Life Prediction Of Fiber-Reinforced Composites: Macro- And Micro-Mechanical Modeling

by

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LIFE PREDICTION OF FIBER-REINFORCED COMPOSITES:
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(ABSTRACT)

In homogenous materials the life of a component is controlled by damage associated with a single crack while that of non-homogenous materials is the result of a distributed damage state. The life prediction of composite materials is thus carried out using damage mechanics two common approaches of which are, macro- and micro-mechanical modeling. The former assumes homogeneity at the lamina level while the latter evaluates failure processes at the fiber-matrix level.

In the first part of this study the remaining strength life prediction methodology MRLife, modified for ceramic composites (CCLife), is integrated into the finite element package CSTEM, to create an integrated design tool for ceramic matrix composites. Using this tool, a case study is carried out to predict the life of a notched Nicalon™/Silicon Carbide 2-D woven laminated composite coupon with a temperature distribution subject to fatigue loading. Global failure of the notched plate is predicted based on a Whitney- Nuismer type average strength criterion.
In the second part of this study, simulation of events occurring at the fiber-matrix level are used to develop micro-mechanical models for the time-dependent behavior of fiber-reinforced composites due to shear creep of the fiber-matrix interface and slow crack growth in the fibers. At first, simulations of the time-dependent failure of the composite are performed using a modified Monte-Carlo fast-fracture model the results of which are then used to validate the analytical models developed for the two mechanisms. Finally, an analytical model for the time-dependent failure of a composite due to the combined effects of the two mechanism, shear creep and slow crack growth is presented. The potential for including the time-dependent failure model into CCLife is evaluated by comparing these results with those form CCLife results under the same conditions.
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1. INTRODUCTION

The need to increase the operating temperatures of future turbo-machinery has directed material scientists towards ceramic materials. Ideally, monolithic ceramics would be suitable candidates for such applications but lack of durability due to poor fracture toughness has limited their use. A common approach to improving the fracture toughness, and hence prolonging the design life of industrial ceramics has been to reinforce the material with fibers to form ceramic matrix composites. The fibers have, in general, improved most of the properties of the monolithic ceramics however, the micro-level inhomogeneity is responsible for the creation of localized damage due to the presence of spatially varying stress states in the material [1].

With the recent emphasis on cost in present material system design philosophy the role of material durability has taken on greater significance. In the present context the definition of durability is the ability to exist or function without deterioration. Ceramic matrix materials are inherently brittle and prone to damage at low loads levels but are capable of functioning well even after such initial damage has occurred. For these material systems engineering expediency requires that we broaden the definition of the word durability to include sustaining a load in a damaged state. In prior design philosophies a
component would be considered for replacement upon the based on the ability to detect significant damage (a large crack), whereas improved damage detection techniques can detect initial damage well before the component needs to be replaced. This requires utilizing the component with the knowledge of it being in a damaged state. The need to understand the response of a material in a damaged state gave rise to the field of fracture mechanics to describe single-crack behavior. However, while earlier engineering materials were almost always isotropic and homogenous such that damage often manifested itself in the form of a single dominant crack, anisotropic and inhomogeneous materials, such as ceramic matrix materials, which experience multiple localized damage can not be analyzed appropriately using fracture mechanics, giving rise to the field of damage mechanics. Experience over the past decade has established the use of damage mechanics to predict the life of ceramic matrix composite components [1].

Two approaches to damage mechanics can be taken: mechanistic and phenomenological. The mechanistic, or micro-level, approach uses the properties of the constituents and established laws of physics and mechanics to account for the global response of the material. This approach is primarily of interest to material scientists as it focuses on events occurring at the micro-level. While the mechanistic approach has shown potential in predicting the response of some composite materials under quasi-static loading, extending its use to predict the time/cycle dependent response requires extensive computation and testing, since it would require analyzing each individual damage event separately. The problem is on one hand somewhat alleviated by the fact that these localized
damaged states are repeatable [1] with the scale of repeatability being dependent on mate-
rrial composition, while on the other, complicated by the statistical variations in the mate-
rrial. In the phenomenological or macro-level, approach, the heterogeneous material is
treated as a continuum at the lamina level and mathematical models are used to link the
external excitation to the material response: relationships between the global response
(stress or strain) and an independent time/cycle unit can be developed. Here the param-
ters of the mathematical models are evaluated from laboratory information at the laminate
level and mechanistic models can be used above the lamina level [2]. This approach is of
particular interest to engineers as the basic element is the lamina which is also the funda-
mental element in component design.

As one would expect, an ideal life predictive methodology based on damage me-
chanics would be purely mechanistic in nature and capable of accommodating even the
most minute changes in the constituents. Such a methodology would be of interest to both
material scientists and engineers. However necessary, given the material development
schedules involved, the pursuit of such purely mechanistic models is not pragmatic. It is
well accepted that the issue of long-term response with regards to composite materials is not
only one of proper representation of the damage, but also the formulation of the problem in
such a manner that it can be solved [3-7]. The practical approach would be a combination
of mechanistic representations of experimental observations and empiricism. The relative
proportion of the two would shift as development of such materials progress; with a move
towards greater contribution of the former. Recent advances in understanding of the dam-

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age mechanisms that control the response of CMC's permits an increase in the contribution of micro-mechanical models to the life prediction methodology [8]. With current knowledge of these materials it is possible to develop expressions for rate equations of a few degradation mechanisms using constituent properties.

Again, there are two approaches to developing a micro-mechanical representation of any degradation process. One is direct mathematical representation of the process and the other is to use numerical modeling or computer simulation of the damage events and then use the simulation results to develop an analytic representation of the damage. The use of computer simulation to represent experimental phenomena is not new, and with the rapid development of computerization offers an alternative to purely mechanistic modeling of composite failure and life prediction. In general the application of simulation not only serves as a means of predicting the evolution of material properties (remaining strength) and life of composite materials, but, could also serves as a benchmark tool for improving material properties through more focused experimentation and validating analytical models as will be shown. In developing a life prediction methodology computer simulations can be used to evaluate the response of the evolving damage and degradation in the material, and also to better understand the mechanisms controlling the failure process. The failure processes are strongly non-linear, in large part due to the time-varying material properties and so the possibility of obtaining closed form solutions for the damage mechanisms that consist of the interaction of those individual processes is remote. Computer simulations provide a mathematical means of circumventing the impending complex mathematical problem and

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yet provides results of competing accuracy. Depending on the level at which the simulation is being performed, macro or micro, it can provide for less compromise by allowing one to incorporate more physics of the problem than would be possible in analytical models.

For a life predictive technique to have validity it must be capable of predicting the failure at the component level. At this level the model is faced with interacting damage mechanisms, complex stress states and competing failure mechanisms. Before the life prediction techniques can accommodate the complexities of component level design they must be capable of replicating a failure envelope for the material under varying conditions of time, temperature and stress. Admittedly, such envelopes have yet to be developed for composites though their use in the metals and ceramic industries, known as fracture mechanism maps, is gaining popularity [9,10]. These maps clearly define the stress and temperature conditions over which each mechanism dominates resulting in materials failure. When using macro-level analysis to determine life, the failure envelope is empirically defined with no reference being made to the specific mechanisms controlling the failure. For detailed information on material behavior a micro-level model would be more desirable. For such a model, to ensure that all possible failure modes have been taken into consideration, a good understanding of fracture maps for monolithic materials is necessary. While the macro-level approach using empirical failure criteria can implicitly or explicitly account for numerous failure mechanisms without clearly defining them, most micro-mechanical models however, relate to specific well-defined failure mechanisms. There are many mechanisms that contribute to composite failure for which there is only a limited understanding, a case in

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point being the oxidation of fibers. Other mechanisms such as fiber and matrix creep are presently under investigation [8].

This study is presented in two parts. In the first part, a life prediction methodology (MRLife®) based on the remaining-strength-degradation approach is presented. In this approach accurate representation of damage phenomena, measured at the macroscopic (lamina) level as a function of some independent variable (time or cycles) is used to calculate the evolution of stress and strength in the material. Remaining strength, which has been shown [3-7] to be a function of the local state of stress and state of the material at any time, is monitored, and failure is considered to occur when the remaining strength falls below the local level of stress. A Fortran code of this remaining strength and life prediction methodology modified for ceramic matrix composites called CCLife, is used as a post-processor to a finite element package CSTEM [11], to create an integrated design tool for ceramic matrix composites. The validity of this design tool will be evaluated by performing a case study on the life prediction of a notched Nicalon/Silicon Carbide 2-D woven laminated composite coupon under a thermal gradient subject to fatigue loading. While in the first part the material evaluation and simulation was performed at the macroscopic level, in the second part of this study computer simulations of events occurring at the micro-level are performed. Here the objective is to provide a better understanding of the mechanisms that result in the creep/stress rupture of ceramic matrix composites. The specific mechanisms that will be simulated are shear creep of the interface and fiber strength degradation by slow crack growth. A Monte-Carlo simulation model used earlier to accurately predict fast-fracture

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strength [12] has been modified to include specific time-dependent behavior. Results from the simulation model, are used to develop analytic models for the two mechanisms. Using the analytic models the impact of each of the mechanisms, shear creep of the interface and slow crack growth in the fibers, acting independently and together, on rupture and remaining strength of the composite is evaluated. Finally, results obtained from the analytic model and CCLife for the two mechanisms acting in concert are compared, and the possibility of using such micro-mechanical models in CCLife discussed.
2. EXPERIMENTAL EVIDENCE OF DAMAGE PHENOMENA

2.1 Introduction

Traditionally, through the use of refractory bricks and oven-ware, the words "ceramics" and "materials for high temperature applications" are almost synonymous. The use of ceramics for such applications was primarily due to their low heat conductivity and high melting point as the loading was largely thermal. The need for new materials to sustain mechanical loads at high temperature, to raise the thermal efficiency of the next generation of heat engines, has focused interest on ceramic materials as an industrial material. Bulk ceramics, commonly referred to as monolithic ceramics, are known to have high strength and hardness, excellent high temperature capability, chemical inertness, wear resistance and low density [13]. However, they are not generally good under tensile and impact loading as they fail in a brittle manner. The property that controls such behavior is fracture toughness. Fracture toughness is the energy-absorbing capability of a material. Monolithic ceramics possess low values of fracture toughness resulting in low reliability in the material which has limited their commercial application. Recognizing that with the existing fracture toughness ceramics are unlikely to gain consideration for industrial use, attempts are being made to incorporate some form of energy-dissipation in the fracture process of these monolithic ceramics i.e. imparting damage-tolerant response [13]. While there has been some gain
in the fracture toughness of ceramic materials by incorporating a second phase (particles) that results in transformation toughening of the material; fiber reinforcement of these materials shows more promise towards attaining this goal. These and other issues related to the industrial use of ceramic and ceramic composites have been investigated for the past three decades. For the development of a life prediction methodology a proper understanding and representation of the damage phenomena associated with the inclusion of fibers is necessary. In this chapter the reader is briefly introduced to the development of ceramic matrix composites and experimental observation of phenomena that have affected the development of these materials. In Section 2.2 the reader is introduced to the class of materials broadly referred to as ceramic matrix composites while in Section 2.3 the response of these materials to tensile monotonic loading is presented. Included in the latter section is the influence of elevated temperatures on the tensile response of these material. In Section 2.4 the response of these materials to cyclic loading at room and elevated temperatures resulting in fatigue are presented, while in Section 2.5 the creep response of these materials is also presented. In each of the sections the influence of the constituents of the composite and the interface characteristics are accentuated.

2.2 Development of Ceramic Matrix Composites

The class of material that results from the inclusion of fibers in a monolithic ceramic (matrix) is referred to as Ceramic Matrix Composites. The discontinuous or
continuous fibers incorporated into a ceramic are intended to produce a light-weight material with vastly improved fracture toughness and reliability over the monolithic ceramics. The fracture toughness is improved by the introduction of energy-dissipative processes such as debonding, crack deflection, fiber bridging and fiber pull-out. Without going into details as to the development of these processes intuition tells us that such processes are energy-dissipation mechanisms. In these materials the correlation between fracture toughness and damage tolerance is not based on the growth of a single crack but on the local micro-level mechanisms that originate at the interface. As will be seen later, the interface plays a key role in ceramic matrix composites and so should be appropriately designed [13]. In polymeric composites the modulus ratio \((E_t/E_m >10-100)\) is high; therefore, to ensure proper load transfer to the fibers (the principal load carrying members), a strong interface is required to resist the high shear stresses developed. In ceramic matrix composites however, the modulus ratio is low \((E_t/E_m >1-10)\) which results in a more equitable distribution of load between the two constituents and so crack deflection rather than load distribution for which a weak interface is desired is the primary function.

When evaluating suitable materials for incorporation into fiber-reinforced ceramics the basic requirements are high strength, good refractoriness, hardness and low density. These requirements can only be fulfilled by materials possessing a high density of strong atomic bonds such as Beryllium, Boron, Carbon, Nitrogen, Magnesium, Aluminum and Silicon all of which have strong covalent bonds and have a tradition of
refractory use [14]. The matrix phase, in general, will consist of existing monolithic ceramics such as alumina, zirconia, silicon carbide, silicon nitride, carbon, glass ceramics and glass. The fibers on the other hand must be superior in strength and refactoriness than the matrix for full advantage and ensure that fiber degradation does not occur during processing [14]. In ceramic composites the two phases are categorized as either oxide or non-oxide. The composites formed from the consolidation of these two phases can be of any combination, i.e. non-oxide fibers/oxide matrix or oxide fibers/non-oxide matrix etc.. However, nonoxide fiber/nonoxide matrix (Silicon Carbide/ Silicon Carbide (SiC/SiC) and SiC/Silicon Nitride (SiC/Si₃N₄) have, of late, received the most attention due to the need for high temperature application, though the lower cost of nonoxide fibers/oxide matrix systems made them more attractive so attracted more research.

Numerous materials may satisfy the broad requirements of the individual components. However, fiber-matrix compatibility which controls the interactive effects, that may result in fiber degradation during processing and the eventual load sharing between fiber and matrix, is an extremely important consideration.

By far the most important properties that control the interface characteristics are fiber diameter and thermal mismatch [14]. The former controls the interface-volume ratio, to which the rate effects are related, while, the later controls the residual stress in the system, and so the level of energy required to precipitate debonding. In addition, chemical compatibility, which determines stability of the system and the interfacial bond, is given serious consideration.

Experimental Evidence of Damage Phenomena
Fabrication of continuous fiber ceramic matrix composites is achieved in many ways depending on how the matrix is incorporated, i.e. either from a powder, liquid or gaseous source. In the powder prepreg process, the fibers are impregnated with fine particles of the matrix to form a prepreg sheet, which, when formed is then consolidated by hot-pressing. The advantage in this method is a high density material that is readily produced; however, the potential of fiber damage during consolidation is extremely high. This damage can be reduced by optimizing the temperature and pressure during processing. When using a liquid precursor, e.g. in a sol-gel technique, multiple impregnations are required before the material is densified by some process to ensure high density is obtained. When using a gaseous precursor, e.g. in a chemical-vapor-impregnation process as used to produce silicon carbide/silicon carbide ceramic composites, the vapor is deposited on a fibrous preform in repeated cycles. The resulting composite will have high porosity and often is restricted to the use of 2-D or 3-D preforms. Unidirectional laminae cannot be produced this way due to the inability to ensure uniform deposition. These composites have the disadvantage of having lower density due to the porosity arising by the deposited matrix sealing off the vapor paths [14].

Early work on ceramic matrix composites consisted of reinforcing ceramic matrices with carbon fibers [15-16]. These composites generated considerable optimism for these materials as they resulted in increase an strength, fracture toughness and fatigue resistance. The optimism however was short-lived, when it was realized that the fibers
readily oxidized in air at elevated temperatures. Such an oxidizing environment is typical in gas-turbine engines for which these materials are intended. This realization resulted in the use of ceramic fibers as reinforcements. While the potential for improving the fracture toughness at room temperature exists the focus now has changed to ensuring that the same improvement can be assured at elevated temperatures.

2.3 Monotonic Loading

Monotonic loading refers to a stress state that is continually increasing in one direction as opposed to cyclic loading. In this section the response of ceramic matrix composites to monotonic loading at room and elevated temperatures is presented. The two common ways of studying the monotonic response of composite materials are by tensile or flexural loading. The pros and cons of each method are debatable, but Davidge and Briggs [17] put it succinctly by stating that while flexural tests are easy to perform they are difficult to interpret, whereas tensile tests are easy to interpret and difficult to perform. This statement is even more true at elevated temperatures. The inclusion of fibers into ceramic materials offers the potential to improve the fracture toughness of the material by increasing the strain to failure of the composite. The increase in strain to failure is the result of the evolving damage due to matrix cracking and subsequent load transfer between the fibers. Figure 1 shows the stress-strain curve of

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Figure 1: Typical stress-strain curve of a ceramic matrix composite [After 12]
a unidirectional glass ceramic containing Silicon Carbide (SiC) fibers, is said to be fairly
typical of this class of materials. At low stress levels, the stress-strain curve is linear with
an elastic response until the matrix cracking stress, \( \sigma_{mc} \), when non-linearity sets in. This
non-linear region is a result of the strain to failure of the matrix being less than the fibers
which leads to the growth of transverse micro-cracks in the matrix. Provided that the
bond between the matrix and fibers has a sufficiently low fracture energy to allow
debonding at the fiber-matrix interface, the composite can sustain further increase in load.
As the matrix micro-cracks saturate, causing a “knee” in the stress-strain curve, the
response is once again linear for some time and is purely due to fiber extension. Shortly
thereafter, fiber failure begins to occur causing non-linearity, once again. A maximum
stress occurs corresponding to bundle failure and is followed by fiber pullout.

As is evident in Figure 1, the difference in stress between the fiber bundle failure
level and the level of first matrix cracking is a good estimate of the contribution made by
the fibers. Also considerable increase in strain occurs after the ultimate stress is reached.
This is the work of fracture due to pullout during rupture, with the resulting energy
dissipation. It must be noted that crack saturation is a function of the matrix material and
interfacial shear stress and may not always occur before the specimen fails.

The effect of the interface on the fiber pullout was observed by Brennan and
Prewo et al. [18-20] in a series of tests while evaluating the response of Nicalon/Lithium-
Alumino-Silicate (LAS) composites under monotonic tensile loading. Using
Transmission Electron Microscopy (TEM), thin foil, and Scanning Auger Microprobe
(SAM) analysis they found the existence of a thin carbon rich layer that was formed
during processing. On performing SAM analysis on some extremely weak and brittle glass matrix composites they noticed the absence of such a layer. Brennan and Prewo [20] attributed the high toughness to the formation of this interfacial layer. They also recognized that this weak interfacial layer was responsible for the occurrence of crack deflection but at the same time allowed load transfer to take place. The kinetics responsible for the formation of the carbon-rich layer was not explained by them, although Cooper and Chyung [21] carried out an exhaustive study to answer this question. Brennan and Prewo [20] concluded that the sliding shear stress after debonding \( \tau \) is an important parameter as it determines the pullout lengths and hence the work of fracture.

In a more recent study on the response of Nicalon/Calcium-Alumino-Silicate (CAS) laminates with increasing number of 90\(^\circ\) plies \([0]_{12},[0/90]_p,[0/90]_3\) and\([0/90]_4\) to tensile loading, Pryce and Smith [22] found that the weaker plies (90\(^\circ\)) cracked first. They showed that as the number of 90\(^\circ\) plies is increased the onset of matrix cracking is at a lower load. To better study the cracks, Pryce and Smith [22] strained the specimens and then carried out their observations under an optical microscope. They observed the matrix damage to consist of an array of cracks spanning the width and thickness of the laminates with the density of the cracks increasing proportionately with the strain. In these specimens they noticed the stress-strain curve became linear at about 0.03\% strain which suggested saturation of matrix cracks which they confirmed by counting. An interesting observation during discontinuous tests on the specimens was that, in contrast to cracked polymer matrix composites specimens which when reloaded showed a linear

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stress-strain response till further cracking occurred, the ceramic matrix composites displayed non-linear stress-strains curves. They attributed this to the fiber-matrix sliding that occurred due to the closing and opening of cracks. Zawada et al. [23-24] also performed tests on unidirectional and cross ply Nicalon/Alumino-Silicate glass laminates. In the quasi-static tests they showed that the proportional limit was much higher than the matrix cracking stress for the unidirectional laminate. For the cross plies laminates they saw two proportional limits with the lower one coinciding with the $\sigma_{mc}$.

The influence of fiber architecture on processing, residual porosity and performance of CVI-processed ceramic composites has been well documented [25,26]. Pluvinage et al. [27] studied the damage mechanisms in Nicalon/Silicon Carbide 2-D woven and 3-D braided composites. They observed the damage to be considerably more complex than that seen in unidirectional laminates. In the 2-D woven and two-step braided material they recorded non-linearity in the stress-strain curve in the longitudinal direction and linearity in the transverse direction suggesting that cracking was only taking place perpendicular to the loading direction. In the 4-step braided material, however, there was non-linearity in the transverse stress-strain curve too. All the composites showed inter-yarn and intra-yarn porosity which could be attributed to the processing technique. The inter-yarn porosity resulted in low strain to failure of the 2-D woven material caused by fiber bundle failure occurring prior to cracking. In the 3-D material they observed multiple cracking in the bundles resulting in the increase of strain-to-failure. According to Pluvinage et al. [27] there are four stages of damage development in these materials as follows: (i) no apparent damage, (ii) inter-yarn cracks, initiated by

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porosity, develop in the matrix-rich region perpendicular to the loading direction, (iii) multiple intra-yarn cracks develop and grow to saturation, (iv) fibers begin to break eventually leading to failure of the composite. In most cases stages (i) and (ii) were not seen to have any macroscopic effect as they could not be identified on the stress-strain curves.

When matrix cracking occurs the fibers become the sole load bearing component and this emphasizes the need to understand the behavior of the reinforcements at elevated temperatures. Pysher et al. [28] evaluated the strength of two Silicon carbide based (Nicalon and Tyranno) and three oxide (Fiber-FP, PRD-166, and Nextel 480) ceramic fibers at temperatures ranging from 25°-1400°C. In comparing the SiC fibers, they observed that the strength of the Tyranno fibers was higher at room temperature, but lower at 1200°C than the Nicalon fibers. Both fibers were seen to degrade with temperature above 1000°C. At 1300° and 1400°C both fibers underwent a significant loss in strength with the formation of large voids near the fiber surfaces. The degradation was attributed to chemical change due to leeching of the oxygen form the interior of the fibers. They concluded since oxidation was the principal reason for strength degradation, the larger diameter of the Nicalon fiber was responsible for its higher strength at the elevated temperatures. Among the oxide fibers Nextel 480 and PRD 166 had similar room temperature strengths with FP slightly lower. Like the SiC fibers, all three fibers started degrading at 800°C but at a faster rate resulting in total degradation at 1200°C. Failure of these fibers was attributed to softening due to creep which resulted in necking at the failure section prior to failure.

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With the improvement of the room temperature fracture toughness of ceramic matrix composites the focus had turned to evaluating the fracture toughness of these materials at elevated temperatures. Prewo et al. [20] studied the behavior of Nicalon/LAS cross-ply laminates under monotonic loading at elevated temperatures in Argon and air. They observed that the stress-strain response remained constant till 1300°C when tested in Argon. They speculated that the change above this temperature was due to the rise in the failure strain of the matrix. This strength retention at 1300°C was surprising as the fibers showed a loss of strength at temperatures less than 1000°C [28]. The sensitivity of strength to environment was very evident. The strength of specimens tested in air was equivalent to the proportional limit stress of the 0° plies. The reason being that the infusion of oxygen into the system and erosion of the interface resulting in brittle failure of the composite. Rousseau [29] studied the response to monotonic loading of cross ply Nicalon/CAS laminate specimens, at 20°C and 815°C. As with Zawada [23,24] before him, Rousseau [29] recorded two proportional limits in the stress-strain curve of the cross-ply laminates. The lower one he attributed to cracking of the 90° plies while the upper one was due to transverse micro-cracks in the 0° plies. He also compared the specimens with matrices modified to result in lower debond energy against the unmodified and found that at both temperatures the specimens with the weaker interface allowed for more debonding and less fiber breakage resulting in higher ultimate stress.

Similar damage mechanisms were observed by Pluvinage et al. [30] who, in order to study the response of Nicalon/Silicon Carbide (SiC) laminates in an oxidative
environment, performed aging tests at various temperatures. Examination of the fracture surfaces of the as-received and oxidized specimens revealed marked differences, in that, while the former showed limited pullout implying ductile failure the latter invariably showed mixed mode failure i.e. brittle accompanied by ductile. Pluvinage et al. [30] presented the events resulting to brittle failure as follows: (i) matrix cracking permits the flow of oxygen to the interior causing oxidation of the carbon interphase, (ii) this leaves interstitial cavities, (iii) subsequent diffusion of oxygen into the interstitial cavities, (iv) growth of silica layers due to oxidation on the matrix and fiber surfaces, (v) the interaction of the two silica layers cuts of the diffusion of oxygen preventing further ingress but also restricts movement of the fiber by increasing the local sliding stress eventually causing the crack to grow through the fiber. To further understand the thermo-mechanical behavior of ceramic matrix composites aging tests were carried out. The effect of heat treatment on a composite was evaluated by soaking the specimens at high temperature for a period of time then testing them to failure. Most tests performed on Nicalon/SiC composites showed degradation in strength when the composite was aged for a period of up to 100 hr. at temperatures as low as 800°C [31]. Others [32,26] showed similar reductions in strength at 900°C and 1000°C. In all cases they attributed the reduction in strength to removal of the carbon layer and the formation of a silica layer at the interface which was confirmed by the change in the pullout length [26]. The reduction in strength, in all cases, reduced to the level of first matrix cracking indicating that matrix cracking was responsible for the eventual failure i.e. the cracks could not be deflected and moved through the specimen.

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To study the role of the interface, Singh [33] compared the response of Boron Nitride (BN) coated and uncoated SiC/Zircon using SCS-6 fibers with monolithic Zircon in flexural tests at temperatures 25-1477°C. The monolithic material underwent brittle failure till 1300°C after which it failed plastically while the composite showed higher strength and toughness at temperatures up to 1100°C above which there was not much difference. Abbe and Chermant [34] also studied changes in the interface of a Nicalon 2-D woven/SiC composite with a pyrolytic carbon interface. They showed that the carbon interface degrades from room temperature till about 1000°C when it all but disappears. This was closely related to the rupture stress. Tortorelli et al. [35] studied the effect of thickness of the pyrolytic carbon layer on the properties of Nicalon/SiC composites. They found that the properties of the composite were significantly affected by the thickness of the interface.

2.4 Cyclic Loading

Early research in ceramic composites was so focused on improving fracture toughness, and understanding the cracking phenomenon in these materials that the response under fatigue was totally ignored under the misconception that it was not important [13]. For structural applications it is necessary that the ceramic composites exhibit the same fracture resistance to cyclic loading that they did to quasi-static loading. Since 1985 several studies on the fatigue behavior of ceramic matrix composites have been carried out [23-25,36-40].

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Early contributions to the understanding of the fatigue response of ceramic matrix composites came from Prewo et al. [36-38] who studied the response of unidirectional LAS glass laminates to cyclic loading at elevated temperatures. In the LAS-II system they observed non-linearity in the stress-strain response when the fatigue stress was above the proportional limit. Prewo et al. [36-38] assumed that the response was due to matrix micro-cracking which began at the proportional limit. Specimens tested below the proportional limit experienced a $10^5$ cycle run-out and showed residual strengths that were practically the same as, or greater, than those in the unfatigued materials. These specimens that experienced run-out did not show any loss in stiffness.

Zawada et al. [23,24] also performed fatigue tests on unidirectional and cross ply Nicalon-Alumino-Silicate glass laminates. In these tests they observed, as the fatigue stress level was increased above the proportional limit, a rapid damage occurred in the first 1000 cycles after which it stabilized. Additional accumulation of damage with further cycling and some modulus recovery at long cycles were observed. The modulus recovery was attributed to debris in the matrix cracks. In fatigue tests on a similar material Rousseau [25] observed that the damage occurring after the first cycle was qualitatively similar to that observed during monotonic loading and remained relatively constant after the first cycle. At 20°C the crack density and inelastic strain were found to depend on the applied stress level. Also, after fatigue cycling at this temperature, the ultimate strength was found to increase while the strain to failure was found to decrease. At the higher temperature, the fatigue lives were found to be shorter than the lower temperature even at fatigue stress levels below matrix cracking stress.

