Chapter 2

POWER NETWORKS, STATE ESTIMATION, & TOPOLOGY ERROR

§ 2.1 Power Networks

Simply described, a power system consists of generators connected to consumers (known as loads) through a network of power lines and transformers. From graph theory, a point in a network where several lines connect is a called a *node*, and in power systems engineering this is commonly called a *bus* (as well as a node). One typically can see these power lines draped from pole to pole along a city or country road, although it is also common that these cables operate underground. In addition, it is a set of three power lines that connect a set of three buses to the next set, one line for each bus. However, in representations of power systems, just one single line and one single bus usually represent these three lines and buses since they are balanced. (*Balanced* means that for the three power lines, each line connected to a bus, that they have the same voltage and current magnitude, and same power). We will assume this throughout this research. At the buses, there are manually or remotely operated circuit breakers that activate or deactivate a line. If a circuit breaker is open then the line is off or deenergized, otherwise a circuit breaker is closed and the line is on or energized. The status of a circuit breaker is usually telesignaled back to a control center, or reported to a control center by someone in the field. The operation of the network is monitored by a control center that maintains the power system at a normal and secure operating state and the role of a power network is to supply electric energy through a region whose demand fluctuates continually and randomly. A very simple network with five buses and seven power lines is depicted in Figure 2.1. A flat, dark bar represents a bus and the dashed lines represent the power lines, and these power lines connect one bus to another, thus forming the network. The arrows on the buses are *injections* of power, either into the system or from the system, the circles are power measurements, which are taken on a power line (called flows), or at a bus (called injections), and the *V*'s represent voltage magnitude measurements. Each power measurement, either a flow on the line or an injection at the bus, is a pair of values; a real power, *P*, and a reactive power, *Q* (to be described later). The power flows are only at the *end* of each line implying that there are at most four measurements per line (two real and two reactive). It is these power and voltage measurements that are used to estimate the state of the network.

