Active Control of Sound Radiation from Fluid Loaded Plates

by

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Abstract

Active control of sound radiation due to subsonic wave scattering from an infinite or
a finite fluid-loaded plate excited below the critical frequency is analytically studied.
The disturbance is caused by a flexural wave in an infinite plate, or by a point force on
a finite plate at subsonic frequencies. The wave scattering is caused by discontinuities
on the plate or by the boundary conditions.

A feed-forward control approach is applied by implementing either point/line forces
or piezoelectric actuators on the plate. The amplitude and phase of control forces are
determined by the optimal solution of a cost function which minimizes the far-field
radiated acoustic power over a prescribed surface in the half space of the fluid field.

The results show that for subsonic excitations, high global reduction in radiated pres-
sure is possible with properly located active control forces. The number and location
of control forces employed in order to obtain high control performance are related to
the excitation frequency. The far-field sound radiation directivity pattern, the modal
amplitudes of the plate vibration, the plate vibration autospectrum in the wave num-
ber domain, and the near-field intensity distribution are extensively studied in order to uncover the mechanisms of control.
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Nomenclature

\{A\} \quad \text{distribution vector of the control forces}

\[A\]_{N_c \times N_c} \quad \text{distribution matrix of } N_c \text{ control forces}

A_j, \hat{A}_j \quad \text{the } j\text{-th element of vector } A

a \quad \text{the length of a rectangular plate in the } x\text{-direction, } m

\{a\} \quad \text{distribution vector of control forces when } V_o \text{ is the disturbance}

a_n \quad \text{the } n\text{-th element of vector } \{a\}

\{B\} \quad \text{distribution vector of the disturbances}

\[B\] \quad \text{distribution matrix of the disturbances}

B_i \quad \text{the } i\text{-th element of vector } \{B\}

b \quad \text{the length of a rectangular plate in the } y\text{-direction, } m

\{b\} \quad \text{distribution vector of the disturbance when } V_o \text{ is the input}

b_1 \quad \text{distribution parameter when } V_o \text{ is the input}

[C] \quad \text{contribution matrix of the disturbance}

c_o, c_f \quad \text{sound velocity in the acoustic medium sea water if}

\text{not otherwise specified, } m/s

c_p \quad \text{flexural wave speed in the plate, } m/s

D \quad \text{flexural rigidity of the plate, } Nm

E \quad \text{plate material modulus of elasticity, } N/m^2

F, F_o, \hat{F}_o \quad \text{amplitude of the excitation line force, } N/m
$F_n$  
amplitude of the n-th line control force, $N/m$

$F_{rm}(u)$  
 auxiliary function to estimate the fluid-loading impedance

$f_{mn}$  
mode coefficient of point force

$G_{sv}(v)$  
 auxiliary function to estimate the fluid loading impedance

$H(r_a, r)$  
Heaviside function

$H_i$  
the first kind of Hankel function of the i-th order

$h$  
thickness of the plate, $m$

$I(r_a, r)$  
integral to estimate plate displacement $w(r)$

$I_{p1, p2}^{+}$  
total impedance including the plate stiffness, the fluid loading, and the attached mass

$I_{p1, p2}$  
branch cut integral to estimate pressure $\bar{p}_P(X, Z)$

$I_{p1, p2}'$  
branch cut integral to estimate pressure $\bar{p}_M(X, Z)$

$J_i$  
Bessel function of the i-th order

$J_r$  
amplitude of the rotational mass inertial due to a rib, $1/m$

$K_0$  
modified Bessel function of the zeroth order

$k_i$  
incident wavenumber, $1/m$

$K_x, K_y$  
directional cosines of the far field

$K_{p1, p2}$  
plate stiffness

$k$  
acoustic wavenumber, $1/m$

$k_p$  
flexural wavenumber, $1/m$

$\bar{k}$  
non-dimensional wavenumber in $r$ direction

$k_x, k_y, k_z$  
wavenumber in the x-, y-, or z-direction, $1/m$

$\bar{k}_x, \bar{k}_y, \bar{k}_z$  
non-dimensional wavenumber in the x-, y-, or z-direction

$\tilde{k}_1, \tilde{k}_2, \tilde{k}_3$  
the roots of the plate characteristic equation

or $\tilde{k}_{r1}, \tilde{k}_{r2,3}$
\( k_{mn} \)  
modal wavenumber, \( 1/m \)

\( l_1, l_2 \)  
distance between the line control force \( F_1 \) or \( F_2 \) and the line constraint, \( 1/m \)

\( M \)  
weight of mass attached on a plate

\( M(x, t) \)  
line moment exerted at \( x = 0 \), \( 1/m \)

\( M_r \)  
amplitude of the line moment due to rib inertia, \( 1/m \)

\( M_0 \)  
amplitude of the line moment \( M(x, t) \), \( 1/m \)

\( M_{ramn}^{++p} \)  
combined mass due to the fluid loading, attached mass, and plate mass

\( m \)  
modal index number in the \( x \)-direction

\( m, m_p \)  
mass per unit area, \( kg/m^2 \)

\( N_c \)  
number of control forces

\( N_s \)  
number of disturbance forces

\( N_t \)  
number of total forces

\( n \)  
modal index number in the \( y \)-direction

\( \hat{P} \)  
spatial variation of pressure \( p(x, y, z, t) \), \( N/m^2 \)

\( P(x, y, z, t) \)  
radiated pressure at position \( (x, y, z) \) and time \( t \), \( N/m^2 \)

\( P_{far}(R, \theta, \phi) \)  
far-field pressure at position \( (R, \theta, \phi) \) in sphere coordinates, \( N/m^2 \)

\( \{p\} \)  
control force vector in cost function

\( p_j \)  
the \( j \)-th element of \( \{p\} \)

\( \hat{p}_F \)  
non-dimensional pressure due to line force \( F_0 \)

\( \hat{p}_M \)  
non-dimensional pressure due to line moment \( M_0 \)

\( \{q\} \)  
disturbance vector in cost function

\( q_i \)  
the \( i \)-th element of \( \{q\} \)

\( R \)  
distance from sound radiation source to the position
where radiation $P_{far}$ is estimated, $m$

$R_{ip}, R_{ip}', R_{iv}, R_{iw}$ residues to estimate Cauchy principles

$r, z$ cylindrical coordinates, when $\phi$ is taken as a constant

$r, s$ modal index numbers

$r_a$ radius of the area where the force is applied, $m$

$S$ area over which the cost function is estimated, $m^2$

$T_{mn}(\theta, \phi)$ modal transmissibility function to calculate $P_{far}(R, \theta, \phi)$

$u, v, u_1, v_1$ transformation variables used to estimate the fluid-loading impedance

$u_n, v_n'$ distribution coefficient relating $F_n$ to $F_0$ or $M_0$

$u', v''$ distribution coefficient relating $V_0$ to $F_0$ or $M_0$

$\tilde{V}$ Fourier transform of the plate velocity $v$

$V_0$ amplitude of the incident wave $v_i, m/s$

$\tilde{v}_F$ non-dimensional plate transverse velocity due to line force $F_0$

$\tilde{v}_M$ non-dimensional plate transverse velocity due to line moment $M_0$

$\hat{V}_F$ Fourier transform of plate transverse velocity due to line force $F_0$

$\hat{V}_M$ Fourier transform of plate transverse velocity due to line moment $M_0$

$v(x, y, t)$ plate transverse velocity, $m/s$

$\hat{W}$ Fourier transform of the plate acceleration, $m/s^2$

$w(x, y, t)$ plate transverse displacement, $m$

$w_n(r)$ non-propagating near field plate displacement, $m$

$X, Y, Z$ non-dimensional Cartesian coordinates, with $Z$ normal to a planar boundary

$x, y, z$ Cartesian coordinates, with $z$ normal to a planar boundary

$Z_x, Z_y$ intermediate parameters used to estimate $Z_{rms}^i$ for beam mass
loading

\( \hat{Z}_p, \hat{Z}_a \) Fourier transform of the structural impedance and the acoustic impedance

\( Z_{rsmn}^i \) cross-modal reactance due to the i-th mass attachment

\( Z_{rsmn}^f \) cross-modal fluid-loading impedance

\( \alpha \) non-dimensional wavenumber ratio, \( \alpha = k_c/k_p \)

\( \beta(p), \hat{\beta}(p) \) cost function

\( \gamma \) Hankel transform variable

\( \Delta_x, \Delta_z \) increment in the x- or z-direction

\( \delta(x) \) delta function

\( \delta_{rm}, \delta_{sn} \) Kronecker delta functions

\( \theta \) direction angle in spherical coordinates, rad

\( \dot{\theta}_F \) non-dimensional plate rotational velocity due to line force

\( \dot{\theta}_M \) non-dimensional plate rotational velocity due to line moment

\( \mu \) Poisson's ratio

\( \xi \) fluid-loading parameter

\( \rho_f \) density of the acoustic medium, \( \text{kg/m}^3 \)

\( \rho_p \) density of the plate, \( \text{kg/m}^3 \)

\( \Omega \) auxiliary function to estimate fluid-loading impedance

\( \omega \) excitation frequency, rad/s

\( \omega' \) non-dimensional excitation frequency

\( \Phi(x, y, z, t) \) velocity potential in the acoustic medium

\( \phi \) azimuth angle in spherical coordinates, rad

\( \phi_{i,j}, \phi_{i,j-1} \) direction angle used to estimate intensity
Chapter 1

Introduction

1.1 Objective

For over fifty years, the reduction of sound radiation from fluid-loaded structures has attracted the attention of the community of acousticians and engineers. One of the most manifest examples of application is the reduction of structure-borne sound from ships and submarines. Other application cases include heat exchanger tubes, off-shore platforms, aircraft, and electrical transmission cables. While some of the structures are only passively excited by the fluid itself, others are excited by prescribed dynamic forces which cause the structure to vibrate. The latter case will be addressed in this dissertation. Fluid loading refers to the forces exerted by an ambient fluid medium on a vibrating structure in reaction to that vibration. The fluid loading can significantly affect the structure and thus change the natural frequencies and the damping and hence the vibration response to excitation forces. Therefore, the acoustic pressure in the ambient fluid corresponding to a prescribed velocity distribution of the boundary must be solved simultaneously with the structural response. The sound radiation mechanism of structures under fluid loading is usually much more complicated than that of structures in vacuo due to the interaction between the vibrating structure
and the fluid. Therefore, it is a challenge to acoustic engineers to control the sound radiation from fluid-loaded structures. Eliminating the sound radiation from a fluid-loaded structure by an active method had received little publicity before this work started.

Attenuating the sound radiation from a real structure, such as a submarine, is the final goal of the research. Problems need to be subdivided in such a way that they can be dealt with in different stages. The investigation of simple structures, such as beams, plates, and shells can provide researchers with a fundamental understanding of the sound radiation and vibration control needed for complicated structures. The plate model is mathematically accessible, and an ideal fluid-loading boundary condition can also be conveniently established. Therefore, a fluid-loaded thin plate has been an idealized plant for decades by acousticians and the sound radiation mechanism from such a plate has been widely studied.

Active structural acoustic control (ASAC) has been applied to different systems, such as beams, plates, and cylinders (Lester and Fuller, 1986; Fuller and Jones, 1987; Fuller, 1988; Wang, 1990; Guigou and Fuller, 1991). All applications were performed to solve the response of the structures in vacuo, which means that the fluid-loading effect was negligible in those circumstances. While ASAC has been proven to work well on structures without fluid-loading, it has yet to be demonstrated in the fluid-loaded case. In many ways, the sound radiation mechanism of fluid-loaded structures has not been as clearly understood as that of structures in vacuo. To apply active control force to such plants no doubt increases the complexity of the use of ASAC. A fundamental theoretical analysis will provide useful insight into both the structural...
response and the acoustic pressure distribution, when ASAC is applied to a fluid-loaded plate. The results will be useful when experiments need to be set up, and the approach can be extended to more general fluid-loaded structures. This is the main thrust of this research. This dissertation will also demonstrate that ASAC can be applied to structures with discontinuities.

The ASAC approach used in this dissertation is based on a feedforward method to minimize a cost function that estimates the acoustic power over a prescribed surface in the acoustic medium while control inputs are applied to the structure. The infinite plate and the simply-supported rectangular plate are the two different plate models to be addressed. The infinite plate model is used to study the sound radiation caused by a line discontinuity of a constraint attached on structures such as a ship's hull. The finite plate model is designed in such a way that the wave equation can be solved with the given boundary conditions. The plate is simply supported and infinitely baffled in a plane where the plate is placed. The modal coupling phenomenon in this case is much more complicated than in the infinite case, and the study of the coupling between the plate vibration modes and the radiated pressure illustrates how the active control is achieved. Line forces are used as actuators in the infinite plate case, and point forces or distributed forces are used in the finite plate case.

1.2 Literature Review

1.2.1 Fluid-Loaded Plates

Before reviewing the literature on sound radiation from fluid-loaded plates, it is worthwhile to study briefly the previous research on infinite and finite plates in vacuo, since
it provides a simplified problem which is closely related to the fluid-loaded case. Rous-
sos (1985) studied a noise transmission problem of a baffled thin rectangular plate. 
The analytical model he used was based on relating the plate vibration to sound 
pressure by using the Rayleigh integral. This is a classical approach summarized by 
The approach is well presented in *Sound and Structural Vibration* by Fahy (1984) 
and in numerous acoustic papers. Basically, the Rayleigh formula was used to study 
the sound pressure caused by the vibration of circular piston and rectangular plate 
sources. The Rayleigh integral provides a relation between the volume acceleration 
of an elemental area of the sound radiation source and the sound pressure in the 
acoustic medium. This approach is also used in the dissertation to find the sound 
pressure generated from a finite fluid-loaded plate. On the other hand, the case of a 
localized force on an infinite plate in *vacuo* has also been addressed by quite a few 
researchers (Heckl, 1959; Skudrzyk, 1968; Junger, 1983; Junger and Feit, 1986; Keltie 
and Peng, 1987). The exact solution for the plate transverse velocity (Junger and 
Feit, 1972) was derived by taking the inverse Fourier transform of the wave number 
spectrum of the plate velocity over an infinite range in wave number domain (if the 
plate is infinite). This kind of integral is usually intractable, so it has to be solved by 
applying such techniques as the Cauchy residue method, the asymptotic method, or 
by direct numerical integration.

When the plate is coupled with heavy fluid loading, the approaches remain similar 
to the above, but the solution becomes much more difficult to obtain due to the 
fluid-loading coupling. While there were many references to cite, the author has only 
selected some of the most important and relevant references as subjects for this re-
view.

In 1920, Lamb (1920) described the phenomenon of the vibration of an elastic plate in contact with water. Since an exact solution was not available at that time, he used a relatively primitive Rayleigh model to investigate the plate vibration under fluid-loading. Since then, much research has been focused on the same subject, and numerous improvements have been made.

Since World War II, the United States and Western allies such as Britain and Germany have spent much effort on developing submarine silencing techniques which has been the drive behind their sound radiation research. One of the main contributors in this field is G. Maidanik. For over thirty years, he has dedicated himself to fluid-loaded structure research and has authored many publications from the early 60's to late 80's. He studied the fluid loading in an infinite plate, ribbed or orthotropic (1962, 1966), at low frequencies (i.e. below coincidence frequency), and in the vicinity of the coincident frequency. He also discussed the acoustic source characteristics in the fluid-loaded case (1966), the transmission of free waves across a rib discontinuity on a fluid-loaded plate, and the compliant coating on the fluid-loaded plate to modify the influence of fluid-loading on the transmission (Maidanik et al., 1976), which can be considered a passive control approach.

When one studies sound radiation from fluid-loaded structures, it is impossible not mention Crighton’s contribution to the understanding of the fluid-loading phenomenon. From 1971 to 1988, he and his colleagues had investigated the sound radiation from panels that were mostly infinite or semi-infinite and loaded with heavy
fluid. For sound scattering caused by line force or line moment on an infinite fluid loaded plate, he developed, with the aid of perturbation theory, a series of asymptotic solutions covering a wide frequency range, from as low as $10^{-4}$ of the coincidence frequency to high above the coincidence frequency. Because the low frequency range is where the fluid-loading effects are greatest, his solution (1972) is very useful as a simple means of accounting for these effects. Crighton also clearly explained the physical meaning of $\xi$, the loading parameter (1980), which is the ratio of the fluid mass to the specific surface mass of the plate in terms of wave length. His findings reveal that heavy fluid-loading not only happens in “heavy” acoustic media such as water, but also exists in “light” media such as air, depending on the relative ratio of the fluid mass to the surface mass. For an infinite fluid-loaded plate, Crighton presented a way to solve the roots of the dispersion equation and discussed the physical meanings of the real pole and the pairs of complex conjugates (1979). His research is considered classical and has often been cited by other later researchers in this field (Junger and Feit, 1986; Liu and Rumerman, 1981; Feit and Liu, 1985, etc.). From the work that Crighton has published, it appears that most of his research was focused on infinite or semi-infinite fluid-loaded plates.

Work similar to Crighton’s has been performed by several other researchers. Leppington (1978) studied the condition under which the fluid loading could be taken as “light” and the scattering from a finite membrane or plate in such circumstances. Smith (1978) provided a physical explanation of the fluid-loading effects on plates through the study of the imaginary part of input admittance. Goyder and White (1980) studied the wave propagation and power flow due to force and torque (moment) excitations at the drive point and in the far field of an infinite plate with a single
line-stiffener. Mace (1981) examined the radiation of sound from infinite fluid-loaded plates when the plates were reinforced with two sets of orthogonal line stiffeners. An expression for the response due to a general excitation was derived, and from this, the acoustic pressure in the far field was determined with particular reference to point force excitation. Eatwell and Butler (1982) also obtained the expressions for the vibration of and sound radiation from a fluid-loaded elastic plate which was stiffened by a finite number of parallel beams. The expressions were evaluated asymptotically in the far field, and the results were presented for point and line excitation of a plate with equally spaced beams. Innes (1982-1988) had co-authored many papers with Crighton on the same subject. His Ph.D research and later work were devoted to the fluid-loading coupling field.

Feit (1966) suggested that the acoustic pressure due to a point force on a plate is relatively nondirectional and insensitive to structural damping below coincidence. Above coincidence, however, the pressure pattern becomes directional and the peak pressures can be significantly reduced by structural damping. Using the Cauchy residue method, Nayak (1970) for the first time presented a model describing a line force exciting an infinite fluid-loaded plate. He studied the physical meaning of the components of the residues and the branch cuts of the integral, and thus provided an insight into the understanding of wave propagation and radiation due to a line force on the plate. His approach provided the tools for the later numerical estimation performed by several other researchers (Liu and Rumerman; 1981, Feit and Liu, 1985) and the author of this dissertation.

Howe and Heckl (1972) were also pioneers who studied the stiffened plate. Their
model was more general, since it included the interaction of bending waves with density and stiffness fluctuations in materials of the plate. The theory was developed in the way that the variations in density and/or bending stiffness might be regarded as random functions of their position on the plate. Stepanishen (1982) was another investigator of the acoustic transmission and scattering characteristics of a plate with line impedance discontinuities. He showed that non-specular contributions to the acoustic field originated at the discontinuities on the plate which he described by using Timoshenko-Mindlin theory. Woolley (1980a, 1980b) presented a theoretical model to calculate the backscattering of a plane sound wave from a submerged rib-stiffened plate and compared the exact solution with two saddle point integral approximations.

By using Nayak’s line force model and Crighton’s solution to the dispersive equation, Liu and Rumeman (1981) extended Nayak and Crighton’s theory to calculate the plate translational and rotational displacement due to a line force or a line moment and the associated far field pressure. Their results suggested that the sound pressure due to multiple line sources can be evaluated by superposing pressure generated by single sources based on a suitable Green function. Feit and Liu (1985) for the first time described the near sound radiation field of a line-force-driven plate by using numerical estimation. Their results indicated that near the radiation source the pressure field is like a monopole when the excitation frequency is well below the coincidence, but it displays a directive lobe in the coincidence direction when the excitation frequency is above coincidence. Feit and Liu also discussed the interference between a cylindrically spreading acoustic wave emanating at the drive line and the evanescent wave associated with the structural flexural motion. The sound radiation near field caused by multiple sources can also be numerically estimated with the same
approach. Waterhouse et al. (1985) presented a numerical algorithm to calculate the energy flow by using the near field pressure gradient. In fact, this numerical approach was equivalent to that presented by Pettersen (1979), who used the pressure gradient to estimate an intensity field. A detailed discussion of the near field intensity distribution was presented by Fahy in his recent book (1988).

It is worthwhile to mention the contributions of Russian scientists to this research field. Lyamshev started the study of the theory of sound radiation from thin elastic shells and plates early in 1959. Gutin presented a model describing a point-force-driven plate (1965), by which he derived a general expression for the acoustic field in the wave range of radiation. He also compared the power radiated into the surrounding medium with the total power developed by the driving force. This work was often cited by later researchers. Lyapunov (1969) studied the flexural wave propagation in a fluid-loaded plate with an obstruction. He showed that a considerable proportion of the energy of the flexural waves incident on the obstruction was transmitted through the fluid contacting the plate, and the energy transfer through the fluid depended on the angle of incidence of the wave and its frequency. Sound generated by beam-reinforced plates was studied by Romanov (1977, 1980), Evseev et al. (1980), Shenderov (1980), Belinskii (1982), Svyatenko (1986), and Aleksandrov and Boev (1988). The random loading and the influence of nonuniformity in a field of random forces on the acoustic pressure were also investigated (Romanov, 1980, 1984). The low frequency asymptotic behavior of the wave numbers of a fluid-loaded plate was addressed by Veshev et al. (1985).

While the above mentioned research was generally based on infinite fluid-loaded
plates, the research on finite fluid-loaded plates has also attracted much attention. In his study of the coupling of sound and panel vibration below the coincidence frequency, Smith (1964) proposed that twice as much power radiates from a clamped edge as from a simply supported edge due to the mode shape difference between these two boundary conditions. Davies (1969, 1971) extensively studied the sound radiation from a fluid-loaded rectangular plate under low frequency excitation. He derived an approximate solution to the set of simultaneous equations for the plate modal velocity amplitudes. The inter-modal coupling coefficients were evaluated by means of the steepest descent method. He also explained that the matching of wave numbers determines the effective coupling between modes. Mkhitarov (1972) also studied the same problem, and his results were consistent with those of Davies. Pope and Leibowitz (1974) presented a revised approximation result for the intermodal coefficients.

Sandman (1975, 1977) for the first time provided numerical and experimental results based on the theory that Davies had used. His model was based on a fluid-loaded and simply supported rectangular plate with a discretely attached mass. The influence of the mass attachment on the plate response and the radiated pressure was also illustrated. Nikiforov (1981) suggested that the radiation of sound from a flexurally vibrating isotropic plate of finite dimensions at frequencies below the coincidence was caused by forces and moments at the edges of the plate and depended on boundary conditions. Lomas and Hayek (1977) developed a similar solution for the steady state vibrations of an elastically supported rectangular plate. According to their interpretation, the fluid-loading impedance has two distinct properties. The real part is a dissipative factor related to the acoustic field in the fluid. By contrast, the imaginary
part represents non-dissipative coupling to purely reactive pressure, which effectively increases the vibrating mass of the plate.

Abrahams (1981) studied the scattering of sound from a heavily loaded finite plate when a plane wave was incident onto the plate. He discovered that waves were present on the flexible surface, and resonance was shown to occur for particular values of the plate half-length. He also examined the sound scattering from a finite nonlinear plate (1987).

1.2.2 Active Control of Sound Radiation

The active control of sound results from destructive interference between the sound field of the original acoustic source and that from a controllable array of secondary acoustic primary sources (Elliott and Nelson, 1990). The concept dates back to the early 30's, but its application became realized only a decade ago, when microprocessor techniques turned the idea into reality. The author of this dissertation does not intend to bring about a historical review of this subject, since it has already been done by other researchers. A more detailed historical review of this subject appears in Nelson and Elliot's new textbook, The Active Control of Sound (1992).

The author does intend, however, to mention some of the development in active control techniques related to this research. Nelson et al. (1987) presented a model designed to express the acoustic power output of the structure as a quadratic function in terms of the complex strengths of control inputs. For a given arrangement of primary and secondary sound sources, this quadratic function has a unique minimum
associated with an optimal solution for secondary control source complex strengths. It is possible to produce source distributions of unexpectedly low radiation efficiency with a relatively small number of secondary sources placed close to the primary source. This optimization technique was fully utilized and further developed by Fuller (1986-1991), Lester (1986), Jones (1987, 1989), and others.

Lester and Fuller (1986) studied the active method for controlling propeller induced noise fields inside a flexible cylinder. The secondary sources were monopole control sources, so this approach was the same as the one proposed by Elliot and Nelson (1986). Later on, Fuller and Jones (1987) investigated experimentally the performance of the active structural acoustic control (ASAC) of a cylinder which simulated the noise level reduction in an aircraft fuselage. They found that by applying control forces directly to the cylinder rather than by using secondary acoustic sources, the number of control inputs can be reduced. This is understandable, since secondary forces change the structural vibration modes to create a destructive sound field. The concept (ASAC) is new compared to that of creating a secondary sound field with acoustic sources. The advantage of ASAC is that it reduces the sound radiation by employing control inputs directly on the structure, thus providing an opportunity for attenuating the sound with fewer control channels. But it is noted that the secondary forces should be controlled so that structure material fatigue does not occur.

The ASAC technique has been used to control the sound transmission/radiation from elastic plates (Fuller, 1988). By using point control forces, global sound attenuation was achieved for both on- and off-resonance cases. It was shown that the efficiency of the control strategy is related to the nature of the coupling between the plate modes
of response and the radiated field. Vyalyshev et al. (1986) also presented a model in which the sound radiation is reduced by applying an auxiliary force to a semi-infinite one-dimensional plate. The parameters of the auxiliary force were determined so that a reduction of the sound transmission coefficient in a given direction and of the acoustic power transmitted through the plate could be achieved. The approach is related to ASAC, but ASAC is more general and the cost function can be modified to fit different requirements.

In recent years, piezoelectric actuators (PZT) have attracted increasing attention because they are light and can be attached or embedded into structure to build a "smart material." A two-dimensional model was developed by Dimitriadis et al. (1989), and its application to the ASAC of a plate system was conducted by Dimitriadis and Fuller (1989), Wang (1990), and Clark (1991). By bonding multiple PZT actuators to the plate, global sound attenuation can be achieved for on- and off-resonance excitation frequencies. PZT's appear to be very suitable actuators for ASAC. The sensors can be either microphones located in the radiation space, where the attenuation is to be achieved, or polyvinylidene fluoride (PVDF) strip sensors bonded on the structure. The former are the best sensors since they directly detect the signal to be minimized, but in reality it is not always possible to put microphones in the radiation space, so their usage is limited. The latter show interesting potential as an alternative. Preliminary experimental results (Clark, 1991) showed that it is possible to design properly located and shaped distributed PVDF sensors to replace microphones. PZT actuators have been shown to work well on plates in vacuo; their application to the fluid-loaded plates will be studied in this dissertation.
1.3 Organization of the Dissertation

Analytical investigations to study the ASAC of fluid-loaded plates are the main goal of this dissertation. First, mathematical models are established to describe the fluid-loaded plate excited by structural disturbances. The legitimacy and exactness of such models are tested by comparing the numerical results with those achieved by previous researchers (Nayak, 1971; Feit and Liu, 1985; Sandman, 1977). The results are found to be very consistent. After this preliminary test of the approach, the optimal control forces are applied to the plates based on a cost function in order to minimize the acoustic power in a prescribed spatial region. The sound radiations before and after ASAC are calculated and compared in different cases.

Chapter 2 presents the results of the study of the ASAC of an infinite fluid-loaded plate. The active control of sound radiation due to subsonic wave scattering from discontinuities represented by a line constraint or by a uniform reinforcing rib positioned on a fluid-loaded infinite plate is also analytically studied. Mathematical models are based on the plate vibration and sound radiation due to a line force or a line moment solved in the spectral $k$ domain. For simplicity, the far-field pressure is estimated by the stationary phase approach. Feed-forward control is achieved by adding secondary line forces applied to the plate near the discontinuity. The amplitudes of control forces are determined by the optimal solution of a cost function, which integrates the far-field radiated acoustic intensity over a semi-cylindrical surface around the discontinuity. The results show that for subsonic incident waves, a high reduction in radiated pressure due to spectral wave scattering at the discontinuities is possible with two active control forces located near the discontinuity. The amount of sound
reduction as well as the residual directivity pattern are shown to depend upon the number and location of the control forces.

In Chapter 3, the active control of sound radiation from a simply-supported uniform rectangular fluid-loaded plate is analytically studied. The plate is assumed to be excited by a point force at subsonic frequencies. The dynamic equation of the plate motion is based on the \textit{in vacuo} eigenfunctions of a homogeneous panel as the basis for the Fourier decomposition of the fluid-loading. A feed-forward control is applied by point forces or by distributed forces employed on the plate. The amplitudes of control forces are determined by the optimal solution of a quadratic cost function, which integrates the far field radiated acoustic pressure over a hemisphere in the radiation half-space. The results show that for a heavily fluid-loaded plate driven by subsonic structural disturbances, a high global reduction in radiated pressure is possible with up to two active control forces properly located when excited at on-resonant frequencies and with up to four forces when driven at off-resonant frequencies. The number and location of the control forces are determined so that the efficient radiating modes can be suppressed. The far-field directivity pattern, the plate power spectrum in the two-dimensional wave number domain, and the near-field pressure distribution are also studied.

Chapter 4 further details the study of the ASAC technique for a fluid-loaded rectangular plate with one or several lumped masses attached to the structure. Besides the fluid loading, the mass discontinuity of the plate also causes intermodal coupling due to the radiation impedance discontinuity. The mass loading inevitably changes the mode shapes of the plate so that optimal control forces need to be relocated when the
mass loading effect becomes substantial. Both discrete point masses and a line mass are considered in the calculations. It is shown that with properly located secondary forces, ASAC still works well in reducing the global sound radiation, even if the mass loading further complicates the modal composition and the sound radiation.

Chapter 5 discussed the investigation of the influence of the non-propagating near-field waves due to a point force or a distributed force driving a fluid-loaded plate. First, a model is presented to describe the wave composition of a distributed force applied to an infinite fluid-loaded plate. The purpose of the wave composition is to study the effect of non-propagating wave components. It is shown that near-field waves have a relatively localized impact on the plate response, but they have a significant impact on the sound radiation. Therefore, an approximate model for calculating the plate response and the sound radiation of a finite plate is developed, and numerical results are presented. The results indicate that the influence of non-propagating wave components cannot be neglected. Second, it is observed that the sound radiation fields generated by propagating wave components and non-propagating wave components are out of phase when ASAC is applied, while the sound radiation fields generated by these two wave components are in phase when ASAC is not applied. This result implies that the ASAC generates a secondary sound radiation field through the non-propagating wave components in order to cancel the sound radiation generated by the propagating wave components. This observation is consistent with the results of other researchers (Guigou and Fuller, 1992).

Chapter 6 summarizes the new main points uncovered in this study of the ASAC of fluid-loaded plates and discusses recommendations for further research.
Chapter 2

Active Control of Sound Radiation Due to Subsonic Wave Scattering from Discontinuities on Infinite Fluid-Loaded Plates

2.1 Introduction

Active control of sound radiation has recently attracted substantial attention from the acoustics research community. For well defined sound radiation patterns, quadratic optimal control theory provides a relatively straightforward mathematical approach to suppress the radiated acoustic power in a spatially prescribed acoustic medium. Although the technique of optimizing a quadratic cost function is well known, its application to the control of sound radiation is still a challenging research subject. This approach was well defined by Nelson et al. (1987) and its application to controlling noise in cylindrical cavities and to controlling radiation from panels was successfully demonstrated (Lester and Fuller, 1986; Fuller, 1988).

For the past several decades, much work has been carried out analyzing the vibration and sound radiation of fluid-loaded plates excited by forces (Nayak, 1971; Crighton,
1979; Liu and Rumerman, 1981; Feit and Liu, 1985). Meanwhile, attention has been also directed to the problem of scattering of waves from discontinuities on fluid-loaded plates (Howe and Heckl, 1972). This problem is important because scattering of subsonic, non-radiating waves from discontinuities on fluid-loaded plates can often lead to radiation of acoustic power (Nayak, 1971). As discontinuities are present on all physical structures, such as marine vessels and aircraft, this phenomenon is important in terms of noise control.

This chapter is concerned with applying active control to reducing the sound radiation due to wave scattering from discontinuities. First, the plate response is studied. The model describing the plate transverse velocity due to a line force or a line moment was well defined by Nayak (1971) and Liu and Rumerman (1981). For this investigation, the analysis is based on an idealized system consisting of infinite thin plate with semi-infinite acoustic fluid on one side and in vacuo on the other. The discontinuities considered consist of a line constraint or a uniform rib attached to the plate. The mathematical approach is to describe the response of the plate system in the spectral $k$ domain and to solve the integral with Cauchy residue method. For the numerical examples presented in the chapter, the acoustic fluid is considered to be water ensuring strong coupling between plate and the fluid. The total plate velocity due to several excitation sources is obtained by superposition based on different discontinuity boundary conditions. The plate transverse motion is highly coupled with the acoustic pressure under heavy fluid loading. To investigate the plate motion in this specific circumstance will certainly help in understanding the sound radiation phenomenon.
It is well described (Fahy, 1985) that in the wavenumber spectrum domain the condition $|k_x| \leq k_o$ (in which $k_o$ is the acoustic wavenumber) determines the radiation power of the plate in a one-dimensional system. This fundamental principle is used to estimate the sound radiation power before and after ASAC is applied. When the optimal control forces are applied to the plate and the far-field pressure is reduced, a diminished power spectrum in the range of $-k_o \leq k_x \leq k_o$ will be observed. In this chapter, the spatial plate velocity illustrates the vibration responses of the plate before and after the control is applied, and the spectral plate velocity provides the key explanation of how the sound radiation power is reduced.

Active control is achieved by applying forces near the discontinuities, and the control approach is based on a steady state feed-forward model (Burcisso and Fuller, 1990) for a single frequency input. The optimal control forces are obtained by minimizing a quadratic cost function which represents the radiated acoustic power. The far-field radiated pressure is found by means of the stationary phase approach. The incident acoustic waves are limited to subsonic waves; thus all noise radiation to the far-field is due to wave scattering phenomena.

After studying the plate transverse vibration response and the far-field pressure, the characteristics of the sound radiation sources are investigated by numerically calculating the near field intensity distribution around the sources. In a conventional analysis method, the intensity vector based on pressure differences at a set of grid is used to describe the near-field intensity field. The near-field pressure is numerically calculated by means of the Cauchy residue method. The intensity vectors have lengths proportional to their magnitudes and directions of energy flow.
Although the infinite plate system is idealized, the study does consider two new themes: active control of sound radiation from fluid-loaded coupled structures as well as control of radiation due to line discontinuities. The results thus add new understanding to this important problem.

