ESSAYS ON TRADE BARRIERS IN IMPERFECTLY COMPETITIVE MARKETS

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(ABSTRACT)

This dissertation contains three essays in strategic trade theory. They focus on the effects of trade barriers on the social welfare of an individual country through the effects of these barriers on the behavior of firms. Our analyses are undertaken in models with imperfectly competitive market structures. The assumption of imperfectly competitive market structures leads to certain conclusions that differ from those in the existing literature, where perfect competition is assumed.

The first essay is a survey sampling recent papers dealing with the topics mentioned above. Specifically, we choose papers on trade barriers in imperfectly competitive markets, according to the types of models—static vs dynamic—and to the types of trade barriers—quotas, tariffs, voluntary export restraints, etc. We consider the case of almost every combination of the previous classifications. From this survey we find that once the assumption of perfect competition is removed, the outcome for a particular trade barrier depends critically on details of the model used.

In the second essay, simple oligopolistic models are used to examine the welfare effects
arising from a quota. The trade-off faced by a policymaker (concerned only with the welfare of his own country) when there is 'competition' between foreign and domestic firms for a domestic market, is highlighted. Moreover, the impact of differing numbers of and cost differences between domestic and foreign firms is investigated.

The third essay considers a repeated game with three players, two quantity-setting firms and one quota-setting government. In this model the differing effects of quotas and VER's on the actions of firms are explored. The focus is on how a quota (or the threat of a quota) can be used not only to break up collusive behavior but also to prevent the firms from colluding in the first place. Further, various ways in which quotas can be used by the government to actually improve on the static Cournot-Nash equilibrium outcome are examined. This last result is somewhat surprising since typically in quantity-setting trade models the static Cournot-Nash equilibrium (though second best) is the benchmark used to judge various outcomes (i.e. the closer to the C-N equilibrium, the better the outcome). Indeed, quotas are usually considered clumsy instruments, inferior to some other industrial policy. However, careful use of the threat potential inherent in quotas is shown to enforce outcomes which approach the competitive solution.
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Chapter 1

Trade Barriers in Imperfectly Competitive Markets:
A Selective Survey

1. **Introduction and Scope**

In this chapter we survey certain papers on quantity controls in international trade. By quantity controls we mean quotas imposed by a government and voluntary export restraints as a result of interaction among firms, etc.

This topic may lead some economists to criticize its relevance since classical trade theory tells us that a tariff is superior to an equivalent quantity control. One response to the criticism is based on theoretical concerns. In imperfectly competitive markets, the superiority of tariffs over quotas is no longer certain, for example, when firms in a particular industry are colluding, it is possible for quotas to be better. Another response is based on the empirical observation: The use of tariffs has declined steadily since the General Agreement on Tariffs and Trade (GATT) was instituted and has reduced the overall level of tariffs. In response to the reduction in tariffs, the use of non-tariff trade barriers has increased, particularly in recent years. While the debate on the theoretical relationship between tariffs and quotas still rages, it is evident that quantity controls are a major part of the world economy and therefore deserving of study.

More precisely, since 1940 the industrial countries' average tariff on manufactured goods has declined from 40% to approximately 5% in 1990 (The Economist, September 22, 1990). Since the mid-1970's, however, the use of non-tariff barriers has grown dramatically. Although the nature of non-tariff trade barriers makes their effect on world trade difficult to determine, some estimates suggest that world trade might increase by $330 billion if non-tariff barriers were
removed. Considering the fact that quantity controls constitute a large portion of all non-tariff trade barriers, the above empirical evidence shows the importance of the investigation of the effects of quantity controls.

Recently, much work has been done investigating the effects of trade barriers including tariffs. The focus here is on the strategic interaction between firms and its impact on the effects of trade barriers and on social welfare in imperfectly competitive markets. Imperfect competition causes firms to behave strategically and changes the effects of trade barriers we commonly expect. Another feature of imperfectly competitive markets is that firms make positive profits. These facts have to be taken into account when we determine the effects of trade barriers on social welfare. The trade models considered here differ from the traditional body of trade theory in that the assumption of perfect competition is dropped and partial equilibrium models are used. This new direction in trade theory came about in response to the unsatisfactory explanation, in the existing body of trade theory, of the persistence of trade barriers in the world economy.

The non-traditional approach relies on game theory and is subdivided into two main parts: static analysis and repeated game analysis. Several papers examine the outcomes occurring in imperfectly competitive markets with trade barriers, for example, Dixit (1984), Helpman and Krugman (1985), McMillan (1986), Helpman and Krugman (1989) (they also provide further references). Yet, the literature lacks a detailed study of the welfare effects of quotas with respect to different numbers of foreign and domestic firms.

Now consider static versus repeated game analysis. Although models for trade barriers in imperfectly competitive markets have become more common in recent years, most of them are static game models. Dixit (1983), Eaton and Grossman (1986), and Krishna (1987) represent a sample of the papers using the static game approach. This static approach prevents us from capturing the dynamic nature of firm interaction. Some work has been done utilizing a repeated
game framework, however. For example, Fung (1987) considers the effects of tariffs on an international duopoly of domestic and foreign firms with product differentiation. Davidson, (1984), considers the effect of tariff policy in an industry with N firms producing a homogeneous good. Both authors determine the effect of tariffs on firms' ability to engage in collusive behavior. Rotemberg and Saloner (1986), examine the effect of quotas on firms' ability to collude in a duopolistic industry producing an identical good.

By analyzing quotas and VER's within the context of a repeated game, economists can consider how these trade barriers affect collusive behavior among firms. With no binding quota in place, firms can mutually enforce collusion by threatening to increase output to more competitive levels if any firm deviates. This increased output would lead to lower profits for both firms. A VER could be an announcement of the foreign firm's collusive output level. By contrast a sufficiently restrictive quota takes away the foreign firms' ability to punish a deviation by the domestic firm, thereby breaking up collusion and leading to higher total output. Under a small quota, foreign firms' threat to expand output isn't sufficiently harsh to deter domestic firms' cheating behavior. The question of how trade barriers affect collusive behavior among firms cannot be addressed using static models, while repeated game models provide a natural framework for considering this question.

There are at least three important issues and ramifications left to future research. First of all, the domestic firms may also export, and then the possibility of retaliation and mutual trade restrictions arises. Second, export opportunities to third countries can matter. Finally, a synthesis of various trade interventions, like quotas, tariffs, cost-reducing subsidies and direct export subsidies should be attempted. Exploratory steps in this direction are taken in Dixit (1984), Brander and Spencer (1984) and Haller and Milam (1991).
2. A Sampling of the Current Literature

Following is a more detailed examination of selected papers:

Brander and Spencer (1981) explore the effects of tariffs using a quantity-setting Stackelberg model of entry deterrence. First they consider the case of a foreign monopolist supplying the domestic market when there is no threat of entry. They find that a tariff can be used to shift surplus (i.e., tariff revenues) from the foreign monopolist to the domestic government. The cost of implementing the tariff is the loss in consumer surplus resulting from the higher price and lower output that the tariff induces; however, in many cases this cost is outweighed by the surplus-shifting effect.

Second, Brander and Spencer consider the effect of a tariff when potential entry into the domestic market by a domestic firm is allowed; they find that the loss in consumer surplus can be eliminated by the threat of entry. This occurs when the foreign firm is practicing the limit output strategy for deterring the potential domestic entrant. The optimum tariff (given no loss in consumer surplus) is the highest tariff such that the profit from exports is positive and the profit from entry deterrence is greater than the profit the foreign firm would receive if it allowed entry. If the second constraint is not binding (i.e., the profit from entry deterrence is strictly greater than the profit when entry occurs), then it is possible to extract all the surplus from the foreign firm with no loss in consumer surplus. If however the second constraint is binding then the domestic country can extract only a portion of the foreign firm’s surplus without reducing consumer surplus. This is because once the second constraint is violated, it will no longer be in the foreign firm’s interest to deter entry by producing a high output level. Therefore the foreign firm will reduce its output and the output produced by the new entrant will only partially offset this reduction.

The separate but related question, whether the government should set the tariff high
enough to induce the incumbent to allow entry, has a less straightforward answer. Whether a small increase in the tariff from slightly below to slightly above the entry-inducing level is welfare increasing depends on the relationship between the domestic potential entrant's variable and fixed costs and the foreign incumbent's variable costs. If the entrant's fixed plus variable costs of producing its equilibrium output under the higher, entry-inducing tariff is less than the variable cost (including transport costs) of the foreign firm producing that same level of output, then the higher tariff is welfare-increasing. If the inequality is reversed, then the lower tariff at which the incumbent deters entry is preferable. In the second case, it is still possible that a large increase in the tariff will be welfare-increasing if the gain in tariff revenue plus domestic firm profits outweighs the loss in consumer surplus.

When the third possibility is allowed, entry into both markets, the results change somewhat. In this case, entry deterrence is more difficult since the potential entrant can earn profits in two markets rather than one. The incumbent has to determine an entry-deterring level of output in both its own market and the domestic market. One effect of this is that any tariff, no matter how small, will cause a reduction in consumer surplus since the incumbent can reduce output in the domestic market while increasing output in its own market and thereby deter entry. Even so, for any tariff, the domestic country is better off when entry into both markets is allowed rather than entry into only one market. This is due to the fact that the entry-deterring output level needed in the home market will be greater when both markets are open to entry rather than just the domestic market.

Dixit and Kyle (1985) consider the effect of trade restrictions on entry promotion and deterrence. They do so in a relatively simple, two-country, two-firm, partial-equilibrium model. The objective of each country (or government) is to maximize the sum of domestic producer and consumer surplus. The basic setup has one incumbent firm located in one country and one
potential entrant located in the other country. The policy tool available to the governments is a choice between free trade and autarky. The firms are assumed to play a basic duopoly game which is not explicitly modeled. The game considered explicitly is a three player game since the incumbent firm is treated as a passive profit maximizer. They find that under certain conditions it may be optimal for a government to protect its market in order to promote entry by the domestic firm or, in the case of the country with an incumbent firm, to prevent (or protect their firm's domestic monopoly in response to) entry of the foreign firm into the market. These results depend on several factors, such as the order in which the players move, the size of the sunk costs of entry, and whether or not the firms play a Bertrand or Cournot duopoly game.

In their 1986 paper, Eaton and Grossman consider the effect of taxes and subsidies on trade in an oligopolistic market. First, they consider a general conjectural variations model of duopoly with one home and one foreign firm that compete outside the home market. In addition the home country government acts as a Stackelberg leader when it chooses the optimal policy (which is to maximize home country welfare). In this case, since all output is exported, there is no difference between an output tax (subsidy) and an export tax (subsidy). Here they find that when the goods are substitutes the optimal policy depends on whether the foreign firm's actual response is greater than or less than the conjectured response. If the actual change in the foreign firm's strategic variable is greater than (less than) the conjectured change, then taxing (subsidizing) exports is optimal. In the special case of the Cournot model, a subsidy is optimal whereas a tax is optimal in the case of the Bertrand model. The intuition is that if the domestic firm overestimates the response of the foreign firm, it will not be aggressive enough, while if it underestimates the response of the foreign firm it will be too aggressive. If the firms have consistent conjectures, then a zero tax or subsidy is optimal.

Next, they consider the case where the foreign government pursues an active trade policy
as well. They model it as a two-stage game where the two governments simultaneously choose policies (tax or subsidy), then the firms compete duopolistically in a third country. This doesn't affect the direction of the optimal policy, just its magnitude. So in Cournot competition, for example, the Nash equilibrium in policies would consist of both countries subsidizing exports. The resulting Nash equilibrium in outputs has greater total output and lower joint profit than would have occurred without subsidies. Thus, the two countries end up with lower welfare than would have occurred without subsidies. By contrast, in the case of Bertrand competition, both countries end up taxing exports and therefore reducing the total output produced. This moves the firms closer to the joint profit maximum, thereby increasing the welfare of both countries. Again, if both firms have consistent conjectures, the Nash equilibrium in policies consists of a zero tax or subsidy.

Eaton and Grossman then consider the same policy question when there is an arbitrary number of symmetric foreign and domestic firms with consistent conjectures and no home consumption. They assume conjectures are consistent to eliminate the profit-shifting motive illustrated in the duopoly model. This allows attention to be shifted to the terms-of-trade motive that occurs in the more general oligopolistic model. They find that with any number of foreign firms and only one domestic firm, the optimal tax is zero; and if the number of domestic firms is greater than one, the optimal tax is positive. This is due to the fact that if the number of domestic firms is greater than one, then a domestic firm increasing its output will cause the profits of all the other domestic firms (as well as foreign ones) to decrease. Therefore, a positive tax at the right level would induce each domestic firm to internalize the externality it imposes on all other domestic firms and to choose an output level such that total domestic profit will be maximized. If conjectures are not consistent, then the profit-shifting and terms of trade motives will be present simultaneously. When firms are not aggressive enough (as in the Cournot model),
the two motives will work against each other and when firms are too aggressive (as in the Bertrand model), the two motives will reinforce each other.

Next Eaton and Grossman examine, in a less formal manner, the effect of these trade policies when entry and exit are endogenously determined. They state that the profit-shifting motive remains as long as at least some of the domestic firms earn positive profits, which will occur if firms are heterogeneous or if fixed entry costs are large.

Finally, they consider the effect of these policies when there is domestic consumption. They return to the duopoly model (with the firms competing in outputs) in order to eliminate the terms of trade motive for taxes and subsidies. In addition, they concentrate on the case of consistent conjectures, which eliminates the profit-shifting motive, in order to solely focus their attention on the impact of taxes or subsidies with respect to domestic consumption. To simplify things even more, they consider only the case of a homogeneous good. As pointed out in their paper, they now have to differentiate between export tax/subsidies and production tax/subsidies (previously, since all output was exported, there was no difference between the two). Regarding a production tax or subsidy, if both firms have constant marginal costs then a zero tax or subsidy is optimal. If, however, the foreign firm has increasing marginal costs then a subsidy is optimal. Concerning a trade tax or subsidy, they are unable to unambiguously determine which is optimal.

Brander and Spencer (1984) examine an importing nation's incentives for the use of tariffs or subsidies and the relationship with demand and cost. Further they examine the benefits of cartelization by the exporting country.

With specific tariffs or subsidies, the relative curvature of demand, measured by \( X \frac{P''}{P} \), determines whether a tariff or subsidy is welfare improving. With constant marginal cost a tariff (subsidy) is preferable if \( X \frac{P''}{P} > -1 \) (\( X \frac{P''}{P} < 1 \)). In words, a subsidy is optimal if demand is sufficiently convex, otherwise a tariff is optimal. They give an example where a subsidy is
optimal by using a constant elasticity demand curve. With ad valorem tariffs or subsidies the change in elasticity along the demand curve is used to determine whether a tariff or subsidy is welfare improving. If demand elasticity is decreasing (increasing) then a tariff (subsidy) improves welfare. These results hold in general, both for ad valorem and specific tariffs or subsidies, irrespective of whether there is a foreign cartel or an oligopolistic foreign industry, and regardless of the number of firms.

When subsidies are used both countries gain; but subsidies only improve domestic welfare for strongly convex demand, which is unlikely. Tariffs decrease the welfare of the exporting country and impose a deadweight loss as well.

From the perspective of the exporting country the strategic variable is the degree of cartelization. This is measured by the number of independent noncooperative decision-making units; the smaller this number the higher the degree of cartelization. When the exporting country consumes none of the exported good, then what’s best for the industry is best for the exporting country as a whole. In this case the best value for the strategic variable is one, or a monopoly cartel, since this maximizes total industry profits. This is true regardless of the level of tariff/subsidy imposed by the importing country. When the exporting country consumes some of the good that it exports the case for cartelization is not so clear. The more important domestic consumption of the good is relative to total exports, the lower the incentive for cartelization and vice versa. With no export market, $n \to \infty$ and (as stated earlier) with no domestic consumption, $n=1$.