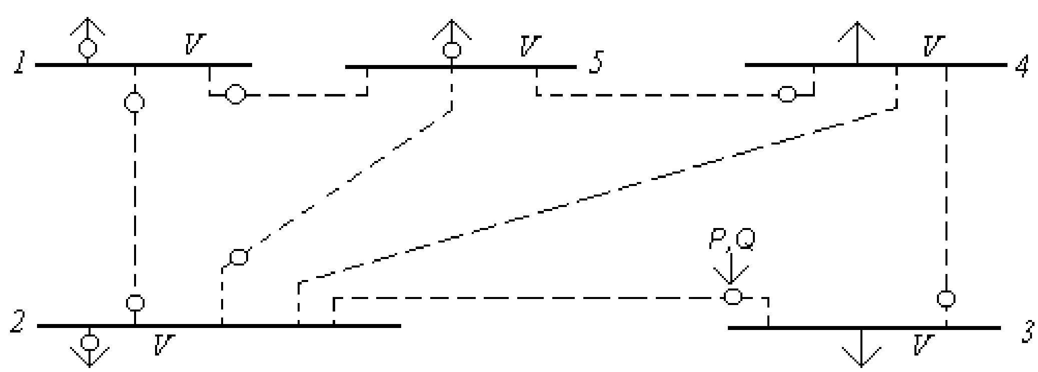


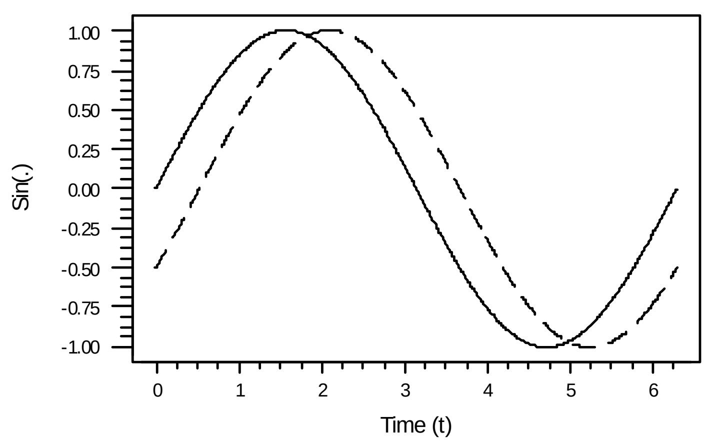
Figure 2.1. IEEE Standard 5-Bus System

§ 2.2 State Estimation

Before describing what the state of a network is, some background information on electrical power is necessary. The voltage in a power network is sinusoidal in nature, and the instantaneous voltage at a time *t*, say, is represented by the equation

$$v_i(t) = V_i \, Sin(\omega t + \theta_{ij}) , \tag{2.1}$$

where V_i is the voltage at a particular bus *i*, ω is the angular frequency which can be rewritten as $2\pi f$, *f* is the frequency in hertz (e.g. 60 Hz), and $\theta_{ij} = \theta_i - \theta_j$ is the phase angle difference in radians between two voltage sine curves at bus *i* and bus *j*. As an example, two voltage sine curves for two different buses can be represented by Figure 2.2 (letting $V_i = 1$ and $\omega = 1$).



Note: $\theta_i = 0$ is solid line, $\theta_j = -\pi / 6$ is dashed line.

Figure 2.2. Plot of Two Voltage Sine Curves

The abscissa is time (t), and the ordinate is $sin(t+\theta)$, where $\theta_i = 0$ radians for the solid line and $\theta_j = -\pi/6$ radians for the dashed line (thus the angular difference in magnitude is $\theta_{ij} = \pi/6$). From (2.1), the unknown quantities are the voltage magnitude at bus i, V_i, and the voltage angle at bus i, θ_i (i=1,…, n for the n buses).

In the model used in power system static state estimation, the generators and loads are represented as constant power injections. Again, there are two types of power, the real power and the reactive power. The power flowing through the network can be described mathematically as well. There are two types of power flows that are measured, real power (denoted by P) and reactive power (denoted by Q). Both powers in per unit are mathematically proportional to

$$P_i \propto V_i I_i \cos(\phi_i) \quad \text{and} \quad Q_i \propto V_i I_i \sin(\phi_i),$$

where V_i is the voltage at bus i, I_i is the current at bus i, and ϕ_i is the angle difference between the voltage and current. It can be shown that the current I_i and ϕ_i can be written as a function of the voltage V_i and θ_i (Appendix A). As with the instantaneous voltage described above, the unknowns are the voltage at bus i, V_i, and the phase angle at bus i, θ_i. In fact, all other equations related to describing the network are in terms of $\underline{V}$ and $\underline{\theta}$, and if these are known, then every quantity about the network is known. Thus, the purpose of state estimation is to statistically estimate these unknown quantities with considerable robustness.

In classical static state estimation, the set of flow and injection measurements is related to the vector of unknowns ($\underline{V}$ and $\underline{\theta}$) through a set of nonlinear equations. The vector of unknowns is called the state vector and for a network of N buses, there are 2N - 1 unknowns; N

bus voltage magnitudes and N-1 bus voltage angles. There are N-1 angles because one is arbitrarily set to zero, since we are only interested in relative differences in angles between buses. Power engineers express the nonlinear equation as

$$\underline{z} = \underline{h}(\underline{x}) + \underline{e}\,, \tag{2.2}$$

where $\underline{z}$ is the vector of measurements, $\underline{h}(\underline{x})$ is the nonlinear vector function relating the measurements to the state vector, and $\underline{e}$ is the measurement error vector. What is commonplace in state estimation is to solve for $\underline{x}$ through a weighted least squares algorithm (WLS). That is, let $J(\underline{x})$ be the objective function of the residuals that needs to be minimized. For the WLS approach,