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Further work on fatigue of unidirectional Nicalon-CAS laminates was performed by Karandikar and Chou [39,40]. They found the growth of damage, due to matrix microcracking, in the unidirectional material to be proportional to the change in the Young’s modulus and Poisson’s ratio and a function of the stress level. No additional damage modes, other than the microcracking were attributed to the cyclic loading and so they proposed a method of predicting modulus reduction due to cyclic loading from crack density data obtained from quasi-static tests. Mall and Tracy [41] tested quasi-isotropic laminates of the same material to monitor the growth of damage at different stress levels. They observed that as the fatigue stress was increased, damage was initiated in the $90^\circ$ ply, and extended from there to the $45^\circ$ and $0^\circ$ plies respectively. Mall and Tracy [41] also inferred that the fatigue limit did not correspond to the proportional limit as was previously observed but that it corresponded to the stress level at which damage in the $0^\circ$ ply was initiated.

Prewo et al. [37,38] and Marshall et al. [42,43] measured hysteresis loops in their respective materials to investigate the mechanisms that contribute to fatigue failure. In earlier work, Marshall and Evans [42] observed the opening and closing of the cracks during the loading-unloading cycle, and associated the hysteresis effect to fiber-matrix interface friction. While Holmes et al. [44] also observed hysteresis in Nicalon-fiber-Silicon Nitride ($\text{Si}_3\text{N}_4$) composites, they noticed considerable strain-ratcheting and hysteresis when the maximum fatigue stress exceeded the proportional limit stress. The reduction in life at stress levels above the proportional limit was attributed to the cyclic propagation of cracks formed during initial loading and the consequently the degradation
of the fiber-matrix interface due to fiber slip. In a related study Holmes [45] found that at load levels above the matrix cracking stress, the R-ratio ($\sigma_{\text{max}}/\sigma_{\text{min}}$) value had a significant impact on fatigue life. He also observed that fatigue life to decreased with the value of R-ratio, which led him to conclude that crack propagation and not mean stress controlled the fatigue life. Holmes [44] also observed that the strain ratcheting was due to creep deformation under mean stress and concluded that for his material, in the absence of initial microcracking, creep damage can be more life-limiting than fatigue.

Summarizing, during cyclic loading of ceramic matrix composites the following have been observed:

a. matrix micro-cracking occurs if the fatigue stress level is above the matrix cracking stress,

b. the matrix cracking strain of off-axis plies is generally lower than that of the same unidirectional material,

c. for fatigue failure to occur, matrix microcracking must take place, though fiber breakage and fiber-matrix debonding has also been observed,

d. the fatigue threshold stress (level at which run-out occurs) may be above the matrix cracking stress though lower than the upper proportional limit,

e. cyclic loading results in opening and closing of the matrix micro-cracks which contributes to sliding at the fiber/matrix interface.
2.5 Creep/Stress Rupture

The primary advantage in the use of ceramic materials is their strength and stiffness at temperatures where most metallic materials creep extensively. The maximum temperature at which a material can be used is limited by the creep rate at that temperature and is often related to its homologous temperature. A rule of thumb used sets this limit at 50% of the homologous temperature [13]. While continuous ceramic fibers can provide substantial toughening to the monolithic ceramics at room temperatures, this toughening has not translated into improved elevated temperature properties. Often the inelastic strain response of ceramic matrix composites at elevated temperatures is not only the result of diffusional or cavitation creep in the constituents, but is also the result of matrix micro-cracking and time-dependent fiber failure. It is therefore necessary, in order to study the response of the composite as a whole, to evaluate the micro-mechanical and micro-structural factors controlling creep and damage evolution at these temperatures.

Ceramic fibers come in either oxide or non-oxide forms. One of the earliest works on non-oxide fibers was performed by Simon and Bunsell [46] on two types of, rice hull derived, Nicalon fibers (NLP 101 and NLM 102). They found that both fibers begin to show degradation of strength and modulus above 1000°C and that there is a threshold stress at about 1200°C below which there was no creep. No such threshold existed at 1300°C and they measured a creep rate stress exponent of 1.5 in air and Argon. Mah et al. [47] evaluated the thermal stability of the same Nicalon fibers up to 1200°C.
They observed that the strength of the fibers subject to high temperature heat treatment depended not only on the temperature but also on the duration of the heat treatment. However, tests at 1200°C indicated that fiber degradation was independent of the duration of heat treatment and they concluded that this was due to the non-stoichiometric structure of the Nicalon fibers.

Pysher and Tressler [48] compared the creep response of two oxide fibers (Alumina based Fiber FP and PRD-166) at temperatures up to 1250°C. At the upper temperature they recorded creep rates of the FP fibers to be 2-10 times higher than the PRD fiber, and on obtaining creep stress exponents of 1.2-2.8, concluded that steady state creep was diffusion controlled. Jakus and Tulluri [49] studied the mechanical behavior of Sumitomo Alumina fibers at room and elevated temperatures and found that these fibers had better creep characteristics than other fibers though, at around 1000°C, the strength in the fibers fell as creep rates increased considerably. At 1200°C the strength was only 42% of room temperature strength. Since the creep of these oxide fibers was related to their polycrystalline structure. Corman [50] studied the behavior of some single crystal oxide fibers such as single crystal stabilized Yttria, Yttria-Stabilized-Zirconia (YSZ), Thoria, Yttrium-Alumina-Garnet (YAG), Beryllia, and Alumina. The fibers were tested at stress levels of 125-400 MPa and temperatures of 1650°C-1850°C. Corman [50] observed that the YSZ and Thoria fibers exhibited poor creep response despite having high melting points. Creep of Alumina and Beryllia was very anisotropic due to the existence of basal planes while the YAG fibers turned out to be the most creep resistant.
Based on the observations presented it can be inferred that the non-oxide fibers perform better at high temperatures though they are extremely susceptible to oxidation due to their non-stoichiometric morphology. The oxide fibers while being immune to oxidation have rather poor creep resistance. This limiting factor of oxides at elevated temperatures is due to their polycrystallinity and better creep characteristics can be expected from single crystal fibers such as sapphire and YAG up to 1600°C.

Studying the effect of temperature on Carbon fiber/glass composite, Prewo [51] noticed that the diffusion rate of oxygen in the glass matrix at elevated temperatures was so high that the oxidation of the graphite fibers could not be prevented. Chermant et al. [52,53] studied the flexural (3 point) creep response of unidirectional SiC/Magnesium-Lithium-Alumino-Silicate (MLAS) composites at 1073°K-1473°K. They recorded steady state creep at temperatures up to 1473°K above which only tertiary creep was observed. Chermant et al. [52,53] concluded that failure at the lower temperatures was caused by matrix cracking parallel to the fibers due to the difference in deformation of the fibers and the matrix at the interface while failure at higher temperatures was attributed to matrix cracking perpendicular to the fibers and fiber fracture. Kervadec and Chermant [53] used this to rationalize the change in the creep rate stress exponent they observed. At the upper temperature the matrix was considered to creep easily so all the load is transferred onto the fibers which then fractured.

More recently Wu and Holmes [54] performed tensile creep tests on unidirectional and cross ply Nicalon/CAS at 1200°C in pure Argon. They observed that at low stresses the two laminates crept at the same rate while the latter showed a greater
recovery of strain which they associated with the transverse fibers pinning the longitudinal fibers. At low stresses and long duration they observed creep due to cavity formation. At moderate stress and long duration creep was characterized by both cavity formation and periodic fiber fracture. What is significant in these two cases was that they observed no matrix cracking. Finally, at high stress, the short duration of creep was attributed to rupture of the bridging fibers. The periodic fiber fracture in the absence of matrix fracture was attributed to the redistribution of stress from the matrix to the creep resistant fibers which is the normal creep damage mode for composites where creep rate of matrix exceeds that of the fibers.

To understand the behavior of nonoxide fiber / non-oxide matrix composites at elevated temperatures, Abbe et al. [53,55] studied the creep behavior of Nicalon fiber woven/ SiC matrix composite in 3-point bend tests at 1473°K and 1673°K, in a vacuum. They observed that these composites experienced creep rates higher than those recorded for monolithic α–SiC which they associated with the high matrix porosity (16%) though the contribution of the high creep rates of the Nicalon fibers could not be discounted. They then extended their work to deconvolute flexural data and put the creep rate data in the form for tensile creep developed by Chuang and Wiederhorn [56] and suggested that the creep was either micro-crack or interfacial shear stress controlled. At the same time Holmes [57] and Hilmas et al. [58] evaluated the tensile creep behavior of SCS-6/Si₃N₄ composites in air at stress levels above and below monotonic proportional limit (84 MPa). Tests were performed at 70-150 MPa and 1350°C. Steady state creep was recorded up to 190 MPa after which only tertiary creep was observed. At 70 MPa
extensive pullout and debonding was observed and found to decrease as the stress was increased. At 120 MPa microcracking was observed during creep and not during the initial loading.

Lamouroux et al. [59] performed experiments to study the damage mechanisms during tensile creep loading of alumina reinforced silicon carbide composite at temperature in vacuum. They observed that matrix micro-cracking inside the fabric tows and some interface debonding in the specimens on cooling from the processing temperature. At low loads they attributed the creep to the interfacial debonding that occurred since they conjectured that matrix microcracking and fiber failure did not occur. Lamouroux et al. [59] also noticed that the creep curves they obtained at 1100°C exhibited only primary and tertiary creep with no secondary creep. They believed that progressive fiber-matrix debonding was the dominant mechanism that could explain the decrease of the longitudinal elastic modulus in the tensile and creep tests, the increase in strain rate during tertiary creep and the large fiber pullout on the creep fracture of surfaces. Lamouroux et al. [59] also observed that creep failure mode was stress dependent.

The above observations show that most fibers improve the creep response of their composites, in a manner that is dependent on the relative creep rates of the fiber and the matrix. Damage can occur during creep although it may not be evident at initial loading. From a design standpoint a fiber creep rate lower than the matrix creep rate is desired as it results in progressive fiber fracture; only matrix micro-cracking would occur due to the fibers shedding their load if the situation were reversed. Prevention of microcracking

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helps protect the fibers from the oxidizing atmosphere and protects the fiber/matrix interface. However this debonding can lead to frictional heating under high frequency fatigue loading [60]. Finally extensive pullout is often associated with creep and that the failure under creep is due to a combination of matrix crack growth, fiber sliding and fiber fracture. It is evident that the interface plays a major role in the creep response of these materials.
3. MACRO-MECHANICAL LIFE PREDICTION MODEL

3.1 Introduction

There has been a significant growth of interest in the life prediction of ceramic matrix composites as the application of these materials approaches reality. The use of these materials for engine components subjects them to thermo-mechanical-fatigue loading (TMF) in an oxidative environment. Under such operating conditions the components are more likely to be subject to low cycle fatigue rather than the traditional high cycle fatigue. Low cycle fatigue is generally associated with high stress and low frequency as opposed to the low stress high frequency associated with high cycle fatigue as such the material is subject to combined fatigue and creep effects. With these materials being used at load levels above the matrix cracking stress, the combination of the ensuing matrix damage and exposure of the fibers to high temperatures severely limit the life of such components. The complex behavior of these materials as a result of the damage sustained has constrained the life prediction techniques that can be used. In this chapter the reader is, in the first section, given a brief overview of life prediction methodologies used for composite materials and in the second section introduced to Macroscopic Life Prediction Methodologies, with particular emphasis being placed on a Remaining Strength Model developed earlier. In the third section interpretation of experimental data presented in chapter 2 with models used in macroscopic life prediction
techniques will be presented while in the fourth section a description of the remaining strength model that will be used as part of this study will be described. Finally, in the fifth section the remaining strength model will be used to predict component failure and in the sixth section a discussion of this procedure will be presented.

3.2 Life Prediction Methodologies

The previous chapter highlighted the existence of the complex damage mechanisms that contribute to the failure of ceramic matrix composites. Such behavior is in contrast to that of homogenous materials in which the response is dominated by the growth of a single crack. Traditional life prediction techniques used for isotropic materials, such as Miner's Rule [61], cannot be used for composite materials as the growth of damage is sensitive to the loading sequence. Even fracture mechanics has not been found suitable for these materials as they exhibit heterogeneity on a scale comparable to the crack size [62]. Life prediction techniques used for composite materials can be categorized as follows: Macro-mechanic, Micro-mechanic, Degradation and Probabilistic. The first provides a general engineering solution for the structural designer whose interest is only in those properties, such as stiffness, that manifest themselves at a macro level. The designer thus avoids attempting to solve the complex micro-mechanic problems of fiber reinforced composites. The second approach uses micromechanics to represent the mechanisms that are responsible for the macro level

Macro-mechanical Life Prediction Model
response and so is more complex. In this approach the properties of the constituents and the interface/interphase are sufficient to recreate the global response of the material. Here failure criterion cannot be defined \textit{a priori}, as it is a function of many competing mechanisms. The Degradation approach makes use of Characteristic Damage Parameters to evaluate the macroscopic damage. Here the failure is defined as the earliest occurrence of a critical damage state, that is related to some macroscopic damage level that has been obtained experimentally [63]. In the Probabilistic life prediction approach statistical representations of the damage parameters are carried out with emphasis on the lower tails of the distribution which are normally responsible for early failure. This is supported by the fact that proof testing of such materials cannot be carried out because of their susceptibility to damage by a single high load.

While the Degradation and Probabilistic methodologies involve added levels of mathematical complexity, and Micro-mechanical models have not been fully developed, the Macroscopic methodology is simple and based on accepted failure criteria and so has attracted considerable attention. In addition the Micro-mechanical methodologies require detailed characterization of the material system at the constituent level, independently and together as a composite, to evaluate the interfacial characteristic and chemical compatibility. The development of some micro-mechanical models will be presented in Chapters 5-7. In the Macro-mechanics methodology homogeneity at the lamina level is assumed so proper characterization of unidirectional and cross ply laminae can provide sufficient information to predict the failure of the off-axis laminates. In this

\textbf{Macro-mechanical Life Prediction Model}
methodology one may either pursue the residual-strength-degradation or modulus-degradation approaches.

3.3 Residual-Strength-Degradation Model

In the residual-strength-degradation approach, failure is considered to occur when the residual strength equals the maximum local stress in the material while, in the stiffness-degradation approach there is no widely accepted failure criteria. Hahn and Kim [64] and O'Brien and Reifsnider [65] assume failure to occur when the fatigue modulus degrades to the static secant modulus. Hwang and Han [66] on the other hand monitor the fatigue strain and use the static strain-to-failure of the material as the fatigue strain limit. Poursartip [67] and Beaumont [68] using the analogy of fracture mechanics mentioned earlier define the critical damage state at which fast fracture will occur. While the residual-modulus approach can claim to have a major advantage over the residual-strength approach since it can be quantified using non-destructive techniques there is no well accepted failure criterion, so defining the point of failure is not quite obvious. To illustrate the importance of using an appropriate failure criterion for a design methodology Salkind [69] schematically compared (Figure 3.1) the fatigue life of two components; one, a spring, whose life is stiffness dependent, and a cable whose life is fracture dependent. He showed the differences in life predicted, depending on whether the component was made of metal or composite, based on stiffness and strength criteria. The

Macro-mechanical Life Prediction Model
Figure 2: Schematic Fatigue Behavior of Metal and Composite Components [After 69]
lives predicted for metals showed little difference between the two criteria while this was not the case for composites. This suggests that the life prediction methodology of composite materials is more sensitive to failure criterion than that for metals.

Residual-strength models are based on the following two assumptions [70]:

a. the residual strength after N cycles of fatigue loading can be related to the quasi-static strength by a deterministic equation; and

b. failure occurs when the residual strength decreases to the level of the maximum applied stress.

None of the many of the cumulative damage theories, such as Miner’s Rule [61], used for metallic structures have been applicable to composite materials. While in metals a large part of the fatigue life is spent initiating a dominant crack, which propagates rapidly after that, in composites there exists a distribution of small cracks that do not propagate since they are either confined to a ply or deflected due to the fibers. One of the first residual-strength cumulative damage models proposed for composites was by Broutman and Sahu [71] who used two-stress level cumulative damage tests to propose the following residual strength failure model:

\[ S_r = S_u^{i-1} - (S_u^0 - \sigma_i)f_i \]  

(3.1)

where \( S_r \) is the residual strength after \( ith \) cycle, \( S_u^{i-1} \) is the ultimate strength before the cycling at \( \sigma_i \), \( \sigma_i \) is stress at \( ith \) cycle and \( f_i \) is \( n/N \) fractional life at the current stress level.
This form is similar to the equations used for modulus degradation that relate the residual property as a function of its present value.

While Broutman and Sahu [71] presented a linear one dimensional degradation equation for residual strength, Hahn and Kim [72] fit the following rate equation to represent change in residual strength:

$$\frac{dS_r}{dt} = -A(\sigma_{app}, t)S_r^{-m}$$  \hspace{1cm} (3.2)

where $A(s,t)$ is a function of the load $\sigma_{app}(t)$, and the exponent $m$ is a material constant. They then integrated the equation with respect to time to obtain

$$S_r^{m+1} = S_o^{m+1} - (m+1)D(t-t_o)$$  \hspace{1cm} (3.3)

where the damage as a function of applied stress is given by

$$D = \frac{\int_{t_o}^{t} A(\sigma_{app}(\tau))d\tau}{t-t_o}$$  \hspace{1cm} (3.4)

Hahn and Kim [72] then related the distribution of quasi-static strength with that of life based on the concept of "equal rank".

The use of statistical theory to determine residual strength has also been proposed by Yang et al. [73,74] based on the Strength-Life Equal Rank assumption. They based their residual strength model on the assumption that stronger specimens will have longer fatigue lives and then use a Weibull distribution to determine quasi-static strength and equate this with residual strength. Their model supported the basis of the Miner’s Rule

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[61], i.e., residual strength was not a function of load sequence. Yang et. al [74] however, corrected this by accounting for the memory effect on fatigue.

In another approach, Hashin and Rotem [75] introduced the concept of “damage contours” (lines of equal damage). These damage contours were assumed to be independent of damage mode and loading history. They are shown to be similar to S-N curves but of different slopes. They based their model on the assumption that the life remaining after certain load cycles is the difference between the S-N curve and the ‘damage curve’. They expressed the damage curves on a log-log scale as:

$$\log(\sigma) = k \log(n) \quad (3.5)$$

where $k$ is the slope that determines the residual fatigue life at each stress level. Rotem et. al [76,77] also defined a curve of slope $k_s$ which is the damage curve that is one cycle short of the S-N curve (failure). They assumed that residual strength degradation begins when the remaining fatigue life is one cycle.

Motivated by the fact that the previous residual-strength-degradation models were all phenomenological, Reifsnider and Stinchcomb [1,3-7] presented a mechanistic residual-strength model. Like Hashin and Rotem's [75] earlier model this relates damage with equivalent residual strength. Being a macroscopic model it is based on parameters measurable in the laboratory that provide representation of the state of damage of the material. Based on their observations of a Characteristic Damaged State [2] in composite materials Reifsnider and Stinchcomb [1,3-7] chose to represent damage by use of a representative volume. This representative volume is a typical element of the global
volume and satisfies the same constitutive law. Having determined the representative volume they defined a boundary value problem on the basis of the failure mechanism. The failure mechanisms being limited, this often involves associating various damage modes with a single failure mechanism. Reifsnider and Stinchcomb [1,3-7] then formulated the problem so that cycles or time could be used as an independent variable depending on the dominant failure mechanism. Again based on experimental observations they introduced the concept of the critical element method (Figure 3). In this approach it is assumed that the representative volume element (RVE) consists of two components; the sub-critical and critical elements. Any damage that takes place in the form of matrix cracking, initial fiber failure etc. is restricted to the sub-critical element. This implies that the damage in the sub-critical element does not result in the failure of the representative volume but causes it to shed load. Since the volume is to be maintained in equilibrium this reduction in the load carrying capacity of the sub-critical element due to damage accumulation causes the load to be transferred onto the critical element. This critical element on the other hand does not experience any damage and remains “intact” till the laminate fails. Thus, the failure of the component is linked to the failure of the critical element. So eventually the state of stress is controlled by the damage while the state of the material is controlled by the degradation of the critical element. The evolution of the remaining strength in the critical element is represented by a “Strength Evolution Integral”.

Macro-mechanical Life Prediction Model
Figure 3: Critical Element Concept
3.3.1 Strength/Damage Evolution Integral

In addition to the Critical Element Approach, Reifsnider and Stinchcomb [1,3-7] also made another assumption to be able to describe the evolution of damage. The physical basis for the development of the Damage Evolution Integral is the concept of Equivalent Damage States. Under this concept the remaining strength of the critical element is given by a non-linear form of the Broutman-Sahu remaining strength equation (3.1),

$$ S_r(n) = 1 - \left[ 1 - \frac{S_a}{S_u} \right]^{n/N} \tag{3.6} $$

where $S_a$ is the constant applied stress, $S_u$ is the ultimate strength and $n/N$ is the life fraction and $j$ is a material parameter that is associated with the shape of the residual strength curve. Equation 3.6 is often represented in the form

$$ Fr = 1 - (1 - Fr)a \tau^j \tag{3.7} $$

where $Fr$ is the normalized remaining strength, $Fa$ is called the failure function, and $\tau$ is the normalized life $n/N$. So a family of remaining strength curves, one for each load level, for a critical element can be visualized as shown in Figure 4. However, the local stress in the critical element is not likely to remain constant in a system undergoing damage and to accommodate this change in local stress of the critical element the Principle of Equivalent Damage is invoked: if at different failure functions (stress states) and different generalized times the material has the same remaining strength then the damage in it is the same. This is shown in Figure 4 as the Line of Equivalent Damage.
$F_r(n) = 1 - \left[ 1 - \frac{S_a}{S_L} \right] \left[ \frac{n}{N} \right]^i$

Figure 4: Line of Equivalent Damage
Figure 5: Principle of Equivalent Damage (After Halverson [78])

\[ F_r(n) = 1 - \left[ 1 - \frac{S_a}{S_L} \right]^n \]

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This line shows that for the same level of damage there are unique pairs of failure functions and generalized times \((Fa_i, \tau_i)\). Putting it differently a material under an applied load \(Fa_1\) for “time” \(\tau_1\) will have a remaining strength of \(Fr\) curve according to equation (15) and represented by \(Fr_1\) which is equal to the same material under load \(Fa_2\) for time \(\tau_2\). Now if a change in load (maybe due to damage) causes the failure function to change to \(Fa_2\) then if the remaining strength were to remain the same the characteristic time will no longer be \(\tau_1\) but \(\tau_2\). The reduction in remaining strength then proceeds along the \(Fr_2\) curve as illustrated in Figure 5.

At this load the remaining strength curve follows \(Fr_2\), whose slope is different from \(Fr_1\). Should the load change again the new remaining strength curve would have the slope of \(Fr_3\). As this continues the cumulative remaining strength curve represents a combination of the remaining strengths from the previous load levels. While in the example used, the Cumulative Remaining Strength Curve may seem to be discontinuous however, if the time and load changes are small then a continuous curve can be attained. The remaining strength equation (Eq. 3.6) of the cumulative curve can then be represented as

\[
Fr(n) = 1 - \int_0^n \left[ 1 - \frac{S_a(n)}{S_u} \right] j^{-1} d\left(\frac{n}{N(n)}\right) 
\]

(3.8)

where the integrand is in effect a summation over the sequential loads experienced by the critical element. Recognizing in the above equation that the \(S_a(n)/S_u\) term is in effect a
one-dimensional maximum stress criterion; they extended the remaining strength equation to a three-dimensional form by rewriting (3.6) as:

\[ F_r(n) = 1 - \Delta S(n) \quad (3.9) \]

where:

\[ \Delta S(n) = \int_0^n \left[ 1 - F_a(n) \right] \left[ \frac{n}{N(n)} \right]^{1-1} \, d\left( \frac{n}{N(n)} \right) \quad (3.10) \]

where \( F_a \) can be any failure criterion. The basis of this transformation is that there are equal damage states and that the contribution of the various non-linear mechanisms can be integrated at small time steps by using a linear superposition approach, which forms the basis for the simulation of this model. The choice of \( n/N \) is chosen as it is a continuous function even when the applied loading spectrum varies with time so equation (3.8) can be used to evaluate the damage accumulation during spectrum loading.

Recently Reifsnider et al. [79] presented arguments to show that Equation 3.9 could be derived from thermodynamic considerations.

### 3.4 Interpretation of Experimental Data

In chapter 2 experimental evidence of various damage mechanisms observed in ceramic matrix composites under different loading conditions was presented. Over the years, Reifsnider and Stinchcomb [1,3-7] have shown that if adequate information
regarding the various damage mechanisms is available then micromechanics can be used to determine the local state of stress and strength at any point in the material. During early stages of material development, this is often not the case. Insufficient knowledge of the response of material system at the micro-level excludes the possibility of using micromechanics and so macro-mechanics is the obvious approach. However, if the failure modes are sufficiently well understood from experimental data, then macroscopic analysis can suitably predict the remaining strength in the material [7]. Since the tensile mode of failure, as a result of fiber fracture, i.e. the tensile rupture of plies of a multi-axial laminate, is the most common form of failure and is commonly used as the criterion for failure in the remaining strength model. To evaluate the local state of stress in the material, models that accurately represent the external response of the lamina as a function of some forcing function need to be developed. In the following sections a literature survey of existing models that serve as inputs to such Macroscopic life prediction methodologies will be presented.

3.4.1 Modeling of Cyclic Effects

The inability to mechanistically model the interaction of the complex damage mechanisms, that occur in the composites due to cyclic loading, has led to the development of many phenomenological damage models. Poursartip et al. [80] allowed for damage accumulation by relating the change in damage with cycles to the present state of damage, in the form shown below:

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\[
\frac{dD}{dN} = f(\Delta \sigma, D) \tag{3.11}
\]

where \(D\) is the damage parameter, \(N\) is cycles and \(\Delta \sigma\) is the stress range. Suggesting that to use cycles as an independent variable in a cumulative damage model implies an empirical basis, Hwang and Han [81] then proposed three models based on fatigue modulus and failure strain while Ye [82] used the analogy of the Paris equation to develop an equation for damage accumulation not unlike the one developed by Poursartip et al. [80] where the rate of damage is related to the damage itself. In a departure from the damage accumulation approach, Ramakrishnan and Jayaraman [83] represented the stiffness loss of unidirectional laminates by using a combined phenomenological and micromechanics approach. They used a combination of logarithmic and linear decay functions of cycles associated with stiffness loss to represent the damage process. This damage evolution model is a combined representation of the three damage processes that result in fatigue failure: matrix cracking, fiber-matrix debonding and finally fiber fracture. The total stiffness drop as a function of fatigue cycles is described by an equation of the form:

\[
\frac{E(N)}{E_c} = 1 - A\left\{(1 - f)\ln(N + 1) + fN\right\} - B\ln\left(1 - \frac{N}{N_f}\right) \tag{3.12}
\]

where \(E_c\) is the modulus of the laminate, \(N_f\) is fatigue life, and \(A, B\) and \(f\) are constants that are related to the constituent properties using the rule of mixtures.

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3.4.2 Modeling of Time Effects

From the literature, the rupture of ceramic matrix composites is seen to be strongly dependent on the time held at temperature and very mildly on stress level. This is in contrast to cyclic effects which openly manifest themselves at early stages of loading in the form of reduction in stiffness, time effects are often not easily detectable and may occur at delayed times. This delay again is largely dependent on the matrix micro-cracking which then exposes the fibers to the aggressive environment. Few studies have been conducted to determine the rupture time of ceramic matrix composites under load at elevated temperatures. Ordinary creep rupture tests are also not easy to come by in the literature [84]. Of the data that has been collected on whisker reinforced ceramic composites the common approach to representing creep rupture is to use the Monkman-Grant [85] form, with an Arrhenius modified Norton-Bailey equation, for time-to-failure as:

\[ t_f = CD^{-m} \sigma_{app}^{-mn} \exp \left( \frac{mQ_c}{RT} \right) \]  \hspace{1cm} (3.13)

where \( \sigma_{app} \) is the constant applied stress, \( Q_c \) is the apparent activation energy for creep, \( R \) is the universal gas constant, \( T \) is temperature in °K and \( C, D, m, n \) are the empirical material constants. In this technique the creep rate of the material is plotted as a linear function of creep rupture time on log-log scale. It is assumed that if all the data fall on the same line for a range of temperatures then there is a common mechanism causing

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rupture. This is most often not the case when at higher temperatures the mechanism may change from crack growth to creep or vice versa and so other models must be considered.

3.5 Life Prediction of a Ceramic Matrix Composite Component

In Section 2 experimental evidence of the complex mechanisms that contribute to the failure of Ceramic Matrix Composites was presented. It is evident that these mechanisms interact in a complex manner particularly when exposed to high temperatures in an oxidative environment. Most of the data presented related to specimens of simple geometry. It is well known that the local state of stress in a specimen is strongly influenced by its geometry. Most real life components have complex geometry and so under even simple mechanical loading will experience a large variations in local stresses. As mentioned in Section 3.3 these variations in local stress are responsible for eventually controlling the life of the component under time-dependent loading. Thus to predict the life of components that have complex geometry a life prediction tool has to be sensitive to the interaction of the various local stresses. The strength-degradation model developed by Reifsnider and Stinchcomb [1,3-7], and described above, has been incorporated into a Turbo-Pascal PC-based computer code called MRLife®. In its present form, this code monitors the remaining strength and predicts life of a component based on local point stresses in the material with the interaction between the various points, due to the geometry, being neglected. With each point being at a different stress and temperature the failure time
of each point is going to be different. The cumulative effect of sequential failure of the different points is referred to as progressive failure. While MRLife® performs progressive failure of a point extending its application to the component level requires an iterative process to simulate progressive failure of the component.

