Essays on the Theory of Tax Evasion

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(ABSTRACT)

Literature on tax evasion has generally ignored the effects of tax evasion by a monopolist in a regulatory environment. When the government is asymmetrically informed about the monopolist’s demand and/or costs, however, the firm may have the opportunity to cheat on its regulatory constraint and tax payments. Adjustments in the regulatory constraint then will directly impact on the tax revenues of the government while alterations in tax policies may alter the effectiveness and efficiency results of a particular regulatory policy. To analyze these issues two forms of regulation, a price ceiling regulation and a fixed profit per unit regulation are considered in an environment where the government is incompletely informed about the monopolist’s cost function.

For the price ceiling regulation (Chapter 2) it is shown that tax evasion decisions are affected by variations in the ceiling in the sense that an increase in the effective price ceiling results in misreporting by a larger proportion. Tax evasion decisions however are found not to affect output decisions of the monopolist. Thus the optimal price ceiling under evasion is set at the same level as without tax evasion, i.e., at the point where price equals expected marginal cost. Optimality in this economy can be achieved in a number of ways. Full compliance is one way but optimality can also be achieved with tax evasion.

When the form of regulation considered is a fixed profit-per-unit regulation (Chapter 3), the results are quite different from above. Because profits of the monopolist are not costlessly observable by the government, firms can cheat on the regulatory constraint itself. Thus output and tax evasion
decisions become inextricably intertwined with one another. A result of this is that the profit tax is no longer neutral with respect to output and alterations in the audit and penalty rates are found to affect the monopolist's output.

Literature on tax evasion has often neglected the fact that income from different sources is taxed at different rates and provides different opportunities for misreporting. Once an individual obtains certain skills, his flexibility in switching jobs to evade taxes on his wage income becomes limited. Also the fact that a large part of the wage income in the U.S. is reported to the government by the employer and often withheld at the source, greatly limits the opportunity for evading wage taxes. However an individual faces many options when deciding on how to invest his savings and the income from at least some of these may not be subject to withholding and reporting. This fact suggests that the savings of an individual can be affected by tax evading opportunities. Chapter 4 examines this problem by considering a dynamic model of tax evasion. The results show that an increase in the penalty rate or audit probability leads to an increase in savings of the individual, given some assumptions on preferences. This fact implies that savings are reduced by the possibility of tax evasion. It also suggests that savings could be increased by stricter enforcement of tax laws.

Because the model used in chapter 4 is fairly complicated, some of the comparative static results are found to be ambiguous under general conditions. It is also not clear from the theory what the optimal policy of the government would be. To address these issues in more details, chapter 5 considers some numerical exercises. A number of results emerge from these exercises. First, savings are found to increase with an increase in either the penalty rate or the audit rate, even when the restrictive assumptions on risk aversion do not hold and labor supply is variable. Second, full compliance seems to be the optimal policy of the government for the specification selected. These results seem to hold both for compensated and uncompensated taxes.
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Chapter 1. Introduction and selective survey

1.1. Introduction

Tax evasion is considered to be a serious problem in many countries nowadays. Each year, a large fraction of potential tax revenues is lost due to evasion activities. Fratenduono [1986] for example, reports that tax evasion reduced tax receipts by 20 percent in 1985. Moreover, there is indication that the problem has been increasing over time. Thus, IRS [1988] reported that the tax gap (the amount of income taxes individuals and corporations owed but did not pay voluntarily) rose to $84.9 billion in 1987 from $30.9 billion in 1973. They also projected that the gap would grow to around $113.7 billion by 1992. While such official estimates might be suspect, the numbers do indicate that tax evasion is quantitatively significant.
The growing awareness of this problem among economists is reflected by the large volume of theoretical and empirical literature on the subject that has appeared in recent years. Inspite of this recent emphasis however, there are still many areas unexplored by this literature. First, the existing literature on tax evasion have generally focused on individual tax evasion. Though in recent years there has been some discussion on tax evasion by a monopolist, the impact of monopoly regulation on tax evasion has generally been neglected. The next two chapters of this dissertation deals with this issue by integrating the decisions of the regulatory authority as well as the tax enforcement authority. Two forms of regulatory policy - a price ceiling regulation (Chapter 2) and a fixed profit per unit regulation (Chapter 3) are considered. The analysis shows that the actions of one authority has definite impact on the outcome of the other authority so that these interactions need to be taken into account when optimal policies are considered. The results also differ significantly from one form of regulation to another.

Second, the literature on tax evasion has mainly been static. However, in a multi-period setting the savings decisions of taxpayers are likely to be affected by evasion opportunities so that alterations in tax enforcement parameters may have an impact on economic growth. Chapters 4 and 5 essay of this dissertation deals with this issue. Chapter 4 provides a model that deals with the theoretical aspects and Chapter 5 provides some numerical results. The results indicate that tax evasion could reduce savings in an economy. So one way of raising savings could be through stricter enforcement of tax laws.

To place the results of the dissertation in perspective, it is useful to provide a discussion of the existing literature on tax evasion. A comprehensive survey of this literature, however, is beyond the scope of this chapter.1 So only a brief discussion of some of the literature that is more relevant to this dissertation is provided here. The discussion is generally confined to theoretical aspects of tax evasion. Thus the literature on tax avoidance is not discussed here. For a survey of this literature

1 See Cowell [1990] for a recent survey of the literature.
the reader is referred to Seldon [1979]. Also the empirical literature on tax evasion is not discussed here. Marrelli [1987] provides a good survey of this literature.

The chapter is organized as follows. Section 2 describes the basic model of individual tax evasion as introduced by Allingham and Sandmo [1972], Srinivasan [1973] and others. Section 3 provides some extensions of this basic model and examines the robustness of the results of the basic model with respect to these extensions. Section 4 discusses the recent contributions discussing various aspects of corporate tax evasion. Section 5 provides a short survey of the literature on policy aspects of tax evasion relating to the government’s optimal choice of parameters. The impact of alternative instruments on revenues is also discussed. The last section provides some concluding remarks.

1.2. The Basic A-S Model

Though economists have been concerned with the problem of tax evasion for a long time, the basic theoretical literature on tax evasion started with the seminal papers by Allingham and Sandmo [1972] (henceforth A-S) and Srinivasan [1973], who extended the standard portfolio theory of choice under risk to explain an individual’s decision to evade income taxes. The basic model as first set out by A-S considered an individual who has a certain amount of exogeneous income that is subject to a proportional income tax. The government cannot observe this income directly, so that the individual has the option of misreporting his income. However, such misreporting puts him at a risk of being audited by the government. If the individual is audited, his income is assumed to be correctly identified and he has to pay all back taxes plus some penalty, which in the A-S model takes the form of a given fraction of the evaded income. The individual perceives a probability of being audited and as an amoral expected-utility maximizer, decides on the amount by which to misreport income so as to maximize his expected utility.
Using this framework, A-S showed that for an interior solution, an increase in either the penalty rate or the individual's perceived probability of being audited increases his reported income. In other words, increased enforcement of tax laws reduces tax evasion, as is expected. However they concluded that a change in the tax rate has an ambiguous effect on the income reported to the tax authorities.

Subsequent researchers found that the effect of the tax rate change on reported income was sensitive to the specific form of the penalty function chosen. Yitzhaki [1974], for example, showed that if the penalty for tax evasion is imposed on evaded taxes instead of evaded income as in A-S, the ambiguity goes away. An increase in the tax rate then leads to an increase in reported income, when the taxpayer's Arrow-Pratt measure of absolute risk aversion is decreasing in income. This result is very interesting because it contradicts the common assertion that higher tax rates encourage evasion.

A-S also looked at the effect of an increase in actual income on reported income but found that the effect is ambiguous in general.

1.3. Some Extensions of the A-S Model

Even though the A-S model was an important contribution to the literature on tax evasion, it was based on a number of unrealistic assumptions. Much of the subsequent literature extended the model by relaxing its assumptions to determine if the A-S results held under more general audit, penalty and tax schedules and when income is endogeneous. Researchers analyzed progressive taxes, audit functions which depend both upon actual and reported income and penalty as some function of reported income or evaded taxes. In many cases however, these extensions have led to inconclusive results.
A-S, in their original paper, attempted to examine whether their results regarding the
effect of changes in the tax enforcement parameters on reported income were still valid for a more
general audit function. They found that if the audit probability is a decreasing function of reported
income, then previous results still hold. Thus an increase in the penalty rate still leads to an increase
in reported income, as does also an upward shift in the audit function (higher probability of audit
for each reported income).

Whereas A-S considered a more general audit function, a number of other economists were interest-
ed in finding out how results change when progressive or regressive taxes are considered.
Srinivasan [1973], for example, considered a general tax and penalty function where both are in-
creasing functions of income. However unlike A-S, he considered a risk neutral individual who
maximizes net expected income and concentrated on the fraction \( \lambda \) of actual income that is re-
ported, rather than total amount misreported. He found that for a proportional or progressive tax,
an increase in the probability of detection leads to a reduction in the \( \lambda \) if the penalty rate is an in-
creasing function of \( \lambda \). Srinivasan also attempted to look at the effect of a change in actual income
on \( \lambda \). However he found that the effect was sensitive to the form of the penalty, tax and audit
function. Thus, the effect was found to be negative when the tax rate is progressive and the penalty
rate is independent of income. On the other hand, the effect was positive if the marginal tax rate
is constant and the probability of detection is an increasing function of actual income. While these
results are not too surprising, given that A-S found the effect to be ambiguous in their model, they
do have some intuitive appeal. In the first case the progressive income tax system acts as an in-
tensive to underdeclare income, provided that there is no offsetting increase in the probability of
punishment. In the second case it is the increased probability of investigation that acts as an in-
tensive to declare more of one’s income, provided there is no counteracting disincentive effect cre-
ated by higher marginal tax rates.

Though Srinivasan considered progressive and regressive taxes and more general penalty functions,
his results are not directly comparable with that of A-S because in his model the agent is risk neutral
and maximizes net income. In fact Penacavel [1979] found that if the basic A-S model is extended
to include nonlinear taxes, a more general penalty function (with the possibility of a large lump sum fine) and endogeneous labor supply, most of the results relating to the impact of a change in the parameters on the endogeneous variables become ambiguous. The ambiguity remains even when the utility function of the individual is assumed to be additive separable in consumption and leisure. In fact, even an increase in the penalty rate has ambiguous effect on reported income. This however is not difficult to understand because the impact of tax rate changes even in a simple model of labor supply turns out to be ambiguous because of opposing income and substitution effects.

The literature discussed so far generally assumed that the way in which tax revenues are spent has no impact on the individual's decision making. However tax revenues are presumably used to provide public goods which benefit the taxpayers. This suggests that individuals' tax evasion decisions are likely to be affected if they are aware of the benefits that arise from their tax payments. This possibility was considered by Falkinger [1988], who assumed that the tax revenues of the government are spent on public goods which provide utility to the taxpayers. Assuming that the that the public good benefits different consumers differently and that the utility function of the consumer is time separable in the public good and income, Falkinger showed that tax evasion is comparatively lower when the taxpayer is aware of the public benefits that he gets in return for his tax payments. However the impact of an increase in the taxpayer’s share of benefits arising from public expenditure on evasion was not clear.

The literature on tax evasion had generally looked at evasion in a static one-period framework where in each period the taxpayer makes his misreporting decisions without regard to the impact of these decisions on future consumption. This however is an unrealistic assumption. It is possible that an individual detected of misreporting in one period could be investigated for previous periods also, so that a multi-period approach might be more appropriate. So far, only A-S considered a dynamic model to address these issues. They considered an infinitely lived agent who gets some exogeneous income each period and decides how much of that to report each period. The individual cannot borrow or lend and if he is audited and found to have cheated in any period $t$, he is investigated for all previous periods. Using this model A-S found that if the individual is myopic
in the sense that he is not concerned about future periods, he will declare all his income in some period T and cheat in periods before that. Also declarations will increase over time. Basically the same results hold when the individual takes the future into account. Only, due to his greater foresight, his tax declarations will be higher than in the previous case.

The results reported above show that many of the results of the A-S model are not robust with respect to extensions in the audit, penalty and tax functions as well as with respect to endogeneous income. So these findings need to be treated with caution.

1.4. Corporate Tax Evasion

The literature on tax evasion have primarily been concerned with the problem of tax evasion at the individual level. In recent years however, there has also been some discussion of the effects of tax evasion at the corporate level. The emphasis here has mainly been on evasion of indirect taxes like sales, excise or commodity taxes. Two exceptions are Kreutzer and Lee [1986] and Wang and Conant [1988], who were concerned with the problem of evasion of corporate profit taxes.

Marrelli [1984] was the first to consider the issue of corporate tax evasion formally. He was concerned with the problem of a monopolist evading payment of ad valorem sales taxes by underreporting its revenues. The monopolist is faced with the choice of reporting the tax or evading it at the risk of being detected and penalized for the evasion. Being risk averse, the monopolist makes its reporting decisions so as to maximize his expected utility of net after-tax (and penalty, if any) profits. Using this framework, Marrelli showed that so long as the penalty rate and the audit rate are independent of actual or reported revenues (as is assumed in the basic A-S model), the presence of the possibility of tax evasion in no way affects the output decisions of the monopolist. Thus alterations in the penalty rate or the probability of detection, leaves the output level of the firm unchanged. Changes in output however have impact on taxes evaded. In fact an increase in
the amount of output produced leads to a reduction in the fraction by which revenues are underreported. Marrelli called this phenomenon a one-sided separation between the tax evasion and production decisions. The fact that an increase in output leads to lower evasion might suggest that big firms evade proportionally less than small firms.

Marrelli found however, that if the probability of detection is taken as a function of reported revenues, the one-sided separation is no longer present and output decisions are found to be affected by tax evasion decisions. With variable probability of detection, the optimal interior rate of declaration will be smaller or greater than that of fixed probability case if the probability of detection is an increasing or decreasing function of reported revenues. The firm's equilibrium output will be smaller or greater than the one the firm would produce if it did not evade taxes (or less than in the fixed probability case) according as whether the probability function is increasing or decreasing with the declared gross revenue. Because both output and tax declarations are higher when the probability of detection is an increasing function of reported revenues, this seems to be a more efficient rule. Also, as usual, reporting was found to be an increasing function of both the audit rate and the penalty rate. Thus, stricter enforcement reduces tax evasion. The effect of alterations in the tax rate on tax evasion however, was found to be ambiguous. Marrelli also compared the indirect tax with that of a profit tax of equal yield without evasion and found that if absolute risk aversion is decreasing, the optimal interior rate of indirect tax declaration will be greater than the corresponding direct tax one.

While Marrelli was concerned with evasion of sales taxes, there has also been some discussion on the effects of evasion of direct corporate taxes. Kreutzer and Lee [1986], for example, considered a model of a monopolist who can evade profit taxes by overstating costs by a certain proportion. Using this model they argued that the monopolist's output decisions might be affected by tax evading decisions.

Kreutzer and Lee's findings were very interesting because it went against the general hypothesis that profit taxes are neutral with respect to output decisions. However their result rested crucially on
the assumption that the monopolist cannot vary the fraction by which costs are overreported. As Wang and Conant [1988] showed, if the monopolist is free to vary this parameter, then Kreutzer and Lee’s result no longer holds and the profit tax is found to be neutral with respect to output.

Whereas the papers cited above were concerned with a tax evading monopolist, Virmani [1989] considered the case of a competitive industry with risk-neutral firms. In his model firms are subject to a commodity tax and maximize expected profits. Firms can evade taxes by misreporting output. However this puts them at a risk of being audited, the probability of detection being an increasing function of output. Evasion activity is costly for the firm because it loses some fraction of its output in such activities. Using this model, Virmani showed that tax evasion may lead to production inefficiency. So the optimal commodity tax rate can be quite different in the presence of tax evasion. The problem is likely to be more serious for industries in which the scale of production is relatively low. Virmani found that if evasion exists in equilibrium, then each firm will produce at less than minimum efficient scale. Moreover, production inefficiency will increase with evasion. As long as equilibrium with partial evasion prevails, evasion will be positively and production negatively related to tax rates.

Though Marrelli and Virmani had looked at corporate tax evasion, their approach was partial equilibrium. Panagariya and Narayana [1988] considered a simple general equilibrium model of excise tax evasion and its implications on welfare. They considered an economy consisting of two sectors (two goods) with an ad valorem (excise) tax on only one good. This good can be produced by two types of firms. One type can evade taxes and the other type cannot. If a firm is caught evading taxes, it has to pay a fraction of its output as penalty. Tax evasion however is costly from the firm’s point of view because it has to spend some resources on evading activities. Firms are assumed to be risk neutral and maximize expected profits. Using this model, Panagariya and Narayana showed that stricter enforcement policies are welfare improving. Moreover, an increase in the excise tax rate reduces welfare unambiguously. However, while an increase in the penalty rate will lower the extent of illegal activities, the welfare effect of this is ambiguous.
Whereas the papers discussed above were concerned with evasion of sales, excise or profit taxes by the firm, Yaniv [1988] pointed to a different form of corporate tax evasion. He argued that use of the withholding system in many countries might provide the firm an opportunity to misreport their wage payments and thus remit less than the total taxes withheld. Since underreporting of wage payments however imply overreporting of profits, it is assumed that if the firm is caught cheating, he will be refunded for the profit tax overpayments. Using this model Yaniv showed that the employer’s hiring (employment) decisions are unaffected by misreporting decisions. In the absence of profit taxation, an increase in the withholding tax rate has a negative effect on declaration when penalty relates to the amount of nonremitted taxes. An increase in the profit tax rate however is found to have a positive effect on declarations.

1.5. Policy Issues

So far issues relating to tax evasion was discussed mainly from the viewpoint of the tax evader. Whereas the tax and audit parameters are exogeneous from the point of view of the isolated taxpayer, these are directly under the control of the government. So it is important to discuss the question of how these parameters ought to be set. A number of authors have addressed these issues. Much of the early literature discussing policy issues, proceeded to find out the least costly ways of eliminating tax evasion. They thus implicitly assumed that full compliance is the optimal policy of the government and that revenue maximization is the relevant policy objective. Subsequently authors have also discussed other objectives of the government, specifically that of welfare maximization.

Srinivasan, in his 1973 paper, compared progressive and proportional taxes in terms of their revenue- generating capacity in the presence of tax evasion. He found that a proportionate tax function that yields the same total revenue as a progressive tax function in the absence of tax eva-
sion, will yield larger expected revenue in the presence of optimal understatement of income, if the penalty rate is an increasing function of the fraction of income that is misreported. Thus a progressive tax function leads to more tax evasion than a proportionate tax.

Since both the penalty rate and the audit rate were found to reduce tax evasion unambiguously in the A-S model, economists tried to find out if one of these two was more “effective” than the other in reducing tax evasion. Thus, Christiansen [1980], was concerned with whether a large fine with a small probability of detection is a stronger deterrent to tax evasion than a small fine and a large probability of detection. He found that if the fine is increased but efforts to detect tax evaders are adjusted to keep the expected monetary gain from tax evasion (to the taxpayer) constant, risk averters will always reduce tax evasion. This suggests that the penalty rate may be more effective in reducing tax evasion as compared to the audit rate.

Of course when the issue relating to the optimal mix of the policy parameters of the government is considered, it is also relevant to consider the costs of adjusting the parameters. Tax evasion literature has generally assumed that the tax rate and the penalty rate can be adjusted costlessly while audits are costly. The idea is that audits require real costs in terms of auditors’ salaries etc., but the tax and the penalty rate can be adjusted through legislature at negligible costs. If this is true, it is not difficult to see that the penalty rate is a “better” policy tool as compared to the audit rate. Thus Kolm [1973] showed that if the objective of the government is to raise a fixed amount of revenue at lowest cost, the optimal policy is to set the penalty rate arbitrarily high and set the audit rate to zero.

McCaleb [1976] went in a slightly different direction to evaluate the effectiveness of the tax rate, the penalty rate and the audit rate in raising actual (as opposed to expected) revenues. He found that while an increase in the penalty rate or the probability of detection unambiguously leads to an increase in tax payments, the effect on an increase in the tax rate on total tax payments is ambiguous. On these grounds he argued that the tax enforcement parameters are “better” policy tools than the tax rates. He noted however that while the tax rate and the penalty rate can be adjusted at minimal
costs, adjustments in the audit rate might prove to be more costly. So the penalty rate might be the best policy.

The fact that tax evasion can be reduced costlessly by increasing the penalty rate of course leads to the question as to why this policy is not followed. However, even though this policy seems to be appealing from the theoretical point of view, there are various practical problems associated with setting the penalty rate very high. First, the punishment should fit the crime. If the penalty for evasion and murder are the same, evaders would have the incentive to murder someone to avoid prosecution. Second, a high penalty rate with low audit probability would be undesirable on equity grounds because detected and undetected tax evaders would be treated very differently. Third, the system would be unjust if the method used in detecting evaders is not perfect. Lastly, it has been shown by many economists that the optimal policy of the government is not always necessarily to ensure full compliance. If the objective of the government is to maximize some measure of social welfare and the utility of the tax evaders is taken into account, then it is not immediately obvious why eliminating tax evasion is the optimal policy.

Kolm was the first to show that the optimal policy of the government may not necessarily imply full compliance. Kolm, in his 1973 paper used a utilitarian approach where the government maximizes the utility of the representative individual subject to a fixed revenue constraint. The individual is also assumed to derive utility from the public good produced from the revenues of the government (net of audit costs). Using this model Kolm showed that the optimal policy of the government might be to allow some tax evasion. Baldry [1984] derived the same conclusions for a wider class of social welfare functions - all Pareto and Rawlsian welfare functions.

The fact that the optimal policy of the government could be to allow some tax evasion was also shown by Weiss [1976] and Sandmo [1981] in two different contexts. Weiss used a model of endogeneous labor supply and argued that tax evasion might reduce distortions associated with the income tax. Sandmo used a model of two markets allowing differing opportunities of tax evasion and argued that tax evasion could be welfare-improving in this framework.
Weiss [1976] considered a model where labor supply is endogeneous and the individuals decide on how much to work and how much income to report. The wage rate is exogeneously given. Regarding the effect of an increase in the audit rate on labor supply, Weiss found that the effect is negative if absolute risk aversion is decreasing in wealth and the utility function is separable in wealth and leisure. The effect of a change in the audit rate on expected tax receipts is ambiguous. From this Weiss suggested that it might be advantageous for both the individuals' and the government to allow some tax evasion. In fact, he found that starting from a position in which no one cheats, it is to the advantage of all agents to have incentives to cheat. He also showed that under certain circumstances these incentives might be provided without a reduction in expected revenues collected. So individuals might prefer a random tax to a certain one that produces the same revenue, even though they are risk averse in the usual sense. Thus the possibility of tax evasion might offset the distortion and inefficiency introduced by an income tax and improve resource allocation.

Sandmo [1981] considered a model with two groups of individuals supplying labor to two markets. One market is regular where the income earned cannot be evaded but the other market allows the possibility of misreporting. One group of individuals supply labor to the regular market only, whereas the other group supplies labor to both markets. The wage rate is the same in both markets and the relative size of the two groups are assumed to be fixed. Also individuals are assumed to have identical preferences but the two groups have different skills. The government maximizes a weighted sum of the expected utility of the two groups, subject to a fixed revenue constraint. Using this model, Sandmo showed that if the weights are such that the evader's utility does not count, then the optimal policy is to control evasion so as to generate the maximum revenue. However under general conditions the optimal policy of the government might be to allow some tax evasion. Sandmo also showed that an increase in the penalty rate must lead to an increase in the supply of labor in the regular market and a decrease in the irregular market.

The literature on optimal government policies as discussed above generally assumed that the taxpayers acts passively in the sense that they make their decisions taking the parameters of the government as given. Greenberg [1984] and Reinganum and Wilde [1985] however have considered the
posibility of interaction between the actions of the government and the taxpayer(s). Also, unlike most of the previous research, they introduced memory in audits so that the probability of being audited depends on the outcome of a previous audit.

Greenberg [1984] considered a game theoretic model of tax evasion where the government takes the tax rate, the penalty rate and the (aggregate) audit probability as given and tried to device an audit scheme (different audit rates for different individuals) that minimizes tax evasion. The individual is assumed to be aware of the government’s policies and acts accordingly. Using this framework Greenberg showed that an audit scheme with three different audit rates for three groups can be devised so that evasion can be reduced to an arbitrarily small level. The question of which individual falls in which group is decided on the basis of whether he is audited that period and found to have cheated or not. Greenberg showed that the no-evasion solution is not feasible so that this scheme is the optimal scheme. It is to be noted that Greenberg’s model holds so long as individuals cannot collude. Once the possibility of collusion is considered, the model breaks down.

Reinganum and Wilde [1985] investigated whether the government can use any cutoff rule. They found that a cut off rule could be that if a taxpayer reports an income below a certain level, then he is audited with probability one. However if the taxpayer reports more than the cutoff income then he is not audited at all. The taxpayer is fully informed about the rule and its consequences.

The advantages of this rule are that (i) it provides an incentive for individuals to report their income accurately, (ii) it is horizontally equitable in both ex ante and ex post sense and (iii) it raises at least as much revenue as a purely random audit policy. However the analysis is based upon the assumptions that all taxpayers are risk neutral, all taxes are lump-sum and that the tax authority’s objective is to maximize net revenue. The results may not hold for more general cases.
1.6. Conclusion

The survey shows that tax evasion is an important and growing problem in many countries including the U.S. The presence of evasion indicates that many of the standard results of public finance need to be re-examined since the framework of an all-knowing all powerful state may not be valid. Evasion could have potentially damaging consequences like loss of revenues, distortions in resource allocation and inefficiency. Economists are only beginning to address these issues so that there is still a lot of scope for research in this area. This dissertation considers two potential areas that has been neglected by the current research. The first is concerned with the effects of tax evasion in a regulatory environment. The second is concerned with the dynamic impact of tax evasion on savings and growth.
Chapter 2. Price ceiling regulation of a tax-evading monopolist

2.1. Introduction

A large volume of literature has emerged, since the seminal paper by Averch and Johnson [1962], focusing on the impact of regulation on a monopoly firm. The basic literature has been concerned with studying the properties of alternative regulatory policies. Recently some work has also been done to characterize optimal regulatory policies. In this recent approach, the regulator or the planner is assumed to be incompletely informed about the monopolist’s demand and/or cost func-

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2. Recent surveys include Caillaud, Guesnerie, Rey and Tirole [1988], Sappington and Stiglitz [1987] and Baron [1989].
tions. The objective of the planner then is to maximize the expected social welfare based on its information about the costs and/or demand functions. Loeb and Magat [1979], Baron and Myerson [1982] and Sappington [1982], for example, study the control of private monopolies, when the planner is not fully informed about the monopolist’s costs.

The existing literature however has generally assumed away the presence of taxes in the model, on the grounds that the regulatory authority acts independently of the tax authority. This however is difficult to justify when optimal regulatory policies are devised using the notion of a social planner maximizing some measure of social welfare. A benevolent dictator should choose both the tax and the regulatory policies simultaneously to maximize social welfare, instead of considering them independently. This is especially true since alterations in regulatory policies may directly impact on the tax revenues of the government, while alterations in tax policies may alter the effectiveness and efficiency results of particular regulatory policy. Moreover, with asymmetry of information between the monopolist and the government, it may also be possible for a monopolist to misreport its costs to evade taxes and/or cheat on the regulatory laws. The government may not always have enough policy variables under its control to eliminate cheating so that this possibility should be taken into account.

This paper addresses these issues by taking a unified approach to tax and regulatory policies. The model considered is that of a monopolist subject to a profit tax and price ceiling regulation. The government can observe the monopolist’s price and quantity costlessly but has to incur audit costs to observe the monopolist’s cost of production. This gives the monopolist an opportunity to misreport costs and evade taxes. Under this framework it is shown that tax evasion decisions are affected by the choice of the regulatory policies. Specifically, an increase in the effective price ceiling is found to result in misreporting by a larger proportion. Tax evasion decisions however are found not to affect output decisions of the monopolist. So the optimal price ceiling is set at the same level as without tax evasion, i.e., at the point where price equals expected marginal cost. The chapter also shows that if the government can adjust all its policy parameters costlessly, optimality can be achieved with or without tax evasion. Thus elimination of tax evasion is not necessarily the best
policy of the government. Because optimality can be achieved in a number of ways the optimal tax, audit and penalty rates are not uniquely determined.

Much of the results of this chapter are found to be sensitive to the specific form of the regulation considered and the nature of the asymmetry of information between the monopolist and the social planner. As the next chapter shows, if instead of a price ceiling regulation the firm is subject to a fixed profit per unit regulation, the outcome turns out to be quite different.

The chapter is organized as follows. Section 2 sets up the problem of the monopolist and looks at the effects of alterations in the regulatory and tax parameters on the monopolist’s output and tax-evading decisions. Section 3 discusses the comparative static results of the model. Section 4 sets up the social planner’s problem and considers the optimal policies of the government. The last section provides some concluding remarks.

2.2. The Monopolist’s Problem

Consider a monopolist producing a single output q at a cost of c(q). Costs are assumed to be increasing in output so that \( c'(q) > 0 \). The inverse demand for the firm’s product is given by \( p(q) \) with \( p'(q) < 0 \). The monopolist is subject to a proportional profit tax at the rate of t. The government also regulates the activity of the monopolist by setting a price ceiling on the monopolist’s product, which takes the form \( p \leq \bar{p} \), where \( \bar{p} \) is the ceiling price specified. It is assumed here that the government is fully informed about the demand for the monopolist’s product and the price that it charges. So it is not possible for the monopolist to misreport its revenues. The government however is incompletely informed about the monopolist’s cost of production. The exact nature of information available to the government will be discussed in details in the next section, when the government’s optimization problem is introduced. The government, if it wants, can perform an audit of the monopolist’s tax return, in which case it can learn the monopolist’s costs accurately.
Auditing a tax return however, involves some costs, so that for budgetary reasons only a certain fraction of tax returns are audited each period, returns being chosen at random for audits. It is assumed that the way the government’s tax revenues are spent, have no impact on the monopolist’s decision making.

The fact that there is asymmetry of information between the monopolist and the government and the fact that only a fraction of all tax returns are audited, provides the monopolist with an opportunity to misreport costs to evade taxes. Thus, the monopolist might overreports cost by a certain fraction \( \mu (\mu \geq 0) \) and report costs as

\[
c^*(q) = (1 + \mu)c(q)
\]

instead of reporting actual costs \( c(q) \). Such overreporting would reduce the tax burden of the monopolist by \( \mu c(q) \) in the event he is not audited. If he is audited however, his true cost is correctly identified and he is forced to meet all his tax obligations and pay a fine of \( (\theta - 1) \) times the evaded tax (\( \theta > 1 \)) as penalty for tax evasion. The government is assumed to announce the penalty rate \( \theta \) before the monopolist makes the production decisions and is also assumed to commit to the announcement.

