Exchange Rate Theory and Practice
Target Zones and Asymmetrical Currency Substitution

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Blacksburg, Virginia

to the memory of my father,
Capt. M. Müfit Özel

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(ABSTRACT)

This dissertation consists of two independent but related chapters on exchange rate theory and practice. The first chapter analyzes target zone exchange rate systems and provides an extension to the standard model of Krugman (1991). The standard model does not allow for imperfect credibility nor for interventions within the band, thence, it cannot explain the empirical evidence from the target zones of the European Monetary System and the Nordic countries. The model in this chapter allows for a realignment in the central parity of the exchange rate while the fundamentals follow a mean-reverting Brownian motion which models the intramarginal interventions of the central banks. We argue that, with these extensions, the model accounts well for the empirical facts.

The second chapter develops a model of asymmetrical currency substitution. The friction for exchange rate determinacy is provided by costly spot market transactions. A stationary equilibrium exists if and only if the cost of spot market transactions is greater than either country's money growth rates, and country 1 money growth rate is lower than that of country 2, given that country 2 has a stronger currency. When the countries cooperate, there are two optimal money growth rates that maximize the steady state utility of the consumers. The optimal money growth rate strictly decreases in currency substitutability and the cost of income taxation. When they do not cooperate, there is a unique positive Nash money growth for country 1 which increases in response to increases in country 2 money growth if the transaction costs are not too high.

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Finally, and the most importantly, I would like to thank GOD for all of the blessings HE bestowed upon me. Without HIM, I would not be where I am today.

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CHAPTER I

TARGET ZONE SYSTEM OF EXCHANGE RATE MANAGEMENT

I.I - INTRODUCTION

The "modern" systems of international monetary history date back to the late nineteenth century in which the gold standard began. Under a gold standard, currencies were defined in terms of their gold values. The participating countries were required to maintain that fixed value of their money relative to gold, hence, national money supplies were constrained by the growth of the gold stock. As long as gold stock grew at a steady state, in other words, without many new discoveries of gold, prices would also follow a steady path.

World War I ended the gold standard. Europe had experienced rapid inflation after the war and, therefore, could not restore the gold standard at the old exchange values. The attempts of the healthier U.S. and U.K. economies to return to the gold standard at prewar values also failed, and the depression years were characterized by competitive devaluations to stimulate domestic economies by boosting exports.

An agreement at the international conference at Bretton Woods, New Hampshire, in 1944, required that each country fix the value of its currency in terms of gold. The U.S. dollar was the anchor of the system, and $1 was defined as being equal in value to 1/35 ounces of gold. Since every currency had a defined gold value, all currencies were again linked together in a fixed system. However, the currencies were allowed to fluctuate within a narrow ± 1% of official parity. This system called Bretton Woods worked well till 1960 when the dollar, the anchor of the system, had a crisis since the U.S. had run large balance of payments deficits. As a result, dollar was devalued, and the currencies of surplus countries were revalued by the "Smithsonian Agreement" in 1971. The system started operating with currencies fluctuating within ± 2.25% of the new official parities.

The Smithsonian agreement was short-lived. The speculative capital flows forced the major currencies to float, in March 1973, i.e. to be freely determined by the free market forces (fundamentals) of supply and demand. However, while some currencies are freely floating today, such as the U.S. dollar, others choose systems like managed float or target zone. The former refers to a system in which a country maintains a fixed value (or peg) of its currency relative to another currency like U.S. dollar, or a basket of currencies if the trade is not heavily concentrated with one country. The basic idea of a ***"target zone system[1]"*** is that a country or group of countries sets explicit margins within which exchange rates fluctuate. While the exchange rate is within the zone limits, policy can be directed toward other macroeconomic goals. When the exchange rate hits the limits, the policy authority intervenes and defends the zone by, for example, selling (buying) reserves.

The best example to the target zone system is the EMS (European Monetary System). EMS currencies are allowed to fluctuate within a narrow target zone of ± 2.25% among themselves while floating jointly against the other countries. The anchor of the system has been the German Mark (DM). Figures 1 and 2 on the next page show the currencies of two members, Ireland and The Netherlands against DM since the system started its inauguration in 1979.

The literature on target zones took off with Krugman (1991). His monetary model (known as the standard model) expresses exchange rate as a deterministic, stationary function of economic fundamentals which evolve according to a continous random walk. The solution takes a simple exponential form, whose parameters are tied down by a tangency condition closely related to the "smooth pasting" condition of options theory. The interventions by the central bank were assumed to be marginal (only at the edges of the band) and certain (perfect credibility). The model's implications were that the target zone was more stabilizing relative to the free float (honeymoon effect) and the fundamentals and exchange rate had a non-linear relationship (smooth pasting). Also, the asymptotic probability distribution of the exchange rate was U shaped, indicating that the target zone exchange rate mostly wanders around the edges, not the central parity of the band. Svensson (1991) and (1992) analyze also the interest rate differentials in a target zone which decrease in fundamentals. The most comprehensive empirical investigation of the implications above can be found in Flood, Rose and Mathieson (1991). Their results, based on the EMS, confirm the honeymoon effect and the smooth pasting property albeit very weak. However, they find a hump-shape probability distribution for the exchange rate, and no clear patterns for the interest rate differentials.

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[1]The terms "target zone" and "currency band" will be used interchangeably throughout the paper for convenience. However, they do not refer exactly to the same thing (see Williamson (1985), pp 64-65).

The empirical failure of the standard model motivated several extensions. Lewis (1990) allowed for interventions by the central banks within the band (intramarginal interventions) by assigning an endogenous probability of stochastic intervention for different random walk processes, and showed that with that extension, honeymoon effect is stronger, and fundamentals fluctuate within wider bands. Bertola and Caballero (1992) allowed for imperfect credibility while assuming marginal interventions and showed that the relationship between the exchange rate and fundamentals had an inverse S shape, the probability distribution was hump shaped, and the interest rate differentials were rising in fundamentals. Similarly, Bertola and Svensson (1993) incorporated a time varying realignment risk into the standard model, and analyzed the relationship of the divergence of exchange rate from the central parity with fundamentals. The results were confirmed by the empirical evidence; any interest rate differentials were possible.

This chapter is intended to provide a thorough analysis of the target zone exchange rate literature and our extensions of the standard model along with the empirical evidence using straightforward econometric tools. We allow for imperfect credibility with an exogenous probability of realignment in the central parity while fundamentals display a mean-reverting continuous random walk. The implications of the extensions account well for the empirical evidence; target zone is stabilizing, smooth pasting is weakly pronounced, the distribution of the exchange rate is hump-shaped, and the exchange rate band shrinks as the realignment probability goes up. The interest rate differentials, however, still decrease in fundamentals. We also show that the fundamentals of major EMS currencies has a strong mean reversion.

The plan of the chapter is as follows. Section I.II provides an extensive, yet selective, survey of recent theoretical and empirical research on the target zone system of exchange rate management, emphasizing the main ideas and the results without too much technical detail. Section I.III is where we incorporate our extensions of the standard model; intramarginal interventions and imperfect credibility. A summary and conclusion is given in section I.IV.

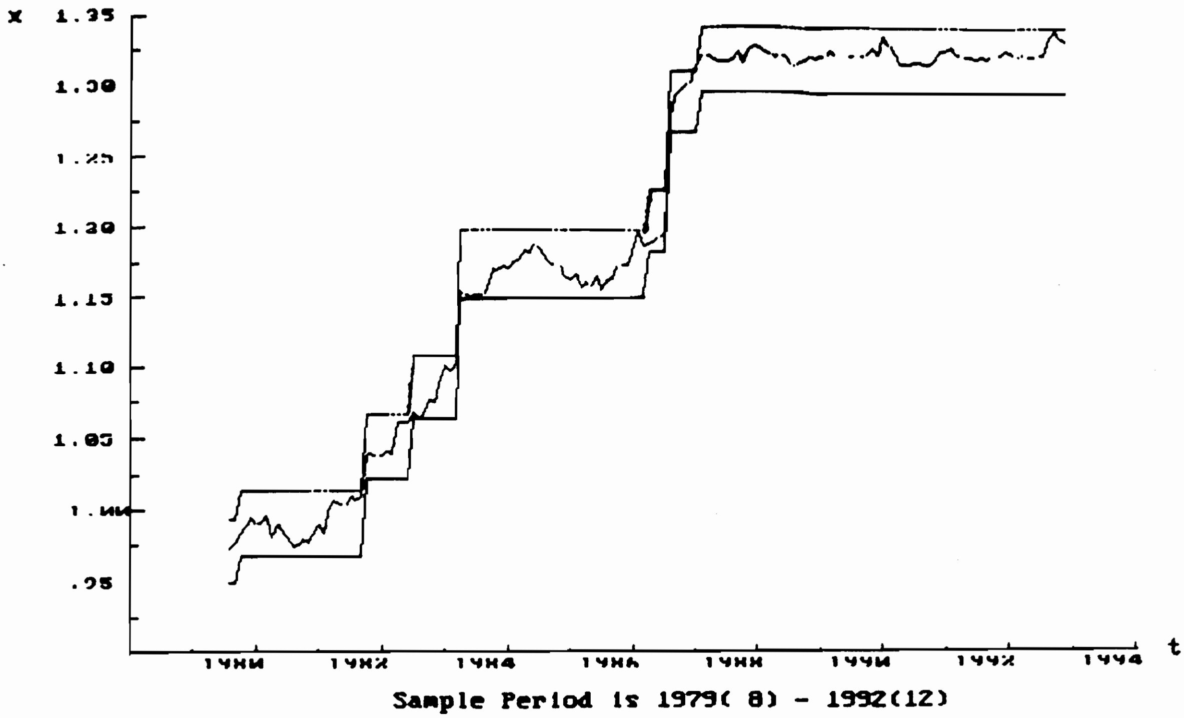

Figure 1: Irish Punt/DM and its target zone

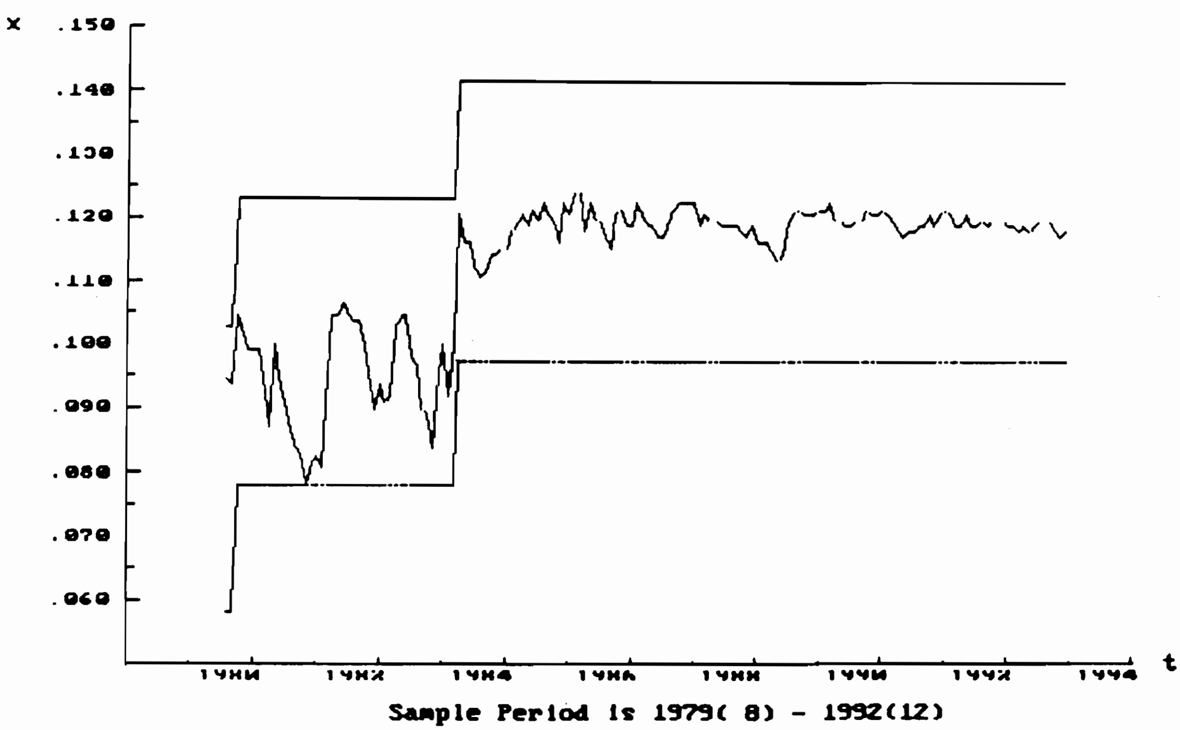

Figure 2: Guilder/DM and its target zone

I.II - A COMPREHENSIVE LOOK AT THE LITERATURE ON THE EXTENSIONS OF THE STANDARD MODEL

In this section, we provide an extensive, yet selective, survey of recent theoretical and empirical research on the target zone system of exchange rate management, emphasizing the main ideas and the results without too much technical detail. The plan of the section is as follows. Section I.II.1 gives the pioneering work of Krugman in this literature; the model's assumptions, set-up, solution, and its implications. Section I.II.2 presents the research which extend the model with introducing imperfect credibility and their empirical support. Section I.II.3 gives the extensions on intervention strategies. Section I.II.4 presents suggestions for future research.

I.II.1 - The Standard Model of Krugman for Target Zones

The recent research on currency bands took off with **Krugman (1991)**[2]. His model assumes the following[3]:

(1) $$i = i^* + \frac{E(dx)}{dt}$$ Uncovered interest rate parity

(2) $$x = p - p^*$$ Purchasing power parity

(3) $$m - p = \gamma + \beta y - \alpha i + \varepsilon$$ Domestic real money demand

(4) $$m^* - p^* = \gamma^* + \beta^* y^* - \alpha^* i^* + \varepsilon^*$$ Foreign real money demand

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[2]First draft is January, 1988.

[3]All variables are in logs, (*) denotes foreign country variables, and time subscript, t, is omitted for convenience; y = output, i = ln(1+i) = nominal interest rate, p = price level, m = money level, E = expectations operator, x = nominal exchange rate, ε = monetary disturbance

Plugging (4), (3) and then (1) into (2) gives the exchange rate[4];

(5) $$x = m + v + \alpha \frac{E(dx)}{dt}$$

where $v = (\beta^* y^* - \beta y) - (\gamma - \gamma^*) - m^* - (\varepsilon^* - \varepsilon)$ and stands for velocity.

If the individuals cannot assign any probability to a devaluation or a revaluation, i.e. E[dx]/dt = 0, the exchange rate is determined solely by the fundamentals, i.e. (m+v); free float case.

Krugman considers v as exogenous and uncontrollable whereas m can be shifted to maintain a target zone. Within the zone limits, v is assumed to follow ***Brownian motion without drift*** [5] such that

(6) $$dv = \sigma dZ$$

where Z is a Wiener process with E[dZ] = 0 , $E[(dZ)^2]$ = dt, and variance, σ^2. The solution of (5) for a symmetric band (see Appendix 1) is

(7) $$x = m + v + A(e^{av} - e^{-av})$$

where a is a distinct real root[6] given by $(2/\alpha\sigma^2)^{1/2}$. Holding m constant, equation (7) suggests a nonlinear S-shaped relationship[7] between the velocity, v, and the exchange rate, x, as in Figure 3 below:

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[4] I assume the interest rate semielasticities, α and α^*, are equal.
[5] See Harrison (1985) chapter 1 for details.
[6] Because $4(-2/\alpha\sigma^2) < 0$.
[7] Provided that $A < 0$ and $a > 0$.

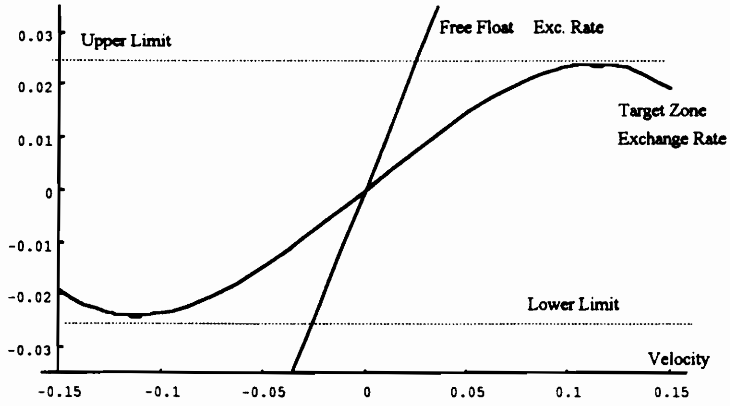

Figure 3: Exchange rate vs. Velocity in a Target Zone

The figure is drawn for a symmetric exchange rate band of limits ± 2.4%.

The straight line shows the 45 degree line which is also the free float case

when A=0, x(v)=m+v.

Whenever the exchange rate hits the upper (lower) limit of the band, i.e. it is too weak (strong), the monetary authority will defend the band. Therefore, the individuals expect appreciation (depreciation) when exchange rate comes close to the upper (lower) boundary.

Clearly, the constant, A, remains to be determined. As long as the band is perfectly credible, the S-curve is flatter than a free float-45 degree line, i.e. the band has a stabilizing effect on the exchange rate variability. This is called "honeymoon effect" by Krugman. Also, this assumption of perfect credibility restricts the path of the S-curve with a tangency to the edges of the band by affecting the expectations. As shown in Figure 4 below, a point like P, close to the upper edge on a curve crossing the edge, constitutes an equilibrium with respect to points O and R and E[dx]/dt=0.

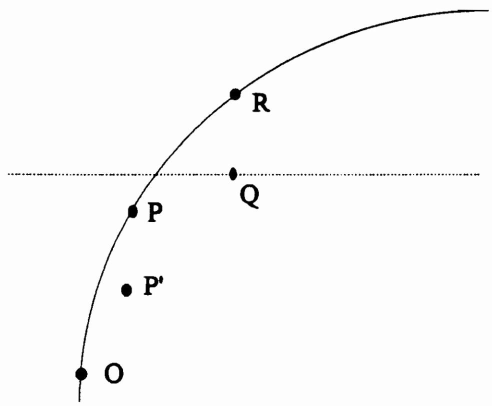

Figure 4: Smooth Pasting Property of

Target Zone Exchange rate

However, with target zone limits in force, E[dx]/dt<0, i.e. there is expected appreciation, because it will not be allowed to go beyond the upper edge to the point R but to a point like Q on the edge passing through P'. In other words, the exchange rate is not sensitive to the changes in velocity at the edges of the band;

(8) $$x_v(\bar{v}) = x_v(\bar{v}) = 0 \quad .$$

Then we may determine A by the following equations, (m=0):

(9) $$\bar{x} = x(\bar{v}) = \bar{v} + A(e^{a\bar{v}} - e^{-a\bar{v}}) \quad ,$$

(10) $$x_v(\bar{v}) = 0 = 1 + Aa(e^{a\bar{v}} + e^{-a\bar{v}}) \quad .$$

Finally, from (10),

$$(11) \qquad A = -\frac{1}{a(e^{\alpha\bar{v}} + e^{-\alpha\bar{v}})} \qquad < 0$$

Clearly, A will take on different values for every change in the process governing (m+v)[8], and the width of the band will also change with different values of A.

In summary, Krugman's model of target zones assumes that

(i) the target zone is perfectly credible, i.e. the monetary authority always defends the band and does not realign the central parity.

(ii) the zone is defended only with marginal and infinitesimal interventions in (m+v). The former means the intervention takes place only at the edges of the band, and the latter refers to very small, nondiscrete interventions.

The model's implications suggest that

(iii) in the long-run, exchange rates should be more frequently near the limits of the band than in the middle of it. Because if, close to the edges, the exchange rate is insensitive to (m+v), it should move slowly (low volatility) and appear most often close to the edges.

(iv) the target zone is stabilizing; the slope of the S-shaped curve is always less than one (honeymoon effect), and flattens to zero at the edges of the band (smooth pasting property).

(v) because of the assumptions of uncovered interest rate parity and the perfect credibility, the interest rate differential equals the expected rate of depreciation, and, therefore, should decrease (increase) as the exchange rate approaches the upper (lower) limit of the band.

These implications of standard Krugman model have been examined empirically by **Flood, Rose, and Mathieson (1991)**[9] with slight modifications, on the EMS data from 30/3/1979 to 16/5/1990[10]. They consider (m+v) as a composite of fundamentals, f, such that;

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[8]This is referred as "stochastic process switching", see Flood and Garber (1983).
[9]Working paper is dated 1990.
[10]Target zone limits of ± 2.25% in the EMS were put into effect on March 30, 1979.

(12) $$x = f + \alpha \frac{E(dx)}{dt}$$

where $f = (\beta^* y^* - \beta y) - (\gamma - \gamma^*) + (m - m^*) - (\varepsilon^* - \varepsilon)$ and stands for fundamentals. Within the zone limits, fundamentals follow Brownian motion ***with drift***;

(13) $$df = \eta dt + \sigma dZ$$

where Z is a Wiener process with E[dZ]=0, E[(dZ)^2] = dt, and variance, σ^2. Then, following the same steps above, the solution for exchange rate in a symmetric band becomes;

(14) $$x = f + \alpha \eta + A(e^{af} - e^{bf})$$

where a and b are given by $-\frac{\eta}{\sigma} \pm \frac{1}{\sigma^2}\sqrt{\eta^2 + \frac{2\sigma^2}{\alpha}}$ .The constant, A, is determined again by the boundary conditions.

Flood et. al. consider mainly the relationship between the exchange rate and the fundamentals. Therefore, their empirical strategy entails obtaining measures of α and E[dx]/dt, and deducing a measure for fundamentals, f. Ignoring the risk premia in their study[11], the uncovered interest rate parity holds, and E[dx]/dt is given by the interest rate differential, $(i - i^*)$. Hence, $f \equiv x - \alpha(i - i^*)$ in every period, and once α is known, f can be calculated for every x. For dt = 1 (1 day), (13) becomes

(15) $$df = \eta + \varepsilon$$

[11]First, Svensson (1990a) argues that both real and nominal risk premia are likely to be small for narrow target zones. Second, Flood et. al. use two-day interest rates whose risk premium is likely to be negligible.

where ε is the disturbance to the process of fundamentals in the form of an integral over one period of σdZ. Substituting (15) into (12) gives

$$(16) \qquad dx = \eta + \alpha[d(i - i^{*})] + \varepsilon \qquad .$$

Finally, by estimating (16) for every 1-day period, η and α can be determined. Looking also to the previous literature[12], Flood et. al report results based on α=.1.

The results of Flood et. al. provide no support for models of credible target zones:

(i) Perfect credibility assumption is rejected easily. Every country in the EMS underwent realignments in the central parities. At first glance, Figures 1 and 2 above show that the Irish and the Dutch target zones lack credibility as the positions of target zones were continuously shifted upwards. However, these target zones (also the other EMS countries) have become increasingly credible in the sense that the periods between realignments have been longer.

(ii) No simple general characterization can be made about x:f relationship while it appears to be approximately linear over the entire sample for every country.

(iii) Consistent with the honeymoon effect, the slope of x:f is often less than unity; target zone is more stabilizing than a free float regime.

(iv) The exchange rate appears most often close to the middle of the band, and, hence, the U-shape of the exchange rate density is rejected.

(v) There are no clear patterns (rather randomness) for the interest differentials.

(vi) The relationship between the volatility of the exchange rate and its position inside the band is occasionally U-shaped, in contrast to the prediction of Krugman's model.

Since the predictions of standard Krugman model are not quite observed empirically, several extensions of the model followed to fit it to the real world data.

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[12] α can be considered as the (-) interest rate semielasticity of money demand which has previously been estimated in the literature (see Goldfeld and Sichel (1990)).

I.II.2 - Introduction of Imperfect Credibility

Imperfect credibility, a dire necessity for the standard model, has been introduced into the model in various ways. **Krugman (1991)** analyzes the case where an imperfectly credible target zone collapses with an exogenous probability, π, and it is defended with probability, $(1-\pi)$ when the exchange rate reaches the edge of the band. When it collapses, the exchange rate jumps to its free float value. Then the solution to the model becomes:

(17) $$x = f + \alpha\,\eta + (1-\pi)A(e^{af} - e^{bf}) \quad .$$

If π=1, namely the market believes for sure that policy authority cannot defend the zone, the stochastic part of the solution above disappears, and the exchange rate equals to its free float value, fundamentals with a drift $(f + \alpha\,\eta)$.

In reality, however, like the EMS, the exchange rate does not revert to a free float when the target zone collapses. Instead, the central parity is realigned, that is a shift from one target zone to another is observed. **Bertola and Caballero (1992)**[13] incorporates such a shift into the model. They further assume that if the target zone is defended, it is defended by discrete marginal intervention in contrast to Krugman's model of infinitesimal marginal intervention, which leaves the band position of the exchange rate at interventions unaltered. More specifically, when the zone is defended, the exchange rate is sent back to the center of the current band, and when the zone cannot be defended, a realignment brings the exchange rate to the center of the new band. Assuming that the exchange rate is a function of not only fundamentals but also their central parity, the driftless solution, i.e. (η=0), is

(18) $$x(f,c) = f + A(e^{a(f-c)} - e^{-a(f-c)})$$

[13]Working paper is dated March (1990).

where $a = \sqrt{\left(2/\alpha\sigma^2\right)}$ .As shown in Figure 5 below, when x hits the upper edge ($f=c_0+I$), either the band is defended with probability ($1-\pi$) and $f=c_0$, or the band position shifts with probability π and $f=c_1=c_0+2I$.

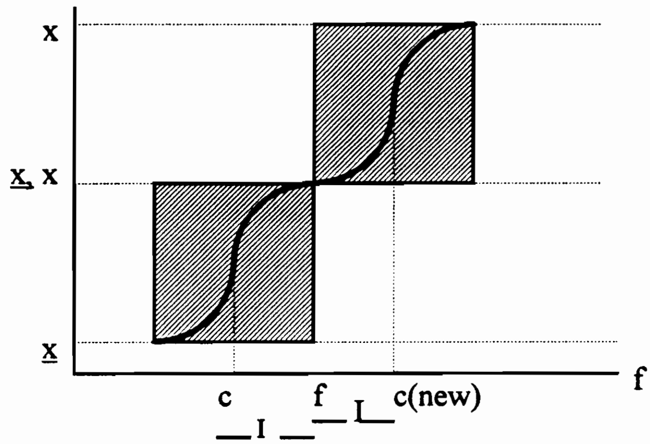

Figure 5: Probability of Realignment

c=center of the current band of fundamentals c(new)=center of the new band of fundamentals I=size of the change in f during a realignment

Speculative arbitrage is eliminated if the two possible moves of the exchange rate weighted with their respective probabilities is equal to the exchange rate when it hits the upper edge:

(19) $$\pi x(c_0+2I, c_0+2I)+(1-\pi)x(c_0,c_0)=x(c_0+I,c_0)$$.

Substituting (18) into (19) gives the new solution:

(20) $$A=\frac{(2\pi-1)I}{e^{aI}-e^{-aI}}$$.

Note that when a>0, as in this model, an S-shape relationship between x and f requires A<0, and that happens only when the policy authority is more likely to defend the target zone, i.e. π<.5. The free float solution emerges when π=.5 and A=0. When the policy authority is more likely to realign the central parity of the exchange rate and shift the band's position, i.e. π>.5, A becomes positive, and an exchange rate function with an ***inverted*** S-shape is obtained. Considering that all the EMS countries have realigned the central parities several times, such an inverted S-shaped relationship between the exchange rate and the fundamentals is closer to the reality. Indeed, with π>.5, the model implies the following, all of which are confirmed by the EMS data except (iv):

(i) The interest rate differential should rise rather than decline as the exchange rate approaches the upper edge of the band.

(ii) The long-run distribution of the exchange rate has more weight in the center than near the edges of the band.

(iii) The slope of the inverted S-shaped curve does not flatten to zero at the edges of the band (smooth pasting property is not valid).

(iv) With π>.5, the exchange rate function is everywhere steeper than it would be under free floating; its slope is greater than unity and the higher likelihood of realignments eliminates the honeymoon effect.

The realignments, however, have not occurred in the EMS only when the exchange rates approach to the edges of the bands. Some bands[14] experienced realignments when the exchange rates have been far from the edges. To account for this, **Bertola and Svensson (1993)**[15] have introduced the time-varying realignment risk into the standard model. They consider the exchange rate and the fundamentals consisting of two parts; the central parity, c, and their deviations (in logs) from c:

(21) $$x = c + x_e \qquad \text{and} \qquad f = c + f_e \quad .$$

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[14]See, for example, the realignments of June 1982 and April 1986 in the French Franc/DM parity, and September 1979 realignment in Irish Punt/DM parity.

[15]Working paper is dated January (1991).

Then the expected rate of (total) depreciation is the sum of two components:

(22) $$\frac{E(dx)}{dt}=\frac{E(dx_e)}{dt}+\frac{E(dc)}{dt}$$.

The first term on the right side of (22) is called the expected rate of depreciation *within the band*, and the second term is considered as the expected rate of realignment (change in the central parity) which works like that of Bertola and Caballero (1992). In other words, when a realignment occurs, the boundaries of the exchange rate band are redefined and both the central parity, exchange rate, and the fundamentals undergo a discrete change. Substitution of (22) into (12) yields

(23) $$x(f,c)=f+\alpha\frac{E(dc)}{dt}+\alpha\frac{E(dx_e)}{dt}$$.

At this point, Bertola and Svensson argue that it should not make sense to plot the exchange rate against the fundamentals to test the implications of the standard model, like Flood et. al. (1991) did. Because the fundamentals, f, omits the expected rate of realignment, E[dc]/dt, which is another state variable and assumed to follow Brownian motion over time. This provides a possible explanation of why the x:f plots of Flood et. al. do not result in a well behaved exchange rate function. Therefore, exchange rate should be plotted to a new state variable which includes expected realignment risk. From (21), one can write (23) as

(24) $$x(f,c)=f_e+c+\alpha\frac{E(dc)}{dt}+\alpha\frac{E(dx_e)}{dt}$$.

Then the new state variable is defined:

(25) $$h \equiv f_e + \alpha \frac{E(dc)}{dt}$$.

Plugging (25) into (24) and subtracting c from both sides gives the deviation of the exchange rate from the central parity as a function of h and expected depreciation rate within the band, so that the model can be solved for x_e:

(26) $$x_e(h) = h + \alpha \frac{E(dx_e)}{dt}$$.

Finally, assuming that h follows Brownian motion with drift, the solution of (26) for a symmetric band is fully analogous to that of Krugman's model:

(27) $$x_e = h + \alpha\,\eta + A(e^{ah} - e^{bh})$$

where a and b are given by $-\frac{\eta}{\sigma} \pm \frac{1}{\sigma^2}\sqrt{\eta^2 + \frac{2\sigma^2}{\alpha}}$.

Furthermore, the extension of Bertola and Svensson (1993) implies that there may not be a deterministic relationship between the interest rate differential and the exchange rate as shown empirically in Flood et. al. (1991). The reason is that the interest rate differential is now the sum of expected depreciation within the band and the expected realignment, i.e.

(28) $$(i - i^*) = \frac{E(dx_e)}{dt} + \frac{E(dc)}{dt}$$.

Even though the expected depreciation within the band is negatively correlated with the exchange rate, depending upon how the expected rate of realignment fluctuates over time and is correlated with the exchange rate, any correlation pattern between the interest rate differential and the exchange rate is possible.

Naturally, it remains to see if the model of Bertola and Svensson (1993) holds empirically. **Rose and Svensson (1991)**[16] for the FF/DM parity during the EMS period, **Svensson (1991)** for the DM parities of the six original EMS countries, and finally, **Lindberg, Svensson and Söderlind (1991)**[17] for the Swedish Krona during 1982-91 period attempt to provide answers to this question. They all use the same strategy of trying to calculate h by estimating the expected rate of realignment, and plotting it against the exchange rate to see if the predictions of the standard model hold.

