Appendix D

Solutions to $f_A(m) - c_\alpha$ and corresponding MRAs

Assume the number of blocks is equivalent to the number of treatments, and the ranks are in a Latin Square configuration.   We can approximate the sum of squared rank sums using equation (5.5), restated below.

$$f_A(m) = \sum_{j=1}^{t} R_{.j}^2 \approx \frac{t}{6}\left(4t\sqrt{m} + 8m^{3/2} - 6m(1+t^2) + t^2(1+3t+2t^2)\right).$$

Define the critical constant $c_\alpha = \frac{\chi^2_{(t-1),\alpha} + 3t(t+1)}{12}\left[t^2(t+1)\right]$.

Then the solution to $f_A(m) - c_\alpha$ which approximates the number of contaminants to force an acceptance is

$$
\begin{aligned}
A_t^F = {} & \frac{9+2t^2+9t^4}{48t^2} + \\
& \Big(\big(1 - I\sqrt{3}\big)\big(-1296t^4 - 576t^6 + 2720t^8 + 6912t^9 + 6336t^{10} + 6912t^{11} + 3312t^{12} - 13824t^5 c_\alpha - 13824t^7 c_\alpha\big)\Big) \Big/ \\
& \Big(192\,2^{2/3} t^4 \big(128t^6 (9+2t^2+9t^4)^3 - 9216t^7 (9+2t^2+9t^4)(7t^3+9t^4+9t^5+9t^6+6t^7-18c_\alpha-18t^2c_\alpha) - \\
& \quad 110592t^8(-t^6-6t^7-13t^8-12t^9-4t^{10}+12t^3c_\alpha+36t^4c_\alpha+24t^5c_\alpha-36c_\alpha^2) + \\
& \quad 6144\sqrt{6}\,t^{15/2}(3t+4t^3+3t^4+2t^5-6c_\alpha)\sqrt{(-45t^3-81t^4+t^5+81t^6+117t^7+405t^8+729t^9+} \\
& \qquad \overline{567t^{10}+162t^{11}+162c_\alpha-162t^2c_\alpha-810t^4c_\alpha-1944t^5c_\alpha-1134t^6c_\alpha+1944tc_\alpha^2)}\big){}^\wedge \\
& \quad (1/3)\Big) - \\
& \frac{1}{384\,2^{1/3}t^4} \\
& \Big(\big(1 + I\sqrt{3}\big)\big(128t^6(9+2t^2+9t^4)^3 - 9216t^7(9+2t^2+9t^4)(7t^3+9t^4+9t^5+9t^6+6t^7-18c_\alpha-18t^2c_\alpha) - \\
& \quad 110592t^8(-t^6-6t^7-13t^8-12t^9-4t^{10}+12t^3c_\alpha+36t^4c_\alpha+24t^5c_\alpha-36c_\alpha^2) + \\
& \quad 6144\sqrt{6}\,t^{15/2}(3t+4t^3+3t^4+2t^5-6c_\alpha)\sqrt{(-45t^3-81t^4+t^5+81t^6+117t^7+405t^8+729t^9+} \\
& \qquad \overline{567t^{10}+162t^{11}+162c_\alpha-162t^2c_\alpha-810t^4c_\alpha-1944t^5c_\alpha-1134t^6c_\alpha+1944tc_\alpha^2)}\big){}^\wedge \\
& \quad (1/3)\Big)
\end{aligned}
$$

and the approximate maximum resistance to acceptance is

$$\alpha_t^F \approx \frac{\left[A_t^F + 1\right]}{t^2},$$

where [.] is the greatest integer function.

Assume the number of blocks is equivalent to a constant multiple of the number of treatments, and the ranks are in a Latin Square configuration. We can approximate the sum of squared rank sums using equation (5.6), restated below.

$$f_A(m) = \sum_{j=1}^{t} R_{.j}^2 \approx \frac{4}{3} mnt^2 \sqrt{m/n} + \frac{1}{6} n^2 t^2 \sqrt{m/n} - mnt^3 + \frac{n^2 t^3}{6} + \frac{n^2 t^4}{2} + \frac{n^2 t^5}{3}.$$

Define the critical constant $c_\alpha = \dfrac{\chi^2_{(t-1),\alpha} + 3t(t+1)}{12}\left[t^2(t+1)\right]$.

