Appendix E

Observability for the Proposed Topology Error Identification Method

For power engineers, to say that a network is observable is equivalent to say that every unknown parameter is estimable. For the topology error identification scheme developed in Chapter 3, we would like to quickly determine if a set of measurements 'observes' the network. As mentioned in Chapter 9, one obvious way to verify observability is to check if the model matrix, X, is of full rank via the determinant of $X'X$. A nonzero determinant implies that the network is observable and equivalently that every unknown quantity is estimable. However, taking a determinant of a (p x p) matrix where p is very large can be computationally intensive. Here, p represents the number of unknown quantities that need to be estimated, which for the topology error methodology equals the total number of branches in the network. And for a network with hundreds of buses, the number of branches can range in the thousands, which implies that taking a determinant is not feasible for real time applications.

A necessary condition for a network to be observable may be derived by making use of graph theory, since any matrix can be represented by an associated graph. It leads to the concept of topological observability, which provides only a necessary condition because the parameters of the lines are not accounted for. Graph theory has two distinct advantages: 1) it is computationally faster than taking a determinant, which is of primary importance to power engineers, and 2) it is more informative than a determinant. Not only will graph theory show if a network is unobservable, but also will show which largest portion of the network is observable. Since we are not really interested in the actual value of the determinant, only whether it is zero or nonzero, the graph theoretic approach becomes a very effective and practical tool.

To invoke graph theory, we need a rule as to what determines topological observability of a network. For this method, we are interested in the true unknown power flow on each branch in the system. In general, if a branch has an observation associated with it, then that branch is observable. More specifically, there are two ways that a branch is observable; either with a flow measurement directly on the branch, or by an injection measurement at one of the adjacent buses. Both of these give information about the true power on that branch, either directly with the flow measurement, or indirectly with the injection measurement by using Kirchhoff's Current Law. As an example, consider a 5-bus network depicted in Figure E.1. There are seven branches, thus seven unknown power flows that need to be estimated. All but one of the branches has a measurement directly associated with it, which is Branch 2-4. However, there is an injection associated with Bus 2, which can be assigned to Branch 2-4. The red arrows in Figure E.1 label a measurement assignment for each branch. Hence, this network is observable.

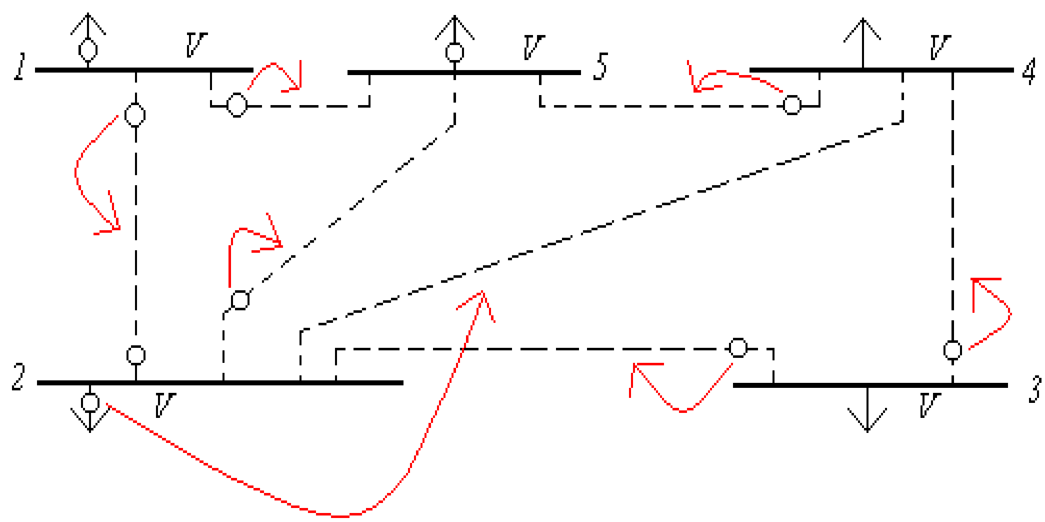

Figure E.1. An Observable 5-Bus System

However, the assignment of a measurement to each one of the branches does not insure that the network is observable, due to Kirchhoff's Current Law. Kirchhoff's Current Law creates natural dependencies among the measurements, which creates natural dependencies within some of the rows of the model matrix. As an example, consider a simple 3-bus system in Figure E.2.

Here, there are three branches and three injections with each injection assigned to a branch. However, this network is unobservable since one injection is a linear combination of the other two via KCL. Thus there needs to exist at least one power flow measurement to observe this system.

