The Optimal Design of Transducers for Active Control of Multiple-Frequency Structural Sound Radiation

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(Abstract)

This study is concerned with the problem of active structural acoustic control (ASAC) of radiation from elastically supported plates under multiple-frequency excitation. The control is achieved by use of piezoelectric (PZT) actuators. An optimization procedure is developed to find the optimal locations of actuators with which the radiated sound power from the plate is minimized. Also, the optimization of the piezoelectric actuator locations has been conducted for the active control of sound radiation from plates under heavy fluid loading.

In this dissertation, two types of approaches have been developed to optimally design the error sensors. One is to design the sensors which can provide information about the radiated sound power. The other is based on the sufficient conditions developed in this work for the error criteria in the linear quadratic optimal control theory. For the second approach, an optimization procedure has been developed to determine the optimal locations of microphone sensors in the sound field or the optimal dimensions and locations of polyvinylidene fluoride (PVDF) structural sensors applied directly to the plates. Moreover, a series of parametric studies have been conducted to evaluate the sensitivity of
the control performance of the optimally designed actuator and sensor systems to the changes in important system parameters, such as the disturbance frequency, the plate support conditions and so on.

The results demonstrate that for a plate under a multiple-frequency excitation, if the disturbance has an equal force amplitude for various frequencies, the optimization of the actuator location can be performed at the highest frequency component only. Through the use of a small number of carefully located error sensors, it is possible to achieve global sound attenuation. The optimization procedure provides not only a technique for control system design in practice, but also knowledge about the potential of active structural acoustic control.
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Chapter 1

Introduction

Noise, by definition, is unwanted sound. Since noises impact human life in a negative way, a significant amount of research has been conducted in attempting to control it. In general, noise control may be classified in two categories: (1) passive control; (2) active control. Both techniques can be applied directly to noise reduction at the noise source, noise control of the transmission path, and noise protection at the receiver. Normally, passive noise control may result in either modifying the noise-making structures or building a heavy barrier or enclosure to separate the receiver from the noise source. Passive noise control has been proved to be an effective technique in many applications. However, for some instances, the restrictions on the size and weight of the control system make this approach less efficient.

Active control of sound, traditionally, can be categorized as either active noise control (ANC) or active structural acoustic control (ASAC). Both of these are based on the principle of the linear superposition. Active noise control involves the use of secondary acoustic sources to suppress the sound from the primary source. The first concept of the method was originally presented by Paul Lueg in about 60 years ago (Lueg, 1936, Guicking 1990). Nowadays, this kind of control strategy has been demonstrated to be very successful in the noise cancellation for the typical systems, such as, plates (Deffayet and
Nelson, 1988), ducts (Tichy et al, 1984), and cylinders (Lester and Fuller 1987 and 1990, Jones and Fuller 1989). Moreover, there are extensive researches conducted to apply active noise control to practical situations like the aircraft fuselages (Elliott et al 1989, Nelson and Elliott, 1992), fans (koopmann et al 1988) and large transformers (Craig and Angevine 1993).

Active structural acoustic control, on the other hand, attacks the sound-radiating structures directly by altering the structure vibration field through the use of force inputs. Investigations of the ASAC strategy have involved with active control of wave propagation in elastic cylinder (Brevart and Fuller, 1994), sound transmission into cylinders (Snyder and Hansen 1991) and rectangular cavities (Pan et al 1990, 1992(a) and 1992(b)), and sound radiation from plates and beams. Most recently, the applications of this technique have been extended to the design of active enclosure (Fuller and White 1993) and active control of sound field inside an aircraft fuselage (Gibbs et al 1994). Since the pioneering work by Fuller (1987), there has been a considerable amount of work reported in the field of ASAC. The research activities and the associated literature are so remarkable in volume that the author will only focus on some of the significant research developments of this work.

1.1 Active Control of Sound Radiation from Finite Plates

Active structural acoustic control applies control inputs directly to the sound-radiating structures while minimizing the sound field. The resulting control output is the overall attenuation of the sound radiation level. Understanding of the structural response and the
associated acoustic behavior provides the key to the design of an effective control system. Hence, it is worthwhile to briefly review the research work on sound radiation from finite plates.

1.1.1 Sound Radiation from Finite Plates

Early work by Maidanik (1962, 1974) presented the mechanism of sound radiation from finite panels responding as in vacuo and the classification of the radiative properties of vibration modes. In terms of the modal radiation efficiency, he cataloged the vibrational modes of the plate into surface, edge, and corner modes. Davies (1969, 1971) studied the acoustic radiation from a simply supported thin rectangular plate with fluid loading on one side. He explicitly addressed the modal coupling coefficients. The real part of the radiation coupling coefficient can be treated as a measure of the contribution from one modal velocity to another modal component of the acoustic field, while the imaginary parts of the modal coupling coefficients characterize the inertia terms acting over the whole plate area. A quantitative evaluation of the radiation efficiencies of the vibration modes has been conducted by Wallace for both beams (1972(a)) and panels (1972(b)). Sandman (1975, 1977), with use of an approach different from that by Davies, also derived the equation of motion for simply supported plates with fluid loading on one side. He provided some numerical and experimental results for the sound radiation from the fluid-loaded plates.

Lomas and Hayek (1977) developed an approach to study the sound radiation from an elastically supported plate. To overcome the problem of the non-zero moment requirement along the plate edges for the elastical supports, they partitioned the problem into two sub-
problems. The first one is concerned with the fundamental solution for a simply supported plate with fluid loading on one side. The second one deals with the response of a fluid loaded plate under the excitation of the line moments distributed along the plate edges. The solution of the problem is obtained by superposing the solutions of the above two sub-problems. Moreover, they performed the numerical evaluation of the modal coupling coefficients ('Radiation Impedance' in their work) and compared their results with the approximations by Maidanik (1962), Davies (1969) and Pope et al. (1974). Their results show that both the radiative and the inertial cross-coupling terms are oscillating functions with respect to the frequency. Also, the radiative cross-coupling terms will be positive as well as negative values. For a certain frequency range, the magnitudes of the cross terms are comparable to those of the self-coupling terms except near the null points.

Keltie and Peng (1987) concluded that the effect of modal coupling on the radiated sound power was important to the degree that the modal coupling terms should not be ignored, especially for low frequency and off-resonant excitation cases. Berry, Guyader and Nicholas (1990) analyzed the sound radiation from a rectangular plate with edges arbitrarily supported. Through a series of numerical analyses, they presented and compared the radiation efficiencies of modes of simply supported, clamped, free, and guided plates. Their analytical model can deal with any local support conditions. Cunefare (1992) presented an optimal velocity (or modal) distribution for simply supported and clamped-clamped beams. With those velocity distributions, the beams would have minimum radiation efficiencies in a set of orthogonal 'radiation' modes. More importantly, the optimal velocity distribution is an inherent property of the beam and is independent of the disturbance and control forces.
1.1.2 Active Control of Sound Radiation from Finite Plates

Fuller (1989(a), 1990) presented an analysis of the active control of sound transmission through a circular plate clamped at the edges. The disturbance source is an acoustic plane wave incident at 45°. One and two secondary forces are applied to the plate, respectively, and the control objective is to minimize a cost function that is proportional to the transmitted sound power. The results show that with carefully positioned control forces, large global sound radiation can be obtained. With control force activated, the plate system will have a lower radiation efficiency. He deduced that the sound reduction is in some cases the result of redistributing the plate response ('modal restructuring') and in others as suppressing plate modal vibration ('modal suppression'). For some cases, sound attenuation may even accompany the case of an increase in the plate vibration amplitudes. Similar results were also obtained by Dimitriadis and Fuller (1989). In their work, piezoelectric actuators are applied to an identical circular plate. Dimitriadis and Fuller suggested that the locations and sizes of the piezoelectric actuators were very important to the control performance.

The experimental study conducted by Fuller et al (1989(b)) investigated the active control of sound transmission through a clamped circular plate. The error sensors used in the experiment were microphones and accelerometers. It was found that the control performance is always significant through the use of the microphones as error sensors. However, with use of the accelerometers as the error sensors, the accelerometer locations were so critical to the control output that, for some instances, the control system resulted in an increase in the sound radiation level.
Wang, Dimitriadis and Fuller (1990, 1991) applied multiple piezoelectric actuators to a simply supported plate for controlling sound radiation. In their analytical model, a cost function that is proportional to the radiated sound power was utilized as the control objective function. They concluded that the control would be much more effective with the use of multiple actuators. Through analyzing different arrangements of the piezoelectric actuators, Wang et al. pointed out that the actuator locations were the key factor to the control effectiveness.

Meirovitch and Thangjitham (1990(a)) employed a large number of control actuators to a simply supported plate. Because of the high excitation frequencies, the number of the modes included in the numerical analysis was 240. The control strategy in their work is, first, to select and suppress a group of vibration modes that are considered to be the most active sound radiators, then to evaluate the attenuation level of the sound field. A feedback control approach is applied to generate suitable control forces to the actuators. The same control scheme was proposed by Meirovitch and Thangjitham (1990(b) and (c)) for controlling sound radiation from a simply supported plate with fluid loading on one side and an orthotropic plate.

With a simply supported rectangular plate as the experimental plant, the experimental comparison made by Fuller, Hansen and Snyder (1991) demonstrates that the control with use of force inputs is more effective than of an array of secondary acoustic sources close to the plate. In general, the number of the control point forces required is less than that of the acoustic sources. They concluded the two basic mechanisms in ASAC strategy, i.e., 'modal suppression' and 'modal restructuring' as discussed previously. For the on-resonant
excitation cases, the control is mainly achieved by the 'modal suppression'. While for the off-resonant cases, 'modal restructuring' will be the dominant mechanism. Snyder and Hansen (1991), through an analytical study, also reached the same conclusion as the above. They also emphasized the importance of the actuator locations on the control performance of the system.

Wang, Burdisso and Fuller (1991) developed an optimization approach for optimal locations of piezoelectric actuators. In their work, the optimization objective function is a finite sum of the mean-squared sound pressures in the far field. It was shown that with optimally located actuators, the control performance of the system is much better than that with the arbitrarily positioned actuators.

Through a series of experimental study, Clark and Fuller (1991, 1992(a) and (b)) investigated the implementation of piezoelectric actuators and PVDF sensors in active control of sound radiation from simply supported plates. Their work covered the scope of shaped PVDF sensors, rectangular PVDF sensors, multiple piezoelectric actuators, and design of the 'smart' structures which can adapt to the environment such that the structural vibration and structure-borne sound can be minimized. The results demonstrate that good attenuation of radiated sound level can be achieved for both on and off resonance with use of shaped PVDF sensors. Their work shows the promise for developing adaptive structures with both actuators and sensors bonded to the structures. In order to accomplish maximum control authority, Clark and Fuller (1992(c)) developed an approach for the optimization of the sizes and locations of rectangular PVDF sensors. The optimized design was experimentally verified. The experimental results illustrate the advantage of the optimally located actuator and sensor in active structural acoustic control.
and demonstrate that the importance of optimally locating the control transducers is of the same importance as the number of control channels.

Burdisso and Fuller (1992(a), (b) and (c)) investigated the dynamic characteristics of feedforward controlled elastic structures in both vibration suppression and acoustic control. Instead of looking at the structural system and controller separately, Burdisso and Fuller observed the structures after control appear as a new system to a primary disturbance input. Their results show that the controlled system has a new set of eigenvalues and associated eigenvectors which are independent of the disturbance forces and only a function of the control force locations and the structural properties.

Maillard and Fuller (1992,1993) developed a new sensing technique for predicting the information related to the acoustic radiation from vibrating structures. Through the use of a set of point structural sensors, such as accelerometers, the spatial wave-number transform is performed in real time with an array of filters and associated signal processing. Both analytical and experimental results demonstrate the ability of this sensing technique to replace the error microphones in the acoustic field. More importantly, good broadband global control of sound radiation can be achieved with only a few point sensors mounted on structures such as beams, plates, and cylinders.

Gu and Fuller (1993) studied the active control of sound radiation from a fluid loaded rectangular plate. The control forces are optimally determined such that the acoustic power radiated from the fluid-loaded plate is minimized. Both point force and piezoelectric actuators are employed as the control inputs. The control performance with use of the point force and piezoelectric actuators is compared. Their results show that the
control performance is dependent on the excitation frequency and the on-resonant excitation case is relatively easy to control than the off-resonant one. Also, in their analysis (Gu 1992), the effect of the localized structural discontinuities, such as the point mass and line mass discontinuities, on the control output has been evaluated. They concluded that ASAC can effectively control the sound radiation from the fluid-loaded structures.

Aiming at reducing the control channels and the complexity of the controller, Carneal and Fuller (1994) developed a biologically inspired (BIO) control approach. In their work, the control forces are divided into 'master' and 'slave' inputs. The 'master' input is directed by the centralized controller, while the 'slave' inputs are driven through the use of a local learning rules. Two local learning rules, the phase variation and the optimal distribution methods, are proposed and analyzed in their work. The experimental and analytical results demonstrated the potential of this BIO control approach in both active sound and vibration control in distributed elastic system.

1.2 Objectives

As reviewed in the above section, considerable research work has been conducted in the field of active control of sound transmission/radiation from finite plates. However, some fundamental investigation about the best location of the control actuators and sensors is still needed, especially with consideration of the total sound power radiated from the plates. Also, the effect of some system parameters, such as, edge support conditions, disturbance location, disturbance frequencies, and the point mass discontinuities, on the control effectiveness of the optimally designed actuators and sensors has not been studied.
Moreover, presently there is no work reported on the optimization of the actuators and sensors for finite plates under multiple-frequency excitation and heavy-fluid loading.

On the basis of the literature review and study, the specific objectives of this dissertation are to develop theory in order to:

- optimize the locations of piezoelectric actuators for controlling sound radiation from plates under multiple-frequency excitation;
- evaluate the effect of the variation of some important system parameters, such as, boundary support conditions, excitation frequencies, disturbance locations, and the point mass discontinuities, on the control performance of the optimized actuators;
- establish sufficient conditions for the error criteria in the linear quadratic optimal control;
- optimize the microphone sensor locations based on the established sufficient conditions for the error criteria;
- optimize the dimensions and locations of the PVDF sensors for controlling sound radiation from plates under multiple-frequency excitation;
- evaluate the effect of design parameters, such as, locations of the optimized sensors and excitation frequencies, on the control performance of the optimally designed sensor system;
- optimize the locations of piezoelectric actuators for controlling sound radiation from plates under heavy fluid loading;
- perform experiments to evaluate the performance of a control system that is optimally designed for controlling multiple-tone sound radiation using the above theories.
1.3 Organization of This Dissertation

The work in this dissertation is broken up into several major sections.

Chapter 1 is concerned with an introduction of the problem and a review of related work.

Chapter 2 is concerned with the analysis of the optimal locations of the piezoelectric actuators for active control of sound radiation from finite plates. The fluid medium considered in Chapter 2 is air. A point force is selected as the disturbance source that is either driven by single or multiple tones. In order to evaluate the effect of the boundary support conditions on the control performance of the optimized actuators, the plate model is set up in such a way that it can deal with any supporting conditions. Arbitrary elastic supports are modeled with massless rotary and deflection springs along the plate edges. The solution to the plate motion is based on either trigonometric or polynomial trial functions. An optimization procedure is developed to find the optimal locations of the piezoelectric actuators for the plate under multiple-frequency excitation. The objective function in the optimization is the total sound power radiated from the plate, while the optimal control force amplitudes to fixed actuators are determined through the use of the linear quadratic optimal control theory. An analytical sensitivity analysis formulation is derived to speed up the optimization procedure. The robustness of the optimized control system with respect to the excitation frequencies, disturbance locations, edge supports, and the discrete point mass is evaluated.

Chapter 3 studies the active control of sound radiation from plates under heavy fluid loading. The solution to the plate motion is based on the *in vacuo* eigenfunctions of a
homogeneous plate. In order to achieve the maximum control authority, an optimization procedure is again developed to find the optimal locations of the piezoelectric actuators. The difference and connection between the equations of plate motion for heavy and light fluid loading are addressed.

Chapter 4 studies the sensor design for the active control of sound radiation from plates. In the first part of Chapter 4, the sensors are designed in such a way that the sensors can provide an approximation for the radiated sound power. Both microphone and accelerometer sensors are considered. For the second part of Chapter 4, the sufficient conditions for the error criteria in the linear quadratic optimal control theory are established. An optimization procedure is developed to obtain the optimal sensor design based on those sufficient conditions. The design variables are the special locations of the error microphones and the dimensions and locations of the PVDF sensors. Also, the optimal design of the PVDF sensors for controlling sound with multiple frequencies is conducted. Upon finishing the optimal design, the sensitivity of the optimized control system with respect to the changes in the excitation frequency and sensor locations is evaluated.

In Chapter 5, first, the adaptive multiple-input-multiple-output Filtered-X LMS control approach for which the work on the design of the piezoelectric actuators and PVDF sensors in Chapter 2, 3, and 4 is proposed is briefly reviewed. Then, Chapter 5 presents the experimental results for one optimally designed control system. The control system is developed to control sound radiation from a plate under three-frequency excitation. The control performance of the experiment is evaluated by the sound power radiated from the plate. The sound power is calculated through the use of the plate velocity field that was
measured with a scanning laser Doppler vibrometer before and after control. A regression analysis is performed to fit the test data. Finally, the analysis and discussion about the experimental results are conducted.

Chapter 6 summarizes the major conclusions deduced from this work.
Chapter 2

Actuator Design for Active Control of Sound Radiation from Plates

2.1 Introduction

As outlined in the introduction of Chapter 1, research into the active structural acoustic control (ASAC) has drawn substantial attention in recent years. The analytical and experimental work on active control of sound radiation from panels by Wang, Dimitriadis and Fuller (1990), Wang, Burdisso, and Fuller (1991), Clark and Fuller (1991(a), 1991(c) and 1992) and Thomas (1992) has shown the advantages of the ASAC strategy. However, no analytical model has so far been developed to optimize the actuator locations for controlling sound radiation from a plate under a multiple-frequency disturbance. Also, the sensitivity of the control performance of optimally located actuators to the other design variables, such as the edge support conditions, disturbance locations, and discontinuities (modeled by a discrete mass attached on the plate), has not yet been analyzed.
This chapter is concerned with the performance of piezoelectric actuators for actively controlling sound radiation from vibrating plates. The analytical model used in this chapter is of an elastically supported thin rectangular plate with an attached discrete point mass. The fluid medium considered here is air, therefore it is assumed only one way coupling is effective, i.e., the influence of the fluid loading on the plate response is neglected. The arbitrary elasical supports are modeled with massless rotary and deflection springs along the edges of the plates, which was used previously by Berry et al (1991) and Thomas (1992). The equations of motion for the plates are derived with use of a variational formulation and the Rayleigh-Ritz method (Shames and Dym, 1985). To conduct extensive numerical analysis, two kinds of Rayleigh-Ritz trial functions are utilized, one consists of trigonometric function, the other polynomial functions. Linear quadratic optimal control theory (LQOCT) is applied to find the optimal amplitudes of the control forces, in which the radiated acoustic power from plate is chosen as the cost function. A procedure is developed to determine the optimal actuator locations for which the control performance is maximum. The objective function in this optimization problem is the radiated sound power, the same as that used in LQOCT. In order to conduct the optimization effectively, an analytical sensitivity analysis formulation is derived and implemented in the optimization procedure. Throughout the analysis, a point force, which contains either a single frequency or multiple frequencies, is chosen as the disturbance. The numerical results demonstrate a superior performance of the optimally located actuators over those located arbitrarily. A series of parameter studies are carried out to evaluate the robustness of the optimized actuators with respect to excitation frequencies, disturbance locations, edge support conditions and the discrete point mass. Some general conclusions about the optimization of actuator locations are obtained for the plate under multiple-frequency excitation. Also, the results show the necessity and importance of
optimizing actuator locations with multiple excitation frequencies considered simultaneously.

2.2 Equations of Motion of the Plate

The analytical model is of a thin rectangular plate having dimensions $L_x$ and $L_y$ with one discrete mass $M_d$ attached to the plate at $(x_d, y_d)$ as shown in Figure 2.1. The effects of fluid loading on the plate are neglected because the fluid media considered in this chapter is air. The dynamic action of the boundary supports is modeled by massless rotary and deflection springs. The bending moment $M_i$ and shear force $F_s$ at point $s$ on the plate contour are given by:

$$M_i(s) = K_i(s) \frac{\partial w(s)}{\partial n_e}$$ (2.1)

$$F_s(s) = -K_s(s) w(s)$$ (2.2)

where $w(s)$ is the flexural out-of-plane displacement of point $s$, $n_e$ is the normal exterior to the edge as shown in Figure 2.2, $K_i(s)$ and $K_s(s)$ are defined as translational stiffness and rotational stiffness per unit length at point $s$ along the edges, respectively. The directions of the moment and shear force are indicated also in Figure 2.2.

The Hamiltonian (Berry et al, 1990) for the response of plate under an external force $f(x, y, t)$ can be written as:
\[ H(w) = \int_{t_0}^{t_1} \left[ T \left( \frac{\partial w}{\partial t} \right) - V(w) + U(w) \right] dt \]  \hspace{1cm} (2.3)

where \( t_0 \) and \( t_1 \) are two arbitrary times, \( T \) is the kinetic energy of the plate response,

\[ T = \int_{-1}^{1} \int_{-1}^{1} \frac{\rho}{8} \frac{L_s^3}{r} \left( \frac{\partial w}{\partial t} \right)^2 \, d\alpha d\beta + \frac{1}{2} M_s \left( \frac{\partial w}{\partial t} \left( x_d, y_d \right) \right)^2 \]  \hspace{1cm} (2.4)

and \( V \) is the potential energy of the plate,

\[ V = \frac{D}{L_s^2} \left[ \int_{-1}^{1} \int_{-1}^{1} \left\{ r^2 \left( \frac{\partial^2 w}{\partial \alpha^2} \right)^2 + r^4 \left( \frac{\partial^2 w}{\partial \beta^2} \right)^2 + 2vr^2 \frac{\partial^2 w}{\partial \alpha^2} \frac{\partial^2 w}{\partial \beta^2} + 2(1-v)r^2 \left( \frac{\partial^2 w}{\partial \alpha \partial \beta} \right)^2 \right\} d\alpha d\beta \right. \]

\[ \left. + \int_{-1}^{1} \frac{k_s}{4} \left[ k_s(\alpha,1)w^2(\alpha,1) + k_s(\alpha,-1)w^2(\alpha,-1) \right] d\alpha \right. \]

\[ \left. + \int_{-1}^{1} \frac{k_s}{4r} \left[ k_s(1,\beta)w^2(1,\beta) + k_s(-1,\beta)w^2(-1,\beta) \right] d\beta \right. \]

\[ \left. + \int_{-1}^{1} \left[ k_s(\alpha,1) \left( \frac{\partial w}{\partial \beta} (\alpha,1) \right)^2 + k_s(\alpha,-1) \left( \frac{\partial w}{\partial \beta} (\alpha,-1) \right)^2 \right] d\alpha \right. \]

\[ \left. + \int_{-1}^{1} \left[ k_s(1,\beta) \left( \frac{\partial w}{\partial \alpha} (1,\beta) \right)^2 + k_s(-1,\beta) \left( \frac{\partial w}{\partial \alpha} (-1,\beta) \right)^2 \right] d\beta \right. \]  \hspace{1cm} (2.5)

and \( U \) is the input work done on the plate by the external force,

\[ U = \frac{L_s^3}{4r} \int_{-1}^{1} \int_{-1}^{1} f(\alpha, \beta) w(\alpha, \beta) d\alpha d\beta \]  \hspace{1cm} (2.6)
where \( \alpha = \frac{2x}{L_x}, \beta = \frac{2y}{L_y}, r = \frac{L_x}{L_y} \), \( D \) is the bending stiffness of the plate, \( \nu \) is the Poisson's ratio. \( k_i(\alpha, \pm 1), k_i(\pm 1, \beta) \) and \( k_j(\alpha, \pm 1), k_j(\pm 1, \beta) \) are non-dimensional edge parameters and are defined as follows:

\[
k_i(\alpha, \beta) = \left. K_i(\alpha, \beta) \frac{L_x^3}{D} \right|_{\alpha = \pm 1 \ or \ \beta = \pm 1}
\]

\[
k_j(\alpha, \beta) = \left. K_j(\alpha, \beta) \frac{L_x}{D} \right|_{\alpha = \pm 1 \ or \ \beta = \pm 1}
\]

Hamilton's principle states that the true displacement of the plate is the displacement field \( w(x, y, t) \) that extremizes the Hamiltonian of Eq. (2.3) during the time interval \( (t_0, t_1) \) (Shames and Dym, 1985). That is:

\[
\delta \left\{ \int_{t_0}^{t_1} \left[ T \left( \frac{\partial w}{\partial t} \right) - V(w) + U(w) \right] dt \right\} = 0
\]

A Rayleigh-Ritz method is used to solve Eq. (2.8). The displacement field \( w(x, y, t) \) is approximated in terms of a series of linearly independent functions \( \phi_{mn}(x, y) \) which satisfy the boundary conditions of Eq. (2.1) and Eq. (2.2). That is, let

\[
w(x, y, t) = \sum_{n} \sum_{m} [a_{mn}(t) \phi_{mn}(x, y)] = \left\{ a_{mn}(t) \right\}^T \left\{ \phi_{mn}(x, y) \right\}
\]

where \( a_{mn} \) are undetermined coefficients. If the size of the series is truncated so that \( m = N, \quad n = N \), then the plate is modeled as a \( N^2 \)-degree-of-freedom system, the \( a_{mn} \) can be considered as generalized coordinates or modal amplitudes.
Substituting Eq. (2.9) into Eq. (2.3) gives:

\[ H(w) = \int_{T}^{t_{n}} [T\{a_{nn}\} - V\{a_{nn}\} + U\{a_{nn}\}]dt \]  

(2.10)

The stationary condition of Eq. (2.8) can be expressed by the Lagrange equations for \( a_{nn} \):

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{a}_{mn}} \right) + \frac{\partial V}{\partial a_{mn}} = \frac{\partial U}{\partial a_{mn}}, \quad \begin{array}{c} m = 1, 2, \ldots, N \\ n = 1, 2, \ldots, N \end{array} \]  

(2.11)

Substituting Equations (2.5), (2.6), (2.7) and (2.9) into Eq.(2.11) results in the equations of motion for the plate:

\[ \begin{bmatrix} m_{mnkl} \end{bmatrix}\{\ddot{a}_{mn}\} + \begin{bmatrix} k_{mnkl} \end{bmatrix}\{a_{mn}\} = \{f_{mn}(t)\} \]  

(2.12)

where the subscripts \( mn \) and \( kl \) are the indices of the trial function orders. The elements of the plate mass matrix are,

\[ m_{mnkl} = m_{mnkl}^{\text{Plate}} + m_{mnkl}^{\text{Mass}}, \]  

(2.13)

where \( m_{mnkl}^{\text{Plate}} \) is the plate's mass contribution to the mass matrix, \( m_{mnkl}^{\text{Mass}} \) is the discrete mass's contribution to the mass matrix,

\[ m_{mnkl}^{\text{Plate}} = \frac{L_{x}^{2}}{4r} \int_{-1}^{1} \int_{-1}^{1} \rho h \phi_{nn}(\alpha, \beta) \phi_{kl}(\alpha, \beta) \, d\alpha d\beta \]  

(2.14)
m_{mnkl}^{mass} = M_c \phi_{mn}(\alpha_d, \beta_d) \phi_{kl}(\alpha_d, \beta_d) \tag{2.15}

and the elements of the stiffness matrix are,

\[ k_{mnkl} = k_{mnkl}^{\text{plate}} + k_{mnkl}^{\text{edge}} \tag{2.16} \]

where

\[
k_{mnkl}^{\text{plate}} = \frac{D}{L_\alpha^2} \int_{-1}^{+1} \int_{-1}^{+1} 2 \left( \frac{\partial^2 \phi_{mn}}{\partial \alpha^2} \frac{\partial^2 \phi_{kl}}{\partial \beta^2} + r^4 \frac{\partial^4 \phi_{mn}}{\partial \alpha^4} \frac{\partial^4 \phi_{kl}}{\partial \beta^4} + 4r^2 \frac{\partial^2 \phi_{mn}}{\partial \alpha^2} \frac{\partial^2 \phi_{kl}}{\partial \beta^2} + \frac{(1-v)r^2}{\partial \alpha^2} \frac{\partial \phi_{mn}}{\partial \beta} \frac{\partial \phi_{kl}}{\partial \alpha} \right) d\alpha d\beta \tag{2.17} \]

is the contribution of the plate stiffness to the stiffness matrix, and

\[
k_{mnkl}^{\text{edge}} = \frac{D}{L_\alpha^3} \left[ \int_{-1}^{+1} \frac{1}{2} \left[ k_s(\alpha,1) \phi_{mn}(\alpha,1) \phi_{kl}(\alpha,1) + k_s(\alpha,-1) \phi_{mn}(\alpha,-1) \phi_{kl}(\alpha,-1) \right] d\alpha 
\right.
\left. + \int_{-1}^{+1} \frac{1}{2} \left[ k_s(1,\beta) \phi_{mn}(1,\beta) \phi_{kl}(1,\beta) + k_s(-1,\beta) \phi_{mn}(-1,\beta) \phi_{kl}(-1,\beta) \right] d\beta 
\right.
\left. + \int_{-1}^{+1} 2r^2 \left( k_s(\alpha,1) \frac{\partial \phi_{mn}}{\partial \beta} (\alpha,1) \frac{\partial \phi_{kl}}{\partial \beta} (\alpha,1) + k_s(\alpha,-1) \frac{\partial \phi_{mn}}{\partial \beta} (\alpha,-1) \frac{\partial \phi_{kl}}{\partial \beta} (\alpha,-1) \right) d\alpha 
\right.
\left. + \int_{-1}^{+1} 2r^2 \left( k_s(1,\beta) \frac{\partial \phi_{mn}}{\partial \alpha} (1,\beta) \frac{\partial \phi_{kl}}{\partial \alpha} (1,\beta) + k_s(-1,\beta) \frac{\partial \phi_{mn}}{\partial \alpha} (-1,\beta) \frac{\partial \phi_{kl}}{\partial \alpha} (-1,\beta) \right) d\beta \right] \tag{2.18} \]

is the edge's contribution to the stiffness matrix.
The elements of model force vector are

\[ f_{mn}(t) = \frac{L_r^2}{4r} \int_{-1}^{1} \int_{-1}^{1} f(\alpha, \beta, t) \phi_{mn}(\alpha, \beta) d\alpha d\beta \]  

(2.19)

It is assumed that the force function is of the following form:

\[ f(\alpha, \beta, t) = \sum_{j=1}^{N_f} \left\{ f_j(\alpha, \beta) e^{i\omega_j t} \right\} \]  

(2.20)

where \( N_f \) is the number of excitation frequencies, \( \omega_j \) is the \( j \)th frequency. Then the modal response of the plate can be written as:

\[ a_{mn}^j(t) = \sum_{j=1}^{N_f} \left( a_{mn}^j e^{i\omega_j t} \right) \]  

(2.21)

where \( a_{mn}^j \) are the solution of the linear system:

\[ (-\omega_j^2 [m_{mn}] + [k_{mn}]) [a_{mn}^j] = \{ f_{mn}^j \} \]  

(2.22)

with

\[ f_{mn}^j = \frac{L_r^2}{4r} \int_{-1}^{1} \int_{-1}^{1} f_j(\alpha, \beta) \phi_{mn}(\alpha, \beta) d\alpha d\beta \]  

(2.23)
2.2.1  Response of a Simply Supported Plate

For a simply supported plate, the trial function of $\phi_{mn}(x,y)$ can be chosen as double sine of the form:

$$\phi_{mn}(x,y) = \sin \left( \frac{m\pi}{2} + \frac{m\pi x}{L_x} \right) \sin \left( \frac{n\pi}{2} + \frac{n\pi y}{L_y} \right), \quad m=1,2,\ldots,N$$
$$n=1,2,\ldots,N \quad (2.24)$$

Because that $k_x = 0$ and $k_y = \infty$ are corresponding to the simply supported boundary condition, the elements of the mass and stiffness matrices in Eq. (2.12) can be derived as:

$$m_{mnkl} = \frac{L_x L_y}{4} \rho h \delta_{mk} \delta_{nl} + M_d \phi_{mn}(x_d,y_d) \phi_{kl}(x_d,y_d) \quad (2.25)$$

$$k_{mnkl} = \frac{D \pi^4 L_x L_y}{4} \left( \frac{m}{L_x} \right)^2 + \left( \frac{n}{L_y} \right)^2 \delta_{mk} \delta_{nl} \quad (2.26)$$

where $\delta_{mk}$ and $\delta_{nl}$ are the Kronecker delta functions.