2.2 Plate Response

2.2.1 Plate Velocity Due to a Line Force

Consider the infinite plate as shown in Figure 2.1 (a) which is excited by a harmonic line force at \( x = 0 \),

\[
F(x,t) = F_0 \delta(x) e^{-i\omega t},
\]

where \( \omega \) is the frequency of excitation, \( F_0 \) is the complex magnitude, and \( \delta(.) \) is the Dirac delta function.

With a wavenumber \( k \)-plane Fourier transform, and associated manipulations, the normalized homogeneous plate transverse velocity can be expressed as the sum of residue terms and two branch cut integrals (Nayak, 1971). These residues and branch cut integrals can be evaluated using the procedure developed by Feit and Liu (1981). Thus, the transverse velocity of the plate is evaluated as

\[
\tilde{v}_F(X) = \frac{v_F(X)}{k_p F_0 / 4 \omega m} = -\frac{2i}{\pi} \int_{-\infty}^{\infty} \frac{(a^2 - k_x^2)^{1/2} e^{i k_x x}}{(k_x^2 - 1)(\alpha^2 - k_x^2)^{1/2} - i \xi / \alpha} d \bar{k}_x
\]

\[
= 4(R_{1v} + R_{2v} + R_{3v}) -
\]

20
Figure 2.1(a): Arrangement of line constraint discontinuity

Figure 2.1(b): Arrangement of rib constraint discontinuity

Figure 2.1: (a) Arrangement of line constraint discontinuity; (b) Arrangement of rib constraint discontinuity.
\[-i \frac{4\xi}{\pi\alpha} \int_0^\infty \frac{(\alpha^2 + u^2)^{1/2} e^{-uX}}{(u^4 - 1)^2(\alpha^2 + u^2) + \xi^2/\alpha^2} du + \frac{4\xi}{\pi\alpha} \int_0^\infty \frac{(\alpha^2 - u^2)^{1/2} e^{iuX}}{(u^4 - 1)^2(\alpha^2 - u^2) + \xi^2/\alpha^2} du, \tag{2.2}\]

where

\[R_{1v} = e^{i\xi_1X} [4\bar{k}_1^3 + (\xi/\alpha)\bar{k}_1(\bar{k}_1^2 - \alpha^2)^{-3/2}]^{-1} \tag{2.3}\]

and

\[R_{2v,3v} = e^{i\xi_2,3X} [4\bar{k}_{2,3}^3 - (i\xi/\alpha)\bar{k}_{2,3}(\alpha^2 - \bar{k}_{2,3}^2)^{-3/2}]^{-1}. \tag{2.4}\]

In Equation (2.2) \(k_p = (\omega^2m/D)^{1/4}\) is the in vacuo flexural wave number, \(m\) is the mass per unit area, \(D = Eh^3/[12(1 - \nu^2)]\) the bending rigidity, \(E\) is the Young's modulus, \(\nu\) is the Poisson's ratio, and \(h\) is the thickness of the plate, respectively. Other important terms are \(\alpha = k_o/k_p, k_o = \omega/c_o\) (the acoustic wavenumber in the fluid), \(\bar{k}_x = k_x/k_p\) (the non-dimensional wave number), \(X = k_p x\) (the non-dimensional distance), \(\xi = \rho_o c_o/\omega_c m\) (the fluid loading parameter, in which \(\rho_o\) is the acoustic fluid density, and \(c_o\) the sound speed in the fluid, and \(\omega_c = c_o^2(m/D)^{1/2}\) the coincidence frequency). The meanings of \(\xi\) and \(\omega_c\) were well defined by Crighton (1979). Finally, \(\bar{k}_1\) and \(\bar{k}_{2,3}\) are roots of the system characteristics equation which are to be discussed next.

Before performing the Cauchy contour integration, it is necessary to find the system poles. The location of the system poles can be obtained by solving the system charac-
teristic equation which has been extensively studied by Crighton (1979) and Junger and Feit (1986). The system characteristic equation is:

\[ (\tilde{k}_z^4 - 1)(\alpha^2 - \tilde{k}_z^2)^{1/2} - i \xi / \alpha = 0. \]  

(2.5)

If \( \tilde{k}_z = (\alpha^2 - \tilde{k}_z)^{1/2} \), the roots of the characteristic equation should be chosen so that Im \( (\tilde{k}_z) \geq 0 \), or when Im \( (\tilde{k}_z) = 0 \), Re \( (\tilde{k}_z) \geq 0 \) in order to satisfy the Sommerfeld radiation condition (see Figure 2.2). The meanings of the poles in Figure 2.2 were well explained by previous researchers (Nayak, 1971; Crighton, 1979).

In addition to the plate transverse velocity, the plate rotational velocity due to a line force can also be obtained by following Nayak’s approach and was given by Liu and Rumerman (1981) as follows:

\[ \dot{\theta}_F(X) = \frac{\partial v(X)}{\partial x} / \frac{F_c k_p^2}{4 \omega m} = -\frac{2i}{\pi} \int_{-\infty}^{\infty} \frac{i \tilde{k}_z (\alpha^2 - \tilde{k}_z)^{1/2} e^{ik_z x}}{(\tilde{k}_z^4 - 1)(\alpha^2 - \tilde{k}_z^2)^{1/2} - i \xi / \alpha} d\tilde{k}_z, \]  

(2.6)

while the residues and branch cuts expressions are omitted here, since they are similar to Equations (2.3) to (2.4).

### 2.2.2 Plate Velocity Due to a Line Moment

If the plate is restrained by a rib instead of being supported by a line constraint, there will be both a line force and a line moment generated by incident wave impinging on the rib.
Figure 2.2: Integration contour in $k_x$ plane and locations of roots of the characteristic equation.
The plate velocity due to a line force has already been derived, and a similar approach will be taken for the line moment excitation. Using a \( k \)-Fourier transform approach, the plate transverse velocity of a plate due to a line moment,

\[
M(x, t) = M_0 \delta'(x) e^{-i\omega t}, \quad (2.7)
\]

can be also derived, where \( M_0 \) is the complex magnitude and \( \delta'() \), the derivative of the Delta function with respect to its argument, is written as \( \delta'(x) = (d/dx)[\delta(x)] \).

The plate transverse velocity due to the line moment described in Equation (2.7) is given as (Liu and Rumernan, 1981):

\[
\tilde{v}_M(X) = \frac{v_M(X)}{M_0 \bar{k}_p^2/4\omega m} = \frac{2i}{\pi} \int_{-\infty}^{\infty} \frac{i\bar{k}_x(\alpha^2 - \bar{k}_x^2)^{1/2} e^{i\bar{k}_x X}}{(\bar{k}_x^2 - 1)(\alpha^2 - \bar{k}_x^2)^{1/2} - i\xi/\alpha} d\bar{k}_x
\]

\[
= -4(R'_{1v} + R'_{2v} + R'_{3v})
\]

\[
-\frac{4\xi}{\pi\alpha} \int_0^{\infty} \frac{u(\alpha^2 + u^2)^{1/2} e^{-uX}}{(u^4 - 1)^2(\alpha^2 + u^2) + \xi^2/\alpha^2} du
\]

\[
-\frac{4\xi}{\pi\alpha} \int_0^{\alpha} \frac{u(\alpha^2 - u^2)^{1/2} e^{iuX}}{(u^4 - 1)^2(\alpha^2 - u^2) + \xi^2/\alpha^2} du, \quad (2.8)
\]

where

\[
R'_{1v} = i\bar{k}_1 e^{i\bar{k}_1 X} \left[ 4\bar{k}_1^3 + (\xi/\alpha)\bar{k}_1(\bar{k}_1^2 - \alpha^2)^{-3/2} \right]^{-1} \quad (2.9)
\]

and

\[
R'_{2v,3v} = i\bar{k}_{2,3} e^{i\bar{k}_{2,3} X} \left[ 4\bar{k}_{2,3}^3 - (i\xi/\alpha)\bar{k}_{2,3}(\alpha^2 - \bar{k}_{2,3}^2)^{-3/2} \right]^{-1}. \quad (2.10)
\]
Comparing Equation (2.8) with Equation (2.2) discloses some similarities. $R_{1\nu}$ in Equation (2.9) corresponds to the unattenuated wave propagating to the infinity. The sum of $R_{2\nu}$ and $R_{3\nu}$ contributes to a decaying near field, since it is always purely imaginary and decreases exponentially without phase change. The first and the second integrals also have similar properties as their counterparts in Equation (2.2).

The plate rotational velocity caused by a line moment is similarly estimated as follows.

$$
\hat{\theta}_M(X) = \frac{\partial v(X)/\partial x}{M_0 k_p^3/4\omega m} = -\frac{2i}{\pi} \int_{-\infty}^{\infty} \frac{\bar{k}_x^2(\alpha^2 - \bar{k}_z^2)^{1/2} e^{i\bar{k}_x X}}{(k_x^4 - 1)(\alpha^2 - \bar{k}_z^2)^{1/2} - i\xi/\alpha} d\bar{k}_x , \quad (2.11)
$$

where the details of residues and branch cuts are omitted.

### 2.2.3 Plate Spectral Velocity $\tilde{V}_F(k_z)$ Due to a Line Force

By using the Fourier transform relation of

$$
f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{F}(k)e^{ikx} dk , \quad \tilde{F}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{-ikx} dx \quad (2.12)
$$

and comparing Equation (2.2) with Equation (2.12), the non-dimensional plate spectral velocity due to a line force is derived as follows:

$$
\tilde{V}_F(\bar{k}_z) = \frac{V_F(\bar{k}_z)}{F_0 k_p/4\omega m} = -\frac{4i}{(2\pi)^{1/2} (k_x^4 - 1)(\alpha^2 - \bar{k}_z^2)^{1/2} - i\xi/\alpha} , \quad (2.13)
$$

where $V_F(\bar{k}_z)$ is the spectral velocity due to a line force. When $\bar{k}_z$ increases along the real axis as wave number velocity autospectrum is evaluated, the two poles, $\bar{k}_z = \pm \bar{k}_1$, 

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will be the two singularities which lead to \( \tilde{V}_P(\tilde{k}_x) \to \infty \) if no structural damping is included. However, singularities of this kind do not affect the accuracy of estimating the acoustic power because the supersonic interval, \( |\tilde{k}_x| \leq \alpha |k_x| \leq k_o \), is well away from the poles \( \tilde{k}_x = \pm k_1 \) for the cases studied here.

When the plate velocity autospectrum is used to evaluate the acoustic power, it is assumed that the contribution of the plate rotational velocity is omitted, so that only the plate transverse velocity is considered.

### 2.2.4 Plate Spectral Velocity \( \tilde{V}_M(\tilde{k}_x) \) Due to a Line Moment

As in the case of a line force, the Fourier transform of Equation (2.5) is taken, and the non-dimensional plate spectral velocity due to a line moment is

\[
\tilde{V}_M(\tilde{k}_x) = \frac{V_M(\tilde{k}_x)}{M_0 k^2 / 4 \omega m} = \frac{4i \tilde{k}_x (\alpha^2 - \tilde{k}_x^2)^{1/2}}{(2\pi)^{1/2} (k_x^4 - 1)(\alpha^2 - \tilde{k}_x^2)^{1/2} - i \xi / \alpha}, \tag{2.14}
\]

where \( V_M(\tilde{k}_x) \) is the spectral velocity due to a line moment. The denominators of Equations (2.13) and (2.14) are the same, since the same plate is excited by a different external force, a line moment instead of a line force; thus, the system characteristic equation, which is based on the homogeneous solution of the governing differential equation (Equation (2.5)), remains unchanged. Therefore, the singularity positions are unchanged, as illustrated in section 2.2.3, and only the magnitudes of the velocity autospectra are different because Equations (2.13) and (2.14) have different numerators.
2.2.5 Plate Response from Line Constraint

When an infinite plate is supported by a line constraint at $X = 0$ and an incident flexural wave is travelling in the plate from $X = -\infty$ to $X = \infty$, the plate velocity has the following components: the subsonic incident wave which is the input of the vibration, the velocity due to a line constraint back force at $X = 0$, $F_o$, and the velocities due to $N_c$ control forces of amplitude $F_n$ (depending on whether control is applied). The total velocity is:

$$v(X) = v_o e^{i\tilde{k}_1 X} + \frac{F_o k_p}{4 \omega m} \bar{\omega}_F(X) + \sum_{n=1}^{N_c} \frac{F_n k_p}{4 \omega m} \bar{\omega}_F(X - \text{sgn}(l_n)|k_p l_n|),$$  \hspace{1cm} (2.15)

in which it should be noted that $F_o$ has different values before and after control forces are added, since different boundary conditions apply.

As the spectral velocity terms have been defined in section 2.2.3, the total plate spectral velocity is the Fourier transform of $v(X)$ expressed as:

$$V(\tilde{k}_z) = V_o(\tilde{k}_z) + \frac{F_o k_p}{4 \omega m} \tilde{\omega}_F(\tilde{k}_z) + \sum_{n=1}^{N_c} \frac{F_n k_p}{4 \omega m} \tilde{\omega}_F(\tilde{k}_z)e^{-\text{sgn}(l_n)\tilde{k}_z|k_p l_n|},$$  \hspace{1cm} (2.16)

where $V_o(\tilde{k}_z)$ is the spectral velocity of the incident wave. Since the incident wave has only one wavenumber $\tilde{k}_1$, $V_o(\tilde{k}_z)$ will appear on the spectrum diagram only as two vertical lines at $\tilde{k}_z = \pm \tilde{k}_1$, so that its influence on the total power spectrum can be neglected. $\tilde{\omega}_F(\tilde{k}_z)e^{-\text{sgn}(l_n)\tilde{k}_z|k_p l_n|}$ is the Fourier transform of $\bar{\omega}_F(X - \text{sgn}(l_n)|k_p l_n|)$ according to the translational theorem of the Fourier transform. The theorem states that if $\tilde{V}(\tilde{k}_z)$ is the Fourier transform of $v(X)$, the Fourier transform of $v(X - X_o)$ will be $\tilde{V}(\tilde{k}_z)e^{-\tilde{k}_z X_o}$. 

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2.2.6 Plate Response from Rib Constraint

In the rib case, the total plate spatial velocity caused by the sum of the incident wave, the rib, and the control forces will be

\[ v(X) = v_0 e^{i k_1 X} + \frac{F_0 k_p}{4 \omega m} \bar{v}_F(X) + \frac{M_0 k_p^2}{4 \omega m} \bar{v}_M(X) + \sum_{n=1}^{N_\xi} \frac{F_n k_p}{4 \omega m} \bar{v}_F(X - \text{sgn}(l_n) k_p l_n), \quad (2.17) \]

where \( F_0 \) and \( M_0 \) are calculated from rib boundary condition at \( X = 0 \), depending on whether control forces are applied.

The total spectral velocity will be

\[ V(\bar{k}_z) = V_0(\bar{k}_z) + \frac{F_0 k_p}{4 \omega m} \bar{V}_F(\bar{k}_z) + \frac{M_0 k_p^2}{4 \omega m} \bar{V}_M(\bar{k}_z) \]
\[ + \sum_{n=1}^{N} \frac{F_n k_p}{4 \omega m} \bar{V}_F(\bar{k}_z) e^{-\text{sgn}(l_n) k_p k_z l_n}. \quad (2.18) \]

2.3 Far-Field Pressure

The infinite plate is considered to be loaded with an acoustic fluid on one side and in vacuo on the other. There are two forms of discontinuities to be studied. The first discontinuity is of a line constraint at which the plate can not move in transverse direction but is free to rotate. When the subsonic incident flexural wave impinges on the constraint, it interacts with the constraint, causing the discontinuity to exert a
reaction line force on the plate. This line force generates additional waves travelling in the plate and the associated sound radiation, known as the scattering phenomenon. The second discontinuity consists of a reinforcing rib attached to the plate. In this case, the rib is assumed to act as a lumped mass which has both translational and rotational inertia, and thus can be approximated as exerting both a reaction moment and a force on the plate while ignoring the dynamics of the rib itself. The amplitude of the reaction force or moment is determined by inertial properties of the rib as well as by the flexural incident wave input. Both of these cases are shown in Figure 2.1.

Thus, before considering control of sound radiation from these discontinuities, it is appropriate to review radiation from plates driven by line forces and moments.

2.3.1 Radiation Due to a Line Force

As the plate response due to a line force, as shown in Fig.2.1(a), is derived in Equation (2.2), the associated acoustic field pressure in the top half plane (Z > 0) is given by (Feit and Liu, 1985)

\[
\tilde{p}_F(X, Z) = \frac{\tilde{p}_F(X, Z)}{F_0 k_o \rho_0 c_o / 4 \omega m} = -\frac{2i}{\pi} \int_{-\infty}^{\infty} \frac{e^{i(k_x X + \sqrt{\alpha^2 - k_z^2} Z)}}{(k_x^4 - 1)(\alpha^2 - k_z^2)^{1/2} - i\xi / \alpha} dk_x,
\]  

(2.19)

where \(\tilde{p}_F\) is the non-dimensional pressure caused by a line force \(F_0\).

Solving the integral of Equation (2.19) directly will follow the same approach which was used in solving Equation (2.2). However, for the far-field pressure a solution can be sought using the stationary phase approach (Junger and Feit, 1986). The approach
is based on the premise that the main contribution to the integral is associated with the region where the phase does not vary rapidly with integration variable $\bar{k}_z$. This means that the resultant contribution of ranges of integration where the modulus varies slowly with $\bar{k}_z$ while the phase fluctuates rapidly is relatively small, because of cancellation between neighboring regions of opposite phases and nearly equal amplitudes. Equation (2.19) is evaluated using the stationary phase approach (Junger and Feit, 1986) as

$$
\tilde{p}_F(R, \theta) = \frac{2\mu \cos \theta e^{i(k_0 R - \pi)}}{1 + i\mu(\alpha^4 \sin^4 \theta - 1) \cos \theta},
$$

(2.20)

where $\tilde{p}_F(R, \theta) = p_F(R, \theta)/c_F$, $c_F = \rho_0 F_0/2\sqrt{2\pi R k_0 m}$, and $R$ is the distance from the line constraint to the position at which the pressure is estimated. Throughout the chapter, the near-field pressure and the far-field pressure due to a line force are evaluated using Equations (2.19) and (2.20), respectively.

### 2.3.2 Radiation Due a Line Moment

The associated acoustic field due to a line moment was derived by Liu and Rumeman (1981) as

$$
\tilde{p}_M(X, Z) = \frac{p_M(X, Z)}{M_0 k_p k_0 \rho_0 c_o/4\omega m} = \frac{2i}{\pi} \int_{-\infty}^{\infty} \frac{i\bar{k}_z e^{i(k_0 X + \sqrt{\alpha^2 - \bar{k}_z^2} Z)}}{i(k_0^2 - 1)(\alpha^2 - \bar{k}_z^2)^{1/2} - i\xi/\alpha} \, d\bar{k}_z,
$$

(2.21)

where $\tilde{p}_M$ is the non-dimensional pressure caused by a line moment $M_o$.

Similarly to the derivation of Equation (2.19), the far-field radiated pressure due to
a line moment is derived with stationary phase approach as

\[ \tilde{p}_M(R, \theta) = \frac{-2i\mu \sin \theta \cos \theta e^{i(k_o R - \frac{\pi}{4})}}{1 + i\mu(\gamma^2 \sin^4 \theta - 1) \cos \theta}. \] (2.22)

where \( \tilde{p}_M(R, \theta) = p_M(R, \theta)/c_M \) and \( c_M = \rho_o M_0 k_p^{1/2}/2\sqrt{2\pi R}\).

### 2.3.3 Radiated Pressure Field from Line Constraint with Control Forces

The configuration of the line constraint system is shown in Figure 2.1(a). When an incident flexural wave impinges on the discontinuity, the line constraint effectively exerts a back reaction force \( F_o \) on the plate. The two other forces, \( F_1 \) and \( F_2 \), represent control forces applied to the plate. Note that all \( F_i (i = 0, 1, 2) \) represent the complex amplitude of force, and the time variation \( e^{-i\omega t} \) is not included.

For a flexural wave input (the disturbance), the incident wave displacement can expressed as:

\[ w_i(x, t) = W_o e^{ik_i x - i\omega t}, \] (2.23)

where \( W_o \) the incident wave amplitude, and \( k_i \) the incident wavenumber. The flexural wave velocity will equal \( v_i(x, t) = V_o e^{ik_i x - i\omega t} \) where \( V_o = -i\omega W_o \), and the wave rotational displacement \( \partial v_i(x, t)/\partial x = ik_i k_p V_o e^{ik_i x - i\omega t} = ik_i k_p v_i(x, t) \).

By superposition, the total plate velocity response to the back reaction force and
control forces can be written, at \( x = 0 \), as

\[
v(0, t) = \left[ \dot{V}_0 + \frac{F_0 k_p}{4\omega m} \bar{v}(0) + \sum_{n=1}^{N_C} \frac{F_n k_p}{4\omega m} \bar{v}(k_p l_n) \right] e^{-i\omega t} \tag{2.24}
\]

where \( n \) is the number of control forces and \( \bar{v}(x) \) is given by Equation (2.2). Applying the boundary condition of the line constraint that transverse displacement is zero at \( x = 0 \) gives

\[
F_0 = \sum_{n=1}^{N_C} F_n v_n + v' , \tag{2.25}
\]

where

\[
v_n = -\bar{v}(k_p l_n)/\bar{v}(0) \tag{2.26}
\]

and

\[
v' = -4\omega m V_0 / k_p \bar{v}(0) , \tag{2.27}
\]

and the time variation is omitted.

Since control forces are located near the line constraint, the distance to the observation point is almost identical from all forces, thus, only the phase difference between control forces is considered. This means that the non-dimensional magnitude of far-field pressure due to each control force is considered the same as that of the constraint reaction force. The pressure field radiated from each control force can thus be ap-
proximately expressed as

\[ \tilde{p}_n(R, \theta) \approx \tilde{p}_F(R, \theta)e^{-i\text{sgn}(l_n)k_0l_n \sin \theta} \]  

(2.28)

for control force \( n \), positioned at \( x = l_n \), and \( \tilde{p}_F(R, \theta) \) is given by Equation (2.20). Note that the approximation holds when \( |l_n| \ll R \) and "\( \text{sgn} \)" is a sign function which takes either +1 or -1 according to positive or negative values of \( l_n \).

Using Equation (2.25) as the solution for the back reaction force, the total plate-radiated pressure can be expressed in terms of the incident wave amplitude (usually assumed known) and unknown control force amplitudes, \( F_n \), as

\[ p^{\text{rad}}_{\text{total}}(R, \theta) = b_1 + \sum_{n=1}^{N_c} a_n F_n , \]  

(2.29)

where

\[ a_n = \frac{\rho_o}{2\sqrt{2\pi Rk_0} m} \tilde{p}_F(R, \theta)[\cos(-i\text{sgn}(l_n)k_0l_n \sin \theta) + v_n] , \]  

(2.30)

and

\[ b_1 = \frac{\rho_o}{2\sqrt{2\pi Rk_0} m} \tilde{p}_F(R, \theta)v_1 , \]  

(2.31)

or be written in vector form as

\[ p^{\text{rad}}_{\text{total}}(R, \theta) = \{a\}^T \{F\} + b \]  

(2.32)

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where \([a]^{T} = [a_{1} \ a_{2} \ldots \ a_{n}]\), \(b = b_{1}\), and \([F]^{T} = [F_{1} \ F_{2} \ldots \ F_{n}]\).

### 2.3.4 Radiated Pressure from Rib Constraint with Control Forces

For the rib-plate, system the derivation is very similar, but the form of the boundary condition is different. In this case, it is assumed that the rib stays attached to the plate; thus, transverse and rotational displacement of the rib (taken as a lumped mass) must be the same as that of the plate at the point of contact, \(x = 0\). Therefore, when an incident flexural wave travels across the rib, it causes both an inertial back reaction force and a moment as depicted in Figure 2.1(b). Note that the transverse displacement at \(x = 0\) is not equal to zero as in the line constraint case. As a result, the plate response will be very different from that of the line constraint case.

Following the procedure of the previous section, the total plate transverse and rotational velocity response of the plate-rib system at \(x = 0\) can be respectively expressed as

\[
v(0, t) = [V_{o} + \frac{F_{0}k_{p}}{4\omega m} \tilde{v}_{F}(0) + \frac{M_{o}k_{p}^{2}}{4\omega m} \tilde{\theta}_{M}(0) + \sum_{n=1}^{N_{c}} \frac{F_{n}k_{p}^{2}}{4\omega m} \tilde{v}_{F}(k_{p}l_{n})]e^{-i\omega t},
\]

\(2.33\)

and

\[
\dot{\theta}(0, t) = \left[ \frac{\partial V_{o}}{\partial x} + \frac{F_{0}k_{p}^{2}}{4\omega m} \tilde{\theta}_{F}(0) + \frac{M_{o}k_{p}^{3}}{4\omega m} \dot{\theta}_{M}(0) + \sum_{n=1}^{N_{c}} \text{sgn}(l_{n}) \frac{F_{n}k_{p}^{2}}{4\omega m} \tilde{\theta}_{F}(k_{p}l_{n}) \right] e^{-i\omega t}.
\]

\(2.34\)

With the approach used by Junger and Feit (1986) of dealing with the scattering of
flexural wave on a plate discontinuity, and with some manipulations, the back reaction force and moment amplitudes can be derived as

\[ F_o = \sum_{n=1}^{N_c} F_n v_n' + v'' , \]  
(2.35)

where

\[ v_n' = \left[ \frac{i M_r k_p \bar{v}_F(0)}{4m - i M_r k_p \bar{v}_F(0)} \right] \frac{\bar{v}_F(k_p l_n)}{\bar{v}_F(0)} , \]  
(2.36)

and

\[ v'' = \left[ \frac{i M_r k_p \bar{v}_F(0)}{4m - i M_r k_p \bar{v}_F(0)} \right] \frac{4\omega_m V_o}{k_p \bar{v}_F(0)} . \]  
(2.37)

\( M_r \) and \( J_r \) are the mass and rotational inertia of the rib per unit length, respectively, and

\[ M_o = \sum_{n=1}^{N_c} F_n u_n + u' \]  
(2.38)

where

\[ u_n = \left[ \frac{i J_r k_p^3 \bar{\theta}_M(0)}{4m - i J_r k_p^3 \bar{\theta}_M(0)} \right] \text{sgn}(l_n) \frac{\bar{\theta}_F(k_p l_n)}{k_p \bar{\theta}_M(0)} , \]  
(2.39)
and

\[
\omega' = \left[ \frac{i J_R k_p^3 \tilde{\theta}_M(0)}{4 \pi m - i J_R k_p^3 \tilde{\theta}_M(0)} \right] \frac{4 \omega m}{k_p^3 \tilde{\theta}_M(0)} \frac{\partial V_o}{\partial x}. \tag{2.40}
\]

The total radiated pressure field due to the rib back reaction force and moment and control inputs can be expressed in terms of amplitudes of control forces as

\[
p_{rad}^{\text{total}}(R, \theta) = b_1 + \sum_{n=1}^{N_k} a_n F_n \tag{2.41}
\]

where

\[
a_n = \frac{\rho_o}{2\sqrt{2\pi R \bar{k}_c m}} [\bar{p}_F(R, \theta) e^{-in\theta} k_p \sin \theta + \bar{p}_F(R, \theta) u_n + k_p \bar{p}_F(R, \theta) u_n], \tag{2.42}
\]

and

\[
b_1 = \frac{\rho_o}{2\sqrt{2\pi R \bar{k}_c m}} [\bar{p}_{FR}(R, \theta) u + k_p \bar{p}_F(R, \theta) u'] \tag{2.43}
\]

Since the form of Equation (2.41) is the same as that of Equation (2.29), it can also be rewritten in a vector form analogous to that of Equation (2.32).

### 2.4 Near-Field Pressure and Intensity

With the same approach used to estimate the plate transverse velocity, the near-field pressure is expressed as the inverse Fourier transform which can be expanded into residues and branch cut integrals (Feit and Liu, 1985). The numerical estimation pro-
procedure remains similar to that used to compute the plate response. As the near-field pressure due to each individual force is known, the total near-field pressure will be the sum of the near-field pressures caused by the reaction force and control forces. Note that the relation between the reaction force and control forces depends on the boundary condition at \( x = 0 \). The intensity field will be formulated on the gradient of the near-field pressure at two adjacent points.

### 2.4.1 Near-Field Pressure Due to a Line Force

With the Cauchy residue method, the expansion result of the near field pressure due to line force is expressed as follows:

\[
\bar{p}_F(X, Z) = \frac{p_F(X, Z)}{\rho_o F_o/4\omega m} = -\frac{2i}{\pi} \int_{-\infty}^{\infty} \frac{e^{i(\kappa_2 X + \sqrt{\alpha^2 - \kappa_2^2} Z)}}{(\kappa_2^4 - 1)(\alpha^2 - \kappa_2^2)^{1/2} - i\xi/\alpha} d\kappa_2 \\
= 4(R_{1p}' + R_{2p}' + R_{3p}') + \left(\frac{4}{\pi}\right)(I_{p1} + iI_{p2}) ,
\]

(2.44)

where

\[
R_{1p}' = \frac{-ie^{i\kappa_1 X - \sqrt{\kappa_1^2 - \alpha^2} Z}}{(\kappa_1^4 - \alpha^2)^{1/2}[4\kappa_1^2 + (\xi/\alpha)\kappa_1(\kappa_1^2 - \alpha^2)^{-3/2}]} ,
\]

(2.45)

\[
R_{2p,3p}' = \frac{e^{i\kappa_{2,3} X + \sqrt{\alpha^2 - \kappa_{2,3}^2} Z}}{(\alpha^2 - \kappa_{2,3}^2)^{1/2}[4\kappa_{2,3}^2 - (i\xi/\alpha)\kappa_{2,3}(\alpha^2 - \kappa_{2,3}^2)^{-3/2}]} ,
\]

(2.46)

\[
I_{p1} = \int_0^{\infty} \frac{(1 - u^4)(\alpha^2 + u^2)^{1/2} \cos(Z\sqrt{\alpha^2 + u^2})e^{-uX}}{(1 - u^4)(\alpha^2 + u^2) + xi^2/\alpha^2} du
\]

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\[ +\left(\frac{\xi}{\alpha}\right) \int_0^\infty \frac{\sin(Z \sqrt{\alpha^2 + u^2}) e^{-uX}}{(1 - u^4)^2(\alpha^2 + u^2) + \xi^2/\alpha^2} du, \quad (2.47) \]

and

\[ I_{p2} = \int_0^\alpha \frac{(1 - u^4)(\alpha^2 - u^2)^{1/2} \cos(Z \sqrt{\alpha^2 - u^2}) e^{iuX}}{(1 - u^4)^2(\alpha^2 - u^2) + \xi^2/\alpha^2} du + \left(\frac{\xi}{\alpha}\right) \int_0^\alpha \frac{\sin(Z \sqrt{\alpha^2 - u^2}) e^{iuX}}{(1 - u^4)^2(\alpha^2 - u^2) + \xi^2/\alpha^2} du. \quad (2.48) \]

In Equation (2.44), \( X = k_x x \) and \( Z = k_z z \) are two normalized distance variables in the \( x - z \) plane.

### 2.4.2 Near-Field Pressure Due to a Line Moment

Similarly to the line force case, the near-field pressure due to a line moment is given as

\[ \bar{p}_M(X, Z) = \frac{p_M(X, Z)}{M_\sigma k_p \rho_\sigma / 4 \omega m} = \frac{2i}{\pi} \int_{-\infty}^{\infty} \frac{i k_x e^{ik_x x + \sqrt{\alpha^2 - k_x^2} Z}}{(k_x^2 - 1)(\alpha^2 - k_x^2)^{1/2} - i \xi/\alpha} dk_x \]

\[ = -4(R''_{1p} + R''_{2p} + R''_{3p}) + (4/\pi)(I_{p1} + I_{p2}), \quad (2.49) \]

where

\[ R''_{1p} = \frac{k_1 e^{ik_1 X - \sqrt{k_1^2 - \omega^2} Z}}{(k_1^2 - \alpha^2)^{1/2}[4k_1^3 + (\xi/\alpha)k_1(k_1^2 - \alpha^2)^{-3/2}]} \quad (2.50) \]
\[ R''_{2p,3p} = \frac{i k_{2,3} e^{i(k_{2,3} x + \sqrt{\alpha^2 - k_{2,3}^2} z)}}{(\alpha^2 - k_{2,3}^2)^{1/2}[4 k_{2,3}^3 + (i \xi / \alpha) k_{2,3}(\alpha^2 - k_{2,3}^2)^{-3/2}]} , \quad (2.51) \]

\[ I'_{p1} = \int_0^\infty \frac{u(1 - u^4)(\alpha^2 + u^2)^{1/2} \cos(Z \sqrt{\alpha^2 + u^2}) e^{-u x}}{(1 - u^4)^2(\alpha^2 + u^2) + x^2 / \alpha^2} \, du 
+ \frac{\xi}{\alpha} \int_0^\infty \frac{u \sin(Z \sqrt{\alpha^2 + u^2}) e^{-u x}}{(1 - u^4)^2(\alpha^2 + u^2) + x^2 / \alpha^2} \, du , \quad (2.52) \]

and

\[ I'_{p2} = \int_0^a \frac{u(1 - u^4)(\alpha^2 - u^2)^{1/2} \cos(Z \sqrt{\alpha^2 - u^2}) e^{iu x}}{(1 - u^4)^2(\alpha^2 - u^2) + x^2 / \alpha^2} \, du 
+ \frac{\xi}{\alpha} \int_0^a \frac{u \sin(Z \sqrt{\alpha^2 - u^2}) e^{iu x}}{(1 - u^4)^2(\alpha^2 - u^2) + x^2 / \alpha^2} \, du , \quad (2.53) \]

where \( M_\circ \) is the amplitude of the line moment and other parameters are defined as in Equation (2.44).

### 2.4.3 The Total Near-Field Pressure and Particle Velocity

Equations (2.44) and (2.49) are two fundamental relations describing individual sound radiation source which will be used to evaluate the near-field pressure. While the far-field pressure is estimated by using the stationary phase expressions, the near-field pressure does not have the luxury of that kind of explicit expression, so it has to be evaluated by means of the Cauchy residue method. The amplitudes of the line force or line moment are determined by the optimal solution of the cost function and the discontinuity boundary conditions.
In the line constraint case, the total near-field pressure is

\[ p_{\text{total}}^{\text{rad}}(X, Z) = \frac{F_o \rho_o}{4m} \bar{p}_F(X, Z) + \sum_{n=1}^{N_n} \frac{F_n \rho_o}{4m} \bar{p}_F(X - \text{sgn}(k_p l_n), Z), \]  

(2.54)

where \( F_o \) and \( F_n \) are the complex amplitudes of the line constraint back force and of the n-th control force, respectively, and \( l_n \) is the distance from \( F_n \) to \( F_o \).

