

ESSAYS ON THE OPTIMUM QUANTITY OF MONEY

by

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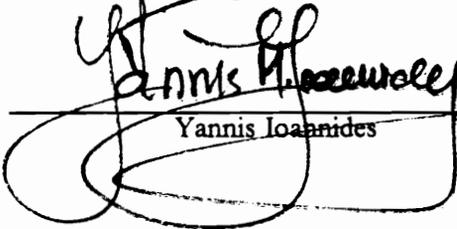
ECONOMICS

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(ABSTRACT)

Milton Friedman's article on the optimum quantity of money has motivated much research since its publication. While most of the research has been on deterministic frameworks, a few models (e.g. Bewley 1983, Taub 1989) have extended the analysis to stochastic environments. The first two essays of the dissertation address the issue in two types of stochastic economies. In both the models, quadratic utility and linear constraints have been used to facilitate the use of Whiteman's techniques (1985). The third essay introduces capital and derives the optimal rate of monetary policy in the presence of financial intermediaries.

In the first essay a pure exchange model in which infinitely lived agents face stochastically varying endowments in each period is considered. In this model individuals can delay payment for purchases into the future with a credit card. It shows that the optimal rate of inflation is the same in a world where individuals are required to pay for their purchases immediately as in a world where they can delay payment with a credit card. Moreover, the optimal inflation rate may be positive or negative depending on the parameters of the model. Therefore, Bewley's (1983) conjecture that deflation should proceed at a rate greater than the rate of time preference in a world of uncertainty is not generally true.

The second essay derives the optimum quantity of money in a stochastic production economy. The optimum quantity of money literature largely ignores the effect of labor supply on money's optimal rate of return. This paper examines the issue in an economy that is subject to stochastic shocks each period. It shows that incorporating production affects the optimal return on money in important ways. If there are individual specific shocks to preferences, then the optimal policy is highly inflationary. When individual preferences are subject to economy wide shocks, however, it is possible for either inflation or deflation to be optimal. The optimal policy depends on the

weight individuals attach to the disutility of work and the weight individuals attach to the utility from holding money. Optimal policy responds positively to increases in the disutility from work and negatively to increases in the weight on consumption in the utility function. The paper therefore shows the sensitivity of the optimal policy on the way labor supply is modeled. Since such considerations do not arise in endowment economies, the optimal policy will generally change as one moves from endowment to production economies.

In the third essay the Tobin effect and optimal monetary policy are analyzed when financial intermediaries develop endogenously. Providing a justification for the development of intermediaries similar to those found in the recent financial intermediation literature, we show that financial intermediation significantly affects investment decisions and monetary policy. In particular, the cost to intermediaries of providing substitutes of outside money play a critical role. Whether a decrease in the return on outside money will increase investment or not is found to depend on how the cost of providing alternative means of payment is affected. It is found that at low and moderate rates of inflation the Tobin effect remains valid. At high rates of inflation, however, the Tobin effect gets reversed. Further, since borrowers have private information regarding the outcome of the investment projects financed by the lenders, credit rationing may occur in equilibrium. We also derive the rate of return on money that maximizes social welfare. This optimal rate of return is not only dependent on the cost of the alternative means of payment, it also depends critically on whether credit is rationed in equilibrium or not. Finally, the paper highlights some of the distributional issues raised by a change in the rate of return on money.

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# Chapter I

## Introduction

Monetary economics studies the influences of money, monetary policy, and financial institutions on the real economy. It is one of the most challenging yet unsettled areas of macroeconomic theory. Milton Friedman in his celebrated paper "The Optimum Quantity of Money" (1969) proposed that the economy's money supply ought to be contracted steadily at the average individual's rate of time preference. He showed that such a deflation allows the individuals to reach their satiation level of real balance holdings. Since money is costless to produce, optimality is reached when the individuals are satiated with it. An implication of this rate of monetary contraction is that it forces the nominal return on alternative assets to zero.

Woodford (1991) surveys in detail the optimum quantity of money literature that was spawned by the work of Friedman. Woodford discussed the robustness of Friedman's rule by considering two forms of interpretation, weak and strong, of his policy prescription. He interprets the weak form of the policy as one that forces the nominal interest rate to zero so that a monetary economy may reach a competitive equilibrium with the most efficient allocation of resources. He interpreted the strong form of the policy to be the one that maximizes the welfare of the individuals by steadily contracting the supply of money so that the nominal interest rate is driven down to zero. On the

basis of these two interpretations Woodford finds that it is much easier to accept Friedman's rule in its weak form rather than its strong form.

Even a cursory observation of the returns of all assets reveals that except money almost all other assets provide a positive nominal return. There are various theories that explain why money coexists with these assets even when it is dominated in rate of return. The common element that runs through all these theories is based on the transactions role of money. In the literature, there are two alternative ways of modelling the transactions services provided by money - money in the utility function and cash-in-advance constraints. Only recently some authors are deriving the necessity of money from first principles. In this effort the works of J. Ostroy, R. Starr, N. Kiyotaki, and R. Wright are most noteworthy. Nonetheless, very little progress has been made in the application of these kinds of models to answer policy questions. Whether money is introduced in a model in the traditional ways or derived from first principles, it is recognized by every monetary economist that if money is not used in transactions, resources from the productive sectors of the economy will be diverted to facilitate transactions. Further recognizing that a positive nominal interest on alternative assets will provide individuals with an incentive to economize on cash, it is argued that the nominal interest ought to be forced down to zero to achieve Pareto optimality in the allocation of resources. Woodford provides a number of examples to show why a zero nominal interest rate is required for optimality. He also shows that the standard theories that explain interest rate differentials, like Sargent and Wallace (1982), Fama (1980), Grossman and Weiss (1983), all yield Pareto inefficient equilibria if the nominal interest is non-zero. Woodford argues on the basis of all these examples that it is very difficult to reject Friedman's rule in its weak form.

It is, however, more difficult to accept Friedman's rule of zero nominal interest when the criterion is welfare maximization. There are a number of examples in the literature that validate Friedman's rule as maximizing welfare. The most notable examples are by Sidrauski (1967) and Brock (1975). Woodford argues that although technically it may be feasible to validate Friedman's rule, in practice an economy is not likely to have at its disposal the wide range of policy instruments that will be required to implement it. This concern is most evident in the criticism of Phelps (1973). He showed how the alternative forms of taxation that will be required to finance the deflation may

be highly distortionary and therefore welfare reducing. There are a number of other considerations which when taken into account invalidate Friedman's rule. Woodford, in the context of an overlapping generations model shows that Friedman's rule is feasible but extremely inequalitarian. Bewley (1983) and Taub (1989) provide examples that point out that under uncertainty Friedman's rule does not provide perfect self insurance against future uncertainties.

This dissertation provides three examples in which Friedman's rule of deflation at the rate of time preference is not optimal when the monetary authority endeavors to maximize social welfare. The first two essays are in the context of economies in which the environment is subject to stochastic shocks each period. Consequently, these essays corroborate the criticisms of Bewley. The last essay, however, is more related to the issues raised by Tobin's monetary growth literature. The conclusions of the papers are, however, quite different from those of Bewley (1983) and Tobin (1965).

In the first essay a pure exchange model in which infinitely lived agents face stochastically varying endowments in each period is considered. This model is closely related to Taub (1989). Unlike Taub's model, individuals in this model can delay payment for purchases into the future with a credit card. Comparison with Taub's model shows that the optimal rate of inflation is the same whether individuals are required to pay for their purchases immediately in cash or can delay payment with a credit card. Moreover, the optimal inflation rate may be positive or negative depending on the parameters of the model. Therefore, Bewley's (1983) conjecture that deflation should proceed at a rate greater than the rate of time preference in a world of uncertainty is not generally true. The essay also shows that the optimal rate of return on money when income is subject to an economy wide shock in Taub's model is incorrect. The corrected result changes his Proposition 2 and shows that Bewley's conjecture holds if the aggregate shock to income is stationary.

The second paper derives the optimum quantity of money in a stochastic production economy. The optimum quantity of money literature has largely ignored production. This essay examines whether optimal monetary policy will be different for pure endowment and production economies under uncertainty. It shows that incorporating production affects the optimal return

on money in important ways. This essay shows that if labor is the only factor of production, money is the only asset, and the good produced is perishable, the return on money is positively correlated with production. If there are individual specific shocks to preferences, then the optimal policy is highly inflationary. When individual preferences are subject to economy wide shocks, however, it is possible for either inflation or deflation to be optimal. The optimal policy depends on the weight individuals attach to the disutility of work and the weight individuals attach to the utility from holding money. Optimal policy responds positively to increases in the disutility from work and negatively to increases in the weight on consumption in the utility function. This therefore shows the sensitivity of the optimal policy to the way labor supply is modeled. Since such considerations do not arise in endowment economies, the optimal policy will generally change as one moves from endowment to production economies.

In the third essay optimal monetary policy is derived when financial intermediaries are present. In the context of an overlapping generations model, reasons for the development of financial intermediaries similar to the ones given by Williamson (1986) are provided. It is shown that if there are two classes of agents, entrepreneurs and lenders, and the outcome of investment projects operated by the entrepreneurs is risky and privately observable by the entrepreneurs, credit rationing may occur in equilibrium. If the investment projects yield higher returns than money, the opportunity cost of holding money becomes high. If a certain amount of liquidity is required each period some amount of money will be held even though it is dominated in rate of return. To reduce money holdings the intermediaries may provide their own medium of exchange. As long as such an alternative medium is costly to produce, money will continue to be held in equilibrium. The cost of the alternative means of payment significantly influences optimal monetary policy. In particular, the cost imposes a limit on the rate of inflation that can increase investment. Even though at moderate rates of inflation the Tobin effect holds, the costs of the alternative means of payment may reverse the Tobin effect at high rates of inflation. The optimal rate of return on money is also derived. Among other factors, the optimal return is dependent on whether credit is rationed or not. Finally, the paper highlights some of the distributional issues raised by a change in the rate of return on money.

## **Chapter II**

# **The Effect of Credit Card and Stochastic Income on Friedman's Rule**

### **1. Introduction**

Milton Friedman's article "The Optimum Quantity of Money" has been very influential in macroeconomics and has inspired a lot of research. Some of the important contributions in the optimum quantity of money literature inspired by the work of Friedman are by Phelps (1970), Grandmont and Younes (1973), Brock (1975), Benhabib and Bull (1983), Lucas and Stokey (1983), and Bewley (1983) to name a few. Except Bewley's paper none of the others considered a purely stochastic economy. The present paper shows that if a credit card can be introduced in an exchange economy where individual income is subject to both individual specific and aggregate shocks, Friedman's rule does not give the optimum. This confirms Bewley's result that under uncertainty welfare is not maximized by deflating at the rate of time preference. The optimal rate of return is, however, found to be different from what Bewley found for his model. The results are similar to those of a model considered by Taub (1989).

Both Bewley (1983) and Taub (1989) showed that Friedman's rule is not optimal when a stochastic economy is considered. The economic environment that Bewley considered in his paper had some differences with the type of economy considered by Friedman, but he conjectured that if uncertainty is introduced in a deterministic version of Friedman's model the rate of deflation should exceed the rate of time preference. Taub proved that this conjecture is true only in special cases. The present paper analyzes Bewley's conjecture in an economy where exchange is made, unlike Taub's paper, with the help of a credit card. In Taub's paper all purchases are made with cash carried from previous periods. Bewley's conjecture is found to be true in a large number of cases when the shock to income is economy wide. When income is subject to purely individual specific shocks, however, Bewley's conjecture does not hold as Taub also showed. A brief discussion of the relevant papers follows.

Friedman considered a very simplified stationary economy with constant immortal individuals. He assumed that tastes, resources and technology remain fixed so that there is no uncertainty in production. Further, the capital goods that are used in production cannot be traded. Individual consumers, however, face some uncertainty regarding the extent of future expenditures. In this sense his model, although stationary, is not static. Since income flows and expenditures do not coincide, individuals are required to hold some money to meet future expenses that cannot be foreseen. In other words, Friedman provided a motivation for a precautionary demand for money to exist. By considering a multi-good economy he justified that the money in the economy also performs a transactions role. Further, he captured the satisfaction individuals derive from holding money by including it as an argument in the utility function. He also included money in the production function by justifying that money holdings provide productive services by releasing resources for production.

In the context of such an economy he investigated the consequences of a helicopter drop of money or having a government that imposes a tax on the individuals and puts the proceeds in a furnace. In other words, he analyzed the consequences of a change in the money supply. He discussed that if a helicopter drops money there can be some welfare losses to the individuals due to the reduction in the amount of real balances held by the individuals and by a reduction in real in-

come brought about by diversion of productive services from the production of goods to substitute for cash balances. These implied that if inflation reduces welfare, deflation ought to increase welfare. He found this to be indeed the case. He then investigated whether there exists any welfare maximizing level of deflation. To determine that level Friedman imposed satiation in real balances for both individuals and firms. The satiation was justified by him by noting that although increased cash balances increases individuals' sense of pride and there is no cost of depreciation of the value of money, increasing levels of money holdings may require guards to protect them. He concluded that considerations of this nature would deter individuals and firms from increasing cash holdings indefinitely. Therefore, the marginal returns from the productive services of money and the marginal return from the non-pecuniary services of money decrease as the level of cash holdings increases. Consequently, equilibrium is reached when the cost of holding a dollar by an individual equals the benefits from holding it at the margin. Therefore, using Friedman's notation, in equilibrium,

$$-\left(\frac{1}{P} \times \frac{dP}{dt}\right)' + MPM + MNPS = IRD$$

where,  $P$  = price level;  $MPM$  = marginal product of money;  $MNPS$  = marginal non-pecuniary services of money;  $IRD$  = internal rate of discount. The left hand side of the above equation gives the marginal benefit of holding money while the right hand side gives the marginal cost. According to Friedman then, optimality is reached when the individuals are satiated with money, that is when  $MPS + MNPS = 0$ . This implies that welfare is maximized if deflation proceeds at the rate of time preference. This result is derived for a particular individual. If all individuals have the same rate of time preference the economy should deflate at the common rate of time preference. This has come to be known as "Friedman's rule".

While some models confirmed this result, papers like Bewley's "A Problem with the Optimum Quantity of Money" did not. He analyzed Friedman's rule in a modified version of Friedman's model. He considered a pure exchange economy where endowments are stochastic. In such a model if there is no transactions demand for money, he showed that a monetary equilibrium may not exist if the rate of deflation equals the rate of time preference. With immortal individuals he

argued that an infinite amount of money would be needed for perfect insurance. Therefore an equilibrium cannot exist. However, if satiation could be introduced it would impose a limit to the amount held. He criticized Friedman's reason for satiation as artificial. He further noted that even if we believe in such a notion, the equilibrium cannot be Pareto optimal. By using a small example in his introduction, Bewley conjectured that if uncertainty can be introduced in a deterministic version of Friedman's model the rate of deflation should exceed the rate of time preference. He however could not prove this "hunch". It was proved to be incorrect by Taub in his paper "The Optimum Quantity of Money in a Stochastic Economy". He showed that Bewley's conjecture would be true only in special cases.

Taub's model is a blend of Friedman's and Bewley's models in the sense that he retained the transactions motive for holding money that Bewley eliminated and introduced uncertainty in income like Bewley. He also considered a one good endowment economy in which endowments are subject to stochastic shocks. To motivate exchange he assumed that individuals cannot directly consume their own endowment, therefore, must exchange with others before consumption. Money is needed to facilitate such trade. Also the good is assumed to be perishable so that the amount that is not consumed cannot be stored. On the average it is assumed that endowment is less than consumption needs so that there is a positive price of the good. Therefore, whenever individuals receive income in excess of their consumption needs they sell the excess for money and hold it to use at future dates. Further, he introduced a penalty term in the utility function that resembles a cash-in-advance restriction. Consequently, all current period purchases are required to be paid in cash. In the future when income is insufficient, individuals use the money to buy more of the good. Due to the stochastic nature of the income process individuals need to insure themselves against future bad lucks. Individuals therefore, unlike Bewley's paper, hold money for both transactions and precautionary reasons in Taub's paper.

Taub used a linear-quadratic framework for the exercise. This framework allowed him to use the solution techniques elaborated by Whiteman in his 1985 paper. In the light of such a model he showed that a monetary equilibrium does exist but Friedman's rule is not optimal. In fact he showed that very high inflation can also be optimal. The important conclusion of the paper was

that the problem of non-existence of equilibrium mentioned by Bewley was an artifact of the absence of a transactions demand for money and not due to the presence of an asset demand.

The present paper follows Taub's model very closely. The only modification is made by introducing a credit card in the model. The credit card is a little different from the ones used in present economies in the sense that individuals cannot buy with it more than their current income can afford. Instead, the usefulness of the card comes from the fact that individuals do not need cash in advance for making transactions any more. This paper demonstrates that it would be always optimal for the government of such an economy to deflate its currency when the stochastic shock to income is economy wide. The inflationary outcome of Taub's model for the economy wide stochastic shock is found to be incorrect. This paper like Taub's paper confirms Bewley's result that Friedman's rule is not optimal and a monetary equilibrium fails to exist if individuals demand money only for precautionary purposes.

The paper is organized as follows. In section 2 the model is specified. Section 3 deals with the individual's optimization problem and the monetary equilibrium. Section 4 derives and solves the monetary authority's problem, section 5 rederives the optimal policy for Taub's model if the shock to income is economy wide, and section 6 concludes the paper.

## **2. The Model**

This model is very similar to Taub's model. The basic difference arises from the introduction of a credit card. Like Friedman's model there are an infinite number of immortal individuals in the economy. These individuals receive some income each period that is subject to stochastic shocks. The shocks have an idiosyncratic component and an economy wide component. Income is in terms of the only good in the economy.

To motivate exchange it is assumed that the income has to be exchanged with the income of others before consumption. Unlike Taub's model, this exchange is made with the help of a credit card so that individuals do not need to hold cash balances before they can purchase the good from

others. Every individual is assumed to sell a portion of his income to another on credit and holds the excess in the form of money. He then buys from another individual the amount he wants to consume on credit. Since we do not use the credit card for insurance purposes, the amount bought on the credit card is restricted to the amount lent on credit. It is assumed that the duration for which dues can be kept is constant both across individuals and over time. For the sake of tractability it is assumed to be 1.

Utility is derived in this model from direct consumption of the single good and from the repayment of credit card bills. There is a consumption bliss implying that the marginal utility of consumption is not positive always. Individuals reach satiation at some level  $\hat{c}$ . If consumption is higher than this level marginal utility of consumption will be negative. We further assume that average income  $\bar{y}$  is less than  $\hat{c}$  so that there is a positive price for the good.

The above discussion can be mathematically summarised as follows: A representative individual maximises

$$-E_t \sum_{s=0}^{\infty} \beta^s [\gamma c_{t+s}^2 + (1-\gamma)(c_{t+s-1} - p_{t+s} M_{t+s-1})^2] \quad (1.1)$$

subject to

$$c_t = y_t - p_t(M_t - M_{t-1}) + p_t H_t \quad (1.2)$$

where,

- $c_{t+s}$  = consumption at time  $t+s$
- $y_{t+s}$  = income at time  $t+s$
- $M_{t+s}$  = nominal balances held at time  $t+s$

- $p_{t+s}$  = inverse of the price level at time  $t + s$
- $M_{t+s-1}$  = nominal balances carried over from time  $t + s - 1$
- $\beta$  = discount factor
- $\gamma$  = positive fraction used as a weight

The above formulation shows that a linear quadratic framework is used. The reason for the choice is purely technical. For solving the model Whiteman's 1985 method will be used. Since a linear quadratic model separates into its deterministic and stochastic parts due to certainty equivalence, application of Whiteman's techniques will be facilitated.

The objective function indicates that utility is derived from present consumption and past consumption less inflation adjusted real balances carried over from the previous period.  $\gamma$  and  $1 - \gamma$  are the respective weights. We have shifted the bliss point from  $\hat{c}$  to zero. This will have no impact on our results because in the absence of non-negativity constraints the location of the bliss point is irrelevant. Since bliss is at zero, both positive and negative consumption would reduce utility - this is captured by the negative sign outside the expectations operator. There is nothing unusual about consumption generating utility but the second term in the utility function needs some explanation.  $c_{t+s-1}$  is the credit card bill that arrives in period  $t + s$  because the individual is assumed to buy the amount he consumes on credit. This has to be repaid from the money balances brought from the previous period. If the bill is greater than the amount of real balances brought over, it would generate a demand for money today to repay the excess. As the formulation indicates, deviation of inflation adjusted real balances carried over from the previous period from the credit card dues would reduce utility. This term therefore forces the individual to repay his dues with the money holdings of the previous period. In a L-Q model non-negativity constraints render a problem intractable. For this reason instead of forcing payment of bills with the help of non-negativity constraints a penalty term in the objective function is used. The reason we are stressing on repayment from the previous period's balances is to impose a limit to the amount of real bal-

ances an individual can hold. This suggests that infinite hoarding for precautionary reasons cannot be optimal for our model.