$$J(\underline{x}) = \frac{1}{2}[\underline{z} - \underline{h}(\underline{x})]' R^{-1} [\underline{z} - \underline{h}(\underline{x})]\,,$$

where $\underline{z}$ and $\underline{h}(\underline{x})$ are defined earlier, and R is a diagonal matrix of given variances for each measurement. The assumption of known variances is not unreasonable since the measurements are metered with instrumentation having a known tolerance or accuracy level. The solution $\underline{\hat{x}}$ that minimizes $J(\underline{x})$ must satisfy

$$\left.\frac{\partial\, J(\underline{x})}{\partial \underline{x}}\right|_{\underline{x}=\underline{\hat{x}}} = -H'(\underline{\hat{x}}) R^{-1} [\underline{z} - \underline{h}(\underline{\hat{x}})] = \underline{0}\,,$$

where $H(\hat{x})$ is called the measurement Jacobian matrix such that

$$H(\underline{\hat{x}}) = \left.\frac{\partial \underline{h}(\underline{x})}{\partial \underline{x}}\right|_{\underline{x}=\underline{\hat{x}}}.$$

Since this is a set of nonlinear equations, the solution $\underline{\hat{x}}$ is usually solved iteratively until convergence via a Newton-Raphson algorithm or some variant. For a complete explanation on the state estimation model and estimation technique, see Schweppe and Wildes (1970), or for a synopsis, see Bose and Clements (1987).

One drawback of the conventional WLS solution is its sensitivity to discordant measurements or outliers, which occur frequently in state estimation. VanSlyck and Allemong (1988) noted that during a 13 year span at American Electric Power, only 5 days were free of outliers and that on the average, 1% to 2% of the data were indeed outliers. These bad data are usually due to model approximations, rounding errors, gross measurement errors, parameter errors (*parameters* in power engineering are known values associated with a particular line or transformer, such as the resistance, reactance or capacitance), and/or topology errors. Due to the sensitivity of the WLS estimates in the presence of outliers, bad data detection on the normalized residuals is employed. Unfortunately, the weighted least squares residuals are prone to a masking effect of multiple bad data. To alleviate this problem, robust techniques are used to minimize the effect of outlying measurements. First advocated in power systems by Merrill and Schweppe (1971), the use of M-estimators became popular due to the downweighting of bad data that is inherent in M-estimation. They encouraged the use of robust estimators due to the bias in the measurements via poor instrumentation, and derived their own M-estimator that is quadratic for small residuals, and of order of a square root for large residuals. However, these techniques had no real time application because they had problems with leverage points. Although M-estimators are robust with respect to outlying data in the response space, they are still apt to have problems in the presence of outlying data in the design space, called leverage points. Leverage points are common in power systems, usually due to flow measurements on short lines or injection measurements at a bus having several incident lines (Mili *et al*. 1990). One way to cope with this problem is to use high breakdown estimators, such as the Least

Median of Squares estimator (LMS). However, estimators, such as the LMS, which are able to handle 'large' amounts of bad data are for the most part very computationally intensive, somewhat impractical, and are not nearly as locally robust in a structured regression model (Mili and Coakley, 1996). A structured regression model has natural or inherent linear dependencies in the rows of the design matrix, which is common in linearized power system models. Because of the sparse, structured design matrix, the high global robustness of these types of estimators is not of great value since the actual robustness is so small. In addition, even though high breakdown estimators such as the LMS and LTS can achieve the maximum global robustness under the structured regression model when properly tuned, these estimators can still break down on a local level. Thus, a more relevant concept is that of the local breakdown point. Work has been done in the area of local robustness (Mili *et al.*, 1994), but the question is still open as to how to implement a high breakdown estimator on a local level to retain its robustness, under a highly structured regression model. Finally, none of the estimators mentioned (WLS, M-, and LMS-type) are robust in the presence of topology errors in the network, the most serious error.