To perform this progressive failure analysis and determine the remaining strength and life of ceramic matrix composite components MRLife® has been adapted into a form called CCLife, which has been integrated with a finite element program, CSTEM [11] with the objective of creating an integrated design tool for ceramic matrix composites. In its present form the integrated design tool still requires human interaction but it is conceivable that this loop can be closed by creating a "batch" file. The development of the integrated design tool for the analysis of a ceramic matrix composite component consists of two major efforts. The first is to modify the finite element code so as to account for damage in the material at the macro-level, while the second effort involves the use of the life prediction code, which for the present effort is called CCLife to account for the effect of the damage mechanisms on the remaining strength and life of the component. By integrating the two well established analytic tools into a single integrated design package a design tool that is capable of accounting for the effects of complex geometry, loading and local changes in material is developed. In this new design tool CSTEM becomes capable of performing stress analyses on a complex structural component that has undergone some damage and CCLife complements it by being able to predict the remaining strength and life at a point under the influence of complex loading and environment.
3.5.1 Design Tool - Overview

The interaction between the various modules of the integrated design tool for CMC's is shown in the flow chart, Figure 6. The basic design of a component is often carried out on a Computer Aided Design (CAD) package that serves as a pre-processor which for this case is PATRAN 2.5 [86]. At this stage the component geometry is defined and meshed and other data such as the temperature distribution, mechanical loads and boundary conditions are input as required. Having designed and meshed the component a "neutral" (ASCII) file is then created by the pre-processor that is then converted into a CSTEM input file format using PASTEM [87]. PASTEM is an interactive package that prompts the user for the kind of analysis that is to be performed by the finite element code such as static, dynamic, thermal, large deflection etc., material properties, and output format. For additional details one may consult Ref. 87. After running PASTEM the input deck is again edited to include the number of cycle/time steps and the number of cycles/time to be run at each step. This may not be necessary in a newer version of PASTEM. In addition the user is required to include the damage parameters to be used in CSTEM. Here, some user discretion is required as this involves the cycle intervals over which CCLife is to be run. The effect of using a number of cycle steps is that CSTEM incorporates the total damage over the big step in small increments. For example if the stresses after 25 cycles of loading are required then the input deck will
Figure 6: Flow Chart of CSTEM - CCLife Integration
be edited to include 5 steps of 5 cycles each step. This is to prevent excessive damage occurring at any one cycle and causing instability in the FEM code. The stiffness of the material is appropriately changed after each cycle step. It must be obvious by now that performing the FEM stress analysis at each single cycle or time step would be prohibitively expensive however, so the use of many cycle or time steps may result in incorrect solutions. Reasonable cycle/time steps must be chosen such that the solution is sensitive to the local changes in stress due to the damage and also to the other non-linear degradation mechanisms that are considered in the life prediction module. Prior research has shown that in CMC’s all the matrix cracking induced damage occurs during the first 100 cycles so initially small steps may need to be used so that the FEM solution can converge. Once this stage has passed then the user needs to focus on the changes that may occur due to creep or rupture effects that, while not directly effected by the FEM analysis, nevertheless are strongly dependent on time. If high local loads resulting in short life are expected then small steps may be used else they can be appropriately increased. It must be understood that the strength degradation integral described in Section 3.3.1 is strongly non-linear and so the shorter the time step the more likely a continuous curve will result. There is of course a minimum cycle/time step that adequately represents the data below which no further accuracy can be obtained. After the input deck has been appropriately edited CSTEM is run. CSTEM was run on a SUN 3/60 workstation. The resulting stresses from the FEM analysis provided us with the stress distribution in the component after the required number of cycles have elapsed.
The stress output file from the CSTEM is then input into the CCLife program to evaluate the remaining strength and life. If the remaining strength at some element is found to fall below the applied stress then the element point is deemed to fail and its stiffness should be appropriately changed to reflect this and is discussed in the next section.

In this type of analysis, the input loading spectrum is simplified so that the load profiles are linear as complexities associated with stress range effects are not considered in this section. From the stress analyses the stress distribution in the component at the peak load of the cycle is obtained and then CCLife simulation is performed to get the degradation of the material. At this initial stage the intent however, is to initially develop a simple integrated tool for the design of CMC components that is based on non-linear elastic analysis where the complex stress state can be determined by the finite element model. As yet there is not any well established way of determining global failure of the component. Approaches range from probabilistic to deterministic criteria [88]. As it is not the intent of this study to focus on a component level failure criterion for convenience a simple averaging technique will be used. The validity of this tool will be checked by performing a case study using data obtained on notched plates under fatigue loading.

3.5.2 Application of Damage Mechanics

When developing an integrated design tool for ceramic matrix composites serious consideration must be given to the existence of notches or stress concentrators in the component. Earlier work on the effect of notches on CMC’s has shown that most such
materials are largely notch insensitive, even for notches of 5 mm length [89]. The notch insensitivity that was found to arise, despite the existence small plastic strains, was associated with the redistribution of stresses due to the damage in the vicinity of the notches. In some class of materials the notch insensitivity was attributed to the damage due to fiber failure and in others due to matrix cracking. In addition work on materials such as Nicalon-LAS/CAS/MLAS and Nicalon-Silicon Carbide [19-24,37-45] has shown that the damage under cyclic loading is a function of the peak stress. In these materials LEFM is violated and so some form of damage mechanics is to be used. When a micro-mechanics approach is used the damage is often analyzed using bridging mechanics along with continuum damage mechanics [8]. However for a macro-mechanical approach only the latter is used as simple phenomenological expressions are used to represent the damage. Whatever the approach the influence of the damage on global stiffness must be taken into consideration. In this design tool simple methods will be used to accommodate the degradation of the matrix and the fibers during the life of the component.

The integrated design tool comprises of two components, a finite element software package (CSTEM) and the life prediction code (CCLife). In the former the material response is evaluated at the laminate/lamina level while in the latter the material evaluated at the lamina level or lower which will be elaborated on later. The damage that occurs in the material does so at the micro-level and the response to it is different at each level. At the laminate level it causes, due to stiffness change, a redistribution of the stresses within the component while at the lamina level it causes change in the distribution of the stresses

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between the fiber and the matrix and so influencing life. Damage in CMC’s has been shown to increase as a function of the stress level, and eventually plateaus when the characteristic damage state is reached, so with the locally varying stress state in the presence of a notch the damage too will be spatially distributed. Though it is not of primary concern to us the manner in which CSTEM evaluates the effect of damage in the material will be briefly discussed.

In the finite element code, CSTEM, the evolution of damage in the material is represented by a change in stiffness at each integration point of each element. Here the change in stiffness, i.e. the damage in the material, can be made to follow some user defined law with the help of what CSTEM defines as “user hooks”. These “user hooks” are user defined sub-routines that can be utilized by the finite element analyses code to perform material degradation [11]. This may take the form of damage, creep, rupture effects, or any other time-dependent degradation process. The convenience of such “hooks” is that the user has to only worry about the transfer of local variables between the call command and the subroutine in which the damage law is defined. There are two common approaches to handling material non-linearity in finite element analyses, the tangent stiffness and initial stiffness methods. In CSTEM the latter, referred to as the Right Hand Side method, is used as it is more efficient from a computational stand-point. In this technique, rather than changing the stiffness matrix on the “left-hand-side”, equilibrium is obtained by using a “fictitious force” on the right-hand-side and iteratively changing it to converge within a specified tolerance to the exact solution [90]. As expected, in using this technique,
limitations on the length of the time-step and the order of convergence were experienced. Since the "hooks" were already incorporated in the original code, to include our damage law into the finite element package a subroutine which passed the damage scalar for reduction of the stiffness matrix back to the main program had to be added. It should also be mentioned that old values of the damage were retained so that the damage is not recovered in the event of a reduction in local stress after each cycle/time step.

While the earlier discussion focused on the influence of damage on the redistribution of stresses at the laminate level, here the focus will be on the influence of damage on failure time. In Section 3.3 the representative volume element (RVE) was defined as that element which had the same constitutive law as the laminate. The local stresses in the component can thus be thought of as acting on the RVE. So now the redistribution of stress within the RVE due to the accumulated damage has to be considered. The RVE is composed of a critical and sub-critical elements. While the actual task of assigning the critical element is largely dependent on the failure mode, in general, the material hierarchy is such that damage to the RVE results in the transfer of load from the sub-critical element to the critical element, so for now the failure mode will be ignored. The critical element does not undergo any damage so the local stress in it monotonically increases. Unlike PMC's in which the $0^\circ$ ply (often the critical element) remains undamaged upon loading, the same does not hold good for in CMC's. Even the $0^\circ$ plies experience matrix cracking in all CMC's. So definition of the critical element in a macro-level analysis for CMC's is a little dubious. If the critical element is to remain undamaged

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during loading then a fictitious element below that of the lamina level has to be identified. In CMC’s, the fibers are the largest component that do not degrade and so can be defined as the critical element and since they don’t actually exist at the lamina level these “fibers” are fictitious. Then in order to retain the integrity of the critical element it is assumed that all the damage that has been observed at the lamina level takes place only in the sub-critical element, i.e. in the fictitious “matrix”. From Rule of Mixtures the modulus of the composite is given by

\[ E_c = fE_f + (1 - f)E_m \]  \hspace{1cm} (3.14)

where \( E_c \) is the modulus of the 0° lamina, \( f \) is the fiber volume fraction and subscripts \( f \) and \( m \) refer to the fiber and matrix respectively. The change in stiffness at the laminate level for a unidirectional material as observed in experiments is given by

\[ \frac{E(n, \sigma)}{E_c} = D(n, \sigma) \]  \hspace{1cm} (3.15)

where \( D \) is a damage scalar which is a function of applied stress and cycles. Now assuming that all the damage observed at the lamina level is attributed to the “fictitious matrix” then Equation 3.14 can be written as

\[ E(n, \sigma) = fE_f + D_m(1 - f)E_m \]  \hspace{1cm} (3.16)

where \( D_m \) is the matrix damage function is given by

\[ D_m = \frac{D(n, \sigma)(fE_f + (1 - f)E_m) - fE_f}{(1 - f)E_m}. \]  \hspace{1cm} (3.17)

In this way the local stresses in the critical element can be evaluated.
The model being based on progressive failure, it is possible for a finite number of elements to fail without any catastrophic effects occurring at the component level. One of the concerns is the need to identify and resolve the effect of failure of a point in an element. Physically, the reduction in remaining strength at a point in an element can be associated with "fiber failure" and so causes a reduction in the stiffness of the local point. There are many approaches to this problem with no one method having a dominant role. In one technique proposed by Gao and Zhao [91] for first ply failure they postulate that the stress in the "failed" ply is limited to its strength value, to maintain this value any increase in global load will result in a reduction of modulus of the failed ply. So the reduction in stiffness, after a point has "failed" or lost its remaining strength has to be changed. This procedure is fairly complex and performing it for many elements may be time consuming. Another approach is to reduce the stiffness by some different damage law and a third is to use a plastic type analysis in which the stiffness is reduced to some minimal value. This third approach has previously been successfully used for lamina failure in laminate analysis and so will be used for this model. Since the possibility of singularity occurring in the finite element analysis due to large stiffness gradients in neighboring elemental points stiffness of a failed point is reduced to 20% of its original value which is lower than that measured during quasi-static test. This value of stiffness loss at a point was settled at because when a stiffness loss of 15% of the original value was used convergence could not be obtained in the finite element analysis.

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3.5.3 Case Study - Life Prediction of Notched Plate

The application of the integrated design tool described above will be demonstrated by performing a case study of a notched plate with a thermal gradient under cyclic loading. This model was chosen for comparison with the analytic predictions because of the local variation in stresses, the existence of a thermal gradient, and the last but not least of all, the availability of experimental data for such specimen geometries. The specimen is a 3” L x 1.5” W plate with a 0.75” diameter center hole. The dimensions of the specimen and the global coordinate system are shown in Figure 7. The plate consists of 8 plies of 2-D woven ceramic matrix composite having ply thickness of 0.01175”, giving a total laminate thickness of 0.094”.

The finite element model was generated by assuming symmetry about the x and y axes and is shown in Figure 8. Displacement constraints were imposed at x = 0 and y = 0, normal to the axes of symmetry and two nodes were constrained in the z direction to resist out-of-plane bending. The displacement boundary conditions imposed on the model are also shown in Figure 8. The specimen tested was held in “cold grips” and so the temperature distribution within the specimen is also shown in Figure 8. Remaining strength analysis was performed at axial loads of 10, 8.75, 7.5 and 5 ksi, at 1Hz and R = 0.05.

3.5.3.1 Material Properties and Failure Criteria

The lamina is made up of a 2-D woven CVD processed Nicalon/Enhanced-Silicon Carbide (E-SiC) composite produced by DuPont Lanxide Corporation (DLC) [92]. For
Figure 7: Case Study Specimen Geometry
Temperature Distribution: $T(y) = 1825 - 25.0 \exp(2.75y)$

Figure 8: Finite Element Mesh and Boundary Conditions for Notched Plate
the finite element analysis the material properties, for convenience are "homogenized" within the laminate and the specific fiber geometry ignored. Most of the properties used in the analysis and are shown in Table 1. The best available estimates for the transverse properties were used.

Table 1: Mechanical Properties of 2-D Woven CVD Nicalon/Enhanced Silicon Carbide [92]

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<th>E_{33} Msi</th>
<th>ν_{12}</th>
<th>ν_{13}=ν_{23}</th>
<th>G_{12} Msi</th>
<th>G_{13}=G_{23} Msi</th>
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</tbody>
</table>

Though the material is “homogenized” at the macro-level (lamina), for modeling purposes a well accepted assumption is that the damage in a CMC is generally restricted to the matrix [12] resulting in the load being shed onto the fibers. Here too this assumption is invoked and in CCLife each layer of material from CSTEM is modeled as a laminate consisting of one layer of fibers and another of matrix bonded together. Using CLT and the assumption that the damage is restricted to the matrix layer, the increase in the local state of stress in the fibers is calculated. The fibers thus constitute the “critical element” in CCLife. For a ceramic matrix composite component to undergo catastrophic failure, the failure of the critical element caused by fiber breakage must occur so the one-dimensional maximum stress in the fibers is the failure criterion. In the event of a multi-axial stress state being a source of concern, the Tsai-Hill failure criterion, for multi-axial stress states has been also

Macro-mechanical Life Prediction Model
incorporated into the code. It is expected that the results from the latter criterion would provide a lower bound for the failure envelope.

The failure criteria mentioned above are point load criteria. There is no well defined criterion for determining the failure of a component that undergoes progressive failure. For notched laminated plate the use of the Whitney-Nuismer [93] average stress criterion

\[
\sigma_o = \frac{1}{d_o} \int_a^{a+d_o} \sigma_f(x,0) dx
\]  

(3.18)

where \(\sigma_o\) is the unnotched strength of the lamina, \(a\) is the hole radius, \(\sigma_f(x,0)\) is the stress at the hole axis and \(d_o\) is the characteristic distance and is the region over which material failure is evaluated and has been shown to provide accurate results [94]. Such a criterion is particularly compatible with the residual strength methodology because the remaining strength is evaluated at each point. If the normalized remaining strength over the characteristic distance falls below the failure function then the component is deemed to have failed. Using equation (3.18) and quasi-static test data on notched plates of the geometry shown in Figure 7 the characteristic distance was found to be 0.0878 in which is approximately equal to one laminate thickness. The criterion used to determine failure of the notched plate will be a modification of the average stress criterion which is failure will have occurred when the average remaining strength falls below the average stress over the characteristic distance.

Macro-mechanical Life Prediction Model
3.5.3.2 Rate Equations

The strength degradation integral in CCLife is very sensitive to representation of the damage mechanisms. The evolution of these damage mechanisms is often referred to as "rate equations". These equations are extremely sensitive to the conditions under which the data were collected. In this section the manner in which the rate equations for the above material are represented is described. Most rate equations fall under one of the two categories; time or cycle dependent. Data used for these rate equations may be obtained from individually performed experiments, experimental data from colleagues, or from the literature.

Cyclic Effects - Fatigue Stiffness Change: CMC's are known to exhibit a larger degree of matrix cracking when subject to a fluctuating load than for an equivalent monotonic loading [19,20]. For this reason stiffness loss due to fatigue is referred to as a cyclic effect. Unlike other composite systems, CMC's experience matrix cracking in not only off axis but also 0° plies. Research on CMC's has shown that the matrix crack density is not only a function of cycles but also depends on the stress level [40]. The effect of temperature on crack density or matrix cracking for this material has not been investigated though observations on other material has shown little influence. For lack of any evidence it is assumed that there is no thermal influence on stiffness loss in this Nicaion/Enhanced-SiC (E-SiC) material. The change in stiffness measured at the laminate level in the laboratory is the same as the variable $D(n,\sigma)$ defined in equation
(3.15) and is cycle and stress dependent and is used to re-distribute the load between the “matrix” (sub-critical element) and the “fiber” (critical element).

Data used in determining the loss in stiffness as a function of cycles was obtained from Ref. 95. In this data hysteresis loops of cyclic loading are represented as bi-linear curves for different load levels. In developing the stiffness loss equation curves are fit to the secant modulus data and in addition it is assumed that no stiffness loss occurs under cyclic loading at stress levels below that of matrix cracking (11-12 ksi). However, no data is available on the load level at which the fatigue damage saturates. For this version of the code it is assumed that the secant modulus saturates at a load of 25 ksi. In keeping with other observations and the data provided stiffness loss is represented as a bilinear curve. This form is used as it implies that most of the damage occurs during the first cycle and only moderate damage growth takes place on subsequent cycling. The change in stiffness is represented by the following equation:

\[ \frac{E(n)}{E_0} = \left( P + Q\sigma + R\sigma^2 \right) + \left( P1 + Q1\sigma + R1\sigma^2 \right) \log(n) \quad (3.19) \]

where: \( E_0 = \text{modulus at room temperature} \),

\[ P = 1.73745 \; Q = -2.93503 \; R = 1.60153; \]

\[ P1 = 0.09466 \; Q1 = -0.41931 \; R1 = 0.34720; \]

A comparison of the analytical representation of the stiffness loss versus the experimental data, as a function of cycles for the model material, is shown in Figure 9.
Figure 9: Stiffness Reduction of 2-D Woven Nicalon/E-SiC Composite at 1800°F
**Time Effects - Creep Rupture:** The creep rupture life of a material is a function of the applied stress and duration of exposure at an elevated temperature and so is referred as a time effect. Creep rupture life is very sensitive to the environment under which the tests are performed. Oxidative environments, in particular, are known to be extremely harmful. Creep rupture test data available on Nicalon/E-SiC is extremely limited as it is expensive to obtain and so may not be available from the same source. The data used in the development of the rate equation for creep rupture were obtained form the following sources, 1800°F data [96], 2010°F data [97]. In addition to these two sets of creep rupture data, data from tests carried out at fixed stress level and temperatures ranging from 925-2000°F from Ref. 98 were also made available to us.

The numerical representation of the creep rupture data was performed by first fitting the 2010°F stress-time data with a log time-stress function. A comparison between the analytical representation and experimental data at two temperatures is shown in Figure 10. Then to develop temperature shift parameters for the stress-time equation the 12 ksi data is used. The data seemed to be symmetric about a $T_{ref}$ which corresponded to the "pest" temperature. The data is also fit with a log time-temperature curve. A comparison between the analytical representation and experimental data at 12 ksi is shown in Figure 11. Finally the failure time as a function of stress and temperature is represented as:

$$\ln(\tau) = \left[ (A + B\sigma + C\sigma^2) + (D + ET_{\text{red}} + FT_{\text{red}}^2) \right]$$  \hspace{1cm} (3.19)
Figure 10: Stress Rupture Data for 2-D Woven Nicalon/E-SiC Composite
Figure 11: Rupture Life vs. Temperature of 2-D Woven Nicalon/E-SiC Composite at 12 ksi
where: \[ T_{red} = \frac{T - T_{pext}}{T_{pext}} \]

\[ T = \text{temperature } ^\circ F. \]

\[ A = 17.95038 \quad B = -18.98970 \quad C = 0.0000; \]

\[ D = -0.611191 \quad E = 0.7337560 \quad F = 18.94490; \quad T_{pext} = 1550 \]

Since the creep-rupture data were obtained from experiments conducted in air, the above expression is used to represent creep-rupture is, in actuality, a representation of the combined effects of creep and oxidation. It must be reiterated, that due to the nature of the mathematical functions used to fit the observed phenomenological behavior, extrapolation of the data is not recommended.

\textbf{S-N Curve:} The S-N curve is the most common rate equation. In CCLife the S-N curve is representation of the life of the critical element under current conditions of stress state and strength state if those conditions were to remain constant. A typical example of such a curve would be the representation of the S-N fatigue strength-life equation for constant amplitude loading fatigue of unidirectional material loaded in the fiber direction at room temperature. Since the model material is of 2-D woven fiber architecture for which no data on the strength loss due to fatigue is available, a nearly flat S-N curve is assumed. The values of the parameters were based on typical S-N curves for this material. The form of the S-N curve used in CCLife is as shown:
\[
\frac{S_d(n)}{S_u(n)} = An - Bn(\log N)^{Pn} \tag{3.20}
\]

where: \( S_d(n) = \text{is the local stress state} \)

\( S_u(n) = \text{is the local strength} \)

\( An = 1; Bn = 0.025, Pn = 1 \)

3.5.3.3 Results

To validate the use of the integrated design tool for ceramic matrix composites in this case study the model is tested at four load levels, 10, 8.75, 7.5, and 5 ksi. These loads were chosen as a compromise between computational time and validating the feasibility of the tool. In performing the analysis, output files are obtained from both the CSTEM and CCLife runs. The results from CSTEM are formatted such that the stresses at the nodal points can be visualized on PATRAN and so the CCLife format had to be compatible with this form. Output from CSTEM consists of stresses, temperature and cycle at each node and integration point of each element while the CCLife output consists of the failure function and remaining strength at each of the same points. The integration point results are used for further analysis, i.e. account for failed material, while the nodal results are used for visual evaluation and also to evaluate the failure condition of the component.

The traditional approach in using finite element analysis is to first develop the model and validate it using a simple load case. Since the case study consists of combined thermal and mechanical loading, to validate the model the mechanical and thermal loads were applied separately. The objective in this case being to validate the response due to the Macro-mechanical Life Prediction Model
mechanical load by comparing the results with some analytic model. The results on the stress analysis, due to a 10 ksi axial load and the thermal distribution specified in Figure 8 are presented in Figure 12a-12b. The results of the finite element model shown in Figure 12a were then compared with an analytic model based on Lekhnitskii's model for anisotropic notched plates. The gross stress concentration factor \( (K_{fg}) \) from the finite element analysis is 5.330 as compared to that of 3.119 based on the Lekhnitskii model [99] and using the properties shown in Table 1.

The difference can be attributed to the finite width and length of the model which are not considered in the Lekhnitskii analysis. The closing in of the boundary conditions is known to increase the stress concentration effect. Additional validation was performed by comparing the model with data for finite length and width quasi-isotropic polymeric matrix composite plates presented by Hong and Crews [100]. For a \( L/w = 2 \) and \( d/w = 0.5 \) Hong and Crews obtained a net stress concentration factor \( (K_{fn}) \) of 2.193 for stress boundary conditions as compared to value of 2.33 obtained here. The disparity here may be due to the difference in properties used. Based on these validations the model seems to accurately represent the state of stress in the material. Other factors such a coarseness of mesh could have some impact on the \( K_f \) values but since the primary objective of this model is to predict life within finite time, the coarse mesh is not in anyway detrimental to the effort.

The stress distribution in the two Figures 12a-12b seem to complement each other in the vicinity of the hole. In Figure 12a the stress concentration at the hole due to the axial load is "balanced" by the compressive stress generated by the thermal gradient as shown in
Figure 12b. The effect of the combined mechanical and thermal stresses is shown in Figure 13a. The thermal stresses reduce the stress concentration at the notch from \( K_{in} = 2.66 \) to 2.00 though in other regions the effect is the opposite. The stress distribution shown in Figure 13a is also the state of stress in the component prior to cycling of the load. The redistribution of stress due to accumulation of damage with cycles can be seen in Figure 13b-13d. In Figure 13b the stresses in the notched plate under an axial load of 10 ksi after 25 cycles of operation are shown. In comparing Figures 13a and 13b the relaxation of stresses in the vicinity of the hole is quite evident. Relaxation is also seen to occur at the edge of the plate which is also under a high stress due to the combined thermal and mechanical loads. When comparing the stresses in Figure 13b with those in Figures 13c and 13d, it can be seen that most of the relaxation occurs during the first 25 cycles subsequent to which the relaxation is very moderate. It is evident that the stress concentration effect is further reduced by the relaxation of the stresses due to damage which contributes to the notch insensitivity of these materials.

As can be expected in such analytical studies, tremendous amount of data is to be stored, verified post-processed, and presented. The best manner in which such information can be presented is by using fringe plots. Accordingly fringe plots have been used in this study to present the results obtained. Some of the results obtained from CCLife were post-processed using PATRAN 2.5 and are shown in Figures 14a-14d. The objective with these plots is to show the evolution of remaining strength in the material due to the load on the notched plate. At each load one remaining strength distribution plot is shown when loss in
Macro-mechanical Life Prediction Model
Figure 14a: Remaining Strength in Notched Plate
LOAD - 7.5 ksi + Thermal
Cycles - 100

Figure 14b: Remaining Strength in Notched Plate
LOAD - 7.5 ksi + Thermal
Cycles - 2000
Figure 14c: Remaining Strength in Notched Plate
LOAD - 10 ksi + Thermal
Cycles - 100

Figure 14d: Remaining Strength in Notched Plate
LOAD - 10 ksi + Thermal
Cycles - 200
remaining strength is first experienced in the material and the other the remaining strength distribution just prior to failure. It is evident from these plots, based on the average strength criterion, that only a little loss in remaining strength is necessary for catastrophic failure of the component to take place. These materials have small a characteristic distance over which failure need occur before the component will fail. In comparing the remaining strength results at the two different load levels in Figure 14a-14d it is interesting to note that the larger amount of loss in remaining strength can be seen along the hole boundary at the higher load level than at the lower load. Though the characteristic distance over which the remaining strength is averaged is the same at both load levels, at the higher load there is a loss in strength even radially along the hole and not only at the axis.

In addition to the post-processing performed above the failure function $F_L$ and remaining strength at the nodes along the characteristic distance of the hole were averaged using Equation (3.18) at each cycle step run and the results are presented in Figure 15a-15d. The predicted cycles to failure of the specimen at each load level is based on the intersection of the two curves, i.e. when the remaining strength over that distance equals the failure function, as shown in the figures. Finally a summary of the failure times (cycles) obtained at each stress level were compared with experimental data for the same material is presented in Figure 16. Though the analytical predictions and experiments were run at different stress levels there is fairly good correlation between the two. The experimental data was run at relatively low load levels which would require extensive computational time.
Figure 15a: Remaining Strength and Failure Function Curves at $\sigma_{app}=10$ ksi

Figure 15b: Remaining Strength and Failure Function Curves at $\sigma_{app}=8.75$ ksi
Figure 15c: Remaining Strength and Failure Function Curves at $\sigma_{app}=7.5$ ksi

Figure 15d: Remaining Strength and Failure Function Curves at $\sigma_{app}=5$ ksi

Macro-mechanical Life Prediction Model
3.6 Discussion of Modeling Approach

It has been shown, albeit using a simple loading case, that it is possible to put together a stress analysis program and a macro-level life prediction tool to complement each other to produce an integrated design tool. While the results obtained in this modeling exercise, as shown in Figure 16, correlated well with experimental data, it may have been a result of the nature of the material and loading conditions rather than a testimony of the capabilities of the design tool developed. Admittedly, in its present form the design tool is not quite sophisticated or user friendly. It is still an open loop system but this can be corrected quite easily. Making the process fully interactive is complicated by the incompatibility of the analysis component (CSTEM) and the post-processing visual component (PATRAN).

While these two components are not necessarily unchangeable and the choice in using these commercially available tools was not of our choosing. Future developments of such a design tool may need to take this into consideration. Most modern finite element packages however, have self contained visual components and as such have the potential for a more efficient design tool.

One of the main points regarding the analysis procedure that needs discussion is the manner in which the stiffness of a failed integration point performed. In this analysis, the failure of an integration point, i.e. having a remaining strength less that its failure function, resulted in its stiffness being set to 26% of its original value.