It is assumed that there are two kinds of external forces acting on the plate, one is the point force which serves as disturbance, the other due to piezoelectric patches for control actuators. The point force disturbance with multiple frequencies located at $(x_0, y_0)$, as shown in Figure 2.3, is of the following form:

$$f_d(t) = \sum_{j=1}^{N_f} (f_d^j e^{i\omega_j t}) \quad (2.27)$$
The modal force for the \( j \)th disturbance frequency can be expressed as (Pilkey et al., 1987):

\[
f_{mn}^{\phi j} = \frac{4f_d^j}{L_x L_y} \phi_{mn}(x_0, y_0)
\]  

(2.28)

For the \( s \)th piezoelectric actuator located at coordinate \( x_s^i, y_s^i \) with dimension \( 2a_s^i, 2b_s^i \), as shown in Figure 2.3, the modal force for the \( j \)th frequency can be written as (Dimitriadis and Fuller, 1991):

\[
f_{mn}^{\phi js} = \frac{16C_0 d_{31} V_s^i (k_m^2 + k_n^2)}{mnt_i \pi^2} \phi_{mn}(x_s^i, y_s^i) \sin(k_m a_s^i) \sin(k_n b_s^i)
\]  

(2.29)

where \( C_0 \) is a function of material properties and dimensions specified by Dimitriadis and Fuller (1991), \( d_{31} \) is the piezoelectric strain constant. In addition, \( t_i \) is the thickness of the piezoelectric actuator, \( V_s^i \) is the voltage applied along the polarization direction for the \( j \)th excitation frequency.

2.2.2 Response of an Elastically Supported Plate

For plates with edges elastically restrained against deflection and rotation, the trial function of Eq. (2.24) will not be compatible with the arbitrary edge conditions. Therefore, trial functions with polynomials are selected for the plates:

\[
\phi_{mn}(x, y) = \left\{ \left( \frac{2x}{L_x} \right)^m \left( \frac{2y}{L_y} \right)^n \right\} \quad m = 0, 1, \ldots, N
\]

\[
n = 0, 1, \ldots, N
\]  

(2.30)
Substituting Eq. (2.33) into Eq. (2.12), the elements of mass and stiffness matrices can expressed

as

\[ m_{mnl} = m_{mnl}^{plate} + m_{mnl}^{mass} \]

with

\[
m_{mnl}^{plate} = \begin{cases} 
\frac{(\rho h)L_s^2}{r(m+k+1)(n+l+1)} & \text{for } m+k = \text{even} \\
0 & \text{otherwise}
\end{cases}
\]

\[ (2.31) \]

\[ m_{mnl}^{mass} = M_d \phi_{nn}(\alpha_d, \beta_d) \phi_{kl}(\alpha_d, \beta_d) \]

\[ (2.32) \]

and

\[ k_{mnl} = k_{mnl}^{plate} + k_{mnl}^{edge} \]

with

\[
k_{mnl}^{plate} = \begin{cases} 
\frac{16D}{L_s^2 r^4} \left( \frac{m(m-1)k(k-1)}{(m+k-3)(n+l+1)} + r^4 \frac{n(n-1)(l-1)}{(m+k+1)(n+l+3)} \right) & \text{for } m+k = \text{even} \\
0 & \text{otherwise}
\end{cases}
\]

\[ (2.33) \]

\[
k_{mnl}^{edge} = \begin{cases} 
\frac{2D}{L_s^2} \left( \frac{k_x}{m+k+1} + \frac{k_y}{r(n+l+1)} + \frac{4k_x n l}{m+k+1} + \frac{4k_y m l}{r(n+l+1)} \right) & \text{for } (m+k) = \text{even} \\
0 & \text{otherwise}
\end{cases}
\]

\[ (2.34) \]
For the point force disturbance expressed in Eq. (2.17), the modal force for the \( j \)th frequency can be written as (Berry \textit{et al}, 1990):

\[
f_m^j = f_d^j \phi_m(x_0, y_0)
\]  

(2.35)

If the piezoelectric patches are used as the actuators, for the \( s \)th actuator as shown in Figure 2.3, the modal force can be expressed as:

\[
f_{mn}^{aj} = \frac{L_s^2 d_{31}}{4rt} V_s^j \left[ \frac{L_y}{L_x} \frac{m}{n+1} (\alpha_2^{m-1} - \alpha_1^{m-1}) (\beta_2^{n+1} - \beta_1^{n+1}) + \frac{L_x}{L_y} \frac{n}{n+1} (\alpha_2^{m+1} - \alpha_1^{m+1}) (\beta_2^{n-1} - \beta_1^{n-1}) \right]
\]  

(2.36)

2.2.3 Total Motion of the Plate

The total modal amplitude of the plate under the action of the disturbance force and the control actuators can be expressed as:

\[
a_m^j(t) = \sum_{j=1}^{N_a} (a_m^j e^{i\omega_j t}) = \sum_{j=1}^{N_a} \left\{ \left[ \sum_{s=1}^{N_s} (Q_{m}^{aj} V_s^j) + Q_{mn}^{aj} f_d^j e^{i\omega_j t} \right] \right\}
\]  

(2.37)

where \( N_a \) is the number of the actuators, and:

\[
\{ Q_{m}^{aj} \} = \left[ -\omega_j^2 [m_{\text{mod}}} + [k_{\text{mod}}] \right]^{-1} \{ \tilde{f}_m^{aj} \}
\]  

(2.38)
\[
\{Q_{mn}^{ij}\} = \left[-\omega_j^2 m_{mnkl} + [k_{mnkl}]^{-1} \{ \tilde{f}_{kl}^{ij} \right\} \quad (2.39)
\]

Where \( \{ \tilde{f}_{kl}^{ij} \} \) and \( \{ \tilde{f}_{kl}^{ij} \} \) are the modal force vectors due to the control and the disturbance with unit force amplitudes, respectively.

The total response of the plate under the action of the disturbance force and the control actuators can be given by:

\[
w(x, y, t) = \sum_{j=1}^{N_f} \sum_{m=1}^{N} \sum_{n=1}^{N} a_{mnj} \phi_{mn}(x, y) \quad (2.40)
\]

It should be noted here that by using the trial functions of Eq. (2.30), the motion of plates with arbitrary boundaries, including simply supporting conditions, can be evaluated. However, in order to find the accurate response, the polynomial order of the trial functions must be significantly large. Usually, the number of the polynomials in Eq. (2.30) should be at least two times of the number of the modes desired. For the two dimensional plates, this factor will be about four. That is why in this work, for simply supported plates, the trial functions of double sine defined in Eq. (2.24) are still used. These trigonometric functions satisfy completely the simply supporting boundary conditions and represent the orthonormal plate modes. For plates with arbitrary boundaries, the orthonormal plate modes and their amplitudes can be obtained from the system eigenvectors and the coefficients \( \{a_{mn}\} \) in Eq. (2.22) (Thomas, 1992). The relevant formulations can be found in Appendix A.
2.3 The Total Sound Power Radiated by the Plate

When the plate is subjected to the excitation of disturbance and control force with multiple frequencies, the total radiated sound pressure at a point \((R, \theta, \phi)\) in the far field for light fluid loading can be written as (Roussos, 1984):

\[
p_i(R, \theta, \phi, t) = \sum_{j=1}^{N_f} \left\{ \frac{-\omega_j^2 \rho_a}{2\pi R} e^{i\omega_j t} \int_{-1}^{1} \int_{-1}^{1} \left[ w(\alpha, \beta) e^{-ik_j \left( \frac{\sin \theta \cos \phi - \sin \theta \sin \phi}{R} \right)} \right] d\alpha d\beta \right\}
\]

(2.40)

where \(k_j = \frac{\omega_j}{c}\) is the wave number for the \(j\)th frequency, \(\rho_a\) is the mass density of air.

The total radiated acoustic power in the far field is defined as:

\[
\Phi_p = \int_{S} \left\{ \frac{1}{T} \int_{s} \left[ |p_i|^2 \frac{1}{\rho_a c} dt \right] ds \right\}
\]

(2.41)

where \(T\) is equal to the minimum factorial of the periods of the \(N_f\) harmonic wave and can be viewed as the common period of the \(N_f\) harmonic waves, \(p_i\) is the total sound pressure and can be rewritten as:

\[
p_i = \sum_{j=1}^{N_f} p_i^j
\]

(2.42)

where \(p_i^j\) is the sound pressure caused by the \(j\)th excitation frequency. It can be shown as derived in Appendix B that:
\[
\frac{1}{T} \int_0^T p_i^* p_i^{*'} \, dt = 0 \quad \text{for} \quad i \neq j.
\] (2.43)

Therefore, the total sound power can be expressed as:

\[
\Phi_p = \sum_{j=1}^{N_f} \left\{ \frac{1}{\rho_a c} \int |p_j|^2 \, ds \right\}
\] (2.44)

Equation (2.44) shows that the total sound power under the multiple-frequency excitation is equal to the summation of the radiated acoustic power at each of the excitation frequencies. By derivation, the total radiated sound power can then be written in the form:

\[
\Phi_p = \sum_{j=1}^{N_f} \left\{ \alpha_{mn}^j \right\}^{'''} \left[ \delta_{mnk}^j \right] \left[ a_{kl}^j \right]
\] (2.45)

where

\[
\delta_{mnk}^j = \frac{\rho_a}{8c} \left( \frac{L_s^2 \omega_j^2}{4r \pi} \right)^2 \int_0^{2\pi} \int_0^{\pi/2} I_{mn}^j (\theta, \phi) I_{kl}^j (\theta, \phi) \sin \theta d\theta d\phi
\] (2.46)

with

\[
I_{mn}^j = \int_{-1}^{+1} \int_{-1}^{+1} \phi_{mn}(\alpha, \beta) e^{i \frac{\alpha}{2c}(L_a \alpha \sin \theta \cos \phi + L_\beta \alpha \sin \theta \sin \phi)} d\alpha d\beta
\] (2.47)

In the above equation, \( \delta_{mnk}^j \) are the modal radiation coefficients of the plate. The properties of the modal radiation coefficients are: (1) \( \delta_{mnk}^j \) are real constants for excitation frequency \( \omega_j \); (2) \( \delta_{mnk}^j = \delta_{klm}^j \), hence are symmetric. Also, it could be easily proved that
\( \delta_{mnkl}^{i} \) are non-zero only when \( m+k = \text{even} \) and \( n+l = \text{even} \) at the same time. Thus, at most, each mode can be coupled to only one quarter of all the other modes.

For simply supported plates with the trial functions of the form in Eq. (2.24), \( \delta_{mnkl}^{i} \) are linearly proportional to the real part of the radiation coefficient defined by Davies (1969). Also, \( \delta_{mmnn}^{i} \) are linearly proportional to the radiation efficiencies \( S_{nn} \) defined by Wallace (1972(b)).

Furthermore, with use of Eq. (2.22), the radiated sound power of Eq. (2.45) can be written as:

\[
\Phi_{p} = \sum_{j=1}^{N_{i}} \left\{ f_{mn}^{q} \right\}^{\eta} \left[ E_{mnkl}^{i} \right] \left[ f_{kl}^{q} \right] \tag{2.48}
\]

where \( [E^{i}] = [D^{i}]^{H}[\delta^{i}][D^{i}] \), and matrix \([D^{i}]\) is defined as:

\[
[D_{mnkl}^{i}] = \left[ -\omega_{j}^{2} [m_{mnkl}] + [k_{mnkl}] \right]^{-1} \tag{2.49}
\]

Similar to the \( \delta_{mnkl}^{i} \), \( E_{mnkl}^{i} \) can be considered as modal force radiation coefficients, which tell the contribution from the \((m, n)\) modal force to the \((k, l)\) modal component of the sound field.
2.4 Linear Quadratic Optimal Control Theory

In the practical implementation of the adaptive filtered-X LMS control algorithm (Elliott et al., 1987), a set of squared variables which are related to the sound radiation properties are selected as the control cost function. By adjusting the filter coefficients, the minimum of that cost function and the associated control force amplitudes can be obtained.

In this analytical work, in conjunction with the adaptive Filtered-X LMS control algorithm, the linear quadratic optimal control theory (LQOCT) is used to calculate the optimal complex voltages applied to the actuators. The cost function in this case is chosen as the radiated sound power in the far field, which is given in Eq. (2.45). The cost function can be expressed with consideration of the weighting coefficients $\lambda_j$ as:

$$
\Phi_p = \sum_{j=1}^{N_r} \lambda_j \left\{ |V^j|^2 H^j |V^j|^H + 2 \text{Re}\{V^j\}^H \{G^j\} + P^j_d \right\}
$$

(2.50)

where $0 < \lambda_j \leq 1$ provide different weights to different frequency components, $[H^j] = [H^j_n]$, $\{G^j\} = \{G^j_i\}$; and

$$
H^j_n = \left\{ \tilde{f}_{mn}^j \right\}^H \left\{ E_{mnkl}^j \right\} \left\{ \tilde{f}_{kl}^j \right\}
$$

(2.51)

$$
G^j_i = f_d^j \left\{ \tilde{f}_{mn}^j \right\}^H \left\{ E_{mnkl}^j \right\} \left\{ \tilde{f}_{kl}^j \right\}
$$

(2.52)

and

$$
P^j_d = f_d^j f_d^* \left\{ \tilde{f}_{mn}^j \right\}^H \left\{ E_{mnkl}^j \right\} \left\{ \tilde{f}_{kl}^j \right\}
$$

(2.53)
The optimal voltage solution for minimization of the cost function $\Phi_p$ can be derived as (Lester and Fuller, 1990):

$$\{V^j\} = -[H^j]^{-1}\{G^j\} \quad j = 1,2,\ldots,N_f \quad (2.54)$$

### 2.5 Sensitivity Analysis

In a typical optimization process, the calculation of the sensitivity of the objective function to the changes in design variables is the major computational cost. Previous work (Wang, Burdisso, and Fuller, 1992) shows that with the use of the finite difference method, the CPU time spent on the sensitivity analysis is approximately 75 percent of the total CPU time for optimization. Therefore, it is important to have an efficient way to evaluate the derivatives of the objective function.

From Eq.(2.45) and (2.54), the objective function can be expressed as:

$$\Phi_p = \sum_{j=1}^{N_f} \lambda_j \phi_j \left( x_s, y_s, \{V^j(x_s, y_s)\} \right) \quad s = 1,2,\ldots,N_a \quad (2.55)$$

It is noted here that the voltages $\{V^j(x_s, y_s)\}$ have been optimized by linear quadratic optimal control theory (LQOCT) as given by Eq.(2.54).
The first derivatives of the objective function to the design variables \( z_s \) (\( z_s = x_s, \) or \( z_s = y_s \)) can be written:

\[
\frac{\partial \Phi_p}{\partial z_s} = \sum_{j=1}^{N_s} \left[ \frac{\partial \phi_p}{\partial z_s} + \left\{ \frac{\partial \phi_p}{\partial \mathbf{v}_q} \right\}^T \frac{\partial \mathbf{v}_q}{\partial z_s} \right]
\]  

(2.56)

where

\[
\frac{\partial \phi_p}{\partial z_s} = 2 \left\{ \frac{\partial f^q}{\partial z_s} \right\}^H \left[ E^j \right] \left[ f^q \right]
\]  

(2.57)

\[
\frac{\partial \phi_p}{\partial \mathbf{v}_q} = 2 \left\{ \mathbf{r}^{\text{up}} \right\}^H \left[ E^j \right] \left[ f^q \right]
\]  

(2.58)

\[
\left\{ \frac{\partial \mathbf{v}_q}{\partial z_s} \right\} = -[H^j]^{-1} \left\{ \frac{\partial \mathbf{G}^j}{\partial z_s} + \frac{\partial \mathbf{H}^j}{\partial z_s} \right\} \left\{ \mathbf{V}^j \right\}
\]  

(2.59)

The expressions for \( \frac{\partial \mathbf{G}^j}{\partial z_s} \), \( \frac{\partial \mathbf{H}^j}{\partial z_s} \) and \( \left\{ \frac{\partial f^q}{\partial z_s} \right\} \) are listed in Appendix C.

\[ \text{2.6 The Optimization Problem} \]

In this work, for simplifying the optimization procedure, the actuator dimension is fixed. Therefore, the design variable is the central location of the actuator, the coordinates of which are denoted as \( x_{0j} \) and \( y_{0j} \) for the \( j \)th actuator. Combined with the physical
constraint conditions to the actuators, the complete problem becomes a nonlinear constrained optimization problem, which is stated as:

**Objective function:**
\[
\Phi_p = \sum_{j=1}^{N_s} \lambda_j \phi_j(x_{0i}, y_{0i})
\]  
(2.60)

**Design variables:**
\[
x_{0i}, y_{0i}, \quad i = 1, 2, \ldots, N_s
\]  
(2.61)

**Constraint conditions:**

1. **constrain actuator inside of the plate boundaries**

\[
\begin{align*}
x_{0i} - a_{0i} & \geq 0 \\
x_{0i} + a_{0i} & \leq L_x \\
y_{0i} - b_{0i} & \geq 0 \\
y_{0i} + b_{0i} & \leq L_y
\end{align*}
\]  
(2.62)

2. **constrain overlap between actuators**

\[
\left( \left( x_{0(i+1)} - x_{0i} \right)^2 + \left( y_{0(i+1)} - y_{0i} \right)^2 \right)^{1/2} - \left( \left( a_{0i}^2 + b_{0i}^2 \right)^{1/2} + \left( a_{0(i+1)}^2 + b_{0(i+1)}^2 \right)^{1/2} \right) > 0
\]  
(2.63)

3. **constrain overlap between actuators and disturbance force**

\[
\left( \left( x_{0i} - X_0 \right)^2 + \left( y_{0i} - Y_0 \right)^2 \right)^{1/2} - \left( a_{0i}^2 + b_{0i}^2 \right)^{1/2} > 0
\]  
(2.64)

4. **constraint on voltage to piezoelectric actuator**

\[
|V_i| \leq V_{\max}
\]  
(2.65)
where $V_{\text{max}}$ is the maximum allowable voltage to be applied to the piezoelectric actuator.

An IMSL subroutine named NCONG (IMSL user's Manual, 1989) is used to compute the optimal solution for the above optimization problem. This subroutine, NCONG, is developed to solve the general nonlinear programming problem by means of the successive quadratic programming method (Hock and Schittkowski, 1981). The gradient of the objective function is evaluated by applying the analytical formula derived in Section 2.5. The optimization algorithm used is reviewed in Appendix D.

2.7 Results and Discussion

The analysis model considered in this work consists of a thin steel plate with a dimension of $L_x=0.38\text{m}$, $L_y=0.30\text{m}$, and $h=2\text{mm}$. The material properties of the plate are listed in Table 2.1. The first 24 natural frequencies of the plate with simply supported boundary conditions are listed in Table 2.2, in which the order of modes is based on their natural frequencies. For some other boundary conditions, the changes of the natural frequencies of plate will be discussed in Section 2.7.1.4. Throughout the numerical analysis, the size of piezoelectric actuator is fixed at $a_0=0.03\text{m}$, $b_0=0.02\text{m}$. It has been shown by Wang et al (1990) that for low excitation frequency, the effect of piezoelectric actuator size on the control performance can be neglected as long as the piezoelectric actuator size is less than half of the wavelength of the dominant radiation modes. Therefore, the design variables in the optimization are the locations of the actuator centers. A point force is used as the disturbance source. The material properties of the piezoelectric actuator are listed in Table 2.3. The reference sound pressure is chosen as $20\mu P_a$, while the reference sound power is
taken as $10^{-12}$ Watt. It is assumed that the far field sound pressure is measured exactly and thus the sensing is perfect.

The efficiency of the theoretical sensitivity analysis over the finite difference method is evaluated. The computer used for the numerical simulation is an IBM 3090 mainframe computer. Typical results are presented in Figure 2.4. From Figure 2.4, it is seen that the analytical gradient analysis takes only about one fourth of the time used by the central finite difference method for the case with only one actuator. It is also apparent that as the number of the piezoelectric actuators increases, this time saving will become very significant.

2.7.1 Single-frequency Excitation

In this section, a number of simulations are conducted for the active control of sound radiation from plates under the disturbance force with a single harmonic excitation frequency. The influence of some system parameters, such as, the excitation frequency changes, the movement of the disturbance force location, and the plate boundary stiffness, on the control performance of the optimized actuators has been evaluated.

2.7.1.1 Effect of Excitation Frequency on the Optimal Location of PZT Actuators and Their Control Performance

Under the action of the disturbance with a fixed location, the control performance and optimal locations of actuators are dependent upon the disturbance frequency. Figure 2.5
gives the radiated sound power from the plate before and after control. It should be noted here that the curves in Figure 2.5 are created by calculating or optimizing the control performance at each frequency and then joining the points to present the results in the frequency domain. The point force disturbance with force amplitude $f_d = 1.0 N$ is located at $(0.08, 0.08)$. The structural damping is neglected for this case and its effect will be discussed later in this section. Also, in order to monitor the influence of the excitation frequency on the optimal actuator location, the third constraint condition of Eq. (2.64) is not included in the optimization procedure.

In Figure 2.5, the dashed line shows the control results of one piezoelectric actuator which is optimized at each frequency, while the dotted line presents the control output by a piezoelectric actuator which is arbitrarily located at $x = 0.119 m$, $y = 0.172 m$. It is seen from Figure 2.5 that regardless of the difference of the radiated sound power over the range of frequency for the uncontrolled plate, the radiated sound power from the optimally controlled plate increases monotonously as excitation frequency increases. Also, it is observed that the uncontrolled plate may radiate nearly the same amount of sound power for different excitation frequencies, but the radiated sound power from the controlled plate is much different. For example, at frequencies $f = 150 Hz$ and $f = 300 Hz$, the sound power of the uncontrolled plate is about $69.2 dB$. But the radiated sound power by optimally controlled plate at $f = 150 Hz$ is $20 dB$ less than that at $f = 300 Hz$. This can be explained by the consideration that at low frequency, the low order modes are the dominant radiators. All the higher order modes do not contribute significantly to the sound radiation due to both the lower radiation efficiencies and the lower modal amplitudes. However, for the higher frequency excitation case, many more modes contribute to the sound radiation and thus reduced attenuation is obtained. Obviously, a small number of actuators are not
effective in controlling a large number of dominant modes. Nevertheless, at least 22dB sound power reduction is obtained with the optimally designed controller over the frequency range considered in Figure 2.5. For the arbitrarily selected piezoelectric actuator, as expected, the control is not nearly that effective. Even though more than 10dB sound power reduction is achieved by this arrangement of control for frequencies less than 200Hz, there is very little control observed for the frequency range from 210Hz to 450Hz. Moreover, at some frequencies, such as \( f = 230Hz \), \( f = 300Hz \), and \( f = 353Hz \), almost no control at all is obtained. By comparison, it is obvious that the control with an optimally located piezoelectric actuator is much better than that with an arbitrarily selected actuator.

Figure 2.6 gives the moving traces of the normalized optimal central location of the piezoelectric actuator as the excitation frequency increases from 50Hz to 450Hz. It is seen that the normalized optimal location moves slightly toward the corner of the plate but not monotonously. The reason is that as the frequency increases, the modal amplitude distributions excited by the disturbance and the control forces are readjusted. The modes whose natural frequencies are close to the excitation frequency will become the important sound-radiating modes in the plate response field. Comparing the modal force distributions of the disturbance point force and the piezoelectric actuators, which are shown in Eq. (2.28) and Eq. (2.29) for the simply supported plate, it is seen that the difference between these distributions is introduced by the following two terms, one is \( \left\{ \sin(k_x a_{0}^k) \sin(k_y b_{0}^k) \right\} \) which is related to the size of the piezoelectric actuator, the other is \( \left\{ \frac{mL_y}{nL_x} + \frac{nL_x}{mL_y} \right\} \) which is a function of the mode order and plate dimension. For the piezoelectric actuator with a smaller size relative to the dimension of the plate, \( (L_x > 12a_0^k \text{ and } L_y = 15b_0^k \text{ in this case}) \), the term of \( \sin(k_x a_{0}^k) \sin(k_y b_{0}^k) \) will be with the same sign for any group of \( m \) and \( n \).
if the mode order is not very high. The existence of these two terms will only make the relative amplitudes of any modes (the first modal amplitude is selected as the reference) excited by the piezoelectric actuator are larger than the corresponding amplitudes excited by the disturbance point force if the center of the piezoelectric actuator is located at the position of the disturbance point force. Therefore, for the very low frequency excitation, the position of the piezoelectric actuator is close to the center of the plate in order to excited a modal distribution with a relative large amplitude of the first mode. As the excitation frequency goes higher, the piezoelectric actuator moves away from the center to the positions that can trace the new disturbed structure response for the maximum sound power reduction. However, without computation, it is still difficult to find a general rule for the determination of the optimal actuator locations.

The introduction of the modal radiation coefficients, especially the coupling terms, marks the major difference between the active structural acoustic control (ASAC) and the active vibration control. As pointed out by Fuller et al (1991), the mechanism of active sound radiation control is essentially what is called 'modal suppression' and 'modal restructuring'. By 'modal suppression', the dominant modes which contribute significantly to the sound radiation, such as the on-resonant modes, are suppressed. By way of 'modal restructuring', the modal amplitudes and phases are rearranged so as to result in a system with less radiation efficiency. Because of the modal coupling effect, complete suppression of a small number of modes may not always provide a quieter system. To illustrate this, Table 2.4 presents the radiated sound power from an uncontrolled plate and a plate without mode (1,1). The plate vibrating without mode (1,1) is equivalent to a plate that is controlled by a modal actuator for mode (1,1) only. It is seen that at excitation frequency $f=150\,\text{Hz}$, if mode (1,1) is controlled completely, the radiated sound power is about $10\,\text{dB}$ less than that
without control. However, at frequency $f = 322\, \text{Hz}$, the plate without mode (1,1) radiates 3$\, \text{dB}$ more sound power than the uncontrolled plate does. This means that the modal coupling terms between mode (1,1) and the other modes has a reducing effect on the sound radiation at frequency $f = 322\, \text{Hz}$.

In Figure 2.7(a) and 2.7(b), the radiated sound power from the plate before and after control is presented for point force disturbance located at (0.14,0.12) and (0.10,0.15) respectively. One actuator which is optimized at each frequency is considered. Because of the fact that plate response is dependent upon the disturbance location, it is seen that the radiated sound power patterns from the uncontrolled plate in Figure 2.7(a) and Figure 2.7(b) are different from that in Figure 2.5. The sound power peaks which correspond to the on-resonant excitation of the even modes in the y-direction are not found in Figure 2.7(b) and this is because the disturbance is located exactly at the nodal line of the even modes in the y-direction. However, the sound power patterns over the frequency range after optimal control in Figure 2.7(a) and Figure 2.7(b) are quite similar to that in Figure 2.5.

Figure 2.8(a) and 2.8(b) show the moving traces of the normalized central optimal location of the piezoelectric actuators for the disturbance of point forces located at (0.14,0.12) and (0.10,0.15), respectively, as the frequency increases from 50$\, \text{Hz}$ to 450$\, \text{Hz}$. It is seen that for a disturbance located at (0.14,0.12), the trace has the same tendency as that in Figure 2.6, but with a different path. With the disturbance located at (0.10,0.15), the optimal central location of the piezoelectric actuator simply moves along the x-axis. This behavior can be understood by the consideration of the fact that this kind of disturbance cannot excite any even modes in y-direction. Therefore, the piezoelectric
actuator is optimally placed at positions where the actuator may not introduce any new modes to the modal response.

The radiated sound power from plate before and after control by two optimally located piezoelectric actuators is plotted against frequency in Figure 2.9. The disturbance considered here is located at (0.08,0.08). Compared with Figure 2.5, it is seen from Figure 2.9 that for frequency range from 50Hz to 300Hz, approximately 6 dB more sound power reduction is achieved at all frequencies with the use of two optimally located actuators than that with one optimally located actuator. However, for frequencies from $f=400Hz$ to $430Hz$, there is very little difference between the radiated sound power controlled by one and two optimally located actuators.

Table 2.5 gives two sets of optimal actuator locations at frequency $f=357Hz$. It is seen from Table 2.5 that even though the locations of the two sets of actuator are completely different, the control performance is very close. The corresponding modal amplitude distributions of plate controlled separately by the two sets of actuators are plotted in Figure 2.10. Some difference is observed for the two kinds of controlled response, especially for mode (3.1). Moreover, a check on the radiation directivity patterns for the two controlled plates, which are plotted in Figure 2.11, shows that the residual pressure distributions are almost the same except at $\theta = \pm 45^\circ$. Obviously, this variation is mainly because of the difference between the amplitudes of the mode (3,1) of the two controlled plate responses, as shown in Figure 2.11. Therefore, it is concluded that the control systems with the different sets of optimally located actuators can result in nearly the same control output.
In the above analysis, the structural damping has been neglected. The inclusion of the structural damping in the equations of the plate motion will have little effect on the forced response of the plate under the off-resonant excitation. However, it will reduce significantly the amplitudes of the plate under the on-resonant excitation. Nevertheless, the numerical results show that the small damping coefficient will have a negligible effect on the optimized actuator locations.