§ 2.3 Topology Error Identification

Topology errors translate into a number of local bad measurements that are conforming. Because of the many centralized discordant measurements that accompany topology errors, any estimator is going to break down because it cannot distinguish where the error occurs. As an example, Figure 2.3 shows a network where a topology error has occurred, depicted in red, on the line 1-2. Because that line has been classified incorrectly, the measurements associated with that line appear as outliers from the design space, also depicted in red, even though they may be perfectly good measurements. In addition, measurements on the adjacent buses appear as outliers from the design space. When this is the case, any estimator will break down in that area of the network when trying to estimate the voltages and phase angle differences. The estimator cannot discriminate between good data and bad data in that area of the network. As mentioned previously, there have been basically three approaches to identify topology errors in a power network; rule-based procedures, robust procedures, and equality constraint procedures. We will look at each type of procedure in more detail and comment on both advantages and disadvantages of each type.

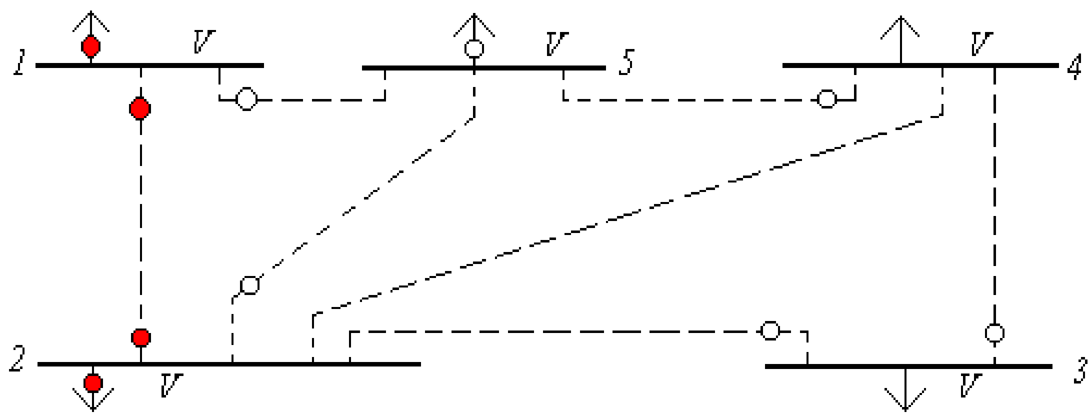


Figure 2.3. IEEE Standard 5-Bus System with Topology Error

§ 2.3.1 Rule-Based Methods

Singh and Oesch (1994) propose a method of topology error identification that does not involve the usual number crunching that is intrinsic in any data analysis. They mention that after the state estimator has been fine tuned to a system, and has been running for a while, that the only errors that occur are in mislabeled circuit breakers. However, an experienced engineer should be able to look at the data, neighboring switch positions, etc., and be able to tell if a reported switch position concurs with what he/she sees in the data. Thus, the rule-based algorithm emulates the experienced engineer by looking at changes in breaker positions, going through a list of applicable rules based on causal principle, and assigns a TRUE, FALSE, or UNDECIDED to the switch position in question based on a majority versus minority point system. Approximately 120 rules are available, not all for every scenario, and are broken into four groups as:

1. Piecewise information available for switch status
2. Interdependent information in present consistent set
    a. Dependence of switch position and measurements
    b. Dependence of switch position to other switch positions.
3. Time dependent relationships
4. Additional information from the specific network not covered above.

This method is very simplistic, simple to implement, robust, and reliable under varying operating conditions. However, the authors mentioned some trouble with bus couplers (short circuits that may split a bus into multiple sub-buses) and with zero injections (buses that neither inject power into nor out of the system, which are data without error).