Macro-mechanical Life Prediction Model
Figure 16: Stress-Life curve for 2-D Nicalon/E-SiC Composite Notched Plate
An interesting feature observed during the analyses procedure was that this condition did not significantly effect the analysis. The local stress at that failed location, prior to failure, was quite high and so was susceptible to a great amount of damage which would cause the stress to be redistributed from it. So when the stiffness is reduced the stresses in each element are already low which reduces the impact of that action. It is possible that under different circumstances and load conditions the response could be significantly different. But as it was discussed earlier there is no well established procedure for effectively handling failed elements or points. The only failure mode that was considered is fiber stress and when other failure modes are given consideration, particularly matrix dominated failures, this feature may have to be reworked and made more sophisticated.

One of the key elements used in the analysis of high temperature components, particularly if made of laminated material, is the ability to predict failure/damage due to delamination. Delamination of the material can give rise to hot spots that are self propagating. The inclusion of the ability to predict delamination is not a simple procedure because of what was discussed above regarding the influence of failure on the stiffness of the element. Physically what is seen in delamination is the reduction in transverse tensile stiffness of the delaminated interface of the material to practically zero. To truly represent such a material would require the use of fracture mechanics or more sophisticated elements.

Composite elements are known to exhibit scale effects. It would thus seem natural that components made of such material would also be susceptible to scale effects. The
component level failure criterion then, being based on coupon tests, may not be applicable as the component sizes increase. In staying with the macro-mechanical methodology it may be necessary to adopt statistical procedures that are capable of incorporating volume scale effects into the analysis. This way there would be some mathematical correlation between the level of discretization and the strength of the local elements.

No discussion on modeling would be complete without a mention of the computation time involved. The prospect of performing life prediction on only a limited number of crucial elements, determined \textit{a priori} or sometime along the process, seems lucrative. In a component with simple geometry such a system could be written into the procedure however, for more complex geometries and if consideration for delamination and tensile failure is to be given the consequences of wrong judgment could be devastating. However, an intelligent algorithm could be developed that would cut down the run time of a such a design package.

The initial objective of performing this exercise, that is, to show the use of a macro-level methodology to predict the life of a CMC component, has been established though the potential for improvement definitely exists. Such a type of modeling approach can be used effectively to study composite components that will be utilized in a damaged state and the remaining strength at any time can be evaluated. Even though complex loading patterns were not used in this exercise, CCLife can easily be modified to evaluate the remaining strength under more complex conditions, if adequate experimental data is available to develop the necessary rate equations. Finally it is well known that most failure modes in

\textbf{Macro-mechanical Life Prediction Model}
composites are the result of micro-level mechanisms. The use of macro-level methodology is only partly capable of incorporating all the subtle influences that occur at the macro-level. This methodology is strongly dependent on the test procedure and operator to determine failure modes and criteria and, with the rapidly changing material development schedules and complex loading sequences desired, these are hard to accurately establish.
4. MICRO-MECHANICAL MODELING

4.1 Introduction

In Chapter 3 a case study was used to show the application of a remaining strength life prediction methodology towards the design of a structural component. The failure of the composite material was predicted in terms of an isotropic failure criterion, at the lamina level. This criterion is an empirical representation of the failure mechanisms that originate at the micro-level and are extremely complex [8]. Also, such failure criteria encompass the sequence of many micro-failure events that, cumulatively, are responsible for the global failure of a component. Rate equations used in simulating the evolution of material properties were obtained from simple laboratory based experiments and were developed phenomenologically. These rate equations represent the growth of the damage mechanisms and are strongly dependent on the load transfer between the fiber, matrix and interface. For this reason, macro-level empirical failure criteria cannot adequately address the subtleties associated with some of the more complex damage phenomena. In addition the equations are only valid for the material actually tested, small changes in the material would require re-characterization of the material to obtain new rate equations for future life predictions. These issues highlight the need for representing the degradation mechanisms present in terms of their constituent properties which is studied under the domain of micro-mechanical modeling. The inter-
relation between that macro-mechanical and micro-mechanical approaches, with regards to component design, is represented in Figure 17. In this representation the key to developing a micro-mechanical methodology that can be used for component design is to be able to develop rate equations for the critical element (lamina or micro-volume) that is dependent only on the constituent properties and degradation mechanisms. These micro-mechanical rate equations should be capable of representing the global response of the composite to the local mechanisms.

While an engineer may not feel the need to possess a working knowledge of the physics of failure that takes place within the microstructure of a ceramic composite, the material designer would like to study the impact of minute changes in the material on component strength and life. Micro-mechanical modeling of degradation mechanisms fundamentally consists of three complex tasks: defining the physics of the process, solving the associated boundary value problem (BVP) and finally relating the local problem to macro-scale (lamina) response. While the first two seem to be relatively attainable tasks limited by ones ability to define and solve the unit cell problem for the given mechanism using the laws of physics, the third requires translating the mechanistic problem into a stochastic one (Figure 18a). It is obvious that the direct mathematical approach to obtaining a closed-form solution to the micro-mechanical problem is mathematically complex and can be prohibitively difficult. An alternate approach is to develop a micro-mechanical closed form solution for a degradation process by using the approach schematically represented in Figure 18b. Here the numerical models take the
MACRO-MECHANICS  |  MICRO-MECHANICS

Figure 17: Relationship Between Macro- and Micro-mechanics in Integrated Design Code
Figure 18a: Relationship between Unit-cell Model and Composite

Boundary Value Problem \rightarrow Numerical Model

\rightarrow Analytical Model Properties \rightarrow Remaining Strength/Life vs. Constituent

Figure 18b: Relationship between Unit-cell Model and Composite
form of simulation techniques, the results of which can be used to fully understand the
macro-level response to the micro-level mechanisms. This technique is similar to
performing an analytical experiment under controlled conditions and can be used to
simulate failure to carefully study the effect of a degradation process or mechanism.
Results obtained from such numerical schemes can then be used to create or guide
analytic solutions for the specific degradation process.

Before any attempt is made to create a micro-mechanical model for a composite
material, a good understanding of the failure mechanisms at the constituent level is
required. The basic element of a composite towards which the study of failure
mechanisms is focused is the unidirectional 0° lamina. Coincidentally (or not), it is often
also the critical element in the macro-mechanical life prediction methodology described
in Chapter 3. It has been shown that the failure of the 0° ply in ceramic matrix
composites is almost always controlled by fiber failure with the matrix, by then, having
shed all its load onto the fibers [42,101]. In most materials two kinds of failure have been
observed; fast-fracture and time-dependent failure. The former controls what is
commonly known as ultimate strength, a fundamental material property determined by
quasi-static loading. Under the domain of time-dependent failure are the degradation
processes such as fatigue, creep (matrix and fiber) and fiber degradation. With regard to
Chapter 2, where oxidation-embrittlement was shown to be a major concern, that
mechanism is being ignored because of its chemical origin; the focus here being on
physical processes. Since the domain of time-dependent failure is quite vast, in this study

Micro-mechanical Modeling
models for time-dependent failure of composites due to slow crack growth in the fibers and the shear creep at the fiber-matrix interface will be presented. One of reasons for pursuing these two specific mechanisms is that they are both time and cycle dependent. Needless to say, a model for fast-fracture is desired before attempting to model time-dependent failure. Fortunately, the lamina level response to the micro-level processes that result in fast-fracture of ceramic matrix composites is well understood, and simulation and analytic models have been developed [12,101-103], both of which will be used during the development of the micro-mechanical models in the following chapters. For the sake of providing an adequate background for further reading, in this chapter, a brief description of a simulation technique and an analytic model developed to predict fast-fracture failure of ceramic matrix composites will be presented. In addition, since time-dependent failure is basically a study of the accumulation of damage, the fast-fracture model for composites with damaged fibers will also be reviewed.

4.2 Monte-Carlo Simulation

The Monte Carlo simulation technique is, in essence, an analytic testing of an idealized composite. It is not meant to replace experimentation but rather to complement it. Some of the benefits of such a simulation scheme are that: with the declining cost of computer time it is an inexpensive way of evaluating a material after initial characterization has been performed; using simplifying assumptions the degradation...
process can be well represented; a physical representation of the failure process under different failure mechanisms can be studied in isolation (something that is very difficult to control in the laboratory); it can be used to validate analytical models; and finally it can address issues such as scale effects that are extremely important and could otherwise only be addressed by extensive component prototyping. The Monte Carlo technique used in the present study is a modification of one used earlier to successfully predict the fast-fracture strength of ceramic matrix composites. In this technique the failure process as seen at the micro-level is mimicked beginning with the breaking of individual fibers, shielding of the broken fibers, transferring of load to neighboring fibers, using global or local load sharing, with the process being repeated until the system is incapable of sustaining the load.

In developing his Monte Carlo simulation model to simulate fast fracture, Curtin [12] made assumptions based on the experimental observations of damage as observed in specimens under monotonic loading as described earlier in Section 2.3. The assumptions are; (i) the ceramic matrix composite is loaded above $\sigma_{mc}$ and that the crack spacing is relatively small, (ii) the composite is reduced to consisting of long fibers holding together relatively thin slabs of matrix by means of the sliding shear stress $\tau$ this means that either the matrix has debonded or that it has a very weak interface, (iii) the matrix being reduced to thin sections does not carry any load so all the load is borne by the fibers which have an average stress of $\sigma_{app}/f$, where $f$ is the fiber volume fraction of the composite. The failure of the composite is thus dependent on the bundle failure of the fibers.

Micro-mechanical Modeling
In the Monte Carlo simulation model developed by Curtin [12,101] for composite fast fracture will be described as this model forms the basis for the simulations of the time-dependent mechanisms that are presented in the following chapters. In the following sub-sections the discretization of the model and the theory used to develop it will be presented and a description of the fast-fracture process will be presented to show its application.

4.2.1 Model Discretization

A key component to optimizing the running time of simulation models is to determine the length scale of the component that is to be simulated. Prior work by Curtin [101], Schweitzer and Stief [102] and Sutcu [103] have shown that the ultimate strength $\sigma_u$, of a composite, is independent of gauge length as long as it is greater than the characteristic length $\delta_c$, for that particular material. The characteristic length $\delta_c$, of the fiber, is length associated with the strength in the single fiber fragmentation problem. Curtin [104] showed that, under the assumption of global load sharing (GLS) that is defined later, a sample greater than $L_s \geq 0.8 \delta_c$, would be adequate to give an accurate $\sigma_u$. Phoenix and Raj [105] however, have confirmed that a safe length to use for such models is $L_{spec} = 2\delta_c$. Next the number of fibers required for accurate simulation needs to be identified, and Curtin [12] and Ilnaabdeljalil [106] have shown that for present ceramic composites at least 200 fibers need to be simulated to provide accurate ultimate strength values.
Figure 19: Discretization of Composite
Having determined the "size" of the composite a discretization of the physical fiber bundle is be carried out. In Curtin's [12] approach the "composite" is discretized into a regular $N_f \times N_m$ mesh as shown in Figure 19. Here, $N_f$ is the widthwise discretization and refers to the number of fibers used while $N_m$ the lengthwise discretization is physically associated with what we refer to as "fiber elements". It is a well known fact that ceramic fibers exhibit a Weibull distribution of strength and that the strength is inversely proportional to the length of the fibers. What this means physically is that every fiber has a distribution of flaws of different size that control the strength of the fiber. The corollary to this being that if a fiber is cut into small segments (fiber elements), each element would have a different strength. Theoretically, the smaller the fiber elements the truer the actual representation of strength in the fiber. However this is not possible and some finite value of discretization must be made, which is the number $N_m$. The fineness of the discretization is associated with the number of fiber elements in the distance $\delta_c$ (characteristic length) of the composite. Too small a number may not provide the model with adequate sensitivity to give accurate results and too many may increase the computational time significantly. The points of intersection between the lengthwise and widthwise discretizations are referred to as nodes, \textit{j,i}. Figure 19 shows a typical discretization for such a simulation.

The model assumes a Weibull distribution of fiber strength. While this controls the global distribution of strength in the fibers, the spatial distribution of the flaws is
assumed to be random [104]. Thus random strengths are assigned to each node from the Weibull distribution of the form

\[ P(s) = 1 - \exp \left[ \left( \frac{s}{\bar{\sigma}} \right)^\rho \right] \]  

(4.1)

where \( P(s) \) is the probability of failure, \( s \) is the node strength, \( \bar{\sigma} \) the Weibull scale parameter based on the element length, \( l_m = \delta_c / N_m \), and \( \rho \) is the Weibull shape factor for the fiber strength. Solving equation (4.1) to obtain \( s \) we get:

\[ s = \bar{\sigma} \left[ -\ln(1 - P(s)) \right]^{1/\rho} \]  

(4.2)

Using a random number generator, random values in the interval [ 0-1], are allocated to \( P(s) \) and used in the above equation to give random normalized strengths of a Weibull distribution to the discretized composite model. The importance of a well established random number cannot be over-emphasized, for this model the subroutine \textit{RAN3} from Press et al. [107] was used.

In developing the fast-fracture simulation procedure Curtin [101] used shear lag theory. This assumes that there is a constant sliding shear stress \( r \) at the interface and the distance over which the stress in the fiber is recovered, starting from a fiber break, referred to as slip length \( l_s \) in ceramic composites, is calculated using force equilibrium as

\[ l_s(\sigma) = \frac{r \sigma}{2 \tau} \]  

(4.3)
where $\sigma$ is the far-field stress reached at distance $l_s$ from the break, and $r$ the fiber radius. The strength associated with the discretized fiber length $l_m$ is $\bar{\sigma}$, such that the mean number of strength-critical flaws in length $l_m$ can be written as

$$\Phi(l_m, \bar{\sigma}) = \frac{l_m}{l_o} \left( \frac{\bar{\sigma}}{\sigma_o} \right)^\rho$$  \hspace{1cm} (4.4)$$

where $\sigma_o$ is the strength of a fiber of length $l_o$ obtained from single fiber tests. Again the slip length of the fiber at stress $\bar{\sigma}$ can be written as

$$l_s(\bar{\sigma}) = \frac{r \bar{\sigma}}{2 \tau}. \hspace{1cm} (4.5)$$

Then in order to create a simulation model independent of fiber properties, Curtin [101] for convenience assumed that at stress $\bar{\sigma}$ the sliding length $l_s$ is an integer multiple of $l_m$ such that it can be written as:

$$l_s(\bar{\sigma}) = l_m(m^* + 1) \hspace{1cm} (4.6)$$

where $m^*$ is an integer. Equations (4.5) and (4.6) can be equated so that the material property $\tau/r$ can be expressed in terms of the integer $m^*$ and written as

$$\frac{\tau}{r} = \frac{\bar{\sigma}}{2(m^* + 1)l_m}. \hspace{1cm} (4.7)$$

Using equations (4.3) and (4.6) the slip length at any stress can be represented by

$$l_s(\sigma) = \frac{\sigma}{\sigma} (m^* + 1)l_m. \hspace{1cm} (4.8)$$

Curtin [108] earlier identified the fundamental properties of a fiber embedded in a matrix as the characteristic length $\delta_c$ and its associated strength $\sigma_c$ which he defined as

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\[ \delta_c = \left( \frac{\sigma_o r l_0}{\tau} \right)^{\frac{1}{\rho}} \quad \sigma_c = \left( \frac{\sigma_o r l_0}{r} \right)^{\frac{1}{\rho+1}} \] (4.9)

and where \( \delta_c = 2l_s(\sigma_c) \) or by (4.3),

\[ \delta_c = \frac{r}{2} \frac{\sigma_c}{\tau} \] (4.10)

Now using equations (4.4), (4.7) and (4.9) \( \delta_c \) and \( \sigma_c \) can be written in terms of \( m^* \) as follows:

\[ \delta_c = l_m(2m^* + 2)^{\frac{1}{\rho}} \] (4.11)

and

\[ \sigma_c = \bar{\sigma} \left[ \frac{1}{(2m^* + 2)} \right]^{\rho+1} \] (4.12)

Graphical representation of the recovery of stress over the slip length, for various stress levels, is shown in Figure 20. The effect of \( m^* \) on the mesh can be seen, i.e. higher the value of \( m^* \) the finer the mesh and vice-versa.

### 4.2.2 Simulation Procedure for Fast-fracture

The simulation procedure is started by applying a small load. In the present form of the algorithm the load is applied in small but equal steps until failure is precipitated. Another approach to the application of load is to apply a load incrementally to break one single fiber at a time [106]. As the load is applied and distributed equally among the
“fiber elements”, at some load the stress in a “fiber element” will be greater than its strength resulting in failure of that “fiber element”. The stress at that “fiber element” is then set to zero due to the local fiber fracture at that “element”. As the “fiber element” breaks a slip length is formed and the fiber “shielded”. The term “shielded” refers to the process by which the sliding length of the fiber, that is the stress recovery zone as shown in Figure 20, is subject to a lower stress than the fiber further away from the break. The stress in the region along the slip length is lower than the far-field stress and so the probability of the fiber breaking in this region is thereby reduced.

The reduction in stress along the slip length of the broken fiber causes the load shed to be taken up by the surrounding unbroken/unslipped fibers. This can be done in two ways; Local Load Sharing (LLS) or Global Load Sharing (GLS). In the former the load shed is distributed locally within a defined region so as to simulate a local stress concentration. In the latter the stress is distributed equally across the full width of the composite specimen such that all the fibers along the plane of the slip length experience equal increments of increased load and also satisfy global force equilibrium. The local traction (stress in a fiber) in the unbroken fibers as defined by GLS is given by

\[
T_m = \frac{N_f \sigma - (load\_carried\_by\_all\_shielded\_fibers\_in\_row\_m)}{(number\_of\_unbroken\_fibers\_in\_row\_m)}. \tag{4.14}
\]

A graphical description of load shedding by a broken fiber is also shown in Figure 20. The load having been shed, and the new fiber tractions calculated, and a search for additional fiber breaks is performed. If no additional fibers breaks are induced the load is
Figure 20: Sliding length of Fiber at Different Stresses [After 105]
increased. The new load causes additional fiber breaks, the broken fibers are shielded, all the fiber tractions recalculated and further damage determined. The process is then repeated. At some load level there will be an insufficient number of surviving fibers to satisfy equilibrium, and the composite is deemed to have failed. That load is the ultimate strength of the composite.

Such a simulation procedure is sensitive to the manner in which the loading occurs. During any load step it is likely that more than one fiber will break upon application of the load. This does not mean that all the fibers will be broken and the slip lengths formed at the same time. Rather, a cumulant scheme in which all the broken fibers are “collected” and then each fiber is broken in sequence based on their strengths, i.e. the weak fibers are broken first and then the strong ones.

The validity of this simulation model to predict the fast fracture failure of ceramic matrix composites has been shown by Curtin [101]. Since the focus of this model is on the manner in which load is shared and shed between the fibers it has the flexibility of being used to represent damage mechanisms, other than fast fracture, that lead to failure of the composite.

4.3 Analytical Model for Fast-fracture

While initial development of micro-mechanical models is based on a proper representation of the response of the damage mechanism at the lamina level using a
simulation procedure the ultimate goal is to develop analytical solutions to those mechanisms. To accomplish this it is essential that there be a reliable analytical form for the fast-fracture process. A well accepted model for the ultimate strength of a multiply cracked ceramic matrix composite based on GLS was proposed by Curtin [101]. This model is well established and since good use of it will be made in the following chapters, a brief description of its salient features will be presented here.

Here too, as was described in Section 4.3.2, under GLS the average stress in a plane perpendicular to the fiber axis is given by equation (4.14), i.e. the local stress in the fiber is a function of breaks "near" the plane being evaluated. Fibers that have no breaks within one slip length of the chosen plane will experience the stress $T$ with the slip length being given by

$$l_s = \frac{rT}{2\tau}.$$  

(4.15)

Those fibers that have breaks within the slip length of the plane experience a stress proportionate to the distance of the break from the plane which Curtin [101] assumed to be equal to $2\tau<\text{L}>/\tau$. where $<\text{L}>$ is the average distance of the breaks from the chosen plane. Then to satisfy global force equilibrium at the chosen plane requires that

$$\frac{\sigma}{f} = (1-q)T + \frac{2\tau}{r} <\text{L}>q$$  

(4.16)

where $q$ is the fraction of fibers with breaks within one slip length $l_s=rT/2\tau$ of the chosen plane. Substituting equation (4.15) in (4.16), (4.16) can be rewritten in the form of a constitutive equation

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Figure 21: Schematic of Composite Showing Slip Length [After 101]
\[
\frac{\sigma}{f} = \left(1 - \left(1 - \frac{\langle L \rangle}{l_s}\right) q\right) T .
\] (4.17)

Curtin then made the two following assumptions

\[
\frac{\langle L \rangle}{l_s} \approx \frac{1}{2} \quad q = \Phi(2l_s, T)
\] (4.18)

where \(\Phi(L, \sigma)\) is given by equation (4.4). Using Equation (4.18) in (4.17) and maximizing with respect to \(T\), the ultimate strength \(\sigma_u\) is:

\[
\sigma_u = f \sigma_c \left(\frac{2}{\rho + 2}\right)^{1/(\rho+1)} \left(\frac{\rho + 1}{\rho + 2}\right)
\] (4.19)

where \(\sigma_c\) is defined in Equation (4.9). In spite of the two assumptions this analytical form for the ultimate strength has been shown to provide accurate estimates of strength for ceramic matrix composites [101].

4.4 Analytical Model for Composite with Damaged Fibers

The analytical model for the ultimate strength of ceramic composites was developed for pristine material and so is based on the assumption that the as-processed specimens are initially undamaged [109]. This however is an unreasonable demand on the composite considering the complex processing they undergo. It is expected that these materials do accumulate processing related damage such as fiber breakage, matrix rich regions and so on. Considering such factors Zhou and Curtin [109] developed a model
capable of predicting the response of composites with initial damage. In addition since the study of time-dependent failure is essentially a study of accumulation of damage over time in the composite it is conceivable that the model can be modified to accommodate time dependent damage growth. While the modification of this model for composites with fiber damage will be shown in the following chapters here a brief description of this model will be presented.

In developing their model, Zhou and Curtin [109] considered the average stress in a multiply broken fiber to satisfy

$$\frac{\sigma}{f} = \frac{1}{L} \int_0^L \sigma(z) dz$$

(4.20)

where L is the length of a fiber (assumed infinite) and $\sigma(z)$ is the stress distribution in the fiber segments of length z. Now if the average distribution of fiber fragments is given by $F(x)$ then Equation (4.20) becomes

$$\frac{\sigma}{f} = \psi \int_0^\infty F(x) dx \int_0^x \sigma(z) dz$$

(4.21)

where $\psi = N/L$ is the number of fragments N per length L. Curtin and Zhou [109] then assumed a Poisson distribution of the fiber fragments such that

$$F(x) = \psi e^{-\psi x}$$

(4.22)

which when substituted into Equation (4.21) and the integration performed produced
\[ \frac{\sigma}{f} = \left( \frac{T}{2\psi l_s} \right) \left[ 1 - e^{-2\psi l_s} \right] \]  

(4.23)

where \( l_s \) is as in Equation (4.15). Furthermore since \( T \) is related to composite strain by \( \varepsilon = T/E_f \) then Equation (4.23) is essentially the constitutive equation for a composite with damaged fibers. Now normalizing Equation (4.23) by length and stress parameters that are relevant in the absence of damage, i.e. \( \delta_c \) and \( \sigma_c \) to obtain

\[ \frac{\bar{\sigma}}{f} = \frac{1}{\bar{\psi}} \left[ 1 - e^{-\bar{\psi} \bar{T}} \right] \]  

(4.24)

where \( \bar{T} = \frac{T}{\sigma_c} \), \( \bar{\psi} = \psi \delta_c \) and \( \bar{\sigma} = \frac{\sigma}{\sigma_c} \). In composites with initial damage Curtin and Zhou [109] showed that

\[ \bar{\psi} = \psi_o + \bar{T}^\rho \]  

(4.25)

where \( \rho_o \) is an initial damage parameter and so if no initial damage exists \( \bar{\psi} = \bar{T}^\rho \).

Curtin and Zhou showed that accurate estimate for the quasi-static ultimate tensile strength can be obtained, from (4.24) with \( \bar{\rho} = \left( \frac{T}{\sigma_c} \right)^\rho \) as the damage parameter [109], as

\[ \sigma_u = f \sigma_c \left( \frac{\rho}{2} \right)^{\rho+1} \left[ 1 - e^{-\left(2/\rho \right)} \right] \]  

(4.26)

A comparison between the analytical estimates of dimensionless ultimate strength (\( \sigma_u/f\sigma_c \)) of a composite as obtained from equations 4.19 and 4.26 and experimental data are presented in Table 2.

Micro-mechanical Modeling
Table 2: Comparison of Estimated Dimensionless Ultimate Strength of Composite with Exact Results [109]

<table>
<thead>
<tr>
<th>Weibull modulus $\rho$</th>
<th>Eq. 4.19</th>
<th>Eq. 4.26</th>
<th>Exact results</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.595</td>
<td>0.639</td>
<td>0.668</td>
</tr>
<tr>
<td>5</td>
<td>0.696</td>
<td>0.708</td>
<td>0.716</td>
</tr>
<tr>
<td>10</td>
<td>0.779</td>
<td>0.783</td>
<td>0.785</td>
</tr>
</tbody>
</table>
5. SHEAR CREEP AT THE FIBER-MATRIX INTERFACE

5.1 Introduction

The need to have proper understanding of the mechanisms that contribute to the degradation materials subject to loading at high temperature, in an oxidizing environment, cannot be over emphasized. Under these loading conditions metals are known to creep which results in reversible and some irreversible strains in the material. Under similar loading conditions, fiber reinforced ceramic composites experience several damage mechanisms which may act simultaneously or independently. In chapter 2 the mechanisms that were shown to contribute to the macro-level creep response of ceramic matrix composites were presented. The accumulation of damage that precipitates the fast-fracture of metal or ceramic matrix composites is now reasonably well-understood. However, mechanisms responsible for creep damage and final rupture of such composites under sustained loading at elevated temperatures is not fully understood. It is essential that the microstructural parameters that control these mechanisms, and eventually the global response of the material be identified. Creep damage in ceramic matrix composites can include the independent or combined effects of micro-cracking, fiber-matrix debonding and time-dependent fiber failure. The inelastic response of CMC’s at elevated temperatures is thus not only the result of normal diffusional or cavitation creep in the constituents but is strongly effected by the presence of the fibers. Wu and Holmes [54] showed that damage caused by the presence of fibers in the
composites is largely a function of the applied load. Lamouroux et al. [59] attributed part of the creep in ceramic composites to the supplementary diffusion path created by the fiber-matrix interface. These considerations indicate that the interface plays a major role in the creep response of these materials and that creep in metal or ceramic matrix composites is initially complicated by the existence of microcracking and subsequently by increases in debonding and fiber fracture. In addition, these mechanisms are always coupled: relaxation of the matrix causes change in the stress state of the fibers which leads to fiber failure or vice-versa; the sequence being dependent on the creep rate mismatch between constituents.

In this chapter the contribution to creep made by the relaxation of shear stress at the fiber-matrix interface along the slip length of a broken fiber is considered, and a simulation approach is used to study its impact on failure of the composite. For the influence of shear stress in the matrix to be significant, the loaded composite must have reached a steady state, i.e. the matrix tensile stress has almost completely relaxed onto the fibers. The focus is then shifted to the relaxation of the shear stress across the fiber-matrix interface around broken fibers. The sequence of events resulting in a time-dependent deformation is as follows. When a fiber breaks, the fiber-matrix interface is debonded (if not already debonded by matrix cracking) and the matrix slides relative to the fibers subject to the constraint of the residual frictional sliding stress $\tau$ along the interface. The matrix in the vicinity of this broken fiber is thus subject to a shear stress $\tau$ at the fiber-matrix interface and the matrix can relax this stress by creep. Decrease in the shear stress then implies that there is additional slip along the fiber to maintain the axial

**Shear Creep At The Fiber-Matrix Interface**
equilibrium condition away from the break. The slip length along the fiber thus changes as a function of time. As the matrix shear decreases and slip increases, the broken fibers carry less and less load and over longer and longer gauge lengths, resulting in increasing load being shed onto unbroken fibers (Figure 22). Eventually additional fibers become overloaded and then break, initiating shear relaxation around their perimeters and increasing even further the stresses on the remaining intact fibers. This process of breakage, slipping, relaxation, and further breaking can culminate in the rupture of the material after a sufficient amount of damage has occurred.

The work in this chapter consists of two parts. First a simplified analytic unit cell model for interfacial shear stress versus time is developed which agrees well with the full results of Ref. 110. Second, composite failure driven by the shear creep at the fiber-matrix interface is modeled using a modified version of a numerical simulation model previously developed by Curtin [101]. The numerical simulations include the same details exhibited in the unit-cell model for each individual fiber break. From these the failure times versus the creep and strength parameters describing the constituent fibers and matrix are mapped out. This model is also used to measure the remaining strength of the composite as a function of time, which is an important design consideration. Furthermore, the interplay between shear creep at the interface and various other non-linear time- and stress-dependent phenomena occurring simultaneously in real composites may be too complex to handle analytically. Hence the simulation results form a benchmark for future calculations of time-dependent composite failure by multiple degradation mechanisms.