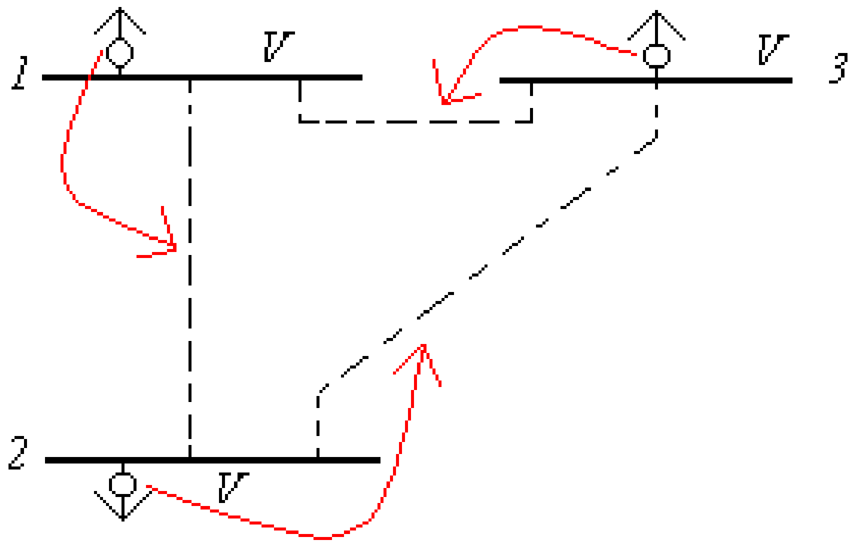

Figure E.2. An Unobservable 3-Bus System

The loop created by the power lines in Figure E.2 is also an example of a *cycle* in graph theory, or a window or *mesh* in power engineering. Within a given network of buses, many meshes are created, with some meshes encompassing others. For observability purposes, we would like to decompose the network into the largest set of unique meshes, with the union of all meshes producing the network. Once this is accomplished, we can check whether each mesh is observable. Figure E.3 identifies the three unique, observable windows for the 5-bus system in Figure E.1. Again, we are interested in the largest set of unique windows, since by definition of a cycle in graph theory, the union of cycle 1 and cycle 2 is itself a cycle.

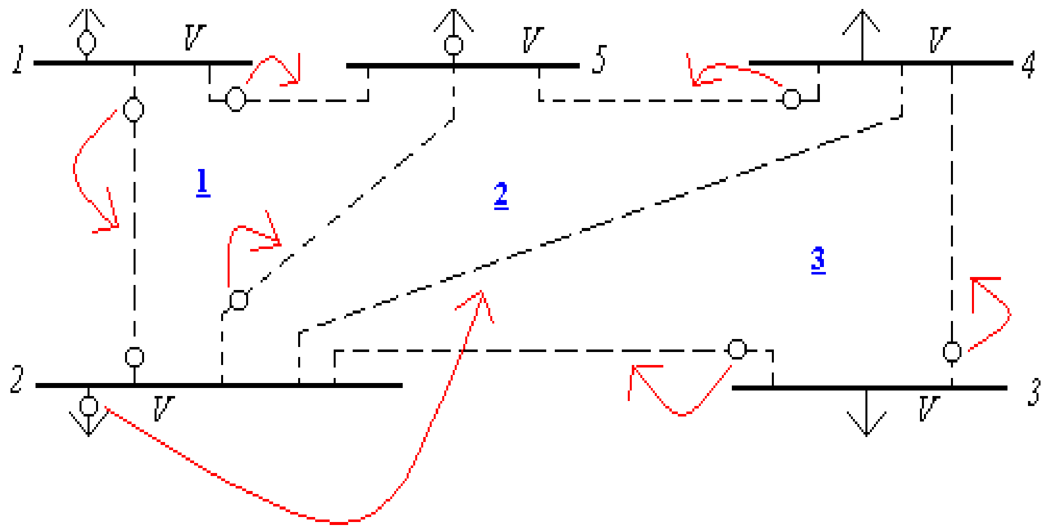

Figure E.3. Identification of Meshes in 5-Bus System

Therefore for the topology error identification method described in Chapter 3, we conjecture that for a given network decomposed into the largest set of w unique windows, a necessary and sufficient condition that the network is topologically observable is that each window j, $j = 1,\ldots w$, is observable. We say window j is observable if and only if each branch has a measurement associated with it, with at least one of the measurements being a power flow. At this time, this is stated without proof.

Assuming that this conjecture holds, there is still a question of implementation. Specifically, the question of where to optimally assign the injection measurements, such that we observe the largest portion of the network, is still an issue. However, we also conjecture that the assignment of injections based on the augmenting sequence proposed by Nucera and Gilles (1991) will easily alleviate this problem. Once this is achieved, coding this in a program such as FORTRAN will be rudimentary and will make a nice addition to the topology error identification program already developed by Greg Steeno.