except for the 2.5° tubes, the normal stress ratio theory using the radius model provides the best agreement with the experimental critical far-field stress for crack initiation. Since the cracks grew from the discontinuity in the slots or from the sides, the theoretical predictions could not account for this behavior. The predicted direction of crack growth agreed with the experimental results (parallel to fibers) in the correct direction for all cases but one which may have been affected by the discontinuity of the notch.

### 5.2.1.2 Compression Loading

Compression tests were conducted on tubes with fiber orientations of 2.5°, 45°, and 87.5°. The 2.5° tubes did not fail when loaded to the testing machine capacity. The 45° tube may have buckled slightly because just prior to failure the notch was observed to taper a small
Figure 32. Schematic of Crack Growth in 45° Tubes in Tension
amount. The failure stress for the 45° tube was -30.7 ksi. The crack initiation site was located at $\phi_c = 4.5^\circ$ (where the slot side intersects the semicircular end) and the crack grew at an angle of 45° parallel to the fibers. The 87.5° tube audibly resonated as it failed at -23.9 ksi with the crack growing parallel to the fibers ($\Omega_e = 87.5^\circ$) at a location of 4.5°. The results of the compression tests are listed on Table 4.

The failure stress predictions for the 45° tube have the best agreement with the semi-axis model inner radius (experimental -30.7 ksi vs. predicted -33.4 ksi). Experimentally, the tubes failed parallel to the fiber direction with the cracks originating at $\phi_c = 4.5^\circ$ (Figure 33). The predicted crack initiation position for the 45° tubes is at the notch tip for the different notch models with the growth predicted to proceed at 47° or 48°. Thus, the predicted crack growth direction agrees with the experimental results and the position of crack growth is influenced by the notch discontinuity which is not reflected in the predictions.

The far-field stress predictions for the 87.5° tube under compression (Table 5) exhibit some large values (104.0 ksi and 887.0 ksi) because the very small tensile stresses at the notch cause the critical normal stress ratio to be extremely small. However, there is good agreement with the outer radius prediction using the radius model (-24.8 ksi vs. -23.9 ksi experimental). The crack growth predictions (Table 6 and Figure 34) for the 87.5° material are interesting in that fiber breakage should occur according to the circle and semi-axis models that have $\Omega_e = 79^\circ$ and 55°, respectively. The radius model again shows better agreement with experiment in that the predicted crack grows almost parallel to the fibers at 274° on the opposite side of the notch at a position of -1°.

The theoretical predictions for compression loading exhibited limited agreement with the experimental results. The semi-axis model provided the best agreement with experimental results for the 45° tube for critical failure stress and the crack growth direction was correctly predicted to occur parallel to the fibers. The location of crack initiation was not predicted correctly for either the 45° or the 87.5° cases since the cracks grew from the notch discontinuity. The 87.5° critical failure stresses predicted using the radius model at the inner radius

Results and Discussion
Figure 33. Schematic of Crack Growth in 45° Tube in Compression
Figure 34. Schematic of Crack Growth in 87.5° Tube in Compression
agreed well with the experiment and the radius model also correctly predicted crack growth parallel to the fibers, but in the opposite direction.

5.2.1.3 Discussion

The 2.5° tubes did not fail when loaded axially (either in tension or compression) even though the predicted critical failure stresses were less than the machine capacity. The predictions may be explained by considering the geometry of the notch with respect to the fibers. Since the notch is small and angled at 0° roughly parallel to the fibers (2.5°), the tube strength is maintained by the fibers. Micro-mechanics becomes dominant in this configuration which is not included in the stress analysis. Furthermore, using classical lamination theory on an unnotched plate in tension, the stresses in the direction of the fibers, $\sigma_1$, for a 0° lamina and a 2.5° lamina are not significantly different; $\sigma_1$ normalized with respect to the applied load for 0° is 1.250 and for 2.5° is 1.247; and for $\sigma_2$ the normalized stresses are 0.0 and 0.002, respectively. Similarly, the difference in the predicted critical failure stress (using NSR) for a 0° composite with a vertical notch is approximately 30.0 ksi for the three models as compared to 26.5 ksi for the 2.5° lamina. Similar trends apply to those tubes loaded in compression.

The predicted critical stress for tensile loading developed using the radius model yields good agreement with the experimental values for all the cases except the 2.5° tubes. This may be explained by the curvature of the radius matching the curvature of the notch yielding a reasonable model of the stress state at the notch tip.

It is interesting to note that Peterson has used a radius model to characterize the stress state of slots, slits with circular holes at the ends, and double holes (two or more connected circular holes) in isotropic materials (steel and aluminum flat coupons) subjected to uniform tensile loading. For these materials, it has been suggested that the free edges along the slot or notch parallel to the applied load are traction free and the radius model correctly focuses on the region of the highest stress concentration. For the isotropic cases, this may be rea-
sonable for small notches in a large plate where the St. Venant's Principle may apply, i.e., the effects of the stress concentrating notch will not affect locations far from the stress concentrator, such as the sides of the slot. However, for anisotropic materials, it has been shown that the stress concentration of a slot can have far reaching effects. In this study for tubes subjected to tension loading, the results indicate that the radius model can be used to determine the stress state of notched anisotropic tubes.

For compression loading, the predicted critical stresses could not be correlated with a particular model. The 87.5° tube results agreed with the radius model prediction and the 45° results correlated with the semi-axis model.

The normal stress ratio theory correctly predicted the side of the notch where the crack initiates in all compression tests but one; however, the exact location of crack initiation was not accurate due to the inability to model the exact geometry of the slot. Most cracks started at the tangent points where the straight sides of the slot meet the semicircular ends. At this point there is a discontinuity which provides additional inducement for failure initiation. The direction of crack growth is predicted correctly along the fibers in all cases and in the appropriate direction from the notch for all but one case.

The modification of the notched infinite plate analysis to include curvature effects for the tube by using unnotched tube stresses as the far-field applied loads does not significantly affect the application of the normal stress ratio theory for tension loading since the plate solution was adequate for most cases. In fact, the spread in critical stress values between inside and outside radii is usually less than 25% with only a few exceptions, notably the ellipse model in tension and the 87.5° compression case. The outside tube radius usually had the lower critical stress, thus the crack should initiate there. However, it could not be determined experimentally if the cracks started at the inside or outside radius. Any bending effects could influence failure; however, with a vertical notch, these effects are minimized. Based on the results presented above, it is concluded that the plate solution was adequate for this study.
<table>
<thead>
<tr>
<th>Fiber Orientation (degrees)</th>
<th>Tube</th>
<th>Ultimate Failure Stress (ksi)</th>
<th>Ultimate Failure Strain (%)</th>
<th>Crack Growth Position $\phi_c$</th>
<th>Crack Growth Angle $\Omega_c$</th>
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</table>

i - Intersection Failure  s - Side Failure  
t - Crack at Top of Notch b - Crack at Bottom of Notch  
1 - Failed Away from Notch  
2 - Not Applicable  * - Tube Did Not Fail  
3 - Slow Crack Growth with the Crack Initiating at 5.9 ksi  
4 - Slow Crack Growth with the Crack Initiating at -4.4 ksi
5.2.2 Torsional Loading

The results of tubes tested in torsion are presented in Table 7. Of the tubes that failed, all failed catastrophically except the two 87.5° tubes; these tubes exhibited slow, stable crack growth. Audible cracking sounds heard during the tests could not be associated with crack growth except in the cases of stable crack growth.

The far-field stress-strain curves for the tests are presented in Appendix C.

5.2.2.1 Positive Torsion Results

Positive torsion tests were conducted for all four fiber orientations; however, the 45° and the 15° tube tests were unsuccessful. The 45° tubes did not fail when loaded to the testing machine capacity. Two 15° tubes were tested and failed at approximately 0.5° from the notch schematically shown in Figure 35. No flaws were evident either on the fracture surfaces or the C-scans of the 15° tubes to account for this behavior. The 2.5° tube failed catastrophically at 8.1 ksi and the 87.5° tube exhibited slow crack growth with the crack initiating at 5.9 ksi and ultimate failure occurring at 8.6 ksi. Both tubes exhibited crack growth parallel to the fibers.

The predicted critical far-field stresses for the 2.5° radius model (5.0 ksi) are slightly better than that predicted using the circle model (4.8 ksi) when compared to the experimental result of 8.1 ksi (Table 8). The predicted crack initiation site is -2° for the semi-axis model and -39° for the radius and circle models. The predicted direction of crack growth is 1° for all models (Table 9). The semi-axis model agrees best with the experimental crack initiation site as shown in Figure 36 and Table 9.

The predicted failure stress for the 45° tube was less than the testing machine capacity (10 kip-in which translates to approximately 15 ksi) for the semi-axis and radius models as indicated in Table 8. The circle model predicted critical far-field stresses of over 20 ksi which is consistent with the experimental results of the tubes not failing. This suggests that the
Figure 35. Schematic of Crack Growth in 15° Tube in Positive Torsion
<table>
<thead>
<tr>
<th>Fiber Orientation</th>
<th>Experimental Stress (ksi)</th>
<th>Notch Model</th>
<th>Inner Radius (ksi)</th>
<th>Plate (ksi)</th>
<th>Outer Radius (ksi)</th>
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</thead>
<tbody>
<tr>
<td>2.5° Positive Torsion</td>
<td>8.1</td>
<td>Circle Semi-Axis Radius</td>
<td>4.87</td>
<td>4.87</td>
<td>4.88</td>
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<td></td>
<td></td>
<td>1.57</td>
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<td></td>
<td></td>
<td>5.01</td>
<td>5.01</td>
<td>5.00</td>
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<td>*</td>
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<td>7.13</td>
<td>6.89</td>
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<td></td>
<td>8.47</td>
<td>8.20</td>
<td>7.95</td>
</tr>
<tr>
<td>45° Positive Torsion</td>
<td>**</td>
<td>Circle Semi-Axis Radius</td>
<td>25.4</td>
<td>22.8</td>
<td>21.0</td>
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<td></td>
<td></td>
<td></td>
<td>1.46</td>
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<td>9.92</td>
<td>10.2</td>
<td>10.3</td>
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<td>4.87</td>
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<td></td>
<td>2.46</td>
<td>2.46</td>
<td>2.44</td>
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<td>Circle Semi-Axis Radius</td>
<td>-4.29</td>
<td>-4.30</td>
<td>-4.32</td>
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<td>Circle Semi-Axis Radius</td>
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<td></td>
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<td>-2.23</td>
<td>-2.27</td>
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</table>