The monopolist is aware of the fact that if he misreports costs he might be audited. Suppose that the monopolist perceives that he is likely to be audited with some probability \( a \). In an economy where the monopolist is fully informed about the government’s actions, \( a \) might be the fraction of tax returns that the government audits. This of course assumes that all tax returns have equal probability of being audited. Arguably, this assumption ignores the fact that monopolies are few in number and may be easier to monitor than other firms. It may also be argued that if monopolists are found to evade taxes, they could be audited each period to eliminate the problem, without having to spend a substantial amount in audit costs. However it is assumed that there are political costs of following such a policy.
The monopolist makes his decisions so as to maximize his expected utility from net (after tax and penalty if any) profits given by

$$EU = \alpha U(\pi^d) + (1 - \alpha) U(\pi^{nd})$$  \[1\]

subject to the price ceiling constraint $p \leq \bar{p}$, where

$$\pi^{nd} = (1 - \theta)[R(q) - c(q)] + t\mu c(q)$$  \[2\]

refers to the net profits of the monopolist in the event that he overreports costs and is not detected, and

$$\pi^d = (1 - \theta)[R(q) - c(q)] - (\theta - 1)t\mu c(q)$$  \[3\]

refers to the net profits of the monopolist in the event that he misreports costs and is detected, where $R(q) \equiv p(q).q$ is the revenue of the monopolist.

Regarding the monopolist's utility function, it is assumed that $U''(\cdot) > 0$. The monopolist is also assumed to be risk averse so that $U''(\cdot) < 0$. Regarding the penalty rate $\theta$ and the audit probability $\alpha$ it is assumed that these are exogeneous (from the monopolist's point of view) and independent of the volume of taxes evaded. For simplicity, it is also assumed that the monopolist cannot adjust the quality of the product. Problems relating to inefficiency arising from quality adjustments are thus ignored.

The problem of the monopolist then is to choose $q$ and $\mu$ so as to maximize his expected utility from profits given by [1] subject to the regulatory constraint $p \leq \bar{p}$ (with $\pi^{nd}$ and $\pi^d$ being given by [2] and [3] respectively).

The Lagrangian for the problem is then given by

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3 The general tradition in the industrial organization literature is to assume that the firm is risk neutral. Risk neutrality in tax evasion models however leads to corner solutions so that following Marrelli [1984], Kreuzer and Lee [1986] and others it is assumed that the monopolist is risk averse.
\[ L = EU + \lambda [\bar{p} - p] \]

where \( \lambda \) is the Lagrange multiplier.

Assuming that the price ceiling is binding and confining attention to interior solutions only, the F.O.C.'s for a maximum are:

\[ \frac{\partial L}{\partial \mu} = (1 - \alpha)tc(q)U'(\pi^{\mu}) - \alpha(\theta - 1)tc(q)U'(\pi^d) = 0 \]  \[ \text{[5]} \]

\[ \frac{\partial L}{\partial q} = \alpha U'(\pi^{d})[(1 - \ell)(R' - c') - (\theta - 1)t\mu c''] \]
\[ + (1 - \alpha)U'(\pi^{nd})[(1 - \ell)(R' - c') + t\mu c'] - \lambda \rho' = 0 \]  \[ \text{[6]} \]

And, \( \frac{\partial L}{\partial \lambda} = \bar{p} - p = 0 \)  \[ \text{[7]} \]

where \( R' \equiv \partial R(q)/\partial q \) represents the marginal revenue of the monopolist,

\( p' \equiv \partial p(q)/\partial q < 0 \) refers to the slope of the monopolist's demand curve,

\( c' \equiv \partial c(q)/\partial q \) refers to the marginal cost of the monopolist

and, \( U'(\pi^i) \equiv \partial U(\pi^i)/\partial \pi^i, i = d, nd \); refers to the marginal utility of the monopolist in the two states of nature.

The solution to equations [5] - [7] give the optimal choice of the monopolist (\( \bar{\mu}, \bar{\eta} \)). It is to be noted that a necessary condition for the monopolist to evade some taxes is that \( 1 > \alpha \theta \). This can be shown easily. The net expected profits of the monopolist can be defined as

\[ E \pi \equiv \alpha \pi^d + (1 - \alpha)\pi^{nd} = (1 - \ell)[R(q) - c(q)] + t\mu(1 - \alpha \theta)c(q) \]  \[ \text{[8]} \]
From [8] it is clear that net expected profits are larger with tax evasion than without tax evasion [in which case the net gain is \((1 - \epsilon)(R(q) - c(q))\) so long as \((1 - \alpha\theta) > 0\). It is assumed here that the parameter values are such that this condition is satisfied.

To look at the second order conditions, equations [5] - [7] are totally differentiated to yield

\[
\begin{bmatrix}
\frac{\partial^2 L}{\partial \mu^2} & \frac{\partial^2 L}{\partial \mu \partial \theta} & 0 \\
\frac{\partial^2 L}{\partial q \partial \mu} & \frac{\partial^2 L}{\partial q^2} & -p' \\
0 & -p' & 0
\end{bmatrix}
\begin{bmatrix}
d\mu \\
dq \\
d\lambda
\end{bmatrix}
= \begin{bmatrix}
A \\
B \\
D
\end{bmatrix}
d\theta + \begin{bmatrix}
E \\
F \\
G
\end{bmatrix}
d\alpha + \begin{bmatrix}
0 \\
0 \\
-1
\end{bmatrix}
d\bar{p}
\]  

\[ [9] \]

where \(A \equiv -\frac{\partial^2 L}{\partial \mu \partial \theta}, \ B \equiv -\frac{\partial^2 L}{\partial \mu \partial \alpha}, \ D \equiv -\frac{\partial^2 L}{\partial q \partial \mu}, \ E \equiv -\frac{\partial^2 L}{\partial q \partial \theta}, \ F \equiv -\frac{\partial^2 L}{\partial q \partial \alpha}, \)

and \(G \equiv -\frac{\partial^2 L}{\partial q} \).

[The exact values of A, B, etc., will be defined later as needed for the comparative static results.]

The determinant of the bordered Hessian is given by

\[ |\Delta| = -\left[ \frac{\partial^2 L}{\partial \mu^2} \right] (pr)^2 \]  

\[ [10] \]

where, \(\frac{\partial^2 L}{\partial \mu^2} = \alpha^2 (\theta - 1)(c(q))^2 U''(\pi^d) + (1 - \alpha)^2 (c(q))^2 U''(\pi^d) < 0\)  

\[ [11] \]

and \(U''(\pi) \equiv \partial U'(\pi)/\partial \pi < 0, \ i = d, nd\).
From equations [10] and [11] it is clear that $|\Delta| > 0$ so long as the monopolist is risk averse. Thus the second order conditions for a maximum are globally satisfied. Note that the second order conditions are satisfied irrespective of any assumptions on the marginal cost function. This is because under an effective ceiling constraint, output is determined solely by the demand function.

Before proceeding to the comparative static results of this model, it is useful to state and prove one result that will be needed to determine the signs of some of the comparative static results.

**Proposition 1:** Assuming that the price ceiling is binding, the monopolist’s optimal output occurs where $R' < c'$.

Proof: Equation [5] implies that $(1 - \alpha)U'(\pi^{d}) = \alpha(\theta - 1)U'(\pi^{t})$. This, along with equation [6] requires that

$$\frac{\partial L}{\partial q} = (1 - t)(R' - c')[(\alpha U'(\pi^{d}) + (1 - \alpha)U'(\pi^{nd}))] - \lambda p' = 0$$  \[12\]

If the price ceiling is binding, $\lambda > 0$ so that $\partial L/\partial q = 0$ only when $R' < c'$.

The proposition implies that the monopolist’s optimum output under regulation is larger than that without regulation. This is of course is true for the no evasion case [See case I, Appendix A]. The exercise here was to show that the same result holds even in the presence of tax evasion.

### 2.3. Comparative Statics Results

The model reveals a number of interesting results. These are summarized in the form of the propositions given below. Propositions 2.1 and 2.2 refer to the effect of changes in the government policy parameters $\theta$, $\alpha$, $t$, and $\bar{p}$ on optimal output produced by the monopolist. Propositions 3.1,
3.2 and 3.3 refer to the effect of the same policies on the optimal proportion $\mu$ by which the monopolist overreports costs.

**Proposition 2.1:** When the price ceiling is binding the monopolist's optimal output choice is unaffected by alterations in the profit tax rate, the penalty rate and the audit probability of the government. That is, $\partial q/\partial t = \partial q/\partial \theta = \partial q/\partial \alpha = 0$

Proof: Follows directly from equation [9].

**Proposition 2.2:** An increase in the effective price ceiling leads to a fall in the monopolist's optimal output. That is, $\partial q/\partial \bar{p} < 0$.

Proof: From [9] and [10] it is clear that

$$\frac{\partial q}{\partial \bar{p}} = - \left[ \frac{\partial^2 L}{\partial \mu^2} \right] \frac{p'}{|\Delta|} = \frac{1}{p'} < 0$$

Propositions 2.1 and 2.2 are intuitively clear. With the price ceiling being effective, the firm's output is obtained solely from the demand curve for the product. Moreover since the monopolist's price and output are both costlessly observed by the government, there is no room for misreporting these.

**Proposition 3.1:** An increase in the penalty rate or the audit probability of the government leads to a reduction in the fraction by which costs are overreported. That is $\partial \mu/\partial \theta < 0$, and $\partial \mu/\partial \alpha < 0$.

Proof: Equation [9] shows that

$$\frac{\partial \mu}{\partial \theta} = - \frac{A(p')^2}{|\Delta|} \quad \text{and,} \quad \frac{\partial \mu}{\partial \alpha} = - \frac{B(p')^2}{|\Delta|}$$

Since

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\[ A \equiv -\frac{\partial^2 L}{\partial \mu \partial \theta} = \alpha tc(q) U'(\pi^d) - \alpha(\theta - 1)\mu\{tc(q)\}^2 U''(\pi^d) > 0 \] \hspace{1cm} [14]

and

\[ B \equiv -\frac{\partial^2 L}{\partial \mu \partial \alpha} = (\theta - 1)tc(q) U'(\pi^d) + tc(q) U''(\pi^{nd}) > 0 \] \hspace{1cm} [15]

it is clear from [12] that \( \partial \mu / \partial \theta < 0 \) and \( \partial \mu / \partial \alpha < 0 \).

This is also an intuitively clear result. It says that stricter enforcement laws reduce tax evasion. This is the usual result observed in tax evasion literature.

Proposition 3.2: A reduction in the price ceiling, other things remaining constant, leads to a reduction in the fraction by which costs are overreported, if the Arrow-Pratt measure of absolute risk aversion of the monopolist is nonincreasing in income. In other words, \( \partial \mu / \partial \overline{p} > 0 \) if, \( R_d(d) \geq R_d(nd) \), where \( R_d(i) = -U''(\pi^i)/U'(\pi^i) \), \( i = d, nd \); is the Arrow-Pratt measure of absolute risk aversion.

Proof: Equation [9] can be simplified to give

\[ \frac{\partial \mu}{\partial \overline{p}} = \left[ \frac{\partial^2 L}{\partial \mu \partial q} \right] \left[ \frac{p'}{|\Delta|} \right] \] \hspace{1cm} [16]

Now, \( \frac{\partial^2 L}{\partial \mu \partial q} = (1 - \alpha)tc(q)U'(\pi^{nd})[(X - \theta \mu c')R_A(nd) - XR_A(d)] \) \hspace{1cm} [17]

where, \( X \equiv (\theta - 1)\mu c' - (1 - \theta)(R' - c') > 0 \)

\( X > 0 \), so long as \( R' - c' < 0 \). Also, because \( \theta \mu c' > 0, (X - \theta \mu c') < X \). Hence from [17] it is clear that a sufficient condition for \( \frac{\partial^2 L}{\partial \mu \partial q} \) to be less than zero is that \( R_d(d) \geq R_d(nd) \), i.e., the Arrow-Pratt measure of absolute risk aversion is nonincreasing in income. In this case [16] implies that \( \partial \mu / \partial \overline{p} > 0 \).
The result is not very difficult to explain. So long as the price ceiling is effective, an increase in the ceiling results in an increase in profits of the monopolist. This would induce him to greater risk taking, if his preferences exhibit decreasing absolute risk aversion. So he tends to misreport costs by a higher fraction.

Proposition 3.2 is very interesting. It indicates that a reduction in the price ceiling in this simple model is not only welfare improving in terms of increasing the output of the monopolist but it has also the added "advantage" of reducing tax evasion. It also indicates that variations in the regulatory policies have impact on the tax evasion and hence tax collections, showing a need for coordinating the activities of the regulatory authority and the tax collecting authority.

Propositions 2.1, 2.2, 3.1 and 3.2 show that changes in tax enforcement parameters $\alpha$ and $\theta$ have no impact on the optimal output choice of the monopolist, this being determined solely by the ceiling constraint $\overline{p}$. Alterations in the ceiling price however have impact on the amount of evasion since $\mu$ depends on $\overline{p}$. The fact that output decisions of firms are unaffected by their tax evasion decisions was first observed by Marrelli [1984] who called this phenomenon a "one sided separation" between output and tax evading decisions. This type of separation have also been observed by Wang and Conant [1987], Marrelli and Martina [1988] and Yaniv [1988].

**Proposition 3.3**: An increase in the proportional profit tax rate leads to a reduction in the fraction by which costs are overreported if the Arrow-Pratt measure of absolute risk aversion of the monopolist is nonincreasing in income. That is, $\partial \mu/\partial t < 0$ if $R_A(d) \geq R_A(nd)$.

Proof: Equation [9] can be simplified to yield

$$\frac{\partial \mu}{\partial t} = \frac{-D(p')^2}{|\Delta|}.$$ Where,

$$D = (1 - \alpha)te(p)U'(\alpha^n)\left[(Z + \theta \mu c(q))R_A(d) - ZR_A(nd)\right]$$

[18]

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where $Z \equiv p.q - (1 + \mu)c(q) > 0$. From [18] it is clear that $D > 0$, so long as the Arrow-Pratt measure of absolute risk aversion is nonincreasing in income. Under these circumstances, it follows that $\frac{\partial \mu}{\partial t} < 0$.

This proposition can be explained in the same way as Proposition 3.2. An increase in the profit tax rate reduces the monopolist’s after-tax profits or net income. This leads him to reduce risk taking (a reduction in $\mu$) if his preferences exhibit decreasing absolute risk aversion.

This result is similar to those obtained by Yitzhaki [1974] for individual income taxes and Wang and Conant [1986] and Kreutzer and Lee [1986] for monopolist’s profit taxes. The result however goes against the usual hypothesis that increasing evasion is a direct consequence of increasing marginal tax rates and the best way to lower tax evasion is to lower these.

From the monopolist’s first order conditions it is possible to find the optimal values of $\mu$ and $q$ as functions of the parameters $\alpha$, $\theta$, $\bar{p}$ and $t$. These can then be used to derive the indirect utility of the monopolist as

$$V(\alpha, \theta, \bar{p}, t) = \alpha U[\pi^d(\alpha, \theta, \bar{p}, t)] + (1 - \alpha) U[\pi^{nd}(\alpha, \theta, \bar{p}, t)]$$  \[19\]

where

$$\frac{\partial V}{\partial \alpha} = U(\pi^d) - U(\pi^{nd}) < 0$$  \[20\]

$$\frac{\partial V}{\partial \theta} = -t \alpha \mu c(q)U'(\pi^d) < 0$$  \[21\]

$$\frac{\partial V}{\partial \bar{p}} = \lambda > 0$$  \[22\]

and

$$\frac{\partial V}{\partial t} = -\alpha U'(\pi^d)[(R(q) - c(q)) + (\theta - 1)\mu c(q)] - (1 - \alpha)U'(\pi^{nd})[R(q) - (1 + \mu)c(q)] < 0$$  \[23\]
It is also of interest to find out how changes in the policy parameters affect the net expected profits of a monopolist. Differentiating equation [8] with respect to the policy variables yields

\[
\frac{\partial E\pi}{\partial \alpha} = - t\mu \theta c(q) + t(1 - \alpha \theta)c(q) \frac{\partial \mu}{\partial \alpha} < 0
\]  \[24\]

\[
\frac{\partial E\pi}{\partial \theta} = - t\mu \alpha c(q) + t(1 - \alpha \theta)c(q) \frac{\partial \mu}{\partial \theta} < 0
\]  \[25\]

\[
\frac{\partial E\pi}{\partial p} = (1 - t)\left[ R' - c' \right] \frac{\partial q}{\partial p} + t\mu(1 - \alpha \theta)c' \frac{\partial q}{\partial p} + t(1 - \alpha \theta)c(q) \frac{\partial \mu}{\partial p}
\]  \[26\]

\[
\frac{\partial E\pi}{\partial t} = - \left[ (R(q) - (1 + \mu)c(q)) + \alpha \theta \mu c(q) \right] + t(1 - \alpha \theta)c(q) \frac{\partial \mu}{\partial t}
\]  \[27\]

Equations [24] and [25] show that expected profits of the monopolist fall with an increase in either the audit rate or the penalty rate (or both). Since actual pre-tax profits do not change with a change in either of these two parameters but reported profits increase, it is not difficult to understand why expected profits fall with increase in the tax enforcement parameters.

As equation [27] shows, the effect of an increase in the price ceiling on expected post-tax profits is ambiguous.

From equation [27] it is clear that so long as \( 1 > \alpha \theta \), and \( R(d) \leq R(d) \), \( \partial E\pi / \partial t < 0 \). An increase in the tax rate, as expected, leads to a fall in expected profits.

### 2.4. The Social Planner’s Problem

The last section set out the problem of a single monopolist and discussed the effects of changes in various government policies on its output, the amount of tax evaded, etc. Suppose the economy
consists of \( N \) firms, firms being indexed by \( i \) (\( i = 1, 2, \ldots, N \)). These may be various local monopolies of the same good or could be monopolies over distinct goods. It is assumed that the \( N \) markets are sufficiently separated from one another so that the action of the \( i \)th firm has no impact on the profits of the \( j \)th firm. The government regulates the activities of these monopolies by imposing a price ceiling \( p_i \) on each of them and through its proportional profit tax \( t \). Whereas the government has costless knowledge about the output of the monopolists, it does not have complete knowledge about the monopolists’ costs. It is aware that the cost of production of the \( i \)th firm takes the form \( c_i(q_i, \gamma_i) \) where \( \gamma_i \) is a parameter that is unknown to the government (but known to the monopolist). The government however has some subjective prior probability distribution for this unknown parameter. It is assumed that \( f_i(\gamma_i) \) represents the density function for the probability distribution. The function is assumed to be continuous in \( \gamma_i \) with \( f_i(\gamma_i) > 0 \) over the interval \([\gamma_i^L, \gamma_i^U]\). The government makes its decisions on the basis of this distribution of \( \gamma_i \) (which may be the true distribution for the parameter or the government’s subjective distribution). The government is also aware that firms can evade taxes due to the asymmetry of information and if it so desires, it can prevent this (at least partially) by auditing a fraction \( \alpha \) of all tax returns and collecting penalty at the rate \( \theta \) from any firm that is found to have evaded taxes. The objective of the government is to set the tax rates, the price ceilings, the audit rate and the penalty rate so as to maximize some measure of social welfare to be defined below. Note that whereas the tax rates, the price ceilings and the penalty rate can be imposed without incurring any administrative costs, auditing tax returns involve some cost which can be given by \( g(\alpha) \) with \( g'(\alpha) > 0 \). [The audit costs should also depend on \( N \), the total number of firms, but this is taken to be constant in our model.]. The government has to raise at least \( \bar{T} \) dollars per period, in (expected) revenues, net of audit costs.\(^4\)

Before formulating the government’s optimization problem, it is of interest to find out how changes in the policy variables affect the expected revenue of the government. The gross expected revenue of the government can be defined as

\(^4\) The number of firms is considered to be sufficiently large that expected revenues reasonably approximate actual collections.
\[ T^g = \sum_{i=1}^{N} \int_{\gamma_i}^{\gamma_i^1} \left[ R_i(q_i) - (1 + \mu_i(1 - \alpha \theta)) c_i(q_i, \gamma_i) \right] f_i(\gamma_i) \, d\gamma_i \]  

[28]

The \( q_i \) and \( \mu_i \) 's in equation [28] actually should be written as \( \tilde{q}_i \) and \( \tilde{\mu}_i \) - the optimal values for the \( i \)th monopolist, but the tilde's are suppressed in [28] and subsequently, for notational convenience. For the same reason, in subsequent equations the limits of the sum and the limits of the integration of the parameter \( \gamma \) are omitted (but understood).

It may be reasonable to assume that "on the average" each monopolist reports nonnegative profits. This then would require that \( \int \left[ R_i(q_i) - (1 + \mu_i) c_i(q_i, \gamma_i) \right] f_i(\gamma_i) \, d\gamma_i \geq 0 \quad \forall i \). Moreover since \( \int c_i(q_i, \gamma_i) f_i(\gamma_i) \, d\gamma_i > 0 \quad \forall i \), it is clear that \( \int \left[ R_i(q_i) - (1 + \mu_i(1 - \alpha \theta)) c_i(q_i, \gamma_i) \right] f_i(\gamma_i) \, d\gamma_i > 0 \quad \forall i \).

The impact of changes in the policy parameters on \( T^g \) is given in the form of the following propositions:

**Proposition 4.1**: The expected revenue of the government, gross of audit costs, increases with increase in either the penalty rate or the probability of detection. That is, \( \partial T^g / \partial \alpha > 0 \) and \( \partial T^g / \partial \theta > 0 \).

**Proof**: Equation [28] can be differentiated with respect to \( \alpha \) and \( \theta \) to give

\[
\frac{\partial T^g}{\partial \alpha} = \theta \sum_{i} \mu_i c_i(q_i, \gamma_i) f_i(\gamma_i) d\gamma_i

- (1 - \alpha \theta) \sum_{i} t c_i(q_i, \gamma_i) \frac{\partial \mu_i}{\partial \alpha} f_i(\gamma_i) d\gamma_i > 0
\]

[29]

because \( \partial \mu_i / \partial \alpha < 0 \) and \( 1 > \alpha \theta \).

Again,

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\[
\frac{\partial T^g}{\partial \theta} = \alpha \sum \int [\mu_c(q_i, \gamma_i) f(\gamma_i) d\gamma_i
\]
\[= (1 - a\theta) \sum \int tc(q_i, \gamma_i) \frac{\partial \mu_i}{\partial \theta} f(\gamma_i) d\gamma_i > 0 \tag{30}
\]

Since a change in the tax enforcement parameters does not affect equilibrium output in this model but increases reported profits, this result follows immediately.

**Proposition 4.2**: An increase in the profit tax rate leads to an increase in the expected revenue of the government so long as the Arrow-Pratt measure of absolute risk aversion of each monopolist is nonincreasing in income. That is, \(\partial T^g/\partial t > 0\) so long as \(R_a(d) \geq R_a(nd)\) for each monopolist.

Proof: Equation [28] can be differentiated with respect to \(t\) to give

\[
\frac{\partial T^g}{\partial t} = \sum \int [R_i(q_i) - (1 + \mu_i(1 - a\theta)) c_i(q_i, \gamma_i)] f(\gamma_i) d\gamma_i
\]
\[= (1 - a\theta) \sum \int tc(q_i, \gamma_i) \frac{\partial \mu_i}{\partial t} f(\gamma_i) d\gamma_i \tag{31}
\]

So long as the Arrow-Pratt measure of absolute risk aversion of each monopolist is nonincreasing in income, \(\partial \mu_i/\partial t < 0\) \(\forall i\). Under these circumstances clearly \(\partial T^g/\partial t > 0\).

The profit tax rate was seen not to affect output levels of the monopolist(s). Also given that the preferences of the ith monopolist exhibit decreasing absolute risk aversion, an increase in the profit tax would lead to a reduction in tax evasion \((\mu)\). So it is clear that expected revenues would increase with an increase in the profit tax of the ith monopolist.

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Proposition 4.3: The effect of a change in the ceiling price on the expected revenue of the government is ambiguous even when the Arrow-Pratt measure of absolute risk aversion is nonincreasing in income.

Proof: Equation [28] can be differentiated with respect to \( \bar{p} \) to yield

\[
\frac{\partial \tau^g}{\partial \bar{p}} = \int \left[ q_i(\bar{p}) + \bar{p} \frac{\partial q_i}{\partial p} - (1 + \mu(1 - \alpha \theta)) c'(q_i, \gamma) \frac{\partial q_i}{\partial p} \right] f_i(\gamma_i) d\gamma_i
\]

\[ - (1 - \alpha \theta) \int t c_i(q_i, \gamma) \frac{\partial \mu_i}{\partial p} f_i(\gamma) d\gamma_i \quad [32] \]

The sign for this is in general ambiguous.

Since an increase in the price ceiling leads to higher profits of the monopolist, it might seem that revenues of the government should be increasing. However a higher price ceiling induces higher misreporting and evasion so that the final outcome is not clear.

The first step towards deriving optimal policies of the government is to derive some measure of social welfare that the government should be maximizing. The literature analyzing the issues of social optimum under regulatory policies usually uses the notion of consumer surplus to characterize social optimum. Such use is justified if it is assumed that all consumers are identical and have utility functions separable in the monopoly good and all other goods, and if the utility function is linear in all other commodities. Under these circumstances, consumer's surplus can be directly identified with utility.\(^5\) Total surplus, which is the sum of consumer surplus and profits of the firm, is then taken as the measure of social welfare which the regulator maximizes. Basically the same measure of social welfare is used here. One difference is that with \( N \) firms, the aggregate consumer surplus (the sum of the consumer surplus from each market) is used. Also with tax evasion the

\(^5\) See Sheshinski [1976].
The aggregate consumer surplus in this model is given by

$$CS = \sum \int_0^{q_i} p(y_i) d\nu(y_i) dy_i - \sum \int R(q_i) f(y_i) dy_i$$  \[33\]

The aggregate expected profits of the monopolists is given by

$$E \pi = \sum \int (1 - \theta) [R(q_i) - c(\theta(q_i, \gamma_i))] f(y_i) dy_i$$

$$+ (1 - \alpha \theta) \sum \int \mu_i c(\theta(q_i, \gamma_i)) f(y_i) dy_i$$  \[34\]

Social welfare can then be written as

$$W = CS + E \pi = \sum \int_0^{q_i} p(y_i) f(y_i) dy_i dy_i - \sum \int c(\theta(q_i, \gamma_i)) f(y_i) dy_i$$

$$- \sum \int t [R(q_i) - (1 + \mu_i (1 - \alpha \theta)) c(\theta(q_i, \gamma_i))] f(y_i) dy_i$$  \[35\]

The expected revenue of the government, net of audit costs is given by

$$T^a = \sum \int t [R(q_i) - (1 + \mu_i (1 - \alpha \theta)) c(\theta(q_i, \gamma_i))] f(y_i) dy_i - g(\alpha)$$  \[36\]

The social planners problem then is to choose $q_i$, $t$, $\alpha$ and $\theta$ so as to maximize $W$ subject to the constraint that $T^a \geq T$. [To be very accurate, the ceiling prices $\bar{p}_i$'s should be taken as the choice
variable of the government instead of the $q_i$ 's. However, in this model both $q$ and $p$ are costlessly observable, so that any one of the two can be considered.]

The Lagrangian for the government's optimization problem then is

$$M = (\phi - 1) \sum \tau \left[ R_i(q_t) - (1 + \mu_t(1 - \alpha \theta))c_t(q_t, \gamma_t) \right] f_t(\gamma_t) d\gamma_t$$

$$+ \sum \int_0^{q_i} p_t(\gamma_t)f_t(\gamma_t) d\gamma_t - \int c_t(q_t, \gamma_t)f_t(\gamma_t) d\gamma_t - \phi [g(\alpha) + T]$$  \[37\]

where $\phi$ is the Lagrange multiplier.

The F.O.C.'s for an interior solution are

$$\frac{\partial M}{\partial t} = (\phi - 1) \sum \left[ R_i(q_t) - (1 + \mu_t(1 - \alpha \theta))c_t(q_t, \gamma_t) \right] f_t(\gamma_t) d\gamma_t$$

$$- (\phi - 1)(1 - \alpha \theta) \sum t c_t(q_t, \gamma_t) \frac{\partial \mu_t}{\partial q_t} f_t(\gamma_t) d\gamma_t = 0$$  \[38\]

$$\frac{\partial M}{\partial q_t} = \int p_t(q_t)f_t(\gamma_t) d\gamma_t + (\phi - 1) \int [ R_i' - (1 + \mu_t(1 - \alpha \theta))c_t'] f_t(\gamma_t) d\gamma_t$$

$$- (\phi - 1)(1 - \alpha \theta) \int t c_t(q_t, \gamma_t) \frac{\partial \mu_t}{\partial q_t} f_t(\gamma_t) d\gamma_t - \int c_t'(q_t, \gamma_t)f_t(\gamma_t) d\gamma_t = 0$$  \[39\]

$$\frac{\partial M}{\partial \alpha} = (\phi - 1) \theta \sum t \mu_t c_t(q_t, \gamma_t) f_t(\gamma_t) d\gamma_t$$

Chapter 2. Price ceiling regulation of a tax-evading monopolist
\[-(\phi - 1)(1 - \alpha \theta) \sum \int c_i(q_i, \gamma_i) \frac{\partial \mu_i}{\partial \alpha} f(\gamma_i) d\gamma_i - \phi g'(\alpha) = 0\]  \[40\]

\[\frac{\partial M}{\partial \theta} = (\phi - 1) \alpha \sum \int t \mu_i c_i(q_i, \gamma_i) f(\gamma_i) d\gamma_i\]

\[-(\phi - 1)(1 - \alpha \theta) \sum \int t c_i(q_i, \gamma_i) \frac{\partial \mu_i}{\partial \theta} f(\gamma_i) d\gamma_i = 0\]  \[41\]

\[\frac{\partial M}{\partial \phi} = \sum \int t \mu_i c_i(q_i, \gamma_i) \frac{\partial \mu_i}{\partial \phi} f(\gamma_i) d\gamma_i - g(\alpha) - \bar{T} = 0\]  \[42\]

Before analyzing the solution characterized by the set of equations [38] - [42], it is important to find out if the optimal policy of the government is to eliminate tax evasion if it can do so costlessly. As it turns out however, it can be shown that while optimality can be achieved by eliminating all tax evasion, it is not the only equilibrium in this model. The same results can also be obtained by allowing some tax evasion.

