TAX TREATMENT OF TRADE IN LIVE CATTLE FUTURES

USING A MEAN-VARIANCE APPROACH:

IMPLICATIONS TO MARKET EFFICIENCY AND WELFARE CHANGES

by

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(ABSTRACT)

Cattle feeders are in a position to incorporate the influence of current and highly specific information into price discovery processes for live cattle and feeder cattle futures. The tax treatment of speculative trades in the cattle futures markets has the potential to effective block participation of cattle feeders, however. To the extent that cattle feeders are effectively blocked from trading in futures in any capacity other than trades that meet the simplistic IRS "equal and opposite" criterion of a hedge, the correction of market imbalances may be slowed. The economic viability of investments in cattle feeding and the wellbeing of the consuming public can be influenced in a significant way by prolonged market imbalances.

A theoretical model is developed based on a mean-variance approach. The model deals with the simultaneous determination of optimal cash and futures positions given tax parameters of marginal tax rate and percent of deductibility of any futures losses. Producers' risk aversion is considered. This research examines the impacts of changes in tax policy

on the optimal cash and futures positions, equilibrium prices, pricing efficiency, welfare changes in both cash and futures markets, and on tax revenues. Comparative static analyses are performed to examine the changes in the measures of efficiency and/or welfare given a unit change in the tax parameters. A revenue-neutral tax scheme is incorporated into the model to analyze the comparative statics when tax receipts are fixed.

The simulation results indicate that given a marginal tax rate, an increase in the probability of deductibility of any futures losses would increase both the optimal cash and futures positions. The adjustments of the optimal cash and futures positions reduce the spreads between expected cash prices and marginal costs for the cash market and between the expected futures prices and current futures price (including trading costs) for the futures market. This implies that pricing efficiency in both the cash and futures markets is improved by an increase in the deduction level for a given tax rate. Both producers' and consumers' welfare increases in response to the increased probability of deductibility for a given tax rate. Expected tax revenues become positive and increase as the deduction level increases.

The comparative static analyses generally support the empirical findings. A revenue-neutral marginal increase in the deductibility of losses for a given tax rate would result in a transfer of risk exposure from the cattle industry to the government sector. This has an important implication for risk-sharing aspects of a revenue-neutral tax policy, and suggests that policies should examine both the potentially variable mean-level tax revenues and variance of that revenue flow under alternative scenarios.

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TABLE OF CONTENTS

CHAPTER FOUR: COMPARATIVE STATICS

CHAPTER FIVE: PROCEDURES FOR EMPIRICAL ANALYSIS

CHAPTER SIX: EMPIRICAL RESULTS AND IMPLICATIONS

LIST OF FIGURES

LIST OF TABLES

CHAPTER ONE : INTRODUCTION

1.1 Introduction

Cattle prices are highly variable. Price changes of $10-15 per hundredweight are common within a 12-month time period. As Purcell (1991) states, variability of that magnitude imposes costs on producers, processors, and consumers in the form of costly adjustments to price risk exposure.

Over time, cattle prices will be determined by long-term supply and demand forces. If there are cyclical and long-run imbalances between supply and demand, the market will eventually correct the imbalances. However, short-run price variations are not caused by the cyclical developments on the supply side or by shifts in demand. Short-run fluctuations in placements of cattle into the feedlots essentially cause the short-run variability in fed cattle prices.

Variability in placements of cattle into feedlots leads to predictable variability in the marketings of fed cattle, given that the weight of feeder cattle placed on feed dictates the length of the feeding program. Variability in the marketing of fed cattle leads to variability in the supply of fed cattle and to variable prices and quantities of beef at the consumer level. Highly volatile prices of feeder and slaughter cattle, in turn, lead to variable placements of cattle on feed.

The volatility of cattle prices is thus related primarily to the widely observed short-run variability on the supply side. Both the volatility of prices and the related variability of supply may increase the uncertainty of profitability of investment in cattle feeding over time. In addition, the price instability means producers face the risk of highly variable revenues in the short run, creating cash flow problems and raising the costs of borrowed funds.

Most analyses indicate a highly inelastic (-0.5) demand for fed cattle at the feedlot level (Chang, 1977; Huang and Haidacher, 1983). This highly inelastic demand means that the changes in supply discussed above will prompt extreme price movements. Thus, for producers, a variable supply of fed cattle means that it is harder to earn profits from cattle feeding.

For consumers, a variable supply leads to volatile prices of beef at higher average prices. Exposure to price risk by cattle feeders and processors carries with it a cost and, over time, that cost will typically be transferred to consumers in the form of higher prices. Research by Brorsen, Chavas, Grant and Schnake (1985) indicates exposure to price risk tends to prompt extraction of larger margins by agricultural processors, pushing consumer level prices up over time.

Taken together, these two types of losses (producer and consumer), in whatever form they exist, constitute social losses which may be possible to measure quantitatively. It can be argued that the social losses could be reduced and the economic viability of the entire beef sector enhanced by stabilizing the prices of feed, feeder cattle, and finished cattle.

1.2 Cattle Feeders' Economic Environment

Cattle feeders turn feeder cattle of varying weights and grades into fed steers and heifers using their facilities and management skills. Part of the feedlot capacity can be utilized for custom feeding. In return for providing its facilities and management services, the operator of the (custom feeding) feedlot is paid by the owners of the cattle.

The decision to place cattle depends on feeder cattle costs, potential selling prices for fed cattle, and the related profit expectations. Figure 1.1 demonstrates the realized net margins on Great Plains feeding activities from 1983 through 1987.[1]

In the calculation of these margins, cattle feeders are assumed to be cash market speculators, implying no forward pricing is being employed. The net margins are calculated by subtracting the total costs of feeder cattle, corn, soybean meal, hay, and interest on the investment in feeder cattle, corn, soybean meal and hay from actual cash prices for fed cattle four months later.[2] The margins are on a per head basis. The cash prices of feeder cattle, corn, soybean meal and hay, and interest rates are assumed to be fixed at the beginning of the feeding program. The typical four-month feeding program involves turning feeder cattle weighing 750 pounds into fed cattle weighing 1,150 pounds.

As shown in Figure 1.1, the net profit series exhibits pronounced

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[1] The 1983-87 time period is used in calculating the margins because it coincides with a unique data base to be employed in this study.

[2] A detailed computation of the variable costs of cattle feeding is provided in Chapter 5.

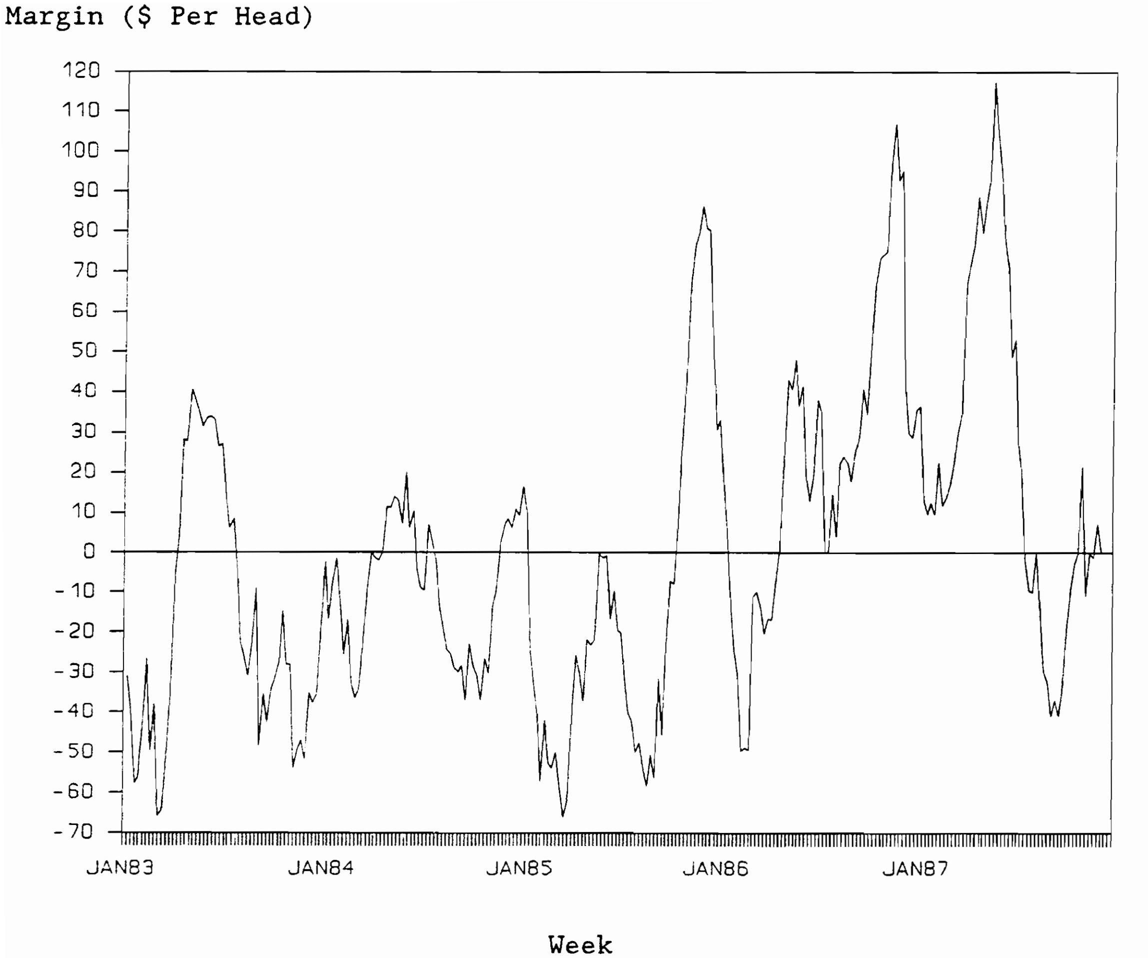


Figure 1.1 Net Profits Per Head in the Great Plains Feeding Area by Weeks, January 1983-December 1987

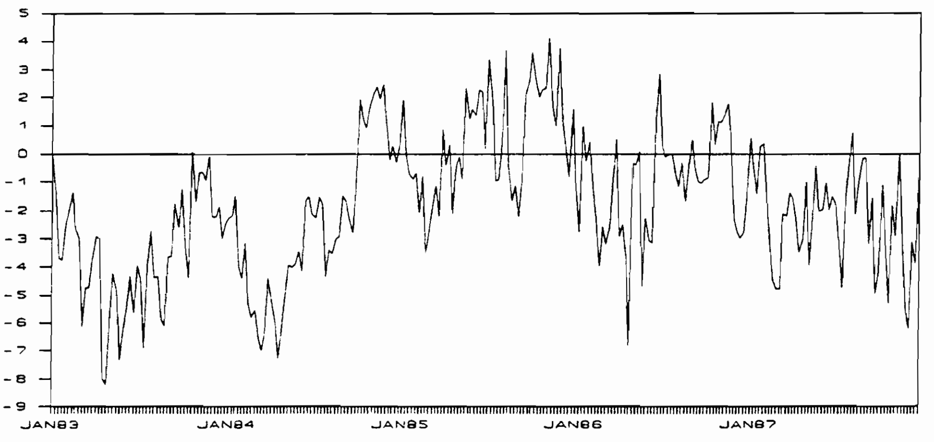
variability. Losses may exceed $65 per head. If an assessment for fixed costs were included in the margin calculation, the illustrations would look even worse. The decision to place cattle into feedlots is apparently accompanied by high levels of risk and uncertainty.

Past research has shown that effectively managed hedging strategies, such as "selective hedging", could reduce the variation in profits without significantly reducing average per head profit over time (Kenyon and Clay, 1987). But even capable market analysts and astute managers of hedging programs are faced with complex and uncertain market situations. Cattle feeders, as potential traders in futures markets, can seldom secure prices above the prevailing daily price for fed cattle of a particular grade and weight in a market area. They are price takers in the cash market for fed cattle. Also, individual cattle feeders have little or no ability to influence feed prices and only limited ability to influence feeder cattle prices on a day-to-day basis.

Figures 1.2 and 1.3 show the difficulties facing the cattle feeders. The available or expected margins are computed by subtracting the projected feeding costs from the weekly closing quotes for the relevant distant live cattle futures contracts. The distant (four months) live cattle futures prices are not adjusted for a basis allowance. The feeding costs used in Figure 1.2 are the same as in the calculation of the realized net margins defined in Figure 1.1. Figure 1.3 illustrates the case where fixed costs are also included.

Some cattle feeders might be able to achieve costs below those estimated in the calculations of the realized net margins (Figure 1.1) and

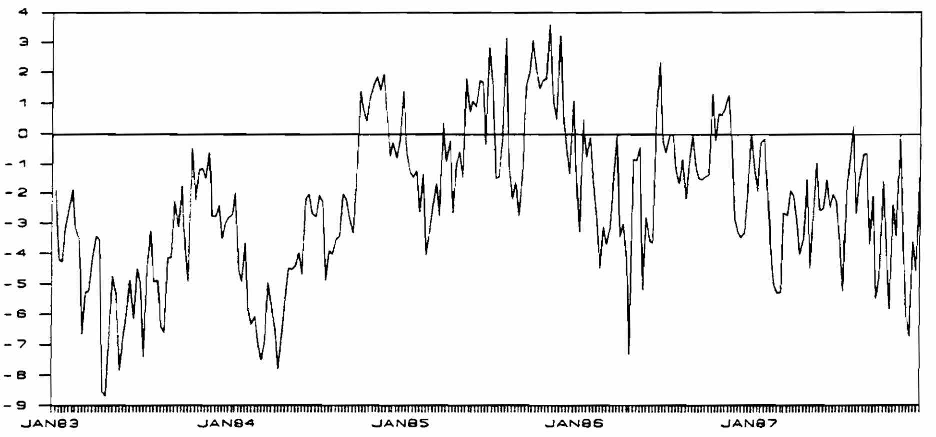
Margin ($/cwt.)



Week

Figure 1.2 Margin over Variable Costs Offered by the Closing Price of Distant Futures, 1983-87

Margin ($/cwt.)



Week

Figure 1.3 Margin over All Costs Offered by the Closing Price of Distant Futures, 1983-87 the expected margins offered by the distant futures prices (Figures 1.2 and 1.3). Also, they might be able to sell futures at more favorable prices. Figure 1.4 demonstrates the case using the highest price offered by the relevant distant futures contract during the month the cattle are placed. From January 1983 to December 1987 (261 weeks in total), the highest available futures prices covered all feeding costs only about 20 percent of the time.

The cattle feeders are apparently faced with a situation in which cattle can seldom be placed and immediately hedged at a profit. Often, even the variable costs (feeder cattle, feed, and interest on those and other variable costs) cannot be covered. In such a case, the cattle feeders must leave the feedlots empty or place the cattle and hope the situation will improve. As a result, placements of cattle on feed are volatile. The volatility in the placements leads to highly variable prices for fed cattle.

There are economic reasons that the futures market will not always offer profitable hedging opportunities to the cattle feeders. The market is extremely competitive, with no significant barriers to entry. Thus, only the most efficient producers would be expected to cover average total production costs in the long run. In the short run, market imbalances between projected costs and available pricing opportunities can persist, resulting in variable supplies of fed cattle and variable prices. The fluctuations at the feedlot level may, in turn, generate variability in prices and in product availability at the consumer level over time. It could therefore be argued that the shorter the duration of any market

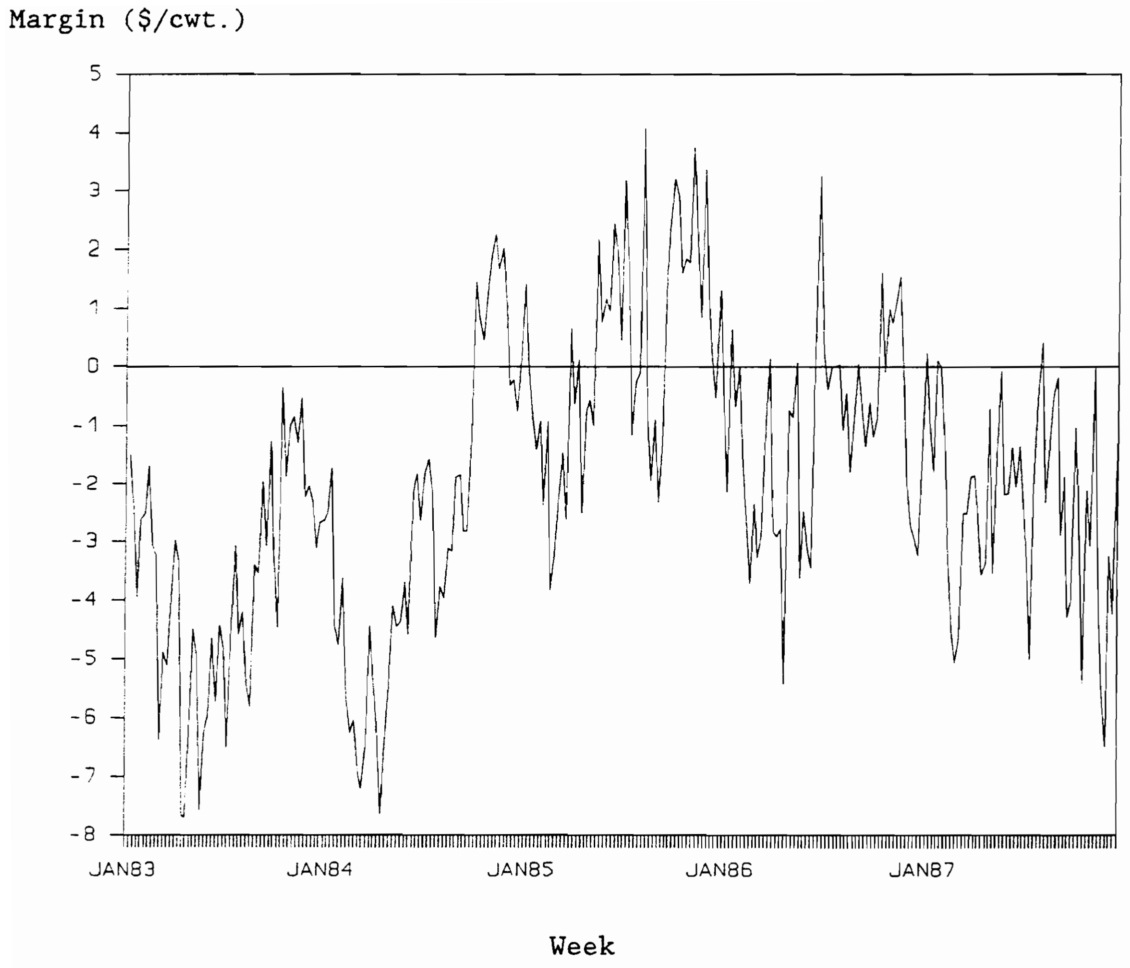


Figure 1.4 Margin over All Costs Offered by the High Price of Distant Futures, 1983-87

imbalances or disequilibriums, the more efficient is the entire system in an economic sense.

As noted earlier in this section, cattle feeders as potential traders of cattle futures can exert little influence on feed prices (corn prices, soybean meal prices and hay prices). Cattle feeders, because they face competition from stocker operators and cow-calf operators seeking breeding stock, cannot dominate the price discovery process for feeder cattle. They are essentially price takers in the fed cattle market, with only limited ability to influence day-to-day prices as they manage the cattle being offered for sale on a "showlist" for the feedlot. Thus, the market imbalances cannot be corrected in the short run by changing the costs of feed or feeder cattle or by changes in the selling prices for fed cattle.

Imbalances from an unknown but underlying equilibrium may be seen as evidence of inefficient markets. However, that view may be too narrow and restrictive. An efficient market in the Fama context is defined as a market that discovers a price that reflects all the available supply and demand information. The efficiency of the feeder cattle and live cattle futures markets is based primarily on the quality of the information base and the effectiveness of the traders.

Publicly available series of price-related information are often weekly or monthly. Information series are available from the USDA and some private services and university extension personnel. Price and projected revenue series reflect some average conversion rate for average cattle under average feedlot conditions. Obviously, the cattle feeders themselves can have access to better (more timely, more accurate, more specific) information on costs of cattle feeding and the related opportunities offered by the futures market. They are directly involved in cattle feeding and have immediate access to proprietary information. They are, perhaps, in a better position to practice any needed arbitrage between cash and futures markets or between nearby and distant futures over time than are other traders in cattle futures.

The length of response lags in the price discovery process to information is closely related to information quality. The following factors may decide the length of time needed for a market to incorporate the relevant information: 1) the time interval between publications and/or the time lag between collection and receipt by traders, and 2) the perceived accuracy and integrity of the information. Purcell and Hudson (1985) suggest that the futures prices appear to be capable of reflecting available information intraday or with a time lag of one day or less. But the authors also documented that short-run volatility persists in the cattle futures and cash markets.

What appears on the surface to be an inefficient market may result from any policy position that blocks well-informed participants from being directly involved in the futures markets. Any policy that constrains the effectiveness of the price discovery process in the markets, generating pricing patterns and market behavior that seem to be evidence of market inefficiency, should be analyzed in terms of net benefits and costs to society. One such policy, perhaps, is the *Treasury/Internal Revenue Service* (IRS) policy on how hedge *vs*. speculative trades in futures or options contracts are defined and taxed.

1.3 IRS Positions

The enactment of the *Economic Recovery Tax Act* of 1981 (ERTA) changed significantly the taxation of transactions in commodity futures contracts. The new provisions were designed to eliminate abusive tax sheltering arrangements. Prior to the enactment of ERTA, there were few provisions in the Internal Revenue Code dealing with the intricacies of commodity futures transactions (Ernst and Tyrrell, 1984).[3]

In general, commodity futures contracts that are not part of hedges are treated as capital assets. The gain or loss from the sale or exchange of such contracts receives capital gain or loss treatment, the deductibility of which is restricted. In a recent case of *Arkansas Best Corp* (1988), the U.S. Supreme Court held that: (1) a tax-payers motivation for purchasing an asset is irrelevant to the question of whether the asset is a "capital asset," (2) the sole exceptions to the "capital asset" definition are those listed in the *Internal Revenue Code*, and (3) stock purchased by a company is subject to capital loss (rather than ordinary loss) treatment at sale regardless of whether it was held for a business purpose.

The *Arkansas Best* court thus rejected the broad interpretations of the earlier 1955 *Corn Products* case. The *Corn Products* doctrine stood for

[3] Prior to the enactment of ERTA, the sale or exchange of a "*long*" commodity futures contracts, held for more than 6 months, would generate a long-term capital gain or loss. The sale or exchange of a "*long*" commodity futures contract, held for less than 6 months, or the sale or exchange of any "*short*" commodity futures contracts would result in short-term capital gain or loss.

the narrow proposition that hedging transactions that are an integral part of a business inventory purchase system fall within the exclusion from the capital asset definition (Moran, 1988). The IRS has failed to provide detailed guidance on its interpretation of what is and is not hedging. The uncertainty engendered by Arkansas Best has raised questions about taxpayers' use of futures markets in controlling the risk of commodity price fluctuations. The IRS, in a July 1994 release, moved to partially clarify the situation via an administrative ruling. The traditional inventory-type short hedge was restored as a legitimate hedge, but there still remains a great deal of uncertainty.

The distinction between hedging and speculation is very important to a business firm involved in agricultural commodities. Hedging is a legitimate business practice and is, therefore, fully deductible as an ordinary business expense. On the other hand, speculation is not a legitimate business expense and loss deductions are limited to $3,000 for individuals and zero for most corporate entities.

A lingering concern, then, among users of the futures markets is the lack of a clear, appropriate hedge definition. The word "hedge" is used in a variety of ways by futures traders, accounts managers, regulators, and there appears to be no generally accepted definitions that is useful in making practical decisions (Financial Accounting Standard Board, 1984). The IRS has not defined hedging. Although three sections of the *Internal Revenue Code* explicitly exempt hedging transactions from general tax rules, only one section describes the tax rules that apply to a hedging transaction. Even that provision fails to define a hedging transaction beyond a transaction that reduces risk (Harris and Slavin, 1991). The July 1994 Treasury Department administrative ruling failed to define hedging. It identified that traditional short hedge to protect inventory value and an option "fence" (buying a put option, selling a higher call option) as appropriate risk reducing and therefore hedging strategies but offered no general definition of a hedge.

Farmers and ranchers using risk management tools will be concerned that IRS auditors may disallow losses resulting from hedging strategies if the strategy involves positions other than the most simple and basic "hedge and hold." The IRS has historically applied a very rigid definition of what is seen as hedging and what is seen as speculative activity in futures markets.[4] A primary criterion of hedging ruled by the IRS is the "equal and opposite" requirement. In other words, the futures position must never exceed the actual or expected position in the cash market (the "equal" requirement) and must be the reverse of the cash position (the "opposite" requirement). For cattle feeders, this criterion restricts them to being long feeder cattle futures (a "long" hedge) and being short live cattle futures (a "short" hedge) in order to benefit from the current tax treatment of a hedge. According to this requirement,

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[4] The *Internal Revenue Code* defines hedging as a transaction entered into by the taxpayer in the normal course of the taxpayer's trade or business, primarily:

(1) To reduce risk of price change or currency fluctuations with respect to property which is held, or to be held, by the taxpayer; and

(2) To reduce risk of interest rate or price changes or currency fluctuations with respect to borrowing made, or to be made, or obligations incurred, or to be incurred, by the taxpayer.

being short the nearby feeder cattle futures and/or long the distant live cattle futures, positions reflecting what would appear to be logical cattle feeders' reactions to large negative feeding margins, would be speculative trades.

Since losses on speculative trade in futures are not deductible for tax purposes, cattle feeders will be reluctant to take positions that might be ruled as speculative by the IRS.[5] When the nearby feeder cattle futures and cash feeder cattle prices are high relative to the distant live cattle futures prices and no profitable hedge is being offered or appears likely, cattle feeders cannot be involved in any economically rational actions that would be defined as hedges given current IRS policy in the live cattle futures markets. They must act as speculators in the cash market and wait for other forces, and other traders, to restore a market balance and more attractive hedging opportunities in the futures market. Their only alternative is to adjust activities in the cash market by changing placements of cattle on feed, an act which will eventually

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[5] A special survey conducted by the *Commodity Futures Trading Commission* (CFTC) on March 13, 1987 indicates feedlots held only 4.5 percent of the short open interest in feeder cattle futures and 4.0 percent of the long open interest in live cattle futures. Since the average open interest for *all* holders of live cattle futures represented only about 30 percent of the on-feed count, it is clear the feedlots are not heavily involved in the markets. Involvement would be much less in the feeder cattle futures where open interest averaged only 17,923 contracts (Kuserk, 1988). In a survey of Kansas and Texas cattle feeders in September of 1991, there was clear implication that many cattle feeders do not enter the cattle futures markets in any but a very basic hedge because of concerns over the IRS treatment of any futures losses that might occur. Purcell concludes that cattle feeders are therefore blocked from participating fully in the price discovery process (Purcell, February 1992).

change the supply/demand situation in the marketplace and influence the price being discovered in the futures, but with an often significant time lag.

Such a situation can accentuate variability in fed cattle supplies. Cattle feeders cannot influence discovered prices directly by taking positions in futures when only negative margins are being offered. This results in unstable and unprofitable margins offered to producers, volatile placements of cattle on feed, more volatile prices of beef to consumers, and higher average prices paid by consumers than might otherwise be possible. Someone must pay for the risk exposure and the variability. As a result, the economic viability of investments in cattle feeding and the beef sector as a whole could be threatened. Purcell (1991) states: "The market relationships between nearby feeder cattle and distant live cattle futures are critically important to the economic viability of feedlot owners' business on a day-to-day basis, but the only legitimate course of action is to wait for the imbalances to be corrected by other participants in the futures markets" (p.7).

1.4 Problem Statement

Above, it was argued that cattle prices are highly variable and that this variability imposes costs on producers, processors, and consumers. The short-term variability is not caused by cyclical developments in supply or shifts in demand, but by short-run fluctuations in placements of cattle into the feedlots.

Live cattle futures markets react to emerging information on changed placements (Koontz and Purcell, 1988). That is, the market is performing a forward pricing role given the periodic releases of supply side information. Cattle feeders *do* adjust their cash activity in response to significant changes in distant live cattle futures. Related to this issue, Purcell (1991) suggests that the average four-month feeding program shows a significant inverse relationship between placements and cash prices four months later.

According to current IRS policy, the only legitimate hedged positions for cattle feeders would be long feeder cattle futures and/or short live cattle futures. Cattle feeders, as potential traders in the futures market, are assumed to have access to better information on the costs of cattle feeding and the related opportunity being presented by the live cattle futures market. If IRS policies deny cattle feeders the opportunity to deal with the market imbalances between feeder cattle and live cattle futures, corrections of the imbalances may be delayed until the changed placements are recognized completely and publicly by other traders. These corrections in the futures are made primarily by traders (speculators) in futures from outside the cattle feeding complex, traders who by definition have no cash-business connections. As a result, it is possible that the IRS position constrains the effectiveness of the price discovery process in the live cattle futures market by denying market access to highly relevant, timely, and proprietary information in the hands of cattle feeders.

Market imbalances involving large negative margins could be corrected by selling the nearby feeder cattle futures and buying the distant live cattle futures. Cattle feeders would be interested in taking such actions when no reasonable prospect for profits are being offered via the traditional short hedging strategies that involve buying cash feeder cattle and selling distant live cattle futures. In spite of their advantageous and well-informed positions, however, cattle feeders are facing strong obstacles to direct participation in this price discovery process via arbitrage activity between feeder cattle and live cattle futures given current IRS policy. Too little is known about how these possible policy-based obstacles influence the price discovery process and how they inflence the economic wellbeing of producers and consumers and/or revenue flows at the IRS.

1.5 Hypotheses

The following general hypothesis is offered:

> The IRS position that blocks cattle feeders' access to the cattle futures markets interferes with the effective and efficient workings of the cattle futures markets and their price discovery functions, perpetuates and accentuates short-run disequilibria in the cash and futures markets for cattle, and imposes unnecessary economic costs on producers and consumers.

Related to the above general hypothesis, the following specific hypotheses will be tested:

1. Tax parameters have a significant effect on the participation of producers/hedgers in live cattle futures market, and for a given marginal tax rate, lower probability of deductibility of futures market losses will constrain or block participation of producers/hedgers in the live cattle futures market; and

2. The behavior of producers/hedgers in reaction to IRS tax policy has an adverse effect on futures market informational efficiency in terms of liquidity, in terms of effectiveness in correcting market imbalances in cash and futures markets, and on social welfare in terms of producers' expected profits, speculators' expected profits, and consumers' economic well-being.

1.6 Objectives

The primary objective of this research is to analyze the impact of changes in tax policy on cattle feeders' behavior, the price discovery process, and the effectiveness of the cattle cash and futures markets in correcting market imbalances or disequilibrium situations. More specific subobjectives are:

1. To develop a conceptual framework to analyze the impact of changes in marginal tax rates and probability of deduction of futures losses on cattle feeders' behavior;
2. To describe the market-clearing process in the cattle industry and live cattle futures markets in response to changes in tax policy;
3. To demonstrate the possible impact of denying cattle feeders' participation in correcting the imbalances between feeder cattle costs and the pricing opportunities in the live cattle futures markets; and
4. To suggest possible implications to the price discovery process and to market effectiveness and efficiency of the IRS policies on hedging versus speculation.

1.7 Overview

Chapter 1 outlines cattle feeders' economic environment and the IRS policies and interpretations. Hypotheses and objectives are also documented.

The literature review in Chapter 2 presents the theory and empirical knowledge concerning mean-variance analysis. In addition, a theoretical framework is developed to evaluate performance of cash and futures markets in terms of pricing efficiency and the price discovery process in the live cattle futures market. The literature in this area is reviewed. Finally, a cattle producer's production decision with futures markets is described and related literature is reviewed.

Based on the conceptual framework discussed in Chapter 2, Chapter 3 presents a theoretical model to analyze the impact of changes in tax parameters on the optimizing behaviors of individual economic agents. This chapter includes the impacts of changes in marginal tax rate and/or probability of deduction of futures losses (called deduction level) on the optimal cash and futures positions and on equilibrium cash and futures prices. Chapter 3 also provides welfare measures for individual economic agents active in the markets.

In Chapter 4, comparative static analyses are performed to examine marginal changes in futures positions and discovered futures prices for a unit change in tax rate and/or deduction level. Chapter 4 is distinguished from Chapter 3 in that Chapter 4 considers the marginal changes in the variables of interest in response to changes in tax parameters (in terms of partial derivatives) while Chapter 4 compares optimal solutions generated by changing the tax parameters in increments. Also, a revenue-neutral tax scheme is introduced and the comparative statics are analyzed under short-run and long-run cases.

Chapter 5 presents the description of data used in a relatively simple empirical analysis. This includes data sources, calculation of feeding costs, margins, etc. An estimation procedure for cash and futures forecasts is presented. A total consumers' demand function for fed cattle is specified and the estimated results are provided. The process employed to compute the empirical results is discussed, and the models are presented as empirical tests of the direction of response to changes in the tax parameters presented in Chapters 3 and 4.

In Chapter 6, the empirically computed results are presented. Based on these empirical results, implications are provided with respect to the arguments made in earlier chapters and the hypotheses presented in Chapter 1.

Chapter 7 summarizes the findings and the related implications. Based on the conclusions, the policy implications of this research are identified and discussed.

CHAPTER TWO : LITERATURE REVIEW

2.1 Introduction

This chapter describes economic theories upon which the conceptual framework to be employed builds. The following theories are thought to be necessary and are reviewed in the literature search:

(1) expected utility theory;

(2) risk attitudes and their measures (risk aversion);

(3) the concept of certainty-equivalent models; and

(4) risk aversion in mean-variance (EV) analysis.

This research focuses on the impacts of changes in tax policy on producers' behavior, social welfare and market performance in terms of efficiency and effectiveness. The following issues are also thought to be necessary to the presentation:

(1) a description of cash market performance, which involves testing whether normal returns are zero or whether selling prices cover variable feeding costs;

(2) a description of futures market performance and related literature, specifically;

    1) forward-pricing efficiency;

    2) the concept of a risk-premium; and

    3) the efficient market hypothesis.

Since the model underlying the research is based on the EV analysis of portfolio selection, the previous studies in the futures area using similar methods are necessary in the literature review. Thus, this chapter includes the description of production decisions with futures markets and reviews the following related literature:

(1) derivation of optimal hedging positions;

(2) simultaneous determination of cash and futures prices; and

(3) the concept of an optimal hedge ratio.

2.2 Expected Utility Theory

Von Neumann and Morgenstern (1944), and later others (Friedman and Savage, 1948; Herstein and Milnor, 1953; Luce and Raiffa, 1957; to name a few) developed the expected utility model (EUM) in an axiomatic system. They asserted that if an individual's behavior conforms to certain postulates, an ordinal utility function can be derived to arbitrarily assign utility values to contingent incomes. The preferred investment decision maximizes the expected value of the ordinal utility function in accordance with the following axioms:

(1) *ordering of choices*: For any two choices X_1 and X_2, the decision maker either prefers X_1 to X_2, prefers X_2 to X_1, or is indifferent;

(2) *transitivity of choices*: If X_1 is preferred to X_2, and X_2 is preferred to X_3, then X_1 must be preferred to X_3;

(3) *substitution of choices*: If X_1 is preferred to X_2, and X_3 is some other choice, then a risky choice $pX_1 + (1 - p)X_3$ is preferred to another risky choice $pX_2 + (1 - p)X_3$, where p is the probability of occurrence of X_1 or X_2; and

(4) *certainty equivalent of choices*: If X_1 is preferred to X_2, and X_2 is preferred to X_3, then some probability p exists that the decision maker is indifferent to having X_2 for certain or receiving X_1 with probability p and X_3 with probability $1 - p$. Thus, X_2 is the certainty equivalent of $pX_1 + (1 - p)X_3$.

If a decision maker obeys these axioms (and several others which are more technical), a utility function can be formulated which reflects the preferences of the decision maker.

In this framework, a single value, $U(x_i)$, is associated with each potential outcome resulting from a particular act. The expected utility of an act is calculated as:

(2.1) $E[U(X)] = \sum_i p_i U(x_i)$,

where

p_i = the probability of the *ith* result; and

$U(x_i)$ = the utility of the *ith* result.

Economists have frequently simplified the analysis of a decision problem by assuming that an individual's utility depends on only one variable, such as income or wealth, because potential outcomes can then be quantified in terms of their impact on these variables. This facilitates determination of the optimal action based on utility maximization.

2.3 Risk Attitudes and Risk Aversion

Marginal utility of income, $U'(y)$, is always positive, implying that individuals are always assumed to prefer more income to less. Risk attitude is indicated by the second derivative of an individual's utility function for income. When the second derivative is negative, implying that the individual derives less utility per increment of income as income increases, the individual is considered to be *risk averse*. A positive second derivative, which implies the individual derives more utility per increment of income as income increases, indicates a *risk loving* or *risk seeking* attitude. A *risk neutral* individual is characterized by the second derivative of the utility function for income being equal to zero. The individual derives the same marginal utility from every added unit of income regardless of income level.

A unique measure of the direction of the bending of U(y) and the rate of change in the slope of the function is the *absolute risk aversion* function. Developed independently by Pratt (1964) and Arrow (1971), it is defined as:

$$(2.2) \quad R(y) = \frac{-U''(y)}{U'(y)} .$$

A related measure is the *relative risk aversion* function $R_r(y)$; it measures the elasticity of marginal utility and is defined as:

$$(2.3) \quad R_r(y) = \frac{-U''(y) \cdot y}{U'(y)} .$$

Neither measure is affected by linear transformations of the utility function. They have positive values for risk averters, a zero value for risk-neutral decision makers, and negative values for risk lovers. Moreover, their uniqueness permits interpersonal comparisons at comparable wealth levels.

The sign of $R'(y)$ indicates how risk attitudes change as income y increases. If $R'(y) < 0$, the most usual assumption, decision makers are said to display *decreasing absolute risk aversion* (DARA). This implies that the risk premium for a lottery decreases as the decision maker moves to higher wealth levels. A logarithmic utility function characterizes DARA:

$$(2.4) \quad R(\ln y) = \frac{1}{y}$$

and

$$(2.4)' \quad R'(\ln y) = \frac{-1}{y^2} < 0.$$

$R'(y) = 0$ implies *constant absolute risk aversion* (CARA). The risk premium for CARA decision makers is constant regardless of changes in the decision maker's wealth. A negative exponential utility function represents CARA:

$$(2.5) \quad R(a - e^{-\lambda y}) = \lambda, \qquad \text{where } a, \lambda > 0,$$

and

$$(2.5)' \quad R'(a - e^{-\lambda y}) = 0.$$

Finally, $R'(y) > 0$ implies *increasing absolute risk aversion* (IARA), suggesting that the risk premium increases for the same lottery with increases in wealth. A quadratic utility function shows IARA:

$$(2.6) \quad R(y - by^2) = \frac{2b}{1 - 2by}, \qquad \text{where } b > 0 \text{ and } (1 - 2by) > 0,$$

and

$$(2.6)' \quad R'(y - by^2) = \frac{(2b)^2}{(1 - 2by)^2} > 0.$$

Since IARA implies such a strong (and rarely observed) response to risk, one can understand why a quadratic utility function is not generally assumed except as a local approximation.

2.4 Certainty Equivalent Model

The absolute risk aversion function $\lambda(\pi)$ indicates both local and global measures of risk aversion. Since $\lambda(\pi)$ is a function defined over π, the measure of risk attitude can occur at any value of π. Pratt (1964) derived the approximate relationship between λ and risk premium *RP*:[6]

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[6] First, consider that there exists a *RP* such that the utility of the certain income $U(\pi - RP)$ is equally preferable to the expected utility of any lottery $EU(\pi + \epsilon)$ where ϵ is a random variable with mean zero and variance σ^2:

$$(1) \quad U(\pi - RP) = EU(\pi + \epsilon).$$

Solving for *RP* by taking the Talor series expansion about π of both sides of the expression above:

$$(2) \quad U(\pi - RP) = U(\pi) - RP \cdot U'(\pi) + 0 \cdot \sigma^2, \text{ and}$$
$$(3) \quad EU(\pi + \epsilon) = E[U(\pi) + \epsilon \cdot U'(\pi) + \tfrac{1}{2}U''(\pi) \cdot \epsilon^2 + \cdots]$$

$$(2.7) \quad RP = \tfrac{1}{2}\cdot\lambda[E(\pi)]\cdot Var(\pi).$$

The risk premium *RP* is equal to one half of the product of the absolute risk aversion, measured at the expected value of π (or any of possible choice outcomes), times the variance of π. The corresponding certainty equivalent profit π_{CE} is found by subtracting *RP* from $E(\pi)$:

$$(2.8) \quad \pi_{CE} = E(\pi) - \tfrac{1}{2}\cdot\lambda[E(\pi)]\cdot Var(\pi).$$

This equation implies that, the greater the measure of risk aversion $\lambda(\pi)$, the larger the required risk premium. In the small range, or at a point (thus in terms of local measure rather than global measure), individuals can be ordered according to their degree of risk aversion either by their absolute risk aversion function valued at a point or by the size of the risk premium (Robinson and Barry, 1984; p. 34).

$$\begin{aligned} &= U(\pi) + E(\epsilon)\cdot U'(\pi) + \tfrac{1}{2}U''(\pi)\cdot E(\epsilon^2) + \cdots \\ &= U(\pi) + \quad 0 \quad + \tfrac{1}{2}U''(\pi)\cdot\sigma^2 \quad + \cdots. \end{aligned}$$

Equating (2) and (3), we can write:

$$U(\pi) - RP\cdot U'(\pi) + 0\cdot\sigma^2 = U(\pi) + \tfrac{1}{2}U''(\pi)\cdot\sigma^2 + \cdots$$

Canceling out $U(\pi)$ from both sides and ignoring all terms beyond the second term in both sides (since they are small enough to be ignored without significantly changing the results), an approximate equality relationship is obtained:

$$-RP\cdot U'(\pi) = \tfrac{1}{2}U''(\pi)\cdot\sigma^2$$

or

$$RP = \tfrac{1}{2}\lambda[E(\pi)]\cdot\sigma^2(\pi)$$

where $\lambda = -U''(\pi)/U'(\pi)$ (Robison and Barry, 1984; p. 40).

2.5 Risk Aversion In EV Analysis

First, note that EV analysis is assumed to hold when (1) $U(\pi)$ is a negative exponential function (thus a constant risk aversion) of π, and (2) the stochastic variable affecting π is normally distributed. Second, if the compensation of the producer is a concave function of π (i.e., $\lambda > 0$ or risk averseness), he will behave as if he were maximizing $EU(\pi)$ described by:

$$(2.9) \quad EU(\pi) = E(\pi) - \tfrac{1}{2}\lambda \cdot Var(\pi).$$

This problem reflects the fact that large losses are penalized more severely than large profits are rewarded. Bankruptcy is to be avoided at a cost related to coefficient λ. The possible binding constraint is generally the availability of capital.

Joint optimization of planned output or expected cash position and the position in the futures market can be given by:

$$(2.10) \quad \partial E(\pi)/\partial x_s = \tfrac{1}{2}\lambda[\partial Var(\pi)/\partial x_s]$$

or

$$(2.10)' \quad MRT(x_s) = \frac{\partial E(\pi)/\partial x_s}{\tfrac{1}{2}[\partial Var(\pi)/\partial x_s]} = \lambda$$

and

$$(2.11) \quad \partial E(\pi)/\partial x_f = \tfrac{1}{2}\lambda[\partial Var(\pi)/\partial x_f]$$

or

$$(2.11)' \quad MRT(x_f) = \frac{\partial E(\pi)/\partial x_f}{\tfrac{1}{2}[\partial Var(\pi)/\partial x_f]} = \lambda$$

Expected return and risk can be traded off against each other in two ways. First, the cash position x_s can be varied, given the position in futures x_f, which is expressed by the marginal rate of transformation MRT(x_s). Second, the futures position x_f can be varied, given a planned cash position x_s, which is shown by the marginal rate of transformation MRT(x_f). In order to simultaneously optimize x_s and x_f, (1) MRT(x_s) must equal MRT(x_f), and (2) the common MRT must equal the coefficient λ of risk aversion. The second order conditions are then satisfied (Stein, 1987; p. 40).

The expression $E(\pi) - \frac{1}{2}\lambda \cdot Var(\pi)$ can also be interpreted as part of the Lagrangian in a maximization problem where expected income is maximized at a given level of variance of income (Rolfo, 1980; p. 102). This interpretation responds to a representation of preferences by an exponential utility function for jointly normal distributions and implies a constant absolute risk aversion at all levels of wealth. The risk parameter, λ, is then the price, measured in units of expected income, paid in order to maintain the same expected utility as the variance of income changes:

$$(2.12) \quad \lambda = \left.\frac{d[E(\pi)]}{d[Var(\pi)]}\right|_{\text{at constant expected utility}} .$$

The higher the required payoff (measured in terms of expected income) to compensate the increase in variance, the more risk averse is the individual. Consequently, a greater (smaller) λ depicts a more risk-averse (less risk-averse) attitude. At $\lambda = 0$, the individual is risk neutral; he or she is indifferent to the variance of income distribution and derives utility only from expected income. Note that the trade-off between expected income and variance of income is independent of the level of expected income. This is a consequence of the underlying assumption of an exponential utility function which implies that individuals' attitude toward risk does not depend on their wealth level.

2.6 Cash Cattle Market Performance

Conceptually, the live cattle futures markets would be expected to offer margins that cover variable costs most of the time (Purcell, 1991). In its simplest form, the markets are in a state of balance when:

$$FCC + FC = LCP, \tag{2.13}$$

where

FCC = cost of feeder cattle that could be placed using cash prices or the nearby futures ($ per head);

FC = cost of inputs other than feeder cattle during the period, reflecting variable and fixed costs where fixed costs include a return on the capital investment ($ per head); and

LCP = per head value of the finished steer using projected weights and available live cattle futures prices ($ per head).

The above equality implies that prices would be expected to approach the average total cost of production for the most efficient producer in the long run.[7] That is, this is the equilibrium position toward which the markets would be expected to move. This prospective is important in moving toward discussion of the performance of the futures markets in the next section. What, precisely, is the futures market expected to do? What price should it discover? The notion that the market might generally, over time, approach the cost of production of the more efficient producers provides a context within which to think about market performance.

The markets are in a state of relative imbalance, then, when:

$$\text{(2.14)} \qquad FCC + FC > LCP, \text{ or } FCC + FC < LCP$$

where the variables have the same definition as defined above. Empirical evidence shown in Chapter 1 suggests that the inequality (FCC + FC > LCP) is present in a majority of the cases and often persists over several months.[8] For example, as shown earlier in Figure 1.2, the market seems to be relatively inefficient or ineffective in that it takes considerable time for the market to restore an equilibrium when the margins being offered are negative. There is a string of 23 consecutive months during which the margins were negative in 1983 and 1984. An imbalance which persists across several months suggests that there may be little or no

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[7] The most efficient producers refer to efficient buyers of feeder cattle and feedlot managers who are well informed and do a good job in managing their feeding programs.

[8] See Section 1.2 of Cattle Feeders' Economic Environment in Chapter 1.

influence from cattle feeders in discovering feeder cattle and live cattle futures prices.

According to Purcell, the economic reasons behind the prolonged market imbalance are as follows:

(1) corn prices are largely beyond the influence of cattle feeders;

(2) cattle feeders cannot dominate the price discovery process for cash feeder cattle and cash fed cattle; and

(3) economic forces external to the cattle feeding and packing subsectors play an important role in moving the markets away from equilibrium.

More than half of the total corn usage (55-75 percent) is feed usage.[9] The variability in total disappearance or use in corn is influenced mainly by export movement, however. Thus, cattle feeders' activities in the cash or futures markets for corn are assumed to exert only small marginal influence on the market imbalances. That is, the imbalance measured as persistent negative feeding margins cannot be corrected by forcing feed costs, especially corn prices, down.

Feeder cattle futures will adjust to the following changes: (1) to projected feeding costs, especially corn prices; (2) to the distant live cattle futures prices; and (3) other (non-feedlot) demand for stocker and feeder cattle and for breeding purposes for heifers. Live cattle futures will adjust to short-run changes in fed cattle numbers, short-run changes

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[9] See **Feed Situation and Outlook Reports** from USDA's Economic Research Service. In the U.S., hogs and poultry are a major users of corn.

in expectations of pork and poultry prices, overall developments in the economy, and changes in consumer incomes. Cattle feeders may have some influence on feeder cattle prices but, as suggested, they face competition for light cattle from stocker operators and from cow-calf operators for heifer for breeding, and there is a significant seasonal pattern in the available supply of feeder cattle. Any negative imbalance cannot, therefore, be easily corrected in the short run by pushing the costs of feeder cattle down in the cash market.

Cash prices for light steers and heifers tend to increase during March-May as stocker operators buy for grazing programs. As the light cattle prices get bid up, prices of the yearling steers and heifers, ready for the feedlot, tend to move up as well. Feeder cattle futures prices move up with an increase in the feeder cattle prices since the cash price movements are positively related to the futures price movements and the cash and futures prices are, arguably, jointly determined. As a result, feeder cattle prices in the cash and in the nearby futures markets can become too high to allow profitable hedges given prevailing distant live cattle futures prices. The imbalance may persist, as documented in Chapter 1, across several months.

When forward pricing opportunities cannot provide cattle feeders with margins covering variable costs, they must either (1) leave the feeding pens empty and absorb the fixed costs, or (2) place the cattle in the hope that profitable prices will be offered during the feeding period. Purcell states: "The net result is often a sporadic pattern of placements and the highly variable prices that comes from volatile placements" (p.5).

From the viewpoint of society, any disequilibrium situation should be short lived and quickly corrected, but such is not the case.

Figure 2.1 portrays the possible policy issue. More efficient producers are assumed to cover variable costs over time. That is, the net margins offered above variable costs for the most efficient producers tend to be positive or zero over time. As previously implied, the moves to positive levels (A to D) may be less sustained than the moves to negative levels (D to G). Any policy position, such as the IRS position, which blocks participation of well-informed participants tends to prolong the moves to negative margins and/or accentuate the price moves over time. As a result, there is a possible social loss which could take the form of excessive variation in fed cattle supplies and in beef prices (Purcell, 1991).

If the market is effective in correcting the imbalances associated with both negative margins (losses) and positive margins (excessive profits), any social loss can be avoided or reduced. The area EFG in Figure 2.1 can be used to represent the loss which may be due to market inefficiency. Excess profits or economic rent, associated with the positive imbalances, is characterized by the area BCD. Consistent with the empirical evidence, Figure 2.1 suggests that the imbalances involving losses are larger than those involving economic rent.

Purcell suggests that it takes more time to correct the market imbalances by changing placement patterns in the cash markets than might be the case by inflencing the price discovery process more directly in the

Net Margin Over

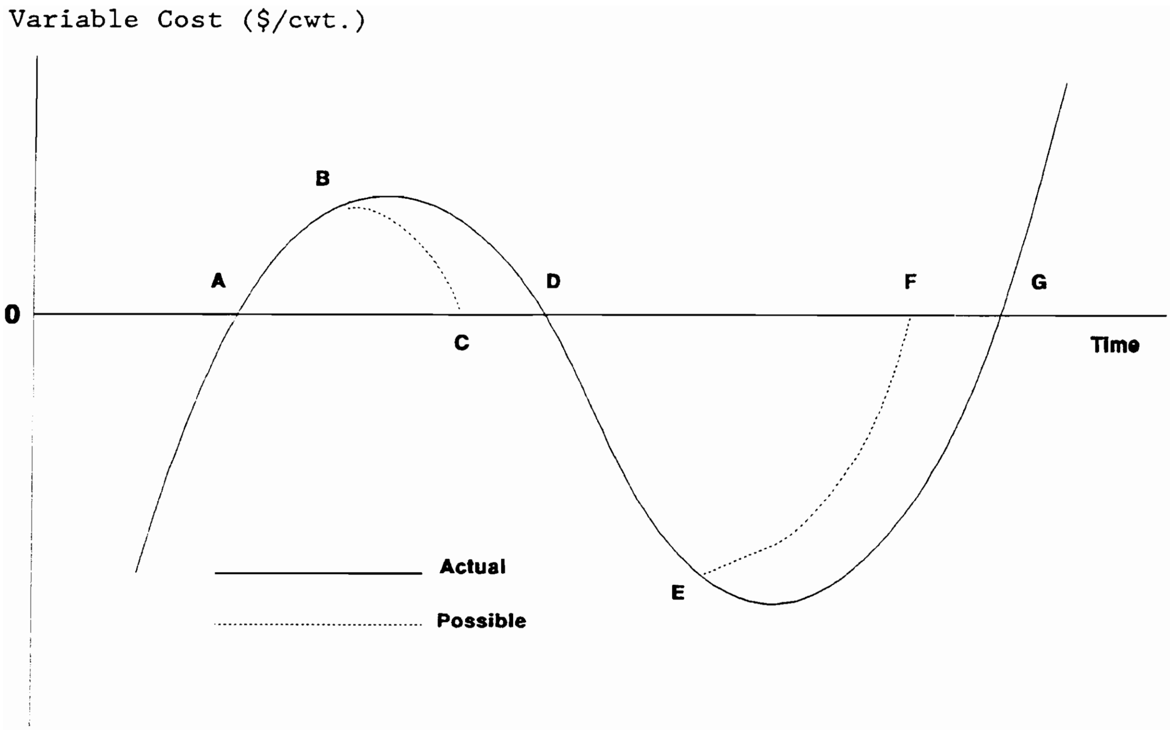

Figure 2.1 Presentation of Actual vs. Possible Market Performance Patterns

futures markets. When excessive positive margins appear, cattle feeders *can* be involved in correcting the situation by selling distant live cattle futures and placing short hedges. This action tends to decrease distant live cattle futures, reducing the excessive margins. But when negative margins appear, the cattle feeder may not take short positions in the nearby feeder cattle and/or long positions in the distant live cattle futures prices for fear such trades will be ruled as speculative and any losses in futures cannot be deducted. The cattle feeders cannot, therefore, be a full and direct participant in the price discovery process and must wait for the situation to correct itself. If theoretical and/or empirical evidence generally supports the hypothesis that cattle feeders would be effective market participants, then the markets are less effective than they could be if cattle feeders were more involved.

2.7 Futures Market Performance

In general, the competitive market model serves as a norm for evaluating market performance. Using the conditions of a (perfectly) competitive market, futures markets can be viewed as a close approximation to the concept of perfectly competitive markets due to the following reasons (Leuthold, Junkus and Cordier, 1989):

(1) *Atomicity of participants*. This condition is, in general, satisfied in futures markets, especially for the liquid, active contracts;

(2) *Homogeneity of product*. This condition is well satisfied on the futures markets as each contract is standardized except for maturity month and price;

(3) *Free mobility of resources.* This condition is nearly fulfilled since futures markets come closer to free exit and entry than most other markets and still maintain the integrity of their contracts; and

(4) *Perfect information.* This condition is essentially achieved as the public nature of futures markets, along with large volumes of publicly supported market information, allow the futures markets to approach the theoretical state of perfect knowledge more closely, perhaps, than other markets.

These characteristics would be consistent with the suggestion in the previous section that, over time, the discovered price will approach average total costs of production by efficient producers--since the markets *are* competitive.

Important concepts of futures market performance have been described and tested using three approaches: (1) forward-pricing efficiency; (2) the issue of a risk premium; and (3) the efficient market hypothesis. However, few studies empirically test the performance of futures markets against alternative market organizations. No researcher has tested the overall performance of futures markets, assessing simultaneously the efficiency of the market to transfer risks, forward price, transmit information, aid firms in obtaining capital, and allocate resources and inventory. Therefore, even if any of the above three enumerated approaches found some unexplained bias or inefficiency in futures markets, this does not imply that some more efficient market alternative does not exist given the competitive organization of the futures markets. Forward-pricing efficiency is only one role of futures markets and its assessment is only one aspect of the performance of the markets.

2.7.1 Forward-Pricing Efficiency

The analysis of price relationships between current cash, current futures, expected cash, and expected futures prices are the domain of research on pricing efficiency. More specifically, forward-pricing efficiency refers to the ability of the futures markets to forecast the expected cash price at the maturity date. It is popular when assessing forward-pricing efficiency to refer to futures prices as forecasts, broadly interpreted.

Cash and futures prices are equally valid expectations of the subsequent cash price since they reflect and contain essentially the same information. A common and traditional model used to test forward-pricing ability is:

$$(2.15) \quad S_t = \alpha + \beta \cdot F_{t-i} + \epsilon_t,$$

where

S_t = the cash price at delivery,

F_{t-i} = the futures price i months before maturity, and

ϵ_t = a random error term.

For an efficient, forward-pricing market, it is hypothesized that $\alpha = 0$ and $\beta = 1$ in an empirical test. Note that if $\beta \neq 1$, then the test for $\alpha = 0$ is no longer appropriate (Martin and Garcia, 1981). Acceptance of this hypothesis implies that the futures price is an unbiased predictor of the expected cash price.[10]

The evidence for this model is mixed in empirical research on agricultural commodities. Continuous inventory commodities usually fail to reject the hypotheses that $\alpha = 0$ and $\beta = 1$. Futures markets for these commodities are unbiased predictors. Discontinuous inventory (e.g. potatoes) and noninventory (e.g. livestock) commodities often fail the hypothesis tests. Goss (1981) suggested that the absence of inventories may bring gaps in the flow of information or increase errors in expectations because of the lack of close ties between cash and futures prices.

When related to the live cattle futures markets, the biased futures price debate is covered by several articles concerning the forecasting performance of the futures price (Leuthold 1974; Just and Rausser 1981; and Martin and Garcia 1981). The agreement among these studies is the existence of a bias in the live cattle futures prices beyond four months prior to maturity. They also find there is some seasonality in the predictive accuracy of the market.

Kofi (1973) concludes that futures markets perform their price

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[10] This model is not a rigorous test of forward-pricing efficiency for the following reasons:

(1) There exist noteworthy statistical problems related to pooling of observations, autocorrelation, and regression technique (Karl and Tomek, 1986);
(2) One does not know if the reported errors are due to a bias in the market, the receipt of new information, or inadequacies in the available information; and
(3) The model cannot measure the influence of new information on changes in prices, nor can it analyze trader behavior.

forecasting role well, and that the performance test statistic, r^2, measures the degree of relative predictability between the various commodity markets. Also, the author notes that the predictive reliability of a futures market improves as more accurate information on supply-demand conditions becomes available.

Koppenhaver (1983) reviews several studies by Leuthold (1972), Cox (1976), Cargill and Rausser (1975), and Helmuth (1981). According to Koppenhaver, the articles reviewed essentially argue that if the live cattle futures market operates efficiently in the context of the random walk model, the futures price must be an unbiased predictor of later cash prices.

Koppenhaver suggests, however, that there is a confusion between the theory of an efficient market and the hypothesis of forward prices as unbiased estimates of future cash prices. The author argues that although there is a bias in live cattle futures prices, it is not inconsistent with certain types of market efficiency. Unbiasedness is not a necessary condition for market efficiency.

Koppenhaver notes that the presence of a risk premium in live cattle futures market can be explained in terms of the nonstorable nature of live cattle. The author also argues that the long speculators play an important role in price discovery. Any risk premium that does exist, he suggests, is necessary to induce participation by the speculators who help to restore market balances.

2.7.2 Risk Premium

The concept of a risk premium in futures markets traditionally relates to the search for a biased futures price with respect to some expected cash price. A bias is defined as a systematic difference between the two prices. The concept of a risk premium, or bias, in futures markets originally rose from the notion of a risk transfer from hedgers to speculators. In this case, the existence of a risk premium is examined by finding a statistically significant gap between two prices:

(2.16) $E(S_t) - F_{t-i}$.

Any statistically significant difference between the expected cash price, $E(S_t)$, and the futures price, F_{t-i}, would be the risk premium. This approach evaluates the price of a futures contract in relation to the cash price and measures its riskiness through price fluctuations in *isolated* market situations. What is governing the riskiness of a futures contract is merely its own variance.

A systematic downward bias in live cattle futures prices might lead to underestimates of the corresponding cash prices and therefore of cash returns. Consequently, expected margins offered by the distant futures will tend to be negative, and profitable hedges will not often be available. Prices may not cover costs, but prices are below costs over time at least partly due to a systematic downward bias.

Helmuth (1981) initiated a debate on efficient market theory by claiming to have discovered a trading technique which "predicted certain live cattle futures price movements with 100-percent accuracy . . ." (p.353). Helmuth thus concludes that the cattle futures prices have a consistent, predictable, and systematic downward bias. The major causes of systematic downward bias in the live cattle futures prices stem, he suggested, from larger selling-side relative to buying-side pressure.

Cash-connected selling pressure is larger than cash-connected buying (short hedges are greater in number than long hedges) in cattle futures. Short speculating adds even more selling pressure when futures price reaches a "signal price", a price speculators feel is too high to be sustained. Helmuth did not examine the dips in futures prices. Long speculative and long hedge positions might be placed when the futures prices are low enough to suggest a significant departure from an underlying equilibrium. These kinds of buying pressure could form support for futures prices to help sustain a normal range and deny the existence of a downward bias as suggested by Helmuth.

Palme and Graham (1981) argued that the report on the systematic downward bias in live cattle futures prices by Helmuth contains unsound economic analysis and offers no acceptable evidence to support his arguments. Palme and Graham emphasize four critical factors. First, the evidence of inefficiency of the live cattle futures market is not supported by the data Helmuth studied. Second, in contrast to the conclusion drawn by Helmuth, the futures market provides profitable hedging opportunities to the majority of cattle feeders at some point during the feeding period. Third, the signal price estimated by Helmuth is not valid because of the unavailability of revised cost data (used by

Helmuth) to cattle feeders. Finally, there is no way to evaluate Helmuth's claim of correlated trading activity among large traders.

Pluhar, Shafer and Sporleder (1985) re-evaluated Helmuth's findings. The authors used USDA reported unrevised cost data and a cash-futures basis adjustment. They expanded the period studied. They hypothesized that if Helmuth's trading technique (HTT) using the unrevised USDA expense estimates and an additional basis adjustment could yield significant gross profits, it could be concluded that the HTT is a robust, useful tool for predicting price changes and that there exists a weak-form inefficiency in live cattle futures markets.

Based on the results of their analysis, Pluhar et al. discuss Palme and Graham's criticisms of the Helmuth trading technique and find the criticisms not to be totally convincing. According to the authors, the success of the HTT implies modest evidence of the existence of weak form inefficiency in live cattle futures.

Kolb and Gay (1983) develop a methodology to evaluate the performance of live cattle futures market in the price discovery process. The authors examine lag-link relatives for 38 live cattle contracts maturing between December 1976 and December 1980. The authors do not focus specifically on the arguments made by Helmuth. But, based on the results of their study, the authors would disagree with Helmuth.

Concerning the mixed results reported by other authors, Kolb and Gay attribute the conflicting arguments to the variety of methodologies and time periods used in different analyses. The high volatility and strong trend of live cattle futures prices, the authors suggest, provide good reasons for the variable findings made by the authors. The authors conclude that all of three statistical tests they performed indicate that live cattle futures prices are good predictors of future spot prices. These test results support the evidence of no bias in live cattle futures prices.

In reviewing other studies, Kamara (1982) concludes that, in general, small speculators do not make substantial profits. In fact, any profits are more than offset by commissions paid. Large speculator profits appear to result from superior forecasting skills. Thus, the notion of a risk premium based on the concept of normal backwardation is not well supported empirically.[11] Kamara suggests that "while it is widely accepted that futures markets are used by risk-averse hedgers, the evidence suggests that they have been able to purchase the insurance very cheaply, so that on average futures prices do not contain a significant risk premium."

More recently, the futures markets are being considered as a diversifying alternative, and risk is, therefore, more than a result of

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[11] Keynes was the first to discuss risk premium in futures markets, and his concept became known as the theory of normal backwardation (Keynes, 1930). Hicks provided further elaboration (Hicks, 1946). Keynes and Hicks advocated that hedgers, primarily short in the futures market, pay a premium to speculators, primarily long in the market, for assuming the price risk. Holders of inventory, desiring to reduce the risk of loss occurring from adverse changes in price, pay a risk premium to speculators to absorb that risk. Also, speculators acting as insurers demand a risk premium as compensation for taking risk. The risk premium is paid through the market by the expected cash price exceeding the futures price by the amount of the premium. That is, futures prices are downward biased estimates of the expected cash price and have a tendency to rise as the contract approaches maturity.

price change. Risk is examined by measuring how highly correlated the futures contracts are with other portfolio assets. To estimate relative risk, the following linear regression model is often used:

(2.17) $\check{R}_f = \alpha + \beta \cdot \check{R}_w + \epsilon$,

where

$\check{R}_f$ = random rate of return for futures in terms of relative price changes of a futures contract, $(F_t - F_{t-i})/F_{t-i}$;

β = a measure of the systematic risk, or risk premium;

$\check{R}_w$ = random rate of return on a portfolio containing all assets; and

ϵ = a random error term.

A beta (β) near one indicates adding the futures contract to the portfolio does not diversify portfolio risk, while a small beta indicates unsystematic risk and that the addition of that futures contract does provide diversification. Contrary to the traditional approach, this alternative approach is to model the pricing of futures contracts in an equilibrium situation within a larger integrated capital market framework. The riskiness of a futures contract depends upon its weighted covariance with all other assets comprising total wealth.

Dusak (1973) was the first to apply the standard Sharpe-Lintner Capital Asset Price Model (CAPM) to commodity futures markets. Dusak utilizes the CAPM to empirically test for a risk premium with futures contracts for wheat, corn, and soybeans during 1952-1967. The author provides evidence against the Keynes-Hicks theory of normal backwardation.

However, results from her model can be highly sensitive to the index of wealth selected to represent the portfolio. Black (1976) argued that futures markets should not have a risk premium nor be biased due to the mark-to-market provision. Chang (1985) questions the reliability of CAPM-based tests. Each model imposes different *a priori* assumptions when estimating the risk premium and researchers therefore debate their appropriateness.