* Failed away from notch
** Tube did not fail
<table>
<thead>
<tr>
<th>Fiber Orientation</th>
<th>Tube</th>
<th>Experimental Crack Growth</th>
<th>Notch Model</th>
<th>Predicted Crack Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Position $\phi_e$</td>
<td>Direction $\Omega_e$</td>
<td>Position $\phi_e$</td>
</tr>
<tr>
<td>Positive Torsion</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5°</td>
<td>T0F</td>
<td>-1.5°</td>
<td>2.5°</td>
<td>Circle Semi-Axis Radius</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>87.5°</td>
<td>T90J</td>
<td>-4.5°i</td>
<td>267.5°</td>
<td>Circle Semi-Axis Radius</td>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Negative Torsion</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5°</td>
<td>T0F2N</td>
<td>2.9°</td>
<td>2.5°</td>
<td>Circle Semi-Axis Radius</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45°</td>
<td>T45B2N</td>
<td>s</td>
<td>45°</td>
<td>Circle Semi-Axis Radius</td>
</tr>
<tr>
<td></td>
<td>T45F1N</td>
<td>4.5°i</td>
<td>45°</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T45E2N</td>
<td>2.3°</td>
<td>45°</td>
<td></td>
</tr>
<tr>
<td>87.5°</td>
<td>T90KN</td>
<td>s</td>
<td>2.5°i</td>
<td>Circle Semi-Axis Radius</td>
</tr>
</tbody>
</table>

s - Side Failure  
i - Intersection Failure  
t - Top of Notch  
b - Bottom of Notch
Figure 36. Schematic of Crack Growth in 2.5° Tube in Positive Torsion
stress concentration effect of the notch in torsion at the load levels in this study is insufficient to cause major damage leading to failure.

For positive torsion loading, crack growth in the 87.5\(^\circ\) tube initiated at 5.9 ksi and grew parallel to the fibers at \(\Omega_c = 267.5\,^\circ\) from \(\phi_c = -4.5\,^\circ\). Comparing these results to those predicted in Table 8 and Table 9 shows that the circle model predicts the critical stress best at 4.9 ksi and the predicted crack direction is 269\(^\circ\) for all models. The location of crack initiation is predicted to occur away from the notch tip at angles of -3\(^\circ\) for the semi-axis model, -20\(^\circ\) for the radius, and -64\(^\circ\) for the circle model. The actual crack initiated at the notch discontinuity which is not modeled adequately (Figure 37).

In summary, the circle model predictions agree best with the experimental critical failure stresses for 45\(^\circ\) and 87.5\(^\circ\). The radius model prediction of critical stress for the 2.5\(^\circ\) tubes was different from the experimental value by almost a factor of 2 while the circle model prediction was slightly worse. The crack initiation site was influenced by the discontinuity in the notch which is not characterized by the model. The direction of crack growth was correctly predicted by the models to be parallel to the fiber direction.

5.2.2.2 Negative Torsion Results

The negative torsion test results for the 2.5\(^\circ\), 45\(^\circ\), and 87.5\(^\circ\) tubes are listed in Table 7. The 2.5\(^\circ\) tube failed at 8.7 ksi parallel to the fibers. The 45\(^\circ\) tubes failed at different locations around the notch, even though they failed at approximately the same failure stress (average 2.65 ksi). The 87.5\(^\circ\) tube exhibited slow crack growth with crack initiation occurring at 4.4 ksi; however, it failed a short distance from the notch tip on one end. This tube may have been improperly supported during notching, thus some damage was incurred on the inside radius of the tube as shown in Figure 38.
Figure 37. Schematic of Crack Growth in 87.5° Tube in Positive Torsion
Figure 38. Damage Incurred During Notching of Tube T90KN
Figure 39. Failure of 45° Tube Showing Fibers Pulled Across the Crack.
The 45° tubes failed in a unique manner when subjected to the negative torsion loading. The fibers did not break but were pulled out of the tubes bridging the gap emanating from the notch. A 45° tube loaded in negative torsion is shown in Figure 39.

The predicted critical failure stresses of the circle model correlated best with the experimental results for the three fiber orientations (Table 8). The experimental critical failure stress for the 2.5° tube was approximately twice that predicted by the circle model (8.7 ksi vs. 4.3 ksi). The 45° and 87.5° tubes failed at approximately the predicted far-field stress using the circle model with plate loading.

The crack growth initiation site and direction predictions (Table 9) show failure away from the notch tip and parallel to the fibers for all cases. The semi-axis model prediction for the position of crack initiation ($\phi_c$) appears best for the 2.5° case as shown in Figure 40. The 45° tubes failed at different positions - one along the side, one at the discontinuity of the notch, and the third at an angle in between (2.3°). The semi-axis model prediction (1°) agrees best with the tube failing at 2.3° (Figure 41). The position of crack growth for the 87.5° tube cannot be fairly compared to the predictions because of the damage induced during notching; however, the crack growth is schematically shown in Figure 42.

With negative torsion loading, the circle model provides the best model for predicting the critical stress while the semi-axis model compares favorably with the crack growth initiation location and direction.

**5.2.2.3 Discussion**

When notched unidirectional tubes are loaded in torsion, the normal stress ratio theory can be used to predict the critical failure stress with the notch modeled as a circular hole. This result is different from those obtained for tensile loading where the radius model predictions correlated better with the critical failure stresses for axial loading. This correlation may be explained by the stress concentration at the ends of the notch dominating the stress field in
Figure 40. Schematic of Crack Growth in 2.5° Tube in Negative Torsion
Figure 41. Schematic of Crack Growth in 45° Tubes in Negative Torsion
Figure 42. Schematic of Crack Growth in 87.5° Tube in Negative Torsion
Figure 43. Normalized Tangential Stress for an 87.5° Tube in Positive Torsion
Figure 44. Normalized Tangential Stress for an 87.5° Tube in Tension
torsion and the straight sides being relatively stress free. To illustrate this change in stress field, Figure 43 and Figure 44 show the normalized stress tangent to the edge of the notch at the edge and 0.05" from the edge around both a circular hole and an elliptical hole with \(a/b = 0.265\) (radius model for an 0.1" notch) for positive torsion and tension loading, respectively. Comparing the stress magnitude at the edge of the holes with that 0.05" from the hole, the stress drops off more rapidly to the far-field load around the circle than the radius model ellipse. In fact, within the first 0.05" from the notch the stresses have decreased almost to the far-field stresses whereas with the radius model the far field stresses are not reached until beyond 0.1" which is critical especially for the notches that have a half notch length of 0.25". Because of this drastic drop in stresses, the stress concentration effects of the notch may be very limited in the 87.5° tubes and others where the circle model predictions are applicable.

As discussed previously for the axial loading cases, the use of the unnotched tube stresses as the far-field applied stresses to a notched infinite plate does not improve correlation with the experimental results. This attempt to include curvature effects is therefore unnecessary. The outside radius usually had the lower critical stress, thus the crack should initiate there. Again, it could not be determined experimentally if the cracks started at the inside or outside radius.

### 5.3 Unidirectional Tubes with 0.25" Notches

Several tubes with notch lengths of 0.25" were tested in tension, compression, and torsion. Specifically, tests were conducted on 87.5° and 2.5° tubes in tension, a 45° tube in compression, 2.5° and 45° tubes in positive torsion, and a 15° tube in negative torsion. The results of these tests are summarized in Table 10. The far-field stress strain curves for these tests are found in Appendix C. Any audible cracking sounds monitored during testing could
### Table 10. 0.25" Notch Experiments

<table>
<thead>
<tr>
<th>Fiber Orientation</th>
<th>Tube</th>
<th>Ultimate Failure Stress (ksi)</th>
<th>Ultimate Failure Strain (%)</th>
<th>Crack Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Position φₗ</td>
</tr>
<tr>
<td>TENSION</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5°</td>
<td>TE0D2</td>
<td>*</td>
<td>*</td>
<td>N/A¹</td>
</tr>
<tr>
<td>87.5°</td>
<td>TE90L1B</td>
<td>2.9</td>
<td>0.19</td>
<td>4.5°i</td>
</tr>
<tr>
<td>COMPRESSION</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45°</td>
<td>C45D2</td>
<td>-24.1</td>
<td>-1.5</td>
<td>st 180°b</td>
</tr>
<tr>
<td>POSITIVE TORSION</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5°</td>
<td>T0C3</td>
<td>4.2</td>
<td>0.36</td>
<td>0°</td>
</tr>
<tr>
<td>45°</td>
<td>T45D1</td>
<td>12.75²</td>
<td>0.83</td>
<td>-0.9°</td>
</tr>
<tr>
<td>NEGATIVE TORSION</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15°</td>
<td>T15CN</td>
<td>-2.84</td>
<td>0.30</td>
<td>3.7°</td>
</tr>
</tbody>
</table>

1 - Intersection Failure  
† - Crack at Top of Notch  
¹ - Not Applicable  
* - Tube did not Fail  
² - Slow Crack Growth with the Crack Initiating at 7.1 ksi
not be associated with crack growth except in the case of slow crack growth of the 45° tube in positive torsion.

5.3.1 Tension

The 2.5° tube did not fail similar to the 0.1° notched tubes loaded axially as described in section 5.2.

One 87.5° tube was tested in tension. The tube failed at 2.9 ksi with a crack growth initiation site, \( \alpha_s = 4.5° \), and crack growth direction, \( \Omega_s = 87.5° \), as shown in Figure 45. Comparing the critical stress to the predicted value using the normal stress ratio, there is good agreement with the circle model as indicated on Table 11. The location of crack initiation was influenced by the discontinuity in the notch. The direction of crack growth was parallel to the fibers as predicted by the NSR theory using all models. Crack growth is schematically shown in Figure 45.