The above utility function is subject to the budget constraint (1.2). This needs some explanation too. The resources available to the individual at time  $t + s$  come from his income at that period, the real balances he brings from the previous period, the payment of the individual he had lent his income to in the previous period and any transfers made by the government. On the expenditure side he has the following- present consumption, present holding of real balances and payments due on the credit card. The budget constraint can be then written as:

$$c_t + p_t M_t + c_t^p = y_t + p_t M_{t-1} + c_{t-1}^c + p_t H_t$$

where,  $c_t^p =$  credit card dues  $c_{t-1}^c =$  repayment made by others for the amount lent by him in the previous period.

The fact that the terms  $c_{t-1}^c$ ,  $c_t^p$  do not appear in (1.2) indicates that an individual is constrained to buy on the credit card the amount he has lent to others. Therefore individuals use their credit cards not for insurance purposes but to just exchange the good. Its only usefulness comes from its ability to eliminate any need for cash in advance. This indicates that even after paying the dues he would effectively have the amount of real balances he carried over from the previous period on hand to spend.

Income has three components - a deterministic component, an idiosyncratic component and an economy wide component. It can be represented as:

$$y_t = \bar{y}_t + A(L)\varepsilon_t + B(L)v_t \tag{1.3}$$

Here  $\varepsilon_t$  is the individual specific shock,  $v_t$  is the economy wide shock.  $A(L)$  and  $B(L)$  are not specified now - their functional forms will depend on the type of representation specified for the income process. The innovations  $\varepsilon_t$  and  $v_t$  are assumed to be i.i.d. across time and individuals. The

distributions of the innovations are:  $\varepsilon \sim N(0, \sigma_\varepsilon^2)$  and  $v \sim N(0, \sigma_v^2)$ . Further the z transforms of the functions A(L) and B(L) are assumed to be analytic in the disk  $\{z: |z| < \beta^{\frac{1}{2}}\}$ , a region smaller than the unit disk.

The gross rate of return on real balances is assumed to be  $\rho_t$ . Therefore,

$$\rho_t = \frac{p_t}{p_{t-1}} \quad (1.4)$$

Real balances and real transfers can then be defined in the following ways:

$$m_t = p_t M_t \quad (1.5)$$

$$h_t = p_t H_t \quad (1.6)$$

where,  $m_t$  and  $h_t$  are real balance and real transfers respectively.

Given these, the objective function and the constraint can be rewritten as:

max

$$-E_t \sum_{s=0}^{\infty} \beta^s [\gamma c_{t+s}^2 + (1-\gamma)(c_{t+s-1} - \rho_{t+s} m_{t+s-1})^2] \quad (1.1')$$

subject to

$$c_t = \bar{y}_t + A(L)\varepsilon_t + B(L)v_t + h_t - (m_t - \rho_t m_{t-1}) \quad (1.2')$$

In this model therefore, individuals can consume more or less than the average for the individual specific shocks, however, the economy on the whole cannot do so. This necessitates the economy wide average resource constraint to be

$$\tilde{c}_t = \bar{y}_t + B(L)v_t \quad (1.7)$$

In the above equation and later,  $\tilde{x}$  would represent the economy wide average of  $x$ . In (1.7) the individual specific part does not enter because the sum of all of them add up to zero. Also, individuals receive money in the form of government transfers. Then according to Walras' law, if the goods market clears the money market should also clear. In other words, the extra money held on the average must equal the average transfers made, that is:

$$\tilde{h}_t = \tilde{m}_t - \rho_t \tilde{m}_{t-1} \quad (1.8)$$

Lastly, we need to equate the demand and supply of average real balance and average transfers. These are given by:

$$\tilde{m}_t = p_t \tilde{M}_t \quad (1.9)$$

$$\tilde{h}_t = p_t \tilde{H}_t \quad (1.10)$$

### 3. Monetary Equilibrium and the Individual's Optimization Problem

The monetary authority must choose the supply of money in such a way that the demand for money generated from the optimisation problem of the individual equals the average supply and satisfies the resource constraints (1.7) - (1.10). In this context a simplification is made. Calvo and others have shown that in cases like these, a monetary authority's objective is generally not dynamically consistent. Since these considerations are not important for the present paper, the following restrictive assumptions are made like Taub and Bewley. It is assumed that the rate of return on real balances is determined in some initial period and fixed thereafter. It cannot be revised later on and individuals take it as given.

The monetary equilibrium can be formally stated as a sequence of  $\{\rho_{t+s}, \tilde{M}_{t+s}, \tilde{H}_{t+s}\}$  and a sequence of demand functions  $m_t$  mapping from  $\mathbb{R} \times H_i(t) \times H_o(t) \rightarrow \mathbb{R}$  and satisfying (1.7) - (1.10).  $H_i(t)$  and  $H_o(t)$  represent the Hilbert spaces spanned by the innovations. Since the solution must lie in the space spanned by the driving processes the above mapping is justified.

The monetary authority's objective is to choose  $\rho$  such that social welfare is maximised. The social welfare function has to be derived from the individual's optimisation problem and the equilibrium in the money market. Our problem first is then to solve the individual's problem and specify the equilibrium.

#### The individual's problem:

As has been noted before, the individual wants to maximise (1.1') subject to (1.2'). Substituting the value of  $c_t$  from the constraint into the objective function we have

$$\begin{aligned}
 -E_t \sum_{s=0}^{\infty} \beta^s & \left[ \gamma \{ \bar{y}_{t+s} + A(L)e_{t+s} + B(L)v_{t+s} + h_{t+s} - (m_{t+s} - \rho m_{t+s-1}) \}^2 \right. \\
 & \left. + (1 - \gamma) \{ \bar{y}_{t+s-1} + A(L)e_{t+s-1} + B(L)v_{t+s-1} + h_{t+s-1} \right. \\
 & \left. - (m_{t+s-1} - \rho m_{t+s-2}) - \rho m_{t+s-1} \}^2 \right]
 \end{aligned} \tag{2.1}$$

The individual then has to maximise (2.1) with respect to  $m_{t+s}$ . Since the innovations fundamental for  $y_t$  and  $h_t$  are the square summable processes  $\varepsilon_t, v_t$  and  $v_t$ , respectively, the solutions of our problem must be found in the Hilbert spaces spanned by them. We also know that quadratic objectives and linear constraints give linear decision functions. These imply the following forms for the solutions:

$$m_t = \bar{m}_t + \mu(L)\varepsilon_t + \lambda(L)v_t \quad (2.2)$$

and

$$\tilde{h}_t = \bar{h}_t + H(L)v_t \quad (2.3)$$

Making these substitutions (2.1) can be rewritten as

$$\begin{aligned} -E_t \sum_{s=0}^{\infty} \beta^s & \left[ \gamma \{ \bar{y}_{t+s} + A(L)\varepsilon_{t+s} + B(L)v_{t+s} + \bar{h}_{t+s} + H(L)v_{t+s} \right. \\ & \left. - (\bar{m}_{t+s} + \mu(L)\varepsilon_{t+s} + \lambda(L)v_{t+s})(1 - \rho L) \right]^2 \\ & + (1 - \gamma) \{ \bar{y}_{t+s-1} + A(L)\varepsilon_{t+s-1} + B(L)v_{t+s-1} \bar{h}_{t+s-1} + H(L)v_{t+s-1} \\ & \left. - (\bar{m}_{t+s} + \mu(L)\varepsilon_{t+s} + \lambda(L)v_{t+s})(1 + \rho - \rho L)L \right]^2 \end{aligned} \quad (2.4)$$

Some rearrangements and taking expectations give us:

$$\begin{aligned} - \sum_{s=0}^{\infty} \beta^s & \left[ \gamma (\bar{y}_{t+s} + \bar{h}_{t+s} - (1 - \rho L)\bar{m}_{t+s})^2 + (1 - \gamma)L^2 (\bar{y}_{t+s} + \bar{h}_{t+s} - (1 + \rho - \rho L)\bar{m}_{t+s})^2 \right. \\ & \left. + \gamma \{ A(L) - (1 - \rho L)\mu(L) \}^2 \sigma_\varepsilon^2 + \gamma \{ B(L) + H(L) - (1 - \rho L)\lambda(L) \}^2 \sigma_v^2 \right. \\ & \left. + (1 - \gamma) \{ LA(L) - L(1 + \rho - \rho L)\mu(L) \}^2 \sigma_\varepsilon^2 + (1 - \gamma) \{ LB(L) + LH(L) - L(1 + \rho - \rho L)\lambda(L) \}^2 \sigma_v^2 \right] \end{aligned}$$

since

- $E_t \varepsilon_{t+s} v_{t+s} = 0$
- $E_t \varepsilon_t = E_t v_t = 0$
- $E_t \varepsilon_{t+s} \varepsilon_{t+j} = \begin{cases} \sigma_\varepsilon^2 & \text{if } s=j \\ 0 & \text{if } s \neq j \end{cases}$
- $E_t v_{t+s} v_{t+j} = \begin{cases} \sigma_v^2 & \text{if } s=j \\ 0 & \text{if } s \neq j \end{cases}$

We now see how the problem has separated into its deterministic and stochastic parts. We can now solve them separately and add the results.

Let us first solve the deterministic part. We must maximise the following with respect to  $\bar{m}_{t+s}$

$$-\sum_{s=0}^{\infty} \beta^s [\gamma(\bar{y}_{t+s} + \bar{h}_{t+s} - (1 - \rho L)\bar{m}_{t+s})^2 + (1 - \gamma)L^2(\bar{y}_{t+s} + \bar{h}_{t+s} - (1 + \rho - \rho L)\bar{m}_{t+s})^2]$$

Let us start from  $s=0$  and write the terms that contain  $\bar{m}_t$  explicitly as follows:

$$\begin{aligned} & [\beta^0 \{ \gamma(\bar{Y}_t - \bar{m}_t + \rho\bar{m}_{t-1})^2 + (1 - \gamma)(\bar{Y}_{t-1} - (1 + \rho)\bar{m}_{t-1} + \rho\bar{m}_{t-2})^2 \} \\ & + \beta^1 \{ \gamma(\bar{Y}_{t+1} - \bar{m}_{t+1} + \rho\bar{m}_t)^2 + (1 - \gamma)(\bar{Y}_t - (1 + \rho)\bar{m}_t + \rho\bar{m}_{t-1})^2 \} + \\ & \beta^2 \{ \gamma(\bar{Y}_{t+2} - \bar{m}_{t+2} + \rho\bar{m}_{t+1})^2 + (1 - \gamma)(\bar{Y}_{t+1} - (1 + \rho)\bar{m}_{t+1} + \rho\bar{m}_t)^2 \} + \dots ] \end{aligned} \quad (2.5)$$

where,  $\bar{Y}_t = \bar{y}_t + \bar{h}_t$ . The stationary state solution for the above is

$$\bar{m} = \left( \frac{\bar{y}}{1 - \gamma} \right) \frac{1}{1 - \rho(1 - \beta)} \left[ \gamma + \beta(1 - \gamma) - \frac{\gamma + \beta(1 - \gamma)(1 + \rho)}{\beta\rho} \right] \quad (2.6)$$

The above solution is derived in Appendix A.

**Solutions of the stochastic parts:**

To solve the stochastic parts Whiteman's techniques will be used. So we need to transfer the problem into the complex domain before solving. The objective then becomes:

$$-\frac{1}{2\pi i} \frac{\beta}{1-\beta} \oint [\sigma_\varepsilon^2 \{ \gamma(A - \mu r)(A_* - r_* \mu_*) + (1-\gamma)(a - \mu k)(a_* - k_* \mu_*) \} + \sigma_\omega^2 \{ \gamma(B + H - \lambda r)(B_* + H_* - r_* \lambda_*) + (1-\gamma)(b + h - \lambda k)(b_* + h_* - k_* \lambda_*) \}] \frac{dz}{z} \quad (2.7)$$

where,  $r = 1 - \rho z$ ,  $k = (1 + \rho - \rho z)z$ ,  $a = zA$ ,  $b = zB$ ,  $h = zH$  and the subscript \* is used to indicate the  $\beta$  conjugates. We want to maximise the above function with respect to  $\mu$  and  $\lambda$ . Let us first find the maximum with respect to  $\mu$ . For this we shall use the method of calculus of variations. Suppose  $\mu^*$  is optimal for  $\mu$  and  $\lambda^*$  is optimal for  $\lambda$ . Let variations of  $\delta\eta$  and  $\zeta\omega$  be taken for  $\mu$  and  $\lambda$  respectively.  $\eta$  and  $\omega$  are assumed to be analytic in the disk. Then they can be expressed as:

$$\mu = \mu^* + \delta\eta$$

$$\lambda = \lambda^* + \zeta\omega$$

Let  $J(\delta, \zeta)$  be the value of the expression in (2.7) when the optimal values of  $\mu$  and  $\lambda$  are substituted. Then  $J(.,.)$  becomes

$$J(\delta, \zeta) = -\frac{1}{2\pi i} \frac{\beta}{1-\beta} \oint [\sigma_\varepsilon^2 \{ \gamma(A - (\mu - \delta\eta)r)(A_* - r_*(\mu_* - \delta\eta_*)) \} + (1-\gamma)(a - (\mu - \delta\eta)k)(a_* - k_*(\mu_* - \delta\eta_*)) \} + \sigma_\omega^2 \{ \gamma(B + H - (\lambda - \zeta\omega)r)(B_* + H_* - r_*(\lambda_* - \zeta\omega_*)) \} + (1-\gamma)(b + h - (\lambda - \zeta\omega)k)(b_* + h_* - k_*(\lambda_* - \zeta\omega_*)) \}] \frac{dz}{z} \quad (2.8)$$

Differentiating with respect to first delta and setting the derivative and delta and zeta equal to zero, we have

$$J(0,0) = 0 = -\frac{\beta}{1-\beta} \frac{1}{2\pi i} \oint [\sigma_\varepsilon^2 \{ \gamma(\eta r(A_* - r_* \mu_*) + (A - \mu r)(r_* \eta_*)) + (1-\gamma)(\eta k(a_* - k_* \mu_*) + (a - \mu k)(k_* \eta_*)) \}] \frac{dz}{z} \quad (2.9)$$

Using  $\beta$  symmetry, the above equation can be rewritten as:

$$\oint 2\sigma_\varepsilon^2 [\gamma(A - r\mu)r_* + (1-\gamma)(a - \mu k)k_*] \eta_* \frac{dz}{z} = 0$$

Since  $\eta$  is an analytic function, its  $\beta$  conjugate  $\eta_*$  contains only non positive powers of  $z$ . Then from Cauchy's theorem, the integral can be zero only if the expression inside the bracket contains strictly negative powers of  $z$ . This gives us then the following Weiner-Hopf equation:

$$\gamma(A - r\mu)r_* + (1-\gamma)(a - \mu k)k_* = \sum_{-\infty}^{-1} \quad (2.10)$$

where,  $\sum_{-\infty}^{-1}$  represents a function whose Laurent expansion contains strictly negative powers of  $z$ .

Rearranging the above equation, we have

$$[\gamma r r_* + (1-\gamma) k k_*] \mu = \gamma A r_* + (1-\gamma) a k_* + \sum_{-\infty}^{-1}$$

We now need to find the analytic solution of  $\mu$ . Since the left hand side contains the product of two terms, we have to use Rozanov's theorem of factorisation. Rozanov's theorem tells us that there exists a unique analytic function  $C(z)$  in the disk  $\{z: |z| < \beta^{\frac{1}{2}}\}$  such that

$$C(z)C(\beta z^{-1}) = [\gamma r r_* + (1 - \gamma) k k_*]$$

Then the Weiner-Hopf equation becomes

$$C(z)C(\beta z^{-1})\mu = \gamma A r_* + (1 - \gamma) a k_* + \sum_{-\infty}^{-1}$$

Multiplying both sides by  $C(\beta z^{-1})^{-1}$ , which contains only negative powers of  $z$ , we have,

$$C(z)\mu = C(\beta z^{-1})^{-1}\{\gamma A r_* + (1 - \gamma) a k_* + \sum_{-\infty}^{-1}\}$$

We now apply the annihilator to get

$$[C(z)\mu]_+ = [C(\beta z^{-1})^{-1}\{\gamma A r_* + (1 - \gamma) a k_*\}]_+ + 0$$

since,  $C(\beta z^{-1})^{-1}\sum_{-\infty}^{-1}$  contains terms with strictly negative powers of  $z$ , the annihilator applied to it annihilates the whole expression. Further noting that  $C(z)$  and  $\mu$  are both analytic functions, the annihilator operator acts like an identity operator and we have

$$\mu = C(z)^{-1}[C(\beta z^{-1})^{-1}\{\gamma A r_* + (1 - \gamma) a k_*\}]_+ \quad (2.11)$$

where,  $a k_* = z A z_*(1 + \rho - \rho_*)$ . This gives the optimal value of  $\mu$ . The optimal solution for  $\lambda$  can be found in exactly the same way. The Weiner-Hopf equation for this case is

$$(1 - \gamma)\rho z k_* \lambda = \gamma B r_* + (1 - \gamma) z B k_* + \sum_{-\infty}^{-1}$$

noting that  $H(z) = (1 - \rho z)\lambda(z)$ . Now,

$$zk_* = zz_*(\rho + 1 - \rho z_*)$$

$$\text{or, } zk_* = \beta(\rho + 1 - \rho\beta z_*^{-1})$$

Since  $zk_*$  contains only negative power of  $z$ , its inverse also contains only negative powers of  $z$ . Multiplying both sides of the Weiner-Hopf equation by the inverse of  $zk_*$  and rearranging after applying the annihilator we have,

$$\lambda = (1 - \gamma)^{-1} \rho^{-1} [(zk_*)^{-1} \{ \gamma Br_* + (1 - \gamma) z B k_* \}]_+ \quad (2.12)$$

The above equation gives the optimal solution of  $\lambda$ .

Let us now compare the polynomial orders of the components of the demand function derived and those of the driving processes. In the solution of  $\mu$  if we put  $A(z) = 1$ ,  $\mu$  becomes autoregressive because  $C(z)$  has the form  $C(z) = c_0(1 - c_1z)$ . This means that the polynomial order of the individual component of the demand function exceeds that of the driving process and this is due to the effects of consumption smoothing. In contrast, the economy wide component is of the same polynomial order. This can be verified by putting  $B(z) = 1$  in (2.12). We see that in this case we will be left with a constant. This is due to the credit card. Although future income and consumption are totally unpredictable, the individual needs to accumulate real balances to pay for the consumption made in the current period.

Substituting the deterministic and stochastic solutions into (2.2) we shall get the money demand function of a representative individual. Equilibrium can be represented by the following equations:

$$m_t = \bar{m}_t + \mu(L)\varepsilon_t + \lambda(L)v_t \quad (2.13)$$

and,

$$h_t = \bar{h}_t + H(L)v_t = (1 - \rho L)\bar{m}_t + (1 - \rho L)\lambda(L)v_t \quad (2.14)$$

Noting that  $\tilde{m}_t = \bar{m}_t + \lambda(L)v_t$  and substituting it in equation (2.3) we have after some simple manipulations  $H(L) = (1 - \rho L)\lambda(L)$ . Having derived the equilibrium, we can now concentrate on the problem of the government. This is done in the next section.