The typical noncooperative equilibrium in this game will consist of a positive tariff and some degree of cartelization. In the simple case with no home consumption of the exported good, a monopoly cartel is best for the exporting country. They emphasize that although this outcome is inefficient in terms of total welfare, it is still preferable to the exporting country than the first-
best outcome because with constant marginal cost the first-best outcome puts all surpluses in the hands of the importing country. They use this observation to support the multilateral approach to trade liberalization in the sense of a country making concessions in some industries in exchange for advantages in others. This is due to the fact that in this model the exporting nation has no incentive for attempting to achieve the first best outcome in its export industry.

Krishna (1989) considers the effects of quotas and VERs in a static duopoly model with Bertrand competition. She does this for both substitute and complementary goods, however, the attention here will be focused on the case of substitute goods since that is where the most interesting results occur. She demonstrates that when the quota or VER (she doesn’t distinguish analytically between the two) is set at or near the free trade level, there will no longer be an equilibrium in pure strategies. Instead, the foreign firm will use a pure strategy in the new equilibrium and the domestic firm will use a mixed strategy. The effects of the VER or quota in this case are to raise (expected) prices and profits for both firms in equilibrium, with the domestic firm attaining the same level of profits it would as a Stackelberg leader. Domestic output may rise or fall but overall output decreases.

Krishna also compares tariffs to quotas which restrict imports to the same level as would occur under the tariff. She finds that prices under a tariff are lower than under a quota, profits to the domestic firm are lower, and that the foreign firm would prefer a quota even if it received the entire tariff revenue as a lump-sum transfer.

Harris (1985) analyses VERs in a model very similar to the one used by Krishna with one distinction. In the presence of a VER he treats the domestic firm as a Stackelberg leader because the VER 'imposes a form of price leadership' on the foreign firm. His claim is that the additional constraint imposed by the VER (which is known by both firms) will induce the domestic firm to act as a Stackelberg leader. However, his results are similar to those of Krishna in that prices and
profits increase for both the foreign and domestic firms.

Mai and Hwang (1988) extend the work of Harris by using a conjectural variations model where firms compete in quantities. They find:

First, at the Cournot equilibrium firms are indifferent to a VER set at the free trade level. Second, at a more collusive equilibrium (i.e. positive conjectures) profits to the home firm increase while profits to the foreign firm decrease. Third, at an equilibrium that is less collusive than the Cournot equilibrium (such as Bertrand competition) both firms benefit from a VER set at the free trade level.

Cooper and Riezman (1989) consider the effects of uncertainty on governments' choice of trade policy. They do this within the framework of two countries exporting to a third, where the governments in the two exporting countries must choose between a policy of quantity controls or a tax/subsidy on exports. 'Quantity controls' as used in this paper must be distinguished from quotas or voluntary export restraints. The latter two would only set an upper bound on the amount of exports while quantity controls set the actual amount of exports. The two exporting countries export to a third so as to simplify the policy-maker's problem by making consumer surplus irrelevant.

Their model is a sequential game that operates as follows: First, the governments choose simultaneously which policy to use (either quantity controls or a tax/subsidy), then they simultaneously determine the level at which to set the policy tool they chose previously (if they picked quantity controls they choose the quantity, if they picked the tax/subsidy policy they choose the amount of the tax/subsidy). Next, the random term in the inverse demand curve ($\theta$) is realized and firms maximize profits within the constraints placed on them by the governments' policy choices and the random realization of demand.

What Cooper and Riezman find is that when the variance of theta (VAR $\theta$) is small,
quantity controls are superior to tax/subsidies because when country 1 (2) imposes quantity controls on its firms it prevents the firms from responding to the policy choice of country 2 (1), thereby reducing the strategic power of country 2's (1's) policy. On the other hand, if a tax/subsidy was used by country 1 (2), country 2 (1) could use its policy choice to influence the output of country 1's (2's) firms. Therefore in equilibrium both countries will use quantity controls.

When VAR $\theta$ is large, tax/subsidies are superior to quantity controls because the benefit of allowing firms to respond to the uncertain realization of demand outweighs the cost imposed by giving up the commitment value of quantity controls and making the firms vulnerable to the influence of the other countries policy. The equilibrium in this case will consist of both countries using a tax/subsidy policy. Whether a tax or a subsidy will be used depends on the number of firms in a country relative to the total number of firms in both exporting countries. A country with a large number of firms will tax exports while a country with few firms will subsidize. This is due to the fact that decreasing output in a country with a large number of firms (which is the result of a tax) will have a large price effect thereby increasing profits while increasing output in a country with few firms (as a result of a subsidy) will have a small price effect and therefore the increased output will increase profits.

For intermediate values of VAR $\theta$ (and under the additional assumption that the number of firms in each country are equal) the situation is more complex. Consider the problem from the perspective of country 1; basically, it faces a tradeoff between the benefit of using a quantity control for the most strategic advantage vs. the benefit of using a tax/subsidy to allow firms the most flexibility to respond to random fluctuations in demand. For high-variance demand the tax/subsidy policy is superior whereas quantity controls are better when the variance in the demand curve is low. At some 'intermediate' variance the government in country 1 would be
indifferent between using a quantity control or a tax/subsidy. What complicates matters is that this break-even point depends on whether country 2 uses a quantity control or a tax/subsidy. Let $V^*(s) = \text{the value for VAR } \theta \text{ such that country 1 (2) is indifferent between a tax/subsidy or a quantity control when country 2 (1) uses a tax/subsidy and } V^*(q) = \text{the value for VAR } \theta \text{ such that country 1 (2) is indifferent between a tax/subsidy or a quantity control when country 2 (1) uses a quantity control. Suppose } V^*(s) > \text{VAR } \theta > V^*(q), \text{ in this case there are asymmetric equilibria. If country 1 (2) uses a tax/subsidy, then country 2 (1) uses a quantity control and if country 1 (2) uses a quantity control, then country 2 (1) uses a tax/subsidy. Suppose now that the inequalities are reversed, } V^*(s) < \text{VAR } \theta < V^*(q), \text{ in this case there are symmetric equilibria. If country 1 (2) uses a tax/subsidy, then country 2 (1) uses a tax/subsidy.}

Syropoulos (1990) analyzes the case where there are only foreign firms producing for the domestic market and no third markets in a repeated game model. For most of the discussion, firms are treated as quantity setters. When analyzing the case of quantitative restrictions, this model is analogous to considering the effect on implicit collusion of varying levels of capacity for the firms supplying the market with, in addition, all firms restricted to the same capacity (where capacity refers to the amount allowed by the quota).

Consider first the case where firms could enforce the monopoly output level at the collusive equilibrium. (It is assumed here and throughout Syropoulos' paper that the firms use 'grim' trigger strategies where the punishment phase involves each firm producing its Cournot-Nash static equilibrium output level in each period after the deviation.) There are three important per-period output levels for the firm which need to be considered here. The first and largest is the output level the firm will produce if it decides to cheat on the optimal collusive agreement. Next is the single-period Cournot-Nash equilibrium output level. Third and smallest is the firm’s share of the collusive output level (here assumed to be $\frac{1}{n} \cdot x^M$ where $x^M$ is the
monopoly output level). Let $x^{CN}$ represent the firm's Cournot-Nash output level and $x^{CH}$ represent the firm's output level when it cheats on the collusive agreement. As the quantitative restriction becomes smaller, it will first affect the firm's ability to cheat since $x^{CH} > x^{CN} > \frac{1}{n} \cdot x^M$, so for $\bar{x}$ between $x^{CH}$ and $x^{CN}$ (where $\frac{1}{n}$ is the quantitative restriction placed on an individual firm when the overall quota is $\bar{x}$) the incentive to cheat will be decreased. This is because the deviating firm would be constrained to produce (or at least sell) less than it would under a less restrictive quota (i.e. $\bar{x} \geq x^{CH}$) and therefore the additional profit gained from cheating would be diminished. For $\bar{x} \in (x^{CN}, x^{CH})$, however, both the profits gained by cheating and the profits lost by the imposition of the punishment will be decreased. The net effect is that the firms have less incentive to collude.

Rotemberg and Saloner (1989) consider the effects of quotas on the ability of one foreign and one domestic firm to collude in a repeated game. They do this for both price-setting and quantity-setting firms. In addition they consider the effects of domestic quotas on competition in foreign markets for two different situations; first when the firms face capacity constraints and second when the firms face fluctuating demand in each market.

They find that when the firms use 'maximal' punishments (i.e. the deviating firm's profits are minimized with respect to the punishing firm's strategic variable), quotas are actually superior to tariffs since regardless of the size of the tariff, the punishing firm can always force the deviating firm to zero profits if it is willing to incur losses to do so. In contrast, when a quota is in place, the most the punishing firm can do is to produce the entire amount of its quota which implies, for sufficiently restrictive quotas, that the home firm can still make positive profits even when receiving the maximal punishment from the foreign firm.

However when the more credible punishment of reverting to the single shot Nash equilibrium is employed, the effects of a quota are similar to the effects of tariffs as described by
Davidson (1984). Initially, the quota will, if anything strengthen the collusive agreement by reducing the foreign firm's ability to cheat. As the quota becomes more restrictive (i.e. less than the firm's one-shot equilibrium output level) the quota weakens collusion since it reduces the punishment the foreign firm is able to inflict on the domestic firm.

Next Rotemberg and Saloner consider the effect of a domestic quota on competition in the foreign market. They find that both when the firms are capacity constrained or when the two markets (foreign and domestic) have perfectly negatively correlated fluctuating demand, that competition in the foreign market may be enhanced by the imposition of a quota in the domestic market.

Carl Davidson (1984) looks at the effects of tariffs in a repeated oligopoly game with N quantity-setting firms. In particular he considers the impact of the tariff on ability of all firms in the industry (both foreign and domestic) to engage in a collusive agreement. Further, after he develops the relationship between tariffs and the incentive for firms to engage in collusion, he shows how this relationship is affected by N, the number of firms in the industry and \( \frac{N_d}{N} \), the proportion of domestic firms in the industry.

What Davidson finds is that small tariffs make it easier for firms to collude (with respect to free trade) while large tariffs make it more difficult. In addition he finds that there is some tariff, \( t^* \), such that the incentive to collude is the same as under free trade. When he investigates the relationship between \( t^* \), N and \( \frac{N_d}{N} \); he finds that \( t^* \) is decreasing in both N and \( \frac{N_d}{N} \). This last result has the implication (given the continuity of \( t^* \) with respect to N and \( \frac{N_d}{N} \)) that the size of the tariff needed to provide some deterrent to collusion decreases as N and \( \frac{N_d}{N} \) increase.

Fung (1987) uses a general quantity-setting duopoly model to investigate the effects of tariffs in a repeated (as well as static) game setting. This general setting is the main factor which differentiates it from some of the other work in the literature. Not only does he consider non-
linear demand functions but asymmetric cost functions and differentiated products as well.

His results in the static game case basically parallel those of Brander and Spencer (1982), i.e., the tariff shifts rents earned by the foreign firm to the home country in the form of tariff revenues accruing to the government and increased rents to the home firm. These effects result from increased demand for the home firm's output in response to a higher price and lower output of the foreign firm.

In the repeated-game setting he examines the ability of a tariff to break up tacit collusion between the foreign and domestic firm. His results are similar in some respects to what is seen in the analysis of simpler linear, symmetric cost models. He finds that (at least in the relevant range) increasing the tariff makes it more difficult for the firms to sustain a collusive agreement and for a sufficiently large tariff the collusive agreement breaks down and the firms revert to the static Cournot-Nash equilibrium. In the paper this is referred to as the antitrust effect.

By introducing a more general framework, some ambiguities are introduced that are not present in the simpler linear model. For example, though it's still the case that the foreign firm's output will increase when the collusive agreement breaks down, under some conditions domestic output will fall. However it's still the case that total output will increase (ignoring the problems associated with adding two differentiated products). Finally, the welfare effects of a tariff when it causes the breakdown of the collusive agreement are ambiguous.

3. Conclusion

Another approach (taken in Chapter 3) to the analysis of quotas within the framework of a repeated game is to treat the government as an active player in the game, which allows the government to develop its own trigger strategies. This opens the door to some interesting trigger strategies by the government such as: the threat of a quota to prevent collusion from occurring, a
zero quota for the foreign firm (or possibly firms) in exchange for a high output level by the domestic firm (or firms) with the threat of opening the market if the domestic output level falls too low, or using the threat of being excluded from the market to induce the foreign firm (or firms) to produce at an output level so high as to give it very low profits (how low depending on the size of the discount factor). Of these three possibilities, the last two would yield (for a discount factor sufficiently large) total output levels higher than would occur at the free trade, non-collusive Cournot-Nash equilibrium. These effects, resulting from treating the government as an active player in the game, differ from most of the literature dealing with quantity-setting firms, since typically the Cournot-Nash equilibrium output level is the benchmark to which other outcomes are compared.

This illustrates the possibilities of trigger strategies which are both a blessing and a curse of the repeated game framework. A blessing because it opens an interesting way of studying the interaction of threats and collusion that can go on between firms in an oligopolistic industry. A curse because the repeated game framework is almost too flexible since many different outcomes can be supported with trigger strategies which makes it difficult for these models to demonstrate the predictive power that economists desire from their theory.

In trying to gain insight into the rapidly growing body of literature which deals with the impacts of international trade policy in imperfectly competitive markets, I attempted to organize this overview according to the game structure used in setting up the various models. Differing game structures lead to different strategic effects for the various policy tools considered in the trade literature. The nature and extent of the effects depend on whether a static or dynamic game framework is used, and within these two broad frameworks, on whether firms set prices or quantities. For example, in some games quotas are seen as promoting collusion while in other models quotas can be used to deter or break up collusive behavior. Therefore, specifying the
particular game structure is crucial.

However these results do not imply that game theory provides useless or contradictory insights into firm behavior, instead it means the precise nature of the strategic interaction between firms in a particular industry (as well as their cost structure) is critically important for a more complete understanding of the effects of various trade policies. Unfortunately, this information is very hard to come by, which brings out one of the major weaknesses of the new trade theory. Even though particular policy tools such as tariffs or quotas, thought to be unequivocally bad under the competitive paradigm, can be beneficial in some cases, it may also be true that the information needed to determine whether a particular policy tool is called for, and precisely how and to what extent it should be used, is not available to policymakers. Seen in this light, the problem becomes one of first: Is it possible to gather enough information to determine the strategic interaction between firms? Second: If it is feasible, what is the best method to do so? The second question leads one to consider two main possibilities: The first is to gather enough basic information to be able to infer from this data what the nature of the relationship between firms consists of; and the second is to use data on pricing and output decisions by firms and how these variables are affected by different policy tools used by the government, which would allow us to determine which game model best describes the data for a particular industry. Attempting to apply the new trade theory in the absence of this detailed information may lead to very undesirable results, therefore, as suggested by Krugman and Helpman (1990), it is probably best to refrain from the use of trade barriers of any sort while our ignorance of the necessary information persists.
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Chapter 2

WELFARE EFFECTS OF QUOTAS VERSUS EXCESSIVE FOREIGN COMPETITION IN AN OLIGOPOLISTIC MARKET

1. Introduction

In a perfectly competitive industry, the domestic social surplus is maximized by the market mechanism regardless of whether the firms are domestic or foreign. Any type of trade barriers reduce the domestic social surplus (there are some minor exceptions such as the infant industry case, etc.). In an imperfectly competitive industry, however, this is not necessarily true. A difference, crucial here, between perfectly and imperfectly competitive industries is that firms necessarily receive zero profits in the former and may make positive profits in the latter. If a foreign firm makes positive profit, it takes that profit to a foreign country, i.e., it reduces the domestic social surplus. Nevertheless, a foreign firm may make a contribution which increases the domestic social surplus by lowering the market price. The presence of a foreign firm has two opposing effects. Thus, in an imperfectly competitive industry, it need not be the case that trade barriers reduce the domestic social surplus.