2.7.3 Efficient Market Hypothesis Tests

An efficient market is defined as one in which prices fully reflect available information (Fama, 1970). Fama classified empirical tests of market efficiency as weak, semi-strong, and strong. Fama's developments provide a base for the analysis of the informational efficiency of market prices. More recently, Blank (1989) has defined the criteria for evaluating price efficiency in futures markets. The forms of pricing efficiency are as follows:

(1) *weak form efficiency*, in which current futures prices reflect all information contained in past price series;

(2) *semi-strong form efficiency*, in which current futures prices reflect all currently available public information; and

(3) *strong form efficiency*, in which current futures prices reflect all currently available public and private information.

Related to the efficient market hypothesis is the random walk hypothesis which implies that the difference between the futures price in

t and the subsequent futures price in $t + 1$ is a random number reflecting the receipt of new information. This hypothesis might be tested by:

$$(2.18) \quad F_{t+1,T} - F_{t,T} = \epsilon_{t+1},$$

where ϵ_{t+1} = a random variable with zero mean in independent drawings.

More often than not, the random walk hypothesis is examined with various tests on the first differences of prices. A more precise test of an efficient market is the Martingale hypothesis in which the market takes all currently available information into account in determining price.[12] An expression for this hypothesis is:

$$(2.19) \quad E(F_{t+1,T}|\Phi_t) = F_{t,T},$$

where Φ_t = current available information. This expression implies that knowledge of current and past price changes contains no information useful for predicting subsequent price changes. Related to this, successive price changes are independent and there is no way to use current information on past prices to earn a return above normal (still, expected returns can exceed zero). That is, mechanical trading rules advocated by technical analysts cannot be used to enhance profits. In this sense,

[12] Both the random walk and the Martingale hypotheses require expected price changes to be serially independent (i.e., no autocorrelation). The latter does not require drawing successive price changes from the same distribution (i.e., identical distribution). That is, the price generating mechanism can have changing variances. Price following a random walk satisfy the requirements for Martingale, but not *vice versa*.

futures prices are a fair game.

The objective of a study by Tomek and Querin (1984) was to demonstrate the possibility of the coexistence of random walks of futures prices and the profitability of technical analysis applied to the prices. They conclude that both logic and empirical results are supportive of the possibility of a profitable technical analysis based on historical data. However, for the technical analysis to be consistently successful, the price series must have systematic components, and the systematic pattern must occur continuously. The authors state: "The speculator clearly should be skeptical of claims that technical analysis of past prices can successfully forecast forthcoming prices" (p.22).

Garcia, Leuthold, Fortenbery and Sarassoro (1988) evaluate the pricing efficiency of live cattle futures market using out-of-sample forecasts from an econometric model, an ARIMA model, and composite forecasting procedures. The relative performance of the forecasting models is compared in terms of a mean-squared error (MSE) criterion. The authors also analyze the pricing efficiency of the live cattle futures market via market simulation procedures. The authors conclude that using only MSE in evaluating pricing efficiency is not a sufficient criterion. In terms of a necessary condition for pricing inefficiency, the overall higher MSE of futures imply that the live cattle futures market is not reflecting all current public information---suggesting semi-strong form inefficiency. The final conclusion made by the authors, after simulation analyses, is that there is no strong evidence of inefficiency in the live cattle futures market.

In his literature review, Rowsell (1991) concludes that there is a strong connection between efficient markets and the price discovery process. According to Rowsell, "Market efficiency is not a requirement for the price discovery process, but efficient markets can be a performance measure of price discovery" (p.36). This statement implies that the market which is most "informationally efficient" will lead (in a time sense) the other markets in discovering the market clearing price.

According to the efficient market theory, if a market is discovering prices efficiently by registering new information quickly, the price changes in the market follow a random walk. In an efficient market, then, it is impossible to predict future prices. Accordingly, if a researcher found any mechanical technique to forecast the future prices with a certain degree of accuracy, such a finding is strong evidence that the price discovery process in a market is not efficient and that the predictability comes from a systematic bias.

2.8 Production Decision with Futures Markets

One of the major decisions faced by large cattle feeders is to select the optimal proportion of cash positions that should be covered by positions on futures markets. This decision involves finding determinants of demand for optimal cash positions and/or hedging instruments. A frequently recommended framework is to find first-order conditions from the maximization problem for expected utility or for profit. Then, the conditional demands for cash and futures positions are derived from the first-order conditions. As a byproduct, this approach provides implications for efficiency of futures markets.

Alternatively, this problem can be addressed by deriving an optimal hedge ratio, an approach which has been extensively studied (Johnson, 1960; Stein, 1961; Heifner, 1972). Researchers solve for the optimal hedge ratio using the ratio of the covariance between cash and futures prices to the variance of cash and futures prices. Myers and Thompson derive alternative methods for empirical estimation of this optimal hedging rule (Myers and Thompson, 1989).

2.8.1 Derivation of Optimal Hedging Positions

When future prices and/or quantity produced are uncertain, agricultural producers face the dual problem of determining the optimal quantities of cash positions and the correct amount of either forward cash contracts or futures contracts to transfer the risk of adverse price movements. The traditional view of hedging in futures markets depicts an agent with a predetermined position in a single cash good seeking to avoid price risk by taking the equal and opposite position in a futures contract involving that cash good (see Hieronymus (1971)). This "routine" hedge has long been recognized as only a simple form of hedging practice. There have been extensive research efforts to deal with some of the deficiencies of the routine hedge prescription by using various hedging models. Working (1962) emphasized that the existence of basis risk prevents the elimination of all risk. He introduced the notions of "anticipatory" and "selective" hedging, where deviations from routine hedge are based on price level expectations. Johnson (1960) and Stein (1961) derive optimal hedges for mean-variance utility maximizers, recognizing the simultaneity of cash and futures decisions. Telser (1955) does the same for hedgers following a safety-first rule of decision. McKinnon (1967) develops a theory of commodity pricing in an attempt to quantify the relationship between output risks and commodity prices. The author was interested in only the behavior of single individuals and assumed the price or the futures contract is equal to the expected future cash price. The thrust of his study was to consider the influence of expected output uncertainty on farmers' hedging decisions, which had been largely ignored in the literature on commodity hedging.

Anderson and Danthine (1981) pointed out some deficiencies of the above studies because they still did not:

(1) explicitly treat cross hedging;

(2) allow for multiple futures markets;

(3) study the impact of hedging on cash decisions;

(4) study the conditions that permit the separation of cash and futures decisions;

(5) recognize that a hedger's position [sometimes] contains a component that is equivalent to pure speculation; and

(6) discuss and analyze the pricing role of futures markets.

The authors examined the joint decisions of production and hedging considering the case of multiple cash goods and multiple futures and allowing for basis risk. Based on the generalized hedging problem, they illustrate the basic theory of an individual agent's choice of position sizes through two examples, the storage of agricultural goods and the optimal use of Treasury bond and bill futures.

Rolfo (1980) developed a model describing the demand for hedging by farmers faced with price and quantity uncertainty. After devising expectational measures of production and price uncertainty, the author presents a model that derives an optional hedging strategy for a producing country that is subject to variability in both the price and the production of its output. Although he does not develop a pricing theory, Rolfo empirically tests this relationship using the futures market for cocoa. Rolfo suggests that limited usage of the futures market may be superior to a full short hedge of expected output when there is production variability. The study may not be correct, however, in maintaining that production uncertainty is the sole or necessarily the main reason for the optimal hedge to deviate from the routine hedge.

Conroy and Rendleman, Jr. (1983) developed a theory of cash and forward commodity pricing under the assumption of uncertainty of future prices and harvest quantity. Their study expands previous pricing models by including the nonagricultural finance assets as well as agricultural commodities. Thus, the authors examine the interactions between farmers and the nonagricultural economy. The authors analyze the simultaneous determination of forward and cash prices. The demand for forward contracting is derived in two markets: the agricultural market and the securities market. The farmer's utility function is assumed to be quadratic. The farmer tries to maximize the expected utility of his wealth. Mathematically, the farmer's optimal hedging problem is developed below. The farmer's objective function is:

(2.20) max: $E[CR + FP)/W] - \lambda \cdot Var[CR + FP)/W]$

where

$CR = \tilde{P}\tilde{Q}$, revenue from cash positions, with $\tilde{P}$ and $\tilde{Q}$ being uncertain cash price and output, respectively, at harvest;

$FP = (F - \tilde{P})X$, profit on forward positions, with F being the forward price at which farmer sells for harvest delivery and X being number of contract on one unit of crop;

λ = farmer's risk aversion; and

W = farmer's initial level of wealth.

The optimal value of X is obtained by taking the derivative of the objective function with respect to X, setting it to zero, and solving for X:

(2.21) $X = [(F - \mu_p)/2\lambda\sigma_p^2]W + \mu_q(1 - \eta)$

where

$\mu_p = E(\tilde{P})$;

$\mu_q = E(\tilde{Q})$;

$\sigma_p^2 = Var(\tilde{P})$;

$\sigma_q^2 = Var(\tilde{Q})$;

ρ_{pq} = the correlation between $\tilde{P}$ and $\tilde{Q}$; and

$\eta = -[(\sigma_q/\mu_q)/(\sigma_p/\mu_p)]\rho_{pq}$, "effective" elasticity of demand from the viewpoint of the individual farmer.

Summing the demands of the various farmers in the economy, the aggregate agricultural demand for forward contracts is derived:

$$(2.22)\quad X^{a*} = \Sigma_a X^a = \Sigma_a\{[(F - \mu_p^a)/2\lambda\sigma_p^2]W^a + \mu_q^a(1 - \eta^a)\}.$$

Rearranging this yields:

$$(2.22)'\quad X^{a*} = [(F - \mu_p^a)/2\lambda\sigma_p^2]W^{a*} + \mu_q^{a*}(1 - \eta^{a*})$$

where

$W^{a*} = \Sigma_a W^a$;

$\mu_p^a = \Sigma_a \mu_p^a (W^a/W^{a*})$;

$\mu_q^{a*} = \Sigma_a \mu_q^a$; and

$\eta^{a*} = \Sigma_a \eta^a (\mu_q^a/\mu_q^{a*})$.

The development of the demand for forward contracts parallels developments used in later chapters.

Related to the above issue, several empirical studies have examined the relationship between traders' activity and volatility of prices. Cornell (1981) examines the relationship between the volume of trading and price variability for futures contracts. A significant, positive, contemporaneous correlation was found between the changes in average daily volume and changes in the standard deviation of daily log price relatives for 14 of 18 commodities in the sample. This work is relevant to this effort because it indicates how markets might react to changes in overall trading volume and liquidity brought about by a policy change.

Martell and Wolf (1987) developed a theoretical model to relate volume in metal futures markets to inter- and intraday volatility and a set of other variables that influence both hedging and speculative behavior. Using daily and monthly regression models, they present empirical results that indicate that the most important variables are the volatility measures with interest rates, open interest, and inflation.

2.8.2 Simultaneous Determination of Cash and Futures Prices

As noted by Kawai (1983a) and Turnovsky (1983), the theoretical studies tend to suffer from a fundamental deficiency. Specifically, they assume that the coefficients of the relevant supply and inventory demand functions remain unchanged with the introduction of the futures markets. But except in the polar case where all individuals are risk neutral, this assumption is invalid. If producers and speculators are risk averse, the slopes of the relevant supply and inventory demand functions depend upon the degree of price stability and this, in turn, varies with the introduction of the futures markets. To incorporate these important aspects adequately, it is necessary to derive the behavioral relationships for firms and speculators from underlying optimizing considerations. Similar optimizing behavior assumptions are adopted by Danthine (1978), Holthausen (1979), Feder, Just and Schmitz (1980), Newbery and Stiglitz (1981), and others.

Baesel and Grant (1982) developed a model of futures prices based on the interactions of farmers, processors, and speculators. Thus, the authors analyze the logical implications for the equilibrium futures price relative to expected cash price if each market participant hedges optimally. Their study does not include the nonagricultural finance sectors. MacMinn, Morgan, and Smith (1982) present a model of futures prices for a world with farmers and producers. They also do not consider the diversification effects provided by nonagricultural finance assets. Their study is distinguished from the others in that it deals with commodity price regulation.

The early contributions to the theory of cash and futures price determination have been made in the context of the price-stabilization role of futures markets. Several economists have tried to provide a theoretical answer to the issue.[13] Peck (1976) examined the effects the forward price might have on the stability of commodity prices. This work is one of only a few works which have rigorously attempted to resolve the issue of the price stabilization function of futures markets from theoretical perspectives (Turnovsky, 1979 , 1981; Kawai, 1983a, 1983b; Sarris, 1980). For this purpose, Peek develops a framework whereby the futures mechanism can be included in the more traditional commodity models. She focuses on long-run stability. However, the potential

[13] The empirical work extends as far back as Emery (1896). The typical approach is to consider two periods, one with, and the other without, futures trading and to compare the variances of the cash prices in the two cases. This has been carried out for a large number of specific commodities markets and the results are not uniform. Several authors find that futures markets definitely reduce price fluctuations: see, e.g., Hieronymus (1960), Working (1960a), Gray (1963), and Cox (1976) for the onions markets; Hooker (1901) and Tomek (1971) for the wheat market; Emery (1896) for the cotton market; Powers (1970), Taylor and Leuthold (1974), and Cox (1976) for the cattle market. Other authors find that there is no essential gain in stability: see, e.g., Johnson (1968) for the onions market; Naik (1970) for the hessian market. This is just a small sample of what has become a vast literature.

effects of futures markets on intrayear price stability are not considered. According to Peck, the finding that cash prices are more stable with futures trading than without that trading, at least in the long run, does not say that they are sufficiently stable from society's point of view, given the nature of recent "exogenous" shocks.

Kawai (1983a) uses an optimizing model with price uncertainty and risk aversion in order to solve equilibrium distributions of prices for nonstorable commodities. The author focuses on the effect of the presence of a commodity futures market upon the price formation process in a stochastic rational expectations framework. Kawai suggests that the existence of futures trading does not affect the degree of short-term cash price fluctuations. However, if the commodity market disturbance that originates from stochastic consumption demand is serially dependent, then the long-term price variation is smaller with a futures market than without it. Futures prices fluctuate less over time than cash and expected prices. In addition, there exists a futures intervention rule whereby the authority can stabilize cash prices and raise the overall welfare of society.

2.8.3 Optimal Hedge Ratio

There are two standard formulas that have been developed for computing the optimal hedge ratio, the optimal futures position relative to the cash position. The minimum risk hedge ratio has been developed by Ederington (1979) and others. Alternatively, there is the utility maximizing optimal ratio. This has been developed by Heifner (1972), Johnson (1960), Ward and Fletcher (1971), and Telser (1958). If the expected profit from holding futures contracts is zero, the utility maximizing optimal hedge then becomes equivalent to the risk minimizing hedge ratio approach (Kahl, 1983).

Deriving the optimal hedge ratio is an empirical issue. It cannot be determined theoretically. Also, a number of versions of the optimal ratio have been specified using different assumptions about objectives of hedgers, and the ratios have been estimated using different methods (Bond and Thompson, 1985, 1986; Nelson and Collins, 1985; Wilson, 1984; Witt, Schroeder and Hayenga, 1987).

Two issues are being debated concerning empirical estimation of optimal hedge ratios: (1) what is the decision maker's objective when hedging, and (2) what type of data should be used? The first issue or question arises from the debate over whether hedging is a risk-minimizing or utility-maximizing activity. The second question comes from the debate over theoretical, statistical, and practical concerns about the estimation process itself (Herbst et al., 1989). Depending on which data are used, the choice of equations to be estimated will vary. Three types of data have been used in estimating hedge ratios: (1) price differences, (2) percentage changes, and (3) price levels (Witt et al., 1987).

Price difference models of hedge ratios vary depending upon the decision maker's goal. If the goal is to minimize the variance of returns, the optimal hedge ratio is:

$$(2.23) \quad \frac{-X_f}{X_c} = \frac{\sigma_{cf}}{\sigma_f^2},$$

where

X_f = quantity of futures commodity;

X_c = quantity of cash commodity;

σ_{cf} = covariance of cash and futures price changes; and

σ_f^2 = variance of futures price changes.

A positive (negative) sign preceding either cash or futures quantity indicates a long (short) position. This hedge ratio can be estimated by regressing cash price changes on futures price changes. On the other hand, if the goal is to maximize expected utility, Kahl (1983) presents the model:

$$(2.24) \quad X_f = \frac{E(F_2 - F_1)}{\gamma\sigma_f^2} - X_c \cdot \frac{\sigma_{cf}}{\sigma_f^2},$$

where

F_1 = futures price expected at the time a hedge is placed;

F_2 = futures price expected at the time a hedge is lifted; and

γ = a risk aversion parameter.

X_f is positive assuming $\gamma > 0$, which will be the case for a risk-averse person.

Models using percentage change data also distinguish between the two possible goals of hedgers. The hedge ratio when minimizing variance is:

$$(2.25) \quad \frac{-V_f}{V_c} = \frac{\sigma_{rcrf}}{\sigma^2_{rf}} ,$$

where

V_f = total value of futures positions;

V_c = total value of cash positions;

r_f = return from period 1 to period 2 on the values of futures positions; and

r_c = return from period 1 to period 2 on the values of cash positions.

In this case, the variances and covariances are of returns rather than prices. This hedge ratio is the slope coefficient of a regression of cash percentage price changes on futures percentage price changes. When maximizing expected utility, the hedge ratio becomes, using the same notation:

$$(2.26) \quad \frac{-V_f}{V_c} = \frac{\sigma_{rcrf}}{\sigma^2_{rf}} - \frac{r_f}{V_c \gamma \sigma^2_{rf}} .$$

An alternative hedging model to the price difference models of variance-minimizing is appropriate for a hedger concerned only with variance about the expected return in an anticipatory hedge (there is no current cash position). The optimal hedge ratio is:

$$(2.27) \quad \frac{-X_f}{X_c} = \frac{\sigma_{c2f2}}{\sigma^2_{f2}} .$$

This equation is similar in form to the price difference models of variance-minimizing. In this case, however, the hedge ratio is the regression coefficient of cash price levels regressed on futures price levels during the period when the hedger would be closing the futures position and entering the cash market.

After considering statistical, theoretical and practical questions about the appropriateness of using one model over another, Witt et al. (1987) point out that;

> In comparing the price difference models with the percentage change models, the gauge is the degree of linearity between the cash price and futures price differences. If the cash price of the commodity to be (cross) hedged responds linearly with the futures price, the price difference model would be preferred because a goal is to keep the model as simple as possible. If a definite nonlinear relationship exists between the parties, the percentage change model may be preferred (p. 141).

They also note that generalized least squares procedures may be needed to produce more efficient estimates of the hedge ratio due to the influence of autocorrelation in the residuals.

Theoretically, the proper hedge ratio estimation technique depends upon the objective function of the hedger and the type of hedge being considered. Witt et al. (1987) conclude that the best method for anticipatory and storage hedges, respectively, is a price-level regression and a price change model. If the hedger's objective is to maximize utility as opposed to minimizing the variance of returns, then none of these estimation techniques will provide the appropriate hedge ratio. In that case, factors in addition to cash and futures price variance will be significant in determining the optimal hedge ratio (Cecchetti et al., 1988).

When presenting the results of optimal hedge position analysis, most empirical studies have generated a mean-variance (EV) efficient frontier to illustrate the relationship between expected returns and risk (Peck, 1975; Chavas and Pope, 1982; Karp, 1987; Levy, 1987). The relationship is often expressed as a preference function such as the one used by Chavas and Pope (1982);

$$(2.28) \quad L = E(\pi) - (\alpha/2)V(\pi),$$

where L is the objective function, E and V denote mean and variance, respectively, π is profit, and α is a measure of risk aversion. It is generally used in the context of expected utility maximization with constant absolute risk aversion and normality of π.

Portfolio models have increasingly been used to estimate optimal hedge ratios when more than one asset is held at a time. Methods for evaluating strategies appropriate in this situation have centered on techniques similar to those developed by Markowitz (1959) and expanded on by Johnson (1960) and Stein (1961) in the early 1960s (Berck, 1981; Wilson, 1984; Berck and Cecchetti, 1985; Peterson and Leuthold, 1987). The Johnson-Stein approach tended to support the traditional theory of hedging which held that the primary motivation for hedging was risk reduction. The Johnson-Stein theoretical results have been reinforced by empirical tests of the portfolio model in which it is found that hedge ratios estimated using simple regression techniques are often less than one. Related to this, Ederington (1979) makes the statement;

> Contrary to the traditional hedging theory . . . our empirical results indicates that even pure risk-minimizers may wish to hedge only a portion of their portfolios. In most cases, the estimated (hedge ratio) was less than one (p. 169).

Applications of portfolio analysis have become much more complicated in the number of products included in the portfolio, and in the estimation techniques used in determining hedge ratios. For example, Peterson and Leuthold (1987) evaluated some multi-product (inputs and outputs) and multiple-time-period hedging strategies available to a cattle feedlot by applying a discrete nonlinear programming routine to the general function. Cross-hedging is a special type of hedge which has been analyzed using traditional mean-variance methods as well as portfolio techniques. The study by Zacharias et al. (1987) typifies recent approaches to this firm-level problem. As an alternative to mean-variance analysis, they used a numerical simulation approach, in combination with stochastic dominance to evaluate a variety of cross-hedging strategies for a rice grower. First-, second-, and third-degree stochastic dominance criteria were used to rank alternatives produced by simulating the equation. They concluded that regression analysis may or may not be a risk-efficient choice depending upon the decision criteria employed.

Most general methods of calculating hedge ratios have focused on cash and futures price movements, i.e., basis. However, basis is just one source of risk affecting portfolios. Duffie (1989, pp. 239-41) describes how daily resettlement requirements and interest on margin necessitate adjusting the size of the "optimal" hedge, a process called "tailing the hedge." Also, Grant (1985) concludes that it is impossible to derive an optimal hedge ratio when both price and quantity uncertainty are present, a conclusion which questions the entire exercise of calculating optimal hedge ratios.

2.9 Chapter Summary

This chapter has investigated selected areas of interest with respect to expected utility theory, market performance, and production decision under uncertainty. First, the section on expected utility theory described the underlying axioms, and related notion of risk aversion. This section also discussed how the expected utility model is related to the mean-variance analysis upon which a theoretical model is built in this research.

Second, the section on market performance defined pricing efficiency in cash market and pricing effectiveness in futures market. This section also presented the possible measures of these notions. Based on these definitions and empirical measures, the following chapters examine the impacts of changes in tax policy on market performance in cash and futures markets. Related to futures market performance, different but related performance criteria were discussed to show alternative empirical measures of futures market performance in terms of pricing efficiency. The literature reviewed generally showed that futures markets are efficient in that markets reflect all publicly available information on the changes in supply-demand conditions. Also, futures markets are assumed to provide forward prices.

Finally, the literature related to production decisions under uncertainty was presented. This section provided a theoretical background for the model underlying the research. Specifically, this section described and documented the simultaneous determination of cash and futures positions and the derivation of an optimal hedge ratio.

CHAPTER THREE : THEORY

3.1 Introduction

The purpose of this chapter is to present a theoretical model describing cattle producers' decision-making process for production and hedging. Based on the studies of simultaneous determination of cash and futures positions reviewed in Chapter 2, cattle producers are assumed to maximize certainty-equivalent profits adjusted by tax considerations. From the first-order conditions for the profit maximization position, the demands for cash and futures positions will be derived as functions of a set of tax parameters and known prices and for the objective expectations and variances of the random output prices. From the derived demand functions, this chapter is to compare changes in the optimal levels of cash and futures positions by altering the tax rate (deduction level) for a given deduction level (tax rate). In addition, assuming the endogeneity of output prices as well as positions, a rational expectation model will solve for equilibrium cash and futures prices along with insurance prices (difference between expected futures prices and current futures prices including trading costs). For this purpose, the optimizing behavior of individual agents (cattle producers, pure speculators, and consumers) will be specified.

Based on these changes in optimal positions, the following economic analyses are to be performed. The first analysis involves the ramifications of a change in tax policy. This examines the changes in the means and variances of producers' profits and government's tax revenues for specific changes in the tax rates or deduction levels. The second analysis examines the changes in welfare of cattle producers, pure speculators, and consumers. In addition, (aggregate) social welfare changes are examined. For this latter purpose, the price adjustments in cash and futures markets are investigated and the results are reported.

3.2 Basic Assumptions

This study makes two major assumptions to deal with the hedging decisions by cattle feeders. First, cattle feeders are assumed to face only price uncertainty. That is, the model is not designed to accommodate producers with a stochastic production technology.[14] Once feeder cattle are purchased, the final output is essentially fixed. This assumption is not unreasonable because death loss is minimal in the feedlot and gains are only occasionally influenced significantly by weather.

This research should be viewed as a partial equilibrium approach because of the second basic assumption. Cattle producers are assumed to feed cattle only. It is assumed they are not diversified into nonagricultural assets. The analysis of the pricing relationship would be more complex, as noted in Chapter 2, if financial assets were considered. Short-run production and hedging decisions are thus dominated by a concern

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[14] This assumption is adopted by Peck (1975) and Myers and Thompson (1989) among others. The analysis developed here uses a model of the situation found more nearly in livestock or egg production rather than in the grains. An alternative framework is required for the case of joint price and output risk (Rolfo, 1980; Grant, 1985).

for short-run profits in cattle feeding operations only.

Producers are also assumed to be price-takers in a probabilistic context since the producers are unable to influence the selling price distributions. This assumption is commonly adopted in the systematic study of the theory of the competitive firm under price uncertainty and risk aversion (Sandmo, 1971; Holthausen, 1979). Consequently, the decision on the volume of cash and futures positions to be taken must be made prior to the sales date, the date at which the market price becomes known. Also, it is assumed that the producers' beliefs about the final sales price can be summarized in an objective probability distribution.

Finally, *a priori*, no particular form of the utility function summarizing the producers' attitude toward risk is specified. However, it is assumed that there are producers whose preferences are sufficiently similar, within the group of producers in the cattle feeding industry, that guarantee the existence of a group preference function.[15]

3.3 Derivation of Optimal Cash and Futures Positions

For expositional purposes, the possible scenarios faced by cattle producers are presented in Appendix 3.1.1 through 3.1.4 depending on the relationship of price expectations toward cash and futures markets. Assume that a cattle feeder is considering selective hedging. In a strict

[15] This may be a strong assumption, because group preferences may not always satisfy the transitivity axiom required for the existence of a utility function, e.g., Neumann-Morgenstern utility function. It is therefore possible that this research implicitly assumes that the producers' reactions to changes the in environment are more predictable and stable than they really are.

sense, selective hedging refers to an approach to hedging where there are time periods when the hedger will assume an unhedged position and times when he will be a cash-market speculator. However, if allowed by tax policy, he might periodically enter the futures markets in a way designed to provide periodic and "selective" price protection and thereby contribute to the price discovery process, rather than assuming either an unhedged cash speculative position or making all adjustments in the quantity of the cash position.

Table 3.1 shows an exemplary situation faced by cattle producers. This is the situation when the producer is faced with a tax scheme comprised of profits that are adjusted by a constant marginal tax rate, $t \in (0, 1)$, and deductions resulting from ordinary losses, $d \in (0, 1)$.[16] At time $t = 0$, suppose that the expected cash price, $E(S_1)$, exceeds the current cash price, S_0. The producer is expected to take long positions

Table 3.1 A Scenario Faced by Cattle Producers

	Cash Market	Futures Market
Price Expectation:	$E(S_1)-S_0 > 0$	$E(F_1)-F_0 > 0$
Action Taken at t=0:	Long Cash	Long Futures
Price Realization:	$S_1 - S_0 > 0$	$F_1 - F_0 < 0$
Realized Profit at t=1:	$\pi_s > 0$	$\pi_f < 0$
Current Tax Treatment:	Ordinary Gain	Capital Loss
Total Profits Adjusted by Current Taxation:		$\pi_s + \pi_f - t\pi_s$
Total Profits Adjusted by Alternative Taxation:		$\pi_s + \pi_f - t(\pi_s + d\pi_f)$

[16] For comparative static purposes it will be useful to preserve the generality of the tax scheme in a two-parameter space. Also, this research focuses on the impact of changes in marginal tax rate and deductibility of futures losses. Another tax parameter could be the level of profits that are exempt from taxation (Holloway, 1990).

in the cash market. When the expected futures price, $E(F_1)$, is greater than the current futures price, F_0, he might also take long positions in futures market subject to a capacity constraint (cash cattle plus long futures $\leq$ feedlot capacity). At time $t = 1$, the cash expectation turns out to be correct, but not the futures expectation. The realized cash profits are positive and the realized futures profits are negative. According to the current tax policy, the futures losses are capital losses (speculative trading) and are not deductible for tax purposes. The total profit function adjusted by taxation will be $\pi_s + \pi_f - t \cdot \pi_s$. However, if deductions for futures losses are allowed, then the total profit function becomes $\pi_s + \pi_f - t(\pi_s + d \cdot \pi_f)$.

From now on, the analysis proceeds using an alternative or "corrected" definition of after-tax total profits from cash and futures positions defined by:

$$(3.1) \quad \Pi = \pi_s + \pi_f - t(\pi_s + d \cdot \pi_f),$$

where

- Π = total profits;
- π_s = cash profits;
- π_f = futures profits;
- t = marginal tax rate, $t \in (0, 1)$; and
- d = deduction level, $d \in (0, 1)$.

Both cash and futures losses are deductible subject to feedlot capacity constraints and profits from both cash and futures activities are taxed at the marginal tax rate, t.

The agent's risky profits for cash and futures positions in period $t = 1$, the date at which the positions are liquidated, are denoted by:

(3.2) $\pi_s = E(S_1|I_0)\cdot x_s - c(x_s) - f$

(3.3) $\pi_f = [E(F_1|I_0) - F_0]x_f - c(x_f)$

where

π_s = cash profit realized at $t = 1$;

$E(S_1|I_0)$ = expected cash price at $t = 1$ conditional on information available at $t = 0$, I_0;

x_s = cash position chosen at $t = 0$ (short if negative, long if positive);

$c(x_s)$ = an increasing and convex cost function, $c(0) = 0$; $c'(x_s) > 0$, and $c(x_s) < 0$ for $x_s < 0$[17]; and

f = fixed costs.

π_f = futures profit realized at $t = 1$;

$E(F_1|I_0)$ = expected futures price at $t = 1$ conditional on information available at $t = 0$, I_0;[18]

F_0 = futures price at $t = 0$;

x_f = futures position chosen at $t = 0$ (short if negative, long if positive);

$c(x_f)$ = an increasing and convex cost function, $c(0) = 0$; and

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[17] In relaxing the assumption of a strict convex cost function, one could assume $c'(x_s) \geq 0$ and $c''(x_s) \geq 0$ for $x_s > 0$ (non-negative and non-decreasing marginal costs). This holds for a hedging cost function of the form $c(x_f)$.

[18] This I_0 is the same as for cash I_0; this assumes the currently available information set in cash and futures is equal and the same. This, in turn, argues that there is no value in the history of price in the futures, i.e., that technical analysis is not useful.

$c'(x_f) > 0$, and $c(x_f) > 0$ for any x_f.

The model can be interpreted as follows.[19] In this framework, a decision maker plans to sell or to buy cash output (fed cattle) in period 0, depending upon the price expectations prevailing in period 1. That is, the agent commits himself in period 0 to an amount x_s, to be sold (or, as will be seen, bought) in period 1 at the prevailing price for period 1. Assume that the agent may also deal in futures. For the futures market, let x_f be the amount of the live cattle futures sold at time 0 ($x_f > 0$ represents a purchase). This position is closed out by an offsetting trade at time 1. The agent would be a cattle producer so that $c(x_s)$ represents production costs. These production costs consist of purchased feeder cattle and feed and of overhead costs with fixed costs of f. Then, $c(x_s)$ is interpreted as the cost, valued at $t = 0$, incurred by producing the cash positions x_s. Implicitly, the cost function is parameterized by the price of inputs. Thus, $c(x_s)$ is treated as non-stochastic reflecting the assumption that production possibilities are certain. In addition to dealing in cash positions, the agent could enter live cattle futures markets with trading costs of $c(x_f)$ which includes commissions and interests on margins. The futures positions will be closed out by an offsetting trade at $t = 1$, assuming negative (positive) x_f means being short (long) in the distant live cattle futures.

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[19] The set of futures markets participants is far more diverse than the traditional dichotomy of hedgers and speculators suggests. However, a single, simple model suffices to characterize the principal features of optimal decisions by these agents.

The after-tax certainty equivalent problem involving expected revenue levels and variances, adjusted by tax parameters, can be stated in the following form:[20]

$$(3.4) \quad \max_{Xs,Xf} (1 - t)\cdot E(\pi_s|I_0) + (1 - td)\cdot E(\pi_f|I_0) - (\lambda/2)[(1 - t)^2\cdot var(\pi_s|I_0) + (1 - td)^2\cdot var(\pi_f|I_0) + 2(1 - t)(1 - td)\cdot cov(\pi_s,\pi_f|I_0)] \ .$$

Changing notation, this expression is equivalent to:

$$(3.5) \quad \max (1 - t)\cdot E(\pi_s|I_0) + (1 - td)\cdot E(\pi_f|I_0) - (\lambda/2)[(1 - t)^2\cdot x_s^2\sigma_s^2 + (1 - td)^2\cdot x_f^2\sigma_f^2 + 2(1 - t)(1 - td)\cdot x_s x_f \sigma_{sf}]$$

where

λ = a measure of the agent's risk aversion;

σ_s^2 = $var(S_1|I_0)$, conditional variance of the cash price;

σ_f^2 = $var(F_1|I_0)$, conditional variance of the futures price; and

σ_{sf} = $cov(S_1,F_1|I_0)$, conditional covariance between cash and futures prices.

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[20] Maximizing the mean-variance objective function is equivalent to maximizing expected utility if one assumes a quadratic (or exponential) utility function or normally distributed profits. Regardless of the utility function and distribution of profits, the maximization of a mean-variance objective function may provide a reasonable approximation to the maximization of the true objective function (Levy and Markowitz, 1979). Further justification for this preference function as well as a statement of conditions when this is compatible with expected utility analysis are contained in Anderson and Danthine (1978, 1980) and Rolfo (1980) among others. The detailed derivation is presented in section 2.4. Since no wealth constraint is imposed, this implicitly assumes the availability of financing of futures positions at a constant rate reflected in F_0.

By taking the derivatives of the objective function with respect to x_s and x_f, respectively, and setting the derivatives to zero, the first-order conditions for this problem are obtained:[21]

$$(3.6) \qquad [E(S_1|I_0) - c'(x_s)] - \lambda[(1 - t)\sigma_s^2 \cdot x_s + (1 - td)\sigma_{sf} \cdot x_f] = 0$$

$$(3.7) \qquad [E(F_1|I_0) - F_0 - c'(x_f)] - \lambda[(1 - td)\sigma_f^2 \cdot x_f + (1 - t)\sigma_{sf} \cdot x_s] = 0 \ .$$

This development assumes that x_s^* and x_f^* are the nonzero, finite, and unique solutions to (3.6) and (3.7) and that second-order conditions are satisfied.[22]

By solving the first-order conditions (3.6) and (3.7) for x_s and x_f, respectively, the beginning-of-the-period demand functions for x_s (conditional on x_f) and x_f (conditional on x_s) are obtained:

$$(3.8) \qquad x_s^* = \frac{[E(S_1|I_0) - c'(x_s)]}{(1 - t)\lambda\sigma_s^2} - \frac{(1 - td)\sigma_{sf} \cdot x_f}{(1 - t)\sigma_s^2}$$

$$(3.9) \qquad x_f^* = \frac{[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - td)\lambda\sigma_f^2} - \frac{(1 - t)\sigma_{sf} \cdot x_s}{(1 - td)\sigma_f^2}$$

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[21] Reference to bounds on the range of x_s is omitted. If inequality constraints apply, then equation (3.5) is appropriate for an interior solution.

[22] Because $c(x_s)$ and $c(x_f)$ are assumed to be convex, these conditions are necessary and sufficient for a maximum, i.e.,

$$\begin{aligned}
\partial^2\Pi/\partial x_s^2 &= -c''(x_s) - \lambda(1 - t)\sigma_s^2 < 0, \\
\partial^2\Pi/\partial x_f^2 &= -c''(x_f) - \lambda(1 - td)\sigma_f^2 < 0, \\
\partial^2\Pi/\partial x_s\partial x_f &= -\lambda(1 - td)\sigma_{sf} < 0, \text{ and} \\
\partial^2\Pi/\partial x_f\partial x_s &= -\lambda(1 - t)\sigma_{sf} < 0.
\end{aligned}$$

Note that the equations (3.8) and (3.9) above are viewed as optimal cash positions given predetermined or fixed futures positions and optimal futures positions given predetermined or fixed cash positions, respectively. This is equivalent to deriving individual cash and futures demand functions separately.[23] Both cash and futures marginal costs are assumed to be fixed so that $c'(x_s)$ and $c'(x_f)$ equal average variable costs and fixed trading costs, respectively. Thus, these marginal costs are not assumed to depend on x_s and x_f and marginal costs are therefore constant over the relevant range of output.

To consider joint determination of cash and futures positions, further steps are needed. For the optimal cash decision[24], the optimal futures position (3.9) is inserted into the first order condition (3.8). For the optimal futures decision, the optimal cash position (3.8) is inserted into the first order condition (3.9). Both are rearranged to obtain:[25]

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[23] It may be useful to consider the cash position as predetermined or fixed and to select futures positions so as to minimize risk. This "pure" hedge is formally equivalent to the position that minimizes variance of expected profit subject to the equation of expected profit shown above. In this case, the predetermined cash position x_s need not be optimal. Generally, as will be shown later, the optimal cash and futures positions are chosen simultaneously.

[24] This is equivalent to obtaining a derived demand function for the cash positions, i.e., feeder cattle placement into the feedlot. Since the projected supply for fed cattle is known given the placement of feeder cattle on feed assuming a non-stochastic production process, the derived demand function for feeder cattle is identical to the [planned] supply function for fed cattle.

[25] Appendix 3.2 shows the detailed derivations according to the different situations.

$$(3.10)\quad x_s^* = \frac{[E(S_1|I_0) - c'(x_s)] - (\sigma_{sf}/\sigma_f^2)[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - t)\cdot\lambda[\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2)]}$$

$$(3.11)\quad x_f^* = \frac{[E(F_1|I_0) - F_0 - c'(x_f)] - (\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - td)\cdot\lambda[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]}$$

As shown in (3.10), the optimal cash position x_s^* turns out to be positive when $E(\pi_s) \geq 0$ and $E(\pi_f) < 0$, which was assumed in the beginning. Specifically, the first term in the numerator, $E(S_1|I_0) = F_0$, exceeds $c'(x_s)$, which is implied by $E(\pi_s) \geq 0$. The inequality $E(\pi_f) < 0$ implies that $E(F_1|I_0) < [F_0 + c'(x_f)]$. The term (σ_{sf}/σ_f^2) is equivalent to r, the hedge ratio.

Now, assume the common situation of positively but not perfectly correlated cash and futures prices, $0 < (\sigma_{sf}/\sigma_f^2) < 1$. Unless (σ_{sf}/σ_f^2) is negative, the second term in the numerator, $(\sigma_{sf}/\sigma_f^2)[E(F_1|I_0) - F_0 - c'(x_f)]$ becomes negative. Thus, the numerator in (3.10) is positive. The denominator in (3.10) is positive because $(1 - t)$ is positive and λ is assumed to be positive, implying a risk averse economic agent. In addition, $[\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2)]$ is positive since $[\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2)] = [\sigma_s^2(1 - \rho^2)] > 0$, denoting ρ to be the correlation coefficient between cash and futures prices, unless the cash and futures prices are perfectly correlated.[26] Thus, the denominator in (3.10) is positive. Put together, x_s^* in (3.10)

[26] Using $\sigma_{sf}^2 = \rho^2\sigma_s^2\sigma_f^2$ from the notion $\rho = \sigma_{sf}/(\sigma_s\sigma_f)$,

$$\begin{aligned}\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2) &= \sigma_s^2 - (\rho^2\sigma_s^2\sigma_f^2/\sigma_f^2)\\ &= \sigma_s^2 - \rho^2\sigma_s^2\\ &= \sigma_s^2(1 - \rho^2).\end{aligned}$$

is typically positive, implying the expected long positions in cash.

Perhaps contrary to initial intuition, x_s^* increases when the marginal tax rate, t, increases. This is because the denominator is smaller when t increases, as shown in (3.10). This results from the fact that when t increases, the perceived risk premium, $(1 - t)\cdot\lambda[\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2)]$, is reduced or discounted by the increase in t. A change in deduction level, d, does not affect the level of x_s^* in this case since the deductibility of futures losses is not a factor in (3.10).

For the optimal futures position, x_f^*, the sign is expected to be negative. Analogous to the above argument, the numerator in (3.11) is negative and the denominator in (3.11) is positive, thereby the total is negative, which implies short positions in the futures.[27] This result conforms to the expectation that when the current futures price exceeds expected futures price, a rational agent would be short in futures. When either t or d increases, x_f^* increases, as shown in (3.11). This result is confirmed by the fact that when t and/or d increase, the perceived risk premium, $(1 - td)\cdot\lambda[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]$, decreases, thereby increasing the short positions in futures.

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[27] In a regression context, the term of σ_{sf}/σ_s^2 is viewed as the estimated coefficient, ß, from the regression:

$$\Delta F_t(\text{futures prices}) = \alpha + \beta\Delta S_t(\text{cash prices}).$$

With manipulation similar to that shown above, $[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)] = \sigma_f^2(1 - \rho^2)$, which is positive but less than one for less than perfectly correlated cash and futures prices.

3.4 Implications of Optimal Cash and Futures Positions

As noted above, changes in deduction levels given a marginal tax rate would alter the optimal levels of cash and futures positions. Specific paths of changes induced by changed deduction levels can be evaluated by using the mathematical notions of the income and substitution effects under risk.[28] Suppose that there is a change in deduction level for a given tax rate. This would affect price volatility (in terms of variances) and profitability (in terms of price-marginal costs relation), and thereby change the perceived risk premiums. The direct or substitution effects on the optimal positions are found by holding λ constant. Similarly, the indirect or income effects are determined by combining the changes in the optimal cash and futures positions for a change in λ with the changes in λ for a change in the price variances (thus risk premiums).[29]

3.4.1 The Optimal Positions in Response to Changes in Tax Rates

Using the derived optimal cash and futures positions adjusted by tax parameters shown above, inferences can be made. Table 3.2 shows the

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[28] One of the reason to assume constant risk aversion is to simplify mathematical developments. In such a case, a negative exponential utility is implicitly assumed and normality of profits suffices for the existence of mathematical solutions. As shown above, assuming constant risk aversion ignores the income effects which sometimes conflict with the qualitative results.

[29] Mathematically, the substitution and income effects are expressed by:

$$\underbrace{dx/dd}_{\text{total effect}} = \underbrace{(\partial x/\partial d)_{\lambda=\text{constant}}}_{\text{substitution effect}} + \underbrace{(\partial x/\partial \lambda)\cdot(\partial \lambda/\partial d)}_{\text{income effect}}$$

Table 3.2 Expected Changes in Positions For Increases in Tax Rates

		DARA[1]		CARA[2]	IARA[3]
Case 1:	$(1-t)E(\pi_s) + (1-t)E(\pi_f)$				
	Δx_s:	(+)	>	(+)	(?)
	Δx_f:	(+)	>	(+)	(?)
Case 2:	$(1-t)E(\pi_s) + (1-td)E(\pi_f)$				
	Δx_s:	(+)	>	(+)	(?)
	Δx_f:	(+)	>	(+)	(?)
Case 3:	$(1-td)E(\pi_s) + (1-t)E(\pi_f)$				
	Δx_s:	(+)	>	(+)	(?)
	Δx_f:	(+)	>	(+)	(?)
Case 4:	$(1-td)E(\pi_s) + (1-td)E(\pi_f)$				
	Δx_s:	(+)	>	(+)	(?)
	Δx_f:	(+)	>	(+)	(?)

1 DARA stands for Decreasing Absolute Risk Aversion,
2 CARA stands for Constant Absolute Risk Aversion, and
3 IARA stands for Increasing Absolute Risk Aversion.

expected changes in the optimal cash and futures positions in responses to an increase in marginal tax rates given a deduction level. These results are comparable to the cases when price variances decrease or the profitability (in terms of expected gains) increases. To illustrate, suppose that there is an increase in marginal tax rates given a deduction level. This increase in tax rates would decrease the perceived risk premiums (RP), both $\lambda[\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2)]$ in cash and $\lambda[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]$ in futures, since either the constant term (1 - t) or (1 - td) decreases for the larger tax rate, t. For constant risk aversion (CARA), the decreased RP will increase both the optimal cash and futures positions because the denominators involving RP in the simultaneously derived optimal positions become smaller in absolute value. The magnitude of decreases in RP are larger when the term (1 - t) is involved than when the term (1 - td) is involved so long as d < 1. This is also true for the case of decreasing risk aversion (DARA) since both the substitution and income effects are negative, implying that the optimal positions in responses to decreased RP are increased. Since the CARA case does not consider the income effect, under the DARA scenario the magnitude of the optimal position responses are expected to be larger than under the CARA. However, for increasing risk aversion (IARA), the responses of the optimal positions to a decrease in RP induced by an increase in tax rates are ambiguous. Because this class of producers is less willing to assume risk as price variances and thus RP decreases, their positive income effect combines with a negative substitution effect to leave the total effect ambiguous.

3.4.2 The Optimal Positions in Response to Changes in Deductibility

Table 3.3 shows the expected changes in the optimal cash and futures positions in responses to an increase in deduction level for a given tax rate. Suppose that the tax authority announces an increase in deduction level given a tax rate. This would decrease the perceived risk premium, either $\lambda[\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2)]$ in cash or $\lambda[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]$ in futures, since (1 - td) decreases for the larger deduction level, d. For constant λ (CARA), the decreased RP will increase either the optimal cash or futures positions because the denominators involving RP in the optimal position equations become smaller. This holds as well for the case of decreasing λ (DARA) since both the substitution and the income effects are negative, implying that the decreased RP increase the optimal positions. Note that

Table 3.3 Expected Changes in Positions For Increases in Deductibility

		DARA[1]		CARA[2]	IARA[3]
Case 1:	$(1-t)E(\pi_s) + (1-t)E(\pi_f)$				
	Δx_s:	(0)		(0)	(0)
	Δx_f:	(0)		(0)	(0)
Case 2:	$(1-t)E(\pi_s) + (1-td)E(\pi_f)$				
	Δx_s:	(0)		(0)	(0)
	Δx_f:	(+)	>	(+)	(?)
Case 3:	$(1-td)E(\pi_s) + (1-t)E(\pi_f)$				
	Δx_s:	(+)	>	(+)	(?)
	Δx_f:	(0)		(0)	(0)
Case 4:	$(1-td)E(\pi_s) + (1-td)E(\pi_f)$				
	Δx_s:	(+)	>	(+)	(?)
	Δx_f:	(+)	>	(+)	(?)

1 DARA stands for Decreasing Absolute Risk Aversion,
2 CARA stands for Constant Absolute Risk Aversion, and
3 IARA stands for Increasing Absolute Risk Aversion.

although the cases of CARA and DARA have the same qualitative results in terms of the expected signs, this is not necessarily true for the quantitative results in terms of the adjusted amounts of the optimal positions. That is, there is no income effect in the case of CARA while there exists an income effect as well as a substitution effect in the case of DARA. Thus, the DARA case would show larger position responses to changes in deduction level than in the CARA case. For increasing λ (IARA), the responses of the optimal positions to a decrease in RP induced by an increase in deduction level are ambiguous. This is again because the positive income effect of decreased RP combines with a negative substitution effect to make the total effect ambiguous.

3.5 Agent's Optimizing Behavior

In previous sections, the microeconomic decision-making problems facing risk-averse, price-taking cattle producers are presented to derive individual cash and futures supply functions for a set of known prices and for the objective expectations and variances of the random prices. Now, assuming the endogeneity of output prices as well as positions, the following sections solve for rational expectations equilibrium cash and futures prices along with insurance prices (the deviation between expected futures price and current futures price including trading costs). For this purpose, the optimizing behavior of individual agents (cattle producer, pure speculator and consumer in cash market) is specified.

3.5.1 Producer

As shown earlier, the risk-averse price-taking cattle producer's supply function of fed cattle and demand for futures contracts are derived as follows:

$$(3.10) \quad x_s = \frac{[E(S_1|I_0) - c'(x_s)] - (\sigma_{sf}/\sigma_f^2)[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - t)\cdot\lambda_s[\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2)]}$$

and

$$(3.11) \quad x_f = \frac{[E(F_1|I_0) - F_0 - c'(x_f)] - (\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - td)\cdot\lambda_s[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]} \ .$$

3.5.2 Pure Speculator

Pure professional speculators assume open positions in live cattle futures without having to commit themselves to the production, handling or processing of the commodity.[30] A speculator has profit function π_f, the product of his position z_f and the difference $[E(F_1|I_0) - F_0]$ between the expiring futures price and the current futures price. If z_f is positive (negative), the speculator holds long (short) positions in live cattle futures. After adjusting by the marginal tax rate t, the objective of a risk-averse pure speculator is to maximize the certainty equivalent profit:[31]

$$(3.12) \quad \underset{Xf}{\text{Max}} \ (1 - t)\cdot E(\pi_f|I_0) - (\lambda_f/2)(1 - t)^2\cdot \text{var}(\pi_f|I_0)$$

where $E(\pi_f|I_0) = [E(F_1|I_0) - F_0]z_f - c(z_f)$.

Changing notation, this expression is equivalent to:

$$(3.12)' \quad \text{Max} \ (1 - t)\cdot E(\pi_f|I_0) - (\lambda_f/2)(1 - t)^2\cdot z_f^2\sigma_f^2.$$

The parameter λ_f measures the degree of the speculator's risk aversion and

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30 At present, futures trading is assumed to be a ("almost") costless operation and hence, the size of a futures contract is not subject to the trader's capital constraint. Thus, entry into the pure speculative activity is facilitated. In fact, all types of individuals are drawn into the futures market from professionals (large speculators) on the floor of the exchanges to rank amateurs (small speculators). Since an analysis of the real effects of the diversity in futures speculators is not the subject of this section, the distinction among the types is not stressed.

31 Under current tax policy, losses from speculators are not deductible. Thus, deductibility is not assumed to matter for the case of pure speculators. The only concern about tax issues by speculators are then the marginal tax rates.

need not bear any particular relation to λ_s, a producer's risk aversion. By taking the derivatives of the objective function with respect to z_f, and setting the derivatives to zero, the first-order condition for this problem is obtained:

$$(3.13) \qquad [E(F_1|I_0) - F_0 - c'(z_f)] - \lambda_f(1 - t)\sigma_f^2 \cdot z_f = 0.$$

Thus, this development assumes that z_f is the nonzero, finite, and unique solution to (3.13) and that the second-order condition is satisfied.[32]

By solving the first-order condition (3.13) for z_f, the futures supply function by pure speculators is obtained:

$$(3.14) \qquad z_f = \frac{[E(F_1|I_0) - F_0 - c'(z_f)]}{(1 - t)\lambda_f\sigma_f^2}.$$

The form of the pure speculator's supply is exactly the same as the speculative component of the producer's demand for futures trading obtained earlier, except for the difference in risk aversion coefficients and related risk premiums.

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[32] Because $c(z_f)$ is assumed to be convex and the objective function with respect to the decision variable z_f is strict (quasi) concave, this condition is necessary and sufficient for a maximum, i.e.,

$$\partial^2\Pi/\partial z_f^2 \qquad = -c''(z_f) - \lambda_f(1 - t)\sigma_f^2 < 0.$$

3.5.3 Consumer

It is not necessary to derive the demand functions for the third group of market participants, consumers, from underlying utility maximization. Instead, it suffices to simply postulate some convenient demand function. A consumer is an agent who makes a purchase decision at the time he faces actual market cash prices without being exposed to price uncertainty. This price-taking consumer maximizes the utility of a commodity consumption subject to a budget constraint and finds his demand for the commodity at time $t = 0$, C_0, as a declining function of the current price:

$$(3.15) \quad C_0 = a - bS_0 + u_0.$$

The demand function is linear; a and b are fixed constants; S_0 is the cash price at $t = 0$; and u_0 is the disturbance term representing the consumers' unique characteristics. At present, in Marshallian fashion, the income effect of the price received by the producers upon market demand is ignored.

3.6 Determination of Cash and Futures Prices

Individual agents' supply and demand functions of cash positions and futures contracts can be aggregated in order to obtain the market supply and demand schedules, which determine equilibrium or market-clearing cash and futures prices. Strictly speaking, the coefficients in the aggregate functions of cash demand and supply and futures demand and supply should be multiplied by the number of corresponding representative agents. However, without essential loss of generality these factors can be set to unity.[33] Individual agents are assumed to be homogeneous within the group: (1) producers possess identical cost functions and identical risk aversion coefficients; (2) pure speculators are alike in their aversion to risk; and (3) consumers are similar in their demand coefficients and disturbances. In addition, producers and pure speculators are assumed to share the identical expectations formation scheme, i.e., a rational expectation in the sense of Muth (1961).[34]

For the producer's expected profit function expressed as $E(\pi_s) + E(\pi_f) - t[E(\pi_s) + d \cdot E(\pi_f)]$, the aggregate relationships describing the cash and futures markets are specified as follows:[35]

[*Cash Demand by Consumers*]

(3.15) $C_1 = a - bS_1 + u_1$

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[33] In such a case, the numbers of agents in the cash and futures markets are assumed to be fixed and to remain unchanged when total amount of demand and supply vary. That is, the possibility of new entry and exit from the cash and futures markets are ignored.

[34] The notion of a Muth rational expectation indicates that individuals form their expectations about the next period's cash and futures prices based upon all currently available information, including the complete structure of the price determination process. Thus, it is assumed there is no asymmetry in the amount of information available to the agents. Stein (1986) illustrated the case in which there exists asymmetry in information among the types of futures participants. Grossman (1977) and Danthine (1978) have assumed that professional or "pure" speculators are endowed with more advanced information about the future than are producers.

[35] Appendix 3.3 shows the derivations of the equilibrium cash and futures prices according to the different situations.

[*Cash Supply by Producers*]

$$(3.10)\qquad X_s = \frac{[E(S_1|I_0) - c'(x_s)] - (\sigma_{sf}/\sigma_f^2)[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - t)\cdot\lambda_s[\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2)]}$$

[*Futures Demand by Producers*]

$$(3.11)\qquad X_f = \frac{[E(F_1|I_0) - F_0 - c'(x_f)] - (\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - td)\cdot\lambda_s[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]}$$

[*Futures Supply by Pure Speculators*]

$$(3.14)\qquad Z_f = \frac{[E(F_1|I_0) - F_0 - c'(z_f)]}{(1 - t)\lambda_f\sigma_f^2}$$

The above four equations can be solved for variables X_s, X_f, Z_f, S_1 and F_0. First, consider the market-clearing futures price F_0. For equilibrium in the futures market to prevail, the (excess) demand for futures contracts by producers must be equal the (excess) supply of futures contracts by pure speculators. This is equivalent to saying that the sum of the excess demand by producers and pure speculators must be zero:

[*Futures Market Clearing*]

$$(3.16)\qquad X_f + Z_f = 0.$$

Substituting (3.11) and (3.14) into (3.16), yields the equilibrium current futures prices, noting that $\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2) = \sigma_f^2(1 - \rho^2)$ and assuming $c'(x_f) = c'(z_f)$:

$$(3.17)\qquad F_0 = E(F_1|I_0) - c'(x_f) - \frac{(1 - t)\lambda_f(\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - t)\lambda_f + (1 - td)\lambda_s(1 - \rho^2)}$$

For the case of $E(S_1|I_0) = F_0$, denoting $A = (1 - t)\lambda_f + (1 - td)\lambda_s(1 - \rho^2)$ and $B = (1 - t)\lambda_f(\sigma_{sf}/\sigma_s^2)$,

$$(3.18)\quad F_0 = \frac{A\cdot[E(F_1|I_0) - c'(x_f)] + B\cdot c'(x_s)}{A + B}$$

$$= \frac{[(1-t)\lambda_f + (1-td)\lambda_s(1-\rho^2)][E(F_1|I_0)-c'(x_f)] + (1-t)\lambda_f(\sigma_{sf}/\sigma_s^2)\cdot c'(x_s)}{(1-t)\lambda_f + (1-td)\lambda_s(1-\rho^2) + (1-t)\lambda_f(\sigma_{sf}/\sigma_s^2)}$$

In order to find the equilibrium insurance price $[E(F_1|I_0) - F_0 - c(x_f)]$, the futures market clearing condition is modified. The futures demand by producers in (3.11) can be divided into two parts:

$$(3.19)\quad X_f = \frac{[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - td)\cdot\lambda_s[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]} - \frac{(\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - td)\cdot\lambda_s[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]}\ .$$

The first term on the right-hand side of (3.19) is called the "speculation" part. This reflects the difference between the futures price expectation and the corresponding futures price, which is an anticipated gain per unit of the commodity purchased in futures. The second term is the "hedging" part, which is related to the positions taken in the cash market. Combined with the futures supply by pure speculators in (3.14), the first term of (3.19) represents the market "speculative" supply of futures contracts:

$$(3.20)\quad SX_f = \frac{[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - td)\cdot\lambda_s[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]} + \frac{[E(F_1|I_0) - F_0 - c'(z_f)]}{(1 - t)\lambda_f\sigma_f^2}$$

Noting that $\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2) = \sigma_f^2(1 - \rho^2)$ and assuming $c'(x_f) = c'(z_f)$,

[Speculative Supply by Producers and Pure Speculators]

$$(3.20)'\quad SX_f = \frac{[E(F_1|I_0) - F_0 - c'(x_f)][(1 - t)\lambda_f + (1 - td)\lambda_s(1 - \rho^2)]}{(1 - t)(1 - td)\lambda_s\lambda_f\sigma_f^2(1 - \rho^2)}$$

Analogous to the cash market, suppose that there exists a "hedge" demand for futures contracts, expressed by a function of the insurance price:

[Hedge Demand by Hedgers]

$$(3.21)\quad DX_f = c - d[E(F_1|I_0) - F_0 - c'(x_f)] + v_t$$

$$\text{if } E(F_1|I_0) > [F_0 - c'(x_f)], \text{ and}$$

$$(3.21)'\quad DX_f = e - f[E(F_1|I_0) - F_0 - c'(x_f)] + w_t$$

$$\text{if } E(F_1|I_0) < [F_0 - c'(x_f)]\ .$$

The two different specifications of the demand schedules by hedgers are due to the fact that long hedge demand when $E(F_1|I_0) > [F_0 + c'(x_f)]$ is not necessarily identical to short hedge demand when $E(F_1|I_0) < [F_0 + c'(x_f)]$. The market-clearing insurance price can be determined when the excess hedging demand by producers is equal to the excess speculative supply by producers and pure speculators:

[*Market-Clearing Insurance Price*]

$$\text{(3.22)} \qquad DX_f = SX_f.$$

Substituting (3.20) and (3.21) into the market-clearing insurance price in (3.22), yields the equilibrium insurance prices, denoting $A = [(1-t)\lambda_f + (1-td)\lambda_s(1-\rho^2)]$ and $E = (1-t)(1-td)\lambda_s\lambda_f\sigma_f^2(1-\rho^2)$:

$$\text{(3.23)} \quad [E(F_1|I_0)-F_0-c'(x_f)] = \frac{(c + v_t)\cdot E}{A + d\cdot E}$$

$$= \frac{(c + v_t)[(1-t)(1-td)\lambda_s\lambda_f\sigma_f^2(1-\rho^2)]}{[(1-t)\lambda_f + (1-td)\lambda_s(1-\rho^2)] + d[(1-t)(1-td)\lambda_s\lambda_f\sigma_f^2(1-\rho^2)]} .$$

This equilibrium insurance price is an important and convenient term in analyzing the demand for and the supply of futures contracts. While the cash demand and supply are determined by cash price level, the futures demand and supply should be determined by the current futures price relative to the expectation of corresponding futures price, i.e., the deviation between the two prices. For the case in which $E(F_1|I_0) > F_0 + c'(x_f)$, as the deviation between $E(F_1|I_0)$ and $F_0 + c'(x_f)$ becomes larger, the supply of long speculation is expected to increase. For $E(F_1|I_0) < F_0 + c'(x_f)$, the larger the deviation between $E(F_1|I_0)$ and $F_0 + c'(x_f)$ becomes, the larger the supply of short speculation.

Second, consider the market-clearing cash price. The equilibrium condition for cash market requires that the (planned) amount produced by cattle producers has to match realized demand by consumers. This is because cattle producers have planned to produce fed cattle at $t = 0$. The cash market-clearing condition is:

$$\text{[\textit{Cash Market Clearing}]} \tag{3.24}$$
$$C_1 = X_s.$$

Substituting (3.10) and (3.15) into (3.24), yields the equilibrium realized cash prices, noting that $\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2) = \sigma_s^2(1 - \rho^2)$:

$$(3.25)\quad S_1 = \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)] - (\sigma_{sf}/\sigma_f^2)[E(F_1|I_0) - F_0 - c'(x_f)]}{b(1 - t)\cdot\lambda_s\sigma_s^2(1 - \rho^2)}$$

This derivation is obtained by equating the cash demand with the cash supply and segregating the futures market equilibrium. This "segregation" result does not take the equilibrium futures prices into account.[36]

3.7 Implications of Cash and Futures Prices and Pricing Efficiency

Figure 3.1 shows the cash market clearing condition. The equilibrium realized cash price is obtained by equating the aggregate consumer demand with the aggregate supply of fed cattle which is simultaneously determined with the futures market. The downward-sloping

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[36] The term "segregation" is distinguished from the "separation" results used by Kawai (1983a, p. 238; 1983b, p. 438) and others like Danthine (1978), Holthausen (1979), and Feder, Just and Schmitz (1980). Contrary to this, Anderson and Danthine (1981) showed the "simultaneous" results which considered the joint optimal determination of cash and futures positions.

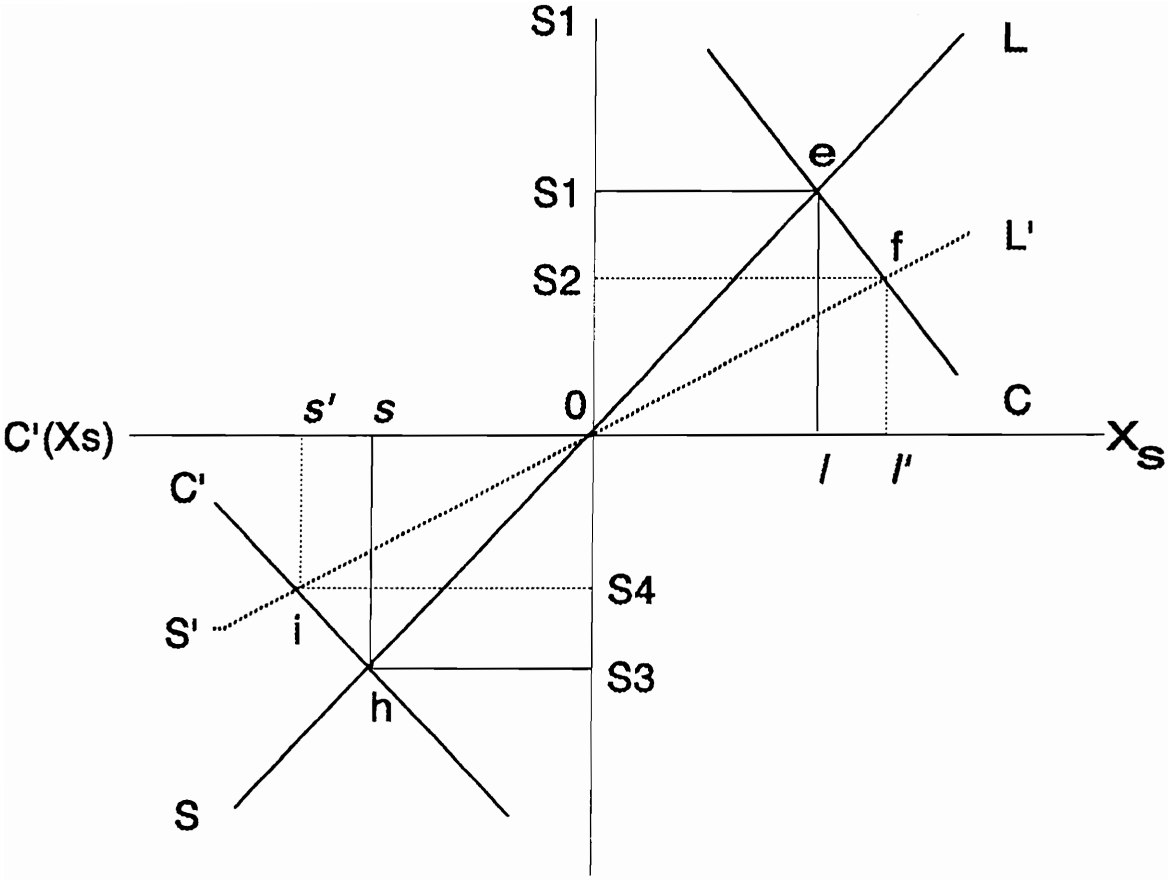

Figure 3.1 Clearing of Cash Market

curve C stands for a consumer demand schedule corresponding to a long supply of cash positions, and C′ denotes a short supply of cash positions. The upward-slope curve L (S) stands for a fed cattle supply schedule of long (short) position, which is the sum of the marginal cost $C'(X_s)$ and risk premium RP adjusted by tax parameters. The reciprocal of the risk premium adjusted by tax parameters, $1/[(1 - t)\cdot\lambda_s\sigma_s^2(1 - \rho^2)]$, represents the cash supply response to the expected cash price. As the tax rate increases, the cash-supply response coefficient increases in absolute value, and the supply curve rotates to the right (from L to L′ for long cash positions and from S to S′ for short cash positions).[37] Consequently, given a cash price, the amount supplied by producers will increase in absolute value from l to l′ for long cash positions (from s to s′ for short cash positions).