2.7.1.2 The Effect of Frequency Uncertainty on Control Performance

As mentioned in the previous sections, the optimized actuator locations are a function of the excitation frequency. The optimization of the piezoelectric actuator locations must be conducted at the required frequency. Once the actuators are fixed on the plate in accordance with the optimal design, the controlled plate radiates the minimum amount of sound power at this design frequency. Therefore, it is necessary to analyze the control performance of the system at frequencies other than the design frequency, in order to study the robustness of the optimization result. In this section, the actuator(s) is(are) first optimized at one particular frequency, then the control performance of the actuator(s) at the other frequencies is calculated through the use of linear quadratic optimal control theory.

In Figure 2.12, the control results are plotted versus frequency for the control system with one actuator designed optimally at frequencies of $f=87\text{Hz}$, $f=272\text{Hz}$, and $f=357\text{Hz}$, respectively. The point force disturbance is located at $(0.08,0.08)$ and the force amplitude is fixed at 1.0N over the complete frequency range. Again, in this section, the structural
damping is not taken into account. From Figure 2.12, it is seen that with one actuator optimally designed at \( f = 87Hz \), which is denoted as dashed line, the control effect is significant in the frequency range from 50Hz to 115Hz. This is due to the fact that for the frequency range the sound power is mainly radiated from the fundamental mode. However, the radiated sound power by the controlled plate with one actuator rises sharply as the frequency increases over 115Hz. For the frequency range from 320Hz to 370Hz, because the natural frequencies of modes (2,2) and (3,1) are very close, the control spillover between the two modes is very significant and almost no control at all is achieved except where the excitation frequencies are equal to the natural frequencies of modes (2,2) and (3,1).

The dotted line in Figure 2.12 shows the control effect for the system in which the actuator is designed optimally at an excitation frequency \( f = 272Hz \). It is seen that for the frequencies less than 210Hz, the radiated sound power by this control is much larger than that denoted by the dashed line which corresponds to the control with one actuator optimally designed at \( f = 87Hz \). But there is at least 10dB sound power reduction obtained for the control with the actuator optimally designed at 272Hz over the frequency range from 50Hz to 310Hz. Moreover, the control performance over the frequency range from 320Hz to 370Hz is improved compared to that designed at \( f = 87Hz \) in Figure 2.12. For example, at a frequency \( f = 353Hz \), which lies between the natural frequencies of modes (2,2) and (3,1), the sound power reduction is 10.69dB for control with actuator designed at \( f = 272Hz \). In comparison, for the system designed at \( f = 87Hz \), the sound power reduction is only of 0.16dB as shown by the dashed line.
For the control system designed optimally at excitation frequency $f = 357\text{Hz}$, the control performance over the range of frequencies from $50\text{Hz}$ to $450\text{Hz}$ is shown by the long dash line in Figure 2.12. Compared with the results for control with the actuator designed at $f = 87\text{Hz}$ and $f = 272\text{Hz}$, much smaller variations on the radiated sound power by the controlled plate are observed when changing the excitation frequency. Furthermore, the radiated sound power reduction by this control is always larger than $10\text{dB}$.

Figure 2.13 gives the control performance by two actuators which are designed optimally for the plate under a single frequency excitation. Two excitation frequencies are considered respectively, one is $f = 272\text{Hz}$, the other is $f = 357\text{Hz}$. Again, the disturbance is located at $(0.08, 0.08)$ and the force amplitude is fixed at $1.0\text{N}$ over the complete frequency range. Comparing Figure 2.13 with Figure 2.12, it is seen that the system with two optimally located actuators provides much more robust performance than that with one optimally designed actuator. For the two different optimized control systems of Figure 2.13, more than $10\text{dB}$ sound power reduction is achieved over the complete frequency range. However, at this stage, it is hard to conclude a general procedure for design of the more robust control systems with respect to the frequency uncertainty.

2.7.1.3 The effect of disturbance location uncertainty on control performance

In the preceding section, the effect of the disturbance frequency on the optimal placement of actuators was analyzed. However, once the location of disturbance shifts, accordingly, the optimal placements will also move to varying extent. The problem now is that, in
practice, a mounted piezoelectric actuator cannot be moved easily to provide maximum performance as the disturbance changes. Hence, for a optimally designed control system, the effect of the disturbance location uncertainty, which is either because of its moving character or inexact measurement, on the control performance is of interest.

In Figure 2.14, the control results are plotted against a disturbance location which moves along the x-direction. The piezoelectric actuators considered in Figure 2.14 are located optimally for disturbance located at (0.08,0.08). It is assumed that as the disturbance moves, the force amplitude of the point disturbance is still 1.0N throughout the analysis. Perfect sensing is also assumed. No structural damping is considered. The excitation frequency is fixed at \( f = 272Hz \), which is a non-resonant excitation case. From Figure 2.14, it is seen that when the disturbance location moves, the radiated sound power by the plate under disturbance only changes slightly. However, for the controlled plate, the farther the disturbance location is from the design position, the less the sound power reduction. When the disturbance is located at (0.06,0.08), that is 0.02m away from the design position, the control effect is reduced by more than 14dB for the one actuator case which is denoted by the dashed line. It also is noticed that the control effect is very sensitive to the change of the disturbance position. When the disturbance location is only 0.004m away from the design point, nearly 6dB more sound power is radiated by the controlled plate with one actuator. Similar variation of the sound power to disturbance changes is observed for the two-actuator case which is denoted by the long dash line in Figure 2.14, even though with two optimally located actuators, much more sound power reduction is obtained. Figure 2.15 gives the control performance of system with one and two actuators, respectively, as the disturbance location moves along y-direction. It is noticed from Figure 2.15 that as the disturbance position moves 0.005m away from the
design point, the radiated sound power increases by about 10\,dB for the system with two actuators. Comparing the results in Figure 2.14 with those in Figure 2.15, it is concluded that the control performance is not only dependent upon the error distance of the disturbance to the design point, but also dependent upon the path of the disturbance offset error.

2.7.1.4 The effect of edge supports on the control performance

In this section, various boundary support conditions are considered for evaluating the effect of edge supports on the control performance. All the results presented in this section are for the point disturbance force which is located at \( x_0 = 0.08m, \, y_0 = 0.08m \). The plate dimension is the same as the previous sections. Different boundary support conditions are modeled by changing the non-dimensional edge parameters \( k_i \) and \( k_s \), which are defined in Eq. (2.7a) and Eq. (2.7b). The clamped boundary, which can be viewed as the upper bound for all the other elastic boundaries, is studied using the values \( k_i = 2.0 \times 10^6 \) and \( k_s = 3.0 \times 10^7 \). The order of the polynomial functions in Eq. (2.30) is chosen as 12 for both \( m \) and \( n \). The resulting linear equations, defined in Eq. (2.22), have 144 unknowns.

The influence of the rotational stiffness \( k_r \) on the control output of piezoelectric actuator is illustrated in Figure 2.16. The excitation frequency in Figure 2.16 is fixed at 200\,Hz. The translational stiffness is fixed at \( k_x = 3.0 \times 10^7 \), which, for \( k_r = 0 \), gives the simply support boundary condition. It is seen that as the rotational stiffness \( k_r \) increases, the radiated sound power from plate before control changes. At \( k_r = 1.8 \), a peak value is observed. This is because that as \( k_r \) increases, the natural frequencies of the plate increase, as shown in
Figure 2.17. For the excitation frequency \( f = 200Hz \), when \( k_i = 18 \), the natural frequency of the second plate mode is very close to 200Hz. This results an on-resonant excitation condition. However, it is noticed that as \( k_i \) increases for \( k_i > 3.0 \), the radiated sound power decreases. This can be understood by checking Figure 2.17 that the excitation frequency of \( f = 200Hz \) is an off-resonant excitation as \( k_i \) is away from \( k_i = 3.0 \). It is also observed from Figure 2.16 that the control performance of the actuator optimized at \( k_x = 3.0 \times 10^7 \) and \( k_i = 0 \) is still significant at other boundary conditions with \( k_i > 0 \), even though less effective than that optimized at each \( k_i \), which is denoted by the dotted line in Figure 2.16.

Figure 2.18 shows the effect of the translational stiffness \( k_x \) on the control effectiveness of the piezoelectric actuator. The rotational stiffness \( k_i \) is chosen to be zero. Again, the excitation frequency is \( f = 200Hz \). It is seen that as the translational stiffness \( k_x \) increases, the radiated sound power by the plate before control goes up and down according to the relationship among the natural frequencies and the excitation frequency, which is illustrated in Figure 2.19. However, for \( k_x > 3.0 \times 10^4 \), the change in \( k_x \) has little effect on the sound radiation. Also shown in Figure 2.18 is the control output by the actuator optimized at \( k_x = 3.0 \times 10^7 \) and \( k_i = 0.0 \), which is corresponding to the simply supported condition. It is seen that when \( k_x \) is far less than \( k_x = 3.0 \times 10^7 \), this actuator does not perform well. The reason is that when the plate support boundaries become less stiffer, the plate natural frequencies are relatively lower as shown in Figure 2.19. Under the same excitation frequency, there are more modes participating actively into the sound radiation. This leads to that the modes which are originally the dominant radiators in the sound radiation become relatively less important. The locations of the control actuators have to be adjusted to control the updated plate response. The plate with less stiffer boundary
supports does not necessarily radiate more sound power than that with stiffer boundaries, which can be seen in Figure 2.18. However, many more control forces are needed to obtain the same control performance. This is confirmed by the dotted line in Figure 2.18, which gives the control output of the actuator optimized at each $k_i$.

2.7.1.5 The effect of a discrete mass on control performance

It is well known that when a lumped mass is attached to a plate, the plate response and associated sound field will be different from that of a uniform plate. Furthermore, the performance of actuators optimized for the uniform plate will be affected by the presence of the lumped mass. Therefore, it is worthy to investigate the effect of a discrete mass on the control performance of optimally located actuators in order to consider realistic systems which have various inhomogeneities. The dimension of plate considered in this section is the same as that in the previous sections. The mass of the plate is $1.7955\text{kg}$. A point force with single harmonic excitation frequency is chosen as the disturbance. The disturbance force amplitude is taken as $1.0\text{N}$.

The effect of a centrally attached point mass, whose weight changes from 0 to $0.975\text{kg}$, on the natural frequencies of the plate is illustrated in Figure 2.20. It is seen that the natural frequencies of the odd-odd modes become smaller as the point mass increases, while the natural frequencies of the even related modes are not affected by the mass changes. This is because the point mass is located on the nodal lines of the even-odd, odd-even, and even-even modes. Obviously, if the point mass is not centrally attached, much more natural frequencies of the plate will become smaller, as shown in Figure 2.21, where the point
mass is located at \((x, y) = (0.08, 0.08)\). It should be noted that the orthonormal mode shapes of the plate changes in conjunction with the natural frequency changes.

Figure 2.22 shows the effect of a point mass on control performance with an actuator whose location is optimally determined for plate without point mass loading. The point mass in Figure 2.22, whose weight changes from 0 to 1\(kg\), is attached at \((x, y) = (0.08, 0.08)\). The excitation frequency is \(f = 340Hz\). It is seen that the radiated sound power from the uncontrolled plate has a peak value for \(M_d = 0.025kg\). This is because that for \(M_d = 0.025kg\), the fourth natural frequency of the plate is very close to \(f = 340Hz\), as observed in Figure 2.21. As the weight of point mass increases, the radiated sound power from the uncontrolled plate decreases. The reason for the decrease is that, as the point mass gets heavier, the plate response becomes relatively smaller, except that near the on-resonant excitation. This is clearly shown in Figure 2.23, where the modal amplitudes of the uncontrolled plate with and without the point mass are compared. An overall decrease in the response amplitudes results in less sound power radiated for this excitation. Also, notice from Figure 2.22 that the sound power reduction is decreasing as the weight of the point mass increases.

Another numerical example is presented in Figure 2.24, where the point mass is attached right at the plate center. The disturbance point force, with an excitation frequency of \(f = 340Hz\), is still located at \((x, y) = (0.08, 0.08)\). The actuator used in Figure 2.24 is optimized for the uniform plate. Again, a peak value, which is corresponding to an on-resonant excitation as indicated in Figure 2.20, is observed in Figure 2.24. However, unlike the case with an off-centrally attached mass, the radiated sound power from the uncontrolled plate with centrally located mass does not change much as the point mass
increases, except for the region near the on-resonant excitation. This can be understood by checking the modal amplitudes of uncontrolled plates with and without the point mass, which are shown in Figure 2.25. From Figure 2.25, it is seen that because the centrally attached mass does not coupled with any even mode related plate motion. Most of the modal amplitudes of the uniform plate, about three fourth of all modes, are not affected by the presence of the point mass. Also, even for the odd-odd modes, the addition of point mass does not mean the reduction of modal amplitudes. For example, in this case, the amplitude of mode (1,1) is about doubled with the presence of the point mass. The influence of the point mass on the control output of one actuator optimized for the uniform plate is also plotted in Figure 2.24. It is seen that for $M_d > 0.2Kg$, the radiated sound power from the plate before and after control is not sensitive to the changes of the weight of the point mass. With the help of Figure 2.20 and Figure 2.25, this can be explained that with increase of the point mass, most of the changes in radiated sound power from the uncontrolled plate are caused by the changes in the modal amplitude of mode (3,1). As the point mass increases, the natural frequency of mode (3,1), noted as the 5th natural frequency in Figure 2.20, moves away from excitation frequency $f = 340Hz$, which results in a smaller amplitude for mode (3,1). However, most of the other modes, such as modes (1,2), (2,1) and (2,2), still keep the same modal amplitudes. This leads to the relatively flat carve for the sound power radiated by the uncontrolled plate. Nevertheless, the actuator optimized for the uniform plate does have a good control authority for the plate with a point mass, even though it is slightly less effective than that by the actuator optimized at each mass point.

It should be pointed out that the modal amplitudes plotted in Figures 2.23 and 2.25 are not for the orthonormal plate modes, instead, for the trial functions defined in Eq. (2.24).
For the plate with discrete point mass attached, the trial functions of Eq. (2.24) are no longer the orthonormal plate modes, even if the plate is simply supported. The determination of the orthonormal plate modes and the relevant formulations are presented in Appendix A.

2.7.2 Multiple-frequency Excitation

In this section, the active control of sound radiation from the plate under a multiple-frequency excitation is studied. The optimization of the piezoelectric actuator locations is performed for the maximum attenuation of the multiple-tone sound. Two types of disturbance forces, one is with equal force amplitudes for different excitation frequencies, the other is with non-equal force amplitudes for the different excitation frequencies, are considered separately.

2.7.2.1 Disturbance with equal force amplitudes for different excitation frequencies

In the following examples, the disturbance is a point force with multiple-frequency components. It is assumed that the force amplitude for the $j$th frequency, $f_{d,j} = 10N$, where $j = 1, 2, ..., N_f$. That is, the force amplitudes are the same for all the disturbance frequencies. The point force is located at $x_o = 0.08m$, $y_o = 0.08m$. The optimization objective is to find the optimal locations of piezoelectric actuators for which the total radiated sound power from the plate is minimum, as defined in Eq. (2.60). Only simply
supported homogenous plate is considered in this section. The weighting coefficients are chosen to be 1.0 for all frequencies. The size of the piezoelectric actuator is fixed in this analysis with a dimension of $0.06 \times 0.04(m)$. The damping factor is set at 0.01 for all plate modes. With considering the damping factor, it is relatively easy to adjust the amplitudes of the sound power at different frequencies. Otherwise, the sharp peaks for the on-resonant excitations may cause a large difference between the sound power from the uncontrolled plate at different excitation frequencies.

Figure 2.26 gives the control performance of one piezoelectric actuator for two on-resonant excitation frequencies. The two on-resonant frequencies are $f_1 = 87.0Hz$ and $f_2 = 358.0Hz$, which are very close to the natural frequencies of plate mode (1,1) and mode (3,1), respectively. Four kinds of actuator locations are considered for this disturbance, as shown in Table 2.6. For the actuator optimized at the frequency of $f_1 = 87.0Hz$, it is seen from Figure 2.26 that the actuator provides very good control at this frequency. But because of the inferior control at $f_2 = 358.0Hz$, the overall sound attenuation for the actuator design is only 9.7dB. On the contrary, the actuator optimized at frequency $f_2 = 358.0Hz$ has the same control output as that optimized simultaneously at both the two excitation frequencies. This is understandable by checking Figure 2.12, which shows the control performance of the actuator optimized at 358.0Hz over the frequency range from 50Hz to 450Hz. It is relatively easy to control the sound radiation at low frequency where fewer modes participating actively in radiating sound. Therefore, the location of actuator optimized at both frequencies is coincident with that optimized at the higher frequency $f_2 = 358.0Hz$, which is shown in Table 2.6. Also, in Figure 2.26 and Table 2.6, it is seen that the arbitrarily located actuator cannot furnish good control of the total sound radiation.
Similar results are obtained for one actuator with two other on-resonant excitation frequencies, as shown in Figure 2.27 and Table 2.7. The two frequencies are $f_1 = 249.0\,Hz$ and $f_2 = 592.0\,Hz$, which are close to the natural frequencies of mode $(1,2)$ and mode $(4,1)$, respectively. It is seen from Figure 2.27 that for the piezoelectric actuator optimized at $f_1 = 249.0\,Hz$, the sound attenuation at this frequency is significant. While at the other frequency $f_2 = 592.0\,Hz$, only about $9.0\,dB$ sound power reduction is gained. This leads to the conclusion that the controlled plate radiates sound with a predominant frequency component of $f_2 = 592.0\,Hz$. The total sound power reduction for this optimal design is $13.76\,dB$. On the other hand, for the actuator optimized at $f_2 = 592.0\,Hz$, the total radiated sound power from the controlled plate contains almost an equal amount of sound power for the two frequencies. Again, it is observed from Figure 2.27 and Table 2.7 that the control effectiveness and location of the actuator optimized at $f_2 = 592.0\,Hz$ is nearly indistinguishable from that optimized simultaneously at both frequencies. Also, it is noticed that sometimes the arbitrarily located actuator is almost useless, such as the one shown in Figure 2.27 and Table 2.7.

The control effectiveness and the locations of two actuators for a disturbance with three on-resonant excitation frequencies are considered next. The results are shown in Figure 2.28 and Table 2.8. The three frequencies are $f_1 = 87.0\,Hz$, $f_2 = 188.0\,Hz$ and $f_3 = 358.0\,Hz$, which are near the natural frequencies of modes $(1,1)$, $(2,1)$ and $(3,1)$, respectively. The radiated sound power before control is $112.6\,dB$ for $87.0\,Hz$, $103.6\,dB$ for $188.0\,Hz$, and $106.6\,dB$ for $358.0\,Hz$. The total sound radiation is $114.0\,dB$. Five kinds of piezoelectric actuator locations are presented in Figure 2.28 and Table 2.8. For the first two sets of actuator designs, which are optimized at $f_1 = 87.0\,Hz$ and $f_2 = 188.0\,Hz$, 

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respectively, the sound power radiated from the controlled plate is dominated by the frequency of 358.0Hz. However, good control is still obtained, the total sound power reductions are 41.7dB and 44.0dB, respectively. Those results are consistent with the discussion in section 2.7.1.2, as illustrated in Figure 2.13. With the actuators optimized at frequency \( f_1 = 358.0\text{Hz} \) and \( f_2 = 188.0\text{Hz} \) is diminished compared with the above designs, the overall sound reduction is up to 58.9dB. Once again, the locations of actuators optimized simultaneously at all the three frequencies are very close to that designed at the highest frequency \( f_3 = 358.0\text{Hz} \). It should be mentioned that even with two actuators, for this relatively lower frequency range (less than 400Hz) and lower mode density (5 active modes below 358Hz), the actuator locations are still very important. As shown in Figure 2.28, the arbitrarily located actuators give only about 8dB sound reduction.

The above examples are for the multiple-frequency excitation where the frequencies are separated by at least 100Hz. There are some cases that the multiple excitation frequencies are very close to each other. Figure 2.29 gives one of such cases. The two excitation frequencies are \( f_1 = 351.0\text{Hz} \), which is close natural frequency of mode (2,2), and \( f_2 = 358.0\text{Hz} \). Three sets of piezoelectric actuator locations are considered in Figure 2.29 and Table 2.9. One is optimized at frequency \( f_1 = 351.0\text{Hz} \). The second is optimized at \( f_2 = 358.0\text{Hz} \). The third one is optimized at the two frequencies simultaneously. It is shown that there is almost no difference among the three actuator locations. The resulting overall sound power reduction is nearly the same for the three designs.
2.7.2.2 Disturbance with non-equal force amplitudes for different excitation frequencies

In the above section, a disturbance of multiple-frequency components is assumed with the same force amplitudes for different frequencies. Some of the general conclusions can be listed as follows:

(1) If the multiple frequencies are apart from each other, the optimal locations of piezoelectric actuators for all frequencies are very close to, if not coincident with, the locations of actuators optimized at the highest frequency. The optimization of the actuator locations for the highest frequency of the multiple excitation frequencies can be used for the multiple-frequency excitation case. Therefore the attention in this case can be paid to the highest frequency component only.

(2) If the multiple frequencies are very close to each other, there is very little difference among the actuator locations optimized at all the frequencies and that optimized at any of the excitation frequencies. In this case, the optimization of actuator locations can be simplified at any single frequency of the multiple excitation frequencies.

However, in the reality, sometimes it is found that the force amplitudes for the different excitation frequencies are different from each other. In this section, the focus is on disturbance with non-equal force amplitudes for different frequencies. Figure 2.30 gives the radiated sound power for a plate under a two-frequency excitation. The force amplitudes for frequencies $f_1=300\text{Hz}$ and $f_2=592\text{Hz}$ are 10.0N and 1.0N, respectively.
One piezoelectric actuator is used in this case. It is noted here that in Fig 2.30, the bars indicate the radiated sound power before and after control, unlike that shown in Figure 2.26 - Figure 2.29 where the sound power reduction is plotted. From Figure 2.30, it is seen that before control, the radiated sound power for frequency $f_1=300Hz$ is close to that of $f_2=592Hz$. When actuator is optimized at frequency $f_1=300Hz$, the sound power radiated from the controlled plate is $55.496dB$ and $72.426dB$ for $f_1=300Hz$ and $f_2=592Hz$, respectively. The total sound power reduction for this case is $11.635dB$. However, for the actuator optimized at frequency $f_2=592Hz$, because of the poor control on the sound of $f_1=300Hz$, the total sound power reduction is only about $3dB$. The control performance of the actuator optimized at the two frequencies simultaneously is almost the same as that optimized at $f_1=300Hz$. Actually, there is only very little difference between the locations of actuator optimized at the two frequencies and that optimized at $f_1=300Hz$, which are shown in Table 2.10. This is completely dissimilar to that discussed in Section 2.7.1, where the force amplitudes are the same for different frequencies.

The control performance of two actuators operating for the same disturbance as above is plotted in Figure 2.31. It is seen that the actuators optimized at both excitation frequencies perform much better than that optimized at one of the two frequencies. Also, the locations of actuators optimized at both frequencies are not close to any of the other two designs, as indicated in Table 2.11. But, it should be mentioned that as shown in Section 2.7.1.1, the optimal locations of multiple actuators which have similar control effectiveness for single frequency excitation may not be unique. Therefore, if the number of actuators is larger than one, the optimization of the actuator locations with multiple frequencies considered simultaneously should be carefully conducted in the design procedure. This is because that
actuators which have the same performance for one frequency may not still behave the same way for the others.

Finally, it is noted that if the weighting coefficients defined in Eq. (2.60) are assigned to
different values for different frequencies, such as the application of the A weighting
network in practice, the assumption of equal force amplitudes for different frequencies will
not be useful. Hence, the optimization of actuator locations with multiple-frequency
components considered at the same time is unavoidable.

2.8 Summary

In this chapter, active control of sound radiation from a plate under single- and multiple-
frequency excitation has been studied. An optimization procedure has been developed to
find optimal actuator locations for maximum control effectiveness. The control objective is
to minimize the total acoustic power radiated by the plate. Some parameter analyses, such
as the control robustness with respect to the changes in excitation frequencies, disturbance
locations, edge supporting conditions, and lumped point mass, have been conducted. By
summarizing the numerical results and analysis, several conclusions can be listed as follows:

1. The control system with optimally located piezoelectric actuators provides much better control performance than that with the actuators located arbitrarily. The optimal locations of the piezoelectric actuators are dependent upon the excitation frequency and disturbance location. It is shown that the control system with optimally located multiple
actuators can provide robust performance to an uncertain excitation frequency, while the control effectiveness of optimized actuators is very sensitive to the changes in the disturbance location. For multiple actuators, the optimal locations may not be unique. Accordingly, different sets of optimally located actuators may result in some difference in the radiation directivity pattern, but the radiated sound power from the controlled plate is nearly the same.

2. For a given disturbance force with fixed force amplitude and excitation frequency, increasing the boundary stiffness makes the active structural acoustic control become relatively easy to implement. On the other hand, decreasing the plate boundary stiffness leads to the need of more control forces to achieve the same control performance. However, slight changes in boundary stiffness from simply supported conditions will not have much effect on the control performance of the actuators optimally located for a simply supported plate. The radiated sound power from the plate under a given disturbance will change as the plate boundary conditions change. But, this change is dominated by the addition or deduction of the on-resonant modes, which is the result of the shift of natural frequencies.

3. The attachment of a point mass on the plate will influence the natural frequencies of the plate. This could make an off-resonant excitation become an on-resonant one. However, the effect of discrete mass on the control performance must be evaluated case by case.

4. For multiple-frequency excitation, two different types of disturbances should be considered separately;
(a) The disturbance has equal force amplitudes for different excitation frequencies. For this kind of excitation, if the multiple frequencies are well apart from each other, the optimal locations of piezoelectric actuators for all frequencies are very close to, if not coincident with, the locations of actuators optimized at the highest frequency. The optimization of the actuator locations for the highest frequency of the multiple excitation frequencies can be used for the multiple-frequency excitation case. Therefore, attention can be paid to the highest frequency component only when designing actuator location for the plate under the multiple-frequency excitation. Moreover, if the multiple frequencies are very close to each other, there is very little difference among the actuator locations optimized at all the frequencies and those optimized at any of the excitation frequencies. In this case, the optimization of actuator locations can be simplified at any single frequency of the multiple excitation frequencies for a disturbance containing frequencies close together in frequency.

(b) The disturbance has non-equal force amplitudes for different excitation frequencies. For this situation, the optimization of the piezoelectric actuator locations must be made with consideration of all excitation frequencies at the same time.
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<th>Property</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Elastic Modules $E$</td>
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<tr>
<td>Density $\rho$</td>
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<td>Poisson's Ratio $\mu$</td>
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<tr>
<td>Dimension</td>
<td>$0.38 \times 0.30 \times 0.002$ (m$^3$)</td>
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Table 2.2: Natural Frequencies of the Plate

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Mode No.</th>
<th>Mode</th>
<th>Frequency (Hz)</th>
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<td>2</td>
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<td>4</td>
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<td>(3,1)</td>
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### Table 2.3: Material Properties of the Piezoelectric Actuator

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<tr>
<td>Elastic Modules $E_{11}$</td>
<td>$6.3 \times 10^{10}$ (N/m²)</td>
</tr>
<tr>
<td>Density $\rho$</td>
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<tr>
<td>Piezoelectric Strain Coefficient $d_{31}$</td>
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### Table 2.4: Effect of modal coupling on control performance

<table>
<thead>
<tr>
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<th>Radiated sound power at 150Hz (dB)</th>
<th>Radiated sound power at 322Hz (dB)</th>
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<td>Plate</td>
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<td>69.489</td>
</tr>
<tr>
<td>Plate without mode (1,1)</td>
<td>58.879</td>
<td>72.993</td>
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</table>
Table 2.5: Two sets of optimal locations of actuators at $f=357\text{Hz}$

<table>
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<tr>
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<th>Normalized Central location</th>
<th>Sound Power Reduction (dB)</th>
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<td>Set I</td>
<td>( X_{01} )</td>
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</tr>
<tr>
<td></td>
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<tr>
<td></td>
<td>( X_{02} )</td>
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</tr>
<tr>
<td></td>
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<tr>
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<td></td>
<td>( Y_{01} )</td>
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<tr>
<td></td>
<td>( X_{02} )</td>
<td>0.3124</td>
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<tr>
<td></td>
<td>( Y_{02} )</td>
<td>0.3634</td>
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Table 2.6: Actuator location for different designs
One actuator for two on-resonant excitation frequencies
Disturbance with equal force amplitudes

<table>
<thead>
<tr>
<th></th>
<th>Actuator Optimized at 87.0 Hz (m)</th>
<th>Actuator Optimized at 358.0 Hz (m)</th>
<th>Actuator Optimized at 87. &amp; 358. Hz (m)</th>
<th>Actuator arbitrarily located (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{a1}$</td>
<td>0.10136</td>
<td>0.06903</td>
<td>0.06903</td>
<td>0.23205</td>
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<td>$x_{a2}$</td>
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<td>0.12903</td>
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<td>$y_{a1}$</td>
<td>0.09801</td>
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<td>0.08711</td>
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<td>$y_{a2}$</td>
<td>0.13801</td>
<td>0.12711</td>
<td>0.12711</td>
<td>0.23819</td>
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</table>

Table 2.7 Actuator location for different designs
One actuator for two on-resonant excitation frequencies
Disturbance with equal force amplitudes

<table>
<thead>
<tr>
<th></th>
<th>Actuator Optimized at 249.0 Hz (m)</th>
<th>Actuator Optimized at 592.0 Hz (m)</th>
<th>Actuator Optimized at 249. &amp; 592. Hz (m)</th>
<th>Actuator arbitrarily located (m)</th>
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<tbody>
<tr>
<td>$x_{a1}$</td>
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<td>0.04903</td>
<td>0.16155</td>
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<td>0.06905</td>
<td>0.06901</td>
<td>0.16657</td>
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</table>
Table 2.8: Actuator locations for different design
Two actuators for three excitation frequencies
Disturbance with equal force amplitudes

<table>
<thead>
<tr>
<th></th>
<th>Actuators Optimized at 87 Hz (m)</th>
<th>Actuators Optimized at 188 Hz (m)</th>
<th>Actuators Optimized at 358 Hz (m)</th>
<th>Actuators Optimized at 87, 188, and 358Hz (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>0.0038 0.1294</td>
<td>0.0038 0.1228</td>
<td>0.0193 0.0916</td>
<td>0.0038 0.0887</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>0.0638 0.1894</td>
<td>0.0638 0.1828</td>
<td>0.0793 0.1516</td>
<td>0.0638 0.1487</td>
</tr>
<tr>
<td>( y_1 )</td>
<td>0.0704 0.1135</td>
<td>0.0712 0.1089</td>
<td>0.0793 0.0897</td>
<td>0.0771 0.0901</td>
</tr>
<tr>
<td>( y_2 )</td>
<td>0.1104 0.1535</td>
<td>0.1112 0.1489</td>
<td>0.1193 0.1297</td>
<td>0.1171 0.1301</td>
</tr>
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</table>
Table 2.9 Actuator location for different designs
One actuator for two on-resonant excitation frequencies
Disturbance with equal force amplitudes

<table>
<thead>
<tr>
<th></th>
<th>Actuator Optimized at 351.0 Hz (m)</th>
<th>Actuator Optimized at 358.0 Hz (m)</th>
<th>Actuator Optimized at 351. &amp; 358. Hz (m)</th>
<th>Actuator arbitrarily located (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{a1}$</td>
<td>0.06956</td>
<td>0.06903</td>
<td>0.06943</td>
<td>0.00372</td>
</tr>
<tr>
<td>$x_{a2}$</td>
<td>0.12956</td>
<td>0.12903</td>
<td>0.12943</td>
<td>0.06372</td>
</tr>
<tr>
<td>$y_{a1}$</td>
<td>0.08688</td>
<td>0.08711</td>
<td>0.08693</td>
<td>0.25700</td>
</tr>
<tr>
<td>$y_{a2}$</td>
<td>0.12688</td>
<td>0.12711</td>
<td>0.12693</td>
<td>0.29700</td>
</tr>
</tbody>
</table>

Table 2.10: Actuator location for different designs
One actuator for two excitation frequencies
Disturbance with nonequal force amplitudes

|        | Actuators Optimized at 300.0 Hz (m) | Actuators Optimized at 592.0 Hz (m) | Actuators Optimized at 300.0 & 592 Hz (m) |
|--------|----------------------------------|----------------------------------|----------------------------------|---------------------------------|
| $x_{a1}$ | 0.0757                           | 0.0476                           | 0.0768                           |
| $x_{a2}$ | 0.1357                           | 0.1076                           | 0.1368                           |
| $y_{a1}$ | 0.0843                           | 0.0291                           | 0.0879                           |
| $y_{a2}$ | 0.1243                           | 0.0691                           | 0.1279                           |
Table 2.11: Actuator locations for different designs
Two actuators for two excitation frequencies
Disturbance with non-equal force amplitudes

<table>
<thead>
<tr>
<th></th>
<th>Actuators Optimized at 300.0 Hz (m)</th>
<th>Actuators Optimized at 592.0 Hz (m)</th>
<th>Actuators Optimized at 300.0 &amp; 592 Hz (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_{a1} )</td>
<td>0.0038</td>
<td>0.1057</td>
<td>0.0887</td>
</tr>
<tr>
<td>( x_{a2} )</td>
<td>0.0638</td>
<td>0.1657</td>
<td>0.1487</td>
</tr>
<tr>
<td>( y_{a1} )</td>
<td>0.0742</td>
<td>0.0981</td>
<td>0.0548</td>
</tr>
<tr>
<td>( y_{a2} )</td>
<td>0.1142</td>
<td>0.1381</td>
<td>0.0948</td>
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</table>
Figure 2.1: Schematic representation of the problem geometry

Figure 2.2: General boundary support condition
Figure 2.3: Arrangement of disturbance and piezoelectric actuator
Figure 2.4: Comparison of time consumed in sensitivity analysis
Figure 2.5: Radiated sound power before and after control.