To see this result note that in the absence of audit costs, [34] and [36] can be combined to give

\[E\pi + T^n = \sum \int [R_i(q_i) - c_i(q_i, \gamma_i)] f(\gamma_i) d\gamma_i\]  \[43\]

Given that the government’s objective is to maximize social welfare subject to the revenue constraint of \(\bar{T}\), its problem can be equivalently given as that of maximizing

\[W = CS + E\pi + T^n - \bar{T} = \sum \int_0^{\gamma_i} p_i(\gamma_i) d\gamma_i f(\gamma_i) d\gamma_i - \sum \int c_i(q_i, \gamma_i) f(\gamma_i) d\gamma_i - \bar{T}\]  \[44\]
Equation [44] shows that welfare in this economy depends only on the equilibrium output. Among the policy parameters of the government, the price ceiling $\bar{p}$ is the only one that affects output of the monopolists. So it is clear that welfare would be independent of the choice of of $\alpha$, $\theta$ and $t$. Regarding the optimal price ceiling it is clear that it should be set where

$$\frac{\partial W}{\partial q_l} = \bar{p}_l - \int c'(q_l, \gamma_l)f(\gamma_l)d\gamma_l = 0$$  \[45\]

Equation [45] says that the price ceiling for the $i$th monopolist should be set at the level where it equals the expected marginal costs of production for that monopolist.

Given that there are no (resource) costs associated with setting the audit and penalty rates, these then could be set high enough to eliminate tax evasion ($\alpha \theta > 1$) and the profit tax rate $t$ could be selected to meet its revenue requirement, so that

$$\bar{T} = t \sum [R_i(q_l) - c_i(q_l, \gamma_l)f(\gamma_l)d\gamma_l$$  \[46\]

Note however that this is not the only solution. The government could also allow tax evasion and compensate for the loss in expected revenues due to evasion by setting a higher profit tax rate. This would imply that $\alpha$ and $\theta$ are set so that $1 > \alpha \theta$, the ceilings $\bar{p}_l$ is set to satisfy [45] and the profit tax rate $t$ is set to satisfy the revenue constraint

$$\bar{T} = t \sum [R_i(q_l) - (1 + \mu(1 - \alpha \theta))c_i(q_l, \gamma_l)f(\gamma_l)d\gamma_l$$  \[47\]

Comparing [46] and [47] it is clear that so long as $1 > \alpha \theta$ the profit tax rate in [46] must be higher than that in [46]. Naturally, the profit tax rate has to be higher when tax evasion is allowed for as compared to the no-evasion case, to compensate for the loss in revenues due to evasion.
The fact that the optimality can be achieved even with tax evasion is not very difficult to understand in the context of this model. Since output is not affected by either the penalty rate, the audit rate or the tax rate, it is clear that the optimal ceiling is set at the level where it equals expected marginal cost. Once this is done however, there are many ways to raise the required revenue $\bar{T}$. Thus the penalty and audit rates could be set low enough to allow tax evasion and the profit tax rate could be set high enough to raise the required revenue. So long as the government sets its instruments to raise just $\bar{T}$ in expected revenues, expected profits of the monopolists remain the same so that social welfare is not affected.

Tax evasion literature however usually assumes that there are some costs associated with eliminating tax evasion. Thus audits are assumed to involve resource costs. Also it is usually assumed that there are social and political costs to raising the penalty rate. So it is assumed here that the penalty rate cannot exceed some upper limit $\tilde{\theta}$. Under these circumstances it is clear that eliminating tax evasion may not be the optimal policy. There are two interesting questions that arise however. The first is what the optimal price ceiling should be in the presence of tax evasion and second, is if there is a trade-off between selecting the audit and the penalty rates. These questions are addressed below.

From the previous analysis it should be clear that the answer to the first question is that the price ceiling for each monopolist should be set where it equals the expected marginal costs of production. To see this from the first order conditions note that so long as firm $i$ reports some positive profits to the government, $[R(q_i) - \{1 + \mu_i(1 - a\theta)\}c_i(q_i)] > 0$. Also from proposition 3.3 it is clear that $(1 - a\theta)c_i(q_i)(\partial \mu_i/\partial t) < 0$, so long as the ith monopolist’s preferences exhibit non-increasing absolute risk aversion. Therefore from [38] it is clear that $\partial M/\partial t = 0$ if and only if $\phi = 1$. Setting $\phi = 1$ in equation [39], the result follows. Because the possibility of tax evasion has no impact on the output decisions of the firm, the optimal output can be achieved by setting the ceiling where it equals the expected marginal cost.
To answer the second question, suppose that the penalty rate is increased and the audit rate is decreased simultaneously to keep expected tax revenues of the government constant. If such a change leads to an unambiguous increase in welfare, then it should be clear that the government will always have the incentive to reduce the audit rate.

Total differential of [36] holding $T^n$, $t$, and $\bar{p}_i$ (i.e., $q_i$) as constant yields

$$\left[\sum \int \left\{ t\alpha \mu_i c_i(q_i, \gamma_i) - t(1 - \alpha \theta) c_i(q_i, \gamma_i) \frac{\partial \mu_i}{\partial \theta} \right\} f(y_i) dy_i \right] d\theta$$

$$= \left[ \sum \int \left\{ t\alpha \mu c_i(q_i, \gamma_i) - t(1 - \alpha \theta) c_i(q_i, \gamma_i) \frac{\partial \mu_i}{\partial \alpha} \right\} f(y_i) dy_i - g'(\alpha) \right] dx$$

[48]

Total differential of [35] yields

$$dW = - \left[ \sum \int \left\{ t\alpha \mu c_i(q_i, \gamma_i) - t(1 - \alpha \theta) c_i(q_i, \gamma_i) \frac{\partial \mu_i}{\partial \theta} \right\} f(y_i) dy_i \right] d\theta$$

$$- \left[ \sum \int \left\{ t\alpha \mu c_i(q_i, \gamma_i) - t(1 - \alpha \theta) c_i(q_i, \gamma_i) \frac{\partial \mu_i}{\partial \alpha} \right\} f(y_i) \right] dx \right]$$

[49]

Equations [48] and [49] can be combined to give

$$dW = - g'(\alpha) dx$$

[50]

From [50] it is clear that welfare can be increased unilaterally by reducing the audit rate (and increasing the penalty rate), so long as audits are costly. Note however that this does not necessarily imply that the government would always set $\theta = \bar{\theta}$. In fact, from equation [41] it is clear that so long as $\phi = 1$, $\partial M/\partial \theta$ is always zero. As seen already for the case where audits are costless, the
government has a number of ways of raising its required revenues. So the constraint on the penalty rate may not necessarily be binding.

From equation [40] it is clear that so long as audits are costly, the government will have an incentive to reduce it. However it is also clear that some audits have to be made if the revenue constraint is to be satisfied.

It is interesting to compare the results on the optimal price ceiling set in this model as well as the profit tax rate, with a model where there is no tax evasion. Regarding the price ceiling, it is shown in Appendix A (Case 2) that the optimal policy in a model where the government is fully informed about monopolists’ price, output and costs, is to set the price ceiling at that level where it equals the marginal cost of production. The difference thus is that with asymmetry of information between the government and the monopolists in this model, price ceiling is set equal to expected marginal cost. This can be taken to be the certainty equivalent of the no tax-evasion case.

Regarding the optimal profit tax rate it is seen that this depends on the choice of the audit and the penalty rates. So long as tax evasion exists, the profit tax has to be higher than in the no evasion case.

2.5. Conclusion

The chapter attempted to bring together the literature on tax evasion and regulation. In spite of the simplistic nature of the model, it managed to highlight a number of interesting problems. One of the interesting results observed in the paper was that a reduction in the effective price ceiling led to a reduction in tax evasion also. So tax revenues are likely to be affected by price ceiling adjustments by the regulatory authorities. However, the analysis showed that optimal regulatory policies are not affected by tax evasion considerations since their optimal price ceiling policies do not change.
in the presence of tax evasion. Lastly, it was also found that optimality in this model can be achieved with or without evasion (even when government can reduce tax evasion costlessly).

Many of the findings of this model however depend crucially on the exact nature of the regulatory policy considered and the nature of the asymmetry of information between the government and the monopolist. The regulatory instrument considered here was the price. Since this could be observed costlessly by the government, there was no scope for cheating on this instrument. While the price ceiling regulation had received a lot of attention by economists in recent years and have been been widely advocated by some economists like Littlechild [1983], because of its advantages over some of the other regulatory policies, this form of regulation has rarely been used in practice in its pure form. Usually, the ceiling is adjusted periodically to allow the monopolist to earn some specified level of profits. Once this is done however, the monopolist has the opportunity to affect the ceiling to be set, by misreporting its costs. The results in this case are quite different from that of this chapter. The next chapter looks at the monopolist’s and the social planner’s problem for the case where the planner attempts to regulate the activity of the monopolist by setting a ceiling on amount of profits the monopolist can earn, per unit of output.

Another point to note is that in practice there could also be asymmetry of information on the demand side. Thus the firm may well have more accurate information on the demand for its product as compared to the regulator. The outcome in that situation could turn out to be quite different from that obtained in this chapter.
Chapter 3. Fixed profit-per-unit regulation and tax evasion

3.1. Introduction

The literature on regulation has considered a large number of regulatory policies. Chapter 2 considered the price ceiling regulation in an environment where the monopolist subject to regulation is also subject to profit taxes and tax evasion is possible. Though theoretically, under the price ceiling regulation, the objective of the government is to maximize some measure of social welfare, in practice this is rarely done without taking into consideration the profit margin of the regulated firm. In fact the price-cap regulation, as it is administered, is adjusted periodically and these adjustments may at least partially reflect changes in profitability of the regulated firm. So it might
be interesting to look at another form of regulation, namely a fixed profit-per-unit regulation, where the firm is allowed to earn profit at a fixed rate for each unit it sells. This chapter considers a model where firms are subject to this form of regulation and examines the impact of such regulation when misreporting of costs is allowed for.

Several interesting results emerge from the analysis. First and most important, unlike in the case of the price ceiling regulation, the firm can cheat on the profit constraint itself, because profits are not costlessly observable by the government. As a result of this, the output decisions and the tax evasion decisions of the firm become inextricably intertwined. Alterations in tax enforcement parameters then have direct impact on the output produced by the monopolist. Second, changes in the profit tax rate results in a change in the equilibrium output of the monopolist. In fact an increase in the profit tax leads to a reduction in equilibrium output of the monopolist under some assumptions on preferences. The profit tax rate is thus not neutral with respect to output as is usually observed for the no evasion case and also in the previous chapter. Third, with regard to optimal policy it is seen that when all the parameters of the government can be freely and costlessly adjusted, optimality can be achieved with full compliance. However this is not the only possibility. Optimality can also be achieved by allowing some evasion. Thus whereas in a model with full information, the price ceiling regulation and the fixed profit-per-unit regulation might be identical in terms of their effects, they are quite different in terms of their effects when there is asymmetry of information between the firm and the government and cheating is allowed for. These results show that the possibility of evasion should be taken into account when considering optimal regulatory policies. Implementation of tax and regulatory policies independently may lead to sub-optimal solutions.

The chapter is organized as follows. Section 2 sets up the problem of the monopolist and looks at the effects of alterations in the government policy parameters on the monopolist’s output and tax evading decisions. Section 3 sets up the social planner’s problem and considers the optimal policies of the government. Section 4 considers some alternative penalty functions and the
monopolist’s optimal choice under these policies. The last section provides some concluding remarks.

3.2. The Monopolist’s Problem

The basic model is set out in the same pattern as in chapter 2, with a monopolist producing a single output at a cost \( c(q) \). This cost is not observable by the government directly. It can only be known accurately, if the government audits the monopolist. Audits however involve expenditure in real resources so that due to budgetary reasons only a certain fraction of all tax returns are audited, returns being chosen at random for audits. The government is assumed to be fully informed about the monopolist’s demand and revenues. The monopolist is subject to a profit tax by the government at the proportional rate of \( t \). It is assumed that the way the tax revenues are spent has no impact on the monopolist’s decision making. The government also regulates the activity of the monopolist by imposing a fixed profit per unit ceiling which requires that the firm cannot earn more than \( \pi \) in profits (before taxes) for each unit sold, \( \bar{\pi} \) being the ceiling set.\(^6\) The profit ceiling is assumed to be effective in the sense that

\[
\pi(\tilde{q}) \equiv R(\tilde{q}) - c(\tilde{q}) > \bar{\pi}\tilde{q}
\]

where \( R(.) \) represents the revenue function of the monopolist and \( \tilde{q} \) represents the monopolist’s optimal output in the absence of regulation. This condition says that the profit ceiling is binding when the government is fully informed about the monopolist’s costs and revenues. If this is not true, the presence of the profit ceiling regulation will have no impact on the monopolist’s choice so that the ceiling would be ineffective. [The situation of a tax evading monopolist without any

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\(^6\) The fixed profit per unit regulation is different from the rate of return regulation where the the firm is allowed to earn a “fair” return on its investment. As Bailey and Malone [1970] has shown, profit per unit regulation is theoretically “superior” to the rate of return regulation since it does not distort input choices but induces the firm to produce more output as the rate of return regulation does.
regulation has been considered by Kreutzer and Lee (1986) and their main results are summa-
in Appendix B (Case 1).]

In this model, due to the asymmetry of information between the monopolist and the government, it is possible for the monopolist to misreport its profits by giving a false report of costs. Thus if the monopolist decides to overreport costs by a certain fraction $\mu$, it will report costs as $c'(q) = (1 + \mu)c(q)$ and hence report profits as $\pi^r = [R(q) - c'(q)]$. Whereas there is no legal restriction on the amount of profits the monopolist can report, it is assumed that if the monopolist reports more profits than the ceiling amount of $\bar{\pi}q$, it has to pay government the excess amount. This effectively eliminates the possibility of reporting more profits than the ceiling level. Thus when reporting profits, the firm is subject to the constraint that

$$\pi^r = R(q) - (1 + \mu)c(q) \leq \bar{\pi}q$$  \[2\]

Before proceeding further, note a number of points.

First, in this model, because costs are not observable by the government, the same is true for profits, so that the monopolist can report less profits than actually earned and thus essentially cheat on the regulatory constraint. In this sense the effect of the profit per unit regulation is quite different from that of the price ceiling regulation discussed in the previous chapter where the monopolist could not cheat on the regulatory constraint itself.

Second, it should be clear that reporting ceiling profits does not imply truthful reporting. The monopolist could report profits as $\pi^r = R(q) - (1 + \mu)c(q) = \bar{\pi}q$ while earning profits of $R(q) - c(q)$.

Third, even though the profit tax and the profit ceiling are levied on actual profits, because of the government cannot observe actual profits before audits, they really become applicable to reported profits of the firm.
Fourth, it should be clear that in this model the less profits the monopolist reports, the lower is his tax liability in the event that he is not audited. Because of this, it is possible that the monopolist might like to report less profits than is required, so that the constraint [2] need not hold with equality.

Whereas the monopolist has the option of misreporting profits by misreporting costs, such misreporting puts him at a risk of being audited. If he is audited, his true profit is identified and he has to pay a penalty for noncompliance. It is assumed here that the monopolist has to pay $\theta$ times profits earned in excess of that reported ($\theta > 1$), as penalty, over and above taxes already paid.\footnote{The tax-penalty scheme chosen here is of course one of the many possible schemes that can be devised. Some alternatives are considered later in section 4.}

Under these circumstances a monopolist who misreports costs faces two possible states of nature. One possibility is that he misreports costs and is not audited. Under these circumstances his net profits are

$$\pi^{nd} = (1 - \theta)[R(q) - c(q)] + \theta u(q)$$

Another possibility is that he misreports costs and is audited. In this situation his net profits are

$$\pi^d = (1 - \theta)[R(q) - c(q)] - (\theta - 1) \mu c(q)$$

The monopolist’s problem then is to choose $q$ and $\mu$ so as to maximize his expected utility from net profits, given by

$$EU = \alpha U(\pi^d) + (1 - \alpha) U(\pi^{nd})$$

where $\alpha$ is the monopolist’s perceived probability of being audited.\footnote{If the monopolist is fully informed about the government’s strategies, this will coincide with the actual probability of being audited.} The monopolist is subject to the constraints [2], [3] and [4]. Regarding the utility function, it is assumed that $U(.) > 0$. The
monopolist is also assumed to be risk averse, so that \( U''(\cdot) < 0 \), where primes denote derivatives. Regarding the penalty rate \( \theta \) and the audit probability \( \alpha \) it is assumed (as in the previous chapter) that these are exogenous (from the monopolist’s point of view) and independent of the volume of taxes evaded.

The Lagrangian for the monopolist’s optimization problem then takes the form

\[
L = \alpha U[(1 - t)(R(q) - c(q)) - (\theta - t)\mu c(q)] + (1 - \alpha)U[(1 - t)(R(q) - c(q)) + t\mu c(q)] \\
+ \lambda[\bar{\pi}q - R(q) + (1 + \mu)c(q)]
\]

where \( \lambda \) is the Lagrange multiplier.

The first order conditions for a maximum require that

\[
\frac{\partial L}{\partial \mu} = (1 - \alpha)tU'(\pi^{nd}) - \alpha(\theta - t)U'(\pi^{d}) + \lambda = 0 \tag{7}
\]

\[
\frac{\partial L}{\partial q} = \alpha[(1 - t)(R' - c') - (\theta - t)\mu c']U'(\pi^{d}) + (1 - \alpha)[(1 - t)(R' - c') + t\mu c']U'(\pi^{nd}) \\
+ \lambda[\bar{\pi} - R' + (1 + \mu)c'] = 0 \tag{8}
\]

and,

\[
\frac{\partial L}{\partial \lambda} = \bar{\pi}q - R(q) + (1 + \mu)c(q) = 0 \tag{9}
\]

where \( R' = \frac{\partial R(q)}{\partial q} \) and \( c' = \frac{\partial c(q)}{\partial q} \).

A number of results follow immediately from the model. These are presented in the form of the following propositions.
Proposition 1.1: The monopolist will always report the maximum amount of profits he is allowed to earn, i.e., \( \pi' = \pi q' \), if, (i) the profit per unit ceiling is effective (so that equation [1] holds), and (ii) \( a\theta > t \).

Proof: Suppose that the monopolist does not report the maximum profits he is allowed to earn, so that \([2]\) does not hold with equality. Then \( \lambda = 0 \).

Now since \( \pi^d \geq \pi^u \) diminishing marginal utility implies that \( U'(\pi^d) \geq U'(\pi^u) \).

Also, if \( a\theta > t \), it is clear that \( a(\theta - t) > (1 - a)t \).

Therefore from \([7]\) it is clear that \( \partial L/\partial \mu < 0 \) so the monopolist would choose \( \mu = 0 \) at the optimum.

Under these circumstances, from \([8]\) it is clear that the firm’s optimal output would be where \( R' = c' \), or \( q = \tilde{q} \).

However it is impossible to have both \( \mu = 0 \) and \( q = \tilde{q} \) if the profit ceiling condition \([1]\) is to be satisfied.

Therefore it must be that \( \lambda > 0 \) and \( \pi' = \pi q' \).

The result can be easily explained by looking at how the net expected profits of the monopolist is affected by reporting less profits than is required. Suppose the monopolist decides to reduce reported profits by \$1. In that case he saves \$t in taxes in both states (because taxes are calculated on reported profits, both when detected and not detected). However with probability \( a \) (when he is detected), the monopolist has to pay \$\theta as penalty for evasion. So the net expected gain from underreporting profits by \$1 is \$\left(t - a\theta\right). This is negative when \( a\theta > t \) so that under these circumstances the monopolist will report just as much profits as required.

Proposition 1.2: At the monopolist’s optimum output choice \( q^* \), \( R' - c' < 0 \) if \( a\theta > t \) and the profit ceiling is effective.

Proof: Let \( \tilde{q} \) denote the level of output for which \( R' = c' \). Then from \([8]\) it is clear that
\[
\frac{\partial EU}{\partial q} \bigg|_{q = \tilde{q}} = (1 - \alpha)\mu c'U'(\pi^{nd}) - \alpha(\theta - t)\mu c'U'(\pi^d) + \lambda[\bar{\pi} + \mu c'] \tag{10}
\]

From (10) it is clear that if \( \mu = 0 \),

\[
\frac{\partial EU}{\partial q} \bigg|_{q = \tilde{q}} = \lambda \bar{\pi} > 0
\]

so that the optimal policy in this case is to produce more output that the no-ceiling case. Therefore \( R' < c' \) in this case. On the other hand if \( \mu > 0 \), then equation (7) implies that

\[
\lambda = \alpha(\theta - t)U'(\pi^d) - t(1 - \alpha)U'(\pi^{nd}) \tag{11}
\]

Substituting this in (10) gives

\[
\frac{\partial EU}{\partial q} \bigg|_{q = \tilde{q}} = \bar{\pi}[\alpha(\theta - t)U'(\pi^d) - (1 - \alpha)tU'(\pi^{nd})] \tag{12}
\]

For \( \mu > 0, \pi_d < \pi_{nd} \), so that assuming diminishing marginal utility, \( U'(\pi_d) > U'(\pi_{nd}) \). From (14) it is clear then, that so long as \( \alpha \theta > t \), \( \partial EU/\partial q|_{\tilde{q}} > 0 \). This implies that the firm's optimal output \( q^* > \tilde{q} \). Since \( R' = c' \) at \( q = \tilde{q} \), it then follows that at \( q = q^*, R' < c' \).

The result that \( R' - c' < 0 \) at the monopolist's optimum is an important one as it is needed subsequently to prove a number of propositions. The result however is also intuitively appealing as it indicates that the monopolist would choose to produce more output than in the absence of regulation, in which case the monopolist would produce where \( R' = c' \). [A proof of the no-regulation case is provided in Appendix B, (Case 1).] Recall that the same result was obtained for the case of the price ceiling regulation.

Propositions 1.1 and 1.2 together show that the firm would produce more output than without regulation and would also report the maximum profits possible. So it remains to show under what circumstances the monopolist would evade some taxes so that \( \mu > 0 \). This is shown in proposition 1.3.
**Proposition 1.3**: A sufficient condition for the monopolist to misreport costs (i.e., choose \( \mu > 0 \)) is that \( 1 > a\theta > t \).

Proof: The monopolist will cheat so long as expected gains from doing so exceeds the gains without cheating. In other words a sufficient condition for cheating requires that

\[
E\pi \equiv \alpha \pi^d + (1 - \alpha)\pi^n d > (1 - t)\pi q
\]

Given that \( a\theta > t \), the firm will report ceiling profits so that the above condition implies that

\[
[R(q) - c(q) - \pi q][1 - a\theta] > 0
\]

[13]

So long as \( 1 > a\theta \), inequality [13] will be satisfied for \( \mu > 0 \). Hence the individual will misreport costs under these circumstances.

Proposition 1.3 is intuitively clear. With profits being reported at the ceiling level, the gain from misreporting costs by \( \mu \) is that it enables the monopolist to enjoy \( \mu c(q) \) more profits than the ceiling allows. However with probability \( \alpha \) the monopolist loses \( \theta \mu c(q) \) for such misreporting, so that the net expected gain from misreporting is \( (1 - a\theta)\mu c(q) \). This is positive so long as \( 1 > a\theta \) in which case the monopolist will misreport costs and \( \mu \) will be positive.

Propositions 1.1 and 1.3 together imply that if \( 1 > a\theta > t \), the monopolist will misreport costs and profits. However he will never report less profits than the ceiling level. So while \( \mu > 0 \), the upper limit is set by the ceiling. Thus, while the government can use the profit-per-unit ceiling to induce the monopolist to produce more output, it cannot control output completely since the monopolist can and does cheat on the regulatory constraint. [Control will be complete only in the case where the government can and does eliminate evasion completely]. Hence, so long as evasion is present, output produced will be less than that in the absence of tax evasion. This, again, is different from the case of the price ceiling regulation where output can be controlled completely by regulation.
For the subsequent analysis, it is assumed that the parameter values are such that the condition 
\( l > \alpha \theta > t \) is satisfied so that \( \pi' = \pi q \) and \( \mu > 0 \).

Given that the monopolist reports profits at the ceiling level, it follows immediately that the monopolist's output and misreporting decisions are completely interdependent. Once \( q \) is decided on, \( \mu \) is automatically determined. In this sense the fixed profit-per-unit regulation is very different from that of the price ceiling regulation where the output and the misreporting decisions are not completely interdependent. With equation [2] holding with equality, it can be totally differentiated to give

\[
\frac{\partial \mu}{\partial q} = \frac{(R' - (1 + \mu)c' - \pi)}{c(q)} \tag{14}
\]

From [14] it is clear \( \partial \mu/\partial q < 0 \) because \( R' - c' \leq 0 \) at the monopolist's optimum.

Under these circumstances, the monopolist's problem boils down to that of choosing \( q \) to maximize his expected utility given by

\[
EU = \alpha U[(\theta - t)\pi q - (\theta - 1)(R(q) - c(q))] + (1 - \alpha)U[R(q) - c(q) - \bar{\pi}q] \tag{15}
\]

with \( \mu \) being determined from the constraint equation [2].

The F.O.C. for a maximum are

\[
\frac{\partial EU}{\partial q} = \alpha U'(\pi^d)[(\theta - t)\pi - (\theta - 1)(R' - c')] + (1 - \alpha)U'(\pi^d)[R' - c' - \pi] = 0 \tag{16}
\]

and the S.O.C. for an interior maximum requires that

\[
\frac{\partial^2 EU}{\partial q^2} = \Delta = (1 - \alpha)(R' - c' - \bar{\pi})^2 U''(\pi^d) + (1 - \alpha)(R'' - c'')U'(\pi^d) \\
+ \alpha[(\theta - t)\pi - (\theta - 1)(R' - c')]^2 U''(\pi^d) - \alpha(\theta - 1)(R'' - c'') U'(\pi^d) < 0 \tag{17}
\]
where $R'' \equiv \partial R' / \partial q$ and $c'' \equiv \partial c' / \partial q$.

It is assumed that the second order condition is satisfied.

**Comparative static results:**

The model can now be used to look at the impact of changes in the parameters of the government on the monopolist’s optimal choice. These comparative static results are summarized in the form of the propositions below.

**Proposition 2.1:** An increase in the probability of detection $\alpha$ leads to an increase in the equilibrium output of the monopolist and a reduction in the fraction by which costs are overreported. In other words $\partial q / \partial \alpha > 0$ and $\partial \mu / \partial \alpha < 0$.

Proof: Total differential of the [16] yields $\frac{dq}{d\alpha} = \frac{A}{\Delta}$ where

$$A \equiv [R' - c' - t\bar{\pi}]U'(\pi^d) - [(\theta - t)\bar{\pi} - (\theta - 1)(R' - c')]U''(\pi^d)$$

[18]

and $\Delta$ is as given by equation [17]. If $R' - c' \leq 0$, it is clear that $A < 0$. Thus, so long as the second order condition is satisfied, $\partial q / \partial \alpha > 0$. The second part of the proof follows directly from equation [14].

**Proposition 2.2:** An increase in the penalty rate $\theta$ leads to an increase in the equilibrium output of the monopolist and a fall in the fraction $\mu$ by which costs are overreported. In other words, $\partial q / \partial \theta > 0$ and $\partial \mu / \partial \theta < 0$.

Proof: Total differential of [16] yields $\frac{dq}{d\theta} = \frac{B}{\Delta}$ where

$$B \equiv \alpha[R(q) - c(q) - \pi q][(\theta - t)\bar{\pi} - (\theta - 1)(R' - c')]U''(\pi^d)$$

$$- \alpha[\pi - (R' - c')]U'(\pi^d)$$

[19]

It is clear that $B < 0$ so long as $R' - c' \leq 0$. Therefore the proposition holds.
Propositions 2.1 and 2.2 show that stricter enforcement leads to lower tax evasion as is expected and obtained in most of the literature on tax evasion, including the case of the price ceiling regulation. However, the propositions also indicate that a change in the penalty rate or the audit rate leads to a change in output. In fact, stricter enforcement while reducing tax evasion on the one hand leads to an increase in output also. In the case of the price ceiling regulation, output was not affected at all by changes in the audit rate or the penalty rate.

An implication of the two propositions is that the profit-per-unit regulation is not completely effective in controlling output in the presence of tax evasion. So the possibility of tax evasion should be taken into account when optimal regulatory policies are to be considered.

**Proposition 2.3**: An increase in the proportional profit tax rate leads to a decrease in the equilibrium output of the monopolist and a rise in the fraction by which costs are overreported, if the preferences of the monopolist exhibit non-decreasing absolute risk aversion. That is, \( \frac{dq}{dt} < 0 \) and \( \frac{\partial \mu}{\partial t} > 0 \) if \( R_A(d) \leq R_A(nd) \).

Proof: Total differential of [16] yields
\[
\frac{dq}{dt} = \frac{C}{\Delta}
\]
where,

\[
C \equiv \pi [\alpha U' (\pi^d) + (1 - \alpha) U' (\pi^{nd})] + (1 - \alpha) [R' - c' - \pi]\left[ R(q) - (1 + \mu)c(q) \right] U'(\pi^{nd})[R_A(d) - R_A(nd)]
\]  \[20\]

From [20] it is clear that \( R_A(d) \leq R_A(nd) \) is a sufficient condition for \( \frac{dq}{dt} \) to be negative. In this case \( \frac{\partial \mu}{\partial t} > 0 \).

The Arrow-Pratt measure of absolute risk aversion however is usually assumed to be non-increasing. In this case, the impact of changes in the profit tax rate on output and tax evasion is ambiguous.

Proposition 2.3 shows however that in this model a proportional profit tax is not neutral in terms of its effects on output as was obtained for the price ceiling regulation.
Proposition 2.4: The effect of an increase in the profit ceiling of the government on output is ambiguous.

Proof: Total differential of \cite{[16]} yields \( \frac{dq}{d\pi} = \frac{E}{\Delta} \) where,

\[
E \equiv (1 - \alpha)tU''(\pi^{nd}) - \alpha(\theta - t)q[(\theta - t)\pi - (\theta - 1)(R' - c')]U''(\pi^d)
\]

\[+ (1 - \alpha)tq[R' - c' - t\pi]U''(\pi^{nd}) - \alpha(\theta - t)U''(\pi^d)\] [21]

From \cite{[21]} it is clear that the sign for \( E \) is ambiguous in general.

Proposition 2.4 is somewhat surprising. A relaxation of an effective ceiling should imply that the monopolist would reduce output and enjoy more profits. This was the result observed for the price ceiling regulation but the same result does not hold here. One explanation for this difference could be that an increase in the profit ceiling, while inducing the monopolist to reduce output to make more profits, could also provide an incentive to reduce evasion activities, so that the net effect becomes ambiguous.