Under the assumptions of uncovered interest rate parity, expected (total) depreciation is given by the interest differential. Then the expected rate of realignment, E[dc]/dt, can be calculated if the expected rate of depreciation within the band, i.e. $E[dx_e]/dt$ is estimated and subtracted from the interest differential. Therefore what is at stake here is to estimate $E[dx_e]/dt$. This is a futile exercise for floating exchange rates[18], however, as Lindberg, Svensson and Söderlind argue, a simple linear regression of future exchange rates within the band on the current exchange rate (within the band) such as

$$(29) \qquad x_{e(t+\tau)} = \beta_0 + \beta_1 x_e + \varepsilon_{(t+\tau)}$$

seems to predict quite well. The reason is that exchange rates within currency bands display, both theoretically and empirically, strong mean reversion especially for longer maturities). In other words, for long maturities, the expected future x_e depends negatively and linearly[19] on x_e as

$$(30) \qquad E_t x_{e(t+\tau)} = \beta_0 + \beta_1 x_e \qquad .$$

16NBER working paper, no other publications.
17NBER working paper, no other publications.
18A simple random walk usually outperforms forecasting models (Meese and Rogoff (1983)).
19Lindberg and Söderlind (1994) show that, for typical parameters, the relationship between expected future x_e and the current x_e is close to linear.

and the parameters β_0 and β_1 can be estimated using a modified version of (29)[20]. Once $E_t x_{e(t+\tau)}$ is estimated this way, $E[dx_e]/dt$, and then $E[dc]/dt$ can be calculated as explained above. Finally, given α=.1 as in Flood et. al. (1991) and assuming uncovered interest rate parity, fundamentals are given by the exchange rate and the interest rate differential adjusted by α, i.e. $f \equiv x - \alpha(i - i^*)$ in every period, and this completes the derivation of the variables to calculate h, the new state variable which includes time varying realignment expectations, as in (25) above.

The plots of x:h relationship confirms the characteristic S-shape (more pronounced with a larger α), the honeymoon effect, and the smooth pasting property all of which were predicted by the standard model of Krugman. What remains to examine is whether equation (28) has empirical support. It is crucial to see if the interest rate differential has the same (or at least similar) time plot as that the estimated $\{E[dx_e]/dt + E[dc]/dt\}$ for the results following therefrom. No author, however, has presented that information.

I.II.3 - Extensions on the Intervention Strategies

As for the interventions, the standard model of Krugman assumes that the policy authorities intervene marginally, i.e. only when the exchange rate reaches the edges of the band, and also infinitesimally. **Flood and Garber (1989)** extended the standard model to allow both a discrete-sized intervention instead of infinitesimal and the possibility that the policy authority intervenes randomly.

First, a discrete intervention refers to a certain (nonzero) size of discontinuous intervention (selling/buying reserves) in fundamentals, to keep x within prescribed bounds as in Figures 6 and 7.

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[20]Rose and Svensson (1991) uses the form of $\Delta x_{e,t}/\Delta t = \Sigma_i \beta_0 d_i + \beta_1(x_{e,t}) + \beta_2(x_{e,t})^2 + \beta_3(x_{e,t})^3 + \Sigma_j \tau_j x_{t-j} + \Sigma_k \varphi_k x_{e,kt} + \varepsilon_{t+22}$ where t is in daily observations; $\Delta t = 1/12$ year which corresponds to 22 daily observations; $\Delta x_{e,t} \equiv x_{e,t+22} - x_{e,t}$; $i = 1,\ldots,7$ refers to each regime of a new realignment; d_i is a dummy for regime i; $j = 1,\ldots,5$; the $x_{e,kt}$ terms denote the log of the bilateral DM exchange rates for five other EMS participants; and ε_{t+22} is uncorrelated with the information at time t by the assumption of rational expectations.

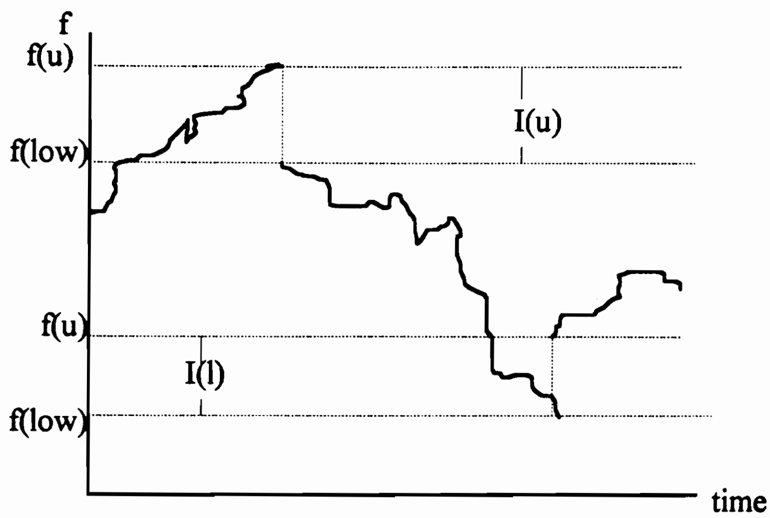

Figure 6: Fundamentals with discrete intervention

Note: I = Intervention size and I(l)<I(u) .

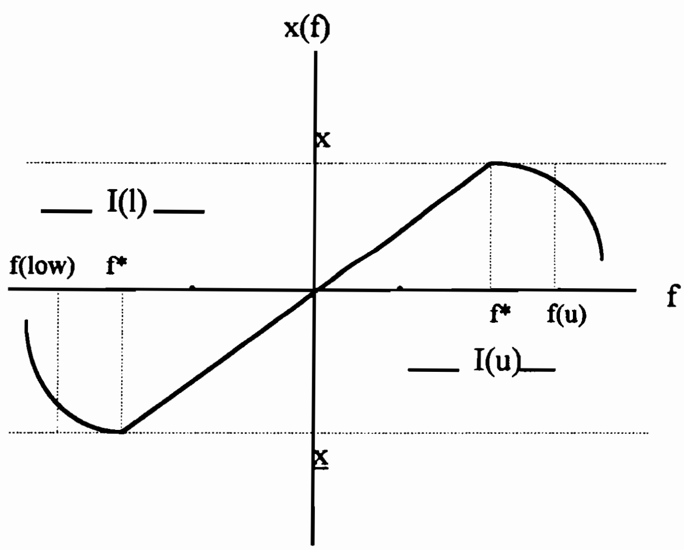

Figure 7: Exchange rate with discrete intervention

f* denotes tangency points; x'(f*)=0, f(u) and f(low) are upper and lower boundaries of the f band, respectively.

If the intervention, I, is let converge to zero, we have the Krugman's case. The addition of a discrete intervention into the model brings it closer to the reality, however, smooth pasting property, now, becomes inapplicable to solve the model (more specifically to find the constant, A, for a symmetric band):

$$(31) \qquad x_f(\bar{f}) \neq x_f(\bar{f}) \neq 0 \qquad .$$

On the other hand, speculative arbitrage would still ensure exchange rate remains the same before and after an intervention:

$$(32) \qquad x(\bar{f}) = x(\bar{f} - \bar{I}) \qquad \text{or} \qquad x(\underline{f}) = x(\underline{f} - \underline{I}) \qquad .$$

Applying (20) to (16) yields the solution for A, given $f = \bar{f}$ and $I = \bar{I}$:

$$(33) \qquad A = \frac{\bar{I}}{(e^{a(\bar{f}-\bar{I})} - e^{b(\bar{f}-\bar{I})}) - (e^{a\bar{f}} - e^{b\bar{f}})} \qquad .$$

The second extension comes by making the intervention policy stochastic. However this time, unlike the first extension, an explicit solution cannot be obtained. A stochastic intervention can be incorporated into the model by affecting the rate of expected depreciation such that

$$(34) \qquad \frac{E(dx)}{dt} = \iint \frac{E[dx|n,y]}{dt} f(n,y)\, dn\, dy$$

where n and y are an intervention strategy and a boundary pair, respectively, and f(n,y) is the probability density function for n,y. For any (n,y), the model can be conditionally solved as above to get the constant, A. Then from (34), the unconditional expected rate of change of x, i.e. E[dx]/dt, can be calculated and substituted into (12) to find the value of the exchange rate with stochastic intervention policy.

As an empirical matter, however, much of the intervention within the EMS has taken place within the bands, not only when the exchange rate reaches the band. Mastropasqua, Micossi and Rinaldi (1988) report that only about 11 percent of gross intervention was at the margin which corresponds to between 20 and 25 percent of EMS related interventions, with the rest taking the form of *intramarginal* operations. Intramarginal interventions are aimed at returning the exchange rate to a specified target level within the band before it reaches the edges. To account for intramarginal interventions in the standard model, **Delgado and Dumas (1991)** considered the interventions as mean reverting towards a central parity. In other words, they specified that the expected rate of change of the fundamentals towards the central parity, i.e. the drift, η in (13) above, is proportional to the distance to central parity:

(35) $$df = -\rho(c-f)dt + \sigma dZ$$

where (f-c) is the distance to central parity, and ρ, positive rate of mean reversion, is the speed with which the process tends towards c. The differential equation implied by combining (35) and (12) is similar to the one in equation (A1.2) in Appendix 1:

(36) $$x = f - \alpha\rho(f-c)x_f(f) + \frac{\alpha\sigma^2}{2}x_{ff}(f) \quad .$$

The solution of (36) requires the use of the confluent hypergeometric functions, M[a,b,Y]:

(37) $$x(y(f)) = \frac{f + c\rho\alpha}{1+\rho\alpha} + W_1 M[\frac{1}{2\rho\alpha}, \frac{1}{2}, y(f)] + W_2 M[\frac{1+\rho\alpha}{2\rho\alpha}, \frac{3}{2}, y(f)] \quad .$$

where W_1 and W_2 are the constants to be determined by the boundary conditions.

Delgado and Dumas solve (37) for both symmetric (i.e. mean reversion point in the middle of the band) and nonsymmetric cases and then analyze the x:f relationship with a change in the width of the band. Furthermore, they find that, for relatively narrow symmetric bands, the degree of fundamentals variability is very large; a result that supports the honeymoon effect. However, this result is lost for asymmetric bands.

Lewis (1990) and **Klein and Lewis (1991)** introduced the intramarginal interventions into the model by assigning an *endogenous* probability of stochastic intervention to the process of fundamentals within the band. The policy authority is assumed to momentarily intervene and to eliminate temporarily the incipient excess supply for foreign exchange. In other words, the fundamentals are not allowed to change. The interventions may occur anytime as the fundamentals within the band deviate from the central parity which yields the target exchange rate:

$$(38) \qquad \begin{aligned} df &= 0 && \text{with probability } \pi(\mathrm{f}-\mathrm{c}) \\ df &= \eta dt + \sigma dZ && \text{with probability } (1-\pi(\mathrm{f}-\mathrm{c})) \end{aligned}$$

and $\pi'(f-c) > 0$. The existence of a probability of intramarginal intervention alters the fundamentals process. In other words, the fundamentals do not follow the same random walk process as specified in (13) with the introduction of stochastic intramarginal interventions. Appendix 2 shows that, when c=0 and f>0, i.e. the currency can only depreciate with an intervention and $\pi(f-c)=\pi(f)$, fundamentals follow a diffusion process given by

$$(39) \qquad df = [\eta(1-\pi(f)) - \sigma^2\pi'(f)]dt + \sigma\sqrt{1-\pi(f)}\,dZ \qquad .$$

Following the same steps in Appendix 1, i.e. solving for E[dx]/dt and substituting into (12), yields

(40) $$x = f + \alpha\{[1-\pi(f)]\eta - \pi'(f)\sigma^2\}x_f(f) + \frac{\alpha\sigma^2}{2}[1-\pi(f)]x_f(f)$$

Different than before, (40) now contains the probability of no intervention, $1\text{-}\pi(f)$, and $\pi'(f)\sigma^2$. The latter term shows that an increase in fundamentals pushes up the probability that intervention will occur and therefore reduces the expected change in the exchange rate. Without this term and a zero probability of intervention within the band, (40) is identical to (A1.4). The analytical solution of (40) requires again the use of hypergeometric functions as in Delgado and Dumas (1991). However, the main implication of the model is still delivered by (40). As shown in Figure 8 below, for a given exchange rate band, the honeymoon effect is stronger than that of the standard model, i.e. fundamentals fluctuate within wider bands when there is intramarginal intervention than when intervention occurs only at the boundary.

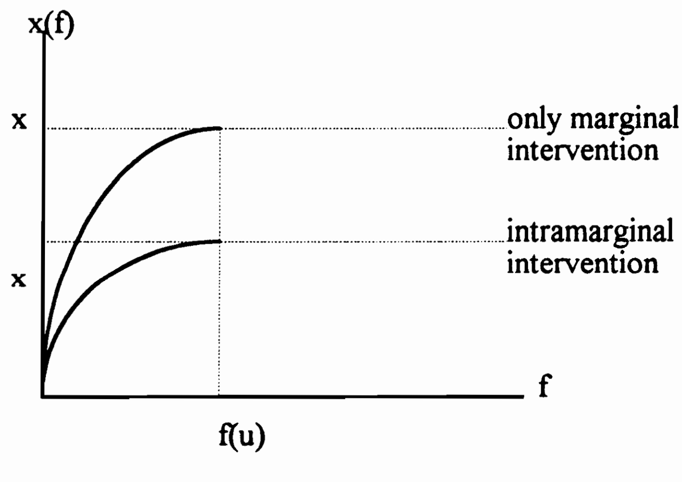

Figure 8: Target zone exchange rate with

without intramarginal intervention

Also, as the probability of intervention within the band increases, fundamentals cluster near the center of the band and so do the corresponding exchange rates, in conformity with the EMS data. However, how much clustering of the exchange rate occurs depends upon how quickly the probability of intervention increases as the fundamentals deviate from the central parity.

Testing the implications of (40) empirically requires estimating $\pi(f)$. It is trivial, however, once the probability of intervention based on the exchange rate, i.e. $\pi(x)$, is estimated because $\pi(f)=\pi(x^{-1}(f))$. Therefore, Lewis (1990) and Klein and Lewis (1991) try to estimate $\pi(x)$ for the implicit bands of the G-3 countries; Japan, Germany, and the United States from 1985 to 1987. Then they convert these estimates into $\pi(x)$, and use them together with parameter values, α, η, and σ, to infer the empirical shape of the equilibrium exchange rate function , $x^*=x(f^*)$, that solves (40). How they estimate $\pi(x)$ is as follows: Three possible intervention events on any given day, t, are designated as

$I_t = 0$ for no intervention,

$I_t = 1$ for intervention to weaken the dollar (dollar sales)

$I_t = -1$ for intervention to strengthen the dollar (dollar purchases).

The probability of these events are regressed against a constant and the closing exchange rate of the previous day as

$$\log[\pi(I_t = -1)/\pi(I_t = 1)] = a_1 + a_2 x_{t-1} \quad \text{and}$$
$$\log[\pi(I_t = 0)/\pi(I_t = 1)] = b_1 + b_2 x_{t-1} \quad \text{and}$$
$$\log[\pi(I_t = -1)/\pi(I_t = 0)] = (a_1 - b_1) + (a_2 - b_2) x_{t-1}$$

where a_1, b_1, a_2, b_2 are parameters to be estimated using the data. A high percentage of the intervention events are correctly predicted by the model indicating that the equations characterize the data fairly well.

The empirical results of Lewis (1990), and Klein and Lewis (1991) indicate a nice S-shape of x(f) for $/DM and $/Yen parities. The smooth pasting property holds at the boundaries. Finally, the observed rates are concentrated far away from the bands.

I.II.4 - Where can we go from here ?

The theoretical target zone consists of only marginal interventions by the policy authority if necessary. However, in practice, exchange rate target zones lack full credibility, and experience not only marginal interventions but also intramarginal interventions and realignments in the central parity. As shown above, several researchers have combined marginal interventions with either intramarginal interventions or realignments in their models. But what we really need is a model which incorporates all types of interventions and the possibility of realignments at the same time to completely account for the real world.

Furthermore, the research above are all on bilateral target zones, however, their empirical tests are not very healthy since the EMS target zone is a multilateral one. In other words, the estimation of fundamentals for an EMS member should involve not only the macroeconomic variables of anchor country but also those of the other countries in the system.

What remains also to see is the impacts of the currency bands on the price and the trade levels. The former is crucial because one major concern of the systems of exchange rate management is to stabilize the price level or to reduce the inflation rate for high-inflation countries. As for the latter, it would be interesting to see, for example, if NAFTA can be more successful with an exchange rate target zone between Canada, Mexico, and the United States with the U.S. dollar being the anchor of the system.

I.III - MODELLING IMPERFECTLY CREDIBLE TARGET ZONES WITH INTRAMARGINAL INTERVENTIONS

In this section we present our modifications on the standard model. These extensions add to the target zone literature in that they allow for an imperfectly credible target zone while assuming mean reverting fundamentals within the band which models the intramarginal interventions of the monetary authority. This means that the devaluation may occur *anytime* within the band whenever fundamentals divert from the central parity. More specifically, the monetary authority which we will name as the central bank announces an exchange rate parity which has a certain band of fluctuation. The implicit policy of the central bank, however, is to reduce the exchange rate variability by leaning against the wind for shocks and intervene whenever the market conditions force fundamentals to divert from the central parity of the implied fundamentals band, and send it back to its central parity. Furthermore, at every attempt of intervention-the argument applies symmetrically for the lower limit-there is a possibility that the central bank may not succeed in its attempt to intervene; it may, for example, be facing a critical level of reserves. Then the central parity is realigned, the target zone shifts position, and the fundamentals continue the mean reverting process to the new central parity.

This section is organized as follows. Section I.III.1 revisits the standard target zone model of Krugman and compares its implications to the empirical evidence. Section I.III.2 extends the model with intramarginal interventions and provides empirical evidence using basic econometric techniques. Imperfect credibility of the target zone along with its empirical evidence is incorporated in section I.III.3. The next section I.IV provides the conclusion and suggestions for further research .

I.III.1 - The Standard Model Revisited

Assuming uncovered interest rate and purchasing power parities, the exchange rate is determined by the fundamendals and the expected change in the exchange rate[21]:

[21] See Flood & Garber (1983) for the derivation of the equation.

(41) $$x = f + \alpha \frac{E(dx)}{dt}$$.

where x is the log of the exchange rate $\in [x^l, x^u]$, f is the log of fundamentals $\in [f^l, f^u]$, and $\alpha>0$ is the semielasticity of money demand to interest rates. Clearly, if there is no expected change in x, fundamentals are the sole determinants of the exchange rate; the free float case.

Fundamentals consist of the money supply and the velocity of money. Whenever the exchange rate hits the upper (lower) limit of the band, i.e. it is too weak (strong), the central bank will defend the band and send x back into the band by decreasing (increasing) the money supply. Within the band, fundamentals follow Brownian motion with drift[22]:

(42) $$df = \mu\, dt + \sigma\, dZ$$.

where Z, independent of f_0, is a Wiener process with $E[dZ]=0$, $E[(dZ)^2]=dt$, and variance, σ^2. By postulating x to be a non-linear, twice continuously differentiable function $x(\cdot)$ of the state f, and applying Ito's Lemma, we get

(43) $$dx = x_f(f)\,df + \frac{1}{2} x_{ff}(f)(df)^2$$.

Inserting (42) into (43), and taking expectations yields

(44) $$E(dx) = \mu x_f(f)\,dt + \frac{1}{2}\sigma^2 x_{ff}(f)\,dt$$.

Then the differential equation implied by substituting (44) into (41) is

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[22] See Harrison (1985), chapter 4.

(45) $$\frac{1}{2}\alpha\sigma^2 x_{ff}(f) + \alpha\mu x_f(f) - x(f) = -f$$.

The general solution to (45)[23] is given by

(46) $$x = f + \alpha\mu + Ae^{af} + Be^{bf}$$.

where a and b are the two distinct real roots given by $-\frac{\mu}{\sigma} \pm \frac{1}{\sigma^2}\sqrt{\mu^2 + \left(\frac{2\sigma^2}{\alpha}\right)}$, and A and B are the constants to be determined by the boundary conditions. If the exchange rate band is assumed to be symmetric, i.e. the upper and lower limits are placed at equal distance from the central parity, then A = -B, and (46) reduces to

(47) $$x = f + \alpha\mu + A(e^{af} - e^{bf})$$.

The constant A is determined by the tangency condition very similar to the smooth pasting condition in option theory[24]. This condition states that the exchange rate is not sensitive to fundamentals at the edges of the band:

(48) $$x_f(f^l) = x_f(f^u) = 0$$.

At the upper edge of the band:

(49) $$x^u = x(f^u) = f^u + \alpha\mu + A\,(e^{af^u} - e^{bf^u})$$.

[23] See Chiang(1984), chapter 15.
[24] See Krugman(1991) for a detailed explanation.

Differentiating (49) and applying (48) yields

(50) $$0 = 1 + A\ (a e^{af^u} - b e^{bf^u})$$.

Finally, from (50) we get

(51) $$A = -\frac{1}{a e^{af^u} - b e^{bf^u}}$$.

Figure 9 below shows the x:f relationship in a standard target zone model. The straight line shows the free float case where x(f)= f + αμ. The x(f) curve is S-shaped since A<0, and flattens to zero at the edges of the band. The exchange rate band is given by $[x^l, x^u]$, and the implied limits on the fundamentals are $[f^l, f^u]$.

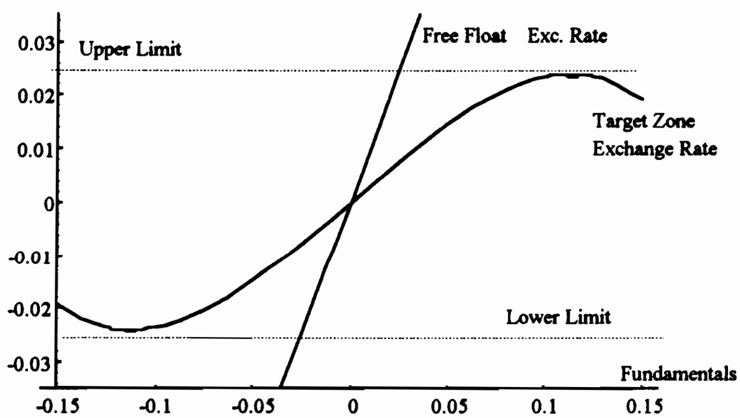

Figure 9: The x:f relationship in a standard target zone model

The figure is drawn for the symmetric (A=-B,a=-b) and no drift (μ=0) case for simplicity. The numerical values of other parameters are α=3, σ=.1, x^u=-x^l=.024, f^u=-f^l=.11. The straight line shows the free float case where x=f.

There are three crucial implications of the standard model for the target zone exchange rate:

(i) The target zone is stabilizing; the slope of the S-shaped curve is always less than that of the free float case (honeymoon effect), and flattens to zero at the edges of the band (smooth pasting property). Flood, Rose, and Mathieson (1991) shows that this result is consistent with the empirical evidence from the European bilateral target zones, however, the smooth pasting of the exchange rate at the edges of the band is not very strong.

(ii) In the long-run, exchange rates should be more frequently near the limits of the band than in the middle of it. Because if, close to the edges, the exchange rate is insensitive to f, it should move slowly (low volatility) and appear most often close to the edges. When the drift is zero, the asymptotic probability density of a regulated Brownian motion is uniform[25], hence the asymptotic probability density of fundamentals, $p^f(f)$, is given by $1/(f^u-f^l)$ for $\mu=0$ and $f\in[f^l,f^u]$. Using the inverse of the exchange rate function, $x^{-1}=x^{-1}(f)=f(x)$, we can get the asymptotic probability density of the exchange rate as $p^x(x) = p^f(f(x)) / x_f(f(x))$[26]:

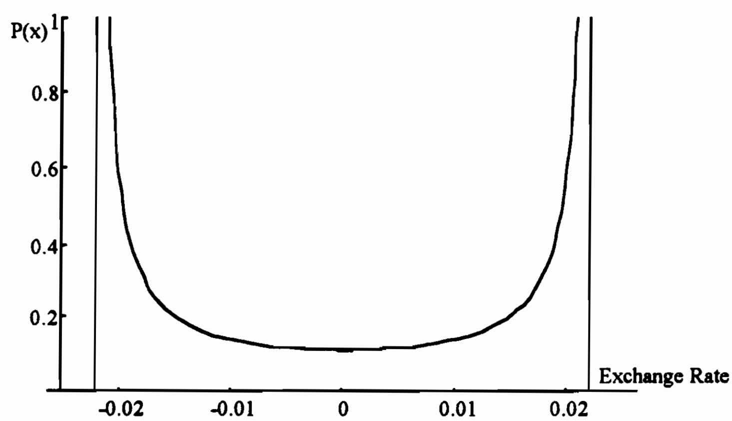

Figure 10: Asymptotic Probability Distribution

of Target Zone Exchange Rate

The figure is drawn for the symmetric, no drift (μ=0) case. The numerical values of other parameters are $\alpha=3$, $\sigma=.1$, $x^u=-x^l=.024$, $f^u=-f^l=.11$.

[25]See Harrison (1985), chapter 5, page 90 for the proof.
[26]See the lemma in Spanos (1986), chapter 6, page 98.

This result fails the empirical tests. Flood et. al. (1991), and Svensson (1992) report that both the European and the Nordic target zone exchange rates appear most often close to the middle of the band. Hence, the U-shape of the probability density of the exchange rate is rejected.

(iii) Because of the assumptions of uncovered interest rate parity and the perfect credibility, the interest rate differential, δ, equals the expected rate of depreciation, and is a function of fundamentals. Therefore, it should decrease (increase) as the exchange rate approaches the upper (lower) limit of the band as shown below:

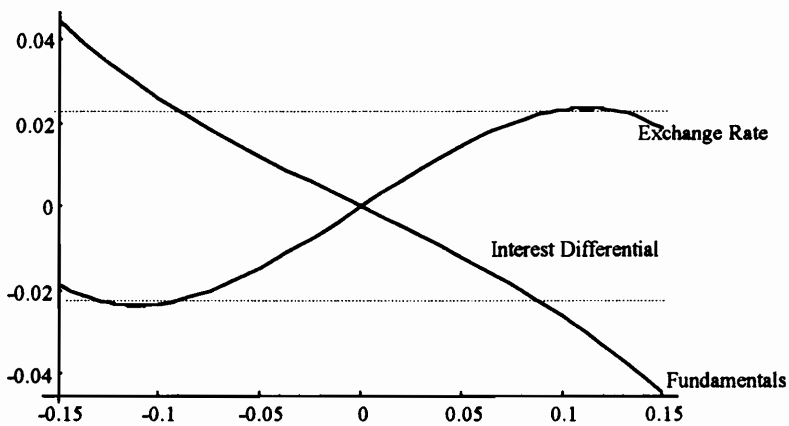

Figure 11: The Interest Rate Differential in a Target Zone

Interest Differential, (f), is given by E[dx]/dt= [x(f)-f]/α. For a symmetric, no drift case, and using equation (47), δ(f)=(A/α)(Exp(af)-Exp(-af)).

As for the empirical evidence, Flood et. al. find no clear patterns (rather randomness) for the interest rate differentials in the European ERM.

I.III.2 - Intramarginal Interventions

Two obvious weaknesses of the standard model are that the central bank always defends the band and intervenes only at the edges of the band. We will now incorporate intramarginal interventions and the possibility that the central bank may fail to defend the band.

I.III.2.a - Incorporating Intramarginal Interventions

Intramarginal interventions refer to interventions of the central bank in the exchange rate any time within the band, not only when the exchange rate hits the limits of the exchange rate band. In the real world, central banks' intervention behaviour is more complicated than simple marginal interventions. They mostly intervene intramarginally to correct deteriorating market conditions. The ERM interventions, for example, are mostly intramarginal and its exchange rates exhibit strong mean reversion[27]. We can incorporate this feature into the model by postulating that the fundamentals within the band, i.e. $f \in (f^l, f^u)$ follow a mean reverting process such that:

(52) $$df = -\ \phi(f - c)dt + \sigma dZ$$

where ϕ is the speed with which the process reverts to the central parity, c. Whenever f diverts from the center of the fundamentals band, the central bank attempts to revert it back to c. Later, in section IV, we will introduce the possibility that the central bank may have to realign the central parity to account for imperfect credibility. Following the same steps above, (52) leads to:

(53) $$\frac{1}{2}\alpha\sigma^2 x_{ff}(f) + \alpha\phi\,(c - f)x_f(f) - x(f) = -f$$

Appendix 3 shows that a change in the variable $y = \phi(c\text{-}f)^2/\sigma^2$ turns this differential equation into a Kummer's equation[28] and that the solution to (53) is given by

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[27]See Svensson, Lindberg and Söderlind (1991) for its practical and empirical importance. Also see Matropasque et. al (1988).
[28]See Abramowitz and Stegun (1972), pp. 504-505.

(54) $$x(y(f)) = \frac{f + c\phi\alpha}{1+\phi\alpha} + W_1\, M[\frac{1}{2\phi\alpha}, \frac{1}{2}, y(f)] + W_2\, \frac{\sqrt{\phi}(c-f)}{\sigma}\, M[\frac{1+\phi\alpha}{2\phi\alpha}, \frac{3}{2}, y(f)]$$

where M[a,b,y] is the confluent hypergeometric function as

$M[a,b,y] = 1 + \frac{a_1 y}{b_1} + \frac{a_2 y^2}{b_2} + \ldots + \frac{a_n y^n}{b_n} + \ldots$ and $a_n = a(a+1)\ldots(a+n-1)$ with $a_0 = 1$. Direct observation of equation (54) shows that choosing a symmetric band where $(x^u - c) = (c - x^l)$ and $(f^u - c) = (c - f^l)$ yields $W_1 = 0$. Then the smooth pasting property at the upper edge implies[29]

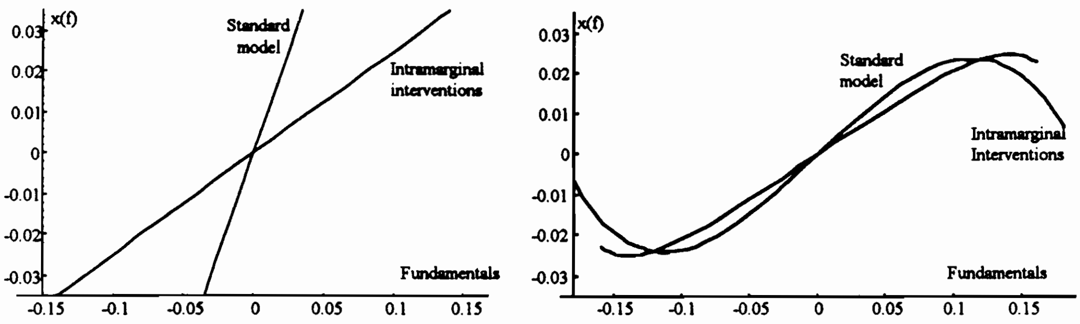
(55) $$x_f(f^U) = 0 = \frac{1}{1+\phi\alpha} - W_2\, \frac{\sqrt{\phi}}{\sigma} M[\frac{1+\phi\alpha}{2\phi\alpha}, \frac{3}{2}, y] + 2y\, \frac{1+\phi\alpha}{3\phi\alpha}\, M[\frac{1+3\phi\alpha}{2\phi\alpha}, \frac{5}{2}, y]$$

where $y = \phi(c - f^u)^2/\sigma^2$. From (55) we get

(56) $$W_2 = \frac{\frac{1}{1+\phi\alpha}}{\frac{\sqrt{\phi}}{\sigma}\, M[\frac{1+\phi\alpha}{2\phi\alpha}, \frac{3}{2}, y] + 2y\frac{1+\phi\alpha}{3\phi\alpha}\, M[\frac{1+3\phi\alpha}{2\phi\alpha}, \frac{5}{2}, y]}$$

which is greater than zero. Figure 12 below shows how the target zone and the free float exchange rates change with intramarginal interventions:

[29] d M[a,b,y(f)]/df = (a/b) y'(f) M[1+a,1+b,y(f)].



(a) (b)

Figure 12: The x:f relationship with intramarginal interventions

(a) Comparison of the free float exchange rates with and without intramarginal interventions. The steeper line is the free float line of the standard model.

(b) Comparison of the target zone exchange rates with and without intramarginal interventions. The more pronounced S-curve is that of the standard model.

The essential difference between the solutions in (47) and (54) is that the central bank is acting to keep the exchange rate near its midpoint. This indicates that the exchange rate moves around the central parity rather than the edges of the band as it does in the standard model; an empirically relevant result. Also, both the free float line where $x(f) = (f+c\phi\alpha)/(1+\phi\alpha)$ and the target zone curve implied by the intramarginal (mean reverting) interventions, are more stabilizing than the those of the standard model; the honeymoon effect is more pronounced with intramarginal interventions.

Another important implication of intramarginal intervention case is that the fundamentals band widens with the intramarginal interventions compared to the standard marginal interventions case. In other words, the ± 2.4% exchange rate band which implied a ± 11% fundamentals band in the standard model, now implies a ± 14% fundamentals band in Figure 12. By the same token, it can be shown that, if the previous limits of the fundamentals band are in effect, the exchange rate implicitly fluctuates in tighter band even tough the fundamentals band implies higher limits. Intuitively, if the market knows that the central bank will intervene intramaginally, the expected change in the exchange rate will be lower.