The disturbance point force is located at (0.08,0.08)
Figure 2.6: Optimal location of actuator with increasing frequency.

The disturbance force is located at (0.08, 0.08)
Figure 2.7(a): Radiated sound power before and after optimal control.

The disturbance point force is located at (0.14, 0.12)
Figure 2.7(b): Radiated sound power before and after optimal control.

The disturbance point force is located at (0.10, 0.15)
Figure 2.8(a): Optimal location of actuator with increasing frequency:
The disturbance force is located at (0.14, 0.12)
Figure 2.8(b): Optimal Location of actuator with increasing frequency.

The disturbance force is located at (0.10, 0.15)
Figure 2.9: Radiated sound power before and after control.

Two optimally located actuators are used.
Figure 2.10: Modal amplitude distribution before and after control, f=357Hz
Figure 2.11: Sound radiation directivity, f=357Hz
Figure 2.12: Control performance of one optimized actuator as the excitation frequency changes from the design frequency
Figure 2.13: Control performance of two actuators.
Figure 2.14: Effect of disturbance location uncertainty on control performance.
Figure 2.15: The effect of disturbance location uncertainty on control performance
Figure 2.16: The effect of the rotational stiffness on control performance
Figure 2.17: The effect of the rotational stiffness on the natural frequencies
Figure 2.18: The effect of the translational stiffness on control performance
Figure 2.19: The effect of the translational stiffness on the natural frequencies
Figure 2.20: The effect of point mass on the natural frequencies of plate with a point mass attached at x=0.19m, y=0.15m
Figure 2.21: The effect of point mass on the natural frequencies of plate with a point mass attached at x=0.08m, y=0.08m
Figure 2.22: The effect of point mass on the control performance at $f=340\text{Hz}$; a point mass is attached at $x=0.08\text{m}, y=0.08\text{m}$. 

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Figure 2.23: Relative modal amplitudes for uncontrolled plate with and without point mass; a point mass is attached at x=0.08m, y=0.08m; the mass weight is 0.975Kg;

Excitation frequency f=340.0Hz
Figure 2.24: The effect of point mass on the control performance at $f=340\text{Hz}$; a point mass is attached at $x=0.19$, $y=0.15$. 
Figure 2.25: Relative modal amplitudes for uncontrolled plate with and without point mass

a point mass is attached at x=0.19m, y=0.15m; the mass weight is 0.975Kg

Excitation frequency f=340Hz
Figure 2.26: Control performance of one actuator for two on-resonant excitation frequencies
Figure 2.27: Control performance of one actuator for two on-resonant excitation frequencies
Figure 2.28: Control performance of two actuators for three on-resonant excitation frequencies
<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>351.0</th>
<th>358.0</th>
<th>351.0 &amp; 358.0</th>
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</thead>
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<tr>
<td>Actuator optimized at 351 Hz</td>
<td>38.475</td>
<td>41.739</td>
<td>40.600</td>
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<td>Actuator optimized at 358 Hz</td>
<td>35.734</td>
<td>42.933</td>
<td>39.636</td>
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<td>Actuator optimized at 351 &amp; 358Hz</td>
<td>38.222</td>
<td>42.243</td>
<td>40.758</td>
</tr>
<tr>
<td>Actuators arbitrarily located</td>
<td>0.253</td>
<td>8.860</td>
<td>4.588</td>
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</table>

Figure 2.29: Control performance of one actuator for two on-resonant excitation frequencies
Figure 2.30: Control performance of one actuator for two excitation frequencies.

The disturbance with nonequal force amplitudes
Figure 2.31: Control performance of two actuators for two excitation frequencies.

The disturbance with nonequal force amplitudes
Chapter 3

Actuator Design for Active Control of Sound Radiation from Fluid-Loaded Plates

3.1 Introduction

In Chapter 2, active structural acoustic control is applied to the structure under light fluid loading. For light fluid-loaded cases, the influence of the fluid pressure on the structural vibration can be neglected in analysis. Therefore, the structural response to other external forces is indeed the in vacuo response. However, in many practical situations, the structure-fluid interaction (or elasto-acoustic coupling) is significant, such as the case of marine structures. With consideration of the structure-fluid interaction, the analysis model of the system response is complicated. Most of the time, it will result in equations of motion with a much higher degree-of-freedom than that needed in the light fluid loading cases. Much research has been conducted on the problem, as reviewed in Chapter 1. Among them, Gu and Fuller (1993) has successfully applied active structural acoustic control technique to heavy fluid-loaded plates. Despite this, so far, no analytical model has
been developed to optimize the piezoelectric actuator locations for minimizing the global sound radiation from the heavy fluid-loaded plates.

In this chapter, attention is paid to the active control of sound radiation from plates mounted in an infinite baffle and with heavy fluid on one side. The plate boundaries are simply supported. The *in vacuo* normal mode shapes of the plate are chosen to express the plate displacement field (Davies, 1969 and 1971; Sandman, 1977; Lomas and Hayek, 1977; Gu, 1992). A point force with a single tone frequency is selected as the disturbance source. Control is achieved with the use of piezoelectric actuators. For actuators located at certain positions, the optimal control forces are determined by linear quadratic optimal control theory (Lester and Fuller, 1987). Furthermore, an optimization procedure is developed to find the optimal actuator locations with which the radiated sound power is minimized to the largest amount.

There are two objectives of the work in this chapter. The first one is to determine the optimal actuator locations for maximum sound radiation attenuation. The second one is to evaluate the effect of certain parameters, such as, excitation frequency, disturbance location, and point mass attached to the plate, on the control performance of the optimally located actuators. This second part of work is also compared with that for the light fluid-loaded plates discussed in Chapter 2. No attempt is made to model more complicated boundary conditions in this chapter, even though it is ready to consider the arbitrary boundary (Berry, 1994) in the optimization procedure.
3.2 Coupled Equations of Motion

The analytical model in this chapter is a thin rectangular plate supported in an infinite, plane rigid baffle. The plate is covered by a heavy fluid field on one side, while the other side is exposed to a vacuum. In addition, there is a discrete mass \( M_d \) attached to the plate at the position \( (x_d,y_d) \) as shown in Figure 3.1. It is assumed that the external force, either disturbance or control force, will not be affected by the plate motion and fluid field. The equation of motion for the plate can be expressed as:

\[
D \nabla^4 w(x,y,z) + \left[ \rho \dot{h} + M_d \delta(x-x_d) \delta(y-y_d) \right] \frac{\partial^2 w(x,y,t)}{\partial t^2} = f(x,y,t) - P_0(x,y,t) \tag{3.1}
\]

where \( P_0(x,y,t) \) is the pressure induced by the motion of the plate, \( \delta \) is the Dirac delta function, \( w(x,y,t) \) is the transverse deflection of the plate, \( f(x,y,t) \) is the external force which includes both disturbance and control actuators, \( \nabla^4 \) is the biharmonic operator, and \( D = Eh^3 / 12(1-\nu^2) \) is the flexural rigidity of the plate.

The acoustic velocity field in the medium with a sound velocity \( c_f \) is governed by the wave equation:

\[
\nabla^2 \psi(x,y,z) - k_0^2 \psi(x,y,z) = 0 \tag{3.2}
\]

where \( \psi(x,y,z) \) is the velocity potential, and \( k_0 = \frac{\omega}{c_f} \) is the acoustic wavenumber. The acoustic pressure can be determined by the velocity potential:

\[
P_0(x,y,z) = -ip_f c_f k_0 \psi(x,y,z) \tag{3.3}
\]
where $\rho_f$ is the mass density of the fluid medium.

The transverse velocity continuity conditions at the plate-fluid interface are:

$$
\frac{\partial \psi(x, y, z)}{\partial z} \bigg|_{z=0} = \begin{cases} 
    i\omega v(x, y) & |x| < \frac{L_x}{2}, |y| < \frac{L_y}{2} \\
    0 & |x| \geq \frac{L_x}{2}, |y| \geq \frac{L_y}{2}
\end{cases}
$$

(3.4)

The complex Fourier transform of the velocity potential can be written as:

$$
\Psi(k_x, k_y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi(x, y, z)e^{i(k_x x + k_y y)} \, dx \, dy
$$

(3.5)

By solving the transformed wave equation of Eq. (3.2) that satisfies transformed interface conditions of Eq. (3.4), the transformed potential field can be obtained:

$$
\Psi(k_x, k_y, z) = -W(k_x, k_y) \frac{e^{-\gamma}}{\gamma}
$$

(3.6)

where

$$
\gamma = (k_x^2 + k_y^2 - k_0^2)^{\frac{1}{2}}
$$

(3.7)

and $W(k_x, k_y)$ is the Fourier transformed plate response $w(x, y)$. 

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The relationship between the transformed pressure field and the plate response at the plate-fluid interface can be expressed as:

\[
P_0(k_x, k_y, z) \bigg|_{z=0} = \frac{-i \rho c k_0 W(k_x, k_y)}{\sqrt{k_x^2 + k_y^2 - k_0^2}}
\]  

(3.8)

For a simply supported plate, the \textit{in vacuo} eigenfunctions are:

\[
\phi_{mn}(x, y) = \sin \left( \frac{m \pi x}{L_x} \right) \sin \left( \frac{n \pi y}{L_y} \right)
\]  

(3.9)

The plate response can be expanded in terms of the \textit{in vacuo} modes:

\[
w(x, y) = \sum_m \sum_n a_{mn} \phi_{mn}(x, y)
\]  

(3.10)

where \(a_{mn}\) are the modal coordinates or modal amplitudes, the same as that defined in Eq. (2.9).

Also, the acoustic pressure at the plate-fluid interface can be expressed as a series based on the \textit{in vacuo} modes:

\[
p_0(x, y, z) \bigg|_{z=0} = \sum_m \sum_n p_{mn} \phi_{mn}(x, y)
\]  

(3.11)

The complex transform of \(\phi_{mn}(x, y)\) is (Lomas and Hayek, 1977):
\[ s_{ma}(k_x, k_y) = \frac{k_n k_s}{(k_x^2 - k_m^2)(k_y^2 - k_m^2)} \left( e^{\frac{-i k_s L_x}{2}} - (-1)^m e^{\frac{i k_s L_x}{2}} \right) \left( e^{\frac{-i k_m L_y}{2}} - (-1)^n e^{\frac{i k_m L_y}{2}} \right) \] (3.12)

It can be shown that \( s_{ma}(k_x, k_y) \) is an orthonormal function over the infinite \((k_x, k_y)\) plane. With the use of the orthogonality of the \( s_{ma}(k_x, k_y) \), the modal amplitudes for the acoustic pressure at the plate-fluid interface can be expressed as:

\[ p_{ma} = i\omega \sum_k \sum_l \zeta_{mkl} a_{kl} \] (3.13)

where the coupling coefficient \( \zeta_{mkl} \) is defined by:

\[ \zeta_{mkl} = \frac{\rho_f c_f k_0}{L_x L_y \pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s_m^*(k_x, k_y) s_{kl}(k_x, k_y) \frac{dk_x dk_y}{\sqrt{k_0^2 - (k_x^2 + k_y^2)}} \] (3.14)

Substituting Equations (3.10), (3.11), and (3.13) into Eq. (3.1), using the orthogonality property of \( \phi_{ma}(x, y) \), Eq. (3.1) becomes:

\[ [-\omega^2 [m_{ma}] + [k_{ma}]] + i\omega [\zeta_{mkl}] [a_{mna}] = \{ f_{ma} \} \] (3.15)

where \([m_{ma}], [k_{ma}]\) and \(\{ f_{ma}\}\) are the same as those defined in Equations (2.25), (2.26) and (2.28) for the simply supported plate under light fluid loading, respectively.
3.3 Complex Impedance $\zeta_{mnl}$ and Radiated Sound Power

Comparing Eq. (3.15) with Eq. (2.22), it is seen that the only difference between the equations of motion of the plate with heavy-fluid loading and that with light-fluid loading is the introduction of the complex impedance $\zeta_{mnl}$. In the heavy fluid case, the complex impedance $\zeta_{mnl}$ describes the efficiency with which two modes, $\phi_{mn}$ and $\phi_{kl}$, are coupled by the interaction of fluid on the plate motion. $\zeta_{mnl}$ is a zero term unless: $m+k = \text{even}$ and $n+l = \text{even}$ at the same time (Davies, 1969 and 1971). Therefore, at most, mode $\phi_{mn}$ is coupled with one-quarter of the other modes. From Eq. (3.14), it can be seen that

$$\zeta_{mnl} = S_{mnl} + iT_{mnl}$$  \hspace{1cm} (3.16)

The imaginary part, $T_{mnl}$, is a pure reactive term effectively leading to the increase of the plate mass, while the real part, $S_{mnl}$, is a dissipative factor related to the sound energy radiated into the acoustic field (Davies, 1969 and 1971). Even though this dissipative factor $S_{mnl}$ and the modal radiation coefficient $\delta_{mnl}$ defined in Section 2.3 are obtained by completely different derivations, it can be proved as shown in Appendix E that term $S_{mnl}$ is linearly proportional to the modal radiation coefficient $\delta_{mnl}$. From Appendix E, it is seen that the relationship between $S_{mnl}$ and $\delta_{mnl}$ is:

$$\delta_{mnl} = \frac{\omega^2 L_x L_y}{8} S_{mnl}$$  \hspace{1cm} (3.17)

Actually, because of the assumption that the fluid medium is an inviscid flow, there is no other dissipation during the propagation of acoustic wave in the fluid medium. Therefore, the evaluation of the sound energy can be made either in the near field, which can be
related to the $S_{mkl}$, or in the far field with use of the modal radiation coefficients $\delta_{mkl}$.

Equation (3.17) can also be considered as an alternative expression of the law of the conservation of energy.

Under the application of external force with a single harmonic frequency, the sound power radiated from the plate can be expressed as:

$$\Phi_p = \frac{i\omega}{2} \text{Re}\left\{ \int_{-L_y/2}^{L_y/2} \int_{-L_x/2}^{L_x/2} P_0(x,y,0)w^*(x,y)dx dy \right\}$$  \hspace{1cm} (3.18)

By substituting Eq. (3.11) into Eq. (3.18), after some manipulation,

$$\Phi_p = \frac{\omega^2 L_x L_y}{8} \text{Re}\left\{ \sum_{n=1}^{N} \sum_{m=-N}^{N} \sum_{k=1}^{N} \sum_{l=-N}^{N} \zeta_{mkl} a_{mn} a_{kl}^* \right\}$$  \hspace{1cm} (3.19)

3.4 The Optimization Problem

In order to achieve the maximum sound attenuation, an optimization procedure is developed for the heavy-fluid loaded plate. The objective function in the optimization is again the radiated sound power from the plate. The design variables are the same as those used in Chapter 2, i.e., the center locations of piezoelectric actuators whose sizes are fixed. As mentioned in Section 3.3, the only difference between the analysis of light and heavy fluid-loaded plates is the coupling terms presented in the equation of motion. This means the statement about the optimization problem is the same as that described in Section 2.6. Here it is briefly reviewed:
Objective function: \[ \Phi_p = \Phi_p(x_0, y_0) \]

Design variables: \[ x_{0i}, y_{0i}; \quad i = 1, 2, \ldots, N_a \]

Subject to the constraint conditions:

1. Constrain actuators inside of the plate;
2. Constrain overlap between actuators and disturbance force;
3. Constrain overlap among actuators;
4. Constraint on the voltages to piezoelectric actuators.

It is also noted that the optimal control voltages to the actuators with certain locations can be obtained with use of linear quadratic optimal control theory, which is formulated in Section 2.4.

### 3.5 Results and Discussion

Numerical simulations are performed for an aluminum rectangular plate with the dimensions 0.5588\( \times \)0.8636\( \times \)0.009525(\( \text{m} \)). The material properties of this plate and the acoustic medium are listed in Table 3.1. Because of the heavy fluid loading, the plate resonant frequencies will be lower than those under light fluid action. Also, the presence of complex impedance terms in the motion equations of plates makes the determination of the natural frequencies much more complicated. Expressions for approximating these natural frequencies have been suggested by Fahy (1985) and shown to be with in reasonable accuracy except for the first mode (Gu, 1992). Exact solutions for the natural frequencies of fluid loaded plates could be found with the use of the equation of plate
motion for free vibration, which, by setting the external force \( \{ f_{ma} \} = 0 \) in Eq. (3.15), can be expressed in the following form:

\[
[-\omega^2[m_{mkl}]+[k_{mkl}]+i\omega[\zeta_{mkl}]]\{a_{ma}\} = 0
\]  (3.20)

Eq. (3.20) is not a typical eigenvalue problem because the terms \( \zeta_{mkl}(\omega) \) are nonlinear functions of the excitation frequency \( \omega \). Extensive numerical computation is needed to solve the above nonlinear homogeneous equations and find the natural frequencies and their associated natural modes of the plate. However, these natural frequencies can be roughly estimated by checking the frequency response of the plate (Gu, 1992). In this work, a method similar to that used by Gu (1992) is applied to locate the natural frequencies of the plate. No attempt is made to determine the natural modes of the fluid-loaded plate due to the fact that for this kind of vibrational system, the natural modes do not hold the orthogonality properties that the modes of a plate in a vacuum show. The natural frequencies of this fluid loaded plate are listed in Table 3.2.

The sound power spectrum of a fluid-loaded plate driven by a point force at (0.01,0.01) is plotted in Figure 3.2. The peaks observed in Figure 3.2 correspond to the resonant excitation and are close to the natural frequencies of the fluid-loaded plate. Because the point force is located next to the plate corner, the lower order modes are all excited and can be found in Figure 3.2.
3.5.1 Control of on-resonant excitation

In this case, the disturbance force is located in the center of the plate. The excitation frequency is set at \( f = 31 \text{Hz} \), which is very close to the natural frequency of mode \((1,1)\). Three kinds of control actuator arrangements are considered: (1) one piezoelectric actuator is located at \( \left( \frac{L_x}{4}, \frac{3L_y}{4} \right) \); (2) two piezoelectric actuators are located at \( \left( \frac{L_x}{5}, \frac{L_y}{2} \right) \) and \( \left( \frac{4L_x}{5}, \frac{L_y}{2} \right) \), respectively; (3) one piezoelectric actuator is optimally located. The sound directivity patterns are plotted in Figure 3.3. It is seen that for one-actuator control cases, the optimally located actuator provides much better control performance than that the one located at \( \left( \frac{L_x}{4}, \frac{3L_y}{4} \right) \) does. However, in this example, the control system with two actuators outperforms that with one optimally located actuator. The main reason for this is that in the optimization procedure, the condition of constraining the overlap between the piezoelectric actuator and the disturbance force is implemented. With this constraint condition, the single actuator will excite some vibration modes which originally do not exist in the plate response field. Thus, the control spillover limits the control performance. This can be clearly seen from Figure 3.4, where the relative modal amplitudes are plotted for the plate before and after control. The displacement field of the disturbed plate contains only odd-odd modes for the centrally located point force. One piezoelectric actuator, unless located at the center of the plate, will always introduce some even-related modes, such as that the actuator located at \( \left( \frac{L_x}{4}, \frac{3L_y}{4} \right) \) excites all the modes considered in Figure 3.4. On the other hand, for the two actuators located symmetrically about the center, only the amplitudes of the odd-odd modes are observed. The optimal control voltages supplied to the two actuators are actually of the same amplitude and phase. This can be verified by the following simple derivation. If two actuators are positioned
symmetrically about the plate center and along one of the two coordinate axes, say, $(-X_a,0)$ and $(+X_a,0)$, the modal force amplitudes can be obtained with use of Eq. (2.29) for a simply supported plate:

\[
f_{mn}^{a1} = \frac{16C_0d_{31}}{mnl\pi^2} V_1(k_m^2 + k_n^2)\phi_{mn}(-X_a,0)\sin(k_m a_0)\sin(k_n b_0)
\]

(3.21)

\[
f_{mn}^{a2} = \frac{16C_0d_{31}}{mnl\pi^2} V_2(k_m^2 + k_n^2)\phi_{mn}(+X_a,0)\sin(k_m a_0)\sin(k_n b_0)
\]

(3.22)

If $V_1 = V_2$, then the total modal force amplitudes

\[
f_{mn}^a = f_{mn}^{a1} + f_{mn}^{a2}
= \frac{16C_0d_{31}}{mnl\pi^2} V_1(k_m^2 + k_n^2)\sin\left(\frac{m\pi}{2}\right)\sin\left(\frac{n\pi}{2}\right)\cos\left(\frac{m\pi}{2} X_a\right)\sin(k_m a_0)\sin(k_n b_0)
\]

(3.23)

From Eq. (3.23), it is seen that only the odd-odd modes will be activated. For this on-resonant excitation, mode (1,1) will be the only dominant radiation mode. Therefore, significant control output is achieved with these two actuators. Also, it is noticed that because the control voltages to the two actuators are the same, the two actuators can be considered as a single unit in the control procedure, thus reducing the number of the control variables or channels. Extensive study about the control with depended or master-slaver related actuators has been conducted by Carneal and Fuller (1994).
3.5.2 Control of off-resonant excitation

In the above on-resonant excitation example, the optimized location for a single actuator should be at the center of the plate given there is no constraint condition on the overlap between actuator and disturbance force. With this constraint activated, the optimal location of the actuator is close to the constraint bound, i.e., the center of the plate. Similar optimization results are obtained at higher excitation frequency, $f = 434 \text{Hz}$, an off-resonant excitation case. Figure 3.5 gives the sound directivity patterns for the plate before and after control. It is noticed that the control performance of the optimized actuator is much better than that of the two-actuator assignment considered in Figure 3.5. The two actuators located symmetrically about the center do not work well at this excitation frequency; only about $6.8dB$ sound power reduction is obtained. The reason can be seen clearly from Figure 3.6(a) and Figure 3.6(b). Unlike the on-resonant excitation at $f = 31\text{Hz}$, where mode $(1,1)$ is the dominant radiator, several modes with comparable amplitudes are excited at the off-resonant excitation frequency $f = 434\text{Hz}$, which falls between the natural frequencies of mode $(3,1)$ and mode $(1,5)$. For the modes considered in Figure 3.6(a), only the amplitude of mode $(1,1)$ is reduced, while the amplitudes of modes $(1,3)$, $(1,5)$ and $(3,1)$ are enlarged notably with control of the two actuators. Even though the control of the two actuators does not introduce any new modes into the plate displacement field, the phase angle mismatch between the relative modal amplitudes by disturbance and control force makes the control less efficient.

As shown in Figure 3.6(b), the phase angle of mode $(1,1)$ excited by the two actuators is almost $180^\circ$ different from that by the disturbance, and the amplitude of mode $(1,1)$ is lowered by a factor of about 3.1 with the control force activated. However, the phase
angle of mode (1,5) of the plate with disturbance only is very close to that with the control force. The resulting modal amplitude of mode (1,5) is about 4.3 times that without control. It should be noted that for a given actuator assignment, the relative modal distribution is fixed for a certain excitation frequency. The control performance is dependent upon not only the radiative properties of the structure modes, but also the relationship between the relative modal distributions excited by control actuators and disturbance. For this two-actuator control, the difference between the modal amplitude distributions by disturbance and control force is significant and limited control is gained. Moreover, it is noted here that the plate controlled by the two actuators has a larger vibration response than that without the control. Hence, active sound control is achieved mainly by the so-called 'modal restructuring' (Fuller 1990) for these two actuators. Nevertheless, the optimally located actuator, despite introducing some new modes into the displacement field, reduces the sound radiation considerably.

3.5.3 The effect of frequency changes on the performance of an optimized actuator

As discussed in Section 2.7.1.2, for light fluid-loaded plate, the control effectiveness of actuators optimized at a particular frequency is not very sensitive to small changes in the excitation frequency, especially for multiple actuators. Here, several examples will be presented to investigate if the above conclusion is still applicable to the heavy fluid-loaded plates. In order to excite most of the lower order modes, in Figure 3.7, the disturbance force is located at (0.149, 0.186). Two kinds of control arrangements are considered, one is with a single actuator optimized at frequency \( f = 161.0\, Hz \), the other is with two
actuators optimized at the same frequency. It is seen that as the frequency changes from 20Hz to 470Hz, the control output by the two kinds of control changes tremendously. For the one-actuator control, the actuator works well only in several small frequency ranges, such as, 20-80Hz, 140-190Hz, and 340-430Hz. It is noticed that there is almost no control at all for some frequency ranges with this actuator. On the other hand, with two optimally located actuators, not only the frequency ranges have been expanded, but also the control effectiveness is much improved compared with that of one actuator. This observation is consistent with that for the light fluid-loaded plates.

Another example is illustrated in Figure 3.8, where the actuators are optimized at frequency \( f = 359.5Hz \), close to the natural frequency of mode (3,1). Similar results as those for actuators optimized at \( f = 161.0Hz \) are also observed in Figure 3.8. Much better control performance is again achieved with two optimized actuators than with one actuator throughout the frequency considered, except at the frequency range from 420 to 440Hz. Comparing the results in Fig. 3.7 with that in Figure 3.8, there is no significant difference in control achievement between the actuators optimized at 161Hz and 359.5Hz. As a whole, control with multiple actuators demonstrates much more robustness with respect to the frequency uncertainty than with a single actuator.

3.5.4 The effect of disturbance location changes on the control performance of optimally located actuators

To study the influence of disturbance uncertainty on the control output of actuators optimized for a fixed disturbance force, two kinds of disturbance location changes are
considered, one is that the disturbance point force moves along the X-axis direction, another is that the point force moves along the Y-axis direction. The first example, as shown in Figure 3.9, is an on-resonant excitation with \( f = 161.0 \text{Hz} \). The disturbance location before changing is pointed at \((0.149,0.186)\). One and two actuators are optimized for this disturbance separately. It is seen from Figure 3.9(a) that as disturbance moves along the X-axis direction, there is little change in the radiated sound power from the uncontrolled plate. With one actuator, the control is not affected much by the disturbance shifts. For disturbance force located at \((0.1,0.186)\), the gap between the sound power before and after control is slightly augmented. Therefore, it is believed that this actuator design is not sensitive to the disturbance changes in the X-axis direction. However, for the two-actuators control, it is seen that the control effectiveness is upset by this disturbance movement. When the disturbance point forces are at \((0.1, 0.186)\) and \((0.195, 0.186)\), respectively, the radiated sound power is increased by more than 15\(dB\). Nevertheless, because of the on-resonant excitation, control is still significant. Figure 3.9(b) shows the control performance of optimized actuators for disturbance changes in the Y-axis direction. It is noticed in Figure 3.9(b) that as the disturbance moves from point \((0.149,0.186)\) to point \((0.149,0.239)\), the radiated sound power from the uncontrolled plate is reduced. The reason is that the on-resonant excitation frequency, \( f = 161.0 \text{Hz} \), is very close to the natural frequency of mode \((1,3)\). As the point force location approaches \((0.149,0.239)\), the participation factor of mode \((1,3)\) into the sound radiation becomes smaller due to the point force near one of the nodal lines of mode \((1,3)\). It is also noticed from Figure 3.9(b) that control of one actuator is less sensitivity to the disturbance changes in the Y-direction than that with two actuators.
Another example is plotted in Figure 3.10(a) and Figure 3.10(b), where the on-resonant excitation frequency is set at $f = 359.5 \text{Hz}$, very close to the natural frequency of mode (3,1). This time, the radiated sound power from uncontrolled plate varies notably as the disturbance location moves along the X-axis direction as shown in Figure 3.10(a). When the disturbance is located at (0.175, 0.189), there is a nadir observed for the sound power radiated for the uncontrolled plate due to the disturbance point is located right on the nodal line of mode (3,1). Therefore, the on-resonant mode cannot be excited and make any contribution to the sound radiation. With one actuator, there is no control at all for some disturbance locations. Again, the two-actuator control is sensitive to the disturbance uncertainty along the X-axis direction. In Figure 3.10(b), it is seen that the curve of the radiated sound power from the plate controlled by one actuator is relatively smooth as the disturbance moves along the Y-axis direction, while the control performance of the two optimized actuators is still very sensitive to the disturbance uncertainty.