In the rib case, the total near-field pressure has a similar expression:

\[ p_{\text{total}}^{\text{rad}}(X, Z) = \frac{F_o \rho_o}{4m} \bar{p}_F(X, Z) + \frac{M_o k_p \rho_o}{4m} \bar{p}_M(X, Z) + \sum_{n=1}^{N_n} \frac{F_n \rho_o}{4m} \bar{p}_F(X - \text{sgn}|k_p l_n|) \]  

(2.55)

in which \( F_o \) and \( M_o \) are the amplitudes of the rib inertia line force and line moment, respectively.

The acoustic particle velocity of the radiated field can be calculated by means of the Euler equation (Kinsler et al., 1982):

\[ v(X, Z) = \left( \frac{1}{i \omega \rho_o} \right) \nabla p, \text{ if } X > 0, \]  

(2.56)

and

\[ v(X, Z) = \left( -\frac{1}{i \omega \rho_o} \right) \nabla p, \text{ if } X < 0. \]  

(2.57)

In Equations (2.56) and (2.57), \( \nabla p \) is the gradient of pressure which is based on the difference between two adjacent values estimated by Equation (2.54) or (2.55). In the
dissertation, to save computation time, the particle velocity is numerically calculated with finite pressure gradients in \( x \) and \( z \) directions instead of using the expansion of the particle velocity in Equations (2.56) and (2.57) and the Cauchy residue method. The results proved quite satisfactory.

### 2.4.4 Time-Averaged Intensity

The time averaged intensity is defined (Fahy, 1988) as

\[
I^a(X, Z) = \frac{1}{2} \text{Re}(pv^*) ,
\]

in which \( p \) is the acoustic pressure and \( v \) is the particle velocity, while "\(^*\)" denotes complex conjugate. With the method described by Petterson (1979) and Kristiansen (1981), the intensity can be approximately evaluated by multiplying the pressure with the particle velocity which is estimated by the pressure gradient. The advantage of this method is to save the computation time, since the near-field pressure has been calculated first. As long as the grid of the intensity field is taken fine enough in terms of the pressure gradient, this approach remains favorable. According to Kristiansen (1981), the intensity at the mid-point between \( X \) and \( X + \Delta_x \) is

\[
\bar{I}^a(X_i, Z_j) \approx \frac{|p(X_{i-1}, Z_j)||p(X_i, Z_j)|}{2\rho_0\omega \Delta_x} \sin(\phi_{i,j} - \phi_{i-1,j})
\]

where

\[
\Delta_x = X_i - X_{i-1}
\]
\[ \phi_{i,j} = \tan^{-1}\left( \frac{\text{Im}[p(X_i, Z_j)]}{\text{Re}[p(X_i, Z_j)]} \right). \]  

(2.61)

In Equation (2.59), it is assumed that \( X > 0 \). If \( X < 0 \), the signs of \( \phi_{i,j} \) and \( \phi_{i-1,j} \) in Equation (2.59) should be exchanged. Similarly, in the orthogonal \( z \) direction

\[ \tilde{I}_z^2(X_i, Z_j) \approx \frac{|p(X_i, Z_{j-1})||p(X_i, Z_j)|}{2\rho_0\omega\Delta_z} \sin(\phi_{i,j} - \phi_{i,j-1}), \]  

(2.62)

where

\[ \Delta_z = Z_j - Z_{j-1}. \]  

(2.63)

Now it is observed that the intensity calculation is based on the pressure difference. Substituting Equation (2.54) or (2.55) into (2.59) and (2.62) results in the time-averaged intensity distribution.

### 2.5 Optimal Solution for Feed-Forward Control

The optimization technique for feed-forward control relies on forming a quadratic function in the control force amplitudes and finding the minimum of that function: the optimal solution. Usually, this is achieved by squaring one variable or a set of appropriate variables which are related to the system power output (Nelson et al., 1987, Fuller, 1988).
Thus, in order to derive the optimal amplitudes of control forces so as to reduce the radiated sound, we need to define a suitable cost function. A suitable cost function in this case is the total radiated power from the plate to the fluid-loaded half-space far field. Hence, the cost function is given by

\[
\beta(\{F\}) = \int_s |p_{total}^{rad}|^2 ds ,
\]  

(2.64)

where \(s\) is the semi-cylindrical surface in the far field at some arbitrary distance \(R\).

Substituting the expressions for total radiated pressure for the line constraint or the rib constraint, i.e. Equation (2.29) or (2.41) into Equation (2.64), respectively, the cost function becomes

\[
\beta(\{F\}) = \{F\}^T[A]\{F\}^* + \{F\}^T[B] + [B]^H\{F\}^* + [C] ,
\]  

(2.65)

where

\[
[A] = \int_s [(a)(a)^H] ds ,
\]  

(2.66)

\[
[B] = \int_s [(b)(a)^H] ds ,
\]  

(2.67)

\[
[C] = \int_s [(b)(b)^H] ds .
\]  

(2.68)
Note that in expressions \([A], [B] \) and \([C], \{a\} \) and \(\{b\}\) have different definitions when line constraint or rib constraint is considered, and in the above equations "\(H\)" denotes the transpose conjugate operator. The cost function is a real scalar function of the complex control vector \(\{F\}\). \([A]\) is a Hermitian matrix, i.e. \([A]^T = [A]^\ast\). When two control forces are used and symmetrically located, \([A]\) becomes a real symmetric matrix.

In order to derive the optimal control force, the cost function is differentiated with respect to the control force vector and set to zero, as outlined by previous researchers (Nelson et al., 1987, Lester and Fuller, 1986).

The optimal solution of Equation (2.65) becomes

\[
\{F\}_{opt} = -[A]^{-1}[B]^\ast, \tag{2.69}
\]

where the optimal control force vector for \(N_c\) forces is given by

\[
\{F\}_{opt}^T = [F_1 \ F_2 \ \ldots \ F_{N_c}]. \tag{2.70}
\]

### 2.6 Results and Discussion

For the system studied, the plate is assumed infinite, thin, elastic and immersed in a fluid on one side. The media chosen were steel and water with material properties
dimensions given in Table 2.1. The incident flexural wave was assumed to have a displacement amplitude \( W_0 = 1 + 0i \) mm. Two subsonic frequencies, 1170.8 rad/s and 11708 rad/s, were considered as calculation examples. These frequencies correspond to values of \( \omega/\omega_c \) equal to 0.02 and 0.2, and the critical frequency for this system is \( \omega_c = 58538 \) rad/s (Feit and Liu, 1985; Junger and Feit, 1986). The plate spatial transverse velocity calculation is based on the superposition of individual velocities due to line force or line moment, expressed in Equations (2.2) and (2.8), respectively. The amplitudes of the discontinuity back forces, \( F_0 \) and \( M_0 \), were determined by solving the boundary conditions expressed in Equations (2.24), or (2.33) and (2.34). The amplitudes of the control forces \( F_1 \) and \( F_2 \) were determined by the optimal solution in Equation (2.69). In all the directivity patterns, it was assumed that the observation distance was \( R = 10 \) m away from the line constraint or rib discontinuity. The rib was assumed to be made of steel, with dimensions and properties also given in Table 2.2.

For the results presented, either one or two control forces were considered as control inputs. When one control force was used, the matrices in the optimal analysis degenerated to scalars; however, the approach remained the same.

The first step in the calculation of radiated pressure with and without control is to evaluate matrices \([A]\) and \([B]\) in the cost function expressed in Equation (2.65). The approach used here follows closely that of Feit and Liu (1986), in which the residue contributions were evaluated explicitly (this also involves solving the system characteristic equation whose poles were discussed by Crighton (1979) and Junger and Feit (1986) in detail) and the branch cut contributions were evaluated numerically using.
a Gauss-Kronrod integral approach. This procedure can be quite difficult, and Feit and Liu (1986) discussed its major pitfalls. They also studied the response of a system identical to the one considered here to as a line force, and detailed the radiation patterns of the far field and near field at the same frequencies. These will serve as the disturbance or primary field and will also be used here to validate the legitimacy of the model derived here.

The plate velocity is plotted in terms of non-dimensional distance \( X = k_p x \) for comparison of different excitation frequencies. The far-field radiation directivity pattern is centered on the discontinuity. For convenience in the radiation plots, negative values of radiation angle \( \theta \) correspond to negative axial coordinate positions in \( z \). The near field is defined to investigate the characteristics of the sound radiation source, but it can still be extended to relatively “large” non-dimensional distances in both \( X \) and \( Z \) directions to study the intensity distribution in the spatial domain.

Since either the line constraint or the rib is assumed to be uniform, the study will be in the plane perpendicular to the discontinuity line on the plate. Hence, the line constraint-plate or rib-plate system actually presents a two-dimensional problem here.

2.6.1 Line Constraint, \( \omega/\omega_c = 0.02 \)

The magnitudes of plate velocities described by Equation (2.2) are numerically evaluated, as shown in Figure 2.3, for two drive frequencies, \( \omega/\omega_c = 0.02 \) and \( \omega/\omega_c = 0.2 \). Figure 2.3 was also previously calculated by Feit and Liu (1985) and the agreement with the present results is good. The fluctuation of the velocity along the normalized
distance shows the interaction between the plate near-field response and the fluid. These normalized transverse plate velocities are fundamental responses for the analysis of the plate vibration with and without active control.

The magnitudes of plate transverse velocities due to a line moment are also numerically evaluated in a similar way, and the results are shown in Figure 2.4 for the same two excitation frequencies calculated in the line force case. It is seen that at the drive point $X = 0$, the integrand of $\bar{v}_M(X)$ will become an odd function with respect to $X$; therefore, the integral over the symmetric interval from $-\infty$ to $+\infty$ is zero, which physically means that a moment does not cause translational velocity at the point where it applies.

Figure 2.5 presents the plate spatial transverse velocities when the plate is excited by a subsonic incident wave only, with one-force control, and with two-force control. When the plate is excited only by the disturbance, the plate velocity consists of two components: the subsonic incident wave, and the velocity due to line constraint back force at $X = 0, F_c$. It is noted that when two control forces are applied, the plate vibration maintains roughly the same pattern, and the magnitudes do not change much, while the sound radiation reduces significantly (referring to Figure 2.7 shown later). Comparing the results obtained without control with those obtained with two-force control in Figure 2.5, the plate velocity magnitude increases in some peaks on the incident side of the constraint. This clearly indicates that control forces which reduce the sound radiation in the far-field do not necessarily reduce the plate response; rather, in some circumstances, they increase the magnitude of the plate response. This is a common phenomenon in ASAC, and it will be seen in many ASAC application cases.
Figure 2.3: Normalized plate transverse velocity magnitude as a function of normalized distance due to a line force for frequencies below coincidence, $\omega/\omega_c = 0.02$ and $\omega/\omega_c = 0.2$. 
Figure 2.4: Normalized plate transverse velocity magnitude as a function of normalized distance due to a line moment for frequencies below coincidence $\omega/\omega_c = 0.02$ and $\omega/\omega_c = 0.2$. 
The cause of this phenomenon is that the effectiveness of the sound radiation ability of a vibrating structure depends not only on the magnitude of the surface acceleration, but on the form of its distribution over that surface as well. The approach of ASAC is aimed at changing the structural vibration response in such a way as to reduce those efficient sound radiation wave components, instead of suppressing the response in general. Comparing the velocity magnitudes obtained by applying one control force and two control forces in Figure 2.5, it is seen that adding an extra control force does not make much difference for the plate vibration magnitude, while sound radiation in the far-field is reduced by about another 20 dB (referring to Figure 2.7).

It is also observed that the spatial velocity magnitude on the incident side shows some standing wave pattern, while on the transmission side of the constraint the propagating velocity becomes low and smooth. This phenomenon indicates that the incident wave and the reflection wave are in opposite direction, and are approximately in phase on the incident side of the constraint. On the transmitted side of the constraint, however, the incident wave and the waves generated by the constraint back force or the control forces are in the same travelling direction (pointing to the right in this case), so their superposition of magnitudes is essentially constant. The slight distortion of the velocity magnitude around the constraint is due to the structural near-field effect around the constraint. Far away from the constraint, the plate response maintains quite consistent vibration patterns. However, the structural near field is important in terms of sound radiation. Later discussion will reveal that the supersonic wave components result from its existence and the interaction of the radiated acoustic wave.
Figure 2.5: Plate spatial velocity magnitude as a function of $k_p x$: the line constraint case, for frequency below coincidence, $\omega/\omega_c = 0.02$. 
The above discussion in the spatial domain illustrates how the plate responds before and after the active control is applied. An explanation why the sound radiation is significantly reduced with little change of the plate spatial response can be found in the analysis in the wavenumber $\vec{k}_x$ domain.

The plate velocity autospectrum is estimated by $|V(\vec{k}_x)|^2$ with Equation (2.16), for the line constraint case. In Figure 2.6, it is found out that supersonic wave component $|V(\vec{k}_x)|^2$ in the range of $|\vec{k}_x| \leq \alpha$ is drastically attenuated after one or two control forces are applied, and that two control forces provide better radiation power reduction than one control force.

The location of applying control forces is also of interest in terms of sound radiation attenuation. In general, the control force magnitudes increase when the control forces are located closer to the constraint. This increase is due to the relatively higher stiffness of the plate because of the line constraint boundary condition. When control forces are moved away from the line constraint, smaller force magnitudes are expected due to the same reason. However, with control forces moving away from the constraint, the effectiveness of the source cancellation between the control forces and the back force of the line constraint decreases. Therefore, these two factors result in a compromise control force location. In this case, when $\omega/\omega_c = 0.02$, the appropriate region to apply control forces is about $1 \leq |k_p l| \leq 4$.

Figure 2.7 presents the sound pressure directivity patterns in the far field for $\omega/\omega_c = 0.02$. It is apparent that in Figure 2.7 the disturbance radiation field is monopole like, being fairly uniform with radiation angle. This is to be expected,
Figure 2.6: Plate velocity autospectrum as a function of wave number $\bar{k}_x$: the line constraint case, for frequencies below coincidence $\omega/\omega_c = 0.02$. 

Nondimensional wavenumber $\bar{k}_x$

- without control
- with one control force
- with two control forces
since the line constraint acts as a single back-reaction force.

When one control force is applied at \( l_1 = 0.36 \, m \) on the right side of the constraint, the radiated pressure is markedly reduced by around 15 to 25 dB. The residual sound field has the shape of a dipole radiation source, due to the control force combining with the back reaction force to create a moment like input to the plate (as they are not collocated). Note that applying the control force at \( x = 0 \) will lead to no reduction, since due to the boundary condition, the input impedance is infinite at this point. Studies have also shown that either increasing or decreasing the control location will lead to a deterioration in the amount of reduction.

When two control forces at \( l_1 = 0.16 \, m \) and \( l_2 = -0.16 \, m \) are applied, there is a further reduction of sound radiation of about 20 dB. The residual radiation field is now symmetric and has the radiation pattern associated with a high order source.

It is interesting to note that there are two alternate ways to view the control action. On the one hand, from the “superposition” point of view, the control action is effectively reducing the back reaction force from the discontinuity, thus reducing the monopole radiation term. Another point of view is that the reaction force, in concert with the control force, creates a radiation source of higher order and lower radiation efficiency, resulting in little significant change in spatially averaged plate response amplitudes, while the radiated sound is decreased.

The sound radiation source characteristics can also be viewed from the sound intensity distribution near the constraint. Figure 2.8 shows that when only the subsonic
Figure 2.7: Directivity pattern of radiation — line constraint case: for frequencies below coincidence $\omega/\omega_c = 0.02$. 
incident wave impinges on the line constraint, the back force of the constraint is the single source responsible for wave scattering. The sound intensity distribution suggests that this is an efficient monopole source. When one control force is applied near the constraint, the source changes to a dipole type, as shown in Figure 2.9. This change is suggested by the following observations: (1) There is a narrow region above the composite sound source in which the radiated pressure is almost reduced to a "in vacuo" state. This means that the sound radiation field produced by the back force of the constraint and the control force almost totally cancel each other in this region as often observed in a dipole source; (2) The directions of outgoing intensity flows become less concentrated, and some of the energy appears flowing parallel to the plate surface; and (3) The pressure magnitude is reduced globally in the areas above the plate. All these observations indicate that the source becomes less efficient. When two control forces are applied symmetrically beside the constraint, the radiation source becomes even less efficient. As shown in Figure 2.10, it is found that the pressure magnitude is further attenuated above the plate surface. Except for a small region near the constraint, the total intensity flows almost parallel to the plate surface, which indicates that the radiated energy is small, and it appears there are two subsonic waves travelling away from the constraint.

Comparing the results shown in Figures 2.5, 2.7, 2.8, 2.9 and 2.10 reveals the correlation between the plate response, the far-field pressure, and the near-field intensity distribution for the sound radiation due to wave scattering from a line constraint before and after control. This approach will be used in the rest of the chapter.
Figure 2.8: Time-averaged intensity $\bar{I}(X, Z)$: the line constraint case, for frequencies below coincidence $\omega/\omega_c = 0.02$, without control.
Figure 2.9: Time-averaged intensity $I^a(X, Z)$: the line constraint case, for frequencies below coincidence $\omega/\omega_s = 0.02$, with one control force.
Figure 2.10: Time-averaged intensity $\bar{I}(X, Z)$: the line constraint case, for frequencies below coincidence $\omega/\omega_c = 0.02$, with two control forces.
2.6.2 Line Constraint, $\omega/\omega_c = 0.2$

For a higher frequency input, the same analysis is repeated. As shown in Figure 2.11, the spatial velocity magnitude is about ten times as large as in the lower frequency case, when the input frequency increases by ten times. Similar wave patterns are observed on both the incident and the transmitted side of the constraint. Again, by simply observing the spatial wave patterns, it is difficult to find a good explanation of the diminishing sound radiation after the control forces are applied. However, in Figure 2.12, the wavenumber domain analysis clearly indicates that a markedly reduced plate velocity autospectrum in the supersonic region ($|k_x| < \alpha$, or $|k_x| < k_e$) matches well the far-field pressure reduction due to control forces (referring to Figure 2.13).

The location to apply control forces, i.e. the distance $k_p l$, was again selected so as to effectively attenuate the sound radiation but not to raise the control force magnitudes too high. Figure 2.13 shows that at this higher frequency the radiation field is also monopole-like, but has significantly increased in amplitude due to the plate mobility increase.

When one control force is employed, located at $x = 0.12 \, m$, there is sound reduction, but not nearly to the degree of the lower frequency, shown in Figure 2.7. In this case, the control force location was again varied to find an optimal location. Note that its axial position is less than the lower frequency, most likely due to the shorter wavelengths for this case. When the number of control forces is increased to two ($l_1 = 0.02 \, m$, $l_2 = -0.02 \, m$), improved control is achieved, with reductions of 25 to 35 dB being observed. It is apparent that as the frequency of excitation is increased,
Figure 2.11: Plate spatial velocity magnitude as a function of $k_p x$: the line constraint case, for frequencies below coincidence $\omega/\omega_c = 0.2$. 
Figure 2.12: Plate velocity autospectrum as a function of wave number $\bar{k}_x$: the line constraint case, for frequencies below coincidence $\omega/\omega_c = 0.2$. 
Figure 2.13: Directivity pattern of radiation — line constraint case: for frequencies below coincidence $\omega/\omega_c = 0.2$.  

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the importance of using an additional second control force also increases.

The intensity distribution is similar to that analyzed in section 2.6.1, as illustrated in Figures 2.14, 2.15, and 2.16. These intensity plots suggest that in this higher frequency case, the intensity distributions have higher level compared to those results obtained in the lower frequency case, in situations without control, with one-force control, and with two-force control respectively.

2.6.3 Rib Constraint, $\omega / \omega_c = 0.02$

The sound radiation from a rib constraint is more complicated, since the noise source consists of a line force as well as a line moment. The combination of such sources will increase the difficulty of control. From Figure 2.17, it is seen that the different boundary condition ($v(X) \neq 0$) causes vibration standing wave magnitudes different from those in the line constraint case. It should be noted that the vibration amplitude of the plate in this case also depends on the physical property of the rib. The size of the stiffened uniform rib determines the level of the sound radiation and the plate vibration amplitude as well. All the spatial and spectral velocity analysis follows the line constraint case, as in section 2.6.1. The velocity responses of the plate are almost coincident when one or two control forces are applied, so it is difficult to recognize the difference in Figure 2.17.

It can also be shown that in the rib case, the amplitude difference between two control forces is considerably larger than that in the line constraint case, when two control forces are symmetrically located near the discontinuity. This is due to the fact that
Figure 2.14: Time-averaged intensity $I^a(X,Z)$: the line constraint case, for frequencies below coincidence $\omega/\omega_c = 0.2$, without control.
Figure 2.15: Time-averaged intensity $\bar{J}(X,Z)$: the line constraint case, for frequencies below coincidence $\omega/\omega_c = 0.2$, with one control force.
Figure 2.16: Time-averaged intensity $\tilde{J}^a(X, Z)$: the line constraint case, for frequencies below coincidence $\omega/\omega_c = 0.2$, with two control forces.
Figure 2.17: Plate spatial velocity magnitude as a function of $k_p x$: the rib case, for frequencies below coincidence $\omega/\omega_c = 0.02$. 

Nondimensional distance $X$

- - - - - without control
- - - - with one control force
- - - - - - - - with two control forces
Figure 2.18: Plate velocity autospectrum as a function of wave number $\bar{k}_x$: the rib case, for frequencies below coincidence $\omega/\omega_c = 0.02$. 

Nondimensional wavenumber $\bar{k}_x$

- Solid line: without control
- Dashed-dotted line: with one control force
- Dotted line: with two control forces
these two optimal forces are necessary to offset the sound radiation caused by the rib inertial line force as well as the rib inertial line moment which is not symmetric to the origin.

Figure 2.19 gives the radiation directivity patterns for an excitation frequency of $\omega/\omega_e = 0.02$. It is apparent that the sound radiation caused by the disturbance is still monopole-like for the rib system. Although the rib exerts both a back reaction moment and a force, the latter is a far more efficient source, and thus its contribution dominates the radiation field.

When control is applied, the sound field is markedly reduced by 60 to 80 dB. Note that the directivity pattern is again dipole-like, since the main action of the control force is to cancel out the rib inertia force $F_\omega$, leaving the dipole-like radiation from the moment term. It is also interesting to note that when the control force is located at $x = 0$, the amount of reduction is not as great. Thus, offsetting the force allows control of the back reaction force as well as creating some control moment input and simultaneously controlling both source terms to some degree.

Thus, one control force located at $l = 0.0054 \ m$ can attenuate the sound pressure level by about 60 dB. When two control forces are symmetrically applied at $l_1 = 0.0013 \ m$ and $l_2 = -0.0013 \ m$, a further sound reduction is achieved. The results also reveal that the closer the two control forces are, the larger the reduction that can be achieved. This is reasonable, since the two control forces are able to cancel the sound radiation from a line force as well as from a line moment. A very short distance between the two control forces will better simulate a control line moment which will lead
Figure 2.19: Directivity pattern of radiation — the rib case, for frequencies below coincidence $\omega/\omega_c = 0.02$. 

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to improved suppression of the moment radiation due to the rib rotational inertia. In practice, it will be difficult to implement two extremely closely spaced control forces, and a design compromise will be necessary.

The wavenumber domain analysis shown in Figure 2.18 illustrates a reduction in the supersonic wavenumber region similar to that in the line constraint case.

The near-field intensity distributions, as illustrated in Figures 2.20, 2.21 and 2.22, are very similar to that of the line constraint case at the same frequency excitation, since the line force due to the rib inertia is the main cause of the sound radiation. Except for some very small areas, such as in the one-force control case, the near-field intensity shows the transition from monopole source to dipole and three-lobe sources when one or two control forces are applied, similarly to the line constraint case.

2.6.4 Rib Constraint, $\omega/\omega_c = 0.2$

For an increased input frequency, the magnitude of plate velocity increases, but the axial response pattern remains the same (referring to Figure 2.23). Again, the attenuated radiation power is clearly illustrated by the wavenumber domain estimation in the supersonic region ($|\vec{k}_x| \leq \alpha$), as shown in Figure 2.24.

A similar sound pressure radiation pattern as that in the lower frequency excitation case is found with one control force located at $l_1 = 0.0092$ m, as shown in Figure 2.25. However, in this case, the amount of reduction achieved is reduced to around 30 to 50 dB. This result tends to indicate that the radiation term from the moment reaction
Figure 2.20: Time-averaged intensity $\bar{I}_a(X, Z)$: the rib case, for frequencies below coincidence $\omega/\omega_c = 0.02$, without control.
Figure 2.21: Time-averaged intensity $\bar{I}^a(X,Z)$: the rib case, for frequencies below coincidence $\omega/\omega_c = 0.02$, with one control force.
Figure 2.22: Time-averaged intensity $\tilde{n}(X, Z)$: the rib case, for frequencies below coincidence $\omega/\omega_c = 0.02$, with two control forces.
Figure 2.23: Plate spatial velocity magnitude as a function of $k_p x$: the rib case, for frequencies below coincidence $\omega/\omega_c = 0.2$. 
Figure 2.24: Plate velocity autospectrum as a function of wave number $\bar{k}_x$: the rib case, for frequencies below coincidence $\omega/\omega_c = 0.2$. 

Nondimensional wavenumber $\bar{k}_x$

- Without control
- With one control force
- With two control forces
becomes more important as the rate of rotational acceleration of the rib increases with $\omega^2$, and plate wavelengths become shorter. With two control forces symmetrically located at $l_1 = 0.0001 \text{ m}$ and $l_2 = -0.0001 \text{ m}$, increased reduction is achieved. Figure 2.25 illustrates some interesting aspects. Although two control forces were used, it was not possible to totally attenuate the radiated sound. This result is most likely due to the fact that the rib theoretically exerts its reaction moment at a line, while the control moment input is distributed over $x$. Thus, the control input cannot completely “cancel” the rib moment unless the two control forces can be implemented on a single line, i.e. as a pure line moment.

The sound radiation intensity distributions are shown from Figure 2.26 to Figure 2.28. It is found that when two control forces are applied, there are small areas where the sound pressure level is very high and sound cancellation does not work well. This fact shows that the line moment can complicate control, and the control forces cannot totally cross out the inertia force and moment effect in the near-field.

### 2.7 Conclusions

The active control of sound radiation from an infinite fluid-loaded plate with a subsonic flexural wave incident upon either a line constraint or a uniform rib has been analytically studied. The control approach is based upon the quadratic optimization of the total acoustic power radiated into the fluid half-space.

The results demonstrate that for both discontinuity cases, significant reductions in the radiated sound pressures can be achieved with two control forces located near the
Figure 2.25: Directivity pattern of radiation — the rib case, for frequencies below coincidence $\omega/\omega_c = 0.2$. 
Figure 2.26: Time-averaged intensity $\bar{I}(X,Z)$: the rib case, for frequencies below coincidence $\omega/\omega_c = 0.2$, without control.
Figure 2.27: Time-averaged intensity $\bar{I}^a(X,Z)$: the rib case, for frequencies below coincidence $\omega/\omega_c = 0.2$, with one control force.
Figure 2.28: Time-averaged intensity $\bar{I}(X, Z)$: the rib case, for frequencies below coincidence $\omega / \omega_c = 0.2$, with two control forces.
discontinuity. The efficiency of sound reduction was demonstrated to be strongly dependent upon the frequency and the location of the control forces. In general, as the frequency is increased, the role of the second control force increases in importance.

The amount of reduction obtainable was also shown to be influenced by the nature of the discontinuity. In general, better control was achieved for fixed boundary conditions, such as the line constraint, than for free boundary conditions such as the rib.

In both discontinuity cases, when optimal control forces are applied, the total plate spatial velocity magnitudes do not significantly change their overall vibration patterns, but the total spectral velocity within the supersonic wavenumber range drastically diminishes. This phenomenon illustrates how important a role the wavenumber domain analysis plays in structural acoustic research. The radiated power is indicated by the wavenumber autospectra in the supersonic region, as well as by the far-field radiated pressure. The observations in these aspects are always consistent.

The results also show that there are some optimal regions to apply the line control forces for both discontinuity cases. For the line constraint case, a distance $1 \leq |k_p| \leq 3$ can generally be considered as the optimal region for applying control forces for an incident wave input of frequency range from $\omega/\omega_c = 0.02$ to $\omega/\omega_c = 0.2$. To avoid excess force amplitude, the control force should not be too close to the line constraint due to the discontinuity stiffness. In the rib case, the situation is different in that a higher sound reduction effect is achieved when two control forces are closely located beside the rib, but in reality a design compromise may be necessary, due to physical space limitations in applying two closely located forces.
The near-field intensity distribution indicates that when secondary control forces are applied near the discontinuity on the plate, the sound radiation efficiency decreases because of the active control forces which change the original efficient radiation source into some form of high order inefficient radiation source. The low efficiency of the multipole radiation sources can also be seen in that a large part of the vibration energy from such sources flows above the plate surface, similar to subsonic wave motion, instead of propagating outwards from the surface.

In this chapter, it is suggested that the ASAC can be studied with the following variables in mind: the plate response, the far-field pressure directivity, the near-field intensity distribution, and the wavenumber domain analysis. The results show that studies of these aspects of the problem complement each other and provide a complete picture of the application of ASAC to an infinite fluid-loaded plate with discontinuities.

This study adds new understanding to the research in scattering from fluid-loaded discontinuities. The results indicate that sound radiation from subsonic waves impinging upon discontinuities can be suppressed by active structural inputs near the discontinuity. This approach can also be extended to other plate boundary conditions and finite plates, as will be studied in the next chapter.
Table 2.1: Material properties of an infinite fluid-loaded plate

<table>
<thead>
<tr>
<th>System</th>
<th>Phase speed (m/s)</th>
<th>Density (kg/m$^3$)</th>
<th>Thickness (m)</th>
<th>Incident wave magnitude (m$m$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel plate</td>
<td>2916</td>
<td>7700</td>
<td>0.0254</td>
<td>1 + 0i</td>
</tr>
<tr>
<td>Water</td>
<td>1500</td>
<td>1026</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.2: Material properties of a rib on an infinite fluid-loaded plate

<table>
<thead>
<tr>
<th>Rib</th>
<th>Density (kg/m$^3$)</th>
<th>Height (m)</th>
<th>Width (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7700</td>
<td>0.127</td>
<td>0.05</td>
</tr>
</tbody>
</table>
Chapter 3

Active Control of Sound Radiation from a Fluid-Loaded Uniform Rectangular Plate

3.1 Introduction

Having applied the ASAC to infinite fluid-loaded plates, the focus now shifts to finite fluid-loaded plates. There are many cases of practical interest to the industry and marine engineering, motivating the study of controlling the sound radiation from a finite plate. Because of the boundary conditions, the vibration behavior of a finite plate is much more complicated than that of an infinite plate, so that controlling the sound radiation becomes more difficult and a further study is needed. Much research has been done on the finite plate vibration response, the modal coupling effects due to the fluid loading, the radiation efficiency, etc. of fluid-loaded plates (Davies, 1977; Sandman, 1977; Lomas and Hayek, 1977; Fahy, 1985; Junger and Feit, 1986). All of the previous work is important in terms of understanding the behavior of sound radiation and dynamic structural response of finite fluid-loaded plates. On the other hand, as mentioned in the last chapter, active structural acoustic control (ASAC) has been recently applied to many structures such as plates (Fuller et al., 1988, 1990a,
1991) and cylinders (Fuller and Jones, 1987, Fuller et al., 1990b), with light fluid loading (i.e. no radiation coupling) as well as to an infinite fluid-loaded plate with discontinuities, as illustrated in the last chapter.

The present study is focussed on ASAC applied to a simply supported rectangular plate located in an infinite baffle and with heavy fluid loading on one side, as shown in Figure 3.1. The simply supported boundary condition is chosen because of the simplicity of the mode shapes and the convenience of comparing the uncontrolled response result with those of previous researchers. The disturbance is selected as a point force operating at a steady state harmonic frequency, while control is achieved by point or distributed forces applied to the plate. The solution to the plate motion is based on the admissible functions for an in vacuo homogeneous plate, which are also the basis for Fourier decomposition of the fluid loading (Sandman, 1977). The amplitudes of the control forces are determined by the optimal solution of a quadratic cost function which integrates the far-field radiated acoustic pressure over a hemisphere in the radiation half space. The results show that for subsonic disturbance inputs applied to plates with heavy fluid loading, high global reduction in radiated pressure is possible with properly located forces. The results thus indicate that the active structural acoustic control approach will provide large attenuations in radiated sound when edge coupling induced by heavy fluid loading is present. The number and location of the control forces are determined so as to suppress those efficiently radiating modes or to restructure those modes to make the plate acoustically less efficient. The far-field directivity pattern, the near-field pressure distribution, the modal amplitudes, and the plate velocity wavenumber autospectrum in the two-dimensional wave number domain are studied. The control objective is to minimize the total radiated power.
which is a quadratic function of the control force amplitudes. The investigation is novel because it introduces for the first time the influence of edge coupling due to the heavy fluid loading into the ASAC technique. It was not known or understood prior to this work how the modal coupling would affect control performance.

3.2 Plate Vibration

The plate is illustrated in Figure 3.1, and the coordinates are shown in Figure 3.2. Note that in Figure 3.1 the position and direction of forces are only illustrative, and control forces do not have to be point force type. In Figure 3.2, the origin of the coordinates is chosen at the corner for comparing the results with those of Sandman (1977).

3.2.1 Plate Motion

With thin plate theory, the governing equation for the transverse deflection of a fluid-loaded uniform plate is (Sandman, 1977)

\[ D \nabla^4 w + \rho_p h \frac{\partial^2 w}{\partial t^2} = q(x, y) - p_o(x, y), \]  

(3.1)

where \( D = Eh^3/12(1 - \nu^2) \) is the flexural rigidity of the plate, with \( \nu \) denoting the Poisson ratio, \( E \) is the plate material modulus of elasticity, \( h \) — the uniform thickness, \( \rho_p \) — the plate mass density. At the same time, \( w(x, y, t) \) is the displacement of point \((x, y)\) at time \( t \), \( q(x, y, t) \) — the directly applied external force, which in this case includes the point disturbance force and the control forces, and \( p_o(x, y, t) \) is the
Figure 3.1: System arrangement of a simply supported fluid-loaded plate.
Figure 3.2: Spatial coordinate definition.
fluid loading pressure.