I.III.2.b - Empirical Evidence

The primary objective of this analysis is to provide with the empirical evidence for the extension of the target zone model in the previous section. Also, because the empirical research have not accompanied the theory in its rapid extension, another motivation of this section is to contribute to that sparse empirical literature on the target zone exchange rate models. Our work is not limited only to econometric analysis; we also provide a visual examination of the data.

Some of the earlier empirical work related to our extension above are as follows. Flood, Rose and Mathieson (1991) provides the most comprehensive empirical analysis on the target zone exchange rates, and it is the closest work to ours in estimating the fundamentals. They measure fundamentals as $f \equiv x - a(i - i^*)$ and the strategy entails obtaining estimates of α - the interest rate semielasticity using daily data. However, the model they test is that of Krugman (1991) with marginal interventions and perfect credibility. Hence, the fundamentals, in their analysis, are assumed to follow a Brownian motion with drift, not a mean-reverting process. They focus on the Netherlands and Italy, and their results which are based on α=.1 provide no support for perfectly credible target zone models with marginal interventions. Similarly, Lindberg and Soderlind (1991) test the implications of the standard target zone model using daily data for the Swedish Krona during the 1980s. They find a significant mean reversion of fundamentals, a hump-shaped frequency distribution of the exchange rate, a positive correlation between the exchange rate and the interest rate differential, and a high volatility of expected exchange rate fluctuations whenever the band lacked credibility. In other words, most implications of the standard model are refuted empirically for the Swedish target zone. In contrast, Pessach and Razin (1990) find statistically significant evidence on the honeymoon effect and discrete marginal interventions using monthly Israeli data in testing the target zone theory for inflation prone economies. They first estimate a standard money demand equation whose parameters help measure the drift and the variance of the fundamentals process. The also find that the effects of the target zone on the managed float weaken as the demand for money increases. Finally, Rose and Svensson (1991) analyze the daily French data, and find strong mean reversion of the exchange rate within the band.

(i) The Data and The Econometric Strategy :

Not surprisingly, our work is also based on the ERM since it constitutes the best example to a target zone system in the real world. The ERM currencies are allowed to fluctuate within a narrow target zone of ± 2.25 % (except Italian Lira which is ± 6.00 %) among themselves while floating jointly against the other currencies. The anchor of the system has been German Mark (DM). We will focus on France, Ireland, Italy, and The Netherlands. The currencies of the first three countries, French Franc (FF), Irish Pound (P), and Italian Lira (L), experienced numerous realignments in the central parities. On the other hand, the Dutch Guilder (G) is a good example to a more credible target zone.

The data consisting of exchange rates and the interest rates were extracted from the International Financial Statistics of the IMF. The observations are monthly and not adjusted seasonally. We will have to use monthly observations for two reasons. First, because of our estimation strategy (given below) of fundamentals, the target zone reflections of fundamentals can bias the coefficient estimates for the process of fundamentals with finely laid out data (data would be persistent). Second, monthly data are simply the highest frequency data on the exchange rates and the interest rates we could retrieve.

The ERM target zone started officially in March 1979, and experienced a major crisis and a temporary breakdown in September 1992. Hence we focus on the period between 79:03 and 92:09 (year:month) which gives us 163 observations. However, for P and L, the data are not complete for that period. We use the period 82:01 and 92:09 for the former and 79:03 and 92:06 for the latter. The exchange rate series are 132.EB.ZF (FF/ECU), 178.ED.ZF (ECU/P), 136.EB.ZF (L/ECU), 138.EB.ZF (G/ECU), and 134.EB.ZF (DM/ECU) supplied monthly as period averages. The mathematical transformations of these series provide the cross rates: namely, FF/DM, P/DM, L/DM, and G/DM. The interest rates are monthly government bond yields (end of month observations) supplied by Central Bank Bulletins and Government Financial Statistic Yearbooks. The names of the IFC series for these rates are 13261.ZF, 13461.ZF, 17861.ZF, 13661.ZF, and 13861.ZF for France, Germany, Ireland, Italy, and The Netherlands, respectively.

Our econometric strategy in this section is as follows. Assuming purchasing power parity and uncovered interest rate parity, we try to estimate fundamentals by regressing the interest differential (i-i*) on the exchange rate, and defining the residual plus the constant to be the fundamentals. In doing this, we will subgroup the data. The reason is that our model does not yet allow for realignments in the central parity (see the next section) and in contrast, every country in the ERM has experienced various realignments. Dividing the data into different sample periods also allows for a sensitivity analysis of the results, and for an examination of policy shifts and the changes in the credibility of a particular country as measured by the expected probability of devaluation. Also it is easier to assume parameter constancy within one-regime periods. It is often noted the credibility of the ERM for every member has increased over its 13 year course. Table IV-2-a in Appendix 4 provides a detailed tabulation of the realignments.

Estimating fundamentals helps us in two ways. First, it enables us to compare the nonlinearity (S-shape in figure 9) of x:f relationship with our results. Second, we can see if the mean-reverting characteristic of the fundamentals which models the intramarginal interventions of the central bank, holds.

<u>(ii) Estimation of Fundamentals :</u>

In this section, we will try to determine fundamentals using straightforward econometric methods. The strategy will be to estimate fundamentals separate for every period; from the beginning until a new parity is announced. FF has experienced six realignments during the ERM period, thus, it gives us seven periods to analyze (see Table IV-2-a in Appendix 4). However, all of these periods are not convenient for an appropriate analysis (to have enough degrees of freedom in regressions) because some of them have short durations. Therefore, we will consider only period 2 (79:10-81:09, 24 observations), period 5 (83:04-86:03, 36 observations), and period 7 (87:01-92:09, 69 observations). Similarly, we have 3 periods for Italian Lira; period 6 (83:04-85:08, 29 observations), period 8 (87:01-90:01, 36 observations), and period 9 (90:02-92:09, 32 observations). There were eight realignments for Irish Pound, but only two periods are long enough for regression analyses; period 5 (83:04-86:03, 36 observations), and period 7 (87:01-92:09, 69 observations). Finally, period 1 (79:10-83:03, 42 observations) and period 2 (83:04-92:09, 114 observations) are used for Dutch Guilder.

Assuming uncovered interest rate parity, expected depreciation of the exchange rate is given by the interest rate differential. Hence the target zone exchange rate is

$$(57) \qquad x_t = f_t + \alpha(i - i^*)_t \quad .$$

In this relationship, the only unknown variable is the fundamentals. Instead of estimating money demand functions, we measure fundamentals by regressing the interest rate differential on the log of the exchange rate and defining the fundamentals to be the residual plus the constant.[30]. In other words, we run the following regression:

$$(58) \qquad x_t = \text{Constant} + \alpha(i - i^*)_t + u_t \quad .$$

The results of the regression for FF are reported in Table III-2-b-1 in Appendix 4. The estimation method is Ordinary Least Squares (OLS). Since our main concern is to find the magnitude of fundamentals by adding the residual to the constant, the goodness of fit (in its linear form) is negligible. We should note, however, the interest differential in (58) is endogenously determined. Therefore, a two stage least squares (2SLS) estimator is a more appropriate choice than the OLS because the former is asymptotically unbiased. The results for estimating (58) with 2SLS using the differentials for money supplies, production, and price levels as instrumentals variables are reported in

[30]This was recommended in Flood, Rose, and Mathieson (1991), footnote 28.

Table III-2-b-2. Because the regression coefficients are very insensitive to the choice of the estimator, we will proceed with the results of the OLS method which seems to estimate quite well and reveals significant coefficients.

An insignificant estimate of α implies that the exchange rate is mainly determined by the fundamentals which is the case for FF in period 2 and P in period 5. Also, the estimates of the constants are very close in size to the central parity in their respective periods, indicating that the reason why exchange rates divert from the central parity is mostly the expectations.

The most important aspect of these results is the magnitude and the significance of the estimates of α. Flood et. al. (1991) interpret α as the absolute value of the interest rate semielasticity of money demand, and present α=.1 as being a reasonable value, and α=1 as being the high end of the range. Similarly, Lindberg and Soderlind (1991) estimate the range for α as [.015, .56]. Our estimates for α which are between [.001, .005] are consistent with the literature. We should note, however, that our estimates for α should be multiplied by 100 to be comparable since we chose interest rate units such that 10 percent a year, for instance, was entered as 10, not .10. Goldfeld and Sichel (1990) which uses interest rates as we do, report short-run semielasticities around .004.

The diagrams in Appendix 5 show the cross plots of exchange rate vs. fundamentals for each exchange rate above. In most graphs, especially FF/DM in period 2 and period 5, P/DM in period 5, L/DM in period 8, and G/DM in both periods, the linearity of the plots can be questioned; we cannot reject the S shape of the x:f relationship by visually examining the cross plots. However, as shown in Figure 12, the S-shape is less pronounced due to the mean reversion in process of fundamentals.

<u>(iii) Mean Reversion</u>

The model in section I.III.2.a assumes that fundamentals follow a mean reverting Brownian motion, rather than the continuous random walk of the standard model. In other words,

(59) $$df_t = -\phi(f_t - c)dt + \sigma dZ_t$$

where the first term on the right hand side of (59) is the mean reverting (to the central parity) part and the second term is the stochastic part of the process. Then it is trivial to test for the significance of mean reversion in their process once the fundamentals are determined. We use the following specification[31]:

(60) $$f_t = \gamma f_{t-1} + \phi(f - c)_{t-1} + \varepsilon_t \quad .$$

A negative sign of ϕ, and a value of γ close to one would indicate mean reversion in the process, in other words, the random walk of the fundamentals would be rejected[32]. Table III-2-c in Appendix 4 provides the results on estimating (60).

The results present a significant mean reversion in the process of fundamentals. The estimates seem to be very insensitive for other methods of estimation than OLS, and most of the coefficents estimated by the OLS method are significant. In every period, the estimated coefficient for the mean reverting term, ϕ, has a negative sign and significant at 5 % except period 2 for FF/DM and G/DM. Even though it was not restricted to be one (by using $f_t - f_{t-1}$ as the dependent variable), the estimated coefficient for the lagged fundamentals are very close to one always significant at 5 %.

It still remains to be seen, however, that the residual in (60) is a white noise as required by the assumption of Brownian motion. the Durban-Watson test should be biased (towards 2.00) towards not detecting autocorrelation because the equation (60) includes a lagged dependent variable. Hence, it would be misleading. Instead, we use the Lagrange Multiplier (LM) test in the form of F statistic which can not reject in any case the null hypothesis that the residual does not have a unit root (the disturbance is white noise). The plots of scaled residuals are also given in Appendix 6. Keeping in mind that we do not have many degrees of freedom, most of the plots except L/DM, display no apparent departures from the distributional assumptions of the residuals. The implied histograms appear to be bell shaped (no obvious asymmetry) and there are only a few outliers beyond two

[31] We also tried the regression with a constant. But it resulted in singular matrices and either the constant or the lagged dependent variable had to be deleted.

[32] Alternatively, we could restrict $\gamma = 1$ by regressing the differenced f_t .

standard deviations from the mean (no suggestion of fat tails). Also, there seem no systematic departures from a constant mean or a variance. For L/DM, residual plots exhibit long cycles and greater variance at the end. Nevertheless, these could be expected since Italian currency is the most volatile ERM currency among the others. Finally, ARCH F-test results show that the variance of the residuals in the regressions do not depend on the size of the preceding residual.

I.III.3 - Imperfect Credibility

In this section, we complete the model above by including imperfect credibility. While doing so, we will consider only devaluations since almost all target zones shift upwards with realignments in the central parity. All our results symmetrically apply to the revaluations.

I.III.3.a - Incorporating Imperfect Credibility

Imperfect credibility, in our context, refers to the probability that the central parity may be realigned during the mean reverting process of fundamentals. In other words, c is now stochastic, and at any time of intervention, i.e. f diverges from c, the central bank may have to realign the central parity, c, and anounce a new central parity, c+I, with probability, π. Put it differently, the central bank intervenes to revert f back to the current central parity, c, with the probability, $(1-\pi)$. We assume that π does not vary over time. In case of devaluation, I is positive, and f continues its mean-reverting process to the new central parity (c+I) which is greater than c. This scenario is illustrated below:

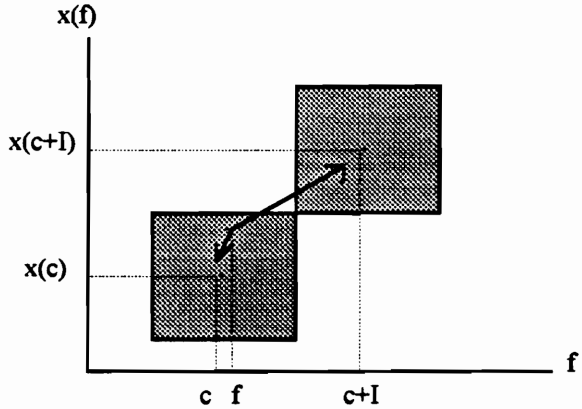


Figure 13: Devaluation to an Adjacent Target Zone

In summary,

$$f^l \le f \le f^u \Rightarrow df = -\phi[f-c]\,dt + \sigma dZ \quad \text{and}$$

$$E(df) = -\phi[f-E(c)]\,dt$$

where E(c) = (1-π)c + π(c+I). Following the steps in Appendix 3, the exchange rate in an imperfectly credible target zone with intramarginal interventions is given by

$$(61)\quad x(f,c) = \frac{f+(c+I\pi)\phi\alpha}{1+\phi\alpha} + W_1\, M[\frac{1}{2\phi\alpha},\frac{1}{2},z(f,c)] + W_2\,\frac{\sqrt{\phi}(c+I\pi-f)}{\sigma}\,M[\frac{1+\phi\alpha}{2\phi\alpha},\frac{3}{2},z(f,c)]$$

where $z(f,c) = \frac{\phi\,(c+I\pi-f)^2}{\sigma^2}$. If the individuals do not expect any devaluation, i.e. the central bank is expected to be able to intervene whenever f diverges from its central parity, then π=0, and we have the same results of section II. However, if π>0, we lose the symmetry of before, and both of the constants, W_1 and W_2 should be calculated. Using the smooth pasting property at the edges, we get a system of four equations:

$$(62)\, x(f^u,c) = \frac{f^u+(c+I\pi)\phi\alpha}{1+\phi\alpha} + W_1\, M[\frac{1}{2\phi\alpha},\frac{1}{2},z(f^u,c)] + W_2\,\frac{\sqrt{\phi}(c+I\pi-f^u)}{\sigma}\,M[\frac{1+\phi\alpha}{2\phi\alpha},\frac{3}{2},z(f^u,c)]$$

$$(63)\quad x(f^l,c)=\frac{f^l+(c+I\pi)\phi\alpha}{1+\phi\alpha}+W_1\, M[\frac{1}{2\phi\alpha},\frac{1}{2},z(f^l,c)]+W_2\,\frac{\sqrt{\phi}(c+I\pi-f^l)}{\sigma}\,M[\frac{1+\phi\alpha}{2\phi\alpha},\frac{3}{2},z(f^l,c)]$$

$$(64)\quad x_f(f^u,c)=0=\frac{1}{1+\phi\alpha}+W_1\left[\frac{1}{\phi\alpha}z_f(f^u,c)M[\frac{1+2\phi\alpha}{2\phi\alpha},\frac{3}{2},z(f^u,c)]\right]+$$

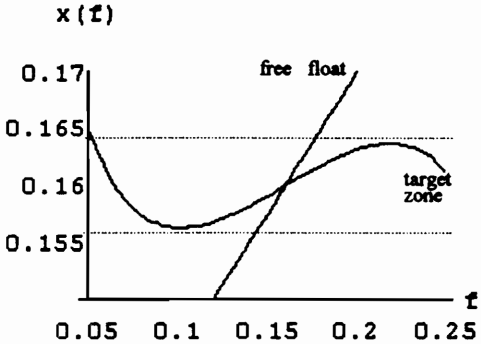
$$W_2\frac{\sqrt{\phi}}{\sigma}\left[-M[\frac{1+\phi\alpha}{2\phi\alpha},\frac{3}{2},z(f^u,c)]+\left(\frac{1+\phi\alpha}{3\phi\alpha}\right)(c+I\pi-f^u)z_f(f^u,c)M[\frac{1+3\phi\alpha}{2\phi\alpha},\frac{5}{2},z(f^u,c)\right]$$

$$(65)\quad x_f(f^l,c)=0=\frac{1}{1+\phi\alpha}+W_1\left[\frac{1}{\phi\alpha}z_f(f^l,c)M[\frac{1+2\phi\alpha}{2\phi\alpha},\frac{3}{2},z(f^l,c)]\right]+$$

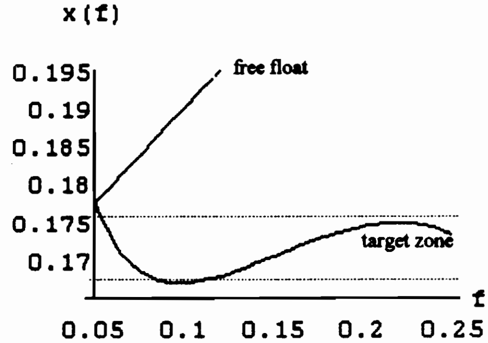
$$W_2\frac{\sqrt{\phi}}{\sigma}\left[-M[\frac{1+\phi\alpha}{2\phi\alpha},\frac{3}{2},z(f^l,c)]+\left(\frac{1+\phi\alpha}{3\phi\alpha}\right)(c+I\pi-f^l)z_f(f^l,c)M[\frac{1+3\phi\alpha}{2\phi\alpha},\frac{5}{2},z(f^l,c)\right].$$

These equations implicitly define f^u, f^l, W_1, and W_2 once the target zone limits x^u, x^l, and I, π, the size and the probability of realignment, respectively, are specified. For example, the realignment rule may be such that the realignment does not change the width of the target zone, and they are adjacent to each other as in Figure 13 above. Hence, I is given by $2(f^u\text{-}c)$. As for the realignment probability, every country can be assigned a different probability; Ireland and the Netherlands could have probabilities of devaluation as .8 and .2, respectively. Then when the target zone limits are anounced, f^u, f^l, W_1, and W_2 can be determined.

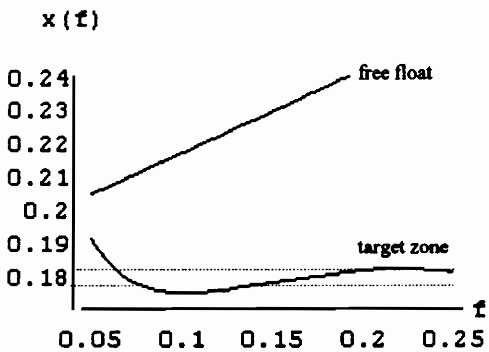
Figure 14 below shows three different cases (π=0, π=.5,π=.8) of x:f relationships in an imperfectly credible target zone with intramarginal interventions. It is drawn for a symmetric fundamentals band of ± 37.5% which is implied by a perfectly credible exchange rate band of ± 2.5% (close to that of the ERM), and the straight lines show the free float cases:



(a) π=0



(b) π=.5



(c) π=.8

Figure 14: Imperfectly Credible Target Zone

with π=0, π=.5, π =.8

(a) π=0 $\Rightarrow$ the zone is perfectly credible, α=3, σ=.1, c=.16, f^u=.22, f^l=.10, x^u=.164, x^l=.156. The exchange rate band is ± 2.5 %. The straight line is the free float case. **(b)** π=.5 $\Rightarrow$ the zone is imperfectly credible, α=3, σ=.1, c=.16, f^u=.22, f^l=.10, x^u=.175, x^l=.167. The exchange rate band is ± 2.3 %. **(c)** π=.8 , α=3, σ=.1, c=.16, f^u=.22, f^l=.10, x^u=.182, x^l=.175. The exchange rate band is ± 1.9 %.

The implications of the results of the model are several. First, the target zone in this model is always stabilizing relative to the free float; the honeymoon effect prevails. In other words, the slope of the S-shaped curve is everywhere less than that of the free float case indicating that target zone exchange rate is less volatility than that of free float. This is an important observation in the sense that although the central parity is very likely to be realigned in the target zone, the exchange rate volatility is still lower relatively, making the target zone a more preferred regime than the free float. The mean reverting characteristic of fundamentals which models the intramarginal intervention of the central bank puts pressure on the exchange rate while an increasing probability of devaluation increases the exchange rate volatility. However, this pressure dominates the latter effect. Second, in the long-run, the exchange rate should still appear more frequently near the middle of the band than the edges; the probability density is hump-shaped. When $\pi=0$, we have the perfect credibility case (the standard model). As the credibility goes down (π increases), the limits of the exchange rate band become tighter. This enhances the effect of intramarginal interventions on the exchange rate which explains why the ERM exchange rate is observed more frequently near its central parity rather than the edges.

Nevertheless, the observation that the exchange rate band shrinks with a higher probability of devaluation decreases not only the uncertainty but also the national monetary independence of the central bank, i.e. some control over the domestic interest rate[33]. Some of the EMS countries, for example, Austria and Belgium, have in recent years kept their exchange rates very close to the central parities which in return eliminated almost completely the interest rate differentials. Therefore, our model implies that there is a trade-off between the credibility and exploiting monetary independence. Fourth, the divergence of the implied exchange rate under both regimes grows as the probability of devaluation goes up. This shows that the more likely is the realignment of the central parity, the greater is the value of the exchange rate implied by the same fundamentals band. In other words, the currency gets weaker with a higher probability of devaluation in the free float relative to the target zone. Note that this model assumes purchasing power parity, therefore the exchange rate always compensates for the divergence in the price levels to avoid speculative arbitrage in the absence of transaction costs. As a result of the greater value of the exchange rate implied by a higher

[33]See Svensson (1992c) for a discussion of target zones and monetary independence.

probability of devaluation, the home country experiences a higher price level. Finally, assuming uncovered interest rate parity, the interest rate differential, δ, is given by the expected depreciation of the exchange rate, and the extended model still implies a negative relationship between δ and f.

I.III.3.b - Empirical Evidence

There are a few empirical work on the imperfectly credible target zone model. Lewis (1990) and Klein and Lewis (1991) estimate the probability of intervention in a model which captures the intramarginal interventions from the Louvre Accord in 1985 to the stock market crash in 1987. In the model, the market's beliefs about the interventions (the probability of intervention) increases the more fundamentals divert from the targeted level. Their results suggest that the market's perceptions of the target zone changed significantly during the period above, indicating that a particular non-linear relationship between the exchange rate and the fundamentals can be illusive. Lindberg, Svensson and Soderlind (!991), and Svensson (1991) estimate devaluation expectations using daily Swedish data. Their model introduces a time-varying realignment risk into the standard model. They consider the exchange rate and the fundamentals consisting of two parts; the central parity, c, and the deviations from c; (x-c) and (f-c). Then the expected rate of total depreciation, E(dx)/dt, is the sum of two components; the expected rate depreciation *within the band*, E[d(x-c)/dt], and the expected rate of realignment (rate of change in the central parity), E(dc)/dt. They argue that it should not make sense to plot the exchange rate against the fundamentals in testing the implications of the standard model, like Flood et. al. (1991) do. Because the fundamentals does not include the expected rate of realignment, E(dc)/dt, which is another state variable and assumed to follow Brownian motion over time. This provides a possible explanation to why the x:f plots of Flood et. al. do not result in a well-behaved exchange rate function. Therefore, both papers use the same strategy of calculating a new state variable which includes expected realignment risk, and plotting it against the exchange rate to see if the predictions of the standard model hold. The results confirm the characteristic S shape (more pronounced with a larger α, the honeymoon effect, and the smooth pasting property of the target zone exchange rate.

It is not difficult to show that the target zone models in practice lack perfect credibility by looking at the ERM policy shifts and realignments in the exchange rate central parities. The data, obtained from the London Currency Report, are presented in Table IV-2-a in Appendix 4, which includes six ERM currencies.

Our strategy, in this section includes, first, the estimation of the point of reversion as specified above by E(c) - the expected central parity of the fundamentals. A good proxy for E(c) is the mean observation (estimate in our case) for fundamentals in every subperiod. For example, the expected central parity becomes higher (lower) than the announced parity in the beginning of a subperiod if the average of fundamentals is greater (lower) than the central parity such that it suggests a depreciation (appreciation) in the exchange rate. We will restrict E(c) to be stationary within each subperiod because we crucially assume that the probability of realignment associated with E(c) does not depend on the process of fundamentals nor does it vary over time until a new parity is announced. Then using the estimated values of the point of mean reversion, i.e. $\hat{E}(c)$, we will calculate the probability of realignment in the central parity for each period. We define the estimated probability of devaluation as $\hat{\pi} \equiv \left| \frac{\hat{E}(c) - c_1}{c_2 - c_1} \right|$ where c_1 and c_2 are the current and the next period central parities (announced at the beginning of each subperiod). Table IV-2-b in Appendix 4 presents these estimated probabilities of realignment for every exchange rate. First, the exchange rates of FF/DM and the L/DM signal a devaluation close to the collapse of the system except P/DM. This shows that a long duration of a target zone is not sufficient for it to be credible as it is considered in most research papers; the investors expected a realignment in the central parities in spite of long no realignment periods towards the September 1992 crisis. Second, expectations are not always in conformity with the realignments. More specifically, a devaluation may follow even when the expected intervention (I) is not positive, i.e. expected revaluation, or when the probability of devaluation is low. For example, for FF/DM in period 2, the mean observation for fundamentals was less than the central parity. In other words, investors were actually expecting a revaluation in the central parity even though a devaluation followed at the end of the period. Also for P/DM in period 7, the expected probability of devaluation was very low although a sizable devaluation of 11 % followed soon. Another interesting observation is that the realignment for the L/DM in period 6 was less than what the fundamentals suggested; the mean of fundamentals was less than the new central parity. Nevertheless, this can be easily explained by the greater width of the Italian target zone (± 6 %) which does not require sizable interventions to keep the exchange rate within the band. Fourth, G/DM seems to be the only consistently credible target zone among the others. Similarly, the Italian target zone in period 8 and that of the Irish Pound in period 7 exhibited credibility even though the durations were not necessarily long.

I.IV - SUMMARY AND CONCLUSION

The model of Krugman (1991) which has long been the starting point for the later research on target zone exchange rates, has provided a great insight into the recent target zone systems, especially the European exchange rate mechanism. However, the empirical evidence from the ERM presented by Flood, Rose, and Mathieson (1991) did not agree with most of the model's implications. While it holds empirically that the target zone is a more stabilizing exchange rate regime than the free float (the honeymoon effect), the smooth pasting property and the nonlinearity of the exchange rates are less pronounced than what the standard model predicts. Also, the asymptotic distribution of the exchange rate is not U-shaped, and the interest rate differentials exhibit no clear patterns.

The reason for this divergence between the results in the standard model and the empirical evidence is that the European target zones, in practice, work different than the assumptions of the standard model. Most importantly, the interventions in the ERM by the member central banks are compulsory but by no means limited to those at the margin[34]. The only motivation for a marginal intervention is that the foreign exchange required for these interventions can be obtained through the very short-term financing facility of the European Monetary System; a network of mutual credit lines. Therefore, although they face explicit fluctuation limits on the exchange rates, the ERM central banks mostly lean against the wind and intervene intramarginally to correct disorderly market conditions. Furthermore, since its inauguration in March 1979, the ERM has functioned with the aid of frequent currency realignments; more than one per year on average[35]. The realignments were inevitable for the system to survive against the divergent macroeconomic policies of the major European countries such as France, Germany, and Italy.

The next step then would be to extend the theory (the standard model of Krugman) by allowing interventions of all nature and imperfect credibility. These features have been incorporated into the standard model in the literature in several ways but always individually and seperate from the

[34]See Mastropasqua, Micossi, and Rinaldi (1988).
[35]See Obstfeld (1988).

effects of each other. The model above closes this gap in the literature, and allows for imperfect credibility while assuming intramarginal (mean reverting) interventions. The implications of the model, not surprisingly, are supported well by the empirical evidence. The stabilizing and the smooth pasting features of the exchange rate are in effect with the latter being less pronounced than that of the standard model. The mean reverting interventions imply that the asymptotic exchange rate distribution is hump-shaped. Nevertheless, the interest rate differentials still display a negative correlation with the fundamentals.

Several extensions of the model above are also possible. It would be interesting to see how the model's implications change when the probability of a realignment in the central parity is endogenous and increases as the fundamentals divert from the central parity as in Lewis (1990). Then the countries will no longer have to be assigned a probability of devaluation over a certain period of time. Also, the effects of the money supply shocks on the sustainability of the target zone could be modeled as temporary or permanent, and be analyzed accordingly. Furthermore, the implications of the model may differ on the length of the period in which the target zone is in effect; a short-run vs. long-run consideration in the analysis may prove useful. The implications about the term structure of the forward premia and its effects on the trade volume, and the balance of payments should also be worth analyzing with the new developments in NAFTA.

APPENDIX 1: Solution of Equation (5)

(5) $$x = m + v + \alpha \frac{E(dx)}{dt} ,$$

(6) $$dv = \sigma dZ .$$

Holding m constant and equal to zero, the exchange rate is a function of the velocity;

(A1.1) $$x = x(v) .$$

(A1.1) can be further differentiated using the Ito's Lemma[36]:

(A1.2) $$dx = x_v(v)dv + \frac{1}{2}x_{vv}(v)(dv)^2 .$$

Taking expectations and applying (6) yields

(A1.3) $$\frac{E(dx)}{dt} = \frac{\sigma^2}{2}x_{vv}(v) .$$

Plugging (A1.3) into (5) gives a second order differential equation:

(A1.4) $$x = m + v + \frac{\alpha\sigma^2}{2}x_{vv}(v) .$$

[36] See Harrison (1985), chapter 4.

The general solution[37] to (A1.4) is given by

(A1.5) $$x = m + v + Ae^{av} + Be^{-av}$$

where a is a distinct real root[38] given by $\sqrt{\frac{2}{\alpha\sigma^2}}$. For notational and conceptual convenience, Krugman assumes a symmetric band, i.e. A=-B. Then

(7) $$x = m + v + A(e^{av} - e^{-av})$$

[37]See Chiang (1984), chapter 15.
[38]Because $4(-2/\alpha\sigma^2) < 0$.

APPENDIX 2: Derivation of Equation (39)

Discrete time processes may be approximated as continuous time processes by taking the limit as the time interval becomes small[39]. Then we need to discretize the time process in (38)

$$(\text{A2.1}) \qquad \begin{array}{ll} f_{t+1} - f_t = \eta + \sigma Z_{t+1} & \text{if intervention occurs} \\ f_{t+1} - f_t = 0 & \text{otherwise} \end{array}$$

Next, we need to specify the probability function of intervention to be able to provide a continuous time approximation of (A2.1). Assume the fundamentals without interventions, k, follow a binomial process and

$$(\text{A2.2}) \qquad k_i - k_{i-1} = \xi$$

and a small time period τ later,

$$(\text{A2.3}) \qquad \begin{array}{ll} k_i = k_{i-1} & \text{with probability p} \\ k_i = k_{i+1} & \text{with probability q} \end{array}$$

where $p=(1/2)(1-(\eta\tau/\xi))$, and $q=(1/2)(1+(\eta\tau/\xi))$. Then the fundamentals incorporating intervention possibilities are given as

$$(\text{A2.4}) \qquad \begin{array}{ll} f_i = f_{i-1} & \text{with probability } p\,(1-\pi(k_{i-1})) \\ f_i = f_{i+1} & \text{with probability } q\,(1-\pi(k_{i+1})) \\ f_i = f_i & \text{with probability } p\,\pi(k_{i-1}) + q\,\pi(k_{i+1}) \end{array}$$

---

[39]See, for example, Nelson (1990), and Karlin & Taylor (1981).

where $\pi(k)$ is the probability of intervention as a function of fundamentals. Having specified the probability function of intervention, we can approximate (A2.1) over the continuous time interval, τ, with the following diffusion process:

(A2.5) $$df = a(f)dt + \sqrt{b(f)}\,dZ$$

where $a(f) = \lim_{\tau\to 0} E\left[\sqrt{\frac{(f_{t+1} - f_t)}{\tau}} | \Omega_t\right]$ and $b(f) = \lim_{\tau\to 0} E\left[\sqrt{\frac{(f_{t+1} - f_t)^2}{\tau}} | \Omega_t\right]$.