In this paper, we consider the effect on domestic social surplus of quotas and tariffs applied to an imperfectly competitive industry. We show that the introduction of quotas almost always affects the domestic social surplus but may increase or decrease it depending upon the configuration of domestic and foreign firms.

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1 This paper was written jointly with Professor Hans Haller.
In many concentrated industries, domestic firms face significant foreign competition in the domestic market, with both domestic and foreign firms exercising market power. Dissipation of market power has been traditionally considered a matter of domestic regulatory policy directed at a domestic monopolist (trust) or a domestic oligopoly (cartel). With both domestic and foreign firms in the very same domestic oligopolistic market, issues of industrial organization and trade are inextricably intertwined. Under imperfect competition, all firms -- irrespective of their residence -- may make positive profits. Tax policy and price regulation would be the conventional instruments to extract oligopoly rents from domestic firms, whereas anti-cartel and anti-trust legislation would be the primary tool to restrict market power per se. However, with both domestic and foreign firms in the domestic market, the domestic firms might favor an asymmetric treatment, which uses trade policy to further their interests at the expense of their foreign competitors. They might argue that with few domestic firms and relatively many foreign competitors of substantial size, too large a share of the oligopoly rents is lost to foreigners. The more "excessive" the foreign competition, the more rent is transferred abroad.

Our key point in simple terms is that the presumption, that "excessive foreign competition" is harmful and should be counteracted by protectionist measures, is flawed. Namely, when there are more foreign competitors, oligopolistic profits tend to become smaller, ceteris paribus. Hence the strategic motive for protection becomes weaker, as there are fewer profits to be shifted.

We demonstrate our key point with a more elaborate analysis which considers the various facets of trade policy. Suitable trade policy, say tariffs or quotas imposed on the foreign firms, could help to enhance domestic welfare in two ways. First, protection from foreign competitors increases the domestic producers' market power and, consequently, their oligopolistic rents. This rent-shifting effect is certainly in the domestic firms' interest. Hence, if domestic producer surplus
is considered a contribution to national welfare, then rent-shifting can be in the national interest. Second, tariffs can directly extract some of the rents from foreign firms. Again, if tariff revenue can be considered a contribution to national welfare, then possibly, tariffs are in the domestic national interest. Anyway, the above arguments rest on the implicit assumption that domestic producer surplus and domestic tariff revenue are ultimately distributed in a welfare improving way. But even in the absence of distributive distortions, which is also assumed in this paper, a further important effect must be taken into account. Namely, if effective, protectionist measures reduce aggregate foreign supply to the domestic market. The loss of foreign supply need not—and typically will not—be fully made up by the increased supply of domestic firms. The resulting net loss in total market supply causes a price increase and a decrease in domestic consumer surplus. Hence there is a potential conflict between domestic producers and domestic consumers. A protectionist trade policy aimed at increasing domestic producer surplus and, perhaps, tariff revenue causes a decline in domestic consumer surplus. The crucial question then is which effect dominates.

In their seminal contribution, Brander and Spencer (1981) study a rent-extracting tariff policy against a foreign monopolist operating in the domestic market and facing potential market entry by a domestic firm, where entry is costly. They show that the threat of entry makes the rent-extracting policy particularly attractive, even if actual entry does not occur in equilibrium. Since then, a growing literature has examined the consequences of trade barriers in imperfectly competitive markets. Major research articles, monographs, and surveys on the subject are, among

\footnote{Under certain instances of static oligopoly, trade restrictions might even facilitate quasi-collusion between domestic and foreign firms and help increase the profits of both. See Harris (1985) and Krishna (1989). Further, in practice, a quantity-restricted foreign firm could attempt to work around the restriction through improvement of the quality and increase of the price of its product, thus maintaining or even expanding the volume of its sales (and supporting, if not boosting its profits).}

\footnote{In another important contribution, Brander and Spencer (1985) investigate the rent-shifting effects of export subsidies.}
Rotemberg and Saloner (1989); they also provide further references.

Here we examine the welfare effects arising from the imposition of a quota in very simple
models of imperfect competition. For the last three decades, quotas have been the predominant
protectionist policy in certain key industries with a more or less oligopolistic market structure,
e.g. automobiles, steel, textiles. More often than not, quotas appear inferior to tariffs, so that
even an optimal quota is merely a second-best instrument. Nonetheless, given the observed
political bias in favor of quotas, further study of the welfare effects of quotas is warranted.

The situation considered is one in which foreign and domestic firms produce output for
domestic consumption, with the main focus on the welfare effects of quotas. We are especially
interested in the welfare impact of quotas in the presence of "excessive foreign competition".
Therefore we perform comparative statics exercises to see how the optimal quota is affected by
differing numbers of foreign and domestic firms.

All firms supplying this domestic market are profit maximizing. They form a static
quantity-setting oligopoly with a linear demand curve. Welfare is measured from the domestic
social planner's point of view by measuring the sum of consumer surplus and domestic firm
profits as a function of the quota imposed on the foreign firms. Increased supply by foreign firms
tends to increase consumer surplus while reducing the profits of domestic firms. So, as indicated
above, the crucial question is which effect dominates? Naturally, domestic firms find the

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4 Anderson (1988) strongly endorses this assessment. As a by-product of our analysis, Section 6
presents another case of the superiority of tariffs over quotas and over quota-tariff combinations. In
contrast, Syropoulos (1990) studies anti-collusive policies against foreign oligopolists in a repeated-
game framework and finds that for some parameter constellations, quotas can be preferable to tariffs.

5 We shall comment on the issue of Cournot competition in quantities versus Bertrand
competition in prices in Section 6, where we shall also assess the virtue of linear-quadratic demand and
cost relations.
imposition of a quota most urgent when there is "excessive foreign competition", meaning a large number of sizable foreign suppliers in the market. Economic analysis as well as economic intuition suggest otherwise. Namely, elimination or domestication of ferocious foreign competitors can cause a loss of consumer surplus which outweighs any rent-shifting or rent-extracting effect.

In Section 2 we present our first model, in which all firms have identical and constant marginal costs, and illustrate the overall welfare effect. We find that with only one domestic firm and less than four foreign firms, the optimal quota from the domestic social planner's point of view is zero, i.e. autarky. However, if there is one domestic firm and more than four foreign firms, a non-binding quota, i.e. free trade, is optimal. (If there is one domestic firm and four foreign firms, autarky and free trade are both optimal.) More generally, the optimal choice turns out to be either free trade or autarky with the choice hinging on the number of foreign firms relative to the number of domestic firms. The implication is that free trade is the (second-)best policy if the foreign sector of the industry is sufficiently competitive relative to the domestic sector; on the other hand, autarky is the best policy if the foreign sector is not sufficiently competitive. In sum, "excessive foreign competition" is desirable under our welfare criterion, whereas modest foreign competition is not.

One would expect that the inherent conflict of interest between domestic consumers and domestic producers is aggravated if the foreign suppliers have a cost advantage. The situation modeled in Section 3 is of this sort: Suppose there is one domestic firm with a quadratic cost function and a variable number of zero-cost foreign firms. With total domestic surplus as the welfare criterion, we find that an increase in total foreign supply is desirable if initially it was neither very small nor very large; an increase of the number of foreign firms leads to a better free-trade Cournot-Nash outcome; and that replacing the particular domestic monopolist by a zero-
cost foreign monopolist improves welfare. Here even modest foreign competition is desirable, while "excessive foreign competition" is still more desirable. Moreover, maintaining the high-cost domestic industry can be too wasteful.

Conversely, Section 4 studies the impact of a cost advantage for domestic firms relative to foreign firms. We first focus on the case of a single domestic firm. Our analysis shows that more foreign competitors are needed to improve upon a domestic monopoly as the cost difference widens. If the cost difference is sufficiently large domestic monopoly is always socially optimal regardless of the number of foreign competitors. Ultimately, when the cost difference becomes large enough, the ban can be lifted, since the very-high-cost foreign firms’ self-interest induces them to withdraw from the domestic market. We have found analogous results for the case of several domestic firms.

The explicit emphasis of this paper lies on the welfare effects of quotas under static Cournot competition. But with little additional effort, we can incorporate a specific tariff into the model of Section 4. This is done in Section 5. For foreign firms, a specific tariff acts like an increase in their constant marginal-cost parameter. Domestically, the tariff provides a new source of revenue. Assuming exogenous marginal costs equal to zero and a single domestic firm, we find that when the number of foreign firms ranges from one to three, the best trade policy is to allow the foreign firms to enter the domestic market while imposing the optimal tariff, whereas autarky was best in a tariff-free world. With five or more foreign firms, free trade was optimal in a tariff-free world; but then it is even better to have all firms in the domestic market and capture some of the foreign firms’ rents through an optimal tariff. These findings support the widely-held belief that tariffs are preferable to quotas. Again the number of foreign firms matters. Total domestic surplus, augmented by tariff revenue, increases with the number of foreign competitors, i.e. increased foreign competition is welfare improving. For simplicity, we focus on the case of one
domestic firm. Analogous results hold for an arbitrary fixed number of domestic firms.

Section 6 concludes the paper and discusses the merits of some of our modeling choices.

2. A Model with Identical Firms

Consider the following **Stackelberg Game** with a domestic government as the leader. There are $n+1$ players in this game, $n$ quantity-setting firms who produce a homogeneous good for the domestic market and the domestic government which sets an aggregate quota, $\bar{x}$, on the foreign firms' output. Of these $n$ firms, $n_f$ are foreign and $n_d$ are domestic, so $n=n_f+n_d$. The number of domestic firms in the industry is fixed, however the proportion of foreign to domestic firms, $n_f/n_d$, may vary as $n_f$ varies.

The firms face a **domestic inverse demand function**

$$p(x)=a-bx,$$

where $a > 0$ and $b > 0$ are given coefficients, $x=\sum_{i=1}^{n} x_i$, $0 \leq x \leq \frac{\bar{x}}{b}$, is the total output supplied to the domestic market and $p=p(x)$ is the resulting market price. For $x > \frac{\bar{x}}{b}$, $p(x)=0$. All firms have constant marginal cost $c<a$. (Therefore firms' output decisions in the domestic market would not be affected by conditions in other markets). The government in this game acts as a Stackelberg leader while the firms, both foreign and domestic, are Stackelberg followers. The payoff functions of the $n$ firms are their profit functions, $\Pi_i$ for firm $i$ with output $x_i$, given by

$$\Pi_i = p \cdot x_i - c \cdot x_i.$$

The government's payoff function is total domestic surplus, the sum of consumer surplus and domestic firm profits,
\[ TS = CS(x) + \Pi_d \]

where

\[ CS(x) = \int_0^x p(t)dt - p(x) \cdot x = \frac{b}{2} \cdot x^2 \text{ for } 0 \leq x \leq \frac{a}{b}, \]

\[ \Pi_d = \sum_{i \in D} \Pi_i, \] and \( D \) is the set of all domestic firms. The strategy spaces for all \( n+1 \) players are identical and equal to \( \mathbb{R}^1. \)

2.1 One Domestic Firm

Our analysis commences with the case of one domestic firm, indexed \( d \), and no collusion among domestic or foreign firms. (In the analysis, without severe loss of accuracy, we sometimes omit the specification of when exactly the first order conditions yield profit maximizers and determine the Cournot-Nash equilibrium. In the later sections of the paper, we become somewhat more pedantic in this respect.) The domestic firm's best reply function is

\[ x_d = \frac{a-c}{2b} - \frac{1}{2} x_F, \]

where \( x_F = \sum_{i \in F} x_i \) and \( F \) is the set of foreign firms. If \( N \) is the set of all firms, \( N \setminus i \) denotes the set of all firms except firm \( i \) and \( x_{N \setminus i} = \sum_{j \in N \setminus i} x_j \), then firm \( i \)'s best reply is

\[ x_i = \frac{a-c}{2b} - \frac{1}{2} x_{N \setminus i} \]

in the absence of a binding quota. The resulting Nash equilibrium is given by the identical outputs

\[ x_i = \frac{1}{n+1} \cdot \frac{a-c}{b}, \quad i. \]

Therefore total equilibrium output is \( x = \frac{n}{n+1} \cdot \frac{a-c}{b} \). Remember, this is the outcome when there is no binding quota. If a binding quota is in place, the best reply function of the home firm is not affected, however the best reply functions for the foreign firms change to

\[ x_i = \min \left\{ \frac{a-c}{2b} - \frac{1}{2} x_{N \setminus i}, \bar{x}/n_f \right\} \]

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where \( \bar{x} \) is the aggregate quota placed on imports. (For simplicity we assume each foreign firm gets an equal share under a binding quota). Under a binding quota, industry output would be

\[
x = \bar{x} + \frac{a-c}{2b} - \frac{1}{2} \bar{x} = \frac{1}{2} (\bar{x} + \frac{a-c}{b}).
\]

**THE QUESTION** is, what quota should the government impose in order to maximize \( TS = CS(x) + \Pi_d \)? To come up with a definitive answer, we rewrite the government's objective function more explicitly as a function of the quota placed on foreign firms' aggregate output, while assuming that the domestic firm chooses a profit-maximizing output level. First, we consider the case where the quota is binding. Recall \( CS(x) = \frac{b}{2} x^2 \) and that total output under a binding quota will be \( x = \frac{1}{2} (\bar{x} + \frac{a-c}{b}) \) so that one can write consumer surplus as

\[
CS(\bar{x}) = \frac{b}{8} \bar{x}^2 + \frac{a-c}{4} \bar{x} + \frac{(a-c)^2}{8b}.
\]

The maximum value of \( \Pi_d \), given \( x_F = \bar{x} \), can be written as

\[
\Pi_d(\bar{x}) = \frac{b}{4} \bar{x}^2 - \frac{a-c}{2} \bar{x} + \frac{(a-c)^2}{4b}.
\]

Adding \( CS(\bar{x}) \) and \( \Pi_d(\bar{x}) \) yields

\[
TS = TS(\bar{x}) = \frac{3b}{8} \bar{x}^2 - \frac{a-c}{4} \bar{x} + \frac{3(a-c)^2}{8b}.
\]

Notice that this formula is valid only for \( 0 \leq \bar{x} \leq \frac{n-1}{n+1} \frac{a-c}{b} \), since the quota is no longer binding for \( \bar{x} > \frac{n-1}{n+1} \frac{a-c}{b} \). When \( \bar{x} > \frac{n-1}{n+1} \frac{a-c}{b} \), the foreign firms' aggregate output will be \( x_F = \frac{n-1}{n+1} \frac{a-c}{b} \).

Therefore with respect to \( \bar{x} > \frac{n-1}{n+1} \frac{a-c}{b} \), the values of \( x_F \), \( x \), and \( TS \) are constant with

\[
TS = \left\{ \frac{3}{8} \left( \frac{n-1}{n+1} \right)^2 - 1, \frac{n-1}{4} \frac{a-c}{n+1} + \frac{3(a-c)^2}{8b} \right\}.
\]

Thus, as a function of \( \bar{x} \geq 0 \), the government's objective function attains

\[
TS = \frac{3b}{8} \bar{x}^2 - \frac{a-c}{4} \bar{x} + \frac{3(a-c)^2}{8b} \quad \text{for} \quad 0 \leq \bar{x} \leq \frac{n-1}{n+1} \frac{a-c}{b}
\]

and
\[
TS = \left\{ \begin{array}{ll}
\frac{3}{8} \left( \frac{n-1}{n+1} \right)^2 - \frac{1}{4} \frac{n-1}{n+1} + \frac{3}{8} \left( \frac{n-1}{n+1} \right) \frac{a-c}{b} & \text{for } \bar{x} > \frac{n-1}{n+1} \frac{a-c}{b} \\
& \\
& \end{array} \right.
\]

Now consider the nature of the function TS with respect to the number of firms in the industry. Recall that, by assumption, the number of domestic firms was fixed at one, so \( n \geq 1 \). \( n = 1 \) is uninteresting, since it implies domestic monopoly.