Figure 3.1 also illustrates how equilibrium (expected) cash prices are adjusted in response to changes in tax policy under the "segregation" results. Suppose that expected cash profitability is positive such that $E(S_1) > C'(X_s)$. In such a case, an increase in tax rate and/or deduction level rotates the long cash supply curve to the right and, given a demand schedule, equilibrium (expected) output increases and equilibrium (expected) cash price decreases from S_1 to S_2. This, in turn, reduces the deviation between cash selling price and fixed marginal costs. For a

[37] When the producer's expected function is expressed as either $E(\pi_s) + E(\pi_f) - t[d\cdot E(\pi_s) + E(\pi_f)]$ or $E(\pi_s) + E(\pi_f) - t[d\cdot E(\pi_s) + d\cdot E(\pi_f)]$, an increase in deduction level as well as tax rate will also rotate this cash supply curve to the same direction, but the magnitude of rotation is expected to be smaller due to the nature of the equation.

negative expected cash profitability of $E(S_1) < C'(X_s)$, increased tax rates and/or deduction levels give rise to a rotation of the short cash supply curve to the left. Note that the impact of an increased tax rate is the same as that of increased deduction level *qualitatively*, but that the former is different from the latter *quantitatively* unless a 100 percent deduction for any losses is allowed, i.e., unless $d = 1$. This increases the short cash supply and thereby increases cash prices from S_3 to S_4, reducing the gap between expected cash selling price and marginal costs. As discussed below, given fixed marginal costs, pushing the equilibrium expected cash prices down (for long cash) or up (for short cash) has an important implication for improvements in pricing efficiency in terms of a competitive market equilibrating process.

As derived above, the equilibrium insurance price is determined by equating the sum of speculative supply by producers and pure speculators with hedge demand by hedgers. Figure 3.2 illustrates the futures price determination process and the supply shifts generated by an increase in either the tax rate or the deduction level. Suppose that there exists a preponderance of buying pressure so that current futures prices including relevant trading costs are higher than the expected (expiring) distant futures prices.[38] In such a case, rational producers and pure speculators would take short positions in the distant live cattle futures. The amount of short positions supplied by them depends on the magnitude of

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[38] This net long pressure can be from either the hedgers' side or the speculators' side. This could be viewed as an excess aggregate demand for long positions given the supply-demand information set.

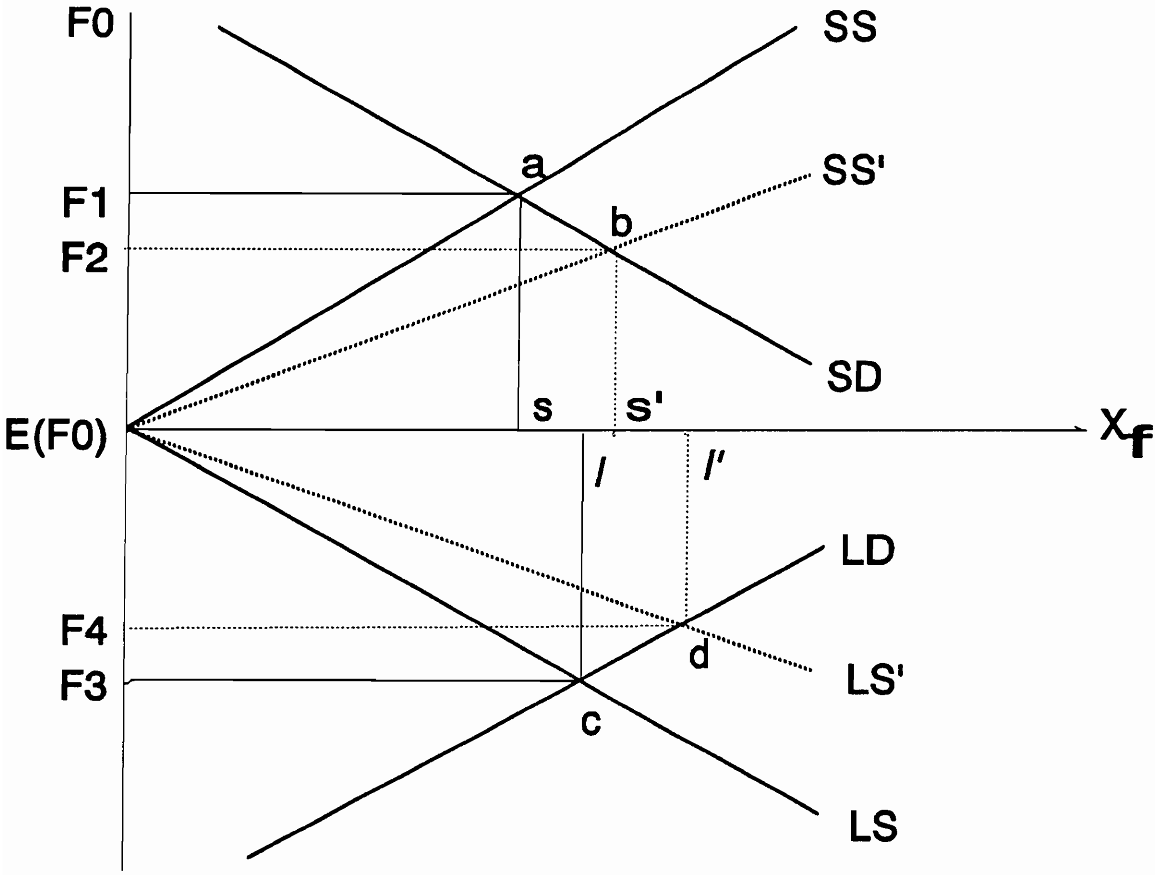

Figure 3.2 Clearing of Futures Market

deviation between the current futures price and the expected futures price in the delivery month. The larger the gap between the two prices, the more they would take (supply) short positions. The curve SS in the upper part of Figure 3.2 stands for this short speculative supply.[39] As derived above, the constant term, $[(1 - t)\lambda_f + (1 - td)\lambda_s(1 - \rho^2)]/[(1 - t)(1 - td)\lambda_s\lambda_f\sigma_f^2(1 - \rho^2)]$, represents the speculative supply response to the deviation between the two prices. As either tax rate or deduction level increases, this coefficient increases and the speculative supply curve rotates to the right (from SS to SS'). Consequently, given a deviation between the two prices, the amount of speculative supply increases with increases in the deviation. Given a short hedge demand schedule (SD), the equilibrium insurance price is reduced from $[F_1 - E(F_0)]$ to $[F_2 - E(F_0)]$, and the equilibrium amount of supply or demand is therefore increased (from s to s').

The lower part of Figure 3.2 illustrates the case in which the current futures market is dominated by short selling pressure. In such a case, the current futures price is lower than the expectation of the expiring futures price. Denoting LS as a long speculative supply schedule, if either tax rate or deduction level increases, then it will increase the speculative-supply response coefficient, and thus the supply curve rotates to the right (from LS to LS'). Given a long hedge demand curve (LD), this rotation will in turn decrease the equilibrium insurance

[39] For exposition purpose, this curve is linear, but it does not have to be. Yun (1992) showed evidence of the nonlinearity of this supply curve.

price from $[E(F_1) - F_3]$ to $[E(F_1) - F_4]$ and increase the equilibrium amount of long positions supplied and demanded (from 1 to 1').

Using the notion of market efficiency introduced in Chapter 2, implications to pricing efficiency in cash and futures markets can be drawn. As shown in Figures 3.1 and 3.2, an announcement of increases in tax rate and/or deduction level have a negative effect on the differences between prices and marginal costs. That is, when an increase in tax rate or deduction level is expected, producers would respond by increasing their planned output (fed cattle). At the same time, the related (distant live cattle) futures traders including hedgers and pure speculators increase their positions by either entering the futures market or increasing existing positions. Given downward-sloping demand schedules and predetermined feeding and hedging costs, the increase in planned output will decrease the expected cash prices and thereby reduce the deviation between cash prices and marginal costs.[40] This corresponds to the change from S_1-$C'(x_s)$ to S_2-$C'(x_s)$ for long cash positions, or from S_3-$C'(x_s)$ to S_4-$C'(x_s)$ for short cash positions, shown in Figure 3.1.

In the futures market, an increase in the positions (in absolute value) taken by hedgers and pure speculators would reduce the deviation between expected futures prices and current futures prices plus hedging costs. This corresponds to the change from F_3-$E(F_0)$ to F_4-$E(F_0)$ for long futures positions or the change from F_1-$E(F_0)$ to F_2-$E(F_0)$ for short futures

[40] As shown in Figure 3.1, an increase in short cash positions (implied by negative profitability of $E(S_1) < c(x_s)$) would involve an increase in the expected cash prices and thereby reduce the gap between prices and marginal costs.

positions, illustrated in Figure 3.2. Defining "unbiasness" as current futures prices being equal to expected futures prices, this response pattern is translated into a situation in which futures prices turn into an unbiased predictor of corresponding expected cash price or, alternatively, the probability of a biased production is reduced. In sum, increasing the tax rate and/or allowing for deductions on losses from futures trade could have a positive impact on restoring market equilibrium in terms of the price-marginal costs relationship and thereby improve pricing efficiency in both the cash and futures markets.

3.8 Welfare Measures

For risk averse economic agents involved in the current model including producers, pure speculators, and consumers, the most appropriate measure of welfare gains or losses due to a change in tax policy is the change in expected utilities generated by the change in tax rates or deduction levels. Under this procedure, clear-cut conclusions cannot be made since it is not directly determined whether the expected utilities of each group increase or decrease. However, the alternative profit maximizing approach used in this study allows for moving away from surplus measures to welfare measures based on means and variances of the profit accruing to the respective groups. As a tractable alternative measure, the expected profits for producers are considered here.[41]

The same measure can be adopted for pure speculators. It has to be

[41] This approach is also adopted by Kawai (1983a, 1983b) and Turnovsky and Campbell (1985).

recognized that expected profits do not correctly capture welfare gains or losses for risk averse agents under uncertainty, so this procedure should be considered as only an approximation. For the consumers' side, without resorting to underlying utility maximization, the traditional consumer's surplus measure is retained as an indicator of welfare gains or losses to consumers.[42] The welfare gains for the three groups are defined in the following sections.

3.8.1 Producers' Welfare Function

To derive the welfare function for the representative producer, $E(\Pi_p)$, the mean and the variance of the profits accruing to the producer should be considered. These have the same form as the certainty equivalent profit functions derived earlier. However, the measure of consumer surplus (CS) does not include the variance and thus the distribution of the relevant prices. The absence of these terms in the CS measure gives a distorted comparison with producers' welfare, as measured by certainty equivalent expected profits. Therefore, the expected profits adjusted by tax parameters are used as the measure of producers' welfare, expressed as:

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[42] The traditional approach in the price stabilization literature has been to evaluate welfare gains and losses in terms of changes in producers' and consumers' surplus. The limitation of these measures are well known, and it is preferable to perform the welfare analysis in terms of the underlying utility function and the qualitative implications from surplus measures are then generally acceptable. Willig (1976) argued that the errors committed by the use of consumer surplus are relatively small.

$$(3.26)\quad E(\Pi_p) = (1 - t)\cdot E(\pi_s|I_0) + (1 - td)\cdot E(\pi_f|I_0),$$

where

$$E(\pi_s|I_0) = [E(S_1|I_0)]x_s - c(x_s) - f; \text{ and}$$

$$E(\pi_f|I_0) = [E(F_1|I_0) - F_0]x_f - c(x_f).$$

The expected gains to producers from the changes in tax policy are defined as:

$$(3.27)\quad \Delta E(\Pi_p) = E(\Pi_p^*) - E(\Pi_p).$$

If this is positive (welfare gain), then producers would be better off due to the changed tax policy, at least, in the aggregate sense.

3.8.2 Pure Speculators' Welfare Function

The procedure for deriving the welfare gains or losses to pure speculators is directly analogous to that obtained earlier. The welfare function of the representative pure speculator, $E(\Pi_f)$, is given by (3.13) with an analogous expression applying in the aggregate:

$$(3.28)\quad E(\Pi_f) = (1 - t)\cdot E(\pi_f|I_0),$$

where $E(\pi_f|I_0) = [E(F_1|I_0) - F_0]z_f - c(z_f)$.

The expected gains to pure speculators from the changes in tax policy are defined as:

(3.29) $\Delta E(\Pi_f) = E(\Pi_f^*) - E(\Pi_f)$.

If this is positive (welfare gain), then pure speculators would be better off due to the changed tax policy, at least, in the aggregate sense.

3.8.3 Consumers' Surplus

As noted above, when considering the gains or losses received by consumers through a changed tax policy, the traditional surplus measure is used. The gain in surplus, CS, from the changed in tax policy is given by:

(3.30) $$\int_{S_1^*}^{S_1} D(S_1) \cdot dS_1.$$

With a linear demand function, and taking expected values, the expected gains to consumers from the changed tax policy are:[43]

(3.31) $\Delta E(CS) = (1/2)E\{[S_1 - S_1^*][D(S_1) + D(S_1^*)]\}$.

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[43] This is simply the area to the left of the demand curve between S_1 and S_1^*. It is derived as follows:

$$\begin{aligned}\Delta E(CS) &= E\{[S_1 - S_1^*]\cdot D(S_1) + ½[S_1 - S_1^*][D(S_1^*) - D(S_1)]\} \\ &= E\{[S_1 - S_1^*][D(S_1) + ½\cdot D(S_1^*) - ½\cdot D(S_1)]\} \\ &= E\{[S_1 - S_1^*][½\cdot D(S_1) + ½\cdot D(S_1^*)]\} \\ &= ½\cdot E\{[S_1 - S_1^*][D(S_1) + D(S_1^*)]\}.\end{aligned}$$

See also Varian (1992, pp. 163-168).

If this is positive (welfare gain), then consumers would be better off due to the changed tax policy, at least, in the aggregate sense.

3.8.4 Social Welfare Function

In the present study, it is assumed that social welfare, W, can be obtained by summing individual agents' (approximate) measures of welfare. The expected value of aggregate welfare is measured as:

$$(3.32) \quad E(W) = E(\Pi_p) + E(\Pi_f) + E(CS).$$

In addition, social welfare gains are measured by the sum of the expected changes in the profits of producers and pure speculators, together with the expected change in consumers surplus, expressed as:

$$(3.33) \quad \Delta E(W) = \Delta E(\Pi_p) + \Delta E(\Pi_f) + \Delta E(CS).$$

Although this is positive, overall economic welfare is not necessarily raised by a change in tax policy.[44] This is because some groups could be worse off as a result of changed tax policy.

3.8.5 Government's Tax Revenue Function

Assuming no tax payments from consumers, the certainty equivalent

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[44] In this sense, a positive social welfare gain does not necessarily imply Pareto improvement. Pareto improvement is possible only when everyone is better off (or some people better off and others no worse off).

for the expected tax revenues generated from producers and pure speculators is expressed as:

$$(3.34)\quad E(G) = t \cdot E(\pi_s | I_0) + t \cdot E(\pi_f | I_0) + t \cdot E(\pi_f | I_0).$$

The expected gains to the government from the changes in tax policy are defined as:

$$(3.35)\quad \Delta E(G) = E(G^*) - E(G).$$

If this is positive, then the government's tax revenues are raised due to the changed tax policy.

3.9 Implications of Welfare Changes

Based on the mathematical results obtained earlier, this section examines the impacts of changes in tax policy on individual groups' welfare changes (producers, pure speculators and consumers). In addition, the impacts on social welfare changes and government's tax revenues are considered. The corresponding impacts are analyzed under two cases: (1) the short-run case in which only optimal cash and futures positions are allowed to vary; and (2) the case for the "segregation" results under which prices as well as positions are allowed to vary, but there is assumed to be no recursive impact of futures price on cash price. To obtain the most complete picture of the overall welfare effects in terms of expected profits, each case considers the effects of tax policy on: (1)

the before-tax and after-tax expected profit;[45] and (2) the one-period before-tax and after-tax variance of profit.[46]

3.9.1 Short-Run Welfare Changes

Table 3.4 shows the expected changes in mean and variance of producers' before-tax and after-tax profits in response to an increase in tax rates, depending upon the assumptions of producers' attitudes toward risk. The expectations are comparable to those of changes in positions in response to changes in tax rates. Suppose that there is an increase in the tax rate. For constant risk aversion (CARA), the mean of cash and futures before-tax profits are expected to increase, regardless of the form of producers' expected profit functions adjusted by tax parameters. The increased tax rate reduces the variance of the profit stream and, as a result, prompts increases in the optimal cash and futures positions with other things remaining constant. In this sense, the increased tax rate can be viewed as "a larger slice (tax receipts) of an even bigger pie."

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[45] The calculation of before-tax profits is comparable to computing the changes in producers' revenues without adjusting tax parameters. Note that the derivation of the optimal positions and equilibrium prices are the same as the after-tax case.

[46] Turnovsky and Campbell (1985, p. 288) argue that the consumer surplus (CS) measure attributes smaller weight to stability, perhaps giving a distorted comparison with producers' and pure speculators' welfare, as measured by certainty equivalent expected profits. The latter involves variances of profits and thus the distribution of the relevant cash price. These are not present in the CS measure, which is analogous to the certainty equivalent expected profits. For this reason, the authors suggest that it is useful to consider the corresponding effects on: (1) the (asymptotic) expected profits; (2) the (asymptotic) one-period variance of profits; and (3) the (asymptotic) welfare as defined by (asymptotic) expectation of the utility function.

Table 3.4 Expected Changes in Mean and Variance of Producer's Profits For Increases in Tax Rates

	DARA[1]	CARA[2]	IARA[3]
Case 1: $(1-t)E(\pi_s) + (1-t)E(\pi_f)$			
$\Delta E(\pi_s)$:	(+)[4] [+][5]	(+)[0]	(?)[?]
$\Delta var(\pi_s)$:	(+)[+]	(+)[0]	(?)[-]
$\Delta E(\pi_f)$:	(+)[+]	(+)[0]	(?)[?]
$\Delta var(\pi_f)$:	(+)[+]	(+)[0]	(?)[-]
Total $\Delta E(\Pi)$:	(+)[+]	(+)[0]	(?)[?]
$\Delta var(\Pi)$:	(+)[+]	(+)[0]	(?)[-]
Case 2: $(1-t)E(\pi_s) + (1-td)E(\pi_f)$			
$\Delta E(\pi_s)$:	(+)[+]	(+)[0]	(?)[?]
$\Delta var(\pi_s)$:	(+)[+]	(+)[0]	(?)[-]
$\Delta E(\pi_f)$:	(+)[+]	(+)[0]	(?)[?]
$\Delta var(\pi_f)$:	(+)[+]	(+)[0]	(?)[-]
Total $\Delta E(\Pi)$:	(+)[+]	(+)[0]	(?)[?]
$\Delta var(\Pi)$:	(+)[+]	(+)[0]	(?)[-]
Case 3: $(1-td)E(\pi_s) + (1-t)E(\pi_f)$			
$\Delta E(\pi_s)$:	(+)[+]	(+)[0]	(?)[?]
$\Delta var(\pi_s)$:	(+)[+]	(+)[0]	(?)[-]
$\Delta E(\pi_f)$:	(+)[+]	(+)[0]	(?)[?]
$\Delta var(\pi_f)$:	(+)[+]	(+)[0]	(?)[-]
Total $\Delta E(\Pi)$:	(+)[+]	(+)[0]	(?)[?]
$\Delta var(\Pi)$:	(+)[+]	(+)[0]	(?)[-]
Case 4: $(1-td)E(\pi_s) + (1-td)E(\pi_f)$			
$\Delta E(\pi_s)$:	(+)[+]	(+)[0]	(?)[?]
$\Delta var(\pi_s)$:	(+)[+]	(+)[0]	(?)[-]
$\Delta E(\pi_f)$:	(+)[+]	(+)[0]	(?)[?]
$\Delta var(\pi_f)$:	(+)[+]	(+)[0]	(?)[-]
Total $\Delta E(\Pi)$:	(+)[+]	(+)[0]	(?)[?]
$\Delta var(\Pi)$:	(+)[+]	(+)[0]	(?)[-]

1 DARA stands for Decreasing Absolute Risk Aversion,
2 CARA stands for Constant Absolute Risk Aversion,
3 IARA stands for Increasing Absolute Risk Aversion,
4 (·) corresponds to before-tax cases, and
5 [·] corresponds to after-tax cases.

This also holds for the case of decreasing risk aversion (DARA). As shown earlier, it should be noted that the increase in total before-tax profits is expected to be larger under the DARA case than under the CARA case since the former involves an income effect as well as substitution effect. For the case of increasing risk aversion (IARA), the effects of increased tax rates on the mean of before-tax profit are ambiguous since the corresponding effects on the optimal positions, as noted earlier, are indeterminate.

For CARA, the mean of cash and futures after-tax profits are expected to remain unchanged. The increased portion of the optimal positions are exactly offset by the increased tax rates and there is thereby no change in the total after-tax profits.[47] For DARA, the mean

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[47] This result is obtained by simple calculus as shown below: From the optimal positions functions, denoting

$$A = \frac{E(S_1) - c'(x_s) - (\sigma_{sf}/\sigma_f^2)[E(F_1) - F_0 - c'(x_f)]}{\lambda_s \sigma_s^2 (1 - \rho^2)} \quad \text{for } X_s,$$

$$= \frac{[E(F_1) - F_0 - c'(x_f)] - (\sigma_{sf}/\sigma_s^2)[E(S_1) - c'(x_s)]}{\lambda_s \sigma_f^2 (1 - \rho^2)} \quad \text{for } X_f.$$

Defining t_h as an after-change (higher) tax rate and t_l as a before-change (lower) tax rate, the changes in the *after-tax* profits are expressed as:

$$\Delta\pi_i = (1 - t_h)[p_i - c(x_i)]x_i^* - (1 - t_l)[p_i - c(x_i)]x_i$$

where i = s (cash) and f (futures) and therefore $p_s = E(S_1)$ and $p_f = [E(F_1 - F_0]$,

$$= \left[(1 - t_h)\frac{A}{(1 - t_h)} - (1 - t_l)\frac{A}{(1 - t_l)} \right] \cdot [p_i - c(x_i)]$$

$$= 0 \qquad (Q.E.D.).$$

of cash and futures after-tax profits (and thus total after-tax profits) will increase since the optimal positions increase more proportionally than the tax-adjusted profits decrease as tax rates increase. For IARA, the changes in the mean of after-tax profits are again indeterminate.

Now, consider the impacts of an increased tax rate on the variances of before-tax and after-tax profits. For constant risk aversion (CARA), the variances of before-tax cash and futures profits will increase since the optimal positions are increased in response to increased variances of the corresponding prices. For decreasing risk aversion (DARA), the magnitude of increases in the variances are bigger than under the CARA case since the DARA case involves the income effect as well as the substitution effect. For increasing risk aversion (IARA), the changes in the variances are indeterminate since the changes in the optimal positions cannot be determined.

For the CARA case, the variances of after-tax profits are expected to remain unchanged. This is because the increases in the optimal positions induced by an increased tax rate are exactly offset by the decreased constant, $(1 - t)^2$ or $(1 - td)^2$, embodied in the variances of profits.[48] This result is not surprising since only positions are

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[48] This result is obtained by simple calculus as shown below: From the optimal positions functions, denoting

$$A = \frac{E(S_1) - c'(x_s) - (\sigma_{sf}/\sigma_f^2)[E(F_1) - F_0 - c'(x_f)]}{\lambda_s \sigma_s^2 (1 - \rho^2)} \quad \text{for } X_s,$$

$$= \frac{[E(F_1) - F_0 - c'(x_f)] - (\sigma_{sf}/\sigma_s^2)[E(S_1) - c'(x_s)]}{\lambda_s \sigma_f^2 (1 - \rho^2)} \quad \text{for } X_f.$$

assumed to vary so that changes in tax parameters are exactly proportionally offset by changes in the optimal positions. For the DARA case, however, the profit variances increase in response to the increased tax rate. The very definition of DARA implies that risk aversion decreases as the expected profits increase. The optimal positions increase more proportionally, and the decreased constant, $(1 - t)^2$ or $(1 - td)^2$, fails to offset the increased portion of the optimal positions.[49]

Defining t_h as after-change (higher) tax rate and t_l as before-change (lower) tax rate, the changes in the optimal positions are expressed as:

$$\Delta X_i = \left[\frac{1}{(1 - t_h)} - \frac{1}{(1 - t_l)} \right] \cdot A$$

$$= \left[\frac{t_h - t_l}{(1 - t_h)(1 - t_l)} \right] \cdot A \quad > 0 \text{ since } t_h > t_l.$$

The changes in the variances of profits are defined as:

$$\Delta \mathrm{var}(\pi_i) = (1 - t_h)^2 \sigma_i^2 x_h^* - (1 - t_l)^2 \sigma_i^2 x_l^*$$

$$= (1 - t_h)^2 \sigma_i^2 [A/(1 - t_h)]^2 - (1 - t_l)^2 \sigma_i^2 [A/(1 - t_l)]^2$$

$$= \sigma_i^2 A^2 - \sigma_i^2 A^2 = 0 \qquad (Q.E.D.).$$

[49] This result is obtained by simple calculus as shown below: From the optimal positions functions, denoting

$$A = \frac{E(S_1) - c'(x_s) - (\sigma_{sf}/\sigma_f^2)[E(F_1) - F_0 - c'(x_f)]}{\sigma_s^2(1 - \rho^2)} \qquad \text{for } X_s,$$

$$= \frac{[E(F_1) - F_0 - c'(x_f)] - (\sigma_{sf}/\sigma_s^2)[E(S_1) - c'(x_s)]}{\sigma_f^2(1 - \rho^2)} \qquad \text{for } X_f.$$

Defining t_h and λ_h as after-change (higher in absolute value) tax rate and (lower in absolute value) risk aversion, and t_l and λ_l as before-change (lower in absolute value) tax rate and (higher in absolute value) risk aversion, the changes in the optimal positions are expressed as:

1 1

For the IARA case, the variances of profits are expected to be reduced. This is obvious in that the substitution effect is positive and the corresponding income effect is negative, which results in a decrease in the optimal positions in absolute value.[50]

With respect to pure speculators' welfare changes, the expectations are analogous to the case of producers' welfare changes. The difference is that the supply responses to the deviation between current and expected futures prices have a different magnitude.[51] For the CARA case, the mean of pure speculators' before-tax profit increases and its variance also increases. For the DARA, the expected before-tax profit of pure

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$$\Delta X_i = \left[\frac{}{(1 - t_h)\lambda_h} - \frac{}{(1 - t_l)\lambda_l}\right] \cdot A$$

$$= \left[\frac{(1 - t_l)\lambda_l - (1 - t_h)\lambda_h}{(1 - t_h)(1 - t_l)\lambda_h\lambda_l}\right] \cdot A \gg 0 \text{ since } t_h > t_l \text{ and } \lambda_h < \lambda_l.$$

The changes in the variances of profits are defined as:

$$\Delta var(\pi_i) = (1 - t_h)^2\sigma_i^2 x_h^* - (1 - t_l)^2\sigma_i^2 x_l^*$$

$$= (1 - t_h)^2\sigma_i^2\{A/[(1 - t_h)\lambda_h]\}^2 - (1 - t_l)^2\sigma_i^2\{A/[(1 - t_l)\lambda_l]\}^2$$

$$= (1/\lambda_h^2)\sigma_i^2 A^2 - (1/\lambda_l^2)\sigma_i^2 A^2 > 0 \text{ since } (1/\lambda_h^2) > (1/\lambda_l^2) \ (Q.E.D.).$$

[50] If the income effect dominates the substitution effect, the sign of the optimal positions will be changed. Unless the (increasing) risk aversion coefficient is unreasonably high, it is unlikely that the optimal positions will be larger with an opposite sign.

[51] Assuming (1) producers and pure speculators have the same risk aversions (i.e., $\lambda_s = \lambda_f$), (2) cash and futures prices are not perfectly correlated (i.e., $\rho^2 \neq 1$), and (3) cash profitability is positive, pure speculators' response to the futures price deviation is larger than that of producers when $E(F_1) > [F_0 + c'(x_f)]$ and thus both would be long in the futures. For the case in which $E(F_1) < [F_0 + c'(x_f)]$ and both groups will be short, pure speculators' response is smaller than that of producers.

speculators increases and its variance also increases. For the IARA, the change in the expected before-tax profits is ambiguous and the corresponding change in the variances is also indeterminate. For CARA, the mean of after-tax profit and the corresponding variance are expected to be unchanged with the same reasoning as that presented for the producers' case. For DARA, both the mean and the variance of after-tax profits increase. For IARA, the change in the mean is indeterminate while its variance decreases when the tax rate increases.

It is tedious to consider the effects of an increase in deduction level given a tax rate since the same expectations as those for the case of tax rate changes should be made, at least in a qualitative sense. The differences between the case of a tax rate change and the case of a deductibility change are: (1) the relevant expectations can be made only when the producers' tax-adjusted expected profit function involves the terms of $(1 - td)E(\pi_s)$ and/or $(1 - td)E(\pi_f)$; and (2) more importantly, the changes in mean and variance of expected profits in response to a change in deduction level are expected to be smaller in absolute value compared to the case of an change in tax rate, unless the marginal tax rate is one-hundred percent. Note that the deduction level, d, is always multiplied by tax rate, t, in the models. In addition, the mean and variance of pure speculators' before-tax and after-tax profits are not influenced by the changes in deduction level. This is because pure speculation itself is not to be involved in any level of futures loss deduction for tax purposes. Table 3.5 summarizes the qualitative expectations of a change in deduction level in a short-run framework.

Table 3.5 Expected Changes in Mean and Variance of Producer's Profits For Increases in Deductibility

	DARA[1]	CARA[2]	IARA[3]
Case 1: $(1-t)E(\pi_s) + (1-t)E(\pi_f)$			
$\Delta E(\pi_s)$:	(0)[4][0][5]	(0)[0]	(0)[0]
$\Delta var(\pi_s)$:	(0)[0]	(0)[0]	(0)[0]
$\Delta E(\pi_f)$:	(0)[0]	(0)[0]	(0)[0]
$\Delta var(\pi_f)$:	(0)[0]	(0)[0]	(0)[0]
Total $\Delta E(\Pi)$:	(0)[0]	(0)[0]	(0)[0]
$\Delta var(\Pi)$:	(0)[0]	(0)[0]	(0)[0]
Case 2: $(1-t)E(\pi_s) + (1-td)E(\pi_f)$			
$\Delta E(\pi_s)$:	(0)[0]	(0)[0]	(0)[0]
$\Delta var(\pi_s)$:	(0)[0]	(0)[0]	(0)[0]
$\Delta E(\pi_f)$:	(+)[+]	(+)[0]	(?)[?]
$\Delta var(\pi_f)$:	(+)[+]	(+)[0]	(?)[-]
Total $\Delta E(\Pi)$:	(+)[+]	(+)[0]	(?)[?]
$\Delta var(\Pi)$:	(+)[+]	(+)[0]	(?)[-]
Case 3: $(1-td)E(\pi_s) + (1-t)E(\pi_f)$			
$\Delta E(\pi_s)$:	(+)[+]	(+)[0]	(?)[?]
$\Delta var(\pi_s)$:	(+)[+]	(+)[0]	(?)[-]
$\Delta E(\pi_f)$:	(0)[0]	(0)[0]	(0)[0]
$\Delta var(\pi_f)$:	(0)[0]	(0)[0]	(0)[0]
Total $\Delta E(\Pi)$:	(+)[+]	(+)[0]	(?)[?]
$\Delta var(\Pi)$:	(+)[+]	(+)[0]	(?)[-]
Case 4: $(1-td)E(\pi_s) + (1-td)E(\pi_f)$			
$\Delta E(\pi_s)$:	(+)[+]	(+)[0]	(?)[?]
$\Delta var(\pi_s)$:	(+)[+]	(+)[0]	(?)[-]
$\Delta E(\pi_f)$:	(+)[+]	(+)[0]	(?)[?]
$\Delta var(\pi_f)$:	(+)[+]	(+)[0]	(?)[-]
Total $\Delta E(\Pi)$:	(+)[+]	(+)[0]	(?)[?]
$\Delta var(\Pi)$:	(+)[+]	(+)[0]	(?)[-]

1 DARA stands for Decreasing Absolute Risk Aversion,
2 CARA stands for Constant Absolute Risk Aversion,
3 IARA stands for Increasing Absolute Risk Aversion,
4 (·) corresponds to before-tax cases, and
5 [·] corresponds to after-tax cases.

Concerning consumers' welfare changes, it would not be appropriate to offer expectations in the short-run analysis. Only positions expected to adjust in response to the changes in tax policy are being considered here. Thus, the relevant analyses for consumers' welfare changes are to be performed under the "segregation" result presented in the next section.

3.9.2 Welfare Changes Under the Segregation Result

As will be shown below, it is impossible to make any kind of general qualitative assessment of the welfare gains for producers and pure speculators without resorting to numerical methods, at least for the after-tax cases under the segregation result.[52] Contrary to the above case in which only the optimal positions are assumed to adjust in response to changes in tax policy, the segregation result is based on the proposition that the positions and prices are simultaneously determined without recursive-type feedback from the futures market. Related to this difficulty is that the amplitude of the changes in the equilibrium prices depends on the elasticities of the related demand functions.

With respect to (aggregate) consumers' surplus, the expectation can be made with certainty since increases in tax rate and/or deduction level always increase cash and speculative optimal positions (in absolute

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[52] Concerning the *before-tax* profits and variances in response to changes in tax policy, the qualitative assessment is equivalent to comparing the area of OS_1el with the area of OS_2fl', shown in Figure 3.1, under the segregation result. The only obvious expectation is that the mean of *before-tax* profits for the short-run case is larger than that under the segregation result. For example, the former *before-tax* cash profit is $[S_1 - C'(X_s)]\cdot l'$ and the latter one is $[S_2 - C'(X_s)]\cdot l'$.

value). Assuming the (linear) demand schedules are downward-sloping, the increased optimal positions involve (proportional) decreases in the corresponding prices, i.e., cash prices in absolute level and futures' insurance prices in deviation form. Thus, consumers' surplus unambiguously increases when there exists an announcement of increases in either tax rate or deduction level. In general, a quantitative assessment depends upon the magnitudes of the corresponding demand elasticities. The differential impacts that depend upon the risk aversion assumptions can be generated, however, based on expectations about the changes in the optimal positions.[53] The adjustments in the optimal positions in response to changes in tax policy are expected to be larger for the decreasing risk aversion (DARA) than those for the constant risk aversion (CARA). Given down-sloping demand schedules, the gains (losses) in consumers' surplus are, therefore, expected to be bigger for the DARA than for the CARA when there exists an increase (decrease) in the tax rate and/or the deduction level.

3.9.3 Government's Tax Revenue Changes

Based on the results of the before-tax cases obtained above, the government's tax revenue change induced by changes in tax policy can be inferred. The above results imply that increases in either tax rate or

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[53] Note that the other differential impacts are found when comparing changes in tax rate with changes in deduction level. Due to the nature of the objective function defined earlier, changes in tax rate have a positive impact on the optimal positions with full strength while changes in deduction level have a positive impact with less-than-full strength.

deduction level have a positive effect on the optimal positions and a negative effect on the equilibrium prices (more precisely, profitability in terms of price-marginal costs deviations). For the short-run case, the mean of before-tax profits will increase in response to an increased tax rate and/or increased deduction level while its variance increases. This implies that the mean of expected government's tax revenue increases and its variance also increases. Analogous to the above discussion, the amplitude of increases in tax revenues is greater when, in aggregate, producers are assumed to have a decreasing risk aversion (DARA) than when they are assumed to possess constant risk aversion (CARA). In addition, the changes in tax revenues are expected to be larger when tax rates are adjusted than when deduction levels are altered.

Contrary to this, the changes in tax revenues are, in general, indeterminate under the segregation result. Note that the short-run change in tax revenues is obviously larger than that under the segregation case. Here again, the elasticities of demand schedules have an important role in determining the amplitude and the direction of changes in tax revenues. The differential impacts between the short-run case and the segregation results also depend upon the demand elasticities.

3.10 Chapter Summary

Chapter 3 started by presenting basic assumptions adopted in the theoretical model: (1) cattle feeders are assumed to face only price uncertainty; (2) they are not diversified into nonagricultural assets; and (3) they are price-takers in the output market. Considering tax parameters, specific tax-adjusted objective functions were specified corresponding to possible scenarios. Based on these objective functions, input demand functions for feeder cattle and demand functions for futures position were derived. The first-part of the analysis included: (1) changes in cash and futures positions as marginal tax rate varies; and (2) changes in cash and futures positions as the deduction level of futures losses varies.

In order to examine the implication of varying risk aversion, conceptual arguments were presented. Direct and indirect effects were considered to show different changes in positions in response to changes in tax policy. Implications to the optimal cash and futures positions were drawn. For the cases of decreasing risk aversion (DARA) and constant risk aversion (CARA), both the optimal cash and futures positions were expected to increase in response to an increase in the tax rate and/or deduction level. For increasing risk aversion (IARA), the responses of the optimal cash and futures positions to an increase in the tax rate and/or deduction level were expected to be ambiguous.

The second part of the analysis examined the determination of cash and futures prices as well as cash and futures positions. For this purpose, optimizing behavior of producers, pure speculators, and consumers was presented under a "segregation" approach which assume the cash and futures prices are determined separately. Implications to equilibrium cash and futures prices were drawn. When $E(S_1) > c'(x_s)$, an increase in the tax rate and/or deduction level was expected to push down the equilibrium cash price to reduce the spreads between $E(S_1)$ and $c'(x_s)$.

When $E(S_1) < c'(x_s)$, the cash price was expected to increase in response to an increased tax rate and/or deduction level. This reduced gap implies that the market moves toward a state of equilibrium. When short (long) futures positions were increased (in absolute value) in response to an increased tax rate and/or deduction level, the current futures prices were expected to decrease (increase) to reduce the spreads between $E(F_1)$ and $F_0 + c'(x_f)$. This reduced gap would increase the effectiveness of the market as a price discovery mechanism.

In order to examine welfare changes induced by tax policy, welfare measures were specified. For producers and pure speculators, the welfare measures are expected profits. For consumers, the measures were the traditional consumers' surplus. Also, changes in tax revenues were considered. For the short-run case when only positions are allowed to vary, producers' and consumers' welfare and expected tax revenue would increase in response to an increased tax rate and/or deduction level. The changes in individual agent's welfare and tax revenues are, in general, indeterminate under the segregation result where output price is also allowed to change.

Implications to pricing efficiency were drawn by considering whether the changes in prices induced by changes in tax policy reduces the difference between the selling price and marginal costs. An increase in the tax rate and/or deduction level could have a positive impact on restoring market equilibrium and thereby improve pricing efficiency in both the cash and futures markets.

CHAPTER FOUR : COMPARATIVE STATICS

4.1 Introduction

This chapter presents comparative statics which analyze the impacts of changes in marginal tax rate and/or deduction level on the optimal cash and futures positions, evaluated at and around equilibrium levels. Chapter 4 is distinguished from Chapter 3 in that Chapter 4 considers marginal changes in the variables of interest in response to changes in tax parameters (in terms of partial derivatives). Chapter 3 compared the changed equilibrium or optimal levels of the variables of interest by changing the tax parameters parametrically. These comparative static analyses examine other theoretical aspects of the issues of interest which could not be evaluated in Chapter 3, changes in the distribution of cash and futures profits, and risk-sharing aspects of tax policy. In addition, Chapter 4 introduces a revenue-neutral tax policy which holds total tax receipts constant.[54]

The specific model used here is for the case of $E(\pi_s) \geq 0$ and $E(\pi_f) < 0$. The basic assumptions adopted are the same as those presented in Chapter 3: (1) cattle feeders are assumed to face only price uncertainty; (2) cattle producers are not diversified into nonagricultural assets; (3) cattle producers are price-takers; and (4) no particular form of the

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[54] A recent example of this tax scheme is the *Tax Reform Act* of 1986. The *Act* was intended to be revenue neutral. That is, the legislation was designed in such a way that the expected levels of net fiscal receipts after the reform equaled the level of those receipts before the reform (Penn, 1984; Rausser and Foster, 1987).

utility function summarizing the producers' attitude toward risk is specified. Based on the first-order conditions derived in Chapter 3, the following comparative static analyses are performed:

(1) marginal changes in cash and futures positions given a unit change in the marginal tax rate;

(2) marginal changes in cash and futures positions given a unit change in deductibility expressed as whole percentage points;

(3) tax revenue-neutral changes in cash and futures positions given a unit change in the marginal tax rate; and

(4) tax revenue-neutral changes in cash and futures positions given a unit change in deductibility expressed as whole percentage points.

This comparative static analysis also examines the relationship between (1) changes in the expected levels of both profits and tax revenue, and (2) between changes in the variance of both profits and tax revenue for a given change in tax parameters.

4.2 A Revenue-Neutral Tax Scheme

Once futures price F_1 and a cash-futures basis are realized, a cattle feedlot industry is comprised of n producers who make tax payments to the treasury through the IRS in an amount, assuming symmetry of producers in the industry, expressed by:

$$(4.1)\quad G^* \equiv ng^* = n(t\cdot\pi_s^* + td\cdot\pi_f^*)$$
$$= n\{t[E(S_1) - c(x_s^*) - f] + t\cdot d[E(F_1) - F_0 - c(x_f^*)]\}$$

where

G^* = total transfer from the industry to the treasury;

n = number of producers in the industry;

g^* = individual tax payment to the treasury;

t = marginal tax rate;

π_s^* = cash profit defined at the optimal cash position, x_s^*;

d = deduction level;

π_f^* = futures profit defined at the optimal futures position, x_f^*;

$E(S_1)$ = expected cash price at $t = 1$ conditional on information avaiable at $t = 0$, I_0;

$c(x_s^*)$ = variable cost, calculated at x_s^*;

f = fixed costs;

$E(F_1)$ = expected futures price at $t = 1$ conditional on information avaiable at $t = 0$, I_0;

F_0 = futures price at $t = 0$; and

$c(x_f^*)$ = futures trading cost, calculated at x_f^*.

Suppose fiscal policy fixes aggregate net tax payments from this industry at level G^*. Within this framework, a feasible tax policy that meets the constraint consists of choices from the set of all feasible combinations of marginal tax rate, t, and deduction levels, d, such that the aggregate level of net payments remain constant at G^*. This subset of policies can be considered as choices from the set $\{(t, d) \mid 0 < t < 1$ and $0 < d < 1,\ dG = 0\}$. When output price is random, the fiscal authority chooses the levels of t and d with less than complete information about prices. Taking expectations and totally differentiating in (4.1), (4.2) $dE_g(G^*) = \pi_s^* \cdot dt + d \cdot \pi_f^* \cdot dt + t \cdot \pi_f^* \cdot dd = 0,$

where E_g denotes the fiscal authority's subjective expectations operator defined over cash and futures prices. Let E denote the objective (mathematical) expectations operator and assume:

(4.3) $E_g(p) = E_p(p) = E(p).$

The expression in 4.3 assumes that the fiscal authority's expectation over prices is identical to producers' expectations. The policy must satisfy the differential equation expressed by:

$$(4.4) \quad (\partial d/\partial t)_{dE(G)=0} = -[(\pi_s^* + d \cdot \pi_f^*)/(t \cdot \pi_f^*)]$$

$$= -(1/t)[(\pi_s^*/\pi_f^*) + d]$$

$$= -(1/t)\left[\frac{(F_0 + B) - c(x_s^*) - f}{E(F_1|I_0) - F_0 - c(x_f^*)} + d \right].$$

The general solution is:

$$(4.5) \quad d(t) = [\kappa/t] - [(\pi_s^* + \pi_f^*)/\pi_f^*]$$

$$= [\kappa/t] - [(\pi_s^*/\pi_f^*) + 1]$$

$$= (\kappa/t)\left[\frac{(F_0 + B) - c(x_s^*) - f}{E(F_1|I_0) - F_0 - c(x_f^*)} + 1 \right].$$

where κ, the constant of integration, is arbitrary. In such a case, (4.5) encompasses a broad agenda of policies that constrain total fiscal receipts to some arbitrary level $G \equiv ng = -n\kappa$. Setting $\kappa = -g^*$ leaves producer profits unchanged and total industry transfers constant at G^*.

4.3 Comparative-Static Analysis of Short-Run Cash and Futures Response to Taxation

In order to examine the short-run effects on x_s and x_f of policies defined by $(\partial d/\partial t)_{dE(G)=0}$, implicitly differentiate in the first-order conditions derived in Chapter 3 (3.6 and 3.7), and evaluate the resulting expression at $x_s = x_s^*$ and $x_f = x_f^*$ (Chiang 1985; pp. 364-368). This short-run analysis assumes that the endogenous variables are cash and futures positions x_s and x_f and that the exogenous variables include cash and futures prices, the degree of risk aversion, policy generated tax parameters, and the number of producers in the industry. In addition, it is assumed that the producer makes production and hedging decisions with sole regard for short-run profits and does not consider the relationship between these decisions and long-run policies for investment and finance.

To make explicit that π_s and π_f are both functions of the endogenous (choice) variables x_s and x_f, rewrite the first-order conditions in the following format:

$$(4.6)\quad F^1(x_s,\ x_f;\ E(S_1|I_0),\ F_0,\ t,\ d) \equiv [(E(S_1) - c'(x_s)] - \lambda[(1 - t)\sigma_s^2 x_s + (1 - td)\sigma_{sf} x_f] = 0$$

$$(4.7)\quad F^2(x_s,\ x_f;\ E(F_1|I_0),\ F_0,\ t,\ d) \equiv [E(F_1|I_0) - F_0 - c'(x_f)] - \lambda[(1 - td)\sigma_f^2 x_f + (1 - t)\sigma_{sf} x_s] = 0.$$

The functions F^1 and F^2 are assumed to possess continuous derivatives so that it would be possible to apply the implicit-function theorem if the Jacobian $|J|$ of this system with respect to the endogenous variables x_s and x_f does not vanish at the initial equilibrium:

$$(4.8)\quad |J| = \begin{vmatrix} (\partial F^1/\partial x_s) & (\partial F^1/\partial x_f) \\ (\partial F^2/\partial x_s) & (\partial F^2/\partial x_f) \end{vmatrix} = |H|$$

$$= \begin{vmatrix} -c''(x_s) - \lambda(1 - t)\sigma_s^2 & -\lambda(1 - td)\sigma_{sf} \\ -\lambda(1 - t)\sigma_{sf} & -c''(x_f) - \lambda(1 - td)\sigma_f^2 \end{vmatrix}$$

where $|H|$ = the Hessian determinant of the first-order conditions.

Assuming the second-order sufficient conditions for profit-maximization are satisfied, the $|H|$ must be positive, and so must $|J|$, at the initial equilibrium or optimum. In this case, the implicit function theorem enables us to write the pair of implicit functions:

$$(4.9)\quad x_s^* = x_s^*[E(S_1|I_0),\ F_0,\ t,\ d]$$

$$(4.10)\quad x_f^* = x_f^*[E(F_1|I_0),\ F_0,\ t,\ d]$$

as well as the pair of identities

$$(4.11)\quad [E(S_1|I_0) - c'(x_s^*)] - \lambda[(1 - t)\sigma_s^2 x_s^* + (1 - td)\sigma_{sf}x_f^*] \equiv 0$$

$$(4.12)\quad [E(F_1|I_0) - F_0 - c'(x_f^*)] - \lambda[(1 - td)\sigma_f^2 x_f^* + (1 - t)\sigma_{sf}x_s^*] \equiv 0.$$

To study the comparative statics of the model, first take the total differential of each identity with respect to x_s, x_f, $E(S_1|I_0)$, $E(F_1|I_0)$, F_0, t and d, allowing all the exogenous variables to vary:

$$(4.13)\quad (\partial F^1/\partial x_s)\cdot dx_s^* + (\partial F^1/\partial x_f)\cdot dx_f^* = -dE(S_1|I_0) - \lambda(\sigma_s^2 x_s + d\cdot\sigma_{sf}x_f)\cdot dt - \lambda t\cdot\sigma_{sf}x_f\cdot dd$$

$$(4.14)\quad (\partial F^2/\partial x_s)\cdot dx_s^* + (\partial F^2/\partial x_f)\cdot dx_f^* = -dE(F_1|I_0) + dF_0 - \lambda(d\cdot\sigma_f^2 x_f + \sigma_{sf}x_s)\cdot dt - \lambda t\cdot\sigma_f^2 x_f\cdot dd.$$

The result yields the following matrix of comparative-static effects:

$$(4.15)\qquad \begin{bmatrix} A & B \\ E & F \end{bmatrix} \begin{bmatrix} dx_s \\ dx_f \end{bmatrix} = \begin{bmatrix} C \\ G \end{bmatrix} dt + \begin{bmatrix} D \\ H \end{bmatrix} dd$$

where

$$\begin{aligned}
A &= (\partial F^1/\partial x_s) &&= -c''(x_s) - \lambda(1 - t)\sigma_s^2 \\
B &= (\partial F^1/\partial x_f) &&= -\lambda(1 - td)\sigma_{sf} \\
C &= &&= -\lambda(\sigma_s^2 x_s + d\cdot\sigma_{sf}x_f) \\
D &= &&= -\lambda t\sigma_{sf}x_f \\
E &= (\partial F^2/\partial x_s) &&= -\lambda(1 - t)\sigma_{sf} \\
F &= (\partial F^2/\partial x_f) &&= -c''(x_f) - \lambda(1 - td)\sigma_f^2 \\
G &= &&= -\lambda(d\cdot\sigma_f^2 x_f + \sigma_{sf}x_s) \\
H &= &&= -\lambda t\cdot\sigma_f^2 x_f \\
I &= (\partial F^1/\partial E(S_1|I_0) &&= -1 \\
J &= (\partial F^2/\partial E(F_1|I_0) &&= -1 \\
K &= (\partial F^2/\partial F_0) &&= 1
\end{aligned}$$

$$|H| = \begin{vmatrix} -c''(x_s) - \lambda(1 - t)\sigma_s^2 & -\lambda(1 - td)\sigma_{sf} \\ -\lambda(1 - t)\sigma_{sf} & -c''(x_f) - \lambda(1 - td)\sigma_f^2 \end{vmatrix}.$$

Using Cramer's rule, we can derive the following comparative static results:[55]

$$(4.16)\quad (\partial x_s/\partial t) = (1/|H|)\begin{vmatrix} C & B \\ G & F \end{vmatrix}$$

$$= (1/|H|)[CF - BG]$$

$$= (1/|H|)\lambda[c''(x_f)(\sigma_s^2 x_s + d\cdot\sigma_{sf}x_f) + \lambda(1 - td)\sigma_s^2\sigma_f^2(1 - \rho^2)x_s]$$

$$(4.17)\quad (\partial x_f/\partial t) = (1/|H|)\begin{vmatrix} A & C \\ E & G \end{vmatrix}$$

$$= (1/|H|)[AG - CE]$$

$$= (1/|H|)\lambda[c''(x_s)(d\cdot\sigma_f^2 x_f + \sigma_{sf}x_s) + \lambda(1 - t)d\cdot\sigma_s^2\sigma_f^2(1 - \rho^2)x_f]$$

$$(4.18)\quad (\partial x_s/\partial d) = (1/|H|)\begin{vmatrix} D & B \\ H & F \end{vmatrix}$$

$$= (1/|H|)[DF - BH]$$

$$= (1/|H|)\lambda t\cdot c''(x_f)\sigma_{sf}x_f$$

$$(4.19)\quad (\partial x_f/\partial d) = (1/|H|)\begin{vmatrix} A & D \\ E & H \end{vmatrix}$$

$$= (1/|H|)[AH - DE]$$

$$= (1/|H|)\lambda t x_f[c''(x_s)\sigma_f^2 + \lambda(1 - t)\sigma_s^2\sigma_f^2(1 - \rho^2)].$$

4.4 Implication of Short-Run Cash and Futures Response to Taxation

Based on the developments in 4.3, the impacts of an increase in both the marginal tax rate and the deduction level seem to be ambiguous, but not indeterminate. The signs of partial comparative analysis are

[55] A detailed explanation for deriving these results is found in Appendix 4.1.

indeterminate in that x_s and x_f evaluated at the equilibrium levels can take on positive or negative values. Due to the structure of the problem, x_s and x_f are jointly determined and, sometimes, negative x_s and/or negative x_f can be found in the solution.

Further economic support for these results can be found by evaluating the effect of such a tax policy on the "distribution" of the producer's profits. This is because introducing tax parameters would affect the distribution of profits in terms of the mean and variance of profits. Let $E(\Pi_1|I_0)$ and $var(\Pi_1|I_0)$ denote, respectively, the mean and variance of total profits.[56] Consider the first two central moments of the distribution of total before- and after-tax profits:

$$(4.20)\quad E(\Pi_1|I_0)_{\text{before-tax}} = E(\pi_s|I_0) + E(\pi_f|I_0) = [S_1x_s - c(x_s) - f] + [(F_1 - F_0)x_f - c(x_f)],$$

$$(4.21)\quad var(\Pi_1|I_0)_{\text{before-tax}} = var(\pi_s|I_0) + var(\pi_f|I_0) + 2\cdot cov(\pi_s,\pi_f|I_0) = x_s^2\cdot\sigma_\epsilon^2 + x_f^2\cdot\sigma_\delta^2 + 2\cdot\sigma_{sf}x_sx_f, \text{ and}$$

$$(4.22)\quad E(\Pi_1|I_0)_{\text{after-tax}} = (1 - t)\cdot E(\pi_s|I_0) + (1 - td)\cdot E(\pi_f|I_0) = (1 - t)\cdot[S_1x_s - c(x_s) - f] + (1 - td)\cdot[(F_1 - F_0)x_f - c(x_f)],$$

$$(4.23)\quad var(\Pi_1|I_0)_{\text{after-tax}} = (1 - t)^2\cdot var(\pi_s|I_0) + (1 - td)^2\cdot var(\pi_f|I_0) + 2(1 - t)(1 - td)\cdot cov(\pi_s,\pi_f|I_0) = (1 - t)^2\cdot x_s^2\cdot\sigma_\epsilon^2 + (1 - td)^2\cdot x_f^2\cdot\sigma_\delta^2 + 2(1 - t)(1 - td)\cdot\sigma_{sf}x_sx_f.$$

[56] A detailed explanation for deriving these results is found in Appendix 4.2.

From (4.20) and (4.21), and (4.22) and (4.23), an increase of magnitude t in the marginal tax rate will decrease positive cash profits by t percent and increase negative expected futures profits by t·d percent.[57] This specific tax scheme has a "semi-full offset provision."[58] Thus, the impact of taxes on total profits is indeterminate. The impact of increased taxes is to reduce the variance of total profits as shown by expression (4.23).

These qualitative results can be more accurately evaluated by finding the effects of a marginal increase in the proportional tax rate on the mean and variance of profits. For this purpose, (4.22) and (4.23) are, respectively, differentiated with respect to t:

$$(4.24) \quad \partial E(\Pi_1|I_0)/\partial t = -E(\pi_s|I_0) - d\cdot E(\pi_f|I_0) = ?, \text{ and}$$

$$(4.25) \quad \partial \mathrm{var}(\Pi_1|I_0)/\partial t = -2(1 - t)\cdot \mathrm{var}(\pi_s|I_0) - 2(1 - td)d\cdot \mathrm{var}(\pi_f|I_0) + 2(2td - d - 1)\cdot \mathrm{cov}(\pi_s,\pi_f|I_0) < 0.$$

As (4.24) and (4.25) show, the impact of an increase in the tax rate on

[57] In the beginning, it is assumed that a cattle feeder makes positive profits in the cash market and negative profits in the futures markets. This would occur when prices are increasing during the feeding period.

[58] In brief, a "full (loss) offset provision" refers to a tax scheme which any loss can be fully deducted, presumably from other income owned by taxpayers. In such a case, both positive and negative incomes are reduced by the same marginal tax rate. A "semi-full offset provision" refers to a tax scheme in which deductions of losses are discounted by deduction level d and losses are therefore not fully deducted. The full offset provision can be viewed as a special case of the semi-full offset provision. That is, when d = 1, the full offset provision is the same as the semi-full offset provision.

the mean of cash profits is negative while it is positive for the mean of futures profits. Thus, the impact of an increase in the marginal tax rate on the mean of total profits is indeterminate. However, the variance of both cash and futures profits are reduced, and thereby the variance of total profits is reduced by the increased tax rate.

Using the comparative-static analysis developed above, the following marginal impact of the level of deduction can be identified:

(4.26) $\partial E(\Pi_1 | I_0)/\partial d = -t \cdot E(\pi_f | I_0) > 0$, and

(4.27) $\partial var(\Pi_1 | I_0)/\partial d = -2(1 - td) \cdot t \cdot var(\pi_f | I_0) + 2(t^2 - t) \cdot cov(\pi_s, \pi_f | I_0) < 0.$

While a change in deduction level has no impact on cash profits, it will increase negative expected futures profits by $t \cdot d$. Any negative profit is made less negative by the deduction. Thus, the mean of total profits increases. Also, increased deductibility will reduce the variance of futures profits and the variance of total profits.

These results are demonstrated in Figures 4.1, 4.2 and 4.3. In Figure 4.1, the distributions of before-tax futures, total, and cash profits are shown by the solid lines from left to right. The dashed lines show the corresponding distributions of after-tax profits. For exposition purposes, these expected profits are assumed to be normally distributed. In addition, it is assumed that the amount of positive expected cash profits is the same as the negative futures profits in absolute value and thereby the mean of total profits is scaled to zero.

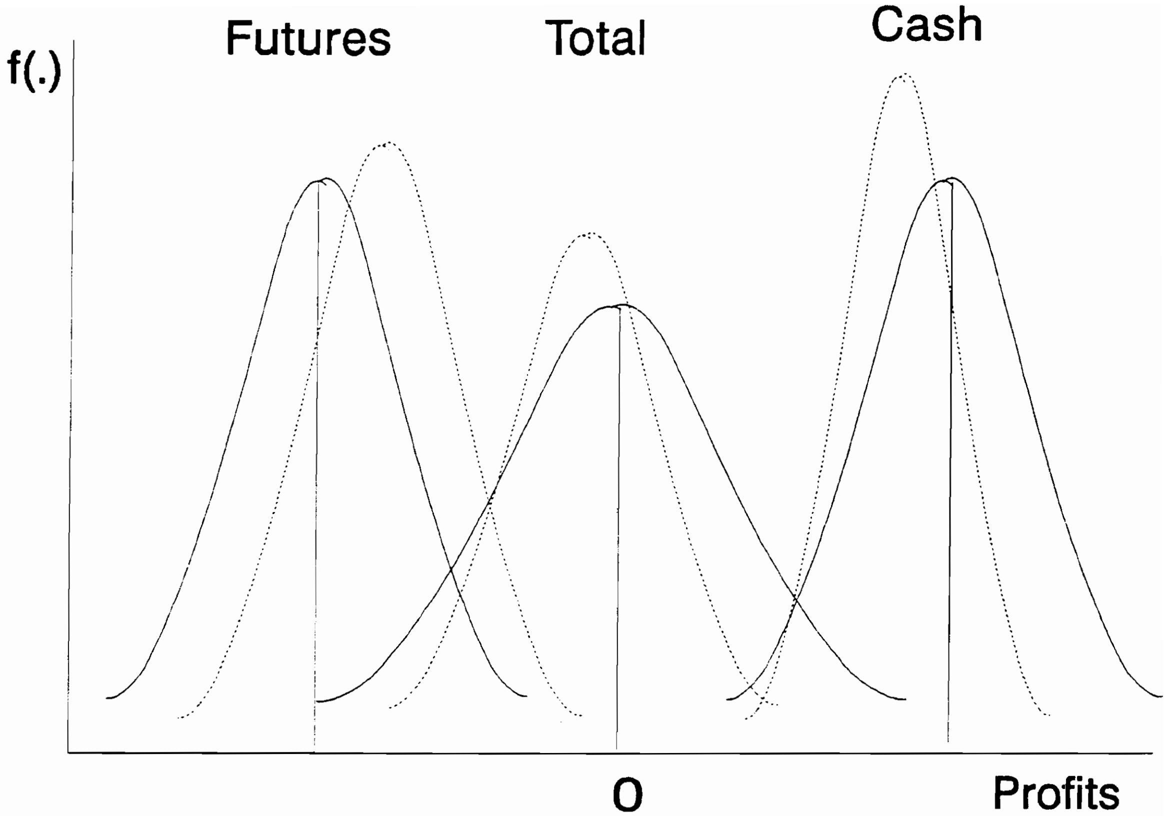

Figure 4.1 Distributions of Before-Tax vs. After-Tax Expected Profits

This figure shows that increases in the marginal tax rate and/or deduction level will reduce the variance of both cash and futures profits and thereby will reduce total variance. The expected mean of total profits is slightly reduced since a decrease in positive cash profits dominates a decrease in negative futures profits so long as $d < 1$. When any futures losses are fully deductible, i.e., $d = 1$, the mean of total profits would be unchanged.

Figure 4.2 demonstrates the impact of an increase in the marginal tax rate for a given deduction level. Assume that the solid lines stand for current after-tax profits and the dashed lines for the changed profits after an increase in the marginal tax rate. An increase in the marginal tax rate results in more peaked distributions, which implies decreases in the variances of profits. The mean of total profits is reduced so long as the futures losses are not fully deducted, or $d < 1$.

In Figure 4.3, the solid lines are the distributions of current after-tax profits and the dashed lines are those of profits after an increase in deduction level for a given marginal tax rate. There is no impact on the mean or variance in the cash markets. The expected value of futures profits increases and its variance decreases. In aggregate, the expectation of total profits is increased and the variance is reduced.

From (4.25) and (4.27), and as shown in Figures 4.2 and 4.3, increases in both tax rates and deduction levels decrease the variance of total profits. It is well known that a risk-averse producer will expand output in response to a reduction in variances of profits when the expected profits remain unchanged or increase (Sandmo, 1971; Ishii, 1977).

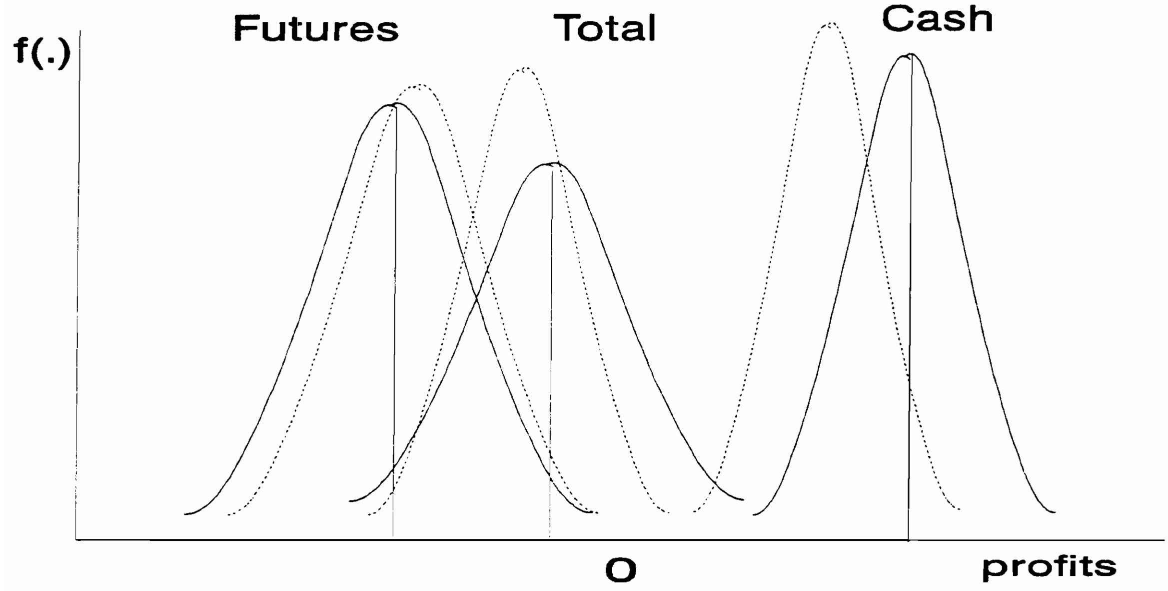

Figure 4.2 Increase in Tax Rate Given a Deduction Level

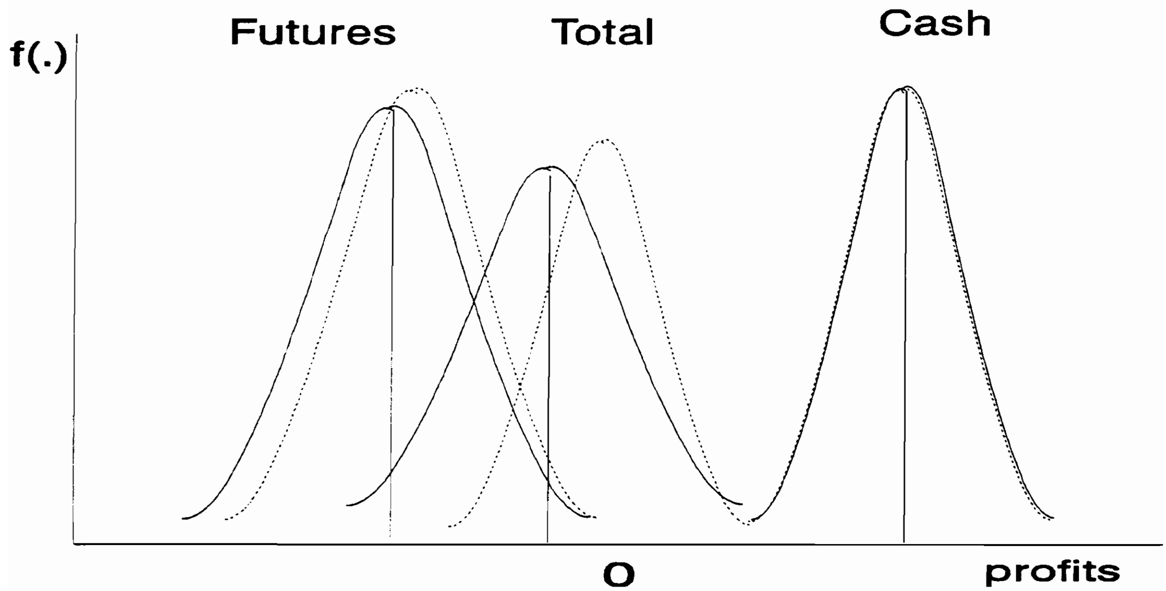

Figure 4.3 Increase in Deduction Level Given a Tax Rate

Similar conclusions are drawn by comparing the implicit model without taxes and deductions with the model featuring those tax parameters. Assuming a constant degree of risk aversion λ[59], the risk in the model without tax parameters is apparently discounted by the tax parameters.