The Ω_t is the family of sigma algebras generated by the probability structure of f_t sequence over the interval, τ, as specified in (A2.4). Next, we need to determine explicitly a(f) and b(f), the conditional mean and variance of the fundamentals process, relatively. From (A2.4), the expected evolution of the probability of intervention during τ can also be written as a function of the expected evolution of market fundamentals (f=k):

(A2.6) $$\pi(f_{t+1}) = \pi(f_t) + \pi'(f_t)(f_{t+1} - f_t) + \phi(\tau)$$

where $\phi(\tau)$ collects all terms that approach zero faster than τ. Then substituting (A2.5) and A(2.1) into a(f) above gives the conditional mean over τ:

(A2.7) $$E\{\tau^{-1}(\eta\tau + \sigma Z_{t+1})[1 - (\pi(f_t) + \pi'(f_t)(\eta\tau + \sigma Z_{t+1}) + \phi(\tau))|\Omega_t]\} \quad .$$

Taking the limit as τ goes to zero yields

(A2.8) $$a(f) = \eta(1 - \pi(f_t)) - \pi'(f_t)\sigma^2 \quad .$$

Following the same steps for the conditional variance yields

(A2.9) $$b(f) = [1 - \pi(f_t)]\sigma^2$$.

Finally, plugging (A2.7) and (A2.8) into (A2.4) gives

(39) $$df = [\eta(1 - \pi(f)) - \sigma^2 \pi'(f)]dt + \sigma\sqrt{1 - \pi(f)}\, dZ$$.

APPENDIX 3: Solution of Equation (53)

(53) $$\frac{1}{2}\alpha\sigma^2 x_{ff}(f)+\alpha\phi(c-f)x_f(f)-x(f)=-f \quad .$$

Changing the variable, f as

(A3.1) $$y(f)=\frac{\phi(c-f)^2}{\sigma^2}$$

turns the equation into a Kummer's equation (Abramowitz and Stegun (1972), page 504), so that it can be solved. Then

(A3.2) $$y_f(f)=-\frac{2\phi(c-f)}{\sigma^2} \quad .$$

(A3.3) $$y_{ff}(f)=\frac{2\phi}{\sigma^2} \quad .$$

(A3.4) $$x_f(f)=x_y(y)\ y_f(f) \quad .$$

(A3.5) $$x_{ff}(f)=x_y(y)\ y_{ff}(f)+x_{yy}(y)[y_f(f)]^2 \quad .$$

Plugging (A3.2) through (A3.5) into (53), and rearranging yields

(A3.6) $$\frac{\phi}{\sigma^2}(c-f)^2 x_{yy}(y)+\left[\frac{1}{2}-\frac{\phi(c-f)^2}{\sigma^2}\right]x_y(y)-\frac{1}{2\phi\alpha}x(y)=-\frac{1}{2\phi\alpha}f$$

.

Inserting (A3.1) into (A3.6) gives a nonhomogeneous Kummer's equation:

(A3.7) $$y\ x_{yy}(y) + \left(\frac{1}{2}-y\right) x_y(y) - \frac{1}{2\phi\alpha}x(y) = \frac{\sigma}{2\phi\alpha}\sqrt{y} - \frac{c}{2\phi\alpha}$$

Guessing the form of particular solution as $K_1\sqrt{y} + K_2$ from the right side of (A3.7), one can find

(A3.8) $$K_1 = -\frac{\sigma}{\sqrt{\phi}+\alpha\phi\sqrt{\phi}} \quad , \quad K_2 = c$$

Two linearly independent solutions for the homogeneous part of (A3.7) are given as

(A3.9) $$x_1 = M[a,b,y]$$

(A3.10) $$x_2 = y^{1-b}\ M[1+a-b,2-b,y]$$

M[a,b,y] is called the confluent hypergeometric function where

$M[a,b,y]=1+\frac{a_1 y}{b_1}+\frac{a_2 y^2}{b_2}+.....+\frac{a_n y^n}{b_n}+.....$and $a_n = a(a+1)...(a+n-1)$ with $a_0 = 1$.

Finally, the complete solution to (53) can be found as:

(54) $$x(y(f)) = \frac{f+c\phi\alpha}{1+\phi\alpha} + W_1\ M[\frac{1}{2\phi\alpha},\frac{1}{2},y(f)] + W_2\ \frac{\sqrt{\phi}(c-f)}{\sigma}\ M[\frac{1+\phi\alpha}{2\phi\alpha},\frac{3}{2},y(f)] \quad .$$

APPENDIX 4: TABLES

Table III-2-b-1: Estimating Fundamentals for Major European Exchange Rates (OLS)

		Obs.	Constant	α	F Statistic	σ	c(x)	c(f)	R^2
FF/DM	**Period 2**	24	.834 (.012)	.004 (.0025)	2.19 [.15]	.011	.857	.845	.10
	Period 5	36	1.14 (.006)	-.005 (.001)	15.45 [.0004]	.006	1.121	1.09	.31
	Period 7	69	1.22 (.001)	-.0023 (.0006)	13.25 [.0005]	.007	1.21	1.204	.17
P/DM	**Period 5**	36	-1.12 (.012)	-.002 (.002)	.86 [.361]	.012	-1.13	-1.143	.03
	Period 7	69	-.98 (.0008)	-.0003 (.0002)	1.31 [.2561]	.005	-.985	-.987	.02
L/DM	**Period 6**	29	6.49 (.025)	-.01 (.003)	9.58 [.005]	.03	6.44	6.334	.26
	Period 8	36	6.55 (.02)	.012 (.005)	6.38 [.016]	.014	6.58	6.546	.15
	Period 9	32	6.62 (.002)	-.004 (.0007)	26.32 [-]	.013	6.62	6.463	.47
G/DM	**Period 1**	42	.103 (.005)	-.006 (.002)	9.52 [.004]	.007	.10	.093	.19
	Period 2	114	.12 (.0003)	-.002 (.0006)	10.54 [.0015]	.002	.12	.119	.09

Equation: $x_t = \text{Constant} + \alpha(i - i^*)_t + u_t$.

Note: OLS estimation, standard errors are given in parentheses for the parameters. F statistic tests $H_0: \alpha = 0$, c(x) and c(f) denote the actual central parity of the exchange rate band and the estimated central parity of the fundamentals band, respectively, and R^2 gives the goodness of fit for the regression equation.

Table III-2-b-2: Estimating Fundamentals for Major European Exchange Rates (2SLS)

		Obs.	Constant	α	F Statistic	σ	c(x)	c(f)
FF/DM	**Period 2**	24	.826 (.014)	.0053 (.003)	65041	.012	.857	.845
	Period 5	36	1.14 (.007)	-.006 (.0015)	577904	.006	1.121	1.09
	Period 7	69	1.224 (.002)	-.0032 (.0007)	1036786	.007	1.21	1.204
P/DM	**Period 5**	36	-1.17 (.05)	-.006 (.0009)	116599	.014	-1.13	-1.143
	Period 7	69	-.98 (.0008)	-.0006 (.0003)	2164968	.004	-.985	-.987
L/DM	**Period 6**	29	6.51 (.027)	-.012 (.0035)	745652	.028	6.44	6.334
	Period 8	36	6.34 (.31)	.07 (.0007)	918382	.030	6.58	6.546
	Period 9	32	6.62 (.003)	-.006 (.0015)	3218201	.015	6.62	6.463
G/DM	**Period 1**	42	.103 (.005)	-.006 (.0034)	3652	.007	.10	.093
	Period 2	114	.12 (.0004)	-.002 (.0006)	10.54 [.0015]	.002	.12	.119

Equation: $x_t = \text{Constant} + \alpha(i - i^*)_t + u_t$.

Note: 2SLS estimation using money supply differentials, GDP differentials, and CPI differentials, standard errors are given in parentheses for the parameters. F statistic tests $H_0: \alpha = 0$, c(x) and c(f) denote the actual central parity of the exchange rate band and the estimated central parity of the fundamentals band, respectively, and R^2 gives the goodness of fit for the regression equation.

Table III-2-c: Significance of Mean Reversion of Fundamentals

		Obs.	γ	ϕ	F Statistic	σ	LM Test F- Form	ARCH F-Test	R^2
FF/DM	**Period 2**	24	1.00 (.003)	-.10 (.11)	236942	.006	.38 [3.52]	2.41 [.12]	.99
	Period 5	36	1.00 (.0009)	-.17 (.08)	2228914	.003	.41 [2.92]	2.54 [.08]	.99
	Period 7	69	1.00 (.0004)	-.18 (.06)	4032565	.004	1.69 [2.36]	4.54 [.0014]	.99
P/DM	**Period 5**	36	1.00 (.0007)	-.17 (.07)	1103591	.005	2.30 [2.92]	10.14 [.0001]	.99
	Period 7	69	1.00 (.0007)	-.142 (.067)	5513188	.003	2.14 [2.36]	6.81 [-]	.99
L/DM	**Period 6**	29	.99 (.002)	-1.58 (.21)	942693	.025	.10 [3.03]	3.99 [.0224]	.99
	Period 8	36	1.00 (.0002)	-.14 (.056)	15775006	.007	.29 [2.91]	6.39 [.0019]	.99
	Period 9	32	1.00 (.0003)	-.67 (.22)	4393041	.012	2.67 [2.89]	35.08 [-]	.99
G/DM	**Period 1**	42	.98 (.009)	-.23 (.096)	10028	.005	1.19 [2.86]	7.02 [.001]	.99
	Period 2	114	1.00 (.001)	-.17 (.05)	461203	.001	1.38 [2.10]	6.54 [-]	.99

Equation: $f_t = \gamma f_{t-1} + \phi(f-c)_{t-1} + \varepsilon_t$

Note: RALS estimation, standard errors are given in parentheses for the parameters. F statistic tests $H_0: \gamma = 0$ and $\theta = 0$, LM (Lagrange Multiplier)tests, in F-form, the null hypothesis that there is no autocorrelation in the residuals (i.e. the errors are white noise), ARCH test (Autoregressive Conditional Heteroscedasticity) is for autocorrelation in the variance of the residual, and R^2 gives the goodness of fit for the regression equation.

Table IV-2-a : Realignments in ERM:

DATES	FRANCE	NETH'DS	BE-LUX	ITALY	DENMARK	IRELAND
79:03:13	-	-	-	-	-	-
79:09:24	+ 2 %	+ 1.9 %	+ 2 %	+ 2 %	+ 5 %	+ 2 %
79:11:30					+ 5 %	
81:03:23				+ 6.4 %		
81:10:05	+ 8.7 %		+ 5.5 %	+ 8.8 %	+ 5.5 %	+ 5.5 %
82:02:21			+ 9.3 %		+ 3.1 %	
82:06:12	+ 10.6 %		+ 4.3 %	+ 7.2 %	+ 4.3 %	+ 4.3 %
83:03:21	+8.2 %	+ 2 %	+ 3.9 %	+ 8.2 %	+ 2.9 %	+ 9.3 %
85:07:22				+ 8.5 %		
86:04:06	+ 6.2 %		+ 2 %	+ 3 %	+ 2 %	+ 3 %
86:08:03						+ 8.7 %
87:01:12	+ 3 %		+ 1 %	+ 3 %	+ 3 %	+ 3 %
90:01:09				+ 3.8 %		
92:09:16				+ 7.3 %		
93:02:01						+ 11.1 %

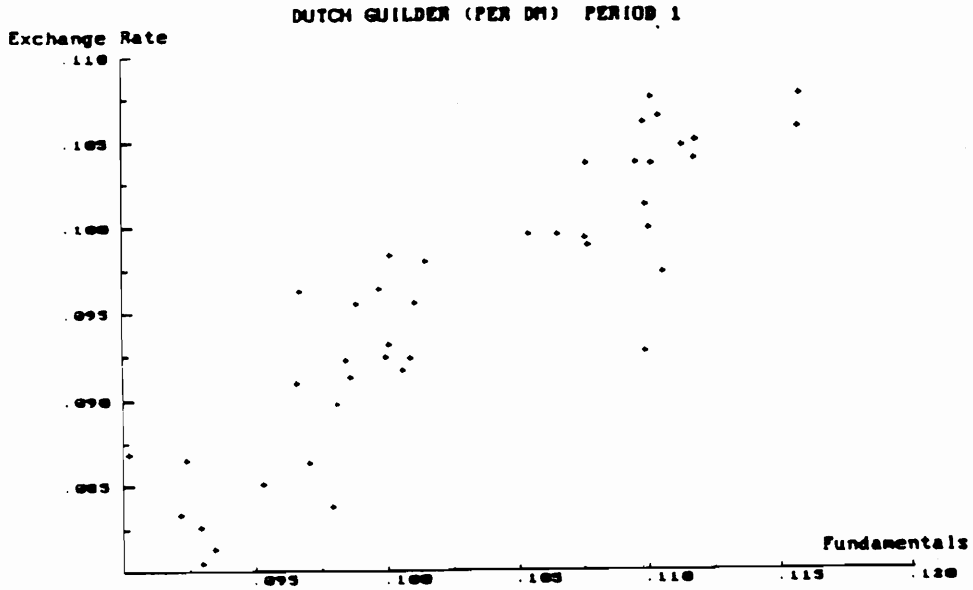
Note: (+) values show devaluation; upward realignment in the central parity. Compiled by the author from London Currency Report.

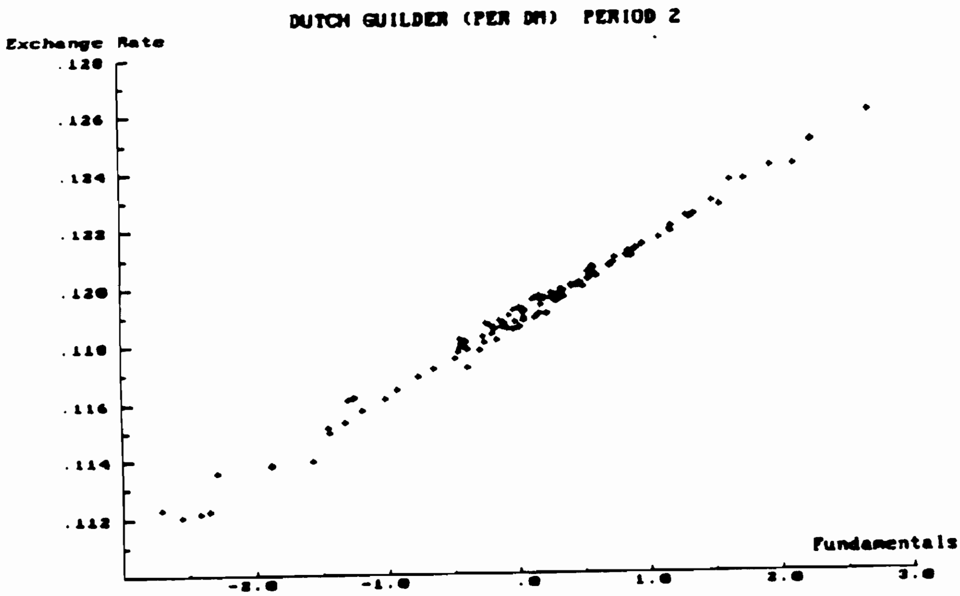
Table IV-2-b : Estimated Probabilities of Realignment:

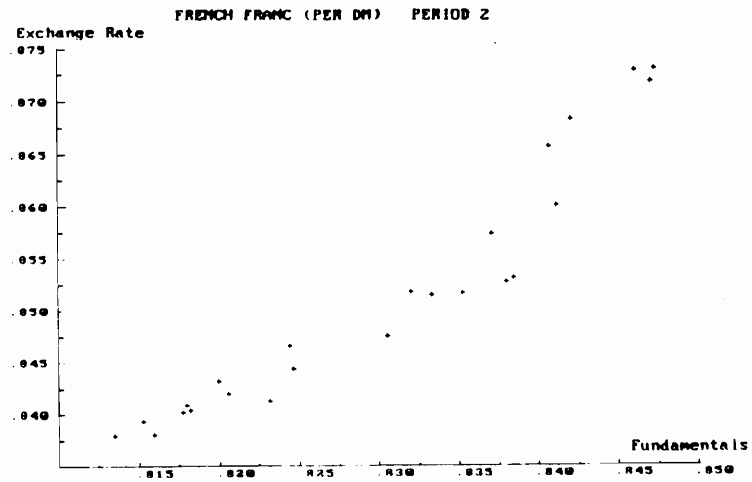
French Franc	Period 2 ⇒	21 % ↓	Italian Lira	Period 6 ⇒	128 % ↑
	Period 5 ⇒	66 % ↑		Period 8 ⇒	66 % ↑
	Period 7 ⇒	69 % ↑		Period 9 ⇒	69 % ↑
Irish Pound	Period 5 ⇒	59 % ↑	Dutch Guilder	Period 1 ⇒	43 % ↑
	Period 7 ⇒	6 % ↑		Period 2 ⇒	43 % ↑

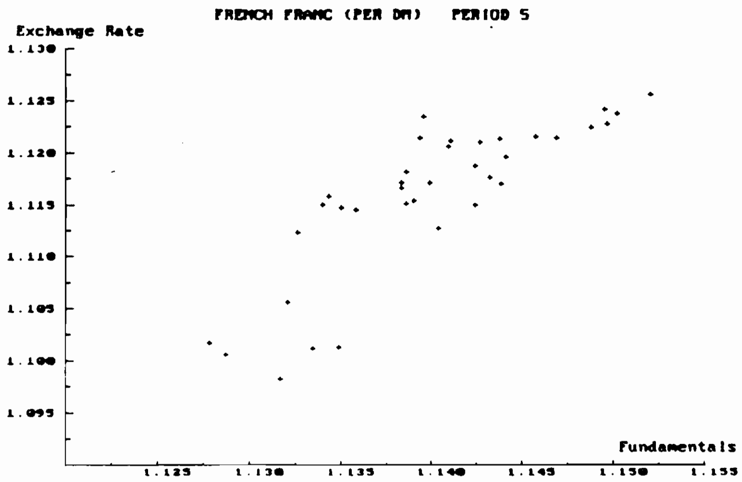
↑(↓) = Probability of devaluation (revaluation).

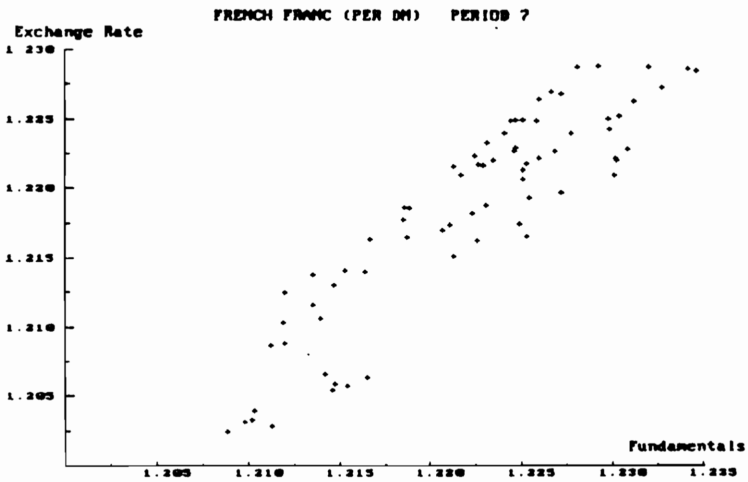
APPENDIX 5: Plots of Exchange Rates vs. Fundamentals from (58)

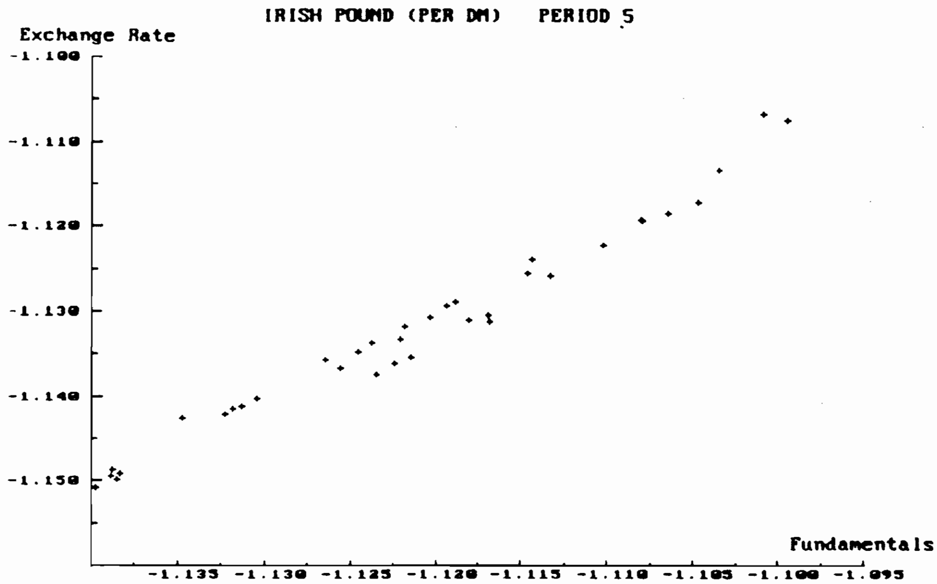


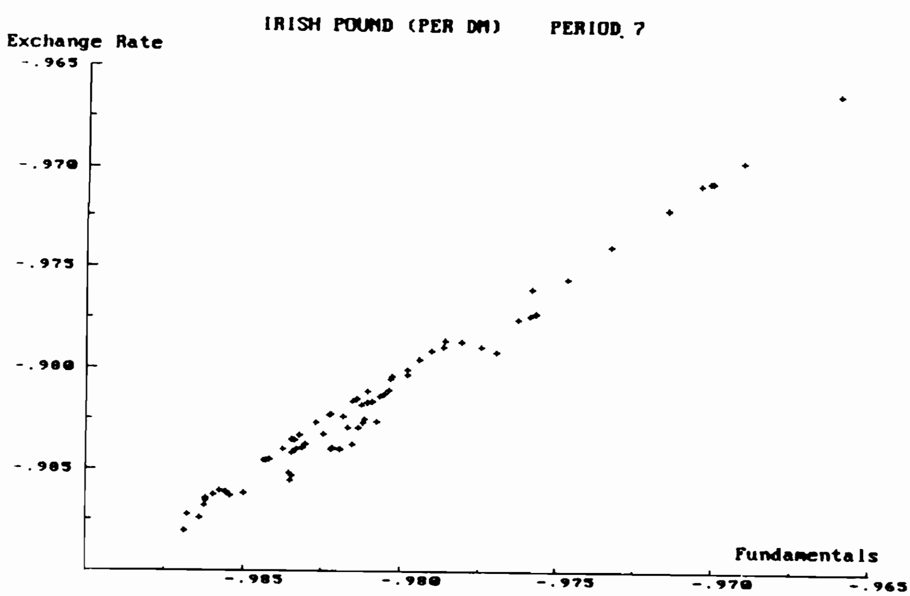


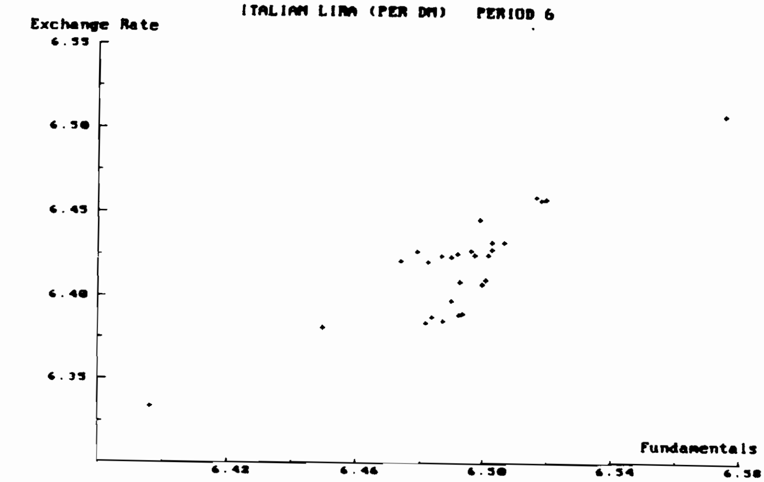

FRENCH FRANC (PER DM) PERIOD 2
Exchange Rate
Fundamentals

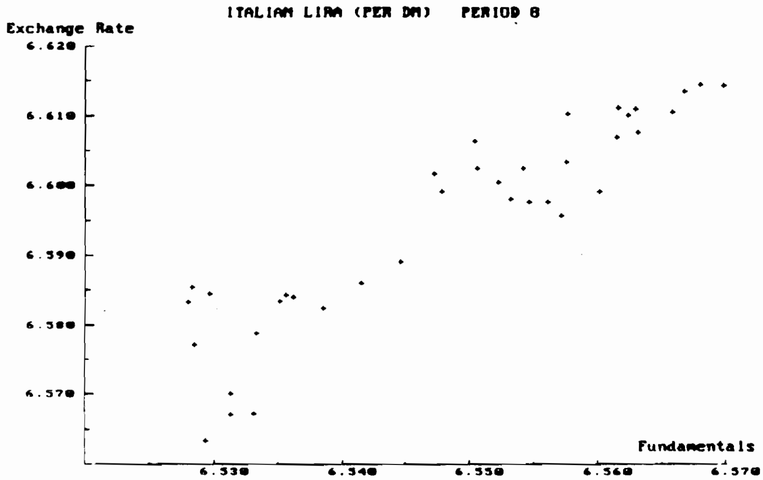

FRENCH FRANC (PER DM) PERIOD 5
Exchange Rate
Fundamentals

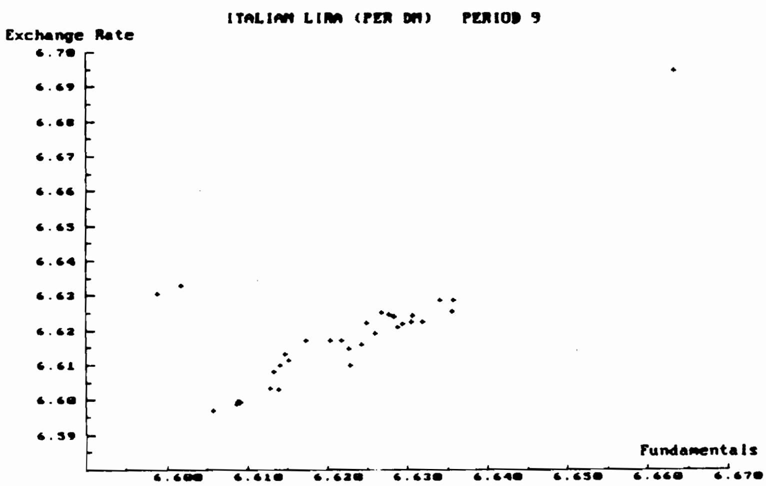

FRENCH FRANC (PER DM) PERIOD 7
Exchange Rate
Fundamentals


IRISH POUND (PER DM) PERIOD 5
Exchange Rate
-1.100
-1.110
-1.120
-1.130
-1.140
-1.150
Fundamentals
-1.135 -1.130 -1.125 -1.120 -1.115 -1.110 -1.105 -1.100 -1.095

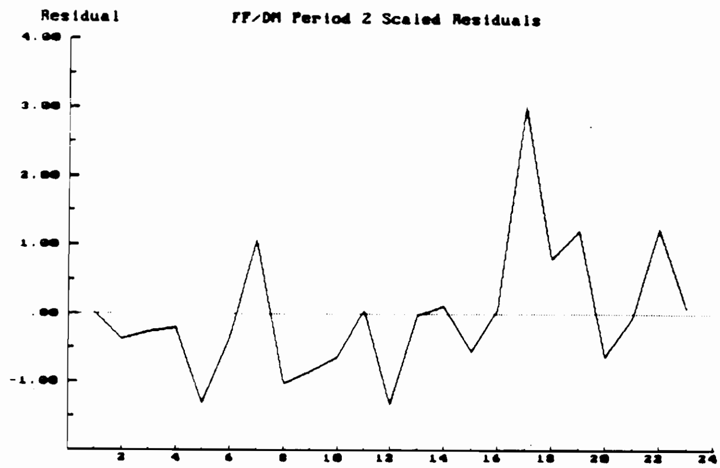

IRISH POUND (PER DM) PERIOD 7
Exchange Rate
-.965
-.970
-.975
-.980
-.985
Fundamentals
-.985 -.980 -.975 -.970 -.965

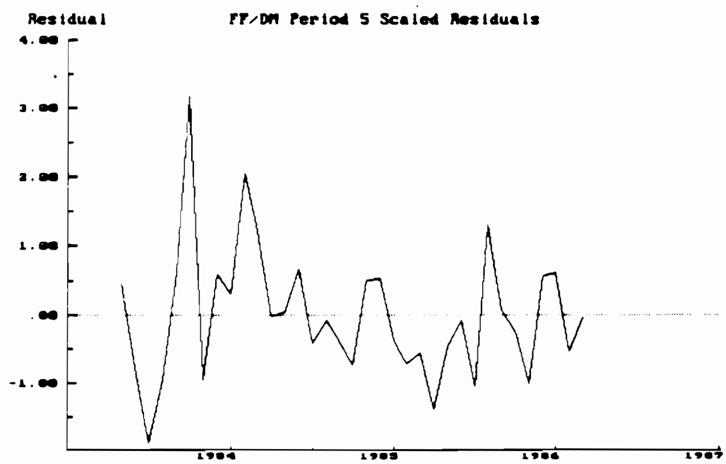

ITALIAN LIRA (PER DM) PERIOD 6
Exchange Rate
6.55
6.50
6.45
6.40
6.35
Fundamentals
6.42
6.46
6.50
6.54
6.58

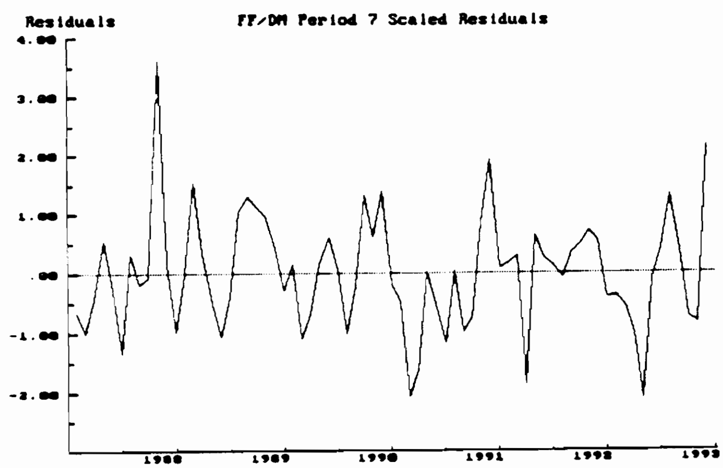

ITALIAN LIRA (PER DM) PERIOD 8
Exchange Rate
6.620
6.610
6.600
6.590
6.580
6.570
Fundamentals
6.530
6.540
6.550
6.560
6.570

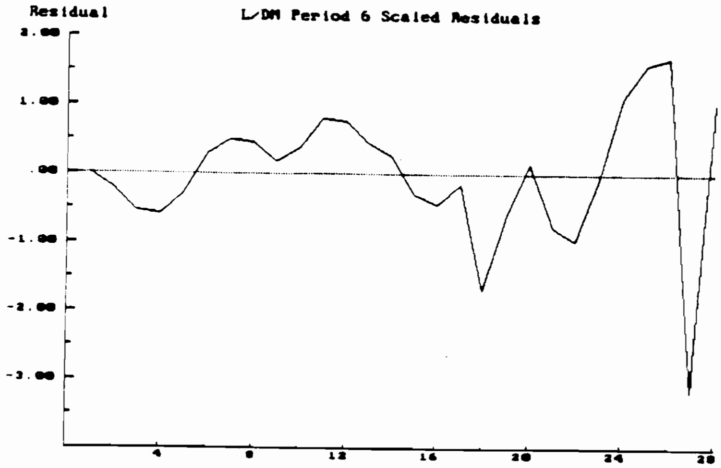

ITALIAN LIRA (PER DM) PERIOD 9
Exchange Rate
6.70
6.69
6.68
6.67
6.66
6.65
6.64
6.63
6.62
6.61
6.60
6.59
Fundamentals
6.600
6.610
6.620
6.630
6.640
6.650
6.660
6.670

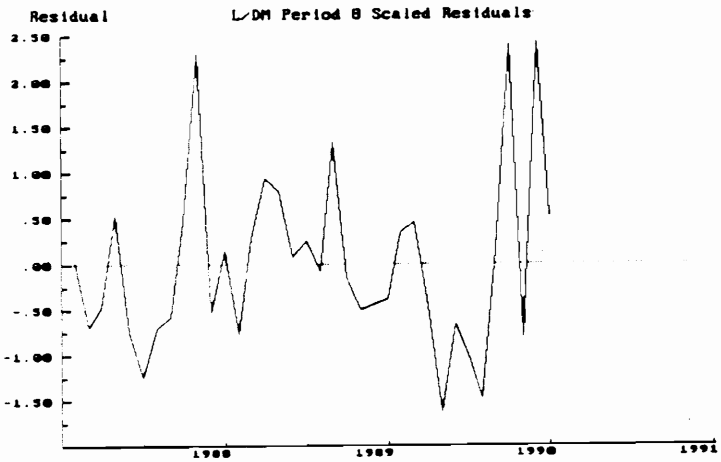
APPENDIX 6: Plots of Scaled Residuals from (60)