When \( n = 2 \) there is one domestic and one foreign firm. In this case, all \( \bar{x} > \frac{n-1}{n+1} \frac{a-c}{b} \) are minimizers of \( TS(\bar{x}) \). Namely, for \( 0 \leq \bar{x} \leq \frac{n-1}{n+1} \frac{a-c}{b} \), TS is a downward sloping quadratic with a minimum at \( \bar{x} = \frac{n-1}{3} \frac{a-c}{b} = \frac{n-1}{n+1} \frac{a-c}{b} \) and for \( \bar{x} > \frac{n-1}{n+1} \frac{a-c}{b} \), \( TS(\bar{x}) = TS(\frac{n-1}{n+1} \frac{a-c}{b}) \). Since \( TS(\bar{x}) \) attains its maximum at \( \bar{x} = 0 \) and is downward sloping from \( \bar{x} = 0 \) to \( \bar{x} = \frac{n-1}{n+1} \frac{a-c}{b} \), we get the interesting implication that any binding quota is superior to free trade, with the optimum quota at \( \bar{x} = 0 \), i.e. autarky.

**THE ANSWER** is that for arbitrary \( n \), the maximizer of \( TS(\bar{x}) \) depends on \( n = n_f + 1 \) as follows.

**Conclusion 1.** For \( n_f < 4 \), \( \bar{x} = 0 \) (autarky) yields the maximum value of \( TS(\bar{x}) \).

For \( n_f > 4 \), any \( \bar{x} > \frac{n-1}{n+1} \frac{a-c}{b} \) (free trade) yields the maximum value of \( TS(\bar{x}) \).

For \( n_f = 4 \), autarky and free trade are optimal.

In discussing the implications of the model so far, we have implicitly assumed that there is no collusion between firms. If we remove this assumption the analysis has further implications. First, if there is only one domestic firm and the foreign firms are colluding among themselves (in an optimal manner), then autarky is best regardless of the number of foreign firms in the industry. Second, if there is more than one firm in the domestic industry and these firms are

\[\text{\textsuperscript{6}Omitte proofs of numbered conclusions are collected in the appendix. Omitted proofs of corollaries are obvious.}\]
colluding in an optimal manner (optimal with respect to the firms' profits), then autarky is preferable whenever (i) the foreign firms are non-colluding and there are at most three of them or (ii) all foreign firms collude among themselves. These are straightforward implications, since in the present case with constant and equal marginal costs, a group of firms colluding in a Pareto-optimal manner with respect to their profits will act (in terms of their output decision) as if they were one firm. In making such statements we ought to be careful to note the casual nature of these observations: in the static model as constructed, it is impossible to describe how a collusive outcome could come about. One method of building a model which could yield collusive outcomes as a Nash equilibrium would be the conjectural variations approach; see McMillan (1986), pp. 19-20. While this approach has the advantage of being relatively simple, it has the disadvantage of being a rather ad hoc method of obtaining certain types of equilibria. The more theoretical and sounder approach would be to explicitly incorporate dynamics into the game, for instance, in a repeated-game model; however, this greatly increases the complexity of the analysis. Syropoulos (1990) has successfully analyzed the special case of a foreign oligopoly in the domestic market within the repeated-game framework.

2.2 Arbitrary Number of Domestic Firms

Next we consider the general case in which there are \( n_d \geq 1 \) domestic firms. To simplify the analysis without endangering the qualitative results, we will normalize the demand curve to \( p = 1 - x \) and the constant marginal costs to \( c = 0 \). In this framework, with \( n_d \geq 1 \) domestic firms and \( n_f \geq 0 \) foreign firms in the market, total domestic surplus is

\[
(i) \quad TS = \frac{1}{2} \cdot \frac{(n_d + n_f)^2}{(n_d + n_f + 1)^2} + \frac{n_d}{(n_d + n_f + 1)^2}.
\]

Hence the change in total domestic surplus from allowing \( n_f \) foreign firms into a market supplied
by \( n_d \) domestic firms is given by the following formula:

\[
\Delta TS = \frac{1}{2} \left( \frac{n_d}{n_d + n_f + 1} - \frac{n_d}{n_d + 1} \right) \left( \frac{1}{n_d + 1} - \frac{1}{n_d + n_f + 1} \right) - \frac{1}{n_d + n_f + 1} \left( \frac{n_d}{n_d + 1} - \frac{n_d}{n_d + n_f + 1} \right)
\]

This formula follows from the above expression for \( TS \). It can be derived more easily from Figure 1. If \( \Delta TS > 0 \), free trade is best; if \( \Delta TS < 0 \), autarky is best. In terms of \( n_d \) and \( n_f \), we have

**Conclusion 2.** If \( n_f > 2n_d^2 + 2n_d \) implies that \( TS(x) \) is maximized under free trade, while \( n_f < 2n_d^2 + 2n_d \) implies that autarky yields the highest value for \( TS(x) \).

The conclusion confirms a conjecture by Laussel et al. (1988) that the more competitive the foreign sector of an industry is, relative to the domestic sector, the stronger the argument for free trade. This goes against the grain of many arguments in favor of protectionism which involve claims of "excessive" foreign competition. As a matter of fact, in the present model "excessive" foreign competition would be an argument in favor of free trade. The intuition is that if a relatively small number of foreign firms enter the market, the beneficial effect they have in terms of lowering the market price and therefore increasing consumer surplus is outweighed by the loss of profits on the part of domestic firms. If the number of foreign firms entering the industry is relatively large, then the gain in consumer surplus that results from the lower price dominates the loss of domestic firm profits to foreign competition. This is not to say that there is not a large loss of domestic firm profits; just that this loss is transferred to consumers in the form of additional consumer surplus.

One interesting point is that, at least within the framework of this partial equilibrium model, there can be an argument in favor of quite restrictive trade barriers -- autarky in fact. What differentiates this approach from the analysis of quotas in a perfectly competitive market is
that we do not assume that profits in the industry are zero and it is the tradeoff between consumer surplus and domestic firm profits which leads to the conclusion that in some cases autarky may be preferable to free trade from the importing country's point of view.

3. Cost Advantage of Foreign Firms

3.1 The Issue

In the domestic market for a homogeneous commodity whose producers form a quantity-setting oligopoly, increased supply by foreign firms tends to increase consumer surplus while reducing the profits of domestic firms. Conversely, limiting the access of foreign firms to the domestic market boosts domestic profits, but at the same time diminishes consumer surplus. The latter effect can dominate if import quotas eliminate a fierce competition which would otherwise exist. For instance, in the model of Section 2.1, the presence of 5 or more unrestricted foreign firms is preferable to the absence of foreign competitors.

One would expect that the conflict of interest between domestic consumers and domestic producers is aggravated if the foreign suppliers have a cost advantage. This intuition is confirmed in the following simple example.

3.2 Example

There is one domestic firm with output $x_d \geq 0$ and cost function $C_d(x_d) = 3x_d^2$. There is a
variable number, \( n_f \), of foreign firms. The generic foreign firm is denoted \( f \), produces a quantity \( x_f \geq 0 \), has costs \( C_f(x_f) = 0 \). The demand on the domestic market is represented by the inverse demand function \( p(x) = 1 - x \) and the corresponding consumer surplus \( CS(x) = \frac{1}{2} \cdot x^2 \) for total supplied output \( x, 0 \leq x \leq 1 \). For \( x > 1 \), \( p(x) = 0 \). Let \( \Pi_d \) denote the profit of the domestic firm and set \( TS = CS + \Pi_d \). If we adopt again \( TS \), the total domestic surplus, as the welfare criterion, then the subsequent analysis yields the following conclusions 3, 4A, 4B, and 5.

With the assumed inverse demand and cost functions, total domestic surplus is

\[
TS = CS(x) + \Pi_d = CS(x) + p(x)x_d - C_d(x_d) = \frac{1}{2} \cdot x^2 + p(x)x_d - 3x_d^2.
\]

The domestic firm maximizes

\[
\Pi_d = p(x)x_d - 3x_d^2 = p(x_F + x_d) - 3x_d^2 \quad \text{where} \quad x_F = \sum_f x_f.
\]

Let \( 0 \leq x_F \leq 1 \) be given. Then the domestic firm's best response is

\[
(2) \quad x_d = \frac{1}{8} \cdot (1 - x_F).
\]

**Conclusion 3.** If initially total foreign supply \( x_F \) satisfies \( \frac{1}{57} \leq x_F < 1 \), then an increase of \( x_F \) increases total domestic surplus.\(^7\)

Notice that if \( x_F \geq 1 \), then an increase of \( x_F \) increases \( x = x_F \), while the domestic firm's best response remains at \( x_d = 0 \); hence neither \( CS = \frac{1}{2} \) nor \( \Pi_d = 0 \) are changed.

**Corollary.** With a domestic monopolist, \( TS = \frac{9}{128} \).

A foreign firm \( f \) maximizes \( \Pi_f = p(x)x_f = p(x_d + x_f + x_f)x_f \) where \( x_f = \sum_{j \neq d, f} x_j \).

\(^7\) For North Americans acquainted with the "Heinz 57" logo, this result is easily remembered as the "Reciprocal Heinz Theorem".
Suppose \(0 \leq x_d + x_f \leq 1\). Then firm \(f\)'s best response is

\[
(3) \quad x_f = \frac{1}{2}(1 - x_d - x_f).
\]

Now we are ready to scrutinize the oligopoly consisting of the domestic firm and \(n_f \geq 1\) foreign firms. At a free trade Cournot-Nash equilibrium, \(x < 1\), so that (2) and (3) hold. This leads to a system of linear equations with a unique solution. Let us try equal treatment of foreign firms. If this approach works, we have gotten the solution. Equal treatment yields

\[
(2A) \quad 8x_d + n_f x_f = 1 \quad \text{and} \quad (3A) \quad x_d + (n_f + 1)x_f = 1
\]

with solution \(x_d = \frac{1}{7n_f + 8}, \quad x_f = \frac{7}{7n_f + 8}, \quad x_f = \frac{7n_f}{7n_f + 8}\).

**Conclusion 4A.** In the presence of the domestic firm, an increase in the number, \(n_f\), of foreign firms leads to a better free-trade Cournot-Nash outcome.

**Proof.** For \(n_f = 1\), \(x_f = \frac{7}{10} > \frac{1}{5}\) and the corresponding total domestic surplus is larger than with a domestic monopolist, i.e. with \(n_f = 0\). The equilibrium value of \(x_f\) increases with \(n_f\). Therefore the assertion follows, by Conclusion 3. Q.E.D.

For the sake of comparison, let us also analyze the oligopoly consisting of \(n_f \geq 1\) foreign firms. Again, at a free-trade Cournot-Nash equilibrium, \(x < 1\), so that (3) with \(x_d = 0\) holds. This yields a system of linear equations with a unique solution. Taking the equal treatment approach yields

\[
(3B) \quad (n_f + 1)x_f = 1
\]

with solution \(x_f = \frac{1}{n_f + 1}, \quad x_f = \frac{n_f}{n_f + 1}\).

**Conclusion 4B.** In the absence of the domestic firm, an increase in the number \(n_f\) of
foreign firms leads to a better free trade Cournot-Nash outcome.

**Proof.** With \( n_f = 0 \), \( x = x_F = 0 \) and \( TS = CS = 9 \). With \( n_f \geq 1 \), \( TS = CS = \frac{1}{2} x_F^2 \). Since the equilibrium value of \( x_F \) is increasing in \( n_f \), the assertion follows. Q.E.D.

**Corollary.** The equilibrium value of \( TS \) converges to \( \frac{1}{2} \) as \( n_f \to \infty \), irrespective of the presence of the domestic firm.

**Conclusion 5.** Replacing the domestic monopolist by a foreign monopolist increases total domestic surplus.

**Proof.** With the domestic monopolist, \( TS = \frac{9}{128} \).

With a foreign monopolist, \( TS = CS = \frac{1}{2} x_F^2 = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8} > \frac{9}{128} \). Q.E.D.

### 3.3 Remarks

Conclusions 3 and 4 are not too surprising in that they reinforce our previous conclusions as expected: now it takes just one additional competitor to enhance welfare relative to domestic monopoly. In contrast, Conclusion 5 leads us beyond our original scope of inquiry. Namely in the present context, not only should a domestic monopolist be exposed to foreign competition, but under certain circumstances, it should go out of business and be replaced by as many foreign firms as possible. In practice, the latter result is obtained if a multinational firm relocates...
domestic production to a lower-cost country. (In that case, total domestic surplus is increased further if the multinational firm has domestic headquarters and repatriates some of the additional profits made by its foreign subsidiaries.)

Although our conclusions are very tempting, one should be wary of premature policy suggestions. After all, we argue in a partial-equilibrium framework. Thus we disregard a possible reduction of domestic purchasing power and, consequently, a shift of the demand curve caused by reduced domestic profits and factor incomes. Similarly, relocating production abroad would, *ceteris paribus*, create a trade deficit which can only be balanced if another domestic product generates an export surplus. Such considerations can be used to justify temporary import restrictions which could allow domestic firms to reposition themselves through cost reducing measures or the development of new products. Going one step further than the "infant industry argument", it has even been argued that protective trade policy furthers an already prosperous economy; see Bairoch (1989). Classical international trade theory, dating back to Ricardo (1817), rejects this point of view: Both countries can engage in mutually beneficial trade, even if one country, say the foreign country, has a cost advantage in all sectors. The only requirement is that the cost advantage be not uniform across sectors so that the domestic country has a "comparative advantage" in at least one sector. (For a reappraisal of the issue of free trade versus interventionism see Krugman (1987).)

What have we illustrated then? First of all, the profits lost by domestic firms to foreign competitors can be outweighed by the increase in domestic consumer surplus. Secondly, if the foreign suppliers have a clear cost advantage, maintaining a domestic industry can be a waste of resources. Finally, trade restrictions are necessarily a controversial issue, even under a purely domestic perspective.
4. Cost Advantage of Domestic Firms

This section investigates to what extent the conclusions of Sections 2 and 3 persist if domestic firms have a cost advantage relative to foreign firms. With a domestic cost advantage, one would expect our earlier conclusions to be weakened or even reversed. For simplicity, we consider the case of a single domestic firm with cost function \( C_d(x_d) = 0 \). Let the generic foreign firm \( f \) be characterized by its cost function \( C_f(x_f) = c_f x_f \) with \( c_f \geq 0 \). The model is completed with a linear demand curve. Namely, inverse demand is \( p(x) = a - bx \) for \( 0 \leq x \leq \frac{a}{b} \) and \( p(x) = 0 \) for \( x > \frac{a}{b} \), where \( a > 0 \) and \( b > 0 \) are fixed from now on. In contrast, the foreign firms’ marginal cost \( c_f \) will be treated as a parameter. Let \( n_f \) denote again the number of foreign firms. Once more, total domestic surplus serves as the welfare criterion.