From the first order condition is possible to obtain the optimal value of \( q \) as a function of the parameter values \( \alpha, \theta, t, \pi \). This value can then be substituted in the monopolist’s utility function to obtain the indirect utility of the monopolist given by

\[
V(\alpha, \theta, t, \pi) \equiv \alpha U[\pi^{d}(\alpha, \theta, t, \pi)] + (1 - \alpha)U[\pi^{nd}(\alpha, \theta, t, \pi)]
\] [22]

where

\[
\frac{\partial EV}{\partial \alpha} = U(\pi^d) - U(\pi^{nd}) < 0
\] [23]

\[
\frac{\partial EV}{\partial \theta} = -\alpha[R(q) - c(q) - \pi q]U''(\pi_d) < 0
\] [24]

\[
\frac{\partial EV}{\partial t} = -\pi q[\pi U''(\pi^d) + (1 - \alpha)U''(\pi^{nd})] < 0
\] [25]
\[
\frac{\partial EV}{\partial \bar{\pi}} = q[\alpha(\theta - t)U'(\pi^d) - t(1 - \alpha)U'(\pi^d)_{\pi}]
\]

[26]

From [26] it is clear that a sufficient condition for \(\partial EV/\partial \bar{\pi} > 0\) if \(\alpha\theta > t\).

It is also of interest to find out how changes in the policy parameters affect the net expected profits of a monopolist. Given that \(\alpha\theta > t\), so that the monopolist reports the maximum ceiling profits, the expected profits of the monopolist can be given by

\[
E\pi = (1 - \alpha\theta)[R(q) - c(q)] + (\alpha\theta - t)\bar{\pi}q
\]

[27]

where

\[
\frac{\partial E\pi}{\partial \alpha} = -\theta[R(q) - c(q) - \bar{\pi}q] + (1 - \alpha\theta)(R' - c') \frac{\partial q}{\partial \alpha} + (\alpha\theta - t)\bar{\pi} \frac{\partial q}{\partial \alpha}
\]

[28]

\[
\frac{\partial E\pi}{\partial \theta} = -\bar{\pi}q + (1 - \alpha\theta)(R' - c') \frac{\partial q}{\partial \theta} + (\alpha\theta - t)\bar{\pi} \frac{\partial q}{\partial \theta}
\]

[29]

\[
\frac{\partial E\pi}{\partial t} = (\alpha\theta - t)q + (1 - \alpha\theta)(R' - c') \frac{\partial q}{\partial t} + (\alpha\theta - t)\bar{\pi} \frac{\partial q}{\partial t}
\]

[30]

The signs of these expressions are in general ambiguous.

3.3. The Social Planner's Problem

The social planner's problem is set out in the same way as in the previous chapter. The economy is assumed to consist of \(N\) firms indexed by \(i (i = 1, 2, \ldots, N)\), \(N\) being large enough that the actual tax collections closely approximate expected collections. The government has at its disposal the pro-
portional profit tax rate $t$ and the profit per unit ceiling $\pi$, for each firm, and the audit and penalty rates $\alpha$ and $\theta$ respectively. As before, the government cannot observe the monopolist's actual costs but is aware that the ith monopolist's costs take the form $c_i(q_i, \gamma_i)$ where $\gamma_i$ is a parameter that is known to the monopolist but unknown to the government. The government believes that $\gamma_i$ takes the density function $f_i(\gamma_i)$ and acts on the basis of this distribution. It is assumed that $f_i(\gamma_i)$ is continuous in $\gamma_i$ with $f_i(\gamma_i) > 0$ over the interval $[\gamma_i^*, \gamma_i^1]$. Also it is assumed that the tax rate and the profit ceiling can be adjusted by the government without incurring any resource costs. However auditing tax returns involve a cost given by $g(\alpha)$ with $g'(\alpha) > 0$.9

Before studying the government's optimization problem explicitly, it is important to find out how alterations in its policy parameters affect its revenue collection. The total collections of the government consists of two parts. One is the tax collection and the other refers to the penalty collections. Regarding tax collections, since $\sum_{i=1}^{N} t_i \pi_i q_i$ is collected in taxes in both states, this is the total tax revenue of the government. It then follows immediately that an increase in the penalty rate and the audit rate increases actual tax collections. The effects of increase in the tax rate and the profit ceiling on tax revenues however are ambiguous.

The government however also gets some penalty revenue when it audits firms. The total (expected) revenue of the government (gross of any audit costs) then is

$$T^g = \sum_{i=1}^{N} \int_{0}^{\gamma_i^1} t_i \pi_i q_i f_i(\gamma_i) d\gamma_i + \sum_{i=1}^{N} \int_{0}^{\gamma_i^1} \alpha \theta [R_i(q_i) - c_i(q_i, \gamma_i) - \pi_i q_i] f_i(\gamma_i) d\gamma_i$$

[The limits of the integration for $\gamma$ and for the sums are dropped subsequently for notational simplicity].

From equation [32], it follows that

9 Audit costs also depend on $N$, the total number of firms to be audited, but because this is constant in this model, it is suppressed in the audit cost function.
\[
\frac{\partial T^g}{\partial \theta} = \sum \int [\alpha(R_i(q_i) - c'\theta) - (\alpha\theta - \theta)\bar{\pi}] \frac{\partial q_i}{\partial \theta} f_1(y_i)dy_i
\]

\[+ \sum \int [\alpha\theta(R_i - c'\theta) - (\alpha\theta - \theta)\bar{\pi}] \frac{\partial q_i}{\partial \theta} f_1(y_i)dy_i \tag{33}\]

\[
\frac{\partial T^g}{\partial \alpha} = \sum \int [\alpha\theta(R_i - c'\theta) - (\alpha\theta - \theta)\bar{\pi}] \frac{\partial q_i}{\partial \alpha} f_1(y_i)dy_i
\]

\[+ \sum \int [\alpha\theta(R_i - c'\theta) - (\alpha\theta - \theta)\bar{\pi}] \frac{\partial q_i}{\partial \alpha} f_1(y_i)dy_i \tag{34}\]

\[
\frac{\partial T^g}{\partial \bar{\pi}} = \sum \int \bar{\pi} q_i f_1(y_i)dy_i + \sum \int [\alpha\theta(R_i - c'\theta) - (\alpha\theta - \theta)\bar{\pi}] \frac{\partial q_i}{\partial \bar{\pi}} f_1(y_i)dy_i \tag{35}\]

and

\[
\frac{\partial T^g}{\partial t} = - \int (\alpha\theta - \theta) q_i f_1(y_i)dy_i + \int [\alpha\theta(R_i - c'\theta) - (\alpha\theta - \theta)\bar{\pi}] \frac{\partial q_i}{\partial \bar{\pi}} f_1(y_i)dy_i \tag{36}\]

From (35) it is clear that \(\partial T^g/\partial t > 0\) so long as \(\alpha\theta \geq t\) and \(R_a(nd) \geq R_a(d)\).

Equation (36) implies that \(\partial T^g/\partial \bar{\pi} > 0\) if \(\partial q_i/\partial \bar{\pi} > 0\). However it is not clear whether \(\partial q_i/\partial \bar{\pi} > 0\) so that in general the impact of changes in the ceiling constraint on tax revenues is ambiguous. Similarly, from (33) and (34) it is clear that the impact of changes in the audit and the penalty rate on revenues is ambiguous.

The objective of the government is to choose its parameters so as to maximize aggregate consumers' surplus plus aggregate expected profits of the firms, subject to a fixed revenue constraint. Social welfare is thus given by
\[ W = CS + E\pi = \sum \int_0^q p(y_i)f(y_i) dy_i \]
\[- \sum \int [\alpha \theta R_i(q_i) + (1 - \alpha \theta) c_i(q_i, \gamma_i) - (\alpha \theta - \iota)\bar{e}_i q_i] f(y_i) dy_i \]

The revenue constraint of the government is

\[ \sum \int (\bar{e}_i q_i f(y_i) dy_i + \sum \int [\alpha \theta R_i(q_i) - c_i(q_i, \gamma_i) - \bar{e}_i q_i] f(y_i) dy_i - g(\alpha) \geq \bar{T} \]

where \( \bar{T} \) refers to the fixed revenue requirement of the government.

The Lagrangian for the maximization problem of the government is then given by

\[ L = \sum \int \int_0^q p(y_i)f(y_i) dy_i \]
\[- \sum \int [\alpha \theta R_i(q_i) + (1 - \alpha \theta) c_i(q_i, \gamma_i) - (\alpha \theta - \iota)\bar{e}_i q_i] f(y_i) dy_i + \sum \int \phi [\bar{e}_i q_i + \alpha \theta (R_i(q_i) - c_i(q_i, \gamma_i)) - a \theta \bar{e}_i q_i - g(\alpha) - \bar{T}] f(y_i) dy_i \]

where \( \phi \) is the Lagrange multiplier.

The first order conditions for a maximum are

\[ \frac{\partial L}{\partial \alpha} = (\phi - 1) \sum \int [\theta [R_i(q_i) - c_i(q_i, \gamma_i) - \bar{e}_i q_i] f(y_i) dy_i + \sum \int [p(q_i) - c_i(q_i)] \frac{\partial q_i}{\partial \alpha} f(y_i) dy_i \]
\[ + (\phi - 1) \sum \int [\alpha \theta (R_i(q_i) - c_i(q_i))] - (\alpha \theta - \iota)\bar{e}_i q_i \frac{q_i}{\partial \alpha} f(y_i) dy_i - \phi g'(\alpha) = 0 \]

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\[
\frac{\partial L}{\partial \theta} = (\phi - 1) \left( \sum \left[ a[R_b(q_i) - c(q_i, y)] - \bar{\pi} q_i \right] f(y_i) dy_i \right) + \sum \left[ p_i(q_i) - c_i \right] \frac{\partial q_i}{\partial \theta} f(y_i) dy_i
\]

\[
+ (\phi - 1) \sum \left[ a \left( R_b'(q_i) - c_i' \right) - (a \theta - t) \bar{\pi} \right] \frac{\partial q_i}{\partial \theta} f(y_i) dy_i = 0
\]  

\[ [41] \]

\[
\frac{\partial L}{\partial t} = \sum \left[ p_i(q_i) - c_i' \right] \frac{\partial q_i}{\partial t} f(y_i) dy_i + (\phi - 1) \sum \bar{\pi} q_i f(y_i) dy_i
\]

\[
+ (\phi - 1) \sum \left[ a \left( R_b'(q_i) - c_i' \right) - (a \theta - t) \bar{\pi} \right] \frac{\partial q_i}{\partial t} f(y_i) dy_i = 0
\]

\[ [42] \]

\[
\frac{\partial L}{\partial \pi_i} = \int \left[ a \left( R_b'(q_i) - c_i' \right) - (a \theta - t) \bar{\pi} \right] \frac{\partial q_i}{\partial \pi_i} f(y_i) dy_i - (\phi - 1)(a \theta - t) \int q_i f(y_i) dy_i
\]

\[
+ (\phi - 1) \int \left[ a \left( R_b'(q_i) - c_i' \right) - (a \theta - t) \bar{\pi} \right] \frac{\partial q_i}{\partial \pi_i} f(y_i) dy_i = 0
\]

\[ [43] \]

And

\[
\frac{\partial L}{\partial \phi} = \sum \bar{\pi} q_i f(y_i) dy_i + \sum a \left[ R_b(q_i) - c(q_i, y) - \bar{\pi} q_i \right] f(y_i) dy_i - g(\alpha) - \bar{T} = 0
\]

\[ [44] \]

It was seen in the previous chapter that when there are no costs associated with setting the penalty and the audit rates, optimality could be achieved with and without evasion. One important characteristic of the price ceiling regulation was that the output level was unaffected by the profit tax, the audit rate and the penalty rate. This however is not true for the fixed-profit-per-unit regulation. So it is important to find out if the optimal policies in the absence of resource costs for audit and penalty, are any different from that for the price ceiling regulation.

Chapter 3. Fixed profit-per-unit regulation and tax evasion
As in the previous chapter, the government’s optimization problem can be reduced to that of maximizing

$$W = CS + E\pi + T^n - \bar{T} = \sum_i \int_0^{q_i} p_i(\gamma_i)f(\gamma_i)d\gamma_i d\gamma_1 - \sum c_i(q_i, \gamma_i)f(\gamma_i)d\gamma_1 - \bar{T} \quad [45]$$

From [45] it is clear that $W$ depends only on output. However, equilibrium output in this model, unlike the case of price ceiling regulation, depends on all parameters of the government, so long as tax evasion is present. So output in this model is also dependent on $\alpha$, $\theta$, and $t$.

From [45] it is clear that $W$ is maximized when

$$p_i(q_i) = \int c_i'\gamma_i, q_i) f(\gamma_i)d\gamma_i \quad \forall i \quad [46]$$

One way of achieving this optimum is to set $\alpha$ and $\theta$ high enough to eliminate tax evasion (i.e., $\alpha \theta > 1$). Under these circumstances output of the monopolists can be affected only by the profit ceilings $\pi_i$. (The profit tax becomes neutral with respect to output when there is no misreporting). So $\pi_i$'s can be set appropriately to ensure that [46] is satisfied. The profit tax is then set to meet the revenue constraint. Thus

$$\bar{T} = I\sum \pi_i q_i \quad [47]$$

This however is not the only equilibrium. It is possible to set the parameters in such a way that the same equilibrium output would be achieved but tax evasion would be present. Such an equilibrium would have to satisfy [46] and the revenue constraint

$$\bar{T} = \sum \pi_i q_i + \sum \alpha \theta \mu_i c_i(q_i, \gamma_i)f(\gamma_i)d\gamma_i \quad [48]$$
Given that the government has $N + 3$ instruments (the $N$ profit ceilings and $\alpha$, $\theta$ and $t$) at its disposal to meet the $N + 1$ constraints, it is not possible to rule out such an equilibrium.

It is clear that an equilibrium with tax evasion would require a lower profit ceiling than for the case of no evasion. For any given profit ceiling, evasion implies that less output is being produced than without evasion. So a lower ceiling is necessary in order to compensate the monopolist’s tendency to reduce output. Regarding the profit tax rate it is not clear whether this should be higher or lower than that without evasion.

It is easy to show how such an equilibrium can be achieved when all firms are assumed to be identical. In this case the parameters could be initially set so that equation [46] is satisfied for all firms. Then if the revenue constraint is not satisfied, $\theta$ and $t$ could be simultaneously adjusted keeping output of each firm constant. The desired revenue can be achieved through such adjustment. The problem of extending the same argument for the case of different firms is that in this case adjustments in $\theta$ and $t$ will affect different firms differently so that the outcome is not clear.

Costless audits and penalty however are assumed away as in most of tax evasion literature. It is usually assumed that for social and political reasons the penalty can be raised only up to some upper limit $\bar{\theta}$. Also audits are costly in resources and these costs are assumed to be an increasing function of the audit rate. Under these circumstances it important to find out if the constraint on $\theta$ is binding. To see this, suppose the penalty rate $\theta$ is increased and the audit rate $\alpha$ decreased simultaneously, to keep the expected revenues of the government constant.

Total differential of [38] holding $\bar{T}$, $\bar{\pi}$, and $t$ constant, yields

\[ [X - g'(\alpha)]d\alpha = Yd\theta \]  

where

Chapter 3. Fixed profit-per-unit regulation and tax evasion
\[ X \equiv \sum \left( \theta (R_i(q_i) - c_i(q_i, \gamma_i) - \bar{\pi} q_i) + \bar{\pi} \frac{\partial q_i}{\partial \alpha} + \alpha \theta (R_i' - c_i' - \bar{\pi}) \frac{\partial q_i}{\partial \theta} \right) f(\gamma_i) d\gamma_i \]  

and

\[ Y \equiv - \sum \left( \alpha (R_i(q_i) - c_i(q_i, \gamma_i) - \bar{\pi} q_i) + \bar{\pi} \frac{\partial q_i}{\partial \alpha} + \alpha \theta (R_i' - c_i' - \bar{\pi}) \frac{\partial q_i}{\partial \theta} \right) f(\gamma_i) d\gamma_i \]

To see the impact of such a change on social welfare, equation [37] can be totally differentiated to yield

\[ dW = Y d\theta - X d\alpha + \sum [p_i(q_i) - c_i'] \left\{ \frac{\partial q_i}{\partial \alpha} d\alpha + \frac{\partial q_i}{\partial \theta} d\theta \right\} f(\gamma_i) d\gamma_i \]  

Combining equations [49] and [52] and noting from propositions 2.1 and 2.2 that

\[ \frac{\partial q_i}{\partial \alpha} = \frac{A_i}{\Delta_i} \quad \text{and} \quad \frac{\partial q_i}{\partial \theta} = \frac{B_i}{\Delta_i} \]

where \( A_i, B_i, \) and \( \Delta_i \) are defined in [18], [19] and [17] respectively, shows that

\[ dW = \sum \frac{[p_i - c_i']}{\Delta_i Y} \left[ A_i Y + B_i X \right] f(\gamma_i) d\gamma_i - \left\{ 1 + \sum \frac{[p_i - c_i'] B_i}{\Delta_i Y} f(\gamma_i) d\gamma_i \right\} \delta'(\alpha) d\alpha \]

It is not possible to come up with an unambiguous sign for this expression because both \( X \) and \( Y \) have ambiguous sign. So it is not possible to conclude that the government will always have an incentive to reduce the audit rate and increase the penalty rate, if this leaves expected revenues unchanged. However it was seen that optimality can be achieved with tax evasion, so that an equilibrium could be achieved satisfying equations [46], [48] and the revenue constraint
\[ \bar{T} + \sum t\pi_i \phi_i + \sum \int \alpha \theta \mu \xi_i(q_i, \gamma) \nu_i(\gamma) d\gamma + g(\alpha) \]

Comparing the optimal policy of the government under the fixed profit per unit regulation with that of the price ceiling regulation considered in the previous chapter, it is clear that there are a number of similarities. First, in both cases, optimality is achieved if the monopolist produces output at the level where price equals expected marginal cost. Also in both cases this optimality can be achieved with and without evasion. The main difference between the two however is that while for the case of the price ceiling regulation the government has potentially infinite set of options in setting the audit, penalty and the tax rates to meet its revenue requirements once the optimal output is achieved by setting the price ceiling; for the profit per unit regulation this is not so. For the price ceiling regulation output depends on the choice of \( \alpha, \theta, \) and \( t, \) so long as evasion is present, the tax, audit and penalty parameters cannot be set independently of the profit-per-unit ceiling.

### 3.4. Alternative Tax-Penalty Schemes

The above analysis was based on a tax-penalty scheme where taxes are based on reported profits and the penalty is a proportion of profits earned in excess of that reported. In this model, however, the firms not only have the option of evading taxes but can also cheat on the regulatory constraint. So it is not very clear as to what the “correct” form of the penalty would be. Theoretically, it is possible to come up with a number of alternative tax-penalty schemes. Thus taxes could be imposed on after-penalty profits, reported profits, actual profits or ceiling profits. The penalty, on the other hand, could be calculated on the basis of actual profits in excess of the ceiling level. Most of these alternatives however result in corner solutions. This fact suggests that the results of the previous sections are quite sensitive to the specific form of the tax-penalty scheme chosen. Two alternative tax-penalty schemes are briefly discussed below to emphasize this point.
Alternative 1: One alternative is to calculate the penalty on the excess of actual profits earned over the ceiling level and tax ceiling profits. In this case the monopolist’s net profits when he evades taxes successfully is the same as before and is given by equation [3]. When he evades taxes and is detected, however, his net profits is given by

\[ \pi^d = (1 - \theta)[R(q) - c(q)] + (\theta - \hat{t}) \hat{\pi} q \]  

[54]

As before, the monopolist’s problem is to choose q and \( \mu \) to maximize his expected utility given by [5], subject to the regulatory constraint [2] and net profits in the two states being given by [3] and [54]. If the Lagrangian is given by \( L \), the monopolist’s optimization problem requires that

\[ \frac{\partial L}{\partial \mu} = (1 - \alpha)t c(q) U'(\pi^{nd}) + \lambda c(q) > 0 \]  

[55]

This shows that under this type of tax-penalty scheme, the monopolist’s optimal policy is to set \( \mu \) as high as possible. In other words, the monopolist’s optimal policy is to declare zero profits.

The result is not difficult to explain. The tax-penalty scheme here is set up in such a way that neither the penalty, nor the tax liability depends on reported profits. So there is no “loss” associated with reporting low or zero profits. For the tax-penalty scheme used previously however, there was an advantage of reporting ceiling profits (smaller tax liability in the case when detected) which induced the firm to report ceiling profits.

Given that the incentive is to report zero profits, it is clear that \( \lambda = 0 \) so that the monopolist’s optimal choice of q is given by

\[ \frac{\partial L}{\partial q} \bigg|_{q = \hat{q}} = \alpha(\theta - \hat{t}) \hat{\pi} U'(\pi^d) + t(1 - \alpha) \mu c' U'(\pi^{nd}) \]  

Therefore, \[ \frac{\partial L}{\partial q} \bigg|_{q = \hat{q}} = \alpha(\theta - \hat{t}) \hat{\pi} U'(\pi^d) + t(1 - \alpha) \mu c' U'(\pi^{nd}) \]  

[56]
Because at $q = \tilde{q}$ there is cheating, $\mu > 0$ so that it is clear from [56] that $\frac{\partial L}{\partial q} \bigg|_{q=\tilde{q}} > 0$. This implies that the monopolist would produce more output that $\tilde{q}$ so at at its equilibrium it must be that $R' - c' < 0$.

*Alternative 2*: Another alternative is to calculate the penalty on the basis of profits earned in excess of the ceiling level and tax actual profits. In this situation

$$\pi^d = (1 - \theta - t)(R(q) - c(q)) + \theta \pi q$$  \hspace{1cm} [57]

Net profits in the event he is not audited is given by [3] as before, and the monopolist is subject to the constraint [2]. Again if the Lagrangian is given by L, the monopolist’s optimization problem would yield

$$\frac{\partial L}{\partial \mu} = t(1 - \alpha)c(q)U'(#\pi^{nd}) + \lambda c(q) > 0$$  \hspace{1cm} [58]

so that the optimal policy of the monopolist is to set $\mu$ as high as possible, as in alternative 1 above. As in Alternative 1, neither the penalty, nor the tax liability depended on reported profits so that the firm has the incentive to report zero profits. Also

$$\frac{\partial L}{\partial q} \bigg|_{q=\tilde{q}} = \alpha(1 - t)(R' - c') + (1 - \alpha)((1 - t)(R' - c') + \mu c')U'(\pi^{nd})$$  \hspace{1cm} [59]

From [57] it is clear that

$$\frac{\partial L}{\partial q} \bigg|_{q=\tilde{q}} = \alpha \theta \pi U'(#\pi^{d}) + (1 - \alpha)\mu c' U'(\pi^{nd}) > 0$$  \hspace{1cm} [60]

so that the monopolist would produce more output than without regulation.
Both Alternatives 1 and 2 then provide incentives for the monopolist to report zero profits. [Assuming that there is is no system of getting subsidies by reporting negative profits, the firm gets no additional benefits by reporting negative profits.]

3.5. Conclusion

The chapter considered a model of a tax evading monopolist who is subject to a fixed profit per unit regulation. Whereas in a full information model, this type of regulation has basically the same effects as that of the price ceiling regulation, it is shown here that there are large differences when asymmetry of information between the government and the monopolist and cheating are allowed for. Whereas for the price ceiling regulation, changes in tax enforcement parameters have no impact on equilibrium output of the monopolist, this is not so for the fixed profit per unit regulation. In fact it is shown that stricter enforcement leads to increased output. Another interesting result of this paper is that adjustments in the profit tax rate affect equilibrium of the monopolist. The profit tax is thus not neutral in terms of resource allocation as is usually obtained. These facts imply that if the tax and the regulatory authorities undertake their policies independently, the final outcome might be sub-optimal inspite of their best efforts. So a coordination of the activities of the two authorities is important. With asymmetry of information, the effectiveness of regulatory policies is reduced so that under these circumstances the audit and penalty rates could be effectively used not only to reduce tax evasion but to enforce regulatory policies more effectively. Regarding the optimal regulatory policies of the government, it is shown that the government should set its parameters in such a way that each monopolist produces output for which price equals expected marginal cost of production.

On comparing the price ceiling and the fixed profit-per-unit ceiling it is clear that the government has more flexibility in the case of price ceiling regulation. Thus for the price ceiling regulation, once
the ceiling is set at the point where it equals marginal cost, there are many ways to raise the required revenues. For the fixed profit-per-unit regulation however the alternatives are not limitless. The basic difference arises from the fact that the price ceiling regulation can be costlessly enforced (because price is costlessly observable) while the fixed profit-per-unit regulation is not. The outcome however will change significantly if the government is not fully informed about revenue and output of the monopolist.
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4.1. Introduction

Though the literature on tax evasion has grown substantially in recent years, a point often ignored in this literature is that income from different sources are taxed at different rates and provide different opportunities for misreporting. Whereas the emphasis in the existing literature has been on wage income, this is often subject to reporting by the employer. This, along with the fact that a worker’s tax liability is often deducted at the source by withholding regulations, greatly limits a wage earner’s scope for evading taxes. Thus IRS [1988] reports:
“...when third parties report to the tax agency the income they pay to individuals, compliance in reporting such income markedly improves. Withholding tax at the source has proven to be an even more effective means of assuring compliance.”

Of course all types of wage income are not subject to withholding and even reporting. However once an individual has acquired some skills his job flexibility becomes limited so that the option to switch jobs to avoid reporting taxes may not always exist.

On the other hand, when investing his savings, an individual faces many options and the income from at least some of these may not be subject to withholding and reporting. Though the tax reform act of 1986 have reduced these opportunities to a large extent, there is evidence that tax evading opportunities still exist. Thus IRS [1988] reports that in 1987 the voluntary reporting percentage for wages and salaries was 99.5 percent while the same figure for capital gains was only 88.3 percent.

This difference in tax evasion opportunities suggest that savings of the individual may at least partially be affected by tax-evading opportunities. This chapter examines this issue by considering a dynamic model of tax evasion. Following the above argument it is assumed that the possibility of underreporting wage income is limited by the fact that this is subject to withholding and reporting regulations. However it is possible to underreport income arising from investment of savings in some assets, since these may not be subject to similar regulations. Using this framework it is shown that alterations in penalty rates or detection schemes of the government lead to changes in the savings of the individual and his reported income from savings. Moreover, it is shown that an increase in the penalty rate or audit probability leads to an increase in savings of the individual, given some assumptions on preferences. This result has important implications for an economy like the U.S., where the rate of savings out of personal income is quite low and the government often contemplates policies that would encourage domestic savings.
The government in this model has a number of policy instruments at its disposal, like the tax rates, the penalty rate and the audit probability. The objective of the government is to choose these policies so as to maximize the social welfare of the economy, subject to a fixed revenue constraint each period. Given this objective it is shown that the optimal policy of the government might be to allow some tax evasion, even if it can be reduced or eliminated costlessly. Thus full compliance may not necessarily be the socially optimal policy in this model. If individuals are compensated for the taxes collected from them each period it is shown that the optimal wage income tax is non-unique. Regarding the optimal profit tax rate it is shown that this must be positive.

The chapter is organized as follows. Section 2 presents the individual’s choice problem and derives the conditions for optimality and tax evasion. Section 3 presents the comparative static results for the state-controlled variables like the tax rates, the penalty rate and the audit rates. Section 4 discusses the objectives of the government and analyzes its optimization problem. The last section provides some concluding remarks.

4.2. The Model

A simple overlapping generations economy along the lines of Samuelson [1958] and Diamond [1965] is considered, where only one good is produced, a good that can be used both for consumption and capital formation. All individuals are identical in this economy and there is no population growth, so that the economy can be thought of as consisting of a single representative individual for each generation.

Each individual lives for two periods, working only in the first life-period, and saving for his second life-period when he retires. Assuming that individuals have no preferences for leisure, labor supply of the young would always equal their total labor endowment, taken to be one unit for simplicity.
The preference ordering of a generation $t$ consumer is assumed to be represented by a utility function which is additive time separable of the form,

$$U(c_t, e_{t+1}) = U(c_t) + \beta V(e_{t+1})$$  \hspace{1cm} [1]

where,

$c_t =$ consumption of a generation $t$ individual when young.

$e_{t+1} =$ consumption of the generation $t$ individual when old.

$\beta =$ the rate of discount. \(0 < \beta < 1\)

Marginal utilities $U'(c_t)$ and $V_{t+1}(\cdot)$ are assumed to be positive while the individual is assumed to be risk averse so that $U''(c_t) < 0$, $V'_{t+1}(\cdot) < 0$.

Production, in this economy, takes place in a single representative firm. Production possibilities are represented by a neoclassical production function of the form

$$y_t = f(k_t, l_t)$$  \hspace{1cm} [2]

assumed to be twice differentiable and quasi-concave, where $y_t$, $k_t$, and $l_t$ represent respectively the amount of output produced, amount of labor employed and amount of capital used for production in period $t$. There is no depreciation in this model and the firm is assumed to be a price taker in the labor market. Taking the single good as the numeraire, the wage rate at time $t$ is given by

$$w_t = \frac{\partial f(k_t, l_t)}{\partial l_t}$$  \hspace{1cm} [3]

At the beginning of period $t$, before production takes place, the old own all the capital. This capital is used in the production process, along with the labor that the old employs, to yield profits defined as

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\[ \pi_t = [f(k_t, l_t) - w_t l_t] \]  

[4]

Note that profits here are defined as (value of) output in excess of labor income. It thus differs from the usual definition where it refers to the residual after wage and capital income are paid.

Total amount of resources available for disposal (or consumption) by the old, after production and wage disbursement then is

\[ \pi_t + k_t = [f(k_t, l_t) - w_t l_t] + k_t \]  

[5]

The old are assumed to consume everything and leave no bequests. Note that savings (or capital) used in the production process at the beginning of period \( t \) remains available for consumption at the end of the period. Output \( f(k_t, l_t) \) thus represents net output. [The process could be interpreted as investing resources in a production process and getting some return for the use of these resources.]

At the beginning of period \( t \) before production takes place, the young supply one unit of labor in- elastically to the old (who owns the firm) in return for wage payments. Part of this wage income is then saved for second period consumption. A generation \( t \) young can save in this model by purchasing physical capital, which in this simple model can be taken to be equivalent to the purchase of shares of the single representative firm. Once the young purchase physical capital, they become the owner of the firm and are entitled to the profits in the next period. It is assumed that no other form of financial market exists through which the individual can save. Hence total savings of the young in period \( t \) constitutes the aggregate capital stock used in production in period \( t+1 \). Thus

\[ s_t = k_{t+1} \]  

[6]
where \( s \) represents the savings of a generation \( t \) young. In the subsequent analysis \( k_{t+1} \) would be used to denote interchangeably both the savings of a generation \( t \) young and the capital stock used in production in period \( t+1 \).