For a simply supported rectangular plate described by Equation (3.1), it is assumed that the solution can be expressed as the sum of admissible functions as (Sandman, 1977)

$$w(x, y, t) = b \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \exp(i\omega'c_p t/a), \quad (3.2)$$

based on the boundary condition of

$$w(x, y) = \frac{\partial^2 w(x, y)}{\partial x^2} = 0, \quad x = 0, a, \quad (3.3)$$

and

$$w(x, y) = \frac{\partial^2 w(x, y)}{\partial y^2} = 0, \quad y = 0, b. \quad (3.4)$$

In Equation (3.2) the double sine summation is the admissible function, $a$ and $b$ are the plate dimensions in the $x$ and $y$ directions, respectively, $W_{mn}$ are the modal amplitudes, $\omega'$ — the non-dimensional excitation frequency, $c_p = [E/(\rho(1 - \mu^2))]^{1/2}$ is the bending wave speed in the plate, and $m$ and $n$ are the modal indices. Note that in Equation (3.2), a term $b$ is introduced in order to non-dimensionalize the modal amplitude $W_{mn}$, and $\omega = \omega'c_p/a$ when $\omega$ is the excitation frequency with unit of rad/s.
3.2.2 The Boundary Condition

The acoustic velocity field in the medium with a sound velocity $c_o$ is governed by the linearized wave equation

$$\nabla^2 \Psi = \frac{1}{c_f^2} \frac{\partial^2 \Psi}{\partial t^2},$$  \hspace{1cm} (3.5)

where $\nabla^2$ is the three dimensional Laplacian operator and $\Psi(x, y, z, t)$ is the velocity potential. The accompanying acoustic pressure is determined by

$$p(x, y, z, t) = -\rho_o \frac{\partial \Psi}{\partial t}.$$  \hspace{1cm} (3.6)

The interface boundary condition for a finite baffled plate located at $z = 0$ is

$$\left( \frac{\partial \Psi}{\partial z} \right)_{z=0} = \frac{\partial w}{\partial t}$$  \hspace{1cm} (3.7)

on the plate, and

$$\left( \frac{\partial \Psi}{\partial z} \right)_{z=0} = 0$$  \hspace{1cm} (3.8)

off the plate but in the baffled plane.

3.2.3 The Forcing Functions

The modal composition of the forcing function depends on the type of force. If an external force is applied to a simply supported plate, it has a form of harmonic
summation expressed as

$$q(x, y, t) = \frac{D}{b^3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} f_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \exp(i\omega'c_p t/a), \quad (3.9)$$

in which $f_{mn}$ is the modal coefficient of the force, $D/b^3$ is a factor used by Sandman to non-dimensionalize the force modal coefficient $f_{mn}$, and the time variation has the same form as in Equation (3.2).

If a point force expressed as

$$F(x, y, t) = F_0 \delta(x_0) \delta(y_0) \quad (3.10)$$

is the external force applied to the plate, then, by substituting Equation (3.10) into Equation (3.9) and taking double sine transform, we obtain the force modal coefficient

$$f_{mn} = \frac{ab}{4D} \sin \frac{m\pi x_a}{a} \sin \frac{n\pi y_b}{b} F_0. \quad (3.11)$$

Substituting Equation (3.11) into Equation (3.9) results in the forcing function expression for a point force.

By using the similar double sine transform for a piezoceramic actuator applied to the plate, the forcing function has the form of (Wang et al., 1991)

$$q(x, y, t) = \frac{D}{b^3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} c_{mn}(\cos \frac{m\pi x_1}{a} - \cos \frac{m\pi x_2}{a})(\cos \frac{n\pi y_1}{b} - \cos \frac{n\pi y_2}{b})$$

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\[ \times \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \exp(i\omega' c_p t/a) \]  

(3.12)

in which \(c_{mn}\) is the property parameter of the piezoceramic actuator and \(x_1, x_2, y_1\) and \(y_2\) are the piezoceramic actuator coordinates on the plate, as illustrated in Figure 3.4.

Point force and piezoceramic patches are two types of control force used as actuators in this study. If there are \(N_s\) disturbance forces and \(N_c\) control forces, the total forcing function \(q(x, y, t)\) will be the sum of the forcing decomposition function, as expressed in Equation (3.9) and/or Equation (3.12).

### 3.2.4 Radiation Impedance Equation

By combining Equations (3.5) and (3.6) and the boundary conditions on and off the plate as expressed in Equations (3.7) and (3.8) (Sandman, 1977), the modal amplitudes of the plate motion can be obtained from a complex non-diagonal matrix equation which reveals the coupled fluid-loading effects. The derivation procedure is illustrated in Appendix A. The matrix impedance equation is expressed as follows:

\[
\begin{bmatrix}
I_{111}^{+f} & I_{121}^{+f} & \cdots & I_{121n}^{+f} \\
I_{211}^{+f} & I_{222}^{+f} & \cdots & \cdot \\
\cdot & \cdot & \cdots & \cdot \\
\cdot & \cdot & \cdots & \cdot \\
I_{r+1m}^{+f} & \cdots & \cdot & I_{r+1mn}^{+f}
\end{bmatrix}
\begin{bmatrix}
W_{11} \\
W_{12} \\
\cdot \\
\cdot \\
W_{mn}
\end{bmatrix}
= 
\begin{bmatrix}
q_{11} \\
q_{12} \\
\cdot \\
\cdot \\
q_{r+1}
\end{bmatrix}
\]  

(3.13)

where \(I_{r+1mn}^{+f} = K_{r+1mn}^{p} + i\omega' R_{r+1mn}^{f} - \omega^2 M_{r+1mn}^{f+q}\) is the total impedance which includes \(K_{r+1mn}^{p}\), the plate stiffness, \(R_{r+1mn}^{f}\), the fluid radiation resistance, and \(M_{r+1mn}^{f+q}\), the com-
Figure 3.3: Point force coordinates on a rectangular plate.

Figure 3.4: Piezoceramic actuator coordinates on a rectangular plate.
bined mass. In general, \( I_{rsmn}^{p+1} \) indicates the coupling between the \((r, s)\) mode and the \((m, n)\) mode. The individual components are defined by relations

\[
K_{rsmn}^p = \{(r\pi)^2 + [s\pi(b/a)]^2\}^2 \delta_{rm} \delta_{sn}, \quad (3.14)
\]

and

\[
R_{rsmn}^f + i\omega M_{rsmn}^{f+p} = 12\left(\frac{a}{h}\right)^2 i\omega' \delta_{rm} \delta_{sn} + 12\frac{\rho_c C_0}{\rho_p C_p} \left(\frac{a}{h}\right)^3 Z_{rsmn}^f, \quad (3.15)
\]

where \( \delta_{rm} \) and \( \delta_{sn} \) are Kronecker delta functions. It is obvious that \( K_{rsmn}^p \) only appear in diagonal elements in the impedance matrix in Equation (3.13). Observation of Equation (3.15) reveals that the off-diagonal elements occur in the second term of \( R_{rsmn}^f + i\omega M_{rsmn}^{f+p} \), which represent the cross-modal coupling between the \((r, s)\) mode and the \((m, n)\) mode due to fluid loading.

### 3.2.5 Fluid-Loading Impedance

\( Z_{rsmn}^f \) defines the direct and cross-modal fluid loading impedance with the detailed form of

\[
Z_{rsmn}^f = \begin{cases} 
  ikb^2 \int_0^1 \int_0^1 F_r m(u) G_{sn}(v) \Omega(u, v) dudv, \\
  \text{when } r + m = \text{even and } s + n = \text{even}; \\
  0, \\
  \text{when } r + m = \text{odd and/or } s + n = \text{odd},
\end{cases} 
\quad (3.16)
\]
with

\[ \Omega(u,v) = \frac{\exp[-ik_0a(u^2 + b^2v^2/a^2)^{1/2}]}{(u^2 + b^2v^2/a^2)^{1/2}} , \quad (3.17) \]

\[ F_{rm}(u) = \int_0^{1-u} \sin[r\pi(u + u_1)] \sin(m\pi u_1) \, du_1 , \quad (3.18) \]

and

\[ G_{sn}(v) = \int_0^{1-v} \sin[s\pi(v + v_1)] \sin(n\pi v_1) \, dv_1 , \quad (3.19) \]

where \( u, v, u_1 \) and \( v_1 \) are determined by the transformation

\[ u = (x - x_1)/a; \quad u_1 = x_1/a \quad (3.20) \]

and

\[ v = (y - y_1)/b; \quad v_1 = y_1/b . \quad (3.21) \]

The analysis of \( Z'_{rsmn} \) depends on numerical estimation, since the integral is intractable.
3.2.6 Spectral Velocity In Wavenumber Domain

As in Chapter 2, the wavenumber domain analysis is performed with the spectral velocity estimation. In the finite plate case, the wavenumber domain velocity has two dimensions instead of one. The two-dimensional wave number domain \((k_x, k_y)\) analysis demonstrates the reduction within the radiating supersonic wavenumber region \((\sqrt{k_x^2 + k_y^2} \leq \omega/c_o)\). The wavenumber expression of the plate velocity is given by the Fourier transform of the plate velocity as follows:

\[
\tilde{V}(k_x, k_y) = \int_{-\infty}^{\infty} v(x, y)e^{i(k_xx + k_yy)} = i\omega'c_p \sum_{i=1}^{N_I} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \times \frac{(m\pi/a)}{k_x^2 - (m\pi/a)^2} \left[(-1)^m e^{-ik_y b} - 1\right] \frac{(n\pi/a)}{k_y^2 - (n\pi/b)^2} \left[(-1)^n e^{-ik_x a} - 1\right], \tag{3.22}
\]

where \(v(x, y) = i\omega w(x, y)\) and \(w(x, y)\) is the solution of Equation (3.13). The velocity autospectrum is used to evaluate the wave number domain energy, and its expression is

\[
|\tilde{V}(k_x, k_y)|^2 = \tilde{V}(k_x, k_y)\tilde{V}^*(k_x, k_y). \tag{3.23}
\]

It should be noted that the velocity wavenumber autospectrum is also a function of excitation frequency.
3.3 Sound Radiation

3.3.1 Radiation Pressure

The solution of Equation (3.13) yields the fluid-loaded plate response \( w(x, y, t) \) and the corresponding pressure field \( p(x, y, z, t) \). The detailed procedure was described by previous researchers (Davies, 1971, Sandman, 1977, Lomas and Hayek, 1977), and the results are summarized here. The steady-state acoustic pressure is

\[
\tilde{P}(x, y, z) = 12 \frac{\rho_0 c_0}{\rho_p c_p} \left( \frac{a}{h} \right)^3 (i\omega j) \frac{i k_o}{2\pi} \times \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \int_0^a \int_0^b \sin \frac{m\pi x_1}{a} \sin \frac{n\pi y_1}{b} \exp \left( -ik_o R \right) dx_1 dy_1
\]

(3.24)

and

\[
p(x, y, z, t) = \frac{D}{a^2} \tilde{P}(x, y, z) \exp(i\omega' c_p t / a)
\]

(3.25)

where \( R = [(x - x_1)^2 + (y - y_1)^2 + z^2]^{1/2} \), and \( k_o \) is the acoustic wave number. \( \tilde{P} \) is the non-dimensional spatial variation. Equation (3.24) is the result of solving Equation (3.5) (the wave equation) and applying Equation (3.6) (the Euler equation). More details are presented in Appendix A. In solving Equation (3.13), a truncation of modes is made by taking \( m = 6 \) and \( n = 6 \) for numerical estimation. To estimate the exactness of this truncation, \( m = 10 \) and \( n = 10 \) were taken to repeat the same calculation of \( w(x, y, t) \). The results obtained from these two truncations are very close (the difference is less than 1%). To save the calculation time, values of \( m = 6 \) and \( n = 6 \) are chosen to perform the numerical estimation. Equation (3.25) is used to calculate the near-field pressure radiated from the plate.
The far-field radiation pressure is expressed as follows (Sandman, 1977)

$$p_{far}(R, \theta, \phi) = \frac{6D\rho_a \omega^3}{\pi R \rho_p h^3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} T_{mn}(\theta, \phi) ,$$  \hspace{1cm} (3.26)

where

$$T_{mn}(\theta, \phi) = \frac{b}{a} (m\pi)(n\pi) \frac{[1 - (-1)^m e^{iK_x}][1 - (-1)^n e^{iK_y}]}{[(m\pi)^2 - K_x^2][n\pi - K_y^2]}$$  \hspace{1cm} (3.27)

is the modal transmissibility function, $K_x = k_c a \sin \theta \cos \phi$ and $K_y = k_c b \sin \theta \sin \phi$ are the direction cosines of the far-field position, and $\theta$ and $\phi$ are defined in Figure 3.2.

### 3.3.2 Total Sound Radiation

The total far-field radiated pressure due to the disturbance and control input is

$$p_{far}^t(R, \theta, \phi) = \sum_{i=1}^{N_d} p_{far}^i(R, \theta, \phi) + \sum_{j=1}^{N_c} p_{far}^j(R, \theta, \phi)$$

$$= \sum_{i=1}^{N_d} B_i q_i + \sum_{j=1}^{N_c} A_j p_j$$

$$= \{B\}^T \{q\} + \{A\}^T \{p\}$$  \hspace{1cm} (3.28)

in which the total pressure $p_{far}^t$ is the sum of $N_d$ disturbance forces and $N_c$ control forces, while $\{q\}$ is the disturbance vector, $\{p\}$ the control force vector, $\{A\}$ the distribution vector for the control force and $\{B\}$ the distribution vector for the distur-
bance. Note that both \( \{A\} \) and \( \{B\} \) are functions of displacement modal amplitude \( \{W_{mn}\} \) and depend on the solution of Equation (3.13). The procedure of deriving vectors \( \{A\} \) and \( \{B\} \) was similar to that used in the infinite plate case, as illustrated in Chapter 2. Substituting the solution of Equation (3.13) into the far-field pressure expression of Equation (3.28) results in

\[
A_j = \frac{6a^4 \rho \omega^2}{\pi R \rho_p h^3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{W_{mnj}}{p_j} T_{mn}(\theta, \phi) \tag{3.29}
\]

and

\[
B_i = \frac{6a^4 \rho \omega^2}{\pi R \rho_p h^3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{W_{mni}}{q_i} T_{nn}(\theta, \phi), \tag{3.30}
\]

where \( T_{mn}(\theta, \phi) \) is defined by Equation (3.27), \( p_j \) is the \( j \)-th element of \( \{p\} \), and \( q_i \) the \( i \)-the element of \( \{q\} \).

### 3.4 Optimal Control

The feed-forward control approach is described in Chapter 2. The difference in this case is that the objective of the optimal control is to minimize the far-field sound radiation over a hemisphere above the plate in the fluid half-space. The cost function used in determining such an optimal control can be expressed as (Fuller, 1988)

\[
\beta(p) = \frac{1}{R^2} \int_S |p_{far}^t|^2 ds = \int_0^{2\pi} \int_0^{\theta} |p_{far}^t|^2 \sin \theta d\theta d\phi, \tag{3.31}
\]
which can be written in matrix form as

\[
\beta(p) = \{p\}[A]\{p\]^* + \{q\}^T[B]\{p\}^* + \{p\}^T[B]^H\{q\}^* + \{q\}^T[C]\{q\}^*,
\] (3.32)

where the superscript "\(T\)" denotes transposition, "\(^*\)" denotes conjugation, and "\(H\)" denotes transposition and conjugation. Matrices \([A]\), \([B]\) and \([C]\) are the results of substituting the vector sum of Equation (3.28) into Equation (3.32) and can be expressed as follows:

\[
[A]_{N_e \times N_e} = \int_0^{2\pi} \int_0^{\pi} \{A\}\{A\}^H \sin \theta d\theta d\phi,
\] (3.33)

\[
[B]_{N_e \times N_e} = \int_0^{2\pi} \int_0^{\pi} \{B\}\{A\}^H \sin \theta d\theta d\phi,
\] (3.34)

and

\[
[C]_{N_e \times N_e} = \int_0^{2\pi} \int_0^{\pi} \{B\}\{B\}^H \sin \theta d\theta d\phi.
\] (3.35)

The optimal solution for the control force vector is (Lester and Fuller, 1991)

\[
\{p\} = -[A]^{-1}[B]\{q\}
\] (3.36)

It is observed that \(-[A]^{-1}[B]\) determines the relation between the disturbance force \(\{q\}\), which is assumed to be known, and the control force \(\{p\}\). Matrix \([A]\) can be considered the distribution matrix of control forces, and matrix \([B]\) the distribution.
matrix relating the control force and the disturbance.

3.5 Results and Discussion

The numerical study is based on an aluminum rectangular plate whose material properties and dimensions are listed in Table 3.1. The center-point-driven response of the plate depicted in Figure 3.5 illustrates the plate resonance with and without heavy fluid loading. For the problem considered here, the excitation frequencies are only those which dominantly excite the low order modes of the plate and are well below the coincidence frequency, \( f_c = c_o^2 (m_p/D)^{1/2} \approx 23,966\, Hz \) for the given plate, where \( c_o \) is the sound velocity in the sea water and \( m_p \) the plate density per area. The disturbance force amplitude is taken as 10 \( N \) in all cases calculated in the following examples. The sound reference pressure is taken as 20 \( \mu Pa \), one of the three reference pressures recommended by Kinsler et al. (1982).

The presence of fluid loading lowers the resonant frequencies of the plate response but does not significantly change the structural mode shapes (Fahy, 1985). The natural frequencies of the first several modes are estimated numerically from the plate displacement diagram and compared with those estimated with the help of the approximate expression provided by Fahy (1985). The far-field pressure is calculated by means of Equation (3.26), in which the optimal control forces are based on Equation (3.36). Near-field pressures are evaluated using a numerical integration of Equation (3.24). With a harmonic point force applied at the center of the plate as the disturbance, the plate is excited at on- and off-resonant frequencies. The near-field pressure distributions illustrate how the control forces modify the sound radiation sources and
Table 3.1: Material properties of an aluminum rectangular plate and the acoustic medium.

<table>
<thead>
<tr>
<th>System</th>
<th>Phase speed $(m/s)$</th>
<th>Density $(kg/m^3)$</th>
<th>Thickness $(m)$</th>
<th>Size $(m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alum. plate</td>
<td>5432</td>
<td>2700</td>
<td>0.009525</td>
<td>0.5588 $\times$ 0.8636</td>
</tr>
<tr>
<td>Sea water</td>
<td>1500</td>
<td>1026</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 3.5: Frequency response of a rectangular plate.
change the features of structural acoustic coupling near the surface of the plate.

3.5.1 Resonance Frequencies

To study the behavior simply due to the fluid loading, a uniform plate is investigated in this chapter. The plate model is shown as Equation (3.1).

For a simply supported rectangular plate in vacuo, the natural frequencies are estimated as

\[ \omega_{mn} = \sqrt{\frac{D}{m_p}} \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right] \]  

(3.37)

where \( m_p = \rho \cdot h \). For a rectangular plate submerged in heavy fluid, it is assumed that the natural frequencies fall below their in vacuo values in proportion to the square root of the ratio of the loaded to unloaded modal masses (Fahy, 1985). According to Fahy, the analysis of reactive loading on structural waves having wavenumbers much greater than an acoustic wavenumber has shown that the effective added mass per unit area is \( \rho_o / k_{mn} \), where \( \rho_o \) is the fluid density and \( k_{mn} \) the primary effective wavenumber component of the vibration. The approximate expression of the fluid-loaded structure natural frequency is

\[ \omega_{mn}^{\text{fl}} \approx \omega_{mn} (1 + \rho_o / m_p k_{mn})^{-1/2}, \]  

(3.38)

where \( \omega_{mn} \) is the correspondent in vacuo natural frequency defined by Equation (3.37).
Hence, there are two ways to determine fluid-loaded plate natural frequencies: one is to observe the peak values from the frequency response diagram such as Figure 3.5, because the nonlinearity of Equation (3.13) makes an explicit solution of eigenvalues unavailable, and the other is to use Equation (3.38). In the following part of the dissertation, the results obtained with the use of these two different methods are compared and found to be very consistent in most situations.

Figure 3.5 illustrates the center-point displacement magnitude of the plate for center-point excitation previously estimated by Sandman (1977). Because of the location of the drive point, it is seen that even numbered modes cannot be excited, so that only the contribution of odd-odd modes appears in the response diagram. The in vacuo resonances are well predicted by Equation (3.37). The natural frequencies of the fluid-loaded case evaluated by two different approaches also converge well (refer to Tables 3.5.1 and 3.5.1). The relative errors between the results are reasonable (8.0 % for $\omega'_{13}$, 9.1 % for $\omega'_{31}$, 4.6 % for $\omega'_{15}$ and 4.3 % for $\omega'_{33}$) except for the first mode (30 % for $\omega'_{11}$). The comparison of these results suggests that Equation (3.38) is a fairly good estimation for fluid-loaded plate lower order modal natural frequencies except for the fundamental frequency.

### 3.5.2 Fundamental Mode Excitation

When the center point disturbance frequency coincides with the first mode (1,1) resonance, a relatively high sound radiation arises. Because of the location and the frequency of the excitation, the fundamental mode dominates the plate vibration and the sound radiation. Since the plate is vibrating like a monopole due to one efficient
Table 3.2: Natural frequencies (Hz) of the fluid-loaded plate estimated with Fahy's approximate formula (Fahy, 1985)

<table>
<thead>
<tr>
<th>mode (m, n)</th>
<th>1</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40.41</td>
<td>173.8</td>
<td>489.7</td>
</tr>
<tr>
<td>3</td>
<td>388.9</td>
<td>555.2</td>
<td>905.7</td>
</tr>
</tbody>
</table>

Table 3.3: Natural frequencies (Hz) of the fluid-loaded plate estimated from numerical frequency response evaluation

<table>
<thead>
<tr>
<th>mode (m, n)</th>
<th>1</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30.94</td>
<td>160.9</td>
<td>467.8</td>
</tr>
<tr>
<td>3</td>
<td>356.4</td>
<td>532.2</td>
<td></td>
</tr>
</tbody>
</table>
radiation mode, the task is to try to suppress the vibration of this mode by secondary forces so as to reduce the sound radiation.

With one control force located at \((x_1, y_1) = (a/4, b/4)\), the controlled plate is observed to radiate like a dipole (referring to Figure 3.6) and the sound radiation is attenuated in the far field by around 65 to 85 dB. From the near-field pressure shown in Figure 3.7(a) and Figure 3.7(b), it is found out that the overall pressure level drops by about 44 dB above the surface of the plate. This drop indicates that the suppression of the vibration of the \((1,1)\) mode leads to a global sound reduction. With two control forces located at \((x_1, y_1) = (a/4, b/4)\) and \((x_2, y_2) = (3a/4, 3b/4)\), a further 25 to 30 dB of far-field pressure attenuation is achieved as observed in Figure 3.6. Comparing Figure 3.7(b) and Figure 3.7(c) reveals that not only a further 15 to 20 dB reduction of pressure level is achieved in the near-field, but the pattern of the radiation source is also changed from a two-lobe type to a three-lobe type. This means that not only the dominant \((1,1)\) mode is suppressed, but the relations between the residual modes are re-adjusted so as to make the overall contribution to sound radiation less efficient. This phenomenon is known as “modal restructuring”. When four control forces are located at one-sixth of the lengths away from the plate edges, even further sound attenuation is observed in the far-field (the residual pressure directivity is localized around the origin in Figure 3.6). The force amplitudes and locations are listed in Table 3.5.2. The control force amplitudes are found to be essentially real (with relatively small imaginary part), so they are either in phase or out of phase with the distance force, which has a real amplitude. It is also observed that the control forces have the same amplitudes if they are located symmetrically with respect to the center of the plate, where the disturbance force is located.
Figure 3.6: Far-field directivity pattern: on-resonance excitation, $f = 31 Hz, \phi = 0$. 
Figure 3.7: Near-field sound pressure level at $z = 0.01m$, $f = 31Hz$. 

(a) Disturbance only
(b) With one control force
(c) With two control forces
(d) With four control forces
Table 3.4: Disturbance and control force amplitudes and locations, $a$ — plate length in the $x$-direction, $b$ — plate length in the $y$-direction

<table>
<thead>
<tr>
<th>Excitation frequency: $f = 31 (Hz)$</th>
<th>Amplitude ($N$)</th>
<th>Location ($x, y$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disturbance force:</td>
<td>$10 + 0 \times i$</td>
<td>$(0.5a, 0.5b)$</td>
</tr>
<tr>
<td>One control force:</td>
<td>$-19.465 - 8.526 \times 10^{-5} \times i$</td>
<td>$(0.25a, 0.25b)$</td>
</tr>
<tr>
<td>Two control forces:</td>
<td>$-9.7325 - 4.198 \times 10^{-5} \times i$</td>
<td>$(0.25a, 0.25b)$</td>
</tr>
<tr>
<td></td>
<td>$-9.7325 - 4.328 \times 10^{-5} \times i$</td>
<td>$(0.75a, 0.75b)$</td>
</tr>
<tr>
<td>Four control forces:</td>
<td>$-9.5444 - 3.1253 \times 10^{-2} \times i$</td>
<td>$(0.1667a, 0.1667b)$</td>
</tr>
<tr>
<td></td>
<td>$-9.5444 - 3.1273 \times 10^{-2} \times i$</td>
<td>$(0.8333a, 0.8333b)$</td>
</tr>
<tr>
<td></td>
<td>$-9.5448 + 3.1131 \times 10^{-2} \times i$</td>
<td>$(0.1667a, 0.8333b)$</td>
</tr>
<tr>
<td></td>
<td>$-9.5448 + 3.1149 \times 10^{-2} \times i$</td>
<td>$(0.1667a, 0.8333b)$</td>
</tr>
</tbody>
</table>
The near-field sound pressure level magnitude distribution shown in Figure 3.7(d) indicates that further modal restructuring is performed, since the overall pressure level does not seem to be lower than that shown in Figure 3.7(c), but different source patterns are observed in these two controlled cases. Note that when comparing the controlled residual pressure distributions shown in Figs. 3.7(b), 3.7(c) and 3.7(d) with the uncontrolled pressure distribution shown in Figure 3.7(a), the modal suppression, i.e. the suppression of the efficient (1,1) mode, seems to remain the main cause of the sound reduction. This observation can be extended to those cases in which only one efficient mode is dominantly excited to radiate sound.

To better explain the sound power reduction, the plate velocity autospectrum in a two dimensional wavenumber domain is calculated, and the results are shown in Figure 3.8. The reference value in Figure 3.8 and Figure 3.11 is arbitrary so that the autospectrum represents the relative values. It is observed that the supersonic region of the wavenumber spectrum, illustrated by the area within the small circle where $\sqrt{k_x^2 + k_y^2} \leq k$, decreases in magnitude with the increase of the number of control forces. This clearly explains that the active control reduces the sound radiation energy through reducing the radiated power in the supersonic region. Meanwhile it is observed that the reduction in sound radiation is not necessarily accompanied by a reduction in plate vibration. For example, the area outside the supersonic region remains at almost the same level in Figures. 3.8(c) and 3.8(d). Only the wavenumber spectrum within the supersonic region is important from the point of view of the far-field sound reduction. On the other hand, the modal suppression is also confirmed by comparing Figures 3.8(b),3.8(c), and 3.8(d) with Figure 3.8(a); the velocity au-
tospectrum is reduced within and outside the supersonic region. This phenomenon indicates not only a reduction in sound radiation level, but a reduction in vibration magnitude level as well.

3.5.3 Off-Resonant Excitation

The off-resonant example is illustrated with the plate centrally driven at a frequency of \( f = 434 \text{ Hz} \). From Figure 3.5, it can be seen that this frequency is higher than in the resonance of mode (3,1) so that more modes are involved in the plate response. The results show that although the sound radiation level due to the disturbance is relatively lower than that of the on-resonant excitation example, reasonable sound reduction is much harder to obtain. Because the control forces need to be coupled into the sound radiation due to several modes in this off-resonant excitation case, the selection of actuator locations is more difficult than the previous case. For example, in this case, the (3,1), (1,5), and (3,3) modes combine to contribute to the plate vibration and sound radiation. It is not an easy job to position one control force to couple into all these three modes in such a way as to reduce their respective sound radiations properly to achieve a global sound reduction. Therefore, there is nothing unusual in observing that one control force reduces the far-field radiation only by around 2 to 4 dB in Figure 3.9. After more than two control forces are added, a reasonable sound reduction is achieved. Figure 3.9 shows that about 15 to 30 dB of attenuation is obtained in the far-field with two control forces, and the directivity patterns suggest that the radiation source is of a multipole type. Four control forces provide a further sound reduction in a global extent of about 10 to 20 dB, although in the region from \( \theta = -25^\circ \) to \( \theta = 25^\circ \) the radiation increases by about 10 to 15 dB
Figure 3.9: Far-field directivity pattern: off-resonant excitation, $f = 434$ Hz, $\phi = 0$. 
Figure 3.10: Near-field sound pressure level at $z = 0.01m$, $f = 434\ Hz$. 
Table 3.5: Disturbance and control force amplitudes and locations, $a$ — plate length in the $x$-direction, $b$ — plate length in the $y$-direction

<table>
<thead>
<tr>
<th>Excitation frequency:</th>
<th>$f = 434$ (Hz)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude ($N$)</td>
<td></td>
<td>Location ($x, y$)</td>
</tr>
<tr>
<td>Disturbance force:</td>
<td>$10 + 0 \times i$</td>
<td>$(0.5a, 0.5b)$</td>
</tr>
<tr>
<td>One control force:</td>
<td>$5.7973 - 2.8288 \times 10^{-3} \times i$</td>
<td>$(0.25a, 0.25b)$</td>
</tr>
<tr>
<td>Two control forces:</td>
<td>$12.981 - 1.3601 \times 10^{-2} \times i$</td>
<td>$(0.5a, 0.25b)$</td>
</tr>
<tr>
<td></td>
<td>$12.981 - 1.3601 \times 10^{-2} \times i$</td>
<td>$(0.5a, 0.75b)$</td>
</tr>
<tr>
<td>Four control forces:</td>
<td>$4.7092 - 3.514 \times 10^{-3} \times i$</td>
<td>$(0.3333a, 0.3333b)$</td>
</tr>
<tr>
<td></td>
<td>$4.7092 - 3.514 \times 10^{-3} \times i$</td>
<td>$(0.6667a, 0.6667b)$</td>
</tr>
<tr>
<td></td>
<td>$4.7092 - 3.514 \times 10^{-3} \times i$</td>
<td>$(0.6667a, 0.3333b)$</td>
</tr>
<tr>
<td></td>
<td>$4.7092 - 3.514 \times 10^{-3} \times i$</td>
<td>$(0.3333a, 0.6667b)$</td>
</tr>
</tbody>
</table>
over the two-control-force case. The control force amplitudes and positions are listed in Table 3.5.3. The force amplitudes are found to have similar behaviors as in Table 3.5.2.

An examination of near-field pressure distribution from Figs. 3.10(a) to 3.10(d) implies that the off-resonant panel source is far more complicated than the on-resonant case, and a far-field sound reduction does not always accompany a significant overall pressure level reduction in the near-field due to "modal restructuring". The near-field pressure distribution in Figure 3.10(a) is the result of radiation of the (3,1), (1,5) and (3,3) modes. When one control force is applied, there is no apparent reduction of the pressure level, as shown in Figure 3.10(b), but there is some change in terms of the source pattern. Meanwhile, it is seen in Figure 3.11(b) that there is some minor radiation reduction, as illustrated by the velocity autospectrum drop in the upper right area in the supersonic circle. This means that ASAC works, although the result is not as good as in the on-resonant case, when one control force is used. Another interesting phenomenon found by observing the subsonic regions in Figures 3.11(a) and 3.11(b) is found that the subsonic region in Figure 3.11(b) has a higher level than that in Figure 3.11(a), which indicates that the plate vibration level may be higher when control is applied (this observation can only be confirmed when the velocity autospectrum is plotted in the full range of \(-\infty < k_x < \infty, -\infty < k_y < \infty\)). The development yields two conclusions: (1) ASAC does not always reduce the structural response; (2) modal restructuring sometimes can play an important role in reducing the sound radiation. When two control forces are applied, it is observed that the overall pressure level is only slightly reduced (Figure 3.10(c)), but a much better sound reduction is observed in the far field (Figure 3.9) as well as in the supersonic region.
of velocity autospectrum (Figure 3.11(c)). From the near-field pressure distribution shown in Figure 3.10(c), it can be concluded that the change of source pattern causes the sound reduction. Finally, when four control forces are applied to the plate, the overall near-field pressure level is reduced by about 10 dB, and the source pattern is further modified (Figure 3.10(d)). A further reduction of velocity autospectrum in supersonic region is also observed in Figure 3.11(d), corresponding to the far-field pressure reduction in Figure 3.9. In this four-force control case, it can be concluded that modal suppression, as well as modal restructuring, is the mechanism for modifying the panel source and reducing the sound radiation.

3.5.4 Piezoceramic Actuators Compared with Point Forces

As mentioned in Chapter 1, piezoceramic patches have become very useful actuators in ASAC. Mainly due to their light weight and accessibility, they have been widely used on light structures for vibration or acoustic control. The performance of PZT actuators on a fluid-loaded structure is worth investigation. In this section, the control performance of PZT actuators will be compared with that of point force actuators. For simplicity, only the uniform rectangular case is studied, but the results extend to many examples. For comparison, the center of each PZT actuator coincides with the position where the point force was located. The forcing function of PZT was presented in section 3.2.3.

The first calculation example is studied in section 3.5.2, where the plate is excited at 31 Hz, the resonant frequency of the (1,1) mode. Figures 3.12 and 3.13 show that when one force is applied, the point force provides slightly better control than
the PZT actuator. The sound reductions observed in both Figures 3.14 and 3.15 are excellent ( > 95 dB). Figure 3.16 demonstrates the modal amplitudes of the plate motion when the plate is excited with the disturbance only, with one or two point forces or PZT actuators. Only three modal amplitudes are shown in Figure 3.16, since the amplitudes of other modes are negligible in magnitude. Figure 3.16 shows that both the point force and the PZT actuator reduce the amplitude of the (1,1) mode by about 17 dB. The PZT has a little bit more spillover on the (1,2) mode, and that is possibly the reason that the residual far-field pressure is slightly higher. In Table 3.6, it is observed that the supersonic power reduction produced with one point force is slightly better than that of one PZT. In the two-control-force case, it seems from Figures 3.14 and 3.15 that the PZT actuators provide better performance, while the power reduction shown in Table 3.6 shows that it is still point forces that outperform the PZT's. Therefore, $\phi = 0$ and $\phi = 90^\circ$ are two planes in which the control results with two PZT's look better than those with two point forces. It is shown that actually four control forces do not provide improved control performance over two control forces (the slightly lower value is associated with numerical precision in the calculation), since the locations of the four forces are not optimal. However, point forces still outperform PZT's. Similar results were found by Wang for a plate with light fluid loading.