As a result, under this kind of semi-full provision, the impact of tax rates on the cash position is shown to be ambiguous. An increase in the marginal tax rate reduces the mean of cash profits but also decreases the variance of cash profits.[60] However, an increase in tax rates will increase futures positions because it will increase the (negative) futures profits and reduce the variance of those profits. An increase in deduction level for futures losses will have no impact on cash positions because the mean and variances of cash profits are not influenced by a change in the deduction level. However, increased deductions will increase the expected futures profits and reduce the variance of futures profits, and thereby cause the hedger to increase futures positions. In

[59] According to Robison and Barry (1987), λ may not stay constant since introducing taxes will shift the EV set. In such a case, it is necessary to consider the indirect effect as well as the direct effect. The direct effect is the output responses from changes in taxes. And the indirect effect is the effect of taxes on the output levels, which causes λ to vary and then affect the output levels. Mathematically,

$$dq/dT = (\partial q/\partial T)_{\lambda \text{constant}} + (\partial q/\partial \lambda)\cdot(\partial \lambda/\partial T),$$

where q = output, T = taxes, and λ = degree of risk aversion. In our case, $(\partial \lambda/\partial T) = 0$ is assumed, implying a "constant" risk aversion.

[60] With full offset provisions, an increase in the marginal tax rate reduces both the expected value and the variance of profits. Assuming risk aversion, which is commonly the case, these simultaneous effects have conflicting impacts on the producer's output, thereby rendering the producer's response to taxation indeterminate (Katz, (1983), p. 453; Robison and Barry, 1987, pp. 95-98).

such a case, it is unclear whether cash positions will increase along with the increased futures positions due to the nature of the analysis presented here.

4.5 Comparative-Static Analysis of a Short-Run Response to Revenue-Neutral Tax Policy

The short-run effects on x_s and x_f of policies defined by $(\partial d/\partial t)_{dE(G)=0} = -[(\pi_s + d\cdot\pi_f)/(t\cdot\pi_f)] = -(1/t)[(\pi_s^*/\pi_f^*) + d]$ can be evaluated by applying a Slutsky equation of the form:

$$(4.28) \quad (\partial a/\partial t)_{dE(G)=0} = (\partial a/\partial t) + (\partial a/\partial d)(\partial d/\partial t)_{dE(G)=0},$$

where $a = x_s$ or x_f, evaluated at $x_s = x_s^*$ and $x_f = x_f^*$. The first term on the right-hand side is the direct (income) effect on cash or futures position of a change in marginal tax rate. The second term is the indirect (substitution) effect induced by the change in deduction level that results from constraining the expected level of receipts in accordance with policies defined by $(\partial d/\partial t)_{dE(G)=0}$.

Substituting $(\partial x_s/\partial t)$, $(\partial x_s/\partial d)$ and $(\partial d/\partial t)_{dE(G)=0}$ into $(\partial x_s/\partial t)_{dE(G)=0}$ yields:[61]

$$(4.29) \quad (\partial x_s/\partial t)_{dE(G)=0} = (\partial x_s/\partial t) + (\partial x_s/\partial d)(\partial d/\partial t)_{dE(G)=0}$$

[61] A detailed explanation for deriving these results is found in Appendix 4.3.

$$= (1/|H|)\lambda\{c''(x_f)[\sigma_s^2 x_s - \sigma_{sf} x_f(\pi_s^*/\pi_f^*)] + \lambda(1 - td)\sigma_s^2\sigma_f^2(1 - \rho^2)x_s\}.$$

Then, substituting $(\partial x_f/\partial t)$, $(\partial x_f/\partial d)$ and $(\partial d/\partial t)_{dE(G)=0}$ into $(\partial x_f/\partial t)_{dE(G)=0}$ yields:

$$(4.30) \quad (\partial x_f/\partial t)_{dE(G)=0} = (\partial x_f/\partial t) + (\partial x_f/\partial d)(\partial d/\partial t)_{dE(G)=0}$$
$$= (1/|H|)\lambda\{c''(x_s)\sigma_{sf}x_s - (\pi_s^*/\pi_f^*)\sigma_f^2 x_f[c''(x_s) - \lambda(1 - t)\sigma_s^2(1 - \rho^2)]\}.$$

Alternatively, the policy can be evaluated to find the impact of deductions on cash and futures positions, as follows:

$$(4.31) \quad (\partial t/\partial d)_{dE(G)=0} = -[(t\cdot\pi_f)/(\pi_s + d\cdot\pi_f)] = -t/[(\pi_s^*/\pi_f^*) + d]$$

The effects on x_s and x_f of policies defined by $(\partial t/\partial d)_{dE(G)=0}$ can be evaluated by applying a Slutsky equation of the form:

$$(4.32) \quad (\partial a/\partial d)_{dE(G)=0} = (\partial a/\partial d) + (\partial a/\partial t)(\partial t/\partial d)_{dE(G)=0},$$

where a can be x_s, x_f or n. Again, the first term on the right-hand side is the direct effect on cash (futures) position of a change in d; the second term is the indirect effect induced by the change in t that results from constraining the expected level of receipts in accordance with policies defined by $(\partial t/\partial d)_{dE(G)=0}$.

First, substituting $(\partial x_s/\partial d)$, $(\partial x_s/\partial t)$ and $(\partial t/\partial d)_{dE(G)=0}$ into $(\partial x_s/\partial d)_{dE(G)=0}$ yields:[62]

$$(4.33)\quad (\partial x_s/\partial d)_{dE(G)=0} = (\partial x_s/\partial d) + (\partial x_s/\partial t)(\partial t/\partial d)_{dE(G)=0}$$
$$= (1/|H|)\lambda t\{c''(x_f)[\sigma_{sf}x_f$$
$$- (\sigma_s^2 x_s + d\cdot\sigma_{sf}x_f)\{1/[(\pi_s^*/\pi_f^*) + d]\}]$$
$$- \lambda(1 - td)\sigma_s^2\sigma_f^2(1 - \rho^2)x_s\{1/[(\pi_s^*/\pi_f^*) + d]\}\}.$$

Second, substituting $(\partial x_f/\partial d)$, $(\partial x_f/\partial t)$ and $(\partial t/\partial d)_{dE(G)=0}$ into $(\partial x_f/\partial d)_{dE(G)=0}$ yields:

$$(4.44)\quad (\partial x_f/\partial d)_{dE(G)=0} = (\partial x_f/\partial d) + (\partial x_f/\partial t)(\partial t/\partial d)_{dE(G)=0}$$
$$= (1/|H|)\lambda t\{c''(x_s)(\sigma_f^2 x_f - (d\cdot\sigma_f^2 x_f + \sigma_{sf}x_s)\cdot$$
$$\{1/[(\pi_s^*/\pi_f^*) + d]\} + \lambda(1 - t)\sigma_s^2\sigma_f^2(1 - \rho^2)x_f]\cdot$$
$$[1 - d\cdot\{1/[(\pi_s^*/\pi_f^*) + d]\}]\}.$$

4.6 Implication of Short-Run Responses to Revenue-Neutral Tax Policy

The signs of comparative-static results in section 4.5 under revenue-neutral tax policy are seemingly indeterminate. As mentioned in section 4.4, these results are based on the initial equilibrium situation, and x_s^* and x_f^* can have positive or negative values. Thus, the impact of such a revenue-neutral taxation on the producer's responses can be examined by considering how such a taxation affects the distribution of

[62] A detailed explanation for deriving these results is found in Appendix 4.4.

profits. First, disaggregate the producer's after-tax profits into total personal revenue and the government's fiscal receipts. Then, consider the first two central moments of the distribution of the firm's profits:

$$(4.45) \quad E(\Pi_1|I_0)_{\text{after-tax}} = (1 - t)\cdot E(\pi_s|I_0) + (1 - td)\cdot E(\pi_f|I_0)$$
$$= E(\pi_s|I_0) + E(\pi_f|I_0) - [t\cdot E(\pi_s|I_0) + td\cdot E(\pi_f|I_0)]$$
$$= E(\pi_s|I_0) + E(\pi_f|I_0) - E(g|I_0)$$
$$= [S_1x_s - c(x_s) - f] + [(F_1 - F_0)x_f - c(x_f)]$$
$$- E(g|I_0)$$

$$(4.46) \quad \text{var}(\Pi_1|I_0)_{\text{after-tax}} = (1 - t)^2\cdot \text{var}(\pi_s|I_0) + (1 - td)^2\cdot \text{var}(\pi_f|I_0)$$
$$+ 2(1 - t)(1 - td)\cdot \text{cov}(\pi_s,\pi_f|I_0)$$
$$= (1 - t)^2\cdot x_s^2\cdot \sigma_\epsilon^2 + (1 - td)^2\cdot x_f^2\cdot \sigma_\delta^2$$
$$+ 2(1 - t)(1 - td)\cdot \sigma_{sf}x_sx_f.$$

From (4.45), a revenue-tax policy does not affect the means of cash and futures profits and thereby does not affect the mean of total profits. This is because the tax parameters t and d are absorbed into the expected level of fiscal revenues, $E(g|I_0)$. However, as (4.46) shows, increases in the marginal tax rate, t, will reduce the variance of cash and futures profits and thereby the variance of total profits, which is proven in section 4.5.

Figure 4.4 shows the impact of increases in the marginal tax rate under a revenue-neutral tax policy. Assuming normal distributions of futures, total, and cash profits from left to right, the solid lines are for after-tax profits and the dashed lines for the distributions after an

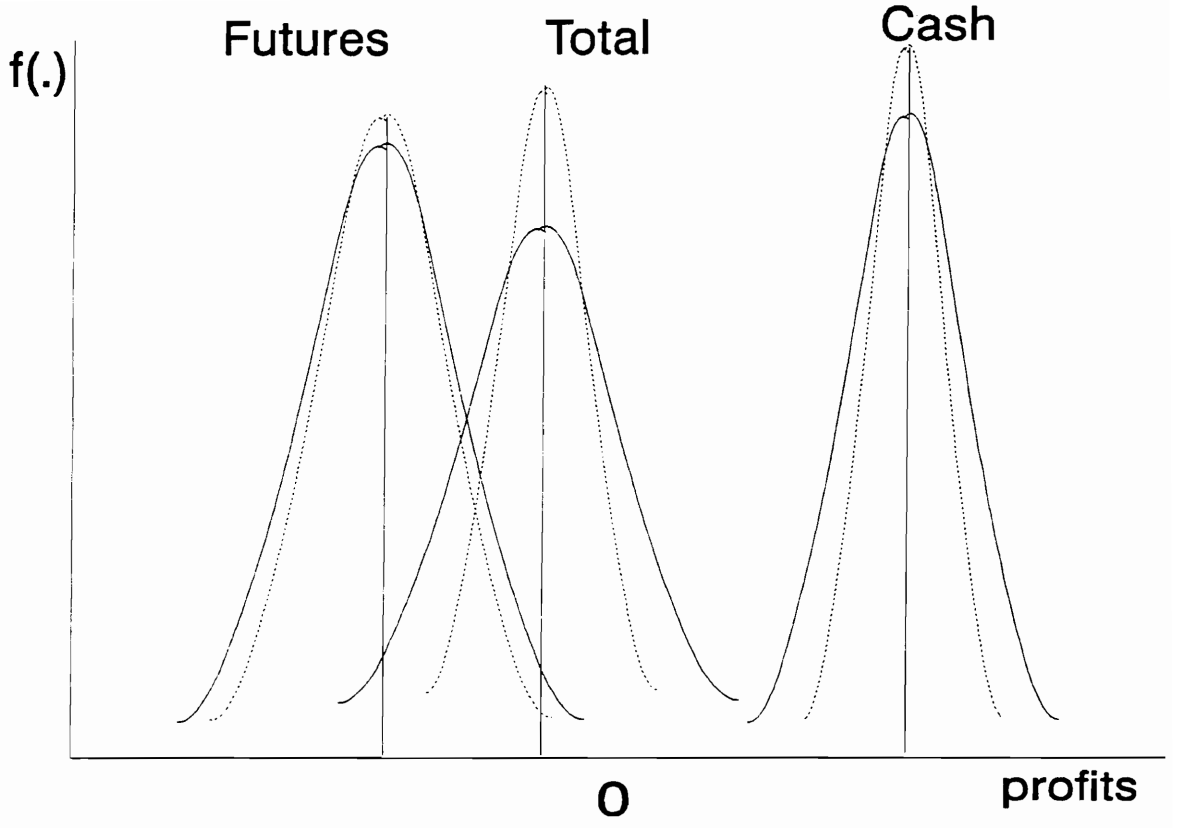

Figure 4.4 Revenue-Neutral Increase in Tax Rate Given a Deduction Level

increase in taxes given a deduction level under a revenue-neutral tax policy. None of the expected levels of the distributions are changed. However, all of the variances are reduced, resulting in more peaked distributions. Note that the magnitude of a percent change in variance of cash profits is larger than that of futures profits. This is because the variance of futures profits is reduced by $(1 - td)^2$ but the variance of cash profits is reduced by $(1 - t)^2$, thereby generating a more peaked distribution.

Now, consider the impact of changes in the level of deduction for futures losses. Increases in the deduction level, d, have no impact on the means of cash and futures profits and, thereby, no impact on the mean of total profits shown in (4.45). As (4.46) implies, however, the impact of increases in the deduction level will have a negative impact on the variance of futures profits and thereby on the variance of total profits. Figure 4.5 demonstrates this impact. The solid lines are the same as those in Figure 4.4. The distribution of cash profits remains unchanged after an increase in the deduction level for a given marginal tax rate. The mean and variance are not altered, as shown by the right-hand dashed line. However, the variance of futures profits is reduced by $(1 - td)^2$ while its mean is unchanged, resulting in a more peaked distribution, as shown by the left-hand dashed line. The resulting distribution of total profits is shown in the middle dashed line, implying its mean is unchanged but its variance is reduced.

Conclusions regarding impacts of tax rates and deduction levels are possible. Note, first, the intuitive result that a risk-averse producer

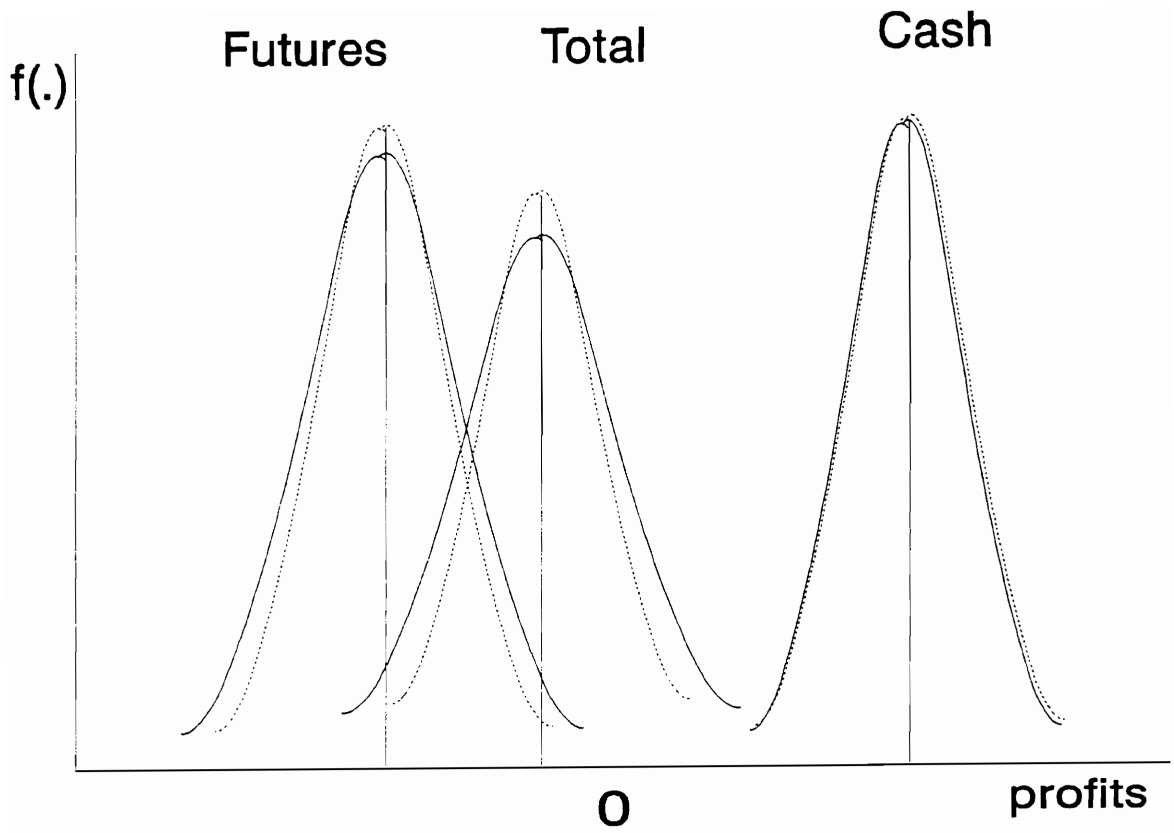

Figure 4.5 Revenue-Neutral Increase in Deduction Level Given a Tax Rate

will increase output in response to a marginal reduction in uncertainty. Increasing the marginal tax rate in a revenue-neutral manner leaves the expected levels of cash, futures and total profits unchanged but reduces the variances of the profits. This will cause a producer/hedger to expand both cash and futures positions.

An increase in the deduction level for futures losses has no impact on the expected levels of cash, futures and thus total profits. The variance of futures profits is reduced while that of cash profits remains unchanged. Thus, when the deduction level increases, a producer/hedger is expected to increase futures positions. Cash positions are expected to remain unchanged. An interesting question (though not answered here) is whether under joint determination of production and hedging decisions, cash positions would still remain unaltered if futures positions are in fact increasing.

4.7 Effect of Revenue-Neutral Tax Policy on Expected Tax Receipts

The effect of a revenue-neutral tax policy constraint on the mean of the equilibrium-level tax receipts is derived in this section.[63] To investigate this qualitative result, it is useful to evaluate the mean of treasury receipts at something other than the equilibrium levels. Before proceeding with the effect of a revenue-neutral tax policy on the mean of

[63] Due to the structure of the problem, the effect of "profit-neutral" tax policy on the expected level of "tax receipts" are identical to that of "revenue-neutral" tax policy. The effect of "profit-neutral" tax policy on the mean of "profits" is expected to be opposite in sign to the effect of "revenue-neutral" tax policy on the mean of "tax receipts" since profits and tax receipts move in opposite directions.

treasury receipts, therefore, consider first the "short-run" effect which does not account for the endogeneity of the number of producers. The number of producers n is assumed to be fixed. Let $E(g|I_0)$ denote the mean of tax receipts for an individual producer and consider the first central moment of the distribution of after-tax receipts, without revenue neutrality:

$$(4.47) \quad E(g|I_0)_{\text{after-tax}} = t\cdot E(\pi_s|I_0) + td\cdot E(\pi_f|I_0)$$
$$= t\cdot[S_1x_s - c(x_s) - f] + td\cdot[(F_1 - F_0)x_f - c(x_f)].$$

From (4.47), the impact of increased tax rates on tax receipts is ambiguous because the increase in the marginal tax rate will increase the positive cash portion and decrease the negative futures portion of the receipts. A negative receipt in the futures markets is made more negative. Thus, the impact of increased tax rates on tax receipt is indeterminate. Using partial derivatives, these results are confirmed:

$$(4.48) \quad \partial E(g|I_0)/\partial t = E(\pi_s|I_0) + d\cdot E(\pi_f|I_0).$$

This result holds for the case of considering the aggregate treasury receipts which is obtained by multiplying the constant or number of producers to (4.47) and (4.48).

Next, consider the short-run effect of tax rates on the mean of tax receipts under a revenue-neutral tax policy. It is redundant to examine this effect because the revenue-neutral scheme itself constrains the expected level of fiscal revenues to remain unchanged. In addition, this effect on the mean of aggregate tax receipts is also unchanged under such a tax scheme.

The impact of deduction levels under such a constrained tax policy on the equilibrium level of treasury receipts can be examined. To evaluate this qualitative result, the mean of tax receipts is examined. First, consider the short-run case without revenue-neutrality. Partially differentiate (4.47) with respect to d, and the result is:

$$(4.49) \quad \partial E(g|I_0)/\partial d \qquad = t \cdot E(\pi_f|I_0)$$

From (4.49), the short-run impact of an increase in the deductibility of futures losses without revenue neutrality is expected to be negative. This holds for the mean of aggregate tax receipts which is obtained by multiplying the constant by (4.49). Under a revenue-neutral tax scheme, the corresponding short-run effects on the mean of individual and aggregate tax receipts become zero. When there is a change in the deduction level, the marginal tax rate is adjusted such that the expected levels of individual and aggregate tax revenues remain unchanged.

4.8 Risk-Sharing Aspect of Revenue-Neutral Tax Policy

By examining the distributions of profits and net fiscal receipts, another interpretation can be given to the revenue-neutral tax policy. By the definition of a revenue-neutral tax scheme, the initial level of expected receipts (thus, expected profits) does not change as a result of the policy. However, the variances in receipts and profits move in opposite directions. This feature of such a tax policy provides changes in taxation (here, t and d) which can shift the shares of the burden of the variance between the government and the cattle-feeding industry. In addition, in the absence of changes in the expected values of profits and tax receipts, changes in the degree of "risk-sharing" would be expected to cause changes in decisions on production and on hedging. The following sections examine the expected changes in the distributions of profits and tax receipts for a given change in tax rates and deduction levels.

4.8.1 Relation between Profits and Receipts For Change in Tax Rate

It is easily shown that the mean of profits and the mean of tax payments move in opposite directions as a result of an increase in the tax rates because:

$$(4.50)\quad \Pi = (1 - t)\cdot\pi_s + (1 - td)\cdot\pi_f, \text{ and}$$

$$(4.51)\quad g = t\cdot\pi_s + td\cdot\pi_f.$$

Taking expectations of profits and tax receipts, we obtain:

$$(4.52)\quad E(\Pi_1|I_0) = (1 - t)\cdot E(\pi_s|I_0) + (1 - td)\cdot E(\pi_f|I_0), \text{ and}$$

$$(4.53)\quad E(g|I_0) = t\cdot E(\pi_s|I_0) + td\cdot E(\pi_f|I_0).$$

Partially differentiating these with respect to t, we obtain:

$$(4.54)\quad \partial E(\Pi_1|I_0)/\partial t = -E(\pi_s|I_0) - d\cdot E(\pi_f|I_0), \text{ and}$$

$$(4.55)\quad \partial E(g|I_0)/\partial t = E(\pi_s|I_0) + d\cdot E(\pi_f|I_0).$$

Thus, given the increase in the marginal tax rate, the change in the mean of profits is shown to be opposite in sign to the change in the mean of tax receipts:

$$(4.56)\quad \partial E(\Pi_1|I_0)/\partial t = -\partial E(g|I_0)/\partial t.$$

Although the signs of $\partial E(\Pi_1|I_0)/\partial t$ and $\partial E(g|I_0)/\partial t$ are individually ambiguous, they move in opposite directions.

Now, consider the relationship between the variance of profits and the variance of tax receipts. Let $var(\Pi_1|I_0)$ and $var(g|I_0)$ denote, respectively, the variance of profit and the variance of tax receipts:

$$(4.57)\quad var(\Pi_1|I_0) = (1 - t)^2\cdot var(\pi_s|I_0) + (1 - td)^2\cdot var(\pi_f|I_0) + 2(1 - t)(1 - td)\cdot cov(\pi_s,\pi_f|I_0), \text{ and}$$

$$(4.58)\quad var(g|I_0) = t^2\cdot var_g(\pi_s|I_0) + t^2d^2\cdot var_g(\pi_f|I_0) + 2t^2d\cdot cov_g(\pi_s,\pi_f|I_0).$$

The variance of profits and the variance of tax payments move in opposite directions in both the cash and futures markets as a result of increased tax rates because:

$$(4.59)\quad \partial var(\Pi_1|I_0)/\partial t = -2(1 - t)\cdot var(\pi_s|I_0) - 2(1 - td)d\cdot var(\pi_f|I_0) + 2(2td - d - 1)\cdot cov(\pi_s,\pi_f|I_0), \text{ and}$$

$$(4.60)\quad \partial var(g|I_0)/\partial t = 2t\cdot var_g(\pi_s|I_0) + 2td^2\cdot var_g(\pi_f|I_0) + 2\cdot 2td\cdot cov_g(\pi_s,\pi_f|I_0).$$

Thus, the relationship is expressed by:

$$(4.61)\quad \partial var(\Pi_1|I_0)/\partial t = var_g(\pi_s|I_0)[(t - 1)/t] + var_g(\pi_f|I_0)[(td - 1)/td] + cov_g(\pi_s,\pi_f|I_0)[(2td - 1)/2td] \approx ratio(t,d)\cdot(\partial var(g|I_0)/\partial t).$$

This result implies that imposing a revenue neutrality causes fixed expected levels of profits and tax receipts, decreased variance of profits, and increased variance of tax receipts. Thus, a revenue-neutral change in tax rate shifts the shares of the risk in profits between the producer and the government. In other words, the risk in profits faced by the producer is transferred to the government sector. The magnitude of this transfer depends upon the marginal tax rate and deduction level given the covariance matrix of expected cash and futures prices.

4.8.2 Relation between Profits and Receipts For A Change in Deduction Level

With respect to changes in the deduction level for futures losses, the mean of profits and the mean of tax payments move in opposite directions because:

$$(4.62)\quad \partial E(\Pi_1|I_0)/\partial d = -t\cdot E(\pi_f|I_0), \text{ and}$$

$$(4.63)\quad \partial E(g|I_0)/\partial d = t\cdot E(\pi_f|I_0).$$

The relationship is given by:

$$(4.64)\quad \partial E(\Pi_1|I_0)/\partial d = -\partial E(g|I_0)/\partial d.$$

For the changes in deduction level, the variance of profits and the variance of tax payments move in opposite directions in both the cash and futures markets because:

$$(4.65)\quad \partial \mathrm{var}(\Pi_1|I_0)/\partial d = -2(1 - td)\cdot t\cdot \mathrm{var}(\pi_f|I_0) + 2(t^2 - t)\cdot \mathrm{cov}(\pi_s,\pi_f|I_0),$$

$$(4.66)\quad \partial \mathrm{var}(g|I_0)/\partial d = 2t^2 d\cdot \mathrm{var}_g(\pi_f|I_0) + 2t^2\cdot \mathrm{cov}_g(\pi_s,\pi_f|I_0).$$

The relationship is expressed by:

$$(4.67)\quad \partial \mathrm{var}(\Pi_1|I_0)/\partial t = \mathrm{var}_g(\pi_f|I_0)[(td - 1)/td] + \mathrm{cov}_g(\pi_s,\pi_f|I_0)[(t - 1)/t] \approx \mathrm{ratio}(t,d)\cdot(\partial \mathrm{var}(g|I_0)/\partial t).$$

Under a revenue-neutral tax policy, an increase in the deduction level causes the variance of profits to decrease and the variance of tax receipts to increase. Thus, the transfer of risk in profits is again from the producer to the government sector, which was the same conclusion as with the case for an increase in the marginal tax rate.

4.9 Chapter Summary

This chapter has presented comparative statics which analyze the marginal impacts of changes in tax rate and/or deduction level for futures losses on the optimal cash and futures positions for the case of positive cash profits and negative futures profits. A revenue-neutral tax scheme was introduced to examine the impacts of constraining expected tax revenue to a fixed level. In addition, these comparative static analyses identified the relationship between (1) changes in the expected levels of both profits and tax revenue, and (2) changes in the variance of both profits and tax revenue for a given change in tax parameters.

The following highlighted results are presented according to the analysis performed in this chapter. First, under a semi-full provision (a tax scheme in which deductions of losses are discounted by tax rate multiplied by deduction level and losses are therefore not fully deducted), the impact of marginal tax rates on the optimal cash position is shown to be ambiguous. An increase in the tax rate reduces the mean of cash profits and decreases the variance of cash profits. However, an increase in the tax rate will increase futures positions because it will improve any negative futures profits and reduce the variance of those profits. An increase in the deduction level for futures losses will have no impact on cash positions because the mean and variances of cash profits are not influenced by a change in deduction levels for futures losses. However, increased deductions will increase the expected futures profits and reduce the variance of futures profits, and thereby cause the hedger to increase futures positions.

Second, revenue-neutral marginal changes in the optimal positions were analyzed. An increase in the marginal tax rate in a tax revenue-neutral manner leaves the expected levels of cash, futures, and total profits unchanged, but reduces the variances of the profits. This will cause a producer/hedger to increase both cash and futures positions. An increase in the deduction level has no impact on the expected levels of a hedged cash program and therefore does not influence total profits. The variance of futures profits is reduced while the variance of cash profits remains unchanged. Thus, a producer/hedger is expected to increase futures positions and to leave cash positions unchanged.

Third, the impact of marginal tax rates on the mean of tax revenues is ambiguous because the increase in the tax rate will increase positive cash receipts and decrease any negative futures receipts. A negative receipt in the futures markets is made more negative. It is redundant to examine the corresponding impact under a revenue-neutral tax policy since the revenue-neutral scheme itself constrains the expected level of fiscal revenues to remain unchanged. The impact of deduction levels on the tax revenues is expected to be negative. A negative futures market receipt becomes more negative. Under a revenue-neutral tax scheme, the corresponding effects on the mean of tax revenues become zero. When there is a change in the deduction level, the marginal tax rate is adjusted such that the expected levels of tax revenues remain unchanged.

Finally, risk-sharing aspects of revenue-neutral tax policy were considered. The mean of total profits and the mean of tax payments move in opposite directions as a result of an increase in the marginal tax rate. In such a case, the variance of total profits and the variance of tax payments also move in opposite directions in both the cash and futures markets. This implies that imposing revenue neutrality causes fixed expected levels of profits and tax receipts, decreased variance of profits, and increased variance of tax receipts. Thus, a revenue-neutral change in the tax rate would transfer part of the risk in profits faced by producers to the government sector in the form of more risky tax revenue flows.

Analogous to the case of increased tax rates, the mean of profits and the mean of tax payments move in opposite directions with respect to changes in the deduction level for futures losses. This holds as well for the variances of profits and tax payments. Under a revenue-neutral tax policy, it is expected that an increase in the deduction level for futures losses causes the variance of profits to decrease and the variance of tax receipts to increase. Thus, the transfer of risk is again from the producer to the government sector.

CHAPTER FIVE : PROCEDURES FOR EMPIRICAL ANALYSIS

5.1 Introduction

This chapter provides an introduction to the data used in an analysis to test the hypotheses of Chapter 1 and the theoretical results of Chapters 3 and 4. The procedure used to compute costs for feeding is discussed. Based on the conceptual framework developed in the previous chapters, this chapter turns to empirical analysis to the extent possible given available data. The following specific issues will be discussed:

(1) the estimation of forecasts of expected cash and futures prices;

(2) the dynamic adjustments of variances and covariance of cash and futures prices;

(3) the operational objective functions used in the calculation of optimal cash and futures positions and equilibrium prices;

(4) the specification of aggregate consumers' demand for fed cattle; and

(5) the process to calculate optimal cash and futures positions, and equilibrium cash and futures prices.

5.2 Data Sources and Calculation of Feeding Costs

To analyze the optimal cash and futures positions along with the equilibrium cash and futures prices, variable costs for feeding over time are computed. All production costs are assumed to be known and fixed at the time of placements of feeder cattle on feed. Also, feedlots are assumed to operate a four-month feeding program which involves feeding feeder steers weighing 750 pounds into fed steers weighing 1,150 pounds.

This scenario generally represents Great Plains (Texas-Oklahoma-New Mexico) custom cattle feeding.

The feed ration used for each animal is 43 bushels of corn, 0.16 tons of soybean meal, and 640 pounds of alfalfa hay.[64] Weekly corn and soybean meal prices are obtained by averaging the midpoints of daily price ranges. Alfalfa hay prices are held constant during each month, then updated monthly. The specific data for the feed costs are as follows:

(1) corn prices --- No. 2, yellow, "truck bid" spot F.O.B. elevator, daily quoted at Omaha, $ per bushel;

(2) soybean meal--- 44% "rail bid" daily quoted at Decatur, $ per ton; and

(3) alfalfa hay --- average price received by farmers plus $30 per ton handling and transportation to feedlots, $ per head.

Other variable costs are obtained from monthly issues of **Livestock and Poultry Situation and Outlook Report.**[65] These costs are held constant during each month, then updated monthly. They are comprised of the following expenses:

---

[64] The feed ration and expense items do not necessarily coincide with experience of individual feedlots. In **Livestock and Poultry Situation and Outlook** from the USDA, Choice feeder steers are assumed to gain 500 pounds in 180 days at 2.8 lb. per day with a feed conversion of 8.4 lb. per pound of gain.

[65] The report represents only what expenses would be if all selected items were paid for during the period indicated. Across different feedlots, expenses and prices for management, production, and locality of operation should be adjusted.

(1) feed handling and management charge;

(2) veterinary and medicine;

(3) interest on feeder cattle and feed;

(4) death loss as a percentage of the cost of the 750-pound feeder steer;

(5) marketing expenses; and

(6) transportation to the feedlot (300 miles) using a 44,000-pound haul and a commission fee, which is associated with the purchase of feeder cattle.

When cattle feeders place animals on feed, the variable costs (VAC) on a per hundredweight basis can be calculated weekly using the following formula:

$$VAC = \frac{(1 + TB6MN)[CAFC \cdot 7.5 + CACORN \cdot 43 + CASM \cdot 0.16 + CAHAY + OVC]}{11.04^{66}}$$

where

TB6MN = U.S. Treasury Bills' yields on 6-month issue per annum, percent;

CAFC = weekly average feeder cattle price, $/cwt.;

CACORN = weekly average cash corn price, $/bu.;

CASM = weekly average soybean meal price, $/ton;

CAHAY = monthly average cash alfalfa hay cost plus $30/ton handling and transportation expenses, $/head; and

OVC = other variable costs defined above.

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[66] Sale weight is assumed to be 1,104 pound (1,150 less 4-percent shrink).

Note that commission fees, interest on margin money, and futures transaction costs are not included with the costs of feeding operation.[67] In order to approximate capital charges, an interest rate is multiplied by the actual costs of the feeder animal and feed. This rate is an annual cost of borrowing capital, adjusted for time.

This formula can be modified to determine the total costs (variable and fixed costs) by including fixed costs with the other variable costs. Assuming a 15-year expected life of the feedlot and 30,000 head capacity, the capital investment of facilities and loan amortization results in a fixed overhead cost of $5.77 per head (Leuthold and Mokler, 1979).[68]

Daily futures prices for live cattle were obtained from either the **Chicago Mercantile Exchange Year Book** or the **Wall Street Journal**. The cash fed cattle price represents the weekly average for 900- to 1,100-pound Choice slaughter steers, direct Kansas trade. The cash feeder-cattle price is the weekly average price for 700- to 800-pound Choice feeder steers at Amarillo. Both of these latter prices are obtained from weekly issues of **Livestock, Meat and Wool Market News---A Weekly Summary**

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[67] In the calculation of optimal positions and equilibrium prices, it is needed to obtain marginal trading costs in futures markets. As an approximation, $25.00 per contract is used to cover commission fees and other transaction costs. However, the interest on margin requirement is not included in the calculation of futures trading costs.

[68] In their study, the authors simulated a cattle feedlot typical of the Midwest and examined 234 feeding periods during 1972-1976. The study used the data for the nonfeed cost items which were taken from either **Beef Cattle Feeding in Iowa 1974: Evaluation of Feedlot Systems**, Iowa State University, or **Farm Management Manual**, issues for 1973-1977, University of Illinois,.

and Statistics.

Trader positions data represent the grouped trading activities for large (reporting) hedgers and speculators. By definition, the large traders refer to those with more than 100 open contracts of a specific commodity. These large traders are responsible for reporting their trading records to the Commodity Futures Trading Commission (CFTC). Each hedging and speculative account is divided into long and short positions. These data are used to construct several variables measuring trading activity in each group. These variables include weekly averages by group and the number in each group each Wednesday for all futures contracts being traded and for the particular futures contract that would be used to hedge the cattle.

5.3 Estimation of Forecasts of Expected Cash and Futures Prices

Using the formulation of the portfolio model in Chapter 3, the intent is to derive a sequence of optimal cash and hedging positions that could have been adopted by a cattle producer/hedger. For cattle producers considering a portfolio of futures for hedging purposes, an important question is how to obtain estimates of the parameters needed to compute the optimal futures position. These estimates must be based on information available at the time the decision is to being made (Peck, 1975; p. 413). The analysis herein is intended to directly estimate the optimal cash and futures positions using observable parameters and estimations from cash and futures series.[69] This approach requires successive forecasts of the expected cash and futures prices during the delivery month.

First, consider the estimates of expected cash price. It may well be that the current futures quote is the best widely available, unbiased predictor of cash price in the delivery month. Thus, current futures prices are forecasts that could be used by cattle producers in making both production and hedging decisions. Thus, the estimates of expected cash prices for period t=1 given information at t=0 are:

$$E(S_1 | I_0) = F_0$$

where F_0 is the current (t=0) estimate of price for t=1.

The choice of which contract to use for current futures prices is problematic. The contracts for live cattle futures expire every other month. The live cattle contracts on the Chicago Mercantile Exchange (CME) are traded for the months of February, April, June, August, October and December. The pertinent futures contracts by time of placement are presented in Table 5.1.

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[69] Anderson and Danthine (1980) suggested two approaches to this question; (1) direct statistical estimation using cash and futures series; and (2) computation of parameters from relations estimated for the cash prices alone. The authors used the former approach using quite rough statistical estimates.

Table 5.1 Pertinent Futures Contracts for Placement of Cattle During Specific Calendar Periods

Placement	Contract
DEC. 3rd Week - FEB. 2nd Week	JUN.
FEB. 3rd Week - APR. 2nd Week	AUG.
APR. 3rd Week - JUN. 2nd Week	OCT.
JUN. 3rd Week - AUG. 2nd Week	DEC.
AUG. 3rd Week - OCT. 2nd Week	FEB.
OCT. 3rd Week - DEC. 2nd Week	APR.

Assume, then, that feeder cattle are placed into feedlots during the period from the third week of December, 1984 to the second week of February, 1985. The closing quotes of the June, 1985 futures contract are used to calculate expected margins for cattle placed during the period week 3 of December through week 2 of February.

Second, consider the forecasts of expected futures prices.[70] As a

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[70] An alternative set of forecasts for futures, or F_1 based on the supply-demand conditions of futures markets was derived. Futures prices during the delivery month were regressed on the positions (open interest) taken by four different large trader groups when the hedges are placed: long commercial position (LONGC); short commercial position (SHORTC); long non-commercial position (LONGN); and short non-commercial position (SHORTN). To incorporate the supply-demand conditions in the cash market, this model was supplemented by adding as independent variables the placements of feeder cattle on feed (PLACE) and the shipment or marketings of fed cattle (SHIP) during the week when the production decision is made. The following equation is then estimated by OLS:

$$F_1 = f(\text{LONGC, SHORTC, LONGN, SHORTN, PLACE, SHIP}).$$

The OLS estimation results in:

approximation, assume that the cash price and the futures price coincide in the delivery period; that is, the expected expiration price of the futures contract maturing in period t = 1, $E(F_1)$, is assumed equal to the cash price expectation for t = 1 held in the same initial period, t = 0.[71] In such a case, the producer could eliminate all his uncertainty about expected returns caused by using a forecast price in his production decision. This could be accomplished by hedging all of his output. Eliminating this residual uncertainty may come at some cost or gain if the current futures prices F_0 does not equal its forecasted expiration value $E(F_1)$, however. Considering this possibility, the analysis can be generalized by relaxing the assumption that cash and futures prices

---

$$
\begin{aligned}
F_1 = \underset{(31.12)}{63.721463} &- \underset{(-8.20)}{0.000853}\cdot LONGC + \underset{(1.23)}{0.000073}\cdot SHORTC \\
&+ \underset{(0.72)}{0.000056}\cdot LONGN + \underset{(5.08)}{0.000699}\cdot SHORTN \\
&+ \underset{(2.23)}{0.000018}\cdot PLACE - \underset{(-0.38)}{0.000005}\cdot SHIP
\end{aligned}
$$

$R^2 = 0.43$ $\quad F$ value = 24.72

where the numbers in parenthesis are t values.

The data for PLACE and SHIP are obtained from the **Cattle Fax** series. The **Cattle Fax** series is a subset of all placements of cattle on feed and marketings of fed cattle in the United States. **Cattle Fax** member feedlots hold about 24% of total U.S. feeding capacity (Koontz, 1985; p. 37). In this research, placement and marketing decisions in the entire cattle feeding sector are represented by scaling the **Cattle Fax** data to match nationwide level.

[71] The assumption that cash and futures prices converge in the delivery month obviates the need for a second forecast (for futures) even though both a cash and a futures market are considered in the analysis.

converge in the delivery period. At a minimum, the cash and futures prices in a delivery month would be expected to differ by the physical costs of delivery, with some discount (or gain) for quality differences, etc. The difference between these two prices in the delivery period, the basis, or B_1, is the new variable which must be considered. The presence of the basis implies that there is really only one price level involved, i.e., there are estimations for the cash price alone. In such a case, the estimates of expected futures prices for t=1 become:

$$E(F_1|I_0) = E(S_1|I_0) + E(B_1|I_0) = F_0 + E(B_1|I_0),$$

assuming constant basis or no basis risk,

$$E(F_1|I_0) = F_0 + E(B_1|I_0) = F_0 + S_1 - F_1.$$

In this case, the basis must be the market-determined price differential between an expiring futures and the cash price. The difference between actual expiring futures and cash prices during the delivery month was used as the measure of the basis. The interesting feature of these forecasts is that they could be available to the cattle industry. Thus, they are forecasts which could have been used to make production, marketing, and hedging decisions.

The estimation of basis is simplified by averaging delivery basis ($S_1 - F_1$) for each contract over the entire 1983-87 sample period. The averaged basis levels for the corresponding contracts are added to the current quotes of futures prices to provide the estimates of expected futures prices for the delivery period. Table 5.2 displays these summary

Table 5.2 Summary Statistics for Basis Estimates For Four-Month Distant Contracts in Dollars per Cwt.

Contract	FEB.	APR.	JUN.	AUG.	OCT.	DEC.
MEAN	-1.92	-2.71	0.43	0.32	-2.58	-2.45
STDR	3.75	2.93	1.92	2.45	3.75	3.62
t-value[1]	-3.39	-6.07	1.48	0.85	-4.51	-4.49
p-value[2]	0.0015	0.0001	0.1455	0.4009	0.0001	0.0001

1 Student's t value for testing the hypothesis that the sample mean of error is zero, and
2 Probability of a greater absolute value of Student's t.

measures, including the sample means (MEAN), sample standard deviations (STDR), and the corresponding *t*-values and *p*-values for each contract. The null hypothesis of a zero basis is not rejected at the 10 percent level for June and August contracts. However, the basis estimates for February, April, October, and December contracts are significantly different from zero at the 1 percent level.

Summary measures of the *ex post* performance of the cash and futures forecasts are presented in Table 5.3. The *t*-values allow for testing the hypothesis of no consistent bias in price forecasting, which implies that the mean values of price forecast errors are not different from zero. This test is a modified version of a test based on the linear regression of realized prices against their respective forecasts.

As shown in Table 5.3, the price forecasts for cash and futures have different mean values (MEAN). The standard errors (STDR) of the cash forecasts and the futures forecast are of similar magnitude. The cash forecast is less variable. The hypothesis of no consistent bias in price

Table 5.3 Summary Statistics for Cash and Futures Forecasts

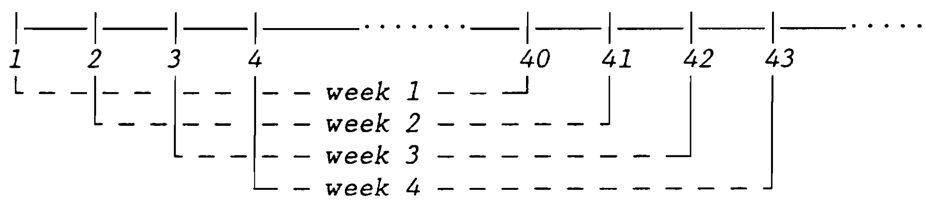
	Cash Forecast	Basis-Adjusted Futures Forecast
MEAN	61.77	59.67
STDR	4.12	5.00
t-value[1]	2.05	2.53E-13
p-value[2]	0.0417	1.0000

1 Student's t value for testing the hypothesis that the sample mean of error is zero, and
2 Probability of a greater absolute value of Student's t.

forecasting is rejected at the 5 percent level for price distributions for the cash forecasts. However, the null hypothesis is not rejected at the 1 percent level for the futures forecast. This is not surprising since the futures forecasts are based on the current futures quote adjusted by the averaged basis estimates defined above.

5.4 The Dynamic Measure of Price Uncertainty

In addition to the simplistic forecasts of cash and futures prices, it is necessary to estimate the variances of cash and futures prices (and the covariances between the two prices) in order to make the current model operational. These estimates are needed such that they would have been available at the time production and hedging decisions are made. Clearly, the choice of a measure of uncertainty will make an important difference in the apparent usefulness of a hedging routine derived from these measures. Considering the dynamics of the hedging decision, these measures must be updated whenever new information is available. To reflect this, the variance of cash and futures prices and the covariance between the two prices are estimated by updating new price information. Suppose that at week one a producer estimates the variances and covariance using the preceding ten month's (forty weeks) price information. At week two, his estimates are made over week two through forty-one. The following time horizon is used to estimate the variances and covariances:



This differs from previous analyses which use the average of price variance over the entire sample period as a measure of price uncertainty. This approach recognizes that the measure of price uncertainty routinely updates as new information arrives.

5.5 Operational Formulae For Certainty Equivalent Model

The operational objective functions for a cattle producer are presented in Appendix 3.2.1 through 3.2.4. As introduced in Chapter 3, the specific (certainty equivalent) profit functions faced by the cattle producer are determined by the relationship of current and expected cash and futures prices adjusted by marginal costs. As shown in Appendix 3.2.1 through 3.2.4, the possible situations resulting from price relationships involve 16 cases. Based on these scenarios, the sample containing all variables necessary to compute the optimal positions and the equilibrium prices is divided into 16 cases. Then, these 16 cases are classified into four categories of corrected objective functions of certainty-equivalent total profit functions adjusted by tax rates and deduction levels. The analyses suggested in Chapter 3 can be based on these data and corresponding objective functions.

5.6 Specification of Consumers' Aggregate Cash Demand Schedule

As shown in Chapter 3, the determination of equilibrium cash prices (under the segregation result) requires identifying an aggregate supply schedule for cattle producers and an aggregate demand schedule for consumers. The aggregate supply schedule was derived by solving the first-order conditions of cattle producers' expected profit maximization functions. The aggregate demand schedule differs from traditional consumer's beef demand schedules which commonly relates per capita quantity of beef consumed to economic variables such as beef prices, other meat prices, income, etc. Such an approach relates the aggregate quantity consumed by a whole group of consumers to the prices perceived by consumers. Here, the prices are for marketed fed cattle rather than for beef.

For this purpose, previous studies related to this area are used to obtain the quantity-coefficient estimates for the expected fed cattle prices. Koontz (1985) analyzed the interaction between the cattle feeding sector and the live cattle futures market using weekly sample over the period 1979-1983. In his work, lead/lag analyses are conducted between the *Cattle Fax* placement series and the live cattle futures price series. The Geweke type model suggested by the minimization of Akaike's final prediction error (FPE) criterion relates the future supply conditions to the live cattle futures prices. The estimates from the first Geweke model which incorporates two placement variables are rewritten (p. 78):

$$
\begin{aligned}
\text{CLOSEL2}(t) = & \underset{(-0.067)}{-0.0071} - \underset{(-1.013)}{0.0064}\cdot \text{DPL}(t) - \underset{(-1.934)}{0.0085}\cdot \text{DPL}(t-1) \\
& + \underset{(11.339)}{0.7320}\cdot \text{CLOSEL2}(t-1) - \underset{(-8.880)}{0.6964}\cdot \text{CLOSEL2}(t-2) \\
& + \underset{(6.692)}{0.5922}\cdot \text{CLOSEL2}(t-3) - \underset{(-5.311)}{0.4954}\cdot \text{CLOSEL2}(t-4) \\
& + \underset{(3.455)}{0.3241}\cdot \text{CLOSEL2}(t-5) - \underset{(-3.284)}{0.2884}\cdot \text{CLOSEL2}(t-6) \\
& + \underset{(2.905)}{0.2236}\cdot \text{CLOSEL2}(t-5) - \underset{(-1.371)}{0.0872}\cdot \text{CLOSEL2}(t-6)
\end{aligned}
$$

$R^2 = 0.3838$ F value $= 14.949$ $D.O.F = 247$ $(\cdot) = t$-values

Ljung-Box Q statistic: $\chi^2 = 2.670$
$D.O.F = 6$
(Prob. > Q) = 0.850

where

CLOSEL2(t) = 4-month distant futures price in time (t-2) minus the futures price in time (t), \$/cwt.; and

DPL(t) = differenced placement series in time (t), 1000 head.

Based on the above estimation result, a 1000-head increase in placements at t = 0 will result in a initial decrease in the 4-month distant live cattle futures price at t = 0 by \$0.0064/cwt., which is then followed by shrinking oscillatory adjustments in the distant futures prices. This price response to a quantity change can be viewed as not being flexible in terms of a price-dependent model or inelastic in terms of a quantity-dependent model.[72]

As a second possible source for the price-quantity estimates, a quarterly forecasting model for fed steer price is used. This model has been adopted by the U.S. Department of Agriculture (USDA). In this model, prices for fed steers are a function of fed and nonfed cattle slaughter representing supplies, and income variables representing derived demand factors. The original estimated model was presented by Westcott and Hull, 1985:

$$\begin{aligned} STEP = -100.72 &- \underset{(7.55)}{0.0095} \cdot STHFFQ - \underset{(9.81)}{0.0128} \cdot STHFNFQ + \underset{(13.96)}{29.34} \cdot LOG(YMA8) \\ &+ \underset{(2.42)}{537.04} \cdot YMA8LC + \underset{(2.25)}{2.60} \cdot D2 + \underset{(2.96)}{3.40} \cdot D3 \end{aligned}$$

$R^2 = 0.95$ $RMSE = 2.95$ $CV = 5.93$ $(\cdot)$ = Standard Errors

where

STEP = fed steer price, $/cwt.;

STHFFQ = fed steer and heifer slaughter, 1000 head;

STHFNFQ = nonfed steer and heifer slaughter, 1000 head;

LOG(YMA8) = log of eight quarter moving average of personal disposal income, bil. $;

YMA8LC = change in LOG(YMA8), bil. $;

D2 = dummy variable equal to 1 in the second quarters; and

[72] The flexibility is calculated to be -1.654, evaluated at the mean values of other explanatory variables. Converted into an own-price elasticity, this value is equivalent to -0.605.

D3 = dummy variable equal to 1 in the third quarters.

According to the above estimation result, a 1000 head increase in fed cattle slaughter at t = 0 will decrease the fed steer price at t = 0 by $0.0095/cwt. This quantity-response coefficient is viewed as being more inelastic than that obtained from Koontz's model.[73]

The above two models indicate that change in price is not flexible in response to a change in quantity.[74] Assuming beef consumption is positively related to the supply of fed cattle, these results are consistent with other studies using price-dependent models for beef consumption (Braschler, 1983; Cornell and Sorenson, 1986; Dahlgran, 1986).

5.7 The Process of Calculating The Variables of Interests

The process of calculating the variables of interests are as follows:

(1) the sample data of 1983-1987 are divided into four different scenarios (defined in detail in Appendix 3.1.1 through Appendix 3.1.4) in order to reflect specific certainty equivalent profit functions adjusted by tax parameters faced by a producer/hedger;

---

[73] The price flexibility is calculated to be -1.734, evaluated at the mean values of the explanatory variables. Converted into an own-price elasticity, this value is equivalent to -0.577.

[74] This has an important implications for the producers/hedgers' welfare changes in response to changes in tax policy, which will be shown in Chapter 6. The increases in the optimal cash positions (supply) induced by increases in tax rate and/or deduction levels will decrease the output prices received by the producers. Given an inflexible (inverse) demand schedule and fixed production costs, percentage increases in quantity are exceeded by percentage decreases in prices, which will decrease the producers' net revenues.

(2) the estimated coefficients are incorporated into the theoretical models, which include basis estimates, forecasts of cash and futures prices, and the quantity-response coefficients cash demand schedule;

(3) marginal tax rate (deduction level) for a given deduction level (marginal tax rate) is increased by 10% increments, starting from 0% and going to 100%;

(4) the risk-aversion coefficients are changed parameterically from 0.000001 to 1000,[75] which would range from almost risk-neutral to highly risk-averse; and

(5) the calculated figures are averaged over the sample period to make tables of 7-by-11 matrices for which rows and columns correspond to deduction levels and tax rates, respectively.

Following the above process, the means of the following variables are computed: (1) optimal producer's cash and futures positions; (2) optimal pure speculator's futures positions; (3) equilibrium expected cash and current futures prices; (4) cash and futures spreads; (5) individual agent's and social welfare changes; and (5) tax revenue changes.

The following example is given to demonstrate how an optimal producer's cash positions in a week can be calculated. Suppose the marginal tax rate, deduction level, and risk aversion coefficient are 30%, 50%, and 0.000001, respectively. Values for the 4-month distant futures contract and marginal cost are 62.00 ($/cwt.) and 60.00 ($/cwt.), respectively. The corresponding basis estimate and fixed trading cost are +2.00 ($/cwt.) and 0.0625 ($/cwt.), respectively. In addition, the

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[75] Peck (1975) used the risk aversion coefficients of $-\infty < \lambda \leq 1$. She reported the estimated results for the cases of λ = -0.1, -0.01 and -0.001. Anderson and Danthine (1980) reported the calculated results for the cases of λ = 0.000005, 0.000010 and 0.000015. Rolfo (1980) used the risk aversion coefficients of $0.00001 \leq \lambda \leq \infty$ by increments of ten-times.

averaged variances of cash and futures prices are 8.00 ($/cwt.) and 6.00 ($/cwt.), respectively. The estimated covariance between two prices is 5.00 ($/cwt.). Incorporating all these observations and estimates into equation (3.10) in Chapter 3 yields:[76]

$$x_s^* = \frac{(62.00 - 60.00) - (5.00/6.00)\cdot[(62.00 + 2.00) - 62.00 - 0.0625]}{(1 - 0.3)\cdot 0.000001\cdot[8.00 - (5.00^2/6.00)]}$$

$$= 143{,}634 \text{ (head)}.$$

Similarly, an optimal futures position is computed based on equation (3.11) in Chapter 3:

$$x_f^* = \frac{[(62.00 + 2.00) - 62.00 - 0.0625] - (5.00/8.00)\cdot(62.00 - 60.00)}{(1 - 0.3\cdot 0.5)\cdot 0.000001\cdot[6.00 - (5.00^2/8.00)]}$$

$$= 281{,}330 \text{ (head)}.$$

An equilibrium expected cash price is obtained by plugging the above computed optimal cash position into the aggregate cash demand schedule from Koontz's study in section 5.6. Except the slope (quantity-response) coefficient, other explanatory variables are evaluated at their mean values. The computed equilibrium expected cash price is obtained as follows:

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[76] These calculations are sensitive to the cash and futures profitability in the numerator, as well as the risk aversion coefficient in the denominator. However, this sensitivity does not alter the *signs* of the optimal positions, but does influence the *magnitudes* of the results.

$$S_1^* = 62.57 + 143.634 \cdot 0.0064$$

$$= 63.49 \text{ (\$/cwt.)}.$$

Repeating the above processes over weeks, 222 optimal values of the variables of interest are calculated and averaged to obtain a number which represents a particular combination of tax rate and deduction level. Tax rate (deduction level) for a given deduction level (tax rate) is changed to present different combinations of tax rates and deduction levels. In addition, the risk-aversion coefficients are changed parametrically after the above averaging processes over different combination of tax rates and deduction levels have been completed.

5.8 Chapter Summary

This chapter has introduced the variables in the data set and the procedure by which variable feeding costs were calculated. For the simulation model to be operational, estimation processes for the expected cash and futures prices were developed. Current futures prices were used for the forecasts of expected cash prices. For the forecasts of expected futures prices, current futures prices adjusted by estimated bases were used. The estimation of basis was simplified by averaging delivery basis (S_1 - F_1) for each contract over the entire sample period. Related to this, the pertinent futures contracts for placement of cattle during specific periods were selected. *Ex post* performance statistics for the cash and futures forecasts were presented.

In addition to the estimation of these forecasts, the estimation procedure for variances of cash and futures prices and covariance between the two prices were documented in order to obtain dynamic measures of price uncertainty in cash and futures markets. This procedure can be considered as a 40-week moving average estimation which updates new information weekly. Operational mean-variance models were identified to reflect different scenarios. The relationship of current and expected cash and futures prices determine the specific profit functions faced by the cattle feeders.

In order to calculate an equilibrium cash price series, it was necessary to obtain an estimate of the quantity coefficient in the context of an inverse aggregate demand function. For this purpose, empirical findings of previous analyses were used. Koontz's model was documented since the data used and the estimated model were relevant to this study. As an alterative model, USDA quarterly forecasting model was presented. This chapter concluded with a presentation of the process to compute the variables of interests and establish the base on which the results of Chapter 6 are to be presented. A numerical example was presented to illustrate the simulation process.

CHAPTER SIX : EMPIRICAL RESULTS AND IMPLICATIONS

6.1 Introduction

This chapter presents empirical results based on the theoretical developments in Chapter 3 and the models developed in Chapter 5. Using data from 1983 to 1987 as described in Chapter 5, a simulation is conducted to generate optimal sizes of cash and futures positions and equilibrium cash and futures prices. The results should be considered as what the optimal positions and equilibrium prices "should have been" in the context of profit maximization given tax parameters and the 1983-87 distributions of cash and futures prices. These results are simulated numbers using observed data (prices and their means and variances) and estimated parameters taken from other studies.

These results should be used and interpreted primarily in terms of direction only. Extreme numbers are sometimes generated at extreme levels of the tax parameters, and such numbers are probably neither relevant nor pertinent. The direction of change in response to changes in the tax parameters is the most important finding. In Chapter 3, expectations were developed for the varying levels of risk aversion, but this chapter reports only the computed results for the case of constant risk aversion. Also, the producer's degree of risk aversion is assumed to be equal to that of a pure speculator, i.e., $\lambda_s = \lambda_f$.

Empirical results reported are descriptive and provided by parameterically changing tax parameters for the various values of degree of risk aversion.[77] The tables of empirical results are formed as 7-by-11 matrices. The rows and the columns of the matrices correspond to deduction levels and marginal tax rates, respectively, which range from 0% to 100% (deduction levels) and from 0% to 60% (tax rates) by increments of 10%. Each cell of the matrix is the average of the optimal solutions (cash and futures positions, equilibrium cash and futures prices, etc.) for the sample period, 1983 to 1987. Policy implications based on the empirical results are presented.

6.2 Optimal Producer's Cash Positions

Table 6.1 shows the mean of the optimal cash positions taken by a cattle producer. With degree of risk aversion (λ) = 0.000001, marginal tax rate (t) = 0 (0%) and deduction level (d) = 0 (0%), the mean of the optimal cash position size over the sample period is 145,790 head.[78] The computed result can be interpreted as follows. Suppose that a cattle producer's degree of risk aversion is 0.000001, which can be viewed as "nearly" risk-neutral. The cattle producer knows that there is no tax

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[77] As shown below, the results are reported mainly for the case of λ = 0.000001, which can be viewed as "nearly" risk neutral. This choice of degree of risk aversion is made for the following reasons. First, a cattle producer is more likely to be characterized as a less risk-averse or nearly risk-neutral economic agent. As discussed in Chapter 1, considering the nature of (highly risky) cattle feeding operations, the above assumption is not unreasonable. Second, a computation of risk aversion parameters based on the equations in Chapter 3 suggests λ = 0.000001 belongs to the range of observed (aggregate) risk aversion.

[78] Appendix 6.1 shows tables for different λ values ranging from 0.00001 to 1000. As λ increases by ten times, the means of optimal values decrease by ten times. As a producer becomes more risk averse, the impact of changes in tax policy is smaller.

Table 6.1 Mean of Optimal Cash Positions by Producer

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	145790	145790	145790	145790	145790	145790	145790	145790	145790	145790	145790
0.1	155548	156134	156731	157341	157964	158599	159248	159911	160589	161282	161989
0.2	167745	168928	170161	171445	172786	174186	175650	177182	178787	180471	182238
0.3	183427	185220	187127	189160	191332	193657	196152	198837	201734	204869	208272
0.4	204336	206752	209377	212241	215378	218829	222643	226881	231617	236945	242984
0.5	233609	236660	240050	243839	248102	252933	258454	264825	272257	281040	291581
0.6	277518	281218	285423	290244	295825	302363	310127	319498	331031	345572	364476

on profits from cattle feeding ($t = 0$) and losses, if any, from speculative trading in futures markets (for example, being short in feeder cattle and/or long in live cattle) are not deductible for tax purposes ($d = 0$). Based on the mean-variance analysis, the cattle producer can simultaneously determine the size of his optimal cash position of 145,790 (the average for the sample period) with the optimal futures positions. The simulation of these results is based on equation (3.10) in Chapter 3, repeated here for convenience:

$$(3.10) \quad x_s^* = \frac{[E(S_1|I_0) - c'(x_s)] - (\sigma_{sf}/\sigma_f^2)[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - t)\cdot\lambda[\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2)]}$$

As concluded in Chapter 3, increases in marginal tax rate and/or deduction level increase the optimal cash positions in absolute value.[79]

[79] The result that an increase in the marginal tax rate will increase the optimal cash positions seems to be contrary to logic and to be a spurious result from a naive mathematical derivation. As mentioned in Chapter 4, there is, however, a good intuitive explanation for this result. Within the mean-variance framework, a risk-averse agent is supposed to be concerned about the variance as well as the mean of expected profits or incomes. An increase in tax rate not only decreases the expected value of profits but also decreases the variance of expected profits. Assuming normally distributed profits, this is equivalent to

Note that the magnitude of the increase in the optimal cash positions in response to an increase in marginal tax rate is larger than that in response to an increase in deduction level. This is due to the nature of the developed model. Deduction levels are discounted (multiplied) by the marginal tax rate in the equations. The response to increases in d are larger for larger values of t. This result arises from the t·d in the profit maximization functions.

Combined with the results of the equilibrium cash prices and spreads between expected cash prices and marginal costs shown later, these increased optimal cash positions (in an absolute sense) have an important implication for pricing efficiency in the cash market and for consumers' welfare improvements. For pricing efficiency, this change in the optimal cash position reduces the gaps between expected cash prices and marginal costs as the quantity increases push selling prices down. This implies that the positive market imbalances demonstrated in Chapter 2 would be corrected more quickly. Concerning consumers' welfare, this increased supply of fed cattle implies a welfare gain for consumers in an aggregate sense. Increased supplies will mean lower consumer prices. The adjustments of equilibrium cash prices in response to changes in the optimal cash positions will also stabilize the retail prices faced by consumers, assuming a positive correlation between fed cattle prices faced by producers and retail prices. This is another benefit to consumers.

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saying the center (mean) of the distribution moves to the left (for positive profits) and its width or dispersion (variance) is narrowed. In order to keep the same utility (or expected profits) as that of the before-tax level, the agent should increase output.

6.3 Optimal Producer's Futures Positions

Table 6.2 shows the mean of the optimal futures positions taken by a cattle producer. With degree of risk aversion (λ) = 0.000001, marginal tax rate (t) = 0 (0%) and deduction level (d) = 0 (0%), the mean of optimal futures position size over the sample period is -597,294 head.[80] Given a marginal tax rate = 0 and a deduction level = 0, the cattle producer would take an average of 597,294 head in short positions in the 4-month distant live cattle futures. This is equivalent to taking 18,099 short contracts, assuming a fed animal weighs an average of 1,200 pounds and the size of one live cattle futures contract is 40,000 pounds. The simulation of these results is based on equation (3.11) in Chapter 3, repeated here:

$$(3.11) \quad x_f^* = \frac{[E(F_1|I_0) - F_0 - c'(x_f)] - (\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - td)\cdot\lambda[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]}$$

Table 6.2 Mean of Optimal Futures Positions by Producer

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	-597295	-597295	-597295	-597295	-597295	-597295	-597295	-597295	-597295	-597295	-597295
0.1	-624978	-628494	-632083	-635745	-639484	-643301	-647200	-651182	-655251	-659410	-663661
0.2	-659582	-666687	-674088	-681804	-689855	-698265	-707056	-716257	-725895	-736004	-746618
0.3	-704073	-714841	-726295	-738505	-751547	-765510	-780495	-796618	-814013	-832839	-853278
0.4	-763395	-777901	-793668	-810869	-829708	-850431	-873335	-898784	-927227	-959226	-995491
0.5	-846445	-864768	-885127	-907882	-933481	-962493	-995649	-1033907	-1078541	-1131290	-1194589
0.6	-971020	-993242	-1018494	-1047442	-1080960	-1120224	-1166851	-1223124	-1292384	-1379711	-1493236

[80] Appendix 6.2 shows tables for different λ values ranging from 0.00001 to 1000. As λ increases by ten times, the means of optimal values decrease by ten times. That is, as a producer/hedger becomes more risk averse, the impact of changes in tax policy is smaller and the optimal futures positions are reduced.