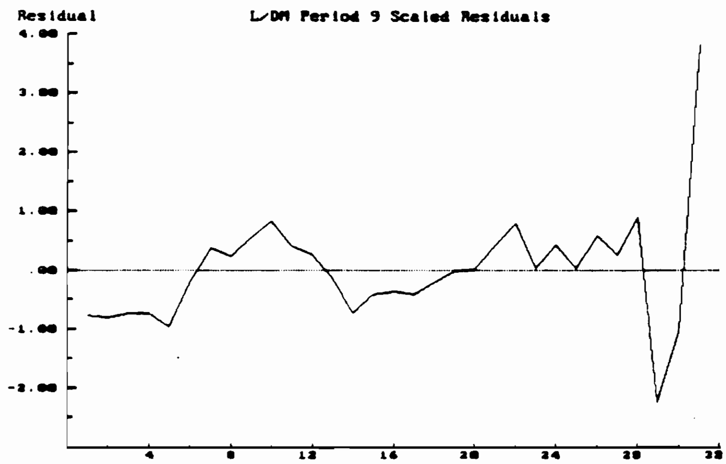


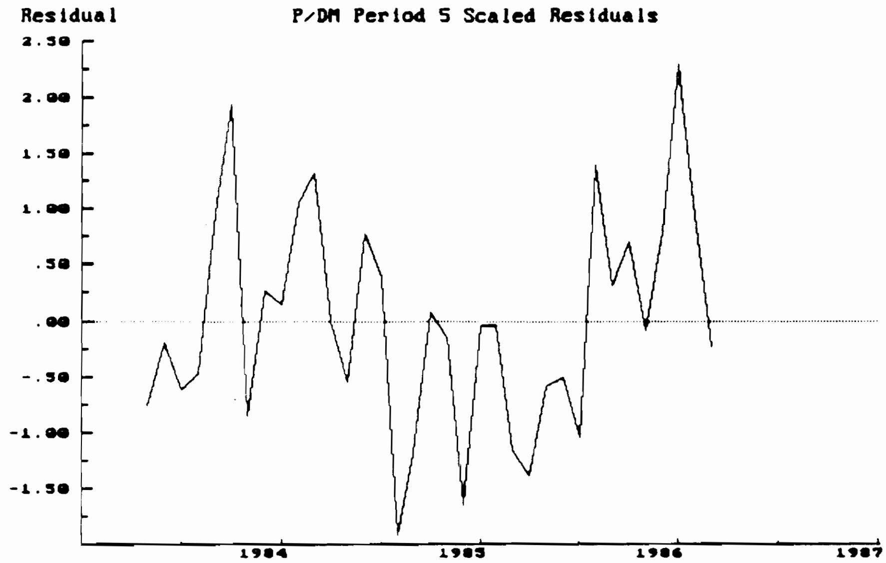


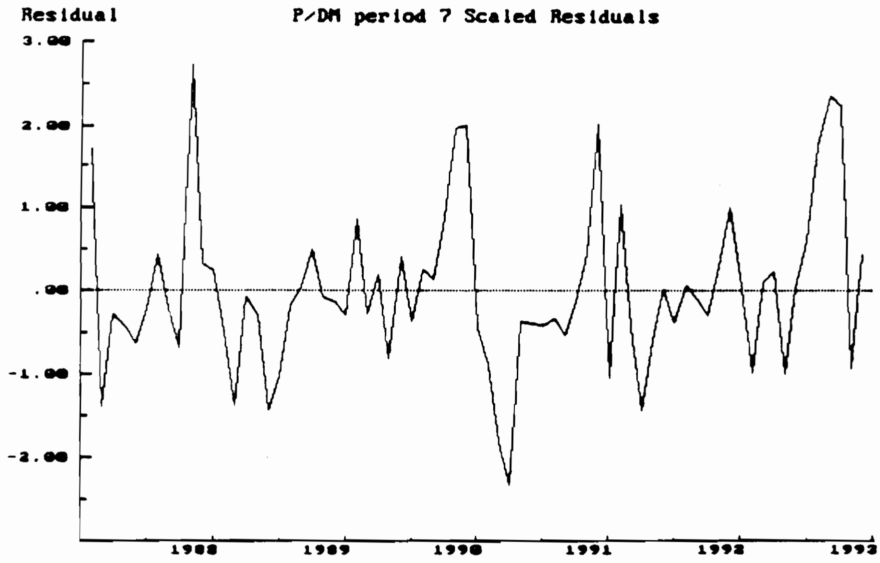


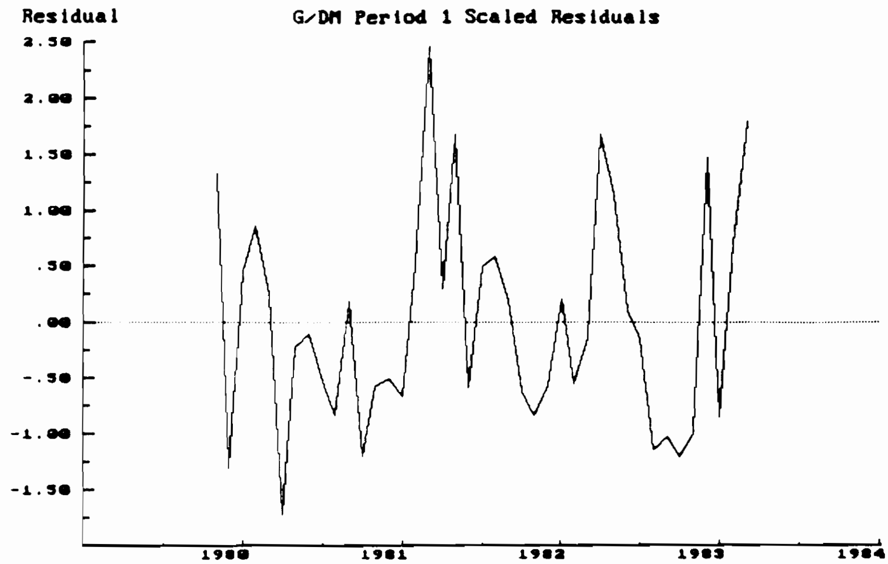

Residual
L/DM Period 6 Scaled Residuals
2.00
1.00
.00
-1.00
-2.00
-3.00
4
8
12
16
20
24
28

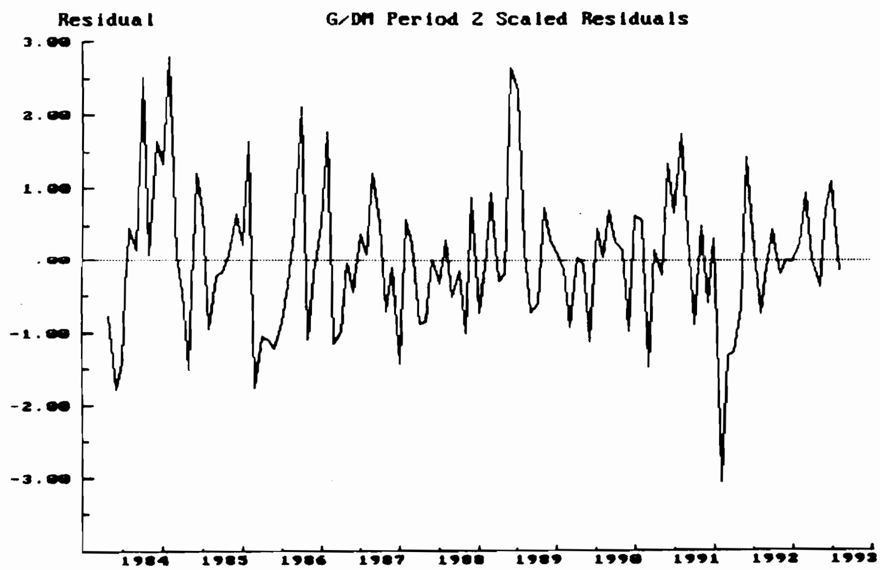

Residual
L/DM Period 8 Scaled Residuals
2.50
2.00
1.50
1.00
.50
.00
-.50
-1.00
-1.50
1988
1989
1990
1991


Residual
L/DM Period 9 Scaled Residuals
4.00
3.00
2.00
1.00
.00
-1.00
-2.00
4
8
12
16
20
24
28
32


Residual
P/DM Period 5 Scaled Residuals
2.50
2.00
1.50
1.00
.50
.00
-.50
-1.00
-1.50
1984
1985
1986
1987


Residual
P/DM period 7 Scaled Residuals
3.00
2.00
1.00
.00
-1.00
-2.00
1988
1989
1990
1991
1992
1993


Residual
G/DM Period 1 Scaled Residuals
2.50
2.00
1.50
1.00
.50
.00
-.50
-1.00
-1.50
1980
1981
1982
1983
1984


Residual
G/DM Period 2 Scaled Residuals
3.00
2.00
1.00
.00
-1.00
-2.00
-3.00
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993

REFERENCES

Abramowitz, M. and Stegun, I., "**Handbook of Mathematical Functions**", NY Dover, 1972, pp. 504-505.

Bertola, G. and R. Caballero, "Target Zones and Realignments", **AER**, June 1992, pp.520-36.

Bertola, G. and L. Svensson, "Stochastic Devaluation Risk and the Empirical Fit of Target Zone Models", **NBER #3576**, January **1991**, pp. 1-33.

Chiang, A. C., "Fundamental Methods of Mathematical Economics", McGraw-Hill, Inc., Third Edition 1984.

De Grauwe, P., "The Economics of Monetary Integration", Oxford University Press, Second Revised Edition, 1994.

Delgado, F. and B. Dumas, "Target Zones, broad and narrow", in P. Krugman and M. Miller, **"Exchange Rate Targets and Currency Bands"**, Cambridge University Press, 1992.

Flood, R. & P. Garber, "A Model of Stochastic Process Switching", **Econometrica**, May 1983, vol. 51, no. 3, pp. 537-551.

Flood, R. and P. Garber, "The Linkage between Speculative Attack and Target Zone Models of Exchange Rates", **NBER #2918**, April 1989, pp. 1-19.

Flood, R., Rose, A., and D. Mathieson, "An Empirical Exploration of Exchange Rate Target Zones", Carnegie-Rochester Conference Series on Public Policy, 1991, pp. 7-66.

Froot, K. and M. Obstfeld, "Stochastic Process Switching: Some Simple Solutions", **Econometrica**, January 1991, pp. 241-250.

Froot, K. and M. Obstfeld, "Exchange Rates Dynamics Under Stochastic Regime Shifts: A Unified Approach", **JIE**, November 1991, pp. 209-229.

Giavazzi, F., "Limiting Exchange Rate Flexibility", MIT Press, 1989.

Goldfeld, S. and D. Sichel, "The Demand For Money", **Handbook of Monetary Economics**, 1990, (Friedman and Hahn eds., Amsterdam, North Holland).

Harrison, M., "Brownian Motion and Stochastic Flow Systems", Wiley Srs., 1985.

Karlin, S. and H. Taylor, "A Second Course in Stochastic Processes", New York: Academic Press, 1981.

Klein, M. and K Lewis, "Learning About Intervention Target Zones", **NBER #3674**, April 1991, pp. 1-33.

Krugman, P., "Target Zones and Exchange Rate Dynamics", **QJE**, August 1991, vol. CVI, issue 3, pp. 669-682.

Krugman, P. and M. Miller, "Exchange Rate Targets and Currency Bands", Cambridge University Press, 1992.

Lewis, K., "Occasional Interventions To Target Rates With a Foreign Exchange Application", **NBER #3398**, July 1990, pp. 1-43.

Lewis, K., "An Empirical Exploration of Exchange Rate Target Zones: A Comment", **Carnegie-Rochester Conference Series on Public Policy**, 1991, pp. 67-78.

Lindberg, H. and P. Soderlind, "Testing theBasic Target Zone Model on Swedish Data: 1982-1990", **European Economic Review"**, August 1994, volume 38, # 7, pp. 1441-69.

Nelson, D., "Arch Models as Diffusion Approximations", **Journal of Econometrics**, 1990, volume 45, pp. 7-38.

Matropasqua, C., Micossi, S., and R. Rinaldi, "Interventions, Sterilization and Monetary Policy in European Monetary System Countries, 1979-87", in F. Giavazzi, S. Micossi, and M. Miller, eds. **"European Monetary System"**, Cambridge University Press, 1988.

Pessach, S. and A. Razin, "Targeting the Exchange Rate: An Empirical Investigation", **NBER #3662**, March 1991, pp. 1-53.

Spanos, A., "Statistical Foundations of Econometric Modelling", Cambridge University Press, Reprinted 1989.

Svensson, L., The Foreign Exchange Rate Risk Premium in a Target Zone with Devaluation Risk", **NBER #3466**, October 1990, pp. 1-27.

Svensson, L., "Assessing Target Zone Credibility: Mean Reversion and Devaluation Expectations in the EMS", **NBER #3795**, July 1991, pp. 1-30.

Svensson, L., "An Interpretation of Recent Research on Exchange Rate Target Zones", **JEP**, Fall 1992, vol.6, no. 4, pp. 119-144.

Svensson, L. and A. Rose, "Expected and Predicted Realignments: The FF/DM Exchange Rate During the EMS", **NBER #3685**, April 1991, pp. 1-37.

Svensson, L., Lindberg, H., and P. Soderlind, "Devaluation Expectations: The Swedish Krona 1982-1991", **NBER #3918**, November 1991, pp. 1-59.

Weber, A., "Exchange Rates, Target Zones and International Trade", **JEL**, December 1992, pp. 1-44.

Williamson, J., "The Exchange Rate System", Institute For International Economics Washington D.C., June 1985.

CHAPTER II

ASYMMETRICAL CURRENCY SUBSTITUTION

II.I - INTRODUCTION

Currency substitution refers to the demand for foreign fiat money by domestic residents of a country. One reason for such a demand is that certain transactions like tourism and foreign trade require the use of foreign currency. Also, the demand for foreign money may be the result of high inflation rates in a country such that the domestic residents start using foreign currency as a store of value. In other words, consumers hedge against high inflation by holding a reliable foreign currency or group of currencies which becomes a part of their investment portfolios. As high inflation continues, currency substitution increases and quoting leases, cars, even salaries in foreign currency becomes a standard practice. Clearly, in such a case, foreign country residents do not have an incentive to invest in foreign currency. They exchange currencies whenever a need arises for transactions. Therefore, this type of unilateral currency substitution is referred to as "asymmetrical" currency substitution vis a vis symmetrical where both foreign and domestic countries simultaneously substitute foreign currency for domestic money.

Asymmetrical currency substitution usually takes place in high inflation countries like Latin American and Eastern European countries, and is also known as "dollarization"[1]. The replacement of domestic currencies by a strong currency is a natural reaction of rational economic agents to the inefficiency of the former as a store of value and as a medium of exchange in the inflationary environment. While dollarization for portfolio reasons, i.e. as a store of value, has a long tradition in Latin America, German Mark has also been equally demanded in Eastern and Southern European countries. It started gaining momentum in Southern European countries with relatively high inflation rates such as Greece, Portugal, Italy, and Spain, as a result of liberalization of capital

[1]See Calvo and Végh (1992) for a detailed note on terminology.

flows and growing financial integration in the European Monetary System (EMS)[2]. The extensive economic reforms in the financial systems of the former communist and socialist European countries such as Russia, Poland[3], Hungary, Bulgaria, and Romania also allow their residents to hedge against high inflation rates by investing in foreign currencies. Furthermore, other European countries such as Turkey[4] and Finland[5] experience a great degree of asymmetric currency substitution. These countries are not yet members but try to cooperate in government policies with the EMS for a smoother transition. Even some Arab countries such as Egypt and Yemen Arab Republic have been experiencing an increasing level of currency substitution[6]. Figure 1, below, shows the inflation rates, exchange rates (against US $), and foreign currency deposits (in US $) for some of these countries above[7]. For each country, there is a clear positive correlation between the foreign currency deposits, inflation rates and the depreciation of the home currencies. This correlation may suggest that the residents tend to replace foreign money with the home money to hedge against inflation.

Asymmetrical currency substitution has many important theoretical and empirical implications. It may cause the real exchange rate to depreciate through its effects on the trade balance. If foreign currency is the only internationally traded asset because of repressed or underdeveloped financial markets, a monetary disturbance may lead to fluctuations in the real exchange rates and then dollarization, i.e. accumulation of foreign

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[2]Bini Smaghi and Vori (1991) and Angeloni, Cottarelli and Levy (1991) document the currency substitution in EMS. Biltoft, Karsten and Karsten (1990) provide evidence for currency substitution in EMS by estimating the traditional money demand equation. Also, Angeloni and Giucca (1991) provides a good analysis for Italy.

[3]Brand (1993) provides an in-depth analysis for Poland, which also shows that inflation rate and the foreign currency deposits in Poland between 1986-1992 had a significantly positive correlation.

[4]Turkey has already signed-up a full customs union treaty with the EMS which is considered to be the first phase of the transition. Also, Selcuk (1994) provides a detailed empirical analysis of the currency substitution in Turkey.

[5]Matti (1990) provides an empirical analysis of currency substitution in Finland.

[6]For an empirical work on these countries, see El-Erian (1988).

[7]Mexico for Latin American countries, Hungary for former socialist countries in Europe, Turkey for Eastern European countries, and Greece for the EMS countries.

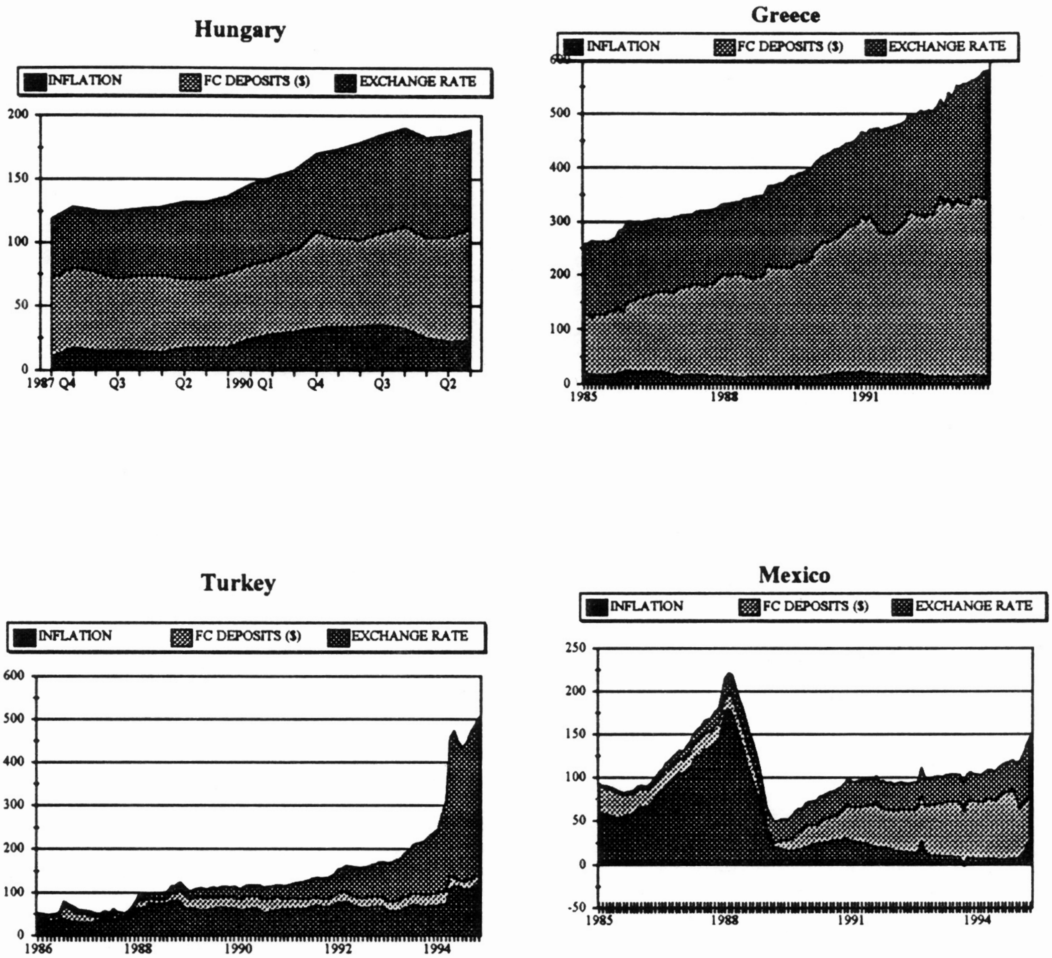

Figure 1: Currency Substitution in Hungary, Greece, Turkey, and Mexico

The main data source is International Financial Statistics by IMF. The data series are 94464..xzf, 944..rf.zf, and 94425..zf for Hungary (quarterly) for the period 1987-1993; 17464..xzf and 174..rf.zf for Greece (monthly) for the period 1985-1984; 18664..xzf, 186..rf.zf, and 18615..zf for Turkey (monthly) for the period 1986-1994; 27364..xzf, 273..rf.zf, and 27325..zf for Mexico (monthly) for the period 1985-1995. The foreign currency deposits for Greece are taken from the Monthly Statistical Bulletin of the Bank of Greece.

currency by domestic residents, will take place through a current account surplus. Then in equilibrium the real exchange rate will depreciate. Also, with a greater degree of dollarization, the governments are constrained to other forms of revenue than inflationary finance (seigniorage revenue) such as higher tax revenues and/or domestic debt, assuming that there are no restrictions on foreign currency holdings. Thus it is argued that it provides the governments with more discipline; for example, the tax system is restructured to eliminate tax evasion and it eventually drives down the inflation rate by generating competitive forces among the governments[8]. Dollarization process usually begins with foreign money substituting the domestic money as store of value to hedge against substitution. This process is common especially in countries with no or inefficient financial markets. Transactions, then, begin to be performed in foreign currency as well as price quotes, and wage and lease contracts. During this process, however, domestic money does not disappear, and retains its functions as unit of account and medium of exchange for most non-durable goods.

Dollarization may also lead eventually to case where the government completely gives up issuing its own currency and adopt the foreign currency[9]. Such a case is called "full dollarization" and has the advantage of higher credibility for a government. On the other hand, it limits the government to the use of fiscal policy, and the banks would be forced to operate without "a lender of last resort"[10].

This chapter is intended to review alternative ways of modeling currency substitution and present a model of asymmetrical currency substitution with an application to seigniorage taxation. The plan of this chapter is as follows. Section II.II provides a selective survey of recent studies on modeling currency substitution emphasizing the theoretical frameworks, strengths and weaknesses without too much technical detail.

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[8]Canzoneri and Diba (1992) analyzes this issue in an environment where governments fight over seigniorage revenues, terms of trade, and lack of credibility.
[9]Panama and Liberia in 1980s are good examples to full dollarization.
[10]See Calvo and Vegh (1992) for a more in-depth discussion.

Some of the major empirical studies on asymmetrical currency substitution are also presented in this section. In Section II.III, we present our model of asymmetrical currency substitution and its implications for government finance. This section also includes a brief review of studies with particular reference to seigniorage taxation. A summary and conclusion is given in section I.IV.

II.II - MODELING CURRENCY SUBSTITUTION

The task of modeling currency substitution has been handled in various ways. Quite a few studies, however, overlap in their assumptions, making it very difficult to distinguish the approaches clearly. Some of the earlier studies such as Kouri (1976), Calvo and Rodriguez (1977), Miles (1978), Girton and Roper (1981), Tanzi and Blejer (1982), McKinnon (1982), and Thomas (1985) view money holdings as foreign assets and analyze optimal portfolio composition in a world with capital mobility. In these models, inflation rate and/or interest rate differentials determine the demand for monies. Alternatively, transactions demand for money leads to currency substitution in Calvo (1985), Végh (1989a,b), Guidotti and Rodriguez (1992), and Guidotti (1993), where monies are imperfect substitutes. Others such as Kim (1985), Hercowitz and Sadka (1987), Handa (1988), Elkhafif and Kubursi (1991), and Sibert and Liu (1994) take the currency substitution to be the use of multiple currencies in a country as a store of value. Yet another approach is to put home and foreign money directly in the utility function as in Liviatan (1981), Melvin (1988), Weil (1991), Canzoneri and Diba (1992), and Sturzenegger (1992)[11]. These models have the advantage of including liquidity services provided by both monies in the utility function.

Asymmetrical currency substitution has also been theoretically and empirically analyzed in several studies such as Ortiz (1981), Tanzi and Blejer (1982), Ramirez-Rojas (1986), El-Erian (1988), Guidotti and Rodriguez (1992), Selcuk (1994), and Krueger and Ha (1995).

In this section, we will review the literature, selectively, on modeling currency substitution under the different approaches summarized above. The emphasis is on the theoretical framework and how the currency substitution arises as a result of the

[11] Sturzenegger (1991) emphasizes the role of currency substitution also at the level of transactions demand for money.

assumptions and the workings of the models. We do not examine necessarily the applications and/or implications of the models unless they contribute tremendously to the understanding of section II.III.

II.II.1 - The Monetary Approach (The Asset View of Money Holdings)

According to this approach, exchange rate, as a relative price of monies, is viewed as a factor which equilibrates the international financial markets. Therefore, behavior of the demands, as well as supplies, for different monies and other financial assets are examined to explain the behavior of the exchange rates. In this regard, Kouri (1976) models currency substitution in analyzing the adjustment path and convergence from momentary to long-run stationary equilibrium in the financial markets, and the dynamic response of the exchange rate (and the balance of payments) to various shocks. The analysis is based on a small economy producing only traded goods with flexible prices. Domestic and foreign money are the only two assets and the rate of interest on foreign assets is zero. There are two equilibrium conditions that summarize the model. The first is that the exchange rate is determined to equilibrate the demand for foreign assets with the existing stock of foreign assets; asset markets equilibrium. The second is that the trade account surplus, given by the excess of domestic output over domestic consumption, equals the change in foreign assets; current account equation. Then the currency substitution arises as a result of the dynamic adjustment of the stock of wealth to its long-run desired level through deficits and surpluses in the current account. This may occur through two channels. One channel is that an increase in the expected rate of depreciation causes an increase in desire to lend abroad which causes the exchange rate to depreciate. The depreciation causes the current account to move to a surplus, and this surplus gradually increases the stock of foreign assets. The other channel is that, with a tax financed increase in government expenditure, the equilibrium in the asset markets remains unchanged and so does the exchange rate. Because of the deteriorating trade account surplus, current account moves to a deficit and this, in return, reduces the stock of foreign assets. An important feature of Kouri's model is that this dynamic adjustment process depends critically on the nature of expectations formation. Substitution of monies intensifies with speculators having a myopic perfect foresight.

The difference between the expected rates of return on domestic and foreign monies leads to currency substitution also in Calvo and Rodriguez (1977). Similar to Kouri (1976), the currency substitution is *assumed* to be asymmetric (we prove in the next section that this assumption holds) in a small open economy with flexible prices where domestic and foreign monies are the only available assets. Both studies also assume that money demands are given, not derived from consumer preferences. Such an extension takes place in by Liviatan (1981) (see section II.II.4). The dynamics in Calvo and Rodriguez (1977) works as follows. Given that the system is initially in steady state, an announced higher rate of monetary expansion results in an instantaneous increase in both the real exchange rate and the price level. This is followed by a transition period during which economy accumulates foreign exchange towards the new steady state with the same real exchange rate and larger holdings of foreign exchange while the total assets remain constant.

In Miles (1978), currency substitution is simply based on the prevailing opportunity costs of holding real balances in domestic and foreign currencies. The opportunity costs are given by the interest rates on each currency. The model is assumed to be initially in equilibrium with total demand for real balances being equal to the money holdings. Furthermore, given the prevailing opportunity costs of holding each money, the individuals are assumed to have adjusted their portfolios so that they are just indifferent between holding more of any currency. In this setup, there are two mechanisms through which currency substitution can occur; a once and for all increase in the supply of one of the currencies, and an increase in the supply of one of the currencies at a constant rate. In the former case, the cost of borrowing the currency (interest rate) will fall in order for the increase in its supply to be absorbed. Then holding this currency becomes relatively more attractive, and the new equilibrium is characterized by greater holdings of one currency in both countries compared to the initial equilibrium. In the latter case, however, the effect of an increase in the supply of one currency will be different. It will create expectations of inflation which will cause the interest rate to rise. Then the new equilibrium will arise with greater holdings of the other currency in both countries.

The frameworks in Girton and Roper (1981) (as well as Girton and Roper (1980)) are also based on two given money demand functions, together with exogenous money supplies and is used to analyze the money exchange rate. Money demands are assumed to depend on differential returns adjusted with the coefficient of substitution between the monies which represents currency substitution. The shifts in the anticipated rate of change of the exchange rate lead to greater currency substitution and thus to greater exchange rate volatility[12]. If the substitutability is perfect, however, the exchange rate is indeterminate. An important implication is that, with substitutable monies, market pressure will force money issuers to make their monies attractive, and a joint monetary policy may stabilize exchange rates without providing money holders lower inflation rates. This result is in contrast to Canzoneri and Diba (1992). The major drawbacks of Girton and Roper (1991), on the other hand, are that it is not clear what feature of the economic environment is causing substitutability, welfare experiments are not possible, and a lack of substitutability is not associated with any real resource loss.

Similarly, McKinnon (1982) begins his analysis defining a two stage money demand function in a two country and two non-interest-bearing money model with a perfectly efficient international bond market (perfect capital mobility). The first stage

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[12] Sibert and Ha (1994) shows that this is true if and only if money shocks are positively autocorrelated. In contrast, Gazioglu (1993) argues, in a model simulating a disturbance in the money market, that high currency substitution reduces exchange rate volatility.

describes the world demand for money and the second stage divides that demand between the two currencies. Currency substitution takes place when the anticipated currency depreciation in one country reduces the share of that currency in the world demand, and the share of the other currency increases commensurately so as to keep the total demand for world money constant for any given interest rate. This may happen through two different channels. First, large commercial banks (and non financial multinationals) shift their non-interest-bearing working balances into foreign currency to reduce direct losses from anticipated home currency devaluation. Second, the perfectly efficient bond market quickly adjusts to the devaluation expectations, moving out from home currency denominated bonds into those denominated in foreign currency. Also, interest rates adjust immediately so as to eliminate international arbitrage. McKinnon's framework is convenient to analyze monetary policy implications of currency substitution, however, his assumptions are too strong to hold empirically. For example, he ignores any direct effect that changes in expected devaluation might have on the world interest rate or on the world demand for money. Moreover, the assumption of perfect capital mobility assumption is far from convincing during much of the 1960-80 period.

Finally, Thomas (1985) argues against the usual conclusions of portfolio theory (monetary approach) and demonstrates that including opportunity cost variables such as foreign interest rates, foreign inflation rates, or expected exchange rate changes in the domestic money demand function fails if investors can borrow in all currencies (through bond market). In other words, if the monies are the only assets, monetary approach models cannot distinguish between the causes of currency substitution and capital mobility. His analysis is based on an intertemporal, continuous-time model of portfolio formation across two monies. Monies do not bear no interest, but do reduce the transaction costs associated with consumption. Borrowing and lending are possible through riskless bonds denominated in each currency. Investors construct portfolios in two steps. First, an investor selects his currency holdings, based on each money's transaction services and interest rate. In other words, optimality requires that investors hold each currency until the marginal currency produces additional transaction services equal in value to its holding cost (interest rate). Then money's expected real return or risk can be ignored in deciding how much money to hold. In the second stage, he borrows and lends to achieve the desired overall portfolio composition. The final portfolio is independent of money demands from the first decision stage. As a result, there is no demand for money as a portfolio asset, and the interest rate, inflation rate or devaluation risks associated with money holdings can be mitigated by adjusting non monetary asset and liability balances. This approach by Thomas (1985) may effectively explain the negative empirical findings of currency substitution in developed countries. However, for the economies with high inflation rates and volatile exchange rates, a strong foreign currency may very well be justified to be a portfolio asset, especially if the secondary financial markets are not very developed (or so volatile that it is not reliable), and the liquidity in investing is of prime importance.