In the second example, the plate is driven at the center at $f = 434 \, Hz$, an off-resonant frequency. When one control force is applied, neither the point force nor the PZT provides a satisfactory sound reduction from the view of the directivity patterns shown in Figures 3.18 and 3.19. Figure 3.24 indicates that modes (1,5), (3,1) and (3,3) contribute to sound radiation when the plate is excited with the disturbance.
Figure 3.12: Far-field directivity pattern: on-resonance excitation, $f = 31\, Hz, \phi = 0$, one-force control, comparison between point force and PZT actuator.
Figure 3.13: Far-field directivity pattern: off-resonance excitation, $f = 31$ Hz, $\phi = 90^\circ$, one-force control, comparison between point force and PZT actuator.
Figure 3.14: Far-field directivity pattern: on-resonance excitation, $f = 31 \text{ Hz}$, $\phi = 0$, two-force control, comparison between point force and PZT actuator.
Figure 3.15: Far-field directivity pattern: on-resonance excitation, $f = 31$ Hz, $\phi = 90^\circ$, two-force control, comparison between point force and PZT actuator.
Figure 3.16: Modal amplitude of the plate, $f = 31 \text{ Hz}$, a comparison between point force and PZT actuator.
Figure 3.17: Wavenumber domain plate velocity autospectrum, $f = 31$ Hz, PZT actuators.
force only. This behavior is expected, since \( f = 434 \, \text{Hz} \) is between the (3,1) and (1,5) modes from Figure 3.5, and two other odd-odd modes, the (1,1) mode and the (1,3) mode resonance points, are well away from the excitation frequency. It seems that the point force does a better job in reducing the (1,5) mode and thus provides a better control performance. Table 3.6 demonstrates that the better power reduction is achieved by one point force than by one PZT actuator. When two control forces are applied, a better control performance is also achieved by point forces (refer to Table 3.6 and Figures 3.11 and 3.25), although in plane \( \phi = 0 \) the residual sound far-field controlled with two PZT's appears better than that with two point forces. However, when four control forces are used, it is demonstrated that the PZT actuators perform almost as well as the point forces. The supersonic power reductions achieved with PZT's and those achieved with point forces are of the same scale (referring to Table 3.6, reductions over 50 dB can be considered the same). This phenomenon is likely due to the increase in the number of actuators.

In general, it is shown that point force actuators outperform PZT actuators in terms of reducing the sound radiation. This finding is consistent with the calculation results of Wang (1990), based on considering a plate radiating into a light fluid medium. The study of the mechanism of this difference should be undertaken on the grounds of mechanics in future research.
Figure 3.18: Far-field directivity pattern: off-resonance excitation, $f = 434 \, \text{Hz}$, $\phi = 0$, one-force control, comparison between point force and PZT actuator.
Figure 3.19: Far-field directivity pattern: off-resonance excitation, $f = 434 \text{ Hz}$, $\phi = 90^\circ$, one-force control, comparison between point force and PZT actuator.
Figure 3.20: Far-field directivity pattern: off-resonance excitation, $f = 434 \text{ Hz}$, $\phi = 0$, two-force control, comparison between point force and PZT actuator.
Figure 3.21: Far-field directivity pattern: off-resonance excitation, $f = 434 \text{ Hz}$, $\phi = 90^\circ$, two-force control, comparison between point force and PZT actuator.
Figure 3.22: Far-field directivity pattern: off-resonance excitation, $f = 434\, \text{Hz}$, $\phi = 0$, four-force control, comparison between point force and PZT actuator.
Figure 3.23: Far-field directivity pattern: off-resonance excitation, $f = 434 \text{ Hz}$, $\phi = 90^\circ$, four-force control, comparison between point force and PZT actuator.
Figure 3.24: Modal amplitude of the plate, $f = 434 \text{ Hz}$, a comparison between point force and PZT actuator.
Figure 3.25: Wavenumber domain plate velocity autospectrum, $f = 434$ Hz, PZT actuators.
Table 3.6: Total reduction of radiated acoustic power (dB)

<table>
<thead>
<tr>
<th>number of actuators</th>
<th>$f = 31\text{Hz}$ point force</th>
<th>$f = 31\text{Hz}$ PZT actuator</th>
<th>$f = 434\text{Hz}$ point force</th>
<th>$f = 434\text{Hz}$ PZT actuator</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.87</td>
<td>3.55</td>
<td>26.87</td>
<td>18.54</td>
</tr>
<tr>
<td>2</td>
<td>6.142</td>
<td>5.764</td>
<td>242.7</td>
<td>90.38</td>
</tr>
<tr>
<td>4</td>
<td>5.933</td>
<td>5.457</td>
<td>334.3</td>
<td>343.9</td>
</tr>
</tbody>
</table>
3.5.5 The Influence of Fluid-Loading Cross-Modal Coefficients

As indicated by the element expressions of matrices $[A]$ and $[B]$ in Equations (3.29) and (3.30), the estimation of $[A]$ and $[B]$ depends on the modal amplitudes of the plate response, $W_{mn}$, which is the solution of Equation (3.13). As presented in section 3.2.4, the impedance matrix on the left side of Equation (3.13) represents the major work of solving the equation. The diagonal elements in that matrix, $I_{iii}$, represent the direct modal coupling due to the plate stiffness and the fluid loading, and the off-diagonal elements in the matrix, $I_{rsmn}$, represent the cross-modal coupling due to fluid loading between the $(r, s)$ mode and the $(m, n)$ mode. It is interesting to investigate the influence of those cross-modal couplings in terms of the estimation of the feedforward controller, i.e. $[A]^{-1}[B]$. If the off-diagonal elements are ignored in the impedance matrix, the modal amplitudes $W_{mn}$ in Equation (3.13) will have a one to one relation with the external forcing function modal component $q_{rs}$. This will make the numerical estimation as easy as in the in vacuo plate case, although the plate is loaded with heavy fluid. A preliminary estimation is made that if the impedance matrix in Equation (3.13) is diagonal, the numerical calculation time of 80% can be saved, based on the first 36 modes ($m = 6, n = 6$). Not including the off-diagonal elements in the impedance matrix is equivalent to ignoring the cross-modal fluid coupling between modes. It should be noted that this simplification of the impedance matrix only applies to the calculating of distribution matrices $[A]$ and $[B]$, which are used to find the optimal control force $\{p\}$, and does not apply to the calculating of the plate response $w(x, y, t)$ and the acoustic pressure $p(x, y, z, t)$. This investigation is to study the influence of the fluid cross-modal coupling on the determination of the optimal control force, and thus on the ASAC performance.
Two numerical calculation examples are presented in this section. The plate is uniform so that only the fluid loading contributes to the cross-modal coupling.

The first example is calculated when the plate is excited at $f = 31 \, Hz$, the fundamental mode of the plate. The optimal control forces are estimated by not including the cross-modal coupling and the control results are compared with those when control were performed by including the cross-modal coupling. From the directivity pattern in the far-field (Figure 3.26; the residual pressures controlled with one and two control forces without including the cross-modal coupling happen to be the same), it is noted that the residual radiated pressure is markedly increased when the cross-modal coupling is ignored in estimating the optimal control force. The residual pressure is up about 58 dB, 100 dB, and 116 dB respectively, for one, two, and four control force cases compared with the result obtained with the control force taking all the cross-modal coupling into consideration (referring to Table 3.7). From the modal amplitudes before and after ASAC, shown in Figure 3.27, it is obvious that control forces estimated without including the cross-modal coupling failed to reduce the dominant and acoustically efficient (1,1) mode, but the control forces estimated including the cross-modal coupling did. In this on-resonance excitation case, the best way to reduce the sound radiation is to suppress the modal amplitude of the (1,1) mode. Since the control forces estimated without including the cross-modal coupling have some problems reducing the (1,1) mode, a decline of the control performance is expected. But it needs to be pointed out that the sound radiation can still be attenuated by about 42 dB with one or two control forces and not including cross-modal terms. This is performance if it was realized in practice.
Figure 3.26: Far-field directivity pattern: on-resonance excitation, $f = 31 \, \text{Hz}, \phi = 0$, a comparison between controllers with and without including cross-modal coupling.
Figure 3.27: Modal amplitude of the plate, $f = 31\text{ Hz}$, a comparison between controllers with and without including cross-modal coupling.
Table 3.7: Far-field pressure reduction with two kinds of control forces (dB), $f = 31 \text{ Hz}, \phi = 0$

<table>
<thead>
<tr>
<th>number of actuators</th>
<th>control force including cross-modal coupling</th>
<th>control force not including cross-modal coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>42</td>
</tr>
<tr>
<td>2</td>
<td>137 $\sim$ 142</td>
<td>42</td>
</tr>
<tr>
<td>4</td>
<td>145</td>
<td>29</td>
</tr>
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</table>
The other example is calculated when the plate is excited at $f = 434\ Hz$, an off-resonant case. Previous work discussed in section 3.5.3 showed that this is a more difficult case. However, it seems that the optimal control forces estimated without including the cross-modal coupling can work better compared with the on-resonant case. From Figures 3.28 and 3.29, it is observed that with one control force, the control result is very good from the view of $\phi = 90^\circ$ plane but is not as good when viewed from $\phi = 0$ plane, in which the residual pressure exceeds the uncontrolled pressure level in regions of $-90^\circ < \theta < -45^\circ$ and $45^\circ < \theta < 90^\circ$. It should be kept in mind that the control result obtained with those forces not including the cross-modal coupling can never outperform that obtained with control forces including the cross-modal coupling, because the optimal function estimated with the former does not include the total acoustic power, since it ignores the cross terms. The controlled pressure achieved with the former force sometimes seems better than that achieved with the latter force, as observed in the one-force control case illustrated in Figure 3.28 and in the two-force control case illustrated in Figure 3.29, only because the selected directivity planes ($\phi = 0$ and $\phi = 90^\circ$) happen to be those where the result achieved with the former forces appears better.

By using two control forces without including the cross-modal coupling, the overall sound radiation seems very good in both planes of $\phi = 0$ (Figure 3.28) and $\phi = 90^\circ$ (Figure 3.29). In Figure 3.28 it is seen that the spillover only occurs in the region from $\theta = -45^\circ$ to $\theta = 45^\circ$, by about 4 to 20 dB. In Figure 3.29, the result seems better than that achieved with two control forces including the cross-modal coupling, but it should be remembered that this happens to the plane in which this improve-
Figure 3.28: Far-field directivity pattern: off-resonance excitation, $f = 434\, \text{Hz}, \phi = 0$, a comparison between controllers with and without including cross-modal coupling.
Figure 3.29: Far-field directivity pattern: off-resonance excitation, $f = 434 \, Hz$, $\phi = 90^\circ$, a comparison between controllers with and without cross-modal coupling.
Figure 4.1: Line mass arrangement
Figure 3.30: Modal amplitude of the plate, $f = 434$ Hz, a comparison between controllers with and without including cross-modal coupling.
Table 3.8: Far-field pressure reduction with two kinds of control forces (dB), \( f = 434 \, Hz \)

<table>
<thead>
<tr>
<th>number of actuators</th>
<th>( \phi = 0 )</th>
<th>( \phi = 90^\circ )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>control force</td>
<td>control force</td>
</tr>
<tr>
<td></td>
<td>including</td>
<td>not including</td>
</tr>
<tr>
<td></td>
<td>cross-modal coupling</td>
<td>cross-modal coupling</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>-5 ~ 8</td>
</tr>
<tr>
<td></td>
<td>10 ~ 15</td>
<td>-10 ~ 15</td>
</tr>
<tr>
<td>2</td>
<td>20 ~ 50</td>
<td>3 ~ 10</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>10 ~ 15</td>
</tr>
<tr>
<td>4</td>
<td>32 ~ 62</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>24 ~ 40</td>
<td>24</td>
</tr>
</tbody>
</table>
ment occurs. However, the overall performance achieved with the forces not including cross-modal coupling is very good in this case.

When four control forces are applied, the control result achieved with the forces without including the cross-modal coupling is also good. From the plane of $\phi = 0$, it is seen that the residual sound radiation is about 10 to 20 dB higher than that controlled with the forces including the cross terms, mostly occurring in $-55^\circ < \theta < 55^\circ$. In the plane of $\phi = 90^\circ$, it is seen that the overall controlled sound radiation is reduced by about 25 dB, i.e. it is up about 9 dB compared with that achieved with forces including the cross-modal coupling. Therefore, the overall performance of the forces not including the cross-modal coupling is quite impressive. The pressure reductions are listed in Table 3.8.

The observation in this off-resonance case suggests that the control forces estimated without including cross-modal coupling may have a better performance in off-resonant cases than in on-resonant cases. It is possibly because more modes are involved in off-resonant vibration so there is more capability for modal restructuring, even if the control forces are optimally located. It is also interesting to note from Figure 3.30 that the control forces estimated with the cross-modal coupling cause a large spillover on the (2,4) mode and have a better performance than those estimated without cross-modal coupling. This fact indicates that the optimal control is achieved by increasing the amplitude of the (2,2) mode. While the sub-optimal control forces do not cause such a spillover, the control results become deteriorated. This means that the structural spillover to some inefficient modes may be necessary when an optimal control is applied, particularly in off-resonant cases.
3.6 Conclusions

Active control of sound radiation from a fluid-loaded rectangular plate excited by a point force at subsonic frequencies has been analytically studied in this chapter. The control forces are chosen so as to minimize the total acoustic power radiated into a hemisphere in the fluid-loaded half-space. The reduction in sound radiation has been shown to depend on the excitation frequency which determines what modes contribute dominantly to the total radiation. In general, off-resonant excitations are more difficult to control than on-resonant ones, since more modes are involved. In the cases studied in this chapter, up to two control forces are needed to effectively control radiation for on-resonant excitation, and up to four control forces for off-resonant excitation.

A two-dimensional wave number domain analysis illustrates how the wavenumber components in the supersonic region decrease when active control is applied. This approach reveals the cause of sound reduction from the point of view of plate vibration radiating components. It is demonstrated that for plates with heavy fluid loading, sound radiation control occurs by two major mechanisms, viz. (1) modal suppression with which the magnitudes of those dominantly radiating modes are controlled and (2) modal restructuring with which the plate-averaged response is little changed but has a lower radiation efficiency due to a more complex residual source. The sound reduction in the on-resonant case is achieved mainly through modal suppression, and the sound reduction in the off-resonant case is achieved through both modal suppression and modal restructuring.
The performance of piezoceramic actuators (PZT) were compared to point forces in controlling the sound radiation from a rectangular fluid-loaded plate. It is demonstrated that both the PZT and point force actuators can control the sound radiation effectively, while the point force actuators slightly outperform the PZT's. This result is consistent with that of Wang (1990), but different from that of Guigou in the one-dimensional case (1991). It suggests that in the two-dimensional case, the PZT does not perform as well as point force when their centers coincide. While the mechanism of this phenomenon needs to be further studied, it should be pointed out, as Wang did (1990), that PZT is still preferable in practice because of its light weight and convenience of implementation.

The influence of the cross-modal fluid-induced coupling on estimating the control forces is also studied. Ignoring the cross-modal coupling due to fluid loading will greatly simplify the control force estimation but cannot guarantee consistently good control performance for all excitation frequencies. If the optimal control force is not absolutely required but the time of calculation is a constraint, the cross-modal coupling can be ignored in the estimation of the control force, and the subsequent controller can still provide reasonable sound attenuation rate, especially in off-resonant cases, when more modes are involved.

This study adds new understanding to the research in controlling the sound radiation from finite fluid-loaded plates. The results indicate that the ASAC feedforward control approach will provide high sound attenuation for vibrating structures submerged in heavy fluids, including edge radiation coupling phenomena.
Chapter 4

Active Control of Sound Radiation from a Fluid-Loaded Rectangular Plate with Mass Discontinuities

4.1 Introduction

In practice, mass discontinuities can exist in many structures. For example, when a uniform plate is excited with a shaker, the force transducer and the stinger can be considered a lumped mass attached to the plate. The knob on the door of an aircraft cabin or the rib on a ship hull should be considered in modelling the vibration and sound radiation of the door or the hull. As investigated in Chapter 2, a mass discontinuity on a structure will cause sound scattering when a subsonic flexural wave is travelling in the structure. When a finite plate is studied, the mass effect will change the mode shapes of planar radiators and modify the sound radiation. Therefore, the mass discontinuity effect is worth investigating.

In the previous chapter, the vibration and sound radiation from a uniform rectangular plate were studied. When a plate has lumped masses attached, the vibration and the associated sound radiation behavior can be different from those of a uniform plate,
just as was shown for an infinite plate in Chapter 2. For a finite plate, the mass attachment produces a mode shape change and causes cross-modal coupling. If the plate is vibrating in vacuo, an explicit eigenvalue solution is still available, as long as the lumped mass is included in the mass matrix expression in the plate motion equation. However, as was illustrated in the last chapter, this kind of an eigenvalue solution is not available for a finite fluid-loaded plate, so a numerical solution is used. The same approach is adopted for a fluid-loaded plate with discrete point or line masses attached. For a numerical solution, the only difference between estimating a uniform plate and a plate with masses attached occurs in the system matrix, denoted by $M_{rsmn}$ in Equation (3.15). This mass effect can be easily incorporated into the model established in section 3.2.4, and the subsequent sound radiation can be calculated by means of the approach described in section 3.3.

### 4.2 Plate Motion Modification

#### 4.2.1 Plate Motion

If there is one discrete mass $M$ attached to the plate at $(x_i, y_i)$, the plate transverse motion is expressed with (Sandman, 1977)

$$D\nabla^4 w + [\rho_p h + M\delta(x - x_i)\delta(y - y_i)]\frac{\partial^2 w}{\partial t^2} = q(x, y) - P_0(x, y) ,$$

(4.1)

in which the mass $M$ effect is added to the plate mass, $\rho_p h$, with the assistance of the Kronecker delta function ($\delta_{mn} = 1, m = n; \delta_{mn} = 0, m \neq n$). Since the rest of Equation (4.1) remains the same as Equation (3.1), the solution procedures for Equation (3.1) derived in section 3.2 are valid in this case and will not be repeated.
4.2.2 Radiation Impedance Modification

If the mass attachment is described in Equation (4.1), the only variation in finding the plate vibration amplitudes $W_{mn}$, in comparison with section 3.2.4, is the modification of the combined mass in the total impedance $I_{rsmn}^{p+f}$. We obtain

$$R_{rsmn}^{f} + i\omega M_{rsmn}^{f+ip} = 12 \left( \frac{a}{h} \right)^2 [i\omega \delta_{rm} \delta_{sn} + 4 \frac{M}{\rho_{phab}} Z_{rsmn}^i]$$

$$+ 12 \frac{p_{cf}}{\rho_{pc} \frac{a}{h}} Z_{rsmn}^f,$$  

\hspace{1cm} (4.2)

in which $M_{rsmn}^{f+ip}$ replaces $M_{rsmn}^f$ from Equation (3.15), indicating the addition of a discrete mass $M$ at point $i$. $Z_{rsmn}^i$ represents the cross-modal coupling between the $(r,s)$ mode and the $(m,n)$ mode due to mass loading, and $Z_{rsmn}^f$ represents the cross-modal coupling due to fluid loading.

4.2.3 The Mass-Loading Impedance

If there is only one discrete mass attached at location $(x_i, y_i)$, by definition, the direct and cross-modal reactances due to the mass attachment can be expressed as

$$Z_{rsmn}^i = (i\omega') \sin \frac{r\pi x}{a} \sin \frac{s\pi y}{b} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}.$$  

(4.3)

If there are $N_M$ point masses attached to the plate, the Kronecker delta term in Equation (4.1) should be replaced by $\sum_{i=1}^{N_M} M_i \delta(x - x_i) \delta(y - y_i)$, in which $M_i$ is the inertia.
of the $i$-th discrete mass located at $(x_i, y_i)$, and the mass-loading term in Equation (4.2) will be replaced by $\sum_{i=1}^{N_M} Z_{rsmn}^i$, while each $Z_{rsmn}^i$ is determined by Equation (4.3).

If the mass attached to the plate is distributed as a line instead of discrete points, as illustrated in Figure 4.1, the Kronecker delta term in equation (4.1) will be replaced by $M \delta(x - x_1)H(x_2 - x)\delta(y_1)$ when the line mass is parallel to the $y$ axis, or by $M \delta(x_1)H(y - y_1)H(y_2 - y)$ when the line mass is parallel to the $x$ axis. In this situation, it should be noted that the line mass is taken as a lumped mass and its dynamics are ignored for the simplicity of calculation.

If a double sine transform is performed on Equation (4.1) when a line mass is attached, the result will be different from the result achieved from discrete masses. It can be illustrated as follows:

$$
\int_{a}^{b} \int_{0}^{a} \sin\left(\frac{m \pi x}{a}\right) \sin\left(\frac{n \pi y}{b}\right) \sin\left(\frac{r \pi x}{a}\right) \sin\left(\frac{s \pi y}{b}\right) H(x - x_1)H(x_2 - x)\delta(y - y_1) dx dy
$$

$$
= \int_{x_1}^{x_2} a \sin\left(\frac{m \pi x}{a}\right) \sin\left(\frac{r \pi x}{a}\right) dx \int_{0}^{b} \sin\left(\frac{n \pi y_1}{b}\right) \sin\left(\frac{s \pi y}{b}\right) \delta(y - y_1) dy
$$

$$
= Z_x Z_y . \tag{4.4}
$$

Then Equation (4.3) should be revised as

$$
Z_{rsmn}^i = (i \omega') Z_x Z_y , \tag{4.5}
$$

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where

\[
Z_x = \frac{a}{(m + r)\pi} \cos\left[\frac{(m + r)\pi(x_1 + x_2)}{a}\right] \sin\left[\frac{(m + r)\pi(x_1 - x_2)}{a}\right] - \frac{a}{(m - r)\pi} \cos\left[\frac{(m - r)\pi(x_1 + x_2)}{a}\right] \sin\left[\frac{(m - r)\pi(x_1 - x_2)}{a}\right],
\]

(4.6)

and

\[
Z_y = \sin\left(\frac{n\pi y_i}{b}\right) \sin\left(\frac{s\pi y_i}{b}\right).
\]

(4.7)

If the line mass is of the form \( M\delta(x - x_i)H(y - y_{i1})H(y_{i2} - y) \), the derivation is as above, and the only difference is the exchange of the subscripts of \( Z_x \) and \( Z_y \) in Equations (4.6) and (4.7). For line masses not parallel to either the \( x \) or the \( y \) axis, similar transform can still be performed, but the lengthy derivation is omitted here.

Both the presence of the fluid and the discretely attached mass produce cross-modal coupling effects. The direct modal coupling is estimated with \( Z^{f}_{rmn} \) and \( Z^{i}_{rmn} \) when \( r = m \) and \( s = n \). The fluid- and mass-induced modal coupling is present when \( r \neq m \) or \( s \neq n \).

### 4.3 Results and Discussion

The fluid-loaded rectangular plate has the same boundary and baffle conditions as those described in Chapter 3, except that there are one or more discrete masses or a line mass attached to the plate. In Figure 4.2, the resonances of the plate with and
Figure 4.2: Frequency response of a mass-attached plate.
without mass loading are plotted. The point mass is attached at the center of the plate and it is of the same weight as the plate. This mass weight is selected in order to compare the calculation results with those of Sandman (1977), and they are found to be very consistent. The most significant mass loading effects occur at a relatively high frequency range \((f > 300 \text{ Hz})\) in Figure 4.1, as pointed out by Sandman (1977), because the heavy fluid loading dominates in the lower frequency range. The disturbance force amplitude is still taken as \(10 \text{ N}\), as in the uniform plate case.

Another example of the mass loading effect is illustrated in Figure 4.3 with an off-centrally attached mass, and the result is also very close to that achieved by Sandman (1977). It is also observed in Figure 4.3 that the asymmetrically attached mass will excite some modes which were not excited by the disturbance force. Because the chosen off-center location is almost at the nodal line of \(n = 3\), the plate responses at resonance \((3,1)\) and \((3,3)\) do not differ much between the case with mass (dashed line in Figure 4.3) and that without mass (solid line in Figure 4.2). However, more modes are excited due to the off-center mass attachment (seven peaks in Figure 4.3 vs. five peaks in Figure 4.2). It seems that the point mass can be considered an additional point excitation, once the plate is excited, due to its back reaction force. This also means that the mode shapes of the plate are changed by the attached mass.

The mass-loading effect on a plate in vacuo was discussed by Snowdon (1975), and the effect on a fluid-loaded plate was numerically calculated by Sandman (1977). Unfortunately, unlike in the case of the uniform plate discussed in section 3.5.1, an approximate expression such as Equation (3.38) is not available for a fluid-loaded plate with masses attached. Hence, the only way to estimate the resonances for such plates is to estimate the plate response numerically, which is a time-consuming pro-
Figure 4.3: Frequency response of a rectangular plate with a point mass attached off-centrally.
The mass-loading effects on ASAC performance will be studied by attaching one or several masses to the plate and calculating the plate vibration in the wavenumber domain and the sound radiation in the near field and far field.

4.3.1 ASAC of the Plate with a Single Mass

The off-center single mass loading discussed in the last section is further studied in this section. This structure is chosen for investigation because its response and sound radiation were studied by Sandman (1977). Here, ASAC is applied to show how the sound can be attenuated. This can be viewed as a continuation of Sandman’s work. When a discrete point mass of the same magnitude as the plate mass is attached to the plate at position \((x_i, y_i) = (0.5a, 0.3b)\), it changes the natural frequencies of the uniform plate and acts as an additional point force exciting the plate. At the off-resonant frequency \(f = 294 \text{ Hz}\), a combination of modes is excited. Here, it is necessary to point out that the admissible modal expansion derived for a uniform plate has a different meaning for a plate with a discontinuity because in that case the mass loading changes the mode shapes. This is different from the case of fluid loading, which does not change the mode shapes. Therefore, it is even more difficult to determine the locations of the control forces, because the mass loading changes the nodal lines, and due to the changed mode shapes, it is not clear which modes are acoustically efficient.

However, it is assumed here that the mode shapes for a uniform plate are still used as
a first guess when considering the location of the control force, and the attached mass is viewed as a force. If the control force location based on this assumption cannot provide satisfactory results, the position of the control force should be adjusted until a good result is achieved. This is not an optimal process, but it is better than random searching. From Figure 4.4, it is observed that without control, the disturbance at the center of the plate, combined with the off-center mass, causes the plate to radiate sound like a monopole from the view in the $\phi = 0$ plane. However, the near-field pressure distribution in Figure 4.5(a) suggests that the sound source is a combination of modes. It is understandable that the near field is symmetric in the $y$ parallel planes and asymmetric in the $x$ plane because both the mass and the disturbance force are located on the nodal line of $(2, \ast)$ modes.

When one control force is applied at $(0.167a, 0.333b)$ to the plate, the far field pressure is reduced by 2 to 9 dB (Figure 4.4). The position of the control force enables the force to cancel some mass loading effects. The source pattern in Figure 4.5 shows little reduction in terms of its overall pressure level, but some changes in the radiation source order occurs. Actually, the near-field pressure level around the control force increases slightly compared with the uncontrolled one shown in Figure 4.5(a). The mechanism of modal restructuring is most likely the reason for the sound reduction in the far field. This control force position may not be the best, but it provides sound reduction, which is confirmed by observing the attenuation in the supersonic wavenumber region in Figure 4.6(b).

When two control forces are applied at $(0.167a, 0.333b)$ and $(0.833a, 0.833)$ respectively, the sound reduction is about 15 dB, as shown in Figure 4.4. The selection of
Figure 4.4: Far-field directivity pattern: off-resonance excitation, $f = 294 \text{ Hz}$, $\phi = 0$, one point mass.
(a) Disturbance only

(b) With one control force

(c) With two control forces

(d) With four control forces

Figure 4.5: Near-field pressure level at $z = 0.01m$, $f = 254$, one point mass.
the control force locations is aimed at reducing the sound radiation generated by the mass-plate system, so that the control forces need to couple into the efficient radiation modes. In Figure 4.5(c), it is noted that the source becomes dipole-like in the y plane (but is still monopole-like in the x plane, and this is why the directivity pattern in Figure 4.4 looks quite uniform). Hence, the source is less efficient compared to the one without control, although the overall near-field pressure does not decrease much. A further sound reduction is also observed in the range of \(|k| < k_o\) in Figure 4.6(c). The modal restructuring mechanism again explains the sound attenuation. It is also noted that in Figures 4.6(b) and 4.6(c) the subsonic wave components increased when compared with those in Figure 4.6(a), which implies that the modal restructuring is achieved at the cost of spillover to some inefficient modes, possibly raising the overall vibration level.

Figure 4.5 shows that there is some near-field sound reduction when four control forces are applied, and the far-field pressure is reduced by about 20 to 44 dB (Figure 4.4). The modal suppression and modal restructuring mechanisms explain this kind of sound reduction. This means that from the near field pressure reduction it can be deduced that some efficient modes are reduced in magnitudes, while their phases are also modified by the control forces so the combination becomes acoustically less efficient. The wavenumber velocity autospectrum shown in Figure 4.6 suggests that there is more sound reduction compared with the two-force control case.

The control force amplitudes and positions are listed in Table 4.3.1. As in the uniform plate case discussed in Chapter 3, the optimal control forces are either in phase or out of phase with respect to the disturbance force. It is also observed that when
Figure 4.6: Wavenumber domain plate velocity autospectrum, f = 294 Hz, one point mass.
two control forces are applied, the amplitude of the second control force is much less than the first one. This suggests that the second control force may not be in a good position.

The sound radiation reduction is confirmed by the examination of the supersonic region in the velocity autospectrum, as shown in Figure 4.6. Comparing Figure 4.6(a), 4.6(b), 4.6(c), and 4.6(d) to Figure 4.5(a), 4.5(b), 4.5(c), 4.5(d), respectively, it is not difficult to observe that the sound reduction is achieved by modal suppression and/or modal restructuring.

4.3.2 ASAC of a Plate with Multiple Point-Mass Loading

To study the point-mass loading effect further, three masses are chosen to be attached to the plate. When the number of discrete point-masses is increased, the basic analysis remains the same as in single-mass loading. However, there will be more cross-modal interaction due to more masses being attached to the plate. In the case studied here, three masses are located at $(x_1, y_1) = (0.167a, 0.167b), (x_2, y_2) = (0.500a, 0.833b),$ and $(x_3, y_3) = (0.667a, 0.500b)$. The disturbance force is at $(x_f, y_f) = (0.500a, 0.833b)$, coincident with the position of the second mass. The mass positions are selected at the antinodal lines of the $(\ast,3)$ modes, if the plate is uniform. The frequency $f = 294\, Hz$ is chosen for comparison reasons, but the plate response at this frequency is expected to be very different from that with one-point-mass loading due to different point mass distributions on the plate. To make the comparison possible, each of the three point masses is chosen to be $1/3$ of the plate mass, so that the total mass is the same as that used in the one-mass loading case discussed in section 4.3.1. The plate response,
Table 4.1: Disturbance and control force amplitudes and locations on a plate with one discrete mass attached at $(0.5, 0.3b)$, $a$ — plate length in the $x$-direction, $b$ — plate length in the $y$-direction

<table>
<thead>
<tr>
<th>Excitation frequency: $f = 294,(Hz)$</th>
<th>Amplitude $(N)$</th>
<th>Location $(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disturbance force:</td>
<td>$10 + 0 \times i$</td>
<td>$(0.5a, 0.5b)$</td>
</tr>
<tr>
<td>One control force:</td>
<td>$-2.2408 - 2.4844 \times 10^{-6} \times i$</td>
<td>$(0.167a, 0.333b)$</td>
</tr>
<tr>
<td>Two control forces:</td>
<td>$-2.7477 + 2.3953 \times 10^{-5} \times i$</td>
<td>$(0.167a, 0.333b)$</td>
</tr>
<tr>
<td></td>
<td>$0.17543 - 1.6783 \times 10^{-6} \times i$</td>
<td>$(0.833a, 0.833b)$</td>
</tr>
<tr>
<td>Four control forces:</td>
<td>$4.3889 - 9.2132 \times 10^{-3} \times i$</td>
<td>$(0.167a, 0.167b)$</td>
</tr>
<tr>
<td></td>
<td>$5.0365 + 1.5414 \times 10^{-2} \times i$</td>
<td>$(0.167a, 0.667b)$</td>
</tr>
<tr>
<td></td>
<td>$3.9903 - 9.5188 \times 10^{-3} \times i$</td>
<td>$(0.833a, 0.833b)$</td>
</tr>
<tr>
<td></td>
<td>$-1.0288 + 1.0751 \times 10^{-2} \times i$</td>
<td>$(0.1667a, 0.8333b)$</td>
</tr>
</tbody>
</table>
such as the one in Figure 4.3, is not calculated due to computation time constraints.

When the point force disturbance excites the plate at 294 Hz, there is a smooth plateau around the $\theta = 0$ region in the directivity pattern in the plane of $\phi = 90^\circ$ ($y$ plane). This indicates that the sound source is not a simple monopole type, as those shown in Figures 3.6 or 3.9. Instead, it seems that the source is a combination of several radiation modes. In spite of the asymmetry of the mass arrangement, the source shows some symmetry in the $y$ planes (Figure 4.8). The near-field pressure level is quite high near both the $y = 0$ and the $y = b$ edges but is low in the middle of the plate (reduced by 10 to 15 dB). This indicates that the plate has some strong edge radiation character at these two edges.

By applying one control force at $(x_{c1}, y_{c1}) = (0.167a, 0.333b)$, the residual sound pressure becomes more like a dipole type in the $\phi = 90^\circ$ plane. Figure 4.8 shows that some source change is found, but the basic shape remains. This is most likely the reason that the overall sound is not significantly reduced. The control force location was selected with the objective of reducing the $(3,*)$ modal effect, but the resulting control force did not couple well with the sound radiation due to $(*,3)$ modes. However, from wavenumber autospectrum (Figure 4.9), it is observed that there is some reduction in the supersonic wavenumber region (Figure 4.9).

When two control forces are applied at $(x_{c1}, y_{c1}) = (0.167a, 0.333b)$ and $(x_{c2}, y_{c2}) = (0.500a, 0.167b)$ respectively, so as to couple into the $(3,*)$ modes as well as $(*,3)$ modes, the residual sound pressure decreases by about 20 dB. But it is also observed that the near-field pressure increases globally, with the changed source pattern sug-
gested by the different distribution style. This phenomenon can be explained by the fact that modal restructuring sometimes leads to overall spillovers so great that the plate response magnitude increases significantly. This observation suggests that the modal restructuring sometimes modifies the source radiation pattern at the cost of slightly increasing the global near-field pressure. The sound reduction and the vibration increase are also confirmed by examining the supersonic wavenumber region and the subsonic wavenumber region in Figure 4.9.