For a given \( c_f \), one of two possibilities arises. Either domestic monopoly is better than any free trade Cournot-Nash outcome, whatever \( n_f \geq 1 \). In that case put \( n_f(c_f) = \infty \). Or domestic monopoly is worse than the free trade Cournot-Nash outcome, whenever \( n_f \geq n_f(c_f) \) where we choose \( n_f(c_f) \) as the smallest (well-determined) number with this property. Our findings are summarized by the following

**Conclusion 6.**

(a) If \( c_f \geq \frac{a}{2} \), then \( n_f(c_f) = \infty \). Whatever \( n_f \), the domestic firm is the only one active in free-trade Cournot-Nash equilibrium, producing its monopoly output.

(b) If \( \frac{a}{2} > c_f \geq \frac{a}{b} \), then \( n_f(c_f) = \infty \). The free-trade Cournot-Nash equilibrium supplies are given as

\[
(4) \quad x_d = \frac{a + n_f c_f}{b(n_f + 2)}, \quad (5) \quad x_f = \frac{a - 2c_f}{b(n_f + 2)}.
\]

(c) If \( \frac{a}{b} > c_f \geq 0 \), then \( n_f(c_f) \) is the smallest integer exceeding \( \frac{4a}{a - 6c_f} \). The free-trade Cournot-
Nash equilibrium is given by (4) and (5). Compared to our earlier results, cases (a) and (b) constitute a reversal. The domestic firm has such a cost advantage, that the presence of foreign firms is undesirable whatever their number. To achieve this goal, no trade policy measure is necessary in case (a), since the foreign firms voluntarily withdraw from the domestic market. In case (b), however, a ban of all foreign firms is required. In case (c), \( n_f(c_f) \) is finite and (weakly) increasing in \( c_f \) with \( n_f(c_f) \) tending to infinity as \( c_f \) approaches \( a/6 \).

**Corollary.** If \( n_f \to \infty \), then \( x_F \to \frac{a-c_f}{b} \) and \( T \to \frac{1}{8b}(4a^2-4ac_f+3c_f^2) \).

**Ramifications**

For arbitrary \( n_d \geq 1 \), analogous results are obtained by a lengthier, but very similar analysis which we omit.

[Specifically, \( x_f^* \) defined in the proof of Conclusion 6 then equals \( \frac{2an_d}{b(2n_d+1)} \).]

(4) and (5) become

\[
(4') \quad x_d = \frac{a+n_f c_f}{b(n_d+n_f+1)}, \quad (5') \quad x_f = \frac{a-(n_d+1)c_f}{b(n_d+n_f+1)}.
\]

Case (a) corresponds to \( c_f \geq \frac{a}{n_d+1} \).

Case (b) corresponds to \( \frac{a}{n_d+1} > c_f \geq \frac{a}{(n_d+1)(2n_d+1)} \).

Case (c) corresponds to \( \frac{a}{(n_d+1)(2n_d+1)} > c_f \geq 0 \)

where now \( n_f(c_f) \) is the smallest integer exceeding \( \frac{2an_d(n_d+1)}{a-(n_d+1)(2n_d+1)c_f} \).

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5. Quotas versus Tariffs

Welfare effects of quotas constitute the core concern of this paper. But as a by-product of our analysis, we can obtain a few results on specific tariffs after a minor modification of the model of the previous section. As a rule, traditional trade theory has assumed perfectly competitive markets and established the equivalence of tariffs and quotas up to the difference in government revenue. With imperfect competition, however, tariffs often are (or are believed to be) superior to quotas. Our analysis confirms this assertion.

Introducing a specific tariff with rate \( t > 0 \) into a model with constant marginal costs \( c \) for both domestic and foreign firms has two effects. First, for a foreign firm, the tariff acts like an increase in the firm's constant marginal cost from \( c \) to \( c + t \). Second, the domestic tariff revenue term \( t \cdot x_F \) enters \( TS \) so that

\[
TS = CS + \Pi_D + t \cdot x_F
\]

where \( CS \) is domestic consumer surplus, \( \Pi_D \) is domestic firm profits, and \( t \cdot x_F \) is domestic tariff revenue.

In the present context of imperfect competition, one would expect that any combination of a tariff and a binding positive quota is welfare-dominated by a higher tariff without a quota. The argument goes as follows. Suppose the prevailing tariff (rate) is \( \bar{t} \geq 0 \) and the binding quota is \( \bar{x} > 0 \). Then there exists a tariff \( t, t > \bar{t} \), such that without a quota, the aggregate foreign equilibrium supply is \( x_F = \bar{x} \). Nothing else has changed; but the augmented total domestic surplus is increased by the additional tariff revenue \((t - \bar{t})\bar{x}\). Thus, as expected, the superiority of tariffs derives from their power to extract more rent. With the tariff \( \bar{t} \) (and symmetric constant marginal costs \( c \)), the equilibrium values \((4')\) and \((5')\) change to
\[ (4'') \quad x_d(t) = \frac{a-c+n_d \bar{t}}{(n+1)b}, \quad (5'') \quad x_f(t) = \frac{a-c-(n_d+1)\bar{t}}{(n+1)b}. \]

If the quota is positive and binding, we have \( x_f(t) > \frac{x}{n_f} > 0 \). Hence there is \( t > \bar{t} \) such that in equilibrium \( x_f(t) = \frac{x}{n_f} \) for all \( f \in F \) and \( x_F = x \). Thus we have

**Conclusion 7.** A tariff together with a binding positive quota is always dominated by some higher tariff without a quota.

Under more restrictive simplifying assumptions, we derive a stronger conclusion.

**Conclusion 8.** Suppose \( a = b = 1 \) and \( c = 0 \). Then for \( n_f \geq 1 \), allowing the \( n_f \) foreign firms in the domestic market without a quota, while imposing the optimal tariff

\[ t^* = \frac{3}{2(n+1)(n_d+1)-3n_f}, \]

uniquely maximizes the augmented total domestic surplus where maximization is over all combinations of tariffs and quotas. Furthermore, the augmented total domestic consumer surplus is increasing in \( n_f \), when the optimal tariff is imposed;

If \( n \to \infty \), then \( t^* \to 0 \) and \( TS \to \frac{1}{2} \) (both without a tariff and with the optimal tariff in place; no quota in both instances).

A quite intuitive implication is that \( t^* \to 0 \) as \( n \to \infty \): as the industry becomes more competitive, the optimal tariff diminishes, becoming zero in the limit.

Having confirmed the superiority of tariffs over quotas, we should ask ourselves why Syropoulos (1990) can arrive at the opposite conclusion. He looks at a domestic market supplied solely by an oligopoly of foreign firms. The main difference is that he considers a repeated game.
In our static framework, collusion, i.e. joint profit maximization, is not an equilibrium outcome and therefore not an issue. Now consider the dynamic context of a repeated game. Syropoulos looks at subgame-perfect equilibria. But for the sake of transparency, let us ignore the subtleties of subgame-perfection and discuss Nash equilibria of the repeated game. Then with sufficiently high discount factors, i.e. sufficiently strong preference for future earnings, continued collusion among the foreign oligopolists is an undesirable equilibrium outcome. It is supported by the threat to punish a deviator by flooding the market. Now a permanent quota would have the following two effects, among others. First, it would shrink the opportunity set for deviators, thus reinforcing collusion. Second, it would narrow the opportunities for retaliation, weakening the incentives to stick to collusion. A severe quota will always break up collusion.\footnote{Though too severe a quota can be detrimental according to the findings of the present paper, which are also relevant in the repeated-game context.} Now what can a tariff achieve? As in the static context before, a tariff is more powerful in extracting oligopoly rents as they arise. But it is powerless in that it does not restrict the physical opportunities of the players. Hence it is plausible that under certain parameter constellations, tariffs constitute an inferior anti-collusive policy and are also inferior with respect to the resulting overall domestic welfare.
6. Concluding Remarks and Qualifications

To conclude, our admittedly very simple models convey a clear and simple message: **Under imperfect competition, protective quotas against "excessive foreign competition" can be detrimental to overall domestic welfare.** There are many important issues and ramifications left to future research. First, the domestic firms may also be exporting. Then the possibility of retaliation and mutual trade restrictions arises. Further, export opportunities to third countries can matter. Finally, a synthesis of various trade interventions, like quotas, tariffs, cost-reducing subsidies and direct export subsidies should be attempted. Exploratory steps in this direction are taken in Dixit (1984).

Since firms almost always compete in prices, if they compete at all, it is very compelling to model imperfectly competitive firms as price setters. Moreover, price-setting oligopoly à la Bertrand is favored in some of the trade-industrial organization literature because Bertrand competition appears to accentuate the impact of trade restrictions in general and the difference between quotas and tariffs in particular relative to Cournot competition. The reason is that prices as the sole strategic variables act as strategic complements (i.e. they move together), whereas quantities act as strategic substitutes (i.e. they move in opposite directions), which makes competition in prices more aggressive.

Nonetheless, we study a price-setting oligopoly à la Cournot. Apart from technical convenience, let us forward two substantial reasons for our choice of strategic variables. Both are related to shortcomings of Bertrand models. First, without capacity constraints, the simplest type of Bertrand model exhibits the "Bertrand paradox": For two or more firms, the Nash equilibrium outcome is equivalent to perfect competition in terms of prices and outputs. Since we want to study the effects of increasing competition, such an insensitive model appears inappropriate.

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Second, with capacity constraints, varying degrees of competition can occur in the Bertrand model; but very frequently, only equilibria in mixed strategies exist, a mixed blessing. Since quantity constraints in trade act like exogenous capacity constraints, this problem resurfaces in the trade-industrial organization literature, e.g. Krishna (1989), Rotemberg and Saloner (1989).\footnote{The pros and cons of static Bertrand and Cournot models are discussed at length in chapter 5 of Tirole (1988) and in Shapiro (1989). In general, a firm decides sequentially or simultaneously on several strategic dimensions, say quality, quantity, and price. Or it selects endogenously its strategic variable, say either quantity or price, as in Singh and Vives (1984).}

Since all the phenomena we want to point out occur already in a static model, there is no need to consider a repeated-game model. The linear-quadratic demand and cost relations assumed in the present paper are admittedly restrictive. They imply a dichotomy in the core of the paper: the optimal quota is either zero or non-binding. However, linear-quadratic relations have the distinct advantage that they yield explicit solutions for the models considered and illustrate the crucial features in a very economical way.
APPENDIX

Proof of Conclusion 1. It remains to consider the case $n > 2$. In that case, for $\bar{x} < \frac{n-1}{n+1} \cdot \frac{a-c}{b}$, TS is a quadratic with an unique minimum at $\bar{x} = \frac{1}{3} \cdot \frac{a-c}{b} < \frac{n-1}{n+1} \cdot \frac{a-c}{b}$ and, for $\bar{x} > \frac{n-1}{n+1} \cdot \frac{a-c}{b}$, TS is constant. Therefore, there are only two values of $\bar{x}$ which could possibly maximize TS($\bar{x}$), namely $\bar{x} = 0$ or $\bar{x} = \frac{n-1}{n+1} \cdot \frac{a-c}{b}$. This means that either autarky or free trade is best. To determine which value of $\bar{x}$ maximizes TS($\bar{x}$), we notice that $\bar{x} = 0$ is a maximizer if and only if $\frac{n-1}{n+1} \cdot \frac{a-c}{b} \leq \frac{2}{3} \cdot \frac{a-c}{b}$. The latter condition is equivalent to $n \leq 5$. $\bar{x} = 0$ is the only maximizer if those inequalities are strict. Q.E.D.

Proof of Conclusion 2. Let $0 \leq x_F \leq 1$. From (1), $x = x_F + x_d = \frac{1}{8} + \frac{7}{8} x_F$, $p(x) = \frac{7}{8} (1 - x_F)$,

$\Pi_d = \frac{1}{16} (1 - x_F)^2$, $TS = \frac{1}{128} (1 + 7x_F)^2 + \frac{1}{16} (1 - x_F)^2 = \frac{1}{128} (9 - 2x_F + 5ix_F^2)$. As a function of $x_F$,

TS is a quadratic minimized at $x_F = \frac{1}{57}$. Q.E.D.

Proof of Conclusion 6. Let us first express total domestic surplus as a function of $x_F$, a given aggregate supply by foreign firms, with $0 \leq x_F \leq \frac{a}{b}$, while the domestic firm is profit maximizing.

The best response of the domestic firm is

$x_d = \frac{1}{2b} (a - bx_F)$

and, consequently,

$x = x_d + x_F = \frac{1}{2b} (a - bx_F + 2bx_F) = \frac{1}{2b} (a + bx_F)$,

$\Pi_d = \frac{1}{2b} (a - bx_F) \cdot \frac{1}{4b} (a - bx_F) = \frac{1}{4b} (a - bx_F)^2$,

$TS = \frac{b}{2} x^2 + \Pi_d = \frac{1}{8b} (a + bx_F)^2 + \frac{1}{4b} (a - bx_F)^2 = \frac{1}{8b} (3a^2 - 2abx_F + 3b^2 x_F^2)$.

As a function of $x_F$, TS is quadratic and minimized at $x_F = \frac{a}{3b}$. To do better than at domestic monopoly, $x_F$ has to exceed $x_F^* = \frac{2a}{3b}$.

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This dichotomy, resulting from a U-shaped objective function, is also exhibited in Laussel et al. (1988), pp. 1555/56.
Fix, for the moment, $c_f$ and $n_f \geq 1$. If there is an interior free-trade Cournot-Nash equilibrium, then it is unique and given by (3) and (4). (Use similar derivations as in Section 3.) Suppose $c_f \geq \frac{a}{2}$. Then (4) would imply $x_f \leq 0$. In that case, there cannot be an interior solution; it follows, by repeating the argument, that there cannot be an equilibrium where some of the foreign firms supply zero output, while others supply a positive amount of output. However, in the case $c_f \geq \frac{a}{2}$, $x_d$ equal to the monopoly output $\frac{a}{2b}$ and $x_f = 0$ for all $f$ constitutes an equilibrium. Hence Conclusion 6(a).

In case $c_f < \frac{a}{2}$, the free-trade Cournot-Nash equilibrium is characterized by (3) and (4). Then by (4), $x_F = \frac{n_f(a-2c_f)}{b(n_f+2)}$. The free-trade Cournot-Nash outcome dominates the domestic monopoly if and only if $x_F > x_F^*$. With the explicit formulas for $x_F$ and $x_F^*$ and $c_f < \frac{a}{2}$, the condition $x_F > x_F^*$ is equivalent to $n_f(a-6c_f) > 4a$. The latter condition cannot be satisfied if $c_f \geq \frac{a}{6}$. It is equivalent to $n_f > \frac{4a}{a-6c_f}$ if $c_f < \frac{a}{6}$. Hence Conclusions 6(b) and 6(c). Q.E.D.