#### 4. The Monetary Authority's Problem

To avoid the problem of dynamic inconsistency, we had assumed that the monetary authority decides on the optimal growth of money in the initial period,  $t=0$ , and follows the rule thereafter. In this section we would want to derive the optimal money supply rule and analyse Bewley's conjecture in the light of our model. For this we substitute the money demand derived in the previous section into the objective function and call it the welfare function. The monetary authority's task will be to maximise the welfare function with respect to  $\rho$ . We first derive the welfare function by substituting the equilibrium obtained in the previous section into the objective function of the individual and call it  $W$ . Then,

$$\begin{aligned}
 W = - \sum_{s=0}^{\infty} \beta^s \left[ \gamma \bar{y}^2 + (1-\gamma)(\bar{y} - \rho(\frac{\bar{y}}{1-\gamma})) \frac{1}{1-\rho(1-\beta)} \left\{ \gamma + \beta(1-\gamma) - \frac{\gamma + \beta(1-\gamma)(1+\rho)}{\beta\rho} \right\} \right]^2 \\
 + \frac{\beta}{2\pi i} \oint \left[ \{ \sigma_\epsilon^2 \{ \gamma(A - \mu^* r)(A_* - r_* \mu_*^*) + (1-\gamma)(a - \mu^* k)(a_* - k_* \mu_*^*) \} \right. \\
 \left. + \sigma_v^2 \{ \gamma BB_* + (1-\gamma)(b + h - \lambda^* k)(b_* + h_* - k_* \lambda_*^*) \} \right] \frac{dz}{z} \quad (3.1)
 \end{aligned}$$

As we have mentioned before the objective of the monetary authority now is to maximise the above welfare function with respect to  $\rho$ . Due to certainty equivalence this problem can also be solved in three parts. Let us first find the optimal  $\rho$  for the deterministic part. Let  $W1$  denote the deterministic part. Then

$$\begin{aligned}
 W1 = \gamma \bar{y}^2 + (1-\gamma)(\bar{y} - \rho(\frac{\bar{y}}{1-\gamma})) \frac{1}{1-\rho(1-\beta)} \left\{ \gamma + \beta(1-\gamma) - \frac{\gamma + \beta(1-\gamma)(1+\rho)}{\beta\rho} \right\}^2 \\
 \text{or, } W1 = \gamma \bar{y}^2 + \frac{\bar{y}^2}{1-\gamma} \left[ (1-\gamma) - \frac{(2\gamma - 1 + \beta(1-\gamma))\beta\rho - (\gamma + \beta(1-\gamma))}{\beta(1-(1-\beta)\rho)} \right]^2
 \end{aligned}$$

Differentiating the above equation with respect to  $\rho$  and setting the derivative equal to zero we have

$$\left[ (1 - \gamma) - \frac{(2\gamma - 1 + \beta(1 - \gamma))\beta\rho - (\gamma + \beta(1 - \gamma))}{\beta(1 - (1 - \beta)\rho)} \right] \left[ \frac{(2\gamma - 1 + \beta(1 - \gamma))\beta - (1 - \beta)(\gamma + \beta(1 - \gamma))}{(1 - (1 - \beta)\rho)^2} \right] = 0$$

The above equation implies that the first factor must be equal to zero because the second factor contains only parameters of the model in the numerator. This gives the following solution for  $\rho$

$$\rho = \frac{1}{\beta} \tag{3.2}$$

This value of  $\rho$  is different from the value in Taub. The difference is arising because of the presence of the credit card. The deterministic part of Taub's model is identical to the framework used by Friedman. In these models individuals buy the good for consumption from the money balances in hand. Therefore as in Taub's model if the consumption in the current period deviates from the money balances carried over from the previous period, the individual is penalised in the current period. In this model however, if his present consumption is different from the balances he carries he is penalised in the next period. All this implies that the preference of the individual for consuming in the current period over holding balances is greater in this model. So the monetary authority would have to deflate the currency more to induce the individuals to insure themselves fully. If  $\gamma$  in the present model equals 1, meaning that there is no demand for money for paying credit cards bills, the optimal rate of return on real balances should equal the rate of time preference. Now there is only a precautionary demand for money and no credit card. So Friedman's rule is applicable.

Let us now derive the optimal rate of return for the stochastic parts. We shall first consider the individual specific part. Let the welfare function for that part be called W2. Then

$$W2 = \int \sigma_i^2 \{ \gamma(A - \mu^* r)(A - r \cdot \mu^*) + (1 - \gamma)(a - \mu^* k)(a - k \cdot \mu^*) \} \frac{dz}{z}$$

Using  $\beta$  symmetry and rearranging we have

$$W2 = \int \sigma_i^2 \{ (\gamma + z(1 - \gamma))AA - 2\mu^*(\gamma r + (1 - \gamma)zz(\rho + r)) \\ + \mu^* \mu^*(\gamma r r + (1 - \gamma)(\rho + r)zz(\rho + r)) \} \frac{dz}{z}$$

Recalling that

$$CC = (\gamma r r + (1 - \gamma)(\rho + r)zz(\rho + r))$$

W2 becomes

$$\int \sigma_i^2 \{ (\gamma + z(1 - \gamma))AA - 2\mu^*(\gamma r + (1 - \gamma)zz(\rho + r)) + \mu^* \mu^* CC \} \frac{dz}{z}$$

We now substitute the value of  $\mu^*$  from equation (2.11) and get after some manipulations the following:

$$W2 = \int \left[ \sigma_i^2 \{ (\gamma + z(1 - \gamma))AA + [C^{-1}A(\gamma r + (1 - \gamma)zz(\rho + r)) - [C(\beta z^{-1})^{-1}\{\gamma Ar + (1 - \gamma)ak\}]_+ ] \right. \\ \left. [C^{-1}A(\gamma r + (1 - \gamma)zz(\rho + r)) - [C(\beta z^{-1})^{-1}\{\gamma Ar + (1 - \gamma)ak\}]_+ ] \right. \\ \left. - C^{-1}A(\gamma r + (1 - \gamma)zz(\rho + r))C^{-1}A(\gamma r + (1 - \gamma)zz(\rho + r)) \} \right] \frac{dz}{z}$$

Using Lemmas C.1 and C.2 in Taub (1986) the above integral simplifies to

$$\oint \sigma_e^2 \{ (\gamma + z(1 - \gamma))AA^* - C^{-1}[C(\beta z^{-1})^{-1}\{\gamma Ar^* + (1 - \gamma)ak^*\}]_+ A \cdot (\gamma r + (1 - \gamma)zz \cdot (\rho + r)) \} \frac{dz}{z}$$

In the integral only the second term contains  $\rho$ . Therefore we have to consider that term only for finding the optimal rate of return. Let  $D = (\gamma r + (1 - \gamma)zz \cdot (\rho + r))$ . Therefore we have to maximise the following integral with respect to  $\rho$

$$\oint \sigma_e^2 C^{-1} A^* D [C^*^{-1} A D^*]_+ \frac{dz}{z} \quad (3.3)$$

Let us now suppose  $A(z)$  has an autoregressive representation  $A(z) = \frac{1}{1 - \alpha z}$ . Then (3.3) integrates to

$$\sigma_e^2 C(\alpha\beta)^{-2} D(\alpha\beta)^2 A(\alpha\beta) \quad (3.4)$$

We have to maximise the above expression with respect to  $\rho$  to derive the optimal  $\rho$ . By maximising we get the following:

**Proposition 1.** The optimum rate of return on real balances held for insuring against the individual specific shock to income is identical to zero.

The proof of the proposition is given in Appendix B.

The intuitive explanation for the result can be given as follows. With  $\rho$  equal to zero real balances do not enter the utility function. Individuals would then want to hold real balances each period such that they can attain consumption bliss. This can be verified from the budget constraint faced by

him. He will adjust real balances such that his consumption is always zero. This would mean that his credit card bill would also be zero. This means that by adjusting real balances in such a way that bliss is reached always, individuals maximise their utility. Now they do not have to worry about real balance holdings deviating from future bills as the high inflation would erode the value of it totally in the next period. This is identical to Taub's result.

We next derive the optimal  $\rho$  for the economy wide shock. Let the welfare function for this part be W3. W3 from equation (3.1) is

$$\oint \sigma_v^2 \{ \gamma BB + (1 - \gamma)(b + h - \lambda^* k)(b + h - k \lambda^*) \} \frac{dz}{z}$$

Noting that  $zr - k = -z\rho$  the above integral becomes

$$\oint \sigma_v^2 \{ \gamma BB + (1 - \gamma)(zB - \lambda^* z\rho)(zB - z\rho \lambda^*) \} \frac{dz}{z}$$

Rearranging and substituting the optimal value of  $\lambda$  from (2.12) we have

$$\begin{aligned} & \oint \sigma_v^2 \{ \gamma BB + \beta(1 - \gamma)(B - (1 - \gamma)^{-1} [(zk^*)^{-1} \{ \gamma Br^* + (1 - \gamma)zBk^* \}]_+) \\ & (B - (1 - \gamma)^{-1} [(zk^*)^{-1} \{ \gamma Br^* + (1 - \gamma)zBk^* \}]_+)^* \} \frac{dz}{z} \end{aligned} \quad (3.5)$$

Derivation of the maximum of the above expression for  $\rho$  gives us the following proposition:

**Proposition 2.** i) If the driving process is autoregressive and the coefficient of autoregression (b) is less than 1 the optimal rate of deflation should be greater than the rate of time preference. ii) If the driving process is a random walk there should be deflation at the rate of time preference. iii) If the driving process is autoregressive but  $b > 1$  the rate of deflation should be slower than the rate of time preference.

The proof of the proposition is given in Appendix C.

The optimal rate of return on real balances for the economy wide component of the shock is given by  $\rho = \frac{1}{b\beta}$ . This is exactly the result obtained by Taub in his paper "Aggregate Fluctuations, Interest Rates and Repeated Insurance under Private Information". In that paper he was interested in determining the optimal rate of interest when individuals can borrow and lend money from others to insure themselves against aggregate and idiosyncratic shocks to income. Given that individual debts could be kept for one period only it is not surprising that the same rate of return on money is needed for optimisation. This shows that the credit card provides insurance in the same way lending does.

This result is interesting because if  $b$  is a fraction,  $\rho$  would be greater than the rate of time preference. This is contrary to Taub's result and confirms Bewley's conjecture. If the driving process is a random walk, implying that  $b = 1$ , the rate of deflation equals the rate of time preference because the shock becomes a permanent shock. If  $b$  is greater than 1 but less than  $\beta^{-1/2}$  then the government should deflate but at a later slower than the rate of time preference. Secondly, the optimal rate of return for the economy wide part is totally independent of the weights attached to the two demands for money in the utility function. So unlike Taub's model the relative importance of these two demands is irrelevant for our model. The rate of return that will maximise the welfare function will be the weighted sum of all the three returns obtained in this section. When  $b$  is less than or equal to 1 the returns for the deterministic and economy wide parts exceed the rate of time preference, the overall rate would exceed the rate of time preference because the return for the individual specific part equals zero.

## 5. A Comment on Taub (1989)

In a recent article, Taub (1989) found a counterexample to Bewley's (1983) conjecture that the optimal rate of deflation in a stochastic economy should exceed the rate of time preference. Bewley

had made the conjecture using a simple example that introduced uncertainty in a simple economy like the one considered by Friedman (1969).

In a linear quadratic framework, Taub showed that his equation (6.5) gives the optimal gross rate of return on real balances when the monetary authority maximizes the economy wide stochastic component of the welfare function of a representative individual. When the driving process has an AR(1) representation, this optimal rate of return is given by his equation (6.6). Due to an algebraic mistake in calculating the first order condition for the economy wide stochastic part, the optimal gross rate of return on real balances would be different from the values given by equations (6.5) and (6.6). As a consequence, Proposition 6.2 of Taub's model should have a modified form.

As in the Appendix of Taub (1989), the portion of the welfare function (given in his equation 6.1) of concern is

$$-\frac{1}{2\pi i} \oint \{ \alpha B B \cdot + (1-\alpha)^{-1} ((1-\alpha)B - z[z^{-1}(1-\rho\alpha\beta z^{-1})B]_{+}) \\ ((1-\alpha)B \cdot - \beta z^{-1}[z^{-1}(1-\rho\alpha\beta z^{-1})B]_{+} \cdot) \} \frac{dz}{z}$$

Differentiation of the above equation with respect to  $\rho$  and the application of  $\beta$  symmetry, yield the following first order condition:

$$-\frac{2}{2\pi i} \oint \{ (1-\alpha)^{-1} (-z[-\alpha\beta z^{-2}B]_{+}) ((1-\alpha)B \cdot - \beta z^{-1}[z^{-1}(1-\rho\alpha\beta z^{-1})B]_{+} \cdot) \} \frac{dz}{z} = 0$$

which differs from the first order condition in Taub (1989). Solving for  $\rho$ ,

$$\rho = \beta^{-2}\alpha^{-1} \frac{\frac{1}{2\pi i} \oint [z^{-2}B]_{+} \{ \beta [z^{-1}B]_{+} \cdot - (1-\alpha)zB \cdot \} \frac{dz}{z}}{\frac{1}{2\pi i} \oint [z^{-2}B]_{+} [z^{-2}B]_{+} \cdot \frac{dz}{z}}$$

A  $z$  and two  $\beta$ 's have been dropped from the numerator in equation (6.5) in Taub (1989). If an AR(1) driving process of the form

$$B(z) = \frac{1}{1 - bz},$$

is considered,

$$\rho = \alpha^{-1} \beta^{-2} \frac{\frac{1}{2\pi i} \oint b^2 B \{ \beta b B \cdot - (1 - \alpha) z B \cdot \} \frac{dz}{z}}{\frac{1}{2\pi i} \oint b^2 B b^2 B \cdot \frac{dz}{z}}$$

Using Cauchy's Integral Formula to evaluate the integrals, the above solution can be simplified as,

$$\rho = \frac{1}{b\beta}$$

The above derivation indicates that the optimal value for  $\rho$  would change when the corrected first order condition is used.

The corrected result indicates that the optimal rate of deflation does in general exceed the rate of time preference. Only in the special case when  $1 < b < 1/\sqrt{\beta}$  will the rate of deflation be less than the rate of time preference. Also, the optimal rate of return on real balances is independent of the weights used in the objective function. Further, since  $b\beta$  is always less than 1, deflation is always the optimal policy. Therefore, in the model under consideration, when the transactions demand for money dominates the precautionary demand, inflation is not the optimal policy contrary to Taub's results. His Proposition 6.2 therefore needs to be modified as follows:

- i) The optimal deflation proceeds at the rate of time preference if the driving process is a random walk.
- ii) If the driving process is a stationary first-order autoregressive process, the deflation proceeds faster than the rate of time preference.
- iii) If the autoregressive process is non-stationary and has autoregressive parameter in the interval  $(1, 1/\sqrt{\beta})$  deflation proceeds slower than the rate of time preference.

The modified Proposition suggests that Bewley's conjecture holds in the stationary case but fails to hold in the non-stationary case. Although the optimal policy is always deflationary for the economy wide shock, it is highly inflationary for the individual specific shock as Proposition 6.1 correctly notes. Being the weighted sum of the optimal returns derived for the two types of shocks, the overall optimal rate of return may be deflationary or inflationary depending on the various parameters of the model and the variances of the two shocks, among other factors.

## 6. Conclusion

This paper demonstrates that in the presence of a credit card and stochastic income, optimal monetary policy differs from Friedman's rule of deflation at the rate of time preference. The paper shows that the policy may be either inflationary or deflationary depending on the nature of the driving processes of the various shocks and the parameters of the model.

In this paper individuals receive some stochastically varying endowment each period. It is assumed that there is a taboo in consuming one's own endowment so that each individual must trade with other individuals in the economy before consumption each period. Such trades require a medium of exchange like money, as Taub's paper shows. In this paper we allow the individuals to use a credit card for the purchases. This credit card permits the individuals to postpone payments for current period purchases to the future. In this paper postponement by only one period is allowed. Potentially, however, the analysis can be generalized for the case of some 'k' periods of postponement. It will be interesting to investigate whether the results of the paper will be robust to such a generalization. I show that if individuals can defer payment for current period purchases by one period, the optimal monetary policy is equivalent to the one in which individuals have a cash-in-advance restriction on current period purchases.

Like Taub's paper, individual income stochastically varies around a deterministic mean. A part of the stochastic shock is idiosyncratic and a part economy wide. Due to the linear-quadratic nature

of the optimization problem, the optimal policy has been analyzed separately for all the different shocks to income. In particular, if individual income is subject to an aggregate shock that affects every individual equivalently, optimal policy is always deflationary. The rate of deflation is shown to be dependent on the rate of time preference and the coefficient of autoregression when the driving process has an autoregressive representation. It is shown that if the shock is stationary, it is optimal to deflate at a rate greater than the rate of time preference. This result confirms Bewley's conjecture that the rate of deflation will exceed the rate of time preference if uncertainty is introduced in Friedman's paper. If the shock is non-stationary, however, Bewley's conjecture is not valid because the rate of deflation is less than the rate of time preference. Nonetheless, the optimal policy is always deflationary when individual income is subject to an aggregate shock.

In section 5 of the paper I show that the optimal policy for an aggregate shock to income in Taub's model is identical to the one found in this paper. The inflationary result derived by him is due to an algebraic mistake. This shows that if individual income is subject to an aggregate shock, the rate of return on money that maximizes welfare is the same whether individuals are required to pay for current period purchases with cash or may delay payment to the next period by using a credit card.

The optimal monetary policy becomes highly inflationary like Taub's paper, however, if individual income is subject to an idiosyncratic shock. In this case since the shock has no aggregate manifestation, the average individual's income is unaffected by this shock. To allow the individuals to achieve their consumption bliss exactly, the monetary authority inflates heavily so that the individual does not get penalized for holding the incorrect amount of money. Comparison of the results of this paper and Taub's paper shows that the optimal policy is robust to changes in the role of money. In other words, in these types of economies, it does not matter for policy as to how individuals pay for their purchases.

# Chapter III

## Inflation and Welfare in a Stochastic Production Economy

### 1. Introduction

Milton Friedman's (1969) article on the optimum quantity of money has motivated much research and controversy. Despite the controversy there remains no doubt about the significance of his policy of deflation at the rate of time preference. Woodford (1990) writes, it

".. - is undoubtedly one of the most celebrated propositions in monetary theory, probably the most celebrated proposition in what one might call "pure" monetary theory."

Theorists have tested Friedman's theory in a wide spectrum of economic environments. If we abstract from the literature of money growth and capital accumulation, we find models that yield Friedman's rule as the optimal solution and an equally large set of models which do not [Stein (1970), Phelps (1973), Grandmont and Younes (1973), Brock (1975), Benhabib and Bull (1983), Lucas Stokey (1983), and Kimbrough (1986) are a few examples]. While the majority of the litera-

ture has concentrated on deterministic endowment economies, Bewley (1983) and Taub (1989) extend the analysis to stochastic endowment economies.

Bewley considered an economy very close to the one discussed by Friedman. In the absence of any transactions motive for holding money, he showed that in an economy where endowments are subject to random shocks, a monetary equilibrium may not exist if Friedman's rule of deflation at the rate of time preference is followed. If Friedman's Rule is followed, individuals are not penalized for holding money as the cost of holding it is exactly offset by the benefit. If, however, future endowments are uncertain, there may arise circumstances in which an individual may suffer from an infinite sequence of bad lucks. To insure him completely against such spells the individual must hold an infinite amount of money. Therefore, Bewley argued, that a monetary equilibrium with a non-zero and finite price level will fail to exist if there is deflation at the rate of time preference.

With the help of a simple example Bewley also conjectured that if uncertainty can be introduced in a deterministic version of Friedman's model, the optimal rate of deflation should proceed at a rate greater than the rate of time preference. Taub (1989) showed that Bewley's conjecture may not be correct. In a linear quadratic model in which individual endowments are subject to aggregate and individual specific shocks each period, he showed that the optimal policy is highly inflationary in response to the individual specific shock. Correcting an error in Taub (1989), Mukherji (1990) showed that Bewley's conjecture holds if a stationary aggregate shock is considered. The optimal return, being the weighted sum of the two shocks, can be either inflationary or deflationary depending on parameter values. Taub's model also shows that the indeterminacy of the monetary equilibrium, as found by Bewley, does not remain if a transactions motive for holding money is introduced. If individuals hold money for transactions purposes, money for precautionary reasons can be diverted from those held for financing transactions. This imposes a bound on the amount of money that individuals want to hold to insure against adverse future shocks.

Both Bewley's paper and Taub's paper consider pure endowment economies. Bewley wrote in this regard

"I define a mathematically precise version of Friedman's model. I assume that the model is of an exchange economy, with no production, although Friedman is not clear on this point."

The purpose of this paper is to show that it is important to make the distinction between endowment and production economies when the optimum quantity of money is to be determined.

In this paper money is the only asset. Unlike the monetary growth literature in which the impact of inflation on output is analyzed via its impact on portfolio choice, in this paper the effect of the return on money on production is directly analyzed. It is shown that moving from an exchange to a production economy changes results in important ways. While most of the optimum quantity of money literature concentrates on endowment economies, Lucas-Stokey (1983) and Kimbrough (1986) are among a few models in which production is considered. But none of these papers have modeled labor supply in a way that can elicit the role it can play on monetary policy.