When four control forces are applied, a further sound radiation reduction of 10 dB is observed in the far field (Figure 4.7). The corresponding near-field pressure level is also found to be reduced globally, accompanying a source pattern change (Figure 4.8). This implies that both modal suppression and modal restructuring are the mechanisms modifying the radiation source. Obviously, a greater number of control forces provide more flexibility for coupling into an increased number of modes. Ideally, the locations of these control forces are chosen based on the consideration of coupling into the efficient radiating modes produced by the disturbance and the three point masses. But in practice, these locations are difficult to find. A further sound reduction is also observed in the supersonic wavenumber region shown in Figure 4.9.

The force and mass locations and the control force amplitudes are listed in Table 4.3.2. All the control forces are almost in phase with respect to the disturbance. When one control force is applied, the magnitude of the control force is quite small (2.9 - i1.06 \times 10^{-7} N) compared to the disturbance force (10 N). This indicates that the control channel number is too small to input enough control authority while not causing significant spillover. As a consequence, the control performance is not as good
Figure 4.7: Far-field directivity pattern: off-resonance excitation, $f = 294 \text{ Hz}$, $\phi = 90^\circ$, three point masses.
Figure 4.8: Near-field pressure level at $z = 0.01 \text{m}$, $f = 294 \text{Hz}$, three point masses
Figure 4.9: Wavenumber domain plate velocity autospectrum, $f = 294$ Hz, three point masses.
as expected. When two control forces are employed, a much better sound reduction is obtained (Figure 4.7). From Table 4.3.2, it is seen that the force amplitudes (43.5 N and 18.1 N) are in the same scale as the disturbance force. This indicates that the control forces have the same authority to excite the plate-mass system as well as the disturbance source. Therefore, a better control result is expected, as long as the location of the two control forces are not too close to the system nodal line. When four control forces are used, it is interesting to find that the magnitude of the first control force is relatively small compared to the other three forces. This means that this force may be taken out without significantly affecting the control performance. The rest of the three force can be considered as being within the same level of the disturbance force, so they should be able to provide reasonable control performance, as shown in Figure 4.7.

4.3.3 ASAC of a Plate with a Line Mass Loading

To explore the control performance when more complicated mass structures are involved, a line-mass-loading case is investigated. The line mass attached on the plate simulates a rib stiffener on a plate. In this example, it is assumed that the mass line is parallel to the $x$ axis, uniformly distributed from $(x_1, y_1) = (0.167a, 0.250b)$ to $(x_2, y_2) = (0.833a, 0.250b)$, as illustrated in Figure 4.1. Once again, the line mass weight is the same as the weight of the plate. The line mass is parallel to the $x$ axis, so that it is expected that $(\ast, 2)$ modes will be excited. For a uniform plate, $(\ast, 2)$ modes are acoustically inefficient because of the symmetry of two adjacent out-of-phase radiators. In this case, however, the mass loading shifts the nodal lines of the radiators and changes the symmetry of adjacent radiators in the $x$ direction.
Table 4.2: Disturbance and control force amplitudes and locations on a plate with three discrete masses attached at \((0.17a, 0.17b),\) \((0.50a, 0.83b),\) and \((0.67a, 0.50b),\) \(a\) — plate length in the \(x\)-direction, \(b\) — plate length in the \(y\)-direction

<table>
<thead>
<tr>
<th>Excitation frequency:</th>
<th>(f = 294, (Hz))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amplitude ((N))</td>
</tr>
<tr>
<td>Disturbance force:</td>
<td>(10 + 0 \times i)</td>
</tr>
<tr>
<td>One control force:</td>
<td>(2.9118 - 1.0624 \times 10^{-1} \times i)</td>
</tr>
<tr>
<td>Two control forces:</td>
<td>(43.556 - 4.7839 \times 10^{-3} \times i)</td>
</tr>
<tr>
<td></td>
<td>(18.095 - 2.3229 \times 10^{-3} \times i)</td>
</tr>
<tr>
<td>Four control forces:</td>
<td>(1.8994 - 1.4012 \times 10^{-1} \times i)</td>
</tr>
<tr>
<td></td>
<td>(8.1838 - 4.1345 \times 10^{-1} \times i)</td>
</tr>
<tr>
<td></td>
<td>(11.020 - 1.7230 \times 10^{-1} \times i)</td>
</tr>
<tr>
<td></td>
<td>(30.828 - 8.5717 \times 10^{-1} \times i)</td>
</tr>
</tbody>
</table>
Therefore, the modal indices in this case do not have the same meanings as in the uniform plate case. They denote only a calculation sequence, rather than indicating a combination of planar radiators of the same geometric size. The control forces should be positioned to couple into the plate motion, which is due to disturbance and mass excitation. The results show that, to achieve satisfactory control performance, it is better not to apply control forces directly to the line mass, but to position them off the mass. This may indicate that the control forces located on the line mass have a problem coupling into the plate motion caused by the disturbance.

The ideal way to select the position of control forces is to calculate the plate-mass system mode shapes and to find out the nodal line formation of different modes first, and then to apply the control force to the best position in order to suppress the modes which are thought to contribute to the sound radiation. This procedure would require much more computation time and should be tried on the in vacuo plate first, so it is not used here. Here, a different approach is used. First, we assume that the plate is uniform and calculate the coefficients of modal expansion ($W_{mn}$ in Equation (3.13)). It should be noted that these modal amplitudes represent the modal amplitudes of a homogeneous plate, not the real modal amplitudes, but they are used here to guide the selection of the control force position. The calculation results indicate that the (1,4) mode is the most dominant one in the vibration; it is assumed that this mode causes the main sound radiation. Hence, the position of the control force is chosen at the antinodal positions of the (1,4) mode of the plate without mass loading. When one control force is used, its position is at $(x_{c1}, y_{c1}) = (0.250a, 0.375b)$. In Figure 4.10, a sound reduction of about 10 dB is observed with $|\theta| > 20^\circ$ in the far field. The near-field pressure distribution shown in Figure 4.11(b) illustrates that the
sound pressure level is reduced globally. The near-field pressure distribution in Figure 4.11(a) suggests that there is strong edge radiation along \(x = 0\) and \(x = a\). This edge radiation in the near field is in agreement with the two lobes observed in the far field directivity in the \(\phi = 90^\circ\) plane (corresponding to \(x = 0\) plane) (Figure 4.10), and is reduced by applying one control force, as shown in Figure 4.11(b). The wavenumber autospectrum shown in Figure 4.12(b) suggests that the wavenumber autospectrum levels in both the supersonic and the subsonic regions have decreased, which suggest that the modal suppression is possibly the primary reason for the sound reduction.

After two control forces are applied at \((x_{c1}, y_{c1}) = (0.250a, 0.375b)\) and \((x_{c2}, y_{c2}) = (0.750a, 0.625b)\), another 15 dB of sound reduction is observed, as shown in Figure 4.10. This further reduction of sound is also demonstrated by the wavenumber autospectrum reduction in the supersonic region, as shown in Figure 4.12(c). The near-field pressure pattern in Figure 4.11(c) suggests that the sound source is very complicated. Comparing Figure 4.11(c) with Figure 4.11(b), the overall near-field pressure level seems not to fall when an extra control force is applied to the plate. But both the far-field residual pressure (refer to Figure 4.10) and the wavenumber autospectrum in the supersonic region (refer to Figure 4.11) indicate that there is a significant reduction of sound after one more control force is added. Hence, modal restructuring is most likely the reason for the extra sound reduction.

When four control forces are applied at \((x_{c1}, y_{c1}) = (0.333a, 0.125b), (x_{c2}, y_{c2}) = (0.667a, 0.375b), (x_{c3}, y_{c3}) = (0.167a, 0.875b)\) and \((x_{c4}, y_{c4}) = (0.833a, 0.833b)\), a much better sound reduction is achieved (Figure 4.10 shows a total reduction of about 30 to 50 dB with respect to the case without control). The modal composition is again
complicated. It seems that modal suppression (a slight overall near-field pressure level reduction) and modal restructuring (more complicated source pattern) together (shown in Figure 4.11(d)) contribute to the sound reduction.

All of the control force amplitudes are listed in Table 4.3.3. The positions of the control forces are chosen after several selective attempts, so they do not represent optimal locations.

4.4 Conclusions

Active control of sound radiation from a fluid-loaded rectangular plate is further studied for the case of the plate having one or more discrete masses attached. Due to a mode shape change caused by the mass loading, the positioning of control forces becomes more difficult. In the cases studied, the positions of control forces are chosen to couple into the plate motion generated by both the attached mass and the disturbance. This force positioning relies on trial and error, rather than on the more scientific method used in the uniform plate case. It appears that this approach works fairly well when one or three point masses, or a line mass, is attached to the plate. The ASAC basic procedures derived for a uniform plate still apply to this mass-loading case.

Similar to fluid loading, the mass loading also causes cross-modal coupling. If the plate is in vacuo, the plate resonance can be explicitly calculated. With the fluid loading, there is no explicit way to estimate the plate response, so a numerical calculation is used. In this chapter, two plate response cases were calculated. One was
Figure 4.10: Far-field directivity pattern: off-resonance excitation, $f = 294 \text{ Hz}$, $\phi = 90^\circ$, a line mass.
(a) Disturbance only
(b) With one control force
(c) With two control forces
(d) With four control forces

Figure 4.11: Near-field pressure level at z = 0.01 m, f = 294 Hz, a line mass.
Figure 4.12: Wavenumber domain plate velocity autospectrum, $f = 294$ Hz, a line mass.
Table 4.3: Disturbance and control force amplitudes and locations on a plate with a line mass attached, $a$ — plate length in the $x$-direction, $b$ — plate length in the $y$-direction

<table>
<thead>
<tr>
<th>Excitation frequency: $f = 294(Hz)$</th>
<th>Amplitude ($N$)</th>
<th>Location $(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disturbance force: $10 + 0 \times i$</td>
<td>($0.500a, 0.833b$)</td>
<td></td>
</tr>
<tr>
<td>One control force: $-8.5019 - 2.4891 \times 10^{-7} \times i$</td>
<td>($0.250a, 0.375b$)</td>
<td></td>
</tr>
<tr>
<td>Two control forces: $24.657 + 1.1533 \times 10^{-3} \times i$</td>
<td>($0.250a, 0.375b$)</td>
<td></td>
</tr>
<tr>
<td>$35.114 + 1.1085 \times 10^{-3} \times i$</td>
<td>($0.750a, 0.625b$)</td>
<td></td>
</tr>
<tr>
<td>Four control forces: $-26.399 - 1.1047 \times 10^{-1} \times i$</td>
<td>($0.333a, 0.125b$)</td>
<td></td>
</tr>
<tr>
<td>$-27.897 - 3.3056 \times 10^{-2} \times i$</td>
<td>($0.667a, 0.375b$)</td>
<td></td>
</tr>
<tr>
<td>$4.0883 - 4.1611 \times 10^{-2} \times i$</td>
<td>($0.167a, 0.875b$)</td>
<td></td>
</tr>
<tr>
<td>$-19.763 - 1.1330 \times 10^{-1} \times i$</td>
<td>($0.833a, 0.833b$)</td>
<td></td>
</tr>
</tbody>
</table>
when the plate was attached with a point mass to the center, and the other was when
the plate was attached with a point mass asymmetric to the geometric center. The
resonances of the fluid-loaded point-mass-plate system were estimated by observing
the maxima of the plate response magnitudes.

The study shows that the interpretations of the wavenumber domain velocity au-
tospectrum and the near-field pressure distribution are more important in this case,
since a clear mode explanation is unavailable. In general, a sound radiation source of
irregular surface has more complicated radiation behavior, and a simple solution as
used before may not be accessible. When applying ASAC to such plants, it should be
noted that the control forces should be coupled into the motion caused by the distur-
bance as well as by the discontinuity. In the calculations performed in this chapter,
it is demonstrated that ASAC can work well on mass-loaded structures, as long as
the control forces are properly located. However, due to the complexity of the struc-
ture and the difficulty in locating the control forces, more effort is needed to achieve
satisfactory control performance in comparison to a uniform structure. The control
effort appears to become more difficult with the increase of the point mass number
or the complexity of the mass distribution. In practice, the system endurance of the
force load should also be considered, in order not to cause the structure breakdown,
which means that there is some upper limit for the control force magnitude.
Chapter 5

Wave Components Generated by a Distributed Force on a Fluid-Loaded Plate

5.1 The Wave Composition Problem

When a point force or a distributed force is applied to a plate, in vacuo or fluid-loaded, finite or infinite, it will generate two wave constituents, propagating waves which travel away from the force and contribute mainly to the global vibration of the plate, and the non-propagating near field, which decays exponentially and does not propagate away from the force location. This phenomenon was mentioned by Keltie (1991) when he studied the sound radiation from simple structural discontinuity on a one dimensional beam. The respective contributions of the propagating and the non-propagating wave components to the vibration and sound radiation from a finite beam were extensively studied by Guigou and Fuller (1991). Their results indicated that in the specific direction where the sensor microphone is positioned, the pressures generated by the propagating wave and by the non-propagating near field tend to cancel each other when ASAC is applied. Now a question arises naturally: how the propagating and non-propagating wave components contribute to the sound
radiation from a fluid-loaded two-dimensional plate? Nayak (1971) illustrated the wave composition in the case of a line-force-driven fluid-loaded infinite plate. His analysis suggested that there are three constituents in a fluid loaded infinite plate excited by a line force: the subsonic propagating wave, the decaying structural near field, and the acoustic wave. These wave components were presented in section 2.2.1 of this dissertation. In fact, the wave composition in a line force driven plate is relatively easier to analyze since the line force driven plate can still be considered a one-dimensional structure. The wave composition generated by a point force or by a distributed force is more complicated. First of all, the amplitude of the propagating wave generated by a point force also decreases with the increase of the distance due to the radial spreading characteristics of the wave. Second, the effort to separate the propagating wave components from the non-propagating near field encounters great difficulty at the drive point. In the in vacuo plate case, the plate transverse displacement due to a point force is expressed as (Junger and Feit, 1986)

\[
\omega(r) = \frac{F_0}{8\omega \sqrt{\rho_p h D}} \left[ H_0^{(1)}(k_pr) + \frac{2i}{\pi} K_0(k_pr) \right] \quad (5.1)
\]

in which \( r \) is the distance from the drive point, \( k_p \) is the flexural wavenumber, \( H_0^{(1)}(k_pr) \) the first kind of Bessel function of order zero, and \( K_0(k_pr) = \frac{i}{2} i H_0(ik_pr) \), the modified first kind of Bessel function of order zero. The first term on the right side of Equation (5.1) is considered the propagating wave, or the structurally travelling wave, and the second term is considered the non-propagating wave, or the structural near field. The problem arises from the singularity at \( r = 0 \) of the Bessel functions.
While \( r = 0 \) does not cause a singularity problem for the total response \( w(r) \) because

\[
\lim_{r \to 0} [H_0^{(1)}(k_p r) + \frac{2i}{\pi} K_0(k_p r)] = 1,
\]

\( r = 0 \) is a singularity for each \( H_0^{(1)}(k_p r) \) and \( K_0(k_p r) \), i.e. \( H_0^{(1)}(0) \to \infty \) and \( K_0(0) \to \infty \). This double singularity indicates that either of the two components, the propagating wave and the non-propagating near field, has an infinite magnitude at the driven point. Hence, separating the wave constituents at the drive point for a point force is extremely difficult. However, in order to study the forced response of a finite fluid-loaded plate, such a wave separation is necessary, based on the assumption that the modal expansion is produced by the propagating wave components. This will be further discussed later.

One possible way to deal with the singularity in the Bessel functions is to consider the point force a distributed force acting over a small area, such as over a circle of a radius of \( r = r_a \). As long as \( r_a \) is relatively small with respect to the plate dimensions, the force can be considered very close to a point force, and physically it makes reasonable sense since a point force actuator is actually applied on a small area. Mathematically this change corresponds to transforming an infinite impulse into a finite rectangular impulse with the same total strength. Then, another question arises: does the displacement \( w(r) \) have the same analytical form when \( r < r_a \) and \( r \geq r_a \)? Only recently was this problem addressed by Martínez and Hofer (1991). They showed that within the area of \( r \leq r_a \) delicate mathematical handling is necessary, because the Hankel transform relation which holds outside this area does not exist within the region. For an in vacuo plate, Martínez and Hofer provided an approach to solve this problem.
and evaluated some numerical examples. For the case of a fluid-loaded plate, the composition of displacement $w(r)$ is more complicated and no solution is available, so this problem will be addressed first.

Assuming that the problem of wave composition of a point-force-driven fluid-loaded plate is solved and the behaviors of individual wave components are understood, it is still a difficult job to study the influence of the non-propagating near field on the response and its associated acoustic radiation field of a finite plate. This is because for a two-dimensional finite structure, an analytical solution such as in the beam case (Guigou and Fuller, 1991) is not possible. Therefore, an approximate solution is suggested. One assumes that the non-propagating near field caused by the excitation force dies out before it reaches the boundary of the simply supported fluid-loaded plate, or the near-field wave impingement on the boundary is negligible when compared with the overall response. The total near fields at the boundary consist of two parts: the impinging near field due to the excitation force, and the near field due to the boundary reaction force. If the former is negligible, the latter must also be close to zero, because according to the simply supported boundary condition, the total displacement due to the near field at the boundary should be zero. Therefore, the non-propagating near field caused by the reaction forces and moments at the boundary can also be neglected, while the total response due to the propagating wave at the boundary is also zero according to the simply supported boundary condition. The total structural response of the plate consists of two parts: the one due to the propagating waves, and the one due to the non-propagating near field of the excitation forces. The first part was found with admissible functions based on modal analysis, as illustrated in section 3.2.1, and the second part, assumed to exist
only locally around the excitation force locations, and possible to find for the infinite plate system, needs to be estimated separately and superposed on the result of the first part. This approximation is illustrated in Figure 5.1.

It should be noted that according to this approximation, the subsonic propagating wave in an infinite plate does not cause sound radiation in the far field, instead, its radiation is localized. However, this subsonic wave will generate far-field sound pressure in a finite plate, because each modal component can be considered a combination of rectangular radiators as described by Maidanik (1974). Another wave component is the acoustic wave generated from the branch cut integral of the displacement, which propagates and radiates sound as well. Its characteristics will be studied later.

The cost function used in Chapter 3 considered only the plate response due to propagating waves because the admissible function formulation is based on the travelling waves; thus, the near field decaying waves were not included. Thus, if the non-propagating near field is to be included in the plate response, the cost function also needs to be modified. In this chapter, the plate response due to the total near-field wave components is superposed on the response due to modal expansion. In this approach, the near-field waves are included into the modal amplitudes based on the same admissible functions used for propagating waves.
Figure 5.1: An approximate expression of the wave composition of a finite plate: the non-propagating wave in an infinite plate, superposed with the propagating wave in a finite plate.
5.2 A Uniformly Distributed Force Applied to an Infinite Fluid-Loaded Plate

In this section, the plate response and the associated radiated sound pressure due to a uniformly distributed force on an infinite fluid-loaded plate are to be studied. The purpose is to study the separate contributions of the propagating structural and non-propagating structural responses, and the $e^{ik_o r}/r^2$ grazing, propagating acoustic wave generated by a uniformly distributed force. It is assumed that the plate is driven well below the coincidence frequency. The approach used to handle the Bessel function follows that used by Martinez and Hofer (1991), and the wave decomposition is based on the Cauchy residue method used in section 2.2.1 for the line force case.

5.2.1 Plate Response Due to a Uniformly Distributed Force

5.2.1.1 The Statement of the Problem

When a uniformly distributed force is applied to an infinite fluid-loaded plate, as shown in Figure 5.2, the partial differential equation of the plate transverse motion is

$$D(\nabla^4 - k_p^4)w(r) = -p(r, 0) + \frac{F_o}{\pi r_a^2} H(r_a - r),$$  \hspace{1cm} (5.3)

where $F_o$ is the total value of the force uniformly distributed over the area $\pi r_a^2$, as shown in Figure 5.2, and its direction is the same as the positive displacement. $D$ is the rigidity of the plate, $k_p$ is the flexural wavenumber, and $p(r, 0)$ is the fluid loading pressure acting on the plate. Equation (5.3) is similar to Equation (3.1).
The difference is that the differential operator $\nabla^4$ is now with respect to the cylindrical coordinates instead of the Cartesian coordinates, and the excitation force is a distributed point force. The Heaviside function $H$ is defined as follows:

$$H(r_a - r) = \begin{cases} 1, & r \leq r_a, \\ 0, & r > r_a. \end{cases} \quad (5.4)$$

A Hankel transform was used by Junger and Feit (1986) to solve Equation (5.3) for $r_a = 0$ (a point force) and was defined as follows:

$$w(r) = \int_0^\infty \tilde{W}(\gamma)J_0(\gamma r)\gamma d\gamma, \quad \tilde{W}(\gamma) = \int_0^\infty w(r)J_0(\gamma r)rdr. \quad (5.5)$$

Applying the Hankel transform to Equation (5.3) results in

$$D(\gamma^4 - k_p^4)\tilde{W}(\gamma) = -\tilde{p}(\gamma, 0) + \frac{F_0}{\pi r_a} \frac{J_1(r_a \gamma)}{\gamma}, \quad (5.6)$$

where $\tilde{W}(\gamma)$ and $-\tilde{p}(\gamma, 0)$ are Hankel transforms corresponding to $w(r)$ and $-p(r, 0)$ respectively, and $\gamma$ is the Hankel transform variable corresponding to $r$ (just as $k_x$ and $z$ in the Fourier transform illustrated in Chapter 2). By defining (Junger and Feit, 1986)

$$\tilde{Z}_p(\gamma) = -i\omega \rho_p h(1 - \gamma^4/k_p^4) \quad (5.7)$$

and

$$\tilde{Z}_a(\gamma) = \frac{\rho f \omega}{\sqrt{k^2 - \gamma^2}}, \quad (5.8)$$
Figure 5.2: A fluid-loaded infinite plate excited by uniformly distributed circular force over $r \leq r_a$. 
Equation (5.6) can be written as

$$\tilde{W}(\gamma) = \frac{iF_o}{\pi r_a \omega} \frac{J_1(r_a \gamma)}{\gamma [\tilde{Z}_a(\gamma) + \tilde{Z}_p(\gamma)]}.$$  \hspace{1cm} (5.9)

By applying the Hankel transform described in Equation (5.5) to Equation (5.9), the plate response in spatial form is

$$w(r) = \frac{iF}{\pi r_a \omega} \int_0^\infty \frac{J_1(r_a \gamma) J_0(\gamma r)}{\tilde{Z}_a(\gamma) + \tilde{Z}_p(\gamma)} \, d\gamma,$$  \hspace{1cm} (5.10)

or can be alternatively expressed as

$$w(r) = \frac{iF}{\pi r_a \omega} I(r_a, r),$$  \hspace{1cm} (5.11)

where

$$I(r_a, r) = \int_0^\infty \frac{J_1(r_a \gamma) J_0(\gamma r)}{\tilde{Z}_a(\gamma) + \tilde{Z}_p(\gamma)} \, d\gamma.$$  \hspace{1cm} (5.12)

In the following sections, the estimation of $I(r_a, r)$ is the main issue. The approach has two parts: (1) selecting the integration contour for a Hankel transform integrand, and (2) handling the two different integrals when $r < r_a$ and $r \geq r_a$. The methods used by previous researchers (Crighton and Innes, 1983, Martinez and Hofer, 1991) were carefully studied, and an approach dealing with an infinite fluid-loaded plate driven by a uniformly distributed force was developed and evaluated.
5.2.1.2 Evaluation of $I(r_a, r)$ for $r \geq r_a$

As explained by Martinez and Hofer (1991), the Hankel function relation (Junger and Feit, 1986),

$$J_0(\gamma r) = \frac{1}{2}[H_0^{(1)}(\gamma r) - H_0^{(1)}(-\gamma r)],$$  \hspace{1cm} (5.13)

holds in the range of $r > r_a$, so that the handling of $I(r_a, r)$ in this case is similar to that used by Junger and Feit for a point force (1986). Substitution of Equation (5.13) into Equation (5.12) and manipulation result in the following expression:

$$I(r_a, r) = \frac{1}{2} \int_{-\infty}^{\infty} \frac{J_1(r_a \gamma)H_0(\gamma r)}{Z_a(\gamma) + Z_p(\gamma)} d\gamma$$

$$= \frac{ik_p}{\omega m} \int_{-\infty}^{\infty} \frac{\sqrt{\alpha^2 - k_r^2}J_1(r_a k_r, k_r)H_0(k_r, k_r, r)d\bar{k_r}}{(k_r^4 - 1)(\alpha^2 - k_r^2)^{1/2} - i\xi/\alpha}. \hspace{1cm} (5.14)$$

The solution of Equation (5.14) will be discussed after the integration contour is selected. It is worth mentioning that when $r_a \rightarrow 0$, $J_1(r_a \gamma) \rightarrow r_a \gamma/2$, Equation (5.14) will converge to Equation (2.2) of the work by Crighton and Innes (1983) if $w(r)$ is multiplied by $i\omega$ to be $v(r)$. Therefore, it is seen that the point-force case studied by Crighton and Innes was a special instance of the distributed case presented here.

5.2.1.3 Evaluation of $I(r_a, r)$ for $r < r_a$

Within the distributed force range $r < r_a$, the Bessel function relation expressed in Equation (5.13) does not hold (Martinez and Hofer, 1991), so $w(r)$ cannot be simply expressed as Equation (5.14). As a result of adopting the approach of Martinez and
Hofer, \( I(r_a, r) \) can be expressed as follows:

\[
I(r_a, r) = \frac{1}{2} \int_{-\infty}^{\infty} \left[ H_1^{(1)}(r_a \gamma) + \frac{2i}{\pi \gamma r_a} \right] \frac{J_0(\gamma r)}{\tilde{Z}_a(\gamma) + \tilde{Z}_p(\gamma)} d\gamma. \tag{5.15}
\]

Note that it can be proved that Equation (5.15) and Equation (5.12) are equal within the range of \( r \leq r_a \). The two integrals resulting from the two terms within square brackets in Equation (5.15) should be handled separately due to the divergence of the second integral in both the upper and lower \( \gamma \) plane. Let

\[
I_1(r_a, r) = \frac{1}{2} \int_{C} \frac{H_1^{(1)}(r_a \gamma) J_0(\gamma r)}{\tilde{Z}_a(\gamma) + \tilde{Z}_p(\gamma)} d\gamma \tag{5.16}
\]

and

\[
I_2(r_a, r) = \frac{i}{\pi r_a} \int_{-\infty}^{\infty} \frac{J_0(\gamma r)}{\gamma [\tilde{Z}_a(\gamma) + \tilde{Z}_p(\gamma)]} d\gamma. \tag{5.17}
\]

Because \( J_0(\gamma r) / \gamma \) only converges when the integral path hugs the real axis for \( |\gamma| \to \infty \) (Martinez and Hofer, 1991), the contour of integration for \( I_2(r_a, r) \) is selected as in Figure 5.3. Note that in Figure 5.3, the contour does not have to hug the real axis between \(-k_1\) and \(k_1\) (\(k_1\) is the propagating wavenumber in the plate), so that the causality condition for a fluid-loaded plate (\( \text{Re} \sqrt{k^2 - \gamma^2} > 0 \)) will not be violated. Note that the denominator of integrand of \( I_2(r_a, r) \) is \( \gamma [\tilde{Z}_a(\gamma) + \tilde{Z}_p(\gamma)] \), and the poles on the real axis are \( \gamma = 0 \) and \( \gamma = \pm k_1 \). Hence,

\[
I_2(r_a, r) = \frac{i}{\pi r_a} \left[ -\pi i \text{ Residues}_{\gamma = -k_1} - \pi i \text{ Residues}_{\gamma = 0} + \pi i \text{ Residues}_{\gamma = k_1} + \right.
\]

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\[ + \int_{-\infty}^{\infty} \frac{J_0(\gamma r) d\gamma}{\gamma [\tilde{Z}_a(\gamma) + \tilde{Z}_p(\gamma)]}. \]  \hspace{1cm} (5.18)

The residues in Equation (5.18) are estimated by means of the Cauchy residue theorem. It should be noted that the equally clockwise half residue at \( \gamma = -k_1 \) cancels its counterclockwise twin at \( \gamma = k_1 \). With an appropriate indentation halfway around and above the pole at \( \gamma = 0 \) (referring to Figure 5.3), the residue is

\[ \text{Residue}_{|\gamma=0} = \frac{J_0(\gamma r)}{\gamma [\tilde{Z}_a(\gamma) + \tilde{Z}_p(\gamma)]} \bigg|_{\gamma=0} = \frac{J_0(0)}{\tilde{Z}_a(0) + \tilde{Z}_p(0)} = \frac{1}{\rho c - i \omega m}. \]  \hspace{1cm} (5.19)

The slash mark on the remaining integral of Equation (5.18) representing the Cauchy principal value signifies that the integral equals zero over the integration range from \(-\infty\) to \(\infty\) because the integrand is odd in \(\gamma\) and the integration range is symmetric with respect to the origin. Thus,

\[ I_a(r_a, r) = \frac{i}{\pi r_a} \frac{\pi i}{\rho c - i \omega m} = \frac{\alpha^2}{r_a \omega m (\xi - i \alpha^2)}. \]  \hspace{1cm} (5.20)

and

\[ I(r_a, r) = \frac{\alpha^2}{r_a \omega m (\xi - i \alpha^2)} + \frac{1}{2} \int_C \frac{H_1^{(1)}(r_a \gamma) J_0(\gamma r)}{\tilde{Z}_a(\gamma) + \tilde{Z}_p(\gamma)} d\gamma. \]  \hspace{1cm} (5.21)

Now the task is to decide how to estimate Equations (5.14) and (5.21). This question is to be discussed in the following context.
Figure 5.3: Integration path for $I_2(r_a, r)$ as expressed in Equation (5.15).
5.2.2 The Contour of Integral $I(r_a, r)$

It is observed that denominator of the integrand of integral $I(r_a, r)$ expressed in Equation (5.10), the system characteristic equation $\tilde{Z}_a(\gamma) + \tilde{Z}_f(\gamma) = 0$, is the same as Equation (2.5) calculated in section 2.2.1 as a result of Fourier transform, if $\gamma$ is replaced with $\tilde{k}_x$. This is understandable, because the only difference is that either the line force or the distributed force is taken as the external disturbance; the plate system remains the same. Hence, one real pole $k_1$ and a pair of complex poles $k_2$ and $k_3$ need to be considered if the integration is carried out in the upper $\gamma$ plane. Now, if the branch cuts are still chosen as those illustrated by Nayak (1971), the singularity of $H^{(1)}_1(\gamma r_a)$ at $\gamma = 0$ will violate the Cauchy integration condition. Therefore, this singularity, as a result of the Hankel transform, requires re-selection of the integration contour. Crighton and Innes (1983) used a different integral contour for a point force driven infinite plate, but their asymptotic estimation method encountered difficulty when the near-field non-propagating term and the branch cut were to be evaluated. As a result, Crighton and Innes did not physically interpret the near field, since the branch cut could not be accurately estimated. In this Chapter, however, it will be shown that the estimation of the branch cut and the near-field non-propagating wave component is possible with numerical calculation, and furthermore, the physical interpretation of the decaying near field is also possible based on the numerical results. This is a development of the point- or distributed-force-driven fluid-loaded infinite plate model.

The integration contour suggested by Crighton and Innes (1983) is illustrated in Figure 5.4. As can be seen, the contour is different from that used by Nayak (1971); the branch cut is from $\gamma = k_o$ to $\gamma = k_o + i\infty$ for the upper $\gamma$ plane. It should be noted
that Crighton’s pole positions of \( k_2 \) and \( k_3 \) were also based on his asymptotic approach (1981, 1983), so they were not as accurate as those presented by Nayak (1971). Hence, the pole positions in this study remain the same as those estimated with Equation (2.5), while the branch cut selection is identical to that used by Crighton.

With the integration contour shown in Figure 5.4, the integral for \( r > r_a \) expressed in Equation (5.14) becomes

\[
I(r_a, r) = \frac{\pi k_p}{\omega m} \left[ \sum_{i=1}^{3} R_{iw} - \frac{\xi}{\alpha \pi} \int_{0}^{\infty} \frac{\sqrt{\alpha^2 - \bar{k}_r^2} J_1(\alpha k_p \bar{k}_r) H_0(k_p \bar{k}_r r) d\bar{k}_r}{(k_r^4 - 1)^2(\alpha^2 - k_r^2) + \xi^2/\alpha^2} \right] \\
= \frac{\pi k_p}{\omega m} \left[ \sum_{i=1}^{3} R_{iw} + w_b(r) \right], \tag{5.22}
\]

where

\[
R_{1w} = \frac{J_1(\alpha k_p \bar{k}_{r1}) H_0(\bar{k}_{r1} k_p r)}{4k_r^3 + (\xi/\alpha) \bar{k}_{r1}(k_r^2 - \alpha^2)^{-3/2}}, \tag{5.23}
\]

\[
R_{2w,3w} = \frac{J_1(\alpha k_p \bar{k}_{r2,3}) H_0(\bar{k}_{r2,3} k_p r)}{4k_r^3 - (i\xi/\alpha) \bar{k}_{r2,3}(\alpha^2 - k_r^2)^{-3/2}}, \tag{5.24}
\]

and

\[
w_b(r) = -\frac{\xi}{\alpha \pi} \int_{0}^{\infty} \frac{\sqrt{\alpha^2 - \bar{k}_r^2} J_1(\alpha k_p \bar{k}_r) H_0(k_p \bar{k}_r r) d\bar{k}_r}{(k_r^4 - 1)^2(\alpha^2 - k_r^2) + \xi^2/\alpha^2}. \tag{5.25}
\]

In Equation (5.22) \( \bar{k}_r = \gamma/k_p, \alpha \) and \( \xi \) are defined in section 2.2.1, and \( \bar{k}_{r1} \) and \( \bar{k}_{r2,3} \) are equivalent to \( \bar{k}_1 \) and \( \bar{k}_{2,3} \) defined in section 2.2, respectively, as poles of the char-
Figure 5.4: The integration contour for $I(r_a,r)$ for a plate excited below coincidence frequency, $k_1 > k_o$.  

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acteristic equation.