Note that in this economy, at any time \( t \) there is only one representative young individual and one representative old individual. Since the young supply one unit of labor inelastically it is clear that this would be the total employment in each period, so that

\[ l = 1 \]  

[7]

The government in this economy uses two types of taxes, a proportional income tax \( \tau \) levied on wage income of the individuals and a proportional profit tax (or capital income tax) \( \lambda \) levied on profits of the firm. The reason for using two taxes is to capture the fact that capital income or profits are usually taxed at a higher rate than wage income. This fact might provide a greater incentive for misreporting this part of the income. Thus it is assumed here that \( \lambda \geq \tau \).

It is assumed in this model that the government can observe wage income of the individual accurately but cannot observe the capital income or profits unless it audits the individual. The wage income is earned in the first life-period and are paid to the wage earners (the current young) by the current old. It is assumed that there are withholding and reporting requirements whereby the current old withholds the wage earner's tax liability and remits it directly to the government. This of course gives the old some opportunity to remit less than the correct amount to the government. This possibility, which has been considered in Yaziv [1988], however is assumed away here. Also the possibility of employer-employee collusion to misreport income is assumed away.

Once savings are used to purchase physical capital, however, the individuals become owners of the firm. Thus in the second life period, there is no second party to report and withhold profit taxes. Therefore the old has some opportunity to misreport profits and thus evade taxes, if they consider this to be a favorable gamble.
To control tax evasion the government can audit the tax returns filed. Once an individual is audited, his true income is assumed to be correctly identified. If the individual is found to have underreported his income, he has to pay a fine (including the unpaid tax) of $\theta$ times the amount of tax evaded ($\theta > 1$). Auditing tax returns however involves some cost so that due to budgetary reasons the government can audit only a certain fraction $\alpha$ of all tax returns filed each period. The government is assumed to choose tax returns to be audited at random. Individuals are assumed to be fully informed about the penalty rate $\theta$ and the audit rate $\alpha$ of the government. Also it is assumed that the way the tax revenue is spent has no effect on the individual’s decision making.

Since an individual cannot cheat in their first life-period, the consumption of a generation $t$ individual in period $t$ is simply given by

$$c_t = (1 - \tau)w_t - k_{t+1}$$  \[8\]

Period $t$ savings of $k_{t+1}$ yields profits in period $t + 1$ of

$$\pi_{t+1} = F(k_{t+1}) - w_{t+1}$$  \[9\]

in period $t + 1$, where

$$F(k_t) = f(k_t, 1)$$

Because this profit is not observable by the government, the individual can report profits as $\pi^*_{t+1}$ ($0 \leq \pi^*_{t+1} \leq \pi_{t+1}$). Such misreporting reduces the individual’s tax burden by

$$T^e = \lambda[\pi_{t+1} - \pi^*_{t+1}]$$  \[10\]

If the individual is audited however his income is correctly identified and he has to pay $\theta T^e$ in back taxes and fines as penalty for evasion. Under these circumstances, the consumption of a generation $t$ individual in period $t + 1$, when he misreports profits and is not detected, can be given by

$$c^m_{t+1} = \pi^*_{t+1} - \lambda \pi^*_{t+1} + k_{t+1}$$  \[11\]
On the other hand, if the individual misreports his second period profits and is detected, his (second period) consumption is given by

\[ e_{t+1}^d = (1 - \theta \lambda) \pi_{t+1} + \lambda (\theta - 1) \pi_{t+1}^r + k_{t+1} \]  \[ 12 \]

An individual born at time \( t \) then decides, at the beginning of period \( t \), on how much to save \( (k_{t+1}) \) and how much profits to report \( (\pi_{t+1}^r) \). In his decision problem, he takes the penalty rate \( \theta \), the audit rate \( \alpha \), the tax rates \( \tau \), and \( \lambda \) and the period \( t \) wages \( w_t \) as exogenously given. The individual is also assumed to have perfect foresight about the wages he has to pay in period \( t + 1 \) and takes this as exogenously given too. The individual makes his decisions so as to maximize his expected utility given by

\[ EU = U(c_t) + \beta [\alpha V(e_{t+1}^d) + (1 - \alpha) V(e_{t+1}^{nd})] \]  \[ 13 \]

subject to the constraints [8], [9], [11] and [12]. The first order conditions for a maximum require that

\[ \frac{\partial EU}{\partial k_{t+1}} = \alpha \beta [(1 - \theta \lambda) \pi_{t+1}^r + 1] V_{e_{t+1}}(d) + (1 - \alpha) \beta [\pi_{t+1}^r + 1] V_{e_{t+1}}(nd) - U'(c_t) = 0 \]  \[ 14 \]

And,

\[ \frac{\partial EU}{\partial \pi_{t+1}^r} = \alpha \beta \lambda (\theta - 1) V_{e_{t+1}}(d) - (1 - \alpha) \beta \lambda V_{e_{t+1}}(nd) = 0 \]  \[ 15 \]

where \( \pi_{t+1}^r \equiv \left\{ \frac{\partial \pi_{t+1}}{\partial k_{t+1}} \right\} = \frac{\partial F(k_{t+1})}{\partial k_{t+1}} = F'(k_{t+1}) > 0 \)

\[ U'(c_t) \equiv \frac{\partial U(c_t)}{\partial c_t} = \text{marginal utility of first period consumption.} \]

while, \( V_{e_{t+1}}(d) \equiv \frac{\partial V(e_{t+1}^d)}{\partial e_{t+1}^d} = \text{marginal utility of second period consumption.} \)
for the two states of nature i = d, nd.

The second order conditions for the individual’s optimization problem are found to be globally satisfied if preferences of the individual are additively time-separable. This is shown in Appendix C.

This completes the specification of the model. The eight endogeneous variables of the model \( k_{t+1}, \pi_{t+1}, \pi_{t+1}^l, w_t, c_t, e_t^l, e_t^d \) and \( y_t \) are determined from the eight equations [2], [3], [4], [8], [11], [12] [14] and [15], keeping in mind that \( l_t \) is exogeneously given to be 1 by equation [7]. Since there is no population growth in this economy, the equilibrium is stationary so that the steady state, if it exists, can be shown by dropping the subscript \( t \).

Before proceeding further, it is of interest to spend some more time on the optimization problem of a generation \( t \) individual. Firstly it is necessary to investigate the circumstances under which the individual will misreport profits. The expected utility maximizing individual would choose to evade taxes iff:

\[
\frac{\partial EU}{\partial \pi_{t+1}^r} \bigg|_{\pi_{t+1}^r = \pi_{t+1}} < 0
\]

This can be combined with [15] to give:

\[
\frac{1 - a}{a(\theta - 1)} > 1 \quad \text{or} \quad (1 - a\theta) > 0
\]

This condition is intuitively clear. It says that the individual will evade taxes if the expected gain from tax evasion is positive. If the individual evades \( T^* \) dollars in taxes, his expected gain from such evasion is \((1 - a\theta)T^* \). This will be positive so long as \((1 - a\theta) > 0 \). As Arrow [1970] has shown, a risk averse individual will never take part in an unfair or actuarially fair gamble. However he will always take some part in a favorable gamble, if gambling opportunities are continuous.

The individual, on the other hand, would report some positive profits so long as

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\[
\frac{\partial EU}{\partial \pi_{t+1}} \mid \pi'_{t+1} = 0 > 0
\]

This requires that

\[
\frac{V_{\pi_{t+1}}(d)}{V_{\pi_{t+1}}(nd)} \mid \pi'_{t+1} = 0 > \frac{1 - \alpha}{\alpha(\theta - 1)}
\]

The condition for an interior solution of \(0 < \pi'_{t+1} < \pi_{t+1}\) then is:

\[
\frac{V_{\pi_{t+1}}(d)}{V_{\pi_{t+1}}(nd)} \mid \pi'_{t+1} = 0 > \frac{1 - \alpha}{\alpha(\theta - 1)} > 1
\]  [16]

Given these restrictions on the parameters, it is relevant to find out if these are really satisfied in practice. While evidence on the audit probability of the government is hard to come by, there have been some IRS studies trying to estimate this value. One IRS study, for example, reports that in 1981, 2 percent of all private returns were audited in the U.S. While the IRS may have elaborate methods of finding suspicious returns, Skinner and Slemrod [1985] argues that the probability of audit for amateur tax evaders cannot exceed 5 percent. Individuals' perception about the probability of audit however are generally higher than these figures. A survey conducted for the IRS in May-July 1984 reported that taxpayers believed about 12 percent of all tax returns are audited. These results of course may not apply directly to this model since one life period in this model may correspond to somewhere around 30 years while the figures reported were for one year. Also the results reported were for private tax evasion and not corporate tax evasion as considered in this paper.

Regarding the penalty assessed in civil fraud and negligence, Skinner and Slemrod suggest that the maximum is about 50 percent of taxes owed. If tax frauds are considered to be criminal frauds, the penalty is of course much higher. However under these circumstances the crime has to be proved
in a court of law, which might be difficult. Hence the probability of being caught and convicted may be lower.

From these figures it is clear that the right hand part of equation [16] (which requires that \( 1 > a \theta \)) is satisfied. The left hand part of course will depend on the the nature of the utility function.

4.3. Comparative Statics

The model can now be used to look at the impact of changes in the parameters of the government on the individual’s savings and reported profits. It is found however that under general conditions it is not possible to predict all the relevant effects. To get unambiguous signs for some of the results, some assumptions about the individual’s risk aversion need to be made. It is seen that if the individual’s preference for second period consumption is assumed to exhibit non-decreasing absolute risk aversion, much of the ambiguity goes away. The Arrow-Pratt measure of absolute risk aversion for second period consumption is given by

\[
R_A(i) = -\frac{\frac{\partial^2 V}{\partial \epsilon^2}}{\frac{\partial V}{\partial \epsilon}} , \quad i = d, \ nd
\]

Thus non-decreasing risk aversion states that \( R_A(d) \leq R_A(nd) \).

The findings on the comparative statics are presented in the form of the propositions below. Because of the rather lengthy nature of the proofs of all the propositions of this chapter, they are provided separately in Appendix C.
Proposition 1.1: The savings of an individual are positively related to the audit probability $\alpha$ of the government if the preference of the individual exhibits non-decreasing absolute risk aversion. That is, $\partial k_{t+1}/\partial \alpha > 0$ if $R_a(d) \leq R_a(nd)$.

Proposition 1.2: The amount of profits reported by the individual is positively related to the audit rate $\alpha$ of the government. That is, $\partial \pi_{t+1}/\partial \alpha > 0$.

Proposition 2.1: The savings of the individual are positively related to the penalty rate $\theta$ of the government if the preferences of the individual exhibit non-decreasing absolute risk aversion. That is, $\partial k_{t+1}/\partial \theta > 0$ if $R_a(d) \leq R_a(nd)$.

Proposition 2.2: The amount of profits reported by the individual is positively related to the penalty rate of the government. That is $\partial \pi_{t+1}/\partial \theta > 0$.

Propositions 1.1 and 2.1 show that in this model policies of the government aimed at regulating tax evasion have direct impact on the savings decision of the individual and the level of output in the economy. If the individual’s preferences exhibit non-decreasing risk aversion, the effect is positive. Of course the usual assumption made about risk aversion is that it is non-increasing in its argument(s). In that case however the impact of the tax enforcement parameters on savings cannot be conclusively determined.

The fact that changes in the tax enforcement parameters affect savings of the individuals, imply that output in this economy is affected by these parameters. This is because an increase in savings leads to more capital being used in production so that output increases. The literature on the effects of tax evasion on output however have usually shown that output is unaffected by tax evasion decisions. While the setting of these papers are quite different from that of this model in that they usually just look at a monopolist evading sales taxes or corporate profit tax [see, for example, Marrelli [1984], Marrelli and Martina [1988] Kreutzer and Lee [1986] ] the finding here does point out the need for integrating the corporate and individuals’ consumption decisions together.
The fact that savings could increase due to an increase in either the penalty rate or audit probability or both, also indicates that the presence of tax evasion could reduce savings of the individuals. If this is true, then savings in the economy could be increased by increasing these parameters. A reduction in the marginal tax rate, a policy commonly thought to reduce tax evasion, might not be necessary. This has important policy implications for an economy like the U.S., where the rate of savings is quite low. As Feldstein [1989] has shown, the U.S. has the lowest rate of savings among most industrialized countries. Savings have been falling, moreover, during the 1980’s. In the first half of 1988, savings comprised only 8.9 percent of the nation’s GNP. Thus alterations in the penalty rates or audit rates could be one way of increasing savings. [Note that if an increase in $\alpha$ or $\theta$ in period $t$ leads to an increase in savings of the current young, this pushes up the wages paid in period $(t+1)$. This in its turn leads to a further increase in savings by period $(t+1)$ young [because $\partial k_{t+1}/\partial w_t > 0$, (see Appendix C for a proof)] reinforcing the initial effect.]

Propositions 1.2. and 2.2. show that reported profits increase with increases in the penalty rate and/or the audit rate. Thus, stricter enforcement reduces tax evasion. This is an intuitively clear result that is obtained in most tax evasion literature.

If an increase in $\alpha$ or $\theta$ leads to an increase in savings, then actual profits of the firm also increase because $\partial \pi_{t+1}/\partial \alpha = (\partial \pi_{t+1}/\partial k_{t+1})(\partial k_{t+1}/\partial \alpha) > 0$. However, an increase in $\alpha$ or $\theta$ also leads to an increase in reported profits. So it is of interest to find out whether the increase in $\alpha$ or $\theta$ leads to a larger fraction of actual profits being reported or not. Proposition 3 provides the answer to this question.

**Proposition 3:** The fraction of total profits reported is positively related to the audit rate and the penalty rate. That is,

$$\frac{\partial}{\partial \alpha} \left\{ \frac{\pi_{t+1}}{\pi_{t+1}} \right\} > 0 \quad \text{and} \quad \frac{\partial}{\partial \theta} \left\{ \frac{\pi_{t+1}}{\pi_{t+1}} \right\} > 0$$

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Thus not only does the level of reported profits increase with $\alpha$ and $\theta$ but the fraction of actual profits reported does too.

One other relevant question is how changes in the two tax rates $\tau$ and $\lambda$ affect reported profits and savings (and hence output) in the economy. For the basic model of individual tax evasion as in A-S it was found that an increase in the (proportional income) tax rate led to an increase in reported income. It is of interest to see if the same effect remains in this model. The effect of changes in the wage income tax on savings and reported profits are given in the form of the next two propositions.

**Proposition 4.1.** The savings of the individual are negatively related to the wage income tax rate. Thus, $\partial s_{t+1}/\partial \tau < 0$.

**Proposition 4.2.** The reported profits of the individual is negatively related to the wage income tax rate, if the individual's Arrow-Pratt measure of absolute risk aversion is non-decreasing in second period consumption. Thus, $\partial \pi_{t+1}/\partial \tau < 0$ if $R_d(d) \leq R_d(nd)$.

Proposition 4.1. thus indicates that an increase in the wage income tax rate reduces savings of the individual unambiguously. The result could be explained as follows. The wage income tax is a tax that cannot be avoided since first period wage income is costlessly observable. Because after tax income in the first period is reduced, savings are reduced to compensate for the reduction in the income available for consumption.

Because an increase in $\tau$ leads to a reduction in both actual profits (as savings fall) and reported profits, under the given assumptions it is interesting to find out what happens to the fraction of profits reported. The answer to this question is provided by the next proposition.

**Proposition 4.3** : An increase in the wage income tax leads to a smaller fraction of profits to be reported if the Arrow-Pratt measure of risk aversion of the taxpayer is non-decreasing in second period consumption. That is,
\[
\frac{\partial}{\partial \tau} \left\{ \frac{\pi_{i+1}}{\pi_{i+1}} \right\} < 0 \quad \text{if} \quad R_A(d) \leq R_A(nd)
\]

**Proposition 4.4.** Changes in the profit tax have ambiguous effect on both savings and reported profits.

Proposition 4.4 however does not come as too much of a surprise because even in a simple overlapping generations model with no tax evasion, the effect of changes in the profit tax rate on savings turn out to be ambiguous.

Given that the government has four parameters under its control, it is of interest to find out how alterations in these parameters affect the tax collections. Of course taxes collected from an individual in this model is a random variable depending on whether he is audited or not. However since the economy can be replicated into one of many individuals it makes sense to look at expected tax collections of the government. The expected tax plus penalty revenue of the government in period $t$ can be given by

\[T_t = \alpha \lambda \theta (\pi_t - \pi^*_t) + \lambda \pi^*_t + \tau w_t \tag{17}\]

The impact of changes in the parameters on expected tax revenues are given in the form of the following propositions.

**Proposition 5.1.** Expected tax revenues of the government are positively related to the audit rate if the preference of the individual exhibit non-decreasing absolute risk aversion. Thus, $\partial T_t/\partial \alpha > 0$, $R_A(d) \leq R_A(nd)$.

**Proposition 5.2.** The expected tax revenue of the government is positively related to the penalty rate if preferences of the individual exhibit non-decreasing absolute risk aversion. Thus $\partial T_t/\partial \theta > 0$ if $R_A(d) \leq R_A(nd)$.
Given the assumption of non-decreasing absolute risk aversion, it is not difficult to explain propositions 5.1 and 5.2. With stricter enforcement, reported profits of the individual increases. Also, with the individual's preferences exhibiting non-decreasing absolute risk aversion, it is clear that savings and actual profit also increase. So expected tax revenues of the government have to increase.

The impact of changes in the tax rates on expected tax revenues cannot be determined. For the wage income tax rate, while an increase results in more revenues collected from the young, it also results in lower actual and reported profits of the old so that the net effect is indeterminate. On the other hand, for changes in the profit tax, the impact on savings and actual profits itself is ambiguous.

These results imply that the tax enforcement parameters $\alpha$ and $\theta$ are not only important tools that can be used to reduce tax evasion, they may also be more effective in raising tax revenues compared to the tax rates in an economy where tax evasion exists. It also shows that the two tax enforcement parameters substitute for one another. Thus an increase in expected revenue due to a reduction in the audit rate can be compensated by an increase in the penalty rate.

4.4. Optimal Government Policy

The last section concentrated only on the effects of various government policies on the individual's saving and tax evading decisions without looking at the optimal policies of the government. Though from the individual's perspective, the parameters $\tau_s$, $\tau_p$, $\alpha$ and $\theta$ of the government are exogenously given, these are directly under the control of the government and can be selected to achieve its own goals. It is assumed that there are no costs involved in setting and adjusting the tax rates and the penalty rate. Auditing tax returns, however, involves some costs, given by $\phi = \phi(\alpha)$
with $\phi'(\alpha) > 0$. [The cost of audit should also depend on the total number of tax returns filed, but population is constant in this model].

The standard literature on optimal taxation usually assumes that the government maximizes some objective measure of welfare, subject to some revenue constraints. Economic theory does not provide any clear-cut answer as to what the government should choose as a welfare measure. Here, following much of the earlier literature on optimal taxation, a utilitarian approach is taken where the government is assumed to maximize the indirect utility of the individual given by

$$G(\tau, \lambda, \alpha, \theta) = U[e_{t+1}^d(\tau, \lambda, \alpha, \theta)] + \alpha \beta V[e_{t+1}^d(\tau, \lambda, \alpha, \theta)] + (1 - \alpha)\beta V[e_{t+1}^n(\tau, \lambda, \alpha, \theta)] \quad [18]$$

The government is subject to the fixed revenue constraint that requires it to raise $\overline{R}$ in revenues each period. Its budget constraint is thus given by

$$T_t = \overline{R} + \phi(\alpha) \quad [19]$$

where $T_t$ is given by equation [17]. Note that in this model since there is no population growth the steady state equilibrium would be stationary, implying that $k_t = k \forall t$; $\pi_t = \pi \forall t$; and $w_t = w \forall t$. So confining attention to steady states only, the optimal policy of the government at any period can be taken to be that of choosing $\tau, \lambda, \theta$ and $\alpha$ to maximize the indirect utility of the representative individual given by [18] subject to the budget constraint [19]. The Lagrangian for the problem is

$$L = G(\tau, \lambda, \alpha, \theta) + \mu[T - \overline{R} - \phi(\alpha)] \quad [20]$$

where $\mu$ is the Lagrange multiplier.

The F.O.C. for a maximum requires that

$$\frac{\partial L}{\partial \alpha} = \beta [V(\alpha) - \gamma(\alpha)] + \mu \left[ \frac{\partial T}{\partial \alpha} - \phi'(\alpha) \right] = 0 \quad [21]$$
\[
\frac{\partial L}{\partial \theta} = -\lambda \alpha \beta V_0(d)[\pi - \pi'] + \mu \left( \frac{\partial T}{\partial \theta} \right) = 0 
\]

\[
\frac{\partial L}{\partial \tau} = -w U'(c) + \mu \left( \frac{\partial T}{\partial \tau} \right) = 0 
\]

\[
\frac{\partial L}{\partial \lambda} = -\alpha \theta V_0(d) + \mu \left( \frac{\partial T}{\partial \lambda} \right) = 0 
\]

and,

\[
\frac{\partial L}{\partial \mu} = \alpha \lambda \theta [\pi - \pi'] + \lambda \pi' + \tau w - \phi(\alpha) - \bar{R} = 0 
\]

where,

\[
\frac{\partial T}{\partial \theta} = \alpha \lambda \theta [\pi - \pi'] + \lambda \lambda \theta \frac{\partial \pi}{\partial \theta} + \lambda (1 - \theta \alpha) \frac{\partial \pi'}{\partial \theta} + \tau \frac{\partial w}{\partial \theta} > 0 
\]

\[
\frac{\partial T}{\partial \alpha} = \lambda \theta [\pi - \pi'] + \alpha \lambda \theta \frac{\partial \pi}{\partial \alpha} + \lambda (1 - \alpha \theta) \frac{\partial \pi'}{\partial \alpha} + \tau \frac{\partial w}{\partial \alpha} > 0 
\]

\[
\frac{\partial T}{\partial \tau} = w + \alpha \lambda \theta \frac{\partial \pi}{\partial \tau} + \lambda (1 - \alpha \theta) \frac{\partial \pi'}{\partial \tau} + \tau \frac{\partial w}{\partial \tau} 
\]

and

\[
\frac{\partial T}{\partial \lambda} = \alpha \theta [\pi - \pi'] + \pi' + \alpha \lambda \theta \frac{\partial \pi}{\partial \lambda} + \lambda (1 - \alpha \theta) \frac{\partial \pi'}{\partial \lambda} + \tau \frac{\partial w}{\partial \lambda} 
\]

The conditions [21] through [24] requires that the government set each instrument at a level where the marginal gains in (net) expected revenues arising from adjusting the instrument equals the marginal reduction in welfare (change in indirect utility of the individual) due to such adjustment.

One important question to discuss in this framework is whether the optimal policy of the government is to eliminate all tax evasion. Now if it is possible to show that \( \partial L/\partial \theta \) is positive, so long as evasion is present, then clearly full compliance would be the optimal policy of the government.
However from equation [22] is clear that the first term is negative while the second term is positive. In fact [22] and [26] can be combined to give

$$\frac{\partial L}{\partial \theta} = \alpha \lambda (\pi - \pi')[\mu - \beta V_d(d)] + \mu \left[ \alpha \lambda \theta \frac{\partial \pi}{\partial \theta} + \lambda (1 - \alpha \theta) \frac{\partial \pi'}{\partial \theta} + \tau \frac{\partial w}{\partial \theta} \right]$$  \[30\]

From equation [30] it is clear that a sufficient condition for $\partial L/\partial \theta$ to be positive is that $\mu \geq \beta V_d(d)$. However there is no reason why this should necessarily be true.

The same conclusion holds for the audit rate. If audits are costless, $g(x) = 0$. In this case expanding $V(nd)$ around $V(d)$ by Taylor’s series and ignoring second and higher order terms, equation [21] can be reduced to

$$\frac{\partial L}{\partial \alpha} = \lambda \theta (\pi - \pi')[\mu - \beta V_d(d)] + \mu \left[ \alpha \lambda \theta \frac{\partial \pi}{\partial \alpha} + \lambda (1 - \alpha \theta) \frac{\partial \pi'}{\partial \alpha} + \tau \frac{\partial w}{\partial \alpha} \right]$$  \[31\]

Equation [31] also implies that a sufficient condition for $\partial L/\partial \alpha$ to be positive is that $\mu \geq \beta V_d(d)$.

Note however that $\partial L/\partial \theta > 0$ is only a sufficient condition for full compliance and not a necessary one. So it is not possible to conclude what the optimal policy of the government would be.

To look at the optimal tax rates of the government, equations [24], [25], [27] and [28] can be combined to give

$$w[1 + (1 - \lambda)\pi'] = \frac{\partial T/\partial \tau}{\partial T/\partial \lambda}$$  \[32\]

Because the expressions for $\partial T/\partial \tau$ and $\partial T/\partial \lambda$ are fairly complicated, it is difficult to get a meaningful expression for the tax rates under general conditions. To simplify the calculations to some extent, suppose that the rate of return on savings is constant and given by $r$. In this case, savings of $k$ yields profits of $rk$ (gross of taxes) in period 2. Also suppose that for notational convenience $c_1$, $c_2$, and $c_3$ refers to the consumption of the individual in the first period, in the second period

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when detected and in the second period when not detected respectively. Under these circumstances equation [32] can be reduced to the form

$$r[c_2 + (\theta - 1)c_3] = [\alpha a \lambda r + (1 - a \theta)(1 + r)]s_{11} + [r \theta \lambda (1 - a) - (1 - a \theta)(1 + r)]s_{21}$$  \[33\]

where

$$s_{ij} \equiv \frac{\partial c_i}{\partial p_j} \bigg|_{E U = \bar{E} U}$$

= the Slutsky substitution terms.

[Calculations leading to the derivation of equation [32] are provided in Appendix D].

Equation [33] gives an expression for the two tax rates in terms of the parameters of the model. The expression is similar to that derived by King [1980] for the case of no evasion and variable labor supply. It is interesting to compare this optimality condition with that for the case of no evasion. As derived in Appendix D, the optimality condition under no tax evasion is given by

$$c_2 = \lambda s_{12}$$  \[34\]

where $s_{12}$ = the Slutsky substitution term between first and second period consumption.

A comparison of equations [33] and [34] shows some similarities between the two. Whereas in the case of no tax evasion, there is only one state of the world in the second period, for the case with tax evasion, there are two possible states of nature. So there are two possible consumption levels for period 2, namely $c_2$ and $c_3$. As a result, there are two Slutsky substitution terms between first and second period, namely $s_{11}$ and $s_{21}$. Equation [34] showed that for the case of no tax evasion, the consumption in the second period is linearly related to the substitution term between first and second period consumption. The right hand side of equation [33] gives a weighted average of the consumption levels for the two states of nature in the second period. This then, is found to equal a linear combination of the two substitution terms between first and second periods.
From the optimality condition, it can be shown that the optimal policy of the government is to have a positive profit tax rate (i.e., $\lambda > 0$). To see this, suppose that $\lambda = 0$. In this case there no taxes on profits in the second period so that the issue of tax evasion disappears. From [33] then it is clear that $\lambda$ cannot be equal to zero at the optimum.

The case of $r = 0$ (golden rule) turns out to be a trivial one because this implies that there is no profit and so no evasion.

**Compensated tax:**

The previous analysis was based on the assumption that the tax revenue collected each period is disposed in a way that does not affect the taxpayers' utility. It is also possible to look at the case of compensated taxes where the taxes collected each period are returned to the individuals in lump-sum form. If this is true, the steady state consumption of the representative taxpayer can be given by

$$c = (1 - \tau)w - k + T_1$$  \hspace{1cm} [35]

$$e^d = (1 - \theta \lambda)\pi + \lambda(\theta - 1)\pi' + k + T_2$$  \hspace{1cm} [36]

and,

$$e^{nd} = \pi - \lambda \pi' + k + T_2$$  \hspace{1cm} [37]

where $T_1$ and $T_2$ are the transfers received in the first and second life-periods respectively (to be specified later). The individual is assumed to take this transfers as exogeneously given so that the first order conditions for a maximum can still be represented by equations [14] and [15].

The objective of the government as before is to maximize the social welfare of the economy given by the the indirect utility of the individual $G(\tau, \lambda, \theta, \alpha, T_1, T_2)$. The government does not have a
fixed revenue constraint but is subject to the balanced budget constraint which requires that the
taxes collected in each period from a certain generation is returned to the same generation as
transfers. This then requires that

\[ T_1 = \tau w \]  \[38\]

and,

\[ T_2 = \alpha \lambda \theta (\pi - \pi') + \lambda \pi' \]  \[39\]

The government's problem then is to maximize \( G(.) \) subject to the constraints [38] and [39]. Now
it should be clear that in this model adjustments in the tax rates may affect the equilibrium wage
and these in turn affect the equilibrium choices of the individuals in future generations. For the case
of compensated changes in \( \tau \) however, these effects are not present because

\[ \left. \frac{\partial k}{\partial \tau} \right|_{\text{com}} = \left. \frac{\partial \pi'}{\partial \tau} \right|_{\text{com}} = 0 \]

[See Appendix C for a proof].

Under these circumstances,

\[ \left. \frac{\partial G}{\partial \tau} \right|_{\text{com}} = \frac{\partial G}{\partial \tau} + \frac{\partial G}{\partial T_2} \frac{\partial T_2}{\partial \tau} = U'(c)[-w] + U'(c)[w] = 0 \]  \[40\]

Equation [40] implies that the optimal wage income tax rate is not unique. In fact any value is
optimal. This result is not difficult to explain. In this model since labor supply is inelastic, the first
period wage income is completely exogeneous. So the compensated tax has no impact on the indi-
vidual's decision making. The same result is obtained for the model without any tax evasion.

The optimal profit tax rate on the other hand is more difficult to determine. This is because the
impact of a change in the profit tax rate on savings and reported profits are themselves indetermi-
nate.
The optimal profit tax rate, requires that

\[
\frac{\partial G}{\partial \lambda} \bigg|_{com} = \frac{\partial G}{\partial \lambda} + \frac{\partial G}{\partial T_2} \frac{\partial T_2}{\partial \lambda} = -\alpha \beta \theta \pi V''(d)
\]

\[+ \beta [\alpha V'(d) + (1 - \alpha) V'(nd)] \frac{\partial T_2}{\partial \lambda} = 0 \tag{41}\]

where

\[
\frac{\partial T_2}{\partial \lambda} = \left[ a\theta (\pi - \pi') + \pi' + \alpha \lambda \theta \left\{ \frac{\partial \pi}{\partial \lambda} - \frac{\partial \pi'}{\partial \lambda} \right\} + \lambda \frac{\partial \pi'}{\partial \lambda} \right] \tag{42}
\]

Because the impact of a change in \( \lambda \) on savings and and reported profits is ambiguous, the sign for \( \partial T_2/\partial \lambda \) is ambiguous. So it is not possible to conclude from [41] what the optimal profit tax would be. However it is possible to say that the optimal profit tax rate would be positive. To see this, note that when \( \lambda = 0 \), there is no evasion (as there are no profit taxes to evade). Therefore equation [41] reduces to

\[
\frac{\partial G}{\partial \lambda} \bigg|_{com (at \lambda = 0)} = (1 - \alpha \theta) \beta \pi V''(\lambda) > 0
\]

Note that the optimal profit tax is ambiguous even in the case of no evasion. In the no-evasion model, the optimal profit tax rate is given by

\[
\frac{\partial V}{\partial \lambda} \bigg|_{com} = \frac{\partial V}{\partial \lambda} + \frac{\partial V}{\partial T_2} \frac{\partial T_2}{\partial \lambda} = \beta \lambda \pi' \frac{\partial k}{\partial \lambda} V_e(e) \tag{43}
\]

Because the effect of a change in the profit tax rate on savings is ambiguous, it is not clear from [43] what the optimal profit tax rate is in the absence of tax evasion.