Kimbrough considered production to show the relationship between the optimal rate of money growth and labor supply. He showed that production responds positively to changes in the return on money. In his model, individuals hold money for transactions purposes only. Instead of the commonly used cash-in-advance constraint, he used a transactions technology and showed that in his deterministic production economy, Friedman's rule of deflation at the rate of time preference holds. Intuition suggests that production ought to change the optimal rate of return on money. Once production is introduced, the monetary authority should not concentrate exclusively on the impact of its policy on intertemporal transfer of purchasing power but also ought to consider the impact of its policy on individual production decisions. This paper makes this point in a stochastic production economy. It is shown that if individuals ascribe different weights to the utility they derive from consumption and leisure, the optimal policy is sensitive to the relative magnitudes of the two weights, among other factors. So Friedman's rule does not hold for the economy considered in this paper. Since disposal is not costless, in equilibrium not only "too little" but even "too much" may be produced and therefore "too much" money may be held. Depending on whether too much or too little is produced, the monetary authority should suitably inflate or deflate the currency.

This paper is organized as follows. Section 2 describes the economic environment. Section 3 solves the individual's problem. Section 4 determines the optimum quantity of money and Section 5 concludes.

## 2. The Model

The economy consists of infinitely many immortal individuals who produce a perishable good each period using their labor services. The utility derived from consumption is subject to stochastic shocks each period. To motivate exchange, I follow Diamond (1984) and Taub in assuming that individuals cannot consume their own output. So after production individuals trade using money that they received by way of government transfers. Barter is forbidden.

A linear quadratic structure is used. It makes possible the application of the solution methods used by Whiteman (1985) and Taub (1989). The fact that individuals require cash to finance their purchases is often captured in the literature by a 'cash-in-advance' constraint. Since inequality constraints are difficult to analyze in linear quadratic models, I capture them using a penalty term in the utility function as in Taub (1989). The individual's problem is then to determine his levels of production, consumption, and real balance holdings.

A typical individual's utility function is

$$-E_t \sum_{s=0}^{\infty} \beta^s [\gamma(\hat{c}_{t+s} - c_{t+s})^2 + \delta l_{t+s}^2 + (1 - \gamma - \delta)\{p_{t+s}M_{t+s-1} - (c_{t+s} - \hat{c}_{t+s})\}^2] \quad (2.1)$$

where  $c_t$ ,  $l_t$ ,  $M_t$  denote consumption, labor, and nominal balances respectively;  $p_t$  is the inverse of the price level. The government transfer payment is represented by  $H_t$ .  $\hat{c}_t$  is a bliss point;  $\gamma$ ,  $\delta$  are the various weights and  $\beta$  is the discount factor.

The individual maximizes his utility subject to the budget constraint:

$$c_t = y_t - p_t(M_t - M_{t-1}) + p_t H_t \quad (2.2)$$

The bliss point  $\hat{c}_t$  is assumed to have an individual specific stochastic component  $\varepsilon_t$  that is assumed to vary both across individuals and over time, and an economy wide stochastic component  $u_t$ . Then

$$\hat{c}_t = A(L)\varepsilon_t + D(L)u_t \quad (2.3)$$

I assume a simple linear production function of the form

$$y_t = l_t \quad (2.4)$$

Finally, as mentioned earlier, a 'cash-in-advance' constraint in the form of a penalty term in the utility function is present in the model. This is captured by the last term in the utility function. The penalty term here shows that the individual uses real balances carried over from the previous period to finance current period purchases that deviate from the bliss level of consumption.

The innovations  $\varepsilon_t$ , and  $u_t$  are assumed to be i.i.d. across time and individuals and have distributions  $N(0, \sigma_\varepsilon^2)$ , and  $N(0, \sigma_u^2)$  respectively.  $A(L)$  and  $D(L)$  are not specified now but I assume their  $z$  transforms have no zeros and are analytic in the disk  $\{z: |z| < \beta^{0.5}\}$ .

Let the gross rate of return on real balances be given by  $\rho_t$ . Noting that the price level is the inverse of  $p_t$ ,

$$\rho_t = \frac{p_t}{p_{t-1}} \quad (2.5)$$

In this economy production like all other variables is driven by the two shocks. Although the individual specific shocks add up to zero for the whole economy, the aggregate shock does not. Since production is affected by these shocks, average consumption in the economy is constrained by the production response to the aggregate shock. Using tildas to represent variables averaged out over individuals, the following economy wide resource constraint is obtained:

$$\tilde{c}_t = \omega(L)u_t \quad (2.6)$$

where the right hand side indicates the production response to the aggregate driving force. Second, to clear the money market, the real transfers made by the government must equal the addition to real balances made by the individuals in the current period. This gives the following money market clearing condition:

$$\tilde{m}_t - \rho \tilde{m}_{t-1} = \tilde{h}_t \quad (2.7)$$

where  $\tilde{h}_t$  denotes the average real transfer made by the government in the current period.

Finally the following equilibrium conditions must be satisfied:

$$\tilde{m}_t = p_t M_t \tag{2.8}$$

$$\tilde{h}_t = p_t H_t \tag{2.9}$$

### 3. The Individual's Problem

I begin by solving the maximization problem faced by the individual. Substitution of the constraints (2.2) to (2.5) in the objective function yields,

$$-E_t \sum_{s=0}^{\infty} \beta^s \left[ \gamma \{ A(L)\varepsilon_{t+s} + D(L)u_{t+s} - (l_{t+s} - m_{t+s} + \rho_{t+s}m_{t+s-1} + h_{t+s}) \}^2 \right. \\ \left. + \delta l_{t+s}^2 + G \{ A(L)\varepsilon_{t+s} + D(L)u_{t+s} - (l_{t+s} - m_{t+s} + H_{t+s}) \}^2 \right] \quad (3.1)$$

In this expression  $G = 1 - \gamma - \delta$  and lower case letters have been used to represent real variables. Following Taub (1989), I assume that the monetary authority addresses the question of the optimum quantity of money at time 0 and fixes it thereafter. This simplification ignores any potential time inconsistency problem. So  $\rho_t$  has been replaced by  $\rho$  in (3.1).

Due to the linear quadratic structure, the objective function separates into its two stochastic parts that can be solved separately. This makes the problem much easier to solve. Further, with quadratic objectives and linear constraints the decision functions are all linear functions of the driving forces of the model. For the problem under consideration, I therefore posit the following decision functions:

$$l_{t+s} = \sigma(L)\varepsilon_{t+s} + \omega(L)u_{t+s} \quad (3.1i)$$

$$m_{t+s} = \mu(L)\varepsilon_{t+s} + \pi(L)u_{t+s} \quad (3.1ii)$$

$$\tilde{h}_{t+s} = H(L)u_{t+s} \quad (3.1iii)$$

Substituting these equations in (3.1), taking expectations, and transferring the problem to the complex domain yield,

$$\begin{aligned}
& -\frac{\beta}{1-\beta} \frac{1}{2\pi i} \oint [\gamma(A - (\sigma - r\mu))(A_* - (\sigma_* - r_*\mu_*)) + \delta\sigma\sigma_* \\
& + G(A - (\sigma - \mu))(A_* - (\sigma_* - \mu_*))\sigma_\varepsilon^2 + \{\gamma(D - \omega + r\pi - H)(D_* - \omega_* + r_*\pi_* - H_*) \\
& + \delta\omega\omega_* + G(D - \omega + \pi - H)(D_* - \omega_* + \pi_* - H_*)\sigma_u^2] \frac{dz}{z}
\end{aligned} \tag{3.2}$$

where,  $z$  is a complex number and  $r = 1 - \rho z$ . For derivation please see Whiteman (1985) and Taub (1989, 1990-Appendix A). I have also represented  $A(z)$  as  $A$ ,  $D(z)$  as  $D$  etc. to economise on notation. The subscript  $*$  has been used to indicate the  $\beta$  conjugate of a function, that is  $A_* = A(\beta z^{-1})$ . Both parts of the problem can now be solved independently.

### 3.1 Solution of the problem for the individual specific shock

The optimal labor supply  $\sigma(\cdot)$  and money demand  $\mu(\cdot)$  functions can be derived from the following part of the objective function:

$$\begin{aligned}
& -\frac{\beta}{1-\beta} \frac{1}{2\pi i} \oint \{\gamma(A - (\sigma - r\mu))(A_* - (\sigma_* - r_*\mu_*)) \\
& + \delta\sigma\sigma_* + G(A - (\sigma - \mu))(A_* - (\sigma_* - \mu_*))\}\sigma_\varepsilon^2 \frac{dz}{z}
\end{aligned} \tag{3.3}$$

To solve I use the method of calculus of variations. Let  $\sigma^*$ , and  $\mu^*$  be the optimal values for  $\sigma$ , and  $\mu$  respectively and let variations of  $\alpha_1\eta_1$ , and  $\alpha_2\eta_2$  respectively be taken from them. I assume that  $\eta_1$ , and  $\eta_2$  are all analytic in the disk. Then  $\sigma$ , and  $\mu$  may be expressed as:

$$\sigma = \sigma^* + \alpha_1\eta_1$$

$$\mu = \mu^* + \alpha_2\eta_2$$

Let  $J(\alpha_1, \alpha_2)$  be the value of the objective function when the optimal values of  $\sigma$  and  $\mu$  are substituted. Then differentiating  $J(\cdot)$  with respect to its arguments, setting  $\alpha_1$ ,  $\alpha_2$  and the derivatives equal to zero, and using  $\beta$  symmetry gives,

$$\frac{-2}{2\pi i} \frac{\beta}{1-\beta} \sigma_\varepsilon^2 \oint \{\gamma(A - \sigma + r\mu)(\eta_{1\cdot}) + \delta\sigma(-\eta_{1\cdot}) + G(A - \sigma + \mu)(\eta_{1\cdot})\} \frac{dz}{z} = 0$$

$$\text{or, } \frac{-1}{2\pi i} \oint \{\gamma(A - \sigma + r\mu) - \delta\sigma + G(A - \sigma + \mu)\}(\eta_{1\bullet}) \frac{dz}{z} = 0$$

Since  $\eta_1$  is analytic it contains only non-negative powers of  $z$ . Therefore,  $\eta_{1\bullet}$  contains only non-positive powers of  $z$ . Then from Cauchy's theorem, the integral can be zero only if the expression in the bracket contains strictly negative powers of  $z$ . This gives the following Weiner-Hopf equation:

$$\gamma(A - \sigma + r\mu) - \delta\sigma + G(A - \sigma + \mu) = \sum_{-\infty_1}^{-1}$$

where,  $\sum_{-\infty_1}^{-1}$  is an unspecified function whose Laurent expansion contains strictly negative powers of  $z$ . The above equation may be rearranged as,

$$\sigma - (\gamma r + G)\mu = (\gamma + G)A - \sum_{-\infty_1}^{-1} \quad (3.4)$$

Similarly differentiating with respect to  $\alpha_2$  the second Weiner-Hopf equation is derived:

$$(\gamma r_* + G)\sigma - (\gamma r r_* + G)\mu = (\gamma r_* + G)A - \sum_{-\infty_2}^{-1} \quad (3.5)$$

These two Weiner-Hopf equations are solved simultaneously to yield the following solutions for the decision variables. The derivations are elaborated in Appendix D. Noting that

$$CC_* = (\gamma r r_* + G) - (\gamma r + G)(\gamma r_* + G) \quad (3.6)$$

$$\mu^* = C^{-1}[C_*^{-1}\{-\delta(\gamma r_* + G)\}A]_+ \quad (3.7)$$

$$\sigma^* = (\gamma + G)A + (\gamma r + G)C^{-1}[C_*^{-1}\{-\delta(\gamma r_* + G)\}A]_+ \quad (3.8)$$

The individual specific shock I consider is stochastic across individuals and has no aggregate manifestation. This generates the possibility of exchange between the members of the economy. The

optimal solutions show that the individuals with high preference shocks produce, consume, and hold real balances that they get from individuals experiencing spells of non-hunger. What one wants to purchase is exactly matched by what someone else has to sell. So for the aggregate economy, these effects exactly wash out and the economy wide resource constraints get satisfied.

### 3.3 Solution of the problem for the aggregate shock

To determine the individual's response to the random aggregate shock the following portion of the objective function (3.2) is maximized:

$$-\frac{\beta}{1-\beta} \frac{1}{2\pi i} \oint \{ \gamma(\omega - r\pi + H - D)(\omega \cdot - r \cdot \pi \cdot + H \cdot - D \cdot) + \delta \omega \omega \cdot \\ + G(\omega - \pi + H - D)(\omega \cdot - \pi \cdot + H \cdot - D \cdot) \} \sigma_u^2 \frac{dz}{z}$$

Using exactly the same techniques as has been followed before, the following Weiner-Hopf equations are obtained:

$$\omega - G(1-r)\pi = (\gamma + G)D - \sum_{-\infty_3}^{-1} \quad (3.9)$$

$$(\gamma r_* + G)\omega - G(1-r)\pi = (\gamma r_* + G)D - \sum_{-\infty_4}^{-1} \quad (3.10)$$

In the Weiner-Hopf equations the economy wide resource constraint (2.7) has been substituted.

Now, (2.7) may be expressed as

$$(1 - \rho L)\pi(L)u_{+s} = H(L)u_{+s}$$

Equating coefficients I get

$$H(L) = (1 - \rho L)\pi(L), \text{ that is } H = r\pi$$

Using this and assuming that the parameter values are such that  $\frac{\gamma\beta\rho}{\delta} > \sqrt{\beta}$ , the following optimal values for  $\omega$  and  $\pi$  are obtained:

$$\omega^* = \gamma\rho \left[ \frac{z^*D}{\delta + \gamma\rho z^*} \right]_+ \quad (3.11)$$

$$\pi^* = \frac{z^{-1}}{G\rho} \{ \omega^* - (\gamma + G)D \} \quad (3.12)$$

These give the general solutions of the choice variables. To analyze how production is dependent on the return on money, the analytic solution for  $\omega$  is considered. To this end I assume an autoregressive representation of the driving process. If

$$D(z) = \frac{1}{1 - dz}$$

where  $d$  is the parameter of autoregression,  $\omega^*$  becomes

$$\omega^* = \frac{\gamma\rho d\beta D(z)}{\delta + \gamma\beta\rho d}$$

using Lemma C.3 in Taub (1986). This solution for  $\omega$  shows that production for the aggregate shock case equals

$$\omega(L)u_{t+s} = \frac{\gamma\beta\rho d}{\delta + \gamma\beta\rho d} D(L)u_{t+s}$$

Therefore,

$$\frac{d}{d\rho} \omega(L)u_{t+s} = \frac{\gamma\delta\beta d}{(\delta + \gamma\beta\rho d)^2} D(L)u_{t+s}$$

This indicates that production responds positively to changes in the return on money. So the higher the return on money,  $\rho$ , the greater is labor supply and therefore production. Kimbrough also found a positively sloped Phillips Curve, although for different reasons.

#### 4. The Optimum Quantity of Money

In the last section, individuals took the rate of return on real balances as a parameter when they solved their optimization problem. The value of this parameter is determined by the monetary authority whose primary objective is to set the rate of return in a way that maximizes individual utility. Friedman showed in a deterministic model that individual utility is maximized when the policy is a deflation at the rate of time preference. This rule however may not hold in stochastic economies as Bewley (1983) and Taub (1989) have shown.

To find the optimal policy for the monetary authority for this model I substitute the optimal solutions derived in the previous section into the objective function of the individual and determine the value of  $\rho$  that maximizes it. The indirect utility function is called the welfare function (W).

$$W = -\frac{\beta}{1-\beta} \frac{1}{2\pi i} \oint \left[ \{y(A - (\sigma^* - r\mu^*))(A - (\sigma^* - r\mu^*))\right. \\ \left. + \delta\sigma^*\sigma^* + G(A - (\sigma^* - \mu^*))(A - (\sigma^* - \mu^*))\} \sigma_z^2 \right. \\ \left. + \{y(\omega^* - r\pi^* + H - D)(\omega^* - r\pi^* + H - D)\} \right. \\ \left. + \delta\omega^*\omega^* + G(\omega^* - \pi^* + H - D)(\omega^* - \pi^* + H - D)\} \sigma_z^2 \right] \frac{dz}{z}$$

The problem may now be separated into the two stochastic parts as follows. The portion of the welfare function for the individual specific shock (W1) is

$$W1 = -\frac{\beta}{1-\beta} \frac{1}{2\pi i} \oint [\gamma(A - (\sigma^* - r\mu^*))(A - (\sigma^* - r\mu^*)) + \delta\sigma^*\sigma^* + G(A - (\sigma^* - \mu^*))(A - (\sigma^* - \mu^*))] \sigma_i^2 \frac{dz}{z}$$

Using the first order condition (3.4) and (3.5), W1 can be simplified as

$$W1 = -\frac{\beta}{1-\beta} \frac{1}{2\pi i} \sigma_i^2 \oint \delta\sigma^* A \cdot \frac{dz}{z}$$

The derivation is shown in Appendix E. If an AR(1) process is assumed for the driving process of the individual specific shock,  $A(z)$  is

$$A(z) = \frac{1}{1-\alpha z},$$

$$\sigma^* = (\gamma + G)A - \delta(\gamma r + G)C^{-1}(\alpha\beta)\{\gamma(1 - \rho\alpha\beta) + G\}C^{-1}A$$

where,  $r = 1 - \rho z$ ,  $C$ , and  $A$  are analytic functions of  $z$ . Substitution of the optimal value of  $\sigma$  in W1 yields,

$$W1 = \frac{\beta}{1-\beta} \frac{1}{2\pi i} \sigma_i^2 \oint \delta \left[ -(\gamma + G)AA \cdot + \delta C^{-1}(\alpha\beta)(\gamma(1 - \rho\alpha\beta) + G) \frac{(\gamma(1 - \rho z) + G)}{C(z)} AA \cdot \right] \frac{dz}{z}$$

Using the AR(1) representation for  $A$ , the integral can be evaluated using Cauchy's Integral Formula as

$$W1 = \frac{\beta}{1-\beta} \sigma_\varepsilon^2 \left[ -\delta(\gamma + G)A(\alpha\beta) + \delta^2 \frac{[\gamma(1 - \rho\alpha\beta) + G]^2}{C^2(\alpha\beta)} A(\alpha\beta) \right] \quad (4.1)$$

The last part of the welfare function (W2) is similarly simplified to

$$W2 = -\frac{\beta}{1-\beta} \frac{1}{2\pi i} \sigma_i^2 \oint \{(-\delta\omega^*)(r\cdot\pi^* - D\cdot)\} \frac{dz}{z}$$

Substituting the optimal value of  $\omega$  yields

$$W2 = \frac{\beta}{1-\beta} \frac{1}{2\pi i} \sigma_u^2 \delta \oint \gamma \rho \left[ \frac{z \cdot D}{\delta + \gamma \rho z} \right]_+ \left\{ \frac{r \cdot}{G \rho z} \left( \gamma \rho \left[ \frac{z \cdot D}{\delta + \gamma \rho z} \right]_+ - (\gamma + G) D \right) - D \right\} \frac{dz}{z}$$

To evaluate the integral an AR(1) representation is used for the driving process. That is,

$$D(z) = \frac{1}{1-dz}$$

Using this driving process and applying the annihilator operator, W2 becomes

$$W2 = \frac{\beta}{1-\beta} \frac{1}{2\pi i} \sigma_u^2 \delta \oint \frac{\gamma \rho \beta d}{\delta + \gamma \rho \beta d} \left\{ \frac{(z - \rho \beta)}{G \rho \beta} \left( \frac{\gamma \rho \beta d}{\delta + \gamma \rho \beta d} - (\gamma + G) \right) - 1 \right\} \frac{D}{z - d\beta} dz$$

Using Cauchy's Integral Formula, the integral can be evaluated as

$$W2 = \frac{\beta}{1-\beta} \sigma_u^2 \delta \left[ \frac{\gamma \rho \beta D(d\beta)}{\delta + \gamma \rho \beta d} \left\{ \frac{(d\beta - \rho \beta)}{G \rho \beta} \left( \frac{\gamma \rho \beta d}{\delta + \gamma \rho \beta d} - (\gamma + G) \right) - 1 \right\} \right] \quad (4.2)$$

The monetary authority's problem is to maximize W1 and W2 with respect to  $\rho$  and determine the overall rate of return using the values obtained from these maximizations.

#### 4.1 Optimum $\rho$ for the individual specific shock

Given the welfare function W1 of the individual for the individual specific shock, the monetary authority faces the problem of choosing the value of  $\rho$  such that the welfare of the individual can be maximized.

To find the optimum  $\rho$  the monetary authority's problem is to calculate the value of  $\rho$  that maximizes equation (4.1). Equation (4.1) gives

$$W1 = -\frac{\beta}{1-\beta} \sigma_u^2 \delta (\gamma + G) A(\alpha\beta) + \frac{\beta}{1-\beta} \sigma_u^2 \delta^2 \frac{[\gamma + G - \gamma \rho \alpha \beta]^2}{C^2(\alpha\beta)} A(\alpha\beta)$$

To find the value of  $\rho$  that maximizes the above equation, the value of  $C^2(\alpha\beta)$  must be determined.