§ 2.3.2 Robust Topology Error Detection

In theory, given enough redundancy and given a robust estimator, the outlying measurements associated with a faulty branch should be rejected and the state estimates should be fairly accurate. *Redundancy* is a measure of the number of observations versus the number of state variables that need to be estimated. Typically, the measurement redundancy is low, implying that just a single mislabeled branch, which would lead to multiple discordant data, would affect the state estimate drastically. Abur *et al.* (1995) proposed a two step plan using the weighted least absolute value estimator (WLAV) that first identifies gross measurements and suspect lines associated with these measurements and then identifies the line whose status is most likely incorrect. Considering the nonlinear equation $\underline{z} = \underline{h}(\underline{x}) + \underline{e}$, the first order Taylor series expansion of $\underline{h}(\underline{x})$ around some point $\underline{x}_0$ can be approximately written as:

$$\Delta\underline{z} = H \cdot \Delta\underline{x} + \underline{e}, \tag{2.3}$$

where $\Delta\underline{z} = \underline{z} - \underline{h}(\underline{x}_0)$, $\Delta\underline{x} = \underline{x} - \underline{x}_0$, and H, $\underline{e}$ are defined as earlier. The WLAV finds the solution $\hat{\underline{x}}$ that minimizes

$$J(\underline{x}) = \sum_{i=1}^{m} w_i |r_i|,$$

where $r_i = z_i - h_i(x)$, w_i is the corresponding weight for the i^{th} residual, and m is the total number of observations. The authors modify (2.3) by adding an additional error vector, since the errors are biased in the presence of topology errors, making the equation now

$$\Delta\underline{z} = H \cdot \Delta\underline{x} + M \cdot \Delta\underline{f} + \underline{e},$$

where M is an $(m \text{ x } b)$ measurement to circuit breaker incidence matrix, such that if measurement i is an injection, then

$$M_{i,j} = \begin{cases} 1 & \text{if the injection is at the to - end of breaker } j \\ -1 & \text{if the injection is at the from - end of breaker } j \\ 0 & \text{else} \end{cases}, \tag{2.4}$$

and if the i^{th} measurement is a flow,

$$M_{i,j} = \begin{cases} -1 & \text{if the flow is at the to-end of breaker } j \\ 1 & \text{if the flow is at the from-end of breaker } j \\ 0 & \text{else} \end{cases}, \tag{2.5}$$

and $\Delta \underline{f}$ is the vector of power flows through the circuit breakers, which is identically $\underline{0}$ if all circuit breakers are open. To describe what the *to-end* and *from-end* are, as mentioned in (2.4) and (2.5), assume a branch links bus *a* to bus *b*. A direction is chosen as to which way the current is flowing (this direction is arbitrary). If the current is flowing from *a* to *b*, then a measurement of the *a* side of the branch is the *from-end* (since the current is coming from *a* to *b*), and a measurement on the *b* side is the *to-end*. On each side of the branch is a circuit breaker, thus we can assign a *to-end* and *from-end* circuit breaker. The scheme is to first estimate the state, then identify large normalized residuals and the corresponding buses associated with those residuals. The state vector is then expanded to include the short circuit lines at the substation level whose status is in doubt. The algorithm is rerun on the entire system, estimating now the extended state vector. Next, the magnitudes of $\Delta \underline{\hat{f}}$ are used to determine the breakers most likely deemed incorrectly. A statistical test is then employed on the elements of $\Delta \underline{\hat{f}}$ using the asymptotic covariance matrix of the WLAV estimator. Elements of $\Delta \underline{\hat{f}}$ that are significantly higher than a critical threshold imply that a circuit breaker is closed. This is a more robust procedure than the conventional weighted least squares approach, but still suffers from possible breakdown due to numerous multiple conforming bad data in the first step.