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4.5. Conclusion

The chapter discussed some of the issues relating to tax evasion in an environment where the government can observe the wage income of the individuals costlessly but can observe their income from savings only by incurring audit costs. Using a dynamic framework, it is shown here that changes in audit rates or penalty rates of the government have direct impact on savings of the individuals, and consequently on the output of the economy. However the optimal policies of the government in this framework is not very clear. Even in the situation when all the policies of the government can be costlessly adjusted, the possibility of an equilibrium with some tax evasion cannot be ruled out.
Chapter 5. Numerical Simulations

5.1. Introduction

The theoretical analysis of the previous chapter showed that the individual's savings and tax evasion decisions are interrelated so that alterations in tax enforcement parameters of the government have a definite impact on savings. However, it was found that under general conditions the optimal policy of the government is not clear. Thus it was not clear what the optimal tax rates would be and whether full compliance was the optimal policy of the government. This chapter considers some numerical examples to shed some more light on these issues. The objective here is to get some idea about the nature of the optimal policy of the government.
To address this issue two types of exercises are considered. One exercise is to look at the impact of compensated changes in the parameters of the government where the individual is compensated in lump-sum form for the taxes (and penalty if any) collected from him each period. Another exercise considers the impact of changes in two parameters of the government that are adjusted simultaneously to keep expected tax revenues of the government constant. The results of these exercises indicate that full compliance may be the optimal policy of the government. When the government is subject to a fixed revenue constraint, a low value of the wage tax rate and a high value of the profit tax rate seems to be socially preferable to a high value of the wage tax coupled with a low value of the profit tax. Also with regard to the audit and the penalty rate the results show that a high penalty with low audits improve social welfare.

The chapter is organized as follows. Section 2 describes the functional form of the utility function and production function chosen and derives the set of optimality conditions for the individual's and the government's problems. Section 3 describes the parameter selection for the problem. Section 4 reports the simulation results for compensated changes in the parameters of the government. Section 5 considers the effect of simultaneous change in two of the parameters of the government keeping its expected revenues as constant. The last section provides some concluding remarks.

5.2. The Model

Among the various possible choices of the utility function, three come to mind - the Cobb-Douglas, CES and constant relative risk aversion (CRRA). Among these three, the CRRA was found to be most suitable since it allowed for variations in the risk aversion of the individual and also was flexible enough to allow for fairly large changes in the parameters. So the CRRA utility function is selected here.
Regarding the production function the Cobb-Douglas case seems to be the most reasonable and commonly used and so this is the one used here.

The (expected) utility of the individual thus takes the form

\[
EU = \frac{c_t^{1-\chi}}{1-\chi} + \frac{\beta\alpha(e_{t+1}^d)^{1-\chi}}{1-\chi} + \frac{\beta(1-\alpha)(e_{t+1}^{nd})^{1-\chi}}{1-\chi}
\]

[1]

where

\[\chi = \text{Arrow-Pratt measure of relative risk aversion.}\]

And,

\[
c_t = (1-\tau)w_t - k_{t+1} + T_1
\]

[2]

\[
e_{t+1}^d = (1-\theta\lambda)\pi_{t+1} + \lambda(\theta - 1)\pi_{t+1} + k_{t+1} + T_2
\]

[3]

\[
e_{t+1}^{nd} = \pi_{t+1} - \lambda\pi_{t+1} + k_{t+1} + T_2
\]

[4]

where \(T_1 = T_2 = 0\) for the case of uncompensated taxes.

For compensated taxes however,

\[
T_1 = \tau w
\]

[5]

and, \(T_2 = \alpha\lambda\theta(\pi - \pi') + \lambda\pi' - \frac{a^2}{2}\)

where, \(a^2/2\) = audit costs of the government.

The expressions for \(w_t\), and \(\pi_{t+1}\) depend on the production function selected and are defined below.

The F.O.C.’s for the individual’s maximization problem are:

\[
\frac{\partial EU}{\partial \pi_{t+1}'} = \alpha(\theta - 1) [e_{t+1}^{nd}]^\chi - (1 - \alpha) [e_{t+1}^d]^\chi = 0
\]

[5]
\[
\frac{\partial EU}{\partial k_{t+1}} = [c_{t+1}^d]^{x}[e_{t+1}^{nd}]^{x} - \beta \alpha [(1 - \theta \lambda)\pi'_{t+1} + 1][c_{t}^{d}]^{x}[e_{t+1}^{nd}]^{x} \\
- \beta(1 - \alpha)[\pi'_{t+1} + 1][c_{t}^{d}]^{x}[e_{t+1}^{d}]^{x}
\tag{6}
\]

The production function takes the Cobb-Douglas form:

\[
y_t = \eta k_t^{\psi} l_t^{1-\psi}
\tag{7}
\]

where

\[\eta = \text{a scaling constant.}\]

\[\psi = \text{capital's share of output.}\]

and, \(l_t (= 1)\) identical employment in period \(t\).

Therefore,

\[
w_t = \eta(1 - \psi)k_t^{\psi}
\tag{8}
\]

and,

\[
\pi_t = y_t - w_t = \psi \eta k_t^{\psi}
\tag{9}
\]

Under the assumption that the individual takes future wages as outside of his control,

\[
\pi'_{t+1} = \eta \psi k_{t+1}^{\psi - 1}
\tag{10}
\]

The equilibrium values of \(w_t, k_t, \pi_t, y_t, c_t, e_t\), and \(e_t^{nd}\) are obtained by solving equations [2]-[10]. The steady state is then obtained by dropping the subscript \(t\).

Chapter 5. Numerical Simulations
The government's objective is to maximize the social welfare of the economy, given by the indirect utility of the representative individual. For the case of uncompensated taxes, the government has a revenue constraint which requires that it raise $\overline{T}$ in revenues net of audit costs of so that

$$T_t = \alpha \theta \lambda [\pi_t - \pi_t'] + \lambda \pi_t' + \tau w_t - \frac{\alpha^2}{2} \geq \overline{T}$$  \[11\]

The government's maximization problem then is to choose $\alpha$, $\theta$, $\tau$ and $\lambda$ so as to maximize \[1\] subject to the constraints \[2\] through \[11\].

5.3. Parameter Selection

To solve the numerical exercises, it is first necessary to specify values for the parameters of the utility and production functions.

The rate of time preference ($\beta$): Empirical research does not provide clear results about the appropriate value of the discount rate. While Browning [1983] suggests that discount rates higher than 6% have been empirically estimated, values used in simulations have typically been around 2%. Thus Auerbach and Kotlikoff [1987] used a value of 1.5% for the for the discount rate. Here a $\beta$ of 0.9 is used, which is close to the values used by previous authors.

Relative risk aversion ($\chi$): Theoretically, $\chi$ can take any positive value. Higher values imply a tendency to smooth consumption in different periods. In the extreme when $\chi = \infty$, the indifference curves become Leontief, with no substitution effect on consumption behavior. The value of 1 is often chosen for this parameter in which case it reduces to the logarithmic utility function. In models of tax evasion it has been observed that for realistic values of the audit probability and
the penalty rate, high values of risk aversion are required to obtain interior solutions.\textsuperscript{10} So here $\chi$ is set to 2.0.

Capital's share of output ($\psi$) : The capital's share in national income in the U.S. has historically been estimated at 25 percent. Thus the value chosen for $\psi$ is 0.25.

The scalar constant ($\eta$) : The scaling constant $\eta$ can be selected arbitrarily. Here the value selected is 3.0.

5.4. Compensated Tax

As a first exercise, effects of changes in the parameters of the government on the steady state of the economy is considered in a situation where the government compensates the taxpayer for the taxes (and penalties) collected from him each period, net of audit costs, in a lump-sum form. Given that the theory predicts that the optimal wage income tax rate is not-unique in this environment, effects of the other three parameters of the government ($\alpha$, $\theta$, and $\lambda$) are considered with the wage tax set at zero. The benchmark values used for the simulations are

\begin{align*}
\alpha &= 0.2 \\
\theta &= 3.5 \\
\lambda &= 0.28
\end{align*}

\textit{Changes in the penalty rate ($\theta$)}

The effects of compensated changes in the penalty rate is shown in Table 1. The results show that as the penalty rate is increased from 3.0 to 4.8 in equal steps of 0.03, the savings of the individual decreases. The decrease is faster for the initial changes. Thus when the penalty rate is increased

\textsuperscript{10} See Skinner and Slemrod [1985].
from 3.0 to 3.3, savings fall by approximately 1.05 percent. On the other hand when the penalty rate is increased from 4.5 to 4.8, savings fall by only about 0.09 percent. The expected utility of the taxpayer however increases with the penalty rate indicating that in an economy with compensated taxes full compliance may be socially optimal. Note however that savings are lower with full compliance than with evasion so that output is also lower. The fact that full compliance seems to be socially optimal can be explained at least partially by the fact that there are audit costs in this model. So in the second period the government is not returning all the taxes and penalty collected but deducts the audit expenses. This indicates a loss due to evasion. Regarding reported profits it is seen to increase with the penalty rate as expected. A penalty rate of 5.0 is seen to result in full compliance while a penalty below 2.7 causes the firm to report no profits at all.

Changes in the audit rate ($\alpha$)

Table 2 reports the effects of changes in the audit rate of the government. It is clear from the table that the effects are very similar to that of changes in the penalty rate. Thus, savings of the individual decreases and the expected utility of the taxpayer increases as $\alpha$ is increased.

Changes in the profit tax rate ($\lambda$)

The effect of changes in the profit tax rate on the steady state of the economy is shown in Table 3. The table shows that as $\lambda$ is increased from 0.15 to 4.2 in equal ateps of 0.03, both the savings and the expected utility of the taxpayer falls. Savings seem to fall at a fairly steady rate as $\lambda$ is increased. While at a first glance these results might seem to suggest that the optimal policy should be to set $\lambda = 0$, note that reported profits become zero as $\lambda$ is lowered beyond 0.15. So it is possible that expected utility is maximized for some positive $\lambda$. In fact theory suggests that $\lambda$ should be positive in equilibrium.
The above analysis suggests that under the parameterization used, a low value of $\lambda$ and full compliance seems to be the optimal policy. This result is quite different from the no-evasion case where the optimal profit tax rate is non-unique and any rate is optimal.

5.5. Changes in the government’s parameters keeping revenue fixed

The last exercise dealt with the effects of changes in the parameters of the government when taxpayers are compensated for the revenues collected from them each period. The previous chapter however modeled the government as maximizing the expected utility of the taxpayer subject to a fixed revenue constraint. So it is important to consider changes in the parameters of the government that leave expected revenues constant. With four parameters under its control the government has many ways of changing these keeping expected revenues constant. Here the discussion is confined to three possibilities involving bilateral changes in the parameters. Even though the government actually sets all its parameters simultaneously to maximize welfare, the discussion may provide some clues as to what the optimal policy of the government will be.

The simulations are done keeping the expected revenues of the government (net of audit costs) constant at 0.45. As before, audit costs are assumed to take the form $\alpha^2/2$.

The three exercises deal with changes in the audit and penalty rate; changes in the two tax rates; and changes in the penalty rate and the profit tax rate.

*Changes in $\alpha$ and $\theta$*

Table 4 shows the effects of increases in $\alpha$ in equal steps of 0.03 with corresponding decreases in $\theta$. The table shows that as $\alpha$ is increased from 0.03 to 0.33, savings of the individual increase. The rate of increase seems to be slightly slower at the initial stages. Thus when $\alpha$ is increased from 0.03
to 0.06, savings increase by 0.11 percent while it increases by 0.17 percent when \( \alpha \) is increased from 0.3 to 0.33. The effect on reported profits however is not very clear. The table shows that it falls as \( \alpha \) is increased up to the value of 0.18. Further increases in \( \alpha \) results in a rise in reported profits. Expected utility of the taxpayer falls steadily as \( \alpha \) is increased. This indicates that a low value of \( \alpha \) coupled with a high penalty rate is welfare improving. This is not very difficult to understand because audits involve resource costs and the penalty does not. Given the fixed revenue requirement of the government, higher \( \alpha \) would imply that the extra audit costs have to be collected from the taxpayers, reducing welfare.

*Effects of changes in \( \tau \) and \( \lambda \)*

Table 5 shows the effects of increases in \( \lambda \) in equal steps of 0.03 matched by decreases in \( \tau \). The results show that savings of the taxpayer increase as \( \lambda \) is increased. Reported profits also increases with \( \lambda \). The increase is very fast for the initial increases but the rate of increase becomes slower as \( \lambda \) is increased further. Expected utility of the taxpayer also increases with \( \lambda \). This implies that the taxpayer would prefer a higher \( \lambda \) to a lower one. This result also holds for the case of no tax evasion. This can be concluded by comparing the figures in Table 5 with that of Table 7 which looks at the exercise for the no-evasion case.

*Changes in \( \theta \) and \( \lambda \)*

Table 6 shows the effect of changes in \( \theta \) in equal steps of 0.03 with a simultaneous reduction in the profit tax rate \( \lambda \). The table shows that savings of the individual fall as \( \theta \) is increased. The decrease is somewhat faster in the initial stages. Thus when the penalty rate is increased from 2.1 to 2.4, savings fall by 0.9 percent. The corresponding decrease is only 0.03 percent when the penalty rate is increased from 4.5 to 4.8. Reported profits increase with \( \theta \) as is expected. Regarding expected utility of the taxpayer it is seen that this increases with the penalty rate suggesting that a higher audit with a lower profit tax is preferable. Thus full compliance could be socially optimal.
5.6. Conclusion

The chapter considered some numerical simulations relating to the model of tax evasion considered in the previous chapter. The results provide some evidence indicating that full compliance may be the optimal policy of the government if all parameters can be adjusted freely. Regarding the two tax rates, it seems that the taxpayer may prefer a lower wage tax to a lower profit tax.


Christiansen, V. “Two comments on tax evasion” *J. Public Econ.* 13 (June 1980) : 389-93.


Samuelson, P.A. “An exact consumption-loan model of interest with or without the social contrivance of money” *J. political Economy* 66 (1958) : 467-82.


This appendix presents some results relating to optimal government policies with price ceiling regulation. The results here are for comparison with that of Chapter 2.

**Case 1. Monopolist under price ceiling regulation (no evasion):**

The monopolist maximizes

\[ U[(1 - \delta)(R(q) - c(q))] \]

subject to the price ceiling constraint which requires that

\[ P(q) \leq \bar{P} \]

where \( \bar{P} \) is the price ceiling set. The Lagrangian for the monopolist’s optimization problem is

\[ L = U[(1 - \delta)(R(q) - c(q))] + \lambda[\bar{P} - p(q)] \]

\[ [A1] \]
F.O.C. for a maximum require that

$$\frac{\partial L}{\partial q} = (1 - t)(R' - c')U'(\pi) - \lambda p'(q)$$

Therefore

$$R' - c' = \frac{\lambda p'(q)}{(1 - t)U'(\pi)}$$

Because the right hand side of equation [A3] is positive, it is clear that at the monopolist's optimum $R' < c'$.

Optimal price ceiling and profit tax with $N$ monopolists and no evasion:

The aggregate consumer surplus is given by

$$CS = \sum \int p_i(v_i)dv_i - \sum R_i(q_i)$$

Aggregate net profits are given by

$$\sum (1 - t_i)[R_i(q_i) - c_i(q_i)]$$

Social welfare, which is the sum of the aggregate consumer surplus and aggregate net profits can then be given by

$$W = \sum \int p_i(v_i)dv_i - \sum t_iR_i(q_i) - \sum (1 - t_i)c_i(q_i)$$

The objective of the government is to maximize $W$ subject to its fixed revenue constraint given by
\[ \sum t_i[R_i(q_i) - c_i(q_i)] \geq \bar{T} \]  

where \( \bar{T} \) represents the fixed revenue requirement of the government. The Lagrangian for the problem is given by

\[ L = \sum \int p_i(v) dv_i - \sum t_i R_i(q_i) - \sum (1 - t_i) c_i(q_i) + \lambda \left[ \sum t_i(R_i(q_i) - c_i(q_i)) - \bar{T} \right] \]

The government selects the profit tax rates and the price ceilings to maximize \( L \). However setting quantity is equivalent to setting price in this model so that \( q_i 's \) are taken as the choice variable. The F.O.C.'s for a maximum requires that

\[ \frac{\partial L}{\partial q_i} = p_i(q_i) - c'_i + t_i(\lambda - 1)[R'_i - c'_i] = 0 \]  

And,

\[ \frac{\partial L}{\partial t_i} = (\lambda - 1)[R_i(q_i) - c_i(q_i)] = 0 \]  

So long as profits of the firm are positive, it is clear from [A10] that \( \lambda = 1 \). Equation [A9] then requires that \( p_i(q_i) = c'_i \).
Appendix B

This appendix provides some results for comparison with the results of chapter 3.

Case 1. Tax evading monopolist with no regulation:

The monopolist is subject to a proportional profit tax at the rate $t$ which it can evade by overreporting costs by a fraction $\mu$ (costs cannot be observed directly by the government). When detected, the government collects a fine (including the unpaid tax) of $\theta$ times the evaded tax. $\alpha$ is the monopolist’s perceived probability of being audited. Under these circumstances the monopolist maximizes his expected utility given by

$$EU = \alpha U(\pi^d) + (1 - \alpha)U(\pi^{nd}) \quad \text{[B1]}$$

subject to the constraints that
\[ \pi^d = (1 - \ell)[R(q) - c(q)] - t\mu(\theta - 1)c(q) \]  

and,

\[ \pi^{nd} = (1 - \ell)[R(q) - c(q)] + t\mu c(q) \]  

The F.O.C.'s for a maximum require that

\[ \frac{\partial EU}{\partial \mu} = t(1 - \alpha)c(q)U'(\pi^{nd}) - \alpha t(\theta - 1)c(q)U'(\pi^d) = 0 \]  

and

\[ \frac{\partial EU}{\partial q} = a[(1 - \alpha)(R' - c') - t\mu(\theta - 1)c']U'(\pi^d) \]

\[ + (1 - \alpha)[(1 - \ell)(R' - c') + t\mu c']U'(\pi^{nd}) = 0 \]  

Equations [B4] and [B5] can be combined to give \( R' = c' \) so that the monopolist's optimal output is unaffected by the possibility of tax evasion.

---

**Case 2. Profit per unit constraint and no evasion:**

The monopolist chooses output \( q \) so as to maximize after tax profits of

\[ (1 - \ell)[R(q) - c(q)] \]  

subject to the profit per unit constraint that requires that

\[ R(q) - c(q) \leq \bar{\pi}q \]  

The Lagrangian for this problem can be given by

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\[ L = (1 - t)[p(q)q - c(q)] + \lambda (\pi - p(q)q + c(q)) \]  

[B8]

The F.O.C’s for a maximum require that

\[
\frac{\partial L}{\partial q} = (1 - t - \lambda)[R_q - c_q] + \lambda \pi = 0 
\]

[B9]

while the S.O.C. requires that

\[
\frac{\partial^2 L}{\partial q^2} = (1 - t - \lambda)[R_{qq} - c_{qq}] < 0 
\]

[B10]

An interior solution requires that \( \lambda \neq 1 - t \). If \( \lambda > 1 - t \), then \( R_q - c_q \) has to be positive for an interior solution. On the other hand \( \lambda < 1 - t \) requires that \( R_q - c_q \) be negative. If it is assumed that \( R_{qq} < 0 \) and \( c_{qq} > 0 \) then the second order condition for a maximum will be satisfied if and only if \( \lambda < 1 - t \). So at the optimum \( R_q - c_q < 0 \). This implies that the optimal output of the monopolist is larger under the fixed profit per unit constraint, than that under no restrictions.
Appendix C

This section provides the calculations for the second order conditions and the proofs of the propositions in chapter 4. Equations that are taken from chapter 4 have the same numberings. Since many of the expressions have a lot of terms and these have to be compared with one another to get the signs of expressions, the terms of an expression are marked using Roman numerals. Other equation numbers start with a C.

First and second order conditions:

The expected utility of the consumer can be given by

\[ EU = U(c_t) + \alpha \beta V(e^{d}_{t+1}) + (1 - \alpha)\beta V(e^{nd}_{t+1}) \]  \hspace{1cm} [13]
where,
\[ c_t = (1 - \tau)w_t - k_{t+1} \]  \hspace{1cm} [8]

\[ e_{t+1}^{nd} = \pi_{t+1} - \lambda \pi_{t+1} + k_{t+1} \] \hspace{1cm} [11]

And,
\[ e_{t+1}^d = (1 - \theta \lambda)\pi_{t+1} + \lambda(\theta - 1)\pi_{t+1} + k_{t+1} \] \hspace{1cm} [12]

The first order conditions for a maximum are
\[
\frac{\partial EU}{\partial k_{t+1}} = \alpha \beta [(1 - \theta \lambda)\pi_{t+1} + 1]V_{e_{t+1}}(d) + (1 - \alpha)\beta[(\pi_{t+1} + 1) V_{e_{t+1}}(nd) - U''(e_t) = 0
\] \hspace{1cm} [14]

And,
\[
\frac{\partial EU}{\partial \pi_{t+1}} = \alpha \lambda \beta(\theta - 1) V_{e_{t+1}}(d) - \lambda(1 - \alpha)\beta V_{e_{t+1}}(nd) = 0
\] \hspace{1cm} [15]

The second order condition for a maximum requires that \([AC - B^2] > 0\), where,

\[
A = \frac{d^2 EU}{dk_{t+1}^2} = \alpha \beta (1 - \theta \lambda) \pi_{t+1} V_{e_{t+1}}(d) + (1 - \alpha)\beta \pi_{t+1} V_{e_{t+1}}(nd) + U''(e_t)
\]

\[
+ \alpha \beta [(1 - \theta \lambda)\pi_{t+1} + 1]^2 V_{e_{t+1}e_{t+1}}(d) + (1 - \alpha)\beta (\pi_{t+1} + 1)^2 V_{e_{t+1}e_{t+1}}(nd) < 0
\] \hspace{1cm} [C1]

\[
C = \frac{d^2 EU}{d\pi_{t+1}^2} = \alpha \beta [(\lambda \theta - 1)]^2 V_{e_{t+1}e_{t+1}}(d) + (1 - \alpha)\beta \lambda^2 V_{e_{t+1}e_{t+1}}(nd) < 0
\] \hspace{1cm} [C2]

And,
\[
B = \frac{d^2 EU}{dk_{t+1} \cdot d\pi_{t+1}^2} = \frac{d^2 EU}{dk_{t+1} \cdot d\pi_{t+1}^2} = \alpha \beta \lambda (\theta - 1)[(1 - \theta \lambda) \pi_{t+1} + 1] V_{e_{t+1}e_{t+1}}(d)
\]
\[-(1 - \alpha)\beta \lambda (\pi'_{t+1} + 1) V_{\epsilon_{t+1} \epsilon_{t+1}}(nd) \quad \text{[C3]}\]

The notation used is

\[\pi''_{t+1} = \frac{\partial \pi'_{t+1}}{\partial k_{t+1}} = F''(k_{t+1}) < 0 \quad \text{(by assumption)}\]

From [C3] it is clear that the sign for B is indeterminate under general conditions as the first term is negative and the second term is positive. However using equation [15], B can be written as

\[B = \alpha \beta \lambda (\theta - 1) V_{\epsilon_{t+1} \epsilon_{t+1}}(d)[(\pi'_{t+1} + 1) R_A(nd) - ((1 - \lambda \theta) \pi'_{t+1} + 1) R_A(d)] \quad \text{[C4]}\]

where,

\[R_A(d) = -\frac{V_{\epsilon_{t+1} \epsilon_{t+1}}(d)}{V_{\epsilon_{t+1}}(d)}\]

and,

\[R_A(nd) = -\frac{V_{\epsilon_{t+1} \epsilon_{t+1}}(nd)}{V_{\epsilon_{t+1}}(nd)}\]

are the Arrow-Pratt measures of absolute risk aversion for second period consumption. From [C4], it is clear that \(B > 0\), if \(R_A(nd) \geq R_A(d)\), i.e., the Arrow-Pratt measure of absolute risk aversion is non-decreasing in its argument.

Now,

\[-B^2 = \left[\alpha \beta \lambda (\theta - 1)\right]^2 (1 - \theta \lambda) \pi'_{t+1} + 1) V_{\epsilon_{t+1} \epsilon_{t+1}}(d) V_{\epsilon_{t+1} \epsilon_{t+1}}(d) \quad \text{[I]}\]

\[-\left[\lambda \beta (1 - \alpha)\right]^2 (\pi'_{t+1} + 1) V_{\epsilon_{t+1} \epsilon_{t+1}}(nd) V_{\epsilon_{t+1} \epsilon_{t+1}}(nd) \quad \text{[II]}\]

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\[ + 2\beta^2 \lambda^2 \alpha(1-\alpha)(\theta - 1)[\pi'_{t+1} + 1][1 - \theta \lambda]\pi'_{t+1} + 1\] \[V_{e_{t+1}e_{t+1}}(d)V_{e_{t+1}e_{t+1}}(nd) \] \[\text{[III]}\]

Again,

\[AC = [\alpha \beta \lambda(\theta - 1)\lambda^2 \pi'_{t+1} + 1][1 - \theta \lambda]\pi'_{t+1} + 1\] \[V_{e_{t+1}e_{t+1}}(d)V_{e_{t+1}e_{t+1}}(nd) \] \[\text{[IV]}\]

\[+ \alpha(1 - \alpha)\lambda^2 \beta^2 \pi'_{t+1} + 1\] \[V_{e_{t+1}e_{t+1}}(d)V_{e_{t+1}e_{t+1}}(nd) \] \[\text{[V]}\]

\[+ \alpha(1 - \alpha)\lambda^2 \beta^2 \pi'_{t+1} + 1\] \[V_{e_{t+1}e_{t+1}}(nd)V_{e_{t+1}e_{t+1}}(d) \] \[\text{[VI]}\]

\[+ [\alpha \beta \lambda(\theta - 1)]^2 \pi'_{t+1} + 1\] \[V_{e_{t+1}e_{t+1}}(d)V_{e_{t+1}e_{t+1}}(nd) \] \[\text{[VII]}\]

\[+ [\alpha \beta \lambda(\theta - 1)]^2 V_{e_{t+1}e_{t+1}}(d) + (1 - \alpha)\beta \lambda^2 V_{e_{t+1}e_{t+1}}(nd) \]

\[\text{[VIII]}\]

\[X[\alpha \beta(1 - \theta \lambda)\pi''_{t+1} + 1V_{e_{t+1}}(d) + (1 - \alpha)\beta \pi''_{t+1}V_{e_{t+1}}(nd) + U''(e_i)] \]

From the calculations it is clear that all terms in the expression \([AC - B^2]\) would be positive, except [I] and [II]. These however get cancelled with [IV] and [VII] respectively, so that \([AC - B^2] > 0\) as required.

[If the utility function is not additively separable, then it is not possible to get an unambiguous sign for \([AC - B^2]\). Also much of the comparative statics come out to be ambiguous. So this assumption is made.]

Proof of propositions 1.1. and 1.2 :
To find the effects of tax evasion on savings of the person and the profit he reports, equations [14] and [15] are totally differentiated to yield

\[ Adk_{t+1} + Bd\pi^r_{t+1} = Dd\alpha \]  

where,

\[ D = \beta[(\pi^r_{t+1} + 1)V_{e_{t+1}} (nd) - ((1 - \theta \lambda)\pi^r_{t+1} + 1)V_{e_{t+1}} (d)] \]

And,

\[ Bdk_{t+1} + Cd\pi^r_{t+1} = Ed\alpha \]

where,

\[ E = -\beta\lambda[(\theta - 1)V_{e_{t+1}} (d) + V_{e_{t+1}} (nd)] \lt 0 \]

Therefore,

\[ \frac{\partial k_{t+1}}{\partial \alpha} = \frac{[CD - BE]}{[AC - B^2]} \]

and,

\[ \frac{\partial \pi^r_{t+1}}{\partial \alpha} = \frac{[AE - BD]}{[AC - B^2]} \]

Looking at the numerator of [C9]

\[ CD = \alpha[\beta \lambda(\theta - 1)]^2[\pi^r_{t+1} + 1]V_{e_{t+1}} (nd)V_{e_{t+1}e_{t+1}} (d) \]

\[ - \alpha[\beta \lambda(\theta - 1)]^2[(1 - \theta \lambda)\pi^r_{t+1} + 1]V_{e_{t+1}} (d)V_{e_{t+1}e_{t+1}} (d) \]

\[ + \lambda^2 \beta^2(1 - \sigma)[\pi^r_{t+1} + 1]V_{e_{t+1}} (nd)V_{e_{t+1}e_{t+1}} (nd) \]
\[-\lambda^2 \beta^2 (1 - \alpha)[(1 - \theta \lambda) \pi'_{t+1} + 1] V_{t+1}^{(d)} V_{t+1}^{e_{t+1}}(nd) \]

[IV]

Again,

\[-\lambda^2 \beta^2 (1 - \alpha)(\theta - 1)[(1 - \theta \lambda) \pi'_{t+1} + 1] V_{t+1}^{e_{t+1}}(d) V_{t+1}^{e_{t+1}}(nd) \]

[V]

\[+ \alpha \lambda^2 \beta^2 (\theta - 1)[(1 - \theta \lambda) \pi'_{t+1} + 1] V_{t+1}^{e_{t+1}}(nd) V_{t+1}^{e_{t+1}}(d) \]

[VI]

\[-\lambda^2 \beta^2 (1 - \alpha)[\pi'_{t+1} + 1] V_{t+1}^{e_{t+1}}(nd) V_{t+1}^{e_{t+1}}(nd) \]

[VII]

Therefore,

\[CD - BE = -\theta \lambda^2 \beta^2 (1 - \alpha)[(1 - \lambda) \pi'_{t+1} + 1] V_{t+1}^{e_{t+1}}(d) V_{t+1}^{e_{t+1}}(nd) \]

[IX]

\[+ \theta(\theta - 1)\alpha \lambda^2 \beta^2 [(1 - \lambda) \pi'_{t+1} + 1] V_{t+1}^{e_{t+1}}(nd) V_{t+1}^{e_{t+1}}(d) \]

[X]

[Terms II & V; III & VIII cancel one another. Terms IV & VI are combined to give IX while terms I and VII are combined to give X.]