From equation (3.6)  $CC = \gamma r r + G - (\gamma r + G)(\gamma r + G)$

where,  $C$  is an unspecified analytic function of  $z$ . Since the right hand side of (3.6) is linear in  $z$ , we can assume a linear function of  $C$  of the form

$$C(z) = c_0(1 - c_1 z)$$

where,  $c_0$  and  $c_1$  are unknown real numbers. Using this expression for  $C(z)$ ,

$$C(z)C(z) = c_0^2(1 - c_1 z - c_1 z + c_1^2 \beta), \text{ noting } z = \beta z^{-1}$$

Substituting  $r = 1 - \rho z$  and expanding, the right hand side of equation (3.6) can be written as

$$\delta(\gamma + G) + \gamma(1 - \gamma)\beta\rho^2 - \gamma\delta\rho(z + z)$$

Therefore,

$$c_0^2(1 - c_1 z - c_1 z + c_1^2 \beta) = \delta(\gamma + G) + \gamma(1 - \gamma)\beta\rho^2 - \gamma\delta\rho(z + z)$$

Equating coefficients,

$$c_0^2(1 + c_1^2 \beta) = \delta(\gamma + G) + \gamma(1 - \gamma)\beta\rho^2 \text{ and}$$

$$c_0^2 c_1 = \gamma\delta\rho$$

These two equations yield

$$\frac{1}{c_1} + c_1 \beta = \frac{(\gamma + G)}{\gamma\rho} + \frac{(1 - \gamma)}{\delta} \beta\rho$$

or,

$$\frac{1}{c_1} + c_1 \beta = \frac{1}{\frac{\gamma}{\gamma + G} \rho} + \frac{\gamma\beta\rho}{\gamma + G} + \frac{G}{\delta(\gamma + G)} \beta\rho \quad (4.3)$$

The above exercise has shown that  $C(\alpha\beta)$  can be expressed in terms of the variables and parameters of the model.  $W1$  can be rewritten as

$$W1 = -\frac{\beta}{1 - \beta} \sigma^2 \delta(\gamma + G) A(\alpha\beta) + \frac{\beta}{1 - \beta} \sigma^2 \delta(\gamma + G) \frac{c_1 \left[ 1 - \frac{\gamma}{\gamma + G} \rho \alpha \beta \right]^2}{\frac{\gamma}{\gamma + G} \rho [1 - c_1 \alpha \beta]^2}$$

Then the relevant part of the welfare function for the maximization problem is

$$\bar{W}_1 = \frac{c_1 \left[ 1 - \frac{\gamma}{\gamma + G} \rho \alpha \beta \right]^2}{\frac{\gamma}{\gamma + G} \rho [1 - c_1 \alpha \beta]^2}$$

Equation (4.3) and  $\bar{W}_1$  show that Lemma A.1 in Taub (1989) is directly applicable. Application of the Lemma directly shows that the optimal solution is

$$\rho = 0$$

The Lemma and proof are given in Appendix F. The optimal rate of return shows that the monetary authority inflates the currency at an infinite rate. Examination of the utility function shows that if  $\rho = 0$ , the current utility becomes independent of money. The individual in this case concentrates on maximizing his consumption and leisure. Any excess money balances that are carried over to the future do not reduce future utility as the inflation completely erodes away its value. This result maximizes individual welfare as the individual specific shock completely washes out when the aggregate economy is considered. The average individual therefore, has a consumption bliss at zero. So he does not require money to make any purchases. The monetary authority helps the average individual to achieve his maximum satisfaction from consumption and leisure by infinitely inflating the currency.

This result is similar to the one in Taub (1989) for an exchange economy. In both the models the average individual does not want to consume anything. Since in both the models average income is zero, for the monetary authority's problem it does not matter whether a production or an endowment economy is being considered. Therefore, it is not surprising that the two models yield identical results. The analysis of the next section will show that the optimal policy starts to differ from the one obtained in the corrected version of Taub's model (Mukherji 1990) once the aggregate shock case is considered. It shows that once there is positive income, the optimal policy changes once we move from pure endowment to production economies.

#### 4.2. Optimum $\rho$ for the aggregate shock

To derive the optimum value of  $\rho$  for the economy wide shock, the monetary authority needs to maximize W2 with respect to  $\rho$ . W2 can be rewritten as

$$W2 = \frac{\beta}{1-\beta} \sigma_v^2 \frac{\delta \beta d}{G} \left[ \frac{\delta \gamma (d-\rho)(\gamma d \beta \rho - (\gamma + G)) - \gamma G \rho (\gamma d \beta \rho + \delta)}{(\delta + \gamma \rho \beta d)^2} \right]$$

The expression for W2 shows that it is a ratio of two quadratic functions of  $\rho$ . It can be written as

$$W2 = -\frac{\beta}{1-\beta} \sigma_v^2 \frac{\delta \gamma \beta d}{G} \left[ \frac{A\rho^2 + B\rho + C}{(E + F\rho)^2} \right]$$

where  $A = \gamma d \beta (\delta + G)$ ,  $B = -\delta \gamma (1 + d^2 \beta)$ ,  $C = \delta d (\gamma + G)$ ,  $E = \delta$ ,  $F = \gamma \beta d$ . Differentiating W2 with respect to  $\rho$  and equating the derivative to zero, the following first order condition is obtained

$$(2AE - 2BF)\rho + BE - 2FC = 0$$

Substituting the values for the constants A, B etc. and solving for  $\rho$

$$\rho = \frac{\delta(1 + d^2 \beta) + 2d^2 \beta (\gamma + G)}{2(d \beta (\delta + G) + \gamma d \beta (1 + d^2 \beta))}$$

I accept those solutions of  $\rho$  that satisfy the restriction on parameters imposed earlier. That is  $\frac{\gamma \beta \rho}{\delta} > \sqrt{\beta}$ . The optimal value for  $\rho$  indicates that it is dependent on the weights individuals give to the utility they derive from consumption ( $\gamma$ ) and leisure ( $\delta$ ). Further, examination of the optimal solution indicates that the optimal policy may be inflationary or deflationary depending on the parameter values. Deflation is optimal when the numerator exceeds the denominator. That is, when

$$\delta > \frac{2d\beta}{(1 - d^2\beta)} (1 - d + \gamma d^2\beta)$$

Inflation is optimal when this inequality is reversed. To examine how the optimal policy responds to changes in  $\delta$  and  $\gamma$ , the following comparative statics exercises are performed:

$$\frac{\partial \rho}{\partial \delta} = \frac{1 - d^2\beta}{2(d\beta(\delta + G) + \gamma d\beta(1 + d^2\beta))}$$

Since the numerator is always positive, the optimal rate of return on money increases as the weight individuals ascribe to the disutility they derive from work increases. This signifies that the more disutility individuals derive from work, the lower the level of production. The monetary authority by increasing the return on money can, not only increase the real value of the money that individuals will carry forward to finance their purchases, but can also stimulate production. This shows how the optimal policy responds to the production decisions of the individuals. The response of the optimal policy on the weight individuals give to the utility they derive from consumption is given by

$$\frac{\partial \rho}{\partial \gamma} = - \frac{d^2\beta(\delta(1 + d^2\beta) + 2d^2\beta(\gamma + G))}{2(d\beta(\delta + G) + \gamma d\beta(1 + d^2\beta))^2}$$

This exercise shows that the optimal return on money decreases as the weight on consumption in the utility function decreases. The result is again intuitive. The more individuals value consumption, every other parameter remaining unchanged, the more they will have an incentive to produce. The more they produce, the less the monetary authority has a role to play in stimulating production and money holding. To discourage too much money being carried forward, the monetary authority may even find it optimal to inflate the currency. The inflationary result obtained in this model are partly due to the linear quadratic structure used. In a L-Q framework, like the one used here, disposal is expensive. This generates the possibility of too much too much money being held.

## 5. Conclusion

This paper derives the optimum quantity of money in a stochastic production economy. The optimum quantity of money literature has largely ignored production. The monetary growth literature considers production, but its results are mainly driven by changes in the relative return on money and capital. Neither of these two literatures concentrates on the role of production on money's optimal rate of return. This paper examines the issue in an economy that is subject to stochastic shocks each period.

When preferences are subject to an individual specific shock, the paper shows that an infinite rate of inflation is optimal. The average individual in this case does not want to consume anything. Therefore, there is no production on the average as well. The monetary authority by inflating the currency infinitely allows the individuals to costlessly dispose off any money that they may have carried over from the past. By making the utility function independent of money, the monetary authority allows the individuals to achieve their consumption and leisure bliss. This result is identical to the one found by Taub for an exchange economy. Since in both the models average income is zero, it does not matter from the point of choosing the optimal policy whether an endowment or a production economy is considered.

When the average individual wants to consume a non-zero quantity of the good, however, as in the case of an aggregate preference shock, the paper shows that it is important from the policy point of view to explicitly take into consideration whether the individuals produce the good using their labor or receive it for free from some exogenous source. It is shown that the optimal policy is sensitive to how labor supply is modeled. The paper shows that the optimal return on money is positively related to the weight on the disutility from labor in the utility function.

Depending on the weights individuals attach to the utility they derive from consumption and leisure, the policy may be inflationary or deflationary. In this case the individual does not produce enough as he values leisure relatively more. The welfare of the individual is increased by a deflation not only because it increases the value of the money carried forward for future purchases but also

stimulates current production. The parameters of the model may also take values that result in overproduction. Since disposal is expensive in linear quadratic models, the monetary authority may have to inflate the currency to maximize welfare. The inflation not only reduces the value of the excess cash carried forward but also discourages production. The paper shows that there exists a direct relationship between the return on money and production. This confirms the positive Phillips curve theory, as Kimbrough also concluded.

The paper has shown that monetary policy not only affects intertemporal transfer of purchasing power and therefore the consumption-saving decisions of individuals, but also production. Like Kimbrough's model this paper links the optimum quantity of money literature with the Phillips curve literature. If we agree that monetary policy can affect production, it is only intuitive to imagine that policy will change as we move from an endowment to a production economy since the impact of policy on output is nonexistent in one and crucial in another.

# **Chapter IV**

## **Optimal Monetary Policy in a World with Risky Investments and Financial Intermediaries**

### **1. Introduction**

The 1980's has witnessed some renewed interest in the influence of financial intermediaries on the macroeconomy. Most importantly, the research in this area has produced a number of papers that endogenise the development of financial intermediaries. This effort has thrown some new light on the process of borrowing and lending in the economy. Some of the notable contributions to the recent financial intermediation literature are by Townsend (1979), Diamond and Dybvig (1983), Chan (1983), Diamond (1984), Ramakrishnan and Thakor (1984), Boyd and Prescott (1985), Williamson (1986), Bernanke and Gertler (1987), Greenwood and Jovanovic (1990), and Bencivenga and Smith (1991).

The recent revival in interest in the role of the financial sector is perhaps due to the increasing realization that the capital market is not perfect. In a perfect capital market, the Modigliani-Miller theorem renders the financial structure irrelevant for the real economy. As Gertler (1988) points

out, the emphasis placed on the financial sector by Gurley and Shaw (1955) started getting ignored because it was not rigorous enough to override the Modigliani-Miller theorem. More recently, however, developments on both the theoretical and empirical fronts have provided a suitable ground for a more thorough analysis of the financial sector. If capital markets are indeed imperfect, the nature of the financial sector may have important implications for the economy. Bernanke and Gertler have written a number of papers that model the role of intermediaries in the propagation of cyclical fluctuations. They conclude, for example, that the severity of the Great Depression was greatly aggravated by the virtual collapse of the financial sector. The recent intermediation literature draws heavily on industrial organization's literature on information. It specifically is based on Akerlof's (1969) "lemon problem". In particular, a large number of models assume that lenders and borrowers are asymmetrically informed so that some inefficiency exists in the allocation of resources. A number of authors argue that these inefficiencies may be alleviated by the introduction of financial intermediaries. For a detailed survey see Gertler (1988).

Papers such as Diamond (1984) and Williamson (1986), assume that investment projects can be operated by entrepreneurs but are financed by lenders. If the outcome of the projects are risky and are the private information of the entrepreneurs, and more than one lender is required to finance one project, monitoring costs can be greatly reduced if financial intermediaries are introduced. As long as the intermediaries pay a fixed return to its depositors, the depositors do not need to monitor the intermediaries. Williamson also provided a link between the intermediation literature and the credit rationing literature similar to the one developed by Stiglitz-Weiss (1981). In the presence of intermediaries he shows how changes in the parameters of the model have different implications under rationing and non-rationing.

Our intention in this paper is not to provide a new theory of the growth of intermediaries. Rather, we extend the analysis of intermediation to investigate what implications financial intermediaries may have for monetary policy. To this end we extend a model similar to the one analyzed by Williamson. Consequently, we do not explicitly prove the development of the financial intermediaries. We specifically investigate how monetary policy influences investment in physical capital in the presence of intermediaries. Money is introduced in the economy primarily to provide liquidity

to the agents. In this context we analyze the possibility of the existence and influence of alternative private substitutes of money. The cost of providing the substitutes of money is found to have significant influence on monetary policy. The paper also finds the possibility of credit rationing in the context of privately informed borrowers. It shows that whether credit rationing exists in equilibrium or not significantly affects the portfolio decision of intermediaries and optimal monetary policy. So the paper links the recent financial intermediation literature with the more traditional monetary policy literature.

We show that although a decrease in the return on money stimulates investment validating the Tobin-Mundell effect, at very high rates of inflation the Tobin effect gets reversed. Consequently, we can derive a rate of return on money that is welfare maximizing. The distributional effects of changes in the return on money are also analyzed. Except a few models like Romer (1986), most models related to the monetary growth literature ignores the issue of optimal monetary policy and the distributional effects of inflation. These models as a result often produce hyperinflationary results as Orphanides and Solow (1990) alluded to.

While there are some papers that examine the issue of monetary policy on the economy in the presence of financial intermediaries, we do not know of any paper that explicitly relates the recent intermediation literature based on asymmetrically informed borrowers and lenders with monetary policy. Wood (1981), Williamson (1987), and Romer (1986) are examples that relate the two issues. While both Williamson and Romer use Baumol-Tobin arguments for the coexistence of outside and inside money, Wood based his argument on diversification. Although Wood analyzed the issue of effectiveness of monetary policy in an economy in which market segmentation justifies the development of intermediaries, the Williamson and Romer papers do not provide any justification for the development of intermediaries.

This paper's conclusions regarding monetary policy are related to the results of the monetary growth literature that was spawned by the works of Mundell and Tobin. Tobin in his 1965 paper argued that investment in physical capital may be stimulated by increasing money supply. Embedding the issue in a growth theoretic framework similar to those of Solow and Swan, he showed that an increase in the rate of inflation brought about by an increase in money supply increases capital

accumulation. Since portfolio decisions are dependent on the rates of return offered by alternative assets, a decrease in the value of money lead investors to move to the alternative asset - capital - that now offers a relatively higher rate of return. This implies higher capital accumulation and therefore higher growth for the economy. Since then the validity of the Tobin-Mundell effect has been thoroughly analyzed in the literature. Some notable contributions are Johnson (1967), Sidrauski (1967a, 1967b), Dornbusch and Frenkel (1973), Stockman (1981), Drazen (1981), and Romer (1985, 1986). Orphanides and Solow (1990) provides a survey of the literature.

Among the papers on monetary growth, this paper is most closely related to Drazen (1981). There are, although, some very significant differences. First, the influence of intermediaries on the Tobin effect we capture are absent in Drazen's paper due to the absence of intermediaries in his paper. Secondly, individuals in our model do not receive any utility from consumption in the first period, implying that the income effect that complements the substitution effect for the Tobin effect in Drazen's paper are absent in ours. The validity of the Tobin effect in our model is not dependent on the income effect highlighted by Drazen. Finally, we have multiple means of payment and show how the cost of inside money crucially affects the Tobin effect. In fact at high levels of inflation if "too little" money is held, the cost of the alternative medium of exchange may get high enough to reverse the Tobin effect.

The paper is organized as follows. Section 2 describes the economy; section 3 solves the intermediary's problem, section 4 deals with some comparative static exercises, section 5 determines the optimal monetary policy, and section 6 concludes.

## 2. Description of the Economy

The economy consists of a finite number of isolated islands, each being inhabited by  $2N$  two-period-lived agents. Each period when the old agents die, an equal number ( $N$ ) of young agents are born to keep the population of every island constant. A fraction  $\alpha$  of the agents of each generation are lenders and the remaining are entrepreneurs.

In the economy there are several intermediaries that manage the borrowing and lending between the lenders and entrepreneurs. The intermediaries accept deposits from the lenders and hold portfolios that maximize the return they pay their depositors. The intermediaries hold their deposits in two alternative forms - finance some risky investment projects and hold money. To invest in the projects the intermediaries have to lend to the entrepreneurs who have exclusive access to the investment projects. These projects yield a return  $R$  with probability  $\pi$  and 0 with probability  $1 - \pi$ . It is assumed that the expected return,  $\pi R$  of the investment projects is greater than  $\rho$ , the real return on money. Although this implies that a risk neutral intermediary will invest only in the risky projects, the intermediaries will also hold money if its depositors want to withdraw some amount in cash. To reduce the quantity of money held, the intermediaries may issue their own money, bank drafts, to their customers. Although these drafts are perfect substitutes of money, they are costly for the intermediaries to issue. Consequently, both money and bank drafts may be held in equilibrium.

The lenders are agents who when young, are endowed with  $e$  amount ( $e < 1$ ) of a good that is perishable and non-transportable to any other island. Although the good can be directly consumed, the lenders do not want to consume when they are young. Due to the perishability of the good after one period, the lenders have to find ways of transferring the value of their endowment to the next period. The following options are available to them. They can invest in the risky projects by lending directly to the entrepreneurs, hold money, or deposit their endowment at an intermediary. The lenders may need money when they are old because they have a probability  $p$  of moving to another island. Since the good is assumed to be non-transportable, the movers can consume only

if they carry money with them. If the lenders have deposits with the intermediaries they can use the bank drafts also to pay for their purchases in case they have to move. If the depositors receive returns  $\hat{r}$  if they move and  $\tilde{r}$  if they stay on their own island, the expected utility of a depositor is given by

$$p \ln \hat{r} + (1 - p) \ln \tilde{r} \tag{1}$$

The logarithmic utility function has been assumed for expositional simplicity.

Unlike the lenders, the entrepreneurs do not receive any endowment at all. As we mentioned earlier, they each, however, have access to an investment project that requires exactly one unit of the endowment good. Since these entrepreneurs do not have any endowment of their own, they will inelastically provide their services for any positive return for their effort if they are risk neutral.

The entrepreneurs can take a costly action that can improve the probability of success,  $\pi$ , of their projects. The true outcome of the project, however, is the private information of the entrepreneur who worked on the project. Any other agent in the economy must incur a cost  $\gamma$  to observe the true outcome. Since the project is risky and the true outcome is not publicly observable, the financiers of the projects will offer contracts to the entrepreneurs which will lay down the returns at the different states of nature and the probability of monitoring in the bad states.

Since the amount of endowment per lender,  $e$ , is insufficient to finance one project which requires exactly one unit of the endowment good, if the lenders lend their resources directly to the entrepreneurs, multiple lenders will monitor one entrepreneur if the project fails. As Williamson (1986) shows, these monitoring costs can be significantly reduced if a financial intermediary intermediates between the borrowers and lenders. Further, due to the law of large numbers, the intermediaries can offer risk-free returns to their depositors. These factors will induce the lenders of the economy to organize intermediaries that will manage their portfolios and monitor the entrepreneurs.

On the basis of the contract offered to him, an entrepreneur determines how much effort to take, or equivalently, what probability of success to achieve so that his expected return can be maximized. If he receives  $\bar{r}$  when he reports the project is successful,  $c$  when his project fails and he is found to be telling the truth when monitored,  $c_0$  if he reports bad luck and is not monitored, and  $t$  is the probability of being monitored, the expected utility of a risk neutral entrepreneur is given by

$$\pi\bar{r} + (1 - \pi)[tc + (1 - t)c_0] - \frac{\beta\pi^2}{2}$$

where,  $\frac{\beta\pi^2}{2}$  is the total disutility of choosing a probability level  $\pi$  and  $\beta > 0$ . It is to be noted here that the contract will be such that the entrepreneur will never misreport the true state of nature. Therefore, the penalties that will be paid in cases of misrepresentation do not appear in the entrepreneur's expected utility function. The entrepreneur chooses  $\pi$  to maximize his expected return. Differentiating the above equation with respect to  $\pi$  yields the optimal value

$$\pi = \frac{\bar{r} - (tc + (1 - t)c_0)}{\beta} \quad (2)$$

Since the entrepreneur can never earn more in the bad state than in the good state,  $\pi$  is always positive. We will show later that a simple assumption regarding the parameters will yield  $\pi < 1$ . The solution for  $\pi$  is quite intuitive. The higher the return in the good state, the harder the entrepreneur works to increase the probability of success; the higher the returns in the bad state, the lower the probability of success chosen; finally the higher the cost of the effort required, the less the entrepreneur works.