5.3.2 Compression Results

One 45° tube was tested in compression. Failure occurred catastrophically at -24.1 ksi and asymmetrically with one crack initiating along the side of the notch center and the other at the bottom of the notch then growing parallel to the fibers (Figure 46). There was no indication of damage in the C-scans of this tube. Comparing the critical stress to those predicted using the normal stress ratio, the semi-axis model yields excellent agreement (24.0 ksi predicted vs. 24.1 ksi experimental) and the radius model yields fair agreement (inner radius at 25.0 ksi and plate at 22.5 ksi) as shown in Table 11. The initiation site of failure does not agree with the predictions due to the asymmetrical crack growth (Table 12). The direction of crack growth is predicted correctly. (Table 12 and Figure 46).
<table>
<thead>
<tr>
<th>Fiber Orientation</th>
<th>Experimental Stress (ksi)</th>
<th>Notch Model</th>
<th>Outer Radius (ksi)</th>
<th>Plate Radius (ksi)</th>
<th>Predicted Critical Stress</th>
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<tbody>
<tr>
<td>2.5° Tension</td>
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<td>Circle</td>
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<td>27.0</td>
<td>28.3</td>
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<tr>
<td>87.5° Tension</td>
<td>-24.1</td>
<td>Circle</td>
<td>27.0</td>
<td>24.2</td>
<td>28.3</td>
</tr>
<tr>
<td>45° Compression</td>
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<td>Circle</td>
<td>27.0</td>
<td>7.70</td>
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<td>2.5° Positive Torsion</td>
<td>4.87</td>
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<td>5.82</td>
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* Tube did not fail
<table>
<thead>
<tr>
<th>Fiber Orientation</th>
<th>Tube</th>
<th>Experimental Crack Growth</th>
<th>Notch Model</th>
<th>Predicted Crack Growth</th>
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<tr>
<td></td>
<td></td>
<td>Position $\phi_c$</td>
<td>Direction $\Omega_c$</td>
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</tr>
<tr>
<td>87.5° Tension</td>
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<td>4.5°i</td>
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<td>Circle</td>
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</tr>
<tr>
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<tr>
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<td>T45D1</td>
<td>-1.9°</td>
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<td>Semi-Axis Radius</td>
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<td></td>
</tr>
<tr>
<td>15° Negative Torsion</td>
<td>T15CN</td>
<td>3.7°</td>
<td>15°</td>
<td>Circle</td>
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</table>

$t^*$ - Crack formed at top of notch only
$s$ - Side Failure
$i$ - Intersection Failure
Figure 45. Schematic of Crack Growth in 87.5° Tube with 0.25° Notch in Tension
Figure 46. Schematic of Crack Growth in 45° Tube with 0.25° Notch in Compression
When comparing the critical stress predictions for the 45° tubes in compression with a 0.1" notch (Table 5) to those with a 0.25" notch, the predicted critical stresses are less than or equal to those for the 0.25" radius and semi-axis models. This is because the elliptical notch solution is dependent on the a/b ratio and not the physical values of a and b. This result also suggests that it may be necessary to include a factor to account for the difference in notch size.

### 5.3.3 Positive Torsion Results

As indicated in Table 10, one 2.5° tube was tested in positive torsion. The 2.5° tube failed at 4.2 ksi with the crack initiating at the tip of the notch and growing parallel to the fibers. The predictions for crack initiation using the radius model shows excellent agreement with the predicted critical far-field stress at 4.29 ksi (Table 11). The predicted crack initiation location of -8° for the radius model is not as good as the -1° location predicted using the semi-axis model shown in Figure 47. The direction of crack growth is predicted to be 1° for all the models which is nearly parallel to the fibers.

The 45° tube exhibited slow, stable asymmetric crack growth which initiated at 7.1 ksi and ultimately failed at 12.7 ksi. The crack initiated at the top of the notch, then grew parallel to the fibers at an angle of 225°. After growing approximately 0.5°, the crack stopped growing in that direction and began to grow at an angle of 45° parallel to the fibers. No crack formed at the bottom of the notch. This crack is shown in Figure 48. There was no indication of damage in the C-scans that could account for this behavior. Comparing the experimental results with those predicted by the normal stress ratio, the radius model yields favorable results with a difference of less than 10% from the experimental critical stress for crack initiation. The radius model predicts that the crack should initiate at -18° from the notch tip and grow parallel to the fibers at 225° as illustrated in Figure 49 and Table 12. Since the crack initiation is asymmetric, there may have been flaws or some other factor affecting failure of this tube.
Figure 47. Schematic of Crack Growth in 2.5° Tube with 0.25° Notch in Positive Torsion
Figure 48. Crack and Growth for Positive Torsion Test of 45° Tube Having Slow Crack Growth.
Figure 49. Schematic of Crack Growth in 45° Tube with 0.25" Notch in Positive Torsion
The results show that the radius model adequately predicts the critical stress for crack initiation as for the 0.1" notches described previously. The predicted location of crack initiation using the models could not be fairly compared with the experimental results due to the asymmetric failure of the 45° tube. For the 2.5° tube, the semi-axis model provided the best agreement.

5.3.4 Negative Torsion

One 15° tube was tested in negative torsion. This tube failed suddenly at 2.84 ksi with the crack initiating at $\phi_c = 3.5°$ and growing parallel to the fibers. The predicted critical stresses for both the circle and the radius models are within 25% of the experimental failure stress with the inside radius prediction of the circle model being the best (Table 11). As for crack growth, the radius model predicts the position of crack initiation at 6° with a growth angle of 15° (Figure 50 and Table 12).

As discussed previously in the negative torsion results for tubes with 0.1" notches (Section 5.2.4), failure of this tube exhibited fibers pulled out of the matrix and bridging the crack as shown in Figure 39. The radius and circle model predictions for critical stress agree best with experimental results due to the matching of notch tip curvature as discussed previously.

5.4 Unidirectional Tubes Loaded Biaxially

Notched tubes were tested in combined compression-negative torsion with load ratios such that $\frac{\sigma}{\tau}$ equal 1.0 and 10.0 as summarized in Table 13. All tubes failed with cracks growing parallel to the fibers in an unstable manner. One tube of each fiber orientation with notch length of 0.1" had a compression/negative torsion ratio of 1.0. One additional 87.5° tube
Figure 50. Schematic of Crack Growth in 15° Tube with 0.25" Notch in Negative Torsion
### Table 13. Combined Loading Experiments

<table>
<thead>
<tr>
<th>Fiber Orientation</th>
<th>Tube</th>
<th>Ultimate Stresses</th>
<th>Ultimate Strains</th>
<th>Crack Growth</th>
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<td>87.5°</td>
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\[
\frac{\sigma_{yy}}{\tau_{xy}} = 1.0
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<th>Ultimate Stresses</th>
<th>Ultimate Strains</th>
<th>Crack Growth</th>
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</table>

i - Intersection Failure  
S - Side Failure  
T - Crack at Top of Notch  
B - Crack at Bottom of Notch  
* - Notch Length = 0.25"
tested at this load ratio did not fail when loaded to the capacity of the testing machine. Both
the 45° and 15° tubes exhibited fibers bridging the crack before final failure as shown in
Figure 39. Two 2.5° tubes were tested under a load ratio of 10.0. One of these tubes had a
notch length of 0.25" and the other 0.1". In addition, one 15° tube and one 45° tube were tested
in combined compression/negative torsion, but the load applied by the testing machine was
not uniform due to electrical noise. Audible cracking sounds heard during testing could not
be associated with crack growth.

The far-field stress-strain curves for the biaxial tests are presented in Appendix C.

5.4.1 One to One Compression to Negative Torsion Loading

The 2.5° tube failed at 8.7 ksi which is about twice any of the predicted critical stresses
indicated in Table 14. The predicted crack initiation sites (ϕc) ranged from 1° to 34° for the
different models as shown in Figure 51 and Table 15. The crack initiated at the notch discon-
tinuity and thus cannot be compared fairly with the predicted positions. The crack growth an-
gles (Ωc) are predicted to be almost parallel to the fibers at 3° or 4°. This is close to the actual
 crack growth direction of 2.5°.

The 15° tube failed at 3.8 ksi with the crack originating at the notch discontinuity and
growing parallel to the fibers at Ωc = 15°. The critical stress is closest to that predicted using
the circle model, plate conditions (Table 14). The crack growth angle is properly predicted
parallel to the fibers and the position of crack growth is predicted to be within the same range
as shown in Figure 52 and Table 15.

The 45° critical stress predictions using the circular hole model agree best with the ex-
perimental results (3.3 ksi experimental vs 3.13 ksi theory, inner radius). The predicted crack
growth direction corresponds to the observed 45° parallel to the fibers. The crack formed at
the discontinuity of the notch.
<table>
<thead>
<tr>
<th>Fiber Orientation</th>
<th>Notch Length (inches)</th>
<th>C/NT Ratio</th>
<th>Experimental Stress (ksi)</th>
<th>Notch Model</th>
<th>Inner Radius (ksi)</th>
<th>Plate (ksi)</th>
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s - Side Failure
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<td>10</td>
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<td>5°</td>
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<td>CNOD2</td>
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<td>10</td>
<td>4.5°i</td>
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<td>5°</td>
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</table>

s - Side Failure
i - Intersection Failure
Figure 51. Schematic of Crack Growth of a 2.5° Tube Loaded Biaxially with a C/NT Ratio = 1.0
Figure 52. Schematic of Crack Growth of a 15° Tube Loaded Blaxially with a C/NT Ratio = 1.0
Figure 53. Schematic of Crack Growth of a 45° Tube Loaded Biaxially with a C/N Ratio = 1.0

Results and Discussion
Figure 54. Schematic of Crack Growth of an 87.5° Tube Loaded Biaxially with a CN/T Ratio = 1.0
For the 87.5° tube, the predictions using the normal stress ratio theory with the circle notch model exhibit the best agreement with experimental critical stresses (Table 14). Crack formation from the notch was not symmetrical (Figure 54). One crack formed along the side of the notch and the other formed at the bottom tip of the notch. There was no evidence of flaws around the notch in this tube that could explain this behavior. The cracks did grow parallel to the fibers in the predicted direction (Table 15).

In summary, the circular hole model critical stress predictions exhibited the best agreement with experimental results for the biaxially loaded tubes. The crack growth directions were correctly predicted in all cases. The location of crack initiation was influenced by the discontinuity in the notch geometry and did not exhibit good correlation with theory.