II.II.2 - Transactions Demand For Money

This approach takes currencies strictly as media of exchange, and the demand for domestic and foreign money is motivated more explicitly by a transactions motive[13]. For example, in Végh (1989a), consumers hold money because it facilitates carrying out transactions by enabling them to save on the time they devote to shopping activities. In other words, holding currencies decreases the effective price of consumption which is given by the market price including consumption tax plus the increase in transaction costs associated with consuming one more unit. The consumers maximize utility (of leisure and consumption) in two stages. In the first stage, they choose the real money balances such

[13]In some studies, for example, Liviatan (1981) and Calvo (1985), money serves both as a medium of exchange and a store of value.

that the marginal productivity of holding an additional unit of relative money balances (in terms of reduced shopping time) equals to its marginal cost (given in terms of the respective interest rate). Then given the optimal real balances, they choose the optimal amounts of consumption and leisure. Currency substitution enters the model in the first stage. A rise in the marginal cost of holding a currency induces consumers to substitute away from that currency towards the other. In such a framework, consumers care only about the effective price of consumption (price inclusive of the transactions costs), not about the particular combination of its elements. Then the importance of holding foreign money comes in reducing the effective price of consumption. This formulation with an addition of a government, which finances revenues through distortionary taxes, is applied to the derivation of optimal inflation tax, as in our model in the next section. The results show that if the foreign interest rate is positive, it is optimal to impose a positive inflation tax. Because a positive domestic interest rate enables the government to offset the increase in the consumption tax which comes from a positive foreign interest rate. Within the same framework, Végh (1989b) adds the possibility that the government may also resort to a labor (income) tax besides consumption and inflation taxes. Then the results show that the higher the degree of currency substitution, the higher is the optimal inflation tax for a given level of government spending and the foreign interest rate.

Giovannini (1991) assumes that domestic and foreign residents can use financial assets as means of transactions in the goods markets while they are obliged by law to use currency on a certain subset of transactions. More specifically, cash is required by law to pay taxes which are returned to the residents in the form of lump-sum transfers. The paper examines the role of substitutability of currencies on the fluctuations of foreign exchange reserves in a fixed exchange rate regime. Thence, the model has an additional agent (central bank) which holds a portfolio of domestic and foreign currency, and manages it to maintain, at the given nominal exchange rate, private demand for each currency equal to supply. At the beginning of each period, the agent receives transfers from the two national governments, dividends from the two firms, monetary transfers from the two central banks, and he carries over any unused money balances from the previous period. These resources are used to buy shares, moneys, and goods. Once goods and assets markets close, the agent uses the money balances accumulated to pay taxes whose amounts in the two currencies are known since the beginning of the period. The main result is that when the exchange rate is credibly fixed, currencies are perfectly substitutable as store of value although they are not so as transactions media. Private agents are indifferent about the composition of their currency portfolio when they hold money balances in excess of their consumption purchases unless currencies are held for their transaction services, not purely for store of value purposes.

Guidotti (1993) also develops a cash-in-advance framework where individuals are required to use domestic (foreign) money to buy domestic (foreign) goods. Currency substitution is generated endogenously through the interaction that the two monies have in affecting the total amount of time devoted to transaction activities. There are two countries inhabited by identical, infinitely lived individuals, two goods, two monies, and two bonds in the model. Bonds pay fixed interest rates. The investors hold two accounts denominated in domestic and foreign currencies. Domestic (foreign) currency is obtained from the stock of domestic (foreign) bonds. In each time period, first consumption (subject to cash-in-advance constraints) takes place, which can be financed by withdrawing money from bonds. These withdrawals involve time costs which can be used for leisure, otherwise. Thus, similar to Végh (1989a), time spent in transactions (cash withdrawals) adds to the price of the consumption good. Second, individuals receive an endowment income, pay taxes, receive interest payments on their bond holdings, an engage in asset trade to choose the new composition of their asset portfolio. An increase in domestic nominal interest rate (the opportunity cost of holding domestic money) generates an increase in the frequency of withdrawals of domestic currency. For a given consumption level, this leads to a fall in domestic real cash balances. Spending more time transacting increases the marginal utility of reducing the frequency of foreign currency withdrawals. Hence, currency substitution arises as a result of this interaction between the domestic and foreign currencies through the amount of time used in financial activities. At this point, however, a careful reader will notice that the transactions demand for money approach, as used in Guidotti (1993), is nothing more than adding an intermediary step to the direct relationship between the interest rate differential and currency substitution of the monetary approach. On the other hand, such a setup becomes very convenient to analyze the macroeconomic effects of a technical progress in the financial system in the presence of currency substitution, and the changes in the regulatory environment. The results in Guidotti (1993) show that a financial innovation in the presence of currency substitution may lead to negative co-movements between the nominal and the real exchange rate through its effects on the relative demand for currencies. Also, the international transmission of financial innovation depends on how it affects the cross-border transfers of seigniorage which occurs because of currency substitution.

II.II.3 - Store of Value

According to this approach, foreign currency is substituted for domestic currency mainly to hedge against inflation; to preserve the value of the money holdings. It is preferred to other forms of assets because of its liquidity. Currency substitution is common especially in countries with underdeveloped or inefficient (very volatile) financial markets where other financial assets than foreign currency cannot be easily and inexpensively traded. Therefore, the store of value function of foreign money becomes crucially important. However, in such a case money may still serve as media of exchange. For example, Handa (1988)[14] considers the degree of substitutability between domestic

[14] This paper actually has a framework of transactions demand for money. However, it emphasizes beautifully the store of value function of foreign money, hence we review it in this section.

and foreign currencies. Foreign currency acts as liquid store of value while the domestic currency, as the most convenient and preferred domestic medium of payments, is the preferred habitat for transactions balances. This is called the "preferred habitat" hypothesis and implies that the degree of substitution between the currencies (for transactions) is very limited. Also, according to the "yield dominance hypothesis", the currencies are highly substitutable only in the absence of bonds. Because bonds have a positive nominal yield while foreign currency has a zero one although they both have generally the same transactions costs. Therefore, in his model of two monies and two bonds, foreign money could be held as store of value but drops out of analysis because of the yield dominance hypothesis which replaces the demand for money as store of value with demand for domestic and foreign bonds. Then the domestic residents demand foreign money only for export/ import payments. The empirical evidence presented on Canadian money demand functions supports the model's implications. The U.S. $ does not serve as medium of payment on a par with the Canadian $ although the former is a very liquid and largely available foreign currency. This implies that the degree of substitution between these currencies, for Canadian residents, is almost zero. This finding receives support from Imrohoroglu (1994) who uses Generalized Method of Moments procedure to directly estimate both the elasticity of substitution between the U. S. and Canadian dollar and the shares of individual currencies in producing money services. The estimates indicate little substitution; the U. S. dollar deposits in Canada do not seem to provide a good substitute for the Canadian dollar in reducing the transactions costs. However, money enters the utility function significantly and the share of U.S. dollar-denominated deposits in the Canadians' portfolio is significantly positive but it is small in magnitude.

Kim (1985) constructs a model of currency substitution in an overlapping generations framework in which the young (workers) supply labor and the old (consumers) consume. He examines the labor supply decisions of the workers when they are allowed to diversify their portfolios in domestic and foreign currencies to finance consumption in old age. Money serves only as a store of value, and the workers maximize expected utility subject to the production technology and the budget constraint. Domestic and foreign inflation rates are assumed to be stochastic but stationary. The first order conditions determine the optimal output decision and the optimal share of foreign currency. The former is an increasing function of the weighted average rate of return on two currencies and the latter increases in both the relative variance of home inflation rate and the excess of the expected domestic inflation over the foreign inflation. In monetary equilibrium, an increase in the variance of the home inflation rate depreciates the exchange rate. This arises from the output effect, which decreases production, and the portfolio balance effect, which increases the proportion of foreign currency held. An increase in the variance of the foreign inflation rate appreciates the exchange rate since the proportion of domestic currency held is increased. But a decrease in production partially offsets the appreciation. Another result is that the imposition of the capital controls, which prevents traders from obtaining efficient portfolio allocation, reduces the welfare of the young as long as demand for foreign assets is strictly positive.

Khan and Ramirez-Rojas (1986) argue that an expected depreciation of the domestic currency causes residents to shift out of domestic currency into foreign money. Therefore, they add the proportional change in the exchange rate to standard money demand which depends negatively on the expected inflation. The model is analyzed in steady state, hence, actual inflation and change in the exchange rate are used in the money demand. They assume that government maximizes revenue from money creation and determine a revenue maximizing inflation rate which depends inversely on the effect of inflation on the change in the exchange rate. In other words, if the change in the exchange rate is not an argument in the money demand, as in the standard case, the revenue maximizing inflation will be higher.

Similarly, Hercowitz and Sadka (1987) deals with the question of financing the government's budget in the presence of currency substitution in a model where both monies can serve as a store of value but only domestic money can serve as a medium of exchange. They assume that the foreign money has a stable real value while the real value of the domestic money depreciates at the rate of inflation. A fixed transactions cost, given for the consumers, is determined by the government. They show that, with any positive inflation rate, individuals would hold only foreign money and convert it to domestic money just before making real transactions. Then foreign money practically becomes the medium of exchange, and the government would have no seigniorage revenue. In this case, government can discourage currency substitution by restricting its exchange. This may cause real resource loss, but increases seigniorage revenue. However, they argue that inflationary finance is never optimal because of the waste of resources in transactions costs.

A similar analysis includes constant proportional transaction costs in Sibert and Liu (1994). In a two-period overlapping generations model where monies are the only assets, goods must be purchased in the seller's currency but currency may be traded before shopping at cost in the second period (in the next section, we allow for transaction costs in the first period). This cost is a measure of currency substitutability. There is no aggregate uncertainty. However, agents do not know their second period preferences, and this causes them to have a precautionary demand for the weaker currency. Then, in equilibrium, consumers hold both monies if the transactions costs are greater than the rate of return to either money. Governments must finance expenditures by levying a costly income tax or by collecting seigniorage, and in deciding the level of inflation, they weigh the costs of income taxation and the distortions created by rising prices. If identical governments cooperate then the optimal rate of money growth is strictly positive. It is decreasing in substitutability of currencies if and only if first and second period consumption are gross substitutes. If governments act independently money growth may be either too high or too low.

Chang (1994) also endogenizes currency substitution with transaction costs in an overlapping generations framework similar to Sibert and Liu (1994). However, transaction cost is fixed, and agents have perfect foresight; no uncertainty. Also, agents can invest in domestic bonds besides the currencies, and the government finances deficit only by printing money; no costly income taxation. Agents receive an endowment when young and consume in both periods. In order to finance the old-age consumption, each young agent must sell part of his endowment and acquire assets. Acquiring foreign currency and/or bonds, however, is costly. Agents maximize logarithmic utility constrained by their budgets. They hold domestic currency if and only if the inflation rate is smaller than some threshold rate which is a decreasing function of the wealth. Chang focuses on two classes of equilibria: equilibria in which all domestic agents hold domestic currency, and equilibria in which all rich agents hold foreign currency and all poor agents hold domestic currency. The results indicate that for a certain range of fiscal deficit, the rich hold foreign currency if the expected inflation is high and domestic currency if the expected inflation is low. In the former case, the base for the inflation tax becomes only the savings of the poor, and the inflation rate needed to finance the deficit becomes large. In the latter case, however, the base for inflation tax becomes larger and the actual inflation rate goes down. These findings support those of Sturzenegger (1992) (see below).

II.II.4 - Money in the Utility Function

Instead of maximizing utility in two stages as an implicit function of money balances, another approach is to put home and foreign money directly in the utility function. For example, Liviatan (1981) includes both monies in the utility function as well as consumption in order to account for differential demand for domestic and foreign monies. He builds his model on Calvo and Rodriguez (1977) but bases the consumers' behavior on utility analysis. Since there are no bonds and borrowing, the only way the consumer can spend in excess of his income is by running down his stocks of both monies. The utility function is separable and hence it allows for the existence of a solution to the consumer's problem. The consumer maximizes utility but constrained by its capability in accumulating assets. Currency substitution results from the dynamics of the model but follows an opposite direction to that of Calvo and Rodriguez (1977). A higher money growth rate decreases the real return on domestic balances which leads to a fall in domestic real money balances. Then this reduces the marginal utility of foreign money balances and thus the steady state demand for foreign currency because of the critical cooperancy assumption; the cross derivative between domestic and foreign money balances is positive. Under long-run perfect foresight, the decumulation of foreign currency must start immediately so that the initial balance of trade deteriorates. This, in turn, requires an appreciation of the real exchange rate in contrast to the result in Calvo and Rodriguez (1977).

The motivation to put monies directly into the utility function is different in Melvin (1988). He argues that concentrating on foreign interest rate effects on the demand for domestic money mixes the effects of substitutions between money and interest bearing assets with substitutions between monies. Hence, it is misleading. To solve this problem, in his model, the demand for money services depend not only on real income and domestic and foreign interest rates but also two "quality" variables related to confidence with regard to the future exchange values of the domestic and foreign monies. They can also be considered to be inflation uncertainty variables. An increase in the quality variables reduces the demand for their respective money balances. The demands for money services are determined by maximizing the utility, as a function of consumption and flows of services provided by the stocks of real money balances, subject to the budget constraint where real income is equal to change in bond holdings plus money holdings. He argues that the estimated responsiveness of money demanders to changes in the quality variables can be examined to infer information regarding substitutions between monies. Proxies for quality variables are created by estimating moving first-order autoregressive processes for each currency's inflation rate. The uncertainty at any period is proxied by the standard error of the residual of the autoregressive (AR) process estimated over the previous twelve quarters. The results show that the money quality variable affects domestic currency demand significantly in both Mexico and Bolivia. In other words, Latin American money demanders are sensitive to changes in the inflation uncertainty associated with the domestic currency relative to the dollar, supporting the notion that dollarization is a significant phenomenon in Latin America.

Weil (1991), on the other hand, bases his analysis on the European Union. He argues that currency competition will not promote the use of low inflation currencies at the expense of high inflation ones in both transactions and deposits, and perfect substitutability will make speculative hyper-inflation more likely. In his two-money and two-bond model, the consumer maximizes utility as a function of consumption and both monies, subject to instantaneous budget constraint. Then he analyzes equilibrium dynamics. When the currencies are perfect substitutes, equilibrium law of motion dictates that, despite divergent money growth, nominal interest rates as the opportunity cost of holding monies be equalized across countries. In an economy with free capital mobility, this leads to the same inflation rate in both countries. Then real balances grow faster in the country in which the nominal money stock grows faster. The equalization of exchange rates, however, leads to a fixed, but indeterminate, exchange rate as in Kareken and Wallace (1981). Weil also shows that if money is included in the utility function, i.e. money is essential, dynamic paths in which both currencies become worthless in real terms cannot be valid equilibria. If money is not essential for utility, however, it is possible to have many equilibria, notably hyper-inflationary, in which both currencies are worthless. In the case of imperfect substitutability, if money is essential, there is a unique equilibrium in which one currency loses its real value.

Canzoneri and Diba (1992), also, develops a model of currency substitution with a money-in-the-utility-function framework, and applies it to inflation tax. In contrast to Weil (1991), they find that a high degree of currency substitution would generate competitive forces among the suppliers of currencies that would drive inflation rates down. But the resulting inflation rate is not necessarily optimal. In a two-country, one-good, two-money model, consumers maximize utility subject to periodic budget constraint. Then they acquire real money balances up to the point where the marginal value of holding money equals the foregone interest on bonds. Currency substitution is parametrized in the utility function. First order conditions determine that as currency substitution increases, marginal utilities of home and foreign holdings of their own currencies converge. Then monetary policies must also converge or one of the currencies will not exist. Governments are added to the model in two different frameworks. They maximize either seigniorage or wealth. In the former case, each period's seigniorage consists of the issues of new real balances and the inflation tax on old balances but a high anticipated inflation lowers demand for money and the base for future seigniorage taxes. The results show that there are incentives to inflate; but as substitutability increases, competition drives down the inflation rate. In the latter case, governments maximize the utility of their citizens. There is again an incentive to inflate because seigniorage tax falls on all who hold national currency; burden is shifted to foreigners. However, inflation still goes down with increasing currency substitutability. They finally show that the inflation rates resulting in games above are not optimal even when currency substitution is high.

Using a similar framework, Sturzenegger (1992) explains how costs of switching from one currency to the other can generate different impacts of the inflation tax on different individuals. He argues that wealthier individuals have preferential access to the technologies such as credit cards, checkable interest bearing accounts, and use of foreign currency whereas the poor usually have a higher fraction of their savings in non-indexed assets or fixed-interest rate deposits. Therefore, high income consumers can better protect themselves from inflationary taxation. In his model, agents face a fixed cost for using foreign currency and must decide if he is going to pay it and avoid the inflation tax on some of his monetary balances or if he is going to use only domestic currency. Similar to Weil (1991), consumers maximize utility as a homothetic function of both currencies and consumption, subject to a dynamic budget constraint. An increase in the rate of money creation implies a steady state increase in the marginal utility of domestic money. The transition to the accumulation of foreign assets along the convergence path is achieved through a downward jump in the level of consumption. Eventually, the level of consumption returns to its steady state level. Sturzenegger conducts the analysis on the level indirect utility functions, given by the level of income times the price index for consumption. In case the agent uses financial adaptation for high inflation, i.e. substitute foreign currency for domestic currency, the indirect utility function is different because the income is discounted with the fixed transaction cost. Since the utility function is homothetic, the indirect utility functions are linear. The slope of the indirect utility function is then greater than that of the indirect utility function of the case in which only domestic currency is used. Because the marginal utility of income has to be greater or equal if the agent has the option of holding some foreign currency. Then he shows that the indirect utility functions have a unique intersection point, a unique income level below which the agent does not find it worthwhile to pay the fixed cost in order to use the foreign currency. The agent has to have enough purchasing power to be able to spread this cost over a sufficiently large holding of foreign assets. These results are then applied to seigniorage taxation. The use of currency substitution entails a cost but allows for a lower price level. Then richer people who can afford this cost use less domestic money and therefore pay a smaller fraction of the inflation tax.

II.II.5 - Modeling Asymmetrical Currency Substitution and Empirical Evidence

Asymmetrical currency substitution has also been theoretically and empirically analyzed in several studies. Tanzi and Blejer (1982) investigates the interest rate and exchange rate policies in an open economy with a higher inflation rate than the trading partner. They argue that asymmetrical currency substitution arises for two reasons. First, if, in transactions, foreign currency were a perfect substitute for domestic currency with no transaction costs, inflation differential induces people to move completely out of domestic currency and into foreign currency, in the absence of uncertainty. The degree of substitution depends on the inflation differential. Second, when the domestic nominal rate of interest falls below the total nominal rate of return from foreign currency holdings (inflation differential plus foreign interest rate), people may satisfy their demand for money requirements by holding foreign currency and other foreign assets. They show that a successful pursuit of financial policies under crawling peg rules becomes difficult in the presence of currency substitution which reduces monetary independence and causes problems in the implementing serious stabilization programs. Ramirez-Rojas (1986) presents a simple model of asymmetrical currency substitution and examines the empirical evidence for some Latin American countries. His model assumes purchasing power parity and consists only of two money demand functions as a function of real wealth. Although it is limited in scope to exchange rate policies, it has interesting implications. The distribution of residents' demand for domestic and foreign currencies is given by the expected rate of devaluation of the domestic currency. Then, an increase in the expected rate of depreciation implies an increase in the holdings of foreign money and a proportional reduction in the holdings of domestic money. The statistical analysis of the currency substitution in Argentina, Mexico, and Uruguay lends support to the model's implications. Dollarization in Latin America is also analyzed in Guidotti and Rodriguez (1992). They model dollarization in a utility maximizing framework as a process in which costs are involved in switching the currency denomination of transactions. The representative individual maximizes a lifetime utility function as a function of consumption subject to domestic and foreign cash-in-advance constraints. Changing the proportion of the consumption, financed by foreign money, involves a cost (cost of dollarization as a function of change in consumption) which represents imperfect substitutability among currencies. Inflation differential induces dollarization but when the domestic inflation goes down, the transaction costs define a band for the inflation differential within which there will be no incentive to switch back to the domestic currency. In other words, dollarization displays irreversibility. They also find that, financial liberalization may generate dollarization even without a significant change in the inflation, and once dollarization is ongoing, reductions in inflation rates may not achieve a significant increase in the degree of monetization of the economy. Finally, Calvo and Végh (1992) provides a survey on policy and analytical issues related to dollarization. They discuss how the presence of asymmetrical currency substitution affects the choice of nominal anchors in inflation stabilization programs, whether currency substitution should be discouraged or not, and the interaction between inflationary finance and currency substitution.

Ortiz (1981) provides an empirical analysis of currency substitution in Mexico. He models dollarization simply by beginning at the level of real money demand formulations as functions of real returns on each currency and an alternative asset, foreign exchange risk, political risk factors, and real wealth. He approximates real return differential by inflation differential, foreign exchange risk by the deviations of the real exchange rate from the trend rate, and political risk by a dummy variable. After some regression experiments, he finds that the relative foreign money demand depends positively on each of the variables above. Similarly, in El-Erian (1988), factors determining the level and character of currency substitution are summarized in a simplified, composite, reduced form demand for money function, and are taken to be exchange rate expectations, interest rate differentials, and political disruptions (institutional changes). The proxies for these determinants are the deviation of the parallel market rate from the spot rate for the exchange rate expectations, the three-month LIBOR U.S. dollar rate relative to the domestic 3-month interest rate for interest rate differential, and a dummy variable for political uncertainty. He presents results for Egypt and Yemen Arab Republic. For both countries, interest differentials were not found to be statistically significant but the increases in foreign currency holdings of residents were significantly associated with higher expectations of exchange rate depreciation and with greater political uncertainty. On the other hand, Savastano (1992) argues against this practice of including expected depreciation of the domestic currency in a demand for money equation and the significance of its negative effect on money demand. In countries with well developed financial and foreign exchange markets and high capital mobility, the expected depreciation of the domestic currency, he argues, tends to be negatively related with any money demand function, even when residents do not demand foreign currency balances. Also, the expected rate of devaluation in addition to the expected inflation hardly adds any information to a money demand function because of the collinear nature of these two opportunity cost variables which makes it very difficult to isolate the different effects they are supposed to capture. Therefore, he estimates a money demand specification that includes the expected inflation rate as the only indicator of the opportunity cost of holding money balances, the industrial production index as an indicator of the evolution of the economic activity, and some dummy variables. The results, presented for Bolivia, Mexico, Peru, and Uruguay, show that there is not much empirical support for the hypothesis that the presence of foreign currency deposits provoke a shift in these countries' demands for domestic money. Also, the identified shifts in the demands for domestic money do not always coincide with the precise periods in which the restrictions on foreign currency deposits are lifted and/or reimposed. This result supports the irreversibility argument of Guidotti and Rodriguez (1992) for dollarization.

Unlike the studies above, Selcuk (1994) uses a multivariate vector autoregression model where each variable is a function of its lagged values as well as the lagged values of other variables, and presents results for Turkey. The currency substitution variable in the system is the ratio of foreign currency deposits by residents to the broad definition of money supply, and is assumed to be mainly influenced by the nominal returns on domestic assets, and the expected change in the exchange rate. The proxies are short-term domestic interest rate for the former, and the trade weighted real exchange rate index and the nominal exchange rate (TL/$) for the latter. According to the estimation results, the first and the third lags of the change in the nominal interest rate and the first lag of the real exchange rate index have a statistically significant negative effect on the growth rate of currency substitution. The lagged values of growth rate of currency substitution are also significant, confirming that the process is persistent as in Guidotti and Rodriguez (1992). The results also indicate that, as in Ramirez-Rojas (1986), an increase in nominal interest rates supported by a real appreciation of the currency, as a result of a consistent and credible stabilization program of government, may induce a shift towards the domestic currency, resulting an increase in real balances, and further appreciation.

Krueger and Ha (1995) provides an analysis on different empirical methodologies as well as the estimation of relative money demands as used in the studies above. These methodologies measure co-circulation; the extent to which foreign currencies are circulated within an economy. The first methodology is the comparison of per capita holdings of national currency which suggests large scale international currency flows. Some countries are estimated to have notably higher per capita balances which implies that some of the currency issued by those countries may be in excess of normal requirements and are in use in other countries. However, the lack of systematic data on the subject and the existence of underground and unrecorded transactions make the results rather unreliable. The second methodology is the constructed estimates of currency flows. This refers to netting of gross outflows and inflows associated with particular types of balance of payments transactions, e.g. tourism and bank reflows for outflows and immigrants' remittances for inflows. Accumulation of the balance of payments flows, however, does not provide information on the distribution of the dollar stock abroad because of subsequent shipments between countries; for example, large flows to Switzerland may be identified but subsequent flows to Eastern Europe may not. Thirdly, the portfolios of businesses and households may be surveyed. One such survey for the U.S. could account for only about one-third of total currency issued, which implies large holdings in other countries. On the other hand, this method is not practical and is subject to problems like costly data collection, unwillingness of respondents to provide accurate information, and difficulty in sampling from a small group holding a large portion of assets. The fourth method is to track actual flows with the serial number of the currency bill. This is possible with highly automated markets, but will fail if flows may be outside of formal institutions. Finally, seasonal adjustment techniques may be used to measure co-circulation. According to this method, the difference between the amplitudes of the estimated true U.S. seasonal factors and the seasonal pattern in the U.S. observed by the U.S. monetary authorities can provide an estimate of currency held abroad. The results show that during the 1980s and the early 1990s, the portion of U.S. currency held abroad grew steadily to equal 60 percent (about $200 billion) or more of U.S. currency.

II.III - ASYMMETRICAL CURRENCY SUBSTITUTION AND GOVERNMENT FINANCE

In this section we present a model which uses a similar approach to Sibert and Liu (1994) and extends their model into asymmetric currency substitution. It adds to the literature by analyzing seigniorage taxation in a model of varying degrees of asymmetrical currency substitution.

We first show that asymmetrical currency substitution arises as a result of transaction costs. The residents of the country with weaker currency (country 1) hold positive balances of each money, while the residents of country 2 hold only their own currency. Currency substitution increases as the transaction costs increase and/or depreciation of money 1 accelerates. We also show that there exists a stationary equilibrium with dollarization if transaction costs are greater than the money growth rates. Then we consider a public finance application and apply the model to a seigniorage tax. The literature on currency substitution focuses almost exclusively on Latin American countries. However, our major motivation for this paper is the sizable asymmetric currency substitution in the European Monetary System (EMS) such as German currency (DM) holdings of Greek, Italian or Portuguese consumers, as a result of liberalization of capital flows and growing financial integration[15]. Also, Eastern European countries who are about to join the EMS such as Turkey[16], Hungary, and Poland experience a great degree of asymmetric currency substitution. These countries are not yet members but try to cooperate in government policies with the EMS for a smoother transition. Even some Arab countries such as Egypt and Yemen Arab Republic have been experiencing an increasing level of currency substitution[17].

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[15]Bini Smaghi and Vori (1991), Angeloni, Cottarelli and Levy (1991) document the currency substitution in EMS. Also, Angeloni and Giucca (1991) provides a good analysis for Italy.
[16]Turkey has already signed-up a full customs union treaty with the EMS which is considered to be the first phase of the transition. Also, Selcuk (1994) provides a detailed empirical analysis of the currency substitution in Turkey.
[17]For an empirical work on these countries, see El-Erian (1988).

In practice, seigniorage has been a substantial source of revenue for many European countries. According to Cukierman et. al. (1989), seigniorage as a fraction of total tax revenue was 12.5 percent for Italy, 9.1 percent for Spain, and 16.6 percent for Portugal during the 1970s and early 1980s[18]. Therefore, we use our model to derive the optimal money growth rates representing the seigniorage tax under cooperative and non-cooperative government policies and then examine its policy implications.

We assume that government must finance expenditures by levying a costly income tax or by collecting seigniorage. In deciding the optimal level of inflation, governments weigh the trade-offs between the costs of administering and complying with income taxes and the distortions created by rising prices. We find that when the countries cooperate, there are two optimal money growth rates that maximize the steady state utility of the consumers. The optimal money growth rate strictly decreases in currency substitutability and the cost of income taxation. For country 2, the higher money growth is the more optimal choice because income taxation is costly, and part of the burden of the seigniorage tax can be shifted to country 1 residents due to asymmetric currency substitution. For country 2, however, the decision depends on which loss is greater; real resource loss in trading currencies and the real resource loss from costly income taxation.

We also consider the case where each country takes the other country's money growth and income tax rates given and chooses a constant money growth rate to maximize the steady state utility of their residents. We assume initially that, in country 2, income taxation is costly and there is no money growth. Then we have a unique positive Nash money growth for country 1 which increases in response to increases in country 2 money growth if the transaction costs are not too high. Finally, we compare the optimal (cooperative) and Nash (non-cooperative) money growth rates and analyze the policy implications for country 1.

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[18] Also see De Grauwe (1994) for a discussion of seignorage revenues in EMS.

In section II.III.1, we present the basic model and analyze the properties of the resulting money demand functions. In section II.III.2, we analyze the equilibrium and in section II.III.3, we apply our model to seigniorage taxation. Summary and conclusion are given in the last section.

II.III.1 - The Model

Our theoretical framework, following Sibert and Liu (1994), is a variant of the cash in advance model. The seller's currency must be used to purchase goods. Prior to purchasing the goods, the currencies may be traded in the spot market at a cost which is a measure of the substitutability of the currencies. This transaction cost makes the model an indeterminate exchange rate model[19] when it is zero and a standard cash-in-advance constraint model when it is infinity. The model has no aggregate uncertainty to keep matters simple. We introduce uncertainty, however, about consumers' future individual preferences which causes them to have a precautionary demand for the weaker currency. This precautionary demand, in return, ensures the possibility that both currencies are valued.

The model has a partial equilibrium, utility maximizing framework where exchange rates may be primarily determined in financial asset markets, but where there is some friction which allows the exchange rate to be uniquely determined. The friction is characterized by a parameter that can be varied in size. This will allow us to examine the effect of policy changes, such as the liberalization of capital flows in the European Community, which make the role of financial asset markets more important in exchange rate determination.

[19]See Kareken and Wallace (1981) for details.

II.III.1.a - The Consumers

We have three key assumptions. First consumers must purchase goods in sellers' currencies. This is similar to Lucas (1982) and Svensson (1985). Second, consumers do not know what their preferences will be in the future. This may reflect stochastic tastes or it may be that the desire to consume a particular good depends upon another exogenous event not captured in the model. It is only when they are old that they learn of their exact preferences. Thus, a consumer must decide how much of each money to hold before knowing his demand for each good[20]. Third, unlike in the Lucas and Svensson models, consumers may trade in the spot market immediately before consumption. The influence of the goods market on the exchange rate can be made arbitrarily small by making the cost to spot market trading sufficiently low.

There are two countries, 1 and 2. Each country is specialized in the exogenous production of a tradable consumption good. The countries are inhabited by overlapping generations of two-period-lived agents[21]. Agents consume both goods and save the two country-specific monies. In Sibert and Liu (1994) consumers must purchase goods with the seller's currency and may trade in the spot market at a constant proportional cost[22], in their second period of life. Since a more realistic model requires transaction costs in every period, we extend their model by adding costly spot market trading for the young. As shown below, this extension generates a very different model. With asymmetric countries, at most one currency will be held by residents of one country while both monies will be demanded by those of the other country.

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[20]The model is related to Goldman (1974). There agents are willing to hold non-interest-bearing money in addition to bonds because they are unsure of their discount rate and portfolio adjustment is costly.

[21]The exchange rate indeterminacy result is associated with the OLG model, but it would arise in any frictionless, optimizing, equilibrium model. The Lucas (1982) and Svensson (1985) financing constraint is associated with the infinitely-lived representative agent model, but it is sufficient to determine the equilibrium in an OLG model. The OLG model is chosen here because money has value endogenously and it is a relatively simple framework given the nature of the friction.

[22]Sturzenegger (1992) assumes a fixed cost to entering currency markets. In that case, only wealthier consumers decide to do so which makes the inflation tax regressive.

The opportunities of the generation-t, country-i consumer when old are given by the kinked budget set below:

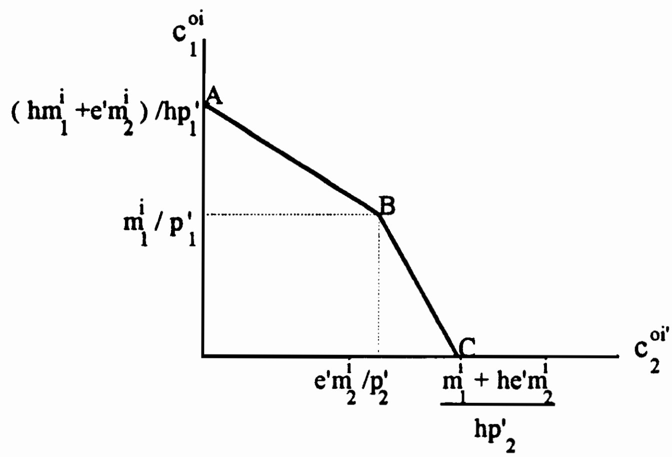


Figure 2: Budget set when old

Let $c^{oi}_{j,t+1}$ be his consumption of good j (j=1,2) at time t+1, e_{t+1} be the time t+1 price of money 2 in terms of money 1, $p_{j,t+1}$ be the time t+1 price of good j in terms of money 1, and h-1>0 be the proportional cost of spot market trading. Suppose the consumer enters his second period of life with $m^i_{j,t}$ units of country-j money. If he does not trade in the spot market for foreign exchange then his consumption is shown by point B. If he engages in costly market trading of money 2 for money 1, he can consume along line segment AB; if he trades money 2 for money 1 he can consume along BC.