Since IX is positive and X is negative, the sign of [CD - BE] is in general ambiguous. However these two terms can be combined to give

\[\theta \alpha(\theta - 1)\lambda^2 \beta^2 [(1 - \lambda) \pi'_{t+1} + 1] V_{t+1}^{e_{t+1}}(d) V_{t+1}^{e_{t+1}}(nd) \left[ \frac{1 - \alpha}{\alpha(\theta - 1)} R_{\delta}(nd) - R_{\delta}(d) \right] \]

This shows that \[CD - BE > 0\] if \(R_{\delta}(nd) \geq R_{\delta}(d)\). Under this assumption, then, \(\partial k_{t+1}/\partial \alpha > 0\).

Looking at the numerator of [Aii1] reveals

\[AE = -\lambda(\theta - 1)\lambda^2 \beta^2 [(1 - \theta \lambda) \pi'_{t+1} + 1] V_{t+1}^{e_{t+1}}(d) V_{t+1}^{e_{t+1}}(d) \]

[I]
\[-\lambda \beta^2(1 - \alpha)[\pi'_{t+1} + 1]^2 V_{\pi_{t+1}}(nd)V_{\pi_{t+1}}(nd) \quad [II]\]

+ some positive terms.

[Because \(A < 0\), and \(E < 0\), all terms in AE are positive.]

Again,

\[-BD = \alpha \lambda(1 - \theta - 1)\beta^2[(1 - \theta \lambda)\pi'_{t+1} + 1]^2 V_{\pi_{t+1}}(d)V_{\pi_{t+1}}(d) \quad [III]\]

+ \(\lambda(1 - \alpha)\beta^2[\pi'_{t+1} + 1]^2 V_{\pi_{t+1}}(nd)V_{\pi_{t+1}}(nd) \quad [IV]\)

+ some positive terms.

III and IV, the only two negative terms in -BD, cancels with [I] and II respectively, so that [AE -BD] is positive. This then implies that \(\partial \pi'_{t+1}/\partial \alpha > 0\).

[Note that the assumption of non-decreasing absolute risk aversion was not necessary to get the sign.]

Proof of propositions 2.1. and 2.2. :

From the F.O.C.'s we get

\[Adk_{t+1} + Bdn^{\pi'_{t+1}} = Gd\theta \quad [C11]\]

and

\[Bdk_{t+1} + Cdn^{\pi'_{t+1}} = Hd\theta \quad [C12]\]
where

\[ G = \alpha \lambda \beta \pi'_{t+1} V_{e_{t+1}}(d) + \alpha \lambda \beta \pi'_{t+1} (1 - \theta \lambda) \pi'_{t+1} + 1] V_{e_{t+1}e_{t+1}}(d) \]  \[ C13 \]

and,

\[ H = \alpha \lambda^2 \beta (\theta - 1)[\pi_{t+1} - \pi'_{t+1}] V_{e_{t+1}e_{t+1}}(d) - \alpha \beta V_{e_{t+1}}(d) < 0 \]  \[ C14 \]

Therefore,

\[ \frac{\partial k_{t+1}}{\partial \theta} = \frac{[GC - BH]}{[AC - B^2]} \]  \[ C15 \]

and

\[ \frac{\partial \pi'_{t+1}}{\partial \theta} = \frac{[AH - BG]}{[AC - B^2]} \]  \[ C16 \]

Now,

\[ GC = \lambda(\alpha \lambda \beta)^2 (\theta - 1)^2 \pi'_{t+1} V_{e_{t+1}}(d) V_{e_{t+1}e_{t+1}}(nd) \]  \[ I \]

\[ + \lambda^2 \beta \alpha (1 - \alpha) \pi'_{t+1} V_{e_{t+1}}(d) V_{e_{t+1}e_{t+1}}(nd) \]  \[ II \]

\[ + \lambda(\alpha \lambda \beta)^2 (\theta - 1)^2 [\pi_{t+1} - \pi'_{t+1}] [(1 - \theta \lambda) \pi'_{t+1} + 1] V_{e_{t+1}e_{t+1}}(d) V_{e_{t+1}e_{t+1}}(nd) \]  \[ III \]

\[ + \lambda^2 \beta \alpha (1 - \alpha) [\pi_{t+1} - \pi'_{t+1}] [(1 - \theta \lambda) \pi'_{t+1} + 1] V_{e_{t+1}e_{t+1}}(nd) V_{e_{t+1}e_{t+1}}(nd) \]  \[ IV \]

And,

\[ - BH = - \lambda(\alpha \lambda \beta)^2 (\theta - 1)^2 [\pi_{t+1} - \pi'_{t+1}] [(1 - \theta \lambda) \pi'_{t+1} + 1] V_{e_{t+1}e_{t+1}}(d) V_{e_{t+1}e_{t+1}}(d) \]  \[ V \]

\[ + \lambda^2 \beta \alpha (1 - \alpha) (\theta - 1)[\pi_{t+1} - \pi'_{t+1}] [\pi'_{t+1} + 1] V_{e_{t+1}e_{t+1}}(d) V_{e_{t+1}e_{t+1}}(nd) \]  \[ VI \]
\[ \lambda^2 \beta^2 \alpha (1 - \alpha) [(1 - \lambda) \pi'_{t+1} + 1] V_{e_{t+1}}(d) V_{e_{t+1}e_{t+1}}(nd) \]  

\[ \text{[XI]} \]

Therefore,

\[ [GC - BH] = (\alpha \lambda \beta)^2 (\theta - 1)[(1 - \lambda) \pi'_{t+1} + 1] V_{e_{t+1}}(d) V_{e_{t+1}e_{t+1}}(d) \]  

\[ \text{[IX]} \]

\[ - \lambda^2 \beta^2 \alpha (1 - \alpha) [(1 - \lambda) \pi'_{t+1} + 1] V_{e_{t+1}}(d) V_{e_{t+1}e_{t+1}}(nd) \]  

\[ \text{[X]} \]

\[ + \lambda^3 \beta^2 \theta \alpha (1 - \alpha) [(\pi_{t+1} - \pi'_{t+1}) [(1 - \lambda) \pi'_{t+1} + 1] V_{e_{t+1}e_{t+1}}(nd) V_{e_{t+1}e_{t+1}}(d) \]  

\[ \text{[XI]} \]

[Terms III and V cancel one another. IX is obtained by combining I and VII, X is obtained by combining II and VIII, XI is obtained by combining IV and VI.]

While X and XI are positive, IX is negative, so that the sign for [GC-BH] is in general ambiguous. However terms IX and X can be combined to give

\[ \lambda^2 \beta^2 \alpha (1 - \alpha) [(1 - \lambda) \pi'_{t+1} + 1] V_{e_{t+1}}(d) V_{e_{t+1}e_{t+1}}(nd) [R_A(nd) - R_A(d)] \]

which is \( \geq 0 \) if \( R_A(nd) \geq R_A(d) \). In that case \( \partial k_t/\partial \theta > 0 \).

To find the sign for \( \partial \pi'_{t+1}/\partial \theta \), the numerator of [A17] can be written as

\[ - BG = - (\alpha \beta \lambda)^2 (\theta - 1)[(\pi_{t+1} - \pi'_{t+1}) [(1 - \lambda) \pi'_{t+1} + 1] V_{e_{t+1}e_{t+1}}(d)]^2 \]  

\[ \text{[II]} \]

\[ + \lambda^2 \beta^2 \alpha (1 - \alpha) [(1 - \lambda) \pi'_{t+1} + 1] V_{e_{t+1}}(d) V_{e_{t+1}e_{t+1}}(nd) \]  

\[ \text{[III]} \]

+ some positive terms.

And,

\[ AH = (\alpha \lambda \beta)^2 (\theta - 1)[(1 - \lambda) \pi'_{t+1} + 1] V_{e_{t+1}e_{t+1}}(d)]^2 \]  

\[ \text{[IV]} \]

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\[- \lambda \alpha (1 - \alpha) \beta^2 [\pi'_{t+1} + 1]^2 V_{e_{t+1}}(d) V_{e_{t+1} e_{t+1}}(nd) \]  
\[ [IV] \]

+ some positive terms.

Therefore,

\[ [AH - BG] = - \lambda \alpha (1 - \alpha) [\pi'_{t+1} + 1] [(1 - \lambda) \pi'_{t+1} + 1] V_{e_{t+1}}(d) V_{e_{t+1} e_{t+1}}(nd) \]  
\[ [V] \]

+ some positive terms.

Terms I and III cancel each other while II and IV are combined to give V, which is positive.

Therefore, [AH - BG] is positive so that \( \frac{\partial \pi'_{t+1}}{\partial \alpha} \) is positive.

[Note that the assumption of non-decreasing risk aversion was not needed to get this result.]

---

**Proof of proposition 3:**

Since both \( \pi_{t+1} \) and \( \pi'_{t+1} \) increase with an increase in \( \alpha \) or \( \theta \), it makes sense to ask what happens to the ratio \( \frac{\pi'_{t+1}}{\pi_{t+1}} \). It is easy to see that

\[ \frac{\partial}{\partial \alpha} \left\{ \frac{\pi'_{t+1}}{\pi_{t+1}} \right\} > 0 \]  
\[ \text{if} \quad \frac{\partial \pi'_{t+1}}{\partial \alpha} > \frac{\partial \pi_{t+1}}{\partial \alpha} \]  
\[ [C17] \]

and

\[ \frac{\partial}{\partial \theta} \left\{ \frac{\pi'_{t+1}}{\pi_{t+1}} \right\} > 0 \]  
\[ \text{if} \quad \frac{\partial \pi'_{t+1}}{\partial \theta} > \frac{\partial \pi_{t+1}}{\partial \theta} \]  
\[ [C18] \]

But,

\[ \frac{\partial \pi'_{t+1}}{\partial \alpha} = \frac{[AE - BD]}{[AC - B^2]} \]
and,

\[
\frac{\partial \pi_{t+1}'}{\partial \alpha} = \pi_{t+1}' \quad \frac{\partial k_{t+1}'}{\partial \alpha} = \pi_{t+1}' \quad \frac{[CD - BE]}{[AC - B^2]}
\]

Therefore,

\[
\frac{\partial \pi_{t+1}'}{\partial \alpha} - \frac{\pi_{t+1}'}{\pi_{t+1}'} = \frac{[AE - BD]}{[AC - B^2]} - \pi_{t+1}' \quad \frac{[CD - BE]}{[AC - B^2]}
\]

This shows that,

\[
\frac{\partial}{\partial \alpha} \left\{ \frac{\pi_{t+1}'}{\pi_{t+1}'} \right\} > 0 \quad \text{if} \quad [AE - BD] - \pi_{t+1}' [CD - BE] > 0 \quad \text{[C19]}
\]

Now,

\[
AE - BD - \pi_{t+1}' CD + \pi_{t+1}' BE
\]

\[
= - \lambda(1 - \alpha)[\pi_{t+1}' + 1]^2 V_{\bar{e}_{t+1}}(nd)V_{\bar{e}_{t+1}^*,\bar{e}_{t+1}^*}(nd) \quad \text{[I]}
\]

\[- \lambda \alpha(\theta - 1)[(1 - \theta \lambda)\pi_{t+1}' + 1]^2 V_{\bar{e}_{t+1}^*}(d)V_{\bar{e}_{t+1}^*,\bar{e}_{t+1}^*}(\bar{d}) \quad \text{[II]}
\]

\[- \lambda(1 - \alpha)(\theta - 1)[\pi_{t+1}' + 1]^2 V_{\bar{e}_{t+1}}(d)V_{\bar{e}_{t+1}^*,\bar{e}_{t+1}^*}(nd) \quad \text{[III]}
\]

\[- \alpha \lambda(\theta - 1)[\pi_{t+1}' + 1][(1 - \theta \lambda)\pi_{t+1}' + 1] V_{\bar{e}_{t+1}}(nd)V_{\bar{e}_{t+1}^*,\bar{e}_{t+1}^*}(d) \quad \text{[IV]}
\]

\[+ \lambda(1 - \alpha)[\pi_{t+1}' + 1]^2 V_{\bar{e}_{t+1}}(nd)V_{\bar{e}_{t+1}^*,\bar{e}_{t+1}^*}(nd) \quad \text{[V]}
\]

\[+ \alpha \lambda(\theta - 1)[(1 - \lambda \theta)\pi_{t+1}' + 1]^2 V_{\bar{e}_{t+1}^*}(d)V_{\bar{e}_{t+1}^*,\bar{e}_{t+1}^*}(d) \quad \text{[VI]}
\]

\[- \lambda(1 - \alpha)[\pi_{t+1}' + 1][(1 - \theta \lambda)\pi_{t+1}' + 1] V_{\bar{e}_{t+1}}(d)V_{\bar{e}_{t+1}^*,\bar{e}_{t+1}^*}(nd) \quad \text{[VII]}
\]
\[ - \alpha \mu (\theta - 1) \beta (\lambda) \pi'_{t+1} + \pi'_{t+1} + 1 \beta \gamma_{t+1} (nd) \gamma_{t+1}\gamma_{t+1} (d) \]  
\[ - \lambda^2 (1 - \alpha) \gamma (\theta - 1) \beta (\lambda) \pi'_{t+1} + \pi'_{t+1} + 1 \beta \gamma_{t+1} (d) \gamma_{t+1}\gamma_{t+1} (d) \]  
\[ + \alpha \mu (\theta - 1) \beta (\lambda) \pi'_{t+1} + \pi'_{t+1} + 1 \beta \gamma_{t+1} (d) \gamma_{t+1}\gamma_{t+1} (d) \]  
\[ + \lambda^2 (1 - \alpha) \gamma (\theta - 1) \beta (\lambda) \pi'_{t+1} + \pi'_{t+1} + 1 \beta \gamma_{t+1} (d) \gamma_{t+1}\gamma_{t+1} (d) \]  
\[ - \alpha \mu (\theta - 1) \beta (\lambda) \pi'_{t+1} + \pi'_{t+1} + 1 \beta \gamma_{t+1} (d) \gamma_{t+1}\gamma_{t+1} (d) \]  
\[ + \lambda^2 (1 - \alpha) \gamma (\theta - 1) \beta (\lambda) \pi'_{t+1} + \pi'_{t+1} + 1 \beta \gamma_{t+1} (d) \gamma_{t+1}\gamma_{t+1} (d) \]  
\[ + \text{ some positive terms.} \]

Therefore,

\[ AE - BD - \pi'_{t+1} CD + \pi'_{t+1} + 1 BE \]

\[ = - \lambda (1 - \alpha) \beta (\theta + 2) (1 - \lambda) \pi'_{t+1} + \pi'_{t+1} + 1 \beta \gamma_{t+1} (d) \gamma_{t+1}\gamma_{t+1} (d) \]  
\[ + \text{ some positive terms.} \]

[Terms I & V; IX & XV; X & XII; II & VI; cancels one another. Also terms IV, VIII and XIV are positive. Term [XVI] is obtained by combining III, VII, XI and XIII.]
\[ \frac{\partial}{\partial \alpha} \left\{ \frac{\pi'_{t+1}}{\pi'_t} \right\} > 0 \]

[Note that the assumption of non-decreasing absolute risk aversion was not needed to get the sign].

To find the sign for \( \frac{\partial}{\partial \theta} \left\{ \frac{\pi'_{t+1}}{\pi'_t} \right\} \) as before we see that

\[\frac{\partial \pi'_{t+1}}{\partial \theta} - \frac{\partial \pi_{t+1}}{\partial \theta} = \pi'_{t+1} \frac{\partial k_{t+1}}{\partial \theta} = \pi'_{t+1} \frac{[GC - BH]}{[AC - B^2]}\]

Therefore,

\[\frac{\partial \pi'_{t+1}}{\partial \theta} - \frac{\partial \pi_{t+1}}{\partial \theta} > 0 \quad \text{if} \quad [AH - BG] - \pi'_t + 1[GC - BH] > 0 \quad \text{[C20]}\]

Now

\[AH - BG - \pi'_t + 1GC + \pi'_t + 1BH\]

\[= -(\alpha \lambda)'(\theta - 1)\pi'_{t+1}[(1 - \theta \lambda)\pi'_t + 1 + 1]V_{\epsilon_{t+1}}(d)\] \[\text{[I]}\]

\[-(\alpha \lambda)'(\theta - 1)\pi'_{t+1}[(1 - \theta \lambda)\pi'_t + 1 + 1]2V_{\epsilon_{t+1}\epsilon_{t+1}}(d)V_{\epsilon_{t+1}\epsilon_{t+1}}(d)\] \[\text{[II]}\]

\[+ \lambda^2(1 - \alpha)\pi'_{t+1}[(\pi'_t + 1 + 1)2V_{\epsilon_{t+1}\epsilon_{t+1}}(d)\] \[\text{[III]}\]

\[+ \lambda^2(1 - \alpha)(\pi'_t + 1 + 1)] \[\text{[IV]}\]

\[- \lambda(\alpha \lambda)(\theta - 1)\pi'_{t+1}V_{\epsilon_{t+1}}(d)\] \[\text{[V]}\]

\[- \lambda(\alpha \lambda)(\theta - 1)\pi'_{t+1}V_{\epsilon_{t+1}}(d)\] \[\text{[VI]}\]

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\[- \alpha(1 - \alpha)\lambda^3 \pi_t^{t + 1} V_{e^{t + 1}_{e + 1}}(nd) \quad [VII]\]

\[- \alpha(1 - \alpha)\lambda^3 \pi_t^{t + 1} [\pi_t^{t + 1} - \pi_t^{t + 1}] [(1 - \theta \lambda)\pi_t^{t + 1} + 1] V_{e^{t + 1}_{e + 1}}(d) V_{e^{t + 1}_{e + 1}}(nd) \quad [VIII]\]

\[+ \lambda [\alpha \lambda (\theta - 1)]^2 \pi_t^{t + 1} [\pi_t^{t + 1} - \pi_t^{t + 1}] [(1 - \theta \lambda)\pi_t^{t + 1} + 1] V_{e^{t + 1}_{e + 1}}(d) V_{e^{t + 1}_{e + 1}}(d) V_{e^{t + 1}_{e + 1}}(d) \quad [IX]\]

\[- (\alpha \lambda)^2 (\theta - 1) [(1 - \theta \lambda)\pi_t^{t + 1} + 1] \pi_t^{t + 1} V_{e^{t + 1}_{e + 1}}(d) V_{e^{t + 1}_{e + 1}}(nd) \quad [X]\]

\[- \lambda^3 \alpha \pi_t^{t + 1} [\pi_t^{t + 1} + \pi_t^{t + 1}] [\pi_t^{t + 1} - \pi_t^{t + 1}] V_{e^{t + 1}_{e + 1}}(d) V_{e^{t + 1}_{e + 1}}(nd) \quad [XI]\]

\[+ \alpha(1 - \alpha)\lambda^2 \pi_t^{t + 1} [\pi_t^{t + 1} + 1] \pi_t^{t + 1} V_{e^{t + 1}_{e + 1}}(d) V_{e^{t + 1}_{e + 1}}(nd) \quad [XII]\]

\[+ (\alpha \lambda)^2 (\theta - 1) [(1 - \theta \lambda)\pi_t^{t + 1} + 1]^2 V_{e^{t + 1}_{e + 1}}(d) V_{e^{t + 1}_{e + 1}}(d) \quad [XIII]\]

\[- \alpha(1 - \alpha)\lambda [\pi_t^{t + 1} + 1]^2 V_{e^{t + 1}_{e + 1}}(d) V_{e^{t + 1}_{e + 1}}(nd) \quad [XIV]\]

\[+ \text{some positive terms.}\]

Therefore,

\[AH - BG - \pi_t^{t + 1} GC + \pi_t^{t + 1} BH\]

\[= \alpha(1 - \alpha) \lambda^3 [\pi_t^{t + 1} - \pi_t^{t + 1}] [\theta(1 - \lambda)^2 (\pi_t^{t + 1})^2 + (2 - \theta \lambda)\pi_t^{t + 1} + 1] V_{e^{t + 1}_{e + 1}}(d) V_{e^{t + 1}_{e + 1}}(nd) [XV]\]

\[- \lambda^2 (1 - \alpha) [(1 - \lambda)^2 (\pi_t^{t + 1})^2 + 2(1 - \lambda)\pi_t^{t + 1} + 1] V_{e^{t + 1}_{e + 1}}(d) V_{e^{t + 1}_{e + 1}}(nd) \quad [XVI]\]

\[+ \text{some positive terms.}\]

[Terms VI & IX; II & XIX cancel one another. Terms IV, VIII & XI are combined to give XV while terms III, VII, XII and XIV are combined to give XVI. All other terms are positive].

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Since XV and XVI are positive, it is clear that

$$\frac{\partial}{\partial \theta} \left\{ \frac{\pi_{t+1}^r}{\pi_{t+1}} \right\} > 0$$

[Note that the assumption of non-decreasing absolute risk aversion was not needed to get the sign.]

**Proof of proposition 4.1., 4.2 and 4.3.:**

The F.O.C.'s can be totally differentiated to yield:

$$Adk_{t+1} + Bd\pi_{t+1}^r = M\delta$$  \[C21\]

and,

$$Bdk_{t+1} + C\pi_{t+1}^r = N\delta$$  \[C22\]

where

$$M \equiv -w_j U''(c_j) > 0$$  \[C23\]

and

$$N \equiv 0$$  \[C24\]

Therefore

$$\frac{\partial k_{t+1}}{\partial \delta} = \frac{[MC - NB]}{[AC - B^2]} < 0$$  \[C25\]

and

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\[
\frac{\partial \pi_{t+1}^r}{\partial \tau} = \frac{[AN - BM]}{[AC - B^2]} < 0 \tag{C26}
\]

if \( R_d(nd) \geq R_d(d) \) in which case \( B > 0 \).

To look at the effect of a change in \( \tau \) on the fraction of profits reported, note that

\[
\frac{\partial}{\partial \tau} \left\{ \frac{\pi_{t+1}^r}{\pi_{t+1}^r} \right\} = \frac{\frac{\partial \pi_{t+1}^r}{\partial \tau} \pi_{t+1}^r - \frac{\partial \pi_{t+1}^r}{\partial \tau} \pi_{t+1}^r}{(\pi_{t+1}^r)^2}
\]

The sign of this expression depends on the sign of the numerator. The numerator can be written as

\[
\frac{[AN - BM] \pi_{t+1}^r - [MC - NB] \pi_{t+1}^r \pi_{t+1}^r}{[AC - B^2]}
\]

The sign of this in turn depends on the numerator which reduces to

\[-M[\pi_{t+1}^r B + \pi_{t+1}^r \pi_{t+1}^r C] \]

Now

\[
\pi_{t+1}^r B + \pi_{t+1}^r \pi_{t+1}^r C
\]

\[
= \alpha \beta \lambda (\theta - 1) \pi_{t+1}^r [(1 - \theta \lambda) \pi_{t+1}^r + 1] V_{t+1} V_{t+1} (d) \tag{I}
\]

\[
- (1 - \alpha) \beta \lambda \pi_{t+1}^r [\pi_{t+1}^r + 1] V_{t+1} V_{t+1} (nd) \tag{II}
\]

\[
+ \alpha \beta [\lambda (\theta - 1)]^2 \pi_{t+1}^r + 1 \pi_{t+1}^r + 1 V_{t+1} V_{t+1} (d) \tag{III}
\]

\[
+ (1 - \alpha) \beta \lambda^2 \pi_{t+1}^r + 1 \pi_{t+1}^r + 1 V_{t+1} V_{t+1} (nd) \tag{IV}
\]

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\[ = \alpha \beta \lambda (\theta - 1) V_\alpha(d) [X R_A(nd) - Y R_A(d)] \]

where

\[ X \equiv \pi_{t+1} \pi_{t+1}^r + \pi_{t+1} - \lambda \pi_{t+1}^r + \lambda^r_{t+1} \]

\[ Y \equiv \pi_{t+1} \pi_{t+1}^r - \theta \lambda \pi_{t+1} \pi_{t+1}^r + \pi_{t+1} + \lambda \theta \pi_{t+1}^r - \lambda \pi_{t+1}^r + \lambda^r_{t+1} \]

and

\[ X - Y = \lambda \theta \pi_{t+1}^r (\pi_{t+1} - \pi_{t+1}^r) > 0 \]

From [C27] it is clear that [\(\pi_{t+1} B + \pi_{t+1}^r \pi_{t+1}^r C\)] > 0 so long as \(R_A(nd) \geq R_A(d)\). In this case clearly

\[ \frac{\partial}{\partial \tau} \left\{ \frac{\pi_{t+1}^r}{\pi_{t+1}} \right\} < 0. \]

Comparative statics for changes in \(\lambda\):

\[ A d k_{t+1} + B d r_{t+1} = R d \lambda \]

\[ B d k_{t+1} + C d r_{t+1} = S d \lambda \]

where

\[ R \equiv \alpha \beta \theta \pi_{t+1}^r + (1 - \alpha) \beta \pi_{t+1}^r \pi_{t+1}^r + 1] V_{e_{t+1}}(nd) \]

\[ + \alpha \beta [\theta (\pi_{t+1} - \pi_{t+1}^r) + \pi_{t+1}^r][1 - (1 - \theta \lambda) \pi_{t+1}^r + 1] V_{e_{t+1}}(nd) \]

\[ S \equiv \alpha \beta \lambda (\theta - 1) [\theta (\pi_{t+1} - \pi_{t+1}^r) + \pi_{t+1}^r] V_{e_{t+1}}(nd) - \lambda \beta (1 - \alpha) \pi_{t+1}^r V_{e_{t+1}}(nd) \]

\[ = \lambda \beta (1 - \alpha) V_{e_{t+1}}(nd) [\pi_{t+1}^r R_A(nd) - (\theta (\pi_{t+1} - \pi_{t+1}^r) + \pi_{t+1}^r)] R_A(d) \]
Thus \( S < 0 \) if \( R_A(d) \geq R_A(nd) \), which is the usual assumption of decreasing risk aversion but the opposite of the one made here.

\[
\frac{\partial k_{i+1}}{\partial \lambda} = \frac{[RC - SB]}{[AC - B^2]} \tag{C35}
\]

and,

\[
\frac{\partial \pi'_{i+1}}{\partial \lambda} = \frac{[AS - BR]}{[AC - B^2]} \tag{C36}
\]

\[
RC = \theta[\alpha \lambda \beta (\theta - 1)]^2 \pi'_{i+1} V_{r_{i+1}}(d) V_{r_{i+1}e_{i+1}}(nd)
\]

\[
+ \alpha \theta \lambda^2 \beta^2 (1 - \alpha) \pi'_{i+1} V_{e_{i+1}}(d) V_{e_{i+1}e_{i+1}}(nd)
\]

\[
+ [\alpha \lambda \beta (\theta - 1)]^2 [\theta (\pi_{i+1} - \pi'_{i+1}) + \pi'_{i+1}][(1 - \theta \lambda) \pi'_{i+1} + 1] V_{e_{i+1}e_{i+1}}(nd) V_{e_{i+1}e_{i+1}}(d)
\]

\[
+ \alpha(1 - \alpha) \theta \lambda^2 \beta^2 \pi'_{i+1} [\theta (\pi_{i+1} - \pi'_{i+1}) + \pi'_{i+1}][(1 - \theta \lambda) \pi'_{i+1} + 1] V_{e_{i+1}e_{i+1}}(nd) V_{e_{i+1}e_{i+1}}(d)
\]

\[
+ \alpha(1 - \alpha) [\lambda \beta (\theta - 1)]^2 \pi'_{i+1} [\pi'_{i+1} + 1] V_{e_{i+1}e_{i+1}}(nd) V_{e_{i+1}e_{i+1}}(d)
\]

\[
- SB = -[\alpha \lambda \beta (\theta - 1)]^2 [\theta (\pi_{i+1} - \pi'_{i+1}) + \pi'_{i+1}][(1 - \theta \lambda) \pi'_{i+1} + 1] V_{e_{i+1}e_{i+1}}(nd) V_{e_{i+1}e_{i+1}}(d)
\]

\[
+ \alpha \lambda^2 \beta^2 (1 - \alpha)(\theta - 1) \pi'_{i+1} [\theta (\pi_{i+1} - \pi'_{i+1}) + \pi'_{i+1}] V_{e_{i+1}e_{i+1}}(nd) V_{e_{i+1}e_{i+1}}(d)
\]

\[
- [\lambda \beta (1 - \alpha)]^2 \pi'_{i+1} [\pi'_{i+1} + 1] V_{e_{i+1}e_{i+1}}(nd) V_{e_{i+1}e_{i+1}}(d)
\]

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III and VI gets cancelled with VII and X respectively. All other terms except I and II are positive. So it is not possible to get an unambiguous sign for $\partial \kappa_{\tau_1}/\partial \lambda$.  

