Chapter 7

MDO Problem Statement

To demonstrate the effectiveness of this design tool, we will perform a sample problem with five design variables. For the purpose of comparison, a portion of the Northrop Grumman MAGLEV design process [28] was redone here using this MDO methodology. Northrop Grumman was one of four companies contracted by the National MAGLEV Initiative (NMI) to develop vehicle designs. For the aerodynamic design they developed a design methodology geared towards minimizing aerodynamic drag using RANS as an analysis tool. They began with over 2000 2-D side view geometries and a separate set of 2-D plan view geometries. The side view geometry definition can be seen in the Fig. 7.1 in terms of the design variables [28]. The design variables are XN, XF, θ, N, and F. L is fixed at 6.0 meters and H is fixed at 3.5 meters. A 12.0 meter long parallel section separates the vehicle nose from its identical tail. This identical nose and tail provide for dual direction capability.

According to the Grumman selection process, fourteen of the 2-D side view geometries were chosen by the designers to represent the whole design space. 2-D RANS calculations were performed for the out of ground effect case, and five side view geometries were chosen based on minimum drag. RANS calculations were then performed on these five geometries for the IGE case. From these five side view geometries, 3-D geometries were constructed using the 2-D plan view designs chosen from a separate selection process. The design process continued using 3-D RANS calculations and the five final designs were chosen for experimental evaluation in the Virginia Tech moving track wind tunnel facility [8] [67].

The five design variable problem dealt with in this work repeats the 2-D, side view design using the MDO design methodology, minimizing drag coefficient. The optimum design is compared to the five 2-D side view designs developed using the Northrop Grumman methodology. These five designs are shown in Fig. 7.2. Optimizations are also performed for additional figures of merit such as acquisition cost, operating cost, life cycle cost, empty weight, and lift to drag ratio. The mission profile for this example is that of an intercity haul of 800 kilometers non stop. The acceleration and braking are ignored for this example problem although it can be incorporated in future studies set at the maximum allowable normal mode value of 0.16g [5]. The analyses are performed for a single, 50 person car although the mission may call for a larger capacity.

The mathematical problem statement for a general optimization problem is, minimize the objective function, which is a function of n design variables, subject to m constraints (Eq. 7.1).

$$\begin{aligned} &\min \quad f(\mathbf{x}) \quad \mathbf{x} \in R^n \\ &\text{subject to} \quad \mathbf{l} \leq \mathbf{c}(\mathbf{x}) \leq \mathbf{u} \quad \mathbf{c} \in R^m \end{aligned} \tag{7.1}$$

Optimization methods fall into two major categories which are calculus-based methods and search methods. Calculus-based methods use gradient information to navigate through the design space to find local optimum points. This area of optimization technology is fairly mature compared to the search methods. These methods involve mathematical criteria to indicate convergence to an optimal point (Kuhn Tucker conditions). These methods have strong theoretical background and converge fairly quickly. Their weakness is that they require gradient information and, therefore, a differentiable design space. In addition to this, they do not search for global optima. There are iterative methods which attempt to find global optima, using calculus-based methods, although, there is no method to prove that the solution is a global optimum point [68].

Search methods available include genetic algorithms, simulated annealing, and neural networks. These methods search for global optima and do not have smoothness requirements on the design space. Although, these methods cannot guarantee the convergence to a global optimum point, they are slow (require many analyses), and can only handle constraints using an augmented penalty function. The work presented here involves a calculus-based method called Sequential Quadratic Programming (SQP). This method was chosen since it is theoretically well founded, converges quickly, and involves a rigorous mathematical requirement for optimality.

Sequential Quadratic Programming is an optimization method for constrained optimizations with nonlinear constraints; it can handle linear constraints and unconstrained problems although methods specifically designed for those problems might be more efficient. In this method, at each iteration step, the optimizer attempts to minimize a quadratic model of the Lagrangian subject to a linear model of the constraints (quadratic subproblem). This problem can be stated as:

$$\begin{aligned} &\min \quad \mathbf{g}^T\mathbf{p} + \frac{1}{2}\mathbf{p}^T H \mathbf{p} \\ &\text{subject to} \quad \mathbf{l} \leq \mathbf{c}(\mathbf{x}) + A\mathbf{p} \leq \mathbf{u} \end{aligned} \tag{7.2}$$

The minimizing function is the linear and quadratic term of the Taylor Series expansion for the Lagrangian. The vector, $\mathbf{g}$, is the gradient of the objective function. The vector, $\mathbf{p}$, is the search direction, and the matrix, H, is an approximation to the Hessian of the Lagrangian. The matrix, A, is the Jacobian of the constraints. At each iteration, the optimality criteria are imposed in order to solve for a new search direction. This optimality criteria states that the gradient of the Lagrangian equals zero and that the constraints are satisfied (Eq. 7.3).

$$\begin{bmatrix} H_k & -A_k^T \\ A_k & 0 \end{bmatrix} \begin{Bmatrix} \mathbf{p}_k \\ \eta_k \end{Bmatrix} = \begin{Bmatrix} -\mathbf{g}_k \\ -\mathbf{c}_k \end{Bmatrix} \tag{7.3}$$

η is a vector of the Lagrange multipliers of the linearly constrained quadratic subproblem. A detailed discussion of SQP methods and of optimization theory in general can be seen in Ref. [69]. A detailed discussion about the specific SQP algorithm used here can be found in the DOT users manual [70].