5.4.2 Ten to One Compression to Negative Torsion Loading

Since 2.5° tubes did not fail in pure compression, several tubes with a high compression to negative torsion loading were tested. One 2.5° tube with a 0.1" notch failed at 41.5 ksi and the tube with the 0.25" notch failed at 31.4 ksi. Both tubes failed at the notch discontinuity with the cracks growing at $\Omega_c = 2.5^\circ$. The predicted crack initiation site and growth direction (shown in Table 15 and sketched in Figure 55) locates the crack between 1° and 9° with a growth angle $\Omega_c = 3^\circ$. The critical stress predictions are presented in Table 14. The circle model critical stress predictions agree best with experimental results for both cases. There is a problem with this because, as stated previously, the elasticity solution used to determine the stresses at the notch tip are independent of the actual notch geometry. To properly apply the predicted stresses, they should be modified using standard fracture mechanics techniques such as dividing the predicted stress by $\sqrt{2\alpha}$. Using this method with the circular hole, plate conditions, the predicted critical stresses become 53.4 ksi for the 0.1" notch and 33.7 ksi for the 0.25" notch which correlate better with the experimental results.
Figure 55. Schematic of Crack Growth of 2.5° Tubes in Biaxial Loading with C/NT = 10.0.
5.4.3 Discussion

For biaxial loading, there was limited agreement between the predicted critical failure stresses, using the circle model, and experiments. The concentration of stresses near the semi-circular ends of the notches similar to that shown in Figure 43 above apply.

The direction of crack growth is correctly predicted to occur parallel to the fibers using any of the notch models. The location of crack initiation is again strongly influenced by the notch discontinuity.

5.5 Fractography of Unidirectional Tubes

Fractographs of the notched specimens were taken from a representative area within 0.1 inch of the notch as illustrated in Figure 56. An SEM (scanning electron microscope) beam angle of 30° or 60° was used for the fractographs. The fracture surfaces of all specimens were studied; however, only representative fractographs are presented.

5.5.1 Notched and Unnotched Tubes

The presence of a notch in unidirectional graphite/epoxy material can alter the failure mode of a tube. This effect is examined using SEM fractographs for 2.5° tubes loaded in positive torsion, and 15°, 45°, and 87.5° tubes loaded in tension.

The effect of a notch on the failure mode for a 2.5° tube loaded in torsion is shown in Figure 57. The failure mode in an unnotched tube is characterized by fiber-matrix interfacial failure as evidenced by the smooth surface of the fibers seen on the fracture surface in
Figure 56. Location of Specimens for Fractography
Figure 57. Fractographs of Unnotched and Notched 2.5° Tubes Failed in Positive Torsion
Table 16. Material Principal Stresses in Notched and Unnotched Tubes

<table>
<thead>
<tr>
<th>Fiber Orientation</th>
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<th>Tension</th>
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<tbody>
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<td>Unnotched</td>
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<tr>
<td>$2.5^\circ$</td>
<td>1.69 ksi</td>
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<tr>
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<td>1.33 ksi</td>
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<td>1.47 ksi</td>
<td>2.15 ksi</td>
</tr>
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<td>Failure Mode</td>
<td>Matrix</td>
<td>Interface</td>
</tr>
<tr>
<td></td>
<td>Interface</td>
<td></td>
</tr>
<tr>
<td>$15^\circ$</td>
<td>0.08 ksi</td>
<td>0.56 ksi</td>
</tr>
<tr>
<td></td>
<td>29.0 ksi</td>
<td>2.08 ksi</td>
</tr>
<tr>
<td></td>
<td>0.25 ksi</td>
<td>7.76 ksi</td>
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<tr>
<td></td>
<td>Interface</td>
<td>Interface</td>
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<td>$45^\circ$</td>
<td>0.11 ksi</td>
<td>0.94 ksi</td>
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<tr>
<td></td>
<td>1.04 ksi</td>
<td>1.03 ksi</td>
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<tr>
<td></td>
<td>0.34 ksi</td>
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<td>Failure Mode</td>
<td>Mostly Interface</td>
<td>Matrix</td>
</tr>
<tr>
<td>$87.5^\circ$</td>
<td>0.009 ksi</td>
<td>2.42 ksi</td>
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<td>0.003 ksi</td>
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<td>0.11 ksi</td>
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</tr>
<tr>
<td>Failure Mode</td>
<td>Mostly Interface</td>
<td>Mostly Matrix</td>
</tr>
</tbody>
</table>

Figure 57. In contrast, with a notch present the failure is predominantly ductile matrix failure as shown by the stringers of matrix between adjacent fibers (Figure 57 b). The few broken fibers observed appear to be tensile failures due to the absence of shear lips or angled breaks.

This observed change in failure morphology is a result of many factors, one in particular is which component of stress dominates the localized region where failure initiates. If one component of stress dominates, then catastrophic interfacial fracture is more likely to occur since the dominant stress will break the bond between fiber and matrix before the matrix is significantly deformed. There is some matrix failure with the interfacial mode as evidenced by the presence of shear hackles in the matrix between the fibers but the weak interfacial bond breaks before the hackles do. With a more uniform state of stress and no component dominating, the matrix may undergo plastic deformation before failure producing the stringers.
To support this explanation of change in failure morphology from interfacial to matrix, it is helpful to consider the stresses in the material principal directions (1-2) at the location of maximum normal stress ratio. Using the radius notch model for a plate in the 2.5° tube tests, ductile matrix failure occurs when $\sigma_2$ is almost equal to $\tau_{12}$ whereas failure of the fiber/matrix interface occurs when these two stresses differ by a factor of 2.0 or more (Table 16). Ductile matrix failure occurs when $\sigma_2$ and $\tau_{12}$ are almost equal because the complex stress state allows the matrix to undergo plastic deformation before final failure.

This observation of the relationship between material principal stresses ($\sigma_2$ and $\tau_{12}$) and fracture morphology can only be considered a trend since the analytical technique for a notched plate smears the anisotropic properties into a homogeneous media. However, it does suggest the possibility of using the normal stress ratio to yield insight into the type of failure that can be anticipated. Also, to fully understand these observations, micromechanical analysis is required.

For 15° tubes loaded in tension, failure occurs primarily along the fiber-matrix interface independent of the presence of a notch. The notched tube fracture surface exhibited more stringers and hackles than the unnotched. In both cases, the fibers were broken in a shearing mode as evidenced by the angled fracture of fibers or the bending of fibers (Figure 58). Using the 1-2 stress comments discussed above, the 15° tubes with and without a notch should, and do, exhibit predominantly fiber/matrix interfacial failure.

The 45° tubes loaded in tension show a difference in failure mode depending upon whether a notch is present or not (Figure 59). In the presence of a notch, failure occurs along the fiber-matrix interface. Without a notch, the composite fractures within the matrix away from the fiber-matrix interface in a ductile manner. The difference in this instance is not as pronounced as in the 2.5° tubes. For the notched 45° tube, the shear and matrix stresses differ by a factor of approximately 3 indicating the possibility of interfacial failure (Table 16). For the unnotched tube, all three material principal stresses are about equal and the morphology exhibits ductile matrix failure.

Results and Discussion
Figure 58. Fractographs of Unnotched and Notched 15° Tubes Failed in Tension
Figure 59. Fractographs of Unnotched and Notched 45° Tubes Failed in Tension
Figure 60. Fractographs of Unnotched and Notched 87.5° Tubes Failed in Tension
Tension loading of 87.5° tubes causes a difference in the fracture surfaces of a notched and unnotched specimen. With a notch, the failure is primarily at the fiber-matrix interface. Without a notch, the failure is mostly ductile matrix with some interfacial characteristics (Figure 50). Accordingly, applying the same stress ratio criteria, the 87.5° tension tests should both exhibit interfacial failure; however, the unnotched specimen does not. For this fiber orientation, it is necessary to consider the micromechanical behavior of the composite in more detail.

### 5.5.2 Notched Tubes Loaded Axially

The fracture of notched unidirectional 15°, 45°, and 87.5° tubes loaded in tension and 87.5° tubes loaded in compression is characterized by fiber-matrix interfacial failure. Fracture of a 45° tube in compression is characterized by matrix failure. Details of each failure are described below.

The notched 15° tube loaded in tension is dominated by fiber-matrix interfacial failure. The shear hackles in the matrix between fibers are almost perpendicular to the fibers (Figure 58) indicating a state of stress other than pure shear between adjacent fibers. In fact, it can be hypothesized that there is a large tensile force acting between fibers near the notch. This is shown to be possible by the large $\sigma_2$ as compared to $\sigma_1$ and $\tau_{12}$ shown in Table 17.

The tensile and compression fracture surfaces of 45° tubes are shown in Figure 61. The tensile loaded tube exhibits primarily fiber-matrix interface failure and has shear hackles that are approximately 90° to the fibers. Similar to the 15° tube described above, $\sigma_2$ is dominant.

The compression fracture surface of the 45° notched tube shows a failure of the matrix by the crushing of the material with few shear hackles present. This mode of failure was not anticipated using the ratios of the material principal stresses calculated at the location where the NSR is a maximum. In fact, the principal stresses indicate that interfacial failure should
Table 17. Stresses Associated with Axial Fracture Morphology

<table>
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<th>Fiber Orientation</th>
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<td>2.71</td>
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dominate. It should be noted that this tube showed a slight indication of buckling prior to failure and that the resulting failure mode could have been affected.

The 87.5° tubes loaded in tension and compression (Figure 62) both exhibit fiber-matrix interface failure; however, there is more adhesion of the matrix to the fiber and ductile matrix failure in the case of tension loading. The shear hackles in these tubes are at angles of less than 10° from the perpendicular to the fiber. This is indicative of the high tensile stresses in the vicinity of the notch. For both loading regimes, the $\sigma_2$ stress component is more than twice the shear stress consistent with the observed trend for interfacial failure.

5.5.3 Notched Tubes Under Torsion

The fracture surfaces of the 2.5° tubes in both positive and negative torsion show ductile matrix failure (Figure 63).

The 15° tube loaded in negative torsion failed in the ductile matrix mode similar to the 45° tubes shown in Figure 64. This failure mode was anticipated because the fibers were pulled out across the open crack in the tube as shown in Figure 39 on page 98.
Figure 61. Fractographs of 45° Tubes Failed in Tension and Compression
Figure 62. Fractographs of 87.5° Notched Tube Loaded Axially
Figure 63. Fractographs of 2.5° Tubes Failed in Torsion
Figure 64. Fractographs of 45° Tubes Failed in Torsion
The fracture surfaces of the 45° tubes loaded in torsion show fiber-matrix interfacial failure in positive torsion (Figure 64 a) and matrix failure in negative torsion (Figure 64 b).

The 87.5° tubes show the opposite behavior to the 45° specimens. The negative torsion loaded specimens had a fracture mode of mostly interface failure while the positive torsion tube exhibits predominantly matrix failure. Figure 65 shows these results.