Again,

$$AS = \alpha^2 \beta^2 \lambda (\theta - 1) [(1 - \theta \lambda) \pi'_{\tau + 1} + 1] \theta (\pi_{\tau + 1} - \pi'_{\tau + 1}) \pi'_{\tau + 1} V_{e_{\tau + 1} + 1} (d) V_{e_{\tau + 1} + 1} (nd)$$  

$$- \alpha (1 - \alpha) \lambda \beta^2 (\theta - 1) \pi'_{\tau + 1} [\pi'_{\tau + 1} + 1] \theta (\pi_{\tau + 1} - \pi'_{\tau + 1}) \pi'_{\tau + 1} V_{e_{\tau + 1} + 1} (d) V_{e_{\tau + 1} + 1} (nd)$$  

$$- \lambda \beta^2 (1 - \alpha) (1 - \theta \lambda) \pi'_{\tau + 1} \pi'_{\tau + 1} V_{e_{\tau + 1} + 1} (d) V_{e_{\tau + 1} + 1} (nd)$$  

$$- \lambda (1 - \alpha)^2 \beta^2 \pi_{\tau + 1} V_{e_{\tau + 1} + 1} (nd) V_{e_{\tau + 1} + 1} (nd)$$  

$$- \lambda \beta (1 - \alpha) \pi'_{\tau + 1} V_{e_{\tau + 1} + 1} (nd) U''(e)$$  

$$+ \alpha^2 \beta^2 \lambda (\theta - 1) (1 - \theta \lambda) \pi''_{\tau + 1} [\theta (\pi_{\tau + 1} - \pi'_{\tau + 1}) + \pi'_{\tau + 1}] V_{e_{\tau + 1} + 1} (d) V_{e_{\tau + 1} + 1} (nd)$$  

$$+ \alpha (1 - \alpha) \lambda \beta^2 \pi''_{\tau + 1} \pi'_{\tau + 1} \theta (\pi_{\tau + 1} - \pi'_{\tau + 1}) \pi'_{\tau + 1} V_{e_{\tau + 1} + 1} (d) V_{e_{\tau + 1} + 1} (nd)$$  

And,

$$- BR = - \alpha^2 \beta^2 \lambda \theta (\theta - 1) \pi'_{\tau + 1} [(1 - \theta \lambda) \pi'_{\tau + 1} + 1] V_{e_{\tau + 1} + 1} (d) V_{e_{\tau + 1} + 1} (nd)$$  

$$+ \alpha (1 - \alpha) \beta \lambda \pi'_{\tau + 1} (1 - \theta \lambda) \pi'_{\tau + 1} \theta (\pi_{\tau + 1} - \pi'_{\tau + 1}) \pi'_{\tau + 1} V_{e_{\tau + 1} + 1} (d) V_{e_{\tau + 1} + 1} (nd)$$
\[- \alpha^2 \beta^2 \lambda (\theta - 1) \left[ (1 - \theta \lambda) \pi'_{t+1} + 1 \right]^2 \left[ \theta (\pi_{t+1} - \pi'_{t+1}) + \pi'_{t+1} \right] V_{e_{t+1} e_{t+1}}(d) V_{e_{t+1} e_{t+1}}(d) \quad [XIII] \]

\[\alpha (1 - \alpha) \lambda \beta^2 \left[ \pi'_{t+1} + 1 \right] \left[ \theta (\pi_{t+1} - \pi'_{t+1}) + \pi'_{t+1} \right] \left[ (1 - \theta \lambda) \pi'_{t+1} + 1 \right] V_{e_{t+1} e_{t+1}}(d) V_{e_{t+1} e_{t+1}}(d) \quad [XIV] \]

\[- \alpha (1 - \alpha) (\theta - 1) \beta^2 \lambda \pi'_{t+1} + 1 \left[ (1 - \theta \lambda) \pi'_{t+1} + 1 \right] V_{e_{t+1} e_{t+1}}(d) V_{e_{t+1} e_{t+1}}(nd) \quad [XV] \]

\[+ (1 - \alpha)^2 \beta^2 \lambda \pi'_{t+1} + 1 \left[ (1 - \theta \lambda) \pi'_{t+1} + 1 \right] V_{e_{t+1} e_{t+1}}(nd) V_{e_{t+1} e_{t+1}}(nd) \quad [XVI] \]

It is not possible to get an ambiguous sign for [AS-BR] and hence for \( \partial \pi'_{t+1} / \partial \lambda \).

**Proof of proposition 5:**

The expected tax revenue of the government at time \( t \) is given by

\[ T_t = \alpha \theta \lambda [\pi_t - \pi'_t] + \lambda \pi'_t + \tau w_t \quad [16] \]

Therefore,

\[ \frac{\partial T_{t+1}}{\partial \alpha} = \theta \lambda [\pi_{t+1} - \pi'_{t+1}] + \lambda [1 - \alpha \theta] \frac{\partial \pi'_{t+1}}{\partial \alpha} + \alpha \theta \lambda \frac{\partial \pi_{t+1}}{\partial \alpha} + \tau \frac{\partial \pi_{t+1}}{\partial k_{t+1}} + \frac{\partial w_{t+1}}{\partial k_{t+1}} \frac{\partial t + 1}{\partial \alpha} \quad [C37] \]

From this expression it is clear that \( \partial T_{t+1} / \partial \alpha > 0 \).

Similarly, it can be shown that

\[ \frac{\partial T_{t+1}}{\partial \theta} = \alpha \lambda [\pi_{t+1} - \pi'_{t+1}] + \alpha \theta \lambda \frac{\partial \pi_{t+1}}{\partial \theta} + \lambda [1 - \alpha \theta] \frac{\partial \pi_{t+1}}{\partial \theta} + \tau \frac{\partial \pi_{t+1}}{\partial k_{t+1}} \frac{dk_{t+1}}{\partial \theta} \quad [C38] \]

so that \( \partial T_{t+1} / \partial \theta > 0 \).

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Sign for \( \partial k_{t+1} / \partial w_t \):

\[ Adk_{t+1} + Bd\pi^T_{t+1} = Jdw_t \]  \[ C39 \]

\[ Bdk_{t+1} + Cd\pi^T_{t+1} = Tdw_t \]  \[ C40 \]

where,

\[ J = (1 - \tau)U''(c_t) \]  \[ C41 \]

and

\[ T = 0 \]  \[ C42 \]

Therefore

\[ \frac{\partial k_{t+1}}{\partial w_t} = \frac{[Jc - BT]}{[Ac - B^2]} > 0 \]  \[ C43 \]

and

\[ \frac{\partial \pi^T_{t+1}}{\partial w_t} = \frac{[Ac - BT]}{[Ac - B^2]} > 0 \]  \[ C44 \]

when \( R_a(na) \geq R_a(d) \)

**Compensated taxes**

When individuals are compensated for the taxes collected from them each period, the consumption for the two periods are given by

\[ c = (1 - \tau)w - k + T_1 \]  \[ 35 \]
\[ e^d = (1 - \theta \lambda) \pi + \lambda (\theta - 1) \pi' + k + T_2 \]  

\[ e^{nd} = \pi - \lambda \pi' + k + T_2 \]  

where \[ T_1 = \tau w \]  

and \[ T_2 = \alpha \lambda \theta (\pi - \pi') + \lambda \pi' \]  

The individual's optimization problem can still be given by equations [14] and [15]. To look at the impact of a compensated change in the two tax parameters on the savings of the individual, note that

\[ \frac{\partial k}{\partial \tau} \bigg|_{\text{com}} = \frac{\partial k}{\partial \tau} + \frac{\partial k}{\partial T_1} \frac{\partial T_1}{\partial \tau} \]  

and

\[ \frac{\partial k}{\partial \lambda} \bigg|_{\text{com}} = \frac{\partial k}{\partial \lambda} + \frac{\partial k}{\partial T_2} \frac{\partial T_2}{\partial \lambda} \]  

where \( \partial k/\partial \tau \) and \( \partial k/\partial \lambda \) are given by [C25] and [C35] respectively. To get the value of \( \partial k/\partial T_1 \), equations [14] and [15] can be totally differentiated to yield

\[ Adk + Bd\pi' = U''(c) dT_1 \]  

and

\[ Bdk + C\pi' = 0 \]
where $A$, $B$ and $C$ are defined by equations [C1], [C2] and [C3] respectively. [C47] and [C48] together imply that

$$\frac{\partial k}{\partial T_1} = \frac{C U''(c)}{A C - B^2}$$  \hspace{1cm} [C49]$$

Therefore equations [C25], [C45] and [C49] can be combined to give

$$\frac{\partial k}{\partial \tau} \bigg|_{\text{com}} = -\frac{w C U''(c)}{A C - B^2} + \frac{w C U''(c)}{A C - B^2} = 0$$  \hspace{1cm} [C50]$$

Again, to obtain the value of $\partial k / \partial \lambda$, equations [14] and [15] can be totally differentiated to give

$$A dk + B d\pi' = WdT_2$$  \hspace{1cm} [C51]$$

$$B dk + C d\pi' = ZdT_2$$  \hspace{1cm} [C52]$$

where

$$W = -[\alpha \beta ((1 - \theta \lambda) \pi' + 1) V_{ee}(d) + \beta (1 - \alpha) (\pi' + 1) V_{ee}(nd)] > 0$$  \hspace{1cm} [C53]$$

and,

$$Z = \lambda (1 - \alpha) \beta V_e(nd)[R_A(d) - R_A(nd)] \leq 0 \quad \text{according as} \quad R_A(nd) \geq R_A(d)$$  \hspace{1cm} [C54]$$

[C51] and [C52] together imply that

$$\frac{\partial k}{\partial T_2} = \frac{W C - B Z}{A C - B^2}$$  \hspace{1cm} [C55]$$

The sign for this expression is indeterminate. The sign for $\partial k / \partial \lambda$, given by [C35] is also indeterminate. So it is not possible to determine what the sign for the compensated change in $\lambda$ would be.

Appendix C
Regarding the effect of a compensated change in the tax rates on reported profits, [C47] and [C48] imply that

\[ \frac{\partial \pi^r}{\partial T_1} = - \frac{BU''(c)}{AC - B^2} \]  \[ \text{[C56]} \]

Therefore [C26] and [C56] together imply that

\[ \frac{\partial \pi^r}{\partial \tau} \bigg|_{\text{com}} = \frac{\partial \pi^r}{\partial \tau} + \frac{\partial \pi^r}{\partial T_1} \frac{\partial T_1}{\partial \tau} = \frac{wBU''(c)}{AC - B^2} - \frac{wBU''(c)}{AC - B^2} = 0 \]  \[ \text{[C57]} \]

The effect of a compensated change in \( \lambda \) on reported profits is ambiguous, as expected.
Appendix D

1. OPTIMAL POLICY WITHOUT EVASION

Individual's problem to choose \( c_1 \) and \( c_2 \) to maximize

\[
U(c_1) + \beta U(c_2)
\]  

subject to the constraints

\[
c_1 = (1 - \tau)w - k
\]  

\[
c_2 = [1 + r(1 - \lambda)]k
\]
where

\( k \) = savings of the individual in the first period.

\( r \) = rate of return on savings.

\( \tau \) = proportional tax rate on wage income.

\( \lambda \) = proportional tax on profits.

\( w \) = exogenous wage rate.

The budget constraint can be written in the form

\[ p_1 c_1 + p_2 c_2 = M \]  \[ \text{[4]} \]

where

\[ p_1 = 1 \]

\[ p_2 = \frac{1}{1 + r(1 - \lambda)} \]  \[ \text{[5]} \]

\[ M = (1 - \tau)w \]  \[ \text{[6]} \]

The F.O.C. for the individual’s optimization problem is

\[ U'(c_1) = [1 + r(1 - \lambda)] \beta U'(c_2) \]  \[ \text{[7]} \]

The indirect utility function for the individual can be written as \( V(\tau, \lambda, w, r) \) where

\[ \frac{\partial V}{\partial \tau} = - wU'(c_1) \]  \[ \text{[8]} \]

and

\[ \frac{\partial V}{\partial \lambda} = - \beta r k U'(c_2) \]  \[ \text{[9]} \]

The government’s objective is to maximize \( V \) subject to its revenue constraint.
\[ T = \tau w + (1 - \tau) \lambda r w - \lambda r c_1 \geq \bar{T} \]

where \( \bar{T} \) is the fixed revenue requirement of the government. From [10] we get

\[ \frac{\partial T}{\partial \tau} = w - \lambda r w + \lambda r w \frac{dc_1}{dM} \]

and

\[ \frac{\partial T}{\partial \lambda} = rk \frac{\lambda r^2 c_2}{(1 + r(1 - \lambda))^2} + \frac{\lambda r^2 c_2}{(1 + r(1 - \lambda))^2} \frac{dc_1}{dM} \]

Now the government’s optimization problem requires that

\[ \frac{\partial V/\partial \tau}{\partial V/\partial \lambda} = \frac{\partial T/\partial \tau}{\partial T/\partial \lambda} \]

This implies that

\[ c_2 = \lambda s_{12} \]

2. OPTIMAL TAX POLICY UNDER TAX EVASION

The individual’s optimization problem can be given as that of maximizing

\[ U(c_1) + \alpha \beta U(c_2) + (1 - \alpha) \beta U(c_3) \]

where \( c_1 = c_n \), \( c_2 = c_{n+1} \) and \( c_3 = c_{n+1} \). If it is assumed that savings of \( k \) in period \( t \) yields profits of \( rk \) in the next period, we have
\[ c_1 = (1 - \tau)w - k \] \hspace{1cm} [2]

\[ c_2 = [1 + r(1 - \theta \lambda)]k + \lambda(\theta - 1)\pi' \] \hspace{1cm} [3]

\[ c_3 = (1 + r)k - \lambda \pi' \] \hspace{1cm} [4]

The constraints can be combined to give

\[ p_1 c_1 + p_2 c_2 + p_3 c_3 = M \] \hspace{1cm} [5]

where

\[ p_1 = 1 \] \hspace{1cm} [6]

\[ p_2 = \frac{1}{\theta [1 + r(1 - \lambda)]} \] \hspace{1cm} [7]

\[ p_3 = \frac{\theta - 1}{\theta [1 + r(1 - \lambda)]} \] \hspace{1cm} [8]

and

\[ M = (1 - \tau)w \] \hspace{1cm} [9]

The individual's problem thus reduces to choosing \( c_1, c_2, \) and \( c_3 \) to maximize \([1]\) subject to the budget constraint \([5]\). The F.O.C.'s for a maximum requires that

\[ \alpha(\theta - 1)U''(c_2) + (1 - \alpha)U''(c_3) = 0 \] \hspace{1cm} [10]

and

\[ U'(c_1) = [1 + r(1 - \lambda)][\alpha \beta U''(c_2) + (1 - \alpha)\beta U''(c_3)] \] \hspace{1cm} [11]

Equations \([10]\) and \([11]\) can be combined to give

Appendix D

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\( U'(c_1) = \alpha \beta \theta [1 + r(1 - \lambda)] U'(c_2) \quad [12] \)

From the individual's optimization problem it is possible to derive the indirect utility of the individual. Suppose this be given by \( V(\tau, \lambda, \alpha, \theta, r, w) \). Then we have

\[ \frac{\partial V}{\partial \tau} = -w U'(c_1) \quad [13] \]

\[ \frac{\partial V}{\partial \lambda} = -\alpha \beta \theta r k U'(c_2) \quad [14] \]

Using [12], [13] and [14] we get

\[ \frac{\partial V/\partial \tau}{\partial V/\partial \lambda} = \frac{[1 + r(1 - \lambda)]w}{rk} \quad [15] \]

The government's problem is to choose \( \tau, \lambda, \theta, \) and \( \alpha \) to maximize \( V \) subject to its revenue constraint

\[ T^n = \tau w + \alpha \theta \lambda r k + \lambda (1 - \alpha \theta) \pi' - g(\alpha) \geq \bar{T} \quad [16] \]

The revenue constraints can be written in terms of \( c \)'s in a number of ways. One way is to write it in terms of \( c_2 \) and \( c_3 \). From [3] and [4] we get

\[ k = \frac{c_2 + (\theta - 1)c_3}{\theta(1 + r(1 - \lambda))} \quad [17] \]

and

\[ \lambda \pi' = \frac{(1 + r)c_2}{\theta(1 + r(1 - \lambda))} - \left[ \frac{1 + r(1 - \theta \lambda)}{\theta(1 + r(1 - \lambda))} \right] c_3 \quad [18] \]

Substituting the values of \( k \) and \( \lambda \pi' \) from [17] and [18] respectively, in [16] we get
\[ T^n = \tau w + \left[ \frac{\alpha \theta \lambda r + (1 - \alpha \theta)(1 + r)}{\theta(1 + r(1 - \lambda))} \right] c_2 + \left[ \frac{\beta \theta \lambda (1 - \alpha) - (1 - \alpha \theta)(1 + r)}{\theta(1 + r(1 - \lambda))} \right] c_3 - g(\alpha) \]  

Therefore, \[ \frac{\partial T^n}{\partial \tau} = w - w \left[ \frac{\alpha \theta \lambda r + (1 - \alpha \theta)(1 + r)}{\theta(1 + r(1 - \lambda))} \right] \frac{dc_2}{dM} - w \left[ \frac{\beta \theta \lambda (1 - \alpha) - (1 - \alpha \theta)(1 + r)}{\theta(1 + r(1 - \lambda))} \right] \frac{dc_3}{dM} \]  

and \[ \frac{\partial T^n}{\partial \lambda} = \frac{r(1 + r)c_2}{\theta(1 + r(1 - \lambda))^2} + \frac{r(1 + r)(\theta - 1)c_3}{\theta(1 + r(1 - \lambda))^2} \]

\[ + \left[ \frac{\alpha \theta \lambda r + (1 - \alpha \theta)(1 + r)}{\theta(1 + r(1 - \lambda))} \right] \left[ \frac{dc_2}{dp_2} \frac{dp_2}{d\lambda} + \frac{dc_2}{dp_3} \frac{dp_3}{d\lambda} \right] \]

\[ + \left[ \frac{\beta \theta \lambda (1 - \alpha) - (1 - \alpha \theta)(1 + r)}{\theta(1 + r(1 - \lambda))} \right] \left[ \frac{dc_3}{dp_2} \frac{dp_2}{d\lambda} + \frac{dc_3}{dp_3} \frac{dp_3}{d\lambda} \right] \]

Now, \[ \frac{dp_2}{d\lambda} = \frac{r}{\theta(1 + r(1 - \lambda))^2} \]  

and \[ \frac{dp_3}{d\lambda} = \frac{r(\theta - 1)}{\theta(1 + r(1 - \lambda))^2} \]  

Also the Slutsky equation implies that \[ \frac{dc_3}{dp_1} = s_y - c_3 \frac{dc_3}{dM} \]  

where \( s_y \) refers to the Slutsky substitution terms.
Equation [21] can then be written as

\[
\frac{\partial T^n}{\partial \lambda} = \frac{r(1 + r)c_2}{\theta(1 + r(1 - \lambda))^2} + \frac{r(1 + r)(\theta - 1)c_3}{\theta(1 + r(1 - \lambda))^2}
\]

\[
+ \frac{r(a \theta \lambda r + (1 - a \theta)(1 + r))}{\theta^2(1 + r(1 - \lambda))^3} \left[ (s_{22} + (\theta - 1)s_{23}) - (c_2 + (\theta - 1)c_3) \frac{dc_2}{dM} \right]
\]

\[
+ \frac{r(\theta \lambda(1 - a) - (1 - a \theta)(1 + r))}{\theta^2(1 + r(1 - \lambda))^3} \left[ (s_{32} + (\theta - 1)s_{33}) - (c_2 + (\theta - 1)c_3) \frac{dc_3}{dM} \right]
\]  

[25]

The optimization problem of the government requires that

\[
\frac{\partial V}{\partial \lambda} = \frac{\partial T^n}{\partial \lambda}
\]

[26]

Upon substitution and simplification, [26] implies,

\[
r[c_2 + (\theta - 1)c_3] + \left[ \frac{a \theta \lambda r + (1 - a \theta)(1 + r)}{\theta(1 + r(1 - \lambda))} \right][s_{22} + (\theta - 1)s_{23}]
\]

\[
+ \left[ \frac{r \theta \lambda(1 - a) - (1 - a \theta)(1 + r)}{\theta(1 + r(1 - \lambda))} \right][s_{32} + (\theta - 1)s_{33}] = 0
\]

or,

\[
r[c_2 + (\theta - 1)c_3] + [a \theta \lambda r + (1 - a \theta)(1 + r)][P_2s_{22} + P_3s_{33}]
\]

\[
+ [r \theta \lambda(1 - a) - (1 - a \theta)(1 + r)][P_2s_{32} + P_3s_{33}] = 0
\]

[27]

Equation [27] could be further simplified by using the condition

\[
\sum_{j=1}^{\eta} s_{ij} p_j = 0
\]

[28]
In this case equation [27] reduces to

\[ r[c_2 + (\theta - 1)c_3] = [a^2\lambda r + (1 - a^2)(1 + r)]s_{31} + [r^2\lambda(1 - a) - (1 - a^2)(1 + r)]s_{21} \] \[ 29 \]

Equation \[29\] shows what the optimal policy of the government regarding the two tax rates \(\tau\), and \(\lambda\) would be.
TABLE 1.

Effect of compensated changes in $\theta$:

$(\chi = 2, \tau = 0.0, \lambda = 0.28, \alpha = 0.2, \beta = 0.9, \eta = 3, \text{ and } \psi = 0.25)$

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$k$</th>
<th>$\pi$</th>
<th>$\pi'$</th>
<th>$EU$</th>
<th>$T_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.8</td>
<td>0.89827994</td>
<td>0.73015353</td>
<td>0.00722262</td>
<td>-1.3478979</td>
<td>0.0953779</td>
</tr>
<tr>
<td>3.0</td>
<td>0.88962028</td>
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<td>0.13651942</td>
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<td>3.3</td>
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<td>0.14188546</td>
</tr>
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<td>0.87378904</td>
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<td>0.40936309</td>
<td>-1.3425529</td>
<td>0.15827928</td>
</tr>
<tr>
<td>3.9</td>
<td>0.86950686</td>
<td>0.72423501</td>
<td>0.50362575</td>
<td>-1.3415992</td>
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</tr>
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<td>4.2</td>
<td>0.86675628</td>
<td>0.72366158</td>
<td>0.57963982</td>
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<td>0.17617307</td>
</tr>
<tr>
<td>4.5</td>
<td>0.86514017</td>
<td>0.72332401</td>
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<td>0.18025015</td>
</tr>
<tr>
<td>4.8</td>
<td>0.86438139</td>
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<td>0.69348293</td>
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<tr>
<td>5.0</td>
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<td>0.72313816</td>
<td>-1.3403909</td>
<td>0.18247868</td>
</tr>
</tbody>
</table>

$\chi$ = the Arrow-Pratt measure of relative risk aversion.
$\delta$ = weight attached to first period leisure.
$\theta$ = the penalty rate.
$\alpha$ = the audit probability.
$\tau$ = tax rate applicable to personal income.
$\lambda$ = tax rate applicable to corporate profits.
$\eta$ = the scalar constant for the Cobb-Douglas production function.
$\psi$ = capital's share of output.
$\beta$ = the rate of discount.
$T_i$ = the compensated tax for the old.
TABLE 2.

Effect of compensated changes in $\alpha$:

$(\chi = 2, \tau = 0.0, \lambda = 0.28, \theta = 3.5, \beta = 0.9, \eta = 3, \text{ and } \psi = 0.25)$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$k$</th>
<th>$\pi$</th>
<th>$\pi^*$</th>
<th>$EU$</th>
<th>$T_2$</th>
</tr>
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<tbody>
<tr>
<td>0.12</td>
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<td>0.73081881</td>
<td>0.00446874</td>
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<td>0.07947002</td>
</tr>
<tr>
<td>0.15</td>
<td>0.88882191</td>
<td>0.72822394</td>
<td>0.14882817</td>
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<td>0.11559307</td>
</tr>
<tr>
<td>0.18</td>
<td>0.8797398</td>
<td>0.72635649</td>
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<td>0.87418387</td>
<td>0.72520696</td>
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<td>0.15804718</td>
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<td>0.70288283</td>
<td>-1.3452209</td>
<td>0.16374212</td>
</tr>
</tbody>
</table>

[Parameters defined on page 147].
TABLE 3.

Effect of compensated changes in $\lambda$:

$(\chi = 2, \tau = 0.0, \alpha = 0.2, \theta = 3.5, \beta = 0.9, \eta = 3.0, \text{and } \psi = 0.25)$

<table>
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</tr>
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<td>0.10303439</td>
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<td>0.24</td>
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<td>-1.3502991</td>
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</table>

[Parameters already defined].

Appendix D
### TABLE 4.

Changes in $\alpha$ and $\theta$ keeping expected revenues of the government constant at 0.45.

$(\chi = 2$, $\tau = 0.15$, $\lambda = 0.28$, $\beta = 0.9$, $\eta = 3.0$, and $\psi = 0.25)$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\theta$</th>
<th>$k_i$</th>
<th>$\pi_i$</th>
<th>$\pi'_i$</th>
<th>$EU_i$</th>
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<tr>
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<td>0.74690592</td>
<td>0.69723269</td>
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</tr>
<tr>
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<td>0.69739122</td>
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</tr>
<tr>
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<td>0.7483167</td>
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</tr>
<tr>
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<td>0.75000364</td>
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<tr>
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</tr>
<tr>
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<td>-1.6867091</td>
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</table>

[Parameters already defined].
TABLE 5.
Changes in $\tau$ and $\lambda$ keeping expected revenues
of the government as constant at 0.45.

$(\chi = 2, \alpha = 0.2, \theta = 3.5, \beta = 0.9, \eta = 3.0, \text{ and } \psi = 0.25)$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\tau$</th>
<th>$k_r$</th>
<th>$\pi_t$</th>
<th>$\pi'_t$</th>
<th>$EU_t$</th>
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<tr>
<td>0.12</td>
<td>0.20408231</td>
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<td>0.67437995</td>
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<td>-1.7598248</td>
</tr>
<tr>
<td>0.15</td>
<td>0.19228324</td>
<td>0.67376219</td>
<td>0.67949759</td>
<td>0.14855027</td>
<td>-1.7400212</td>
</tr>
<tr>
<td>0.18</td>
<td>0.1805321</td>
<td>0.69403016</td>
<td>0.68455106</td>
<td>0.24067111</td>
<td>-1.7209897</td>
</tr>
<tr>
<td>0.21</td>
<td>0.16882662</td>
<td>0.71449564</td>
<td>0.68954269</td>
<td>0.30790055</td>
<td>-1.7026855</td>
</tr>
<tr>
<td>0.24</td>
<td>0.15716467</td>
<td>0.73515776</td>
<td>0.69447465</td>
<td>0.35955018</td>
<td>-1.6850676</td>
</tr>
<tr>
<td>0.27</td>
<td>0.14554427</td>
<td>0.75601564</td>
<td>0.699349</td>
<td>0.40079401</td>
<td>-1.6680978</td>
</tr>
<tr>
<td>0.3</td>
<td>0.13396357</td>
<td>0.77706837</td>
<td>0.70416765</td>
<td>0.43473723</td>
<td>-1.6517411</td>
</tr>
<tr>
<td>0.33</td>
<td>0.12242084</td>
<td>0.79831503</td>
<td>0.70893242</td>
<td>0.46335651</td>
<td>-1.6359649</td>
</tr>
</tbody>
</table>

[Parameters already defined].
TABLE 6.
Changes in $\theta$ and $\lambda$ keeping expected revenues
of the government constant at 0.45.

($x = 2$, $\alpha = 0.2$ $\tau = 0.15$, $\beta = 0.9$, $\eta = 3.0$, and $\psi = 0.25$)

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\lambda$</th>
<th>$k_r$</th>
<th>$\pi_r$</th>
<th>$\pi'_r$</th>
<th>$EU_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>0.49156562</td>
<td>0.76638506</td>
<td>0.70173481</td>
<td>0.032762588</td>
<td>-1.7097361</td>
</tr>
<tr>
<td>2.4</td>
<td>0.40086839</td>
<td>0.75917754</td>
<td>0.70007908</td>
<td>0.09718045</td>
<td>-1.6957903</td>
</tr>
<tr>
<td>2.7</td>
<td>0.34076054</td>
<td>0.75446988</td>
<td>0.69899125</td>
<td>0.17117768</td>
<td>-1.6867912</td>
</tr>
<tr>
<td>3.0</td>
<td>0.29997115</td>
<td>0.75127447</td>
<td>0.69824996</td>
<td>0.25097921</td>
<td>-1.6807319</td>
</tr>
<tr>
<td>3.3</td>
<td>0.27201803</td>
<td>0.74906652</td>
<td>0.69773637</td>
<td>0.33231147</td>
<td>-1.6765679</td>
</tr>
<tr>
<td>3.6</td>
<td>0.25292499</td>
<td>0.74753913</td>
<td>0.69738041</td>
<td>0.41205979</td>
<td>-1.6736983</td>
</tr>
<tr>
<td>3.9</td>
<td>0.24013098</td>
<td>0.7465</td>
<td>0.69713794</td>
<td>0.48671323</td>
<td>-1.6717511</td>
</tr>
<tr>
<td>4.2</td>
<td>0.23192157</td>
<td>0.74582216</td>
<td>0.69697963</td>
<td>0.55454349</td>
<td>-1.6704832</td>
</tr>
<tr>
<td>4.5</td>
<td>0.22711252</td>
<td>0.74541827</td>
<td>0.69688525</td>
<td>0.61455931</td>
<td>-1.6697285</td>
</tr>
<tr>
<td>4.8</td>
<td>0.22486426</td>
<td>0.74522624</td>
<td>0.69684036</td>
<td>0.66652676</td>
<td>-1.6693699</td>
</tr>
<tr>
<td>5.0</td>
<td>0.22448051</td>
<td>0.745193</td>
<td>0.69683259</td>
<td>0.69683259</td>
<td>-1.6693078</td>
</tr>
</tbody>
</table>

[Parameters defined on page 147].
TABLE 7.
Changes in $\tau$ and $\lambda$ keeping expected revenues of the government as constant at 0.45.

No tax evasion

$(\chi = 2, \beta = 0.9, \eta = 3.0, \text{ and } \psi = 0.25)$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\tau$</th>
<th>$k_t$</th>
<th>$\pi_t$</th>
<th>$EU_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td>0.17789612</td>
<td>0.7097706</td>
<td>0.68840141</td>
<td>-1.6962235</td>
</tr>
<tr>
<td>0.15</td>
<td>0.1659626</td>
<td>0.73553919</td>
<td>0.69456472</td>
<td>-1.6764402</td>
</tr>
<tr>
<td>0.18</td>
<td>0.1540914</td>
<td>0.76159325</td>
<td>0.70063533</td>
<td>-1.6574335</td>
</tr>
<tr>
<td>0.21</td>
<td>0.14227921</td>
<td>0.78793451</td>
<td>0.70661653</td>
<td>-1.6391544</td>
</tr>
<tr>
<td>0.24</td>
<td>0.13052295</td>
<td>0.81455839</td>
<td>0.7125114</td>
<td>-1.6215581</td>
</tr>
<tr>
<td>0.27</td>
<td>0.11881975</td>
<td>0.84146038</td>
<td>0.71832285</td>
<td>-1.6046035</td>
</tr>
<tr>
<td>0.3</td>
<td>0.10716697</td>
<td>0.86863612</td>
<td>0.72405363</td>
<td>-1.5882528</td>
</tr>
<tr>
<td>0.33</td>
<td>0.09556214</td>
<td>0.89608134</td>
<td>0.72970634</td>
<td>-1.5724715</td>
</tr>
</tbody>
</table>

[Parameters defined on page 147].
Vita

Partha Sengupta was born in Calcutta, India. In 1983 he completed his B.Sc. from the University of Calcutta with economics as the major subject and statistics and mathematics as minor subjects. In fall 1984 he joined the State University of New York at Stony Brook and completed his M.A. in economics in December 1985. In April 1986 he joined Virginia Polytechnic Institute and State University and completed his Ph.D in Economics in August, 1991.

Partha had taught at Virginia Polytechnic Institute and State University from Summer 1987 through Spring 1988. His research and teaching interests are in the area of public finance, economic development and international trade.