The 5 design variable problem uses the five geometry variables outlined in Fig. 7.1. The problem also involves two constraints. The first constraint requires XF to be greater than or equal to XN. The second constraint requires the drag coefficient to be positive. This constraint is necessary to avoid areas in the design space where the aerodynamic model fails. For low-order methods, “kinks” in the surface can result in singularities and, therefore, very large negative pressure coefficients which could result in a thrust. These solutions are mathematically viable but physically unrealistic. This problem can also be lessened by choosing proper bounds on the design variables.

A 7 design variable problem is adopted as an alternative way to perform the optimizations. The reasons for this higher dimensional problem and the benefits of this change in the problem statement will be discussed in the Results section. The problem formulation requires 7 constraints and a separate scaling scheme.

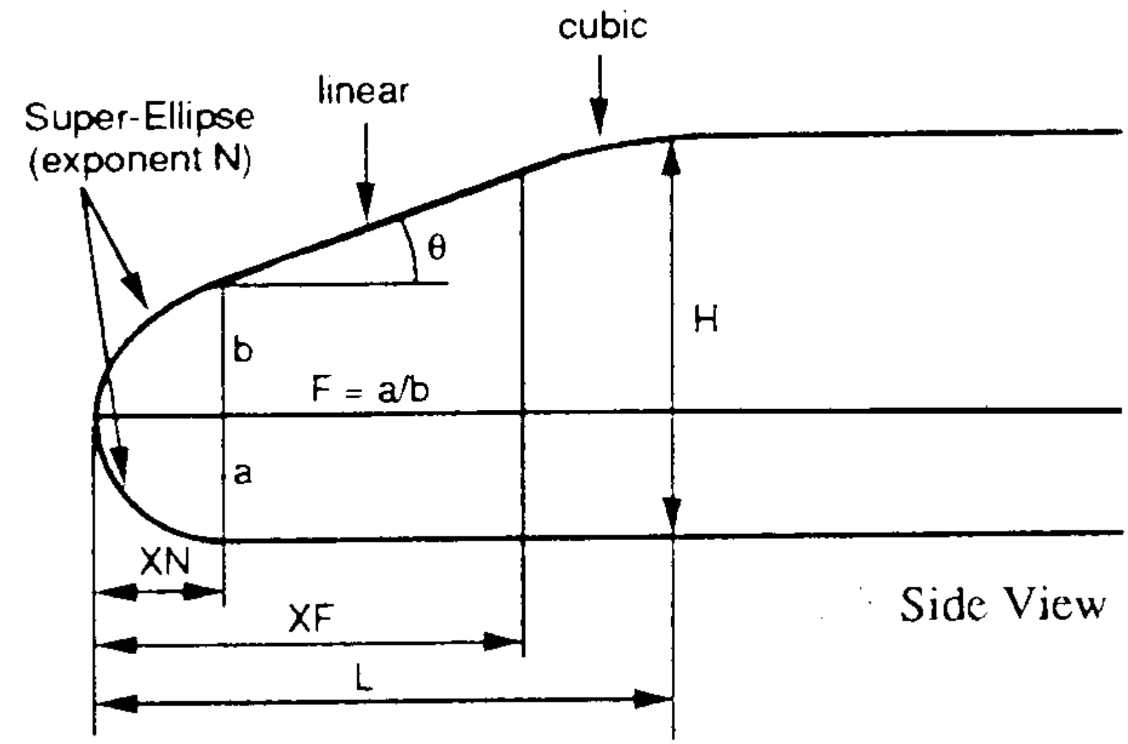

Figure 7.1: Geometry Definition [28]

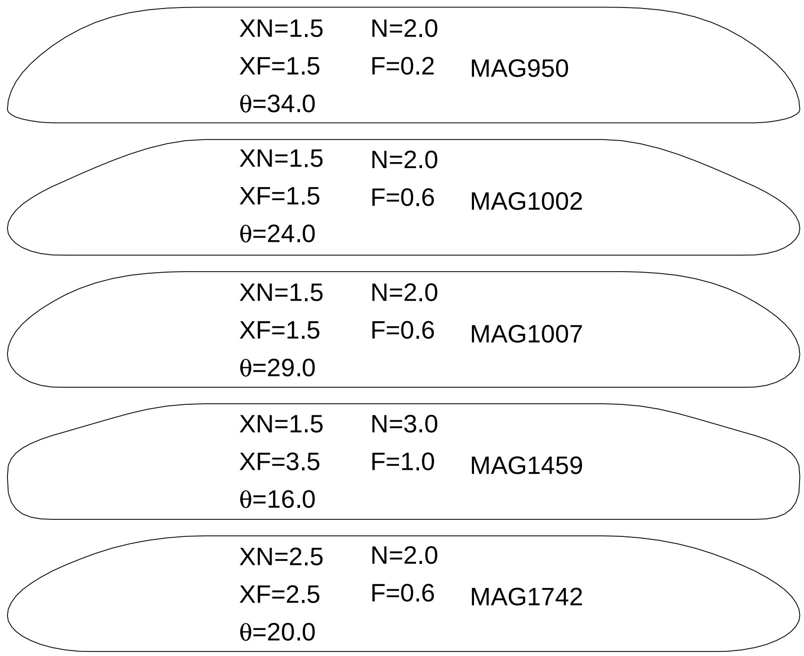

Figure 7.2: Northrop Grumman 2-D Side View Designs