Overall, it is seen that similar to the results for the light fluid-loaded plate, the disturbance uncertainty has a certain degree influence on the control performance of actuators optimized for a fixed disturbance. The extent of this influence is dependent upon the excitation frequency, the original disturbance location, and the path of the change in the disturbance force location.
3.5.5 The effect of a point mass on control performance of optimized actuators

As displayed in Section 2.7.1.5, the existence of point mass on plate will cause the natural frequencies of the system to become smaller. However, the influence of the point mass on specific natural frequencies will be dependent upon the point mass position and its weight. Because of the coupling of the complex impedance $\zeta_{n,m}$ into the equation of plate motion, there is no typical eigenvalue problem for a heavy fluid-loaded plate system. Therefore, it is relatively difficult to find the trace of natural frequencies with respect to the point mass changes as discussed in Chapter 2 for in vacuo plates. At this time, the problem has to be evaluated case by case. Figure 3.11 and Figure 3.12 give two examples of the radiated sound power from a plate with and without a point mass. In Figure 3.11, the point mass is located at the center of the plate and is the same weight as the plate. The disturbance is located at (0.01,0.01), a corner point from which most of the lower order modes can be excited. It is seen from Figure 3.11 that because the point mass is on the nodal lines of all the even-related modes, it can only lower the natural frequencies of the odd-odd modes and leave the others unchanged. As for the point mass attached off-center, shown in Figure 3.12, almost every natural frequency of the plate has been reduced to some degree. Among them, the natural frequencies of the (3,1), (3,2), (1,3), and (2,3) dominant modes are found to be only slightly changed by the point mass attached at (0.167,0.268). This is because the point mass position is close to the nodal lines of the (3,3) mode. It is noted here that even though in Figure 3.12 the mode number of the plate with the point mass is larger than that without the point mass, the presence of the point mass does not introduce any additional modes. The attachment of the point mass to the plate thus does not increase
the degree-of-freedom of the plate system; it only adjusts the distribution of the natural frequencies and their associated mode shapes.

Figure 3.13 illustrates the effect of a point mass on the control performance of actuators optimized for the uniform plate. The disturbance considered in Figure 3.13 is positioned at (0.01,0.01) with an excitation frequency of \( f = 280.0 \text{Hz} \). One point mass is attached to the plate at point (0.167,0.268), the same as that in Figure 3.12. It is seen from Figure 3.13 that as the weight of the point mass increases, a peak appears for the radiated sound power from the uncontrolled plate. This is because when the ratio of the point mass weight to the plate weight, \( \mu_m \), is about 0.2, the natural frequency of mode \( (1,4) \) is very close to \( f = 280.0 \text{Hz} \), which results in an on-resonant excitation. The one-actuator control does not have any attenuation on the sound radiation as the ratio \( \mu_m \) is larger than 0.5. Similar results are found for the two-actuator control, even though the control performance of the two actuators is slightly better than that of one actuator as the point mass becomes heavier.

Another case is presented in Figure 3.14, where the actuators are also optimized for a uniform plate. The disturbance force is located at the same point as that in Figure 3.13, but with an excitation frequency of \( f = 340.0 \text{Hz} \). The point mass is attached at point (0.167,0.268). It is seen that for this case, the weight of the point mass has almost no effect on the control performance of the two optimized actuators considered in Figure 3.14. However, the control performance of the one optimized actuator degrades as the weight of the point mass increases. Because there is no natural frequency crossing the excitation frequency \( f = 340.0 \text{Hz} \) for this range of point mass weight, the curve of the radiated sound power from the uncontrolled plate is relatively smooth. Nevertheless, the
problem the effect of a point mass effect on the control performance of actuators is so complicated that it is hard to deduce any general conclusion with use of limited numerical examples. Several factors played important roles in the analysis, such as, the disturbance location, the point mass position, the weight of the point mass, the excitation frequency, the eigenproperties of the plate, and the locations of the actuators. Because those factors are coupled to each other in the problem of active structural acoustic control, further research is necessary.

3.6 Summary

Active control of sound radiation from the plate under heavy fluid loading has been studied in this chapter. In order to achieve maximum sound attenuation with a certain number of control actuators, an optimization procedure has been developed to find the optimal locations of piezoelectric actuators. Due to the presence of the dense fluid loading, the plate response characteristics are different from those in vacuo which has been investigated in Chapter 2. The fluid loading effects are: (1) causing a damping force to the plate which is related to the acoustic energy; (2) exerting a reaction force which effectively increases the inertia of the plate. It is the second of the above two effects that has the dominant influence on the optimal locations of actuators. The introduction of the fluid loading lowers the natural frequencies of the plate and results in many more modes actively participating in the sound radiation for a certain excitation frequency. By summarizing the numerical results and related discussion, the following conclusions are deduced:
1. For a plate with heavy fluid loading, the effect of the excitation frequency change on the control performance of actuators optimized for a certain frequency is similar to that in the light fluid loading cases. Control systems with multiple actuators present much more robustness with respect to the frequency uncertainty than those with a single actuator.

2. As observed in Chapter 2, the control effectiveness of the actuators optimized for a certain disturbance is also sensitive to the shift of the disturbance location for a plate with heavy fluid loading.

3. Because of the difficulty in tracing the change of the natural frequencies with respect to the point mass for the dense fluid-loaded plates, the analysis of the influence of the point mass on control performance of optimized actuators is much more complicated than that for the light fluid-loaded plates. The numerical study must be conducted case by case.

Overall, the optimally located actuators demonstrate the potential of the active structural acoustic control. For the structures with dense fluid loading, because their mode densities are heavier than that with light fluid loading, the optimization of the actuator locations must be considered in the design procedure.
Table 3.1: Material properties and dimension of the aluminum plate

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus $E$</td>
<td>$7.1 \times 10^{10}$ (N/m²)</td>
</tr>
<tr>
<td>Density $\rho$</td>
<td>2700 (kg/m³)</td>
</tr>
<tr>
<td>Poisson Ratio $\nu$</td>
<td>0.33</td>
</tr>
<tr>
<td>Dimension</td>
<td>$0.5588 \times 0.8636 \times 0.009525$ (m³)</td>
</tr>
</tbody>
</table>

Table 3.2: The natural frequencies of the fluid-loaded plate

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Mode</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1,1)</td>
<td>31</td>
</tr>
<tr>
<td>2</td>
<td>(1,2)</td>
<td>78</td>
</tr>
<tr>
<td>3</td>
<td>(2,1)</td>
<td>143</td>
</tr>
<tr>
<td>4</td>
<td>(1,3)</td>
<td>161</td>
</tr>
<tr>
<td>5</td>
<td>(2,2)</td>
<td>200</td>
</tr>
<tr>
<td>6</td>
<td>(1,4)</td>
<td>291</td>
</tr>
<tr>
<td>7</td>
<td>(2,3)</td>
<td>296</td>
</tr>
<tr>
<td>8</td>
<td>(3,1)</td>
<td>359</td>
</tr>
<tr>
<td>9</td>
<td>(3,2)</td>
<td>428</td>
</tr>
<tr>
<td>10</td>
<td>(2,4)</td>
<td>437</td>
</tr>
</tbody>
</table>
Figure 3.1: Schematic representation of the problem geometry
Figure 3.2: Radiated sound power from the fluid-loaded plate.

The disturbance force is located at (0.01, 0.01)
Figure 3.3(a): Far-field radiation directivity pattern.

On-resonant excitation, $f=31$ Hz; $\phi = 0^\circ$. 
Figure 3.3(b): Far-field radiation directivity pattern.

On-resonant excitation, f=31 Hz; $\phi = 90^\circ$. 
<table>
<thead>
<tr>
<th>Mode</th>
<th>(1,1)</th>
<th>(1,2)</th>
<th>(1,3)</th>
<th>(1,4)</th>
<th>(1,5)</th>
<th>(2,1)</th>
<th>(2,2)</th>
<th>(2,3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without control</td>
<td>1.0000</td>
<td>0.0000</td>
<td>0.0159</td>
<td>0.0000</td>
<td>0.0008</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>One PZT</td>
<td>0.0009</td>
<td>0.0000</td>
<td>0.0087</td>
<td>0.0000</td>
<td>0.0043</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Two PZT</td>
<td>0.0062</td>
<td>0.0352</td>
<td>0.0159</td>
<td>0.0001</td>
<td>0.0054</td>
<td>0.0186</td>
<td>0.0200</td>
<td>0.0101</td>
</tr>
<tr>
<td>One PZT (Optimized)</td>
<td>0.0033</td>
<td>0.0000</td>
<td>0.0094</td>
<td>0.0000</td>
<td>0.0046</td>
<td>0.0052</td>
<td>0.0000</td>
<td>0.0029</td>
</tr>
</tbody>
</table>

**Figure 3.4**: Relative modal amplitudes of plate, f=31Hz
Figure 3.5: Far-field radiation directivity pattern.

Off-resonant excitation, f=434 Hz.
Figure 3.6(a): Relative modal amplitudes of plate, $f=434\text{Hz}$
Figure 3.6(b): Modal amplitudes and phase angles of plate.

Off-resonant excitation $f=434\text{Hz}$
Figure 3.7: The effect of frequency changes on the performance of actuators optimized at frequency $f=161.0\text{Hz}$.

The disturbance location is $(0.149,0.186)$.
Figure 3.8: The effect of frequency changes on the performance of actuators optimized at frequency $f=359.5$ Hz.

The disturbance location is (0.149, 0.186).
Figure 3.9: The effect of disturbance location on the control performance of actuators optimized for disturbance point force located at (0.149,0.186)
On-resonant excitation frequency f=161.0Hz
Figure 3.10: The effect of disturbance location on the control performance of actuators optimized for disturbance point force located at (0.149,0.186)
On-resonant excitation frequency $f=359.5$ Hz
Figure 3.11: Radiated sound power from the fluid-loaded plate.

The disturbance force is located at (0.01,0.01)
Figure 3.12: Radiated sound power from fluid-loaded plate with and without a point mass.

The disturbance force is located at (0.01,0.01).
Figure 3.13: The effect of the point mass on the control performance of actuators optimized for the uniform plate at $f=280\text{Hz}$.

The disturbance force is located at $(0.01,0.01)$. 
Figure 3.14: The effect of the point mass on the control performance of actuators optimized for a uniform plate, $f=340Hz$.

The disturbance force is located at $(0.01,0.01)$
Chapter 4

Sensor Design for Active Control of Sound Radiation

4.1 Introduction

In Chapter 2 and Chapter 3, the control objective is to make the sound source as quiet as possible. Therefore, the cost function used in the previous two chapters was the radiated sound power from the plates. To obtain the sound power in the far field with Eq. (2.41), the sound pressure at every point in the sound field must be measured. This is equivalent to putting an infinite number of microphone sensors in the acoustic field during the control implementation. In practical application, obviously, it is impossible to achieve this type of measurement. Hence, to establish an applicable index, which holds clear physical meaning and is based on a finite number of measurements, is the first priority job in designing a control system.

As reviewed in Chapter 1, considerable progress has been made by the previous researchers to the work of sensor design for active structural acoustical control. Among them, Wang (1991(c)) has compared the performance of distributed and discreted microphone error sensors in the active control of sound radiation from plates. Baumann,
Saunders and Robertshaw (1991) have developed a method for estimating the sound energy from the measured structural response. Clark and Fuller (1992) have demonstrated both experimentally and analytically that the effectiveness of the Polyvinylidene Fluoride (PVDF) films as the error sensors in ASAC approaches. Snyder and Tanaka (1993) have shown the feasibility of using structural vibration measurements as error sensors in adaptive active control systems. The analytical and experimental results by Maillard and Fuller (1994(a), 1994(b)) show that the sound pressure in the far field can be estimated by sensing the structural response with a set of accelerometers and processing the output with an array of filters. Their work reveals the inherent advantage of the ASAC strategy, that is to build and implement the control transducers on or embedded in the sound-radiating structures, while the ultimate control goal is the radiated sound field which is far away from the control site. However, so far, little research has been done about designing discreted microphone error sensor systems while considering the total sound power as the control objective. Also, very little work has been reported concerning the sensitivity of the control performance to the system parameters, such as, the sensor positions, disturbance frequencies, and so on, for active structural acoustic control.

This chapter is concerned with the sensor design for the active control of sound radiation from structures. It contains two major parts. One is concerned with the sensor arrangements which can provide the information about the sound field directly to the controller. The other is concerned with sensor design based on the requirements on the error criteria in the linear quadratic optimal control. In the first part, both microphone and accelerometer sensors are considered. The emphasis is concentrated on solving the integration equation for the sound power radiated from structures and relating the mathematical solutions to the practical implementation. For the second part, an
optimization procedure is developed to find the optimal sensor locations which maximize the sound power attenuation. Numerical results for the optimal designs of the microphone and PVDF sensors are presented. The optimal design of PVDF sensors for controlling the sound with multiple-frequency components is also conducted. Furthermore, a series of parametric studies are carried out to examine the sensitivity of the control system with optimized sensors to the changes in significant system parameters, such as the excitation frequency and the sensor position.

4.2 Sensor Design Based on the Quadrature Formula

In this section, the locations and number of microphone and accelerometer sensors are determined through the use of the Gaussian quadrature formula. The output from the sensors can be used to approximate the sound power radiated from the structures. The idea behind of this design is that the numerical integration scheme used in the analytical work can be directly applied to the measurement procedure in practice.

4.2.1 Microphone sensors

The work in Chapter 2 and Chapter 3 provides the optimized piezoelectric actuator system for which the sound power radiated from the structures will be minimum. However, the performance of the optimized piezoelectric actuators is highly dependent upon the accurate measurement of the radiated acoustic pressure (or sound power) in the practical implementation. Therefore, how to make the arrangement of the acoustic sensors, such as
the microphone sensors, to insure a global sound attenuation is just as important as the design of the control actuators.

Recalling from Chapter 2, the sound power radiated through a hemisphere above the plate is expressed in Eq. (2.41) and rewritten here for a single-frequency excitation as:

\[
\Phi = \frac{1}{\rho c} \int_0^{2\pi} \int_0^\pi |p|^2 R^2 \sin \theta d\theta d\phi
\]  

(4.1)

It can be seen that the problem in obtaining the radiated sound power with a finite number of measurements is how to integrate Eq. (4.1) with properly selected integral points. Those integral points will correspond to the locations and number of the measured pressure points in the sound field.

By applying the Gaussian integral technique, Eq. (4.1) can be written as:

\[
\Phi = \frac{1}{\rho_0 c} \sum_{r=0}^{N_r} \sum_{s=0}^{N_s} |p(\theta_r, \varphi_s)|^2 R^2 \sin \theta_r \lambda_r
\]

(4.2)

\[
= \sum_{r=0}^{N_r} \sum_{s=0}^{N_s} |p(\theta_r, \varphi_s)|^2 \vartheta_r
\]

where \(\theta_r, \varphi_s\) are the points of subdivision of interval \((0, 2\pi)\) for \(\varphi\) and \((0, \pi/2)\) for \(\theta\), respectively; as shown in Figure 4.1, \(\lambda_r\) is the Gaussian weight factor corresponding to the subdivision \((r, s)\), \((N_r + 1)\) is the order of the Gaussian integral, and

\[
\vartheta_r = \frac{1}{\rho_0 c} R^2 \lambda_r \sin \theta_r
\]
From Eq. (4.2), it is seen that the sum of the squared pressure values $|p(\theta_1, \varphi)|^2$ timed with the associated weight factors $\theta^2 n$ provides the approximated sound power. The microphone sensors are located at the positions $(\theta_1, \varphi)$ which are determined by the Gaussian integral. With use of Eq. (4.2), the estimated sound power $\Phi$ is much more accurate, in general, than that obtained from the other method, such as the Simpson's one-third rule, provided the number of measured pressure points are the same. The determination of the integral points is independent on the excitation frequency. Precisely, the selection of the subdivision of the interval and the values of the function at these points in numerical integration can be directly applied to the measurement procedure in practice.

4.2.2 Accelerometer Sensors

In section 4.2.1, attention is focused on the locations of microphone sensors in acoustic field. However, it is often desired to use sensors bonded on or embedded in the sound-radiating structures. The advantages of these structure-bonded sensors are obvious, such as, a compact control design, easy maintenance, etc.. This section is aimed at the application of accelerometers as error sensors to provide sound power radiated from plates with a reasonable accuracy.

The sound pressure at an acoustic field point $(x', y', z')$, as shown in Figure 4.1, can be expressed by Rayleigh's integral (Junger and Feit, 1993):

$$p_r(x', y', z', t) = \int_{-L_z/2}^{L_z/2} \int_{-L_y/2}^{L_y/2} \frac{i \rho c \omega}{2\pi R} w(x, y, t) e^{-ikR} \, dx \, dy$$

(4.3)
where \( R^2 = (x'-x)^2 + (y'-y)^2 + z'^2 \), \( k_0 \) is the acoustic wavenumber of the sound field, and \( \rho_0 \) is the mass density of the acoustic media.

For a far-field point \((\theta, \varphi, R)\), Eq. (4.3) can be simplified as:

\[
p_i(\theta, \varphi, R) = i \rho_0 \omega \frac{e^{-ik_0 R}}{2\pi i R} \hat{w}(\theta, \varphi) \tag{4.4}
\]

where \( \hat{w}(\theta, \varphi) \) is the double Fourier transform of the plate displacement:

\[
\hat{w}(\theta, \varphi) = \frac{L_z^2}{4r} \int_{-1}^{1} \int_{-1}^{1} \hat{w}(\xi, \eta) e^{i\left(\frac{L_z}{L_x} \xi \alpha + \frac{L_z}{L_y} \beta \eta\right)} d\xi d\eta \tag{4.5}
\]

where \( \alpha = k_0 \sin \theta \cos \varphi, \ \beta = k_0 \sin \theta \sin \varphi, \ \xi = \frac{2x}{L_x}, \ \eta = \frac{2y}{L_y}, \ \text{and} \ r = \frac{L_z}{L_y}. \)

By applying the Gaussian quadrature formula to Eq. (4.5), the transformed velocity field \( \hat{w}(\theta, \varphi) \) can be expressed approximately as:

\[
\hat{w}(\theta, \varphi) = \frac{L_z^2}{4r} \sum_{r=0}^{N_x} \sum_{s=0}^{N_y} (\lambda_r \hat{w}(\xi_r, \eta_s)) e^{i\left(\frac{L_z}{L_x} \xi_r \alpha + \frac{L_z}{L_y} \beta \eta_s\right)} \tag{4.6}
\]

where \((N_z + 1)\) is the order of the Gaussian quadrature, \( \lambda_r \) is the Gaussian weight factor which is identical to that defined in Eq. (4.2), and \( \xi_r, \eta_s \) are the Gaussian integral coordinates.

Substituting Eq. (4.6) into Eq. (4.4), the sound pressure at point \((\theta, \varphi, R)\) can be expressed as:
\[ p_i(\theta, \varphi, R) = \sum_{i=1}^{N_G} \tilde{w}_i A_i = [\tilde{w}]^T \{ A \} \]  

(4.7)

where \( N_G = (N_x + 1)^* (N_y + 1) \), \( \tilde{w}_i = \tilde{w}(\xi_i, \eta_i) \) with \( i = 1, 2, \ldots, N_G \), and

\[ A_i(\theta, \varphi, R) = \rho_0 \frac{e^{-ikR}}{2\pi R} \frac{L_x^2}{4r} \lambda_n e^{i(\xi_i \omega_x + \eta_i \omega_y)} \]  

(4.8)

Combining Eq. (4.7) and Eq. (4.1), the radiated sound power in the far field can be approximated as:

\[ \Phi = [\tilde{w}]^T [B] [\tilde{w}]^* \]  

(4.10)

where:

\[ [B] = \frac{R^2}{\rho_0 c} \int_0^{2\pi} \int_0^{\pi/2} \{ A \} \{ A \}^H \sin \theta d\theta d\varphi \]  

(4.11)

When the measurement of the plate acceleration field has been performed at the plate points \((\xi_i, \eta_i)\), the sound power radiated from the plate can be evaluated with the use of Eq. (4.10). The accuracy of this evaluation is dependent upon the order of Gaussian integral formula used. Once the order of the Gaussian integral is fixed, the locations of the measured points are independent on the excitation frequency. Matrix \([B]\) in Eq. (4.11) is a function of the excitation frequency and can be pre-calculated for a certain excitation frequency and order of Gaussian integral. The implementation of the evaluation technique can be summarized in the following steps;
(1). Determine the number of the measurement to be performed on the plate;
(2). Calculate the element values of matrix \([B]\) through the use of Eq. (4.11);
(3). Determine the locations of the plate points where the acceleration values to be measured;
(4). Measure the acceleration at the selected points;
(5). Perform the sound power evaluation through the use of Eq. (4.10).

4.3 Sensor Design Based on the Requirements for Error Criteria in Quadratic Optimal Control

In chapters 2 and 3, linear quadratic optimal control theory is employed to determine the optimal control magnitudes. The cost function and the optimal solution to the cost function can be found in Eq. (2.50) and Eq. (2.54), respectively. In order to evaluate Eq. (2.50) and (2.54), the information of the radiated sound pressure field must be obtained first. Section 4.2 is devoted to the approximation of the radiated sound power through the use of a finite number of measurements. Even though the number of error sensors can be reduced with the introduction of the Gaussian integral formula, a certain number of measurements are still needed for the estimation of the radiated sound power with a certain accuracy. If it is possible to establish an error function which has the same minimum characteristics as the control cost function Eq. (2.50) with respect to the control variables, the demand for accurate sensing of sound field will not be required. In this way, the information from the error sensors can only lead to suitable control force amplitudes for the corresponding actuators. The disadvantage of this type of error function is that no
knowledge about the sound radiation can be directly supplied. This could be a reasonable trade-off due to the fact that accurate sensing of the sound field is hard to achieve for some practical situations.

4.3.1 The requirements for error criteria in LQOCT

Based on the above discussion, the sufficient conditions for the error criterion $\Phi_e$ in the linear quadratic optimal control theory are presented here:

1. $\Phi_e$ has the same minimum (or maximum) value characteristics as the control cost function $\Phi_o$;
2. The number of the error sensors must be larger than or equal to the number of control actuators.

Figure 4.2 illustrates graphically Condition 1 of above. In Figure 4.2, two function, $\Phi_c$ and $\Phi_e$ are the quadratic functions of the control variable $V$. Their minimum values are $\Phi_{c_{\text{min}}}$ and $\Phi_{e_{\text{min}}}$, respectively, while $\Phi_{c_{\text{min}}} \neq \Phi_{e_{\text{min}}}$. However, if the value $V_c$ of the control variable, which makes the function $\Phi_c$ reach its minimum, is equal to $V_e$, then the function $\Phi_e$ is also minimized. Then, the function $\Phi_e$ can be used to replace $\Phi_c$, if the aim is to determine the control value $V_c$. Nevertheless, with use of $\Phi_e$, no further information about $\Phi_c$ can be obtained due to the fact that these two functions may represent completely different physical meanings.

Condition 2 is applied to insure a unique solution to Eq. (2.50).
4.3.2 Optimal Locations of Microphone Sensors

In this section, the locations of the microphone sensors are optimally determined based on the sufficient conditions discussed in Section 4.3.1. Due to the fact that a sensing system, at most, can only lead to the control performance which the actuators are capable of, the sensor design is considered as a step associated to a completed actuator design. With use of the sufficient conditions for the error criteria, the number of the error sensors can be further reduced to equal the number of the control actuators.

4.3.2.1 Cost function for error microphones

Assuming that there are \( N_e \) microphones placed in the radiated sound field as error sensors, the pressure at the \( j \)th point can be obtained from Eq. (2.40):

\[
p_j(\theta_j, \varphi_j) = K(\theta_j, \varphi_j) \sum_{m=1}^{N} \sum_{n=1}^{N} a_{mn}^i I_{mn}(\theta_j, \varphi_j)
\]  \hspace{1cm} (4.11)

where \( I_{mn}(\theta_j, \varphi_j) \) are the transmissibilities as defined in Eq. (2.47), \( a_{mn}^i \) are the modal amplitudes defined in Eq. (2.9), \( N^2 \) is the number of the plate modes considered, and

\[
K(\theta_j, \varphi_j) = \frac{-\omega^2 \rho J_z L_y}{2nR} \left\{ -i \alpha \left[ \frac{\theta_j}{2 \alpha} \left( L_x \cos \varphi_j + L_y \sin \varphi_j \right) \right] \right\}
\]  \hspace{1cm} (4.12)

The sum of the squares of the pressure can be chosen as the cost function:
\[ \Phi_e = \sum_{j=1}^{N} \left| p_j(\theta_j, \varphi_j) \right|^2 \]  \hspace{1cm} (4.13)

Let:

\[ \zeta_{max} (\theta_j, \varphi_j) = I_{mn} (\theta_j, \varphi_j) I^*_n(\theta_j, \varphi_j) K(\theta_j, \varphi_j) K^*(\theta_j, \varphi_j) \]  \hspace{1cm} (4.14)

Then the cost function can be expressed as:

\[ \Phi_e = \sum_{m=1}^{N} \sum_{n=1}^{N} \sum_{k=1}^{N} \sum_{l=1}^{N} \left\{ G_{mnkl} a_{ml}^* a_{nl}^* \right\} \]  \hspace{1cm} (4.15)

where:

\[ G_{mnkl} = \sum_{j=1}^{N} \zeta_{mnkl} (\theta_j, \varphi_j) \]  \hspace{1cm} (4.16)

It should be noted that unlike the modal radiation coefficients \( \delta_{mnkl} \) of Eq. (2.46), \( G_{mnkl} \) of Eq. (4.16) may be of complex values. To avoid the problem of a complex \( G_{mnkl} \), the concept of microphone pair is presented here.

For two microphones, A1 and B1 as shown in Figure 4.3, located at points \((\theta, \varphi, R)\) and \((\theta, \varphi + \pi, R)\), respectively, the modal transmissibilities \( I_{mn} (\theta, \varphi) \) and \( I_{mn} (\theta, \varphi + \pi) \) can be written as:

\[ I_{mn} (\theta, \varphi) = \frac{mn \pi^2 \left[ 1 - (-1)^n e^{i\alpha_1} \right] \left[ 1 - (-1)^n e^{i\beta_1} \right]}{\left( (m\pi)^2 - \alpha_1^2 \right) \left( (n\pi)^2 - \beta_1^2 \right)} \]  \hspace{1cm} (4.17)
\[ I_{mn}(\theta, \varphi + \pi) = \frac{m n \pi^2 \left[ 1 - (-1)^m e^{i \alpha_2} \right] \left[ 1 - (-1)^n e^{i \beta_2} \right]}{(m \pi^2 - \alpha_2^2)(n \pi^2 - \beta_2^2)} \] (4.18)

where:
\[
\begin{align*}
\alpha_1 &= k_0 L_x \sin \theta \cos \varphi \\
\beta_1 &= k_0 L_y \sin \theta \sin \varphi \\
\alpha_2 &= k_0 L_x \sin \theta \cos(\varphi + \pi) \\
\beta_2 &= k_0 L_y \sin \theta \sin(\varphi + \pi)
\end{align*}
\] (4.19)

It can be shown that:
\[ I_{mn}(\theta, \varphi) I_{k\ell}^*(\theta, \varphi) = \left[ I_{mn}(\theta, \varphi + \pi) I_{k\ell}^*(\theta, \varphi + \pi) \right]^* \] (4.20)

Therefore, if we let
\[ \xi_{m\ell k}(\theta, \varphi) = \xi_{m\ell k}(\theta, \varphi) + \xi_{m\ell k}(\theta, \varphi + \pi), \] (4.21)

then the properties of \( \xi_{m\ell k}(\theta, \varphi) \) can be listed as follows:

1. \( \xi_{m\ell k}(\theta, \varphi) \) are real constants once the position of microphone sensor and excitation frequency are fixed;
2. \( \xi_{m\ell k}(\theta, \varphi) = 2 \text{Re}\{\xi_{m\ell k}(\theta, \varphi)\}; \)
3. \( \xi_{m\ell k}(\theta, \varphi) = \xi_{k\ell m}(\theta, \varphi), \) therefore they are symmetric.

With the similar procedure as that used for Eq. (4.15), the cost function \( \Phi_e \) is written as:
\[ \Phi_e = \sum_{m=1}^{N} \sum_{n=1}^{N} \sum_{k=1}^{N} \sum_{l=1}^{N} \left\{ G_{m kl} a_m^l a_k^l \right\} \] (4.22)

where:

\[ G_{m kl} = \sum_{j=1}^{N_s} \xi_{m kl}(\theta_j, \phi_j) \] (4.23)

Finally, by substituting Eq.(2.37) into Eq.(4.22) with considering only single excitation frequency, the cost function \( \Phi_e \) can be expressed in matrix form:

\[ \Phi_e = \{ V \}^H [U] \{ V \} + \{ V \}^H \{ T \} + E \] (4.24)

where \([ U ] = [ u_n ]\), \([ T ] = [ t_s ]\), \([ V ] = [ V_s ]\), and:

\[ u_n = \sum_{m=1}^{N} \sum_{n=1}^{N} \sum_{k=1}^{N} \sum_{l=1}^{N} \left\{ G_{m kl} Q_m^a Q_k^a \right\} \] (4.25)

\[ t_s = 2 \text{ Re} \left[ \sum_{m=1}^{N} \sum_{n=1}^{N} \sum_{k=1}^{N} \sum_{l=1}^{N} \left\{ G_{m kl} Q_m^a Q_k^d \right\} \right] \] (4.26)

\[ E = f_d \sum_{m=1}^{N} \sum_{n=1}^{N} \sum_{k=1}^{N} \sum_{l=1}^{N} \left\{ G_{m kl} Q_m^d Q_k^d \right\} \] (4.27)

here: \( Q_m^a \) and \( Q_m^d \) are defined in Eq. (2.38) and Eq. (2.39), respectively, and \( V_s \) is the voltage applied to the \( s \)th actuator, while \( f_d \) is the force amplitude of the disturbance.
Then, the optimal solution for the cost function can be found as (Lester and Fuller, 1990):

$$\{V\}_e = -[U]^{-1}\{T\}$$  \hspace{1cm} (4.28)

4.3.2.2 Optimization of microphone sensor locations

Upon establishing the cost function for error microphones, the determination of the optimal microphone locations can be made with the use of the optimization procedure. As discussed in Section 4.3.1, the design objective is to find the locations of error microphones with which the optimal solution \(\{V\}_e\) of Eq. (4.28) is as close to the optimal control voltages \(\{V\}_e\) of Eq. (2.54) as possible. This objective function is formulated as:

$$|\vec{V}_e - \vec{V}_c| \to \min$$  \hspace{1cm} (4.29)

The optimal design variables for the objective function are the locations of microphone pairs, \(\theta_j, \varphi_j\); \(j = 1, 2, \cdots, N_e\). The constraint conditions for the design variables are listed below:

1. Constrain overlap between microphone pairs

$$\left(\theta_j - \theta_{(j,j)}\right)^2 + \left(\varphi_j - \varphi_{(j,j)}\right)^2 > 0$$  \hspace{1cm} (4.30)

2. Constrain the design variables inside the corresponding quadrants
\[ 0 \leq \phi_j \leq \pi \]  

(3). Constrain voltages to the piezoelectric actuators

\[ |v_j| \leq 400\text{(volt)} \]  

The above nonlinear optimization problem is solved by an IMSL subroutine named N0ONF, which uses the successive quadratic programming algorithm and a finite different gradient. The optimization method is reviewed in Appendix D. The only difference between subroutine N0ONF and NCONG used in Chapter 2 is that subroutine NCONG uses a user-supplied gradient. For the objective function of Eq. (4.29), it is not possible to do an efficient analytical sensitivity analysis.