On average, then, the futures contracts taken are more than four times as large as the cash positions planned by the cattle producer. Compared to the empirical results from the literature on optimal hedging ratios, the magnitude obtained from this direct calculation method seems to be surprising.[81] The optimal futures positions computed based on this model, however, involve "speculative" components as well as the "hedging" components.[82] The studies estimating an optimal hedging ratio only estimate the "hedging" components or "risk-minimum" factors. Thus, the traditional optimal hedging ratio does not consider the "speculative" profits which could be generated by taking futures positions. In this sense, the optimal hedging ratio approach is for "risk-reducing" purposes only, while this approach of direct calculation of optimal futures

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[81] These results stand in obvious contrast to the practice of routine hedging which is obtained by applying the rules for optimal choice of a live cattle futures to the data of the present example. For example, suppose a cattle producer were to hedge a cash position (fed cattle) in the live cattle futures. By calculating the appropriate simple regression, it is found the pure hedge ("risk-minimizing") proportions would range from -0.03 (long) to +0.97 (short) head of live cattle futures for every long position of 1 head of cash fed cattle.

[82] Anderson and Danthine (1980) show the optimal cash and futures positions using the direct calculation method, similar to the approach used here. For oats for the period of June 1975 through February 1976, the following optimal cash futures position sizes are as follows (thousand bushels):

λ	Cash	Futures
0.000005	225	1001
0.000010	113	500
0.000015	75	334

Note that the futures positions are nearly 5 times the cash positions.

positions is for purposes of "risk-managing." Risk management includes pursuing "speculative" profits as well as "hedging" or protection against adverse price movements.

As indicated in Chapter 3, increases in the marginal tax rate and/or deduction level increase the optimal futures positions in absolute value. Analogous to the optimal cash positions, the magnitude of increase in the optimal futures positions in response to an increase in marginal tax rate is larger than the increase in response to an increase in deduction level. As would be expected, the impact of increases in d is much larger for larger values of t. Positions should increase when a larger percentage of any futures losses are deductible.

Combined with the results of the equilibrium futures prices and computed futures gains in terms of $E(F_1) - F_0 - c'(x_f)$ shown later, these increased optimal futures positions (in an absolute sense) have an important implication for improvements in liquidity and pricing efficiency in the futures market. These increased futures positions in response to increases in the tax rate and/or deduction level would provide more liquidity in the live cattle futures market.[83] The futures market literature suggests that increased liquidity should improve informational availability. Pricing effectiveness as defined in Chapter 2 would be improved with any departures from the underlying equilibrium being of

[83] This statement does not mean that current live cattle futures market has a liquidity problem. For the current live cattle futures market, liquidity in terms of open interests and trading volume is not a serious problem. As some argue, there may be a problem associated with less than desirable involvement of cattle producers or commercial processing firms.

smaller amplitude and/or of lesser duration in a time context.

6.4 Optimal Pure Speculators' Positions

Table 6.3 shows the mean of the optimal futures positions taken by a pure speculator. The pure speculator's degree of risk aversion is assumed to be the same as the cattle producer's. With degree of risk aversion (λ_f) = 0.000001, marginal tax rate (t) = 0 (0%) and deduction level (d) = 0 (0%), the mean of optimal futures position size taken by the pure speculator is over the sample period -300,144 head.[84] Given a marginal tax rate = 0 and a deduction level = 0, the pure speculator would take an average of 300,144 head in short positions in the 4-month distant live cattle futures. This is equivalent to taking 9,095 short contracts, assuming a fed animal weighs an average of 1,200 pounds and the size of one live cattle futures contract is 40,000 pounds. The simulation of these results is based on equation (3.14) in Chapter 3, repeated here:

$$(3.14) \quad z_f = \frac{[E(F_1|I_0) - F_0 - c'(z_f)]}{(1 - t)\lambda_f \sigma_f^2} \quad .$$

On average, the futures contracts taken by the pure speculator is 50 percent of the optimal futures contracts taken by the cattle producer.

[84] Appendix 6.3 shows tables for different λ values ranging from 0.00001 to 1000. As λ increases by ten times, the means of optimal values decrease by ten times. That is, as a pure speculator becomes more risk averse (still being constant for different levels of expected incomes), the impact of changes in tax policy is smaller; the optimal pure speculative positions are reduced.

Table 6.3 Mean of Optimal Futures Positions by Pure Speculator

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	-300145	-300145	-300145	-300145	-300145	-300145	-300145	-300145	-300145	-300145	-300145
0.1	-333494	-333494	-333494	-333494	-333494	-333494	-333494	-333494	-333494	-333494	-333494
0.2	-375181	-375181	-375181	-375181	-375181	-375181	-375181	-375181	-375181	-375181	-375181
0.3	-428779	-428779	-428779	-428779	-428779	-428779	-428779	-428779	-428779	-428779	-428779
0.4	-500242	-500242	-500242	-500242	-500242	-500242	-500242	-500242	-500242	-500242	-500242
0.5	-600290	-600290	-600290	-600290	-600290	-600290	-600290	-600290	-600290	-600290	-600290
0.6	-750362	-750362	-750362	-750362	-750362	-750362	-750362	-750362	-750362	-750362	-750362

This difference in magnitude results from the fact that the pure speculator has no cash position comparable to the cattle producer and that he considers only the "speculative" component. The cattle producer has a hedging component as well in determining his optimal futures positions. The signs of optimal speculative positions taken by the pure speculator are not always the same as the optimal futures positions taken by the cattle producers for each week across 1983-87. The cattle producer's optimal futures positions are determined by both futures gain, $E(F_1) - F_0 - c'(x_f)$, and cash gain, $E(S_1) - c'(x_s)$. In a few cases, the cash gain exceeds the futures gain to occasionally reverse the sign of the optimal futures position taken by the cattle producer.

As discussed in Chapter 3, it is assumed that under current tax policy, losses from pure speculation are not deductible. Thus, deductibility is not supposed to matter for the case of the pure speculator. The only concerns about tax issues for the pure speculator is the marginal tax rate. As shown in Table 6.3, an increase in the marginal tax rate increases the optimal speculative position taken by the pure speculators. The variance of returns is reduced at higher tax rates. However, an increase in deduction level does not change the optimal speculative positions since the deduction level does not enter into the equation for the optimal speculative positions.

Analogous to the producer's optimal futures positions, this adjustment in the optimal (pure speculative) futures positions also provides more liquidity in the live cattle futures market. There is an increased supply of speculative positions offsetting any increased hedging demand by cattle producers. These increased futures positions and increased liquidity should enhance pricing effectiveness in the futures market.

6.5 Expected Cash Price

Table 6.4 shows the mean of the expected (equilibrium) cash prices determined by the cash supply-demand condition shown in Chapter 3. As shown in Chapter 5, the (aggregate) demand schedule by consumers is estimated by regressing 4-month distant live cattle futures prices on placements of feeder cattle into feedlots and the lagged dependent variables. The estimated demand coefficient in response to cash price is -156,250 head. That is, when cash price increases by one dollar, the aggregate demand by consumers is expected to decrease by 156,250 head. The supply of fed cattle is computed directly based on equation (3.10) in Chapter 3 and the simulation results were shown earlier in Table 6.1.

With degree of risk aversion (λ) = 0.000001, marginal tax rate (t) = 0 (0%) and deduction level (d) = 0 (0%), the mean of the expected cash

Table 6.4 Mean of Equilibrium Expected Cash Prices

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	65.871	65.871	65.871	65.871	65.871	65.871	65.871	65.871	65.871	65.871	65.871
0.1	65.827	65.825	65.822	65.819	65.816	65.814	65.811	65.808	65.805	65.801	65.798
0.2	65.772	65.767	65.762	65.756	65.750	65.743	65.737	65.730	65.723	65.715	65.707
0.3	65.702	65.694	65.685	65.676	65.666	65.656	65.645	65.633	65.620	65.606	65.590
0.4	65.608	65.597	65.585	65.572	65.558	65.543	65.526	65.507	65.485	65.461	65.434
0.5	65.476	65.463	65.447	65.430	65.411	65.389	65.365	65.336	65.303	65.263	65.216
0.6	65.279	65.262	65.243	65.222	65.197	65.167	65.132	65.090	65.038	64.973	64.888

prices at t = 1 is 65.87 (\$/cwt.) over the sample period.[85] This result can be interpreted as follows. Given the tax parameters and the distribution of cash and futures prices, a risk-averse cattle producer is expected to produce fed cattle of (average) 145,790 head four month later. Assuming a fixed demand schedule, the planned production of fed cattle gives rise to an average equilibrium price of 65.87 (\$/cwt.) over the sample period.

As expected in Chapter 3, increases in the marginal tax rate and/or deduction level decrease (but only slightly) the mean of the expected equilibrium cash price level over the sample period. The decrease in the expected cash price has an important implication for improvement in consumers' welfare, discussed later. However, this does not mean that the equilibrium cash price computed each week is decreased. Using Figure 3.1 dealing with the clearing of long and short cash positions, the following explanation is possible. For the weeks when $E(S_1) > c'(x_s)$, an increase

[85] Appendix 6.4 shows tables for different λ values ranging from 0.00001 to 1000. As λ increases from 0.000001 up to 0.01, the means of optimal values increase. The changes in the equilibrium cash prices become smaller since the changes in the optimal cash position are reduced as λ increases. Beyond λ value of 0.01, the changes in the optimal positions are so small that the changes in the equilibrium prices are negligible.

in the marginal tax rate and/or deduction level increases the optimal long cash positions taken by the cattle producer. This increased cash position is comparable to rotating the cash supply schedule to the right (from L to L' in Figure 3.1) from the origin, a small increase in supply.[86] Given a downward-sloping demand schedule, the rightward rotation of supply schedule gives rise to a decreased equilibrium price (from S1 to S2 in Figure 3.1). When a price-marginal cost relation is given as $E(S_1) < c'(x_s)$, an increase in the tax rate and/or deduction level increases the optimal short cash positions in absolute value. In such a case, the supply curve of short cash positions rotates to the left (from S to S' in Figure 3.1). This rotation of supply curve increases the equilibrium cash price (from S3 to S4 in Figure 3.1).

As Table 6.4 shows, the magnitude in response to changes in the tax rate and in the deduction level are quite small. These small changes in the equilibrium cash prices in response to tax parameters are not a surprising result. Remember that a single cattle producer is assumed to change (plan) his production schedule and the impact of this changed firm-level production (supply) on the averaged equilibrium prices of the 13 major cattle feeding states is supposed to be trivial. Aggregating across producers would cause bigger price changes. Recall that as noted in Chapter 3, the changes in the equilibrium cash prices are sensitive to the estimated price-quantity coefficients for aggregate demand. The

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[86] Note that this rotation from origin is mathematically shown in section 3.7. The possibility of rotating supply curve around an equilibrium price is thus eliminated.

qualitative results still hold, however, when considering the impact of changes in tax parameters on the individual producer.

Combined with the optimal cash positions, the above finding has an important implication for pricing efficiency in the cash market. When the cash market is in a disequilibrium state in which $E(S_1) > c'(x_s)$, an increase in the tax rate and/or deduction level "pushes down" via supply increases the equilibrium cash price to reduce the gap between $E(S_1)$ and $c'(x_s)$. When the cash market is characterized by a disequilibrium of $E(S_1) < c'(x_s)$, an increase in the tax parameters "pushes up" via supply decreases the equilibrium cash price to reduce the gap between $E(S_1)$ and $c'(x_s)$. As defined in Chapter 2, a cash market is in a state of relative imbalance when the output price either significantly exceeds or is well below the average variable costs of feeding feeder cattle. The "push down" or "push up" actions induced by increases in the marginal tax rate and/or deduction level make the gap between $E(S_1)$ and $c'(x_s)$ narrow. This narrowed gap between $E(S_1)$ and $c'(x_s)$ implies that the expected (positive or negative) margins of cattle feeding become smaller in absolute value and that the market moves toward a state of relatively balance or equilibrium. When the market is effective in quickly correcting and/or reducing the amplitude of the imbalances associated with both negative margins (losses) and positive margins (excessive profits), the price discovery process is more effective and any social loss can be avoided or reduced.

Table 6.5 shows the mean of the differences between the expected cash prices and marginal costs. As discussed above, an increase in the marginal tax rate and/or deduction level decreases the gap between $E(S_1)$ and $c'(x_s)$ in absolute value. With degree of risk aversion (λ_f) = 0.000001, marginal tax rate (t) = 0 (0%), and deduction level (d) = 0 (0%), the mean of the gap between $E(S_1)$ and $c'(x_s)$ is 2.70 ($/cwt.) over

Table 6.5 Mean of Expected Cash Spreads ($E(S_1)$ - $c'(x_s)$)

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	2.702	2.702	2.702	2.702	2.702	2.702	2.702	2.702	2.702	2.702	2.702
0.1	2.658	2.655	2.652	2.650	2.647	2.644	2.641	2.638	2.635	2.632	2.629
0.2	2.603	2.598	2.592	2.586	2.580	2.574	2.567	2.560	2.553	2.546	2.538
0.3	2.532	2.524	2.516	2.507	2.497	2.486	2.475	2.463	2.450	2.436	2.421
0.4	2.438	2.428	2.416	2.403	2.389	2.373	2.356	2.337	2.316	2.292	2.265
0.5	2.307	2.293	2.278	2.261	2.242	2.220	2.195	2.166	2.133	2.094	2.046
0.6	2.109	2.093	2.074	2.052	2.027	1.998	1.963	1.921	1.869	1.804	1.719

the sample period.[87] The impact of increases in the marginal tax rate on the change in the gap of $E(S_1)$ - $c'(x_s)$ is larger than that of an increase in deduction level. For t=.3, increases in d to .9 to 1.0 reduce the spreads by $.10 per cwt., a significant amount in an economic context.

6.6 Current Futures Price

Table 6.6 shows the mean of the equilibrium current futures prices determined by equating the sum of speculative supply offered by producers and pure speculators with the hedge demand by hedgers. With a degree of risk aversion (λ) = 0.000001, a marginal tax rate (t) = 0 (0%) and a

[87] Appendix 6.5 shows tables for different λ values ranging from 0.00001 to 1000. As λ increases from 0.000001 up to 0.01, the means of the optimal values increase. The changes in the difference of cash prices-marginal costs become smaller since the changes in the optimal cash position are reduced as λ increases. Beyond λ value of 0.01, the changes in the optimal positions are so small that the changes in the differences of price-marginal cost are negligible.

deduction level (d) = 0 (0%), the mean of the equilibrium current futures prices at t = 0 is 60.53 ($/cwt.) over the sample period.[88]

Table 6.6 Mean of Equilibrium Current Futures Prices

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	60.526	60.526	60.526	60.526	60.526	60.526	60.526	60.526	60.526	60.526	60.526
0.1	60.520	60.521	60.521	60.522	60.522	60.523	60.524	60.524	60.525	60.526	60.526
0.2	60.513	60.514	60.515	60.517	60.518	60.519	60.521	60.522	60.523	60.525	60.526
0.3	60.504	60.506	60.508	60.510	60.512	60.514	60.517	60.519	60.521	60.524	60.526
0.4	60.493	60.496	60.499	60.502	60.505	60.508	60.512	60.515	60.519	60.522	60.526
0.5	60.481	60.484	60.488	60.492	60.496	60.501	60.505	60.510	60.515	60.521	60.526
0.6	60.465	60.469	60.474	60.479	60.484	60.490	60.496	60.503	60.510	60.518	60.526

The simulation of these results is based on equation (3.18) in Chapter 3, repeated here:

$$(3.18) \quad F_0 = \frac{A\cdot[E(F_1|I_0) - c'(x_f)] + B\cdot c'(x_s)}{A + B},$$

where $A = (1 - t)\lambda_f + (1 - td)\lambda_s(1 - \rho^2)$ and $B = (1 - t)\lambda_f(\sigma_{sf}/\sigma_s^2)$.

This result can be interpreted as follows. Given the tax parameters and the distributions of cash and futures prices, a risk-averse cattle producer is expected to produce on average 145,790 head of fed cattle and

[88] As λ increases from 0.000001 up to 1000, the means of optimal values remain unchanged. This is because λ_s and λ_f in equation (3.18) in section 3.6 change proportionately to give rise to constant equilibrium futures prices. Considering the empirical futures demand schedules, the means of optimal values slightly increase as λ increases from 0.000001 up to 0.01. The changes in the equilibrium futures prices become smaller since the changes in the optimal futures position are reduced as λ increases. Beyond λ value of 0.01, the changes in the optimal futures positions are so small that the changes in the equilibrium prices are negligible.

to take average short positions of 597,294 head in the four-month distant live cattle futures. Given a hedge demand schedule, the supply of optimal future positions gives rise to the equilibrium current futures price of 60.53 ($/cwt.) over the sample period.

As discovered in Chapter 3, an increase in the marginal tax rate decreases the mean of the equilibrium current futures prices over the sample period. Analogous to the equilibrium expected cash prices, this result does not imply that the equilibrium current futures prices are decreased each week. When the current futures price including trading costs $[F_0 + c'(x_f)]$ is higher than the expected (expiring) distant futures price $[E(F_1)]$, producers and pure speculators are supposed to take short positions in the distant live cattle futures.[89] In such a case, an announcement of an increase in the marginal tax rate, which reduces the variance of futures revenue streams, would increase the short futures positions taken by hedgers and speculators. Given a hedge demand schedule, this increased supply of short positions decreases the equilibrium futures prices. Using Figure 3.2 dealing with the clearing of the futures market, the increased supply of short positions is comparable to rotating the short supply curve from SS to SS' from the origin, and the

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[89] More precisely, producers/hedgers could take the opposite positions as those taken by pure speculators. This is because producers/hedgers are assumed to determine their futures positions in terms of signs and magnitudes based on futures gains or profitability $[E(F_1)- F_0-c(x_f)]$ along with cash gains or profitability $[E(S_1)-c'(x_s)]$. In contrast, pure speculators are supposed to decide the signs and the magnitudes of their futures positions solely based on futures gains. In some cases, cash gains exceed futures gains so that the sign (long and short) of futures positions taken by producers can be reversed as opposed to that taken by pure speculators.

decrease in the equilibrium current futures price is to reduce the price level from F1 to F2. When the price relationship of cash and futures markets implies that producers would take long positions in the distant live cattle futures markets, an increase in the marginal tax rate induces the producers to take even more long positions in the futures market. Given the hedge demand schedule, this increased supply of long positions (rotation from LS to LS' in Figure 3.2) will increase the equilibrium current futures price (from F3 to F4 in Figure 3.2). Both cases of changes in the equilibrium current futures prices reduce the futures gains in terms of $E(F_1) - F_0 - c'(x_s)$.

In contrast to the above case of an increase in the marginal tax rate, an increase in the deduction level given a tax rate increases the mean of the equilibrium current futures prices. Here again, this result does not imply that every weekly equilibrium current futures price is increased in response to an increase in deduction level. Analogous to the case of an increase in the marginal tax rate, an increase in the deduction level increases the optimal sizes of futures positions taken by the producer in absolute value. When short (long) futures positions are increased in absolute value in response to an increase in the deduction level, the equilibrium current futures price decreases (increases) to reduce the futures gains, the gap between the expected futures price and the current futures price. Along with the case of an increase in the tax rate, this result has an important implication for pricing efficiency (more precisely, pricing effectiveness) in the futures market, discussed below.

As shown in Table 6.6, the magnitude of change in the equilibrium current futures prices in response to changes in the marginal tax rate is larger (in absolute value) than that in response to changes in the deduction level, especially at lower levels of d. The term (1-td) appears in the functions employed, and it increases at small levels of d for any level of t.

Table 6.7 shows the mean of the difference between the expected futures prices and the current futures prices including relevant trading costs. As a consequence of changes in the equilibrium current futures prices, an increase in the marginal tax rate decreases the mean of the gap between $E(F_1)$ and $F_0 + c'(x_f)$ while an increase in the deduction level increases the mean of the differences. With a degree of risk aversion (λ_f) = 0.000001, a marginal tax rate (t) = 0 (0%) and a deduction level (d) = 0 (0%), the mean of the gap between $E(F_1)$ and $F_0 + c'(x_f)$ is 0.30 ($/cwt.) over the sample period.[90] Here again, the impact of an increase in marginal tax rate on changes in the gap of $E(F_1) - F_0 - c'(x_f)$ is larger than that of an increase in the deduction level. Multiplying by any level of t less than 1.0 dampens the impact of a certain level of d when the (1-td) term is employed in the calculations.

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[90] As λ increases from 0.000001 up to 1000, the means of the optimal values remain unchanged. This is because λ_s and λ_f in equation (3.18) in section 3.6 change proportionately to give rise to constant equilibrium futures prices. Considering the empirical futures demand schedules, the means of optimal values slightly increase as λ increases from 0.000001 up to 0.01. The changes in the insurance prices become smaller since the changes in the optimal futures position are reduced as λ increases. Beyond a λ value of 0.01, the changes in the optimal futures positions are so small that the changes in the insurance prices are negligible.

Table 6.7 Mean of Expected Futures Spreads ($E(F_1) - F_0 - c'(x_f)$)

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	-0.298	-0.298	-0.298	-0.298	-0.298	-0.298	-0.298	-0.298	-0.298	-0.298	-0.298
0.1	-0.291	-0.292	-0.292	-0.293	-0.294	-0.294	-0.295	-0.296	-0.296	-0.297	-0.298
0.2	-0.284	-0.285	-0.286	-0.288	-0.289	-0.291	-0.292	-0.293	-0.295	-0.296	-0.298
0.3	-0.275	-0.277	-0.279	-0.281	-0.284	-0.286	-0.288	-0.290	-0.293	-0.295	-0.298
0.4	-0.265	-0.268	-0.270	-0.273	-0.277	-0.280	-0.283	-0.286	-0.290	-0.294	-0.298
0.5	-0.252	-0.256	-0.259	-0.263	-0.268	-0.272	-0.277	-0.281	-0.286	-0.292	-0.298
0.6	-0.236	-0.240	-0.245	-0.250	-0.256	-0.261	-0.268	-0.274	-0.281	-0.289	-0.298

to that of the optimal cash and futures positions.

Combined with the result of the optimal futures positions, the above finding provides an important implication for pricing effectiveness in futures markets. First, an increase in the sizes of the optimal futures positions would increase the liquidity of the market. Second, analogous to the situation in the cash market, the decreases in the magnitude of futures gains or profitability in terms of $E(F_1) - F_0 - c'(x_f)$ implies that the futures market is moving toward to state of relative balance or equilibrium.[91]

6.7 Welfare Analysis

In the previous sections, it is demonstrated that given the price relationship of cash and futures markets, the optimal cash and futures positions increase in absolute value in response to increases in marginal tax rates and/or deduction levels. In addition, the equilibrium expected

[91] Zero futures gains or profitability does not necessarily imply that the futures market is in a state of equilibrium. In fact, the case of $E(F_1) = F_0 + c'(x_f)$ implies that there is no incentive for speculation. For a futures market to exist or succeed, prices must be variable and the size of possible futures gains be large enough to induce speculators to enter the futures market.

cash prices and the equilibrium futures prices are changed such that the spreads between the expected cash prices and marginal costs (for cash markets) and between the expected futures prices and current futures price including trading costs (for futures market) are reduced. This result is confirmed by examining the computed differences $E(S_1) - c'(x_s)$ and $E(F_1) - F_0 - c'(x_f)$. This finding has a clear implication for pricing efficiency in the cash market and pricing effectiveness in the futures market. This result also has an important welfare implication for those who wish to stabilize their income streams and/or commodity consumption flows over a long-run horizon.

The rest of this section highlights some of the welfare implications for the cattle producer, pure speculator and consumers. Changes in social welfare, the sum of each agent's welfare, and changes in tax revenues are also discussed.

6.7.1 Producer's Welfare Change

Table 6.8 shows the mean of the producer's welfare in terms of expected profits adjusted by the tax parameters. The number in each cell of Table 6.8 is the level of expected profits. The changes (gains or losses) in producer's welfare could be obtained by subtracting the welfare measured with t = 0 and d = 0 (-\$3,401,072 of the first cell in Table 6.8) from the corresponding cell. For example, when t = 0.3 (30%) and d = 1 (100% deduction allowed for futures losses), the producer's expected profit adjusted by the tax parameters is -\$5,634,995 and thus the welfare change in response to changes in tax rate and deduction level becomes

-$2,233,923.[92] As argued in Chapter 3, the most appropriate measure of producer's welfare gains or losses due to changes in tax parameters is the change in his expected utilities generated by changes in those tax parameters. As an alternative measure, this study uses simply the expected profits adjusted by the tax parameters. Thus, it has to be recognized that expected profits do not perfectly capture welfare gains or

Table 6.8 Mean of Expected Producer's Welfare

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	-3401072	-3401072	-3401072	-3401072	-3401072	-3401072	-3401072	-3401072	-3401072	-3401072	-3401072
0.1	-3717391	-3741296	-3765687	-3790579	-3815987	-3841927	-3868417	-3895474	-3923117	-3951364	-3980237
0.2	-4112844	-4161163	-4211485	-4263939	-4318662	-4375806	-4435536	-4498033	-4563493	-4632134	-4704194
0.3	-4621365	-4694628	-4772542	-4855567	-4944225	-5039111	-5140911	-5250409	-5368518	-5496299	-5634995
0.4	-5299520	-5398283	-5505583	-5622585	-5750673	-5891508	-6047102	-6219913	-6412977	-6630091	-6876063
0.5	-6249150	-6374001	-6512635	-6667483	-6841582	-7038779	-7264019	-7523770	-7826654	-8184430	-8613558
0.6	-7673989	-7825558	-7997650	-8194764	-8422820	-8689766	-9006531	-9388565	-9858447	-1.0E+07	-1.1E+07

losses for a risk averse producer under certainty, so this method should be considered as only an approximation.

As shown in Table 6.8, the welfare measured by the expected profits adjusted by tax parameters have negative means and are of large magnitude.[93] This empirical result is obtained due to the following

[92] Appendix 6.6 shows tables for different λ values ranging from 0.00001 to 1000. As λ increases from 0.00001 up to 1000, the "positive" means of the optimal values decrease. The changes in the producer's welfare become smaller since the changes in the optimal positions are reduced as λ increases. Beyond a λ value of 0.01, the changes in the optimal positions are so small that the changes in producer's welfare are negligible.

[93] This result is obtained for the segregation result under which the equilibrium prices are assumed to adjust in response to changes in the optimal positions. For the short-run case in which only positions are assumed to change in response to changes in tax parameters, the producer's welfare has a positive mean and remains unchanged with changed tax parameters. As discussed in Chapter 3, increased portions of the optimal positions are exactly offset by the increased tax parameters in computing

reasons. First, for the period 1985-1986, higher-than-normal price variances and correlations between cash and futures prices periodically generate abnormal sizes of cash and futures positions. Given cash and hedge demand schedules, these abnormal changes in the optimal positions give rise to greater-than-normal changes in the equilibrium cash and futures prices. Consequently, both the abnormal equilibrium prices and the optimal positions reduce the mean of the expected profits. Second, the cash marginal costs obtained before changes in the cash position and prices are assumed to be equal to those after changes in the cash position and prices. That is, the marginal costs in the calculation of expected cash profits are not adjusted to reflect the changed market conditions. Third, the expected cash prices estimated using observed data are not flexible with respect to changes in the optimal cash positions induced by changes in tax parameters.[94] The changes in the optimal cash positions adjust ("push down" or "push up") the equilibrium cash prices to make the cash revenues (in terms of changed price times changed position) smaller

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after-tax profits, thereby making welfare gain unchanged.

[94] Not being flexible means price is inelastic. Thus, a percent change in quantity induces a percent change in own-price that is larger than the percent change in quantity.

Instead of using the estimated quantity-response coefficients from either Koontz's or USDA models shown in section 5.6, an arbitrary coefficient is chosen to reflect a flexible (in terms of price-dependent model) or an elastic (in terms of quantity-dependent model) demand schedule. For the coefficient > -0.000555 (1000 head), the means of producer's welfare become positive. In terms of flexibility (elasticity), this coefficient is equivalent to -0.901 (-1.109). More importantly, an increase in tax rate and/or deduction level increases the positive means of producer's welfare. Appendix 6.7 shows tables for different λ assuming the inverse demand coefficient is -0.0004.

than before-tax cash revenues.

As a consequence of the changed optimal positions and the adjusted equilibrium prices, increases in the marginal tax rates and/or deduction levels decrease the mean of producer's welfare in terms of expected profits. This result is confirmed by examining a producer's expected cash profits and futures profits individually. As discussed above, the result is sensitive to the inverse demand coefficient estimated by regressing 4-month distant live cattle prices on the quantities of placements of feeder cattle into feedlots and the lagged dependent variables. If the coefficient estimated is flexible (or elastic in the context of a quanity-dependent model), then the producer's welfare changes could be positive, implying that the producer gains in terms of the expected profits when there are increases in the marginal tax rate and/or deduction level.

6.7.2 Pure Speculator's Welfare Change

Table 6.9 shows the mean of the pure speculator's welfare in terms of expected profits adjusted by marginal tax rate only. The number in each cell of Table 6.9 is the level of the expected profits. Analogous to producer' welfare changes, the changes (gains or losses) in pure speculator's welfare could be obtained by subtracting the first cell (+$43,341, the base number with $t = 0$ and $d = 0$) from each corresponding cell. As argued earlier, this measure of pure speculator's welfare is an alternative to utility changes. This study uses simply the expected profits adjusted by tax parameters. Thus, these measures should be considered as only an approximation.

Table 6.9 Mean of Expected Pure Speculator's Welfare

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	43341	43341	43341	43341	43341	43341	43341	43341	43341	43341	43341
0.1	41767	41920	42074	42229	42385	42542	42700	42859	43019	43179	43341
0.2	39925	40247	40573	40903	41237	41576	41920	42268	42621	42978	43341
0.3	37743	38249	38766	39294	39834	40386	40950	41528	42118	42722	43341
0.4	35112	35819	36549	37301	38079	38882	39713	40573	41463	42385	43341
0.5	31878	32799	33761	34767	35819	36922	38079	39294	40573	41920	43341
0.6	27801	28940	30148	31432	32799	34258	35819	37493	39294	41237	43341

Contrary to the producer's welfare changes shown above, the mean of the pure speculator's welfare is positive except for t = 1 (100%). Remembering that the changes in the equilibrium current futures prices is relatively smaller than those in the expected equilibrium cash prices, this result is not surprising. In other words, the changes in the equilibrium current futures prices in response to changes in the optimal cash positions induced by changes in tax parameters is less elastic or less sensitive than the corresponding changes in the optimal cash prices.[95] This result could be altered if any estimated hedge demand has a more elastic coefficient of price in response to changes in the positions.[96]

[95] Analogous to the producer's welfare changes, this result is obtained under the segregation result. Considering the short-run case, the pure speculator's welfare remains unchanged.

Appendix 6.8 shows tables for different λ values ranging from 0.00001 to 1000. As λ increases from 0.000001 up to 1000, the "positive" means of optimal values decrease. The changes in the pure speculator's welfare become smaller since the changes in the optimal positions are reduced as λ increases. Beyond λ values of 0.01, the changes in the optimal positions are so small that the changes in pure speculator's welfare are negligible.

[96] Efforts were made to empirically estimate (rather than compute) the short and long hedge demand functions by using OLS procedure. This specification is based on the preposition that the market-clearing insurance prices ($E(F_1)$ - F_0 - $c'(x_f)$) are determined by equating the excess speculative supply schedule computed from the current mean-variance model with the excess hedging demand by producers/hedgers. The different

As shown in Table 6.9, an increase in the marginal tax rate decreases the mean of pure speculator's welfare in terms of the expected profits adjusted by tax rate over the sample period. However, an increase in deduction level increases the mean of his welfare. Note that although

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specification of the hedging demands results from the presumption that short and long hedging demands are not symmetric, i.e., they would have different elasticities with respect to insurance prices. The estimated short hedging demand is represented as:

$$
\begin{aligned}
IP_S &= \underset{(6.66)}{13.910120} - \underset{(-5.85)}{0.0000148 \cdot SHORTC} \\
&\quad - \underset{(-4.67)}{0.328337 \cdot T_1} + \underset{(3.89)}{0.003229 \cdot T_2} - \underset{(-2.66)}{0.000007 \cdot T_3}
\end{aligned}
$$

$R^2 = 0.51$ F value $= 21.39$ t values $= (\cdot)$

where

IP_S = insurance price when $E(F_1) < F_0 + c'(x_f)$;

SHORTC = commercial (hedging) short positions, scaled to 13 major cattle-feeding states level; and

T_i = time trend variables up to power three.

Similarly, the long hedging demand is estimated as:

$$
\begin{aligned}
IP_L &= \underset{(7.93)}{11.298495} - \underset{(-2.63)}{0.0000113 \cdot LONGC} \\
&\quad - \underset{(-4.80)}{0.236170 \cdot T_1} + \underset{(5.24)}{0.002739 \cdot T_2} - \underset{(-4.91)}{0.000008 \cdot T_3}
\end{aligned}
$$

$R^2 = 0.31$ F value $= 13.01$ t values $= (\cdot)$

where

IP_L = insurance price when $E(F_1) > F_0 + c'(x_f)$;

LONGC = commercial (hedging) long positions, scaled to 13 major cattle-feeding states level; and

T_i = time trend variables up to power three.

Based on these estimated hedging demand functions, the pure speculator's welfare was computed. Contrary to the results shown in Table 6.9, the mean of the expected profits adjusted by marginal tax rate only is negative. This result comes from the fact that the coefficients estimated here are rather elastic and thus the changes in the current futures prices are larger in magnitude than those computed from the theoretical model developed in this study.

deduction level does not matter for the case of the pure speculator, the changes in the equilibrium current futures prices induced by changes in the deduction level perceived by the producer/hedger are incorporated in computing pure speculators welfare. Thus, the pure speculator's welfare computed by using after-tax measures are changed with changes in deduction levels.

6.7.3 Consumers' Welfare Change

Table 6.10 shows the mean of the aggregate consumers' welfare changes in terms of expected consumers' surplus. The number in each cell of Table 6.10 reflects the (average) increment of consumers' gains relative to the case of t = 0 and d = 0 as a base. For example, given the marginal tax rate of 0.3 (30%), if the deduction level increases from 0% to 100%, then aggregated consumers' welfare increases by $2,157,211 ($10,435,423 - $8,278,212).[97] As argued earlier, this measure of

Table 6.10 Mean of Expected Consumers' Welfare Changes

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	5865338	5865338	5865338	5865338	5865338	5865338	5865338	5865338	5865338	5865338	5865338
0.1	6440476	6486345	6533398	6581679	6631234	6682112	6734365	6788047	6843214	6899925	6958244
0.2	7209033	7301955	7399791	7502922	7611771	7726801	7848527	7977523	8114424	8259942	8414871
0.3	8278212	8419415	8572102	8737662	8917707	9114123	9329122	9565319	9825825	10114362	10435423
0.4	9845597	10036355	10248335	10485091	10750988	11051435	11393209	11784906	12237567	12765589	13388049
0.5	12312213	12553849	12829980	13148123	13518051	13952685	14469424	15092187	15854665	16805650	18018153
0.6	16607745	16901634	17247239	17658654	18155357	18764956	19527737	20504556	21791208	23546273	26049243

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[97] Appendix 6.9 shows tables for different λ values ranging from 0.00001 to 1000. As λ increases from 0.00001 up to 0.01, the "*negative*" means of optimal values decrease in absolute value, still preserving the qualitative results shown above. The changes in the consumers' welfare become smaller since the changes in the optimal positions are reduced as λ increases. Beyond λ value of 0.01, the changes in the optimal positions are so small that the changes in consumers' welfare are negligible.

consumers' welfare in terms of the expected consumers' surplus can be viewed as a suitable welfare criterion since the errors committed by the use of consumers' surplus are relatively small (Willig, 1976).

As shown in Table 6.10, increases in the marginal tax rate and/or deduction levels increase the mean of consumers' welfare. This result, however, does not mean that every consumers' surplus computed each week is increased. When the expected cash price exceeds the marginal costs of feeding, the cattle producer is supposed to take long cash positions, to bring more feeder cattle into feedlot. Given a downward-sloping demand schedule, the expected cash price at t = 1 would drop. In such a case, the consumers' welfare gains are always positive, implying consumers become better off, in the aggregate sense, due to the increases in tax rate and/or deduction level.

Contrary to this case, suppose that the expected cash price does not cover marginal costs. Then, the cattle producer would take a short cash position by reducing the number of feeder cattle placed into the feedlot. Consequently, the equilibrium (expected) cash price is expected to increase relative to the marginal costs. In this case, consumers' welfare gains are negative and they become worse off in the aggregate. Note that pricing efficiency in the cash market may still be improved in this case of increased equilibrium cash price and decreased cash positions.

As Table 6.10 shows, the magnitude of changes in consumers' welfare in response to changes in the marginal tax rate is larger than that in response to changes in the deduction level. The changes due to d are always reduced by the discounting impact of t so long as t is less than 1.0.

6.7.4 Social Welfare Change

Table 6.11 shows the mean of the social welfare over the period of 1983-1987.[98] As presented in Chapter 3, social welfare is obtained by summing individual agent's (approximate) measures of welfare (here, the cattle producer's, pure speculator's and consumers' welfare changes).

Table 6.11 Mean of Expected Social Welfare Changes

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	2507607	2507607	2507607	2507607	2507607	2507607	2507607	2507607	2507607	2507607	2507607
0.1	2764851	2786968	2809784	2833328	2857632	2882727	2908648	2935432	2963116	2991740	3021348
0.2	3136114	3181038	3228878	3279886	3334346	3392571	3454911	3521758	3593552	3670787	3754018
0.3	3694590	3763036	3838326	3921389	4013317	4115397	4229161	4356437	4499424	4660786	4843770
0.4	4581189	4673891	4779300	4899808	5038394	5198809	5385821	5605566	5866053	6177883	6555328
0.5	6094941	6212647	6351106	6515406	6712288	6950829	7243484	7607711	8068584	8663140	9447936
0.6	8961557	9105016	9279737	9495322	9765336	10109448	10557025	11153484	11972055	13136975	14872783

As shown in Table 6.11, increases in the marginal tax rate and/or deduction level increase the mean of social welfare. That is, if the tax authority announces an increase in marginal tax rate and/or deduction level, this announcement could generate a social welfare gain for cattle producers, pure speculators, and consumers viewed collectively. Here again, this finding does not necessarily imply that every social welfare computed each week is increased. In addition, note that this is only an

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[98] Appendix 6.10 shows tables for different λ values ranging from 0.00001 to 1000. As λ increases from 0.0001 up to 0.01, the "negative" means of optimal values decrease in absolute value, still preserving the qualitative results shown above. The changes in the social welfare become smaller since the changes in the optimal positions are reduced as λ increases. Beyond a λ value of 0.01, the changes in the optimal positions are so small that the changes in social welfare are negligible.

approximate measure of potential benefits, since nothing guarantees that those who lose are compensated by those who gain. In such a case, some groups could be worse off as a result of the changed tax policy.

The magnitude of changes in social welfare in response to changes in the marginal tax rate is again larger than that in response to changes in deduction level.

6.7.5 Tax Revenue Change

Table 6.12 shows the mean of the total tax revenues over the period of 1983-1987.[99] This result is obtained by assuming that only the optimal cash and futures positions are adjusted in response to changes in marginal tax rate and/or deduction level. In addition, it is assumed that there is no tax on consumers' consumption and thus the tax receipts are generated only from producers' and pure speculators' expected profits.

First, consider the short-run case in which only cash and futures positions are assumed to change in response to changes in tax parameters. As shown in Table 6.12, increases in the marginal tax rate and/or deduction level increase the (positive) mean of the total tax receipts. As presented earlier, increases in the marginal tax rate and/or deduction level increases the optimal cash and futures positions in absolute value.

---

[99] Appendix 6.11 shows tables for different λ values ranging from 0.00001 to 1000. As λ increases from 0.000001 up to 0.01, the "*positive*" means of optimal values decrease in absolute value, still preserving the qualitative results shown above. The changes in the expected tax revenues become smaller since the changes in the optimal positions are reduced as λ increases. Beyond λ value of 0.01, the changes in the optimal positions are so small that the changes in the expected tax revenues are negligible.

Table 6.12 Mean of Expected Tax Revenues For Short-Run Case

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0	0	0	0	0	0	0	0	0	0	0
0.1	353583	355254	356958	358698	360474	362288	364140	366032	367965	369941	371960
0.2	754213	760964	767996	775327	782977	790967	799321	808062	817220	826825	836910
0.3	1222053	1237399	1253724	1271126	1289714	1309615	1330971	1353950	1378743	1405573	1434703
0.4	1790708	1818273	1848236	1880923	1916723	1956103	1999628	2047989	2102039	2162846	2231761
0.5	2520666	2564191	2612553	2666603	2727410	2796325	2875084	2965961	3071983	3197282	3347641
0.6	3532907	3596250	3668230	3750744	3846287	3958209	4091115	4251520	4448941	4697863	5021462

For given cash and futures prices and marginal costs, these increases in the optimal positions increase the total expected revenues and thus the expected profits of cattle producers and pure speculators. In this case, it is obvious that tax receipts by the tax authority increase in response to an increase in the marginal tax rate and/or deduction level. The magnitude of changes in the total tax revenues in response to changes in marginal tax rate is larger than that in response to changes in deduction level.

Second, consider the segregation result under which the equilibrium prices are supposed to adjust in response to changes in the optimal positions induced by changed tax parameters. Compared to the above case, when the equilibrium cash and futures prices as well as the optimal cash and futures positions are assumed to adjust in response to the changes in tax rate and/or deduction level, the result is reversed. As shown in Table 6.13, increases in marginal tax rate and/or deduction level decrease the (negative) mean of the total tax revenues.[100] These negative values

[100] Appendix 6.12 shows tables for different λ values ranging from 0.00001 to 1000. As λ increases from 0.00001 up to 0.01, the "positive" means of optimal values decrease in absolute value, still preserving the qualitative results shown above. The changes in the expected tax revenues become smaller since the changes in the optimal positions are reduced as

Table 6.13 Mean of Expected Tax Revenues Under Segregation Result

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0	0	0	0	0	0	0	0	0	0	0
0.1	-391743	-396094	-400583	-405217	-410001	-414942	-420046	-425321	-430774	-436414	-442249
0.2	-932439	-950118	-968951	-989039	-1010494	-1033440	-1058017	-1084381	-1112705	-1143187	-1176048
0.3	-1734570	-1774982	-1819458	-1868552	-1922913	-1983310	-2050655	-2126039	-2210773	-2306446	-2414998
0.4	-3023437	-3096437	-3179497	-3274517	-3383865	-3510522	-3658279	-3832019	-4038113	-4284999	-4584042
0.5	-5295367	-5411277	-5547723	-5709760	-5904082	-6139713	-6429037	-6789419	-7245809	-7835080	-8613558
0.6	-9798331	-9967956	-1.0E+07	-1.0E+07	-1.1E+07	-1.1E+07	-1.2E+07	-1.2E+07	-1.3E+07	-1.5E+07	-1.7E+07

of the expected tax revenues is viewed as potential tax losses by the tax authority, which results from negative expected profits of the cattle producer on average. As argued earlier, this result can be altered if the elasticities being used for cash and futures demand were to change.

6.8 Chapter Summary

This chapter presented empirical results to test the hypotheses developed in Chapter 1. In this section, the following highlighted results are summarized. First, optimal positions in response to changes in tax policy were considered. An increase in the marginal tax rate and/or deduction level increases the optimal cash positions taken by producers/hedgers in absolute value, *ceteris paribus*. As risk aversion increases, the sizes of the optimal cash positions are reduced, but the qualitative results in terms of direction of impact are preserved. Similarly, an increase in the marginal tax rate and/or deduction level increases the optimal futures positions taken by producers/hedgers in absolute value, *ceteris paribus*. As risk aversion increases, the sizes of

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λ increases. Beyond a λ value of 0.01, the changes in the optimal positions are so small that the changes in the expected tax revenues are negligible.

the optimal futures positions are reduced, but the qualitative results are preserved. These results have an important implication for pricing efficiency and effectiveness in cash and futures markets. With increased activity and liquidity, information from a broader sample of cash firms is involved in the price discovery process.

Second, equilibrium prices in response to changes in tax policy were considered. An increase in the marginal tax rate and/or deduction level decreases the means of expected equilibrium cash prices over the sample period. As price responses become more flexible, the impacts of changes in tax policy become smaller. The decrease in the expected cash price has an important implication for improvement in consumers' welfare since consumer buying prices are reduced. Similarly, an increase in the marginal tax rate decreases the means of expected equilibrium current futures prices over the sample period, while an increase in the deduction level slightly increases the means of current futures prices.

Third, welfare changes in response to changes in tax policy were considered. For producers, an increase in the marginal tax rate and/or deduction level decreases the negative means of producer's welfare over the sample period, given a non-flexible inverse demand schedule (inelastic in the context of a quantity-dependent model). For a flexible (elastic demand) inverse demand schedule, the positive means of producer's welfare increases in response to an increase in the tax rate and/or deduction level. As price responses become more flexible, the impacts of changes in tax policy become smaller. For consumers, an increase in the marginal tax rate and/or deduction level increases the positive means of consumers' welfare over the sample period whether the inverse demand schedule is flexible or not. As price responses become more flexible, the consumers' welfare changes are still positive but become smaller. For society, an increase in the marginal tax rate and/or deduction level increases the positive means of social welfare (sum of cattle producers', pure speculators', and consumers' welfare changes) over the sample period.

Finally, tax revenue changes in response to tax policy were considered. An increase in the marginal tax rate and/or deduction level decreases the negative means of expected tax revenues over the sample period, given a not flexible (inelastic) demand schedule. If an inverse demand becomes flexible (elastic quantity-dependent demand), then the result will be altered. The negative means of expected tax revenues become positive and increase in response to an increase in the tax rate and/or deduction level. These changes in the means come from positive producers' profits, the expected result for a flexible inverse demand structure.

CHAPTER SEVEN : SUMMARY AND CONCLUSIONS

7.1 Introduction

This research focused on the examination of the impact of changes in tax policy on cattle feeders' behavior, the price discovery process, and the effectiveness of the cash cattle and futures markets in correcting market imbalances or disequilibrium situations. The research began with a description of cattle feeders' economic environment and IRS tax policy. Chapter 2 reviewed the literature on the issues related to mean-variance analysis, performances of cash and futures markets, and production decisions in conjunction with futures markets. In Chapter 3, a theoretical model was presented to analyze the impact of changes in tax parameters on the optimizing behaviors of individual economic agents and on their welfare changes. In Chapter 4, under a revenue-neutral tax scheme, comparative static analyses were performed to examine marginal impacts of changes in tax parameters on the optimal positions and the distributions of expected profits. Chapter 5 presented the description of data used in an empirical analysis and procedures to compute the variables of interest. In Chapter 6, empirically computed results were presented.

In this chapter, the empirical results from Chapter 6 and the theoretical results from Chapter 3 are summarized. Based on these results, implications and conclusions are drawn. A brief policy implication statement is also provided.

7.2 Simulation Analysis In Chapter 6 Based on Chapter 3

This section summarizes the simulation results shown in Chapter 6. Based on the results, implications are presented to support the hypotheses developed in Chapter 1.

Optimal Positions In response to Changes in Tax Policy:

- An increase in the marginal tax rate and/or deduction level increases the optimal cash positions taken by producers/hedgers in absolute value, *ceteris paribus*. As risk aversion increases, the sizes of the optimal cash positions are reduced, preserving the qualitative results. For pricing efficiency in the cash market, this adjustment of the optimal cash position reduces the spreads between expected cash prices and marginal costs, correcting the type of market imbalances defined in Chapter 2. Concerning consumers' welfare, the increased supply of fed cattle along with reduced prices implies a welfare gain for consumers in an aggregate sense.

- An increase in the marginal tax rate and/or deduction level increases the optimal futures positions taken by producers/hedgers in absolute value, *ceteris paribus*. As risk aversion increases, the sizes of the optimal futures positions are reduced, preserving the qualitative results. For informational efficiency, these increased futures positions (in absolute value) provide more liquidity in live cattle futures market. For pricing effectiveness in the futures market, this adjustment of the optimal futures positions reduces the futures spreads, $[E(F_1) - F_0 - c'(x_f)]$, correcting the imbalances or disequilibrium in the futures market described in detail in Chapter 2.

Equilibrium Prices In response to Changes in Tax Policy:

- An increase in the marginal tax rate and/or deduction level decreases the means of expected equilibrium cash prices over the sample period. As price responses become more flexible, the impacts of changes in tax policy become smaller. The decrease in the expected cash price has an important implication for improvement in consumers' welfare since consumer buying prices are reduced. Combined with the result of the optimal cash positions, this finding also has an important implication for pricing efficiency in the cash market. For the weeks during which $E(S_1) > c'(x_s)$, an increase in the tax rate and/or deduction level pushes down the equilibrium cash price to reduce the spreads between $E(S_1)$ and $c'(x_s)$. When $E(S_1) <$

$c'(x_s)$, the cash price is boosted by increases in the tax rate and/or deduction level. This reduced gap implies that the expected margins (price *vs.* costs) of cattle feeding become smaller in absolute value, and that the market moves toward a state of relatively balance or equilibrium more effectively over time.

- An increase in the marginal tax rate decreases the means of expected equilibrium current futures prices over the sample period, while an increase in the deduction level slightly increases the means. Combined with the result of the optimal futures positions, this finding has an important implication for pricing effectiveness in the futures market. When short (long) futures positions are increased in absolute value, the current futures price decreases (increases) to reduce or constrain futures price moves above equilibrium levels. Futures prices turn into an unbiased predictor of corresponding expected cash price, or the futures market predictors become more unbiased. This increases the effectiveness of the market as a price discovery mechanism.

Pricing Efficiency In response to Changes in Tax Policy:

- An increase in the marginal tax rate and/or deduction level decreases the means of the spreads between the expected cash prices and marginal costs over the sample period. As price responses become more flexible, the impacts of changes in tax policy become smaller. Concerning the futures spreads, an increase in the tax rate decreases the gaps, while an increase in the deduction level slightly increases the gaps. However, these results do not imply that all weekly values are increased or decreased in response to increases in the tax rate and/or deduction level. The adjustments of the optimal cash and futures positions reduce the magnitudes of departures from equilibrium and thereby improve pricing efficiency and effectiveness in both cash and futures markets.

Welfare Changes In response to Changes in Tax Policy:

- An increase in the marginal tax rate and/or deduction level decreases the negative means of producer's welfare over the sample period, given a non-flexible demand schedule (inelastic in the context of a quantity-dependent model). For a flexible (elastic demand) inverse demand schedule, the positive means of producer's welfare increases in response to an increase in the tax rate and/or deduction level. As price responses become more flexible, the impacts of changes in tax policy become smaller.

- An increase in the marginal tax rate and/or deduction level increases the positive means of consumers' welfare over the sample period whether the inverse demand schedule is flexible or not. As price responses become more flexible, the consumers' welfare changes are still positive but become smaller.

- An increase in the marginal tax rate and/or deduction level increases the positive means of social welfare (sum of cattle producers', pure speculators', and consumers' welfare changes) over the sample period. For a less flexible inverse demand schedule, the welfare losses from a producer are translated into welfare gains by consumers. For a flexible inverse demand, the producer's welfare changes become positive and increase in response to an increase in the tax rate and/or deduction level, while the consumers' welfare changes are still positive but become smaller. Thus, individual agent's (producers or consumers) welfare changes are sensitive to the flexibility of quantity demanded.

Tax Revenues In response to Changes in Tax Policy:

- An increase in the marginal tax rate and/or deduction level decreases the negative means of expected tax revenues over the sample period, given a less flexible demand schedule. This negative values of the expected tax revenues are viewed as potential tax losses by the tax authority, which results from negative expected profits of the producers on average.

- If an inverse demand becomes flexible, then the above result will be altered. That is, the negative means of expected tax revenues become positive and increase in response to an increase in the tax rate and/or deduction level. These changes in the means result from positive producers' profits which are the expected result for a flexible inverse demand.

7.3 Comparative Static Analysis In Chapter 4

This section summarizes the comparative analyses presented in Chapter 4. Note that these analyzes are based on the exemplary case of $E(\pi_s) \geq 0$ and $E(\pi_f) < 0$. Some implications are drawn based on the theoretical analyses. These comparative static analyses examine other theoretical aspects of the issues of interest which could not be evaluated from the empirical results in Chapter 6.

Marginal Changes in the Optimal Positions:

- Under a kind of semi-full provision[101], the impact of marginal tax rates on the optimal cash position is shown to be ambiguous, since an increase in the tax rate reduces the mean of cash profits and decreases the variance of cash profits. However, an increase in the tax rate will increase futures positions because it will improve the negative futures profits and reduce the variance of those profits.

- For deductibility, an increase in deduction level will have no impact on cash positions because the mean and variances of cash profits are not influenced by a change in deduction levels. However, increased deductions will increase the expected futures profits and reduce the variance of futures profits, and thereby cause the hedger to increase futures positions.

Revenue-Neutral Marginal Changes in the Optimal Positions:

- An increase in the marginal tax rate in a tax revenue-neutral manner leaves the expected levels of cash, futures, and total profits unchanged, but reduces the variances of the profits. This will cause a producer/hedger to increase both cash and futures positions.

- An increase in the deduction level has no impact on the expected levels of cash, futures and thus total profits. The variance of futures profits is reduced while the variance of cash profits remains unchanged. Thus, a producer/hedger is expected to increase futures positions and to leave cash positions unchanged.

Marginal Changes in the Tax Revenues:

- The impact of marginal tax rates on the mean of tax revenues is ambiguous because the increase in the tax rate will increase positive cash receipts and decrease negative futures receipts. A negative receipt in the futures markets is made more negative. It is redundant to examine the corresponding impact under a revenue-neutral tax policy since the revenue-neutral scheme itself constrains the expected level of fiscal revenues to remain unchanged.

- The impact of deduction levels on the tax revenues is expected to be negative. A negative futures market receipt becomes more negative. Under a revenue-neutral tax scheme, the corresponding effects on the

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[101] As noted in section 4.4, a "semi-full offset provision" refers to a tax scheme in which deductions of losses are discounted by deductions, and losses are therefore not fully deducted.

mean of tax revenues become zero. When there is a change in the deduction level, the marginal tax rate is adjusted such that the expected levels of tax revenues remain unchanged.

Risk-Sharing Aspect of Revenue-Neutral Tax Policy:

- The mean of total profits and the mean of tax payments move in opposite directions as a result of an increase in the marginal tax rate. In such a case, the variance of total profits and the variance of tax payments also move in opposite directions in both the cash and futures markets. This implies that imposing revenue neutrality causes fixed expected levels of profits and tax receipts, decreased variance of profits, and increased variance of tax receipts. Thus, a revenue-neutral change in the tax rate would transfer part of the risk in profits faced by producers to the government sector in the form of more risky tax revenue flows.

- Analogous to the case of increased tax rate, the mean of profits and the mean of tax payments move in opposite directions with increases in the deduction level. This holds as well for the variances of profits and tax payments. Under a revenue-neutral tax policy, it is expected that an increase in the deduction level causes the variance of profits to decrease and the variance of tax receipts to increase. Thus, the transfer of risk in profits is again from the producer to the government sector.

7.4 Conclusions

It was implicitly assumed that cattle feeders have access to superior information related to cattle feeding such as feeding costs. In addition, they are in a position to use that information without extended time delay. Cattle feeders would therefore be able to inject the influence of very current and specific information on costs of feeding numbers of cattle on feed, when cattle will be ready for market, etc. into trading levels for cattle futures. This is not to imply that they would be controlling price, but that collectively their behavior may make price discovery process more efficient and more effective.

Well-informed producers can establish forward prices by hedging their slaughter cattle through the Chicago Mercantile Exchange (CME) live cattle futures contracts. Also, since they have a strong business-related interest in the markets, producers could act to minimize the duration and magnitude of the market imbalances when no profitable hedges are being offered or could be reasonably anticipated. They would thus help the markets restore equilibrium, with their trading activity moving the markets back toward equilibrium.

In spite of the immediate availability of the information and incentives to participate in the price discovery process, cattle feeders are effectively denied the opportunity to get involved in the markets to correct market imbalances and disequilibrium situations. This is true especially in those instances when only negative margins are being offered and the feeders cannot enter the markets as short hedgers. The obstacles to participation arise from the tax treatment of what would be seen as speculative trades by the IRS.

The empirical results of this study have shown that given a marginal tax rate, an increase in the probability of deduction for losses from speculation (as IRS now enforces tax policy) would increase both the optimal cash and futures positions. For the cash market, this adjustment of the optimal cash positions reduces the spreads between the expected cash prices and marginal costs. This implies that pricing efficiency in the cash market is improved in response to an increase in the deduction level for a given tax rate. Analogous to the cash market, the adjustment of the optimal futures positions has an important implication for pricing effectiveness in the futures market. The amplitude and duration of departures from the underlying equilibrium would be reduced if more of the now-defined speculative losses were deductible.

Assuming an inflexible inverse demand schedule, an increase in the deduction level for a given tax rate would increase consumers' welfare and decrease producers' welfare. The expected changes in tax revenues are negative and mean levels of tax revenue decrease as the deduction level increases. Considering a more flexible inverse demand, producers' welfare increases in response to an increased deduction level for a given tax rate. The expected tax revenues become positive and increase as the deduction level increases.

The theoretical comparative static analyses supported the above more nearly empirical findings. Based on the comparative static analyses, it could be argued that a revenue-neutral marginal increase in the deduction level for a given tax rate would result in the transfer of risk from the cattle industry to the government sector. This has an important implication for risk-sharing aspects of a revenue-neutral tax policy. Tax revenue flows would become more variable, but the mean levels increase as deduction levels increase.

If cattle feeders were able to participate using trading programs that react to departures from the underlying (and unknown) equilibrium, they would tend to establish positions that would push the market back toward the equilibrium position more quickly and more effectively. They will, it could be argued, be able to recognize the departure from equilibrium more quickly as the feeding margins become more negative or more positive. Cattle feeders are involved in the market related activities daily, have access to timely and proprietary information, and are in, perhaps, a superior position to correct a market imbalance. When extreme negative margins are being offered and cash cattle feeding programs do not appear economically viable or sensible, cattle feeders would tend to sell nearby feeder cattle futures and buy the distant live cattle futures if they were allowed by changed tax policy to be fully involved in the price discovery process. These actions would tend to block continued moves to negative margins and decrease the duration and/or magnitude of any string of negative margins.

As discussed in Chapter 1, prolonged market imbalances tend to prompt highly variable placements of cattle and create variability in fed cattle prices. Such variability implies a possible social loss associated with any policy position, such as the IRS position, which could impose costs on every economic agent in the markets, from producer to consumer. Also, the economic viability of producers' business investments is influenced in a significant and negative way by those market imbalances. Therefore, more active participation by cattle feeders could be important to the overall effectiveness and efficiency of trade in cattle futures.

7.5 Policy Implications

Prolonged imbalances between feeder cattle costs and the pricing opportunities offered by live cattle futures would be seen as evidence of inefficient markets by many research analysts. But that view may be too narrow and too restrictive. As prices are being discovered, the quality of the information base and the effectiveness of the traders as market analysts are closely related to the efficiency of the cattle futures markets. What appears to be an inefficient market may result from any policy position, such as the IRS position, that blocks participation of well-informed participants such as feedlot owners/managers in correcting market imbalances. As a result, the policy could constrain the effectiveness of the price discovery process in the cattle futures markets. It also generates pricing patterns and market behavior that can be presented as empirical evidence of market inefficiency.

To the extent that cattle feeders are effectively blocked from trading in futures other than hedging trades as ruled by the IRS position, they are not allowed to participate in correcting the market imbalances. More research is needed to confirm that the markets are more volatile and less efficient in the price discovery process than they could be. But it is logical to expect that cattle feeders are concerned about the tax treatments over losses on speculative trades constrained by the IRS policy and this concern was verified by a 1991 survey conducted by Purcell. IRS policy thus appears to have at least a marginal effect on constraining and/or reducing the efficiency of the price discovery process for cattle. That is, the IRS position has a "chilling impact[102]" on cattle feeders

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[102] Although cattle feeders are concerned about the tax treatment over the losses from speculative trades, they, in practice, would be and should be forced to participate as speculators when the futures markets provide no profitable opportunities from cattle feeding (Purcell, February 1992). In other words, cattle feeders are reluctant to take "reverse" positions (long live cattle futures and/or short feeder cattle futures) which are treated as speculative trades under the current IRS interpretations.

participation as speculators under current IRS position and interpretations. If this is the case, legislative or administrative action to correct the current policies of the IRS should be considered.

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Appendix 3.1.1 A Scenario of $E(S_1) - S_0 > 0$ and $E(F_1) - F_0 > 0$

	Cash Market	Futures Market
Price Expectation:	$E(S_1) - S_0 > 0$	$E(F_1) - F_0 > 0$
Action Taken at t=0:	Long Cash	Long Futures
Price Realization:	$S_1 - S_0 > 0$	$F_1 - F_0 > 0$
Realized Profit at t=1:	$\pi_s > 0$	$\pi_f > 0$
Current Tax Treatment:	Ordinary Gain	Capital Gain
Total Profits Adjusted by Current Taxation:		$\pi_s + \pi_f - t(\pi_s + \pi_f)$
Total Profits Adjusted by Corrected Taxation:		$\pi_s + \pi_f - t(\pi_s + \pi_f)$

Price Expectation:	$E(S_1) - S_0 > 0$	$E(F_1) - F_0 > 0$
Action Taken at t=0:	Long Cash	Long Futures
Price Realization:	$S_1 - S_0 > 0$	$F_1 - F_0 < 0$
Realized Profit at t=1:	$\pi_s > 0$	$\pi_f < 0$
Current Tax Treatment:	Ordinary Gain	Capital Loss
Total Profits Adjusted by Current Taxation:		$\pi_s + \pi_f - t\pi_s$
Total Profits Adjusted by Corrected Taxation:		$\pi_s + \pi_f - t(\pi_s + d\pi_f)$

Price Expectation:	$E(S_1) - S_0 > 0$	$E(F_1) - F_0 > 0$
Action Taken at t=0:	Long Cash	Long Futures
Price Realization:	$S_1 - S_0 < 0$	$F_1 - F_0 > 0$
Realized Profit at t=1:	$\pi_s < 0$	$\pi_f > 0$
Current Tax Treatment:	Ordinary Loss	Capital Gain
Total Profits Adjusted by Current Taxation:		$\pi_s + \pi_f - t(d\pi_s + \pi_f)$
Total Profits Adjusted by Corrected Taxation:		$\pi_s + \pi_f - t(d\pi_s + \pi_f)$

Price Expectation:	$E(S_1) - S_0 > 0$	$E(F_1) - F_0 > 0$
Action Taken at t=0:	Long Cash	Long Futures
Price Realization:	$S_1 - S_0 < 0$	$F_1 - F_0 < 0$
Realized Profit at t=1:	$\pi_s < 0$	$\pi_f < 0$
Current Tax Treatment:	Ordinary Loss	Capital Loss
Total Profits Adjusted by Current Taxation:		$\pi_s + \pi_f - td\pi_s$
Total Profits Adjusted by Corrected Taxation:		$\pi_s + \pi_f - t(d\pi_s + d\pi_f)$

where t = marginal tax rate, $t \in (0, 1)$, and
d = deduction rate, $d \in (0, 1)$.

Appendix 3.1.2 A Scenario of $E(S_1) - S_0 > 0$ and $E(F_1) - F_0 < 0$

	Cash Market	Futures Market
Price Expectation:	$E(S_1) - S_0 > 0$	$E(F_1) - F_0 < 0$
Action Taken at t=0:	Long Cash	Short Futures
Price Realization:	$S_1 - S_0 > 0$	$F_1 - F_0 > 0$
Realized Profit at t=1:	$\pi_s > 0$	$\pi_f < 0$
Current Tax Treatment:	Ordinary Gain	Ordinary Loss
Total Profits Adjusted by Current Taxation:		$\pi_s + \pi_f - t(\pi_s + d\pi_f)$
Total Profits Adjusted by Corrected Taxation:		$\pi_s + \pi_f - t(\pi_s + d\pi_f)$

Price Expectation:	$E(S_1) - S_0 > 0$	$E(F_1) - F_0 < 0$
Action Taken at t=0:	Long Cash	Short Futures
Price Realization:	$S_1 - S_0 > 0$	$F_1 - F_0 < 0$
Realized Profit at t=1:	$\pi_s > 0$	$\pi_f > 0$
Current Tax Treatment:	Ordinary Gain	Ordinary Gain
Total Profits Adjusted by Current Taxation:		$\pi_s + \pi_f - t(\pi_s + \pi_f)$
Total Profits Adjusted by Corrected Taxation:		$\pi_s + \pi_f - t(\pi_s + \pi_f)$

Price Expectation:	$E(S_1) - S_0 > 0$	$E(F_1) - F_0 < 0$
Action Taken at t=0:	Long Cash	Short Futures
Price Realization:	$S_1 - S_0 < 0$	$F_1 - F_0 > 0$
Realized Profit at t=1:	$\pi_s < 0$	$\pi_f < 0$
Current Tax Treatment:	Ordinary Loss	Ordinary Loss
Total Profits Adjusted by Current Taxation:		$\pi_s + \pi_f - t(d\pi_s + d\pi_f)$
Total Profits Adjusted by Corrected Taxation:		$\pi_s + \pi_f - t(d\pi_s + d\pi_f)$

Price Expectation:	$E(S_1) - S_0 > 0$	$E(F_1) - F_0 < 0$
Action Taken at t=0:	Long Cash	Short Futures
Price Realization:	$S_1 - S_0 < 0$	$F_1 - F_0 < 0$
Realized Profit at t=1:	$\pi_s > 0$	$\pi_f > 0$
Current Tax Treatment:	Ordinary Loss	Ordinary Gain
Total Profits Adjusted by Current Taxation:		$\pi_s + \pi_f - t(d\pi_s + \pi_f)$
Total Profits Adjusted by Corrected Taxation:		$\pi_s + \pi_f - t(d\pi_s + \pi_f)$

where t = marginal tax rate, $t \in (0, 1)$, and
d = deduction rate, $d \in (0, 1)$.