In the next section we describe and solve the intermediary's problem.

### 3. The Intermediary's Problem

Each intermediary in this model solves two problems. First, it must choose its portfolio in a way that maximizes the utility of its depositors and second it determines the optimal contract that it must offer the entrepreneurs for their services. We begin by solving the second problem.

#### The Optimal Contract

We begin by assuming that there is a competitive market for the services of the intermediaries. Consequently, they compete in the resource market as well for the services of the entrepreneurs. So each intermediary takes the minimum utility that it must offer each entrepreneur as given and designs its optimal contract accordingly.

Given the optimal solution for the probability of success from equation (2), an intermediary offers a contract  $\{\bar{r}, c, c_0, c_1, t\}$  to each entrepreneur to maximize the intermediary's expected return. If  $q$  is the net return to the intermediary per investment project,  $c_1$  is the penalty the entrepreneur pays if he reports bad luck and is caught lying,  $\pi$ ,  $R$ , and  $\gamma$  are the probability of success of the project, the return when the project succeeds, and the cost of monitoring respectively, as has been mentioned earlier, substituting the optimal solution for  $\pi$ , the intermediary selects a contract to maximize,

$$q = \left[ \frac{\bar{r} - (tc + (1-t)c_0)}{\beta} \right] R - \gamma \left[ 1 - \frac{\bar{r} - (tc + (1-t)c_0)}{\beta} \right] t - \left[ \frac{\bar{r} - (tc + (1-t)c_0)}{\beta} \bar{r} + \left\{ 1 - \frac{\bar{r} - (tc + (1-t)c_0)}{\beta} \right\} (tc + (1-t)c_0) \right] \quad (3)$$

subject to

$$\left[ \frac{\bar{r} - (tc + (1-t)c_0)}{\beta} \right] \bar{r} + \left[ 1 - \frac{\bar{r} - (tc + (1-t)c_0)}{\beta} \right] [tc + (1-t)c_0] - \frac{\beta}{2} \left[ \frac{\bar{r} - (tc + (1-t)c_0)}{\beta} \right]^2 \geq \bar{U} \quad (4)$$

$$\bar{r} \geq (1-t)(R + c_0) + t(R - c_1) \quad (5)$$

$$\bar{r} \geq 0; \quad c \geq 0; \quad c_0 \geq 0; \quad R \geq c_1; \quad t \geq 0; \quad 1 \geq t$$

The first constraint indicates the minimum market determined utility level  $\bar{U}$  that the intermediary must offer each entrepreneur; the second constraint is the incentive compatibility constraint that ensures that it does not pay the entrepreneur to declare bad luck when the project succeeds, and the rest are some usual non-negativity restrictions. For completeness we must also specify a second incentive compatibility constraint that indicates that it is not in the interest of the entrepreneur to declare good luck when the project has actually failed. Since it is obvious that this constraint will never bind we can safely ignore it. Substituting the optimal solution for the probability of success of the project, the Lagrangean for the intermediary's problem becomes

$$\begin{aligned} L = & \left[ \frac{\bar{r} - (tc + (1-t)c_0)}{\beta} \right] (R - \bar{r}) - \left[ 1 - \frac{\bar{r} - (tc + (1-t)c_0)}{\beta} \right] [\gamma t + tc + (1-t)c_0] \\ & + \lambda_1 \left[ \left( \frac{\bar{r} - (tc + (1-t)c_0)}{\beta} \right) \bar{r} + \left( 1 - \frac{\bar{r} - (tc + (1-t)c_0)}{\beta} \right) [tc + (1-t)c_0] \right. \\ & \quad \left. - \frac{\beta}{2} \left( \frac{\bar{r} - (tc + (1-t)c_0)}{\beta} \right)^2 - \bar{U} \right] \\ & + \lambda_2 [\bar{r} - (1-t)(R + c_0) - t(R - c_1)] + \lambda_3 [R - c_1] + \lambda_4 [1 - t] \end{aligned}$$

Here,  $\lambda_1, \lambda_2, \lambda_3, \lambda_4$  are the Lagrange multipliers. From this problem the following proposition is derived.

**Proposition 1:** (i)  $c = c_0 = 0$ , that is, the entrepreneur receives no payment in the bad state, (ii) the entrepreneur receives  $\bar{r} = \sqrt{2\beta\bar{U}}$  in the good state, and (iii) the probability of audit is  $t = \frac{R - \bar{r}}{R}$ .

**Proof:** The solutions are derived easily by writing down the Kuhn-Tucker conditions and solving them. Part (i) is intuitively quite obvious because the best contract will never pay a risk neutral agent anything in the bad state. Since a risk neutral agent is only concerned with the expected return, agency costs are minimized when the agent earns nothing in the bad state. Further, the solution of  $\pi$  shows that the entrepreneur's effort decreases in the returns in the bad states. Therefore, it is in the best interest of the intermediary to pay nothing in the bad states.

Substituting  $c = c_0 = 0$  in the utility constraint (4), the solution of  $\bar{r}$  directly follows. Since it is obvious that the intermediary will not pay the entrepreneur when he cheats and is caught lying, the penalty for cheating is to confiscate the entire amount that is concealed. Therefore,  $c_1 = R$ . Further, since it is always in the best interest of the intermediary to offer a contract in which the incentive compatibility constraint binds, the solution for the probability of audit follows from this constraint.  $R - \bar{r}$  is what the entrepreneur pays the intermediary when the project succeeds. This is, therefore, the loan rate. The solution of the probability of audit shows that the probability increases in the loan rate. This is because the higher the loan rate the lower the return to the entrepreneur. As the solution of  $\pi$  shows, this reduces the probability of success, and increases the probability of misreports.

**Corollary:** The net return to the intermediary per unit of investment,  $q$ , is a concave function of the loan rate and reaches a maximum at the loan rate  $r_R = \frac{R^2 + \gamma(R - \beta)}{2(R + \gamma)}$ .

**Proof:** From Proposition 1, the probability of success of the project is  $\frac{\bar{r}}{\beta}$ . Substituting this and the value of the probability of audit from Proposition 1 in the expression for the profit of the intermediary per unit investment, we observe the return per unit of investment  $q$  to be

$$q = \frac{\bar{r}}{\beta} (R - \bar{r}) - \gamma \left(1 - \frac{\bar{r}}{\beta}\right) \left(1 - \frac{\bar{r}}{R}\right) = -\frac{r^2}{\beta R} [R + \gamma] + \frac{r}{\beta R} [R^2 + \gamma(R - \beta)] \quad (6)$$

In equation (6),  $r = R - \bar{r}$  is the loan rate. The quadratic nature of the solution of  $q$  shows that the return to the intermediary is a concave function of the loan rate. The concavity of the return function is due to the two opposing effects a change in the loan rate has on the intermediary's net return. As the loan rate increases the net return directly increases because the intermediary earns more per unit invested. On the other hand, an increase in the loan rate decreases the return to the entrepreneur reducing the probability of success of the project. This reduction reduces the expected return to the intermediary. As a result the net change in intermediary's return due to a change in the loan rate depends on which effect dominates. Differentiating equation (6) with respect to  $r$  and setting the derivative to zero gives the value of the loan rate that maximizes the return of the intermediary. This yields,

$$r_R = \frac{R^2 + \gamma(R + \beta)}{2(R + \gamma)} \quad (7)$$

Substituting in the solution for  $\bar{r}$  in Proposition 1 we can calculate the value of  $\bar{U}$ . This also implies

$$q_R = \frac{1}{(R + \gamma)R\beta} \left[ \frac{R^2 + \gamma R - \gamma\beta}{2} \right]^2 \quad (8)$$

Here  $q_R$  is the net return to the intermediary associated with the loan rate  $r_R$ . It is interesting to note that the intermediary's net return will be maximized at the same loan rate if the intermediary were a monopsonist instead of a competitor. The loan rate that will maximize the net return of the intermediary is also  $r_R$ . In this model, if the loan rate  $r_R$  is achievable, it will be in the interest of

every intermediary to charge it. If any intermediary tries to attract more entrepreneurs by offering a higher return to them it will hurt its depositors. As long as there are enough entrepreneurs available for the other intermediaries to offer the loan rate  $r_R$ , the intermediary that lends at a lower rate will not find any depositors.

We next determine the intermediary's optimal investment level given the return  $q$  it obtains per unit of investment.

### The Intermediary's Optimal Portfolio Decision

Once the intermediary solves the optimal contract that it offers the entrepreneurs, it makes its portfolio decision to maximize the utility of its depositors. As has been mentioned earlier, money is an alternative asset available in the economy. In the presence of an asset like capital, money is introduced mainly for transactions purposes. Recalling that a fraction of the lenders have to move to another island in their old age, money increases the welfare of the lenders by providing them with the much needed liquidity. It is introduced by means of transfers (or tax) to the old each period. We assume that the rate of return on money is fixed unlike the return on the investment project. If the investment level is large enough the uncertainty of the return can be reduced to a minimum due to the law of large numbers. In this case if  $q$ , the net return on investment, is less than the return on money,  $\rho$ , no investment will take place as money will dominate capital not only because of its medium of exchange characteristic but also because it will be a better store of value. So the interesting case, the one we study, is when  $q$  dominates  $\rho$ .

To reduce the quantity of money to be held for liquidity purposes, the intermediary can establish a market for their bank drafts. If such a market is established, the movers can use their bank drafts instead of money, to buy the consumption good on the other island. If such a market can be established costlessly, money will cease to be used as a means of payment. We, however, assume that to sell  $\theta$  number of claims, the intermediary must incur a cost of  $s(\theta)$ . The only assumptions we make about the cost function are that  $s'(\theta) > 0$ ,  $s''(\theta) > 0$ . This cost will induce the intermediary to hold some money in equilibrium. The intermediary's problem then is to determine its optimal

holdings of outside money, investments, and number of bank drafts to issue such that the utility of its representative depositor can be maximized.

As has been mentioned earlier,  $\hat{r}$  is the return to the depositor if he moves,  $\tilde{r}$  is the return to the depositor if he stays on the island,  $p$  is the probability that an agent will move. Further, let  $i$  denote per capita investment,  $m$  denote the real value of money held,  $s(\theta)$  denote the cost to the intermediary to set up a market for its claims of size  $\theta$ . If  $\rho$  is the gross real rate of return on money,  $q$  is the return on investment,  $h$  is the real quantity of government transfers made in the second period, and  $e$  is the amount each depositor deposits at the intermediary, the intermediary solves the following problem:

$$\text{maximize } p \ln \hat{r} + (1 - p) \ln \tilde{r}$$

subject to

$$e = i + m + s(\theta) \tag{9}$$

$$qi + \rho m + h = p\hat{r} + (1 - p)\tilde{r} \tag{10}$$

$$\theta + \rho m + h = p\hat{r} \tag{11}$$

$$i, m, \theta \geq 0; \hat{r}, \tilde{r} > 0$$

The problem shows that the intermediary maximizes the expected utility of its representative depositor subject to three resource constraints. The first constraint, which is the intermediary's budget constraint, indicates that the cost of the liquidity service provided by it is incurred in the first period. The second constraint gives the total return from the portfolio held by the intermediary that

can be distributed to its depositor and expresses that this is possible due to the law of large numbers. The third constraint indicates the resources that can be distributed to the depositor if he is required to move in the second period. The Lagrangean for this problem,

$$L = p \ln \hat{r} + (1-p) \ln \tilde{r} + \mu_1 [e - i - m - s(\theta)] \\ + \mu_2 [qi + \rho m + h - p\hat{r} - (1-p)\tilde{r}] + \mu_3 [\theta + \rho m + h - p\hat{r}]$$

yields the following Kuhn-Tucker conditions:

$$\frac{\partial L}{\partial \hat{r}} = \frac{p}{\hat{r}} - \mu_2 p - \mu_3 p = 0 ;$$

$$\frac{\partial L}{\partial \tilde{r}} = \frac{(1-p)}{\tilde{r}} - \mu_2 (1-p) = 0 ;$$

$$\frac{\partial L}{\partial i} = -\mu_1 + \mu_2 q \leq 0; \quad i \geq 0; \quad i \frac{\partial L}{\partial i} = 0 ;$$

$$\frac{\partial L}{\partial m} = -\mu_1 + \mu_2 \rho + \mu_3 \rho \leq 0; \quad m \geq 0; \quad m \frac{\partial L}{\partial m} = 0 ;$$

$$\frac{\partial L}{\partial \theta} = -s'(\theta)\mu_1 + \mu_3 \leq 0; \quad \theta \geq 0; \quad \theta \frac{\partial L}{\partial \theta} = 0.$$

In these conditions  $\mu_1, \mu_2, \mu_3$  are the Lagrange multipliers. Using the money market clearing condition,  $h = m - \rho m$ , we derive the following optimal solutions from the first order conditions for  $q, \rho > 0$

$$s'(\theta) = \frac{1}{\rho} - \frac{1}{q} \tag{12}$$

$$\hat{r} = \frac{\rho [q(e + \theta - s(\theta)) - \theta]}{q[p\rho + 1 - p]} \tag{13}$$

$$\tilde{r} = \frac{q\hat{r}}{\rho} \tag{14}$$

$$i = e + \theta - s(\theta) - p \frac{\rho[q(e + \theta - s(\theta)) - \theta]}{q[p\rho + 1 - p]} \quad (15)$$

Equation (12) implicitly determines  $\theta$ . Under the assumption that  $s'(\theta), s''(\theta) > 0$ , equation (12) implies that  $\theta$  is an increasing function of the return on the investment project,  $q$  and a decreasing function of the return on money,  $\rho$ . This result is obvious because the higher the return on investment, the higher will be the opportunity cost of holding money, encouraging higher expenditure on the market for claims on other islands. Similarly, an increase in the return on money discourages expenditures on the liquidity service provided by it.

Equation (14) shows that the return to the depositors who stay exceeds the return to the depositors who move to compensate for the loss in resources due to expenditure on the liquidity services that need to be provided to the movers.

Finally, equation (15) gives the optimal level of investment by the intermediary. To analyze the investment demand function, we differentiate equation (15) with respect to  $q$  to get

$$\begin{aligned} \frac{\partial i}{\partial q} &= [1 - s'(\theta)] \frac{\partial \theta}{\partial q} - \frac{p\rho}{p\rho + 1 - p} \left[ \frac{q(q - qs'(\theta) - 1)}{q^2} \frac{\partial \theta}{\partial q} + \frac{\theta}{q^2} \right] \\ &= \left[ 1 - s'(\theta) - \frac{p(\rho - 1)}{p\rho + 1 - p} \right] \frac{\partial \theta}{\partial q} - \frac{p\theta\rho}{q^2(p\rho + 1 - p)} \\ &= \frac{(\rho - 1)[q(1 - p) + p\rho] + \rho - p\rho^2 q\theta s''(\theta)}{\rho q^3 s''(\theta)(p\rho + 1 - p)} \end{aligned} \quad (16)$$

The result of this exercise is summarized in the following proposition:

**Proposition 2:** Investment as a function of its rate of return,  $q$ , is backward bending.

**Proof:** Differentiating the numerator of equation (16) with respect to  $q$  we obtain  $(\rho - 1)(1 - p) - p\rho^2\theta s''(\theta) - p\rho^2qs''(\theta) \frac{\partial \theta}{\partial q} - p\rho^2\theta qs'''(\theta) \frac{\partial \theta}{\partial q}$ . This shows that even if  $s'''(\theta) = 0$

and  $\rho > 1$  high values of  $q$  will make the sum negative because  $\frac{\partial \theta}{\partial q}$  is positive. If  $\rho < 1$  the derivative is unambiguously negative. Therefore, high values of  $q$  will make the numerator of equation (16) negative.

The intuition behind the backward bending investment function is the following. In this economy an increase in  $q$  increases the return per investment thereby increasing the incentive to invest more. However, as equation (12) shows an increase in  $q$  also increases  $\theta$ . Although this increase in  $\theta$  reduces the amount of money held by the intermediary, it also increases its cost of operation. In other words the net effect depends on the substitution and wealth effects. If the substitution of money by investment is large enough, the increased cost of providing the bank's liquidity service may divert so much resources that the intermediary will have less to invest. Even then, the total revenue from investment,  $qi$ , may still increase. In the case of investment then, once  $q$  becomes high enough, the increased cost of providing the liquidity service may motivate the intermediary to invest less to save on liquidity costs.

### The Market Equilibrium

Since the investment projects are operated by the entrepreneurs who each has the capacity to operate only one project, the total investment level equals the number of entrepreneurs who get their projects financed. If this number is less than the number of entrepreneurs available in the economy, some entrepreneurs will be credit rationed.

In this economy credit rationing may occur due to any of the following two reasons. First, as we have seen, the net return to the intermediary from each investment project is a concave function of the loan rate. Therefore, there exists a maximum loan rate,  $r_R$ , given by equation (7) that the intermediaries will be willing to charge the entrepreneurs per loan. If at the loan rate  $r_R$  the quantity of loans supplied is less than the quantity of loans demanded, given the inelastic demand curve, excess demand will exist. The vertical demand curve for loans is due to the lack of endowment and other investment opportunities of the entrepreneurs. Since the probability of success of the invest-

ment projects decreases as the loan rate increases, as the Corollary to Proposition 1 shows, the entrepreneurs cannot induce the intermediaries to increase the quantity of loans supplied by increasing the loan rate after  $r_R$  is reached. As a result some of the entrepreneurs may not have their projects financed in equilibrium. The equilibrium is described in Figure 1 (a).

Secondly, Proposition 2 shows that the investment function bends backwards as the net return to the intermediary becomes large. We know that the net return  $q$  is a function of the loan rate  $r$ . If the return at which the investment function starts to bend backwards,  $r_{max}$ , is lower than  $r_R$ , some entrepreneurs will be credit rationed because any further increase in the return to the intermediary will reduce rather than increase the quantity of investment demanded because of the high cost of providing the liquidity service. The rationing in this model is completely random. Figure 1 (b) describes the equilibrium.