For the integral expressed in Equation (5.21) where \( r \leq r_a \), a similar formulation is developed as follows:

\[
I(r_a, r) = \frac{\alpha^2}{r_a \omega \eta (\xi - i\alpha^2)} + \frac{\pi k_p}{\omega m} \left[ \sum_{i=1}^{3} R'_{iw} \right] - \frac{\xi}{\alpha \pi} \int_{\alpha}^{\alpha+\infty} \frac{\sqrt{\alpha^2 - k_r^2 H_1(r_a, k_p, k_r) J_0(k_p, k_r, r)} d(k_r)}{(k_r^4 - 1)^2 (\alpha^2 - k_r^2) + \xi^2 / \alpha^2} 
\]

\[
= \frac{\alpha^2}{r_a \omega \eta (\xi - i\alpha^2)} + \frac{\pi k_p}{\omega m} \left[ \sum_{i=1}^{3} R'_{iw} + w_i'(r) \right], \tag{5.26}
\]

where

\[
R'_{1w} = \frac{H_1(r_a, k_p, k_{r,1}) J_0(k_{r,1}, k_p, r)}{4 k_r^3 + (\xi/\alpha) k_{r,1} (k_{r,1}^2 - \alpha^2)^{-3/2}}, \tag{5.27}
\]

\[
R'_{2w,3w} = \frac{H_1(r_a, k_p, k_{r,3}) J_0(k_{r,3}, k_p, r)}{4 k_r^3 - (i\xi/\alpha) k_{r,3} (\alpha^2 - k_{r,3}^2)^{-3/2}}, \tag{5.28}
\]

and

\[
w_i'(r) = -\frac{\xi}{\alpha \pi} \int_{\alpha}^{\alpha+\infty} \frac{\sqrt{\alpha^2 - k_r^2 H_1(r_a, k_p, k_r) J_0(k_p, k_r, r)} d(k_r)}{(k_r^4 - 1)^2 (\alpha^2 - k_r^2) + \xi^2 / \alpha^2}. \tag{5.29}
\]

The wave composition in Equation (5.22) or in Equation (5.26) is similar to those in Equation (2.2), as illustrated in section 2.2.1 in a line force case. But it is noted that in the distributed force case, the amplitude of the traveling wave also decreases with the distance \( r \), due to the \( 1/\sqrt{r} \) spreading of the subsonic wave (\( R_{1w} \) or \( R'_{1w} \)) and the
$1/r^2$ of the acoustic wave ($w_b(r)$ or $w_b'(r)$). The first residue, $R_{1w}$ or $R'_{1w}$, is the subsonic propagating wave component. Note that in both $R_{1w}$ and $R'_{1w}$, as expressed in Equations (5.23) and (5.27), respectively, the only factor that varies with distance, $r$, is the Bessel function $H_0$ or $J_0$, and the remainder is a complex constant. The Bessel function in $R_{1w}$, $H_0$, is a cyclical complex value with a decreasing amplitude, when $r \to \infty$, because $H_0 = J_0 + i Y_0$, $J_0 \to \frac{1}{\sqrt{r}} \cos(r)$, and $Y_0 \to \frac{1}{\sqrt{r}} \sin(r)$ when $r \to \infty$.

In the case of $R'_{1w}$, $J_0$ has the same form as in $R_{1w}$. This kind of traveling wave will impinge on the boundary or discontinuity of a finite plate and will be reflected to form standing waves or mode shapes. The sum of $R_{2w}$ and $R_{3w}$, or $R'_{2w}$ and $R'_{3w}$, is purely imaginary, decaying without changing the phase, so it is the non-propagating structural near field. Here, it is found out that this near field comprises a pair of residues, and its estimation does not cause any significant problem. According to Crighton’s definition (1983), the branch cut, $w_b(r)$ or $w_b'(r)$, in Equation (5.25) or (5.29) is called the “acoustic component”. The behavior of this acoustic component is very interesting. When $k_p r \sim O(1)$ ("$\sim O(M)$" means "of the order of value $M"), w_b(r)$ or $w_b'(r)$ defines an almost purely imaginary acoustic field, and there is no energy transferring into the fluid or plate from the force for an infinite plate because the acoustic impedance is purely imaginary. Within the range of $k_p r$ the branch cut (acoustic component) is described as the “acoustic near field”. When $k_o r \sim O(1)$ or larger, this branch will produce a propagating wave, but it will decay in $1/r^2$ (Crighton, 1983).

Crighton also suggested that at distance $k_o r \gg O(1)$ the incompressible fluid approximation is untenable. This suggests the radiation behavior of this wave changes with the increase of $k_o r$. In other words, when estimating the acoustic wave $w_b(r)$ or $w_b'(r)$, the greater $k_o r$ is, the more contribution to the branch integral comes from near the acoustic wavenumber $k_o$, i.e., $w_b(r)$ or $w_b'(r)$ becomes more radiating. For smaller $k_o r$, 

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the branch cut integral yields larger imaginary contributions from wavenumbers away from the acoustic wavenumber $k_a$. Since those wavenumbers are complex and larger in magnitude than $k_a$, their contributions are highly non-acoustic. Therefore, they form a near field (acoustic near field, in Crighton’s words (1983)). This theory will be confirmed by the numerical results presented next. Up to now, we have defined the meaning of each wave component.

5.2.3 The Continuity of Integral $I(r_a, r)$ at $r = r_a$

In sections 5.2.1 and 5.2.2, two different integral expressions were given for estimating the plate response within the distributed force area ($r < r_a$) and outside the area ($r \geq r_a$). Obviously, these two integrals must be equal at $r = r_a$, i.e. $I(r_a, r \to r_a^-) = I(r_a, r \to r_a^+)$). For a plate in vacuo, this was proved (Martinez and Hofer, 1991) by using the Wronskian relationship. In this case, the same relation of

$$H_1^{(1)}(k_pr_a\tilde{k}_r)J_0(k_pr_a\tilde{k}_r) = \frac{2i}{\pi k_pr_a\tilde{k}_r} + J_1(k_pr_a\tilde{k}_r)H_0(k_pr_a\tilde{k}_r) \tag{5.30}$$

is used. By substituting Equation (5.30) into Equations (5.22) and (5.26) and letting $I(r_a, r \to r^-) = I(r_a, r \to r^+)$, the following relation of

$$\frac{\alpha^2}{r_\alpha \omega m (\xi - i\alpha^2) + \frac{\pi k_p}{\omega m} \{ - \frac{2i}{\pi r_\alpha k_pr_1 [4k_r^3 + (\xi/\alpha)k_r (k_r^3 - \alpha^2)^{-3/2}] } \}}$$

$$- \frac{\pi r_\alpha k_pr_2 [4k_r^3 - (i\xi/\alpha)k_r (\alpha^2 - k_r^3)^{-3/2}] }{2i}$$

$$+ \frac{\xi}{\alpha \pi} \int_{\alpha}^{\alpha+i\infty} \frac{2i}{\pi r_\alpha k_pr_r (k_r^4 - 1)^2 (\alpha^2 - k_r^2) + \xi^2/\alpha^2} = 0 \tag{5.31}$$

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should hold. In fact, the integral in Equation (5.31) can be expanded and expressed with the rest part on the left side of the equation, if a delicate evaluation is performed. The validity of Equation (5.31) can also be proven numerically. Although the left side of the equation cannot be exactly zero due to numerical estimation errors, the residual value is of $O(10^{-2})$ of the smallest real or imaginary part of the individual term on the left, so it is considered convergent. Careful observation of Equation (5.31) indicates that the continuity condition at $r = r_a$ is not related to the value of $r_a$. This is reasonable, since the two $I(r_a, r)$'s shown in Equations (5.22) and (5.26) must converge regardless of the value of $r_a$.

5.2.4 Radiated Pressure Due to a Uniformly Distributed Force

As in the line-force-driven plate case, the associated pressure generated by a distributed force will be analyzed. Note that many researchers (Gutin, 1965; Feit, 1966; Crighton, 1971, 1979) have studied the mathematical model describing a fluid-loaded infinite plate driven by a point force. The difference here is that the force is distributed and the contribution of each wave component is to be studied, while the previous research estimated the pressure asymptotically.

If the pressure directly above the drive location at $(R, 0, 0)$ is not estimated, the pressure can be calculated in the range of $r \geq r_a$, so only the integral of Equation (5.22) will be considered. By using the second relation between $\tilde{W}$ and $\tilde{P}$ (Junger
and applying a Hankel transform to \( \tilde{P}(\gamma; z) \), the pressure can be evaluated as

\[
\begin{align*}
    p(r, z) &= \int_0^\infty \tilde{P}(\gamma; z) J_0(\gamma r) \gamma d\gamma \\
    &= \frac{\rho f F}{i\pi r_a m} \int_0^\infty \frac{J_1(r_a k_p \bar{k}_r) J_0(k_p \bar{k}_r r) e^{ik_p z \sqrt{\alpha^2 - k_r^2}}}{(k_r^4 - 1)^2(\alpha^2 - k_r^2) + \xi^2/\alpha^2} d\bar{k}_r.
\end{align*}
\]  

(5.33)

Note that Equation (5.13) holds here, since \( r \geq r_a \) is to be considered. Hence, the above equation becomes

\[
\begin{align*}
    p(r, z) &= -\frac{i\rho f F}{2\pi r_a m} \int_{-\infty}^\infty \frac{J_1(r_a k_p \bar{k}_r) H_0(k_p \bar{k}_r r) e^{ik_p z \sqrt{\alpha^2 - k_r^2}}}{(k_r^4 - 1)^2(\alpha^2 - k_r^2) + \xi^2/\alpha^2} d\bar{k}_r.
\end{align*}
\]  

(5.34)

As in a line-force-driven case studied in section 2.3.1, there are two ways to estimate \( p(r, z) \) expressed in Equation (5.34): one using a stationary phase approach, basically for evaluating the far field response; and the other using the Cauchy residue method for studying the acoustic near field and individual contributions of different wave components. These two approaches are presented below.

5.2.4.1 Radiated Pressure Estimated by Stationary Phase Approach

As mentioned in Chapter 2, with stationary phase approach, it is assumed that the main contribution to the integral is associated with the region where the phase does
not vary rapidly with the integration variable \( \bar{k}_r \). By changing to the spherical coordinates of

\[
    r = R \sin \theta, z = R \cos \theta
\]  

(5.35)

for a planar source displaying cylindrical symmetry \( R = \sqrt{r^2 + z^2} \), the result of applying the stationary phase approach is (Junger and Feit, 1986)

\[
    p(R, \theta) = \rho_f \tilde{W}(k_o \sin \theta) \frac{e^{ik_o R}}{R}.
\]  

(5.36)

This relation has the advantage that the spectral acceleration distribution, \( \tilde{W} \), can assume any form. Combining Equations (5.9) and (5.36) yields

\[
    p(R, \theta) = -\frac{i \omega \rho_f F_o}{\pi r_a R k_o} \frac{J_1(r_a k_o \sin \theta) e^{ik_o R}}{\sqrt{[Z_a(k_o \sin \theta) + \tilde{Z}_p(k_o \sin \theta)]}}.
\]  

(5.37)

Using the definitions of \( \tilde{Z}_a \) and \( \tilde{Z}_p \) to estimate Equation (5.37) explicitly, the far field pressure is expressed as

\[
    p(R, \theta) = -\frac{i F_o}{\pi r_a R \sin \theta [1 + i \mu \cos \theta (\alpha^4 \sin^4 \theta - 1)]} \left[ \frac{\cos \theta J_1(r_a k_o \sin \theta) e^{ik_o R}}{R} \right].
\]  

(5.38)

If \( r_a k_o \sin \theta \ll 1, J_1(r_a k_o \sin \theta) \approx r_a k_o \sin \theta / 2 \). Equation (5.38) becomes

\[
    p(R, \theta) = -\frac{i F_o k_o}{2 \pi R} \frac{\cos \theta e^{ik_o R}}{[1 + i \mu \cos \theta (\alpha^4 \sin^4 \theta - 1)]},
\]  

(5.39)
which is the point force model derived by Junger and Feit (1986). Hence, it is seen that Equation (5.39) is a special case of Equation (5.38).

\textbf{5.2.4.2 Radiated Pressure Estimated with Cauchy Residue Method}

Starting from Equation (5.34) and applying the Cauchy residue method with the contour defined in section 4.2.2 result in the following expression of the radiated pressure due to a distributed force:

\[
p(r, z) \quad = \quad \frac{\rho f F_o}{r a m} \left[ \sum_{i=1}^{3} R_{ip} \right] + i \int_{\alpha}^{\alpha + i \infty} \frac{J_{1}(r_o k_p \tilde{k}_p) H_0(k_p \tilde{k}_p r)(\tilde{k}_r^4 - 1)\sqrt{\alpha^2 - \tilde{k}_p^2} \cos(k_p z \sqrt{\alpha^2 - \tilde{k}_p^2}) d\tilde{k}_r}{(\tilde{k}_r^4 - 1)^2(\alpha^2 - k_p^2) + \xi^2 / \alpha^2}
\]

\[
- \frac{\xi}{\alpha \pi} \int_{\alpha}^{\alpha + i \infty} \sqrt{\alpha^2 - \tilde{k}_r^2} J_1(r_o k_p \tilde{k}_p) H_0(k_p \tilde{k}_p r) \sin(k_p z \sqrt{\alpha^2 - \tilde{k}_p^2}) d\tilde{k}_r
\]

\[
= \frac{\rho f F_o}{r a m} \left[ \sum_{i=1}^{3} R_{ip} + \tilde{p}_b(r) \right], \tag{5.40}
\]

where

\[
R_{1p} = \frac{e^{-k_p z \sqrt{k_r^2 - \alpha^2}}}{-i(k_r^2 - \alpha^2)^{1/2}} R_{1w}, \tag{5.41}
\]

\[
R_{2,3p} = \frac{e^{i k_p z \sqrt{\alpha^2 - R_{2,3}^2}}}{(\alpha^2 - k_{r,2,3}^2)^{1/2}} R_{2,3w}, \tag{5.42}
\]
\[ \tilde{p}_b(r, z) = \frac{i}{\pi} \int_\alpha^{+\infty} \frac{J_1(k_p \tilde{k}_r r) H_0(k_p \tilde{k}_r r) (\tilde{k}_r^4 - 1) \sqrt{\alpha^2 - \tilde{k}_r^2 \cos(k_p z \sqrt{\alpha^2 - \tilde{k}_r^2})}}{(\tilde{k}_r^4 - 1)^2 (\alpha^2 - \tilde{k}_r^2) + \xi^2/\alpha^2} d\tilde{k}_r \\
- \frac{\xi}{\alpha \pi} \int_\alpha^{+\infty} \frac{\sqrt{\alpha^2 - \tilde{k}_r^2} J_1(k_p \tilde{k}_r r) H_0(k_p \tilde{k}_r r) \sin(k_p z \sqrt{\alpha^2 - \tilde{k}_r^2})}{(\tilde{k}_r^4 - 1)^2 (\alpha^2 - \tilde{k}_r^2) + \xi^2/\alpha^2} d\tilde{k}_r. \] (5.43)

Behavior similar to the case of a line driven force is observed here. The first residue, \( R_{1p} \), corresponds to the subsonic travelling wave component, \( R_{1w} \). For an infinite plate, it does not radiate to the far field, but for a finite plate, it is the main contributor to the formulation of modes (or standing waves). It is also observed that this component decays exponentially with the increase of vertical distance from the plate, \( z \). The sum of \( R_{2p} + R_{3p} \) has a purely real value corresponding to the structural near field \( R_{2w} + R_{3w} \). Compared to the sound pressure of \( R_{1p} \), the sum of \( R_{2p} + R_{3p} \) diminishes rapidly with the increase of \( z \), so the structural near field generates a near-field pressure. The branch cut integral expressed in Equation (5.40), therefore, corresponds to the acoustic wave component \( \omega_b(r) \) in Equation (5.25), and thus is the only component that contributes to the far-field radiated pressure for an infinite plate. In fact, in the far field \( (R \to \infty) \), the pressure caused by this branch cut acoustic wave should be the same as the one estimated with the stationary phase approach, as expressed in Equation (5.37), because \( R_{2p} + R_{3p} \) only comprises a near-field pressure when \( k_1 > k_o \). Hence, it can be concluded that the structural poles produce a structural wave travelling in the plate and a secondary acoustic impact, whereas the branch cut contains the primary acoustic impact of the structural deformation.

The above discussion means that for subsonic excitation frequencies below coinci-
dence: (1) in the far field, the radiated pressure $p(r, z)$ is caused solely by the branch cut integral, and its corresponding wave component is the acoustic wave $u_b(r)$, as expressed in Equation (5.25) or Equation (5.29); and (2) $p_b(r, z) = \frac{2F_c}{\lambda_{mn}} p_b(r, z)$ can be approximated with the stationary phase expression in the far field, because the remainder of $p(r, z)$ approaches zero in the far field. With the stationary phase expression, the radiated pressure behaves like a monopole propagating with the phase of $e^{ik_oR}$ and decaying with $1/R$. When applying these results to a finite plate in an approximate way, it is necessary to consider carefully which component contributes to the mode shape formulation and which component contributes to the structural near field. The propagating waves generated by each force are considered to cause mode shapes, and non-propagating waves are believed to produce localized near field. However, this kind of near field can be decomposed into a modal expansion expressed in admissible functions, and their impact can be evaluated. The procedures will be discussed in the next sections. Here, it is assumed that the far-field pressure due to $R_{1p}$ has been included in the modal expansion derived before because of the propagating characteristics of the first residue, and the impact of $R_{2p} + R_{3p}$ and $p_b(r)$ will be further investigated.

5.3 Modified Optimal Control

When a finite plate is driven by a localized force, as shown in Figure 5.1, it is assumed that the plate response has two components: the propagating standing waves based on modal analysis for the finite plate system, and the structural and acoustic near fields based on an infinite plate analysis. The near fields will produce a localized decaying response and affect the mode shape, and the standing wave will form modal
components which can be viewed as a combination of radiator cells. The behavior of such radiators was described by Maidanik (1974) and Fahy (1985). Hence, both the plate response and the associated radiation pressure based on modal functions as established in sections 3.2 and 3.3 need to be modified.

5.3.1 Modified Plate Response

As shown in Figure 5.1, the plate response is approximately expressed as the superposition of the standing waves and the near fields. The standing waves are the result of modal expansion as expressed in Equation (3.2), and the near fields are localized near each of the forces exciting the plate, and they can also be decomposed with the same modal functions as will discussed later. This approximation is based on: (1) in a finite two-dimensional structure, an exact description of a point-force-induced structural near field is too difficult to obtain because of the unknown boundary reaction forces; (2) the near fields at the boundary of a simply supported finite plate is negligible compared to the near fields in the vicinity of the excitation force due to the boundary conditions; and (3) the modal functions used to describe the plate motion as in Equation (3.2) only include the structurally travelling wave component generated by the excitation force when the modal summation is truncated, so the structural near field and the acoustic near field should be superimposed on the result of the modal expansion based on the standing waves. The reasons of these assumptions will be seen in later calculation results.

The plate response due to modal component is assumed to be caused by the struc-
nurally propagating wave components only, and its expression is

\[ w(x, y) = b \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \]  

(5.44)

for a simply supported plate. It is assumed that in Equation (5.44), \( W_{mn} \) is the plate modal amplitude due to the subsonic travelling wave component \( R_{1w} \) generated by \( N_s \) disturbances and \( N_c \) control forces.

The near fields exist in the vicinity of each of excitation forces. For a uniformly distributed force on area \( r \leq r_a \), the near fields are computed, as shown below, by evaluating the non-propagating structural near field, \( R_{2w} + R_{3w} \), and the acoustic near field, \( w_b(r) \), in Equation (5.22), or \( R_{2w} + R_{3w} \) and \( w_b'(r) \) in Equation (5.26):

\[
w_n(r) = -\frac{F_0\kappa_p}{\pi r_a \omega^2 \mu} \left[ R_{2w} + R_{3w} \right. \\
- \frac{\xi}{\alpha \pi} \int_{\alpha}^{\alpha+i\infty} \frac{\sqrt{\alpha^2 - k_r^2} J_1(r_a k_p k_r) H_0(k_p k_r r) d k_r}{(k_r^4 - 1)^2(\alpha^2 - k_r^2) + \xi^2/\alpha^2} \left], \ r > r_a, \right.
\]

(5.45)

and

\[
w_n(r) = \frac{i F_0 \alpha^2}{\pi r_a^2 \omega^2 \mu (\xi - i\alpha^2)} - \frac{F_0 \kappa_p}{r_a \omega^2 \mu} \left[ R_{2w} + R_{3w}' \right. \\
- \frac{\xi}{\alpha \pi} \int_{\alpha}^{\alpha+i\infty} \frac{\sqrt{\alpha^2 - k_r^2} H_1(r_a k_p k_r) J_0(k_p k_r r) d k_r}{(k_r^4 - 1)^2(\alpha^2 - k_r^2) + \xi^2/\alpha^2} \left], \ r < r_a. \right.
\]

(5.46)

Note that for \( k_p a \sim O(1) \), the acoustic wave component \( w_b(r) \) does not directly transfer energy into the fluid, and thus becomes an “acoustic near field” (Crighton, 1983). For a plate excited with \( N_s \) disturbances and \( N_c \) control forces, the total
response due to near-field wave components, \( w_n^i(x, y) \), will be approximately expressed as

\[
 w_n^i(x, y) = \sum_{i=1}^{N_x} w_n^i(x, y) + \sum_{j=1}^{N_x} w_n^j(x, y),
\]  

(5.47)

in which \( w_n^i(x, y) \) denotes the structural and acoustical near fields generated by the i-th disturbance, \( w_n^j(x, y) \) denotes the near field generated by the j-th control force, and \( w_n^i(r) = w_n^i(x, y) = w_n^i(\sqrt{x^2+y^2}) \) while \( r \) must be within the finite plate, i.e. \( 0 < x < a \) and \( 0 < y < b \).

This a priori near-field response (by virtue of Equation (5.45)) is then decomposed by using modal decomposition technique (refer to Appendix B) as

\[
 w_n^i(x, y) = b \sum_{m=1}^{M} \sum_{n=1}^{N} W_{mn}^n \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}
\]  

(5.48)

based on the same admissible function and modal indices as in Equation (5.44), in which \( M \) and \( N \) are modal truncation numbers. Then the modal coefficients in Equations (5.44) and (5.48) are superposed so that the total plate response can be considered as

\[
 w^t(x, y) = b \sum_{m=1}^{M} \sum_{n=1}^{N} (W_{mn}^n + W_{mn}^n) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}.
\]  

(5.49)

which therefore includes the effects due to the structural near field plus the acoustic near field (which will be further discussed next) in \( W_{mn}^n \), and due to the subsonic travelling waves in \( W_{mn}^n \), for a plate below coincidence. Equation (5.49) is the basis of
the approximation proposed in section 5.1 and depicted in Figure 5.1. When \( M \to \infty \) and \( N \to \infty \), however, we expect the near field effects will eventually be included in the modal expansion as expressed in Equation (5.44), i.e. \( w_n^t(x, y) \to w(x, y) \).

### 5.3.2 Modified Radiated Pressure

The radiated pressure is of more interest than the plate response, since sound radiation control is the main objective. In the acoustic field, the sound pressure has two parts: the pressure due to the structurally propagating wave components that travel slower than the acoustic speed since the plate is below coincidence, and the pressure due to the structural plus acoustic near fields localized around the disturbance or control forces. The pressure in terms of modal expansion was expressed as in Equations (3.27) and (3.28) in section 3.3.1, and here it is rewritten as

\[
p(x, y, z) = 12\frac{D\rho_f c_f}{\rho_p c_p a^3} (i\omega)^{ik_a} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \times \int_0^a \int_0^b \sin \frac{m\pi x_1}{a} \sin \frac{n\pi y_1}{b} e^{-ik_a R} dx_1 dy_1,
\]

in which \( W_{mn} \) is the summed modal component if \( N_s \) disturbances and \( N_c \) control forces are included in \( q \) on the right side of the coupled impedance matrix equation expressed as Equation (3.9).

Observing Equations (5.49) and (5.50) reveals that if \( W_{mn} \) is replaced by \( W_{mn} + W_{mn}^t \), the total radiated pressure caused by the subsonic travelling wave, the non-propagating structural near field, and the acoustic near field can be estimated as
follows:

\[ p_t(x, y, z) = 12 \frac{D \rho f c_f}{\rho_f c_p a^3} (i \omega') \frac{i k_o}{2\pi} \sum_{m=1}^{M} \sum_{n=1}^{N} (W_{mn} + W_{mn}^*) \]

\[ \times \int_0^a \int_0^b \sin \frac{m \pi x_1}{a} \sin \frac{n \pi y_1}{b} e^{-ik_o R} dx_1 dy_1. \]

(5.51)

If Equation (5.51) is taken to calculate the acoustic power, the interaction between the propagating wave and the structural and acoustic near fields will be included. The approximation of Equation (5.51) is similar to that used in section 3.3, so it is not repeated here.

Usually, the distance considered as far field is much larger than the characteristic dimension of the finite plate, i.e. \( R \gg a \), if \( a \) is the typical plate dimension. But, it is worthwhile to discuss in what situation the approximation of the structural near-field estimation in a finite plate will be possible.

According to Crighton's analysis (1983), the acoustic wave \( w_b(r) \) is an "acoustic near field" at the distance of \( k_p r \sim O(1) \), so it does not radiate energy into the fluid within this distance. However, with the increase of the distance from the force (at low subsonic excitation \( k_p > k_o \)), \( w_b(r) \) becomes an acoustic propagating wave when \( k_p r \sim O(1) \). In the finite plate approximation, as shown in Figure 5.1, the acoustic near field is taken as a part of the non-propagating wave composition and included in the near field wave components.

Based on the above consideration, there are three situations that should be consid-
ered in different approximation approaches. In the following, it is assumed that the finite simply supported plate is baffled with a rigid plane beyond the boundary, and $a$ is the typical dimension of the finite plate.

(1) The plate is very large compared with the acoustic wave length, $k_a a \gg 1$. As mentioned in section 5.1, the subsonic travelling wave component generated by a localized force (point force or distributed force) also decays with distance $r$. If this propagating wave diminishes to a very small value even before it reaches the boundary, there will be no reflection from the boundary and no significant mode shapes will exist. In this situation, the boundary can be assumed to be nonexistent. The physical explanation is that the acoustic energy dissipates into the fluid due to the damping of the fluid loading, and therefore the propagating wave diminishes before it reaches the boundary. Therefore, the plate response and the associated pressure radiation can be estimated as if the plate were infinite. Equations (5.10) and (5.38) are recommended to perform such estimations. This assumption should be used carefully in case the standing waves are overlooked. It is recommended to estimate the plate response at the drive point and compare it with that near the boundary.

(2) The plate is relatively large compared with the structural wave length ($k_a a \gg 1$) and of the same scale or larger than the acoustic wave length ($k_a a \sim O(1)$ or $k_a a > 1$). According to Crighton (1983), the acoustic wave $u_k(r)$ becomes radiating beyond the range of $k_a a \sim O(1)$ or $k_a a > 1$. The approximate solution derived in section 5.3.2 was basically designed for dealing with this situation. The subsonic propagating wave generated by the localized force reaches the boundary and forms standing waves as modal components. But it is assumed that the acoustic wave $u_k(r)$ was not counted.
in the modal expansion of the propagating waves because its reflection at the boundary is negligible, so its influence is considered a part of near field rather than a part of propagating wave. This assumption is based on the fact that the acoustic wave propagates with $1/r^2$ rather than with $1/r$, as the subsonic propagating wave does. Therefore, $w_b(r)$ decays much faster than the subsonic propagating wave. Later the numerical estimation indicates that when $k_pr > 4$ the influence of $w_b(r)$ is negligible compared with $R_{1w}$. The acoustic near field $w_b(r)$ and the structural near field $R_{2w} + R_{3w}$ are considered the non-propagating near fields, and their influences are estimated by modal decomposition as described in Equation (5.49).

(3) The plate is relatively small compared to the acoustic wave length ($k_pa \ll 1$) and of the same scale or smaller than the structural wave length ($k_pa \sim O(1)$). In this situation, the previous assumption that the non-propagating wave does not impinge on the boundary does not hold anymore, because the boundary is too close to the force location. The modal expansion based on the reflection of waves on the boundary has captured all the wave components, the subsonic wave, the structural near field, and the acoustic wave, and thus is well associated with the sound radiation. This method is the traditional approach and was used in Chapters 3 and 4.

As the result of the above discussion, the computation of the far-field pressure is based on the kinds of waves which contribute to the plate motion. The approximate expressions are as follows.

In situation (1), the finite plate is assumed to be without a boundary because the reflection is negligible, so the pressure is caused by the acoustic wave and has a
monopole characteristics:

\[ p_t(R, \theta) \approx -\sum_{i=1}^{N_t} \frac{iF_i \cos \theta J_1(r_a k_\theta \sin \theta) e^{ik_\rho R}}{\pi r_a R \sin \theta[1 + i\mu \cos \theta(\alpha^4 \sin^4 \theta - 1)]}, \tag{5.52} \]

in which \( N_t = N_s + N_c \) includes all the disturbances and control forces, while \( F_i \) corresponds to the force amplitude. Note that \( p_t \) is a bivariate function of \( R \) and \( \theta \) because of the symmetry of an infinite plate.

In situation (2), the pressure is caused by the structurally subsonic propagating wave as well as by the structurally non-propagating plus acoustic near fields. The former is estimated with the modal expansion in Equation (5.50), and the latter is that due to the infinite plate response and incorporated into the known modes of the finite domain by using modal decomposition technique. The result was expressed in Equation (5.51). The far-field pressure \( p_t(R, \theta, \phi) \) is derived with the same Rayleigh approximation as in Equations (3.26) and (3.27). The difference is that \( W_{mn} \) are replaced by \( W_{mn} + W^*_m \).

In situation (3), the far field pressure is caused by the modal expansion expression derived in Chapter 3, which includes the influence of all the wave components, propagating and non-propagating. Therefore, the far-field pressure expression described by Equation (3.28) is not repeated here.

With the total far-field pressure estimated in one of these three situations with appropriate approximation, the total sound power in the far field can also be estimated.
5.3.3 Modified Optimal Cost Function

The cost function is of the same form as that expressed in Equation (3.31); the difference is that the computation of the far-field pressure is based on the evaluation of \( k_p a \) and \( k_c a \) as discussed in section 5.3.2, which determines which situation describes the vibration and sound radiation in the most appropriate way. Hence, the cost function is rewritten as

\[
\hat{\beta}(p) = \frac{1}{R} \int_S |p_t(R, \theta, \phi)|^2 d\sigma = \int_0^{2\pi} \int_0^\pi |p_t(R, \theta, \phi)|^2 \sin \theta d\theta d\phi ,
\]  

(5.53)

which can also be expressed in a matrix form as

\[
\hat{\beta}(p) = \{p\}[\hat{A}]{\{p\}}^* + \{q\}^T[\hat{B}]{\{p\}}^* + \{p\}^T[\hat{B}^H]{\{q\}}^* + \{q\}^T[\hat{C}]{\{q\}}^* .
\]  

(5.54)

In situation (1) described in section 5.3.2, the \([\hat{A}], [\hat{B}]\) and \([\hat{C}]\) are estimated as:

\[
[\hat{A}]_{N_c \times N_c} = 2\pi \int_0^\pi [\{A\}^H\{A\}] \sin \theta d\theta ,
\]  

(5.55)

\[
[\hat{B}]_{N_c \times N_c} = 2\pi \int_0^\pi [\{B\}^H\{A\}] \sin \theta d\theta ,
\]  

(5.56)

and

\[
[\hat{C}]_{N_c \times N_c} = 2\pi \int_0^\pi [\{B\}^H\{B\}] \sin \theta d\theta ,
\]  

(5.57)
in which

\[ A_j = -\frac{i}{\pi r_a R_j} \frac{\cos \theta J_1(r_a k_0 \sin \theta) e^{ik_0 R_j}}{\sin \theta[1 + i\mu \cos \theta(\alpha^4 \sin^4 \theta - 1)]}, \quad (5.58) \]

and

\[ B_i = -\frac{i}{\pi r_a R_i} \frac{\cos \theta J_1(r_a k_0 \sin \theta) e^{ik_0 R_i}}{\sin \theta[1 + i\mu \cos \theta(\alpha^4 \sin^4 \theta - 1)]}. \quad (5.59) \]

Correspondingly, in situation (2) described in section 5.3.2, the \([\hat{A}], [\hat{B}], \) and \([\hat{C}]\) are of the following forms:

\[ [\hat{A}]_{N_x \times N_y} = \int_0^{2\pi} \int_0^{\pi} [\{A\} \{A\}^H] \sin \theta d\theta d\phi, \quad (5.60) \]

\[ [\hat{B}]_{N_x \times N_y} = \int_0^{2\pi} \int_0^{\pi} [\{B\} \{A\}^H] \sin \theta d\theta d\phi, \quad (5.61) \]

and

\[ [\hat{C}]_{N_x \times N_y} = \int_0^{2\pi} \int_0^{\pi} [\{B\} \{B\}^H] \sin \theta d\theta d\phi, \quad (5.62) \]

in which

\[ A_j = \frac{6a^4 \rho f \omega^2}{\pi R_f \rho p h^3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(W_{mnj} + W_{mn})}{p_j} T_{mn}(\theta, \phi) \quad (5.63) \]
and

\[ B_i = \frac{6a^4 \rho J \omega^2}{\pi R_i R_p h^3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(W_{mn} + W_{mn})}{q_i} T_{mn}(\theta, \phi). \] (5.64)

Note that the forms of \( A_j \) and \( B_i \) are exactly the same as those defined in Equations (3.29) and (3.30), but the modal coefficients are changed to include the influence of the structural and acoustic near fields.

In situation (3), \( \hat{A} \), \( \hat{B} \), and \( \hat{C} \) have the same forms as those defined in Equations (5.60), (5.61) and (5.62) respectively. \( A_j \) and \( B_i \) have already been defined by Equations (3.29) and (3.30), respectively, so they are omitted.

For the cost function defined by Equation (5.54) for all the above situations, the optimal solution for the control force vector is

\[ \{p\} = -[\hat{A}]^{-1}[\hat{B}]\{q\}. \] (5.65)

Comparing Equation (5.65) with Equation (3.36), it is found that \( \hat{A} \) and \( \hat{B} \) replaced \( [A] \) and \( [B] \), respectively.

### 5.4 Results and Discussion

As was analyzed in section 5.3.2, the far-field pressure radiated from a finite plate is evaluated for three situations defined by different acoustic wavelengths compared with the plate dimension. In the first situation, the plate is assumed to be infinite,
and in the third situation, the non-propagating near field is included in the modal expansion. In the line-force-driven plate analysis performed in Chapter 2, an infinite plate model was analyzed. An infinite plate driven by a point force is very similar to an infinite plate driven by a line force. Therefore, for brevity, situation (1) will not be discussed here. Situation (3) has already been extensively studied in Chapter 3, in which the plate response is solely based on the modal components. Thus, only situation (2) will be investigated in this section.