Appendix 3.1.3 A Scenario of $E(S_1) - S_0 < 0$ and $E(F_1) - F_0 > 0$

	Cash Market	Futures Market
Price Expectation:	$E(S_1) - S_0 < 0$	$E(F_1) - F_0 > 0$
Action Taken at t=0:	Short Cash	Long Futures
Price Realization:	$S_1 - S_0 > 0$	$F_1 - F_0 > 0$
Realized Profit at t=1:	$\pi_s < 0$	$\pi_f > 0$
Current Tax Treatment:	Ordinary Loss	Capital Gain
Total Profits Adjusted by Current Taxation:		$\pi_s + \pi_f - t(d\pi_s + \pi_f)$
Total Profits Adjusted by Corrected Taxation:		$\pi_s + \pi_f - t(d\pi_s + \pi_f)$

Price Expectation:	$E(S_1) - S_0 < 0$	$E(F_1) - F_0 > 0$
Action Taken at t=0:	Short Cash	Long Futures
Price Realization:	$S_1 - S_0 > 0$	$F_1 - F_0 < 0$
Realized Profit at t=1:	$\pi_s > 0$	$\pi_f < 0$
Current Tax Treatment:	Ordinary Loss	Capital Loss
Total Profits Adjusted by Current Taxation:		$\pi_s + \pi_f - td\pi_s$
Total Profits Adjusted by Corrected Taxation:		$\pi_s + \pi_f - t(d\pi_s + d\pi_f)$

Price Expectation:	$E(S_1) - S_0 < 0$	$E(F_1) - F_0 > 0$
Action Taken at t=0:	Short Cash	Long Futures
Price Realization:	$S_1 - S_0 < 0$	$F_1 - F_0 > 0$
Realized Profit at t=1:	$\pi_s > 0$	$\pi_f > 0$
Current Tax Treatment:	Ordinary Gain	Capital Gain
Total Profits Adjusted by Current Taxation:		$\pi_s + \pi_f - t(\pi_s + \pi_f)$
Total Profits Adjusted by Corrected Taxation:		$\pi_s + \pi_f - t(\pi_s + \pi_f)$

Price Expectation:	$E(S_1) - S_0 < 0$	$E(F_1) - F_0 > 0$
Action Taken at t=0:	Short Cash	Long Futures
Price Realization:	$S_1 - S_0 < 0$	$F_1 - F_0 < 0$
Realized Profit at t=1:	$\pi_s > 0$	$\pi_f < 0$
Current Tax Treatment:	Ordinary Gain	Capital Loss
Total Profits Adjusted by Current Taxation:		$\pi_s + \pi_f - t\pi_s$
Total Profits Adjusted by Corrected Taxation:		$\pi_s + \pi_f - t(\pi_s + d\pi_f)$

where t = marginal tax rate, $t \in (0, 1)$, and
d = deduction rate, $d \in (0, 1)$.

Appendix 3.1.4 A Scenario of $E(S_1) - S_0 < 0$ and $E(F_1) - F_0 < 0$

	Cash Market	Futures Market
Price Expectation:	$E(S_1) - S_0 < 0$	$E(F_1) - F_0 < 0$
Action Taken at t=0:	Short Cash	Short Futures
Price Realization:	$S_1 - S_0 > 0$	$F_1 - F_0 > 0$
Realized Profit at t=1:	$\pi_s < 0$	$\pi_f < 0$
Current Tax Treatment:	Ordinary Loss	Ordinary Loss
Total Profits Adjusted by Current Taxation:		$\pi_s + \pi_f - t(d\pi_s + d\pi_f)$
Total Profits Adjusted by Corrected Taxation:		$\pi_s + \pi_f - t(d\pi_s + d\pi_f)$
Price Expectation:	$E(S_1) - S_0 < 0$	$E(F_1) - F_0 < 0$
Action Taken at t=0:	Short Cash	Short Futures
Price Realization:	$S_1 - S_0 > 0$	$F_1 - F_0 < 0$
Realized Profit at t=1:	$\pi_s < 0$	$\pi_f > 0$
Current Tax Treatment:	Ordinary Loss	Ordinary Gain
Total Profits Adjusted by Current Taxation:		$\pi_s + \pi_f - t(d\pi_s + \pi_f)$
Total Profits Adjusted by Corrected Taxation:		$\pi_s + \pi_f - t(d\pi_s + \pi_f)$
Price Expectation:	$E(S_1) - S_0 < 0$	$E(F_1) - F_0 < 0$
Action Taken at t=0:	Short Cash	Short Futures
Price Realization:	$S_1 - S_0 < 0$	$F_1 - F_0 > 0$
Realized Profit at t=1:	$\pi_s > 0$	$\pi_f < 0$
Current Tax Treatment:	Ordinary Gain	Ordinary Loss
Total Profits Adjusted by Current Taxation:		$\pi_s + \pi_f - t(\pi_s + d\pi_f)$
Total Profits Adjusted by Corrected Taxation:		$\pi_s + \pi_f - t(\pi_s + d\pi_f)$
Price Expectation:	$E(S_1) - S_0 < 0$	$E(F_1) - F_0 < 0$
Action Taken at t=0:	Short Cash	Short Futures
Price Realization:	$S_1 - S_0 < 0$	$F_1 - F_0 < 0$
Realized Profit at t=1:	$\pi_s > 0$	$\pi_f > 0$
Current Tax Treatment:	Ordinary Gain	Ordinary Gain
Total Profits Adjusted by Current Taxation:		$\pi_s + \pi_f - t(\pi_s + \pi_f)$
Total Profits Adjusted by Corrected Taxation:		$\pi_s + \pi_f - t(\pi_s + \pi_f)$

where t = marginal tax rate, $t \in (0, 1)$, and
d = deduction rate, $d \in (0, 1)$.

Appendix 3.2 Derivation of Optimal Cash and Futures Positions

Case 1: $E(\pi_s) \geq 0$ and $E(\pi_f) \geq 0$

The expected profit function is $E(\pi_s) + E(\pi_f) - t[E(\pi_s) + E(\pi_f)]$

$$\begin{aligned} \text{Max } & (1 - t)\cdot E(\pi_s|I_0) + (1 - t)\cdot E(\pi_f|I_0) \\ & - (\lambda/2)[(1 - t)^2\cdot x_s^2\sigma_s^2 + (1 - t)^2\cdot x_f^2\sigma_f^2 \\ & + 2(1 - t)(1 - t)\cdot x_s x_f \sigma_{sf}] \end{aligned}$$

By taking the derivatives of the objective function with respect to x_s and x_f, respectively, and setting the derivatives to zero, the first-order conditions for this problem are obtained:

$$\begin{aligned} (1) \quad & (1 - t)[E(S_1|I_0) - c'(x_s)] \\ & - \lambda[(1 - t)^2\sigma_s^2\cdot x_s + (1 - t)(1 - t)\sigma_{sf}\cdot x_f] = 0 \\ (2) \quad & (1 - t)[E(F_1|I_0) - F_0 - c'(x_f)] \\ & - \lambda[(1 - t)^2\sigma_f^2\cdot x_f + (1 - t)(1 - t)\sigma_{sf}\cdot x_s] = 0 \end{aligned}$$

After factoring out the nonzero constants $(1 - t)$ in (1) and (2), the expressions become

$$(1)' \quad [E(S_1|I_0) -'(x_s)] - \lambda[(1 - t)\sigma_s^2\cdot x_s + (1 - t)\sigma_{sf}\cdot x_f] = 0$$

$$(2)' \quad [E(F_1|I_0) - F_0 - c'(x_f)] - \lambda[(1 - t)\sigma_f^2\cdot x_f + (1 - t)\sigma_{sf}\cdot x_s] = 0$$

Because $c(x_s)$ and $c(x_f)$ are assumed to be convex, these conditions

are necessary and sufficient for a maximum, i.e.,

$$\partial^2\Pi/\partial x_s^2 = -c''(x_s) - \lambda(1 - t)\sigma_s^2 < 0,$$

$$\partial^2\Pi/\partial x_f^2 = -c''(x_f) - \lambda(1 - t)\sigma_f^2 < 0,$$

$$\partial^2\Pi/\partial x_s\partial x_f = -\lambda(1 - t)\sigma_{sf} < 0, \text{ and}$$

$$\partial^2\Pi/\partial x_f\partial x_s = -\lambda(1 - t)\sigma_{sf} < 0.$$

This development assumes that x_s^* and x_f^* are the nonzero, finite, and unique solutions to (1) and (2) and that second-order conditions are satisfied.

By solving the first-order conditions (1) and (2) for x_s and x_f, respectively, the beginning-of-the-period demand function for x_s and x_f are obtained:

$$(3) \quad x_s^* = \frac{[E(S_1|I_0) - c'(x_s)]}{(1 - t)\lambda\sigma_s^2} - \frac{(1 - t)\sigma_{sf}\cdot x_f}{(1 - t)\sigma_s^2}$$

$$(4) \quad x_f^* = \frac{[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - t)\lambda\sigma_f^2} - \frac{(1 - t)\sigma_{sf}\cdot x_s}{(1 - t)\sigma_f^2}$$

The optimal futures position (4) is inserted into the first order condition (3) and rearrange to obtain:

$$(5) \quad x_s^* = \frac{[E(S_1|I_0) - c'(x_s)] - (\sigma_{sf}/\sigma_f^2)[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - t)\cdot\lambda[\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2)]}$$

$$(6) \quad x_f^* = \frac{[E(F_1|I_0) - F_0 - c'(x_f)] - (\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - t)\cdot\lambda[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]}$$

Appendix 3.2 Continued.

Case 3: $E(\pi_s) < 0$ and $E(\pi_f) \geq 0$

The expected profit function is $E(\pi_s) + E(\pi_f) - t[d \cdot E(\pi_s) + E(\pi_f)]$

$$\text{Max } (1 - td) \cdot E(\pi_s | I_0) + (1 - t) \cdot E(\pi_f | I_0)$$
$$- (\lambda/2)[(1 - td)^2 \cdot x_s^2 \sigma_s^2 + (1 - t)^2 \cdot x_f^2 \sigma_f^2$$
$$+ 2(1 - td)(1 - t) \cdot x_s x_f \sigma_{sf}]$$

By taking the derivatives of the objective function with respect to x_s and x_f, respectively, and setting the derivatives to zero, the first-order conditions for this problem are obtained:

$$(1) \quad (1 - td)[E(S_1 | I_0) - c'(x_s)]$$
$$- \lambda[(1 - td)^2 \sigma_s^2 \cdot x_s + (1 - td)(1 - t)\sigma_{sf} \cdot x_f] = 0$$

$$(2) \quad (1 - t)[E(F_1 | I_0) - F_0 - c'(x_f)]$$
$$- \lambda[(1 - t)^2 \sigma_f^2 \cdot x_f + (1 - td)(1 - t)\sigma_{sf} \cdot x_s] = 0$$

After factoring out the nonzero constants (1 - td) in (1) and (1 - t) in (2), the expressions become

$$(1)' \quad [E(S_1 | I_0) - c'(x_s)] - \lambda[(1 - td)\sigma_s^2 \cdot x_s + (1 - t)\sigma_{sf} \cdot x_f] = 0$$
$$(2)' \quad [E(F_1 | I_0) - F_0 - c'(x_f)] - \lambda[(1 - t)\sigma_f^2 \cdot x_f + (1 - td)\sigma_{sf} \cdot x_s] = 0$$

Because $c(x_s)$ and $c(x_f)$ are assumed to be convex, these conditions

are necessary and sufficient for a maximum, i.e.,

$$\partial^2\Pi/\partial x_s^2 = -c''(x_s) - \lambda(1 - td)\sigma_s^2 < 0,$$

$$\partial^2\Pi/\partial x_f^2 = -c''(x_f) - \lambda(1 - t)\sigma_f^2 < 0,$$

$$\partial^2\Pi/\partial x_s\partial x_f = -\lambda(1 - t)\sigma_{sf} < 0, \text{ and}$$

$$\partial^2\Pi/\partial x_f\partial x_s = -\lambda(1 - t)\sigma_{sf} < 0.$$

This development assumes that x_s^* and x_f^* are the nonzero, finite, and unique solutions to (1) and (2) and that second-order conditions are satisfied.

By solving the first-order conditions (1) and (2) for x_s and x_f, respectively, the beginning-of-the-period demand function for x_s and x_f are obtained:

$$(3) \quad x_s^* = \frac{[E(S_1|I_0) - c'(x_s)]}{(1 - td)\lambda\sigma_s^2} - \frac{(1 - t)\sigma_{sf}\cdot x_f}{(1 - td)\sigma_s^2}$$

$$(4) \quad x_f^* = \frac{[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - t)\lambda\sigma_f^2} - \frac{(1 - td)\sigma_{sf}\cdot x_s}{(1 - t)\sigma_f^2}$$

The optimal futures position (4) is inserted into the first order condition (3) and rearrange to obtain:

$$(5) \quad x_s^* = \frac{[E(S_1|I_0) - c'(x_s)] - (\sigma_{sf}/\sigma_f^2)[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - td)\cdot\lambda[\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2)]}$$

$$(6) \quad x_f^* = \frac{[E(F_1|I_0) - F_0 - c'(x_f)] - (\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - t)\cdot\lambda[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]}$$

Appendix 3.2 Continued.

Case 4: $E(\pi_s) < 0$ and $E(\pi_f) < 0$

The expected profit function is $E(\pi_s) + E(\pi_f) - t[d \cdot E(\pi_s) + Ed(\pi_f)]$

$$\begin{aligned} \text{Max } & (1 - td) \cdot E(\pi_s | I_0) + (1 - td) \cdot E(\pi_f | I_0) \\ & - (\lambda/2)[(1 - td)^2 \cdot x_s^2 \sigma_s^2 + (1 - td)^2 \cdot x_f^2 \sigma_f^2 \\ & + 2(1 - td)(1 - td) \cdot x_s x_f \sigma_{sf}] \end{aligned}$$

By taking the derivatives of the objective function with respect to x_s and x_f, respectively, and setting the derivatives to zero, the first-order conditions for this problem are obtained:

$$\begin{aligned} (1) \quad & (1 - td)[E(S_1 | I_0) - c'(x_s)] \\ & - \lambda[(1 - td)^2 \sigma_s^2 \cdot x_s + (1 - td)(1 - td)\sigma_{sf} \cdot x_f] = 0 \\ (2) \quad & (1 - td)[E(F_1 | I_0) - F_0 - c'(x_f)] \\ & - \lambda[(1 - td)^2 \sigma_f^2 \cdot x_f + (1 - td)(1 - td)\sigma_{sf} \cdot x_s] = 0 \end{aligned}$$

After factoring out the nonzero constants (1 - td) in (1) and (2), the expressions become

$$(1)' \quad [E(S_1 | I_0) - c'(x_s)] - \lambda[(1 - td)\sigma_s^2 \cdot x_s + (1 - td)\sigma_{sf} \cdot x_f] = 0$$

$$(2)' \quad [E(F_1 | I_0) - F_0 - c'(x_f)] - \lambda[(1 - td)\sigma_f^2 \cdot x_f + (1 - td)\sigma_{sf} \cdot x_s] = 0$$

Because $c(x_s)$ and $c(x_f)$ are assumed to be convex, these conditions

are necessary and sufficient for a maximum, i.e.,

$$\partial^2\Pi/\partial x_s^2 \quad = -c''(x_s) - \lambda(1 - td)\sigma_s^2 < 0,$$

$$\partial^2\Pi/\partial x_f^2 \quad = -c''(x_f) - \lambda(1 - td)\sigma_f^2 < 0,$$

$$\partial^2\Pi/\partial x_s\partial x_f \quad = -\lambda(1 - td)\sigma_{sf} < 0, \text{ and}$$

$$\partial^2\Pi/\partial x_f\partial x_s \quad = -\lambda(1 - td)\sigma_{sf} < 0.$$

This development assumes that x_s^* and x_f^* are the nonzero, finite, and unique solutions to (1) and (2) and that second-order conditions are satisfied.

By solving the first-order conditions (1) and (2) for x_s and x_f, respectively, the beginning-of-the-period demand function for x_s and x_f are obtained:

$$(3) \quad x_s^* = \frac{[E(S_1|I_0) - c'(x_s)]}{(1 - td)\lambda\sigma_s^2} - \frac{(1 - td)\sigma_{sf}\cdot x_f}{(1 - td)\sigma_s^2}$$

$$(4) \quad x_f^* = \frac{[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - td)\lambda\sigma_f^2} - \frac{(1 - td)\sigma_{sf}\cdot x_s}{(1 - td)\sigma_f^2}$$

The optimal futures position (4) is inserted into the first order condition (3) and rearrange to obtain:

$$(5) \quad x_s^* = \frac{[E(S_1|I_0) - c'(x_s)] - (\sigma_{sf}/\sigma_f^2)[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - td)\cdot\lambda[\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2)]}$$

$$(6) \quad x_f^* = \frac{[E(F_1|I_0) - F_0 - c'(x_f)] - (\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - td)\cdot\lambda[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]}$$

Appendix 3.3 Derivation of Equilibrium Cash and Futures Prices

Case 1: For the producer's expected profit function of $E(\pi_s) + E(\pi_f) - t[E(\pi_s) + E(\pi_f)]$, the aggregate relationships describing the cash and futures markets are specified as follows:

[*Cash Demand by Consumers*]

(1) $C_1 = a - bS_1 + u_1$

[*Cash Supply by Producers*]

(5) $$X_s = \frac{[E(S_1|I_0) - c'(x_s)] - (\sigma_{sf}/\sigma_f^2)[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - t)\cdot\lambda_s[\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2)]}$$

[*Futures Demand by Producers*]

(6) $$X_f = \frac{[E(F_1|I_0) - F_0 - c'(x_f)] - (\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - t)\cdot\lambda_s[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]}$$

[*Futures Supply by Pure Speculators*]

(4) $$Z_f = \frac{[E(F_1|I_0) - F_0 - c'(z_f)]}{(1 - t)\lambda_f\sigma_f^2}$$

[*Futures Market Clearing*]

(7) $X_f + Z_f = 0.$

Substituting (4) and (6) into (7), yields the equilibrium current futures prices:

$$\frac{[E(F_1|I_0) - F_0 - c'(x_f)] - (\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - t)\cdot\lambda_s[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]} + \frac{[E(F_1|I_0) - F_0 - c'(z_f)]}{(1 - t)\lambda_f\sigma_f^2} = 0$$

Noting that $\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2) = \sigma_f^2(1 - \rho^2)$,

$$\frac{[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - t)\cdot\lambda_s\sigma_f^2(1 - \rho^2)} + \frac{[E(F_1|I_0) - F_0 - c'(z_f)]}{(1 - t)\lambda_f\sigma_f^2}$$

$$- \frac{(\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - t)\cdot\lambda_s\sigma_f^2(1 - \rho^2)} = 0$$

Assuming $c'(x_f) = c'(z_f)$,

$$\frac{[E(F_1|I_0) - F_0 - c'(x_f)][(1 - t)\lambda_f + (1 - t)\lambda_s(1 - \rho^2)]}{(1 - t)(1 - t)\cdot\lambda_s\lambda_f\sigma_f^2(1 - \rho^2)}$$

$$- \frac{(\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - t)\cdot\lambda_s\sigma_f^2(1 - \rho^2)} = 0$$

(10) $$F_0 = E(F_1|I_0) - c'(x_f) - \frac{(1 - t)\lambda_f(\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - t)\lambda_f + (1 - t)\lambda_s(1 - \rho^2)}$$

$$= E(F_1|I_0) - c'(x_f) - \frac{\lambda_f(\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{\lambda_f + \lambda_s(1 - \rho^2)} .$$

For the case of $E(S_1|I_0) = F_0$ as assumed before, denoting $A = \lambda_f + \lambda_s(1 - \rho^2)$ and $B = \lambda_f(\sigma_{sf}/\sigma_s^2)$,

$$F_0\cdot\frac{A + B}{A} = \frac{A\cdot[E(F_1|I_0) - c'(x_f)] + B\cdot c'(x_s)}{A}$$

$$F_0 = \frac{A\cdot[E(F_1|I_0) - c'(x_f)] + B\cdot c'(x_s)}{A + B}$$

$$= \frac{[\lambda_f + \lambda_s(1-\rho^2)][E(F_1|I_0)-c'(x_f)] + \lambda_f(\sigma_{sf}/\sigma_s^2)\cdot c'(x_s)}{\lambda_f + \lambda_s(1-\rho^2) + \lambda_f(\sigma_{sf}/\sigma_s^2)} .$$

In order to find the equilibrium insurance price (or risk-premium) $[E(F_1|I_0) - F_0 - c(x_f)]$, the futures market clearing condition is modified. The futures demand by producers in (6) can be divided into two parts:

$$(6)' \quad X_f = \frac{[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - t)\cdot\lambda_s[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]} - \frac{(\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - t)\cdot\lambda_s[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]}$$

Combined with the futures supply by pure speculators in (4), the first term of (6)′ represents the market "speculative" supply of futures contracts:

$$SX_f = \frac{[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - t)\cdot\lambda_s[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]} + \frac{[E(F_1|I_0) - F_0 - c'(z_f)]}{(1 - t)\lambda_f\sigma_f^2}$$

Noting that $\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2) = \sigma_f^2(1 - \rho^2)$ and assuming $c'(x_f) = c'(z_f)$,

[*Speculative Supply by Producers and Pure Speculators*]

$$(15) \quad SX_f = \frac{[E(F_1|I_0) - F_0 - c'(x_f)][(1 - t)\lambda_f + (1 - t)\lambda_s(1 - \rho^2)]}{(1 - t)(1 - t)\lambda_s\lambda_f\sigma_f^2(1 - \rho^2)}$$
$$= \frac{[E(F_1|I_0) - F_0 - c'(x_f)][\lambda_f + \lambda_s(1 - \rho^2)]}{(1 - t)\lambda_s\lambda_f\sigma_f^2(1 - \rho^2)} .$$

Analogous to the cash market, suppose that there exist a "hedge" demand for futures contracts, expressed by a function of insurance price:

[*Hedge Demand by Hedgers*]

$$(16) \quad DX_f = c - d[E(F_1|I_0) - F_0 - c'(x_f)] + v_t$$

The market-clearing insurance price or risk premium can be determined when the excess hedging demand by producers is equal to the excess speculative supply by producers and pure speculators:

[*Market-Clearing Insurance Price*]

(17) $DX_f = SX_f$.

Substituting (15) and (16) into the market-clearing insurance price in (17), yields the equilibrium insurance prices:

$$c - d[E(F_1|I_0) - F_0 - c'(x_f)] + v_t$$

$$= \frac{[E(F_1|I_0) - F_0 - c'(x_f)][\lambda_f + \lambda_s(1 - \rho^2)]}{(1 - t)\lambda_s\lambda_f\sigma_f^2(1 - \rho^2)}$$

Denoting $A = [\lambda_f + \lambda_s(1 - \rho^2)]$ and $E = (1 - t)\lambda_s\lambda_f\sigma_f^2(1 - \rho^2)$,

$$c - d[E(F_1|I_0) - F_0 - c'(x_f)] + v_t = (A/E)[E(F_1|I_0) - F_0 - c'(x_f)]$$

$$[d + (A/E)][E(F_1|I_0) - F_0 - c'(x_f)] = c + v_t$$

(18) $$[E(F_1|I_0)-F_0-c'(x_f)] = \frac{(c + v_t)\cdot E}{A + d\cdot E}$$

$$= \frac{(c + v_t)[(1-t)\lambda_s\lambda_f\sigma_f^2(1-\rho^2)]}{[\lambda_f + \lambda_s(1-\rho^2)] + d[(1-t)\lambda_s\lambda_f\sigma_f^2(1-\rho^2)]}$$

The cash market-clearing condition is:

[*Cash Market Clearing*]

(8) $C_1 = X_s$.

Substituting (1) and (5) into (8), yields the equilibrium realized cash prices:

$$a - bS_1 + u_1 = \frac{[E(S_1|I_0) - c'(x_s)] - (\sigma_{sf}/\sigma_f^2)[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - t)\cdot\lambda_s[\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2)]}.$$

Noting that $\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2) = \sigma_s^2(1 - \rho^2)$,

$$(11) \quad S_1 = \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)] - (\sigma_{sf}/\sigma_f^2)[E(F_1|I_0) - F_0 - c'(x_f)]}{b(1 - t)\cdot\lambda_s\sigma_s^2(1 - \rho^2)}.$$

Substituting the equilibrium insurance price given in (18) into this (11) and denoting $C = (1 - t)\cdot\lambda_s\sigma_s^2(1 - \rho^2)$ and $D = (\sigma_{sf}/\sigma_f^2)$,

$$(12) \quad S_1 = \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)]}{b\cdot C} - \frac{\dfrac{D\cdot(c + v_t)\cdot E}{A + d\cdot E}}{b\cdot C}$$

$$= \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)]}{b\cdot C} - \frac{D\cdot(c + v_t)\cdot E}{b\cdot C(A + d\cdot E)}$$

$$= \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)]}{b\cdot(1 - t)\cdot\lambda_s\sigma_s^2(1 - \rho^2)}$$

$$- \frac{(c + v_t)(\sigma_{sf}/\sigma_f^2)[(1-t)\lambda_s\lambda_f\sigma_f^2(1-\rho^2)]}{b(1-t)\lambda_s\sigma_s^2(1-\rho^2)[\lambda_f + \lambda_s(1-\rho^2) + d(1-t)\lambda_s\lambda_f\sigma_f^2(1-\rho^2)]}$$

$$= \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)]}{b\cdot(1 - t)\cdot\lambda_s\sigma_s^2(1 - \rho^2)}$$

$$- \frac{(c + v_t)\lambda_f\sigma_{sf}}{b\sigma_s^2[\lambda_f + \lambda_s(1-\rho^2) + d(1-t)\lambda_s\lambda_f\sigma_f^2(1-\rho^2)]}.$$

Substituting the equilibrium current futures price given in (10) into this (11) and denoting $C = (1 - t)\cdot\lambda_s\sigma_s^2(1 - \rho^2)$ and $D = (\sigma_{sf}/\sigma_f^2)$,

$$(11)' \quad S_1 = \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)] - D(B/A)[E(S_1|I_0) - c'(x_s)]}{b\cdot C}$$

$$= \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)][1 - (D\cdot B)/A]}{b\cdot C}$$

$$= \frac{a + u_1}{b} - [E(S_1|I_0) - c'(x_s)]\cdot\frac{A - B\cdot D}{b\cdot A\cdot C}$$

$$= \frac{a + u_1}{b}$$

$$- [E(S_1|I_0) - c'(x_s)]\cdot\frac{\lambda_f[1 - \sigma_{sf}^2/(\sigma_s^2\sigma_f^2)] + \lambda_s(1-\rho^2)}{b[\lambda_f + \lambda_s(1-\rho^2)][(1-t)\lambda_s\sigma_s^2(1-\rho^2)]}$$

$$= \frac{a + u_1}{b}$$

$$- [E(S_1|I_0) - c'(x_s)]\cdot\frac{(1-\rho^2)[\lambda_f + \lambda_s]}{b[\lambda_f + \lambda_s(1-\rho^2)][(1-t)\lambda_s\sigma_s^2(1-\rho^2)]} \quad .$$

Appendix 3.3 Continued.

Case 3: For the producer's expected profit function of $E(\pi_s) + E(\pi_f) - t[d \cdot E(\pi_s) + E(\pi_f)]$, the aggregate relationships describing the cash and futures markets are specified as follows:

[*Cash Demand by Consumers*]

(1) $C_1 = a - bS_1 + u_1$

[*Cash Supply by Producers*]

(5) $$X_s = \frac{[E(S_1|I_0) - c'(x_s)] - (\sigma_{sf}/\sigma_f^2)[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - td) \cdot \lambda_s[\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2)]}$$

[*Futures Demand by Producers*]

(6) $$X_f = \frac{[E(F_1|I_0) - F_0 - c'(x_f)] - (\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - t) \cdot \lambda_s[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]}$$

[*Futures Supply by Pure Speculators*]

(4) $$Z_f = \frac{[E(F_1|I_0) - F_0 - c'(z_f)]}{(1 - t)\lambda_f\sigma_f^2}$$

[*Futures Market Clearing*]

(7) $X_f + Z_f = 0.$

Substituting (4) and (6) into (7), yields the equilibrium current futures prices:

$$\frac{[E(F_1|I_0) - F_0 - c'(x_f)] - (\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - t) \cdot \lambda_s[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]}$$

$$+ \frac{[E(F_1|I_0) - F_0 - c'(z_f)]}{(1 - t)\lambda_f\sigma_f^2} = 0$$

Noting that $\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2) = \sigma_f^2(1 - \rho^2)$,

$$\frac{[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - t)\cdot\lambda_s\sigma_f^2(1 - \rho^2)} + \frac{[E(F_1|I_0) - F_0 - c'(z_f)]}{(1 - t)\lambda_f\sigma_f^2}$$

$$- \frac{(\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - t)\cdot\lambda_s\sigma_f^2(1 - \rho^2)} = 0$$

Assuming $c'(x_f) = c'(z_f)$,

$$\frac{[E(F_1|I_0) - F_0 - c'(x_f)][(1 - t)\lambda_f + (1 - t)\lambda_s(1 - \rho^2)]}{(1 - t)(1 - t)\cdot\lambda_s\lambda_f\sigma_f^2(1 - \rho^2)}$$

$$- \frac{(\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - t)\cdot\lambda_s\sigma_f^2(1 - \rho^2)} = 0$$

$$(10) \quad F_0 = E(F_1|I_0) - c'(x_f) - \frac{(1 - t)\lambda_f(\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - t)\lambda_f + (1 - t)\lambda_s(1 - \rho^2)}$$

$$= E(F_1|I_0) - c'(x_f) - \frac{\lambda_f(\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{\lambda_f + \lambda_s(1 - \rho^2)} .$$

For the case of $E(S_1|I_0) = F_0$ as assumed before, denoting $A = \lambda_f + \lambda_s(1 - \rho^2)$ and $B = \lambda_f(\sigma_{sf}/\sigma_s^2)$,

$$F_0\cdot\frac{A + B}{A} = \frac{A\cdot[E(F_1|I_0) - c'(x_f)] + B\cdot c'(x_s)}{A}$$

$$F_0 = \frac{A\cdot[E(F_1|I_0) - c'(x_f)] + B\cdot c'(x_s)}{A + B}$$

$$= \frac{[\lambda_f + \lambda_s(1-\rho^2)][E(F_1|I_0)-c'(x_f)] + \lambda_f(\sigma_{sf}/\sigma_s^2)\cdot c'(x_s)}{\lambda_f + \lambda_s(1-\rho^2) + \lambda_f(\sigma_{sf}/\sigma_s^2)} .$$

In order to find the equilibrium insurance price (or risk-premium) $[E(F_1|I_0) - F_0 - c(x_f)]$, the futures market clearing condition is modified. The futures demand by producers in (6) can be divided into two parts:

$$(6)' \quad X_f = \frac{[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - t)\cdot\lambda_s[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]} - \frac{(\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - t)\cdot\lambda_s[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]}$$

Combined with the futures supply by pure speculators in (4), the first term of (6)' represents the market "speculative" supply of futures contracts:

$$SX_f = \frac{[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - t)\cdot\lambda_s[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]} + \frac{[E(F_1|I_0) - F_0 - c'(z_f)]}{(1 - t)\lambda_f\sigma_f^2}$$

Noting that $\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2) = \sigma_f^2(1 - \rho^2)$ and assuming $c'(x_f) = c'(z_f)$,

[*Speculative Supply by Producers and Pure Speculators*]

$$(15) \quad SX_f = \frac{[E(F_1|I_0) - F_0 - c'(x_f)][(1 - t)\lambda_f + (1 - t)\lambda_s(1 - \rho^2)]}{(1 - t)(1 - t)\lambda_s\lambda_f\sigma_f^2(1 - \rho^2)}$$

$$= \frac{[E(F_1|I_0) - F_0 - c'(x_f)][\lambda_f + \lambda_s(1 - \rho^2)]}{(1 - t)\lambda_s\lambda_f\sigma_f^2(1 - \rho^2)} .$$

Analogous to the cash market, suppose that there exist a "hedge" demand for futures contracts, expressed by a function of insurance price:

[*Hedge Demand by Hedgers*]

$$(16) \quad DX_f = c - d[E(F_1|I_0) - F_0 - c'(x_f)] + v_t$$

The market-clearing insurance price or risk premium can be determined when the excess hedging demand by producers is equal to the excess speculative supply by producers and pure speculators:

[Market-Clearing Insurance Price]

(17) $DX_f = SX_f$.

Substituting (15) and (16) into the market-clearing insurance price in (17), yields the equilibrium insurance prices:

$$c - d[E(F_1|I_0) - F_0 - c'(x_f)] + v_t$$

$$= \frac{[E(F_1|I_0) - F_0 - c'(x_f)][\lambda_f + \lambda_s(1 - \rho^2)]}{(1 - t)\lambda_s\lambda_f\sigma_f^2(1 - \rho^2)}$$

Denoting $A = [\lambda_f + \lambda_s(1 - \rho^2)]$ and $E = (1 - t)\lambda_s\lambda_f\sigma_f^2(1 - \rho^2)$,

$$c - d[E(F_1|I_0) - F_0 - c'(x_f)] + v_t = (A/E)[E(F_1|I_0) - F_0 - c'(x_f)]$$

$$[d + (A/E)][E(F_1|I_0) - F_0 - c'(x_f)] = c + v_t$$

(18) $$[E(F_1|I_0)-F_0-c'(x_f)] = \frac{(c + v_t)\cdot E}{A + d\cdot E}$$

$$= \frac{(c + v_t)[(1-t)\lambda_s\lambda_f\sigma_f^2(1-\rho^2)]}{[\lambda_f + \lambda_s(1-\rho^2)] + d[(1-t)\lambda_s\lambda_f\sigma_f^2(1-\rho^2)]}$$

The cash market-clearing condition is:

[Cash Market Clearing]

(8) $C_1 = X_s$.

Substituting (1) and (5) into (8), yields the equilibrium realized cash prices:

$$a - bS_1 + u_1 = \frac{[E(S_1|I_0) - c'(x_s)] - (\sigma_{sf}/\sigma_f^2)[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - td)\cdot\lambda_s[\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2)]}.$$

Noting that $\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2) = \sigma_s^2(1 - \rho^2)$,

$$(11)\quad S_1 = \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)] - (\sigma_{sf}/\sigma_f^2)[E(F_1|I_0) - F_0 - c'(x_f)]}{b(1 - td)\cdot\lambda_s\sigma_s^2(1 - \rho^2)}.$$

Substituting the equilibrium insurance price given in (18) into this (11) and denoting $C = (1 - td)\cdot\lambda_s\sigma_s^2(1 - \rho^2)$ and $D = (\sigma_{sf}/\sigma_f^2)$,

$$(12)\quad S_1 = \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)]}{b\cdot C} - \frac{\dfrac{D\cdot(c + v_t)\cdot E}{A + d\cdot E}}{b\cdot C}$$

$$= \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)]}{b\cdot C} - \frac{D\cdot(c + v_t)\cdot E}{b\cdot C(A + d\cdot E)}$$

$$= \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)]}{b\cdot(1 - td)\cdot\lambda_s\sigma_s^2(1 - \rho^2)}$$

$$- \frac{(c + v_t)(\sigma_{sf}/\sigma_f^2)[(1-t)\lambda_s\lambda_f\sigma_f^2(1-\rho^2)]}{b(1-td)\lambda_s\sigma_s^2(1-\rho^2)[\lambda_f + \lambda_s(1-\rho^2) + d(1-t)\lambda_s\lambda_f\sigma_f^2(1-\rho^2)]}$$

$$= \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)]}{b\cdot(1 - td)\cdot\lambda_s\sigma_s^2(1 - \rho^2)}$$

$$- \frac{(c + v_t)(1-t)\lambda_f\sigma_{sf}}{b(1-td)\sigma_s^2[\lambda_f + \lambda_s(1-\rho^2) + d(1-t)\lambda_s\lambda_f\sigma_f^2(1-\rho^2)]}.$$

Substituting the equilibrium current futures price given in (10) into this (11) and denoting $C = (1 - \text{td})\cdot\lambda_s\sigma_s^2(1 - \rho^2)$ and $D = (\sigma_{sf}/\sigma_f^2)$,

$$(11)' \quad S_1 = \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)] - D(B/A)[E(S_1|I_0) - c'(x_s)]}{b\cdot C}$$

$$= \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)][1 - (D\cdot B)/A]}{b\cdot C}$$

$$= \frac{a + u_1}{b} - [E(S_1|I_0) - c'(x_s)]\cdot\frac{A - B\cdot D}{b\cdot A\cdot C}$$

$$= \frac{a + u_1}{b}$$

$$- [E(S_1|I_0) - c'(x_s)]\cdot\frac{\lambda_f[1 - \sigma_{sf}^2/(\sigma_s^2\sigma_f^2)] + \lambda_s(1-\rho^2)}{b[\lambda_f + \lambda_s(1-\rho^2)][(1-\text{td})\lambda_s\sigma_s^2(1-\rho^2)]}$$

$$= \frac{a + u_1}{b}$$

$$- [E(S_1|I_0) - c'(x_s)]\cdot\frac{(1-\rho^2)[\lambda_f + \lambda_s]}{b[\lambda_f + \lambda_s(1-\rho^2)][(1-\text{td})\lambda_s\sigma_s^2(1-\rho^2)]} \quad .$$

Appendix 3.2 Continued.

Case 4: For the producer's expected profit function of $E(\pi_s) + E(\pi_f) - t[d \cdot E(\pi_s) + d \cdot E(\pi_f)]$, the aggregate relationships describing the cash and futures markets are specified as follows:

[*Cash Demand by Consumers*]

(1) $C_1 = a - bS_1 + u_1$

[*Cash Supply by Producers*]

(5) $$X_s = \frac{[E(S_1|I_0) - c'(x_s)] - (\sigma_{sf}/\sigma_f^2)[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - td)\cdot\lambda_s[\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2)]}$$

[*Futures Demand by Producers*]

(6) $$X_f = \frac{[E(F_1|I_0) - F_0 - c'(x_f)] - (\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - td)\cdot\lambda_s[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]}$$

[*Futures Supply by Pure Speculators*]

(4) $$Z_f = \frac{[E(F_1|I_0) - F_0 - c'(z_f)]}{(1 - t)\lambda_f\sigma_f^2}$$

[*Futures Market Clearing*]

(7) $X_f + Z_f = 0.$

Substituting (4) and (6) into (7), yields the equilibrium current futures prices:

$$\frac{[E(F_1|I_0) - F_0 - c'(x_f)] - (\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - td)\cdot\lambda_s[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]}$$

$$+ \frac{[E(F_1|I_0) - F_0 - c'(z_f)]}{(1 - t)\lambda_f\sigma_f^2} = 0$$

Noting that $\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2) = \sigma_f^2(1 - \rho^2)$,

$$\frac{[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - td)\cdot\lambda_s\sigma_f^2(1 - \rho^2)} + \frac{[E(F_1|I_0) - F_0 - c'(z_f)]}{(1 - t)\lambda_f\sigma_f^2}$$

$$- \frac{(\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - td)\cdot\lambda_s\sigma_f^2(1 - \rho^2)} = 0$$

Assuming $c'(x_f) = c'(z_f)$,

$$\frac{[E(F_1|I_0) - F_0 - c'(x_f)][(1 - t)\lambda_f + (1 - td)\lambda_s(1 - \rho^2)]}{(1 - t)(1 - td)\cdot\lambda_s\lambda_f\sigma_f^2(1 - \rho^2)}$$

$$- \frac{(\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - td)\cdot\lambda_s\sigma_f^2(1 - \rho^2)} = 0$$

(10) $$F_0 = E(F_1|I_0) - c'(x_f) - \frac{(1 - t)\lambda_f(\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - t)\lambda_f + (1 - td)\lambda_s(1 - \rho^2)} .$$

For the case of $E(S_1|I_0) = F_0$ as assumed before, denoting $A = (1 - t)\lambda_f + (1 - td)\lambda_s(1 - \rho^2)$ and $B = (1 - t)\lambda_f(\sigma_{sf}/\sigma_s^2)$,

$$F_0\cdot\frac{A + B}{A} = \frac{A\cdot[E(F_1|I_0) - c'(x_f)] + B\cdot c'(x_s)}{A}$$

$$F_0 = \frac{A\cdot[E(F_1|I_0) - c'(x_f)] + B\cdot c'(x_s)}{A + B}$$

$$= \frac{[(1-t)\lambda_f + (1-td)\lambda_s(1-\rho^2)][E(F_1|I_0) - c'(x_f)] + (1-t)\lambda_f(\sigma_{sf}/\sigma_s^2)\cdot c'(x_s)}{(1-t)\lambda_f + (1-td)\lambda_s(1-\rho^2) + (1-t)\lambda_f(\sigma_{sf}/\sigma_s^2)} .$$

In order to find the equilibrium insurance price (or risk-premium) $[E(F_1|I_0) - F_0 - c(x_f)]$, the futures market clearing condition is modified. The futures demand by producers in (6) can be divided into two parts:

$$(6)' \quad X_f = \frac{[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - td)\cdot\lambda_s[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]} - \frac{(\sigma_{sf}/\sigma_s^2)[E(S_1|I_0) - c'(x_s)]}{(1 - td)\cdot\lambda_s[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]}$$

Combined with the futures supply by pure speculators in (4), the first term of (6)' represents the market "speculative" supply of futures contracts:

$$SX_f = \frac{[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - td)\cdot\lambda_s[\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2)]} + \frac{[E(F_1|I_0) - F_0 - c'(z_f)]}{(1 - t)\lambda_f\sigma_f^2}$$

Noting that $\sigma_f^2 - (\sigma_{sf}^2/\sigma_s^2) = \sigma_f^2(1 - \rho^2)$ and assuming $c'(x_f) = c'(z_f)$,

[*Speculative Supply by Producers and Pure Speculators*]

$$(15) \quad SX_f = \frac{[E(F_1|I_0) - F_0 - c'(x_f)][(1 - t)\lambda_f + (1 - td)\lambda_s(1 - \rho^2)]}{(1 - t)(1 - td)\lambda_s\lambda_f\sigma_f^2(1 - \rho^2)}.$$

Analogous to the cash market, suppose that there exist a "hedge" demand for futures contracts, expressed by a function of insurance price:

[*Hedge Demand by Hedgers*]

$$(16) \quad DX_f = c - d[E(F_1|I_0) - F_0 - c'(x_f)] + v_t$$

The market-clearing insurance price or risk premium can be determined when the excess hedging demand by producers is equal to the excess speculative supply by producers and pure speculators:

[*Market-Clearing Insurance Price*]

(17) $DX_f = SX_f$.

Substituting (15) and (16) into the market-clearing insurance price in (17), yields the equilibrium insurance prices:

$$c - d[E(F_1|I_0) - F_0 - c'(x_f)] + v_t$$

$$= \frac{[E(F_1|I_0) - F_0 - c'(x_f)][(1 - t)\lambda_f + (1 - td)\lambda_s(1 - \rho^2)]}{(1 - t)(1 - td)\lambda_s\lambda_f\sigma_f^2(1 - \rho^2)} .$$

Denoting $A = [(1 - t)\lambda_f + (1 - td)\lambda_s(1 - \rho^2)]$ and $E = (1 - t)(1 - td)\lambda_s\lambda_f\sigma_f^2(1 - \rho^2)$,

$$c - d[E(F_1|I_0) - F_0 - c'(x_f)] + v_t = (A/E)[E(F_1|I_0) - F_0 - c'(x_f)]$$

$$[d + (A/E)][E(F_1|I_0) - F_0 - c'(x_f)] = c + v_t$$

$$(18) \quad [E(F_1|I_0)-F_0-c'(x_f)] = \frac{(c + v_t)\cdot E}{A + d\cdot E}$$

$$= \frac{(c + v_t)[(1-t)(1-td)\lambda_s\lambda_f\sigma_f^2(1-\rho^2)]}{[(1-t)\lambda_f + (1-td)\lambda_s(1-\rho^2)] + d[(1-t)(1-td)\lambda_s\lambda_f\sigma_f^2(1-\rho^2)]} .$$

The cash market-clearing condition is:

[*Cash Market Clearing*]

(8) $C_1 = X_s$.

Substituting (1) and (5) into (8), yields the equilibrium realized cash prices:

$$a - bS_1 + u_1 = \frac{[E(S_1|I_0) - c'(x_s)] - (\sigma_{sf}/\sigma_f^2)[E(F_1|I_0) - F_0 - c'(x_f)]}{(1 - td)\cdot\lambda_s[\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2)]}.$$

Noting that $\sigma_s^2 - (\sigma_{sf}^2/\sigma_f^2) = \sigma_s^2(1 - \rho^2)$,

$$(11)\quad S_1 = \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)] - (\sigma_{sf}/\sigma_f^2)[E(F_1|I_0) - F_0 - c'(x_f)]}{b(1 - td)\cdot\lambda_s\sigma_s^2(1 - \rho^2)}.$$

Substituting the equilibrium insurance price given in (18) into this (11) and denoting $C = (1 - td)\cdot\lambda_s\sigma_s^2(1 - \rho^2)$ and $D = (\sigma_{sf}/\sigma_f^2)$,

$$(12)\quad S_1 = \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)]}{b\cdot C} - \frac{\dfrac{D\cdot(c + v_t)\cdot E}{A + d\cdot E}}{b\cdot C}$$

$$= \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)]}{b\cdot C} - \frac{D\cdot(c + v_t)\cdot E}{b\cdot C(A + d\cdot E)}$$

$$= \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)]}{b\cdot(1 - td)\cdot\lambda_s\sigma_s^2(1 - \rho^2)}$$

$$- \frac{(c + v_t)(\sigma_{sf}/\sigma_f^2)[(1-t)(1-td)\lambda_s\lambda_f\sigma_f^2(1-\rho^2)]}{b(1-td)\lambda_s\sigma_s^2(1-\rho^2)[(1-t)\lambda_f + (1-td)\lambda_s(1-\rho^2) + d(1-t)(1-td)\lambda_s\lambda_f\sigma_f^2(1-\rho^2)]}$$

$$= \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)]}{b\cdot(1 - td)\cdot\lambda_s\sigma_s^2(1 - \rho^2)}$$

$$- \frac{(c + v_t)(1-t)\lambda_f\sigma_{sf}}{b\sigma_s^2[(1-t)\lambda_f + (1-td)\lambda_s(1-\rho^2) + d(1-t)(1-td)\lambda_s\lambda_f\sigma_f^2(1-\rho^2)]} .$$

Substituting the equilibrium current futures price given in (10) into this (11) and denoting $C = (1 - td)\cdot\lambda_s\sigma_s^2(1 - \rho^2)$ and $D = (\sigma_{sf}/\sigma_f^2)$,

$$(11)' \quad S_1 = \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)] - D(B/A)[E(S_1|I_0) - c'(x_s)]}{b\cdot C}$$

$$= \frac{a + u_1}{b} - \frac{[E(S_1|I_0) - c'(x_s)][1 - (D\cdot B)/A]}{b\cdot C}$$

$$= \frac{a + u_1}{b} - [E(S_1|I_0) - c'(x_s)]\cdot\frac{A - B\cdot D}{b\cdot A\cdot C}$$

$$= \frac{a + u_1}{b}$$

$$- [E(S_1|I_0) - c'(x_s)]\cdot\frac{(1-t)\lambda_f[1 - \sigma_{sf}^2/(\sigma_s^2\sigma_f^2)] + (1-td)\lambda_s(1-\rho^2)}{b[(1-t)\lambda_f + (1-td)\lambda_s(1-\rho^2)][(1-td)\lambda_s\sigma_s^2(1-\rho^2)]}$$

$$= \frac{a + u_1}{b}$$

$$- [E(S_1|I_0) - c'(x_s)]\cdot\frac{(1-\rho^2)[(1-t)\lambda_f + (1-td)\lambda_s]}{b[(1-t)\lambda_f + (1-td)\lambda_s(1-\rho^2)][(1-t)\lambda_s\sigma_s^2(1-\rho^2)]} .$$

Appendix 4.1 Derivation of Optimal Position Changes

$$(\partial x_s/\partial t) = (1/|H|)\begin{vmatrix} C & B \\ G & F \end{vmatrix} = (1/|H|)[CF - BG]$$

$$= (1/|H|)\{\lambda(\sigma_s^2 x_s + d\cdot\sigma_{sf}x_f)[c''(x_f) + \lambda(1 - td)\sigma_f^2]$$

$$-\lambda^2(1 - td)\sigma_{sf}(d\cdot\sigma_f^2 x_f + \sigma_{sf}x_s)\}$$

$$= (1/|H|)\lambda[\sigma_s^2 x_s\cdot c''(x_f) + \sigma_s^2 x_s\cdot\lambda(1 - td)\sigma_f^2 + d\cdot\sigma_{sf}x_f\cdot c''(x_f)$$

$$+ d\cdot\sigma_{sf}x_f\lambda(1 - td)\sigma_f^2$$

$$- \lambda(1 - td)\sigma_{sf}\cdot d\cdot\sigma_f^2 x_f - \lambda(1 - td)\sigma_{sf}\cdot\sigma_{sf}x_s]$$

$$= (1/|H|)\lambda[c''(x_f)(\sigma_s^2 x_s + d\cdot\sigma_{sf}x_f)$$

$$+ \lambda(1 - td)(\sigma_s^2\sigma_f^2 - \sigma_{sf}^2)x_s]$$

$$= (1/|H|)\lambda[c''(x_f)(\sigma_s^2 x_s + d\cdot\sigma_{sf}x_f)$$

$$+ \lambda(1 - td)\sigma_s^2\sigma_f^2(1 - \rho^2)x_s]$$

using $\sigma_{sf}^2 = \sigma_s^2\sigma_f^2\rho^2$, where ρ = correlation coefficient.

$$|H| = \begin{vmatrix} -c''(x_s) - \lambda(1 - t)\sigma_s^2 & -\lambda(1 - td)\sigma_{sf} \\ -\lambda(1 - t)\sigma_{sf} & -c''(x_f) - \lambda(1 - td)\sigma_f^2 \end{vmatrix}.$$

$$(\partial x_f/\partial t) = (1/|H|)\begin{vmatrix} A & C \\ E & G \end{vmatrix} = (1/|H|)[AG - CE]$$

$$= (1/|H|)\{\lambda[c''(x_s) + \lambda(1 - t)\sigma_s^2](d\cdot\sigma_f^2 x_f + \sigma_{sf}x_s)$$

$$-\lambda^2(\sigma_s^2 x_s + d\cdot\sigma_{sf}x_f)(1 - t)\sigma_{sf}\}$$

$$= (1/|H|)\lambda[c''(x_s)(d\cdot\sigma_f^2 x_f + \sigma_{sf}x_s)$$

$$+ \lambda(1 - t)\sigma_s^2\cdot d\cdot\sigma_f^2 x_f + \lambda(1 - t)\sigma_s^2\cdot\sigma_{sf}x_s$$

$$- \lambda(1 - t)\sigma_{sf}\cdot\sigma_s^2\cdot x_s - \lambda(1 - td)d\cdot\sigma_{sf}\cdot\sigma_{sf}x_f]$$

$$= (1/|H|)\lambda[c''(x_s)(d\cdot\sigma_f^2 x_f + \sigma_{sf}x_s)$$

$$+ \lambda(1 - t)d\cdot(\sigma_s^2\sigma_f^2 - \sigma_{sf}^2)x_f]$$

$$= (1/|H|)\lambda[c''(x_s)(d\cdot\sigma_f^2 x_f + \sigma_{sf}x_s)$$

$$+ \lambda(1 - t)d\cdot\sigma_s^2\sigma_f^2(1 - \rho^2)x_f]$$

$$(\partial x_s/\partial d) = (1/|H|)\begin{vmatrix} D & B \\ H & F \end{vmatrix} = (1/|H|)[DF - BH]$$

$$= (1/|H|)\{\lambda t\sigma_{sf}x_f[c''(x_f) + \lambda(1 - td)\sigma_f^2]$$

$$- \lambda^2(1 - td)\sigma_{sf}\cdot t\cdot\sigma_f^2 x_f\}$$

$$= (1/|H|)\lambda t\sigma_{sf}x_f[c''(x_f) + \lambda(1 - td)\sigma_f^2 - \lambda(1 - td)\sigma_f^2]$$

$$= (1/|H|)\lambda t\cdot c''(x_f)\sigma_{sf}x_f$$

$$(\partial x_f/\partial d) = (1/|H|)\begin{vmatrix} A & D \\ E & H \end{vmatrix} = (1/|H|)[AH - DE]$$

$$= (1/|H|)\{\lambda[c''(x_s) + \lambda(1 - t)\sigma_s^2]\cdot t\sigma_f^2 x_f$$

$$- \lambda^2 t\sigma_{sf}x_f\cdot(1 - t)\sigma_{sf}\}$$

$$= (1/|H|)\lambda t x_f[c''(x_s)\sigma_f^2 + \lambda(1 - t)\sigma_s^2\sigma_f^2 - \lambda(1 - t)\sigma_{sf}^2]$$

$$= (1/|H|)\lambda t x_f[c''(x_s)\sigma_f^2 + \lambda(1 - t)(\sigma_s^2\sigma_f^2 - \sigma_{sf}^2)]$$

$$= (1/|H|)\lambda t x_f[c''(x_s)\sigma_f^2 + \lambda(1 - t)\sigma_s^2\sigma_f^2(1 - \rho^2)]$$

Appendix 4.2 Derivation of Variances and Covariances of Profits

Considering only price risks or uncertainties, let S_1 and F_1 be the expected cash and futures prices, respectively such that:

$$E(S_1 + \epsilon | I_0) = S_1$$

$$E(F_1 + \delta | I_0) = F_1$$

where ϵ = a random variable $E(\epsilon) = 0$ and $E(\epsilon^2) = 0$, and

δ = a random variable $E(\delta) = 0$ and $E(\delta^2) = 0$.

Since, cash and futures prices are known to be (positively) related, the random variables ϵ and δ are assumed to be serially correlated, i.e., $E(\epsilon \cdot \delta) \neq 0$.

First, consider before-tax risky (or uncertain) cash and futures profits separately. Before-tax risky cash profit is:

$$\begin{aligned}\pi_s &= E(S_1|I_0)x_s - c(x_s) - f \\ &= (S_1 + \epsilon)x_s - c(x_s) - f.\end{aligned}$$

Expected cash profit is:

$$E(\pi_s|I_0) = S_1 x_s - c(x_s) - f.$$

and variance of cash profit is:

$$\mathrm{var}(\pi_s|I_0) = x_s^2 \cdot \sigma_\epsilon^2.$$

Similary, before-tax risky futures profit is:

$$\begin{aligned}\pi_f &= [E(F_1|I_0) - F_0]x_f - c(x_f) \\ &= [(F_1 + \delta) - F_0]x_f - c(x_f).\end{aligned}$$

Expected futures profit is:

$$E(\pi_f|I_0) = (F_1 - F_0)x_f - c(x_f).$$

and variance of futures profit is:

$$\mathrm{var}(\pi_f|I_0) = x_f^2 \cdot \sigma_\delta^2.$$

The corresponding means and variances of after-tax cash and futures profits are, respectively:

$$E(\pi_s|I_0)_{\text{after-tax}} = (1 - t)[S_1x_s - c(x_s) - f]$$

$$\text{var}(\pi_s|I_0)_{\text{after-tax}} = (1 - t)^2x_s^2\cdot\sigma_\epsilon^2, \text{ and}$$

$$E(\pi_f|I_0)_{\text{after-tax}} = (1 - td)[(F_1 - F_0)x_f - c(x_f)]$$

$$\text{var}(\pi_f|I_0)_{\text{after-tax}} = (1 - td)^2x_f^2\cdot\sigma_\delta^2.$$

Second, the mean of total before-tax profits of cash and futures positions is given:

$$\begin{aligned} E(\Pi_1|I_0) &= E(\pi_s|I_0) + E(\pi_f|I_0) \\ &= [S_1x_s - c(x_s) - f] + [(F_1 - F_0)x_f - c(x_f)], \end{aligned}$$

and the variance of total before-tax profits is given:

$$\begin{aligned} \text{var}(\Pi_1|I_0) &= \text{var}(\pi_s|I_0) + \text{var}(\pi_f|I_0) + 2\cdot\text{cov}(\pi_s,\pi_f|I_0) \\ &= x_s^2\cdot\sigma_\epsilon^2 + x_f^2\cdot\sigma_\delta^2 + 2\cdot\sigma_{sf}x_sx_f \end{aligned}$$

where I have used:

using $\text{cov}(X,Y)=E\{[X - E(X)][Y - E(Y)]\}$ and ignoring I_0 for a moment,

$$\begin{aligned} \text{cov}(\pi_s,\pi_f|I_0) &= E\{[\pi_s - E(\pi_s)][\pi_f - E(\pi_f)]\} \\ &= E[\pi_s\cdot\pi_f - \pi_s\cdot E(\pi_f) - E(\pi_s)\cdot\pi_f + E(\pi_s)\cdot E(\pi_f)], \\ &= E[\epsilon\cdot\delta\cdot x_sx_f + \epsilon\cdot x_sE(\pi_f) + \delta\cdot x_fE(\pi_s) + E(\pi_s)\cdot E(\pi_f) \\ &\quad - \epsilon\cdot x_sE(\pi_f) - E(\pi_s)\cdot E(\pi_f) - \delta\cdot x_fE(\pi_s) + E(\pi_s)\cdot E(\pi_f) \\ &\quad + E(\pi_s)\cdot E(\pi_f)] \\ &= E(\epsilon\cdot\delta)\cdot x_sx_f + E(\epsilon)\cdot x_sE(\pi_f) + E(\delta)\cdot x_fE(\pi_s) \\ &\quad + E(\pi_s)\cdot E(\pi_f) - E(\epsilon)\cdot x_sE(\pi_f) - E(\pi_s)\cdot E(\pi_f) \\ &\quad - E(\delta)\cdot x_fE(\pi_s) - E(\pi_s)\cdot E(\pi_f) + E(\pi_s)\cdot E(\pi_f)] \\ &= E(\epsilon\cdot\delta)\cdot x_sx_f \\ &= \sigma_{sf}x_sx_f \end{aligned}$$

where I have used:

$$
\begin{aligned}
\pi_s \cdot \pi_f &= [(S_1 + \epsilon)x_s - c(x_s) - f] \cdot \{[(F_1 + \delta) - F_0]x_f - c(x_f)\} \\
&= \{\epsilon \cdot x_s + [S_1 x_s - c(x_s) - f]\} \cdot \\
&\quad \{\delta \cdot x_f + [(F_1 - F_0)x_f - c(x_f)]\} \\
&= [\epsilon \cdot x_s + E(\pi_s)] \cdot [\delta \cdot x_f + E(\pi_f)] \\
&= \epsilon \cdot \delta \cdot x_s x_f + \epsilon \cdot x_s E(\pi_f) + \delta \cdot x_f E(\pi_s) + E(\pi_s) \cdot E(\pi_f) \\
\pi_s \cdot E(\pi_f) &= \{\epsilon \cdot x_s + [S_1 x_s - c(x_s) - f]\} \cdot E(\pi_f) \\
&= \epsilon \cdot x_s E(\pi_f) + E(\pi_s) \cdot E(\pi_f) \\
E(\pi_s) \cdot \pi_f &= E(\pi_s) \cdot \{\delta \cdot x_f + [(F_1 - F_0)x_f - c(x_f)]\} \\
&= \delta \cdot x_f E(\pi_s) + E(\pi_s) \cdot E(\pi_f),
\end{aligned}
$$

and remembering $E(\epsilon) = 0$, $E(\delta) = 0$, and $E(\epsilon \cdot \delta) = \text{cov}(S_1, F_1) = \sigma_{sf} \neq 0$, because:

$$
\begin{aligned}
\text{cov}[E(S_1), E(F_1)] &= E\{[S_1 - E(S_1)] \cdot [F_1 - E(F_1)]\} \\
&= E\{[S_1 - (S_1 + \epsilon)] \cdot [F_1 - (F_1 + \delta)]\} \\
&= E(\epsilon \cdot \delta).
\end{aligned}
$$

Third, the mean of total after-tax profits of cash and futures positions is given:

$$
\begin{aligned}
E(\Pi_1 | I_0)_{\text{after-tax}} &= (1 - t) \cdot E(\pi_s | I_0) + (1 - td) \cdot E(\pi_f | I_0) \\
&= (1 - t) \cdot [S_1 x_s - c(x_s) - f] \\
&\quad + (1 - td) \cdot [(F_1 - F_0)x_f - c(x_f)],
\end{aligned}
$$

and the variance of total after-tax profits is given:

$$
\begin{aligned}
\text{var}(\Pi_1 | I_0)_{\text{after-tax}} &= (1 - t)^2 \cdot \text{var}(\pi_s | I_0) + (1 - td)^2 \cdot \text{var}(\pi_f | I_0) \\
&\quad + 2(1 - t)(1 - td) \cdot \text{cov}(\pi_s, \pi_f | I_0) \\
&= (1 - t)^2 \cdot x_s^2 \cdot \sigma_\epsilon^2 + (1 - td)^2 \cdot x_f^2 \cdot \sigma_\delta^2 \\
&\quad + 2(1 - t)(1 - td) \cdot \sigma_{sf} x_s x_f.
\end{aligned}
$$

Appendix 4.3 Derivation of Revenue-Neutral Optimal Position Changes

Substituting $(\partial x_s/\partial t)$, $(\partial x_s/\partial d)$ and $(\partial d/\partial t)_{dE(G)=0}$ into $(\partial x_s/\partial t)_{dE(G)=0}$ yields:

$$\begin{aligned}(\partial x_s/\partial t)_{dE(G)=0} &= (\partial x_s/\partial t) + (\partial x_s/\partial d)(\partial d/\partial t)_{dE(G)=0}\\
&= (1/|H|)\lambda[c''(x_f)(\sigma_s^2 x_s + d\cdot\sigma_{sf}x_f)\\
&\qquad + \lambda(1 - td)\sigma_s^2\sigma_f^2(1 - \rho^2)x_s]\\
&\quad + (1/|H|)\lambda t\cdot c''(x_f)\sigma_{sf}x_f\{-(1/t)[(\pi_s^*/\pi_f^*) + d]\}\\
&= (1/|H|)\lambda[\sigma_s^2 x_s\cdot c''(x_f) + d\cdot\sigma_{sf}x_f\cdot c''(x_f)\\
&\qquad + \lambda(1 - td)\sigma_s^2\sigma_f^2(1 - \rho^2)x_s - d\cdot c''(x_f)\sigma_{sf}x_f\\
&\qquad - c''(x_f)\sigma_{sf}x_f(\pi_s^*/\pi_f^*)]\\
&= (1/|H|)\lambda\{c''(x_f)[\sigma_s^2 x_s - \sigma_{sf}x_f(\pi_s^*/\pi_f^*)]\\
&\qquad + \lambda(1 - td)\sigma_s^2\sigma_f^2(1 - \rho^2)x_s\}\end{aligned}$$

Next, substituting $(\partial x_f/\partial t)$, $(\partial x_f/\partial d)$ and $(\partial d/\partial t)_{dE(G)=0}$ into $(\partial x_f/\partial t)_{dE(G)=0}$ yields:

$$\begin{aligned}(\partial x_f/\partial t)_{dE(G)=0} &= (\partial x_f/\partial t) + (\partial x_f/\partial d)(\partial d/\partial t)_{dE(G)=0}\\
&= (1/|H|)\lambda[c''(x_s)(d\cdot\sigma_f^2 x_f + \sigma_{sf}x_s)\\
&\qquad + \lambda(1 - t)d\cdot\sigma_s^2\sigma_f^2(1 - \rho^2)x_f]\\
&\quad + (1/|H|)\lambda t x_f[c''(x_s)\sigma_f^2\\
&\quad + \lambda(1 - t)\sigma_s^2\sigma_f^2(1 - \rho^2)]\{-(1/t)[(\pi_s^*/\pi_f^*) + d]\}\\
&= (1/|H|)\lambda[c''(x_s)\{d\cdot\sigma_f^2 x_f + \sigma_{sf}x_s - [(\pi_s^*/\pi_f^*) + d]\sigma_f^2 x_f\}\\
&\qquad + \lambda(1 - t)\sigma_s^2\sigma_f^2(1 - \rho^2)x_f[d - (\pi_s^*/\pi_f^*) - d]\\
&= (1/|H|)\lambda\{c''(x_s)[\sigma_{sf}x_s - (\pi_s^*/\pi_f^*)\sigma_f^2 x_f]\\
&\qquad - \lambda(1 - t)\sigma_s^2\sigma_f^2(1 - \rho^2)x_f(\pi_s^*/\pi_f^*)\}\\
&= (1/|H|)\lambda\{c''(x_s)\sigma_{sf}x_s - (\pi_s^*/\pi_f^*)\sigma_f^2 x_f[c''(x_s)\\
&\qquad - \lambda(1 - t)\sigma_s^2(1 - \rho^2)]\}\end{aligned}$$

Appendix 4.4 Derivation of Revenue-Neutral Optimal Position Changes

Substituting $(\partial x_s/\partial d)$, $(\partial x_s/\partial t)$ and $(\partial t/\partial d)_{dE(G)=0}$ into $(\partial x_s/\partial d)_{dE(G)=0}$ yields:

$$
\begin{aligned}
(\partial x_s/\partial d)_{dE(G)=0} &= (\partial x_s/\partial d) + (\partial x_s/\partial t)(\partial t/dt)_{dE(G)=0} \\
&= (1/|H|)\lambda t\cdot c''(x_f)\sigma_{sf}x_f + (1/|H|)\lambda[c''(x_f)(\sigma_s^2 x_s \\
&\quad + d\cdot\sigma_{sf}x_f) + \lambda(1 - td)\sigma_s^2\sigma_f^2(1 - \rho^2)x_s]\{-t/[(\pi_s^*/\pi_f^*) + d]\} \\
&= (1/|H|)\lambda t[c''(x_f)\sigma_{sf}x_f \\
&\qquad - c''(x_f)(\sigma_s^2 x_s + d\cdot\sigma_{sf}x_f)\{1/[(\pi_s^*/\pi_f^*) + d]\} \\
&\qquad - \lambda(1 - td)\sigma_s^2\sigma_f^2(1 - \rho^2)x_s\{1/[(\pi_s^*/\pi_f^*) + d]\}] \\
&= (1/|H|)\lambda t\{c''(x_f)[\sigma_{sf}x_f \\
&\qquad - (\sigma_s^2 x_s + d\cdot\sigma_{sf}x_f)\{1/[(\pi_s^*/\pi_f^*) + d]\}] \\
&\qquad - \lambda(1 - td)\sigma_s^2\sigma_f^2(1 - \rho^2)x_s\{1/[(\pi_s^*/\pi_f^*) + d]\}\}
\end{aligned}
$$