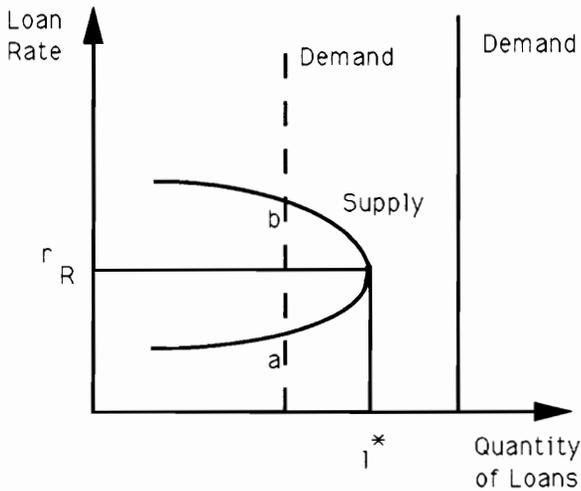


Figure 1(a)

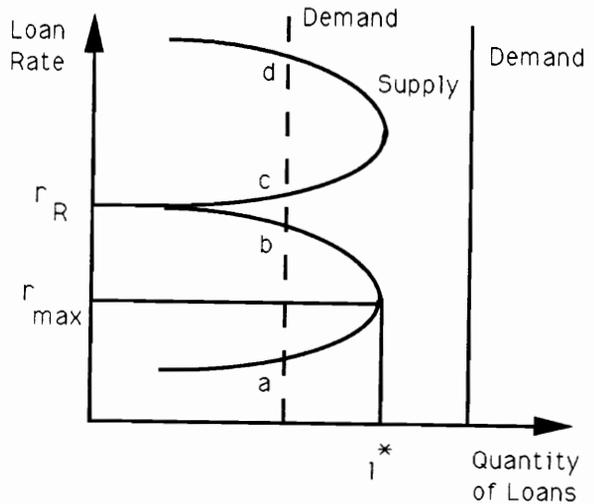


Figure 1(b)

Figure 1 (a) is drawn under the assumption that  $r_{max} > r_R$ . Consequently, once  $r_R$  is reached, the return to the intermediary will continue to fall monotonically, ruling out any other point of

inflection. In this case too we see that credit rationing is possible. If the demand curve is given by the dashed line, there will be no rationing in equilibrium. We notice that there are two candidates for equilibrium, 'a' and 'b', but only 'a' gives stability. So we obtain only one stable equilibrium here. It is to be noted that the equilibrium loan rate in this case, under non-rationing is necessarily less than  $r_R$ . In this paper we will concentrate on this type of credit rationing only by assuming that the parameter values are such that  $r_R < r_{\max}$ .<sup>1</sup>

It is to be noted that in the second case  $r_{\max}$  is below  $r_R$ . As the loan rate is increased further, till it reaches  $r_R$ , the net return to the intermediary,  $q$ , is increasing. Consequently, till this point the supply of loans curve continues to have a negative slope. After this point has been reached, however, any further increase in the loan rate will decrease the net return to the intermediary. This decrease in  $q$  will start stimulating investment making the supply of loans curve to have a positive slope. Again when the net return falls to  $q_{\max}$ , the net return at which the investment curve starts to bend back, the supply curve will start bending backwards. In this case if the demand curve lies to the right of the supply curve some entrepreneurs will be rationed. If, however, the demand curve is as shown by the dashed line, all the entrepreneurs will be employed. We observe four intersection points in this case. Only points 'a' and 'c' give stable equilibria and either of them can be chosen by the intermediary. So we get multiple equilibria if rationing is due to the backward bending investment function. This discussion is summarized in the following proposition:

**Proposition 3:** There may be credit rationing in equilibrium in this economy because (i) the return to the intermediary per investment project is a concave function of the loan rate, or (ii) the investment function is backward bending. When there is rationing due to (i),

$$i = e + \theta - s(\theta) - p \frac{\rho[q_R(\alpha Ne + \theta - s(\theta)) - \theta]}{q_R[p\rho + 1 - p]} < \frac{(1 - \alpha)N}{\alpha N}. \text{ If}$$

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<sup>1</sup> Preliminary analysis indicates that the results of the paper will change if the other type of credit rationing is considered.

$i = e + \theta - s(\theta) - p \frac{\rho[q_R(e + \theta - s(\theta)) - \theta]}{q_R[p\rho + 1 - p]} \geq \frac{(1 - \alpha)N}{\alpha N}$ , there will be no rationing in equilibrium and the level of investment will equal  $(1 - \alpha)N$ . The rate of return per project  $q_{NR}$  is the one that clears the capital market.

#### 4. Comparative Statics

For the comparative statics we intend to perform, we have to consider the rationing and non-rationing cases separately. In the rationing cases the variable of most interest to us is investment. Changes in other variables can be easily deduced once the change in investment is calculated. Under non-rationing, however, investment is constant. Consequently, any change will affect the return on investment that clears the capital market. We will concentrate on this change in most of the cases. We will consider changes in the return on money, cost of monitoring, and entrepreneur's utility cost of effort. We begin by considering a change in the return on money.

##### (i) Change in $\rho$ :

###### Rationing

To examine how a change in the gross real rate of return on money affects investment, we differentiate equation (15) with respect to  $\rho$  to obtain

$$\begin{aligned} \frac{\partial i}{\partial \rho} &= \left[ 1 - s'(\theta) - \frac{p\rho - p}{p\rho + 1 - p} \right] \frac{\partial \theta}{\partial \rho} - \frac{p(1-p)[q_R(e + \theta - s(\theta)) - \theta]}{q_R(p\rho + 1 - p)^2} \\ &= \left[ \frac{(1-p)(\rho - 1)}{\rho(p\rho + 1 - p)} + \frac{1}{q_R} \right] \frac{\partial \theta}{\partial \rho} - \frac{p(1-p)\hat{r}}{\rho(p\rho + 1 - p)} \end{aligned} \quad (17)$$

Since we are concentrating on credit rationing due to the concavity of the return function, the net return  $q_R$  is given by equation (8). Therefore, it is independent of  $\rho$ .

As in the case of a change in  $q$  the overall effect depends crucially on the sign of  $\frac{(1-p)(\rho-1)}{\rho(p\rho+1-p)} + \frac{1}{q_R}$ . This factor determines the net effect of a change in  $\theta$  on investment. The magnitude of  $\rho$  significantly affects this term. It shows that if  $\rho$  is greater than 1 the net effect of an increase in  $\theta$  on investment is positive.

When  $\theta$  increases it has two opposing effects on the resources available for investment. On the one hand a higher  $\theta$  implies increased expenditure due to the direct cost to the intermediary of providing more liquidity, on the other hand relatively fewer money balances may be held releasing resources for investment. When the return on money is high (greater than 1), the second effect may dominate the first effect increasing investment. If  $\rho$ , however, is very low the first effect may dominate the second. The intuition for this lies in the fact that when  $\rho$  is low money holding is low and  $\theta$  is high. This implies that total cost of providing the  $\theta$  is also high. Under such circumstances, any further increase in  $\theta$  may increase the cost more than it releases resources for investment. To save on these liquidity service costs, the intermediaries may even start investing less. Since the net effect of a change in the return on money on investment is the difference of the two terms in equation (17), the derivative can be positive if the first term exceeds the second. In this case therefore, an increase in the return on money increases investment implying that the Tobin effect may get reversed. Consequently, we find a limit on the rate of inflation that can increase investment. As long as the economy does not have an alternative asset that performs the medium of exchange role costlessly like money, there will exist a limit to the rate of inflation that can increase investment. Therefore, the effect of an increase in the real return on money on investment remains ambiguous. The result depends on the magnitudes of  $q_R$ ,  $\rho$ , among other factors.

In terms of the loans market equilibrium then, an increase in the return on money may shift the supply of loans curve in either direction. This is because the supply of loans is directly related to the amount of investment by the intermediaries. Since a change in  $\rho$  does not change the terms of the contract, the shifted supply curve bends backwards at exactly the same loan rate as before. So the only change in the loans market is in the equilibrium quantity of loans.

### Non-Rationing

To determine the effect of a change in  $\rho$  on the market clearing rate of return on investment,  $q$ , when there is no rationing, we differentiate the following equation with respect to  $\rho$

$$i = e + \theta - s(\theta) - p\hat{r} = \frac{(1 - \alpha)}{\alpha}$$

Recalling the solution of  $\hat{r}$  from equation (13) and noting that it is a function of  $q_{NR}$ ,  $\rho$ , and  $\theta$ , we derive the following after differentiation:

$$\left[ (1 - s'(\theta)) \frac{\partial \theta}{\partial q_{NR}} - p \frac{\partial \hat{r}}{\partial q_{NR}} \right] \frac{\partial q_{NR}}{\partial \rho} = - \frac{(1 - p)\hat{r}}{\rho(p\rho + 1 - p)} + \left[ \frac{(1 - p)(\rho - 1)}{\rho(p\rho + 1 - p)} + \frac{1}{q_{NR}} \right] \frac{\partial \theta}{\partial \rho}$$

The term multiplying  $\frac{\partial q_{NR}}{\partial \rho}$  on the left hand side of the above equation equals  $-\frac{\partial i}{\partial q}$  from equation (16). As we mentioned earlier we assume that this term is negative. As in the rationing case, whether  $q_{NR}$  increases or decreases as the return on money increases depends on the whether the cost of higher  $\theta$  outweighs the resources that are released from a reduction in the quantity of money held.

In the loans market therefore, if investment by the intermediaries decreases due to an increase in  $\rho$ , the supply of loans curve will shift to the left increasing the market clearing loan rate, thereby increasing the net return to the intermediary. The equilibrium loan rate may, however, decrease if the supply curve shifts to the right due to the reversal of the Tobin effect.

**Proposition 4:** (i) Under rationing, an increase (decrease) in  $\rho$  decreases (increases) investment by intermediaries if  $\rho > 1$ , but may increase (decrease) investment if  $\rho < 1$  and the resources released (necessary) from a reduction (increase) in the liquidity service is large enough. (ii) Under non-rationing, an increase (decrease) in  $\rho$  increases (decreases) the net return,  $q_{NR}$ , to the intermediaries if  $\rho > 1$ , but may decrease (increase)  $q_{NR}$  if  $\rho < 1$  and the resources released (necessary) from a reduction (increase) in the liquidity service is large enough.

(ii) Change in  $\gamma$ :

Rationing

Under rationing, to determine the effect of a change in the cost of monitoring the entrepreneurs, we first determine how such a change affects the optimal rate of return on investment. To this end we first differentiate equation (8) with respect to  $\gamma$  to get

$$\frac{\partial q_R}{\partial \gamma} = \frac{1}{4R\beta(R + \gamma)^2} [R^2 + \gamma R - \gamma\beta - 2R\beta][R^2 + \gamma R - \gamma\beta]$$

If  $R > \beta$ , the sign of the above derivative depends on the sign of  $R^2 + \gamma R - \gamma\beta - 2R\beta$ . Recalling the solution of the loan rate  $r_R$  from equation (7) and noting that  $\bar{r} = R - r_R$ , some algebra yields  $R^2 + \gamma R - \gamma\beta - 2R\beta = 2(\bar{r} - \beta)(R + \gamma)$ . Recalling that the probability of success  $\pi = \frac{\bar{r}}{\beta} < 1$ ,  $\bar{r}$  must be less than  $\beta$ . Therefore,

$$\frac{\partial q_R}{\partial \gamma} < 0$$

$$\text{Since } \frac{\partial i}{\partial \gamma} = \frac{\partial i}{\partial q_R} \frac{\partial q_R}{\partial \gamma}$$

investment will decrease as  $\gamma$  increases if the relationship between investment and its rate of return is positive and vice versa. Although the rate of return to the intermediary per unit of investment decreases as the cost of monitoring increases, the return to the entrepreneur increases as the following derivative shows:

$$\frac{\partial \bar{r}}{\partial \gamma} = \frac{R\beta}{2(R + \gamma)^2}$$

When  $\gamma$  increases, the per unit cost of monitoring the entrepreneurs increases. The intermediaries can reduce some of their monitoring costs if they can reduce the probability with which they

monitor. An increase in  $\bar{r}$  allows them to precisely do that because an increase in  $\bar{r}$  increases the probability of success of the projects.

In the loans market, therefore, an increase in  $\gamma$  reduces  $q_R$ , the net return to the intermediary. As a result, the supply curve of loans bends backward at a lower loan rate. Consequently, the equilibrium quantity of loans and therefore investment decreases as the cost of monitoring increases.

### Non-Rationing

When all the entrepreneurs are employed,  $q_{NR}$  is the return that clears the capital market. Given that the demand for capital by the entrepreneurs is inelastic, and the supply of capital by the intermediaries is independent of  $\gamma$  as Proposition 3 shows, any change in the cost of monitoring will fail to change the equilibrium rate of return on investment,  $q_{NR}$ . It, however, changes the loan rate as differentiation of the equation expressing  $q$  as a function of  $\bar{r}$  in Proposition 1 shows

$$0 = [(R - 2\bar{r})(R + \gamma) + \gamma\beta] \frac{\partial \bar{r}}{\partial \gamma} + (R - \bar{r})(\bar{r} - \beta)$$

$$\text{Therefore, } \frac{\partial \bar{r}}{\partial \gamma} = - \frac{(R - \bar{r})(\bar{r} - \beta)}{(R - 2\bar{r})(R + \gamma) + \gamma\beta}$$

The denominator of the above expression is negative. This is because under non-rationing the equilibrium rate of return on capital is below the rate of return  $q_R$ . When the return is below this level, an increase in the loan rate increases the net return  $q$  implying that an increase in the return to the entrepreneur  $\bar{r}$  decreases the return to the intermediary. The denominator is the derivative of the return to the intermediary with respect to  $\bar{r}$ . Since this derivative is negative for any return to the intermediary that is below  $q_R$ , the denominator must be negative.

Further, since the loan rate must be positive and  $\bar{r} < \beta$  it follows that  $\frac{\partial \bar{r}}{\partial \gamma} < 0$ . So although the net return to the intermediary per investment remains unchanged, its loan rate increases. In the loans market then, the supply of loans curve shifts to the left. To restore equilibrium at the non-

rationed quantity of loans, the equilibrium loan rate increases. We therefore have the following result

**Proposition 5:** An increase (decrease) in the cost of monitoring (i) decreases (increases) investment by the intermediaries, increases (decreases) the return to the entrepreneurs, and decreases (increases) the loan rate and the return to the intermediary under rationing, (ii) decreases (increases) the return to the entrepreneur, increases the loan rate but does not affect investment and the net return to the intermediary under non-rationing.

(iii) Change in  $\beta$ :

Rationing

We know that  $\bar{r} = \frac{R^2 + \gamma(R + \beta)}{2(R + \gamma)}$ . Differentiating with respect to  $\beta$  we get

$$\frac{\partial \bar{r}}{\partial \beta} = \frac{\gamma}{2(R + \gamma)} > 0$$

$$\text{and } \frac{\partial \pi}{\partial \beta} = \frac{\partial(\bar{r}/\beta)}{\partial \beta} = -\frac{R^2 + \gamma R}{2\beta^2(R + \gamma)} < 0.$$

$$\text{Finally, } \frac{\partial q_R}{\partial \beta} = \frac{(R^2 + \gamma R - \gamma\beta)(-2\beta\gamma - R^2 - \gamma R + \gamma\beta)}{4(R + \gamma)R\beta^2} < 0.$$

Therefore,  $\frac{\partial i}{\partial \beta} = \frac{\partial i}{\partial q_R} \frac{\partial q_R}{\partial \beta}$  is negative if  $\frac{\partial i}{\partial q_R} > 0$ , positive otherwise. The exercise shows that when the cost of improving the probability of success of the investment projects increases, the entrepreneurs must be compensated more to induce them to take proper actions. Given that the probability of success still declines, the return to the intermediary decreases. This decline reduces the demand for investment as long as the return on capital positively affects investment.

In the loans market the reduction in the profitability of loans reduces the supply of loans shifting the supply curve to the left. Since  $\bar{r}$  increases  $r_R$  decreases. As a result the loan rate at which

the supply curve bends backwards decreases. So the final equilibrium will have lower quantity of loans and a lower loan rate.

### Non-Rationing

For reasons similar to those given for the case of a change in the cost of monitoring,

$$\frac{\partial q_{NR}}{\partial \beta} = 0.$$

$$\text{and } \frac{\partial \bar{r}}{\partial \beta} = \frac{q_R R + \gamma(R - \bar{r})}{(R - 2\bar{r})(R + \gamma) + \gamma\beta} < 0.$$

As has been explained in the previous case, the denominator is negative making the derivative negative.

Therefore,  $\frac{\partial \pi}{\partial \beta} = \frac{\beta \frac{\partial \bar{r}}{\partial \beta} - \bar{r}}{\beta^2} < 0$ . We summarize the effects of a change in  $\beta$  in the following result:

**Proposition 6:** An increase (decrease) in  $\beta$  (i) decreases (increases) investment, increases (decreases) the return to the entrepreneurs while decreasing (increasing) the probability of success and the return to the intermediary under rationing, (ii) decreases (increases) the return to the entrepreneur and the probability of success while keeping the net return to the intermediary and investment constant under non-rationing.

## 5. Optimal Monetary Policy

In the previous section we analyzed the optimal solutions derived for the economy by investigating how the solutions changed as the parameters of the model changed. One such parameter was the rate of return on money. In this section we investigate whether there exists any optimal rate of return on money.

To determine the optimal rate of return on money, we construct a social welfare function which is the weighted sum of the welfare of the depositors and entrepreneurs. Recognizing that it is possible to construct other welfare functions, we try to provide some insight into the question of what type of policy will maximize overall welfare by considering a specific function. As will become evident later, the function we consider is sufficient to illustrate the point that optimal policies may be very different for the cases of rationing and non-rationing. The welfare function we consider is

$$W = \delta\alpha[p \ln \hat{r} + (1-p) \ln \tilde{r}] + (1-\delta) \frac{i}{(1-\alpha)} \left[ \pi\bar{r} - \frac{\beta\pi^2}{2} \right] \quad (18)$$

In this equation,  $W$  is the weighted sum of the welfares of a representative lender and a representative entrepreneur.  $\delta$  is an arbitrary weight attached to the welfare of the depositor. Given that the number of entrepreneurs who are employed equals the level of investment, the probability of being employed is  $\frac{i}{(1-\alpha)}$ . Further, each entrepreneur has an expected return equal to  $\pi\bar{r}$ . The last term in equation (18) then gives the expected welfare of each entrepreneur. To determine the optimal monetary policy, we begin with the rationing case.

### Rationing

Substituting the optimal solutions derived earlier in the paper and differentiating equation (18) with respect to  $\rho$  we obtain

$$\begin{aligned}
\frac{\partial W}{\partial \rho} &= \delta \alpha \left[ \frac{1}{\hat{r}} \frac{\partial \hat{r}}{\partial \rho} + \frac{(1-p)}{q_R} \frac{\partial q_R}{\partial \rho} - \frac{(1-p)}{\rho} \right] + \frac{(1-\delta)}{(1-\alpha)} \frac{\bar{r}^2}{2\beta} \frac{\partial i}{\partial \rho} \\
&= \delta \alpha \left[ \frac{1}{\hat{r}} \frac{\partial \hat{r}}{\partial \rho} - \frac{(1-p)}{\rho} \right] + \frac{(1-\delta)}{(1-\alpha)} \frac{\bar{r}^2}{2\beta} \frac{\partial i}{\partial \rho} \quad \text{since } \frac{\partial q_R}{\partial \rho} = 0 \\
&= -\frac{\delta \alpha (\rho - 1)}{(p\rho + 1 - p)} \left[ \frac{1}{\hat{r} \rho^2 s''(\theta)} + \frac{p(1-p)}{\rho} \right] + \frac{(1-\delta)}{(1-\alpha)} \frac{\bar{r}^2}{2\beta} \frac{\partial i}{\partial \rho}
\end{aligned}$$

From Proposition 4 we know that  $\frac{\partial i}{\partial \rho}$  is unambiguously negative for any  $\rho > 1$ . The first term in the above derivative is also unambiguously negative for any  $\rho > 1$ . Therefore, any  $\rho > 1$  cannot make the sum of the two terms in the above derivative zero.

The first term in the above derivative becomes positive if  $\rho < 1$ . Proposition 4 indicates that the change in investment to a change in the return on money remains ambiguous when  $\rho < 1$ . So it is possible to find a value of  $\rho$  that will make the derivative of  $W$  with respect to  $\rho$  zero if  $\rho$  is less than 1. The value of  $\rho$  cannot, however, decrease so much that it will decrease investment. So the optimal level must lie between this level and 1. Therefore, under rationing optimal policy must be inflationary. Further, if  $\delta = 1$ , implying that the monetary authority does not put any weight on the welfare of the entrepreneurs, optimal monetary policy becomes a constant money supply rule, that is  $\rho = 1$ . If, however,  $\delta = 0$  the optimal return will be the one that maximizes investment. From Proposition 4 it directly follows that this can be reached when  $\rho < 1$ . This exercise points out that the optimal policy is sensitive to the distributional effects of a change in the return on money.

### Non-Rationing

Recalling that under non-rationing the level of investment equals the number of entrepreneurs and the return to the investment project is less than  $q_R$ , we get

$$\begin{aligned}
\frac{\partial W}{\partial \rho} &= \delta \alpha \left[ \frac{1}{\hat{r}} \frac{\partial \hat{r}}{\partial \rho} + \frac{(1-p)}{r} \frac{\partial q_{NR}}{\partial \rho} - \frac{(1-p)}{\rho} \right] + \frac{(1-\delta)}{(1-\alpha)2\beta} i \frac{\partial \bar{r}^2}{\partial \rho} \\
&= \delta \alpha \frac{(1-\rho)}{(p\rho + 1-p)} \left[ p \frac{(1-p)}{\rho} - \frac{1}{\hat{r}} \frac{\partial \theta}{\partial \rho} \right] \\
&\quad + \left[ \frac{\delta \alpha}{q_{NR}} \left\{ \frac{\rho\theta + (\rho-1)}{\hat{r} q_{NR}(p\rho + 1-p) s''(\theta)} + 1-p \right\} + \frac{(1-\delta)\bar{r}}{\alpha\beta} \frac{\partial \bar{r}}{\partial q_{NR}} \right] \frac{\partial q_{NR}}{\partial \rho}
\end{aligned}$$

since  $i = \frac{(1-\alpha)}{\alpha}$ . As has been discussed earlier,  $\frac{\partial \bar{r}}{\partial q} < 0$  for any  $r < r_R$ . Like the rationing case the first term is negative for any  $\rho > 1$ . Although Proposition 4 indicates that  $\frac{\partial q_{NR}}{\partial \rho} > 0$  for  $\rho > 1$ , the term that multiplies it in the above equation is not necessarily negative. If the sum of the two terms in brackets is positive it may be possible to find an optimal policy when  $\rho > 1$ . Depending on parameter values, it is also possible to find a solution when  $\rho < 1$ . We therefore have the following proposition

**Proposition 7:** If an optimal policy exists (i) it must be inflationary under rationing but (ii) either inflationary or deflationary under non-rationing.