### 4.3.3 Optimal locations and sizes of PVDF sensors

In this section, structural error sensors are made of Polyvinylidene Fluoride (PVDF) material which can be bonded to the surface of the structures. The electric response of the PVDF sensor is the integral of strain within the volume of the sensor which is caused by the structural response. The advantages of this material are its high sensitivity and low mass density and compliance. Also, it is easy to tailor the shape of PVDF film in order to satisfy design requirements. The material properties of PVDF are listed in Table 4.1.
4.3.3.1 Cost function for PVDF error sensors

For a simply supported plate under multiple-frequency excitation, the expression for the electrical response of a PVDF film of rectangular shape bonded to the surface of the plate can be written as follows [Lee and Moon, 1987; Clark, 1992]:

\[ q_r(t) = \sum_{j=1}^{N_f} q_r^j(t) \]  

(4.33)

where \( N_f \) is the number of excitation frequencies, and

\[ q_r^j(t) = 2(h_x + t_p) \omega_j^2 \omega_j \sum_{n=1}^{N_s} V_s^j(\omega_j) \left\{ \sum_{n=1}^{N} \sum_{m=1}^{N} \left( Q_{nm}^{\omega_j} R_{nm} \right) \right\} \]  

(4.34)

where

\[ R_{nm} = \left[ e_{11}^{(0)} \frac{mL_y}{nL_x} + e_{32}^{(0)} \frac{nL_x}{mL_y} \right] \sin(k_m x'_s) \sin(k_n x'_s) \sin(k_m x'_s) \sin(k_n x'_s) \sin(k_m y'_s) \sin(k_n y'_s) \]  

(4.35)

and \( e_{11}^{(0)} \) and \( e_{32}^{(0)} \) are the piezoelectric stress/charge constants; \( t_p \) is the thickness of the PVDF film; \( (x'_s, y'_s) \) and \( (x'_s, y'_s) \) are the size and central location of the \( r \)th rectangular PVDF sensor, as shown in Figure 4.4.; \( Q_{nm}^{\omega_j} \) is the modal amplitude defined in Eq. (2.38), \( \omega_j \) is the \( j \)th excitation frequency; \( . \) and \( V_s^j \) is the control voltage applied to the \( s \)th actuator.

The cost function is selected as a sum of the squares of the electrical response at each PVDF sensor, and can be written as:
\[
\Phi_{ep} = \sum_{r=1}^{N_e} \left\{ \frac{1}{T} \int_0^T q_r q_r^* dt \right\} 
\]  
(4.36)

where \( N_{ep} \) is the number of the PVDF sensors. By the arrangement similar to the derivation in Appendix A, Eq. (4.36) can be expressed as:

\[
\Phi_{ep} = \sum_{j=1}^{N_f} \left( \{V_s^j\}^H [T_A^j] \{V_s^j\} + 2 \text{Re} \{V_s^j\}^H \{T_B^j\} + T_C^j \right) 
\]  
(4.37)

where

\[
[T_A^j] = \sum_{r=1}^{N_e} \left\{ \sum_{m=1}^N \sum_{a=1}^N Q_{mn}^{ja} R_{mn} \right\} \left\{ \sum_{k=1}^N \sum_{l=1}^N Q_{kl}^{ja} R_{kl} \right\}^H 
\]  
(4.38)

\[
\{T_B^j\} = \sum_{r=1}^{N_e} \left( \sum_{m=1}^N \sum_{a=1}^N Q_{mn}^{ja} R_{mn} \right)^* \left( \sum_{k=1}^N \sum_{l=1}^N Q_{kl}^{ja} R_{kl} f_d^j \right) 
\]  
(4.39)

\[
T_C^j = \sum_{r=1}^{N_e} \left( \sum_{m=1}^N \sum_{a=1}^N Q_{mn}^{ja} R_{mn} f_d^j \right) \left( \sum_{k=1}^N \sum_{l=1}^N Q_{kl}^{ja} R_{kl} f_d^j \right)^* 
\]  
(4.40)

Finally, the optimal solution for Eq. (4.37) can be found as:

\[
\{V_{ep}^j\} = -[T_A^j]^{-1} \{T_B^j\} 
\]  
(4.41)
4.3.3.2 Optimization of PVDF error sensor locations and sizes

The optimization procedure for the determination of PVDF sensor locations and sizes is similar to that discussed in Section 4.3.2.2. However, with consideration of multiple-frequency excitation, the objective function expressed in Eq. (4.29) may not be suitable. The reason is that even though \( |V_{ep}^{i} - V_{c}^{i}| = 0 \) is the ultimate solution for the optimization problem on the ideal condition, the sensitivities of the total sound power to the control voltages at different frequencies are different. Hence, weighting coefficients must be added to the objective function of Eq. (4.29) for multiple-frequency excitation cases. Because the final goal is to control radiated sound power from the plate, an alternative way is to substitute Eq. (4.41) into Eq. (2.50) and choose the radiated sound power as the objective function (Clark 1992). Therefore, the objective function is formulated as:

\[
\Phi_p = \sum_{j=1}^{N_f} \phi_p \left( x_a^r, y_a^r, x_b^r, y_b^r, \{V_{ep}^j(x_a^r, y_a^r, x_b^r, y_b^r) \} \right) \tag{4.42}
\]

where \( \{V_{ep}^j \} \) is obtained by solving Eq. (4.41). The design variables are the locations and sizes of the PVDF film, \( x_a^r, y_a^r, x_b^r, y_b^r \), with \( r = 1, 2, \ldots, N_{ep} \). The typical constraint conditions for the design variables are then:

1. Constrain overlap between PVDF sensors;
2. Constrain the sizes of the PVDF sensors;
3. Constrain the PVDF sensors inside of the plate;
4. Constrain the overlap among piezoelectric actuators and PVDF sensors.
It is proved to be a difficult task to establish the mathematical forms for constraint conditions (1) and (3) of above due to the optimization sequence. Once the optimal locations of the piezoelectric actuators have been determined, the available space on the plate for the PVDF sensors has been divided into several small broken areas. In this work, the problem is overcome by partitioning the problem. Figure 4.5 illustrates the definition of each sub-problem. For the plate shown in Figure 4.5, the presence of the piezoelectric actuator has separated the plate into four major parts, i.e., areas (1). ABIJ; (2). ADEL; (3). CDGH; (4). KFGJ. Therefore, the optimization problem can be sectioned into four sub-problems. In each sub-problem, the constraint conditions of the following are activated:

(1). Constrain the sizes of the PVDF sensors:

\[
\begin{align*}
X_{\text{min}} & \leq x'_a & \leq X_{\text{max}} \\
Y_{\text{min}} & \leq y'_a & \leq X_{\text{max}} \\
\end{align*}
\]

(4.43)

(2). Constrain the PVDF sensors inside of one of the four areas:

\[
\begin{align*}
x'_b - x'_a & \geq L_{\text{bound}} \\
y'_b - y'_a & \geq L_{\text{bound}} \\
x'_b + x'_a & \leq U_{\text{bound}} \\
y'_b + y'_a & \leq U_{\text{bound}} \\
\end{align*}
\]

(4.44)

where the determination of the values for \(L_{\text{bound}}, L_{\text{bound}}, U_{\text{bound}}, U_{\text{bound}}\), are based on the area considered, as indicated in Figure 4.5, for the current sub-problem. It should be noted that for a multiple-actuator case, many more sub-problems will be created, and the resulted optimization procedure becomes an exhausting process. However, numerical study shows that for the plate on which the actuator number is not more than three,
optimization can still be relatively easy to accomplish. Nevertheless, for the case with four or more actuators, the above optimization problem has been proved to be inefficient. Further research should be conducted to overcome the kind of obstacle.

Similar to the optimization in Section 4.3.2.2, this nonlinear optimization problem is solved with use of an IMSL subroutine named N0ONF, which has been introduced in the above Sections and also reviewed in Appendix D.

4.4 Results and Discussion

The numerical simulations are performed for a simply supported steel rectangular plate of dimensions $0.38 \times 0.30 \times 0.002(m)$. The material properties of the steel plate were previously listed in Table 2.1. A point force is chosen as the disturbance input whose location will be given in the following examples. The control is achieved with the use of the piezoelectric actuators

4.4.1 Microphone Sensors

In Sections 4.4.1.1 and 4.4.1.2, two types of approaches are applied to determine the locations of microphone error sensors. The first method is based on the Gaussian Quadrature technique which was discussed in Section 4.2.1. The second method is based on the sufficient conditions established for the error criterion in the linear quadratic optimal control theory which is described in Section 4.3.2.
4.4.1.1 Microphone Sensors for Far Field Sound Power

In this section, microphone sensors are distributed in the acoustic far field. The sound power is estimated with the use of those microphone sensors. Active control of sound radiation from a plate is achieved through the use of the piezoelectric actuators and the optimal control forces to the piezoelectric actuators are determined based on the information of the microphone sensors distributed in the acoustic field.

Figure 4.6 shows the estimation of the radiated sound power from a plate disturbed by a point force. The point force is positioned at (0.08,0.08) and has a single harmonic excitation frequency. Two sets of microphone sensors are considered in Figure 4.6, the number and positions of the microphones in each set are determined with the use of Gaussian quadrature technique discussed in Section 4.2.1 and listed in Table 4.2. The solid line in Figure 4.6, which represents the accurate sound power at each frequency, is obtained by integrating Eq. (4.1) with Simpson's one third rule of 200 points for each variable and considered as the ideal sensor. It is seen from Figure 4.6 that the prediction of the radiated sound power by nine microphones and four microphones is good. Only slight difference of less than 2dB is observed for the frequency range considered. When the two sets of microphone sensors are used in active control of sound radiation, the key question is: can those sensors lead to a reasonable estimation of the control forces to the actuators. Figure 4.7 illustrates the performance of the two sets of microphone sensors for the control with one piezoelectric actuator. The piezoelectric actuator is optimized at an excitation frequency \( f = 357 \text{Hz} \) with ideally sensed sound field information. It is seen that for most of the frequencies regarded in Figure 4.7, the two sets of microphone sensors
work very well. The estimated control voltages are very close to that for the ideal sensor. However, for the frequency range from 300 to 360Hz, clear differences, especially in the imaginary parts of the control force amplitudes, is noticed. Nevertheless, the control performance with use of the two sets of microphone sensors is close to that with the ideal sensor, which is shown in Figure 4.8.

The performance of the two sets of microphone sensors at some higher frequencies is displayed in Figure 4.9. It is seen from Figure 4.9 that as the frequency goes higher, the estimation of the sound power with those microphones becomes less accurate. For example, at frequency $f = 520\,\text{Hz}$, the estimated sound power by four microphones is only about half of that by the ideal sensor. Figure 4.10 shows the control performance of one piezoelectric actuator which is optimized at $f = 357\,\text{Hz}$. It should be noted that because the actuator is designed for the disturbance with frequency $f = 357\,\text{Hz}$, the control is less effective at frequency higher than $520\,\text{Hz}$. Notice from Figure 4.10 that the control with four microphones as a sensor does not always reduce the sound output from the plate. At some frequencies, such as from 520 to 550Hz and 575 to 585Hz, the sound power reduction after control becomes negative and thus the control system at those frequencies increases the total sound radiation level. Obviously, the poor control performance of the four-microphone sensor is caused by the error in the prediction of the control force amplitudes, which is clearly shown in Figure 4.11. However, the control with nine microphones still work well in the range of frequencies considered in Figure 4.11.
4.4.1.2 Microphone Sensors Based on the Requirement for the Error Criteria in LQOCT

The establishment of the sufficient conditions for the error criteria in linear quadratic optimal control can simplify the sensor design procedure. In this section, the positions of microphone sensors are determined by the optimization procedure outlined in Section 4.3.2. It should be noted here that the optimized microphone positions are the points in the sound field at which the pressure signals reach their minimum values. In this case, the sound power radiation will also be at its minimum.

Table 4.3 gives the positions and performance of three sets of microphone pairs which are defined in Section 4.3.2.1. The disturbance is a point force with excitation frequency of $f = 87\text{Hz}$ and located at $x=0.08m$ and $y=0.08m$. The control force is an optimally mounted piezoelectric actuator. It is seen that with only one properly located microphone pair, the control system has achieved the maximum sound power reduction which the optimal actuator can offer. Also, the optimized locations of the microphone pairs are not unique. However, there are still some positions in the sound field at which the microphone pairs cannot bring out the best performance of the optimized piezoelectric actuator, such as the position of $\theta = 5.73^\circ$ and $\phi = 110.36^\circ$. Nevertheless, the control performance with this arbitrarily selected microphone pair is still significant because of the on-resonant excitation.

Another example for an off-resonant excitation with frequency $f = 272\text{Hz}$ is listed in Table 4.4. This time, the single piezoelectric actuator is arbitrarily located at a position close to the disturbance point (0.08, 0.08). It is seen that the two optimized microphone
sensors in Table 4.4 do provide the same control performance as the ideal sensor does. However, with the arbitrarily selected sensor, the control may not be very effective. It is shown that the arbitrary one listed in Table 4.4 leads to a sound power reduction which is about 15dB less than that with the ideal sensor.

Figure 4.12 shows the effect of the microphone position changes on the control output at $f = 357\text{Hz}$. The control force is an optimally located piezoelectric actuator. The optimal position of the microphone pair is $\theta = 49^\circ$ and $\varphi = 45^\circ$. It is seen from Figure 4.12 that as the microphone moves in the $\theta$ direction while keeping $\varphi = 45^\circ$, the sound power reduction will decrease slowly. When the microphone arrives at $\theta = 74^\circ$ and $\varphi = 45^\circ$, the sound power reduction is about 4.5dB less than that with the optimal position. One the other hand, for this case, the control performance of the system will not be affected much by the microphone position changes in the $\varphi$ direction. As the position of this microphone moves from $\varphi = 35^\circ$ to $\varphi = 175^\circ$ while keeping $\theta = 49^\circ$ unchanged, the variation in the sound power reduction is less than 0.5dB. Hence, it is believed that the control system with the optimal sensor is not very sensitive to the sensor position changes for this case. However, at some arbitrary position, the microphone pair does not work very well. Table 4.5 shows an arbitrarily placed microphone sensor for which the sound power reduction is about 12dB less than that with an ideal or optimal sensor. Therefore, an optimally positioned microphone pair is so critical to the control performance that is must be considered in the control system design.

The effect of the microphone position changes on the control performance is further investigated for an off-resonant excitation frequency. In Figure 4.13, two optimized microphone pairs are considered. It is seen that for both microphone pairs, the position
changes in the $\phi$ direction will have almost no influence on the control effectiveness, while the position changes in the $\theta$ direction will only slightly affect the sound power reduction.

4.4.2 PVDF Sensor

In this section, the locations and dimensions of the PVDF sensors are optimized through the use of the optimization procedure outlined in Section 4.3.3. The disturbance is chosen as a point force with either a single excitation frequency or multiple frequencies. The positions of the piezoelectric actuators are optimally determined for the corresponding disturbance force. The optimization procedure for the piezoelectric actuators was previously described in Chapter 2.

4.4.2.1 PVDF sensor for single frequency excitation

Figure 4.14 shows the optimal location and size of PVDF sensor for an excitation frequency of $f=272\,\text{Hz}$. Because the requirement on the PVDF error sensor is relative loose when the optimization design is based on the sufficient conditions established in Section 4.3.3, the optimal location and size of the PVDF sensor is not unique. Three different optimization results are plotted in Figure 4.14 for $f=272\,\text{Hz}$. It should be pointed out here that the objective of the optimization is to find the location and size of the PVDF sensor with which the function $\Phi_{ep}$ in Eq. (4.37) has the same minimum characteristics as the objective function $\Phi_{p}$ in Eq. (4.42). Therefore, no attempt is made to explain any physical meaning about the specific locations and dimensions of the PVDF sensors optimally designed in this section. For an excitation frequency of $f=357\,\text{Hz}$, three optimal
locations and sizes of PVDF sensor are plotted in Figure 4.15(a), (b) and (c), respectively. The piezoelectric actuator shown in Figure 4.15 is optimized at frequency of $f=357\,Hz$. Note that in Figure 4.14(b) and Figure 4.15(b), the PVDF sensors overlap with their corresponding actuators, respectively. This can be avoided with the application of the constraint conditions discussed in Section 4.3.3.2 but in concept the PVDF sensors could be overlaid by the piezoelectric actuators.

The performance of the optimized PVDF sensors at excitation frequencies other than the design one is examined in Figure 4.16. In Figure 4.16, the piezoelectric actuator and PVDF sensor considered are optimized at frequency $f=272\,Hz$ and the locations are shown in Figure 4.14(a). It is seen from Figure 4.16 that in the frequency range from 170 to $280\,Hz$, the PVDF sensor provides similar performance as the ideal sensor. However, in the range from 60 to $140\,Hz$ and 355 to $380\,Hz$, the performance of the PVDF sensor is much inferior to that of the ideal sensor. Of even more concern, however, at the frequencies around $f=150\,Hz$ and $f=440\,Hz$, the control system with this PVDF sensor configuration is so poor that the controlled plate radiates more sound energy than the plate without control. Especially, at frequency $f=150\,Hz$, there is a peak value for the radiated sound power from the controlled plate. The reason for this can be explained with help of Figure 4.17, where the optimal control voltages with which the radiated sound power $\Phi_p$ in Eq. (2.50) and the cost function $\Phi_{re}$ in Eq. (4.37) are a minimum, respectively, are plotted against the excitation frequency. As seen in Figure 4.17 at frequency around $f=150\,Hz$, the predicted control voltages goes to the infinity. This singularity is due to the inversion of a very small term $T_\alpha$ in Eq. (4.41) for the particular excitation frequency. Physically, it means that the PVDF error sensor system will have a very small amount of output to the excitation of the piezoelectric actuator at the
frequency. Therefore, the cost function of Eq. (4.37) cannot be used to predict the correct control voltages to the actuators at this moment.

Figure 4.18 gives the performance of the PVDF sensor and piezoelectric actuator, which are optimized for excitation frequency $f=357\,Hz$ and whose positions are shown in Figure 4.15(a), for a frequency range from 60 to 450Hz. The optimized PVDF sensor works well in the range of frequencies 185-270Hz, 310-345Hz and 352-380Hz. However, it is seen from Figure 4.18 that, similar to the case that illustrated in Figure 4.16, this PVDF sensor leads to several sharp peaks which are originally not excited by the disturbance force. At the frequencies corresponding to these sharp peaks, the PVDF sensor could not provide a reasonable estimation for the control voltages, as shown in Figure 4.19. Again, these peaks are caused by the very small values of terms $T_A$ at the corresponding frequencies. Figure 4.20 graphically illustrates the amplitudes of terms $T_A$ and $T_B$ in Eq. (4.38) and Eq. (4.39) with respect to the frequency change. It is clearly seen that at frequency $f=291\,Hz$, $T_A$ is so small that the estimated control voltage is significant large.

Figure 4.21 shows the effect of the change in the PVDF sensor size on the control performance. The piezoelectric actuator and PVDF sensor considered in Figure 4.21 are optimized, in sequence, at an excitation frequency of $f=272\,Hz$. The original optimized position and size of the PVDF sensor are denoted by ABCD in Figure 4.21. It is seen that as the side A-B of the PVDF sensor moves to the position E-F while keeping the side C-D of the PVDF sensor unchanged, the control performance of the system changes but not monotonously. The sound power reduction decreases as side A-B of the PVDF sensor moves away from its optimal position $x=0.2011m$ to the position $x=0.1367m$. At position $x=0.1367m$, the sound power reduction reaches a stationary value. As side A-B moves to
the position $x=0.0937m$, the sound power reduction is up to the same value as that with the original (optimal) PVDF sensor. After that, as the $x$-coordinate of the A-B side approaches to $x=0.002m$, the sound power reduction with this PVDF sensor decreases tremendously. The sound power reduction with use of PVDF sensor, marked with CDEF, is about $22dB$ less than that with the optimized PVDF sensor indicated by ABDC in Figure 4.21.

Figure 4.22 shows the effect of the changes to the PVDF sensor position on the control performance. The dimension of the PVDF sensor is fixed during the position changes. The original optimized PVDF sensor is denoted by ABCD in Figure 4.22. It is seen from Figure 4.22 that at some positions, the PVDF sensor leads to a negative sound power reduction. Also, for a range of the positions from $x=0.002m$ to $x=0.09m$, the control system can produce very small amount of sound reduction.

The effect of the PVDF sensor configuration changes on the control performance is further examined with the following case where the side B-D of the optimized PVDF sensor ABCD moves to increase the sensor's dimension while keeping side A-C unchanged, as shown in Figure 4.23. It is seen from Figure 4.23 that the control performance of the system is not very sensitive to the small changes, such as from $x=0.1836m$ to $x=0.15m$, of the position of the B-D side. However, when side B-D of the PVDF sensor shifts to the positions from $x=0.1m$ to $x=0.065m$ in the Y direction, the control actuator combined with the PVDF sensor cannot be used to attenuate the sound radiation. Moreover, there is only limited control performance achieved through the use of the PVDF sensor with side B-D falling in the range from $x=0.15m$ to $x=0.065m$. 

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In general, it can be seen from the above results and discussion that the sensor arrangement is so important to a control system that the design must be carefully made. Arbitrarily selected actuators combined with an ideal sensor, at least, will not amplify the original annoying sound. However, a carelessly designed sensor may become a "noise-maker" and result in higher sound radiation, even though the control actuators are optimally designed.

4.4.2.2 PVDF sensor for multiple-frequency excitation

In section 4.2.1, it has been shown that the optimized PVDF sensors are very sensitive to frequency change. If improperly tailored and positioned, the PVDF sensors may lead to a control system which will amplify the sound power radiated from the structures. Therefore, it is worth making an effort to design the PVDF sensors which can work well at several excitation frequencies. This is especially important for some practical situations where the disturbance may simultaneously contain several dominant tones.

Figure 4.24 shows the performance of one PVDF sensor, which is optimally designed for two excitation frequencies $f_1=87\,Hz$ and $f_2=189\,Hz$ simultaneously. The disturbance is a point force located at (0.08,0.08), while the control input is one piezoelectric actuator which is optimized for the two frequencies. Also plotted in Figure 4.24 is the performance of the control system with an ideal sensor which is obtained through the use of the Simpson's one-third rule. It is seen that the control system works well at frequency close to $87\,Hz$ and the frequency range from 180 to $260\,Hz$. However, the control system with the PVDF sensor makes the control performance much less effective over the frequency
range from 90 to 170Hz than that with the ideal sensor. Moreover, due to the reasons mentioned in the above section, there are several regions where the control system with the PVDF sensor amplifies the sound radiation from the plate. Nevertheless, with consideration of two frequencies in the design, the frequency range over which the control system works well has been expended.

Another example is illustrated in Figure 4.25, where the two excitation frequencies are $f_1 = 87Hz$ and $f_2 = 272Hz$. One piezoelectric actuator and one PVDF sensor are optimized, in sequence, for the two frequencies. It is noticed that the control system performs well not only at frequencies close to the two design points, but also over the frequency ranges from 175 to 260Hz and 270 to 350Hz. However, the sound power redaction of the system with PVDF sensor over the frequency ranges of 90-170Hz and 351-450Hz is much less than that with ideal sensor. Also, there are three sharp peaks observed in the frequencies ranges considered in Figure 4.25.

Figure 4.26 shows the performance of a two-actuator-two-sensor system over the frequency range from 60Hz to 580Hz. The piezoelectric actuators and the PVDF sensors are optimized, in sequence, for a three-frequency excitation case with $f_1 = 200Hz$, $f_2 = 300Hz$, and $f_3 = 500Hz$. It is seen from Figure 4.26 that at frequencies $f_1 = 200Hz$ and $f_2 = 300Hz$, the control with the PVDF sensors is not as effective as that with the ideal sensor. This is because that when the control objective is to control the sound with three tones simultaneously, a compromise must be made among the different frequency components to achieve the overall optimal control performance. For this case, it is seen good control performance is obtained at frequency $f_3 = 500Hz$, while only mediocre control performance can be provided at the other two frequencies. It is also seen from
Figure 4.26 that the sound power radiated from the controlled plate is relatively flat over the frequency range from 60Hz to 300Hz, except a sharp peak at frequency $f = 231.5Hz$. However, for frequencies from 330-500Hz, the control effectiveness of the system is very limited. At frequencies around 347Hz and 498Hz, there are two sharp peaks observed for the controlled plate. Overall, it is seen that with multiple frequencies considered in the optimization procedure, the resulting control system can provide much balanced performance at the designed frequencies. This can be used to design a system which works well at several frequencies. However, the poor control performance at some frequency points is still a problem for the PVDF sensors which are optimized based on the sufficient conditions for error criteria of LQOCT. Much more research should be made to overcome this obstacle.

4.5 Summary

In this chapter, the sensor design has been studied for active control of sound radiation from the simply supported plates. The sufficient conditions for the error criteria in the linear quadratic optimal control theory have been established. The emphasis of the study is on the sensor design based on those sufficient conditions. An optimization approach is developed for determining the optimal locations of the error sensors, which could be microphones, PVDF films, or accelerometers. The optimization of the PVDF sensor locations and sizes has been conducted for the control of the sound radiation from the plates under multiple-frequency excitation. By summarizing the numerical results and analysis, several conclusions can be deduced as follows:
1. For the control system, sensor design plays a very important role in achieving the projected control performance. To some extent, the sensor design is more important than the actuator design. A carelessly designed sensor system may lead to the control system cannot provide any control authority over the sound radiation from the structures. However, in this work, the sensor design is considered as a second step associated with a completed actuator design. This is because that the sensors cannot promote the performance of the control system over the capacity of the actuators.

2. With the use of the sufficient conditions set up for the error criteria in the linear optimal control, the constraints on the sensors are relatively loose. The sensor design problem might have multiple solutions.

3. The microphone pairs, if optimized based on the sufficient conditions, are not very sensitive to the position changes. However, improperly located microphone sensors may leads to the control performance well below the capability of the control actuators.

4. The optimized PVDF sensors may be extremely sensitive to the excitation frequency changes. Sometimes, if working at the frequencies other than the designed ones, the PVDF sensors may lead to that the controlled structures radiate more sound power than that without control. However, the optimization of the PVDF sensors with multiple frequencies considered simultaneously will result in the PVDF sensors working well at several excitation frequencies.

5. The changes in the position and dimensions of the optimized PVDF sensors may have a strong influence on the control effectiveness. Because of the multiple-solution
property of the problem, it is possible to find a design for which the control system can
provide robust control performance with respect to those parameters. This could be done
through a secondary optimization with consideration of the robustness of the design to the
changes of those parameters.
Table 4.1: Material properties of PVDF film

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density ($\rho_p$)</td>
<td>1780 (kg/m³)</td>
</tr>
<tr>
<td>Elastic Modulus (E)</td>
<td>$2 \times 10^6$ (N/m²)</td>
</tr>
<tr>
<td>Thickness ($h_p$)</td>
<td>$16 \times 10^{-6}$ (m)</td>
</tr>
<tr>
<td>Strain per charge ($d_{31}$)</td>
<td>$23 \times 10^{-12}$ (C/N)</td>
</tr>
<tr>
<td>Strain per charge ($d_{32}$)</td>
<td>$3 \times 10^{-12}$ (C/N)</td>
</tr>
<tr>
<td>Strain per charge ($d_{33}$)</td>
<td>$-33 \times 10^{-12}$ (C/N)</td>
</tr>
</tbody>
</table>

Table 4.2(a): Microphone positions determined with use of Gaussian quadrature (Four points)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$ (rad)</td>
<td>$\theta_1, \varphi_1$</td>
<td>$\theta_1, \varphi_2$</td>
<td>$\theta_2, \varphi_1$</td>
<td>$\theta_2, \varphi_2$</td>
</tr>
<tr>
<td>$\varphi$ (rad)</td>
<td>0.332</td>
<td>0.332</td>
<td>1.238</td>
<td>1.238</td>
</tr>
</tbody>
</table>

Table 4.2(b): Microphone positions determined with use of Gaussian quadrature (Nine points)

<table>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$ (rad)</td>
<td>$\theta_1, \varphi_1$</td>
<td>$\theta_1, \varphi_2$</td>
<td>$\theta_1, \varphi_3$</td>
<td>$\theta_2, \varphi_1$</td>
<td>$\theta_2, \varphi_2$</td>
<td>$\theta_3, \varphi_1$</td>
<td>$\theta_3, \varphi_2$</td>
<td>$\theta_3, \varphi_3$</td>
<td></td>
</tr>
<tr>
<td>$\varphi$ (rad)</td>
<td>0.177</td>
<td>0.177</td>
<td>0.177</td>
<td>0.785</td>
<td>0.785</td>
<td>0.785</td>
<td>1.394</td>
<td>1.394</td>
<td>1.394</td>
</tr>
<tr>
<td></td>
<td>0.708</td>
<td>3.141</td>
<td>5.575</td>
<td>0.708</td>
<td>3.142</td>
<td>5.575</td>
<td>0.708</td>
<td>3.141</td>
<td>5.575</td>
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Table 4.3  Microphone sensor positions and control performance, excitation frequency $f=87\text{Hz}$

<table>
<thead>
<tr>
<th>Sensor Position</th>
<th>Ideal Sensor</th>
<th>Optimized Sensor I</th>
<th>Optimized Sensor II</th>
<th>Arbitrary Sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
<td>Simpson's Rule</td>
<td>$\theta = 73.27^\circ$</td>
<td>$\theta = 41.43^\circ$</td>
<td>$\theta = 5.73^\circ$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\varphi = 81.15^\circ$</td>
<td>$\varphi = 23.18^\circ$</td>
<td>$\varphi = 110.36^\circ$</td>
</tr>
<tr>
<td>Sound Power Reduction (dB)</td>
<td>101.5</td>
<td>101.5</td>
<td>101.5</td>
<td>95.84</td>
</tr>
</tbody>
</table>

Table 4.4  Microphone sensor positions and control performance, excitation frequency $f=272\text{Hz}$

<table>
<thead>
<tr>
<th>Sensor Position</th>
<th>Ideal Sensor</th>
<th>Optimized Sensor I</th>
<th>Optimized Sensor II</th>
<th>Arbitrary Sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
<td>Simpson's Rule</td>
<td>$\theta = 16.58^\circ$</td>
<td>$\theta = 84.27^\circ$</td>
<td>$\theta = 6.80^\circ$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\varphi = 25.23^\circ$</td>
<td>$\varphi = 68.89^\circ$</td>
<td>$\varphi = 151.34^\circ$</td>
</tr>
<tr>
<td>Sound Power Reduction (dB)</td>
<td>21.18</td>
<td>21.18</td>
<td>21.18</td>
<td>5.28</td>
</tr>
</tbody>
</table>

Table 4.5: Microphone sensor positions and control performance, excitation frequency $f=357\text{Hz}$

<table>
<thead>
<tr>
<th>Sensor Position</th>
<th>Ideal Sensor</th>
<th>Optimized Sensor I</th>
<th>Optimized Sensor II</th>
<th>Arbitrary Sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
<td>Simpson's Rule</td>
<td>$\theta = 59.38^\circ$</td>
<td>$\theta = 44.45^\circ$</td>
<td>$\theta = 84.30^\circ$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\varphi = 83.49^\circ$</td>
<td>$\varphi = 12.08^\circ$</td>
<td>$\varphi = 179.29^\circ$</td>
</tr>
<tr>
<td>Sound Power Reduction (dB)</td>
<td>78.92</td>
<td>78.92</td>
<td>78.92</td>
<td>66.93</td>
</tr>
</tbody>
</table>
Figure 4.1: Sound pressure coordinate point in the far field
Figure 4.2: Requirement on the error criteria in LQOCT
Figure 4.3: The schematic representation of the microphone pair
Figure 4.4: Arrangement of the PVDF sensor

Figure 4.5: Sub-spaces for the constraint conditions in the optimization
Figure 4.6: Prediction of the radiated acoustic power
from plate under point force disturbance
Figure 4.7: Prediction of control voltage amplitude
Figure 4.8: Control performance of system with microphone sensors
Figure 4.9: Prediction of the radiated acoustic power from plate under point force disturbance with different sensor arrangements
Figure 4.10: Control performance of actuator with different sensor arrangements
Figure 4.11: Prediction of control voltage amplitudes
Figure 4.12: The effect of the microphone location on the control performance at f=357Hz.
(One actuator and one microphone pair)
Figure 4.13: The effect of the microphone position changes on the control performance.