Proof of Conclusion 8. Let $a = b = 1$, $c = 0$, $n_d \geq 1$, and $n_f \geq 1$. If $t > -\frac{1}{n_d+1}$, the equilibrium outcome is autarky with $x_d = \frac{1}{n_d+1}$ and $x_f = 0$. Let us proceed with $0 \leq t < -\frac{1}{n_d+1}$. Then the Nash equilibrium outputs $(3''')$ and $(4''')$ assume the simpler form

$$(3''')x_d(t) = \frac{i+n_ft}{n+1}, \quad (4''')x_f(t) = \frac{1-(n_d+1)t}{n+1},$$

and total output amounts to $x = n_d x_d + n_f x_f = \frac{n-n_ft}{n+1}$. Consequently, one obtains domestic

consumer surplus $CS = \frac{1}{2}x^2$, total domestic firm profits $\Pi_D = \frac{(1+n_ft)^2}{(n+1)^2}$, and tariff revenue

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\[ t \cdot x_F = \frac{t \cdot n_f \cdot [1 - (n_d + 1)t]}{n + 1} \]. Therefore total domestic surplus is

\[ TS = CS + \Pi_D + t \cdot x_F = \frac{1}{2} \frac{(n - n_f t)^2}{(n + 1)^2} + \frac{(1 + n_f t)^2}{(n + 1)^2} + \frac{t \cdot n_f \cdot [1 - (n_d + 1)t]}{n + 1} \]

\[ = \frac{1}{2} \frac{n^2 + 2 + 6n_f t - n_f \cdot [2(n + 1)(n_d + 1) - 3n_f] \cdot t^2}{(n + 1)^2}. \]

To find the \( t \) that maximizes \( TS \), simply differentiate \( TS \) with respect to \( t \), then set the derivative equal to 0 and solve for \( t \). (The second-order conditions are met by inspection). Now

\[ \frac{\partial TS}{\partial t} (t^*) = 0 \iff 6n_f - n_f [2(n + 1)(n_d + 1) - 3n_f] \cdot 2t^* = 0 \iff t^* = \frac{3}{2(n + 1)(n_d + 1) - 3n_f}. \]

Since \( 0 < t^* < \frac{1}{n_d + 1} \), we can conclude that this positive tariff which allows some imports is preferable to both free trade and autarky. Because of Conclusion 7, \( t^* \) with no binding quota is preferable to any other combination of \( t > 0 \) and a binding quota.

Furthermore, \( t^* = \frac{3}{[2(n_d + 1) - 3n_f + 2(n_d + 1)]^2} \) is decreasing in \( n_d \) and \( n_f \) with \( t^* \to 0 \) as \( n \to \infty \).

And \( TS \) evaluated at \( t^* \), assumes the form \( TS = \frac{1}{2} \frac{n^2 + 2 + 3n_f \cdot t^*}{(n + 1)^2} \), with \( \frac{\partial TS}{\partial n_f} < 0 \) and \( TS \to \frac{1}{2} \) as \( n \to \infty \).

For the case without a tariff, \( TS \to \frac{1}{2} \) as \( n \to \infty \) follows from (1). Q.E.D.
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Chapter 3

The Effects of Quotas and Voluntary Export Restraints in an
Imperfectly Competitive Market

Section I: Introduction

In this paper I have two main objectives. First, to show how the effects of quotas and voluntary export restraints (VER's) may differ in an imperfectly competitive market. Second, to examine how a government can use quotas or the threat of quotas to increase total output in its domestic market. To achieve these two objectives I perform partial equilibrium analysis in a repeated-game setting. This allows me to demonstrate how it could be in the foreign firm's best interest to subject itself to a VER, while a quota allowing the foreign firm to export the same amount would make it worse off. Further I show that the threat of quotas is a powerful strategic tool the government can use to affect the behavior of both the foreign and domestic firm.

In recent years, many papers have analyzed trade barriers in imperfectly competitive markets using static game models. The papers by Dixit (1983), Eaton and Grossman (1986), and K. Krishna (1987) exhibit this approach. However trade barriers often emerge as a result of dynamic interaction among firms and government. For example, quotas are determined by interactions between firms and government, and VER's may occur as a result of interaction among firms. Since these interactions have a highly dynamic nature, the static approach doesn't capture such dynamic effects. Therefore we need the repeated game approach for the consideration of the endogenous formation of trade barriers in imperfectly competitive market.

Some work has been done in a repeated game framework. For example, Fung (1987), considers the effects of tariffs on an international duopoly where the domestic and foreign firm
produce a differentiated product. Davidson (1984) considers the effect of tariff policy in an industry with N firms and a homogeneous good. Both authors determine the effect of tariffs on firms' ability to engage in collusive behavior. Rotemberg and Saloner (1986) examine the effect of quotas on firms' ability to collude in a duopolistic industry producing an identical good. Syropoulos (1996) analyzes the case where there are n foreign firms producing differentiated output for the domestic market and how quotas can be used to break up collusion. When analyzing quotas and VER's within the context of a repeated game, one can consider the effects of these trade barriers on the nature of collusion between firms.

By collusion, I am referring to a (possibly tacit) agreement between the two firms to limit their total output in such a way as to maximize joint profits. Now what share of profits each firm will receive needs to be determined. Given the symmetric cost structure of the two firms, I assume that each firm gets an equal share of the profits. In this setting, with no quota in place, firms can enforce collusion by threatening to increase output to more competitive levels if any firm deviates. To be more explicit, if one firm deviates from the agreement by increasing its output above the agreed-upon level, the other firm retaliates by reverting to its higher, non-collusive output level. In the jargon of game theory, this type of strategic behavior is known as a trigger strategy and the resulting equilibrium that it enforces is a trigger-strategy equilibrium. For simplicity, I restrict attention to the class of 'grim' trigger strategies. A grim trigger strategy is one in which once the punishment phase is implemented it stays in place for the rest of the game.

A VER could be viewed as an announcement by the foreign firm of its collusive output level. By contrast a quota, if sufficiently restrictive, would take away the foreign firm's ability to punish a deviation by the domestic firm, thereby breaking up collusion and leading to higher total output in the repeated game equilibrium. This result (and model) is similar to the one developed
by Rotemberg and Saloner (1986). They consider a given quota and determine which, if any, divisions of output (and therefore profit) would allow collusion to continue. My paper considers the effects of different quotas on a given collusive agreement. In particular, I determine the least restrictive quota necessary to break up this collusive agreement. The result is that a quota which is non-binding in the presence of collusion is sufficient to cause the firms to act competitively. The intuitive distinction between the two papers at this point would be that the two firms are able to communicate more easily in the model devised by Rotemberg and Saloner and therefore are able to devise a more complex trigger strategy where the output allocations are determined by the level of the quota and can change when the quota changes.

I also go beyond Rotemberg and Saloner by introducing a government as a player in the game. I develop a strategy for the government which, by using the threat of a quota, would make the foreign firm unwilling to collude. Thus, the government prevents collusion from occurring. Furthermore, I consider the possibility that the government can actually improve on the single-shot Cournot-Nash equilibrium output level. The government can achieve this by an agreement, possibly implicit, with the domestic firm to exclude the foreign firm from the market; in exchange the domestic firm promises to produce a suitably determined output level which guarantees higher profits to the domestic firm than it would otherwise receive and is higher than the resulting total output if the foreign firm were allowed to operate in the market. The government enforces this agreement by using the threat of allowing the foreign firm back into the market.

Further, I consider a trigger strategy, which involves an agreement between the government and the foreign firm, which induces the foreign firm to produce an output level so large that it yields a stream of profits whose present value is near zero, under the threat of being excluded from the market. This trigger strategy, developed more fully in the paper, leads to the
highest total output levels of the scenarios scrutinized in the paper and has the property that as firms place more weight on future payoffs, output gets closer to the efficient output level (the output level such that price equals marginal cost). In the extreme case where firms place as much weight on future payoffs as on the present payoff, the total output would be within an arbitrary \( \epsilon \) of the efficient output level.

One point I want to emphasize at the outset is that when first writing this paper I was thinking in terms of foreign and domestic firms. One problem with this is that the objective function I assign the government is consumer surplus alone, when the most natural objective function would be the sum of consumer surplus and domestic firm profits. I did this for technical reasons\(^{11}\), however, the reader who is bothered by this assumption is encouraged to think of the two firms as both being foreign where each could still be singled out as subject to a quota. When using this conceptual framework my paper could be considered as dealing with 'discriminatory' trade barriers under imperfect competition. This viewpoint may yield some insight into the incentives that could lead rational policymakers into using discriminatory trade practices, although GATT officially discourages their use.

I organize the paper as follows: In Section II I present the model in its infinite-horizon form. I analyze the model in Section III, first by examining different possible outcomes within a single period, then by looking at what can happen when the multi-period nature of the game is taken into account. In Section IV I conclude the paper by summarizing the results and suggesting avenues for further research.

\(^{11}\)With only two firms in the model, one foreign and one domestic, the optimal quota in the linear Cournot model with no collusion turns out to be zero when the government's objective function is domestic consumer surplus plus home firm profits. As it turns out this result is not necessarily true when there are \( n \) firms in the market. With \( n \) firms and no collusion, the optimal quota for the government will be either zero or non-binding (i.e. autarky or free trade) depending on the number of foreign firms relative to the number of domestic firms. These topics (along with others) are discussed in detail in Haller and Milam (1991).
Section II: An Infinite Horizon Repeated Game Model With Three Players

The model is of a quantity-setting duopoly with one home and one foreign firm. In addition, the home country’s government is a player in the game with the power to set quotas for the foreign firm. The marginal cost of each firm is constant and equal to that of the other firm. The output of both firms is sold exclusively in the home country and demand for the good is linear. Firms are profit maximizers while the government is interested in maximizing consumer surplus. A more natural objective function for the government might be the sum of consumer surplus and home firm profits, or total domestic surplus. However, the result is that\(^{12}\), even without collusion, total domestic surplus is maximized by eliminating the foreign firm from the market and allowing the home firm to act as a monopolist. This result is strongly related to the assumption that there are only two firms in the industry. To avoid this uninteresting outcome I use the simpler and somewhat less intuitive payoff function for the government. Consumer surplus is not so dramatically affected by the number of foreign and domestic firms. Again I want to remind the reader of the alternative interpretation in which both firms are foreign and the quota placed on only one firm is a discriminatory trade barrier. More explicitly the game structure is as follows:

*Strategy Spaces* (at any time period \(t\))

\(^{12}\text{Haller and Milam (1991)}\)
Home firm: \( x_i^h \in \mathbf{X}^h = \mathbb{R}^+ \).

Foreign firm: \( x_i^f \in \mathbf{X}^f = \mathbb{R}^1 \setminus [\bar{x}^f, \infty) = [0, \bar{x}^f] \).

Government: \( \bar{x}_i^f \in \mathbf{X}^f = \mathbb{R}^+ \).

**Payoff Functions**

Home firm: \( \Pi_i^h = \sum_{t=0}^{\infty} \delta^t \Pi_i^h(x_i^h, x_i^f) \).

Foreign firm: \( \Pi_i^f = \sum_{t=0}^{\infty} \delta^t \Pi_i^f(x_i^h, x_i^f) \).

Government: \( G = \sum_{t=0}^{\infty} \delta^t CS_i(x_i) \).

Where:

- \( CS_i = \) consumer surplus at time \( t \),
- \( x_i^h = \) home firm's output at time \( t \),
- \( x_i^f = \) foreign firm's output at time \( t \),
- \( \bar{x}_i^f = \) quota set by government at time \( t \),

\[
\Pi_i^i = x_i^i \cdot p_i(x_i) - c \cdot x_i^i, \quad i = h, f, \text{ with } x_i = x_i^h + x_i^f \text{ and } p_t(x_i) = \begin{cases} a - b \cdot x_i & \text{if } a - b \cdot x_i \geq 0 \\ 0 & \text{otherwise} \end{cases}.
\]

At any time \( t \), each player knows the previous moves of all the players. Let the history of the game at time \( t \) be defined as,

\[
h_t = [(x_0^h, x_0^f), (x_1^h, x_1^f), \ldots, (x_{t-1}^h, x_{t-1}^f)].
\]
The history of the government’s actions is omitted for simplicity.

Section III: Analysis

Here I investigate the different effects of a quota and a VER on the two firms’ output levels. I will ignore the government’s payoffs temporarily, with the quota level being chosen for convenience.

III.1 Single-shot Game

First, I examine the potential actions of the two firms in the single-shot game and then introduce repetition. While in the context of the single-shot game, I suppress the time subscripts and the firms will be concerned only with their one-period profits.

No binding quota

Firm $i$ seeks to solve the following problem,

$$\max_{x^i \in X^i} \prod^i (x^i, x^j) \quad i = h, f.$$ 

This yields the best-reply functions, which are:

$$x^i = B_r^i(x^j) = \arg \max_{x^i} \prod^i (x^i, x^j) = \frac{a - c}{2b} - \frac{1}{2} x^j$$

with $i, j = h, f$ and $i \neq j$. 

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First consider the case where no binding quota is placed on the foreign firm. Solving the best-reply functions simultaneously yields the Cournot-Nash equilibrium, \( x^h = x^f = \frac{a-c}{3b} \). Therefore total output equals \( \frac{2}{3} \frac{a-c}{b} \). However this outcome is not Pareto optimal with respect to the firms. To show this, combine the two firm's profit functions to form the joint profit function,

\[
\Pi = x \cdot p(x) - c \cdot x
\]

then maximize this function w. r. t. \( x \), which yields:

\[
\arg\max_{x \in \mathbb{R}^1} \Pi = \frac{a-c}{2b}.
\]

This implies that if the two firms restrict total output to \( x = \frac{a-c}{2b} \), an appropriate division of the profits could make both firms better off. Figure 2 illustrates the relationship in quantity space.

Here simplifying assumptions regarding the nature of collusion between the two firms are made. First, if the firms collude, they will always pick a jointly Pareto-optimal outcome. This seems reasonable for rational players. The second assumption is more arbitrary. It is that when they collude, the firms will divide output (and profits) symmetrically. The analysis would still hold for other divisions of output, although some allocations might be sustainable while others were not.\(^{13}\)

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\(^{13}\) This statement is not technically correct since, in the single-shot game, "sustainability" has no meaning because it involves the concept of punishment in future periods for actions taken in the current period and in a single-shot game there is only one period. Indeed, none of the Pareto optimal allocations are a stable equilibrium, since at least one player will always have an incentive to deviate.

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Binding quota

Next consider the case where a binding quota, \( \bar{x}' \), is placed on the foreign firm. Since the (unrestricted) Cournot-Nash equilibrium output for the foreign firm is \( x' = \frac{a-c}{3b} \), \( \bar{x}' \) must be in the interval \([0, \frac{a-c}{3b}]\) in order to be binding. The best response of the domestic firm is,

\[
BR^h(\bar{x}') = \frac{a-c}{2b} - \frac{1}{2} \bar{x}'
\]

because the foreign firm will always choose to produce output equal to the maximum allowed by the quota for \( \bar{x}' \in [0, \frac{a-c}{3b}] \). Total output is

\[
x = BR^h(\bar{x}') + \bar{x}' = \frac{a-c}{2b} + \frac{1}{2} \bar{x}'.
\]

The (restricted) Cournot-Nash equilibrium profits of the firms as a function of \( \bar{x}' \) are

\[
\Pi^h = \frac{(a-c)^2}{4b} - \frac{a-c}{2} \bar{x}' + \frac{b}{4} (\bar{x}')^2
\]

and

\[
\Pi' = \frac{a-c}{2} \bar{x}' - \frac{b}{2} (\bar{x}')^2.
\]

Notice that \( \Pi^h \) is decreasing in \( \bar{x}' \) and \( \Pi' \) is increasing in \( \bar{x}' \) for all \( \bar{x}' \in [0, \frac{a-c}{3b}] \).

As an example, let the quota be \( \bar{x}' = \frac{a-c}{4b} \). The Cournot-Nash outcome in the presence of the quota would then be, \( x' = \frac{a-c}{3b} \) and \( x = \frac{a-c}{3b} \). At this point the choice of the quota is made for convenience and should just be considered as an illustration of a certain type of outcome. Later, when the government's strategy is considered in the framework of a repeated game, an optimal quota will be derived.

Table 1 illustrates that whereas total output is highest at the unrestricted Cournot-Nash equilibrium, when the Cournot-Nash outcome under the quota is compared to the collusive
outcome both total output and home-firm profits are higher.

III.2 Repeated Game

At this point I introduce repetition and consider the effects of VER’s and quotas in this context. Would the foreign firm have any incentive in this model to adopt a VER and, if adopted, to abide by it once adopted? The answer is yes if the VER is part of a trigger strategy being played by the two firms. In other words, if the two firms have an agreed-upon collusive strategy, the VER could just be a public announcement of the collusive output level for the foreign firm.