The uncertainty faced by consumers is represented formally as follows. In each period, t = ..., -1, 0, 1, ... , a unit interval of agents is born in each country. Agents are indexed by their location in the unit interval. An agent's location, α, is unknown to him when young; however, he believes that it is distributed uniformly on (0,1).

The preferences of a country-i consumer are given by

(1) $$W^i = U(c_1^{yi}, c_2^{yi}; .5) + \beta E_t[U(c_1^{oi'}, c_2^{oi'}; \alpha)]$$

where c_j^{yi} and $c_j^{oi'}$ are consumption good j, j=1,2, when young and when old, respectively, and

(2) $$U(c;\alpha) = \alpha \ln(c_1) + (1-\alpha)\ln(c_2) \quad .$$

Also variables without primes are evaluated at time t, variables with a prime are evaluated at time t+1.

When a country-i resident is young, he is endowed with x units of his country's consumption good. He pays a real tax, $\tau \in (0,x)$, and allocates his after tax income between consumption and savings of home and foreign money. His budget constraint when young is

(3) $$\begin{aligned} &p_1 c_1^{yi} + hp_2 c_2^{yi} + m_1^i + hem_2^i = p_i(x-\tau^i) \quad if \quad i=1 \\ &hp_1 c_1^{yi} + p_2 c_2^{yi} + hm_1^i + em_2^i = p_i(x-\tau^i) \quad if \quad i=2 \end{aligned} \quad .$$

When the agent is old, he uses his savings to purchase the consumption goods. Hence, the budget constraint when old is

(4) $$\begin{aligned} &hp_1' c_1^{oi'} + p_2' c_2^{oi'} = hm_1^i + e'm_2^i \qquad if \quad p_1' c_1^{oi'} \geq m_1^i \\ &p_1' c_1^{oi'} + hp_2' c_2^{oi'} = m_1^i + he'm_2^i \qquad otherwise \end{aligned} \quad .$$

At time t+1, each generation-t agent learns his value of α and maximizes $U(c_1^{oi'}, c_2^{oi'}; \alpha)$ subject to (4). The solution has

$$
(5)\quad c_1^{oi'} = \begin{cases} \bar{c}_1^{oi'} = \dfrac{\alpha m_1^j}{\bar{\alpha}^{i'} p_1'} & \text{if } \alpha > \bar{\alpha}^{i'} \\ \underline{c}_1^{oi'} = \dfrac{\alpha m_1^j}{\underline{\alpha}^{i'} p_1'} & \text{if } \alpha < \underline{\alpha}^{i'} \\ \dfrac{m_1^j}{p_1'} & \text{if } \underline{\alpha}^{i'} \le \alpha \le \bar{\alpha}^{i'} \end{cases} \quad , \quad c_2^{oi'} = \begin{cases} \bar{c}_2^{oi'} = \dfrac{(1-\alpha) e' m_2^j}{(1-\bar{\alpha}^{i'}) p_2'} & \text{if } \alpha > \bar{\alpha}^{i'} \\ \underline{c}_2^{oi'} = \dfrac{(1-\alpha) e' m_2^j}{(1-\underline{\alpha}^{i'}) p_2'} & \text{if } \alpha < \underline{\alpha}^{i'} \\ \dfrac{e' m_2^j}{p_2'} & \text{if } \underline{\alpha}^{i'} \le \alpha \le \bar{\alpha}^{i'} \end{cases}
$$

and

$$
(6)\qquad \underline{\alpha}^{i'} = \frac{m_1^j}{m_1^j + h e' m_2^j} \quad ; \quad \bar{\alpha}^{i'} = \frac{h m_1^j}{h m_1^j + e' m_2^j} \quad .
$$

Consumption when young satisfies

$$
(7)\qquad \begin{aligned} & c_1^{yi} = \frac{p_i(x-\tau^j) - m_1^j - hem_2^j}{2p_1} \quad , \quad c_2^{yi} = \frac{p_i(x-\tau^j) - m_1^j - hem_2^j}{2hp_2} \qquad \text{if} \quad i=1 \\ & c_1^{yi} = \frac{p_i(x-\tau^j) - hm_1^j - em_2^j}{2hp_1} \quad , \quad c_2^{yi} = \frac{p_i(x-\tau^j) - hm_1^j - em_2^j}{2p_2} \qquad \text{if} \quad i=2 \end{aligned} \quad .
$$

Thus the problem of the consumers to maximize

$$
(8)\qquad W^i = .5(\ln c_1^{yi} + \ln c_2^{yi}) \quad + \quad \beta\left[\int_0^{\underline{\alpha}^{i'}} [\alpha \ln \underline{c}_1^{oi'} + (1-\alpha)\ln \underline{c}_2^{oi'}] d\alpha \quad + \right.
$$

$$
\left. \int_{\underline{\alpha}^{i'}}^{\bar{\alpha}^{i'}} [\alpha \ln c_1^{oi'} + (1-\alpha)\ln c_2^{oi'}] d\alpha + \int_{\bar{\alpha}^{i'}}^{1} [\alpha \ln \bar{c}_1^{oi'} + (1-\alpha)\ln \bar{c}_2^{oi'}] d\alpha \right] \quad .
$$

by choosing optimal amounts of foreign (money 2) and home (money 1) money balances subject to (5) - (7).

Employing Leibnitz's rule and noting that $U(\underline{c}_1^{oi},\underline{c}_2^{oi};\alpha) \to U(m_1^i / p_1',e'm_2^i / p_2';\alpha)$ when $\alpha \to \underline{\alpha}^{i'}$ and $U(\bar{c}_1^{oi},\bar{c}_2^{oi};\alpha) \to U(m_1^i / p_1',e'm_2^i / p_2';\alpha)$ when $\alpha \to \bar{\alpha}^{i'}$ gives the money demand functions for the each country's residents

$$(9) \quad \frac{1}{2\beta h p_2 c_2^{y1}} \geq \frac{1}{hp_2'}\int_0^{\underline{\alpha}^{1'}}\left(\frac{(1-\alpha)}{\underline{c}_2^{o1'}}\right)d\alpha + \frac{1}{p_1'}\int_{\underline{\alpha}^{1'}}^{\bar{\alpha}^{1'}}\left(\frac{\alpha}{c_1^{o1'}}\right)d\alpha + \frac{h}{p_2'}\int_{\bar{\alpha}^{1'}}^{1}\left(\frac{(1-\alpha)}{\bar{c}_2^{o1'}}\right)d\alpha \qquad \text{, with equality if } m_1^1 > 0$$

$$(10) \quad \frac{e}{2\beta p_2 c_2^{y1}} \geq \frac{e'}{p_2'}\int_0^{\underline{\alpha}^{1'}}\left(\frac{(1-\alpha)}{\underline{c}_2^{o1'}}\right)d\alpha + \frac{e'}{p_2'}\int_{\underline{\alpha}^{1'}}^{\bar{\alpha}^{1'}}\left(\frac{(1-\alpha)}{c_2^{o1'}}\right)d\alpha + \frac{e'}{p_2'}\int_{\bar{\alpha}^{1'}}^{1}\left(\frac{(1-\alpha)}{\bar{c}_2^{o1'}}\right)d\alpha \qquad \text{, with equality if } m_2^1 > 0$$

$$(11) \quad \frac{h}{2\beta p_2 c_2^{y2}} \geq \frac{1}{hp_2'}\int_0^{\underline{\alpha}^{2'}}\left(\frac{(1-\alpha)}{\underline{c}_2^{o2'}}\right)d\alpha + \frac{1}{p_1'}\int_{\underline{\alpha}^{2'}}^{\bar{\alpha}^{2'}}\left(\frac{\alpha}{c_1^{o2'}}\right)d\alpha + \frac{h}{p_2'}\int_{\bar{\alpha}^{2'}}^{1}\left(\frac{(1-\alpha)}{\bar{c}_2^{o2'}}\right)d\alpha \qquad \text{, with equality if } m_1^2 > 0$$

$$(12) \quad \frac{e}{2\beta p_2 c_2^{y2}} \geq \frac{e'}{p_2'}\int_0^{\underline{\alpha}^{2'}}\left(\frac{(1-\alpha)}{\underline{c}_2^{o2'}}\right)d\alpha + \frac{e'}{p_2'}\int_{\underline{\alpha}^{2'}}^{\bar{\alpha}^{2'}}\left(\frac{(1-\alpha)}{c_2^{o2'}}\right)d\alpha + \frac{e'}{p_2'}\int_{\bar{\alpha}^{2'}}^{1}\left(\frac{(1-\alpha)}{\bar{c}_2^{o2'}}\right)d\alpha \qquad \text{, with equality if } m_2^2 > 0$$

Next we consider the properties of money demand functions (9) - (12).

Proposition 1. Suppose that prices and exchange rates are strictly positive and finite, and h>1. Then there exists a unique solution to the consumer's problem in which the residents of the country with the weaker currency hold both monies, others only hold their own currency; asymmetrical currency substitution. In particular, if and only if $h^2 > s > 1$, country 2 residents hold only money 2 while country 1 residents demand both monies, and vice versa if and only if $h^2 > 1/s > 1$, where $s = e'/e$ which gives the return to money 2..

Proof. See the Appendix.

The intuition is as follows. If $h^2 > s > 1$, country 2 residents hold only money 2 because the return to money 2 is higher than the return to money 1, and they will trade for money 1 only if $\alpha > \bar{\alpha}$ in the second period because cost of trading money in both periods is greater than the return to money 2. Country 1 residents want to hold money 2 for two reasons; to hedge against inflation, i.e. to take advantage of higher return, and for precautionary purposes to minimize the transaction costs in the second period. In this sense, country 1 can be considered as developing and country 2 as developed. The intuition for the case of $h^2 > 1/s > 1$ is also similar; the only difference then is that the country 2 money is the depreciating currency. If h=1, then an interior solution requires non-asset dominance; the exchange rate will be constant (Kareken and Wallace (1981)). These implications of the money demands confirm the results of Khan and Ramirez (1986) and Tanzi and Blejer (1992) where the expected depreciation of the domestic currency, in the former, and the inflation differential in the latter cause residents to shift out of domestic currency into foreign money.

We now consider the response of money demand to changes in transaction costs and the rate of depreciation of money 1.

Proposition 2. Suppose h>1 and $h^2 > s > 1$. Then as the transaction costs increase and/or depreciation of money 1 accelerates, currency substitution increases (see Figure 3).

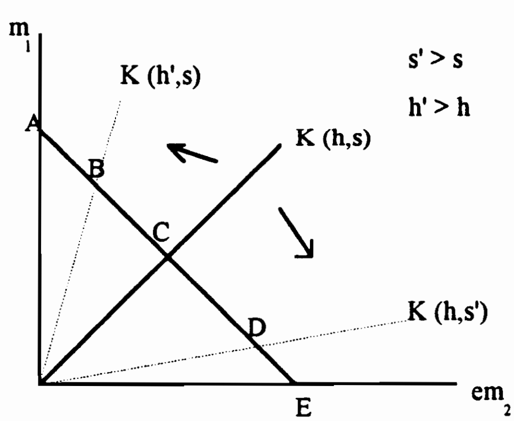


Figure 3: Shifts of currency portfolio

$\kappa = m_1^1 / em_2^1 = \kappa(h, s)$ is the proportional money holdings of the country 1 resident.

Proof : See the Appendix.

The composition of the country 1 resident's portfolio shifts in favor of the domestic currency to the point B when the transaction costs increase, and in favor of the foreign currency to the point D when the expected depreciation goes up. |AE| denotes the budget line.

Proposition 3. The economy of the country 1 is inflationary if $h^2 > s > 1$:

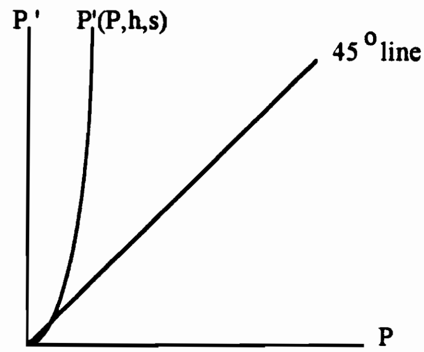


Figure 4: Inflation in the Country 1 economy

Proof : See the Appendix.

II.III.1.b - The Governments

Each period government i produces g^i units of the public good from the country's private consumption good via a linear production technology. Each unit of the private good can be costlessly transformed into a unit of the public good. The government finances production by costly taxation and by collecting seigniorage.

We assume that income taxation is costly to administer and comply with. If residents of country i forego τ^i units of resources to pay their taxes, the government ends up with only $c\tau^i$

units of revenue with which to purchase inputs for the public good, where c < 1. Thus the governments' budget constraints are

$$(13) \qquad g^1 = \frac{M_1' - M_1}{p_1'} + c\tau^{1'} \qquad ; \qquad g^2 = e' \frac{M_2' - M_2}{p_2'} + c\tau^{2'}$$

II.III.2 - Equilibrium

In equilibrium, goods and money markets clear:

$$(14) \qquad \sum_{i=1}^{2}\left\{c_1^{yi'} + \int_0^{\underline{\alpha}^{i'}} \underline{c}^{\alpha i'} d\alpha + \int_{\underline{\alpha}^{i'}}^{\overline{\alpha}^{i'}} \frac{m_1^i}{p_1'} d\alpha + \int_{\overline{\alpha}^{i'}}^{1} \overline{c}^{\alpha i'} d\alpha\right\} + g^1 + (1-c)\tau^1 +$$

$$(h-1)\left\{\left[\frac{p_2 x + hem_2^1 - em_2^2}{2hp_1}\right] + \sum_{i=1}^{2}\int_{\overline{\alpha}^{i'}}^{1}\left(\overline{c}^{oi'} - \frac{m_1^i}{p_1'}\right)d\alpha\right\} = x$$

$$(15) \qquad m_i^1 + m_i^2 = M_i \quad i = 1,2$$

Equation (14) gives the market clearing condition for good 1; spending on good 1 by the young, the old, and the government, plus real resources lost in costly tax collection and costly trading of money in both periods equal the amount of good 1 available. The money market equilibrium is given by (15).

The first step in analyzing the equilibrium is to define it in terms of real variables where $\gamma_j^i = m_j^i / M_j$,the fraction of money j held by the country i residents; $z = M_1/M_1'$, the reciprocal of country 1 money growth; $q = eM_2/M_1$, ratio of money 2 real balances to money 1 real balances; $r_j = p_j/M_1$, the reciprocal of the real value of country 1 money in terms of good j .

It is well known that monetary rational expectations models can have a plethora of equilibria; we focus on the one we find most believable. The current state of the world is summarized by the current policy variables; hence it is natural to consider equilibria which are stationary in that real variables depend only on these policies. We will be restricting attention to constant government policies; hence we will only consider constant real variables. Then rewriting equations (5)-(7), (9),(10), and (13)-(15) yields

$$c_1^{y1} = \frac{r_1(x-\tau^1)-\gamma_1^1-hq\gamma_2^1}{2r_1} \quad , \quad c_2^{y1} = \frac{r_1(x-\tau^1)-\gamma_1^1-hq\gamma_2^1}{2hr_2}$$

(16)

$$c_1^{y2} = \frac{r_2(x-\tau^2)-h\gamma_1^2-q\gamma_2^2}{2hr_1} \quad , \quad c_2^{y2} = \frac{r_2(x-\tau^2)-h\gamma_1^2-q\gamma_2^2}{2r_2}$$

(17) $$c_1^{oi} = \begin{cases} \bar{c}_1^{oi} = \dfrac{\alpha\gamma_1^i z_1}{\bar{\alpha}^i r_1} & if\ \alpha > \bar{\alpha}^i \\ \underline{c}_1^{oi} = \dfrac{\alpha\gamma_1^i z_1}{\underline{\alpha}^i r_1} & if\ \alpha < \underline{\alpha}^i \\ \dfrac{\gamma_1^i z_1}{r_1} & if\ \underline{\alpha}^i \le \alpha \le \bar{\alpha}^i \end{cases} \quad , \quad c_2^{oi} = \begin{cases} \bar{c}_2^{oi} = \dfrac{(1-\alpha)q\,\gamma_2^i z_2}{(1-\bar{\alpha}^i)r_2} & if\ \alpha > \bar{\alpha}^i \\ \underline{c}_2^{oi} = \dfrac{(1-\alpha)q\,\gamma_2^i z_2}{(1-\underline{\alpha}^i)r_2} & if\ \alpha < \underline{\alpha}^i \\ \dfrac{q\,\gamma_2^i z_2}{r_2} & if\ \underline{\alpha}^i \le \alpha \le \bar{\alpha}^i \end{cases}$$

(18) $$\underline{\alpha}^i = \frac{\gamma_1^i z_1}{\gamma_1^i z_1 + h\gamma_2^i q z_2} \quad ; \quad \bar{\alpha}^i = \frac{h\gamma_1^i z_1}{h\gamma_1^i z_1 + \gamma_2^i q z_2}$$

(19) $$\frac{1}{2\beta h z_1 c_2^{y1}} = \frac{1}{h}\int_0^{\underline{\alpha}^1}\left(\frac{(1-\alpha)}{\underline{c}_2^{o1}}\right)d\alpha + \frac{r_2}{r_1}\int_{\underline{\alpha}^1}^{\bar{\alpha}^1}\left(\frac{\alpha}{c_1^{o1}}\right)d\alpha + h\int_{\bar{\alpha}^1}^{1}\left(\frac{(1-\alpha)}{\bar{c}_2^{o1}}\right)d\alpha$$

(20) $$\frac{1}{2\beta z_2 c_2^{y1}} = \int_0^{\underline{\alpha}^1}\left(\frac{(1-\alpha)}{\underline{c}_2^{o1}}\right)d\alpha + \int_{\underline{\alpha}^1}^{\bar{\alpha}^1}\left(\frac{(1-\alpha)}{c_2^{o1}}\right)d\alpha + \int_{\bar{\alpha}^1}^{1}\left(\frac{(1-\alpha)}{\bar{c}_2^{o1}}\right)d\alpha$$

$$(21) \qquad g^1 = \frac{(1-z_1)}{r_1} + c\tau^1 \quad ; \quad g^2 = q\frac{(1-z_2)}{r_2} + c\tau^2$$

$$(22) \qquad \sum_{i=1}^{2}\left\{c_1^{yi} + \int_0^{\underline{\alpha}^j} \underline{c}^{oi} d\alpha + \int_{\underline{\alpha}^j}^{\overline{\alpha}^j} \frac{\gamma_1^j z_1}{r_1} d\alpha + h\int_{\overline{\alpha}^j}^{1} \overline{c}^{oi} + (h-1)\int_{\overline{\alpha}^j}^{1} \frac{\gamma_1^j z_1}{r_1} d\alpha\right\} + g^1 +$$

$$(1-c)\tau^1 + (h-1)\left\{\left[\frac{r_2 z_1 x + q(h\gamma_2^1 - \gamma_2^2)}{2hr_1}\right]\right\} = x$$

$$(23) \qquad \gamma_1^1 = 1 \quad , \quad \gamma_2^1 + \gamma_2^2 = 1$$

Definition : Given $\{z_i, \tau^i\}_{i=1,2}$, an equilibrium is a vector

$\left\{\left[(\gamma_1^1, \gamma_2^1, \gamma_2^2), (c_j^y, \overline{c}_j^o, \underline{c}_j^o, c_1^o)_{j=1,2}, (\overline{\alpha}, \underline{\alpha}, r_i, g)_{i=1,2}\right], q\right\}$ of positive, finite numbers such that

$r_i > 0$ and $q > 0$, and the equations (16) - (23) hold.

Proposition 4. Given the asymmetrical currency substitution and country 1 is the developing country (money 1 is weaker), a stationary equilibrium exists if and only if $h^2 > z_2 > z_1$.

Proof : See the Appendix.

The intuition is as follows. The return to country 2 money is greater than the return to country 1 money and the transaction costs are less than the return to money 2. Then no countries will hold money 1; they can always do better holding money 2 and exchanging currencies whenever the need arises. Precautionary demand for money 1 will go to zero while transactions demand will still be positive due to the assumption that the seller's currency must be used to purchase goods. Without this assumption, however, country 2 money would drive out country 1 money. The model may also be used to analyze the phenomenon of " full dollarization", as a policy of inflation stabilization (see Cukierman et. al. (1992) below).

Proposition 5. If $h^2 < (z_2 / z_1)$, there is no stationary equilibrium where at least one country holds both monies.

Proof. Since h >1, $z_2 > z_1$. Then the proof follows from (19), (20), and L.

In the next section, we use our model to analyze the interaction between currency substitution, which we view as inversely related to h, and seigniorage collection.

II.III.3 - Government Finance

In this section, we consider a public finance application. Before we proceed, we briefly discuss some of the studies on the interaction between inflationary finance and currency substitution, and review the implications of their results. Tanzi and Blejer (1982) argue that when the domestic interest rate falls below the total rate of return from foreign money holdings, people may satisfy some or most of their demand-for-money requirements by holding foreign currency and other foreign assets. Then a reduction in the stock of domestic money reduces the revenues that the government can raise through deficit financing and money creation. Similarly, Khan and Ramirez-Rojas (1986) find that, in a basic money demand model extended by allowing for currency substitution, such an extension has a strong negative effect on the revenue maximizing rate of inflation. For the typical developing country, the revenue maximizing rate of inflation would be significantly lower than the values obtained from the standard models. Hercowitz and Sadka (1987) uses foreign money as a store of value, and examine the optimality of restrictions on foreign money for policymakers in case they resort to inflation tax. They find that, with a positive inflation rate and no restrictions on foreign money, consumers hold only foreign money and convert it into domestic currency right before carrying out transactions. Since they assume costless income taxation and the inflation tax generates waste of resources, the optimal inflation tax is zero, and the optimality of restrictions is indeterminate.

Végh (1989a) also examines the optimality of the inflation tax in the presence of currency substitution. He introduces currency substitution in a small, open economy through transactions technology approach; both domestic and foreign money serve as media of exchange. More specifically, transactions require the use of a scarce resource, time. Therefore, consumers hold money because money enables them to save on the time they devote to shopping activities, and they equate the marginal productivity of holding an additional unit of relative domestic (or foreign) balances (in terms of reduced shopping time) to its marginal cost (foregone interest). The government uses consumption and inflation taxes to finance a constant path of public expenditure. Then if the foreign interest rate is positive, it is optimal to impose an inflation tax. Because inflation tax, through a positive domestic interest rate, lowers the necessity to tax consumption and decreases consumption costs. Using the same framework, Végh (1989b) adds one more alternative to the government finance instruments; labor income tax. He finds that the greater is substitutability, the greater is the distortion caused by the positive foreign interest rate and the higher is the optimal inflation tax. In contrast to Végh (1989a,b), Canzoneri and Diba (1992) argues that currency substitution should be encouraged since it provides an inflation discipline for the government. They have a model with two countries, one good, and two monies. Consumers maximize utility as a function of consumption and real money balances, and acquire real money balances up to the point where the marginal value is equal to marginal cost, similar to Végh (1989a). Seigniorage, in each period, consists of the issues of new real balances and the inflation tax on old balances, and they show that a high anticipated inflation lowers demand for money and the base for future seigniorage taxes. First, they analyze a game in which each government tries to maximize its own seigniorage revenue. They find that there are incentives to inflate; but as substitutability increases, competition drives down the inflation rate. They also consider a game in which governments maximize the utility of their citizens. There is again an incentive to inflate because seigniorage tax falls on all who hold national currency; burden is shifted to foreigners. However, inflation still goes down with increasing currency substitutability. They finally show that the inflation rates resulting in games above are not optimal even when currency substitution is high.

Full dollarization, as the complete substitution of foreign currency for domestic currency as the only legal tender, is examined in Cukierman, Kiguel, and Liviatan (1992). This arrangement arises as one of the strongest commitments to stabilize high inflation, hence, it more difficult to deviate from it. By accepting full dollarization, government gives up the privilege to collect seigniorage, and the ability to devalue. However, the authors show that, even though the commitment is serious, it may be optimal to deviate from the rule when the economy is hit by a large shock. Calvo and Végh (1992) discusses the two main approaches to studying inflationary finance under currency substitution; level and variability of the inflation tax, and public finance principles. In the former case, currency substitution arises as the endogenous response of money holders to a high inflation tax. This causes a decrease in the seigniorage revenues forcing the government to inflate more. Also, inflation becomes more volatile. In the latter case, governments always optimally choose income and inflation taxes to finance an exogenously given level of spending. The results, however, are conflicting, and the optimal inflation may or may not be positive[23]. Finally, Végh (1994) analyzes the impact of currency substitution on inflationary finance in a public finance framework. He shows that the optimal inflation tax is zero if foreign balances can be taxed. Otherwise, it is positive whenever government runs budget deficit. He also finds that the optimal inflation tax is an increasing function of government spending and the foreign nominal interest rate.

We also assume, following the literature, that governments optimally choose an endowment (income) tax and an inflation tax to finance an exogenously given level of government spending. In other words, governments produce g^i units of the public good, financed by income taxation and seigniorage and in doing so, they choose a constant money growth rate and tax to maximize the steady state utility of its residents given by

[23] Hercowitz and Sadka (1987) and Kimbrough (1991) argue that optimal inflation tax is zero, whereas Vegh (1989a,b), optimal inflation tax becomes positive with currency substitution.

$$(24)\quad W^i = .5(\ln c_1^{yi} + \ln c_2^{yi}) \; + \; \beta\left[\int_0^{\underline{\alpha}^i}[\alpha \ln \underline{c}_1^{oi} + (1-\alpha)\ln \underline{c}_2^{oi}]d\alpha \; + \right.$$

$$\left.\int_{\underline{\alpha}^i}^{\overline{\alpha}^i}[\alpha \ln c_1^{oi} + (1-\alpha)\ln c_2^{oi}]d\alpha + \int_{\overline{\alpha}^i}^{1}[\alpha \ln \overline{c}_1^{oi} + (1-\alpha)\ln \overline{c}_2^{oi}]d\alpha\right]$$

where $c^{yi}, \underline{c}^{oi}, c^{oi}$, and $\overline{c}^{oi}$ are given by (18) and (19), and, $\underline{\alpha}^i$ and $\overline{\alpha}^i$ are given by (20).

II.III.3.a - Cooperative Policies (Optimal Money Growth)

As a bench mark, we consider the case where the two countries cooperate. In other words, both governments choose the same constant money growth and the tax and maximize $W = \lambda W^1 + (1-\lambda) W^2$ where λ denotes the relative importance of the developing country's wealth in choosing z and τ. Differentiating W and substituting (16) - (23), we get

$$(25)\qquad z^o = \frac{4\beta c(1+\lambda)r_1 r_2}{(r_2 + r_1 q)\left[\lambda\left(\frac{1}{c_1^{y1}} + \frac{r_1}{hr_2}\frac{1}{c_2^{y1}}\right) + (1-\lambda)\left(\frac{1}{c_1^{y2}}\frac{r_2}{hr_1} + \frac{1}{c_2^{y1}}\right)\right]}.$$

Substituting from (16) for $c_1^{y1}, c_2^{y1}, c_1^{y2}$, and c_2^{y2} , and (21) for τ turns (25) into a second degree polynomial:

$$K \equiv -z^2\left[((r_2 + r_1 q)/(2cr_1 r))^2(\beta(1+\lambda)+4)\right]+$$

(26) $$z((r_2 + r_1 q)/(2cr_1 r))\left[2(\lambda(x-M)+(1-\lambda)(x-V))+\beta(1+\lambda)[(x-M)+(x-V)]\right]-$$

$$\beta(1+\lambda)(x-M)(x-V)=0$$

where $V = \left[(g^1+g^2)/2c+(1+hq\gamma_2^1)/r_1+(r_2+r_1 q)/(2cr_1 r_2)\right] < x$, and $M = \left[(g^1+g^2)/2c+(1+q\gamma_2^2)/r_2+(r_2+r_1 q)/(2cr_1 r_2)\right] < x$. From (26) we have $\partial^2 K/\partial z^2 < 0$, i.e., there is a maximum for all z , $z|_{\frac{\partial K}{\partial z}=0} > 0$, $K|_{z=0} = -\beta(1+\lambda)(x-M)(x-V) < 0$, and $K(z|_{\frac{\partial K}{\partial z}=0}) > 0$. Then there are two optimal positive money growth rates that maximize the steady state utility in the coordinated case:

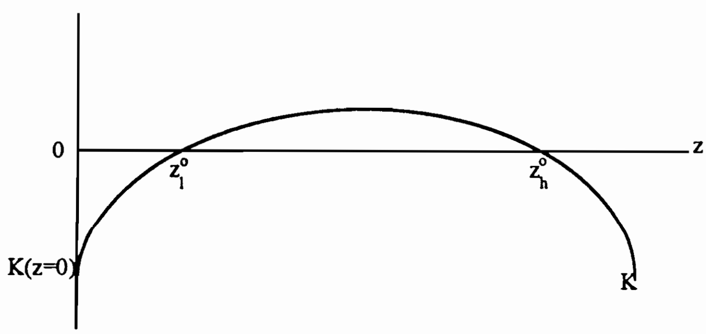


Figure 5: Optimal money growth

The intuition for two optimal money growth rates is provided by the conditions that affect the choice among them. Both money growth rates maximize the steady state utility of the consumers and the taxes are positive. The choice will then depend on which money growth rate corresponds to a lower real resource cost, i.e. the real resource loss in trading currencies given by $(h-1)q\gamma_2^1$ and the real resource loss from costly income taxation given by $(1-c)\tau$. Country 2 government relies on seigniorage revenue rather than taxing income and chooses a higher money growth for two reasons. Income taxation is costly, and their residents do not substitute foreign currency. Hence, z_l^o (higher money growth) is the more optimal choice for country 2. It is also optimal for country 1 government because as long as income taxation is costly enough such that $(h-1)q\gamma_2^1 < (1-c)\tau$, seigniorage revenue corresponds to a lower real resource loss and their residents can shift the part of the burden of seigniorage to country 2 residents.

On the other hand, country 1 residents hedge against inflation by substituting foreign currency but in doing so, they incur a real resource loss because of transactions costs. Then if income taxation is not very costly such that $(h-1)q\gamma_2^1 > (1-c)\tau$, z_h^o (lower money growth) is the more optimal choice for the country 1 government. It is also optimal for country 2 government to rely less on seigniorage because seigniorage burden can not be shifted to country 1 residents. As a result, both money growth rates are optimal for the countries while z_l^o (z_h^o) is the more optimal choice for country 2 (1). Then the relative importance of the developing country's wealth, λ, determines the choice of the optimal money growth and tax rates. In contrast, Hercowitz and Sadka (1987) argue that inflationary finance is never optimal because of the waste of resources in transactions costs. However, in their analysis, income taxation is not costly.

Proposition 6. Optimal money growth is strictly decreasing in currency substitutability and the cost of taxation.

Proof. See the Appendix.

These results are quite intuitive. Transaction costs, h, is a measure of the substitutability of currencies. When h is high, currencies become less substitutable. Therefore, the dampening effect of currency substitution on the seigniorage revenue is lower and the governments can collect more seigniorage simply by choosing a higher money growth. This is in contrast to Végh (1989a,b) where greater substitutability leads to a greater distortion caused by the foreign interest rate and a higher optimal home inflation. We should note, however, our model does not include interest bearing assets. On the other hand, this result confirms that of Girton and Roper (1981) which argues that with substitutable monies a joint monetary policy may not provide money holders lower inflation rates.

Also, as c decreases, i.e. income taxation gets more costly, the governments resort to inflation tax rather than income taxation. For a given h, choosing a lower money growth increases seigniorage revenue with less real resource cost.

Proposition 7. If the transactions cost is sufficiently high, optimal money growth increases as the wealth of the developing country becomes relatively more important.

Proof. See the Appendix.

The intuition is as follows. The domestic good consumption of country 1 consumers when young is strictly decreasing in cost of spot market trading because of currency substitution, unlike that of country 2 consumers. If transactions costs are high, the value of foreign currency holdings of country 1 consumers goes down and so does the loss in the seigniorage revenue of country 1 government. Then the government relies more on seigniorage revenue because income taxation is costly, and thence, the optimal z goes down. If transaction costs are low, however, such that $c_2^{y2} < c_1^{y1}$ the seigniorage base in country 1 is lower due to the asymmetrical currency substitution and z increases in λ.