The simply supported rectangular fluid-loaded plate with properties given in Table 3.1 is to be studied here. As explained in section 5.3.2, for the special purpose in this chapter, the fluid-loaded plate system is viewed as the superposition of two plate systems: (1) an infinite fluid-loaded plate excited by one or a few distributed forces which only generate the structural and acoustic near fields; and (2) a rectangular fluid-loaded plate with the same thickness, but constrained with simply supported boundaries in an infinite baffle, excited by the same forces which only generate the subsonic travelling waves. Note that an exact analytical solution is unavailable, and this model represents an approximate approach. The system response, therefore, can be viewed as in Figure 5.1. The limitation of this approximation was discussed in section 5.3.1.

Based on the analysis in section 5.3.2, it should be stressed that this approximation is based on the assumption that the plate dimension is of the same scale or larger than the acoustic wave length \((k_o \ a \sim O(1)) \) or \((k_o \ a > O(1))\), which means that the distance between the excitation force and the boundary is large enough to let the non-propagating wave impinging on the boundary be negligible. With the plate dimension

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defined in Table 3.1, this condition requires that the driving frequency be above 800 Hz. However, for the purpose of comparison, \( f = 434 \text{ Hz} \) is chosen as the excitation frequency, and the results based on the modified plate response and the associated far-field pressure will be compared with those estimated in section 3.5.1. Note that this frequency is still considered as \( k_c a \sim O(1) \), so the approximation discussed in situation (2) in section 4.3.2 is appropriate.

### 5.4.1 Wave Components Due to a Uniformly Distributed Force

The individual contribution of each wave component due to a distributed force is to be estimated and discussed in this section. With Equations (5.10), (5.26), (5.22), (5.25) and (5.29), the plate response due to a distributed force \( F_o \) over the area \( r \leq r_a \) can be expressed as

\[
w(r) = \frac{iF_0 k_p}{r_a \omega^2 m} \left[ R_{1w} + R_{3w} + R_{3w} + w_b(r) \right], \quad r > r_a, \tag{5.66}
\]

and

\[
w(r) = \frac{iF_0 k_p}{r_a \omega^2 m} \left[ R'_{1w} + R'_2 + R'_3 + w'_b(r) \right], \quad r \leq r_a. \tag{5.67}
\]

Note that all the detailed expressions of \( R_{1w}, R_{2w} + R_{3w} \), and \( w_b(r) \) (\( R'_{1w}, R'_2 + R'_3 \), and \( w'_b(r) \)) were presented in section 5.2, so they are not repeated here.

Equation (5.66) or (5.67) can also be rewritten as \( w(r) = \text{Real}(w(r)) + \text{Imag}(w(r)) \)
to estimate the real and imaginary parts separately. Numerical estimation shows that $R_{2w} + R_{3w}$ ($R'_{2w} + R'_{3w}$) is purely imaginary, $w_1(r)$ ($w'_1(r)$) is also almost purely imaginary within $k_p r < 4$, and $R_{1w}$ is the dominating propagating wave component.

The respective contributions of $R_{1w}, R_{2w} + R_{3w}$, and $w_1(r)$ ($R'_{1w}, R'_{2w} + R'_{3w}$, and $w'_1(r)$ when $r < r_a$), as well as the real and imaginary part of $w(r)$, are plotted in Figures 5.5 to Figure 5.7. The plate is assumed to be excited at $f = 434$ Hz.

Note that all the curves shown in Figures 5.5, 5.6 and 5.7 are normalized with $r_a \omega^2 m/(ik_p F_a)$. The distributed force area (which is too small to be seen in Figure 5.5 and Figure 5.6, but can be seen in Figure 5.7) changes from $r_a = 0.005$ m ($k_p r_a = 0.0676$) to $r_a = 0.05$ m ($k_p r_a = 0.676$). In Figures 5.5 to Figure 5.7, it is seen that the main contribution to $w(r)$ is the subsonic propagating wave component $R_{1w}$ ($R'_{1w}$), since $\text{Real}(R_{1w})$ almost coincides with $\text{Real}(w(r))$, and $\text{Imag}(R_{1w})$ converges with $\text{Imag}(w(r))$ in the range of $k_p r > 1$. Hence, it is appropriate to assume that $R_{1w}$ contributes to the modal expansion due to the reflection on a finite boundary. This phenomenon can be observed by considering the wave amplitudes at $k_p r = 4$. Both the structural near field $R_{2w} + R_{3w}$ and the acoustic near field $w_1(r)$ are almost purely imaginary so they do not appear in the real part of $w(r)$. From the imaginary parts shown in Figures 5.5, 5.6, and 5.7, it is observed that both $R'_{2w} + R'_{3w}$ and $w'_1(r)$ rise when $k_p r$ decreases, but with opposite signs, so their combination tends to cancel the effects from each other within the distance $k_p r < 1$. The peak of either $R'_{2w} + R'_{3w}$ or $w'_1(r)$ becomes more obvious when $r_a$ decreases from 0.05 m (corresponding to $k_p r_a = 0.676$ when $f = 434$ Hz) to 0.005 m (corresponding to $k_p r_a = 0.0676$). If $r_a \to 0$, both the structural near field and the acoustic near field will eventually
Figure 5.5: The residues and the branch cut, the real and imaginary parts of $r_o \omega^2 m \omega(r)/i k_p F_o$ versus $k_p r$; $f = 434 Hz, r_o = 0.005(m)$, the uniformly distributed force is loaded where $k_p r < k_p r_o = 0.0676$. 

$k_p r_o = 0.0676$
$k_p r_a = 0.135$

Figure 5.6: The residues and the branch cut, the real and imaginary parts of $\tau_a \omega^2 m w(r)/i k_p F_o$ versus $k_p r$; $f = 434 \text{Hz}, r_a = 0.01 \text{m}$, the uniformly distributed force is loaded where $k_p r < k_p r_a = 0.135$. 

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Figure 5.7: The residues and the branch cut, the real and imaginary parts of 
\( r_a \omega^2 m w(r) / i k_p F_o \) versus \( k_p r \); \( f = 434 \text{Hz}, r_a = 0.05(m) \), the uniformly distributed force is loaded where \( k_p r < k_p r_a = 0.676 \).
go to infinity due to the singularity of Hankel function $H_1$ at $r_a = 0$. It should be
noted that in section 5.2, the integral separation between $r < r_a$ and $r \geq r_a$ solved
the singularity problem caused by $r = 0$ (response distance), and the singularity of
$r_a = 0$ (distributed force radius). It should be reminded here that, $r_a$ is assumed to
be greater than zero in order to investigate the near-field effects and perform wave
separation. If $r_a = 0$ (corresponding to a point force), we return to the point force
problem and the resulting singularity will prevent the wave separation effort. $R'_{1w}$ also
rises when $r_a$ decreases because of the influence of $H_1$. The physical interpretation of
the interaction between $R'_{1w}$, $R'_{2w} + R'_{3w}$ and $w_b(r)$ needs to be studied further.

5.4.2 The Influence of the Structural and Acoustic Near
Fields on Control Performance at $f = 434\ Hz$

The influence of the structural and acoustic waves on the active structural control
of sound radiation from a fluid-loaded finite plate is illustrated with a uniform plate
driven at $434\ Hz$; the same case is discussed in section 3.5.1.3. For comparison, the
disturbance and control force locations are not changed. The primary difference is
the modal composition, as presented in Equations (5.44) and (5.49). The near-field
wave components, $R_{2w} + R_{3w}$ and $w_b(r)$, are decomposed to form modal components,
$W_{mn}^n$, as expressed in Equation (5.48). The number of modes taken to calculate the
near fields are chosen as $m = 6$ and $n = 6$, the same as the truncation numbers
used to calculate the plate response in Chapter 3. The ideal approach would be to
make $m$ and $n$ very large and take the first $6 \times 6$ pairs of indices. The numerical
calculation in this chapter indicates that even with a truncation of $m = 6$ and $n = 6$,
a fairly good approximate of the near fields is obtained. Therefore, this pair numbers
of modal truncation is chosen. These additional modal components change the plate response, the associated radiated pressure, the cost function, and the optimal control force. The results presented in the following generally show the impact of the near field wave components.

The plate response due to the structural and acoustic near field waves is estimated when the plate is excited by a harmonic force located at the center of the plate at 434 Hz. The focus of the investigation, therefore, is on the far-field pressure and the control performance based on the new cost function. It is apparent from Table 5.4.2 that the control force amplitudes are different from those estimated in Chapter 3, when only the subsonic travelling wave components were considered. This means that the near-field wave components have a significant impact on the determination of the control force amplitudes, so they will ultimately affect the control performance.

Comparing Figure 5.8 with Figure 3.9 shows that the pressure caused by the disturbance force at the center of the plate rises by about 3 dB globally when the near-field waves are included in the modal components. The individual contributions of the propagating and the non-propagating waves are estimated, respectively, by including only $W_{mn}$ or $W^*_m$ as it appears in Equation (5.51) in the pressure estimation. It should be noted that later the optimal control force estimation still takes all the wave components into consideration, i.e. Equations (A.2) and (B.2) are still applied to estimate the control force \{p\}. Hence, the control force amplitudes remain as listed in Table 5.4.2. It is seen in Figures 5.8 and 5.9 that the radiated pressures due to the propagating wave and the non-propagating waves are of almost the same magnitude. For a single force located at the center of the plate, their combination tends
to produce a total pressure about 5 dB up from their individual radiation level. This means that the pressures caused by the propagating wave and by the near fields are nearly in phase in this case.

Due to the computation time constraint, only the two-force control case is calculated here. When two control forces are positioned at the same locations as those illustrated in Chapter 3, their amplitudes are different when the modal components include the structural and acoustic near fields. It is also found out that the control result is very different from that obtained without including the structural and acoustic near fields. In the plane of $\phi = 0$, it appears that the source contains more dipole components compared to Figure 3.9. A more interesting phenomenon is that the pressure due to subsonic travelling waves generated by the disturbance and two control forces and the pressure due to the structural and acoustic near fields produced by the same forces are almost of the same magnitude (in Figure 5.11 they have different magnitudes where $|\theta| > 45^\circ$, and the difference is from 2 to 8 dB), but their combined pressure radiation is lower in most radiation areas shown in Figures 5.10 and 5.11. In Figure 5.10, it is seen that the total controlled pressure is about 10 to 20 dB lower than the pressure caused either by the subsonic travelling wave or by the structural and acoustic near fields alone. In Figure 5.11, a similar reduction is observed in the region of $|\theta| < 45^\circ$. This indicates that the pressures generated by the subsonic travelling wave component and by the structural and acoustic near fields are nearly out of phase after ASAC is applied. The pressure due to the near fields wave components tends to cancel the counterpart caused by the propagating wave component. Therefore, the near fields wave components play an important role in terms of pressure radiation and active control. This observation is similar to that of Guigou and Fuller in a beam.
control case (1991) for light fluid loading and uncoupled response equations.

With the control force positions illustrated in Figures 5.12 and 5.13, a reasonable global sound reduction is obtained when two optimal control forces are applied to the plate. Comparing Figure 5.12 and Figure 3.9, it is observed that the controlled pressure patterns are different. This observation can also be confirmed by comparing the near-field pressure distributions shown in Figures 5.14(b) and Figure 3.10(c). In Figure 3.10(c), it is observed that in \( x \) plane the near-field pressure distribution has three lobes with roughly the same magnitude in the middle of plate. This explains why the far-field directivity has three lobes in the plane of \( \phi = 0 \) (one of \( x \) planes). When the structural and acoustic near fields are included, as shown in Figure 5.14(b), the center lobe of the near-field pressure is largely reduced, so from \( x \) plane the pressure distribution has two lobes instead of three. This suggests that the sound radiation source is more like a dipole from the view of \( x \) plane, and in the far-field directivity (Figure 5.12) a relatively lower radiation level is observed around \( \theta = 0 \).

Further observation of the near-field pressure is made by comparing Figures 3.10(a) and 3.10(c) (corresponding to the uncontrolled and controlled near-field pressure without including the structural plus acoustic waves) with Figures 5.14(a) and 5.14(b) (corresponding to the uncontrolled and controlled sound sources including the non-propagating waves). It is seen that in general the near-field pressure is higher when the structural and acoustic near fields are included. This finding is consistent with the observation in the far-field directivities in Figures 5.12 and 5.13. The uncontrolled pressure distribution in Figure 5.14(a) looks more uniform. After ASAC is applied, Figure 5.14(b) indicates that the modal restructuring is the primary mechanism for

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reducing the sound radiation in the far field. The modal restructuring mechanism remains the same as that discussed in Chapter 3.

The wavenumber investigation also suggests that sound radiation is affected by including the structural and acoustic near fields wave components (Figures 5.15(a) and 5.15(b)). It is also observed that the plate vibration level is higher when near fields are included. This is understandable since including the near fields in the plate response will have a great impact on the plate vibration power. After the control forces are applied to the plate, the reduction in the supersonic region is different from that without considering the near fields (compare Figure 5.15(b) with Figure 3.11(c)). This means that the control performance is affected. However, it shows that the reduction in the supersonic wavenumber region still looks very good. This is consistent with the results shown in the far-field directivity patterns.

5.5 Conclusions

The structural and acoustic near field waves caused by a distributed force on a fluid-loaded plate were investigated and numerically calculated. Similar to studying a plate in vacuo, the wave components due to a distributed force on a fluid-loaded plate were separated, and the behavior of each component was studied. Previous researchers (Sandman, 1977; Lomas and Hayek; 1977, Wang, 1990) used admissible functions to estimate the finite plate response based on in vacuo plate modes. The approach used in this chapter is to study the influence of the structural and acoustic waves whose reflection at the boundary is negligible, so they are not captured in the admissible functions.
Figure 5.8: Far-field directivity pattern: $f = 434 \text{ Hz, } \phi = 0$, disturbance.
Figure 5.9: Far-field directivity pattern: $f = 434 \text{ Hz}, \phi = 90^\circ$, disturbance.
Figure 5.10: Far-field directivity pattern: $f = 434 \, Hz, \phi = 0$, control forces.
Figure 5.11: Far-field directivity pattern: $f = 434 \, Hz, \phi = 90^\circ$, control forces.
Figure 5.12: Far-field directivity pattern: \( f = 434 \, \text{Hz}, \phi = 0 \), pressure due to the subsonic travelling wave and the structural and acoustic near fields.
Figure 5.13: Far-field directivity pattern: $f = 434 \ Hz, \phi = 90^\circ$, pressure due to the subsonic travelling wave and the structural and acoustic near fields.
Figure 5.14: Near-field pressure level at $z = 0.01\ m, f = 434\ Hz$, including the structural and acoustic near fields.
Figure 5.15: Wavenumber domain plate velocity autospectrum including the non-propagating wave components, $f = 434$ Hz.
This approach assumes that the plate response comprises two parts: one due to the subsonic travelling waves caused by the distributed forces, and the other due to the structural and acoustic near-field waves. The contribution of the latter was incorporated into the modal components with a modal decomposition technique. It is also assumed that the acoustic wave \( w_b(r) \) is a part of the near fields under certain subsonic excitation condition. From the calculation results, it is shown that the acoustic wave decreases with \( 1/r^2 \), so its reflection on the boundary is negligible compared to the impinging of the structural travelling wave.

It has been proven that for a fluid-loaded infinite plate driven by a distributed force, careful mathematical handling is necessary to model the response within and outside the force driven area, \( r < r_a \) and \( r \geq r_a \), similar to the plate in vacuo case. A detailed derivation introduced, for the first time, a model to estimate the respective contributions from the subsonic propagating wave, the structural near field, and the acoustic wave. This model provides a tool for studying the structural near field generated by distributed forces applied to a fluid-loaded structure.

The numerical calculation results show that the structural and acoustic near fields have a significant impact on the sound radiation and the control performance. When one disturbance force is applied to the plate, the pressures generated by the subsonic propagating wave and the near fields are approximately in phase. When ASAC is applied, the pressures generated by these two waves are close to out of phase. Hence, ASAC basically generates a sound radiation field from the structural and acoustic near fields to cancel the sound radiation generated from the subsonic propagating
wave components. This conclusion is important in understanding the sound reduction mechanism. On the other hand, including the structural and acoustic near fields increases the vibration level of the plate globally.

The approach used in this chapter views a finite plate as both finite and infinite: finite in terms of the subsonic propagating waves and infinite in terms of the structural and acoustic near fields. The modal decomposition of the near fields and the modal modification to incorporate their influence are crucial in the estimation of sound radiation. While this approach adds new understanding to the analysis of the sound radiation from a two-dimensional structure driven by localized forces, further study of the phenomenon is needed.
Table 5.1: Comparison of the control force amplitudes with and without including the structural and acoustic near fields

<table>
<thead>
<tr>
<th>Without including the near fields</th>
<th>Location ((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Amplitude (N)</strong></td>
<td></td>
</tr>
<tr>
<td>Disturbance force:</td>
<td></td>
</tr>
<tr>
<td>(10 + 0 \times i)</td>
<td>((0.5a, 0.5b))</td>
</tr>
<tr>
<td>Two control forces:</td>
<td></td>
</tr>
<tr>
<td>(12.981 - 1.3601 \times 10^{-2} \times i)</td>
<td>((0.50\alpha, 0.25\beta))</td>
</tr>
<tr>
<td>(12.981 - 1.3601 \times 10^{-2} \times i)</td>
<td>((0.50\alpha, 0.75\beta))</td>
</tr>
<tr>
<td>Including the near fields</td>
<td></td>
</tr>
<tr>
<td><strong>Amplitude (N)</strong></td>
<td></td>
</tr>
<tr>
<td>Disturbance force:</td>
<td></td>
</tr>
<tr>
<td>(10 + 0 \times i)</td>
<td>((0.5a, 0.5b))</td>
</tr>
<tr>
<td>Two control forces:</td>
<td></td>
</tr>
<tr>
<td>(30.416 - 5.9577 \times i)</td>
<td>((0.50\alpha, 0.25\beta))</td>
</tr>
<tr>
<td>(30.416 - 5.9577 \times i)</td>
<td>((0.50\alpha, 0.75\beta))</td>
</tr>
</tbody>
</table>
Chapter 6

Overall Conclusions and Recommendations

6.1 Overall Conclusions

This dissertation studied the active structural acoustical control (ASAC) of fluid loaded plate structures excited by a subsonic flexural wave or a point force disturbance below the coincidence frequency. The analysis showed that ASAC can significantly reduce the global sound radiation of heavy fluid-loaded plates, and the active control technique is very effective when the excitation is below coincidence frequencies. The control was achieved by applying secondary forces directly to the structure, while radiated pressure variables were minimized. The major conclusions are highlighted as follows.

For an infinite fluid loaded plate:

- Sound radiation from discontinuities such as line constraints and ribs can be controlled by applying active forces located near the discontinuities. The control forces are determined by the optimization of the sound radiation power in the fluid-loaded upper-half space.
• The reduction in sound radiation is not necessarily accompanied by reduction in plate displacement level. Rather, the plate velocity autospectrum within the radiation range \( k_x < |k_o| \) (supersonic region) was markedly reduced after control forces were applied. Therefore, the wavenumber domain analysis reveals the cause of sound reduction effect from the point of view of plate vibration radiating components.

• In the spatial domain, it is observed that the control forces change the original efficient sound radiation source into an inefficient sound radiation source and thus decreases the radiation. This radiation source pattern modification can be recognized from the near-field pressure and intensity distribution.

For a finite fluid loaded plate:

• Radiation from a finite fluid-loaded plate can be effectively controlled by the use of active forces when the plate is excited by a subsonic disturbance force.

• A reduction in sound radiation depends on the input frequency that determines which modes will mostly contribute to the total radiation. Generally, off-resonant excitations are more difficult to control than on-resonant ones, since more modes are involved and the radiation source is more complicated. In the cases studied, up to two control forces are needed for on-resonant excitations and up to four control forces for off-resonant excitations, if a significant sound
reduction effect is expected.

- A two-dimensional wave number domain analysis illustrates the reduction of radiated power in the supersonic region when active control is applied. This emphasizes the fact that the wavenumber analysis is one of the important tools in acoustical research and has proven to be very effective in examining the ASAC performance.

- Mass loading causes cross-modal coupling as well as the fluid loading. ASAC can still eliminate the sound radiation from the mass-plate system, while the complexity of the structure requires more delicate positioning of the control forces.

- Ignoring the cross-modal fluid-induced coupling greatly simplifies the control force calculation at the cost of lowering the control performance. A simplified control force design can still provide reasonably good control results.

For the wave components study:

- The investigation of wave components generated by a point force or a distributed force applied to a fluid-loaded plate is important in understanding sound radiation. The proposed model suggests that the plate response be estimated with delicate mathematical handling within and beyond the force loading area. The results show that the structural non-propagating near field and the acoustic near
field have a significant impact on the sound radiation from the structure. This result indicates that sound radiation estimated solely by the standing waves in a finite structure is inappropriate.

- ASAC generates a sound radiation field from the structural non-propagating wave and the acoustic near field, which is out of phase with respect to the sound radiation generated from the structural propagating wave components. This phenomenon may have revealed the ASAC mechanism.

6.2 Recommendations

Although this dissertation has extensively studied the ASAC of fluid-loaded plates under a harmonic subsonic excitation, there is still a lot of room for furthering the research on this topic, and some recommendations are as follows:

- The excitation frequency is a steady single harmonic throughout the study in this dissertation. While the final goal is to extend the ASAC to white noise excitation, it is advisable to investigate a multiple harmonic input situation as an intermediate step. This recommendation is based on the fact that the model presented in this dissertation is available for such an investigation. In real application, some noises can be approximated as the sum of several harmonic disturbances, so this extension is of practical use.
• In this dissertation, the computation time imposed by the numerical estimation of the fluid-loading impedance and the optimal control force is a main constraint that hinders a detailed investigation of the location of the control forces. One way to decrease the computation time is to use a perturbation theory to derive an asymptotic expression for the intractable integrals instead of directly using numerical calculation. This approach requires much mathematical work, but, if the computation time needs to be reduced, it may be the only way to accomplish the goal.

• The cost function used throughout this dissertation is the sound power over a semi-cylindrical or a hemispherical surface. In real cases, it is hard to measure a great many points in the acoustic medium. Hence, cost functions based on discrete microphone measurements should be studied, and the model of a fluid-loaded plate needs to be modified to accommodate this change. It is also recommended that the application of the polyvinylidene fluoride sensors bonded on the structure be studied.
Appendix A

The Derivation of the Matrix Impedance Equation for a Fluid-Loaded Rectangular Plate

If the plate has a mass $M$ attached at $(x_i, y_i)$, the governing equation for the transverse deflection of a fluid loaded plate is (Sandman, 1977)

$$D \nabla^4 w + [\rho_p h + M \delta(x - x_i) \delta(y_i)] \frac{\partial^2 w}{\partial t^2} = q(x, y) - p_v(x, y) .$$  \hspace{1cm} (A.1)

For a simply supported rectangular plate, as described in Equation (A.1), it is assumed that the harmonic forcing function takes the form of

$$q(x, y, t) = \frac{D}{a} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b} \exp(\iota \omega' c_p t/a) ,$$  \hspace{1cm} (A.2)

and the plate displacement takes the form of (Sandman, 1977)

$$w(x, y, t) = b \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b} \exp(\iota \omega' c_p t/a) .$$  \hspace{1cm} (A.3)
Sandman (1977) used the assumed mode approach to solve a partial differential equation. But it should be remembered that the orthogonality between modes as in the case of a plate in vacuo does not hold here.

The fluid loading pressure has the form of

\[ p_o(x, y, t) = \frac{D}{a} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \tilde{P}_o(x, y) \exp(i\omega'c_p t/a) \]  \hspace{1cm} (A.4)

in which \( \tilde{P}_o \) is the spatial variation of \( p_o \).

The operator in Equation (A.1) has the form of \( \nabla^4 w = (\partial^4 / \partial x^4 + 2 \partial^2 / \partial x^2 \partial y^2 + \partial^2 / \partial y^4)w \) in Cartesian coordinates. When \( w \) takes the form of Equation (A.3), the following expressions are the result of differentiation:

\[ \nabla^4 w = \frac{D}{a} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right] W_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \exp(i\omega'c_p t/a) \]  \hspace{1cm} (A.5)

\[ \frac{\partial^2 w}{\partial t^2} = -\frac{\omega'^2 c_p^2}{a} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \exp(i\omega'c_p t/a) \]  \hspace{1cm} (A.6)

Substituting Equations (A.2), (A.3), (A.5) and (A.6) into Equation (A.1) results in

\[ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Da \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right] W_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \]

\[ -\left[ \rho_p h + M \delta(x-x_i) \delta(y-y_i) \right] \left( \frac{\omega'^2 c_p^2}{a} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \right) \]
\[ = \frac{D}{a^3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b} - \frac{D}{a^3} \tilde{P}_0(x, y). \]  \hspace{1cm} (A.7)

The orthogonality of the admissible function is expressed as

\[ \int_0^b \int_0^1 \sin \frac{r \pi x}{a} \sin \frac{s \pi y}{b} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b} \, dx \, dy = \begin{cases} \frac{ab}{4} & (r = m \text{ and } s = n) \\ 0 & \text{(else)} \end{cases} \]  \hspace{1cm} (A.8)

After applying the double sine transform to Equation (A.7) and multiplying both sides of the equation by \(4 \alpha^2/Db\), Equation (A.7) becomes

\[ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left\{ \left[ (r \pi)^2 + \left( \frac{s \pi b}{b} \right)^2 - 12 \left( \frac{a}{h} \right)^2 \omega^2 \right] \delta_{rm} \delta_{sn} \right. \\
-48 \mu \left( \frac{a}{h} \right)^2 \sin \frac{r \pi x_i}{a} \sin \frac{s \pi y_i}{b} \sin \frac{m \pi x_i}{a} \sin \frac{n \pi y_i}{b} \right\} W_{mn} \\
= a_{rs} - \frac{4}{ab} \int_0^b \int_0^1 \tilde{P}_0(x, y) \sin \frac{r \pi x}{a} \sin \frac{s \pi y}{b} \, dx \, dy , \]  \hspace{1cm} (A.9)

where \( \mu = M/\rho_p hab \) is the weight ratio of point mass to plate mass.

The acoustic pressure is determined by

\[ p(x, y, z, t) = -\rho_0 \frac{\partial \Psi}{\partial t}, \]  \hspace{1cm} (A.10)

where \( \Psi(x, y, z, t) \) is the velocity potential and can be expressed as

\[ \Psi(x, y, z, t) = a c_p \tilde{\Psi}(x, y, z) \exp(i \omega' c_p t/a). \]  \hspace{1cm} (A.11)
\( \tilde{\Psi} \) is the spatial variation of \( \Psi \) and it satisfies the wave equation

\[
\nabla^2 \tilde{\Psi} + k^2 \tilde{\Psi} = 0.
\tag{A.12}
\]

The boundary conditions of the velocity potential are

\[
\left. \frac{\partial \tilde{\Psi}}{\partial z} \right|_{z=0} = \frac{\partial \varphi}{\partial t}
\tag{A.13}
\]
on the plate, and

\[
\left. \frac{\partial \tilde{\Psi}}{\partial z} \right|_{z=0} = 0
\tag{A.14}
\]
off the plate, but in the baffled plane. Taking differentiation of Equations (A.3) and (A.11) and substituting the results into Equation (A.13) and (A.14), respectively, the boundary condition can be rewritten as

\[
\left. \frac{\partial \tilde{\Psi}}{\partial z} \right|_{z=0} = f(x, y) = \begin{cases} 
\frac{i k \varepsilon_a}{k_p} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin \frac{mnx}{a} \sin \frac{mny}{b}, & \text{on the plate.} \\
0, & \text{off the plate}
\end{cases}
\tag{A.15}
\]

After performing Fourier transform on Equation (A.12) and Equation (A.15), the results are (Sneddon, 1951)

\[
\frac{d^2 \tilde{\Psi}^*}{dz^2} = \gamma \tilde{\Psi}^*, \quad \gamma = (K_x^2 + K_y^2 - k^2)^{1/2}
\tag{A.16}
\]
and

\[
\left( \frac{d\tilde{\psi}^*}{dz} \right)_{z=0} = f^*(K_x, K_y),
\]  

(A.17)

respectively. Here "*" denotes Fourier transform. \(K_x\) and \(K_y\) are the Fourier transform variables of \(x\) and \(y\), respectively. The solution to Equation (A.16) (which is an ordinary differential equation) with a boundary condition as in Equation (A.17) is

\[
\tilde{\psi}^* = -f^*(K_x, K_y) \frac{\exp(-\gamma z)}{\gamma}.
\]  

(A.18)

By employing the conjugate Fourier kernel function and the methods of inversion, the convoluted integral solution is (Sandman and Vieira, 1975)

\[
\tilde{\psi}(x, y, z) = -\frac{ikc_0}{2\pi c_p} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \int_0^a \int_0^b \sin \frac{m\pi x_1}{a} \sin \frac{n\pi y_1}{b} \frac{\exp(-ik_R)}{R} dx_1 dy_1,
\]  

(A.19)

with \(R = [(x - x_1)^2 + (y - y_1)^2 + z_0^2]^{1/2}\) and \(k = \omega/c_p\). By applying the relation of Equation (A.10), the pressure is derived as

\[
P(x, y, z) = \frac{2 \rho_p c_2}{\rho_p c_p} \left( \frac{a}{h} \right)^3 (i\omega) \frac{ikc_0}{2\pi} \times \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \int_0^a \int_0^b \sin \frac{m\pi x_1}{a} \sin \frac{n\pi y_1}{b} \frac{\exp(-ik_R)}{R} dx_1 dy_1.
\]  

(A.20)

Equation (A.20) is the Rayleigh integral used to estimate the sound radiation. Substituting Equation (A.20) into Equation (A.9) and expressing \(p_o(x, y, t)\) in terms of
\( w(x, y, t) \) yields the following:

\[
\begin{align*}
\left[ \Gamma_r^2 - 12\left(\frac{a}{h}\right)^3 \omega^2 \right] W_{mn} &+ 12\frac{\rho \omega^2 c_o}{\rho_p c_p} \left(\frac{a}{h}\right)^3 i\omega' \frac{4}{2\pi ab} \\
\times \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} &\int_0^b \int_0^b \sin \frac{m\pi x_1}{a} \sin \frac{n\pi y_1}{b} \exp(-ikR) \frac{dz_1 dz_2 x_1 y_1}{R} dx_1 dy_1 \sin \frac{r\pi x_1}{a} \sin \frac{s\pi y_1}{b} dx_1 dy_1 W_{mn} \\
+ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} 48 \mu \left(\frac{a}{h}\right)^2 i\omega' \sin \frac{r\pi x_1}{a} \sin \frac{s\pi y_1}{b} \sin \frac{m\pi x_1}{a} \sin \frac{n\pi y_1}{b} \omega' W_{mn} = a_{rs}.
\end{align*}
\]  
(A.21)

According to Sandman (1977), the second term in Equation (A.21), the fluid-loading impedance \( Z_{\text{fmn}}' \), can be evaluated in a modified integral transform which was discussed in Chapter 3. \( Z_{\text{fmn}}' \) is the mass-loading term, and it was discussed in Chapter 4. Equation (A.21) can be rewritten as

\[
\begin{align*}
\left[ \Gamma_r^2 - 12\left(\frac{a}{h}\right)^3 \omega^2 \right] W_{mn} &+ 12\frac{\rho \omega^2 c_o}{\rho_p c_p} \left(\frac{a}{h}\right)^3 i\omega' \frac{4}{2\pi ab} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Z_{\text{rsmn}}' W_{mn} \\
+ 48 \mu \left(\frac{a}{h}\right)^2 (i\omega') \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Z_{\text{rsmn}}' W_{mn} = a_{rs},
\end{align*}
\]  
(A.22)

or in a compact form of

\[
\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (K_{\text{rsmn}}' + i\omega R_{\text{rsmn}}' - \omega^2 M_{\text{rsmn}}' + i\omega \rho_p c_p) W_{mn} = a_{rs},
\]  
(A.23)

where \( \Gamma_r^2 = (r\pi)^2 + [s\pi(b/a)]^2 \). The matrix form of Equation (A.23) was presented as Equation (3.13) in Chapter 3, and the detailed expressions of the modal coefficients of plate stiffness \( K_{\text{rsmn}}' \) and the combined fluid and mass resistance \( R_{\text{rsmn}}' + i\omega M_{\text{rsmn}}' \) were also presented in Chapter 3 and Chapter 4, respectively.

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Appendix B

Modal Decomposition

The modal decomposition method is introduced to incorporate the plate response generated by the non-propagating wave components into the modal expansion, as expressed in Equation (5.44). For a simply supported rectangular plate, the plate displacement takes the form of (Sandman, 1977)

\[ w(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \exp(i\omega' c_p t/a). \] (B.1)

The double sine function in Equation (B.1) is the admissible function describing the mode shapes of the plate vibration.

The near-field plate response estimated with Equation (5.47) is a function of variables \( x \) and \( y \), so it can be assumed to be the sum of modal components of admissible functions, as expressed in Equation (B.1):

\[ w_n^f(x, y) = b \sum_{m=1}^{M} \sum_{n=1}^{N} W_{mn}^n \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}, \] (B.2)

where \( W_{mn}^n \) are the modal coefficients due to the plate response caused by the non-
propagating waves. The total near-field response \( w^f_n(x, y) \) can be calculated with Equations (5.46) and (5.47) derived in Chapter 5. Hence, the estimation of \( W^n_{mn} \) is the task of modal decomposition.

There are different ways of performing modal decomposition. Here, a method similar to Fourier transform is used to solve for the modal coefficients \( W^n_{mn} \). Taking double sine transform to Equation (B.2) and using the orthogonality condition of Equation (A.8), \( W^n_{mn} \) is derived as follows:

\[
W^n_{mn} = \frac{4}{ab} \int_0^a \int_0^b w^f_n(x, y) \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b} \, dx \, dy. \tag{B.3}
\]

Ideally, \( w^f_n(x, y) \) should be calculated continuously over the plate. With a numerical method, this is accomplished by setting a grid of \( 20 \times 20 \) and estimating all the values on the nodes in order to perform the integration. It should be noted that the result is an approximation due to the limited nodes of the grid. Hence, the resulting \( w^f_n(x, y) \) estimated with Equation (B.1) is an approximation of that estimated with Equations (5.45) and (5.46).

The purpose of this modal decomposition is to incorporate the modal response due to the non-propagating waves, \( W^n_{mn} \), into the modal response due to the propagating wave, \( W_{mn} \), as expressed in Equation (5.49).
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Vita

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