Nonbinding or no quota

Consider the following trigger strategy:

\[
x_i^t(h_t) = \begin{cases} 
  x_i^{t,p} & \text{if } h_t = h_i^*, \\
  x_i^{t,c} & \text{if } h_t \neq h_i^*
\end{cases} \quad t=1,2,3,4,\ldots \ldots \quad i=h,f
\]

and \[x_i^{t}=x_0^{i,p} \quad i=h,f\]

where \( h_i^* \) = the history of the game at time \( t \) along the desired equilibrium path

\[= [(x_0^{h,p},x_0^{f,p},x_1^{h,p},x_2^{f,p},\ldots, x_{t-1}^{h,p},x_{t-1}^{f,p})] = (x_0^{p},x_1^{p},\ldots, x_{t-1}^{p})\]

\[x_i^{p} = (x_i^{h,p},x_i^{f,p})\]

\[x_i^{i,p} = \frac{a-c}{4h} \quad \text{is the collusive, Pareto optimal output level}\]
\[ x_i^{c} = \frac{a - c}{3b} \] is the Cournot-Nash single-shot equilibrium output level for \( t = 0, 1, 2, 3, \ldots \); \( i = h, f \)

The trigger strategy is this:

The firms agree to produce the collusive level of output in period zero. Each firm promises to produce the collusive output level in every period thereafter, so long as the other firm does the same. However, if either firm deviates, the other firm will increase its output to the single-shot Nash equilibrium level in the following period and continue producing at that level forever. Since the punishment phase of the trigger strategy involves playing the Cournot-Nash single-shot equilibrium strategy, the above trigger strategy is subgame perfect as shown by Friedman (1986).

Let:

\[ \Pi_{p,0}^i = \Pi_i^i(h_i^p, x_i^{p,0}) \] firm \( i \)'s profit at time \( t \) when both firms produce at the collusive Pareto-optimal output level,

\[ \Pi_c^i = \Pi_i^i(h_i^{c}, x_i^{c}) \] firm \( i \)'s profit at time \( t \) in the Cournot-Nash equilibrium, and

\[ \Pi_{c,h}^i = \Pi_i^i(BR_i^i(h_i^c, x_i^{c}), x_i^{p,0}) \] firm \( i \)'s profit when it cheats on the collusive agreement at time \( t \) while firm \( j \) abides by the agreement

\[ t = 0, 1, 2, 3, 4, \ldots \; ; \; i, j = h, f; \; i \neq j. \]

This trigger-strategy pair yields the collusive outcome discussed in the context of the single-shot game. To determine if the trigger-strategy pair constitutes a subgame-perfect equilibrium, I need to compare the payoff to each firm when it abides by the agreement with the payoff it could get
by cheating. The payoff to firm $i$ of abiding by the agreement when $j$ does is:

$$\sum_{t=0}^{\infty} \delta^t \Pi_{p^o_i} = \frac{\Pi_{p^o_i}}{1-\delta} ; \quad i = h, f$$

However, if firm $i$ chooses to deviate in period $t'$ by producing $x^{i'}_{t'} = BR_{t'}(x^{j,p^o}_{t'})$ while firm $j$ produces $x^{j,p^o}_{t'}$, then the payoff to firm $i$ will be:

$$\sum_{t=0}^{t'-1} \delta^t \Pi_{p^o_i} + \delta^{t'} \Pi_{c_i} + \sum_{t=t'+1}^{\infty} \delta^t \Pi_{c} = \frac{1-\delta^{t'}}{1-\delta} \Pi_{p^o_i} + \delta^{t'} \Pi_{c_i} + \frac{\delta^{t'+1}}{1-\delta} \Pi_{c} ; \quad i,j = h,f; \quad i \neq j.$$

So in order for the collusive outcome to be sustainable by a subgame-perfect trigger strategy, the payoff to either firm of abiding by the agreement must be greater than the payoff it could receive by cheating, i.e.:

$$\frac{\Pi_{p^o_i}}{1-\delta} > \frac{1-\delta^{t'}}{1-\delta} \Pi_{p^o_i} + \delta^{t'} \Pi_{c_i} + \frac{\delta^{t'+1}}{1-\delta} \Pi_{c} ; \quad i = h,f$$

which is equivalent to,

$$\delta > \frac{\Pi_{c_i} - \Pi_{p^o_i}}{\Pi_{c_i} - \Pi_{c_i}} ; \quad i = h,f.$$

**Effect of binding ex-ante quota**

Now consider the role of government in this model. The above analysis hinged on the unstated assumption that the government-imposed quota was nonbinding. Recall the government's objective function is:
\[ G = \sum_{t=0}^{\infty} \delta^t CS_t \quad \text{where} \quad CS_t = \int_{0}^{x_t} P(x_t) \, dx_t - x_t P(x_t) = \frac{b}{2} x_t^2 \]

so, \[ G = \frac{b}{2} \sum_{t=0}^{\infty} \delta^t x_t^2. \]

Consider two sequences of output \( \{x_t\}_0^\infty \) and \( \{x'_t\}_0^\infty \).

Define \( \{x_t\} \geq \{x'_t\} \iff \forall t \exists t^* \text{ s.t. } x_{t^*} > x'_{t^*} \).

Clearly, \( \{x_t\} \geq \{x'_t\} \implies G(\{x_t\}) \geq G(\{x'_t\}) \). Therefore, the government would prefer \( \{x_t\} \) to \( \{x'_t\} \) where \( x_t^* = \frac{\bar{a} - \bar{c}}{\bar{b}} \) and \( x'_t^* = \frac{\bar{a} - c}{2\bar{b}} \). So obviously the government would prefer that the two firms didn’t collude. ("Pareto optimal" symbolized by \( p_o \) refers exclusively to the firms' welfare.)

By assumption, the only policy tool the government has at its disposal is its choice of quota. To understand how the quota will affect the behavior of the firms, the sustainability condition,

\[ \delta > \prod_{i=0}^{h} \frac{\Pi_{i}^h - \Pi_{i}^{p_o}}{\Pi_{i}^h - \Pi_{i}^f} \quad i = h, f \]

needs to be considered,\(^{14}\) in particular the inequality for the home firm becomes,

\[ \delta > \prod_{i=0}^{h} \frac{\Pi_{i}^h - \Pi_{i}^{p_o}}{\Pi_{i}^h - \Pi_{i}^f}. \]

Notice that each of the \( \Pi \)'s is a function of \( \bar{x}_t' \). For the situation here, \( \bar{x}_t' \) will be in the interval \( [\frac{\bar{a} - \bar{c}}{4\bar{b}}, \infty) \).\(^{15}\) When \( \bar{x}_t' < \frac{\bar{a} - \bar{c}}{4\bar{b}} \) the story becomes more complicated without generating any additional insight. For \( \bar{x}_t' \in [\frac{\bar{a} - \bar{c}}{4\bar{b}}, \frac{\bar{a} - \bar{c}}{3\bar{b}}] \), only \( \Pi_{i}^h \) will vary and for \( \bar{x}_t' > \frac{\bar{a} - \bar{c}}{3\bar{b}} \) all the \( \Pi \)'s are

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\(^{14}\)There is some ambiguity at the point \( \bar{x}_t'^* \). For \( \beta(\bar{x}_t') \) as defined on the following page, notice that, \( \beta(\bar{x}_t') = \delta \) when \( \bar{x}_t' = \bar{x}_t'^* \). I assume here that for a trigger strategy to be sustainable, \( \beta(\bar{x}_t') \) must be strictly less than \( \delta \). Therefore, for any \( \bar{x}_t' \) such that \( \beta(\bar{x}_t') \geq \delta \), the trigger strategy is not sustainable and the two firms will revert to a non-cooperative Cournot-Nash equilibrium. Technically, this is not correct since when \( \beta = \delta \) the domestic firm would be indifferent between abiding by the agreement and cheating, therefore collusion would still be a (weak) equilibrium. To break up collusion I would have to make \( \beta = \delta + \epsilon \) for some arbitrarily small \( \epsilon > 0 \).
constant. Assume the initial situation at time zero is for $\bar{x}_t' \geq \frac{a-c}{3b}$ (i.e. a non-binding quota).

Define a function $\beta: \left[ \frac{a-c}{4b}, \infty \right) \rightarrow \mathbb{R}^+$ s. t.

$$\beta(\bar{x}_t') = \frac{\prod_{i \in \Lambda} h_{i}^{b} - \prod_{i \in \Lambda} h_{b}^{d}}{\prod_{i \in \Lambda} h_{i}^{b} - \prod_{i \in \Lambda} h_{b}^{d}(\bar{x}_t')}$$

where $\prod_{i \in \Lambda} h_{i}^{b}(\bar{x}_t') =$

$$= \left\{ \begin{array}{ll}
\frac{(a-c)^2}{4b} - \frac{a-c}{2} \cdot \bar{x}_t' + \frac{b}{4} \bar{x}_t'^2 & \text{MAX} \prod_{i \in \Lambda} h_{i}^{b}(x_i^b, \bar{x}_t') \text{ for } \bar{x}_t' \in (0, \frac{a-c}{3b}) ; \\
\frac{a-c}{3b} & \text{MAX} \prod_{i \in \Lambda} h_{i}^{b}(x_i^b, \frac{a-c}{3b}) \text{ otherwise.}
\end{array} \right.$$ 

If by varying $\bar{x}_t'$, $\beta(\bar{x}_t')$ becomes greater than or equal to $\delta$, then the trigger strategy will no longer be a subgame-perfect equilibrium. Consider the graph of $\beta(\bar{x}_t')$ in Figure 3. $\frac{9}{17}$ is the value of $\beta(\bar{x}_t')$ when $\bar{x}_t'$ is non-binding. As $\bar{x}_t'$ falls from $\frac{a-c}{3b}$ to $\frac{a-c}{4b}$, $\prod_{i \in \Lambda} h_{i}^{b}$ decreases causing $\beta(\bar{x}_t')$ to rise.

The $\bar{x}_t'$ of interest would be that $\bar{x}_t'^*$ such that $\beta(\bar{x}_t'^*) = \delta$, since that would be the maximum $\bar{x}_t'$ that would prevent the trigger strategy from being sustainable. It can be shown that for any $\delta \in (\frac{9}{17}, 1]$, $\exists 1 \bar{x}_t'^* \in (\frac{a-c}{4b}, \frac{a-c}{3b})$ s. t. $\beta(\bar{x}_t'^*) = \delta$. (Proof in appendix).

**Government quota trigger strategy**

While imposing an ex-ante quota can prevent equilibrium collusion, the actual imposition

\[ \text{restrict } \bar{x}_t' \text{ to this interval for simplicity without any loss of generality since, given the fact that the government is an output maximizer, it will always be in its interest to choose the least restrictive quota that will break up the firms' collusive agreement and this will always occur for } \bar{x}_t' \geq \frac{a-c}{4b}. \]
of the quota is not necessary. The threat of a quota may suffice to deter collusive behavior. Namely, since \( \bar{F}^* \) exists, a government trigger strategy to prevent collusion can be developed. First I make some assumptions about the sequential nature of the game. Each time period \( t \) will be subdivided into two parts, \( t_1 \) and \( t_2 \). \( t_1 \) occurs before \( t_2 \) and \( t_1 \) is when the government announces its quota. At \( t_2 \) both firms are aware of the quota announced at \( t_1 \) and they simultaneously choose their output levels. Let \( \bar{F}_t(h_t) \) be the quota the government chooses as a function of the history of the game at time \( t \).

Define:
\[
\bar{F}_t(h_t) = \begin{cases} 
\frac{a-c}{ab} & \text{if } h_t = h_t^c \\
\bar{F}^* & \text{if } h_t \neq h_t^c 
\end{cases} \quad t=1,2,3, \ldots \ldots 
\]
and
\[
\bar{F}_0(h_0) = \frac{a-c}{3b}
\]
where
\[
h_t^c = (x_0^c, \ldots, x_{t-1}^c) \quad t=1,2,3, \ldots
\]
and
\[
x_t^c \in \mathbb{R}^2
\]
is the equilibrium output that occurs in the absence of both collusion and binding quotas at time period \( t \).

A verbal description of how the trigger strategy operates is as follows. Suppose at some time \( t-1 \) the government observes collusion on the part of the two firms (i.e. \( h_t \neq h_t^c \)). Next period, the government would impose the quota, \( \bar{F}_t(h_t) = \bar{F}^* \). In the previous plays of the game, when the quota for the foreign firm was equal to or greater than its Cournot-Nash level of output, the collusive strategy yielding the symmetric Pareto-optimal outcome was a subgame-perfect equilibrium. However, with the quota set at \( \bar{F}^* \), the ability of the foreign firm to retaliate in response to cheating by the home firm has been restricted to the point that the deterrent effect is
not enough to prevent the home firm from deviating from the agreed-upon collusive strategy. Therefore the two firms will shift back to a non-cooperative Cournot-Nash equilibrium. The output at this equilibrium will be greater for both firms than what they produced while colluding, however, the total output will be less than would occur at the Cournot-Nash equilibrium with a non-binding quota. Figure 4 illustrates the three outcomes discussed previously. The best reply functions \( BR^h \) and \( BR^f \) describe the best replies for the one-shot game, just as the Cournot-Nash points refer to single-shot Cournot-Nash equilibria. It is easy to see that total output is greatest at \( C_{eq} \) and least at \( PO \).

One additional note concerns the effect of a government quota on the sustainability condition for the foreign firm. The condition can be written as:

\[
\delta > \frac{\prod_{ch} (x^f_t) - \prod_{po}^f}{\prod_{ch} (x^f_t) - \prod_{c}^f (x^f_t)}
\]

The effect of a binding quota of the type I consider here would be to decrease the ratio, at least for interesting values of \( x^f_t \). Thus the imposition of the quota would only strengthen the foreign firm’s incentive to abide by the collusive agreement and therefore has no relevance.

Recall that the government’s objective function is increasing in total output. Now I examine each firm’s reaction to a given government quota. Figures 5 & 6 describe the reaction of the two firms to any government quota given that the firms have agreed to follow the symmetric Pareto-optimal collusive strategy. Figure 5 shows the response of the home firm, while Figure 6 gives the response of the foreign firm. When the government sets \( x^f_t = x^f^* \), both firms are aware of this and realize that the trigger strategy is no longer sustainable. At this point firms move immediately to the Cournot equilibrium and stay there.\(^{16} \) The Cournot output level is greater than the output that would occur with the collusive outcome, therefore the government is better
For the government's trigger strategy to be sufficient to prevent collusion, it has to be sustainable. For sustainability, the following inequality must hold:

$$\delta > \frac{\prod_{i=0}^{t} \prod_{j=0}^{t} \prod_{k=0}^{t}}{\prod_{i=0}^{t} \prod_{j=0}^{t} \prod_{k=0}^{t}}$$

where \(\prod_{i=0}^{t} \prod_{j=0}^{t} \prod_{k=0}^{t}\) is equal to the profits earned by the foreign firm when the government quota is \(q^*\) and the firms are producing at the restricted Cournot-Nash equilibrium. As it turns out, the government's trigger strategy is sustainable for values of \(\delta \in \left[ \frac{169}{225}, 1 \right]\) (proof in appendix). The reason it is not sustainable for \(\delta \in \left[ \frac{9}{17}, \frac{169}{225} \right]\) is that when the foreign firm discounts the future rapidly enough, the initial gain from colluding with the domestic firm will outweigh the loss the foreign firm incurs when the government imposes the quota which both breaks down the collusion and restricts the foreign firm's output to less than it would produce at the non-collusive, free trade equilibrium.