Next, substituting $(\partial x_f/\partial d)$, $(\partial x_f/\partial t)$ and $(\partial t/\partial d)_{dE(G)=0}$ into $(\partial x_f/\partial d)_{dE(G)=0}$ yields:

$$
\begin{aligned}
(\partial x_f/\partial d)_{dE(G)=0} &= (\partial x_f/\partial d) + (\partial x_f/\partial t)(\partial t/\partial d)_{dE(G)=0} \\
&= (1/|H|)\lambda t x_f[c''(x_s)\sigma_f^2 + \lambda(1 - t)\sigma_s^2\sigma_f^2(1 - \rho^2)] \\
&\quad + (1/|H|)\lambda[c''(x_s)(d\cdot\sigma_f^2 x_f + \sigma_{sf}x_s) \\
&\quad + \lambda(1 - t)d\cdot\sigma_s^2\sigma_f^2(1 - \rho^2)x_f]\{-t/[(\pi_s^*/\pi_f^*) + d]\} \\
&= (1/|H|)\lambda t\{c''(x_s)\sigma_f^2 x_f + \lambda(1 - t)\sigma_s^2\sigma_f^2(1 - \rho^2)x_f \\
&\qquad - [c''(x_s)(d\cdot\sigma_f^2 x_f + \sigma_{sf}x_s) \\
&\qquad + \lambda(1 - t)d\cdot\sigma_s^2\sigma_f^2(1 - \rho^2)x_f]\{1/[(\pi_s^*/\pi_f^*) + d]\}\} \\
&= (1/|H|)\lambda t\{c''(x_s)(\sigma_f^2 x_f - (d\cdot\sigma_f^2 x_f + \sigma_{sf}x_s)\{1/[(\pi_s^*/\pi_f^*) + d]\} \\
&\qquad + \lambda(1 - t)\sigma_s^2\sigma_f^2(1 - \rho^2)x_f][1 - d\cdot\{1/[(\pi_s^*/\pi_f^*) + d]\}]\}
\end{aligned}
$$

Appendix 6.1 Mean of Optimal Cash Positions by Producer

λ = 0.00001											
t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	14579	14579	14579	14579	14579	14579	14579	14579	14579	14579	14579
0.1	15555	15613	15673	15734	15796	15860	15925	15991	16059	16128	16199
0.2	16775	16893	17016	17145	17279	17419	17565	17718	17879	18047	18224
0.3	18343	18522	18713	18916	19133	19366	19615	19884	20173	20487	20827
0.4	20434	20675	20938	21224	21538	21883	22264	22688	23162	23695	24298
0.5	23361	23666	24005	24384	24810	25293	25845	26482	27226	28104	29158
0.6	27752	28122	28542	29024	29583	30236	31013	31950	33103	34557	36448
0.7	35070	35506	36014	36611	37324	38192	39268	40640	42448	44941	48597
0.8	49706	50211	50811	51537	52435	53571	55058	57085	60013	64614	72895
0.9	93616	94189	94888	95760	96877	98359	100421	103487	108523	118330	145790
1	1.05E20	1.05E20	1.05E20	1.05E20	1.05E20	1.05E20	1.05E20	1.05E20	1.05E20	1.05E20	1.4E20
λ = 0.0001											
	1458	1458	1458	1458	1458	1458	1458	1458	1458	1458	1458
	1555	1561	1567	1573	1580	1586	1592	1599	1606	1613	1620
	1677	1689	1702	1714	1728	1742	1757	1772	1788	1805	1822
	1834	1852	1871	1892	1913	1937	1962	1988	2017	2049	2083
	2043	2068	2094	2122	2154	2188	2226	2269	2316	2369	2430
	2336	2367	2401	2438	2481	2529	2585	2648	2723	2810	2916
	2775	2812	2854	2902	2958	3024	3101	3195	3310	3456	3645
	3507	3551	3601	3661	3732	3819	3927	4064	4245	4494	4860
	4971	5021	5081	5154	5243	5357	5506	5708	6001	6461	7290
	9362	9419	9489	9576	9688	9836	10042	10349	10852	11833	14579
	1.05E19	1.05E19	1.05E19	1.05E19	1.05E19	1.05E19	1.05E19	1.05E19	1.05E19	1.05E19	1.4E19
λ = 0.001											
	146	146	146	146	146	146	146	146	146	146	146
	156	156	157	157	158	159	159	160	161	161	162
	168	169	170	171	173	174	176	177	179	180	182
	183	185	187	189	191	194	196	199	202	205	208
	204	207	209	212	215	219	223	227	232	237	243
	234	237	240	244	248	253	258	265	272	281	292
	278	281	285	290	296	302	310	319	331	346	364
	351	355	360	366	373	382	393	406	424	449	486
	497	502	508	515	524	536	551	571	600	646	729
	936	942	949	958	969	984	1004	1035	1085	1183	1458
	1.05E18	1.05E18	1.05E18	1.05E18	1.05E18	1.05E18	1.05E18	1.05E18	1.05E18	1.05E18	1.4E18
λ = 0.01											
	14.5790	14.5790	14.5790	14.5790	14.5790	14.5790	14.5790	14.5790	14.5790	14.5790	14.5790
	15.5548	15.6134	15.6731	15.7341	15.7964	15.8599	15.9248	15.9911	16.0589	16.1281	16.1989
	16.7745	16.8928	17.0161	17.1445	17.2786	17.4186	17.5650	17.7182	17.8787	18.0471	18.2238
	18.3427	18.5220	18.7127	18.9160	19.1332	19.3657	19.6152	19.8837	20.1734	20.4869	20.8272
	20.4336	20.6752	20.9377	21.2241	21.5378	21.8829	22.2643	22.6881	23.1617	23.6945	24.2984
	23.3609	23.6660	24.0050	24.3839	24.8102	25.2933	25.8454	26.4825	27.2257	28.1040	29.1581
	27.7518	28.1218	28.5423	29.0244	29.5825	30.2363	31.0127	31.9498	33.1031	34.5572	36.4476
	35.0700	35.5064	36.0138	36.6110	37.3245	38.1916	39.2680	40.6399	42.4483	44.9409	48.5968
	49.7064	50.2105	50.8107	51.5371	52.4345	53.5712	55.0577	57.0847	60.0126	64.6135	72.8952
	93.6157	94.1890	94.8882	95.7599	96.8766	98.3588	100.421	103.487	108.523	118.330	145.790
	1.05E17	1.05E17	1.05E17	1.05E17	1.05E17	1.05E17	1.05E17	1.05E17	1.05E17	1.05E17	1.4E17
λ = 0.1											
	1.45790	1.45790	1.45790	1.45790	1.45790	1.45790	1.45790	1.45790	1.45790	1.45790	1.45790
	1.55548	1.56134	1.56731	1.57341	1.57964	1.58599	1.59248	1.59911	1.60589	1.61281	1.61989
	1.67745	1.68928	1.70161	1.71445	1.72786	1.74186	1.75650	1.77182	1.78787	1.80471	1.82238
	1.83427	1.85220	1.87127	1.89160	1.91332	1.93657	1.96152	1.98837	2.01734	2.04869	2.08272
	2.04336	2.06752	2.09377	2.12241	2.15378	2.18829	2.22643	2.26881	2.31617	2.36945	2.42984
	2.33609	2.36660	2.40050	2.43839	2.48102	2.52933	2.58454	2.64825	2.72257	2.81040	2.91581
	2.77518	2.81218	2.85423	2.90244	2.95825	3.02363	3.10127	3.19498	3.31031	3.45572	3.64476
	3.50700	3.55064	3.60138	3.66110	3.73245	3.81916	3.92680	4.06399	4.24483	4.49409	4.85968
	4.97064	5.02105	5.08107	5.15371	5.24345	5.35712	5.50577	5.70847	6.00126	6.46135	7.28952
	9.36157	9.41890	9.48882	9.57599	9.68766	9.83588	10.0421	10.3487	10.8523	11.8330	14.5790
	1.05E16	1.05E16	1.05E16	1.05E16	1.05E16	1.05E16	1.05E16	1.05E16	1.05E16	1.05E16	1.4E16

Appendix 6.1 **Continued.**

$\lambda = 1$											
t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	.145790	.145790	.145790	.145790	.145790	.145790	.145790	.145790	.145790	.145790	.145790
0.1	.155548	.156134	.156731	.157341	.157964	.158599	.159248	.159911	.160589	.161281	.161989
0.2	.167745	.168928	.170161	.171445	.172786	.174186	.175650	.177182	.178787	.180471	.182238
0.3	.183427	.185220	.187127	.189160	.191332	.193657	.196152	.198837	.201734	.204869	.208272
0.4	.204336	.206752	.209377	.212241	.215378	.218829	.222643	.226881	.231617	.236945	.242984
0.5	.233609	.236660	.240050	.243839	.248102	.252933	.258454	.264825	.272257	.281040	.291581
0.6	.277518	.281218	.285423	.290244	.295825	.302363	.310127	.319498	.331031	.345572	.364476
0.7	.350700	.355064	.360138	.366110	.373245	.381916	.392680	.406399	.424483	.449409	.485968
0.8	.497064	.502105	.508107	.515371	.524345	.535712	.550577	.570847	.600126	.646135	.728952
0.9	.936157	.941890	.948882	.957599	.968766	.983588	1.00421	1.03487	1.08523	1.18330	1.45790
1	1.05E15	1.05E15	1.05E15	1.05E15	1.05E15	1.05E15	1.05E15	1.05E15	1.05E15	1.05E15	1.4E15
$\lambda = 10$											
	.014579	.014579	.014579	.014579	.014579	.014579	.014579	.014579	.014579	.014579	.014579
	.015555	.015613	.015673	.015734	.015796	.015860	.015925	.015991	.016059	.016128	.016199
	.016775	.016893	.017016	.017145	.017279	.017419	.017565	.017718	.017879	.018047	.018224
	.018343	.018522	.018713	.018916	.019133	.019366	.019615	.019884	.020173	.020487	.020827
	.020434	.020675	.020938	.021224	.021538	.021883	.022264	.022688	.023162	.023695	.024298
	.023361	.023666	.024005	.024384	.024810	.025293	.025845	.026482	.027226	.028104	.029158
	.027752	.028122	.028542	.029024	.029583	.030236	.031013	.031950	.033103	.034557	.036448
	.035070	.035506	.036014	.036611	.037324	.038192	.039268	.040640	.042448	.044941	.048597
	.049706	.050211	.050811	.051537	.052435	.053571	.055058	.057085	.060013	.064614	.072895
	.093616	.094189	.094888	.095760	.096877	.098359	.100421	.103487	.108523	.118330	.145790
	1.05E14	1.05E14	1.05E14	1.05E14	1.05E14	1.05E14	1.05E14	1.05E14	1.05E14	1.05E14	1.4E14
$\lambda = 100$											
	.001458	.001458	.001458	.001458	.001458	.001458	.001458	.001458	.001458	.001458	.001458
	.001555	.001561	.001567	.001573	.001580	.001586	.001592	.001599	.001606	.001613	.001620
	.001677	.001689	.001702	.001714	.001728	.001742	.001757	.001772	.001788	.001805	.001822
	.001834	.001852	.001871	.001892	.001913	.001937	.001962	.001988	.002017	.002049	.002083
	.002043	.002068	.002094	.002122	.002154	.002188	.002226	.002269	.002316	.002369	.002430
	.002336	.002367	.002401	.002438	.002481	.002529	.002585	.002648	.002723	.002810	.002916
	.002775	.002812	.002854	.002902	.002958	.003024	.003101	.003195	.003310	.003456	.003645
	.003507	.003551	.003601	.003661	.003732	.003819	.003927	.004064	.004245	.004494	.004860
	.004971	.005021	.005081	.005154	.005243	.005357	.005506	.005708	.006001	.006461	.007290
	.009362	.009419	.009489	.009576	.009688	.009836	.010042	.010349	.010852	.011833	.014579
	1.05E13	1.05E13	1.05E13	1.05E13	1.05E13	1.05E13	1.05E13	1.05E13	1.05E13	1.05E13	1.4E13
$\lambda = 1000$											
	.000146	.000146	.000146	.000146	.000146	.000146	.000146	.000146	.000146	.000146	.000146
	.000156	.000156	.000157	.000157	.000158	.000159	.000159	.000160	.000161	.000161	.000162
	.000168	.000169	.000170	.000171	.000173	.000174	.000176	.000177	.000179	.000180	.000182
	.000183	.000185	.000187	.000189	.000191	.000194	.000196	.000199	.000202	.000205	.000208
	.000204	.000207	.000209	.000212	.000215	.000219	.000223	.000227	.000232	.000237	.000243
	.000234	.000237	.000240	.000244	.000248	.000253	.000258	.000265	.000272	.000281	.000292
	.000278	.000281	.000285	.000290	.000296	.000302	.000310	.000319	.000331	.000346	.000364
	.000351	.000355	.000360	.000366	.000373	.000382	.000393	.000406	.000424	.000449	.000486
	.000497	.000502	.000508	.000515	.000524	.000536	.000551	.000571	.000600	.000646	.000729
	.000936	.000942	.000949	.000958	.000969	.000984	.001004	.001035	.001085	.001183	.001458
	1.05E12	1.05E12	1.05E12	1.05E12	1.05E12	1.05E12	1.05E12	1.05E12	1.05E12	1.05E12	1.4E12

Appendix 6.2 Mean of Optimal Futures Positions by Producer

λ = 0.00001											
t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	-59729	-59729	-59729	-59729	-59729	-59729	-59729	-59729	-59729	-59729	-59729
0.1	-62498	-62849	-63208	-63575	-63948	-64330	-64720	-65118	-65525	-65941	-66366
0.2	-65958	-66669	-67409	-68180	-68986	-69826	-70706	-71626	-72590	-73600	-74662
0.3	-70407	-71484	-72630	-73850	-75155	-76551	-78050	-79662	-81401	-83284	-85328
0.4	-76339	-77790	-79367	-81087	-82971	-85043	-87333	-89878	-92723	-95923	-99549
0.5	-84644	-86477	-88513	-90788	-93348	-96249	-99565	-103391	-107854	-113129	-119459
0.6	-97102	-99324	-101849	-104744	-108096	-112022	-116685	-122312	-129238	-137971	-149324
0.7	-117864	-120485	-123532	-127119	-131403	-136611	-143075	-151314	-162174	-177143	-199098
0.8	-159390	-162417	-166021	-170384	-175773	-182599	-191526	-203699	-221282	-248912	-298647
0.9	-283965	-287408	-291607	-296841	-303548	-312449	-324834	-343243	-373487	-432384	-597294
1	-299E18	-299E18	-299E18	-299E18	-299E18	-299E18	-299E18	-299E18	-299E18	-299E18	-508E18
λ = 0.0001											
	-5973	-5973	-5973	-5973	-5973	-5973	-5973	-5973	-5973	-5973	-5973
	-6250	-6285	-6321	-6357	-6395	-6433	-6472	-6512	-6553	-6594	-6637
	-6596	-6667	-6741	-6818	-6899	-6983	-7071	-7163	-7259	-7360	-7466
	-7041	-7148	-7263	-7385	-7515	-7655	-7805	-7966	-8140	-8328	-8533
	-7634	-7779	-7937	-8109	-8297	-8504	-8733	-8988	-9272	-9592	-9955
	-8464	-8648	-8851	-9079	-9335	-9625	-9956	-10339	-10785	-11313	-11946
	-9710	-9932	-10185	-10474	-10810	-11202	-11669	-12231	-12924	-13797	-14932
	-11786	-12048	-12353	-12712	-13140	-13661	-14307	-15131	-16217	-17714	-19910
	-15939	-16242	-16602	-17038	-17577	-18260	-19153	-20370	-22128	-24891	-29865
	-28396	-28741	-29161	-29684	-30355	-31245	-32483	-34324	-37349	-43238	-59729
	-299E17	-299E17	-299E17	-299E17	-299E17	-299E17	-299E17	-299E17	-299E17	-299E17	-508E17
λ = 0.001											
	-597	-597	-597	-597	-597	-597	-597	-597	-597	-597	-597
	-625	-628	-632	-636	-639	-643	-647	-651	-655	-659	-664
	-660	-667	-674	-682	-690	-698	-707	-716	-726	-736	-747
	-704	-715	-726	-739	-752	-766	-780	-797	-814	-833	-853
	-763	-778	-794	-811	-830	-850	-873	-899	-927	-959	-995
	-846	-865	-885	-908	-933	-962	-996	-1034	-1079	-1131	-1195
	-971	-993	-1018	-1047	-1081	-1120	-1167	-1223	-1292	-1380	-1493
	-1179	-1205	-1235	-1271	-1314	-1366	-1431	-1513	-1622	-1771	-1991
	-1594	-1624	-1660	-1704	-1758	-1826	-1915	-2037	-2213	-2489	-2986
	-2840	-2874	-2916	-2968	-3035	-3124	-3248	-3432	-3735	-4324	-5973
	-299E16	-299E16	-299E16	-299E16	-299E16	-299E16	-299E16	-299E16	-299E16	-299E16	-508E16
λ = 0.01											
	-59.729	-59.729	-59.729	-59.729	-59.729	-59.729	-59.729	-59.729	-59.729	-59.729	-59.729
	-62.498	-62.849	-63.208	-63.575	-63.948	-64.330	-64.720	-65.118	-65.525	-65.941	-66.366
	-65.958	-66.669	-67.409	-68.180	-68.986	-69.826	-70.706	-71.626	-72.590	-73.600	-74.662
	-70.407	-71.484	-72.630	-73.850	-75.155	-76.551	-78.050	-79.662	-81.401	-83.284	-85.328
	-76.339	-77.790	-79.367	-81.087	-82.971	-85.043	-87.333	-89.878	-92.723	-95.923	-99.549
	-84.644	-86.477	-88.513	-90.788	-93.348	-96.249	-99.565	-103.39	-107.85	-113.13	-119.46
	-97.102	-99.324	-101.85	-104.74	-108.10	-112.02	-116.69	-122.31	-129.24	-137.97	-149.32
	-117.86	-120.48	-123.53	-127.12	-131.40	-136.61	-143.07	-151.31	-162.17	-177.14	-199.10
	-159.39	-162.42	-166.02	-170.38	-175.77	-182.60	-191.53	-203.70	-221.28	-248.91	-298.65
	-283.96	-287.41	-291.61	-296.84	-303.55	-312.45	-324.83	-343.24	-373.49	-432.38	-597.29
	-299E15	-299E15	-299E15	-299E15	-299E15	-299E15	-299E15	-299E15	-299E15	-299E15	-508E15
λ = 0.1											
	-5.9729	-5.9729	-5.9729	-5.9729	-5.9729	-5.9729	-5.9729	-5.9729	-5.9729	-5.9729	-5.9729
	-6.2498	-6.2849	-6.3208	-6.3575	-6.3948	-6.4330	-6.4720	-6.5118	-6.5525	-6.5941	-6.6366
	-6.5958	-6.6669	-6.7409	-6.8180	-6.8986	-6.9826	-7.0706	-7.1626	-7.2590	-7.3600	-7.4662
	-7.0407	-7.1484	-7.2630	-7.3850	-7.5155	-7.6551	-7.8050	-7.9662	-8.1401	-8.3284	-8.5328
	-7.6339	-7.7790	-7.9367	-8.1087	-8.2971	-8.5043	-8.7333	-8.9878	-9.2723	-9.5923	-9.9549
	-8.4644	-8.6477	-8.8513	-9.0788	-9.3348	-9.6249	-9.9565	-10.339	-10.785	-11.313	-11.946
	-9.7102	-9.9324	-10.185	-10.474	-10.810	-11.202	-11.669	-12.231	-12.924	-13.797	-14.932
	-11.786	-12.048	-12.353	-12.712	-13.140	-13.661	-14.307	-15.131	-16.217	-17.714	-19.910
	-15.939	-16.242	-16.602	-17.038	-17.577	-18.260	-19.153	-20.370	-22.128	-24.891	-29.865
	-28.396	-28.741	-29.161	-29.684	-30.355	-31.245	-32.483	-34.324	-37.349	-43.238	-59.729
	-299E14	-299E14	-299E14	-299E14	-299E14	-299E14	-299E14	-299E14	-299E14	-299E14	-508E14

Appendix 6.2 Continued.

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\lambda = 1$											
0	-.59729	-.59729	-.59729	-.59729	-.59729	-.59729	-.59729	-.59729	-.59729	-.59729	-.59729
0.1	-.62498	-.62849	-.63208	-.63575	-.63948	-.64330	-.64720	-.65118	-.65525	-.65941	-.66366
0.2	-.65958	-.66669	-.67409	-.68180	-.68986	-.69826	-.70706	-.71626	-.72590	-.73600	-.74662
0.3	-.70407	-.71484	-.72630	-.73850	-.75155	-.76551	-.78050	-.79662	-.81401	-.83284	-.85328
0.4	-.76339	-.77790	-.79367	-.81087	-.82971	-.85043	-.87333	-.89878	-.92723	-.95923	-.99549
0.5	-.84644	-.86477	-.88513	-.90788	-.93348	-.96249	-.99565	-1.0339	-1.0785	-1.1313	-1.1946
0.6	-.97102	-.99324	-1.0185	-1.0474	-1.0810	-1.1202	-1.1669	-1.2231	-1.2924	-1.3797	-1.4932
0.7	-1.1786	-1.2048	-1.2353	-1.2712	-1.3140	-1.3661	-1.4307	-1.5131	-1.6217	-1.7714	-1.9910
0.8	-1.5939	-1.6242	-1.6602	-1.7038	-1.7577	-1.8260	-1.9153	-2.0370	-2.2128	-2.4891	-2.9865
0.9	-2.8396	-2.8741	-2.9161	-2.9684	-3.0355	-3.1245	-3.2483	-3.4324	-3.7349	-4.3238	-5.9729
1	-299E13	-299E13	-299E13	-299E13	-299E13	-299E13	-299E13	-299E13	-299E13	-299E13	-508E13
$\lambda = 10$											
	-.05973	-.05973	-.05973	-.05973	-.05973	-.05973	-.05973	-.05973	-.05973	-.05973	-.05973
	-.06250	-.06285	-.06321	-.06357	-.06395	-.06433	-.06472	-.06512	-.06553	-.06594	-.06637
	-.06596	-.06667	-.06741	-.06818	-.06899	-.06983	-.07071	-.07163	-.07259	-.07360	-.07466
	-.07041	-.07148	-.07263	-.07385	-.07515	-.07655	-.07805	-.07966	-.08140	-.08328	-.08533
	-.07634	-.07779	-.07937	-.08109	-.08297	-.08504	-.08733	-.08988	-.09272	-.09592	-.09955
	-.08464	-.08648	-.08851	-.09079	-.09335	-.09625	-.09956	-.10339	-.10785	-.11313	-.11946
	-.09710	-.09932	-.10185	-.10474	-.10810	-.11202	-.11669	-.12231	-.12924	-.13797	-.14932
	-.11786	-.12048	-.12353	-.12712	-.13140	-.13661	-.14307	-.15131	-.16217	-.17714	-.19910
	-.15939	-.16242	-.16602	-.17038	-.17577	-.18260	-.19153	-.20370	-.22128	-.24891	-.29865
	-.28396	-.28741	-.29161	-.29684	-.30355	-.31245	-.32483	-.34324	-.37349	-.43238	-.59729
	-299E12	-299E12	-299E12	-299E12	-299E12	-299E12	-299E12	-299E12	-299E12	-299E12	-508E12
$\lambda = 100$											
	-.00597	-.00597	-.00597	-.00597	-.00597	-.00597	-.00597	-.00597	-.00597	-.00597	-.00597
	-.00625	-.00628	-.00632	-.00636	-.00639	-.00643	-.00647	-.00651	-.00655	-.00659	-.00664
	-.00660	-.00667	-.00674	-.00682	-.00690	-.00698	-.00707	-.00716	-.00726	-.00736	-.00747
	-.00704	-.00715	-.00726	-.00739	-.00752	-.00766	-.00780	-.00797	-.00814	-.00833	-.00853
	-.00763	-.00778	-.00794	-.00811	-.00830	-.00850	-.00873	-.00899	-.00927	-.00959	-.00995
	-.00846	-.00865	-.00885	-.00908	-.00933	-.00962	-.00996	-.01034	-.01079	-.01131	-.01195
	-.00971	-.00993	-.01018	-.01047	-.01081	-.01120	-.01167	-.01223	-.01292	-.01380	-.01493
	-.01179	-.01205	-.01235	-.01271	-.01314	-.01366	-.01431	-.01513	-.01622	-.01771	-.01991
	-.01594	-.01624	-.01660	-.01704	-.01758	-.01826	-.01915	-.02037	-.02213	-.02489	-.02986
	-.02840	-.02874	-.02916	-.02968	-.03035	-.03124	-.03248	-.03432	-.03735	-.04324	-.05973
	-299E11	-299E11	-299E11	-299E11	-299E11	-299E11	-299E11	-299E11	-299E11	-299E11	-508E11
$\lambda = 1000$											
	-.00060	-.00060	-.00060	-.00060	-.00060	-.00060	-.00060	-.00060	-.00060	-.00060	-.00060
	-.00062	-.00063	-.00063	-.00064	-.00064	-.00064	-.00065	-.00065	-.00066	-.00066	-.00066
	-.00066	-.00067	-.00067	-.00068	-.00069	-.00070	-.00071	-.00072	-.00073	-.00074	-.00075
	-.00070	-.00071	-.00073	-.00074	-.00075	-.00077	-.00078	-.00080	-.00081	-.00083	-.00085
	-.00076	-.00078	-.00079	-.00081	-.00083	-.00085	-.00087	-.00090	-.00093	-.00096	-.00100
	-.00085	-.00086	-.00089	-.00091	-.00093	-.00096	-.00100	-.00103	-.00108	-.00113	-.00119
	-.00097	-.00099	-.00102	-.00105	-.00108	-.00112	-.00117	-.00122	-.00129	-.00138	-.00149
	-.00118	-.00120	-.00124	-.00127	-.00131	-.00137	-.00143	-.00151	-.00162	-.00177	-.00199
	-.00159	-.00162	-.00166	-.00170	-.00176	-.00183	-.00192	-.00204	-.00221	-.00249	-.00299
	-.00284	-.00287	-.00292	-.00297	-.00304	-.00312	-.00325	-.00343	-.00373	-.00432	-.00597
	-2992E9	-2992E9	-2992E9	-2992E9	-2992E9	-2992E9	-2992E9	-2992E9	-2992E9	-2992E9	-5083E9

Appendix 6.3 Mean of Optimal Futures Positions by Pure Speculator

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
λ = 0.00001											
0	-30014	-30014	-30014	-30014	-30014	-30014	-30014	-30014	-30014	-30014	-30014
0.1	-33349	-33349	-33349	-33349	-33349	-33349	-33349	-33349	-33349	-33349	-33349
0.2	-37518	-37518	-37518	-37518	-37518	-37518	-37518	-37518	-37518	-37518	-37518
0.3	-42878	-42878	-42878	-42878	-42878	-42878	-42878	-42878	-42878	-42878	-42878
0.4	-50024	-50024	-50024	-50024	-50024	-50024	-50024	-50024	-50024	-50024	-50024
0.5	-60029	-60029	-60029	-60029	-60029	-60029	-60029	-60029	-60029	-60029	-60029
0.6	-75036	-75036	-75036	-75036	-75036	-75036	-75036	-75036	-75036	-75036	-75036
0.7	-100048	-100048	-100048	-100048	-100048	-100048	-100048	-100048	-100048	-100048	-100048
0.8	-150072	-150072	-150072	-150072	-150072	-150072	-150072	-150072	-150072	-150072	-150072
0.9	-300145	-300145	-300145	-300145	-300145	-300145	-300145	-300145	-300145	-300145	-300145
1	-36E19	-36E19	-36E19	-36E19	-36E19	-36E19	-36E19	-36E19	-36E19	-36E19	-36E19
λ = 0.0001											
	-3001	-3001	-3001	-3001	-3001	-3001	-3001	-3001	-3001	-3001	-3001
	-3335	-3335	-3335	-3335	-3335	-3335	-3335	-3335	-3335	-3335	-3335
	-3752	-3752	-3752	-3752	-3752	-3752	-3752	-3752	-3752	-3752	-3752
	-4288	-4288	-4288	-4288	-4288	-4288	-4288	-4288	-4288	-4288	-4288
	-5002	-5002	-5002	-5002	-5002	-5002	-5002	-5002	-5002	-5002	-5002
	-6003	-6003	-6003	-6003	-6003	-6003	-6003	-6003	-6003	-6003	-6003
	-7504	-7504	-7504	-7504	-7504	-7504	-7504	-7504	-7504	-7504	-7504
	-10005	-10005	-10005	-10005	-10005	-10005	-10005	-10005	-10005	-10005	-10005
	-15007	-15007	-15007	-15007	-15007	-15007	-15007	-15007	-15007	-15007	-15007
	-30014	-30014	-30014	-30014	-30014	-30014	-30014	-30014	-30014	-30014	-30014
	-36E18	-36E18	-36E18	-36E18	-36E18	-36E18	-36E18	-36E18	-36E18	-36E18	-36E18
λ = 0.001											
	-300	-300	-300	-300	-300	-300	-300	-300	-300	-300	-300
	-333	-333	-333	-333	-333	-333	-333	-333	-333	-333	-333
	-375	-375	-375	-375	-375	-375	-375	-375	-375	-375	-375
	-429	-429	-429	-429	-429	-429	-429	-429	-429	-429	-429
	-500	-500	-500	-500	-500	-500	-500	-500	-500	-500	-500
	-600	-600	-600	-600	-600	-600	-600	-600	-600	-600	-600
	-750	-750	-750	-750	-750	-750	-750	-750	-750	-750	-750
	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000
	-1501	-1501	-1501	-1501	-1501	-1501	-1501	-1501	-1501	-1501	-1501
	-3001	-3001	-3001	-3001	-3001	-3001	-3001	-3001	-3001	-3001	-3001
	-36E17	-36E17	-36E17	-36E17	-36E17	-36E17	-36E17	-36E17	-36E17	-36E17	-36E17
λ = 0.01											
	-30.014	-30.014	-30.014	-30.014	-30.014	-30.014	-30.014	-30.014	-30.014	-30.014	-30.014
	-33.349	-33.349	-33.349	-33.349	-33.349	-33.349	-33.349	-33.349	-33.349	-33.349	-33.349
	-37.518	-37.518	-37.518	-37.518	-37.518	-37.518	-37.518	-37.518	-37.518	-37.518	-37.518
	-42.878	-42.878	-42.878	-42.878	-42.878	-42.878	-42.878	-42.878	-42.878	-42.878	-42.878
	-50.024	-50.024	-50.024	-50.024	-50.024	-50.024	-50.024	-50.024	-50.024	-50.024	-50.024
	-60.029	-60.029	-60.029	-60.029	-60.029	-60.029	-60.029	-60.029	-60.029	-60.029	-60.029
	-75.036	-75.036	-75.036	-75.036	-75.036	-75.036	-75.036	-75.036	-75.036	-75.036	-75.036
	-100.05	-100.05	-100.05	-100.05	-100.05	-100.05	-100.05	-100.05	-100.05	-100.05	-100.05
	-150.07	-150.07	-150.07	-150.07	-150.07	-150.07	-150.07	-150.07	-150.07	-150.07	-150.07
	-300.14	-300.14	-300.14	-300.14	-300.14	-300.14	-300.14	-300.14	-300.14	-300.14	-300.14
	-36E16	-36E16	-36E16	-36E16	-36E16	-36E16	-36E16	-36E16	-36E16	-36E16	-36E16
λ = 0.1											
	-3.0014	-3.0014	-3.0014	-3.0014	-3.0014	-3.0014	-3.0014	-3.0014	-3.0014	-3.0014	-3.0014
	-3.3349	-3.3349	-3.3349	-3.3349	-3.3349	-3.3349	-3.3349	-3.3349	-3.3349	-3.3349	-3.3349
	-3.7518	-3.7518	-3.7518	-3.7518	-3.7518	-3.7518	-3.7518	-3.7518	-3.7518	-3.7518	-3.7518
	-4.2878	-4.2878	-4.2878	-4.2878	-4.2878	-4.2878	-4.2878	-4.2878	-4.2878	-4.2878	-4.2878
	-5.0024	-5.0024	-5.0024	-5.0024	-5.0024	-5.0024	-5.0024	-5.0024	-5.0024	-5.0024	-5.0024
	-6.0029	-6.0029	-6.0029	-6.0029	-6.0029	-6.0029	-6.0029	-6.0029	-6.0029	-6.0029	-6.0029
	-7.5036	-7.5036	-7.5036	-7.5036	-7.5036	-7.5036	-7.5036	-7.5036	-7.5036	-7.5036	-7.5036
	-10.005	-10.005	-10.005	-10.005	-10.005	-10.005	-10.005	-10.005	-10.005	-10.005	-10.005
	-15.007	-15.007	-15.007	-15.007	-15.007	-15.007	-15.007	-15.007	-15.007	-15.007	-15.007
	-30.014	-30.014	-30.014	-30.014	-30.014	-30.014	-30.014	-30.014	-30.014	-30.014	-30.014
	-36E15	-36E15	-36E15	-36E15	-36E15	-36E15	-36E15	-36E15	-36E15	-36E15	-36E15

Appendix 6.3 Continued.

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
λ = 1											
0	-.30014	-.30014	-.30014	-.30014	-.30014	-.30014	-.30014	-.30014	-.30014	-.30014	-.30014
0.1	-.33349	-.33349	-.33349	-.33349	-.33349	-.33349	-.33349	-.33349	-.33349	-.33349	-.33349
0.2	-.37518	-.37518	-.37518	-.37518	-.37518	-.37518	-.37518	-.37518	-.37518	-.37518	-.37518
0.3	-.42878	-.42878	-.42878	-.42878	-.42878	-.42878	-.42878	-.42878	-.42878	-.42878	-.42878
0.4	-.50024	-.50024	-.50024	-.50024	-.50024	-.50024	-.50024	-.50024	-.50024	-.50024	-.50024
0.5	-.60029	-.60029	-.60029	-.60029	-.60029	-.60029	-.60029	-.60029	-.60029	-.60029	-.60029
0.6	-.75036	-.75036	-.75036	-.75036	-.75036	-.75036	-.75036	-.75036	-.75036	-.75036	-.75036
0.7	-1.0005	-1.0005	-1.0005	-1.0005	-1.0005	-1.0005	-1.0005	-1.0005	-1.0005	-1.0005	-1.0005
0.8	-1.5007	-1.5007	-1.5007	-1.5007	-1.5007	-1.5007	-1.5007	-1.5007	-1.5007	-1.5007	-1.5007
0.9	-3.0014	-3.0014	-3.0014	-3.0014	-3.0014	-3.0014	-3.0014	-3.0014	-3.0014	-3.0014	-3.0014
1	-36E14	-36E14	-36E14	-36E14	-36E14	-36E14	-36E14	-36E14	-36E14	-36E14	-36E14
λ = 10											
	-.03001	-.03001	-.03001	-.03001	-.03001	-.03001	-.03001	-.03001	-.03001	-.03001	-.03001
	-.03335	-.03335	-.03335	-.03335	-.03335	-.03335	-.03335	-.03335	-.03335	-.03335	-.03335
	-.03752	-.03752	-.03752	-.03752	-.03752	-.03752	-.03752	-.03752	-.03752	-.03752	-.03752
	-.04288	-.04288	-.04288	-.04288	-.04288	-.04288	-.04288	-.04288	-.04288	-.04288	-.04288
	-.05002	-.05002	-.05002	-.05002	-.05002	-.05002	-.05002	-.05002	-.05002	-.05002	-.05002
	-.06003	-.06003	-.06003	-.06003	-.06003	-.06003	-.06003	-.06003	-.06003	-.06003	-.06003
	-.07504	-.07504	-.07504	-.07504	-.07504	-.07504	-.07504	-.07504	-.07504	-.07504	-.07504
	-.10005	-.10005	-.10005	-.10005	-.10005	-.10005	-.10005	-.10005	-.10005	-.10005	-.10005
	-.15007	-.15007	-.15007	-.15007	-.15007	-.15007	-.15007	-.15007	-.15007	-.15007	-.15007
	-.30014	-.30014	-.30014	-.30014	-.30014	-.30014	-.30014	-.30014	-.30014	-.30014	-.30014
	-36E13	-36E13	-36E13	-36E13	-36E13	-36E13	-36E13	-36E13	-36E13	-36E13	-36E13
λ = 100											
	-.00300	-.00300	-.00300	-.00300	-.00300	-.00300	-.00300	-.00300	-.00300	-.00300	-.00300
	-.00333	-.00333	-.00333	-.00333	-.00333	-.00333	-.00333	-.00333	-.00333	-.00333	-.00333
	-.00375	-.00375	-.00375	-.00375	-.00375	-.00375	-.00375	-.00375	-.00375	-.00375	-.00375
	-.00429	-.00429	-.00429	-.00429	-.00429	-.00429	-.00429	-.00429	-.00429	-.00429	-.00429
	-.00500	-.00500	-.00500	-.00500	-.00500	-.00500	-.00500	-.00500	-.00500	-.00500	-.00500
	-.00600	-.00600	-.00600	-.00600	-.00600	-.00600	-.00600	-.00600	-.00600	-.00600	-.00600
	-.00750	-.00750	-.00750	-.00750	-.00750	-.00750	-.00750	-.00750	-.00750	-.00750	-.00750
	-.01000	-.01000	-.01000	-.01000	-.01000	-.01000	-.01000	-.01000	-.01000	-.01000	-.01000
	-.01501	-.01501	-.01501	-.01501	-.01501	-.01501	-.01501	-.01501	-.01501	-.01501	-.01501
	-.03001	-.03001	-.03001	-.03001	-.03001	-.03001	-.03001	-.03001	-.03001	-.03001	-.03001
	-36E12	-36E12	-36E12	-36E12	-36E12	-36E12	-36E12	-36E12	-36E12	-36E12	-36E12
λ = 1000											
	-.00030	-.00030	-.00030	-.00030	-.00030	-.00030	-.00030	-.00030	-.00030	-.00030	-.00030
	-.00033	-.00033	-.00033	-.00033	-.00033	-.00033	-.00033	-.00033	-.00033	-.00033	-.00033
	-.00038	-.00038	-.00038	-.00038	-.00038	-.00038	-.00038	-.00038	-.00038	-.00038	-.00038
	-.00043	-.00043	-.00043	-.00043	-.00043	-.00043	-.00043	-.00043	-.00043	-.00043	-.00043
	-.00050	-.00050	-.00050	-.00050	-.00050	-.00050	-.00050	-.00050	-.00050	-.00050	-.00050
	-.00060	-.00060	-.00060	-.00060	-.00060	-.00060	-.00060	-.00060	-.00060	-.00060	-.00060
	-.00075	-.00075	-.00075	-.00075	-.00075	-.00075	-.00075	-.00075	-.00075	-.00075	-.00075
	-.00100	-.00100	-.00100	-.00100	-.00100	-.00100	-.00100	-.00100	-.00100	-.00100	-.00100
	-.00150	-.00150	-.00150	-.00150	-.00150	-.00150	-.00150	-.00150	-.00150	-.00150	-.00150
	-.00300	-.00300	-.00300	-.00300	-.00300	-.00300	-.00300	-.00300	-.00300	-.00300	-.00300
	-3605E9	-3605E9	-3605E9	-3605E9	-3605E9	-3605E9	-3605E9	-3605E9	-3605E9	-3605E9	-3605E9

Appendix 6.4 Mean of Equilibrium Expected Cash Prices

λ = 0.00001

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	66.4609	66.4609	66.4609	66.4609	66.4609	66.4609	66.4609	66.4609	66.4609	66.4609	66.4609
0.1	66.4565	66.4563	66.4560	66.4557	66.4554	66.4552	66.4549	66.4546	66.4543	66.4539	66.4536
0.2	66.4510	66.4505	66.4500	66.4494	66.4488	66.4481	66.4475	66.4468	66.4461	66.4453	66.4445
0.3	66.4440	66.4432	66.4423	66.4414	66.4404	66.4394	66.4383	66.4371	66.4358	66.4344	66.4328
0.4	66.4346	66.4335	66.4323	66.4310	66.4296	66.4281	66.4264	66.4245	66.4223	66.4199	66.4172
0.5	66.4214	66.4201	66.4185	66.4168	66.4149	66.4127	66.4103	66.4074	66.4041	66.4001	66.3954
0.6	66.4017	66.4000	66.3981	66.3960	66.3935	66.3905	66.3870	66.3828	66.3776	66.3711	66.3626
0.7	66.3688	66.3668	66.3646	66.3619	66.3587	66.3548	66.3499	66.3438	66.3356	66.3244	66.3080
0.8	66.3030	66.3007	66.2980	66.2948	66.2907	66.2856	66.2790	66.2698	66.2567	66.2360	66.1988
0.9	66.1056	66.1031	66.0999	66.0960	66.0910	66.0843	66.0750	66.0613	66.0386	65.9945	65.8711
1	-474E12	-474E12	-474E12	-474E12	-474E12	-474E12	-474E12	-474E12	-474E12	-474E12	-631E12

λ = 0.0001

	66.5199	66.5199	66.5199	66.5199	66.5199	66.5199	66.5199	66.5199	66.5199	66.5199	66.5199
	66.5195	66.5194	66.5194	66.5194	66.5193	66.5193	66.5193	66.5193	66.5192	66.5192	66.5192
	66.5189	66.5189	66.5188	66.5187	66.5187	66.5186	66.5185	66.5185	66.5184	66.5183	66.5183
	66.5182	66.5181	66.5180	66.5179	66.5178	66.5177	66.5176	66.5175	66.5174	66.5172	66.5171
	66.5173	66.5172	66.5170	66.5169	66.5168	66.5166	66.5164	66.5162	66.5160	66.5158	66.5155
	66.5159	66.5158	66.5157	66.5155	66.5153	66.5151	66.5148	66.5145	66.5142	66.5138	66.5133
	66.5140	66.5138	66.5136	66.5134	66.5131	66.5129	66.5125	66.5121	66.5116	66.5109	66.5101
	66.5107	66.5105	66.5103	66.5100	66.5097	66.5093	66.5088	66.5082	66.5074	66.5062	66.5046
	66.5041	66.5039	66.5036	66.5033	66.5029	66.5024	66.5017	66.5008	66.4995	66.4974	66.4937
	66.4844	66.4841	66.4838	66.4834	66.4829	66.4822	66.4813	66.4799	66.4777	66.4733	66.4609
	-474E11	-474E11	-474E11	-474E11	-474E11	-474E11	-474E11	-474E11	-474E11	-474E11	-631E11

λ = 0.001

	66.5258	66.5258	66.5258	66.5258	66.5258	66.5258	66.5258	66.5258	66.5258	66.5258	66.5258
	66.5257	66.5257	66.5257	66.5257	66.5257	66.5257	66.5257	66.5257	66.5257	66.5257	66.5257
	66.5257	66.5257	66.5257	66.5257	66.5257	66.5257	66.5257	66.5256	66.5256	66.5256	66.5256
	66.5256	66.5256	66.5256	66.5256	66.5256	66.5256	66.5256	66.5256	66.5255	66.5255	66.5255
	66.5255	66.5255	66.5255	66.5255	66.5255	66.5255	66.5254	66.5254	66.5254	66.5254	66.5254
	66.5254	66.5254	66.5254	66.5253	66.5253	66.5253	66.5253	66.5253	66.5252	66.5252	66.5251
	66.5252	66.5252	66.5252	66.5251	66.5251	66.5251	66.5251	66.5250	66.5250	66.5249	66.5248
	66.5249	66.5248	66.5248	66.5248	66.5248	66.5247	66.5247	66.5246	66.5245	66.5244	66.5243
	66.5242	66.5242	66.5242	66.5241	66.5241	66.5240	66.5240	66.5239	66.5237	66.5235	66.5232
	66.5222	66.5222	66.5222	66.5221	66.5221	66.5220	66.5219	66.5218	66.5216	66.5211	66.5199
	-4741E9	-4741E9	-4741E9	-4741E9	-4741E9	-4741E8	-4741E9	-4741E9	-4741E9	-4741E9	-6306E9

λ = 0.01

	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
	66.5264	66.5264	66.5264	66.5263	66.5263	66.5263	66.5263	66.5263	66.5263	66.5263	66.5263
	66.5263	66.5263	66.5263	66.5263	66.5263	66.5263	66.5263	66.5263	66.5263	66.5263	66.5263
	66.5263	66.5263	66.5263	66.5263	66.5263	66.5263	66.5263	66.5263	66.5263	66.5263	66.5263
	66.5263	66.5263	66.5263	66.5263	66.5263	66.5263	66.5263	66.5263	66.5263	66.5262	66.5262
	66.5262	66.5262	66.5262	66.5262	66.5262	66.5262	66.5262	66.5262	66.5262	66.5262	66.5261
	66.5260	66.5260	66.5260	66.5260	66.5260	66.5260	66.5260	66.5260	66.5260	66.5259	66.5258
	-474E9	-474E9	-474E9	-474E9	-474E9	-474E9	-474E9	-474E9	-474E9	-474E9	-631E9

λ = 0.1

	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
	-474E8	-474E8	-474E8	-474E8	-474E8	-474E8	-474E8	-474E8	-474E8	-474E8	-631E8

Appendix 6.4 Continued.

λ = 1

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
0.1	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
0.2	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
0.3	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
0.4	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
0.5	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
0.6	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
0.7	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
0.8	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
0.9	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
1	-4.74E9	-4.74E9	-4.74E9	-4.74E9	-4.74E9	-4.74E9	-4.74E9	-4.74E9	-4.74E9	-4.74E9	-6.31E9

λ = 10

66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
-4.74E8	-4.74E8	-4.74E8	-4.74E8	-4.74E8	-4.74E8	-4.74E8	-4.74E8	-4.74E8	-4.74E8	-6.31E8

λ = 100

66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
-4.74E7	-4.74E7	-4.74E7	-4.74E7	-4.74E7	-4.74E7	-4.74E7	-4.74E7	-4.74E7	-4.74E7	-6.31E7

λ = 1000

66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264	66.5264
-4.74E6	-4.74E6	-4.74E6	-4.74E6	-4.74E6	-4.74E6	-4.74E6	-4.74E6	-4.74E6	-4.74E6	-6.31E6

Appendix 6.5 Mean of Expected Cash Spreads $(E(S_1) - c'(x_s))$

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
λ = 0.00001											
0	3.2914	3.2914	3.2914	3.2914	3.2914	3.2914	3.2914	3.2914	3.2914	3.2914	3.2914
0.1	3.2870	3.2867	3.2865	3.2862	3.2859	3.2856	3.2853	3.2850	3.2847	3.2844	3.2841
0.2	3.2815	3.2810	3.2804	3.2798	3.2792	3.2786	3.2780	3.2773	3.2765	3.2758	3.2750
0.3	3.2745	3.2737	3.2728	3.2719	3.2709	3.2699	3.2687	3.2675	3.2662	3.2648	3.2633
0.4	3.2651	3.2640	3.2628	3.2615	3.2601	3.2585	3.2568	3.2549	3.2528	3.2504	3.2477
0.5	3.2519	3.2505	3.2490	3.2473	3.2454	3.2432	3.2407	3.2379	3.2345	3.2306	3.2258
0.6	3.2322	3.2305	3.2286	3.2264	3.2239	3.2210	3.2175	3.2133	3.2081	3.2016	3.1931
0.7	3.1993	3.1973	3.1950	3.1923	3.1891	3.1852	3.1804	3.1742	3.1661	3.1549	3.1385
0.8	3.1335	3.1312	3.1285	3.1252	3.1212	3.1161	3.1094	3.1003	3.0871	3.0665	3.0292
0.9	2.9361	2.9335	2.9304	2.9265	2.9214	2.9148	2.9055	2.8917	2.8691	2.8250	2.7016
1	-474E12	-474E12	-474E12	-474E12	-474E12	-474E12	-474E12	-474E12	-474E12	-474E12	-631E12
λ = 0.0001											
	3.3504	3.3504	3.3504	3.3504	3.3504	3.3504	3.3504	3.3504	3.3504	3.3504	3.3504
	3.3499	3.3499	3.3499	3.3498	3.3498	3.3498	3.3498	3.3497	3.3497	3.3497	3.3496
	3.3494	3.3493	3.3493	3.3492	3.3491	3.3491	3.3490	3.3489	3.3489	3.3488	3.3487
	3.3487	3.3486	3.3485	3.3484	3.3483	3.3482	3.3481	3.3480	3.3478	3.3477	3.3475
	3.3477	3.3476	3.3475	3.3474	3.3472	3.3471	3.3469	3.3467	3.3465	3.3463	3.3460
	3.3464	3.3463	3.3461	3.3459	3.3458	3.3455	3.3453	3.3450	3.3447	3.3443	3.3438
	3.3444	3.3443	3.3441	3.3439	3.3436	3.3433	3.3430	3.3425	3.3420	3.3414	3.3405
	3.3411	3.3409	3.3407	3.3405	3.3401	3.3397	3.3393	3.3386	3.3378	3.3367	3.3351
	3.3346	3.3343	3.3341	3.3337	3.3333	3.3328	3.3322	3.3312	3.3299	3.3279	3.3241
	3.3148	3.3146	3.3143	3.3139	3.3134	3.3127	3.3118	3.3104	3.3081	3.3037	3.2914
	-474E11	-474E11	-474E11	-474E11	-474E11	-474E11	-474E11	-474E11	-474E11	-474E11	-631E11
λ = 0.001											
	3.3563	3.3563	3.3563	3.3563	3.3563	3.3563	3.3563	3.3563	3.3563	3.3563	3.3563
	3.3562	3.3562	3.3562	3.3562	3.3562	3.3562	3.3562	3.3562	3.3562	3.3562	3.3562
	3.3562	3.3562	3.3561	3.3561	3.3561	3.3561	3.3561	3.3561	3.3561	3.3561	3.3561
	3.3561	3.3561	3.3561	3.3561	3.3560	3.3560	3.3560	3.3560	3.3560	3.3560	3.3560
	3.3560	3.3560	3.3560	3.3560	3.3559	3.3559	3.3559	3.3559	3.3559	3.3558	3.3558
	3.3559	3.3558	3.3558	3.3558	3.3558	3.3558	3.3557	3.3557	3.3557	3.3556	3.3556
	3.3557	3.3556	3.3556	3.3556	3.3556	3.3556	3.3555	3.3555	3.3554	3.3554	3.3553
	3.3553	3.3553	3.3553	3.3553	3.3552	3.3552	3.3551	3.3551	3.3550	3.3549	3.3547
	3.3547	3.3547	3.3546	3.3546	3.3546	3.3545	3.3544	3.3543	3.3542	3.3540	3.3536
	3.3527	3.3527	3.3526	3.3526	3.3526	3.3525	3.3524	3.3523	3.3520	3.3516	3.3504
	-4741E9	-4741E9	-4741E9	-4741E9	-4741E9	-4741E9	-4741E9	-4741E9	-4741E9	-4741E9	-6306E9
λ = 0.01											
	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568
	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568
	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568
	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568
	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568
	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568
	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3568	3.3567
	3.3568	3.3568	3.3567	3.3567	3.3567	3.3567	3.3567	3.3567	3.3567	3.3567	3.3567
	3.3567	3.3567	3.3567	3.3567	3.3567	3.3567	3.3567	3.3567	3.3566	3.3566	3.3566
	3.3565	3.3565	3.3565	3.3565	3.3565	3.3565	3.3565	3.3564	3.3564	3.3564	3.3563
	-474E9	-474E9	-474E9	-474E9	-474E9	-474E9	-474E9	-474E9	-474E9	-474E9	-631E9
λ = 0.1											
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3568
	-474E8	-474E8	-474E8	-474E8	-474E8	-474E8	-474E8	-474E8	-474E8	-474E8	-631E8

Appendix 6.5 Continued.

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\lambda = 1$											
0	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
0.1	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
0.2	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
0.3	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
0.4	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
0.5	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
0.6	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
0.7	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
0.8	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
0.9	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
1	-4.74E9	-4.74E9	-4.74E9	-4.74E9	-4.74E9	-4.74E9	-4.74E9	-4.74E9	-4.74E9	-4.74E9	-6.31E9
$\lambda = 10$											
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	-4.74E8	-4.74E8	-4.74E8	-4.74E8	-4.74E8	-4.74E8	-4.74E8	-4.74E8	-4.74E8	-4.74E8	-6.31E8
$\lambda = 100$											
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	-4.74E7	-4.74E7	-4.74E7	-4.74E7	-4.74E7	-4.74E7	-4.74E7	-4.74E7	-4.74E7	-4.74E7	-6.31E7
$\lambda = 1000$											
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569	3.3569
	-4.74E6	-4.74E6	-4.74E6	-4.74E6	-4.74E6	-4.74E6	-4.74E6	-4.74E6	-4.74E6	-4.74E6	-6.31E6

Appendix 6.6 Mean of Expected Producer's Welfare

$\lambda = 0.00001$

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	129017	129017	129017	129017	129017	129017	129017	129017	129017	129017	129017
0.1	125974	125723	125467	125206	124940	124669	124392	124109	123820	123526	123225
0.2	122165	121656	121127	120576	120002	119405	118781	118129	117447	116733	115985
0.3	117260	116485	115663	114789	113859	112865	111803	110662	109435	108111	106677
0.4	110707	109656	108520	107285	105938	104463	102840	101042	99042	96799	94267
0.5	101511	100175	98699	97058	95223	93156	90805	88107	84975	81292	76892
0.6	87679	86042	84197	82099	79687	76882	73574	69609	64760	58684	50829
0.7	64541	62587	60337	57715	54613	50878	46284	40480	32891	22511	7392
0.8	18040	15740	13038	9808	5868	938	-5432	-14021	-26295	-45396	-79483
0.9	-122507	-125187	-128407	-132360	-137347	-143863	-152788	-165852	-187002	-227636	-340107
1	-343E18	-343E18	-343E18	-343E18	-343E18	-343E18	-343E18	-343E18	-343E18	-343E18	-485E18

$\lambda = 0.0001$

	17593	17593	17593	17593	17593	17593	17593	17593	17593	17593	17593
	17575	17571	17567	17563	17559	17555	17552	17547	17543	17539	17535
	17551	17543	17535	17527	17519	17510	17501	17492	17483	17473	17463
	17520	17508	17495	17482	17469	17454	17439	17423	17406	17389	17369
	17477	17460	17443	17424	17404	17382	17359	17335	17308	17278	17245
	17415	17393	17369	17344	17316	17286	17253	17216	17174	17127	17072
	17319	17290	17259	17226	17188	17147	17100	17046	16982	16906	16811
	17149	17113	17073	17029	16980	16923	16857	16778	16680	16553	16377
	16787	16742	16691	16633	16567	16489	16395	16277	16120	15891	15508
	15598	15539	15473	15396	15305	15195	15056	14870	14595	14115	12902
	-343E16	-343E16	-343E16	-343E16	-343E16	-343E16	-343E16	-343E16	-343E16	-343E16	-485E16

$\lambda = 0.001$

	1806	1806	1806	1806	1806	1806	1806	1806	1806	1806	1806
	1807	1807	1807	1807	1807	1806	1806	1806	1806	1806	1806
	1808	1808	1808	1807	1807	1807	1806	1806	1806	1805	1805
	1810	1809	1809	1808	1808	1807	1807	1806	1805	1805	1804
	1812	1811	1810	1809	1808	1808	1807	1806	1805	1804	1803
	1814	1813	1812	1811	1810	1808	1807	1806	1804	1803	1801
	1817	1816	1814	1813	1811	1809	1807	1805	1803	1801	1798
	1822	1820	1818	1816	1813	1811	1808	1805	1802	1798	1794
	1829	1826	1823	1820	1816	1813	1809	1805	1799	1793	1785
	1838	1835	1830	1826	1821	1815	1809	1802	1792	1780	1759
	-343E14	-343E14	-343E14	-343E14	-343E14	-343E14	-343E14	-343E14	-343E14	-343E14	-485E14

$\lambda = 0.01$

	181	181	181	181	181	181	181	181	181	181	181
	181	181	181	181	181	181	181	181	181	181	181
	181	181	181	181	181	181	181	181	181	181	181
	182	182	181	181	181	181	181	181	181	181	181
	182	182	182	182	182	181	181	181	181	181	181
	182	182	182	182	182	182	182	181	181	181	181
	183	182	182	182	182	182	182	182	181	181	181
	183	183	183	183	182	182	182	182	182	181	181
	184	184	184	184	183	183	183	182	182	181	181
	187	186	186	185	185	184	184	183	183	182	181
	-343E12	-343E12	-343E12	-343E12	-343E12	-343E12	-343E12	-343E12	-343E12	-343E12	-485E12

$\lambda = 0.1$

	18.1136	18.1136	18.1136	18.1136	18.1136	18.1136	18.1136	18.1136	18.1136	18.1136	18.1136
	18.1270	18.1257	18.1243	18.1230	18.1217	18.1203	18.1190	18.1176	18.1163	18.1149	18.1136
	18.1431	18.1402	18.1374	18.1344	18.1315	18.1286	18.1256	18.1226	18.1196	18.1165	18.1135
	18.1631	18.1583	18.1536	18.1487	18.1439	18.1389	18.1339	18.1289	18.1238	18.1186	18.1134
	18.1884	18.1814	18.1743	18.1671	18.1598	18.1524	18.1448	18.1372	18.1293	18.1214	18.1133
	18.2217	18.2119	18.2019	18.1917	18.1813	18.1706	18.1597	18.1485	18.1370	18.1252	18.1131
	18.2678	18.2544	18.2406	18.2264	18.2117	18.1966	18.1811	18.1649	18.1482	18.1309	18.1128
	18.3362	18.3179	18.2989	18.2791	18.2586	18.2371	18.2147	18.1912	18.1664	18.1402	18.1124
	18.4505	18.4252	18.3985	18.3705	18.3408	18.3092	18.2756	18.2396	18.2007	18.1583	18.1115
	18.6890	18.6537	18.6155	18.5740	18.5287	18.4788	18.4235	18.3614	18.2909	18.2087	18.1089
	-3433E9	-3433E9	-3433E9	-3433E9	-3433E9	-3433E9	-3433E9	-3433E9	-3433E9	-3433E9	-4847E9

Appendix 6.6 Continued.

$\lambda = 1$

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	1.81141	1.81141	1.81141	1.81141	1.81141	1.81141	1.81141	1.81141	1.81141	1.81141	1.81141
0.1	1.81275	1.81262	1.81248	1.81235	1.81222	1.81208	1.81195	1.81181	1.81168	1.81154	1.81141
0.2	1.81437	1.81408	1.81379	1.81350	1.81321	1.81291	1.81261	1.81232	1.81202	1.81171	1.81141
0.3	1.81637	1.81589	1.81542	1.81493	1.81445	1.81395	1.81346	1.81295	1.81244	1.81193	1.81141
0.4	1.81890	1.81821	1.81750	1.81678	1.81605	1.81531	1.81455	1.81379	1.81301	1.81221	1.81141
0.5	1.82225	1.82127	1.82027	1.81925	1.81821	1.81714	1.81605	1.81493	1.81379	1.81261	1.81140
0.6	1.82687	1.82552	1.82415	1.82273	1.82127	1.81976	1.81820	1.81660	1.81493	1.81320	1.81140
0.7	1.83373	1.83190	1.83000	1.82803	1.82597	1.82383	1.82159	1.81924	1.81677	1.81416	1.81140
0.8	1.84520	1.84267	1.84001	1.83720	1.83424	1.83109	1.82773	1.82414	1.82026	1.81604	1.81139
0.9	1.86917	1.86565	1.86184	1.85769	1.85316	1.84818	1.84265	1.83646	1.82942	1.82123	1.81136
1	-343E8	-343E8	-343E8	-343E8	-343E8	-343E8	-343E8	-343E8	-343E8	-343E8	-485E8

$\lambda = 10$

	.181141	.181141	.181141	.181141	.181141	.181141	.181141	.181141	.181141	.181141	.181141
	.181275	.181262	.181249	.181235	.181222	.181209	.181195	.181182	.181168	.181155	.181141
	.181437	.181408	.181380	.181350	.181321	.181292	.181262	.181232	.181202	.181172	.181141
	.181637	.181590	.181542	.181494	.181445	.181396	.181346	.181296	.181245	.181193	.181141
	.181891	.181821	.181751	.181679	.181606	.181532	.181456	.181379	.181301	.181222	.181141
	.182225	.182128	.182028	.181926	.181821	.181715	.181606	.181494	.181379	.181262	.181141
	.182687	.182553	.182415	.182274	.182127	.181977	.181821	.181661	.181494	.181321	.181141
	.183374	.183191	.183001	.182804	.182598	.182384	.182160	.181926	.181679	.181418	.181141
	.184521	.184268	.184002	.183722	.183425	.183110	.182775	.182415	.182027	.181606	.181141
	.186920	.186568	.186187	.185772	.185319	.184821	.184268	.183649	.182945	.182127	.181141
	-3.43E8	-3.43E8	-3.43E8	-3.43E8	-3.43E8	-3.43E8	-3.43E8	-3.43E8	-3.43E8	-3.43E8	-4.85E8

$\lambda = 100$

	.018114	.018114	.018114	.018114	.018114	.018114	.018114	.018114	.018114	.018114	.018114
	.018128	.018126	.018125	.018124	.018122	.018121	.018120	.018118	.018117	.018115	.018114
	.018144	.018141	.018138	.018135	.018132	.018129	.018126	.018123	.018120	.018117	.018114
	.018164	.018159	.018154	.018149	.018145	.018140	.018135	.018130	.018125	.018119	.018114
	.018189	.018182	.018175	.018168	.018161	.018153	.018146	.018138	.018130	.018122	.018114
	.018223	.018213	.018203	.018193	.018182	.018171	.018161	.018149	.018138	.018126	.018114
	.018269	.018255	.018242	.018227	.018213	.018198	.018182	.018166	.018149	.018132	.018114
	.018337	.018319	.018300	.018280	.018260	.018238	.018216	.018193	.018168	.018142	.018114
	.018452	.018427	.018400	.018372	.018343	.018311	.018278	.018242	.018203	.018161	.018114
	.018692	.018657	.018619	.018577	.018532	.018482	.018427	.018365	.018295	.018213	.018114
	-3.43E6	-3.43E6	-3.43E6	-3.43E6	-3.43E6	-3.43E6	-3.43E6	-3.43E6	-3.43E6	-3.43E6	-4.85E6

$\lambda = 1000$

	.001811	.001811	.001811	.001811	.001811	.001811	.001811	.001811	.001811	.001811	.001811
	.001813	.001813	.001812	.001812	.001812	.001812	.001812	.001812	.001812	.001812	.001811
	.001814	.001814	.001814	.001814	.001813	.001813	.001813	.001812	.001812	.001812	.001811
	.001816	.001816	.001815	.001815	.001814	.001814	.001813	.001813	.001812	.001812	.001811
	.001819	.001818	.001818	.001817	.001816	.001815	.001815	.001814	.001813	.001812	.001811
	.001822	.001821	.001820	.001819	.001818	.001817	.001816	.001815	.001814	.001813	.001811
	.001827	.001826	.001824	.001823	.001821	.001820	.001818	.001817	.001815	.001813	.001811
	.001834	.001832	.001830	.001828	.001826	.001824	.001822	.001819	.001817	.001814	.001811
	.001845	.001843	.001840	.001837	.001834	.001831	.001828	.001824	.001820	.001816	.001811
	.001869	.001866	.001862	.001858	.001853	.001848	.001843	.001836	.001829	.001821	.001811
	-34334	-34334	-34334	-34334	-34334	-34334	-34334	-34334	-34334	-34334	-48467

Appendix 6.7 Mean of Expected Producer's Welfare For Flexible Demand

λ = 0.000001

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	1911107	1911107	1911107	1911107	1911107	1911107	1911107	1911107	1911107	1911107	1911107
0.1	2047807	2048443	2049038	2049588	2050089	2050538	2050933	2051267	2051539	2051742	2051873
0.2	2207640	2208871	2209922	2210766	2211372	2211706	2211728	2211394	2210652	2209443	2207698
0.3	2395113	2396894	2398238	2399048	2399201	2398551	2396916	2394073	2389747	2383593	2375177
0.4	2613528	2615810	2617260	2617617	2616541	2613586	2608163	2599479	2586462	2567641	2540963
0.5	2858738	2861470	2862804	2862176	2858796	2851544	2838803	2818201	2786174	2737228	2662627
0.6	3094053	3097178	3098141	3095856	3088687	3074117	3048165	3004337	2931616	2810389	2603543
0.7	3132908	3136368	3136662	3131868	3118858	3092370	3043192	2954351	2792442	2485758	1860855
0.8	1885609	1889341	1888622	1880244	1858543	1813044	1723172	1545144	1174985	323475	-2.04E6
0.9	-1.25E7	-1.25E7	-1.25E7	-1.25E7	-1.25E7	-1.26E7	-1.27E7	-1.31E7	-1.4E7	-1.69E7	-3.31E7
1	-459E35	-459E35	-459E35	-459E35	-459E35	-459E35	-459E35	-459E35	-459E35	-459E35	-553E35

λ = 0.00001

243292	243292	243292	243292	243292	243292	243292	243292	243292	243292	243292
263675	264217	264770	265333	265908	266494	267092	267702	268324	268960	269609
289044	290139	291277	292461	293693	294978	296317	297715	299176	300704	302303
321481	323139	324897	326765	328754	330874	333141	335568	338175	340981	344010
364415	366647	369063	371687	374546	377673	381108	384898	389101	393787	399045
423916	426734	429849	433309	437175	441522	446445	452067	458546	466092	474988
511843	515259	519115	523502	528536	534372	541215	549349	559172	571263	586488
654854	658880	663525	668941	675338	683005	692360	704022	718947	738683	765878
927627	932276	937758	944320	952313	962257	974956	991719	1014802	1048377	1100502
1639944	1645228	1651604	1659444	1669316	1682120	1699364	1723773	1760695	1821255	1911107
-459E33	-459E33	-459E33	-459E33	-459E33	-459E33	-459E33	-459E33	-459E33	-459E33	-553E33

λ = 0.0001

24851	24851	24851	24851	24851	24851	24851	24851	24851	24851	24851
26956	27015	27076	27137	27200	27264	27329	27396	27464	27534	27605
29587	29706	29831	29960	30095	30236	30383	30537	30699	30868	31046
32968	33148	33340	33545	33764	33998	34249	34518	34809	35124	35466
37472	37715	37980	38268	38584	38930	39314	39739	40215	40749	41354
43772	44079	44421	44802	45230	45716	46270	46909	47654	48533	49586
53209	53581	54005	54489	55050	55707	56485	57424	58577	60029	61910
68901	69340	69851	70452	71168	72038	73116	74488	76292	78769	82386
100153	100661	101265	101995	102896	104035	105522	107544	110453	114998	123096
192850	193428	194131	195007	196127	197611	199670	202719	207695	217272	243292
-459E31	-459E31	-459E31	-459E31	-459E31	-459E31	-459E31	-459E31	-459E31	-459E31	-553E31

λ = 0.001

2490	2490	2490	2490	2490	2490	2490	2490	2490	2490	2490
2702	2707	2714	2720	2726	2733	2739	2746	2753	2760	2767
2966	2978	2990	3003	3017	3031	3046	3061	3078	3095	3113
3305	3323	3343	3363	3385	3409	3434	3461	3491	3523	3557
3758	3782	3809	3838	3870	3905	3943	3986	4035	4089	4150
4391	4422	4456	4495	4538	4587	4643	4708	4783	4873	4979
5341	5379	5421	5470	5527	5593	5672	5767	5884	6032	6224
6924	6969	7020	7081	7153	7241	7350	7490	7673	7926	8297
10089	10140	10201	10275	10366	10482	10632	10838	11135	11601	12440
19574	19632	19703	19791	19905	20055	20264	20575	21086	22079	24851
-459E29	-459E29	-459E29	-459E29	-459E29	-459E29	-459E29	-459E29	-459E29	-459E29	-553E29

λ = 0.01

249	249	249	249	249	249	249	249	249	249	249
270	271	271	272	273	273	274	275	275	276	277
297	298	299	300	302	303	305	306	308	310	311
331	332	334	336	339	341	344	346	349	352	356
376	378	381	384	387	391	394	399	404	409	415
439	442	446	450	454	459	464	471	479	487	498
534	538	542	547	553	560	567	577	589	603	623
693	697	702	708	716	724	735	749	768	793	830
1010	1015	1021	1028	1037	1049	1064	1085	1114	1161	1245
1960	1966	1973	1982	1993	2008	2029	2061	2112	2211	2490
-459E27	-459E27	-459E27	-459E27	-459E27	-459E27	-459E27	-459E27	-459E27	-459E27	-553E27

Appendix 6.7 Continued.