§ 2.3.3 WLS with Equality Constraints

Clements and Costa introduced a method based on modeling circuit breakers as branches whose statuses are treated as *operational* constraints. These constraints are defined such that if the circuit breaker on a branch between bus *i* and bus *j* is closed (the line is active), then $V_i = V_j$ and $\theta_i = \theta_j$. Otherwise, the breaker is open and $P_{ij} = 0$ and $Q_{ij} = 0$. Also modeled are physical or *structural* constraints. Specifically, buses that neither inject power into nor out of the system are called zero injections, and they are perfect measurements with no error. They obey the constraint that the real and reactive measurements at those buses are identically zero. So, a constrained weighted least squares algorithm finds the solution $\underline{\hat{x}}$ that minimizes

$$J(\underline{x}) = \frac{1}{2}[\underline{z} - \underline{h}(\underline{x})]' R^{-1} [\underline{z} - \underline{h}(\underline{x})],$$

subject to:

$$\underline{r} = \underline{z} - \underline{h}_m(\underline{x}), \ \underline{h}_s(\underline{x}) = \underline{0}, \ \underline{h}_o(\underline{x}) = \underline{0},$$

where $\underline{z}$ and R^{-1} are defined previously, $\underline{h}_m(\underline{x})$, $\underline{h}_s(\underline{x})$ and $\underline{h}_o(\underline{x})$ are the nonlinear vector functions relating the measurements to the state vector, the structural constraints and operational constraints, respectively, and $\underline{h}(\underline{x}) = \begin{pmatrix} \underline{h}_m(\underline{x}) & \underline{h}_s(\underline{x}) & \underline{h}_o(\underline{x}) \end{pmatrix}'$. Again, using a first order Taylor series expansion about the point $\underline{x}_0$, we obtain

$$\underline{h}(\underline{x}) \approx \underline{h}(\underline{x}_0) + H \cdot \Delta \underline{x}_0,$$

where $\underline{h}(\underline{x}_0)$ and H are equal to

$$\underline{h}(\underline{x}_0) = \begin{pmatrix} \underline{h}_m(\underline{x}_0) \\ \underline{h}_s(\underline{x}_0) \\ \underline{h}_o(\underline{x}_0) \end{pmatrix} \text{ and } H = \begin{pmatrix} \partial \underline{h}_m(\underline{x}) / \partial \underline{x} \\ \partial \underline{h}_s(\underline{x}) / \partial \underline{x} \\ \partial \underline{h}_o(\underline{x}) / \partial \underline{x} \end{pmatrix}.$$

The state estimate is then calculated by iteratively solving the linear system of equations which can be represented at the k^{th} iteration as

$$\begin{pmatrix} R & H \\ H' & 0 \end{pmatrix} \cdot \begin{pmatrix} \lambda \\ \Delta\hat{x} \end{pmatrix} = \begin{pmatrix} r^{(k)} \\ 0 \end{pmatrix},$$

where

$$R = \begin{pmatrix} R_m & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \underline{\lambda} = \begin{pmatrix} \underline{\lambda}_m \\ \underline{\lambda}_s \\ \underline{\lambda}_o \end{pmatrix}, \quad \underline{r}^{(k)} = \begin{pmatrix} \underline{z}_m - \underline{h}_m(\hat{\underline{x}}^{(k)}) \\ -\underline{h}_s(\hat{\underline{x}}^{(k)}) \\ -\underline{h}_o(\hat{\underline{x}}^{(k)}) \end{pmatrix}.$$

Here, R_m is the known variance-covariance matrix of the measurements, and $\Delta x = \hat{x}^{(k+1)} - \hat{x}^{(k)}$.

At convergence, the following relationships hold true:

$$R\underline{\lambda} = \underline{r}$$
$$H'\underline{\lambda} = \underline{0}.$$

Clements and Costa show that the vector of Langrage multipliers associated with the aforementioned constraints is a random vector with mean zero and variance-covariance matrix *V*. They normalize by letting $\lambda_i^N = \lambda_i / \sqrt{V_{ii}}$ and compare this to a critical threshold to determine errors in circuit breaker statuses, which induce topology errors. While this routine is sound intuitively and mathematically, one concern is that it expands the state vector considerably, on the order of doubling it, thereby greatly increasing the number of measurements necessary to have adequate estimates.