Two actuators and two microphone pairs, f=272Hz.
Figure 4.14: Optimized PVDF sensors for excitation frequency $f=272\text{Hz}$.

The disturbance is located at $(0.08, 0.08)$. 
Figure 4.15: Optimized PVDF sensors for excitation frequency $f=357$ Hz.

The disturbance is located at $(0.08,0.08)$. 
Figure 4.16: The effect of frequency change on the performance of the PVDF sensor optimized at a frequency $f=272\text{Hz}$
Figure 4.17: Prediction of the relative control voltages with an ideal sensor and a PVDF sensor optimized at frequency f=272Hz
Figure 4.18: The effect of frequency change on the performance of the PVDF sensor optimized at frequency f=357Hz
Figure 4.19: Prediction of the relative control voltages with an ideal sensor and a PVDF sensor optimized at frequency $f=357\,\text{Hz}$.
Figure 4.20: Illustration of the amplitudes of terms in Eq. (4.38) and Eq. (4.39).

The disturbance force with $f=357Hz$ is located at (0.08,0.08).
Figure 4.21: The effect of the change of one PVDF side on the control performance.

Excitation frequency $f=272Hz$. 
Figure 4.22: The effect of the position change of the PVDF sensor on the control performance at $f=272$Hz.
Figure 4.23: The effect of the PVDF sensor location and dimension on the control performance at $f=272\text{Hz}$.
Figure 4.24: The effect of frequency change on the performance of PVDF sensor. One PZT actuator and one PVDF sensor are optimized, in turn, for two-frequency excitation $f_1=87\,\text{Hz}$ and $f_2=189\,\text{Hz}$.
Figure 4.25: The effect of frequency change on the performance of PVDF sensor. One PZT actuator and one PVDF sensor are optimized, in turn, for two-frequency excitation $f_1=87Hz$ and $f_2=272Hz$. 
Figure 4.26: The effect of frequency change on the performance of PVDF sensors.

Two PZT actuators and two PVDF sensors are optimized for three-frequency excitation $f_1=200\,\text{Hz}, f_2=300\,\text{Hz}$ and $f_3=550\,\text{Hz}$. 
Chapter 5

Experimental Investigation

5.1 Introduction

In the above three chapters, analytical models and the numerical analyses have been made to design control system with optimal actuators and sensors for active control of sound radiation from plates. The effectiveness of the design approaches has been shown by the experiments conducted by Smith, Burdisso and Fuller (1994). It was also demonstrated in the work by Smith et al (1994) that the designs can be extended to the control of broadband acoustic radiation from plates. This chapter presents the experimental results for a 212O control system in which piezoelectric actuators and PVDF error sensors were optimally designed to control sound radiation from the simply supported plate under three-frequency excitation. The filtered-X adaptive LMS algorithm (Elliott et al, 1987) was used in the experiment. As compared to a conventional experiment on ASAC in which multiple microphones are used to monitor the control performance, the radiated sound power attenuation in this experimental work was evaluated with use of the out-of-plane velocity fields of the test plate before and after control. Those plate velocity fields were obtained with a scanning laser Doppler vibrometer. A least square regression analysis, which used both polynomial functions and double-sine functions as basis functions, was performed to
extract the noise content from the tested velocity data. The results were then compared to the previously derived analytical values for similar situations. In order to analyze the disagreement between the analytical and the experimental results, two types of boundary support models were presented. Through the experimental and analytical study, the importance of the exact structural model to the optimal design of control system is addressed.

5.2 Filtered-X Adaptive LMS Control Algorithm

The filtered-X adaptive LMS algorithm was used in this experiment study. Figure 5.1 shows a schematic diagram of the control loop. The algorithm that follows was previously presented by Elliott et al (1987). For a controller with \( M \) actuators and \( L \) sensors, the \( l \)th error signal at the \( k \)th time step can be expressed as:

\[
e_l(k) = d_l(k) + y_l(k)
\]

(5.1)

where \( d_l(k) \) is the \( l \)th error signal due to the disturbance, and \( y_l(k) \) is the \( l \)th error signal caused by the control forces,

\[
y_l(k) = \sum_{m=1}^{M} \sum_{j=0}^{N-1} T_{mj} \sum_{i=1}^{N-1} w_{mi}(k-j)x(k-i-j)
\]

(5.2)

where \( T_{mj} \) is the \( j \)th coefficient of the transfer function between the output of the \( m \)th actuator and the \( l \)th error sensor, \( w_{mi}(k-j) \) is the \( i \)th filter coefficient at the \((k-j)\)th sample
for the $m$th actuator, $x(k - i - j)$ is the reference signal, and $N_p$ is the order of the FIR filter. The cost function in the LMS algorithm is defined as the mean square error at every sample time:

$$C(w_{mi}) = \sum_{i=1}^{L} e_i^2(k)$$  \hspace{1cm} (5.3)

The cost function $C(w_{mi})$ is a quadratic function of the unknown filter coefficients $w_{mi}$. If the filtered reference signal $x(k - i - j)$ is of the form:

$$\hat{x}_{ln}(k - i) = \sum_{j=0}^{N_r-1} T_{lnq} x(k - i - j)$$  \hspace{1cm} (5.4)

The coefficients of the adaptive FIR filter are adjusted at every sample time point according to the filter update equation:

$$w_{mi}(k + 1) = w_{mi}(k) - \mu \sum_{i=1}^{L} e_i(k) \hat{x}_{ln}(k - i)$$  \hspace{1cm} (5.5)

where $\mu$ is the convergence parameter.

5.3 Experiment Arrangement

The steel plate used for the experiment was measured $380\text{mm} \times 300\text{mm} \times 1.96\text{mm}$. The simply supported boundary conditions were approximated by connecting thin, flexible steel shims to the edges of the plate. These shims were then rigidly attached to a heavy
support stand. A shaker was used to approximate the point force disturbance and attached to the plate at $x = 0.08m$, $y = 0.08m$ by a stinger.

5.3.1 Control actuators and error sensors

Piezoelectric actuators (the properties of which were previously listed in Table 2.3) were used as control inputs in the experiment. The positions of the actuators were optimized to control sound radiation from the plate under three-frequency excitation using the theory of Chapter 2. In the design process, the plate was assumed to have a simply supported boundary. The three frequencies considered were $f_1 = 200\,Hz$, $f_2 = 300\,Hz$, and $f_3 = 550\,Hz$. All of the frequencies correspond to off-resonant excitation. The optimal locations of the two actuators are plotted in Figure 5.2. It should be noted here that each piezoelectric actuator consisted of two piezoelectric patches. The two piezoelectric patches were attached to the front and back sides of the plate at the same location and driven 180 degree out-of-phase, respectively, for consistency with the analytical model to generate uniform line moments along the edges of the piezoelectric patches (Dimiriadis and Fuller, 1989). These patches were glued to the plate with use of the M-Bond 200 strain gauge glue (Clark and Fuller, 1991(a)).

As pointed out in Chapter 4, the sensor design is critical to the performance of a control system and, to some extent, is more important than the actuator design. In this experiment, distributed PVDF films were used as the error sensors. The optimization procedure for the dimensions and locations of the PVDF sensors can be found in Chapter 4 and the properties of the PVDF material were previously listed in Table 4.1. The optimal
dimensions and locations of the PVDF sensors, which are associated with the optimal actuator design mentioned above, are also shown in Figure 5.2. The PVDF film sensors were attached to the plate with use of the Tuck carpet tape (Clark and Fuller, 1992), while a copper tape was applied to serve as an electric connection. It should be noted that the PVDF film sensor was attached only on one side of the plate. More details about the attachment technique can be found in the reference by Clark and Fuller (1992) and Smith (1993).

5.3.2 Experimental setup

A schematic of the experimental setup is shown in Figure 5.3. The reference signal was generated by a B&K signal generator. The reference signal was then amplified to provide the input signal to the point force shaker. To prevent the shaker from damage, the amplified voltage amplitude was limited to 1 volt, which was monitored with a digital voltmeter. In addition, the reference signal was fed to a DSP board that resided in a 386 PC. To avoid damage to the DSP system, the amplitude of the reference signal input to the DSP board was regulated to be less than ±2 volts.

The control inputs, which were generated by the DSP through the use of the filtered-X LMS algorithm described in Section 5.2, were first filtered through a low pass filter. The control signals were amplified with a power amplifier and then stepped up using a transformer so that the control input voltages matched that of the operation ranges of the piezoelectric actuators.
The error signals from the PVDF sensors were filtered with low pass filters. The filtered error signals were then amplified and fed to the DSP board. In order to protect the DSP system, each channel of the amplified error signal was limited to be less than ±2 volts, which was closely monitored with an oscilloscope. The types of the instruments used in the experiment are noted in Figure 5.3.

5.3.3 Evaluation of the control performance

With use of the piezoelectric actuators and PVDF sensors, all the key control elements are mounted on the structure. Therefore, during the control implementation, no information about the ultimate objective, the sound radiation, is fed directly back to the controller. It has been assumed that once the error criterion, which was the sum of the squares of the PVDF signals in this case, reaches its minimum, the attenuation of the sound radiation will be maximum as discussed in the previous chapters.

In order to evaluate the control performance, a monitoring system must be established. There are two basic types of the monitoring methods. One method is to apply a number of microphones in the sound field (Clark and Fuller, 1992; Smith, Burdisso and Fuller 1994) and check the control performance through the comparison of the pressure values before and after control which are usually measured in an anechoic chamber. The other method is to measure the out-of-plane response field, such as the normal velocity and acceleration, of the sound-radiating surface and process the structure response data to calculate the desired sound radiation properties. In this experiment work, the second strategy was
implemented and was thought to provide more accurate results than the direct microphone method.

The velocity field of the plate was measured with the use of a scanning laser Doppler vibrometer. A scanning laser Doppler vibrometer is essentially a velocity transducer. Compared with the conventional accelerometer in dynamic measurements, the scanning laser Doppler vibrometer (SLDV) has many advantages, such as, high speed, high sensitivity, high accuracy and high spatial resolution. Also, because of the non-contact feature, the SLDV will not distort the response field of the structures. This is especially important for the light-weight structures as the plates considered in this experiment.

For ease of implementation, the test plate and the control system were placed outside of the anechoic chamber. This is based on the assumption that the light-fluid (air) loading has a negligible effect on the structure vibration response. Therefore, the change of the surrounding acoustic field, which is caused by moving the test plate out of or into the anechoic chamber, will have little effect on the plate normal velocity field (or the plate response field). The baffle in which the plate is usually mounted can be also removed on the basis of the same assumption.

5.3.4 Regression analysis with the least square method

With the use of the scanning laser Doppler vibrometer (SLDV), the plate velocity field can be measured with a very high spatial resolution. Normally, the test data contain two typical pieces of information, one is the deterministic results of the test, the other is related
to random noise. In order to only obtain the deterministic results, the random content must be extracted from the test data. This was done by using a least square regression to fit the test data to a set of basis functions, the output of which was a set of regression coefficients. Because of the spatial characteristics of the plate velocity field, either the trigonometric functions, which are expressed in Eq. (2.24), or the polynomial functions written in Eq. (2.30) can be used in the regression model. Both of these functions satisfy the boundary support conditions of a simply supported plate.

The regression model can be of the following form:

\[ \{v\} = [\Psi] \{\gamma\} + \{\varepsilon\} \]  

(5.7)

where \(\{v\}\) is the vector of the measured normal velocity field with \(N_v = N_s \times N_s\) elements, \(\{\gamma\}\) is the vector of the regression coefficients with \(N_\delta = N_R \times N_R\) elements, \(N_s\) is the number of the scanned points in \(X\) or \(Y\) direction, \(N_R\) is the order of the regression model in variable \(x\) or \(y\), \(\{\varepsilon\}\) is the vector of the residuals associated with each measurement, the matrix \([\Psi]\) is a \(N_v \times N_\delta\) matrix and can be expressed as:

\[
[\Psi] = 
\begin{bmatrix}
\phi_{11}(x_1, y_1) & \phi_{12}(x_1, y_1) & \cdots & \phi_{N_sN_\delta}(x_1, y_1) \\
\phi_{11}(x_2, y_2) & \phi_{12}(x_2, y_2) & \cdots & \phi_{N_sN_\delta}(x_2, y_2) \\
\vdots & \vdots & \ddots & \vdots \\
\phi_{11}(x_{N_v}, y_{N_v}) & \phi_{12}(x_{N_v}, y_{N_v}) & \cdots & \phi_{N_sN_\delta}(x_{N_v}, y_{N_v})
\end{bmatrix}
\]

(5.8)

where \((x_i, y_i)\) is the \(i\)th measurement point.

The sum of the squared errors (residuals) (SSE) is:
The least square estimates of the regression coefficients can be found by:

\[
\{\hat{y}\} = (\{\Psi\}[\Psi]^\top)^{-1}\{\Psi\}\{v\}
\]

(5.10)

5.4 Estimation of the Radiated Sound Power

Upon obtaining the regression coefficients \(\{\hat{y}\}\) through the use of the least square method, the deterministic content of the test data can be used for the further analysis about the plate response and associated sound radiation. Because of the utilization of the double-sine functions of Eq. (2.24) and polynomial functions of Eq. (2.30) in the regression models, the regression coefficients \(\{\hat{y}\}\) can be viewed as the modal normal velocity amplitudes. Thus, it is easy to estimate the sound radiation with the use of Eq. (2.40) and Eq. (2.45). For the plate excited under the frequency of \(\omega\), the radiated sound power can be calculated through the use of Eq. (2.45):

\[
\Phi(\omega) = \frac{1}{\omega^2} \{\hat{y}\}^H \{\delta_{\text{modl}}(\omega)\} \{\hat{y}\}
\]

(5.11)

where \(\delta_{\text{modl}}(\omega)\) are the modal radiation coefficients defined in Eq. (2.46).

If let \(\{\hat{y}\}_B\) and \(\{\hat{y}\}_A\) represent the modal velocity amplitudes of the uncontrolled and controlled plate, respectively, the sound power reduction can be expressed as:
\[ R = \Phi_B(\omega) - \Phi_A(\omega) \]
\[
= \frac{1}{\omega^2} \left[ \{ \hat{\gamma} \}_B^H [\delta_{\text{exact}}(\omega)] [\hat{\gamma} \}_B - \{ \hat{\gamma} \}_A^H [\delta_{\text{exact}}(\omega)] [\hat{\gamma} \}_A \right] \tag{5.12}
\]

5.5 Results and Discussion

In this section, first, the experimental results for the 2120 control system described in Section 5.3 are presented. Then, the effect of the support conditions on the control performance of the optimized piezoelectric actuators and PVDF sensors is evaluated with the use of two types of boundary models. Finally, the importance of an exact structural model to the control system design is discussed.

5.5.1 Experimental results

The optimal design for a 2120 system has been plotted in Figure 5.2. The three frequencies selected were \( f_1 = 200Hz \), \( f_2 = 300Hz \) and \( f_3 = 550Hz \). To evaluate the velocity field of the plate with more accuracy, the control was conducted at each design frequency, separately, i.e., each time, the disturbance signal contained only one of the three design frequencies, which is valid due to the principle of superposition.

The plate surface was scanned by the laser vibrometer with an equal increment of \( \Delta x = L_x / 29 \) and \( \Delta y = L_y / 29 \), which resulted in a total of 900 measured velocity values with which the least square regression analysis was performed. The order of the scanned
points is plotted in Figure 5.4. Because the simply supported condition was not perfect, both polynomials and trigonometric functions were used in the regression models. The number of the polynomials in the X or Y direction was set to 14, while the number of the trigonometric functions was 10.

Figure 5.5 and Figure 5.6 show the graphic plots of the original test data and the fitted results with use of the polynomials and double-sine functions, respectively, for the plate before and after control at frequency $f_1 = 200Hz$. It is seen from Figure 5.5 that there are noticeable velocity amplitudes along the plate boundaries. This was the result of two broken screws, which were used to attach the thin shims to the plate, as indicated in Figure 5.7 and not noticed till after the experiments were completed. The supports around the broken screws were closer to free support boundary conditions than simply supported boundary conditions. The measured natural frequencies of the plate are listed in Table 5.1 along with the natural frequencies of the plate with all screws attached, which were previously reported by Smith (1993). An overall decrease in the natural frequencies is observed due to the broken screws. Clearly, the support conditions of the plate are less stiffer than its original configuration. Due to the boundary movement, the double-sine functions with the form of Eq. (2.24) no longer satisfy the plate boundary conditions. Therefore, the resulted velocity field fitted with these double-sine functions does not agree with the original data. On the other hand, with use of the polynomials, the fitted surface results agree with the original results as shown in Figure 5.5. However, for the controlled plate, because the overall response of the plate is relatively small, the movement along the plate boundaries is hardly noticeable, as seen in Figure 5.6.
The test data and the corresponding fitted results for the plate before and after control at excitation frequency \( f_2 = 300 \text{Hz} \) are shown graphically in Figure 5.8 and Figure 5.9, respectively. It is again clearly seen that, for the uncontrolled plate response, the regression model with the polynomials reveals the deterministic contents of the original test data, while the double-sine functions are not suitable for fitting the response field of the plate. The uncontrolled plate response also shows that the mode pattern is close to that of mode (2,2) with some boundary movements.

Figure 10 and 11 show the plate response of the plate before and after control at excitation frequency \( f_3 = 550 \text{Hz} \), respectively. For this frequency, due to the higher density of the important modes, the relative phase angles for the response are so complicate that it is hard to accurately separate out one dominant mode. The plate response appears to be composed of at least two dominant modes.

The estimated sound power reduction from the experiment data is listed in Table 5.2 along with the sound power reduction from the theoretical analysis for the plate with perfectly simply supported boundaries for the three frequencies. The results show that the sound power reduction obtained from experiments is less than that by the theoretical predicted. In particular, at frequency \( f_3 = 550 \text{Hz} \), the sound power radiated from the controlled plate is slightly more than that from the uncontrolled plate. The main reason for this is believed to be the result of the imperfect boundary support conditions. To address the importance of the support conditions (or the exact structural model) to the control performance of the optimized actuators and sensors, the following analysis is conducted.
5.5.2 The influence of imperfect boundary conditions

In order to evaluate the effect of the boundary condition changes on the control performance, two types of boundary support models are presented. The first model, denoted as G1, is shown in Figure 5.12(a), where all the boundary supports are assumed to be with perfectly simply supported condition, except for the supporting regions around the broken screws. For those regions, the free support condition is applied. In the second model, noted as G2, it is assumed that all the support points along the boundary are with the free supports, except for the small areas close to the screws, as illustrated in Figure 5.12(b). The influence of those changes to the support conditions on the stiffness matrix of the system and the corresponding mathematical expressions are presented in Appendix 5.A. The estimated natural frequencies of the two types of plate models are listed in Table 5.3. From Table 5.3, it is seen that as the plate support changes from the perfectly simply-supported condition to the condition of model G1 or model G2, the changes to the natural frequencies of the plate are not the same for the different modes. For some modes, the changes are significant, while for some others, those changes are insignificant. Nevertheless, comparing Table 5.3 with Table 5.1, it is seen that from the point of view of the natural frequencies, the model with boundary G1 is slightly stiffer than the actual boundaries, while the model with boundary G2 is very close to that of the test plate.

The analytical models with boundaries G1 and G2 were used to check the plate response at different excitation frequencies. The predicted response fields of the plate before and after control are plotted in Figure 5.13 to Figure 5.18 for the excitation frequencies of \( f_1 = 200\text{Hz}, f_2 = 300\text{Hz}, \) and \( f_3 = 550\text{Hz}, \) respectively. Also plotted in Figures 5.13 to 5.18 are the corresponding plate normal displacements along the boundaries. Comparing
the response field of the plates of model G1 and G2 with the tested results at each excitation frequency, it is seen that even though the two models reflect the movement of the plate boundaries, the response patterns still do not match with the test data well, especially for the highest excitation frequency of \( f_3 = 550\, \text{Hz} \). For the lower frequency excitation, such as the cases of \( f_1 = 200\, \text{Hz} \) and \( f_2 = 300\, \text{Hz} \), the predicted response patterns are close to the tested ones, but not exactly the same. For model G1, the constraints on the plate boundaries are relatively stiffer than that of the test plate. While the boundary supports of model G2 are slightly softer than the actual boundary conditions. With the high excitation frequency of \( f_3 = 550\, \text{Hz} \), because of the higher modal density, the calculated plate response is different than the tested results. For model G1, this is understandable by checking the natural frequencies of the plates in Table 5.1 and Table 5.3. The difference between the natural frequencies of the test plate and the analytical model G1 of Table 5.3 is notable at frequencies around \( f_3 = 550\, \text{Hz} \). However, even though the natural frequencies of model G2 are very close to that of the test plate, it is believed the slight response difference is caused by: (1) the damping effect of the sealing material attached along the plate edges, (2) the rotational stiffness along the plate support boundaries which was not included in the analytical models. Both of these two factors, even through with insignificant values, may cause the movement of the plate boundaries to be less than that predicted by boundary model G2.

The sound power reduction with control implementation is listed in Table 5.4 for the plates with: (1) perfectly simply-supported condition, (2) boundary G1, (3) boundary G2, respectively. The actuators used are located at the positions shown in Figure 5.2. It is seen that with boundary G1, the control performance at \( f_2 = 300\, \text{Hz} \) and \( f_3 = 550\, \text{Hz} \) is better than that with the perfectly simply supported boundaries. It should be noted that Table 5.4
shows the effect of the boundary changes on the control capability of the designed actuators. The control attenuation in Table 5.4 is achieved by assuming the complete information in the sound field can be exactly sensed, which is performed by integrate Eq. (4.1) through the use of the Simpson's one-third rule. The performance of the optimally designed control system with use of the PVDF sensors shown in Figure 5.2 is listed in Table 5.5. It is clearly seen that the overall performance of the control system is determined mainly by the error sensors. Even though the changes to the plate supporting condition do not have significant effect on the control capacity of the designed actuators in this case, the PVDF sensors optimized for the simply supported condition is so sensitive to these boundary changes that the resulted control performance of the system is very poor at frequency \( f_3 = 550Hz \).

5.6 Summary

In this chapter, active control of sound radiation from a plate was experimentally studied. The piezoelectric actuators and PVDF sensors are optimally designed through the use of the optimization procedures outlined in chapters 2 and 4. The velocity field of the plate before and after control has been measured with a scanning laser Doppler vibrometer. By processing the measured velocity data, the control performance has been evaluated. The effect of the imperfect boundary conditions on the control performance has been analyzed with the use of two types of boundary support models. The experimental results and associated analyses demonstrate that the importance of an exact structural model to the optimal design of actuators and sensors.
There are several reasons that lead to a control performance that is not as significant as the predicted. (1) The noise floor level plays an important role in the control evaluation. As observed in the test example, when the controlled plate response becomes smaller, the noise content in the measured response is so significant that misfitting of the test data may be resulted, especially for the use of high order polynomial functions (Liu, 1993). (2) The influence of the shaker weight on the plate response. Even though in the analytical model the influence of the shaker weight on the plate response has been considered, the determination of the actual values of the weight is still difficult. (3) The effect of the support conditions on the locations of the PZT actuators and PVDF sensors. Through the use of the two analytical models, it is concluded that the inferior performance of the optimally designed control system at $f_3 = 550Hz$ is mainly due to the less robustness of the PVDF sensors to the boundary changes. Also, it should be pointed out that the relative excitation condition may be different (off-resonant excitation to on-resonant excitation or vice versa) when the plate boundary conditions changes that will modify the eigenproperties of the plates.

Even though the design approaches for the optimal actuators and sensors have been shown to be effective for simply supported plates by the previous research (Smith 1993), it is still a difficult work to design a control system with optimally located actuators and sensors for complex structures. It is seen that the error sensor implemented with the PVDF films is more sensitive to the changes to the plate supporting conditions than the optimally located piezoelectric actuators.
Table 5.1: Measured natural frequencies of the tested plates with shaker attached

<table>
<thead>
<tr>
<th></th>
<th>Current tested plate (Hz)</th>
<th>Previously reported (From Smith (1993)) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>83</td>
<td>86</td>
</tr>
<tr>
<td>2</td>
<td>177</td>
<td>182</td>
</tr>
<tr>
<td>3</td>
<td>214</td>
<td>248</td>
</tr>
<tr>
<td>4</td>
<td>295</td>
<td>331</td>
</tr>
<tr>
<td>5</td>
<td>320</td>
<td>350</td>
</tr>
<tr>
<td>6</td>
<td>373</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>445</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>523</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>570</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2: The estimated sound power reduction from experimental and theoretical analysis

<table>
<thead>
<tr>
<th></th>
<th>Sound power reduction from the theoretical analysis (dB)</th>
<th>Sound power reduction from the experimental data (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1 = 200Hz$</td>
<td>16.661</td>
<td>9.69</td>
</tr>
<tr>
<td>$f_2 = 300Hz$</td>
<td>16.492</td>
<td>13.42</td>
</tr>
<tr>
<td>$f_3 = 550Hz$</td>
<td>19.905</td>
<td>-0.494</td>
</tr>
</tbody>
</table>
Table 5.3: Natural frequencies of plates with different boundary supports (with point mass attached at (0.08,0.08))

<table>
<thead>
<tr>
<th></th>
<th>Natural frequencies of simply supported plate (Hz)</th>
<th>Natural frequencies of plate with boundary G1 (Hz)</th>
<th>Natural frequencies of plate with boundary G2 (Hz)</th>
<th>Natural frequencies of current tested plate (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>85.96</td>
<td>83.96</td>
<td>82.33</td>
<td>83</td>
</tr>
<tr>
<td>2</td>
<td>184.97</td>
<td>179.33</td>
<td>176.23</td>
<td>177</td>
</tr>
<tr>
<td>3</td>
<td>244.82</td>
<td>230.08</td>
<td>215.67</td>
<td>214</td>
</tr>
<tr>
<td>4</td>
<td>343.83</td>
<td>324.13</td>
<td>305.54</td>
<td>295</td>
</tr>
<tr>
<td>5</td>
<td>350.00</td>
<td>335.84</td>
<td>319.12</td>
<td>320</td>
</tr>
<tr>
<td>6</td>
<td>508.85</td>
<td>429.37</td>
<td>378.88</td>
<td>373</td>
</tr>
<tr>
<td>7</td>
<td>509.58</td>
<td>491.56</td>
<td>456.43</td>
<td>445</td>
</tr>
<tr>
<td>8</td>
<td>581.02</td>
<td>559.96</td>
<td>526.46</td>
<td>523</td>
</tr>
<tr>
<td>9</td>
<td>608.60</td>
<td>598.02</td>
<td>568.28</td>
<td>570</td>
</tr>
<tr>
<td>10</td>
<td>739.88</td>
<td>652.01</td>
<td>587.84</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.4: Estimated sound power reduction for plates with different boundaries (Ideal Sensor)

<table>
<thead>
<tr>
<th></th>
<th>S-S Boundary (dB)</th>
<th>Boundary G1 (dB)</th>
<th>Boundary G2 (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200Hz</td>
<td>31.022</td>
<td>27.386</td>
<td>29.137</td>
</tr>
<tr>
<td>300Hz</td>
<td>24.398</td>
<td>27.769</td>
<td>23.731</td>
</tr>
<tr>
<td>550Hz</td>
<td>21.904</td>
<td>22.725</td>
<td>15.895</td>
</tr>
</tbody>
</table>

Table 5.5: Sound power reduction for plates with different boundaries (PVDF sensors)

<table>
<thead>
<tr>
<th></th>
<th>Tested Results (dB)</th>
<th>Boundary G1 (dB)</th>
<th>Boundary G2 (dB)</th>
<th>S-S Boundary (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300Hz</td>
<td>13.42</td>
<td>17.942</td>
<td>15.357</td>
<td>16.492</td>
</tr>
<tr>
<td>550Hz</td>
<td>-0.494</td>
<td>2.685</td>
<td>-6.569</td>
<td>19.905</td>
</tr>
</tbody>
</table>
Figure 5.1: Block diagram of filtered-x adaptive LMS control algorithm

Figure 5.2: Optimally designed PZT actuators and PVDF sensors for a 2I2O plate system. The plate is under a three-frequency excitation.
Figure 5.3: Schematic of the experimental setup

LPF: Frequency Device Type 9002 low pass Filter
AMP1: lithaco model 455 amplifier
AMP2: NAD power amplifier 2600A
T: Transformer with a factor of 17:1
Figure 5.4: The order of the scanned points on the test plate
Real-part fitted with Polynomials

Real-part Original Data

Real-part fitted with Double-sine

Figure 5.5: Uncontrolled plate normal velocity response at frequency $f=200\text{Hz}$ (Normalized by the highest velocity value of the plate)
Figure 5.6: Controlled plate normal velocity response at frequency f=200Hz (Normalized by the highest velocity value of the plate)
Figure 5.7: Indication of the plate supports and screw positions
Figure 5.8: Uncontrolled plate normal velocity response at frequency $f=300\text{Hz}$
(Normalized by the highest velocity value of the plate)
Figure 5.9: Controlled plate normal velocity response at frequency f=300Hz (Normalized by the highest velocity value of the plate)
Figure 5.10: Uncontrolled plate normal velocity response at frequency f=550Hz (Normalized by the highest velocity value of the plate)
Figure 5.11: Controlled plate normal velocity response at frequency f=550Hz (normalized by the highest velocity value of the plate)
(a) Boundary G1

(b) Boundary G2

Figure 5.12: Plate with Boundaries G1 and G2
Figure 5.13: Normal displacement field of the plate with Boundary G1 at f=200Hz
(Normalized by the highest displacement value of the plate)
Figure 5.14: Normal displacement field of plate with Boundary G2 at f=200Hz (Normalized by the highest displacement value of the plate)
Figure 5.15: Normal displacement field of the plate with Boundary G1 at f=300Hz (Normalized by the highest displacement value of the plate)
Figure 5.16: Normal displacement field of the plate with Boundary G2 at f=300Hz (Normalized by the highest displacement value of the plate)
Figure 5.17: Normal displacement field of the plate with Boundary G1 at f=550Hz (Normalized by the highest displacement value of the plate)
Figure 5.18: Normal displacement field of the plate with Boundary G2 at f=550Hz (Normalized by the highest displacement value of the plate)
Chapter 6

Conclusions and Recommendations

6.1 Overall Conclusions

This dissertation studies the active control of sound radiation from elastic plates under multiple-frequency excitation. The emphasis is laid on the transducer design for maximum sound attenuation. The major conclusions are highlighted as follows.