Shear Creep At The Fiber-Matrix Interface
Figure 22: Effect of $\tau(t)$ on Stress Recovery
The rest of the chapter is organized in the following manner. In the second section of this chapter a historical perspective on the models developed to predict the creep response of fiber reinforced metal and ceramic reinforced composites are be presented. In Section 5.3 an analytic relationship for shear stress and slip length as a function of time is developed. In Section 5.4 the simulation model for the time-dependent evolution of damage as a result of the shear creep at the interface is described. In Section 5.5 the results of the simulation are presented, and compared with corresponding results obtained by Du and McMeeking [110]. In Section 5.6 the results are applied to a Ti-Metal Matrix Composite (MMC).

5.2 Micro-mechanics Models for Creep of Fiber Reinforced Materials

Literature dealing with micro-mechanical modeling of creep of continuous fiber ceramic matrix composites is extremely limited and most of the earlier attempts at modeling creep were based on data obtained from discontinuous fiber metal matrix composites. No review on the modeling of creep of ceramic matrix materials will be complete without acknowledging the contribution made by research in metal matrix composites. One of the earliest efforts to understand the behavior of creep in fiber reinforced composites was a study undertaken by Kelly and Tyson [111,112]. They investigated the creep behavior of silver specimens containing 40%, by volume, of tungsten wires at temperatures between 400°C and 600°C. Noting the contribution made
by fibers to the increase in strength of the silver under quasi-static loading, Kelly and Tyson [111,112] studied the contribution of the fibers to the silver under creep loading. A considerable improvement in the strain rate data of the composite over the pure silver matrix was observed. The existence of a threshold stress, at which point the creep rate stress exponent changed from 3 to 14, and very little fiber breakage was observed. No voids were recorded in specimens, during steady state creep, though some were during the tertiary creep stage. Kelly and Tyson [111,112] suggested that the matrix shear stress \( \tau \), was reduced by the relaxation of the matrix which resulted in creep of the composite. They related the formation of voids to the short length of the fibers preventing them from fully allowing the stress recovery to take place finally concluding that the creep rate of the composite was determined by the rate of creep of a portion of the matrix close to the fibers subject to this shear stress.

Silva [113] questioned the conclusions reached by Kelly and Tyson [112] that the matrix axial stress \( \sigma_m \), could be ignored and that the creep of the composite was interface shear \( \tau \) controlled. Using a variational approach he asserted that the relaxation of the matrix tensile stress \( \sigma_m \) would result in a change in the matrix shear stress \( \tau \) with time that would oppose the original relaxation of \( \tau \). Silva [113] also considered the effects of the following: a decreasing shear stress \( \tau \), relaxation of \( \sigma_m \) in the matrix; and creep of the fibers; on the creep composite. He concluded that the three processes are inter-related so none could really be ignored.
Mileiko [114] recognized that the time dependent mechanical response of fibrous composite materials was controlled at the local level but lacking the information on the interface he used the constitutive properties of the matrix. He represented the composite as two parallel plates separated by a viscous liquid, with one moving relative to the other to create shear in the matrix. Mileiko [114] represented the constitutive law of the matrix by the Norton-Bailey creep equation. Assuming flow in the matrix he used the St. Venant condition $\sigma_m = 2\tau_m$ to relate axial and shear stresses and strains. He also showed the results of his model to exhibit void formation as observed by Kelly and Tyson [112].

Using Mileiko's [114] plate model of the composite McLean [115] considered the energy dissipated during flow of the matrix. Again, assuming conservation of flow past the fiber ends; i.e. $\tau_0 = \sigma_0/2$ and $\gamma_0 = \varepsilon_0/2$, he showed that the end stress was proportional to the diameter of the fiber. Based on this he concluded that smaller fibers were better than thicker fibers for creep rupture. In his analysis, McLean [115], also assumed that the applied tensile load was equally distributed between the fiber and the matrix i.e. it was independent of the fiber volume.

As early as 1972, Kelly and Street [116] recognized the role and the existence of slip at the interface, which they included in their model to describe the steady state creep behavior of a discontinuous fiber of lead phosphor bronze composite. They, too, assumed conservation of flow to relate axial stress and strain with shear and accounted for slip by assuming lack of displacement continuity at the interface. The slip term, which is a fraction ranging from 0 to 1, was included in their displacement function. Using a power law steady state creep rate equation for the matrix they considered the following three

Shear Creep At The Fiber-Matrix Interface
cases: rigid fiber with perfect bond, creeping fiber with rigid bond and rigid fiber with imperfect bond.

In attempting to model experimental data obtained for γ-γ' Cr₂-C₂ alloy at 825°C, McLean [117,118] considered the case of creeping matrix/elastic fibers and creeping fibers/creeping matrix. To better represent the creep response McLean [118] modeled the transient creep due to relaxation of the axial stress in the matrix. To accomplish this he represented the creep using the Norton-Bailey power law strain rate equation, and using the rule of mixtures he obtained the constitutive law for the composite. Different creep rate stress exponents were used for the fiber and matrix. McLean [118] then recognized that the equation assumed that the stresses were homogenous in each phase and that this would result in discontinuity at the interface. To overcome this problem he assumed that the fiber-matrix interface could be regarded as a separately identifiable phase of thickness, δ and so he accounted for the interface region by considering three phases rather than two in the manner shown below:

\[ V_f = \text{const.}, \quad V'_m = 1 - V_f - \frac{2V_f \delta}{\lambda}; \quad V_b = \frac{2V_f \delta}{\lambda} \]  

(5.1)

and

\[ \sigma_c = \sigma_f V_f + \sigma_m V'_m + \sigma_b V_b \]  

(5.2)

Later McLean and Goto [119,120] used the above formulation to improve on the Kelly and Street [117] model to include the elastic part of the creep curve. Assuming continuity of displacement at the interface they obtained a solution to the interface creep problem in the form of three coupled differential equations, which they solved using the
Runge-Kutta scheme. They showed that primary creep was dominant as a result of the non-linear visco-elastic deformation of the highly-stressed central region of the fiber; this in turn led to a steady state creep due to matrix flow around the fibers. The tertiary creep was a result of fiber fracture and a reduction of the fiber aspect ratio. McLean and Goto [119,120] also evaluated the impact of two types of interfaces on the response of short fiber composites: one which had a shear strength less than that of the matrix and the other that was greater. From their solutions they concluded that: weak interfaces have no significant effect on creep performance and creep life (defined as the time required to attain a given strain) is increased by increase in fiber volume fractions, applied stress fiber diameter ratio, fiber modulus applied stress ratio and interphase to matrix strain rate ratio. When evaluating rupture time they did not consider the impact of creep at the broken fiber ends.

In developing a model to predict the creep rupture time of whisker reinforced metal matrix composites Deahn and Gonzalez-Doncel [121] assumed the existence of a complex stress state in the matrix which was dependent on the location along the fiber, i.e. the stress state in the matrix near the end of the whisker is different than in regions constrained between two closely spaced whiskers. In developing their model they made the following assumptions: perfect bonding between matrix and whiskers, whisker end effects were neglected, the strength of the composite could be determined by the octahedral shear stress criterion and is the rate limiting step in composite deformation, all the applied load is transferred through the whiskers, and finally the fibers are elastic and

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the matrix visco-plastic. The problem was adequately analyzed using a two-dimensional problem.

Seeing the need to understand the behavior of composites with weak chemical bonds at the interfaces, Chou et al. [122,123] studied the creep of short fiber ceramic matrix composites using an advanced shear lag theory. In the unit cell the fiber was assumed to be elastic and the while the matrix responded to the Norton-Bailey power law creep. As opposed to most other unit cell models that assumed the fiber and matrix to support only axial or shear load respectively, Chou et al. [122,123] assumed based on the comparative stiffness of the fiber and matrix in ceramic composites that the axial and shear loads would be borne by both the constituents. To account for the multi-axial stress state in the matrix they used the creep rate equations developed by Johnson [124]. They also incorporated the fiber-matrix interface sliding parameter as proposed by Kelly and Street [116]. Due to mathematical complexities they had to use a fourth order Runge-Kutta numerical method to solve the governing equations however they did not corroborate their model with experimental data.

McMeeking [125] analyzed a unit cell model of a high aspect ratio short fiber aligned with the load axis. He assumed, since the aspect ratio of the fiber was large, compared to the distance between the neighboring fibers, that the matrix flow around the fibers, used in prior analysis [119,120], could be neglected. He further assumed that the fiber and matrix were well bonded and that the bond consisted of a thin layer of interphase that had a power law rheology of its own which caused that matrix to slip relative to the fiber. His solutions gave results similar to those obtained by Kelly and

Shear Creep At The Fiber-Matrix Interface
Street [117], such as linearity of the axial velocity and the dominance of creep strength by
the shearing flow in the matrix. McMeeking [125] attributed the differences in creep
strength of the two models to the stress averaging technique used by Kelly and Street
[116] since they used shear lag.

As research interest grew from discontinuous to continuous fiber composites,
most creep models still considered only the effects of axial stress in the matrix and did
not accurately represent the response [122,123] since the contribution of damage to creep
was largely ignored. Concurrently with the development of the creep models for
continuous fiber composites were those for the progressive fracture of the fibers
[103,101]. Mason et al. [126] presented an analysis for creep due to the presence of a
broken fiber in a three-fiber model (one broken two intact). Their focus being on
polymeric composites, they included memory effects in the development of the steady
state response due to the inelastic strains in the composite.

Lamouroux et al. [127] performed experimental and analytical studies on the
damage mechanisms during tensile creep loading of alumina reinforced silicon carbide
composite at elevated temperatures in a vacuum. They observed that matrix micro-
cracking inside the fabric tows and some interface debonding in the specimens on cooling
from the processing temperature. They associated the change in modulus, at temperature,
only to the interfacial debonding that they observed and they conjectured that matrix
microcracking and fiber failure did not occur. Lamouroux et al. [127] also noticed that
the creep curves they obtained at 1100°C exhibited only primary and tertiary creep with
little or no secondary creep. They believed that progressive fiber-matrix debonding was

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the dominant mechanism that could explain the decrease of the longitudinal elastic modulus in the tensile and creep tests, the increase in strain rate during tertiary creep and the large fiber pullout on the creep fracture of surfaces. They also observed that creep failure mode was stress dependent.

To analytically represent their experimental observations of creep response of alumina reinforced silicon carbide composite, at elevated temperatures, in a vacuum, Lamouroux et al. [59,127] created a unit cell model of the fiber and matrix. In this model they used a “block” of the composite that included a debonded and a bonded regions. They defined the creep of the block as follows:

\[
\dot{\varepsilon}_i(t) = \dot{\varepsilon}_f(t)X_i(t)
\]

where \( \dot{\varepsilon}_i \) is the strain rate of block \( i \) and \( X_i \) is the debonded fraction \( \delta_i \lambda_i \). Assuming a debonding rate, Lamouroux et al. [127], obtained the creep strain of the microcomposite as sum of the elongation of the debonded and undebonded regions divided by the original length of the block. Lamouroux et al. [127] also obtained an expression for the change in modulus during tertiary creep and used a Weibull modulus approach to predict rupture of the fibers.

\[
t_2 = t_1 \left( \frac{X_{o1}}{X_{o2}} \right)^{\frac{1}{b}} \left( \frac{\sigma_1}{\sigma_2} \right)^{\frac{\rho}{b}}
\]

where \( \rho, b \) are the Weibull modulus for strength and time to failure. Using such a model to account for fiber debonding and fiber failure they were able to model the tertiary creep strain and predict the time to rupture of these composites.

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Combining a creep model with a progressive fiber fracture model, Du and McMeeking [110] presented a model for the axial creep response of a composite, with similar work carried out by Fabeny and Curtin [128]. Recognizing that McLean et al. [118-120] had only included the axial stress in the matrix and did not consider damage effects due to fiber failure in their creep model, Du and McMeeking integrated Curtin’s composite strength model with McLean’s creep model to develop the “Curtin-McLean” model. Two versions of this model were presented, one based on a maximum strain, and the other on the maximum stress criterion. These models give good results for “short” time scales during which the matrix axial stresses relax and fibers undergo damage due to the increased axial stresses. The additional increase in strain due to relaxation of the matrix shear stress at the fiber-matrix interface was ignored. To overcome this Du and McMeeking [110] also presented a displacement formulation for the creep in a metal matrix composite brought about by the combined relaxation of the tensile and shear stresses in the matrix. This model provided an accurate representation of creep in a unit cell (single broken fiber and matrix) and then was used with a fiber damage evolution model, but only in an approximate, average sense. Du and McMeeking [110] recognized that to fully consider the actual case of many distributed fiber breaks would require solving the partial differential equation for shear stress many times, and would be computationally exhausting. In rationalizing the results, Du and McMeeking [110] suggest that the stress relaxation due to the axial stress in the matrix stabilizes quickly as compared to the relaxation of the interface shear due to the broken fibers whose relaxation time is large.

Shear Creep At The Fiber-Matrix Interface
5.3. Shear Relaxation at Fiber-matrix Interface in a Single Fiber

Recent studies on the creep response of ceramic matrix composites have focused on the impact of damage and resulting degradation. Before the composite degradation can be studied numerically or analytically, a form for the interfacial shear stress along a broken fiber versus time must be obtained that incorporates the shear creep behavior of the matrix and the slip along the fiber-matrix interface. Such a shear stress must also be expressible in a closed form so that it can easily be employed in the numerical simulations. Here, the nice work of Du and McMeeking [110] is elaborated and followed, then one physical approximation is made to permit obtaining an analytic solution.

Du and McMeeking [110] addressed the problem of matrix shear stress relaxation around a broken fiber in a composite for a power law creep constitutive law for the matrix subject to a complex stress state given by

\[ \dot{\gamma} = \frac{\dot{\tau}}{G_m} + 3B\sigma_e^{n-1}\tau \]  

(5.5)

where \( G_m \) is the matrix shear modulus, \( \sigma_e^2 = (\sigma_{ij}\sigma_{ij}) \) is the equivalent stress, \( \tau \) and \( \gamma \) are the shear stress and strain, and \( \dot{\tau} \) and \( \dot{\gamma} \) their derivatives with respect to time. \( B \) is the creep constant in the constitutive law for the matrix tensile stress \( \sigma_m \) under axial creep,

\[ \dot{\varepsilon} = \frac{\dot{\sigma}_m}{E_m} + B\sigma_m^n. \]  

(5.6)

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Using the force equilibrium equation at a broken fiber

$$\frac{\partial \sigma_f (z)}{\partial z} = \frac{2\tau}{r}, \quad (5.7)$$

the fiber axial stress-displacement relationship

$$\sigma_f (z) = E_f \frac{\partial u_f (z,t)}{\partial z}, \quad (5.8)$$

and the matrix shear strain-displacement relationship

$$\gamma_m = \frac{u_c(z,t) - u_f(z,t)}{w}, \quad (5.9)$$

where $u_f$ and $u_c$ are the axial displacements of the fiber and composite respectively,

$$w = 2r\zeta, \quad \text{and} \quad \zeta = \left( \frac{\pi}{\sqrt{2f \sqrt{3}}} \right)$$

for hexagonal packed fibers, Du and McMeeking [110]

obtained the partial differential equation for fiber displacement

$$\frac{1}{w} \left[ \frac{\partial u_c(z,t)}{\partial z} - \frac{\partial u_f(z,t)}{\partial z} \right] = -\frac{DE_f}{4G_m} \frac{\partial^3 u_f(z,t)}{\partial z^3} - \frac{3}{4}BD\sigma_e^{n-1}E_f \frac{\partial^2 u_f(z,t)}{\partial z^2}. \quad (5.10)$$

Eq. 5.10 is strongly non-linear and full solutions of this differential equation are not obtainable analytically. Moreover, the resulting shear stress (from Eq. 5.7) is explicitly dependent on the axial location $z$, which adds greatly to the complexity of the problem.

Numerical solutions to Eq. 5.10 for various cases by Du and McMeeking [110] demonstrate that the shear stress is only weakly dependent on $z$. This, along with the assumption that the fibers carry all the load after the matrix has relaxed, motivate two simplifications to Eq. 5.5 and 5.10. First it is envisioned that because the fiber is free at

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the broken end, the displacement of the slipping part of the fiber occurs in a rigid manner and the magnitude of the displacement is controlled entirely by the displacement at the end of the slip zone \( z = l_s \) where compatibility with the matrix is established. Mathematically this is expressed by the condition

\[
\dot{u}_f(z,t) = \hat{l}_s \varepsilon_c \quad 0 < z < l_s \tag{5.11}
\]

where \( \varepsilon_c \) is the composite strain. This is in contrast to the strain-displacement relationship \( \dot{u}_f = \dot{\varepsilon}_m l_s \), used by Kelly and Street [116] in which the slip length was constant and the rate of change in fiber displacement was proportional to the rate of change of strain in the matrix. The second assumption is that the matrix tensile stress has completely relaxed which leads to \( \sigma_e = \sqrt{3} \tau \) so that Eq. 5.5 reduces to

\[
\dot{\gamma} = \frac{\dot{\tau}}{G_m} + 3^{(n+1)/2} B \tau^n \tag{5.12}
\]

In addition, the ansatz of Eq. 5.11 leads to a \( z \)-independent \( \tau \) so that the force equilibrium relationship at the end of the slip zone of a broken fiber becomes

\[
\frac{\sigma_{\text{app}}}{f} = \frac{l_s}{6} \frac{2 \tau(t)}{r} \, dz = \frac{2 \tau(t) l_s}{r} \tag{5.13}
\]

Combining Eqs. 5.11 and 5.13 with Eqs. 5.8 and 5.9 and setting \( \dot{u}_c = 0 \) since the matrix has completely relaxed and the fibers are completely elastic, leads to a simplified version of Eq. 5.10 given by:

\[
\frac{r \sigma_{\text{app}}^2}{2 \omega f E f} \frac{1}{\tau(t)^2} \, d\tau = -\frac{1}{G_m} \frac{d\tau}{dt} - 3^{(n+1)/2} B \tau(t)^n. \tag{5.14}
\]

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Eq. 5.14 is an ordinary differential equation for \( \tau(t) \) independent of \( z \) which can be integrated analytically for integer values \( n \geq 1 \), yielding a polynomial equation for \( \tau(t) \).

For \( n = 2 \) and 3, which are relevant to many materials \( \tau(t) \) satisfies

\[
\frac{r \sigma_{\text{app}} (n-1)}{2w f(n+1) \tau(0)^2} \left[ \frac{1 - \left( \frac{\tau(t)}{\tau(0)} \right)^{-n+1}}{2(1+\nu)E_f} \right] \left[ 1 - \left( \frac{\tau(t)}{\tau(0)} \right)^{-n+1} \right] = -3^{(n+1)/2} BE_f \tau(0)^{n-1} t. (5.15)
\]

where \( \tau(0) \) is the initial value of interfacial shear stress. To obtain \( \tau(t) \) from Eq. 5.15, for \( n = 3 \) Eq. 5.15 results in a bi-quadratic (harmonic) equation while if \( n = 2 \) then the result is a cubic equation, for which the roots are to be found. However, if \( n = 1 \) then Eq. 5.15 reduces to a simple transcendental equation. In most cases, then, a closed form solution can be obtained for the interfacial shear stress versus time under conditions of full relaxation of the matrix tensile stress. Using these solutions for the time-dependence of shear stress and the force equilibrium Eq. 5.13 similar polynomials for the time-dependence of slip length \( l_s \) can be obtained.

The simple analytic model for \( \tau(t) \) has been validated by comparing results for axial stress \( \sigma(z) \) (from Eq. 5.13) with those of Du and McMeeking [110] at a constant applied stress and \( n = 3 \). To do that normalizing quantities for stress, length, and time as defined by Du and McMeeking [110] were introduced to obtain:

\[
\hat{\sigma} = \frac{\sigma_{\text{app}}}{\sigma_c}, \quad \hat{\tau} = \frac{\tau(0)}{\sigma_c}, \quad \hat{L} = \frac{l_f(0)}{l_f(t)}, \quad \hat{E} = \frac{E_f}{E_m} \quad \text{and} \quad \hat{t} = B E_f \sigma_c^{n-1} t
\]

where \( \sigma_c \) is a reference stress important in fiber damage evolution. The axial fiber stress versus time at \( \hat{\sigma} = 0.20 \), \( \hat{\tau} = 0.01 \), and at normalized times of \( 0, 1x10^5, 1x10^6, 1x10^7 \) and
$5 \times 10^7$ are shown in Figure 23. The trends predicted by the simplified model is consistent with those predicted by Du and McMeeking [110]. At $t=0$, Du and McMeeking have a smaller total slip length because the matrix axial stress has not completely relaxed, as assumed in the above analytic model. However, after a “short time” of $1 \times 10^5$ the axial stress has nearly completely relaxed and so at times $\geq 1 \times 10^5$ the results are consistent with the trends found by Du and McMeeking [110]. The results predict a slip length less than 20% larger over a broad range of time scales, indicating that the approximate form of Eq. 5.15 accurately represents the non-linear time evolution of slip due to shear creep.

The influence of the creep rate exponent $n$ on slip length is shown in Figure 24 for $n = 1, 2$ and 3 using the same normalizations as above. The slip lengths are strongly dependent on $n$ but, since the time scales also contain the creep exponent $n$, equal normalized times do not equate to equal real time. It is interesting to note that after some transient time the slip length becomes a power law in time, $l_s \sim t^\alpha$ where $\alpha = 1.7, 0.78$ and 0.64 for $n = 1, 2$ and 3 respectively, this from is similar to the results of Mason et al. [126].

5.4 Modeling Composite Creep, Damage and Failure

Now turning to the composite as a whole, as a first approximation, the effect of the decreasing $\tau(t)$ by using $\tau(t)$ in the analytical expression for strength of a composite given in Eq. 4.19 and 4.9 is considered. From these equations it can be seen that as the fiber-matrix

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Figure 23: Comparison of slip lengths at different times with Du and McMeeking model [110]

\[ n = 3, \rho = 5, E_r/E_m = 3, \tau(0)/\sigma_c = 0.01, \sigma_{app}/\sigma_c = 0.20 \]
Figure 24: Comparison of slip lengths for $n = 1, 2$ and $3$

at different normalized times

$p = 5, E_f/E_m = 3, \sigma_{app}/\sigma_c = 0.20$
interfacial shear stress $\tau$, and hence $\sigma_C$, decreases the ultimate strength of the composite decreases and failure occurs when the strength reaches the level of the applied load. This is the present manner in which cyclic fatigue behavior of CMC’s is understood, because the major fatigue mechanism at low temperatures is interface wear and a concomitant decrease in $\tau$ with cycles [130]. So, given a form of $\tau(t)$, the failure time can then be estimated directly. The above simple estimate is formally incorrect for the following reasons: first, the value of $\tau$ decreases around any individual fiber only after that fiber has broken, and hence in a real composite there is, at any instant of time, a broad spectrum of $\tau$ values that depend on the history of the fiber breakage. Second, the ultimate strength expression given by Eq. 4.19 is based on the existence of only a few breaks per characteristic slip length $\delta_c$. Since $l_s$ now depends on time, it is possible that this condition is violated during the damage evolution. Finally, finite-length effects come into play as the slip length becomes comparable to the sample gauge length. The analytic model suggested by Eqs. 4.19 and 4.9 however, is found to be quite accurate for “long” sample lengths.

First, the numerical simulations of composite failure, described in Chapter 4, will be used to study this overall degradation behavior. Modifications of the basic simulation model by Ilnabadeljalil and Phoenix [131] and Zhou and Curtin [109] were used to predict the creep rupture of composites and failure of composites due to initial damage, respectively. This is because the Monte Carlo simulation model has the capability of simulating fiber breakage, independent of the mechanism responsible for the breaks. If the micromechanics of a damage mechanism that will result in fiber breaks is known, *Shear Creep At The Fiber-Matrix Interface*
then it can be incorporated into the simulation model to predict failure. For the present creep rupture problem the load is held constant and the local stresses in the fibers change with time due to the relaxation of the shear stress according to Eq. 5.15. The simulation proceeds as follows. A constant load $\sigma_{app}$, some predetermined fraction $\sigma_{app} / \sigma_u$ of the fast-fracture load, is applied to the composite at a rate such that negligible shear relaxation occurs at the broken fibers. At that applied stress some weak fiber elements may fail, followed by slip at the fiber-matrix interface along each break, and the load in the broken/slipped regions is redistributed globally until the system is mechanically stable (each fiber element is stronger than the local stress on it). The slip length $l_s$ and interfacial shear $\tau_o$ are at their initial values at $t = 0$. The load is then held constant for the remainder of the test and the time-dependent slip of the interface is commenced. Time is incremented and the slip along the interface near the existing broken fibers is updated according to the $\tau(t)$ obtained from the solution of Eq. 5.14. As time elapses the reduced slip, or increase in slip length $l_s$, increases the stress on the other fibers along each section resulting in a sequence of fiber breaks at times $t_i$. Each of the break times $t_i$ results in the initiation of interfacial slip and relaxation around that fiber element which evolves as $\tau(t-t_i)$. The cumulative effect of the slip and fiber damage determines the creep strain (which does not include the explicit fiber creep contribution) and failure occurs quite naturally when the stress on the remaining fibers exceeds their collective strength and rapid failure occurs.
5.5 Results and Discussion

No discussion on numerical modeling of the failure of composites can be initiated without first addressing scale effects on the model. The sensitivity of fast-fracture tensile strength of composites to the gauge length of the sample, under GLS, has been studied extensively [12,104]. The strength of “long” samples under fast-fracture conditions is very mildly dependent on gauge length where “long” samples are those longer than \( \delta_c \). Specimens with lengths less than \( \delta_c \) are stronger than “long” samples. While initially the composite specimen length may be greater than \( \delta_c \), as time under load elapses, the \( \ell_s \) increase with time and can very well exceed the length of the specimen. This mechanism can, in effect, transition the specimen from being a “long” sample to a “short” one. Once in the short-length regime the sample strength does not decrease as fast with time and the failure time is thus longer than expected if the length effect were ignored. To determine the minimum composite length required for different load levels and Weibull moduli \( \rho \), parametric analysis of failure time versus composite length must be performed.

Results of simulations of composite failure time due to shear relaxation at the fiber-matrix interface are presented in Figures 25-27 for various composite lengths, for creep exponents of \( n = 2 \) and 3, Weibull moduli \( \rho = 3, 5 \) and 7, and load levels \( \sigma_{app}/\sigma_u = 0.5 - 0.9 \). In these figures the composite length is normalized by \( \delta_c \) (Eq. 4.9) while time is normalized by \( B\ell (n-1)t \). In all cases the time-to-failure is a strong function of specimen length for lengths below a certain length which depends on \( \sigma_{app}, \rho \) and \( n \). This
Figure 25: Comparison of Failure Times with Composite Length and influence of failure spectrum for $\sigma_{\text{app}}/\sigma_u = 0.50 - 0.90$

$n = 3$, $\rho = 5$, $E_f/E_m = 3$
Figure 26: Comparison of Failure Times with Composite Length and influence of failure spectrum for $\sigma_{\text{app}}/\sigma_u = 0.50 - 0.90$
$n = 2, \rho = 5, E'/E_m = 3$
Figure 27: Comparison of Failure Times with Composite Length and influence of failure spectrum for $\sigma_{ap}/\sigma_u = 0.70$

$n = 3, \rho = 3, 5, 7, \ E_f/E_m = 3$
is the minimum model length needed to perform numerical simulations relevant to long-length composites and will henceforth be referred to as the minimum composite length. For the data presented in Figures 25-27 the minimum composite length is close to the largest length studied in each case. The minimum composite length is inversely proportional to, and a non-linear function of, the load level because failure time increases with decreasing $\sigma_{app}$ and the typical slip lengths at failure are much longer at lower $\sigma_{app}$.

In Figure 27, the simulation results of times-to-failure versus length for $\sigma_{app}/\sigma_u = 0.7$ and $n = 3$ and in which for several Weibull moduli are compared. The change in $\rho$ is seen to impact both the failure time of the composite due to time-dependent slip at the interface and also the minimum length of the composite. An increment of 2 in $\rho$ results in an order of magnitude increase in the time-to-failure for the range of loads evaluated. Such behavior can be rationalized intuitively since an increase in $\rho$ is associated with a reduction in damage and so a corresponding increase in lifetime. It must be noted that the ultimate strength of the composite (Eq. 4.19) is also a function of and increases with $\rho$. Change in the Weibull modulus $\rho$ also results in a change in the minimum length of the composite needed for simulation of this time-dependent mechanism: the minimum composite length required to simulate the stress relaxation effect changes from $4\delta_c$ for $\rho = 3$ to $\sim20\delta_c$ for $\rho = 7$ at $\sigma_{app}/\sigma_u = 0.7$. Needless to say, it is possible that for higher values of $\rho$ and low load levels, standard specimen dimensions used in the laboratory

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may not be adequate for obtaining accurate failure times of larger components when under the influence of interfacial shear creep.