## 6. Conclusion

We have addressed the issue of optimal monetary policy and its effect on investment in a model in which financial intermediaries exist.

When savers and investors are different groups of agents, and the outcomes of the investment projects are the private information of the investors, problems of informational asymmetry exist. In these cases, the entrepreneurs are monitored in equilibrium to elicit truth-telling. Monitoring costs are reduced if the monitoring is conducted by a central authority like a financial intermediary.

Although the return of the investment projects are risky, the entrepreneurs in this paper can increase the probability of success of their projects by taking some costly actions. To induce them

to take proper actions, the intermediaries ought to give them sufficient incentives in the form of high returns. Noting that the loan rate obtained by an intermediary declines as the entrepreneur receives a higher return, the net return to the intermediary per investment becomes a concave function of the loan rate of return. This concavity in the return function imposes an upper bound on the rate of return per project that is profitable for the intermediary. Therefore even an increase in the rate of return to eliminate any excess supply of projects will not increase the number of projects demanded by the intermediaries. We consequently derive cases for credit rationing in this model. We show that the possibility of credit rationing has significant effects on the solutions of the model.

The intermediaries in this economy also hold some money. Each period a fraction of the old agents have to move to different islands. Money is primarily introduced to provide liquidity to these movers. Since the good is non-transportable, money is the only asset that can be used in transactions on other islands.

Money, however, has a high opportunity cost when the return on capital is high. This motivates the intermediaries to provide some liquidity service themselves. The intermediaries provide such a service by organizing markets for their claims on other islands. In these markets their depositors use deposit slips instead of money to purchase goods. It is shown that if it is costly to organize such markets, money will continue to be held in equilibrium. It is interesting to note that the effect of a change in the return on money on investment depends significantly on how the change affects the liquidity service provided by the intermediary.

Proposition 4 shows that when the rate of return on money becomes very low, compared to the return on capital, there may be so much resources used in the provision of the liquidity service by the intermediaries that any further attempt to increase investment at the cost of lower money holdings may increase the costs by more than the benefits. Under such conditions any further decrease in the rate of return on money may even reduce investment. This result indicates that Tobin's (1965) argument that a decrease in the return on money will increase capital accumulation is reversed under high rates of inflation.

This result of the reversal of the Tobin effect in our model highlights the importance of money as a medium of exchange. It continues to exist in even the most financially developed countries

because no other financial asset performs the medium of exchange role as effectively and costlessly. This paper shows that as long as it is costly for the economy to provide an asset that is universally acceptable, money will continue to exist. Therefore, although most of the monetary growth literature concludes that a decrease in the return on money will increase capital accumulation and thereby economic growth, this paper shows that excessive inflation can be harmful for the economy. It explicitly takes into consideration the costs of providing any alternative medium of exchange. So this paper perhaps dispels the concern raised by Orphanides and Solow (1991) regarding the inherent hyperinflationary tendency of the majority of the monetary growth literature.

In the last section of the paper we derive the optimal rate of return on money. To this end we constructed a social welfare function that is the weighted sum of the welfare of the depositors and entrepreneurs. We show in the context of this welfare function that the optimal policy is an inflation if credit rationing exists in equilibrium but may be either inflationary or deflationary under non-rationing. The optimal rate of growth of money depends on the weights attached to the welfares of the two classes of agents. Consequently, the distributional aspects of a change in monetary policy is also highlighted. Specifically, in the case of rationing, as the weight on the welfare of the entrepreneurs is reduced, the policy tends toward a constant money supply rule, while as the weight on the lenders is made significantly small the optimal policy becomes highly inflationary.

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## Appendix A

To obtain the solution given in (2.6) we have to differentiate (2.5) with respect to  $\bar{m}_t$ . Differentiating and setting the derivative equal to zero we have,

$$\begin{aligned}
 & -\gamma(\bar{Y}_t - \bar{m}_t + \rho\bar{m}_{t-1}) + \beta\rho\gamma(\bar{Y}_{t+1} - \bar{m}_{t+1} + \rho\bar{m}_t) \\
 & - \beta(1-\gamma)(1+\rho)(\bar{Y}_t - (1+\rho)\bar{m}_t + \rho\bar{m}_{t-1}) + (1-\gamma)\beta^2\rho(\bar{Y}_{t+1} - (1+\rho)\bar{m}_{t+1} + \rho\bar{m}_t) = 0
 \end{aligned}$$

Substituting for  $\bar{Y}_t$  and rearranging we have

$$\beta\rho\bar{m}_{t+1} + (1-\rho)\bar{m}_t = \frac{1}{1-\gamma} [(\gamma + \beta(1-\gamma))\bar{y}_{t+1} - \frac{\gamma + \beta(1-\gamma)(1+\rho)}{\beta\rho}\bar{y}_t]$$

Putting the stationary state values  $\bar{y}_{t+1} = \bar{y}_t = \bar{y}$  and  $\bar{m}_{t+1} = \bar{m}_t = \bar{m}$  in the above equation we get (2.6).

## Appendix B

We prove Proposition 1 in this section. We need to derive the  $\rho$  that would maximise the expression in equation (3.4).  $C(\cdot)$  and  $D(\cdot)$  are functions of complex variables but are evaluated at real numbers in the above expression. Like Taub's paper, we begin by assuming that  $C(z)$  has the following form:

$$C(z) = c_0(1 - c_1z)$$

$$\text{Then, } C(z)C(\beta z) = c_0(1 - c_1z)c_0(1 - c_1\beta z)$$

Recalling the expression for  $CC$  from the text we have

$$CC = \gamma r r + (1 - \gamma)\beta(\rho + r)(\rho + r)$$

Equating the coefficients for the two expressions for  $CC$  we get

$$c_1^2(1 + c_1^2\beta) = \gamma(1 + \rho^2\beta) + \beta(1 - \gamma)[(1 + \rho)^2 + \rho^2\beta]$$

and  $c_1^2 c_1 = [\gamma + \beta(1 - \gamma)(1 + \rho)]\rho$

Combining the above two equations we have

$$\begin{aligned} \frac{1}{c_1} + c_1 \beta &= \frac{\gamma + \beta(1 - \gamma)(1 + \rho)^2}{[\gamma + \beta(1 - \gamma)(1 + \rho)]\rho} + \frac{[\gamma + \beta(1 - \gamma)]\rho\beta}{\gamma + \beta(1 - \gamma)(1 + \rho)} \\ &= \frac{\gamma + \beta(1 - \gamma)(1 + \rho)}{[\gamma + \beta(1 - \gamma)(1 + \rho)]\rho} + \frac{[\gamma + \beta(1 - \gamma)]\rho\beta}{\gamma + \beta(1 - \gamma)(1 + \rho)} + \frac{\gamma(1 - \gamma)\beta\rho}{[\gamma + \beta(1 - \gamma)(1 + \rho)][\gamma + \beta(1 - \gamma)]} \end{aligned}$$

$$\text{Now, } \frac{D^2(\alpha\beta)}{C^2(\alpha\beta)} = [\gamma + \beta(1 - \gamma)(1 + \rho)]^2 \frac{\left[1 - \frac{\gamma + \beta(1 - \gamma)}{\gamma + \beta(1 - \gamma)(1 + \rho)} \rho\alpha\beta\right]}{c_1^2(1 - c_1\alpha\beta)^2}$$

Now this expression exactly resembles the expression for  $V(\rho)$  in the Appendix of Taub's paper. consequently we prove the proposition in exactly the same way and obtain the  $\rho = 0$ .

## Appendix C

Differentiating the expression in (3.5) with respect to  $\rho$  and setting the derivative equal to zero and using  $\beta$  symmetry, we have after some manipulations

$$-\oint \sigma_0^2 \frac{d}{d\rho} \left\{ \left[ \gamma B \frac{(1-\rho z_*)}{\beta(1+\rho-\rho z_*)} \right]_+ \left[ \gamma B \frac{(1-\rho z_*)}{\beta(1+\rho-\rho z_*)} \right]_{+*} \right\} \frac{dz}{z} = 0$$

Since by assumption  $B(z)$  is an analytic function it was taken out of the annihilator when it was not associated with another function of  $z$ . Differentiation now yields

$$\oint \left[ \frac{B}{(1+\rho-\rho z_*)^2} \right]_+ \left[ B \frac{(1-\rho z_*)}{(1+\rho-\rho z_*)} \right]_{+*} \frac{dz}{z} = 0$$

Let  $G_*$  indicate  $\frac{1}{1+\rho-\rho z_*}$ , then after some manipulations we get,

$$\oint [BG^*]_+ [B - \rho BG_*]_{+*} \cdot \frac{dz}{z} = 0$$

The above equation can be rewritten as

$$\oint B_*[BG_*^2]_+ \frac{dz}{z} - \oint \rho[BG_*^2]_+[BG_*]_* \frac{dz}{z} = 0 \quad (C.1)$$

To calculate the optimal value of  $\rho$  let us first calculate the integrals. Let us start with the first integral. The integrand expands in the following way

$$\sum_i B_*[BG_*^2 \beta^{2i} z^{-2i}]_+$$

$$\text{or, } \sum_i B_* G_*^2 \beta^{2i} [Bz^{-2i}]_+$$

The expression in the brackets annihilates to  $b^{2i}B(z)$  if  $B(z)$  is assumed to have an autoregressive representation  $B(z) = \frac{1}{1 - bz}$ . Substituting this the integral becomes  $G(\beta b)^2 \oint BB_* \frac{dz}{z}$

We can again use Cauchy's integral formula to calculate the integral and the above expression integrates to

$$G(\beta b)^2 B(\beta b) \quad (C.2)$$

Using the same techniques the second integral integrates to

$$G(\beta b)^3 B(\beta b) \quad (C.3)$$

We now substitute the values of the integrals from equations (C.2) and (C.3) to (C.1) to get

$$G(\beta b)^2 B(\beta b) = \rho G(\beta b)^3 B(\beta b) \text{ This simplifies to}$$

$$\rho = \frac{1}{\beta b} \quad (3.8)$$

## Appendix D

To derive the second Wiener-Hopf equation, the  $J(\dots)$  function is first differentiated with respect to  $\alpha_2$  and the derivative and the  $\alpha$ 's are equated to zero. This gives the following first order condition using  $\beta$  symmetry,

$$\frac{-2}{2\pi i} \frac{\beta}{1-\beta} \sigma^2 \oint \{ \gamma(A - \sigma + r\mu)(-r\eta_2) + G(A - \sigma + \mu)(-\eta_2) \} \frac{dz}{z} = 0$$

$$\text{or, } \frac{1}{2\pi i} \oint \{ \gamma(A - \sigma + r\mu)r + G(A - \sigma + \mu) \} (\eta_2) \frac{dz}{z} = 0$$

The following Wiener-Hopf equation is derived:

$$\gamma(A - \sigma + r\mu)r + G(A - \sigma + \mu) = \sum_{-\infty_2}^{-1}$$

This equation can be rearranged to give

$$(\gamma r + G)\sigma - (\gamma r r + G)\mu = (\gamma r + G)A - \sum_{-\infty_2}^{-1}$$

There are now two Wiener-Hopf equations to solve for the two variables. To solve,  $\sigma$  is determined from the first equation and substituted into the second to give

$$(\gamma r + G) \left[ (\gamma r + G)\mu + (\gamma + G)A - \sum_{-\infty_1}^{-1} \right] - (\gamma r r + G)\mu = (\gamma r + G)A - \sum_{-\infty_2}^{-1}$$

$$\text{or, } [(\gamma r + G)(\gamma r + G) - (\gamma r r + G)]\mu = [(\gamma r + G) - (\gamma r + G)(\gamma + G)]A + \sum_{-\infty_3}^{-1}$$

where,  $\sum_{-\infty_3}^{-1} = \sum_{-\infty_1}^{-1} - \sum_{-\infty_2}^{-1}$

The function multiplying  $\mu$  can be expressed as the product of two rational functions  $C(z)$  and  $C(z)$ . Then

$$CC\mu = [(\gamma r + G) - (\gamma r + G)(\gamma + G)]A + \sum_{-\infty_3}^{-1}$$

Multiplying both sides by  $C^{-1}$  and applying the annihilator I obtain,

$$\mu^* = C^{-1}[C^{-1}\{\delta(\gamma r + G)\}A]_+$$

## Appendix E

Substituting the optimal solutions in the utility function, I obtain

$$\begin{aligned}
 W = & -\frac{\beta}{1-\beta} \frac{1}{2\pi i} \oint \left[ \{ \gamma(A - (\sigma^* - r\mu^*))(A - (\sigma^* - r\mu^*)) \right. \\
 & + \delta\sigma^*\sigma^* + G(A - (\sigma^* - \mu^*))(A - (\sigma^* - \mu^*)) \} \sigma_i^2 \\
 & + \{ \gamma(\omega^* - r\pi^* + H - D)(\omega^* - r\pi^* + H - D) \} \\
 & \left. + \delta\omega^*\omega^* + G(\omega^* - \pi^* + H - D)(\omega^* - \pi^* + H - D) \} \sigma_i^2 \right] \frac{dz}{z}
 \end{aligned}$$

If the portion of the welfare function associated with the idiosyncratic shock is called W1 and that associated with the aggregate shock is called W2, they can be expressed as follows:

$$\begin{aligned}
 W1 = & -\frac{\beta}{1-\beta} \frac{1}{2\pi i} \oint \left[ \{ \gamma(A - (\sigma^* - r\mu^*))(A - (\sigma^* - r\mu^*)) \right. \\
 & \left. + \delta\sigma^*\sigma^* + G(A - (\sigma^* - \mu^*))(A - (\sigma^* - \mu^*)) \} \sigma_i^2 \right] \frac{dz}{z}
 \end{aligned}$$

This equation can be rearranged as

$$W1 = -\frac{\beta}{1-\beta} \frac{1}{2\pi i} \sigma_i^2 \oint \{ \gamma(A - (\sigma^* - r\mu^*))A + G(A - (\sigma^* - \mu^*))A \\ - [\gamma(A - (\sigma^* - r\mu^*)) - \delta\sigma^* + G(A - (\sigma^* - \mu^*))] \sigma^* \\ + [\gamma(A - (\sigma^* - r\mu^*))r + G(A - (\sigma^* - \mu^*))] \mu^* \} \frac{dz}{z}$$

Using the first order conditions (3.4) and (3.5) W1 can be simplified to:

$$W1 = -\frac{\beta}{1-\beta} \frac{1}{2\pi i} \sigma_i^2 \oint \{ \gamma(A - (\sigma^* - r\mu^*))A + G(A - (\sigma^* - \mu^*))A \\ - \sigma^* \sum_{-\infty_1}^{-1} + \mu^* \sum_{-\infty_2}^{-1} \} \frac{dz}{z}$$

Using Cauchy's theorem, the above equation further simplifies to

$$W1 = -\frac{\beta}{1-\beta} \frac{1}{2\pi i} \sigma_i^2 \oint \{ \gamma(A - (\sigma^* - r\mu^*)) + G(A - (\sigma^* - \mu^*)) \} A \cdot \frac{dz}{z}$$

If an autoregressive driving process with parameter  $\alpha$  is used the integral can be evaluated by substituting the optimal solutions. This is given by equation (4.1) in the paper. W2 can be simplified in a similar way.

$$W2 = -\frac{\beta}{1-\beta} \frac{1}{2\pi i} \sigma_i^2 \oint \{ \gamma(\omega^* - r\pi^* - D + H)(\omega^* - r\pi^* D + H) \\ + \delta\omega^* \omega^* + G(\omega^* - \pi^* - D + H)(\omega^* - \pi^* - D + H) \} \frac{dz}{z}$$

Expanding in away similar to the first part of the problem and using the Weiner-Hopf equations (3.8) and (3.9) and the equilibrium condition (2.7) I have,

$$W2 = -\frac{\beta}{1-\beta} \frac{1}{2\pi i} \sigma_i^2 \oint \{ \gamma(\omega^* - D) + G(\omega^* - \pi^* - D + H) \} (H - D) \cdot \frac{dz}{z}$$

Using (3.8) again the above equation further simplifies to

$$W2 = -\frac{\beta}{1-\beta} \frac{1}{2\pi i} \sigma_i^2 \oint ( -\delta\omega^* ) (H - D) \cdot \frac{dz}{z}$$

If an autoregressive driving process is assumed again, substituting the optimal values from equations (3.10) and (3.11) the above integral can be evaluated using Cauchy's theorem.

## Appendix F

The relationship between  $c_1$  and  $\rho$  is given by equation (4.3) in the text. It gives

$$\frac{1}{c_1} + c_1\beta = \frac{1}{\frac{\gamma}{\gamma+G}\rho} + \frac{\gamma\beta\rho}{\gamma+G} + \frac{G}{\delta(\gamma+G)}\beta\rho$$

This equation can be expressed as

$$f(c_1) = f\left(\frac{\gamma\rho}{\gamma+G}\right) + \frac{G}{\delta(\gamma+G)}\beta\rho$$

Since  $\gamma/(\gamma+G)$  is less than 1,

$$\frac{\gamma}{\gamma+G}\rho < \rho$$

Using this relationship and equation (4.3) the following Lemmas of Taub (1989) (expressed in terms of the notations of this model) are directly applicable for the smaller root of  $c_1$

**Lemma**

i)  $c_1 < \frac{\gamma}{\gamma+G}\rho$

ii)  $\lim_{\rho \rightarrow 0} c_1 = 0$

iii)  $\lim_{\rho \rightarrow 0} \left(\frac{\gamma}{\gamma+G}\right)^{-1} c_1 = 1$

For a proof of the Lemma please see Taub (1989). Application of i) of the Lemma proves that

$$\overline{W}1 = \frac{c_1 \left[ 1 - \frac{\gamma}{\gamma + G} \rho \alpha \beta \right]^2}{\frac{\gamma}{\gamma + G} \rho [1 - c_1 \alpha \beta]^2}$$

is between 0 and 1 for non-negative values of  $\rho$ . To find the optimal solution for  $\rho$ ,  $\overline{W}1$  is differentiated with respect to  $\rho$ . It yields the following first order condition:

$$\left( \frac{1 + c_1 \alpha \beta}{1 - c_1 \alpha \beta} \right) \frac{dc_1}{c_1} = \left( \frac{\gamma + G + \gamma \alpha \beta \rho}{\gamma + G - \gamma \alpha \beta \rho} \right) \frac{d\rho}{\rho}$$

This is a differential equation that has the following solution

$$\frac{\rho(1 - c_1 \alpha \beta)}{c_1(\gamma + G - \gamma \alpha \beta \rho)} = K$$

where K is the constant of integration. Evaluating at  $\rho$  tending to zero and applying ii) and iii) yields

$$K = \frac{1}{\gamma}$$

Substitution of this value of K results in the following relationship between  $\rho$  and  $c_1$

$$c_1 = \frac{\gamma}{\gamma + G} \rho$$

But equation (4.3) gives the general relationship between  $\rho$  and  $c_1$ . Substituting the value of  $c_1$  in terms of  $\rho$  from the one derived from the first order condition in equation (4.3) gives an equation expressing  $\rho$  in terms of the parameters. This equation is solved to yield

$$\rho = 0$$

This proves that the first order condition is satisfied for  $\rho = 0$ . Instead of differentiating again to show that the second order condition is satisfied for this value of  $\rho$ , application of ii) and iii) of the Lemma directly implies that

$$\lim_{\rho \rightarrow 0} \overline{W}1 = 1$$

Since  $\bar{W}1$  always lies between 0 and 1, the limit proves that for  $\rho = 0$   $\bar{W}1$  reaches a maximum. Since the first order condition is satisfied for only  $\rho = 0$  it is the unique solution. This completes the proof.

## Vita

Nivedita Mukherji was born in New Delhi, India, on September 8, 1963. She received her Bachelor's degree in Economics from Calcutta University, India, in 1986. She joined Virginia Polytechnic Institute and State University, USA, in 1987 and received her Master's degree in Economics in 1989 and Ph.D. in Economics in 1992.

A handwritten signature in black ink that reads "Nivedita Mukherji". The signature is written in a cursive style with a large, sweeping initial 'N'.