For tubes loaded in torsion, the primary failure mode is matrix failure except for positive torsion failure of the 45° tube and the 87.5° tube in negative torsion. The correlation between the material principal stresses $\sigma_2$ and $\tau_{12}$ being nearly equal and ductile matrix failure applies to these results except for the 87.5° tubes in negative torsion. The stresses are tabulated on Table 18.

5.5.4 Biaxial Loading of Notched Tubes

The 2.5° tubes were loaded with compression/negative torsion ratios of 10.0 and 1.0. The fracture surfaces are shown in Figure 66. When the load ratio is 1.0, the fracture surface is quite different. Failure occurs primarily along the fiber/matrix interface with the matrix having been sheared perpendicular to the fibers. Where the sheared matrix material has dropped out, there are gaps left between the fibers. The overall fracture surface has a very clean look with little or no debris. For a compression/torsion ratio of 10.0, there is some adhesion of the matrix to the fibers and shear hackles appear to have no specific orientation (Figure 66 b).

The 15° tubes with a loading ratio of 1.0 show fiber/matrix interface failure with the matrix failing in a ductile manner between fibers. Broken fibers are either crushed or break perpendicular to the axis of the fiber.

The failure surfaces of the 45° tubes loaded biaxially with a compression/torsion ratio of 1.0 show primarily interfacial failure with some brittle matrix failure, i.e., few or no shear lips. Fibers tend to fail perpendicular to their longitudinal axis. These details can be seen in Figure 68.
Figure 65. Fractographs of 87.5° Tubes Failed in Torsion
Table 16. Stresses Associated with Torsional Fracture Morphology

<table>
<thead>
<tr>
<th>Fiber Orientation</th>
<th>Loading</th>
<th>$\sigma_1$ (ksi)</th>
<th>$\sigma_2$ (ksi)</th>
<th>$\tau_{12}$ (ksi)</th>
<th>Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5°</td>
<td>Positive Torsion</td>
<td>1.69</td>
<td>1.33</td>
<td>-1.47</td>
<td>Matrix</td>
</tr>
<tr>
<td>2.5°</td>
<td>Negative Torsion</td>
<td>1.55</td>
<td>1.09</td>
<td>1.27</td>
<td>Matrix</td>
</tr>
<tr>
<td>15°</td>
<td>Negative Torsion</td>
<td>2.64</td>
<td>1.05</td>
<td>1.66</td>
<td>Matrix</td>
</tr>
<tr>
<td>45°</td>
<td>Positive Torsion</td>
<td>2.51</td>
<td>6.35</td>
<td>-3.10</td>
<td>Interface</td>
</tr>
<tr>
<td>45°</td>
<td>Negative Torsion</td>
<td>2.64</td>
<td>1.67</td>
<td>2.10</td>
<td>Matrix</td>
</tr>
<tr>
<td>87.5°</td>
<td>Positive Torsion</td>
<td>1.09</td>
<td>1.49</td>
<td>-1.25</td>
<td>Matrix</td>
</tr>
<tr>
<td>87.5°</td>
<td>Negative Torsion</td>
<td>1.08</td>
<td>1.47</td>
<td>1.23</td>
<td>Interface</td>
</tr>
</tbody>
</table>

The 87.5° tubes exhibited a combination of interface and matrix failure. Even though the crack growth direction was parallel to the fibers, some fibers were broken and tended to be sheared and/or crushed. The fractographs are shown in Figure 69.

The material principal stresses for the biaxial tests are listed in Table 19. For biaxially loading regimes, there is no clear correlation between the stresses in the 1-2 directions at the site of failure as predicted by the normal stress ratio and the observed fracture morphology. Only in the case of the 2.5° orientation with a C/NT ratio of 10.0 is there an order of magnitude difference between $\sigma_1$ and the other two stress components. As evidenced by the failure mode, this case was dominated by compressive failure. There is also evidence of local fiber failure in the biaxially loaded tubes. With varying load ratios, the mode of the localized fiber failure changes. Analytical characterization of these results requires a micromechanical evaluation.
Figure 66. Fractographs of 2.5° Tubes Failed in Compression/Negative Torsion
Figure 67. Fractographs of 15° Tubes Failed in Compression/Negative Torsion
Figure 68. Fractograph of 45° Tube Failed in Compression/Negative Torsion
Figure 69. Fractographs of 87.5° Tubes Failed Biaxially
Table 19. Stresses Associated with Biaxial Fracture Morphology

<table>
<thead>
<tr>
<th>Fiber Orientation</th>
<th>C/NT Ratio</th>
<th>$\sigma_1$ (ksi)</th>
<th>$\sigma_2$ (ksi)</th>
<th>$\tau_{12}$ (ksi)</th>
<th>Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5°</td>
<td>1.0</td>
<td>1.51</td>
<td>0.72</td>
<td>1.03</td>
<td>Interface</td>
</tr>
<tr>
<td>2.5°</td>
<td>10.0</td>
<td>0.30</td>
<td>0.01</td>
<td>0.04</td>
<td>Interface and Matrix</td>
</tr>
<tr>
<td>15°</td>
<td>1.0</td>
<td>1.38</td>
<td>0.68</td>
<td>0.94</td>
<td>Interface</td>
</tr>
<tr>
<td>45°</td>
<td>1.0</td>
<td>1.90</td>
<td>0.58</td>
<td>1.05</td>
<td>Interface</td>
</tr>
<tr>
<td>87.5°</td>
<td>1.0</td>
<td>3.47</td>
<td>0.79</td>
<td>1.87</td>
<td>Interface and Matrix</td>
</tr>
</tbody>
</table>

5.6 Notched Angle-ply Tubes

5.6.1 Torsion of Angle-ply Tubes

Three $\pm 87.5^\circ$ tubes were tested in torsion. One tube in negative torsion and two with different notch sizes in positive torsion. The location of crack initiation was at the slot discontinuity for both positive and negative loadings (Figure 30). The positive torsion tests exhibited stable crack growth and failed at $\Omega_c = -70^\circ$, clearly breaking fibers (Figure 70). After growing about one half inch, the cracks turned and grew at approximately $90^\circ$. The negative torsion test also exhibited slow crack growth with cracks growing from the opposite side of the notch at $\Omega_c = 90^\circ$ (Figure 71). Figure 72 displays the crack at the top of the notch of tube T90D2 loaded in positive torsion and the cracks associated with failure of the negative torsion test. The results of the tests are presented in Table 20. It is also noted that similar to the unidirectional tube tests for torsion, the cracks formed on opposite sides of the notch for positive and negative torsion.

Results and Discussion
Figure 70. Schematic of Crack Growth in Angle-ply Tubes in Positive Torsion
Figure 71. Schematic of Crack Growth in Angle-ply Tube in Negative Torsion
Figure 72. Crack Growth in Angle-ply Tubes Loaded in Torsion
Table 20. Torsion Tests of Angle-ply Tubes

<table>
<thead>
<tr>
<th>Notch Length</th>
<th>Tube</th>
<th>Ultimate Failure Stress (ksi)</th>
<th>Ultimate Failure Strain ( % )</th>
<th>Crack Initiation Stress (ksi)</th>
<th>Crack Growth Position $\phi_c$</th>
<th>Crack Growth Angle $\Omega_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25&quot;</td>
<td>T90D2</td>
<td>7.20</td>
<td>0.82</td>
<td>3.6</td>
<td>-4.5°*</td>
<td>-70°</td>
</tr>
<tr>
<td>0.1&quot;</td>
<td>T90D3 †</td>
<td>9.80</td>
<td>1.13</td>
<td>7.1</td>
<td>-4.5°*</td>
<td>-70°</td>
</tr>
<tr>
<td>0.1&quot;</td>
<td>T90D1N</td>
<td>-6.70</td>
<td>1.11</td>
<td>-5.9</td>
<td>4.5°*</td>
<td>90°</td>
</tr>
</tbody>
</table>

† - Damage near notch prior to testing
i - Intersection Failure

Failure of tube T90D3 (0.1" notch) was interesting as shown in Figure 74. Two cracks formed initially - one at the bottom of the notch at 176.5° and the other at the top of the notch some distance from the tip. As loading progressed, the crack near the top of the notch closed and a new crack formed at a position of -4.5°. The second crack grew in a symmetric manner with the crack at the bottom of the notch until failure. This asymmetric crack initiation may be related to the damage that was induced on the inside radius of the tube during notching as shown in Figure 73.

The predictions of the normal stress ratio theory are presented in Table 21 and Table 22. Only the plate solution predictions are listed since the difference in the critical stress between the inside and outside radius is less than 10% of the plate value.

The predicted critical stress and crack growth parameters using NSR theory for negative torsion loading are very different from the experimental results. The critical stress is off by 50% or more for all of the notch models. The crack growth location does predict the correct side of the notch. The predicted growth direction is perpendicular to the fibers while the material fails along the tow orientation. The layer in which first failure occurs is predicted to be the -87.5° ply on the outside radius for all models.

For the case of positive torsion, the critical stress for the 0.5 notch is 29% larger than the radius model prediction (3.6 ksi vs 2.56 ksi). The crack grows from the same side of the notch.
Damage on Inner Radius of Tube

Figure 73. Damage Associated with the Notch in Tube 90D3
Figure 74. Asymmetric Crack Growth in Angle-ply Tube
as the prediction but at a different location. Instead of breaking fibers at a 90° angle as predicted, the fibers are broken at an angle of 20° with respect to the fibers. The ply for failure initiation is the +87.5° ply nearest the outside radius indicating that failure is layer dependent.

The experimental critical stresses for the three cracks formed in angle-ply tube with the 0.1" notch corresponds fairly well with the semi-axis model prediction. Ignoring the asymmetry of the cracks, the critical stress for initiating the mid-notch crack is 17% less than the semi-axis model (7.1 ksi vs. 8.29 ksi); the stress for initiating the bottom crack is 5% lower (7.9 ksi); and the top crack's initiation stress is 7% higher (8.9 ksi). The location of crack growth is on the same side of the notch albeit at a different angle than predicted and as with the 0.25" notch, the experimental growth angle is 20° instead of the 90° predicted.