The failure or stress rupture times associated with the minimum composite lengths for each load level are presented as stress versus time-to-failure in Figure 28-29 for $\rho = 5$ and $n = 2, 3$, and $\rho = 3, 5, 7$ and $n = 3$. The effect of $n$ and $\rho$ on predicted times-to-failure as a function of applied load can be seen. The results for $\rho = 5$ show that reduction in $n$ has the effect of reducing the normalized failure time and also of changing the slope of the stress-failure time curve. As seen in Figure 27 earlier, an increase in $\rho$ results in an increase in the time-to-failure. At the higher loads initial damage that occurs during loading is high relative to the damage that occurs at lower loads, resulting in shorter life times. The effect of initial damage diminishes as the load level is decreased and lifetime is governed by the subsequent growth of damage. As the load level approaches the fast-fracture limit the failure times tend to converge and must, of course, all become zero at $\sigma_{app} = \sigma_u$.

An interesting feature of Fig. 25-27 is an apparent divergence in failure time for sufficiently small composites. This limiting case of “short” lengths for which $t_f \rightarrow \infty$, can be understood analytically as follows. When the slip length becomes much greater than the specimen length, the fiber stress recovery in the gauge section is nil so that fiber breaks are equivalent to completely removing the fibers from the composite.
Figure 28: Comparison of Failure Times and Applied Stress for Analytical, Simulation and Du-McMeeking Models [110]

$\rho = 5$, $E_f/E_m = 3$, $\sigma_{app}/\sigma_c = 0.20$
Figure 29: Comparison of Failure Times and Applied Stress between Analytical and Simulation Models

$n = 3, \frac{E_f}{E_m} = 3, \frac{\sigma_{app}}{\sigma_c} = 0.20$
The fibers in the composite under these conditions thus tend to replicate a “dry bundle” of fibers. The strength of a “dry bundle” of fibers of length \( L_0 \) is [132]

\[
\sigma_b = f \sigma_c \left( \frac{\delta_c}{L_0} \right)^{1/\rho} (e \rho)^{-1/\rho} .
\]  

(5.16)

If the applied load is smaller than \( \sigma_b \) then the composite will never fail even though all the broken fibers completely relax as \( t \to \infty \). Hence, for a given load there is a critical length below which \( t_f \to \infty \), which is, using Eq. 4.19 and 4.9,

\[
L_c = \delta_c \frac{e \rho}{(S \varphi(\rho))^{\rho}}
\]

(5.17)

where \( S = \sigma_{app} / \sigma_u \), and \( \varphi(\rho) = \left( \frac{\rho+1}{\rho+2} \right)^{1/(\rho+1)} \). The critical length \( L_c \) predicted by Eq. 5.17 for each case simulated is shown in Figures 25 and 26 and agrees very well with the length at which the failure times diverge in the simulation studies.

The sensitivity of the failure time to the spectrum \( \{t_i\} \), of fiber failure times, has been tested. Specifically, simulations have been performed in which the slip around every fiber, independent of when it actually broke, is governed by \( \tau(t) \) where \( t = 0 \) at the start of the test. In other words, each broken fiber is treated as if it has been broken for the entire test time \( t \) rather than only \( t - t_i \). Figures 25-27 show the results of these simulations as open symbols. The open symbols nearly overlay the more precise results using \( \tau(t - t_i) \) in all cases. The fact that the time at which the shear at the fiber-matrix interface begins to relax is inconsequential allows us to use the analytic Eq. 5.17 for the composite strength in which \( \tau(t) \) is given by Equation 5.15. These results suggest that for Shear Creep At The Fiber-Matrix Interface
long lengths, failure times can be assessed analytically using Eq. 4.19 and 4.9. Specifically, at applied stress $\sigma_{ap}$ the time-to-failure approximately satisfies [129]

$$\frac{\tau(t_f)}{\tau_o} = \left(\frac{\sigma_{ap}}{\sigma_u}\right)^{\rho+1}$$

(5.18)

as long as the composite length satisfies

$$\frac{L}{\delta_c} \geq \left(\frac{\tau_o}{\tau(t_f)}\right)^{\frac{\rho+1}{\rho}}$$

(5.19)

This analytic prediction are shown in Figures 28 and 29. These results show reasonably good agreement between our simulation and analytic results and those of Du and McMeeking [110]. The analytic results show smaller failure times than the simulation results which is attributed to the fact that the analytic model under-predicts the ultimate strength of ceramic matrix composites particularly at low values of $\rho$. The Du-McMeeking model shows a good correlation with the analytic result because they use Eq. 4.9 and 4.19 in their analysis. A very significant point to note is that, our slip lengths are only 20% greater than those estimated by Du-McMeeking, and this would imply that our analytic model for failure times should be shorter than theirs by a factor of $1.2^{1/(\rho+1)}$. However, the simulated failure times are a factor of ~2 longer than Du and McMeeking. This shows that the benefit of the complex unit-cell analysis for a detailed $\tau(t,z)$ is somewhat negated by the use of analytic strength model. The analytic model with our simplified estimate of $\tau(t)$ provides, in general, a good estimate of failure times over a wide range of loads and other parameter values.

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Large pullout lengths at fracture surfaces of specimens subject to creep at low load levels have also been observed [59]. The simulation model was used to predict pullout lengths at the fracture surfaces created by composite failure due to matrix shear relaxation. The results of the normalized pullout lengths $L_f/\delta_c$ predicted for the different load levels is shown in Figure 30. From the data the change in pullout length was seen to be solely dependent on the relaxation of the shear stress and not on the creep rate exponent $n$ so only data for $n = 3$ has been shown. The value of $n$ only relates to the rate of the process and not to the amount of damage accumulated, while the pullout length is strongly dependent on the Weibull modulus $\rho$ of the fibers. Again this is due to the amount of damage in the fibers which is directly related to the size of the specimen and so to the minimum composite length required for the simulation.

5.6 Applications and Summary

In developing the model, the fundamental assumption made was that the matrix has reached its characteristic damage state or has completely relaxed such that it is completely relieved of all its axial stress. Few ceramic matrix composites presently under development can satisfy this assumption as they are known to fail prematurely, e.g. due to oxidation-embrittlement, before this state can be reached. Recent work by Weber et al. [129] on the deformation and fracture of SiC fiber reinforced Ti-6Al-4V matrix MMC’s at high temperatures provide rupture data on specimens that did not fail prematurely due to oxidation-embrittlement. In that work Weber et al. [129] performed constant stress

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Figure 30: Comparison Pullout Lengths with Applied Stress for different Weibull Modulii, $\rho = 1-3$
loading at 600°C with the load being applied rapidly. The properties of the constituents at 600°C, were obtained by Weber et al. [129] by extracting the fibers from the composite and from transverse tensile creep data. They obtained fiber strengths after processing and after additional 100 hrs and 500 hrs of aging at 600°C. Since long-term response (say ≥50 hrs) is the focus of the research only the aged fiber strengths, which were the same after 100 and 500 hrs, will be used. The composite constituent properties are shown in Table 3.

Table 3: Constituent Properties of Ti-6Al-4V MMC at 600°C

<table>
<thead>
<tr>
<th>Fibers</th>
<th>$E_f$ (GPa)</th>
<th>$r$ (µm)</th>
<th>$\sigma_o$ (GPa)</th>
<th>$L_o$ (m)</th>
<th>$\rho$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BP Sigma</td>
<td>360</td>
<td>50</td>
<td>1.29</td>
<td>1</td>
<td>5</td>
<td>0.32</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Matrix</th>
<th>$E_m$ (GPa)</th>
<th>$\sigma_{my}$ (MPa)</th>
<th>$B$ (Pa$^{-3}$ s$^{-1}$)</th>
<th>n</th>
<th>$\tau_0$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti-6Al-4V</td>
<td>71.76</td>
<td>280</td>
<td>4.12×10$^{-31}$</td>
<td>3</td>
<td>20</td>
</tr>
</tbody>
</table>

Using the values from Table 3 and Eq. 4.9 the characteristic fiber stress $\sigma_c$ is 3356 MPa and the corresponding predicted fast-fracture strength of the composite using Eq. 4.19 is 747 MPa. The predicted fast fracture strength is lower than measured because aged fiber strengths were used and also Eq. 4.19 completely ignores the load carried by the yielding matrix in the MMC. Since after creep the matrix has relaxed, the appropriate $\sigma_u$ for the calculations should not include the matrix contribution and so $\sigma_u = 747$ MPa is appropriate.
The measurements made by Weber et al. [129] are all at stress $\geq 750$ MPa. Thus, failure is predicted to occur by tensile stress relaxation only. However, we show the predictions of our model to indicate the trend expected in these materials. The present model is valid after relaxation of the matrix, so we also need to find the time required for the matrix to relax that will correspond to an effective $t = 0$ for our model. Here we define the matrix relaxation as the point at which the tensile stress in the matrix equals the shear stress $\tau_0$ at the interface, beyond which $\tau$ dominates in the effective stress in Eq. 5.5. Using the Faben-Curtin model, the time required for the matrix to relax is found to be 13.5 hr. This time is then added to the time obtained from our model.

Using the constituent properties from Table 3 as input into the simulation model, the failure times predicted by our model are then compared with the 600°C constant stress data of Weber et al. [129] in Figure 31. Also shown is the strength versus time during matrix tensile stress relaxation as predicted by Faben and Curtin [128]. There is good correlation between the Faben-Curtin model and the experimental data, indicating that the composites failed before the matrix completely relaxed. As expected the predicted failure times are longer than those observed by Weber et al. [129] because the model is valid only at lower stresses. However, the trend of the data, particularly the last point of the 25 mm data which is in the shear relaxation domain in Figure 31 appears consistent with the predicted results. Also, since these materials are likely to be used at lower stresses, it is important to perform further testing which will also validate the above predictions.
Another interesting phenomenon presented by Weber et al. [129] is the effect of length on the time-to-failure of the Titanium MMC's. The two composite lengths they tested were 25 and 100 mm. The predicted fast-fracture strength of 1023 MPa is slightly lower than the measured value of the 25 mm specimen (1120 MPa) and almost equal to the measured strengths of the 100 mm specimen. This is contrary to expectations as both the specimens are at least twice as long as the characteristic fiber length which from Eq. 4.19 is 9.33 mm long. From the earlier discussion in Section 5.5 it was shown that specimen length had significant effect on failure time due to shear creep at the interface. The length chosen for the simulation was the same as that of the specimens which were \( \sim 2.7 \) and \( \sim 10.75 \) times \( \delta_c \).

In summary, a model for composite failure driven by relaxation of the shear stress at the fiber-matrix interface has been presented. This mechanism occurs on a much longer time scale than the tensile stress relaxation in the matrix, a process that has been well researched, but becomes important at low load levels. The relaxation of the shear stress results in an increase in the "slip" length. The increase in slip length, coupled with the Weibull nature of the fibers and their length-strength relationship, causes more fibers to fail, additional damage to accumulate, and eventually composite failure if the stress on the composite is above the "dry bundle limit".

A simple closed-form solution for the relaxation of the shear stress for \( n = 1, 2, \) and 3 has been presented. The predicted slip lengths versus time were validated by comparing to the results of Du-McMeeking [110], and the predictions over-estimated the values by only 20%. The analytic expressions for \( \tau(t) \) were then incorporated into a Shear Creep At The Fiber-Matrix Interface
Figure 31: Comparison of Predicted Failure Times versus Ti-MMC Experimental Data [129]
numerical simulation technique to predict the time-to-failure as a function of $n$, $\rho$ and
applied load. The insensitivity of failure times to the spectrum $\{t_i\}$ of fiber failure times
has been shown, where $t_i$ is time for the $i$th fiber break with slip relaxation then evolving
as $\tau(t-t_i)$. This allows all local changes in $\tau$ to be approximately represented by $\tau(t)$ where
$t=0$, such that an analytic relationship between applied stress and failure time can be
obtained (Eq. 5.18 and 5.19).

The model has been applied to simulate the failure of Titanium MMCs under
creep loading. In comparison with available data longer failure times have been predicted
because the high loads used in the experiments cause failure in short time spans, which
are inadequate time to allow the matrix to relax its tensile stress. An assessment of the
accuracy of the present model cannot be made until data is obtained at lower loads.
6. SLOW CRACK GROWTH

6.1 Introduction

It has been speculated that the highest pay-off from aerospace applications of ceramic matrix composites will be at high temperature, extended time, and long term load bearing service at temperatures greater than 1000°C [133]. This requires that the fibers exhibit thermo-mechanical and thermo-oxidative stability. The literature review in Chapter 2 clearly identified the susceptibility of ceramic matrix composites to degradation when exposed to a thermo-oxidative environment, however, the cause of composite rupture has invariably been associated with the chemical breakdown of the fibers at those elevated temperatures. In order to study this problem of thermo-oxidative stability DiCarlo [133] recommends that the failure of monolithic ceramics be studied, because, ceramic fibers in effect are monolithic ceramics with high aspect ratios. Ceramics have been known to fail due to brittle fracture which is often preceded by slow crack growth. This results in delayed failure and reveals the time dependence of strength [134]. This delayed failure is known to occur without warning, and can do so within weeks, months or years after the first application of the load. This type of failure must be understood to avoid failure [133]. The detailed understanding of the micromechanical mechanisms of failure in fiber-reinforced composites has been an active area of research in recent years due to the desirable mechanical properties of such composites under quasi-static tensile loads applied parallel to the reinforcing fibers. The fast-fracture ultimate
tensile strength in metal and ceramic matrix composites is now fairly well-understood, including the effects of the fiber statistical strength distribution and the sliding resistance between fibers and matrix [110]. For composites intended for use at elevated temperatures, one of the most critical remaining issues is the lifetime of the composite. Present materials do not possess oxidation-resistant interface coatings and so are susceptible to oxidation-driven embrittlement. The development of new interface coatings, external component coatings (as for C/C composites) and/or oxidation-inhibiting matrices are clearly needed to improve composite durability under high temperature, oxidative conditions. However, supposing this major problem were overcome, what would be the expected life of such a "protected" component? It is well-established that the load-bearing fibers in ceramic matrix composites, and even in some metal matrix composites, lose their strength over time at elevated temperatures. The loss in fiber strength is supported by recent strength data on various ceramic fibers obtained at elevated temperatures and subject to various atmospheres, via single-fiber testing in the laboratory at convenient gauge lengths [135]. Given such fiber degradation, what is the composite lifetime relative to the measured single fiber lifetimes? The answer is not immediately clear because simple concepts such as the rule-of-mixtures do not strictly apply to ceramic composites. The composite failure depends on accumulated fiber damage at a critical gauge length different from that investigated in the laboratory. Large differences in the failure time between individual fibers and composites can thus arise, depending on the constituent material properties.

Slow Crack Growth
In this Chapter, the slow-crack growth model of individual fiber degradation in
time is used and the consequences of such a constituent degradation mechanism on the
overall composite failure is analyzed. Slow crack growth is a well-established strength-
degradation mechanism in monolithic ceramics such as Silicon Carbide, and the rate of
degradation can be very sensitive to the applied stress level. While loss in fiber strength
is known to occur, the mechanisms are not always clearly identified. Here, a slow crack
growth model is assumed because of the tractability of the resulting model and the clean
association between fast-fracture strength and time-dependent strength in such a model.
Experimentally, fiber strength degradation can always be fit to a slow-crack growth
(SCG) model and the resulting parameters utilized to predict expected composite lifetime;
it is anticipated that the predictions will not be too far from reality even if the SCG model
is not applicable.

The high non-linearity in the rate of strength loss observed in many ceramic fibers
is an important issue in a composite because individual fibers experience increasing stress
as other fibers fail. The interplay between the statistical distribution of flaws which are
growing in time and the stresses driving the growth of these flaws determines the overall
damage evolution and final failure in the material. Here, an equation for composite strain
is developed that when solved with the equation for damage in a composite described
earlier in Section 4.5 leads to predictions of composite failure as a function of all of the
underlying micromechanical parameters of the material. Furthermore, a simple
approximate relationship is obtained between the single fiber life at a laboratory gauge
length $L_o$ and composite life, a relationship that should be a useful tool for fiber

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developers and testers. Strength degradations as measured by Yun et al. [136] are used to assess composite lifetime versus fiber lifetime for Nicalon fibers at various applied loads.

Composite failure by the SCG mechanism is also modeled by numerical simulations of composite failure using an appropriately modified version of a simulation model previously developed by Curtin [12]. The numerical simulations capture the same details that were exhibited by the analytic model as shown by the results described below. The two models are also used to measure the remaining strength of the composite as a function of time, which is also an important design consideration.

The remainder of the chapter is organized as follows. In the next section experimental evidence of fiber degradation and specifically the existence of slow crack growth in fibers is presented. In Section 6.3, a review of the well-established slow crack growth model for ceramics and its application to fibers having a statistical strength distribution is presented. In Section 6.4 the time-dependent damage evolution occurring in the composite and its application to the stress-strain-damage constitutive relation that allows the full deformation to be determined is presented. A variety of results for different parameters are then presented and discussed, the relevant time scale for failure is demonstrated, and a simple expression for the time to failure relative to that of a laboratory fiber is presented. In Section 6.5, the simulation procedure for the composite failure process is described and results are presented and compared to the analytic results. Section 6.6 contains an application of the results to a Nicalon fiber composite, further discussion, and a summary.
6.2 Literature Review

Early work by Weiderhorn [134] has shown that slow crack growth in ceramics can be described using crack velocity and the stress intensity factor for a given microstructure and corrosive species. Slow crack growth in a ceramic can be affected by the environment, temperature and stress level. The number of studies on slow crack growth in monolithic ceramics are far too many to even mention and vary greatly depending on the material processing technique, densification additive used, testing environment and geometry. It will suffice to say that there are two main mechanisms that promote slow crack growth in ceramics they being chemical reaction at a stress concentrator resulting in stress corrosion cracking and the other high temperature viscous/diffusive creep damage [134].

Most ceramics exhibit a threshold stress intensity factor ($K_{th}$) below which no crack growth occurs. Minford and Tressler [136] investigated the $K_{th}$ for two silicon carbide ceramics in two different environments. In the non-oxidizing environment they measured $K_{th}$ values of 2.25 and 1.75 MPa√m, at 1200°C and 1400°C, respectively, while at the same temperatures in an oxidizing environment the values dropped to 1.75 and 1.25 MPa√m. Yavuz and Tressler [137] studied the threshold stress for three polycrystalline silicon carbide materials between 1200°C-1400°C and used Weibull distributions at the static load temperature to determine the $K_{th}$ values. For the CVD SiC they could not determine the threshold value due to low Weibull modulus of the material but for the other two, sintered and HIPed silicon carbides, they obtained values of $K_{th}$ between 3.30
and 1.70 MPa\(\sqrt{\text{m}}\) which they correlated well with the diffusive crack growth model proposed by Chuang [138]. In the same manner as threshold stress intensity manifests itself in rupture tests on monolithic materials; creep tests performed on ceramic composites show the existence of threshold stress on creep rates [51,57,139]. The authors did not directly ascribe this threshold stress to the onset of the slow crack growth mechanism in the fibers, since inspection of the fractured surfaces was not performed one cannot ignore the possibility of the existence of such a mechanism. However, they did recognize that fiber failure was responsible for this change in the rate of failure.

Having determined that rupture of the fibers limits the applicability of such composite materials; recent interest in evaluating the slow crack growth in ceramic fibers has grown. Giannuzzi et al. [140] were the first to investigate high temperature creep and strength of Carborundum's polycrystalline sintered 30 \(\mu\text{m}\) \(\alpha\)-SiC fiber heat treated for 100 hr at 1400\(^{\circ}\)C, air and Argon. Testing was performed on the Advanced Fiber Tester (AFT) developed at Penn State. Using a TEM to examine the fiber fracture surfaces they found evidence of failure due to slow crack growth. The used intermittent static loading and dynamic fatigue tests to obtain the \(K_{th}\) value and threshold stress below which no crack growth occurred. Creep rates were obtained for fibers tested over 20,000s. They used the following relationship along with published values of \(K_{Ic}\) to obtain a \(K_{th} = 2.1\) MPa\(\sqrt{\text{m}}\) which was quite close to the value obtained from tests on monolithic Silicon Carbide.

\[
\frac{\sigma_{\text{static}}}{\sigma_{\text{fast fracture}}} = \frac{K_{\text{stat}}}{K_{Ic}}
\]  

(6.1)

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Following up on the same fibers, Rugg et al. [141] investigated time dependent behavior at high temperature, 1200°C. They compared the strengths of fibers pre-oxidized in air for 100 hr at 1200°C against new fibers. The strength of the former was 782 MPa for a 90mm gage length and 944 MPa for a 210mm gage length for the latter. Rugg et al. [141] ran the same test as Giannuzzi et al. [140] before them and used the following relationships to determine rupture time.

\[ v = AK_f^\beta \]  \hspace{1cm} (6.2)

\[ \sigma_f = \left[ B(\beta + 1) \frac{d\sigma}{dt} S_i^{\beta - 2} \right]^\frac{1}{\beta + 1} \]  \hspace{1cm} (6.3)

\[ t_f = B\sigma_A^{-\beta} S_i^{\beta - 2} \]  \hspace{1cm} (6.4)

In addition, using dynamic fatigue tests, Rugg et al. [141] obtained the following values, \( \beta = 16.4 \) and 26.8 in Nitrogen and air respectively for the pre-oxidized fibers. While for the new fibers \( \beta = 14.5 \) in air which correlated well with that of the bulk material under similar conditions.

In another study Tressler et al. [142] studied the time-to-failure in c-axis sapphire and sintered silicon carbide fibers from 800°C-1500°C and 1200°C-1400°C respectively. They related this delayed failure to slow crack growth, regions of which were evident on the fracture surfaces. The sapphire fibers showed the planar growth of an annular crack from an internal pore bounded by a fracture mirror, and on the silicon carbide, a region of intergranular fracture bounded by a region of transgranular fracture were seen.
Studying the stress-rupture behavior of polycrystalline Alumina fibers (Nextel 610 and Fiber FP), Yun et al. [143], found that below 1000°C and 100 hr the Nextel fibers with a smaller grain size had greater fast fracture and rupture-strength than the FP fibers. However, the stress-time rupture exponent $\beta$ obtained was found to fall from 13 at 900°C to near 3 at 1050°C for both the fibers. They suggested that the reduction could be attributed to a change in the controlling mechanism from slow crack growth to creep rupture. They also used a modified Arrhenius equation to obtain an apparent stress rupture activation energy of 690 KJ/mol. In another study Yun et al. [144] investigated the tensile creep and stress-rupture of polymer derived Nicalon, Hi-Nicalon and SiC/BN-coated Nicalon SiC fibers at 1200°C-1400°C from 100-1600 MPa for up to 200 hr. in air, argon and vacuum. The ranking of the fibers in terms of maximum rupture time at temperature were Hi-Nicalon, coated Nicalon and Nicalon. At 1200°C-1315°C and temperatures below 500°C most fibers did not rupture in less than 100 hr. They related the initial drop with time to follow the $t^{1/\beta}$ dependence with $\beta$ of about 5-6. They estimated the creep strength of the Hi-Nicalon and Nicalon at 0.2 and 1% and when tabulated the data in the form $t = \sigma^p$ obtained values of $p=4$ and 5.1 at 1200°C for Hi-Nicalon and 2.5 at 1315°C-1400°C for Nicalon. The influence of the air environment seemed to effect the creep and creep rupture of the Nicalon fibers.

In an effort to evaluate the possibility of using single-crystal fibers Sayir [145], investigated the creep resistance of Sapphire at 1500°C. He noticed that the strength at high temperatures was of considerable concern which they attributed to decrease in fracture toughness $K_{IC}$ at elevated temperatures. Sayir [145] also considered the Slow Crack Growth.
possibility of slow crack growth in these fibers and suggested that the fiber strength was a function of loading rate and application time. Fibers 23 cm long with a furnace length of 2.5 cm. were tested up to 1400°C in vacuum. He also found that the fiber tests at 1000°C more dependent on strain rate than those at 25°C. The failures at 25°C and 1000°C were very similar. Repeating the tests at 800°, 900°, 1000°, 1200°, and 1400°C Sayir [145] showed that the slow crack growth stress intensity exponent fell from 28 at room temperature to 10 at 1400°C with most of the decrease occurring at 1000°C and over. He also showed a substantial increase in Weibull modulus \( \rho \) from 5 at 800°C to 18 at 1400°C and attributed the degradation of the fibers at elevated temperatures on the slow crack growth mechanism.

In spite of there not being any direct evidence from experimental data on ceramic matrix composites, of the slow crack growth in the fibers contributing to failure of the material the above literature survey indicates that such a mechanism should not be ignored. The lack of evidence may be due to the fact that it is not a common procedure to study the fracture surface of the fiber when the composite fails. Recent investigations into the slow crack growth phenomena in fibers leaves open the opportunity to relate this mechanism to the failure of the composite. The only study that has been performed to date in attempting to relate the slow crack growth in the fibers to the failure of the composite was that presented by Curtin and Srinivasan [146]. In trying to compare the lifetimes of the composite as against that of the fibers obtained in the laboratory Srinivasan and Curtin found that the fibers in the composite lasted a lot longer than those in the single fiber tests.

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6.3 Slow Crack Growth in Ceramic Fibers

A crack in a brittle ceramic can undergo steady growth in the presence of an applied load and an embrittling environment. Even ceramic fibers have been observed to fail due to the growth of these cracks [135]. Such flaw growth is generally represented by a crack growth rate versus stress intensity $K$ prevailing at the crack tip equation of the form

$$\frac{dc}{dt} = AK^\beta \quad K < K_{lc} \tag{6.5a}$$

$$K = Y\sigma_{app}(t)c^\frac{1}{2} \tag{6.5b}$$

Here, $K$ is the stress intensity factor, $Y$ a geometrical loading factor, and $\sigma_{app}(t)$ is the "applied" stress on the flaw at time $t$. The flaw grows until $K=K_{lc}$, at which point fast-fracture occurs. Applying this to an individual fiber of gauge length $L_i$, it is first recognized that the initial largest flaw size $c_i$ in the fiber is also the flaw that determines the initial, or fast-fracture, tensile strength $\sigma_i$ of the fiber through the relationship $K_{lc} = Y\sigma_i c_i^{1/2}$. Starting with a flaw of size $c_i$, then, and under a time-dependent load $T(t)$, the crack growth rate can be integrated in time to obtain a flaw size $c(t)$ and an associated fiber strength $\sigma(t)$ given by

$$\sigma(t) = \left[ \sigma_i^{\beta-2} - (\frac{\beta}{2} -1)(AY^2\pi K_{lc}^{\beta-2}) \int_0^t dt' T(t')^\beta \right]^{1/\beta-2} \tag{6.6}$$
For a single fiber of length $L_i$ and fast-fracture strength $\sigma_i$ tested in a single fiber tension test at a constant tensile load $\sigma_{app}$, the time to failure $t_f$ is determined as the time at which the fiber strength is equal to the applied load, $\sigma(t_f) = \sigma_{app}$. This leads to the well-known time to failure

$$t_f = \left[\left(\frac{\beta}{2} - 1\right)\left(AY^2 \pi K_{lc}^{\beta-2}\right)\right]^{-1} \left[\frac{\sigma_i^{\beta-2}}{\sigma_{app}^{\beta}} - \frac{1}{\sigma_{app}^{2}}\right] \tag{6.7}$$

Although each individual fiber of a fixed gauge length has a particular strength and time to failure at any load, there are actually many flaws along the fiber which are degrading simultaneously but are not necessarily the strength-limiting flaws at the selected gauge length. The composite is also composed of many fibers and the distribution of fiber strengths at any fixed gauge length must be described statistically. These two features of fiber strength are summarized by defining the number of flaws in a length $L$ which will fail under an instantaneous load $\sigma$ as

$$N(L, \sigma) = \frac{L}{L_o} \left(\frac{\sigma}{\sigma_o}\right)^\rho \tag{6.8}$$

There is thus typically one flaw in length $L_o$ that will fail at stress $\sigma_o$, and the distribution of fiber strengths around the characteristic value $\sigma_o$ is described by the Weibull modulus $\rho$. From standard weak-link statistical arguments the fraction of failed fibers at stress $\sigma$ is $1 - \exp(-N(L, \sigma))$. At any other gauge length $\delta$, the associated characteristic fiber strength $\sigma_0$ must satisfy the Weibull relationship.
\[ \sigma_\delta = \sigma_0 \left( \frac{L}{\delta} \right)^{\frac{1}{\rho}} \]  

(6.9)

Therefore, the characteristic time-to-failure of a typical fiber tested at length \( \delta \) is given by Equation 6.7 above with \( \sigma_\delta \) in place of \( \sigma_t \).