- An optimization procedure has been developed to find the optimal actuator locations for maximum control effectiveness. The control objective in the optimization is the total acoustic power radiated from the plate. The optimal locations of the piezoelectric actuators are dependent upon the excitation frequency and disturbance force location. For multiple-actuator system, there may be several different arrangements of the actuators on the plate that can result in the same or very similar control performance. The control system with optimally located actuators is not very sensitive to small changes in the excitation frequency. However, the control of the optimally designed actuator system is very sensitive to changes in the disturbance force location.
For plates under a multiple-frequency excitation, the optimal locations of actuators can be determined through the use of the optimization procedure developed in this dissertation. If the disturbance has an equal force amplitude for different excitation frequencies, the optimization of the actuator locations can be simplified to optimization at the highest frequency component only. However, if the disturbance has non-equal force amplitudes for different excitation frequencies, the optimization of the piezoelectric actuator locations must be made at all the excitation frequencies simultaneously.

Changes to the plate boundary conditions and the attachment of a point mass on the plate will modify the plate eigenproperties. This may result in that an on-resonant excitation condition becomes an off-resonant condition or vice versa. Given the same disturbance condition, increasing the boundary stiffness makes the active structural acoustic control become relatively easy to implement. However, decreasing the plate boundary stiffness may result in the need for more control forces to achieve the same control performance. The effect of a discrete point mass on the control performance must be studied case by case.

The response characteristics of a plate under a dense fluid loading are different from that in vacuo. The dense fluid loading results in a reaction force which effectively increases the inertia of the plate and decreases the plate natural frequencies. Thus because of this reaction force, the optimal locations of the actuators will be different from that optimized for the light fluid loading case. However, the effect of the design parameters, such as, the excitation frequency changes, the disturbance location
changes and point mass discontinuities, on the control performance of the optimized actuators is similar to that in the light fluid lading cases.

- For a control system, the sensor design plays a very important role in achieving the projected control performance. For some cases, an improperly designed sensor system may result in a controlled structure that radiates more sound power than that without control, even though the control actuator system is optimally designed. The sensor design can be based on: (1) The Gaussian integral formula which generally gives much more accurate results than the other methods, such as the Simpson's rule, provided the numbers of the integral points are the same; (2) The sufficient conditions for the error criteria in the linear quadratic optimal control theory (LQOCT). The sufficient conditions state that as long as that the number of the error sensors is larger than or equal to the number of the control actuators and the associated cost function has the same minimum value characteristics as the control objective function, the error sensors can be used to estimate the optimal control forces applied to the actuators.

- The establishment of the sufficient conditions for the error criteria in the LQOCT clearly explains the mechanism of the control implementation with a small number of error sensors. With use of the sufficient conditions in the sensor design, the constraints on the sensors are relatively loose. This may lead to multiple solutions for the optimal sensor system. The microphone sensors, if optimized based on the sufficient conditions, are not very sensitive to the changes in the positions of the microphone sensors. On the other hand, the PVDF sensors optimized based on the sufficient conditions may be extremely sensitivity to the changes of the dimensions and locations of the PVDF sensors. Also, changes to the disturbance frequency and plate boundary
support conditions may result in a very poor control performance of a control system with the optimized PVDF sensors. For a disturbance force with multiple frequency components, the optimization of the PVDF sensors must be made for all the excitation frequencies simultaneously.

- For a structure with a relatively simple geometric shape, the sound radiation properties can be evaluated through the use of a scanning laser Doppler vibrometer. This can not only simplify the experimental procedure, but also provide more accurate results than the direct microphone method.

6.2 Recommendations

There are many topics of interest for the future work. Some recommendations are made as follows:

- The PVDF sensors have been used in this research work. It is found that the performance of the PVDF sensors is very sensitive to the changes in some system parameters, such as, the excitation frequency, the dimensions and locations of the PVDF sensors, and the plate support conditions. Therefore, it would be worthwhile to make some effort to design a PVDF sensor system which has a robust performance with respect to the system parameters mentioned above.

- In this dissertation, the sensors are located at the Gaussian integral points for approximating the total sound power radiated from plates. Because the integral
scheme used in the numerical analysis work can be directly applied to a practical measurement, it is recommended to investigate the expression of the radiated sound power and find an integral method that is more suitable to the special properties of the integration.

- In this work, the disturbance with multiple frequency components has been considered. It is advisable to extend the work to the case of a random noise excitation. This is especially important for the design of the PVDF sensors.
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Appendix A
Modal Decomposition

For plate with arbitrary boundaries, trial functions in the forms of Eq. (2.30) are used to approximate the plate response. The amplitudes and phase angles of the orthonormal modes can be obtained from the system eigenvectors and the modal amplitudes \( \{a_{mn}\} \) in Eq. (2.22) (Thomas, 1992).

In the absence of external forces, Eq. (2.12) can written as the following form for stable free vibration:

\[
(-\omega^2 [m_{mod}] + [k_{mod}])\{u\} = 0
\]  \hspace{1cm} (A.1)

Equation (A.1) represents the eigenvalue problem associated with \([m_{mod}]\) and \([k_{mod}]\). The square roots of the eigenvalues are the plate natural frequencies, while the eigenvectors represent the natural modes. The orthonormal modes of plate can be obtained with the normalization scheme:

\[
\{u_r\}'' [m_{mod}]\{u_s\} = \delta_{rs} \hspace{1cm} (A.2)
\]

where \(\{u_r\}\) are the normalized eigenvectors, and \(\delta_{rs}\) is the Kronecker delta function.

The modal matrix can be defined as:
\[ [U] = \{u_1, u_2, ..., u_{N^2}\} \] \hspace{1cm} (A.3)

The normalized eigenvectors \( \{u_r\} \), \( r = 1, 2, ..., N^2 \), are linear independent. Physically, any plate response can be expressed by the linear combination of \( \{u_1\}, \{u_2\}, ..., \{u_{N^2}\} \) with the orthonormal mode amplitudes \( \{c\} \):

\[ \{a_{ma}\} = [U]\{c\} \] \hspace{1cm} (A.4)

By solving linear equations of Eq. (A.4), the orthonormal mode amplitudes \( \{c\} \) can be found.
Appendix B

Total Sound Power

If the sound pressure in the far field consists multiple-frequency components, the total sound pressure can be expressed as:

\[ p_i = \sum_{j=1}^{N_j} p_i^j(\omega_j) \]  \hspace{1cm} (B.1)

where \( N_j \) is the number of the frequencies, \( \omega_j \) is the \( j \)th excitation frequency. Therefore, the total sound power can be written as (Nelson and Elliott, 1992):

\[ \Phi_i = \int_{s} \left\{ \frac{1}{T} \int_{0}^{T} |p_i|^2 \frac{1}{\rho_s c} \, dt \right\} ds \]  \hspace{1cm} (B.2)

By substituting Eq. (B.1) into Eq. (B.2), the total sound power can be rewritten as:

\[ \Phi_i = \sum_{j=1}^{N_j} \Phi^j + \sum_{r=1}^{N_j} \sum_{s=1}^{N_j} \Phi^r_s \]  \hspace{1cm} (B.3)

where

\[ \Phi^j = \int_{0}^{\pi/2} \int_{0}^{2\pi} \frac{R^2}{\rho_s c} |p_i^j(\omega_j)|^2 \, d\theta d\phi \]  \hspace{1cm} (B.4)
\[
\Phi_{zs} = \frac{1}{T} \int_0^T \left\{ \frac{R^2}{\rho_c c} \int_0^{2\pi} 2 \Re\{p_i^r(\omega_r)p_i^s(\omega_s)\} \sin\theta d\theta d\phi \right\} dt \quad r \neq s \quad (B.5)
\]

Because \( p_i^r(\omega_r) = \overline{p_i^r} e^{i\omega t} \) and \( p_i^s(\omega_s) = \overline{p_i^s} e^{i\omega t} \), it is easy to show that

\[
\int_0^T \Re\{p_i^r(\omega_r)p_i^s(\omega_s)\} dt = 0 \quad \text{for} \quad s \neq r \quad (B.6)
\]

Therefore, the total sound power can be expressed as

\[
\Phi = \sum_{j=1}^{N_i} \left\{ \frac{R^2}{\rho_c c} \int_0^{2\pi} \int_0^{\pi/2} |p_i^r|^2 \sin\theta d\theta d\phi \right\} \quad (B.7)
\]
Appendix C

Sensitivity Analysis

In Section 2.5, the analytical formula about the sensitivity of the objective function to the changes in the design variables has been derived. Listed below are the expressions for some terms appearing in Eq. (2.56).

The first derivatives of the total sound power to the design variables \( z_s \) (\( z_s = x_s \), or \( z_s = y_s \)) have been shown in Equations (2.56), (2.57), (2.58), and (2.59) and are rewritten here:

\[
\frac{\partial \Phi_p}{\partial z_s} = \sum_{j=1}^{N_x} \left[ \frac{\partial \phi^j_p}{\partial z_s} \left( \frac{\partial \phi^j_p}{\partial V^j_q} \right)^T \left( \frac{\partial V^j_q}{\partial z_s} \right) \right] \tag{C.1}
\]

where

\[
\frac{\partial \phi^j_p}{\partial z_s} = 2 \left\{ f^i_{mn} \right\}^H \left[ E^j \right] \left\{ f^i_{kl} \right\} \tag{C.2}
\]

\[
\frac{\partial \phi^j_p}{\partial V^j_q} = 2 \left\{ \tilde{f}^i_{mn} \right\}^H \left[ E^j \right] \left\{ f^i_{kl} \right\} \tag{C.3}
\]

\[
\left\{ \frac{\partial V^j_q}{\partial z_s} \right\} = -[H^j]^{-1} \left\{ \frac{\partial [G^j]}{\partial z_s} + \frac{\partial [H^j]}{\partial V^j_q} \right\} \tag{C.4}
\]
where \([H^i] = [H^i_{pq}]\) and \(\{G^i\} = \{G^i_p\}\). Therefore,

\[
\frac{\partial H^i_{pq}}{\partial z_s} = \frac{\partial \left( \left\{ f_{pq} \right\}^H \left[ E^i_{mnk} \right] \left\{ \tilde{f}_{qip} \right\} \right)}{\partial z_s} = \left\{ \frac{\partial \tilde{f}_{mn}^{qi}}{\partial z_s} \right\}^H \left[ E^i_{mnk} \right] \left\{ \tilde{f}_{qip} \right\} + \left\{ f_{mn} \right\}^H \left[ E^i_{mnk} \right] \left\{ \frac{\partial \tilde{f}_{qip}}{\partial z_s} \right\}
\]

(C.5)

\[
\frac{\partial G^i_{L_s}}{\partial z_s} = f^i_d \left\{ \tilde{f}_{qip} \right\}^H \left[ E^i_{mnk} \right] \left\{ \frac{\partial \tilde{f}_{qip}}{\partial z_s} \right\}
\]

(C.6)

For the trial functions of the double sine forms, as expressed in Eq. (2.24), the first derivatives of the modal forces to the design variables are written as:

\[
\frac{\partial \tilde{f}_{mn}^{qi}}{\partial x_s} = \begin{cases} 
\frac{m\pi}{2} \frac{\cos\left(\frac{m\pi}{2} + \frac{m\pi x_p}{L_x}\right)}{L_x} \tilde{f}_{mn}^{qi} & \text{if } s = p \text{ and } z_s = x_s \\
\frac{n\pi}{2} \frac{\cos\left(\frac{n\pi}{2} + \frac{n\pi y_p}{L_y}\right)}{L_y} \tilde{f}_{mn}^{qi} & \text{if } s = p \text{ and } z_s = y_s \\
0 & \text{if } s \neq p
\end{cases}
\]

(C.7)

While for the trial functions of the polynomials, as shown in Eq. (2.30), the first derivatives of the modal forces to the design variables can be written as:
\[
\frac{d}{dt} \left[ \frac{L_y}{L_x} \frac{m(m-1)}{n+1} (\alpha_2^{m-2} - \alpha_1^{m-2}) (\beta_2^{n+1} - \beta_1^{n+1}) \right] + \frac{L_y}{L_x} \frac{n(n-1)}{m+1} (\alpha_2^{n+1} - \alpha_1^{n+1}) (\beta_2^{n-2} - \beta_1^{n-2}) \quad s = r \text{ and } z = x,
\]

\[
\frac{\partial \bar{F}_{\text{ipp}}}{\partial z_s} = \begin{cases} 
rd_1 \left[ \frac{L_y}{L_x} m(\alpha_2^{m-1} - \alpha_1^{m-1}) (\beta_2^n - \beta_1^n) \right] + \\
\frac{L_y}{L_x} \frac{n(n-1)}{m+1} (\alpha_2^{n+1} - \alpha_1^{n+1}) (\beta_2^{n-2} - \beta_1^{n-2}) \quad s = r \text{ and } z = y,
\end{cases}
\]

0 \quad s \neq p

(C.8)
Appendix D

Optimization Algorithm

The nonlinear optimization problem is solved through the use of the successive quadratic programming algorithm by means of an IMSL (IMSL, 1989) subroutine named N0ONF (or N0ONG).

The optimization problem is stated as follows:

$$\begin{align*}
\min & \quad f(\bar{x}) = f(x_1, x_2, \ldots, x_n) \\
\text{subject to} & \quad g_j(\bar{x}) = 0, \quad \text{for} \quad j = 1, \ldots, m_e \\
& \quad g_j(\bar{x}) > 0, \quad \text{for} \quad j = m_{e+1}, \ldots, m \\
& \quad \bar{x}_L \leq \bar{x} \leq \bar{x}_U
\end{align*}$$

(D.1)

where the objective function $f(\bar{x})$ and the constraint functions $g_j(\bar{x})$ are assumed to be continuously differentiable, $m_e$ is the number of the equality constraints, and $(m - m_e)$ is the number of the inequality constraints. The method is to form a sub-problem by using a quadratic approximation of the Lagrangian and linearizing the constraints as follows:

$$\begin{align*}
\min & \quad \frac{1}{2} \bar{d}^T \{B_k\} \bar{d} + \nabla f(\bar{x}_k)^T \bar{d} \\
\text{subject to} & \quad \nabla g_j(\bar{x}_k)^T \bar{d} + g_j(\bar{x}_k) = 0, \quad \text{for} \quad j = 1, \ldots, m_e \\
& \quad \nabla g_j(\bar{x}_k)^T \bar{d} + g_j(\bar{x}_k) > 0, \quad \text{for} \quad j = m_{e+1}, \ldots, m \\
& \quad \bar{x}_L - \bar{x}_k \leq \bar{d} \leq \bar{x}_U - \bar{x}_k
\end{align*}$$

(D.2)
where \( \{B_k\} \) is a positive definite approximation of the Hessian, \( \bar{x}_k \) is the current iterate, and \( \bar{d}_k \) is the solution of the sub-problem. A line search is used to find the new point \( \bar{x}_{k+1} \)

\[
\bar{x}_{k+1} = \bar{x}_k + \lambda \bar{d}_k, \quad 0 < \lambda < 1
\]  

(D.3)

Such that a merit function will have a lower value at the new point. The merit function is the augmented Lagrange function.
Appendix E

Relationship between the complex impedance $\zeta_{mkl}$ and the modal radiation coefficients $\delta_{mkl}$

In Chapters 2 and 3, the sound power radiated by plate is evaluated and used in optimization procedure. The modal radiation coefficients $\delta_{mkl}$ and complex impedance $\zeta_{mkl}$ by two totally different approaches. The objective of Appendix E is to show the relationship between the modal radiation coefficients $\delta_{mkl}$ and the complex impedance $\zeta_{mkl}$.

In Chapter 2, the modal radiation coefficients in Equation (2.46) are derived based on the radiated sound pressure in the acoustic field and can be rewritten in the following form:

$$\delta_{mkl} = \frac{\rho_s \omega^4}{8 c \pi^2} \int_0^{2\pi} \int_0^{\pi/2} I_{mn}(\theta, \varphi) I_{kl}^*(\theta, \varphi) \sin \theta d\theta d\varphi$$  \hspace{1cm} (E.1)$$

where

$$I_{mn}(\theta, \varphi) = \int_{-L_x/2}^{L_x/2} \int_{-L_y/2}^{L_y/2} \Phi_{mn}(x, y) e^{i(k_x x \sin \theta \cos \varphi + k_y y \sin \theta \cos \varphi)} dx dy$$  \hspace{1cm} (E.2)$$

with $l_1 = \frac{L_x}{2}$ and $l_2 = \frac{L_y}{2}$.

For simply supported plate, the trial function $\Phi_{mn}(x, y)$ can be chosen as the double sine of the form:
\[ \phi_{mn}(x, y) = \sin \left( \frac{m\pi}{2L_x} + \frac{m\pi x}{L_x} \right) \sin \left( \frac{n\pi}{2L_y} + \frac{n\pi y}{L_y} \right) \]  \hspace{1cm} (E.3)

If two new variables are defined

\[ k_x = k_0 \sin \theta \cos \varphi \]  \hspace{1cm} (E.4)

\[ k_y = k_0 \sin \theta \sin \varphi \]  \hspace{1cm} (E.5)

and substituted into Eq. (E.2), then \( I_{mn} \) can be expressed as the function of \( k_x \) and \( k_y \):

\[ I_{mn}(k_x, k_y) = \int_{-h}^{h} \int_{-l}^{l} \phi_{mn}(x, y)e^{i(k_x x + k_y y)} \, dx \, dy \]  \hspace{1cm} (E.6)

It is seen from Eq. (E.6) that \( I_{mn}(k_x, k_y) \) are a complex Fourier transform of \( \phi_{mn}(x, y) \) and the same as \( s_{mn}(k_x, k_y) \) defined in Eq. (3.12). Also, \( k_x \) and \( k_y \) are, respectively, the corresponding components of the wavenumber \( k_0 \) in \( x \) and \( y \) directions at the plate surface, as shown in Fig. E-1.

The differential areas \( d\theta \, d\varphi \) and \( dk_x \, dk_y \) can be related with use of the determination of the Jacobian:

\[ d\theta \, d\varphi = \left| \det J \right| dk_x \, dk_y \]  \hspace{1cm} (E.7)

where

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\[
[J] = \begin{bmatrix}
\frac{\partial \theta}{\partial k_x} & \frac{\partial \phi}{\partial k_x} \\
\frac{\partial \theta}{\partial k_y} & \frac{\partial \phi}{\partial k_y}
\end{bmatrix}
\] (E.8)

The determinant of the Jacobian \([J]\) can be derived with use of Eq. (E.4) and Eq. (E.5):

\[
\det[J] = \frac{1}{k_0^2 \cos \theta \sin \theta} = \frac{1}{\sin \theta} \frac{1}{k_0 \sqrt{k_0^2 - k_x^2 - k_y^2}}
\] (E.9)

The integral ranges for \(k_x\) and \(k_y\) are

\[
-k_0 \leq k_x \leq k_0 \quad (E.10a)
\]

\[
-k_0 \sqrt{k_0^2 - k_x^2} \leq k_y \leq k_0 \sqrt{k_0^2 - k_x^2} \quad (E.10b)
\]

Substituting Equations (E.6), (E.7) and (E.9) into Eq. (E.1) results in:

\[
\delta_{\text{mkl}} = \frac{\rho \omega^4}{8c\pi^2} \int_{-k_0}^{k_0} \int_{-\sqrt{k_0^2 - k_x^2}}^{\sqrt{k_0^2 - k_x^2}} I_{mn}(k_x, k_y) I_{kl}^*(k_x, k_y) dk_x dk_y
\] (E.11)

Comparing \(\delta_{\text{mkl}}\) in Eq. (E.11) with the real parts of \(\zeta_{\text{mkl}}\) defined in Eq. (3.14), it is seen that:

\[
\delta_{\text{mkl}} = \frac{\omega^2 L_x L_y}{8} \text{Re}\{\zeta_{\text{mkl}}\}
\] (E.12)
Figure E.1: Wavenumber decomposition
Appendix F

Contribution of the edge boundary condition to the Stiffness Matrix for Models G1 and G2

F.1 Contribution of the edge boundary condition to the Stiffness Matrix of Plate of Model G1

It is assumed in model G1 that all the boundary supports are perfectly simply-supported, except the regions around the broken screws. For those regions, a free edge support condition is assumed. Figure F.1 shows the geometric representation about model G1.

Figure F.1: Geometric representation of the local supports of model G1

For edge AB, the translational stiffness $k_s(\alpha,l)$ can be expressed as:

$$k_s(\alpha,l) = \begin{cases} 
  k_s & \text{for } -1 \leq \alpha \leq -a \text{ and } a \leq \alpha \leq 1 \\
  0 & \text{for } -a \leq \alpha \leq a 
\end{cases}$$  \hspace{1cm} (F.1)
The contribution of edge AB to the stiffness matrix is:

\[
k_{AB}^{\text{mod}} = \begin{cases} 
\frac{D}{L_x^2} \frac{\alpha^{\alpha+1} k_A}{m + k + 1} \left[ 2 - 2a^{n+k+1} \right] & \text{for } m + k = \text{even} \\
0 & \text{for } m + k = \text{odd}
\end{cases}
\]  

(F.2)

For edge CD, the translational stiffness \( k_{s}(\alpha,-1) \) is:

\[
k_{s}(\alpha,-1) = k_A \quad \text{for } -1 \leq \alpha \leq 1
\]  

(F.3)

The contribution of edge CD to the stiffness matrix can be written as:

\[
k_{CD}^{\text{mod}} = \begin{cases} 
\frac{D}{L_x^2} \frac{(-1)^{\alpha+1} k_A}{m + k + 1} & \text{for } m + k = \text{even} \\
0 & \text{for } m + k = \text{odd}
\end{cases}
\]  

(F.4)

For edge BC, the translational stiffness \( k_{s}(1,\beta) \) is assumed to be:

\[
k_{s}(1,\beta) = k_A \quad \text{for } -1 \leq \beta \leq 1
\]  

(F.5)

The contribution of edge BC to the stiffness matrix can be expressed as:

\[
k_{BC}^{\text{mod}} = \begin{cases} 
\frac{D}{L_x^2} \frac{(+1)^{n+k} k_A}{n + l + 1} & \text{for } n + l = \text{even} \\
0 & \text{for } n + l = \text{odd}
\end{cases}
\]  

(F.6)

For edge AD, the translational stiffness is assumed to be:
\[ k_{i}(-1, \beta) = \begin{cases} 
    k_A & \text{for } -1 \leq \beta \leq 0 \text{ and } b \leq \beta \leq 1 \\
    0 & \text{for } 0 \leq \beta \leq b
\end{cases} \quad (F.7) \]

The contribution of edge AD to the stiffness matrix can be written as:

\[ k_{mnkl}^{AD} = \begin{cases} 
    \frac{D L_y (1)^{m+k} k_A}{2 L_x^3} \left[ 2 - b^{n+l+1} \right] & \text{for } n + l = \text{even} \\
    \frac{D L_y (1)^{m+k} k_A}{2 L_x^3} \left[ -b^{n+l+1} \right] & \text{for } n + l = \text{odd}
\end{cases} \quad (F.8) \]

Finally, the contribution of the edges to the system stiffness matrix can be summarized as:

\[ k_{mnkl}^{edge} = \begin{cases} 
    \frac{D k_A}{L_x^2} \left[ \frac{(2 - a^{m+k+1})}{m + k + 1} + \frac{L_y (4 - b^{n+l+1})}{2 L_x (n + l + 1)} \right] & \text{for } m + k = \text{even and } n + l = \text{even} \\
    \frac{D k_A}{L_x^2} \left[ \frac{(2 - a^{m+k+1})}{m + k + 1} + \frac{L_y (4 - b^{n+l+1})}{2 L_x (n + l + 1)} \right] & \text{for } m + k = \text{even and } n + l = \text{even} \\
    \frac{D k_A}{L_x^2} \left[ \frac{L_y (-b^{n+l+1})}{2 L_x (n + l + 1)} \right] & \text{for } m + k = \text{odd}
\end{cases} \quad (F.9) \]
F.2 Contribution of the edge boundary condition to the Stiffness Matrix of Plate of Model G2

In model G2, it is assumed that all the boundaries are free edges, except the areas close to the broken screws. For those regions, a perfectly simply-supported condition is assumed. Figure F.2 shows the geometric representation of model G2.

![Diagram of model G2 with labels for dimensions and forces](image)

Figure F.2: Geometric Representation of the local supports of model G2

For edge AB, the translational stiffness \( k_s(\alpha,1) \) can be expressed as:

\[
 k_s(\alpha,1) = \begin{cases} 
 k_A & \text{for } -d \leq \alpha \leq -c \text{ and } c \leq \alpha \leq d \\
 -b \leq \alpha \leq -a \text{ and } a \leq \alpha \leq b \\
 -1 \leq \alpha \leq -d \text{ and } d \leq \alpha \leq 1 \\
 0 & \text{for } -c \leq \alpha \leq -b \text{ and } b \leq \alpha \leq c \\
 -q \leq \alpha \leq q 
\end{cases} \tag{F.10}
\]
The contribution of edge AB to the stiffness matrix can be written as:

\[
k_{AB} = \begin{cases} 
\frac{D}{L^2} \frac{(-1)^{n+1}}{m+k+1} k_A \left[ d^{m+k+1} + b^{m+k+1} - c^{m+k+1} - a^{m+k+1} \right] & \text{for } m+k = \text{even} \\
0 & \text{for } m+k = \text{odd}
\end{cases}
\]

(F.11)

For edge CD, the translational stiffness can be expressed as:

\[
k_s(\alpha, -1) = \begin{cases} 
-1 \leq \alpha \leq -d & \text{and} & d \leq \alpha \leq 1 \\
0 & \text{for } -c \leq \alpha \leq -b & \text{and} & b \leq \alpha \leq c \\
-a \leq \alpha \leq -q & \text{and} & q \leq \alpha \leq a \\
-d \leq \alpha \leq -c & \text{and} & c \leq \alpha \leq d \\
k_A & \text{for } -b \leq \alpha \leq -a & \text{and} & a \leq \alpha \leq b \\
-q \leq \alpha \leq q
\end{cases}
\]

(F.12)

The contribution of edge CD to the plate stiffness matrix can be written as:

\[
k_{CD} = \begin{cases} 
\frac{D}{L^2} \frac{(-1)^{n+1}}{m+k+1} k_A \left[ d^{m+k+1} + b^{m+k+1} - a^{m+k+1} - c^{m+k+1} + q^{m+k+1} \right] & \text{for } m+k = \text{even} \\
0 & \text{for } m+k = \text{odd}
\end{cases}
\]

(F.13)

For edge BC, the translational stiffness can be expressed as:

\[
k_s(1, \beta) = \begin{cases} 
-1 \leq \beta \leq -h & \text{and} & h \leq \beta \leq 1 \\
0 & \text{for } -g \leq \beta \leq -f & \text{and} & f \leq \beta \leq g \\
-e \leq \beta \leq -p & \text{and} & p \leq \beta \leq e \\
-h \leq \beta \leq -g & \text{and} & g \leq \beta \leq h \\
k_A & \text{for } -f \leq \beta \leq -e & \text{and} & e \leq \beta \leq f \\
-p \leq \beta \leq p
\end{cases}
\]

(F.14)

The contribution of edge BC to the plate stiffness matrix is:
\[
\begin{align*}
K_{mnl}^{CD} &= \begin{cases} 
\frac{D}{L_x^2 n + l + 1} \left[ e^{n+l+1} + f^{n+l+1} - g^{n+l+1} - e^{n+l+1} + p^{n+l+1} \right] & \text{for } n + l = \text{even} \\
0 & \text{for } n + l = \text{odd}
\end{cases} 
\end{align*}
\]  
\text{(F.15)}

For edge AD, the translational stiffness is assumed to be of the following form:

\[
k_z(-1, \beta) = \begin{cases} 
-1 \leq \beta \leq -h \text{ and } h \leq \beta \leq 1 \\
0 & \text{for } -g \leq \beta \leq -f \\
-e \leq \beta \leq -p \text{ and } p \leq \beta \leq g \\
k_A & \text{for } -h \leq \beta \leq -g \text{ and } g \leq \beta \leq h \\
-f \leq \beta \leq -e \text{ and } -p \leq \beta \leq p
\end{cases}
\]  
\text{(F.16)}

The contribution of edge AD to the plate stiffness matrix is:

\[
\begin{align*}
K_{mnl}^{CD} &= \begin{cases} 
\frac{D}{2L_x^2 n + l + 1} k_A \left[ 2e^{n+l+1} + f^{n+l+1} - 2g^{n+l+1} - e^{n+l+1} + 2p^{n+l+1} \right] & \text{for } n + l = \text{even} \\
\frac{D}{L_x^2 n + l + 1} k_A \left[ e^{n+l+1} - f^{n+l+1} \right] & \text{for } n + l = \text{odd}
\end{cases} 
\end{align*}
\]  
\text{(F.17)}
Vita

Tao Song was born in Zhangjiakou City, Hebei Province, People's Republic of China on December 23, 1962. He grew up in Beijing. He attended Beijing University of Aeronautics and Astronautics in 1980. He received his Bachelor of Science in Mechanical Engineering degree in 1984, and his Master of Science in Mechanical Engineering degree in 1987 from the Department of Jet Propulsion of the same university. After doing research work in his old school for two and half years, he enrolled in the graduate school at Virginia Polytechnic Institute and State University, Blacksburg, Virginia in August, 1989. In January, 1995, he completed the requirements for the degree of Doctor of Philosophy in Mechanical Engineering.

Tao Song