**Government Collusion with the Domestic Firm**

The above discussion of a government trigger strategy contains an implicit assumption that free trade with no collusion is the best obtainable outcome. This assumption rules out the

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16 One possible alternative would be that when \(q^*\) is imposed, both firms attempt to cheat simultaneously. This has the same effect on the foreign firm's action as previously, since it is restricted to produce no more than \(q^*\). However, in the case of the home firm, when it attempts to cheat on the agreed upon collusive strategy, it produces more than it would at the Cournot equilibrium. Then in period \(t+1\) the home firm moves to the Cournot equilibrium level of output while the foreign firm's actions remain unchanged. In this case the response of the home firm is illustrated in Figures 7 & 8. The problem with looking at alternative adjustment processes such as the one above is that it opens a can of worms with respect to the firms' beliefs about the actions of the other. For example, the above case implies that there is a divergence in the perceptions of the two firms. The domestic firm anticipates the response of the foreign firm, while the foreign firm believes the domestic firm's behavior will remain unchanged.
interesting possibility of some sort of collusive agreement between the government and the home firm. By ignoring this possibility (and another to be considered later) we implicitly restrict the government to a somewhat passive role in the game. Removing this restriction allows us to consider outcomes that improve on free trade with no collusion by yielding higher total output levels.

The form of a collusive agreement between the government and the domestic firm will be dependent on the assumptions made about the behavior of the players if the agreement breaks down. Three possibilities are: First, the firms could revert to playing the single-shot Nash equilibrium strategies with no binding quota in place. Second, the firms could revert to the collusive output level, again with no binding quota in place. Third, the government could put in place the previously discussed quota $\bar{x}^f$ which would be just sufficient to prevent collusion between the firms and therefore they would revert to the restricted single-shot Nash equilibrium.

I'll assume that the firms will revert to the single-shot Cournot-Nash equilibrium if the agreement breaks down. This is equivalent to assuming that the government can prevent collusion from occurring (i.e. $\delta \in \{ 100, 1 \}$). In general, the agreement would take the form of the government eliminating the foreign firm from the market ($\bar{x}^f = 0$) in return for the domestic firm promising to produce output above some specified amount. The amount chosen would have to be greater than the amount that would be produced in a single-shot Nash equilibrium in order for the government to be willing to enter into the agreement; in addition, the output would have to be small enough to make the domestic producer willing to abide by the agreement. The reason output would fall within this range is because the government could use the previously described trigger strategy to insure the Nash total output level and the home producer would most prefer to produce the monopoly output level (less than the Nash total output level) when the foreign firm is excluded from the market. Since the government is a Stackelberg leader, it will use its strategic
advantage to induce the home firm to produce as much as it would be willing to without giving it a strong enough incentive to break the agreement. This output level can be found by solving the inequality below for $\hat{x}^h$:

$$\prod_{r}^{o} \geq \prod_{c}^{r} + \delta \prod_{c}^{t}.$$  

Where:

$\prod_{r}^{o} =$ the profit to the home firm when it abides by the agreement and $x' = 0$,

$\prod_{c}^{r} =$ the profit to the home firm when it cheats on the agreement and $x' = 0$,

(this is just the monopoly profit)

$\prod_{c}^{t} =$ the profit to the home firm at the single-shot Cournot-Nash equilibrium, and

$\hat{x}^h =$ the output level agreed to by the home firm and the government.

The above inequality must hold in order for the trigger strategy to be sustainable. It can easily be seen that only $\prod_{r}^{o}$ will be affected by the choice of $\hat{x}^h$ while the other profit values remain constant. What this inequality represents is just the sustainability condition faced by the home firm under this agreement. When I solve the inequality for $\hat{x}^h$ the value for $\hat{x}^h$ will represent the largest output that the home firm would be willing to produce and not decide to cheat on the agreement. The solution to this inequality yields:

$$\hat{x}^h = a - c \left( 1 + \left( \frac{5\delta}{9} \right)^{\frac{1}{2}} \right)$$

as the maximum output the home firm would be willing to produce in order to keep the foreign firm out of the market. This output will be greater than the total output produced at the single-shot Cournot-Nash equilibrium as long as $\delta > \frac{1}{5}$. I don't have to consider whether the government would cheat since, given the structure of this model, lifting the quota would not have any beneficial effect for the government because the domestic firm would be able to adjust its output
in the same period, thereby eliminating any potential gain to the government.

Two comments with respect to the effects of the structure of the model are in order here: First, the assumption that the government’s objective function includes only domestic consumer surplus can be criticized as somewhat unrealistic, however the inclusion of domestic firm profits in the objective function would only strengthen the government’s incentive to exclude the foreign firm. A second, and more damaging, criticism concerns the effect of the constant-marginal-cost assumption on this conclusion. It’s obvious that in the presence of rising marginal cost, the home firm would not be willing to agree to as high an output level as it would with constant marginal costs and thus the likelihood that its output level would be high enough to satisfy the government under the agreement (more output than would occur under free trade) is reduced.

Government Collusion with the Foreign Firm

The reasons given why the government might not want a collusive agreement with the home firm suggest the intriguing possibility of a collusive agreement between the government and the foreign firm in which the government requires that the foreign firm produce so much output that it makes almost no profit and, to insure compliance, the government uses the threat of completely excluding the foreign firm from the market. However the government’s threat is not subgame-perfect since imposing the punishment would in effect make the domestic firm a monopolist in the market. This still leaves open the possibility of a more complex agreement which would make the deal with the foreign firm stick. For example, the government could threaten the foreign firm with the implementation of the previously discussed agreement with the domestic firm which involved the domestic firm producing more output than would occur under the single-shot Cournot-Nash equilibrium. I’ll examine this in more detail for the simple case where $a = b = 1$ and $c = 0$. 

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In order to ensure that the government has the ability to carry out the threat involved in the agreement, assume $\delta \in \left[\frac{15}{22}, 1\right]$. The first step in developing the trigger strategy is to determine whether the government's threat of completely excluding the foreign firm from the market is sufficient to deter the foreign firm from deviating from the agreement. Let $\Pi_{ch} = $ the profit to the foreign firm when it cheats on the agreement and $\Pi_a = $ the profit (per period) to the foreign firm from abiding by the agreement. Since the punishment the foreign firm would incur from cheating is exclusion from the market, the sustainability condition is just

$$\sum_{t=0}^{\infty} \delta^t \Pi_a \geq \Pi_{ch} \Leftrightarrow \frac{1}{2} \frac{1}{1-\delta} (x^f - x^* )^2 \geq \frac{1}{\delta} ,$$

since the profit to the foreign firm after cheating will be zero. The competitive market solution for this model in terms of total output is $x = 1$. It can be seen that $x^f$, the maximum output by the foreign firm that can be induced using the trigger strategy, converges to 1 as $\delta \rightarrow 1$; furthermore, the output by the domestic firm determined by its best reply function will be $1 - \frac{x^h}{2}$ making total output even higher. The next relevant question is, for what values of the discount factor will this trigger strategy induce higher total output than would occur using the previous trigger strategy which excluded the foreign firm in exchange for a high output level by the domestic firm. As it turns out, this is true for all $\delta \in (0,1)$. Intuitively this is not surprising since the punishment inflicted by the latter trigger strategy in response to deviations is much harsher and since abiding by the agreement leads to positive output levels by both firms. By contrast, the preceding trigger strategy would still allow the deviating firm to receive positive profits in the punishment phase (a harsher punishment would not be subgame perfect in the class of grim trigger strategies), and would induce a high output level by the domestic firm in exchange for the exclusion of the foreign firm from the market.

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Section IV: Conclusion

First, I have shown that quotas and VER's may affect the actions of the two firms differently. A VER can be a symptom of collusion between the two firms, in the sense that if the VER is truly voluntary it can be a 'signal' or announcement to the other firm of the importing firm's willingness to produce the collusive output level. Alternatively a quota, if restrictive enough, can break up a collusive agreement between the two firms by taking away from the firm subject to the quota the ability to punish deviations by the other firm.

Second, I have developed a unilateral trigger strategy on the part of the government which, for large enough values of the discount factor, would prevent collusion from occurring. It does this by making the (credible) threat of imposing a quota on one of the firms if it observes evidence of collusion, a quota which strangely enough will not be binding if imposed. In other words, when the government uses this trigger strategy on firm one, the quota, $\bar{Q}/\kappa$, if implemented, would be greater than firm one's actual output level. However, the quota is set low enough so that the firm subject to it would not have the ability to sell a sufficient amount of output to deter the other firm from breaking the collusive agreement.

Third, I have considered the implementation of an agreement which, by excluding one firm, induces the other to produce more than would be produced at the single-shot Cournot-Nash equilibrium. It would do this by excluding one firm from the market in exchange for the other firm more than doubling its output. The producing firm would be induced to abide by this agreement, as opposed to producing the monopoly output level, by the threat of having the quota on the excluded firm lifted.

Fourth, I discussed the possibility of implementing an agreement which would induce the one firm to produce at an output level which would give it almost no profit under the threat of being completely excluded from the market. This agreement, if sustainable, would lead to the
highest total output. In fact, for discount factors equal to 1 (which means the firms care as much about the future as they do the past), the enforceable output would be virtually equal to that under perfect competition.

As explained in the introduction, the model can be reinterpreted as one with two foreign firms and provide a rationale for discriminatory trade policy. On the other hand, if we stick to the original interpretation, with one domestic and one foreign firm, then the near efficiency result illustrates the possible role of trade policy as a substitute for industrial policy. On a more general note, one would expect an interdependence of the effects of trade and industrial policies in models with imperfect competition.
Appendix

A.1

Prove that $\forall \delta \in (\frac{9}{17}, 1) \exists ! \quad \bar{x}_t' \in \left(\frac{a-c}{4b}, \frac{a-c}{3b}\right)$ s.t. $\beta(\bar{x}_t') = \delta$.

$$\beta(\bar{x}_t') = \delta \iff \frac{1}{4} \bar{x}_t'^2 - \frac{a-c}{2} \bar{x}_t' + \frac{(a-c)^2}{4b} + \frac{1-\delta}{\delta} \cdot \prod_{ch} b - \frac{1}{\delta} \cdot \prod_{po} b = 0$$

This quadratic equation yields two possible solutions:

$$\bar{x}_t'' = \frac{a-c}{b} + \sqrt{\frac{1}{b} \cdot (\prod_{po} b - (1-\delta) \prod_{ch} b)}$$

$$\bar{x}_t''' = \frac{a-c}{b} - \sqrt{\frac{1}{b} \cdot (\prod_{po} b - (1-\delta) \prod_{ch} b)}$$

These solutions are real as long as,

$$\delta \geq \frac{\prod_{ch} b - \prod_{po} b}{\prod_{ch} b}$$

which always holds, since

$$\frac{\prod_{ch} b - \prod_{po} b}{\prod_{ch} b} \geq \frac{\prod_{ch} b - \prod_{po} b}{\prod_{ch} b}$$

and, by assumption,

$$\frac{\prod_{ch} b - \prod_{po} b}{\prod_{ch} b} \geq \frac{\prod_{ch} b - \prod_{po} b}{\prod_{ch} b}.$$  Since the term under the radical is always non-negative, $\bar{x}_t''$ can't be the solution, because $\bar{x}_t'' \geq a-c > \frac{a-c}{3b}$. For $\bar{x}_t'''$ to be a viable solution, it has to be true that

$$\frac{a-c}{4b} < \bar{x}_t''' < \frac{a-c}{3b}.$$
By substituting in the values for $\Pi_{P0}^h$ and $\Pi_{ch}^h$, it can be shown that

$$\overline{x}_t''' = \frac{a-c}{b} \left( 1 - \frac{1}{4} \sqrt{9 - \frac{1}{\delta}} \right) .$$

Therefore the inequality involving $\overline{x}_t'''$ is true for all $\delta \in (\frac{9}{17}, 1)$. So $\overline{x}_t' = \overline{x}_t'''$ is the unique $\overline{x}_t'$ that solves $\beta(\overline{x}_t') = \delta$. QED

A.2.

Determine for what range of values of the discount factor, $\delta$, the government's trigger strategy for preventing collusion is sustainable.

In order for the trigger strategy to be sustainable, the following inequality must hold:

$$\delta > \frac{\Pi_{P0}'}{\Pi_{P0}^h - \Pi_{c}'}$$

Taking the values for $\Pi_{P0}' = \frac{23}{8} \alpha = \frac{1}{4} \alpha$ and $\Pi_{c} = \frac{64}{876} \alpha = \frac{1}{3} \alpha$ given in the table below, we need to determine an expression for $\Pi_{c}'$. Recall $\Pi_{c}'$ is just the profit to the foreign firm when it produces at an output level equal to $\overline{x}_t''$, the largest quota that will break up the collusive agreement. Plugging in $\overline{x}_t''$ and the domestic firm's best reply to this output yields a profit to the foreign firm of

$$\Pi_{c}' = \left( \frac{1}{8} \sqrt{9 - \frac{1}{\delta}} - \frac{1}{32}(9 - \frac{1}{\delta}) \right) \alpha .$$
Substituting these values into the above inequality yields:

\[
\delta > \frac{\frac{1}{\delta^2}}{\frac{13}{32} - \frac{1}{6} \sqrt{9 - \frac{1}{\delta^2} - \frac{1}{32}}}
\]

\[
\Leftrightarrow \text{for } \delta \in (0, 1]\frac{1}{\delta} \quad \delta \left(\frac{13}{32} - \frac{1}{6} \sqrt{9 - \frac{1}{\delta^2} - \frac{1}{32}}\right) > \frac{1}{\delta^2}
\]

\[
\Leftrightarrow 234 \delta - 72 \sqrt{9 \delta^2} - \delta - 26 > 0
\]

So to show that the government's trigger strategy is sustainable, I need to find for what values of \( \delta \) the above inequality is true. Furthermore I limit my search to \( \delta \in (\frac{5}{11}, 1) \) since these are the feasible values such that the firms have an incentive to collude. Let

\[
\mathcal{f}(\delta) = 234 \delta - 72 \sqrt{9 \delta^2} - \delta - 26
\]

\( \mathcal{f}(\delta) \) has a local minimum at \( \frac{1}{6} \) and is increasing thereafter; in addition \( \mathcal{f}(\delta) = 0 \) when \( \delta = \frac{169}{228} \). Therefore the inequality is true for all \( \delta \in (\frac{169}{228}, 1] \quad \text{QED} \)
References


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### Table 1

<table>
<thead>
<tr>
<th>Cournot</th>
<th>Collusion</th>
<th>Quota</th>
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<tbody>
<tr>
<td>$\Pi^h$</td>
<td>$\frac{64}{576} \alpha$</td>
<td>$\frac{72}{576} \alpha$</td>
</tr>
<tr>
<td>$\Pi'^h$</td>
<td>$\frac{64}{576} \alpha$</td>
<td>$\frac{72}{576} \alpha$</td>
</tr>
<tr>
<td>$\sigma^h$</td>
<td>$\frac{8}{24} \gamma$</td>
<td>$\frac{6}{24} \gamma$</td>
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<td>$\sigma'^h$</td>
<td>$\frac{8}{24} \gamma$</td>
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</tr>
<tr>
<td>$\sigma$</td>
<td>$\frac{16}{24} \gamma$</td>
<td>$\frac{12}{24} \gamma$</td>
</tr>
</tbody>
</table>

where $\alpha = \frac{(a-c)^2}{b}$ and $\gamma = \frac{a-c}{b}$
Figure 1
symmetric Pareto optimal outcome

FIGURE 2
Figure 3
Cournot-Nash equilibrium (with binding quota of $x_i^*$) = $C_i$

Cournot-Nash equilibrium (with non-binding quota) = $C_{n\ell}$

Pareto optimal outcome = P. O.

FIGURE 4
Vita

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