Then the solution to $f_A(m) - c_\alpha$ which approximates the number of contaminants to force an acceptance is (here $c = c_\alpha$)

$$\begin{aligned}
A^F_{nt} = {} & \frac{1}{48} n\left(-4 + 9t^2\right) + \\
& \Big(\left(1 + I\sqrt{3}\right)\left(-13824 c n^2 t^7 - 64 n^4 t^8 + 3456 n^4 t^{10} + 6912 n^4 t^{11} + 3312 n^4 t^{12}\right)\Big) \Big/ \Big(192\, 2^{2/3} n t^4 \\
& \quad \Big(128 n^6 t^{12} \left(-4 + 9t^2\right)^3 - 2304 n^4 t^{11} \left(-4 + 9t^2\right)\left(-72c + n^2 t + 12 n^2 t^3 + 36 n^2 t^4 + 24 n^2 t^5\right) - 110592 n^2 \\
& \qquad t^8 \left(-36c^2 + 12 c n^2 t^3 + 36 c n^2 t^4 + 24 c n^2 t^5 - n^4 t^6 - 6 n^4 t^7 - 13 n^4 t^8 - 12 n^4 t^9 - 4 n^4 t^{10}\right) + \\
& \qquad 768\sqrt{3}\, n^2 t^8 \left(24c - 7 n^2 t^3 - 12 n^2 t^4 - 8 n^2 t^5\right) \sqrt{\big(15552 c^2 - 6480 c n^2 t^3 - 15552 c n^2 t^4 +} \\
& \qquad\quad \overline{8 n^4 t^4 - 9072 c n^2 t^5 + 639 n^4 t^6 + 3240 n^4 t^7 + 5832 n^4 t^8 + 4536 n^4 t^9 + 1296 n^4 t^{10}\big)}\Big) \,\hat{}\, \\
& \quad (1/3)\Big) - \\
& \frac{1}{384\, 2^{1/3} n t^4} \Big(\left(1 - I\sqrt{3}\right) \\
& \quad \Big(128 n^6 t^{12} \left(-4 + 9t^2\right)^3 - 2304 n^4 t^{11} \left(-4 + 9t^2\right)\left(-72c + n^2 t + 12 n^2 t^3 + 36 n^2 t^4 + 24 n^2 t^5\right) - 110592 n^2 \\
& \qquad t^8 \left(-36c^2 + 12 c n^2 t^3 + 36 c n^2 t^4 + 24 c n^2 t^5 - n^4 t^6 - 6 n^4 t^7 - 13 n^4 t^8 - 12 n^4 t^9 - 4 n^4 t^{10}\right) + \\
& \qquad 768\sqrt{3}\, n^2 t^8 \left(24c - 7 n^2 t^3 - 12 n^2 t^4 - 8 n^2 t^5\right) \sqrt{\big(15552 c^2 - 6480 c n^2 t^3 - 15552 c n^2 t^4 + 8 n^4 t^4 -} \\
& \qquad\quad \overline{9072 c n^2 t^5 + 639 n^4 t^6 + 3240 n^4 t^7 + 5832 n^4 t^8 + 4536 n^4 t^9 + 1296 n^4 t^{10}\big)}\Big) \,\hat{}\, \\
& \quad (1/3)\Big)
\end{aligned}$$

and the approximate maximum resistance to acceptance is

$$\alpha^F_{nt} \approx \frac{\left[A^F_{nt} + 1\right]}{nt^2},$$

where [.] is the greatest integer function.

The expression for the Friedman maximum resistance to acceptance for a given number of treatments, *t*, is written as

$$\lim_{n\to\infty} MRA = g(t) =$$

$$\frac{1}{96t^6}\left(\begin{array}{l} 2t^4\left(9t^2-4\right)+I\left(I+\sqrt{3}\right)\left(t^{11}\left(8t+54t^3-27t^7-6\left(1+2t^2\right)\sqrt{24t^4-27t^6}\right)\right)^{1/3} \\ -\dfrac{6I\left(-I+\sqrt{3}\right)\left(1+2t^2\right)\sqrt{24t^4-27t^6}\left(t^{11}\left(8t+54t^3-27t^7-6\left(1+2t^2\right)\sqrt{24t^4-27t^6}\right)\right)^{2/3}}{t^5(9t^4+4)^2} \\ +\dfrac{\left(1+I\sqrt{3}\right)\left(-8+27t^2\left(t^4-2\right)\right)\left(t^{11}\left(8t+54t^3-27t^7-6\left(1+2t^2\right)\sqrt{24t^4-27t^6}\right)\right)^{2/3}}{t^4(4+9t^4)^2} \end{array} \right)$$