The critical stress predictions for the positive torsion tests show fair agreement with the semi-axis model for the 0.1" notch and the radius model for the 0.25" notch. The circular hole model does not apply since the predicted stress is too low. The differences in the predicted stresses arise from the different a/b ratios of the notches. The a/b ratio for the 0.1" semi-axis
Table 22. Crack Growth Geometry of Angle-ply Tubes

<table>
<thead>
<tr>
<th>Tube</th>
<th>Notch Length</th>
<th>Experimental Crack Growth</th>
<th>Notch Model</th>
<th>Predicted Crack Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Position $\phi_c$</td>
<td>Angle $\Omega_c$</td>
<td>Position $\phi_c$</td>
</tr>
<tr>
<td>T90D1N</td>
<td>0.1&quot;</td>
<td>4.5°</td>
<td>87.5°</td>
<td>Circle</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14°</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1°</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4°</td>
</tr>
<tr>
<td>T90D2</td>
<td>0.25&quot;</td>
<td>-4.5°</td>
<td>-70°</td>
<td>Circle</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-14°</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-1°</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-2°</td>
</tr>
<tr>
<td>T90D3</td>
<td>0.1&quot;</td>
<td>-4.5°</td>
<td>-70°</td>
<td>Circle</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5°</td>
<td>-90°</td>
<td>-14°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>274.5°</td>
<td>110°b</td>
<td>-1°</td>
</tr>
</tbody>
</table>

s Crack starting at side of notch
b Crack starting at bottom of notch
t Crack starting at top of notch

The model is 0.035 and for the radius model of the 0.25" notch, it is 0.167. The results of the 0.1" notch are not good for comparison with the theory since the failure was asymmetric due to the damage on the inside of the tube at the notch. Also, it is unrealistic that the failure stress for smaller notch is less than that for a larger notch (radius model 0.1" notch critical stress is 1.83 ksi and for the 0.25" notch it is 2.56 ksi). In order to account for this, the critical stress should be modified to account for the finite size of the specimen. Using a typical fracture mechanics scheme, if the value for the radius model 0.25" notch is divided by the square root of half the notch length ($\sqrt{\frac{b}{2}}$), the resulting predicted critical stress is 8.1 ksi which is in good agreement with the experimental results for the 0.1" notch (7.1 to 8.9 ksi).

The negative torsion case exhibits poor agreement with experiment but again this could be due to the use of the unmodified notch edge stress.

The position of crack growth around the edge of the notch is predicted on the correct side of the notch for both positive and negative torsion cases but the exact location is incorrect.
due to failure occurring at the intersection of the circular ends and the straight sides of the slot which is not accounted for in the models.

The predicted crack growth angle is perpendicular to the fibers indicating fiber breakage for both loading configurations. With the negative torsion test, failure was nearly parallel to the fibers at an angle of approximately 90°. For positive torsion, the crack grew at an angle of -70° breaking fibers. In this case, the normal stress ratio theory correctly predicted fiber breakage although at the wrong angle.

5.6.2 Fractography of Angle-ply Tubes

These tubes were loaded in positive or negative torsion. As stated previously, the tubes loaded in positive torsion failed at an angle of -70°, then curved perpendicular to the notch. The failure mode changed corresponding to the change in crack angle. Initially, the fibers were torn from the bundles unevenly; then, as the crack leveled off, bundles of fibers were broken uniformly. The negative torsion case exhibited a crack growth angle of almost 90° from its initiation site. In this case, the fracture surface shows bundles of fibers broken straight across the tows in both plies. Figure 75 shows macrophotographs of the fracture surface of both loading cases. Figure 76 shows the details of the failure mode on a microscopic level showing mainly failure at the fiber-matrix interface.

The failure surfaces of the angle-ply tubes loaded in positive torsion showed fiber breakage in the vicinity of the notch. Away from the notch, bundles of fibers broke allowing the fracture process to follow roughly parallel to the tows. The cause of this change in fracture mode occurred due to the concentration of stresses near the notch within a small region where the $\sigma_1$ stress was large enough to break individual fibers. As the distance from the notch increased, the stress concentration of the notch diminishes and the crack tip stress concentration was all that remains. Here the material broke along the path of least resistance - between the tows. The tows that broke uniformly along the fracture surface possibly did so.
Figure 75. Macrographs of Angle-ply Tubes Loaded in Torsion
Figure 76. Fracture Surfaces of Angle-Ply Tubes Loaded in Torsion
due to flaws at those locations. An attempt was made to view microscopic damage or inclusions on the broken tows but no clear cause of failure could be identified.

When loaded in negative torsion, the failure mode was interfacial between the tows with random tows in both plies breaking evenly across their width identical to positive torsion fracture approximately an inch away from the notch. This was an interesting observation since the tubes are symmetric through the thickness. The reason for this behavior requires additional testing to determine if it is indeed a correct observation or just an isolated case. Micromechanical analyses should aid in the understanding of the fracture morphology. Obvious reasons such as flaws induced during manufacture or notching were not found in the C-scans and the notches in the tubes appeared uniform.

A three dimensional analysis is necessary to determine the effect of the interlaminar stresses at the notch. These stresses could play a dramatic role in determining the mode of failure. 3-D finite element analyses could be used to analyze these results for further insight.
6.0 Conclusions

This study examined crack growth in notched unidirectional graphite/epoxy tubes under a variety of loading conditions. Experimental results were compared with predictions of the normal stress ratio theory. To further the understanding of the failure process, fracture surfaces of the failed tubes were characterized using scanning electron microscopy. The observed failure modes were correlated with the material principal stresses at the point of maximum normal stress ratio. Also, a comparison was made of failure modes for selected notched and unnotched unidirectional tubes. Additionally, notch $\pm 87.5^\circ$ tubes tested in torsion and the resulting failure compared to that predicted by the normal stress ratio. The main conclusions of this study are:

1. Crack growth in the notched unidirectional tubes is summarized as follows:

   - The notched unidirectional tubes under axial, torsional, and biaxial loading failed along the fiber direction.

   - In most cases, independent of loading, the cracks initiated at the discontinuity of the notch where the semi-circular end intersects the straight side. Crack growth pro-
ceeding in an unstable manner in all cases except one 45° tube and the 87.5° tubes in torsion.

- Both notched and unnotched tubes failed parallel to the fibers. Examination of the fracture surfaces with an SEM showed that the failure mode was either along the fiber-matrix interface or within the matrix. For the biaxially loaded tubes, the complexity of the stress state in the vicinity of the notch was indicated by the varying angles of shear hackles.

2. The normal stress ratio theory was used to predict the failure of notched unidirectional graphite/epoxy tubes using an infinite flat plate elasticity solution with an elliptical hole. The application of the normal stress ratio theory is summarized as follows:

- For axial and torsional loading of tubes, the normal stress ratio theory was used to adequately predict the far-field failure stress when the experimental notch was modeled by an ellipse with the same notch tip radius as the experimental slot.

- For the tubes loaded biaxially and the 87.5° tubes, the agreement between predicted failure stress and experiment was better when the notch was modeled as a circular hole. It is hypothesized that this is due to the semi-circular ends of the notch dominating the stress concentration in the material.

- The normal stress ratio theory correctly predicted the direction of crack growth parallel to the fibers in all tubes.

- The normal stress ratio theory could not predict the location of crack extension because the discontinuity in the notch (where the flat sides intersected the semi-circular ends) was not accounted for in the model.
3. The normal stress ratio theory was employed to evaluate the failure mode of unidirectional composite tubes. Material principal stresses at the location of maximum normal stress ratio were correlated with either fiber-matrix interface failure or pure matrix failure. It was shown that interfacial failure is usually associated with a difference in the material principal shear stress and stress perpendicular to the fibers by a factor of 2.0 or more. Matrix failure occurs when these stresses are nearly the same.

4. Three ±87.5° angle-ply tubes were failed in torsion. The fracture modes and the application of the normal stress ratio theory are summarized as follows:

- In the case of positive torsion, cracks emanating from the notch discontinuity grew at an angle of about -70° with respect to the tube axis breaking the fibers independent of the tows until the crack turned perpendicular to the tube axis where the tows were uniformly broken.

- For negative torsion, the cracks grew perpendicular to the tube axis starting from the notch discontinuity and broke tows uniformly.

- The normal stress ratio theory was applied to a notched laminated tube and predicted that fiber breakage would occur. The predicted stress for crack initiation exhibited poor agreement with experimental results. The location of crack initiation was influenced by the notch discontinuity that was not included in the model.

**Recommendations for Future Work**

1. A detailed micromechanics analysis of the stress state at and near the notch could yield insight into several facets of this study. First, the observed change of fracture morphology from fiber/matrix interface failure to matrix failure with different loading regimes could
be evaluated in detail. The micromechanics analysis could also clarify the relationship between the shear hackle angle and stress state and hence, ultimate failure. Lastly, the effect that the high volume fraction of fibers (74% vs the more common 58%) has on the stress state and failure modes of the tube as compared to more conventional graphite/epoxy material could be examined in detail.

2. Modeling the notch as a slot with semi-circular ends would permit a better determination of stress state. This would affect the crack growth initiation sites and stresses predicted by the normal stress ratio. The notch could be modeled using finite element analysis or conformal mapping.  

3. The extent of out-of-plane bending in a unidirectional tube at the notch should be analyzed and the effects of curvature should be included in the stress analysis. These factors could be examined by 3-D finite element analyses.

4. Additional experiments should be conducted with unidirectional tubes or coupons having the same notch configurations to confirm the relationship between fracture morphology and stresses at the point of maximum normal stress ratio. Other notch configurations (different notch lengths and angles) would provide further verification of the normal stress ratio theory.

5. The stress state of notched angle-ply tubes should be determined using more sophisticated analytical techniques such as 3-D finite element analysis. The interplanar stresses could then be included in the normal stress ratio theory.
7.0 References


Sundaresan, M.J., Personal Communication.


Gurdal, Z., Personal Communication.

Appendix A. Programming of Complex Potentials

The use of Fortran on both mainframe and personal computers to calculate the stress state utilizing Lekhnitski's complex potential solutions has presented problems to many investigators. (The Fortran and computers used in this study are IBM Professional Fortran Version 1.30 on an IBM PC AT and IBM VS Fortran Version 2 on an IBM 3090 mainframe.) The difficulty is that the calculated stresses are not continuous at the edge of an elliptical notch or at a given distance from a crack tip. The discontinuities arise when branch cuts in the solution are incorrectly taken by the computer program. The three steps required to correct the problem are discussed in this appendix.