Now consider the flaws described statistically by Equation 6.8 to each undergo the slow crack growth degradation. After a time \( t \), the number of locations at which the flaws make the fiber weaker than the present "applied" stress \( T(t) \) is to be determined. The number of such flaws is exactly the number of flaws that were initially weaker than that initial strength \( \sigma_i \) for which the strength has degraded to exactly \( T(t) \) in the time \( t \).

That is, the number of fibers is \( N(L, \sigma_i) \) where \( \sigma_i \) is the "initial" strength that satisfies, following Equation 6.2,

\[ T(t) = \left[ \sigma_i^{\beta-2} - \left( \frac{\beta}{2} - 1 \right) \left( AY^2 \pi K \right) \int_0^t x^{\beta-2} \, dx \right]^{\frac{1}{\beta-2}} \]  

(6.10)

which is easily inverted to obtain \( \sigma_r \).

### 6.4 Damage and Failure in the Composite

#### 6.4.1 The Model and Constitutive Relations

In Section 4.4 a brief description of a constitutive relationship for a composite with damaged fibers was presented. The applicability of this damage formulation law (Equation 4.24) has been previously demonstrated for time-independent uniaxial tensile
strength in both ceramic and metal matrix composites [109]. In the composite, the fibers experience a time-varying load $T(t)$ which arises from the load transfer from previously failed fibers being cast onto the remaining fibers. Under a general time-varying load, the damage parameter $\tilde{\psi}$ (the number of fiber failures per length $\delta_c$ at stress $T$ and time $t$), is then simply $N(\delta_c, \sigma_l)$ which, when combined with Eq. 6.10 for $\sigma_l$, can be expressed as

$$\tilde{\psi}(T, t) = \left[ T(t)^{\beta - 2} + \left( \frac{\beta}{2} - 1 \right) \left( AY^2 \pi K_{lc}^{\beta - 2} \right) \int_0^t dt' T(t')^{\beta - 2} \right]^\frac{\rho}{\sigma_c^\beta} \frac{1}{\sigma_c^\rho} \quad (6.11)$$

The constitutive relation of Eq. 4.24 combined with the damage parameter evolution of Eq. 6.11 represents an integral equation for the time varying stress $T(t)$ in terms of the underlying micromechanical parameters describing the slow crack growth and the fiber strength and statistics. Failure occurs when no solutions to the integral equation exist, i.e. the damage has reached such a level that Eq. 4.24 cannot be satisfied for any value of $T$. The natural stress scale for this problem is $\sigma_C$, and the associated time scale for degradation is

$$t_c = \left[ \left( \frac{\beta}{2} - 1 \right) \left( AY^2 \pi K_{lc}^{\beta - 2} \sigma_c^2 \right) \right]^{-1} \quad (6.12)$$

Expressing Eqs. 4.24 and 6.11 in terms of normalized stresses $\tilde{\sigma} = \sigma / \sigma_c$ and normalized times $\tilde{t} = t / t_c$ leads to the coupled non-dimensional equations

$$\frac{\tilde{\sigma}_{app}}{f} = \frac{1}{\tilde{\psi}} \left[ 1 - e^{-\tilde{\psi}\tilde{t}} \right] \quad (6.13a)$$

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\[ \tilde{\psi}(\tilde{T}, \tilde{\tau}) = \left[ \tilde{T}(\tilde{\tau})^{\beta-2} + \int_0^{\tilde{\tau}} d\tilde{\tau} \tilde{T}(\tilde{\tau})^\beta \right]^{\frac{\rho}{\beta-2}} \]  

(6.13b)

### 6.4.2 Results of Analytical Model

Eqs. 6.13 cannot be solved analytically, although some progress on the closely-related problem of time-dependent failure of fiber bundles with no matrix has been studied by McCartney and Kelly [147]. Numerical solutions to the integral equation are easy to obtain, however. At a fixed \( \sigma_{app} \), the initial conditions for Eqs. 6.13 can be established by solving Eq. 4.26 for \( \tilde{T}_0 \) with \( \tilde{\rho} = \tilde{T}_0^\rho \). Incrementing time by an amount \( dt \) at constant \( T \) then yields a similar change to the damage in Eq. 6.13b. With this new damage, Eq. 6.13a is then solved for the value of \( T \) required to satisfy Eq. 6.13a. Continually applying this incremental procedure thus gives the time-dependent stress \( \tilde{T}(t) \), composite strain \( \tilde{e}(t) = fE_t \), and damage \( \tilde{\psi} \) as functions of time. The failure time \( t_f \) is reached when there are no solutions to Eq. 6.13a - i.e. enough damage has formed that the damaged fiber bundle can no longer support the applied stress.

Now the applied stress creates an initial slip length \( l_f = r\sigma_{app} / 2\tau \) and the composite damage evolution is controlled by failures in the fibers over twice this length scale \( \delta = r\sigma_{app} / \tau \). Hence, the best time scale to characterize the composite failure process is the time required to fail a "typical" individual fiber of length \( \delta \) under stress \( \sigma_{app} \). Eq. 6.9 gives the typical fiber strength \( \sigma_\delta \) at this length and so the time scale for

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fiber failure at this length, \( t_\delta \) is given by Eq. 3 but with the substitution of \( \sigma_\delta \) for \( \sigma_r \). Introducing the reference time \( t_c \) and normalizing the stresses by the characteristic fiber strength \( \sigma_c \), as indicated by the tildes over the stresses, leads to the time

\[
t_\delta = t_c \left[ \frac{\beta^2}{\sigma_{\tilde{\delta}}^2} - \frac{1}{\sigma_{\tilde{app}}^2} \right]
\]  

(6.14)

The composite failure time is generally related to the characteristic time \( t_\delta \) more closely than to any other time scale. Examples of the evolution of strain and failure in a composite for several values of \( \beta \) and Weibull modulus \( \rho \), at applied stress of 0.5 \( \sigma_u \), are shown versus time normalized by \( t_\delta \) in Figure 32. Figure 33 shows the normalized failure times predicted for a wide range of \( \beta \), \( \rho \), and applied stress.

Considering all of the results for failure time obtained from Eqs. 6.13a,b for combinations of the key parameters \( \beta \) and \( \rho \), an empirical relationship between the time to failure of the composite and the characteristic time \( t_\delta \) has been obtained. The relationship is simply

\[
\frac{t_f(\text{composite})}{t_\delta} = 5 \left( \frac{\beta - 2}{\rho} \right)
\]  

(6.15)

Figure 34 shows the ratio of \( t_f(\text{composite})/t_\delta \) raised to the \( \rho/(\beta-2) \) power for all of the data of Figure 33, which according to Eq. 6.15 should equal 0.2. The accuracy of the simple empirical expression of Eq. 6.15 for applied loads below about 0.5 \( \sigma_c \) (typically

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Figure 32: Normalized strain ($\varepsilon \sigma / E_t$) versus normalized time ($t / t_\delta$) as predicted from analytical model at $\sigma_{app} / \sigma_0 = 0.5$ for $\rho = 3, 5, 10$ and $\beta = 5, 10, 20$. 

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Figure 33: Normalized failure time ($t_f/t_o$) versus SCG exponent $\beta$ as predicted from analytical model for $\rho=3,5,10$
Figure 34: Scaled failure time \( (t/t_b)^{(\rho/\beta-2)} \) versus normalized stress \( \sigma_{app}/\sigma_c \) as predicted from analytical model for \( \rho=3,5,10 \) and \( \beta=5,10,20 \).
<70% σu ) is quite good, with only a factor 2 difference in spite of the four orders of magnitude difference exhibited in Figure 33.

To make the empirical relationship most useful to designers, the composite failure time \( t_f \) (composite) is normalized by the time to failure \( t_f(\text{fiber}; L_o, \sigma_{app} / f) \) of a single typical fiber of gauge length \( L_o \) and fast-fracture strength \( \sigma_o \), tested at the effective stress \( \sigma_{app} / f \). Using Eq. 6.7, the result is, with stresses normalized by \( \sigma_o \):

\[
\frac{t_f(\text{composite})}{t_f(\text{fiber}; L_o, \sigma_{app} / f)} = \left[ \left( \frac{\sigma_{app}}{\sigma_o} \right)^{\frac{1}{\rho}} \frac{\sigma_o}{\tilde{\sigma}_o} \right]^{-(\beta-2)}
\]  
(6.16)

Prediction of composite life is thus obtainable quite simply and rapidly from experimental data on fibers tested in the laboratory, independent of many other details of the failure process, if this mechanism dominates.

6.5 Simulation of Rupture

6.5.1 Simulation Model

The analytical model results in Figure 33 show a variation of four orders of magnitude in failure times of the composites for a range of values of Weibull modulus \( \rho \) and crack growth exponent \( \beta \). The analytic solution, however, is based on the assumption that the damage parameter \( \tilde{\gamma} \) is small and the damage is randomly distributed. Hence, with the strong dependence of failure time on \( \beta \), it is possible that small errors in the

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representation of damage will be greatly magnified, resulting in larger differences between the actual and predicted times-to-failure of the composite. To verify the accuracy of the failure times predicted by the analysis the numerical model described in Section 4.3 that has been shown to provide accurate strengths under quasi-static conditions [12] will be used. Using that same basic model Ibnabdelljalil and Phoenix [131] and Zhou and Curtin [109] modified it to predict the creep rupture of composites and failure of composites due to initial damage, respectively. In this section it will be shown how, with minor modifications, the same approach can be used to simulate time-dependent failure of a composite by fiber SCG. The simulation results will be presented, and then compared to analytic model and the implications discussed.

The Monte Carlo simulation model described in Section 4.3 has the capability of simulating fiber breakage, independent of the mechanism producing the breaks. If the micromechanics of a damage mechanism that will result in fiber breaks is known, then such a technique can be used to predict failure of a composite. Accordingly this technique can be used to simulate time-dependent failure due to slow crack growth. In contrast to the fast fracture problem where the local fiber strengths are fixed and the stress increased to create additional damage, in the time-dependent simulation the load is held constant with the fiber strengths undergoing a time dependent degradation according to the slow crack growth equations presented in Section 6.3.

The slow crack growth simulation of damage evolution and failure proceeds as follows. A constant load, some predetermined fraction of the ultimate load $\sigma_{app}/\sigma_*$, is applied to the composite. The load is applied at a rate that is fast enough such that no
time-dependent degradation occurs. At that applied stress $\sigma_{app}$, a number of weak fiber elements fail, followed by slip at the fiber-matrix interface along each break, and the load in the broken/slipped regions is redistributed globally until the system is mechanically stable (each fiber element is stronger than the local stress on it). The load is then held constant for the remainder of the test and the time-dependent SCG in the fibers is commenced. As time elapses, the strength of each fiber element decreases at the rate given by Eq. 6.10. The stronger fiber elements correspond to elements with smaller flaws, and so if the fiber elements are under the same stress, the weaker elements will fracture in a shorter period of time than the stronger elements. However, as fiber elements fail, and redistribution of stress takes place, the stress in each fiber element varies according to the precise (statistical) configuration of damage at any instant of time, so stronger elements under higher stress can degrade rapidly and fail sooner than the initially weaker elements under lower stress. The simulation algorithm proceeds as follows: (i) finding the next element to fail under the current stress distribution, (ii) calculating the time required for that element to fail $\Delta t$, (iii) degrading strengths of all other fibers for the time $\Delta t$. (iv) breaking the failed element and redistributing the stresses, and then returning to step (i) above. This is operationally carried out by finding the time required for each element to fail under its current load, $\tilde{T}_{m,n}$, using the following form of Eq. 6.10.

$$\tilde{T}(\tau + \Delta \tau)_{m,n}^{\beta-2} = \tilde{\sigma}(\tau)_{m,n}^{\beta-2} - \tilde{T}_{m,n}^{\beta} \Delta \tau$$ (6.17)

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where $\sigma(t)_{m,n}$ is the strength of the element $m,n$ after time $\tau$, and $\tilde{T}(t)_{m,n}$ is the average stress on the unbroken/slipping fiber element $m,n$. The local stresses $\tilde{T}_{n,n}$ are known at any point in the simulation and so the only unknown is $\Delta \tau$. Starting at $\tau=0$, the time $\Delta \tau$ required to fail each element is calculated. The fiber element $m,n$ associated with the minimum $\Delta \tau$ is the element that is the one that will fail next. The fiber at this element location is broken and the neighboring elements are accordingly shielded, and the load redistributed, as described above. In addition during the time $\Delta \tau$ the growth of the cracks in all the other elements results in a proportionate loss in strength of the remaining elements. The new strength of the elements can then be calculated using

$$\bar{\sigma}_{m,n}(\tau + \Delta \tau)^{\beta - 2} = \bar{\sigma}_{m,n}(\tau)^{\beta - 2} - \tilde{T}^{\beta} \Delta \tau$$

(6.18)

with $\Delta \tau$ being the fixed increment calculated above. The same procedure is repeated to find the next time increment $\Delta \tau$ and location $m,n$ of the next break. In order to provide comparison with the results obtained from the analytical model in Section 6.4 the time scale $t_f$ obtained from the simulations is normalized by the reference time $t_{ref}$.

### 6.5.2 Simulation Results

In this section the results obtained from the numerical model are compared with those from the analytical model, for values of $\rho = 3,5,10$ and $\beta = 5,10,20$. These values were chosen so as to represent the extremes and mid-value that are typical of $\rho$ and $\beta$ for fibers of interest. In Figure 35-37 analytical and numerical results for normalized strain
Figure 35: Normalized strain ($\varepsilon \sigma_f / E_f$) versus normalized time ($t/t_\delta$) at $\sigma_{app}/\sigma_u = 0.5$ and $\beta=5,10,20$, as predicted by simulation and analytical models.
Figure 36: Normalized strain ($\varepsilon \sigma_c / E_c$) versus normalized time ($t/t_0$) at $\sigma_{app}/\sigma_u = 0.5$ and $\beta=5,10,20$, as predicted by simulation and analytical models.
Figure 37: Normalized strain ($\varepsilon \sigma_c/E_I$) versus normalized time ($t/t_0$) at $\sigma_{app}/\sigma_u = 0.5$ and $\beta=5,10,20$, as predicted by simulation and analytical models.
versus normalized time \((t/t_0)\), for varying \(\beta\) at constant \(\rho\) are presented. In general, there is good correlation between the results from the two models with the steady state and tertiary phases of the fiber strain being well characterized. However, the results in Figure 35-37 show there to be a better agreement at lower values of \(\beta\) than higher, with the maximum difference in failure times being a factor of 2 at \(\rho=10\) and \(\beta=20\). In this figure the analytic results show a lower failure time than numerical results at low \(\rho\) while as \(\rho\) increases the situation is reversed. The difference in the initial steady state non-dimensionalized strain at \(\rho=3\) are due to the slight differences in ultimate strengths predicted by the analytic and simulation models. The analytic model is known to predict accurate values only at low levels of damage, but as the value of \(\rho\) falls below 5 the level of damage increases causing increasing deviations from actual values of strength.

Finally, in Figure 35-37 it is seen that the maximum strain at failure from the analytical model is higher than those obtained from the simulations. The difference in strain also tends to increase as \(\rho\) increases. The likely cause is due to the difference in damage that will result in failure of the composite using the two models. The maximum strain is not, however, of major concern as it occurs during the tertiary phase with little or no increase in elapsed time.

The failure times, for the analytic and numerical models, as a function of \(\beta\) for different \(\rho\)'s, are presented in Figure 38. Again, comparing the results of the two models in Figure 38 good correlation is again found, with the maximum difference in failure times occurring when \(\rho=10\). At this \(\rho\) value the failure times are larger than those

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Figure 38: Normalized failure time ($t_f/t_0$) versus SCG exponent $\beta$ for $\rho=3,5,10,20$, as predicted by simulation and analytical models.
predicted for $\rho=5$ and $\rho=20$ which leads us to believe that the $\rho=10$ results are due to statistical variations in the simulation procedure and are not a testimony of the deviation of the analytic model. Performing more simulations would probably rectify this anomaly. It is interesting to note that while Figure 38 confirms one's intuition of the failure time being strongly dependent on $\beta$, it shows very mild differences between analytic and simulation results with increasing $\beta$. Thus it is seen that the difference in the results from the two models do not change substantially with increasing $\rho$ or $\beta$.

### 6.5.3 Simulation of Remaining Strength

So far, most of the emphasis has been on the effect of SCG on the strain and life of the composite. While in Eq. 6.6 an analytic form for the change in strength of the fiber with time was presented, its impact on the change in strength of the individual fibers due to SCG on the strength of the composite needs to be evaluated.

This is done by monitoring the change in remaining strength of the composite in time where remaining strength is defined as the failure stress that is measured when a loading sequence is interrupted (prior to failure) and the composite is then loaded rapidly to failure. In the analytic model the remaining strength is obtained using the following procedure: after the required time has elapsed, the integral in Eq. 6.13b is kept constant, implying no additional increment in damage due to SCG, and the stress ($\bar{T}$) increased till Eq. 6.13a cannot be satisfied, i.e., the composite fails. This process is repeated for all the different times at which the remaining strength is required so that a remaining strength-

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time curve can be obtained. Similarly, in the simulation model after the summations of \( \Delta t \) has reached the time at which remaining strength is desired, the SCG mechanism is interrupted (no change in \( \bar{\sigma}(\tau) \)) and the load increased until the composite fails. The remaining strength will then always be somewhere between the quasi-static value and the level of the applied load. As the cracks in the fibers grow they accordingly degrade the strength of the fibers which then will cause a reduction in strength of the composite with time. However, as was shown in other studies on the effect of damage on strength [109,131], measurable changes in strength occur only after a certain amount of damage has occurred. A similar trend is seen due to the SCG phenomenon i.e. at less than 75% of the life there is limited change in the remaining strength but following that the change becomes more rapid. Having seen good correlation between the analytical and simulation models in Figures 35-38, the expected good correlation for normalized remaining strength versus normalized time (\( \psi / \rho \)), for different values of \( \rho \) and \( \beta \) is demonstrated in Figure 39-41.

6.6 Applications and Discussion

Few CMC's presently under development can satisfy the assumption used in this model that the failure not be driven by oxidation-embrittlement. However, the recent development of a Glass-Ceramic Matrix Composite made of Nicalon fibers with dual-layer CVD coated SiC/BN interfaces and a BMAS matrix, by Brennan et al. [148], shows evidence of thermo-oxidative stability provided by the multi-layer coating. Nicalon fiber Slow Crack Growth
Figure 39: Normalized remaining strength ($\sigma/\sigma_c$) versus normalized time $t/t_s$ at $\sigma_{app}/\sigma_u = 0.5$ and $\beta=5,10,20$, as predicted by simulation and analytical models for $\rho=3$. 
Figure 40: Normalized remaining strength ($\sigma/\sigma_c$) versus normalized time ($t/t_\delta$) at $\sigma_{yy}/\sigma_u = 0.5$ and $\beta=5,10,20$, as predicted by simulation and analytical models for $\rho=5$.
Figure 41: Normalized remaining strength ($\sigma/\sigma_c$) versus normalized time ($t/t_\beta$) at $\sigma_{app}/\sigma_u = 0.5$ and $\beta=5,10,20$, as predicted by simulation and analytical models, $\rho=10$. 
stress-rupture data presented by DiCarlo [133] and stress-rupture data on the Nicalon/BMAS composite [148] will be used in an example to demonstrate the application of this time-dependent model.

The data on the fast-fracture and stress rupture failure of Nicalon/BMAS material presented by Brennan et al. [148] has been summarized in Tables 4 and 5.

Table 4: Fast-fracture Properties of Nicalon/BMAS [148]

<table>
<thead>
<tr>
<th>Layup</th>
<th>Temp (°C)</th>
<th>$\sigma_u$ (MPa)</th>
<th>Proportional Limit (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0°/90°]</td>
<td>20</td>
<td>237</td>
<td>95</td>
</tr>
<tr>
<td>[0°/90°]</td>
<td>1100</td>
<td>272</td>
<td>88</td>
</tr>
</tbody>
</table>

Table 5: Rupture Properties of Nicalon/BMAS [148]

<table>
<thead>
<tr>
<th>Layup</th>
<th>Temp (°C)</th>
<th>Load (MPa)</th>
<th>Test Time (hr)</th>
<th>Remaining Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0°/90°]</td>
<td>1100</td>
<td>138</td>
<td>14685</td>
<td>248</td>
</tr>
</tbody>
</table>

Stress-rupture data for Nicalon fibers in air has been presented by DiCarlo et al. [149] in a "Larson-Miller" type form using the following relationships:

$$\ln(\sigma_{app}) = \left( 2.3E - \frac{\phi}{R}Q \right)$$  \hspace{1cm} (6.19)

$$Q = RT\left( \ln(t_f) - D \right)$$  \hspace{1cm} (6.20)

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where $\phi$ and $E$ are material parameters, $R$ is the universal gas constant, $f_r$ is the rupture time, $T = \text{temperature in Kelvin}$, and $D = 42$ for $f_r$ in seconds. From the "Larson-Miller" equations (6.19 and 6.20) it can be seen that $f_r \propto \sigma^{-\phi T}$ which, when equated with Eq. 6.7, gives the relationship between the slope of the "Larson-Miller" curve and the SCG exponent to be

$$\beta = \frac{1}{\phi T}$$  \hspace{1cm} (6.21)

Using equation (6.21) for Nicalon fibers at 1100°C gives a SCG exponent $\beta = 7.283$.

The load level during the rupture test was clearly above the matrix cracking stress. While Brennan et al. [148] reported ultimate strengths at high temperatures they did not present any strength statistics for the fibers. However, the statistical parameters for other, similar hot-pressed material Nicalon-Glass matrix composites have shown fiber Weibull modulus of $\rho=3.4$ [110]. Since the primary objective in this section is to demonstrate the application rather than the validation of the model an $\rho=3.5$ is assumed. Fitting the measured ultimate strength with $\sigma_u$ from Table 1 to the predicted strength from Eq. 4.19 using $f=0.225$ (45% total fiber fraction) and $\rho=3.5$, a $\sigma_c=1850$ MPa is obtained. Then using an interfacial sliding resistance of $\tau = 90$ MPa measured by Sun et al. [150] Eq. 4.9 is inverted to get $\sigma_0=425$ MPa. Using the values so obtained above for $\beta$, $\rho$, $\sigma_c$, and $\sigma_0$ and the failure times measured by DiCarlo et al. [149], the applied stress versus failure time (analytical model Eq. 6.13 and Eq. 6.15 approximation) for this composite at 1100°C is shown in Figure 42. Also shown is a single experimental data point for a specimen which did not fail after 14685 hours at 138

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Figure 42: Predicted applied stress $\sigma_{\text{app}}$ versus failure time for Nicalon-reinforced BMAS matrix composite (solid lines). Also shown are predictions for several other combinations of material parameters $\rho$ and $\tau$ (dashed lines); the single fiber Nicalon data; and the one experimental data point which had not yet failed.
MPa. The predicted failure time at this load level is 60300 hours, which is longer than the duration of the experiment. The predicted remaining strength after 14865 hours at 138 MPa is 255 MPa, which agrees very well with the measured value of 248 MPa. Also included in Figure 42 the rupture time of single fibers obtained from Eqs. 6.19 and 6.20 at the effective stress of $\sigma_{\text{app}}/f$ to emphasize the difference in lifetimes experienced by the fibers and composite. For the same load as the composite the single fiber rupture data predicts a life of 35.5 hours. Finally, since the composite life predictions are sensitive to variations in $\tau$ and $\rho$, lifetime curves for other combinations of those two parameters are also included in Figure 42.

In summary, a model for time-dependent failure in fiber-reinforced composites by fiber degradation has been presented. The validity of the analytic model has been confirmed using a numerical simulation technique. It has been shown that the rupture life of a composite is a function of the applied stress $\sigma_{\text{app}}$, fiber Weibull modulus $\rho$, and crack-growth exponent $\beta$. At applied stress levels < 0.70 $\sigma_{\text{u}}$, and for the ranges of $\rho$ and $\beta$ chosen, the lifetime predictions are shown to approximately collapse onto a single
7. TIME-DEPENDENT FAILURE MODELS

7.1 Introduction

In the macro-mechanical methodology described in Section 3.5.3.1 a deterministic criterion was used to determine the material failure. This approach is largely a carry over from design techniques used for homogenous materials. Such criteria should be used carefully when incorporated into life prediction methodology for composite materials because i) these materials are NOT homogenous at that the lamina level, and ii) they exhibit statistical variations in strength. Developing strength theories for homogenous material itself must have been a formidable task but to extend it to composite orthotropic material would be even more difficult. Failure in these materials, even under fast-fracture conditions is precipitated by the accumulation of damage at the micro-level that alters the load distribution within the material. So while a single, all encompassing, criterion cannot easily be established, one solution to this problem would be to use a constitutive law for a damaged material, such as presented in Section 4.4. While the foregoing explanation adequately addresses the fast-fracture problem, the combination of the statistical nature of fiber strength and the time dependence of the damage mechanisms in these materials complicates the efforts to predict time-dependent failure.

Most composites are fiber dominated, so it is evident that if the fibers exhibit time-dependent strength then so to will the composites. Time-dependent failure of
composites, often referred to as stress rupture, has been investigated for quite some time [151-157]. The stress rupture of composites has long been recognized to be due to the combination of at least two factors: the statistical and time-dependent strength of the fibers and the "viscoelastic" behavior of matrix [158]. The term "viscoelastic" traditionally relates to stress relaxation of the matrix due to either material diffusional or cavitation effects though it may also be used to refer to micro-level damage. In presenting the damage processes controlling the response of ceramic matrix composites (Chapter 2) it is observed that the response of a ceramic composite to micro-level damage is similar to the viscoelastic behavior in polymer matrix composites.

In Chapters 5 and 6, models were presented for the time-dependent failure of a composite due to shear creep at the fiber-matrix interface and fiber degradation due to slow crack growth, respectively. In Chapter 5 a simple closed form solution for the relaxation of the shear stress was presented and its use with the Curtin strength equation (Eq. 4.19) was shown to give results comparable to the more complicated Du-McMeeking model [110]. In Chapter 6 a rate equation based on slow crack growth in the fibers was used with a constitutive law for a damaged composite to evaluate the-dependent failure due to fiber degradation. In this chapter a study of the time-dependent failure of a composite due the combined effect of both the above damage mechanisms will be presented.

The complexities of the interrelated damage phenomena and the lack of a thorough understanding has limited the ability to adequately model time-dependent

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failure (stress rupture) of fiber reinforced ceramic matrix composites. As mentioned in Section 2.3.2, presently the most common means of representing stress rupture is through the empirical Monkman-Grant equation [85]. An analytical model that couples the two mechanisms that will provide a basis towards the development of a more complete model for stress rupture is presented in this section. In addition an analytical “experiment” will be used to illustrate how such micro-mechanical models can be integrated together to obtain a collective response of the composite. This exercise provides the basis for later models that can be developed with the view of being “integrated” into the same format so that the cumulative response under additional mechanisms can be determined. The method used to study the combined response of the two mechanisms, which independently can have vastly different time scales, will consist of changing the time scales of one of the mechanisms relative to the other and studying its impact on the remaining strength and failure-times of the composite. The results of the analysis will be compared with the results from CCLife for the same damage mechanisms. This exercise is presently only of theoretical interest to demonstrate the potential of this approach. While the overall interest of this study is focused on life prediction, it is accepted that the life or failure-time is only the extreme condition of the evolution of remaining strength. To better evaluate the applicability of the micro-mechanical models developed and to provide fruitful comparison with the CCLife approach the remaining strength results from the two models will be compared. In this manner even subtle differences in the two approaches are bound to stand out.

Time-Dependent Failure Models
The rest of the chapter is organized in the following manner. In the second section of this chapter a brief literature survey on models, relevant to our effort, developed to predict the creep rupture of fiber reinforced metal and ceramic reinforced composites are presented. In Section 7.3 results of the integration of the shear creep and fiber degradation models, from Chapters 5 and 6 respectively, to create a stress rupture model at different time scales presented. In Section 7.4 the same model parameters used in Section 7.3 will be incorporated into the CCLife code and results from this will be compared with results obtained in the previous section. Finally, in Section 7.5 the conclusions of this investigation will be presented.