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
λ = 0.1											
0	24.9089	24.9089	24.9089	24.9089	24.9089	24.9089	24.9089	24.9089	24.9089	24.9089	24.9089
0.1	27.0219	27.0814	27.1421	27.2041	27.2674	27.3320	27.3980	27.4654	27.5343	27.6046	27.6766
0.2	29.6630	29.7833	29.9085	30.0391	30.1754	30.3177	30.4665	30.6223	30.7854	30.9565	31.1362
0.3	33.0588	33.2410	33.4349	33.6416	33.8623	34.0986	34.3523	34.6252	34.9196	35.2382	35.5842
0.4	37.5865	37.8320	38.0989	38.3900	38.7089	39.0596	39.4473	39.8780	40.3594	40.9010	41.5140
0.5	43.9252	44.2354	44.5800	44.9651	45.3984	45.8894	46.4506	47.0981	47.8536	48.7464	49.8178
0.6	53.4333	53.8095	54.2369	54.7268	55.2941	55.9587	56.7479	57.7004	58.8726	60.3507	62.2721
0.7	69.2802	69.7237	70.2394	70.8465	71.5717	72.4531	73.5472	74.9416	76.7798	79.3134	83.0293
0.8	100.974	101.486	102.096	102.834	103.747	104.902	106.413	108.473	111.449	116.126	124.544
0.9	196.053	196.636	197.347	198.233	199.368	200.874	202.971	206.086	211.205	221.174	249.084
1	-459E25	-459E25	-459E25	-459E25	-459E25	-459E25	-459E25	-459E25	-459E25	-459E25	-553E25
λ = 1											
	2.49090	2.49090	2.49090	2.49090	2.49090	2.49090	2.49090	2.49090	2.49090	2.49090	2.49090
	2.70219	2.70814	2.71422	2.72042	2.72674	2.73321	2.73980	2.74655	2.75343	2.76047	2.76767
	2.96631	2.97833	2.99086	3.00392	3.01755	3.03178	3.04666	3.06223	3.07855	3.09566	3.11362
	3.30589	3.32411	3.34350	3.36417	3.38624	3.40987	3.43524	3.46253	3.49197	3.52383	3.55843
	3.75866	3.78321	3.80990	3.83901	3.87090	3.90597	3.94474	3.98781	4.03596	4.09012	4.15150
	4.39254	4.42355	4.45801	4.49652	4.53985	4.58896	4.64508	4.70983	4.78538	4.87466	4.98180
	5.34335	5.38097	5.42371	5.47270	5.52944	5.59589	5.67481	5.77006	5.88729	6.03510	6.22725
	6.92805	6.97240	7.02398	7.08469	7.15721	7.24535	7.35476	7.49420	7.67802	7.93139	8.30299
	10.0974	10.1487	10.2097	10.2835	10.3747	10.4903	10.6414	10.8474	11.1450	11.6127	12.4545
	19.6056	19.6639	19.7350	19.8236	19.9371	20.0877	20.2973	20.6089	21.1208	22.1177	24.9089
	-459E23	-459E23	-459E23	-459E23	-459E23	-459E23	-459E23	-459E23	-459E23	-459E23	-553E23
λ = 10											
	.249090	.249090	.249090	.249090	.249090	.249090	.249090	.249090	.249090	.249090	.249090
	.270219	.270814	.271422	.272042	.272675	.273321	.273981	.274655	.275343	.276047	.276767
	.296631	.297833	.299086	.300392	.301755	.303178	.304666	.306224	.307855	.309566	.311362
	.330589	.332411	.334350	.336417	.338624	.340987	.343524	.346253	.349197	.352383	.355843
	.375866	.378321	.380990	.383901	.387090	.390597	.394474	.398782	.403596	.409012	.415150
	.439254	.442355	.445801	.449652	.453985	.458896	.464508	.470983	.478538	.487466	.498180
	.534336	.538097	.542371	.547271	.552944	.559590	.567482	.577006	.588729	.603510	.622725
	.692805	.697241	.702398	.708469	.715721	.724535	.735476	.749421	.767802	.793139	.830300
	1.00974	1.01487	1.02097	1.02835	1.03747	1.04903	1.06414	1.08474	1.11450	1.16127	1.24545
	1.96056	1.96639	1.97350	1.98236	1.99371	2.00878	2.02974	2.06090	2.11209	2.21178	2.49090
	-459E21	-459E21	-459E21	-459E21	-459E21	-459E21	-459E21	-459E21	-459E21	-459E21	-553E21
λ = 100											
	.024909	.024909	.024909	.024909	.024909	.024909	.024909	.024909	.024909	.024909	.024909
	.027022	.027081	.027142	.027204	.027267	.027332	.027398	.027465	.027534	.027605	.027677
	.029663	.029783	.029909	.030039	.030175	.030318	.030467	.030622	.030785	.030957	.031136
	.033059	.033241	.033435	.033642	.033862	.034099	.034352	.034625	.034920	.035238	.035584
	.037587	.037832	.038099	.038390	.038709	.039060	.039447	.039878	.040360	.040901	.041515
	.043925	.044236	.044580	.044965	.045399	.045890	.046451	.047098	.047854	.048747	.049818
	.053434	.053810	.054237	.054727	.055294	.055959	.056748	.057701	.058873	.060351	.062272
	.069281	.069724	.070240	.070847	.071572	.072453	.073548	.074942	.076780	.079314	.083030
	.100975	.101487	.102097	.102835	.103748	.104903	.106414	.108474	.111450	.116127	.124545
	.196056	.196639	.197350	.198236	.199371	.200878	.202974	.206090	.211209	.221178	.249090
	-459E19	-459E19	-459E19	-459E19	-459E19	-459E19	-459E19	-459E19	-459E19	-459E19	-553E19
λ = 1000											
	.002491	.002491	.002491	.002491	.002491	.002491	.002491	.002491	.002491	.002491	.002491
	.002702	.002708	.002714	.002720	.002727	.002733	.002740	.002747	.002753	.002760	.002768
	.002966	.002978	.002991	.003004	.003018	.003032	.003047	.003062	.003079	.003096	.003114
	.003306	.003324	.003343	.003364	.003386	.003410	.003435	.003463	.003492	.003524	.003558
	.003759	.003783	.003810	.003839	.003871	.003906	.003945	.003988	.004036	.004090	.004151
	.004393	.004424	.004458	.004497	.004540	.004589	.004645	.004710	.004785	.004875	.004982
	.005343	.005381	.005424	.005473	.005529	.005596	.005675	.005770	.005887	.006035	.006227
	.006928	.006972	.007024	.007085	.007157	.007245	.007355	.007494	.007678	.007931	.008303
	.010097	.010149	.010210	.010284	.010375	.010490	.010641	.010847	.011145	.011613	.012454
	.019606	.019664	.019735	.019824	.019937	.020088	.020297	.020609	.021121	.022118	.024909
	-459E17	-459E17	-459E17	-459E17	-459E17	-459E17	-459E17	-459E17	-459E17	-459E17	-553E17

Appendix 6.8 Mean of Expected Pure Speculator's Welfare

λ = 0.00001											
t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	4334	4334	4334	4334	4334	4334	4334	4334	4334	4334	4334
0.1	4177	4192	4207	4223	4238	4254	4270	4286	4302	4318	4334
0.2	3993	4025	4057	4090	4124	4158	4192	4227	4262	4298	4334
0.3	3774	3825	3877	3929	3983	4039	4095	4153	4212	4272	4334
0.4	3511	3582	3655	3730	3808	3888	3971	4057	4146	4238	4334
0.5	3188	3280	3376	3477	3582	3692	3808	3929	4057	4192	4334
0.6	2780	2894	3015	3143	3280	3426	3582	3749	3929	4124	4334
0.7	2250	2383	2527	2684	2855	3043	3249	3477	3730	4014	4334
0.8	1530	1673	1833	2011	2213	2443	2708	3015	3376	3808	4334
0.9	506	625	765	931	1129	1371	1673	2060	2571	3280	4334
1	-942	-942	-942	-942	-942	-942	-942	-942	-942	-942	3188
λ = 0.0001											
	433	433	433	433	433	433	433	433	433	433	433
	418	419	421	422	424	425	427	429	430	432	433
	399	402	406	409	412	416	419	423	426	430	433
	377	382	388	393	398	404	410	415	421	427	433
	351	358	365	373	381	389	397	406	415	424	433
	319	328	338	348	358	369	381	393	406	419	433
	278	289	301	314	328	343	358	375	393	412	433
	225	238	253	268	286	304	325	348	373	401	433
	153	167	183	201	221	244	271	301	338	381	433
	51	63	77	93	113	137	167	206	257	328	433
	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	319
λ = 0.001											
	43.3412	43.3412	43.3412	43.3412	43.3412	43.3412	43.3412	43.3412	43.3412	43.3412	43.3412
	41.7665	41.9198	42.0739	42.2290	42.3850	42.5419	42.6998	42.8587	43.0185	43.1794	43.3412
	39.9253	40.2469	40.5728	40.9029	41.2373	41.5763	41.9198	42.2679	42.6208	42.9785	43.3412
	37.7426	38.2488	38.7659	39.2942	39.8342	40.3861	40.9504	41.5276	42.1181	42.7225	43.3412
	35.1122	35.8192	36.5486	37.3014	38.0789	38.8824	39.7131	40.5728	41.4628	42.3850	43.3412
	31.8781	32.7992	33.7611	34.7667	35.8192	36.9220	38.0789	39.2942	40.5728	41.9198	43.3412
	27.8012	28.9401	30.1482	31.4320	32.7992	34.2583	35.8192	37.4934	39.2942	41.2373	43.3412
	22.4961	23.8295	25.2733	26.8422	28.5531	30.4267	32.4878	34.7667	37.3014	40.1393	43.3412
	15.3039	16.7326	18.3259	20.1137	22.1337	24.4340	27.0775	30.1482	33.7611	38.0789	43.3412
	5.05835	6.25462	7.65265	9.30645	11.2910	13.7134	16.7326	20.5953	25.7081	32.7992	43.3412
	-9.4163	-9.4163	-9.4163	-9.4163	-9.4163	-9.4163	-9.4163	-9.4163	-9.4163	-9.4163	31.8781
λ = 0.01											
	4.33412	4.33412	4.33412	4.33412	4.33412	4.33412	4.33412	4.33412	4.33412	4.33412	4.33412
	4.17665	4.19198	4.20739	4.22290	4.23850	4.25419	4.26998	4.28587	4.30185	4.31794	4.33412
	3.99253	4.02469	4.05728	4.09029	4.12373	4.15763	4.19198	4.22679	4.26208	4.29785	4.33412
	3.77426	3.82488	3.87659	3.92942	3.98342	4.03861	4.09504	4.15276	4.21181	4.27225	4.33412
	3.51122	3.58192	3.65486	3.73014	3.80789	3.88824	3.97131	4.05728	4.14628	4.23850	4.33412
	3.18781	3.27992	3.37611	3.47667	3.58192	3.69220	3.80789	3.92942	4.05728	4.19198	4.33412
	2.78012	2.89401	3.01482	3.14320	3.27992	3.42583	3.58192	3.74934	3.92942	4.12373	4.33412
	2.24961	2.38295	2.52733	2.68422	2.85531	3.04267	3.24878	3.47667	3.73014	4.01393	4.33412
	1.53039	1.67326	1.83259	2.01137	2.21337	2.44340	2.70775	3.01482	3.37611	3.80789	4.33412
	.505835	.625462	.765265	.930645	1.12910	1.37134	1.67326	2.05953	2.57081	3.27992	4.33412
	-.94163	-.94163	-.94163	-.94163	-.94163	-.94163	-.94163	-.94163	-.94163	-.94163	3.18781
λ = 0.1											
	.433412	.433412	.433412	.433412	.433412	.433412	.433412	.433412	.433412	.433412	.433412
	.417665	.419198	.420739	.422290	.423850	.425419	.426998	.428587	.430185	.431794	.433412
	.399253	.402469	.405728	.409029	.412373	.415763	.419198	.422679	.426208	.429785	.433412
	.377426	.382488	.387659	.392942	.398342	.403861	.409504	.415276	.421181	.427225	.433412
	.351122	.358192	.365486	.373014	.380789	.388824	.397131	.405728	.414628	.423850	.433412
	.318781	.327992	.337611	.347667	.358192	.369220	.380789	.392942	.405728	.419198	.433412
	.278012	.289401	.301482	.314320	.327992	.342583	.358192	.374934	.392942	.412373	.433412
	.224961	.238295	.252733	.268422	.285531	.304267	.324878	.347667	.373014	.401393	.433412
	.153039	.167326	.183259	.201137	.221337	.244340	.270775	.301482	.337611	.380789	.433412
	.050584	.062546	.076526	.093064	.112910	.137134	.167326	.205953	.257081	.327992	.433412
	-.09416	-.09416	-.09416	-.09416	-.09416	-.09416	-.09416	-.09416	-.09416	-.09416	.318781

Appendix 6.8 Continued.

λ = 1

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	.043341	.043341	.043341	.043341	.043341	.043341	.043341	.043341	.043341	.043341	.043341
0.1	.041767	.041920	.042074	.042229	.042385	.042542	.042700	.042859	.043019	.043179	.043341
0.2	.039925	.040247	.040573	.040903	.041237	.041576	.041920	.042268	.042621	.042978	.043341
0.3	.037743	.038249	.038766	.039294	.039834	.040386	.040950	.041528	.042118	.042722	.043341
0.4	.035112	.035819	.036549	.037301	.038079	.038882	.039713	.040573	.041463	.042385	.043341
0.5	.031878	.032799	.033761	.034767	.035819	.036922	.038079	.039294	.040573	.041920	.043341
0.6	.027801	.028940	.030148	.031432	.032799	.034258	.035819	.037493	.039294	.041237	.043341
0.7	.022496	.023829	.025273	.026842	.028553	.030427	.032488	.034767	.037301	.040139	.043341
0.8	.015304	.016733	.018326	.020114	.022134	.024434	.027077	.030148	.033761	.038079	.043341
0.9	.005058	.006255	.007653	.009306	.011291	.013713	.016733	.020595	.025708	.032799	.043341
1	-.00942	-.00942	-.00942	-.00942	-.00942	-.00942	-.00942	-.00942	-.00942	-.00942	.031878

λ = 10

.004334	.004334	.004334	.004334	.004334	.004334	.004334	.004334	.004334	.004334	.004334
.004177	.004192	.004207	.004223	.004238	.004254	.004270	.004286	.004302	.004318	.004334
.003993	.004025	.004057	.004090	.004124	.004158	.004192	.004227	.004262	.004298	.004334
.003774	.003825	.003877	.003929	.003983	.004039	.004095	.004153	.004212	.004272	.004334
.003511	.003582	.003655	.003730	.003808	.003888	.003971	.004057	.004146	.004238	.004334
.003188	.003280	.003376	.003477	.003582	.003692	.003808	.003929	.004057	.004192	.004334
.002780	.002894	.003015	.003143	.003280	.003426	.003582	.003749	.003929	.004124	.004334
.002250	.002383	.002527	.002684	.002855	.003043	.003249	.003477	.003730	.004014	.004334
.001530	.001673	.001833	.002011	.002213	.002443	.002708	.003015	.003376	.003808	.004334
.000506	.000625	.000765	.000931	.001129	.001371	.001673	.002060	.002571	.003280	.004334
-.00094	-.00094	-.00094	-.00094	-.00094	-.00094	-.00094	-.00094	-.00094	-.00094	.003188

λ = 100

.000433	.000433	.000433	.000433	.000433	.000433	.000433	.000433	.000433	.000433	.000433
.000418	.000419	.000421	.000422	.000424	.000425	.000427	.000429	.000430	.000432	.000433
.000399	.000402	.000406	.000409	.000412	.000416	.000419	.000423	.000426	.000430	.000433
.000377	.000382	.000388	.000393	.000398	.000404	.000410	.000415	.000421	.000427	.000433
.000351	.000358	.000365	.000373	.000381	.000389	.000397	.000406	.000415	.000424	.000433
.000319	.000328	.000338	.000348	.000358	.000369	.000381	.000393	.000406	.000419	.000433
.000278	.000289	.000301	.000314	.000328	.000343	.000358	.000375	.000393	.000412	.000433
.000225	.000238	.000253	.000268	.000286	.000304	.000325	.000348	.000373	.000401	.000433
.000153	.000167	.000183	.000201	.000221	.000244	.000271	.000301	.000338	.000381	.000433
.000051	.000063	.000077	.000093	.000113	.000137	.000167	.000206	.000257	.000328	.000433
-.00009	-.00009	-.00009	-.00009	-.00009	-.00009	-.00009	-.00009	-.00009	-.00009	.000319

λ = 1000

.000043	.000043	.000043	.000043	.000043	.000043	.000043	.000043	.000043	.000043	.000043
.000042	.000042	.000042	.000042	.000042	.000043	.000043	.000043	.000043	.000043	.000043
.000040	.000040	.000041	.000041	.000041	.000042	.000042	.000042	.000043	.000043	.000043
.000038	.000038	.000039	.000039	.000040	.000040	.000041	.000042	.000042	.000043	.000043
.000035	.000036	.000037	.000037	.000038	.000039	.000040	.000041	.000041	.000042	.000043
.000032	.000033	.000034	.000035	.000036	.000037	.000038	.000039	.000041	.000042	.000043
.000028	.000029	.000030	.000031	.000033	.000034	.000036	.000037	.000039	.000041	.000043
.000022	.000024	.000025	.000027	.000029	.000030	.000032	.000035	.000037	.000040	.000043
.000015	.000017	.000018	.000020	.000022	.000024	.000027	.000030	.000034	.000038	.000043
.000005	.000006	.000008	.000009	.000011	.000014	.000017	.000021	.000026	.000033	.000043
-.00001	-.00001	-.00001	-.00001	-.00001	-.00001	-.00001	-.00001	-.00001	-.00001	.000032

Appendix 6.9 Mean of Expected Consumers' Welfare Changes

λ = 0.00001											
t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	-83606	-83606	-83606	-83606	-83606	-83606	-83606	-83606	-83606	-83606	-83606
0.1	-83248	-82759	-82257	-81743	-81215	-80673	-80116	-79545	-78957	-78354	-77734
0.2	-82305	-81314	-80271	-79172	-78014	-76790	-75496	-74126	-72673	-71130	-69489
0.3	-80281	-78775	-77149	-75387	-73473	-71387	-69107	-66604	-63848	-60798	-57410
0.4	-76165	-74131	-71874	-69357	-66534	-63349	-59731	-55593	-50819	-45260	-38719
0.5	-67679	-65103	-62165	-58785	-54863	-50264	-44808	-38247	-30234	-20265	-7589
0.6	-48995	-45862	-42186	-37820	-32561	-26124	-18090	-7832	5638	23949	49967
0.7	-1971	1733	6207	11706	18601	27461	39192	55320	78593	114426	174852
0.8	151637	155928	161268	168070	176983	189082	206270	232201	274736	353708	533214
0.9	1088948	1093842	1100122	1108430	1119853	1136362	1161892	1205344	1291018	1508830	2477048
1	2.06E36	2.06E36	2.06E36	2.06E36	2.06E36	2.06E36	2.06E36	2.06E36	2.06E36	2.06E36	2.49E36
λ = 0.0001											
	-15062	-15062	-15062	-15062	-15062	-15062	-15062	-15062	-15062	-15062	-15062
	-15598	-15590	-15582	-15573	-15565	-15556	-15547	-15538	-15529	-15519	-15509
	-16263	-16246	-16230	-16212	-16193	-16174	-16153	-16131	-16109	-16084	-16059
	-17109	-17085	-17058	-17030	-17000	-16967	-16931	-16892	-16849	-16802	-16751
	-18224	-18191	-18154	-18114	-18070	-18020	-17964	-17900	-17828	-17744	-17647
	-19757	-19715	-19668	-19614	-19553	-19482	-19398	-19299	-19180	-19035	-18853
	-21997	-21946	-21888	-21819	-21737	-21639	-21518	-21366	-21171	-20912	-20553
	-25572	-25512	-25441	-25355	-25248	-25115	-24941	-24708	-24381	-23892	-23097
	-32126	-32057	-31972	-31866	-31730	-31550	-31300	-30935	-30356	-29326	-27098
	-47024	-46945	-46846	-46717	-46544	-46302	-45939	-45344	-44224	-41533	-30415
	2.06E34	2.06E34	2.06E34	2.06E34	2.06E34	2.06E34	2.06E34	2.06E34	2.06E34	2.06E34	2.49E34
λ = 0.001											
	-1573	-1573	-1573	-1573	-1573	-1573	-1573	-1573	-1573	-1573	-1573
	-1633	-1632	-1632	-1631	-1631	-1630	-1630	-1630	-1629	-1629	-1628
	-1707	-1706	-1705	-1704	-1703	-1702	-1701	-1700	-1699	-1698	-1697
	-1802	-1801	-1799	-1798	-1796	-1795	-1793	-1791	-1790	-1787	-1785
	-1928	-1927	-1925	-1923	-1921	-1919	-1916	-1913	-1910	-1907	-1902
	-2106	-2104	-2101	-2099	-2096	-2093	-2089	-2085	-2080	-2074	-2066
	-2371	-2368	-2365	-2362	-2359	-2354	-2349	-2342	-2334	-2324	-2311
	-2811	-2808	-2805	-2801	-2796	-2790	-2783	-2773	-2760	-2743	-2715
	-3686	-3682	-3678	-3673	-3667	-3660	-3649	-3635	-3614	-3580	-3514
	-6262	-6258	-6253	-6247	-6240	-6230	-6215	-6193	-6156	-6077	-5823
	2.06E32	2.06E32	2.06E32	2.06E32	2.06E32	2.06E32	2.06E32	2.06E32	2.06E32	2.06E32	2.49E32
λ = 0.01											
	-158	-158	-158	-158	-158	-158	-158	-158	-158	-158	-158
	-164	-164	-164	-164	-164	-164	-164	-164	-164	-164	-164
	-171	-171	-171	-171	-171	-171	-171	-171	-171	-171	-171
	-181	-181	-181	-181	-181	-180	-180	-180	-180	-180	-180
	-194	-194	-194	-193	-193	-193	-193	-193	-192	-192	-192
	-212	-212	-211	-211	-211	-211	-210	-210	-210	-209	-208
	-239	-239	-238	-238	-238	-237	-237	-236	-236	-235	-234
	-284	-283	-283	-283	-282	-282	-281	-280	-279	-278	-276
	-373	-373	-373	-372	-372	-371	-370	-369	-367	-364	-359
	-642	-641	-641	-640	-640	-639	-638	-636	-633	-627	-610
	2.06E30	2.06E30	2.06E30	2.06E30	2.06E30	2.06E30	2.06E30	2.06E30	2.06E30	2.06E30	2.49E30
λ = 0.1											
	-15.806	-15.806	-15.806	-15.806	-15.806	-15.806	-15.806	-15.806	-15.806	-15.806	-15.806
	-16.405	-16.402	-16.398	-16.395	-16.391	-16.387	-16.384	-16.380	-16.376	-16.372	-16.368
	-17.154	-17.147	-17.140	-17.133	-17.125	-17.117	-17.108	-17.099	-17.090	-17.080	-17.070
	-18.117	-18.107	-18.096	-18.084	-18.071	-18.058	-18.043	-18.027	-18.011	-17.992	-17.973
	-19.401	-19.387	-19.372	-19.355	-19.337	-19.317	-19.295	-19.270	-19.242	-19.211	-19.176
	-21.199	-21.181	-21.161	-21.139	-21.114	-21.086	-21.054	-21.017	-20.974	-20.923	-20.861
	-23.895	-23.873	-23.849	-23.821	-23.789	-23.750	-23.705	-23.651	-23.584	-23.499	-23.389
	-28.389	-28.363	-28.334	-28.299	-28.258	-28.207	-28.144	-28.065	-27.959	-27.814	-27.601
	-37.375	-37.346	-37.311	-37.269	-37.217	-37.151	-37.064	-36.946	-36.776	-36.507	-36.025
	-64.331	-64.298	-64.257	-64.206	-64.141	-64.055	-63.935	-63.756	-63.463	-62.891	-61.286
	2.06E28	2.06E28	2.06E28	2.06E28	2.06E28	2.06E28	2.06E28	2.06E28	2.06E28	2.06E28	2.49E28

Appendix 6.9 Continued.

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
λ = 1											
0	-1.5807	-1.5807	-1.5807	-1.5807	-1.5807	-1.5807	-1.5807	-1.5807	-1.5807	-1.5807	-1.5807
0.1	-1.6406	-1.6402	-1.6399	-1.6395	-1.6392	-1.6388	-1.6384	-1.6380	-1.6377	-1.6373	-1.6368
0.2	-1.7155	-1.7148	-1.7141	-1.7133	-1.7126	-1.7117	-1.7109	-1.7100	-1.7091	-1.7081	-1.7071
0.3	-1.8118	-1.8108	-1.8097	-1.8085	-1.8072	-1.8059	-1.8044	-1.8028	-1.8012	-1.7993	-1.7974
0.4	-1.9402	-1.9388	-1.9373	-1.9356	-1.9338	-1.9318	-1.9296	-1.9271	-1.9244	-1.9213	-1.9178
0.5	-2.1200	-2.1182	-2.1163	-2.1141	-2.1116	-2.1088	-2.1056	-2.1019	-2.0975	-2.0924	-2.0863
0.6	-2.3897	-2.3875	-2.3851	-2.3823	-2.3790	-2.3752	-2.3707	-2.3653	-2.3586	-2.3501	-2.3391
0.7	-2.8391	-2.8366	-2.8336	-2.8302	-2.8260	-2.8210	-2.8147	-2.8068	-2.7962	-2.7818	-2.7605
0.8	-3.7380	-3.7351	-3.7316	-3.7274	-3.7222	-3.7156	-3.7069	-3.6951	-3.6781	-3.6514	-3.6033
0.9	-6.4347	-6.4313	-6.4273	-6.4222	-6.4157	-6.4071	-6.3951	-6.3773	-6.3480	-6.2910	-6.1314
1	2.06E26	2.06E26	2.06E26	2.06E26	2.06E26	2.06E26	2.06E26	2.06E26	2.06E26	2.06E26	2.49E26
λ = 10											
	-.15807	-.15807	-.15807	-.15807	-.15807	-.15807	-.15807	-.15807	-.15807	-.15807	-.15807
	-.16406	-.16403	-.16399	-.16395	-.16392	-.16388	-.16384	-.16381	-.16377	-.16373	-.16368
	-.17155	-.17148	-.17141	-.17134	-.17126	-.17118	-.17109	-.17100	-.17091	-.17081	-.17071
	-.18118	-.18108	-.18097	-.18085	-.18072	-.18059	-.18044	-.18029	-.18012	-.17994	-.17974
	-.19402	-.19388	-.19373	-.19356	-.19338	-.19318	-.19296	-.19271	-.19244	-.19213	-.19178
	-.21200	-.21182	-.21163	-.21141	-.21116	-.21088	-.21056	-.21019	-.20976	-.20925	-.20863
	-.23897	-.23875	-.23851	-.23823	-.23791	-.23753	-.23707	-.23653	-.23586	-.23502	-.23392
	-.28391	-.28366	-.28337	-.28302	-.28261	-.28210	-.28148	-.28068	-.27963	-.27818	-.27606
	-.37381	-.37351	-.37317	-.37274	-.37222	-.37156	-.37070	-.36952	-.36782	-.36515	-.36033
	-.64348	-.64315	-.64274	-.64224	-.64159	-.64073	-.63953	-.63775	-.63482	-.62912	-.61317
	2.06E24	2.06E24	2.06E24	2.06E24	2.06E24	2.06E24	2.06E24	2.06E24	2.06E24	2.06E24	2.49E24
λ = 100											
	-.01581	-.01581	-.01581	-.01581	-.01581	-.01581	-.01581	-.01581	-.01581	-.01581	-.01581
	-.01641	-.01640	-.01640	-.01640	-.01639	-.01639	-.01638	-.01638	-.01638	-.01637	-.01637
	-.01716	-.01715	-.01714	-.01713	-.01713	-.01712	-.01711	-.01710	-.01709	-.01708	-.01707
	-.01812	-.01811	-.01810	-.01808	-.01807	-.01806	-.01804	-.01803	-.01801	-.01799	-.01797
	-.01940	-.01939	-.01937	-.01936	-.01934	-.01932	-.01930	-.01927	-.01924	-.01921	-.01918
	-.02120	-.02118	-.02116	-.02114	-.02112	-.02109	-.02106	-.02102	-.02098	-.02092	-.02086
	-.02390	-.02388	-.02385	-.02382	-.02379	-.02375	-.02371	-.02365	-.02359	-.02350	-.02339
	-.02839	-.02837	-.02834	-.02830	-.02826	-.02821	-.02815	-.02807	-.02796	-.02782	-.02761
	-.03738	-.03735	-.03732	-.03727	-.03722	-.03716	-.03707	-.03695	-.03678	-.03651	-.03603
	-.06435	-.06432	-.06427	-.06422	-.06416	-.06407	-.06395	-.06377	-.06348	-.06291	-.06132
	2.06E22	2.06E22	2.06E22	2.06E22	2.06E22	2.06E22	2.06E22	2.06E22	2.06E22	2.06E22	2.49E22
λ = 1000											
	-.00158	-.00158	-.00158	-.00158	-.00158	-.00158	-.00158	-.00158	-.00158	-.00158	-.00158
	-.00164	-.00164	-.00164	-.00164	-.00164	-.00164	-.00164	-.00164	-.00164	-.00164	-.00164
	-.00172	-.00171	-.00171	-.00171	-.00171	-.00171	-.00171	-.00171	-.00171	-.00171	-.00171
	-.00181	-.00181	-.00181	-.00181	-.00181	-.00181	-.00180	-.00180	-.00180	-.00180	-.00180
	-.00194	-.00194	-.00194	-.00194	-.00193	-.00193	-.00193	-.00193	-.00192	-.00192	-.00192
	-.00212	-.00212	-.00212	-.00211	-.00211	-.00211	-.00211	-.00210	-.00210	-.00209	-.00209
	-.00239	-.00239	-.00239	-.00238	-.00238	-.00238	-.00237	-.00237	-.00236	-.00235	-.00234
	-.00284	-.00284	-.00283	-.00283	-.00283	-.00282	-.00281	-.00281	-.00280	-.00278	-.00276
	-.00374	-.00374	-.00373	-.00373	-.00372	-.00372	-.00371	-.00370	-.00368	-.00365	-.00360
	-.00643	-.00643	-.00643	-.00642	-.00642	-.00641	-.00640	-.00638	-.00635	-.00629	-.00613
	2.06E20	2.06E20	2.06E20	2.06E20	2.06E20	2.06E20	2.06E20	2.06E20	2.06E20	2.06E20	2.49E20

Appendix 6.10 Mean of Expected Social Welfare Changes

$\lambda = 0.00001$											
t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	49744	49744	49744	49744	49744	49744	49744	49744	49744	49744	49744
0.1	46902	47156	47417	47686	47964	48250	48546	48850	49165	49490	49825
0.2	43853	44367	44913	45494	46112	46772	47476	48229	49036	49901	50831
0.3	40753	41534	42390	43332	44369	45517	46791	48211	49799	51585	53602
0.4	38053	39107	40301	41659	43213	45003	47079	49507	52369	55778	59881
0.5	37020	38351	39910	41750	43942	46584	49805	53789	58798	65219	73637
0.6	41464	43074	45026	47422	50405	54184	59066	65527	74328	86756	105130
0.7	64820	66702	69071	72105	76069	81382	88725	99276	115214	140950	186578
0.8	171207	173341	176139	179890	185065	192464	203546	221195	251817	312119	458065
0.9	966947	969280	972480	977001	983635	993871	1010777	1041552	1106587	1284474	2141275
1	2.06E36	2.06E36	2.06E36	2.06E36	2.06E36	2.06E36	2.06E36	2.06E36	2.06E36	2.06E36	2.49E36
$\lambda = 0.0001$											
	2964	2964	2964	2964	2964	2964	2964	2964	2964	2964	2964
	2394	2400	2406	2412	2418	2425	2431	2438	2445	2452	2459
	1688	1699	1712	1724	1738	1752	1767	1783	1800	1818	1837
	788	806	825	845	867	891	918	947	979	1014	1052
	-395	-372	-346	-317	-285	-248	-207	-160	-105	-42	32
	-2023	-1994	-1961	-1923	-1879	-1827	-1765	-1691	-1601	-1489	-1348
	-4400	-4367	-4327	-4279	-4221	-4149	-4060	-3946	-3796	-3594	-3308
	-8198	-8161	-8115	-8057	-7983	-7887	-7759	-7582	-7328	-6938	-6286
	-15186	-15148	-15098	-15032	-14942	-14816	-14634	-14356	-13899	-13055	-11157
	-31376	-31343	-31296	-31228	-31126	-30969	-30715	-30268	-29372	-27090	-17080
	2.06E34	2.06E34	2.06E34	2.06E34	2.06E34	2.06E34	2.06E34	2.06E34	2.06E34	2.06E34	2.49E34
$\lambda = 0.001$											
	276	276	276	276	276	276	276	276	276	276	276
	216	217	217	218	218	218	219	219	220	220	221
	142	143	143	144	145	146	147	148	149	150	151
	46	47	48	50	51	53	54	56	58	60	62
	-82	-80	-78	-77	-75	-72	-70	-67	-64	-60	-56
	-260	-258	-256	-253	-251	-248	-244	-240	-235	-229	-222
	-526	-523	-521	-518	-515	-511	-506	-500	-492	-482	-469
	-967	-964	-962	-958	-954	-949	-942	-933	-921	-904	-878
	-1842	-1840	-1837	-1833	-1829	-1822	-1813	-1800	-1781	-1748	-1685
	-4418	-4417	-4415	-4412	-4408	-4401	-4389	-4371	-4337	-4264	-4020
	2.06E32	2.06E32	2.06E32	2.06E32	2.06E32	2.06E32	2.06E32	2.06E32	2.06E32	2.06E32	2.49E32
$\lambda = 0.01$											
	27	27	27	27	27	27	27	27	27	27	27
	21	21	21	22	22	22	22	22	22	22	22
	14	14	14	14	14	14	14	14	15	15	15
	4	4	4	5	5	5	5	5	5	6	6
	-9	-8	-8	-8	-8	-8	-7	-7	-7	-7	-6
	-27	-26	-26	-26	-26	-25	-25	-25	-24	-24	-23
	-53	-53	-53	-53	-52	-52	-52	-51	-50	-49	-48
	-98	-98	-98	-97	-97	-96	-96	-95	-94	-93	-90
	-187	-187	-187	-187	-186	-186	-185	-184	-182	-179	-174
	-455	-455	-454	-454	-454	-453	-452	-451	-448	-442	-425
	2.06E30	2.06E30	2.06E30	2.06E30	2.06E30	2.06E30	2.06E30	2.06E30	2.06E30	2.06E30	2.49E30
$\lambda = 0.1$											
	2.74115	2.74115	2.74115	2.74115	2.74115	2.74115	2.74115	2.74115	2.74115	2.74115	2.74115
	2.13954	2.14315	2.14684	2.15061	2.15446	2.15839	2.16240	2.16650	2.17069	2.17497	2.17935
	1.38826	1.39548	1.40301	1.41088	1.41909	1.42767	1.43665	1.44606	1.45591	1.46625	1.47710
	.423371	.434135	.445624	.457907	.471060	.485173	.500345	.516693	.534350	.553471	.574237
	-.86161	-.84747	-.83199	-.81499	-.79628	-.77560	-.75265	-.72708	-.69843	-.66616	-.62956
	-2.6582	-2.6410	-2.6217	-2.5998	-2.5749	-2.5465	-2.5137	-2.4757	-2.4311	-2.3783	-2.3148
	-5.3492	-5.3297	-5.3070	-5.2803	-5.2488	-5.2112	-5.1660	-5.1109	-5.0424	-4.9557	-4.8426
	-9.8275	-9.8071	-9.7822	-9.7515	-9.7134	-9.6657	-9.6049	-9.5257	-9.4199	-9.2725	-9.0553
	-18.772	-18.754	-18.729	-18.697	-18.655	-18.597	-18.518	-18.405	-18.237	-17.968	-17.480
	-45.591	-45.581	-45.565	-45.539	-45.500	-45.439	-45.344	-45.189	-44.915	-44.354	-42.744
	2.06E28	2.06E28	2.06E28	2.06E28	2.06E28	2.06E28	2.06E28	2.06E28	2.06E28	2.06E28	2.49E28

Appendix 6.10 Continued.

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\lambda = 1$											
0	.274094	.274094	.274094	.274094	.274094	.274094	.274094	.274094	.274094	.274094	.274094
0.1	.213931	.214292	.214661	.215038	.215422	.215815	.216216	.216625	.217044	.217472	.217909
0.2	.138799	.139521	.140274	.141059	.141880	.142738	.143636	.144575	.145560	.146593	.147678
0.3	.042304	.043380	.044528	.045756	.047070	.048481	.049997	.051631	.053395	.055306	.057381
0.4	-.08620	-.08479	-.08324	-.08154	-.07967	-.07761	-.07531	-.07276	-.06990	-.06667	-.06302
0.5	-.26588	-.26416	-.26223	-.26004	-.25755	-.25471	-.25144	-.24764	-.24318	-.23791	-.23157
0.6	-.53501	-.53306	-.53079	-.52812	-.52497	-.52122	-.51670	-.51119	-.50435	-.49569	-.48440
0.7	-.98290	-.98086	-.97837	-.97530	-.97150	-.96673	-.96065	-.95275	-.94218	-.92746	-.90578
0.8	-1.8775	-1.8757	-1.8733	-1.8701	-1.8658	-1.8600	-1.8521	-1.8409	-1.8241	-1.7973	-1.7485
0.9	-4.5604	-4.5594	-4.5578	-4.5552	-4.5513	-4.5452	-4.5357	-4.5202	-4.4929	-4.4370	-4.2767
1	2.06E26	2.06E26	2.06E26	2.06E26	2.06E26	2.06E26	2.06E26	2.06E26	2.06E26	2.06E26	2.49E26
$\lambda = 10$											
	.027409	.027409	.027409	.027409	.027409	.027409	.027409	.027409	.027409	.027409	.027409
	.021393	.021429	.021466	.021504	.021542	.021581	.021621	.021662	.021704	.021747	.021791
	.013880	.013952	.014027	.014106	.014188	.014274	.014363	.014457	.014556	.014659	.014767
	.004230	.004338	.004452	.004575	.004707	.004848	.004999	.005163	.005339	.005530	.005738
	-.00862	-.00848	-.00832	-.00815	-.00797	-.00776	-.00753	-.00728	-.00699	-.00667	-.00630
	-.02659	-.02642	-.02622	-.02600	-.02576	-.02547	-.02514	-.02476	-.02432	-.02379	-.02316
	-.05350	-.05331	-.05308	-.05281	-.05250	-.05212	-.05167	-.05112	-.05044	-.04957	-.04844
	-.09829	-.09809	-.09784	-.09753	-.09715	-.09667	-.09607	-.09528	-.09422	-.09275	-.09058
	-.18776	-.18757	-.18733	-.18701	-.18658	-.18601	-.18522	-.18409	-.18242	-.17973	-.17486
	-.45606	-.45596	-.45579	-.45553	-.45514	-.45453	-.45359	-.45204	-.44930	-.44372	-.42769
	2.06E24	2.06E24	2.06E24	2.06E24	2.06E24	2.06E24	2.06E24	2.06E24	2.06E24	2.06E24	2.49E24
$\lambda = 100$											
	.002741	.002741	.002741	.002741	.002741	.002741	.002741	.002741	.002741	.002741	.002741
	.002139	.002143	.002147	.002150	.002154	.002158	.002162	.002166	.002170	.002175	.002179
	.001388	.001395	.001403	.001411	.001419	.001427	.001436	.001446	.001456	.001466	.001477
	.000423	.000434	.000445	.000458	.000471	.000485	.000500	.000516	.000534	.000553	.000574
	-.00086	-.00085	-.00083	-.00082	-.00080	-.00078	-.00075	-.00073	-.00070	-.00067	-.00063
	-.00266	-.00264	-.00262	-.00260	-.00258	-.00255	-.00251	-.00248	-.00243	-.00238	-.00232
	-.00535	-.00533	-.00531	-.00528	-.00525	-.00521	-.00517	-.00511	-.00504	-.00496	-.00484
	-.00983	-.00981	-.00978	-.00975	-.00972	-.00967	-.00961	-.00953	-.00942	-.00927	-.00906
	-.01878	-.01876	-.01873	-.01870	-.01866	-.01860	-.01852	-.01841	-.01824	-.01797	-.01749
	-.04561	-.04560	-.04558	-.04555	-.04551	-.04545	-.04536	-.04520	-.04493	-.04437	-.04277
	2.06E22	2.06E22	2.06E22	2.06E22	2.06E22	2.06E22	2.06E22	2.06E22	2.06E22	2.06E22	2.49E22
$\lambda = 1000$											
	.000274	.000274	.000274	.000274	.000274	.000274	.000274	.000274	.000274	.000274	.000274
	.000214	.000214	.000215	.000215	.000215	.000216	.000216	.000217	.000217	.000217	.000218
	.000139	.000140	.000140	.000141	.000142	.000143	.000144	.000145	.000146	.000147	.000148
	.000042	.000043	.000045	.000046	.000047	.000048	.000050	.000052	.000053	.000055	.000057
	-.00009	-.00008	-.00008	-.00008	-.00008	-.00008	-.00008	-.00007	-.00007	-.00007	-.00006
	-.00027	-.00026	-.00026	-.00026	-.00026	-.00025	-.00025	-.00025	-.00024	-.00024	-.00023
	-.00054	-.00053	-.00053	-.00053	-.00052	-.00052	-.00052	-.00051	-.00050	-.00050	-.00048
	-.00098	-.00098	-.00098	-.00098	-.00097	-.00097	-.00096	-.00095	-.00094	-.00093	-.00091
	-.00188	-.00188	-.00187	-.00187	-.00187	-.00186	-.00185	-.00184	-.00182	-.00180	-.00175
	-.00456	-.00456	-.00456	-.00456	-.00455	-.00455	-.00454	-.00452	-.00449	-.00444	-.00428
	2.06E20	2.06E20	2.06E20	2.06E20	2.06E20	2.06E20	2.06E20	2.06E20	2.06E20	2.06E20	2.49E20

Appendix 6.11 Mean of Expected Tax Revenues For Short-Run Case

λ = 0.00001

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0	0	0	0	0	0	0	0	0	0	0
0.1	35358	35525	35696	35870	36047	36229	36414	36603	36797	36994	37196
0.2	75421	76096	76800	77533	78298	79097	79932	80806	81722	82683	83691
0.3	122205	123740	125372	127113	128971	130961	133097	135395	137874	140557	143470
0.4	179071	181827	184824	188092	191672	195610	199963	204799	210204	216285	223176
0.5	252067	256419	261255	266660	272741	279632	287508	296596	307198	319728	334764
0.6	353291	359625	366823	375074	384629	395821	409112	425152	444894	469786	502146
0.7	510971	519686	529819	541747	555995	573312	594809	622207	658323	708104	781116
0.8	809793	821298	834996	851577	872059	898003	931930	978195	1045021	1150034	1339057
0.9	1673178	1687900	1705853	1728234	1756909	1794969	1847921	1926634	2055949	2307772	3012877
1	2.03E21	2.03E21	2.03E21	2.03E21	2.03E21	2.03E21	2.03E21	2.03E21	2.03E21	2.03E21	3.03E21

λ = 0.0001

0	0	0	0	0	0	0	0	0	0	0
3536	3553	3570	3587	3605	3623	3641	3660	3680	3699	3720
7542	7610	7680	7753	7830	7910	7993	8081	8172	8268	8369
12221	12374	12537	12711	12897	13096	13310	13539	13787	14056	14347
17907	18183	18482	18809	19167	19561	19996	20480	21020	21628	22318
25207	25642	26126	26666	27274	27963	28751	29660	30720	31973	33476
35329	35962	36682	37507	38463	39582	40911	42515	44489	46979	50215
51097	51969	52982	54175	55600	57331	59481	62221	65832	70810	78112
80979	82130	83500	85158	87206	89800	93193	97819	104502	115003	133906
167318	168790	170585	172823	175691	179497	184792	192663	205595	230777	301288
2.03E20	2.03E20	2.03E20	2.03E20	2.03E20	2.03E20	2.03E20	2.03E20	2.03E20	2.03E20	3.03E20

λ = 0.001

0	0	0	0	0	0	0	0	0	0	0
354	355	357	359	360	362	364	366	368	370	372
754	761	768	775	783	791	799	808	817	827	837
1222	1237	1254	1271	1290	1310	1331	1354	1379	1406	1435
1791	1818	1848	1881	1917	1956	2000	2048	2102	2163	2232
2521	2564	2613	2667	2727	2796	2875	2966	3072	3197	3348
3533	3596	3668	3751	3846	3958	4091	4252	4449	4698	5021
5110	5197	5298	5417	5560	5733	5948	6222	6583	7081	7811
8098	8213	8350	8516	8721	8980	9319	9782	10450	11500	13391
16732	16879	17059	17282	17569	17950	18479	19266	20559	23078	30129
2.03E19	2.03E19	2.03E19	2.03E19	2.03E19	2.03E19	2.03E19	2.03E19	2.03E19	2.03E19	3.03E19

λ = 0.01

0	0	0	0	0	0	0	0	0	0	0
35	36	36	36	36	36	36	37	37	37	37
75	76	77	78	78	79	80	81	82	83	84
122	124	125	127	129	131	133	135	138	141	143
179	182	185	188	192	196	200	205	210	216	223
252	256	261	267	273	280	288	297	307	320	335
353	360	367	375	385	396	409	425	445	470	502
511	520	530	542	556	573	595	622	658	708	781
810	821	835	852	872	898	932	978	1045	1150	1339
1673	1688	1706	1728	1757	1795	1848	1927	2056	2308	3013
2.03E18	2.03E18	2.03E18	2.03E18	2.03E18	2.03E18	2.03E18	2.03E18	2.03E18	2.03E18	3.03E18

λ = 0.01

0	0	0	0	0	0	0	0	0	0	0
3.53583	3.55254	3.56958	3.58698	3.60474	3.62288	3.64140	3.66032	3.67965	3.69941	3.71960
7.54213	7.60964	7.67996	7.75327	7.82977	7.90967	7.99321	8.08062	8.17220	8.26825	8.36910
12.2205	12.3740	12.5372	12.7113	12.8971	13.0961	13.3097	13.5395	13.7874	14.0557	14.3470
17.9071	18.1827	18.4824	18.8092	19.1672	19.5610	19.9963	20.4799	21.0204	21.6285	22.3176
25.2067	25.6419	26.1255	26.6660	27.2741	27.9632	28.7508	29.6596	30.7198	31.9728	33.4764
35.3291	35.9625	36.6823	37.5074	38.4629	39.5821	40.9112	42.5152	44.4894	46.9786	50.2146
51.0971	51.9686	52.9819	54.1747	55.5995	57.3312	59.4809	62.2207	65.8323	70.8104	78.1116
80.9793	82.1298	83.4996	85.1577	87.2059	89.8003	93.1930	97.8195	104.502	115.003	133.906
167.318	168.790	170.585	172.823	175.691	179.497	184.792	192.663	205.595	230.777	301.288
2.03E17	2.03E17	2.03E17	2.03E17	2.03E17	2.03E17	2.03E17	2.03E17	2.03E17	2.03E17	3.03E17

Appendix 6.11 Continued.

$\lambda = 1$

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0	0	0	0	0	0	0	0	0	0	0
0.1	.353583	.355254	.356958	.358698	.360474	.362288	.364140	.366032	.367965	.369941	.371960
0.2	.754213	.760964	.767996	.775327	.782977	.790967	.799321	.808062	.817220	.826825	.836910
0.3	1.22205	1.23740	1.25372	1.27113	1.28971	1.30961	1.33097	1.35395	1.37874	1.40557	1.43470
0.4	1.79071	1.81827	1.84824	1.88092	1.91672	1.95610	1.99963	2.04799	2.10204	2.16285	2.23176
0.5	2.52067	2.56419	2.61255	2.66660	2.72741	2.79632	2.87508	2.96596	3.07198	3.19728	3.34764
0.6	3.53291	3.59625	3.66823	3.75074	3.84629	3.95821	4.09112	4.25152	4.44894	4.69786	5.02146
0.7	5.10971	5.19686	5.29819	5.41747	5.55995	5.73312	5.94809	6.22207	6.58323	7.08104	7.81116
0.8	8.09793	8.21298	8.34996	8.51577	8.72059	8.98003	9.31930	9.78195	10.4502	11.5003	13.3906
0.9	16.7318	16.8790	17.0585	17.2823	17.5691	17.9497	18.4792	19.2663	20.5595	23.0777	30.1288
1	2.03E16	2.03E16	2.03E16	2.03E16	2.03E16	2.03E16	2.03E16	2.03E16	2.03E16	2.03E16	3.03E16

$\lambda = 10$

0	0	0	0	0	0	0	0	0	0	0
.035358	.035525	.035696	.035870	.036047	.036229	.036414	.036603	.036797	.036994	.037196
.075421	.076096	.076800	.077533	.078298	.079097	.079932	.080806	.081722	.082683	.083691
.122205	.123740	.125372	.127113	.128971	.130961	.133097	.135395	.137874	.140557	.143470
.179071	.181827	.184824	.188092	.191672	.195610	.199963	.204799	.210204	.216285	.223176
.252067	.256419	.261255	.266660	.272741	.279632	.287508	.296596	.307198	.319728	.334764
.353291	.359625	.366823	.375074	.384629	.395821	.409112	.425152	.444894	.469786	.502146
.510971	.519686	.529819	.541747	.555995	.573312	.594809	.622207	.658323	.708104	.781116
.809793	.821298	.834996	.851577	.872059	.898003	.931930	.978195	1.04502	1.15003	1.33906
1.67318	1.68790	1.70585	1.72823	1.75691	1.79497	1.84792	1.92663	2.05595	2.30777	3.01288
2.03E15	2.03E15	2.03E15	2.03E15	2.03E15	2.03E15	2.03E15	2.03E15	2.03E15	2.03E15	3.03E15

$\lambda = 100$

0	0	0	0	0	0	0	0	0	0	0
.003536	.003553	.003570	.003587	.003605	.003623	.003641	.003660	.003680	.003699	.003720
.007542	.007610	.007680	.007753	.007830	.007910	.007993	.008081	.008172	.008268	.008369
.012221	.012374	.012537	.012711	.012897	.013096	.013310	.013539	.013787	.014056	.014347
.017907	.018183	.018482	.018809	.019167	.019561	.019996	.020480	.021020	.021628	.022318
.025207	.025642	.026126	.026666	.027274	.027963	.028751	.029660	.030720	.031973	.033476
.035329	.035962	.036682	.037507	.038463	.039582	.040911	.042515	.044489	.046979	.050215
.051097	.051969	.052982	.054175	.055600	.057331	.059481	.062221	.065832	.070810	.078112
.080979	.082130	.083500	.085158	.087206	.089800	.093193	.097819	.104502	.115003	.133906
.167318	.168790	.170585	.172823	.175691	.179497	.184792	.192663	.205595	.230777	.301288
2.03E14	2.03E14	2.03E14	2.03E14	2.03E14	2.03E14	2.03E14	2.03E14	2.03E14	2.03E14	3.03E14

$\lambda = 1000$

0	0	0	0	0	0	0	0	0	0	0
.000354	.000355	.000357	.000359	.000360	.000362	.000364	.000366	.000368	.000370	.000372
.000754	.000761	.000768	.000775	.000783	.000791	.000799	.000808	.000817	.000827	.000837
.001222	.001237	.001254	.001271	.001290	.001310	.001331	.001354	.001379	.001406	.001435
.001791	.001818	.001848	.001881	.001917	.001956	.002000	.002048	.002102	.002163	.002232
.002521	.002564	.002613	.002667	.002727	.002796	.002875	.002966	.003072	.003197	.003348
.003533	.003596	.003668	.003751	.003846	.003958	.004091	.004252	.004449	.004698	.005021
.005110	.005197	.005298	.005417	.005560	.005733	.005948	.006222	.006583	.007081	.007811
.008098	.008213	.008350	.008516	.008721	.008980	.009319	.009782	.010450	.011500	.013391
.016732	.016879	.017059	.017282	.017569	.017950	.018479	.019266	.020559	.023078	.030129
2.03E13	2.03E13	2.03E13	2.03E13	2.03E13	2.03E13	2.03E13	2.03E13	2.03E13	2.03E13	3.03E13

Appendix 6.12 Mean of Expected Tax Revenues Under Segregation Result

λ = 0.00001											
t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0	0	0	0	0	0	0	0	0	0	0
0.1	13774	13769	13763	13757	13750	13742	13734	13724	13714	13704	13692
0.2	29527	29506	29480	29448	29410	29365	29311	29249	29177	29093	28996
0.3	47622	47572	47504	47415	47300	47155	46973	46749	46473	46134	45719
0.4	68277	68184	68047	67852	67585	67228	66754	66132	65317	64248	62844
0.5	90979	90830	90587	90218	89682	88919	87846	86339	84220	81215	76892
0.6	112145	111924	111532	110897	109914	108425	106186	102804	97622	89487	76244
0.7	114315	114014	113424	112399	110709	107974	103540	96229	83753	61273	17248
0.8	-2594	-2968	-3798	-5357	-8119	-12937	-21453	-37100	-67892	-135784	-317932
0.9	-1.31E6	-1.31E6	-1.31E6	-1.31E6	-1.32E6	-1.33E6	-1.34E6	-1.38E6	-1.46E6	-1.71E6	-3.06E6
1	-412E34	-412E34	-412E34	-412E34	-412E34	-412E34	-412E34	-412E34	-412E34	-412E34	-497E34
λ = 0.0001											
	0	0	0	0	0	0	0	0	0	0	0
	1907	1911	1915	1918	1922	1927	1931	1935	1939	1944	1948
	4180	4196	4212	4228	4246	4264	4282	4302	4322	4343	4366
	6973	7008	7045	7084	7126	7170	7218	7268	7323	7381	7444
	10534	10597	10665	10738	10818	10906	11001	11107	11223	11352	11497
	15303	15403	15512	15634	15769	15921	16092	16287	16510	16769	17072
	22134	22280	22443	22629	22841	23085	23370	23707	24111	24604	25217
	32927	33128	33360	33628	33944	34319	34774	35336	36047	36971	38212
	52896	53166	53482	53859	54313	54874	55582	56503	57747	59497	62032
	102357	102717	103148	103671	104322	105155	106257	107780	109998	113345	116115
	-412E32	-412E32	-412E32	-412E32	-412E32	-412E32	-412E32	-412E32	-412E32	-412E32	-497E32
λ = 0.001											
	0	0	0	0	0	0	0	0	0	0	0
	196	196	197	197	198	198	199	199	200	200	201
	430	432	434	436	438	440	442	444	446	449	451
	719	723	727	732	737	742	747	753	759	766	773
	1090	1097	1105	1113	1122	1132	1143	1156	1169	1185	1202
	1592	1603	1616	1629	1645	1662	1682	1705	1732	1763	1801
	2323	2339	2357	2378	2403	2431	2465	2505	2555	2617	2698
	3508	3530	3556	3587	3623	3667	3722	3791	3881	4005	4186
	5821	5851	5887	5930	5983	6049	6135	6252	6420	6680	7141
	12571	12610	12659	12719	12795	12894	13032	13233	13559	14178	15834
	-412E30	-412E30	-412E30	-412E30	-412E30	-412E30	-412E30	-412E30	-412E30	-412E30	-497E30
λ = 0.01											
	0	0	0	0	0	0	0	0	0	0	0
	20	20	20	20	20	20	20	20	20	20	20
	43	43	44	44	44	44	44	45	45	45	45
	72	73	73	73	74	74	75	76	76	77	78
	109	110	111	112	113	114	115	116	117	119	121
	160	161	162	164	165	167	169	171	174	177	181
	233	235	237	239	241	244	248	252	257	263	272
	353	355	358	361	365	369	375	382	391	404	422
	587	590	594	598	604	611	619	631	648	675	724
	1280	1284	1289	1295	1303	1313	1327	1348	1381	1446	1626
	-412E28	-412E28	-412E28	-412E28	-412E28	-412E28	-412E28	-412E28	-412E28	-412E28	-497E28
λ = 0.1											
	0	0	0	0	0	0	0	0	0	0	0
	1.96561	1.96989	1.97425	1.97870	1.98324	1.98788	1.99262	1.99746	2.00240	2.00745	2.01262
	4.31673	4.33403	4.35204	4.37081	4.39040	4.41085	4.43223	4.45459	4.47802	4.50259	4.52837
	7.21834	7.25771	7.29958	7.34419	7.39183	7.44281	7.49750	7.55632	7.61976	7.68839	7.76288
	10.9452	11.0161	11.0931	11.1770	11.2689	11.3699	11.4814	11.6053	11.7437	11.8993	12.0755
	15.9919	16.1041	16.2286	16.3677	16.5240	16.7011	16.9032	17.1362	17.4078	17.7286	18.1131
	23.3463	23.5103	23.6964	23.9095	24.1558	24.4440	24.7858	25.1977	25.7040	26.3415	27.1693
	35.3126	35.5399	35.8035	36.1133	36.4824	36.9302	37.4848	38.1902	39.1180	40.3943	42.2623
	58.7959	59.1001	59.4610	59.8964	60.4323	61.1086	61.9899	63.1871	64.9102	67.6079	72.4461
	128.274	128.677	129.167	129.773	130.544	131.559	132.962	135.031	138.402	144.911	162.980
	-412E26	-412E26	-412E26	-412E26	-412E26	-412E26	-412E26	-412E26	-412E26	-412E26	-497E26

Appendix 6.12 Continued.

$\lambda = 1$

t\d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0	0	0	0	0	0	0	0	0	0	0
0.1	.196566	.196994	.197430	.197875	.198330	.198794	.199268	.199752	.200246	.200751	.201268
0.2	.431686	.433415	.435217	.437094	.439053	.441098	.443236	.445473	.447816	.450273	.452852
0.3	.721856	.725794	.729981	.734443	.739207	.744305	.749775	.755658	.762003	.768867	.776317
0.4	1.09456	1.10165	1.10935	1.11774	1.12693	1.13703	1.14819	1.16058	1.17442	1.18998	1.20760
0.5	1.59925	1.61047	1.62293	1.63684	1.65247	1.67018	1.69039	1.71370	1.74086	1.77294	1.81140
0.6	2.33474	2.35114	2.36976	2.39106	2.41570	2.44452	2.47871	2.51990	2.57054	2.63431	2.71710
0.7	3.53148	3.55420	3.58057	3.61155	3.64847	3.69325	3.74872	3.81927	3.91208	4.03973	4.22659
0.8	5.88013	5.91055	5.94664	5.99018	6.04378	6.11142	6.19956	6.31932	6.49166	6.76152	7.24555
0.9	12.8297	12.8701	12.9190	12.9796	13.0567	13.1583	13.2986	13.5055	13.8427	14.4940	16.3023
1	-412E24	-412E24	-412E24	-412E24	-412E24	-412E24	-412E24	-412E24	-412E24	-412E24	-497E24

$\lambda = 10$

	0	0	0	0	0	0	0	0	0	0	0
	.019657	.019699	.019743	.019788	.019833	.019879	.019927	.019975	.020025	.020075	.020127
	.043169	.043342	.043522	.043710	.043905	.044110	.044324	.044547	.044782	.045027	.045285
	.072186	.072580	.072998	.073444	.073921	.074431	.074978	.075566	.076201	.076887	.077632
	.109457	.110165	.110935	.111774	.112693	.113703	.114819	.116058	.117442	.118998	.120761
	.159925	.161048	.162293	.163684	.165248	.167018	.169040	.171370	.174087	.177295	.181141
	.233475	.235115	.236977	.239107	.241571	.244454	.247872	.251992	.257056	.263432	.271712
	.353150	.355423	.358060	.361157	.364850	.369328	.374875	.381930	.391211	.403977	.422663
	.588018	.591060	.594670	.599024	.604383	.611148	.619962	.631938	.649173	.676159	.724565
	1.28299	1.28703	1.29193	1.29799	1.30570	1.31586	1.32988	1.35058	1.38430	1.44943	1.63027
	-412E22	-412E22	-412E22	-412E22	-412E22	-412E22	-412E22	-412E22	-412E22	-412E22	-497E22

$\lambda = 100$

	0	0	0	0	0	0	0	0	0	0	0
	.001966	.001970	.001974	.001979	.001983	.001988	.001993	.001998	.002002	.002008	.002013
	.004317	.004334	.004352	.004371	.004391	.004411	.004432	.004455	.004478	.004503	.004529
	.007219	.007258	.007300	.007344	.007392	.007443	.007498	.007557	.007620	.007689	.007763
	.010946	.011017	.011094	.011177	.011269	.011370	.011482	.011606	.011744	.011900	.012076
	.015993	.016105	.016229	.016368	.016525	.016702	.016904	.017137	.017409	.017730	.018114
	.023348	.023512	.023698	.023911	.024157	.024445	.024787	.025199	.025706	.026343	.027171
	.035315	.035542	.035806	.036116	.036485	.036933	.037488	.038193	.039121	.040398	.042266
	.058802	.059106	.059467	.059902	.060438	.061115	.061996	.063194	.064917	.067616	.072457
	.128299	.128703	.129193	.129799	.130570	.131586	.132989	.135058	.138430	.144943	.163027
	-412E20	-412E20	-412E20	-412E20	-412E20	-412E20	-412E20	-412E20	-412E20	-412E20	-497E20

$\lambda = 1000$

	0	0	0	0	0	0	0	0	0	0	0
	.000197	.000197	.000197	.000198	.000198	.000199	.000199	.000200	.000200	.000201	.000201
	.000432	.000433	.000435	.000437	.000439	.000441	.000443	.000445	.000448	.000450	.000453
	.000722	.000726	.000730	.000734	.000739	.000744	.000750	.000756	.000762	.000769	.000776
	.001095	.001102	.001109	.001118	.001127	.001137	.001148	.001161	.001174	.001190	.001208
	.001599	.001610	.001623	.001637	.001652	.001670	.001690	.001714	.001741	.001773	.001811
	.002335	.002351	.002370	.002391	.002416	.002445	.002479	.002520	.002571	.002634	.002717
	.003532	.003554	.003581	.003612	.003648	.003693	.003749	.003819	.003912	.004040	.004227
	.005880	.005911	.005947	.005990	.006044	.006111	.006200	.006319	.006492	.006762	.007246
	.012830	.012870	.012919	.012980	.013057	.013159	.013299	.013506	.013843	.014494	.016303
	-412E18	-412E18	-412E18	-412E18	-412E18	-412E18	-412E18	-412E18	-412E18	-412E18	-497E18

VITA

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