One technique to maintain continuous stresses is to convert the complex numbers from the \((x, iy)\) form to the complex plane form \((r, e^{i\theta})\) and do all arithmetic with the \(r, \theta\) form. Since the intrinsic complex arithmetic function are for the \((x, iy)\) form, appropriate subroutines must be written. In transforming the variables from \((x, iy)\) to the complex plane form, if one uses the intrinsic arctan function, discontinuities occur because the arctan function only outputs angles in the first two quadrants (from 0° to 180°) thus limiting the stress functions.

The next step in the analysis is to consider how the stresses are calculated. There are subtleties in the manipulation of complex numbers that can have a drastic effect on the resulting
solution. The critical example in the elliptical notch solution is in the following equation for the derivatives of the complex functions:

\[ \phi'_{1e} = K_1 \left\{ 1 - \sqrt{\frac{Z_1^2}{Z_1^2 - (a^2 + \mu \lambda^2 b^2)}} \right\} \]  

(3.17)

This equation can be rewritten:

\[ \phi'_{1e} = K_1 \left\{ 1 - \frac{Z_1}{\sqrt{Z_1^2 - (a^2 + \mu \lambda^2 b^2)}} \right\} \]  

(E.1)

This second version causes the stresses to have severe discontinuities. Mathematically, the two functions do not have identical imaginary components. The correct method is the first since the function tends to remain continuous on the complex plane.

The third requirement for continuous stresses is to perform a check on values by comparing the two previous calculations beginning at a position along the real axis. By monitoring the complex arithmetic at each calculation in the program, it was found that the function in braces of Eqn. 3.20 is critical. The imaginary part within the braces must be compared to previous steps to avoid a branch cut, i.e. the result jumps into the wrong quadrant. When the imaginary angle is near 2.5°, 180°, or 360°, the check on previous values is necessary to adjust the current value accordingly. Limits were also set on how much of an numerical error could be tolerated at these three angles. For double precision arithmetic, an error of less that $10^{-9}$ was found to be sufficient.

By implementing the three steps above, the complex potential solution was made almost fool proof in yielding continuous stresses as per Leknitskii's solution. In all cases tested - including material properties different from those used in this study - only, one case exhibited a discontinuity and that could be attributed to machine round off error.
Appendix B. Material Property Determination

Since the composite used for this study has an unusually high fiber volume fraction (74% versus the more common 58%), it was necessary to determine the appropriate material properties, namely: $E_1$, $E_2$, $v_{12}$, $G_{12}$, $X_1$, $Y_1$, and $S$. Several tubes without notches were used to determine these properties and were subjected to the same experimental procedure as described previously. These tubes had 2 or 4 strain gages mounted on them as shown in Figure 77. The additional gages were used to assess the state of strain in the tube and to determine the extent of the effects of gripping on the strain state in the tubes. It was found that the end effects were negligible approximately 1.5" from the grips yielding a gage length of 3" at the center of the tubes. A typical distribution of strains are shown in Figure 78.

B.1 Shear Properties

The determination of shear modulus and shear strength were determined using the $2.5^\circ$, the $87.5^\circ$, $45^\circ$, and $15^\circ$ tubes. $2.5^\circ$ and $87.5^\circ$ tubes were loaded in pure torsion in both the positive and negative sense. A $45^\circ$ tube and a $15^\circ$ tube were loaded in pure tension to compare the
Figure 77. Placement of Gages for Evaluating Material Properties
Figure 78. Strains measured at Different Locations on the Tube
values determined using an off axis type test. The conundrum encountered was that three unnotched 2.5° tubes failed at far field stresses lower than the notched tubes, i.e., 7 ksi vs 8 ksi, and at shear stresses lower than the 87.5° tube which failed at 12.8 ksi. The 2.5° tubes appeared to first fail in the grips. The ultrasonic c-scans were checked for flaws and only tube P0A1S had some indications of damage near where the tube failed. Since the three 2.5° tubes were all out of round by approximately 0.040", it is hypothesized that the oval shape was the cause of premature failure in the following manner: the oval shape was reproduced by the grip adhesive which debonded from the tube and caused the tube to be split by the rotation of the adhesive/plug.

The shear property data are summarized in Table 23. The shear modulus, $G_{12}$ for this material is 0.937 Msi and the shear failure stress, $S$, is 12 ksi. The oval tubes and some preliminary tests on tubes that were not loaded to failure are noted.

### B.2 Transverse Properties

The transverse modulus was determined by testing a 87.5° tube in tension. The ultimate stress, $Y_u$, was 7.2 ksi. Another tube was tested to check the strain distribution using 4 gages but was not loaded to failure. The two notched 87.5° tension tests are also included in the determination of the modulus. These data are listed on Table 24 which shows an average modulus, $E_2$, of 1.44 Msi. These values are consistent with others having a lower fiber volume fraction.\(^4\) It has been shown that the strength and modulus perpendicular to the fibers is relatively insensitive to the fiber volume fraction since it is more a function of adhesion between the fibers and the matrix.\(^9\)
Table 23. Shear Properties

<table>
<thead>
<tr>
<th>Tube ID</th>
<th>Fiber Angle</th>
<th>$G_{12}$ (Msi)</th>
<th>$S$ (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0A1S ¹</td>
<td>2.5°</td>
<td>0.90</td>
<td>5.9 ²</td>
</tr>
<tr>
<td>P0A2S ¹</td>
<td>2.5°</td>
<td>0.97</td>
<td>-8.0 ²</td>
</tr>
<tr>
<td>P0A3S ¹</td>
<td>2.5°</td>
<td>0.91</td>
<td>7.6 ²</td>
</tr>
<tr>
<td>P0E2S</td>
<td>2.5°</td>
<td>0.94</td>
<td>11.2</td>
</tr>
<tr>
<td>T0B1 ³</td>
<td>2.5°</td>
<td>0.94</td>
<td>--</td>
</tr>
<tr>
<td>P90A2S</td>
<td>87.5°</td>
<td>0.74</td>
<td>12.8</td>
</tr>
<tr>
<td>P90J2S</td>
<td>87.5°</td>
<td>0.99</td>
<td>-8.6 ²</td>
</tr>
<tr>
<td>P45C ⁴</td>
<td>45°</td>
<td>1.05</td>
<td>12.0</td>
</tr>
<tr>
<td>P15A ⁴</td>
<td>15°</td>
<td>0.99</td>
<td>6.4 ²</td>
</tr>
</tbody>
</table>

Averages 0.94 12.0

¹ Oval Tube Failed Prematurely
² Not Used in Average
³ Preliminary Test Not Loaded to Failure
⁴ Tested in Tension

B.3 Axial Properties

The modulus and Poisson’s ratio determined from some preliminary tests were averaged with the data from the notched 2.5° tubes tested in tension which were not loaded beyond the linear region and did not fail. Table 25 shows these results. The average modulus, $E_t$, is 19.76 Msi and the average Poisson’s ratio, $v_{12}$, is 0.276.

Major difficulty was encountered in determining the ultimate strength parallel to the fibers, $X_t$. Several techniques were tried.

First strips approximately 0.375" wide were cut from a previously tested and failed 2.5° tube. The ends were potted with Hysol 934 epoxy so that the curved specimen could be gripped
in standard flat coupon grips. A strain gage 0-45-90 rosette (Texas Measurements FRA-2) was put on one side to monitor strains and a single gage (FLA-2) on the other side to detect bending. This coupon was unsuccessful in determining the ultimate strength since the specimens slipped in the grips or split when grip pressure was increased. The modulus measured is not reported below but was within the range shown on Table 25.

Another technique attempted was to cut a thin slice from a tube approximately 0.08" wide yielding a square cross-section for the coupon. The slices were then endtabbed using standard fiberglass/epoxy tabs. The tabs were 0.08" wide and care was taken to insure that the tabs and the specimen were properly aligned. The tabs were bonded onto the specimen using Hysol 934 adhesive with glass beads to maintain a 3-5 mil layer of adhesive. The procedure for bonding the tabs to the specimen was the same used to bond the tubes to the aluminum plugs. After curing, the specimens were tested using a United Testing Machine with an IBM PC-XT and Data Translations Data Acquisition Board as described by Beuth, et al. The average $X_f$ was 259 ksi as shown on Table 26.

Both the modulus and failure strength parallel to the fibers compare well with other material having lower fiber volume fractions.

---

**Table 24. Transverse Properties**

<table>
<thead>
<tr>
<th>Tube ID</th>
<th>Fiber Angle</th>
<th>$E_t$ (Msi)</th>
<th>$Y_t$ (Ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P90C3</td>
<td>87.5°</td>
<td>1.40</td>
<td>7.2</td>
</tr>
<tr>
<td>P90C2 (^1)</td>
<td>87.5°</td>
<td>1.37</td>
<td>--</td>
</tr>
<tr>
<td>TE90L1B (^2)</td>
<td>87.5°</td>
<td>1.56</td>
<td>--</td>
</tr>
<tr>
<td>TE90K2 (^2)</td>
<td>87.5°</td>
<td>1.43</td>
<td>--</td>
</tr>
<tr>
<td>Averages</td>
<td></td>
<td>1.44</td>
<td>7.2</td>
</tr>
</tbody>
</table>

\(^1\) Preliminary Test Not Loaded to Failure  
\(^2\) Notched Tube
### Table 25. Axial Properties

<table>
<thead>
<tr>
<th>Tube ID</th>
<th>Fiber Angle</th>
<th>$E _s$ (Msi)</th>
<th>$v _{12}$ (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T0B3 $^1$</td>
<td>2.5°</td>
<td>19.5</td>
<td>0.231</td>
</tr>
<tr>
<td>T0B1 $^1$</td>
<td>2.5°</td>
<td>19.1</td>
<td>0.295</td>
</tr>
<tr>
<td>TE0D2 $^2$</td>
<td>2.5°</td>
<td>19.7</td>
<td>0.276</td>
</tr>
<tr>
<td>TE0F1 $^2$</td>
<td>2.5°</td>
<td>20.7</td>
<td>0.300</td>
</tr>
<tr>
<td>Averages</td>
<td></td>
<td>19.75</td>
<td>0.276</td>
</tr>
</tbody>
</table>

$^1$ Preliminary Test Not Loaded to Failure  
$^2$ Notched Tube

### Table 26. Axial Failure Stress

<table>
<thead>
<tr>
<th>Slice ID</th>
<th>$X _s$ (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S0A1-3</td>
<td>253</td>
</tr>
<tr>
<td>S0A1-4</td>
<td>241</td>
</tr>
<tr>
<td>S0A1-5</td>
<td>283</td>
</tr>
<tr>
<td>Average</td>
<td>259</td>
</tr>
</tbody>
</table>
Appendix C. Stress-Strain Curves