7.2 Literature Survey

The study of time-dependent failure of composite materials is not new. As early as 1957 Coleman [151-153] studied the time-dependent statistical response of polymeric fibers. Phoenix et al. [154-156] adapted Coleman's theory to Weibull type fibers and used asymptotic analysis to study the creep rupture of fibers. Later, interest turned towards relating the life of the fibers to that of the composite by considering the manner in which load is "shared" after a fiber break. Phoenix and Tierney [157], Lagoudas et al. [159] and Mason et al. [126] studied rupture times of a composite, due to a creeping matrix, by considering "local load sharing" due to fiber breaks, in the composite. In their study Mason et al. [126] used the model of a three-fiber composite. While the earlier
work leaned more towards the visco-elastic response of the composite due to tensile and shear relaxation of the matrix, because of interest in polymeric matrix composites, Ibnabdeljalil and Phoenix [147] studied the statistical aspects of time dependent failure of a brittle fibers in a brittle matrix composite. They used Coleman’s power law [151-153] representation of damage with the Weibull statistical model to study the time dependent failure of fibers. In their model, the probability distribution function for the time to failure of a fiber of length, $l$, under stress history $\sigma(t)$ is given by:

$$
\tilde{F}_l(t|\sigma(\cdot)) = 1 - \exp \left\{ -\lambda_l \left( \int_0^t \kappa(\sigma(s))\, ds \right) \right\},
$$

(7.1)

where: $\kappa(\cdot)$ and $\lambda_l(\cdot)$ are known as the breakdown rule and shape function. The breakdown rule is represented by Coleman’s [151-153] the power law representation of creep:

$$
\kappa(x) = \gamma x^\beta
$$

(7.2)

where they used Phoenix and Tierney’s [157] relationship of $\gamma$ and $\beta$ to represent Arrhenius type rate effects as shown below:

$$
\gamma = \frac{1}{\tau_o} \bar{U}_o \sigma \quad \text{and} \quad \beta = \frac{\bar{U}_o}{kT}
$$

(7.3)

using a Monte-Carlo simulation scheme similar to the one presented in Section 3.3.2, and assuming a constant applied load, Ibnabdeljalil and Phoenix [147] predicted the

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probability distribution of the failure of composites. They also established an empirical relationship tying the failure time of the fibers to that of the composite.

While the work of Ibrabdeljalil and Phoenix [147] has the same general focus as this effort, their work concentrated on the long term creep of fibers under low load. This study is focused on the response of the composites to shorter term creep at load levels above matrix cracking. In their study, using the activation energy approach, Ibrabdeljalil and Phoenix [147] specified the exact degradation mechanism that was the cause of the composite stress rupture while, in this study two specific mechanisms, shear creep at the interface and slow crack growth are modeled. These mechanisms are thought to significantly contribute to stress rupture and are study is performed by varying the relative time scales of fiber degradation and shear creep and evaluating the impact on the overall response of rupture time.

7.3 Analytical Model for Combined Damage Mechanisms

The literature review on creep response of ceramic matrix composites, in Chapter 2, showed evidence that the damage induced in these materials is extremely complex and a function of both stress level and loading rate [54,60]. Each of the damage processes, matrix cracking, interface debonding and fiber breakage result in a continually changing stress state in the specimen even when subject to a static load. However, the contribution of each to the load redistribution is expected to vary significantly. The response of the

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composite to the damage is then determined by the redistribution of the load between the matrix and fibers and subsequently between the broken and unbroken fibers. Such behavior highlights the significant role played by the matrix in influencing the "long term" strength of the composite as it controls the manner in which load is transferred between the fibers and the matrix. This is in contrast to fast-fracture strength in which the role played by the matrix has been shown, in many ceramic matrix composites, to be relatively limited [13].

The relative contribution of each of the damage processes to time-dependent failure of the material is yet to be determined. However, two factors that greatly contribute to the failure are redistribution of stress within the material, and the degradation of the material itself. Admittedly the two are not mutually exclusive. Given the time scale and stress level over which stress rupture is likely to occur (Figure 31 and 42), it is reasonable to consider the combined role of shear creep at the interface and fiber degradation in the time-dependent failure of the composite. In this section a time-dependent failure model, based on the integration of the two micro-mechanical models developed, will be demonstrated. Here the rate expressions for the two processes will be reformulated so that the constitutive law for a composite with damaged can be made a function of interface shear as was done earlier for slow crack growth in the fibers, in Section 6.5.

In Chapters 5 and 6 analytical models for shear creep of the interface and fiber degradation were obtained by taking into account the time-dependence of the

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characteristic fiber strength ($\sigma_c$) and the local stress $T$ respectively. The relationship between these two parameters is shown in Eq. 6.13. From the analytic expression for the failure of a composite subject to shear creep at the fiber-matrix interface, presented in Section 5.5 (Eq. 5.18), it is seen that $\sigma_c(t) \propto \tau(t)^{1/(\rho+1)}$. The constitutive law for a damaged composite presented in Section 4.4 was based on the number of flaws in a length $\delta_c$, with the equation being normalized by $\sigma_c$. However, with a time-dependent change in slip length, both $\delta_c = \delta_c(t)$ and $\sigma_c = \sigma_c(t)$ which requires Eq. 4.24 be rewritten as

$$\frac{\hat{\sigma}}{f} = \frac{1}{\hat{\psi}} \left[ 1 - e^{-\hat{\psi} \hat{T}} \right]$$

(7.4)

where $\hat{T} = \frac{T}{\sigma_c(0)}$, $\hat{\psi} = \frac{\Psi}{\Theta(t)}$, and $\hat{\sigma} = \frac{\sigma}{\sigma_c(0)}$. The relationship between the time-dependent characteristic fiber strength ($\sigma_c(t)$) and its value at $t=0$ ($\sigma_c(0)$) is given by

$$\Theta(t) = \left( \frac{\sigma_c(t)}{\sigma_c(0)} \right)^{\rho+1} = \left( \frac{\tau(t)}{\tau(0)} \right),$$

(7.5)

where $\tau(t)$ for $n = 1, 2$ and 3 can be obtained from Eq. 5.15. Assuming there is no initial damage such $\rho = \hat{T}^w$, the coupled Eq. 7.4 and 7.5 can be solved to obtain the failure time of the composite due to shear creep at the interface. To include the influence of fiber degradation due to slow crack growth into Eq. 7.4, Eq. 6.13b is replaced by

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\[
\hat{\psi}(\hat{T}, \hat{i}) = \left[ \hat{T}(\hat{i})^{\beta-2} + \int_0^\hat{i} d\hat{i}' \hat{T}(\hat{i}')^{\beta} \right]^{\frac{\rho}{\beta-2}}.
\]

(7.6)

Using Eq. 7.4-7.6 the time-dependent failure of a composite due to shear creep at the fiber-matrix interface and slow crack growth in the fibers can be obtained.

To validate the predictive capability of a model under the influence of the two mechanisms acting in concert, it is necessary that the time scales of the two mechanisms be similar in magnitude. If there is considerable difference in the two time scales then one mechanism is likely to dominate and the influence of the less dominant mechanism may not be evident. The material parameters used along with Eq. 7.4-7.6 for the following analytical exercise are presented in Table 5.

Table 6: Parameters used in Analytical Model

<table>
<thead>
<tr>
<th>Weibull Modulus</th>
<th>SCG Exponent</th>
<th>Creep Exponent</th>
<th>(E_fE_m)</th>
<th>(\tau(0)/\sigma_c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho)</td>
<td>(\beta)</td>
<td>(n)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>7.283</td>
<td>3</td>
<td>3</td>
<td>0.01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(E_f)</th>
<th>(\sigma_c)</th>
<th>B</th>
<th>(f)</th>
<th>(t_c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>360</td>
<td>1850</td>
<td>4.494\times10^{28}</td>
<td>0.32</td>
<td>34300</td>
</tr>
</tbody>
</table>

For this exercise the above properties were chosen so that the failure time for each mechanism, acting independently at a load of \(\sigma_{app}/\sigma_u = 0.6\), would be \(1\times10^7\) sec. The failure time under shear creep is directly proportional to \(B\) while due to slow crack

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growth it is a proportional to $t_C$. So to obtain the required failure time of $1 \times 10^7$ sec these two parameters had to be manipulated.

Before analyzing the response of the composite to the combined effect of the two mechanisms it is necessary to validate the expected response of the composite using Eq. 7.4 and 7.6 due to each mechanism separately. Figure 43 shows remaining strength versus time curves for the composite due to fiber degradation and shear creep at the interface at $\sigma_{app}/\sigma_u = 0.60$. The influence of each of the mechanisms on the remaining strength in the material is significantly different.

While fiber degradation produces little reduction in remaining strength up to $\sim 1 \times 10^6$ sec. the effect of shear creep on the material is a little more immediate with the reduction in remaining strength commencing within $\sim 1 \times 10^2$ sec. of loading. The remaining strength due to shear creep shows a steady decline up to failure while, the fiber degradation shows little initial reduction in remaining strength for up to 10% of the failure time after which the change is rapid. The failure time though, due to both mechanisms is the same, $\sim 1 \times 10^7$ sec., which, as mentioned earlier, was obtained by choosing appropriate values for $t_C$ and $B$, for the fiber degradation and shear creep mechanisms, respectively.

Having shown the influence of each mechanism, acting independently on the time-dependence of strength of the composite, analyses will now be performed to evaluate the effect of the two mechanisms acting in concert. The result of the combined effect of slow crack growth and shear creep acting together at equal time scales (slip:scg, 1:1), on the remaining strength, are shown in Figure 44. The remaining strength curve
Figure 43: Remaining strength versus time curves for fiber degradation (scg) and shear creep at the interface (slip) acting independently.
representing the result of the combination of the two mechanisms initially shows characteristics of the shear creep curve while only later, towards the end of life, does it show the effect of the slow crack growth degradation process on strength. While in Figure 43 the failure times were seen to be $\sim 1 \times 10^7$ sec. the combined effect of the two mechanisms reduces the failure time to $\sim 5 \times 10^5$ sec. To better understand the manner in which the two mechanisms interact to create the cumulative remaining strength curve (1:1) in Figure 44 two simple damage accumulation procedures are used. So the actual change in failure time is 4 times more than predicted by the above equations. This exercise shows that in spite of knowing the failure time due to the individual mechanisms, the failure time due to a process that had, say even equal parts, of the two mechanisms could not be obtained intuitively.

To further study the combined effect of the two mechanisms, analyses are performed to evaluate the change in strength and failure time that can be expected when they interact in varying time scale ratios. The term “time scale ratio” refers to the rate of one mechanism relative to the other, e.g. scg:slip 1:1, means that the time scales allocated to each process is the same while, scg : slip, 100 : 1 means that the time scale of the slow crack growth is 100 times the time scale for the shear creep or slip process. Again this is accomplished by appropriately changing $B$ and $t_C$ as required. Multiplying $B$ by 100 will increase the failure time under slip by 100 and vice versa. Similarly, multiplying $t_C$ by 100 will increase the failure time under scg by 100.
Figure 44: Remaining strength versus time curves for fiber degradation and shear creep at the interface acting together in the ratio 1:1
Physically this represents a deceleration of the slip process while emphasizing the role of the slow crack growth effect. This analytic "experiment" is performed to evaluate the relative influence of each of the active mechanisms over a span of time. The shape of the resulting remaining strength curve is strongly influenced by the time scale ratio used. Such an exercise can also be used to check the robustness of a model under varying conditions and to create remaining strength curves for hypothetical situations. The results of such an exercise are presented in Figure 45-46.

In Figure 45 results of the two mechanisms acting in concert at time scale ratios of scg:slip of 1:1, 10:1 and 100:1 are shown. It is immediately evident that as the time scale ratio of the slow crack growth mechanism is increased the remaining strength curve of the combination begins to look like that of the shear creep acting independently. This extreme condition is as would have been expected. Also seen is that an order of magnitude change (1:1 to 10:1) in the slow crack growth rate increases the failure time by a magnitude of 4, while when the ratio is changed from 10:1 to 100:1 the failure time is increased by a magnitude of only 2.75. In addition, the shape of the 1:1 curve shows characteristics of both the independent (scg and slip) curves but as the scg time scale ratio is increased the curves show a transition to the slip curve. The results of the time scale ratios being reversed, that is, when the shear creep time scale is increased relative to the slow crack growth the response, are shown in Figure 46. Here again, it is seen that as the slip time scale ratio is increased the 1:1 remaining strength curve transforms to the scg curve. An increase of one order of magnitude in the time scale ratio (slip:scg, 1:1 to 10:1) increases the failure time by a magnitude of 2.5 while the next order of magnitude

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Figure 45: Remaining strength versus time curves for fiber degradation and shear creep at the interface acting together in time scale ratios slip:scg = 1:1, 1:10 and 1:100.
Figure 46: Remaining strength versus time curves for fiber degradation and shear creep at the interface acting together in time scale ratios scg:slip = 1:1, 1:10 and 1:100
increase (slip:scg, 10:1 to 100:1) results in an increase in the failure time by a magnitude of only 2. Like the results shown in Figure 45 these results too are extremely non-linear.

By performing these analytical “experiments” it has been shown that it is possible to integrate the analytical models for two or more mechanisms and use them with the constitutive law for composites with damage above to evaluate the effect of cumulative damage mechanisms. Such an analytical model would serve as a good foundation for a more complete stress rupture model.

7.4 Comparison of Results from Analytical Model and CCLife

In proposing modifications or replacement to existing methodology it is essential that the new approach be shown to provide the same if not better results than an existing approach. Since CCLife traditionally obtains its input from laboratory experiments on laminates, in this section parameters obtained from the life prediction effort in Section 7.3, for each mechanism acting independently, are used as input into CCLife in the form of stress-time expressions. Remaining strength validation tests are then performed. The data obtained from the analytical model when each damage mechanism was activated independently was curve fit to the following form

\[
\log \left( \frac{\sigma_{app}}{\sigma_u} \right) = A + B \log(t) \tag{7.9}
\]

where for the present analysis

\[ A = 0.52764 \text{ and } B = -0.10800 \text{ for the slow crack growth degradation process and} \]

Time-Dependent Failure Models
\[ A = 0.21296 \text{ and } B = -0.06211 \text{ for the shear creep at the interface process.} \]

One of the main features in CCLife is the manner in which data is input. It is not the intent in this section to provide a detailed description of the application of CCLife which may be obtained from Ref. 160; for the present analyses it will suffice to say that CCLife accounts for changes in strength due to two processes, creep and fatigue. The way these two processes are handled internally however, is quite different. For this analyses Eq. 3.8 is rewritten as

\[
F_r(t) = 1 - \int_0^t \left[ 1 - \frac{S_d(t)}{S_u(t)} \right]^{j-1} d\left(\frac{t}{\gamma}\right), \tag{7.10}
\]

which is the commonly used form when the dominant mechanisms are time dependent. Here \( S_u(t) \) is the change in strength due to a “fatigue” effect, \( \gamma \) is the rupture time at stress \( S_d \), and \( t \) is the elapsed time under load. From Eq. 7.10 it can be seen that a change in strength due to fatigue will alter the failure function \( F_d(t) \) in Eq. 3.10 while change in rupture time will alter the ratio \( t/\gamma \). Both the mechanisms modeled (slip and scg) were shown in Section 7.3 to affect a change in the strength of the composite. Since CCLife can account for change in strength in two ways, for the present analyses it is decided that the change in strength due to shear creep could resemble a “fatigue” process and so will be input as a change in fatigue strength to alter \( S_u \). The effect of the slow crack growth model will be input as a “stress rupture” strength change and so will alter \( \gamma \). The allocation of one mechanism as a “fatigue” strength change and the other as a “stress rupture” process is quite subjective but this exercise is for demonstration purpose only.

**Time-Dependent Failure Models**
To reiterate, though the overall aim of a life prediction methodology is to predict failure time of a composite material, the damage metric is remaining strength which is the property that is monitored. When comparing two models it becomes necessary to define the one that will provide the basis against which the other will be compared. For the present investigation it is decided to use the micro-mechanical model as the base model and compare the CCLife results against it. This decision was made based on the fact that the micro-mechanical model has been shown (Chap. 5 and 6) to give good results for these mechanisms. The macro-mechanical model (CCLife) cannot be used as a base model as it uses parameters that have not been determined for specific mechanism. One such parameter is the non-linear remaining strength exponent \( j \). Traditionally a value of 1.2 has been used for \( j \) principally because of the fact that it worked well for polymeric composites than any other reason. The value of \( j \) however has little bearing on the failure time of the material however, is important in defining the true shape of the remaining strength curve. At any rate this analytical exercise will be used to show correlation and some differences between the two models and generate some questions that will assist future efforts in the development of life prediction methodology.

As was done in the previous section, to show that each of the mechanisms modeled in CCLife using Eq. 7.9 does indeed replicate the analytic model results shown in Figure 43. The results of this analysis for each mechanism acting independently is shown in Figure 47. Figure 47, as expected, shows good correlation for failure time with the slow crack growth model since CCLife is just reproducing the data input. It is however, evident that CCLife shows a slight delayed reduction in remaining strength when compared to the Time-Dependent Failure Models
results of the analytical model though the ultimate failure time is the same. Also shown in Figure 47 is the effect of the exponent $j$ on the normalized remaining strength curve from CCLife. As mentioned earlier the shape of the normalized remaining strength curve is sensitive to the value of $j$. Higher values of $j$ tend to produce normalized remaining strength curves that exhibit a more “sudden death” form than those with low values. For slow crack growth in the fibers $j=1.2$ shows the best correlation with the analytic model. In Figure 48 the representation of remaining strength is a little more complex. Here, the remaining strength obtained from CCLife is represented in two ways for $j=1.2$. Under this mechanism, if the direct output of Eq. 7.8 is used there is little or no change in remaining strength, because reduction in fatigue strength changes $S_u(t)$ the denominator of $F_a = S_{\alpha}/S_u(t)$. Thus under this mechanism there is little change in the normalized remaining strength with all the change being manifested in $F_a$. Since the change in fatigue strength only alters the failure function, the value of $j$ has not impact on the results. Since the analytical model results are shown relative to a constant $S_{\alpha}/S_u$ the normalized remaining strength from CCLife is to modified by

$$res\_str\_new = \frac{F_a(0)}{F_a(t)} \ast rem\_str$$  \hspace{1cm} (7.11)

where $F_a(0)$ and $F_a(t)$ are the failure function $(S_{\alpha}/S_u)$ at cycle 0 and $t$ respectively, to represent the change in normalized remaining strength with $S_{\alpha}/S_u$ is kept constant. The results with using Eq. 7.11 show relatively good comparison with the analytical model result. So, it is interesting to see that depending on how the mechanism is input, into CCLife, the predicted normalized remaining strength can show either very good or little

**Time-Dependent Failure Models**
Figure 47: Comparison of CCLife and analytical model results of change in remaining strength due to fiber degradation only.
Figure 48: Comparison of CCLife and analytical model results for change in remaining strength due to shear creep at the interface only.
correlation with the analytical models. To be consistent, in all future analysis the normalized remaining strength will be obtained using Eq. 7.10. The use of Eq. 7.11 was only meant to demonstrate its impact on the result.

Next, CCLife simulations were then performed to replicate the remaining strength obtained in Section 7.3 and the results are presented in Figures 50-52. It is interesting to note that in all the Figures 50-52 there is good correlation between the failure times predicted by CCLife and the analytic model. This is in spite of the fact that the remaining strength at failure as predicted from the two models is significantly different. In Section 7.3, the complex manner in which the damage accumulated in the system was discussed, so the good correlation between the CCLife and analytic model failure times is an authentication of the “summation” procedure being carried out in CCLife.

7.5 Discussion of Models

One of the advantages in performing such analytical experiments is that each mechanism can be modeled independently and the “experiment” performed under extremely controlled conditions. However, it must be kept in mind that the results of an analytical exercise can at best be only as good as the original model. In the preceding exercise it was shown that even when two models have been “calibrated” to provide the same results when mechanisms are acting independently, can show variations in failure times when the mechanisms are integrated, depending on the time scale ratios chosen. Under certain conditions the slight variations in remaining strength can have significant

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Figure 49: Comparison of CCLife and analytical model results for remaining strength with the fiber degradation and shear creep at the interface acting together with time scales seg:slip, 1:1.
Figure 50: Comparison of CCLife and analytical model results for remaining strength with the fiber degradation and shear creep at the interface acting together with time scales sce:slip, 1:100
Figure 51: Comparison of CCLife and analytical model results for remaining strength with the fiber degradation and shear creep at the interface acting together with time scales $scg:slip, 100:1$
effect when evaluating the combined response, largely due to the manner in which the interaction occurs. In the foregoing analyses relatively good correlation was seen when comparing results of the analytical model with those from CCLife.

Prior use of CCLife has been shown to give good results when predicting failure time as is evidenced in the case study performed in Chapter 3 and Ref. 161. However, those analyses were performed on material in which failure was invariably dominated by oxidation embrittlement. When there is a single dominant failure mechanism then prediction is fairly easy as there is little interaction involved and the characterization of only one mechanism is required. A true test of a damage accumulation model is its ability to predict remaining strength and failure times in situations described in Section 7.3 and 7.4. In the analyses performed in Section 7.3, the failure times of the two mechanisms was intentionally chosen to be the same, to evaluate the prediction of remaining strength by the macro- and micro-mechanical models due to the strong and weak interaction of the two mechanisms. The results showed that, while there was good correlation of the failure times predicted for the different cases studied, the prediction of remaining strength in most cases showed significant differences. This characteristic represents the fundamental difference between the two models and requires some discussion as the definition of remaining strength in the two models is not the same.

While the analytic model presents the results as remaining strength versus time the results from CCLife are, as can be seen in Eq. 7.10, that of normalized remaining strength versus time. If the strength of the material were constant, as is seen in Figure 47 then the two would give similar results, i.e. the normalized remaining strength and

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remaining strength would be the same. This is because the strength $S_u(t)$ is independent of time and so a constant. However, in this case the state of the material is changing, i.e. $S_u = S_u(t)$, so the normalizing value of remaining strength is changing so a direct comparison with the analytic model result cannot be made. This fundamental disparity, in the respective definitions of remaining strength, will persist and make comparison difficult. However, since this only effects the remaining strength value and not the failure time which was shown to correlate well with the analytic model if absolute values of remaining strength were to be output by the two models then good correlation can be expected. The issue then may turn to finding the correct value of $j$.

One source of conflict regarding the true definition of remaining strength may lie in the manner in which the two models calculate the remaining strength. In the micromechanical model failure of the lamina is determined when the unbroken fibers are no longer capable of satisfying global (lamina) force equilibrium. This accounts for the fact that at failure the remaining strength is equal to the applied load. In CCLife however, a critical element is defined such that failure is deemed to occur when the remaining strength in the critical element equals the stress in the critical element. As discussed in Section 7.4 and shown in Figure 48, depending on how the problem is formulated, the stress in the critical element generally increases, not due to damage in the sub-critical elements but rather due to change in strength of the critical element, and so at failure the remaining strength does not fall to the level of the applied load as in seen in Figure 50-52. So here the question that arises is, what exactly constitutes the critical element, is the whole 0° lamina or is it some part of it. This is an on-going topic of discussion that was
initiated and discussed by Halverson [58]. These fundamental issues need to be addressed as the use of such macro-mechanical models begin to be used together with micro-mechanical models.
8. SUMMARY AND CONCLUSIONS

The first part of this investigation consisted of demonstrating the use of an integrated design tool for the design of ceramic matrix composite components. The design tool has two components: one the stress analysis part consisting of the finite element code CSTEM, and the other the life prediction part containing the code CCLife. CCLife is a modified version of the remaining strength and life simulation code MRLife® used for ceramic composites. The design tool has been developed to evaluate the possibility of using an existing remaining strength macro-mechanical life prediction technique to predict the life of ceramic matrix composite components. The applicability of the design tool has been demonstrated using a case study of a notched Nicalon™/Enhanced-SiC [92] plate with a temperature gradient under fatigue loading.

In the case study of the notched plate, stress analysis using CSTEM, has been performed at various predefined cycle steps. The existing version of CSTEM has been modified to be capable of simulating damage as a loss in material stiffness. During the first cycle of loading the notch in the plate causes a stress concentration which results in local damage in the region around the notch. The damage induced in the plate is a function of the local stress. This spatially distributed damage causes local changes in stiffnesses resulting in a corresponding redistribution of stresses.

Following each finite element analyses, the stress file is post-processed using CCLife to evaluate the remaining strength in each element. If at any point in an element the

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remaining strength is lower than the local stress the point was deemed to have failed. When a point failed its stiffness is set to 20% its original value, the stiffness of the element is recalculated and the calculation proceeds. Component failure was considered to occur when the normalized remaining strength, averaged over a characteristic distance (Whitney-Nuismier scheme), fell below the failure function (local stress/local strength) over the same distance.

Good correlation was obtained when the results of the analyses were compared with experimental data. This has been attributed to the dominant role played by the rupture process in the failure of the material. Some of the limitations of the macro-mechanical methodology are associated with the use of an empirical failure criterion and the fact that the rate equations used are specific to the particular batch of material. Small changes even in the interface properties require re-characterization of the material properties.

Most failure in composites is the result of micro-level mechanisms. The use of a macro-level methodology is only partly capable of incorporating all the subtle influences that occur at the macro-level. This methodology is strongly dependent on the test procedure and operator to determine failure modes and criteria and, with the rapidly changing material development schedules and complex loading sequences desired, these are hard to accurately establish.

While the micro-mechanics of fast-fracture of fiber-composites is fairly well understood the effect of damage on the time-dependent response of these materials has not been well identified. As part of the contribution of this investigation to the understanding
of composites, analytic models for the time-dependent failure due to shear creep at the fiber-matrix interface and fiber degradation due to slow crack growth are developed. Both these mechanisms can be the result of cyclic or time dependent processes.

Unlike traditional diffusional or cavitation creep mechanisms, shear creep at the interface is the result of damage in the composite. The relaxation of the shear stress at the fiber-matrix interface, due to the existence of a broken fiber, results in an increase in the “slip” length along the fiber. The increase in slip length, coupled with the Weibull nature of the fibers and their length-strength relationship, causes more fibers to fail, additional damage to accumulate, and eventually composite failure if the stress on the composite is above the “dry bundle limit”. A simple closed-form solution for the relaxation of the shear stress for stress exponent $n = 1, 2,$ and $3$ has been presented. The predicted slip lengths vs. time were validated by comparing to the results of Du-McMeeking [110], and the predictions over-estimated the values by only 20%. The analytic expressions for $\tau(t)$ were then incorporated into a numerical simulation technique to predict the time-to-failure as a function of $n$, $\rho$ and applied load. The insensitivity of failure times to the spectrum $\{t_i\}$ of fiber failure times has been shown, where $t_i$ is time for the $i$th fiber break with slip relaxation then evolving as $\tau(t-t_i)$. This allowed all local changes in $\tau$ to be approximately represented by $\tau(t)$ where $t=0$, such that an analytic relationship between applied stress and failure time could be obtained.

The model has been applied to simulate the failure of Titanium MMCs under creep loading. In comparison with available data longer failure times have been predicted.
because the high loads used in the experiments cause failure in short time spans, which are inadequate time to allow the matrix to relax its tensile stress. An assessment of the accuracy of the present model cannot be made until data is obtained at lower loads.

Slow crack growth is a well-established strength-degradation mechanism in monolithic ceramics such as Silicon Carbide, and the rate of degradation can be very sensitive to the applied stress level. Here the fiber is assumed to be a large aspect ratio monolith in which the interplay between the statistical distribution of flaws which are growing in time and the stresses driving the growth of these flaws determines the overall damage evolution and final failure in the material. Here, an equation for composite strain was developed that when solved with the equation for damage in a composite that allows prediction of composite failure as a function of all of the underlying micro-mechanical parameters of the material. Furthermore, a simple approximate relationship is obtained between the single fiber life at a laboratory gauge length $L_0$ and composite life, a relationship that should be a useful tool for fiber developers and testers. Since this model was developed on the assumption that composite rupture was not influenced by the environment and in reality most ceramic composite specimens have been known to fail due to oxygen embrittlement only one source of data was available to validate the results.

In the last part of this investigation analytic "experiments" were carried out to evaluate the composite response under the combined effect of shear creep at the fiber-matrix interface and slow crack growth in the fibers. To do this the constitutive law for a composite with damage had to be modified to account for the change in $\sigma_C$ due to the

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shear creep at the interface. To fully study the impact of the interaction of the two mechanisms the material parameters were chosen such that the failure times of each mechanism acting independently at $S_a/S_u = 0.6$ was $\sim 1 \times 10^7$. In the analytic "experiments" the combined effect of the two damage mechanisms was evaluated by varying their time scale relative to each other.

Next the stress-failure time results of the analytic model were input into CCLife with the objective being to evaluate the response of CCLife under mechanism that strongly interact with one another. The stress-time data from the shear creep model was input into CCLife as a fatigue strength change input while the similar data from slow crack growth model was input into CCLife as a stress rupture change. The failure time results obtained showed very good correlation between the two models for the various conditions tested. The remaining strength, however, showed considerable differences. This was attributed to the difference in the definition of remaining strength in the two models. The analytic model calculates the remaining strength in the material while CCLife evaluates the normalized remaining strength. In CCLife when fatigue strength is changed the ultimate strength of the material changes so the normalized value is not the same in the two cases.

In summary it can be said that there is good correlation between the results from the two models this is in spite of having shown that the manner in which the damage is accumulated in the composite is quite complex. Attempts to replicate the accumulation process using simple additive and multiplicative schemes were not successful.

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