II.III.3.b - Non-cooperative Policies (Nash Money Growth)

We now suppose that countries do not cooperate. Country 1 government takes the other country's money growth and tax as given and chooses a constant money growth rate to maximize the steady state utility. We restrict our attention to the policies of developing country and assume no money growth in country 2 (z_2=1). Differentiating (24) and substituting (16) → (23), we find

$$(27) \qquad \frac{(hz_1+\gamma_2^1 q)^2(z_1+h\gamma_2^1 q)^2}{z_1(hz_1+\gamma_2^1 q)^2+(2h\gamma_2^1 q+h^2 z_1)(z_1+h\gamma_2^1 q)^2}=\beta c c_1^{y1}$$

The left-hand side (LHS) of (27) is strictly increasing in z>0, with two negative roots. If

$LHS = 0 \Rightarrow z_{11} = -\frac{\gamma_2^1 q}{h}$ *and* $z_{12} = -h\gamma_2^1 q$, $LHS(z_1 = 0) = \frac{\gamma_2^1 q}{2h} > 0$, $\frac{\partial LHS}{\partial z_1} = 0$ for each root and

$\frac{\partial LHS}{\partial z_1}|_{z_1=0} > 0$. The right-hand side (RHS) is strictly decreasing in z>0. If

$RHS = 0 \Rightarrow z_1 = r_1(cx - g^1) + 1 - c(1 + hq\gamma_2^1)$, and $\frac{\partial RHS}{\partial z_1} = -\frac{\beta}{2r_1} < 0 \Rightarrow z_1|_{RHS=0} > 0$. Also,

$RHS(z_1 = 0) > 0$. Then we have a unique Nash money growth if $c_1^{y1} > \frac{\gamma_2^1 q}{2\beta chr_1}$:

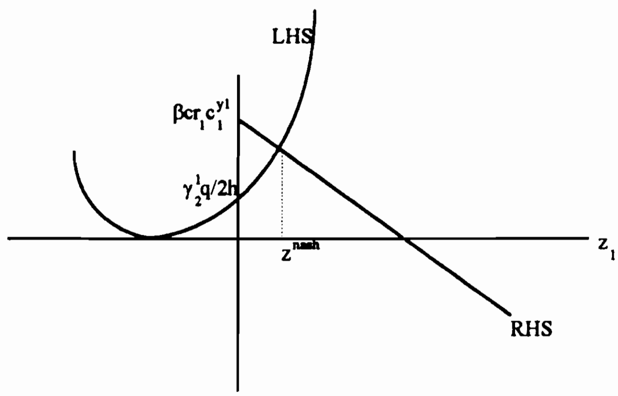


Figure 6: Nash money growth

Next, we examine how the Nash money growth depends on the rate of growth of foreign money supply.

Proposition 8. The Nash money growth of country 1 increases in response to increases in the country 2 money growth if the transaction costs are not too high.

Proof. See the Appendix.

The intuition is as follows. Suppose initially, country 2 has no money growth. As the money supply grows, the depreciation of money 1 will slow, and the return to money 2 will go down. Therefore, the dampening effect of asymmetric currency substitution on the country 1 seigniorage revenue will decrease. Because the transaction cost is small, this allows the country 1 government to use more inflationary financing with low real resource loss. The same intuition follows with increasing transaction costs, however, the real resource loss will then be higher, and the adjustment in money growth will depend on the trade-off between the costly currency trading and income taxation[24]. This result is in contrast to Canzoneri and Diba (1992) where it is shown that, with increasing substitutability of currencies, competition between the currencies drives down the inflation rate. We agree, however, with Weil (1991) where he argues that increasing substitutability between the currencies will make speculative hyper-inflations more likely.

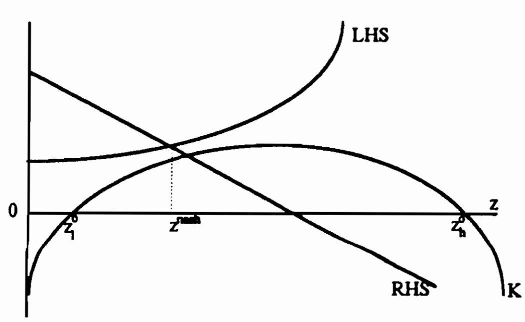
II.III.3.c - Policy Implications

In both cases of cooperative and non-cooperative policies, the steady state utility of consumers are maximized. Then suppose that governments weigh the trade-offs between two costs in deciding the optimal level of inflation; the cost of administering and complying with income taxes, and the distortions created by rising prices and real resource loss from costly currency trading. By putting the Nash and optimal money growth rates together, we have two cases to analyze the policy implications for country 1:

Case 1 $\rightarrow$ $z^n \in (z_l^o, z_h^o)$ or $z^n > z_h^o$: In both of these cases, country 1 should cooperate with the developed country and choose z_l^o . The seigniorage revenue increases with higher money growth, and the integration of monetary and fiscal policies enables country 1 government to optimally choose a higher money growth (lower z). Then for any given level of g, the need for costly income taxation and hence real resource loss is lower. Although the country 1 residents

[24] This result is similar to Kimbrough (1993), however, the key determinant there to solve ambiguity is the elasticity of labor supply. It also provides an argument against discouraging currency substitution because, with a low foreign money growth, the decrease in the demand for domestic money will reduce the government's incentive to inflate for the purpose of raising more revenues (See Barro (1983)).

incur a welfare loss due to asymmetric currency substitution, the effect is weakened compared to the Nash case where there is no money growth in country 2. Because, with a positive money growth in country 2, the income tax can be lowered; hence the wealth of the country 2 young rises and so does their demand for domestic money holdings to buy more of country 1 good. Thus the real resource loss is the lowest by choosing the lower optimal z with cooperative policies. Moreover, cooperating with developed countries in an economic union, for example, will require the new entrants to harmonize their fiscal and monetary policies with the union. The tax systems are more efficient (more use of computers, ethical issues, etc.) and collection is less costly in developed countries. Entering the union will force the developing country to restructure the tax system and decrease the costs of collecting taxes, and hence the need for inflation tax. After all, this is, perhaps, the most important reason to why the Eastern European countries are lined up for a full membership into European Union where the costs of such restructuring are compensated by the special funds of the union.



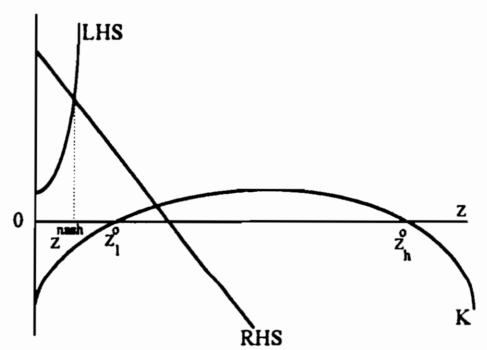


Figure 7: Optimal and Nash money growth

(cases of $z^n \in (z_l^o, z_h^o)$ and $z_l^o > z^n$)

Case 2 $\rightarrow z_l^o > z^n$: In this case, country 1 should not cooperate with the developed country. The intuition is similar to the previous case. Since trading currencies is costly, the Nash money growth will allow the country 1 government to collect more seigniorage and incur lower resource loss from costly taxation than that of the coordinated case while maximizing the steady state utility of its residents. In the longer run, however, such a policy may very well lead to full dollarization. If country 1 residents do not expect that the policy will be reversed in the future foreign money will not be used only as store of value but also for transactions. Because the assumption that seller's currency must be used to buy goods can be justified only if country 1 residents still have confidence in their money for the future, and use foreign money only temporarily to hedge against inflation. Although its credibility will be higher in this case, the government would completely give up its monetary sovereignty. To fight against, government may discourage the use of foreign currency either by frequent and puzzling interventions or by legal restrictions on foreign currency holdings. This, however, may result in an inefficient barter equilibrium with less liquidity, and reduced domestic trade and output. Also, legal restrictions do not always work; underground economy becomes more dollarized, cross-border and the under-the-pillow deposits increase.

II.IV - SUMMARY AND CONCLUSION

In this chapter, we have extended Sibert and Liu (1994) and developed a model of asymmetrical currency substitution. It was shown that if transaction costs are greater than return to the stronger currency (money 2), country 2 residents hold only money 2 because the return to money 2 is higher than the return to money 1, and they trade for money 1 whenever a need arises. Country 1 residents want to hold money 2 for two reasons; to hedge against inflation, i.e. to take advantage of higher return, and for precautionary purposes to minimize the transaction costs in the second period. A stationary equilibrium with asymmetrical currency substitution exists if and only if the cost of spot market transactions is greater than either country's money growth rates, and country 1 money growth rate is lower than that of country 2, given that country 2 has a stronger currency. In this sense, country 1 and 2 may also be considered as "developing" and "developed", respectively. If there are no transaction costs (h=1), then an interior solution requires non-asset dominance; the exchange rate will be constant (Kareken and Wallace (1981)). An increase in the transaction costs and/or an increase in the pace of depreciation of money 1 lowers the share of portfolios allocated to money 1.

We applied the model to seigniorage taxation. The governments are assumed to finance expenditures by levying a costly income tax or by collecting seigniorage. In deciding the optimal level of inflation, governments weigh the trade-offs between the costs of administering and complying with income taxes and the distortions created by rising prices. When the countries cooperate, there are two optimal money growth rates that maximize the steady state utility of the consumers. The optimal money growth rate strictly decreases in currency substitutability and the cost of income taxation. For country 2, the higher money growth is the more optimal choice because income taxation is costly, and part of the burden of the seigniorage tax can be shifted to country 1 residents due to asymmetric currency substitution. For country 2, however, the decision depends on which loss is greater; real resource loss in trading currencies and the real resource loss from costly income taxation.

We also considered the case where each country takes the other country's money growth and income tax rates given and chooses a constant money growth rate to maximize the steady state utility of their residents. We assume initially that, in country 2, income taxation is costly and there is no money growth. Then we have a unique positive Nash money growth for country 1 which increases in response to increases in country 2 money growth if the transaction costs are not too high. Finally, our results suggested that country 1 should cooperate with the developed country and choose the higher optimal money growth rate if it is greater than the Nash money growth rate. Higher money growth increases the seigniorage revenue increases with higher money growth, and the integration of monetary and fiscal policies enables country 1 government to optimally choose a higher money growth (lower z). Country 1 residents may incur a welfare loss due to asymmetrical currency substitution, but the effect is weakened compared to the Nash case where there is no money growth in country 2. Because, with a positive money growth in country 2, the income tax can be lowered; hence the wealth of the country 2 young rises and so does their demand for domestic money holdings to buy more of country 1 good. Thus the real resource loss is the lowest by choosing the lower optimal z when the countries cooperate.

APPENDIX: Proofs of Propositions

Proof of Proposition 1 - Dividing both sides of (10) by *eh* and subtracting (9) from the result yields

$$L \equiv \left(\frac{s-1}{hp_2'}\right)\int_0^{\underline{\alpha}^{1'}}\left(\frac{(1-\alpha)}{\underline{c}_2^{o1'}}\right)d\alpha + \int_{\underline{\alpha}^{1'}}^{\overline{\alpha}^{1'}}\left[\frac{s}{hp_2'}\left(\frac{(1-\alpha)}{c_2^{o1'}}\right)-\left(\frac{\alpha}{c_1^{o1'}}\right)\frac{1}{p_1'}\right]d\alpha+\left(\frac{s-h^2}{hp_2'}\right)\int_{\overline{\alpha}^{1'}}^{1}\left(\frac{(1-\alpha)}{\overline{c}_2^{o1'}}\right)d\alpha = 0$$

If $\delta^1 \equiv \dfrac{m_1^1}{m_1^1+em_2^1} \to 0 \quad \Rightarrow \quad \begin{cases} m_1^1 \to 0 \\ m_2^1 > 0 \end{cases}$ and $\begin{cases} \underline{\alpha}^{1'} \to 0 \\ \overline{\alpha}^{1'} \to 0 \end{cases}$

then $L \to \displaystyle\int_0^1\left[\frac{s-h^2}{hp_2'}\left(\frac{(1-\alpha)}{\overline{c}_2^{o1'}}\right)\right]d\alpha \geq 0$ if and only if $s \geq h^2$.

If $\delta^1 \equiv \dfrac{m_1^1}{m_1^1+em_2^1} \to 1 \quad \Rightarrow \quad \begin{cases} m_1^1 > 0 \\ m_2^1 \to 0 \end{cases}$ and $\begin{cases} \underline{\alpha}^{1'} \to 1 \\ \overline{\alpha}^{1'} \to 1 \end{cases}$

then $L \to \displaystyle\int_0^1\left[\frac{s-1}{hp_2'}\left(\frac{(1-\alpha)}{\underline{c}_2^{o1'}}\right)\right]d\alpha \leq 0$ if and only if $s \leq 1$.

Thus country 1 residents hold positive balances of each money (unique solution to (9) and (10) with equality) if and only if $h^2 > s > 1$. The case of $h^2 > 1/s > 1$ is similar.

Proof of Proposition 2 - Let $\kappa = \dfrac{m_1^1}{em_2^1} = \kappa(h,s)$ is the proportional money holdings of the country 1 resident. Evaluating the marginal utilities in L and multiplying both sides of the result by (em_2^1) yields $\frac{\kappa(s-1)}{(\kappa+hs)^2}+\frac{(s-h^2)s}{(h\kappa+s)^2}+\frac{(s-h)}{2hs}\left[\frac{h\kappa}{(h\kappa+s)}-\frac{\kappa}{(\kappa+hs)}\right]=0$ where

$\kappa=\frac{-4hs+\sqrt{(16h^2s^2-4(1+h^2)s^2}}{2(1+h^2)}$ and $\frac{\partial\kappa}{\partial h}>0$ and $\frac{\partial\kappa}{\partial s}<0$ for $h>1$ and $h^2>s>1$.

Proof of Proposition 3 - Differentiating (8) and employing (2) - (7), we get

$$p_1'=p_1\,s\frac{\beta[h\int_0^{\underline{\alpha}^{1'}}\left(\frac{\alpha}{\underline{c}_1^{o1'}}\right)d\alpha+\int_{\underline{\alpha}^{1'}}^{\overline{\alpha}^{1'}}\left(\frac{\alpha}{c_1^{o1'}}\right)d\alpha+(1/h)\int_{\overline{\alpha}^{1'}}^{1}\left(\frac{\alpha}{\overline{c}_1^{o1'}}\right)d\alpha]}{h/2c_1^{y1}}$$ where $\frac{\partial p_1'}{\partial p_1}>0$. Assuming

that the young and old consumption are gross substitutes, the arbitrage condition in equilibrium assures that the domestic prices in both periods should adjust to equalize the expected marginal utilities of consuming when old and young (the numerator and the denominator of RHS above, respectively) in the presence of transaction costs. Since s >1 in equilibrium, we have P'(P,s,h) rising above the 45 degree line so that the slope is greater than one (Figure 4).

Proof of Proposition 4 - Equations (19) and (20) can be solved to find

$$\frac{\gamma_1^i}{r_1}=f^i\left(\frac{r_2}{r_1}\right)\in\begin{cases}(0,(x-\tau^1)) & if\ i=1\\ 0 & if\ i=2\end{cases}$$

$$\frac{q\gamma_1^i}{r_2}=g^i\left(\frac{r_2}{r_1}\right)\in\begin{cases}(0,(x-\tau^1)) & if\ i=1\\ (0,(r_2/r_1)(x-\tau^2)) & if\ i=2\end{cases}$$

where f and g are continuous functions. Then by (23),

(A) $$\frac{1}{r_1}=\sum_{i=1,2}f^i\left(\frac{r_2}{r_1}\right)\in(0,\infty);\ \frac{q}{r_2}=\sum_{i=1,2}g^i\left(\frac{r_2}{r_1}\right)\in(0,\infty)\ .$$

Let D denote aggregate demand for good 1, given by the left hand side of (22). By the above results and (16) $\rightarrow$ (22), D is a continuous function of (r_2 / r_1). Then we can show that $D \rightarrow \infty$ as $(r_2 / r_1) \rightarrow \infty$, and $D < x$ as $(r_2 / r_1) \rightarrow 0$ and $h^2 > z_2 > z_1$. Suppose $r_2 / r_1 \rightarrow \infty$:

<u>Case (a) -</u> $r_1 \rightarrow 0$ and $r_2 > 0$. Then $q\gamma_2^2/r_2 \rightarrow (x - \tau^2)$ and

$$D \geq h \int_{\bar{\alpha}^2}^{1} \bar{c}^{o2} d\alpha + (h-1) \int_{\bar{\alpha}^2}^{1} \frac{\gamma_1^2 z_1}{r_1} d\alpha \rightarrow \infty \quad .$$

<u>Case (b) -</u> $r_2 \rightarrow \infty$ and $r_1 > 0$. Then $q\gamma_2^2/r_2 \nrightarrow (x - \tau^2)$ and $D \geq c_1^{y2} \rightarrow \infty$.

Now suppose $r_2 / r_1 \rightarrow 0$:

<u>Case (a) -</u> $r_1 \rightarrow \infty$ and $r_2 > 0$. Then $\gamma_1^1/r_1 \rightarrow 0$ and $c_1^{y1} < (x - \tau^1)/2$. Then

$$D \rightarrow c_1^{y1} + c\tau^1 + (1-c)\tau^1 = \frac{x - \tau^1}{2} + \tau^1 < x$$

<u>Case (b) -</u> $r_2 \rightarrow 0$ and $r_1 > 0$. Then $\gamma_1^1/r_1 \nrightarrow 0$. Following from (19), (20), and L, $h^2 > (z_2 / z_1)$ ensures that country 1 demand for good 1 is less than if only money 1 were demanded and if $(z_2/z_1) > 1$, γ_1^2/r_1 remains zero. Country 1 has strictly positive income and the relative price of good 2 is finite. Hence, a strictly positive amount of good 2 must be demanded by country 1 and the demand for good 1 remains strictly less than if this were not so. Thus, $D \rightarrow c_1^{y1} + c_1^{01} + g^1 + (1-c)\tau^1 < x$.

Finally, (22) can be solved for a strictly positive, finite r_2 / r_1. This and (A) above ensure strictly positive, finite values of r_2, r_1, and q. Then the existence of other equilibrium values is straightforward.

Proof of Proposition 6 - Rewriting (25) we get, $z^o = \dfrac{2\beta c}{D}$ where

$$D = \left(\frac{r_1 q + r_2}{r_1 r_2} \right) \left[\frac{\lambda r_1}{r_1(x-\tau) - 1 - hq\gamma_2^1} + \frac{(1-\lambda) r_2}{r_2(x-\tau) - q\gamma_2^2} \right] > 0. \text{ Then } \frac{\partial z}{\partial h} = -\frac{2\beta c(\partial D / \partial h)}{D^2} \text{ and}$$

$$\frac{\partial D}{\partial h}=(\lambda r_1)\left(\frac{r_1\dfrac{\partial\tau}{\partial z}\dfrac{\partial z}{\partial h}+q\gamma_2^1}{\left[r_1(x-\tau)-1-hq\gamma_2^1\right]^2}\right)+(1-\lambda)r_2\left(\frac{r_2\dfrac{\partial\tau}{\partial z}\dfrac{\partial z}{\partial h}}{\left[r_2(x-\tau)-q\gamma_2^2\right]^2}\right)$$. Then

$$\frac{\partial z}{\partial h}=-\frac{2\beta c\lambda r_1 q\gamma_2^1/\left[r_1(x-\tau)-1-hq\gamma_2^1\right]^2}{D^2+\left(\dfrac{r_1q+r_2}{r_1r_2}\right)\left(\dfrac{\lambda r_1^2}{\left[r_1(x-\tau)-1-hq\gamma_2^1\right]^2}+\dfrac{(1-\lambda)r_2^2}{\left[r_2(x-\tau)-q\gamma_2^2\right]^2}\right)}<0.$$ Also,

$$\frac{\partial z}{\partial c}=\frac{2\beta}{D^2}\left(D-c\frac{\partial D}{\partial c}\right)$$ and $$\frac{\partial D}{\partial c}=\frac{(r_1q+r_2)}{r_1r_2}\frac{\partial\tau}{\partial z}\frac{\partial z}{\partial c}\left(\frac{\lambda(r_1)^2}{[r_1(x-\tau)-1-hq\gamma_2^1]^2}+\frac{(1-\lambda)(r_2)^2}{[r_2(x-\tau)-hq\gamma_2^1]^2}\right).$$

Then $$\frac{\partial z}{\partial c}=\frac{2\beta}{D^2}\left(D-c\frac{(r_1q+r_2)}{r_1r_2}\frac{\partial\tau}{\partial z}\frac{\partial z}{\partial c}\left(\frac{\lambda(r_1)^2}{[r_1(x-\tau)-1-hq\gamma_2^1]^2}+\frac{(1-\lambda)(r_2)^2}{[r_2(x-\tau)-hq\gamma_2^1]^2}\right)\right)$$ where

$\frac{\partial\tau}{\partial z}>0$. Finally, $$\frac{\partial z}{\partial c}=\frac{2\beta}{D\left[1+\dfrac{2\beta c}{D^2}\dfrac{\partial\tau}{\partial z}\left(\dfrac{\lambda(r_1)^2}{[r_1(x-\tau)-1-hq\gamma_2^1]^2}+\dfrac{(1-\lambda)(r_2)^2}{[r_2(x-\tau)-hq\gamma_2^1]^2}\right)\right]}>0$$.

Proof of Proposition 7 - Following the equations in proposition 6, we get
$\partial z/\partial\lambda=-2\beta c(\partial D/\partial\lambda)/D^2$ where

$\frac{\partial D}{\partial\lambda}=\frac{\partial D}{\partial z}\frac{\partial z}{\partial\lambda}$. Then $$\frac{\partial z}{\partial\lambda}=-\frac{\beta\left(\dfrac{1}{c_1^{y1}}-\dfrac{1}{c_2^{y2}}\right)}{D^2+\dfrac{\partial\tau}{\partial z}\left[\lambda\left(\dfrac{1}{c_1^{y1}}\right)^2+(1-\lambda)\left(\dfrac{1}{c_2^{y2}}\right)^2\right]}<0$$ if $c_2^{y2}>c_1^{y1}$. With λ

increasing, country 1 government becomes more influential in choosing optimal policy variables. From (16), we have $c_1^{y1} = .5[(x-\tau)-(1+hq\gamma_2^1)/r_1]$ and $c_2^{y2} = .5[(x-\tau)-q\gamma_2^2/r_2]$. It is trivial to see that if transactions costs are high enough such that $h > (r_1\gamma_2^2/r_2\gamma_2^1)$ then $c_2^{y2} > c_1^{y1}$, and hence

$$\frac{\partial z}{\partial \lambda} < 0.$$

Proof of Proposition 8 - In this case, $z_2<1$. Then (27) becomes

$$\frac{(hz_1+\gamma_2^1 qz_2)^2(z_1+h\gamma_2^1 qz_2)^2}{z_1(hz_1+\gamma_2^1 qz_2)^2+(2h\gamma_2^1 qz_2+h^2z_1)(z_1+h\gamma_2^1 q_2z_2)^2} - \beta c c_1^{y1} = 0 \ .$$

Total differentiation of the equation above with respect to z_2 yields

$$\frac{\partial z_1}{\partial z_2} = \frac{AB\,2\gamma_2^1 q[B^3(2h\gamma_2^1 qz_2+h^2z_1)+hA^3z_1-hAB^3]}{D^2(\beta/2)+2hAB^4(2h\gamma_2^1 qz_2+h^2z_1)+2z_1BA^4-A^2B^2(A^2+h^2B^2)}$$

where $D = z_1(hz_1+\gamma_2^1 qz_2)^2+(2h\gamma_2^1 qz_2+h^2z_1)(z_1+h\gamma_2^1 q_2z_2)^2$, $A=(hz_1+\gamma_2^1 qz_2)$, and $B=(z_1+h\gamma_2^1 qz_2)$. It is the developing country's reaction function which describes how the developing country adjusts its monetary policy in response to changes in foreign monetary policy to maximize steady state utility of its consumers. The sign of the reaction function is ambiguous for any level of h. However, if transaction cost is small, i.e. as $h \to 1$, $B \to A, D \to 2A^3$, the numerator goes to $2\gamma_2^1 qA^6 > 0$, and the denominator goes to $2(1+\beta)A^6 > 0$. Then country 1 increases its Nash money growth rate in response to increases in the country 2 money growth or vice versa.

REFERENCES

Angeloni, I. and P. Giucca, "The Role of Monetary Aggregates in the Coordination of Monetary Policies in the EEC", **Giornale deli Economosti e Anali di Economia**, 50, Jan-Feb 1991, pp. 21-49.

Angeloni, I., C. Cottarelli and A. Levy, "Cross Border Deposits and Monetary Aggregates in the Transition to EMU", mimeo, Bank of Italy.

Bini Smaghi, L. and S. Vori, "Currency Substitution, Monetary Stability and European Monetary Unification", **Rivista Internazionale di Scienze Sociali**, January 1991.

Barro, R. and D. Gordon, "Rules, Discretion and Reputation in a Model of Monetary Policy", **Journal of Monetary Economics**, 12, 1983, pp. 101-121.

Biltoft, J., P. Karsten and N. Karsten, "Currency Substitution in EMS member Countries", **Nationalokonomisk-Tidsskrift**, 128 (1), 1990, pp. 97-104.

Brand, D., "Currency Substitution in Developing Countries", Weltforum Verlag, 1993.

Calvo, G. A., "Currency Substitution and the Real Exchange Rate: The Utility Maximization Approach", **Journal of International Money and Finance**, Vol. 4, 1985, pp. 175-188.

Calvo, G. A. and C. A. Rodriguez, "A Model of Exchange Rate Determination under Currency Substitution and Rational Expectations", **Journal of Political Economy**, Vol. 85, 1977, pp. 617-625.

Calvo, G. A., and C. A. Végh, "Currency Substitution in Developing Countries: An Introduction", **IMF Working Paper**, May 1992, pp. 1-36.

Canzoneri M. B. and B. T. Diba, "The Inflation Discipline of Currency Substitution", **EER**, 36, 1992, pp. 827-845.

Chang, R., "Endogeneous Currency Substitution, Inflationary Finance, and Welfare", **Journal Money, Credit and Banking**, 26 (4), November 1994, pp. 903-16.

Cukierman, A., S. Edwards and G. Tabellini, "Seigniorage and Political Instability", **NBER #** 3199, 1989.

Cukierman, A., M. A. Kiguel and N. Liviatan, "How much to commit to an Exchange Rate Rule? Balancing Credibility and Flexibility", **Revista de Analisis Economico**, Vol. 7, No. 1, January 1992, pp. 73-89.

De Grauwe, P., "The Economics of Monetary Integration", Oxford University Press, Second Revised Edition, 1994.

El-Erian, M., "Currency Substitution in Egypt and the Yemen Arab Republic", **IMF Staff Papers**, Vol. 35, 1988, pp. 85-103.

Gazioglu, S., "Monetarization of the Budget Constraint", **Economic Modeling**, Vol. 10 (2), April 1993, pp. 163-172.

Giovannini, A. and B. Turtelboom, "Currency Substitution", **NBER** # 4232, December 1992.

Girton, L. and D. Roper, "The Theory of Currency Substitution and Monetary Unification", **Economie Apliquee,** 33 (1), 1980, pp. 135-60.

Girton, L. and D. Roper, "Theory and Implications of Currency Substitution", **Journal Money, Credit and Banking**, 13 (1), February 1981, pp. 12-30.

Goldman, S. M., "Flexibility and the Demand for Money", **Journal of Economic Theory** 9, 1974, pp. 203-222.

Guidotti, P. E., "Currency Substitution and Financial Innovation", **IMF Working Paper** 89/39,1989.

Handa, J., "Substitution Among Currencies: A Preferred Habitat Hypothesis", **International Economic Journal**, Vol. 2, No. 2, Summer 1988, pp. 41-61.

Hercowitz, Z. and E. Sadka, "On Optimal Currency Substitution Policy and Public Finance", in A. Razin and E. Sadka, eds, **Economic Policy in Theory and Practice**, St. Martin Press, New York, 1987, pp. 147-164.

Imrohoroglu, S., "GMM Estimates of Currency Substitution between the Canadian Dollar and the U.S. Dollar", **Journal Money, Credit and Banking**, 26 (4), November 1994, pp. 792-807.

Khan, M. S. and C. L. Ramirez-Rojas, "Currency Substitution and Government Revenue From Inflation", **Revista de Analisis Economico**, Vol. 1, No. 1, November 1986, pp. 79-88.

Kareken, J. and N. Wallace, "On the Indeterminacy of Equilibrium Exchange Rates", **Quarterly Journal of Economics,** 96, May 1981, pp. 207-222.

Kouri P. J. K., "The Exchange Rate and the Balance of Payments in the Short Run and in the Long Run: A Monetary Approach", **Scandinavian Journal of Economics,** Vol. 78, pp. 280-304.

Kim, K. S., "Currency Substitution in a Production Economy", **Journal of International Economics**, Vol. 18 (1-2), 1985, pp. 141-158.

Kimbrough, K. P., "Optimal Monetary Policies and Policy Interdependence in the World Economy", **Journal of International Money and Finance**, 12, 1993, pp. 227-248.

Liviatan, N., "Monetary Expansion and Real Exchange Rate Dynamics", **Journal of Political Economy**, Vol. 89, 1981, pp. 1218 - 1227.

Lucas, R. E. Jr., "Interest Rates and Currency Prices in a Two Country World", **Journal of Monetary Economics** 10, (1982) pp. 335-359.

Matti, V., "McKinnon's Currency Substitution Hypothesis: Some New Evidence", **Economia Internazionale**, 43 (2-3), May-August 1990, pp. 226-34.

McKinnon, R., "Currency Substitution and Instability in the World Standard", **American Economic Review**, Vol. 72, no. 3, June 1982, pp. 320-330.

Miles, M. A., "Currency Substitution, Flexible Exchange Rates, and Monetary Independence", **American Economic Review**, Vol. 68, no. 3, June 1978, pp. 428-436.

Ortiz, G., "Dollarization in Mexico: Causes and Consequences", in A.A. Pedro, R. Dornbusch and M. Obstfeld, eds, **Financial Policies and the World Capital Market**, The University of Chicago Press, Chicago, 1983, pp. 71-95.

"Proceedings of a Conference on Currency Substitution and Currency Boards", ed. by Nissan Liviatan, **World Bank Discussion Papers**, 207, October 1993, pp. 1-105.

Ramirez-Rojas, C. L., "Currency Substitution in Argentina, Mexico, and Uruguay", **International Monetary Fund Staff Papers**, 32(4), 1986, pp. 629-667.

Savastano, M. A., "The pattern of Currency Substitution in Latin America: An Overview", **Revista de Analisis Economico**, Vol. 7, No. 1, January 1992, pp. 29-72.

Selcuk, F., "Currency Substitution in Turkey", **Applied Economics**, 26, 1994, pp. 509-518.

Sibert, A. and J. Ha, "Portfolio Substitution and Exchange Rate Volatility", mimeo, January 1994, pp.1-32.

Sibert, A. and L. Liu, "Government Finance in a Model of Currency Substitution", **IMF Working Paper**, October 1993, pp. 1-24.

Sibert, A., "Government Finance in a Common Currency Area", **Journal of International Money and Finance**, 11, 1992, pp. 567-578.

Sturzenegger, F. A., "Currency Substitution and the Regressivity of Inflationary Taxation", **UCLA Working Paper** 656, April 1992.

Svensson, L. E. O., "Currency Prices, Terms of Trade, and Interest Rates", **Journal of International Economics** 18, (1985), pp. 17-41.

Tanzi, V. and M. I. Blejer, "Inflation, Interest Rate Policy, and Currency Substitutions in Developing Economies: A Discussion of Some Major Issues", **World Development** 10 (9), 1982, pp. 781-789.

Thomas, L. R., "Portfolio Theory and Currency Substitution", **Journal of Money, Credit and Banking**, Vol. 17, no. 3, August 1985, pp. 347-357.

Végh, C. A., "The Optimal Inflation Tax in the Presence of Currency Substitution", **Journal of Monetary Economics**, March 1989a, pp. 139-146.

Végh, C. A., "Currency Substitution and the Optimal Inflation Tax under Labor Income Taxation", Unpublished Manuscript, IMF, 1989b.

Végh, C. A., "Inflationary Finance and Currency Substitution in a Public Finance Framework", **Journal of International Money and Finance**, September 1994, pp. 1-26.

Weil, P., "Currency Competition and the Transition to Monetary Union: Currency Competition and the Evolution of Multi-Currency Regions", in A Giovannini and C. Maier, eds., **European Financial Integration**, Cambridge University Press, 1991, pp. 257-289.

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