Tables describing experiments and far field stress-strain curves are presented in this appendix. Most stress-strain curves include the stress-strain relation calculated using the exact anisotropic cylinder elasticity solution.
## Table 27. Tension Experiments

<table>
<thead>
<tr>
<th>Fiber Orientation (degrees)</th>
<th>Tube</th>
<th>Notch Length (in)</th>
<th>$\sigma_{fy}$ Failure Stress (ksi)</th>
<th>$\epsilon_{fy}$ Failure Strain (%)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>TE0D2</td>
<td>0.488</td>
<td>--</td>
<td>--</td>
<td>Tube did not fail</td>
</tr>
<tr>
<td></td>
<td>TE0F1</td>
<td>0.2</td>
<td>--</td>
<td>--</td>
<td>Tube did not fail</td>
</tr>
<tr>
<td>15</td>
<td>TE15C1</td>
<td>0.2</td>
<td>26.8</td>
<td>.33</td>
<td>Adhesive Grip Failed</td>
</tr>
<tr>
<td></td>
<td>TE15C1B</td>
<td>0.2</td>
<td>26.0</td>
<td>.26</td>
<td>Retest of TE15C1</td>
</tr>
<tr>
<td></td>
<td>TE15A2</td>
<td>0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td></td>
<td>26.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>TE45F3</td>
<td>0.2</td>
<td>6.1</td>
<td>0.26</td>
<td>s</td>
</tr>
<tr>
<td></td>
<td>TE45C3</td>
<td>0.2</td>
<td>7.2</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td></td>
<td>6.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>87.5</td>
<td>TE90L1B</td>
<td>0.477</td>
<td>2.9</td>
<td>0.19</td>
<td>Tube failed at grip</td>
</tr>
<tr>
<td></td>
<td>TE90K2B</td>
<td>0.2</td>
<td>2.9</td>
<td>0.17</td>
<td>Previously tested C/NT</td>
</tr>
</tbody>
</table>

s - Side Failure

## Table 28. Compression Experiments

<table>
<thead>
<tr>
<th>Fiber Orientation (degrees)</th>
<th>Tube</th>
<th>Notch Length (in)</th>
<th>$\sigma_{fy}$ Failure Stress (ksi)</th>
<th>$\epsilon_{fy}$ Failure Strain (%)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>C0B2</td>
<td>0.2</td>
<td>--</td>
<td>--</td>
<td>Tube did not fail</td>
</tr>
<tr>
<td>45</td>
<td>C45D2</td>
<td>0.5</td>
<td>-24.1</td>
<td>-1.5</td>
<td>s, Asymmetric Failure</td>
</tr>
<tr>
<td></td>
<td>C45E3</td>
<td>0.2</td>
<td>-30.7</td>
<td>-2.5</td>
<td></td>
</tr>
<tr>
<td>87.5</td>
<td>C90L1</td>
<td>0.2</td>
<td>-23.9</td>
<td>-1.87</td>
<td></td>
</tr>
</tbody>
</table>

s - Side Failure
### Table 29. Positive Torsion Experiments

<table>
<thead>
<tr>
<th>Fiber Orientation (degrees)</th>
<th>Tube</th>
<th>Notch Length (in)</th>
<th>$\tau_{fy}$ Failure Stress (ksi)</th>
<th>$\tau_{fy}$ Failure Strain (%)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>T0F</td>
<td>0.2</td>
<td>8.1</td>
<td>1.18</td>
<td>Improper Machine Set Up</td>
</tr>
<tr>
<td></td>
<td>T0D</td>
<td>0.2</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T0C3</td>
<td>0.5</td>
<td>4.2</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>T15B1</td>
<td>0.2</td>
<td>10.80</td>
<td>1.44</td>
<td>Failed away from Notch</td>
</tr>
<tr>
<td></td>
<td>T15B2</td>
<td>0.2</td>
<td>12.00</td>
<td>1.60</td>
<td>Failed away from Notch</td>
</tr>
<tr>
<td>45</td>
<td>T45D1</td>
<td>0.505</td>
<td>12.75</td>
<td>0.83</td>
<td>$t$, Crack Growth Began at 7.1 Ksi</td>
</tr>
<tr>
<td></td>
<td>T45B2</td>
<td>0.2</td>
<td>--</td>
<td>--</td>
<td>Tube did not Fail</td>
</tr>
<tr>
<td></td>
<td>T45F1</td>
<td>0.2</td>
<td>--</td>
<td>--</td>
<td>Tube did not Fail</td>
</tr>
<tr>
<td>87.5</td>
<td>T90J</td>
<td>0.2</td>
<td>8.60</td>
<td>1.35</td>
<td>Crack Growth Began at 5.9 Ksi</td>
</tr>
</tbody>
</table>

$t$ - Failed at top of notch only

### Table 30. Negative Torsion Experiments

<table>
<thead>
<tr>
<th>Fiber Orientation (degrees)</th>
<th>Tube</th>
<th>Notch Length (in)</th>
<th>$\tau_{fy}$ Failure Stress (ksi)</th>
<th>$\tau_{fy}$ Failure Strain (%)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>T0F2N</td>
<td>0.2</td>
<td>-8.70</td>
<td>1.16</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>T15CN</td>
<td>0.477</td>
<td>-2.84</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T45B2N</td>
<td>0.2</td>
<td>-3.00</td>
<td>0.22</td>
<td>$s$, Previously loaded in Pos Torsion</td>
</tr>
<tr>
<td></td>
<td>T45F1N</td>
<td>0.2</td>
<td>-2.49</td>
<td>0.19</td>
<td>Previously loaded in Pos Torsion</td>
</tr>
<tr>
<td></td>
<td>T45E2N</td>
<td>0.2</td>
<td>-2.45</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td></td>
<td>-2.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>87.5</td>
<td>T90KN</td>
<td>0.2</td>
<td>-6.50</td>
<td>0.83</td>
<td>Slow Crack Growth at 4.4Ksi</td>
</tr>
</tbody>
</table>

$s$ - Side Failure

Appendix C. Stress-Strain Curves 183
<table>
<thead>
<tr>
<th>Fiber Orientation (degrees)</th>
<th>Tube</th>
<th>Notch Length (in)</th>
<th>$\sigma_{yy}$ Failure Stress (ksi)</th>
<th>$\gamma_{yy}$ Failure Stress (ksi)</th>
<th>$\epsilon_{yy}$ Failure Strain (%)</th>
<th>$\gamma_{yy}$ Failure Strain (%)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>CNOB2</td>
<td>0.2</td>
<td>-41.4</td>
<td>-3.9</td>
<td>-22</td>
<td>.53</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CN6D2</td>
<td>0.488</td>
<td>-31.4</td>
<td>-3.7</td>
<td>-23</td>
<td>.65</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CN0F1</td>
<td>0.2</td>
<td>-5.7</td>
<td>-8.7</td>
<td>-.017</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>CN15A1</td>
<td>0.2</td>
<td>-13.4</td>
<td>-5.4</td>
<td>-.033</td>
<td>.21</td>
<td>N</td>
</tr>
<tr>
<td></td>
<td>CN15B2</td>
<td>0.2</td>
<td>-3.8</td>
<td>-3.8</td>
<td>-.046</td>
<td>.30</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>CN45E1</td>
<td>0.2</td>
<td>-0.4</td>
<td>-2.4</td>
<td>+.077</td>
<td>.174</td>
<td>N</td>
</tr>
<tr>
<td></td>
<td>CN45F2</td>
<td>0.2</td>
<td>-3.29</td>
<td>-3.24</td>
<td>-.047</td>
<td>.13</td>
<td></td>
</tr>
<tr>
<td>87.5</td>
<td>CN90K1</td>
<td>0.2</td>
<td>-14.2</td>
<td>-13.97</td>
<td>-1.1</td>
<td>3.25</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>CN90K2</td>
<td>0.2</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>F</td>
</tr>
</tbody>
</table>

| A - Asymmetric Failure     |
| N - Electronic Noise Caused Nonuniform Loading |
| F - Tube Did Not Fail      |
Figure 79. Tension Test of 2.5° Tubes
Figure 80. Tension Test of 15° Tubes
Figure 21. Tension Test of 45° Tubes
Figure 82. Tension Test of 87.5° Tubes
Figure 83. Compression Test of 2.5° Tube
Figure 84. Compression Test of 45° Tubes
Figure 85. Compression Test of 97.5° Tube
Figure 86. Positive Torsion Test of 2.5° Tubes
Figure 87. Positive Torsion Test of 15° Tubes
Figure 98. Positive Torsion Test of 45° Tubes
Figure 89. Positive Torsion Test of 87.5° Tube
Figure 90. Negative Torsion Test of 2.5° Tube
Figure 91. Negative Torsion Test of 15° Tube
Figure 92. Negative Torsion Test of 45° Tubes
Figure 93. Negative Torsion Test of 87.5° Tube

Appendix C. Stress-Strain Curves
Figure 94. Compression/Negative Torsion Test of 2.5° Tube (C/NT=1)

Appendix C. Stress-Strain Curves
Figure 95. Compression/Negative Torsion Test of 2.5° Tube (C/NT = 10)
Figure 96. Compression/Negative Torsion Test of 15° Tube (C/NT = 1)

Appendix C. Stress-Strain Curves
Figure 97. Compression/Negative Torsion Test of 15° Tube
Figure 98. Compression/Negative Torsion Test of 45° Tube (C/NT = 1)
Figure 99. Compression/Negative Torsion Test of 87.5° Tubes (C/NT = 1)
Figure 106. Positive Torsion Test of 87.5° Angleply Tubes
Figure 101. Negative Torsion Test of 87.5° Angleply Tube
Deidre A. Hirschfeld was born in Jamaica, New York, U.S.A., on April 28, 1953. She graduated from Fort Cherry High School, McDonald, PA in 1971 then received her B.S. in Metallurgy and Materials Science from Carnegie-Mellon University in 1975. After getting a Master of Applied Science in Metallurgical Engineering from the University of British Columbia, Vancouver, BC, Canada, in 1977, she was employed as a research engineer at Lukens Steel Co., Coatesville, PA. Beginning in 1981, she worked as a process metallurgist at Roanoke Electric Steel Corp., Roanoke, VA before pursuing a Ph.D. in Materials Engineering Science at Virginia Polytechnic Institute and State University.

[Signature]